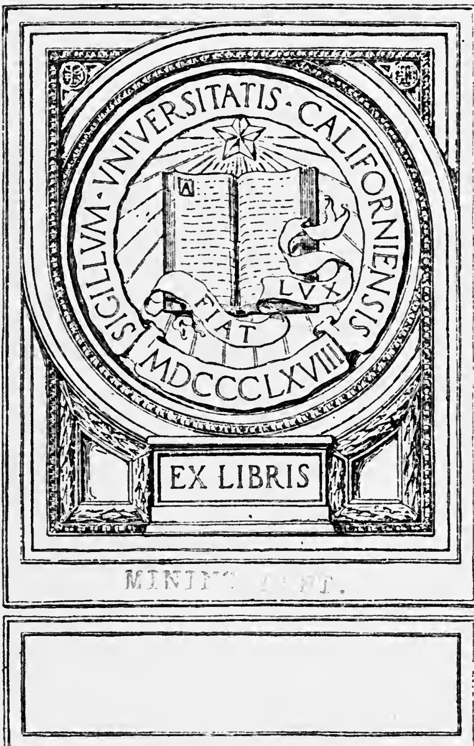
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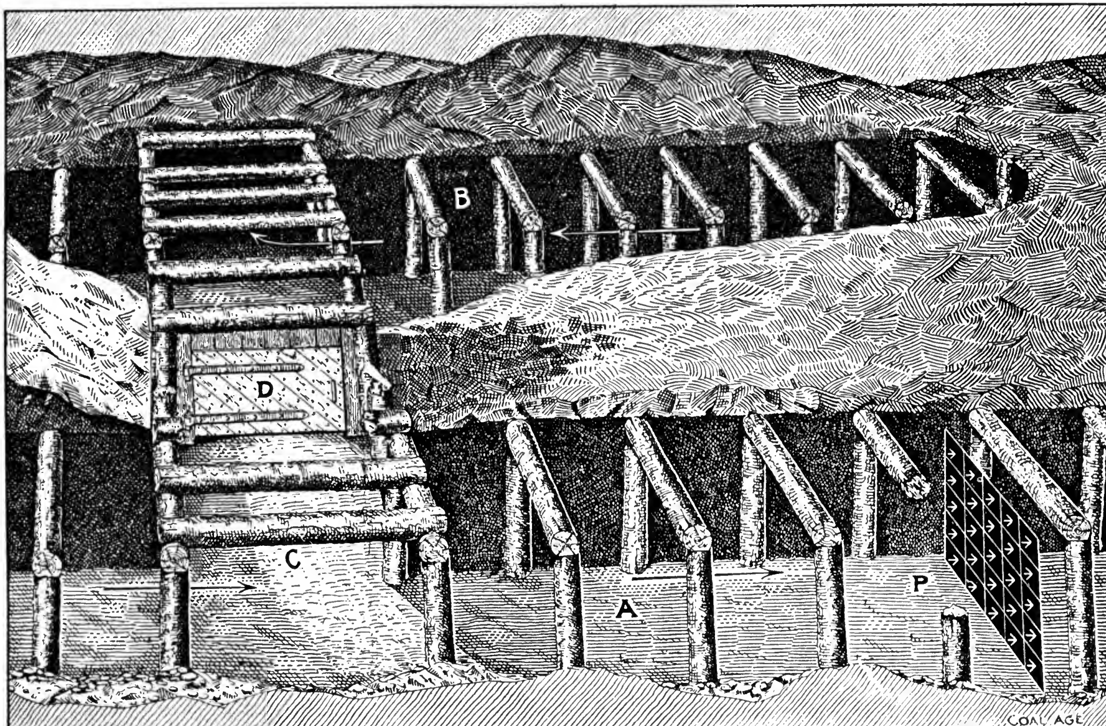
MINE GASES AND VENTILATION



(Frontispiece)

MINE GASES AND VENTILATION

TEXTBOOK FOR STUDENTS OF MINING, MINING ENGINEERS AND CANDIDATES PREPARING FOR MINING EXAMINATIONS

Designed for Working Out the Various Problems That Arise in the Practice of Coal Mining, as They Relate to the Safe and Efficient Operation of Mines

BY

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PREFACE TO SECOND EDITION

Any one who has been closely associated with the practical operation of coal mines will realize quickly the need of technical knowledge relating to the safe and economical production of coal. In no department of the work is this need more urgent than in the ventilation of the mine.

A knowledge of the properties and behavior of the gases found or generated in the mine, and the means for effecting their safe removal or rendering them harmless are of chief importance, requiring careful study combined with practical experience in the operation of mines.

Experience, without a knowledge of the theory of mining, is little better than is the possession of such knowledge by one who has had no experience in the practical work. Experience and knowledge must go hand in hand.

The problems relating air, gases, ventilation, safety lamps, breathing apparatus, rescue work, gas and dust explosions in mines are treated in a thoroughly practical manner, while at the same time showing their correct solution. Formulas must always play an important part in mine ventilation and their treatment is made as simple as possible.

No effort has been spared to make this volume a standard of ventilating practice. With this end in view, the various constants used have been carefully selected and are those most generally adopted. Particularly is this true of the tables of weight and measures and the conversion tables relating to the common and metric systems given in the Addenda. Their use is recommended.

The present volume, which replaces the little booklet issued by *Coal Age*, some time previous, under the same title, will be recognized as a second edition of that handbook, though greatly enlarged by the addition of whole new sections on Safety Lamps, Oils, Breathing Apparatus, Rescue Work and numerous tables, making it a complete treatise on the subject. The author desires to thank those who have generously lent their aid in the work, among whom he would particularly mention James W. Paul, Mining Engineer, Federal Bureau of Mines, and J. T. Ryan, Vice-president and General Manager, Mine Safety Appliances Co., Pittsburgh, Pa.

James T. Beard.

New York City,
June, 1920.

PREFACE TO FIRST EDITION

In March, 1913, there was started in *Coal Age* a department entitled "Study Course in Coal Mining," and each week following that date there have appeared two pages of matter in pocket-book form, which were intended to be later compiled and published as "The Coal Age Pocket Book."

The publication of these weekly pages was not confined to a consecutive order, which gave to that department of *Coal Age* an increasing and widening interest among readers and students of technical mining subjects. The matter treated was in response to the requests of coal-mining men, who were seeking to know the development of formulas, the explanation of principles, and the most approved and generally adopted methods in the practice of coal mining. The requests that have been received from publishers of similar technical matter, asking for the privilege of reproducing many of the pages already published in *Coal Age,* is sufficient evidence of the technical value of the work.

Recently, so many letters have come from mining men and from several mining classes who have been studying the pages as they have appeared each week, asking that the matter already prepared be published at once in suitable book form, it has been decided to issue the following sections on the atmosphere, gases and ventilation of mines. Although it is not assumed that these sections are in their final form, they contain much valuable matter that will be appreciated by practical mining men and students of coal mining.

Coal Age particularly commends this work to mining students, engineers, mine foremen, assistant foremen and firebosses, superintendents and managers. The book contains only original matter, prepared at great expense of time and labor, involving much careful research and experiment. The author does not hesitate to say that many of the practical problems in the ventilation of mines, which cannot be solved by the usual methods employed, are easily worked by the potential methods explained fully in these pages. No mine official or mine employee can afford to be without this edition in his reference file or library.

James T. Beard.

New York City,
July, 1916.

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MINE GASES AND VENTILATION

SECTION I

AIR

THE ATMOSPHERE—THE BAROMETER—PHYSICS OF AIR AND GASES—MATTER—MEASUREMENT—DENSITY AND VOLUME—SPECIFIC GRAVITY—OCCLUSION, EMISSION, DIFFUSION OF GASES

Little was known of the aërial envelope that surrounds the earth, until the researches of Cavendish and Priestley in England and Lavoisier in France, in the latter part of the 18th century showed that air was **not an element,** as had been supposed, but a mechanical mixture of gases.

Up to this time, air and all combustible material was believed to contain a certain substance called **"phlogiston,"** which escaped as flame when the substance was burned. Both Cavendish and Priestley held this phlogistic theory even after they discovered the complex nature of air. Hence, the name "dephlogisticated air" was applied to oxygen; while hydrogen was called "inflammable air" and carbon dioxide "fixed air."

It remained for Lavoisier to expose this fallacy by showing that no matter was lost, but the weight of the products of a combustion was equal to that of the combustibles burned. A large number of carefully made analyses showed a practically constant proportion of the two chief gases of which air is formed. This seemed to suggest that the oxygen and nitrogen of the air were chemically united, although the proportion of each gas did not correspond to its combining power as determined by the analyses of well-known chemical compounds. The character of air as a **mechanical mixture** thus became definitely established.

Besides the two principal gases oxygen and nitrogen that constitute the air we breathe, there are other gases whose presence in the atmosphere is of much vital importance, although their proportion is small. Of these may be mentioned carbon dioxide, water vapor, ammonia, argon and ozone.

Carbon dioxide is most important, because of its toxic effect on the human system. This effect, it is stated on the highest authority, increases with the barometric pressure. Thus, for example, air containing but 1 per cent. carbon dioxide, at a pressure of 4, 5 or 6 atmospheres produces the same effect on the respiratory organs as air containing 4, 5 or 6 per cent. of the gas at a pressure of 1 atmosphere. In other words, **the true gage** of the effect of this gas in inspired air is the percentage of the gas multiplied by the number of atmospheres.

Water vapor present in the atmosphere breathed has a marked effect on the vital activities and the consequent development of physical energy in the body. In what manner the relative humidity of the inspired air operates to impair the physical force has not been fully explained; but experience has shown that a high degree of humidity in a warm atmosphere or climate has an extremely weakening effect on the human system.

The association of **high humidity and temperature** marks a comparatively large amount of water per unit volume of air and, to that extent, it may be assumed impairs the respiratory functions of the lungs. The result is to incapacitate men exposed to such conditions and render them wholly or in part unfit to perform the required manual or mental labor. These effects are continually observed in the warm moist atmosphere of deep mine workings and other similar places.

The Respiratory System.—Respiration is the prime means of maintaining the vital action in animal organisms. Its objects are twofold: 1. The **oxidation** of the organic matter of the animal tissues with the resulting development of vital energy. 2. The **removal** of the carbon dioxide produced in the process of oxidation. Both of these processes are performed through the medium of the blood.

The Circulation.—Under the action of the respiratory system, the blood flows from the heart into and through the arteries of the body, as water flows through a circulating pipe system under the action of a pump. The pulsations of the heart, corresponding to the strokes of the pump, force the blood through a complex system of arteries and veins to every portion of the body and limbs.

All the blood does not flow in a continuous circuit, but the arteries branch, forming separate channels leading to different parts of the body. The time required to complete a circuit and return to the heart is obviously widely different, varying from 20 or 30 sec. to one-fourth as many minutes. This is of interest in relation to the time required for poison entering the blood to be disseminated throughout the system.

Respiratory Action.—The action known as "breathing" originates, or, at least, is regulated by a nerve center at the base of the brain from which impulses are transmitted through the spinal column to the respiratory muscles. By this means air enters the air cells of the lungs and oxygen, absorbed therefrom by the red corpuscles (hæmoglobin) of the blood, is carried by the circulation to the tissues of the body, where it is consumed with the production of carbon dioxide. This gas is absorbed by the blood and carried back through the veins to the heart and lungs, where it gives up a portion of its gas, which enters the lungs and is expelled by each succeeding exhalation.

While air expired by a healthy adult, at rest, contains from 2 to 3 per cent. carbon dioxide, careful determinations show a constant production of 5.6 per cent. of this gas in the lungs when the person is at rest.

Quantity of Oxygen Consumed in Breathing.—A man at rest consumes 263 $cm.^3$ of oxygen per min., or $263 \times 0.06102 = 16$ cu. in. per min. and exhales an equal volume of carbon dioxide. Air exhaled from the lungs contains 2.6 per cent. carbon dioxide, 18.3 per cent. oxygen, 79.1 per cent. nitrogen. In violent exercise, a man consumes from eight to nine times the amount of oxygen required when at rest; or, say 128 to 144 cu. in. per min. The exhaled breath may then contain 6.6 per cent. carbon dioxide and only 14.3 per cent. oxygen.

Depletion of Oxygen in Air, Effect on Life.—Air containing 3 per cent. carbon dioxide can be breathed without discomfort, even when the oxygen content has been reduced to 16 per cent.; but 5 per cent. carbon dioxide causes headache, dizziness and nausea, after a short time. When no carbon dioxide is present in the air the oxygen content may fall as low as 14 per cent. before much difficulty is experienced in breathing; but air containing but 10 per cent. is no longer breathable; but will cause death quickly by suffocation.

Composition of Air.—Normal air is composed chiefly of oxygen and nitrogen, which are invariably mixed in the following proportions expressed as percentage by volume and by weight of each of these gases:

TABLE SHOWING COMPOSITION OF NORMAL AIR

	By Volume	By Weight
Oxygen	20.9 per cent.	23.0 per cent.
Nitrogen	79.1 per cent.	77.0 per cent.
	100.0 per cent.	100.0 per cent.

Air also contains 0.04 per cent. of carbon dioxide (CO_2), together with smaller amounts of argon, ammonia and water vapor. Atmospheric air, it may be said, is never absolutely dry or free of moisture. The term **"dry air"** in respect to the atmosphere is only a relative expression, meaning that such air is comparatively dry.

Weight of Dry Air.—The weight of dry air, per unit volume, varies directly with the pressure it supports, and inversely as its absolute temperature. There are two formulas for finding the weight of 1 cu. ft. of air, one being expressed in terms of the barometer (B), in inches, and the other in terms of the pressure (p) in pounds per square inch.

By the barometer, $$w = \frac{1.3273\,B}{460 + t}$$

By the pressure, $$w = \frac{p}{0.37\,(460 + t)}$$

Moisture in Air.—This subject is fully treated under "Hygrometry," and it is sufficient here to say that the water absorbed or held by the air is an invisible vapor that resembles a gas in its behavior, until a sufficient amount is present to fully saturate the space it occupies. This point of saturation is called the **"dew point,"** because at that point any excess of vapor condenses and appears as a mist or cloud. The condensation is more rapid in contact with a cold surface.

Normal Air.—The term "normal air" in respect to its composition refers to air containing a normal percentage of oxygen (20.9 per cent.) as given above. When the percentage of oxygen present is less than normal the air is said to be "depleted" of its oxygen. This frequently occurs in poorly ventilated places in mines. The depletion of oxygen is the result of the various forms of combustion or oxidation that are constantly taking place in mines, and is also caused by the absorption of oxygen from the air by the coal.

Mine Air.—Except when diluted with other gases, the air in a well-ventilated mine never shows any appreciable depletion of its oxygen content. Even in poorly ventilated places it is exceptional to find less than 20 per cent. of oxygen except where other gases are being generated in considerable volume whereby the air is diluted and the percentage of oxygen correspondingly diminished. This fact has been well established by innumerable tests of mine air made at different mines and under varying conditions of ventilation.

THE ATMOSPHERE

The atmosphere is the aërial envelope surrounding the earth. The term is also used to describe the air or gaseous mixture filling any given space; as, for example, the mine atmosphere is the air and gases filling the mine or any portion of the workings.

Atmospheric Pressure.—The weight of the air surrounding the earth causes a pressure, which decreases as the height above the surface increases; and the density of the air decreases in like manner, with the elevation above sea level.

Variation of Atmospheric Pressure.—Atmospheric pressure at any given place varies irregularly with the condition in respect to storms; the storm center being always an area of lower pressure than that surrounding the storm. In this country, a variation of 2 in. of mercury (say 1 lb. per sq. in.) in atmospheric pressure, in 48 hr., is not uncommon.

There is also a regular daily variation, the pressure attaining a maximum about 10 o'clock and a minimum at 4 o'clock, morning and evening. There is, likewise, a yearly variation, the general pressure reaching a maximum, in the northern hemisphere, in January and a minimum in July.

THE BAROMETER

The Mercurial Barometer.—The pressure of the atmosphere is measured by the height of mercury column it will support against a vacuum. The mercurial barometer is a glass tube. about 36 in. long, closed at one end. This is first filled with mercury and then inverted. The open end being immersed in a basin of the same liquid, the mercury in the tube will fall to a height above the surface of that in the basin, such that the pressure of the atmosphere acting on the surface of the liquid in the basin will support the mercury column in the tube.

Barometric Pressure.—The pressure of the atmosphere expressed in inches of mercury is called the barometric pressure. For example, at sea level, the atmospheric pressure will commonly support 30 in. of mercury column; or is equivalent to a barometric pressure of 30 in.

Calculation of Barometric Pressure.—One cubic inch of mercury (32°F.) weighs 0.49 lb. A barometric pressure of 30 in., therefore, indicates an atmospheric pressure of

$$0.49 \times 30 = 14.7 \textit{ lb. per sq. in.}$$

which is the normal pressure at sea level.

Calculation of Water Column.—The height of water column, in feet, the atmospheric pressure will support is found by multiplying the pressure (lb. per sq. in.) by 2.3; or dividing the same by 0.434. Or the barometric pressure, in inches, multiplied by one and one-eighth will give the equivalent water column, in feet. For example, at sea level,

$$14.7 \times 2.3 = 33.8, \textit{ say } 34 \textit{ ft.}$$
$$30 \times 1\tfrac{1}{8} = 33.75, \textit{ say } 34 \text{ ft.}$$

Principle of the Barometer.—In the mercurial barometer the pressure of the atmosphere supports the column of mercury in the tube. The weight of the atmosphere counterbalances the weight of the mercury column, which rises as the atmospheric pressure increases and falls as it decreases. The height of the mercury column is therefore a true index of the pressure of the atmosphere at the surface of the earth, at the moment of taking the observation.

The principle of the balance pressure between the air and the mercury is clearly illustrated in Fig. 1, where a glass tube, closed at one end, is shown supported in a basin of mercury. The surface of the liquid in the basin is shown as divided into imaginary squares, by lines one inch apart; and the small arrow-heads represent the pressure of the atmosphere exerted on each square inch of surface.

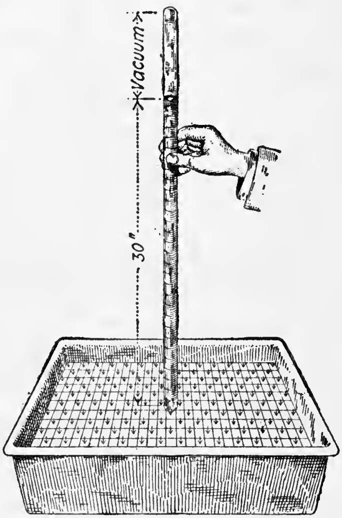


Fig. 1.

Suppose for a moment, that the column of mercury in the tube is exactly one square inch in cross-section; it is evident, in that case, that the mercury column takes the place of the atmospheric pressure on one square inch of surface; and, since there is perfect equilibrium, its weight is equal to the pressure of the atmosphere per square inch.

Furthermore, whatever the sectional area of the mercury column, it is clear that its weight will always equal the atmospheric pressure for the same area of surface. Hence, the area of mercury column is not important, but its height only.

If the weight of one cubic inch of mercury (0.4911 lb.) be multiplied by the observed height of the column of mercury measured in inches, the product will be the pressure of the atmosphere, in pounds per square inch, at the place where the observation was taken. This assumes, that the barometric reading has been reduced to a standard reading, at a temperature of 32 deg. (Fahr.), which must be done when making accurate determinations.

Standard Barometric Readings.—Owing to the fact that the mercury in the tube expands and contracts more rapidly than the glass of the tube, the reading of the barometer will vary slightly for the same pressure, at different temperatures.

In comparing barometric readings taken at different times and at varying temperatures, it is necessary to carefully note the temperature when the reading was taken and reduce the observed reading to a so-called **standard reading** at 32 deg. F.

Calling the standard reading H, the observed reading h and the temperature t (Fahr.), the corrected reading is found by the formula,

$$H = h(1 - 0.0002\ t)$$

For example, the standard reading corresponding to 30 in. of barometer, observed at a temperature of 60 deg. is

$$30\ (1 - 0.0002 \times 60) = 29.64\ in.$$

It is even possible, owing to the more rapid expansion or contraction of the mercury than of the glass, that an observed fall of barometer may correspond to an actual rise in atmospheric pressure, or *vice versa*, within about 0.4 in.

Description of the Instrument.—In the illustration, Fig. 2, is shown the common form of the standard mercurial barometer. The glass tube that contains the mercury column is here inclosed in the metal case A, to the bottom of which is attached a somewhat larger casing B. The latter holds a glass cylinder G terminated at the bottom with a chamois-skin bag, the whole forming the basin that holds the mercury.

The entire case AB is hung in a truly vertical position, supported on a substantial base, as shown in the figure. The top of the mercury column is observed through the opening *O*, in the upper end of the case. In this opening, is arranged a sliding vernier *V*, which can be adjusted, by means of the thumbscrew *D*, so that its lower edge exactly corresponds with the top of the mercury column. The position of the vernier is then read on the scale *S* marked on the sides of the opening in the case. This scale is graduated in inches, but only extends an inch or two above and an equal distance below the normal barometric reading. The normal reading at sea level is about 30 in., and the scale extends from 26 to 32 inches.

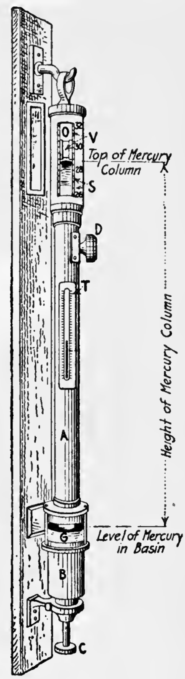


Fig. 2.

Before setting the vernier, however, it is necessary to adjust the level of the mercury in the basin so that it corresponds exactly with what would be the zero of the extended scale. To enable this to be done with precision, there is attached to the scale a long rod that extends downward inside the casing. The lower end of the rod is drawn to a fine point that marks the zero of the scale.

To adjust the level of the mercury in the basin, the thumb-screw *C* is turned. This screw bears against the bottom of the chamois-skin bag and operates to raise or lower the level of the surface of the mercury in the glass cylinder. The adjustment is complete when the fine pointed end of the rod is seen to just prick the surface of the mercury. The point of the rod is observed through the glass cylinder above the surface of the mercury.

A thermometer *T* is shown attached to the metal case. In making accurate observations it is necessary to reduce all readings to standard readings.

The Aneroid Barometer.—The aneroid barometer consists of a metallic case, having a flexible vacuum box within, which is sensitive to the slightest change in atmospheric pressure.

The corrugated diaphragm forming the back of the vacuum box is supported against the pressure of the atmosphere by a steel spring, and its movement under changes of pressure is communicated to the index hand or needle that registers the pressure on a dial calibrated to read inches of mercury corresponding to the readings of the mercurial barometer under the same pressures (Fig. 3).

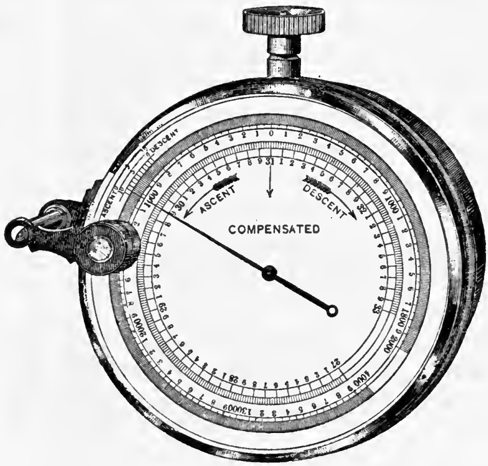


Fig. 3.

The aneroid being portable is very useful in ascertaining quickly differences in elevation of two or more points in mines and on the surface. The dial of mining aneroids has two concentric scales. The inner scale of the aneroid shown in the accompanying figure is graduated to read inches of mercury, while the outer scale reads feet of elevation. It has always been the custom, in arranging the graduation of these two scales, to make the altitude scale read

Table Showing Atmospheric Pressure at Different Elevations and Corresponding Density of Air for Different Temperatures

Eleva. above or below sea level (ft.)	Height of barom. (in.)	Atmos. press. (lb. p. sq. in.)	Mean temp. observed (F.)	Temperature (deg. F.)							
				−20	0	32	60	100	200	300	400
				Weight of dry air (lb. per cu. ft.)							
25,000	11.343	5.571	0.0	.0342	.0327	.0306	.0290	.0269	.0228	.0198	.0175
20,000	13.874	6.814	8.0	.0418	.0400	.0373	.0354	.0329	.0279	.0242	.0214
15,000	16.948	8.323	17.0	.0511	.0489	.0457	.0433	.0402	.0341	.0296	.0262
14,000	17.626	8.656	18.8	.0532	.0509	.0475	.0450	.0418	.0354	.0308	.0272
13,000	18.328	9.000	20.7	.0553	.0529	.0494	.0468	.0434	.0369	.0320	.0283
12,000	19.053	9.357	22.7	.0575	.0550	.0514	.0486	.0452	.0383	.0333	.0294
11,000	19.805	9.726	24.8	.0597	.0571	.0534	.0505	.0469	.0398	.0346	.0306
10,000	20.582	10.107	27.0	.0621	.0594	.0555	.0525	.0488	.0414	.0359	.0318
9,000	21.392	10.505	29.4	.0645	.0617	.0577	.0546	.0507	.0430	.0374	.0330
8,000	22.229	10.916	32.0	.0670	.0641	.0600	.0567	.0527	.0447	.0388	.0343
7,000	23.088	11.339	34.8	.0696	.0666	.0623	.0589	.0547	.0464	.0403	.0356
6,000	23.975	11.774	37.8	.0723	.0692	.0647	.0612	.0568	.0482	.0419	.0370
5,000	24.890	12.224	41.0	.0751	.0718	.0671	.0635	.0590	.0500	.0435	.0384
4,500	25.360	12.455	42.7	.0765	.0732	.0684	.0647	.0601	.0510	.0443	.0391
4,000	25.837	12.689	44.4	.0779	.0745	.0697	.0659	.0612	.0520	.0451	.0399
3,500	26.322	12.927	46.2	.0794	.0759	.0710	.0672	.0624	.0529	.0460	.0406
3,000	26.813	13.169	48.0	.0809	.0774	.0723	.0684	.0635	.0539	.0468	.0414
2,500	27.315	13.415	49.9	.0824	.0788	.0737	.0697	.0647	.0549	.0477	.0422
2,000	27.824	13.665	51.8	.0839	.0803	.0751	.0710	.0659	.0559	.0486	.0429
1,500	28.339	13.918	53.8	.0855	.0818	.0764	.0723	.0672	.0570	.0495	.0437
1,000	28.861	14.174	55.8	.0871	.0833	.0778	.0737	.0684	.0580	.0504	.0445
900	28.966	14.225	56.1	.0874	.0836	.0781	.0739	.0686	.0582	.0506	.0447
800	29.072	14.277	56.4	.0877	.0839	.0784	.0742	.0689	.0585	.0508	.0449
700	29.178	14.329	56.7	.0880	.0842	.0787	.0745	.0691	.0587	.0510	.0450
600	29.296	14.387	57.0	.0884	.0845	.0790	.0748	.0694	.0589	.0512	.0452
500	29.390	14.433	57.4	.0886	.0848	.0793	.0750	.0696	.0591	.0513	.0454
400	29.496	14.486	57.8	.0890	.0851	.0796	.0753	.0699	.0593	.0515	.0455
300	29 603	14.538	58.3	.0893	.0854	.0799	.0756	.0702	.0595	.0517	.0457
200	29.710	14.591	58.8	.0896	.0857	.0801	.0758	.0704	.0597	.0519	.0458
100	29.818	14.643	59.4	.0899	.0860	.0804	.0761	.0707	.0600	.0521	.0460
Sea level } **0**	**29.925**	**14.696**	**60.0**	**.0903**	**.0863**	**.0807**	**.0764**	**.0709**	**.0602**	**.0523**	**.0462**
−500	30.469	14.963	....	.0919	.0879	.0822	.0778	.0722	.0613	.0532	.0470
−1,000	31.022	15.235	....	.0936	.0895	.0837	.0792	.0735	.0624	.0542	.0479
−1,500	31.582	15.510	....	.0953	.0911	.0852	.0806	.0749	.0635	.0552	.0487
−2,000	32.151	15.789	....	.0970	.0928	.0867	.0821	.0762	.0647	.0561	.0496
−2,500	32.727	16.072	....	.0987	.0944	.0883	.0835	.0776	.0658	.0572	.0505
−3,000	33.312	16.359	....	.1005	.0961	.0899	.0850	.0790	.0670	.0582	.0514
−3,500	33.903	16.650	....	.1023	.0978	.0915	.0865	.0804	.0682	.0592	.0523
−4,000	34.504	16.945	....	.1041	.0996	.0931	.0881	.0818	.0694	.0603	.0533
−4,500	35.113	17.244	....	.1059	.1013	.0947	.0896	.0832	.0706	.0613	.0542
−5,000	35.730	17.547	....	.1078	.1031	.0964	.0912	.0847	.0719	.0624	.0551

The table on the preceding page is deduced from the determinations of atmospheric density and pressure, under normal conditions, at different elevations above and below sea level, as established by the celebrated British astronomer royal, Sir George Biddle Airy (1840), and the aëronautic observations of Herschel and Glaisher.

The **atmospheric pressures** in the third column of the table are the mean of many direct observations taken at different altitudes, under normal conditions, and constitute what are generally known as "Airy's tables."

The **temperatures** in the fourth column correspond to the mean observed temperatures, at different altitudes and are based on a sea-level temperature of 60 deg. F. They are suggestive of the rate of cooling or fall of temperature with respect to increase of altitude.

The following table shows the mean observed temperatures of the atmosphere at different altitudes, the rate of fall (deg. per 1000 ft.) and the estimated average temperature of air column extending from sea level to each respective altitude given:

Table Showing Relation of Mean Temperature to Altitude, in the Atmosphere

Altitude or elevation above sea level, ft.	Mean observed temperature, deg. F.	Rate of fall in temperature, deg. per 1000 ft.	Mean average temperature of air column, deg. F.
25,000	0	1.6	24
20,000	8	1.8	29
15,000	17	2.0	35
10,000	27	2.5	42
8,000	32	3.0	45
5,000	41	3.5	50
3,000	48	4.0	54
0	60		

The mean average temperature of air column extending from sea level to any altitude given in the above table makes it possible to calculate the normal barometric pressure for that altitude, by means of the following formula:

The application of this formula requires the use of a table of seven-place logarithms or more. It serves to check the temperature observations at these altitudes.

$$B_h = 29.925\left[1 \pm \frac{1}{144\ (0.37T)}\right]^h$$

in which

B_h = barometric pressure, at altitude h (in.);
T = average absolute temperature of air column, extending from sea level to altitude h (deg. F.);
h = altitude above sea level (ft.).

The sign $\pm$, in the formula, relates to the altitude h, as being above or below sea level. For altitudes above sea level, the second term within the brackets is negative and the minus $(-)$ sign must be used. For altitudes below sea level, this term is positive and the plus $(+)$ sign is employed.

Relation of Drop in Temperature to Altitude.—Approximately, the fall in temperature (t), in the atmosphere, varies as the 1.4 root of the height (h) above the sea level; thus,

$$\frac{t_2}{t_1} = \sqrt[1.4]{\frac{h_2}{h_1}}$$

Applying this principle and assuming a temperature drop of 6 deg. at an altitude of 1000 ft. above sea level, disregarding the effect of the radiation of heat from the earth, the mean average temperature (t), for any altitude (h), expressed in thousands of feet, can be calculated approximately thus:

$$t = 60 - 6\sqrt[1.4]{h} + \frac{2}{(h-2)^2}$$

This formula assumes a normal sea-level temperature of 60 deg. F., which is the first term in the second member of the equation. The second term of this member accounts for the fall of temperature corresponding to the increase of altitude; while the third term expresses the effect of the radiation of heat from the earth, which varies inversely as the square of the altitude factor $h - 2$, probably owing to the influence of clouds or vapor in the lower atmosphere.

Example.—Let it be required to find the temperature, at an elevation of 8000 ft. above sea level, corresponding to a normal temperature of 60 deg. at sea level.

Solution.—In this case, the altitude expressed in thousands of feet is $h = 8$; which substituted in the formula gives:

$$t = 60 - 6\sqrt[1.4]{8} + \frac{2}{(8-2)^2} = 33.4 \text{ deg. F.}$$

The mean observed temperature for this altitude as given in the table is 32 deg. F.

Average Temperature of Air Column.—The average temperature of the air column extending from sea level to any altitude h, expressed in thousands of feet, can be calculated with close approximation by the formula

$$\text{Average temp.} = 60 - 3\sqrt[1.28]{h}$$

The mean average air-column temperature, as calculated by this formula, can be used to find the corresponding normal atmospheric pressure by substituting its value, reduced to absolute temperature (T), in the formula

$$p_h = 14.696\left(1 - \frac{1}{53.28T}\right)^h$$

The use of this formula will require a table of seven-place logarithms or more. In the solution of the following example, a ten-place logarithmic table was employed.

Example.—Find the mean average air-column temperature corresponding to a sea-level temperature of 60 deg. F., for an elevation of 12,000 ft. above the sea.

Solution.—In this case, $h = 12$, which gives for the mean average air-column temperature

$$\text{Average temp.} = 60 - 3\sqrt[1.28]{12} = 39 \text{ deg. F.}$$

The absolute temperature is 460 + 39 = 499 deg.F., abs.

Example.—Calculate the normal atmospheric pressure for an altitude of 12,000 ft., using the mean average air-column temperature found in the last example, $T = 499$ deg. F. abs.

Solution.—Substituting the given values in the formula gives for the normal atmospheric pressure at this altitude,

$$p_{12,000} = 14.696\left(1 - \frac{1}{53.28 \times 499}\right)^{12,000} = 9.359 \text{ lb. per sq. in.}$$

The diagram shown on the following page compiles the data relating to average observed temperatures at different elevations, and the calculated heights of the corresponding water and mercury columns, weight and pressure of air, of interest to the student of atmospheric conditions.

Elevation above Sea Level (Feet)	Mean Observed Temperature (Fahrenheit)	Weight of Air (lb. per cu. ft.)	Atmospheric Pressure Lb. per sq. ft.	Lb. per sq. in.	Water Column Maximum Density (Feet)	Mercury Column (Inches)
25,000	0°	0.0327	802.2	5.571	12.85	11.343
20,000	8°	0.0393	981.2	6.814	15.70	13.874
15,000	17°	0.0472	1198.5	8.323	19.17	16.948
10,000	27°	0.0561	1455.4	10.107	23.30	20.582
5,000	41°	0.0659	1760.3	12.224	28.20	24.890
1,000	55°	0.0744	2041.1	14.174	32.70	28.861
Sea Level	60°	0.0764	2116.2	14.696	33.90	29.925

Air Column — 1 Ft. Square

The Differential Method.—The pressure of the atmosphere, per unit area, at any altitude x is due to the weight of air column above such point of observation. Air being compressible, any increment of pressure (δp_x), causes a corresponding minus increment of height $(-\delta x)$; and, calling the unit weight of air w_x at the altitude x, we have

$$\delta p_x = -w_x \delta x \tag{1}$$

But the unit weight of air varies with the pressure it supports. Hence, calling this unit weight and pressure at sea level w_0 and p_0, respectively, and that at any altitude x, w_x, and p_x, we have

$$\frac{w_x}{w_0} = \frac{p_x}{p_0}; \text{ and } w_x = \frac{w_0}{p_0} p_x \qquad (2)$$

Substituting this value in equation 1 and dividing both members of the equation by p_x, gives

$$\frac{\delta p_x}{p_x} = -\frac{w_0}{p_0} \delta_x \qquad (3)$$

But, the differential of a quantity divided by the quantity is equal to the differential of its Naperian logarithm.

Hence, $$\delta \log p_x = -\frac{w_0}{p_0} \delta_x; \text{ or } \delta_x = -\frac{p_0}{w_0} \delta \log p_x \qquad (4)$$

Then integrating between the limits $x = 0$, and $x = h$, remembering that when $x = 0$, $p_x = p_0$; and when $x = h$, $p_x = p_h$ and subtracting the lower integral from the higher,

$$h - 0 = \frac{p_0}{w_0} (\log p_0 - \log p_h) \qquad (5)$$

But the unit weight of dry air at sea level, normal atmospheric pressure (lb. per sq. ft.), is

$$w_0 = \frac{p_0}{53.28T}; \text{ and } \frac{p_0}{w_0} = 53.28T \qquad (6)$$

which, substituted in equation 5, gives for the altitude corresponding to any pressure, under normal conditions,

$$h = 53.28T (\log p_0 - \log p_h) \qquad (7)$$

Or, expressed in common logarithms,

$$h = 122.68T(\log p_0 - \log p_h) \qquad (8)$$

For normal atmospheric pressure, at sea level, $p_0 = 14.696$ lb. per sq. in., and $\log 14.696 = 1.1672$; hence

$$h = 122.68T (1.1672 - \log p_h)$$

Or, $$\log p_h = 1.1672 - \frac{h}{122.68\,T} \qquad (10)$$

PHYSICS OF AIR AND GASES

The volume of any given weight of air or gas depends on two factors—the temperature of the gas and the pressure it supports.

Effect of Temperature.—For any given weight of air or gas, its volume varies directly as its absolute temperature, assuming the pressure remains constant.

Effect of Pressure.—For any given weight of air or gas, its volume varies inversely as the pressure it supports, assuming the temperature remains constant.

Expansion and Contraction of Air or Gas.—Any change in temperature or pressure causes a corresponding change in the volume of the air or gas, as follows:

Increase of temperature causes expansion.
Decrease of temperature causes contraction.
Increase of pressure causes contraction.
Decrease of pressure causes expansion.

Coefficient of Expansion or Contraction.—The coefficient of expansion is the same as that of contraction. This coefficient relates to change in volume due to change in temperature and is practically the same for all gases and air and independent of the pressure.

The coefficient of expansion of air or gas is the ratio of the increase in volume to the original volume, for an increase of one degree in temperature. Since a degree of the Fahrenheit scale is $\frac{5}{9}$ of a degree of the centigrade scale, it is evident that the Fahrenheit coefficient of expansion will be exactly $\frac{5}{9}$ of the centigrade coefficient. These coefficients are as follows: Centigrade, 0.003663; Fahrenheit, 0.002035.

Illustration.—Let it be required to find the increase in volume in an air current of 100,000 cu. ft. entering a mine at a temperature of 32 deg. F. and discharged at a temperature of 68 deg. F.

Solution.—The rise in temperature is 68 − 32 = 36 deg. F. The increase in volume, calculated by the Fahrenheit scale, is

$$100{,}000 \times 0.002035 \times 36 = 7326 \text{ cu. ft.}$$

Or, since 68 and 32 deg. F. correspond to 20 and 0 deg. C., the rise in temperature is 20 − 0 = 20 deg. C., and the increase in volume, calculated by the centigrade scale, is

$$100{,}000 \times 0.003663 \times 20 = 7326 \text{ cu. ft.}$$

2

Note.—Instead of multiplying by these coefficients, it is possible to divide by their reciprocals, which are

Fahrenheit, $\frac{1}{0.002035} = 491.4$, say 492

Centigrade, $\frac{1}{0.003663} = 273$

These numbers, being divisors, show that air or gas expands or contracts $\frac{1}{273}$ of its volume, for each degree rise or fall in temperature (centigrade); or $\frac{1}{492}$ of the same volume for each degree rise or fall in temperature (Fahrenheit). The figures point to what has been called the "absolute zero" of temperature scales as being 273 deg. below freezing (−273°C.) or 492 deg. below freezing (−460°F.).

Absolute Zero.—The so-called "absolute zero" of temperature scales is based on the observed rate of expansion and contraction of all gases and air. This rate is practically $\frac{1}{273}$ of the volume, per degree centigrade; or $\frac{1}{492}$ of the volume, per degree Fahrenheit. It is clear that if this rate continued unchanged a fall in temperature of 273 deg. C., or 492 deg. F., below the freezing point of water, would reduce the volume of the gas to zero, when all molecular vibrations would cease, indicating a total absence of heat and pressure.

The absolute zero has therefore been fixed at 273 deg. below the common zero of the centigrade scale (−273°C.), which corresponds to 460 deg. below zero on the Fahrenheit scale. The fixing of this point is purely arbitrary, its chief value being the facility it affords in the calculation of gaseous volumes with respect to temperature.

Absolute Temperature.—Absolute temperatures differ from common temperatures only in being estimated from the absolute zero. Hence the absolute temperature is obtained from the common temperature by adding 273 in the centigrade or 460 in the Fahrenheit scale; thus,

30 deg. C. = 273 + 30 = 303 deg., absolute.
60 deg. F. = 460 + 60 = 520 deg. absolute.

Relation of Volume and Absolute Temperature of Air and Gas.—The law commonly known as **Gay Lussac's or Charles'**

law makes the volume of all gases and air, under constant pressure, vary directly as the absolute temperature.

This relation is clearly illustrated in Fig. 4, which assumes a volume of 460 cu. ft. of air or gas at 0 deg. F., corresponding to the absolute temperature at that point. It will be observed that this volume expands and contracts exactly as the absolute temperature rises or falls, except at the lowest temperatures approaching the liquefaction of the air or gas where the law naturally fails, owing to the changing state of the matter.

Relation of Volume and Pressure of Air and Gas.—For a constant temperature, the volume of air and gases varies inversely as the pressure supported. In this connection, pressure is often estimated as one, two, three, etc., **atmospheres,** meaning that the pressure supported by the air or gas is one, two, three, etc., times the normal atmospheric pressure at that place. This is commonly known as **Boyle's** or **Mariotte's law of volume.**

An "atmosphere" is sometimes incorrectly taken to mean normal sea-level pressure (14.7 lb. per sq. in.). Such a meaning of the term, however, would manifestly limit its application to sea level, or furnish an arbitrary standard inconvenient for use.

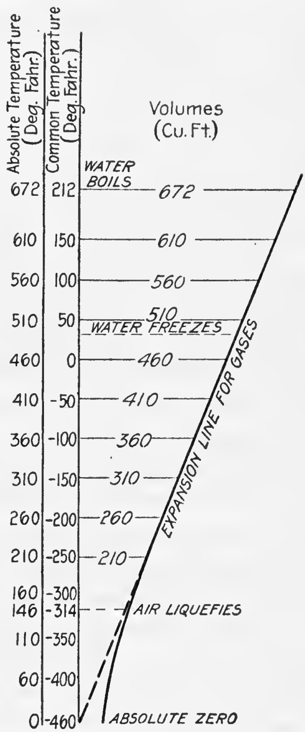


Fig. 4.

The term **"free air"** relates to atmospheric air at any elevation and for any condition. According to the above rule, when free air is compressed to two, three or four atmospheres its volume is reduced to $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ of the original volume, assuming the temperature remains constant. At the same time, the pressure or **tension of the air** is increased to two, three or four times the atmospheric or free-air pressure, whatever that may have been, assuming always a constant temperature of the air.

The **expansion of air,** by the same law, is accompanied by a fall of pressure, the **volume ratio** being equal to the inverse **pressure ratio,** for the same temperature. The pressure referred to here is the **absolute pressure,** or the pressure above a vacuum or zero.

Relation of Absolute Temperature and Pressure of Air and Gas.—For a constant volume, the absolute temperature of air and gases varies directly as the absolute pressure.

Volume, Temperature, Pressure of Air and Gas.—The relation of the volume (v), pressure (p) and absolute temperature (T), for a given weight of air or gas is expressed simply by the following formulas:

Constant pressure	Constant temperature	Constant volume
$\frac{v_2}{v_1} = \frac{T_2}{T_1}$	$\frac{v_2}{v_1} = \frac{p_1}{p_2}$	$\frac{p_2}{p_1} = \frac{T_2}{T_1}$

The relations of volume, temperature and pressure of air and gas depend on two main conditions: 1. The gas may or may not be free to expand. 2. Heat may or may not be added or taken from the gas.

Addition of Heat.—Two cases may arise, as follows:

(*a*) If the air is confined (constant volume) the rise in temperature is more rapid, since all the heat is then transformed into heat energy or **internal work,** and the pressure rises accordingly.

(*b*) If the air is free to expand (constant pressure) the rise in temperature, for the same addition of heat, is much less rapid. In this case, the air in expanding performs **external work** against the pressure it supports. A part of the heat added is thus absorbed in doing outside work while the remainder, only, is available for internal work and manifest as heat energy, thus causing a lesser rise of temperature.

Work of Expansion of Air.—When air is expanded by the addition of heat the external work performed can be calculated in two ways, as follows:

1. On a heat-unit basis, by subtracting the heat absorbed,

per pound of air, per degree rise in temperature, for constant volume, from the heat, per pound, per degree, for constant pressure; and multiplying this difference, which is the heat converted into external work, by the foot-pounds per heat unit; thus, since 1 B.t.u. = 778 ft.-lb.,

Heat, per lb.-deg. (sp. heat, const. pressure)	0.2374 B.t.u.
Heat, per lb.-deg. (sp. heat, const. volume)	0.1689 B.t.u.
Heat, per lb.-deg., available for external work	0.0685 B.t.u.
External work, per lb.-deg. . . . . 0.0685 × 778 =	53.29 ft.-lb.

2. The external work performed in the expansion of air, per pound, per degree, can be calculated, also, very simply by multiplying the volume of 1 lb. of dry air, at 1 deg. F., absolute, and 1 lb. per sq. in. pressure (0.37 cu. ft.), by 144, the number of square inches in 1 sq. ft.; thus,

External work, per lb.-deg. . . . 0.37 × 144 = 53.28 ft.-lb.

Adiabatic Expansion and Compression.—When there is no addition of heat in the expansion, or no loss of heat in the compression of air or gas, the relations of volume, temperature and pressure follow other laws than those previously given. Such expansion or compression is described as **"adiabatic,"** meaning no passage (of heat) in or out of the gas.

In **adiabatic expansion,** there being no addition of heat, the increase in volume is at the expense of the internal energy and a fall of temperature is the result, which is accompanied also by a fall of pressure.

In **adiabatic compression,** there being no loss of heat, the internal energy is augmented by the heat of compression, and the result is an increase of both temperature and pressure.

Adiabatic Formulas.—The following formulas express the relation of volume (v), pressure (p) and absolute temperature (T), for any given weight of air or gas, when expanded or compressed without gain or loss of heat. In actual practice it is only possible to approximate adiabatic expansion or compression:

$$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{0.7117} \qquad \frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{2.469} \qquad \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{3.469}$$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{1.405} \qquad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{0.405} \qquad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{0.288}$$

It is important to observe that adiabatic expansion or compression always involves a change in temperature. Where the temperature is maintained constant, by adding heat in expanding, or extracting heat (cooling) in compressing, the change in volume is described as **"isothermal"** expansion or compression. In practice, it is only possible to approximate isothermal conditions in the expansion or compression of air or gas.

The application of the above formulas necessitates the use of logarithms.

MATTER

Definition.—Matter is the tangible substance occupying space and endowed with properties that give to it form, motion and other distinguishing characteristics, by virtue of an all-pervading or impressed subtle force generally described as electrical.

Divisions of Matter.—Until recently, the ultimate or smallest conceivable division of matter was assumed to be the **atom** (Dalton, 1808). Later researches of radio-active substances have developed the infinitely smaller particles which have been termed **"electrons"** (Stoney, 1891) and **"corpuscles"** (Thomson, 1897). The electron is assumed to be a minute particle of matter having a negative charge of electricity; and its mass is variously estimated at from $1/1700$ to $1/2000$ of the mass of the atom of hydrogen.

The chemical divisions of matter are the familiar **atoms** and **molecules.**

Properties of Matter.—The universal attribute of all matter is that described as **"mass,"** which may be simply defined as amount of matter. By virtue of its assumed electrical state or condition, all matter is endowed with certain tangible and measurable qualities or properties, such as **weight, inertia, density, elasticity, cohesion, divisibility, impenetrability, expansion, contraction.**

Matter undergoes many changes but is absolutely indestructible.

Law of Attraction.—The universal law of attraction is that every particle of matter attracts every other particle of matter, the force of attraction varying inversely as the square of the distance between the particles.

Terrestrial attraction is the attraction that the mass of the earth exerts on the mass of a body. This is commonly called **"gravitation"** and the attractive force, the **"force of gravity"** or simply **"gravity."**

Form or State of Matter.—All matter exists in one of three different forms, namely, **solid, liquid,** or **gaseous.** The same matter may pass from one form or state to another owing to a change in density.

Molecular State.—The molecular theory assumes that all matter, solid, liquid or gaseous, in respect to its physical condition, is composed of molecules, each complete in itself. It is assumed that these molecules are subject to two opposite or opposing forces known as the **"molecular forces"** of attraction and repulsion.

Molecular attraction, acting to bind the molecules of matter together, is in obedience to the common law of attraction in all matter.

Molecular Repulsion, acting to drive the molecules of matter apart, is the result of a state of incessant molecular vibration, which produces the effect called **"heat."**

Solids.—Matter in the solid state is characterized by a greater or less rigidity of its molecules. The force of molecular attraction is here stronger than that of repulsion, and the molecules are held in a firmer grasp.

Liquids.—In the liquid state, the forces of attraction and repulsion are about evenly balanced, and the molecules move freely among each other.

Gases.—In the gaseous state, the repulsive forces are in the ascendency and the molecules are driven so far apart that the density of the matter is reduced to that of a gas.

Liquids and gases are both **fluids**, which is a general term applied to any form of matter other than a solid.

Illustration.—Ice, water and steam furnish a good illustration of how the same matter can pass successively from the solid to the liquid and gaseous states. In the passage from one state to another, there is no change in the matter itself, the difference being due to the heat condition of the mass.

In the passage from solid to liquid, or from liquid to gas or vapor, heat is given out; and, vice versa, heat is absorbed when a gas or vapor becomes a liquid, or a liquid becomes a solid. The change is thus a heat condition only.

Vapors and Gases.—The term **vapor** properly describes the gaseous condition of most substances that, at ordinary temperatures, exist as liquid or solid; or a gas at or near its point of liquefaction. The term thus has a suggestive meaning of the possible liquid or solid state of the substance now in the gaseous state.

The term **gas,** on the other hand, is a general term that relates solely to the gaseous condition of matter; and is thus more properly applied to those substances that, at ordinary temperatures, exist as gas; although they may be liquefied or solidified by a decrease of temperature and an increase of pressure.

Thus, we speak of air, oxygen, hydrogen, nitrogen, carbon dioxide, methane, etc., as "gases," in contrast to steam (water vapor) and the vapors of such volatile liquids and solids as naphtha, benzine, camphor and other similar substances.

Vaporization takes place at all temperatures; and in many instances, a substance will pass directly from the solid to the gaseous condition, without becoming liquid.

Mass, Volume, Density.—Since mass is amount of matter, the mass (M) of a body is the quantity of matter it contains, which is determined by the volume (V) of the body and the density (D) of the matter. The relation of these elements is expressed by the formula

$$M = VD$$

Then, considering a unit volume ($V = 1$), it is evident that the **"unit of mass"** is equal to the **"unit of density."** In other words, whatever is taken as the accepted unit or standard of density is also the unit and standard for the measurement of mass, which is the ultimate unit.

MEASUREMENT

The valuation and comparison of the various forms and conditions of matter and the estimation of physical phenomena are made by reference to three general standards of measurement, namely, **distance, force** and **time.** There are many modifications and combinations of these three elemental standards.

Distance.—This includes the measurement of length, surface and volume, all of which are derived from the same standard of measure.

Force.—All measurement of force is based on the attractive force exerted by the earth on an assumed **unit of mass** at the surface (sea level), in any given latitude. Mass thus becomes the true unit in this measurement; but being intangible, the adopted unit is the **pound,** which represents a certain definite mass, taken as the "unit of mass," for purposes of measurement. A force is measured by the effect of its action on a known mass. There are two conditions: 1. Static condition (mass fixed, immovable), force applied to a body produces pressure, weight. 2. Dynamic condition (mass free to move) force produces motion, velocity.

Under these two conditions, there are, therefore, two units of force. The unit of measure for **static force** is the pound, while the unit of measure for **dynamic forces** is the force that will produce a unit of velocity in a unit of mass, in a unit of time. In other words, the force that will increase the rate of motion of a unit mass, by a unit distance, in a unit time.

Application.—Applying these units of measure, the weight (W) of a body, expressed in pounds, is the static force (F) acting on the body, due to gravity.

Hence, in statics,

$$F = W \qquad (1)$$

In dynamics, the force (F_1) producing motion is measured by the mass (M) of the body and the velocity (v) produced per unit of time. Hence, in dynamics,

$$F_1 = Mv \qquad (2)$$

The velocity produced may be constant or accelerated. Constant velocity is the distance passed over in a unit of time. **Acceleration** is the gain in velocity per unit of time. A constant force, as gravity, acting on a body free to move produces a uniform acceleration; that is to say the gain in velocity, each unit of time, is constant.

Assuming a falling body, the force producing motion is the weight (W) of the body, and the gain in velocity per unit of time (acceleration due to gravity, **g**) is the velocity produced in the mass (M). Hence, in falling bodies,

$$W = Mg \tag{3}$$

and

$$M = \frac{W}{g} \tag{4}$$

which enables the calculation of the mass of a body from its weight.

Combining formulas (2) and (3),

$$\frac{F_1}{W} = \frac{v}{g} \tag{5}$$

Hence a force acting to produce motion in a body bears the same ratio to the weight of the body, as the acceleration due to the force bears to the acceleration due to gravity. Or, expressed as a proportion,

$$F_1 : W :: v : g \tag{6}$$

Time.—The element of time is important in the estimation of velocity and power. For example, to traverse the same distance in one-half the time will require twice the velocity. Likewise, to perform the same work in one-half the time will require twice the power.

Special Units.—There are numerous other units of limited significance; such as units of capacity, pints, quarts, gallons, barrels, etc.; units of currency, cents, dimes, dollars, etc.; circular units, degrees, radians, etc.; electrical units, amperes, volts, ohms, watts, etc.

Compound Units.—Many units of measure are composed of two or more simple units. The following are examples:

Unit velocity—(distance) ÷ (time) Ft. per sec., or ft. per min.
Unit work—(distance) × (force)................Ft.-lb.
Unit power—(distance) × (force) ÷ (time)...Ft.-lb. per min.

The above are only given as samples of many similar compound units; such as inch-pounds; miles per hour; gallons per hour; cubic feet per minute; pounds per cubic foot; tons per acre; foot-acres, etc.

All of these, it will appear, are derived from the simple units of distance, force, time, or the special units to which reference has been made.

Energy.—Energy, in physics, is **capacity to perform work.** It is the vitalizing force that is manifested in matter by the familiar agencies of heat, light, electricity, magnetism, molecular attraction, chemical affinity, etc., all of which are equally convertible, one into the other, without loss.

The physical agencies or forms of energy just mentioned are each and all convertible into mechanical motion, which, again, can be reconverted into heat, light, electricity, and magnetism. This fact gives rise to what is called the **"mechanical equivalent"** in reference to heat.

Forms of Energy.—Energy is of two kinds that differ from each other only in the sense that one (kinetic) is actual and present, while the other (potential) is possible only.

Kinetic energy (E) is the energy possessed by a body by virtue of its motion. The force producing an acceleration (f) in a mass (M), or the "living force" in the body (**momentum**), is Mf. The acceleration (f) being uniform or the velocity increasing uniformly, the distance increase, per unit of time is $f/2$, and the work performed in producing this acceleration is stored in the body as "kinetic energy," by virtue of which the body would continue to move at the velocity imparted, till opposed by some force. The energy stored per second is calculated by the formula

Kinetic energy, $$E = Mf \times \frac{f}{2} = \frac{1}{2} Mf^2$$

Potential energy is the energy that is possessed by a body by virtue of the position or state in which it is held or restrained so that motion cannot take place till the restraining force is removed. Examples of bodies having potential energy are, a suspended ball, a confined spring, etc.

A common method of making physical measurements for the estimation of weight, volume, heat, etc., is by reference to some adopted standard. All such measurements are relative and are frequently termed "specific." Such, for example, are **specific gravity, specific volume, specific heat,** etc. The atomic weight of elements is often called **specific weight.**

The Elements.—An element is a substance that has not, as yet, been resolved into parts of a different nature and is, therefore, regarded as being composed wholly of one kind of matter or **simple,** in contrast with a **compound,** which is composed of two or more elements or kinds of matter.

The following table gives the more important elements, together with their chemical symbols and specific or atomic weights:

Table of the More Important Elements
International Committee (1910)

Elements	Symbols	Atomic weights		Elements	Symbols	Atomic weights	
		H = 1	O = 16			H = 1	O = 16
Aluminum	Al	26.9	27.1	Manganese	Mn	54.49	54.93
Antimony	Sb	119.3	120.2	Mercury	Hg	198.4	200.0
Argon	A	39.6	39.9	Molybdenum	Mo	95.23	96.0
Arsenic	As	74.36	74.96	Nickel	Ni	58.21	58.68
Barium	Ba	136.27	137.37	**Nitrogen**	**N**	13.9	**14**.01
Bismuth	Bi	206.34	208.0	Osmium	Os	189.37	190.9
Boron	B	10.91	11.0	**Oxygen**	**O**	15.88	**16.0**
Bromine	Br	79.28	79.92	Palladium	Pd	105.9	106.7
Cadmium	Cd	111.5	112.4	Phosphorus	P	30.77	31.0
Cæsium	Cs	131.75	132.81	Platinum	Pt	193.44	195.0
Calcium	Ca	39.77	40.09	Potassium	K	38.78	39.1
Carbon	C	11.9	**12.0**	Radium	Ra	224.6	226.4
Cerium	Ce	139.13	140.25	Rhodium	Rh	102.08	102.9
Chlorine	Cl	35.18	35.46	Selenium	Se	78.6	79.2
Chromium	Cr	51.58	52.0	Silicon	Si	28.1	28.3
Cobalt	Co	58.5	58.97	Silver	Ag	107.02	107.88
Columbium	Cb	92.75	93.5	Sodium	Na	22.82	23.0
Copper	Cu	63.06	63.57	Strontium	Sr	86.92	87.62
Fluorine	F	18.85	19.0	**Sulphur**	**S**	31.81	**32**.07
Gold	Au	195.62	197.2	Tellurium	Te	126.48	127.5
Helium	He	3.97	4.0	Thallium	Tl	202.37	204.0
Hydrogen	H	**1.0**	1.008	Tin	Sn	118.05	119.0
Iodine	I	125.9	126.92	Titanium	Ti	47.72	48.1
Iridium	Ir	191.56	193.1	Tungsten	W	182.53	184.0
Iron	Fe	55.4	55.85	Uranium	U	236.59	238.5
Lead	Pb	205.44	207.1	Vanadium	V	50.79	51.2
Lithium	Li	6.94	7.0	Zinc	Zn	64.85	65.37
Magnesium	Mg	24.13	24.32	Zirconium	Zr	89.88	90.6

The preceding table contains only 56 out of the 80 or more elements that have been discovered, many of which are so rare as to be of little practical importance. The values of the atomic weights are given referred both to hydrogen as unity and oxygen as 16. The heavy type indicates the values commonly used in the study of mine gases.

DENSITY AND VOLUME

Density Defined.—The term "density" refers to the amount of matter in a given volume or space. The commonly adopted measure of density is the ratio of the weight of a body to its volume or the space it occupies, as expressed by the formula:

$$Density = \frac{weight}{volume}$$

In a general sense, the term density has thus come to mean the weight per unit volume. For example, the density of water is commonly understood to mean its weight per cubic foot (62.4283 lb., max. dens., 4°C.).

Specific or Atomic Volume.—These terms have reference to an assumed unit volume for all gases, which **unit** is the assumed vo'ume of a single gaseous atom.

Avogadro's Law of Gaseous Volume.—This law may be stated briefly and clearly as follows:

At the same temperature and pressure all gaseous molecules are assumed to be of the same size.

With a few unimportant exceptions, this law applies to all gases, whether simple or compound. It holds true for all mine gases and is important in the calculation of the relative volume of gases concerned in chemical reactions.

Molecular Volume.—Chemical hypothesis assumes that the molecules of simple substances each contain two atoms only, while the molecules of a compound substance may contain any number of atoms, but never less than two. Notwithstanding this multiplicity of atoms, Avogadro's law makes all gases, with a few unimportant exceptions, to contain the same number of molecules, per unit volume, when measured at the same temperature and pressure. In other words, measured at the same temperature and pressure, all gaseous molecules are of the same size.

Calculation of Density.—The elements form the basis of all relative measurements with respect to volume, density and weight. For example, the density of air, referred to hydrogen as unity (H = 1), can be calculated from the relative weights and volumes of oxygen and nitrogen, which are the chief constituents of air. The composition of pure air, by volume, is practically, oxygen (O), 20.9 per cent.; nitrogen (N), 79.1 per cent. Then, since the atomic weight of oxygen is 16 and that of nitrogen 14, the relative weight of 100 volumes of air, referred to hydrogen as unity, is found as follows:

Oxygen,	20.9×16	$= 334.4$
Nitrogen,	79.1×14	$= 1107.4$
Air,	100 vol's	$= 1441.8$

Therefore, one volume of air is $1441.8 \div 100 = 14.418$ times as heavy as the same volume of hydrogen; or, the density of air referred to hydrogen is 14.418.

The percentage composition of pure air, by weight, is readily calculated from the above figures; thus:

Oxygen, $(334.4 \times 100) \div 1441.8 =$ say 23.2 *per cent.*
Nitrogen, $(1107.4 \times 100) \div 1441.8 =$ say 76.8 *per cent.*

SPECIFIC GRAVITY

The specific gravity of a substance—solid, liquid, or gas—is the ratio of the weight of that substance to the weight of another substance taken as a standard, volume for volume;

$$Sp.\ gr. = \frac{wt.\ of\ unit\ vol.\ of\ substance}{wt.\ of\ unit\ vol.\ of\ standard}$$

Comparison of Standards.—Hydrogen, air and water are the three standards commonly used in the determination of the specific gravity of gases, liquids and solids. The relative densities of these standards are as follows:

Air (dry) is 14.418 times as heavy as hydrogen, at the same temperature and pressure, volume for volume.

Water (max. density, 4°C.) is 773 times as heavy as dry air at 32 deg. F., bar. 29.92 in.; and 815 times as heavy as dry air at 60 deg. F., bar. 30 in., volume for volume.

Standard for Gases.—The standard adopted for gases is air or hydrogen, of the same temperature and pressure as the gas.

Standard for Liquids and Solids.—The standard adopted for liquids and solids is water at maximum density. Except where great accuracy is desired, the weight of 1 cu. ft. of water is taken as 62.5 lb. Exactly, 1 cu. ft. of pure water, at maximum density weighs 62.4283 lb.; or 1 cu. in. weighs 252.89 grains = 0.03613 lb.

Calculation of the Specific Gravity of Gases.—Since air is 14.4 times as heavy as hydrogen, at the same temperature and pressure, the specific gravity of a gas, referred to air as unity, can be calculated by dividing one-half of its molecular weight by 14.4. For example, the molecular weight of carbon dioxide is 44; therefore, 44 ÷ 2 = 22, and 22 ÷ 14.4 = 1.528. The actual specific gravity is 1.529.

Finding Specific Gravity of Gases.—A glass globe, any convenient size, is first weighed empty (air exhausted), w; then full of air, w_1; and, lastly, filled with the gas, w_2: the temperature and pressure remaining constant.

$$Sp.\ gr. = \frac{w_2 - w}{w_1 - w}$$

Finding Specific Gravity of Liquids.—A glass-stoppered bottle is first weighed empty, w; then filled with water w_1; and, lastly, filled with the liquid, w_2. The specific gravity is then calculated by the above formula for gases. Or, the specific gravity is determined by a graduated float (hydrometer).

Finding Specific Gravity of Solids.—Weight of the solid in air, w; weight immersed in water w_1. The weight of the water displaced is then $w - w_1$, which has the same volume as that of the solid.

$$Sp.\ gr. = \frac{w}{w - w_1}$$

Specific Gravities and Unit Weights of Solids and Liquids

Substance	Average specific gravity (water = 1)	Average weight (lb. per cu. ft.)
Alcohol, pure	0.793	49.5
commercial	0.834	52.1
Aluminum	2.66	166.0
Asphalt (1 to 1.8)	1.4	87.0
Brass, cast (7.8 to 8.4)	8.1	506.0
rolled	8.4	525.0
Brick, pressed	2.4	150.0
common, hard	2.0	125.0
Brickwork, masonry (1.8 to 2.3)	....	110 to 140
Bronze (8.7 to 8.9)	8.8	550.0
Clay (1.8 to 2.6)	2.2	137.5
Coal, anthracite (1.3 to 1.7)	1.5	93.75
bituminous (1.2 to 1.5)	1.3	81.25
cannel, gas coal (1.18 to 1.28)	1.23	76.88
lignite, brown coal	1.1	68.75
Coke, loose piled	....	20 to 25.0
Concrete	2.3	144.0
Copper, cast (8.6 to 8.8)	8.7	543.0
rolled (8.8 to 9)	8.9	556.0
Earth, dry, loose to well rammed	....	76 to 95.0
moist, loose to well rammed	....	78 to 96.0
wet, flowing mud	....	105 to 115.
Granite (2.56 to 2.88)	2.72	170.0
Gold, cast (18.29 to 19.37)	18.83	1176.0
Gravel, loose	....	95 to 100.
Gypsum, ground or calcined, loose	....	56.0
well shaken	....	64.0
Ice	0.92	57.5
Iron, cast (6.9 to 7.4)	7.2	450.0
rolled	7.68	480.0
wrought, sheet (7.6 to 7.9)	7.8	485.0
Lead (11.3 to 11.47)	11.38	710.0
Lime (quicklime)	1.5	93.75
ground, loose (66 lb. per bus.)	....	53.0
Limestone	2.7	168.0
Marble (2.5 to 2.8)	2.65	165.0
Mercury (32 deg. F.)	13.593	850.0
(62 deg. F.)	13.555	847.0

Pitch	1.155	72.0
Platinum	21.6	1348.0
Rosin	1.1	68.67
Sand, dry		100.0
wet		130.0
Sandstone (2.1 to 2.7)	2.4	150.0
Shale (2.4 to 2.8)	2.6	162.0
Silver	10.5	655.0
Slate (2.7 to 2.9)	2.8	175.0
Steel (7.8 to 7.9)	7.85	490.0
Sulphur	2.0	125.0
Tallow	0.94	58.7
Tar	1.0	62.5
Tin, cast (7.2 to 7.5)	7.35	459.0
Traprock	3.0	187.0
Water (max. density, 4°C.)	**1.0**	**62.428**
(pure, 62°F.)	0.999	62.366
(pure, 212°F.)	0.958	59.806
sea, average	1.028	64.176

Weight of Woods (Dry, Seasoned)

	Lb. per cu. ft.
Ash, white	38
Birch	41
Cedar, white	23
red	35
Cherry	42
Chestnut	41
Elm	35
Ebony	76
Hemlock	25
Hickory	53
Mahogany, Spanish	53
Honduras	35
Maple	49
Oak, live	59
white	48
black, jack, etc.	35 to 45
Pine, white	25
yellow, Northern	34
Southern	45
Poplar (cottonwood)	33
Spruce	25
Sycamore	37
Walnut	37

Specific Gravities and Weights of Oils

	Sp. Gr.	Lb. per Gal.
Animal—lard	0.916	7.64
sperm (pure)	0.880	7.34
whale	0.925	7.72
Vegetable—cottonseed	0.923	7.70
linseed (raw)	0.933	7.79
(boiled)	0.780	6.51
olive	0.917	7.65
rape (colza)	0.915	7.63
Mineral—petroleum (crude)	0.77–1.06	
gasoline	0.700	5.84
kerosene (coal oil)	0.800	6.68
naphtha	0.730	6.09

Use of Specific Gravity.—To find the weight of any volume of a substance, multiply the unit weight of the standard, by the specific gravity of the substance, and that product by the given volume; or, expressed as a formula,

$$Wt. = unit\ weight\ of\ standard \times sp.\ gr. \times vol.$$

For example, taking the average specific gravity of anthracite coal as 1.5 the weight of this coal underlying 1 acre (43,560 sq. ft.) of land, for a thickness in the seam of 1 ft.; or, as we say, per foot-acre, in long tons (2240 lb.) is

$$\frac{62.5 \times 1.5 \times 43{,}560}{2240} = 1823\ long\ tons$$

Or, taking the weight of 1 cu. ft. of air (60°F., bar. 30 in.) as 0.0766 lb., since the specific gravity of carbon dioxide (CO_2) referred to air as unity is 1.529, the weight of 100 cu. ft. of this gas, at the same temperature and pressure, is

$$0.0766 \times 1.529 \times 100 = 11.712+\ lb.$$

OCCLUSION, EMISSION, DIFFUSION OF GASES

Occlusion of Gases.—The occlusion of gases in coal or other solid substances is the result of the absorptive power of the substance for that particular gas. For example, platinum, palladium, gold and other metals, as well as coal (carbon), absorb varying quantities of hydrogen, nitrogen, oxygen, the hydrocarbon and other gases.

The most common **examples of occlusion** are the absorption of hydrogen by platinum; and of methane, nitrogen, oxygen and carbon dioxide by coal and coal dust. The law that governs this absorption is unknown. The occluded gas is often held very strongly by the substance with which, however, it is not combined.

The occluded gases of coal seams **were probably produced** in the metamorphic processes that formed the coal; and their absorption (occulsion) in the solid formation may have resulted in the oxidation, to a limited extent, of the carbonaceous matter that was being transformed into coal. Such reactions, if taking place in the measures, together with the consolidation that accompanied the formation, would naturally give rise to the observed pressures of occluded gases.

The **pressure of occluded gases** in coal formations is very variable, depending not only on the conditions attending the occlusion; but to an even greater extent on the impermeability of the infolding strata, which has prevented the escape of the gases from the measures where they are formed.

Transpiration, Emission of Gases from Coal.—The gases occluded in coal exude from its exposed surface in the same manner as perspiration exudes from the pores of the skin. The term "transpiration" relates to the motion of a gas through a capillary tube and thus describes the emission of gas from coal.

The **velocity of transpiration** is according to a different law from that governing the rate of the diffusion of gases. For the same gas, the rate of transpiration varies directly as its pressure or density, and inversely as the length of the tubes through which it must pass. The velocity of transpiration is independent of the material that forms the tube, but is affected by temperature, being less for a higher temperature, and vice versa.

RELATIVE VELOCITY OF GASES (AIR = 1)

Gas	Rel. Veloc.	Gas	Rel. Veloc.
Hydrogen	2.066	Carbon dioxide	1.237
Olefiant gas	1.788	Carbon monoxide	1.034
Methane	1.639	Nitrogen	1.030
Hydrogen sulphide	1.458	Oxygen	0.903

The above table gives the relative rates or velocities with which the common mine gases transpire, referred to the rate for air as unity. The actual rate of emission of gas from coal, however, will depend chiefly on the pressure of the gas in the coal. Any sudden fall in barometric pressure is always accompanied with an increase in the emission of gas from the coal, but the increase is almost inappreciable.

Diffusion of Air and Gases.—If the molecules of all matter are assumed to be in a constant state of vibration, it naturally follows that the vibratory movement or force will vary with the density of the matter. In the case of fluids—air, gas, or liquid—the molecules are free to move among themselves, which is not true of solids, whose molecules, normally, hold fixed relations to each other.

If the densities of two fluids are equal, the vibratory force is equal in each fluid; and, at the plane of contact of the two fluid bodies, action and reaction are equal between the vibrating molecules and there is no tendency of these fluids to mix. The laws governing the mixture of liquids is not as simple as in the case of gases, owing chiefly to numerous physical properties of liquids that modify and retard the diffusive action. While the diffusion of gases into each other and into air is extremely rapid, the diffusion of liquids is often very slow and in some cases does not take place at all because of the counteracting forces.

Gases of different densities diffuse into each other and into air. The action is extremely rapid and conforms very closely to certain well defined laws. The diffusion of mine gases into the mine air and into the air current is an important feature of mine ventilation.

Law of Diffusion of Air and Gases.—By a similar experiment, showing the diffusion of hydrogen into oxygen, Graham found that for every volume of oxygen that passed into the hydrogen, four volumes of the hydrogen passed into the oxygen, the ratio thus being 4:1, in this case. But, calling the density of hydrogen unity or 1, that of oxygen is 16 and $\sqrt{16} = 4$. This and other similar experiments, all confirming the first, led Graham to propound the following law:

Graham's Law.—The velocity or rate of diffusion of air and gases varies inversely as the square roots of their densities or specific gravities, density being referred to hydrogen as unity, and specific gravity to air.

This law is simply expressed by the following formulas:

$$\textit{Rel. vel. of diffusion (hydrogen : gas)} = \frac{1}{\sqrt{\textit{density of gas}}}$$

$$\textit{Rel. vel. of diffusion (air : gas)} = \frac{1}{\sqrt{\textit{sp. gr. of gas}}}$$

Experiment.—The diffusion of air and gases has been shown to take place through certain substances with practically the same rapidity as when they are in direct contact. The diffusion of hydrogen into air is well shown by the following simple experiment. A glass tube, say 18 or 20 in. long, 1-in. bore, is closed at one end with a plug of plaster. The tube is first filled with the gas and the open end then immersed beneath the surface of a basin of mercury. At once the mercury is observed to rise slowly in the tube to take the place of the hydrogen that is passing out through the plug and escaping into the air. Investigation shows, however, that while hydrogen has passed out of the tube, some air has passed into the tube, as there remains in the tube a mixture of hydrogen and air.

Illustration of Graham's Law.—The relative velocities or rates of diffusion of different gases (hydrogen = 1) are calculated from their respective densities referred to hydrogen as unity; thus,

$$\text{Methane } (CH_4)\text{; density, 8; Rel. vel.} = \frac{1}{\sqrt{8}} = \frac{1}{2.828} = 0.354 \ (H = 1)$$

In like manner, the relative velocities or ratio of diffusion of different gases (air = 1) are calculated from their respective specific gravities, referred to air as unity; thus,

$$\left.\begin{array}{ll} \text{Carbon dioxide } (CO_2)\text{; sp. gr., 1.529;} & v = \dfrac{1}{\sqrt{1.529}} = 0.808 \\ \text{Methane } (CH_4)\text{; sp. gr., 0.559;} & v = \dfrac{1}{\sqrt{0.559}} = 1.337 \end{array}\right\} (\text{Air} = 1)$$

Experiment Showing Effect of Diffusion.—An interesting experiment, showing the relative increase or decrease of the volume of gas contained in a vessel owing to diffusion, is illustrated in Fig. 5. The velocity of diffusion of methane being greater than that of carbon dioxide, when the latter is contained in the inner jar and the former in the outer bell-jar the bladder is expanded, because the methane passing into the small jar is greater in volume than the carbon dioxide passing out. Again, the bladder is depressed when the gases change places.

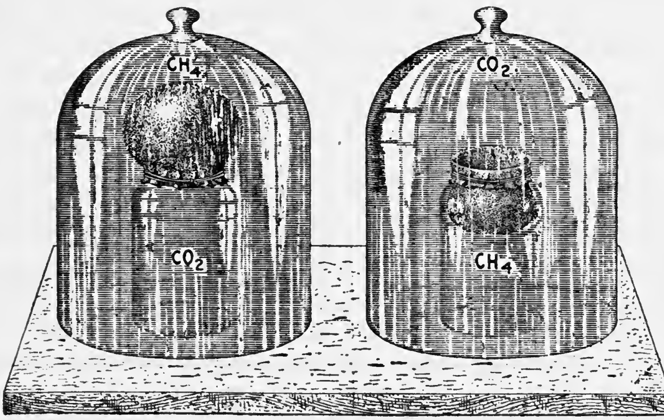


FIG. 5.

Composition of Gases.—Gas, like other material substances, is composed of the elements of matter. A **simple** or **elementary gas** is composed wholly of one kind of matter; as hydrogen (H), oxygen (O), nitrogen (N), etc.

Many gases, like many solids and liquids, are **compound.** The molecule of such a gas is formed by the chemical union of two or more atoms of different elements; as methane (CH_4), carbon monoxide (CO), carbon dioxide (CO_2), etc. A **gaseous mixture** is a mechanical mixture of different gases, simple or compound. These gases are mixed together in any proportion, but are not chemically united.

Firedamp is a mechanical mixture of a combustible gas or gases with air in such proportions as to render the mixture inflammable or explosive. The term, however, is generally understood to mean an inflammable or explosive mixture of methane (CH_4) and air. In English and other foreign text-books, the term "firedamp" is improperly applied to any mixture of explosive gas and air, without regard to whether the proportions are within the inflammable or explosive limits of the gas. Such a mixture will not inflame or explode and is not, properly speaking, a firedamp mixture.

Percentage Composition by Weight.—By the "percentage composition" of a compound is generally meant the percentage, by weight, of each element composing the substance. This is calculated from the ratio of the relative weight of each constituent element to its molecular weight. The term "percentage composition" may refer, however, to the percentage by volume of each constituent element.

For example, a **molecule of methane** (CH_4) contains one atom of carbon and four atoms of hydrogen. Then, since the atomic weight of carbon is 12 and that of hydrogen 1, the molecular weight of methane is $12 + (4 \times 1) = 16$, and the percentage composition of this gas is calculated as follows:

Carbon (C); atomic weight, 12; relative weight	12
Hydrogen (H_4); atomic weight, 1; relative weight, $4 \times 1 =$	4
Molecular weight of gas	16

The percentage of each constituent element is then:

Carbon	$\frac{12}{16}$ (100) =	75 per cent.
Hydrogen	$\frac{4}{16}$ (100) =	25 per cent.
		100 per cent.

In like manner, a **molecule of carbon dioxide** (CO_2) contains one atom of carbon and two atoms of oxygen. The atomic weight of carbon being 12 and that of oxygen 16, the molecular weight of carbon dioxide is $12 + (2 \times 16) = 44$, and the percentage composition of the gas is found as follows:

Carbon (C); atomic weight, 12; relative weight.	12
Oxygen (O_2); atomic weight, 16; relative weight, $2 \times 16 =$	32
Molecular weight of gas	44

The percentage composition is then:

Carbon	$^{12}/_{44}$ (100) =	27.27 per cent.
Oxygen	$^{32}/_{44}$ (100) =	72.73 per cent.
		100.00 per cent.

Percentage by Volume.—When applied to a gaseous mixture the term "percentage composition" is usually taken as referring to the **percentage by volume** of the several gases forming the mixture, unless otherwise stated. The method of making this calculation is given on page 102.

Specific Gravity of Mixtures of Gases.—When different volumes of gases of different densities are uniformly mixed the density of the mixture is determined by dividing the combined weight of the mixed gases by the total volume of the mixture, which will give the unit weight or the weight per unit of volume of the mixture.

The actual weights of the gases may not be known, but only the volume of each gas and its density or specific gravity. In that case, multiply the density of each gas by its volume, add the products together and divide the sum by the total volume of the mixture; the quotient obtained will be the required density of the mixture.

Or, in like manner, multiply the specific gravity of each gas by its volume, and divide the sum of these products by the total volume of the mixture, and the quotient obtained will be the specific gravity of the mixture.

Calculation.—For illustration, let it be required to calculate the specific gravity of flashdamp, which has a theoretical composition of 1658 volumes of methane (CH_4) to each 1000 volumes of carbon dioxide (CO_2). The process is as follows:

	Volume		Sq. gr.		Relative wt. (air = 1)
Methane	1658	×	0.559	=	926.8
Carbon dioxide	1000	×	1.529	=	1529.0
	2658				2455.8

The specific gravity of the flashdamp is then calculated, in accordance with the above rule, as follows:

$$Sp. gr. = \frac{relative\ wt.\ (air\ =\ 1)}{relative\ total\ vol.} = \frac{2455.8}{2658} = 0.924,\ nearly$$

Calculation Based on the Law of Diffusion of Gases.—If two gases diffuse into each other, directly, without being diluted with air, the volumes of the gases are inversely proportional to the square roots of their densities or specific gravities. This law makes it possible to calculate the density or specific gravity of such an undiluted mixture of two gases directly from their densities or specific gravities, without reference to their relative volumes. This is accomplished by means of the formula

$$D = \frac{a\sqrt{b} + b\sqrt{a}}{\sqrt{a} + \sqrt{b}}$$

in which D = density or specific gravity of the mixture; a and b = the corresponding densities or specific gravities of the two gases, respectively.

Calculation.—For illustration, let it be required to calculate the specific gravity of flashdamp (undiluted mixture of methane and carbon dioxide) directly from the specific gravities of these gases; methane = 0.559 and carbon dioxide = 1.529. The process is as follows:

$$Sp.gr. = \frac{0.559\sqrt{1.529} + 1.529\sqrt{0.559}}{\sqrt{0.559} + \sqrt{1.529}} = 0.924$$

SECTION II

HEAT

Sources and Measurement of Heat—Chemistry of Gases—Thermochemistry—Hygrometry—Steam

Definition.—Heat is now understood to be a form of motion. All matter is assumed to be in a state of molecular vibration. The rapidity of the vibration depends on the degree of heating of the mass. The theory assumes that the amplitude of the vibrations or the swing of the molecules is greater as the density of the mass is less. This would lead naturally to the conclusion that pressure, which increases the density of matter, will decrease the amplitude and increase the rapidity of vibration.

Heat is thus assumed to be a form of energy, the amplitude and rapidity of the vibrations being functions, respectively, of pressure and velocity, the factors of energy, in mechanics. The theory is well supported by observed facts, as the blow of a hammer or the friction of rubbing surfaces alike develop heat.

Heat in Bodies.—Assuming that heat is a form of molecular vibration, which varies in different kinds of matter, it is clear that each kind of matter has its own peculiar capacity for heat. This is shown to be the case by the fact that different bodies when exposed to the same source of heat are heated differently. For example, when equal weights of water and mercury are exposed, for the same time, to the same heat it is found that the mercury becomes much hotter than the water. When water and mercury at the same temperature are allowed to cool in the atmosphere, the air absorbing the same heat from each, the mercury is found to cool much quicker than the water. It is evident that the water absorbs more heat and gives out more heat, per pound, than the mercury, for the same change in temperature. In other words, water has a greater heat capacity.

Temperature.—The temperature of any body or mass of matter is the degree of heat it can radiate or impart to other bodies or matter with which it is in contact; or, in other words, the degree of sensible heat of the body. It is not the amount of heat in the body; as water contains 20 times the quantity of heat contained in an equal weight of mercury, at the same temperature.

The temperature of a body depends on the quantity of heat the body contains, per unit weight, and its heat capacity. A body or matter having a large heat capacity will have a comparatively low temperature.

How Temperature is Measured. Temperature is measured by the thermometer, an instrument so common as to need no description. The principle involved is that the expansion of the liquid contained in the bulb of the thermometer is much magnified in the capillary stem. Any rise of temperature is thus clearly indicated by a corresponding rise of the liquid in the stem and a fall of temperature is likewise accompanied by the contraction of the liquid, which drops in the stem.

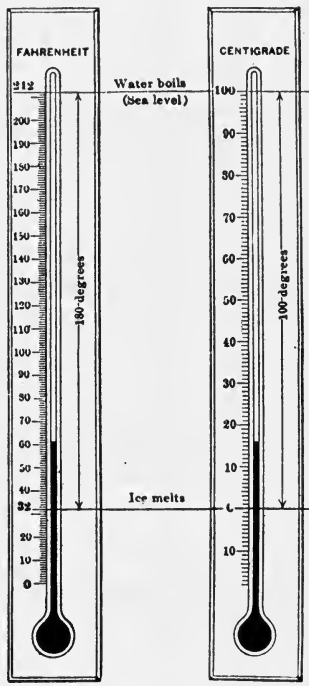


FIG. 6.

Two Scales.—There are two principal thermometer scales, the Fahrenheit and the centigrade. These are each calibrated with reference to the melting of ice and boiling of water. As shown in the illustration, Fig. 6, these points are marked 32 and 212 deg., respectively, in the Fahrenheit, and 0 and 100 deg., respectively, in the centigrade scale. Thus, 180 deg. of the former correspond to 100 deg. of the latter; or the ratio is 9 : 5.

TABLE SHOWING CORRESPONDING VALUES OF THE FAHRENHEIT SCALE FOR EACH FIVE DEGREES OF THE CENTIGRADE SCALE

C.	F.	C.	F.	C.	F.	C.	F.	C.	F.
−50	−58	200	392	450	842	700	1292	950	1742
−45	−49	205	401	455	851	705	1301	955	1751
−40	−40	210	410	460	860	710	1310	960	1760
−35	−31	215	419	465	869	715	1319	965	1769
−30	−22	220	428	470	878	720	1328	970	1778
−25	−13	225	437	475	887	725	1337	975	1787
−20	− 4	230	446	480	896	730	1346	980	1796
−15	+ 5	235	455	485	905	735	1355	985	1805
−10	14	240	464	490	914	740	1364	990	1814
− 5	23	245	473	495	923	745	1373	995	1823
0	32	250	482	500	932	750	1382	1000	1832
+5	41	255	491	505	941	755	1391	1005	1841
10	50	260	500	510	950	760	1400	1010	1850
15	59	265	509	515	959	765	1409	1015	1859
20	68	270	518	520	968	770	1418	1020	1868
25	77	275	527	525	977	775	1427	1025	1877
30	86	280	536	530	986	780	1436	1030	1886
35	95	285	545	535	995	785	1445	1035	1895
40	104	290	554	540	1004	790	1454	1040	1904
45	113	295	563	545	1013	795	1463	1045	1913
50	122	300	572	550	1022	800	1472	1050	1922
55	131	305	581	555	1031	805	1481	1055	1931
60	140	310	590	560	1040	810	1490	1060	1940
65	149	315	599	565	1049	815	1499	1065	1949
70	158	320	608	570	1058	820	1508	1070	1958
75	167	325	617	575	1067	825	1517	1075	1967
80	176	330	626	580	1076	830	1526	1080	1976
85	185	335	635	585	1085	835	1535	1085	1985
90	194	340	644	590	1094	840	1544	1090	1994
95	203	345	653	595	1103	845	1553	1095	2003

C.	F.	C.	F.	C.	F.	C.	F.	C.	F.
100	212	350	662	600	1112	850	1562	1100	2012
105	221	355	671	605	1121	855	1571	1105	2021
110	230	360	680	610	1130	860	1580	1110	2030
115	239	365	689	615	1139	865	1589	1115	2039
120	248	370	698	620	1148	870	1598	1120	2048
125	257	375	707	625	1157	875	1607	1125	2057
130	266	280	716	630	1166	880	1616	1130	2066
135	275	385	725	635	1175	885	1625	1135	2075
140	284	390	734	640	1184	890	1634	1140	2084
145	293	395	743	645	1193	895	1643	1145	2093
150	302	400	752	650	1202	900	1652	1150	2102
155	311	405	761	655	1211	905	1661	1155	2111
160	320	410	770	660	1220	910	1670	1160	2120
165	329	415	779	665	1229	915	1679	1165	2129
170	338	420	788	670	1238	920	1688	1170	2138
175	347	425	797	675	1247	925	1697	1175	2147
180	356	430	806	780	1256	930	1706	1180	2156
185	365	435	815	685	1265	935	1715	1185	2165
190	374	440	824	690	1274	940	1724	1190	2174
195	383	445	833	695	1283	945	1733	1195	2183

To convert Fahrenheit (F.) readings into centigrade (C) or vice versa, the following formulas are useful:

$$F = \tfrac{9}{5}\,C + 32$$
$$C = \tfrac{5}{9}\,(F - 32)$$

Example—(*a*) What are the readings of the Fahrenheit scale corresponding to 40°, and − 10° centigrade?

Solution—

$$F = \tfrac{9}{5} \times 40 + 32 = 104°\text{F.}$$
$$F = \tfrac{9}{5}\,(-10) + 32 = 14°\text{F.}$$

Example—Convert − 4 F. and 50 F. into centigrade readings.

Solution—

$$C = \tfrac{5}{9}\,(-4 - 32) = -20\ \text{C.}$$
$$C = \tfrac{5}{9}\,(50 - 32) = 10\ \text{C.}$$

Readings above zero are plus (+) and those below zero minus (−).

Sources and Measurement of Heat

Sources of Heat.—In a sense the sun is the original source of most of the heat of the solar system—in other words, the sun is the power house of that system. It may be said that much of the terrestrial life and activity emanates from the sun. The source of the sun's heat is understood to be the chemical and possibly electrical activities that are constantly developed in its huge mass and radiating heat, light and electrical energy.

The same chemical and possibly electrical activities are taking place to a less degree in the mass of the earth, creating internal heat. Both the radiated heat of the sun and the internal heat of the earth are **natural sources of heat.**

Besides these natural or physical sources of heat, there are the **mechanical sources of heat,** such as **friction, impact** and **pressure.** These each develop heat as the result of force applied mechanically.

Sensible Heat.—The heat that is accompanied by a change of temperature when absorbed or given out by a body is called "sensible heat," because it is manifest to the senses.

Latent Heat.—When matter passes from the solid to the liquid state, or from the liquid to the gaseous state, the change is always accompanied by the absorption of a considerable amount of heat, although the temperature remains constant. The heat thus absorbed is called "latent heat," it being absorbed in performing the work of driving the molecules of matter farther apart than they were in the previous state. This heat is again given out when the matter passes from a gas to a liquid, or from a liquid to a solid.

Chemical Heat.—Theory assumes that chemical heat is the result of the **chemical affinity** of material atoms for each other, by which they are drawn and held in more or less close contact and union. This condition is in harmony with the notion of "atomic heat," explained elsewhere, and suggests the estimation of the **heat of formation,** or **heat of combination,** as the result of chemical union.

In contrast with atomic heat, **molecular heat** is akin to specific heat and representative of the heat capacity of a substance, or the quantity of heat a particular substance will absorb, per unit weight, per degree of rise in its temperature. Theory assumes that all heat of any nature is a vibratory state of atoms or molecules and, as such, is convertible into or created by other forms of energy.

The **molecular heat** of a substance is found by multiplying a gram-molecule (page 54) of the substance by its specific heat.

Combining Heat.—All matter is assumed to possess a certain definite heat energy peculiar to itself, which is expressed in heat units, per unit weight of substance and called the "combining heat" of the substance.

Heat of Formation.—In the combining of atoms to form compound molecules, a neutralization of the energies of the combining atoms causes either an evolution or an absorption of heat, the molecule formed then possessing an amount of heat called "heat of formation" or "heat of combination."

Heat Due to Friction.—Friction is caused by one body rubbing against another, whereby a molecular vibration is set up in the two bodies, as manifested by the heat generated.

Heat Due to Impact.—The impact of one body against another likewise sets up a molecular vibration in the bodies, which is manifested by the heat generated.

Heat Due to Pressure.—Pressure applied to a body having a degree of elasticity, or being compressible, forces the molecules of matter closer together, which reduces the intermolecular space and, as a result, there being no loss of molecular energy, the speed of vibration is increased in proportion as the space is diminished and heat is developed.

Transformation of Heat Energy.—Heat energy of any nature, whether chemical or physical, is convertible, without loss, into mechanical energy measured in foot-pounds, which is the "mechanical equivalent of heat."

At each change of state in matter heat is either absorbed and becomes **latent in the mass,** or is given out and becomes **sensible,** causing a rise of temperature in the surrounding medium. **Heat is absorbed** when a solid becomes a liquid or a liquid becomes a gas, the change being one in which the density of the mass is made less. On the other hand, **heat is given out** when a gas is condensed to a liquid or a liquid to a solid, the density of the mass being then increased.

Heat of Fusion.—The change from a solid to a fluid state is described as **"liquefaction"** when solution takes place, or **"fusion"** if the solid is melted. The heat absorbed in the latter case is called "heat of fusion."

Liquefaction may take place as the result of the absorption of moisture from the air, the substance dissolving either wholly or in part in the water absorbed. Such a substance is said to be **"deliquescent."**

Solution takes place when a solid disappears in a liquid in which it is immersed. The solid is "dissolved," in the liquid, which is called the **"solvent."**

In any case of liquefaction or fusion heat is absorbed and becomes latent in the liquid, causing a seeming loss or **disappearance of heat.** When a solid is dissolved in a liquid the liquid is cooled provided no chemical reaction takes place, which might produce heat.

Heat of Vaporization.—The formation of vapor or the change from a solid or liquid to a gaseous state is known as **"vaporization"** and the heat absorbed and rendered latent in the vapor is called "heat of vaporization" or frequently "heat of evaporation," especially when the vapor is formed by boiling the liquid.

Heat of Condensation.—When a gas or vapor is condensed to a liquid or a liquid is frozen or condensed to a solid the latent heat of the gas, vapor or liquid is given out and appears as sensible heat, which causes a rise of temperature. The heat given out is called "heat of condensation" and is exactly equal to the heat of vaporization or the heat of fusion or liquefaction, as the case may be.

Total Heat in a Body.—By this is meant the total heat absorbed by a body in a given change of temperature or state. For example, the total heat in 1 lb. of water, in passing from ice at 32 deg. F. to steam at 212 deg. F. is as follows:

Latent heat of fusion of ice, from and at 32°F......	144	B.t.u.
Sensible heat absorbed by water, 32° to 212°F.....	180	B.t.u.
Latent heat of vaporization, from and at 212°F...	970.4	B.t.u.
Total heat absorbed........................	1294.4	B.t.u.

The total heat of steam at any temperature or pressure is usually estimated from water at 32 deg. F.; thus the total heat in steam (water vapor) at 212 deg. F. is 180 + 970.4 = 1150.4 B.t.u. This is the heat in steam at atmospheric pressure at sea level (14.7 lb. per sq. in.). When steam is generated in a boiler, its temperature increases with the pressure.

Effect of Pressure on Fusion.—Pressure acts to oppose increase of volume. Some substances, as water, for example, expand when passing from the liquid to the solid state and an increase of pressure therefore **lowers the freezing point** of such substances. The decrease of atmospheric pressure at high altitudes facilitates the formation of ice, though to a less degree than other more potent causes.

On the other hand, some substances, as wax, contract when solidifying, and an increase of pressure then acts to **raise the freezing point** or point of solidifying. In other words, an increase of pressure acts to assist the melting of wax and similar substances, while it retards that of ice.

Melting Points of Substances.—The melting point of substances depends largely on their purity and treatment. For this reason different authorities often give different values for the same substance. The table on the following page gives the approximate melting points and the heat of fusion, in British thermal units, per pound, for the substances named.

Difference Between Melting and Freezing Points.—The melting point of a substance does not always correspond exactly with its freezing point, even at the same pressure. The melting point of ice is more uniformly constant than the freezing point of water, and for this reason is taken to indicate the zero of the centigrade scale (32°F.).

The solidification of a liquid is generally accompanied with crystallization, and the formation of the crystals is often delayed in a quiet medium, so that the temperature of water free of air may fall as low as 5 deg. F. when perfectly quiet and not freeze. But if the water at this low temperature be stirred or jarred the whole will instantly change to ice or become solid.

MELTING POINTS AND HEATS OF FUSION OF SUBSTANCES

Substance	Melting point, deg. Fahr.	Heat of fusion, B.t.u. per lb.
Aluminum	1211	138.6
Beeswax	148	76.1
Copper	1980	77.4
Gold	1947	
Ice	32	144.0
Iron, cast (white)	2000	41.4
Iron, cast (gray)	2400	59.4
Iron, wrought	2820	
Lead	620	9.0
Nickel	2600	8.3
Platinum	3100	48.6
Silver	1764	37.9
Spermaceti	120	66.5
Steel	2462	36.0
Sulphur	235	16.2
Tallow	92	
Tin	450	25.6
Zinc	786	50.4

To express heat of fusion in calories per kilogram:
B.t.u. per lb. × $\frac{5}{9}$ = cal. per kg.

Effect of Pressure on Vaporization.—Pressure acts to retard vaporization. An increase of pressure, therefore, **raises the boiling point** of water and other liquids. For the same reason a decrease of pressure lowers the boiling point of liquids. At an elevation of 10,000 ft. above sea level, under normal atmospheric conditions, pure water boils at 193 deg. F., and at an elevation of 15,000 ft. the boiling point, for the same normal atmospheric conditions, is reduced to 185 deg. F.

Vaporization, Evaporation, Boiling.—Vaporization is a general term relating to the formation of vapor, or the change from a solid or liquid state to a vaporous or gaseous condition, without regard to whether the change is slow or rapid.

The term "evaporation" relates to the slow vaporizing of a solid or liquid that takes place at its surface when the latter is exposed to an atmosphere that is not fully saturated.

The evaporation of a liquid may also be caused by the application of heat.

The term "boiling" refers to the violent ebullition that takes place throughout the mass of a liquid, caused by the formation of vapor in the liquid and its escape to the surface. Boiling results from the application of heat to the liquid, or may result from a sudden decrease of pressure.

Boiling Points of Liquids.—A liquid boils when raised to such a temperature that the tension of its vapor is equal to the pressure at its surface. At this point the liquid becomes vapor. The term "boiling point," as commonly used, however, refers to atmospheric pressure at sea level, unless otherwise stated. The following table gives both the freezing and the boiling points of a few liquids of interest in mining:

FREEZING AND BOILING POINTS OF LIQUIDS

Liquid	Freezing Point, Deg. Fahr.	Boiling Point, Deg. Fahr.
Alcohol (ethyl)	−202	172
Ammonia	−106	140
Linseed oil	−18	597
Mercury	−38	676
Nitroglycerine	45	

Measurement of Heat.—Although heat, as already explained, is a **condition of matter** and not a tangible quantity, it is possible to measure its intensity or degree through the effect it produces, referred to certain established **standards of measurement.** The most convenient standard is the heat energy that will cause a rise of one degree in the temperature of a unit weight of pure distilled water at its point of maximum density. This is called a "heat unit" or "thermal unit" and is a quantity capable of exact measurement.

Heat or Thermal Units.—There are several heat units in common use, the principal ones being the British unit and the French unit. A third unit that is largely used combines these two units.

The British Thermal Unit.—The British thermal unit (B.t.u.) is the quantity of heat required to raise the temperature of 1 lb. of pure distilled water at maximum density, 1 deg. of the Fahrenheit scale.

The French Thermal Unit or Calorie.—This is the quantity of heat required to raise the temperature of 1 kg. of pure distilled water at maximum density, 1 deg. of the Centigrade scale.

The Pound Calorie.—This is the quantity of heat required to raise the temperature of 1 lb. of pure distilled water, at maximum density, 1 deg. of the Centigrade scale.

Conversion Formulas—

B.t.u.	$\times$ 0.252	=	Calories
B.t.u.	$\times$ $\frac{5}{9}$	=	Pound-calories
Calories	$\times$ 3.968	=	B.t.u.
Calories	$\times$ 2.2046	=	Pound-calories
Pound-calories	$\times$ $\frac{9}{5}$	=	B.t.u.
Pound-calories	$\times$ 0.4536	=	Calories

Note.—Since 1 lb. (avoirdupois) = 0.4536 kg.; and
1 deg. (Fahr.) = $\frac{5}{9}$ deg. (Cent.),
1 B.t.u. = 0.4536 $\times$ $\frac{5}{9}$ = 0.252 cal.

Again, since 1 kilogram = 2.2046 lb. (avoir.); and
1 deg. (Cent.) = $\frac{9}{5}$ deg. (Fahr.),
1 cal. = 2.2046 $\times$ $\frac{9}{5}$ = 3.968 B.t.u.

These simple calculations show the derivation of the constants used in the above formulas.

Transmission of Heat.—The condition known as "heat" is transmitted in any one of the three following ways: 1. By **radiation.** 2. By **conduction.** 3. By **convection.**

Heat is radiated in straight lines in all directions from its source and is then called "radiant heat." It is transmitted through the vibrations of the ether that fills all space and the radiated heat is imparted in varying degree to all matter in its path. Heat so imparted to a body is said to be "absorbed" by the body.

When heat travels through a body the process of transmission is known as "conduction." Heat thus spreads throughout the **mass as a solid.**

The spread of heat **in any fluid** (liquid or gas) is through the circulation caused by the unequal distribution of the heat. This mode of transmission is known as "convection."

Mechanical Equivalent of Heat.—Since heat is assumed to be a form of energy, it must be capable of performing work, which is expressed in foot-pounds. This has given rise to what is properly called the "mechanical equivalent of heat." It is the theoretical amount of work expressed in foot-pounds or kilogram-meters per unit of heat absorbed.

The values of the several heat units are as follows:

	Foot-pounds	Kilogram-meters
1 British thermal unit......	778	107.5
1 calorie..................	3087	426.8
1 pound-calorie...........	1400	193.5

The reverse of these values is as follows:

	B.t.u.	Calories	Lb.-cal.
1000 foot-pounds..........	1.285	0.324	0.714
100 kilogram-meters.......	9.297	2.343	5.168

Atomic Heat.—An important relation has been found to exist between the atomic weights of the elements and their specific heats. Dulong and Petit (1819) found that the specific heats (relative heat capacity) of most of the solid elements vary inversely as their atomic weights, so that the product of these two factors is a constant quantity (6.4), which has been properly called the **"atomic heat."** Thus, taking the specific heats of iron, lead and mercury, respectively, as 0.1190, 0.0305 and 0.0333, gives the value for the atomic heat in each case as follows:

	At. wt.		Sp. ht.		At. ht.
Iron........	55.40	×	0.1190	=	6.59 heat units.
Lead........	205.44	×	0.0305	=	6.27 heat units.
Mercury....	198.40	×	0.0333	=	6.61 heat units.

The average value for the atomic heat of the elements may be taken as 6.4, though it is sometimes given as low as 6.25 (Remsen). **Atomic heat** may be briefly defined as the heat capacity of matter per unit-weight atom.

A **gram-atom** of any elementary substance is a weight of that substance, in grams, equal to the atomic weight of the element. Thus, the atomic weight of iron being 55.4 (H = 1), a gram-atom of iron is 55.4 grams of that substance; and its heat capacity is the atomic heat value (6.4 heat units).

This average value of **atomic heat** often assists the determination of the specific heat from the atomic weight of an elementary substance, or, vice versa, its atomic weight when the specific heat is known. For example, since the heat capacity of 55.4 grm. of iron is 6.4 heat units, the average specific heat of iron is $6.4 \div 55.4 = 0.1155$.

In like manner, a **gram-molecule** of any compound substance is a weight of that substance, in grams, equal to the molecular weight of the substance.

Specific Heat.—Investigation has shown that the same quantity of heat imparted to equal weights of different substances does not produce the same rise of temperature in each substance. Also, equal weights of different substances when cooling give out different quantities of heat for each degree the temperature falls. These facts show that different substances have different capacities for absorbing and holding heat as **sensible heat** causing a rise of temperature.

The "specific heat" of any substance is its **relative heat capacity,** or its heat capacity referred to that of an equal weight of pure water. The **unit of heat** is the amount of heat required to raise the temperature of a unit weight of water one degree. Therefore, the specific heat of a substance being referred to water expresses the heat units required to raise the temperature of a unit weight of the substance one degree.

The **specific heat of a solid** or liquid always refers to the heat per unit weight. The **specific heat of a gas** may be referred to the unit weight or unit volume, as desired. The specific heat of air and gases is different according as the air or gas is confined (constant volume) or is allowed to expand (constant pressure). The specific heat of a gas for "equal volumes" is the heat capacity of the gas referred to that of an equal volume of air at the same temperature and pressure.

The following table gives the specific heats of a few of the common solids and liquids of interest in mining:

SPECIFIC HEATS OF SOLIDS AND LIQUIDS

Substance	Temperature, deg. Fahr.	Specific heat
Aluminum	60–1150	0.2145–0.3077
Copper	32–1650	0.0933–0.1259
Iron	32–1100	0.1050–0.1989
Lead	60– 600	0.0299–0.0338
Lead (at melting point, 610°F.)	610– 680	0.0356–0.0410
Mercury	32– 500	0.0334–0.0320
Platinum	60– 210	0.0324
Silver	32–1200	0.0559–0.0750
Tin	32– 210	0.0545
Zinc	32– 700	0.0935–0.1220

The following table gives the specific heats of the common mine gases, for equal weights at constant pressure and constant volume, and for equal volumes under constant pressure:

SPECIFIC HEATS OF AIR, MINE GASES AND VAPORS

Substance	Equal weights		Equal volumes
	Const. pres.	Const. vol.	Const. pres.
Air	0.2374	0.1689	0.2374
Methane	0.5929	0.4219	0.3314
Olefiant gas	0.4040	0.2875	0.3951
Carbon monoxide	0.2450	0.1743	0.2369
Carbon dioxide	0.2163	0.1539	0.3307
Hydrogen sulphide	0.2432	0.1731	0.2897
Oxygen	0.2175	0.1548	0.2405
Nitrogen	0.2438	0.1735	0.2368
Hydrogen	3.4090	2.4260	0.2361
Water vapor	0.4805	0.3419	0.2996
Ammonia	0.5080	0.3615	0.2992

When gas, air or vapor is free to expand (constant pressure) heat is absorbed and becomes latent. For this reason more heat is required to produce the same rise of temperature when expansion occurs than when the volume remains constant, and the specific heats in the first column are therefore higher than those in the second column of the table given above.

The values given in the first column of this table have been determined by experiment directly, while those in the second column have been derived from the first by dividing the latter by 1.405, the ratio of the specific heat of gases at constant pressure to that at constant volume. Likewise, the values given in the third column have been derived from those in the first by multiplying the latter by the specific gravity of the gas or vapor referred to air.

The specific heat of all substances varies more or less with the temperature as appears in the above table. In the case of gases, the increase per degree (Fahr.) above zero is roughly estimated as follows: Air, nitrogen, carbon monoxide, 0.000012; oxygen, 0.00001; carbon dioxide, 0.00006; hydrogen, 0.0002; and water vapor, 0.0001; etc.

CHEMISTRY OF GASES

The chemistry of all matter treats of the interchange of the atoms constituting molecules, by virtue of which interchange the character and nature of the matter is wholly altered. In other words, the matter is transformed and a new substance created having properties that vary widely from those of the original substance.

Chemical Reaction.—The change that takes place when matter is thus transformed is a **chemical change,** and the action is described as a "chemical reaction." It assumes an intimate contact between two unlike substances, under conditions that favor an interchange of atoms. The reaction that takes place is the direct result of different affinities of the atoms for each other.

Chemical Affinity.—The theory of chemical change supposes that all atoms constituting matter have various affinities or **degrees of attraction** for each other. By reason of this difference in the affinities of atoms, an interchange may or may not occur when two unlike substances are brought into intimate relation with each other, according as the atoms of the original substances possess a less or a greater affinity for each other in their present state or grouping. If the atoms of one of these substances possess a greater affinity for atoms of the other substance an interchange of atoms will take place and new substances will be formed that will be wholly different from the original substances.

Influence of Heat to Produce Chemical Change.—The theory of heat assumes a wider separation of the particles of matter as the amount of heat in a substance is increased. Thus, it naturally follows that a higher temperature invites a more intimate mingling of two different gases in contact with each other. This intermingling of the gaseous molecules greatly assists a chemical reaction that otherwise would not take place.

Examples of Chemical Change.—The most common and familiar examples of chemical change are those due to the strong affinity of the oxygen of the air for most other matter. The resulting reaction is described as **oxidation.** The more familiar forms of oxidation are the rusting of iron and some other metals in a damp atmosphere. The action results in the "corrosion" or eating away of the metal and the formation of an oxide, which is quite different in its character and properties from the original metal.

Combustion.—In a general sense, any form of oxidation is combustion, and the latter term does not relate alone to oxidation, but describes generally any chemical reaction in which one substance is consumed either slowly or rapidly by reason of the presence of another substance whose atoms possess an affinity for those of the first that invites reaction.

The substance consumed is termed the **combustible** and the other the **supporter of the combustion,** while the substances produced are the **products of the combustion.** The products of a combustion may be gaseous, vaporous or solid, the last named being the **ash** of an active combustion.

Slow Combustion.—This term implies a slow but continuous wasting away of the substance consumed, the conditions being unfavorable or the affinities of the atoms being insufficient to support a more rapid reaction. Slow combustion is characterized by the generation of **heat** without the production of **flame.**

Active or Rapid Combustion.—Active combustion is generally accompanied by the production of flame. The same amount of heat is generated in less time, resulting in a higher temperature, which in turn frequently modifies the products of the combustion.

