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IN A MONOCENTRIC CITY

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A Model of Non-Central Production in a Monocentric City

by Jan K. Brueckner*

The standard microeconomic model of an urban area postulates that residents commute to the central business district, where they produce some commodity. Housing production is the only production activity occuring outside the CBD. Since workers are identical and incur commuting costs which increase with the distance of their residence from the CBD, unit housing prices must decline with distance to insure that the utility level is uniform across all households. Since the price of the other consumer good is constant over space by assumption, relatively more housing per household is consumed at greater distances from the CBD. Spatial equilibrium for housing producers requires that land rent decrease with distance, which means that land is used more intensively in the production of housing closer to the CBD. In conjunction with consumer substitution in favor of housing, this effect results in declining population density as distance from the CBD increases, which is the main testable implication of the

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standard model. The model has been extensively developed and tested by Muth [3].

The purpose of the model presented in this paper is to increase the realism of the standard model by adding more structure to the non-housing consumer good production activity. The modification introduced was suggested by the apparent fact that consumer shopping trips are short compared to commuting trips, with consumers travelling close to home to acquire daily necessities such as food. Of course, shopping travel for infrequent purchases of items such as consumer durables may be more extensive, but this is ignored in the model. The production process we envision is a "retailing" process; the output is goods sold in a particular spot, and the inputs are produced goods, labor, and building space. Retail producers locate at every distance from the CBD and consumers shop "locally" by making costless circumferential shopping trips to nearby producers.

Section I contains the assumptions of the model and preliminary analysis. Section II explores relationships among price gradients and develops results on observable quantities. Section III discusses estimating equations, and Section IV presents empirical results. Section V presents modifications of the model, while Section VI contains conclusions.

I

We begin by enumerating the assumptions of the model:

A1) Production which requires labor input occurs in the CBD, and CBD commuters live at every distance in the city from the CBD.

A2) Housing, h, and the retail good, g, are the only arguments of utility functions.

A3) Consumers are identical.

A4) The money cost of radial travel is exogenous and is increasing and concave in radial distance, while circumferential travel is costless.

A5) Retail production uses labor, L, commercial real estate, R, and wholesale goods, Q, as inputs.

A6) Housing and commercial real estate production use labor, land, ℓ, and non-land capital, N as inputs.

A7) The unit prices of N and Q are invariant over space, while all other prices may vary.

A8) Perfect competition prevails in all markets.

A9) Consumers make the same number of commuting and shopping trips per period.

A10) Consumers acquire the retail good at the distance at which they live.

While most of these assumptions are in the spirit of the standard model, A4, A9, and A10 require comment. We ignore the time cost of travel because with identical consumers this cost will not vary. The money cost of radial travel is $T(k)$, where k is distance travelled, and $T' > 0$, $T'' \leq 0$. Exogeneity of the function T requires zero congestion at all traffic levels and requires that the transportation system uses no resources whose prices are endogenous. The zero-cost assumption for circumferential travel is artificial, but in conjunction with A10, it allows us to construct a model where consumers make "short" shopping trips with zero cost. In the real world, shopping trips appear to be short relative to CBD commuting trips, but while shopping costs may be negligible, they are not zero. Thus, consumers value

accessibility to retail producers, and a realistic model must be multicentered, with each retail producer a different center. A4 and A10 generate short shopping trips while allowing us to avoid the intractable multicentered problem generated by small positive shopping costs. Since shopping transportation costs are zero, the need for A9 is not obvious but will become apparent below in the demonstration of the consistency of all the other assumptions. A9 seems to be a natural assumption, although shopping travel would be endogenous in a more realistic model.

We now develop some basic implications of the assumptions. First, if γ is the number of commute trips per period, $2\gamma T(k) \equiv t(k)$ is commuting transport cost per period from a residence at distance k from the CBD. Since g is a retail good, shipping of the produced good does not make sense in that the selling of the good at a particular location is the essence of the production process. A1 and A2 imply that housing is produced at every distance from the CBD, and A1, A2, and A10 imply that retail production occurs at every distance, which in conjunction with A5 means commercial real estate is produced at every distance. Clearly, labor is also employed at every distance from the CBD.

We may show that these laborers must reside at the distance of their place of employment, travelling circumferentially to work and incurring no commuting cost, as follows. The disposable income per period of a CBD commuter living at distance k is $y-t(k)$, where y is the exogenous CBD wage per period. If the local wage rate at k exceeded $y-t(k)$, all CBD commuters at k would switch to local employment, violating A1. Hence $w(k) \leq y - t(k)$, where $w(k)$ is the local wage

at k. Competition among firms bids up the local wage until it reaches $y-t(k)$, and hence $w(k) \equiv y - t(k)$. Since $t' > 0$, the wage of locally employed workers declines with k.

We may now ask if a worker will work outside the CBD and commute radially to his place of work. Outward commuting results in a lower wage and extra costs, and it will never occur. A worker will not commute inward from k_0 to k_1, $0 < k_1 < k_0$, when

$$w(k_1) - t(k_0-k_1) < w(k_0) \tag{1}$$

which requires $t(k_0-k_1) > t(k_0) - t(k_1)$.[1] This inequality holds when $t' > 0$ and $t'' \leq 0$ with the exception of the case $t(k) \equiv \beta k$, in which case equality holds.[2] When transport costs have the latter form, consumers are indifferent between zero radial commuting and commuting any distance inward. Otherwise, inward commuting lowers disposable income and will not occur.

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[1]Notice that (1) relies on the implicit assumption that the cost of travel from $k = k_0$ to $k = k_1$ equals the cost of travel from $k = k_0-k_1$ to $k = 0$, which is somewhat unrealistic. We have assumed congestion is zero, which eliminates one reason why the cost of traveling a mile might increase as the point of origin moves closer to the CBD. Another reason why this might happen is that the quality of roads may decline as distance to the CBD decreases. The results which follow can be derived, however, using a transportation cost function that has both distance travelled and point of origin as arguments.

[2]Consider the function $x(k) \equiv t(k+\alpha) - t(k)$ where $\alpha > 0$. Now $x'(k) = t'(k+\alpha) - t'(k) \leq 0$ since $t'' \leq 0$. Let $\alpha = k_0 - k_1$. Substituting $k = 0$ and $k = k_1$ in $x(k)$ and noting $x' \leq 0$, we have $t(k_0) - t(k_1) \leq t(k_0-k_1) - t(0) \leq t(k_0-k_1)$. The only way equality can hold all the way through this relation is when $t'' = 0$ and $t(0) = 0$, that is if $t(k) \equiv \beta k$, $\beta > 0$. Otherwise, $t(k_0) - t(k_1) < t(k_0-k_1)$.

The salient features of the model based on A1 - A10 are thus:

B1) CBD workers and locally employed workers, who travel circumferentially to work, reside at every distance from the CBD in the city.

B2) The disposable income at k of both types of workers equals w(k).

B3) Both types of workers make circumferential shopping trips at zero cost.

B4) Producers of g, h, and R locate at every distance from the CBD in the city.

B5) Perfect competition prevails in all markets.

