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Jan K. Brueckner, Assistant Professor of Economics

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January 23, 1978

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Summary:

This paper analyzes the incidence of the property tax in a general equilibrium model under four different mobility assumptions for economic agents. The model has workers, landowners, and entrepreneurs who produce bread or housing, and two cities. An increase in the property tax rate in one city usually benefits housing producers at the expense of other agents in the economy.

Mobility and General Equilibrium Incidence of the Property Tax

by Jan K. Brueckner[1]

I. Introduction

In Mieszkowski's well-known 1972 paper, modern tax incidence theory, as enunciated by Harberger (1962), was first applied to analysis of the incidence of the property tax. The paper's unconventional conclusions have now become accepted doctrine, embodying what has been referred to as the "new view" of the property tax.[2] The main idea of the new view is that an increase in the property tax rate in a community has two effects: first, it depresses the rate of return on capital in the economy as a whole; second, it increases the gross price of capital in the given community relative to its price elsewhere.

The present paper reflects dissatisfaction with the previous analysis on three grounds. First, the mobility assumption implicit in Mieskowski's analysis is that capital, but not labor, is mobile among communities. This is hardly an appropriate assumption for long-run analysis, in which all economic agents must be freely mobile. Our paper remedies this deficiency by analyzing the incidence of the property tax under various mobility assumptions appropriate to the short, intermediate, and long run. Our goal is to discover how the incidence of the tax depends on the mobility of agents in a properly constructed model of the economy.

Our second criticism of Mieszkowski's analysis is really a criticism of the underlying Harberger model. In that model, the stock of capital in the economy is assumed to be fixed, which Harberger justified by

appealing to the empirical observation that aggregate saving is unresponsive to the rate of interest. A true general equilibrium model must make the capital stock endogenous, and we should be suspicious of theoretical results which do not incorporate this endogeneity.

A related criticism of the Harberger framework concerns the migration equilibrium condition, which states that rates of return on capital should be equal in all communities. Our view is that the correct equilibrium condition states that the utility level of the owners of capital, not the rate of return on it, should be uniform throughout the economy. The difference between these conditions is obvious when it is realized that the prices of consumer goods will vary across communities due to differences in property tax rates. Equal returns will not make owners of capital indifferent to where they locate when the prices of the goods they consume vary across communities.

In this paper, we analyze a stylized model of the economy which does not suffer from the above defects. Our principal innovation is the assumption of a fixed number of entrepreneurs in the economy rather than a fixed stock of capital. Each entrepreneur uses factors of production to produce an output according to a given production function. The entrepreneur's endowment is his entrepreneurial skill; other economic agents do not have the ability to manage production. Since entrepreneurship takes the place of the capital input in Harberger's analysis, production in our model does not use intermediate goods (capital). The other economic agents, workers and landowners, are endowed respectively with identical labor skills, which are supplied inelastically to producers,

and with land, which is also a factor of production. The analysis does not require that we specify how much land each owner controls.

Our economy has two cities with equal fixed land areas, ℓ (they can be thought of as islands). Two kinds of entrepreneurs exist: bread producers and housing producers. Entrepreneurial skills are not transferable between production processes; individuals endowed with the ability to produce housing are incapable of producing bread, and vice versa. The four types of economic agents all have the same utility function, which depends on the consumption of bread and housing. We assume that labor and land are the sole inputs into bread and housing production, respectively. The use of one-factor production functions eliminates factor substitution as a response to changes in property tax rates. While it would be easy to write down equilibrium conditions for a model where both land and labor are inputs to housing and bread production, analysis of such a model would be prohibitively difficult . We view our model as plausible and suggestive, and feel that it represents a reasonable framework for analysis of tax incidence in a general equilibrium setting.

It is also assumed that bread, which is numeraire, may be traded between cities at zero cost, but that housing is a non-traded good. In addition, it is assumed that workers and entrepreneurs are potentially mobile at zero cost between cities, but that landowners may never move. This is made plausible by the fact that the endowments of workers and entrepreneurs are mobile, while the endowment of landowners is not. Migration equilibrium will be characterized by equal utility levels in both cities for those agents which are assumed to be mobile. This does not mean that

the profits of mobile entrepreneurs will be the same in both communities (as in Harberger), because the price of housing will also be an argument of the indirect utility function.

The strategy of the analysis will be to perform comparative static calculations on the general equilibrium solution of the model under four mobility assumptions: complete immobility (short run), producer immobility (intermediate run), worker immobility (Mieszkowski's case), and full mobility (long run). The tax rate change we impose is an increase in the property tax rate in city 1 matched by a lump-sum rebate of the tax revenues to housing producers, who we assume pay the tax. This change is equivalent to reducing an existing head tax on housing producers while increasing property tax payments such that tax revenue is unchanged. Indeed, we can imagine that a uniform head tax, levied on all the agents in the economy, is used to support public expenditures which are uniform across and within cities. The property tax rate increase accompanied by the lump-sum rebate merely changes the way in which the fixed amount of public expenditure is financed. Without loss of generality, we set the level of the pre-existing head tax and the level of public expenditure equal to zero to analyze the incidence question. We assume no property tax is levied in city 2.

