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MISCELLANEOUS

EXAMPLES IN ALGEBRA

BY

J. W. COLENSO, D.D.

Price Half-a-Crown

MISCELLANEOUS EXAMPLES

IN

ALGEBRA

WITH EQUATION-PAPERS

ORIGINALLY PROPOSED AT ST. JOHN'S COLLEGE, CAMBRIDGE

BY THE

RIGHT REV. J. W. COLENSO, D.D.

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MISCELLANEOUS EXAMPLES: PART I.

1. Multiply $a^2-2ax-b^2+bx$ by b^2+ax.
2. Divide $3x^3+4abx^2-6a^2b^2x-4a^3b^3$ by $2ab+x$.
3. If $x=1$, $y=-2$, $z=3$, find the value of

$$\frac{3x^2-2xy+5y^2+5z^2+2yz+2xz}{4x^2+2xy+3y^2+2z^2+yz-xz}.$$

4. Reduce $\dfrac{m^3a^2+n^3a^2}{a(m^2+n^2)-man}$ and $\dfrac{x^4+x^2y^2+y^4}{x^4+2x^3y+3x^2y^2+2xy^3+y^4}$.
5. Extract the square roots of $1\frac{56}{169}$ and 66.455104.
6. Simplify $\dfrac{(1\frac{1}{2}-x)-\frac{1}{2}(x-1\frac{1}{3})}{(1\frac{1}{3}-x)-\frac{1}{3}(x-1\frac{1}{2})}$ and $\dfrac{1}{x-1}-\dfrac{1}{2x+2}-\dfrac{x+3}{2x^2+2}$.
7. Sum the A. P. $7+8\frac{1}{2}+$&c. to 8 and to n terms.
8. Insert an H. mean between $1\frac{1}{2}$ and $1\frac{1}{3}$.
9. Reduce to their simplest forms $\sqrt{125}$, $\sqrt{98a^2x}$, $\frac{4}{7}\sqrt[3]{21\frac{7}{16}}$.
10. Expand $(1-2x)^{-\frac{1}{2}}$ to five terms.
11. (i) $\frac{1}{3}(5x-7)-\frac{1}{5}(4x-9)=3\frac{4}{5}$. (ii) $x+7=\sqrt{5x^2+19}$.

    (iii) $\left.\begin{array}{l}\frac{1}{2}x-\frac{1}{3}y=1\\ 6(x+y)-3(x-y)=13(x-1)\end{array}\right\}$. (iv) $\left.\begin{array}{l}x^2+y^2=13\\ xy\ \ =6\end{array}\right\}$.
12. A certain fraction becomes 1 when 3 is added to the num^r^, and $\frac{1}{2}$ when 2 is added to the den^r^: find it.

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13. Write down the square of $1+2x-x^2-\frac{1}{2}x^3$.
14. Divide $51x^2y^2+10x^4-48x^3y-15y^4+4xy^3$ by $4xy-5x^2+3y^2$.
15. Find the value of $x^4-2a(a-b)x^2+(a^2+b^2)(a-b)x-a^2b^2$, when $a=1$, $b=-2$, $x=3$.
16. Find the G. C. M. of

    $3x^4-x^2y^2-2y^4$ and $10x^4+15x^3y-10x^2y^2-15xy^3$.
17. Extract the cube roots of 1953125 and 5.
18. Simplify $\dfrac{2(x^2-\frac{1}{4})}{2x+1}+\dfrac{1}{2}$ and $\dfrac{a^3+3a^2x+3ax^2+x^3}{x^3-y^3}\div\dfrac{(a+x)^2}{x^2+xy+y^2}$.
19. Sum the G. P. $3-1+$&c. to 5 terms and *ad infinitum*.
20. Simplify $\{(a^{\frac{1}{2}}b^{-\frac{2}{3}}c^{\frac{3}{4}})^{-\frac{1}{2}}\}^{-6}$ and $x^{-1}y^{-\frac{1}{2}}z\sqrt{(xyz^{-\frac{2}{3}})}$.
21. Expand to five terms $\dfrac{1}{\sqrt[3]{a-3x}}$.

22. Express 3000 (*quaternary*) in the quinary scale, and 3000 (*quinary*) in the quaternary, and all four in the septenary.

23. (i) $\frac{3x-2}{2x-5}-\frac{21-3x}{5}=\frac{6x+13}{10}$. (ii) $\frac{3}{4}x=1+\frac{1}{x}$.

(iii) $\left.\begin{array}{l}1\frac{1}{2}-5(\frac{1}{2}x-1)=2-\frac{3}{2}(y+1)\\ \frac{2}{3}x+8-\frac{1}{2}(y-5)=11x-3\frac{1}{3}(3x-2)\end{array}\right\}$.

24. A can do a piece of work in $10\frac{1}{2}$ days, which A and B can do together in $5\frac{3}{5}$ days: how long would B take to do it alone?

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25. Find the product of x^2-a, $x^2-a^{\frac{1}{2}}x+a$, and $x^2+a^{\frac{1}{2}}x+a$.

26. Divide $\frac{3}{4}x^5-4x^4+\frac{77}{8}x^3-\frac{43}{4}x^2-\frac{33}{4}x+27$ by $\frac{1}{2}x^2-x+3$.

27. If $x=1$, $y=-2$, $z=3$, find the value of

$$\tfrac{1}{2}[x-\tfrac{1}{3}\{y-\tfrac{1}{4}(z-\overline{x-2y})\}].$$

28. Reduce $\frac{x^4+x^2y}{x^4-y^2}$ and $\frac{27a^5x^2-18a^4x^2-9a^3x^2}{36a^6x^2-18a^5x^2-27a^4x^2+9a^3x^2}$.

29. Simplify $\frac{1-\frac{1}{2}\{1-\frac{1}{3}(1-x)\}}{1-\frac{1}{3}\{1-\frac{1}{2}(1-x)\}}$ and $\frac{x+2}{2(x+1)}+\frac{2-x}{2(x-1)}-\frac{x}{x^2+1}$

30. Find the square roots of 19321, 1.9321, and 19.321.

31. Obtain a fourth proportional to $\frac{2}{7}$, $\frac{3}{4}$, $\frac{5}{6}$, and a mean proportional to .017 and .153.

32. Sum the G. P. $\frac{4}{5}-\frac{2}{3}+$&c. to n terms and *ad infinitum*.

33. Expand $(ax-x^2)^{-\frac{3}{5}}$ to five terms.

34. In how many ways may a sum of 40 guineas be paid in dollars (4*s.* 6*d.*) and doubloons (13*s.*)? and how may it be paid with fewest coins?

35. (i) $\frac{x-2}{3}-\frac{1-\frac{1}{2}x}{6}=87\frac{1}{4}-\frac{27(x-2)}{5}$.

(ii) $\left.\begin{array}{l}\frac{1}{2}x-12=\frac{1}{4}y+8\\ \frac{1}{5}(x+y)+\frac{1}{3}x=\frac{1}{4}(2y-x)+35\end{array}\right\}$. (iii) $\frac{13}{x+2}+\frac{4}{x}=3\frac{14}{15}$.

36. How many square feet are in a rectangular surface, of which the diagonal is 53 inches, and the length and breadth $2a+13$ and $2a-4$ inches respectively?

37. Multiply $(a+b+c)(a+b-c)$ by $(a-b+c)(b+c-a)$.

38. Divide $1-\frac{1}{2}x$ by $1-\frac{1}{3}x-\frac{1}{4}x^2$ to five terms.

39. If $a=-x=\frac{1}{2}$, $b=0$, find the numerical value of

$$x^4-(a-b)x^3+(a-b)b^2x-b^4.$$

40. Reduce to its lowest terms $\frac{2x^3-x^2+x+1}{2x^3+3x^2+3x+1}$.

41. Find the cube roots of 2685619 and $\frac{1}{8}$.

42. Simplify the fraction $\dfrac{\frac{1}{2}(x+1\frac{1}{3})-\frac{2}{3}(1-\frac{3}{4}x)}{1\frac{3}{4}-\frac{1}{3}(x+4\frac{1}{4})}$.

43. Expand $(a^4-4a^2x^2)^{\frac{7}{4}}$ to five terms.

44. Reduce to their simplest forms $\dfrac{2a}{3}\sqrt[3]{\dfrac{9}{4a^2}}$ and $\dfrac{3x}{2}\sqrt[4]{\dfrac{80y^2}{81x^2}}$.

45. Sum the A. P. $\frac{1}{2}+\frac{2}{3}+$&c. to 31 and to $n-2$ terms.

46. Transform 1828 into the septenary scale, and square it; reduce the result to the nonary, and extract the square root; and express the latter two results in the denary.

47. (i) $3x-\frac{1}{2}(x-1\frac{1}{2})=9-\frac{1}{4}(5x-7)$.

(ii) $\left.\begin{aligned} x-y-z&=6\\ 3y-x-z&=12\\ 7z-y-x&=24\end{aligned}\right\}$. (iii) $\left.\begin{aligned} a(x+y)-b(x-y)&=2a^2\\ (a^2-b^2)(x-y)&=4a^2b\end{aligned}\right\}$.

48. Two men can do a piece of work in 12 days, and one of them can do half as much again in 24 days: in what time could the other do a third as much again?

---

49. Simplify $\frac{1}{4}\{\frac{1}{5}a-(b-a)\}-\frac{1}{2}[(b-\frac{1}{3}a)-\frac{2}{3}\{a-\frac{3}{4}(b-\frac{4}{5}a)\}]$.

50. If $a=1$, $b=3$, $c=5$, find the numerical value of

$$\{a-(b-c)\}^2+\{b-(c-a)\}^2+\{c-(a-b)\}^2.$$

51. Expand and simplify the quantities in the preceding question.

52. Find the G. C. M. of

$7x^3-2x^2y-63xy^2+18y^3$ and $5x^4-3x^3y-43x^2y^2+27xy^3-18y^4$.

53. Extract the square roots of 1110916 and $9+2\sqrt{14}$.

54. Simplify

$$\left(a+b+\frac{b^2}{a}\right)\div\left(a+b+\frac{a^2}{b}\right) \text{ and } \left(a-b+\frac{b^2}{a+b}\right)\div\left(a+b+\frac{b^2}{a-b}\right).$$

55. Sum .2+.02+.002 to n terms and *ad infinitum.*

56. How many terms of the series 17, 15, &c. will make 72?

57. Expand $(a^2-bx)^{-\frac{2}{5}}$ to five terms.

58. How many different throws can be made with two dice?

59. (i) $\dfrac{3}{x+1}=8-2\left(\dfrac{4x+3}{x+3}\right)$. (ii) $\left.\begin{aligned} 5x+7y&=43\\ 11x+9y&=69\end{aligned}\right\}$.

(iii) $x^2y-xy^2=6=2xy$.

60. In a concert room 800 persons are seated on benches of equal length. If there were 20 fewer benches, 2 persons more would have to sit on each bench. Find the number of benches.

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61. Multiply $2y+3x^{\frac{1}{4}}y^{\frac{1}{2}}-x^{\frac{1}{2}}$ by $7x^{\frac{1}{4}}-5y^{\frac{1}{2}}$.

62. Divide x^4+4x+3 by x^2-2x+3.

63. If $a=1$, $b=2$, $c=3$, find the value of $\sqrt[3]{a(b^3+ac)-\frac{1}{2}b^2c}$.

64. Find the G. C. M. of $a^3(b^4-b^2c^2)$ and $b^3(ab+ac)^2$.

65. Obtain the fourth root of $16x^3(x-2)-8x^2(x^{-1}-3)+1$.

66. Simplify $\dfrac{x+1}{2x-1}-\dfrac{x-1}{2x+1}-\dfrac{1-3x}{x(1-2x)}$.

67. Find the G. mean between $12\frac{1}{2}$ and 13, to 3 places of decimals.

68. Expand $\left(\dfrac{1}{a^2x-ax^2}\right)^{\frac{1}{2}}$ to five terms.

69. What number is that, which is just as much below 35 as its half is above its third part?

70. Convert 297 to radix 11: square and cube it in that scale, extract the roots, and reconvert them to the common scale.

71. (i) $\frac{1}{8}(3x+5)-\frac{1}{3}(21+x)=39-5x$.
(ii) $2x^2+x=28$. (iii) $2x-9y+2=0=3x-12y+2\frac{1}{2}$.

72. *A* and *B* can reap a field in 10 hrs., *A* and *C* in 12 hrs., *B* and *C* in 15 hrs: in what time can they do it *jointly* and *separately?*

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73. Obtain the quotient of $6\sqrt[3]{x^4}-96\sqrt[3]{x^{-4}}$ by $\sqrt[3]{x}-2\sqrt[3]{x^{-1}}$.

74. If $x=\frac{1}{2}$, and $x+y=x+y+z=0$, find the value of
$$(y^2-z^2)\ \{y^2+z^2-y(x-z)\}.$$

75. Reduce $\dfrac{x(x^3+y^3)\ (x-y)}{(x^2-y^2)\ (x^2+y^2-xy)}$ and $\dfrac{x^4+3x^3+x+3}{x^3-8x+3}$.

76. Add together $7\sqrt{63}+2\sqrt{252}+11\sqrt{28}$.

77. Find $\sqrt{3.14159}$, and the fourth root of $x^4-\frac{1}{2}x+\frac{3}{2}x^2+\frac{1}{16}-2x^3$.

78. Show by the Bin. Theor. that $\sqrt{2}=1+\frac{1}{2}-\frac{1}{8}+\frac{1}{16}-\frac{5}{128}+$&c.

79. Sum the A. P. $\frac{4}{5}+2+$&c. to 9 and to n terms.

80. Form the equation whose roots are 2, -2, $1+\sqrt{5}$, $1-\sqrt{5}$.

81. What number is that which is the same multiple of 7, that its excess above 20 is of its defect from 30?

82. How many different arrangements can be made of the letters of the word *Novogorod?* How many with two o's at the beginning and two at the end?

83. (i) $\frac{1}{3}(7x+5)-\frac{4}{5}(x+4)+6=\frac{3}{2}(x+3)$.
(ii) $x+y-8=0=\frac{1}{2}(x-y)+\frac{2}{3}(x-\frac{1}{2}y+2)$.
(iii) $x+\sqrt{5x+10}=8$.

84. Out of £5000, a person leaves £20 to an old servant, and the remainder among three societies, *A*, *B*, and *C*, so that *B* may have twice as much as *C*, and *A* three times as much as *B*: how much does each receive?

---

85. Multiply $\sqrt{x^3}+1+\frac{1}{\sqrt{x^3}}$ by $\sqrt{x^3}-1+\frac{1}{\sqrt{x^3}}$.

86. Divide $\frac{1}{2}a^3+\frac{3}{2}a^2x-2x^3$ by $\frac{1}{2}a+x$.

87. If $a=1$, $b=\frac{2}{3}$, $x=7$, $y=8$, find the numerical value of

$$5(a-b)\sqrt[3]{(a+x)y^2}+a-b\sqrt{(a+x)y}-\sqrt[4]{\tfrac{1}{2}y^2-\{a-\sqrt{3(x+2b)}\}}$$

88. Simplify

$$1\tfrac{1}{2}-\tfrac{3}{4}\{1-\tfrac{2}{3}(x-\tfrac{1}{2})\} \text{ and } \{a-\tfrac{1}{2}(a-\tfrac{2}{3}b\}\div\{b-\tfrac{1}{3}(a+\tfrac{3}{2}b)\}.$$

89. Write down the quotient of $ax^{-1}+b^2$ by $a^{\frac{1}{3}}x^{-\frac{1}{3}}+b^{\frac{2}{3}}$.

90. Find the square root of $(x+x^{-1})-2(x^{\frac{1}{2}}-x^{\frac{1}{2}})-1$.

91. Sum the A. and G. P. $\frac{4}{5}+2+$&c., each to n terms. Can the latter series be summed *ad infinitum?*

92. Expand $\sqrt[4]{1+4x}$ to five terms, and square the result.

93. Find two numbers in the ratio of $1\frac{1}{2} : 2\frac{2}{3}$, such that, when increased each by 15, they shall be in the ratio of $1\frac{2}{3} : 2\frac{1}{2}$.

94. In how many ways may £24 16*s*. be paid in guineas and crowns?

95. (i) $\frac{1}{2}(9x+7)-\{x-\frac{1}{7}(x-2)\}=36$.

(ii) $\left.\begin{array}{l} x+1 : y :: 5 : 3 \\ \frac{2}{3}x-\frac{1}{2}(5-y)=3\frac{5}{12}-\frac{1}{4}2x-1) \end{array}\right\}$.

(iii) $\frac{8-x}{2}-\frac{2x-11}{x-3}=\frac{x-2}{6}$.

96. If at a fixed rate per cent, for a certain time, 21 francs and 54 shillings equal the simple interest of £58 17*s*. 1*d*., and 12 francs and 38 shillings equal the interest of 1000 francs; what is the supposed worth of the franc in English money?

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97. Simplify

$$\tfrac{1}{6}\{x(x+1)(x+2)+x(x-1)(x-2)\}+\tfrac{2}{3}(x-1)x(x+1).$$

98. Divide $a^4-\frac{19}{6}a^2b^2+\frac{1}{3}ab^3+\frac{1}{6}b^4$ by $a^2+2ab+\frac{1}{3}b^2$.

99. Find the G. C. M. of $3x^3+4x^2-3x-4$ and $2x^4-7x^2+5$.

100. Reduce $\frac{(x^4-b^4)(x-b)}{(x^2+b^2-2bx)(bx+x^2)}$ and $\frac{x+\sqrt[4]{a^2x^2y}}{x-a\sqrt{y}}$.

101. Find the cube root of 69.426531.

102. Multiply $1+a^{\frac{2}{3}}-x^{-\frac{2}{3}}-a^{\frac{4}{3}}+x^{-\frac{4}{3}}+a^{\frac{2}{3}}x^{-\frac{2}{3}}$ by $x^{-\frac{2}{3}}-a^{\frac{2}{3}}+1$.

103. Find the common difference of an A. P., when the first term is 1, the last term 50, and the sum 204.

104. If $a : b :: c : d$, show that $7a+b : 3a-5b :: 7c+d : 3c-5d$.

105. Divide 100 into two parts, so that $\frac{1}{3}$ the greater may be greater than $\frac{1}{2}$ the less by $\frac{1}{4}$ their difference.

106. Employ the septenary scale to find the side of a square which contains a million square feet.

107. (i) $\frac{1}{6}(x+3)-\frac{1}{7}(11-x)=\frac{2}{5}(x-4)-\frac{1}{21}(x-3)$.

(ii) $\frac{2x-1}{2x+1}+\frac{2x+1}{2x-1}=3$.

(iii) $\left.\begin{array}{l} 3x-y+z=17, \\ 5(x+y-2)=2(y+z), \\ 4(x+y+z)=3(1-x+3z) \end{array}\right\}$.

108. A and B engaged in trade, A with £275, B with £300; A lost half as much again as B, and B had then remaining half as much again as A: how much did each lose?

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109. If $a-b=x=3$ and $a+b+x=2$, find the value of

$$(a-b)\{x^3-2ax^2+a^2x-(a+b)b^2\}.$$

110. Show that $(2a+b^{-1})(2b+a^{-1})=(2a^{\frac{1}{2}}b^{\frac{1}{2}}+a^{-\frac{1}{2}}b^{-\frac{1}{2}})^2$.

111. Find the L. C. M. of $6x^2-13x+6$, $6x^2+5x-6$, and $9x^2-4$.

112. Obtain the square root of $\frac{1}{4}x^4+\frac{1}{9}a^4-\frac{1}{3}ax(2a^2+3x^2-4ax)$.

113. Obtain $\sqrt{6}$ to four places, and thence find $\sqrt{\frac{1}{6}}$, $\sqrt{\frac{2}{3}}$, $\sqrt{1\frac{1}{2}}$.

114. Simplify $\dfrac{a^{\frac{1}{4}(3m-1)}b^{-1}}{a^{\frac{1}{4}(m-3)}b^{\frac{1}{2}(m-1)}}$ and $\dfrac{b}{b+x}+\left(\dfrac{x}{b+x}\right)^2+\dfrac{bx}{(b+x)^2}$.

115. Square $a-2b-3c$ and $2a-\frac{1}{2}bx-\frac{1}{4}cx^2+2dx^3$.

116. Sum the G. P. $5+2+$&c. to n terms and *ad infinitum*.

117. The trinomial ax^2+bx+c becomes 8, 22, 42 respectively, when x becomes 2, 3, 4: what does it become when $x=-\frac{1}{3}$?

118. Expand $\sqrt{1-4x}$ to five terms, and obtain the same by Evol^n.

119. (i) $\frac{1}{9}(4x-21)+3\frac{3}{4}+\frac{1}{4}(57-3x)=241-\frac{1}{12}(5x-96)-11x$.

(ii) $11x^2+1=4(2-x)^2$.

(iii) $\left.\begin{array}{l} \frac{1}{5}(3x-2y+1)-\frac{1}{3}(x-y)=\frac{4}{9}y \\ \frac{5}{x}-\frac{3}{2y}=\frac{15}{2xy} \end{array}\right\}$.

120. A and B sold 130 ells of silk, of which 40 were A's and 90 B's, for 42 crowns; and A sold for a crown $\frac{1}{3}$ an ell more than B did. How many ells did each sell for a crown?

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121. Write down the quotient of $16-81a$ by $2+3\sqrt[4]{a}$.

122. Multiply $a^2+\frac{2}{3}(a+b)x-\frac{1}{2}x^2$ and $a^2-\frac{2}{3}(a-b)x+\frac{1}{2}x^2$.

123. Reduce to its lowest terms $\dfrac{9x^3+6x^2-2x-4}{12x^3-5x^2+4x-4}$.

124. Find the L. C. M. of $a^2\underline{+}x^2$, $(a\underline{+}x)^2$, and $a^3\underline{+}x^3$.

125. Obtain the square root of $1\frac{1}{3}$ and of $12+6\sqrt{3}$.

126. Simplify $a-(b-c)-\{b-(a-c)\}-[a-\{2b-(a-c)\}]$, and show that $\dfrac{a+c}{(a-b)(x-a)}-\dfrac{b+c}{(a-b)(x-b)}=\dfrac{x+c}{(x-a)(x-b)}$.

127. Sum the A. P. $\frac{1}{2}+\frac{1}{3}+$&c. to 7 and to n terms.

128. How could a sum of £24 16*s*. be paid from *A* to *B* with the use of fewest coins, if *A* have only guineas and *B* crowns?

129. Simplify

$$\sqrt{8(a^3x+ax^3)-16a^2x^2} \text{ and } (\sqrt{a})^{\frac{2}{3}-\frac{1}{6}}-\tfrac{3}{4}(a^{\frac{5}{2}}b\sqrt{a^{-3}b^{-2}})^{\frac{1}{4}}.$$

130. Compare the numbers of combinations of 24 different letters, when taken 7 and 11 together; and also when the letters a, b, c occur in each of such combinations.

131. (i) $\dfrac{6x+18}{13}-4\frac{5}{6}-\dfrac{11-3x}{36}=5x-48-\dfrac{13-x}{12}-\dfrac{21-2x}{18}$.

(ii) $\left.\begin{array}{l} 1+\frac{5}{6}(y+5)-\frac{1}{3}(7x-6)=10-\frac{1}{12}(3x-10+7y) \\ \frac{1}{9}(12-x) : 5x-\frac{1}{3}(14+y) :: 1 : 8 \end{array}\right\}$.

(iii) $5x-\dfrac{3(x-1)}{x-3}=2x+\dfrac{3(x-2)}{2}$.

132. A party at a tavern had a bill of £4 to pay between them, but, two having sneaked off, those who remained had each 2*s*. more to pay: how many were there at first?

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133. Show that $(ac\underline{+}bd)^2+(ad\mp bc)^2=(a^2+b^2)(c^2+d^2)$, and exemplify this identity when $a=1=-d$, $b=2=-c$.

134. Obtain the product of $x+2\sqrt[4]{x^2y}+2\sqrt{y}$ by $x-2\sqrt[4]{x^2y}+2\sqrt{y}$.

135. Divide $x^4-(a^2-b-c)x^2-(b-c)ax+bc$ by x^2-ax+c.

136. Reduce $\dfrac{6a^2-13ay+6y^2}{10a^2-9ay-9y^2}$ and $\dfrac{x^3+11x^2+30x}{9x^3+53x^2-9x-18}$.

137. Find the L. C. M. of

$$m^3n-mn^3,\ m^2+mn-2n^2, \text{ and } m^2-mn-2n^2.$$

138. Obtain the square root of $a^{\frac{1}{3}}-2a^{\frac{1}{4}}+3a^{\frac{1}{6}}-2a^{\frac{1}{12}}+1$.

139. If $a : b :: c : d$, express $(b+d)(c+d)$ in terms of a, b, c.

140. Find $\sqrt{24}$, and thence deduce the values of

$$\frac{5}{\sqrt{54}},\quad \frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}},\quad \frac{2\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}},\quad \frac{1+\sqrt{216}}{7-\sqrt{6}}.$$

141. Insert two A. and two H. means between 1 and 3.

142. Expand $(1-4x)^{-\frac{1}{4}}$ and $(1-4x)^{-\frac{1}{2}}$ to five terms; and show that the former series, when squared, coincides with the latter.

143. (i) $\frac{1}{2}x-\frac{1}{3}(x-2)=\frac{1}{4}\{x-\frac{2}{3}(2\frac{1}{2}-x)\}-\frac{1}{3}(x-5)$.

(ii) $\dfrac{x}{x-2}+\dfrac{x-9}{x-7}=\dfrac{x+1}{x-1}+\dfrac{x-8}{x-6}$.

(iii) $\dfrac{7x+1}{6\frac{1}{2}-3x}=\dfrac{80}{3}\left(\dfrac{x-\frac{1}{2}}{x-\frac{2}{3}}\right)$.

144. A farmer bought 5 oxen and 12 sheep for £63, and for £90 could have bought four more oxen than he could have bought sheep for £9: what did he pay for each?

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145. Find the continued product of $(x+a)(x+b)(a-2x)(b-x)$.

146. Write down the square and fourth powers of $a-\frac{1}{2}\sqrt{ax}-2x$.

147. Simplify $\dfrac{(x^2-4x)(x^2-4)^2}{(x^2-2x)^2}$ and $\dfrac{(a^2-1)(a^6-1)}{(a+1)^2(a^2-a)^2}$.

148. Reduce to its lowest terms $\dfrac{3a^2+2b^2+c^2-5ab-3bc+4ac}{3a^2+4b^2+3c^2-8ab-8bc+10ac}$.

149. Extract $\sqrt[3]{.01}$ to four places of decimals.

150. Obtain the square root of $(x+1)^2-4\sqrt{x}(x-\sqrt{x}+1)$.

151. Determine which is the greater $\sqrt{2}\div\sqrt[3]{3}$ or $\sqrt{3}\div\sqrt[3]{5}$.

152. Sum the G. P. $\frac{1}{2}+\frac{1}{3}+$&c. to n terms and *ad infinitum.*

153. Given -1 to be a root of the equation $x^4-7x^2-6x=0$, find the other three roots.

154. In how many different ways could a farmer lay out a sum of £63, in buying sheep and oxen at 30*s.* and £9 respectively?

155. (i) $a(x-b)=b(a-x)-(a+b)x$.

(ii) $\dfrac{3}{1-3x}+\dfrac{5}{1-5x}+\dfrac{4}{2x-1}=0$.

(iii) $\left.\begin{aligned}2x^2+3xy&=26\\3y^2+2xy&=39\end{aligned}\right\}$.

156. *A* and *B* can do a piece of work together in 4 days: *A* works alone for 2 days, and then they finish it in $2\frac{1}{2}$ days more: in what time could they have done it separately?

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157. Find the value of $\sqrt{\dfrac{1+a}{1-b}}+\sqrt{\dfrac{3(1+2a^2)}{1-b^2}}+\sqrt{a^2-2ab+4b^2}$,

when $a=\frac{1}{4}$, $b=\frac{1}{5}$.

158. Divide $a^2+2ab^{\frac{3}{2}}+b^3$ by $a^{\frac{2}{3}}+2a^{\frac{1}{3}}b^{\frac{1}{2}}+b$.

159. Find the G. C. M. of $x^4+7x^3+7x^2-15x$ and $x^3-2x^2-13x+110$.

160. Simplify $\frac{2}{x-2}-\frac{1}{x+2}-\frac{x+6}{x^2+4}$ and $\frac{\frac{3}{4}(1+\frac{2}{5}x)}{1\frac{1}{2}-\frac{2}{3}(1-\frac{1}{2}x)}$.

161. Multiply together $x-1+\sqrt{2}$, $x+2+\sqrt{3}$, $x-1-\sqrt{2}$, and $x+2-\sqrt{3}$.

162. Find the 7th term of $5+5\frac{2}{3}+6\frac{1}{3}+$&c., and its sum to 16 terms.

163. If $a:b::b:c::c:d$, show that $a:b::\sqrt[3]{a}:\sqrt[3]{d}$; and express $(a+b)(c+d)$ in terms of b and c.

164. Find the least number which, when divided by 47 and 74, shall leave remainders 26 and 62 respectively.

165. Expand $(1+2x^2)^{-3}$ and $(a+2b)^{\frac{3}{2}}$, each to five terms.

166. Express a million in the senary scale, extract its square and cube roots in that scale, and reduce the results to the denary.

167. (i) $\frac{2}{3}(x-5)-\frac{3}{11}(x-13\frac{1}{3})=5-\frac{1}{5}(7-x)$.

(ii) $\frac{x-ay}{b}=1=\frac{ax+y}{c}$. (iii) $\frac{3x+8}{x-4}-\frac{5(12-x)}{2x+3}=11$.

168. If A's money were increased by half of B's, it would amount to £54; and, if B's present sum were trebled, it would exceed three times the difference of their original sums by £6. What had each at first?

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169. Write down the expression for the product of the square root of the sum of the cubes of the square roots of a and b, by the square of the cube root of the sum of their squares: and find its value approximately, when $a=4$, $b=1$.

170. Multiply $x^{-\frac{3}{2}}+2x^{-\frac{3}{4}}y^{\frac{1}{2}}+3y$ by $x^{-\frac{3}{2}}-2x^{-\frac{3}{4}}y^{\frac{1}{2}}+y$.

171. Simplify $\left\{\left(\frac{a^{-m}}{b^{-n}}\right)^{\frac{p}{m}}\right\}^{-\frac{q}{n}}$, and reduce $\frac{x^5-4x^3+3x}{2x^4-5x^2-3}$.

172. Obtain the square root of $1-ax^{\frac{1}{2}}-\frac{15}{4}a^2x+2a^3x^{\frac{3}{2}}+4a^4x^2$.

173. Find the sum of $\frac{a+b}{x-a}-\frac{b}{x-b}$, and of $1+\frac{2x+1}{2(x-1)}-\frac{4x+5}{2(x+1)}$.

174. Extract $\sqrt{15}$, and thence obtain the square roots of $\frac{5}{3}$, $\frac{3}{5}$, $2\frac{2}{5}$, $41\frac{2}{3}$.

175. Sum the A. P. $13+11\frac{1}{2}+$&c. to 5 and to n terms, beginning in each case with the *ninth*.

176. If $x=\frac{\sqrt{3}+1}{\sqrt{3}-1}, y=\frac{\sqrt{3}-1}{\sqrt{3}+1}$, find the value of x^2+xy+y^2.

177. Expand $(1+\sqrt{x})^{-2}$ to five terms, and obtain from the result the series for $(1+\sqrt{x})^{-4}$.

178. Find three numbers in the proportion of $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$, the sum of whose squares is 724.

179. (i) $6x-a : 4x-b :: 3x+b : 2x+a$. (ii) $3(x-\frac{1}{4})-\frac{x-1}{x+2}=5$
(iii) $(x+5)^2+(y+6)^2=2\{xy+12(x-1)\}, y=x+1$.

180. *A* does $\frac{3}{5}$ of a piece of work in 6 days, when *B* comes to help him; they work at it together for $\frac{7}{8}$ of a day, and then *B* by himself just finished it by the end of the day: in what time could they have each done it separately?

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181. Find the continued product of $a+x$, $a+\frac{1}{2}y$, $a-\frac{1}{2}z$; and deduce from the result the value of $(a+b)^3$.

182. Multiply $\frac{5}{2}x+3a^{-\frac{1}{3}}x^{\frac{1}{2}}-\frac{7}{3}a^{-\frac{2}{3}}$ by $2x-a^{-\frac{1}{3}}x^{\frac{1}{2}}-\frac{1}{2}a^{-\frac{2}{3}}$.

183. Simplify $x-\frac{1}{2}\{(1\frac{1}{3}-x)-\frac{1}{3}(2\frac{1}{2}-x)-\frac{2}{3}(1\frac{1}{2}-\overline{x-2\frac{1}{4}})\}$.

184. Reduce to its lowest terms $\frac{x^4-x^3+3x^2-2x+2}{x^4-5x^2+6x-5}$.

185. Find the L. C. M. of $ax^3-a^{\frac{1}{3}}x$, ax^3-1, and ax^3+1.

186. Extract the square root of $a^2b^{-2}+\frac{1}{4}a^{-2}b^2-a^{-1}b+2ab^{-1}$.

187. Find the sum of $\frac{1}{2(x-1)^2}+\frac{1}{2(x-1)}-\frac{x}{2(x^2+1)}$.

188. Multiply together $3\sqrt{8}, 2\sqrt{6}, \sqrt{15}, \sqrt{20}$; and find $\sqrt{2+\frac{1}{2}\sqrt{7}}$.

189. Sum the G. P. $6-2+$&c. to 7 and to n terms.

190. The present worth of £1463 is £1400, but were the term of credit 1 year more, and the rate of interest £1 less, the present worth would be £1393$\frac{1}{3}$; find the rate of interest.

191. (i) $\frac{1}{14}(3x+\frac{2}{3})-\frac{1}{7}(4x-6\frac{2}{3})=\frac{1}{2}(5x-6)$.
(ii) $5x+4y=38\frac{1}{4}+\frac{1}{4}(3x-y)$, $x=5\frac{5}{24}-\frac{1}{4}\{\frac{1}{2}(x+y)-\frac{1}{3}(x-y)\}$.
(iii) $\frac{2}{x-4}+\frac{3}{x-6}=\frac{5}{x-2}$.

192. A man and his wife would empty a cask of beer in 16 days: after drinking together 6 days, the woman alone drank for 9 days more, and then there were 4 gallons remaining, and she had drank altogether $3\frac{3}{4}$ gallons. Find the number of gallons in the cask at first.

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193. If $4a = 5b = 1$, find the value of

$$\{\tfrac{1}{3}(a^{\frac{3}{2}} + a^{-\frac{1}{2}}b^{-1})\}^{\frac{2}{3}} - \sqrt{[\tfrac{1}{3}\{1 + a^{-\frac{3}{2}} - (1 + ab^{-1})^{-\frac{1}{2}}\}]}.$$

194. Find the sum of $\dfrac{x^{3n}}{x^n - 1} - \dfrac{x^{2n}}{x^n + 1} - \dfrac{1}{x^n - 1} + \dfrac{1}{x^n + 1}$.

195. Simplify the surd expressions

$$\frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}}, \quad \frac{3\sqrt{2} - \sqrt{3}}{\sqrt{3} - \sqrt{\frac{1}{2}}}, \quad \frac{3\sqrt{\frac{1}{3}} + 2\sqrt{\frac{1}{2}}}{\frac{1}{2}\sqrt{\frac{1}{3}} - \frac{1}{3}\sqrt{\frac{1}{2}}}.$$

196. Reduce $\dfrac{(m^2 - 4a^2)(m^2 + am - 2a^2)}{(m^2 - a^2)(m^2 - am - 2a^2)}$ and $\dfrac{a^3 - a^2x - ax^2 - 2x^3}{a^5 - 2a^4x - ax^4 + 2x^5}$.