The analysis below concerns the equilibrium conditions for an urban economy characterized by B1 - B5. One question which might occur to the reader is: Does the equilibrium which arises out of A1 - A10 have the property that people have no incentive to deviate from the seemingly arbitrary behavior postulated in A10? The answer is affirmative: prices generated by A1 - A10 imply that the behavior in A10 is optimal, which is shown as follows. Analysis in Section II of the model based on A1 - A10 yields housing and retail price functions which decline with distance. Hence, consumers will never travel radially inward to purchase g because extra transport costs are incurred and g is more expensive closer to the center. We can show also that outward shopping travel also reduces utility. Let V be the utility level of a worker living at k_0 and shopping at $k_1 > k_0$. His shopping costs are given by $t(k_1-k_0)$, since by A9 the number of shopping trips per period equals γ, the number of commute trips per period. His disposable income is $w(k_0) - t(k_1-k_0)$, which, from above, is less than $w(k_1)$, the disposable income of a worker

living and shopping at k_1. Since the latter worker faces the same retail price, a lower housing price, and has a higher disposable income than the worker who travels radially to shop, his utility level, $U(k_1)$, exceeds V. But locational equilibrium in the model with circumferential shopping requires $U(k_1) = U(k_0)$, where $U(k_0)$ is the utility level of a worker living and shopping at k_0. Hence $U(k_0) > V$ and the radial shopper is better off shopping circumferentially at k_0. Thus, assumptions A1 - A10 are validated by the equilibrium they generate. While other assumptions on consumer travel may have this property, the attractiveness of assumptions A1 - A10 lies in their apparent realism and in the simplicity of the structure they generate.

II

The utility function and the production functions for retail producers, housing producers, and commercial real estate producers are respectively

$$U = U(g, h)$$

$$G = G(Q, L, R)$$

$$H = H(N, \ell, L)$$

$$R = R(N, \ell, L).$$

The unit price of g is s(k), the unit rental prices of H, R, and ℓ are p(k), z(k), and r(k) respectively, and the unit prices of Q and N are q and n, which are constant by A7. The consumer's Lagrangean and producers' profits at k are

$$u(g,h) - \lambda(s(k)g + p(k)h - w(k))$$

$$s(k)G(Q,L,R) - qQ - w(k)L - z(k)R$$

$$p(k)H(N,\ell,L) - nN - r(k)\ell - w(k)L$$

$$z(k)R(N,\ell,L) - nN - r(k)\ell - w(k)L.$$

Each agent solves a maximization problem which involves choosing an optimal distance k from the city center. Since B1 - B5 require each agent to be present at all k, the locational equilibrium conditions must hold at all k. After some manipulation, these conditions are

$$m_g \frac{s'(k)}{s(k)} + m_h \frac{p'(k)}{p(k)} - \frac{w'(k)}{w(k)} = 0$$

$$\frac{s'(k)}{s(k)} - g_L \frac{w'(k)}{w(k)} - g_R \frac{z'(k)}{z(k)} = 0$$

$$\frac{p'(k)}{p(k)} - \rho_L \frac{w'(k)}{w(k)} - \rho_\ell \frac{r'(k)}{r(k)} = 0 \qquad (2)$$

$$\frac{z'(k)}{z(k)} - \mu_L \frac{w'(k)}{w(k)} - \mu_\ell \frac{r'(k)}{r(k)} = 0,$$

where m_g and m_h are budget shares for the consumer and $g_.$, $\rho_.$, $\mu_.$ are factor shares for producers. For example, $g_L = w(k)L/s(k)G$. These budget and factor shares embody optimal consumption, input, and output levels which come from solution of the entire optimization problem for each agent. Clearly, these shares are by no means constant but are implicitly functions of k. The system (2) is actually a four-equation,

first-order, non-linear differential equation system in the four prices s(k), z(k), p(k), and r(k), since w(k) is given. While a general solution is unachievable, imposing Cobb-Douglas utility and production functions allows easy solution. However, (2) is a linear system in the four price gradients s'/s, z'/z, p'/p, and r'/r which may be solved for in terms of w'/w as follows:

$$\frac{r'(k)}{r(k)} = c_0 \frac{w'(k)}{w(k)}$$

$$\frac{z'(k)}{z(k)} = (\mu_L + \mu_\ell c_0) \frac{w'(k)}{w(k)} \equiv c_1 \frac{w'(k)}{w(k)}$$

$$\frac{p'(k)}{p(k)} = (\rho_L + \rho_\ell c_0) \frac{w'(k)}{w(k)} \equiv c_2 \frac{w'(k)}{w(k)}$$

$$\frac{s'(k)}{s(k)} = (g_L + g_R c_1) \frac{w'(k)}{w(k)} \equiv c_3 \frac{w'(k)}{w(k)}, \tag{3}$$

where

$$c_0 = \frac{1 - g_L m_g - \rho_L m_h - g_R \mu_L m_g}{g_R \mu_\ell m_g + \rho_\ell m_h}.$$

This solution tells how the relative rates of change of the urban prices must be related to the relative rate of change of the local wage at any distance k so that neutral locational equilibrium obtains for each agent. The c_i are implicitly functions of k. With Cobb-Douglas utility and production functions, however, the c_i will be constant and we may integrate the expressions in (3) with results such as $r(k) = b\, w(k)^{c_0}$, where b is an integration constant.

General results are available without an appeal to special functional forms. We have $c_0 > 1$, which, since $w'(k) < 0$, implies

$$r'(k) < 0 \qquad z'(k) < 0 \qquad p'(k) < 0 \qquad s'(k) < 0 \tag{4}$$

and

$$\left|\frac{r'(k)}{r(k)}\right| > \left|\frac{w'(k)}{w(k)}\right| . \tag{5}$$

All urban prices decline with distance and the relative rate of decline of land rent exceeds the relative rate of decline of the local wage at all k. From (3), $c_0 > 1$ when

$$1 > m_h(\rho_L + \rho_\ell) + m_g(g_L + g_R(\mu_L + \mu_\ell)) . \tag{6}$$

We know that $m_h + m_g = 1$, $\rho_L + \rho_\ell + \rho_N \leq 1$, $\mu_L + \mu_\ell + \mu_N \leq 1$, and $g_L + g_R + g_Q \leq 1$, with equality holding in the last three cases when profits are zero for all producers. So $\rho_L + \rho_\ell < 1$, $\mu_L + \mu_\ell < 1$, and $g_L + g_R(\mu_L + \mu_\ell) < 1$, and thus the second inequality in (6) holds, giving $c_0 > 1$.

Similar manipulations establish $c_0 > c_1$, $c_0 > c_2$, $c_0 > c_3$, which yield

$$\left|\frac{r'(k)}{r(k)}\right| > \left|\frac{z'(k)}{z(k)}\right|, \quad \left|\frac{p'(k)}{p(k)}\right|, \quad \left|\frac{s'(k)}{s(k)}\right| .$$

The relative rate of decline of land rent exceeds the relative rates of decline of the commercial real estate price, the housing price, and the retail good price.