Spontaneous Combustion.—Under certain favorable conditions, combustion may start in a mass of combustible material without the application of flame or other exciting cause. This is due to the natural **generation of heat** within the mass, owing to chemical reaction taking place between the substances. The action is explained as being chiefly due to the **absorption of oxygen** from the air by the substance, when the ensuing oxidation generates sufficient heat to ignite both the gas produced by the combustion and the material. The combustion, which is at first slow, may, in time, develop actively and inflame and consume the material.

Chemical Symbols.—A chemical symbol is a letter or letters used to designate an element or simple substance. The symbols of the more common elements together with their atomic or specific weights have been given in a table, previously. The symbol written alone expresses a single atom of the substance; but, since an atom is not conceived to exist alone, the symbol of an element should always be written as a molecule.

Symbol of a Molecule.—A molecule is assumed to be the smallest chemical division of matter that can exist in a free state. A molecule of any simple or elementary substance is assumed to contain two atoms only. Its symbol is expressed by writing the symbol for that element with a subscript ($_2$) to indicate two atoms; thus for the molecule of carbon, write C_2; oxygen, O_2; etc.

The molecule of a compound substance may contain any number of atoms and is expressed by writing the symbols of its elements each with a subscript figure indicating the number of atoms of that element in the molecule. A single atom of an element is indicated by the symbol only, omitting the subscript figure.

The following examples will serve to illustrate the fact that, while a molecule of any simple substance is taken to contain two atoms only, the molecule of a compound may contain any number of atoms:

Substance	Composition	Symbol
Carbon monoxide,	carbon, 1 atom; oxygen, 1 atom = 2 atoms;	CO
Carbon dioxide,	carbon, 1 atom; oxygen, 2 atoms = 3 atoms;	CO_2
Ammonia,	nitrogen, 1 atom; hydrogen, 3 atoms = 4 atoms;	NH_3
Methane,	carbon, 1 atom; hydrogen, 4 atoms = 5 atoms;	CH_4
Olefiant gas,	carbon, 2 atoms; hydrogen, 4 atoms = 6 atoms;	C_2H_4

All these gaseous molecules are of equal size, though containing different numbers of atoms.

Molecular Theory of Matter.—Chemical investigations have led to the accepted conclusion that all matter is composed of minute particles called **molecules**, the molecule being considered the smallest division of which the matter is capable without destroying its identity.

Theory further assumes that the molecule is composed of two or more **atoms**, like or unlike, but bound together by a force of attraction for each other known as **affinity.** Each of these combined atoms represents an **element** or a particular kind of matter and their combination as molecules diversifies matter and creates substances of various nature and kind.

Atomic Weight.—Atomic weight is simply relative. The atom of each element has a weight peculiar to that element, referred to the weight of the hydrogen atom as unity.

Molecular Weight.—The molecular weight of a substance is equal to the sum of the atomic weights of the elements of which it is composed. These elements combine in **fixed proportions,** which are determined by the number of atoms that saturate each other or the "**valences**" of the elements.

Valence or Valency.—The valence of an element is a term used to express its **combining power** in relation to the number of atoms of hydrogen (the assumed **unit**) or its equivalent required to satisfy the affinity. For example, two atoms of hydrogen are required to saturate a single atom of oxygen, and the valence of hydrogen being one, the valence of oxygen is two. The reaction is expressed by the chemical equation

$$2H_2 + O_2 = 2H_2O.$$

There are many elements, however, that do not unite with hydrogen and to determine their valency it is necessary to compare them with other elements that combine with them and whose valence is known. For this purpose the elements **oxygen and chlorine** are most convenient. The valence of oxygen, as shown above is **two**. The valence of chlorine is **one**, since one atom of hydrogen completely saturates one atom of chlorine.

$$H_2 + Cl_2 = 2HCl.$$

The element calcium combines both with oxygen and with chlorine but not with hydrogen alone. Its valence is two as shown by the following equations:

$$Ca_2 + O_2 = 2CaO$$
$$Ca_2 + 2Cl_2 = 2CaCl_2.$$

The **valence** of most elements is **not absolute** but changes, often by two and frequently by successive units. For example, calcium has a valence of two and four; gold, one and three; copper, one and two; iron, two, three, four and six; while nitrogen forms the following series of oxides:

$$N_2O,\ N_2O_2,\ N_2O_3,\ N_2O_4,\ N_2O_5.$$

Classification of Elements by Valence.—Owing to the change in valency exhibited by many elements it is not possible to make an unvarying classification in this respect. For the sake of convenience, however, many of the elements are designated as **univalent, bivalent, trivalent, quadrivalent,** etc.; or as **monads, dyads, triads, tetrads, pentads, hexads,** etc., according as they exhibit valencies of one, two, three, four, five, six, etc., in combining with other elements.

A Chemical Compound.—A chemical compound is a substance composed of molecules formed by the chemical union of two or more unlike atoms. In a chemical compound the elements are always combined in **fixed proportions** and the substance has fixed properties that are always the same.

A Mechanical Mixture.—A mechanical mixture is composed of unlike substances mixed together in any proportion and not chemically combined. The properties of such a mixture vary with the kind and proportion of the substances of which it is formed.

The **atmosphere** is a mechanical mixture of oxygen and nitrogen. Although the proportion of these gases is practically always the same in pure air, the gases are only mixed and do not combine with each other.

Acids, Bases and Salts.—Chemistry considers three general classes or conditions of matter, which make the substance either an acid, a base, or a salt.

Briefly and plainly stated, an **acid** is a substance that dissociates in aqueous solution yielding hydrogen ions.

A **base** is a compound capable of reacting with an acid to produce a salt. It is an alkaline metallic oxide.

A **salt** is a generally neutral compound formed by the union of an acid and a base.

In general the **nature of an acid** is the direct opposite to that of a base. In combination they neutralize each other, forming a neutral salt and water. The **distinguishing characteristics** of all acids are: 1. The sour taste. 2. The turning of blue litmus red. 3. The evolution of hydrogen by contact with a metal.

A number of acids are formed by the direct union of hydrogen with another element; as hydrochloric acid (HCl); hydrogen sulphide (H_2S). Other acids are formed by the union of two radicals—the hydrogen radical or hydroxyl (HO) and an acid radical; or they may be considered as the result of the addition of water (H_2O) to an anhydrous acid (anhydride).

In the first instance, the formation is as follows:

Hydrogen radical (hydroxyl)	2(HO)
Acid radical	SO_2
Sulphuric acid	H_2SO_4

Or, again, the formation may be regarded thus:

Water	H_2O
Sulphuric anhydride	SO_3
Sulphuric acid	H_2SO_4

Oxides.—Nearly all the elements unite with oxygen to form oxides, but the affinity for oxygen is stronger in some cases than in others. When the affinity of the elements for each other is strong the compound formed is more stable than when the affinity is weak.

A **monoxide** is formed when the molecule contains but one atom of oxygen; as for example, carbon monoxide (CO).

A **dioxide** is formed when the molecule contains two atoms of oxygen, as carbon dioxide (CO_2).

A **trioxide** contains three atoms of oxygen.

Chemical Change, Reaction.—Any interchange of atoms between two substances, or a combination of two unlike substances, by which one or more new substances are formed, is a chemical change and the process is called a "chemical reaction."

A Chemical Equation.—It is a natural law that no matter is ever lost or destroyed. **Matter is Indestructible.** As a result of chemical change both the form and nature of the matter may be altered—a solid may become a liquid or gas, or vice versa; but the weight of the resulting products is the same as that of the original substances that are involved in the reaction.

Since there is **no change** in the weight of matter before and after chemical reaction takes place, it is possible to express the reaction by an equation showing the equality of matter. This is called a **chemical equation.** It is formed by writing in the first member the chemical symbols of all the substances entering or involved in the reaction, connecting these together with a plus (+) sign. Likewise, in the second member of the equation, write the chemical symbols of the several products of the reaction, connecting them together, as before, with a plus (+) sign. Then complete the equation by writing the sign (=) of equality between the two members.

For reasons that will be better understood when discussing molecular volume, when writing a chemical equation each substance should be expressed by its molecular formula. This means that any elementary or simple substance as carbon (C), hydrogen (H) nitrogen (N), etc., should be expressed as a molecule; thus, C_2, H_2, N_2, etc.

Illustration.—When carbon (C) is completely burned in a plentiful supply of oxygen (O) there is produced carbon dioxide (CO_2). The reaction is expressed by the equation

$$C_2 + 2O_2 = 2CO_2$$

The expression $2CO_2$ should be interpreted to mean two molecules of CO_2, each comprising one atom of carbon and two atoms of oxygen.

Observe there are the same number of atoms of carbon and the same number of oxygen on each side of the equation. **Not an atom is lost** in the reaction, although these are grouped differently. In this case the solid carbon unites with the oxygen (gas) and carbon dioxide (gas) is produced. Also, the weight of the carbon dioxide is equal to the sum of the weights of the carbon burned and the oxygen consumed. There is **no loss in weight.**

It is important to note that the **atoms** involved in any reaction represent the **weights** of the substances they form, while the **molecules** or molecular formulas of the several substances represent their respective **volumes.** Hence, when each substance is expressed by its molecular formula the chemical equation shows both the **relative weights** of all the substances and the **relative volumes** of the gases.

In the reaction represented by the above equation each atom of the carbon molecule (C_2) takes up two atoms of oxygen to form the molecule of carbon dioxide (CO_2), the valence of carbon being four and that of oxygen two. The reaction in this case is complete, the affinity of the carbon for oxygen being fully satisfied.

Use of Chemical Equations.—As previously stated, when properly written a chemical equation shows both the relative weights and relative gaseous volumes of each respective substance involved in a chemical reaction. The relative weights are indicated by the molecular weights of the substances as shown by the completed equation.

In estimating relative gaseous volumes, **the volume of a**

gaseous atom is taken as unity and since, as previously explained, an elementary molecule is assumed to contain two atoms and all gaseous molecules at the same temperature and pressure are of equal size regardless of the number of atoms they contain, it follows that **the relative volume of all gaseous molecules is two.**

Application of the Law of Volumes.—The law of molecular volume as just explained finds important application in calculating the volumes of gases that are involved in a chemical reaction. While there is never any change in the weight or amount of matter due to chemical reaction, there frequently results a **change in the volume** of the gases concerned in the reaction.

To illustrate such change of gaseous volume, write the chemical equation representing the dissociation of ammonia gas (NH_3) by electrolysis, forming free nitrogen (N) and hydrogen (H) gases, placing below each molecular formula its relative or molecular volume; thus,

	$2NH_3$ =	N_2 +	$3H_2$
Mol. vol.	2	1	3

It is evident that two molecules of ammonia gas, in dissociation, yield one molecule of nitrogen and three molecules of hydrogen, making four volumes in all. In other words, two volumes become four. The volume of the gases resulting from the breaking up of the molecule of ammonia is, therefore, double that of the original gas.

There is no chemical change of volume when methane or marsh gas (CH_4) is exploded in a plentiful supply of normal air, and the methane is completely burned, forming only carbon dioxide (CO_2) and water (H_2O). The nitrogen of the air being unchanged it may be omitted in writing the equation expressing this reaction, which is as follows:

	CH_4 +	$2O_2$ =	CO_2 +	$2H_2O$
Mol. vol.	1	2	1	2

The equation shows that the complete combustion of methane requires twice its volume of oxygen; and there is produced an equal volume of carbon dioxide and two volumes of aqueous vapor.

On the other hand, when carbon monoxide (CO) is burned in air, producing carbon dioxide (CO_2), there results a reduction in volume, as shown by the following equation:

	$2CO$	$+ O_2$	$+ 4N_2$	$= 2CO_2$	$+ 4N_2$
Mol. vol......	2	1	4	2	4

Normal air consists of practically one-fifth oxygen and four-fifths nitrogen. The equation shows that two volumes of carbon monoxide, in burning, consume five volumes of air, and there remain two volumes of carbon dioxide and four volumes of unchanged nitrogen. The seven volumes of the original gas and air are thus reduced to six volumes of burned gases.

THERMOCHEMISTRY

Thermochemistry treats of the heat changes that accompany all chemical reactions. A knowledge of such heat changes is of the greatest importance in the study of explosive phenomena.

Heat Changes.—In a chemical reaction, when combination takes place, the heat energy of the compound or compounds formed is the **heat of formation** or **combination.**

Chemical reaction may also be accompanied by dissociation or decomposition of a compound, its heat of formation being then **heat of decomposition,** which neutralizes or is neutralized by the heats of formation of the products of the reaction. The heat of decomposition of a substance is always equal to its heat of formation.

The **heat of elements,** in a reaction, is always zero, there being no combination or dissociation in the element.

When the sum of the heats of formation of the products of a reaction is greater than the total heat of decomposition **heat is liberated** and the reaction is "exothermic." When the total heat of decomposition is the greater, heat is absorbed and the reaction is then "endothermic."

Heat of Combustion.—This term is generally applied to the heat liberated in the oxidation of a combustible. The reaction is exothermic; and, in general,

$$\textit{Heat of combustion} = \begin{matrix}\textit{Heat of formation}\\ \textit{of products}\end{matrix} - \begin{matrix}\textit{Heat of formation}\\ \textit{of combustible}\end{matrix}$$

The heat of combustion of a substance, like combining heat and heats of formation or decomposition, is expressed in heat units, per unit weight of substance. The following table gives the heats of combustion of some of the more important combustibles in mining:

TABLE OF HEATS OF COMBUSTION
(Favre & Silbermann)

Combustible			Heat of combustion, B.t.u. per lb.
Methane, to carbon dioxide and water at 32 deg. F.			23,513
Olefiant gas, to carbon dioxide and water at 32 deg. F.			21,344
Carbon, to carbon dioxide			14,544
Carbon, to carbon monoxide			4,451
Carbon monoxide, to carbon dioxide			4,325
Hydrogen, to water at 32 deg. F.			62,032
Hydrogen, to steam at 212 deg. F.			51,717
Sulphur, to sulphur dioxide			4,000
Petroleum, heavy (sp. gr. 0.886)			19,000
Petroleum, light (sp. gr. 0.833)			18,200
Coal (average values)	State	Fixed carbon, per cent.	
Anthracite	Pennsylvania	84.3	14,200
Bituminous	Pennsylvania	57.0	14,900
Bituminous	West Virginia	65.8	14,240
Bituminous	Illinois	46.4	14,460
Bituminous	Ohio	51.5	14,400
Bituminous	Kentucky	50.1	12,700
Bituminous	Alabama	59.3	13,700
Bituminous	Indiana	44.3	14,140

The above are average values for each entire state, as taken from Government analyses and do not represent mining districts.

Heat Calculation.—The calculation of the heat of combustion from the heats of combination of the combustible and the several products, formed, will be best understood by a practical illustration following the statement of a few fundamental principles that always govern the operation. Briefly stated these are as follows:

1. No heat energy is lost, but the heat of an element, in any reaction, is zero, there being neither combination nor dissociation possible in the element as in a compound.

2. Total heat of formation of products is the **positive** (+) **heat** developed in the reaction.

3. Heat of decomposition (same as heat of formation) of the combustible is the **negative** (−) **heat** or the heat absorbed in the reaction.

4. The heat of combustion is the **net heat,** or the difference between the total heat in the products and the heat in the combustible.

5. The reaction generates heat, or is **exothermic,** when there is an excess of positive (+) heat.

6. The reaction absorbs heat, or is **endothermic,** when there is an excess of negative (−) heat.

Note.—The **chemical equation** expressing a reaction shows the equivalence of weight of matter before and after reaction, but does not show the thermal effect.

A **thermochemical equation** is written by adding to the chemical equation a positive or a negative term indicating the heat generated or absorbed in the reaction. This heat may be expressed as "gram-calories" "kilogram-calories" or "pound-calories," according as the weight of the combustible taken is a gram-molecule, a kilogram-molecule or a pound-molecule. Or, the heat of the reaction may be given as B.t.u. per pound, or other denomination. The weight-unit is immaterial, since the **heat of the reaction** is always that due to the molecular weight of the combustible expressed in the same weight-unit.

The amount of heat corresponding to the molecular weight of the combustible (expressed in any weight-unit) is frequently called the **"molecular heat"** of the reaction.

The molecular heat of a chemical reaction, divided by the molecular weight of the substance consumed, gives the heat of the reaction per unit weight of substance, which is the heat of the combustion expressed in the same denomination as the weight of the substance.

Illustration.—The heat of combustion of methane (CH_4), as determined by Favre and Silbermann (See Table), is 23,513 B.t.u. per lb.; or $23,513 \times 5/9 = 13,063$ lb.-cal. per lb.; or 13,063 kg.-cal. per kg. or grm.-cal. per grm of the gas.

The molecular heat of this reaction is therefore

$$16 \times 23,513 = 376,208 \text{ B.t.u.}$$

or

$$16 \times 13,063 = 209,008 \text{ cal.}$$

It is observed, thus, that the molecular heat, in the combustion of methane, is the heat (B.t.u.) generated by 16 lb. of the gas; or the heat (lb.-cal.) generated by the 16 lb.; or the heat (kg.-cal.) due to 16 kg.; or the heat (grm.-cal.) due to 16 grm. of this gas. Different authorities have obtained slightly varying heat values of the gases.

Heats of Formation of Substances.—The heats of formation of a few substances that are of interest in mining are given in the following table. The heats are given as molecular heats for convenience of substitution in equations.

TABLE OF HEATS OF FORMATION OF SUBSTANCES

Substance	Symbol	Molecular heats of formation	
		B.t.u.	Cal.
Methane	CH_4	39,060	21,700
Acetylene	C_2H_2	98,550	54,750
Ethene (olefiant gas)	C_2H_4	−20,250	−11,250
Ethane	C_2H_6	47,970	26,650
Carbon monoxide	CO	52,200	29,000
Carbon dioxide	CO_2	174,600	97,000
Hydrogen sulphide	H_2S	8,640	4,800
Sulphur dioxide	SO_2	124,668	69,260
Ice (32°F.)	H_2O	128,880	71,600
Water (32°F.)	H_2O	126,288	70,160
Water (212°F.)	H_2O	123,048	68,360
Steam (212°F.)	H_2O	105,660	58,700

For the most part, the heat values in the above table have been determined by experiment, by means of the calorimeter. The values of the heats of combustion, as calculated from these molecular heats of formation, by substitution in the chemical equation expressing the reaction, will not be found to check the earlier determinations of Favre and Silbermann; but the variation is slight.

For example, writing the thermochemical equation for the combustion of methane, indicating the required heat of combustion by x, we have

$$CH_4 + 2\,O_2 = CO_2 + 2\,H_2O - x$$
$$39{,}060 + 0 = 174{,}600 + 2(126{,}288) - x$$
$$x = 174{,}600 + 2(126{,}288) - 39{,}060 = 388{,}116\ \textit{B.t.u.}$$

Then, the molecular weight of methane being 16, the unit heat of combustion is $388{,}116 \div 16 = 24{,}257$, instead of 23,513 B.t.u.

Writing a Thermochemical Equation.—The thermochemical equation expressing the reaction that takes place and the heat that is generated in the combustion of methane (CH_4) is written thus:

$$CH_4 + 2O_2 = CO_2 + 2H_2O - 388{,}116\ \textit{B.t.u.}$$

Or, in the French system,

$$CH_4 + 2O_2 = CO_2 + 2H_2O - 215{,}620\ \textit{cal.}$$

The reaction is exothermic, or generates heat, which is the excess of the heats of formation of the products of the combustion (carbon dioxide and water), over the heat of formation of the combustible (methane).

Likewise, for the combustion of carbon to carbon dioxide, which generates 14,550 B.t.u. per lb., or $14{,}500 \times 5/9 = 8083$ cal., the molecular heat of the reaction is $12 \times 14{,}550 = 174{,}600$ B.t.u., or $12 \times 8083 =$ say 97,000 cal. The thermochemical equation expressing this combustion is

$$C + O_2 = CO_2 - 174{,}600\ \textit{B.t.u.}$$

or

$$C + O_2 = CO_2 - 97{,}000\ \textit{cal.}$$

In these equations, the heat of combustion is equal to the heat of formation of the product (carbon dioxide), the heats of the elements (carbon and oxygen) being zero.

HYGROMETRY

Hygrometry is the measurement of the amount of vapor in the air, at any given time. The capacity of the air for holding moisture varies with the temperature. For example, at 32 deg. F., a cubic foot of air will hold or has a capacity of only 2.13 grains of water; while at 60 deg. the capacity is 5.77 gr. per cu. ft.; at 100 deg., 19.84 gr. per cu. ft.; and at 212 deg. F., air fully saturated with moisture holds about 258 gr. per cu. ft.

Hygrometric State of Air.—Air absorbs moisture from bodies in contact with it, and thus exerts a drying action, which is of great importance in mining. The absorptive power of the air varies with its degree of saturation. For example, air at 60 deg. F., containing, say 2.9 gr. per cu. ft., is only about half saturated and is then said to contain 50 per cent. of moisture. In this condition, the air will readily absorb more moisture. The degree of saturation of air is called its "hygrometric state."

Air is said to be "dry" or "wet," according to the degree of its saturation. It is important to observe that these terms have no reference to the actual amount of vapor present in a given volume of air; but only express how nearly the air is saturated. For example, air fully saturated at 32 deg. F. contains 2.13 gr. of moisture per cubic foot and is "wet" because it is full of water vapor; but if the temperature now rises to, say 60 deg., the vapor capacity of the air is thereby increased to 5.77 gr. per cu. ft., and its degree of saturation or humidity" is then $2.13/5.77 \times 100 = 36.9$ per cent. In other words, the air at this temperature contains only 36.9 per cent. of its capacity, and is therefore comparatively speaking, "dry" air. Owing to the rise of temperature, from 32 to 60 deg., the air is capable of absorbing $5.77 - 2.13 = 3.64$ gr. of moisture per cubic foot.

Calculation of Weight of Moisture in Air.—In order to calculate the weight (w), in pounds, of moisture contained in one cubic foot of air, it is necessary to know the degree of saturation of the air (c), its temperature (t), and the vapor pressure (p_v) corresponding to that temperature. This last must be taken from tables known as psychrometric tables. Calling the absolute temperature $T = 460 + t$, the formula is

$$w = 0.6235 \frac{cp_v}{0.37T}$$

The constant 0.6235 is the specific gravity of water vapor, and the constant 0.37 is the reciprocal of the weight of one cubic foot of dry air, at a temperature of 1 deg. F. (absolute) and a pressure of 1 lb. per sq. in.

Example.—Calculate the weight of water vapor carried in an air current of 100,000 cu. ft. when the saturation is 80 per cent. and the temperature 70 deg. F., if the vapor pressure at the given temperature is $t_v = 0.3602$ lb. per sq. in. (see Table, p. 77).

Solution.—The absolute temperature, in this case, is $T = 460 + 70 = 530$; and the total weight of vapor is

$$100{,}000 \times 0.6235 \frac{0.80 \times 0.3602}{0.37 \times 530} = 91.62 \, lb.$$

How Humidity is Measured.—The humidity of the air is commonly measured by an instrument called the "hygrometer" or "psychrometer." This is the "wet-and-dry-bulb hygrometer."

Other forms of hygrometer have been employed depending on the absorption of the moisture from the air by certain hygroscopic substances, and dew-point hygrometers; but these are less simple and not as portable as the wet-and-dry-bulb hygrometer, which indicates the humidity by the difference in the reading of the wet- and dry-bulb thermometers.

The Hygrometer or Psychrometer.—A neat and portable form of the wet-and-dry-bulb hygrometer, designed by the Davis Instrument Manufacturing Co., is shown in the Fig. 7. Two delicate thermometers are mounted on springs on the inside of a light cylindrical folding metallic case, the dry bulb on the door and the wet bulb in the case. To the latter bulb is attached a fine silk or muslin sack, which forms a wick that extends downward to the small vessel which holds the water that keeps this bulb wet.

Still another form of this instrument is that known as the "Swing psychrometer," from the manner of its use. As shown in Fig. 8, it consists of two thermometers mounted on a metal support, which is firmly attached to a handle on which it is arranged to swing. The left-hand thermometer has a dry bulb and its reading indicates the actual tempera-

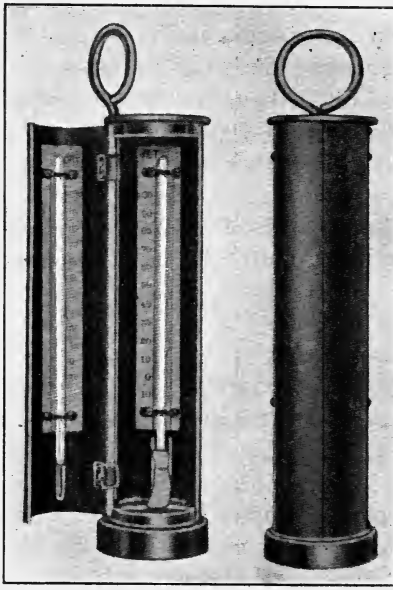


Fig. 7.

ture of the air; while the bulb of the right-hand glass is covered with a sack that is wet with water when an observation is to be taken.

Holding the handle in a firm grasp, the operator swings the instrument so that the metal support holding the two thermometers rotates rapidly on the handle as an axis. The swift movement accelerates the evaporation from the wet sack and cools the bulb of that thermometer, whose reading enables the calculation of the degree of saturation by difference with the dry-bulb reading.

The swing psychrometer is a popular form of the wet- and dry-bulb hygrometer, because of its portability and the reliability of its indications, which are generally assumed to be more representative of the actual state of the air, because of its movement when an observation is being taken.

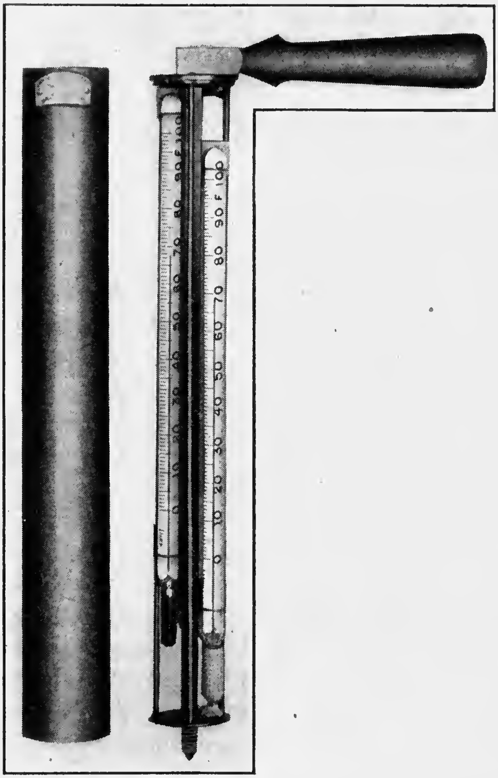


FIG. 8.

Principle of Hygrometer.—Unsaturated vapors, like gases, obey Boyle's law; and, for any given temperature, the ratio of the quantity or volume of vapor is equal to the pressure ratio, or the relative humidity (H), is expressed by the formula.

$$H = \frac{\text{Actual vapor pressure}}{\text{Saturated vapor pressure}}$$

The saturated vapor pressure (dry-bulb temp.) is given in the tables. The actual vapor pressure, at the time of observation, is equal to the saturated vapor pressure of the tables, for the dew-point temperature, which, if known, would make the calculation easy by the use of the above formula. In the use of the wet-and-dry-bulb hygrometer, however, the relative humidity is calculated by the formula

$$H = \frac{p_w - \frac{B}{30}\left(\frac{t_d - t_w}{88}\right)}{p_d}$$

in which H = relative humidity; p_w and p_d the respective saturated vapor pressures of the tables, for the corresponding wet-and-dry-bulb temperatures t_w and t_d; and B the barometric pressure, in inches.

What the Wet-and-dry-bulb Hygrometer Indicates.—The wet-and-dry-bulb hygrometer shows the difference between the readings of the two thermometers. The dry-bulb thermometer, of course, indicates the actual temperature of the air. The reading of the wet-bulb thermometer is lowered by the evaporation of the water from the little sack surrounding this bulb, and which is kept moist by the water drawn up through the wick from the vessel below.

The difference of temperature indicated by these two thermometers depends on the rapidity of the evaporation of the water from the wet bulb. The evaporation is more rapid in dry than in wet air; and the difference of reading is, thus, an index or measure of the degree of saturation of the air. When the air is fully saturated with moisture there is no evaporation from the wet bulb and the readings of the two thermometers are the same. The difference increases with the dryness of the air.

Relative Humidity of Air.—As previously explained the relative humidity of air is expressed by the ratio of the actual vapor pressure in the air at the time, to the saturated vapor pressure. The following table gives the percentage of saturation or the hygrometric state of air for various differences of readings, at different temperatures.

Difference Between Dry and Wet Bulbs

Reading of dry-bulb ther., deg. F	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	17.5	18°	18.5	19°	19.5	20°	20.5	21°
	Relative humidity																								
65	95	90	85	80	75	70	66	62	57	53	48	44	40	36	32	28	25	23	21	19	17	15	13	12	10
66	95	90	85	80	76	71	66	62	58	53	49	45	41	37	33	29	26	24	22	20	18	17	15	13	11
67	95	90	85	80	76	71	67	62	58	54	50	46	42	38	34	30	27	25	23	21	20	18	16	15	13
68	95	90	85	81	76	72	67	63	59	55	51	47	43	39	35	31	28	26	24	23	21	19	17	16	14
69	95	90	86	81	77	72	68	64	59	55	51	47	44	40	36	32	29	27	25	24	22	20	19	17	15
70	95	90	86	81	77	72	68	64	60	56	52	48	44	40	37	33	30	28	26	25	23	21	20	18	17
71	95	90	86	82	77	73	69	64	60	56	53	49	45	41	38	34	31	29	27	26	24	22	21	19	18
72	95	91	86	82	78	73	69	65	61	57	53	49	46	42	39	35	32	30	28	27	25	23	22	20	19
73	95	91	86	82	78	73	69	65	61	58	54	50	46	43	40	36	33	31	29	28	26	24	23	21	20
74	95	91	86	82	78	74	70	66	62	58	54	51	47	44	40	37	34	32	30	29	27	25	24	22	21
75	96	91	87	82	78	74	70	66	63	59	55	51	48	44	41	38	34	33	31	30	28	26	25	23	22
76	96	91	87	83	78	74	70	67	63	59	55	52	48	45	42	38	35	34	32	30	29	27	26	24	23
77	96	91	87	83	79	75	71	67	63	60	56	52	49	46	42	39	36	34	33	31	30	28	27	25	24

To use the table, find the observed temperature of the air, in the left-hand column, and the difference of the observed readings of the wet- and dry-bulb thermometers, at the top of the table; the corresponding number in the table is the percentage of saturation which expresses the degree of humidity of the air. For example, if the dry-bulb temperature is 70 deg. and the wet-bulb 64 deg. F. the difference of readings is 6 deg. and the corresponding humidity as taken from the above table is 72 per cent.

Actual Vapor Pressure.—The pressures given in the table below are the pressures the vapor exerts when the space it occupies is fully saturated; they are called the "saturated vapor pressures." When the weight of vapor in the air is not sufficient for saturation the vapor pressure will be exactly proportional to the degree of saturation. For example, if 50 per cent. of moisture is present or the air only half saturated, at, say 70°F., the "actual vapor pressure," as it is called, is one-half of the saturated vapor pressure, in the table given later; or $\frac{1}{2} \times 0.3602 = 0.1801$ lb. per sq. in.

To calculate the actual vapor pressure from the difference of the wet- and dry-bulb temperatures $(t_d - t_w)$ and the barometric pressure (B), in inches of mercury, first find the saturated vapor pressure (p_w), in inches of mercury, corresponding to the wet-bulb temperature (t_w), from the table; and substitute this and the given values in the formula

$$\textit{Actual vapor pressure at temperature } t_d = p_w - \frac{B}{30}\left(\frac{t_d - t_w}{88}\right)$$

Example.—Find the actual vapor pressure when the dry bulb reads 60° and the wet bulb 54°F., the barometric pressure being $B = 30$ in., and the saturated vapor pressure for the wet-bulb temperature (54°F.) being 0.4178 in. of mercury.

Solution.—

$$p_v = 0.4178 - \frac{30}{30}\left(\frac{60 - 54}{88}\right) = 0.3497 \textit{ in. of mercury}$$

Since the saturated vapor pressure (see table) for the dry-bulb temperature (60°F.) is 0.5183 in., the relative humidity in the above example is

$$H = \frac{p_v}{p_d} \times 100 = \frac{0.3497 \times 100}{0.5183} = 67.4 \textit{ per cent.}$$

The Dew Point.—What is called the "dew point," in hygrometry, is the temperature below which the moisture contained in the air begins to be deposited. For example, the weight of moisture, in grains per cubic foot, contained in the air, in the above example is (1 lb. = 7000 grs.)

$$w = 7000 \times 0.6235 \frac{0.674 \times 0.2545}{0.37(460 + 60)} = 3.9 \textit{ gr. per cu. ft.}$$

The temperature at which this weight of moisture will fully saturate a cubic foot of air is the dew point, because the slightest fall of temperature below that point will cause a deposition of moisture from the air.

The dew-point temperature is ascertained, in any given case, by first calculating the actual vapor pressure of the moisture in the air, as in the above example; and then, by referring to the table of saturated vapor pressures, find the temperature orresponding to that vapor pressure. This is true, because, as previously stated, the actual vapor pressure, at any given time, is equal to the saturated vapor pressure for the dew-point temperature. Thus, the actual vapor pressure for dry bulb 60° and wet bulb 54° was found to be 0.3497 in., which corresponds to a saturated vapor pressure or dew point of about 49 deg. F.

TABLE SHOWING SATURATED VAPOR PRESSURES FOR DIFFERENT TEMPERATURES

Degrees, Fahr	Barometric pressure, mercury (32°F.) in.	Pressure, pounds per square inch	Degrees, Fahr.	Barometric pressure, mercury (32°F.) in.	Pressure, pounds per square inch
−30	0.0099	0.0049	70	0.7335	0.3602
−20	0.0168	0.0082	71	0.7587	0.3726
−10	0.0276	0.0136	72	0.7848	0.3854
0	0.0439	0.0216	73	0.8116	0.3986
5	0.0551	0.0271	74	0.8393	0.4122
10	0.0691	0.0339	75	0.8678	0.4262
15	0.0865	0.0425	76	0.8972	0.4406
20	0.1074	0.0527	77	0.9275	0.4555
26	0.1397	0.0686	78	0.9587	0.4708
32	0.1815	0.0891	79	0.9906	0.4865
34	0.1961	0.0963	80	1.024	0.5027
36	0.2122	0.1042	81	1.058	0.5194
37	0.2205	0.1083	82	1.092	0.5365
38	0.2293	0.1126	83	1.128	0.5542
39	9.2382	0.1170	84	1.165	0.5723
40	0.2476	0.1216	85	1.203	0.5910
41	0.2574	0.1264	86	1.243	0.6102
42	0.2674	0.1313	87	1.283	0.6299
43	0.2777	0.1364	88	1.324	0.6502
44	0.2885	0.1417	89	1.367	0.6711
45	0.2995	0.1471	90	1.410	0.6925
46	0.3111	0.1528	95	1.647	0.8090
47	0.3229	0.1586	100	1.918	0.9421
48	0.3352	0.1646	105	2.227	1.0938
49	0.3478	0.1708	110	2.578	1.2663
50	0.3610	0.1773	115	2.977	1.4618
51	0.3745	0.1839	120	3.427	1.6828
52	0.3885	0.1908	125	3.934	1.9318
53	0.4030	0.1979	130	4.504	2.2119
54	0.4178	0.2052	135	5.144	2.5261
55	0.4333	0.2128	140	5.859	2.8774
56	0.4492	0.2206	145	6.658	3.2696
57	0.4657	0.2287	150	7.547	3.7063
58	0.4826	0.2370	155	8.535	4.1914
59	0.5001	0.2456	160	9.630	4.7292
60	0.5183	0.2545	165	10.841	5.324
61	0.5370	0.2637	170	12.179	5.981
62	0.5561	0.2731	175	13.651	6.704
63	0.5760	0.2829	180	15.272	7.500
64	0.5964	0.2929	185	17.050	8.373
65	0.6176	0.3033	190	18.954	9.330
66	0.6394	0.3140	195	21.130	10.377
67	0.6618	0.3250	200	23.457	11.520
68	0.6850	0.3364	205	25.993	12.765
69	0.7086	0.3481	212	29.925	14.696

Caution.—It is absolutely necessary in the use of such formulas as embrace terms or constants of a given denomination to use only values of that denomination. For example, the formula for finding the weight of moisture that will saturate a cubic foot of air at a temperature of t degrees, is

$$w = 0.6235 \frac{p_v}{0.37\,(460 + t)}$$

This is recognized as being derived from the formula previously given (p. 71) to find the weight of a cubic foot of dry air at a pressure p and temperature t, by substituting for the atmospheric pressure p (lb. per sq. in.), the saturated vapor pressure for p_v (lb. per sq. in.); and multiplying the formula by the specific gravity of water vapor (0.6235) referred to air.

In these formulas, the pressure must always be expressed in pounds per square inch, because the constant 0.37 is in that denomination; and the temperature must be given in Fahrenheit degrees, for a like reason. Also, the weight will be found in pounds per cubic foot and, if desired in grains per cubic foot, must be multiplied by 7000, as there are 7000 gr. in a pound (avdp.).

On the other hand, the formulas given for calculating the relative humidity of the air, or the actual vapor pressure contain the constant 88, which is based on barometric pressure (in. of mercury) and Fahrenheit temperatures. The constant 88 is used for all temperatures above 32 deg., and 96 for any temperature below 32 deg.

The table of saturated vapor pressures, on the preceding page, gives the pressure or tension of water vapor for different temperatures (Fahr. scale), from −30 deg. to 212 deg. The pressures are given both in inches of mercury and pounds per square inch.

Example.—Find the actual vapor pressure, the relative humidity, dew point and weight of moisture present, in grains per cubic foot, when the readings of the dry- and wet-bulb thermometers are 62 deg. and 54 deg. F., respectively, and the barometric pressure is 28.2 in.

Solution.—The actual vapor pressure, in this case, as calculated from the saturated vapor pressure corresponding to the wet-bulb reading (p_{54} = 0.4178 in.), is

$$p_v = 0.4178 - \frac{28.2}{30}\left(\frac{62 - 54}{88}\right) = 0.33235\ in.$$

The saturated vapor pressure for the given temperature (see Table) is p_{62} = 0.5561 in. and the relative humidity,

$$H = \frac{0.33235 \times 100}{0.5561} \times 59.7 \textit{ per cent.}$$

The dew-point temperature corresponding to a saturated vapor pressure of 0.3323 (see Table) is 47.7 deg. F.

The actual weight of vapor the saturated vapor pressure corresponding to the dry-bulb temperature 62 deg. F. (see Table) being 0.2731 lb. per sq. in., is

$$w = 7000 \times 0.6235 \frac{0.597 \times 0.2731}{0.37(460 + 62)} = 3.6 \textit{ gr. per cu. ft.}$$

Dry and Wet Air Compared.—Strange as it may at first appear, wet air is lighter than dry air, volume for volume. This is because the water vapor in the air is much lighter than the same volume of air which it displaces. The specific gravity of water vapor referred to air as a standard or unity is 0.6235.

The weights, per cubic foot, of water vapor and dry, partly-saturated and fully-saturated air, respectively, are calculated by the following formulas:

Water vapor, $$w = 0.6235 \frac{cp_v}{0.37T} \quad (1)$$

Dry air, $$w = \frac{p_a}{0.37T} \quad (2)$$

Air partly saturated, $$w = \frac{p_a - 0.3765cp_v}{0.37T} \quad (3)$$

Air fully saturated, $$w = \frac{p_a - 0.3765p_v}{0.37T} \quad (4)$$

w = weight (lb. per cu. ft.)
c = degree of saturation, expressed as a decimal
p_a = atmospheric pressure (lb. per sq. in.)
p_v = saturated-vapor pressure (lb. per sq. in.)
T = absolute temperature (deg. Fahr.)

It is readily seen, from Formulas 2, 3 and 4, that perfectly dry air is always heavier than air containing water vapor, and that the weight of air decreases as its degree of saturation increases. The weight of moisture in air is usually estimated in grains instead of pounds, per cubic foot, and it is necessary to multiply the results obtained from the above formulas by 7000 (1 lb. = 7000 gr.).

The same formulas expressing the atmospheric pressure and the vapor pressure in inches of barometer B, instead of pounds per square inch, are as follows:

Water vapor, $$w = \frac{0.82757cp_v}{T} \tag{5}$$

Dry air, $$w = \frac{1.3273B}{T} \tag{6}$$

Air partly saturated, $$w = \frac{1.3273}{T}(B - 0.3765cp_v) \tag{7}$$

Air fully saturated, $$w = \frac{1.3273}{T}(B - 0.3765p_v) \tag{8}$$

It is evident that when air is fully saturated, $c = 1$, and disappears from the formula. The values of p_v are given in a preceding table, in pounds per square inch and inches of mercury.

Formula 3 is obtained by the addition of Formulas 1 and 2, making $p_a = p_a - cp_v$; and Formula 5 is derived from Formula 1, by reducing the pressure (lb. per sq. in.) to pressure (in. barom.), since 1 in. barom. = 0.4911 lb. per sq. in. and (0.6235 × 0.4911) ÷ 0.37 = 0.82757. But the value of p_v, in Formulas 5, 7, 8, must be given in inches of barometer, instead of pounds per square inch as in Formulas 1, 3, 4.

Important.—Properly speaking, a vapor does not saturate the air, but the space it occupies; since, for any given temperature, the same weight of vapor serves to fill a given space whether that space is full or void of air. Commonly speaking, vapor is said to be saturated or unsaturated according as the space it occupies is saturated or otherwise.

Laws of Vapors.—The following laws express the chief characteristics of vapors:

1. Vaporization takes place at the surface of all volatile liquids, at all temperatures, till the space surrounding the liquid is saturated or the critical temperature is reached.

2. Vapor pressure (different for different vapors) depends on the temperature and the degree of saturation.

3. For any given temperature, the weight and pressure of a vapor saturating a given space is the same whether that space is full or void of air or other gas.

4. Saturated vapor pressures increase with the temperature and when equal to the pressure above the liquid vaporizing, the ebullition of the liquid begins, which marks the **boiling point** of the liquid for that pressure.

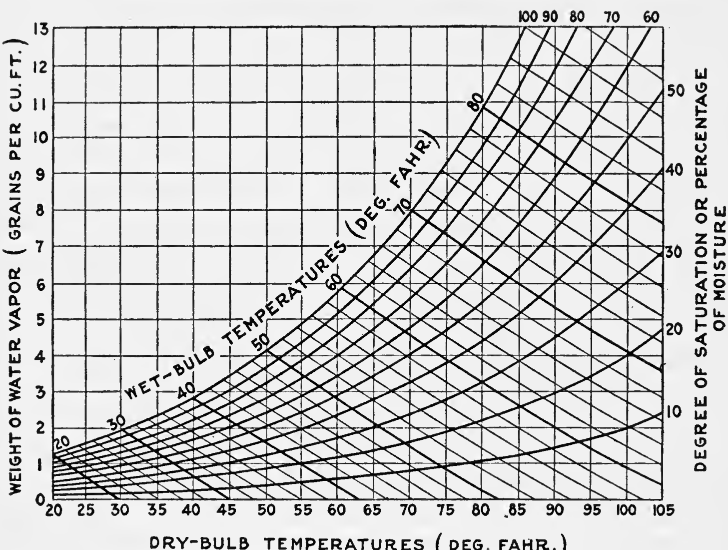


Fig. 9.

5. In a confined space, a further addition of heat to the liquid causes a rise of both temperature and vapor pressure till an equilibrium of densities of the liquid and vapor stops further vaporization and marks the so-called **"critical temperature"** for that liquid.

The diagram, Fig. 9, is useful in showing at a glance the weight of water vapor that will saturate a cubic foot of space at any temperature from 20 to 105 deg. F. and the degree of humidity for different dry- and wet-bulb readings of the psychrometer.

STEAM

Steam is the vapor of water formed at any temperature at or above the boiling point of the water. It is a certain vaporized or gaseous state of water. Water vaporizing below its boiling point forms vapor but not steam. Thus, while all steam is vapor, correctly speaking, all vapor is not steam.

Steam in its natural state or when saturating a given space, has a temperature corresponding to the pressure it supports. This will be more clearly understood by taking an example of a given volume of steam in contact with the water from which it was formed. For instance, the steam in a steam boiler, at a pressure of 65 lb. gage (sea level) or, say 80 lb. absolute, has a temperature of 312 deg. F. But any increase of pressure will be accompanied with a corresponding increase in its temperature, so that, at a pressure of 155 lb. absolute, the temperature of the steam will have increased to 361 deg. F.

Again, assuming a given volume of steam in contact with the water from which it was formed, such steam can neither be compressed nor expanded without a corresponding change taking place in its temperature. For example, for the same temperature, any increase of pressure would cause some of the steam to condense, while a decrease of pressure would cause more steam to form, as the water would vaporize under the decreased pressure. Thus, the space above the water is always saturated by a weight of steam corresponding to the temperature, which is fixed for any given pressure.

Saturated Steam.—Saturated steam may be defined as steam in contact with water. From the foregoing, it will be understood that saturated steam is in its natural state, having a temperature corresponding to its pressure. Saturated steam may be either dry or wet, according as it does or does not hold any entrained water. The density of dry saturated steam is always the same for the same temperature.

Superheated Steam.—When steam is not in contact with water, any addition of heat causes an increase in both temperature and pressure, the pressure increasing with the absolute temperature. The steam is no longer saturated, and is said to be "superheated." Superheated steam is always dry.

Unlike saturated steam, superheated steam follows the laws of a perfect gas. For a constant volume, its pressure increases with the absolute temperature; and, for a constant temperature, the pressure increases inversely as its volume. Steam is superheated, therefore, whenever its temperature exceeds that of saturated steam, for any given pressure.

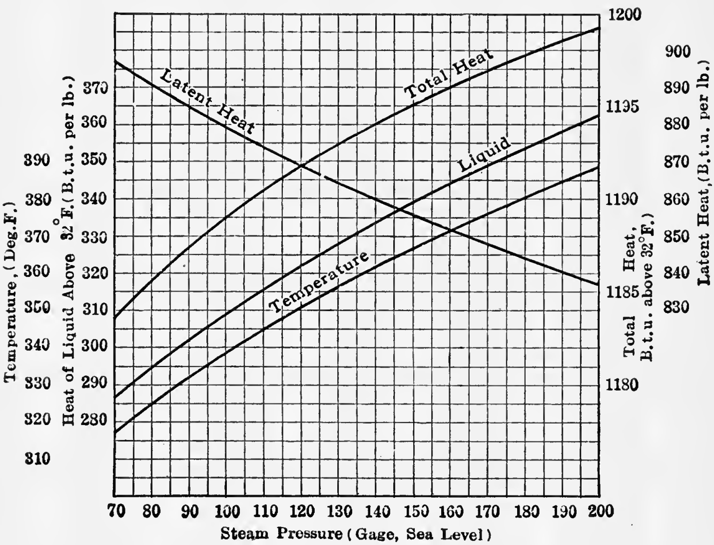


Fig. 10.

Steam Tables.—The table, in the following pages, gives the temperature, specific volume, heat of the liquid above 32 deg. F., latent heat of evaporation, and the total heat in the steam, for different absolute pressures, as taken from Marks & Davis Steam Tables, which are the generally accepted values, today. The diagram, Fig. 10, was compiled by J. T. Beard, Jr. from the same source, and will be found convenient for use in connection with the tables.

PRESSURE TABLE FOR DRY SATURATED STEAM
(Condensed from Marks and Davis, by Permission)

Absolute pressure, lb. per sq. in.	Temp., deg. F.	Sp. vol., cu. ft. per lb.	Heat of the liquid B.t.u.	Latent heat of evap. B.t.u.	Total heat of steam B.t.u.
p	t	v or s	h or q	l or r	h
1	101.83	333.0	69.8	1034.6	1104.4
2	126.15	173.5	94.0	1021.0	1115.0
3	141.52	118.5	109.4	1012.3	1121.6
4	153.01	90.5	120.9	1005.7	1126.5
5	162.28	73.33	130.1	1000.3	1130.5
6	170.06	61.89	137.9	995.8	1133.7
7	176.85	53.56	144.7	991.8	1136.5
8	182.86	47.27	150.8	988.2	1139.0
9	188.27	42.36	156.2	985.0	1141.1
10	193.22	38.38	161.1	982.0	1143.1
11	197.75	35.10	165.7	979.2	1144.9
12	201.96	32.36	169.9	976.6	1146.5
13	205.87	30.03	173.8	974.2	1148.0
14	209.55	28.02	177.5	971.9	1149.4
15	213.0	26.27	181.0	969.7	1150.7
16	216.3	24.79	184.4	967.6	1152.0
17	219.4	23.38	187.5	965.6	1153.1
18	222.4	22.16	190.5	963.7	1154.2
19	225.2	21.07	193.4	961.8	1155.2
20	228.0	20.08	196.1	960.0	1156.2
21	230.6	19.18	198.8	958.3	1157.1
22	233.1	18.37	201.3	956.7	1158.0
23	235.5	17.62	203.8	955.1	1158.8
24	237.8	16.93	206.1	953.5	1159.6
25	240.1	16.30	208.4	952.0	1160.4
26	242.2	15.72	210.6	950.6	1161.2
27	244.4	15.18	212.7	949.2	1161.9
28	246.4	14.67	214.8	947.8	1162.6
29	248.4	14.19	216.8	946.4	1163.2
30	250.3	13.74	218.8	945.1	1163.9
31	252.2	13.32	220.7	943.8	1164.5
32	254.1	12.93	222.6	942.5	1165.1
33	255.8	12.57	224.4	941.3	1165.7
34	257.6	12.22	226.2	940.1	1166.3
35	259.3	11.89	227.9	938.9	1166.8
36	261.0	11.58	229.6	937.7	1167.3
37	262.6	11.29	231.3	936.6	1167.8
38	264.2	11.01	232.9	935.5	1168.4
39	265.8	10.74	234.5	934.4	1168.9
40	267.3	10.49	236.1	933.3	1169.4
41	268.7	10.25	237.6	932.2	1169.8
42	270.2	10.02	239.1	931.2	1170.3
43	271.7	9.80	240.5	930.2	1170.7
44	273.1	9.59	242.0	929.2	1171.2
45	274.5	9.39	243.4	928.2	1171.6
46	275.8	9.20	244.8	927.2	1172.0
47	277.2	9.02	246.1	926.3	1172.4
48	278.5	8.84	247.5	925.3	1172.8
49	279.8	8.67	248.8	924.4	1173.2
50	281.0	8.51	250.1	923.5	1173.6
52	283.5	8.20	252.6	921.7	1174.3
54	285.9	7.91	255.1	919.9	1175.0
56	288.2	7.65	257.5	918.2	1175.7
58	290.5	7.40	259.8	916.5	1176.4

PRESSURE TABLE FOR SATURATED STEAM—(*Continued.*)

Absolute pressure, lb. per sq. in.	Temp., deg. F.	Sp. vol., cu. ft. per lb.	Heat of the liquid	Latent heat of evap.	Total heat of steam
p	t	v or s	h or q	l or r	h
60	292.7	7.17	262.1	914.9	1177.0
62	294.9	6.95	264.3	913.3	1177.6
64	297.0	6.75	266.4	911.8	1178.2
66	299.0	6.56	268.5	910.2	1178.8
68	301.0	6.38	270.6	908.7	1179.3
70	302.9	6.20	272.6	907.2	1179.8
72	304.8	6.04	274.5	905.8	1180.4
74	306.7	5.89	276.5	904.4	1180.9
76	308.5	5.74	278.3	903.0	1181.4
78	310.3	5.60	280.2	901.7	1181.8
80	312.0	5.47	282.0	900.3	1182.3
82	313.8	5.34	283.8	899.0	1182.8
84	315.4	5.22	285.5	897.7	1183.2
86	317.1	5.10	287.2	896.4	1183.6
88	318.7	5.00	288.9	895.2	1184.0
90	320.3	4.89	290.5	893.9	1184.4
92	321.8	4.79	292.1	892.7	1184.8
94	323.4	4.69	293.7	891.5	1185.2
96	324.9	4.60	295.3	890.3	1185.6
98	326.4	4.51	296.8	889.2	1186.0
100	327.8	4.429	298.3	888.0	1186.3
105	331.4	4.230	302.0	885.2	1187.2
110	334.8	4.047	305.5	882.5	1188.0
115	338.1	3.880	309.0	879.8	1188.8
120	341.3	3.726	312.3	877.2	1189.6
125	344.4	3.583	315.5	874.7	1190.3
130	347.4	3.452	318.6	872.3	1191.0
135	350.3	3.331	321.7	869.9	1191.6
140	353.1	3.219	324.6	867.6	1192.2
145	355.8	3.112	327.4	865.4	1192.8
150	358.5	3.012	330.2	863.2	1193.4
160	363.6	2.834	335.6	858.8	1194.5
170	368.5	2.675	340.7	854.7	1195.4
180	373.1	2.533	345.6	850.8	1196.4
190	377.6	2.406	350.4	846.9	1197.3
200	381.9	2.290	354.9	843.2	1198.1
225	391.9	2.046	365.5	834.4	1199.9
250	401.1	1.850	375.2	826.3	1201.5
300	417.5	1.551	392.7	811.3	1204.1
350	431.9	1.334	408.2	797.8	1206.1
400	444.8	1.17	422.0	786.0	1208.0
450	456.5	1.04	435.0	774.0	1209.0
500	467.3	0.93	448.0	762.0	1210.0
550	477.3	0.83	459.0	751.0	1210.0
600	486.6	0.76	469.0	741.0	1210.0

SECTION III

MINE GASES

Geological Conditions—Common Mine Gases—Hydrocarbon Gases—Properties and Behavior of Mine Gases—Methane—Firedamp—Carbon Monoxide—Carbon Dioxide—Blackdamp—Afterdamp—Inflammable and Explosive Mine Gases.

GEOLOGICAL CONDITIONS

Gas, Oil and Water.—The strata of the earth's crust form a great natural reservoir for gas, oil and water. These collect in the formations, in the order of their relative densities. As illustrated in the Fig. 11, which represents an ideal geo-

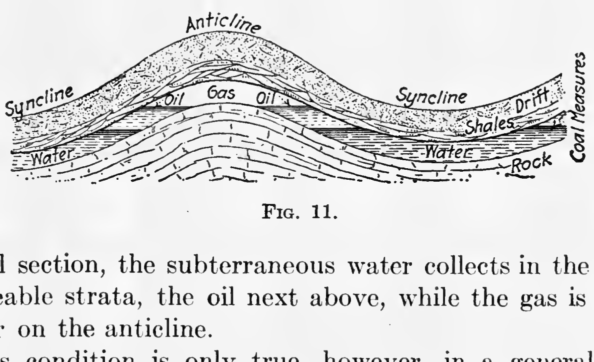


Fig. 11.

logical section, the subterraneous water collects in the lower permeable strata, the oil next above, while the gas is found higher on the anticline.

This condition is only true, however, in a general way, depending on the nature of the strata and their power to absorb and hold these elements. Water, and oil to a less extent, find their way by gravity to a "hard-pan" or stratum impervious to them; while gas drains to the surface and escapes, unless confined by an overlying stratum of clay or oil, from the overlying rocks into the synclinal basins, creates enormous pressures, which are exerted more or less equally on the water, oil and gas.

Water Level.—In every geological section, there is a more or less defined "water level" or depth at which water is found in quantity. Wells or boreholes sunk to this general level strike a usually abundant supply of water. The same is true, but to a less extent, of oil, in oil regions. The flow of oil, in oil-bearing rocks, however, is not as free as that of water, owing to its viscosity and limited supply.

The water level is not constant, but varies according to the changing supply or surface drainage, being higher in wet seasons and lower in seasons of drought. As the oil floats on the water any change in water level is accompanied by a similar change in the oil supply. It is due to this fact that exhausted oil wells often become productive in a season of flood, and producing wells frequently cease to flow in a prolonged season of drought.

Natural Gas.—All gas formed and contained in the strata is called "natural gas," in distinction from gas manufactured in the industries. Natural gas commonly occurs in large volume, in coal formations, where it accumulates in cavities or pockets and in crevices in the strata. It is very largely composed of what are commonly known as the "hydrocarbon" gases.

Effect of Faults.—Fault lines and other geological disturbances of the strata have opened channels by which the gas confined in certain strata escape to other strata or into the mine workings or to the surface. For this reason, the near approach of the working face to a fault line or a disturbed condition of the strata is often accompanied by a marked change in the gaseous condition of the mine air. The percentage of gas common to the mine may then either increase or decrease depending on the location of the gas and the nature of the fault.

Gas Feeders, Blowers.—Any continuous flow of gas from a crack or crevice in the strata is called a "gas feeder," or simply a "feeder." The gas flowing from the crevice is known as "feeder gas."

When a gas feeder is under high pressure so that the gas issues with considerable velocity, the feeder is called a "blower" and the gas "blower gas."

Occluded Gases.—The gases commonly occluded in the coal formations are methane, ethane, nitrogen, carbon dioxide and oxygen. They are the result of the chemical changes that took place in the formation of the coal; or are produced by the action of acid waters on certain limestones or other carbonates. Occluded gases are held in the pores of the coal and other strata, from which they drain into the mine openings, or work upward through such pervious strata as shale and sandstone. The process is called "emission" or "transpiration" of gases.

Pressure of Occluded Gas.—At times, the gas is confined in the coal or other strata by an overlying stratum of clay or impervious limerock that prevents its escape to the surface, and the pressure of the gas is then often very great, varying from 500 and 600 lb. per sq. in. to four or five times that amount. This pressure is manifested in different ways. As the mine workings are extended the flow of gas into the mine increases with the exposure of fresh faces of coal, except where the conditions are such as to allow the gas to drain off and reach the surface.

Effect of Gas Pressure in Mining.—The pressure of gas confined in the coal is often sufficient to splinter the coal in its effort to escape, the fine coal being thrown into the face of the miner at work. At times, the gas escapes from the coal with a peculiar hissing sound known as the "singing of the coal." The pressure of gas in the roof frequently causes heavy roof falls, and gas in the floor causes the bottom to heave. In some instances, the gas pressure assists the extraction of the coal and lessens the work of the miner by helping to break down the coal.

Outbursts of Gas.—In the mining of gaseous seams, it is not uncommon for gas to work in the strata as the coal is extracted. As a result, the gas often accumulates in pockets as shown in the ideal section, Fig. 12. The settlement of the roof incident to the removal of the coal affords opportunity for the gas to expand and work forward toward the opening. The working of the gas in the strata is often accompanied by severe "poundings" or "bumps," due to sudden displacement of the gas. Such sounds often continue for several days previous to a sudden outburst of the gas into the mine workings. The continuance of these poundings are a sufficient warning to experienced miners to vacate that part of the mine till the strata have become more quiet by the gradual draining off of some of the gas.

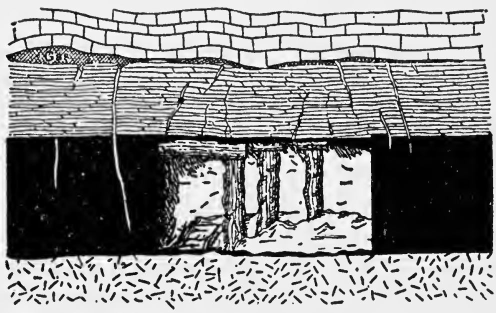


Fig. 12.

In many cases, where the gas works down into the coal, either at the face or in the "ribs," as shown in the figure above, the pressure of the gas becomes distributed over a considerable surface, and is sufficiently great to throw down the coal. This is called an "outburst" of gas, since large volumes of gas escape and often hundreds of tons of coal are thrown violently into the opening.

THE COMMON MINE GASES

The gases of most importance in coal mining, together with their chemical symbols, molecular weights, densities referred to hydrogen and specific gravities referred to air of the same temperature and pressure, are the following:

Gas	Symbol	Molecular weight	Density $H = 1$	Spec. gravity air = 1
Methane (marsh gas)	CH_4	16	8	0.559
Ethene, Ethylene (olefiant gas)	C_2H_4	28	14	0.978
Ethane	C_2H_6	30	15	1.0366
Carbon monoxide	CO	28	14	0.967
Carbon dioxide	CO_2	44	22	1.529
Hydrogen sulphide	H_2S	34	17	1.1912
Oxygen	O_2	32	16	1.1056
Nitrogen	N_2	28	14	0.9713
Hydrogen	H_2	2	1	0.06936

Occurrence of Mine Gases.—Aside from the oxygen and nitrogen of the air, the gases commonly occurring in coal mines are methane, carbon dioxide, carbon monoxide, and less frequently or in less quantity, hydrogen sulphide and olefiant gas. These gases are produced by the processes of decomposition or combustion constantly going on in the mine, or they emanate from the coal or other strata, where they exist as natural gases.

Condition of Gas Confined in Coal.—The results of careful experimental study of coal indicate (Chamberlin) that gas may exist in coal in three different ways: 1. The gas is occluded, in a true sense, or absorbed (possibly condensed) by the coal. 2. The gas is entrapped or held mechanically in the cavities, cracks or pores of the coal. 3. The gas may result from chemical changes going on in the coal.

Escape of Gas from Coal.—Experiments made by the Bureau of Mines, by crushing weighed samples of different coals in closed vessels of known capacity, show that coal continues to give off gas for a long time after it is mined.

Coal exposed to the atmosphere loses much of its occluded gas, but the gas is liberated more freely by crushing the coal, which would indicate that much of the gas is held mechanically within the mass. It is also shown that the coal continues to absorb oxygen from the air, during the same period.

The following table gives the percentages, by volume, of the constituents of natural gases obtained from various coals, in different localities.

Table Showing the Composition of Gas Evolved from Coals at 212 Deg. F., in Vacuo

Locality	CH_4	N_2	CO_2	O_2	C_2H_6	Remarks
South Wales........	.....	62.78	36.42	0.80	....	Bituminous
South Wales........	63.76	29.75	5.44	1.05	....	Bituminous
South Wales........	87.30	7.33	5.04	0.33	....	Steam coal
South Wales........	93.13	4.25	2.62	....	....	Anthracite
Lancashire..........	80.69	8.12	6.44	....	4.75	Cannel
Lancashire..........	77.19	5.96	9.05	....	7.80	Cannel
Westphalia.........	....	89.91	7.50	2.59	....	Gas coal
Westphalia.........	34.85	58.48	2.56	4.11	....	Gas coal

Composition of Feeder or Blower Gas.—A large number of analyses of gas issuing from coal seams as "feeders" or "blowers" have been made. Gas has also been obtained by drilling holes several feet into the face of the coal. These analyses show a wide variation in the composition of the gas in different localities. Moreover, since the rate of emission of gases varies, the composition of feeder gas is only suggestive of the contamination of the mine air.

The following table gives the composition, by volume, of blower gas in different localities, which shows in a general way a higher percentage of methane, in comparison with that of nitrogen. This may be due, to a large extent, to the higher rate of transpiration of the methane, as compared with nitrogen, which tends to increase its percentage in blower gas over what actually exists in the pores of the coal:

Table Giving Composition of Blower Gas in Different Localities

Locality	CH_4	N_2	CO_2	O_2	CO	C_2H_4
Austria	88.9	10.8	1.0	0.3	...	...
Austria	99.1	0.7	0.2	...	...	...
Austria	90.0	9.2	0.2	0.6	...	...
Germany	87.2	11.7	1.1	...	...	...
Germany	77.7	18.5	3.7	0.1	...	...
South Wales	96.7	2.8	0.5	...	...	...
Wallsend, England	92.8	6.9	0.3	...	...	...
Jarrow, England	83.1	14.2	2.1	0.6	...	...
Oakwellgate, England	98.2	1.3	0.5	...	...	...
Wilkes-Barre, Penn	94.2	3.3	1.1	0.9	0.1	0.4

It is important to remember that the occluded gases of coal are not chemically combined with the constituents of the coal as shown by analysis, and do not form a part of the coal itself, although adding much to its inflammability and heat value.

HYDROCARBON GASES

General Formulas of Hydrocarbon Gases.—Carbon (C) and hydrogen (H) unite in different ways to form groups of compounds, having certain distinct characteristics. Such are the "paraffins," represented by the general formula C_nH_{2n+2}; the "olefines," C_nH_{2n}; the "acetylenes," C_nH_{2n-2}; and other compounds of less importance in mining, as the "benzenes," "naphthalines," etc.

Occurrence and Formation.—Methane or light carbureted hydrogen (CH_4) and ethane (C_2H_6), belong to the paraffin or fatty group, while olefiant gas (C_2H_4) belongs to the olefine or oily group. These are all products of the destructive distillation of organic matter. Methane is often seen bubbling up from the bottom of stagnant pools, in marshes, which fact suggested the name "marsh gas." It is the result of the slow decay of the vegetable matter (in the presence of water and absence of air), at the bottom of the pool.

On the other hand, olefiant gas is the result of the dry distillation of gas from organic matter, which takes place less frequently in the strata, owing to the almost invariable presence of moisture. The character of these hydrocarbon gases, moreover, varies, also, with the kind of organic matter that undergoes decomposition.

Of the hydrocarbon gases, the paraffins (methane and ethane) are the ones chiefly occluded in the coal measures; while olefiant gas, belonging to the olefine group is rarely found even in minute quantity. Beside the hydrocarbon gases occluded in coal, as has been stated, varying quantities of nitrogen, oxygen and carbon dioxide have been absorbed.

The Heavy Hydrocarbon Gases.—The heavy hydrocarbons occur in the coal measures as occluded gases, only to a limited extent. Of these, there are but two that are worthy of mention; they are

Olefiant gas, ethene or ethylene, (C_2H_4); sp. gr., 0.978;

Ethane, (C_2H_6); sp. gr., 1.0366.

Both of these gases are colorless and odorless; they occur but to a limited extent in association with methane; and their chief importance lies in the fact that they each have a wider explosive range and a lower temperature of ignition than pure methane. The analyses of the gases exuded from coal rarely show any appreciable quantity of olefiant gas (ethene); but ethane (C_2H_6) occurs more frequently as an occluded gas.

PROPERTIES AND BEHAVIOR OF MINE GASES

The symbols, molecular weights, densities and specific gravities of the common mine gases have been given in another place. The properties and behavior of these gases in the mine will be treated here from a practical, rather than a theoretical standpoint.

METHANE

This gas is commonly known as "marsh gas" or "light carbureted hydrogen," it being the lightest of the hydrocarbon gases. It is a colorless, odorless and tasteless gas. It is combustible, burning with a pale-blue flame, in the air or in oxygen. It contains no oxygen and is not, therefore, a supporter of combustion, in the generally accepted meaning of the term. A lamp flame is quickly extinguished by this gas unmixed with air. Mixed with air in certain proportions, the gas becomes explosive, the mixture being known as "firedamp." Marsh gas is not poisonous, but when unmixed with air suffocates by excluding oxygen from the lungs. The diluted gas can be breathed for a long time with no ill effects, except a slight dizziness, which quickly passes away on return to fresh air.

Marsh gas is the most common of the occluded gases of the coal formations. It seldom, if ever, occurs pure, but is mixed in varying proportions with other hydrocarbons (olefiant gas and ethane) and often with nitrogen. These mixed gases greatly modify the character and properties of the pure gas.

Marsh gas issues from the strata into the mine workings where it accumulates in quantity, unless removed by a copious air current. The most gaseous seams are those that are overlaid with a compact rock, slate, or shale that is impervious to gas and not traversed by faults, which would allow the gas to escape. Gas is generated most freely from a virgin seam and from a freshly exposed face of coal. Hence, new workings generate more gas than old workings; because, in the old workings, the gas has mostly drained from the strata and escaped.

Marsh gas diffuses rapidly into the air and other gases, the rate of diffusion depending on the relative densities of the two mediums. The question is often asked, if the diffusion of gas is so rapid how is it possible for a large body of gas to accumulate in a void place in the mine. The reason is that diffusion only takes place at the surface of contact, and is therefore limited, and the gas is being generated faster than it passes away.

Marsh gas being lighter than air tends to accumulate at the roof and at the head of steep pitches and in rise workings. It is found in such places where the air current is not sufficiently strong to sweep away the gas and in other poorly ventilated or abandoned places. Gas can generally be found at the roof or close to the face of the coal in chambers generating gas. It is detected by observing the flame of a safety lamp. If gas is present in sufficient quantity in the air a faint nonluminous cap will appear surmounting the flame of the lamp. The gas also lengthens and enlarges the flame.

FIREDAMP

All gases were formerly known to the miner as "damps," which is a word of Dutch or German origin meaning vapor or fumes. Later, as the characters of the different gases became known, they were named according to their several characteristics. The term "firedamp" was applied to any inflammable or explosive mixture of gas and air.

The word firedamp, today, in this country, means any inflammable or explosive mixture of marsh gas and air, with or without other gases. In England, the word is taken to mean any mixture of marsh gas and air without regard to whether or not the mixture was inflammable or explosive, which, however, is not its logical meaning.

When but a small amount of marsh gas is mixed with pure air the gas is so diluted that the mixture is not inflammable. In contact with flame, this small percentage of gas in the air adds to the combustion and lengthens and enlarges the flame; but the flame is not propagated throughout the mixture, as the absorption of the heat by the air is too great to maintain the temperature necessary for combustion.

Lower Inflammable Limit.—As more gas is added to the air, a point is soon reached where the combustion of the gas develops sufficient heat to raise the temperature of the air to that required to maintain the combustion. When this point is reached the flame causing the ignition is extended or propagated through the mixture. In other words, the mixture becomes inflammable, because the combustion is supported in the mixture independent of any other source. The theoretical percentage of gas in the firedamp at this point, as calculated, is slightly above 2 per cent., for dry air or saturated air. The heat absorbed by the water of saturation is so slight in comparison that it can be ignored without appreciable error. There are heat losses, however, that cannot be calculated, which fact raises the lower inflammable limit of pure marsh gas to between 4 and 5 per cent.

Effect of Dust and Other Gases.—Owing to the fact that marsh gas is rarely, if ever, found pure, but is generally mixed with dust or other gases or both, it is never safe to work with open lights, in air containing more than 1 per cent. of gas, in bituminous mines; or 2½ per cent. in anthracite mines.

Gases are divided into two general classes, in respect to the effect they produce on the inflammability of firedamp. Gases having a lower ignition point than marsh gas, as for example, carbon monoxide, hydrogen sulphide, ethane and olefiant gas, lower the inflammable limit of firedamp, as given above. Fine coal dust floating in the mine air has a similar effect, in proportion as the dust is highly inflammable. On the other hand, extinctive gases such as nitrogen and carbon dioxide raise the limit given above.

In the working of bituminous mines, coal dust is a most dangerous factor, especially when the coal is highly inflammable. In many cases, the finely divided dust produces an explosive atmosphere even when no gas is present. The presence of such dust in the mine air, acted on by the flame of a blownout shot, is certain to cause trouble.

To Calculate the Lower Inflammable Limit.—In order to calculate the proportion of gas (methane) and air when the firedamp mixture first becomes inflammable, it must be assumed that all the heat generated by the combustion of the gas is absorbed by the products of the combustion and the remaining unburned air. Owing, however, to there being a certain amount of heat lost by radiation or otherwise that cannot be estimated or accounted for, the calculated inflammable limit will only approach the actual, to the extent that the conditions are fully realized in the calculation. The process is as follows:

The weight of oxygen necessary to burn 1 lb. of methane or marsh gas (CH_4) is shown by the relative weights of these gases in the following reaction:

	CH_4	+ $2O_2$	= CO_2	+ $2H_2O$
Molecular weights.......	16	64	44	36
Relative weights.........	1	4	2¾	2¼

But oxygen forms 23 per cent., by weight, of the air, the remaining 77 per cent. being practically all nitrogen. The weight of nitrogen concerned in burning 1 lb. of this gas in air is then calculated as follows:

$$23 : 77 :: 4 : N$$

and

$$N = \frac{77 \times 4}{23} = 13.39 \ lb.$$

The table giving the heats of combustion of different substances (p. 66) shows that methane, burned in air or oxygen, gives out 23,513 heat units (B.t.u.). The temperature of ignition of this gas is 1200° F.

Now, since the specific heat of a substance is the heat (B.t.u.) absorbed by 1 lb. of that substance, during a rise of 1 deg. F. in its temperature, the heat absorbed by the products of combustion of 1 lb. methane, for each degree rise in temperature, is found by multiplying the specific heat of each of the products, including the nitrogen of the air, by the relative weight of each product, respectively. The total heat is then found by multiplying that result by the number of degrees rise in temperature; and adding the latent heat in the steam or water vapor, as follows:

The specific heats of the several products of combustion, referred to water as unity (1), are carbon dioxide, 0.2163; nitrogen, 0.2438; water vapor, 0.4805; and air, 0.2374. The latent heat of the water vapor (steam) or the heat absorbed when 1 lb. water becomes steam at 212°F. is 970.4 B.t.u. The heat absorbed by the products of combustion, for a rise of 1200 − 32 = 1168°F., is therefore

Carbon dioxide,	$0.2163 \times 2.75 \times 1168 =$	694.7264	
Nitrogen,	$0.2438 \times 13.39 \times 1168 =$	3812.9360	4507.6624 *B.t.u.*
Water,	$1.0000 \times 2.25 \times 180 =$	405.0000	
Latent heat,	$970.4000 \times 2.25 =$	2183.4000	
Water vapor,	$0.4805 \times 2.25 \times 988 =$	1068.1515	3656.5515 *B.t.u.*
Total heat absorbed by products			8164.2139 *B.t.u.*

Having found the heat absorbed by the products, the next step is to find the heat absorbed by the unburned air. Let x = weight of air required to make 1 lb. of the gas inflammable; and, since 1 lb. CH_4 consumes 4 lb. O + 13.39 lb. N = 17.39 lb. air, the unburned air is $x - 17.39$ lb. The original temperature of the air being 60°F., the rise is 1200 − 60 = 1140 deg. and the heat absorbed is $0.2374(x - 17.39)1140 = 270.636x - 4706.36$ B.t.u., which makes the total heat absorbed

$$8164.2139 + 270.636x - 4706.36 = 270.636x + 3457.8539 \, B.t.u.$$

Since the heat absorbed is assumed equal to the heat generated,

$$270.636x + 3457.8539 = 23{,}513 \, B.t.u.$$

and
$$x = \frac{23{,}513 - 3457.8539}{270.636} = 74.10 \, lb. \, air.$$

This is the total weight of air required to make 1 lb. of methane (CH_4) inflammable. In other words, the weight ratio of gas to air, at the lower inflammable limit, is 1 : 74.10. But since the specific gravity of methane, referred to air as unity, is 0.559, the volume ratio of gas to air, at this point, is 1 : 0.559 × 74.10; or 1 : 41.42. That is to say, a mixture of pure methane and air first becomes inflammable when 1 volume of this gas is mixed with 41.42 volumes of air.

The percentage of gas in this mixture is

$$\frac{1}{1 + 41.42} \times 100 = \frac{100}{42.42} = 2.3 \textit{ per cent.}$$

Lower Explosive Limit.—The continued addition of gas to the air causes the firedamp mixture to become more and more inflammable till a point is reached when the combustion of the gas is so rapid that the mixture is explosive. As this condition is approached, in practice, owing to the mixture of the gas and air not being uniform, the ignited gas often snaps and cracks in the combustion chamber of a safety lamp.

In the same manner, an accumulation of firedamp, in the mine, when ignited, may burn with greater or less energy or violence and small explosions may occur here and there, followed perhaps by the general explosion of the entire body of the firedamp. The explosion depends not alone on the proportion of gas and air in the mixture, although that is important, but on the intensity and volume of the igniting flame. Thus, it happens that a firedamp mixture ignited in the narrow confines of the mine workings may, after burning for a brief period with more or less energy, suddenly develop a violent explosion.

The lower explosive limit of pure methane has been determined, by experiment, to occur when 1 volume of the gas is mixed with 13 volumes of air; or the percentage of gas in the mixture is

$$\frac{1}{1 + 13} \times 100 = \frac{100}{14} = 7.14 \textit{ per cent.}$$

This limit, however, is considerably modified by any conditions that tend to increase or decrease the amount of heat developed.

Maximum Explosive Point.—The maximum explosive force of a combustible gas is developed when the proportion of gas to air is just sufficient for complete combustion. If the gas in the mixture is in excess of this proportion the full heat energy is not developed, owing to the incomplete combustion of the gas. On the other hand, if the air is in excess of what is required for complete combustion, the unburned air absorbs a portion of the heat generated by the combustion, which thus becomes latent.

The maximum explosive force of methane is developed when the proportion of gas to air is 1 : 9.57. It is calculated in the following manner: Write, again, the chemical equation expressing the reaction that takes place when this gas burns in oxygen, forming carbon dioxide and water; thus,

	CH_4	+ $2O_2$	= CO_2	+ $2H_2O$
Molecular volumes,	1	2	1	2

It should be observed that when the symbol of each gas is written as a molecule (oxygen = O_2) the prefix or number written before the symbol, indicating the number of molecules of that gas taken, shows also the relative volume of the gas concerned in the reaction; because the volume of all gaseous molecules at the same temperature and pressure is the same.

The above equation shows that two volumes of oxygen ($2O_2$) are required to completely burn one volume of methane (CH_4); and there are formed one volume of carbon dioxide (CO_2) and two volumes of water ($2H_2O$).

But, oxygen forms 20.9 per cent., by volume, of the atmosphere. Therefore, when methane is burned in air, the volume of air required to completely burn two volumes of the gas is

$$\frac{2\ \textit{volumes}}{0.209} = 9.569, \text{ say } 9.57\ \textit{vol.}$$

Hence the proportion of gas to air that will develop, in explosion, the maximum force is 1 : 9.57. The percentage of gas in the mixture, at this point, is

$$\frac{1}{1 + 9.57} \times 100 = \frac{100}{10.57} = 9.46\ \textit{per cent.}$$

Higher Explosive Limit.—The continued addition of gas after the maximum explosive point is reached, causes the explosion of the firedamp mixture to be less and less violent, till a point is finally reached where the proportion of air is so reduced that explosion ceases and the mixture becomes simply inflammable.

The point at which explosion ceases is called the "higher explosive limit." For pure methane, this point is practically reached when the proportion of gas to air is 1 : 5, although the position and character of the igniting flame, may vary this proportion slightly. The percentage of gas in the firedamp, at this point, is practically

$$\frac{1}{1+5} \times 100 = \frac{100}{6} = 16.67 \textit{ per cent.}$$

Higher Inflammable Limit.—By the continued addition of gas, the firedamp having ceased to be explosive, now becomes less and less inflammable. The mixture not only ignites less readily, but when ignited burns less regularly and quietly than did the same firedamp mixture, in the lower inflammable stage when less gas and more air were present.

The higher inflammable stage of the gas is more dangerous, in mining practice, than the lower inflammable stage of the same gas, because the slightest addition of air, which is liable to occur at any moment in the mine, causes the mixture to approach the maximum explosive point. The addition of air to firedamp in the lower explosive or inflammable stages makes the mixture less explosive or inflammable.

Another important distinction between the lower and higher stages of firedamp mixtures is the relative ease with which the flame cap may be detected in the two stages. While the flame of a safety lamp burns steadily and yields a good cap that is easily detected, in the lower inflammable stage; the lamp flame is unsteady and the flame cap generally hard to discern in the higher inflammable stage. The reason is probably to be found in the uncertain and varying amount of air in the mixture feeding the flame, which makes the gas continually approach the explosive point The gas in this (higher) stage is said to be "sharp."

The following table will make the several stages of firedamp more clear; but it must be remembered the proportions of gas to air and percentages of gas given as marking the dividing line between the different stages or the inflammable and explosive limits are only suggestive and vary with the degree of purity of the gas; the volume, intensity and position of the igniting flame, and the pressure and temperature of the surrounding atmosphere.

FIREDAMP MIXTURES (METHANE AND AIR)

Lower inflammable stage	Explosive stages			Higher inflammable stage
	Lower stage	Maximum point	Higher stage	
		Proportion of Gas to Air		
1 : 40	1 : 13	1 : 9.57	1 : 5	1 : 2.4
		Percentage of Gas		
2.5%	7.14%	9.46%	16.67%	29.5%

The continued addition of gas thus renders the firedamp extinctive of its own flame and therefore noninflammable. The proportions and percentages given in the table denote more or less closely the limits of the several stages.

Flashdamp.—This is a mixture composed almost wholly of marsh gas (CH_4) and carbon dioxide (CO_2), mixed in the proportion in which these gases diffuse into each. It is formed under special conditions, in mines, where carbon dioxide from the old workings of an abandoned seam becomes mixed with the undiluted marsh gas generated in the strata. The mixture is lighter than air and possesses the peculiar and misleading property of extinguishing the lamp at the roof of the seam or the face of a steep pitch.

Calculation of Composition of Flashdamp.—According to the law of diffusion, gases diffuse into each other in the inverse ratio of the square roots of their densities or specific gravities. For example, the specific gravities of methane and carbon dioxide are 0.559 and 1.529, respectively; and the ratio of the velocities of diffusion of these two gases into each other is then the inverse ratio of the square roots of these numbers.

$$\frac{CH_4}{CO_2} = \frac{\sqrt{1.529}}{\sqrt{0.559}} = \frac{1.236}{0.747} = 1.65$$

which can be written 1.65 : 1 ; or 1650 : 1000. This ratio shows that when these gases diffuse into each other, directly, before dilution with air takes place, the mixture will contain 1650 volumes of methane for each 1000 volumes of carbon dioxide. The same result is obtained by stating the law thus: The ratio of diffusion is equal to the square root of the inverse ratio of the densities or specific gravities of the gases; or, as follows:

$$\frac{CH_4}{CO_2} = \sqrt{\frac{1.529}{0.559}} = \sqrt{2.735} = 1.653$$

A slightly different, though theoretically more correct result is obtained when the calculation is based on the densities of these gases, referred to hydrogen as unity (1). The process is as follows:

Methane (CH_4):

$$\begin{aligned} C &= 1 \times 12 = 12 \\ H_4 &= 4 \times 1 = \underline{\ 4\ } \end{aligned}$$

Molecular wt. = 16; density, 16 ÷ 2 = 8

Carbon dioxide (CO_2):

$$\begin{aligned} C &= 1 \times 12 = 12 \\ O_2 &= 2 \times 16 = \underline{32} \end{aligned}$$

Molecular wt. = 44; density, 44 ÷ 2 = 22

The ratio of diffusion is then equal to the square root of the inverse ratio of these densities; or

$$\frac{CH_4}{CO_2} = \sqrt{\frac{22}{8}} = \sqrt{2.75} = 1.658$$

Calculation of Percentage Composition, by Volume.—The mixture is estimated to contain

Methane (CH_4)	1658 volumes;
Carbon dioxide (CO_2)	1000 volumes;
Total	2658 volumes.

Percentage, by volume,

Methane, $\frac{1658 \times 100}{2658} = 62.38$ *per cent.*

Carbon dioxide, $\frac{1000 \times 100}{2658} = 37.62$ *per cent.*

100.00 *per cent.*

CARBON MONOXIDE

This gas, formerly known in mining textbooks as "carbonic oxide," or "whitedamp," is the product of the combustion of carbon in a limited supply of pure air. Because the supply of oxygen is limited the combustion of the carbon is incomplete and the **monoxide** is formed instead of the **dioxide.**

Carbon monoxide is a **colorless** gas. It is extremely **poisonous,** owing to its being absorbed very rapidly by the hæmoglobin or red coloring matter of the blood, from which it is separated slowly and with difficulty. The effect on the system is therefore **cumulative** when exposed to the smallest percentage of this gas in the atmosphere breathed. The affinity of carbon monoxide for the hæmoglobin is from 250 to 400 times as great as that of oxygen, so that the blood corpuscles are quickly rendered inert and **death** is the sure result. The gas is not displaced by the oxygen administered in treatment, but is eliminated slowly by natural processes that take place in the system, unless the latter is too weak or the percentage of the gas absorbed is too great for such result to take place.

The **treatment** for carbon-monoxide poisoning is the enforced inhalation of pure oxygen, by the use of the **pulmotor.** This is a device that consists essentially of a small portable tank containing compressed oxygen, which is pumped into the lungs by a bellows, while another belows withdraws the same from the lungs after use. The pressure of the gas in the oxygen tank automatically operates the bellows at a rate of 16 strokes per minute as in normal breathing. A face mask completes the equipment. It is important to draw the tongue forward with tongs provided for that purpose, and to close the gullet leading to the stomach, by a gentle pressure of the thumb on the throat, in order to avoid the gas filling the stomach.

The presence of the smallest percentage of carbon monoxide in the atmosphere breathed is **dangerous** to health and life because of its cumulative tendency, its possible toxic effect on the nervous system and the impairment of the vital organs of the body. The **fatal percentage** of this gas cannot be definitely stated because of numerous other factors that together determine a fatal effect. The more important of these are the following: The depletion of the oxygen of the air breathed; the length of the time of exposure to the poisonous atmosphere; the energy expended in physical work in such atmosphere; the state of health and the normal physical condition of the person.

Some persons are more sensitive to gas poisoning than others, owing to a less vigorous constitution, a temporarily weakened condition, a more nervous temperament, or previous exposure to gas poisoning, the baneful effects being hard to eradicate from the system. For these reasons, what would prove a fatal percentage in some instances of less purity of atmosphere, longer exposure, more difficult work, or physical ailment of any nature, would not necessarily produce fatal results under better conditions and more robust health of the individual exposed to the gas.

Relative Rate of Absorption by Blood.—The experiments of Dr. J. S. Haldane and others have shown that 0.02 per cent. of carbon monoxide in otherwise pure air produces about 20 per cent. of saturation in a brief period of time (20 min.?). Since pure air contains 20.9 per cent. of oxygen, the ratio of carbon monoxide to oxygen, in the air breathed, is 2:2090, or 1:1045. But the ratio of absorption, carbon monoxide to oxygen, in this case, is 20:80, or 1:4, the blood showing only 20 per cent. carbon monoxide and 80 per cent. oxygen. Hence, the relative **rate of absorption** by the blood, carbon monoxide to oxygen, is about 260:1, since $1045 \div 4 =$ say 260. In other words, the blood in this experiment absorbed carbon monoxide about 260 times as rapidly as it absorbed oxygen, under the same conditions.

Another **experiment** showed 50 per cent. saturation in the blood when the air breathed contained 0.08 per cent. of carbon monoxide. In this case, the ratio of carbon monoxide to oxygen in the air breathed is 8:2090, or 1:260. But the corresponding ratio of absorption is 1:1, the blood showing 50 per cent. of saturation, or equal quantities of these two gases. Hence, in this case also, the relative **rate of absorption** of carbon monoxide and oxygen is the same as before, namely, 260 : 1.

Another **experiment** showed 50 per cent. saturation in the blood when the air breathed contained 0.05 per cent. of carbon monoxide. Here the ratio of carbon monoxide to oxygen in the air breathed being 5 : 2090, or 1 : 418, and the ratio of absorption, as before, 1 : 1, the relative **rate of absorption** is 418 : 1, showing that the blood absorbed carbon monoxide, in this case, about 400 times as rapidly as it absorbed oxygen, under like conditions, in the two previous experiments.

The experiments suggest not only the variation in the rapidity of the absorption of carbon monoxide by the blood of different individuals, with varying constitutions and degrees of health; but show clearly the great affinity of the hæmoglobin of the blood for carbon monoxide as compared with oxygen. These facts demonstrate forcibly the danger of working in a mine atmosphere containing the smallest possible percentage of this gas even when the worker is in robust health.

Production of Carbon Monoxide in Mines.—Carbon monoxide does not occur naturally in mines, but may be and often is produced in dangerous quantities under the practically unavoidable conditions and occurrences incident to coal mining.

This gas is produced in considerable quantities by any combustion, on a large scale, commonly occurring in the limited confines of mine workings. Examples of this are mine fires and explosions of gas or dust. This gas is also produced by the explosion of powder in blasting. It is produced in dangerous quantities by the slow combustion of fine coal and slack thrown in the waste, in poorly ventilated places and abandoned areas void of circulation. Carbon monoxide is the deadly component of afterdamp, which renders the latter so quickly fatal to life, as shown by the fatal results that follow many mine explosions.

Detection of Carbon Monoxide in Mines.—There is no reliable **flame test** for the detection of carbon monoxide as it occurs in mines. The lamp flame is, no doubt, lengthened when fed with air containing the gas, but this effect is imperceptible in a percentage that would be **fatal to life.**

The **lengthening of the flame** is plainly noticeable when the fine dust of an inflammable coal is suspended in considerable quantity in the still air of a mine entry or chamber. This is the result of the increased combustion owing to the dust-laden air feeding the flame. It is possible that a barely perceptible cap may be discerned at times under particularly favorable conditions. This, however, would be a **dust cap** and would not indicate the presence of the gas.

What is known as the **"blood test"** will reveal the presence of very small percentages (0.01 per cent., Haldane*) in the air. The delicacy of this test, however, is greatly impaired by the difficulty of correctly judging of the change in the color of the blood solution employed in making the test. The difficulty is increased by the dim, artificial light of the mine and the impaired eyesight and possible partial color-blindness of the observer. The blood test also requires time and care in its making, which together with the necessary apparatus do not recommend its use in the mine.

The **experiments** of Dr. J. S. Haldane† to ascertain the extent to which animal life is affected by the presence of carbon monoxide in the atmosphere breathed into the lungs led him, first, to suggest the use of small animals as a plainly visible and thoroughly reliable index of the presence of gas in quantity dangerous to human life. Dr. Haldane observed that **mice and small birds,** preferably canaries, were prostrated by the gas in a much briefer period than is required to produce the same effect on a man.

Exposed to an atmosphere containing 0.1 per cent. of carbon monoxide, a mouse became giddy in 12 min., while a man experienced a like effect only after breathing the same atmosphere for a period of two hours. Again, three small

*Trans. I. M. E., Vol. 38, p. 275.

†Trans. I. M. E., Vol. 38, pp. 267–280.

mice and a canary were exposed to an atmosphere containing 0.6 per cent. of this gas. In 4 min. the canary fell from its perch and died, and the mice became helpless, but recovered quickly in fresh air. A man continued to breathe the same atmosphere and, at the expiration of 10 min., was unaffected, a test of his blood showing but one-fourth saturation.

Dr. Haldane's **conclusions,** based on his experiments, are briefly as follows:

1. Noticeable symptoms are never produced by less than about 0.02 per cent. of carbon monoxide in otherwise pure air.

2. The poisonous effect is decreased somewhat by a moderate addition of carbon dioxide; but increased by depletion of the oxygen of the air.

3. Small animals recover quickly and do not exhibit the after effects of the poisoning so often fatal to man.

4. The analyses of the blood of victims of the afterdamp of mine explosions usually show 80 per cent. saturation.

A series of **experiments** made at the Pittsburgh testing station to determine the effect of **repeated exposure** of mice and canaries corroborates the conclusion of Dr. Haldane in respect to the complete rapid recovery of these small animals from the effects of carbon-monoxide poisoning.

As previously explained, men who have been once overcome by this gas are more sensitive to its effects again. This is not the case, however, with mice and birds, which fact makes them the more useful in mining practice. A bird or a mouse that has been exposed to the gas and overcome a great number of times shows no more sensitiveness to its poisonous effects than one never poisoned by the gas.

Following is the record of eight exposures of a canary to an atmosphere containing 0.25 per cent. carbon monoxide, as given on p. 8, Technical Paper 62, of the U. S. Bureau of Mines, each exposure, except the last, being made immediately upon the recovery of the bird from the previous one. The table shows the time, in minutes intervening between the moment of exposure, first signs of distress, collapse of the bird and recovery in fresh air.

Table 1.—Effect of Repeated Exposure on Canary

No. of exposures	Time in minutes		
	Distress	Collapse	Recovery
1	3	1	7
2	3	1	8
3	1	3	8
4	2	3	7
5	2	2	7
6	2	2	7
7	2	1	12
	(a 2-min. interval)		
8	1	1	8

The above record shows earlier signs of distress after the first two exposures. This may naturally be attributed to the alarm and expectancy of the bird arising from its previous experience; but the total interval to collapse was uniform (4 min.), except in the fourth and the two last exposures, which were 5, 3 and 2 min., respectively.

The following table shows the same data recorded in four successive exposures of a mouse to a 0.3-per cent. mixture of carbon monoxide and pure air:

Table 2.—Effect of Repeated Exposure on Mouse

No. of exposure	Time in minutes		
	Distress	Collapse	Recovery
1	3	6	17
2	3	10	23
3	4	12	34
4	3	14	not given

A similar series of experiments, performed by exposing a canary at irregular intervals and on different days to atmospheres containing from 0.18 to 0.24 per cent. of carbon monoxide and numbering 14 exposures in all, extending over a period of nine days, showed practically the same results.

CARBON DIOXIDE

This gas, often called "carbonic acid gas" or "chokedamp" is a colorless and odorless gas, having a distinctly acid taste. It is not combustible and will not support combustion in any ordinary form.

How Produced.—Carbon dioxide is the product of the complete combustion of carbon or carbonaceous matter in a plentiful supply of air or oxygen. It is produced, in mines, by the breathing of men and animals; burning of lamps; explosion of powder slow combustion of fine coal and slack in the gob; and other forms of combustion taking place.

Effect on Flame.—Carbon dioxide has a similar effect on flame to that caused by an exces of nitrogen; or, what is the same thing, a dep etion of oxygen in the air. The presence of carbon dioxide in the air tends to reduce the activity of combustion. It dims the flame of a lamp and extinguishes it when present in sufficient quantity.

The percentage of carbon dioxide that will extinguish flame depends on both the **nature of the flame** and the **amount of oxygen** in the air feeding the flame. A **gas-fed flame,** as the hydrogen flame of the Clowes lamp, or the acetylene flame of a carbide lamp, is less susceptible to extinction from this cause than is an **oil-fed flame.**

The flame of a lamp burning sperm or cottonseed oil is extinguished in an **artificial atmosphere** (which is the usual condition in a mine) containing 14 per cent. of carbon dioxide. But, in a **residual atmosphere** formed by allowing the lamp to burn in a closed place till extinguished, only 3 per cent. of carbon dioxide is required for extinction of the flame.

Effect on Life.—Carbon dioxide is not classed as one of the poisonous mine gases, although it exerts a **toxic effect** on the human system. It is irrespirable when unmixed with air and if breathed produces death by suffocation. In smaller quantities, it causes headache, nausea and pains in the back and limbs.

According to Dr. Haldane, no appreciable effect is produced by breathing air containing carbon dioxide, until there is about 3 per cent. of this gas present. Breathing then becomes slightly more difficult; 5 or 6 per cent. of the gas causes decided **panting;** and 18 per cent. **suffocation and death.** The effect of the gas is much increased if the oxygen content of the air is below the normal.

For example, with 18 per cent. carbon dioxide present, there is 0.209 (100 − 18) = 17.14 per cent. oxygen and 0.791 (100 − 18) = 64.86 per cent. nitrogen, under normal conditions. This is a **fatal atmosphere.**

But, if the oxygen of the air has been depleted so that the ratio, oxygen : nitrogen, is less than 20.9 : 79.1; then a less percentage of carbon dioxide than that named above (18%) would be **fatal to life.**

Treatment when Overcome.—Remove promptly to fresh air; apply alternately cold and lukewarm bandages to the chest; rub the limbs and body briskly to start circulation; and, if necessary, use artificial respiration. When consciousness is restored put the patient to bed and keep him quiet for several days.

BLACKDAMP

It is a common mistake, in mining practice, to regard carbon dioxide as another name for "blackdamp," which is found in such quantities in many poorly ventilated mines. Carbon dioxide is one constituent only of blackdamp.

The term **blackdamp** describes a variable mixture of air deficient in oxygen, and carbon dioxide. It consists therefore of carbon dioxide, nitrogen and oxygen, in varying quantities. The percentage of **oxygen** in the mixture will determine its respirable quality. The **nitrogen** is wholly inert and acts only to dilute the mixture and thus reduce the percentage of oxygen present. The **carbon dioxide** not only dilutes the mixture but produces also a toxic effect on the human system, although this effect is not of such a nature as to class carbon dioxide as a poisonous gas.

The **production of blackdamp** in coal mines is due to two chief causes: 1. The absorption of the oxygen of the air by the coal. 2. The generation of carbon dioxide by the various forms of combustion or oxidation continually taking place in the workings of the mine.

The **absorption of oxygen** from the mine air by the freshly exposed surfaces of coal is more rapid than what is generally supposed. Experiment has shown that a certain freshly mined bituminous coal absorbed from one-eighth to one-seventh of its volume of oxygen from the surrounding air, in 24 hr.; while only about one-tenth of this oxygen was converted into carbon dioxide. It is suggested that the remaining nine-tenths of the oxygen absorbed unites chemically with certain unsaturated hydrocarbons in the coal.

The **effect** of this rapid absorption of oxygen, in the still air of badly ventilated places, in coal mines, as can be readily imagined, is to **deplete the oxygen** content of the air. This is especially the case where tons of coal are shot down at night and left to be loaded out the following day and the ventilation during the night is much diminished in the mine.

On the other hand, where the ventilation is adequate and there is still blackdamp produced in quantity, it is the result of the generation of carbon dioxide from some cause, generally a mine fire or the slow combustion of fine coal.

AFTERDAMP

The term "afterdamp," as the word implies, is used to describe the variable mixture of noxious gases that remains after any explosion of gas, dust or powder in a mine.

Composition.—It is impossible to give the composition of afterdamp, except in the most general way; because the gases formed depend on so many **varying conditions,** in respect to the character of the gas or dust burned; the relative volume of available oxygen; the size of the workings where the explosion takes place, as determining the temperature and pressure developed; and the condition of the mine with respect to gas, dust and moisture.

Afterdamp may contain variable quantities of nitrogen, carbon dioxide, carbon monoxide, water vapor and, at times, lesser amounts of nitrous oxide gas and possibly some unburned methane. The mixture is extremely dangerous, being fatal to life and often highly explosive.

INFLAMMABLE AND EXPLOSIVE MINE GASES

The presence of combustible gases in the atmosphere of a mine is always an element of danger for three principal reasons. 1. The percentage of gas in the mine air may be sufficient to form an explosive mixture known as **firedamp.** 2. The temperature of ignition of most of these gases is lower than that of methane, which is usually the chief constituent of firedamp, and the latter is rendered **more readily ignitable** by reason of their presence. 3. The presence of the smallest percentage of a combustible gas assists to that extent the ignition of a **dust-laden atmosphere,** and increases the violence of its explosion when ignited.

The Inflammable Gases.—The inflammable or combustible mine gases, in the order of their importance, are methane (CH_4), carbon monoxide (CO), ethane (C_2H_6), ethene or olefiant gas (C_2H_4), hydrogen (H_2) and hydrogen sulphide (H_2S). Each of these gases is not only combustible but forms an **explosive mixture** when mixed with air in certain proportions.

Inflammable Range of Gases.—The combustion of an inflammable gas, under mining conditions, requires the presence of air or available oxygen. The relative proportion of air and gas in the mixture determines the character and completeness of the combustion and the range of inflammability of the gas.

The maintenance of flame throughout a gaseous mixture requires that the **heat of combination** between the combustible and the atmosphere supporting the combustion shall be equal to that lost by **radiation, conduction** and **absorption** by the air and gaseous products formed. Two conditions are possible.

1. The proportion of gas to air may be such as to give a low **rate of combination** and a correspondingly small generation of heat, which is insufficient to raise the adjacent gaseous molecules to an equal temperature, resulting in a still lower rate of combination and a lesser generation of heat as the action proceeds through the mass till it finally ceases.

2. Again, the proportion of air to gas may be such as to cause an **absorption of heat** greater than that generated when the condition will likewise be a falling one and there can result no general extension of flame throughout the mass.

The first of these two conditions (excess of gas) determines the **higher inflammable limit** of the gas, while the second condition mentioned (excess of air) marks the **lower inflammable limit.** Beyond these two limits the gaseous mixture is not inflammable. In mining practice, mixtures above the higher limit are more dangerous than those below the lower limit, as more air will make them explosive.

Explosive Range of Gases.—A combustible gas is always inflammable in proportions of gas to air outside of the explosive range of the gas. In other words, the **range of inflammability** is wider than and embraces the **range of explosibility.** The same principles, however, apply in respect to each of these conditions.

The **degree of explosiveness** of a gaseous mixture is increased as the rate of combination is more rapid and the loss of heat less; or decreased as the rate of combining is slower and the loss of heat greater.

Maximum Explosive Point.—It is quite generally assumed that the maximum explosive force of a gas is developed when the proportion of air or oxygen is just sufficient for the **complete combustion** of the gas. While this is sufficiently close for all practical purposes, it is stated (Emich) that the explosibility is not necessarily greatest at this point.

Inflammable and Explosive Limits.—The following table gives the lower and higher inflammable and explosive limits and the maximum explosive point of the three most important combustible mine gases, except only the higher inflammable limit of carbon monoxide, which has not been determined, but is probably about 80 per cent. The table shows the **percentage of gas** present in the mixture, at each of the five stages given. The lower inflammable limit and the maximum explosive point have been calculated for each of these gases, while the other data are the results of experiment. A normal condition of the air is assumed:

TABLE GIVING THE INFLAMMABLE AND EXPLOSIVE LIMITS AND THE MAXIMUM EXPLOSIVE POINT OF METHANE, HYDROGEN AND CARBON MONOXIDE

Gas	Lower inflam. limit	Lower explo. limit	Maximum explo. point	Higher explo. limit	Higher inflam. limit
Methane.............	4.5	7.1	9.5	16.7	29.5
Carbon monoxide.....	8.4	16.5	29.5	75.0	
Hydrogen............	5.0	9.5	29.5	66.3	72.0

The same data, in reference to olefiant gas (ethene or ethylene), C_2H_4, are: Lower explosive limit, 4.0 per cent.; maximum explosive point, 6.5 per cent.; and higher explosive limit, 22 per cent. These, however, have only a relative importance in respect to mining, because the percentage of this gas present in mines is very small.

Peculiarities of Explosion.—A peculiarity in the explosion of a mixture of **methane and air** is that, at the temperature of ignition (1200°F.), about 10 sec. are required before the gas will ignite (Mallard and Le Chatelier), while both hydrogen and carbon monoxide ignite at once, upon contact with the flame. The **time required** for the ignition of methane grows rapidly less as the temperature is increased.

The same authorities also claim that mixtures of methane and air in any proportion are explosive at **high temperatures,** and the same effect has been observed at **high pressures.** In other words, an increase of temperature or pressure has the effect to widen the explosive range of a gas

A mixture of **carbon monoxide and air** will not explode in the **absence of moisture.** The explosion, in this case, seems to require two stages, the carbon monoxide taking the oxygen from the water, which is replaced immediately by the oxygen of the air, as represented by the following equations:

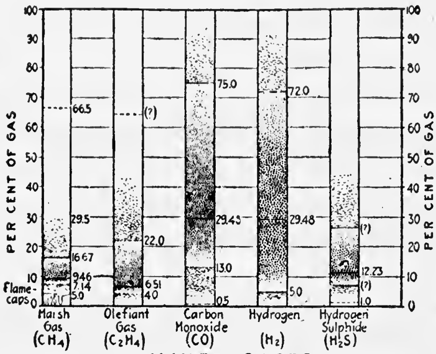
$$CO + H_2O = CO_2 + H_2$$

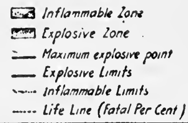
and

$$2H_2 + O_2 = 2H_2O$$

It has been argued that, since carbon monoxide, which is distilled from coal dust floating in the mine air, is not explosive in dry air, the safest condition is a dry mine atmosphere, which, however, is practically impossible.

Explosive Mine Gases.—The diagram, Fig. 13, given below combines, in a compact form, most of the important reactions and data, relating to the combustion and explosion of those mine gases that form explosive mixtures with air. In the upper left-hand corner is a graphic illustration of the relative extent





SPECIFIC HEATS OF MINE GASES AND VAPORS REFERRED TO WATER AS UNITY		
GAS OR VAPOR	EQUAL WEIGHTS	
	CONSTANT VOLUME	CONSTANT PRESSURE
AIR	0.1689	0.2374
METHANE	0.4219	0.5929
OLEFIANT GAS	0.2875	0.4040
CARBON MONOXIDE	0.1743	0.2450
CARBON DIOXIDE	0.1539	0.2163
HYDROGEN SULPHIDE	0.1731	0.2432
HYDROGEN	2.4260	3.4090
OXYGEN	0.1548	0.2175
NITROGEN	0.1735	0.2438
WATER VAPOR	0.3419	0.4805

MINE GASES

GAS	SYMBOL	MOL. WEIGHT	DENSITY H=1	SP. GRAVITY AIR=1	WEIGHT LB. PER CU. FT. 60° F. BAR. 30"	VOLUME CU. FT. PER POUND	EQUATION SHOWING COMBUSTION OF GAS IN OXYGEN	HEAT OF COMBUSTION IN OXYGEN B.T.U. PER POUND
METHANE LIGHT CARBURETED HYDROGEN MARSH GAS	CH_4	16	8	0.559	0.0428	23.36	REACTION $CH_4 + 2O_2 = CO_2 + 2H_2O$ MOLECULAR WEIGHT 16 64 44 36 RELATIVE WEIGHT 1 4 $\frac{11}{4}$ $\frac{9}{4}$ RELATIVE VOLUME 1 2 1 2	BURNED TO CO_2 AND H_2O AT 32° F. 23,513
ETHENE ETHYLENE OLEFIANT GAS	C_2H_4	28	14	0.978	0.0743	13.35	REACTION $C_2H_4 + 3O_2 = 2CO_2 + 2H_2O$ MOLECULAR WEIGHT 28 96 88 36 RELATIVE WEIGHT 7 24 22 9 RELATIVE VOLUME 1 3 2 2	BURNED TO CO_2 AND H_2O AT 32° F. 21.344
CARBON MONOXIDE CARBONIC OXIDE	CO	28	14	0.967	0.0740	13.51	REACTION $2CO + O_2 = 2CO_2$ MOLECULAR WEIGHT 56 32 88 RELATIVE WEIGHT 7 4 11 RELATIVE VOLUME 2 1 2	BURNED TO CO_2 4,325
HYDROGEN	H_2	2	1	0.06926	0.0053	188.68	REACTION $2H_2 + O_2 = 2H_2O$ MOLECULAR WEIGHT 4 32 36 RELATIVE WEIGHT 1 8 9 RELATIVE VOLUME 2 1 2	BURNED TO H_2O AT 32° F 62.032
HYDROGEN SULPHIDE SULPHURETED HYDROGEN	H_2S	34	17	1.912	0.0912	10.96	REACTION $2H_2S + 3O_2 = 2SO_2 + 2H_2O$ MOLECULAR WEIGHT 68 96 128 36 RELATIVE WEIGHT 17 24 32 9 RELATIVE VOLUME 2 3 2 2	BURNED TO SO_2 AND H_2O AT 32° F. 7.239

Fig. 13.

of the explosive and inflammable zones of each of these gases when mixed with air. The horizontal lines, in each gas column, mark, approximately, the maximum explosive point and the lower and upper explosive and inflammable limits; also the fatal percentage is indicated by the dotted lines. These marks are explained by the legend in the upper right-hand corner The specific heats are given for equal weights of the gases, for constant volume and constant pressure, referred to water as unity.

SECTION IV

EXPLOSIONS IN MINES

Definition, Gas Explosion, Dust Explosion—Inflammation of Gas—Nature and Temperature of Flame—Explosion of Gas—Coal Dust, Its Inflammability and Influence, Effect of Stone Dust—Mine Explosion, Development, Causes, Mixed Lights, Electric Mine Lamps, Prevention of Mine Explosions.

Definition.—A mine explosion is understood to be a violent disturbance of the atmosphere within a mine, as manifested by a destructive blast or rush of air accompanied by more or less flame, and is the result of the ignition and combustion with explosive rapidity of gas and dust or either accumulated in the mine.

Gas Explosion.—An explosion produced and maintained chiefly by gas accumulated in the mine workings and passages or mixed with the air current is described as a "gas explosion," although practically every mine explosion involves the combustion of both gas and dust.