197. Find the L. C. M. of $3x^2 - 2x - 1$ and $4x^3 - 2x^2 - 3x + 1$.

198. Sum the series $3 - 2 + 1\frac{1}{3} -$ &c. to n terms and *ad infinitum*.

199. Prove that the sum of any number, n, of consecutive odd numbers, beginning with unity, is a square number.

200. Given $y^2 \propto a^2 - x^2$, and when $x = \sqrt{a^2 - b^2}$, $ay = b^2$, find the value of x when $y = \frac{4}{5}b$.

201. A person distributed £2 1*s.* 8*d.* among some poor people, giving $9\frac{1}{2}$*d.* to each man and $6\frac{1}{2}$*d.* to each woman: how many men were there, it being known that the whole number was a multiple of 10?

202. Expand $(1 + \sqrt[3]{x})^{-6}$ to five terms, and obtain from the result by Evolution the series for $(1 + \sqrt[3]{x})^{-3}$.

203. (i) $\frac{1}{4}\{1 + \frac{3}{2}(x + 2)\} - \frac{2}{7}\{1\frac{1}{3} - (1\frac{1}{2} - x)\} = 1\frac{3}{28}$.
(ii) $abx^2 - (a + b)x + 1 = 0$. (iii) $\frac{1}{2}(x + y) = x - y = \sqrt{x + 2y} - 1$.

204. A and B lay out equal sums in trade; A gains £100, and B loses so much, that his money is now only $\frac{2}{3}$ of A's; but if each gave the other $\frac{1}{3}$ of his present sum, B's loss would be diminished by one half. What had each at first, and what would A's gain be now?

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205. Show that $\frac{1}{4}(x^2 + y^2) + z^2 - \frac{1}{2}xy + xz - yz$ and $(y \sim z)^2$ become identical when $-x = y = a$.

206. Divide $mpx^3 + (mq - np)x^2 - (mr + nq)x + nr$ by $mx - n$.

207. Multiply $a^{\frac{2}{3}} + a^{-\frac{2}{3}} + 2 - a^{\frac{1}{3}} + a^{-\frac{1}{3}}$ by $a^{\frac{1}{3}} - a^{-\frac{1}{3}} + 1$.

208. Reduce to its lowest terms $\dfrac{x^3 + a^2x^2 - ax - a^3}{x^2 - ax + a^{\frac{1}{2}}x - a^{\frac{3}{2}}}$.

209. Obtain the sum of $\dfrac{1 - 2x}{3(x^2 - x + 1)} + \dfrac{1 + x}{2(x^2 + 1)} + \dfrac{1}{6(x + 1)}$.

210. Find the square root of 49.14290404 and the cube root of 8242408.

211. The 3rd and 13th terms of an A. P. are 3 and $\frac{1}{3}$: find the 14th term, and the sum of 20 terms.

212. Simplify the surd expression $\{ab^{-2}.\sqrt{ab^3}.\sqrt[3]{ab^4}.\sqrt[4]{ab^5}\}^{\frac{1}{5}}$.

213. The fore-wheel of a carriage makes 6 revolutions more than the hind wheel in 120 yards, and the circumference of one is a yard less than that of the other: find that of each.

214. Transform 1000000 from the quinary to the septenary scale; and extract its square and cube roots in the latter.

215. (i) $\frac{1}{2}(x-1)(x-2)=(x-2\frac{2}{3})(x-1\frac{3}{4})$.
(ii) $2x+3y=5=-(2y+3x)$. (iii) $x^2+xy=a^2$, $y^2+xy=b^2$.

216. Find the time in which A and B can do together a piece of work, which they can do separately in m and n days. How long must A work to do what B can in m days?

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217. Find the difference between $(n+2)(n+3)(n+4)$ and $24\{n-\frac{1}{2}(n-1)\}\{n-\frac{2}{3}(n-2)\}\{n-\frac{3}{4}(n-1\frac{1}{3})\}$.

218. Divide $a+b^2+c^3-3\sqrt[3]{ab^2c^3}$ by $a^{\frac{1}{3}}+b^{\frac{2}{3}}+c$.

219. Find the sum of $\frac{x-a}{x-b}+\frac{x-b}{x-a}-\frac{(a-b)^2}{(x-a)(x-b)}$.

220. Find the L. C. M. of $x^3+x^2y+xy^2+y^3$ and $x^3-x^2y+xy^2-y^3$.

221. Obtain $\sqrt{10}$, and thence derive the values of $\frac{1}{5}\sqrt{\frac{2}{5}}$, $\sqrt{4\frac{4}{9}}$, $\sqrt{2\frac{1}{2}}$, $(\sqrt{5}+\sqrt{2})\div(\sqrt{5}-\sqrt{2})$, and $(\sqrt{5}-\sqrt{2})\div(5\sqrt{2}-2\sqrt{5})$.

222. Sum $(1\frac{1}{2})^{-1}+2^{-1}+(2\frac{2}{3})^{-1}+$&c. to n terms and *ad infinitum*.

223. Expand $(a^2+2x^2)^{-\frac{3}{2}}$ and $(2a-3x)^{-2}$ each to five terms.

224. A servant agrees with a master for 12 months, on the condition of receiving a farthing the first month, a penny the second, fourpence the third, and so on: what would his wages amount to in the course of the year?

225. Given two roots of the equation $x^5+4x=5x^3$ to be 1 and -2, find the other three roots.

226. A person changed a sovereign for 25 pieces of foreign coin, some of them going 30 to the £, the others 15: how many did he get of each?

227. (i) $2ax^2+(a-2)x-1=0$. (ii) $ax+1=by+1=ay+bx$.
(iii) $\frac{x}{x-1}+\frac{x+2}{x+1}=\frac{8x-13}{4(x-2)}$.

228. Find the time in which A, B, and C can together do a piece of work, which A can do in m days, B in n days, and C in $\frac{1}{2}(m+n)$ days.

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229. Divide $5y^4+\frac{7}{2}ay^3-\frac{107}{12}a^2y^2+\frac{5}{6}a^3y+\frac{7}{6}a^4$ by $\frac{5}{2}y^2+3ay-\frac{7}{3}a^2$.

230. Obtain the products of $\sqrt{x^3}+a\sqrt[4]{x^3+a^2}$
(i) by $\sqrt{x^3}-a\sqrt[4]{x^3+a^2}$, (ii) by $\sqrt{x^3}+a\sqrt[4]{x^3-a^2}$,
(iii) by $\sqrt{x^3}-a\sqrt[4]{x^3-a^2}$.

231. Find the G. C. M. of

$$3a^4-a^2b^2-2b^4 \text{ and } 10a^4+15a^3b-10a^2b^2-15ab^3.$$

232. Find the L. C. M. of x^3-3x^2+3x-1, x^3-x^2-x+1, x^4-2x^3+2x-1, and $x^4-2x^3+2x^2-2x+1$.

233. Simplify $\dfrac{\frac{1}{2}\sqrt{\frac{1}{2}}}{\sqrt{2}+3\sqrt{\frac{1}{2}}}$ and $\dfrac{\sqrt[3]{x}+\sqrt{a}}{\sqrt[3]{x}-\sqrt{a}}-\dfrac{2a^{\frac{1}{2}}x^{\frac{1}{3}}}{x^{\frac{2}{3}}-a}$.

234. Extract the fourth root of

$$\frac{1}{16}x^{\frac{20}{3}}-\frac{5}{2}x^5y^{\frac{4}{5}}+\frac{75}{2}x^{\frac{10}{3}}y^{\frac{8}{5}}-250x^{\frac{5}{3}}y^{\frac{12}{5}}+625y^{\frac{16}{5}}.$$

235. Sum $16\frac{1}{3}+14\frac{2}{3}+13+$&c. to 11 terms, and $\frac{5}{6}+\frac{5}{9}+\frac{10}{27}+$&c. to n terms and *ad inf.*; and insert 3 H. means between 1 and 2.

236. Given $y^2-b^2 \propto x+a$, and when $x=b$, $y=a$, find the value of y when $x=3a$.

237. Four places lie in the order of the letters A, B, C, D. A is distant from D 34 miles, and the distance from A to B is $\frac{2}{3}$ of that from C to D; also $\frac{1}{4}$ of the distance from A to B is less than thrice the distance from B to C by $\frac{1}{2}$ of the distance from C to D. Find the respective distances.

238. If $(1+x)^n=1+A_1x+$&c., and $(1+x)^{-n}=1+B_1x+$&c., show, by finding the actual values of A_1, B_1, &c. that

$$A_3+A_2B_1+A_1B_2+B_3=0.$$

239. (i) $3x+20=7-\frac{1}{2}\{3-\frac{4}{3}(x-1)\}$. (ii) $\dfrac{m}{x}+\dfrac{n}{y}=a$, $\dfrac{n}{x}+\dfrac{m}{y}=b$.

(iii) $\dfrac{6y-4x}{3z-7}=\dfrac{5z-x}{2y-3z}=\dfrac{y-2z}{3y-2x}=1$.

240. If in (228) A work for $\frac{1}{4}(3m-2n)$ days and B for $\frac{1}{4}(3n-2m)$ days, in what time will C finish the work?

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241. Write down the quotient of x^2-y^{-3} by $x^{\frac{1}{2}}+y^{-\frac{3}{4}}$, and divide $x^3-2ax^2+(a^2+ab-b^2)x-a^2b+ab^2$ by $x-a+b$.

242. If $a=16$, $b=10$, $x=5$, $y=1$, find the value of $(x-b)(\sqrt{a}-b)+\sqrt{(a-b)(x+y)}$ and $(a-x)^2-(b-x^2)-\sqrt{(a-x)(b+y)}$.

243. Find the G. C. M. of $300x^3+265x^2+50x+24$ and $60x^2+53x+4$.

244. Simplify $\dfrac{a^{-1}+a^{-1}b^{-3}}{b-1+b^{-1}}$ and $\left\{\dfrac{1}{3}+\dfrac{2x}{3(1-x)}\right\}\times\left\{\dfrac{3}{4}-\dfrac{3}{2(1+x)}\right\}$.

245. Find $\sqrt{\cdot 6}$, and obtain by means of it the values of $\sqrt{2\frac{2}{3}}$, $\sqrt{4\frac{1}{6}}$, $(\sqrt{3}-\sqrt{2})^2$, and $(2\sqrt{3}+3\sqrt{2})\div(3\sqrt{3}-2\sqrt{2})$.

246. Show that $\sqrt{\{a^2+\sqrt[3]{a^4b^2}\}}+\sqrt{\{b^2+\sqrt[3]{a^2b^4}\}}=(a^{\frac{2}{3}}+b^{\frac{2}{3}})^{\frac{3}{2}}$.

247. Divide 48 into nine parts, so that each may just exceed that which precedes it by $\frac{1}{2}$.

248. Given the coefficients of the 4th and 6th terms of $(1+x)^{n+1}$ equal to one another: find n.

249. In the permutations of the first eight letters of the alphabet how many begin with ab?

250. Express 12345654321 in the scale of 12, and extract its square root in that scale.

251. (i) $\frac{2}{3}(x-5)-\frac{3}{11}(x-13\frac{1}{3})=15-\frac{3}{5}(19-\frac{1}{3}x)$.

(ii) $\left.\begin{array}{l} ax-by=a^2 \\ bx-ay=b^2 \end{array}\right\}$. (iii) $\left(\dfrac{8x-3}{4x-1}\right)^2=\dfrac{4x-5}{x-1}$.

252. Find the time in which A, B, C can together do a piece of work, which (i) A can do in m days, and B and C together in $\frac{1}{2}(m+n)$ days, or (ii) A can do in m days, A and B in n, and A and C in $\frac{1}{2}(m+n)$ days.

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253. Find the coefficient of x in $(x+2)(x-6)(x+10)(x-5)$, and of x^4 in $(1+\frac{1}{2}x+\frac{1}{3}x^2+\frac{1}{4}x^3+\&c.)\times(1-\frac{1}{3}x+\frac{1}{5}x^2-\frac{1}{7}x^3+\&c.)$.

254. Divide $x^{-2}+y$ by $x^{-\frac{2}{3}}+y^{\frac{1}{3}}$ and $x^{\frac{3}{4}}-ma^{\frac{1}{2}}x^{\frac{1}{2}}+max^{\frac{1}{4}}-a^{\frac{3}{2}}$ by $x^{\frac{1}{4}}-a^{\frac{1}{2}}$.

255. Find the L. C. M. of $6x^3-11x^2+5x-3$ and $9x^3-9x^2+5x-2$.

256. Simplify $\dfrac{\frac{2}{3}(1-\frac{1}{2}x)-x}{1-\frac{1}{2}(1+2x)}$, and reduce $\dfrac{2a^2+ab-b^2}{a^3+a^2b-a-b}$.

257. Find the sum of

$$\frac{a}{b}-\frac{ac}{b(b+c)}-\frac{a}{b}\cdot\frac{b-2c}{b-c}, \text{ and } \frac{2-2x^2}{\sqrt{(1-x^2)^5}}-\frac{1}{\sqrt{1-x^2}}.$$

258. A walks at the rate of 3 miles an hour, B starts 2 hours after him at 4 miles an hour: how many miles will A have walked before B overtakes him? Find also how long B should start after A, in order that A, when overtaken, may have walked 6 miles.

259. Simplify $b\sqrt[3]{8a^6b}+4a\sqrt[3]{a^3b^4}-\sqrt{125a^6b^4}$.

260. If the first term of an A. P. be 6, and the sum of 7 terms 105, find the common difference, and show that the sum of n terms : sum of $n-3$ terms : : $n+3 : n-3$.

261. Which is the greater of the ratios

$$a+2x : a+3x \text{ and } a^2+2ax+2x^2 : a^2+3ax+3x^2 ?$$

262. Of 12 white and 6 black balls how many different collections can be made, each composed of 4 white and 2 black balls?

263. (i) $(x-1\frac{2}{3})(x-2\frac{1}{2})=\frac{1}{2}(1+\frac{1}{3}x)(x-1)$.

(ii) $\frac{1}{5}x-\frac{1}{3}y+z=7$, $\frac{1}{2}x+y-\frac{1}{4}z=1$, $\frac{1}{3}y+\frac{1}{4}z-x+10=0$.

264. A market-woman bought eggs at two a penny, and as many more at three a penny; and, thinking to make her money again, she sold them at five for twopence. She lost, however, 4*d.* by the business: how much did she lay out?

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265. Show that $(x+x^{-1})^2-(y+y^{-1})^2=(xy-x^{-1}y^{-1})(xy^{-1}-x^{-1}y)$, and exemplify this result numerically when $x=\frac{1}{2}$, $y=-\frac{2}{3}$.

266. Find the G. C. M. of $4a^2x^4+9a^{\frac{3}{2}}x^3+2ax^2-2a^{\frac{1}{2}}x-4$ and $3a^{\frac{3}{2}}x^3+5ax^2-a^{\frac{1}{2}}x+2$.

267. Find by Evolution $\sqrt{a+6x}$ to five terms, and square the result.

268. Simplify $3a-[b+\{2a-(b-x)\}]+\frac{1}{2}-\dfrac{\frac{1}{2}-2x^2}{2x+1}$.

269. Find the sum of $\dfrac{1}{2x+2}-\dfrac{4}{x+2}+\dfrac{9}{2(x+3)}-\dfrac{x-1}{(x+2)(x+3)}$.

270. A gamester loses $\frac{1}{3}$ of his money, and then wins 10*s.*; he loses $\frac{1}{3}$ of this, and then wins £1, when he leaves off as he began. What had he at first?

271. The sum of n terms of the series $21+19+17+$&c. is 120; find the n^{th} term and n.

272. Divide 100 into two parts so that one shall be a multiple of 7 and the other of 11.

273. Into how many different triangles may a polygon of n sides be divided, by joining its angular points?

274. Convert 85 and 257 to the quaternary scale; multiply them in that scale, and reduce the result back to the denary.

275. (i) $\frac{1}{2}x+\frac{1}{3}x-1=\frac{1}{4}\{3x-\frac{1}{3}(x-1)\}$.

(ii) $ax+y=x+by=\frac{1}{2}(x+y)+1$. (iii) $3x^2y=144=4xy^2$.

276. A and B can reap a field of wheat in m days, B and C in n days, and A can do p times as much as C in the same time: in what time would the three reap it together?

277. Find the value of $ax+by-c$ when

$$x=\frac{mc-nb}{ma-lb} \text{ and } y=\frac{lc-na}{lb-ma}.$$

278. When $a=4$, $x=-8$, $y=1$, show that

$$a^{-\frac{3}{2}}x^2+y^{\frac{9}{4}}=(a^{-\frac{1}{2}}x^{\frac{2}{3}}+y^{\frac{3}{4}})\ (a^{-1}x^{\frac{4}{3}}-a^{-\frac{1}{2}}x^{\frac{2}{3}}y^{\frac{3}{4}}+y^{\frac{3}{2}}).$$

279. Reduce to its simplest form

$$\frac{3a^{-2}x^2+5a^{-1}x-12}{a^{-3}x^3-8a^{-2}x^2-12a^{-1}x+63}.$$

280. Find the L. C. M. of

$$ax^2-1,\ ax^2+1,\ (a^{\frac{1}{2}}x-1)^2,\ (a^{\frac{1}{2}}x+1)^2,\ a^{\frac{3}{2}}x^3-1,\ a^{\frac{3}{2}}x^3+1.$$

281. Obtain the square root of $x^{\frac{4}{3}}-4x+8x^{\frac{1}{3}}+4$.

282. Simplify $\sqrt[3]{40}-\frac{1}{2}\sqrt[3]{320}+\sqrt[3]{135}$, and $8\sqrt{\frac{3}{4}}-\frac{1}{2}\sqrt{12}+4\sqrt{27}-2\sqrt{\frac{3}{16}}$.

283. Show that the sum of the cubes of any three consecutive numbers is divisible by three times the middle number.

284. If $a:b::c:d$, show that $2a^2-3b^2:2c^2-3d^2::a^2+b^2:c^2+d^2$.

285. Two-thirds of a certain number of poor persons received 1*s.* 6*d.* each, and the rest 2*s.* 6*d.* each: the whole sum spent being £2 15*s.*, how many poor persons were there?

286. The No. of Comb^ns^ of n letters taken 5 and 5 together, in all of which a, b, and c occur, is 21: find the No. of Comb^ns^ of them taken 6 and 6 together, in all of which a, b, c, d, occur.

287. (i) $\sqrt{\frac{2}{3}x+(1-x)^2}=\frac{1}{3}-x$. (ii) $\frac{1}{x+3}+\frac{2}{x+6}=\frac{3}{x+9}$.
(iii) $x^2+xy+y^2=37$, $x+y=7$.

288. A certain number of sovereigns, shillings, and sixpences amount together to £8 6*s.* 6*d.*, and the amount of the shillings is a guinea less than that of the sovereigns and $1\frac{1}{2}$ guinea more than that of the sixpences: how many were there of each?

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289. What is the difference of $a(b+c)^2+b(a+c)^2+c(a+b)^2$ and $(a+b)(a-c)(b-c)+(a-b)(a-c)(b+c)-(a-b)(a+c)(b-c)$?

290. Prove the preceding result when $a=-\frac{1}{2}$, $b=\frac{1}{3}$, $c=-\frac{1}{4}$.

291. Multiply $1+\frac{1}{2}a^{-\frac{1}{2}}x+\frac{1}{4}a^{-1}x^2$ by $1-\frac{1}{2}a^{-\frac{1}{2}}x+\frac{1}{8}a^{-\frac{3}{2}}x^3-\frac{1}{16}a^{-2}x^4$.

292. Obtain the coefficient of x^6 in $(1-2x+3x^2-4x^3+\&c.)^2$.

293. Extract the square roots of $7\frac{9}{16}$, .064, and $31-10\sqrt{6}$.

294. Simplify

$$\{(a-b)^2+4ab\}^{\frac{1}{2}}\times\{(a+b)^2-4ab\}^{\frac{3}{2}}\times\left\{\frac{a^4-b^4}{a-b}+2ab(a+b)\right\}^{\frac{2}{3}}.$$

295. Given two numbers such that the difference of their squares is double of their sum, show that their product will be less than the square of the greater by the double of it.

296. Sum to n terms $\frac{a}{n^2}+\frac{2a}{n^2}+\frac{3a}{n^2}+$&c. and $\frac{a}{n}+1+\frac{n}{a}+$&c.

297. Required two numbers whose sum shall be triple of their difference, and less than 50 by the greater of the two.

298. The No. of Comb^{ns} of $n+1$ things, taken $n-1$ together, is 36: find the number of Permutations of n things.

299. (i) $(a+x)(b+x)-a(b+c)=a^2cb^{-1}+x^2$.
(ii) $\sqrt{x}+\sqrt{a-x}=2\{\sqrt{x}-\sqrt{a-x}\}$.
(iii) $2x^2+3y^2=5=-5(2x+3y)$.

300. A can do a piece of work in 2 hours which B can do in 4 hours, and B and C together in $1\frac{1}{2}$ hour: in what time could they do it, working all three together?

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301. Divide

$$12x-20x^{\frac{3}{4}}y^{-\frac{1}{3}}+27x^{\frac{1}{2}}y^{-\frac{2}{3}}-18x^{\frac{1}{4}}y^{-1}+4y^{-\frac{4}{3}} \text{ by } 4x^{\frac{1}{2}}-4x^{\frac{1}{4}}y^{-\frac{1}{3}}+y^{-\frac{2}{3}}.$$

302. Find the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$, when $x=\frac{4ab}{a+b}$.

303. Extract the square roots of

$$18945044881 \text{ and } (x+x^{-1})^2-4(x-x^{-1}).$$

304. Find the G. C. M. of $(b-c)x^2+2(ab-ac)x+a^2b-a^2c$ and $(ab-ac+b^2-bc)x+(a^2c+ab^2-a^2b-abc)$.

305. Simplify

$$\sqrt{128}-2\sqrt{50}+\sqrt{72}-\sqrt{18}, \text{and } (5\sqrt{5}-7\sqrt{2})\div(\sqrt{5}-2\sqrt{2})^2.$$

306. Find the sum of $\frac{x+y}{2x-2y}-\frac{y-x}{2x+2y}-\frac{x^2-y^2}{x^2+y^2}$.

307. When are the hour and minute hands of a watch first together after 12 o'clock?

308. Expand $(3a^{-\frac{2}{3}}-2a^{-1}x^{\frac{1}{3}})^{-3}$ to five terms.

309. Sum $\frac{1}{12}+\frac{1}{8}+\frac{1}{6}+$&c. to 8 and to $3n$ terms; and insert four H. means between $\frac{2}{3}$ and $\frac{3}{2}$.

310. The No. of Comb^{ns} of 10 letters, $r-1$ together : No. of Comb^{ns} of them, $r+1$ together :: 21 : 10 : find r.

311. (i) $.03x^2 - 2.7x = 30$.

(ii) $\left.\begin{array}{l}(x+a)(y-b)+c=(x-a)(y+b)-c \\ (x+b)(y-a)=(x+a)(y-b)\end{array}\right\}$.

(iii) $x+y=ax+by=ax^2-by^2$.

312. Supposing in (300) A to begin by himself, how long after must B and C begin to help him, so that, when the work is finished, A may have done upon the whole twice as much as C?

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313. Obtain the product of $\sqrt{a}+\sqrt[4]{ax}+\sqrt{x}$,

(i) by $\sqrt{a}-\sqrt[4]{ax}+\sqrt{x}$, (ii) by $\sqrt{x}-\sqrt[4]{ax}-\sqrt{a}$.

314. Find the value of $\dfrac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}$ when $x=$ (i) $\frac{24}{25}a$, (ii) $\dfrac{2ab}{1+b^2}$.

315. Write down the quotient of $16a^3x^2-y$ by $2a^{\frac{3}{4}}x^{\frac{1}{2}}+y^{\frac{1}{4}}$.

316. Extract the square root of $\frac{9}{4}-\sqrt{5}$, and of

$$25\tfrac{3}{7}-\tfrac{20}{7}xy^{-1}+\tfrac{9}{16}x^{-2}y^2-\tfrac{15}{2}x^{-1}y+\tfrac{4}{49}x^2y^{-2}.$$

317. Expand $\sqrt{\dfrac{a^2}{a-x}}$ and $\sqrt[3]{\dfrac{ax}{a+x}}$ each to five terms.

318. Multiply together

$1+2\sqrt{2}$, $4-\sqrt{3}$, $\sqrt{2}+\sqrt{3}$, $4+\sqrt{3}$, $2\sqrt{2}-1$, $\sqrt{3}-\sqrt{2}$.

319. Find the n^{th} term and the sum of n terms of the A. P.

$$\frac{a-n}{n}+\frac{a-2n}{n}+\frac{a-3n}{n}+\&c.$$

320. If the sum or difference of two numbers be 1, show that the difference of their squares is the difference or sum of the numbers respectively.

321. A servant agreed to live with his master for £8 a year and a livery, but was turned away at the end of 7 months, and received only £2 13*s.* 4*d.* and his livery: what was it worth?

322. How many different sums might be made of a sovereign, half-sovereign, crown, half-crown, shilling, and sixpence? and what would be the value of them all?

323. (i) $\dfrac{2x+a}{b}-\dfrac{x-b}{a}=\dfrac{3ax+(a-b)^2}{ab}$.

(ii) $\dfrac{x+1\frac{1}{3}}{x+2}-\dfrac{x+12}{2(x+19)}=\dfrac{1}{2}$. (iii) $ax-cy=0=ay+bx-cxy$.

324. Two girls carried between them 25 eggs to market: they sold at different prices, but each received the same amount upon the whole: the first would have sold them *all* for 1*s.*, the second for 13*d.*: how many did they each sell?

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325. Write down the square of $1-\frac{1}{2}x+\frac{1}{3}x^2$, and square the result.

326. Divide $-2x^5y^{-8}+17x^6y^{-4}-5x^7-24x^8y^4$ by $-x^2y^{-5}+7x^3y^{-1}+8x^4y^3$.

327. Find $\sqrt{7}$, and thence $\sqrt{\frac{1}{7}}$, $\sqrt{1\frac{3}{4}}$, $\sqrt{3\frac{1}{2}}+\sqrt{4\frac{1}{2}}$, $2\div(4-\sqrt{7})$.

328. Find the value of $\dfrac{a}{2na-2nx}+\dfrac{b}{2nb-2nx}$, when $x=\frac{1}{2}(a+b)$.

329. Simplify $\sqrt[5]{32a^6-96a^5x}$, and $\dfrac{3\sqrt{8}-2\sqrt{12}+\sqrt{20}}{3\sqrt{18}-2\sqrt{27}+\sqrt{45}}$.

330. Find the sum of $\dfrac{1}{x^3}+\dfrac{1}{x^2}-\dfrac{1}{x}-\dfrac{1}{(x^2+1)^2}+\dfrac{x-1}{x^2+1}-\dfrac{3}{x^2(x^2+1)^2}$.

331. If $u=p+q+r$, where p is constant, $q \propto xy$, and $r \propto xy^{-1}$, and when $x=y=1$, $u=0$, when $x=y=2$, $u=6$, and when $x=0$, $u=1$, find u in terms of x and y.

332. Show by the Bin. Theorem that $\sqrt[3]{3}=1+\frac{2}{3}-\frac{4}{9}+\frac{40}{81}-\frac{160}{243}+\&c.$

333. In how many ways could I distribute exactly 55*s.* among the poor of a parish, by giving 1*s.* 6*d.* to some and 2*s.* 6*d.* to others?

334. How many words can be formed of 4 consonants and 2 vowels, in a language of 24 letters, of which 5 are vowels?

335. (i) $\dfrac{c}{a-c}\left(x+\dfrac{1}{x}\right)=1+\dfrac{a+c}{(a-c)x}+\dfrac{b}{a-c}\left(1+\dfrac{1}{x}\right)$.

(ii) $4x-5y+mz=7x-11y+nz=x+y+pz=3$.

336. A boat's crew rowed $3\frac{1}{2}$ miles down a river and up again in 100′: supposing the stream to have a current of 2 miles an hour, find at what rate they would row in still water.

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337. If $x=\dfrac{2ac}{b(1+c^2)}$, find the value of $\dfrac{\sqrt{a+bx}+\sqrt{a-bx}}{\sqrt{a+bx}-\sqrt{a-bx}}$.

338. Reduce to its lowest terms $\dfrac{3ax^3-2a^{\frac{2}{3}}x^2-a^{\frac{1}{3}}}{6a^{\frac{2}{3}}x^2-a^{\frac{1}{3}}x-1}$.

339. Find the coefficient of x^6 in $(1+\frac{1}{2}x+\frac{2}{3}x^2+\frac{3}{4}x^3+\&c.)^3$.

340. Find the sum of $\dfrac{a^3+a^2b}{a^2b-b^3}-\dfrac{a(a-b)}{(a+b)b}-\dfrac{2ab}{a^2-b^2}$.

341. Simplify $a^3bc\sqrt[5]{a^{-9}bc}-b^2c\sqrt[5]{a^6b^{-4}c}+a^2b^4c^2\sqrt[5]{243a^{-4}b^{-14}c^{-4}}$.

342. Obtain the cube roots of 51.064811, and $1-6x+21x^2-44x^3+63x^4-54x^5+27x^6$.

343. The prime cost of 38 gallons of wine is £25, and 8 gallons are lost by leakage: at what price per gallon should the remainder be sold, to gain 10 per cent. upon the outlay?

344. If $a : b :: c : d$, show that

$$a^2\{(a^2+d^2)-(b^2+c^2)\}=(a^2-b^2)(a^2-c^2).$$

345. Expand $\{2a-3\sqrt{ax}\}^{\frac{5}{3}}$ and $\{3a-2\sqrt[3]{a^2x}\}^{-\frac{5}{2}}$, each to five terms.

346. From a company of 50 men, 5 are draughted off every night on guard: on how many different nights can a different selection be made? and on how many of these will two given soldiers be found upon guard?

347. (i) $\dfrac{a(a^2+x^2)}{a+x}=ax+b^2.$ (iii) $\left.\begin{aligned}axy&=c(bx+ay)\\ bxy&=c(ax-by)\end{aligned}\right\}.$

(ii) $5x-11y^{\frac{1}{2}}+13z^{\frac{1}{3}}=22,\ 4x+6y^{\frac{1}{2}}+5z^{\frac{1}{3}}=31,\ x-y^{\frac{1}{2}}+z^{\frac{1}{3}}=2.$

348. A person, having to walk 10 miles, finds that, by increasing his speed half a mile an hour, he might reach his journey's end $16\frac{2}{3}$ minutes sooner than he otherwise would: what time will he take, if he only begin to quicken his pace halfway?

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349. Divide $(x^3-1)a^3-(x^3+x^2-2)a^2+(4x^2+3x+2)a-3(x+1)$ by $(x-1)a^2-(x-1)a+3$.

350. Multiply $\sqrt[3]{a^{-\frac{1}{2}}}+\sqrt{(a^{\frac{1}{2}}c)^{\frac{1}{3}}}$ by $\sqrt{a^{-\frac{1}{3}}}-\sqrt[3]{(a^{\frac{1}{2}}c)^{\frac{1}{2}}}$.

351. If $x=\sqrt[3]{\{-\frac{1}{2}r+\sqrt{(\frac{1}{4}r^2-\frac{1}{27}q^3)}\}}$, find the value of $x^6+rx^3+\frac{1}{27}q^3$.

352. Extract the square root of $\frac{9}{4}x^3-5x^{\frac{5}{2}}y^{\frac{1}{2}}+\frac{179}{45}x^2y-\frac{4}{3}x^{\frac{3}{2}}y^{\frac{3}{2}}+\frac{4}{25}xy^2$.

353. Add together $\dfrac{\frac{1}{2}(1+\sqrt{5})x-2}{x^2-\frac{1}{2}(1+\sqrt{5})x+1}$ and $\dfrac{\frac{1}{2}(1-\sqrt{5})x-2}{x^2-\frac{1}{2}(1-\sqrt{5})x+1}$.

354. Find the sum to n terms and *ad inf.* of the G. P., whose first two terms are the A. and H. means between 1 and 2.

355. What is the least number which is divisible by 7 and 11 with remainders 6 and 10 respectively?

356. A privateer, running at the rate of 10 miles an hour, discovers a ship 18 miles off, making away at the rate of 8 miles an hour: how long will the chase last?

357. Expand $\{2a-3\sqrt{ax}\}^{-\frac{5}{3}}$ and $\{3a-2\sqrt[3]{a^2x}\}^{\frac{5}{2}}$, each to five terms.

358. In what scale will the common number 803 be expressed by 30203? What are the greatest and least common numbers that can be expressed with five digits in it?

359. (i) $\dfrac{a^2}{b+x}+\dfrac{a^2}{b-x}=c.$ (ii) $\dfrac{a^2}{b+x}-\dfrac{a^2}{b-x}=c.$

(iii) $\dfrac{x}{a}+\dfrac{y}{b}=1=\dfrac{x-a}{b}+\dfrac{y-b}{a}.$

360. A, B, C reaped a field together in a certain time: A could have done it alone in $9\frac{7}{8}$ hrs. more, B in half the time that A could, and C in an hour less than B. What time did it take them?

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361. Divide $\sqrt[12]{x^{10}y^9} - z\sqrt[6]{x^7y^5} - \frac{3}{2}x\sqrt[4]{y^3} + \frac{3}{2}x^2yz\sqrt[6]{x^{-4}y^{-1}}$ by $\sqrt{xy} - \frac{3}{2}\sqrt[6]{x^4y^3}$.

362. The edges of three cubes are a, b, $a+b$; show that the greatest : difference between it and the sum of the others $:: (a^{\frac{1}{2}}b^{-\frac{1}{2}} + a^{-\frac{1}{2}}b^{\frac{1}{2}})^2 : 3$.

363. Extract the square root of $x + 1 - 2\sqrt[4]{x}(1 + \sqrt{x}) + 3\sqrt{x}$.

364. Simplify $\sqrt[3]{72} - 3\sqrt[3]{\frac{1}{3}}$ and $\sqrt{2ax^2} - \sqrt{2ax^2 - 4ax + 2a}$.

365. If $x = \frac{1}{2}(\sqrt{3} + 1)$, find the value of $4(x^3 - 2x^2) + 2x + 3$.

366. A's money with $\frac{1}{2}$ of B's would be $\frac{1}{3}$ as much again as before; and if 2*s*. be taken from A's present sum and added to B's, the latter amount will be $\frac{1}{3}$ of the former. What had they each at first?

367. Find the value of $\sqrt[3]{a + 6x}$, and square the result.

368. If the difference of two fractions be mn^{-1}, show that m times their sum $= n$ times the difference of their squares.

369. The first term of an A. P. is $n^2 - n + 1$, the common difference 2 : find the sum of n terms, and thence show that $1 = 1^3$, $3 + 5 = 2^3$, $7 + 9 + 11 = 3^3$, &c.

370. Find the area of a court 250 ft. long by 200 ft. broad, (i) by the senary, (ii) by the duodenary scale.

371. (i) $\dfrac{1}{ab - ax} + \dfrac{1}{bc - bx} = \dfrac{1}{ac - ax}$. (ii) $nx + \dfrac{b}{x} = na + ba^{-1}$.
(iii) $x^2 + y^2 = 2a^2$, $x + y : x - y :: m : n$.

372. A cistern has three pipes A, B, and C: by A and B together it can be filled in 36′, and emptied by C in 45′; whereas, if A and C were opened together, it would be emptied in $1\frac{1}{2}$ hr.: in what time would it be filled, by A, by B, or by all together?