Three additional assumptions are made in what follows: profits are zero so that factor shares sum to one; $\mu_L = \rho_L = 0$, indicating labor is not among the inputs of the G and R producers; and $\rho_\ell > \mu_\ell$, or the share of land in the housing industry is greater than in the commercial real estate industry at all distances. The first and second assumptions are standard, while the third follows from zero profits and

$$\left(\frac{N}{\ell}\right)_H < \left(\frac{N}{\ell}\right)_R ,$$

which says that non-land capital per acre is higher in commercial real estate than in housing at each distance, an assumption which conforms to intuition. These extra assumptions yield $c_2 > 1$ and $c_3 < 1$. From (3), $c_2 - 1$ is $\rho_\ell c_0 - 1$, which has the same sign as

$$m_g(\rho_\ell(1-g_L) - \mu_\ell g_R). \tag{7}$$

Since $1-g_L > g_R$ and $\rho_\ell > \mu_\ell$, (7) is positive and $c_2 > 1$. The result $c_3 < 1$ is established similarly. Also, $c_2 > c_1$ follows immediately from (3). These facts yield

$$\left|\frac{r'(k)}{r(k)}\right| > \left|\frac{p'(k)}{p(k)}\right| > \left|\frac{w'(k)}{w(k)}\right| > \left|\frac{s'(k)}{s(k)}\right| ; \left|\frac{p'(k)}{p(k)}\right| > \left|\frac{z'(k)}{z(k)}\right| . \tag{8}$$

The additional assumption $g_Q = 0$ allows us to complete the hierarchy of price gradients:

$$c_0 > c_2 > 1 > c_3 > c_1,$$

or

$$\left|\frac{r'(k)}{r(k)}\right| > \left|\frac{p'(k)}{p(k)}\right| > \left|\frac{w'(k)}{w(k)}\right| > \left|\frac{s'(k)}{s(k)}\right| > \left|\frac{z'(k)}{z(k)}\right| . \tag{9}$$

This factor assumption means that wholesale goods are not an input to retail production, which is unrealistic but simplifies the subsequent analysis considerably. We relax the assumption in Section V.

Since urban prices are hard to observe, further analysis is required to deduce the behavior of observables. Zero profits in the commercial real estate industry means

$$zR \equiv nN + r\ell$$

Differentiation yields

$$\frac{d(\frac{zR}{\ell})^*}{dk} = \mu_N \frac{d(\frac{N}{\ell})^*}{dk} + \mu_\ell \frac{dr^*}{dk} \tag{10}$$

where the * refers to the natural logarithm of a given variable (for example, $dr^*/dk = r'(k)/r(k)$). Now

$$\frac{d(\frac{N}{\ell})^*}{dk} = \frac{d(\frac{N}{\ell})^*}{d(\frac{r}{n})^*} \frac{d(\frac{r}{n})^*}{dk} = \sigma^R_{N\ell} \frac{dr^*}{dk} , \tag{11}$$

where $\sigma^R_{N\ell}$ is the elasticity of substitution between N and ℓ in R production. Unless the production function is CES, $\sigma^R_{N\ell}$ will vary with k due to its dependence on the factor price ratio. Substituting (11) in (10) yields, using (3)

$$\frac{d(\frac{zR}{\ell})^*}{dk} = c_0(\mu_\ell + \mu_N\sigma^R_{N\ell}) \frac{dw^*}{dk} .$$

Since $w' < 0$, commercial real estate output per unit land input declines with distance. This is caused by declining land rent and the falling N/ℓ ratio which declining land rent generates. Also,

$$\frac{d(\frac{R}{\ell})^*}{dk} = \frac{d(\frac{zR}{\ell})^*}{dk} - \frac{dz^*}{dk} = c_0 \mu_N \sigma^R_{N\ell} \frac{dw^*}{dk} . \qquad (12)$$

Commercial real estate output per unit land input also declines with distance.

Also, we have

$$\frac{d(\frac{sG}{L})^*}{dk} = g_L \frac{dw^*}{dk} + g_R \frac{dz^*}{dk} + g_R \frac{d(\frac{R}{L})^*}{dk}$$

$$= [g_L + g_R(c_1 + \sigma^G_{RL}(1 - c_1))] \frac{dw^*}{dk} . \qquad (13)$$

Since $c_1 < 1$, retail sales per unit labor input decline with distance. This result is guaranteed by the decline of the R/L ratio, which occurs because the wage declines faster than the price of commercial real estate. Similarly,

$$\frac{d(\frac{sG}{R})^*}{dk} = [g_R c_1 + g_L(\sigma^G_{RL} c_1 + (1 - \sigma^G_{RL}))] \frac{dw^*}{dk} . \qquad (14)$$

If $\sigma^G_{RL} \leq 1$, sG/R declines with distance. Since L/R increases while all prices decline, sG/R decreases only if L/R does not increase too fast, which requires low substitutability between L and R in retail production, as seems reasonable.

Also,

$$\frac{d(\frac{sG}{\ell})^*}{dk} = \frac{d(\frac{sG}{R})^*}{dk} + \frac{d(\frac{R}{\ell})^*}{dk}$$

$$= (g_R c_1 + g_L(\sigma^G_{RL} c_1 + (1 - \sigma^G_{RL})) + c_0 \mu_N \sigma^R_{N\ell}) \frac{dw^*}{dk} , \tag{15}$$

which is negative when $\sigma^G_{RL} \leq 1$. In addition

$$\frac{d(\frac{L}{\ell})^*}{dk} = \frac{d(\frac{sG}{\ell})^*}{dk} - \frac{d(\frac{sG}{L})^*}{dk}$$

$$= [(c_1 - 1)\sigma^G_{RL} + c_0 \mu_N \sigma^R_{N\ell}] \frac{dw^*}{dk} , \tag{16}$$

which is ambiguous in sign. Retail sales per unit land input (which is an indirect input embodied in R) declines when R and L are not too substitutable, while retail labor input per unit land input may increase or decrease.

Although these expressions pertain to individual firms, they also describe the behavior of aggregate quantities. Suppose j retail firms are located at the same distance from the center. Total sales divided by total labor input, for example, for these firms will be sjG/jL = sG/L since the firms are identical and locate at the same distance.