Dust Explosion.—An explosion in which the fine coal dust accumulated in the mine or suspended in the air current plays a prominent part is commonly called a "dust explosion," although it may have originated in a local explosion of gas, which is true of most mine explosions.

Few if any mine explosions are wholly due to gas or dust, but combine both of these elements in varying proportions the character of the explosion as "gas" or "dust" being determined by the later evidences.

INFLAMMATION OF GAS

Theory of Inflammation.—The inflammation of a combustible gas involves, at least, two main conditions that are essential to the reaction. They are as follows:

1. The presence of another gas that will **support the combustion** by reason of the different affinities of the elements of the gases that invite dissociation and recombination to form other compounds.

2. A **rise of temperature,** at the point of contact of the two gases, sufficient to start the reaction.

The ignition of a combustible gas in some cases (carbon monoxide) requires, besides the above, the presence of **water vapor.**

Temperature of Ignition.—At the same pressure and under the same conditions of ignition, the temperature at which a given gas inflames or the temperature of ignition for that gas is **fixed.** The following table gives the average temperatures of ignition of the principal mine gases, as determined by experiment:

AVERAGE TEMPERATURES OF IGNITION OF THE COMBUSTIBLE MINE GASES IN NORMAL AIR

Gas	Symbol	Temperature of ignition (deg. F.)
Carbon monoxide	CO	1240
Methane	CH_4	1212
Ethane	C_2H_6	1140
Ethene (olefiant gas)	C_2H_4	1124
Hydrogen	H_2	1077
Acetylene	C_2H_2	970

NATURE AND TEMPERATURE OF FLAME

The Nature of Flame.—Flame, as here considered, is burning gas. It may be **luminous** or **nonluminous,** according to the presence or absence of carbon either free or combined as hydrocarbons. The incandescence of the carbon particles when present renders the flame luminous. This is the case with most oil-fed flames and flames burning in a dusty atmosphere. The flame of hydrogen burning in clear, pure air is practically nonluminous. Methane produces an almost non-luminous flame, but the flame of the heavy hydrocarbon gases is always more or less luminous.

The Temperature of Flame.—The temperature of flame is variable, owing to numerous conditions that affect the combustion of the gas both as to its **rapidity and completeness.** The temperature will vary in different parts of the same flame, because of a variable supply of air that not only affects the combustion of the gas but absorbs much of the heat developed and lowers the temperature of the flame.

Owing to these varying conditions it is clearly impossible to calculate the actual flame temperature of a burning gas. This is often roughly assumed to be about one-half of the **theoretical value** as calculated from the heat of combustion per pound of gas and the heat absorbed by the corresponding products of combustion, for each degree rise in temperature.

It is important not to confuse the **flame temperature** of a combustible gas with its **temperature of ignition,** as they have no connection with each other.

Calculation of the Theoretical Flame Temperature.—The theoretical temperature of the flame of a burning gas is the highest possible temperature that results from its complete combustion, assuming (what is never the case in an open-burning flame) that only sufficient air is present for the complete combustion of the gas.

There is always an **excess of air** in the outer envelope or zone of a flame exposed to the air, and this excess of air beyond what is required for the combustion absorbs heat and lowers the temperature of the flame in the **outer zone.**

The temperature within or in the body of the flame more nearly approaches the **theoretical maximum,** which can be calculated. This maximum temperature is found by dividing the total **heat of combustion** above 32 deg. F., per pound of combustible, less the heat rendered latent in the water vapor produced, by the heat required to raise the temperature of the **products of combustion** one degree. The quotient obtained gives the rise of temperature above 32 deg. F., which must therefore be added in order to find the theoretical temperature of the flame.

Flame Temperature of Methane Burning in Air.—The first portion of the process is similar to that explained in the calculation of the lower inflammable limit of methane and need not be repeated here. It was found that for every pound of methane burned there was produced **carbon dioxide,** 2¾ lb.; **water vapor,** 2¼ lb.; and **nitrogen,** 13.39 lb. So far the two operations are the same. (Page 96.)

As before, one pound of methane, burning to carbon dioxide and water at 32 deg. F., develops 23,513 B.t.u. From this must be subtracted the heat required to convert 2¼ lb. of water at 32 deg. into steam at 212 deg., which is absorbed in the **formation of the water vapor;** thus,

$$23{,}513 - 2\tfrac{1}{4}\,(212 - 32 + 970.4) = 20{,}924.6 \text{ B.t.u.}$$

The result obtained is the **net heat available** for raising the temperature of the products of combustion, which constitute the larger portion of the body of the flame.

It is necessary now to calculate the heat required to **raise the temperature** of the respective weights of the products of combustion **one degree.** The weight of each of these products, as previously given, is multiplied by its specific heat for constant pressure and the sum of these products is the total heat required for each degree of rise in temperature; thus,

	Sp. heat		Weight		B.t.u.
Carbon dioxide	0.2163	×	2.75	=	0.5948
Water vapor	0.4805	×	2.25	=	1.0811
Nitrogen	0.2438	×	13.39	=	3.2645
Heat absorbed, per degree rise...					4.9404

Finally, the **rise of temperature** in the body of the flame that is possible, in this case, assuming that all of the heat developed is absorbed by the products of the combustion only, is as follows:

Rise of temperature, 20,924.6 ÷ 4.9404 = 4235 deg. F.

This rise of temperature, like the heat developed by the combustion, is estimated from 32 deg. F. The theoretical flame temperature is therefore 4235 + 32 = 4267 deg. F.

Flame Temperature of Carbon Monoxide.—The first step in calculating the flame temperature of this gas is to write the chemical equation expressing the reaction that takes place when carbon monoxide burns to carbon dioxide, ignoring for the present the nitrogen in the air; thus,

	$2CO$ +	O_2 =	$2CO_2$
Molecular weights,	56	32 =	88
Relative weights,	1	4/7	11/7

Since oxygen forms 23 per cent. of normal air, by weight, and nitrogen 77 per cent., the ratio of nitrogen to oxygen is 77 : 23, and the relative weight of nitrogen involved here is

$$\frac{4}{7} \times \frac{77}{23} = \frac{44}{23} = 1.91+ \text{ lb.}$$

Hence, for every pound of carbon monoxide burned, there is produced carbon dioxide, 11/7 lb.; and nitrogen, 1.91 lb.

The heat of combustion of carbon monoxide burning to carbon dioxide, as taken from a table giving the heat of combustion of various substances, is 4325 B.t.u. per lb. of gas burned. There being no water vapor formed in this reaction, the above is the actual heat available for raising the temperature of the products of the combustion, which form the body of the flame, disregarding radiation and conduction losses.

Now, calculating, as before, the heat required to raise the temperature of the respective weights of the products of this combustion **one degree,** by multiplying the weight of each product by its specific heat for constant pressure and finding the sum of those products, we have

	Sp. heat	Weight	B.t.u.
Carbon dioxide....................	0.2163 ×	11/7 =	0.3399
Nitrogen.........................	0.2438 ×	1.91 =	0.4657
Heat absorbed, per degree rise....			0.8056

The resulting **rise of temperature** above 32 deg. F., in the body of the flame, which determines the theoretical flame temperature, is then 4325 ÷ 0.8056 = 5369 deg. F. and the corresponding temperature, 5369 + 32 = say 5400 deg. F.

Although the **presence of moisture** (water vapor, H_2O) is necessary to the ignition of carbon monoxide, it is not required to take this into account in making the above calculation, for the reason that the **heat of dissociation** is balanced by the **heat of recombination** in the molecule of water and no loss of heat is assumed to occur. It has been suggested that the water only serves to start the reaction by effecting the ionization of the elements.

The theoretical flame temperature as calculated above, however, both for methane and carbon monoxide, is considerably modified by the **humidity of the air** supporting the combustion.

Volume of Flame.—It is frequently estimated roughly that the volume of a flaming gas is proportional to its absolute temperature. For example, assuming the original temperature of the gas as 0 deg. F., the theoretical flame volumes of methane and carbon monoxide are, respectively,

Methane,	$460 + 4267 \div 460 =$ say, 10	volumes.
Carbon monoxide,	$460 + 5400 \div 460 =$ say, $12\frac{3}{4}$	volumes.

EXPLOSION OF GAS

Influence of Temperature on Explosion.—A rise of the initial temperature of an explosive mixture slightly extends the lower inflammable limit, but has no appreciable effect on the higher limit, owing to the small relative value of the increase as compared with the high temperature developed in the explosion.

Influence of Pressure on Explosion.—Pressure exerted on an explosive mixture increases its density and temperature and renders it more readily ignitable. In other words, an increase of pressure lowers the lower inflammable limit of an explosive gaseous mixture. An increase of pressure, likewise increases the **velocity of propagation** of explosion in the mixture, raises the temperature developed and extends the higher inflammable limit. In other words, an increase of pressure **widens the explosive range** of a combustible gas.

Influence of Relative Humidity on Explosion.—While the presence of moisture (water vapor) in a gaseous mixture is often necessary to secure its explosion, as explained in reference to carbon monoxide, the water vapor absorbs much of the heat and lowers the temperature developed, thereby reducing the **rate of combination** and the **force of the explosion,** except where fine coal dust is suspended in the air, when partial dissociation may take place in the water vapor and result in increasing the energy of the reaction.

Influence of Catalysis to Cause Explosion.—Catalysis is the effect produced by a foreign substance to assist chemical reaction between two other substances, while the substance itself undergoes no change—first discovered by Berzelius. Much difference of opinion exists as to the suggested **catalytic action** of fine incombustible dust suspended in mine air, to assist the explosion of combustible gases. Finely powdered **stone dust** has been shown to retard the ignition of coal dust by mixing with and diluting the latter. This effect, however, is wholly physical and not related to the possible catalytic action referred to by Sir Frederick Abel and others who have studied the subject closely.

Influence of Character of Initial Impulse.—The manner in which the gas is ignited or the character of the initial impulse determines largely the explosion of gaseous mixtures. For example, a firedamp mixture ignited by a **lamp flame** may not explode, while if fired by the flame of a **blownout** or **windy shot,** the greater volume and intensity of the flame may cause an explosion.

The **volume of the flame** is important, because it envelops a larger portion of the gaseous mixture and ignition is thus started generally throughout the mass, causing a greater development of heat and reducing the percentage of loss by radiation, convection and conduction.

The **intensity** of the initial impulse or the higher temperature of the igniting flame will often cause the explosion of a gaseous mixture that would burn quietly if ignited by a less intense source of heat energy. The dissipation of heat is so rapid and general in a burning gas that the transition from **inflammation** to **explosion** requires a conservation of heat or greater local energy than can often be realized in the large open workings of a well-ventilated mine.

COAL DUST

Influence of Coal Dust on Explosion.—The fine dust of an inflammable coal when floating in the mine air may render the air explosive in the entire **absence of explosive gas.** Under such conditions, however, the ignition and explosion will only take place when the floating dust is acted upon by a flame of considerable volume and intensity.

When a **small percentage of methane** is present, insufficient of itself to make the air explosive, the presence of the dust floating in the air is more dangerous than when no gas is present. The dust-laden air is **more easily ignited** and the force of the resulting explosion is increased in proportion to the inflammability of the mixture.

The **purity, fineness, humidity** and **inflammability** of the dust are important factors in determining the character of the explosion, since these with oxygen are the chief elements that promote the rapidity of the combustion, which is the necessary condition of any explosion.

The suspended dust **feeds the flame** of an explosion that is started in a mine, and thus serves to propagate the blast and extend what would otherwise have proved only a local explosion. This **action is cumulative** in a dry and dusty mine. The dust lying on the roads and clinging to the sides and timbers of the passageways is blown into the air by the force of the rushing wind that precedes the explosive wave, producing what has well been called a **"pioneering cloud"** of dust that is itself highly explosive.

The **weight of fine bituminous coal dust** required to render normal air explosive has been variously estimated. Tests made at the Pittsburgh Experiment Station with dust from a 200-mesh sieve showed explosion took place in a density of 32 grm. per cu. m. (0.032 oz. per cu. ft.) or, say 1 lb. of dust in 500 cu. ft. of air. The Taffanel experiments (Liévin) gave explosion in 70 grm. per cu. m. (0.07 oz. per cu. ft.) or, say

1 lb. of dust in 230 cu. ft. of air. In one instance only, explosion occurred in 23 grm. per cu. m. (0.023 oz. per cu. ft.), or 1 lb. of dust in about 700 cu. ft. of air.

It is quite evident, as experiments also show, that conditions in respect to the **purity, humidity** and particularly the **inflammability** of the dust are so variable that the question of the density of the dust cloud has only an experimental value. The size of the workings, as determining the conservation of heat and pressure, will also modify the results in the mine.

Theoretically, since the atomic weights of carbon and oxygen are 12 and 16, respectively, 1 lb. of carbon will yield

$$\frac{12 + 16}{12} = \frac{28}{12} = 2\tfrac{1}{3} \text{ lb. carbon monoxide.}$$

But, carbon monoxide measures 13.5 cu. ft. per lb., at normal temperature and pressure. Hence, $2\frac{1}{3}$ lb. of this gas produced by 1 lb. of coal dust makes $2\frac{1}{3} \times 13.5 = 31.5$ cu. ft. Then, since the lower inflammable limit is reached when the mixture of gas and air contains 8.4 per cent. of the gas, **inflammation** might be expected when the dust present was 1 lb. in $31.5 \div 0.084 = 375$ cu. ft. of air. Also, the lower explosive limit of the gas occurring when 16.5 per cent. of gas is present, **explosion** might be expected to take place when there was 1 lb. of dust in $31.5 \div 0.165 = 190$ cu. ft. of air.

Inflammability of Coal Dust.—The inflammation of a dust cloud in mine workings, under like conditions, depends largely on the inflammable nature of the coal. The experiments at different testing stations have demonstrated that the **volatile combustible matter** contained in coal is a fair index of its susceptibility to inflammation when held in suspension as fine dust in the air.

Experiments performed with **anthracite dust** seem to indicate that the fine dust of that coal is not capable of propagating an explosion in a mine, under ordinary mining conditions. This fact points significantly to the conclusion previously stated that the volatile combustible matter in a coal is an important index of its explosibility. It is not asserted or claimed that anthracite dust cannot be exploded under favorable conditions. However, the conditions that would cause anthracite dust floating in the air to explode are not liable to occur in ordinary mining practice.

Influence of Shale or Stone Dust.—Shale or other soft rock of the coal formations have been ground to a fine powder for use in mines and, in this form, have been sprinkled on the roads in a manner to form **stone-dust zones**, or distributed on shelves hung across and overhead in the entries to form so-called **stone-dust "barriers."**

The purpose of these dust zones and barriers is to **arrest the progress** of an explosion should one occur in the mine. Their use, however, has not been attended with **unvarying success**, which is due in part to the different conditions of temperature, humidity, air space or volume of mine workings available for expansion, inflammability of the gas- and dust-laden air and the initial intensity of the explosion; also, in part to the limited extent or adequacy of the dust zone or barrier as compared with the strength developed by the explosion.

Notwithstanding the **apparent failure** of these means for preventing the spread of an explosion in a mine in many observed instances, there is no question but that finely powdered shale or stone dust blown into the path of an explosive wave by the pioneering impulse, or suspended in the air with the inflammable coal dust has a most decided effect and **reduces explosive conditions.**

The **action of incombustible dust**, suspended in an otherwise explosive atmosphere, to allay the explosiveness of the mixture or reduce the violence of the blast should ignition and explosion occur, is **wholly physical.** The incombustible particles disseminated through a dust-laden atmosphere separate more widely the inflammable particles of coal dust and dilute the air necessary for combustion. In other words, the percentage of inflammable matter in the mixture is reduced and the **liability to inflame** diminished in the same proportion.

Also, by its **absorption of heat**, the incombustible matter lessens the heat available for ignition and decreases the heat energy developed when ignition has taken place. The action is entirely similar to that of the inert nitrogen of air depleted of its oxygen, or to the extinctive effect of carbon dioxide when present in firedamp mixtures, both of which conditions act to diminish the explosibility of gaseous mixtures.

MINE EXPLOSION

Development of a Mine Explosion.—Explosion does not necessarily follow the ignition of gas in mine entries and workings. The firedamp mixture must, of course, be within the explosive range, as determined by the conditions in that portion of the mine. But even then a mine explosion will only take place when the **conservation of heat** is sufficient to render the explosive action self-supporting. Otherwise, a local explosion of gas or dust will expend its energy within a limited area and the disturbance will not be propagated throughout the mine.

The ignition of an inflammable mixture of gas or dust in the mine air may produce a considerable body of flame that, within the narrow confines of the mine, may gather force and generate sufficient heat to cause an explosion. **Experiment** has shown that an explosive mixture of gas and air placed in a tube and ignited at one end will burn quietly at first, then flutter or vibrate with increasing energy as the combustion penetrates deeper in the tube, the contending forces being the entering air and the escaping products of the combustion. This action, however, quickly develops sufficient energy to produce an explosion, which darts through the entire length of the tube.

This experiment illustrates more or less closely the development of an explosion in a mine entry or chamber. Investigation has shown that the **explosion gathers force** and probably **develops characteristic energy** within a few yards of its origin or the point where the ignition of the gas took place. This may vary from 10 to 30 yd. or more, depending on many conditions—chiefly the size or volume of air space available for the expansion of the gases of the explosion, the intensity of the igniting flame and inflammability of the mixture.

All of these factors determine severally the initiation as well as the character of the explosion and its limitations in the mine workings.

Causes of Mine Explosions.—The causes of mine explosions may be generally stated as the ignition of gas or dust by one of the following causes:

1. By the use of **open lights** or **defective safety lamps** in mines where the air current is charged with gas or dust, or where gas has accumulated in void or abandoned places in sufficient quantities to be dangerous.
2. By the use of **mixed lights** in mines generating gas.
3. By the **inexperienced** or **careless use** of a safety lamp, or by **fooling** or **tampering** with the lamp, or **exposing it to gas** too long or to a **strong gas blower** or **strong current** or blast of air, or carrying **too high a flame.**
4. By the use of a **dirty lamp** or one that has been improperly assembled or injured by a fall or other accidental cause.
5. By the **explosion of powder** in blasting or the accidental explosion of a keg of powder, or the flame of a **blownout shot** or a **windy shot.**
6. By the use of matches or other means of lighting.
7. By the sparking of **electric wires, switches** or **brushes,** or the blowing out of an **electric fuse**, or the breaking of an **incandescent lamp.**
8. By the **spontaneous ignition** of oily waste carelessly thrown aside, or of fine coal or slack in the gob.
9. By the fall of certain hard roof **rock striking sparks,** as claimed in the Bellevue mine explosion (1910), Alberta, Canada.
10. By the possible **generation of heat** due to concussion of the mine air in contracted workings in thin seams.

Mixed Lights in Mines.—By "mixed lights" is meant the use of **open lights** in one or more sections of a mine in which gas is generated in other portions of the mine in sufficient quantity to require **safety lamps** being employed therein.

The expression does not refer, however, to the use of open lights by drivers, triprunners or motormen whose duties are confined to the main intake haulage roads and shaft or slope bottom of a mine worked on safety lamps, provided there are lamp stations beyond which these men may not pass.

The use of mixed lights is a **dangerous practice.** The danger does not consist wholly in a man carrying an open light into the safety-lamp section, or to a foreman or fire-boss forgetting that he has an open light on his head while carrying a "safety" at his side. These are possibilities that can be prevented by properly safeguarding the entrances to the gaseous section.

The real danger lies in a heavy fall of roof occurring in the safety-lamp section and driving out the gas into other parts of the mine where open lights are in use. Or, a squeeze may develop in any part of the mine and permit the gas to find its way without warning into an open-light section and cause an explosion.

Electric Mine Lamps.—Any installation of electricity in a mine worked on safety lamps is necessarily accompanied with more or less danger. Whether the installation is for the purpose of lighting, hauling, coal cutting or drilling, pumping or ventilation, it should be made by a competent electrician. The entire system of wiring should be closely inspected at frequent intervals and tested to insure freedom from short-circuiting or grounding of the current, which are not only wasteful of power, but may start combustion and result in an explosion of gas.

The use of **incandescent lamps** in mines has become so common that the Bureau of Mines has made a careful investigation to determine their safety. Their experiments show that **ignition of gas** may follow the breaking of the glass bulb of a lamp in an explosive mixture. The experiments also seem to indicate that the liability of ignition increases with the cross-section of the filament of the lamp. In the breaking of an incandescent lamp **two conditions** may arise that materially affect the possibility of the ignition of the gas. The same blow that breaks the bulb may or may not break the filament. The result in either case may be briefly explained as follows:

1. If the **filament is broken** and its parts do not short-circuit the current ignition of the gas is not likely to occur. If the broken parts, however, fall across each other in such manner as to again close the circuit their burning out in the air will generally ignite any gas present.

2. If the **filament remains intact** when the bulb is broken it will burn out more or less rapidly, according to the manner of fracture and consequent inrush of air and gas. A small hole due to the breaking of the tip may admit the air so slowly that the gas is consumed without explosive violence. In that case there may occur a slight explosion within the bulb, which is not broken but only pierced. This feeble explosion, however, may not be communicated to the outside gas.

Prevention of Mine Explosions.—No means has yet been devised that will insure absolute freedom from mine explosions. But the tendency to explosion and the frequency of these occurrences can and has been greatly reduced by studying their causes and adopting measures to remove them.

The following points are of chief importance:

1. Effective mine regulations and discipline.
2. Operation in accordance with the state mining law.
3. Enforcing by suitable penalties all mine regulations.
4. Thorough frequent inspection by competent men.
5. Education and training of all men employed in any capacity in the mine, in respect to the proper performance of their duties, the dangers to which they are exposed and the mining law and mine regulations in force.
6. Eternal vigilance of mine officials and a regard for safety greater than the desire for increasing the daily output of the mine.
7. Coöperation of employers and employed in increasing the safety of mine work.
8. Coöperation of all coal companies in respect to mining requirements.

Aside from the above general outline there is the necessity for each company to study carefully the conditions existing in its own mines, and to adopt a system of inspection and methods of ventilating the mine and mining and hauling the coal that will produce the best results and insure the greatest freedom from accumulations of gas and dust on the roads and in the workings. Immunity from explosion can only be secured by removing the cause.

SECTION V

MINE RESCUE WORK AND APPLIANCES

Preliminary, Entering a Mine After Explosion, First-aid Suggestions—Breathing Apparatus, Principle, Action and Requirements in Respiration, Development, Design and Testing of Breathing Apparatus—Types of Breathing Apparatus, Draeger, Fleuss Proto, Gibbs, Paul—Bureau of Mines, Permissible Breathing Apparatus—Specifications by the Bureau of Mines—First-aid Work.

PRELIMINARY

Entering a Mine after an Explosion.—Prompt action and intelligent and effective measures are necessary for the rescue of any possible survivors of a mine explosion. The nature of the work and the great risk incurred in its undertaking demand that it shall be performed by the most experienced of the volunteers, of whom there is never any lack.

Immediately after an explosion in a mine, the following procedure is important:

1. Call for volunteers and from them choose those who are more experienced and familiar with the mine and the work to be performed.

2. At the same time, observe the mine entrances and judge of the probable effect of the explosion in the mine; examine the ventilating apparatus and have any necessary repairs made at once.

3. Collect the necessary safety lamps, tools, timber, canvas, brattice boards, nails, etc. Caged canaries or mice should also be provided, and two or more sets of breathing apparatus should make up the equipment.

4. Divide the rescuers into three parties, as follows:

(*a*) Apparatus men to explore in advance;

(*b*) Repair gang and rescuers;

(*c*) Supply gang to render every possible assistance.

Organize each party under a competent leader who shall be in absolute control while underground.

5. Enter the mine at the earliest possible moment—the apparatus men proceeding first and keeping from 100 to 200 yd. in the lead of the others, who must not advance ahead of the air.

6. Each section of the mine should be explored by the apparatus men to discover any possible fire therein, before restoring the circulation in that section.

As quickly as any survivors are found they must be promptly removed to fresh air and the proper restoratives applied. At the surface, physicians should be in attendance and ambulances provided for the prompt removal of those brought out of the mine.

Suggestions on First-aid to Explosion Victims.—Those trained in first-aid work are the ones who should assume charge and have absolute control of the care of any survivors as quickly as found, until the arrival of a physician. The following brief suggestions are important:

1. Be calm and quiet; act promptly but not in a hurry; keep cool and observe closely every symptom and condition.
2. Remove promptly but carefully to fresh air.
3. Do everything possible to stop bleeding.
4. Examine for broken bones before moving far.
5. Use aromatic spirits of ammonia if stimulant is needed.
6. If overcome by gas, give artificial respiration.
7. If unconscious, loosen clothing, warm and stimulate by rubbing the limbs; give no stimulant if face is flushed and pulse strong, but sprinkle cold water on face and chest. If the body and limbs are cold, use warm applications; keep the patient covered with blanket or other coverings; apply smelling salts or spirits of ammonia cautiously to the nostrils.

BREATHING APPARATUS

Principle of Breathing Apparatus.—The principle of all breathing apparatus is that the wearer breathes the same air over and over again, the carbon dioxide exhaled in the breath being absorbed after each expiration while, at the same time, the requisite amount of oxygen is restored, thus rendering the expired air pure and fit to be again inhaled.

Action in Respiration.—In the act of inhalation, the air enriched with oxygen passes from the breathing bag in the bottom of the cooler, up through the latter and is drawn through the inhalation valve and tube into the lungs.

In exhalations, the air, deprived of some of its oxygen and containing from ½ to 4 per cent. of carbon dioxide, depending on the amount of the exertion, is discharged through the exhalation tube and valve into the exhalation side of the cooler where it meets the oxygen supply, as previously stated, and passes into the regenerator where it is to give up its carbon dioxide, by contact with the absorbent caustic soda.

Requirements in Respiration.—The average full capacity of the lungs of an adult person is about 300 cu. in. This volume, however, is never utilized in the act of breathing; that is to say, all of the air contained in the lungs is never exhaled or the lungs would collapse, which would be fatal. There is a certain volume of residual air, about 100 cu. in., that remains in the lungs after a deep expiration. In the ordinary act of breathing, the average person expires only about 20 or 30 cu. in. of air at a single breath. This has been called "tidal air." In the performance of work or when undergoing any extra exertion, a larger quantity of air is expelled from the lungs at each breath and a corresponding quantity again inhaled.

The ordinary rate of respiration is 16 breaths per minute when a person is at rest, making the volume inhaled, from 300 to 500 cu. in. per min. When making violent exertion in the performance of work, breathing is more rapid and a much larger volume of air is respired. This quantity will vary with the person and the exertion made or the work performed. When doing strenuous work a man may inhale 200 cu. in. of air at a single breath.

Approximately, the volume of carbon dioxide exhaled is equal to that of the oxygen breathed into the lungs, the ratio of carbon dioxide to oxygen being slightly less when the person is at rest, than it is in the performance of work. However, for the purposes of ordinary estimate, it may be assumed that a man, at rest, will inhale from 25 to 30 cu. in. of air at a single breath and this may be increased to 150 or possibly 200 cu. in. when making violent exertion. Practically, one-fifth of this volume of air is oxygen; but, in the act of breathing, only one-third or one-half of this oxygen is consumed.

The standard supply of oxygen, in mine breathing apparatus, has been fixed, therefore, at 2 liters per min. (122 cu. in.). Compressed to 120 atmospheres, this rate of supply of oxygen, for a 2-hr. period, will require a cylinder capacity of $2(122 \times 60) \div 120 = 122$ cu. in. Again, assuming that the average amount of carbon dioxide produced in breathing is equal to the volume of oxygen consumed, it appears that the quantity of the former gas required to be absorbed by the caustic soda in the regenerator, in a 2-hr. period, is $2(2 \times 60) = 240$ liters, or 8.47 cu. ft.

The following table gives carefully compiled data and the results of actual tests regarding the oxygen consumed, carbon dioxide produced, quantity of air breathed and number of respirations per minute, under different conditions of rest and exertion. These data were compiled by James M. Stewart, Instructor at the Brazeau Rescue Station, Alberta, Canada.*

Data Regarding Air Respired when Walking and at Rest

Condition of subject	Oxygen consumed per minute in liters	CO_2 expired per minute in liters	Air breathed per minute in liters	Average volume of each breath	Number of breaths per minute
At rest in bed	0.237	0.197	7.7	0.457	16.8
At rest, standing	0.328	0.264	10.4	0.612	17.1
Walking, 2 mi. per hr.	0.780	0.662	18.6	1.270	14.7
Walking, 3 mi. per hr.	1.065	0.992	24.8	1.530	16.2
Walking, 4 mi. per hr.	1.595	1.395	37.3	2.060	18.2
Walking, 4½ mi. per hr.	2.005	1.788	46.5	2.520	18.5
Walking, 5 mi. per hr.	2.543	2.386	60.9	3.140	19.5

* *Bulletin,* November, 1916, Rocky Mountain Branch of the Canadian Mining Institute.

It is evident from the table that more than the standard supply of oxygen allowed in the design of breathing apparatus may be consumed by a person under great physical exertion. Mr. Stewart suggests, therefore, that it is of the utmost importance that the captain of a rescue team observe carefully that his men do not overexert themselves while in the performance of their duties in the mine. He also suggests that, in the use of the nose-clip, greater comfort and security is obtained by inserting a cotton-wool plug in each nostril, before adjusting the clip.

Development of Breathing Apparatus.—The development of breathing apparatus, during the past few years, since the Government took up the work of improving mining conditions (1907) has been rapid. In the earlier types of apparatus, a helmet was employed to cover the head and oxygen was supplied through rubber tubes that connected the helmet with a gas cylinder or bag containing the gas. Owing to the danger of these connecting tubes being broken in the rough service to which they are subjected in the mine, the first attempt to improve the apparatus resulted in the adoption of a form that was self-contained, so as to eliminate, as far as practicable, the tube connections.

Mining practice quickly demonstrated that the substitution of a simple mouthpiece, and noseclip to close the nostrils, gave better service underground than the clumsy helmet, although the latter afforded more comfort in breathing and enabled the wearer to talk to his comrades with greater facility than when the mouthpiece was used and a noseclip closed the nostrils. However, these disadvantages were largely outweighed by the greater facility offered for work by this form of apparatus.

Design of Breathing Apparatus.—Breathing apparatus is designed to supply the wearer with a perfectly respirable air independent of the atmosphere in which he may be placed. The design of the apparatus is to enable the wearer to work in an irrespirable atmosphere for a limited period of two hours. The principal features of the device consist in maintaining a sufficient supply of oxygen to replace that consumed by the wearer of the apparatus, and absorbing the carbon dioxide he exhales.

Oxygen, compressed to 120 atmospheres, is contained in a strong steel cylinder. The quantity is sufficient to afford a supply of 2 liters of this gas (122 cu. in., normal temperature and pressure) per minute. A pressure of 120 atmospheres, at sea level, corresponds to about 1800 lb. per sq. in. A reducing valve is employed to control this pressure and reduce it to the normal pressure of the atmosphere, for breathing. An air-tight breathing bag filled with pure air and equipped with a release valve, forms part of the apparatus and is connected directly with the oxygen supply cylinder and the helmet or mouthpiece.

Another important feature of breathing apparatus is the regenerator, holding a supply of 4 or 5 lb. of caustic soda or caustic potash. This minimum weight of caustic soda (4 lb.) will absorb, if fully utilized, 532 liters of carbon dioxide and is ample for all contingencies. By the absorption of the carbon dioxide, the caustic soda is converted into sodium carbonate and some water is produced according to the equation

$$2NaOH + CO_2 = Na_2CO_3 + H_2O$$

The molecular weight of the caustic soda or sodium hydroxide is $2(23 + 16 + 1) = 80$, while the molecular weight of the carbon dioxide is $12 + 2 \times 16 = 44$. The ratio of the weight of carbon dioxide absorbed to that of the caustic used is, therefore, $^{44}/_{80} = {}^{11}/_{20}$; and the 4 lb. of caustic soda, if completely utilized, would absorb $4(^{11}/_{20}) = 2.2$ lb., or 18.78 cu. ft. of carbon dioxide (532 liters), at normal temperature and pressure.

In the absorption of carbon dioxide, however, the caustic soda becomes encrusted with the sodium carbonate formed, which prevents or at least impedes the action of absorption. The shaking of the regenerator helps to break up this crust and restore the absorptive power of the caustic.

Testing Breathing Apparatus.—All breathing apparatus should be regularly tested to insure its perfect condition. Especially should this be done by the wearer before he enters an irrespirable atmosphere. The apparatus may be defective from any one of a number of such causes as negative pressure; leaks in joints, tubes, breathing bag or other container; obstructed valves or tubes, imperfect regeneration, owing to insufficient absorption of carbon dioxide or inadequate supply of oxygen; etc

Before putting on the apparatus, the wearer should examine and test its various parts to ascertain that it is tight, the valves and tubes free from obstruction and the supply of oxygen and caustic soda adequate. Each tube, the bag and the assembled apparatus should be tested for leaks, by means of the pressure gage and observing the constant water level in the U tube kept for that purpose. The old habit of immersing apparatus in water to show leakage is harmful.

TYPES OF BREATHING APPARATUS

The principal types of breathing apparatus now in use in this country are the Draeger breathing apparatus, the Fleuss Proto apparatus, the Paul type of apparatus and the more recent and highly improved Gibbs apparatus, which combines all of the best features of other types and many improvements.

Draeger Breathing Apparatus.—There are two general types of this apparatus, one employing the helmet and the other the noseclip and mouthpiece. These two types are shown in Fig. 14 together with side and rear views of the apparatus as worn by the rescuer. Owing to its bulkiness the helmet type is not so well adapted to mine work as that equipped with the noseclip and mouthpiece.

Since its introduction in 1903 the Draeger apparatus has undergone various marked improvements and is at present one of the standard types of rescue appliances in use. The canvas breathing bags, one for inhalation and the other for exhalation, are rubber-lined. The oxygen cylinder is supplied with a perfected high-pressure valve that enables the wearer to shut off the pressure at any moment desired, by a simple thumb pressure. These together with the safety locked couplings securing all tube connections, and the time recorder and pressure gage, always ready for inspection by the wearer, insure both safety and comfort.

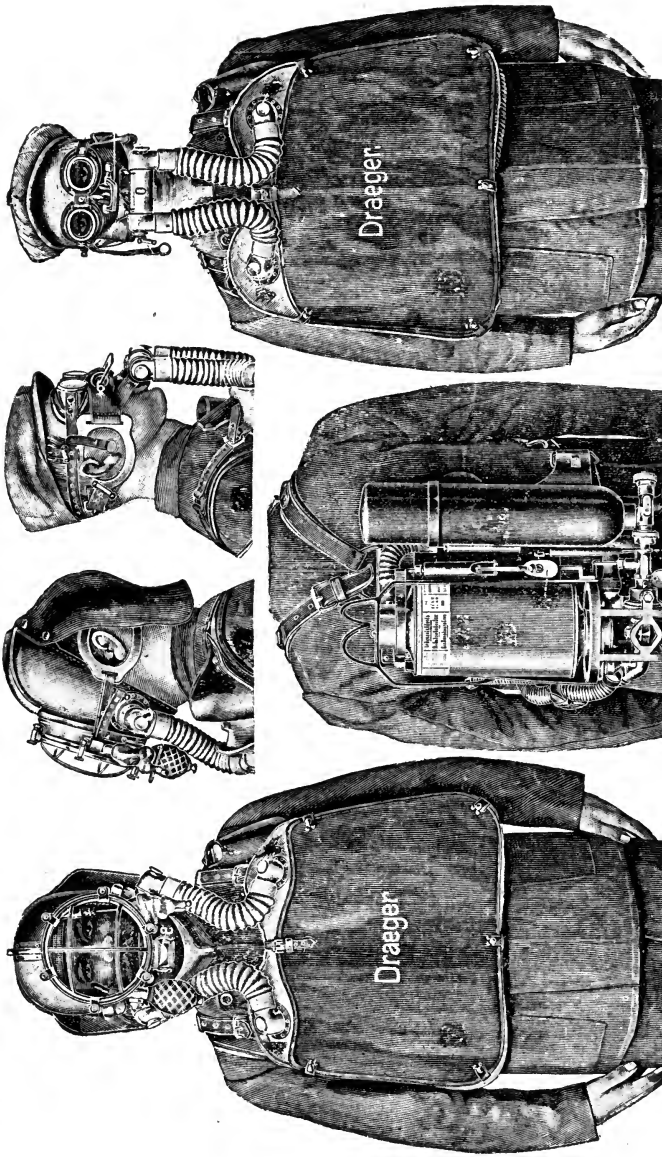


FIG. 14.

Essential Parts.—The diagram, Fig. 15, shows the arrangement of the several parts of the apparatus for the purpose of making clear their relation and the circulation of the system. The diagram shows the helmet *H*, the expiration valve V_2, the exhalation bag L_2, that receives the exhaled air, the regenerator *R*, the cooler *K*, the aspiration pipe *C*, the inhalation bag L_1, holding the purified air and the inspiration valve V_1. The oxygen cylinder and pressure gage also appear, the former has withstood an official test of 225 atmospheres and is commonly charged to a pressure of 120 atmospheres.

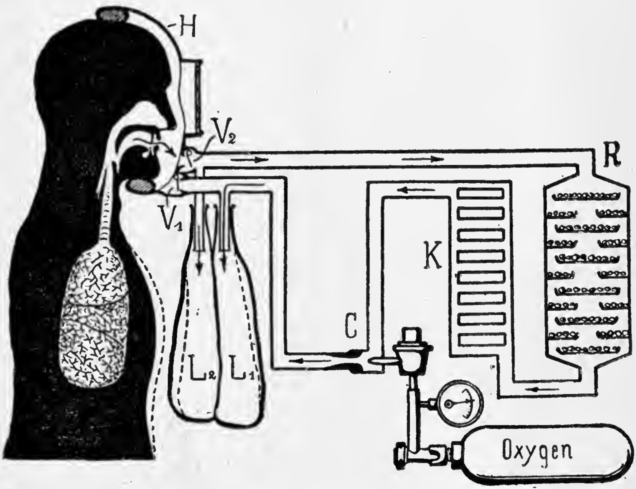


Fig. 15.

Capacity of the Apparatus.—This apparatus will purify about 3000 liters (105 cu. ft.) of air per hour, besides supplying 120 liters (4.2 cu. ft.) of oxygen, and absorb 50 liters (1¾ cu. ft.) of carbon dioxide. This is claimed to enable the wearer of the apparatus to perform 260,000 ft.-lb. of work. While an untrained man will generally do less than this, the work done in one instance amounted to 398,000 ft.-lb.

Fleuss Proto Apparatus.—This apparatus is designed to supply the user with a perfectly respirable air, entirely independent of any communication with the outside atmosphere for at least two hours at a time. It has been designed to withstand the severe conditions to which it must be subjected in mining use and insure the safety of the wearer while engaged in the dangerous work of rescuing men from mine workings filled with poisonous or irrespirable gases.

FIG. 16.

Front and rear views of the apparatus are shown in the Fig. 16, in the position in which it is worn, the large, double-compartment breathing bag being in front and the oxygen cylinder in the rear of the wearer. A diagrammatic view is shown on the opposite page (Fig. 17) explaining the various parts of the apparatus.

Essential Parts.—The principal features are the oxygen cylinder *B;* the reducing valve *C*; the breathing bag *D* with inhaling and exhaling divisions; inspiratory and expiratory valves *T* and *S*; mouthpiece and noseclip *R* and *Y*.

The wearer exhales through valve *S*, the air passing down one side of the partition of the breathing bag and through the caustic soda, which absorbs the carbon dioxide, and thence up

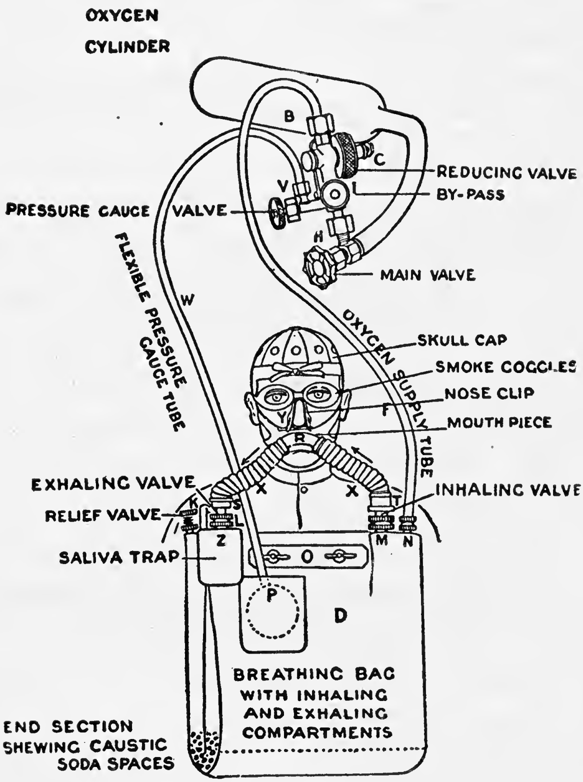


Fig. 17.

the other side of the partition to valve *T* to be again inhaled, after mixing with fresh oxygen, which is being constantly delivered at the rate of two liters per minute from the oxygen cylinder through the reducing valve *C*. Connected to a flexible tube *W* is a pressure gage *P* indicating the quantity of oxygen in the cylinders and the duration of supply. An emergency by-pass *I* is for use in case the reducing valve fails; it enables the wearer to fill his breathing bag direct from the oxygen cylinders. A saliva trap *Z* prevents the saliva from entering the breathing bag.

The **steel cylinder** contains about 10 cu. ft. of oxygen, compressed to 120 atmospheres, which gives a two-hours' supply when the reducing valve is passing two liters per minute. The cylinder can be charged to 150 atmospheres if desired, which will give a 2½ hour-supply.

A **reducing valve** *C* is fitted to the bottle nipple and is so adjusted as to pass a regular supply of from 2 to 2½ liters of oxygen per minute, no matter what the pressure may be in the cylinder. This valve can be readily adjusted to deliver any flow from one to three liters per minute, as desired. The valve is fitted with a by-pass, having a small wheel valve *I* so that should it from any cause fail to act properly the wearer of the apparatus can supply himself with what oxygen he requires direct from the cylinder by turning the small valve. Also, by the same means, the automatic supply of two liters per minute can be increased at any time by the wearer if desirable. When working in an excessively hot atmosphere it is possible to cool the hot air by exhausting all the air from the bag through the relief valve *K*, and then filling the bag with pure, cool oxygen from the cylinder, by means of this by-pass.

The reducing valve delivers the oxygen through the flexible tube *F* to the breathing bag *D*, carried on the wearer's chest. Another connection at *V*, made through a flexible high pressure tube *W* with a **pressure gage** *P*, carried in a pocket of the canvas cover, enables the wearer to ascertain the available supply and duration of oxygen. Each division of the pressure gage indicates 10 atmospheres of pressure, or 10 minutes of time, assuming the valve to be passing two liters per minute. The connection *V* is also fitted with a small valve, to enable the wearer to shut off the oxygen should the gage or its flexible tube become damaged.

The **breathing bag** *D* is of strong vulcanized India rubber and contained in an outer strong canvas bag. The rubber bag has two compartments, connected, however, at the bottom of the bag. The bag is fitted at the upper left-hand corner with a **saliva trap** *Z* and relief valve *K* to allow the escape of any excess oxygen that might be delivered by the reducing valve. At the upper right-hand corner is a small connection *N* for the oxygen supply from the cylinder. The mouth of the bag is closed with metal clamps and wing nuts *O*.

The **mouthpiece** is of soft vulcanized India rubber, fitted to a German silver connection *R* and shaped to fit comfortably between the lips and the gums. To the connecting piece *R* are also fitted strong flexible corrugated tubes *XX*, sometimes called "bellows tubes," to the opposite ends of which are fitted the exhaling and inhaling valves *S* and *T*, respectively. These valves are of mica and extremely sensitive. They are screwed into their respective connections *L* and *M*. The **noseclip** *Y* is made to fit any nose comfortably. The **skull cap** has a back apron to which the mouthpiece can be securely buckled, which supports it comfortably.

One feature of the Fleuss Proto apparatus is the fact that the caustic soda is held in a bag instead of a rigid container and the movements of the wearer when walking or at work automatically rubs off the carbonated surface of the soda, and constantly exposes a fresh surface for the absorption of carbon dioxide. The bag is easily emptied after use, and a fresh supply of soda added at once, thus making the apparatus ready for use again in two or three minutes. The bag is so constructed that external pressure on it does not impede the wearer's breathing. In fact, a man may lie flat upon the bag and still be able to breathe freely.

Gibbs Breathing Apparatus.—This form of apparatus was developed by W. E. Gibbs, of the Federal Bureau of Mines, who sought to improve on the older types of English makes of breathing apparatus in mining use.

The general requirements sought to be fulfilled in this design were: (1) Automatic control of oxygen supply in rest or exertion. (2) Adequate absorption of carbon dioxide. (3) Freedom of respiration under constant positive pressure. (4) Avoiding collapse of breathing bag from any cause. (5)

Efficient heat radiation and cooling to avoid high temperature. (6) Simplicity, durability and strength and tight joints in every part.

The position of the apparatus when in use is shown by the side and rear views in the Fig. 18. For the better protection of the parts from injury, in the mine, a cover is provided as a

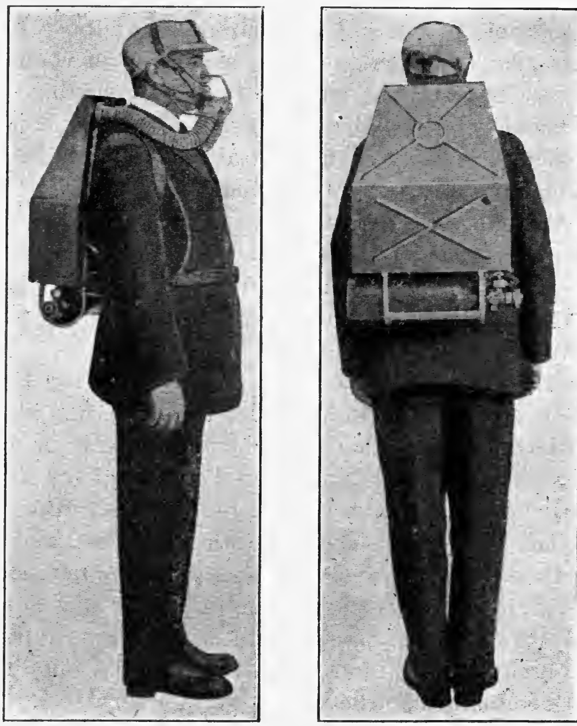


Fig. 18.

shield. The general arrangement of the parts is shown by the Fig. 19 in which the several elements are numbered to correspond to their description in the text.

Circulation in the Apparatus.—Oxygen from the bottle (1) in which it is compressed to 135 atmospheres, passes through the closing valve (2) to the reducing valve (3); thence, under normal pressure, by rubber tube connection, it passes through a metal tube surrounded by a cooler; through an admission valve into another metal tube inclosed in cooler, being then discharged into the exhalation side of the cooler where it meets the exhaled air and passes downward with it into the regenerator; then upward into the inhalation side of the cooler, where

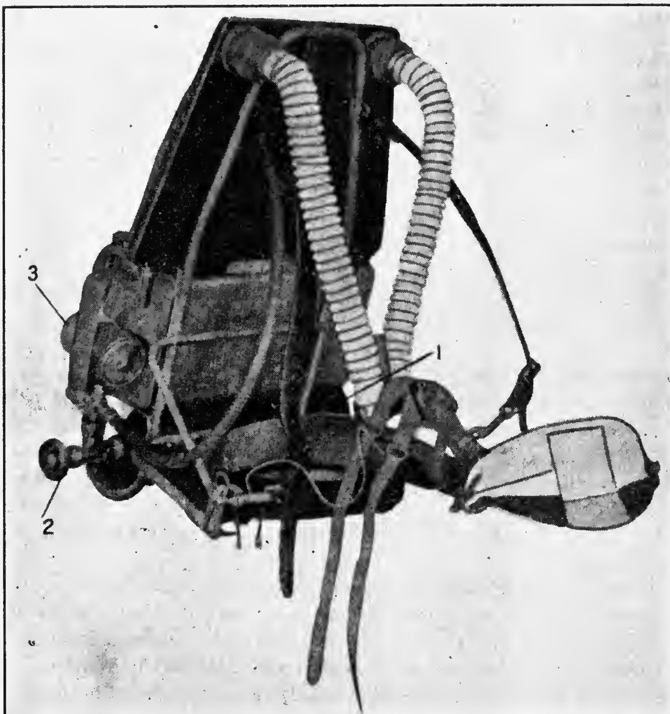


Fig. 19.

it enters the breathing bag in the cooler. From the breathing bag the air passes through an inhalation valve and enters the lungs, from which it is discharged through the exhalation tube into the exhalation side of the cooler.

Testing Gibbs Apparatus.—The following series of tests of the Gibbs breathing apparatus are recommended by its manufacturers:

1. Oxygen bottle should be charged to 135 atmospheres. The oxygen cylinder being tested under water for leaks, with main valve both open and closed. The cylinder is first tested with valve closed, then cap is placed on cylinder and tested with valve open. Connect oxygen bottle to reducing valve, using wrench in order to make tight connections.

2. Examine seals of regenerators in order to see that they are not broken. Connect regenerator to cooler, being sure that gaskets are in place between the connections. Screw down screws by hand and tighten with screw driver.

3. Lift breathing bag from bumper on admission valve, then turn on main oxygen valve.

Observe mica inhalation valve—if admission valve leaks the mica inhalation valve will raise and let oxygen escape.

Turn pressure tube valve on and observe the number of atmospheres indicated by the pressure gage. Pressure gage valve should always be left open. Squeeze bellows of reducing valve in order to open seat over orifice; this approximately increases the pressure to five pounds in rubber tube and metal tube. Safety valve will whistle at the above pressure if working properly. Try all connections from oxygen bottle to cooler for leaks by using brush and soap suds. Turn off main oxygen valve.

4. Blow into exhalation valve and observe air returning by way of inhalation valve, showing circulation of air through exhalation side of cooler, regenerator, inhalation side of cooler, and breathing bag. Next, close inhalation valve either by cupping hand over valve or by special connection, then blow into exhalation valve until bag is fully inflated. Exhalation valve seat and mica should make an air tight connection, keeping bag fully inflated. Test all connections for leaks, using brush and soap suds.

5. Connect mouthpiece to cooler, seeing that gaskets are in place. Inflate breathing bag and test mouthpiece connections for leaks, using brush and soap suds. Try release valve and saliva pumps for leaks.

6. After apparatus has been tested and adjusted to wearer, before adjusting noseclip, it is essential that the wearer turn on main oxygen valve, inhale from apparatus, exhale into open air several times before readjusting the clip. In this way a high percentage of oxygen and a low percentage of nitrogen will be contained in breathing apparatus. While inhaling from the apparatus the wearer will observe whether the whole apparatus is functioning properly. After noseclip is adjusted, the wearer is ready for a preliminary test in room filled with fumes. After remaining in room for five (5) minutes and no leaks being observed, the wearer can feel assured that his apparatus is in good working condition for doing work in poisonous gases and irrespirable air.

7. Under no circumstances should grease or oil be used on apparatus parts.

The Paul Breathing Apparatus.—This type of apparatus was designed by James W. Paul, long in charge of the mine-rescue work, as engineer of the Federal Bureau of Mines, at Pittsburgh, Penn. The apparatus is manufactured by the old Draeger Company, now known as the American Atmos Corporation, Mr. Paul having disposed of his right and title in the apparatus to that company.

One of the highly essential improvements of the Paul apparatus, which is modeled chiefly after the Gibbs, is the combination of the self-adjusting oxygen-feed valve with a low-pressure oxygen-control valve, at the intake of the circulatory system. This device regulates the supply of oxygen and proportions it to the rate of consumption, which varies with the work performed by the wearer. Also, a pressure slightly in excess of 1 cm. of water column is automatically maintained in the system and minimizes the liability of an outside poisonous atmosphere penetrating within the apparatus.

BUREAU OF MINES

The Federal Bureau of Mines recommends that the circulation in breathing apparatus be under positive pressure throughout and that the apparatus be equipped with mouthpiece and noseclip and provided with a by-pass valve. The helmet, for mining use, is objectionable and dangerous, not only because of the difficulty of obtaining a perfectly airtight joint around the face, but also because it is easily dislodged and greatly cuts down the range of vision. Also, the large dead-air space in the helmet permits an excessive accumulation of carbon dioxide.

The injector used in some types of breathing apparatus is complicated and liable to be out of order when needed. Any slight particle is sufficient to choke the orifice and cut off the supply of oxygen. The use of the injector also involves a negative pressure, which would cause an inflow of the surrounding atmosphere into the apparatus should there be any leak in the joints or tube connections.

Permissible Breathing Apparatus.—Owing to the grave importance of securing safe types of mining appliances manufactured in this country, an act of Congress (37 Stat., 681), approved Feb. 25, 1913, authorized the director of the Bureau of Mines to prescribe rules and regulations for testing such appliances as may be submitted to the bureau for that purpose.

Acting under this authority the Federal Bureau of Mines has prepared and published, Mar. 5, 1919, "Schedule 13," defining the requirements necessary to establish a list of so-called "Permissible" self-contained, mine-rescue, breathing apparatus. Following are the more important specifications contained in that schedule.

Definition.—The Bureau of Mines considers a self-contained mine-rescue breathing apparatus to be permissible for use in irrespirable and poisonous gases if all the details of construction and materials are the same in all respects as those of the self-contained mine-rescue breathing apparatus that met the requirements and passed the tests for safety, practicability and efficiency made by the bureau and hereinafter described.

Conditions of Testing.—The conditions under which the Bureau of Mines will examine and test self-contained mine-rescue breathing apparatus to establish their permissibility are as follows:

1. The examination, inspection, and test shall be made at the experiment station of the Bureau of Mines at Pittsburgh, Pa.

2. Applications for inspection, examination, and test shall be made to the Director, Bureau of Mines, Washington, D. C., and shall be accompanied by a complete written description of the self-contained mine-rescue breathing apparatus including the regenerator, and a set of drawings showing full details of construction of both the regenerator and the apparatus.

3. The applicant submitting the self-contained mine-rescue breathing apparatus for inspection, examination, and test will be required to furnish the apparatus in duplicate, which shall be sent prepaid to the mine-safety engineer, Bureau of Mines, 4800 Forbes Street, Pittsburgh, Penn. In the event of the apparatus successfully passing all of the Bureau of Mines tests and requirements hereinafter specified, one set will be retained by the Bureau of Mines as a laboratory exhibit and the other set will be returned to the owner. In the event that an apparatus does not pass all of the bureau's tests or requirements, both sets will be returned to the owner.

4. Each self-contained mine-rescue breathing apparatus shall have marked on it in a distinct manner the name of the manufacturer and the name, letter, or number by which the type is designated for trade pur-

poses, and a written statement shall be made whether or not the apparatus is ready to be marketed.

5. The applicant will supply the regenerators or regenerating material for the test. For tests of self-contained mine-rescue, oxygen breathing apparatus dependent on a supply of compressed gaseous oxygen, the oxygen will be supplied by the Bureau of Mines and will be of the purity specified by the bureau in contracts for the supply of its safety cars and stations; namely, 98 or more per cent. oxygen and not more than 0.2 of 1 per cent. hydrogen; other impurity to consist of nitrogen only.

6. Upon receipt of the self-contained mine-rescue breathing apparatus for which application has been made for examination, inspection, or test, the mine-safety engineer in charge of breathing-apparatus testing will advise the applicant whether additional spare parts are deemed necessary to facilitate a proper test of the apparatus, and the applicant will be required to furnish such parts as may be necessary.

7. No self-contained mine-rescue breathing apparatus will be tested unless the type submitted is in the complete form in which it is to be placed on the market.

8. Only the Bureau of Mines mine-safety engineer in charge of breathing-apparatus testing, his assistants and one representative of the applicant will be permitted to be present during the conduct of the tests.

9. The conduct of the tests shall be entirely under the direction of the bureau's mine-safety engineer in charge of the testing.

10. As soon as possible after the receipt of the formal application for test, the applicant will be notified of the date on which the test of his self-contained mine-rescue breathing apparatus will begin and the amount and character of the additional material, if any, it will be necessary for him to submit.

11. The tests will be made in the order of the receipt of the applications for test, provided the necessary apparatus and material are submitted at the proper time.

12. The details of the results of the tests shall be regarded as confidential by all present at the tests, and shall not be made public in any way prior to their official announcement by the Bureau of Mines.

13. The results of tests of the breathing apparatus that fail to pass the requirements shall not be made public but shall be kept confidential, except that the person submitting the apparatus will be informed with a view to possible remedy of defects in future mine-rescue breathing apparatus submitted, but such changes will not be permitted while testing is in progress.

14. Tests will be made for manufacturers or accredited manufacturers' agents and for inventors.

15. A list of permissible self-contained mine-rescue breathing apparatus and the results of their tests will be made public, from time to time, by the Bureau of Mines.

Character of Tests.—After the self-contained mine-rescue breathing apparatus under test for permissibility has been thoroughly inspected for mechanical principles, a series of fifteen (15) working tests, each of two (2) hours' duration, will be made. At the beginning of the series of tests, if an oxygen bottle is used on the apparatus it shall be first charged with oxygen to a pressure of 10 atmospheres and the oxygen permitted to escape into the air. The bottle used in the tests shall be charged for the tests at a pressure prescribed by the manufacturer of the apparatus and shall be fully charged at the beginning of each test. At the beginning of each test the breathing bag or bags shall be deflated to expel any nitrogen contained within.

A single test must be continuous, without removal of the apparatus from the wearer during the test.

Samples of air will be obtained from the apparatus on the inhalation side of the circulatory system and as near to the mouthpiece or the face attachment as possible. The first sample will be taken from the oxygen bottle to be used and just prior to the beginning of the test. The second sample will be taken immediately after the apparatus has been adjusted to the wearer and oxygen has been turned on. Samples will be taken every half-hour thereafter during the test. The physiological effects of the apparatus on the wearer will be noted in each test.

Not more than one test of 2 hours' duration will be made on any one day. The tests will be completed within 60 days from date of beginning, unless prevented by conditions arising which are beyond the control of the mine-safety engineer in charge of the tests.

All tests of apparatus will be conducted in a specially equipped gallery filled with an irrespirable atmosphere, at the Pittsburgh experiment station of the Bureau of Mines.

Before beginning each test the apparatus shall be examined and tested to insure that there is no air leakage under working conditions.

SPECIFICATIONS BY THE BUREAU OF MINES

In order to receive the approval of the Bureau of Mines, self-contained mine-rescue breathing apparatus must pass satisfactorily each of the 15 tests required by the bureau and meet the following requirements:

1. The amount of oxygen supplied by the apparatus must meet the needs of the wearer at all times during the tests.

2. The regenerating material shall absorb, from the expired air, carbon dioxide to the extent that not more than 2½ per cent. shall at any time be present in the inspired air. The average shall not exceed 1 per cent. for any of the two-hour periods of test. This average is to be determined by the analyses of air samples taken as near the point of inspiration as practicable and at uniform intervals of time.

3. The apparatus shall be free from mechanical obstructions in order that the wearer may breathe freely at all times.

4. The temperature of the inspired air must not exceed a maximum of 110 deg. F. when that of the external air does not exceed 85 deg. F. A much lower temperature than 110 deg. F. for the inspired air is desirable. Temperature readings will be taken at regular intervals.

5. The apparatus shall be sufficiently rugged in construction and all vital parts so protected as to prevent material damage or wear to the apparatus during the period of tests to which it will be subjected.

Construction

1. The apparatus shall be designed to meet the needs of the wearer for not less than a period of two hours when worn in irrespirable air without recharging. The apparatus shall be of a design using a mouth-breathing device or other face attachment that when properly adjusted to the face of the wearer, has a capacity of not more than 250 c.c. of dead space inside the face attachment or mouth-breathing device, exclusive of tubes or connections thereto.

Preferably the apparatus shall not weigh more than 36 pounds complete with headpiece and fully charged, and no apparatus weighing more than 40 pounds, complete with headpiece and fully charged, will be accepted for final test.

2. The mechanical construction of the apparatus shall be such that every part can be tested, inspected and repaired by persons skilled in such work, and all parts which require sterilizing shall be readily accessible for this purpose.

3. All parts of the apparatus subject to or liable to be subjected to pressures in excess of 5 pounds per square inch shall be of such construction or equipped with such safety devices as shall insure the safety of the wearer, as determined by the 15 tests.

4. In apparatus equipped with breathing bag or bags, or their equivalent, the inhalation and exhalation compartments shall have a combined capacity of at least 8 liters. If a single breathing bag is used it shall have a capacity of at least 5 liters.

5. The apparatus shall not have in its circulating system any zone of constant negative pressure.

6. The apparatus shall be provided with a release valve, operated by hand or automatically, placed at some point in the circulatory system of the apparatus. The function of this valve shall be to permit the escape to the outside air of a part of the air in the circulatory system of the machine.

7. Where apparatus is equipped with high-pressure oxygen cylinders, such cylinders shall be tested in accordance with the Interstate Commerce Commission specifications No. 3-*A*. Such tests shall be made prior to submitting the apparatus to the Bureau of Mines for test and the applicant submitting the apparatus shall furnish the necessary certificate of test as issued by the Interstate Commerce Commission or submit evi-

dence satisfactory to the bureau's mine-safety engineer in charge of the testing of the apparatus, that such oxygen cylinders have been tested in accordance with Interstate Commerce Commission specifications No. 3-*A*.

8. Where apparatus is equipped with high-pressure oxygen cylinders the safety cap attached to the closing valve shall, in addition to the usual copper disk provided, be filled with a metal (such as Roses metal) fusing at a temperature of approximately 94 deg. C. Such fusible metal shall not extrude from the safety cap under a pressure of 150 atmospheres.

9. The closing valve of such oxygen cylinders shall be provided with the necessary device to prevent the wearer of the apparatus from screwing the stem entirely out of the valve. The closing valve shall also be provided with such a device as will enable the wearer to lock the valve stem when the valve has been opened to the desired point.

10. When apparatus is equipped with gages for recording time or pressures of oxygen supply, such gages will be tested for accuracy of calibration by the Bureau of Mines. A toleration of three atmospheres will be allowed in comparison with the Bureau of Mines standard pressure gage.

11. The apparatus shall be supplied with a valve that will cut off the oxygen supply from the gage; this valve shall be so placed that it can be readily manipulated by the wearer and at the same time not interfere with the flow of oxygen from the oxygen container to the circulatory system of the apparatus.

12. The gage shall be placed on the apparatus at such a point that it can easily be read by the wearer.

13. Apparatus equipped with a reducing valve giving a constant flow of oxygen shall be provided with a by-pass valve which will permit a free flow of oxygen from the oxygen container to the circulatory system of the apparatus independent of the reducing valve.

14. When the oxygen supply of the apparatus is controlled by automatic devices, such devices shall readily adjust themselves to the needs of the wearer.

15. When an apparatus is equipped with mouth-breathing device, such apparatus shall be provided with an adequate saliva trap. The adequacy of the saliva trap will be determined by the tests to which the apparatus will be subjected.

16. When an apparatus is equipped with mouth-breathing attachment, a suitable noseclip shall be provided and properly attached to the apparatus. The suitability of the nose clip will be determined by the tests to which the apparatus will be subjected.

The apparatus under test will be worn during each and all of the 2-hour periods of the 15 tests by the Bureau of Mines safety engineer in charge of the testing or by one or more of his assistants. Immediately before participation in any or all of these tests the prospective wearer of the apparatus under test shall pass, in a satisfactory manner, physical examination by a qualified physician. If it is impossible to carry any one of these tests to completion solely on account of the physical condition of the wearer, where such condition has been brought about through no fault of the apparatus under test, such test shall be disregarded and the apparatus under test shall not be penalized or disqualified thereby.

At the conclusion of each test a note shall be made of the general physical condition of the apparatus and the amount of oxygen, if any, remaining in the container. The schedule of work to be performed by the wearer of the apparatus in each one of the 15 working tests is as follows:

Detail of Procedure in Tests.—Following is an outline of the manner of proceeding in the making of each successive test of breathing apparatus submitted to the bureau.

Test 1.—The wearer of the apparatus shall walk continuously, except for time necessary to take air samples and temperature readings, over a level measured course at the rate of 3½ miles per hour. At the end of each 30-minute period, 2 minutes shall be allowed for taking air samples and temperature readings.

Tests 2, 3, *and* 4 will be repetitions of Test 1.

Test 5.—In Test 5 the wearer of the apparatus shall—

(*a*) Walk over a level measured course at a rate of 3 miles per hour for a period of 10 minutes.

(*b*) Carry a sack of bricks weighing 50 pounds over an overcast ten times, making one complete trip in 2 minutes.

(*c*) Allow two minutes for taking of air samples and temperature readings.

(*d*) Walk at the rate of 3 miles per hour over a level measured course for a period of 10 minutes.

(*e*) Carry a 45-pound weight a distance of 1000 feet, consuming 5 minutes while doing this work.

(*f*) Raise a 45-pound weight through a vertical distance of 5 feet 75 times, consuming 5 minutes while doing this work.

(*g*) Saw wood for a period of 10 minutes.

(*h*) Allow two minutes for taking of air samples and temperature readings.

(*i*) Carry a sack of bricks weighing 50 pounds over an overcast 10 times, making one complete trip in 2 minutes.

(*j*) Walk at the rate of 3 miles per hour over a level measured course until the end of the 2 hours allowed for this test, air and temperature readings to be taken in 2-minute periods at 1½ and 2 hours after start of test.

Tests 6, 7, *and* 8 will be repetitions of Test 5.

Test 9.—In Test 9 the wearer of the apparatus shall—

(*a*) Walk at the rate of 3 miles per hour over a level measured course for a period of 10 minutes.

(*b*) Crawl for a distance of 100 feet, consuming 5 minutes while doing this work.

(*c*) Lie down on side for 5 minutes.

(*d*) Lie down on back for 5 minutes.

(*e*) Allow 2 minutes for taking of air samples and temperature readings.

(*f*) Walk at the rate of 3 miles per hour over a level measured course for a period of 10 minutes.

(*g*) Run 600 feet at a rate of 6 to 8 miles per hour over a level measured course, consuming 2 minutes while doing this work.

(*h*) Walk 1000 feet over a level measured course at the rate of approximately 3 miles per hour, consuming 4 minutes while doing this work.

(*i*) Walk at the rate of 3 miles per hour over a level measured course until end of the 2 hours allowed for this test. Air and temperature readings to be taken in 2-minute periods at one hour, 1½ hours and two hours after the beginning of the test.

Tests 10 *and* 11 will be repetitions of Test 9.

Test 12.—In Test 12 the wearer of the apparatus shall—

(*a*) Walk 1000 feet at the rate of approximately 3 miles per hour over a level measured course, consuming 4 minutes while doing this work.

(*b*) Run 600 feet at a rate of 6 to 8 miles per hour over a level measured course, consuming 2 minutes while doing this work.

(*c*) Walk 1000 feet at the rate of 3 miles per hour over a level measured course, consuming 4 minutes while doing this work.

(*d*) Raise a 45-pound weight 75 times through a vertical distance of 5 feet, consuming 5 minutes while doing this work.

(*e*) Carry a 45-pound weight over a level measured course 1000 feet, consuming 5 minutes while doing this work.

(*f*) Carry a sack of bricks weighing 50 pounds over an overcast 5 times, making one complete trip in 2 minutes.

(*g*) Allow 2 minutes for taking of air samples and temperature readings.

(*h*) Raise a 45-pound weight 75 times through a vertical distance of 5 feet, consuming 5 minutes while doing this work.

(*i*) Walk over a measured course at rate of 3 miles per hour for a period of 10 minutes.

(*j*) Carry a sack of bricks weighing 50 pounds over an overcast 10 times, making one complete trip in 1½ minutes.

(*k*) Allow 2 minutes for taking of air samples and temperature readings.

(*l*) Walk 1000 feet at rate of approximately 3 miles per hour over a level measured course, consuming 4 minutes while doing this work.

(*m*) Raise a 45-pound weight 75 times through a vertical distance of 5 feet consuming 5 minutes while doing this work.

(*n*) Walk at the rate of 3 miles per hour over a level measured course until the end of the two hours allowed for this test. Air and temperature readings are to be taken in 2-minute periods at 1½ and 2 hours after the start of the test.

Tests 13 *and* 14 will be repetitions of Test 12.

Test 15.—This test will be made to determine the maximum length of time that the apparatus will supply the needs of the wearer when in a quiescent state. The wearer will remain as far as possible in a sitting posture throughout the test and perform no work. He will be allowed to manipulate the devices controlling the oxygen supply with a view to conserving such oxygen supply to the greatest advantage.

At the end of each 30-minute period, 2 minutes shall be allowed for taking of air samples and temperature readings.

NOTE.—Self-contained mine-rescue breathing apparatus in course of development may be submitted by manufacturers and inventors for preliminary test or inspection with the view of ascertaining defective construction or the misapplication of safety principles. The nature of such tests or inspection will be determined by the bureau's mine-safety engineer in charge of the testing of such apparatus.

Approval of Apparatus.—The manufacturers of such types of self-contained mine-rescue breathing apparatus as have passed the tests of the bureau will be required to attach to each apparatus a plate containing the following inscription:

Permissible Mine-Rescue Breathing Apparatus,
U. S. Bureau of Mines Approval No. __________.

The use of the plate will not be required if the same inscription is stamped or cast into the metal of the apparatus.

Manufacturers shall, before claiming the bureau's approval for any modification of a permissible self-contained mine-rescue breathing apparatus, submit to the Bureau drawings or parts that shall show the extent and nature of such modifications, in order that the bureau may decide whether test of the remodeled apparatus will be necessary for approval. If it is decided by the bureau that testing of the remodeled apparatus is necessary, the word "permissible" shall not be used on the remodelled apparatus until it has again passed the complete schedule of tests or such part of these tests as the bureau's engineer in charge of the tests shall deem necessary.

The bureau will, on application, make separate tests, identical with the foregoing tests, of regenerators manufactured for use in connection with any mine-rescue breathing apparatus that has been approved by the bureau under the provisions of this schedule.

Regenerators that fulfill the requirements of the foregoing tests will be approved for use only in connection with that particular type of apparatus for which they are designed and which has previously received the bureau's approval.

The listing by the Bureau of Mines, as "permissible," any self-contained mine-rescue breathing apparatus shall be construed as applying only to apparatus of that specific type, class, form and rating, made by the same manufacturer, which have the same construction in all details directly or indirectly affecting the safety features of the apparatus.

The bureau reserves the right to rescind for cause, at any time, any approval granted under the conditions herein set forth. Cause for rescinding of approval shall be considered to be the use of the bureau's issuance of approval in an unauthorized manner; that is, placing the approval stamp on apparatus that has not been approved by the bureau, or on apparatus certain parts of which have been altered in construction or material without submittal to the bureau for test.

Notification to Manufacturer.—As soon as the mine-safety engineer of the Bureau of Mines is satisfied that a self-contained mine-rescue breathing apparatus has passed all the tests herein set forth in a satisfactory manner, the manufacturer or inventor shall be formally notified to that effect.

When two or more applications for tests on different apparatus are received within a period of 10 days, the announcement of approval for each shall not exceed the interval of time between the receipt of the applications.

When a manufacturer or inventor receives this formal notification he shall be free to advertise this type of successfully tested self-contained mine-rescue breathing apparatus as permissible according to the Bureau of Mines standards and may attach approval plates to this type of breathing apparatus.

Fees for Testing.—Careful investigation has been made regarding the necessary expenses involevd in testing mine-rescue breathing apparatus, at the Pittsburgh experiment station of the bureau. The following schedule of fees to cover expenses to be charged on and after March 5, 1919 has been established and approved by the Secretary of the Interior, in accordance with the provisions of the statute previously quoted,

Complete mine-rescue breathing apparatus test	$100
Separate preliminary inspection and test	$10
Separate regenerator test	$5
Separate inspection and test of reducing valves	$10

The fees specified above may be increased to cover the cost of testing an unusually complicated type of mine-rescue breathing apparatus, and are also subject to change upon the recommendation of the Director of the Bureau of Mines and the approval of the Secretary of the Interior.

Application for Test of Apparatus.—1. Application for tests should be addressed to the director of the Bureau of Mines, Washington, D. C. This application must be accompanied by check or draft made payable to the Secretary of the Interior, and by a complete written description of the mine-rescue breathing apparatus to be tested, and a set of the drawings as specified in the Conditions of Testing, page 148, and marked "Drawings of Approved Mine-Rescue Breathing Apparatus to be Filed." Duplicate copies of the application and drawings should be sent to the mine-safety engineer, Bureau of Mines, Pittsburgh, Penn.

2. As soon as the application is received by the bureau's mine-safety engineer, the applicant will be notified of the date the tests will begin.

3. After the applicant has received this notification, he should send the material required to the mine-safety engineer, Bureau of Mines, Pittsburgh, Penn. This material should be delivered not less than one week in advance of the date set for the beginning of the tests.

4. The tests will be begun on the date set and continued until the mine-rescue breathing apparatus has been approved, rejected or withdrawn.

5. After the bureau's mine-safety engineer has considered the results of the tests, a formal report of the approval of the self-contained mine-rescue breathing apparatus will be made to the applicant, in writing, by the director of the Bureau of Mines. No verbal report will be made, and the details of the test will be regarded as confidential by all present.

Approved March 5, 1919.

S. G. Hopkins, *Assistant Secretary.*

Van H. Manning, *Director.*

FIRST-AID WORK

Practical Use of Breathing Apparatus.—It is of the greatest importance that all breathing apparatus should be carefully examined and tested before the wearer proceeds to enter an irrespirable atmosphere. First, it is necessary to observe the gage or meter to see that the proper supply of oxygen is contained in the oxygen cylinder. Observe also that the required quantity of oxygen (2 liters) is being delivered each minute, as indicated by a registering meter. The breathing bag must be carefully tested and all valves examined to see that they are in good working condition and to ascertain that the breathing bag contains no airleaks.

In use, always inflate the bag with pure air when ready to put on the apparatus and before turning on the supply of oxygen. It is well for the wearer, then, to take the precaution of going into a smoke chamber, for a short period before entering the mine. This will enable him to ascertain that there are no leaks in the apparatus and that breathing is normal.

Resuscitation.—To resuscitate is to revive, or to restore animation in an unconscious person or one who is seemingly dead. A person may be apparently lifeless as the result of any one of several causes; (1) Fainting from overexertion. 2. The result of a nervous shock. 3. An electric shock, received by contact with a live wire. 4. Suffocation, by reason of inhaling irrespirable gases, or the lungs being filled with water, as in drowning. 5. A blow on the head. In fact, unconsciousness may result from any accidental occurrence affecting directly or indirectly the nervous system on which respiration and animation depends.

In the work of resuscitation, due regard must always be had to the cause of suspended animation. Where the lungs have filled with water, as in drowning, or with gas inhaled in the mine or elsewhere, immediate steps must be taken to drive the water or gas from the lungs and permit the entry of fresh air through artificial respiration applied vigorously and continued till the person revives, or it is absolutely certain that life is extinct. If the trouble arises from the inhalation of gas, the victim must be removed promptly to fresh air before treatment is administered, loosen the clothing about the neck and chest and give artificial respiration, at the same time chafing the limbs, rubbing them toward the body to assist the flow of the venous blood back to the heart.

Smelling salts applied to the nostrils assist to quicken animation. As soon as the victim is able to swallow and on the first signs of returning life, give a stimulant, hot coffee or tea, or half a teaspoonful of aromatic spirits of ammonia in a half-glass of water, administered in small doses at slight intervals. Where shock has resulted from injury and loss of blood, however, stimulants should not be given, as these will assist the action of the heart and increase the flow of blood from the wound. In all other cases, return of animation will be assisted by any means that will assist the circulation of blood and revive the respiratory system. Keep the patient warm with blankets and give plenty of fresh air during treatment for resuscitation.

Artificial Respiration.—There are two general methods of applying artificial respiration. In the **Sylvester method,** which is now little used, the patient is laid on his back, while the operator kneeling at his head grasps the wrists of both arms and proceeds to alternately swing the arms, first forward on the chest and then back to a position above the head, at the normal rate of breathing or, say 16 times a minute. In the forward movement, the arms are doubled at the elbow and pressed down firmly against the sides of the chest so as to compress the lungs and force out the gas therefrom. This is followed by the backward movement, which has the effect of expanding the lungs and inducing inhalation. These movements are continued alternately, first compressing the lungs and then expanding them in turn. While doing this, it is

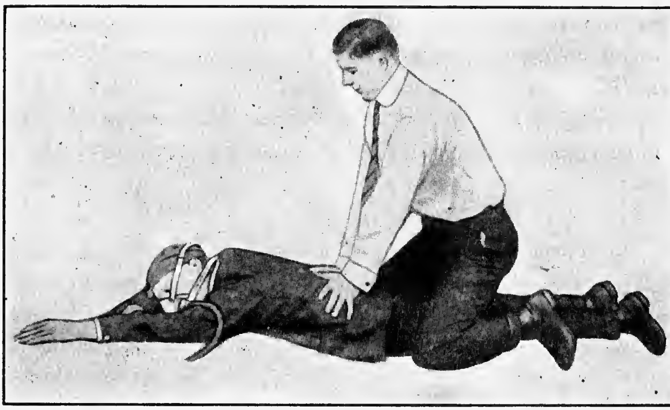


Fig. 20.

important to secure the tongue and hold it forward in the mouth so that it will not impede the access of air to the lungs. A handkerchief covering the fingers will help to hold the tongue forward, or a clip must be used for that purpose.

The common method of resuscitation now most generally employed is that known as the **"Schaefer method,"** or the "prone method" of resuscitation. By this method, the patient is laid prone on his face, except that the head is turned to one side to facilitate breathing. The operator, having made sure that the tongue is drawn forward in the mouth so as to give free access of air to the lungs, straddles the patient's thigh, as shown in Fig. 20, and rests the palms of his hands on the person's loins with the two thumbs together and the fingers reaching well down on each side, in a manner to bring pressure on the short ribs and across the small of the back.

In this position, the operator first swings forward so as to throw his weight on the patient's body compressing the lungs to drive out the gas or water they contain. Then, swinging backward, he gives opportunity for the expansion of the lungs, which induces the inhalation of fresh air. As in the Sylvester method, this forward and backward movement must be continued alternately, for a period of an hour or two, until there are signs of returning life or it is absolutely necessary that life is extinct. There are instances on record where the victim has been revived after several hours of hard work. It is often necessary for the operator to be relieved for a time by another, but the process must be continued without cessation, until a doctor gives it as his opinion that life has fled. In every case, send for a doctor while giving first-aid to the patient.

SECTION VI

THEORY OF VENTILATION

Mine Ventilation—Problems—Flow of Air in Airways—Ventilating Pressure, How Produced and Measured, The Water Gage—Velocity of air Currents—Quantity of Air, Requirements—Work or Power on the Air—Equivalents in Measurement—Examples for Practice—Mine Airways—Symbols and Formulas—Mine Potential Methods—Measurement of Air Currents—Examples for Practice—Tandem Circulations—Splitting the Air Current—Natural Division of Air—Examples in Natural Division—Proportionate Division of Air, Regulators—Secondary Splitting—Theoretical Considerations in Splitting—Practical Problem

MINE VENTILATION

The ventilation of a mine, as the term implies, involves the supply and maintenance of a sufficient current of air throughout the mine to render the same healthful and safe.

Requirements of Ventilation.—The quantity of air in circulation must be sufficient to comply with the state mining law, and to dilute, render harmless and sweep away the gases that would otherwise accumulate in the mine. The air current must be conducted so as to sweep the entire working face and all void places with a moderate velocity sufficient to remove the gas without danger from the lamps or inconvenience to the workmen.

The Circulating System.—In order to circulate a current of air through a mine, it is necessary to provide two separate openings, one for the air to enter, called the "intake opening," and the other for it to leave the mine, called the "return" or "discharge opening." Two distinct air passages or airways are also required, leading from these openings into the mine, in order to conduct the air current to and from the working face. These are called, respectively, the "intake" and "return" airways. These openings and airways form a part of the circulating system in the mine, similar to the arteries and veins of the human body.

Kinds of Ventilation.—There are three different kinds of ventilation, in mining practice, known as "natural ventilation," "furnace ventilation" and mechanical or "fan ventilation," according to the agency employed for its production.

Natural Ventilation.—Ventilation is natural when it is produced by any natural agency, such as surface winds, falling water or the natural heat of the mine. The accompanying Fig. 21 illustrates the manner in which the natural heat of the mine produces a warm upcast air column, in either a drift mine or a shaft mine.

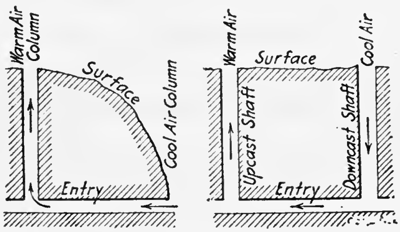


FIG. 21.

In the drift mine shown on the left, the warmer air column in the shaft only partly balances the cooler outside air. Above the level of the top of the shaft the two air columns are of equal temperature and equal weight, and, therefore, need not be considered since they balance each other. The same is true in the shaft mine shown on the right, whenever the two shafts have the same elevation at the surface.

Natural Ventilation in Slope Mines and Dip Workings.—A similar condition in respect to the natural heat of the mine producing or modifying the circulation of the air, holds in all slope mines and dip workings, the same as in shafts and drifts. Whenever the mine temperature is much below or above that of the outside atmosphere, the difference in temperature makes the return air heavier or lighter than the intake air; and the difference in weight of these two air columns destroys the equilibrium of the mine air and creates a current in the airways throughout the mine.

A considerable difference of temperature is often observed between the dip and rise air currents in particular sections of a mine. It is this difference in the temperatures of the intake and return currents that often makes dip workings harder to ventilate in summer than in winter. For the same reason, rise workings are frequently found to be more easily ventilated in the summer season.

Air Columns.—The term "air column," like water column, always refers to a vertical column. The air column, in ventilation, is an imaginary vertical column of air, of unit section (commonly, 1 sq. ft.) and of such height that its weight, in pounds, is equal to the pressure it measures (lb. per sq. ft.). The density of the air (wt. per cu. ft.) is either stated or understood, so that when the height of air column is given the pressure it indicates is readily calculated.

In mining practice, it is common to express ventilating pressure in feet of air column or, as we say, "head of air." Calling the weight of 1 cu. ft. of air w (lb.) and the head of air column h (ft.), the pressure p (lb. per sq. ft.) is calculated by the formula

$$p = wh$$

Or the air column corresponding to any given pressure is found by transposing this formula; thus,

$$h = \frac{p}{w}$$

Example.—What is the head of air column corresponding to a ventilating pressure of 10 lb. per sq. ft., assuming a temperature of 60 deg. F. and a barometric pressure of 30 in.?

Solution.—The weight of 1 cu. ft. of air, at the given temperature and pressure is

$$w = \frac{1.3273B}{460 + t} = \frac{1.3273 \times 30}{460 + 60} = 0.0766 \textit{ lb., nearly}$$

The required head of air is then

$$h = \frac{p}{w} = \frac{10}{0.0766} = 130.5 \textit{ ft.}$$

Example.—Find the ventilating pressure and water gage corresponding to 80 ft. of air column, at the same density.

Solution.—

$$p = wh = 0.0766 \times 80 = 6.128 \textit{ lb. per sq. ft.}$$
$$w.g. = 6.128 \div 5.2 = 1.18 \textit{ in., nearly}$$

Furnace Ventilation.—When the circulation of air throughout a mine is created and maintained by means of a furnace built in the mine the system is known as "Furnace ventilation."

Principle of Furnace Ventilation.—The heat of the furnace imparted to the air in the furnace shaft makes it lighter, volume for volume, which causes it to rise in obedience to the law of the equilibrium of fluids. The cooler and heavier outside air, in obedience to the same law, flows into the mine by way of another opening, to take the place of the air displaced. The action is continuous as long as the furnace is in operation. There is thus created and maintained a constant flow of air into and through the mine.

Location of a Mine Furnace.—The furnace is built in the main-return airway about 20 or 25 yd. back from the foot of the upcast or furnace shaft, so as to reduce the danger of the fire damaging or destroying the shaft.

Construction of Furnace.—The essential details to be considered in the construction of an efficient mine furnace are the following:

1. Beginning, say 50 yd. back from the foot of the shaft, the main-return airway should be gradually widened and its height increased so that the unobstructed sectional area at the furnace will not be less than 25 per cent. greater than that of the original airway.

2. The roof of the enlarged airway should then be secured by steel rails or I beams supported on posts or concrete walls, as illustrated in Fig. 22, which represents a well built mine furnace.

3. As shown in the figure, both the concrete walls and the brick walls supporting the arch are started on a good firm bottom below the floor line. The thickness of the concrete walls will vary from 10 or 12 in. to 2 ft., depending on depth

of cover and other roof conditions. The brick walls and arch will vary in thickness from 8 to 12 in. A good quality of vitrified brick should be used, except where the arch and walls are exposed to the direct action of the flame they should be lined with the best firebrick. All bricks should be first soaked in water before being laid and only the best cement mortar should be used.

4. The brick walls and arch should be started about 2 yd. in front of the furnace proper and extended to the face of the shaft. The clear width between the walls should equal the width of the fire-grate, and should be such as to leave a clear passageway between the brick and concrete walls.

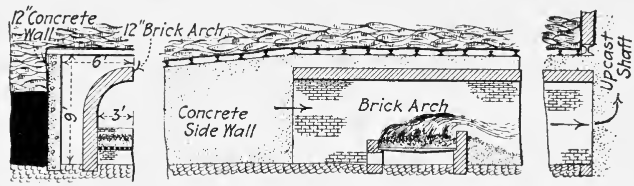


FIG. 22.

The arch is semicircular and sprung at such a height above the floor as to leave not less than 12 in. of space between the crown of the arch and the rails that support the roof. The purpose of this air space around the furnace is to isolate the heat, which is thus more completely utilized in heating the air current.

5. The area of the grate or the grate surface must be sufficient to burn the weight of coal per hour required to heat the volume of air passing the furnace in that time, to a temperature that will create the air column, in a given depth and condition of shaft, necessary to circulate such volume of air against a specified mine potential.

The theoretical problem of determining the weight of coal burned per hour, per volume of air circulated, is thus seen to depend on many factors. In ordinary mining practice, however, a safe estimate is to assume that each pound of coal burned per hour will cause a rise in temperature of from 10 to 15 deg. F., per 1000 cu. ft. of air in circulation. Or, calling the weight of coal burned W (lb. per hr.); the volume of air passing Q_m (1000 cu. ft. per min.); the rise in temperature t (deg. F.), and the temperature constant c = 10 to 15 deg. F.,

$$W = \frac{Q_m t}{c}$$

Example.—Find the weight of coal required per hour, to produce a rise of temperature of 360 deg. F., in a furnace shaft when a current of 100,000 cu. ft. of air per minute is passing, under fair mining conditions.

Solution.—The weight of coal required is

$$W = \frac{Q_m t}{c} = \frac{100 \times 360}{12} = 3000 \textit{ lb. per hr.}$$

In very deep or wet shafts or a comparatively small mine resistance, giving a larger air volume and greater loss of heat, the constant 10 deg. should be used; while in dry shafts of less depth, especially if the mine resistance is considerable, a temperature constant of 15 or even 16 may be employed to find the necessary weight of coal.

6. The grate area necessary to burn any required weight of coal W (lb. per hr.) varies with the hardness and the inflammability of the coal. A mine furnace will commonly burn from 15 to 20 lb. of anthracite, or from 20 to 25 lb. of bituminous coal, per square foot of grate, per hour. Hence the weight of coal required, divided by such constant will give the necessary area of grate surface, in square feet.

Example.—What grate area will be required to burn, say 3000 lb. of a very soft, inflammable coal per hour?

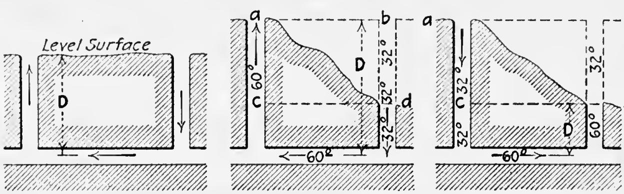
Solution.—In this case, the coal being a free-burning, inflammable coal, the constant 25 should be used; and the required area is 3000 ÷ 25 = 120 *sq. ft.*

Estimation of Air Columns in Practice.—In the ventilation of shaft or slope mines or rise and dip workings in inclined seams, the weight of each respective downcast and upcast column is sometimes calculated separately, by multiplying the weight of 1 cu. ft. of air, at a barometric pressure B and a temperature t equal to the average temperature of the column, by the height or depth D of the same column, as expressed by the formula

$$w = D\frac{1.3273B}{460 + t}$$

All air columns are of unit cross-section (1 sq. ft.) and the calculated weight of the column, therefore, gives the corresponding pressure in pounds per square foot.

Positive and Negative Air Columns.—An air column that acts to assist the circulation in the mine or airway is called a "positive" column; while one that acts to oppose the circulation is termed a "negative" column. In fan ventilation, a negative air column may exist in the downcast shaft by reason of its temperature being greater than that of the upcast, which frequently happens in the summer season.



a *b* *c*

FIG. 23.

Conditions.—The height or depth D of air column, in any particular case, can only be determined by carefully considering the conditions. It is important to remember that, with few exceptions, the temperature of a downcast-shaft column will closely approximate that of the outer air with which this shaft is constantly filled; while the temperature of the upcast column is practically determined by that of the mine or, in furnace ventilation, by the furnace.

When two shafts, upcast and downcast, Fig. 23, (*a*), are sunk from a level surface or, in other words, have the same surface elevation it is evident that this level marks the upper limit of both columns.

When, however, the two shafts are sunk on a hillside and have different surface elevations, two cases may arise, as illustrated in Fig. 22, (*b*) and (*c*), in which, for the sake of clearness, the outside temperature is assumed as 32 deg. F. and that of the mine as 60 deg. F.

The two cases are as follows:

1. When the shaft having the higher surface elevation is made the upcast, as is usually done, that elevation marks the upper limit of both shaft columns; because the downcast shaft has practically the same temperature as the outer air.

2. When the shaft having the lower surface elevation is made the upcast this elevation marks the upper limit of both shaft columns; because the air in the other (downcast) shaft above this level is balanced by the corresponding column of outside air.

These two conditions, therefore, are simply expressed by the statement that, in either case, the upper limit of both shaft columns is the surface level of the upcast shaft.

In the same manner it can be shown that the lower limit of both shaft columns is the bottom of the downcast shaft when the seam has a general inclination. Hence, the length (D) of both shaft columns is measured, in any case, from the top of the upcast to the bottom of the downcast shaft. This rule does not apply to slopes.

Ventilating Pressure and Shaft Columns.—Since the weight of an air column, in pounds, expresses the corresponding pressure, in pounds per square foot; and since ventilating pressure (lb. per sq. ft.) is the difference of pressure between the intake and return; the unit pressure p, in any given case, is found by subtracting the weight of the upcast-shaft column from that of the downcast column; thus,

Downcast-shaft column, $$w_d = \frac{1.3273B}{460 + t} D$$

Upcast-shaft column, $$w_u = \frac{1.3273B}{460 + T} D$$

Unit pressure, $$p = 1.3273B \left(\frac{1}{460 + t} - \frac{1}{460 + T} \right) D$$

which can be written

$$p = \frac{1.3273B\ (T - t)\ D}{(460 + T)(460 + t)}$$

Calculation of Air Column.—The air column corresponding to the above unit ventilating pressure can be expressed in terms of either the downcast or upcast air. The air in the downcast being heavier than that in the upcast, gives a shorter air column for the same pressure.

To find the air column (h_d) in terms of the **downcast air,** divide the above expression for unit ventilating pressure by the weight (w_d) of 1 cu. ft. of downcast air (temp. = t), which gives

$$h_d = \frac{p}{w_d} = \frac{(T - t)\,D}{460 + T}$$

To find the corresponding air column (h_u) in terms of the **upcast air,** divide the same expression for unit ventilating pressure by the weight of 1 cu. ft. of upcast air (temp. = T), which gives

$$h_u = \frac{p}{w_u} = \frac{(T - t)\,D}{460 + t}$$

Effective Depth of Air Column.—It has been shown that in all shaft ventilation the effective "head of air column" D is the difference in elevation of the top of the upcast and the bottom of the downcast. This applies equally to all forms of natural, furnace or fan ventilation, in shaft mines, where a positive or negative air column may exist.

Likewise, in drift or slope mines, the same law will apply, except where a long slope causes an appreciable rise in the temperature of the downcast air; and in the furnace ventilation of a slope mine. In either of these two cases, three temperatures may be concerned: (1) average upcast temperature in the shaft; (2) average downcast temperature in the slope; (3) outside temperature.

In furnace ventilation, in inclined seams, also, three temperatures must be considered: (1) average temperature of the furnace (upcast) shaft; (2) mine temperature, rise or dip of seam; (3) average downcast temperature. In a few cases, a fourth (4) outside temperature may require consideration. In all cases where more than two temperatures are concerned it is necessary to calculate the column for each separate temperature and corresponding depth and take their algebraic sum.

In practice, the arrangement of the circulation in the mine may be such that the rise or dip column is eliminated by a balance of intake and return columns of equal temperature.

PROBLEMS

Example.—A shaft mine, in a level seam, is ventilated by a furnace. The furnace shaft is 900 ft. deep and has an average temperature of 300 deg. F.; the downcast shaft is 600 ft. deep. Calculate the air column producing circulation in this mine and the corresponding ventilating pressure and water gage when the temperature of the outside air is 20 deg. F. and the barometer 30 in.

Solution.—The effective head of air, in this case, is D = 900 ft. and, assuming that the temperature of the downcast shaft is practically the same as that of the outside air, which is commonly true, the air column, expressed in terms of the downcast air, is

$$h_d = \frac{(T - t)D}{460 + T} = \frac{(300 - 20)\ 900}{460 + 300} = \frac{280 \times 900}{760} = 331.5\ ft.$$

Expressed in terms of the upcast air the air column, in this mine, is

$$h_u = \frac{(T - t)D}{460 + t} = \frac{(300 - 20)\ 900}{460 + 20} = \frac{280 \times 900}{480} = 525\ ft.$$

The pressure is found by multiplying either of these air columns by the corresponding weight of downcast or upcast air.

Thus (downcast), $$p = \frac{1.3273 \times 30}{460 + 20} \times 331.5 = 27.5\ lb.\ per\ sq.\ ft.$$

Or (upcast), $$p = \frac{1.3273 \times 30}{460 + 300} \times 525 = 27.5\ lb.\ per\ sq.\ ft.$$

The corresponding water gage is, then,

$$w.g. = 27.5 \div 5.2 = 5.3\ in.,\ nearly$$

Example.—A slope mine is ventilated by means of a blowing or force fan located at the top of an air shaft 800 ft. deep. The slope is the main return airway and the elevation at its mouth is 275 ft. below that of the top of the air shaft. What natural air column exists, assuming the temperature of the mine is 60 deg. and that of the outside air 10 deg. below zero (−10°F.); and is this positive or negative?

Solution.—The effective head of air, in this case, is $D = 800 - 275 = 525$ ft.; because the downcast fan shaft has the same temperature as the outside air column, which therefore balances 275 ft. of the shaft column. The downcast air in the shaft being colder and heavier than the upcast or return air in the slope, the resulting air column assists the circulation produced by the fan and is, therefore, a positive air column. It is

$$h_d = \frac{[60 - (-10)] \times 525}{460 + 60} = \frac{(60 + 10)\ 525}{520} = \frac{70 \times 525}{520} = 70.67\ ft.$$

This air column is in terms of the downcast air, which weighs, assuming a barometric pressure $B = 30$ in.,

$$w_d = \frac{1.3273 \times 30}{460 + (-10)} = \frac{39.819}{450} = 0.0885\, lb., \textit{ nearly}$$

The natural pressure due to this air column is then

$$p_n = 70.67 \times 0.0885 = 6.25 \textit{ lb. per sq. ft.}$$

Ques.—If the fan, in this example, were to be reversed so as to exhaust air from the mine, thereby making the slope the intake and the fan shaft the upcast, what air column would result, if the average slope temperature is then 40° F.?

Ans.—In this case, three air columns exist, two assisting and one opposing the circulation induced by the fan. They are as follows:

Outside column (positive), $w_o = \frac{1.3273 \times 30}{460 - 10} \times 275$

Slope column (positive), $w_s = \frac{1.3273 \times 30}{460 + 40} \times 525$

Shaft column (negative), $w_u = \frac{1.3273 \times 30}{460 + 60} \times 800$

The net air column, expressed in terms of, say the slope air, is now found by dividing the algebraic sum of these positive (+) and negative (−) columns by the weight of 1 cu. ft. of the slope air, which gives after simplifying,

$$h_s = (460 + 40)\left(\frac{275}{460 - 10} + \frac{525}{460 + 40} - \frac{800}{460 + 60}\right)$$

$$= 500\left(\frac{275}{450} + \frac{525}{500} - \frac{800}{520}\right) = 61.3\, ft. \textit{ (positive)}$$

The weight of 1 cu. ft. of slope air is

$$w_s = \frac{1.3273 \times 30}{460 + 40} = \frac{39.819}{500} = 0.0796\, lb.$$

The natural pressure assisting the circulation is then

$$p_n = 61.3 \times 0.0796 = 4.88 \textit{ lb. per sq. ft.}$$

Example.—To show the effect of natural air columns in fan ventilation, assume a shaft mine ventilated by means of a fan; the seam is practically level; the fan shaft is 800 ft. deep and the hoisting shaft 600 ft. deep.

(*a*) Assume the fan is exhausting and produces a circulation of 200,000 cu. ft. of air against a water gage of 2 in., in the winter when the outside temperature is 30 deg. and that of the mine 60 deg. F., and calculate the resulting water gage and the volume of air that the fan will circulate, running at the same speed in the summer season when the outside temperature is 70 deg. and that of the mine, as before, 60 deg. F.

(*b*) Assume the same conditions in the mine and the same respective temperatures and calculate the water gage and volume of air this fan will produce when running at the same speed and blowing instead of exhausting the air, for the winter and summer seasons, respectively.

Solution.—(*a*) When the fan is exhausting, the fan shaft being the upcast, the effective depth of air column is $D = 800$ ft. The natural water gage due to this depth (barom., B. = 30 in.) is

$$\text{Winter,} \quad w.g._n = \frac{1.3273 \times 30(60 - 30)800}{(460 + 60)(460 + 30)5.2} = 0.72 \textit{ in. (positive)}$$

$$\text{Summer,} \quad w.g._n = \frac{1.3273 \times 30(70 - 60)800}{(460 + 70)(460 + 60)5.2} = 0.22 \textit{ in. (negative)}$$

In the circulation of 200,000 cu. ft. of air, under a 2-in. water gage, as stated in the question, therefore, the water gage due to the action of the fan is 2 − 0.72 = 1.28 in., the natural water gage, in this case, assisting circulation, being positive. In the summer season, the fan exhausting at the same speed as before will create the same ventilating pressure and water gage (1.28 in.); but, the natural air column now being negative (0.22 in.), the effective water gage producing circulation is 1.28 − 0.22 = 1.06 in. Then, since the circulation in any given mine or airway varies as the square root of the pressure or water gage, the quantity ratio is equal to the square root of the water-gage ratio.

$$\frac{x}{200{,}000} = \sqrt{\frac{1.06}{2}} = \sqrt{0.53} = 0.728$$

Summer (exhausting), $x = 200{,}000 \times 0.728 = 145{,}600$ *cu. ft. per min.*

(*b*) When the fan is blowing the hoisting shaft is the upcast and the effective depth of air column is then $D = 600$ ft. The natural water gage is then 600/800 = ¾ of the value previously found; or ¾ × 0.72 = 0.54 in. (winter), and ¾ × 0.22 = 0.165 in. (summer). As before, the natural gage is positive in winter and negative in summer, which makes the effective gage 1.28 + 0.54 = 1.82 in. (winter) and 1.28 − 0.165 = 1.115 in. (summer). The circulation is then

$$\text{Winter (blowing),} \quad x = 200{,}000\sqrt{\frac{1.82}{2}} = \text{say } 190{,}800 \textit{ cu. ft. per min.}$$

$$\text{Summer (blowing),} \quad x = 200{,}000\sqrt{\frac{1.115}{2}} = \text{say } 149{,}400 \textit{ cu. ft. per min.}$$

FLOW OF AIR IN AIRWAYS

The flow of air in a conduit or airway is in obedience to an excess of pressure at one end of the conduit over that at the other end. Air always moves from a point of higher pressure toward a point of lower pressure. The moving air is called the air current.

Velocity of Air Currents.—The rate of motion or the distance traveled per unit of time is called the velocity of the air current. The velocity is commonly expressed in feet per second or feet per minute, as most convenient.

Relation of Pressure and Velocity.—To double the velocity of air in an airway or conduit requires four times the pressure; and since $2 = \sqrt{4}$, the velocity v varies as the square root of the pressure p; thus

$$v \text{ varies as } \sqrt{p}$$

or, vice versa,

$$p \text{ varies as } v^2$$

For example, if an airway in a mine is of such size and length that the pressure per square foot at the intake is 3 lb. greater than that at the discharge opening, and this difference of pressure produces a velocity of 5000 ft. per min.; it will require a difference of pressure of $4 \times 3 = 12$ lb. per sq. ft. to produce a velocity of 1000 ft. per min. in the same airway.

Solution by Ratios.—Expressed as ratios, the solution is always simpler and shorter, because the method admits of ready cancellation, thereby keeping the numbers small and reducing the amount of necessary work. For example, when quantities are proportional their ratios are equal. Or, in this case, the velocity ratio is equal to the square root of the pressure ratio. Calling the first velocity v_1, second velocity v_2; the first pressure p_1 and the second pressure p_2, we have

$$\frac{v_2}{v_1} = \sqrt{\frac{p_2}{p_1}}$$

or, vice versa,

$$\frac{p_2}{p_1} = \left(\frac{v_2}{v_1}\right)^2$$

Example.—What difference of pressure per square foot will be required to produce a velocity of 1200 ft. per min. in an airway where the air is moving at the rate of 500 ft. per min., under a moving pressure of 3.5 lb. per sq. ft.?

Solution.—Let x = the required difference of pressure; then

$$\frac{x}{3.5} = \left(\frac{1200}{500}\right)^2 = \left(\frac{12}{5}\right)^2 = \frac{144}{25} = 5.76$$

$$x = 3.5 \times 5.76 = 20.16 \textit{ lb. per sq. ft.}$$

Example.—If a difference of pressure between the two ends of an airway, of 8 lb. per sq. ft., produces a velocity of 600 ft. per min., what will be the velocity in the same airway when the difference of pressure is only 2 lb per sq. ft.?

Solution.—In this case, calling the required velocity x,

$$\frac{x}{600} = \sqrt{\frac{2}{8}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$x = 600 \times \tfrac{1}{2} = 300 \textit{ ft. per min.}$$

VENTILATING PRESSURE

Pressure Producing Circulation.—In mine ventilation, the term "ventilating pressure" is the pressure exerted to move the air. It is the difference between the intake pressure and the discharge pressure. Since the pressure of the atmosphere is equal at both ends of the airway it may be disregarded, as far as the movement of the air is concerned.

The Blowing System of Ventilation.—To move the air or cause it to circulate in an airway or a mine, an extra pressure must be created at one end of the airway, so as to overcome the resistance of the mine due to friction. This is called the **"blowing"** system of ventilation, because the air is blown through the airway by the pressure created.

The Exhaust System of Ventilation.—The same difference of pressure may be caused by decreasing the atmospheric pressure at one end of the airway, when the full pressure of the atmosphere at the other end will cause the air to move toward the point where the pressure is less. The principle is that commonly called "suction;" but this system is known as the **"exhaust"** system of ventilation.

How Pressure is Produced.—Various means have been used to cause a circulation of air in mine airways. The wind cowl, waterfall and steam jet are useful under favorable conditions and where a limited air supply only is needed. The mine furnace, built in the mine near the bottom of the upcast shaft, is often used in nongaseous mines, especially in deep shafts (see Furnace Ventilation). The most reliable means of creating pressure in mine ventilation, however, is the mine fan, which is generally erected at the surface, either at the top of the downcast shaft, as a blower; or at the top of the upcast, as an exhaust fan (see Fan Ventilation). The blowing fan creates a pressure above that of the atmosphere, while the exhaust fan reduces the atmospheric pressure.

How Pressure is Estimated.—In mine ventilation, the pressure producing circulation is estimated in height of air column, as in natural ventilation and often in furnace ventilation. The more common method, however, is to state the pressure in pounds per square foot or ounces per square inch. Pressure is also stated in inches of water gage. These all refer to the unit of ventilating pressure or simply "unit pressure."

Atmospheric pressure is given in pounds per square inch, or, as barometric pressure (which is the same as atmospheric pressure), in inches of mercury.

1 in. water gage	= 5.2 *lb. per sq. ft.*
1 in. mercury	= 0.491 *lb. per sq. in.*
1 oz. per sq. in.	= 9 *lb. per sq. ft.*
1 in. mercury	= 13.6 *in. water gage*

How Pressure is Measured.—In mine ventilation, the pressure producing circulation is commonly measured by means of the water gage; or, in case of high pressures a special form of manometer is sometimes used. The manometer differs from the water gage in having one end of the bent tube closed so that the rise of the water level in that arm of the tube compresses the air above the water, which lessens the rise of water level and gives a greater range of readings.

The Mine Water Gage.—This consists of a glass tube of about 3/8-in. bore, bent to the shape of the letter **U** and mounted on a solid base. Three styles of water gage are shown in Fig. 24. These differ only in the kind of scale. The first two on the left have the zero at the center of the scale and read up and down to the respective water levels. The first of these scales is graduated to full-length inches, and to obtain a correct reading it is necessary to add the two readings together, or double either of them, as they are equal. To avoid this necessity the second scale is made of half-length inches, so that either the upper or the lower reading gives the full gage required, which, in this case, is 3 inches. As shown in the figure, the scale is adjustable by means of the screw rod on which it is mounted.

When the zero of the scale is at the middle and the scale reads up and down, it is evident that the scale must be adjusted so that its zero will correspond with the two water levels, before the pressure acts on the gage. When the pressure acts it depresses the water level in one arm while that in the other arm rises an equal amount. The difference between these two levels is the actual water column supported by the difference in the pressures acting on the water in the two arms. As will be explained later, one arm of the gage when in position is open to the intake pressure and the other to the return. The difference between these two pressures is the pressure that circulates the air between these two points.

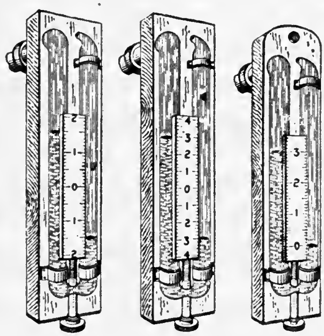


FIG. 24.

The scale shown on the right has its zero at the bottom and reads upward. This scale must evidently be set, after the gage is in position, so that the zero will correspond with the lower water level, which is always that in the arm open to the intake pressure, as that pressure is always greater than the return pressure. The reading of the scale at the upper level is then the required gage.

The reading of each of the three gages shown in the figure is 3 in., which indicates a ventilating pressure of $3 \times 5.2 = 15.6$ lb. per sq. ft.

Reading the Water Gage.—In the common use of the water gage, in mine practice, the scale is not read closer than ⅛ in. On the left of Fig. 25, is shown a portion of a water column and scale graduated to eighths of an inch. The scales shown in Fig. 24 are decimal scales, being graduated to tenths of an inch for greater accuracy. In all engineering practice, therefore, and whenever accuracy is desired the decimal scale shown in Fig. 24 is used and the reading taken to tenths or hundredths of an inch.

There are several sources of possible error in reading the mine gage. If the gage is not truly vertical the reading will not be correct. Error often occurs from the cupping of the surface of the water in the tube. As shown in Fig. 25, the

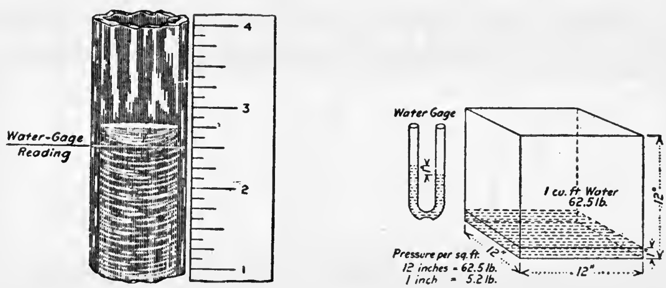


Fig. 25.

reading of the gage should be taken at the bottom of the concave or bowl. This will give greater uniformity in the results obtained.