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373. Find $\dfrac{1}{mn - mz} + \dfrac{1}{np - nz} + \dfrac{1}{mz - mp}$, when $z = \dfrac{n}{m}(m - n + p)$.

374. Multiply $max^{\frac{5}{3}} + (m-1)a^2x^{\frac{2}{3}} + (m-2)a^3x^{-\frac{1}{3}}$ by $a^{-1}\sqrt[3]{x^4} - \sqrt[3]{x}$.

375. Extract the square root of $1 + m^3 + 2(1 - m^2)\sqrt{m} + 3m - m^2$.

376. Simplify $\dfrac{xy}{x-y} \pm \sqrt{\dfrac{x^2y^2}{(x-y)^2}+\dfrac{x^2y}{x-y}} - \dfrac{xy}{x \mp \sqrt{xy}}$.

377. Find a number of two digits such that its quotient by their sum exceeds the first digit by 1, and equals the other.

378. How many terms of the series $-7-5-3-$&c. amount to 9200? and how many of $6+4+2\frac{2}{3}+$&c. amount to $14\frac{4}{9}$?

379. A certain number of men mowed 4 acres of grass in 3 hours, and a certain number of others mow 8 acres in 5 hours: how long would they be in mowing 11 acres, all working together?

380. If a, b, c, d are in G. P., show that

$$(a+b+c+d)^2=(a+b)^2+(c+d)^2+2(b+c)^2.$$

381. The No. of Var[ns] of n things, r together : the No., $r-1$ together :: 10 : 1, and the corresponding Nos. of Comb[ns] are as 5 : 3; find n and r.

382. A person makes 20 lbs. of tea at 4*s.* 9*d.*, by mixing three kinds at 3*s.* 6*d.*, 4*s.* 6*d.*, and 5*s.*: how can this be done?

383. (i) $\frac{1}{2}(x-1\frac{25}{26})-\dfrac{1-3x}{6\frac{1}{2}}=x-\frac{1}{39}\{5x-\frac{5}{2}(1-.3x)\}$.

(ii) $x+a+b+c=\dfrac{x^2+a^2+b^2+c^3}{a+b-c+x}$. (iii) $\left.\begin{array}{r}\left(\dfrac{x}{a}\right)^2+\left(\dfrac{y}{b}\right)^2=2\\ ay+bx=0\end{array}\right\}$.

384. A trader maintained himself for 3 years at an expense of £50 a year, and in each of these years increased that part of his stock which was not so expended by $\frac{1}{3}$ thereof: at the end of 3 years his original stock was doubled; find it.

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385. Divide $(6a^2-7ab+2b^2)\,x^3+(5a^3-3a^2b-5ab^2+3b^3)\,x^2+(a^2-b^2)^2x$ by $(2a-b)x+a^2-b^2$.

386. Find the L. C. M. of

$x^4-(p^2+1)\,x^2+p^2$ and $x^4-(p+1)^2\,x^2+2(p+1)px-p^2$.

387. Obtain the values of (i) $x-\sqrt{xy}+y$, and (ii) of x^2+xy+y^2, when $x=\frac{1}{16}(4\frac{1}{3}+\sqrt{7\frac{2}{3}})$, $y=\frac{1}{16}(4\frac{1}{3}-\sqrt{7\frac{2}{3}})$.

388. Simplify $(a-b)\left\{\dfrac{1}{(x+a)^2}+\dfrac{1}{(x+b)^2}\right\}+2\left\{\dfrac{1}{x+a}-\dfrac{1}{x+b}\right\}$.

389. Obtain the square roots of

$$2+a^{2\sqrt{2}}+a^{-2\sqrt{2}} \text{ and } \frac{a^2c}{b}+cf-2ac\sqrt{\frac{f}{b}}.$$

390. The n^{th} term of an A. P. is $\frac{1}{2}n-\frac{1}{6}$: find the sum of n terms.

391. The diagonal of a cube is a foot longer than each of the sides: find the solid content.

392. Find the first time after noon when the hour and minute hands of a watch point exactly in opposite directions.

393. In how many ways may £10 be paid in crowns, sevenshilling pieces, and moidores (27*s.*), thirty coins being used?

394. Out of 5 white, 7 red, and 8 black balls, how many different sets of 6 balls could be drawn, (i) two of each colour, (ii) one white, two red, three black, (iii) three red, three black?

395. (i) $x+\sqrt{x^2-2ax+b^2}=a+b.$

(ii) $\frac{a}{x+a}-\frac{c}{x-c}=\frac{a-c}{x+a-c}.$ (iii) $\sqrt{1+\frac{bx}{a^2}}+\sqrt{1-\frac{bx}{a^2}}=1\frac{3}{5}.$

396. Two vessels, A and B, contain each a mixture of water and wine, A in the ratio of $2:3$, B in that of $3:7$. What quantity must be taken from each, to form a mixture which shall consist of 5 gallons of water and 11 of wine?

397. Show that $(ay-bx)^2+(cx-az)^2+(bz-cy)^2$

$$=(a^2+b^2+c^2)\,(x^2+y^2+z^2)-(ax+by+cz)^2.$$

398. Find the G. C. M. of $3x^2+(4a-2b)x-2ab+a^2$ and

$$x^3+(2a-b)x^2-(2ab-a^2)x-a^2b.$$

399. From $\frac{1}{2}(x^{\frac{1}{2}}+3x^{-\frac{1}{2}})\,(x^{\frac{1}{2}}-2x^{-\frac{1}{2}})$ take $\frac{1}{3}(x^{\frac{1}{2}}+2x^{-\frac{1}{2}})\,(x^{\frac{1}{2}}-3x^{-\frac{1}{2}})$. and multiply the result by $6\,(1-x^{\ 1})^{-1}$.

400. Extract the square root of $x^{\frac{2}{3}}+2x^{\frac{1}{2}}+3x^{\frac{1}{3}}-2x^{-\frac{1}{6}}+x^{-\frac{1}{3}}-1.$

401. Multiply together

$$\sqrt[2n]{(a+b)^{m+1}},\ \sqrt{(a+b)^{\frac{n+1}{m}}},\ \sqrt{(a+b)^{\frac{m-1}{n}}},\ \sqrt[m]{(a+b)^{\frac{n-1}{2}}}.$$

402. Simplify

$$\left\{\frac{1+x}{1-x}+\frac{4x}{1+x^2}+\frac{8x}{1+x^4}-\frac{1-x}{1+x}\right\}\div\left\{\frac{1+x^2}{1-x^2}+\frac{4x^2}{1+x^4}-\frac{1-x^2}{1+x^2}\right\}.$$

403. Sum $(a+x)^2+(a^2+x^2)+(a-x)^2+$&c. to 5 and to n terms.

404. Suppose that A, B, C, start at one point in a circumference of 42 miles, to travel round it in one direction, A at the rate of 5, B $5\frac{1}{3}$, and C $6\frac{1}{4}$ miles an hour; in what time will they all be together at one point, if B set out 3 hrs. 45 min. after A, and C 9 minutes before B?

405. Apply the Bin. Theor. to find $(1.01)^{-\frac{3}{2}}$ to nine places.

406. Find the least integer which divided by 125 leaves the cube root of 729, and divided by 729 leaves the cube root of 125.

407. (i) $x+3=\sqrt{2(x+3)}+4$. (ii) $abx^2-(a+b)cx+c^2=0$.

(iii) $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1,\ \frac{x}{b}-\frac{y}{a}=0$.

408. A person bought 38 sheep for £57; but, having lost a certain number, n, of them, he sold the remainder for n shillings a head more than they cost him, and so gained upon the whole 16*s.*: how many sheep did he lose?

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409. Show that $(a^2+b^2-1)^2+(a'^2+b'^2-1)^2+2(aa'+bb')^2$
$=(a^2+a'^2-1)^2+(b^2+(b'^2-1)^2+2(ab+a'b')^2$.

410. Find the G. C. M. of $xy+2x^2-3y^2+4yz+xz-z^2$ and
$2x^2-9xz-5xy+4z^2-8yz-12y^2$.

411. Find the fourth term of $(\sqrt{2}+\sqrt{3})^6$, correct to four places.

412. Obtain the square root of $1+x-\frac{3}{2}\sqrt[3]{x}(1+\sqrt{x})$
$+\sqrt{x}(2+\frac{9}{16}\sqrt[6]{x})$.

413. If the r^{th} term of a series be $ar^{-1}-r$, show that the sum of the m^{th} and n^{th} terms exceeds the $(m+n)^{th}$ by $\frac{m^2+mn+n^2}{mn(m+n)}a$.

414. If $x^{-1}=(a-c)(b-c)$, $y^{-1}=(a-b)(b-c)$, $z^{-1}=(a-b)(a-c)$, find the values of $x-y+z$ and $abx-acy+bcz$.

415. If P, Q, R, be the p^{th}, q^{th}, and r^{th} terms of any H. P, show that $(p-q)PQ+(q-r)QR+(r-p)RP=0$.

416. Two parcels of cotton, weighing 9 lbs. and 16 lbs., cost 11*s.* 6*d.* and £1 0*s.* 4*d.* respectively, and the charge for carriage was proportional to the square root of the weight: how much per lb. was paid for the purchase of the cotton?

417. If $a:b::b:c$, show that $a+b:b+c::a^2(b-c):b^2(a-b)$.

418. Find the least number which being divided by 36, or 100, or 169, shall leave in each case the square root of the divisor.

419. (i) $(x-1)+2(x-2)+3(x-3)+$&c. to six terms $=14$.

(ii) $\frac{2x(a-x)}{3a-2x}=\frac{1}{4}a$. (iii) $\frac{x}{a}+\frac{y}{b}=1=\frac{x}{a}+\frac{z}{c},\ yz=bc$.

420. A square court-yard has a rectangular walk around it; the side of the court wants 2 yds of being six times the breadth of the walk, and the no. of sq. yds. in the walk exceeds by 92 the no. of yds in the periphery of the court: find its area.

MISCELLANEOUS EXAMPLES: PART II.

1. Find the L. C. M. of $2x^3 + (2a - 3b)x^2 - (3a + 2)bx + 3b^2$ and $2x^2 - (3b - 2c)x - 3bc$.

2. Divide $ax^{-1} + a^{-1}x + 2$ by $a^{\frac{1}{3}}x^{-\frac{1}{3}} + a^{-\frac{1}{3}}x^{\frac{1}{3}} - 1$.

3. Shew that $(a - b)^2 + (b - c)^2 + (c - a)^2$
$$= 2\{(a - b)(a - c) + (b - a)(b - c) + (c - a)(c - b)\}.$$

4. A starts from a certain place, and travels a miles the first day, $2a$ the second, $3a$ the third, &c.; after 4 days, B starts to overtake him, travelling $9a$ miles per day. After how many days will he come up with him?

5. Solve in positive integers $4x + 5y + 14z = 49$.

6. If α, β, are the roots of $x^2 + px + q = 0$, find the value of $\alpha^2 + \alpha\beta + \beta^2$, $\alpha^3 + \beta^3$, and $\alpha^4 + \alpha^2\beta^2 + \beta^4$.

7. Divide a given number a into n parts in the ratio of 1, 2, 3, &c.

8. The No. of Variations of $m + n$ things, two together, is 56, and of $m - n$ things is 12: find the No. of Combinations of m things, n together.

9. Solve the equation $\sqrt{(a^2 + cx)} + \sqrt{(a^2 - cx)} = \sqrt{(2acx)}$.

10. £P is left among A, B, C, so that, at the end of a, b, c, years respectively, when they come to age, they will possess equal sums: find the present share of each at Comp. Interest.

11. Reduce $\dfrac{y^3 - (2a + b)y^2 + (2ab + a^2)y - a^2b}{3y^2 - (4a + 2b)y + 2ab + a^2}$.

12. If $2m = x + x^{-1}$ and $2n = y + y^{-1}$, express $mn + \sqrt{\{(m^2 - 1)(n^2 - 1)\}}$ in terms of x and y.

13. Find the n^{th} term of an A. P., when the sum of $n + 1$ terms is $(n + 1)(n + 1\frac{1}{3})$.

14. A lb. of tea and 3 lbs. of sugar cost together 6*s*; but, if sugar were to rise 50 *per cent* and tea 10 *per cent*, they would cost 7*s*: find their prices.

15. If $a_1 : a_2 :: a_2 : a_3 :: \text{\&c.} :: a_{n-1} : a_n$, then $a_1 : a_2 :: \sqrt[n-1]{a_1} : \sqrt[n-1]{a_n}$.

16. A, counting a box of oranges, which was known to contain under 200, observed that when he told them by 2, 3, 4, 5, 6 at a time he had none over, but when by 7, he had 5 over: how many had he?

17. Two vessels contain each a mixture of wine and water. In A the wine : water :: 1 : 3, in B :: 3 : 5; how much must be taken from each to make 5 gals. of wine and 9 of water?

18. Obtain the square root of $1+(1-c^2)^{-\frac{1}{2}}$ in the form $\alpha^{\frac{1}{4}}+\beta^{\frac{1}{4}}$.

19. Solve the equation $\dfrac{x}{a+x}+\dfrac{a}{\sqrt{(a+x)}}=\dfrac{b}{x}$.

20. A flag-staff (c) stands at the top of a tower, whose height is a. Find the distance from the foot of the tower, at which the flag-staff subtends the greatest angle.

21. Decompose $1-\left\{\dfrac{a^2+b^2-c^2-d^2}{2(ab+cd)}\right\}^2$ into simple factors.

22. Form the equation whose roots are $\sqrt{m}\div\{\sqrt{m}\pm\sqrt{(m-n)}\}$.

23. If $a:b::c:d$, then $a+b:c+d::a^2(c-d):c^2(a-b)$.

24. Shew that in the series 1, 3, 5, &c., the first half of any even No. of terms has to the second half a fixed ratio.

25. If A had travelled half-a-mile an hour faster, he would have finished his journey in $\frac{4}{5}$ of the time: whereas, if he had travelled half-a-mile an hour slower, he would have been $2\frac{1}{2}$ hrs longer on the road. How many miles did he travel?

26. Find the No. of Combinations that can be made of the letters in the word *Notation*, taken 3 together.

27. A sum of £8 6s 6d is made up of sovereigns, shillings, and sixpences; find the No. of coins of each kind, it being known that the amount of the shillings is a guinea less than that of the sovereigns, and a guinea and a half more than that of the sixpences.

28. Find the equated time of payment, at 5 per cent Simp. Int., for sums of £400 and £2100, due at the end of 2 years and 8 years respectively.

29. Solve the equations $(x+y)(x^3+y^3)=76$, $(x+y)^3=64(x-y)$.

30. In a certain country, the births in a year amount to an m^{th} of the whole population, and the deaths to an n^{th}: in how many years will the population be doubled?

31. Express in the form of the sum of two simple surds the roots of the equations,

    (i) $x^4-2ax^2+b^2=0$, (ii) $4x^4-4(1+n^2)a^2x^2+n^2a^4=0$.

32. Find the relation between p, q, r, s, (i) when px^3+qx^2+rx+s is a perfect *cube*, (ii) when $x^4+px^3+qx^2+rx+s$ is a square.

33. A train, an hour after starting, meets with an accident which detains it an hour, after which it proceeds at $\frac{3}{5}$ of its former rate, and arrives 3 hrs behind time: but had the accident happened 50 miles farther on the line, it would have arrived $1\frac{1}{3}$ hr sooner. Find the length of the line.

34. If $x + 2\left\{x - \frac{1}{n-1}\right\} + 3\left\{x - \frac{2}{n-1}\right\} + 4\left\{x - \frac{3}{n-1}\right\} + \&c.$ to n terms $= 0$, shew that $x = \frac{2}{3}$.

35. Find the No. which when divided by 7, 8, 9, leaves rem^{rs} 1, 2, 3, and such that the sum of the three quotients is 570.

36. Find the coefficient of x^n in $\log_e \frac{1}{1 - ax + x^2}$.

37. Given the No. of Var^{ns} of $2n + 1$ things, $n - 1$ together : No. of Var^{ns} of $2n - 1$ things, n together, :: 3 : 5; determine n.

38. Given $\log 1\frac{1}{4} = .0969100$, and $\log .\dot{1} = \bar{1}.0457575$, find the logs of $2\frac{1}{2}$, $2\frac{1}{4}$, $.\dot{2}$, $\sqrt{\frac{1}{2}}$, $\sqrt[3]{\frac{1}{3}}$, and $\frac{3}{40}\sqrt[5]{(.0027)^3} \div \sqrt[3]{(.015)^5}$.

39. Solve the equations $\sqrt{(ax)} + \sqrt{(by)} = \frac{1}{2}(x + y) = a + b$.

40. What is the chance of throwing (i) 10, (ii) 20, in three throws with two dice?

41. Shew that $N(N^4 - 1)$ or $N(N^2 + 20)$ will be divisible by 48, according as N is an *odd* or *even* number.

42. Find the value of $\sqrt{\left(\frac{1}{x^2 - x} + \frac{1}{4x^2}\right)} - \frac{1}{2x}$, when $x = 0$.

43. A person distributed p shillings among n persons, giving $9d$ to some and $15d$ to the rest. How many were there of each?

44. Obtain the square root of

$$(\sqrt[x]{\sqrt[y]{a^{-1}}})^2 + \sqrt[z]{(b\sqrt[y]{a})} + 2\sqrt{(\sqrt[z]{b})}\,.\,\sqrt{\sqrt[x]{\sqrt[y]{\sqrt[z]{a^{x-2z}}}}}.$$

45. Divide 75 into two parts so that, when divided by 5 and 6 respectively, they may each give the same remainder 4.

46. Write down the general terms of $(a^2 - x^2)^{\frac{2}{3}}$ and $(a^2 + x^2)^{-\frac{2}{3}}$.

47. If $\frac{a}{b} = \frac{c}{d}$, shew that $\frac{a^2 + b^2}{c^2 + d^2} = \frac{ab}{cd} = \left(\frac{a + b}{c + d}\right)^2 = \frac{ma^2 - nab + pb^2}{mc^2 - ncd + pd^2}$.

48. A country trebles its population in a century: what is the increase in one year per million, given $\log 3 = .4771213$, $\log 101\frac{1}{10} = 2.0047512$, $\log 101\frac{11}{100} = 2.0047941$.

49. Solve (i) $\left(\frac{a + x}{a - x}\right)^2 = 1 + \frac{cz}{ab}$, (ii) $\frac{a^3x}{y^2z^2} = \frac{b^3y}{x^2z^2} = \frac{c^3z}{x^2y^2} = 1$.

50. Compare the chances of throwing 4 with one die, 8 with two, and 12 with three dice, having two throws in each case.

51. If $(a^2+bc)^2 . (b^2+ac)^2 . (c^2+ab)^2 = (a^2-bc)^2 . (b^2-ac)^2 . (c^2-ab)^2$, shew that either $a^3+b^3+c^3+abc=0$, or $a^{-3}+b^{-3}+c^{-3}+a^{-1}b^{-1}c^{-1}=0$.

52. If N, N' be two consecutive numbers, neither of them a multiple of 3, shew that $N^3+N'^3$ is a multiple of 9 and $(N^3 \sim N'^3) - 7$ of 54.

53. Evaluate $\dfrac{x^3-1}{x^3-2x^2+2x-1}$ and $\dfrac{x^2(y+1)-xy-1}{x^2(y-1)-x(y-2)-1}$, when $x=1$.

54. Given $\log \frac{1}{2} = \bar{1}.6989700$, $\log \frac{1}{3} = \bar{1}.5228787$, find the logs of $\sqrt{3}$, $\sqrt[3]{2}$, $\sqrt[4]{6}$, $\sqrt[5]{(1.44)^2}$, $\frac{2}{3}\sqrt[3]{.05}$, $\frac{1}{2}\frac{2}{5}\sqrt[4]{(1.6)^3} \times \sqrt[3]{(21.6)^4}$.

55. Find the n° of terms in $(a+b^2+c^3)^{12}$ and the coeff. of $a^8b^4c^6$.

56. Expand $(1+x)^{\frac{3}{2}}$ to 5 terms by reversion of series.

57. The sum of n terms of an A. P. is $pn+qn^2$: find the m^{th} term.

58. A gives B a bill for £a, due at the end of m years, in discharge of a bill for £b, due at the end of n years: for what sum should B give A a bill due at the end of p years, to balance the account at Comp. Int.?

59. Solve the equation $\left(\dfrac{x}{x-1}\right)^2 + \left(\dfrac{x}{x+1}\right)^2 = n(n-1)$.

60. In a bag are five red balls and one white: find the chance that in three drawings (replacing the ball drawn after each) there will have been drawn (i) three white balls, (ii) three red balls, (iii) two red and one white.

61. Find the G. C. M. of

$$(ax+by)^2 - (a-b)(x+z)(ax+by) + (a-b)^2 xz$$

and $(ax-by)^2 - (a+b)(x+z)(ax-by) + (a+b)^2 xz$.

62. Evaluate $\left(\dfrac{x^3-a^3}{x^2-a^2}\right)^2$ and $\dfrac{1-e^{1-\frac{x}{a}}}{x^4-a^4}$, when $x=a$.

63. Find the *maximum* or *minimum* value of $\dfrac{4m^2x^2+1}{(4m^2+1)x}$.

64. A person bought for £100 a hundred head of cattle, for which he paid 30*s*, 10*s*, and £10 a head respectively: how many did he buy of each, it being known that the sheep and oxen were together under 20, the rest being pigs?

65. Determine m and n in terms of a and b, so that $\dfrac{ma+nb}{m+n}$ may be the Arithmetic mean between m and n, and the Geometric mean between a and b.

66. Find the coefficients of $x^{\frac{3}{2}}$ and x^2 in $(a - bx^{\frac{1}{2}} + cx - dx^{\frac{1}{4}})^7$.

67. The mail from A starts for B at p hrs P.M., and that from B for A at q hrs A.M. Now, if the mail from B, n hrs after starting, meet a mail from A, and again at a miles from A meet another mail from A, what is the distance from A to B?

68. If $x^2 = ay - y^2 + a^{-1}y^3 - \&c.$, express y in terms of x.

69. Solve $\sqrt{(x^2 + 2bx + a^2)} - \sqrt{(x^2 - 2bx + a^2)} = 2\sqrt{(x^2 - b^2)}$.

70. Find the last two chances in [60], if the balls are *not* replaced.

71. Simplify $(x+1)(x^2+x+1)^{-1}+(x-1)(x^2-x+1)^{-1}+2(x^4+x^2+1)^{-1}$.

72. Shew that $\dfrac{2+\sqrt{3}}{\sqrt{2}+\sqrt{(2+\sqrt{3})}} + \dfrac{2-\sqrt{3}}{\sqrt{2}-\sqrt{(2-\sqrt{3})}} = \sqrt{2}$.

73. Shew that $a : b :: a^2 + ac + c^2 : b^2 + bc + c^2 :: (a + c)^2 : (b + c)^2$, if c be a mean proportional between a and b.

74. Write the general terms of $(1+x)^{-3}$, $(1-x)^{-\frac{1}{3}}$, and $(a^2 - ax)^{\frac{3}{4}}$.

75. Find what values of x make $\dfrac{x^3 - 2ax^2 - a^2x + 2a^3}{x^3 - ax^2 - 4a^2x + 4a^3}$ a vanishing fraction, and evaluate it in those cases.

76. Given S the sum, and s^2 the sum of the squares, of the terms of an infinite G.P., shew that its sum to n terms

$$= S\left\{1 - \left(\frac{S^2 - s^2}{S^2 + s^2}\right)^n\right\}.$$

77. A and A' can separately produce effects a and a' in times t and t': in what time could they together produce an effect c? Shew that if $c = a + a'$, this time will be the A. or G. mean between t and t', according as $t : t' = a : a'$ or $a^2 : a'^2$.

78. Shew that the n° of different Combinations of n things taken 1, 2, 3, &c. n together, of which p are of one sort, q of another, r of another, &c., is $(p+1)(q+1)(r+1)\ldots - 1$.

79. Solve the equations $ax + \dfrac{b^3}{y} = a^2 = (a^2 - bx)\dfrac{y}{a}$.

80. A had in his pocket a sovereign and four shillings; taking out two coins at random, he promises to give them to B and C: what is the worth of C's expectation?

81. Shew that $(a + b - c)^3 + (b + c - a)^3 + (c + a - b)^3 > 3abc$.

82. Eliminate x and y from $ax+by=c\sqrt{(x^2+y^2)}$, $a'x+b'y=c'\sqrt{(x^2+y^2)}$.

83. Simplify $\dfrac{7+\sqrt{-3}}{2-\sqrt{-3}} + \dfrac{8+3\sqrt{-3}}{2+\sqrt{-3}} - \dfrac{4(2-\sqrt{-3})}{1-\sqrt{-3}}$.

84. Resolve $\dfrac{x^2+x+1}{(x-1)(x-2)(x-3)}$ and $\dfrac{x^2-x+1}{x^2(x-1)^2}$ into partial fractions.

85. If a, b, c are in H.P., then $a : a-b :: a+c : a-c$, and $a^2+c^2>2b^2$.

86. The sides of a square are bisected and joined, the side of the square thus formed being a foot less than that of the former; the sides of the second square are bisected and joined, and so on for ever: find the sums of the perimeters and areas of all the squares.

87. If $y^3-axy-b^3=0$, expand y in terms of x.

88. Two bells, 3 miles asunder, toll together for an hour, one 235 times, and the other 243 times, the first and last tolls of each coinciding at the beginning and end of the hour. Find the strokes that are *most nearly* coincident in the course of the hour; and determine a person's distance from the first bell in a line between them, that these may appear to be absolutely coincident, if sound travels 4860 feet in a second.

89. Solve the equation $\frac{1}{a}\sqrt{(a+x)}+\frac{1}{x}\sqrt{(a+x)}=\frac{1}{b}\sqrt{x}$.

90. Find the worth of C's expectation in [80], (i) if it is seen that one of the two coins which A has drawn is a shilling, (ii) if B, having received his coin, finds it to be a shilling, (iii) if *both* these suppositions are made together.

91. Find the n° of div^{rs} of 2160, and the n° of Nos. less than it, and prime to it.

92. Shew that $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80}=\log 2$.

93. Expand by the Bin. Theor. $(1-ax+bx^2)^{-\frac{1}{2}}$ to five terms.

94. Shew that $\sqrt[3]{N}=a\,\frac{2N+a^3}{N+2a^3}$ nearly, a being the integer next greater than $\sqrt[3]{N}$.

95. In every G.P. of an odd number of terms, the sum of the squares of the terms = product of sum of all the terms by excess of the odd terms above the even.

96. If the G. mean between x and y : the H. mean :: $m : n$, then $x : y :: m+\sqrt{(m^2-n^2)} : m-\sqrt{(m^2-n^2)}$.

97. Find the sum of all the numbers of n places which can be made with n digits p_0, p_1, &c. in the scale of r. What is the greatest possible value of that sum for the given radix?

98. The weight of a spherical shell is $\frac{7}{8}$ of what it would have been if wholly solid. Given that the weight of a sphere $\propto$ (diam)^3, compare the inner and outer radii: and, if the inner be increased by one-half, find in what ratio the present weight will be reduced.

99. Solve the equation

$$\sqrt{(x+\sqrt{x})} - \sqrt{(x-\sqrt{x})} = a\sqrt{\frac{x}{x+\sqrt{x}}}.$$

100. The veracities of A, B, C, are as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$: a shilling is tossed, and A and B assert that it has fallen *head*, C that it has fallen *tail:* what is the chance that it fell *head?*

101. If $ax^2+bx+c=0$ and $a'x^2+b'x+c'=0$ have a common root, prove that $(ac'-a'c)^2+(ab'-a'b)(cb'-bc')=0$. What is the condition that they may have both roots common?

102. Shew that the sum of any two consecutive triangular numbers is a square.

103. If a, b, c, d, &c. are in G. P., find the sum of n terms of the series $(a^3+b^3)^{-1}+(b^3+c^3)^{-1}+(c^3+d^3)^{-1}+$&c. in terms of a and b.

104. If $2a=3b$, find the numerical values of

$$\frac{a-b}{a+b},\ \frac{a^2-b^2}{a^2+b^2},\ \frac{a^3+3b^3}{2a^3-\frac{3}{8}b^3},\ \frac{5a^2b^2}{2a^4+3b^4},\ \frac{ab(a+b)}{(2a+3b)(3a-2b)^2}.$$

105. Evaluate $\{(1-x)^{-\frac{3}{2}}-(1+x)^{\frac{3}{2}}\}\div(x^3-x^4)^{\frac{2}{3}}$, when $x=0$.

106. If m shillings in a row reach as far as n sovereigns, and a pile of p shillings be as high as a pile of q sovereigns, compare the values of equal bulks of gold and silver.

107. S_1, S_2, &c. are the sums of m AR. series, each to n terms, the first terms being 1, 2, 3, &c. and the differences 1, 3, 5, &c.: shew that S_1+S_2+ &c. $=\frac{1}{2}mn(mn+1)$.

108. If $a\div b$ be an irreducible fraction, and b any number prime to 3, shew that, when the fraction is converted to a decimal, the period will be divisible by 9, and the sum of the remainders will be a multiple of b.

109. (i) $\dfrac{a+x+\sqrt{(a^2-x^2)}}{a+x-\sqrt{(a^2-x^2)}}=\dfrac{b}{x}$; (ii) $\dfrac{x^2-a^2}{cx}+\dfrac{cx}{x^2-a^2}=b$.

110. The veracities of A, B, C, being $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, C asserts that I have won a prize of £10 in a certain raffle, where there are 10 tickets, of which I hold *two:* what is the value of my expectation, if I find that C has only heard a report of the case from B, who again had only heard from A?

111. Given log 2 = .3010300, find those of 5, .016, $\frac{1}{2}$, 6.25, $1\frac{3}{5}$, $15\frac{5}{8}$.

112. Shew that $x+\dfrac{m}{nx}>1+\dfrac{m}{n}$, unless x lies between 1 and $\dfrac{m}{n}$.

113. Write down the general terms of

$$(a^2-5ax)^{-\frac{3}{5}},\ (a^3+3a^2x)^{\frac{5}{3}},\ \text{and}\ (ax+x^2)^{-4}.$$

114. Separate $\dfrac{x^3+a^3}{x^2(x-a)^2}$ and $\dfrac{a^2x^2}{x^3+a^3}$ into partial fractions.

115. Find the coeff. of x^{13} in the expansion of $(ax-bx^3+cx^5-\&c.)^5$.

116. If N and n be nearly equal,

$$\text{then } \sqrt{\frac{N}{n}} = \frac{N}{N+n} + \frac{1}{4}\,\frac{N+n}{n} \text{ very nearly.}$$

117. If in [116] $\dfrac{N}{N+n}$ and $\dfrac{1}{4}\cdot\dfrac{N+n}{n}$ have their first p decimal places the same, shew that the approximation may be relied on to $2p$ decimals at least; and hence find $\sqrt{30}$ to eight decimal places.

118. If the m^{th} term of an A. P. be n, and the n^{th} term m, how many terms must be taken so as to give the sum $\frac{1}{2}(m+n)(m+n-1)$? and what will be the last of them?

119. Solve the equations

$$\left.\begin{aligned} &1+\tfrac{1}{5}x^2y^{-2}+\tfrac{1}{25}x^4y^{-4}+\&c.\ ad\ inf. = 5x^2y^{-2} \\ &1+\tfrac{1}{2}(x+y)+\frac{1.3}{2.4}(x+y)^2+\frac{1.3.5}{2.4.6}(x+y)^3+\&c.\ ad\ inf. = \sqrt{1.25} \end{aligned}\right\}$$

120. There are 7 balls in a bag, one of them a white one. *A* and *B* stake each 3*s* 6*d*, the whole to be won by whichever shall first draw the white ball, the balls not being replaced when drawn. *A* has the first draw: what are their expectations? and what should *B* have staked, that *A*'s drawing first might give him no advantage?

121. Find the n° of divisors of 140, and the n° of numbers less than 140 and prime to it: express generally the rational values of x and y which satisfy the equation $140x = y^3$, and find how many integral solutions there are of $xy = 10^{2n}$.

122. Given that $e^x = y + \sqrt{(1+y^2)}$, shew that $y = \frac{1}{2}(e^x - e^{-x})$; and prove that $\sqrt{\{c+\sqrt{(2ac-a^2)}\}} + \sqrt{\{c-\sqrt{(2ac-a^2)}\}} = \sqrt{(2a)}$.

123. Find a series of fractions converging to $\frac{351}{965}$ and $\sqrt{28}$.

124. If $c = a - b$, and is very small compared with a and b, then

$$a^2b^2(a^2-a^2x^2+b^2x^2)^{-\frac{3}{2}} = a - 2c + 3cx^2 \text{ nearly.}$$

125. Resolve $\dfrac{x+a}{x(x-a)^2}$, $\dfrac{x^2+2ax+3a^2}{(x^2-a^2)^2}$, $\dfrac{x^2-ax+a^2}{x(x^3-a^3)}$ into partial fractions.

126. If α, β, γ, be the roots of $x^3-2x^2+3x-4=0$, find the values of

$$\alpha^{-1}+\beta^{-1}+\gamma^{-1},\ \alpha^2+\beta^2+\gamma^2,\ \alpha^{-2}+\beta^{-2}+\gamma^{-2}.$$

127. The value of diamonds $\propto$ (weight)^2, and the (value)^2 of rubies $\propto$ (weight)^3. A diamond of a carats is worth m times a ruby of b carats, and both together are worth £c. Find the values of a diamond and ruby, each of x carats.

128. A vessel (A) contains a gallons of wine, another (B) contains b gallons of water: c gallons are taken from each and poured into the other, and this operation is continually repeated. Shew that, if $c = ab \div (a + b)$, the quantity of wine in each vessel will remain always the same after the first operation.

129. (i) $\left(\frac{x+a}{x+b}\right)^{\frac{1}{2}} - \frac{a-b}{2(x+c)} = 1$: (ii) $c^2x^4 - 2c^2x^3 + 2x - 1 = 0$.

130. A bets B 3s to 1s that, with two dice at one cast, he will throw *seven* before B throws *four*, each having a pair of dice and throwing together: what is the value of B's expectation, and what odds ought A to have given?

131. Prove that, if $x = \sqrt{(a^2 + s^2)}$ and $a\,e^{\frac{y}{a}} = x + \sqrt{(x^2 - a^2)}$, then

$$2x = a\left(e^{\frac{y}{a}} + e^{-\frac{y}{a}}\right),\quad 2s = a\left(e^{\frac{y}{a}} - e^{-\frac{y}{a}}\right).$$

132. Represent $\sqrt{(2n\sqrt{-1})}$ in the form of a binomial surd; and shew also that $\sqrt{(4 + 3\sqrt{-20})} + \sqrt{(4 - 3\sqrt{-20})} = 6$.

133. Given $\log 1\frac{2}{5} = .1461280$, $\log .144 = \bar{1}.1583625$, and $\log .0441 = \bar{2}.6444386$, find the logs of the nine digits.

134. Eliminate x and y from each of these sets of equations;

$$\text{(i)}\quad x + y = z,\quad x^2 + y^2 = a^2,\quad x^3 + y^3 = b^3,$$
$$\text{(ii)}\quad x + y = a,\quad xy = z^2,\quad x^7 + y^7 = b^7.$$

135. Find the convergents to $\frac{421}{972}$ and $\frac{1096}{9119}$, and the first four to the value of $\sqrt{11}$.

136. If a be the first of n GEOM. means between a and b, then $a : b :: a^{n+1} : a^{n+1}$.

137. In [128] shew that generally the quantity of wine in B after r operations will be $\frac{ab}{a+b}(1 - p^r)$, where $c = \frac{ab}{a+b}(1-p)$.