While data on land used in retail activity are rare, it is possible to deduce the behavior of, say, retail sales per total land area as the distance to the land area in question changes. Consider a narrow ring of inner radius k. Let ℓ_T be the area of the ring and sG_T total retail sales in the ring. If j firms operate in the ring

$$\frac{sG_T}{\ell_T} = \frac{sjG}{j\ell}\,\frac{j\ell}{\ell_T} \equiv \frac{sG}{\ell}\,\lambda \ ,$$

where G and ℓ pertain to individual firms and λ is the fraction of the ring land used in retail production. Since

$$\frac{d(\frac{sG_T}{\ell_T})^*}{dk} = \frac{d(\frac{sG}{\ell})^*}{dk} + \frac{d\lambda^*}{dk} \ , \tag{17}$$

we need only to calculate $d\lambda^*/dk$. In order that the market for retail goods clear in a ring of inner radius k, it must be true that

$$\lambda(\frac{G}{\ell})\varepsilon = (1 - \lambda)(\frac{g}{h})(\frac{H}{\ell})\varepsilon \tag{18}$$

where G/ℓ is retail output per unit land input at k, g/h is the retail goods-housing consumption ratio, and H/ℓ is housing output per unit land input, and ε is the fraction of the land at k available for use. Now (18) implicitly defines the market-clearing λ, and after differentiating with respect to k, much manipulation results in

$$\frac{d\lambda^*}{dk} = (1 - \lambda)[\frac{d(\frac{g}{h})^*}{dk} + \frac{d(\frac{H}{\ell})^*}{dk} - \frac{d(\frac{G}{\ell})^*}{dk}]$$

$$= (1 - \lambda)[\sigma_{gh}(c_2 - c_3) + c_0(\rho_N\sigma^H_{N\ell} - \mu_N\sigma^R_{N\ell})$$

$$+ g_L\sigma^G_{LR}(1 - c_1)]\ \frac{dw^*}{dk} \ , \tag{19}$$

where σ_{gh} is the elasticity of substitution between g and h in consumption. The sign of (19) is ambiguous, so the fraction of land used for retail production may increase or decrease with distance. Using (17), (19),

and (15), we get

$$\frac{d\left(\frac{sG_T}{\ell_T}\right)^*}{dk} = [g_R c_1 + g_L(\sigma^G_{LR} c_1 + (1-\sigma^G_{RL})) + (1-\lambda)(\sigma_{gh}(c_2 - c_3) + c_0\rho_N\sigma^H_{N\ell} + g_L\sigma^G_{LR}(1-c_1)) + \lambda c_0\mu_N\sigma^R_{N\ell}]\,\frac{dw^*}{dk}, \tag{20}$$

which is negative when $\sigma^G_{LR} \leq 1$. Similarly,

$$\frac{d\left(\frac{R_T}{\ell_T}\right)^*}{dk} = [(1-\lambda)(\sigma_{gh}(c_2 - c_3) + c_0\rho_N\sigma^H_{N\ell} + g_L\sigma^G_{LR}(1-c_1)) + \lambda c_0\mu_N\sigma^R_{N\ell}]\,\frac{dw^*}{dk}, \tag{21}$$

which is negative. Also,

$$\frac{d\left(\frac{L_T}{\ell_T}\right)^*}{dk} = [(g_R + \lambda g_L)\sigma^G_{RL}(c_1 - 1) + (1-\lambda)(\sigma_{gh}(c_2 - c_3) + c_0\rho_N\sigma^H_{N\ell}) + \lambda c_0\mu_N\sigma^R_{N\ell}]\,\frac{dw^*}{dk}. \tag{22}$$

which is ambiguous in sign. Retail sales and commercial real estate output per unit total land area decline with distance, while retail employment per unit total land area may increase or decrease. We may state the following _a priori_ predictions, which are tested below:

$$\frac{d\left(\frac{R}{\ell}\right)^*}{dk},\quad \frac{d\left(\frac{R_T}{\ell_T}\right)^*}{dk},\quad \frac{d\left(\frac{sG}{L}\right)^*}{dk} < 0$$

$$\frac{d\left(\frac{sG}{R}\right)^*}{dk}, \quad \frac{d\left(\frac{sG_T}{\ell_T}\right)^*}{dk} < 0 \qquad \text{when } \sigma_{LR}^G \leq 1$$

$$\frac{d\left(\frac{L_T}{\ell_T}\right)^*}{dk} \qquad \text{ambiguous in sign.} \tag{23}$$

III.

Equations 12-16 and 20-22 are of the form

$$\frac{dS^*}{dk} = B(k)\,\frac{dw^*}{dk}\;, \tag{24}$$

where B(k) represents the coefficient expressions which depend on k. Assume for the moment that B(k) is constant, as it will be with Cobb-Douglas utility and production functions. Integrating (24) yields

$$\log S = B \log w + c, \tag{25}$$

where c is an integration constant. We assume (25) does not hold exactly in the observable variables due to, say, measurement error in S. Expanding log w in a first-order Taylor series, we have

$$\begin{aligned} \log S &= B \log y - \frac{B}{y}\, t(k) + R(k) + c + \varepsilon, \\ &\equiv \alpha + \beta\, t(k) + v \end{aligned} \tag{26}$$

where R(k) is the remainder and ε is a random term with mean zero and variance which is constant across k. The Taylor series approximation will be nearly exact since t(k) will be quite small compared to y. We take k to be a non-stochastic quantity.

Now S is a ratio, say A/E, but we have observations on $\Sigma A_i / \Sigma E_i$, for example, total retail sales divided by total labor input, from some areal unit of observation. This must be related to (26), which describes A_j/E_j for a firm j. After a sequence of Taylor series approximations, which are likely to be fairly exact, and much manipulation, we arrive at

$$\log X \equiv \log \frac{\Sigma A_i}{\Sigma E_i} = \alpha + \beta\, t(k) + u \tag{27}$$

where k is now interpreted as the distance to the geographic center of the area of observation, and u includes remainders from the Taylor series approximations, a term arising from assuming all firms locate at the center of the area, and the sum of the stochastic terms for firms in the area. The approximation terms will be small because the distance variation within census tracts, which determines the accuracy of the approximations used, will be negligible. More importantly, there is no reason to expect correlation between k and the approximation terms, which would induce bias in $\hat{\beta}$. Thus we are justified in ignoring the approximation terms and claiming that an OLS regression from (27) yields an unbiased and efficient estimate of $\beta = -B/y$, which is negative in all the unambiguous cases in (23).

Even if B(k) is not constant, the expectation of the OLS estimator from a regression equation of the form of (27) is negative if we ignore approximation terms. Integrating (24), we have

$$\log S = \int B(k)\, \frac{dw^*}{dk} + c \equiv Z(k) + c, \tag{28}$$

where the integral sign refers to the antiderivative operator. Assuming

(28) does not hold exactly and following the same approximation procedure as before, we get $\log X = Z(k) + v$, where k is now again the distance to the geographic center of the area of observation and v contains approximation terms and a random part. The expectation of the $\hat{\beta}$ estimator from a regression equation of the form of (27), ignoring the approximation terms in v, is

$$\frac{\Sigma\, Z(k_i)(t(k_i) - \bar{t})}{\Sigma(t(k_i) - \bar{t})^2}\ , \tag{29}$$

where the summations run over the T areas of observation and $\bar{t} = \Sigma t(k_i)/T$. Since $Z'(k) < 0$ when $B(k) > 0$, it is easily shown that (29) is negative. Since the expectation of $\hat{\beta}$ is negative whether or not B(k) is constant, we assume for simplicity in what follows that B(k) does not vary with k.

Fitting (27) to the data requires specification of a functional form for t(k). We used $a_0 + b_0 \log k$, $a_1 + b_1\sqrt{k}$, and $a_2 + b_2 k$, which result in equations which are linear in log k, $\sqrt{k}$, or k. Since $b_i > 0$, the slope estimates from the regressions still have negative expectations. We turn now to the results.

IV.