In fan ventilation, especially when the reading is taken in the fan drift, there is a constant oscillation of the water level, which makes it difficult to decide on the true reading. The oscillation is much reduced when the tube of the gage is contracted at the bend. The best gages are provided with a stop-cock in the bend by which the connection between the two arms can be closed. The gage can then be carried to a more convenient place to be read.

Unit of Ventilating Pressure.—In mine ventilation, the unit of ventilating pressure, or the unit pressure producing the circulation, is estimated in pounds per square foot This is calculated from the reading of the water gage by multiplying that reading, in inches, by 5.2.

On the right, in Fig. 25, is shown clearly how the constant 5.2 is derived. The weight of 1 cu. ft. of water is, practically, 62.5 lb. The figure represents a cube that measures 12 in. on each edge; the base of the cube being 1 sq. ft. Since the weight of 12 in. of water, resting on this square foot, is 62.5 lb., the weight of 1 in. of water covering the same area is 62.5 ÷ 12 = 5.2 lb., which represents the pressure, in pounds per square foot, due to 1 in. of water column. The principle involved is that the unit pressure on a given area of surface depends only on the height of water column the pressure supports.

The Water Gage in the Mine.—As used in the mine, the reading of the water gage shows the difference of pressure

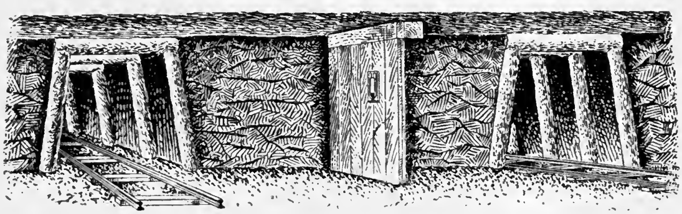


FIG. 26.

between the intake and return airways, at the point where the reading is taken. The intake pressure is always greater than the return pressure and this excess or difference of pressure is what moves the air or creates the current.

The use of the instrument is clearly illustrated in Fig. 26 where two parallel airways are shown leading into the mine, one of these being the intake and the other the return airway of that section of the mine. It makes no difference on which side of the brattice the instrument is placed; the water will always be depressed in that arm of the gage which is open to the intake, because the pressure on the intake is always greater than that on the return airway.

What the Water Gage Shows.—The water-gage reading indicates the ventilating pressure required to circulate the air, and is therefore equal to the resistance of the airways between the two points on the intake and the return; or, in other words, the resistance inby from the point of observation. The nearer this reading is taken to the head of a pair of entries, the closer it will approach zero, while at the next to the last crosscut it would be practically zero.

The use of the water gage in mining practice is of great importance. In connection with the observed velocity of the air, it shows the "power on the air" or the power producing the circulation. What is required in the practical ventilation of a mine is the production of the necessary velocity and volume of air, with the smallest expenditure of power. The most economical circulation is obtained when the required air volume is circulated by the least power, which means a comparatively low water gage.

The circulation of a comparatively large quantity of air under a low gage indicates ideal economic conditions, as far as the circulation is concerned. On the other hand, a small air volume and a comparatively high water gage shows a needless waste of power. In practice, an unusual reduction of the quantity of air passing in a mine or entry, accompanied by a similarly uncommon rise of gage pressure would indicate an obstruction of the airways.

VELOCITY OF AIR CURRENTS

The velocity of the air current is one of the most important factors in the practice of mine ventilation. If the velocity of the air current is too low the ventilation of the mine is inefficient, as the air will not sweep away the accumulating gases from their lurking places in the mine. On the other hand, if the air moves with too great a velocity, not only do the workmen suffer inconvenience, but the high velocity of the current is often dangerous.

Danger of High Velocity.—A rapid air current carries a great quantity of dust, and, by supplying large quantities of oxygen, maintains an unnecessarily active condition of the mine atmosphere that favors the ignition of the gas and dust. The high wind creates a draft that greatly intensifies the flame of lamps or of a blast of powder and increases the possibility of ignition.

How Velocity is Estimated.—In mine ventilation the velocity of the ventilating current is commonly estimated in feet per minute, or feet per second.

How Velocity is Measured.—A simple method of ascertaining, with more or less accuracy, the average velocity of the air current passing in an airway is to measure off a distance of, say 300 ft. along a straight portion of the airway; and

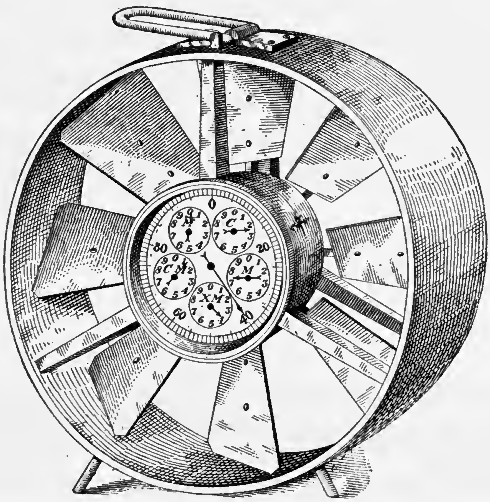


FIG. 27.

note the exact time between the observed flash of powder at one end and the smell of smoke at the other end of this distance. The distance (300 ft.) divided by the time will give the velocity of the air in the center of the entry. The average velocity of the current may then be taken as $\frac{4}{5}$ of this observed velocity. For example, if the observed time is 30 sec., the center velocity is $300 \div 30 = 10$ ft. per sec.; and the average velocity $\frac{4}{5} \times 10 = 8$ ft. per sec. or $8 \times 60 = 480$ ft. per min.

The Anemometer.—The common method of measuring the velocity of the air in airways is by the use of the anemometer, one form of which is shown in Fig. 27. The dial hands record the number of revolutions of the vane. The instrument is so calibrated that each revolution of the vane corresponds to 1 ft. of air travel. The reading of the dial, therefore, shows the distance the air traveled during the time that the instrument was exposed to the current. Hence, this reading divided by the time of exposure, in minutes, will give the velocity of the current in feet per minute. A single revolution of the large hand corresponds to 100 revolutions of the vane. The small dials register the total reading.

QUANTITY OF AIR

The term "quantity," in mine ventilation, refers to the volume of air passing in an airway, estimated in cubic feet per minute. This is often spoken of as the "circulation" of the airway or mine.

How Quantity is Estimated.—As stated above, the quantity of air circulated in an airway or mine, or the "circulation," as it is called, is always estimated, in this country, in cubic feet per minute.

How Quantity is Measured.—To measure the quantity, in ventilation, it is necessary (1) to measure the sectional area of the airway at the point of observation and (2) to carefully measure the average velocity of the air current at the same point. From these measurements, the volume of air passing or the circulation is calculated by means of the formula,

$$Quantity = area \times velocity$$
$$q = av$$

Example.—Calculate the circulation in an airway having a sectional area of 50 sq. ft., the average velocity of the air current being 600 ft. per min.

Solution.—Substituting the given values in the formula for quantity in terms of velocity and area,

$$q = av = 50 \times 600 = 30{,}000 \textit{ cu. ft. per min.}$$

Quantity of Air Required.—In determining the required circulation of a mine, it is necessary to consider (1) the requirements of the mining law of the state in which the mine is located and (2) the requirements of the mine as determined by the natural conditions existing in the seam and the enfolding strata.

Requirements of the Mining Law.—These vary somewhat in different states. Owing to the numerous and changing conditions, in mines, mining laws are of necessity arbitrary standards, which must, however, be met, except in cases where the law specially confers discretionary powers upon the mine inspector or the mine foreman, thereby authorizing them to decrease the circulation in any mine or section of the mine, as conditions may require or their judgment dictate.

The mining law commonly specifies from 100 to 150 cu. ft. per man, per min., for nongaseous, and 200 cu. ft. per min., for gaseous mines. In addition, some of the laws require from 500 to 600 cu. ft. per min., for each animal employed underground.

Natural Requirements.—Gaseous mines naturally require more air than nongaseous mines. The rise workings of seams generating marsh gas or the dip workings of mines giving off quantities of blackdamp are often difficult to ventilate and require a circulation greater than what the law specifies, in order to keep the workings free from gas and healthful and safe for work. Slips and faults often give off much gas when least expected and require, therefore, a larger circulation of air than would otherwise be necessary in the same mine.

WORK OR "POWER ON THE AIR"

The terms "work" and "power" as used in mine ventilation, are synonymous, because the work performed in moving the air through the mine airways is based on a unit of time, both the velocity and the quantity being rated per minute of time.

Power on the Air.—The air current in an airway or mine is moved by a pressure called the "ventilating pressure." The ventilating pressure or the pressure producing the circulation is the total pressure pa exerted on the entire sectional area of the airway, as illustrated in Fig. 28. The small arrowheads in the figure represent the unit pressure or the pressure p on each square foot of cross-section. The large arrow shown at A represents the total pressure $P = pa$.

It is a law of mechanics that when a force pa moves or is exerted through a distance v the work performed is equal to the product pav of the force and the distance. But in this case, the force pa moves through the distance v in one minute. The work (pav) is, therefore, performed in one minute and is the "power on the air." The work performed per minute or the power on the air is expressed in foot-pounds

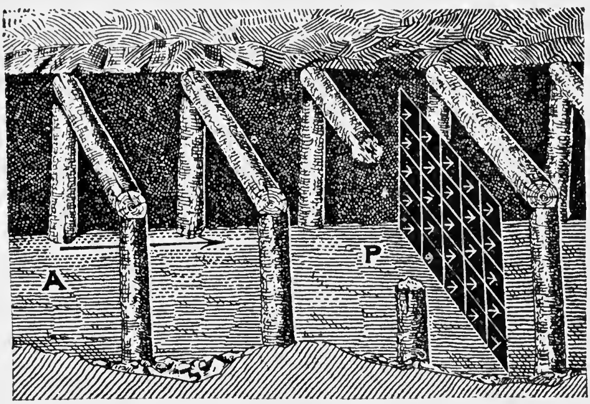


Fig. 28.

per minute. Calling this work per minute or power on the air u, the formula for power is

$$Power = unit\ pres. \times area \times vel.$$
$$u = pav$$

Again, since $q = av$, the formula for power on the air may be written:

$$Power = quantity \times unit\ pres.$$
$$u = qp$$

The formula for horsepower of the circulation is, therefore, since 1 hp. = 33,000 ft.-lb. per min.

$$H = \frac{qp}{33,000}$$

The power formulas, in ventilation, make it possible to calculate the power required to produce any given circulation, against any given pressure or water gage when the efficiency of the venilator is known or assumed.

EQUIVALENTS IN MEASUREMENT

Air Column and Water Gage.—Since water is practically 815 times as heavy as air at normal temperature and pressure, 1 ft. of water column measures the same pressure as 815 ft. of ordinary air column; and 1 in. of water gage is therefore equal to 815 ÷ 12 = say 68 ft. of air column, which gives the following:

Rule.—To reduce feet of air column to inches of water gage, divide by 68.

To reduce inches of water gage to feet of air column, multiply by 68.

Air Column and Unit Ventilating Pressure.—Since air at a normal temperature and pressure weighs, practically, 13 cu. ft. to the pound, every 13 ft. of air column represents, approximately, a ventilating pressure of 1 lb. per sq. ft., which gives the following:

Rule.—To reduce feet of air column to unit pressure, divide by 13.

To reduce unit pressure (lb. per sq. ft.) to feet of air column, multiply by 13.

Air Column and Barometric Pressure.—Since 1 cu. in. of mercury weighs 0.491 lb., each inch of mercury column indicates a pressure of 0.491 lb. per sq. in.; 0.491 × 144 = 70.7 lb. per sq. ft.; and since each pound per square foot of pressure corresponds to 13 ft. of air column, approximately, 1 in. of barometer = 70.7 × 13 = say 920 ft. of air column, which gives the following:

Rule (Approximate).—To reduce feet of air column to inches of barometer, divide by 920.

To reduce barometric pressure (inches) to feet of air column, multiply by 920.

Barometric and Unit Ventilating Pressure.—Barometric pressure is always expressed in inches of mercury column.

Unit ventilating pressure is expressed in pounds per square foot, ounces per square inch, or inches of water gage.

Rule.—To reduce barometric pressure (inches) to ventilating pressure (lb. per sq. ft.), multiply by 70.7; or to ventilating pressure (oz. per sq. in.), multiply by 0.491 × 16 = 7.856; or to water gage (in.), multiply by 70.7 ÷ 5.2 = 13.6, which is the specific gravity of mercury referred to water as a standard.

Since 13 ft. air column represents a pressure of 1 lb. per sq. ft., a pressure of 1 oz. per sq. in. corresponds to an air column of (13 × 144) ÷ 16 = 117 ft.

EQUIVALENTS IN PRESSURE

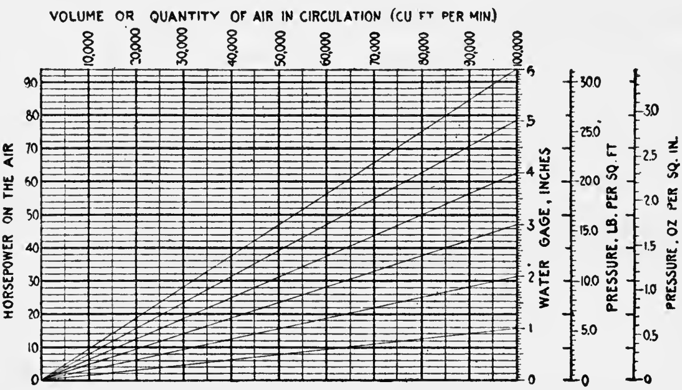


Fig. 29.

Air column (ft.)	=	68	× water gage (in.);
	=	13	× pressure (lb. per sq. ft.);
	=	117	× pressure (oz. per sq. in.);
	=	920	× barometric pressure (in.);
Pressure (lb. per sq. ft.)	=	5.2	× water gage (in.);
	=	70.7	× barometric pressure (in.);
Pressure (oz. per sq. in.)	=	0.58	× water gage (in.);
	=	7.86	× barometric pressure (in.);
Water gage (in.)	=	13.6	× barometric pressure (in.).

Power-Volume-Pressure Diagram.—The diagram shown in Fig. 29 is convenient as showing at a glance the power required to circulate a given quantity of air against a certain pressure, in pounds per square foot, ounces per square inch, or inches of water gage. In order to find the power required to pass any given volume of air against any given pressure or water gage, follow the diagonal line corresponding to the given water gage to its intersection with the vertical line corresponding to the given volume and read this point of intersection on the power scale at the left of the diagram.

For example, it requires 50 hp. to pass 80,000 cu. ft. of air per minute, under a 4-inch water gage or, reversing the order, 30 hp. will pass about 96,000 cu. ft. per minute under a 2-inch gage. Since the power is proportional to the quantity and pressure alike, in order to deal with higher values than those given in the diagram, it is only necessary to treat these as multiples of the values given in the diagram. Thus, 100 hp would pass 160,000 cu. ft. under a 4-inch gage; or 320,000 cu. ft. under a 2-inch gage. The horsepower in this diagram is the power on the air, which is commonly, in fan practice, 60 per cent. of the horsepower of the engine or the indicated horsepower.

EXAMPLES FOR PRACTICE

1. How many feet of air column is equivalent to a mine water gage of three inches?

Solution.—Under ordinary or normal conditions water weighs 815 times as heavy as the same volume of air; hence,

1 ft. (12 in.) water column = 815 *ft. air column*
1 in. water gage = 815 ÷ 12 = 68 *ft. air column*
3 in. water gage = 3 × 68 = 204 *ft. air column*

2. Express the pressure equivalent to 200 ft. of ordinary air column, in pounds per square ft.; ounces per square inch; inches of barometer; inches of water gage.

Solution.—

200 ÷ 13 = 15.39 *lb. per sq. ft., nearly*
200 ÷ 117 = 1.71 *oz. per sq. in., nearly*
200 ÷ 920 = 0.22 *in. of mercury, nearly*
200 ÷ 68 = 2.94 *in. of water gage.*

3. What is the pressure of the atmosphere, in pounds per square inch, corresponding to a barometric pressure of 30 in.?

Solution.—

$$30 \times 7.86 = 235.8 \text{ oz. per sq. in.}$$
$$235.8 \div 16 = 14.74 \text{ lb. per sq. in., nearly}$$

4. Find the pressure in ounces per square inch corresponding to a water gage of 2.5 in.

Solution.—

$$2.5 \times 0.58 = 1.45 \text{ oz. per sq. in.}$$

5. Find the barometric pressure in inches of mercury corresponding to a water gage of 3.4 in.

Solution.—

$$3.4 \div 13.6 = 0.25 \text{ in.}$$

6. If an aneroid barometer gives a reading of 29.65 in. on the surface, what should be the reading at the bottom of a downcast shaft 500 ft. deep where the ventilating pressure caused by a blowing fan gives a water gage of 2.85 in., assuming all readings are taken at about the same time?

Solution.—The air column in this shaft will increase the barometric pressure $500 \div 920 = 0.54$ in. The water gage due to the blower will still further increase the barometric pressure, at the foot of the downcast shaft, $2.85 \div 13.6 = 0.21$ in. The reading of the aneroid, therefore, should be $29.65 + 0.54 + 0.21 = 30.4$ in., approximately.

7. In a mine ventilated by an exhaust fan, giving a water gage of 2.33 in., if aneroid readings taken on the surface and at the bottom of the upcast shaft show a difference of 0.77 in., what is the calculated depth of the shaft?

Solution.—The action of the exhaust fan makes the aneroid reading at the shaft bottom lower than it would be if the fan were not running, and decreases the difference of the surface and underground readings $2.33 \div 13.6 = 0.17$ in. of mercury. The difference of reading due to the depth of the shaft only is, therefore, $0.77 + 0.17 = 0.94$ in. of mercury. Reducing this barometric difference to air column gives for the approximate depth of the shaft $920 \times 0.94 =$ say 865 ft. under ordinary conditions.

MINE AIRWAYS

Definition of Terms.—The term "airway," in mining, generally relates to a passageway for the circulation of the air current, in distinction from a haulage road or travelingway, although these entries may serve also as airways. The entry by which the air current enters the mine is called the main "intake," and that by which it is carried out, the main "return." In like manner, the two shaft or slope openings in a mine are called, respectively, the "downcast" and the "upcast."

The "perimeter" of an airway is the distance measured around the circumference of its cross-section. The "area" or "sectional area" of an airway is the area of its cross-section.

The "rubbing surface" s of an airway is the entire inner surface of the same; and is found by multiplying the perimeter o by the length l, of the airway; thus,

$$s = lo$$

Essential Features of Mine Airways.—Airways in mines should be as straight as possible and avoid all sharp bends and other obstructions that increase the resistance of the airway to the flow of air. The shape of the airway is important as affecting the pressure required to pass a given quantity of air.

Shape of Airways.—The cross-section of an airway may be a circle, square, rectangle, ellipse, or any combination of these that best meets the needs or conditions. For the purpose of ventilation, that form of airway is best that has the shortest length of perimeter, for the same area of section.

In this respect, the circular airway is first; the ellipsoidal airway next, until the major axis exceeds 2.73 times the minor axis when, for the same area, the perimeter is equal to that of a square airway. The square airway is then third in the series and the rectangular and trapezoidal forms last.

There are, however, other requirements than those of ventilation. Haulage requires a level bottom for the roadway. Roof conditions or economy of driving entries may put an arched roof out of the question, making it necessary to adopt the square, rectangular, or trapezoidal shape. Again, a weak coal and heavy side pressure may demand an ellipsoidal shape of section or a special type of timbering approaching the same. It is not uncommon to arch the roof of airways for a distance, using either a semicircle or a semiellipse to form the arch, the latter being called a "flat arch."

The closer the ellipse approaches the circle or the nearer a rectangle comes to being a square, the less is the perimeter of the airway, for the same area of section. For the same length of airway, the perimeter is proportional to the rubbing surface of the airway.

Similar Airways.—Two airways are similar to each other when their cross-sections are similar; the term "similar" has no reference to the length of the airway.

The cross-sections of airways are similar when their corresponding dimensions are proportional, each to each, and their perimeters parallel throughout or can be so placed.

Illustration.—All circular or square airways are similar, because they have but one dimension, the diameter of the circle or the side of the square, and these dimensions are, therefore, always proportional.

For example, one circular airway may have a diameter twice or three times as great as that of another circular airway; or the side of a square airway may be two or three times that of another square airway; and their perimeters can always be placed so that their circumferences will be concentric or their sides parallel, each to each.

On the other hand, the rectangle, trapezoid and ellipse each have two dimensions; and while one of these dimensions may be two, three, etc., times as great as the corresponding dimension of another airway of the same form, it does not follow that the other dimensions of the two airways have the same proportion; and unless they do the airways are not similar. Thus, a 6 × 8-ft. airway and a 9 × 12-ft. airway are similar, because their corresponding sides have the same ratio, or are proportional and may be written

$$\frac{6}{9} = \frac{8}{12}; \qquad \text{or } 6:9::8:12$$

A 6 × 8-ft. airway and a 3 × 16-ft. airway, however, are not similar airways, though they have equal sectional areas 6 × 8 = 48 sq. ft., and 3 × 16 = 48 sq. ft.); because the second airway is twice as wide but only half as high as the first.

It is important to observe that in all similar airways, the ratio of the sectional areas of the airways is equal to the square of the ratio of the corresponding dimensions. For example, in Figs. 30 and 31 showing two similar trapezoidal sections, the top, bottom and sides of the larger airway are each twice those of the smaller, and the area of the larger section is, therefore, $2^2 = 4$ times that of the smaller.

Principle of Similar Airways.—Since corresponding dimensions of similar airways have a fixed ratio, which is the same

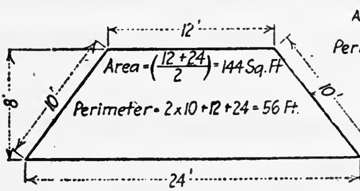


FIG. 30.

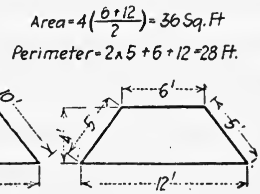


FIG. 31.

for each dimension (diameter, side, height or width) it is possible to compare similar airways with respect to any of these dimensions.

Application.—Assume, for example, the same pressure (p) is applied to each of two similar circular airways, and it is required to find how the quantity of air will vary in the two airways. First write the formula for the quantity (q), in terms of the pressure (p) and the dimensions, area (a), perimeter (o) and length (l) of the airway, and the coefficient of friction (k); thus,

$$q^2 = \frac{pa^3}{klo}$$

Now, if the two airways have the same length, and are under the same pressure, p, l and k are all constant and

$$q^2 \text{ varies as } \frac{a^3}{o}$$

But, the area of a circle varies as the square of its diameter (d^2) and the perimeter varies as the diameter (d); hence,

$$\frac{a^3}{o} \text{ varies as } \frac{d^6}{d}, \text{ or simply as } d^5$$

Hence,

$$q^2 \text{ varies as } d^5$$

In the same manner, it can be shown in respect to all similar airways of any form, that the square of the quantity varies as the fifth power of any corresponding dimension (d), whether diameter, side, height, or width.

Rule.—In comparing similar airways of equal length, for the same unit pressure, the square of the quantity ratio is equal to the fifth power of the dimension ratio; and, for the same power on the air, the cube of the quantity ratio is equal to the fifth power of the dimension ratio.

Example.—If 100,000 cu. ft. of air is passing per minute, in a 6 × 9-ft. airway under a given pressure, what quantity of air will the same pressure circulate in an airway 8 × 12 ft. of the same length? What quantity will the same power circulate?

Solution.—These airways are similar because their corresponding dimensions are proportional 6 : 8 : : 9 : 12. Therefore, calling the required quantity x,

$$\left(\frac{x}{100{,}000}\right)^2 = \left(\frac{8}{6}\right)^5 = \left(\frac{4}{3}\right)^5 = \frac{1024}{243} = 4.214$$

$$\frac{x}{100{,}000} = \sqrt{4.214} = 2.0528$$

$$x = 100{,}000 \times 2.0528 = 205{,}280 \textit{ cu. ft. per min.}$$

Assuming a constant power on the air:

$$\frac{x}{100{,}000} = \sqrt[3]{4.214} = 1.6152$$

$$x = 100{,}000 \times 1.6152 = 161{,}520 \textit{ cu. ft. per min.}$$

Resistance of Airways.—The resistance that an airway offers to the passage of air is of two kinds: frictional resistance due to the rubbing of the air on the inner surface of the airway, and the resistance due to the air striking against obstructions such as timbers, roof falls, sharp bends, etc.

How Resistance Varies.—In mine ventilation, the entire resistance of airways is estimated on a frictional basis, according to the extent of rubbing surface and the velocity of the air. It is assumed that when the velocity of the air current is doubled, each resisting particle in the airway is struck twice as often and twice as hard, by the passing air, which makes the resistance offered by each particle $2 \times 2 = 4$ times as great as before. If the velocity is increased three times, the resistance of each particle is increased $3 \times 3 = 9$ times, etc. On this assumption, the resistance of an airway varies as the extent of rubbing surface (s) and the square of the velocity ($v \times v = v^2$), or as the expression sv^2 for that airway.

Unit Resistance or Coefficient of Friction.—The amount of resistance, per unit of rubbing surface (1 sq. ft.), for a unit velocity (1 ft. per min.) is called the unit of resistance or the coefficient of friction. The values most commonly adopted for this unit are

$$k = 0.00000002\ lb.\ \text{(Atkinson, revised)}$$
$$k = 0.00000001\ lb.\ \text{(Fairley)}$$

Calculation of Resistance of Airways.—To find the resistance of an airway for any given velocity, multiply the unit resistance (k) by the rubbing surface in square feet (s), and that product by the square of the velocity in feet per minute (v^2); the final product will be the total resistance (R), in pounds, as expressed by the formula

$$R = ksv^2$$

Example.—Find the resistance of an airway having 60,000 sq. ft. of rubbing surface, when the velocity of the air current is 800 ft. per min.

Solution.—The resistance, in this case, is

$$R = 0.00000002 \times 60{,}000 \times 800^2 = 768\ lb.$$

SYMBOLS AND FORMULAS

Most of the rules of mine ventilation are expressed by means of formulas, which show at a glance the relation of the several factors to each other, and make possible many transformations and developments.

Symbols.—As far as practicable, the same symbols are used throughout to designate the same factors; and these are, for the most part, those symbols commonly employed in ventilation, as being the initial letter of the word for which they stand. For example, p = pressure; v = velocity; q = quantity, etc. The following table gives the more important symbols used:

TABLE OF COMMON SYMBOLS, MINE VENTILATION

A	= area of regulator,	*sq. ft.*
a	= area of airway,	*sq. ft.*
B	= height of barometer,	*in.*
C	= Centigrade reading,	*deg.*
c	= constant,	
D	= depth of shaft,	*ft.*
d	= diam. or side of airway,	*ft.*
F	= Fahrenheit reading,	*deg.*
g	= gravity,	*ft. per sec.*
H	= horsepower,	33,000 *ft.-lb. per min.*
h	= height of air column,	*ft.*
K	= Efficiency of fan,	*per cent.*
k	= coefficient of friction,	0.00000002
l	= length of airway,	*ft.*
n	= number of revolutions,	*r.p.m.*
o	= perimeter of airway,	*ft.*
P	= total pressure,	*lb.*
p	= unit pressure,	*lb. per sq. ft.*
Q	= total circulation of air,	*cu. ft. per min.*
q	= single current,	*cu. ft. per min.*
R	= resistance of mine or airway,	*lb.*
r	= any ratio,	
s	= rubbing surface of airway,	*sq. ft.*
T	= absolute or higher temperature,	*deg.*
t	= actual or lower temperature,	*deg.*
U	= total power on air,	*ft.-lb. per min.*
u	= power, single current,	*ft.-lb. per min.*
v	= velocity of air,	*ft. per sec., or ft. per min.*
V	= volume of air or gas,	*cu. ft.*
W	= total weight of body,	*lb.*
w	= unit weight,	*lb. per cu. ft.*
X	= potential of mine or airway,	
X_p	= pressure potential,	
X_u	= power potential	
x	= the unknown quantity whose value is sought	
w.g.	= water gage reading,	*in.*
Sp. gr.	= specific gravity,	

Small subscript letters and figures are frequently written immediately after any symbol to show its reference to a particular kind or thing. For example, q_1, q_2, q_3, etc., indicate the quantities of air passing in three or more airways; q_a, q_b, q_c, etc., indicate the quantities passing in Splits A, B, C, etc. In like manner, the potential values of different airways and splits are indicated by X_1, X_2, X_3, etc.; or X_a, X_b, X_c, etc., as the case may be.

In some cases, two or more subscript letters or figures are used after a single symbol to indicate its reference; as for example, the pressure potential for Split A is written X_{pa} or the power potential X_{ua}. The general potential, in a split circulation, is written X_0; or X_{p0} and X_{u0} to indicate the general pressure and power potentials, respectively.

It is often necessary to indicate the summation of a number of items of the same kind, for which purpose the character Σ is written before the symbol indicating the kind. For example, ΣX_{abc} indicates the sum of the potential values for the splits A, B and C, instead of writing $X_a + X_b + X_c$.

In a complex circulation, consisting of a main airway and two or more splits, it is often necessary to indicate the general split potentials by X_0, X_{a0}, X_{b0}, etc., and the mine potential by X. (See Fig. 33, p. 236.)

Use of Formulas.—A comparatively few formulas form the basis from which practically all the other formulas of mine ventilation are derived. These few basal formulas also show the true relation, one to the other, of the principal factors of ventilation, such as pressure, velocity, quantity, power, rubbing surface and the sectional area of mine airways.

The understanding of these formulas makes it unnecessary to learn and remember a large number of rules of ventilation. A formula is written as an algebraic equation in which each factor is expressed by its proper symbol. The equation shows the equality of certain factors grouped in the form of an expression. For example, the formula

$$pa = ksv^2$$

shows the equality of the total ventilating pressure **pa** and the resistance of the airway when the rubbing surface is **s** and the velocity of the air current **v**.

How Factors Vary.—It is evident, from the inspection of a formula, that:

1. Any factor in one member of the equation varies **directly** as any like factor in the other member, provided the other factors remain constant and none of the quantities expressed in the formula are connected by the signs plus (+) or minus (−).

2. Any factor in either member varies **inversely** as any like factor in the same member, with the provisions just stated (1) above.

For example, the formula previously given shows that:

The total ventilating pressure (pa) for airways varies as the resistance (ksv^2) of the airway.

For any given airway, **a**, **s** and **k** being constant, the unit pressure (p) varies directly as the square of the velocity (v^2) of the air current.

For the same total pressure (pa), in an airway, **k** being constant, the square of the velocity (v^2) varies inversely as the rubbing surface (s). Or, in other words, the velocity (v) of the air current varies inversely as the square root of the rubbing surface ($\sqrt{s}$).

For the same velocity (v) of air and the same rubbing surface (s) in an airway, **k** being constant, the unit pressure (p) always varies inversely as the sectional area (a) of the airway.

3. Again, transposing the formula for total pressure, the formula for unit pressure producing a given velocity in a given airway or mine is

$$p = \frac{ksv^2}{a}$$

An inspection of this formula shows that:

The other factors remaining constant and none of the quantities being connected by the signs plus (+) or minus (−), any factor in the denominator of a fractional term forming either member of the equation varies **directly** as any factor in the numerator of that fraction; and likewise as any similarly placed factor in the other member.

Basal Formulas.—There are, in fact, but two truly basal formulas, in mine ventilation; the one expressing the **resistance** that an airway offers to the passage of an air current having a certain velocity; the o'her expressing the **power on the air** producing a certain velocity in an airway, against a certain resistance. These formulas are as follows:

Resistance of airway, $R = pa = ksv^2$
Power on the air, $u = pav = ksv^3$

From these two simple formulas as a basis, with the aid of a few other recognized formulas and principles for determining the quantity, horsepower, water gage, rubbing surface, etc., all ventilation formulas are derived.

Mine Potential Methods

An Important Principle.—One of the most important principles of mine ventilation may be stated briefly as follows:

Every airway or mine possesses a certain definite **resisting power,** which is determined by the ratio of its area of passage to rubbing surface. For this reason, a given power will produce a certain velocity and develop a certain resistance, in a given airway; the velocity of the air current varying inversely as the resistance. **Ventilating pressure** is caused by and equal to the resistance developed. **Power,** then, creates **velocity,** which in the airway develops **resistance;** and the resistance produces **pressure.**

The conclusion is, therefore, evident that it is the resisting power of a mine or airway that determines the velocity and pressure a given power will produce in that airway. The airway, it is clear only possesses this resisting power potentially, its development requiring the passage of an air current. Hence, it is proper to term such resisting power, expressed in terms of the airway, the **"potential of the airway"** or the **"mine potential,"** in respect to a mine.

As has been explained, the equivalent of the mine potential, expressed in terms of the power, quantity or pressure, is properly called the "potential of the circulation."

Illustration of Formulas.—To illustrate the use of formulas in mine ventilation, and to make clear their application, the following table is given, in which most of the formulas in common use are classified under their proper heads. Many of these formulas, as will be observed, are simple transpositions of another formula or obtained by substitution. The calculations, in the table, all refer to an airway 5 × 10 ft. in cross-section and 4000 ft. long, passing an air current of, say 25,000 cu. ft. per min. against a pressure of 12 lb. per sq. ft.

The Airway.—

Perimeter,	$o = 2(5 + 10) = 30\ ft.$
Length,	$l = 4000\ ft.$
Rubbing surface, $(s = lo)$	$s = 4000 \times 30 = 120{,}000\ sq.\ ft.$
Sectional area,	$a = 5 \times 10 = 50\ sq.\ ft.$

Power potential of airway or mine,

$$X_u = \frac{a}{\sqrt[3]{ks}} \qquad X_u = \frac{50}{\sqrt[3]{0.00000002 \times 120{,}000}} = 373.45$$

The Air Current.—

Velocity,

$$v = \frac{q}{a} \qquad v = \frac{25{,}000}{50} = 500\ ft.\ per\ min.$$

$$v = \sqrt{\frac{pa}{ks}} \qquad v = \sqrt{\frac{12 \times 50}{0.00000002 \times 120{,}000}} = 500\ ft.\ per\ min.$$

$$v = \sqrt[3]{\frac{u}{ks}} \qquad v = \sqrt[3]{\frac{300{,}000}{0.00000002 \times 120{,}000}} = 500\ ft.\ per\ min.$$

$$v = \frac{u}{pa} \qquad v = \frac{300{,}000}{12 \times 50} = 500\ ft.\ per\ min.$$

Power potential of the circulation,

$$X_u = \frac{q}{\sqrt[3]{u}} \qquad X_u = \frac{25{,}000}{\sqrt[3]{300{,}000}} = 373.45$$

The square of the pressure potential can always be used instead of the cube of the power potential since these are equal, as expressed by the formula

$$X_p^2 = X_u^3$$

Thus, $X_p = X_u\sqrt{X_u} = 373.45\sqrt{373.45} = 7217$, *nearly*

Pressure potential,

$$X_p = q\sqrt{\frac{q}{u}} \qquad X_p = 25{,}000\sqrt{\frac{25{,}000}{300{,}000}} = 7217, \textit{ nearly}$$

$$X_p = \frac{q}{\sqrt{p}} \qquad X_p = \frac{25{,}000}{\sqrt{12}} = 7217, \textit{ nearly}$$

Quantity, $q = av \qquad q = 50 \times 500 = 25{,}000$ *cu. ft. per min.*

$$q = a\sqrt{\frac{pa}{ks}} \qquad q = 50\sqrt{\frac{12 \times 50}{0.00000002 \times 120{,}000}} = 25{,}000 \textit{ cu. ft. per min.}$$

$$q = a\sqrt[3]{\frac{u}{ks}} \qquad q = 50\sqrt[3]{\frac{300{,}000}{0.00000002 \times 120{,}000}} = 25{,}000 \textit{ cu. ft. per min.}$$

$$q = \frac{u}{p} \qquad q = \frac{300{,}000}{12} = 25{,}000 \textit{ cu. ft. per min.}$$

$$q = X_u\sqrt[3]{u} \qquad q = 373.45\sqrt[3]{300{,}000} = 25{,}000 \textit{ cu. ft. per min.}$$

$$q = X_p\sqrt{p} \qquad q = 7217\sqrt{12} = 25{,}000 \textit{ cu. ft. per min}$$

Pressure, $p = \frac{ksv^2}{a} \qquad p = \frac{0.00000002 \times 120{,}000 \times 500^2}{50} = 12$ *lb. per sq. ft.*

$$p = \frac{ksq^2}{a^3} \qquad p = \frac{0.00000002 \times 120{,}000 \times 25{,}000^2}{50^3} = 12 \textit{ lb. per sq. ft.}$$

$$p = \frac{u}{q} \qquad p = \frac{300{,}000}{25{,}000} = 12 \textit{ lb. per sq. ft.}$$

$$p = \left(\frac{q}{X_p}\right)^2 \qquad p = \left(\frac{25{,}000}{7217}\right)^2 = 12 \textit{ lb. per sq. ft.}$$

$$p = 5.2 \textit{ w.g.} \qquad p = 5.2 \times 2.307 = 12 \textit{ lb. per sq. ft.}$$

Resistance, $R = pa \qquad R = 12 \times 50 = 600$ *lb.*

$$R = ksv^2 \qquad R = 0.00000002 \times 120{,}000 \times 500^2 = 600 \textit{ lb.}$$

$$R = \frac{u}{v} \qquad R = \frac{300{,}000}{500} = 600 \textit{ lb.}$$

Water gage, $w.g. = \frac{p}{5.2}$ $w.g. = \frac{12}{5.2} = 2.307 + in.$

Power on air, $u = ksv^3$ $u = 0.00000002 \times 120{,}000 \times 500^3$
$= 300{,}000$ *ft.-lb. per min.*

$u = \frac{ksq^3}{a^3}$ $u = \frac{0.00000002 \times 120{,}000 \times 25{,}000^3}{50^3}$
$= 300{,}000$ *ft.-lb. per min.*

$u = qp$ $u = 25{,}000 \times 12$
$= 300{,}000$ *ft.-lb. per min.*

$u = \left(\frac{q}{X_u}\right)^3$ $u = \left(\frac{25{,}000}{373.45}\right)^3 = 300{,}000$ *ft.-lb. per min.*

$u = pav$ $u = 12 \times 50 \times 500$
$= 300{,}000$ *ft.-lb. per min.*

Horsepower, $H = \frac{u}{33{,}000}$ $u = \frac{300{,}000}{33{,}000} = 9.09\ hp.$

MEASUREMENT OF AIR CURRENTS

The measurement of air currents, in mining practice, involves the careful observation of the velocity and pressure of the current and the accurate measurement of the sectional area of the airway. From these data the volume and power of the air current are determined.

Requirements.—The mining laws of the state, in most cases, require a specified volume of air per man, per minute, circulated throughout the mine. In order to meet this requirement, it is necessary to estimate the power that will produce such quantity in a given mine.

The Mine Potential.—Every airway and every mine has a certain resisting power, in respect to the circulation of air. For this reason, the same power will circulate different quantities of air through airways that differ in respect to either their size or length.

The formulas of mine ventilation show the following relation of the quantity of air circulated to the power producing the circulation, and the sectional area to the rubbing surface of the airway.

$$\frac{Quantity}{\sqrt[3]{Power}} \text{ varies as } \frac{sectional\ area}{\sqrt[3]{rubbing\ \ surface}}$$

Or, say: quantity (cu. ft. per min.) = q; power (ft. lb. per min.) = u; sectional area (sq. ft.) = a; and rubbing surface (sq. ft.) = s; the unit resistance being k, we have

$$\frac{q}{\sqrt[3]{u}} = \frac{a}{\sqrt[3]{ks}}$$

The first of these expressions, being given in terms of the power and quantity of air circulated, may be called, properly, the "potential of the circulation;" while the second expression, being given in terms of the airway, is the "potential of the airway," or the "mine potential." The significance of the term "potential," in this connection, is apparent since it describes the capacity of an airway or mine in respect to the volume of air it will pass, per unit of power.

Values of the Potential.—Calling the potential factor X, its value for any given mine or airway is calculated by the formula

$$X = \frac{a}{\sqrt[3]{ks}}$$

The value of the potential for the circulation of any quantity (q), by any power (u) or pressure (p), is found by the formula

$$X = \frac{q}{\sqrt[3]{u}} = \sqrt[3]{\frac{q^2}{p}}$$

The value of the potential lies in the fact that it gives to every mine, or air split, a definite value that enables a correct comparison to be made between them, and the proper type of ventilator and system of ventilation to be chosen.

Potential of Airway.—Calculate the potential of an airway 6 × 10 ft., in cross-section, and 2000 ft. long.

Solution—The sectional area of this airway is 6 × 10 = 60 sq. ft.; the rubbing surface is 2(6 + 10)2000 = 64,000 sq. ft. The potential of the airway is, therefore,

$$X = \frac{a}{\sqrt[3]{ks}} = \frac{60}{\sqrt[3]{0.00000002 \times 64{,}000}} = 552.6$$

Potential of Circulation.—What is the value of the potential factor in the circulation of 60,000 cu. ft. of air, by 10 hp.?

Solution.—The potential of this circulation is

$$X = \frac{q}{\sqrt[3]{u}} = \frac{60{,}000}{\sqrt[3]{10 \times 33{,}000}} = 868.2$$

Find the potential value for the same volume of air when circulated under a pressure of 8 lb. per sq. ft.

Solution.—The potential value, in this case, is

$$X = \sqrt[3]{\frac{q^2}{p}} = \sqrt[3]{\frac{60{,}000^2}{8}} = 766.3$$

Power, Pressure, Quantity.—By transposing the formulas for potential, it is possible to calculate the power or pressure required to circulate any given quantity of air against any given mine potential; or to find the air volume a given power or pressure will produce, for any given mine potential.

Example.—Find the (1) power, and (2) pressure required to circulate 24,000 cu. ft. of air through an airway 5 × 14 ft. in section and 3000 ft. long?

Solution.—The area and rubbing surface of the airway are: a = 5 × 14 = 70 sq. ft.; and s = 2(5 + 14)3000 = 114,000 sq. ft. The potential factor of this airway is then

$$X = \frac{a}{\sqrt[3]{ks}} = \frac{70}{\sqrt[3]{0.00000002 \times 114{,}000}} = 531.8$$

(1) Power, $u = \left(\frac{q}{X}\right)^3 = \left(\frac{24{,}000}{531.8}\right)^3 = 91{,}900$ *ft.-lb. per min.*

(2) Pressure, $p = \frac{q^2}{X^3} = \frac{24{,}000^2}{531.8^3} = 3.83$ *lb. per sq. ft.*

or, $p = \frac{u}{q} = \frac{91{,}900}{24{,}000} = 3.83$ *lb. per sq. ft.*

Example.—Find the volume of air circulated in the same mine, by (1) 10 hp.; (2) a pressure of 7.8 lb. per sq. ft.

Solution.—

(1) By 10 hp.,

$$q = X\sqrt[3]{u} = 531.8\sqrt[3]{10 \times 33{,}000} = 36{,}750 \text{ cu. ft. per min.}$$

(2) By 7.8 lb.,

$$q = X\sqrt{Xp} = 531.8\sqrt{531.8 \times 7.8} = 34{,}250 \text{ cu. ft. per min.}$$

Potential Values of Different Airways.—In order to show the resisting power of airways of different lengths, for those sizes in more common use, the following table has been prepared, showing the potential value of each airway, as calculated by the formula

Potential of airway, $$X = \frac{a}{\sqrt[3]{ks}}$$

Following this is another table giving the potential values of different circulations, by which is meant the circulation of different volumes of air under different pressures or water gages. A comparison of the potential values in these two tables will serve to show what circulation can be obtained in airways of given size and length when properly arranged and unobstructed.

TABLE.—POTENTIAL VALUES FOR DIFFERENT AIRWAYS

Size of airway, feet	Length of Airway, Including Return (ft.)					
	1,000	2,000	3,000	5,000	8,000	10,000
	Potential value of airway					
4 × 10	485.3	385.0	336.5	283.8	242.6	225.2
4 × 12	557.0	441.9	386.2	325.7	278.5	258.5
4 × 14	624.8	495.7	433.2	365.4	312.4	290.0
5 × 8	497.4	394.6	344.9	290.9	248.7	230.9
5 × 10	592.8	470.3	411.0	346.7	296.4	275.2
6 × 8	582.3	462.0	403.8	340.6	291.2	270.3
6 × 10	696.2	552.4	482.7	407.2	348.1	323.2
7 × 8	664.0	526.7	460.4	388.3	332.0	308.2
7 × 10	796.0	631.5	552.0	465.5	398.0	369.5
8 × 10	892.6	708.1	618.9	522.0	446.3	414.3

Potential Values of Different Circulations.—The circulation of a given quantity of air in a certain airway or mine requires a certain pressure or water gage, which determines the "potential of the circulation."

In the following table, the potential of the circulation is calculated by the formula

$$\text{Potential of circulation,} \quad X = \sqrt[3]{\frac{Q^2}{p}} = \sqrt[3]{\frac{Q^2}{5.2\,w.g.}}$$

TABLE.—POTENTIAL VALUES FOR DIFFERENT CIRÇULATIONS

Water gage (in.)	Pressure (lb. per sq. ft.)	Volume of air circulated (cu. ft. per min.)					
		10,000	15,000	25,000	50,000	75,000	100,000
		Potential value of circulation					
6	31.2	147.4	193.2	271.6	431.1	564.9	684.4
5	26.0	156.7	205.3	288.6	458.1	600.3	727.2
4	20.8	168.8	221.2	310.9	493.5	646.7	783.4
3	15.6	185.8	243.4	342.2	543.2	711.8	862.2
2½	13.0	197.4	258.7	363.6	577.2	756.3	916.3
2	10.4	212.6	278.6	391.7	621.8	814.8	987.0
1½	7.8	234.0	306.7	431.1	684.3	896.8	1,086.4
1	5.2	267.9	351.1	493.5	783.4	1,026.5	1,243.6
½	2.6	337.6	442.3	621.8	987.0	1,293.4	1,566.8

Comparing this table with that on the preceding page shows that to pass a current of 25,000 cu. ft. per min. through an airway of 5 × 8 ft., 3000 ft. long, including the return will require, practically, a 3-in. water gage. This is ascertained by observing that the potential value of an airway 5 × 8 ft., 3000 ft. long, as given in the first table, is, say 345. Then find the water gage corresponding as nearly as possible to this value, in the second table, in the vertical column for 25,000 cu. ft. per min. The potential of the circulation of this air volume under a 3-in. gage is, say 342, showing that a 3-in. gage is a little in excess of what is required to circulate 25,000 cu. ft. of air per minute in a 5 × 8-ft. airway, 3000 ft. long, including the return.

Effect of Splitting on Mine Potential.—As a mine is developed and its airways extended, it becomes impracticable to carry the air in a single current throughout the entire length of the airways, as the water gage then increases directly as the length or distance of air travel. To avoid this difficulty, the air must be divided or "split" one or more times; so that there will be two or more separate currents in the mine. Each of these currents is called a "split of air," or simply a "split" (see p. 219).

It should be observed that dividing the current does not change the total rubbing surface (s) in the mine; but the area of passage is increased in proportion to the number of splits or currents. Calling the number of equal splits n, the **area of passage** (p. 211), in splitting an air current is na, and the formula for the potential can be written:

Split potential, $$X = \frac{na}{\sqrt[3]{ks}}$$

Since the rubbing surface (s), the sectional area (a) and the coefficient (k) are constant, the potential (X) varies as n, or as the number of equal splits or currents. Therefore, any of the airway potentials of the first table can be multiplied 2, 3, 4, etc. times according to the number of splits or currents employed.

For illustration, suppose the airways of a mine are 5×10 ft. and have a total length, including return, say 10,000 ft.; and the required circulation is 100,000 cu. ft. per min. The velocity of the air should not exceed, say 500 ft. per min., in the airways. This will require a total area of passage of $100{,}000 \div 500 = 200$ sq. ft. But the sectional area of these airways is $5 \times 10 = 50$ sq. ft.; and there must, therefore, be $200 \div 50 = 4$ splits or currents to comply with the conditions named. The potential value, as given in the table, for a single current, is, say 275; and the mine potential for four splits is, therefore, $4 \times 275 = 1100$. By referring, now, to the second table giving the values of the potential of circulation, it is found that a potential value of 1100, in the circulation of 100,000 cu. ft. per min. shows a water gage between 1 and 1½ in. The true value may be found by interpolation, if desired, and is 1.46 in.

The potential value of any desired circulation of air, as compared with the potential value or "potential factor" of the proposed mine or airway is thus seen to have an important practical value that commends it to all students of mining.

Example.—It is proposed to open a mine in a 6-ft. seam of coal and provide for a capacity of 1000 tons a day. A general estimate is desired of the requirements for the proper ventilation of the mine, under working conditions. In other words, what volume of air will be required and what will be the approximate water gage and horsepower necessary for the circulation of such quantity in this mine?

Solution.—Assuming an average daily output of 2.5 tons of coal per miner, the number of miners working will be 1000 ÷ 2.5 = 400. Then allowing for, say 150 loaders and 50 company men including bosses, the total number of men in the mine will be 600, for whom the quantity of air specified by law must be provided.

Assume that the mine generates considerable gas and to cover all requirements, estimate on supplying 200 cu. ft. of air per man, per minute, which gives a total required air volume of 200 × 600 = 120,000 cu ft. per min.

In order to estimate approximately what water gage will result in the circulation of this quantity of air, it is necessary to decide on the size of the entries; and make the sectional area such as will allow of a safe maximum velocity of the air current in the cross-headings and find the number of splits required to meet these conditions.

In this case, suppose all entries to be 6 × 10 ft., giving a sectional area of 60 sq. ft.; and the mine being gassy, say the velocity of the air current on all cross-headings or splits must not exceed 360 ft. per min. This condition will require a total area of passage or the sum of the sectional areas of all the splits, 120,000 ÷ 360 = 333 sq. ft. But the area of the entries being each 60 sq. ft., the number of splits required to give this area of passage and thus keep the velocity of the air currents in the splits within the specified limit is 333 ÷ 60 = 5.5, say 6 splits or pairs of cross-headings.

The next step is to decide on the distance each pair of cross-headings will be driven, from which the extent of rubbing surface can be approximately estimated. For example, assume the cross-headings to be driven, say 2000 ft. on each side of the main heading, making 4000 ft. of entry, including the return, in each split. The total length of entry for the six splits is then 6 × 4000 = 24,000 ft. Assume the main headings are driven four abreast, so as to provide two intake haulage roads affording separate tracks for the empty and loaded trips; and two return airways.

If the cross-entries are turned to the right and left of the main headings, every 500 ft., the length of these headings may be taken as 3 × 500 = 1500 ft., giving a total length for the four headings 4 × 1500 = say 6000 ft. The total length of all entries in the mine may thus be assumed as 24,000 + 6000 = 30,000 ft.

The estimated rubbing surface is then $s = 2(6 + 10) \times 30{,}000 = 960{,}000$ sq. ft.; and the mine potential is

$$X = \frac{6a}{\sqrt[3]{ks}} = \frac{6 \times 60}{\sqrt[3]{0.00000002 \times 960{,}000}} = 1344$$

This is only an approximately correct value for this mine, because the six splits do not start from the shaft bottom.

The **water gage** required is then calculated from the mine potential and the air volume; thus,

$$w.g. = \frac{Q^2}{5.2X^3} = \frac{120{,}000^2}{5.2 \times 1344^3} = 1.14 \ in.$$

It will be safe to assume, from the above calculation, that the proposed mine can be properly ventilated by the circulation of 120,000 cu. ft. of air per minute under a water gage of say 1.5 in., providing for six main air splits as described, and making due allowance for possible conditions.

The **horsepower** required to produce this circulation, assuming a general efficiency of $K = 60$ per cent. is

$$H = \frac{Q(5.2\,w.g.)}{K33{,}000} = \frac{120{,}000\ (5.2 \times 1.5)}{0.60 \times 33{,}000} = 47+, \text{ say } 50\ hp.$$

Example.—Find the unit pressure, water gage and horsepower required to circulate 80,000 cu. ft. of air per minute in a mine in two equal splits. The airways are all 8 × 10 ft., and have a total length of 12,000 ft., including the return airways.

Solution.—The sectional area of the airways, in this case, is $a = 8 \times 10 = 80$ sq. ft.; the perimeter is $o = 2(8 + 10) = 36$ ft. The potential of the airway for two splits is then

$$X = \frac{na}{\sqrt[3]{klo}} = \frac{2 \times 80}{\sqrt[3]{0.00000002 \times 12{,}000 \times 36}} = 780$$

The unit pressure is

$$p = \frac{Q^2}{X^3} = \frac{80{,}000^2}{780^3} = say\ 13.5\ lb.\ per\ sq.\ ft.$$

The corresponding water gage is 13.5 ÷ 5.2 = say 2.6 *in.*

The horsepower on the air, as calculated from the above unit pressure, is

$$H = \frac{Qp}{33{,}000} = \frac{80{,}000 \times 13.5}{33{,}000} = 32.7\ hp.$$

Or, the horsepower may be found directly from the mine potential, as follows:

$$H = \frac{1}{33,000}\left(\frac{Q}{X}\right)^3 = \frac{1}{33,000}\left(\frac{80,000}{780}\right)^3 = 32.7 \, hp.$$

Example.—Find the quantity of air in circulation in four equal splits in a mine, when the size of the airways is 5 × 14 ft. and the total length of airways in all the splits, including the returns in each case, is 40,000 ft.; the water gage at the shaft bottom where the air is divided being 3 in.

Solution.—The rubbing surface, in this case, is s = 40,000 × 2(5 + 14) = 1,520,000 sq. ft., and the sectional area of each airway 5 × 14 = 70 sq. ft. The mine potential for four splits is then

$$X = \frac{na}{\sqrt[3]{ks}} = \frac{4 \times 70}{\sqrt[3]{0.00000002 \times 1,520,000}} = 897$$

The quantity of air in circulation under a 3-in. water gage is then

$$Q = X\sqrt{X(5.2\,w.g.)}$$
$$= 897\sqrt{897 \div 5.2 \times 3} = \text{say } 106,000 \textit{ cu. ft. per min.}$$

Caution.—In the calculation of all problems in mine ventilation, regard must be had to the conditions with respect to the power and the pressure producing or resulting from the circulation of the air in the mine.

Both the power and the pressure are commonly said to produce the circulation; but, as a matter of fact, it is the power that produces the circulation, while the pressure is the result and measured by the resistance of the mine or airway.

Unfortunately, these factors do not vary alike, but the cube root of the power varies as the square root of the pressure; or, more simply, the cube root of the power ratio, in any mine or airway, is equal to the square root of the pressure ratio, for the same circulation; thus,

$$\sqrt[3]{\frac{u_1}{u_2}} = \sqrt{\frac{p_1}{p_2}}$$

For example, in what proportion must the power be increased in order to double the pressure ($p_2/p_1 = 2$)?

$$\sqrt[3]{\textit{power ratio}} = \sqrt{2} = 1.414$$
$$\textit{power ratio} = 1.414^3 = 2.828$$

In other words, if 10 hp. on the air produces a given pressure or water gage in a certain mine or airway, it will require $2.828 \times 10 = 28.28$ hp. to double that pressure or gage.

Use of Potential Factors.—Attention has been drawn to the potentiality of an airway or mine, in respect to the resistance it can offer to the passage of air, by virtue of its rubbing surface (s) and its sectional area (a). The potential of an airway or mine is the factor that determines the quantity of air such airway or mine will pass, for any given power or pressure. It is important, in the use of the potential, therefore, to consider whether the pressure or power is in question.

For every airway or mine, therefore, there is a power potential (X_u) and a pressure potential (X_p). The cube of the power potential is equal to the square of the pressure potential, for the same mine or airway, giving the equal values.

$$X_u^3 = X_p^2 = \frac{q^3}{u} = \frac{q^2}{p} = \frac{a^3}{klo}$$

An inspection of these equal values shows that:

1. The quantity of air a given power will circulate varies as the power potential of the airway or mine.

2. The quantity of air a given pressure will circulate varies as the pressure potential of the airway or mine.

Hence, in comparing the circulations in different airways or mines, a constant power requires the use of the power potential, and a constant pressure, the pressure potential.

Other Potential Formulas.—Transposing the values given above makes it possible to calculate the power or pressure required to circulate a given quantity of air in a certain airway or mine directly from its potential factor.

$$u = \left(\frac{q}{X_u}\right)^3 = \frac{q^3}{X_p^2}$$

$$p = \left(\frac{q}{X_p}\right)^2 = \frac{q^2}{X_u^3}$$

It is, likewise, possible to calculate the quantity of air a given power or pressure will circulate against any given potential factor representing a certain airway or mine, by simply multiplying the cube root of the power or the square root of the pressure by the corresponding potential of the airway or mine as expressed by the following formulas:

$$q = X_u \sqrt[3]{u}$$
$$q = X_p \sqrt{p}$$

A few examples will serve to make the use of these formulas clear and to show their practical application, in the rapid estimation of what is required in the proposed development of mines, in order to make suitable provision for their proper ventilation.

EXAMPLES FOR PRACTICE

1. If 25 hp. produces a water gage of 1.5 in., in a certain mine, what water gage will 40 hp. produce in the same mine?

Solution.—Since the square root of the pressure or water-gage ratio is equal to the cube root of the power ratio, calling the required water gage x,

$$\sqrt{\frac{x}{1.5}} = \sqrt[3]{\frac{40}{25}} = \sqrt[3]{\frac{8}{5}} = \sqrt[3]{1.6} = 1.17$$
$$\frac{x}{1.5} = 1.17^2 = 1.37, \textit{ nearly}$$
$$x = 1.5 \times 1.37 = 2.05 \textit{ in.}$$

2. It is proposed to provide for the circulation of 75,000 cu. ft. of air, in two generally equal splits, the airways including the return in each split being 6 × 10 ft. in section and about 8000 ft. long. (*a*) Find the power potential for the entire mine; and (*b*) calculate from that both the power and the water gage of the circulation.

Solution.—(*a*) The sectional area of the airways, in this case, is $a = 6 \times 10 = 60$ sq. ft.; the total rubbing surface in the mine, $s = 2 \times 2(6 + 10)8000 = 512{,}000$ sq. ft. Substituting these values and that for the coefficient of resistance $k = 0.00000002$ in the formula for power potential of mine,

$$X_u = \frac{na}{\sqrt[3]{ks}} = \frac{2 \times 60}{\sqrt[3]{0.00000002 \times 512{,}000}} = 552.6$$

The power on the air required to circulate 75,000 cu. ft. of air against this potential is, then,

$$u = \left(\frac{q}{X_u}\right)^3 = \left(\frac{75{,}000}{552.6}\right)^3 = 2{,}500{,}000 \textit{ ft.-lb. per min.}$$

The water gage, as calculated directly from the power potential, $X_u = 552.6$, is

$$w.g. = \frac{q^2}{5.2 X_u^3} = \frac{75{,}000^2}{5.2 \times 552.6^3} = 6.41 \textit{ in.}$$

Or, the water gage may be found thus

$$w.g. = 2{,}500{,}000 \div (5.2 \times 75{,}000) = 6.41 \textit{ in.}$$

3. (*a*) Calculate the value of the pressure potential for the entire mine mentioned in the preceding question, the airways being 6×10 ft. in section and about 16,000 ft. long, including the return, assuming as before two equal splits; and (*b*) calculate from this pressure potential the power that will produce the desired circulation of air; namely 75,000 cu. ft. per min. and the resulting water gage.

Solution.—(*a*) The total rubbing surface is $s = 2(6 + 10)\ 16{,}000 = 512{,}000$ sq. ft. For two equal splits, the area of passage in this mine is $a = 2(6 \times 10) = 120$ sq. ft. The mine pressure potential is then

$$X_p = a\sqrt{\frac{a}{ks}} = 120\sqrt{\frac{120}{0.00000002 \times 512{,}000}} = \textit{say}\ 13{,}000$$

(*b*) The power on the air, calculated from the pressure potential, is then,

$$u = \frac{q^3}{X^2_p} = \frac{75{,}000^3}{13{,}000^2} = \textit{say}\ 2{,}500{,}000\ \textit{ft.-lb. per min.}$$

The water gage, calculated in the same manner, is

$$w.g. = \frac{1}{5.2}\left(\frac{q}{X_p}\right)^2 = \frac{1}{5.2}\left(\frac{75{,}000}{13{,}000}\right)^2 = 6.41\ in.$$

4. What volume of air will 10 hp. circulate in an airway 6 × 8 ft., in section, and 2500 ft. long?

Solution.—The sectional area of this airway is $a = 6 \times 8 = 48$ sq. ft.; the rubbing surface $2(6 + 8)\ 2500 = 70{,}000$ sq. ft. The power potential is therefore

$$X_u = \frac{a}{\sqrt[3]{ks}} = \frac{48}{\sqrt[3]{0.00000002 \times 70{,}000}} = 429.1,\ \textit{nearly.}$$

For 10 hp. on the air, the quantity of air in circulation in this airway is

$$q = X_u\sqrt[3]{u} = 429.1\sqrt[3]{10 \times 33.000} = \textit{say}\ 30{,}000\ \textit{cu. ft. per min.}$$

5. (*a*) What quantity of air will be circulated, in the airway, in the last example, under a 3-in. water gage; and what power on the air will be necessary to develop this quantity and gage? (*b*) What was the original water gage when 10 hp. circulated 30,000 cu. ft. of air, in this mine?

Solution.—(*a*) Since the square of the pressure potential is equal to the cube of the power potential

$$X_p = \sqrt{X^3_u} = \sqrt{429.1^3} = 8890,\ \textit{nearly}$$

Then, $q = X_p\sqrt{p} = 8890\sqrt{5.2 \times 3} = \textit{say}\ 35{,}000\ \textit{cu. ft. per min.}$

The power required to produce a 3-in. water gage is

$$H = \frac{35{,}000 \times 3 \times 5.2}{33{,}000} = \textit{say}\ 17\ hp.$$

(b) The previous water gage due to the circulation of 30,000 cu. ft., in this mine, under 10 hp. can be calculated in several ways; but most simply, thus,

$$w.g. = \frac{10 \times 33{,}000}{5.2 \times 30{,}000} = 2.1\ in.$$

The calculation may also be made from the potential; thus,

$$w.g. = \frac{q^2}{5.2X^3_u} = \frac{30{,}000^2}{5.2 \times 429.1^3} = 2.1\ in.$$

Area of Passage.—It is important to notice that the potential value for any mine is determined by its area of passage with respect to the resisting power of its rubbing surface. For a single air current the area of passage is the sectional area (a) of the airway. For 2, 3, etc., equal splits the area of passage is $2a$, $3a$, etc.; for n equal splits the area of passage, for the mine, is na.

The unit of resistance being k, the resisting power of the entire airway or mine is indicated by ks; and the potential values of the mine with respect to power and pressure, respectively, are thus expressed

Mine power potential, $$X_u = \sqrt[3]{\frac{(na)^3}{ks}} = \frac{na}{\sqrt[3]{ks}}$$

Mine pressure potential, $$X_p = \sqrt{\frac{(na)^3}{ks}} = na\sqrt{\frac{na}{ks}}$$

It should be observed that the mine power potential varies as the number of equal splits or currents, which is not true of the pressure potential of a mine. This fact has an important application, since, for the same mine, the rubbing surface being constant, the number of splits (n) is equal to the power-potential ratio. An example will serve to make this clear.

Example.—Suppose it is desired to ascertain quickly how many equal splits would pass the same quantity of air (75,000 cu. ft. per min.), under a 2-in. water gage, in Example 3, previously given where two splits of air gave a water gage of 6.41 in., the power remaining constant.

Solution.—From the equations expressing the potential values previously given (p. 208), it appears, for the same quantity of air in circulation, the pressure or water gage varies inversely as the cube of the power potential. But, since the power potential varies as the number of splits in a mine, it follows that, for the same quantity of air in circulation, the power remaining constant, the pressure or water gage varies inversely as the cube of the number of splits.

In other words, for the same quantity of air, and constant power, the pressure or water-gage ratio is equal to the cube of the inverse ratio of the number of splits. In this case, calling the required number of splits n, the split ratio is $n/2$, and the corresponding water-gage ratio 2/6.41, and we write

$$\left(\frac{n}{2}\right)^3 = \frac{6.41}{2} = 3.205$$

$$n = 2\sqrt[3]{3.205} = 2.95, \textit{ say 3 splits.}$$

The reference, thus far, has been to equal division of the air current and the rules and formulas given above apply strictly, only to mines in which the air current is divided at or near the main entrance and passes through the mine in two or more separate and equal splits.

Part Potential Value.—The part potential value is found by omitting k in the calculation, and writing it outside the parenthesis. The **relative potential** obtained by canceling common factors cannot be used here. The relative potential, so much used in the calculation of the splitting of air currents, can only be employed when the potential appears as a ratio (see p. 221.)

General Potential of a Mine.—An important application of the potential method, in mine ventilation, is the calculation of the potential value for the entire mine when the airways and shafts are of various dimensions.

Example.—Calculate the general mine power potential in the following mine, shafts 1250 ft. deep:

		Area	Rub. Sur.
Shafts, upcast and downcast,	8 × 10 ft., 2500 ft.	80	90,000
Main airway ("A" seam),	6 × 10 ft., 3750 ft.	60	120,000
Cross-headings ("A" seam),	6 × 8 ft., 2500 ft.	48	70,000
Tunnel to "B" seam,	5 × 8 ft., 500 ft.	40	13,000
Return air course ("B" seam),	5 × 14 ft., 5500 ft.	70	209,000

Solution.—The total power producing a given circulation, is clearly equal to the sum of the powers absorbed in the several sections of the mine, as expressed by the following general formula:

$$H = \frac{kQ^3}{K33{,}000}\left(\frac{1}{X_1^3} + \frac{1}{X_2^3} + \frac{1}{X_3^3} + \text{etc.}\right)$$

It will be readily observed that this general formula, for a mine of various sections (K being the coefficient of efficiency of the ventilator), is derived from the power formula

$$H = \frac{1}{K33{,}000}\left(\frac{Q}{X_u}\right)^3 = \frac{Q^3}{K33{,}000}\,\frac{1}{X_u^3}$$

But, since $1/X_u^3 = k\,s/a^3$ and k being constant, it is much simpler in using the above general formula, to factor and write k outside of the parenthesis, which makes each of the potential values within the parenthesis what may be called a "part potential" whose value is, omitting k,

Part potential $\quad X_u = \frac{a}{\sqrt[3]{s}}$; and $\frac{1}{X_u^3} = \frac{s}{a^3}$

Now, calculating the value of $1/X_u^3 = s/a^3$, for each separate section of air passage in the mine given above,

Shafts, $\quad \frac{1}{X_1^3} = \frac{90{,}000}{80 \times 80 \times 80} = 0.1758$

Main airway ("A" seam), $\quad \frac{1}{X_2^3} = \frac{120{,}000}{60 \times 60 \times 60} = 0.5555$

Cross-headings ("A" seam), $\quad \frac{1}{X_3^3} = \frac{70{,}000}{48 \times 48 \times 48} = 0.6330$

Tunnel to "B" seam, $\quad \frac{1}{X_4^3} = \frac{13{,}000}{40 \times 40 \times 40} = 0.2031$

Return air course ("B" seam), $\quad \frac{1}{X_5^3} = \frac{209{,}000}{70 \times 70 \times 70} = 0.6093$

Potential factor for entire mine $\frac{1}{X_0^3}$ 2.1767

The part power potential for this mine is therefore

$$X_0 = \frac{1}{\sqrt[3]{2.1767}} = 0.7716$$

Example.—(*a*) From the part power potential calculated for the mine, in the preceding example, find the horsepower required to circulate 30,000 cu. ft. of air per minute in a single current, assuming the ventilating fan to have a mechanical efficiency $K = 60$ per cent. (*b*) What water gage will be produced by the resistance in the mine, for this circulation?

Solution.—(*a*) The required horsepower of the ventilator is

$$H = \frac{kQ^3}{K33{,}000}\,\frac{1}{X_0^3} = \frac{0.00000002 \times 30{,}000^3}{0.60 \times 33{,}000} \times \frac{1}{0.7716^3} = \text{say } 60\ hp.$$

(*b*) The mine water gage due to this circulation is

$$w.g. = \frac{kQ^2}{5.2X_u^3} = \frac{0.00000002 \times 30{,}000^2}{5.2 \times 0.7716^3} = 7.5\ in.$$

General Mine Potential, Equal Splits.—It is possible to calculate the general mine potential when there are two or more airways of equal dimensions, by simply multiplying the common sectional area by the number of airways, as shown by the following example:

Example.—A drift mine is opened on the triple-entry system. It is proposed to drive the main intake 7×10 ft. in section, a distance of 3000 ft. to the boundary. The cross-entries are to be driven double, 5×12 ft. in section and 1500 ft. to the side lines on each side of the main road, making in all 6000 ft. of cross-entries, including the returns. The main-return airways, on each side of the main intake are each 7×12 ft. in section and 3000 ft. long. Calculate (*a*) the horsepower on the air; and (*b*) the water gage produced, for a circulation of 50,000 cu. ft. of air in this mine, in two equal parts.

Solution.—The first step is to calculate the value $1/X_u{}^3 = s/a^3$ for each sectional division; thus

Main intake, 7×10 ft., 3000 ft. long: $a =$ 70 sq. ft.; $s =$ 102,000 sq. ft.
Cross-entries 5×12 ft., 6000 ft. long: $a =$ 120 sq. ft.; $s =$ 204,000 sq. ft.
Main returns, 7×12 ft., 3000 ft. long: $a =$ 168 sq. ft.; $s =$ 228,000 sq. ft.

Substituting these values in the formula for finding the part potential factor for each section,

Main intake, $$\frac{1}{X_1{}^3} = \frac{102{,}000}{70 \times 70 \times 70} = 0.2974$$

Two splits, $$\frac{1}{X_2{}^3} = \frac{204{,}000}{120 \times 120 \times 120} = 0.1181$$

Two main returns, $$\frac{1}{X_3{}^3} = \frac{228{,}000}{168 \times 168 \times 168} = 0.0480$$

Potential factor for entire mine $1/X_0{}^3$ 0.4635

For the horsepower and water gage, we have

$$H = \frac{kQ^3}{33{,}000}\,\frac{1}{X_0{}^3} = \frac{0.00000002 \times 50{,}000^3}{33{,}000} \times 0.4635 = \text{say } 35\ hp.$$

$$w.g. = \frac{kQ^2}{5.2}\,\frac{1}{X_0{}^3} = \frac{0.00000002 \times 50{,}000^2}{5.2} \times 0.4635 = 4.46\ in.$$

Tandem Circulations

Summation of Potentials.—When an air current passes in succession through two or more airways of different section, the total unit pressure (lb. per sq. ft.) due to the circulation is equal to the sum of the unit pressures of the several sections. The arrangement, in this case, may be described as "tandem."

Likewise, in a tandem circulation, the total power on the air (ft.-lb. per min.) producing the circulation is equal to the sum of the powers absorbed in the several sections through which the current passes.

Indicating the potentials of the respective sections of the air-course in a tandem circulation by X_1, X_2, X_3, etc.; and the corresponding unit pressures and powers on the air by p_1, p_2, p_3, etc.; and u_1, u_2, u_3, etc., respectively, remembering that the square of the pressure potential is equal to the cube of the power potential, as expressed by the formula

$$X_p^2 = X_u^3$$

we can write the following:

For tandem circulations, calling the general mine pressure p_0 and the total power on the air u_0.

Mine pressure, $$p_0 = Q^2\left(\frac{1}{X^2_{p1}} + \frac{1}{X^2_{p2}} + \text{etc.}\right)$$

or $$p_0 = Q^2\left(\frac{1}{X^3_{u1}} + \frac{1}{X^3_{u2}} + \text{etc.}\right)$$

These formulas may be written more simply by indicating the summation of the potential factors by the character Σ; thus,

Mine pressure, $$p_0 = Q^2\,\Sigma\left(\frac{1}{X^2_p}\right)$$

or $$p_0 = Q^2\,\Sigma\left(\frac{1}{X^3_u}\right)$$

In like manner, the total power on the air or power producing tandem circulation in a mine is expressed by the formula,

Power on the air, $$u_0 = Q^3\left(\frac{1}{X^3_{u1}} + \frac{1}{X^3_{u2}} + \text{etc.}\right)$$

or $$u_0 = Q^3\left(\frac{1}{X^2_{p1}} + \frac{1}{X^2_{p2}} + \text{etc.}\right)$$

These formulas may be expressed by indicating the summation of the potential factors by Σ, as before; thus,

Power on the air $$u_0 = Q^3\,\Sigma\left(\frac{1}{X^3_u}\right)$$

or $$u_0 = Q^3\,\Sigma\left(\frac{1}{X^2_p}\right)$$

In a tandem circulation, if desired, the general mine potentials for power (X_{uo}) and for pressure (X_{po}) can be calculated by the formulas

$$X_{uo} = \frac{1}{\sqrt[3]{\Sigma(1/X^3_u)}};\ \text{and}\ X_{po} = \frac{1}{\sqrt{\Sigma(1/X^2_p)}}$$

To illustrate the formulas that apply to a tandem circulation where a single air current is carried continuously through shafts and airways of different size or cross-section, assume the following mine is passing 30,000 cu. ft. of air in a single undivided current:

1. Downcast shaft................	8 × 12 ft.,	600 ft. deep
2. Main road and return, each.....	6 × 10 ft.,	1200 ft. long
3. Cross-tunnel and return, each...	6 × 8 ft.,	200 ft. long
4. Upper seam and return, each....	5 × 14 ft.,	2000 ft. long
5. Upcast shaft..................	10 × 10 ft.,	2250 ft. deep

The sectional areas are 96, 60, 48, 70 and 100 sq. ft.; and the rubbing surfaces, 24,000, 76,800, 11,200, 152,000 and 90,000 sq. ft., respectively.

Part potential factors, $\frac{1}{X_u^3} = \frac{s}{a^3};\ \frac{1}{X^3_1} = \frac{24,000}{96^3} = 0.0271$

$$\frac{1}{X^3_2} = \frac{76,800}{60^3} = 0.3556$$

$$\frac{1}{X^3_3} = \frac{11,200}{48^3} = 0.1013$$

$$\frac{1}{X^3_4} = \frac{152,000}{70^3} = 0.4430$$

$$\frac{1}{X^3_5} = \frac{90,000}{100^3} = 0.0900$$

Potential factors for entire mine, $\Sigma\left(\frac{1}{X^3_u}\right) = 1.0170$

Mine part potentials, $X_{uo} = \frac{1}{\sqrt[3]{\Sigma(1/X_u^3)}} = \frac{1}{\sqrt[3]{1.0170}} = 0.9944$

$$X_{po} = \frac{1}{\sqrt{\Sigma(1/X_p^2)}} = \frac{1}{\sqrt{1.0170}} = 0.9916$$

Pressure, $p = \frac{kQ^2}{X^3_{uo}} = \frac{0.00000002 \times 30,000^2}{0.9944^3} = 18.3$ *lb. per sq. ft.*

Water gage, $w.g. = p/5.2 = 18.3 \div 5.2 = 3.5$ *in.*

Power on the air, $u = \frac{kQ^3}{X^3_{uo}} = \frac{0.00000002 \times 30{,}000^3}{0.9944^3} = 549{,}000$ *ft.-lb. per min.*

Horsepower, $H = \frac{u}{33{,}000} = \frac{549{,}000}{33{,}000} = 16.6$ *hp.*

Example.—A shaft mine has been opened on the triple-entry system. The downcast and upcast shafts are each 600 ft. deep and 8 × 20 ft. in section. The main headings have been driven a distance of 2000 ft. from the shaft bottom. The center one of these headings is the intake and is 7 × 14 ft. in section, while the two side headings are the return airways for the respective sides of the mine and are each 6 × 12 ft. in section. On each side of the main headings, cross-headings, 6 × 10 ft. in section, have been driven 500 ft., including the return in each.

If the intake air divides at the face of the main heading and equal currents ventilate the two sides of the mine, what power on the air will be required to circulate a total of 60,000 cu. ft. per min. in this mine, and what water gage will be produced in the fan drift?

Solution.—The first step is to calculate the potential values of the two shafts, main intake, two cross-headings and two return airways, as follows, remembering that these being equal splits, it is only necessary to double the potentials of the cross-headings and return airways by taking twice the sectional area, in each case:

Shafts,	8 × 20 ft.,	600 ft.;	a = 160 sq. ft.;	s = 67,200 sq. ft.
Main intake,	7 × 14 ft.,	2000 ft.;	a = 98 sq. ft.;	s = 84,000 sq. ft.
Two cross-headings,	6 × 10 ft.,	500 ft.;	$2a$ = 120 sq. ft.;	s = 32,000 sq. ft.
Two return airways,	6 × 12 ft.,	2000 ft.;	$2a$ = 144 sq. ft.;	s = 144,000 sq. ft

The part potential factors are then as follows, omitting k

Shafts, $\frac{1}{X_u^3} = \frac{s}{a^3} = \frac{67{,}200}{160^3} = 0.0164$

Main intake, $\frac{1}{X_u^3} = \frac{s}{a^3} = \frac{84{,}000}{98^3} = 0.0892$

Two cross-headings, $\frac{1}{X_u^3} = \frac{s}{(2a)^3} = \frac{32{,}000}{120^3} = 0.0185$

Two return airways, $\frac{1}{X_u^3} = \frac{s}{(2a)^3} = \frac{144{,}000}{144^3} = \underline{0.0482}$

Sum of potential factors, $\Sigma\left(\frac{1}{X_u^3}\right)$ 0.1723

The horsepower on the air in the fan drift, in this case, is found by substituting this general potential factor. in the formula for finding the power in a tandem circulation; thus,

$$H = \frac{kQ^3}{33{,}000}\,\Sigma\left(\frac{1}{X_u^3}\right) = \frac{0.00000002 \times 60{,}000^3 \times 0.1723}{33{,}000} = 22\,55\ hp.$$

The water gage, in the fan drift, due to this circulation, can be calculated in like manner, independently, from the same general potential factor, by substituting the same in the formula for finding the unit pressure and water gage in a tandem circulation; thus,

$$w.g. = \frac{kQ^2}{5.2}\Sigma\left(\frac{1}{X_u{}^3}\right) = \frac{0.00000002 \times 60{,}000^2 \times 0.1723}{5.2} = 2.38+ \text{ in.}$$

The same result is obtained when the water gage is calculated from the power and the quantity of air in circulation.

$$w.g. = \frac{u}{5.2Q} = \frac{22.55 \times 33{,}000}{5.2 \times 60{,}000} = 2.38 \text{ in.}$$

SPLITTING THE AIR CURRENT

Early Practice, Coursing the Air.—In the early practice of mine ventilation, the method commonly adopted was that known as "coursing the air." In this method the air was conducted throughout the mine in one continuous current, from the intake opening to the point where it was again discharged into the atmosphere.

Single Current Not Adequate.—Experience has shown, however, that a single air current is not adapted to the ventilation of a large mine, for many reasons. As a mine is developed and the workings extended, more men are employed and greater quantities of air are required to ventilate the mine and dilute and carry away the gases generated.

Need of Dividing the Air Current.—The division of the air into two or more currents provides separate ventilation districts in the mine and brings the ventilation under better control, since the quantity of air can then be proportioned to the requirements in each district

A larger volume of air can be circulated by the same power, and the velocity of the current is kept low.

The smoke and gases generated in one section of the mine are not carried by the current into another section, but pass directly into the main return airway and are conducted out of the mine.

A local explosion of gas or dust, in one portion of the mine, is not as liable to extend throughout the mine.

Method of Splitting the Air-current.—Whenever two or more passages or airways are provided by which the air current can travel in passing through the mine, the air will always divide between them in proportion to their several potential values. Hence, all that is required to split an air-current is to provide two or more separate routes for its passage. Each separate current is called an "air split" or simply a "split."

Natural Splitting.—When all the airways are open to the free passage of the air-current through them, the air divides naturally between them, each airway or split taking a quantity of air in proportion to its potential value. In other words, the potential of the airway is an index of the quantity of air that airway will pass, in natural splitting.

Proportionate Splitting.—When any other division of the air is desired than the natural division, it is necessary to introduce regulators in one or more of the airways so as to obstruct the flow in those splits that naturally take more than the desired proportion, and thereby increase the quantity passing in the other airways till the desired proportion is reached.

Primary and Secondary Splits.—A branch or split off the main air current is called a "primary split." If a primary air split be again divided, the result is a "secondary split." When the air current is equally divided between two or more airways the splits are said to be "equal;" but when each airway passes a different volume of air the splits are "unequal."

Increase of Quantity Due to Splitting.—The quantity of air in circulation is proportional to the general mine potential. In other words, the quantity ratio is always equal to the mine-potential ratio; the power potential being used for a constant power, and the pressure potential for a constant pressure; always remembering, however, that the cube of the power potential is equal to the square of the pressure potential. Denoting the original quantity of air in circulation, by Q_1 and the original mine potentials for power and pressure by X_{u1} and X_{p1}, respectively; and designating these factors after splitting, by Q_2, X_{u2} and X_{p2}, respectively, we have the following formulas:

$$\text{Power constant,}\quad Q_2 = Q_1 \frac{X_{u2}}{X_{u1}};\ \text{or}\ Q_2 = Q_1 \sqrt[3]{\left(\frac{X_{p2}}{X_{p1}}\right)^2}$$

$$\text{Pressure constant,}\quad Q_2 = Q_1 \frac{X_{p2}}{X_{p1}};\ \text{or}\ Q_2 = Q_1 \sqrt{\left(\frac{X_{u2}}{X_{u1}}\right)^3}$$

An illustration of the use of these formulas is to be found in the solution of the example given under Secondary Splitting. In that example (p. 242), the power on the air remained constant before and after splitting the current. The pressure potential was used, which before splitting was $X_{p1} = 0.6708$, and after splitting $X_{p2} = 0.8554$:

$$\text{Hence,}\ Q_2 = Q_1 \sqrt[3]{\left(\frac{X_{p2}}{X_{p1}}\right)^2} = 120{,}000 \sqrt[3]{\left(\frac{0.8554}{0.6708}\right)^2} = 141{,}100$$

cu. ft. per min.

NATURAL DIVISION OF AIR

In all splitting calculations, it is assumed that the unit pressure (lb. per sq. ft.) is the same at the mouth of each split starting from the same point. Therefore, writing the formula for unit pressure,

$$p = \frac{kloq^2}{a^3};\ \text{and}\ q^2 = \frac{p}{k}\left(\frac{a^3}{lo}\right)$$

Then, since p and k are both constant, q^2 varies as a^3/lo

$$\text{and} \qquad q \text{ varies as } a\sqrt{\frac{a}{lo}}$$

This expression, as previously explained is the pressure potential of the airway. It must be remembered that the square of the pressure potential (X_p) is equal to the cube of the power potential (X_u); thus,

$$X_p{}^2 = X_u{}^3$$

It is the pressure potential that is always used in splitting calculations; because, as stated above, the unit pressure is the same for all splits at one point. The calculation of the quantity of air passing in any one of two or more splits starting from the same point in a mine, is based on the following simple rule:

Rule.—The ratio of the quantity of air passing in a single split, to the total quantity for all the splits, is equal to the ratio of the pressure potential of that split, to the sum of the pressure potentials for all the splits.

Calling the quantities passing in the several splits, q_1, q_2, q_3, etc., and the corresponding split potentials X_1, X_2, X_3, etc.; the total quantity of air in circulation in all the splits Q, and indicating the sum of the split potentials by ΣX;

$$Q = q_1 + q_2 + q_3 + \text{etc.}$$

and

$$\Sigma X = X_1 + X_2 + X_3 + \text{etc.}$$

Then, according to the rule given above,

$$\frac{q_1}{Q} = \frac{X_1}{\Sigma X}$$

The work of calculation is much simplified and shortened by using what may be called the "relative potential" values, instead of finding the actual pressure potential for each split. This is only possible in splitting calculations, where the potentials are used as ratios, and the value of the ratio is not changed by the cancellation of any like factors in all the potentials.

Relative Potential Values.—Whenever the potential is used as a ratio, as in splitting air currents, the relative values should be used. These are calculated from the lowest relative values for the areas, perimeters and lengths of the several airways or splits. For example, if the areas are 48, 60 and 72 sq. ft., the lowest relative values, canceling the common factor 12, are 4, 5 and 6, respectively Likewise, instead of the perimeters, 28, 32, 34; use the lowest relative perimeters 14, 16, 17, canceling the common factor 2 from each.

The use of the "relative potential" value, in all calculations to determine the natural division of air between two or more airways, is one of the most important considerations in the saving of time and labor and avoiding unnecessary multiplicity of figures, which increases the opportunities for error and yields less accurate results. An example or two will serve to make this fact plain.

Summation of Split Potentials.—The circulation of air in two or more splits or currents, in a mine, differs from a tandem circulation in the fact that the same unit pressure circulates the air in each and all the splits, which are thus separate currents moved by one pressure; while in a tandem circulation one continuous current passes in succession through different airways or sections of the mine.

While in a tandem circulation the mine pressure is equal to the sum of the pressures for the several sections through which the current passes; and, likewise, the total power for the mine is equal to the sum of the powers absorbed in the sections; in a split circulation, the total power for the mine is equal to the sum of the powers absorbed in the splits, but there is but one pressure, which is the same for all the splits starting from the same point in the mine. As before, indicate the several split pressure potentials by X_{p1}, X_{p2}, X_{p3}, etc.; the corresponding powers on the air by u_1, u_2, u_3, etc.; and the total power on the air by u_0, remembering that it is necessary, in all splitting calculations, to use the pressure potential, which has the value

$$X_p = a\sqrt{\frac{a}{klo}}$$

The work is simplified by using the part potential value, as previously stated, omitting k when finding the potential values and multiplying the final result by that coefficient.

The following shows the development of the formulas for the summation of the potentials in split circulations where the splits all start from one point in the mine:

$$u_0 = u_1 + u_2 + etc. \qquad (1)$$

But,

$$u_1 = \frac{q^3{}_1}{X^2{}_{p1}};\ u_2 = \frac{q^3{}_2}{X^2{}_{p2}};\ etc. \qquad (2)$$

By the principle of splitting air currents,

$$q_1 = \frac{X_{p1}}{\Sigma X_p} Q; \; q_2 = \frac{X_{p2}}{\Sigma X_p} Q; \; etc. \tag{3}$$

Combining equations 2 and 3 and simplifying,

$$u_1 = \left(\frac{Q}{\Sigma X_p}\right)^3 X_{p1}; \; u_2 = \left(\frac{Q}{\Sigma X_p}\right)^3 X_{p2}; \; etc. \tag{4}$$

Finally, substituting these values (4) in equation 1, and factoring,

$$u_0 = k\left(\frac{Q}{\Sigma X_p}\right)^3 (X_{p1} + X_{p2} + \text{etc.}) = \frac{kQ^3}{(\Sigma X_p)^2} \tag{5}$$

From Equation 5 is obtained the formula for calculating the horsepower on the air at the point of split, by the summation of the part pressure split potentials:

Horsepower on the air, $$H = \frac{kQ^3}{33{,}000(\Sigma X_p)^2} \tag{6}$$

The formula for calculating the water gage, in like manner, is

Water gage, $$w.g. = \frac{kQ^2}{5.2(\Sigma X_p)^2} \tag{7}$$

Equal Splits.—When an air current is divided naturally between two or more equal splits, the calculation of the mine potentials, velocity, pressure, power, etc., is the same as for a single undivided current, except that the sectional area (a) of the airways must be multiplied by the number of splits (n) to obtain the total **area of passage** (na).

To illustrate the application of the formulas in this case, assume an air current of 60,000 cu. ft. of air is circulated in three equal splits, the size and total length of the airways, including the returns being 5×8 ft. and 10,000 ft. long.

Velocity, $$v = \frac{Q}{na} = \frac{60{,}000}{3(5 \times 8)} = 500 \; ft. \; per \; min.$$

Mine part potentials, $$X_u = \frac{na}{\sqrt[3]{s}} = \frac{3(5 \times 8)}{\sqrt[3]{260{,}000}} = 1.880$$

$$X_p = na\sqrt{\frac{na}{s}} = 120\sqrt{\frac{120}{260{,}000}} = 2.578$$

Pressure, $$p = \frac{kQ^2}{X^2_p} = \frac{0.00000002 \times 60{,}000^2}{2.578^2} = 10.83 \; lb. \; per \; sq. \; ft.$$

Water gage, $w.g. = p/5.2 = 10.83 \div 5.2 = 2.08$ *in.*

Power on the air,

$$u = \frac{kQ^3}{X_u{}^3} = \frac{0.00000002 \times 60{,}000^3}{1.88^3} = 650{,}000 \text{ ft.-lb. per min.}$$

Horsepower,

$$H = \frac{u}{33{,}000} = \frac{650{,}000}{33{,}000} = 19.7 \text{ hp.}$$

Unequal Splits.—To illustrate the formulas used in the calculation of the natural division of an air current between two or more airways and the pressure and power on the air, assume a current of 75,000 cu. ft. per min. is passing in the following three splits, starting from the same point of the main airway or at or near the intake opening. The lengths given for the several splits include the return, in each case; and the pressure and power are for the circulation in the splits only.

Split *A*,	6 × 10 ft.;	2000 ft. long;	a = 60 sq. ft.;	o = 32 ft.;	l = 2000 ft.
Split *B*,	6 × 8 ft.;	1500 ft. long;	a = 48 sq. ft.;	o = 28 ft.;	l = 1500 ft.
Split *C*,	4 × 12 ft.;	2500 ft. long;	a = 48 sq. ft.;	o = 32 ft.;	l = 2500 ft.

The lowest relative values are as follows: Areas, 5, 4, 4; perimeters, 8, 7, 8; lengths, 4, 3, 5.

Relative split pressure potentials,

$$X_p = a\sqrt{\frac{a}{lo}}; \qquad X_{pa} = 5\sqrt{\frac{5}{4 \times 8}} = 5\sqrt{0.1562} = 1.976$$

$$X_{pb} = 4\sqrt{\frac{4}{3 \times 7}} = 4\sqrt{0.1905} = 1.746$$

$$X_{pc} = 4\sqrt{\frac{4}{5 \times 8}} = 4\sqrt{0.1000} = 1.265$$

Sum of potentials, ΣX_p 4.987

Natural division of air current,

$$q_a = \frac{X_a}{\Sigma X_p} Q; \qquad q_a = \frac{1.976}{4.987} \times 75{,}000 = 29{,}720 \text{ cu. ft.}$$

$$q_b = \frac{1.746}{4.987} \times 75{,}000 = 26{,}260 \text{ cu. ft.}$$

$$q_c = \frac{1.265}{4.987} \times 75{,}000 = 19{,}020 \text{ cu. ft.}$$

Total circulation, Q . . . . 75,000 *cu. ft.*

To calculate the pressure and power of the circulation, it is necessary to employ the part-potential values, instead of the relative values; thus,

Part potential values,

$$X_p = a\sqrt{\frac{a}{lo}};\quad X_{pa} = 60\sqrt{\frac{60}{2000 \times 32}} = 1.837$$

$$X_{pb} = 48\sqrt{\frac{48}{1500 \times 28}} = 1.623$$

$$X_{pc} = 48\sqrt{\frac{48}{2500 \times 32}} = 1.176$$

General part potential for splits, ΣX_p. 4.636

Pressure, $$p = \frac{kQ^2}{(\Sigma X_p)^2} = k\left(\frac{Q}{\Sigma X_p}\right)^2$$

$$p = 0.00000002\left(\frac{75{,}000}{4.636}\right)^2 = 5.2 \textit{ lb. per sq. ft.}$$

Horsepower on the air,

$$H = \frac{kQ^3}{33{,}000(\Sigma X_p)^2} = \frac{0.00000002 \times 75{,}000^3}{33{,}000 \times 4.636^2} = 11.9 \textit{ hp.}$$

EXAMPLES IN NATURAL DIVISION

Example.—An air current of 100,000 cu. ft. per min. is divided at the foot of the downcast shaft, between the following four air-courses or splits, thereby providing two separate ventilation districts on each side of the shaft:

Split A,	8 × 12 ft., 6000 ft. long
Split B,	6 × 20 ft., 12,000 ft. long
Split C,	6 × 12 ft., 8000 ft. long
Split D,	4 × 6 ft., 1000 ft. long

All the splits are open to the free passage of the air, no regulators being used. (*a*) Find the natural division of the main air current or the quantity of air passing in each split. (*b*) What is the pressure due to this circulation? (*c*) What is the horsepower on the air?

Solution.—(*a*) The first step is to calculate the relative pressure potential for each of the four air splits. The area, perimeter and length of each airway are as follows:

Split A,	a = 96 sq. ft.;	o = 40 ft.;	l = 6,000
Split B,	a = 120 sq. ft.;	o = 52 ft.;	l = 12,000
Split C,	a = 72 sq. ft.;	o = 36 ft.;	l = 8,000
Split D,	a = 24 sq. ft.;	o = 20 ft.;	l = 1,000

Instead of using these full values as when finding the true potential value of an airway, the lowest relative values for the areas, perimeters and lengths are used. These relative values are obtained by canceling the common factors in the areas, perimeters and lengths separately, which gives the following:

Split A,	$a = 4$;	$o = 10$;	$l = 6$
Split B,	$a = 5$;	$o = 13$;	$l = 12$
Split C,	$a = 3$;	$o = 9$;	$l = 8$
Split D,	$a = 1$;	$o = 5$;	$l = 1$

The relative split potentials are then found as follows:

Split A, $4\sqrt{\frac{4}{6 \times 10}} = 4\sqrt{\frac{1}{15}} = 4\sqrt{0.06666} = 1.033$

Split B, $5\sqrt{\frac{5}{12 \times 13}} = 5\sqrt{\frac{5}{156}} = 5\sqrt{0.03205} = 0.895$

Split C, $3\sqrt{\frac{3}{8 \times 9}} = 3\sqrt{\frac{1}{24}} = 3\sqrt{0.04166} = 0.612$

Split D, $1\sqrt{\frac{1}{1 \times 5}} = \sqrt{\frac{1}{5}} = \sqrt{0.2} = 0.447$

Sum of relative potentials........................ 2.987

Since the quantity of air passing in each split, in natural division is proportional to the corresponding potential, the quantity ratio is equal to the potential ratio, which is true also for the sum of the quantities and the sum of the potentials. Thus, the ratio of the quantity (q) passing in any split, to the total quantity (Q) in circulation, is equal to the ratio of the corresponding split pressure potential (X_p), to the sum of all the split potentials (ΣX_p).

$$\frac{q}{Q} = \frac{X_p}{\Sigma X_p}; \text{ which gives } q = \frac{X_p}{\Sigma X_p} Q$$

Therefore, substituting the relative potential values just found in this formula gives the following:

Split A, $q_a = \frac{1.033}{2.987} \times 100{,}000 = 34{,}570$ *cu. ft. per min.*

Split B, $q_b = \frac{0.895}{2.987} \times 100{,}000 = 29{,}960$ *cu. ft. per min.*

Split C, $q_c = \frac{0.612}{2.987} \times 100{,}000 = 20{,}500$ *cu. ft. per min.*

Split D, $q_d = \frac{0.447}{2.987} \times 100{,}000 = 14{,}970$ *cu. ft. per min.*

Total quantity................ 100,000 *cu. ft. per min.*

(*b*) Since the pressure is the same for all the splits, it can be calculated from any one of the given splits, by substituting the values for that split in the formula

$$p = \frac{kloq^2}{a^3}$$

Thus, taking split *A*,

$$p = \frac{0.00000002 \times 6000 \times 40 \times 34{,}570^2}{96 \times 96 \times 96} = 6.48 \textit{ lb. per sq. ft.}$$

(*c*) The horsepower on the air in the main entry, or the horsepower producing this circulation is, then,

$$H = \frac{Qp}{33{,}000} = \frac{100{,}000 \times 6.48}{33{,}000} = 19.6 \textit{ hp.}$$

As an illustration of the usefulness of the summation of potential values, we give below the calculation of the horsepower on the air, unit pressure and water gage developed in the circulation of 100,000 cu. ft. of air per minute, in four splits, previously calculated by the usual method in the last example, where it was necessary, first, to find the natural division of the air.

Example.—An air current of 100,000 cu. ft. per min. is divided, at the foot of the downcast shaft, between the following four splits:

Split *A*, 8 × 12 ft.,	6,000 ft., long;	a = 96 sq. ft.;	s = 240,000 sq. ft.
Split *B*, 6 × 20 ft.,	12,000 ft., long;	a = 120 sq. ft.;	s = 624,000 sq. ft.
Split *C*, 6 × 12 ft.,	8,000 ft., long;	a = 72 sq. ft.;	s = 288,000 sq. ft.
Split *D*, 4 × 6 ft.,	1,000 ft., long;	a = 24 sq. ft.;	s = 20,000 sq. ft.

Calculate the horsepower on the air, unit pressure and water gage concerned in producing this circulation, using the part potential values and employing the method by summation of potentials; no regulators being used in the mine, but the division of air being natural.

Solution.—The part potential values for the several splits are as follows:

Split *A*, $X_{p1} = a\sqrt{\frac{a}{s}} = 96\sqrt{\frac{96}{240{,}000}} = 1.920$

Split *B*, $X_{p2} = 120\sqrt{\frac{120}{624{,}000}} = 1.664$

Split *C*, $X_{p3} = 72\sqrt{\frac{72}{288{,}000}} = 1.138$

Split *D*, $X_{p4} = 24\sqrt{\frac{24}{20{,}000}} = 0.831$

Sum of part pressure potentials (ΣX_p) 5.553

Substituting this value for ΣX_p, in the formulas for finding the horsepower on the air and water gage, in natural splitting,

Horsepower on air, $H = \dfrac{0.00000002 \times 100{,}000^3}{33{,}000 \times 5.553^2} = 19.6\ hp$

Unit pressure, $p = \dfrac{0.00000002 \times 100{,}000^2}{5.553^2} = 6.48\ lb.\ per\ sq.\ ft.$

Water gage, $w.g. = \dfrac{0.00000002 \times 100{,}000^2}{5.2 \times 5.553^2} = 1.24\ in.$

In natural splitting or when no regulators are employed the general mine potential is always equal to the sum of the several split potentials, which is true for either power or pressure.

General Mine Potential.—The power potential for the combined splits can be calculated from the total quantity of air in circulation and the resulting pressure, using the formula

$$X^3_u = \frac{Q^2}{p};\ \text{or}\ X_u = \sqrt[3]{\frac{Q^2}{p}}$$

Example.—What is the general power potential for all the splits combined, in the example given above, where 100,000 cu. ft. of air was circulated under a pressure of 6.48 lb. per sq. ft.?

Solution.—The general power potential for these combined splits is

Mine power potential, $X_u = \sqrt[3]{\dfrac{Q^2}{p}} = \sqrt[3]{\dfrac{100{,}000^2}{6.48}} = 1155$

Example.—An air current of 60,000 cu. ft. per min. is passing in an airway 8 × 10 ft. in section, to a point 1500 ft. distant from the foot of the downcast shaft, where it divides naturally between the following four airways or splits:

Split *A*,	5 × 6 ft., 900 ft. long
Split *B*,	6 × 6 ft., 825 ft. long
Split *C*,	4 × 6 ft., 840 ft. long
Split *D*,	4 × 5 ft., 720 ft. long

What is the quantity of air passing in each split; and what will be the water-gage reading for the entire mine and power on the air, at the foot of the downcast shaft?

Solution.—Since the water gage is required in this case, the relative potential values cannot be used; but, instead, the part potential value (omitting *k*) is found for the main airway and for each split separately;

thus, taking the length of the main airway including the return as $2\times 1500 = 3000$ ft.:

Main airway, $a = 80;\ o = 36;\ l = 3000;\ X_1 = 80\sqrt{\dfrac{80}{3000\times 36}} = 2.177$

Split A, $a = 30;\ o = 22;\ l = 900;\ X_a = 30\sqrt{\dfrac{30}{900\times 22}} = 1.168$

Split B, $a = 36;\ o = 24;\ l = 825;\ X_b = 36\sqrt{\dfrac{36}{825\times 24}} = 1.531$

Split C, $a = 24;\ o = 20;\ l = 840;\ X_c = 24\sqrt{\dfrac{24}{840\times 20}} = 0.907$

Split D, $a = 20;\ o = 18;\ l = 720;\ X_d = 20\sqrt{\dfrac{20}{720\times 18}} = 0.786$

The general split potential (X_0) is equal to the sum of the potentials for the four splits; thus,

$$X_0 = \Sigma X_{abcd} = 4.392$$

The quantity of air that will pass in each of these splits is proportional to the corresponding split potential, assuming that no regulators are employed but all the airways are free and unobstructed. The natural division of the air between the four splits is therefore calculated in the usual manner, as follows:

Split A, $q_a = 60{,}000\dfrac{1.168}{4.392} = 15{,}950$ *cu. ft. per min.*

Split B, $q_b = 60{,}000\dfrac{1.531}{4.392} = 20{,}920$ *cu. ft. per min.*

Split C, $q_c = 60{,}000\dfrac{0.907}{4.392} = 12{,}390$ *cu. ft. per min.*

Split D, $q_d = 60{,}000\dfrac{0.786}{4.392} = 10{,}740$ *cu. ft. per min.*

Total circulation 60,000 *cu. ft. per min.*

In order to find the water-gage reading at the foot of the downcast shaft, for this circulation, it is necessary to calculate the general mine potential X_p by combining, in tandem, the main-airway potential (X_1) and the general split potential (X_0) previously found, using the formula (p. 215).

Mine water gage, $w.g. = \dfrac{kQ^2}{5.2}\Sigma\left(\dfrac{1}{X_p{}^2}\right)$

Substituting the values of the potential factors previously found.

Main airway, $\dfrac{1}{X^2_1} = \dfrac{1}{2.177^2} = 0.2109$

Split section, $\dfrac{1}{X^2_0} = \dfrac{1}{4.392^2} = 0.0518$

Sum of values, $\Sigma(1/X_p{}^2)$ 0.2627

Finally, substituting this value in the above formula for finding the mine water gage,

$$w.g. = \frac{0.00000002 \times 60{,}000^2 \times 0.2627}{5.2} = 3.64 \ in.$$

In like manner, the power on the air, at the foot of the shaft is calculated by the formula

$$H = \frac{kQ^3}{33{,}000}\Sigma\left(\frac{1}{X_p{}^2}\right) = \frac{0.00000002 \times 60{,}000^3 \times 0.2627}{33{,}000} = 34.39 \ hp.$$

PROPORTIONATE DIVISION OF AIR

Every large and well managed mine is, now, divided into two or more separate ventilation districts. The natural division of the air current between these several districts is not generally in proportion to their respective needs.

The longer entries, working more men and requiring the most air for their ventilation are the ones that have the greater resisting power and, as a result, receive a lesser proportion of the air, in natural division; while, on the other hand, the shorter air-courses where fewer men are working and less air is required, have a smaller resisting power and naturally pass the larger quantity of air.

To Regulate the Air.—In order to overcome these natural conditions, in mine ventilation, and divide the main air current so as to give each district of the mine the required proportion of air, it is necessary to employ some means that will produce this result.

Two methods have been used to divide the air proportionately; they are as follows:

1. The flow of air is obstructed in those airways that take naturally more than the desired porportion.

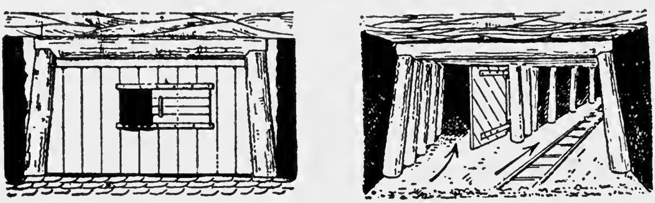
2. The power on the air, at the mouth of each split, is proportioned to the work to be performed in that split.

The former of these two methods has been in common use for many years; the latter was suggested (Mine Ventilation, Beard, 1894, p. 93) as an improvement and has been put in use since in many mines where practical considerations would permit.

The Box Regulator.—This form of regulator is shown in Fig. 32 (*a*), and consists of a brattice built in the return airway or haulway. As shown in the figure, an opening is provided in the brattice and a sliding shutter is used to regulate the size of the opening so as to control the flow of air in that airway or split. If more air is needed the shutter is pushed back so as to enlarge the opening; or the shutter can be partially closed to decrease the quantity of air passing in the split.

The Door Regulator.—Wherever the conditions will permit this form of regulator to be employed it will be found an improvement over the common "box regulator," just described.

As shown in Fig. 32 (*b*), the door regulator consists of a door hung at the mouth of an entry or split and swung into the wind. The door should be arranged so that it will fall naturally against a set-stop, and when not in use will assume a position whereby the air current will be divided in the desired proportion, between the two airways or splits.



a *b*

Fig. 32.

Effect of Regulator.—Any regulation of the air current in a mine, to accomplish a distribution of air other than what is natural, causes an increase of both the power producing the circulation and the resulting pressure or water gage. This is true in every case, whatever form of regulator is employed, provided the total quantity of air in circulation is not decreased. The reason that an increase of power is necessary in proportionate splitting, is that an increase in the circulation in any split causes a corresponding increase in pressure; and this pressure is the same for all splits starting from the same point in the mine. To circulate the same quantity of air against this higher pressure requires a corresponding increase of power.

Illustration.—Let it be required to find the horsepower and the pressure per square foot, in the following distribution of the air current between the following four splits; the natural distribution of air, as previously calculated (p. 225), being repeated here, for sake of comparison:

		Nat. div. (cu. ft. p. m.)	Reqd. div. (cu. ft. p. m.)
Split *A*,	8 × 12 ft., 6,000 ft. long,	34,570	20,000
Split *B*,	6 × 20 ft., 12,000 ft. long,	29,960	40,000
Split *C*,	6 × 12 ft., 8,000 ft. long,	20,500	30,000
Split *D*,	4 × 6 ft., 1,000 ft. long,	14,970	10,000
Total circulation,		100,000	100,000

Solution.—The first step is to calculate the natural pressure for each split when passing the required quantity of air per minute, by substituting the following values for the area, perimeter and length of each split, in the formula for finding the unit pressure:

Split *A*,	$a = 96$ sq. ft.;	$o = 40$ ft.;	$l = 6{,}000$ ft.
Split *B*,	$a = 120$ sq. ft.;	$o = 52$ ft.;	$l = 12{,}000$ ft.
Split *C*,	$a = 72$ sq. ft.;	$o = 36$ ft.;	$l = 8{,}000$ ft.
Split *D*,	$a = 24$ sq. ft.;	$o = 20$ ft.;	$l = 1{,}000$ ft.

The natural pressure in each split is then calculated as follows:

$$\text{Split } A,\ p = \frac{0.00000002 \times 6000 \times 40 \times 20{,}000^2}{96 \times 96 \times 96} = 2.17\ lb.\ per\ sq.\ ft.$$

$$\text{Split } B,\ p = \frac{0.00000002 \times 12{,}000 \times 52 \times 40{,}000^2}{120 \times 120 \times 120} = 11.55\ lb.\ per\ sq.\ ft.$$

$$\text{Split } C,\ p = \frac{0.00000002 \times 8000 \times 36 \times 30{,}000^2}{72 \times 72 \times 72} = 13.89\ lb.\ per\ sq.\ ft.$$

$$\text{Split } D,\ p = \frac{0.00000002 \times 1000 \times 20 \times 10{,}000^2}{24 \times 24 \times 24} = 2.98\ lb.\ per\ sq.\ ft.$$

The highest natural pressure is developed in Split *C*, in the required distribution of air, and that is, therefore, the "open" or "free" split, regulators being necessary in each of the other splits, to raise the pressure to the same amount.