138. Find the number of men in a hollow equilateral wedge, the ranks being r deep, and the outer one containing n men.

139. Solve the equations $\frac{x^2}{y^2} + \frac{2x+y}{\sqrt{y}} = 20 - \frac{y^2+x}{y}$, $x + 8 = 4y$.

140. In [130] determine B's expectation, and the odds A should have given, (i) if A, and (ii) if B, has the *first* throw.

141. If $c = a\sqrt{(1-b^2)} + b\sqrt{(1-a^2)}$, then

$$(a+b+c)(a+b-c)(a-b+c)(b+c-a) = 4a^2b^2c^2.$$

142. Sum the series $1 - 2n + 3\frac{n(n+1)}{1.2} - 4\frac{n(n+1)(n+2)}{1.2.3} + \&c.$; and, if $n < 1$, shew that the terms continually decrease to the r^{th}, so long as $rn < 1$, and after that increase.

143. Find the first four convergents to 3.14159, and also to the ratio of 5 h. 48 min. 51 sec. to 24 h.

144. If the equation $x^2 + px + q = 0$ have equal roots, shew that $ax^2 + p(a+b)x + q(a+2b) = 0$ has one of them, and find the other.

145. Divide unity into four parts in A. P., so that the sum of their cubes may be $\frac{1}{10}$.

146. Shew that $(a_1b_1 + a_2b_2 + \&c.)^2 < (a_1^2 + a_2^2 + \&c.)(b_1^2 + b_2^2 + \&c.)$, unless $a_1 : b_1 = a_2 : b_2 = \&c.$

147. Find the coefficient of x^n in $(a + bx + cx^2)\, e^{-x}$.

148. Given A, my income, a the premium for assuring £100, r the rate of Int. per cent per annum: find what sum I must lay out in insuring my life, so that my executors may receive a sum, whose Int. shall equal my reduced income.

149. (i) $nx = \{\sqrt{(1+x)} - 1\}\{\sqrt{(1-x)} + 1\}$.

(ii) $x^2 = ax + by,\ y^2 = bx + ay$.

150. There are two bags, in one of which are one white ball and two black, in the other three white and one black. Find the chance of a person drawing (i) a white ball, (ii) white and black in two trials, balls replaced, (iii) three white in three trials, balls not replaced.

151. Divide an odd number $2n+1$ into two integers, so that their product may be the greatest possible.

152. Find the value of $1 - n^2 + \left\{\frac{n(n-1)}{1.2}\right\}^2 - \left\{\frac{n(n-1)(n-2)}{1.2.3}\right\}^2 + \&c.$ when n is a positive integer.

153. If $(a+b)(p+q) = 2(ab+pq)$, $(c+d)(p+q) = 2(cd+pq)$, shew that $\frac{(p-q)^2}{2} = \frac{(a-c)(a-d)(b-c)(b-d)}{(a+b-c-d)^2}$.

154. In any G. P. the sum of any two terms is > sum of any two between them, equally distant from the extremes.

155. Determine which is the greatest term of $(3+5x)^8$, when $x = \frac{1}{2}$, and the greatest coefficient of $(1+x)^{\frac{11}{\sqrt{2}}}$.

156. Shew that the product of $n+1$ quantities, each of which is the sum of two squares, can be expressed as the sum of two squares in 2^n different ways.

157. There are a number of series in A.P., whose common differences are 1, 2, 3, &c.: shew that, if the sum of n terms of each of these be n^2, their first terms will form a decreasing A.P., whose first term is $\frac{1}{2}(n+1)$ and common diff. $\frac{1}{2}(n-1)$.

158. If n be very great, shew that

$$a^r + (a+b)^r + (a+2b)^r + \&c. \text{ to } n \text{ terms} = \frac{n^{r+1}b^r}{r+1}$$

159. (i) $\dfrac{\sqrt{(1+a^2)} - a\sqrt{(1+x^2)}}{\sqrt{(1+x^2)} - x\sqrt{(1+a^2)}} = a.$ (ii) $\left(\dfrac{x-a}{x-b}\right)^3 = \dfrac{x-2a+b}{x+a-2b}.$

160. If in [150] the first drawn ball be *white*, find the chance that it has been drawn out of the first bag. What is it, if also the second be *black*, the former not having been replaced?

161. The diff. between any No. and that No. inverted is div. by 9.

162. Find the sum of the cubes of the roots of $x^2 - x + 1 = 0$.

163. If $n = a^2 + b$, where b is very small, $\sqrt{n} = a + \dfrac{2ab}{4a^2+b}$ nearly.

164. Evaluate $\dfrac{a}{x-a}\sqrt[3]{\log_e \dfrac{x}{a}}$, and $\dfrac{(x^2-a^2)^{\frac{3}{2}} + (x-a)}{(1+x-a)^3 - 1}$, when $x = a$.

165. Write down the general terms of

$$(a^2+x^2)^{-2},\quad (a^2-x^2)^{-\frac{1}{5}},\quad \text{and}\quad (a^2-x^2)^{\frac{3}{2}}.$$

166. Prove that 12321, 1234321, 123454321, &c., are squares whatever be the scale of notation.

167. To complete a piece of work, A takes m times as long as B and C together, B, n times as long as A and C together, C, p times as long as A and B together:

shew that $\dfrac{1}{m+1} + \dfrac{1}{n+1} + \dfrac{1}{p+1} = 1.$

168. If p_r denote the coefficient of x^r in $(a_0 + a_1x + a_2x^2 + \&c.)^{\frac{1}{2}}$, shew that $p_0p_r + p_1p_{r-1} + p_2p_{r-2} + \&c. + p_rp_0 = a_r$.

169. (i) $\sqrt{\left(x - \dfrac{1}{x}\right)} + \sqrt{\left(1 - \dfrac{1}{x}\right)} = x.$ (ii) $(x^2-1)^{\frac{1}{3}} + (x^4-1)^{\frac{1}{3}} = x^3$

170. The skill of A is double of that of B: find the odds against A's winning four games before B wins two.

171. Given log 2 = .3010300, log 3 = .4771213, find the logs of 3.2, $1\frac{1}{2}$, $\frac{2}{3}$, 15, .0054, $14\frac{2}{5}$, 1.8, 8.1.

172. If $a : b = c : d = e : f$, prove that $(a^2 + c^2) f^2 = (b^2 + d^2) e^2$, and

$$a^{\frac{3}{2}} - \sqrt{(ace)} + e^{\frac{3}{2}} : (\sqrt{a} + \sqrt{c} + \sqrt{e})^3 :: b^{\frac{3}{2}} - \sqrt{(bdf)} + f^{\frac{3}{2}} : (\sqrt{b} + \sqrt{d} + \sqrt{f})^3.$$

173. If n A., G., or H. means be inserted between a and c, obtain the m^{th} mean in each case.

174. Ev. $\dfrac{x-a+\sqrt{(2ax-2a^2)}}{\sqrt{(x^2-a^2)}}$ and $\dfrac{x\sqrt{(3a^3x-2x^4)}-ax\sqrt[5]{(a^4x)}}{a-\sqrt[3]{(ax^2)}}$, when $x=a$.

175. Obtain the general terms of

$$(a^2-x^2)^{-\frac{1}{4}},\ (a^2-ax)^{\frac{1}{3}} \text{ and } (a+5x)^{\frac{2}{5}}.$$

176. If $A \propto B$, $B^4 \propto AC$, and $C \propto \sqrt[3]{A^2D}+\sqrt[3]{AB^2}$, shew that $m\sqrt[4]{AD}-n\sqrt[4]{BC} \propto p\sqrt{A}+q\sqrt{D}$.

177. Shew that $abc>(2a-b)(2b-c)(2c-a)$, unless $a=b=c$.

178. A bequeaths to his 1st child an n^{th} of his property $+£P$, to the 2nd an n^{th} of the remainder$+£2P$, and so on. They all share alike: how many were they, and what did each receive?

179. Solve $x^3+y^3+xy(x+y)=13$, $(x^2+y^2)x^2y^2=468$.

180. A put a handful of money into a purse and gave it to B. B saw that A had at least 2 sovereigns and 2 shillings in his hand, and, in dropping the money into the purse, he had let fall a coin, which turned out also to be a sovereign. He feels that there are 6 coins in the purse, and knows by the size that they are either sovereigns or shillings: what is the value of B's reasonable expectation?

181. Shew that any common number N is divisible by 7, when $p_0+3p_1+9p_2+$ &c. is so divisible, where p_0, p_1, &c. are the digits, reckoning from the end of the number.

182. The present value of an annuity A to continue for n years is a, and for $2n$ years is a': find n and the rate of Interest.

183. Find five n[os] in A.P., whose sum shall be 25 and product 2520.

184. Write down the general terms of $(a^{\frac{1}{3}}-x^{-1})^{-6}$ and $(c+c^{\frac{2}{3}}x^{\frac{1}{3}})^{-1}$.

185. Resolve into partial fractions $\dfrac{x^3+x^2+2}{x(x+1)^2(x-1)^2}$ and $\dfrac{3x^2-2x+1}{x^6-1}$.

186. A square is divided into 16 equal squares by vertical and horizontal lines. In how many different ways can 4 of these be painted white, 4 black, 4 red, and 4 blue, without repeating the same colour in any vertical, horizontal, or diagonal line?

187. Find the sum of

$$pa^m \pm m(p+q)a^{m-1}b+\tfrac{1}{2}m(m-1)(p+2q)a^{m-2}b^2 \pm \text{\&c.}$$

188. Between each of $m+1$ pairs of quantities, (x, y), $(x, 2y)$, $(x, 4y)$, &c. are inserted m GEOM. means, and M_1, M_2, M_3, &c. are the m^{th} means respectively: prove that

$$\frac{M_1}{M_2}+\frac{M_2}{M_3}+\text{\&c.}=\sqrt[m+1]{\frac{m^{m+1}}{2^m}}.$$

189. Solve the equation $\sqrt{\frac{\sqrt[n]{a} - \sqrt[n]{x}}{\sqrt[n]{x^2}}} - \sqrt{\frac{\sqrt[n]{a} - \sqrt[n]{x}}{\sqrt[n]{a^2}}} = \sqrt[m]{\frac{x}{b^2}}$.

190. A and B play at a game, in which their skills are as $3 : 5$; find the chance of A's winning at least three games out of four.

191. If $m>3$, then $\sqrt[3]{m}>\sqrt[4]{(m+1)}$: also $\sqrt{(-\frac{1}{12}+\sqrt{-\frac{1}{3}})}+\sqrt{(-\frac{1}{12}-\sqrt{-\frac{1}{3}})}=1$.

192. If $a_1 : a_2 :: a_2 : a_3 :: a_3 : a_4$ &c., then

$$(a_1^2 + a_2^2 + \&c.)\,(a_2^2 + a_3^2 + \&c.) = (a_1a_2 + a_2a_3 + \&c.)^2.$$

193. Shew that $\{(1+x)^{\frac{1}{2}} + (1-x)^{\frac{2}{3}}\} \div \{(1+x) + \sqrt{(1+x)}\} = 1 - \frac{5}{6}x$, nearly, when x is *small:* and find its approximate value when x is *large*.

194. Given $\log \frac{1}{2} = \bar{1}.69897$, find x from the equation $20^x = 100$.

195. Expand $\dfrac{x - bx^3 + dx^5}{1 - ax^2 + cx^4}$ to five terms by Ind. Coefficients.

196. Find the limits between which lie all real values of x and y, which satisfy the equation $(n^2 + 1)(x^2 + y^2 - a^2) = 4nxy$.

197. If S_n denote the sum of n terms of an A. P., shew that $S_n + S_{n+1} + \&c.$ to n terms $= \frac{1}{2}n(3n-1)a + \frac{1}{6}n(n-1)(7n-2)b$.

198. If the prices of p and q cubic feet of timber be £a and £b, find the price of a tree containing r cubic feet, the values of timber and bark being proportional to the m^{th} and n^{th} powers of the quantity of timber in the tree.

199. Solve the equations $x + y = 5$, $(x^2 + y^2)(x^3 + y^3) = 455$.

200. A, B, C, D, in order cut a pack of cards, replacing them after each cut, on condition that the first who cuts a heart shall win. What are their respective chances of success?

201. Shew that

$$1 \pm 2n + 3\,\frac{n(n-1)}{1.2} \pm 4\,\frac{n(n-1)(n-2)}{1.2.3} + \&c. = (n+2)\,2^{n-1} \text{ or } 0,$$

according as we use the upper or lower signs.

202. Write down the general terms of

$$(1 - x^2)^{-\frac{1}{2}},\quad (a^4 - a^2x^2)^{\frac{7}{3}},\quad (a - x)^{-\frac{1}{3}}.$$

203. Given $\log \frac{1}{16} = \bar{2}.7958800$ and $\log 20.001 = 1.3010517$, find the logs of 2.000037 and .02000073: and determine the numbers whose logs are 3.3010395, .3010426, $\bar{2}.3010479$.

204. A farm is let for n years at a fixed rent and a fine of £P. When p years of the lease remain, what fine must be paid to extend these p years to q, at Compound Interest?

205. A and B are playing with two dice, each having staked 1*s*, the highest throw to win. A has thrown 6; what is B's expectation?

206. If $x = 1 + h$, where h is small, then (approximately)

$$\sqrt{\frac{2x - x^2 + a^2x^2}{a^2 + 1}} = 1 + \frac{a^2h}{a^2+1}, \text{ and } a^px^{-m} + (b^q - a^p)\,x^{-n} = b^qx^{(n-m)a^pb^{-q}-n}.$$

207. Eliminate a and b from the equations

$$\frac{a^3 - x^3}{b^3 - y^3} = \frac{2x + 3y}{3x + 2y},\quad a^3 - b^3 = (x - y)^3,\quad a^{\frac{3}{2}} + b^{\frac{3}{2}} = z^{\frac{3}{2}}.$$

208. S_1, S_2, S_3, &c. are the sums, to n terms, of n Geom. series, whose first terms are each unity, and common ratios, 1, 2, 3, &c.: shew that

$$S_1 + S_2 + 2S_3 + 3S_4 + \text{\&c.} + (n-1)\,S_n = 1^n + 2^n + 3^n + \text{\&c.} + n^n.$$

209. Shew that

$$\frac{1}{1.4} + \frac{1}{2.5} + \frac{1}{3.6} + \text{\&c. to } n \text{ terms} = \frac{n}{3\,(n+1)} + \frac{n}{6\,(n+2)} + \frac{n}{9\,(n+3)}.$$

210. There are three balls in a bag, one white, another black, and the third either white or black: if two be drawn, find the chance of their being (i) two black ones, (ii) one black and one white.

211. Shew that $\dfrac{1}{1 + x^{m-n} + x^{m-p}} + \dfrac{1}{1 + x^{n-m} + x^{n-p}} + \dfrac{1}{1 + x^{p-m} + x^{p-n}} = 1.$

212. If the difference of a and b be small, compared with either, then $\sqrt[m]{a} - \sqrt[m]{b} : \sqrt[n]{a} - \sqrt[n]{b} :: n\sqrt[n]{b^{n-1}} : m\sqrt[m]{a^{m-1}}$, nearly.

213. From $\log 1\frac{1}{5} = .0791812$ and $\log 2\frac{2}{5} = .3802112$, find the value of $\sqrt[5]{(3.6)^3} \times \sqrt[4]{\frac{1}{25}} \div \sqrt[3]{8\frac{49}{72}}$, given the *mant.* for 45323 and 45324 to be 6563186 and 6563282.

214. Resolve into partial fractions $\dfrac{a + bx + cx^2}{x^2\,(x^2 + a^2)}$.

215. Compare the chances of throwing a single ace in one trial with two dice and in two trials with three.

216. Find what value of b will make $b^2 - 4ac$ a complete square.

217. At a contested election, the n° of candidates was one more than the n° of persons to be elected, and each elector, by voting for one, two, &c. or as many as were to be elected, could dispose of his votes n ways. Find the n° of candidates.

218. In bringing an irreducible fraction to a circulating decimal, shew that, when any two rem[rs] give the div[r] for their sum, the two consecutive rem[rs] will give the same sum, and the sum of the two figures in the period, which correspond to those rem[rs], will be 9.

219. Sum to n terms

(i) $\frac{1}{3.6} - \frac{1}{6.8} + \frac{1}{9.10} -$ &c. (ii) $\frac{1}{1.2.4} + \frac{1}{2.3.5} + \frac{1}{3.4.6} +$ &c.

220. A, tossing a coin, is to pay B 1*s* if it fall *heads* the first time, 2*s* if the second, 3*s* if the third, and so on for n throws, the game to cease as soon as it falls heads. Find B's expectation.

221. Eliminate x, y, z, from the equations

$$\frac{a^m}{x^{m+n}} = \frac{b^m}{y^{m+n}} = \frac{c^m}{z^{m+n}}, \quad \frac{a^m}{x^m} + \frac{b^m}{y^m} + \frac{c^m}{z^m} = 1 = \frac{x^n + y^n + z^n}{k^n}.$$

222. Find the coefficients of x^3 and x^4 in $(1 + x^{\frac{1}{2}} + x^{\frac{3}{2}} + x^{\frac{5}{2}} - x^{\frac{7}{2}})^5$.

223. Find n integers in A.P. whose sum shall be n^2, whatever n may be.

224. If $\sqrt[n]{x}+m\sqrt[n]{y} : \sqrt[n]{x}-m\sqrt[n]{y} :: \sqrt[2n]{x}+m\sqrt[2n]{(x-y)} : \sqrt[2n]{x}-m\sqrt[2n]{(x-y)}$, shew that $x : y :: 1 \pm \sqrt{5} : 2$.

225. If on the average 9 ships out of 10 return safe to port, find the chance that out of 5 ships expected, at least 3 will arrive.

226. Find three square numbers whose sum shall be a given square (a^2). Ex. 81.

227. In Ex. 217, find the n° of candidates, if it was one more than twice the number of persons to be elected.

228. If A_m denote the middle term of $(1 + x)^{2m}$, then

$$A_0 + A_1 + A_2 + \text{\&c.} = (1 - 4x)^{-\frac{1}{2}}.$$

229. Sum $1 + \frac{1}{2} n + \frac{1}{3} \frac{n(n-1)}{1.2} +$ &c., and (to n terms)

$$1.2^2 + 2.3^2 + 3.4^2 + \text{\&c.}$$

230. Shew that in taking a handful of shot from a bag, it is more probable that an odd number will be drawn than an even one.

231. If $ax^3 = by^3 = cz^3$, and $x^{-1} + y^{-1} + z^{-1} = k^{-1}$, then

$$(ax^2 + by^2 + cz^2)^{\frac{1}{3}} = (a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}})\, k^{\frac{2}{3}}.$$

232. Shew that, if $pq = r$, the equation $x^3 + px^2 + qx + r = 0$ will have two roots equal and of opposite signs.

233. Find three numbers with prime den^rs, whose sum shall be $1\frac{86}{315}$.

234. Resolve into partial fractions

$$\frac{7x + 8}{(x^2 + 1)(x + 1)^2} \text{ and } \frac{7x + 8}{(x^2 + x + 1)(x + 1)^2}.$$

235. If the chance for an event A : that for B :: $m : n$, then in $r(m + n)$ trials it is most likely that A will happen rm times and B rn times.

236. If $aX + bY + cZ = 0$ and $a'X + b'Y + c'Z = 0$, where $X = ax + a'x' + a''$, $Y = bx + b'x' + b''$, $Z = cx + c'x' + c''$, then $X^2 + Y^2 + Z^2 = \dfrac{\{a''(bc' - b'c) + b''(a'c - ac') + c''(ab' - a'b)\}^2}{(bc' - b'c)^2 + (a'c - ac')^2 + (ab' - a'b)^2}$.

237. If from a vessel, containing a gallons of wine, b gallons be drawn off, and the vessel filled up with water, and this be repeated n times, find the quantity of wine remaining.

238. Shew that $\log_e x = \dfrac{m}{n}\{(1 - \sqrt[m]{x^{-n}}) + \frac{1}{2}(1 - \sqrt[m]{x^{-n}})^2 + \frac{1}{3}(1 - \sqrt[m]{x^{-n}})^3 + \&c.\}$.

239. Shew that $1 + 2^2 + 3 + 4^2 + 5 + 6^2 + \&c.$ to n terms
$= \frac{1}{12}(n+1)(2n^2 + n + 3)$ or $\frac{1}{12}n(n+4)(2n+1)$,
according as n is odd or even.

240. A bets B 10s to 1s that he will throw heads at least once in three trials: what is B's expectation, and what would have been a fair bet?

241. Prove that
$$(Aa + Bb + Cc + \&c.)^2 = (A + B + C + \&c.)(Aa^2 + Bb^2 + Cc^2 + \&c.) - AB(a-b)^2 - AC(a-c)^2 - BC(b-c)^2 - \&c.$$

242. If $x = 1$ nearly, then
$mx^m - nx^n = (m-n)x^{m+n}$, and $x^{-e} = \sqrt{\{1 - e^x + \sqrt[x]{e}\}}$ nearly.

243. Find the coefficients of x^3, x^5, and x^7, in $(1 - 2x - 3x^2)^{-2}$.

244. Find an A. P., beginning with unity, in which the sum of the first half of any even number of terms shall have to the second half a constant ratio. Shew that there is but one such series.

245. Compare the chances of throwing two aces only in two trials with three dice and in three trials with four.

246. If $2x_r = y^{(2r-1)z} - y^{(1-2r)z}$, then $x_1(x_1 + x_2 + \&c. + x_{2n-1}) = x_n^2$.

247. Shew that $x^4 + px^3 + qx^2 + rx - s^2$ can be resolved into rational quadratic factors, if $s^2 = \dfrac{r^2}{p^2 - 4q}$; and hence solve the equation
$$x^4 - 6x^3 + 5x^2 + 8x - 4 = 0.$$

248. Given $x = z - \frac{1}{3}z^3 + \frac{1}{5}z^5 - \&c.$ and $y = z\sqrt{(1 - y^2)}$, find y in terms of x by reversion of series.

249. Solve in positive integers $2xy - 3x^2 + y = 1$.

250. A is allowed to draw two coins from a bag, containing five sovereigns and four shillings. What is his expectation? and what if B draws a coin *before* or *after* A's first draw?

251. If $\dfrac{ad-bc}{a-b-c+d} = \dfrac{ac-bd}{a-b-d+c}$, then either of these fractions $= \frac{1}{4}(a+b+c+d)$.

252. If a, b, c, are in G. P., shew that $a^2-b^2+c^2 > (a-b+c)^2$; and if $a^2+b^2+c^2 = 1 = a'^2+b'^2+c'^2$, then $aa'+bb'+cc' < 1$.

253. Find the fifth term of $(a^3+b^3\sqrt{-1})^{-\frac{2}{3}}$, the fifth and the greatest terms of $(1-\frac{3}{7})^{-\frac{8}{3}}$, and the fifth term of $(2-5x-7x^2)^5$.

254. If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} =$ &c., then $\dfrac{a^{3n}-c^{3n}}{b^{3n}-d^{3n}} = \dfrac{a^n c^n e^n - (a^n - c^n + e^n)^3}{b^n d^n f^n - (b^n - d^n + f^n)^3}$.

255. In Ex. 250, what will be *A*'s chance in each case, if *B*'s coin, being looked at, is found to be a sovereign, *A not* looking at his, till he has drawn them both?

256. Shew that $\left(\dfrac{x-x'}{a}\right)^2 + \left(\dfrac{y-y'}{b}\right)^2 + \left(\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-1\right)\left(\dfrac{x'^2}{a^2}+\dfrac{y'^2}{b^2}-1\right) = 0$ resolves itself into the two equations $\dfrac{xx'}{a^2}+\dfrac{yy'}{b^2} = 1$ and $\dfrac{x}{x'} = \dfrac{y}{y'}$.

257. Find three numbers such that the sum of all three and of every two may be squares.

258. If $\dfrac{a+(a+y)x+(a+2y)x^2+\&\text{c. } ad\ inf.}{a+(a-y)x+(a-2y)x^2+\&\text{c. } ad\ inf.} = b$, and if x receive values in H.P., shew that the corresponding values of y will be in A.P.

259. Shew that the series $x - \dfrac{x^3}{1.2.3} + \dfrac{x^5}{1.2.3.4.5} -$ &c. converges at the n^{th} term, if $n > \frac{1}{2}x$; and find the greatest term of $(x-x^{-1})^n$, when $x=2$, $n=5\frac{1}{2}$.

260. There are three black and four white balls in a bag, and three persons draw one each in succession, not replacing them. Find (i) each person's chance of drawing black, (ii) the chance of first and third drawing black and second white, (iii) of all three drawing black.

261. Find the value of $\dfrac{x+2a}{x-2a} + \dfrac{x+2b}{x-2b}$, if $x = \dfrac{12ab}{a+b+\sqrt{\{(a+b)^2+12ab\}}}$.

262. Resolve into factors of the first degree

$$2x^2 - 21xy - 11y^2 + 34y - x - 3.$$

263. Shew that $n^2-n+1 : n^2+n+1$ lies between 3 and $\frac{1}{3}$.

264. If $\{f(x)\}^2 = 1 + \{f'(x)\}^2$, and $a^x = f(x) + f'(x)$, shew that $f(x\pm y) = f(x)f(y) \pm f'(x)f'(y)$, and $f'(x\pm y) = f'(x)f(y) \pm f(x)f'(y)$.

265. From a bag containing 2 guineas, 3 sovereigns, and 7 shillings, *A* is allowed to draw three coins. What is his expectation? and what if one of the guineas should be known to be base?

266. Shew that $\frac{1}{55}\{3.10^x - 25(-1)^x\}$ is a positive integer, when x is so. Find the fifth term of the series of which it is the x^{th}, and sum the series to n terms.

267. Find a series of numbers which shall be at the same time of the forms $n^2 - 1$ and $10m^2$.

268. How small must x be taken so that the third term of $1 + 3x + 5x^2 + 7x^3 + \&c.$ may contain the sum of all that follow at least 500 times.

269. A person devised his estate among n persons in the following manner. A was to receive £P + 1-n^{th} of the remainder, B £$2P$ + 1-n^{th} of the remainder, C £$3P$ + 1-n^{th} of the remainder, and so on: find the value of the estate.

270. In one of two purses there are three sovereigns and a shilling, in the other three shillings and a sovereign. A coin is taken from one (it is not known which) and dropped into the other; and then, on drawing a coin from each bag, they are found to be two shillings. What is the chance that this will occur again, if two more are drawn, one from each purse?

271. Shew that $(n+1)(n+2)(n+3)\ldots$ to n factors $= 1.3.5\ldots(2n+1)2^n$; and if $a^1.a^3.a^5\ldots = p$, find the number of the factors a^1, a^3, &c.

272. If $\dfrac{a+bx}{a-bx} = \dfrac{b+cx}{b-cx} = \dfrac{c+dx}{c-dx} = \&c.$, and if also $a^x = b^y = c^z = \&c.$, then will a, b, c, &c. be in G.P., and x, y, z, &c. in H.P.

273. Find the coefficient of x^n in $(1 + x + 2x^2 + 3x^3 + \&c.)^2$.

274. What value of y will make $2(y^2+y)x^2 + (11y-2)x + 4$ and $2(y^3+y^2)x^3 + (11y^2-2y)x^2 + (y^2+5y)x + 5y - 1$ commensurable?

275. From a bag, containing $2n+1$ balls, $2n$ are taken out, and are found to be alternately white and red. Shew that it is equally likely that the remaining one is either red or white; and find the chance that it is neither the one nor the other.

276. Shew that any triangle will have its area expressed in rational terms, if its sides be proportional to

$$gh(k^2+l^2),\quad kl(g^2+h^2),\quad (hk+gl)(hl-gk).$$

277. If p be prime, and neither a nor $a-1$ a mult. of p, and m a positive integer, then each of the sums $a^{m+1} + a^{m+2} + \&c. + a^{m+p-1}$ and $a^{m+1} + a^{m+2} + \&c. + a^{mp}$ is a multiple of p.

278. Sum to n terms and *ad inf.* $\dfrac{2}{3.5} - \dfrac{3}{5.7} + \dfrac{4}{7.9} - \dfrac{5}{9.11} + \&c.$

279. If one of the roots of $x^2 - px + q = 0$ be large, compared with the other, shew that p, $p - \frac{q}{p}$, $\frac{p(p^2 - 2q)}{p - q}$, are closer and closer approximations to it.

280. A is to receive a certain number of farthings, expressed, he knows, by the digits 1, 2, 3, 4, 5, but in what order he is not aware. Find his expectation.

281. What value of x makes $(4x + 1)(2z + 1)^2 = 5(3x + 1)(z + 3)^2$ for large values of z? What values of x and y make the fraction $\frac{2z^2 + (x - a)z + 2b(x - 2c)}{3z^2 + (y - b)z + 3a(y - 3c)}$ independent of z?

282. If $x + c$ be the G. C. M. of $x^2 + ax + b$ and $x^2 + a'x + b'$, their L. C. M. will be $x^3 + (a + a' - c)x^2 + (aa' - c^2)x + (a - c)(a' - c)c$.

283. Extract the square roots of $x^m + \frac{1}{4}\sqrt[np]{(b^{2p}x^{4n})} - \sqrt[n]{b}\,.\,\sqrt[2p]{x^{mp+4}}$, and of $4\{(a^2 - b^2)cd + ab(c^2 - d^2)\}^2 + \{(a^2 - b^2)(c^2 - d^2) - 4abcd\}^2$.

284. Find the maximum or minimum value of $\frac{a^2}{x^2} + \frac{b^2}{a^2 - x^2}$.

285. There are three white and five black balls in a bag, and three persons draw a ball in succession (the balls drawn not being replaced) until a white one is drawn: shew that their respective chances are as 27 : 18 : 11.

286. Find two numbers such that, if their sum be added to the square of each, the results shall be squares.

287. Sum to n terms and *ad inf.*, when $x < 1$, the series

$$1 + 2x^2 + 2x^3 + 6x^4 + 10x^5 + 22x^6 + 42x^7 + 86x^8 + \&c.;$$

and find also the coefficient of x^n.

288. Shew that the series for $(1 - x)^{-x}$ diverges or converges from the first term, according as $x \gtrless 1$. From what term does the series for $(1 + x)^x$ converge?

289. If n be a prime number and N a number prime to n, then, when the square numbers N^2, $4N^2$, $9N^2$, &c., $\{\frac{1}{2}(n - 1)\}^2 N^2$ are divided by n, they will each leave a different remainder.

290. Find the chance that, if a halfpenny be tossed, it will neither fall heads nor tails three times successively in five trials, but will fall heads the sixth and tails the three following times.

291. Shew that $\frac{cd - ab \pm \sqrt{\{(a - c)(a - d)(b - c)(b - d)\}}}{a + b - c - d}$ is always a possible quantity, if a, b, c, d, are the roots of a biquadratic with rational coefficients.

292. Given $2(x^2 + y^2 - x - y) + 1 = 0$, find x and y: and given $\sqrt{x} + \sqrt{y} : \sqrt{(2x)} - \sqrt{(3y)} :: a : b$, find the value of

$$\sqrt{x} - \sqrt{y} : \sqrt{(2x)} + \sqrt{(3y)}.$$

293. If the difference between the $(n-1)^{\text{th}}$ and n^{th} terms of an H. P. be $\dfrac{1}{an^2 + bn + c}$, find the relation between a, b, and c.

294. Solve in positive integers $2xy - 3x^2 + y = 1$.

295. An even n° (n) of pieces of money being thrown, shew that it is $2^{n-1}+1$ to $2^{n-1}-1$ against there being an even n° of heads.

296. A person spends in the first year m times the interest of his property, in the second $2m$ times that of the remainder, in the third $3m$ times that of what is now remaining, and so on; and at the end of $2p$ years has nothing left. Shew that in the p^{th} year he spends as much as he had left at the end of that year.

297. If N be any number, which differs from the square numbers next greater and less than it by a and b respectively, prove that $N - ab$ is a square number.

298. Find two integers such that if unity be added to each of them, as also to their sum and difference, the four results shall be squares.

299. If $x = 1 + n^{-1}$, shew that the sum of n terms of the series $1 + 2x + 3x^2 +$ &c. is n^2.

300. Shew that it is *probable* that, in 25 throws with two dice, sixes will be thrown at least once.

301. Shew that 12345654321 is divisible by 12321 in any scale, where the radix exceeds 6.

302. Find the greatest term in the expansion of $(\sqrt{2} + \sqrt{3})^{12\sqrt{6}}$.

303. Find two integers, such that the sum or difference of their squares shall each exceed unity by a square number.

304 Sum to n terms and *ad infinitum*, when $x < 1$,

$$1 + 8x + 27x^2 + 64x^3 + 125x^4 + 216x^5 + 343x^6 + \text{\&c.}$$

305. Four cards being drawn from a common pack, find the chance that they are marked one, two, three, four, of different suits.

306. If $\phi(n, m) = \dfrac{1}{m} - n\dfrac{1}{m+p} + \dfrac{n(n-1)}{1.2}\dfrac{1}{m+2p} -$ &c., shew that $\phi(n, m) = \dfrac{np}{m}\phi(n-1, m+p)$; and thence deduce the sum of the series, when n is a positive integer.

307. Shew that if any number, N, can be resolved into the sum of n squares, then $2(n-1)N$ can be resolved into the sum of $n(n-1)$ squares.

308. If $z^2 + z + 1 = 0$, shew that the sum of those terms of the expansion of $(1+x)^n$, in which the index of x is a mult. of 3,

$$= \tfrac{1}{3}\{(1+xz)^n + (1+x)^n + (1+xz^{-1})^n\}.$$

309. Sum to n terms and *ad infinitum* $\dfrac{2}{1.3.3} + \dfrac{3}{3.5.9} + \dfrac{4}{5.7.27} + \&c.$

310. If p witnesses concur in their statement of a fact, which they have heard from another individual, shew that the chance of its being true is $\dfrac{v^p - mv^{p-1} + m^2v^{p-2} - \&c.}{v^p + m^p}$, where v = veracity of each of the $p+1$ persons, and $m = 1 - v$.

311. Shew that $n > \log_e(1+n)$, and that $4xy - 3(x^2-y^2)^{\frac{2}{3}} \gtrless 1$, according as $(x+y)^{\frac{2}{3}} - (x-y)^{\frac{2}{3}} \gtrless 1$.

312. If p_r be the coeff. of x^r in $(1+x)^n$, (n a positive int.,) shew that

$$\frac{p_1}{p_0} + 2\frac{p_2}{p_1} + 3\frac{p_3}{p_2} + \&c. + n\frac{p_n}{p_{n-1}} = \frac{n(n+1)}{2},$$

and $(p_0+p_1)(p_1+p_2)(p_2+p_3)\ldots(p_{n-1}+p_n) = \dfrac{(n+1)^n}{\lfloor n} p_1p_2p_3\ldots p_n$.

313. A circular field is divided into an odd number of equal areas by the circumferences of concentric circles; and the radius of the outer circle is p times the breadth of the middle area. Find the number of circles.

314. Eliminate x, y, z from the equations

$$x^2(y+z) = a^3,\ y^2(x+z) = b^3,\ z^2(x+y) = c^3,\ xyz = abc.$$

315. A's skill is to B's as $1:3$, to C's as $3:2$, and to D's as $4:3$; find the probability that A in three trials, one with each person, will succeed (i) twice *exactly*, (ii) twice *at least*.