The regression results for the different forms of t(k) were qualitatively very similar, and we report results for the logarithmic form only. Small-area retail trade data were available for six cities: Chicago, Oklahoma City, Honolulu, Seattle, Baltimore, and Atlanta. Four different sources were available for Chicago, and the samples are denoted Chicago (CATS), Chicago (WWW), Chicago (CCRH), and Chicago,

where the letters in parentheses are abbreviations of the source title. The appendix lists for each sample the source, year, and size of the areal units of observation, as well as the units of measurement of variables. To the best of the author's knowledge, the data collected exhaust all generally available sources of small-area retail data.

The tables report the OLS estimates $\hat{\alpha}$ and $\hat{\beta}$ from the equation $\log X = \alpha + \beta \log k + u$, where X is the variable identified at the top of each table and k is the straight line distance from the CBD to the geographic center of the area from which the X observation was taken. The t-ratios appear in parentheses under the estimates and R^2 and degrees of freedom are listed. When the t-ratio exceeded 4 in absolute value, it was rounded off to the nearest integer to prevent the tables from looking too number-heavy. While R^2 adjusted for degrees of freedom is appropriate when comparing regression equations from different size samples, relatively large sample sizes result in little difference between the ordinary and adjusted R^2's when only two right-hand variables are present. When a coefficient is not significantly different from zero at the one percent level in a one-tailed test but is significant at the five percent level, the estimate is marked with a *. When the coefficient is not significantly different from zero at the five per cent level, it is marked with **. Although the model pertains to production outside the CBD, we report regressions with CBD included in, as well as deleted from, the sample.

In Table 1, X equals floor space per unit land. Both Chicago (CATS)results, which use different floorspace definitions, conform to expectations. The CBD-excluded $\hat{\beta}$'s are both negative and the absolute t-ratios and the R^2's are high. Both Oklahoma $\hat{\beta}$'s are negative and

highly significant when the CBD is included in the sample, but the slope estimate in the aggregated Oklahoma Sectors sample is insignificant when the CBD is deleted. The Oklahoma City and Oklahoma Sectors R^2's are low. Since $\beta = -b_0 B/y$, there is no reason to expect the $\hat{\beta}$ estimates to be comparable across cities. The components of B may well vary across urban areas, and variation in b_0 and y is also to be expected.

In Table 2, X is retail or grocery sales per unit total land area. Slopes are significantly negative in all cases, confirming a priori expectations, and R^2's are uniformly excellent. For each city except Honolulu, the absolute value of the CBD-included slope exceeds that of the CBD-excluded slope, suggesting that CBD production is characterized by higher sales per unit total land area than would be generated by an extrapolation of the CBD-excluded regression line.

In Table 3, X is retail employment per unit total land area, and the estimated $\hat{\beta}$'s are uniformly negative and significant and R^2's are fairly good. Recall that the model did not predict the sign of β in this case.

In Tables 4 and 5, the dependent variables are retail sales per retail worker and retail sales per square foot, respectively. One difficulty with the Chicago (CCRH) data is that the sales figures refer to establishments on both ground and upper floors, while square footage data pertain only to ground floor establishments. Letting SLS represent sales and FA, GFA, UFA represent total floor area, ground floor area, and upper floor area, we have

$$\log\left(\frac{SLS}{GFA}\right) = \log\left(\frac{SLS}{FA}\right) - \log\left(\frac{GFA}{FA}\right) = \alpha + \beta \log k + \log\left(1 + \frac{UFA}{GFA}\right) + u, \quad (29)$$

which can be approximated by

$$\alpha + \beta \log k + \gamma \text{ UFR} + u, \tag{30}$$

where UFR = N^U/N^G, with N^U and N^G the number of firms on upper and ground floors respectively, and where $\gamma = FA^U/FA^G$, with FA^U and FA^G the floor areas used by upper and ground floor firms. We estimate γ, which is assumed to be constant over k. Failure to account for $\log (1 + \frac{UFA}{GFA})$ in the error term of (29) could lead to downward bias in the estimate of β due to the likely negative correlation between k and $\log (1 + \frac{UFA}{GFA})$.

In Table 4, $\hat{\beta}$ is not significantly different from zero in Honolulu, while the estimate is significantly positive in Chicago, results which hold whether or not the CBD is excluded from the samples. In Chicago (CCRH), we get a significantly positive estimate in the samples with all retail centers and major centers only, while the estimate for neighborhood centers only is not significantly different from zero. All of these results are impossible under the model, which calls for declining sales per worker.

In Table 5, where (30) has been fitted, we see that $\hat{\beta}$ is never significantly different from zero. The lack of significance of $\hat{\gamma}$ is probably due to the fact that most centers had no upper floor establishments. For β to be zero, the B expression in (14) must be zero, which requires $\sigma_{RL}^G = 17.3$ given plausible values of 0.6, 0.85, 0.4, and 0.3 for g_L, $\dot{m}_g$, ρ_ℓ, and μ_ℓ respectively. This number is implausibly large, suggesting that the Table 5 results are inconsistent with the model. In the next section we explore various modifications of the model in an attempt to account for these contradictory empirical results.

V.

1. Variation in Store Mix over Distance

Suppose we have m retail goods, each produced at every distance. Then in place of (2), we have

$$\frac{p'}{p} - \rho_L \frac{w'}{w} - \rho_\ell \frac{r'}{r} = 0$$

$$\frac{z'}{z} - \mu_L \frac{w'}{w} - \mu_\ell \frac{r'}{r} = 0$$

$$\frac{s_i'}{s_i} - g_{Li} \frac{w'}{w} - g_{Ri} \frac{z'}{z} = 0 \qquad i=1,2,\ldots,m$$

$$\frac{w'}{w} - m_h \frac{p'}{p} - \sum m_{gi} \frac{s_i'}{s_i} = 0, \tag{31}$$

where s_i is the ith retail price, and g_{Li}, g_{Ri}, and m_{gi} are factor shares and the budget share for the ith retail good. Now (31) is an (m + 3)-equation system in the m + 3 unknown price gradients, and it may be solved for each price gradient in terms of w'/w. We may also compute expressions such as $d(s_i G_i/L_i)^*/dk$. It can be shown that when the individual $B_i(k)$ are constant and X is sales per worker,

$$\log X = \bar{\alpha} + \bar{\beta} \log k + \Sigma p_j \tau_j + \Sigma p_j \theta_j \log k + v,$$

where k is the distance to the geographic center of the census tract, $\bar{\beta} = \Sigma \beta_j/m$, $\bar{\alpha} = \Sigma \alpha_j/m$, $\tau_j = \alpha_j - \bar{\alpha}$, $\theta_j = \beta_j - \bar{\beta}$, and p_j is the proportion of firms in the census tract of type j. As before, v will contain approximation terms and a stochastic part. Suppose there is a higher concentration of high-α firms at large distances than at small distances.

Then $\Sigma p_j \tau_j$ will be positively correlated with k, and the estimate of $\bar{\beta}$ will be upward-biased. Since our prior belief is that the level differences (α) are more important than slope differences (β) across firm types, we report only estimates from the equation $\log X = \alpha + \beta \log k + \Sigma p_j \tau_j + v$, with X equal to sG/L and sG/R.