The horsepower producing this circulation is then

$$H = \frac{100{,}000 \times 13.89}{33{,}000} = 42.09\ hp.$$

Pressure Due to Box Regulator.—The primary effect of this regulator is to increase the pressure on its intake side, by obstructing the flow of air in the airway or split that it controls. This increase of ventilating pressure is necessary to accomplish the desired increase of circulation in another airway, which remains open or unobstructed and which, for that reason, is called the **"free split."**

The increase of pressure is the pressure due to the regulator; and is equal to the difference between the natural pressure of the free split and that of the split in which the regulator is placed, calculated for the required distribution of air. For example, in the illustration previously given, the natural pressure required to circulate 30,000 cu. ft. of air in Split C was 13.89 lb. per sq. ft., while that required to circulate 20,000 cu. ft. in Split A was only 2.17 lb. per sq. ft. The pressure due to the regulator in Split A is, therefore,

$$13.89 - 2.17 = 11.72 \textit{ lb. per sq. ft.}$$

Velocity of Air Passing Regulator.—The velocity of the air flowing through the regulator is determined by the difference of pressure on its two sides or the pressure due to the regulator. This velocity is calculated from the well known formula

$$v = \sqrt{2gh}$$

In the case of a regulator, the pressure head is equal to the pressure (p_r) due to the regulator, divided by the weight of 1 cu. ft. of air ($w = 0.0766$ lb.); and taking $2g = 2 \times 32.16 = 64.32$ ft. per sec., the theoretical velocity of the air due to this pressure is

$$v = \sqrt{\frac{64.32p_r}{0.0766}} = \text{say } 29\sqrt{p_r}$$

By this formula, the theoretical velocity corresponding to the pressure due to the regulator in Split A is

$$v = 29\sqrt{11.72} = 99.28 \textit{ ft. per sec.}$$

Quantity Passing Regulator.—Owing to the vena contracta, at the opening in a box regulator, the effective area of the opening is only 0.62 of the actual area A; and the quantity (Q), in cubic feet per minute, passing through the opening, is

$$Q = 60(0.62Av) = 37.2Av$$

Or, substituting the value of v, as given above,

$$Q = 37.2 \times 29A\sqrt{p_r} = 1078A\sqrt{p_r}$$

Or, since $p = 5.2\ w.g.$

$$Q = 1078A\sqrt{5.2\,w.g.} = \text{say } 2460A\sqrt{w.g.}$$

Area of Opening, Box Regulator.—The area of the opening required to pass any given quantity of air, in splitting, is found by solving the last formula given above, with respect to A, as follows:

$$A = \frac{Q}{2460\sqrt{w.g.}} = \frac{0.0004Q}{\sqrt{w.g.}}$$

Example.—Calculate the size of opening in each of the regulators in Splits A, B and D, in the illustration previously given where the required circulation was as follows:

	Required circulation	Natural pressure	
Split A,	20,000 cu. ft.;	2.17 lb. per sq. ft.	Regulator
Split B,	40,000 cu. ft.;	11.55 lb. per sq. ft.	Regulator
Split C,	30,000 cu. ft.;	13.89 lb. per sq. ft.	Free split
Split D,	10,000 cu. ft.;	2.98 lb. per sq. ft.	Regulator

Solution.—The first step is to find the pressure due to the regulator and reduce that to water gage, in each case. The pressure due to the regulator is found by subtracting the natural pressure for the given split from that of the free split, which is always the one having the greatest natural pressure. Thus,

	Pressure due to regulator	Water gage
Split A, 13.89 − 2.17 =	11.72 lb. per sq. ft.;	11.72 ÷ 5.2 = 2.25 in.
Split B, 13.89 − 11.55 =	2.34 lb. per sq. ft.;	2.34 ÷ 5.2 = 0.45 in.
Split D, 13.89 − 2.98 =	10.91 lb. per sq. ft.;	10.91 ÷ 5.2 = 2.10 in.

Substituting these values for the water gage due to regulator in the formula for finding the area of opening,

Split A, $$A_a = \frac{0.0004Q}{\sqrt{w.g.}} = \frac{0.0004 \times 20{,}000}{\sqrt{2.25}} = 5.33\ sq.\ ft.$$

Split B, $$A_b = \frac{0.0004 \times 40{,}000}{\sqrt{0.45}} = 23.85\ sq.\ ft.$$

Split D, $$A_d = \frac{0.0004 \times 10{,}000}{\sqrt{2.10}} = 2.76\ sq.\ ft.$$

Use of the Door Regulator.—In the use of the door regulator, the same general formulas apply, except that in estimating the quantity of air that will pass the regulator, for a given gage or pressure; or the area of opening necessary to pass a given quantity under such gage, no allowance should be made for vena contracta, which gives the following:

Quantity of air passing through an area of opening A, in a door regulator under a water gage $w.g.$,

$$Q = 3960A\sqrt{w.g.}$$

Area of opening required to pass a quantity of air Q, in a door regulator, under a water gage $w.g.$,

Area, $$A = \frac{0.00025Q}{\sqrt{w.g.}}$$

Example.—What must be the width of opening of a regulator door where the height of the entry is 5 ft. in the clear, in order to pass 40,000 cu. ft. per min., if the natural pressure for the required circulation produces a water gage of 1.25 in. for this split and 1.75 in. for the free split?

Solution.—The difference of pressure, in this case, is equivalent to a water gage of $1.75 - 1.25 = 0.50$ in.; hence,

$$A = \frac{0.00025 \times 40{,}000}{\sqrt{0.50}} = 14.14 \textit{ sq. ft.}$$

Width of opening, $14.14 \div 5 = 2.83$ *ft.*, or 2 *ft.* 10 *in.*

SECONDARY SPLITTING

Secondary splitting involves the principles of both tandem and split circulations. The tandem portion consists of one airway of the primary split and the two airways branching from this and forming the secondary split section.

It is necessary to first find the general pressure potential for the secondary split section. This is equal to the sum of the pressure potentials of the airways forming that section. This general potential for the secondary split is then combined with the corresponding primary potential, according to the method employed for a tandem circulation, which is then regarded as one branch of the primary split.

The diagram Fig. 33 shows clearly the method of naming the splits and indicating them by symbols. The **primary splits,** branching from the point where the air current is first divided, are designated by the letters A, B, C, etc., and the corresponding potentials by X_a, X_b, X_c, etc.

Secondary splits are designated A_1, A_2, etc., and B_1, B_2, etc., depending on the primary split from which they branch; and the corresponding split potentials by X_{a1}, X_{a2}, X_{b1}, X_{b2}, etc. The general potential for a primary split is designated X_0, and for a secondary split X_{ao}, X_{bo}, etc.

In secondary splitting the operation is much simplified by calculating the general potential for each consecutive point or section, beginning always at the inby end of the system and finding first the general potential for the secondary split;

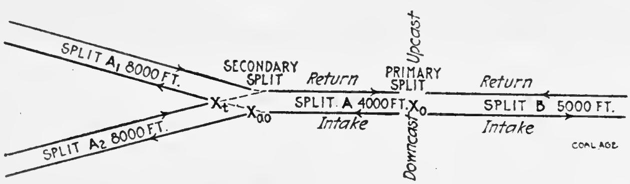


FIG. 33.

then combining this in tandem with the corresponding primary potential; and using this result to find the general potential for the primary split, in the same manner as for the secondary split. Two formulas only are necessary; the one expressing the summation of the potential values for a split circulation, and the other a similar summation for a tandem circulation. They as as follows:

General split potential, $X_{p0} = \Sigma X_p$

General tandem potential (see p. 216), $X_{pt} = \dfrac{1}{\sqrt{\Sigma(1/X_p^2)}}$

In all splitting calculations it will generally be found more convenient to use the pressure potential, for the reason that the calculation of the distribution of the air is based on equal pressures, for all splits starting from one point.

Illustration.—Primary splits are best indicated by the large letters, as Splits A, B, C, etc. Secondary splits are named after the primaries in which they occur; thus A_1, A_2, etc., or B_1, B_2, etc.

The corresponding split potentials are indicated thus:

Primary potentials,	X_a, X_b, X_c, etc.
Secondary potentials,	X_{a1}, X_{a2}; X_{b1} X_{b2}; X_{c1} X_{c2}; etc.
General split potentials,	X_{ao}, X_{bo}, X_{co}
General mine potentials,	X_o

To illustrate the calculation of the effect of making a secondary split in the circulation calculated under "Unequal Splits" (p. 224), assume the air is again split in C, at a point 500 ft. inby from the main or primary split.

Splits A and B are the same as before, while Split C is now 500 ft. long; Split C_1, 1200 ft. long; and Split C_2, 800 ft. long. The part potential values for the splits are, then,

Split A, $\quad X_p = a\sqrt{\frac{a}{lo}}$; $\quad X_a$ (as before) $= 1.837$

Split B, $\quad X_b$ (as before) $= 1.623$

Split C, $\quad X_c = 48\sqrt{\frac{48}{500 \times 32}} = 2.629$

Split C_1, $\quad X_{c1} = 48\sqrt{\frac{48}{1200 \times 32}} = 1.697$

Split C_2, $\quad X_{c2} = 48\sqrt{\frac{48}{800 \times 32}} = 2.078$

General split potential, $\Sigma X_c = 1.697 + 2.078 = 3.775$

Combining this general potential for Splits C_1 and C_2 with the potential for Split C, in tandem, we have,

Part potential factors, (*Tandem circulation*) $\quad \frac{1}{X^2_c} = \frac{1}{2.629^2} = 0.1447$

$$\frac{1}{(\Sigma X^2_c)} = \frac{1}{3.775^2} = 0.0702$$

Tandem value, $\quad X_{co} = \Sigma\,(1/X^2_c) \ldots\ldots \quad 0.2149$

General part potential,

(*Primary split C*) $\quad X_{co} = \frac{1}{\sqrt{0.2149}} = 2.157$

Part potential, Split A, $\quad X_a = 1.837$

Part potential, Split B, $\quad X_b = 1.623$

Mine pressure potential, $X_{po} \ldots\ldots$ 5.617

Mine power potential, (After splitting)

$$X_{u2} = \sqrt[3]{5.617^2} = 3.160$$

Mine power potential, (Before splitting, p. 225)

$$X_{u1} = \sqrt[3]{4.636^2} = 2.780$$

For a constant power, the quantity ratio is equal to the power-potential ratio; thus,

$$\frac{Q_2}{Q_1} = \frac{X_{u2}}{X_{u1}}; \text{ and } \frac{Q_2}{75{,}000} = \frac{3.16}{2.78};$$

$$Q_2 = \frac{75{,}000 \times 3.16}{2.78} = 85{,}240 \textit{ cu. ft. per min.}$$

Mine pressure, $p = k\left(\frac{Q}{X_p}\right)^2 = 0.00000002\left(\frac{85{,}240}{5.617}\right)^2 = 4.6$ *lb. per sq. ft.*

Power on the air, $u = k\left(\frac{Q}{X_u}\right)^3 = 0.00000002\left(\frac{85{,}240}{3.16}\right)^3 = 11.9$ *hp.*

The **natural division** of the main air current of 85,240 cu. ft. between the three primary splits, A, B, C; and the two secondary splits C_1, C_2, in the last example, is calculated first for the primary division, and then for the secondary, as follows: Primary splits,

Part pressure potentials		Natural	Required (cu. ft. per min.)
$X_a = 1.837$;	$q_a = \frac{1.837}{5.617} \times 85{,}240 =$	27,880	29,240
$X_b = 1.623$;	$q_b = \frac{1.623}{5.617} \times 85{,}240 =$	24,630	16,000
$X_{co} = 2.157$;	$q_c = \frac{2.157}{5.617} \times 85{,}240 =$	32,730	40,000
$\Sigma X_p = 5.617$	$Q = \ldots\ldots$	85,240	85,240

Secondary splits,

$X_{c1} = 1.697$;	$q_{c1} = \frac{1.697}{3.775} \times 32{,}730 =$	14.710	25,000
$X_{c2} = 2.078$;	$q_{c2} = \frac{2.078}{3.775} \times 32{,}730 =$	18.020	15,000
$\Sigma X_p = 3.775$		32,730	40,000

The natural pressures are then calculated for the required circulation of air in each split. The highest pressure of the secondary splits determines the **secondary pressure,** which must be added to the natural pressure of the tandem airway, to obtain the effective primary pressure for Split *C*. Finally, the highest primary pressure determines the **primary pressure,** which is the pressure for the entire split circulation. The process is as follows:

Secondary pressures,

$$p_c = k\left(\frac{q}{X_p}\right)^2;$$

$$p_{c1} = 0.00000002\left(\frac{25{,}000}{1.697}\right)^2 = \mathbf{4.341}\ lb.\ per\ sq.\ ft.$$

$$p_{c2} = 0.00000002\left(\frac{15{,}000}{2.078}\right)^2 = 1.042\ lb.\ per\ sq.\ ft.$$

Tandem $$p_c = 0.00000002\left(\frac{40{,}000}{2.629}\right)^2 = 4.630\ lb.\ per\ sq.\ ft.$$

Primary pressures, $$p_{c0} = \Sigma p_c \quad \mathbf{8.971}\ lb.\ per\ sq.\ ft.$$

$$p_a = 0.00000002\left(\frac{29{,}240}{1.837}\right)^2 = 5.067\ lb.\ per\ sq.\ ft.$$

$$p_b = 0.00000002\left(\frac{16{,}000}{1.623}\right)^2 = 1.944\ lb.\ per\ sq.\ ft.$$

Horsepower, $$H = \frac{Qp}{33{,}000};$$

$$H = \frac{85{,}240 \times 8.971}{33{,}000} = 23.17\ hp.$$

The secondary pressure, as determined by the highest natural pressure in those splits, is that in Split C_1, which is 4.341 lb. per sq. ft. Likewise the primary pressure (the highest of those splits) is that of the tandem split C_0, which is 8.971 lb. per sq. ft. These pressures are indicated above by the heavy type.

Regulators.—The difference between the secondary pressure and the natural pressure in any secondary split is the pressure due to the regulator or the **regulator pressure** for that split. The same is true for primary splits.

The pressures due to the regulators required in Splits A, B and C_2, in order to accomplish the required distribution of air are, therefore, as follows:

Split A, $8.971 - 5.067 = 3.904$ lb. per sq. ft. (0.751 in. w.g.)
Split B, $8.971 - 1.944 = 7.027$ lb. per sq. ft. (1.351 in. w.g.)
Split C_2, $4.341 - 1.042 = 3.299$ lb. per sq. ft. (0.634 in. w.g.)

The necessary area of opening in a regulator to pass the required quantity of air, under the given water gage is calculated as follows:

Box regulator, $A = \frac{0.0004q}{\sqrt{w.g.}}$;

$$A_a = \frac{0.0004 \times 29{,}240}{\sqrt{0.751}} = 13.5 \text{ sq. ft.}$$

$$A_b = \frac{0.0004 \times 16{,}000}{\sqrt{1.351}} = 5.5 \text{ sq. ft.}$$

$$A_{c2} = \frac{0.0004 \times 15{,}000}{\sqrt{0.634}} = 7.5 \text{ sq. ft.}$$

If door regulators are used the openings have the following areas:

Door regulator, $A = \frac{0.00025q}{\sqrt{w.g.}}$;

$$A_a = \frac{0.00025 \times 29{,}240}{\sqrt{0.751}} = 8.4 \text{ sq. ft.}$$

$$A_b = \frac{0.00025 \times 16{,}000}{\sqrt{1.351}} = 3.4 \text{ sq. ft.}$$

$$A_{c2} = \frac{0.00025 \times 15{,}000}{\sqrt{0.634}} = 4.7 \text{ sq. ft.}$$

The results of making the secondary split in Primary C may therefore be summarized as follows:

The above comparison shows: (1) The increase in the quantity of air in circulation and the decrease in the unit pressure and water gage, for the same power on the air, caused by making a small secondary split, in one of the original primaries. (2) The large increase of power on the air and pressure and water gage necessary to make the required distribution of air, in this case.

	Distribution of air		
	Natural circulation (No regulators)		Required (Regulators)
Split A (cu. ft. per m.)	29,720	27,880	29,240
Split B	26,260	24,630	16,000
Split C	19,020	(32,730)	(40,000)
Split C_1	......	14,710	25,000
Split C_2	......	18,020	15,000
Totals	75,000	85,240	85,240
Pressure (lb. per sq. ft.)	5.2	4.6	8.97
Water gage (in.)	1.0	0.88	1.72
Horsepower on air (hp.)	11.9	11.9	23.17

Example.—An air current of 120,000 cu. ft. per min. is passing in a mine in two splits, as follows:

Split A, 5 × 10 ft., 20,000 ft. long; 40,000 cu. ft. per min.
Split B, 5 × 10 ft., 5,000 ft. long; 80,000 cu. ft. per min.

More air being required, a careful investigation shows that Split A can be again divided at a point 2000 ft. inby from the foot of the downcast shaft, thereby forming two secondary air splits, each 5 × 10 ft., 8000 ft. long, including the return. This would make Split A 4000 ft. long including the return. With the same power on the air, what quantity of air will be circulated in this mine after dividing Split A?

Solution.—The first step is to calculate the potential values of the different sections or splits, both before and after dividing Split A to form the two secondary Splits A_1 and A_2. This being a comparison of two circulations, it is possible to use the relative potentials, reducing the areas, perimeters and lengths to their lowest relative values, which gives the following:

Before dividing Split A:

		(Relative values)
Split A,	$a = 50; o = 30; l = 20{,}000$	$a = 1; o = 1; l = 20$
Split B,	$a = 50; o = 30; l = 5{,}000$	$a = 1; o = 1; l = 5$

After dividing Split A:

Split A,	$a = 50; o = 30; l = 4{,}000$	$a = 1; o = 1; l = 4$
Split B,	$a = 50; o = 30; l = 5{,}000$	$a = 1; o = 1; l = 5$
Split A_1,	$a = 50; o = 30; l = 8{,}000$	$a = 1; o = 1; l = 8$
Split A_2,	$a = 50; o = 30; l = 8{,}000$	$a = 1; o = 1; l = 8$

Relative potentials, before division:

$$\left.\begin{aligned} X_a &= a\sqrt{\frac{a}{lo}} = 1\sqrt{\frac{1}{20 \times 1}} = \frac{1}{\sqrt{20}} = 0.2236 \\ X_b &= 1\sqrt{\frac{1}{5 \times 1}} = \frac{1}{\sqrt{5}} = 0.4472 \end{aligned}\right\} X_1 = 0.6708$$

Relative potentials, after division:

$$X_a = a\sqrt{\frac{a}{lo}} = 1\sqrt{\frac{1}{4 \times 1}} = \frac{1}{\sqrt{4}} = 0.5000$$

$$X_b = \textit{Same as before} = 0.4472$$

$$\left.\begin{aligned} X_{a1} &= 1\sqrt{\frac{1}{8 \times 1}} = \frac{1}{\sqrt{8}} = 0.3535 \\ X_{a2} &= 1\sqrt{\frac{1}{8 \times 1}} = \frac{1}{\sqrt{8}} = 0.3535 \end{aligned}\right\} X_{a0} = 0.707$$

Tandem summation (X_a and X_{a0}):

$$X_t = \frac{1}{\sqrt{1/X^2_a + 1/X^2_{ao}}} = \frac{1}{\sqrt{1/0.5^2 + 1/0.707^2}} = 0.4082$$

$$X_2 = X_t + X_b = 0.4082 + 0.4472 = 0.8554$$

Since the power is the same, before and after division and calling these respective general potentials X_1, X_2, we have

$$\frac{Q_1^3}{(X_1)^2} = \frac{Q_2^3}{X_2^2}; \text{ and } \frac{120{,}000^3}{0.6708^2} = \frac{Q_2^3}{0.8554^2}$$

$$Q_2 = 120{,}000\sqrt[3]{\left(\frac{0.8554}{0.6708}\right)^2} = 141{,}100 \textit{ cu. ft. per min.}$$

THEORETICAL CONSIDERATIONS IN SPLITTING

Theory assumes that when an air current traveling in an airway divides, at a certain point called the "point of split," into two separate currents or "splits," the unit pressure (p) at the point of split is common to each split. In other words, two splits starting from the same point in a mine have the same unit pressure (p) and, for the same sectional area (a), the resistance ($R = pa$) is the same for each split. The same holds true for any number of splits (n) of equal area.

Whether the unit pressure (p) or the unit work (pv) is the factor common to each of two or more splits starting from the same point will not be discussed here. The law of dynamic equilibrium of fluids points to the equality of unit work for each split. The comparison of the relation of the quantity of air (q), the rubbing surface (s) and the sectional area (a), on these two bases of reasoning, is as follows:

Unit pressure	Unit work
$p = \frac{ksq^2}{a^3}$	$\frac{u}{a} = \frac{ksq^3}{a^4}$

For constant unit pressure:

$$q \text{ varies as } a\sqrt{\frac{a}{s}}$$

For constant unit work:

$$q \text{ varies as } a\sqrt[3]{\frac{a}{s}}$$

Practical Conditions.—In considering the practical results of splitting the air current in a mine, it may be assumed that the power on the air (U) at the mouth (intake) of the mine remains constant. Assuming a number of splits (n), starting from the same point in the mine, at or near the shaft bottom or mine entrance, the total area of passage is na and the formula for power is then

$$U = \frac{ksQ^3}{(na)^3}$$

which shows that, since in any case U, k, s and a are each constant, Q^3 varies as n^3, or Q varies as n, which is the number of equal splits, each having an area a.

In other words, the quantity of air circulated in a given mine, by a given power on the air (effective power), is proportional to the number of splits, assuming the splits all start from the mine entrance or so near to it that the resistance of the main intake entry, slope or shaft may be ignored. Under these conditions, splitting the air has no effect to alter the velocity or the resistance in the mine.

When the point of split, however, is some distance inby from the mouth of the mine or "daylight" the effect of splitting the air, in that case, is to cause a disproportion. The quantity of air circulated by a given power no longer varies as the number of splits; but the ratio of increase in volume is less, because the power on the air at the mouth of the splits is decreased by splitting.

Assuming, as before, a constant power on the air at the mouth of the mine, since the quantity has been increased by splitting, both the velocity and resistance have been increased in the main airway, which absorbs more power thus decreasing the power on the splits.

Effect of Splitting on Velocity.—In order to show the general effect of splitting the air current, at any point in a mine, on the velocity (v_0) in the shaft or main airway and the velocity (v_1) in the splits, it is necessary to know the ratio (m) of the rubbing surface (s_1) in the splits, to that of (s_0) in the shaft or main airway; also, the ratio (n) of the total area (A_1) of the splits, to that of (A_0) in the shaft or main airway.

Then, $$s_1 = ms_0; \quad \text{and } A_1 = nA_0; \tag{1}$$

and, since for a given quantity the velocity varies inversely as the area,

$$v_1 = \frac{v_0}{n}. \tag{2}$$

But, the power on the air (u) at the mouth of the mine is equal to the power (u_0) absorbed in the shaft or main airway, or both, plus the power (u_1) absorbed in the splits.

$$u = u_0 + u_1 \tag{3}$$

or, expressed in terms of the mine, since $u = ksv^3$,

$$u = k(s_0v_0^3 + s_1v_1^3) \tag{4}$$

Substituting for s_1 and v_1^3 the values given in Equations 1 and 2, gives after simplifying

$$u = ks_0v_0^3\left(1 + \frac{m}{n^3}\right) = ks_0v_0^3\left(\frac{n^3 + m}{n^3}\right) \tag{5}$$

Equation 5 shows clearly that, for a constant power on the air at the mouth of a mine, in splitting,

$$v_0 \text{ varies as } \frac{n}{\sqrt[3]{n^3 + m}} \tag{6}$$

and, observing Equation 2,

$$v_1 \text{ varies as } \frac{1}{\sqrt[3]{n^3 + m}} \tag{7}$$

It appears from the last two equations that as the ratio of the split area to the shaft or main-intake area, represented by n, is increased the main-intake velocity (v_0) is increased, while the split velocity (v_1) is decreased, the increase and decrease of velocity, however, being less rapid than the change in the area ratio.

Effect of Splitting on Quantity.—The quantity of air in circulation varies directly as the intake velocity v_0; or, for a constant power (u) on the air,

$$Q \text{ varies as } \frac{n}{\sqrt[3]{n^3 + m}} \tag{8}$$

Effect of Splitting on the Mine Resistance.—The total mine resistance is the sum of the main-intake and split resistances. Thus,

$$R = k(s_0 v_0^2 + s_1 v_1^2) \tag{9}$$

and

$$R = k s_0 v_0^2 \left(\frac{n^2 + m}{n^2}\right) \tag{10}$$

Finally, from Equations 6 and 10 is derived

$$R \text{ varies as } \frac{n^2 + m}{\sqrt[3]{(n^3 + m)^2}} \tag{11}$$

PRACTICAL PROBLEM

Example.—A current of 25,000 cu. ft. per min. is passing in a shaft mine. The shafts are 8 × 12 ft. in section and 250 ft. deep. The airways are 6 × 10 ft. and 15,000 ft. long, including the return. (*a*) What is the water gage due to this circulation? (*b*) Assuming the power applied to the fan shaft remains unchanged and the current is divided into two equal splits, at a point 1500 ft. inby from the foot of the shaft, what volume of air may be expected to be passing? (*c*) What will be the water-gage reading on the fan drift and at the bottom of the shaft, after splitting?

Solution.—The rubbing surface and sectional area of the shafts and airways are, respectively, as follows:

		Sq. Ft.
Shafts—		
Rubbing surface	2(8 + 12)2 × 250 =	20,000
Sectional area	8 × 12 =	96
Airways (total)—		
Rubbing surface	2(6 + 10)15,000 =	480,000
Sectional area	6 × 10 =	60

Main airway—
Rubbing surface.................... $2(6 + 10)2 \times 1500 =$ 96,000
Sectional area.............................. $6 \times 10 =$ 60
Two equal splits—
Rubbing surface...................... $2(6 + 10)12{,}000 =$ 384,000
Sectional area............................ $2(6 \times 10) =$ 120
The relative part potential factors are then:
Before splitting—

Shafts .... $\left(\frac{1}{X_p{}^2} \text{ or } \frac{1}{X_u{}^3}\right) = \frac{s}{a^3} = \frac{20{,}000}{96^3} = 0.0226$

Airways (total)............................ $= \frac{480{,}000}{60^3} = 2.2223$

General relative mine potential factor $\left(\Sigma \frac{1}{X_p{}^2}\right)$ 2.2449

After splitting—
Shafts (as before)..................................... 0.0226

Main airway......................... $\frac{1}{X_p{}^2} = \frac{s}{a^3} = \frac{96{,}000}{60^3} = 0.4444$

Splits................................ $= \frac{384{,}000}{120^3} = 0.2222$

General relative mine potential factor $\left(\Sigma \frac{1}{X_p{}^2}\right)$ 0.6892

(*a*) **Water gage** (before splitting)—

$$w.g. = \frac{1}{5.2}\left(Q^2 \times \frac{1}{X_p{}^2}\right) = \frac{0.00000002 \times 25{,}000^2 \times 2.2449}{5.2} = 5.4 \text{ in.}$$

(*b*) For a constant power on the air, the quantity varies directly as the mine power potential; but, for a constant power applied to the fan shaft, owing to the efficiency of the fan varying inversely as the 3/5 power of the potential X_u the quantity varies as the 4/5 power of that potential.

The mine potentials, in this case, are,

Before splitting—Since $1/X^3{}_{u1} = 2.2449$; $X_{u1} = \frac{1}{\sqrt[3]{2.2449}} = 0.7637$

After splitting—Since $1/X^3{}_{u2} = 0.6892$; $X_{u2} = \frac{1}{\sqrt[3]{0.6892}} = 1.1321$

Then, for a constant power applied to the fan shaft, the quantity of air in circulation varies as the 4/5 power of the power potential, which gives for the circulation after splitting

$$Q_2 = Q_1 \left(\frac{X_{u2}}{X_{u1}}\right)^{\frac{4}{5}} = 25{,}000 \left(\frac{1.1321}{0.7637}\right)^{\frac{4}{5}} = 34{,}250 \text{ cu. ft. per min.}$$

(*c*) **Water gage** (after splitting).—In the fan drift the gage is

$$w.g. = \frac{0.00000002 \times 34{,}250^2 \times 0.6892}{5.2} = 3.1 \text{ in.}$$

To find the gage at the shaft bottom it is necessary to deduct the potential factor for the two shafts from the total potential factor for the mine after splitting; thus

$$\frac{1}{X_p^2} - \frac{1}{X_{po}^2} = 0.6892 - 0.0226 = 0.6666$$

Then, since the gage is proportional to this potential factor, the gage at the bottom of the shaft is

$$w.g. = 3.1 \times \frac{0.6666}{0.6892} = 3.0\,in.$$

Relative Variation of Factors.—The following relation of some of the more important factors in the ventilation of mines by means of centrifugal fans is based on the results of many experiments:

Power on Air Constant ($KU = u$)—

Unit pressure, p varies inversely as Q

p varies as $\frac{1}{X_u} = \frac{\sqrt[3]{s}}{a}$

Quantity, Q varies as $X_u = \sqrt[3]{\frac{a^3}{s}} = \frac{a}{\sqrt[3]{s}}$

Power Applied to Fan Shaft Constant (U)—

Efficiency, $1/K^5$ varies as $X_u^3 = X_p^2 = a^3/s$
Effective power, u varies as K
Quantity, Q^5 varies as X_u^4

Mine Potential Constant ($X_u^3 = X_p^2 = a^3/s$)—

Effective power, u varies as Q^3
Quantity, Q^5 varies as n^4
Water gage, $(w.g.)^5$ varies as n^8

SECTION VII

PRACTICAL VENTILATION

Conducting Air Currents, Air Bridges—General Plan of Mine—Distribution of Air in the Mine—Splitting Air Currents—Systems of Ventilation—Systems of Mine Airways.

The first step, in the practical ventilation of a mine, is to determine the volume of air that will be required in order to maintain a pure and wholesome atmosphere in the mine workings. This will depend on conditions, such as the size and depth of the mine; thickness and inclination of the seam; character and quality of the coal; kind and quantity of gas generated; methods of working the seam and mining the coal. Aside from these conditions the volume of air must always be sufficient to meet the requirements of the mine law.

The second question to be determined is the general ventilating pressure or water gage, under which the mine is to be ventilated. This will depend on the possible extent and size of the workings and the power available. The water gage, in mining practice, varies from a fraction of an inch to 3 or 4 in., in this country; and higher gages are in use in the deep mines of Belgium and other countries. The best practice, however, employs such a system of mining that the required volume of air can be circulated under, say 1 or 2 in. of water gage. This can only be accomplished by so planning the mine, in the start, that it can be divided into separate ventilation districts. The number of ventilation districts should increase with the development of the mine. Each district is thus ventilated by a separate air split or current, which insures good air, besides reducing the water gage necessary for the ventilation of the mine.

Power Required to Produce a Given Circulation.—Having decided on the volume of air required and the water gage, these factors determine the power that will be necessary to produce the circulation. The power on the air may, generally, be safely taken as 60 per cent. of the indicated horsepower of the engine driving the ventilating fan. For example, the circulation of 75,000 cu. ft. of air against a water gage of 2 in. will require, with a safe margin, an engine capable of developing

$$\frac{75{,}000 \times 2 \times 5.2}{0.60 \times 33{,}000} = 39.4, \textit{ say } 40 \textit{ hp.}$$

The above calculation assumes a properly designed ventilating fan, since a poorly designed fan, or a fan working under conditions for which it is not adapted, may give an efficiency of only 40 or 50 per cent.; or at times this may not exceed 25 per cent., under particularly adverse conditions. An unsuspected negative air column existing in some portion of the mine may be the hidden cause of the low efficiency of a ventilating fan.

Conducting Air Currents

Conducting Air Currents in Mines.—To conduct the air on its course through the mine, **doors, stoppings, brattices, air-crossings,** or **bridges**—either **overcasts** or **undercasts**—are employed to deflect the air current. When the air is divided and made to travel in two or more splits **regulators** are used to proportion the quantity of air to the requirements in each split.

In Fig. 34 are shown two forms of self-closing doors used in mines. There are many different methods in use to prevent a mine door standing open, but these are as practical as any. The door on the left is shown with canvas flaps to stop the leakage of air. Both doors swing either way, being heavy enough to overcome the pressure of the ventilating current.

Stoppings, in mine ventilation, are built in entries or in crosscuts for the purpose of closing the passage. When built in crosscuts they serve to carry the air current forward to the head of the entry. A common form of stopping consists of two walls of slate or rock built 10 or 12 in. apart and the space between them filled tight with road dirt or sand. More substantial stoppings are built of brick laid in cement, or of concrete.

In Fig. 34 is also shown the right and the wrong way of erecting a line of brattice in a pair of headings. As shown in

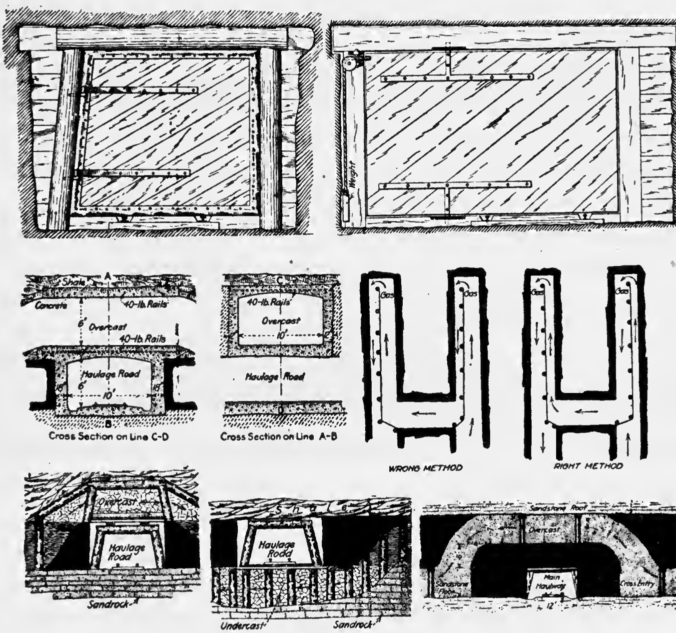


FIG. 34.

each of the figures, a row of posts is set, one at a time, and canvas or brattice boards nailed to them on the intake side. The posts are stood 18 in. or 2 ft. from the right rib if the intake is on the right, or the left rib if on the left. The same order is followed on the return airway or heading. The work of nailing the canvas or boards to the post is done on the fresh-air side and the brattice extended as the current sweeps away the gas accumulated in these headings. The arrows show the course of the air as it circulates around the brattice in each heading.

Air Bridges.—In Fig. 34 are also shown different methods of constructing air bridges in mines, for the purpose of conducting one air current across another. First is shown a standard type of overcast built of reinforced concrete. Immediately below this is shown two common types of air bridges, an overcast and an undercast. In the "**undercast**" shown on the right, the cross-current of air is conducted under the main road or heading, the bridge in that case forming the floor of the roadway. A safer and more serviceable form of air bridge, however, is the "**overcast**" shown on the left, by which the cross-current is carried over the haulage road. The undercast possesses the disadvantage that it cannot be drained and may become flooded and cut off the air current completely.

Natural Overcast.—Owing to the difficulty of keeping air bridges air-tight, and for the further reason that the possible destruction of an air bridge by an explosion would cut off the circulation of air in the district fed by that means, a **natural overcast** is frequently referred.

In the lower right-hand corner of Fig. 34 is shown a natural overcast as driven in the upper portion of a thick coal seam, although the same form of overcast is often driven in the rock strata overlying a thinner seam. Such a natural overcast is formed by starting an uprise in the roof of the cross-entry, a short distance on either side of the main heading, and then driving a crosscut in the solid formation above and across the main roadway, thereby forming a wholly separate air passage for the intake and return air currents.

Regulators.—As described previously and illustrated in Fig. 32, regulators are used to divide an air current in any desired proportion between two entries or splits. The "box" regulator is commonly placed on the return airway, where it offers no obstruction to haulage, while the "door" regulator is always placed on the intake. The use and effect of these two forms of regulators are fully treated under "Proportionate Division of Air," page 231, in the section "Theory of Ventilation."

GENERAL PLAN OF MINE

Requirements.—In the planning or laying cut of a mine the most careful consideration must be given to the questions of **ventilation, drainage** and **haulage,** as these arrangements, to a great degree, determine the successful operation of the mine.

In order to insure the safe and economic extraction of the coal, the same careful consideration must be given to ascertaining the extent and character of the seam, its depth below the surface, inclination and thickness, the character of the roof and floor and the hardness of the coal; its cleavages and faults, impurities, etc.

The information thus gained will be of the first importance in deciding on the most suitable method of mining to adopt, in order to secure the largest returns on the investment, the most complete extraction of the coal and the greatest safety in mining the same.

Economy and Efficiency.—The **economic ventilation** of a mine premises the circulation of the required air volume, with the least expenditure of power. **Efficient ventilation** requires the circulation and proportionate distribution in the mine workings, of such a volume of air as will not only meet the requirements of the law, but, likewise, produce the necessary velocity in all roads and passageways and at the working faces of all headings and chambers, so as to sweep away the smoke and gases that would otherwise accumulate therein; and to so ventilate all waste, void and abandoned places as to prevent them from becoming a menace to the safety of the mine as reservoirs for the accumulation of gas.

Drainage.—Economic mine drainage requires such a disposition of the openings driven in the seam for the extraction of the coal, including all passageways, headings and chambers, that the water coming from the strata will flow by gravity, either to the main sump at the shaft or slope bottom, or to certain gathering centers from which it can be readily siphoned to the main sump or pumped directly to the surface.

In practically level seams or seams having slight inclination, the question of drainage does not materially affect the general mine plan. In this case, good roadside ditches afford the necessary waterways by which the underground water flows to the sumps provided to receive it Such sumps or catch basins are located at one or more convenient low places or "swamps," in the mine, where it is possible to install a pump of sufficient size to handle the water of that section at all times.

The rooms or chambers, in practically level seams, are turned off both entries of a pair, which greatly reduces the expense of entry driving and necessary maintenance of roadways and air-courses.

In inclined seams the direction and amount of pitch are controlling factors in determining the general plan of the mine, in respect to the course of main roads, cross-headings and rooms or chambers. In respect to drainage, it is important to drive all such openings to the rise, in order to avoid the annoyance and expense of providing artificial means of draining the working faces.

Haulage.—Economic mine haulage requires that the coal, like water must gravitate, as far as practicable, from the coal face where it is mined, to the foot of the shaft or slope opening from whence it is hoisted to the surface.

In level seams, the question of haulage does not affect the plan of mine; but, in seams of more or less inclination, it becomes a matter of first consideration.

In inclined seams, it is always possible to drive the main haulage roads in such a direction that the grade of the road will not only favor the movement of the loaded cars, but will be such that the power required to haul the loaded trip out of the mine will be equal to that necessary for hauling the empty trip back into the mine. This is called the "economical grade."

The grade of any road, or the road grade, in an inclined seam, may be calculated when the angle of inclination of the seam and the angle the road makes with the strike of the seam are known, by the following rule:

Rule.—The tangent of the grade angle is equal to the tangent of the angle of inclination of the seam, multiplied by the sine of the angle the road makes with the strike of the seam.

Or, calling the angle between the road and the strike of the seam, the "road angle," this angle is calculated by the use of the formula

$$\sin \text{ road angle} = \frac{\tan \text{ grade angle}}{\tan \text{ inclination}}$$

There is shown clearly in Fig. 35, a perspective plan of a pair of entries with rooms turned off the haulage road. The

Fig. 35.

left-hand entry is the return air-course, while the haulage road is the intake. A canvas or curtain hung on the entry just inside of the mouth of the first room deflects the intake air mostly into the rooms, where it passes through the breakthroughs from room to room. Better results are generally obtained when the breakthroughs are staggered or not driven directly opposite each other, as shown in the figure. The

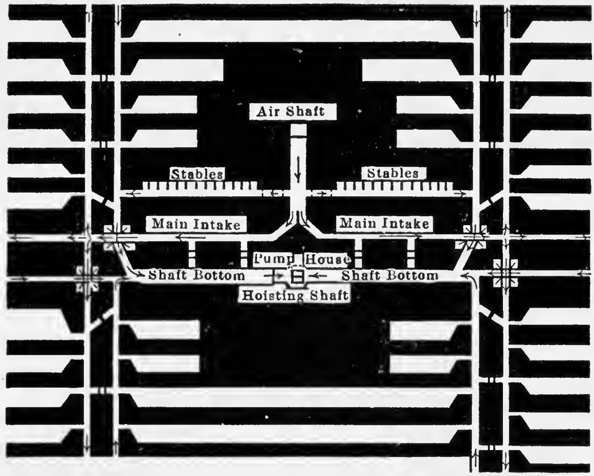

Air Shaft
Stables
Stables
Main Intake
Main Intake
Pump House
Shaft Bottom
Shaft Bottom
Hoisting Shaft

Fig. 36.

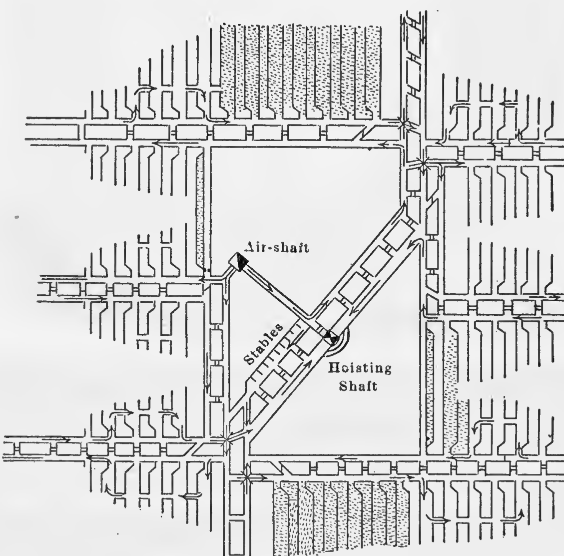

Air-shaft
Stables
Hoisting
Shaft

Fig. 37.

crosscuts on the entries are closed by substantial stoppings, except the last crosscut where the intake air passes into the return, as shown by the arrows.

General Plan, Level Seam.—In Fig. 36 is illustrated the general plan of a mine shaft bottom in a level seam. At times, it may be necessary to drive the shaft bottom at an angle with the main and cross-entries, as shown in Fig. 37, in order to square the hoisting shaft with the loading tracks

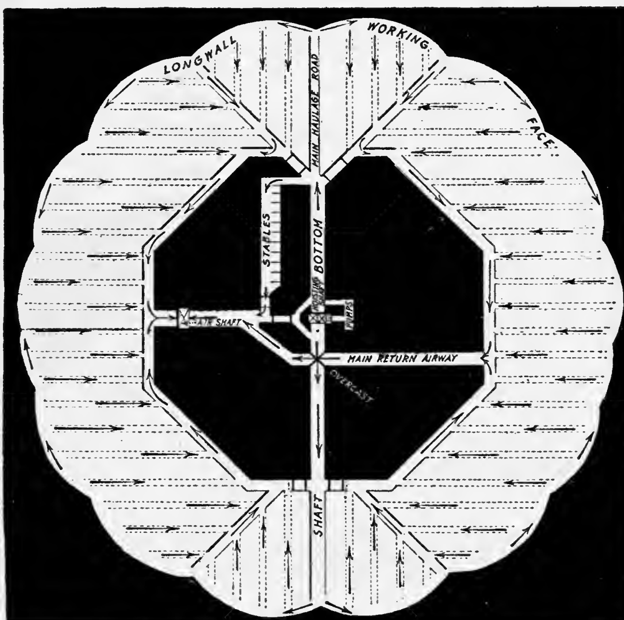
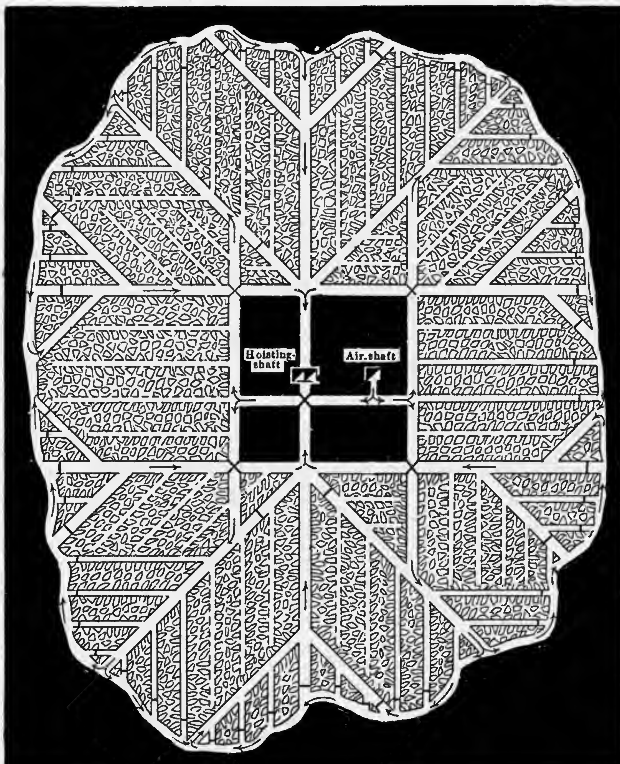


Fig. 38.

on the surface. In each of these figures the intake current is divided, forming two main splits of air near the foot of the downcast or air shaft and these main splits are again divided two or more times to ventilate different sections of the mine, as indicated by the arrows.

Ventilation of Longwall Workings.—Figs. 38 and 39 are two general plans of longwall workings, showing the main air current carried, in two or more splits, from the bottom of the downcast shaft directly to the working face, where it is again divided and made to sweep the entire face, returning by the numerous roads to the main-return airways, by which it is conducted to the foot of the upcast shaft. Fig. 39 shows



Overcasts shown thus: ✕ Curtains shown thus: —

FIG. 39.

a more extended development of the mine, on a slightly different plan from that given in the preceding figure.

DISTRIBUTION OF AIR

Ventilating a Mine.—Small mines are generally or often ventilated by a single current of air passing in one continu-

17

ous circuit around the mine. In larger mines the main current entering the mine is divided into two or more currents or "**air splits,**" as they are called.

The current flowing into a mine or section of a mine is called the "**intake current**" and that passing out from the mine the "**return.**" Likewise, these airways are termed the "**intake**" and the "**return**" airways respectively.

The figures previously given show clearly the general arrangement of the circulation in a mine, as indicated by the arrows. In a gassy mine, the hoisting shaft is made the downcast and the main-haulage road is then the intake airway. The mine is ventilated by an exhaust fan located at the upcast shaft, because it is impracticable to use a blower fan whenever the main-haulage road is made the intake. A blower fan would require doors placed on the haulage road, at the shaft bottom, to prevent the air short-circuiting and passing out through the hoisting shaft. All crosscuts, except those through which the air must pass, are closed by stoppings or doors. By this means, the air current is forced to travel certain airways from the downcast to the upcast shaft.

In Figs. 36 and 37, the hoisting shaft is the upcast and the haulage road the return. The air is first split near the foot of the downcast shaft. One current or split travels north to ventilate that side of the mine, while the other current travels in the opposite direction to ventilate the south side of the mine. Each of these currents is shown returning to the upcast shaft by the main return air-course. Double doors are used in the crosscut at the shaft bottom to prevent the air current from being broken or staggered, when it is necessary to pass through this crosscut. Only one of these doors is open at a time, and the air is thus prevented from short-circuiting at this point.

On the main south (Fig. 37), the air is divided into three separate splits or currents, which ventilate respectively the main south headings, the first and second east and the first and second west. In order to do this, two overcasts are required, one to conduct the main-south intake current over the first-west haulage road, and the other to carry the second-east intake current over the main-south haulway. It should be observed that the stables, in both Fig. 36 and 37, are ventilated by a separate scale of air, which is then carried directly into the main return and passes out of the mine as indicated by the arrows.

Ventilation of Cross-entries.—In the illustration (Fig. 40) are shown two ways of ventilating a pair of cross-entries turned off the main headings. As shown on the left, the main-intake current is deflected into the cross-entries by placing a door on the main heading. The total current is thus made to pass down the first cross-entry and, returning through the second by a crosscut at the face, continues on its way up the main heading, thus forming one continuous current.

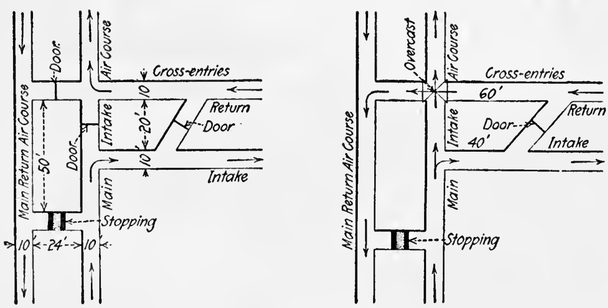


Fig. 40.

In the plan shown on the right in the same figure, the main-intake current is divided at the mouth of the first cross-entry. Part of the air enters the first cross-entry and returning by the second passes over the air bridge at its mouth and through the crosscut into the main-return air-course. The remainder of the main intake current continues up the main heading, passing under the air bridge on its way. This method furnishes a separate current for each district of the mine and leaves the main haulage road unobstructed by any doors. As shown in the figure, an inclined crosscut, called a "crossover," connects the two cross-entries near their mouth, which permits the coal from the back entry to reach the main haulage road by passing through the door on the crossover. This door divides the intake from the return on these entries.

Ventilation of the Mine Stable.—The mine stable, as previously stated, should be ventilated by a small split commonly caled a "**scale**" of air, taken from the main intake current. This current, after ventilating the stables, passes directly to the upcast shaft, without contaminating the air of the mine. It is important to locate underground stables so that they can be ventilated (Figs. 36, 37) with a small scale of air that is conducted at once into the main return air-course. To make possible the rescue of the animals in case of accident, and to

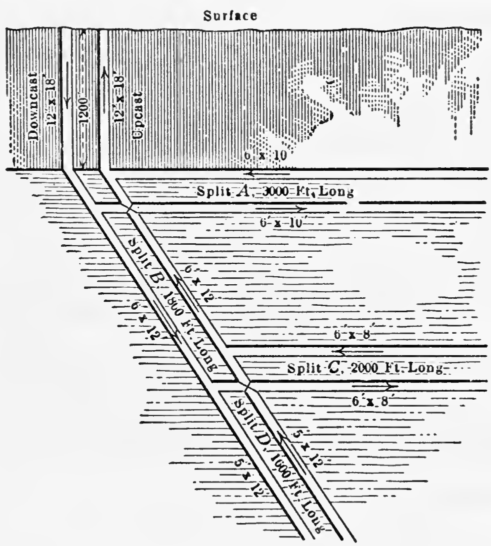


FIG. 41.

facilitate the handling of feed and refuse to and from the surface, the stables should be located near the bottom of the hoisting shaft or other opening.

SPLITTING AIR CURRENTS

Illustration of Air Splitting.—Fig. 41 gives a diagrammatic perspective of a mine ventilated by two primary air splits, *A* and *B*, and two secondary splits, *C* and *D*. In this case either the downcast or the upcast may be made the hoisting shaft, as desired. In gassy mines where haulage is performed on the intake air, the downcast becomes the hoisting shaft, which avoids the use of doors on the shaft bottom. In that case, the air bridges are constructed to conduct the return air over the intake current, thus leaving the haulage road unobstructed.

Systems of Ventilation

Exhaust vs. Blowing System of Ventilation.—The natural or physical conditions that exist in a mine will generally determine whether it should be ventilated on the exhaust or the blowing system. A mine generating gas in sufficient quantity to make the main-return airway unsafe for haulage will require the exhaust system, in order to leave the hoisting shaft, which would then be the downcast, and the shaft bottom unobstructed by doors.

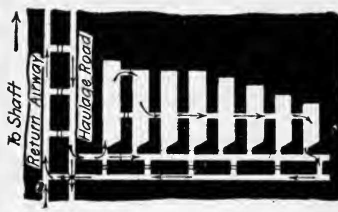


Fig. 42.

The **exhaust system** of ventilation is illustrated in Fig. 42, which shows the circulation in a section or district where

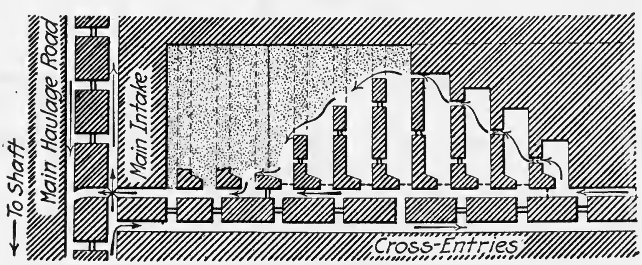


Fig. 43.

the future development of a pair of cross-entries warrants the building of an overcast on the main headings, and haulage must be performed on the intake air.

As indicated by the arrows in the figure, a curtain hung on the first cross-entry, just inby from the mouth of the first room working, deflects the air into the rooms so that the major portion of the current sweeps the face of each room. It is necessary also to hang canvas at the mouth of each room except the last to keep the air at the working face.

The **blowing system** of ventilation is illustrated in Fig. 43 which shows the general arrangement under conditions similar to those just described, except that here the haulage is performed on the return air, the hoisting shaft being the upcast. As indicated by the arrows, the air is carried directly to the head of the cross-entries and returned through the crosscuts in the rooms.

Systems of Mine Airways

The Main Airways.—While two airways, an intake and a return airway of sufficient size, furnish the necessary means

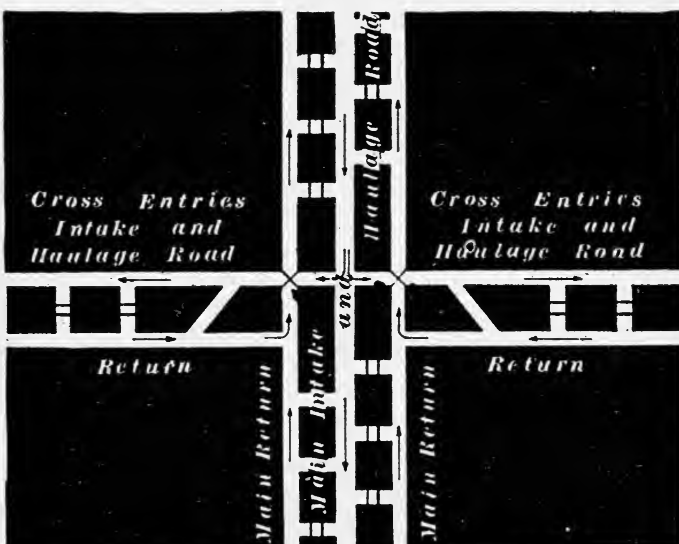


Fig. 44.

for conducting the air current to and from the working faces of the mine, there are other considerations of economy and safety of operation that frequently demand a larger number of main airways.

Single-entry System.—In the early days of mining and in some small mines, today, supplying local trade, the plan is adopted of driving a single entry, which serves the double purpose of haulage road and air-course, the air being returned through the rooms. The single-entry system is unsafe and no longer used in scientific mining.

Double-entry System.—In this system, all entries are driven in pairs, one entry being made the **intake** and the other the return, in each pair. This system is commonly employed in a large majority of coal mines and is shown on the cross-entries in Fig. 44.

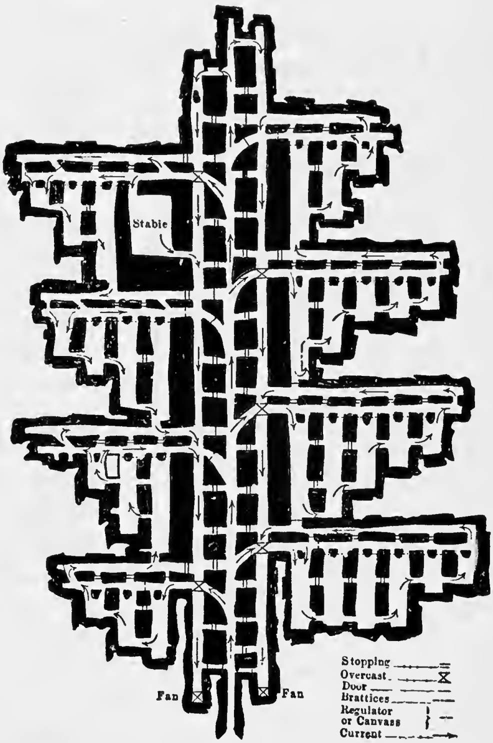


FIG. 45.

Triple-entry System.—In this system, three parallel entries are driven abreast, as for example the main entries in Fig. 44, and the same in Fig. 45, which illustrates the workings in a slope mine. The main slope haulage road being the intake for the entire mine, and the air-course on either side being the return for that respective side of the mine. In the use of the triple-entry system, the center entry is generally made the intake and haulage road, while the two side entries are the return air-courses for each respective side of the mine.

In the slope mine illustrated in Fig. 45, the rooms are driven to the rise of each pair of gangway headings. The mine is equipped with two ventilating fans operating on the exhaust

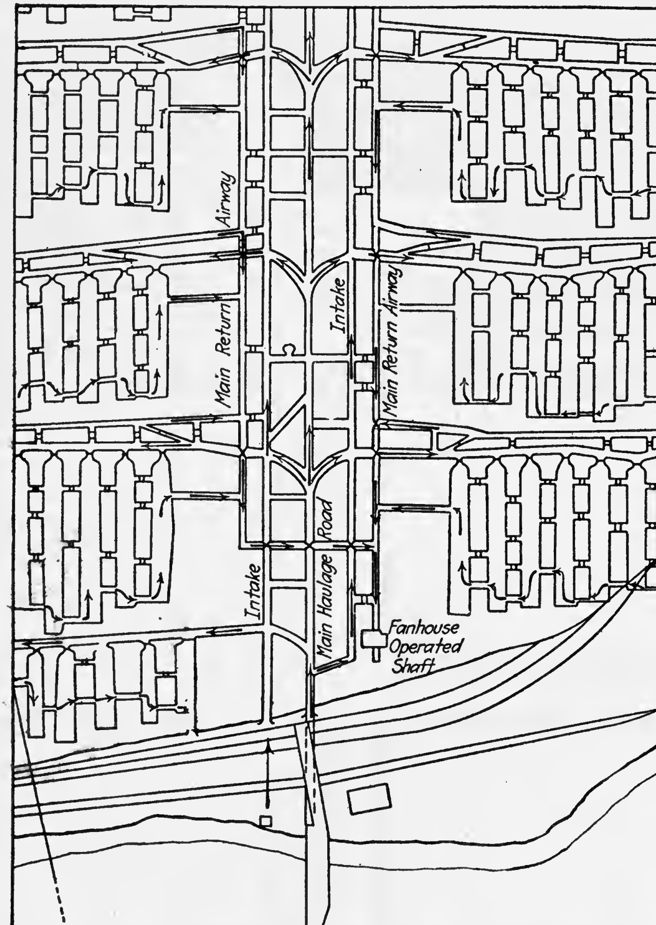


Fig. 46.

system. The air is split and overcast at each pair of headings on the right of the slope, except the last; while there are but two air splits ventilating the levels on the left of the slope headings. Unfortunately for purposes of rescue and handling feed and refuse, the mine stable is located far in the workings, probably to avoid the necessity of driving the mules to and from the working face.

Multiple-entries.—In Fig. 46 is shown a mine opened on the five-entry system for the main headings, thus providing three intake airways and two separate return airways, one for each side of the mine.

The **number** of main airways required, in any case, is determined by their size and the necessary volume of air that must pass through them. The limiting factor in this calculation is the safe and economic **velocity** of the air current traveling the main airways.

While too low a velocity of the air is dangerous because of its failure to remove the accumulating gases, too high a velocity, on the other hand, is dangerous by reason of its increasing explosive conditions in the mine air, by raising and carrying in suspension fine dust, and by furnishing an excessive supply of oxygen that invites active and explosive combustion.

The velocity of main air currents in mines can safely vary between 250 and 1200 ft. per min.: and for short distances a velocity of 2000 ft. per min. may be permitted, although high velocities rapidly increase the power producing the circulation. Where the main intake airways are used for haulage roads, it will not be possible or advisable to employ a velocity much exceeding 400 or 500 ft. per min., owing to the annoyance and danger of drivers losing their lights.

Economy of Multiple Main Airways.—The economy of driving a multiple system of main airways will not be questioned in the planning of large operations. The same plan should be applied to the opening of mines on a smaller scale, the objective point being to keep the velocity of the main air current so that it will not exceed 1200 ft. per min., for any considerable distance.

The saving in power (fuel consumption, equipment and attendance) will pay for the increased expense of upkeep of entries; and the system affords a large increase in safety by reducing explosive conditions and providing additional avenues of escape in case of accident. There is afforded, besides, room for a double-track haulage system, which will prove a great advantage in the operation of the mine.

Assuming that one-half the power on the air is consumed in the main airways, which more or less closely approximates the fact, and taking the general efficiency of the fan and engine as 60 per cent., a double-entry system, for the main intake and return airways, would effect a saving in fuel of 11.25 per cent.; a triple-entry system, 13.32 per cent., and a 4-entry system, 14.10 per cent.

Illustration.—In the planning of a mine for an output of, say 2000 tons of coal per working day, in a 6-ft. seam of more or less inflammable bituminous coal (shaft, slope or drift openings), the following data may be assumed as approximating possible conditions, but must be modified to suit known facts that have been determined, in special cases:

Output per man per day (average)	2½ tons
Number of miners employed (2000 ÷ 2.5)	800
Number of loaders or helpers	400
Number of drivers, trackmen, timbermen, etc.	60
Foreman, assistant foremen and firebosses	20
Total number of men and boys	1280
Number of mules	25

Assuming a gaseous mine requiring, by law, say 150 cu. ft. of air per man, and 600 cu. ft. per mule, per minute, the necessary circulation based on these data would be $(1280 \times 150) + (25 \times 600) = 207{,}000$ cu. ft. per min.; or, to allow for certain leakage, say the necessary air volume is, in this case, 225,000 cu. ft. per min.

Driving 10-ft. openings in a 6-ft. seam and allowing for necessary timbering would leave an unobstructed effective area of, say 50 sq. ft. In this case adopting a 4-entry system for the intake and the same for the return, would give for the total effective intake and return areas, each $4 \times 50 = 200$ sq. ft., which would make the velocity of the intake air current 225,000 ÷ 200 = 1125 ft. per min., which is a safe and economical velocity, provided these airways are not used as haulage roads.

To provide for the expansion of the return air, owing to rise of temperature and addition of mine gases, which may altogether amount to 6 or 8 per cent., the return airways should be driven, say 8 or 10 in. wider than the intake airways.

SECTION VIII

MINE LAMPS AND LIGHTING

Principles of Construction, Classification of Safety Lamps, Requirements—Characteristic Types of Lamps—Special Types of Safety Lamps—Permissible Mine Safety Lamps—Use and Care of Safety Lamps—Testing for Gas by Indicators—The Flame Test—Illuminants for Safety Lamps, Oils, etc.—Miners' Carbide Lamps—Electric Mine Lamps—Permissible Portable Electric Mine Lamps.

A volume could be written on the development of the so-called "safety lamp." It is not proposed to give, here, the history of that development further than to say that it began with the discovery of the two most important and essential principles of all mine safety lamps. Strange to say, these two principles were discovered at practically the same time and by two men of different education and calling.

PRINCIPLES OF CONSTRUCTION

Principle of Protecting Shield.—George Stephenson was a practical miner of considerable mechanical ability, which led him into the practice of cleaning and repairing watches and clocks, running engines and performing other similar services. It was at the Killingworth colliery, Oct. 21, 1815, that he made the first trial of a lamp he had devised for use in mines generating gas.

The **principle of the Stephenson lamp** consisted in confining the burnt air and products of combustion in the upper portion of the lamp chimney or bonnet, the idea being that this would furnish an extinctive atmosphere at the top of the lamp and prevent the flame of the burning gases passing out of the chimney and igniting the gas-charged air surrounding the lamp. This, today, is one of the important principles of all mine safety lamps, though the method of its application differs from that employed by Stephenson.

Principle of Wire Gauze.—The principle of the isolation of a lamp flame, by means of a wire gauze envelope or chimney, was discovered by Sir Humphry Davy, an eminent chemist. As the result of a series of experiments, Davy was able, Dec. 15, 1815, to announce to the world the fact, that an ordinary lamp flame will not pass through the mesh of cool wire gauze. The idea was suggested to the mind of Davy by observing that a flame, as shown in Fig. 47, never comes in direct contact with cool metal. The reason is that the temperature of the burning gas is reduced, in close proximity to the metal, below the point of ignition. He showed that the burning gas, on passing through the mesh of a wire gauze, is broken up into tiny streamlets, which are so cooled by contact with the metal of the gauze that the flame is extinguished. As the gauze becomes heated by the close proximity of the flame, however, it loses its cooling effect and the flame then passes through the mesh.

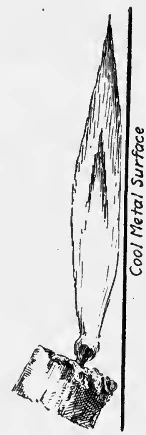


FIG. 47.

The **effect of cool wire gauze** to prevent the passage of flame through its mesh is shown in the lower half of Fig. 48. In the upper half, appears the later passage of the flame through the mesh of the gauze when the wire has become heated so that it is unable to absorb sufficient heat from the burning gas to extinguish the flame. This **isolation of the flame** of a safety lamp by means of a wire gauze chimney found its earliest application in the Davy lamp. A careful study of the problem and the experiments performed showed that the greatest safety was secured by the adoption of a **standard mesh** formed by 28 steel wires, No. 28 B.w.g., making 784 openings per square inch. This standard mesh is still used in England and in this country, today. It was also found that the **volume of the chimney,** including the combustion chamber of the lamp, should bear a certain relation to the surface of the gauze in order to produce the best results and insure the greatest security of the lamp when burning in the presence of gas. There is, however, no fixed value for this ratio, which controls the circulation of the air and gas passing in and out of the lamp and varies with the type of construction.

Classification of Safety Lamps.—Mine safety lamps are divided into two general classes, according to their use in the mine, as follows: (*a*) Lamps for testing for gas. (*b*) Lamps for general use at the working face. A good working lamp does not make a good lamp for testing for gas, neither does a good testing lamp answer for work at the face. Each of these lamps is designed for the particular service or work to be performed and the requirements of each are widely different.

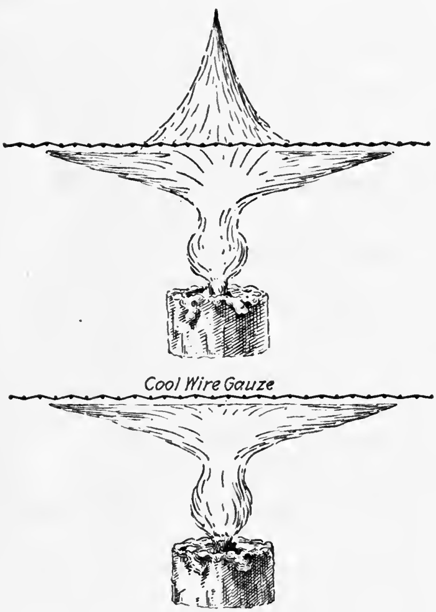


FIG. 48.

Requirements of a Good Testing Lamp.—A good lamp for testing for gas must be sensitive to small percentages of gas present in the mine air and must possess, as nearly as practicable, the same conditions with respect to gas in the combustion chamber as exist in the air surrounding the lamp. Otherwise, the test for gas observed within the lamp will not correctly represent the gaseous condition of the outer air.

The **sensitiveness of a lamp to gas** depends on both the character of the oil burned and the freedom of circulation within the combustion chamber. A lamp burning hydrogen gas (Clowes' hydrogen lamp) is more sensitive than a lamp burning oil, which is true in general of a gas-fed flame. The Clowes lamp is the only safety lamp burning gas, however, and has but a limited use in testing for gas in mines. There are two general types of **oil-burning lamps,** according as the illuminant is a non-volatile or a volatile oil, the former being derived from animal or vegetable sources, while the latter are chiefly derivatives of mineral oil or petroleum distilled below 300 deg. F., such as naphtha, benzine, etc. Coal oil (kerosene) is a distillate of petroleum between 300 and 500 deg. F., and is not classed as a volatile oil. It is frequently mixed with twice its volume or more of a vegetable oil to improve the illuminating power of the latter.

The **volatile oils,** while more sensitive to the presence of gas, possess the disadvantage of giving a more pronounced oil or fuel cap that is frequently mistaken for a gas cap. Moreover, the height of the flame cap, for any given percentage of gas, is always greater in a lamp burning a volatile oil and allowance must be made for this fact, in estimating the percentage of gas present when making the test with such a lamp.

In order that a lamp shall present the same condition with respect to gas, within as exists without the lamp, two conditions must be fulfilled: (1) The air must enter the combustion chamber at a point below the flame. (2) There must be a free circulation within the lamp and it must always be ascensional so as to avoid the contamination of the atmosphere in the combustion chamber with the products of combustion in the chimney, which are apt to descend from the upper portion of the lamp if the chimney is too closely bonneted and the circulation in the lamp is not wholly ascensional.

Other requirements of a good testing lamp are some means of accurately **measuring the height of the flame cap** formed in the lamp and, if possible, making the cap more **plainly discernible** by means of a good background and the absence of a reflection that would interfere with the observation. A good testing lamp should also be provided with a **shield** or suitable **bonnet** to protect the lamp against strong air currents and as an added protection against slight explosions that may occur within the lamp, owing to a body of strong gas.

Requirements of a Good Working Lamp.—Unlike the testing lamp, a lamp designed for general work in the mine must not be too sensitive to gas. Its chief requirements are the following:

1. The lamp must give a **good light** that will enable the miner to perform his work readily and discover any dangers that may exist in the roof or about him.

2. The lamp should be **simple in construction,** portable and light and, at the same time, capable of resisting rough usage that is liable to break the glass, injure the gauze or otherwise damage the lamp. There should be as few parts as practicable, and these should be assembled in such a manner that no single part can be accidentally omitted when putting the lamp together in the lamproom.

3. A good working lamp must be secure against **strong air currents.** It should be suitably protected by a shield or bonnet of such construction as will not unduly obstruct the circulation within the lamp. The best type of lamp admits the air to the combustion chamber, at a point below the flame, and allows the products of combustion to pass out through tangential openings in the bonnet. A shield protects the top of the bonnet from dust and falling fragments of the roof.

4. It is important that every working lamp should be provided with a **lock fastening** that will betray any attempt on the part of the miner to tamper with the lock. **Magnetic locks,** it is claimed can only be opened by means of a strong magnet in the lamproom, but the claim has been questioned in numerous instances, especially where a mine is equipped with electrical installation. The fastening that has given, perhaps, the greatest amount of satisfaction because of its simplicity and security is the old **lead lock** that is fastened in the lamproom with a steel die of special design.

Working lamps are supplied with both round and flat burners, as desired. When a flat burner is used the illumination is much improved by the simple device illustrated in Fig. 49, consisting of a semi-circular cut made in the center of the top of the burner. This simple artifice has the effect of producing a rounder and less smoky flame, besides giving a hotter flame when the latter is reduced, in testing for gas.

The illuminating power of a safety lamp is greatly influenced by the way in which the air supply is brought into contact with the flame and the volume of air supplied to the combustion chamber of the lamp. The light-giving power of the flame is also increased by the use of duplex flat-wick tubes, or triplex round-wick tubes. Tin or aluminum tubes produce a better light than either brass or copper, and porcelain is far better than any metal, in this respect.

Increased light does not mean an increased cost in oil. Petroleum having a high flashing point, such as mineral colza oil, is probably best adapted for use in high-powered lamps. The illuminating power of vegetable oils is greatly increased by the admixture of one-third part of pretroleum (coal oil) having a flashing point of 80 deg. F., although the lamp flame will then have a greater tendency to smoke and will require a better circulation of air in the lamp.

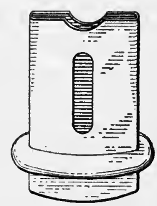


FIG. 49.

The safety of gauze-protected lamps is much increased by a suitable restriction of both the inlet and the outlet openings, which is a prominent feature of many lamps of high illuminating power. Another important feature of these lamps and one that affords increased protection at the top of the chimney is the inner metal bonnet surmounted by a truncated cone. Still another feature that adds to the protection of the lamp and increases its illuminating power, by the concentration of the heat in the combustion chamber, is the conical glass. All of these features originated in the Ashworth-Gray lamp, a type of which was later styled the Ashworth-Hepplewhite-Gray lamp.

A working lamp must be of a design that will make it most convenient for the use of the miner. The base of the lamp should be sufficiently broad to enable the lamp to be set on the mine bottom, in a position to throw a good light where the coal is being undercut or mined. It is often necessary for the miner to hang his lamp on a timber or post. For that reason, some lamps are furnished with a short hook instead of the usual ring forming the handle. The hook is not commonly used in this country, the miner preferring to hang his lamp on a nail driven in the timber.

An important feature of a working lamp is a good pricker, which will enable the miner to remove the crust that forms on the top of the wick of an oil-burning lamp. The pricker must be of such a form that the wick can be cleaned without danger of extinguishing the light.

A lamp burning a volatile oil, the most common form being those of the Wolf type, requires some kind of **igniter**, in the combustion chamber, to enable the lamp to be relit when accidentally extinguished. Lamps burning a volatile oil are more subject to extinction, either from a sudden jar or from gas, than those burning a non-volatile oil. The chief objection to lamp igniters is the opportunity that they afford the curious miner of fooling with his lamp.

The old form of igniter consisted of a narrow ribbon of waxed paper containing little nubs of fulminate, which were ignited by a rod-scraper that extended up through the oil vessel of the lamp. This form of igniter has now largely given place to one in which ignition is caused by the sparks from a cerium compound. The objection to the wax-taper igniter is the flame of the burning taper and the charred remains that often proves an annoyance in the lamp, especially when one or more of the nubs fail to ignite, which is frequently the case.

Specifications by the Bureau of Mines.—In January, 1915, the Federal Bureau of Mines, acting under the authorization of an act of Congress (37 Stat., 681), approved Feb. 25, 1913, issued "Schedule 7, entitled "Procedure for Establishing a List of Permissible Miners' Safety Lamps." Following are the more important announcements and specifications contained in that schedule, which is still in force in relation to so-called "Permissible" safety lamps for mining use.

The Bureau of Mines is prepared, at its Pittsburgh experiment station, to conduct tests of miners' flame safety lamps for the purpose of establishing a list of permissible safety lamps for use in mines in which explosive gas is liberated. This schedule of tests is submitted for the information of those who may desire to submit a type of lamp for test, which must fulfill the following general requirements. (See also, p. 288.)

1. The lamp must be provided with double gauzes or with some other adequate arrangement serving the same purpose. Every gauze must be of steel or best charcoal-annealed iron wire, not larger than 27 Brown & Sharpe gage (0.014 in. in diameter), with 28 meshes to the lineal inch (784 to the square inch), nor less than 29 Brown & Sharpe gage (0.01125 in. in diameter) with 29 meshes to the lineal inch (841 to the square inch).

2. If lamp standards are used, the standards must be so arranged that a straight line touching the exterior part of any two consecutive standards will not touch the glass.

3. The lamp must be so constructed that it will not be possible without easy detection to assemble the component parts of the lamp without the gauze.

4. The lamp must be provided with an efficient locking device to prevent the fuel vessel, glass, or bonnet from being removed by unauthorized persons, or being loosened to such an extent that the safety of the lamp is impaired. Provision shall also be made for taking up the play due to wear of the screw threads.

5. The glass globes shall have their two ends as nearly parallel as it is practicable to make them.

6. The lamp will be examined in respect to its general design, strength, and general character of construction.

CHARACTERISTIC TYPES OF LAMPS

The purpose, in this volume, is to show the general development of the safety lamp, by explaining those characteristic features that form the most essential elements of all safety lamps. It would be useless to attempt to describe in detail the construction of the many different lamps now on the market, as such a description would not be instructive in the way of demonstrating what features are essential in securing the highest efficiency and a maximum degree of security in the lamp. While the number of different safety lamps in use are legion, there are a comparatively few that are characteristic of the essential features that promote safety in the use of the lamp.

The Davy Lamp.—This is one of the early types of safety lamps that still survives. The common, unbonneted Davy is shown in the illustration, Fig. 50 and consists of a brass or aluminum oil vessel surmounted by a wire-gauze chimney of standard mesh. Three round iron or brass rods, called the "standards" of the lamp, are attached to the oil vessel and carry a brass ring that furnishes the upper support of the gauze chimney. Above the ring is a cap or shield of brass to which is attached the handle for holding the lamp.

There are several forms of the Davy lamp known, respectively, as the "fireboss Davy," "pocket Davy," etc. The common Davy has a single, gauze chimney, in the form of a straight cylinder $1\frac{9}{16}$ in. in diameter and varying from $4\frac{1}{2}$ to 6 in. in height. The type known as the "pocket Davy" is somewhat smaller and the height of its gauze is reduced to 4 in. One form of the Davy lamp that was much used in England had a glass cylinder surrounding the lower portion of the gauze chimney, while a steel bonnet enclosed the top of the chimney. Openings were provided in the top of the bonnet for the escape of the gases and burnt air formed in the lamp. Other forms used in England were the "tin-can Davy," having a metal shield covering the entire gauze chimney. This shield was provided with openings for the circulation of the air and a glass window for observing the indications of the lamp. In the "Davy with glass shield" the metal shield was replaced with a glass cylinder that extended the full height of the gauze chimney. The "jack Davy" was a small sized lamp corresponding to the pocket Davy used in this country.

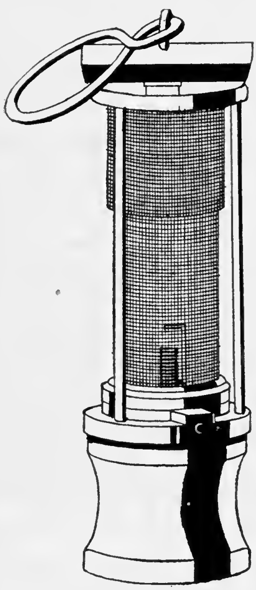


Fig. 50.

The Davy lamp is designed to burn sperm, cottonseed, or lard oil. Owing to the free circulation of air passing in and out of the lamp, the unbonneted Davy is a favorite among firebosses in this country. It is extremely sensitive to gas, and, on this account, flames readily when exposed to a considerable body of gas. Owing to its sensitiveness to gas and the dim light afforded, the Davy is not a safe or suitable working lamp. Its use for that purpose is prohibited by the mining laws of some states. The unbonneted Davy lamp is unsafe in a current having a velocity exceeding 6 ft. per second.

The Clanny Lamp.—The illustration, Fig. 51, shows the common form of Clanny lamp, unbonneted and bonneted.

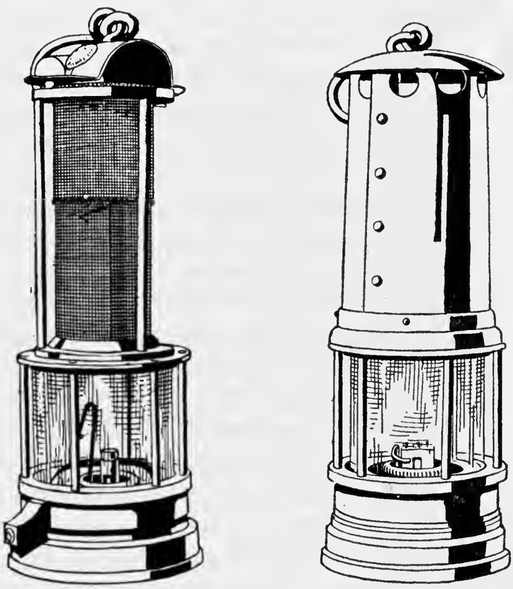


FIG. 51.

In this lamp the brass oil vessel is surmounted by a glass cylinder above which is the wire-gauze chimney. The glass of the Clanny lamp enables it to give a better light than the Davy. The lamp is less sensitive to gas and more or less liable to smoke, however, because the air must enter the lamp above the glass, through the lower portion of the gauze chimney and descend to the flame, which causes a conflict of the descending and ascending currents of air, in the combustion chamber of the lamp.

Owing to the simplicity of its construction, the bonneted Clanny lamp is largely used as a working lamp, in many mining districts. Improved types of the Clanny lamp have been introduced, from time to time, by different manufacturers. Some of these have adopted the principle of the early Eloin lamp, by which the air entered the combustion chamber of the lamp at a point below the flame. This construction is known as the "Eloin principle" of safety lamps. By this means, the tendency of the lamp to smoke is reduced to a minimum.

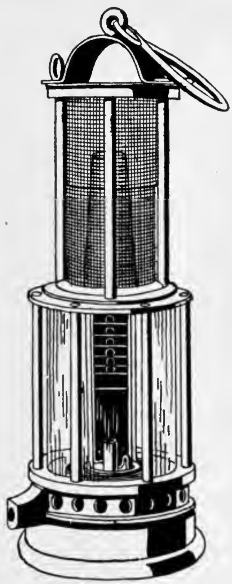


Fig. 52.

The Clanny lamp is designed to burn sperm, cottonseed, or lard oil. It is equipped either with the round or the flatwick burner and the usual pricker for cleaning and raising or lowering the wick in the wick tube. The illuminating power of different types of Clanny lamps varies from 0.25 to 0.50 cp. While the unbonneted Clanny lamp becomes unsafe in a current velocity exceeding 8 ft. per sec., different types of this lamp when bonneted have been able to withstand current velocities varying from 1200 to 1500 ft. per min., and, in a few cases, certain lamps of this type have not failed when the velocity has been increased to 2000 ft. per min., but this must be regarded as exceptional.

The Marsaut Lamp.—This lamp differs in no respect from the Clanny lamp just described, with the one exception that the single-gauze chimney of the Clanny lamp is here replaced by two or three concentric conical gauzes forming the chimney of the lamp. This feature is clearly seen in the illustration, Fig. 52, which shows an unbonneted Marsaut lamp having a conical gauze within the cylindrical gauze forming the chimney of the lamp. The double-gauze chimney is the characteristic feature of the Marsaut type.

The multiple gauzes give protection to the upper portion of the lamp. The top of a lamp chimney, where the heat is concentrated, always presents the greatest danger of the transmission of the flame through the gauze. This fact is recognized in the construction of both the Davy and Clanny lamps by providing a gauze cap, which serves as a means for the better protection of that point.

The lamp shown here is a modified type of Marsaut, designed on the "Eloin" principle of admitting the air below the glass, which improves the circulation and the illuminating power of the lamp. This type is known as the "Beard Deputy" and contains the Beard-Mackie Sight Indicator, described later (see p. 297).

The Marsaut principle of multiple wire-gauze chimneys has been found particularly applicable to lamps designed on the Eloin principle, where the air is admitted to the combustion chamber of the lamp at a point below the flame, which increases the air column or the upward draft in the lamp.

One type of double-gauze Marsaut lamp, bonneted, when tested, was found to be safe in an explosive mixture having a velocity of 2600 ft. per min., while a triple-gauze lamp of this type withstood a current velocity of 3100 ft. per min.

The illuminating power of the double-gauze lamp, burning sperm oil, was found to be 0.70 cp.; but, in the triple-gauze Marsaut, this was reduced to 0.50 cp.

The Mueseler Lamp.—The special feature of this lamp that is characteristic is the central conical sheet-iron chimney, supported with its mouth a short distance above the tip of the flame of the lamp and concentric within the wire-gauze chimney, as shown in the illustration, Fig. 53. The other features of the Mueseler lamp are similar to those of the Clanny lamp, except that the height of the glass cylinder is somewhat reduced and the lamp is provided with a deflector surrounding and supporting the metal chimney and directing the air as it enters the lower portion of the wire-gauze chimney.

The chief effect of the metal chimney of the Mueseler lamp is the increased protection afforded against explosion within the lamp, by separating the descending and ascending air currents. Although the inner chimney improves the circulation, the illuminating power of the lamp is decreased.

The Mueseler principle, however, presents the advantage of increasing the security of the lamp against internal explosions. The shape of the central chimney is conical, corresponding to that of the gauze chimney above it. When the lamp is exposed to a body of sharp gas, and slight explosions occur in the combustion chamber of the lamp, the force of these explosions is broken by the solid metal chimney, and the danger of flame being transmitted through the wire gauze is much less than where the gauze chimney must withstand the full force of the explosion within the lamp. This has always been considered as an important principle in safety lamp construction. For some reason, however, the Mueseler principle has not been generally adopted in the manufacture of safety lamps in this country.

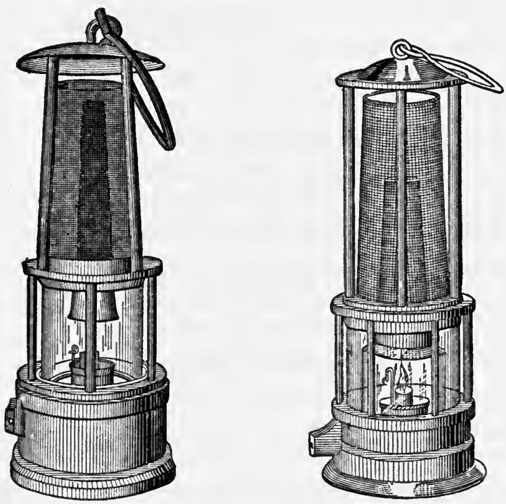


Fig. 53.

There are two types of the Mueseler lamp, known as the English Mueseler, shown on the right in Fig. 53, and the Belgian Mueseler, shown on the left. These types differ only in the dimensions of the central sheet-iron chimney. The Belgian chimney is taller and narrower than that of the English type. The tests of these two types of Mueseler have shown that the Belgian lamp is superior to the English type. The former successfully withstood a current velocity of over 2800 ft. per min., while the English lamp failed at a velocity of 1000 ft per min., the explosive condition of the current being the same in each case.

The original Mueseler type of safety lamp has a horizontal wire-gauze diaphragm, at the base of the gauze chimney. This diaphragm separates the air in the combustion chamber from that within the gauze chimney above, except for the opening provided through the central metal chimney. The failure of the English Mueseler at a comparatively low velocity was probably due to the short and broad metal chimney of that lamp, which provided an ample passage between the combustion chamber and the gauze chimney above. The effect of this was to counterbalance the protection afforded by the gauze diaphragm separating these two compartments of the lamp.

The Mueseler chimney, as stated, in spite of its advantage in increasing the security of the lamp, possesses the disadvantage of decreasing its illuminating power, which is only from 0.20 to 0.40 cp. This type of lamp also possesses the disadvantage that it must be held in an erect position, as only a slight deviation from the vertical interferes so seriously with the circulation through the central chimney as to give opportunity for gas that accumulates between the gauze chimney and the central tube, to enter the combustion chamber. From this cause, explosions have resulted within the lamp and caused its failure. Owing to the same conditions requiring the lamp to be held in a vertical position, its flame is easily extinguished by the burnt air and gases drawn into the combustion chamber from the gauze-chimney above.

SPECIAL TYPES OF SAFETY LAMPS

Under the head of Special Lamps may be classed those designed for a special purpose only, such as testing for gas for example, the Pieler, the Chesneau, the Ashworth, Stokes, and the Clowes hydrogen lamps, besides lamps of the Wolf type designed to burn a volatile oil and the Beard-Deputy, with the B-M sight indicator attachment for measuring small percentages of gas with accuracy. These lamps will be treated briefly, being modifications of the original types of safety lamp described previously.

The Pieler Lamp.—This is a special Davy lamp designed to burn alcohol and used for the purpose of testing for gas. The alcohol flame, as is well known, is sensitive to gas to a high degree The presence of $\frac{1}{4}$ of 1 per cent. of gas in the air entering the lamp elongates the alcohol flame to a height of 3.2 in., while $1\frac{1}{2}$ per cent. of gas lengthens the flame in the Pieler lamp to a height close to 7 in. Larger percentages of gas than this cause the lamp to flame and makes its use very dangerous in coal-mining practice. In making a test for gas with this lamp the flame is first adjusted so that its tip reaches the top of the conical shield that surrounds the flame. The height of this flame is 2 in.

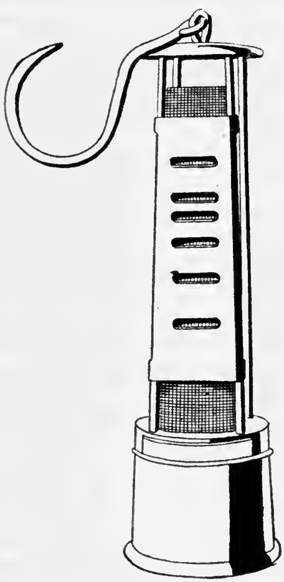


Fig. 54.

Owing to the free circulation of air in the Pieler lamp, as in the original Davy, and the lengthening of the alcohol flame, the gauze-chimney of the Pieler lamp, as shown in the illustration, Fig. 54, is increased to a height of 7.5 in. and made slightly conical. The lamp has four standards and is provided with a screen having horizontal slots through which the height of the flame cap is observed and measured. This screen is attached to two of the standards of the lamp in a fixed position.

A slightly conical metal hood surrounds the flame of the lamp and is of such height that the tip of the ordinary alcohol flame just reaches the top of this hood. At times, the Pieler lamp is bonneted, in which case a glass window is provided extending the full height of the bonnet and marked with a scale for measuring the observed height of the flame in gas.

The Chesneau Lamp.—This lamp is very similar to the Pieler lamp just described, except in a few details of construction. The lamp is bonneted and the air enters the lamp through double-gauze openings at the bottom of the chimney. A hollow sheet-metal cylinder surrounds the flame and supports the small gauze chimney, its purpose being similar to that of the metal one in the Pieler lamp. Like the Pieler, the Chesneau lamp is designed to burn alcohol. In both of these lamps cotton is inserted in the oil vessel for the purpose of absorbing the alcohol and preventing leakage in case the lamp is overturned. However, the absorptive power of the cotton is sufficiently strong to modify the height of the flame and affect the accuracy of the determination of percentage.

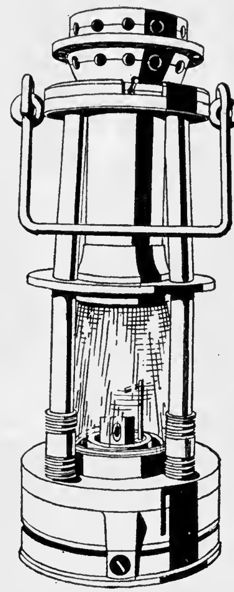


Fig. 55.

Ashworth-Hepplewhite-Gray Lamp.—This is a special form of lamp designed to be used both as a working and a testing lamp and which, at one time, attained a considerable popularity in this country. It is designed after the Gray lamp, so widely used in England. As appears in the illustration, Fig. 55, its principal features are: The hollow brass tubes that serve as standards for the support of the cylindrical brass bonnet surrounding the gauze chimney. These standards are arranged to draw the air from the top of the lamp when testing for a thin stratum of air at the roof of a mine airway or room. There are openings at the bottom of these hollow standards that can be closed by sliding muffs when it is desired to test for gas Otherwise, these openings are exposed to the free admission of the air to the bottom of the lamp. At thet op of the lamp, the standards are affixed to a brass plate to which the bale or handle of the lamp is attached. Another sliding plate fits closely over the first and is arranged to close the open ends of the standards when the lamp is used as a working lamp.

The A.-H.-G. lamp is designed to burn ordinary sperm, cottonseed or lard oil. The conical glass chimney has the advantage of throwing the light upward on the roof. The illuminating power of the lamp is 0.79 cp. When tested, this lamp has withstood a current velocity of 6000 ft. per min., which is one of the features that strongly recommended its use in this country.

Fig. 56.

Stokes Alcohol Lamp.—This lamp is designed by an English mine inspector, whose purpose was to supply an alcohol flame in an oil burning lamp, the oil flame to be used when the miner was working at the face, and the alcohol flame to be used for testing for gas. The lamp is an Ashworth-Hepplewhite-Gray lamp having a small vessel for holding the alcohol when the lamp is to be used for testing for gas. As shown in the illustration, Fig. 56, this alcohol vessel is screwed into the bottom of the regular oil vessel of the lamp, its long slim wick tube passing up through a hollow tube fixed in the oil vessel of the lamp. In no other respect does the lamp differ from an A.-H.-G. lamp. When the Stokes lamp is to be used for testing for gas, the alcohol vessel is screwed in place beneath the oil vessel. The oil flame is drawn down and the lamp tilted slightly to ignite the wick of the alcohol lamp, after which the oil flame is extinguished. The lamp is then ready for testing for gas.

The Clowes Hydrogen Lamp.—This lamp is also a modified Ashworth-Hepplewhite-Gray lamp. Like the Stokes lamp, it is provided with an oil vessel and burner and a second burner to which hydrogen gas is supplied from the strong brass cylinder shown in the illustration, Fig. 57, and which can be attached to or detached from the lamp, as desired. There are but few of this type of lamp in the country where it has seldom been used, as it is heavy and cumbersome. The hydrogen flame, though extremely sensitive to gas, is easily extinguished when testing and the use of the lamp for that purpose requires extreme care and caution. A small scale with crossbars is attached to the oil vessel for the purpose of observing and estimating more accurately the height of the flame in testing.

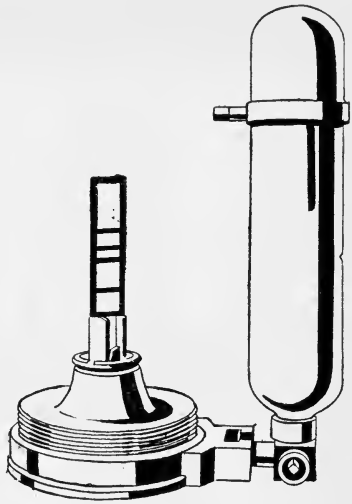


Fig. 57.—Oil Vessel and Hydrogen Cylinder Removed from Lamp.

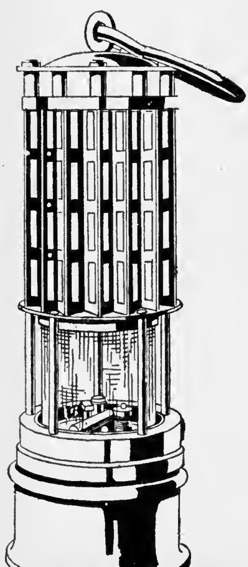


Fig. 58.

Hydrogen gas is compressed to 120 atmospheres or a pressure of 1800 lb. per sq. in. at sea level. This furnishes an ample supply for making a large number of tests in the mine. The gas cylinder is attached to the side of the oil vessel by a screw joint or union. A valve controls the flow of gas into the lamp when it is desired to make a test in the mine. The oil flame is then drawn down and extinguished after the hydrogen has been turned on and ignited in the lamp.

The Wolf Lamp.—The original Wolf lamp shown in the illustration, Fig. 58, is a German product that was widely introduced into this country and

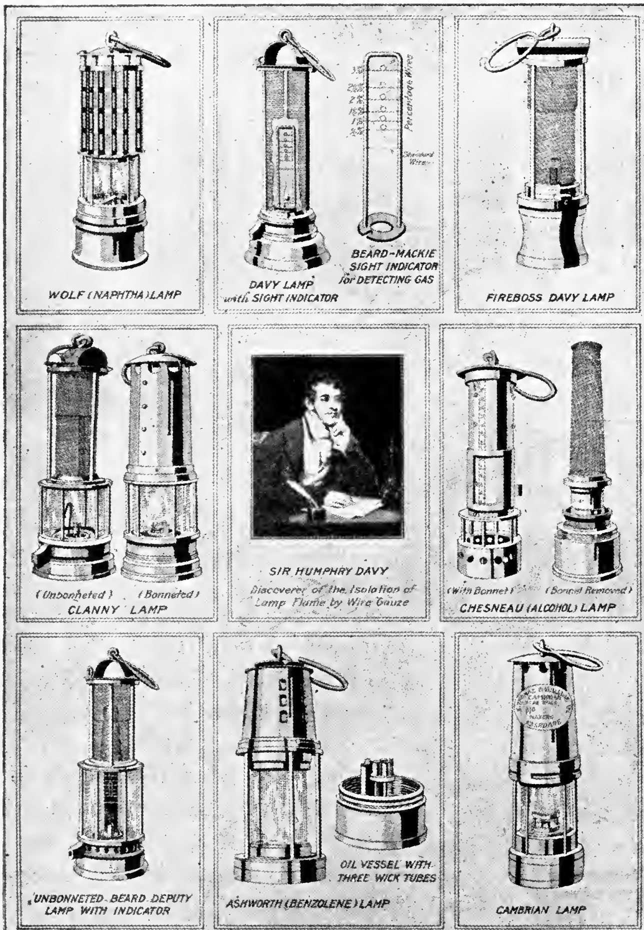

WOLF (NAPHTHA) LAMP
DAVY LAMP with SIGHT INDICATOR
BEARD-MACKIE SIGHT INDICATOR for DETECTING GAS
FIREBOSS DAVY LAMP
(Unbonneted) (Bonneted)
CLANNY LAMP
SIR HUMPHRY DAVY
Discoverer of the Isolation of Lamp Flame by Wire Gauze
(With Bonnet) (Bonnet Removed)
CHESNEAU (ALCOHOL) LAMP
UNBONNETED BEARD DEPUTY LAMP WITH INDICATOR
ASHWORTH (BENZOLENE) LAMP
OIL VESSEL WITH THREE WICK TUBES
CAMBRIAN LAMP

Fig. 59.

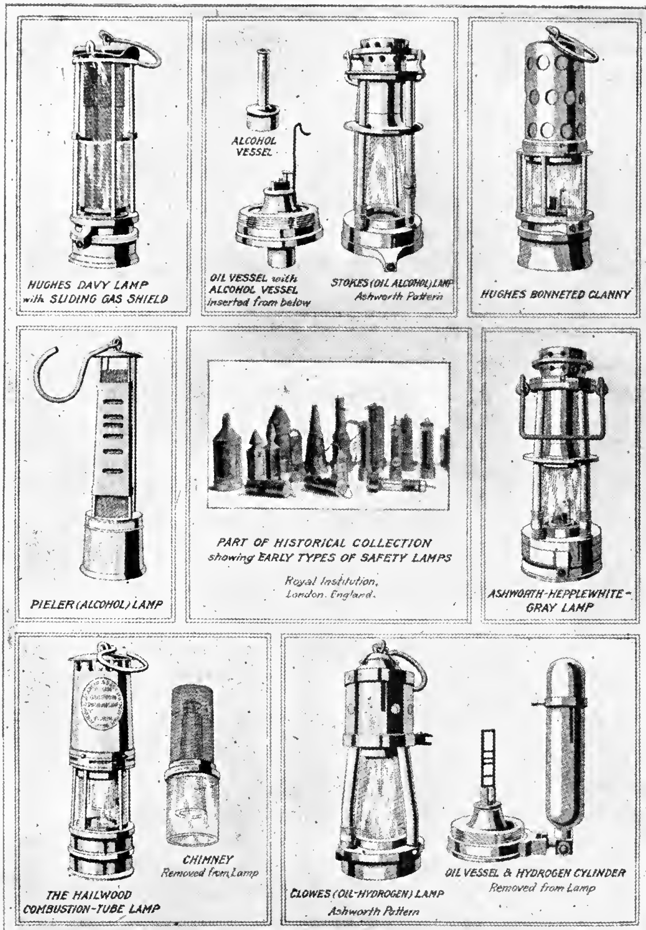

HUGHES DAVY LAMP
with SLIDING GAS SHIELD
ALCOHOL VESSEL
OIL VESSEL with ALCOHOL VESSEL inserted from below
STOKES (OIL ALCOHOL) LAMP
Ashworth Pattern
HUGHES BONNETED CLANNY
PIELER (ALCOHOL) LAMP
PART OF HISTORICAL COLLECTION
showing EARLY TYPES OF SAFETY LAMPS
Royal Institution, London, England.
ASHWORTH-HEPPLEWHITE-GRAY LAMP
THE HAILWOOD COMBUSTION-TUBE LAMP
CHIMNEY
Removed from Lamp
CLOWES (OIL-HYDROGEN) LAMP
Ashworth Pattern
OIL VESSEL & HYDROGEN CYLINDER
Removed from Lamp

Fig. 60.

became very popular as a working lamp. At the present time, there are a number of lamps of this type in use and manufactured in this country, among which may be mentioned the Koehler, the American deputy, the Hughes acetylene lamp, and many others. All of these, like the Wolf lamp, are designed to burn a volatile oil contained in a strong oil vessel of pressed steel, in which absorbent cotton is placed to retain the oil and minimize the danger of leaking should the lamp be overturned.

The volatile oil flame is particularly sensitive to gas, which enables this lamp to show gas when less than 1 per cent. is present in the mine air. A volatile oil, however, cannot be recommended for the purpose of testing for gas, owing to the fuel cap that is often mistaken for a gas cap when no gas is present. Owing to the ease with which a volatile oil flame is extinguished in the mine, all such lamps are provided with igniters. The original Wolf lamp is claimed to have an illuminating power of 1.45 cp., while the average of this type of lamp will but slightly exceed a single candlepower.

On the two pages preceding will be found most of the important types of mine safety lamps grouped in a historical setting that cannot fail to be of interest in connection with the subject. These appear as Figs. 59 and 60.

PERMISSIBLE MINE SAFETY LAMPS

In "Schedule 7, issued by the Federal Bureau of Mines, the engineers of the bureau have defined what is to be understood as a "permissible" miners' safety lamp in the following words:

Definition.—The Bureau of Mines considers a miners' safety lamp to be permissible for use in gaseous mines if the details of the construction of the lamp are the same as those of the type of lamp that has passed the tests made by the bureau and hereinafter described.

Conditions of Testing.—The conditions under which the Bureau of Mines will examine, inspect, and conduct tests on miners' safety lamps are as follows:

1. The examination, inspection and tests will be made at the experiment station of the Bureau of Mines, at Pittsburgh, Pa.

2. Applications for inspection, examination and test shall be made to the Director, Bureau of Mines, Washington, D. C., and shall be accompanied by a complete description of the lamp and a set of drawings showing all the details of the lamp's construction.

3. The applicant for the inspection, examination and test will be required to furnish two lamps of each type, which shall be sent prepaid to the Engineer in Charge of Lamp Testing, Bureau of Mines, Fortieth and Butler Streets, Pittsburgh, Pa., and will be retained by the bureau as a laboratory exhibit.

Each lamp shall have marked on it in a distinct manner the name of the manufacturer and the name, letter or number by which the type is designated for trade purposes, and a statement shall be made whether or not the lamp is ready to be marketed; also a statement describing the fuel used, its trade name and properties. The applicant may supply the fuel for the test if he so desires.

4. Upon the receipt of a lamp for which application has been made for examination, inspection or test, the engineer in charge of lamp testing will advise the applicant whether additional spare parts are deemed necessary to facilitate a proper test of the lamp, and the applicant will be required to furnish such parts as may be requested.

5. No lamp will be tested unless the type submitted is in the completed form in which it is to be placed on the market.

6. Only the engineer in charge of lamp testing, his assistants and one representative of the applicant will be permitted to be present during the conduct of the tests.

7. The conduct of the tests shall be entirely under the direction of the bureau's engineer in charge of the investigation. The tests will be made in accordance with a predetermined schedule, which is outlined herein.

8. As soon as possible after the receipt of the formal application for test, the applicant will be notified of the date on which his lamp will be tested and the amount and character of additional material it will be necessary for him to submit.

9. The tests will be made in the order of the receipt of applications for test, provided the necessary lamps and material are submitted at the proper time.

10. The details of the results of the tests shall be regarded as confidential by all present at the tests and shall not be made public in any way prior to their official announcement by the Bureau of Mines.

11. The results of tests made on lamps that fail to pass the requirements shall not be made public but shall be kept confidential, except that the person submitting the lamp will be informed with a view of possible remedy of defects in future lamps submitted; but such changes other than changing the glass globe or chimney, will not be permitted while the testing is in progress.

12. Tests will be made for manufacturers, manufacturers' agents, state mine inspectors and mine operators.

13. A list of permissible lamps and the results of their tests will be made public, from time to time, by the Bureau of Mines.

14. The glass globe or chimney shall be marked in a distinct manner by a name or design by which its type is designated for trade purposes.

Mechanical Tests.—The following mechanical tests will be applied to every lamp submitted to the bureau to ascertain its strength and resistance under the rough usage common to mining work.

1. The lamp is dropped, by means of a mechanical arrangement, onto a wooden floor, from a height of 6 ft. measured from the floor to the bottom of the lamp, which has been fitted together complete with the glass, a component part of the lamp.

Five successive trials are made, the lamp being fitted with a different glass each time. The lamp passes the test if the glass is broken in not more than one of the five trials. Should the glass be broken in two but not more than two of the five trials, the lamp is submitted to five more trials with fresh glasses and if the glass breaks in two of them the lamp will be considered as having failed to pass the test.

2. A weight of 5 lb. is dropped, from a height of 6 ft., onto the lamp standing vertically on a wooden platform beneath the weight.

The height of 6 ft. is measured between the bottom of the weight and the top of the lamp. The weight is a lead disk 3 in. in diameter and 1¾ in. thick and is dropped mechanically.

Should the glass of the lamp break, two more trials are made, each with a different glass, and if the glass breaks in either the second or third trial the lamp will be considered as having failed to pass the test.

3. A weight of 10 lb., attached to a cord the other end of which is secured to the bottom of the lamp, is dropped a distance of 6 ft., the lamp being suspended at a height of 7 ft., from the ground.

The lamp is gripped by means of claws, or slung by means of straps fastened around its upper part, above the standards protecting the glass. A plate is fastened to the bottom of the lamp and the cord is attached to the center of this plate. The weight is a lead disk 4¾ in. in diameter and 1½ in. thick. It is dropped mechanically.

This test is repeated three times. If, as the result of any one of these three trials, the security of the lamp is found to be defective in any way the lamp will be considered as having failed to pass the test.

Tests 1, 2, and 3 are to be made in succession on one lamp. Cracking of the glass will be regarded as a breakage.

Photometric Test.—The lamp is required to give a minimum candlepower of 0.30, as compared with a pentane standard, during a period of 10 hours.

Explosion Test.—After a lamp has passed the mechanical tests, it will be tested by placing the lighted lamp in an explosive mixture of gas and air, as follows:

1. In currents of air and gas containing 8½ per cent. of natural gas drawn from the Pittsburgh gas mains. In a gallery (lamp gallery No. 1) a lamp which has passed the mechanical tests is tested, with a

fresh glass if necessary, in horizontal, inclined and vertical currents of the explosive mixture of gas and air:

a. In a horizontal current, velocity 600 to 2500 ft. per min.

b. In a 45 deg. descending current, velocity 600 to 2500 ft. per min.

c. In a 45 deg. ascending current, velocity 600 to 2500 ft. per min.

d. In a vertical descending current, velocity 600 to 2500 ft. per min.

e. In a vertical ascending current, velocity 600 to 2500 ft. per min.

Trials will be made at velocities of 600, 800, 1000, 1200, 1500, 2000, and 2500 ft. per min. Into the horizontal current moving at 1500 ft. per min., the lamp will be suddenly thrust from below.

The duration of each trial is two minutes and each trial is repeated three times. An ignition exterior to the lamp will cause the lamp to be rejected.

2. In a still atmosphere (lamp gallery No. 3) containing 8½ per cent. of natural gas. The lamp is placed, with a fresh glass if necessary, in this inflammable atmosphere for three minutes. Five separate determinations will be made. An ignition exterior to the lamp will cause the lamp to be rejected.

Tests of Glasses.—1. A weight of 1 lb. is dropped by means of a mechanical arrangement, from a height of 4 ft., upon the glass placed in a vertical position on a wooden floor. The weight is a lead disk 2½ in. in diameter ½ in. thick. Twenty glasses of any one kind will be tested. Two failures in the twenty will cause the glasses to be rejected.

2. Ten glasses are heated in an air bath to a temperature of 212 deg. F. and when at that temperature are removed from the bath and plunged into water at a temperature of 60 deg. to 65 deg. F. One failure in ten will cause the glasses to be rejected.

If the lamp has two glasses the outer glass will be tested by mechanical means only and the inner glass by heating only.

Igniter Tests.—Lamps having internal igniters will be tested to determine the safety and permissibility of the igniter device. The permissibility of the lamp will be dependent in part on the result of the tests of the igniter device.