316. If $f(p, q)$ denote the coeff. of x^q in $(1 + x + x^2 + \&c. + x^n)^p$, shew that $f(1, q) - \frac{1}{2}f(2, q-1) + \frac{1}{3}f(3, q-2) - \&c. = \dfrac{1}{q+1}$, unless $q+1$ should be a multiple of $n+2$.

317. If a_1, a_2, &c. are in G. P., and x_1, x_2, &c. in A. P., shew that

$$a_1^{x_1} \cdot a_2^{x_2} \ldots a_n^{x_n} = \{a_1^{(2n-1)x_1+(n-2)x_n} \times a_n^{(2n-1)x_n+(n-2)x_1}\}^{\frac{n}{6(n-1)}}.$$

318. Sum to n terms and *ad infinitum* each of the series,

$$\frac{5}{1.2.3.2} + \frac{6}{2.3.4.4} + \frac{7}{3.4.5.8} + \&c. \text{ and } \frac{5}{1.2.3} + \frac{6}{2.3.4} + \frac{7}{3.4.5} + \&c.$$

319. Investigate the general forms of x and y, which rationalize $(ax^2 + by^2)^{\frac{1}{n}}$, when $n = 3$ or any *odd* number.

320. Fifteen persons sit down at a round table. Shew that it is 6 to 1 against two particular persons sitting next each other; and that, generally, for n persons, the odds against the same event are $n - 3 : 2$.

321. If $\frac{p_{n-1}}{q_{n-1}}, \frac{p_n}{q_n}, \frac{p_{n+1}}{q_{n+1}}$, be consecutive convergents to x, then each of the fractions $\frac{p_n + p_{n-1}}{q_n + q_{n-1}}, \frac{2p_n + p_{n-1}}{2q_n + q_{n-1}}, \frac{3p_n + p_{n-1}}{3q_n + q_{n-1}}$, &c. $\frac{p_{n+1}}{q_{n+1}}$, approach more and more nearly to the value of x than any fraction with smaller denominator.

322. If a, b, c, &c. k, be n unequal numbers, and $m < n - 1$, then

$$\frac{a^m}{(a-b)(a-c)\dots(a-k)} + \frac{b^m}{(b-a)(b-c)\dots(b-k)} + \&c. + \frac{k^m}{(k-a)(k-b)\dots} = 0.$$

323. Shew that $\log_e x = (x - 1)\frac{2}{x^{\frac{1}{2}} + 1} \cdot \frac{2}{x^{\frac{1}{4}} + 1} \cdot \frac{2}{x^{\frac{1}{8}} + 1} \dots$ *ad inf.*

324. Find two integers such that their difference, the diff. of their squares, and the diff. of their cubes, may all be squares.

325. The corners of a common die are filed away, till the faces, which before were squares, become regular octagons. Compare the chances of its falling, when thrown, upon a triangular or octagonal face, neglecting all mechanical considerations.

326. If $s = a^2 + b^2$, $p = 2ab$, $P = (a+b)^p$, shew that $P.P^{\frac{1}{2}}.P^{\frac{1}{4}}.P^{\frac{1}{8}}\dots$ *ad inf.*

$$= s^p + p^2 s^{p-1} + p^3 . \frac{p-1}{2} s^{p-2} + p^4 . \frac{p-1}{2} . \frac{p-2}{3} s^{p-3} + \&c.$$

327. Sum *ad infinitum*

$$\frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \&c., \quad \frac{1^3}{3} + \frac{2^3}{3^2} + \frac{3^3}{3^3} + \&c., \quad \frac{1^3}{5} - \frac{2^3}{5^2} + \frac{3^3}{5^3} - \&c.$$

328. How many triangular pyramids can be formed, whose edges are six given lines, any two of which are > than the third?

329. Eliminate m, n, p, q from the equations

$$\frac{x-p}{m} + \frac{y+q}{n} = \frac{pm}{a^2} + \frac{qn}{b^2} = \frac{m^2}{a^2} - \frac{n^2}{b^2} = \frac{p^2}{a^2} + \frac{q^2}{b^2} - 1 = 0.$$

330. Find the *probable* sum of the series $1 + x + x^2 +$ &c., when the number of its terms is known to be not greater than q nor less than p.

331. If $x = 1$ nearly, shew that x, $1 - x + x^2$, $\frac{1}{2}(1 + x - x^2 + x^3)$, are nearer and nearer approximations to the value of x^x.

332. If y be the Harmonic mean between x and z, and x and z respectively the Arithmetic and Geometric means between a and b, shew that $y = 2(a+b) \div \left\{\left(\frac{a}{b}\right)^{\frac{1}{4}} + \left(\frac{b}{a}\right)^{\frac{1}{4}}\right\}$.

333. Of $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \&c.$, $1 + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \&c.$, the former series is *divergent* and the latter *convergent.*

334. If $x \div \{(1-x)^2 - cx\}$ be expanded in a series of ascending powers of x, shew that the coefficient of x^r is
$$r\left\{1 + \frac{r^2-1}{2.3}c + \frac{(r^2-1)(r^2-4)}{2.3.4.5}c^2 + \frac{(r^2-1)(r^2-4)(r^2-9)}{2.3.4.5.6.7}c^3 + \&c.\right\}.$$

335. There are a n° of tickets, marked with some one of the numbers from 1 to n^2+1. Every one of these numbers, (such as r,) is marked upon the same number (r) of tickets; and every ticket marked with a square number (m^2) confers a prize of m shillings. A draws one ticket: find his expectation.

336. Shew that the product of any number of factors of the form $x^2 + axy + by^2$, $x'^2 + ax'y' + by'^2$, &c., may be put into the same form, $X^2 + aXY + bY^2$.

337. Apply the preceding to find a series of positive integral values for x and y, which shall make $x^2 + 3xy + 5y^2$ a square number.

338. If $s = a + b$, $p = ab$, and $q = a \div b$, prove that
$$p^2 = s^4\left(q^2 - 4q^3 + \frac{4.5}{1.2}q^4 - \frac{4.5.6}{1.2.3}q^5 + \&c.\right),$$
and $a^n + b^n = s^n - nps^{n-2} + \frac{1}{2}n(n-3)p^2s^{n-4} - \&c.$

339. Sum to n terms and *ad infinitum* $\frac{1}{3.8} + \frac{1}{6.12} + \frac{1}{9.16} + \&c.$

340. A bag contains 50 balls, 5 of which are drawn at a time, and replaced after each drawing. Two persons draw alternately, the prize being won by him who first draws two particular balls. Find the odds in favour of the first drawer?

341. $$\frac{1}{\log_e x} = \frac{1}{2}\cdot\frac{x+1}{x-1} - \frac{1}{2}\left\{\frac{1}{2}\cdot\frac{\sqrt{x}-1}{\sqrt{x}+1} + \frac{1}{4}\cdot\frac{\sqrt[4]{x}-1}{\sqrt[4]{x}+1} + \frac{1}{8}\cdot\frac{\sqrt[8]{x}-1}{\sqrt[8]{x}+1} + \&c.\right\}.$$

342. Shew that every term of the preceding series (within the brackets) is greater than $\frac{1}{4}$th of the term next before it, and less than a third proportional to the two next before it.

343. Shew that the sum of the products of n quantities c, c^2, c^3, &c. taken m and m together, is $c^{\frac{m(m+1)}{2}} \times \frac{(c^n-1)(c^{n-1}-1)\ldots(c^{n-m+1}-1)}{(c-1)(c^2-1)\ldots(c^m-1)}$.

344. If a, b, c, &c. are any n quantities, shew that $a^n + b^n + c^n +$ &c. $> n(abc\ldots)$; and thence prove that $1.2.3\ldots n < \{\frac{1}{2}(n+1)\}^n$.

345. A plays at a game, in which he reckons his chance of success to be e. If it be an even chance that he has made an error e', or not, in his calculation, shew that this does not affect his chance of success in a single trial, but increases his chance of continual success in any number of repeated trials.

346. Eliminate x from $x^4 + rx + s = 0$, $x^2 + vx = y$; and shew that the equation in y will be a biquadratic, wanting its second and fourth terms, if $rv^3 + 4sv^2 - r^2 = 0$. Ex. Solve $x^4 + 3x = 2$.

347. If n be a prime n°, the expression $A_0x^m + A_1x^{m-1} +$ &c. $+ A_m$, cannot admit of more than m different values of x, less than n, which will render it divisible by n.

348. Apply the preceding to shew that, if n be a prime number, each of the quantities $N^{\frac{n-1}{2^p}} \pm 1$ has $\frac{n-1}{2^p}$ integral values of N less than n, which make it divisible by n.

349. Shew that the number of different ways, in which the letters of the expression $p^{m+r}q^n$ can be written at length, so that at least r p's may always follow each other, is $(n+1)\dfrac{\lfloor m+n}{\lfloor m \lfloor n}$.

350. Apply the preceding to find the chance that, on tossing a shilling 12 times, it will fall heads at least 6 times successively.

351. Prove that, when the expression in Ex. 347 admits of exactly m such different values of x, that render it a multiple of n, then the quantities $A_0S_1 + A_1$, $A_0S_2 - A_2$, &c. $A_0S_m - (-1)^m A_m$, are all multiples of n, where S_r denotes the sum of the products of those values of x, taken r together.

352. Apply the preceding to shew that, when n is a prime number, $1.2.3\ldots(n-1) + 1$ and $1.2.3\ldots(n-1)\left(1 + \frac{1}{2} + \frac{1}{3} + \text{\&c.} + \frac{1}{n-1}\right)$ are each multiples of n.

353. The chance of an event, whose prob^y is p, occurring at least r times successively in $n + r$ trials, is $\{1 + n(1-p)\}p^r$.

354. Two persons are known to have passed over a piece of road in opposite directions within the time $a + b + c$, in the times a and b respectively. Find the chance that they will meet.

EXAMPLES: PART III.

EQUATION PAPERS OF ST. JOHN'S COLLEGE, CAMBRIDGE.

1.

1. $\frac{3x+7}{14} - \frac{2x-7}{21} + 2\frac{3}{4} = \frac{x-4}{4}$

2. $\left.\begin{aligned} 3x + 6y + 1 &= \frac{6x^2 + 130 - 24y^2}{2x - 4y + 3} \\ 3x - \frac{151 - 16x}{4y - 1} &= \frac{9xy - 110}{3y - 4} \end{aligned}\right\}$

3. $\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$

4. $5y + \frac{1}{5}\sqrt{(x^2 - 15y - 14)} = \frac{1}{3}x^2 - 36$
$\frac{x^2}{8y} + \frac{2x}{3} = \sqrt{\left(\frac{x^3}{3y} + \frac{x^2}{4}\right)} - \frac{y}{2}$

5. A brewer, from ingredients worth £20, brews 500 gallons of ale, (on which there is a duty of 6*d* *per* gallon,) and sells it at 2*s* a gallon. From the same ingredients he afterwards brews the same n° of gals, part strong beer, (on which he pays ale-duty,) and the rest small beer, (on which he pays $\frac{1}{4}$ ale-duty.) By mixing, and selling the mixture as ale, his gains are increased in the ratio of 10 : 7. Find the n° of gals of strong beer.

6. The n° of deaths in a besieged garrison was 6 daily, and, allowing for this, their provisions would just have lasted 8 days. But on the 6th evening, 100 men were killed in a sally, and afterwards the mortality increased to 10 daily. At the end of the 6th day there was stock enough remaining to support 6 men for 61 days: how many will be alive when the whole is exhausted?

7. A man buys a guinea at the market-price of standard gold; but an Act passing, which makes it illegal to sell the coin of the realm, he clips off $\frac{1}{25}$th part. He may now legally sell it as a light guinea; and finds that by reason of the rise of pure gold in the ratio of 239 : 249, he just gains the clippings by his purchase. Find the ratio of pure gold and alloy in the guinea, and also the relative value of equal quantities of pure gold and alloy, having given that the sum of the squares of the two ratios exceeds eleven times their sum by $233\frac{9}{121}$.

2.

1. $\frac{2x+1}{29} - \frac{402-3x}{12} = 9 - \frac{471-6x}{2}$

2. $\left.\begin{array}{l} \frac{1}{3}(3x-5y) - \frac{1}{12}(2x-8y-9) = \frac{1}{2}y + \frac{1}{3} + \frac{1}{4} \\ \frac{1}{7}x + \frac{1}{4}y + 1\frac{1}{3} : 4x - \frac{1}{8}y - 24 :: 3\frac{1}{3} : 3\frac{1}{2} \end{array}\right\}$

3. $\frac{x+3}{2} + \frac{16-2x}{2x-5} = 5\frac{1}{5}$

4. $\left.\begin{array}{l} \frac{x+y+\sqrt{(x^2-y^2)}}{x+y-\sqrt{(x^2-y^2)}} = \frac{9(x+y)}{8y} \\ (x^2+y)^2 + x - y = 2x(x^2+y) + 506 \end{array}\right\}$

5. At the review of an army, the troops were drawn up in a solid mass, 40 deep, when there were just $\frac{1}{4}$ as many men in front as there were spectators. Had the depth, however, been increased by five, and the spectators drawn up with the army, the n° of men in front would have been 100 fewer than before. Find the force of the army.

6. A n° of persons bought a field for £345, the youngest paying a certain sum, the next £5 more, and so on in A. P. The younger half took a portion of the field proportional to the sum they had subscribed; and this they agreed to divide equally, by equalizing their contributions to £22 each. How many persons were there in all?

7. *A* and *B* travelled on the same road and at the same rate from H to L. At the 50th milestone from L, *A* overtook a drove of geese, which were proceeding at the rate of 3 miles in 2 hrs; and 2 hrs afterwards he met a waggon, moving at the rate of 9 miles in 4 hrs. *B* overtook the geese at the 45th milestone, and met the waggon just 40 minutes before he came to the 31st milestone. How far was *B* from L when *A* reached it?

3.

1. $\frac{4x-34}{17} - \frac{258-5x}{3} = \frac{69-x}{2}$

2. $\left.\begin{array}{l} \frac{1}{3}(4x-2y+3) - \frac{1}{7}(18-x+5y) = \frac{1}{4}x - \frac{1}{5}y - \frac{1}{7} - 7\frac{1}{10} \\ 2x - y + 15 : y - 2x + 15 :: \frac{1}{8}x - \frac{1}{4}y + \frac{3}{4} : \frac{1}{4}y - \frac{1}{3}x + \frac{1}{12} \end{array}\right\}$

3. $\frac{2x}{x-4} + \frac{2x-5}{x-3} = 8\frac{1}{3}$

4. $\left.\begin{aligned} \frac{y}{x} - \frac{9\sqrt{x}}{y} - \frac{81}{xy} &= (2y+9)\frac{\sqrt{x}}{y} \\ \frac{\sqrt{y}}{x} + 3\sqrt{\frac{x}{y}} &= \frac{9}{x\sqrt{y}} + \sqrt{x} \end{aligned}\right\}$

5. A packet from Dover reaches Calais in 2 hrs; but, on the return voyage, proceeds at first 6 miles an hour slower than it went. The wind, however, changing halfway, it sails 2 miles an hour faster, and reaches Dover sooner than it would have done, if the wind had not changed, in the proportion of 6 : 7. Find the distance between Dover and Calais.

6. From the middle of a town two streets branched off, and crossed a straight river by bridges *A* and *B*. From their junction, a sewer, equally inclined to both streets, led to a point in the river distant 6 chains from *A*, and from *B* 11 chains less than the length of the sewer, the expense of making which was as many £'s per chain as there were chains in the street leading to *A*. The sewer proving insufficient, a drain was made from a point in this street, distant 4 chains from *A*, which entered the river at the same point with the sewer, and was equally inclined to the river and sewer. Now a drain down each street, at £9 per chain, would have cost only £54 more than the sewer. Find the lengths of the streets and sewer.

7. A labourer, with his wife and children, saved each a certain n° of pence in A.P. The whole monthly saving was less by 3*s* 3*d* than the cost of $\frac{1}{6}$ as many bushels of wheat, as the seventh child saved pence, the price of wheat being such that the savings of the eldest and fifth child, increased by 10*s*, would buy two bushels. But wheat rising 2*s* a bushel, and work being scarce, they find that their savings will not buy as much wheat as before by two bushels; and that, at this rate, their annual savings would be less by 5 guineas than before. At this time, the two youngest died; and it was calculated that, if the others saved each 1*s* less than the eldest child had done before the rise of wheat, their monthly savings would not be affected by this event. How many were they in family?

4.

1. $\frac{1}{2}(5x - 1) - \frac{1}{10}(7x - 2) = 6\frac{3}{5} - \frac{1}{2}x$

2. $\left.\begin{array}{l} \sqrt{y} - \sqrt{(a - x)} = \sqrt{(y - x)} \\ \sqrt{(y - x)} + \sqrt{(a - x)} : \sqrt{(a - x)} :: 5 : 2 \end{array}\right\}$

3. $\frac{5(3x - 1)}{1 + 5\sqrt{x}} + \frac{2}{\sqrt{x}} = 3\sqrt{x}$

4. $\left.\begin{array}{l} \frac{y}{2x} + \frac{2}{3} \cdot \frac{y - \sqrt{(x - 1)}}{y^2 - 2\sqrt{(x^2 - 1)}} = \frac{\sqrt{(x + 1)}}{x} \\ \frac{1}{2}y^4 = y^2 x - 1 \end{array}\right\}$

5. A farmer laid up a stock of corn, expecting to sell it in 6 months at $3s$ a bushel more than he gave. But by that time, instead of having risen, corn had fallen $1s$ a bushel, and he found that he should, by selling now, lose the cost price of 5 bushels. He therefore kept it to the year's end; and then, being obliged to sell at $2s$ a bushel under prime cost, lost just $10s$ less than he had expected to gain. Find the cost price per bushel, allowing 5 per cent simple interest.

6. A ship, with a crew of 175, set sail with water enough to last the voyage. But, at the end of 30 days, the scurvy began to carry off 3 men daily, and a storm protracted the voyage 3 weeks. They just reached port, however, without the water falling short. Required the time of passage.

7. The hold of a vessel partly full of water, (which is uniformly increased by a leak,) has two pumps worked by A and B, of whom A takes 3 strokes to 2 of B's, but 4 of B's throw out as much water as 5 of A's. B works for the time in which A alone would have emptied the hold; A then pumps out the rest, and the hold is cleared in $13\frac{1}{3}$ hours. Had they worked together, the hold would have been emptied in $3\frac{3}{4}$ hours, and A would have pumped 100 gals more than he did. Find the influx per hour at the leak.

5.

1. $\frac{1}{7}(4x - 21) + 7\frac{5}{6} + \frac{1}{3}(7x - 28) = x + 3\frac{3}{4} - \frac{1}{8}(9 - 7x) + \frac{1}{12}$

2. $\left.\begin{array}{l} \frac{1}{3}(3x - 2y) + 1 + \frac{1}{8}(11y - 10) = \frac{1}{7}(4x - 3y + 5) + \frac{1}{5}(45 - x) \\ 45 - \frac{1}{3}(4x - 2) = \frac{1}{18}(55x + 71y + 1) \end{array}\right\}$

3. $x^{\frac{7}{3}} + \frac{41\sqrt[3]{x}}{x} = \frac{97}{\sqrt{x^2}} + x^{\frac{5}{3}}$

4. $\left.\begin{array}{l} \dfrac{x^2y^2}{2} + 4 - 40y^2 = 140 - y^2\sqrt{\left(x^2 - \dfrac{272}{y^2}\right)} \\ x^2 - \dfrac{2}{y}\left(\dfrac{3}{y} + 15x\right) = \dfrac{30}{y^2} + \dfrac{5x}{y} \end{array}\right\}$

5. On Jan. 1, 1799, a beggar received from *A* as many groats as *A* was years old, and a similar donation in each of the seven following years, during the last of which *A* died, his alms having in all amounted to £7 18*s* 8*d*. In what year was he born?

6. *A* entered into a canal speculation with 14 others, the profits of which were in all £595 more than five times the price of each share. Seven of his partners afterwards joined him in a scheme for navigating the canal with steam-boats, each venturing a sum less than his former gains by £173. But this concern failed, and *A* lost £419 by it; for they not only never recovered their outlay, but lost all their former gains and £368 besides. Find the cost price of shares in each concern.

7. *A*, *B*, *C* were architects. *A* and *B* built four warehouses with flat roofs, each a large and each a small one, the width of the large ones being the same, and likewise of the small ones. *A* built his as long and as high as they were broad; but *B* made the length and height of his small one the same as the breadth of his large one, and the difference between the contents of *A*'s and *B*'s buildings was 73728 feet. *C* also built upon a square plot of ground, whose area was the difference of those on which *A* built; and would have been in fact just 2688 square feet, if he had added to it eight times as many square feet as there were feet in its width. Find the width of the buildings.

6.

1. $\dfrac{2x}{3} - \dfrac{1 - \frac{1}{2}x}{4x} = \dfrac{x - 1}{2} + \dfrac{x}{6} + \dfrac{7}{12}$

2. $\left.\begin{array}{l} x + 2y + 3z = 17 \\ 2x + 3y + z = 12 \\ 3x + y + 2z = 13 \end{array}\right\}$

3. $a^2b^2x^{\frac{1}{n}} - 4(ab)^{\frac{3}{2}}x^{\frac{m+n}{2mn}} = (a - b)^2x^{\frac{1}{m}}$

4. $\left.\begin{array}{l} x^4 + y^4 = 1 + 2xy + 3x^2y^2 \\ x^3 + y^3 = 2y^2x + 2y^2 + x + 1 \end{array}\right\}$

5. *A* can walk forwards four times as fast as he can backwards, and undertakes to walk a certain distance ($\frac{1}{4}$ of it backwards) in a certain time. But the ground being bad, he finds that his rate per hour backwards is $\frac{1}{5}$ of a mile less than he had reckoned, and that to win his wager he must walk forwards two miles an hour faster. What is his rate per hour backwards?

6. *A* lent *B* a sum at a certain rate of Int., taking as security such an amount of Spanish 5 *per cents* as would produce the same interest as the debt. At the year's end, *B* proved insolvent; and Spanish bonds having fallen 40 per cent, *A* lost £400. Had they not fallen, he would have been repaid with a surplus of £250; and if he had been at liberty to have sold them out at the half-year's end, when they were at 50, (which was before the int. upon them was due,) he would have lost only £300. Required the amount of the debt.

7. The builder of a treadmill agreed to take for payment the produce of n weeks' labour of the convicts then in prison, and to supply them with food during that period. His estimate was a shillings for the weekly expense of each man: but, finding this sufficient for the first week only of a man's labour, he increased it the second week by ra, the third, in addition, by r^2a, and so on. Now at the beginning of each week after the first, c fresh convicts were sent to the mill, when he found that, had his contract included these, he would have gained as much as he had calculated on at first. Supposing each man's weekly labour to be worth pa shillings, find the number of convicts at first.

7

1. $$\frac{11x-13}{25}+\frac{19x+3}{7}-\frac{5x-25\frac{1}{3}}{4}=28\tfrac{1}{7}-\frac{17x+4}{21}$$

2. $$\frac{3}{x}-\frac{4}{5y}+\frac{1}{z}=7\tfrac{3}{5},\quad \frac{1}{3x}+\frac{1}{2y}+\frac{2}{z}=10\tfrac{1}{6},\quad \frac{4}{5x}-\frac{1}{2y}+\frac{4}{z}=16\tfrac{1}{10}$$

3. $$\frac{9\frac{3}{5}-\frac{3}{2}\sqrt{x}-x}{5\sqrt{x}-8}+\tfrac{31}{50}=\tfrac{4}{5}\cdot\frac{\frac{74}{5}\sqrt{x}-x^{\frac{3}{2}}}{4x-7}$$

4. $$\left.\begin{aligned}&\left(\sqrt{\frac{x}{y}}-\sqrt{\frac{y}{x}}\right)^2+\sqrt{y}\left(\sqrt{x}-\frac{\sqrt{y}}{2}\right)=\frac{x}{\sqrt{y}}\left(\sqrt{x}-\frac{y}{\sqrt{2}}\right)\\&9\sqrt{\frac{x}{y}}+3\sqrt{\frac{y}{x}}=\frac{21\sqrt{(2x)}-1}{2}\sqrt{\frac{y}{x}}+\frac{1}{2\sqrt{(xy)}}\end{aligned}\right\}$$

5. *A* sends his agent money to buy pimento, calculating the price at £8 for 5 bags. But the price having risen, the money sent would not buy as much by 18 bags as *A* intended: and it was found that $5\frac{1}{4}$ bags more than $\frac{1}{3}$ of the original quantity would now cost just £10 7*s* more than before. How many bags were purchased?

6. *A* and *B* set out from C and D, *A* starting 3 hrs before *B*. They meet at 20 miles from D, and *A* reaches D one hour before *B* reaches C. The next day *B* starts early, meeting *A*, who had then gone $\frac{1}{7}$ of his journey back; and, though delayed 3 hours, *B* reaches D in time to have gone 28 miles further before *A* reaches C. Required their rates per hour of journeying.

7. *A* called a meeting of his creditors, whose claims increased in A.P.; when it was found that his effects would have paid as many shillings in the £, as there were £'s in the common difference of the claims, and would in fact have exactly sufficed to pay the claims of the *third* and the *highest* creditors. But the latter failed to make good his claim, and the others consequently received 2*s* 8*d* in the £ more than they would have done; and the third and highest dividends were in consequence increased together by £9 12*s*. Find the assets.

8.

1. $$\frac{2x+8\frac{1}{2}}{9} - \frac{13x-2}{17x-32} + \frac{x}{3} = \frac{7x}{12} - \frac{x+16}{36}$$

2. $$\left.\begin{aligned} \frac{9}{8}\,\frac{\sqrt[3]{(x+y)}}{y} + \frac{9}{8}\,\frac{\sqrt[3]{(x+y)}}{x} &= 1\tfrac{1}{7} \\ \frac{7}{4}\,\frac{\sqrt[3]{(x-y)}}{y} - \frac{7}{4}\,\frac{\sqrt[3]{(x-y)}}{x} &= \frac{1}{9} \end{aligned}\right\}$$

3. $$\sqrt[2pq]{x^{p+q}} - \frac{1}{2}\,\frac{a^2-b^2}{a^2+b^2}\{\sqrt[p]{x} + \sqrt[q]{x}\} = 0$$

4. $$\left.\begin{aligned} 3x - x\sqrt{(\tfrac{5}{4}x^2 - 2y + 8)} &= 2 - y \\ \frac{\sqrt{(x+y)}}{2x} - \frac{3}{4}x &= \frac{2x-3}{\sqrt{(x+y)}} - \frac{3y}{2x} \end{aligned}\right\}$$

5. A farmer's rent was £50, and his expenditure (of which $\frac{1}{6}$ was in payment of assessed taxes) was such that he could only pay his landlord £30. The next year his rent was lowered 20 per cent, the taxes also were reduced one-half, and farm produce

increased in value one-third; and now, after paying his rent and former debts, he had £5 over. Find his expenditure.

6. *A* starts from Newmarket to London at the same time that *B* and *C* leave Hockeril and London for Newmarket. *A* meets *B* 4 hours before *C* overtakes *B*; but, on his return, *A* meets *C* one hour before he meets *B*, on their return also, all three having rested the same time at their destinations. Now *A* rode 10 miles an hour, and met *B* at the same place going and returning. Find the distance from London to Newmarket, Hockeril lying midway between them.

7. Two vessels, *P* and *Q*, contain fluids in the ratio of 4 : 21, which consist of different mixtures of wine and spirits. *A* pumps out of *P* into *Q*, and then *B* pumps into *Q* $\frac{3}{4}$ of what remains, and now the mixture *Q* is found to have only $\frac{12}{13}$ of its original strength. Now if, when *A* stopped, *B* had pumped as much as before from *Q* into *P* instead of from *P* into *Q*, the strength of *P* would have been a mean proportional between the original strengths of *P* and *Q*; and *B* would have pumped the same quantity of wine as before of spirits. Compare the quantities of fluid pumped by *A* and *B*, the strength of spirits being three times that of wine.

9.

1. $$\frac{25 - \frac{1}{3}x}{x + 1} + \frac{16x + 4\frac{1}{5}}{3x + 2} = 5 + \frac{23}{x + 1}$$

2. $$\left.\begin{aligned} \frac{4x - 8y + 5}{2} &= \frac{10x^2 - 12y^2 - 14xy + 2x}{5x + 3y + 3} + 2 \\ \sqrt{(6 + x)} : \sqrt{(6 - y)} &:: 3 : 2 \end{aligned}\right\}$$

3. $$(a^{4b} + 1)\,(x^{\frac{1}{2}} - 1)^2 = 2\,(x + 1)$$

4. $$\left.\begin{aligned} 2x + \sqrt{(x^2 - y^2)} &= \frac{14}{y}\left\{\sqrt{\frac{x + y}{2}} + \sqrt{\frac{x - y}{2}}\right\} \\ (x + y)^{\frac{3}{2}} + (x - y)^{\frac{3}{2}} &= 18\sqrt{2} \end{aligned}\right\}$$

5. There was a run on two bankers, *A* and *B*. After 3 days, *B* stopped payment, by reason of which the daily demand on *A* was tripled, and he failed also after 2 days more. But if *A* and *B* had joined their capitals, they might both have stood the run as it was at first for 7 days, when *B* would have owed £4000 to *A*. What was the daily drain at first on *A*'s bank?

6. The gas-contractors undertake to light a shop with 5 large and 3 small burners; but, having by them only one large burner, supply the deficiency with 5 small ones. The shopkeeper, not finding this light sufficient, procures two more small burners, and agrees for all the lights to burn double the usual time on Saturday nights; and for the additional gas he paid 31*s* *per annum*. What did he pay altogether?

7. A man, who is not aware that his watch gains uniformly, engages to ride from Cambridge to London in 9 hours, and sets his watch by St. Mary's at starting. Upon looking at it after having gone halfway, he supposes it necessary to increase his pace in the ratio of 4 : 3, and consequently reaches London 15′ within the time. But if the watch had lost at the same rate, and he had looked at it at the end of the 14th mile, and then regulated his pace accordingly, he would have been in London too late by 7′. Find the distance from Cambridge to London.

10.

1. $$\frac{x - 1\frac{25}{26}}{2} - \frac{2 - 6x}{13} = x - \frac{5x - \frac{1}{4}(10 - 3x)}{39}$$

2. $$\left.\begin{aligned} \sqrt{y} - \sqrt{(y - x)} &= \sqrt{(20 - x)} \\ \sqrt{(y - x)} : \sqrt{(20 - x)} &:: 3 : 2 \end{aligned}\right\}$$

3. $$8\sqrt{(3x)} + \frac{243 + 324\sqrt{(3x)}}{16x - 3} = 16x + 3$$

4. $$\left.\begin{aligned} x + \sqrt{(3y^2 - 11 + 2x)} &= 7 + 2y - y^2 \\ \sqrt{(3y - x + 7)} &= \frac{x + y}{x - y} \end{aligned}\right\}$$

5. From each of two bags, containing different numbers of balls, *A* draws a handful; and now the number in the larger bag is the cube of that in the less, and just the square of one handful. He then draws out of the larger until the number left is the square of that in the less, and now empties the larger into the less, and finds its original number increased by two-thirds. Find the number of balls in each bag at first.

6. *A* and *B* are towns beside a river, which runs at the rate of 4 miles an hour. A waterman rows from *A* to *B* and back again, and takes 39′ more to do it than if there had been no stream. The next day he does the same with another waterman, with

whose help he can row half as fast again: and they are now only 8′ longer than if there had been no stream. At what rate would the waterman row by himself without any stream?

7. Two master bricklayers undertake to lay the foundation of a new court, each taking a part and beginning together. If they had worked together till the whole was finished, it would have taken only $\frac{4}{5}$ of the time it actually took to finish it; and *B* would have done enough to occupy *A* three months, and *A* enough to occupy *B* twelve months, which is 36 yds more than *A* actually did. How many yards were there in all?

11.

1. $\frac{6x - 7\frac{1}{3}}{13 - 2x} + 2x + \frac{1 + 16x}{24} = 4\frac{5}{12} - \frac{12\frac{5}{8} \quad 8x}{3}$

2. $\left.\begin{array}{l} x\,(bc - xy) = y\,(xy - ac) \\ xy\,(ay + bx - xy) = abc\,(x + y - c) \end{array}\right\}$

3. $16\,(x^2 + 2)^{\frac{3}{2}} + \frac{3}{\sqrt{(x^2 + 2)}} = 32x^2 + 48$

4. $\left.\begin{array}{l} 30\sqrt{\dfrac{x^{\frac{2}{3}} + y^{\frac{2}{3}}}{x^{\frac{4}{3}}y^{\frac{4}{3}}}} + 40\sqrt{\dfrac{x^{\frac{2}{3}}y^{\frac{2}{3}}}{x^{\frac{2}{3}} + y^{\frac{2}{3}}}} = 241 \\ \left\{1 + \left(\frac{y}{x}\right)^{\frac{2}{3}}\right\} \cdot \{3x^{\frac{4}{3}}y^{\frac{2}{3}} + \frac{91}{216}\sqrt{(x^2 + x^{\frac{4}{3}}y^{\frac{2}{3}})}\} = \left(\frac{5}{6}\right)^3 - x^2 - y^2 \end{array}\right\}$

5. Two clocks strike the hour, and are heard to strike 19 times. They differ 2″ in time, and one strikes every 3″, the other every 4′. When they strike *together*, it cannot be distinguished whether one or both are striking; and this is the case with the last stroke of the faster clock. What hour did they strike?

6. A revenue cutter observes a smuggler *q* leagues directly to windward; and gives chase, sailing at $5\frac{1}{3}$ points from the wind, and making tacks of $4p$ miles. The smuggler lies off on the other tack at $2\frac{2}{3}$ points, making tacks of $\frac{1}{3}p\sqrt{3}$ miles, its rate of sailing being to the cutter's as $1 : 4\sqrt{3}$. They sail half the above distances before the first tack. In what tack will the smuggler, while lying in the eye of the wind, first be within range of the cutter's guns, which carry *r* miles?

7. In the first and least considerable irruption of the Thames into the Tunnel, the water rose in the vertical shaft 8 times as fast as in the horizontal levels in the second. If the levels at the second influx had been 110 feet longer, the velocities of the water ascending in them in the first and second irruptions, and when thus increased would have formed an A.P., the common difference being $\frac{1}{9}$ of the difference of the velocities with which the water rose in the shaft in the two irruptions; and, if the levels had been of the same length on both occasions, the first time of filling would have been half as long again.

The tunnel consisted of two equal levels, terminated by a vertical shaft of twice the breadth of either. The sections of the shaft and levels are supposed to be squares; and the height of the shaft above the upper surface of the levels to be double of its breadth. Given the first time of filling to be 10′ less than the second, find the duration of the latter.

12.

1. $$\frac{7x+6}{28} - \frac{2x+4\frac{2}{7}}{23x-6} + \frac{x}{4} = \frac{11x}{21} - \frac{x-3}{42}$$

2. $$\left.\begin{aligned} \sqrt{(x-y)} + \tfrac{1}{2}\sqrt{(x+y)} &= \frac{x-1}{\sqrt{(x-y)}} \\ x^2 + y^2 : xy :: 34 &: 15 \end{aligned}\right\}$$

3. $$\frac{x^{(m-n)^2} + x^{-4mn}}{x^{(m-n)^2} - x^{-4mn}} = a^{\frac{r}{s}}$$

4. $$\left.\begin{aligned} 5 - 2\sqrt{(y+2)} &= \tfrac{9}{64}x^2 - (\sqrt{x} - 3\sqrt{y})^2 \\ \frac{7}{y} - 10\sqrt{\frac{x}{y}} &= x - 16 \end{aligned}\right\}$$

5. A question was lost on which 600 persons had voted. The same persons having voted again on the same question, it was carried by twice as many as it was before lost by, and the new majority was to the former :: 8 : 7. How many changed their minds?