In the Chicago sample, two store categories were deleted to avoid multicollinearity problems, while in the Chicago(CCRH) sample it was necessary to delete 16 out of 26 classifications to achieve significant estimates. Most of the latter deleted categories had so few establishments in most centers that their deletion seemed defensible. We present Chicago estimates with the CBD included since previous results changed little when the CBD was excluded.

In Table 6, we see that the $\hat{\beta}$ estimate has decreased to 0.061 from its previous value of 0.108, indicating some upward bias was eliminated by controlling for store mix. The R^2 improves dramatically. However, since $\hat{\beta}$ is still positive and significant, the results of section IV were not due to a failure to account for a variation in store mix across distance. In Table 7, $\hat{\beta}$ is still not significantly different from zero, although the R^2 improves considerably. As above, $\hat{\gamma}$ is the estimated coefficient of UFR. Control for store mix does not change the conclusion that sG/R is constant over distance. Examination of the τ estimates indicates which types of establishments have comparatively low or high sales per worker or sales per square foot.

2. Income Variation over Distance

When income stratification is present, consumers live in annular areas segregated by income level. Suppose that retail labor is of the

same skill class as that in the poorest income group, with workers of this class living in residences scattered throughout all income areas. In this kind of situation, which seems to approximate reality, the qualitative results are the same as those in the simpler model. A modification that may lead to different results has retail production occurring according to different production functions in different income areas. This could reflect, for instance, the higher quality of retail goods in high than in low income areas. Our basic equation would then become

$$\log X = \alpha(y) + \beta(y) \log k + u, \tag{32}$$

where y is the level of income in the area of observation. Since we believe that the level effects of y are more important than the slope effects, we drop the dependence of β on y in the regressions.

Suppose X is sales per worker, which we postulate is higher in a high income than a low income area, holding k fixed. Since y is usually positively correlated with k, computing regressions using (27) when the true model is (32) with $\beta(y) \equiv \beta$ leads to an upward biased estimate of β. A similar argument holds when we control for store mix but $\alpha_j = \alpha_j(y) \equiv \alpha(y) + \tau_j$.

The choice of a functional form for $\alpha(y)$ is arbitrary, and we computed regressions using $\alpha(y) = \alpha_0 + \alpha_1 y$ and $\alpha(y) = \alpha_0 + \alpha_1 \log y$, where y is the median income of the area of observation, and we report the latter. Table 8 reports estimates from fitting $\log (sG/L) = \alpha_0 + \alpha_1 \log y + \beta \log k + u$, while Table 9 reports estimates from $\log (sG/L) = \alpha_0 + \alpha_1 \log y + \beta \log k + \Sigma p_j \tau_j + v$.

A lack of median income figures required deletion of about 150 Chicago census tracts. An unreported regression on this restricted sample with control for store mix but not for income level gave results very similar to those in Table 6, suggesting that the restricted sample, which contains only inhabited census tracts, was not fundamentally different from the unrestricted sample.

In Table 8, $\hat{\alpha}_1$ is significantly positive for the Chicago sample but is insignificant for Honolulu. Also, regressions for Honolulu with sales per acre and retail employment per acre show no significant effect of median income on these variables. Controlling for store mix in Chicago makes $\hat{\alpha}_1$ insignificant, suggesting that the effects of income variation are felt principally through store mix. However, in each of these cases the β estimate retains its former sign and significance, which means that the results of section IV are not attributable to a failure to account for income variation over distance.

3. Infinitely Durable Structures

Suppose that housing and commercial real estate are infinitely durable. Factor inputs embodied in structures are frozen in place. Current owners of commercial real estate, for example, receive

$$\pi_R = z(k)\bar{R}(k) - r(k)\bar{\ell}_R(k) - n\bar{N}_R(k), \tag{33}$$

where $\bar{R}(k)$, $\bar{\ell}_R(k)$, $\bar{N}_R(k)$ are the fixed values of firm output and inputs at distance k and $\bar{R}(k) = R(\bar{N}_R(k), \bar{\ell}_R(k))$. Uniform profits over distance requires

$$\frac{z'}{z} - \mu_\ell \frac{r'}{r} = -[(zR_N - n)\bar{N}'_R + (zR_\ell - r)\bar{\ell}'_R]. \tag{34}$$

where the term on the LHS of (34) results from differentiation of $\bar{R}$, $\bar{\ell}_R$, $\bar{N}_R$, which are no longer choice variables for the firm. The equivalent equation for housing producers is

$$\frac{p'}{p} - \rho_\ell \frac{r'}{r} = -[pH_N - n)\bar{N}'_H + (pH_\ell - r)\bar{\ell}'_R]. \tag{35}$$

If structures embodied optimal inputs, then $zR_N = n$, etc. and the LHS of both (34) and (35) would equal zero. However, in a world where incomes and transport costs are continually changing, there is no reason to expect frozen inputs to be optimal. If a producer chooses to operate some fraction δ of the structures initially constructed by one firm, then his profits are $\delta\pi_R$, where π_R is given by (33). As long as profits are zero, (34) still is necessary because $d(\delta\pi_R)/dk = \pi_R d\delta/dk + \delta\ d\pi_R/dk = 0$ requires $d\pi_R/dk = 0$, regardless of the value of $d\delta/dk$, when $\pi_R = 0$.

The equilibrium conditions for retail producers and consumers are the same as in (2), since these agents are free to adjust their use of structures. It can be shown that solution of the modified equilibrium system yields no predictions whatsoever about the sign of the urban price gradients. The spatial behavior of observables is similarly ambiguous. The results of section IV may be due to the kind of disequilibrium situation we have sketched, but the disequilibrium model has little predictive power in an urban setting, and taking refuge in its ambiguity does not seem to be a satisfactory way to rationalize our empirical results.

4. Retail Production with Three Factors

Above, we admitted the unrealism of excluding Q, the wholesale good input, from retail production. The model becomes much more complicated and ambiguous when we introduce it, however. Following Hicks [2], we

can formulate the retail profit maximization problem using the Lagrangean expression

$$sG - wL - zR - qQ - \lambda(G - G(L, R, Q)).$$

Symmetry of the bordered Hessian matrix allows us to deduce $dx/dp_y = dy/dp_x$ where x and y are two of the factors L, R, Q and p_x and p_y are their prices. In addition, symmetry results in

$$\frac{dG}{dp_x} = - \frac{dx}{ds} \qquad (36)$$

for $x \equiv L, R, Q$; $p_x \equiv w, z, q$. It always is true that

$$s\frac{dG}{dp_x} = w\frac{dL}{dp_x} + z\frac{dR}{dp_x} + q\frac{dQ}{dp_x} \qquad (37)$$

since $SG_L = w$, etc. From (36) and the symmetry of factor substitution (37) can be expressed

$$\frac{dx^*}{dw^*} + \frac{dx^*}{dz^*} + \frac{dx^*}{dq^*} + \frac{dx^*}{ds^*} = 0, \qquad (38)$$

for $x \equiv L, R, Q$. Since the own price effect is negative we must have $dx^*/dp_y^* > 0$ for at least one $y \neq x$, $x \equiv L, R, Q$. Letting $dx^*/dp_y^* = \phi_{xy}$ and $dx^*/ds^* = \phi_x$ we have[3]