These tests will be made to determine the liability of external ignition when the igniter device is operated in the presence of inflammable mixtures of gas and air under such conditions as may be determined by the engineer in charge of lamp testing, for each type of igniting device. Tests will be made to determine:

1. If external ignition is possible when the igniter is operated in still and moving currents of gas and air mixtures.

2. To determine if the residue left in the lamp after working the igniter device is a source of danger in subsequent use of the lamp in inflammable mixtures of gas and air.

3. To determine the nature of the material used in the igniter device.

The igniter will have passed the tests if no external ignition is caused by manipulating the igniter when in position within a double-gauze safety lamp, or if no external ignition is caused by the use of the lamp in inflammable mixtures of gas and air after the igniter has been in service.

Applicants for tests will be required to furnish two complete igniter devices and 5 dozen igniter refills, which shall be shipped in sealed boxes or packages with the trade name written on the outside and addressed to the Engineer in Charge of Lamp Testing, Bureau of Mines, Pittsburgh, Pa. When known by the applicant, the proximate chemical composition of the igniter tape or point should be furnished and the place of its manufacture.

Note.—The inflammable gas used in these series of tests will be the natural gas supplied to the city of Pittsburgh The composition of this gas is approximately: Methane, 83.1 per cent.; ethane, 16 per cent.; nitrogen, 0.9 per cent.; carbon dioxide, a trace.

Lamps in the course of development may be submitted by manufacturers for inspection and preliminary tests, with a view to ascertaining defective construction or the misapplication of safety principles. The nature of such inspection and tests will be determined by the engineer in charge of lamp testing.

Approval of Safety Lamps.—The manufacturers of such types of lamps as have passed the tests of the bureau may attach a plate containing, or stamp into the metal of the lamp, the following inscription:

PERMISSIBLE MINERS' SAFETY LAMP.

U. S. BUREAU OF MINES APPROVAL NO.—.

Before claiming the bureau's approval of any modification of any approved type of lamp, the manufacturer shall submit to the bureau drawings that show the extent and nature of such modifications. Each approval of a permissible lamp will be given a serial number, and approvals of modified types will bear the same serial number as the original, with the addition of the letters *a*, *b*, *c*, etc.

The bureau will, on application, make separate tests of glasses manufactured for use in connection with any lamp that has been approved by the bureau under the provisions of this schedule. Glass globes that fulfill the requirements of the tests will be approved for types manufactured in every particular like those submitted that passed the test.

The bureau will, on application, make separate tests of internal igniter devices for use with any type of lamp that has been approved by the bureau under the provisions of this schedule. Igniters that fulfill the requirements of the tests will be approved for types manufactured in every particular like those submitted that passed the test.

The bureau's approval of any lamp shall be construed as applying to all lamps of the same type as tested, made by the same manufacturer and having the same construction in detail, but to no other lamp. The bureau reserves the right to rescind, for cause, at any time, any approval granted under the conditions herein set forth.

Notification to Manufacturer.—As soon as the bureau's engineers are satisfied that a lamp is permissible the manufacturer, agent or applicant and the mine inspection departments of the several states shall be notified to that effect. As soon as a manufacturer receives formal notification that his lamp has passed the tests prescribed by the Bureau of Mines, he shall be free to advertise such lamp as permissible.

Fees for Testing.—Careful investigation has been made regarding the necessary expenses involved in testing miners' safety lamps at the Pittsburgh experiment station, and the following schedule of fees to be charged on and after February 15, 1915, has been established and approved by the Secretary of the Interior, in accordance with the provisions of the statute previously quoted:

Preliminary inspection and test	$10.00
Complete lamp test	50.00
Candlepower test	5.00
Separate glass globe tests	5.00
Separate igniter tests	10.00

The fees specified above may be increased to cover the cost of testing an unusually complicated type of lamp, and are also subject to change upon the recommendation of the Director of the Bureau of Mines and the approval of the Secretary of the Interior.

USE AND CARE OF SAFETY LAMPS

No safety lamp, however perfect, is safe when improperly used; nor has the safety lamp yet been devised that is foolproof. For these reasons, a safety lamp should never be entrusted to an incompetent or an unreliable person. With the single exception of the lamps used by the mine examiners or firebosses, all lamps used in a mine should be the property and care of the operator.

The Lamphouse or Station.—A lamphouse or lampstation should be established convenient to the mine entrance, where the miners can secure their lamps when entering the mine and return the same on coming to the surface. Each lamp should be stamped with a number and, as far as practicable, the same lamp should be given to the same man, each day, and he be made responsible for its use and condition.

The lamphouse should be in charge of a competent man and one or more assistants, whose duties would be to receive and deliver all lamps in return for checks bearing the lamp number. No lamp must be given out, except in return for this check, which should be placed in the pigeonhole from which the lamp is taken or hung on its hook ready to be given back to the man when his lamp is returned at the close of the shift.

A properly organized and arranged lamphouse will have one or more lampracks with holes or hooks for the lamps. Each hole or hook has a number corresponding to that on the lamp. Tables are provided where the lamps can be taken apart, cleaned, filled and trimmed, after which they are carefully assembled, inspected and returned to their respective places in the rack.

The oil for filling the lamps should be drawn from a tank or reservoir outside of the building. No oil container other than the lamp vessels should be permitted in the lamphouse or station, which should be of fireproof construction and kept free from all accumulations of oily waste or other material liable to spontaneous combustion. The presence of a man's lamp or check on the lamprack will indicate whether he has come out or is still in the mine and will thus serve the same purpose as a checking board, in that respect.

No one must be permitted in the lamphouse other than those in charge. All lamps should be delivered through one or more windows opening on a passageway. The work of delivering and receiving lamps, where a large number of men are employed, will be greatly expedited if there are several windows, each corresponding to a division in the numbering of the lamps. A further advantage in such an arrangement is that each division can be in charge of a man who is responsible for the lamps in that division.

Handling of Safety Lamps.—A safety lamp must never be given to a man who has not been instructed and drilled in respect to its use. Before being entrusted with a safety lamp, a man must show his ability to determine the presence of gas, by observing the flame cap formed in his lamp. He should be taught how to proceed when he has observed a cap in his lamp, and cautioned to carefully lower his lamp and withdraw quietly but promptly from the place.

The man should be shown how his lamp may **flame** should a larger proportion of gas be present in the air. He should be instructed, in that case, as to the necessity of maintaining his presence of mind and making no quick movement with the lamp, which must be withdrawn promptly but cautiously from the gas, by lowering the lamp toward the floor. The man should be further cautioned in regard to the danger of disturbing a body of gas, which may then surround him and make it difficult for him to escape with safety.

A safety lamp must always be held in an upright position and protected against a rush of air such as follows a blast in the mine. It is necessary to protect the lamp when walking against a strong air current. A lamp should never be swung, but should be held quietly at one's side when going from place to place in the mine. Care must be taken not to drop the lamp or permit it to fall. Under no circumstances must a man tamper with his lamp or attempt any experiment. If the lamp is accidentally extinguished, the man's duty is to proceed at once to the nearest relighting station, which should be provided at a convenient point in the mine.

TESTING FOR GAS BY INDICATORS

The work of testing for gas is the most important work to be performed in the operation of a gaseous mine and can only be safely entrusted to a mine examiner, fireboss or deputy who has had experience both in the testing and the handling of gas. The examination of a mine for gas and other dangers must be performed conscientiously and faithfully. The work will not permit of the taking of chances, as the life of every worker in the mine depends on the thoroughness and capability of the examiner.

From time to time, different means have been employed in making the test for gas in mine workings. These consist in various forms of indicators and detectors especially designed to reveal the presence of gas in mine air and ascertain its percentage. Besides these appliances, a few of which will be described briefly, there is the old-established flame test, made by the use of the Davy or other safety lamp, and which is still the most largely employed by mine examiners and fire-bosses.

Numerous Gas Indicators.—Perhaps the earliest attempt to devise a means of indicating the percentage of gas present in air consisted of a glass tube into which had been fused a platinum wire that could be rendered incandescent by an electric current. A sample of the air to be tested was drawn into the tube where the gas contained in the air was consumed by the incandescent wire. The volume of the remaining gases was then measured. Comparing this with the original volume of gas and air gave the percentage of gas present in the air. Devices of this nature, however, were never of practical value, until the recent design of such a gas detector by George A. Burrell, of the Federal Bureau of Mines, which will be described later (see p. 299).

Another device depended on the increase of pressure in an air container that was separated from a similar container of gas and air by a porous partition through which diffusion of the gas into the air took place. The resulting increase of pressure in the first container was an index of the percentage of gas present in the sample tested, but the device had no practical value for use in mines. Still another device depended on the rise in temperature caused by the absorption of gas by platinum black, which coated the bulb of one of two thermometers. The rise in temperature thus indicated furnished the means of determining approximately the percentage of gas present. Again, another device depended on the compression of a sample of gas-charged air contained in a strong glass tube into which was fitted a piston. The rapid compression of the air in the tube would ignite the gas and cause a flash when not less than 5 per cent. of gas was present.

The **Liveing indicator** was a more accurate means of determining percentages of gas, but this also never came largely into use. Two platinum wires of equal resistance were rendered incandescent by an electric current. One of these wires was inclosed in a tube containing a sample of the air to be tested, while the other wire was in pure air. An ingenious sliding arrangement of the two tubes containing the wires provided a means of comparing their relative brilliancy, which furnished a suggestion of the percentage of gas present in the air tested. None of these devices, however, can be considered of any practical importance in coal mining.

The Shaw Gas Machine.—This machine, though not of portable form, on which account it could not be taken into the mine but samples of air to be tested must be brought to the surface, furnished a means of correctly determining the explosibility of samples of air collected in the mine workings. For this purpose, it was formerly used at many large collieries. The disadvantage in its use lay in the fact that a test could not be made on the spot and time must elapse between the taking of the sample of air and knowing the results of the test. In that time, conditions in the mine might materially change, which rendered the test valueless for the purpose intended.

The Shaw machine consists of two cylinders whose volume ratio is known. Both cylinders are fitted with air-tight pistons operated by a single lever arm. By this means exact proportions of gas and air can be pumped into a combustion chamber where they are ignited when the mixture becomes explosive. A graduated scale indicates the volume percentage of air and gas present when explosion occurs.

In the operation of this machine, it is first necessary to standardize an artificial gas supply to ascertain the lower explosive limit of the gas. To do this the machine was arranged so that the larger cylinder would pump pure air while the smaller one pumped gas, and the point noted when explosion occurred. This having been done, the tube that formerly supplied pure air to the larger cylinder is now connected with the bag containing the sample of mine air to be tested, while the smaller cylinder continues to pump its proportion of the standard gas. Evidently, a less ratio of the supply from the two cylinders will now be required to produce an explosion, should the air pumped by the larger cylinder contain some gas. The difference shown on the graduated scale gives the percentage of gas present in the air tested.

The Beard-Mackie Sight Indicator.—This is a simple and extremely practical device designed to be attached to the burner of a safety lamp burning sperm, cottonseed or lard oil but not a voltaile oil. As shown on the right, in the illustration, Fig. 61, the device consists of a ∩-shaped support mounted on a small brass disk that fits over the burner and is held in place by the screw nipple of the lamp. On this support are arranged fine platinum wires at fixed heights above the lamp flame.

The lower straight **standard wire** is for the purpose of standardizing the flame, which is raised to a height just sufficient to incandesce that wire. This must be done in pure air, although a slight alteration in the height of the flame produces no practical effect in determining the percentage of gas by the incandescence of the successive percentage wires when the lamp is taken into the mine. Indeed, the standardizing of the flame is generally done after entering the mine when the examiner has once become acquainted with the use of this indicator.

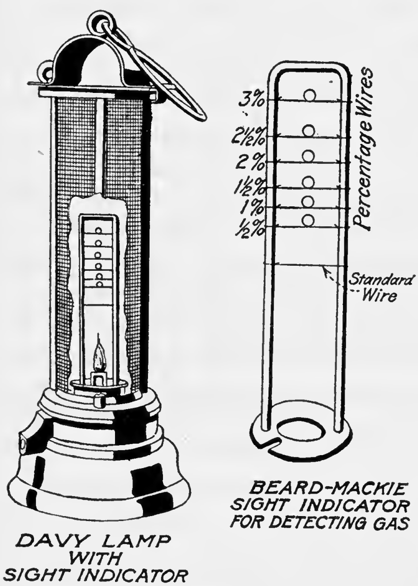


Fig. 61.

The **percentage wires** are each looped at the center, the purpose being to make their incandescence more perceptible when observed through the gauze of the Davy lamp, as shown on the left of the figure. The incandescence mounts higher in the percentage wires as the proportion of gas in the mine air increases and the uppermost wire incandesced determines the percentage of explosibility of the mine air.

The use of the **sight indicator** furnishes the means of determining with considerable accuracy the explosibility of mine air, at the point and at the moment the test is made. Its use eliminates the necessity of the fireboss guessing the percentage from the height of the flame cap observed in his lamp. It enables a just comparison to be made between the reports of different firebosses whose judgment may differ, or who may not be equally capable of discerning the caps formed in their lamps.

With proper care, the sight indicator can be used, for a year or more, by a fireboss when making his morning examination of the mine. Its construction is naturally somewhat delicate, which requires it to be carefully handled when being inserted or taken out of the lamp. A careless fireboss will often permit his lamp to smoke and carbonize the wires, which interferes with their delicacy. The same effect is caused by burning a poor quality of oil or oil mixed with kerosene, which increases the smokiness of the flame.

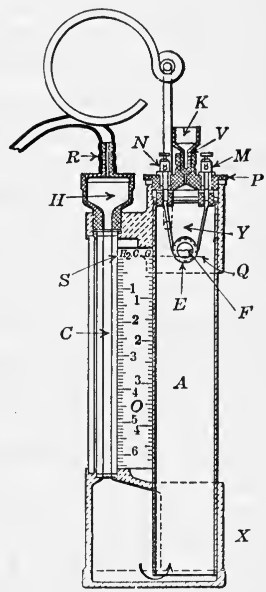


Fig. 62.

The **advantages** derived by the use of the indicator are that it standardizes all tests for gas, making them comparable. It eliminates the guessing of the height of a flame cap and the percentage of gas indicated thereby. It indicates the presence of gas as low as one-half of 1 per cent. The indications are plainly visible by the incandescence of the looped wires. The presence of an indicator in a lamp has often avoided the extinction of the lamp in gas and reduces the tendency to internal explosion in the lamp. Finally, all indications are made with a normal flame, which not only saves time but avoids the necessity of lowering the flame and possibly extinguishing it when making a test.

The Burrell Gas Detector.—This device, which is shown in section in the illustration, Fig. 62, consists of a brass tube *A* surmounted by a screw cap *P* equipped with a valve *V*, a little cup *K* and two binding posts *M* and *N*. Connected with and supported by the latter is a fine platinum-wire bridge *F*, which can be rendered incandescent by the current from an electric battery. A stout gage-glass *C* is surmounted by a brass reservoir or cap *H* to which a rubber tube *R* is attached. Both the gage-glass *C* and the brass tube *A* are set into an aluminum base *X*, by which they are connected, forming a U-tube after the manner of a water gage. A graduated scale *O* provides the means of measuring the height of water column in the gage-glass.

In the use of this instrument for the detection of mine gas in the workings, the brass cap *P* is unscrewed and water poured into *A*, until it rises in the gage-glass to a level indicated by the zero of the scale at *S*. This level corresponds to the level *Q* in the brass tube *A*, just below the platinum wire *F*.

When a test is to be made in the mine the valve *V* is first opened and the operator blows gently into the rubber tube *R*, depressing the water level in the gage-glass and causing it to rise in the brass tube, until it appears in the little cup *K*, or until a slight click of the valve *V* tells that the water has completely filled the combustion space *Y*, in the top of the brass tube. The rubber tube attached at *R* is now pinched with the fingers and the instrument raised to the roof or into the cavity where it is desired to test the air for gas. In that position, the rubber tube is released and the water level at once falls in *A* and rises in *C* to where it originally stood at zero of the scale. By this action, the air to be tested is drawn in through the open valve *V* and fills the combustion space *Y* above the water level *Q*.

When equilibrium is established, the valve *V* is closed and the battery current switched on, causing the incandescence of the wire bridge *F*, which is plainly observed through the small glass window *E*. About 1½ min. is required to consume all the gas present in the air contained in the combustion space above the water. The current is now turned off and the instrument shaken, for the purpose of cooling the air and gaseous products of the combustion, and permit of their volume being measured at the original temperature. As cooling takes place, the water rises in *A* and falls in the gage-glass, until it becomes stationary at a certain level. The graduation at that point will show the percentage of gas that was present in the air tested. The aluminum scale *O* is easily removable and is graduated for the detection of any combustible gas or vapor. The two scales that appear in the figure are for hydrogen (H) and carbon monoxide (CO).

This instrument has proved quite effective for the purpose intended in its design. There is no doubt but that some of the carbon dioxide produced by the combustion of the gas is absorbed in the water when the instrument is shaken; but this is probably largely compensated by the slightly higher water level in *A* above that in the gage-glass *C*, at the time the measurement is taken. This difference of level is, moreover, rendered extremely slight by reason of the relatively larger diameter of the tube *A*, as compared with the bore of the gage-glass *C*. Actual tests of the results obtained in the mine, by comparison with the analysis of the same air, in the laboratory, show the following percentages which are not exceptional.

By detector.........	0.4	0.7	0.9	1.6	1.9	1.5	2.5	2.0	2.2
By analysis........	0.45	0.57	1.11	1.61	1.23	1.93	2.52	1.46	2.54

For all practical purposes, the slight differences shown by these figures between the tests made in the mine and the analyses made in the laboratory are immaterial.

THE FLAME TEST

From the earliest time, the most universal method of testing for gas in mines has been that of observing the effect of the gas on the flame of a safety lamp. As is well known, in every candle, or lamp flame burning oil, there are three zones as indicated in the illustration, Fig. 63. The inner zone *A* is dark, being filled with the hydrocarbon vapors formed by the vaporization of the oil. There is no combustion taking place in this zone. The heat of the flame dissociates the hydrogen and carbon of these vapors, and the second zone *B* is rendered luminous by the incandescent carbon particles, which there undergo combustion. The remaining hydrogen and the carbon monoxide resulting from this combustion pass into the outer zone *C* where they burn with a non-luminous flame, supported by the surrounding air which here has free access to the flame. Owing to the brightness of the second zone *B*, caused by the incandescence of the carbon particles, it is difficult to discern the non-luminous envelope surrounding it and forming the third zone *C*.

Flame Caps.—When a lamp flame is lowered, almost to its point of extinction, the surrounding air so closely approaches the wick that the hydrocarbon vapors are consumed without the incandescence of the carbon. The dark zone is here greatly reduced, while the second luminous zone is practically eliminated, leaving a small non-luminous flame covering the wick, as shown in the lower right-hand corner of the figure. Just above, in the upper right-hand corner, the flame is shown as slightly increased in size by raising the wick a trifle. There now appears a small luminous zone surmounted by a non-luminous cap, which can be readily discerned. This cap is known as a **"fuel cap,"** being due solely to the combustion of the vaporized oil. This fuel cap is often mistaken for a gas cap when testing for gas with a reduced flame.

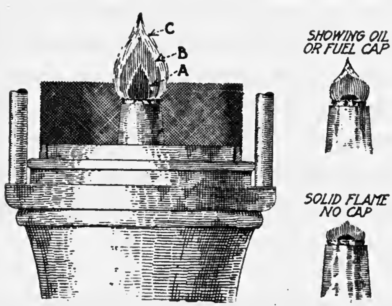


FIG. 63.

The description given thus far refers to a flame burning in pure air. Now, when a lamp flame is burning in air charged with a small percentage of a combustible gas, as methane for example, the gas in contact with the flame is consumed. At the same time, the outer zone of the flame is lengthened and rendered more luminous than before because of its increased size, and there now appears what is known as the **"gas cap"** or more commonly **"flame cap."**

The height of the flame cap varies with the percentage of gas present in the air, the kind of lamp employed and the oil or luminant burned therein. The visibility of the cap is greatly assisted by the free access of air to the combustion chamber of the lamp. The air should enter the lamp at a point below the flame; in other words, the ventilation in the combustion chamber should be ascensional. Any other arrangement interferes decidedly with the clear observance of the cap.

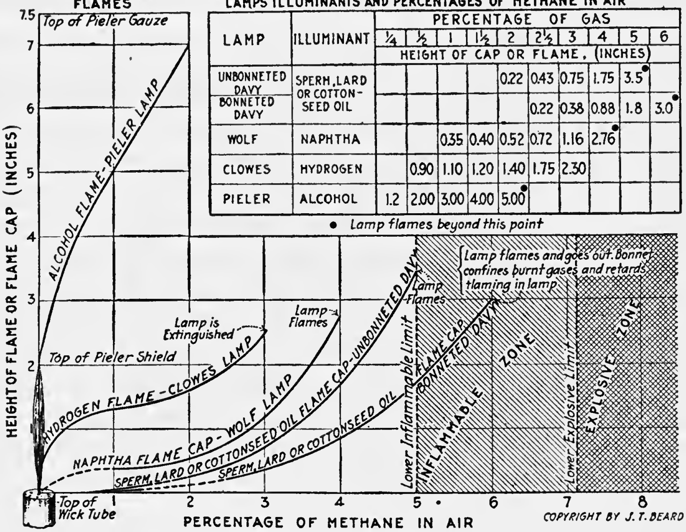


TABLE GIVING HEIGHT OF FLAME CAP OR ELONGATION OF FLAME FOR DIFFERENT LAMPS ILLUMINANTS AND PERCENTAGES OF METHANE IN AIR

LAMP	ILLUMINANT	PERCENTAGE OF GAS — HEIGHT OF CAP OR FLAME, (INCHES)									
		¼	½	1	1½	2	2½	3	4	5	6
UNBONNETED DAVY	SPERM, LARD OR COTTON-SEED OIL					0.22	0.43	0.75	1.75	3.5•	
BONNETED DAVY	SPERM, LARD OR COTTON-SEED OIL						0.22	0.38	0.88	1.8	3.0•
WOLF	NAPHTHA			0.35	0.40	0.52	0.72	1.16	2.76•		
CLOWES	HYDROGEN		0.90	1.10	1.20	1.40	1.75	2.30			
PIELER	ALCOHOL	1.2	2.00	3.00	4.00	5.00•					

• Lamp flames beyond this point

Fig. 64.

A dark background in the lamp also renders a cap more plainly visible.

The effect of the form of the lamp and the illuminant burned, to produce a given height of cap, for a given percentage of gas, is clearly shown in the lamp diagram, Fig. 64. The tall gauze chimney, free access of air and the alcohol burned in the Pieler lamp very greatly increase the height of the flame, in the use of that lamp, for the same percentage of gas present. On the other hand, the bonnet of the Clowes lamp burning hydrogen, or the Wolf lamp burning naphtha, materially reduce the height of flame cap formed in these lamps, notwithstanding the volatile nature of the illuminants burned. The effect of the bonnet in the Davy lamp burning sperm, lard or cottonseed oil is clearly shown to reduce the height of the cap, for the same percentage of gas, as compared with that obtained in the unbonneted Davy.

The preceding diagram is of interest in connection with the use of different types of safety lamps burning hydrogen, alcohol, naphtha, or a non-volatile oil, as sperm, lard or cottonseed oil, in testing for gas. The height of flame cap, or the elongation of the flame, produced by different percentages of gas, in the use of different lamps is tabulated in the upper right-hand corner of the diagram.

The heights of flame cap given in the diagram, for the Davy and Wolf lamps, are the minimum caps produced by drawing down the flame to its lowest point. The heights given for the Clowes (hydrogen) lamp and the Pieler (alcohol) lamp are for the elongation of the flame due to the gas. The original flame of the Clowes lamp is 0.3 in., while the flame of the Pieler lamp is adjusted so that its tip just reaches the top of the shield, at a height of 2 in., as shown in Fig. 64. (See description of Pieler lamp, p. 282.)

The presence of other gases or dust will, of course, modify the results shown in this diagram. The effect of carbon dioxide is to diminish the length of the flame and obstruct the formation of the cap. On the other hand, carbon monoxide and dust when present in the air lengthen the flame and assist the formation of a cap.

Calculation of Height of Flame Cap.—For a Davy lamp, burning sperm or cottonseed oil of good quality, in an atmosphere charged with pure methane or marsh gas, experiments have shown that the height of flame cap varies as the cube of the percentage of gas present. Using a bonneted Davy burning colza oil, William Galloway has estimated the height of flame cap to be $\frac{1}{70}$ of the cube of the percentage of gas present in the air surrounding the lamp

In a long series of experiments under favorable conditions, the author found when using an unbonneted Davy lamp burning sperm oil the height of flame cap was $1/36$ of the cube of the percentage of gas present in the feed air entering the lamp. The height of cap was accurately measured by a scale in the lamp, and the percentage of gas in the air was obtained by the use of a Shaw gas machine, which drew the air from the testing chamber in which the lamp was placed and which was ventilated by a continuous current of air charged with the gas. The arrangement eliminated the effects that would otherwise have been produced by accumulation of the products of combustion in the lamp chamber.

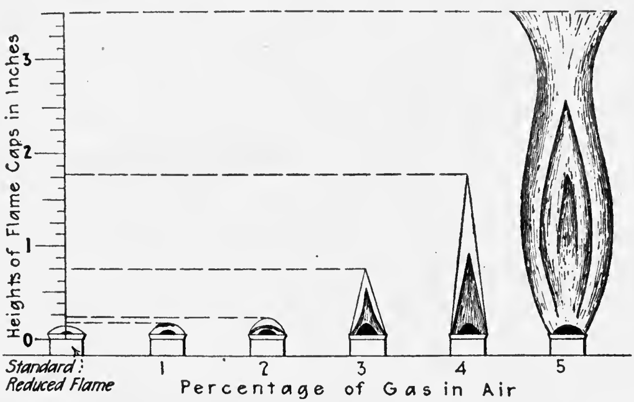


Fig. 65.

The results are expressed by the following formulas, giving the height of flame cap h for any percentage of gas J:

$$\text{Unbonneted Davy, sperm oil (Beard), } h = \frac{J^3}{36}$$

$$\text{Bonneted Davy, colza oil (Galloway), } h = \frac{J^3}{70}$$

The appearance of the flame and the height of cap, for different percentages of gas, as derived from the author's experiments, are shown in the illustration, Fig. 65. These tests were made with the flame reduced to a height of $3/16$ in. It will be observed that, as the height of the flame increases, its volume is enlarged. At about 3.5 per cent. of gas, the flame became unsteady and, as the percentage of gas was increased above that point, the flame became more voluminous, rotating in a wierd manner about the gauze, then expanding at the top into a fan-shape and finally filling the gauze chimney with flame.

Beyond this point, the flame has been frequently seen to leave the lampwick, while the gas continued to burn in the upper portion of the chimney. When this occurred with a sight indicator in the lamp, the flame would relight the wick as the percentage of gas was reduced, all of the percentage wires of the indicator being then brightly incandescent. The same action has been observed by the author when holding an unbonneted Davy, equipped with sight indicator, exposed to a strong gas feeder. At that time, slight explosions occurred within the gauze, but the lamp was not extinguished when carefully withdrawn from the gas.

Making a Test for Gas in the Mine.—When approaching a place where gas is suspected, one must move quietly so as not to unnecessarily disturb the gas from its lodgment at the roof or in a cavity. Having lowered the flame, the lamp is cautiously raised into the gas and watched for the first appearance of a cap or the lengthening of the flame. As quickly as this is observed the lamp should be promptly but cautiously withdrawn from the gas.

On finding a body of sharp gas that has caused the lamp to flame, danger occurs when, in withdrawing the lamp, fresh air enters the combustion chamber, creating a highly explosive mixture within the lamp. For this reason, the lamp must be withdrawn from such a mixture slowly and with great caution, which often requires much presence of mind. One should never trifle with gas he has found in a cavity of the roof or on the falls.

Gas issuing from the coal, at the face of a chamber, will often pass out in a thin film or layer at the roof, and may be unobserved by a fireboss until he is well within the chamber. His movement beneath the layer of gas may cause it to descend as he passes and he finds, too late, that he is enveloped in gas from which he is able to escape with difficulty. Under such circumstances, a fireboss will frequently smother his lamp beneath his coat, while he retraces his steps cautiously.

A thin layer of gas at the roof of a chamber can often be detected by holding the lamp erect toward the roof and blowing a slight puff against the roof, so as to cause the gas to descend on the lamp. This is a practice followed by many experienced firebosses. Without doing so, it is possible for a fireboss to miss the gas and report the place safe for work when it is quite unsafe.

ILLUMINANTS FOR SAFETY LAMPS

The principal illuminants used in safety lamps are the various kinds of vegetable, animal and mineral oils. Hydrogen gas is used in the Clowes hydrogen lamp, but this is the only lamp burning gas. For practical purposes, the oils burned in mine safety lamps can be designated as volatile and non-volatile oils. A few testing lamps are designed to burn alcohol (spirits of wine), which is also a highly volatile illuminant.

Non-volatile Oils Used in Safety Lamps.—These are mostly derived from the vegetable and animal kingdom. Among the **vegetable oils** largely used in mining practice may be mentioned cottonseed and colza or rapeseed oil. The principal **animal oils,** which are also non-volatile, are the sperm, lard, seal and whale oils. Of these, sperm and lard oils are most commonly used in safety lamps today.

Both vegetable and animal oils possess less illuminating power than mineral oils, and have a greater tendency to incrust the wick of the lamp. They are more stable, however, and the flame is not as readily extinguished in the mine as when mineral oil is burned in the lamp. The addition of about one-half of their volume of coal oil (kerosene) greatly improves the illuminating power of these oils but increases their tendency to smoke. The rate of burning is slightly increased and the mixture does not incrust the wick as rapidly as when a pure vegetable or animal oil is burned.

Mineral Oils.—All mineral oils are classed under the general term, "petroleum," which is derived in a crude state from the oil-bearing strata. When the crude petroleum or "rock oil," as it is sometimes called, is distilled, the more readily vaporized hydrocarbon vapors condense on cooling to what are termed light or volatile oils. These are distilled at temperatures below 300 deg. F. **Coal oil,** or **kerosene,** is the product distilled between 300 and 570 deg. F., while the heavy lubricating oils are distilled at still higher temperatures. These last products contain **paraffin,** which is separated from the heavy oils by its solidifying at 130 deg. F., in cooling. Of the light oils, **gasoline** is distilled below 140 deg., **naphtha,** below 230 deg., and **benzine,** below 300 deg. F.

Light, Volatile Oils.—The danger in the use of light volatile oils, as illuminants in safety lamps, arises from their low flashing points. The ready vaporization of the oil, as the lamp heats in gas, renders the test for gas unreliable in the use of a lamp burning such an oil. The storing of a highly volatile oil at a mine and the filling of the lamps in the lamphouse requires extra precautions to be taken to avoid accident. In order to reduce the danger of its use in the lamp, the oil vessel is filled with absorbent or filling cotton. A light volatile oil is not as stable as a vegetable or animal oil, and its flame is more easily extinguished when such an oil is used in the mine. A volatile oil flame, however, is more sensitive to gas and has a higher illuminating power than other oils, which has favored its use in many mining districts.

MINERS' CARBIDE LAMPS

The acetylene or carbide lamp that has come into such extensive use in coal mining, within the past few years, is an **open-flame lamp** constructed to burn acetylene gas, generated within the lamp by the slow feeding of water onto the carbide. The water and the carbide are contained in two separate compartments of the lamp.

The supply of water to the carbide is regulated by a valve having a screw adjustment at the top of the lamp. The water is contained in the upper half of the lamp and the carbide in the compartment below. The latter should not be more than half filled with the carbide, which swells when moistened with the water. A charge of $2\frac{1}{2}$ oz. of carbide will supply gas sufficient to maintain a flame $1\frac{1}{8}$ in. in length during a half-shift or more but then it will be necessary to recharge the lamp.

Owing to the brightness of the acetylene flame, the carbide lamp has very largely replaced the old open-flame torch so commonly used in mines generating no gas. The general form of carbide lamp in common use is shown in Fig. 66, although there are different styles of this lamp manufactured, some having no reflectors behind the flame and differing in other details. The lamp shown in the figure is a type very largely used in the anthracite district. Most of these lamps in use differ only in slight details.

FIG. 66.

Generation of Acetylene Gas.—Carbide (CaC_2) is a product of the action of coke on quicklime, calcium oxide (CaO). The lime and coke are finely ground, thoroughly mixed and heated to a white heat in an electric furnace. Under the high heat of this furnace a portion of the carbon unites with the calcium to form calcium carbide (CaC_2), the remainder of the carbon taking up the oxygen and passing off as carbon dioxide (CO_2), according to the reaction,

$$4CaO + 5C_2 = 4CaC_2 + 2CO_2$$

When water comes in contact with calcium carbide, calcium hydroxide, $Ca(OH)_2$, is formed and acetylene gas (C_2H_2) is set free according to the equation.

$$CaC_2 + 2H_2O = Ca(OH)_2 + C_2H_2$$

The acetylene gas is highly inflammable and when ignited in the air burns, producing carbon dioxide and water vapor. Ignoring the inert nitrogen of the air, this reaction is expressed by the following equation:

$$2C_2H_2 + 5O_2 = 4CO_2 + 2H_2O$$

One ounce of pure crystallized calcium carbide will generate 622 cu. in. of acetylene gas, measured at a normal temperature of 60 deg. F., barometer 30 in. Commercial carbide, however, will commonly yield only from 400 to 500 cu. in. per ounce of carbide used, depending on the completeness of its consumption in the lamp.

Burning Acetylene Gas.—For the purpose of estimate, it may be assumed that an average miner's carbide lamp consumes ½ oz. of carbide per hour and generates 250 cu. in. of acetylene gas. Then, since one volume of this gas, in burning, consumes 2½ volumes of oxygen or, say 12½ volumes of air and produces 2 volumes of carbon dioxide and 1 volume water vapor, the burning of a carbide lamp may be estimated as producing 500 cu. in. of carbon dioxide and half that volume of water vapor, per hour. In the same time, the lamp takes from the air 625 cu. in. of oxygen, leaving practically 2500 cu. in. of excess nitrogen.

The effect of the burning of a carbide lamp to vitiate the mine air is thus seen to be inappreciable and far less than the breathing of a man, who consumes little short of 1000 cu. in. of oxygen, per hour, when at rest, and over 8000 cu. in. per hr., in violent exercise, and exhales an equal volume of air containing from 2½ to 6½ per cent. of carbon dioxide.

Calculation.—The molecular weight of calcium carbide (CaC_2) being $40 + 2(12) = 64$; and that of acetylene (C_2H_2), $2(12 + 1) = 26$; and the specific gravity of this gas referred to air being 0.92, we have the following:

Weight of 1 cu. ft. air (60 deg. F., bar. 30 in.) . . 0.0766 *lb.*
Weight of 1 cu. ft. acetylene, 0.92(0.0766) . . . . 0.07047 *lb.*
Volume of 1 lb. acetylene
(60 deg. F., bar. 30 in.) $1/0.07047$ . . . . 14.19 *cu. ft.*

Volume of 1 oz. acetylene $\dfrac{14.19 \times 1728}{16}$ . . . . 1532.5 *cu. in.*

Then, since 64 parts, by weight, of calcium carbide yield 26 parts, by weight, of acetylene gas, one ounce of the pure crystallized carbide will generate

$$\frac{26}{64}(1532.2) = 622 + \mathit{cu.\ in.\ acetylene},$$

measured at 60 deg. F., bar. 30 in.

Properties of Acetylene Gas.—The gas is colorless and has a strong pungent odor, due to the presence of some sulphureted and phosphureted hydrogen, as generated in the carbide lamp, by the action of water on the carbide. It has a specific gravity of 0.92, referred to air at the same temperature and pressure. Under atmospheric pressure, the gas liquefies at − 115 deg. F., the volume of the liquid being $\frac{1}{400}$ of that of the original gas.

Acetylene gas is combustible, igniting, in contact with air, at a temperature of 900 deg. F. When the gas is largely in excess and the supply of air limited the acetylene is smoky and deposits soot, but when a fine stream of the gas is spurted into the air, as in the carbide lamp, a flame of exceeding brilliancy is the result. Owing to its low temperature of ignition, the gas can be ignited by a lighted cigar.

Mixed with air the gas becomes highly explosive its explosive range being wider than that of any other gas. While the inflammable range of hydrogen extends from 5 to 72 per cent., that of acetylene ranges from 3 to 82 per cent., as determined by Clowes. This high value for the upper explosive limit has not been obtained by other investigators, whose results vary from 50 per cent. (Federal Bureau of Mines) to 65 per cent. (LeChatelier).

The Carbide Lamp in Blackdamp.—What is known as "blackdamp" in mining is a variable mixture of carbon dioxide and air deficient in oxygen; in other words, an atmosphere of blackdamp consists of nitrogen, oxygen and carbon dioxide in varying proportions. When carbon dioxide is generated in a mine ventilated by an ample air current containing a normal percentage (20.9%) of oxygen the addition of any considerable amount of carbon dioxide to this normal air reduces the oxygen content by the dilution of the air with the gas. The air is then said to be "deficient in oxygen," which is due solely to its dilution with the carbon dioxide.

On the other hand a much greater reduction of the oxygen content often occurs when a portion of the oxygen has been consumed by the various forms of combustion that are constantly taking place in the mine. It is this reduction of the oxygen content, or the "depletion of oxygen" in the mine air that is most harmful to life and affects the burning of the lamps.

It is a well known fact that the carbide lamp will continue to burn in air deficient in oxygen when oil-fed flames and the hydrogen flame are quickly extinguished. The acetylene gas burned in the carbide lamp is generated, in the lamp, by the action of water on the carbide of calcium, the calcium taking the oxygen and some of the hydrogen, while the carbon takes the remaining portion of the hydrogen.

We cannot say but that, in the dissociation of the hydrogen and oxygen of the water (H_2O), some oxygen may go to support the combustion of the acetylene gas (C_2H_2), instead of the flame being wholly dependent on the oxygen of the air for support. However, it is safe to say that an atmosphere in which a carbide continues to burn may be dangerous to life and therefore unsafe for work.

In an atmosphere containing no carbon dioxide, the oxygen content may fall as low as 14 per cent. before much difficulty is experienced in breathing; but air containing but 10 per cent. is no longer breathable and will cause death quickly by suffocation."

The toxic effect of carbon dioxide is clearly shown by the fact that the depletion of the oxygen content of air, by the addition of carbon dioxide, produces a fatal atmosphere when the oxygen is reduced to but 17 per cent.; while, if no carbon dioxide is present, a fatal atmosphere is produced only when the depletion of the oxygen reaches 10 per cent.

In the former of these two cases, there is but 83 per cent. of noxious gases present—carbon dioxide, 18 per cent. and nitrogen, 65 per cent.; while, in the latter case, there is 90 per cent. of nitrogen present. In the former case a depletion of oxygen to 17 per cent. marks a fatal atmosphere; while in the latter case, a depletion of oxygen to 10 per cent. is necessary to produce the same result.

It is quite doubtful if a carbide lamp is extinguished when the oxygen of the atmosphere is reduced to 14 per cent., as is frequently assumed.

Precautions to be Taken.—In the use of carbide lamps in mines, suitable rules and regulations should be made and enforced limiting the supply of carbide that a miner may carry into the mine to what is ample for his purpose in a single shift and prohibiting its careless use. A supply of carbide should never be permitted to be stored in a miner's box or elsewhere in a mine. With proper care and precautions there need be little fear of trouble. The carbide light being an open-flame lamp should not be used in a mine generating gas.

ELECTRIC MINE LAMPS

The electric mine lamp is now almost universally used in all up-to-date mines in the states and Canada, there being at present 150,000 of these lamps installed by the Edison Storage Battery Co. alone. Of this number, 80,000 of the lamps are in daily use in the mines of Western Pennsylvania.

Selecting a Suitable Battery.—In the endeavor to provide a portable electric mine lamp that would meet the requirements of mine service, the chief difficulty was to find a battery that would be sufficiently light and have the necessary watt-hour capacity to furnish a good light a full 8-hr. shift.

All forms of primary batteries that depend on the chemical reaction set up between certain elements immersed in a solution, as well as the lead-sulphuric acid storage battery, proved unsuited to service in the mine. The lead-lead battery was too heavy, besides failing in other ways to meet the requirements of mining use. Even the substitution of a gelatinous electrolyte proved ineffectual, owing to the hardened jelly not absorbing the water when once dried and the crack becoming filled with sediment short-circuiting the cells and weakening the battery.

The Edison Storage Battery.—The difficulties just mentioned have been practically overcome in the Edison storage battery designed for mine use. This battery employs as elements nickel hydroxide and iron oxide immersed in a potash solution. The battery cells are incased in a strong nickel-plated steel container, which is tightly sealed except for one small vent being left for the escape of the harmless gases that result in the charging of the battery.

The illustration, Fig. 67, shows the two cells of the Edison mine-lamp battery removed from the nickeled-steel case. The steel container of one cell is cut away to show the interior arrangement. The positive plates (steel tubes of nickel hydrate) and the negative plates (steel pockets of iron oxide) are assembled on steel poles and intermeshed, which gives an exceptionally strong and compact construction entirely of steel, there being no acid to cause corrosion.

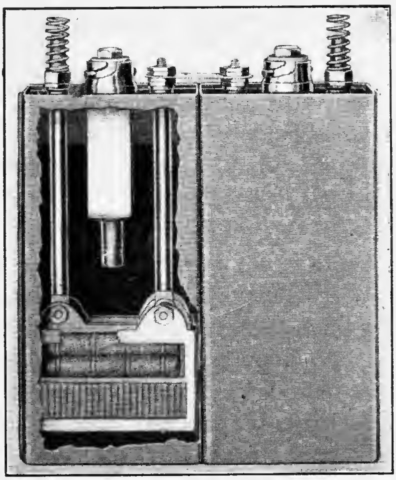


Fig. 67.

The construction of this battery is such that it is practically impossible for the solution to find its way out, even should the battery be turned upsidedown; and no injury can result from a possible overcharging, or from leaving the cell in a charged, semi-charged or discharged condition, for an indefinite period. While the cell must be charged in the right direction to be fit for service, no injury can result from accidentally reversing this direction. The steel container is proof against rough usage, and no insulation troubles can occur. Specific gravity tests are not required as the potash solution is renewed after 9 or 10 months of use in continuous daily service.

Cap Lamp and Connecting Cable.—The illustration, Fig. 68, shows the electric cap lamp and the nickeled-steel carrying case holding two cells. The cover of the case is removed to show the steel contact plates affixed to but insulated from the cover. These plates connect with the contact springs shown mounted on the two terminals of the battery. The cover is secured to the case by a strong hasp and padlock. To this cover is attached a twin-conductor, rubber-covered cable, armored at both ends to prevent injury where sharp bending is liable to occur. If injured the cable is easily replaced.

The supporting base of the lamp is a nickel-plated reflector having a highly finished surface and provided with a hook to fit into the regulation miner's cap. The angle of distribution is considerably greater than the 130 deg. specified by the government. (see p. 322). A tungsten lamp is forced into a spring socket by means of a clip at its tip in such a way that if the lamp should be broken the base is immediately disconnected and the lamp extinguished. This safety feature has been thoroughly tested by the Bureau of Mines and un-

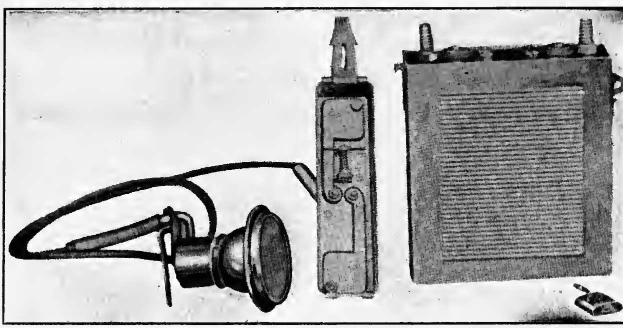


Fig. 68.

qualifiedly approved under Schedule 6A. In place of a lens is a plain glass that is easily replaced if broken. The entire design is such as to afford the greatest possible headroom clearance.

Charging Miners' Lamp Batteries.—The recharging of a large number of lamp batteries, between shifts, calls for a special design of equipment that will provide at once for the charging of the batteries and enumerating them so that any individual battery can be found without delay.

A convenient form of charging rack that meets these requirements is one built up on the unit system, corresponding to the sectional bookcase idea. The illustration, Fig. 69, is a view of such a rack, designed and built by the Cutler-Hammer Mfg.

Co., Milwaukee, Wis. The figure shows four units, but the system can plainly be extended indefinitely to accommodate an increasing number of lamps as the development of the mine proceeds. The recharging room must be well ventilated and open lights should not be permitted.

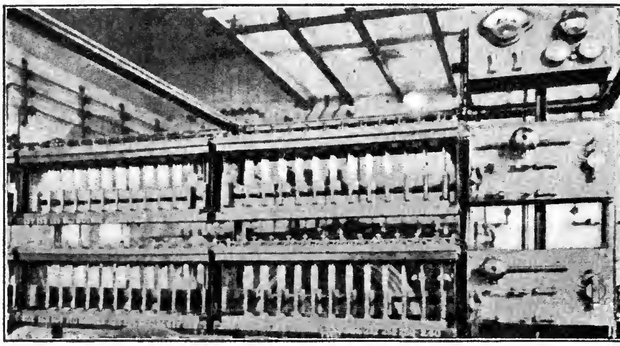


FIG. 69.

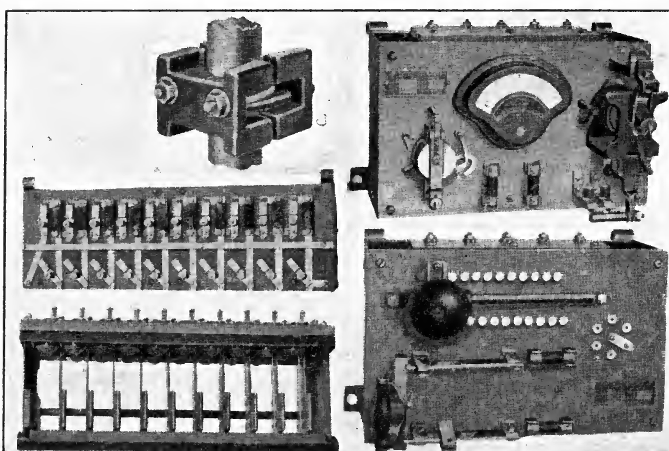


FIG. 70.

On the right of the figure are shown two rheostat panels and a meter panel above. These panels are shown in greater detail in the Fig. 70, together with front and top views of a single unit capable of holding ten lamp batteries for charging.

The contact parts supported by the upper slab are pressed down in contact with the battery by the coil springs above the slab. The batteries are charged in series and provision is made for interpolating resistances to take the place of one or more absent batteries.

Pipe columns to which are clamped supporting brackets, as shown in this figure, form the framework of the rack on which are hung the several battery units and panels by means of the strong hooks shown attached to each.

Each rheostat panel is designed to control the current in the corresponding line of units, and is equipped with a sliding arm for adjusting the charging rate to any desired value. The double-pole knife switch shown on this panel is so arranged that when partly closed the ammeter on the meter panel is thrown into circuit; but when closed completely the ammeter is cut out and the current passed through the charging racks.

The meter panel not only holds the ammeter for measuring the strength of the current and regulating it in accordance with the number of units to be charged; but is also provided with a magnetic switch and compound relay, which prevents a reversal of current from the partially charged batteries taking place should the charging current be interrupted for a time. This device automatically opens and closes the circuit as the current is broken and again restored. The breaking of the current is immediately announced by the signal bell on each rheostat panel.

Edison mine-lamp batteries require a pressure of 40 volts, which makes it possible to charge six 10-battery units, on a 250-volt circuit. However, it is generally advisable to install but five such units on this circuit, which would allow the pressure to drop to 200 volts without interrupting the charging.

Use of the Electric Cap Lamp.—The need of a reliable source of illumination in mining work has long been sought but with limited success. Open-flame lamps or torches are necessarily restricted to non-gaseous mines, or where the conditions are such as not to require the exclusive use of safety lamps. On the other hand, the relatively dim light of a safety lamp and its lack of adaptation to the requirements of mining work make it always desirable to find a suitable substitute that will be both convenient and safe for general work.

The electric cap lamp with storage battery equipment similar to that shown in the illustration, Fig. 71, has apparently solved the problem, and furnished the miner with a good light that is convenient and safe. The principal objections that have been urged against the miners' electric lamp are the slightly increased cost of the equipment, and the fact that an

Fig. 71.

electric lamp affords no indication of the presence of gas, either methane or blackdamp, and gives the miner no warning of danger in that respect.

Notwithstanding these disadvantages, the electric lamp has steadily grown in favor among miners, as shown by its general adoption and successful use. In daily practice, the miner straps the battery case to his back, by his ordinary belt. The lamp is attached to the leather support in his cap, leaving his arms entirely free of lamp, cord and battery case. When the case is locked and the equipment handed to the miner charged and ready for use there can be no safer or surer means of illumination.

PERMISSIBLE PORTABLE ELECTRIC MINE LAMPS

Schedule 6A, issued by the Federal Bureau of Mines, defines what is to be understood as included under the appellation "Permissible," in reference to portable electric mine lamps, in the following words:

The Bureau of Mines considers a portable electric lamp to be permissible for use in mines if all the details of the lamp's construction are the same, in all respects, as those of the lamp that passed the inspection and the tests for safety, practicability, and efficiency made by the bureau and hereinafter described.

Conditions of Testing.—The conditions under which the Bureau of Mines will examine and test portable electric lamps to establish their permissibility are as follows:

1. The tests will be made at the experiment station of the Bureau of Mines at Pittsburgh, Pa.

2. Applications for tests shall be addressed to the Director, Bureau of Mines, Washington, D. C., and shall be accompanied by a complete description of the lamp to be tested and a full set of the drawings mentioned below.

A drawing or drawings clearly showing the size and general appearance of the lamp mounting.

A drawing or drawings clearly showing the character, size and relative arrangement of the parts of the lamp mounting and the principle of operation of the safety devices.

Any other drawings that may be necessary to identify the safety devices or to explain how they accomplish their purpose.

A copy of the description, a duplicate of the drawings and one complete lamp shall be sent to the Electrical Engineer, Bureau of Mines, Fortieth and Butler Streets, Pittsburgh, Pa.

3. As soon as possible, after the receipt of his application for test, the lamp manufacturer will be notified of the date on which his lamps will be tested and the amount of material that it will be necessary for him to submit.

4. All material for test shall be delivered by the manufacturer to the Electrical Engineer, Bureau of Mines, Fortieth and Butler Streets, Pittsburgh, Pa., not less than one week prior to the date set for the test.

5. No lamp equipment will be tested, unless it is in the completed form in which it is to be put on the market.

6. Lamps so constructed that they can be used both as cap lamps and as hand lamps must pass the tests for both cap lamps and hand lamps or they will not be approved for either class of service.

7. No one is to be present at these tests, except the necessary government officers, their assistants, and one representative of the manufacturer of the lamp to be tested, who shall be present in the capacity of an observer only.

The conduct of the tests shall be entirely in the hands of the bureau's engineer in charge of the investigation. While the tests are in progress the manufacturer's representative shall not make unsolicited suggestions or criticisms of the method of conducting the test.

8. The tests will be made in the order of the receipt of application for test, provided that the necessary lamp equipment is submitted at the proper time.

9. The details of the results of the tests shall be regarded as confidential by all present at the tests, and shall not be made public, in any way, prior to their official publication by the Bureau of Mines.

Requirements for Approval.—The requirements that a portable electric-lamp equipment must have, to pass successfully the inspection and tests required by the bureau, are stated below:

1. The lamp equipment must comply with the following requirements for mechanical and electrical construction:

The construction of permissible portable electric-lamp equipment shall be especially durable. All parts shall be constructed of suitable material of the best quality and shall be assembled in a thorough workmanlike manner. Current-carrying parts shall be well insulated from parts of opposite polarity and from parts not intended to carry current.

The battery shall be inclosed in a locked or sealed box so constructed as to preclude the possibility of anyone meddling with the electrical contacts or making an electrical connection with them while the box cover is closed.

The leads connecting the battery with the headpiece shall be made up in a single cable efficiently insulated and provided, where it leaves the battery casing and enters the headpiece, with a reinforcement of flexible metallic tubing. The flexible metallic tubing will not be required if other equally durable means of reinforcement are provided.

It is recommended, but not required, that the headpiece be so designed that it can be sealed or locked. The battery terminals and leads connecting thereto, and the gas vent of the battery shall be so designed and constructed as to prevent corrosion of the battery terminals or of the essential metallic parts mounted in the cover of the battery casing.

The following qualities will be considered in determining the excellence of the mechanical and electrical construction of lamps covered by these specifications:

Simplicity of design; mechanical strength of parts and fastenings; suitability of material used; design of moving and removable parts; design and construction of terminals and contacts, for permanence and electrical efficiency; and ease of repair.

2. The lamp equipment must be provided with a safety device or devices as follows:

Permissible portable electric lamps shall be so designed and constructed that whenever the bulb of a completely assembled lamp equipment is broken the lamp filament shall, at once and under all circumstances, cease to glow at a temperature that will ignite explosive mixtures of mine gas and air.

The mounting of the bulb may be designed so that a blow sufficient to break the bulb will short-circuit it, open the electric circuit of the lamp or otherwise insure that the filament will be wholly or practically extinguished. All safety devices with which the lamps are provided shall be so completely protected from injury or disturbance as to insure that the devices will always be in condition to perform their functions.

The design of the safety features shall be such that their action can not readily be hindered or prevented. The design of the safety devices shall be such that they will not act to extinguish the lamp unnecessarily.

3. The lamp equipment must be provided with a battery having a short-circuit current not in excess of the values here specified.

The bureau's engineers have made tests (reported in Technical Paper 47 of the bureau), which have satisfied them that mine gas can not be ignited by the sparks from portable electric-lamp equipments if the batteries used with such equipments are made so that their maximum short-circuit current can not exceed the following values: For batteries giving 2.5 volts or less, 125 amperes; for batteries giving more than 2.5 volts but not more than 4 volts, 85 amperes; for batteries giving more than 4 volts but not more than 5 volts, 65 amperes; for batteries giving more than 5 volts but not more than 6 volts, 45 amperes. Therefore, lamps whose short-circuit current does not exceed these values will be considered satisfactory in that respect.

4. The lamp equipment must meet the following requirements for time of burning, flux of light, intensity of light and distribution of light:

All portable electric lamps offered for test under the provisions of this schedule shall produce, for 12 consecutive hours, on one charge of battery, a light stream having an averge intensity of light not less than four-tenths of a candlepower. The total flux of light produced by cap lamps shall not fall below 1½ lumens during the 12 hours, and the total flux of light produced by hand lamps shall not fall below 3 lumens during the 12 hours.

The distribution of light, by lamps that use reflectors, shall be determined both by observation and by photometric measurement. The lamps shall be placed so that the filaments are 20 in. away from a plane surface that is perpendicular to the axis of the light stream of the lamp. When so placed the lamp shall illuminate a circular area not less than 7 ft. in diameter.[a] All observations and measurements of distribution shall be referred to this 7-ft. circle regardless of how large an area the lamp may illuminate. As observed with the eye, there shall be no "black spots" within the 7-ft. circle, nor any sharply contrasting areas of bright and faint illumination anywhere. As measured with a photometer, the distribution of light diametrically across the circle shall fulfill the following requirements:

The curve of light distribution along the diameter of the circle shall be obtained by rotating the lamp and thus obtaining the average distribution curve.

The average illumination in foot-candles, on the best illuminated one-tenth of the diameter, shall be not more than three times the average illumination throughout the diameter; and, for at least 40 per cent. of the diameter, the illumination shall be not less than the average.

5. The lamp equipment must be provided with lamp bulbs that meet the following requirements, for variation in current consumption, variation in candlepower and length of life:

The bulbs submitted for test shall be identified by the name of the manufacturer and by a number or symbol with reference to which approval will be granted.

The current consumption of at least 95 per cent. of the bulbs tested shall not exceed, by more than 6 per cent., the average current consumption of all the bulbs examined.

The candlepower of at least 90 per cent. of the bulbs tested shall not fall short of the average candlepower, by more than 30 per cent.

The life of a lamp bulb will be considered as the number of hours that the bulb can be burned, under normal conditions of voltage, before it becomes so depreciated that when used with an average, standard, freshly charged equipment it fails to produce, for 12 consecutive hours, the flux and intensity of light specified in paragraph 4.

The average life of lamp bulbs shall be not less than 300 hours, for acid storage batteries, and not less than 200 hours, for primary batteries and for alkaline storage batteries. Not more than 5 per cent. of the bulbs examined shall give less than 250 hours' life, with acid batteries, nor less than 150 hours' life, with primary batteries and alkaline batteries.

6. The lamp equipment must comply with the following requirements as to leakage of electrolyte:

Lamps shall be so designed and constructed that they will not spill nor leak electrolyte throughout an 8-hour test, during which they will be placed in any position or sequence of positions that, in the opinion of the bureau's engineers, will be most likely to prove whether or not the electrolyte can be spilled.

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[a] This requirement will be met by lamps that have an angle of light stream of 130° or more.

Tests of Design and Construction.—The excellence of the mechanical and electrical features of the design and construction of the lamps will be carefully determined.

The following tests will also be made: Hand lamps and the headpieces of cap lamps will be dropped 10 times, upon a concrete floor from a point 6 ft. above it. As the result of these dropping tests, there must be no breakage of the battery jar or material distortion of the casing of the battery or of the shell of the headpiece. The engineers in charge of the investigation shall be the sole judges of whether or not material distortion occurs. The dropping tests of the headpiece must demonstrate that the safety devices will not operate unnecessarily.

Cap lamps will be dropped 10 times, upon a wooden floor, from a point 3 ft. above it. There must be no breakage of the battery jar or material distortion of the casing.

Tests of Safety Devices.—In making tests of the safety devices, it will be assumed that if the short-circuit current of the battery does not exceed a certain value, stated previously, the glowing filament of the lamp is the only source of danger.

It will also be assumed (based on tests reported in Technical Paper 23) that the glowing filament presents an element of danger, in the presence of mine gas, if the bulb of the lamp can be broken without causing the filament to become wholly or practically extinguished as the result of the action of the safety devices with which the lamp is provided.

The tests will therefore be made with a view to determining whether or not the lamp bulb may be broken without causing the safety device of the lamp to extinguish the lamp or cause the filament to glow at a temperature that is not high enough to ignite explosive mixtures of mine gas and air.

If the safety devices are designed to extinguish the lamp before the bulb is broken it will not be necessary to make the tests in gas, unless the safety devices do not completely extinguish the lamp. It will then be necessary to determine whether or not the filament is glowing at a temperature sufficient to ignite gas.

If the safety devices are designed to extinguish the lamp at the same time that the bulb is broken it will be desirable to make the tests in explosive mixtures of gas and air.

Gas, if used, will be the natural gas supplied to the city of Pittsburgh. The composition of this gas, as determined from recent analyses, is approximately 83.1 per cent. methane, 16 per cent. ethane, 0.9 per cent. nitrogen and a trace of carbon dioxide.

The details of conducting the tests will, manifestly, not be the same for all lamps submitted, because different lamps will no doubt have safety devices differing in design, construction and basic principles. The bureau proposes to determine, for each lamp separately, a schedule of tests that, after due examination of the lamp and its safety devices, seem best adapted to ascertaining the merits of the equipment submitted. This schedule may be examined and discussed by the manufacturer's representative before the tests are begun.

In general, the tests will consist of striking the mounting or holder of the lamp bulb, in an attempt to break the bulb without extinguishing the lamp.

If the safety devices are designed to extinguish the lamp (as, by disconnecting the bulb from circuit, or by opening the circuit at some other point) the devices will be considered to have acted:

1. If, after the blow has been delivered, the lamp bulb, whether broken or not, is clearly disconnected from circuit.

2. If, after the blow has been delivered:

(*a*) When the lamp filament is not broken by the blow and does not glow;

(*b*) When the lamp filament is broken by the blow a sound filament, replacing the broken filament, does not glow.

If the safety devices are designed to decrease the temperature of the filament (by short-circuiting the filament or by other means), the devices will be considered to have acted if, after the blow has been delivered:

(*a*) When the lamp filament is not broken by the blow it does not glow at a temperature sufficient to ignite gas;

(*b*) When the lamp filament is broken by the blow a sound filament, replacing the broken filament, does not glow at a temperature sufficient to ignite gas.

If there is any question as to whether or not a filament is glowing at a dangerous temperature the point will be settled by surrounding the filament with an explosive mixture of gas and air.

If, after the blow has been delivered, the bulb has not been broken and the safety devices have not acted the test will be repeated with the same equipment, or with a different equipment, at the discretion of the bureau's engineers.

The bureau believes that approximately 50 tests will be necessary to determine whether or not the safety devices of a lamp are permissible for use in gaseous mines; but more or fewer tests may be made at the discretion of the engineer in charge of the tests.

To Determine Maximum Short-circuit Current.—The short-circuit current of the battery will be measured under conditions that will give the same current that would flow through a short-circuit between the conductors of the flexible cord, at the point in the cord nearest to the battery casing.

Tests of Lighting.—The tests to determine the time of burning, flux, intensity and distribution of light will be made, for not less than 20 batteries, 6 reflectors or lamp mountings, and 100 lamp bulbs.

The average performance of the various equipments will be taken as the average performance of the lamp. The measurements of flux and intensity of light will be made after the bulbs have been burned for about 10 hours in order to season them somewhat.

Tests of Current Consumption, Candlepower, Life of Bulb.—Measurements of current consumption and candlepower will be made with bulbs that have been burned about 10 hours.

Measurements of current consumption will be made at approximately the average potential given by the lamp battery, after having been used for one hour.

Measurements of bulb candlepower will be made in one direction only. Usually the direction that gives the largest exposure of filament will be selected.

Determination of bulb-life will be made with batteries that have the same voltage characteristics as those used with the lamp. Tests will be made with the bulbs in a fixed position.

Although, as stated in Technical Paper 75, Bureau of Mines, the bureau considers that the batteries of portable electric mine lamps should give 3600 hours of service (300 12-hour shifts) without requiring repairs or replacements of any part, it is manifestly impracticable for the bureau to carry out the 3600-hour test upon each battery submitted for approval. Therefore, the requirements of the bureau, with respect to the durability of batteries, will be considered as satisfied if the batteries shall perform their functions without repair while being used by the bureau, in accordance with the written instructions of the lamp manufacturer, to conduct the bulb-life tests; and, at the completion of these tests, the condition of the batteries shall give no evidence of weakness that indicates the early failure of any part of the battery.

Test of Leakage of Electrolyte.—The lamps will be tested for leakage and spilling of electrolyte, by placing the batteries for various lengths of time, totaling eight hours, in various positions that seem most likely to cause the cells to leak or spill. If a battery does not leak or spill more than one full drop of electrolyte during the eight-hour test the battery casing will be regarded as non-spilling.

Approval of Electric Mine Lamps.—The manufacturers will be required to attach to the battery casing of each permissible lamp equipment a plate bearing the seal of the Bureau of Mines and inscribed as follows:

PERMISSIBLE PORTABLE ELECTRIC MINE LAMP. APPROVAL NO.—.

Issued for safety and for practicability and efficiency in general service to the ————Co.

The use of the plate will not be required if the same inscription is stamped or cast into the casing of the battery.

Manufacturers shall, before claiming the bureau's approval for any modification of any approved lamp, submit to the bureau drawings that shall show the extent and nature of such modifications, in order that the bureau may decide whether or not it should test the remodeled lamp before approving it. Each approval of a permissible lamp will be given a serial number. Approvals of modified forms of a previously approved lamp will bear the same number as the original approval with the addition of the letters *a*, *b*, *c*, etc.

The bureau will, upon request, make tests of lamp bulbs to determine whether or not they will comply with the bureau's requirements when used in connection with any lamp that has been approved by the bureau under the provisions of this schedule. Lamp bulbs that fulfill the requirements will be specifically approved for use with stated lamps. Applications for tests of bulbs should be made in a manner similar to application for tests of lamps.

The bureau's approval of any lamp shall be construed as applying to all lamps made by the same manufacturer that have the same construction in the details considered by the bureau, but to no other lamps. The bureau reserves the right to rescind, for cause, at any time, any approval granted under the conditions herein set forth.

Notification of Manufacturer.—As soon as the bureau's engineers are satisfied that a lamp is permissible, the manufacturer of the lamp and the mine-inspection departments of the several states shall be notified to that effect. As soon as a manufacturer receives formal notification that his lamp has passed the tests prescribed by the bureau, he shall be free to advertise such lamp as permissible.

Fees for Testing.—The necessary expenses involved in testing portable electric mine lamps have been determined, and the following schedule of fees to be charged, on and after the date of issue of this schedule, has been established and approved by the Secretary of the Interior:

1. For a complete official investigation leading to the formal approval of a portable electric mine lamp, the investigation to include tests of the safety devices and the determination of the time of burning, flux of light, intensity of light, distribution of light, bulb characteristics, leakage of electrolyte, and durability $150.00
2. For tests of the safety devices only $30.00
   For additional necessary tests, under the same investigation (for each five tests or fraction thereof) $2.50
3. For tests to determine only the time of burning, flux of light, intensity of light, distribution of light, bulb characteristics, and leakage of electrolyte $120.00
4. For tests to determine only bulb life, variation in bulb candlepower and variation in bulb current consumption:
   If such tests involve making discharge-voltage determinations $75.00
   If such tests do not involve making discharge-voltage determinations $50.00

5. The following charges will be made for individual tests included under item 3:

Discharge-voltage tests	$25.00
Reflector tests	$20.00
Time-of-burning tests	$10.00
Light-distribution tests	$5.00
Electrolyte-spilling tests	$3.00
Short-circuit tests of battery	$1.00
Mechanical tests of cord	$6.00
Bulb-life tests	$35.00
Bulb-uniformity tests	$15.00

6. Special tests that circumstances shall render necessary, during the course of the investigation, will be made at the request of the lamp manufacturer and will be charged for in accordance with the amount of work involved.

ADDENDA

LOGARITHMS—CIRCULAR FUNCTIONS, SINES AND COSINES, TANGENTS AND COTANGENTS—SQUARES, CUBES, ROOTS AND RECIPROCALS OF NUMBERS—CIRCUMFERENCES AND AREAS—DENOMINATE NUMBERS—WEIGHTS AND MEASURES—UNITED STATES AND BRITISH SYSTEMS—METRIC SYSTEMS OF WEIGHTS AND MEASURES—CONVERSION TABLES—CONVERSION OF COMPOUND UNITS.

LOGARITHMS

The treatment of logarithms here will be simple and practical and such as to enable their use to be clearly understood. Much time and labor are saved when multiplying and dividing, or when extracting the roots of numbers, or raising a number to a given power by the use of logarithms.

Definition.—The logarithm of a number is the exponent of the power to which it is necessary to raise a fixed number called the **"base"** to produce the given number.

Systems of Logarithms.—There are two systems of logarithms in use: 1. The Briggs or **common system** employs 10 as a base. 2. The Naperian or hyperbolic or **natural system** is derived from 2.71828+ as a base. The common logarithms (log) are those generally used, while the natural logarithms (nat. log) are often employed in theoretical analyses.

The Naperian or natural logarithm of a number can always be found by multiplying the common logarithm of the number by 2.302585, which is expressed thus:

$$\text{Nat. log.} = 2.302585 \text{ com. log.}$$

In any system of logarithms, the logarithm of 1 is zero, and the logarithm of the base of the system is always 1.

The Logarithm.—Every logarithm is composed of two distinct parts separated by a decimal point The number preceding the decimal point, or the integer of the logarithm is called the characteristic," while the decimal portion of the logarithm is the "mantissa." These two parts of a logarithm must be regarded separately. The mantissa is always positive, but the characteristic may be either positive or negative, according as the given number is greater or less than 1, in a system whose base is greater than 1.

The **characteristic** is always 1 less than the number of figures in the integral portion of the given number; or 1 greater than the number of ciphers following the decimal point when the given number is wholly decimal. In the former case the characteristic is positive; in the latter case it is negative. The following examples will make this clear:

log 325.00 = 2.51188	log 0.325 = $\bar{1}$.51188
log 32.50 = 1.51188	log 0.0325 = $\bar{2}$.51188
log 3.25 = 0.51188	log 0.00325 = $\bar{3}$.51188

The **mantissa,** as is readily observed from the above examples, is determined by the sensible figures of a number, without regard to the decimal point. Also, the mantissa of the logarithm of a number is unchanged when the number is multiplied or divided by 10, 100, 1,000, etc. For example, the mantissa of the logarithm of 3, which is 0.47712, is the same for 30, 300, 3,000 or for 0.3, 0.03, 0.003, etc.

A table of the common logarithms of numbers from 0 to 10,000 follows and will be found useful. In this table the mantissas only are given and, to avoid unnecessary repetition, the first two figures are not repeated. An asterisk * appearing before the remaining three figures of the mantissa indicates that the first two figures must be taken from the line below. Bars are employed to mark the division by tens, which facilitates the finding of the mantissa of any desired number given in the left-hand column. In this table, the differences are given as proportion parts and placed in the right-hand column marked "P. P.," which avoids the necessity of multiplying by the decimal as will be explained.

To Find the Logarithm of a Number.—From the table of logarithms, find the mantissa corresponding to the given number, ignoring the decimal point. To do this, the first three figures on the left of the given number are found in the left-hand column of the table, and the fourth figure in the line at the top. The required mantissa is then taken from the line and column thus indicated.

But if the given number contains five or more figures, write the excess figures as a decimal and multiply the difference between the mantissa found and the one next following by this decimal; point off and add the integral portion of the result to the mantissa already found. If desired this logarithm can be extended by annexing the decimal portion of the same result, but this is not commonly necessary. When there is but one excess figure, as when finding the mantissa of a number having five figures, the difference to be added to complete the mantissa is taken from the corresponding proportional part, in the right-hand column without multiplying.

Having found the mantissa, prefix a decimal point preceded by a characteristic one less than the number of integral figures in the given number. If there is but one integral figure the characteristic of the logarithm will be zero.

If the given number is a decimal, having no integral figures, the characteristic will be negative and numerically one greater than the number of ciphers that follow the decimal point.

Illustrations.—The following examples will illustrate the method of finding the logarithms of numbers under different conditions and make clear the use of the table.

1. Suppose it is required to find the logarithm of the number 4,657. Opposite 465, in the column under 7, is found 811, and this annexed to 66 found at the left gives for the mantissa of this number the decimal 0.66811. The characteristic, in this case, is 3, since there are four integral figures in the given number. Hence, log 4,657 = 3.66811.

2. To find the logarithm of 32.567, ignoring the decimal point, opposite 325 in the column under 6, is found the mantissa, 0.51268; but there is still another figure 7 in the given number. Therefore, to complete this mantissa subtract it from the one following, giving the difference 14 found in the right-hand column. The proportional part of this difference corresponding to the fifth figure 7 is 9.8 or, say 10. Then 51,268 + 10 = 51,278 and the complete mantissa is therefore 0.51278. In this case, the given number contains but two integral figures, which makes the characteristic 1; hence, log 32.567 = 1.51278.

3. To find the logarithm of 0.509065, ignoring the decimal point, opposite 509, in the column under 0, is found the mantissa 0.70672. To complete this mantissa subtract it from the one next following, thus, 680 − 672 = 8, and multiply the remaining figures of the given number written as a decimal, by the difference 8 and add the integral of the result to the mantissa already found.

Thus, $$70{,}672 + 0.65 \times 8 = 70{,}672 + 5 = 70{,}677.$$

Now, since the given number is a decimal, the characteristic of its logarithm is negative; and its numerical value is 1, as there are no ciphers immediately following the decimal point. The complete logarithm is, therefore, log 0.509065 = $\bar{1}.70677$, the minus sign being written over the characteristic, since the characteristic only is negative.

Use of Logarithms.—By the use of logarithms the processes of multiplication, division, involution and evolution are greatly shortened and simplified. The two latter processes are in fact a repetition of the two former; while division and evolution are the reverse operations of multiplication and involution, respectively.

It is important to observe that the use of logarithms enables the finding of decimal powers and decimal roots of numbers, which is impossible by other means. When the index of a power or root of a number can be expressed as a fraction the numerator and denominator of such fraction express, respectively, the indices of the power and root or the root and power, as the case may be. A decimal index, therefore, expresses in one operation the extraction of any given root of any given power of a number, which will be better understood later.

The application of this principle is shown in numerous instances where quantities vary in their relation to each other according to different powers. For example, in fan ventilation, the fourth power of the speed

(n^4) of the fan varies as the fifth power of the quantity (q^5) of air in circulation; which is expressed as follows:

	n^4	varies as	q^5
or	n	varies as	$q^{\frac{5}{4}}$; or $q^{1.25}$
and	q	varies as	$n^{\frac{4}{5}}$; or $n^{0.8}$

The expression $n^{\frac{4}{5}}$ or the fourth-fifths power of n is identical with $\sqrt[5]{n^4}$ or the fifth root of the fourth power of n. Hence, to extract the root of a power, divide the exponent of the power by the index of the desired root and the quotient will be the new exponent, which combines the two operations in a single transaction.

Rules for the Use of Logarithms.—The following four simple rules cover all the operations of logarithms:

1. Multiplication: To find the product of two or more numbers, add their logarithms; the number corresponding to this logarithmic sum is the desired product.

In other words, the logarithm of the product of two or more numbers is equal to the sum of the logarithms of the numbers.

2. Division: To divide one number by another, subtract the logarithm of the divisor from that of the dividend; the number corresponding to this logarithmic remainder is the required quotient.

In other words, the logarithm of the quotient is equal to the logarithm of the dividend minus that of the divisor.

3. Involution: To find any given power of a number, multiply the logarithm of the number by the exponent of the power; the number corresponding to the resulting logarithm is the required power of the given number.

4. Evolution: To find any given root of a number, divide the logarithm of the number by the index of the root; the number corresponding to the resulting logarithm is the required root of the given number.

Arithmetical Complement.—The arithmetical complement of a logarithm is the remainder found by subtracting the log from 10; the logarithm of 3 is 0.47712, and its arithmetical complement is, therefore, $10 - 0.47712 = 9.52288$. Its use involves subtracting from the final result as many tens as have thus entered the solution. The antilog is more convenient for use.

The Antilog.—The solution of problems frequently involves the multiplication and division of many quantities. In the use of logarithms, the sum of the logs of the divisors would be subtracted from the sum of the logs of the multipliers, to obtain the log of the final result. By the use of what is called the "antilog" of each divisor, it is possible to complete such a solution in a single operation, by adding together the logs of the multipliers and the antilogs of the divisors.

The **antilog** of a number is obtained as follows: Subtract the mantissa of its log from 1, for the mantissa of the antilog. Then, add 1 to the characteristic of the log and change its sign, the addition being always algebraic. The following examples will make the process understood:

1. To find the antilog of 800: Log 800 = 2.90309
Mantissa of antilog, 1 − 0.90309 = 0.09691
Characteristic of antilog, 2 + 1 = 3; and changing sign = −3
Hence.. Antilog 800 = $\bar{3}.09691$

2. To find the antilog of 2: log 2 = 0.30103
Mantissa of antilog, 1 − 0.30103 = 0.69897
Characteristic of antilog, 0 + 1 = 1; giving − 1
Hence.. Antilog 2 = $\bar{1}.69897$

3. To find the antilog of 0.4: Log 0.4 = $\bar{1}.60206$
Mantissa of antilog, 1 — 0.60206 = 0.39794
Characteristic of antilog, − 1 + 1 = 0 (zero has no sign)
Hence.. Antilog 0.4 = 0.39794

4. To find the antilog of 0.00125: Log 0.00125 = $\bar{3}.09691$
Mantissa of antilog, 1 − 0.09691 = 0.90309
Characteristic of antilog, − 3 + 1 = − 2; giving + 2
Hence.. Antilog 0.00125 = 2.90309

Note.—The use of the antilog accomplishes the same purpose as the arithmetical complement and requires no correction of the final result as explained in reference to the latter. It should be observed that the antilog of a number is always the log of the reciprocal of that number.

Thus, Log 800 = antilog 1/800 or 0.00125

As shown above, log 800 = 2.90309; antilog 0.00125 = 2.90309.

Example.—Solve the following by the use of logarithms:

$$p = \frac{ksq^2}{a^3} = \frac{0.00000002 \times 40{,}000 \times 50{,}000^2}{50^3} = ?$$

Solution.— log 0.00000002.......................... $\bar{8}.30103$
log 40,000.......................... 4.60206
log $50{,}000^2$ (4.69897 × 2).......... 9.39794
antilog 50^3, (log 50^3 = 1.69897 × 3 = 5.09691)............ $\bar{6}.90309$

Log p.......................... 1.20412

Hence p = 16 lb. per sq. ft.

LOGARITHMIC TABLES

COMMON LOGARITHMS OF NUMBERS

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
0	— ∞	**20**	30 103	**40**	60 206	**60**	77 815	**80**	90 309
1	00 000	21	32 222	41	61 278	61	78 533	81	90 849
2	30 103	22	34 242	42	62 325	62	79 239	82	91 381
3	47 712	23	36 173	43	63 347	63	79 934	83	91 908
4	60 206	24	38 021	44	64 345	64	80 618	84	92 428
5	69 897	25	39 794	45	65 321	65	81 291	85	92 942
6	77 815	26	41 497	46	66 276	66	81 954	86	93 450
7	84 510	27	43 136	47	67 210	67	82 607	87	93 952
8	90 309	28	44 716	48	68 124	68	83 251	88	94 448
9	95 424	29	46 240	49	69 020	69	83 885	89	94 939
10	00 000	**30**	47 712	**50**	69 897	**70**	84 510	**90**	95 424
11	04 139	31	49 136	51	70 757	71	85 126	91	95 904
12	07 918	32	50 515	52	71 600	72	85 733	92	96 379
13	11 394	33	51 851	53	72 428	73	86 332	93	96 848
14	14 613	34	53 148	54	73 239	74	86 923	94	97 313
15	17 609	35	54 407	55	74 036	75	87 506	95	97 772
16	20 412	36	55 630	56	74 819	76	88 081	96	98 227
17	23 045	37	56 820	57	75 587	77	88 649	97	98 677
18	25 527	38	57 978	58	76 343	78	89 209	98	99 123
19	27 875	39	59 106	59	77 085	79	89 763	99	99 564
20	30 103	**40**	60 206	**60**	77 815	**80**	90 309	**100**	00 000

N.	L. 0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
131	727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
141	922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	056	085	114	143	173	202	231	260	289
149	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	44	**43**	**42**
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	**40**	**39**
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	**37**	**36**
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29 6	28.8
9	34.2	33.3	32.4

	35	**34**	**33**
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	**31**	**30**
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

N.	L. 0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
151	898	926	955	984	*013	*041	*070	*099	*127	*156
152	18 184	213	241	270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	257	285
156	312	340	368	396	424	451	479	507	535	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	*003	*030	*058	*085	*112
159	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
161	683	710	737	763	790	817	844	871	898	925
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21 219	245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	748	775	801	827	854	880	906	932	958	985
166	22 011	037	063	089	115	141	167	194	220	246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	*005	*030
174	24 055	080	105	130	155	180	204	229	254	279
175	304	329	353	378	403	428	452	477	502	527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	*018
178	25 042	066	091	115	139	164	188	212	237	261
179	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	031	055	079	102	126	150	174	198	221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	717	741	764	788	811	834	858	881	905	928
186	951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27 184	207	231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
191	28 103	126	149	171	194	217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	026	048	070	092	115	137	159	181	203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	23
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325	345	365	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40 140	157	175	192	209	226	243	261	278	295
253	312	329	346	364	381	398	415	432	449	466
254	483	500	518	535	552	569	586	603	620	637
255	654	671	688	705	722	739	756	773	790	807
256	824	841	858	875	892	909	926	943	960	976
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41 162	179	196	212	229	246	263	280	296	313
259	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
261	664	681	697	714	731	747	764	780	797	814
262	830	847	863	880	896	913	929	946	963	979
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144
264	42 160	177	193	210	226	243	259	275	292	308
265	325	341	357	374	390	406	423	439	455	472
266	488	504	521	537	553	570	586	602	619	635
267	651	667	684	700	716	732	749	765	781	797
268	813	830	846	862	878	894	911	927	943	959
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	521	537	553	569	584	600
273	616	632	648	664	680	696	712	727	743	759
274	775	791	807	823	838	854	870	886	902	917
275	933	949	965	981	996	*012	*028	*044	*059	*075
276	44 091	107	122	138	154	170	185	201	217	232
277	248	264	279	295	311	326	342	358	373	389
278	404	420	436	451	467	483	498	514	529	545
279	560	576	592	607	623	638	654	669	685	700
280	716	731	747	762	778	793	809	824	840	855
281	871	886	902	917	932	948	963	979	994	*010
282	45 025	040	056	071	086	102	117	133	148	163
283	179	194	209	225	240	255	271	286	301	317
284	332	347	362	378	393	408	423	439	454	469
285	484	500	515	530	545	561	576	591	606	621
286	637	652	667	682	697	712	728	743	758	773
287	788	803	818	834	849	864	879	894	909	924
288	939	954	969	984	*000	*015	*030	*045	*060	*075
289	46 090	105	120	135	150	165	180	195	210	225
290	240	255	270	285	300	315	330	345	359	374
291	389	404	419	434	449	464	479	494	509	523
292	538	553	568	583	598	613	627	642	657	672
293	687	702	716	731	746	761	776	790	805	820
294	835	850	864	879	894	909	923	938	953	967
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47 129	144	159	173	188	202	217	232	246	261
297	276	290	305	319	334	349	363	378	392	407
298	422	436	451	465	480	494	509	524	538	553
299	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	18	17	16	15	14
1	1.8	1.7	1.6	1.5	1.4
2	3.6	3.4	3.2	3.0	2 8
3	5.4	5.1	4.8	4.5	4.2
4	7.2	6.8	6.4	6.0	5.6
5	9.0	8.5	8.0	7.5	7.0
6	10.8	10.2	9.6	9.0	8.4
7	12.6	11.9	11.2	10.5	9.8
8	14.4	13.6	12.8	12.0	11.2
9	16.2	15.3	14.4	13.5	12.6

N.	L. 0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
301	857	871	885	900	914	929	943	958	972	986
302	48 001	015	029	044	058	073	087	101	116	130
303	144	159	173	187	202	216	230	244	259	273
304	287	302	316	330	344	359	373	387	401	416
305	430	444	458	473	487	501	515	530	544	558
306	572	586	601	615	629	643	657	671	686	700
307	714	728	742	756	770	785	799	813	827	841
308	855	869	883	897	911	926	940	954	968	982
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
311	276	290	304	318	332	346	360	374	388	402
312	415	429	443	457	471	485	499	513	527	541
313	554	568	582	596	610	624	638	651	665	679
314	693	707	721	734	748	762	776	790	803	817
315	831	845	859	872	886	900	914	927	941	955
316	969	982	996	*010	*024	*037	*051	*065	*079	*092
317	50 106	120	133	147	161	174	188	202	215	229
318	243	256	270	284	297	311	325	338	352	365
319	379	393	406	420	433	447	461	474	488	501
320	515	529	542	556	569	583	596	610	623	637
321	651	664	678	691	705	718	732	745	759	772
322	786	799	813	826	840	853	866	880	893	907
323	920	934	947	961	974	987	*001	*014	*028	*041
324	51 055	068	081	095	108	121	135	148	162	175
325	188	202	215	228	242	255	268	282	295	308
326	322	335	348	362	375	388	402	415	428	441
327	455	468	481	495	508	521	534	548	561	574
328	587	601	614	627	640	654	667	680	693	706
329	720	733	746	759	772	786	799	812	825	838
330	851	865	878	891	904	917	930	943	957	970
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101
332	52 114	127	140	153	166	179	192	205	218	231
333	244	257	270	284	297	310	323	336	349	362
334	375	388	401	414	427	440	453	466	479	492
335	504	517	530	543	556	569	582	595	608	621
336	634	647	660	673	686	699	711	724	737	750
337	763	776	789	802	815	827	840	853	866	879
338	892	905	917	930	943	956	969	982	994	*007
339	53 020	033	046	058	071	084	097	110	122	135
340	148	161	173	186	199	212	224	237	250	263
341	275	288	301	314	326	339	352	364	377	390
342	403	415	428	441	453	466	479	491	504	517
343	529	542	555	567	580	593	605	618	631	643
344	656	668	681	694	706	719	732	744	757	769
345	782	794	807	820	832	845	857	870	882	895
346	908	920	933	945	958	970	983	995	*008	*020
347	54 033	045	058	070	083	095	108	120	133	145
348	158	170	183	195	208	220	233	245	258	270
349	283	295	307	320	332	345	357	370	382	394
350	407	419	432	444	456	469	481	494	506	518
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	15	14	13	12
1	1.5	1.4	1.3	1.2
2	3.0	2.8	2.6	2.4
3	4.5	4.2	3.9	3.6
4	6.0	5.6	5 2	4.8
5	7.5	7.0	6.5	6.0
6	9.0	8.4	7.8	7.2
7	10.5	9.8	9.1	8.4
8	12.0	11.2	10.4	9.6
9	13.5	12.6	11.7	10.8

N.	L. 0	1	2	3	4	5	6	7	8	9
350	54 407	419	432	444	456	469	481	494	506	518
351	531	543	555	568	580	593	605	617	630	642
352	654	667	679	691	704	716	728	741	753	765
353	777	790	802	814	827	839	851	864	876	888
354	900	913	925	937	949	962	974	986	998	*011
355	55 023	035	047	060	072	084	096	108	121	133
356	145	157	169	182	194	206	218	230	242	255
357	267	279	291	303	315	328	340	352	364	376
358	388	400	413	425	437	449	461	473	485	497
359	509	522	534	546	558	570	582	594	606	618
360	630	642	654	666	678	691	703	715	727	739
361	751	763	775	787	799	811	823	835	847	859
362	871	883	895	907	919	931	943	955	967	979
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098
364	56 110	122	134	146	158	170	182	194	205	217
365	229	241	253	265	277	289	301	312	324	336
366	348	360	372	384	396	407	419	431	443	455
367	467	478	490	502	514	526	538	549	561	573
368	585	597	608	620	632	644	656	667	679	691
369	703	714	726	738	750	761	773	785	797	808
370	820	832	844	855	867	879	891	902	914	926
371	937	949	961	972	984	996	*008	*019	*031	*043
372	57 054	066	078	089	101	113	124	136	148	159
373	171	183	194	206	217	229	241	252	264	276
374	287	299	310	322	334	345	357	368	380	392
375	403	415	426	438	449	461	473	484	496	507
376	519	530	542	553	565	576	588	600	611	623
377	634	646	657	669	680	692	703	715	726	738
378	749	761	772	784	795	807	818	830	841	852
379	864	875	887	898	910	921	933	944	955	967
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58 092	104	115	127	138	149	161	172	184	195
382	206	218	229	240	252	263	274	286	297	309
383	320	331	343	354	365	377	388	399	410	422
384	433	444	456	467	478	490	501	512	524	535
385	546	557	569	580	591	602	614	625	636	647
386	659	670	681	692	704	715	726	737	749	760
387	771	782	794	805	816	827	838	850	861	872
388	883	894	906	917	928	939	950	961	973	984
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59 106	118	129	140	151	162	173	184	195	207
391	218	229	240	251	262	273	284	295	306	318
392	329	340	351	362	373	384	395	406	417	428
393	439	450	461	472	483	494	506	517	528	539
394	550	561	572	583	594	605	616	627	638	649
395	660	671	682	693	704	715	726	737	748	759
396	770	780	791	802	813	824	835	846	857	868
397	879	890	901	912	923	934	945	956	966	977
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60 097	108	119	130	141	152	163	173	184	195
400	206	217	228	239	249	260	271	282	293	304
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	13	12	11	10
1	1.3	1.2	1.1	1.0
2	2.6	2.4	2.2	2.0
3	3.9	3.6	3.3	3.0
4	5.2	4.8	4.4	4.0
5	6.5	6.0	5.5	5.0
6	7.8	7.2	6.6	6.0
7	9.1	8.4	7.7	7.0
8	10.4	9.6	8.8	8.0
9	11.7	10.8	9.9	9.0

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
400	60 206	217	228	239	249	260	271	282	293	304	
401	314	325	336	347	358	369	379	390	401	412	
402	423	433	444	455	466	477	487	498	509	520	
403	531	541	552	563	574	584	595	606	617	627	
404	638	649	660	670	681	692	703	713	724	735	
405	746	756	767	778	788	799	810	821	831	842	
406	853	863	874	885	895	906	917	927	938	949	
407	959	970	981	991	*002	*013	*023	*034	*045	*055	
408	61 066	077	087	098	109	119	130	140	151	162	
409	172	183	194	204	215	225	236	247	257	268	
410	278	289	300	310	321	331	342	352	363	374	
411	384	395	405	416	426	437	448	458	469	479	
412	490	500	511	521	532	542	553	563	574	584	
413	595	606	616	627	637	648	658	669	679	690	
414	700	711	721	731	742	752	763	773	784	794	
415	805	815	826	836	847	857	868	878	888	899	
416	909	920	930	941	951	962	972	982	993	*003	
417	62 014	024	034	045	055	066	076	086	097	107	
418	118	128	138	149	159	170	180	190	201	211	
419	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
421	428	439	449	459	469	480	490	500	511	521	
422	531	542	552	562	572	583	593	603	613	624	
423	634	644	655	665	675	685	696	706	716	726	
424	737	747	757	767	778	788	798	808	818	829	
425	839	849	859	870	880	890	900	910	921	931	
426	941	951	961	972	982	992	*002	*012	*022	*033	
427	63 043	053	063	073	083	094	104	114	124	134	
428	144	155	165	175	185	195	205	215	225	236	
429	246	256	266	276	286	296	306	317	327	337	
430	347	357	367	377	387	397	407	417	428	438	
431	448	458	468	478	488	498	508	518	528	538	
432	548	558	568	579	589	599	609	619	629	639	
433	649	659	669	679	689	699	709	719	729	739	
434	749	759	769	779	789	799	809	819	829	839	
435	849	859	869	879	889	899	909	919	929	939	
436	949	959	969	979	988	998	*008	*018	*028	*038	
437	64 048	058	068	078	088	098	108	118	128	137	
438	147	157	167	177	187	197	207	217	227	237	
439	246	256	266	276	286	296	306	316	326	335	
440	345	355	365	375	385	395	404	414	424	434	
441	444	454	464	473	483	493	503	513	523	532	
442	542	552	562	572	582	591	601	611	621	631	
443	640	650	660	670	680	689	699	709	719	729	
444	738	748	758	768	777	787	797	807	816	826	
445	836	846	856	865	875	885	895	904	914	924	
446	933	943	953	963	972	982	992	*002	*011	*021	
447	65 031	040	050	060	070	079	089	099	108	118	
448	128	137	147	157	167	176	186	196	205	215	
449	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

P. P.

	11
1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

N.	L. 0	1	2	3	4	5	6	7	8	9
450	65 321	331	341	350	360	369	379	389	398	408
451	418	427	437	447	456	466	475	485	495	504
452	514	523	533	543	552	562	571	581	591	600
453	610	619	629	639	648	658	667	677	686	696
454	706	715	725	734	744	753	763	772	782	792
455	801	811	820	830	839	849	858	868	877	887
456	896	906	916	925	935	944	954	963	973	982
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077
458	66 087	096	106	115	124	134	143	153	162	172
459	181	191	200	210	219	229	238	247	257	266
460	276	285	295	304	314	323	332	342	351	361
461	370	380	389	398	408	417	427	436	445	455
462	464	474	483	492	502	511	521	530	539	549
463	558	567	577	586	596	605	614	624	633	642
464	652	661	671	680	689	699	708	717	727	736
465	745	755	764	773	783	792	801	811	820	829
466	839	848	857	867	876	885	894	904	913	922
467	932	941	950	960	969	978	987	997	*006	*015
468	67 025	034	043	052	062	071	080	089	099	108
469	117	127	136	145	154	164	173	182	191	201
470	210	219	228	237	247	256	265	274	284	293
471	302	311	321	330	339	348	357	367	376	385
472	394	403	413	422	431	440	449	459	468	477
473	486	495	504	514	523	532	541	550	560	569
474	578	587	596	605	614	624	633	642	651	660
475	669	679	688	697	706	715	724	733	742	752
476	761	770	779	788	797	806	815	825	834	843
477	852	861	870	879	888	897	906	916	925	934
478	943	952	961	970	979	988	997	*006	*015	*024
479	68 034	043	052	061	070	079	088	097	106	115
480	124	133	142	151	160	169	178	187	196	205
481	215	224	233	242	251	260	269	278	287	296
482	305	314	323	332	341	350	359	368	377	386
483	395	404	413	422	431	440	449	458	467	476
484	485	494	502	511	520	529	538	547	556	565
485	574	583	592	601	610	619	628	637	646	655
486	664	673	681	690	699	708	717	726	735	744
487	753	762	771	780	789	797	806	815	824	833
488	842	851	860	869	878	886	895	904	913	922
489	931	940	949	958	966	975	984	993	*002	*011
490	69 020	028	037	046	055	064	073	082	090	099
491	108	117	126	135	144	152	161	170	179	188
492	197	205	214	223	232	241	249	258	267	276
493	285	294	302	311	320	329	338	346	355	364
494	373	381	390	399	408	417	425	434	443	452
495	461	469	478	487	496	504	513	522	531	539
496	548	557	566	574	583	592	601	609	618	627
497	636	644	653	662	671	679	688	697	705	714
498	723	732	740	749	758	767	775	784	793	801
499	810	819	827	836	845	854	862	871	880	888
500	897	906	914	923	932	940	949	958	966	975
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P. P.

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9	7.2

P. P.

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500	69 897	906	914	923	932	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	157	165	174	183	191	200	209	217	226	234	
504	243	252	260	269	278	286	295	303	312	321	
505	329	338	346	355	364	372	381	389	398	406	
506	415	424	432	441	449	458	467	475	484	492	
507	501	509	518	526	535	544	552	561	569	578	**9** 1 0.9 2 1.8 3 2.7 4 3.6 5 4.5 6 5.4 7 6.3 8 7.2 9 8.1
508	586	595	603	612	621	629	638	646	655	663	
509	672	680	689	697	706	714	723	731	740	749	
510	757	766	774	783	791	800	808	817	825	834	
511	842	851	859	868	876	885	893	902	910	919	
512	927	935	944	952	961	969	978	986	995	*003	
513	71 012	020	029	037	046	054	063	071	079	088	
514	096	105	113	122	130	139	147	155	164	172	
515	181	189	198	206	214	223	231	240	248	257	
516	265	273	282	290	299	307	315	324	332	341	
517	349	357	366	374	383	391	399	408	416	425	
518	433	441	450	458	466	475	483	492	500	508	
519	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
521	684	692	700	709	717	725	734	742	750	759	**8** 1 0.8 2 1.6 3 2.4 4 3.2 5 4.0 6 4.8 7 5.6 8 6.4 9 7.2
522	767	775	784	792	800	809	817	825	834	842	
523	850	858	867	875	883	892	900	908	917	925	
524	933	941	950	958	966	975	983	991	999	*008	
525	72 016	024	032	041	049	057	066	074	082	090	
526	099	107	115	123	132	140	148	156	165	173	
527	181	189	198	206	214	222	230	239	247	255	
528	263	272	280	288	296	304	313	321	329	337	
529	346	354	362	370	378	387	395	403	411	419	
530	428	436	444	452	460	469	477	485	493	501	
531	509	518	526	534	542	550	558	567	575	583	
532	591	599	607	616	624	632	640	648	656	665	
533	673	681	689	697	705	713	722	730	738	746	
534	754	762	770	779	787	795	803	811	819	827	
535	835	843	852	860	868	876	884	892	900	908	
536	916	925	933	941	949	957	965	973	981	989	**7** 1 0.7 2 1.4 3 2.1 4 2.8 5 3.5 6 4.2 7 4.9 8 5 6 9 6.3
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
538	73 078	086	094	102	111	119	127	135	143	151	
539	159	167	175	183	191	199	207	215	223	231	
540	239	247	255	263	272	280	288	296	304	312	
541	320	328	336	344	352	360	368	376	384	392	
542	400	408	416	424	432	440	448	456	464	472	
543	480	488	496	504	512	520	528	536	544	552	
544	560	568	576	584	592	600	608	616	624	632	
545	640	648	656	664	672	679	687	695	703	711	
546	719	727	735	743	751	759	767	775	783	791	
547	799	807	815	823	830	838	846	854	862	870	
548	878	886	894	902	910	918	926	933	941	949	
549	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
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550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	8
561	896	904	912	920	927	935	943	950	958	966	1 0.8
562	974	981	989	997	*005	*012	*020	*028	*035	*043	2 1.6
563	75 051	059	066	074	082	089	097	105	113	120	3 2.4
564	128	136	143	151	159	166	174	182	189	197	4 3.2
565	205	213	220	228	236	243	251	259	266	274	5 4.0
566	282	289	297	305	312	320	328	335	343	351	6 4.8
567	358	366	374	381	389	397	404	412	420	427	7 5.6
568	435	442	450	458	465	473	481	488	496	504	8 6.4
569	511	519	526	534	542	549	557	565	572	580	9 7.2
570	587	595	603	610	618	626	633	641	648	656	
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	7
581	418	425	433	440	448	455	462	470	477	485	1 0.7
582	492	500	507	515	522	530	537	545	552	559	2 1.4
583	567	574	582	589	597	604	612	619	626	634	3 2.1
584	641	649	656	664	671	678	686	693	701	708	4 2.8
585	716	723	730	738	745	753	760	768	775	782	5 3.5
586	790	797	805	812	819	827	834	842	849	856	6 4.2
587	864	871	879	886	893	901	908	916	923	930	7 4.9
588	938	945	953	960	967	975	982	989	997	*004	8 5.6
589	77 012	019	026	034	041	048	056	063	070	078	9 6.3
590	085	093	100	107	115	122	129	137	144	151	
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
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N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
600	77 815	822	830	837	844	851	859	866	873	880
601	887	895	902	909	916	924	931	938	945	952
602	960	967	974	981	988	996	*003	*010	*017	*025
603	78 032	039	046	053	061	068	075	082	089	097
604	104	111	118	125	132	140	147	154	161	168
605	176	183	190	197	204	211	219	226	233	240
606	247	254	262	269	276	283	290	297	305	312
607	319	326	333	340	347	355	362	369	376	383
608	390	398	405	412	419	426	433	440	447	455
609	462	469	476	483	490	497	504	512	519	526
610	533	540	547	554	561	569	576	583	590	597
611	604	611	618	625	633	640	647	654	661	668
612	675	682	689	696	704	711	718	725	732	739
613	746	753	760	767	774	781	789	796	803	810
614	817	824	831	838	845	852	859	866	873	880
615	888	895	902	909	916	923	930	937	944	951
616	958	965	972	979	986	993	*000	*007	*014	*021
617	79 029	036	043	050	057	064	071	078	085	092
618	099	106	113	120	127	134	141	148	155	162
619	169	176	183	190	197	204	211	218	225	232
620	239	246	253	260	267	274	281	288	295	302
621	309	316	323	330	337	344	351	358	365	372
622	379	386	393	400	407	414	421	428	435	442
623	449	456	463	470	477	484	491	498	505	511
624	518	525	532	539	546	553	560	567	574	581
625	588	595	602	609	616	623	630	637	644	650
626	657	664	671	678	685	692	699	706	713	720
627	727	734	741	748	754	761	768	775	782	789
628	796	803	810	817	824	831	837	844	851	858
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630	934	941	948	955	962	969	975	982	989	996
631	80 003	010	017	024	030	037	044	051	058	065
632	072	079	085	092	099	106	113	120	127	134
633	140	147	154	161	168	175	182	188	195	202
634	209	216	223	229	236	243	250	257	264	271
635	277	284	291	298	305	312	318	325	332	339
636	346	353	359	366	373	380	387	393	400	407
637	414	421	428	434	441	448	455	462	468	475
638	482	489	496	502	509	516	523	530	536	543
639	550	557	564	570	577	584	591	598	604	611
640	618	625	632	638	645	652	659	665	672	679
641	686	693	699	706	713	720	726	733	740	747
642	754	760	767	774	781	787	794	801	808	814
643	821	828	835	841	848	855	862	868	875	882
644	889	895	902	909	916	922	929	936	943	949
645	956	963	969	976	983	990	996	*003	*010	*017
646	81 023	030	037	043	050	057	064	070	077	084
647	090	097	104	111	117	124	131	137	144	151
648	158	164	171	178	184	191	198	204	211	218
649	224	231	238	245	251	258	265	271	278	285
650	291	298	305	311	318	325	331	338	345	351
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P. P.

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P. P.

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650	81 291	298	305	311	318	325	331	338	345	351	
651	358	365	371	378	385	391	398	405	411	418	
652	425	431	438	445	451	458	465	471	478	485	
653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
658	823	829	836	842	849	856	862	869	875	882	
659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	**7**
661	82 020	027	033	040	046	053	060	066	073	079	1 | 0.7
662	086	092	099	105	112	119	125	132	138	145	2 | 1.4
663	151	158	164	171	178	184	191	197	204	210	3 | 2.1
664	217	223	230	236	243	249	256	263	269	276	4 | 2.8
665	282	289	295	302	308	315	321	328	334	341	5 | 3.5
666	347	354	360	367	373	380	387	393	400	406	6 | 4.2
667	413	419	426	432	439	445	452	458	465	471	7 | 4.9
668	478	484	491	497	504	510	517	523	530	536	8 | 5.6
669	543	549	556	562	569	575	582	588	595	601	9 | 6.3
670	607	614	620	627	633	640	646	653	659	666	
671	672	679	685	692	698	705	711	718	724	730	
672	737	743	750	756	763	769	776	782	789	795	
673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	**6**
681	315	321	327	334	340	347	353	359	366	372	1 | 0.6
682	378	385	391	398	404	410	417	423	429	436	2 | 1.2
683	442	448	455	461	467	474	480	487	493	499	3 | 1.8
684	506	512	518	525	531	537	544	550	556	563	4 | 2.4
685	569	575	582	588	594	601	607	613	620	626	5 | 3.0
686	632	639	645	651	658	664	670	677	683	689	6 | 3.6
687	696	702	708	715	721	727	734	740	746	753	7 | 4.2
688	759	765	771	778	784	790	797	803	809	816	8 | 4.8
689	822	828	835	841	847	853	860	866	872	879	9 | 5.4
690	885	891	897	904	910	916	923	929	935	942	
691	948	954	960	967	973	979	985	992	998	*004	
692	84 011	017	023	029	036	042	048	055	061	067	
693	073	080	086	092	098	105	111	117	123	130	
694	136	142	148	155	161	167	173	180	186	192	
695	198	205	211	217	223	230	236	242	248	255	
696	261	267	273	280	286	292	298	305	311	317	
697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
699	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
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700	84 510	516	522	528	535	541	547	553	559	566
701	572	578	584	590	597	603	609	615	621	628
702	634	640	646	652	658	665	671	677	683	689
703	696	702	708	714	720	726	733	739	745	751
704	757	763	770	776	782	788	794	800	807	813
705	819	825	831	837	844	850	856	862	868	874
706	880	887	893	899	905	911	917	924	930	936
707	942	948	954	960	967	973	979	985	991	997
708	85 003	009	016	022	028	034	040	046	052	058
709	065	071	077	083	089	095	101	107	114	120
710	126	132	138	144	150	156	163	169	175	181
711	187	193	199	205	211	217	224	230	236	242
712	248	254	260	266	272	278	285	291	297	303
713	309	315	321	327	333	339	345	352	358	364
714	370	376	382	388	394	400	406	412	418	425
715	431	437	443	449	455	461	467	473	479	485
716	491	497	503	509	516	522	528	534	540	546
717	552	558	564	570	576	582	588	594	600	606
718	612	618	625	631	637	643	649	655	661	667
719	673	679	685	691	697	703	709	715	721	727
720	733	739	745	751	757	763	769	775	781	788
721	794	800	806	812	818	824	830	836	842	848
722	854	860	866	872	878	884	890	896	902	908
723	914	920	926	932	938	944	950	956	962	968
724	974	980	986	992	998	*004	*010	*016	*022	*028
725	86 034	040	046	052	058	064	070	076	082	088
726	094	100	106	112	118	124	130	136	141	147
727	153	159	165	171	177	183	189	195	201	207
728	213	219	225	231	237	243	249	255	261	267
729	273	279	285	291	297	303	308	314	320	326
730	332	338	344	350	356	362	368	374	380	386
731	392	398	404	410	415	421	427	433	439	445
732	451	457	463	469	475	481	487	493	499	504
733	510	516	522	528	534	540	546	552	558	564
734	570	576	581	587	593	599	605	611	617	623
735	629	635	641	646	652	658	664	670	676	682
736	688	694	700	705	711	717	723	729	735	741
737	747	753	759	764	770	776	782	788	794	800
738	806	812	817	823	829	835	841	847	853	859
739	864	870	876	882	888	894	900	906	911	917
740	923	929	935	941	947	953	958	964	970	976
741	982	988	994	999	*005	*011	*017	*023	*029	*035
742	87 040	046	052	058	064	070	075	081	087	093
743	099	105	111	116	122	128	134	140	146	151
744	157	163	169	175	181	186	192	198	204	210
745	216	221	227	233	239	245	251	256	262	268
746	274	280	286	291	297	303	309	315	320	326
747	332	338	344	349	355	361	367	373	379	384
748	390	396	402	408	413	419	425	431	437	442
749	448	454	460	466	471	477	483	489	495	500
750	506	512	518	523	529	535	541	547	552	558
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P. P.