6. Three towns *A*, *B*, *C*, lie at the angles of a right-angled triangle, *B* at the right angle, and the distance *AB* being the least of the three. A pedestrian finds that the time of his going from *A* to *B*, and then from *B* to *C*, exceeds the time from *A* to *C*

direct by $2\frac{2}{3}$ hours. A coach, which left *A* four hours after him, and travels thrice as fast, overtakes him 8 miles from *B* in the way to *C*; and after passing through *C* to *A*, and waiting there $6\frac{2}{3}$ hours, it makes the same circuit, and reaches *A* again at the same time with the pedestrian, who had rested four hours at *C*. Find the rate at which he walks.

7. *A*, *B*, *C*, *D*, are rough diamonds. The value of *C* in £'s is less by 52 than the weight of *A* in carats, and the value of *C* and *D* in £'s is equal to the weight of *B* in carats. Each loses half its weight by cutting: but the dust from *A* and *B* is worth £85; and the value of *A* : that of *C*, *D*, and the dust from *A* :: half that of *B* : that of the dust from *B*. A diamond, weighing one carat when rough, is worth £3 when cut, and £2 when uncut: the value $\propto$ the square of the weight, and the dust is worth £1 per carat. Find the value of *D* when cut.

13.

1. $$\frac{6-5x}{15} - \frac{7-2x^2}{14(x-1)} = \frac{1+3x}{21} - \frac{2x-2\frac{1}{5}}{6} + \frac{1}{105}$$

2. $$\left.\begin{aligned} 3x + \tfrac{2}{3}\sqrt{(xy^2+9x^2y)} &= (x-\tfrac{1}{3})\,y \\ 6x+y : y :: x+5 &: 3 \end{aligned}\right\}$$

3. $$\sqrt{x} - \frac{8}{x} = \frac{7}{\sqrt{x}-2}$$

4. $$\left.\begin{aligned} x^2y - 4 &= 4x^{\frac{1}{2}}y - \tfrac{1}{4}y^3 \\ x^{\frac{3}{2}} - 3 &= x^{\frac{1}{2}}y^{\frac{1}{2}}\,(x^{\frac{1}{2}} - y^{\frac{1}{2}}) \end{aligned}\right\}$$

5. *A* and *B* start for a race which lasts 6′. At the end of 4′, the distance between them is $\frac{1}{440}$ of the length of the course. After 1′ more, *B*, who is last, quickens his horse 20 yds a minute, and comes in 2 yds before *A*, who has gone uniformly throughout. Find the length of the course.

6. The revenue of a state was increased for war in the ratio of $2\frac{1}{4}$: 1; and, after deducting the expense of collecting and the interest of the national debt, the net income was augmented in the ratio of $3\frac{1}{2}\frac{2}{3}$: 1. If, however, the revenue had been reduced in the ratio of $1\frac{7}{9}$: 1, the net income would have been diminished in the ratio of $7\frac{2}{3}$: 1, and would in fact have just amounted to 4 millions. Find the revenue before the increase, supposing the

expense of collecting to vary as the square root of the sum collected.

7. Three boats, *A*, *B*, and *C*, start in a race, *B* being 20 yards behind *A*, and *C* the same behind *B*. *A* and *C* set off at an uniform rate, *C* making a yard less *per* stroke than *A*. But *B* took 7 strokes to 6 of *A* or *C*, and increased its speed besides by 3 in. every stroke; so that, when *A* had taken 42 strokes, *B*, which had lost 16 yards by steering, was only a yard behind *A*. At this point *B*'s speed decreased twice as fast as it had increased before; while *C*, quickening its strokes at the same instant in the ratio of 6 : 7, and gaining each stroke as much speed as *B* lost, at the end of 28 strokes overtook *B*, which had lost 11 yds more by steering. Compare the velocities with which they started.

14.

1. $\left. \begin{aligned} x - \frac{2y - x}{23 - x} &= 20 - \frac{59 - 2x}{2} \\ y + \frac{y - 3}{x - 18} &= 30 - \frac{73 - 3y}{3} \end{aligned} \right\}$

2. $\{a^2 - a\sqrt{(b^2 + bx)} + \frac{1}{2}bx - \frac{1}{4}b^2 + \frac{1}{2}b\sqrt{(x^2 - bx + b^2)}\} \times$
$\{a^2 + a\sqrt{(b^2 + bx)} + \frac{1}{2}bx - \frac{1}{4}b^2 - \frac{1}{2}b\sqrt{(x^2 - bx + b^2)}\} = a^4 - \frac{3}{16}b^4$

3. $2x\sqrt{(1 - x^4)} = a(1 + x^4)$

4. $\left. \begin{aligned} (2 + 4xy - 3x^2)^2 &= 2 - 4x^2y^2 + 3x^4 \\ (x^2 - 1)^2 &= (2y^2 + x^2 + 1)(2y^2 - x^2 - 1) \end{aligned} \right\}$

5. The owner of a balloon calculated that, if he filled the enclosure, which he had hired for the day at £5, with spectators at 2*s* each, and two persons ascended with him, he should make a profit of 140 per cent on his outlay. But, the gas and the weather proving bad, he pays but half the price of inflating, and ascends alone with the enclosure a fourth part full, losing on the whole $\frac{1}{3}$ of his outlay. On the next day he ascends again with a full balloon, the enclosure $\frac{3}{4}$ filled, and one companion, and by the whole speculation gained £10. What was the cost of inflation?

6. *A* and *B* row between two places, *B* in a certain time by his watch: but *A*, when he has gone by his watch the same time, relaxes his speed, and moves only $\frac{2}{3}$ as fast as before. Now, if this take place in going down the stream, the first part of the distance will take *A* six times as long as the last, but, if in going up, only

the same time; and this would also be the case in going upwards, if *A*, instead of relaxing, were even to increase his speed in the ratio of 7 : 5, provided that he exchange watches with *B* at starting. Supposing their watches to gain uniformly, compare the rates of rowing of *A* and *B*.

7. A regiment, in which there are between 10 and 100 officers and twice as many serjeants, in clearing the streets during a revolution, loses 2 officers; and, after storming a barricade in which three more fall, is obliged to retreat, taking in a volunteer as officer, but, in so doing, loses other three. While clearing the streets, the liability of an officer to fall is half that of a serjeant or private; but at the barricade as 4 : 3, and in the retreat as 3 : 4. Also, on their leaving the barracks, the number, whose two left-hand digits express the number of serjeants and its other digits that of officers, exceeds by 20 ten times the number of privates; but, on their return, (having parted with the volunteer,) it exceeds it only by 12, the number of officers being still above 10. Find the state of the regiment at first.

15.

1. $\dfrac{x^2+1}{4x^2-1} = \dfrac{x}{1+2x} - \dfrac{1}{4}$

2. $\left.\begin{aligned} \sqrt{x} - \sqrt{y} &= \sqrt{x}\,(\sqrt{x}+\sqrt{y}) \\ (x+y)^2 &= 2\,(x-y)^2 \end{aligned}\right\}$

3. $\sqrt{x}\,\sqrt{(x^3+a^3)} = x^2 - a^2 + 3ax$

4. $\left.\begin{aligned} \sqrt[3]{\frac{27y^{\frac{3}{2}}-1}{x^3+3y^2-2xy^{\frac{3}{2}}}} &= 3\sqrt{\frac{x}{y}} \\ 3x^2+42xy+16y^2 &= 4\sqrt{(xy)}\,(5x+11y) \end{aligned}\right\}$

5. A farm was rated at 3*s* an acre, and the tenant, on receiving back 10 per cent of his rent, found that the sum returned was £6 more than the whole rate. The next year the rates were doubled, and he received back 15 per cent of his rent; but now the sum returned only just paid for the rate. What was his rent?

6. *A* starts to walk from Cambridge to London at the rate of $3\frac{1}{2}$ miles per hour. In $2\frac{1}{2}$ hrs the Times passes him, and the Fly at 10′ to 10; he rests $2\frac{1}{2}$ hrs on the road, and again meets the Times on its return, and half-a-mile farther the Fly, at 20′ after 5. The Times left Cambridge at 6, and the Fly at $\frac{1}{2}$-past 7, and both started from London at 3: find the distance from Cambridge to London.

7. At an election of one member to Parliament, one-third of the electors gave plumpers for *C*, and those given for *A* and *B* were $\frac{4}{21}$ of the whole number of votes given. Of those electors who gave single votes to *C*, twice as many voted for *B* as for *A*: and *B* stood at the head of the poll with a majority of 110 over *C*. A scrutiny being demanded, it appeared that those who had split their votes between *A* and *B* had no legal right to vote, and *C* is now returned with a majority of 200 over *A*. Find the final state of the poll, it being observed that *A* has now as many single votes as plumpers.

16.

1. $\frac{3x-2}{4} + \frac{x}{2} - 11\frac{5}{6} = \frac{x - \frac{1}{3}(4x-9)}{6} - 5$

2. $\left.\begin{aligned} 3y + 11 &= \frac{4x^2 - y(x+3y)}{x-y+4} + 31 - 4x \\ (x+y)(y-2) + 3 &= 2xy - (y-1)(x+1) \end{aligned}\right\}$

3. $x - 2\sqrt{(x+2)} = 1 + \sqrt[4]{(x^3 - 3x + 2)}$

4. $\left.\begin{aligned} (1-x^2)^2(1+y^2) - (1+x^2)^2(1-y^2) &= 4x^2\sqrt{(1+y^4)} \\ 4xy &= \sqrt{2}(1-x^2)(1-y^2) \end{aligned}\right\}$

5. *A* and *B* engaged to reap equal quantities of wheat, and *A* began half-an-hour before *B*. They stopped at noon to rest an hour, and observed that just half the work was done. *B*'s part was finished at 7 o'clock, and *A*'s at $\frac{1}{4}$ to 10. At what hour did *A* begin?

6. Two boys start from the right angle of a triangular field, and run along the sides with velocities in the ratio of 13 : 11. They meet first in the middle of the opposite side, and again 30 yards from the starting point. Find the length round the field.

7. A stable-keeper bought two horses for £50, and sold them, one for double, and the other for half, of what he gave for it. The former had produced for its hire only half of what it cost him, and cost in keep as much per cent on its price as the hire of the other produced on its price, the latter being kept for $\frac{5}{8}$ as many guineas as it sold for pounds. The keep of the two cost £33, and he made by them, upon the whole transaction, nine times his profit on the sale. What did each cost?

17.

1. $\frac{1}{3} - \frac{7x-1}{6\frac{1}{2} - 3x} = \frac{8}{3} \cdot \frac{x - \frac{1}{2}}{x-2}$

2. $\left.\begin{array}{l} x^m a^n + y^n b^m = 2(ax)^{\frac{m}{2}} \cdot (by)^{\frac{n}{2}} \\ xy = ab \end{array}\right\}$

3. $x^2 - 2x + 4 = 2\sqrt{(x^3 - 1)}$

4. $a^2(x^4 + y^4) = y^6 - 2a^2xy\sqrt{(x^4 - y^4)}$
$a^2(x^4 - y^4) = x^2y^2(2a^2 - x^2)$

5. *A* and *B* run a race to a post and back again. *A* returning meets *B* 90 yards from the post, and reaches the starting-place 3′ before him. If he had then returned, he would have met *B* at a distance from the starting-place equal to $\frac{1}{6}$ of the whole distance. Find the length of the course, and duration of the race.

6. The upper spokes *R* and *r* of the hind and fore wheels of a carriage are vertical at starting. When *r* has revolved once. it is at right angles to the spoke next before *R*; and when *R* has made $\frac{3}{8}$ of a revolution, *r* ascending again makes the same angle with an horizontal line as the spoke next before it. Given diam. of forewheel : diff. of heights of axles : : number of spokes in forewheel : 2, find the number of spokes in each.

7. *A* agrees with his steward to allow a per centage on the rents he collected, on condition of his returning half the same on the rents not paid. The first year the steward's income amounts to 6 per cent on the whole rental; but in the next, in order to obtain the same income, he makes a return of rents received £270 under their value. In the third year, though the rents are reduced $7\frac{1}{2}$ per cent, the amount not paid is the same as in the second year; the steward's income is only $\frac{3}{4}$ of his first year's, and, to make it up, he doubles his last year's fraud. Required the rental of the estate.

18.

1. $\frac{1}{2}\left(\frac{2}{3}x + 4\right) - \frac{7\frac{1}{2} - x}{3} = \frac{x}{2}\left(\frac{6}{x} - 1\right)$

2. $\left.\begin{array}{l} \frac{1}{2}x + \frac{1}{4}y + \frac{1}{5}z = 1\frac{1}{3} \\ \frac{1}{3}y + \frac{1}{4}x + \frac{1}{10}z = \frac{17}{18} \\ \frac{1}{3}z + \frac{1}{5}x + \frac{1}{6}y = \frac{169}{180} \end{array}\right\}$

3. $\frac{1 + x^5}{(1 + x)^5} = a$

4. $\left.\begin{array}{l} y + 3\sqrt[3]{y}\,\{\sqrt[3]{(a + bx)} - \sqrt[3]{y}\}\sqrt[3]{(a + bx)} = 2a \\ \frac{y - 3\sqrt[3]{y}\sqrt[3]{(a^2 - b^2x^2)}}{\sqrt[3]{y} - \sqrt[3]{(a + bx)}} = 2a\sqrt[3]{(a - bx)} \end{array}\right\}$

5. Bacchus caught Silenus asleep by the side of a full cask, and seized the opportunity of drinking, which he did for two-thirds of the time, in which Silenus would have emptied the cask. After that Silenus wakes, and drinks what the other had left. Had they both drunk together, the cask would have been emptied two hours sooner, and Bacchus would have drunk only half of what he left for Silenus. Find the time in which each by himself would have emptied the cask.

6. A, B, C, whose powers are in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, engage to reap a field of wheat at so much per acre, to be proportioned among them according to their work. After two days a quarrel arises and C withdraws, receiving for his labour 5*s*, the farmer having made a deduction from the stipulated price per acre, on account of the delay thus occasioned. A and B then engage to finish the work on the same terms as at first, which takes them another day. A receives upon the whole 1*s* 6*d* less than he would have done if C had not withdrawn; while the farmer saves in money half as much as he agreed to pay per acre. Find the number of acres.

7. P, Q, R, represent three candidates at an election. Q polled as many plumpers wanting one as the split votes between P and R exceeded those between himself and R; and the number of split votes between Q and R was one more than twice the number between Q and P. If P had not voted for himself and R, but for R only, and if five others who split betwixt P and Q had voted for Q only, Q would have just beaten P, and would have been 48 below R. The number of voters was 1341, of which 565 gave plumpers. Find the final state of the poll.

19.

1. $$\frac{7x - 13\frac{1}{2}}{11} - \frac{2}{3} \cdot \frac{x - 15}{7} = \frac{15}{14}(x - 1)$$

2. $$\left.\begin{aligned} \frac{2x}{x+y} - \frac{y^2}{x^2 - y^2} &= \frac{13x + 16y}{8(x + 2y)} \\ \sqrt{x} + \sqrt{y} &= \sqrt{(x + y + \sqrt{3})} \end{aligned}\right\}$$

3. $$\sqrt{\{2(x + 2)\}} - 2\sqrt{(2 - x)} = \frac{12x - 8}{\sqrt{(9x^2 + 16)}}$$

4. $$\left.\begin{aligned} ax^2 + by^2 &= a(x + b) \\ x^4 + x^2(y^2 - 2x) - x(y^2 - 1) + y^4 &= a^2 \end{aligned}\right\}$$

5. In a race between two boats a spectator, walking at the rate of 5 miles an hour, is $\frac{1}{8}$ of a mile a-head of the first boat at starting; and, when it passes him, he observes that the interval between the boats, which at first was 30 yards, is reduced to 20. At $1\frac{1}{4}$ mile from where it started, the first boat is overtaken by the second: how long did the race last?

6. When wax candles are 2*s* 6*d* per lb, a composition is invented, such that a candle made of it will burn $\frac{2}{3}$ of the time in which a wax candle, of the same thickness and $\frac{1}{4}$ as heavy again, would burn. If the two give an equally bright light, what must be charged per lb for the composition, that it may be as *cheap* as wax?

7. Into a cubical cistern, 8 feet deep, and having an unknown leak, water is poured from two pumps worked by A and B. They pump together till it is half filled, when B falls asleep, but A goes on pumping till it is three-fourths filled, and then goes away B upon waking finds it still half full, and, after pumping till it is again three-fourths filled, departs also to look for A. They return together, and find the water $1\frac{1}{2}$ in. lower than when B left. The leak is now discovered and stopped; and the vessel is filled by them in half the time in which they had worked together at first. Now $10\frac{1}{3}$ hours had elapsed since they first began pumping, and B had worked alone twice as long as A had. Given that a cubic foot contains $15\frac{5}{8}$ gals, find the quantity of water thrown in per hour by each pump.

20.

1. $\dfrac{4x-17}{9} - \dfrac{3\frac{2}{3}-22x}{33} = x - \dfrac{6}{x}\left(1 - \dfrac{x^2}{54}\right)$

2. $x(y+z)^2 = 1 + a^3, \quad x + y = \frac{3}{2} + z, \quad yz = \frac{5}{16}$

3. $4\{(x^2-16)^{\frac{3}{4}} + 8\} = x^2 + 16(x^2-16)^{\frac{1}{4}}$

4. $(x^6+1)y = (y^2+1)x^3, \quad (y^6+1)x = 9(x^2+1)y^3$

5. A at his death leaves a certain property in money and sums due to him. The executors invest the money in the funds at 96. Of the debts $\frac{1}{6}$ is not recovered; and, when the stock is sold out at 92, the heir (it is found) receives less by £140 than he would have done, if the debts had been completely recovered. His loss is also $\frac{1}{8}$ of the sum he receives. Find the amount of the debts and money.

6. A ferry-boat was about to cross a river, when it was upset by a party leaping in, who increased the n° of persons in the boat in the ratio of 4 : 5. The n° who got out without help (including the ferryman) was $\frac{5}{6}$ of the increased n° of passengers, and the n° helped out was $\frac{1}{3}$ of the n° of minutes the last man was in the water. The numbers extricated in both ways in each of the first three minutes form a series of fractions, whose num^rs increase in A.P., and den^rs in G.P., the common ratio and difference being the same as the n° of persons in the water at the end of the three minutes, and the first num^r (with den^r *unity*) being the n° of minutes remaining till the last man was out. Had, however, the n° of intruders been less by four, and still increased the n° of persons in the boat in the same ratio, the increased n° of passengers would have been the same as twice the common ratio of the G. P. How long was the last man in the water?

7. Towards the end of a cricket match, the second party were a certain n° of notches behind, and had still three men, *A*, *B*, *C*, remaining. *A* and *B* are in, and after $\frac{5}{8}$ of the n° have been gained, *A* is struck out, and *C* takes his place. Now *B* scores as many notches in *C*'s innings, as there were bye-balls in *A*'s, and as many in *A*'s as were gained altogether in *C*'s. If also the byes in *A*'s innings be added to *B*'s notches in it, and the byes in *C*'s innings to *C*'s notches, these quantities will be inversely proportional to the corresponding n^os of byes. *C* gets one more notch than *B* in their common innings, and the party loses by 3: but, if *B*'s scoring be reversed, that is, if he be supposed to get as many notches in *A*'s innings as the n° of byes in *C*'s, and as many in *C*'s as the whole n° now gained in *A*'s, the three would have scored between them, (not reckoning the bye-balls,) just as many as their whole former number. How many notches did *A* score?

21.

1. $\frac{3x}{2} + \frac{81x^2 - 9}{(3x-1)(x+3)} = 3x - \frac{3}{2} \cdot \frac{2x^2 - 1}{x+3} - \frac{57 - 3x}{2}$

2. $xy + z = 5, \ xyz = 4, \ 2(x^2 - y) = (y^2 - x)^2$

3. $(x+3)^2 - 2(x^2 + 3) = 2x(x+1)^2$

4. $\left.\begin{array}{l}(x+y)^2 = x^4 + x^2y^2 + y^4 \\ x^4 + 4y^4 = 4xy(2y^2 - x^2)\end{array}\right\}$

5. In a tithe commutation, the rent-charge was paid at 3*s* an acre, and the tithe-owner found the first year that his rates wanted £6 of being 10 per cent on his receipts. The next year the rates were doubled, and amounted to 15 per cent on his receipts. What was the number of acres in the parish?

6. An omnibus starts with a certain n° of passengers, and takes up four more on the road, whose fare is the same as that paid by the others. On deducting $\frac{1}{12}$ of the whole fare for expences, there remains a profit of 4*s* 7*d*. But if those who were last taken in had paid half as many pence as there were passengers altogether, the money received would have exceeded by 2*s* 8*d* double the difference of the sums actually paid by the two sets of passengers. With how many did the omnibus start?

7. *A* and *B*, having a single horse, travel between two milestones distant an even n° of miles in $2\frac{62}{63}$ hrs, riding alternately mile and mile, and each leaving the horse tied to a milestone until the other comes up. The horse's rate is twice that of *B*: *B* rides first, and they come together to the 7th milestone. Finding it necessary to increase their speed, each man after this walks half-a-mile an hour faster than before, and the horse's rate is now twice that of *A*, *B* again riding first. Find the distance they travelled.

22.

1. $$\frac{1}{x^2+11x-8}+\frac{1}{x^2+2x-8}+\frac{1}{x^2-13x-8}=0$$

2. $(x-2)^2+(y-3)^2+(z-1)^2=24,\quad xy+yz+xz=63,\quad 2x+3y+z=30$

3. $$\frac{(n-1)\,(a^4+a^2x^2+x^4)}{(n+1)\,(a^4-a^2x^2+x^4)}=\left(2-\frac{1}{n}\right)\left(\frac{ax}{a^2-x^2}\right)^2$$

4. $$\left.\begin{array}{c}(x^4+2bx^2y+a^2y^2)\,(y^4+2bxy^2+a^2x^2)=4\,(a^2-b^2)\,(b+c)^2x^2y^2\\ x^3+y^3=2cxy\end{array}\right\}$$

5. A merchant, travelling from St. Petersburg to Moscow, had provided himself with notes of the Bank of Russia, amounting in all to 540 roubles. The paper at first bore the value marked on it; but south of Torjok, a town on the road, and in Moscow itself, a premium of 20 per cent was allowed on each note. On reaching Moscow, he received 432 roubles for the notes that remained: he spent there 237 roubles, and had just enough left to pay his expences back, supposing them the same as before. How many roubles did he spend between St. Petersburg and Moscow?

6. From a quantity of gold, silver, and copper, weighing in all 20300 oz., two alloys were formed. In the one gold and copper were mixed in the proportion of 11 : 1, in the other silver and copper, in the proportion of 37 : 3; and there were 288 oz. of copper over. The alloy of gold and copper was coined at the rate of £3 17*s* 10½*d* per oz., and that of silver and copper at the rate of 5*s* 6*d* per oz. The whole sum thus produced was £5546 14*s* 6*d*. How many ounces were there of each metal?

7. A body of 6048 soldiers was divided into a number of equal detachments, and sent to occupy as many fortresses. In the course of the campaign as many as two whole garrisons and half of another died of an epidemic, and all the rest except 84 invalids, who returned to head-quarters, were equally divided among the fortresses as before. But, the reduced garrisons proving too weak for their defence, all the fortresses fell into the hands of the enemy, and the men, with the exception of four whole garrisons and 210 fugitives, were killed or made prisoners. The loss thus sustained, together with that caused by the epidemic, amounted to 4186 men. Required the number of fortresses.

23.

1. $$\frac{x-4\frac{2}{3}}{3} - \frac{2x-3\frac{2}{3}}{4} = \frac{3}{2}\left\{x - \frac{x-1\frac{1}{2}}{2}\right\} + \frac{4x}{3}\left\{x - 3 - \frac{(x-1)(x-2)}{x}\right\}$$

2. $$\left.\begin{aligned} \frac{2}{3}\left\{x - \frac{3}{5}y\right\} + \frac{x+\frac{1}{5}y}{6} &= \frac{1}{3} - \frac{1}{2}\left\{\frac{\frac{4}{5}y-2}{6} - (x-y)\right\} \\ x - 2y - \frac{3y-5x}{2} &= \frac{11}{2}(x+y) + 3(x-y) \end{aligned}\right\}$$

3. $$\left(x-\tfrac{1}{3}\right)^2 - \tfrac{25}{9} = \frac{3x^2+\frac{4}{9}}{2\left(x-\frac{1}{3}\right) + \sqrt{\left\{x\left(x-\frac{8}{3}\right)\right\}}}$$

4. $$\left.\begin{aligned} \frac{3+2x^2-4x^4}{x^2-1} &= y^2(1-2y^2) \\ (2x^2-1)(2y^2-1) &= 3 \end{aligned}\right\}$$

5. A person leaves London for Derby by the Birmingham Railway at 10 A.M., intending to get upon the Midland Counties line at Rugby, and allowing for a delay of 30′ in changing trains, but expecting to travel the 48 miles from thence to Derby at the same rate at which he had come down, he calculated to reach Derby at 4 P.M. On reaching Rugby, however, he finds that there will be no train for Derby till too late for his purpose; but that by going

on upon the first line to Hampton ($\frac{1}{4}$ as far again as he had come already) he might start immediately by the Derby Junction line, and though the whole distance by this route would be 13 miles longer than by the other, yet the speed on the second line being one mile an hour quicker, he would reach Derby just $1\frac{1}{2}'$ before 4. What is the distance from London to Rugby?

6. Two cubical boxes A, B, of which A is larger by 1216 cubic inches, are filled with balls, there being 12 more along an edge of B than in an edge of A, and the number of balls in the faces of A being to the number in the edges of B as 7 : 22. Also the difference between the areas enclosed by the balls of B, (defined by a thread passing round them,) when they are spread out first into a hollow and then into a solid square, is to the same difference with respect to the balls of A as $129\frac{181}{900}$ 1. Find the radii of the balls.

7. The income of a schoolmaster arises partly from ten pupils residing in his house; and partly from an endowment, which produces a certain number of quarters of wheat each year. When wheat sells for 60*s* the expenditure of his family (£249) is less than his savings by a number, which when divided by twice the number of his pupils expresses the proportion which the clear gain bears to the whole charge for each pupil. In the following year wheat falls to 55*s*, and a tax of 8*d* in the pound is laid upon income, payable upon the net income of the preceding year; but the cost of living for his pupils being diminished, (so that, in fact, the amount of income-tax he has to pay, with 10*s* added, would just support one pupil,) he finds that his savings are greater than in the year previous by a sum equal to the difference of his net income in the two years, which is $\frac{1}{18}$th of the expenditure of his family in the second year, besides allowing for an outlay of £15 in repairs. The net income from pupils in the first year being £330, find that from the endowment in the same year and the ratio of the costs of living in the two years.

24.

1. $\sqrt{\{(x-1)(x-2)\}} + \sqrt{\{(x-3)(x-4)\}} = \sqrt{2}$
2. $x(\sqrt{x}+1)^2 = 102(x+\sqrt{x}) - 2576$
3. $\left(\dfrac{2x+3}{2x-3}\right)^{\frac{1}{3}} + \left(\dfrac{2x-3}{2x+3}\right)^{\frac{1}{3}} = \dfrac{8}{13} \cdot \dfrac{4x^2+9}{4x^2-9}$.
4. $a^2 - x^2 = 3xy$, $(\sqrt{y}-\sqrt{x})(a-x) = 3\sqrt{x}(x+y)$.

5. *A* and *B* embark in trade for 5 years; *B* is to have $\frac{7}{16}$ of the net annual profits for the first half of the time, and half of them for the remainder. After $3\frac{1}{2}$ years the annual profits, by a lowering of the tariff, were increased in the proportion of 6 to 5, and at the same time became liable to a reduction of 7 pence in the pound by the laying on of the Income-tax. At the termination of the partnership, *B*'s share of the total net profits amounted to £987. Required the annual profits of the business before the duties were reduced.

6. A cubical vessel is filled with water and has its surface exposed to the sky. The temperature of the atmosphere is 30° on the first day, and every successive day it is increased by 1°. Suppose that a temperature of 15° would evaporate 1 inch in one day, and other temperatures in the same proportion. Every evening there are showers; on the first evening 3 inches of rain fall, and the depth of rain falling each successive evening decreases in an A.P. whose common difference is $\frac{1}{60}$th of what fell on the first day. At the end of 41 days the vessel is found to be empty; required its content.

7. *A* and *B* set out together, and walk from Keswick over Helvellyn to Ambleside. *B* arrives at the foot of Helvellyn, which is 5 miles from Keswick, in an hour, and then slackens his pace so that he just reaches the top at the end of the second hour. Here he sits down to rest, until *A* passes him and gets as much before him (*B*) as he was behind when *B* first sat down. *B* then starts at an increased pace, and passes *A* at the end of the third hour from the time of starting. *B* walks on at the same pace for another hour, and then waits for *A* to come up, whose distance behind him was $\frac{2}{3}$ of what it was at the end of the first hour. When *A* overtakes him, he starts again and walks at the same pace until he reaches Ambleside; the time of this last stage being to that of his first rest in the ratio of 20 : 7. On his arrival, he fires a pistol for the information of *A* who, having hitherto kept up a uniform pace without stopping, now diminishes it in the ratio of 5 : 7, and reaches Ambleside 10 minutes after *B*. What is the distance from Keswick to Ambleside by this route?

25.

1. $$\frac{\frac{3}{x}-1}{2} - \frac{9\left(\frac{1}{2x}-1\right)-\frac{2}{5}\left(\frac{9}{2x}-4\right)}{\frac{3}{x}-4} = \frac{\frac{9}{x}+19}{6}$$

2. $$\sqrt{(a+x)}+\sqrt{(a-x)}=\sqrt{\left(\frac{3b^2+x^2}{a+b}\right)}$$

3. $$\left.\begin{aligned} x^3 &= 31x^2-4y^2 \\ y^3 &= 31y^2-4x^2 \end{aligned}\right\}$$

4. $$(x+y)^{\frac{1}{3}}+(x-y)^{\frac{1}{3}}=a^{\frac{1}{3}},\ (x^2+y^2)^{\frac{1}{3}}+(x^2-y^2)^{\frac{1}{3}}=a^{\frac{2}{3}}$$

5. A railway train travels from A to C passing through B where it stops 7 minutes. Two minutes after leaving B, it meets an express train which started from C when the former was 28 miles on the other side of B. The express travels at double the rate of the other, and performs the journey from C to B in $1\frac{1}{2}$ hours; and, if on reaching A it returned at once to C, it would arrive 3 minutes after the first train. Find the distances between A, B, and C, and the speed of each train.

6. To meet a deficiency of (m) millions in the revenue, an *additional* tax of a per cent. was laid on exports, and the tax on imports was diminished (c) per cent.: the value of the imports was in consequence increased so as to be n times as great as the exports, and the deficiency was made up. Now, if the additional tax on exports had been a' per cent, and the tax on imports diminished c' per cent, the values of the articles being altered as before, the deficiency would not have been made up by m' millions. Find the exports and imports after the alteration.

7. Fifty thousand voters, who have to return a member to an assembly, are divided into sections of equal size, and each section chooses an elector, the member being returned by the majority of such electors. There are two candidates, A and B. In those sections which return electors favourable to A, the majority is double the minority, while, in those favourable to B, the minority forms only a tenth of the whole. After the primary elections C comes forward, and is returned by a majority of 3 over A, and 14 over B. If C had not come forward, A would have been returned by a majority 19 less than the whole number of C's votes, and if the 50,000 had voted *directly* between A and B, B would have had a majority of 6000. Find the number of sections.

ANSWERS

TO

MISCELLANEOUS EXAMPLES.

PART I.

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1. $(a^2-b^2)b^2+(a^3-3ab^2+b^3)x-(2a-b)ax^2$.

2. $3x^2-2abx-2a^2b^2$. 3. $3\frac{1}{7}$. 4. $(m+n)a, \dfrac{x^2-xy+y^2}{x^2+xy+y^2}$.

5. $1\frac{2}{13}$, 8.152. 6. $\dfrac{13-9x}{11-8x}, \dfrac{x+3}{x^4-1}$. 7. 98, $\frac{1}{4}n(3n+25)$.

8. $1\frac{7}{17}$. 9. $5\sqrt{5}, 7a\sqrt{2x}, \sqrt[3]{4}$. 10. $1+x+\frac{3}{2}x^2+\frac{5}{2}x^3+\frac{35}{8}x^4+$&c.

11. (i) $x=5$; (ii) $x=5$ or $-1\frac{1}{2}$; (iii) $x=4, y=3$;
(iv) $x=\pm 3, y=\pm 2$, or $x=\pm 2, y=\pm 3$. 12. $\frac{5}{8}$.

13. $1+4x+2x^2-5x^3-x^4+x^5+\frac{1}{4}x^6$. 14. $-2x^2+8xy-5y^2$.

15. 68. 16. x^2-y^2. 17. 125, 1.709. 18. $x, \dfrac{a+x}{x-y}$.

19. $2\frac{7}{27}, 2\frac{1}{4}$. 20. $a^{\frac{3}{2}}b^{-2}c^{\frac{9}{4}}, x^{-\frac{1}{2}}z^{\frac{2}{3}}$.

21. $a^{-\frac{1}{3}}\{1+a^{-1}x+2a^{-2}x^2+\frac{14}{3}a^{-3}x^3+\frac{35}{3}a^{-4}x^4+\text{\&c.}\}$.

22. 1232, 11313, 363, 1044.