To facilitate analysis of the model, specific functional forms are imposed. The production function for bread producers is $f(L) = L^{\alpha}$, $0 < \alpha < 1$, where L is labor input. The production function for housing producers is $h(\ell) = \ell^{\theta}$, $0 < \theta < 1$, where ℓ is land input. The utility function is Cobb-Douglas, which means (remembering that bread is numeraire) that the indirect utility function is proportional to $Ip^{-\beta}$ where I is income, p is the housing price, and β is the exponent of

housing in the utility function divided by the sum of the exponents. Demand functions for bread and housing are $(1-\beta)I$ and $\beta I/p$ respectively. In the appendix, analysis of models with a general utility function and unspecified concave production functions is undertaken, but results are incomplete.

In the next section, comparative-static analysis of the general equilibrium solutions is performed. Subscripts on variables refer to the cities, 1 or 2. Variables are wages, w_i; bread profits, π_{ci}; labor inputs, L_i; land rents, r_i; housing profits, π_{Hi}; land inputs, ℓ_i; housing prices, p_i; property tax rates, τ_i ($\tau_2 \equiv 0$); housing entrepreneurs, E_{Hi}; and bread entrepreneurs, E_{ci}. In the analysis, we ignore the requirement that the L_i, E_{ci}, E_{Hi} be integer-valued.

II. Analysis

Equilibrium conditions which must hold with perfect competition under all mobility assumptions are as follows for i=1, 2:

$$\alpha L_i^{\alpha-1} = w_i \tag{1}$$

$$\pi_{ci} = (1-\alpha)L_i^{\alpha} \tag{2}$$

$$p_i(1-\tau_i)\theta \ell_i^{\theta-1} = r_i \tag{3}$$

$$\pi_{Hi} = (1-\theta(1-\tau_i))p_i \ell_i^{\theta} \tag{4}$$

$$E_{Hi}\ \ell_i^{\theta} = \frac{\beta}{p_i}(E_{ci}L_i^{\alpha} + \ell p_i \ell_i^{\theta-1}) \tag{5}$$

$$E_{c1}L_1^{\alpha} + E_{c2}L_2^{\alpha} = (1-\beta)(E_{c1}L_1^{\alpha} + E_{c2}L_2^{\alpha} + \ell p_1 \ell_1^{\theta-1} + \ell p_2 \rho_2^{\theta-1}) \tag{6}$$

Condition (1) states that the labor inputs of bread producers are profit -maximizing in both cities, and (2) gives the maximized value of profit. Condition (3) says that the land inputs of housing producers are profit -maximizing in both jurisdictions. Condition (4) incorporates the assumption that the property tax revenue is returned to housing producers in a lump-sum grant. The value of the lump-sum grant is $\tau_i p_i \ell_i^{\theta} = s_i$ and profit is $p_i(1-\tau_i)(1-\theta)\ \ell_i^{\theta} + s_i$, which equals (4). Equation (5) states that the supply of housing in each city equals the demand for it. Aggregate city income is $E_{ci}\pi_{ci} + E_{ci}L_i w_i + E_{Hi}\pi_{Hi} + r_i \ell$, which, in view of (1)-(4) reduces to the expression in parentheses in (5). Since bread is traded between cities, total output must equal total demand in both cities, a condition which is

expressed in (6). Further conditions are needed to close the model, but the form of these conditions depends on the particular mobility assumptions. In the next four sections, we present the additional conditions and the solutions for each mobility case.

A. Complete Immobility

When bread and housing entrepreneurs and workers are immobile, equilibrium may involve unequal utility levels for a given agent in the two cities. For simplicity, we assume equal division of the populations of the agents between cities. The conditions which emerge from this assumption are

$$\ell_1 = \ell_2 = \frac{2\ell}{E_H} \tag{7a}$$

$$L_1 = L_2 = \frac{N}{E_c} , \tag{8a}$$

where E_H and E_c are the total numbers of housing and bread entrepreneurs, respectively, in the economy, and N is total worker population. The land input of a housing producer is equal to total land area, ℓ, divided by the number of producers, $E_H/2$, yielding (7a). The labor input of a bread producer is the size of the labor force, N/2, divided by the number of producers, $E_c/2$, yielding (8a). The system (1)-(6), (7a), (8a) has fifteen equations, while the unknowns L_i, ℓ_i, w_i, r_i, p_i, π_{ci}, π_{Hi} are fourteen in number. A general equilibrium system where no numeraire has been specified has one redundant equation by Walras' law, which means that the level of prices is indeterminate. Specifying the numeraire eliminates one unknown, but the redundancy of the extra equation means the system is not overdetermined. In our system, a solution which

satisfies any fourteen equations necessarily satisfies the fifteenth. Recall that we assume $\tau_2=0$ (no property tax is levied in city 2) and analyze the effect of increases in τ_1.

Analysis of the complete immobility case is particularly simple. Solutions for ℓ_i and L_i are given directly by (7a) and (8a), w_i and π_{ci} are given by (1) and (2), and the p_i are given by (5). None of these solutions depends on τ_1, and since populations are symmetrical, the solutions for each of these variables are the same in both cities. From (3) and (4), r_2 and π_{H2} do not depend on τ_1. But (3) and (4) also show that $\partial r_1/\partial\tau_1 < 0$ and $\partial\pi_{H1}/\partial\tau_1 > 0$. Land rent decreases and housing profits increase in city 1 when τ_1 is increased. Since utility levels of the agents depend only on income and the housing price, we see immediately that in city 1, the landowners' utility level falls and the housing producers' utility level rises when the property tax rate increases, while other agents are unaffected. The same result holds in the general case (see appendix).