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[3]We have used

$$\frac{dR^*}{dk} = \frac{dR^*}{dw^*}\,\frac{dw^*}{dk} + \frac{dR^*}{dz^*}\,\frac{dz^*}{dk} + \frac{dR^*}{ds^*}\,\frac{ds^*}{dk}$$

$$= (\phi_{RL} + c_1\phi_{RR} + c_3\phi_R)\,\frac{dw^*}{dk} \quad ,$$

and proceeded similarly for dL^*/dk and dQ^*/dk.

$$\frac{d(\frac{R}{L})^*}{dk} = [(\phi_{RL} - \phi_{LL}) + c_1(\phi_{RR} - \phi_{LR}) + c_3(\phi_R - \phi_L)] \frac{dw^*}{dk}$$

$$\equiv \theta_{RL} \frac{dw^*}{dk}$$

$$\frac{d(\frac{Q}{L})^*}{dk} = [(\phi_{QL} - \phi_{LL}) + c_1(\phi_{QR} - \phi_{LR}) + c_3(\phi_Q - \phi_L)] \frac{dw^*}{dk}$$

$$\equiv \theta_{QL} \frac{dw^*}{dk}$$

$$\frac{d(\frac{Q}{R})^*}{dk} = [(\phi_{QL} - \phi_{RL}) + c_1(\phi_{QR} - \phi_{RR}) + c_3(\phi_Q - \phi_R)] \frac{dw^*}{dk}$$

$$\equiv \theta_{QR} \frac{dw^*}{dk} \tag{39}$$

Clearly, θ_{RL}, θ_{QL}, and θ_{QR} are ambiguous in sign even when reasonable assumptions such as $\phi_{xy} > 0$, $x \neq y$, and $\phi_x > 0$ are made. In place of (13) and (14), we have

$$\frac{d(\frac{sG}{L})^*}{dk} = [g_L + g_R(c_1 + \theta_{RL}) + g_Q\theta_{QL}]\frac{dw^*}{dk}$$

$$\frac{d(\frac{sG}{R})^*}{dk} = [g_L(1 - \theta_{RL}) + g_R c_1 + g_Q\theta_{QR}]\frac{dw^*}{dk} \tag{40}$$

which are ambiguous in sign.[4] Similarly, $d(sG/\ell)^*/dk$ and $d(L/\ell)^*/dk$ are of indeterminate sign. The ambiguity of factor substitution when three factors are present eliminates many of the a priori predictions of the simpler model, suggesting that the results of section IV could be rationalized by appeal to this more realistic model of retail

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[4]Substitution of (39) into (40) does not lessen the ambiguity.

production. However, the predictive powerlessness of the three-factor model makes it less than attractive as a framework for analysing urban structure.

VI.

The goal of this study was to extend the standard neoclassical urban model and to ascertain whether the empirical robustness of the standard approach survives at a higher level of realism. The study shows that a simple model embodying assumptions A1 - A10 fails to capture reality, and that rescuing the basic approach requires appeal to a disequilibrium model of frozen structures or to a model with three retail factors of production. The widespread success of equilibrium urban models suggests that our contradictory results are not due to the durability of structures, while the viability of the three-factor rationalization must be explored by further work.

Another explanation for our results which is perhaps most plausible is that the spatial equilibrium generated by A1 - A10 is not the one that obtains in the world. While assumptions A1 - A10 were shown to be internally consistent, it may be that another set of sensible, self-consistent assumptions would lead to a spatial equilibrium with much different properties than the one that has been analysed. Theoretical work exploring other possibilities for spatial equilibria, especially with regard to consumer shopping travel, deserves high priority in future research in urban economics. Eventually a reasonable, empirically-supported model of retail production outside the CBD may emerge.

Appendix

Units of measurement:

retail sales -- $1000

employment -- actual values

floor space -- 1000's of square feet

land -- acres

distance -- miles

Sample descriptions (sample, year, source number, size of areal units):

Chicago (CATS) -- 1956, [8], grid zones (much larger than census tracts)

Chicago (WWW) -- 1968, [10], zip code areas

Chicago (CCRH) -- 1961, [7], retail centers at major street intersections

Chicago -- 1948, [11], census tracts

Oklahoma City -- 1965, [12], traffic zones (smaller than census tracts)

Oklahoma Sectors -- 1965, [12], aggregations of traffic zones (same sample as Oklahoma City)

Honolulu -- 1948, [9], census tracts

Seattle -- 1958, [13], clusters of usually two or three census tracts

Baltimore -- 1948, 1953, [6], planning zones each about 10 percent of the size of city

Atlanta -- 1961, [4] and [5], census tracts

Table 1

FLOOR SPACE PER UNIT LAND

	$\hat{\alpha}$	$\hat{\beta}$	R^2	d.f.
With CBD				
CHICAGO (CATS)				
1)[a]	4.73 (15)	-1.30 (-10)	.6950	40
2)[b]	5.84 (20)	-1.48 (-12)	.7777	40
OKLAHOMA CITY[c]	2.22 (29)	-.36 (-7)	.1429	306
OKLAHOMA SECTORS[c, d]	2.80 (11)	-.51 (-3.60)	.4982	13
Without CBD				
CHICAGO (CATS)				
1)	4.79 (13)	-1.32 (-9)	.6498	39
2)	5.75 (17)	-1.45 (-10)	.7235	39
OKLAHOMA CITY	1.93 (18)	-.17 (-2.40)	.0201	283
OKLAHOMA SECTORS	2.30 (9)	-.24** (-1.61)	.1784	12

[a] The dependent variable is log retail floor space per unit commercial land.

[b] The dependent variable is log retail + wholesale + service floorspace per unit commercial land.

[c] The dependent variable is log retail floorspace per unit retail land.

[d] This sample is based on an aggregation of Oklahoma City traffic zones into sectors.

**Estimated coefficient not significantly different from zero at the 5 percent level, one-tailed test.

Table 2

SALES PER UNIT TOTAL LAND AREA

	$\hat{\alpha}$	$\hat{\beta}$	R^2	d.f.
With CBD				
HONOLULU	3.17 (10)	-1.89 (-6)	.4476	48
SEATTLE	4.33 (14)	-1.41 (-7)	.5475	41
BALTIMORE[a]	5.20 (11)	-2.43 (-6)	.7860	10
BALTIMORE[b]	3.13 (9)	-1.40 (-5)	.6742	10
Without CBD				
HONOLULU	3.22 (8)	-1.94 (-5)	.3751	46
SEATTLE	4.10 (13)	-1.28 (-7)	.5155	40
BALTIMORE[a]	4.40 (10)	-1.83 (-5)	.7378	9
BALTIMORE[b]	2.87 (6)	-1.21 (-3.23)	.5365	9

[a]The dependent variable is log retail sales per unit total land area, 1948.

[b]The dependent variable is log grocery sales per unit total land area, 1953.