23. (i) $x=3\frac{3}{16}$; (ii) $x=2$ or $-\frac{2}{3}$; (iii) $x=5$, $y=4\frac{1}{3}$.
24. 12 days. 25. x^6-a^3. 26. $\frac{3}{2}x^3-5x^2+\frac{1}{4}x+9$. 27. $\frac{3}{4}$.
28. $\frac{x^2}{x^2-y}$, $\frac{3a+1}{4a^2+2a-1}$. 29. $\frac{4-x}{5-x}$, $\frac{2x}{x^4-1}$.
30. 139, 1.39, 4.3955. 31. $2\frac{3}{16}$, .051. 32. $\frac{24}{55}\{1-(-\frac{5}{6})^n\}$, $\frac{24}{55}$.
33. $(ax)^{-\frac{3}{5}}$ $1+\frac{3}{5}a^{-1}x+\frac{12}{25}a^{-2}a^2+\frac{52}{125}a^{-3}x^3+\frac{234}{625}a^{-4}x^4+$&c.}.
34. 7; 22 dollars and 57 doubloons.
35. (i) $x=17$; (ii) $x=60$, $y=40$; (iii) $x=3$ or $-\frac{40}{59}$.
36. $8\frac{3}{4}$ sq. ft. 37. $2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4$.
38. $1-\frac{1}{6}x+\frac{7}{36}x^2+\frac{5}{216}x^3+\frac{73}{1296}x^4+$&c. 39. $\frac{1}{8}$.
40. $\frac{x^2-x+1}{x^2+x+1}$. 41. 139, .6933. 42. $\frac{3x}{1-x}$.
43. $a^7\{1-7a^{-2}x^2+\frac{21}{2}a^{-4}x^4+\frac{7}{2}a^{-6}x^6+\frac{35}{8}a^{-8}x^8+$&c.}.
44. $\frac{1}{3}\sqrt[3]{18a}$, $\sqrt[4]{5x^2y^2}$. 45. 93, $\frac{1}{12}(n^2+n-6)$.
46. 5221, 40255141, 6252711, 2451, 3341584, 1828.
47. (i) $x=2\frac{2}{3}$; (ii) $x=39$, $y=21$; $z=12$;
(iii) $x=a\cdot\frac{a+b}{a-b}$, $y=a\cdot\frac{a-b}{a+b}$. 48. 64 days. 49. $a-b$.
50. $9+1+49=59$. 51. $3(a^2+b^2+c^2)-2(ab+ac+bc)$.
52. x^2-9y^2. 53. 1054, $\sqrt{7}+\sqrt{2}$.
54. $\frac{b}{a}$, $\frac{a-b}{a+b}$. 55. $\frac{2}{9}\{1-(\frac{1}{10})^n\}$, $\frac{2}{9}$. 56. 6.
57. $a^{-\frac{4}{5}}\{1+\frac{2}{5}a^{-2}bx+\frac{7}{25}a^{-4}b^2x^2+\frac{28}{125}a^{-6}b^3x^3+\frac{119}{625}a^{-8}b^4x^4+$&c.}
58. 21. 59. (i) $x=-\frac{3}{5}$; (ii) $x=3$, $y=4$; (iii) $x=3$, $y=1$;
(iv) $x=-1$, $y=-3$.
60. 100. 61. $26x^{\frac{1}{2}}y^{\frac{1}{2}}-x^{\frac{1}{4}}y-7x^{\frac{3}{4}}-10y^{\frac{3}{2}}$. 62. x^2+2x+1. 63. 1.
64. $a^2b^2(b+c)$. 65. $2x-1$. 66. $\frac{1-x}{x(4x^2-1)}$. 67. 12.747.
68. $(a^2x)^{-\frac{1}{2}}\{1+\frac{1}{2}a^{-1}x+\frac{3}{8}a^{-2}x^2+\frac{5}{16}a^{-3}x^3+\frac{35}{128}a^{-4}x^4+$&c.}.
69. 30. 70. 250, 60300, 13874000.
71. (i) $x=9$; (ii) $x=3\frac{1}{2}$ or -4; (iii) $x=\frac{1}{2}$, $y=\frac{1}{3}$.
72. 8 hrs.; $17\frac{1}{7}$ hrs., 24 hrs., 40 hrs. 73. $6(x+2x^{\frac{1}{3}}+4x^{-\frac{1}{3}}+8x^{-1})$.
74. $\frac{1}{8}$. 75. x, $\frac{x^3+1}{x^2-3x+1}$. 76. $55\sqrt{7}$. 77. 1.772452, $x-\frac{1}{2}$.
79. $50\frac{2}{5}$, $\frac{1}{5}n(3n+1)$. 80. $x^4-2x^3-8x^2+8x+16=0$. 81. 28.
82. 15120, 120. 83. (i) $x=1$; (ii) $x=2\frac{2}{3}$, $y=5\frac{1}{3}$; (iii) $x=3$.
84. £$553\frac{1}{3}$, £$1106\frac{2}{3}$, £3320. 85. x^3+1+x^{-3}. 86. $a^2+ax-2x^2$.
87. 7. 88. $\frac{1}{2}(1+x)$, $\frac{3a+2b}{3b-2a}$. 89. $a^{\frac{2}{3}}x^{-\frac{2}{3}}-a^{\frac{1}{3}}x^{-\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}$.
90. $x^{\frac{1}{2}}-1-x^{-\frac{1}{2}}$. 91. $\frac{1}{5}n(3n+1)$, $\frac{8}{15}\{(\frac{5}{2})^n-1\}$.

92. $1+x-\frac{3}{2}x^2+\frac{7}{2}x^3-\frac{77}{8}x^4+$&c., $1+2x-2x^2+4x^3-10x^4+$&c.
93. 27, 48. 95. (i) $x=9$; (ii) $x=4$, $y=3$; (iii) $x=6$ or $\frac{1}{2}$.
94. 5. 96. $9\frac{1}{2}d$. 97. x^3. 98. $a^2-2ab+\frac{1}{2}b^2$.
99. x^2-1. 100. $\frac{x^2+b^2}{x}$, $\frac{\sqrt{x}}{\sqrt{x}-\sqrt[4]{a^2y}}$. 101. 4.11.
102. $1-a^2+x^{-2}+3a^{\frac{2}{3}}x^{-\frac{2}{3}}$. 103. 7. 105. 75, 25.
106. $\sqrt{11333311}$ *septenary* $=2626=1000$ *denary*.
107. (i) $x=9$; (ii) $x=\pm\frac{1}{2}\sqrt{5}$; (iii) $x=4$, $y=0$, $z=5$.
108. £135, £90. 109. 48. 111. $36x^4-97x^2+36$.
112. $\frac{1}{2}x^2-ax+\frac{1}{3}a^2$. 113. 2.4494, .4082, .8164, 1.2247.
114. $(ab^{-1})^{\frac{1}{2}(m+1)}$, 1. 115. $a^2-4ab-6ac+4b^2+12bc+9c^2$,
$4a^2-2abx-(ac-\frac{1}{4}b^2)x^2+(8ad+\frac{1}{4}bc)x^3-(2bd-\frac{1}{16}c^2)x^4$
$-cdx^5+4d^2x^6$. 116. $\frac{25}{3}\{1-(\frac{2}{5})^n\}$, $8\frac{1}{3}$.
117. $-1\frac{1}{3}$. 118. $1-2x-2x^2-4x^3-10x^4-$&c.
119. (i) $x=21$; (ii) $x=-3$ or $\frac{5}{7}$; (iii) $x=5$, $y=3$.
120. $3\frac{1}{3}$, 3. 121. $8-12a^{\frac{1}{4}}+18a^{\frac{1}{2}}-27a^{\frac{3}{4}}$.
122. $a^4+\frac{4}{3}a^2bx-\frac{4}{9}(a^2-b^2)x^2+\frac{2}{3}ax^3-\frac{1}{4}x^4$.
123. $\frac{3x^2+4x+2}{4x^2+x+2}$. 124. $a^{10}-a^6x^4-a^4x^6+x^{10}$.
125. 1.2247, $3+\sqrt{3}$. 126. c. 127. 0, $\frac{1}{12}n(7-n)$.
128. A gives 26 guineas and receives 10 crowns.
129. $2(a-x)\sqrt{2ax}$, $\frac{1}{4}\sqrt[4]{a}$. 130. 33 : 238, 1 : 34.
131. (i) $x=10$; (ii) $x=3$, $y=7$; (iii) $x=4$ or -1. 132. 10.
133. With *upper* signs, $16+9=5\times5$; with *lower*, $0+25=5\times5$.
134. x^2+4y. 135. x^2+ax+b. 136. $\frac{3a-2y}{5a+3y}$, $\frac{x(x+5)}{9x^2-x-3}$.
137. $mn(m^2-n^2)(m^2-4n^2)$.
138. $a^{\frac{1}{6}}-a^{\frac{1}{12}}+1$. 139. $\frac{bc}{a^2}(a+b)(a+c)$.
140. 4.8989, .6803, 4.4494, 1.5506, 3.4494. 141. $1\frac{2}{3}$, $2\frac{1}{3}$; $1\frac{2}{7}$, $1\frac{4}{5}$.
142. $1+x+\frac{5}{2}x^2+\frac{15}{2}x^3-\frac{195}{8}x^4+$&c., $1+2x+6x^2+20x^3+70x^4+$&c.
143. (i) $x=7$; (ii) $x=4$; (iii) $x=2$ or $\frac{43}{87}$.
144. £9, 30s. 145. $a^2b^2-ab^2x-(a^2+2b^2)x^2+ax^3+2x^4$.
146. $a^2-a^{\frac{3}{2}}x^{\frac{1}{2}}-\frac{15}{4}ax+2a^{\frac{1}{2}}x^{\frac{3}{2}}+4x^2$, $a^4-2a^{\frac{7}{2}}x^{\frac{1}{2}}-\frac{13}{2}a^3x+\frac{23}{2}a^{\frac{5}{2}}x^{\frac{3}{2}}$
$+\frac{289}{16}a^2x^2-23a^{\frac{3}{2}}x^{\frac{5}{2}}-26ax^3+16a^{\frac{1}{2}}x^{\frac{7}{2}}+16x^4$.
147. $x^2-12-16x^{-1}$, a^2+1+a^{-2}. 148. $\frac{a-b+c}{a-2b+3c}$. 149. .2154.
150. $x-2\sqrt{x}+1$. 151. $\sqrt{3}+\sqrt[3]{5}$. 152. $\frac{3}{2}\{1-(\frac{2}{3})^n\}$, $1\frac{1}{2}$.
153. 0, 3, -2. 154. 6. 156. $5\frac{1}{3}$ days, 16 days.
155. (i) $x=\frac{ab}{a+b}$; (ii) $x=\frac{2}{7}$; (iii) $x=\pm2$, $y=\pm3$. 157. $3\frac{19}{40}$.

158. $a^{\frac{4}{3}}-2ab^{\frac{1}{2}}+3a^{\frac{2}{3}}b-2a^{\frac{1}{3}}b^{\frac{3}{2}}+b^2$. 159. $x-5$. 160. $\frac{8(x+6)}{x^4-16}$, $\frac{9}{10}$.
161. $x^4+2x^3-8x^2-6x-1$. 162. 9, 160. 163. $(b+c)^2$.
164. 2282. 165. $1-6x^2+24x^4-80x^6+240x^8-$&c., $a^{\frac{3}{2}}\{1+3a^{-1}b+\frac{3}{2}a^{-2}b^2-\frac{1}{2}a^{-3}b^3+\frac{3}{8}a^{-4}b^4-\text{\&c.}\}$.
166. 33233344, 4344 = 1000 *den.*, 244 = 100 *den.*
167. (i) $x=17$; (ii) $x=\frac{ac+b}{a^2+1}$, $y=\frac{c-ab}{a^2+1}$; (iii) $x=\pm 6$.
168. £40, £28, or £28, £52, according as *A* had more or less at first than *B*.
169. $\sqrt{\{a^{\frac{3}{2}}+b^{\frac{3}{2}}\}}\times(a^2+b^2)^{\frac{2}{3}}=3\sqrt[3]{289}=19.834$.
170. $x^{-3}-4x^{-\frac{3}{4}}y^{\frac{3}{2}}+3y^2$. 171. $a^{\frac{pq}{n}}b^{-\frac{pq}{m}}$, $\frac{x(x^2-1)}{2x^2+1}$.
172. $1-\frac{1}{2}ax^{\frac{1}{2}}-2a^2x$. 173. $\frac{ax-b^2}{(x-a)\ (x-b)}$, $\frac{x+2}{x^2-1}$.
174. 3.8729, 1.2909, .7745, 1.5491, 6.4549.
175. -10, $\frac{1}{4}n(7-3n)$. 176. 15.
177. $1-2x^{\frac{1}{2}}+3x-4x^{\frac{3}{2}}+5x^2-$&c., $1-4x^{\frac{1}{2}}+10x-20x^{\frac{3}{2}}+35x^2-$&c.
178. 12, 16, 18. 179. (i) $\frac{a^2-b^2}{4a-b}$; (ii) 2 or $-1\frac{3}{4}$; (iii) $x=49, y=50$.
180. 10 days, $3\frac{1}{5}$ days. 181. $a^3+\frac{1}{2}a^2(2x+y-z)+\frac{1}{4}a(2xy-2xz-yz)-\frac{1}{4}xyz$, which becomes $a^3+3a^2b+3ab^2+b^3$, by putting $x=b=\frac{1}{2}y=-\frac{1}{2}z$, or $x=b$, $y=2b$, $z=-2b$.
182. $5x^2+\frac{7}{2}a^{-\frac{1}{3}}x^{\frac{3}{2}}-\frac{107}{12}a^{-\frac{2}{3}}x+\frac{5}{6}a^{-1}x^{\frac{1}{2}}+\frac{7}{6}a^{-\frac{4}{3}}$. 183. $x+1$.
184. $\frac{x^2+2}{x^2+x-5}$. 185. $a^{\frac{1}{3}}x(a^2x^6-1)$. 186. $ab^{-1}-\frac{1}{2}a^{-1}b+1$.
187. $\frac{x^2}{(x^2+1)\ (x-1)^2}$. 188. 720, $\frac{1}{2}(1+\sqrt{7})$.
189. $4\frac{122}{243}$, $\frac{9}{2}\{1+(-\frac{1}{3})^n\}$. 190. 3 per cent.
191. (i) $1\frac{2}{5}$; (ii) $x=4$, $y=5$; (iii) $4\frac{1}{2}$. 192. 10. 193. $\frac{7}{12}$.
194. $x^{2n}+2$. 195. $5+2\sqrt{6}$, $\sqrt{6}$, $6(5+2\sqrt{6})$.
196. $\left(\frac{m+2a}{m+a}\right)^2$, $\frac{a^2+ax+x^2}{a^4-x^4}$. 197. $12x^4-2x^3-11x^2+1$.
198. $\frac{9}{5}\{1-(-\frac{2}{3})^n\}$, $1\frac{4}{5}$. 199. n^2. 200 $\pm\frac{3}{5}a$. 201. 15.
202. $1-6x^{\frac{1}{3}}+21x^{\frac{2}{3}}-56x+126x^{\frac{4}{3}}-$&c., $1-3x^{\frac{1}{3}}+6x^{\frac{2}{3}}-10x+15x^{\frac{4}{3}}-$&c.
203. (i) $\frac{2}{3}$; (ii) a^{-1} or b^{-1}; (iii) $x=3$, $y=1$, or $x=\frac{3}{4}$, $y=\frac{1}{4}$.
204. £800, 0. 206. px^2+qx-r. 207. $a-a^{-1}+4$.

208. $\frac{x^2+(a^2-a^{\frac{1}{2}})x-a^{\frac{5}{2}}}{x-a}$. 209. $\frac{1}{(x^2+1)(x^3+1)}$.

210. 7.0102, 202. 211. $\frac{1}{15}$, 20. 212. $(ab)^{\frac{5}{12}}$.

213. 4 yds., 5 yds. 214. 63361, 236, 34.

215. (i) 4 or $1\frac{5}{6}$; (ii) $x=-5, y=5$; (iii) $x=\frac{a^2}{\sqrt{a^2+b^2}}, y=\frac{b^2}{\sqrt{a^2+b^2}}$.

216. $\frac{mn}{m+n}$ days, $\frac{m^2}{n}$ days. 217. $2(n+4)$.

218. $a^{\frac{2}{3}}+b^{\frac{4}{3}}+c^2-a^{\frac{1}{3}}b^{\frac{2}{3}}-a^{\frac{1}{3}}c-b^{\frac{2}{3}}c$. 219. 2. 220. x^4-y^4.

221. 3.1622, .12649, 2.1081, 1.5811, 4.4414, .31622.

222. $\frac{8}{3}\{1-(\frac{3}{4})^n\}$, $2\frac{2}{3}$.

223. $a^{-3}\{1-3a^{-2}x^2+\frac{15}{2}a^{-4}x^4-\frac{35}{2}a^{-6}x^6+\frac{315}{8}a^{-8}x^8-\&c.\}$,
$\frac{1}{4}a^{-2}\{1+3a^{-1}x+\frac{27}{4}a^{-2}x^2+\frac{27}{2}a^{-3}x^3+\frac{405}{16}a^{-4}x^4+\&c.\}$.

224. £5825 8*s.* $5\frac{1}{4}$*d.* 225. 0, −1, 2. 226. 20, 5.

227. (i) $x=a^{-1}$ or $-\frac{1}{2}$; (ii) $x=\frac{b}{a^2-ab+b^2}, y=\frac{a}{a^2-ab+b^2}$; (iii) $x=3$ or $\frac{1}{5}$. 228. $\frac{mn(m+n)}{m^2+4mn+n^2}$ days.

229. $2y^2-ay-\frac{1}{2}a^2$. 230. $x^3+a^2x^{\frac{3}{2}}+a^4$, $x^3+2ax^{\frac{9}{4}}+a^2x^{\frac{3}{2}}-a^4$, $x^3-a^2x^{\frac{3}{2}}-2a^3x^{\frac{3}{4}}-a^4$. 231. a^2-b^2.

232. $x^6-2x^5+x^4-x^2+2x-1$. 233. $\frac{1}{10}$, $\frac{\sqrt[3]{x^2+a}}{\sqrt[3]{x^2-a}}$.

234. $\frac{1}{2}x^{\frac{5}{3}}-5y^{\frac{4}{5}}$. 235. 88, $\frac{5}{2}\{1-(\frac{2}{3})^n\}$, $2\frac{1}{2}$, $1\frac{1}{7}$, $1\frac{1}{3}$, $1\frac{3}{5}$.

236. $2a\sim b$. 237. 12, 4, 18 miles. 240. $\frac{m^3+n^3}{4mn}$ days.

239. (i) $x=-6\frac{1}{2}$; (ii) $x=\frac{m^2-n^2}{am-bn}, y=\frac{m^2-n^2}{bm-an}$; (iii) $x=10, y=7, z=3$. 241. $x^{\frac{3}{2}}-xy^{-\frac{3}{4}}+x^{\frac{1}{2}}y^{-\frac{3}{2}}-y^{-\frac{9}{4}}$, $x^2-(a+b)x+ab$.

242. 36, 125. 243. $5x+4$. 244. $\frac{1+b}{ab^2}$, $\frac{1}{4}$.

245. .8164, 1.6329, 2.0412, .1010, 3.2549. 247. $3\frac{1}{3}$, $3\frac{5}{6}$, $4\frac{1}{3}$, &c.

248. 7. 249. 720. 250. 248664*et*69, 54373.

251. (i) $x=17$; (ii) $x=\frac{a^2+ab+b^2}{a+b}, y=\frac{ab}{a+b}$; (iii) $x=\frac{4}{13}$.

252. $\frac{m(m+n)}{3m+n}$ days, $\frac{mn(m+n)}{m^2+2mn-n^2}$ days. 253. 140, $\frac{281}{1260}$.

254. $x^{-\frac{4}{3}}-x^{-\frac{2}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}$, $x^{\frac{1}{2}}-(m-1)a^{\frac{1}{2}}x^{\frac{1}{4}}+a$. 256. $1\frac{1}{3}$, $\frac{2a-b}{a^2-1}$.

255. $18x^4-45x^3+37x^2-19x+6$. 257. $\dfrac{2ac^2}{b(b^2-c^2)}$, $\dfrac{1+x^2}{\sqrt{(1-x^2)^3}}$.

258. 24 miles, $\frac{1}{2}$ hr. 259. $a^2b^{\frac{4}{3}}$. 260. 3. 261. $a+2x : a+3x$.

262. 7425. 263. (i) $x=4$ or $1\frac{2}{5}$; (ii) $x=10$, $y=-3$, $z=4$.

264. 8*s*. 4*d*. 265. $\frac{25}{4}-\frac{169}{36}=\frac{14}{9}=\frac{8}{3}\times\frac{7}{12}$. 266. $a^{\frac{1}{2}}x+2$.

267. $a^{\frac{1}{2}}+3a^{-\frac{1}{2}}x-\frac{9}{2}a^{-\frac{3}{2}}x^2+\frac{27}{2}a^{-\frac{5}{2}}x^3-\frac{405}{8}a^{-\frac{7}{2}}x^4+$&c., $a+6x$.

268. a. 269. $\dfrac{1}{(x+1)(x+2)(x+3)}$. 270. £2 8*s*.

271. $n=10$ or 12, $l=3$ or -1. 272. 56, 44.

273. $\frac{1}{6}n(n-1)(n-2)$. 274. $1111\times10001=11111111=21845$ *den.*

275. (i) $x=6\frac{1}{2}$; (ii) $x=\dfrac{b-1}{ab-\frac{1}{2}(a+b)}$, $y=\dfrac{a-1}{ab-\frac{1}{2}(a+b)}$;
(iii) $x=4$, $y=3$. 276. $\dfrac{(p-1)mn}{np-m}$ days. 277. 0.

278. $\frac{1}{8}\times64+1=9=(\frac{1}{2}\times4+1)(\frac{1}{4}\times16-\frac{1}{2}\times4\times1+1)$.

279. $\dfrac{3a^{-1}x-4}{a^{-2}x^2-11a^{-1}x+21}$. 280. $a^5x^{10}-a^3x^6-a^2x^4+1$.

281. $x^{\frac{2}{3}}-2x^{\frac{1}{3}}-2$. 282. $3\sqrt[3]{5}$, $\frac{29}{2}\sqrt{3}$. 285. 30. 286. 15.

287. (i) $x=1\frac{1}{3}$; (ii) $x=-4\frac{1}{2}$; (iii) $x=4$, $y=3$, or $x=3$, $y=4$.

288. 4, 59, 55. 289. $12abc$.

290. $(-\frac{1}{288}+\frac{3}{16}-\frac{1}{144})-(\frac{7}{288}+\frac{5}{288}-\frac{35}{96})=\frac{1}{2}=12(-\frac{1}{2}\times\frac{1}{3}\times-\frac{1}{4})$.

291. $1-\frac{1}{64}a^{-3}x^6$. 292. 84. 293. $2\frac{3}{4}$, .25298, $5-\sqrt{6}$.

294. $(a^2-b^2)^3$. 296. $\dfrac{n+1}{2n}a$, $\dfrac{n^n-a^n}{n(n-a)a^{n-2}}$. 297. 10, 20.

298. 40320. 299. (i) $x=acb^{-1}$; (ii) $x=\frac{9}{10}a$; (iii) $x=1$, $y=-1$, or $x=-1\frac{2}{5}$, $y=\frac{3}{5}$. 300. $\frac{6}{7}$ hr.

301. $3x^{\frac{1}{2}}-2x^{\frac{1}{4}}y^{-\frac{1}{3}}+4y^{-\frac{2}{3}}$.

302. 2. 303. 137641, $x-2-x^{-1}$. 304. $b-c$.

305. $\sqrt{2}$, $\sqrt{5}+\sqrt{2}$. 306. $\dfrac{4x^2y^2}{x^4-y^4}$. 307. 1 hr. $5\frac{5}{11}'$.

308. $\frac{1}{27}a^2\{1+2a^{-\frac{1}{3}}x^{\frac{1}{3}}+\frac{8}{3}a^{-\frac{2}{3}}x^{\frac{2}{3}}+\frac{80}{27}a^{-1}x+\frac{80}{27}a^{-\frac{4}{3}}x^{\frac{4}{3}}+$&c.$\}$.

309. $1\frac{5}{6}$, $\frac{3}{16}n(n+1)$; $\frac{3}{4}$, $\frac{6}{7}$, 1, $1\frac{1}{5}$. 310. 6.

311. (i) $x=100$ or -10; (ii) $x=\dfrac{c}{a+b}$, $y=-\dfrac{c}{a+b}$;
(iii) $x=\dfrac{1-b}{1-ab}$, $y=\dfrac{a-1}{1-ab}$. 312. $\frac{4}{9}$ hr. 314. $1\frac{1}{3}$, b or b^{-1}.

313. $a+a^{\frac{1}{2}}x^{\frac{1}{2}}+x$, $x-2a^{\frac{3}{4}}x^{\frac{1}{4}}-a^{\frac{1}{2}}x^{\frac{1}{2}}-a$.

315. $8a^{\frac{9}{4}}x^{\frac{3}{2}} - 4a^{\frac{3}{2}}xy^{\frac{1}{4}} + 2a^{\frac{3}{4}}x^{\frac{1}{2}}y^{\frac{1}{2}} - y^{\frac{3}{4}}$. 316. $\frac{1}{2}(\sqrt{5}-2)$, $\frac{2}{7}xy^{-1} + \frac{3}{4}x^{-1}y - 5$.

317. $a^{\frac{1}{2}}\{1 + \frac{1}{2}a^{-1}x + \frac{3}{8}a^{-2}x^2 + \frac{5}{16}a^{-3}x^3 + \frac{35}{128}a^{-4}x^4 + \&c.\}$,
$x^{\frac{1}{3}}\{1 - \frac{1}{3}a^{-1}x + \frac{2}{9}a^{-2}x^2 - \frac{14}{81}a^{-3}x^3 + \frac{35}{243}a^{-4}x^4 - \&c.\}$. 318. 91.

319. $an^{-1} - n$, $a - \frac{1}{2}n(n+1)$. 321. £4 16*s*. 322. 63, £62 8*s*.

323. (i) $x = \dfrac{2ab}{a+b}$; (ii) $x = 2$; (iii) $x = \dfrac{a^2+bc}{ac}$, $y = \dfrac{a^2+bc}{c^2}$.

324. 13, 12. 325. $1 - x + \frac{11}{12}x^2 - \frac{1}{3}x^3 + \frac{1}{9}x^4$, $1 - 2x + \frac{17}{6}x^2 - \frac{5}{2}x^3 + \frac{83}{48}x^4 - \frac{5}{6}x^5 + \frac{17}{54}x^6 - \frac{2}{27}x^7 + \frac{1}{81}x^8$. 326. $2x^3y^{-3} - 3x^4y$.

327. 2.64575, .37796, 1.32287, .88191, 1.47683. 328. n^{-1}.

329. $2a\sqrt[5]{a-3x}$, $\frac{2}{3}$, $\sqrt[6]{x^{-1}y}$. 330. $\dfrac{(x-1)^2}{x^3(x^2+1)^2}$.

331. $1 + 2xy - 3xy^{-1}$. 333. 7. 334. 27907200.

335. (i) $x = \dfrac{a+b}{c}$ or -1; (ii) $x = 2$, $y = 1$, $z = 0$; but indeterminate, if $2m = n+p$. 336. 5 miles an hour. 337. c or c^{-1}.

338. $\dfrac{x\sqrt[3]{a}(x\sqrt[3]{a}-1)}{2x\sqrt[3]{a}-1}$. 339. $4\frac{99}{560}$. 340. $\dfrac{3a}{a+b}$.

341. $3(abc)^{\frac{6}{5}}$. 342. 3.71, $1 - 2x + 3x^2$. 343. 18*s*. 4*d*.

345. $(2a)^{\frac{5}{3}}\{1 - \frac{5}{2}a^{-\frac{1}{2}}x^{\frac{1}{2}} + \frac{5}{4}a^{-1}x + \frac{5}{24}a^{-\frac{3}{2}}x^{\frac{3}{2}} + \frac{5}{48}a^{-2}x^2 + \&c.\}$,
$(3a)^{-\frac{5}{2}}\{1 + \frac{5}{3}a^{-\frac{1}{3}}x^{\frac{1}{3}} + \frac{35}{18}a^{-\frac{2}{3}}x^{\frac{2}{3}} + \frac{35}{18}a^{-1}x + \frac{385}{216}a^{-\frac{4}{3}}x^{\frac{4}{3}} + \&c.\}$.

346. 2118760, 17296. 347. (i) $x = a\dfrac{a^2-b^2}{a^2+b^2}$; (ii) $x = 1$, $y = 4$, $z = 27$; (iii) $x = \dfrac{(a^2+b^2)c}{a^2-b^2}$, $y = \dfrac{(a^2+b^2)c}{2ab}$. 348. 2 hrs. $21\frac{2}{3}'$.

349. $(x^2+x+1)a - (x+1)$. 350. $a^{-\frac{1}{3}} - a^{\frac{1}{6}}c^{\frac{1}{3}}$. 351. 0.

352. $\frac{3}{2}x^{\frac{3}{2}} - \frac{5}{3}xy^{\frac{1}{2}} + \frac{2}{5}x^{\frac{1}{2}}y$. 353. $\dfrac{x^3-2x^2+3x-4}{x^4-x^3+x^2-x+1}$.

354. $\frac{27}{2}\{1 - (\frac{8}{9})^n\}$, $13\frac{1}{2}$. 355. 76. 356. 9 hrs.

357. $(2a)^{-\frac{5}{3}}\{1 + \frac{5}{2}a^{-\frac{1}{2}}x^{\frac{1}{2}} + 5a^{-1}x + \frac{55}{6}a^{-\frac{3}{2}}x^{\frac{3}{2}} + \frac{385}{24}a^{-2}x^2 + \&c.\}$,
$(3a)^{\frac{5}{2}}\{1 - \frac{5}{3}a^{-\frac{1}{3}}x^{\frac{1}{3}} + \frac{5}{6}a^{-\frac{2}{3}}x^{\frac{2}{3}} - \frac{5}{54}a^{-1}x - \frac{5}{648}a^{-\frac{4}{3}}x^{\frac{4}{3}} - \&c.\}$.

358. 4; 1023, 256. 360. $2\frac{2}{9}$ hrs. 361. $x^{\frac{1}{3}}y^{\frac{1}{4}} - x^{\frac{2}{3}}y^{\frac{1}{3}}z$.

359. (i) $\pm\sqrt{\dfrac{b(bc-2a^2)}{c}}$; (ii) $\dfrac{a^2 \pm \sqrt{a^4+b^2c^2}}{c}$; (iii) $x = \dfrac{a^2}{a-b}$, $y = \dfrac{b^2}{b-a}$.

363. $x^{\frac{1}{2}} - x^{\frac{1}{4}} + 1$. 364. $\sqrt[3]{9}$, $\sqrt{2a}$. 365. 1. 366. 24*s*., 16*s*.

367. $a^{\frac{1}{3}}\{1+2a^{-1}x-4a^{-2}x^2+\frac{40}{3}a^{-3}x^3-\frac{160}{3}a^{-4}x^4+\&c.\}$,
$a^{\frac{2}{3}}\{1+4a^{-1}x-4a^{-2}x^2+\frac{32}{3}a^{-3}x^3-\frac{112}{3}a^{-4}x^4+\&c.\}$.
369. n^3. 370. 1023252 *sen.* = 24e28 *duod.* = 50,000 sq. ft.
371. (i) $x=\frac{b}{a}(a-b+c)$; (ii) $x=a$ or $\frac{b}{na}$; (iii) $x=\pm a\frac{m+n}{\sqrt{m^2+n^2}}$,
$y=\pm a\frac{m-n}{\sqrt{m^2+n^2}}$. 372. $1\frac{1}{2}$h, 1h, 3h. 373. 0.
375. $m^{\frac{3}{2}}-m-m^{\frac{1}{2}}-1$.
374. $mx^3-ax^2-a^2x-(m-2)a^3$. 376. $\pm\sqrt{xy}$. 377. 45.
378. 100, 4. 379. $3\frac{3}{4}$ hrs. 381. 15, 6. 382. 1, 7, 12, or 2, 4, 14, or 3, 1, 16.
383. (i) $x=11$; (ii) $x=\frac{c^2-ab}{a+b}$; (iii) $x=\pm a, y=\mp b$.
384. £740. 385. $(3a-2b)x^2+(a^2-b^2)x$. 387. $\frac{1}{3}$, $\frac{1}{4}$.
388. $\frac{(a-b)^3}{(x+a)^2(x+b)^2}$.
386. $x^6+(p+1)x^5-(p^2+p+1)x^4-(p+1)(p^2+1)x^3+(p^2+p+1)px^2+(p+1)p^2x-p^3$.
389. $a^{\sqrt{2}}-a^{-\sqrt{2}}$, $a\sqrt{cb^{-1}}-\sqrt{cf}$.
390. $\frac{1}{12}n(3n+1)$. 391. 2.549038 cub. ft. 392. 12 hr. $32\frac{8}{11}'$.
393. 2. 394. 5880, 5880, 1960. 396. 2 gals., 14 gals.
395. (i) $x=\frac{a(a+2b)}{2b}$; (ii) $x=\frac{1}{2}(c-a)$; (iii) $x=\pm\frac{24a^2}{25b}$.
398. $x+a$. 399. $x+6$. 400. $x^{\frac{1}{3}}+x^{\frac{1}{6}}+1-x^{-\frac{1}{6}}$.
401. $(a+b)^{\frac{m}{n}+\frac{n}{m}}$. 402. $2x^{-1}$. 403. $5(a-x)^2$, $n\{(a+x)^2-(n-1)ax\}$. 404. 186 hrs. from *A*'s starting.
405. .985185312. 406. 73634.
407. (i) 5 or -1; (ii) ca^{-1} or cb^{-1};
(iii) $x=\pm\frac{ab^2}{\sqrt{a^4+b^4}}$, $y=\pm\frac{a^2b}{\sqrt{a^4+b^4}}$.
408. 4. 410. $2x+3y-z$. 411. 293.9387.
412. $1-\frac{3}{4}x^{\frac{1}{3}}+x^{\frac{1}{2}}$. 414. 0, 1. 416. 15*d*.
418. 69810. 420. 256 sq. yds.
419. (i) 5; (ii) $\frac{1}{2}a$ or $\frac{3}{4}a$; (iii) $x=0, y=b, z=c$, or $x=2a, y=-b, z=-c$.

ANSWERS

TO

MISCELLANEOUS EXAMPLES.

PART II.

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1. $2x^4 + (2a - 3b + 2c)\,x^3 - (3ab - 2ac + 3bc + 2b)\,x^2 - (3abc - 3b^2 + 2bc)\,x + 3b^2c.$

2. $a^{\frac{2}{3}}x^{-\frac{2}{3}} + a^{-\frac{2}{3}}x^{\frac{2}{3}} + a^{\frac{1}{3}}x^{-\frac{1}{3}} + a^{-\frac{1}{3}}x^{\frac{1}{3}}.$ **4.** 4.

5. $x = 5,\ y = 3,\ z = 1$; or $x = 4,\ y = 1,\ z = 2$.

6. $p^2 - q,\ p\,(3q - p^2),\ (p^2 - q)\,(p^2 - 3q).$

7. $\frac{2a}{n\,(n+1)},\ \frac{4a}{n\,(n+1)},\ \frac{6a}{n\,(n+1)}$, &c. **8.** 15.

9. 0 or $\frac{2a^3}{(a^2+1)\,c}$. **10.** A's share is $\frac{PR^{b+c}}{R^{a+b} + R^{a+c} + R^{b+c}}$.

11. $\frac{y^2 - (a+b)\,y + ab}{3y - (a+2b)}$. **12.** $\frac{1}{2}\left(xy + \frac{1}{xy}\right)$. **13.** $2\,(n - \frac{1}{3})$.

14. $5s$ and $4d$ per lb. **16.** 180. **17.** 2 gals, 12 gals.

18. $\sqrt[4]{\frac{1+c}{4\,(1-c)}} + \sqrt[4]{\frac{1-c}{4\,(1+c)}}$.

19. $\frac{1}{2}\{p^2 \pm p\sqrt{(p^2+4a)}\}$, where $p = \frac{1}{2}\{-a \pm \sqrt{(a^2+4b)}\}$. **20.** $\sqrt{\{a(a+c)\}}$.

21. $\frac{(a+b+c-d)\,(b+c+d-a)\,(c+d+a-b)\,(d+a+b-c)}{4\,(ab+cd)^2}$.

22. $nx^2 - 2mx + m = 0$. **24.** 1 : 3. **25.** 15. **26.** 22.

27. 4, 59, 55. **28.** 7 years. **29.** $x = \pm 2\frac{1}{2},\ y = \pm 1\frac{1}{2}$.

30. $\frac{\log 2}{\log\,(mn - m + n) - \log mn}$.

31. (i) $\pm\sqrt{\{\frac{1}{2}(a+b)\}} \pm \sqrt{\{\frac{1}{2}(a-b)\}}$; (ii) $\pm\frac{1}{2}a\,\{\sqrt{(1+n+n^2)}\} \pm \sqrt{(1-n+n^2)}\}$.

32. (i) $pr^3 = q^3s,\ q^2 = 3pr$; (ii) $p^2s = r^2,\ pq = \frac{1}{4}p^3 \pm 2r$.

33. 100 miles. **35.** 1506.

36. $\frac{1}{n}\{a^n - n\,a^{n-2} + \frac{n\,(n-3)}{1.2}\,a^{n-4} - \frac{n\,(n-4)\,(n-5)}{1.2.3}\,a^{n-6} + \text{\&c.}\}$.