In this example, city 2 is insulated from the effect of the tax change because all agents are immobile. The city 1, bread producers' profits and wages are independent of τ_1 because the fixed work force and fixed number of bread producers yield labor input per firm which is independent of τ_1. Since land use by housing producers is also determined directly by the fixed number of producers, aggregate income in (5), and hence the price of housing, is independent of τ_1. Thus, the effects of a change in τ_1 are felt only through the solutions for π_{H1} and r_1 in (3) and (4).

B. Producers Immobile

While complete immobility is a short-run assumption, our view is that the appropriate intermediate-run assumption makes producers immobile while allowing workers to move. In reality, it appears that individuals change locations more easily than firms, which means that in the intermediate run, it is appropriate to treat producers as immobile.

Since housing producers are immobile, the solution for ℓ_1 and ℓ_2 is the same as in the previous case:

$$\ell_1 = \ell_2 = \frac{2\ell}{E_H} . \tag{7b}$$

However, the labor force of each city is now endogenous, so the full -employment condition becomes

$$\frac{Ec}{2} (L_1 + L_2) = N , \tag{8b}$$

which states that when we add the numbers of workers in each city, which equal $E_c L_i/2$ since bread entrepreneurs are divided equally, the sum equals total worker population. Since workers are mobile, their utility levels must be the same in both cities in equilibrium. From the Cobb-Douglas indirect utility function, this requires

$$w_1 p_1^{-\beta} = w_2 p_2^{-\beta} . \tag{9b}$$

As in the previous case, this model has fifteen equations and fourteen unknowns, and as before, the solution is relatively simple. Equation (7b) gives ℓ_1 and ℓ_2, and using (1) and (5) in conjunction with this solution gives

$$p_i \frac{E_H}{2} \left(\frac{2\ell}{E_H}\right)^{\theta} \frac{1-\beta}{\beta} = \frac{E_c}{2} L_i^{\alpha} = \frac{E_c}{2} \left(\frac{w_i}{\alpha}\right)^{\alpha/\alpha-1} \tag{10b}$$

which implies that p_i is proportional to $w_i^{\alpha/\alpha-1}$. Using (9b), this fact implies that w_1=w_2, which means L_1=L_2. From (8b) we get L_1=L_2=N/E_c, which gives w_i, and the (equal) π_{ci} and p_i are given by (2) and (10b). In city 2, (3) and (4) give r_2 and π_{H2}. None of the above solution values depends on τ_1. However, we see from (3) and (4) that $\partial r_1/\partial\tau_1<0$ and $\partial\pi_{H1}/\partial\tau_1>0$, and since p_1 does not change, the utility levels of landowners and housing producers in city 1 fall and rise respectively; the outcome is exactly the same as in the complete immobility case. In fact, the worker mobility assumption does not lead to any migration in this model. The equal-wage result yields equal labor inputs by bread producers, which, since producers are evenly divided, implies equal division of the work force regardless of the level of τ_1. Thus city 2 is insulated from the effect of changes in τ_1, and since aggregate income in city 1 is independent of τ_1, the price of housing is independent of τ_1 and effects of a change in the tax rate are felt only through (3) and (4). Similar results hold in the general case, as shown in the appendix.

C. Workers Immobile

In our view, a model with mobile producers and immobile workers does not realistically represent differential mobility over any time horizon. We analyze this model, however, because it was analyzed by Mieszkowski and implicitly underlies the "new view" of the property tax. Our results surprisingly contradict Mieszkowski's conclusion that the level of the price of housing in a city is positively related to its property tax rate.

The distribution of entrepreneurs is endogenous under our assumptions, and the additional equilibrium conditions are

$$\ell_1 = \ell/E_{H1} \tag{7c}$$

$$\ell_2 = \ell/E_{H2} \tag{8c}$$

$$L_1 = N/2E_{c1} \tag{9c}$$

$$L_2 = N/2E_{c2} \tag{10c}$$

$$E_{c1} + E_{c2} = E_c \tag{11c}$$

$$E_{H1} + E_{H2} = E_H \tag{12c}$$

$$\pi_{c1}\, p_1^{-\beta} = \pi_{c2}\, p_2^{-\beta} \tag{13c}$$

$$\pi_{H1}\, p_1^{-\beta} = \pi_{H2}\, p_2^{-\beta} \tag{14c}$$

Note that (9c) and (10c) reflect the assumption that the size of the (immobile) labor force in each city is N/2. Equations (13c) and (14c) constrain the utility levels of bread and housing producers in the two cities to be equal. Our system now has nineteen equations, and the number of unknowns has increased to eighteen with the addition of the E_{Hi} and E_{ci}. Solution of model is considerably more complicated than in the previous cases.

Equations (5), (9c), and (10c) yield

$$p_i\, \ell\, \ell_i^{\theta-1} \left(\frac{1-\beta}{\beta}\right) = E_{ci} L_i^{\alpha} = N L_i^{\alpha-1}/2, \tag{15c}$$

and (13c) and (14c) in conjunction with (2) and (4) give

$$L_1^{\alpha}\, p_1^{-\beta} = L_2^{\alpha}\, p_2^{-\beta} \tag{16c}$$

$$(1-\theta(1-\tau_1))\ell_1^{\theta}\, p_1^{1-\beta} = (1-\theta)\ell_2^{\theta}\, p_2^{1-\beta} . \tag{17c}$$