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P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
750	87 506	512	518	523	529	535	541	547	552	558
751	564	570	576	581	587	593	599	604	610	616
752	622	628	633	639	645	651	656	662	668	674
753	679	685	691	697	703	708	714	720	726	731
754	737	743	749	754	760	766	772	777	783	789
755	795	800	806	812	818	823	829	835	841	846
756	852	858	864	869	875	881	887	892	898	904
757	910	915	921	927	933	938	944	950	955	961
758	967	973	978	984	990	996	*001	*007	*013	*018
759	88 024	030	036	041	047	053	058	064	070	076
760	081	087	093	098	104	110	116	121	127	133
761	138	144	150	156	161	167	173	178	184	190
762	195	201	207	213	218	224	230	235	241	247
763	252	258	264	270	275	281	287	292	298	304
764	309	315	321	326	332	338	343	349	355	360
765	366	372	377	383	389	395	400	406	412	417
766	423	429	434	440	446	451	457	463	468	474
767	480	485	491	497	502	508	513	519	525	530
768	536	542	547	553	559	564	570	576	581	587
769	593	598	604	610	615	621	627	632	638	643
770	649	655	660	666	672	677	683	689	694	700
771	705	711	717	722	728	734	739	745	750	756
772	762	767	773	779	784	790	795	801	807	812
773	818	824	829	835	840	846	852	857	863	868
774	874	880	885	891	897	902	908	913	919	925
775	930	936	941	947	953	958	964	969	975	981
776	986	992	997	*003	*009	*014	*020	*025	*031	*037
777	89 042	048	053	059	064	070	076	081	087	092
778	098	104	109	115	120	126	131	137	143	148
779	154	159	165	170	176	182	187	193	198	204
780	209	215	221	226	232	237	243	248	254	260
781	265	271	276	282	287	293	298	304	310	315
782	321	326	332	337	343	348	354	360	365	371
783	376	382	387	393	398	404	409	415	421	426
784	432	437	443	448	454	459	465	470	476	481
785	487	492	498	504	509	515	520	526	531	537
786	542	548	553	559	564	570	575	581	586	592
787	597	603	609	614	620	625	631	636	642	647
788	653	658	664	669	675	680	686	691	697	702
789	708	713	719	724	730	735	741	746	752	757
790	763	768	774	779	785	790	796	801	807	812
791	818	823	829	834	840	845	851	856	862	867
792	873	878	883	889	894	900	905	911	916	922
793	927	933	938	944	949	955	960	966	971	977
794	982	988	993	998	*004	*009	*015	*020	*026	*031
795	90 037	042	048	053	059	064	069	075	080	086
796	091	097	102	108	113	119	124	129	135	140
797	146	151	157	162	168	173	179	184	189	195
798	200	206	211	217	222	227	233	238	244	249
799	255	260	266	271	276	282	287	293	298	304
800	309	314	320	325	331	336	342	347	352	358
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
800	90 309	314	320	325	331	336	342	347	352	358
801	363	369	374	380	385	390	396	401	407	412
802	417	423	428	434	439	445	450	455	461	466
803	472	477	482	488	493	499	504	509	515	520
804	526	531	536	542	547	553	558	563	569	574
805	580	585	590	596	601	607	612	617	623	628
806	634	639	644	650	655	660	666	671	677	682
807	687	693	698	703	709	714	720	725	730	736
808	741	747	752	757	763	768	773	779	784	789
809	795	800	806	811	816	822	827	832	838	843
810	849	854	859	865	870	875	881	886	891	897
811	902	907	913	918	924	929	934	940	945	950
812	956	961	966	972	977	982	988	993	998	*004
813	91 009	014	020	025	030	036	041	046	052	057
814	062	068	073	078	084	089	094	100	105	110
815	116	121	126	132	137	142	148	153	158	164
816	169	174	180	185	190	196	201	206	212	217
817	222	228	233	238	243	249	254	259	265	270
818	275	281	286	291	297	302	307	312	318	323
819	328	334	339	344	350	355	360	365	371	376
820	381	387	392	397	403	408	413	418	424	429
821	434	440	445	450	455	461	466	471	477	482
822	487	492	498	503	508	514	519	524	529	535
823	540	545	551	556	561	566	572	577	582	587
824	593	598	603	609	614	619	624	630	635	640
825	645	651	656	661	666	672	677	682	687	693
826	698	703	709	714	719	724	730	735	740	745
827	751	756	761	766	772	777	782	787	793	798
828	803	808	814	819	824	829	834	840	845	850
829	855	861	866	871	876	882	887	892	897	903
830	908	913	918	924	929	934	939	944	950	955
831	960	965	971	976	981	986	991	997	*002	*007
832	92 012	018	023	028	033	038	044	049	054	059
833	065	070	075	080	085	091	096	101	106	111
834	117	122	127	132	137	143	148	153	158	163
835	169	174	179	184	189	195	200	205	210	215
836	221	226	231	236	241	247	252	257	262	267
837	273	278	283	288	293	298	304	309	314	319
838	324	330	335	340	345	350	355	361	366	371
839	376	381	387	392	397	402	407	412	418	423
840	428	433	438	443	449	454	459	464	469	474
841	480	485	490	495	500	505	511	516	521	526
842	531	536	542	547	552	557	562	567	572	578
843	583	588	593	598	603	609	614	619	624	629
844	634	639	645	650	655	660	665	670	675	681
845	686	691	696	701	706	711	716	722	727	732
846	737	742	747	752	758	763	768	773	778	783
847	788	793	799	804	809	814	819	824	829	834
848	840	845	850	855	860	865	870	875	881	886
849	891	896	901	906	911	916	921	927	932	937
850	942	947	952	957	962	967	973	978	983	988
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	6
1	0.6
2	1.2
3	1.8
4	2.4
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9	5.4

	5
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3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P.P.
850	92 942	947	952	957	962	967	973	978	983	988	
851	993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93 044	049	054	059	064	069	075	080	085	090	
853	095	100	105	110	115	120	125	131	136	141	
854	146	151	156	161	166	171	176	181	186	192	
855	197	202	207	212	217	222	227	232	237	242	
856	247	252	258	263	268	273	278	283	288	293	
857	298	303	308	313	318	323	328	334	339	344	**6**
858	349	354	359	364	369	374	379	384	389	394	1 0.6
859	399	404	409	414	420	425	430	435	440	445	2 1.2
860	450	455	460	465	470	475	480	485	490	495	3 1.8 4 2.4
861	500	505	510	515	520	526	531	536	541	546	5 3.0
862	551	556	561	566	571	576	581	586	591	596	6 3.6 7 4.2
863	601	606	611	616	621	626	631	636	641	646	8 4.8
864	651	656	661	666	671	676	682	687	692	697	9 5.4
865	702	707	712	717	722	727	732	737	742	747	
866	752	757	762	767	772	777	782	787	792	797	
867	802	807	812	817	822	827	832	837	842	847	
868	852	857	862	867	872	877	882	887	892	897	
869	902	907	912	917	922	927	932	937	942	947	
870	952	957	962	967	972	977	982	987	992	997	
871	94 002	007	012	017	022	027	032	037	042	047	**5**
872	052	057	062	067	072	077	082	086	091	096	1 0.5
873	101	106	111	116	121	126	131	136	141	146	2 1.0 3 1.5
874	151	156	161	166	171	176	181	186	191	196	4 2.0
875	201	206	211	216	221	226	231	236	240	245	5 2.5
876	250	255	260	265	270	275	280	285	290	295	6 3.0
877	300	305	310	315	320	325	330	335	340	345	7 3.5
878	349	354	359	364	369	374	379	384	389	394	8 4.0
879	399	404	409	414	419	424	429	433	438	443	9 4.5
880	448	453	458	463	468	473	478	483	488	493	
881	498	503	507	512	517	522	527	532	537	542	
882	547	552	557	562	567	571	576	581	586	591	
883	596	601	606	611	616	621	626	630	635	640	
884	645	650	655	660	665	670	675	680	685	689	
885	694	699	704	709	714	719	724	729	734	738	
886	743	748	753	758	763	768	773	778	783	787	**4**
887	792	797	802	807	812	817	822	827	832	836	1 0.4
888	841	846	851	856	861	866	871	876	880	885	2 0.8
889	890	895	900	905	910	915	919	924	929	934	3 1.2 4 1.6
890	939	944	949	954	959	963	968	973	978	983	5 2.0
891	988	993	998	*002	*007	*012	*017	*022	*027	*032	6 2.4 7 2.8
892	95 036	041	046	051	056	061	066	071	075	080	8 3.2
893	085	090	095	100	105	109	114	119	124	129	9 3.6
894	134	139	143	148	153	158	163	168	173	177	
895	182	187	192	197	202	207	211	216	221	226	
896	231	236	240	245	250	255	260	265	270	274	
897	279	284	289	294	299	303	308	313	318	323	
898	328	332	337	342	347	352	357	361	366	371	
899	376	381	386	390	395	400	405	410	415	419	
900	424	429	434	439	444	448	453	458	463	468	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
900	95 424	429	434	439	444	448	453	458	463	468	
901	472	477	482	487	492	497	501	506	511	516	
902	521	525	530	535	540	545	550	554	559	564	
903	569	574	578	583	588	593	598	602	607	612	
904	617	622	626	631	636	641	646	650	655	660	
905	665	670	674	679	684	689	694	698	703	708	
906	713	718	722	727	732	737	742	746	751	756	
907	761	766	770	775	780	785	789	794	799	804	
908	809	813	818	823	828	832	837	842	847	852	
909	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	5
911	952	957	961	966	971	976	980	985	990	995	1 | 0.5
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	2 | 1.0
913	96 047	052	057	061	066	071	076	080	085	090	3 | 1.5
914	095	099	104	109	114	118	123	128	133	137	4 | 2.0
915	142	147	152	156	161	166	171	175	180	185	5 | 2.5
916	190	194	199	204	209	213	218	223	227	232	6 | 3.0
917	237	242	246	251	256	261	265	270	275	280	7 | 3.5
918	284	289	294	298	303	308	313	317	322	327	8 | 4.0
919	332	336	341	346	350	355	360	365	369	374	9 | 4.5
920	379	384	388	393	398	402	407	412	417	421	
921	426	431	435	440	445	450	454	459	464	468	
922	473	478	483	487	492	497	501	506	511	515	
923	520	525	530	534	539	544	548	553	558	562	
924	567	572	577	581	586	591	595	600	605	609	
925	614	619	624	628	633	638	642	647	652	656	
926	661	666	670	675	680	685	689	694	699	703	
927	708	713	717	722	727	731	736	741	745	750	
928	755	759	764	769	774	778	783	788	792	797	
929	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	4
931	895	900	904	909	914	918	923	928	932	937	1 | 0.4
932	942	946	951	956	960	965	970	974	979	984	2 | 0.8
933	988	993	997	*002	*007	*011	*016	*021	*025	*030	3 | 1.2
934	97 035	039	044	049	053	058	063	067	072	077	4 | 1.6
935	081	086	090	095	100	104	109	114	118	123	5 | 2.0
936	128	132	137	142	146	151	155	160	165	169	6 | 2.4
937	174	179	183	188	192	197	202	206	211	216	7 | 2.8
938	220	225	230	234	239	243	248	253	257	262	8 | 3.2
939	267	271	276	280	285	290	294	299	304	308	9 | 3.6
940	313	317	322	327	331	336	340	345	350	354	
941	359	364	368	373	377	382	387	391	396	400	
942	405	410	414	419	424	428	433	437	442	447	
943	451	456	460	465	470	474	479	483	488	493	
944	497	502	506	511	516	520	525	529	534	539	
945	543	548	552	557	562	566	571	575	580	585	
946	589	594	598	603	607	612	617	621	626	630	
947	635	640	644	649	653	658	663	667	672	676	
948	681	685	690	695	699	704	708	713	717	722	
949	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
950	97 772	777	782	786	791	795	800	804	809	813	
951	818	823	827	832	836	841	845	850	855	859	
952	864	868	873	877	882	886	891	896	900	905	
953	909	914	918	923	928	932	937	941	946	950	
954	955	959	964	968	973	978	982	987	991	996	
955	98 000	005	009	014	019	023	028	032	037	041	
956	046	050	055	059	064	068	073	078	082	087	
957	091	096	100	105	109	114	118	123	127	132	
958	137	141	146	150	155	159	164	168	173	177	
959	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	**5**
961	272	277	281	286	290	295	299	304	308	313	1 0.5
962	318	322	327	331	336	340	345	349	354	358	2 1.0
963	363	367	372	376	381	385	390	394	399	403	3 1.5
964	408	412	417	421	426	430	435	439	444	448	4 2.0
965	453	457	462	466	471	475	480	484	489	493	5 2.5
966	498	502	507	511	516	520	525	529	534	538	6 3.0
967	543	547	552	556	561	565	570	574	579	583	7 3.5
968	588	592	597	601	605	610	614	619	623	628	8 4.0
969	632	637	641	646	650	655	659	664	668	673	9 4.5
970	677	682	686	691	695	700	704	709	713	717	
971	722	726	731	735	740	744	749	753	758	762	
972	767	771	776	780	784	789	793	798	802	807	
973	811	816	820	825	829	834	838	843	847	851	
974	856	860	865	869	874	878	883	887	892	896	
975	900	905	909	914	918	923	927	932	936	941	
976	945	949	954	958	963	967	972	976	981	985	
977	989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99 034	038	043	047	052	056	061	065	069	074	
979	078	083	087	092	096	100	105	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	**4**
981	167	171	176	180	185	189	193	198	202	207	1 0.4
982	211	216	220	224	229	233	238	242	247	251	2 0.8
983	255	260	264	269	273	277	282	286	291	295	3 1.2
984	300	304	308	313	317	322	326	330	335	339	4 1.6
985	344	348	352	357	361	366	370	374	379	383	5 2.0
986	388	392	396	401	405	410	414	419	423	427	6 2.4
987	432	436	441	445	449	454	458	463	467	471	7 2.8
988	476	480	484	489	493	498	502	506	511	515	8 3.2
989	520	524	528	533	537	542	546	550	555	559	9 3.6
990	564	568	572	577	581	585	590	594	599	603	
991	607	612	616	621	625	629	634	638	642	647	
992	651	656	660	664	669	673	677	682	686	691	
993	695	699	704	708	712	717	721	726	730	734	
994	739	743	747	752	756	760	765	769	774	778	
995	782	787	791	795	800	804	808	813	817	822	
996	826	830	835	839	843	848	852	856	861	865	
997	870	874	878	883	887	891	896	900	904	909	
998	913	917	922	926	930	935	939	944	948	952	
999	957	961	965	970	974	978	983	987	991	996	
1000	00 000	004	009	013	017	022	026	030	035	039	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

CIRCULAR FUNCTIONS

SINES AND COSINES

′	0°		1°		2°		3°		4°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.00000	1.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	1.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	1.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	1.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	1.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	1.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	1.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	1.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	1.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	1.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	1.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	89°		88°		87°		86°		85°		

′	5° Sine	5° Cosine	6° Sine	6° Cosine	7° Sine	7° Cosine	8° Sine	8° Cosine	9° Sine	9° Cosine	′
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	79°		78°		77°		76°		75°		′

′	15°		16°		17°		18°		19°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	74°		73°		72°		71°		70°		′

′	20°		21°		22°		23°		24°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93502	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	69°		68°		67°		66°		65°		

′	25°		26°		27°		28°		29°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	64°		63°		62°		61°		60°		′

′	30°		31°		32°		33°		34°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	59°		58°		57°		56°		55°		′

′	35° Sine	35° Cosine	36° Sine	36° Cosine	37° Sine	37° Cosine	38° Sine	38° Cosine	39° Sine	39° Cosine	′
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	54° Cosine	54° Sine	53° Cosine	53° Sine	52° Cosine	52° Sine	51° Cosine	51° Sine	50° Cosine	50° Sine	′

′	40°		41°		42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	49°		48°		47°		46°		45°		′

TANGENTS AND COTANGENTS

′	0°		1°		2°		3°		4°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infin.	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	.07022	14.2411	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	.07051	14.1821	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	.07080	14.1235	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	.07110	14.0655	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	.07139	14.0079	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	.07168	13.9507	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	.07197	13.8940	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	.07227	13.8378	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	.07256	13.7821	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	.07285	13.7267	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	.07314	13.6719	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	.07344	13.6174	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	.07373	13.5634	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	.07402	13.5098	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	.07431	13.4566	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	.07461	13.4039	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	.07490	13.3515	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	.07519	13.2996	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	.07548	13.2480	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	.07578	13.1969	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	.07607	13.1461	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	.07636	13.0958	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	.07665	13.0458	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	.07695	12.9962	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	.07724	12.9469	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	.07753	12.8981	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	.07782	12.8496	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	.07812	12.8014	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	.07841	12.7536	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	.07870	12.7062	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	.07899	12.6591	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	.07929	12.6124	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	.07958	12.5660	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	.07987	12.5199	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	.08017	12.4742	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	.08046	12.4288	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	.08075	12.3838	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	.08104	12.3390	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	.08134	12.2946	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	.08163	12.2505	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	.08192	12.2067	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	.08221	12.1632	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	.08251	12.1201	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	.08280	12.0772	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	.08309	12.0346	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	.08339	11.9923	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	.08368	11.9504	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	.08397	11.9087	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	.08427	11.8673	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	.08456	11.8262	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	.08485	11.7853	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	.08514	11.7448	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	.08544	11.7045	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	.08573	11.6645	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	.08602	11.6248	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	.08632	11.5853	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	.08661	11.5461	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	.08690	11.5072	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	.08720	11.4685	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	.08749	11.4301	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
′	89°		88°		87°		86°		85°		′

′	5° Tang	5° Cotang	6° Tang	6° Cotang	7° Tang	7° Cotang	8° Tang	8° Cotang	9° Tang	9° Cotang	′
0	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	60
1	.08778	11.3919	.10540	9.48781	.12308	8.12481	.14084	7.10038	.15868	6.30189	59
2	.08807	11.3540	.10569	9.46141	.12338	8.10536	.14113	7.08546	.15898	6.29007	58
3	.08837	11.3163	.10599	9.43515	.12367	8.08600	.14143	7.07059	.15928	6.27829	57
4	.08866	11.2789	.10628	9.40904	.12397	8.06674	.14173	7.05579	.15958	6.26655	56
5	.08895	11.2417	.10657	9.38307	.12426	8.04756	.14202	7.04105	.15988	6.25486	55
6	.08925	11.2048	.10687	9.35724	.12456	8.02848	.14232	7.02637	.16017	6.24321	54
7	.08954	11.1681	.10716	9.33155	.12485	8.00948	.14262	7.01174	.16047	6.23160	53
8	.08983	11.1316	.10746	9.30599	.12515	7.99058	.14291	6.99718	.16077	6.22003	52
9	.09013	11.0954	.10775	9.28058	.12544	7.97176	.14321	6.98268	.16107	6.20851	51
10	.09042	11.0594	.10805	9.25530	.12574	7.95302	.14351	6.96823	.16137	6.19703	50
11	.09071	11.0237	.10834	9.23016	.12603	7.93438	.14381	6.95385	.16167	6.18559	49
12	.09101	10.9882	.10863	9.20516	.12633	7.91582	.14410	6.93952	.16196	6.17419	48
13	.09130	10.9529	.10893	9.18028	.12662	7.89734	.14440	6.92525	.16226	6.16283	47
14	.09159	10.9178	.10922	9.15554	.12692	7.87895	.14470	6.91104	.16256	6.15151	46
15	.09189	10.8829	.10952	9.13093	.12722	7.86064	.14499	6.89688	.16286	6.14023	45
16	.09218	10.8483	.10981	9.10646	.12751	7.84242	.14529	6.88278	.16316	6.12899	44
17	.09247	10.8139	.11011	9.08211	.12781	7.82428	.14559	6.86874	.16346	6.11779	43
18	.09277	10.7797	.11040	9.05789	.12810	7.80622	.14588	6.85475	.16376	6.10664	42
19	.09306	10.7457	.11070	9.03379	.12840	7.78825	.14618	6.84082	.16405	6.09552	41
20	.09335	10.7119	.11099	9.00983	.12869	7.77035	.14648	6.82694	.16435	6.08444	40
21	.09365	10.6783	.11128	8.98598	.12899	7.75254	.14678	6.81312	.16465	6.07340	39
22	.09394	10.6450	.11158	8.96227	.12929	7.73480	.14707	6.79936	.16495	6.06240	38
23	.09423	10.6118	.11187	8.93867	.12958	7.71715	.14737	6.78564	.16525	6.05143	37
24	.09453	10.5789	.11217	8.91520	.12988	7.69957	.14767	6.77199	.16555	6.04051	36
25	.09482	10.5462	.11246	8.89185	.13017	7.68208	.14796	6.75838	.16585	6.02962	35
26	.09511	10.5136	.11276	8.86862	.13047	7.66466	.14826	6.74483	.16615	6.01878	34
27	.09541	10.4813	.11305	8.84551	.13076	7.64732	.14856	6.73133	.16645	6.00797	33
28	.09570	10.4491	.11335	8.82252	.13106	7.63005	.14886	6.71789	.16674	5.99720	32
29	.09600	10.4172	.11364	8.79964	.13136	7.61287	.14915	6.70450	.16704	5.98646	31
30	.09629	10.3854	.11394	8.77689	.13165	7.59575	.14945	6.69116	.16734	5.97576	30
31	.09658	10.3538	.11423	8.75425	.13195	7.57872	.14975	6.67787	.16764	5.96510	29
32	.09688	10.3224	.11452	8.73172	.13224	7.56176	.15005	6.66463	.16794	5.95448	28
33	.09717	10.2913	.11482	8.70931	.13254	7.54487	.15034	6.65144	.16824	5.94390	27
34	.09746	10.2602	.11511	8.68701	.13284	7.52806	.15064	6.63831	.16854	5.93335	26
35	.09776	10.2294	.11541	8.66482	.13313	7.51132	.15094	6.62523	.16884	5.92283	25
36	.09805	10.1988	.11570	8.64275	.13343	7.49465	.15124	6.61219	.16914	5.91236	24
37	.09834	10.1683	.11600	8.62078	.13372	7.47806	.15153	6.59921	.16944	5.90191	23
38	.09864	10.1381	.11629	8.59893	.13402	7.46154	.15183	6.58627	.16974	5.89151	22
39	.09893	10.1080	.11659	8.57718	.13432	7.44509	.15213	6.57339	.17004	5.88114	21
40	.09923	10.0780	.11688	8.55555	.13461	7.42871	.15243	6.56055	.17033	5.87080	20
41	.09952	10.0483	.11718	8.53402	.13491	7.41240	.15272	6.54777	.17063	5.86051	19
42	.09981	10.0187	.11747	8.51259	.13521	7.39616	.15302	6.53503	.17093	5.85024	18
43	.10011	9.98931	.11777	8.49128	.13550	7.37999	.15332	6.52234	.17123	5.84001	17
44	.10040	9.96007	.11806	8.47007	.13580	7.36389	.15362	6.50970	.17153	5.82982	16
45	.10069	9.93101	.11836	8.44896	.13609	7.34786	.15391	6.49710	.17183	5.81966	15
46	.10099	9.90211	.11865	8.42795	.13639	7.33190	.15421	6.48456	.17213	5.80953	14
47	.10128	9.87338	.11895	8.40705	.13669	7.31600	.15451	6.47206	.17243	5.79944	13
48	.10158	9.84482	.11924	8.38625	.13698	7.30018	.15481	6.45961	.17273	5.78938	12
49	.10187	9.81641	.11954	8.36555	.13728	7.28442	.15511	6.44720	.17303	5.77936	11
50	.10216	9.78817	.11983	8.34496	.13758	7.26873	.15540	6.43484	.17333	5.76937	10
51	.10246	9.76009	.12013	8.32446	.13787	7.25310	.15570	6.42253	.17363	5.75941	9
52	.10275	9.73217	.12042	8.30406	.13817	7.23754	.15600	6.41026	.17393	5.74949	8
53	.10305	9.70441	.12072	8.28376	.13846	7.22204	.15630	6.39804	.17423	5.73960	7
54	.10334	9.67680	.12101	8.26355	.13876	7.20661	.15660	6.38587	.17453	5.72974	6
55	.10363	9.64935	.12131	8.24345	.13906	7.19125	.15689	6.37374	.17483	5.71992	5
56	.10393	9.62205	.12160	8.22344	.13935	7.17594	.15719	6.36165	.17513	5.71013	4
57	.10422	9.59490	.12190	8.20352	.13965	7.16071	.15749	6.34961	.17543	5.70037	3
58	.10452	9.56791	.12219	8.18370	.13995	7.14553	.15779	6.33761	.17573	5.69064	2
59	.10481	9.54106	.12249	8.16398	.14024	7.13042	.15809	6.32566	.17603	5.68094	1
60	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	.17633	5.67128	0
′	84° Cotang	84° Tang	83° Cotang	83° Tang	82° Cotang	82° Tang	81° Cotang	81° Tang	80° Cotang	80° Tang	′

′	10°		11°		12°		13°		14°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.17633	5.67128	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	60
1	.17663	5.66165	.19468	5.13658	.21286	4.69791	.23117	4.32573	.24964	4.00582	59
2	.17693	5.65205	.19498	5.12862	.21316	4.69121	.23148	4.32001	.24995	4.00086	58
3	.17723	5.64248	.19529	5.12069	.21347	4.68452	.23179	4.31430	.25026	3.99592	57
4	.17753	5.63295	.19559	5.11279	.21377	4.67786	.23209	4.30860	.25056	3.99099	56
5	.17783	5.62344	.19589	5.10490	.21408	4.67121	.23240	4.30291	.25087	3.98607	55
6	.17813	5.61397	.19619	5.09704	.21438	4.66458	.23271	4.29724	.25118	3.98117	54
7	.17843	5.60452	.19649	5.08921	.21469	4.65797	.23301	4.29159	.25149	3.97627	53
8	.17873	5.59511	.19680	5.08139	.21499	4.65138	.23332	4.28595	.25180	3.97139	52
9	.17903	5.58573	.19710	5.07360	.21529	4.64480	.23363	4.28032	.25211	3.96651	51
10	.17933	5.57638	.19740	5.06584	.21560	4.63825	.23393	4.27471	.25242	3.96165	50
11	.17963	5.56706	.19770	5.05809	.21590	4.63171	.23424	4.26911	.25273	3.95680	49
12	.17993	5.55777	.19801	5.05037	.21621	4.62518	.23455	4.26352	.25304	3.95196	48
13	.18023	5.54851	.19831	5.04267	.21651	4.61868	.23485	4.25795	.25335	3.94713	47
14	.18053	5.53927	.19861	5.03499	.21682	4.61219	.23516	4.25239	.25366	3.94232	46
15	.18083	5.53007	.19891	5.02734	.21712	4.60572	.23547	4.24685	.25397	3.93751	45
16	.18113	5.52090	.19921	5.01971	.21743	4.59927	.23578	4.24132	.25428	3.93271	44
17	.18143	5.51176	.19952	5.01210	.21773	4.59283	.23608	4.23580	.25459	3.92793	43
18	.18173	5.50264	.19982	5.00451	.21804	4.58641	.23639	4.23030	.25490	3.92316	42
19	.18203	5.49356	.20012	4.99695	.21834	4.58001	.23670	4.22481	.25521	3.91839	41
20	.18233	5.48451	.20042	4.98940	.21864	4.57363	.23700	4.21933	.25552	3.91364	40
21	.18263	5.47548	.20073	4.98188	.21895	4.56726	.23731	4.21387	.25583	3.90890	39
22	.18293	5.46648	.20103	4.97438	.21925	4.56091	.23762	4.20842	.25614	3.90417	38
23	.18323	5.45751	.20133	4.96690	.21956	4.55458	.23793	4.20298	.25645	3.89945	37
24	.18353	5.44857	.20164	4.95945	.21986	4.54826	.23823	4.19756	.25676	3.89474	36
25	.18384	5.43966	.20194	4.95201	.22017	4.54196	.23854	4.19215	.25707	3.89004	35
26	.18414	5.43077	.20224	4.94460	.22047	4.53568	.23885	4.18675	.25738	3.88536	34
27	.18444	5.42192	.20254	4.93721	.22078	4.52941	.23916	4.18137	.25769	3.88068	33
28	.18474	5.41309	.20285	4.92984	.22108	4.52316	.23946	4.17600	.25800	3.87601	32
29	.18504	5.40429	.20315	4.92249	.22139	4.51693	.23977	4.17064	.25831	3.87136	31
30	.18534	5.39552	.20345	4.91516	.22169	4.51071	.24008	4.16530	.25862	3.86671	30
31	.18564	5.38677	.20376	4.90785	.22200	4.50451	.24039	4.15997	.25893	3.86208	29
32	.18594	5.37805	.20406	4.90056	.22231	4.49832	.24069	4.15465	.25924	3.85745	28
33	.18624	5.36936	.20436	4.89330	.22261	4.49215	.24100	4.14934	.25955	3.85284	27
34	.18654	5.36070	.20466	4.88605	.22292	4.48600	.24131	4.14405	.25986	3.84824	26
35	.18684	5.35206	.20497	4.87882	.22322	4.47986	.24162	4.13877	.26017	3.84364	25
36	.18714	5.34345	.20527	4.87162	.22353	4.47374	.24193	4.13350	.26048	3.83906	24
37	.18745	5.33487	.20557	4.86444	.22383	4.46764	.24223	4.12825	.26079	3.83449	23
38	.18775	5.32631	.20588	4.85727	.22414	4.46155	.24254	4.12301	.26110	3.82992	22
39	.18805	5.31778	.20618	4.85013	.22444	4.45548	.24285	4.11778	.26141	3.82537	21
40	.18835	5.30928	.20648	4.84300	.22475	4.44942	.24316	4.11256	.26172	3.82083	20
41	.18865	5.30080	.20679	4.83590	.22505	4.44338	.24347	4.10736	.26203	3.81630	19
42	.18895	5.29235	.20709	4.82882	.22536	4.43735	.24377	4.10216	.26235	3.81177	18
43	.18925	5.28393	.20739	4.82175	.22567	4.43134	.24408	4.09699	.26266	3.80726	17
44	.18955	5.27553	.20770	4.81471	.22597	4.42534	.24439	4.09182	.26297	3.80276	16
45	.18986	5.26715	.20800	4.80769	.22628	4.41936	.24470	4.08666	.26328	3.79827	15
46	.19016	5.25880	.20830	4.80068	.22658	4.41340	.24501	4.08152	.26359	3.79378	14
47	.19046	5.25048	.20861	4.79370	.22689	4.40745	.24532	4.07639	.26390	3.78931	13
48	.19076	5.24218	.20891	4.78673	.22719	4.40152	.24562	4.07127	.26421	3.78485	12
49	.19106	5.23391	.20921	4.77978	.22750	4.39560	.24593	4.06616	.26452	3.78040	11
50	.19136	5.22566	.20952	4.77286	.22781	4.38969	.24624	4.06107	.26483	3.77595	10
51	.19166	5.21744	.20982	4.76595	.22811	4.38381	.24655	4.05599	.26515	3.77152	9
52	.19197	5.20925	.21013	4.75906	.22842	4.37793	.24686	4.05092	.26546	3.76709	8
53	.19227	5.20107	.21043	4.75219	.22872	4.37207	.24717	4.04586	.26577	3.76268	7
54	.19257	5.19293	.21073	4.74534	.22903	4.36623	.24747	4.04081	.26608	3.75828	6
55	.19287	5.18480	.21104	4.73851	.22934	4.36040	.24778	4.03578	.26639	3.75388	5
56	.19317	5.17671	.21134	4.73170	.22964	4.35459	.24809	4.03076	.26670	3.74950	4
57	.19347	5.16863	.21164	4.72490	.22995	4.34879	.24840	4.02574	.26701	3.74512	3
58	.19378	5.16058	.21195	4.71813	.23026	4.34300	.24871	4.02074	.26733	3.74075	2
59	.19408	5.15256	.21225	4.71137	.23056	4.33723	.24902	4.01576	.26764	3.73640	1
60	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
′	79°		78°		77°		76°		75°		′

′	15°		16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.26795	3.73205	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.26826	3.72771	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.26857	3.72338	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.26888	3.71907	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.26920	3.71476	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.26951	3.71046	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.26982	3.70616	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.27013	3.70188	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.27044	3.69761	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.27076	3.69335	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.27107	3.68909	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.27138	3.68485	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.27169	3.68061	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.27201	3.67638	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.27232	3.67217	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.27263	3.66796	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.27294	3.66376	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.27326	3.65957	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.27357	3.65538	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.27388	3.65121	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.27419	3.64705	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.27451	3.64289	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.27482	3.63874	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.27513	3.63461	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.27545	3.63048	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.27576	3.62636	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.27607	3.62224	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.27638	3.61814	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.27670	3.61405	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.27701	3.60996	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.27732	3.60588	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.27764	3.60181	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.27795	3.59775	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.27826	3.59370	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.27858	3.58966	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.27889	3.58562	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.27921	3.58160	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.27952	3.57758	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.27983	3.57357	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.28015	3.56957	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.28046	3.56557	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.28077	3.56159	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.28109	3.55761	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.28140	3.55364	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.28172	3.54968	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.28203	3.54573	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.28234	3.54179	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.28266	3.53785	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.28297	3.53393	.30192	3.31216	.32106	3.11464	.34043	2.93743	.36002	2.77761	12
49	.28329	3.53001	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.28360	3.52609	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.28391	3.52219	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.28423	3.51829	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.28454	3.51441	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.28486	3.51053	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.28517	3.50666	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.28549	3.50279	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.28580	3.49894	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.28612	3.49509	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.28643	3.49125	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
′	74°		73°		72°		71°		70°		′

′	20°		21°		22°		23°		24°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	.44558	2.24428	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	.44593	2.24252	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	.44627	2.24077	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	.44662	2.23902	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	.44697	2.23727	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	.44732	2.23553	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	.44767	2.23378	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	.44802	2.23204	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	.44837	2.23030	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	.44872	2.22857	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	.44907	2.22683	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	.44942	2.22510	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	.44977	2.22337	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	.45012	2.22164	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	.45047	2.21992	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	.45082	2.21819	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	.45117	2.21647	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	.45152	2.21475	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	.45187	2.21304	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	.45222	2.21132	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	.45257	2.20961	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	43205	2.31456	.45292	2.20790	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	.45327	2.20619	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	.45362	2.20449	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	.45397	2.20278	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	.45432	2.20108	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	.45467	2.19938	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	.45502	2.19769	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	.45538	2.19599	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	.45573	2.19430	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	.45608	2.19261	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	.45643	2.19092	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	.45678	2.18923	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	.45713	2.18755	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	.45748	2.18587	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	.45784	2.18419	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	.45819	2.18251	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	.45854	2.18084	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	.45889	2.17916	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	.45924	2.17749	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	.45960	2.17582	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	.45995	2.17416	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	.46030	2.17249	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	.46065	2.17083	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	.46101	2.16917	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	.46136	2.16751	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	.46171	2.16585	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	.46206	2.16420	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	.46242	2.16255	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	.46277	2.16090	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	.46312	2.15925	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	.46348	2.15760	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	.46383	2.15596	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	.46418	2.15432	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	.46454	2.15268	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	.46489	2.15104	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	.46525	2.14940	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	.46560	2.14777	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	.46595	2.14614	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	.46631	2.14451	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
′	69°		68°		67°		66°		65°		′

′	25°		26°		27°		28°		29°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	60
1	.46666	2.14288	.48809	2.04879	.50989	1.96120	.53208	1.87941	.55469	1.80281	59
2	.46702	2.14125	.48845	2.04728	.51026	1.95979	.53246	1.87809	.55507	1.80158	58
3	.46737	2.13963	.48881	2.04577	.51063	1.95838	.53283	1.87677	.55545	1.80034	57
4	.46772	2.13801	.48917	2.04426	.51099	1.95698	.53320	1.87546	.55583	1.79911	56
5	.46808	2.13639	.48953	2.04276	.51136	1.95557	.53358	1.87415	.55621	1.79788	55
6	.46843	2.13477	.48989	2.04125	.51173	1.95417	.53395	1.87283	.55659	1.79665	54
7	.46879	2.13316	.49026	2.03975	.51209	1.95277	.53432	1.87152	.55697	1.79542	53
8	.46914	2.13154	.49062	2.03825	.51246	1.95137	.53470	1.87021	.55736	1.79419	52
9	.46950	2.12993	.49098	2.03675	.51283	1.94997	.53507	1.86891	.55774	1.79296	51
10	.46985	2.12832	.49134	2.03526	.51319	1.94858	.53545	1.86760	.55812	1.79174	50
11	.47021	2.12671	.49170	2.03376	.51356	1.94718	.53582	1.86630	.55850	1.79051	49
12	.47056	2.12511	.49206	2.03227	.51393	1.94579	.53620	1.86499	.55888	1.78929	48
13	.47092	2.12350	.49242	2.03078	.51430	1.94440	.53657	1.86369	.55926	1.78807	47
14	.47128	2.12190	.49278	2.02929	.51467	1.94301	.53694	1.86239	.55964	1.78685	46
15	.47163	2.12030	.49315	2.02780	.51503	1.94162	.53732	1.86109	.56003	1.78563	45
16	.47199	2.11871	.49351	2.02631	.51540	1.94023	.53769	1.85979	.56041	1.78441	44
17	.47234	2.11711	.49387	2.02483	.51577	1.93885	.53807	1.85850	.56079	1.78319	43
18	.47270	2.11552	.49423	2.02335	.51614	1.93746	.53844	1.85720	.56117	1.78198	42
19	.47305	2.11392	.49459	2.02187	.51651	1.93608	.53882	1.85591	.56156	1.78077	41
20	.47341	2.11233	.49495	2.02039	.51688	1.93470	.53920	1.85462	.56194	1.77955	40
21	.47377	2.11075	.49532	2.01891	.51724	1.93332	.53957	1.85333	.56232	1.77834	39
22	.47412	2.10916	.49568	2.01743	.51761	1.93195	.53995	1.85204	.56270	1.77713	38
23	.47448	2.10758	.49604	2.01596	.51798	1.93057	.54032	1.85075	.56309	1.77592	37
24	.47483	2.10600	.49640	2.01449	.51835	1.92920	.54070	1.84946	.56347	1.77471	36
25	.47519	2.10442	.49677	2.01302	.51872	1.92782	.54107	1.84818	.56385	1.77351	35
26	.47555	2.10284	.49713	2.01155	.51909	1.92645	.54145	1.84689	.56424	1.77230	34
27	.47590	2.10126	.49749	2.01008	.51946	1.92508	.54183	1.84561	.56462	1.77110	33
28	.47626	2.09969	.49786	2.00862	.51983	1.92371	.54220	1.84433	.56501	1.76990	32
29	.47662	2.09811	.49822	2.00715	.52020	1.92235	.54258	1.84305	.56539	1.76869	31
30	.47698	2.09654	.49858	2.00569	.52057	1.92098	.54296	1.84177	.56577	1.76749	30
31	.47733	2.09498	.49894	2.00423	.52094	1.91962	.54333	1.84049	.56616	1.76629	29
32	.47769	2.09341	.49931	2.00277	.52131	1.91826	.54371	1.83922	.56654	1.76510	28
33	.47805	2.09184	.49967	2.00131	.52168	1.91690	.54409	1.83794	.56693	1.76390	27
34	.47840	2.09028	.50004	1.99986	.52205	1.91554	.54446	1.83667	.56731	1.76271	26
35	.47876	2.08872	.50040	1.99841	.52242	1.91418	.54484	1.83540	.56769	1.76151	25
36	.47912	2.08716	.50076	1.99695	.52279	1.91282	.54522	1.83413	.56808	1.76032	24
37	.47948	2.08560	.50113	1.99550	.52316	1.91147	.54560	1.83286	.56846	1.75913	23
38	.47984	2.08405	.50149	1.99406	.52353	1.91012	.54597	1.83159	.56885	1.75794	22
39	.48019	2.08250	.50185	1.99261	.52390	1.90876	.54635	1.83033	.56923	1.75675	21
40	.48055	2.08094	.50222	1.99116	.52427	1.90741	.54673	1.82906	.56962	1.75556	20
41	.48091	2.07939	.50258	1.98972	.52464	1.90607	.54711	1.82780	.57000	1.75437	19
42	.48127	2.07785	.50295	1.98828	.52501	1.90472	.54748	1.82654	.57039	1.75319	18
43	.48163	2. 7630	.50331	1.98684	.52538	1.90337	.54786	1.82528	.57078	1.75200	17
44	.48198	2.07476	.50368	1.98540	.52575	1.90203	.54824	1.82402	.57116	1.75082	16
45	.48234	2 07321	.50404	1.98396	.52613	1.90069	.54862	1.82276	.57155	1.74964	15
46	.48270	2.07167	.50441	1.98253	.52650	1.89935	.54900	1.82150	.57193	1.74846	14
47	.48306	2.07014	.50477	1.98110	.52687	1.89801	.54938	1.82025	.57232	1.74728	13
48	.48342	2.06860	.50514	1.97966	.52724	1.89667	.54975	1.81899	.57271	1.74610	12
49	.48378	2.06706	.50550	1.97823	.52761	1.89533	.55013	1.81774	.57309	1.74492	11
50	.48414	2.06553	.50587	1.97681	.52798	1.89400	.55051	1.81649	.57348	1.74375	10
51	.48450	2.06400	.50623	1.97538	.52836	1.89266	.55089	1.81524	.57386	1.74257	9
52	.48486	2.06247	.50660	1.97395	.52873	1.89133	.55127	1.81399	.57425	1.74140	8
53	.48521	2.06094	.50696	1.97253	.52910	1.89000	.55165	1.81274	.57464	1.74022	7
54	.48557	2.05942	.50733	1.97111	.52947	1.88867	.55203	1.81150	.57503	1.73905	6
55	.48593	2.05790	.50769	1.96969	.52985	1.88734	.55241	1.81025	.57541	1.73788	5
56	.48629	2.05637	.50806	1.96827	.53022	1.88602	.55279	1.80901	.57580	1.73671	4
57	.48665	2.05485	.50843	1.96685	.53059	1.88469	.55317	1.80777	.57619	1.73555	3
58	.48701	2.05333	.50879	1.96544	.53096	1.88337	.55355	1.80653	.57657	1.73438	2
59	.48737	2.05182	.50916	1.96402	.53134	1.88205	.55393	1.80529	.57696	1.73321	1
60	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	.57735	1.73205	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
′	64°		63°		62°		61°		60°		′

′	30°		31°		32°		33°		34°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.57735	1.73205	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	60
1	.57774	1.73089	.60126	1.66318	.62527	1.59930	.64982	1.53888	.67493	1.48163	59
2	.57813	1.72973	.60165	1.66209	.62568	1.59826	.65024	1.53791	.67536	1.48070	58
3	.57851	1.72857	.60205	1.66099	.62608	1.59723	.65065	1.53693	.67578	1.47977	57
4	.57890	1.72741	.60245	1.65990	.62649	1.59620	.65106	1.53595	.67620	1.47885	56
5	.57929	1.72625	.60284	1.65881	.62689	1.59517	.65148	1.53497	.67663	1.47792	55
6	.57968	1.72509	.60324	1.65772	.62730	1.59414	.65189	1.53400	.67705	1.47699	54
7	.58007	1.72393	.60364	1.65663	.62770	1.59311	.65231	1.53302	.67748	1.47607	53
8	.58046	1.72278	.60403	1.65554	.62811	1.59208	.65272	1.53205	.67790	1.47514	52
9	.58085	1.72163	.60443	1.65445	.62852	1.59105	.65314	1.53107	.67832	1.47422	51
10	.58124	1.72047	.60483	1.65337	.62892	1.59002	.65355	1.53010	.67875	1.47330	50
11	.58162	1.71932	.60522	1.65228	.62933	1.58900	.65397	1.52913	.67917	1.47238	49
12	.58201	1.71817	.60562	1.65120	.62973	1.58797	.65438	1.52816	.67960	1.47146	48
13	.58240	1.71702	.60602	1.65011	.63014	1.58695	.65480	1.52719	.68002	1.47053	47
14	.58279	1.71588	.60642	1.64903	.63055	1.58593	.65521	1.52622	.68045	1.46962	46
15	.58318	1.71473	.60681	1.64795	.63095	1.58490	.65563	1.52525	.68088	1.46870	45
16	.58357	1.71358	.60721	1.64687	.63136	1.58388	.65604	1.52429	.68130	1.46778	44
17	.58396	1.71244	.60761	1.64579	.63177	1.58286	.65646	1.52332	.68173	1.46686	43
18	.58435	1.71129	.60801	1.64471	.63217	1.58184	.65688	1.52235	.68215	1.46595	42
19	.58474	1.71015	.60841	1.64363	.63258	1.58083	.65729	1.52139	.68258	1.46503	41
20	.58513	1.70901	.60881	1.64256	.63299	1.57981	.65771	1.52043	.68301	1.46411	40
21	.58552	1.70787	.60921	1.64148	.63340	1.57879	.65813	1.51946	.68343	1.46320	39
22	.58591	1.70673	.60960	1.64041	.63380	1.57778	.65854	1.51850	.68386	1.46229	38
23	.58631	1.70560	.61000	1.63934	.63421	1.57676	.65896	1.51754	.68429	1.46137	37
24	.58670	1.70446	.61040	1.63826	.63462	1.57575	.65938	1.51658	.68471	1.46046	36
25	.58709	1.70332	.61080	1.63719	.63503	1.57474	.65980	1.51562	.68514	1.45955	35
26	.58748	1.70219	.61120	1.63612	.63544	1.57372	.66021	1.51466	.68557	1.45864	34
27	.58787	1.70106	.61160	1.63505	.63584	1.57271	.66063	1.51370	.68600	1.45773	33
28	.58826	1.69992	.61200	1.63398	.63625	1.57170	.66105	1.51275	.68642	1.45682	32
29	.58865	1.69879	.61240	1.63292	.63666	1.57069	.66147	1.51179	.68685	1.45592	31
30	.58905	1.69766	.61280	1.63185	.63707	1.56969	.66189	1.51084	.68728	1.45501	30
31	.58944	1.69653	.61320	1.63079	.63748	1.56868	.66230	1.50988	.68771	1.45410	29
32	.58983	1.69541	.61360	1.62972	.63789	1.56767	.66272	1.50893	.68814	1.45320	28
33	.59022	1.69428	.61400	1.62866	.63830	1.56667	.66314	1.50797	.68857	1.45229	27
34	.59061	1.69316	.61440	1.62760	.63871	1.56566	.66356	1.50702	.68900	1.45139	26
35	.59101	1.69203	.61480	1.62654	.63912	1.56466	.66398	1.50607	.68942	1.45049	25
36	.59140	1.69091	.61520	1.62548	.63953	1.56366	.66440	1.50512	.68985	1.44958	24
37	.59179	1.68979	.61561	1.62442	.63994	1.56265	.66482	1.50417	.69028	1.44868	23
38	.59218	1.68866	.61601	1.62336	.64035	1.56165	.66524	1.50322	.69071	1.44778	22
39	.59258	1.68754	.61641	1.62230	.64076	1.56065	.66566	1.50228	.69114	1.44688	21
40	.59297	1.68643	.61681	1.62125	.64117	1.55966	.66608	1.50133	.69157	1.44598	20
41	.59336	1.68531	.61721	1.62019	.64158	1.55866	.66650	1.50038	.69200	1.44508	19
42	.59376	1.68419	.61761	1.61914	.64199	1.55766	.66692	1.49944	.69243	1.44418	18
43	.59415	1.68308	.61801	1.61808	.64240	1.55666	.66734	1.49849	.69286	1.44329	17
44	.59454	1.68196	.61842	1.61703	.64281	1.55567	.66776	1.49755	.69329	1.44239	16
45	.59494	1.68085	.61882	1.61598	.64322	1.55467	.66818	1.49661	.69372	1.44149	15
46	.59533	1.67974	.61922	1.61493	.64363	1.55368	.66860	1.49566	.69416	1.44060	14
47	.59573	1.67863	.61962	1.61388	.64404	1.55269	.66902	1.49472	.69459	1.43970	13
48	.59612	1.67752	.62003	1.61283	.64446	1.55170	.66944	1.49378	.69502	1.43881	12
49	.59651	1.67641	.62043	1.61179	.64487	1.55071	.66986	1.49284	.69545	1.43792	11
50	.59691	1.67530	.62083	1.61074	.64528	1.54972	.67028	1.49190	.69588	1.43703	10
51	.59730	1.67419	.62124	1.60970	.64569	1.54873	.67071	1.49097	.69631	1.43614	9
52	.59770	1.67309	.62164	1.60865	.64610	1.54774	.67113	1.49003	.69675	1.43525	8
53	.59803	1.67198	.62204	1.60761	.64652	1.54675	.67155	1.48909	.69718	1.43436	7
54	.59849	1.67088	.62245	1.60657	.64693	1.54576	.67197	1.48816	.69761	1.43347	6
55	.59888	1.66978	.62285	1.60553	.64734	1.54478	.67239	1.48722	.69804	1.43258	5
56	.59928	1.66867	.62325	1.60449	.64775	1.54379	.67282	1.48629	.69847	1.43169	4
57	.59967	1.66757	.62366	1.60345	.64817	1.54281	.67324	1.48536	.69891	1.43080	3
58	.60007	1.66647	.62406	1.60241	.64858	1.54183	.67366	1.48442	.69934	1.42992	2
59	.60046	1.66538	.62446	1.60137	.64899	1.54085	.67409	1.48349	.69977	1.42903	1
60	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
′	59°		58°		57°		56°		55°		′

′	35°		36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.70021	1.42815	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.70064	1.42726	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.70107	1.42638	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.70151	1.42550	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.70194	1.42462	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.70238	1.42374	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.70281	1.42286	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.70325	1.42198	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.70368	1.42110	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.70412	1.42022	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.70455	1.41934	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.70499	1.41847	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.70542	1.41759	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.70586	1.41672	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.70629	1.41584	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.70673	1.41497	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.70717	1.41409	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.70760	1.41322	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.70804	1.41235	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.70848	1.41148	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.70891	1.41061	.73547	1 35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.70935	1.40974	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.70979	1.40887	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.71023	1.40800	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.71066	1.40714	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.71110	1.40627	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.71154	1.40540	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.71198	1.40454	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.71242	1.40367	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.71285	1.40281	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.71329	1.40195	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.71373	1.40109	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.71417	1.40022	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.71461	1.39936	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.71505	1.39850	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.71549	1.39764	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.71593	1.39679	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.71637	1.39593	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.71681	1.39507	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.71725	1.39421	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.71769	1.39336	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.71813	1.39250	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.71857	1.39165	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.71901	1.39079	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.71946	1.38994	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.71990	1.38909	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.72034	1.38824	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.72078	1.38738	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.72122	1.38653	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.72167	1.38568	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.72211	1.38484	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.72255	1.38399	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.72299	1.38314	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.72344	1.38229	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.72388	1.38145	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.72432	1.38060	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.72477	1.37976	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.72521	1.37891	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.72565	1.37807	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.72610	1.37722	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
′	54°		53°		52°		51°		50°		′

′	40°		41°		42°		43°		44°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	.96625	1.03493	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	.96681	1.03433	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	.96738	1.03372	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	.96794	1.03312	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	.96850	1.03252	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	.96907	1.03192	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	.96963	1.03132	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	.97020	1.03072	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	.97076	1.03012	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	.97133	1.02952	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	.97189	1.02892	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	.97246	1.02832	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	.97302	1.02772	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	.97359	1.02713	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	.97416	1.02653	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	.97472	1.02593	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	.07529	1.02533	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	.97586	1.02474	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	.97643	1.02414	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	.97700	1.02355	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	.97756	1.02295	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	.97813	1.02236	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	.97870	1.02176	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	.97927	1.02117	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	.97984	1.02057	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	.98041	1.01998	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	.98098	1.01939	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	.98155	1.01879	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	.98213	1.01820	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	.98270	1.01761	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	.98327	1.01702	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	.98384	1.01642	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	.98441	1.01583	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	.98499	1.01524	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	.98556	1.01465	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	.98613	1.01406	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	.98671	1.01347	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	.98728	1.01288	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	.98786	1.01229	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	.98843	1.01170	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	.98901	1.01112	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	.98958	1.01053	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	.99016	1.00994	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	.99073	1.00935	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	.99131	1.00876	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	.99189	1.00818	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	.99247	1.00759	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	.99304	1.00701	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	.99362	1.00642	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	.99420	1.00583	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	.99478	1.00525	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	.99536	1.00467	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	.99594	1.00408	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	.99652	1.00350	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	.99710	1.00291	5
56	86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	.99768	1.00233	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	.99826	1.00175	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	.99884	1.00116	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	.99942	1.00058	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	1.00000	1.00000	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		45°		

SQUARES, CUBES, ROOTS AND RECIPROCALS OF NUMBERS, CIRCUMFERENCES AND AREAS OF CIRCLES

SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
1	1	1	1.0000	1.0000	1.000000000	3.1416	0.7854
2	4	8	1.4142	1.2599	.500000000	6.2832	3.1416
3	9	27	1.7321	1.4422	.333333333	9.4248	7.0686
4	16	64	2.0000	1.5874	.250000000	12.5664	12.5664
5	25	125	2.2361	1.7100	.200000000	15.7080	19.635
6	36	216	2.4495	1.8171	.166666667	18.850	28.274
7	49	343	2.6458	1.9129	.142857143	21.991	38.485
8	64	512	2.8284	2.0000	.125000000	25.133	50.266
9	81	729	3.0000	2.0801	.111111111	28.274	63.617
10	100	1,000	3.1623	2.1544	.100000000	31.416	78.540
11	121	1,331	3.3166	2.2240	.090909091	34.558	95.033
12	144	1,728	3.4641	2.2894	.083333333	37.699	113.10
13	169	2,197	3.6056	2.3513	.076923077	40.841	132.73
14	196	2,744	3.7417	2.4101	.071428571	43.982	153.94
15	225	3,375	3.8730	2.4662	.066666667	47.124	176.71
16	256	4,096	4.0000	2.5198	.062500000	50.265	201.06
17	289	4,913	4.1231	2.5713	.058823529	53.407	226.98
18	324	5,832	4.2426	2.6207	.055555556	56.549	254.47
19	361	6,859	4.3589	2.6684	.052631579	59.690	283.53
20	400	8,000	4.4721	2.7144	.050000000	62.832	314.16
21	441	9,261	4.5826	2.7589	.047619048	65.973	346.36
22	484	10,648	4.6904	2.8020	.045454545	69.115	380.13
23	529	12,167	4.7958	2.8439	.043478261	72.257	415.48
24	576	13,824	4.8990	2.8845	.041666667	75.398	452.39
25	625	15,625	5.0000	2.9240	.040000000	78.540	490.87
26	676	17,576	5.0990	2.9625	.038461538	81.681	530.93
27	729	19,683	5.1962	3.0000	.037037037	84.823	572.56
28	784	21,952	5.2915	3.0366	.035714286	87.965	615.75
29	841	24,389	5.3852	3.0723	.034482759	91.106	660.52
30	900	27,000	5.4772	3.1072	.033333333	94.248	706.86
31	961	29,791	5.5678	3.1414	.032258065	97.389	754.77
32	1,024	32,768	5.6569	3.1748	.031250000	100.53	804.25
33	1,089	35,937	5.7446	3.2075	.030303030	103.67	855.30
34	1,156	39,304	5.8310	3.2396	.029411765	106.81	907.92
35	1,225	42,875	5.9161	3.2717	.028571429	109.96	962.11
36	1,296	46,656	6.0000	3.3019	.027777778	113.10	1,017.88
37	1,369	50,653	6.0828	3.3322	.027027027	116.24	1,075.21
38	1,444	54,872	6.1644	3.3620	.026315789	119.38	1,134.11
39	1,521	59,319	6.2450	3.3912	.025641026	122.52	1,194.59
40	1,600	64,000	6.3246	3.4200	.025000000	125.66	1,256.64
41	1,681	68,921	6.4031	3.4482	.024390244	128.81	1,320.25
42	1,764	74,088	6.4807	3.4760	.023809524	131.95	1,385.44
43	1,849	79,507	6.5574	3 5034	.023255814	135.09	1,452.20
44	1,936	85,184	6.6332	3 5303	.022727273	138.23	1,520.53
45	2,025	91,125	6.7082	3.5569	.022222222	141.37	1,590.43
46	2,116	97,336	6 7823	3 5830	.021739130	144.51	1,661.90
47	2,209	103,823	6 8557	3 6088	.021276600	147.65	1,734.94
48	2,304	110,592	6.9282	3.6342	.020833333	150.80	1,809.56
49	2,401	117,649	7.0000	3 6593	.020408163	153.94	1,885.74
50	2,500	125,000	7.0711	3.6840	.020000000	157.08	1,963.50
51	2,601	132,651	7.1414	3.7084	.019607843	160.22	2,042.82
52	2,704	140,608	7.2111	3.7325	.019230769	163.36	2,123.72
53	2,809	148,877	7.2801	3.7563	.018867925	166.50	2,206.18
54	2,916	157,464	7.3485	3.7798	.018518519	169.65	2,290.22
55	3,025	166,375	7.4162	3.8030	.018181818	172.79	2,375.83

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum	Area
56	3,136	175,616	7.4833	3.8259	.017857143	175.93	2,463.01
57	3,249	185,193	7.5498	3.8485	.017543860	179.07	2,551.76
58	3,364	195,112	7.6158	3.8709	.017241379	182.21	2,642.08
59	3,481	205,379	7.6811	3.8930	.016949153	185.35	2,733.97
60	3,600	216,000	7.7460	3.9149	.016666667	188.50	2,827.43
61	3,721	226,981	7.8102	3.9365	.016393443	191.64	2,922.47
62	3,844	238,328	7.8740	3.9579	.016129032	194.78	3,019.07
63	3,969	250,047	7.9373	3.9791	.015873016	197.92	3,117.25
64	4,096	262,144	8.0000	4.0000	.015625000	201.06	3,216.99
65	4,225	274,625	8.0623	4.0207	.015384615	204.20	3,318.31
66	4,356	287,496	8.1240	4.0412	.015151515	207.34	3,421.19
67	4,489	300,763	8.1854	4.0615	.014925373	210.49	3,525.65
68	4,624	314,432	8.2462	4.0817	.014705882	213.63	3,631.68
69	4,761	328,509	8.3066	4.1016	.014492754	216.77	3,739.28
70	4,900	343,000	8.3666	4.1213	.014285714	219.91	3,848.45
71	5,041	357,911	8.4261	4.1408	.014084517	223.05	3,959.19
72	5,184	373,248	8.4853	4.1602	.013888889	226.19	4,071.50
73	5,329	389,017	8.5440	4.1793	.013698630	229.34	4,185.39
74	5,476	405,224	8.6023	4.1983	.013513514	232.48	4,300.84
75	5,625	421,875	8.6603	4.2172	.013333333	235.62	4,417.86
76	5,776	438,976	8.7178	4.2358	.013157895	238.76	4,536.46
77	5,929	456,533	8.7750	4.2543	.012987013	241.90	4,656.63
78	6,084	474,552	8.8318	4.2727	.012820513	245.04	4,778.36
79	6,241	493,039	8.8882	4.2908	.012658228	248.19	4,901.67
80	6,400	512,000	8.9443	4.3089	.012500000	251.33	5,026.55
81	6,561	531,441	9.0000	4.3267	.012345679	254.47	5,153.00
82	6,724	551,368	9.0554	4.3445	.012195122	257.61	5,281.02
83	6,889	571,787	9.1104	4.3621	.012048193	260.75	5,410.61
84	7,056	592,704	9.1652	4.3795	.011904762	263.89	5,541.77
85	7,225	614,125	9.2195	4.3968	.011764706	267.04	5,674.50
86	7,396	636,056	9.2736	4.4140	.011627907	270.18	5,808.80
87	7,569	658,503	9.3274	4.4310	.011494253	273.32	5,944.68
88	7,744	681,472	9.3808	4.4480	.011363636	276.46	6,082.12
89	7,921	704,969	9.4340	4.4647	.011235955	279.60	6,221.14
90	8,100	729,000	9.4868	4.4814	.011111111	282.74	6,361.73
91	8,281	753,571	9.5394	4.4979	.010989011	285.88	6,503.88
92	8,464	778,688	9.5917	4.5144	.010869565	289.03	6,647.61
93	8,649	804,357	9.6437	4.5307	.010752688	292.17	6,792.91
94	8,836	830,584	9.6954	4.5468	.010638298	295.31	6,939.78
95	9,025	857,375	9.7468	4.5629	.010526316	298.45	7,088.22
96	9,216	884,736	9.7980	4.5789	.010416667	301.59	7,238.23
97	9,409	912,673	9.8489	4.5947	.010309278	304.73	7,389.81
98	9,604	941,192	9.8995	4.6104	.010204082	307.88	7,542.96
99	9,801	970,299	9.9499	4.6261	.010101010	311.02	7,697.69
100	10,000	1,000,000	10.0000	4.6416	.010000000	314.16	7,853.98
101	10,201	1,030,301	10.0499	4.6570	.009900990	317.30	8,011.85
102	10,404	1,061,208	10.0995	4.6723	.009803922	320.44	8,171.28
103	10,609	1,092,727	10.1489	4.6875	.009708738	323.58	8,332.29
104	10,816	1,124,864	10.1980	4.7027	.009615385	326.73	8,494.87
105	11,025	1,157,625	10.2470	4.7177	.009523810	329.87	8,659.01
106	11,236	1,191,016	10.2956	4.7326	.009433962	333.01	8,824.73
107	11,449	1,225,043	10.3441	4.7475	.009345794	336.15	8,992.02
108	11,664	1,259,712	10.3923	4.7622	.009259259	339.29	9,160.88
109	11,881	1,295,029	10.4403	4.7769	.009174312	342.43	9,331.32
110	12,100	1,331,000	10.4881	4.7914	.009090909	345.58	9,503.32
111	12,321	1,367,631	10.5357	4.8059	.009009009	348.72	9,676.89
112	12,544	1,404,928	10.5830	4.8203	.008928571	351.86	9,852.03
113	12,769	1,442,897	10.6301	4.8346	.008849558	355.00	10,028.75
114	12,996	1,481,544	10.6771	4.8488	.008771930	358.14	10,207.03
115	13,225	1,520,875	10.7238	4.8629	.008695652	361.28	10,386.89
116	13,456	1,560,896	10.7703	4.8770	.008020690	364.42	10,568.32
117	13,689	1,601,613	10.8167	4.8910	.008547009	367.57	10,751.32
118	13,924	1,643,032	10.8628	4.9049	.008474576	370.71	10,935.88

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
119	14,161	1,685,159	10.9087	4.9187	.008403361	373.85	11,122.02
120	14,400	1,728,000	10.9545	4.9324	.008333333	376.99	11,309.73
121	14,641	1,771,561	11.0000	4.9461	.008264463	380.13	11,499.01
122	14,834	1,815,848	11.0454	4.9597	.008196721	383.27	11,689.87
123	15,129	1,860,867	11.0905	4.9732	.008130081	386.42	11,882.29
124	15,376	1,906,624	11.1355	4.9866	.008064516	389.56	12,076.28
125	15,625	1,953,125	11.1803	5.0000	.008000000	392.70	12,271.85
126	15,876	2,000,376	11.2250	5.0133	.007936508	395.84	12,468.98
127	16,129	2,048,383	11.2694	5.0265	.007874016	398.98	12,667.69
128	16,384	2,097,152	11.3137	5.0397	.007812500	402.12	12,867.96
129	16,641	2,146,689	11.3578	5.0528	.007751938	405.27	13,069.81
130	16,900	2,197,000	11.4018	5.0658	.007692308	408.41	13,273.23
131	17,161	2,248,091	11.4455	5.0788	.007633588	411.55	13,478.22
132	17,424	2,299,968	11.4891	5.0916	.007575758	414.69	13,684.78
133	17,689	2,352,637	11.5326	5.1045	.007518797	417.83	13,892.91
134	17,956	2,406,104	11.5758	5.1172	.007462687	420.97	14,102.61
135	18,225	2,460,375	11.6190	5.1299	.007407407	424.12	14,313.88
136	18,496	2,515,456	11.6619	5.1426	.007352941	427.26	14,526.72
137	18,769	2,571,353	11.7047	5.1551	.007299270	430.40	14,741.14
138	19,044	2,628,072	11.7473	5.1676	.007246377	433.54	14,957.12
139	19,321	2,685,619	11.7898	5.1801	.007194245	436.68	15,174.68
140	19,600	2,744,000	11.8322	5.1925	.007142857	439.82	15,393.80
141	19,881	2,803,221	11.8743	5.2048	.007092199	442.96	15,614.50
142	20,164	2,863,288	11.9164	5.2171	.007042254	446.11	15,836.77
143	20,449	2,924,207	11.9583	5.2293	.006993007	449.25	16,060.61
144	20,736	2,985,984	12.0000	5.2415	.006944444	452.39	16,286.02
145	21,025	3,048,625	12.0416	5.2536	.006896552	455.53	16,513.00
146	21,316	3,112,136	12.0830	5.2656	.006849315	458.67	16,741.55
147	21,609	3,176,523	12.1244	5.2776	.006802721	461.81	16,971.67
148	21,904	3,241,792	12.1655	5.2896	.006756757	464.96	17,203.36
149	22,201	3,307,949	12.2066	5.3015	.006711409	468.10	17,436.62
150	22,500	3,375,000	12.2474	5.3133	.006666667	471.24	17,671.46
151	22,801	3,442,951	12.2882	5.3251	.006622517	474.38	17,907.86
152	23,104	3,511,008	12.3288	5.3368	.006578947	477.52	18,145.84
153	23,409	3,581,577	12.3693	5.3485	.006535948	480.66	18,385.39
154	23,716	3,652,264	12.4097	5.3601	.006493506	483.81	18,626.50
155	24,025	3,723,875	12.4499	5.3717	.006451613	486.95	18,869.19
156	24,336	3,796,416	12.4900	5.3832	.006410256	490.09	19,113.45
157	24,649	3,869,893	12.5300	5.3947	.006369427	493.23	19,359.28
158	24,964	3,944,312	12.5698	5.4061	.006329114	496.37	19,606.68
159	25,281	4,019,679	12.6095	5.4175	.006289308	499.51	19,855.65
160	25,600	4,096,000	12.6491	5.4288	.006250000	502.65	20,106.19
161	25,921	4,173,281	12.6886	5.4401	.006211180	505.80	20,358.31
162	26,244	4,251,528	12.7279	5.4514	.006172840	508.94	20,611.99
163	26,569	4,330,747	12.7671	5.4626	.006134969	512.08	20,867.24
164	26,896	4,410,944	12.8062	5.4737	.006097561	515.22	21,124.07
165	27,225	4,492,125	12.8452	5.4848	.006060606	518.36	21,382.46
166	27,556	4,574,296	12.8841	5.4959	.006024096	521.50	21,642.43
167	27,889	4,657,463	12.9228	5.5069	.005988024	524.65	21,903.97
168	28,224	4,741,632	12.9615	5.5178	.005952381	527.79	22,167.08
169	28,561	4,826,809	13.0000	5.5288	.005917160	530.93	22,431.76
170	28,900	4,913,000	13.9384	5.5397	.005882353	534.07	22,698.01
171	29,241	5,000,211	13.0767	5.5505	.005847953	537.21	22,965.83
172	29,584	5,088,448	13.1149	5.5613	.005813953	540.35	23,235.22
173	29,929	5,177,717	13.1529	5.5721	.005780347	543.50	23,506.18
174	30,276	5,268,024	13.1909	5.5828	.005747126	546.64	23,778.71
175	30,625	5,359,375	13.2288	5.5934	.005714286	549.78	24,052.82
176	30,976	5,451,776	13.2665	5.6041	.005681818	552.92	24,328.49
177	31,329	5,545,233	13.3041	5.6147	.005649718	556.06	24,605.74
178	31,684	5,639,752	13.3417	5.6252	.005617978	559.20	24,884.56
179	32,041	5,735,339	13.3791	5.6357	.005586592	562.35	25,164.94
180	32,400	5,832,000	13.4164	5.6462	.005555556	565.49	25,446.90
181	32,761	5,929,741	13.4536	5.6567	.005524862	568.63	25,730.43

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
182	33,124	6,028,568	13.4907	5.6671	.005494505	571.77	26,015.53
183	33,489	6,128,487	13.5277	5.6774	.005464481	574.91	26,302.20
184	33,856	6,229,504	13.5647	5.6877	.005434783	578.05	26,590.44
185	34,225	6,331,625	13.6015	5.6980	.005405405	581.19	26,880.25
186	34,596	6,434,856	13.6382	5.7083	.005376344	584.34	27,171.63
187	34,969	6,539,203	13.6748	5.7185	.005347594	587.48	27,464.59
188	35,344	6,644,672	13.7113	5.7287	.005319149	590.62	27,759.11
189	35,721	6,751,269	13.7477	5.7388	.005291005	593.76	28,055.21
190	36,100	6,859,000	13.7840	5.7489	.005263158	596.90	28,352.87
191	36,481	6,967,871	13.8203	5.7590	.005235602	600.04	28,652.11
192	36,864	7,077,888	13.8564	5.7690	.005208333	603.19	28,952.92
193	37,249	7,189,017	13.8924	5.7790	.005181347	606.33	29,255.30
194	37,636	7,301,384	13.9284	5.7890	.005154639	609.47	29,559.25
195	38,025	7,414,875	13.9642	5.7989	.005128205	612.61	29,864.77
196	38,416	7,529,536	14.0000	5.8088	.005102041	615.75	30,171.86
197	38,809	7,645,373	14.0357	5.8186	.005076142	618.89	30,480.52
198	39,204	7,762,392	14.0712	5.8285	.005050505	622.04	30,790.75
199	39,601	7,880,599	14.1067	5.8383	.005025126	625.18	31,102.55
200	40,000	8,000,000	14.1421	5.8480	.005000000	628.32	31,415.93
201	40,401	8,120,601	14.1774	5.8578	.004975124	631.46	31,730.87
202	40,804	8,242,408	14.2127	5.8675	.004950495	634.60	32,047.39
203	41,209	8,365,427	14.2478	5.8771	.004926108	637.74	32,365.47
204	41,616	8,489,664	14.2829	5.8868	.004901961	640.88	32,685.13
205	42,025	8,615,125	14.3178	5.8964	.004878049	644.03	33,006.36
206	42,436	8,741,816	14.3527	5.9059	.004854369	647.17	33,329.16
207	42,849	8,869,743	14.3875	5.9155	.004830918	650.31	33,653.53
208	43,264	8,998,912	14.4222	5.9250	.004807692	653.45	33,979.47
209	43,681	9,129,329	14.4568	5.9345	.004784689	656.59	34,306.98
210	44,100	9,261,000	14.4914	5.9439	.004761905	659.73	34,636.06
211	44,521	9,393,931	14.5258	5.9533	.004739336	662.88	34,966.71
212	44,944	9,528,128	14.5602	5.9627	.004716981	666.02	35,298.94
213	45,369	9,663,597	14.5945	5.9721	.004694836	669.16	35,632.73
214	45,796	9,800,344	14.6287	5.9814	.004672897	672.30	35,968.09
215	46,225	9,938,375	14.6629	5.9907	.004651163	675.44	36,305.03
216	46,656	10,077,696	14.6969	6.0000	.004629630	678.58	36,643.54
217	47,089	10,218,313	14.7309	6.0092	.004608295	681.73	36,983.61
218	47,524	10,360,232	14.7648	6.0185	.004587156	684.87	37,325.26
219	47,961	10,503,459	14.7986	6.0277	.004566210	688.01	37,668.48
220	48,400	10,648,000	14.8324	6.0368	.004545455	691.15	38,013.27
221	48,841	10,793,861	14.8661	6.0459	.004524887	694.29	38,359.63
222	49,284	10,941,048	14.8997	6.0550	.004504505	697.43	38,707.56
223	49,729	11,089,567	14.9332	6.0641	.004484305	700.58	39,057.07
224	50,176	11,239,424	14.9666	6.0732	.004464286	703.72	39,408.14
225	50,625	11,390,625	15.0000	6.0822	.004444444	706.86	39,760.78
226	51,076	11,543,176	15.0333	6.0912	.004424779	710.00	40,115.00
227	51,529	11,697,083	15.0665	6.1002	.004405286	713.14	40,470.78
228	51,984	11,852,352	15.0997	6.1091	.004385965	716.28	40,828.14
229	52,441	12,008,989	15.1327	6.1180	.004366812	719.42	41,187.07
230	52,900	12,167,000	15.1658	6.1269	.004347826	722.57	41,547.56
231	53,361	12,326,391	15.1987	6.1358	.004329004	725.71	41,909.63
232	53,824	12,487,168	15.2315	6.1446	.004310345	728.85	42,273.27
233	54,289	12,649,337	15.2643	6.1534	.004291845	731.99	42,638.48
234	54,756	12,812,904	15.2971	6.1622	.004273504	735.13	43,005.26
235	55,225	12,977,875	15.3297	6.1710	.004255319	738.27	43,373.61
236	55,696	13,144,256	15.3623	6.1797	.004237288	741.42	43,743.54
237	56,169	13,312,053	15.3948	6.1885	.004219409	744.56	44,115.03
238	56,644	13,481,272	15.4272	6.1972	.004201681	747.70	44,488.09
239	57,121	13,651,919	15.4596	6.2058	.004184100	750.84	44,862.73
240	57,600	13,824,000	15.4919	6.2145	.004166667	753.98	45,238.93
241	58,081	13,997,521	15.5242	6.2231	.004149378	757.12	45,616.71
242	58,564	14,172,488	15.5563	6.2317	.004132231	760.27	45,996.06
243	59,049	14,348,907	15.5885	6.2403	.004115226	763.41	46,376.98
244	59,536	14,526,784	15.6205	6.2488	.004098361	766.55	46,759.47

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
245	60,025	14,706,125	15.6525	6.2573	.004081633	769.69	47,143.52
246	60,516	14,886,936	15.6844	6.2658	.004065041	772.83	47,529.16
247	61,009	15,069,223	15.7162	6.2743	.004048583	775.97	47,916.36
248	61,504	15,252,992	15.7480	6.2828	.004032258	779.11	48,305.13
249	62,001	15,438,249	15.7797	6.2912	.004016064	782.26	48,695.47
250	62,500	15,625,000	15.8114	6.2996	.004000000	785.40	49,087.39
251	63,001	15,813,251	15.8430	6.3080	.003984064	788.54	49,480.87
252	63,504	16,003,008	15.8745	6.3164	.003968254	791.68	49,875.92
253	64,009	16,194,277	15.9060	6.3247	.003952569	794.82	50,272.55
254	64,516	16,387,064	15.9374	6.3330	.003937008	797.96	50,670.75
255	65,025	16,581,375	15.9687	6.3413	.003921569	801.11	51,070.52
256	65,536	16,777,216	16.0000	6.3496	.003906250	804.25	51,471.85
257	66,049	16,974,593	16.0312	6.3579	.003891051	807.39	51,874.76
258	66,564	17,173,512	16.0624	6.3661	.003875969	810.53	52,279.24
259	67,081	17,373,979	16.0935	6.3743	.003861004	813.67	52,685.29
260	67,600	17,576,000	16.1245	6.3825	.003846154	816.81	53,092.92
261	68,121	17,779,581	16.1555	6.3907	.003831418	819.96	53,502.11
262	68,644	17,984,728	16.1864	6.3988	.003816794	823.10	53,912.87
263	69,169	18,191,447	16.2173	6.4070	.003802281	826.24	54,325.21
264	69,696	18,399,744	16.2481	6.4151	.003787879	829.38	54,739.11
265	70,225	18,609,625	16.2788	6.4232	.003773585	832.52	55,154.59
266	70,756	18,821,096	16.3095	6.4312	.003759398	835.66	55,571.63
267	71,289	19,034,163	16.3401	6.4393	.003745318	838.81	55,990.25
268	71,824	19,248,832	16.3707	6.4473	.003731343	841.95	56,410.44
269	72,361	19,465,109	16.4012	6.4553	.003717472	845.09	56,832.20
270	72,900	19,683,000	16.4317	6.4633	.003703704	848.23	57,255.53
271	73,441	19,902,511	16.4621	6.4713	.003690037	851.37	57,680.43
272	73,984	20,123,648	16.4924	6.4792	.003676471	854.51	58,106.90
273	74,529	20,346,417	16.5227	6.4872	.003663004	857.65	58,534.94
274	75,076	20,570,824	16.5529	6.4951	.003649635	860.80	58,964.55
275	75,625	20,796,875	16.5831	6.5030	.003636364	863.94	59,395.74
276	76,176	21,024,576	16.6132	6.5108	.003623188	867.08	59,828.49
277	76,729	21,253,933	16.6433	6.5187	.003610108	870.22	60,262.82
278	77,284	21,484,952	16.6783	6.5265	.003597122	873.36	60,698.71
279	77,841	21,717,639	16.7033	6.5343	.003584229	876.50	61,136.18
280	78,400	21,952,000	16.7332	6.5421	.003571429	879.65	61,575.22
281	78,961	22,188,041	16.7631	6.5499	.003558719	882.79	62,015.82
282	79,524	22,425,768	16.7929	6.5577	.003546099	885.93	62,458.00
283	80,089	22,665,187	16.8226	6.5654	.003533569	889.07	62,901.75
284	80,656	22,906,304	16.8523	6.5731	.003522127	892.21	63,347.07
285	81,225	23,149,125	16.8819	6.5808	.003508772	895.35	63,793.97
286	81,796	23,393,656	16.9115	6.5885	.003496503	898.50	64,242.43
287	82,369	23,639,903	16.9411	6.5962	.003484321	901.64	64,692.46
288	82,944	23,887,872	16.9706	6.6039	.003472222	904.78	65,144.07
289	83,521	24,137,569	17.0000	6.6115	.003460208	907.92	65,597.24
290	84,100	24,389,000	17.0294	6.6191	.003448276	911.06	66,051.99
291	84,681	24,642,171	17.0587	6.6267	.003436426	914.20	66,508.30
292	85,264	24,897,088	17.0880	6.6343	.003424658	917.35	66,966.19
293	85,849	25,153,757	17.1172	6.6419	.003412969	920.49	67,425.65
294	86,436	25,412,184	17.1464	6.6494	.003401361	923.63	67,886.68
295	87,025	25,672,375	17.1756	6.6569	.003389831	926.77	68,349.28
296	87,616	25,934,836	17.2047	6.6644	.003378378	929.91	68,813.45
297	88,209	26,198,073	17.2337	6.6719	.003367003	933.05	69,279.19
298	88,804	26,463,592	17.2627	6.6794	.003355705	936.19	69,746.50
299	89,401	26,730,899	17.2916	6.6869	.003344482	939.34	70,215.38
300	90,000	27,000,000	17.3205	6.6943	.003333333	942.48	70,685.83
301	90,601	27,270,901	17.3494	6.7018	.003322259	945.62	71,157.86
302	91,204	27,543,608	17.3781	6.7092	.003311258	948.76	71,631.45
303	91,809	27,818,127	17.4069	6.7166	.003301330	951.90	72,106.62
304	92,416	28,094,464	17.4356	6.7240	.003289474	955.04	72,583.36
305	93,025	28,372,625	17.4642	6.7313	.003278689	958.19	73,061.66
306	93,636	28,652,616	17.4929	6.7387	.003267974	961.33	73,541.54
307	94,249	28,934,443	17.5214	6.7460	.003257329	964.47	74,022.99

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
308	94,864	29,218,112	17.5499	6.7533	.003246753	967.61	74,506.01
309	95,481	29,503,629	17.5784	6.7606	.003236246	970.75	74,990.60
310	96,100	29,791,000	17.6068	6.7679	.003225806	973.89	75,476.76
311	96,721	30,080,231	17.6352	6.7752	.003215434	977.04	75,964.50
312	97,344	30,371,328	17.6635	6.7824	.003205128	980.18	76,453.80
313	97,969	30,664,297	17.6918	6.7897	.003194888	983.32	76,944.67
314	98,596	30,959,144	17.7200	6.7969	.003184713	986.46	77,437.12
315	99,225	31,255,875	17.7482	6.8041	.003174603	989.60	77,931.13
316	99,856	31,554,496	17.7764	6.8113	.003164557	992.74	78,426.72
317	100,489	31,855,013	17.8045	6.8185	.003154574	995.88	78,923.88
318	101,124	32,157,432	17.8326	6.8256	.003144654	999.03	79,422.60
319	101,761	32,461,759	17.8606	6.8328	.003134796	1,002.17	79,922.90
320	102,400	32,768,000	17.8885	6.8399	.003125000	1,005.31	80,424.77
321	103,041	33,076,161	17.9165	6.8470	.003115265	1,008.45	80,928.21
322	103,684	33,386,248	17.9444	6.8541	.003105590	1,011.59	81,433.22
323	104,329	33,698,267	17.9722	6.8612	.003095975	1,014.73	81,939.80
324	104,976	34,012,224	18.0000	6.8683	.003086420	1,017.88	82,447.96
325	105,625	34,328,125	18.0278	6.8753	.003076923	1,021.02	82,957.68
326	106,276	34,645,976	18.0555	6.8824	.003067485	1,024.16	83,468.98
327	106,929	34,965,783	18.0831	6.8894	.003058104	1,027.30	83,981.84
328	107,584	35,287,552	18.1108	6.8964	.003048780	1,030.44	84,496.28
329	108,241	35,611,289	18.1384	6.9034	.003039514	1,033.58	85,012.28
330	108,900	35,937,000	18.1659	6.9104	.003030303	1,036.73	85,529.86
331	109,561	36,264,691	18.1934	6.9174	.003021148	1,039.87	86,049.01
332	110,224	36,594,368	18.2209	6.9244	.003012048	1,043.01	86,569.73
333	110,889	36,926,037	18.2483	6.9313	.003003003	1,046.15	87,092.02
334	111,556	37,259,704	18.2757	6.9382	.002994012	1,049.29	87,615.88
335	112,225	37,595,375	18.3030	6.9451	.002985075	1,052.43	88,141.31
336	112,896	37,933,056	18.3303	6.9521	.002976190	1,055.58	88,668.31
337	113,569	38,272,753	18.3576	6.9589	.002967359	1,058.72	89,196.88
338	114,244	38,614,472	18.3848	6.9658	.002958580	1,061.86	89,727.03
339	114,921	38,958,219	18.4120	6.9727	.002949853	1,065.00	90,258.74
340	115,600	39,304,000	18.4391	6.9795	.002941176	1,068.14	90,792.03
341	116,281	39,651,821	18.4662	6.9864	.002932551	1,071.28	91,326.88
342	116,964	40,001,688	18.4932	6.9932	.002923977	1,074.42	91,863.31
343	117,649	40,353,607	18.5203	7.0000	.002915452	1,077.57	92,401.31
344	118,336	40,707,584	18.5472	7.0068	.002906977	1,080.71	92,940.88
345	119,025	41,063,625	18.5742	7.0136	.002898551	1,083.85	93,482.02
346	119,716	41,421,736	18.6011	7.0203	.002890173	1,086.99	94,024.73
347	120,409	41,781,923	18.6279	7.0271	.002881844	1,090.13	94,569.01
348	121,104	42,144,192	18.6548	7.0338	.002873563	1,093.27	95,114.86
349	121,801	42,508,549	18.6815	7.0406	.002865330	1,096.42	95,662.28
350	122,500	42,875,000	18.7083	7.0473	.002857143	1,099.56	96,211.28
351	123,201	43,243,551	18.7350	7.0540	.002849003	1,102.70	96,761.84
352	123,904	43,614,208	18.7617	7.0607	.002840909	1,105.84	97,313.97
353	124,609	43,986,977	18.7883	7.0674	.002832861	1,108.98	97,867.68
354	125,316	44,361,864	18.8149	7.0740	.002824859	1,112.12	98,422.96
355	126,025	44,738,875	18.8414	7.0807	.002816901	1,115.27	98,979.80
356	126,736	45,118,016	18.8680	7.0873	.002808989	1,118.41	99,538.22
357	127,449	45,499,293	18.8944	7.0940	.002801120	1,121.55	100,098.21
358	128,164	45,882,712	18.9209	7.1006	.002793296	1,124.69	100,659.77
359	128,881	46,268,279	18.9473	7.1072	.002785515	1,127.83	101,222.90
360	129,600	46,656,000	18.9737	7.1138	.002777778	1,130.97	101,787.60
361	130,321	47,045,881	19.0000	7.1204	.002770083	1,134.11	102,353.87
362	131,044	47,437,928	19.0263	7.1269	.002762431	1,137.26	102,921.72
363	131,769	47,832,147	19.0526	7.1335	.002754821	1,140.40	103,491.13
364	132,496	48,228,544	19.0788	7.1400	.002747253	1,143.54	104,062.12
365	133,225	48,627,125	19.1050	7.1466	.002739726	1,146.68	104,634.67
366	133,956	49,027,896	19.1311	7.1531	.002732240	1,149.82	105,208.80
367	134,689	49,430,863	19.1572	7.1596	.002724796	1,152.96	105,784.49
368	135,424	49,836,032	19.1833	7.1661	.002717391	1,156.11	106,361.76
369	136,161	50,243,409	19.2094	7.1726	.002710027	1,159.25	106,940.60
370	136,900	50,653,000	19.2354	7.1791	.002702703	1,162.39	107,521.01

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
371	137,641	51,064,811	19.2614	7.1855	.002695418	1,165.53	108,102.99
372	138,384	51,478,848	19.2873	7.1920	.002688172	1,168.67	108,686.54
373	139,129	51,895,117	19.3132	7.1984	.002680965	1,171.81	109,271.66
374	139,876	52,313,624	19.3391	7.2048	.002673797	1,174.96	109,858.35
375	140,625	52,734,375	19.3649	7.2112	.002666667	1,178.10	110,446.62
376	141,376	53,157,376	19.3907	7.2177	.002659574	1,181.24	111,036.45
377	142,129	53,582,633	19.4165	7.2240	.002652520	1,184.38	111,627.86
378	142,884	54,010,152	19.4422	7.2304	.002645503	1,187.52	112,220.83
379	143,641	54,439,939	19.4679	7.2368	.002638521	1,190.66	112,815.38
380	144,400	54,872,000	19.4936	7.2432	.002631579	1,193.81	113,411.49
381	145,161	55,306,341	19.5192	7.2495	.002624672	1,196.95	114,009.18
382	145,924	55,742,968	19.5448	7.2558	.002617801	1,200.09	114,608.44
383	146,689	56,181,887	19.5704	7.2622	.002610966	1,203.23	115,209.27
384	147,456	56,623,104	19.5959	7.2685	.002604167	1,206.37	115,811.67
385	148,225	57,066,625	19.6214	7.2748	.002597403	1,209.51	116,415.64
386	148,996	57,512,456	19.6469	7.2811	.002590674	1,212.65	117,021.18
387	149,769	57,960,603	19.6723	7.2874	.002583979	1,215.80	117,628.30
388	150,544	58,411,072	19.6977	7.2936	.002577320	1,218.94	118,236.98
389	151,321	58,863,869	19.7231	7.2999	.002570694	1,222.08	118,847.24
390	152,100	59,319,000	19.7484	7.3061	.002564103	1,225.22	119,459.06
391	152,881	59,776,471	19.7737	7.3124	.002557545	1,228.36	120,072.46
392	153,664	60,236,288	19.7990	7.3186	.002551020	1,231.50	120,687.42
393	154,449	60,698,457	19.8242	7.3248	.002544529	1,234.65	121,303.96
394	155,236	61,162,984	19.8494	7.3310	.002538071	1,237.79	121,922.07
395	156,025	61,629,875	19.8746	7.3372	.002531646	1,240.93	122,541.75
396	156,816	62,099,136	19.8997	7.3434	.002525253	1,244.07	123,163.00
397	157,609	62,570,773	19.9249	7.3496	.002518892	1,247.21	123,785.82
398	158,404	63,044,792	19.9499	7.3558	.002512563	1,250.35	124,410.21
399	159,201	63,521,199	19.9750	7.3619	.002506266	1,253.50	125,036.17
400	160,000	64,000,000	20.0000	7.3681	.002500000	1,256.64	125,663.71
401	160,801	64,481,201	20.0250	7.3742	.002493766	1,259.78	126,292.81
402	161,604	64,964,808	20.0499	7.3803	.002487562	1,262.92	126,923.48
403	162,409	65,450,827	20.0749	7.3864	.002481390	1,266.06	127,555.73
404	163,216	65,939,264	20.0998	7.3925	.002475248	1,269.20	128,189.55
405	164,025	66,430,125	20.1246	7.3986	.002469136	1.272.35	128,824.93
406	164,836	66,923,416	20.1494	7.4047	.002463054	1,275.49	129,461.89
407	165,649	67,419,143	20.1742	7.4108	.002457002	1,278.63	130,100.42
408	166,464	67,917,312	20.1990	7.4169	.002450980	1,281.77	130,740.52
409	167,281	68,417,929	20.2237	7.4229	.002444988	1,284.91	131,382.19
410	168,100	68,921,000	20.2485	7.4290	.002439024	1,288.05	132,025.43
411	168,921	69,426,531	20.2731	7.4350	.002433090	1,291.19	132,670.24
412	169,744	69,934,528	20.2978	7.4410	.002427184	1,294.34	133,316.63
413	170,569	70,444,997	20.3224	7.4470	.002421308	1,297.48	133,964.58
414	171,396	70,957,944	20.3470	7.4530	.002415459	1,300.62	134,614.10
415	172,225	71,473,375	20.3715	7.4590	.002409639	1,303.76	135,265.20
416	173,056	71,991,296	20.3961	7.4650	.002406846	1,306.90	135,917.86
417	173,889	72,511,713	20.4206	7.4710	.002398082	1,310.04	136,572.10
418	174,724	73,034,632	20.4450	7.4770	.002392344	1,313.19	137,227.91
419	175,561	73,560,059	20.4695	7.4829	.002386635	1,316.33	137,885.29
420	176,400	74,088,000	20.4939	7.4889	.002380952	1,319.47	138,544.24
421	177,241	74,618,461	20.5183	7.4948	.002375297	1,322.61	139,204.76
422	178,084	75,151,448	20.5426	7.5007	.002369668	1,325.75	139,866.85
423	178,929	75,686,967	20.5670	7.5067	.002364066	1,328.89	140,530.51
424	179,776	76,225,024	20.5913	7.5126	.002358491	1,332.04	141,195.74
425	180,625	76,765,625	20.6155	7.5185	.002352941	1,335.18	141,862.54
426	181,476	77,308,776	20.6398	7.5244	.002347418	1,338.32	142,530.92
427	182,329	77,854,483	20.6640	7.5302	.002341920	1,341.46	143,200.86
428	183,184	78,402,752	20.6882	7.5361	.002336449	1,344.60	143,872.38
429	184,041	78,953,589	20.7123	7.5420	.002331002	1,347.74	144,545.46
430	184,900	79,507,000	20.7364	7.5478	.002325581	1,350.88	145,220.12
431	185,761	80,062,991	20.7605	7.5537	.002320186	1,354.03	145,896.35
432	186,624	80,621,568	20.7846	7.5595	.002314815	1,357.17	146,574.15
433	187,489	81,182,737	20.8087	7.5654	.002309469	1,360.31	147,253.52

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
434	188,356	81,746,504	20.8327	7.5712	.002304147	1,363.45	147,934.46
435	189,225	82,312,875	20.8567	7.5770	.002298851	1,366.59	148,616.97
436	190,096	82,881,856	20.8806	7.5828	.002293578	1,369.73	149,301.05
437	190,969	83,453,453	20.9045	7.5886	.002288330	1,372.88	149,986.70
438	191,844	84,027,672	20.9284	7.5944	.002283105	1,376.02	150,673.93
439	192,721	84,604,519	20.9523	7.6001	.002277904	1,379.16	151,362.72
440	193,600	85,184,000	20.9762	7.6059	.002272727	1,382.30	152,053.08
441	194,481	85,766,121	21.0000	7.6117	.002267574	1,385.44	152,745.02
442	195,364	86,350,888	21.0238	7.6174	.002262443	1,388.58	153,438.53
443	196,249	86,938,307	21.0476	7.6232	.002257336	1,391.73	154,133.60
444	197,136	87,528,384	21.0713	7.6289	.002252252	1,394.87	154,830.25
445	198,025	88,121,125	21.0950	7.6346	.002247191	1,398.01	155,528.47
446	198,916	88,716,536	21.1187	7.6403	.002242152	1,401.15	156,228.26
447	199,809	89,314,623	21.1424	7.6460	.002237136	1,404.29	156,929.62
448	200,704	89,915,392	21.1660	7.6517	.002232143	1,407.43	157,632.55
449	201,601	90,518,849	21.1896	7.6574	.002227171	1,410.58	158,337.06
450	202,500	91,125,000	21.2132	7.6631	.002222222	1,413.72	159,043.13
451	203,401	91,733,851	21.2368	7.6688	.002217295	1,416.86	159,750.77
452	204,304	92,345,408	21.2603	7.6744	.002212389	1,420.00	160,459.99
453	205,209	92,959,677	21.2838	7.6801	.002207506	1,423.14	161,170.77
454	206,116	93,576,664	21.3073	7.6857	.002202643	1,426.28	161,883.13
455	207,025	94,196,375	21.3307	7.6914	.002197802	1,429.42	162,597.05
456	207,936	94,818,816	21.3542	7.6970	.002192982	1,432.57	163,312.55
457	208,849	95,443,993	21.3776	7.7026	.002188184	1,435.71	164,029.62
458	209,764	96,071,912	21.4009	7.7082	.002183406	1,438.85	164,748.26
459	210,681	96,702,579	21.4243	7.7188	.002178649	1,441.99	165,468.47
460	211,600	97,336,000	21.4476	7.7194	.002173913	1,445.13	166,190.25
461	212,521	97,972,181	21.4709	7.7250	.002169197	1,448.27	166,913.60
462	213,444	98,611,128	21.4942	7.7306	.002164502	1,451.42	167,638.53
463	214,369	99,252,847	21.5174	7.7362	.002159827	1,454.56	168,365.02
464	215,296	99,897,344	21.5407	7.7418	.002155172	1,457.70	169,093.08
465	216,225	100,544,625	21.5639	7.7473	.002150538	1,460.84	169,822.72
466	217,156	101,194,696	21.5870	7.7529	.002145923	1,463.98	170,553.92
467	218,089	101,847,563	21.6102	7.7584	.002141328	1,467.12	171,286.70
468	219,024	102,503,232	21.6333	7.7639	.002136752	1,470.27	172,021.05
469	219,961	103,161,709	21.6564	7.7695	.002132196	1,473.41	172,756.97
470	220,900	103,823,000	21.6795	7.7750	.002127660	1,476.55	173,494.45
471	221,841	104,487,111	21.7025	7.7805	.002123142	1,479.69	174,233.51
472	222,784	105,154,048	21.7256	7.7860	.002118644	1,482.83	174,974.14
473	223,729	105,823,817	21.7486	7.7915	.002114165	1,485.97	175,716.35
474	224,676	106,496,424	21.7715	7.7970	.002109705	1,489.11	176,460.12
475	225,625	107,171,875	21.7945	7.8025	.002105263	1,492.26	177,205.46
476	226,576	107,850,176	21.8174	7.8079	.002100840	1,495.40	177,952.37
477	227,529	108,531,333	21.8403	7.8134	.002096486	1,498.54	178,700.86
478	228,484	109,215,352	21.8632	7.8188	.002092050	1,501.68	179,450.91
479	229,441	109,902,239	21.8861	7.8243	.002087683	1,504.82	180,202.54
480	230,400	110,592,000	21.9089	7.8297	.002083333	1,507.96	180,955.74
481	231,361	111,284,641	21.9317	7.8352	.002079002	1,511.11	181,710.50
482	232,324	111,980,168	21.9545	7.8406	.002074689	1,514.25	182,466.84
483	233,289	112,678,587	21.9775	7.8460	.002070393	1.517.39	183,224.75
484	234,256	113,379,904	22.0000	7.8514	.002066116	1,520.53	183,984.23
485	235,225	114,084,125	22.0227	7.8568	.002061856	1,523.67	184,745.28
486	236,196	114,791,256	22.0454	7.8622	.002057613	1,526.81	185,507.90
487	237,169	115,501,303	22.0681	7.8676	.002053388	1,529.96	186,272.10
488	238,144	116,214,272	22.0907	7.8730	.002049180	1,533.10	187,037.86
489	239,121	116,930,169	22.1133	7.8784	.002044990	1,536.24	187,805.19
490	240,100	117,649,000	22.1359	7.8837	.002040816	1,539.38	188,574.10
491	241,081	118,370,771	22.1585	7.8891	.002036660	1,542.52	189,344.57
492	242,064	119,095,488	22.1811	7.8944	.002032520	1,545.66	190,116.62
493	243,049	119,823,157	22.2036	7.8998	.002028398	1,548.81	190,890.24
494	244,036	120,553,784	22.2261	7.9051	.002024291	1,551.95	191,665.43
495	245,025	121,287,375	22.2486	7.9105	.002020292	1,555.09	192,442.18
496	246,016	122,023,936	22.2711	7.9158	.002016129	1,558.23	193,220.51

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
497	247,009	122,763,473	22.2935	7.9211	.002012072	1,561.37	194,000.41
498	248,004	123,505,992	22.3159	7.9264	.002008032	1,564.51	194,781.89
499	249,001	124,251,499	22.3383	7.9317	.002004008	1,567.65	195,564.93
500	250,000	125,000,000	22.3607	7.9370	.002000000	1,570.80	196,349.54
501	251,001	125,751,501	22.3830	7.9423	.001996008	1,573.94	197,135.72
502	252,004	126,506,008	22.4054	7.9476	.001992032	1,577.08	197,923.48
503	253,009	127,263,527	22.4277	7.9528	.001988072	1,580.22	198,712.80
504	254,016	128,024,064	22.4499	7.9581	.001984127	1,583.36	199,503.70
505	255,025	128,787,625	22.4722	7.9634	.001980198	1,586.50	200,296.17
506	256,036	129,554,216	22.4944	7.9686	.001976285	1,589.65	201,090.20
507	257,049	130,323,843	22.5167	7.9739	.001972387	1,592.79	201,885.81
508	258,064	131,096,512	22.5389	7.9791	.001968504	1,595.93	202,682.99
509	259,081	131,872,229	22.5610	7.9843	.001964637	1,599.07	203,481.74
510	260,100	132,651,000	22.5832	7.9895	.001960785	1,602.21	204,282.06
511	261,121	133,432,831	22.6053	7.9948	.001956947	1,605.35	205,083.95
512	262,144	134,217,728	22.6274	8.0000	.001953125	1,608.50	205,887.42
513	263,169	135,005,697	22.6495	8.0052	.001949318	1,611.64	206,692.45
514	264,196	135,796,744	22.6716	8.0104	.001945525	1,614.78	207,499.05
515	265,225	136,590,875	22.6936	8.0156	.001941748	1,617.92	208,307.23
516	266,256	137,388,096	22.7156	8.0208	.001937984	1,621.06	209,116.97
517	267,289	138,188,413	22.7376	8.0260	.001934236	1,624.20	209,928.29
518	268,324	138,991,832	22.7596	8.0311	.001930502	1,627.34	210,741.18
519	269,361	139,798,359	22.7816	8.0363	.001926782	1,630.49	211,555.63
520	270,400	140,608,000	22.8035	8.0415	.001923077	1,633.63	212,371.66
521	271,441	141,420,761	22.8254	8.0466	.001919386	1,636.77	213,189.26
522	272,484	142,236,648	22.8473	8.0517	.001915709	1,639.91	214,008.43
523	273,529	143,055,667	22.8692	8.0569	.001912046	1,643.05	214,829.17
524	274,576	143,877,824	22.8910	8.0620	.001908397	1,646.19	215,651.49
525	275,625	144,703,125	22.9129	8.0671	.001904762	1,649.34	216,475.37
526	276,676	145,531,576	22.9347	8.0723	.001901141	1,652.48	217,300.82
527	277,729	146,363,183	22.9565	8.0774	.001897533	1,655.62	218,127.85
528	278,784	147,197,952	22.9783	8.0825	.001893939	1,658.76	218,956.44
529	279,841	148,035,889	23.0000	8.0876	.001890359	1,661.90	219,786.61
530	280,900	148,877,001	23.0217	8.0927	.001886792	1,665.04	220,618.34
531	281,961	149,721,291	23.0434	8.0978	.001883239	1,668.19	221,451.65
532	283,024	150,568,768	23.0651	8.1028	.001879699	1,671.33	222,286.53
533	284,089	151,419,437	23.0868	8.1079	.001876173	1,674.47	223,122.98
534	285,156	152,273,304	23.1084	8.1130	.001872659	1,677.61	223,961.00
535	286,225	153,130,375	23.1301	8.1180	.001869159	1,680.75	224,800.59
536	287,296	153,990,656	23.1517	8.1231	.001865672	1,683.89	225,641.75
537	288,369	154,854,153	23.1733	8.1281	.001862197	1,687.04	226,484.48
538	289,444	155,720,872	23.1948	8.1332	.001858736	1,690.18	227,328.79
539	290,521	156,590,819	23.2164	8.1382	.001855288	1,693.32	228,174.66
540	291,600	157,464,000	23.2379	8.1433	.001851852	1,696.46	229,022.10
541	292,681	158,340,421	23.2594	8.1483	.001848429	1,699.60	229,871.12
542	293,764	159,220,088	23.2809	8.1533	.001845018	1,702.74	230,721.71
543	294,849	160,103,007	23.3024	8.1583	.001841621	1,705.88	231,573.86
544	295,936	160,989,184	23.3238	8.1633	.001838235	1,709.03	232,427.59
545	297,025	161,878,625	23.3452	8.1683	.001834862	1,712.17	233,282.89
546	298,116	162,771,336	23.3666	8.1733	.001831502	1,715.31	234,139.76
547	299,209	163,667,323	23.3880	8.1783	.001828154	1,718.45	234,998.20
548	300,304	164,566,592	23.4094	8.1833	.001824818	1,721.59	235,858.21
549	301,401	165,469,149	23.4307	8.1882	.001821494	1,724.73	236,719.79
550	302,500	166,375,000	23.4521	8.1932	.001818182	1,727.88	237,582.94
551	303,601	167,284,151	23.4734	8.1982	.001814882	1,731.02	238,447.67
552	304,704	168,196,608	23.4947	8.2031	.001811594	1,734.16	239,313.96
553	305,809	169,112,377	23.5160	8.2081	.001808318	1,737.30	240,181.83
554	306,916	170,031,464	23.5372	8.2130	.001805054	1,740.44	241,051.26
555	308,025	170,953,875	23.5584	8.2180	.001801802	1,743.58	241,922.27
556	309,136	171,879,616	23.5797	8.2229	.001798561	1,746.73	242,794.85
557	310,249	172,808,693	23.6008	8.2278	.001795332	1,749.87	243,668.99
558	311,364	173,741,112	23.6220	8.2327	.001792115	1,753.01	244,544.71
559	312,481	174,676,879	23.6432	8.2377	.001788909	1,756.15	245,422.00

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
560	313,600	175,616,000	23.6643	8.2426	.001785714	1,759.29	246,300.86
561	314,721	176,558,481	23.6854	8.2475	.001782531	1,762.43	247,181.30
562	315,844	177,504,328	23.7065	8.2524	.001779359	1,765.58	248,063.30
563	316,969	178,453,547	23.7276	8.2573	.001776199	1,768.72	248,946.87
564	318,096	179,406,144	23.7487	8.2621	.001773050	1,771.86	249,832.01
565	319,225	180,362,125	23.7697	8.2670	.001769912	1,775.00	250,718.73
566	320,356	181,321,496	23.7908	8.2719	.001766784	1,778.14	251,607.01
567	321,489	182,284,263	23.8118	8.2768	.001763668	1,781.28	252,496.87
568	322,624	183,250,432	23.8328	8.2816	.001760563	1,784.42	253,388.30
569	323,761	184,220,009	23.8537	8.2865	.001757469	1,787.57	254,281.29
570	324,900	185,193,000	23.8747	8.2913	.001754386	1,790.71	255,175.86
571	326,041	186,169,411	23.8956	8.2962	.001751313	1,793.85	256,072.00
572	327,184	187,149,248	23.9165	8.3010	.001748252	1,796.99	256,969.71
573	328,329	188,132,517	23.9374	8.3059	.001745201	1,800.13	257,868.99
574	329,476	189,119,224	23.9583	8.3107	.001742164	1,803.27	258,769.85
575	330,625	190,109,375	23.9792	8.3155	.001739130	1,806.42	259,672.27
576	331,776	191,102,976	24.0000	8.3203	.001736111	1,809.56	260,576.26
577	332,929	192,100,033	24.0208	8.3251	.001733102	1,812.70	261,481.83
578	334,084	193,100,552	24.0416	8.3300	.001730104	1,815.84	262,388.96
579	335,241	194,104,539	24.0624	8.3348	.001727116	1,818.98	263,297.67
580	336,400	195,112,000	24.0832	8.3396	.001724138	1,822.12	264,207.94
581	337,561	196,122,941	24.1039	8.3443	.001721170	1,825.27	265,119.79
582	338,724	197,137,368	24.1247	8.3491	.001718213	1,828.41	266,033.21
583	339,889	198,155,287	24.1454	8.3539	.001715266	1,831.55	266,948.20
584	341,056	199,176,704	24.1661	8.3587	.001712329	1,834.69	267,864.76
585	342,225	200,201,625	24.1868	8.3634	.001709402	1,837.83	268,782.89
586	343,396	201,230,056	24.2074	8.3682	.001706485	1,840.97	269,702.59
587	344,569	202,262,003	24.2281	8.3730	.001703578	1,844.11	270,623.86
588	345,744	203,297,472	24.2487	8.3777	.001700680	1,847.26	271,546.70
589	346,921	204,336,469	24.2693	8.3825	.001697793	1,850.40	272,471.12
590	348,100	205,379,000	24.2899	8.3872	.001694915	1,853.54	273,397.10
591	349,281	206,425,071	24.3105	8.3919	.001692047	1,856.68	274,324.66
592	350,464	207,474,688	24.3311	8.3967	.001689189	1,859.82	275,253.78
593	351,649	208,527,857	24.3516	8.4014	.001686341	1,862.96	276,184.48
594	352,836	209,584,584	24.3721	8.4061	.001683502	1,866.11	277,116.75
595	354,025	210,644,875	24.3926	8.4108	.001680672	1,869.25	278,050.58
596	355,216	211,708,736	24.4131	8.4155	.001677852	1,872.39	278,985.99
597	356,409	212,776,173	24.4336	8.4202	.001675042	1,875.53	279.922.97
598	357,604	213,847,192	24.4540	8.4249	.001672241	1,878.67	280,861.52
599	358,801	214,921,799	24.4745	8.4296	.001669449	1,881.81	281,801.65
600	360,000	216,000,000	24.4949	8.4343	.001666667	1,884.96	282,743.34
601	361,201	217,081,801	24.5153	8.4390	.001663894	1,888.10	283,686.60
602	362,404	218,167,208	24.5357	8.4437	.001661130	1,891.24	284,631.44
603	363,609	219,256,227	24.5561	8.4484	.001658375	1,894.38	285,577.84
604	364,816	220,348,864	24.5764	8.4530	.001655629	1,897.52	286,525.82
605	366,025	221,445,125	24.5968	8.4577	.001652893	1,900.66	287,475.36
606	367,236	222,545,016	24.6171	8.4623	.001650165	1,903.81	288,426.48
607	368,449	223,648,543	24.6374	8.4670	.001647446	1,906.95	289,379.17
608	369,664	224,755,712	24.6577	8.4716	.001644737	1,910.09	290,333.43
609	370,881	225,866,529	24.6779	8.4763	.001642036	1,913.23	291,289.26
610	372,100	226,981,000	24.6982	8.4809	.001639344	1,916.37	292,246.66
611	373,321	228,099,131	24.7184	8.4856	.001636661	1,919.51	293,205.63
612	374,544	229,220,928	24.7386	8.4902	.001633987	1,922.65	294,166.17
613	375,769	230,346,397	24.7588	8.4948	.001631321	1,925.80	295,128.28
614	376,996	231,475,544	24.7790	8.4994	.001628664	1,928.94	296,091.97
615	378,225	232,608,375	24.7992	8.5040	.001626016	1,932.08	297,057.22
616	379,456	233,744,896	24.8193	8.5086	.001623377	1,935.22	298,024.05
617	380,689	234,885,113	24.8395	8.5132	.001620746	1,938.36	298,992.44
618	381,924	236,029,032	24.8596	8.5178	.001618123	1,941.50	299,962.41
619	383,161	237,176,659	24.8797	8.5224	.001615509	1,944.65	300,933.95
620	384,400	238,328,000	24.8998	8.5270	.001612903	1,947.79	301,907.05
621	385,641	239,483,061	24.9199	8.5316	.001610306	1,950.93	302,881.73
622	386,884	240,641,848	24.9399	8.5362	.001607717	1,954.07	303,857.98