37. 4. **38.** .3979400, .3521825, $\bar{1}$.3467875, $\bar{1}$.8494850, $\bar{1}$.8409595, .3737275. **39.** $x = (\sqrt{a} \pm \sqrt{b})^2,\ y = (\sqrt{a} \mp \sqrt{b})^2$.

40. $\frac{7}{2592},\ \frac{469}{5184}$. **42.** -1. **43.** $\frac{1}{2}\,(5n - 4p),\ \frac{1}{2}\,(4p - 3n)$.

44. $a^{-\frac{1}{xy}} + a^{\frac{1}{2yx}}b^{\frac{1}{2z}}$. **45.** 59 and 16, or 29 and 46.

46. $-\frac{2.1.4.7\ldots(3r-5)}{1.2.3.4\ldots\ r.3^r}\,a^{\frac{4}{3}-2r}x^{2r}$, $(-1)^r.\frac{2.5.8\ldots(3r-1)}{1.2.3\ldots\ r.3^r}\,a^{-\frac{4}{3}-2r}x^{3r}$.

48. 11046. 49. (i) 0 or $a(1\pm 2\sqrt{\frac{b}{c}})$; (ii) $x=\sqrt[3]{\frac{b^2c^2}{a}}$, &c.

50. 324 : 105 : 38. 53. 3; $1+2y^{-1}$.

54. .2385606, .1003433, .1945378, .0633450, $\bar{1}$.3902320, 1.6136032. 55. 91, 2970. 56. $1+\frac{3}{2}x+\frac{3}{8}x^2-\frac{1}{16}x^3+\frac{3}{128}x^4-$&c.

57. $p+(2m-1)q$. 58. $aR^{p-m}-bR^{p-n}$.

59. $\pm\sqrt{\frac{n}{n-2}}$ or $\pm\sqrt{\frac{n-1}{n+1}}$. 60. $\frac{1}{216}$, $\frac{125}{216}$, $\frac{75}{216}$.

61. $b(x+y)$. 62. $\frac{9}{4}a^2$, $\frac{1}{4}a^{-4}$. 63. $\frac{4m}{4m^2+1}$ a *min.*

64. 12, 84, 4. 65. $\frac{2b\sqrt{a}}{\sqrt{a}+\sqrt{b}}$, $\frac{2a\sqrt{b}}{\sqrt{a}+\sqrt{b}}$.

66. $-42a^5bc+105a^4cd^2-35a^4b^3+210a^3b^2d^2-105a^2bd^4+7ad^6$; $21a^5c^2+105a^4b^2c-420a^3bcd^2+105a^2cd^4+35a^3b^4-210a^2b^3d^2+105ab^2d^4-7bd^6$.

67. $a\{\frac{n+12}{n-p+q}+1\}$ miles. 68. $y=\frac{x^2}{a}+\frac{x^4}{a^3}+\frac{x^6}{a^5}+$&c.

69. $\pm\frac{b}{a}\sqrt{(a^2+b^2)}$. 70. $\frac{1}{2}$, $\frac{1}{2}$. 71. $\frac{2(x+1)}{x^2+x+1}$.

74. $(-1)^r.\frac{(r+1)\ (r+2)}{1.2}x^r$, $\frac{1.4.7\ldots(3r-2)}{1.2.3\ldots\ r.3^r}x^r$, $-\frac{3.1.5.9\ldots(4r-7)}{1.2.3.4\ldots\ r.4^r}a^{\frac{3}{2}-r}x^r$.

75. $x=a$ gives $\frac{2}{3}$, $x=2a$ gives $\frac{3}{4}$. 77. $\frac{ctt'}{at'+a't}$.

79. $x=\frac{a^2(a^3-b^3)}{a^4-b^4}$, $y=\frac{a^4-b^4}{a^2(a-b)}$. 80. $4\frac{4}{5}s$.

82. $(ab'-a'b)^2=(ac'-a'c)^2+(bc'-b'c)^2$. 83. $\frac{1}{7}$.

84. $\frac{3}{2(x-1)}-\frac{7}{x-2}+\frac{13}{2(x-3)}$, $\frac{1}{x^2}+\frac{1}{x}+\frac{1}{(x-1)^2}-\frac{1}{x-1}$.

86. 46.6 ft, 23.3 sq. ft. nearly.

87. $y=b+\frac{1}{3}ab^{-1}x-\frac{1}{81}a^3b^{-5}x^3+\frac{1}{243}a^4b^{-7}x^4-$&c.

88. The 88th and 91st tolls respectively; $\frac{182}{1034}$ mile.

89. $ab^{\frac{2}{3}}\div(a^{\frac{2}{3}}-b^{\frac{2}{3}})$. 90. $3\frac{3}{8}s$, $5\frac{3}{4}s$, $3\frac{5}{7}s$. 91. 40, 576.

93. $1+\frac{1}{2}ax+(\frac{3}{8}a^2-\frac{1}{2}b)x^2+(\frac{5}{16}a^3-\frac{3}{4}ab)x^3+(\frac{35}{128}a^4-\frac{15}{16}a^2b+\frac{3}{8}b^2)x^4+$&c.

97. $\lfloor n-1\ \{p_0+p_1+\text{\&c.}\}\frac{r^n-1}{r-1}$; $\frac{1}{2}(r^r-1)\lfloor r$.

98. 1 : 2; 56 : 37. 99. 0, $\frac{1}{4}\left\{a-1+\frac{1}{a-1}\right\}^2$

100. $\frac{3}{5}$. 101. $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$. 103. $\frac{a^{3n} - b^{3n}}{b^{3n-3}(a^6 - b^6)}$.

104. $\frac{1}{5}$, $\frac{5}{13}$; 1, $\frac{6}{7}$, $\frac{1}{10}$. 105. $1\frac{1}{2}$. 106. $20n^2q : m^2p$.

109. (i) 0, $\pm\sqrt{(2ab - b^2)}$; (ii) $\frac{1}{2}\{cp \pm \sqrt{(c^2p^2 + 4a^2)}\}$, where $p = \frac{1}{2}\{b \pm \sqrt{(b^2 - 4)}\}$. 110. £$2\frac{16}{47}$.

111. .6989700, $\bar{2}$.2041200, $\bar{1}$.6989700, .7958800, .2041200, 1.1938200.

113. $\frac{3.8.13\ldots(5r-2)}{1.2.\ 3\ldots\ r}\, a^{-\frac{6}{5}-r}x^r$, $(-1)^r . \frac{5.2.1.4.7\ldots(3r-8)}{1.2.3.4.5\ldots\ r}\, a^{b-r}x^r$, $(-1)^r . \frac{(r+1)(r+2)(r+3)}{1.2.3}\, a^{-4-r}x^{-4+r}$.

114. $\frac{a}{x^2} + \frac{2}{x} + \frac{2a}{(x-a)^2} - \frac{1}{x-a}$, $\frac{a^2(2x-a)}{3(x^2 - ax + a^2)} + \frac{a^3}{3(x+a)}$.

115. $5(a^4e + 4abd + 2a^3c^2 + 6a^2b^2c + ab^4)$. 117. 5.477225575

118. $m + n$ or $m + n - 1$; 1 or 0.

119. $x = \frac{1}{30}\{3 \pm 2\sqrt{3} \pm \sqrt{21}\}$; $y = \frac{1}{30}\{3 \mp 2\sqrt{3} \mp \sqrt{21}\}$.

120. A, 4s, B, 3s; 2s $7\frac{1}{2}d$.

121. 12, 48; $x = 2450m^3$, $y = 70m$; $(2n+1)^2$.

123. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{4}{11}$; 5, $5\frac{1}{3}$, $5\frac{2}{7}$, $5\frac{7}{24}$, $5\frac{72}{247}$.

125. $\frac{1}{ax} - \frac{1}{a(x-a)} + \frac{2}{(x-a)^2}$; $\frac{3}{2(x-a)^2} - \frac{1}{2a(x-a)} + \frac{1}{2(x+a)} + \frac{1}{2a(x+a)}$; $\frac{1}{3a(x-a)} - \frac{1}{ax} + \frac{2(x+2a)}{3a(x^2+ax+a^2)}$.

126. $\frac{3}{4}$, -2, $-\frac{7}{16}$. 127. $\frac{mc}{m+1} . \frac{x^2}{a^2}$, $\frac{c}{m+1} . \left(\frac{x}{b}\right)^{\frac{3}{2}}$.

129. (i) $\frac{(b-2c)^2 - ab}{a + 3b - 4c}$; (ii) $\frac{1}{2}[(1 \pm q) \pm \sqrt{\{(1 \pm q)^2 - 4(p \mp r)\}}]$, where $p = \sqrt[3]{\frac{1-c^2}{2c^4}}$, $q = \sqrt{(1+2p)}$, $r = \sqrt{(p^2 + \frac{1}{c^2})}$.

130. $4d$; 2 to 1. 132. $\sqrt{n} + \sqrt{-n}$.

133. 0, .30103, .4771213, .60206, .69897, .7781513, .845098, .90309, .9542425. 134. $z^3 - 3a^2z + 2b^3 = 0$; $a^7 - b^7 = 7az^2(a^2 - z^2)^2$.

135. $\frac{1}{2}$, $\frac{3}{7}$, $\frac{13}{30}$, $\frac{68}{157}$; $\frac{1}{8}$, $\frac{3}{25}$, $\frac{25}{208}$, $\frac{153}{1273}$; 3, $3\frac{1}{3}$, $3\frac{6}{19}$, $3\frac{19}{60}$.

138. $\frac{3}{2}r\{2n - 3r + 1\}$.

139. $x = 8$, -4, $152 \pm 16\sqrt{6}$; $y = 4$, 1, $40 \pm 16\sqrt{6}$.

140. $2\frac{2}{17}d$, 12 to 5; $4\frac{16}{17}d$, 11 to 6. 142. $\frac{2-n}{2^{n+1}}$.

143. 3, $\frac{22}{7}$, $\frac{333}{106}$, $\frac{355}{113}$; $\frac{1}{4}$, $\frac{7}{29}$, $\frac{8}{33}$, $\frac{39}{161}$. 144. $-\frac{p}{2}(1 + \frac{2b}{a})$.

145. $\frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}$. 147. $(-1)^n \cdot \frac{a - nb + n(n-1)c}{1.2.3 \ldots n}$. 148. $\frac{Aa}{a+r}$.

149. (i) $0, \frac{4n(1-n^2)}{(1+n^2)^2}$; (ii) $x = 0,\ a+b,\ \frac{1}{2}[(a-b) \pm \sqrt{\{(a-b)(a+3b)\}}]$

$y = 0,\ a+b,\ \frac{1}{2}\{(a-b) \mp \&c.\}$.

150. $\frac{13}{24}, \frac{143}{288}, \frac{3}{32}$. 151. n and $n+1$.

152. $(-2)^{\frac{n}{2}} \cdot \frac{1.3.5 \ldots (n-1)}{1.2.3 \ldots \frac{1}{2}n}$, n even; 0, n odd.

155. The 5th in both cases.

159. (i) $x = \frac{1}{a}$ or $\frac{1-3a^2}{a(a^2-3)}$; (ii) $\frac{1}{2}(a+b)$.

160. $\frac{4}{13}, \frac{5}{14}$. 162. -2. 164. $\frac{1}{3}, \frac{1}{3}$.

165. $(-1)^r(r+1)a^{-4-2r}x^{2r}$, $\frac{1.6.11 \ldots (5r-4)}{1.2.3 \ldots r.5^r} a^{-\frac{2}{5}-2r}x^{2r}$,

$\frac{3.1.1.3.5 \ldots (2r-5)}{1.2.3.4.5 \ldots r.2^r} a^{3-2r}x^{2r}$.

169. (i) $\pm\frac{2}{3}\sqrt{3}$; (ii) $\pm\sqrt{\{\frac{1}{2}(1 \pm \sqrt{5})\}}$. 170. 131 to 112.

171. .50515, .1760913, $\bar{1}$.8239087, 1.1760913, $\bar{3}$.7323939,
1.1583626, .2552726, .9084852. 174. $1, \frac{81}{20}a^2$.

173. $\frac{(n-m+1)a + mc}{n+1}$, $\sqrt[n+1]{(a^{n-m+1}c^m)}$, $\frac{(n+1)ac}{ma + (n-m+1)c}$.

175. $\frac{1.5.9 \ldots (4r-3)}{1.2.3 \ldots r.4^r} a^{-\frac{1}{2}-2r}x^{2r}$, $-\frac{1.2.5.8 \ldots (3r-4)}{1.2.3.4 \ldots r.3^r} a^{\frac{2}{3}-r}x^r$,

$(-1)^{r-1} \cdot \frac{2.3.8 \ldots (5r-7)}{1.2.3.4 \ldots r} a^{\frac{2}{5}-r}x^r$.

178. $n-1$, £nP. 179. $x = 3, -2$; $y = -2, 3$.

180. £3 7s $2\frac{2}{3}d$.

182. $r = A \cdot \frac{2a - a'}{a^2}$; $n = \frac{\log a - \log(a' - a)}{\log(1+r)}$.

183. 3, 4, 5, 6, 7; or $-6, -\frac{1}{2}, 5, 10\frac{1}{2}, 16$.

184. $\frac{(r+1)(r+2) \ldots (r+5)}{1.2 \ldots 5} a^{-2-\frac{r}{3}}x^{-\frac{r}{3}}$, $(-1)^r \cdot c^{-1-\frac{r}{3}}x^{\frac{r}{3}}$.

185. $\frac{2}{x} - \frac{1}{2(x+1)^2} - \frac{5}{4(x+1)} + \frac{1}{(x-1)^2} - \frac{3}{4(x-1)}$,

$\frac{1}{3(x-1)} - \frac{1}{x+1} - \frac{x-1}{2(x^2-x+1)} + \frac{7x-1}{6(x^2+x+1)}$.

186. 384. 187. $\{pa \pm (p+mq)b\}(a \pm b)^{m-1}$.

189. $ab^{\frac{2}{3}} \div (a^{\frac{2}{3n}} + b^{\frac{2}{3n}})^n$. 190. $\frac{621}{4096}$. 193. $x^{-\frac{1}{3}}$. 194. 1.537.

195. $x + (a-b)\,x^3 + (a^2 - ab - c + d)\,x^5 + (a^3 - a^2b - 2ac + ad + bc)\,x^7$
$+ \{a^4 - a^3b + a^2(b - 2c + d) + a(c-d)\}\,x^9 + \&c.$

196. $\pm a^{\frac{n^2+1}{n^2-1}}$. 198. $\dfrac{(aq^n - bp^n)\,r^m + (bp^m - aq^m)\,r^n}{p^mq^n - p^nq^m}$.

199. $x = 3,\ 2,\ \frac{1}{6}(15 \pm \sqrt{-309})$; $y = \&c.$ 200. $\frac{64}{175}, \frac{48}{175}, \frac{36}{175}, \frac{27}{175}$.

202. $\dfrac{1.3.5 \ldots (2r-1)}{1.2.3 \ldots\ r.2^r}\,x^{2r}$, $-\dfrac{7.4.1.2.5 \ldots (3r-10)}{1.2.3.4.5 \ldots\ r.3^r}\,a^{\frac{28}{3}-2r}x^{2r}$,

$\dfrac{1.4.7 \ldots (3r-2)}{1.2.3 \ldots\ r.3^r}\,a^{-\frac{1}{3}-r}x^r$. 204. $\dfrac{P}{1-R^{-n}}(R^{-p} - R^{-q})$.

203. .3010380, $\bar{2}$.3010458; 2000.043, 2.000058, .02000082.

205. $3\frac{2}{3}d$. 207. $x^{\frac{1}{2}} + y^{\frac{1}{2}} = z^{\frac{1}{2}}$. 210. $\frac{1}{6}, \frac{2}{3}$. 213. .3053377.

214. $\dfrac{1}{ax^2} + \dfrac{b}{a^2x} - \dfrac{bx + a - a^2c}{a^2(x^2 + a^2)}$. 215. 432 : 625.

216. $am + \dfrac{c}{m}$. 217. $\dfrac{\log(n+2)}{\log 2}$.

219. $\dfrac{1}{24} - \dfrac{(-1)^n}{12(n+1)(n+2)}$; $\dfrac{n}{3(n+1)} - \dfrac{n}{12(n+2)} - \dfrac{n}{18(n+3)}$.

220. $2 - \dfrac{n+2}{2^n}$ shillings. 221. $a^{\frac{mn}{m+n}} + b^{\frac{mn}{m+n}} + c^{\frac{mn}{m+n}} = k^{\frac{mn}{m+n}}$.

222. 50, 50. 223. $1 + 3 + 5 + \&c.$ 225. .99144.

226. $\dfrac{m^4a^2}{(m^2+2)^2}, \dfrac{4m^2a^2}{(m^2+2)^2}, \dfrac{4a^2}{(m^2+2)^2}$; 64+16+1. 227. $\dfrac{\log 2(n+1)}{\log 2}$.

229. $\dfrac{2^{n+1}-1}{n+1}$, $\frac{1}{12}n(n+1)(n+2)(3n+5)$. 233. $\frac{2}{5}, \frac{3}{7}, \frac{4}{9}$.

234. $\dfrac{1}{2(x+1)^2} + \dfrac{4}{x+1} - \dfrac{8x-7}{2(x^2+1)}$; $\dfrac{1}{(x+1)^2} + \dfrac{8}{x+1} - \dfrac{8x+1}{x^2+x+1}$.

237. $\dfrac{(a-b)^n}{a^{n-1}}$ gallons. 240. $4\frac{1}{2}d$, 7*s* to 1*s*.

243. 68, 888, 10456. 244. 1, 3, 5, &c. 245. $6^6 : 22.5^5$.

247. 2, -1, or $\frac{1}{2}(5 \pm \sqrt{17})$. 248. $x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \&c.$

249. $x = 3,\ y = 4$. 250. $23\frac{1}{9}s$ in each case.

253. $\frac{110}{243}a^{-14}b^{12}$, $\frac{374}{1029}$, $1\frac{1}{7}$, $-10830x^4$. 255. 21*s* in each case.

257. $\frac{2}{3}(m^2-1)$, $\frac{1}{3}(m^2+2)$, $\frac{1}{36}(m^2-1)(m^2-25)$: 80, 41, 320.

259. The 2nd $= -44\sqrt{2}$. 260. Each $\frac{3}{7}$; $\frac{36}{343}$; $\frac{27}{343}$. 261. 1.

262. $(2x + y - 3)(x - 11y + 1)$. 265. £1 7*s* 3*d*, £1 2*s*.

266. 5455, $\frac{1}{66}\{4.10^x - 15(-1)^x + 11\}$.

267. 360, 519840, 57168810, &c. 268. $\frac{1}{702}$.

269. $(n-1)P\left\{\left(\frac{n}{n-1}\right)^n+(n-1)\right\}$. 270. $\frac{1}{60}$. 271. $\sqrt{\frac{\log p}{\log a}}$.

273. $\frac{1}{6}n(n^2+11)$. 274. 5; $5x+4$. 275. $\frac{1}{2n-3}$.

278. $\frac{1}{12}-\frac{(-1)^n}{4(2n+3)}$; $\frac{1}{12}$. 280. £34 14*s* $5\frac{1}{4}$*d*.

281. 1; $x=a+2c$, $y=b+3c$.

282. $\sqrt{x^m}-\frac{1}{2}\sqrt[np]{(b^px^2)}$; $(a^2+b^2)(c^2+d^2)$. 284. $\left(\frac{a\pm b}{a}\right)^2$.

286. $\frac{1}{4}$, $\frac{5}{4}$.

287. $\frac{1}{3}\cdot\frac{(2x)^n-1}{2x-1}-\frac{2}{3}\cdot\frac{(-x)^n-1}{x+1}$, $\frac{x-1}{(x+1)(2x-1)}$, $\frac{2}{3}\{2^{n-1}+(-1)^n\}$.

288. From the r^{th}, where $r>x$. 290. $\frac{1}{82}$.

292. $x=\frac{1}{2}=y$; $\frac{a(\sqrt{3}-\sqrt{2})+2b}{2a\sqrt{6}+b(\sqrt{2}-\sqrt{3})}$.

293. $a^2+4ac=b^2$. 294. $x=3$, $y=4$.

298. $x=m^2(m^4+1)^2-1$, $y=4m^4(m^4-1)$; if $m=2$, $x=1155$, $y=960$.

302. The 17^{th}. 303. 8 and 9. 304. $\frac{1+4x+x^2}{(1-x)^4}$. 305. $\frac{24}{270725}$.

306. $\frac{\lfloor n\, p^n}{m(m+p)\ldots(m+np)}$. 309. $\frac{1}{4}-\frac{1}{4(2n+1)3^n}$, $\frac{1}{4}$.

313. $\frac{p}{\sqrt{(2p^2-1)}}$. 314. $a^3+b^3+c^3+abc=0$. 315. $\frac{53}{140}$, $\frac{13}{28}$.

318. $\frac{1}{2}-\frac{1}{(n+1)(n+2)}$, $\frac{1}{2}$; $\frac{3}{2}-\frac{n+3}{(n+1)(n+2)}$, $\frac{3}{2}$.

319. $x=a^{\frac{n-1}{2}}\{p^n-\frac{n(n-1)}{1.2}p^{n-2}q^2\frac{b}{a}+\&c.\}$, $y=a^{\frac{n-1}{2}}\{np^{n-1}q-\&c.\}$.

324. 10 and 6. 325. $\sqrt{2}-1:2\sqrt{3}$. 327. 6, $1\frac{1}{2}$, $\frac{5}{54}$.

328. 30. 329. $\frac{x}{a}+\frac{y}{b}=\sqrt{2}$. 330. $\frac{x^p(x^{q-p+1}-1)}{(q-p+1)(x-1)^2}-\frac{1}{x-1}$.

335. $\frac{n^2(n+1)^2}{2(n^2+1)(n^2+2)}s$. 339. $\frac{n}{12(n+1)}$, $\frac{1}{12}$.

337. $x=4, 11, 20, 31$, &c, $y=9, 11, 13, 15$, &c.

340. 245 : 243. 346. $x=\frac{1}{2}\{-1\pm\sqrt{5}\}$ or $\frac{1}{2}\{1\pm\sqrt{-7}\}$. 350. $\frac{1}{16}$.

354. $\frac{ab+ac+bc}{(a+c)(b+c)}$.

ANSWERS

TO

THE EQUATION PAPERS.

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1. 1. 35. 2. $x = 9,\ y = 2.$ 3. $3,\ -\frac{4}{5}.$
4. $x = \pm 12,\ \pm 9\frac{1}{2};\ y = 2,\ -1\frac{7}{12}.$ 5. 100. 6. 26. 7. 11 : 1.

2. 1. 72. 2. $x = 7,\ y = 4.$ 3. $5,\ 6\frac{9}{10}.$
4. $x = 5,\ -4\frac{3}{5};\ y = 3,\ -2\frac{19}{25}.$ 5. 180,000.
6. 10. 7. 25 miles.

3. 1. 51. 2. $x = 18,\ y = 24.$ 3. $6,\ 3\frac{1}{13}.$
4. $x = 0,\ 4;\ y = 9,\ 25.$ 5. 22 miles.
6. 18 chains, 30 chains, 21 chains. 7. 10.

4. 1. 3. 2. $x = \frac{4}{5}a,\ y = \frac{5}{4}a.$ 3. $4,\ \frac{1}{9}.$ 4. $x = 1\frac{1}{4};\ y = \pm 2.$
5. 10*s*. 6. 79 days. 7. 120 gallons.

5. 1. 7. 2. $x = 5,\ y = 6.$ 3. $4,\ -\sqrt[3]{49}.$
4. $x = \pm 9,\ \pm 11\frac{1}{4};\ y = \pm 4,\ \pm 3\frac{1}{5}.$ 5. 1742.
6. £700, £100. 7. 50 ft, 20 ft, 48 ft.

6. 1. 6. 2. $x = 1,\ y = 2,\ z = 4.$ 3. $x = (a^{-\frac{1}{2}} + b^{-\frac{1}{2}})^{\frac{4mn}{m-n}}.$
4. $x = 2$ or -1; $y = 1.$ 5. A mile. 6. £1071 17*s* 6*d*.
7. $\frac{c}{1-r}\left\{1 + \frac{n(n-1)(1-r)^2\{p(1-r)-1\}}{2r\{n(1-r)-(1-r^n)\}}\right\}.$

7. 1. 8. 2. $x = \frac{1}{2},\ y = \frac{1}{3},\ z = \frac{1}{4}.$ 3. $4,\ \frac{196}{841}.$
4. $x = 2,\ \frac{1}{18},\ \frac{1}{32};\ y = 1,\ \frac{1}{21},\ \frac{1}{36}.$ 5. 432.
6. 7 miles, $9\frac{1}{3}$ miles. 7. £96.

8. 1. 4. 2. $x = \pm 4\frac{1}{2},\ \pm 3\frac{1}{2};\ y = \pm 3\frac{1}{2},\ \pm 4\frac{1}{2}.$
3. $\left(\frac{a+b}{a-b}\right)^{\frac{2pq}{p\sim q}}.$ 4. $x = 2,\ -2,\ \frac{34}{81};\ y = 2,\ 6,\ \frac{1446}{961}.$
5. £60. 6. 60 miles. 7. 4 : 5.

9. 1. $3\frac{3}{8}.$ 2. $x = 3,\ y = 2.$ 3. $\left(\frac{a^{2b}+1}{a^{2b}-1}\right)^{\pm 2}.$ 4. $x = 5,\ y = 3.$
5. £2000. 6. £5 5*s* *per annum.* 7. 60 miles.

10. 1. 11. 2. $x = 16,\ y = 25.$ 3. $3,\ \frac{3}{16}.$
4. $x = 6,\ 4,\ 0;\ y = 0,\ 2,\ -2.$ 5. 72, 12.
6. 6 miles per hour. 7. 432.

11. 1. $1\frac{1}{2}.$ 2. $x = \pm\sqrt{(ac)}$ or $\frac{1}{2}[(a+c-b) \pm \sqrt{\{(a+c-b)^2 - 4ac\}}]$;
$y = \pm\sqrt{(bc)}$ or $\frac{1}{2}[(b+c-a) \pm \sqrt{\{(b+c-a)^2 - 4bc\}}].$

3. $x=\pm\frac{1}{2}$. 4. $x=\pm\frac{8}{27}$ or $\pm\frac{1}{8}$; $y=\pm\frac{1}{8}$ or $\pm\frac{8}{27}$.
5. 11 o'clock. 6. The number of the tack is the integer equal to or next greater than $\frac{2q}{p}-\frac{2r}{3p}$. 7. 2 hrs.

12. 1. 4. 2. $x=5, 1, \frac{1}{29}(15\pm 6\sqrt{-1})$; $y=3, \frac{3}{5}, \frac{1}{29}(25\pm 10\sqrt{-1})$.
3. $\left\{\frac{a^{\frac{r}{s}}+1}{a^{\frac{r}{s}}-1}\right\}^{\frac{1}{(m+n)^2}}$. 4. $x=4, -2\pm 2\sqrt{-3}$; $y=\frac{1}{4}, -\frac{1}{8}(13\pm 3\sqrt{-3})$.
5. 150. 6. 3 miles an hour. 7. £27.

13. 1. 4. 2. $x=4$ or $\frac{4}{17}$; $y=12$ or $1\frac{17}{19}$.
3. 1, 16, or $\frac{1}{2}(1\pm 3\sqrt{-7})$.
4. $x=1$ or $1\pm\sqrt{-2}$; $y=4$ or -2.
5. 3 miles. 6. £64,000,000. 7. 24 : 21 : 22.

14. 1. $x=21$; $y=20$. 2. $\sqrt[3]{(\frac{9}{4}a^2b-b^3)}$.
3. $\pm\sqrt{[\frac{1}{2}\{p\pm\sqrt{(p^2-4)}\}]}$, where $a^2p=-2\{1\pm\sqrt{(1-a^4)}\}$.
4. $x=\pm\frac{1}{2}\{(4\pm\sqrt{6})\pm\sqrt{(18\pm 8\sqrt{6})}\}$;
$y=\pm\frac{1}{2}\{(2\pm\sqrt{6})\pm\sqrt{(18\pm 8\sqrt{6})}\}$.
5. £20. 6. 2 : 3. 7. 20*O*., 40*S*., 400*P*.

15. 1. $-\frac{1}{2}, -\frac{3}{4}$. 2. $x=(\sqrt{2}+1)^2$ or $(\sqrt{2}-1)^2$; $y=1$ or $(\sqrt{2}-1)^4$.
3. $\frac{1}{2}a$ or $\frac{1}{6}(-5\pm\sqrt{37})a$. 4. $x=4$ or $\frac{1}{16}$; $y=9$ or $\frac{9}{64}$.
5. £240. 6. $54\frac{5}{6}$ miles. 7. *A*, 30; *B*, 163; *C*, 230.

16. 1. 6. 2. $x=4$; $y=3$. 3. $9\pm 4\sqrt{7}$, $\frac{1}{2}(3\pm\sqrt{13})$.
4. $x=\sqrt{\frac{m^2-1}{m^2+1}}$, $y=\sqrt{\frac{n^2-1}{n^2+1}}$, where $n^2=\frac{1}{2}\{1\pm\sqrt{3}\pm\sqrt{\pm 2\sqrt{3}}\}$, $m^2=\sqrt{\frac{n^4+1}{n^4-1}}$. 5. $4\frac{1}{2}$ A.M. 6. 360 yds. 7. £18, £32.

17. 1. $1\frac{1}{13}$. 2. $x=[b^n\{a^{\frac{1}{2}(m-n)}\pm\sqrt{(a^{m-n}-b^{m-n})}\}]^{\frac{2}{m+n}}$;
$y=[a^m\{b^{\frac{1}{2}(n-m)}\mp\sqrt{(b^{n-m}-a^{n-m})}\}]^{\frac{2}{m+n}}$.
3. $4\pm\sqrt{6}$, $\pm\sqrt{-2}$. 4. $x=y=\pm a\sqrt{2}$.
5. 1080 yds; $16\frac{1}{2}'$. 6. 8, 12. 7. £30600.

18. 1. 3. 2. $x=1\frac{1}{2}$, $y=1\frac{1}{3}$, $z=1\frac{1}{4}$.
3. -1, $\frac{1}{2}\{p\pm\sqrt{(p^2-4)}\}$, where $p=\frac{-(4a+1)\pm\sqrt{\{5(4a+1)\}}}{2(a-1)}$.
4. $x=\frac{a-1}{b}$, $y=\{1+\sqrt{(2a-1)}$.
5. 6 hrs, 3 hrs. 6. 3 acres. 7. 693, 688, 736.

19. 1. $2\frac{2}{5}$. 2. $x = \pm\frac{3}{2}$ or $\pm\frac{3}{2}\sqrt{-1}$; $y = \pm\frac{1}{2}$ or $\mp\frac{1}{2}\sqrt{-1}$.
3. $x = \frac{2}{3}$, $\pm\frac{4}{3}\sqrt{2}$, $\frac{4}{9}(-2 \pm \sqrt{-14})$.
4. $x=0$ or 1, $y=\pm\sqrt{a}$; or $x=\frac{1}{2}\{1\pm\sqrt{(1+4p)}\}$, $y=\pm\sqrt{\left(\frac{a}{b}(b-p)\right)}$,
where $p = \dfrac{\{2a^2 - (a-1)b\}b}{a^2 - ab + b^2}$.
5. $10\frac{1}{2}$ minutes. 6. 2*s* 1*d*. 7. 3300 gals, 1800 gals.

20. 1. 3. 2. $x = 1 + a$; $y = \frac{1}{2}\{\sqrt{(1 - a + a^2)} + \frac{1}{2} - a\}$;
$z = \frac{1}{2}\{\sqrt{(1 - a + a^2)} - \frac{1}{2} + a\}$.
3. $4\sqrt{2}$ 4. $x = \frac{1}{2}\{\sqrt{(\sqrt[3]{3} + 3)} \pm \sqrt{(\sqrt[3]{3} - 1)}\}$;
$y = \frac{1}{2}\{\sqrt[3]{3}\sqrt{(\sqrt[3]{3} + 3)} \pm \sqrt{(3\sqrt[3]{9} - 1)}\}$.
5. Money £672, debts £840. 6. 15′. 7. 10.

21. 1. $-2, \frac{1}{3}$. 2. $x=\pm 1, 2, -2\sqrt[3]{4}$; $y=\pm 1, 2, -\sqrt[3]{2}$; $z=4, 1, 1$.
3. $1, -3, -\frac{1}{2}$. 4. $x=\pm\sqrt{(\frac{8}{13}+\frac{2}{13}\sqrt{3})}$; $y=\pm\sqrt{(\frac{11}{13}+\frac{6}{13}\sqrt{3})}$.
5. 1600. 6. 6. 7. 16 miles.

22. 1. ± 1 or ± 8. 2. $x=6$ or $3\frac{1}{3}$, $y=5$ or $6\frac{1}{3}$, $z=3$ or $4\frac{1}{2}$.
3. $\pm\dfrac{a}{2}\left\{\sqrt{\dfrac{1+2n}{n}} \pm \sqrt{\dfrac{1-2n}{n}}\right\}$ or $\pm\dfrac{a}{2}\left\{\sqrt{\dfrac{5n-3}{n-1}} \pm \sqrt{\dfrac{n+1}{n-1}}\right\}$.
4. $x^3 = (a^2 - 2bc - 2b^2)\{c \pm \sqrt{(c^2 - a^2 + 2bc + 2b^2)}\}$;
$y^3 = (a^2 - 2bc - 2b^2)\{c \mp \&c.\}$.
5. 195. 6. 11, 18500, 1789. 7. 12.

23. 1. $\frac{65}{66}$. 2. $x = 3$, $y = -2\frac{1}{2}$. 3. $3, -\frac{1}{3}, \frac{1}{3}(4 \pm \sqrt{-2})$.
4. $x = \pm\frac{1}{2}\sqrt{5}$ or 0, $y = \pm\sqrt{\frac{3}{2}}$ or $\pm\sqrt{-1}$;
$x = \pm\frac{1}{2}\sqrt{\{\frac{1}{2}(1 \pm \sqrt{33})\}}$, $y = \pm\frac{1}{2}\sqrt{(5 \pm \sqrt{33})}$.
5. 84 miles. 6. $1\frac{1}{2}$ in., $\frac{1}{4}$ in. 7. £180; 44 : 35.

24. 1. 2 or 3. 2. $49, 64, \frac{1}{2}(93 \pm \sqrt{185})$.
3. $\pm 1\frac{1}{6}$, $\pm 1\frac{13}{14}$, $-1\frac{1}{2}$, $\pm\frac{3}{2}\sqrt{-1}$.
4. $x = \pm\frac{1}{2}a$ or $\pm a\sqrt{-2}$; $y = \pm\frac{1}{2}a$ or $\mp a\sqrt{-\frac{1}{2}}$.
5. £400. 6. 41 cub. in. 7. 20 miles.

25. 1. 3. 2. $\pm\sqrt{\{a^2 - (a-b)^2\}}$ or $\pm\sqrt{\{a^2 - (a+3b)^2\}}$.
3. $x = \frac{7}{2}(3 \pm \sqrt{33})$; $y = \frac{7}{2}(3 \mp \sqrt{33})$.
4. $x=\frac{1}{2}a(1\pm\sqrt{3})$; $y=\frac{1}{6}a\sqrt{(36\pm 22\sqrt{3})}$. 5. $31\frac{1}{2}$.
6. $\dfrac{m'}{(a-a')+n(c'-c)}$, $\dfrac{m'n}{(a-a')+n(c'-c)}$. 7. 100.