Expressing p_i in terms of ℓ_i and L_i from (15c), (16c) and (17c) become

$$(L_2/L_1)^e \; (\ell_2/\ell_1)^b = 1 \tag{18c}$$

$$(L_2/L_1)^{e-1} \; (\ell_2/\ell_1)^{b+1} = z \tag{19c}$$

where

$$e = \alpha + \beta(1 - \alpha)$$

$$b = (\theta - 1)\beta$$

$$z = \frac{1 - \theta(1 - \tau_1)}{1 - \theta} \quad . \tag{20c}$$

Using (7c) - (10c), (18c) and (19c) become

$$\left(\frac{E_c}{E_{c2}} - 1\right)^e \left(\frac{E_H}{E_{H2}} - 1\right)^b = 1 \tag{21c}$$

$$\left(\frac{E_c}{E_{c2}} - 1\right)^{e-1} \left(\frac{E_H}{E_{H2}} - 1\right)^{b+1} = z \tag{22c}$$

Solving for E_{c2} gives

$$E_{c2} = \frac{E_c}{1 + z^A} \tag{23c}$$

and

$$E_{c1} = \frac{E_c}{1 + z^{-A}} \tag{24c}$$

follows from (11c), where

$$A = \frac{-b}{b + e} = \frac{(1 - \theta)\beta}{\alpha(1 - \beta) + \beta\theta} > 0 \tag{25c}$$

From (23c), (24c), (9c), and (10c), we have

$$L_1 = \frac{N(1 + z^{-A})}{2E_c} \tag{26c}$$

$$L_2 = \frac{N(1 + z^{A})}{2E_c} \tag{27c}$$

Solving (21c) and (22c) for E_{H2} yields

$$E_{H2} = \frac{E_H}{1 + z^B} \tag{28c}$$

and

$$E_{H1} = \frac{E_H}{1 + z^{-B}} \tag{29c}$$

follows from (12c), where

$$B = \frac{e}{b + e} = \frac{\alpha + \beta(1 - \alpha)}{\alpha(1 - \beta) + \beta\theta} > 0. \tag{30c}$$

We also find

$$\ell_2 = \frac{\ell(1 + z^B)}{E_H} \tag{31c}$$

$$\ell_1 = \frac{\ell(1 + z^{-B})}{E_H} \tag{32c}$$

From (15c), (3), (26c), and (27c), we have

$$r_2 = k(1 + z^A)^{\alpha-1} \tag{33c}$$

$$r_1 = k(1 - \tau_1)(1 + z^{-A})^{\alpha-1}, \tag{34c}$$

where $k = \theta\beta(N/2)^{\alpha} E_c^{1-\alpha}/(1-\beta)\ell$. Also, substituting the solutions for ℓ_i and L_i into (15c) yields

$$P_2 = m(1 + z^A)^{\alpha-1} (1 + z^B)^{1-\theta} \tag{35c}$$

$$p_1 = m(1 + z^{-A})^{\alpha-1} (1 + z^{-B})^{1-\theta}, \tag{36c}$$

where $m = (k/\theta)(\ell/E_H)^{1-\theta}$. Using (4), (31c), (32c), (35c), and (36c), we have

$$\pi_{H2} = q(1 - \theta)(1 + z^A)^{\alpha-1} (1 + z^B) \tag{37c}$$

$$\pi_{H1} = q(1 - \theta(1 - \tau_1))(1 + z^{-A})^{\alpha-1} (1 + z^{-B}) \tag{38c}$$

where $q = (k/\theta)(\ell/E_H)$.

Now $\partial z/\partial\tau_1 = \theta/(1 - \theta) > 0$, and given $A, B > 0$, the above results immediately imply

$$\frac{\partial E_{c1}}{\partial\tau_1} > 0, \quad \frac{\partial E_{c2}}{\partial\tau_1} < 0$$

$$\frac{\partial L_1}{\partial\tau_1} < 0, \quad \frac{\partial L_2}{\partial\tau_1} > 0 \tag{39c}$$

$$\frac{\partial E_{H1}}{\partial\tau_1} > 0, \quad \frac{\partial E_{H2}}{\partial\tau_1} < 0$$

$$\frac{\partial \ell_1}{\partial\tau_1} < 0, \quad \frac{\partial \ell_2}{\partial\tau_1} > 0.$$

From (1) and (2), variations in the L_i from (39c) imply

$$\frac{\partial w_1}{\partial\tau_1} > 0, \quad \frac{\partial w_2}{\partial\tau_2} < 0 \tag{40c}$$

$$\frac{\partial \pi_{c1}}{\partial\tau_1} < 0 \quad \frac{\partial \pi_{c2}}{\partial\tau_1} > 0$$

Since $\alpha - 1 < 0$, we have $\partial r_2/\partial\tau_1 < 0$. It may be shown that $\partial r_1/\partial\tau_1 < 0$

as long as

$$\frac{(1-\alpha)(1-\tau_1)\,\theta\beta}{\theta\beta+\alpha(1-\beta)} - z < 0\ . \tag{41c}$$

This inequality holds because $z > 1$ and the first expression is less than unity. Hence $\partial r_1/\partial\tau_1 < 0$. It may be shown that $\partial p_1/\partial\tau_1$ has the sign of

$$-z^{-A-B-1}\frac{\alpha(1-\theta)}{\alpha(1-\beta)+\theta\beta} - (1-\theta)Bz^{-B-1}\left(1-\frac{\beta(1-\alpha)}{\beta(1-\alpha)+\alpha}\,z\right) \tag{42c}$$