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
623	388,129	241,804,367	24.9600	8.5408	.001605136	1,957.21	304,835.80
624	389,376	242,970,624	24.9800	8.5453	.001602564	1,960.35	305,815.20
625	390,625	244,140,625	25.0000	8.5499	.001600000	1,963.50	306,796.16
626	391,876	245,314,376	25.0200	8.5544	.001597444	1,966.64	307,778.69
627	393,129	246,491,883	25.0400	8.5589	.001594896	1,969.78	308,762.79
628	394,384	247,673,152	25.0599	8.5635	.001592357	1,972.92	309,748.47
629	395,641	248,858,189	25.0799	8.5681	.001589825	1,976.06	310,735.71
630	396,900	250,047,000	25.0998	8.5726	.001587302	1,979.20	311,724.53
631	398,161	251,239,591	25.1197	8.5772	.001584786	1,982.35	312,714.92
632	399,424	252,435,968	25.1396	8.5817	.001582278	1,985.49	313,706.88
633	400,689	253,636,137	25.1595	8.5862	.001579779	1,988.63	314,700.40
634	401,956	254,840,104	25.1794	8.5907	.001577287	1,991.77	315,695.50
635	403,225	256,047,875	25.1992	8.5952	.001574803	1,994.91	316,692.17
636	404,496	257,259,456	25.2190	8.5997	.001572327	1,998.05	317,690.42
637	405,769	258,474,853	25.2389	8.6043	.001569859	2,001.19	318,690.23
638	407,044	259,694,072	25.2587	8.6088	.001567398	2,004.34	319,691.61
639	408,321	260,917,119	25.2784	8.6132	.001564945	2,007.48	320,694.56
640	409,600	262,144,000	25.2982	8.6177	.001562500	2,010.62	321,699.09
641	410,881	263,374,721	25.3180	8.6222	.001560062	2,013.76	322.705.18
642	412,164	264,609,288	25.3377	8.6267	.001557632	2,016.90	323,712.85
643	413,449	265,847,707	25.3574	8.6312	.001555210	2,020.04	324,722.09
644	414,736	267,089,984	25.3772	8.6357	.001552795	2,023.19	325,732.89
645	416,125	268,336,125	25.3969	8.6401	.001550388	2,026.33	326,745.27
646	417,316	269,585,136	25.4165	8.6446	.001547988	2,029.47	327,759.22
647	418,609	270,840,023	25.4362	8.6490	.001545595	2,032.61	328,774.74
648	419,904	272,097,792	25.4558	8.6535	.001543210	2,035.75	329,791.83
649	421,201	273,359,449	25.4755	8.6579	.001540832	2,038.89	330,810.49
650	422,500	274,625,000	25.4951	8.6624	.001538462	2,042.04	331,830.72
651	423,801	275,894,451	25.5147	8.6668	.001536098	2,045.18	332,852.53
652	425,104	277,167,808	25.5343	8.6713	.001533742	2,048.32	333,875.90
653	426,409	278,445,077	25.5539	8.6757	.001531394	2,051.46	334,900.85
654	427,716	279,726,264	25.5734	8.6801	.001529052	2,054.60	335,927.36
655	429,025	281,011,375	25.5930	8.6845	.001526718	2,057.74	336,955.45
656	430,336	282,300,416	25.6125	8.6890	.001524390	2,060.88	337,985.10
657	431,639	283,593,393	25.6320	8.6934	.001522070	2,064.03	339,016.33
658	432,964	284,890,312	25.6515	8.6978	.001519751	2,067.17	340,049.13
659	434,281	286,191,179	25.6710	8.7022	.001517451	2,070.31	341,083.50
660	435,600	287,496,000	25.6905	8.7066	.001515152	2,073.45	342,119.44
661	436,921	288,804,781	25.7099	8.7110	.001512859	2,076.59	343,156.95
662	438,244	290,117,528	25.7294	8.7154	.001510574	2,079.73	344,196.03
663	439,569	291,434,247	25.7488	8.7198	.001508296	2,082.88	345,236.69
664	440,896	292,754,944	25.7682	8.7241	.001506024	2,086.02	346,278.91
665	442,225	294,079,625	25.7876	8.7285	.001503759	2,089.16	347,322.70
666	443,556	295,408,296	25.8070	8.7329	.001501502	2,092.30	348,368.07
667	444,899	296,740,963	25.8263	8.7373	.001499250	2,095.44	349,415.00
668	446,224	298,077,632	25.8457	8.7416	.001497006	2,098.58	350,463.51
669	447,561	299,418,309	25.8650	8.7460	.001494768	2,101.73	351,513.59
670	448,900	300,763,000	25.8844	8.7503	.001492537	2,104.87	352,565.24
671	450,241	302,111,711	25.9037	8.7547	.001490313	2,108.01	353,618.45
672	451,584	303,464,448	25.9230	8.7590	.001488095	2,111.15	354,673.24
673	452,929	304,821,217	25.9422	8.7634	.001485884	2,114.29	355,729.60
674	454,276	306,182,024	25.9615	8.7677	.001483680	2,117.43	356,787.54
675	455,625	307,546,875	25.9808	8.7721	.001481481	2,120.58	357,847.04
676	456,976	308,915,776	26.0000	8.7764	.001479290	2,123.72	358,908.11
677	458,329	310,288,733	26.0192	8.7807	.001477105	2,126.86	359,970.75
678	459,684	311,665,752	26.0384	8.7850	.001474926	2,130.00	361,034.97
679	461,041	313,046,839	26.0576	8.7893	.001472754	2,133.14	362,100.75
680	462,400	314,432,000	26.0768	8.7937	.001470588	2,136.28	363,168.11
681	463,761	315,821,241	26.0960	8.7980	.001468429	2,139.42	364,237.04
682	465,124	317,214,568	26.1151	8.8023	.001466276	2,142.57	365,307.54
683	466,489	318,611,987	26.1343	8.8066	.001464129	2,145.71	366,379.60
684	467,856	320,013,504	26.1534	8.8109	.001461988	2,148.85	367,453.24
685	469,225	321,419,125	26.1725	8.8152	.001459854	2,151.99	368,528.45

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
686	470,596	322,828,856	26.1916	8.8194	.001457726	2,155.13	369,605.23
687	471,969	324,242,703	26.2107	8.8237	.001455604	2,158.27	370,683.59
688	473,344	325,660,672	26.2298	8.8280	.001453488	2,161.42	371,763.51
689	474,721	327,082,769	26.2488	8.8323	.001451379	2,164.56	372,845.00
690	476,100	328,509,000	26.2679	8.8366	.001449275	2,167.70	373,928.07
691	477,481	329,939,371	26.2869	8.8408	.001447178	2,170.84	375,012.70
692	478,864	331,373,888	26.3059	8.8451	.001445087	2,173.98	376,098.91
693	480,249	332,812,557	26.3249	8.8493	.001443001	2,177.12	377,186.68
694	481,636	334,255,384	26.3439	8.8536	.001440922	2,180.27	378,276.03
695	483,025	335,702,375	26.3629	8.8578	.001438849	2,183.41	379,366.95
696	484,416	337,153,536	26.3818	8.8621	.001436782	2,186.55	380,459.44
697	485,809	338,608,873	26.4008	8.8663	.001434720	2,189.69	381,553.50
698	487,204	340,068,392	26.4197	8.8706	.001432665	2,192.83	382,649.13
699	488,601	341,532,099	26.4386	8.8748	.001430615	2,195.97	383,746.33
700	490,000	343,000,000	26.4575	8.8790	.001428571	2,199.11	384,845.10
701	491,401	344,472,101	26.4764	8.8833	.001426534	2,202.26	385,945.44
702	492,804	345,948,408	26.4953	8.8875	.001424501	2,205.40	387,047.36
703	494,209	347,428,927	26.5141	8.8917	.001422475	2,208.54	388,150.84
704	495,616	348,913,664	26.5330	8.8959	.001420455	2,211.68	389,255.90
705	497,025	350,402,625	26.5518	8.9001	.001418440	2,214.82	390,362.52
706	498,436	351,895,816	26.5707	8.9043	.001416431	2,217.96	391,470.72
707	499,849	353,393,243	26.5895	8.9085	.001414427	2,221.11	392,580.49
708	501,264	354,894,912	26.6083	8.9127	.001412429	2,224.25	393,691.82
709	502,681	356,400,829	26.6271	8.9169	.001410437	2,227.39	394,804.73
710	504,100	357,911,000	26.6458	8.9211	.001408451	2,230.53	395,919.21
711	505,521	359,425,431	26.6646	8.9253	.001406470	2,233.67	397,035.26
712	506,944	360,944,128	26.6833	8.9295	.001404494	2,236.81	398,152.89
713	508,369	362,467,097	26.7021	8.9337	.001402525	2,239.96	399,272.08
714	509,796	363,994,344	26.7208	8.9378	.001400560	2,243.10	400,392.84
715	511,225	365,525,875	26.7395	8.9420	.001398601	2,246.24	401,515.18
716	512,656	367,061,696	26.7582	8.9462	.001396648	2,249.38	402,639.08
717	514,089	368,601,813	26.7769	8.9503	.001394700	2,252.52	403,764.56
718	515,524	370,146,232	26.7955	8.9545	.001392758	2,255.66	404,891.60
719	516,961	371,694,959	26.8142	8.9587	.001390821	2,258.81	406,020.22
720	518,400	373,248,000	26.8328	8.9628	.001388889	2,261.95	407,150.41
721	519,841	374,805,361	26.8514	8.9670	.001386963	2,265.09	408,282.17
722	521,284	376,367,048	26.8701	8.9711	.001385042	2,268.23	409,415.50
723	522,729	377,933,067	26.8887	8.9752	.001383126	2,271.37	410,550.40
724	524,176	379,503,424	26.9072	8.9794	.001381215	2,274.51	411,686.87
725	525,625	381,078,125	26.9258	8.9835	.001379310	2,277.65	412,824.91
726	527,076	382,657,176	26.9444	8.9876	.001377410	2,280.80	413,964.52
727	528,529	384,240,583	26.9629	8.9918	.001375516	2,283.94	415,105.71
728	529,984	385,828,352	26.9815	8.9959	.001373626	2,287.08	416,248.46
729	531,441	387,420,489	27.0000	9.0000	.001371742	2,290.22	417,392.79
730	532,900	389,017,000	27.0185	9.0041	.001369863	2,293.36	418,538.68
731	534,361	390,617,891	27.0370	9.0082	.001367989	2,296.50	419,686.15
732	535,824	392,223,168	27.0555	9.0123	.001366120	2,299.65	420,835.19
733	537,289	393,832,837	27.0740	9.0164	.001364256	2,302.79	421,985.79
734	538,756	395,446,904	27.0924	9.0205	.001362398	2,305.93	423,137.97
735	540,225	397,065,375	27.1109	9.0246	.001360544	2,309.07	424,291.72
736	541,696	398,688,256	27.1293	9.0287	.001358696	2,312.21	425,447.04
737	543,169	400,315,553	27.1477	9.0328	.001356852	2,315.35	426,603.94
738	544,644	401,947,272	27.1662	9.0369	.001355014	2,318.50	427,762.40
739	546,121	403,583,419	27.1846	9.0410	.001353180	2,321.64	428,922.43
740	547,600	405,224,000	27.2029	9.0450	.001351351	2,324.78	430,084.03
741	549,081	406,869,021	27.2213	9.0491	.001349528	2,327.92	431,247.21
742	550,564	408,518,488	27.2397	9.0532	.001347709	2,331.06	432,411.95
743	552,049	410,172,407	27.2580	9.0572	.001345895	2,334.20	433,578.27
744	553,536	411,830,784	27.2764	9.0613	.001344086	2,337.34	434,746.16
745	555,025	413,493,625	27.2947	9.0654	.001342282	2,340.49	435,915.62
746	556,516	415,160,936	27.3130	9.0694	.001340483	2,343.63	437,086.64
747	558,009	416,832,723	27.3313	9.0735	.001338688	2,346.77	438,259.24
748	559,504	418,508,992	27.3496	9.0775	.001336898	2,349.91	439,433.41

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
749	561,001	420,189,749	27.3679	9.0816	.001335113	2,353.05	440,609.16
750	562,500	421,875,000	27.3861	9.0856	.001333333	2,356.19	441,786.47
751	564,001	423,564,751	27.4044	9.0896	.001331558	2,359.34	442,965.35
752	565,504	425,259,008	27.4226	9.0937	.001329787	2,362.48	444,145.80
753	567,009	426,957,777	27.4408	9.0977	.001328021	2,365.62	445,327.83
754	568,516	428,661,064	27.4591	9.1017	.001326260	2,368.76	446,511.42
755	570,025	430,368,875	27.4773	9.1057	.001324503	2,371.90	447,696.59
756	571,536	432,081,216	27.4955	9.1098	.001322751	2,375.04	448,883.32
757	573,049	433,798,093	27.5136	9.1138	.001321004	2,378.19	450,071.63
758	574,564	435,519,512	27.5318	9.1178	.001319261	2,381.33	451,261.51
759	576,081	437,245,479	27.5500	9.1218	.001317523	2,384.47	452,452.96
760	577,600	438,976,000	27.5681	9.1258	.001315789	2,387.61	453,645.98
761	579,121	440,711,081	27.5862	9.1298	.001314060	2,390.75	454,840.57
762	580,644	442,450,728	27.6043	9.1338	.001312336	2,393.89	456,036.73
763	582,169	444,194,947	27.6225	9.1378	.001310616	2,397.04	457,234.46
764	583,696	445,943,744	27.6405	9.1418	.001308901	2,400.18	458,433.77
765	585,225	447,697,125	27.6586	9.1458	.001307190	2,403.32	459,634.64
766	586,756	449,455,096	27.6767	9.1498	.001305483	2,406.46	460,837.08
767	588,289	451,217,663	27.6948	9.1537	.001303781	2,409.60	462,041.10
768	589,824	452,984,832	27.7128	9.1577	.001302083	2,412.74	463,246.69
769	591,361	454,756,609	27.7308	9.1617	.001300390	2,415.88	464,453.84
770	592,900	456,533,000	27.7489	9.1657	.001298701	2,419.03	465,662.57
771	594,441	458,314,011	27.7669	9.1696	.001297017	2,422.17	466,872.87
772	595,984	460,099,648	27.7849	9.1736	.001295337	2,425.31	468,084.74
773	597,529	461,889,917	27.8029	9.1775	.001293661	2,428.45	469,298.18
774	599,076	463,684,824	27.8209	9.1815	.001291990	2,431.59	470,513.19
775	600,625	465,484,375	27.8388	9.1855	.001290323	2,434.73	471,729.77
776	602,176	467,288,576	27.8568	9.1894	.001288660	2,437.88	472,947.92
777	603,729	469,097,433	27.8747	9.1933	.001287001	2,441.02	474,167.65
778	605,284	470,910,952	27.8927	9.1973	.001285347	2,444.16	475,388.94
779	606,841	472,729,139	27.9106	9.2012	.001283697	2,447.30	476,611.81
780	608,400	474,552,000	27.9285	9.2052	.001282051	2,450.44	477,836.24
781	609,961	476,379,541	27.9464	9.2091	.001280410	2,453.58	479,062.25
782	611,524	478,211,768	27.9643	9.2130	.001278772	2,456.73	480,289.83
783	613,089	480,048,687	27.9821	9.2170	.001277139	2,459.87	481,518.97
784	614,656	481,890,304	28.0000	9.2209	.001275510	2,463.01	482,749.69
785	616,225	483,736,625	28.0179	9.2248	.001273885	2,466.15	483,981.98
786	617,796	485,587,656	28.0357	9.2287	.001272265	2,469.29	485,215.84
787	619,369	487,443,403	28.0535	9.2326	.001270648	2,472.43	486,451.28
788	620,944	489,303,872	28.0713	9.2365	.001269036	2,475.58	487,688.28
789	622,521	491,169,069	28.0891	9.2404	.001267427	2,478.72	488,926.85
790	624,100	493,039,000	28.1069	9.2443	.001265823	2,481.86	490,166.99
791	625,681	494,913,671	28.1247	9.2482	.001264223	2,485.00	491,408.71
792	627,264	496,793,088	28.1425	9.2521	.001262626	2,488.14	492,651.99
793	628,849	498,677,257	28.1603	9.2560	.001261034	2,491.28	493,896.85
794	630,436	500,566,184	28.1780	9.2599	.001259446	2,494.42	495,143.28
795	632,025	502,459,875	28.1957	9.2638	.001257862	2,497.57	496,391.27
796	633,616	504,358,336	28.2135	9.2677	.001256281	2,500.71	497,640.84
797	635,209	506,261,573	28.2312	9.2716	.001254705	2,503.85	498,891.98
798	636,804	508,169,592	28.2489	9.2754	.001253133	2,506.99	500,144.69
799	638,401	510,082,399	28.2666	9.2793	.001251364	2,510.13	501,398.97
800	640,000	512,000,000	28.2843	9.2832	.001250000	2,513.27	502,654.82
801	641,601	513,922,401	28.3019	9.2870	.001248439	2,516.42	503,912.25
802	643,204	515,849,608	28.3196	9.2909	.001246883	2,519.56	505,171.24
803	644,809	517,781,627	28.3373	9.2948	.001245330	2,522.70	506,431.80
804	646,416	519,718,464	28.3549	9.2986	.001243781	2,525.84	507,693.94
805	648,025	521,660,125	28.3725	9.3025	.001242236	2,528.98	508,957.64
806	649,636	523,606,616	28.3901	9.3063	.001240695	2,532.12	510,222.92
807	651,249	525,557,943	28.4077	9.3102	.001239157	2,535.27	511,489.77
808	652,864	527,514,112	28.4253	9.3140	.001237624	2,538.41	512,758.19
809	654,481	529,475,129	28.4429	9.3179	.001236094	2,541.55	514,028.18
810	656,100	531,441,000	28.4605	9.3217	.001234568	2,544.69	515,299.74
811	657,721	533,411,731	28.4781	9.3255	.001233046	2,547.83	516,572.87

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
812	659,344	535,387,328	28.4956	9.3294	.001231527	2,550.97	517,847.57
813	660,969	537,367,797	28.5132	9.3332	.001230012	2,554.11	519,123.84
814	662,596	539,353,144	28.5307	9.3370	.001228501	2,557.26	520,401.68
815	664,225	541,343,375	28.5482	9.3408	.001226994	2,560.40	521,681.10
816	665,856	543,338,496	28.5657	9.3447	.001225490	2,563.54	522,962.08
817	667,489	545,338,513	28.5832	9.3485	.001223990	2,566.68	524,244.63
818	669,124	547,343,432	28.6007	9.3523	.001222494	2,569.82	525,528.76
819	670,761	549,353,259	28.6182	9.3561	.001221001	2,572.96	526,814.46
820	672,400	551,368,000	28.6356	9.3599	.001219512	2,576.11	528,101.73
821	674,041	553,387,661	28.6531	9.3637	.001218027	2,579.25	529,390.56
822	675,584	555,412,248	28.6705	9.3675	.001216545	2,582.39	530,680.97
823	677,329	557,441,767	28.6880	9.3713	.001215067	2,585.53	531,972.95
824	678,976	559,476,224	28.7054	9.3751	.001213592	2,588.67	533,266.50
825	680,625	561,515,625	28.7228	9.3789	.001212121	2,591.81	534,561.62
826	682,276	563,559,976	28.7402	9.3827	.001210654	2,594.96	535,858.32
827	683,929	565,609,283	28.7576	9.3865	.001209190	2,598.10	537,156.58
828	685,584	567,663,552	28.7750	9 3902	.001207729	2,601.24	538,456.41
829	687,241	569,722,789	28.7924	9.3940	.001206273	2,604.38	539,757.82
830	688,900	571,787,000	28.8097	9.3978	.001204819	2,607.52	541,060.79
831	690,561	573,856,191	28.8271	9.4016	.001203369	2,610.66	542,365.34
832	692,224	575,930,368	28.8444	9.4053	.001201923	2,613.81	543,671.46
833	693,889	578,009,537	28.8617	9.4091	.001200480	2,616.95	544,979.15
834	695,556	580,093,704	28.8791	9.4129	.001199041	2,620.09	546,288.40
835	697,225	582,182,875	28.8964	9.4166	.001197605	2,623.23	547,599.23
836	698,896	584,277,056	28.9137	9.4204	.001196172	2,626.37	548,911.63
837	700,569	586,376,253	28.9310	9.4241	.001194743	2,629.51	550,225.61
838	702,244	588,480,472	28.9482	9.4279	.001193317	2,632.65	551,541.15
839	703,921	590,589,719	28.9655	9.4316	.001191895	2,635.80	552,858.26
840	705,600	592,704,000	28.9828	9.4354	.001190476	2,638.94	554,176.94
841	707,281	594,823,321	29.0000	9.4391	.001189061	2,642.08	555,497.20
842	708,964	596,947,688	29.0172	9.4429	.001187648	2,645.22	556,819.02
843	710,649	599,077,107	29.0345	9.4466	.001186240	2,648.36	558,142.42
844	712,336	601,211,584	29.0517	9.4503	.001184834	2,651.50	559,467.39
845	714,025	603,351,125	29.0689	9.4541	.001183432	2,654.65	560,793.92
846	715,716	605,495,736	29.0861	9.4578	.001182033	2,657.79	562,122.03
847	717,409	607,645,423	29.1033	9.4615	.001180638	2,660.93	563,451.71
848	719,104	609,800,192	29.1204	9.4652	.001179245	2,664.07	564,782.96
849	720,801	611,960,049	29.1376	9.4690	.001177856	2,667.21	566,115.78
850	722,500	614,125,000	29.1548	9.4727	.001176471	2,670.35	567,450.17
851	724,201	616,295,051	29.1719	9.4764	.001175088	2,673.50	568,786.14
852	725,904	618,470,208	29.1890	9.4801	.001173709	2,676.64	570,123.67
853	727,609	620,650,477	29.2062	9.4838	.001172333	2,679.78	571,462.77
854	729,316	622,835,864	29.2233	9.4875	.001170960	2,682.92	572,803.45
855	731,025	625,026,375	29.2404	9.4912	.001169591	2,686.06	574,145.69
856	732,736	627,222,016	29.2575	9.4949	.001168224	2,689.20	575,489.51
857	734,449	629,422,793	29.2746	9.4986	.001166861	2,692.34	576,834.90
858	736,164	631,628,712	29.2916	9.5023	.001165501	2,695.49	578,181.85
859	737,881	633,839,779	29.3087	9.5060	.001164144	2,698.63	579,530.38
860	739,600	636,056,000	29.3258	9.5097	.001162791	2,701.77	580,880.48
861	741,321	638,277,381	29.3428	9.5135	.001161440	2,704.91	582,232.15
862	743,044	640,503,928	29.3598	9.5171	.001160093	2,708.05	583,585.39
863	744,769	642,735,647	29.3769	9.5207	.001158749	2,711.19	584,940.20
864	746,496	644,972,544	29.3939	9.5244	.001157407	2,714.34	586,296.59
865	748,225	647,214,625	29.4109	9.5281	.001156069	2,717.48	587,654.54
866	749,956	649,461,896	29.4279	9.5317	.001154734	2,720.62	589,014.07
867	751,689	651,714,363	29.4449	9.5354	.001153403	2,723.76	590,375.16
868	753,424	653,972,032	29.4618	9.5391	.001152074	2,726.90	591,737.83
869	755,161	656,234,909	29.4788	9.5427	.001150748	2,730.04	593,102.06
870	756,900	658,503,000	29.4958	9.5464	.001149425	2,733.19	594,467.87
871	758,641	660,776,311	29.5127	9.5501	.001148106	2,736.33	595,835.25
872	760,384	663,054,848	29.5296	9.5537	.001146789	2,739.47	597,204.20
873	762,129	665,338,617	29.5466	9.5574	.001145475	2,742.61	598,574.72
874	763,876	667,627,624	29.5635	9.5610	.001144165	2,745.75	599,946.81

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
875	765,625	669,921,875	29.5804	9.5647	.001142857	2,748.89	601,320.47
876	767,376	672,221,376	29.5973	9.5683	.001141553	2,752.04	602,695.70
877	769,129	674,526,133	29.6142	9.5719	.001140251	2,755.18	604,072.50
878	770,884	676,836,152	29.6311	9.5756	.001138952	2,758.32	605,450.88
879	772,641	679,151,439	29.6479	9.5792	.001137656	2,761.46	606,830.82
880	774,400	681,472,000	29.6648	9.5828	.001136364	2,764.60	608,212.34
881	776,161	683,797,841	29.6816	9.5865	.001135074	2,767.74	609,595.42
882	777,924	686,128,968	29.6985	9.5901	.001133787	2,770.88	610,980.08
883	779,689	688,465,387	29.7153	9.5937	.001132503	2,774.03	612,366.31
884	781,456	690,807,104	29.7321	9.5973	.001131222	2,777.17	613,754.11
885	783,225	693,154,125	29.7489	9.6010	.001129944	2,780.31	615,143.48
886	784,996	695,506,456	29.7658	9.6046	.001128668	2,783.45	616,534.42
887	786,769	697,864,103	29.7825	9.6082	.001127396	2,786.59	617,926.93
888	788,544	700,227,072	29.7993	9.6118	.001126126	2,789.73	619,321.01
889	790,321	702,595,369	29.8161	9.6154	.001124859	2,792.88	620,716.66
890	792,100	704,969,000	29.8329	9.6190	.001123596	2,796.02	622,113.89
891	793,881	707,347,971	29.8496	9.6226	.001122334	2,799.16	623,512.68
892	795,664	707,932,288	29.8664	9.6262	.001121076	2,802.30	624,913.04
893	797,449	712,121,957	29.8831	9.6298	.001119821	2,805.44	626,314.98
894	799,236	714,516,984	29.8998	9.6334	.001118568	2,808.58	627,718.49
895	801,025	716,917,375	29.9166	9.6370	.001117818	2,811.73	629,123.56
896	802,816	719,323,136	29.9333	9.6406	.001116071	2,814.87	630,530.21
897	804,609	721,734,273	29.9500	9.6442	.001114827	2,818.01	631,938.43
898	806,404	724,150,792	29.9666	9.6477	.001113586	2,821.15	633,348.22
899	808,201	726,572,699	29.9833	9.6513	.001112347	2,824.29	634,759.58
900	810,000	729,000,000	30.0000	9.6549	.001111111	2,827.43	636,172.51
901	811,801	731,432,701	30.0167	9.6585	.001109878	2,830.58	637,587.01
902	813,604	733,870,808	30.0333	9.6620	.001108647	2,833.72	639,003.09
903	815,409	736,314,327	30.0500	9.6656	.001107420	2,836.86	640,420.73
904	817,216	738,763,264	30.0666	9.6692	.001106195	2,840.00	641,839.95
905	819,025	741,217,625	30.0832	9.6727	.001104972	2,843.14	643,260.73
906	820,836	743,677,416	30.0998	9.6763	.001103753	2,846.28	644,683.09
907	822,649	746,142,643	30.1164	9.6799	.001102536	2,849.42	646,107.01
908	824,464	748,613,312	30.1330	9.6834	.001101322	2,852.57	647,532.51
909	826,281	751,089,429	30.1496	9.6870	.001100110	2,855.71	648,959.58
910	828,100	753,571,000	30.1662	9.6905	.001098901	2,858.85	650,388.22
911	829,921	756,058,031	30.1828	9.6941	.001091695	2,861.99	651,818.43
912	831,744	758,550,825	30.1993	9.6976	.001096491	2,865.13	653,250.21
913	833,569	761,048,497	30.2159	9.7012	.001095290	2,868.27	654,683.56
914	835,396	763,551,944	30.2324	9.7047	.001094092	2,871.42	656,118.48
915	837,225	766,060,875	30.2490	9.7082	.001092896	2,874.56	657,554.98
916	839,056	768,575,296	30.2655	9.7118	.001091703	2,877.70	658,993.04
917	840,889	771,095,213	30.2820	9.7153	.001090513	2,880.84	660,432.68
918	842,724	773,620,632	30.2985	9.7188	.001089325	2,883.98	661,873.88
919	844,561	776,151,559	30.3150	9.7224	.001088139	2,887.12	663,316.66
920	846,400	778,688,000	30.3315	9.7259	.001086957	2,890.27	664,761.01
921	848,241	781,229,961	30.3480	9.7294	.001085776	2,893.41	666,206.92
922	850,084	783,777,448	30.3645	9.7329	.001084599	2,896.55	667,654.41
923	851,929	786,330,467	30.3809	9.7364	.001083423	2,899.69	669,103.47
924	853,776	788,889,024	30.3974	9.7400	.001082251	2,902.83	670,554.10
925	855,625	791,453,125	30.4138	9.7435	.001081081	2,905.97	672,006.30
926	857,476	794,022,776	30.4302	9.7470	.001079914	2,909.11	673,460.08
927	859,329	796,597,983	30.4467	9.7505	.001078749	2,912.26	674,915.42
928	861,184	799,178,752	30.4631	9.7540	.001077586	2,915.40	676,372.33
929	863,041	801,765,089	30.4795	9.7575	.001076426	2,918.54	677,830.82
930	864,900	804,357,000	30.4959	9.7610	.001075269	2,921.68	679,290.87
931	866,761	806,954,491	30.5123	9.7645	.001074114	2,924.82	680,752.50
932	868,624	809,557,568	30.5287	9.7680	.001072961	2,927.96	682,215.69
933	870,489	812,166,237	30.5450	9.7715	.001071811	2,931.11	683,680.46
934	872,356	814,780,504	30.5614	9.7750	.001070664	2,934.25	685,146.80
935	874,225	817,400,375	30.5778	9.7785	.001069519	2,937.39	686,614.71
936	876,096	820,025,856	30.5941	9.7829	.001068376	2,940.53	688,084.19
937	877,969	822,656,953	30.6105	9.7854	.001067236	2,943.67	689,555.24

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
938	879,844	825,293,672	30.6268	9.7889	.001066098	2,946.81	691,027.86
939	881,721	827,936,019	30.6431	9.7924	.001064963	2,949.96	692,502.05
940	883,600	830,584,000	30.6594	9.7959	.001063830	2,953.10	693,977.82
941	885,481	833,237,621	30.6757	9.7993	.001062699	2,956.24	695,455.15
942	887,364	835,896,888	30.6920	9.8028	.001061571	2,959.38	696,934.06
943	889,249	838,561,807	30.7083	9.8063	.001060445	2,962.52	698,414.53
944	891,136	841,232,384	30.7246	9.8097	.001059322	2,965.66	699,896.58
945	893,025	843,908,625	30.7409	9.8132	.001058201	2,968.81	701,380.19
946	894,916	846,590,536	30.7571	9.8167	.001057082	2,971.95	702,865.38
947	896,808	849,278,123	30.7734	9.8201	.001055966	2,975.09	704,352.14
948	898,704	851,971,392	30.7896	9.8236	.001054852	2,978.23	705,840.47
949	900,601	854,670,349	30.8058	9.8270	.001053741	2,981.37	707,330.37
950	902,500	857,375,000	30.8221	9.8305	.001052632	2,984.51	708,821.84
951	904,401	860,085,351	30.8383	9.8339	.001051525	2,987.65	710,314.88
952	906,304	862,801,408	30.8545	9.8374	.001050420	2,990.80	711,809.50
953	908,209	865,523,177	30.8707	9.8408	.001049318	2,993.94	713,305.68
954	910,116	868,250,664	30.8869	9.8443	.001048218	2,997.08	714,803.43
955	912,025	870,983,875	30.9031	9.8477	.001047120	3,000.22	716,302.76
956	913,936	873,722,816	30.9192	9.8511	.001046025	3,003.36	717,803.66
957	915,849	876,467,493	30.9354	9.8546	.001044932	3,006.50	719,306.12
958	917,764	879,217,912	30.9516	9.8580	.001043841	3,009.65	720,810.16
959	919,681	881,974,079	30.9677	9.8614	.001042753	3,012.79	722,315.77
960	921,600	884,736,000	30.9839	9.8648	.001041667	3,015.93	723,822.95
961	923,521	887,503,681	31.0000	9.8683	.001040583	3,019.07	725,331.70
962	925,444	890,277,128	31.0161	9.8717	.001039501	3,022.21	726,842.02
963	927,369	893,056,347	31.0322	9.8751	.001038422	3,025.35	728,353.91
964	929,296	895,841,344	31.0483	9.8785	.001037344	3,028.50	729,867.37
965	931,225	898,632,125	31.0644	9.8819	.001036269	3,031.64	731,382.40
966	933,156	901,428,696	31.0805	9.8854	.001035197	3,034.78	732,899.01
967	935,089	904,231,063	31.0966	9.8888	.001034126	3,037.92	734,417.18
968	937,024	907,039,232	31.1127	9.8922	.001033058	3,041.06	735,936.93
969	938,961	909,853,209	31.1288	9.8956	.001031992	3,044.20	737,458.24
970	940,900	912,673,000	31.1448	9.8990	.001030928	3,047.34	738,981.13
971	942,841	915,498,611	31.1609	9.9024	.001029866	3,050.49	740,505.59
972	944,784	918,330,048	31.1769	9.9058	.001028807	3,053.63	742,031.62
973	946,729	921,167,317	31.1929	9.9092	.001027749	3,056.77	743,559.22
974	948,676	924,010,424	31.2090	9.9126	.001026694	3,059.91	745,088.39
975	950,625	926,859,375	31.2250	9.9160	.001025641	3,063.05	746,619.13
976	952,576	929,714,176	31.2410	9.9194	.001024590	3,066.19	748,151.44
977	954,529	932,574,833	31.2570	9.9228	.001023541	3,069.34	749,685.32
978	956,484	935,441,352	31.2730	9.9261	.001022495	3,072.48	751,220.78
979	958,441	938,313,739	31.2890	9.9295	.001021450	3,075.62	752,757.80
980	960,400	941,192,000	31.3050	9.9329	.001020408	3,078.76	754,296.40
981	962,361	944,076,141	31.3209	9.9363	.001019168	3,081.90	755,836.56
982	964,324	946,966,168	31.3369	9.9396	.001018330	3,085.04	757,378.30
983	966,289	949,862,087	31.3528	9.9430	.001017294	3,088.19	758,921.61
984	968,256	952,763,904	31.3688	9.9464	.001016260	3,091.33	760,466.48
985	970,225	955,671,625	31.3847	9.9497	.001015228	3,094.47	762,012.93
986	972,196	958,585,256	31.4006	9.9531	.001014199	3,097.61	763,560.95
987	974,169	961,504,803	31.4166	9.9565	.001013171	3,100.75	765,110.54
988	976,144	964,430,272	31.4325	9.9598	.001012146	3,103.89	766,661.70
989	978,121	967,361,669	31.4484	9.9632	.001011122	3,107.04	768,214.44
990	980,100	970,299,000	31.4643	9.9666	.001010101	3,110.18	769,768.74
991	982,081	973,242,271	31.4802	9.9699	.001009082	3,113.32	771,324.61
992	984,064	976,191,488	31.4960	9.9733	.001008065	3,116.46	772,882.06
993	986,049	979,146,657	31.5119	9.9766	.001007049	3,119.60	774,441.07
994	988,036	982,107,784	31.5278	9.9800	.001006036	3,122.74	776,001.66
995	990,025	985,074,875	31.5436	9.9833	.001005025	3,125.88	777,563.82
996	992,016	988,047,936	31.5595	9.9866	.001004016	3,129.03	779,127.54
997	994,009	991,026,973	31.5753	9.9900	.001003009	3,132.17	780,692.84
998	996,004	994,011,992	31.5911	9.9933	.001002004	3,135.31	782,259.71
999	998,001	997,002,999	31.6070	9.9967	.001001001	3,138.45	783,828.15
1000	1,000,000	1,000,000,000	31.6228	10.0000	.001000000	3,141.59	785,398.16

CIRCUMFERENCES AND AREAS OF CIRCLES

CIRCUMFERENCES AND AREAS OF CIRCLES FROM 1-64 to 100

Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area
$\frac{1}{64}$	.0491	.0002	6	18.8496	28.2744	$13\frac{1}{8}$	41.2335	135.297
$\frac{1}{32}$	.0982	.0008	$6\frac{1}{8}$	19.2423	29.4648	$13\frac{1}{4}$	41.6262	137.887
$\frac{1}{16}$	.1963	.0031	$6\frac{1}{4}$	19.6350	30.6797	$13\frac{3}{8}$	42.0189	140.501
$\frac{1}{8}$	.3927	.0123	$6\frac{3}{8}$	20.0277	31.9191	$13\frac{1}{2}$	42.4116	143.139
$\frac{3}{16}$	.5890	.0276	$6\frac{1}{2}$	20.4204	33.1831	$13\frac{5}{8}$	42.8043	145.802
$\frac{1}{4}$	.7854	.0491	$6\frac{5}{8}$	20.8131	34.4717	$13\frac{3}{4}$	43.1970	148.490
$\frac{5}{16}$	.9817	.0767	$6\frac{3}{4}$	21.2058	35.7848	$13\frac{7}{8}$	43.5897	151.202
$\frac{3}{8}$	1.1781	.1104	$6\frac{7}{8}$	21.5985	37.1224	14	43.9824	153.938
$\frac{7}{16}$	1.3744	.1503	7	21.9912	38.4846	$14\frac{1}{8}$	44.3751	156.700
$\frac{1}{2}$	1.5708	.1963	$7\frac{1}{8}$	22.3839	39.8713	$14\frac{1}{4}$	44.7678	159.485
$\frac{9}{16}$	1.7671	.2485	$7\frac{1}{4}$	22.7766	41.2826	$14\frac{3}{8}$	45.1605	162.296
$\frac{5}{8}$	1.9635	.3068	$7\frac{3}{8}$	23.1693	42.7184	$14\frac{1}{2}$	45.5532	165.130
$\frac{11}{16}$	2.1598	.3712	$7\frac{1}{2}$	23.5620	44.1787	$14\frac{5}{8}$	45.9459	167.990
$\frac{3}{4}$	2.3562	.4418	$7\frac{5}{8}$	23.9547	45.6636	$14\frac{3}{4}$	46.3386	170.874
$\frac{13}{16}$	2.5525	.5185	$7\frac{3}{4}$	24.3474	47.1731	$14\frac{7}{8}$	46.7313	173.782
$\frac{7}{8}$	2.7489	.6013	$7\frac{7}{8}$	24.7401	48.7071	15	47.1240	176.715
$\frac{15}{16}$	2.9452	.6903	8	25.1328	50.2656	$15\frac{1}{8}$	47.5167	179.673
1	3.1416	.7854	$8\frac{1}{8}$	25.5255	51.8487	$15\frac{1}{4}$	47.9094	182.655
$1\frac{1}{8}$	3.5343	.9940	$8\frac{1}{4}$	25.9182	53.4563	$15\frac{3}{8}$	48.3021	185.661
$1\frac{1}{4}$	3.9270	1.2272	$8\frac{3}{8}$	26.3109	55.0884	$15\frac{1}{2}$	48.6948	188.692
$1\frac{3}{8}$	4.3197	1.4849	$8\frac{1}{2}$	26.7036	56.7451	$15\frac{5}{8}$	49.0875	191.748
$1\frac{1}{2}$	4.7124	1.7671	$8\frac{5}{8}$	27.0963	58.4264	$15\frac{3}{4}$	49.4802	194.828
$1\frac{5}{8}$	5.1051	2.0739	$8\frac{3}{4}$	27.4890	60.1322	$15\frac{7}{8}$	49.8729	197.933
$1\frac{3}{4}$	5.4978	2.4053	$8\frac{7}{8}$	27.8817	61.8625	16	50.2656	201.062
$1\frac{7}{8}$	5.8905	2.7612	9	28.2744	63.6174	$16\frac{1}{8}$	50.6583	204.216
2	6.2832	3.1416	$9\frac{1}{8}$	28.6671	65.3968	$16\frac{1}{4}$	51.0510	207.395
$2\frac{1}{8}$	6.6759	3.5466	$9\frac{1}{4}$	29.0598	67.2008	$16\frac{3}{8}$	51.4437	210.598
$2\frac{1}{4}$	7.0686	3.9761	$9\frac{3}{8}$	29.4525	69.0293	$16\frac{1}{2}$	51.8364	213.825
$2\frac{3}{8}$	7.4613	4.4301	$9\frac{1}{2}$	29.8452	70.8823	$16\frac{5}{8}$	52.2291	217.077
$2\frac{1}{2}$	7.8540	4.9087	$9\frac{5}{8}$	30.2379	72.7599	$16\frac{3}{4}$	52.6218	220.354
$2\frac{5}{8}$	8.2467	5.4119	$9\frac{3}{4}$	30.6306	74.6621	$16\frac{7}{8}$	53.0145	223.655
$2\frac{3}{4}$	8.6394	5.9396	$9\frac{7}{8}$	31.0233	76.589	17	53.4072	226.981
$2\frac{7}{8}$	9.0321	6.4918	10	31.4160	78.540	$17\frac{1}{8}$	53.7999	230.331
3	9.4248	7.0686	$10\frac{1}{8}$	31.8087	80.516	$17\frac{1}{4}$	54.1926	233.706
$3\frac{1}{8}$	9.8175	7.6699	$10\frac{1}{4}$	32.2014	82.516	$17\frac{3}{8}$	54.5853	237.105
$3\frac{1}{4}$	10.2102	8.2058	$10\frac{3}{8}$	32.5941	84.541	$17\frac{1}{2}$	54.9780	240.529
$3\frac{3}{8}$	10.6029	8.9462	$10\frac{1}{2}$	32.9868	86.590	$17\frac{5}{8}$	55.3707	243.977
$3\frac{1}{2}$	10.9956	9.6211	$10\frac{5}{8}$	33.3795	88.664	$17\frac{3}{4}$	55.7634	247.450
$3\frac{5}{8}$	11.3883	10.3206	$10\frac{3}{4}$	33.7722	90.763	$17\frac{7}{8}$	56.1561	250.948
$3\frac{3}{4}$	11.7810	11.0447	$10\frac{7}{8}$	34.1649	92.886	18	56.5488	254.470
$3\frac{7}{8}$	12.1737	11.7933	11	34.5576	95.033	$18\frac{1}{8}$	56.9415	258.016
4	12.5664	12.5664	$11\frac{1}{8}$	34.9503	97.205	$18\frac{1}{4}$	57.3342	261.587
$4\frac{1}{8}$	12.9591	13.3641	$11\frac{1}{4}$	35.3430	99.402	$18\frac{3}{8}$	57.7269	265.183
$4\frac{1}{4}$	13.3518	14.1863	$11\frac{3}{8}$	35.7357	101.623	$18\frac{1}{2}$	58.1196	268.803
$4\frac{3}{8}$	13.7445	15.0330	$11\frac{1}{2}$	36.1284	103.869	$18\frac{5}{8}$	58.5123	272.448
$4\frac{1}{2}$	14.1372	15.9043	$11\frac{5}{8}$	36.5211	106.139	$18\frac{3}{4}$	58.9050	276.117
$4\frac{5}{8}$	14.5299	16.8002	$11\frac{3}{4}$	36.9138	108.434	$18\frac{7}{8}$	59.2977	279.811
$4\frac{3}{4}$	14.9226	17.7206	$11\frac{7}{8}$	37.3065	110.754	19	59.6904	283.529
$4\frac{7}{8}$	15.3153	18.6555	12	37.6992	113.098	$19\frac{1}{8}$	60.0831	287.272
5	15.7080	19.6350	$12\frac{1}{8}$	38.0919	115.466	$19\frac{1}{4}$	60.4758	291.040
$5\frac{1}{8}$	16.1007	20.6290	$12\frac{1}{4}$	38.4846	117.859	$19\frac{3}{8}$	60.8685	294.832
$5\frac{1}{4}$	16.4934	21.6476	$12\frac{3}{8}$	38.8773	120.277	$19\frac{1}{2}$	61.2612	298.648
$5\frac{3}{8}$	16.8861	22.6907	$12\frac{1}{2}$	39.2700	122.719	$19\frac{5}{8}$	61.6539	302.489
$5\frac{1}{2}$	17.2788	23.7583	$12\frac{5}{8}$	39.6627	125.185	$19\frac{3}{4}$	62.0466	306.355
$5\frac{5}{8}$	17.6715	24.8505	$12\frac{3}{4}$	40.0554	127.677	$19\frac{7}{8}$	62.4393	310.245
$5\frac{3}{4}$	18.0642	25.9673	$12\frac{7}{8}$	40.4481	130.192	20	62.8320	314.160
$5\frac{7}{8}$	18.4569	27.1086	13	40.8408	132.733	$20\frac{1}{8}$	63.2247	318.099

Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area
20¼	63.6174	322.063	28⅛	88.3575	621.264	36	113.098	1,017.878
20⅜	64.0101	326.051	28¼	88.7502	626.798	36⅛	113.490	1,024.960
20½	64.4028	330.064	28⅜	89.1429	632.357	36¼	113.883	1,032.065
20⅝	64.7955	334.102	28½	89.5356	637.941	36⅜	114.276	1,039.195
20¾	65.1882	338.164	28⅝	89.9283	643.549	36½	114.668	1,046.349
20⅞	65.5809	342.250	28¾	90.3210	649.182	36⅝	115.061	1,053.528
21	65.9736	346.361	28⅞	90.7137	654.840	36¾	115.454	1,060.732
21⅛	66.3663	350.497	29	91.1064	660.521	36⅞	115.846	1,067.960
21¼	66.7590	354.657	29⅛	91.4991	666.228	37	116.239	1,075.213
21⅜	67.1517	358.842	29¼	91.8918	671.959	37⅛	116.632	1,082.490
21½	67.5444	363.051	29⅜	92.2845	677.714	37¼	117.025	1,089.792
21⅝	67.9371	367.285	29½	92.6772	683.494	37⅜	117.417	1,097.118
21¾	68.3298	371.543	29⅝	93.0699	689.299	37½	117.810	1,104.469
21⅞	68.7225	375.826	29¾	93.4626	695.128	37⅝	118.203	1,111.844
22	69.1152	380.134	29⅞	93.8553	700.982	37¾	118.595	1,119.244
22⅛	69.5079	384.466	30	94.2480	706.860	37⅞	118.988	1,126.669
22¼	69.9006	388.822	30⅛	94.6407	712.763	38	119.381	1,134.118
22⅜	70.2933	393.203	30¼	95.0334	718.690	38⅛	119.773	1,141.591
22½	70.6860	397.609	30⅜	95.4261	724.642	38¼	120.166	1,149.089
22⅝	71.0787	402.038	30½	95.8188	730.618	38⅜	120.559	1,156.612
22¾	71.4714	406.494	30⅝	96.2115	736.619	38½	120.952	1,164.159
22⅞	71.8641	410.973	30¾	96.6042	742.645	38⅝	121.344	1,171.731
23	72.2568	415.477	30⅞	96.9969	748.695	38¾	121.737	1,179.327
23⅛	72.6495	420.004	31	97.3896	754.769	38⅞	122.130	1,186.948
23¼	73.0422	424.558	31⅛	97.7823	760.869	39	122.522	1,194.593
23⅜	73.4349	429.135	31¼	98.1750	766.992	39⅛	122.915	1,202.263
23½	73.8276	433.737	31⅜	98.5677	773.140	39¼	123.308	1,209.958
23⅝	74.2203	438.364	31½	98.9604	779.313	39⅜	123.700	1,217.677
23¾	74.6130	443.015	31⅝	99.3531	785.510	39½	124.093	1,225.420
23⅞	75.0057	447.690	31¾	99.7458	791.732	39⅝	124.486	1,233.188
24	75.3984	452.390	31⅞	100.1385	797.979	39¾	124.879	1,240.981
24⅛	75.7911	457.115	32	100.5312	804.250	39⅞	125.271	1,248.798
24¼	76.1838	461.864	32⅛	100.9239	810.545	40	125.664	1,256.640
24⅜	76.5765	466.638	32¼	101.3166	816.865	40⅛	126.057	1,264.510
24½	76.9692	471.436	32⅜	101.7093	823.210	40¼	126.449	1,272.400
24⅝	77.3619	476.259	32½	102.1020	829.579	40⅜	126.842	1,280.310
24¾	77.7546	481.107	32⅝	102.4947	835.972	40½	127.235	1,288.250
24⅞	78.1473	485.979	32¾	102.8874	842.391	40⅝	127.627	1,296.220
25	78.5400	490.875	32⅞	103.280	848.833	40¾	128.020	1,304.210
25⅛	78.9327	495.796	33	103.673	855.301	40⅞	128.413	1,312.220
25¼	79.3254	500.742	33⅛	104.065	861.792	41	128.806	1,320.260
25⅜	79.7181	505.712	33¼	104.458	868.309	41⅛	129.198	1,328.320
25½	80.1108	510.706	33⅜	104.851	874.850	41¼	129.591	1,336.410
25⅝	80.5035	515.726	33½	105.244	881.415	41⅜	129.984	1,344.520
25¾	80.8962	520.769	33⅝	105.636	888.005	41½	130.376	1,352.660
25⅞	81.2889	525.838	33¾	106.029	894.620	41⅝	130.769	1,360.820
26	81.6816	530.930	33⅞	106.422	901.259	41¾	131.162	1,369.000
26⅛	82.0743	536.048	34	106.814	907.922	41⅞	131.554	1,377.210
26¼	82.4670	541.190	34⅛	107.207	914.611	42	131.947	1,385.450
26⅜	82.8597	546.356	34¼	107.600	921.323	42⅛	132.340	1,393.700
26½	83.2524	551.547	34⅜	107.992	928.061	42¼	132.733	1,401.990
26⅝	83.6451	556.763	34½	108.385	934.822	42⅜	133.125	1,410.300
26¾	84.0378	562.003	34⅝	108.778	941.609	42½	133.518	1,418.630
26⅞	84.4305	567.267	34¾	109.171	948.420	42⅝	133.911	1,426.990
27	84.8232	572.557	34⅞	109.563	955.255	42¾	134.303	1,435.370
27⅛	85.2159	577.870	35	109.956	962.115	42⅞	134.696	1,443.770
27¼	85.6086	583.209	35⅛	110.349	969.000	43	135.089	1,452.200
27⅜	86.0013	588.571	35¼	110.741	975.909	43⅛	135.481	1,460.660
27½	86.3940	593.959	35⅜	111.134	982.842	43¼	135.874	1,469.140
27⅝	86.7867	599.371	35½	111.527	989.800	43⅜	136.267	1,477.640
27¾	87.1794	604.807	35⅝	111.919	996.783	43½	136.660	1,486.170
27⅞	87.5721	610.268	35¾	112.312	1,003.790	43⅝	137.052	1,494.730
28	87.9648	615.754	35⅞	112.705	1,010.822	43¾	137.445	1,503.300

Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area
43⅞	137.838	1,511.910	51¾	162.578	2,103.35	59⅝	187.318	2,792.21
44	138.230	1,520.530	51⅞	162.970	2,113.52	59¾	187.711	2,803.93
44⅛	138.623	1,529.190	52	163.363	2,123.72	59⅞	188.103	2,815.67
44¼	139.016	1,537.860	52⅛	163.756	2,133.94	60	188.496	2,827.44
44⅜	139.408	1,546.56	52¼	164.149	2,144.19	60⅛	188.889	2,839.23
44½	139.801	1,555.29	52⅜	164.541	2,154.46	60¼	189.281	2,851.05
44⅝	140.194	1,564.04	52½	164.934	2,164.76	60⅜	189.674	2,862.89
44¾	140.587	1,572.81	52⅝	165.327	2,175.08	60½	190.067	2,874.76
44⅞	140.979	1,581.61	52¾	165.719	2,185.42	60⅝	190.459	2,886.65
45	141.372	1,590.43	52⅞	166.112	2,195.79	60¾	190.852	2,898.57
45⅛	141.765	1,599.28	53	166.505	2,206.19	60⅞	191.245	2,910.51
45¼	142.157	1,608.16	53⅛	166.897	2,216.61	61	191.638	2,922.47
45⅜	142.550	1,617.05	53¼	167.290	2,227.05	61⅛	192.030	2,934.46
45½	142.943	1,625.97	53⅜	167.683	2,237.52	61¼	192.423	2,946.48
45⅝	143.335	1,634.92	53½	168.076	2,248.01	61⅜	192.816	2,958.52
45¾	143.728	1,643.89	53⅝	168.468	2,258.53	61½	193.208	2,970.58
45⅞	144.121	1,652.89	53¾	168.861	2,269.07	61⅝	193.601	2,982.67
46	144.514	1,661.91	53⅞	169.254	2,279.64	61¾	193.994	2,994.78
46⅛	144.906	1,670.95	54	169.646	2,290.23	61⅞	194.386	3,006.92
46¼	145.299	1,680.02	54⅛	170.039	2,300.84	62	194.779	3,019.08
46⅜	145.692	1,689.11	54¼	170.432	2,311.48	62⅛	195.172	3,031.26
46½	146.084	1,698.23	54⅜	170.824	2,322.15	62¼	195.565	3,043.47
46⅝	146.477	1,707.37	54½	171.217	2,332.83	62⅜	195.957	3,055.71
46¾	146.870	1,716.54	54⅝	171.610	2,343.55	62½	196.350	3,067.97
46⅞	147.262	1,725.73	54¾	172.003	2,354.29	62⅝	196.743	3,080.25
47	147.655	1,734.95	54⅞	172.395	2,365.05	62¾	197.135	3,092.56
47⅛	148.048	1,744.19	55	172.788	2,375.83	62⅞	197.528	3,104.89
47¼	148.441	1,753.45	55⅛	173.181	2,386.65	63	197.921	3,117.25
47⅜	148.833	1,762.74	55¼	173.573	2,397.48	63⅛	198.313	3,129.64
47½	149.226	1,772.06	55⅜	173.966	2,408.34	63¼	198.706	3,142.04
47⅝	149.619	1,781.40	55½	174.359	2,419.23	63⅜	199.099	3,154.47
47¾	150.011	1,790.76	55⅝	174.751	2,430.14	63½	199.492	3,166.93
47⅞	150.404	1,800.15	55¾	175.144	2,441.07	63⅝	199.884	3,179.41
48	150.797	1,809.56	55⅞	175.537	2,452.03	63¾	200.277	3,191.91
48⅛	151.189	1,819.00	56	175.930	2,463.01	63⅞	200.670	3,204.44
48¼	151.582	1,828.46	56⅛	176.322	2,474.02	64	201.062	3,217.00
48⅜	151.975	1,837.95	56¼	176.715	2,485.05	64⅛	201.455	3,229.58
48½	152.368	1,847.46	56⅜	177.108	2,496.11	64¼	201.848	3,242.18
48⅝	152.760	1,856.99	56½	177.500	2,507.19	64⅜	202.240	3,254.81
48¾	153.153	1,866.55	56⅝	177.893	2,518.30	64½	202.633	3,267.46
48⅞	153.546	1,876.14	56¾	178.286	2,529.43	64⅝	203.026	3,280.14
49	153.938	1,885.75	56⅞	178.678	2,540.58	64¾	203.419	3,292.84
49⅛	154.331	1,895.38	57	179.071	2,551.76	64⅞	203.811	3,305.56
49¼	154.724	1,905.04	57⅛	179.464	2,562.97	65	204.204	3,318.31
49⅜	155.116	1,914.72	57¼	179.857	2,574.20	65⅛	204.597	3,331.09
49½	155.509	1,924.43	57⅜	180.249	2,585.45	65¼	204.989	3,343.89
49⅝	155.902	1,934.16	57½	180.642	2,596.73	65⅜	205.382	3,356.71
49¾	156.295	1,943.91	57⅝	181.035	2,608.03	65½	205.775	3,369.56
49⅞	156.687	1,953.69	57¾	181.427	2,619.36	65⅝	206.167	3,382.44
50	157.080	1,963.50	57⅞	181.820	2,630.71	65¾	206.560	3,395.33
50⅛	157.473	1,973.33	58	182.213	2,642.09	65⅞	206 953	3,408.26
50¼	157.865	1,983.18	58⅛	182.605	2,653.49	66	207.346	3,421.20
50⅜	158.258	1,993.06	58¼	182.998	2,664.91	66⅛	207.738	3,434.17
50½	158.651	2,002.97	58⅜	183.391	2,676.36	66¼	208.131	3,447.17
50⅝	159.043	2,012.89	58½	183.784	2,687.84	66⅜	208.524	3,460.19
50¾	159.436	2,022.85	58⅝	184.176	2,699.33	66½	208.916	3,473.24
50⅞	159.829	2,032.82	58¾	184.569	2,710.86	66⅝	209.309	3,486.30
51	160.222	2,042.83	58⅞	184.962	2,722.41	66¾	209.702	3,499.40
51⅛	160.614	2,052.85	59	185.354	2,733.98	66⅞	210.094	3,512.52
51¼	161.007	2,062.90	59⅛	185.747	2,745.57	67	210.487	3,525.66
51⅜	161.400	2,072.98	59¼	186.140	2,757.20	67⅛	210.880	3,538.83
51½	161.792	2,083.08	59⅜	186.532	2,768.84	67¼	211.273	3,552.02
51⅝	162.185	2,093.20	59½	186.925	2,780.51	67⅜	211.665	3,565.24

Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area
67½	212.058	3,578.48	75⅜	236.798	4,462.16	83¼	261.538	5,443.26
67⅝	212.451	3,591.74	75½	237.191	4,476.98	83⅜	261.931	5,459.62
67¾	212.843	3,605.04	75⅝	237.583	4,491.81	83½	262.324	5,476.01
67⅞	213.236	3,618.35	75¾	237.976	4,506.67	83⅝	262.716	5,492.41
68	213.629	3,631.69	75⅞	238.369	4,521.56	83¾	263.109	5,508.84
68⅛	214.021	3,645.05	76	238.762	4,536.47	83⅞	263.502	5,525.30
68¼	214.414	3,658.44	76⅛	239.154	4,551.41	84	263 894	5,541.78
68⅜	214.807	3,671.86	76¼	239.547	4,566.36	84⅛	264.287	5,558.29
68½	215.200	3,685.29	76⅜	239.940	4,581.35	84¼	264.680	5,574.82
68⅝	215.592	3,698.76	76½	240.332	4,596.36	84⅜	265.072	5,591.37
68¾	215.985	3,712.24	76⅝	240.725	4,611.39	84½	265.465	5,607.95
68⅞	216.378	3,725.75	76¾	241.118	4,626.45	84⅝	265.858	5,624.56
69	216.770	3,739.29	76⅞	241.510	4,641.53	84¾	266.251	5,641.18
69⅛	217.163	3,752.85	77	241.903	4,656.64	84⅞	266.643	5,657.84
69¼	217.556	3,766.43	77⅛	242.296	4,671.77	85	267.036	5,674.51
69⅜	217.948	3,780.04	77¼	242.689	4,686.92	85⅛	267.429	5,691.22
69½	218.341	3,793.68	77⅜	243.081	4,702.10	85¼	267.821	5,707.94
69⅝	218.734	3,807.34	77½	243.474	4,717.31	85⅜	268.214	5,724.69
69¾	219.127	3,821.02	77⅝	243.867	4,732.54	85½	268.607	5,741.47
69⅞	219.519	3,834.73	77¾	244.259	4,747.79	85⅝	268.999	5,758.27
70	219.912	3,848.46	77⅞	244.652	4,763.07	85¾	269.392	5,775.10
70⅛	220.305	3,862.22	78	245.045	4,778.37	85⅞	269.785	5,791.94
70¼	220.697	3,876.00	78⅛	245.437	4,793.70	86	270.178	5,808.82
70⅜	221.090	3,889.80	78¼	245.830	4,809.05	86⅛	270.570	5,825.72
70½	221.483	3,903.63	78⅜	246.223	4,824.43	86¼	270.963	5,842.64
70⅝	221.875	3,917.49	78½	246.616	4,839.83	86⅜	271.356	5,859.59
70¾	222.268	3,931.37	78⅝	247.008	4,855.26	86½	271.748	5,876.56
70⅞	222.661	3,945.27	78¾	247.401	4,870.71	86⅝	272.141	5,893.55
71	223.054	3,959.20	78⅞	247.794	4,886.18	86¾	272.534	5,910.58
71⅛	223.416	3,973.15	79	248.186	4,901.68	86⅞	272.926	5,927.62
71¼	223.839	3,987.13	79⅛	248.579	4,917.21	87	273.319	5,944.69
71⅜	224.232	4,001.13	79¼	248.972	4,932.75	87⅛	273.712	5,961.79
71½	224.624	4,015.16	79⅜	249.364	4,948.33	87¼	274.105	5,978.91
71⅝	225.017	4,029.21	79½	249.757	4,963.92	87⅜	274.497	5,996.05
71¾	225.410	4,043.29	79⅝	250.150	4,979.55	87½	274.890	6,013.22
71⅞	225.802	4,057.39	79¾	250.543	4,995.19	87⅝	275.283	6,030.41
72	226.195	4,071.51	79⅞	250.935	5,010.86	87¾	275.675	6,047.63
72⅛	226.588	4,085.66	80	251.328	5,026.56	87⅞	276.068	6,064.87
72¼	226.981	4,099.84	80⅛	251.721	5,042.28	88	276.461	6,082.14
72⅜	227.373	4,114.04	80¼	252.113	5,058.03	88⅛	276.853	6,099.43
72½	227.766	4,128.26	80⅜	252.506	5,073.79	88¼	277.246	6,116.74
72⅝	228.159	4,142.51	80½	252.899	5,089.59	88⅜	277.629	6,134.08
72¾	228.551	4,156.78	80⅝	253.291	5,105.41	88½	278.032	6,151.45
72⅞	228.944	4,171.08	80¾	253.684	5,121.25	88⅝	278.424	6,168.84
73	229.337	4,185.40	80⅞	254.077	5,137.12	88¾	278.817	6,186.25
73⅛	229.729	4,199.74	81	254.470	5,153.01	88⅞	279.210	6,203.69
73¼	230.122	4,214.11	81⅛	254.862	5,168.93	89	279.602	6,221.15
73⅜	230.515	4,228.51	81¼	255.255	5,184.87	89⅛	279.995	6,238.64
73½	230.908	4,242.93	81⅜	255.648	5,200.83	89¼	280.388	6,256.15
73⅝	231.300	4,257.37	81½	256.040	5,216.82	89⅜	280.780	6,273.69
73¾	231.693	4,271.84	81⅝	256.433	5,232.84	89½	281.173	6,291.25
73⅞	232.086	4,286.33	81¾	256.826	5,248.88	89⅝	281.566	6,308.84
74	232.478	4,300.85	81⅞	257.218	5,264.94	89¾	281.959	6,326.45
74⅛	232.871	4,315.39	82	257.611	5,281.03	89⅞	282.351	6,344.08
74¼	233.264	4,329.96	82⅛	258.004	5,297.14	90	282.744	6,361.74
74⅜	233.656	4,344.55	82¼	258.397	5,313.28	90⅛	283.137	6,379.42
74½	234.049	4,359.17	82⅜	258.789	5,329.44	90¼	283.529	6,397.13
74⅝	234.442	4,373.81	82½	259.182	5,345.63	90⅜	283.922	6,414.86
74¾	234.835	4,388.47	82⅝	259.575	5,361.84	90½	284.315	6,432.62
74⅞	235.227	4,403.16	82¾	259.967	5,378.08	90⅝	284.707	6,450.40
75	235.620	4,417.87	82⅞	260.360	5,394.34	90¾	285.100	6,468.21
75⅛	236.013	4,432.61	83	250.753	5,410.62	90⅞	285.493	6,486.04
75¼	236.405	4,447.38	83⅛	261.145	5,426.93	91	285.886	6,503.90

Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area
91⅛	286.278	6,521.78	94⅛	295.703	6,958.26	97⅛	305.128	7,408.89
91¼	286.671	6,539.68	94¼	296.096	6,976.76	97¼	305.521	7,427.97
91⅜	287.064	6,557.61	94⅜	296.488	6,995.28	97⅜	305.913	7,447.08
91½	287.456	6,575.56	94½	296.881	7,013.82	97½	306.306	7,466.21
91⅝	287.849	6,593.54	94⅝	297.274	7,032.39	97⅝	306.699	7,485.37
91¾	288.242	6,611.55	94¾	297.667	7,050.98	97¾	307.091	7,504.55
91⅞	288.634	6,629.57	94⅞	298.059	7,069.59	97⅞	307.484	7,523.75
92	289.027	6,647.63	95	298.452	7,088.24	98	307.877	7,542.98
92⅛	289.420	6,665.70	95⅛	298.845	7,106.90	98⅛	308.270	7,562.24
92¼	289.813	6,683.80	95¼	299.237	7,125.59	98¼	308.662	7,581.52
92⅜	290.205	6,701.93	95⅜	299.630	7,144.31	98⅜	309.055	7,600.82
92½	290.598	6,720.08	95½	300.023	7,163.04	98½	309.448	7,620.15
92⅝	290.991	6,738.25	95⅝	300.415	7,181.81	98⅝	309.840	7,639.50
92¾	291.383	6,756.45	95¾	300.808	7,200.60	98¾	310.233	7,658.88
92⅞	291.776	6,774.68	95⅞	301.201	7,219.41	98⅞	310.626	7,678.28
93	292.169	6,792.92	96	301.594	7,238.25	99	311.018	7,697.71
93⅛	292.562	6,811.20	96⅛	301.986	7,257.11	99⅛	311.411	7,717.16
93¼	292.954	6,829.49	96¼	302.379	7,275.99	99¼	311.804	7,736.63
93⅜	293.347	6,847.82	96⅜	302.772	7,294.91	99⅜	312.196	7,756.13
93½	293.740	6,866.16	96½	303.164	7,313.84	99½	312.589	7,775.66
93⅝	294.132	6,884.53	96⅝	303.557	7,332.80	99⅝	312.982	7,795.21
93¾	294.525	6,902.93	96¾	303.950	7,351.79	99¾	313.375	7,814.78
93⅞	294.918	6,921.35	96⅞	304.342	7,370.79	99⅞	313.767	7,834.38
94	295.310	6,939.79	97	304.735	7,389.83	100	314.160	7,854.00

DENOMINATE NUMBERS

A **denominate number** is one expressed in units of a certain kind; as, for example, 5 days, 8 men, etc.

A **compound denominate number** is one expressed in two or more units; as 3 hr. 20 min., 8-ton mi., 4-acre-ft., etc. The terms ft. per sec., mi. per hr., rev. per min., etc., are all compound units.

An **abstract number** is any number not expressed in units of a kind; as 3, 5, 8, etc.

Kinds of Units.—The principal kinds of units may be classed as follows:

1. Units of **weight**; as tons, pounds, ounces, grains, etc.
2. Units of **length** or distance; as miles, feet, inches, etc.
3. Units of **volume**; as cubic yards, cubic feet, etc.
4. Units of **capacity**; as gallons, quarts, pints, etc.
5. Units of **surface** or area; as square miles, square feet, etc.
6. Units of **time**; as years, months, days, hours, etc.
7. Units of **circular measure**; as degrees, minutes, etc.
8. Units of **currency**; as dollars, dimes, cents, etc.

WEIGHTS AND MEASURES

Systems in Use.—There are two systems of weights and measures in general use, known as the "English, United States or British," and the "French or metric" systems.

The **basis of comparison** of the English and French systems is expressed by the following established values:

Weight.—The pound (7,000 grs.) is the same in the United States and Great Britain. The pound avoirdupois is equal to 453.5924277 grams in the French system.

Length.—(United States) The length of the meter, by act of Congress, is 39.37 in. (Great Britain) The length of the meter, by act of Parliament, is 39.37079 in.

The slight difference in the length of the meter, as established by law in the United States and in Great Britain, makes the English inch and yard proportionally shorter than the same units in the United States.

Capacity.—The gallon and liter are the accepted units of comparison in the English and French systems, respectively. The United States or "Winchester gallon," however, is quite different from the "Imperial gallon" of Great Britain, which was made the volume of 10 lb. of distilled water, at maximum density (4 deg. C.), weighed with brass weights in air at 62 deg. F., barometer 30 in.

Since 1 cu. in. pure water, under the same conditions, weighs 252.458

grs. and 1 lb. = 7,000 grs., the volume of the imperial gallon of Great Britain is

$$\frac{10 \times 7000}{252.458} = 277.274 \text{ cu. in.}$$

The volume of the Winchester gallon of the United States is 231 cu. in. The French liter is the volume of 1 kg. of distilled water, at 4 deg. C., weighed in a vacuum, or 1,000 c.c., which gives

Winchester gallon (United States), 231 cu. in. = 3.78543 liters.
Imperial gallon (Great Britain), 277.274 cu. in. = 4.54346 liters.

UNITED STATES AND BRITISH SYSTEMS

Following are the more useful of the tables of weights and measures in the English system:

Avoirdupois Weight

(United States)

16 drams	= 1 ounce	437.5	pounds
16 ounces	= 1 pound	7,000	grains
25 pounds	= 1 quarter	400	ounces
4 quarters	= 1 hundredweight	100	pounds
20 hundredweight	= 1 short ton	2,000	pounds

(Great Britian)

28 pounds	= 1 quarter	448	ounces
4 quarters	= 1 hundredweight	112	pounds
20 hundredweight	= 1 long ton	2,240	pounds

The short ton (2,000 lb.) is more generally used in the United States, although the long ton (2240 lb.) is used at times.

Troy Weight

24 grains	= 1 pennyweight	
20 pennyweights	= 1 ounce	480 grains
12 ounces	= 1 pound	5,760 grains

Apothecaries Weight

20 grains	= 1 scruple	
3 scruples	= 1 dram	60 grains
8 drams	= 1 ounce	480 grains
12 ounces	= 1 pound	5,760 grains

The grain (troy) is the same as the grain (apothecaries) and is the basis of comparison of these and avoirdupois weights. Thus,

1 lb. avoirdupois = 7,000/5,760 = 1.21528 lb. troy.
1 lb. troy = 5,760/7,000 = 0.822857 lb. avoirdupois.
1 oz. avoirdupois = 437.5/480 = 0.911458 oz. troy.
1 oz. troy = 480/437.5 = 1.097143 oz. avoirdupois.

Long Measure

12 inches	= 1 foot	
3 feet	= 1 yard	36 inches
$5\frac{1}{2}$ yards	= 1 rod, perch, or pole	$16\frac{1}{2}$ feet
40 rods	= 1 furlong	660 feet
8 furlongs	= 1 mile	5,280 feet
3 miles	= 1 league	

The old surveyor's chain of 100 links (1 link = 7.92 in.) was 66 ft. long, making 80 chains = 1 mi. Chains now in common use are 50,100 and 300 ft. long, made up of 1-ft. links.

A fathom is 6 ft. or 2 yd., used in estimating depth.

Square Measure

144 sq. inches	= 1 square foot	
9 square feet	= 1 square yard	1296 square inches
$30\frac{1}{4}$ square yards	= 1 square rod	$272\frac{1}{4}$ square feet
40 square rods	= 1 rood	10,890 square feet
4 roods	= 1 acre	43,560 square feet
640 acres	= 1 square mile	102,400 square rods

An acre contains 43,560 sq. ft. and measures 208.7 ft. on each side; $\sqrt{43,560} = 208.7$ ft.

Cubic Measure

1728 cubic inches	= 1 cubic foot	
27 cubic feet	= 1 cubic yard	46,656 cubic inches
16 cubic feet	= 1 cord foot	27,648 cubic inches
8 cord feet	= 1 cord	128 cubic feet

A **cord of wood** is a pile 8 ft. long, 4 ft. wide and 4 ft. high, and contains $8 \times 4 \times 4 = 128$ cu. ft.

A **cord foot** is one foot of the length of the pile that makes a cord, and contains $1 \times 4 \times 4 = 16$ cu. ft.

A ton of **round timber** (green) is taken as 50 cu. ft.

A ton of **squared timber** (green) is 40 cu. ft., it being assumed that hewed or squared timber has lost one-fifth of its original volume in squaring.

A long ton (2,240 lb.) of **anthracite** or a short ton (2,000 lb.) of bituminous coal broken (mine-run) occupies about 40 cu. ft.

There are two measures of capacity, known as "Liquid" and "Dry" measures, having like denominations but of different values. The old English wine gallon (231 cu. in.) was replaced in England, in 1824, by the imperial gallon (277.274 cu. in.), but is still the standard "Winchester" gallon in the United States. The "Dry" gallon, now practically obsolete, contained 268.8 cu. in.

Liquid Measure (U. S.)

4 gills	= 1 pint	28.875	cubic inches
2 pints	= 1 quart	57.75	cubic inches
4 quarts	= 1 gallon	231	cubic inches
31½ gallons	= 1 barrel	4.21	cubic feet
2 barrels	= 1 hogshead	63	gallons
2 hogsheads	= 1 pipe	126	gallons
2 pipes	= 1 tun	8	barrels

Dry Measure (U. S.)

2 pints	= 1 quart	67.2	cubic inches
8 quarts	= 1 peck	537.6	cubic inches
4 pecks	= 1 bushel	2150.4	cubic inches
36 bushels	= 1 chaldron	44.8	cubic feet
Or, 4 quarts	= 1 gallon	268.8	cubic inches
8 gallons	= 1 bushel	2150.4	cubic inches

The standard bushel, in the United States, is the old Winchester bushel, which is a circular measure 18½ in. in diameter and 8 in. deep, containing $8\ (0.7854 \times 18.5^2) = 2150.4$ cu. in. This was replaced in England, in 1826, by the imperial bushel (2218.192 cu. in.), which was then made the legal bushel.

Liquid and Dry Measure (Great Britain)

4 gills	= 1 pint	34.659	cubic inches
2 pints	= 1 quart	69.318	cubic inches
4 quarts	= 1 gallon	277.274	cubic inches
2 gallons	= 1 peck	554.548	cubic inches
4 pecks	= 1 bushel	2218.192	cubic inches

There is no separate standard for liquid and dry measures in Great Britain, both being referred to the same unit or standard, which is the imperial gallon (277.274 cu. in.).

Measure of Time

60 seconds	= 1 minute
60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week
365 days	= 1 common year
366 days	= 1 leap year
12 calendar months	= 1 calendar year
100 years	= 1 century

Commonly speaking, **a day** is marked by one complete revolution of the earth on its axis, and **a year** by one revolution of the earth in its orbit about the sun. Unfortunately, however, the earth does not make an even number of turns on its axis, while making one complete revolution in its orbit. There are approximately 365¼ revolutions on the axis to a single revolution in the orbit.

In order to compensate for this eccentricity and make the calendar year conform as closely as possible to the solar year, so as to preserve uniformity in the return of the seasons, it was necessary to add one day to the calendar every fourth year, except the closing year of the century. Thus, the **common year** of 365 days was supplemented by a **leap year** containing 366 days.

The "Gregorian" calendar, established by Pope Gregory XIII (1582) and generally adopted in Great Britain and elsewhere (1752), replaced the "Julian" calendar and, in dropping 10 days by making Oct. 5, Oct. 15, 1582, restored the equinoxes to their proper date. To obtain closer correspondence of the calendar and solar years, the closing year of each century, 1600, 1700, etc., was made a common year, although these would be leap years in the regular course.

The Day.—A day is the interval of time marked by two successive transits of a heavenly body across a given meridian, caused by the revolution of the earth on its axis.

The **solar day** (24 hr., 0 min.) is the time interval marked by two successive transits of the sun across the meridian.

The **sidereal day** (23 hr., 56 min.) is the time interval marked by two successive transits of a fixed star across a given meridian.

The Month.—The calendar year has been arbitrarily divided into 12 months, in correspondence to the "number of moons" or the revolutions of the moon about the earth in a solar year. But, since 365 days are not equally divisible by 12, it was necessary to make an unequal division, as follows:

January	31 days	May	31 days	September	30 days
February	28 days	June	30 days	October	31 days
March	31 days	July	31 days	November	30 days
April	30 days	August	31 days	December	31 days

The extra day required in a leap year is added to the month of February, making 29 days in that month every leap year, instead of 28 as in the common year.

The Year.—A year is the period of time in which the earth completes one revolution in its orbit.

The **solar year** (365 d., 5 hr., 48 min., 45.51 sec.) marks a complete revolution about the sun.

The **sidereal year** (365 d., 6 hr., 9 min., 8.97 sec.) marks a complete revolution with respect to a fixed star.

Circular Measure

60 seconds	= 1 minute	
60 minutes	= 1 degree	3,600 seconds
15 degrees	= 1 hour angle	900 minutes
30 degrees	= 1 sign	1,800 minutes
12 signs	= 1 great circle or circumference	360 degrees

The "**sign**" is one of the twelve divisions of the zodiac, which correspond to the twelve calendar months of the year. The sign has no practical value technically.

It is often convenient to express the length of an arc, or the angle it subtends, in terms of the radius of the circle. In that case, the unit of length is called a "radian." A **radian** is a length of arc equal to the describing radius. Its value expressed in degrees is $180° \div \pi = 180/3.14159 = 57.2958$ deg., or $57° 17' 44.88''$. Since the length of the circumference of a circle is $2\pi r$, there are 2π radians in a circumference or 360 deg.

Circular measure is used in the measurement of angles and in the estimation of **latitude, longitude** and **solar** or **sun time,** which varies from **standard time** according to the location of the observer.

Measurement of Time.—The passing of time is measured by the revolution of the earth on its axis, as determined by the observation of the sun or one of the fixed stars when crossing the meridian of a place. A single revolution of the earth marks a period of 24 hr. or one day.

Sun Time.—Owing to the inclination of the earth's axis to the plane of its orbit and the eccentricity of the orbit, the sun's apparent motion in the celestial sphere is not wholly uniform, on which account **solar time** is referred to a "**mean sun**" having an assumed uniform motion.

Equation of Time.—The difference between the mean sun and the true or observed sun, expressed in hours, minutes and seconds, is called the "equation of time." This is found for any date in the "Ephemeris" or Nautical Almanac.

Sidereal Time.—The apparent movement of the fixed stars, unlike that of the sun, is uniform, which makes the sidereal day correspond precisely with one complete revolution of the earth on its axis. About Mar. 21, or at the vernal equinox, sidereal time agrees with mean sun or solar time.

Local Time.—When the 24-hr. cycle is referred to the local meridian as zero (noon or midnight) the indicated hour is the local time, or the time for that place only. Since there are 360 deg. in a circle, which marks 1 day or 24 hr. of the celestial equator, 1 hr. corresponds to $360 \div 24 = 15$ deg. Hence, a difference of 15 deg. marks a difference of 1 hr. in local time.

Longitude, Latitude.—Longitude is the distance either east or west of the **meridian of Greenwich,** which is marked by the Royal Observatory, and measured in degrees, minutes and seconds, on the equator. There are thus 180 deg. of **east longitude** and 180 deg. of **west longitude.**

Latitude is likewise distance north or south of the equator, measured in degrees, minutes and seconds, on any meridian or great circle passing through the poles. There are thus 90 deg. of north latitude and 90 deg. of south latitude.

Standard Time.—To obviate the confusion caused by the difference in local time, a system of "standard time" has been adopted. Starting from the meridian of Greenwich, standard time is 1 hr. later for each 15 deg. of east longitude, and 1 hr. earlier for each 15 deg. of west longitude. Calling the equatorial circumference of the earth 25,000 mi., a degree of longitude represents a distance on the equator of 25,000 ÷ 360 = 69.4 mi. One hour (15 deg.) corresponds to a distance of practically 1,000 mi. at the equator.

In the United States and Canada, there are four divisions of standard time, known as **Eastern, Central, Mountain** and **Pacific time,** which are exactly 1 hr. apart. These are all referred to the observatory at Greenwich, which marks the zero of longitude.

Eastern time is the solar time of the meridian 75 deg. west longitude, and is the standard time for all places within 7½ deg. on either side of that meridian. Eastern time is therefore 75 ÷ 15 = 5 hr. earlier than Greenwich time.

Central time is solar time for the meridian 90 deg. west longitude, and is likewise standard for all places within 7½ deg. east or west of that meridian. Central time is 1 hr. earlier than Eastern time.

Mountain time is solar time for the meridian 105 deg. west longitude and standard for all places within 7½ deg. east or west of that meridian. Mountain time is 1 hr. earlier than Central time.

Pacific time is solar time for the meridian 120 deg. west longitude and standard for all places within 7½ deg. east or west of that meridian. Pacific time is 1 hr. earlier than Mountain time.

When it is noon at the observatory at Greenwich it is 7 a.m. at New York, 6 a.m. at Chicago, 5. a.m. at Denver and 4 a.m. at San Francisco. At the same time it is 1 p.m. at Berlin and Rome, 2 p.m. at Petrograd and 8 p.m. in the Philippines.

Civil Time.—The day, for all common purposes of reckoning, begins and ends at midnight. The 24 hr. are divided into two periods of 12 hr. each. The hours from midnight to noon are designated by the letters a.m. (ante meridian), and those from noon to midnight by the letters p.m. (post meridian).

Astronomical Time.—The astronomical day is reckoned from noon to noon, the hours being counted from 1 to 24. The astronomical day begins 12 hr. later than the civil day, as the following comparisons will show:

Civil time,	Nov. 6, 3 a.m.; Nov. 6, 3 p.m.; Nov. 7, 3 a.m.
Astronomical time,	Nov. 5, 15 hr.; Nov. 6, 3 hr.; Nov. 6, 15 hr

METRIC SYSTEM OF WEIGHTS AND MEASURES

The units of the metric system are the **gram, meter** and **liter.** The system, unlike that of the United States and Great Britain is wholly a **decimal system** and, for that reason, is more convenient for use.

Denominations.—The higher denominations of weight, length and capacity are obtained by multiplying each respective unit by 10, 100, 1000, etc., while lower denominations than the unit are likewise obtained by dividing the same by 10, 100 or 1000.

The denominations of the metric system are expressed by the Latin and Greek prefixes, the former being used to indicate divisions of the unit, while the latter are employed to express multiples of the same unit. These prefixes and their respective values are as follows:

Milli,	1/1000	1 milligram (mg.)	=	0.001	gram
Centi,	1/100	1 centigram (cg.)	=	0.01	gram
Deci,	1/10	1 decigram (dg.)	=	0.1	gram
		Unit of Weight		**1**	**gram**
Deca,	10	1 decagram	=	10	grams
Hecto,	100	1 hectogram	=	100	grams
Kilo,	1000	1 kilogram (kg.)	=	1000	grams
Myria,	10,000	1 myriagram	=	10,000	grams

The same prefixes are used to express similar divisions and multiples of the units of length and capacity. Area and volume are expressed by the words square and cubic preceding the same denominations of length. Following are the tables of the metric system and equivalents:

METRIC WEIGHT

10 milligrams	= 1 centigram	0.15432356	gr. (troy)
10 centigrams	= 1 decigram	1.54323564	gr.
10 decigrams	= 1 **gram**	15.43235639	gr.
		0.03527396	oz. (avdp.)
10 grams	= 1 decagram	0.35273957	oz.
10 decagrams	= 1 hectogram	3.52739575	oz.
10 hectograms	= 1 kilogram	35.27395746	oz.
		2.20462234	lb.
10 kilograms	= 1 myriagram	22.04622341	lb.
		0.22046223	cwt.
10 myriagrams	= 1 quintal	2.20462234	cwt.
10 quintals	= 1 tonne	1.10231117	tons

The French tonne (2204.6 lb.) differs but slightly from the British long ton (2240 lb.)

METRIC LENGTH

10 millimeters	= 1 centimeter	0.3937	inches
10 centimeters	= 1 decimeter	3.937	inches
10 decimeters	= 1 **meter**	39.37	inches
		3.2808	feet
10 meters	= 1 decameter	32.8083	feet
10 decameters	= 1 hectometer	328.0833	feet
		0.0621	miles
10 hectometers	= 1 kilometer	0.6214	miles

The Austrian, Prussian, Danish and Norwegian mile is equal to about 4.7 American miles; the Swedish, to about 6⅔ American miles; while the Russian "verst" is 3500 ft.

Metric Area

100 sq. millimeters	= 1 sq. centimeter	0.155	sq. in.
100 sq. centimeters	= 1 sq. decimeter	15.500	sq. in.
100 sq. decimeters	= 1 **sq. meter (centare)**	1549.997	sq. in.
		10.764	sq. ft.
100 centares	= 1 sq. decameter (are)	1076.387	sq. ft.
		0.025	acres
100 ares	= 1 sq. hectometer (hectare)	2.471	acres
100 hectares	= 1 sq. kilometer	247.104	acres
		0.386	sq. mi.
100 sq. kilometers	= 1 sq. myriameter	38.610	sq. mi.

The unit of area is the square meter or centare.

Metric Volume

1000 cu. millimeters	= 1 cu. centimeter	0.061	cu. in.
1000 cu. centimeters	= 1 cu. decimeter	61.023	cu. in.
1000 cu. decimeters	= 1 cu. meter	35.314	cu. ft.
		1.308	cu. yd.

The weight of 1 cu. centimeter of distilled water at maximum density (4°C.), weighed in a vacuum, is 1 gram; or 1 cu. decimeter of same under like conditions is 1 kilogram.

Metric Capacity

10 milliliters	= 1 centiliter	0.610	cu. in.
10 centiliters	= 1 deciliter	6.102	cu. in.
10 deciliters	= 1 **liter**	61.023	cu. in.
		0.035	cu. ft.
10 liters	= 1 decaliter (centistere)	0.353	cu. ft.
10 centisteres	= 1 hectoliter (decistere)	3.531	cu. ft.
10 decisteres	= 1 kiloliter (stere)	35.314	cu. ft.
10 steres	= 1 myrialiter (decastere)	353.145	cu. ft.

The liter is the unit of capacity in the metric system. Its volume is 1000 cu. centimeters or 1 cu. decimeter. It contains 61.02338189 cu. in., or 0.26417 gal. (Winchester). Or a single Winchester gallon contains 3.785434 liters.

The Fluid Ounce.—What is known as the "fluid ounce" is a quantity of any liquid equal to that of pure water at maximum density (4°C.) and weighing exactly 1 oz. avoirdupois. The volume of the fluid ounce is calculated as follows:

1 cubic centimeter of water (4°C.) = 1 gram.
1 ounce avoirdupois = 437.5 grains.
1 gram = 15.43236 grains.

Hence, since the volume of 1 gram (water) is 1 c.c. and the fluid ounce has a volume based similarly on the avoirdupois ounce, the value of the fluid ounce is

Fluid ounce (fl. oz.), $\frac{437.5}{15.43236} = 28.3495$ *c.c.*

The **minim** (a drop), the smallest liquid measure, is $\frac{1}{60}$ of a fluid dram or the equivalent in volume of 1 grain, which is $1 \div 15.43236 = 0.0648$ c.c.; or $28.3495 \div 437.5 = 0.0648$ c.c.

Metric Abbreviations.—The following are the common abbreviations used in the metric system:

Milligram, mg.;	millimeter, mm.;	milliliter, ml.
Centigram, cg.;	centimeter, cm.;	centiliter, cl.
Decigram, dg.;	decimeter, dm.;	deciliter, dl.
Gram, g.;	meter, m.;	liter, l.
Kilogram, kg.;	kilometer, km.;	kiloliter, kl.

Square millimeter, mm^2; cubic millimeter, mm^3.
Square centimeter, cm^2; cubic centimeter, cm^3.
Square decimeter, dm^2; cubic decimeter, dm^3.
Square meter, m^2; cubic meter, m^3.
Square kilometer, km^2.

Compound Units.—It is often convenient to express values involving two or more denominations in terms of a single compound unit. The following are examples of such compound units:

Work is expressed as a force (pounds) exerted through a distance (feet) and its unit, therefore, combines both of these denominations, giving foot-pounds (ft.-lb.), or inch-pounds (in.-lb.), as the case may be.

Power is expressed as work performed per unit of time, as foot-pounds per minute (ft.-lb. p.m.), or per second (ft.-lb. p.s.).

In like manner, the speed of rotation is given in revolutions per minute (r.p.m.); or the speed of a train as miles per hour (mi. p. hr.); or the velocity of an air current as cubic feet per minute (cu. ft. p. m.).

It is common to estimate the value of coal lands in tons per acre, or **acre-tons**; or to express the amount of underlying coal in **acre-feet**, which combines in a single unit both the acreage of the seam and the average thickness of the coal in feet.

CONVERSION TABLES

Numerous forms of tables are in use for converting denominations of the United States system into the corresponding denominations of the metric system and vice versa, but the following are believed to best serve the purpose. For the sake of more ready reference, the denominations of weight, length, area, volume and capacity are here given in separate tables, and the values given in the tables are simple multipliers:

Avoirdupois (Metric to U. S.)

	Drams	Ounces	Pounds	Tons
1 milligram =	0.00056			
1 centigram =	0.0056			
1 decigram =	0.0564			
1 gram =	0.564	0.035	0.0022	
1 decagram =	5.644	0.353	0.022	
1 hectogram =	56.438	3.527	0.220	
1 kilogram =	564.38	35.274	2.205	0.0011
1 myriagram =			22.046	0.0110
1 quintal =			220.46	0.1102
1 tonne =			2204.62	1.1023

When closer determinations are desired the values given in the metric tables should be employed.

Avoirdupois (U. S. to Metric)

	Milligrams	Grams	Kilograms	Tonne
1 dram =	1771.8	1.77		
1 ounce =		28.35	0.02835	
1 pound =		453.59	0.4536	
1 ton =			907.184	0.90718

Troy (Metric to U. S.)

	Grains	Penny weights	Ounces	Pounds
1 milligram =	0.0154			
1 centigram =	0.154	0.006		
1 decigram =	1.54	0.064	0.0032	
1 gram =	15.43	0.643	0.032	
1 decagram =		6.430	0.322	0.0268
1 hectogram =		64.302	3.215	0.2679
1 kilogram =			32.151	2.679
1 myriagram =				26.79

Troy (U. S. to Metric)

	Milligrams	Grams	Kilograms
1 grain =	64.8	0.065	
1 pennyweight =		1.555	
1 ounce =		31.103	0.031
1 pound =			0.373

Apothecaries (Metric to U. S.)

	Grains	Scruples	Drams	Ounces	Pounds
1 milligram =	0.0154				
1 centigram =	0.154	0.0077			
1 decigram =	1.54	0.077	0.026		
1 gram =	15.43	0.772	0.257	0.032	
1 decagram =		7.72	2.57	0.322	
1 hectogram =				3.215	0.268
1 kilogram =				32.15	2.679

APOTHECARIES (U. S. TO METRHC)

	Milligrams	Grams	Kilograms
1 grain =	64.8	0.065	
1 scruple =		1.296	
1 dram =		3.888	
1 ounce =		31.103	0.031
1 pound =			0.373

LINEAR (METRIC TO U. S.)

	Inches	Feet	Yards	Rods	Miles
1 millimeter =	0.039				
1 centimeter =	0.39	0.033			
1 decimeter =	3.94	0.33			
1 meter =	39.37	3.28	1.094	0.199	
1 decameter =		32.81	10.936	1.988	0.0062
1 hectometer =			109.36	19.884	0.0621
1 kilometer =					0.6214
1 myriameter =					6.2137

The old surveyor's chain (66 ft.) contains 20.1168 meters, and one kilometer (3280.83 ft.) is 49.71 of such chains.

LINEAR (U. S. TO METRIC)

	Millimeters	Centimeters	Meters	Kilometers
1 inch =	25.400	2.540	0.0254	
1 foot =	304.800	30.480	0.3048	
1 yard =		91.440	0.914	
1 rod =			5.029	0.005
1 furlong =			201.168	0.201
1 mile =			1609.347	1.609

SQUARE (METRIC TO U. S.)

	Sq. in.	Sq. ft.	Sq. rods	Acres	Sq. mi.
1 sq. millimeter =	0.0015				
1 sq. centimeter =	0.155				
1 sq. decimeter =	15.500	0.108			
1 sq. meter (centare) =		10.764	0.040		
1 sq. decameter (are) =		1076.387	3.954	0.025	
1 sq. hectometer (hectare) =			395.367	2.471	
1 sq. kilometer =				247.104	0.386
1 sq. myriameter =					38.61

Square (U. S. to Metric)

	Sq. mm.	Sq. cm.	Centares	Ares	Hectares
1 sq. inch =	645.16	6.45			
1 sq. foot =		929.03	0.093		
1 sq. yard =			0.836		
1 sq. rod =			25.293	0.253	
1 acre =				40.469	0.405
1 sq. mile =					259.

Cubic (Metric to U. S.)

	Cu. inches	Cu. feet	Cu. yards
1 cu. millimeter =	0.00006		
1 cu. centimeter =	0.06102		
1 cu. decimeter =	61.0235	0.0353	0.0013
1 cu. meter =		35.3145	1.308

Cubic (U. S. to Metric)

	Cu. mm.	Cu. cm.	Cu. dm.	Cu. m.
1 cu. inch =	16,387	16.387	0.016	
1 cu. foot =		28,316.84	28.317	0.028
1 cu. yard =			764.555	0.765

Capacity (Metric to U. S., Liquid)

	Gills	Pints	Quarts	Gallons	Barrels	Hhd.
1 milliliter =	0.008					
1 centiliter =	0.085	0.021				
1 deciliter =	0.845	0.211	0.106			
1 liter =	8.453	2.113	1.057	0.264		
1 decaliter =			10.567	2.642	0.084	
1 hectoliter =				26.417	0.839	0.419
1 kiloliter =				264.170	8.386	4.193
1 myrialiter =					83.864	41.932

One myrialiter contains 10.48295 tuns.

Capacity (Metric to U. S., Dry)

	Pints	Quarts	Gallons	Pecks	Bushels
1 centiliter =	0.018				
1 deciliter =	0.182	0.091			
1 liter =	1.816	0.908	0.227	0.114	0.028
1 centistere =		9.081	2.270	1.135	0.284
1 decistere =			22.702	11.351	2.838
1 stere =					28.378
1 decastere =					283.777

The decastere is equal to 7.88269 chaldrons.

CAPACITY (U. S. TO METRIC)

(Liquid)		Ml.	Cl.	Dl.	L.	Kl.
1 gill	=	118.29	11.829	1.183	0.118	
1 pint	=		47.318	4.732	0.473	
1 quart	=			9.464	0.946	
1 gallon	=			37.854	3.785	
1 barrel	=				119.241	0.119
1 hogshead	=				238.482	0.238
1 pipe	=				476.965	0.477
1 tun					953.929	0.954
(Dry)						
1 pint	=	550.61	55.061	5.506	0.551	
1 quart	=		110.122	11.012	1.101	
1 gallon	=			44.049	4.405	
1 peck	=			88.097	8.810	
1 bushel	=				35.239	0.035
1 chaldron	=					1.269

CAPACITY (METRIC TO BRITISH)

(Wet and dry)		Gills	Pints	Quarts	Gallons	Pecks	Bushels
1 milliliter	=	0.007					
1 centiliter	=	0.070	0.018				
1 deciliter	=	0.704	0.176	0.088	0.022		
1 liter	=	7.043	1.761	0.880	0.220	0.110	0.028
1 decaliter	=			8.803	2.201	1.100	0.275
1 hectoliter	=				22.008	11.004	2.751
1 kiloliter	=				220.083	110.042	27.510
1 myrialiter	=						275.104

CAPACITY (BRITISH TO METRIC)

(Wet and dry)		Ml.	Cl.	Dl.	L.	Kl.
1 gill	=	142.0	14.199	1.420	0.142	
1 pint	=		56.797	5.680	0.568	
1 quart	=			11.359	1.136	
1 gallon	=			45.437	4.544	
1 peck	=			90.875	9.087	
1 bushel	=				36.350	0.036

The conversion factors in these tables have been derived independently from the following standards:

1 meter (U. S.) = 39.37 in. (1 in. = 25.4 mm.);
1 sq. meter = $39.37^2 \div 144$ = 10.76386736 sq. ft.;
1 cu. meter = $39.37^3 \div 1728$ = 35.31445447 cu. ft.;
1 liter = 61.02338189 cu. in.;
1 U. S. (Winchester) bushel = 2150.4 cu. in.;
1 British (Imperial) bushel = 2218.192 cu. in.

CONVERSION OF COMPOUND UNITS

In the conversion of compound units from the United States to the metric system, and vice versa, it is more convenient and saves much time and frequently avoids error arising from confusion of terms to employ a single factor. The following are the more common conversion factors:

Weight per Unit Length

1 lb. per ft.	(0.4536 × 3.28)	= 1.488 kg. per m.
1 lb. per yd.	(0.4536 × 1.0936)	= 0.496 kg. per m.
1 ton per mi.	(0.9072 × 0.6214)	= 0.5637 tonnes per km.
1 long ton per mi.	(1.016 × 0.6214)	= 0.6313 tonnes per km.

Weight per Unit Area

1 lb. per sq. ft.	(0.4536 × 10.764)	= 4.882 kg. per m^2
1 ton per sq. ft.	(0.9072 × 10.764)	= 9.765 tonnes per m^2
1 ton per sq. yd.	(0.9072 × 1.196)	= 1.085 tonnes per m^2
1 ton per acre.	(0.9072 × 2.471)	= 2.2417 tonnes per hectare
1 long ton per acre.	(1.016 × 2.471)	= 2.5105 tonnes per hectare

Weight per Unit Volume

1 oz. per cu. in . . .	(28.35 × 0.06102)	= 1.73 g. per cm^3
1 oz. per cu .ft.	(0.0283 × 35.3145)	= 1.00 kg. per m^3
1 lb. per cu. ft.	(0.4536 × 35.3145)	= 16.0184 kg. per m^3
1 lb. per cu. yd. . . .	(0.4536 × 1.308)	= 0.5933 kg. per m^3
1 ton per cu. yd . .	(0.9072 × 1.308)	= 1.1866 tonnes per m^3
1 ton per acre-ft. . .	(0.9072 × 8.106)	= 7.3538 tonnes per hectare-m.
1 long ton per acre-ft	(1.016 × 8.106)	= 8.2357 tonnes per hectare-m.

It is worthy of note that **ounces per cubic foot** are equivalent to **kilograms per cubic meter,** or **grams per liter,** since 1 m^3 = 1000 liters.

Weight per Unit Capacity—Liquid

1 gr. per gal.—U. S.	(64.8 × 0.264)	= 17.107 mg. per l.
1 oz. per gal.	(28.35 × 0.264)	= 7.484 g. per l.
1 lb. per gal.	(453.59 × 0.264)	= 119.748 g. per l.
1 gr. per gal.—Gt. Br.	(64.8 × 0.22)	= 14.256 mg. per l.
1 oz. per gal.	(28.35 × 0.22)	= 6.237 g. per l.
1 lb. per gal.	(453.59 × 0.22)	= 99.790 g. per l.

Weight per Unit Capacity—Dry

1 lb. per bu.—U. S.	(0.4536 × 28.378)	= 12.872 kg. per stere
1 lb. per bu.—Gt. Bt.	(0.4536 × 27.51)	= 12.479 kg. per stere

Pressure

1 oz. per sq. in.	(28.35 × 0.155)	= 4.394 g. per cm^2
1 lb. per sq. in.	(453.59 × 0.155)	= 70.306 g. per cm^2
1 lb. per sq. ft.	(0.4536 × 10.764)	= 4.882 kg. per m^2

Work

1 inch-pound.........	(2.54 × 453.59)	= 1152.1 gram-centimeters
1 foot-pound.........	(0.3048 × 0.4536)	= 0.1383 kilogram meters
1 ton-pound..........	(0.3048 × 0.9072)	= 0.2765 tonne-meters

Work in Heat Units

1 B.t.u.—778 ft.-lb.............	(778 × 0.1383)	= 107.564 kg.-m.
1 pound-calorie................	(107.564 × 1.8)	= 193.615 kg.-m.
1 calorie.....................	(193.615 × 2.2046)	= 426.844 kg.-m.

Calorific or Heating Value

1 B.t.u. per lb..........	(0.252 × 2.2046)	= 0.55556 cal. per kg.
1 B.t.u. per lb..........	5/9(2.2046)	= 1.22478 lb.-cal. per kg.
1 B.t.u. per cu. ft.......	(0.252 × 35.3145)	= 8.89925 cal. per m^3
1 lb.-cal. per lb.........	(0.4536 × 2.2046)	= 1.00000 cal. per kg.
1 lb.-cal. per lb		= 2.20462 lb.-cal. per kg.
1 lb.-cal. per cu. ft.....	(0.4536 × 35.3145)	= 16.01866 cal. per m^3

Power

The metric horsepower (force de cheval), which for convenience may be abbreviated "**cheval,**" is the power capable of performing 75 kg.-m. of work per second, or 75 × 60 = 4500 kg.-m. per min.

1 horsepower...........	(33,000 × 0.1383)	= 4563.9 kg.-m. per min.
1 horsepower...........	(4563.9 ÷ 4500)	= 1.0142 chevals
1 cheval................	(4500 ÷ 4563.9)	= 0.986 hp.

Power Factors

1 sq. ft. per hp..............	(0.093 × 0.986)	= 0.0937 m^2 per cheval
1 cu. ft. per hp..............	(0.028 × 0.986)	= 0.0276 m^3 per cheval

Fuel or Water Consumption

1 lb. per hp.-hr..........	(0.4536×0.986)	=0.4472 kg. per cheval-hr.
1 ton per hp.-hr.........	(0.9072×0.986)	=0.8945 tonnes per cheval-hr.
1 gal. (U. S.) per hp.-hr..	(3.785×0.986)	=3.7320 liters per cheval-hr.
1 gal. (Gt. Bt.) per hp.-hr.	(4.544×0.986)	=4.4804 liters per cheval-hr.

Evaporation Factors

1 gal. per sq. ft.—U. S......	(3.785 × 10.764)	= 40.7417 l. per $m.^2$
1 gal. per lb. fuel...........	(3.785 × 2.2046)	= 8.3444 l. per kg.
1 gal. per B.t.u.............	(3.785 × 3.968)	= 15.0189 l. per cal.
1 gal. per B.t.u.............	(3.785 × 1.8)	= 6.8130 l. per lb.-cal.
1 gal. per sq. ft.—Gt. Bt.....	(4.544 × 10.764)	= 48.9116 l. per m^2
1 gal. per lb. fuel............	(4.544 × 2.2046)	= 10.0177 l. per kg.
1 gal. per B.t.u.............	(4.544 × 3.968)	= 18.0306 l. per cal.
1 gal. per B.t.u.............	(4.544 × 1.8)	= 8.1792 l. per lb.-cal

Equivalents in Air Measurements

Atmospheric pressure, sea, level, normal, 14.696 lb. per sq. in.

$(14.696 \times 0.0703) = 1.033$ kg. per cm^2

$(14.696 \div 0.4911) = 29.925$ in. mercury

$(29.925 \times 25.4) = 760$ mm. mercury

$\left(\frac{29.925 \times 13.6}{12}\right) = 33.9$ ft. water column

$(33.915 \times 0.3048 = 10.34$ m. water column

The specific gravity of mercury (32 deg. F.) being 13.593, 1 in. barometer (standard reading) corresponds to 13.6 in. water gage and, roughly, to $(13.6 \times 815) \div 12 =$ say 900 ft. air-column.

Pressure, in fan ventilation is frequently expressed in ounces per square inch, instead of in pounds per square inch. The following table giving the equivalent values in these denominations and inches of water gage.

Water gage	Lb. per sq. ft.	Oz. per sq. in.	Water Gage	Lb. per sq. in.	Oz. per sq. in.
0	.....	.....	3	15.60	1.733
⅛	0.65	0.072	¼	16.90	1.878
¼	1.30	0.144	½	18.20	2.022
⅜	1.95	0.216	¾	19.50	2.167
½	2.60	0.289	4	20.80	2.311
⅝	3.25	0.361	¼	22.10	2.456
¾	3.90	0.433	½	23.40	2.600
⅞	4.55	0.505	¾	24.70	2.744
1	5.20	0.578	5	26.00	2.889
¼	6.50	0.722	¼	27.30	3.033
½	7.80	0.867	½	28.60	3.178
¾	9.10	1.011	¾	29.90	3.322
2	10.40	1.156	6	31.20	3.467
¼	11.70	1.300	¼	32.50	3.611
½	13.00	1.444	½	33.80	3.756
¾	14.30	1.589	¾	35.10	3.900

The table on the following page will be found convenient in comparing short and long tons. It expresses the decimal equivalent of the short and long ton, per hundredweight, to 20,000 lb. or 10 short tons.

Table of Comparative Values of the Short and Long Ton

Pounds	Tons Short	Tons Long
100	0.05	0.05
200	0.10	0.10
300	0.15	0.15
400	0.20	0.20
500	0.25	
600	0.30	0.25
700	0.35	0.30
800	0.40	0.35
900	0.45	0.40
1000	0.50	0.45
1100	0.55	0.50
1200	0.60	0.55
1300	0.65	0.60
1400	0.70	
1500	0.75	0.65
1600	0.80	0.70
1700	0.85	0.75
1800	0.90	0.80
1900	0.95	0.85
2000	1.00	

Pounds	Tons Short	Tons Long
		0.90
2100	1.05	0.95
2200	1.10	1.00
2300	1.15	
2400	1.20	1.05
2500	1.25	1.10
2600	1.30	1.15
2700	1.35	1.20
2800	1.40	1.25
2900	1.45	1.30
3000	1.50	1.35
3100	1.55	1.40
3200	1.60	1.45
3300	1.65	
3400	1.70	1.50
3500	1.75	1.55
3600	1.80	1.60
3700	1.85	1.65
3800	1.90	1.70
3900	1.95	1.75
4000	2.00	

Pounds	Tons Short	Tons Long
		1.80
4100	2.05	1.85
4200	2.10	
4300	2.15	1.90
4400	2.20	1.95
4500	2.25	2.00
4600	2.30	2.05
4700	2.35	2.10
4800	2.40	2.15
4900	2.45	2.20
5000	2.50	2.25
5100	2.55	
5200	2.60	2.30
5300	2.65	2.35
5400	2.70	2.40
5500	2.75	2.45
5600	2.80	2.50
5700	2.85	2.55
5800	2.90	2.60
5900	2.95	2.65
6000	3.00	

Pounds	Tons Short	Tons Long
		2.70
6100	3.05	
6200	3.10	2.75
6300	3.15	2.80
6400	3.20	2.85
6500	3.25	2.90
6600	3.30	2.95
6700	3.35	3.00
6800	3.40	3.05
6900	3.45	3.10
7000	3.50	
7100	3.55	3.15
7200	3.60	3.20
7300	3.65	3.25
7400	3.70	3.30
7500	3.75	3.35
7600	3.80	3.40
7700	3.85	3.45
7800	3.90	3.50
7900	3.95	
8000	4.00	3.55

Pounds	Tons Short	Tons Long
		3.60
8100	4.05	
8200	4.10	3.65
8300	4.15	3.70
8400	4.20	3.75
8500	4.25	3.80
8600	4.30	3.85
8700	4.35	3.90
8800	4.40	3.95
8900	4.45	
9000	4.50	4.00
9100	4.55	4.05
9200	4.60	4.10
9300	4.65	4.15
9400	4.70	4.20
9500	4.75	4.25
9600	4.80	4.30
9700	4.85	4.35
9800	4.90	
9900	4.95	4.40
10,000	5.00	4.45

Pounds	Tons Short	Tons Long
		4.50
10,100	5.05	
10,200	5.10	4.55
10,300	5.15	4.60
10,400	5.20	4.65
10,500	5.25	4.70
10,600	5.30	4.75
10,700	5.35	
10,800	5.40	4.80
10,900	5.45	4.85
11,000	5.50	4.90
11,100	5.55	4.95
11,200	5.60	5.00
11,300	5.65	5.05
11,400	5.70	5.10
11,500	5.75	5.15
11,600	5.80	5.20
11,700	5.85	
11,800	5.90	5.25
11,900	5.95	5.30
12,000	6.00	5.35

Pounds	Tons Short	Tons Long
12,100	6.05	5.40
12,200	6.10	5.45
12,300	6.15	5.50
12,400	6.20	5.55
12,500	6.25	5.60
12,600	6.30	
12,700	6.35	5.65
12,800	6.40	5.70
12,900	6.45	5.75
13,000	6.50	5.80
13,100	6.55	5.85
13,200	6.60	5.90
13,300	6.65	5.95
13,400	6.70	6.00
13,500	6.75	
13,600	6.80	6.05
13,700	6.85	6.10
13,800	6.90	6.15
13,900	6.95	6.20
14,000	7.00	6.25

Pounds	Tons Short	Tons Long
14,100	7.05	6.30
14,200	7.10	6.35
14,300	7.15	6.40
14,400	7.20	6.45
14,500	7.25	
14,600	7.30	6.50
14,700	7.35	6.55
14,800	7.40	6.60
14,900	7.45	6.65
15,000	7.50	6.70
15,100	7.55	6.75
15,200	7.60	6.80
15,300	7.65	6.85
15,400	7.70	
15,500	7.75	6.90
15,600	7.80	6.95
15,700	7.85	7.00
15,800	7.90	7.05
15,900	7.95	7.10
16,000	8.00	

Pounds	Tons Short	Tons Long
		7.15
16,100	8.05	7.20
16,200	8.10	7.25
16,300	8.15	
16,400	8.20	7.30
16,500	8.25	7.35
16,600	8.30	7.40
16,700	8.35	7.45
16,800	8.40	7.50
16,900	8.45	7.55
17,000	8.50	7.60
17,100	8.55	7.65
17,200	8.60	7.70
17,300	8.65	
17,400	8.70	7.75
17,500	8.75	7.80
17,600	8.80	7.85
17,700	8.85	7.90
17,800	8.90	7.95
17,900	8.95	8.00
18,000	9.00	

Pounds	Tons Short	Tons Long
		8.05
18,100	9.05	8.10
18,200	9.10	
18,300	9.15	8.15
18,400	9.20	8.20
18,500	9.25	8.25
18,600	9.30	8.30
18,700	9.35	8.35
18,800	9.40	8.40
18,900	9.45	8.45
19,000	9.50	8.50
19,100	9.55	
19,200	9.60	8.55
19,300	9.65	8.60
19,400	9.70	8.65
19,500	9.75	8.70
19,600	9.80	8.75
19,700	9.85	8.80
19,800	9.90	8.85
19,900	9.95	8.90
20,000	10.00	

INDEX

NOTE.—Numbers refer to pages. Letters are used to abbreviate words in the same line or in the heading in which they stand. Other abbreviations are those in common use.

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