Table 3

RETAIL EMPLOYMENT PER UNIT TOTAL LAND AREA

With CBD	$\hat{\alpha}$	$\hat{\beta}$	R^2	d.f.
CHICAGO (WWW)	2.28 (10)	-1.30 (-11)	.7327	46
ATLANTA	.48 (2.54)	-1.45 (-6)	.3752	70
HONOLULU	.53** (1.59)	-1.91 (-6)	.4463	48
Without CBD				
CHICAGO (WWW)	1.43 (2.67)	-.90 (-3.43)	.2316	39
ATLANTA	.16** (.70)	-1.07 (-4)	.2037	67
HONOLULU	.57** (1.43)	-1.96 (-5)	.3719	44

**Estimated coefficient not significantly different from zero at the 5 percent level, one-tailed test.

Table 4

RETAIL SALES PER UNIT RETAIL LABOR

	$\hat{\alpha}$	$\hat{\beta}$	R^2	d.f.
With CBD				
HONOLULU	2.64 (42)	.024** (.42)	.0035	48
CHICAGO[a]	2.66 (98)	.105 (6)	.0367	891
Without CBD				
HONOLULU	2.65 (35)	.017** (.24)	.0013	46
CHICAGO	2.66 (88)	.108 (6)	.0321	889
CHICAGO (CCRH)[b]				
MAJOR CENTERS	3.17 (44)	.133* (2.02)	.0920	40
NEIGHBORHOOD CENTERS	3.21 (52)	.112** (1.55)	.1192	18
MAJOR PLUS NEIGHBORHOOD CENTERS	3.19 (69)	.116 (2.56)	.0976	60

[a]The Chicago sample has the best retail employment data of any sample that contains such data. While other samples present "retail employment" magnitudes, the Chicago sample has full-time workers, part-time workers, and active proprietors in retail establishments. Our labor measure is proprietors plus full-time workers plus 1/2 times part-time workers.

[b]The observations for this sample are from retail store clusters at the intersections of major streets. Neighborhood centers have a narrower representation of store types than major centers.

*Estimated coefficient significantly different from zero at the 5 percent level but not at the one percent level, one-tailed test.

**Estimated coefficient not significantly different from zero at the 5 percent level, one-tailed test.

Table 5

RETAIL SALES PER UNIT RETAIL FLOOR SPACE, CHICAGO (CCRH)

	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	R^2	d.f.
MAJOR CENTERS	-2.09 (-14)	.072** (.55)	-.141** (-.63)	.0209	39
NEIGHBORHOOD CENTERS	-1.69 (-7)	-.170** (-.58)	1.36** (1.08)	.0685	17
MAJOR PLUS NEIGHBORHOOD CENTERS	-1.84 (-14)	-.081** (-.63)	-.129** (-.45)	.0104	59

**Estimated coefficient not significantly different from zero at the 5 percent level, one-tailed test.

Table 6

SALES PER UNIT LABOR CONTROLLING FOR STORE MIX--CHICAGO

Estimate	Value	t-ratio	Store-type
$\hat{\alpha}$	2.79	48	--
$\hat{\beta}$	.061	3.76	--
$\hat{\tau}_1$	-.053**	-.86	grocery
$\hat{\tau}_2$	-.459	-5	eating and drinking
$\hat{\tau}_3$	-2.54*	-2.08	general stores
$\hat{\tau}_4$	.763	2.35	general merchandise
$\hat{\tau}_5$	-.317	-2.44	apparel
$\hat{\tau}_6$	.262**	1.28	furniture
$\hat{\tau}_7$	2.17	13	automotive
$\hat{\tau}_8$	.023**	.17	gas
$\hat{\tau}_9$	.813	3.05	lumber
$\hat{\tau}_{10}$	.089**	.40	drugs
$\hat{\tau}_{11}$	-.296**	-1.43	liquor

$R^2 = .2546$ d.f. = 874

*Estimated coefficient significantly different from zero at the 5 percent level but not at the one percent level, one-tailed test.

**Estimated coefficient not significantly different from zero at the 5 percent level, one-tailed test.

Table 7

SALES PER UNIT FLOORSPACE CONTROLLING FOR STORE MIX--

CHICAGO (CCRH)--MAJOR PLUS NEIGHBORHOOD CENTERS

Estimate	Value	t-ratio	Store-type
$\hat{\alpha}$	-.945**	-.86	--
$\hat{\beta}$	.075**	.44	--
$\hat{\tau}_1$	-4.61**	-1.52	department, general stores
$\hat{\tau}_2$	.433**	.21	grocery
$\hat{\tau}_3$	-7.55**	-1.10	automotive and gas
$\hat{\tau}_4$	-1.03**	-.83	apparel
$\hat{\tau}_5$	-1.01**	-.68	furniture and appliances
$\hat{\tau}_6$	.505**	.29	eating and drinking
$\hat{\tau}_7$	-1.30**	-.70	miscellaneous
$\hat{\tau}_8$	-6.00*	-1.90	banks
$\hat{\tau}_9$	-.812**	-.51	misc. personal services
$\hat{\tau}_{10}$	-12.85	-2.57	supermarkets
$\hat{\gamma}$	-.021**	-.08	--

$R^2 = .3158$ d.f. = 49

*Estimated coefficient significantly different from zero at the 5 percent level but not at the one percent level, one-tailed test.

**Estimated coefficient not significantly different from zero at the 5 percent level, one-tailed test.

Table 8

SALES PER UNIT LABOR CONTROLLING FOR INCOME LEVEL

	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}$	R^2	d.f.
CHICAGO[a]	2.55	.114	.091	.0905	720
	(67)	(2.86)	(4)		
HONOLULU	2.50	.126**	.001**	.0080	46
	(7)	(.38)	(.02)		
Sales per Unit Land					
HONOLULU	2.27**	.860**	-2.06	.4476	46
	(1.27)	(.49)	(-4)		
Retail Employment per Unit Land					
HONOLULU	-.234**	.734**	-2.06	.4455	46
	(-.13)	(.41)	(-4)		

[a]All samples include the CBD.

**Estimated coefficient not significantly different from zero at the 5 percent level, one-tailed test.

Table 9

SALES PER UNIT LABOR CONTROLLING FOR INCOME LEVEL AND STORE MIX--CHICAGO

Estimate	Value	t-ratio
$\hat{\alpha}_0$	2.71	45
$\hat{\alpha}_1$	.044**	1.18
$\hat{\beta}$	.066	3.43
$\hat{\tau}_1$	-.012**	-.21
$\hat{\tau}_2$	-.404	-4
$\hat{\tau}_3$	-3.10	-2.84
$\hat{\tau}_4$	.486**	1.30
$\hat{\tau}_5$	-.123**	-.98
$\hat{\tau}_6$	.290**	1.46
$\hat{\tau}_7$	2.60	14
$\hat{\tau}_8$	.008**	.05
$\hat{\tau}_9$	-.060**	-.29
$\hat{\tau}_{10}$	-.207**	-.93

R^2 = .3182 d.f. = 709

**Estimated coefficient not significantly different from zero at the 5 percent level, one-tailed test.

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