Since $z > 1$ and its coefficient in the last term of (42c) is less than unity, the last term in parentheses, and hence the entire expression, is ambiguous in sign. However as τ_1 approaches zero, z approaches unity and (42c) becomes negative; for "small" values of τ_1, $\partial p_1/\partial\tau_1 < 0$. It may be shown that $\partial p_2/\partial\tau_1$ has the sign of

$$z^{A+B-1}\frac{\alpha(1-\theta)}{\alpha(1-\beta)+\theta\beta} + (1-\theta)Bz^{B-1}\left(1-\frac{\beta(1-\alpha)}{\beta(1-\alpha)+\alpha}\,\frac{1}{z}\right) \tag{43c}$$

Since $1/z$ and its coefficient in (43c) are both less than unity, the term in parentheses in (43c), and hence the entire expression, is positive. Thus $\partial p_2/\partial\tau_1 > 0$. Similar arguments establish $\partial\pi_{H2}/\partial\tau_1 > 0$. Unfortunately, the sign of $\partial\pi_{H1}/\partial\tau_1$ is ambiguous.

A noteworthy feature of our results is that they contradict Mieszkowski's well-known conclusions. When the property tax rate is increased from a low level in city 1, the price of housing in city 1 declines while the housing price in city 2 increases. Mieszkowski believed that cities with above-average property tax rates would have above-average housing prices. Both housing and bread entrepreneurs migrate from city 2 to city 1 in response to the change in τ_1. Land rents fall in both cities, while the wage increases

in city 1 and decreases in city 2. The profits of bread producers fall in city 1 and rise in city 2, while the profits of housing producers increase in city 2.

The utility level of workers rises in city 1 because the wage rises and the price of housing falls, while the fall in the wage and the increase in the housing price in city 2 means the utility level of workers there falls. Since land rent falls in city 2 and the price of housing increases, the utility level of city 2 landowners falls. Determining utility changes for other agents requires additional computation. Calculations show that the τ_1 - derivative of the indirect utility function of bread producers in city 1 (see 13c) with substitution of the solutions for π_{c1} and p_1 has the same sign as $1 - z$, which is non-positive since $z \geq 1$. Thus the utility level of bread producers in city 1 falls, and since bread producers' utility is always equal in both cities, their utility level in city 2 also falls. Similar calculations show that the utility level of housing producers increases in city 2 when τ_1 is "small", which also implies an increase in city 1. In addition, for τ_1 "small", it may be shown that the utility level of city 1 landowners falls when τ_1 increases. All the results for this model are summarized in Table 1. Unfortunately, analysis of the model in the general case is intractable, as discussion in the appendix shows.

D. Complete Mobility

The appropriate long-run assumption is that all agents aside from landowners are mobile. The additional equilibrium conditions under this assumption are

$$\ell_1 = \ell/E_{H1} \tag{7d}$$

$$\ell_2 = \ell/E_{H2} \tag{8d}$$

$$E_{c1}L_1 + E_{c2}L_2 = N \tag{9d}$$

$$E_{c1} + E_{c2} = E_c \tag{10d}$$

$$E_{H1} + E_{H2} = E_H \tag{11d}$$

$$w_1 p_1^{-\beta} = w_2 p_2^{-\beta} \tag{12d}$$

$$\pi_{c1} p_1^{-\beta} = \pi_{c2} p_2^{-\beta} \tag{13d}$$

$$\pi_{H1} p_1^{-\beta} = \pi_{H2} p_2^{-\beta} \ . \tag{14d}$$

While most of these conditions are familiar, (9d) is the new worker full employment condition.

The solution is simpler in this model than in the previous case. Dividing (13d) by (12d) yields $\pi_{1c}/w_1 = \pi_{2c}/w_2$, which from (1) and (2) yields $L_1 = L_2$, $w_1 = w_2$, and $\pi_{c1} = \pi_{c2}$. From (9d) we have $L_1 = L_2 = N/E_c$, which also gives the w_i and π_{ci}. None of these solutions depends on τ_1, so we have immediately

$$\frac{\partial w_1}{\partial \tau_1} = \frac{\partial w_2}{\partial \tau_1} = \frac{\partial L_1}{\partial \tau_1} = \frac{\partial L_2}{\partial \tau_1} = \frac{\partial \pi_{c1}}{\partial \tau_1} = \frac{\partial \pi_{c2}}{\partial \tau_1} = 0 \ . \tag{15d}$$

Wages, bread producer profits and labor input per firm are unresponsive to changes in τ_1 in both cities. Now (11d), (7d), and (8d) yield

$$\ell_1 + \ell_2 = \ell_1 \ell_2 E_H/\ell \ , \tag{16d}$$

and (14d) and (4) yield, in view of $p_1 = p_2$,

$$\ell_1 = \left(\frac{1 - \theta}{1 - \theta(1 - \tau_1)}\right)^{1/\theta} \ell_2 \equiv a\ell_2. \tag{17d}$$

Together (16d) and (17d) give

$$\ell_1 = \frac{(a + 1)\ell}{E_H} \tag{18d}$$

$$\ell_2 = \frac{(a + 1)\ell}{a\ E_H} \tag{19d}$$

and

$$E_{H1} = \frac{E_H}{a + 1} \tag{20d}$$

$$E_{H2} = \frac{aE_H}{a + 1} \tag{21d}$$

follow from (7d) and (8d). Using (18d) and (19d) and the solution for L_1 and L_2, (6) yields

$$p = y\ (a + 1)^{1-\theta}\ (1 + a^{1-\theta})^{-1} \tag{22d}$$

where $y = \beta E_c^{\ 1-\alpha}\ N^\alpha/(1 - \beta)\ell^\theta\ E_H^{\ 1-\theta}$. Also, (4) with (19d) and (22d) yield

$$\pi_{H1} = \pi_{H2} = t(a + 1)(a^\theta + a)^{-1}, \tag{23d}$$

where $t = \beta(1 - \theta)\ E_c^{\ 1-\alpha}\ N^\alpha/(1 - \beta)\ E_H$. Similarly,

$$r_1 = n(1 - \tau_1)(1 + a^{1-\theta})^{-1} \tag{24d}$$

$$r_2 = n(1 + a^{\theta-1})^{-1}, \tag{25d}$$

where $n = \beta\theta E_c^{\ 1-\alpha}\ N^\alpha/(1 - \beta)\ell$. Finally, (5) yields

$$E_{c1} = E_c(1 + a^{1-\theta})^{-1} \tag{26d}$$

$$E_{c2} = E_c(1 + a^{\theta-1})^{-1}. \tag{27d}$$

Since $\partial a/\partial\tau_1 = -a/(1-\theta(1-\tau_1)) < 0$, the following results are immediate:

$$\frac{\partial \ell_1}{\partial \tau_1} < 0 \qquad \frac{\partial \ell_2}{\partial \tau_1} > 0$$

$$\frac{\partial E_{H1}}{\partial \tau_1} > 0 \qquad \frac{\partial E_{H2}}{\partial \tau_1} < 0 \tag{28d}$$

$$\frac{\partial E_{c1}}{\partial \tau_1} > 0 \qquad \frac{\partial E_{c2}}{\partial \tau_1} < 0$$

$$\frac{\partial r_2}{\partial \tau_1} < 0.$$

Computation shows that

$$\frac{\partial r_1}{\partial \tau_1} = \frac{-r_1}{(1+a^{1-\theta})(1-\tau_1)} \left(1 + \frac{\tau_1 a^{1-\theta}}{1-\theta(1-\tau_1)}\right) < 0 \tag{29d}$$

$$\frac{\partial p}{\partial \tau_1} = \frac{-p}{(a+1)^{\theta-1}\,(1+a^{1-\theta})}\,(a+1)^{\theta-2}\,\theta\tau_1\,\frac{\partial a}{\partial \tau_1} > 0, \tag{30d}$$

where p refers to the common housing price. Since $\pi_{H1} = \pi_{H2} = p(1-\theta)\ell_2^{\theta}$ and both p and ℓ_2 increase with τ_1, we have

$$\frac{\partial \pi_H}{\partial \tau_1} > 0, \tag{31d}$$

where π_H refers to the common housing profit level.

When τ_1 increases, both types of entrepreneurs migrate to city 1, the price of housing rises in both cities, land rents fall in both cities, and housing producers' profits rise in both cities. However, wages and bread producers' profits do not change.

The utility levels of workers, bread producers, and landowners fall in both cities because the wage, profit, and rent levels either fall or remain

unchanged while the housing price rises in both cities. The utility level of housing producers is $\pi_H p^{-\beta} = (1 - \theta)\ell_2^{\theta} p^{1-\beta}$, which increases with τ_1 because both ℓ_2 and p increase. The results of this model are summarized in Table 1.

Requiring equal utility levels across cities for both bread producers and workers yields equal wages, implying equal labor inputs by bread producers, which (9d) shows to be independent of τ_1. This establishes that wages and bread producer profits are independent of τ_1 in both cities; part of the economy is insulated from the effect of changes in τ_1 by the simultaneous satisfaction of the worker and bread producer equal-utility conditions. The appendix shows that this result always follows with a general utility function and a concave bread production function. Other results from this section also hold in the general case, as is shown in the appendix.

III. Conclusions

Examination of Table 1 suggests several broad conclusions. Regardless of mobility assumptions, housing producers are never hurt by and usually benefit from an increase in city 1's property tax rate. Landowners and bread producers never benefit from the tax rate increase, and are usually made worse off by it. Workers in city 2 never benefit from the increase, while workers in city 1 benefit only under the assumption of worker immobility. Roughly speaking, an increase in the property tax rate in city 1 benefits housing producers at the expense of other agents in the economy regardless of the mobility assumption.

A striking change in the incidence of the tax occurs between the first two and last two lines of Table 1. The difference between these sets of cases is, of course, the mobility assumption for producers. Producer immobility means that incidence is localized, with effects felt only by housing producers and landowners in city 1, but when producers are mobile, all agents in the economy are affected by a change in the property tax rate. It is interesting to note that for a given assumption on producer mobility, changing the worker mobility assumption has little effect on the qualitative incidence results. When producers are immobile, no change in incidence follows from allowing consumers to move. When producers are mobile, allowing worker mobility only changes the direction of the effect on workers in city 1. The utility increase under worker immobility turns into a decrease under mobility as workers migrate from city 2 to city 1 to take advantage of better opportunities there. What these observations indicate is that in terms of qualitative incidence

results, Mieszkowski's case is a good approximation to the long run, full mobility case.

It is never possible to give a causal interpretation of comparative static results in a fully simultaneous equation system because everything depends on everything else. Thus it is not possible to say "why" our model generates many of the results we have derived. It is clear, however, that the model's unconventional structure is responsible for results which seem at variance with the new view of the property tax. But precisely because this structure provides a more realistic representation of the economy than the Harberger framework, we feel that our incidence results must be taken more seriously than the new view results. Of course, there is considerable room for improvement on our analysis. Future work could be directed toward adding detail and more realism to the model. The general framework, however, appears to be an appropriate one for the analysis of property tax incidence.

Appendix

This appendix presents analysis using general concave bread and housing production functions, $f(L)$ and $h(\ell)$, and a general indirect utility function $V(p, I)$. The general conditions analogous to (1) - (6) are

$$f'(L_i) = w_i \tag{1'}$$

$$\pi_{ci} = f(L_i) - f'(L_i)L_i \tag{2'}$$

$$p_i(1 - \tau_i)\, h'(\ell_i) = r_i \tag{3'}$$

$$\pi_{Hi} = p_i(h(\ell_i) - (1 - \tau_i)\, h'(\ell_i)\ell_i) \tag{4'}$$

$$E_{Hi}h(\ell_i) = D_H(p_i, E_{ci}f(L_i) + \ell p_i h(\ell_i)/\ell_i) \tag{5'}$$

$$E_{c1}f(L_1) + E_{c2}f(L_2) = D_c(p_1, E_{c1}f(L_1) + \ell p_1 h(\ell_1)/\ell_1) + D_c(p_2, E_{c2}f(L_2) + \ell p_2 h(\ell_2)/\ell_2), \tag{6'}$$

where D_H and D_c are aggregate demand functions for housing and bread respectively. The price of housing appears in the bread demand function with a general utility function. The second argument of the demand functions is aggregate income, which is calculated using (1') - (4'). Condition (4') incorporates the lump sum rebate $s_i = p_i\tau_i h(\ell_i)$. We now consider the different mobility cases.

A. Complete Immobility

The additional conditions for this case are (7a) and (8a) as before. Solutions for the ℓ_i and L_i are given directly by (7a) and (8a) and the (equal) p_i are then given by (5'). Profits are given by (2') and (4')

and land rent by (3'). As before the only solutions which depend on τ_1 are r_1 and π_{H1}, and $\partial r_1/\partial \tau_1 < 0$, $\partial \pi_{H1}/\partial \tau_1 > 0$.

B. Producers Immobile

Additional conditions for this case are (7b), (8b), and

$$V(w_1, p_1) = V(w_2, p_2). \tag{9b'}$$

If we substitute $w_i = f'(L_i)$ in (9b') and substitute the solution for the ℓ_i into (5'), then the four-equation system composed of (5'), (9b'), and (8b) solves for the four unknowns L_i and p_i, and the solutions do not depend on τ_1. Thus, as before, the only variables which depend on τ_1 are r_1 and π_{H1}, and we have $\partial r_1/\partial \tau_1 < 0$, $\partial \pi_{H1}/\partial \tau_1 > 0$.

C. Consumers Immobile

Additional conditions are (7c) - (12c) and

$$V(\pi_{c1}, p_1) = V(\pi_{c2}, p_2) \tag{13c'}$$

$$V(\pi_{H1}, p_1) = V(\pi_{H2}, p_2). \tag{14c'}$$

Substituting π_{ci} and π_{Hi} in (13c') and (14c') from (2') and (4'), eliminating E_{ci} and E_{Hi} in (5') using (7c) - (10c), and using (7c) - (12c) to derive $E_c L_1 L_2 = N(L_1 + L_2)$ and $E_H \ell_1 \ell_2 = \ell(\ell_1 + \ell_2)$ results in a six-equation system in the six unknowns p_i, ℓ_i, and L_i. Since τ_1 appears in this system, all the solutions depend on it, and performing comparative status requires totally differentiating the system. We assumed this computation would be intractable and did not attempt it.

D. Full Mobility

Additional conditions in this case are (7d) - (11d) and

$$V(w_1, p_1) = V(w_2, p_2) \tag{12d'}$$

$$V(\pi_{c1}, p_1) = V(\pi_{c2}, p_2) \tag{13d'}$$

$$V(\pi_{H1}, p_1) = V(\pi_{H2}, p_2) \tag{14d'}$$

We may show $w_1 = w_2$, $p_1 = p_2$ as follows: Suppose $p_1 > p_2$. Then $w_1 > w_2$ and $\pi_{c1} > \pi_{c2}$ are necessary from (12d') and (13d'). But $w_1 > w_2$ implies $L_1 < L_2$ given the concavity of f. However, since $d\pi_{ci}/dL_i = -f''(L_i)L_i > 0$, $L_1 < L_2$ implies $\pi_{c1} < \pi_{c2}$. Hence $w_1 > w_2$ and $\pi_{c1} > \pi_{c2}$ are incompatible. Assuming $p_1 < p_2$ also leads to a contradiction, and hence $p_1 = p_2$ is necessary. This implies $w_1 = w_2$ and $L_1 = L_2$, and using (9d) we get $L_1 = L_2 = N/E_c$. This solution determines the (equal) w_i and π_{ci}, and it is clear that none of the above solution values depends on τ_1. Since $p_1 = p_2$, (14d) requires $\pi_{H1} = \pi_{H2}$, or

$$h(\ell_1) - (1 - \tau_1)h'(\ell_1)\ell_1 - h(\ell_2) + h'(\ell_2)\ell_2 = 0 \tag{15d'}$$

From (7d), (8d), and (11d), we get

$$\ell_1 + \ell_2 - \ell_1\ell_2 E_H/\ell = 0 \tag{16d'}$$

These equations yield

$$\frac{\partial \ell_1}{\partial \tau_1} = h'(\ell_1)\ell_1 \, (\ell_1 E_H/\ell - 1)/D < 0 \tag{17d'}$$

$$\frac{\partial \ell_2}{\partial \tau_1} = h'(\ell_1)\ell_1 \, (1 - \ell_2 E_H/\ell)/D > 0 \tag{18d'}$$

where $D = \tau_1(h'(\ell_1) - (1 - \tau_1)h''(\ell_1)\ell_1)(1 - \ell_1 E_H/\ell) - h''(\ell_2)\ell_2(1 - \ell_2 E_H/\ell)$ < 0 and $\ell_i E_H/\ell = E_H/E_{Hi} > 1$. Equations (17d) and (18d) imply

$$\frac{\partial E_{H1}}{\partial \tau_1} > 0 \quad \frac{\partial E_{H2}}{\partial \tau_1} < 0 \tag{19d'}$$

in view of (7d) and (8d).

Since $p_1 = p_2$, (6) may be written

$$E_c f(N/E_c) = D_c\left(p,\ E_c f(N/E_c) + \ell p(h(\ell_1)/\ell_1 + h(\ell_2)/\ell_2)\right), \tag{20d'}$$

where p is the common value of the p_i. Since $(\partial D_c/\partial p)/(\partial D_c/\partial I) > 0$, it turns out that $\partial p/\partial \tau_1$ has the sign of

$$-\left(\frac{h'(\ell_1)\ell_1 - h(\ell_1)}{\ell_1^{\,2}}\ \frac{\partial \ell_1}{\partial \tau_1} + \frac{h'(\ell_2)\ell_2 - h(\ell_2)}{\ell_2^{\,2}}\ \frac{\partial \ell_2}{\partial \tau_1}\right). \tag{21d'}$$

Using (17d') and (18d') we find that $(\partial \ell_2/\partial \tau_1)/\ell_2^{\,2} = -(\partial \ell_1/\partial \tau_1)/\ell_1^{\,2}$ and hence that (21d') has the sign of

$$-\left(h'(\ell_1)\ell_1 - h(\ell_1) + h(\ell_1) - (1 - \tau_1)\ h'(\ell_1)\ell_1\right)\frac{\partial \ell_1}{\partial \tau_1}$$
$$= -\tau_1 h'(\ell_1)\ell_1 \frac{\partial \ell_1}{\partial \tau_1} > 0, \tag{22d'}$$

where (15d') has been used. Hence $\partial p/\partial \tau_1 > 0$. From (4'), $\pi_{H2} = p(h(\ell_2) - h'(\ell_2)\ell_2)$, and the expression in parentheses is increasing in τ_1 by the concavity of h and by (18d'). Since $\partial p/\partial \tau_1 > 0$, this means $\partial \pi_{H2}/\partial \tau_1 > 0$, and $\partial \pi_{H1}/\partial \tau_1 > 0$ follows also. Analyzing changes in the r_i is complicated, but it may be shown that $\partial r_1/\partial \tau_1 < 0$ while $\partial r_2/\partial \tau_1$ is ambiguous in sign. In addition, $\partial E_{ci}/\partial \tau_1$ cannot be signed unambiguously.

All we can say about utility changes in the general model is that, workers, bread producers, and city 1 landowners suffer a utility decrease because their incomes fall or remain unchanged while the price of housing increases. Since housing profits increase, the utility change of housing producers is ambiguous, and since the change in r_2 is uncertain, the utility change for city 2 landowners is also uncertain.

The similarity of the results in the general full mobility case to those derived using specific functional forms leads us to believe there is nothing pathological about the special solution presented in the text.

Table I

	w_1	w_2	r_1	r_2	π_{H1}	π_{H2}	π_{c1}	π_{c2}	p_1	p_2	E_{c1}^{*}	E_{H1}^{*}	$N_1^{\dagger}$	u_{L1}	u_{L2}	u_{c1}	u_{c2}	u_{H1}	u_{H2}	$u_{\ell 1}$	$u_{\ell 2}$
Complete Immobility Producers Immobile	0	0	-	0	+	0	0	0	0	0	0	0	0	0	0	0	0	+	0	-	0
Workers Immobile	+	-	-	-	?	+	-	+	-	+	+	+	0	+	-	-	-	+	+	-	-
Full Mobility	0	0	-	-	+	+	0	0	+	+	+	+	+	-	-	-	-	+	+	-	-

* The direction of change for E_{c2} is opposite to that of E_{c1} and similarly for E_{H2}.

† N_1 represents the worker population in city 1. N_2 changes in the opposite direction.

Footnotes

[1] I am indebted to Ron Harstad for invaluable assistance in developing the basic structure of the model analyzed in this paper.

[2] For other papers in this tradition see McClure (1970), (1977), Courant (1978). See also Mieszkowski (1969). McClure (1970) addresses the issue of mobility, but his model only has one good and suffers from the defects discussed below.

References

Courant, Paul N., 1978. A general equilibrium model of heterogeneous local property taxes, Journal of Public Economics, 8, forthcoming.

Harberger, A. C., 1962. The incidence of the corporation income tax, Journal of Political Economy, 70, 215-240.

McClure, Charles, 1970. Taxation, substitution and industrial location, Journal of Political Economy, 78, 112-132.

McClure, Charles, 1977. The new view of the property tax: A caveat, National Tax Journal, 30, 69-76.

Mieszkowski, Peter, 1969. Tax incidence theory: The effects of taxes on the distribution of income, Journal of Economic Literature, 7, 1103-1124.

Mieszkowski, Peter, 1972. The property tax: An excise tax or a profits tax, Journal of Public Economics, 1, 73-96.