Modern Commercial Arithmetic

MODERN

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PREFACE

A course in business arithmetic should train the pupil to figure correctly, easily, and rapidly, and should fit him to solve the problems that arise in the ordinary course of business. To this end Modern Commercial Arithmetic gives a brief review of the fundamental operations, fractions and decimals, introducing short practical methods. The mechanical part of arithmetic is illustrated and explained by diagrams, examples, operations, and notes. The intellectual part is developed in the pupil's mind by mental problems, questions, and statements. A student should solve a problem from his knowledge of the facts or conditions of the problem and the principles involved; therefore, rules and cases are superseded by development exercises which will make him thoughtful and independent.

In the business office problems do not come tabbed with article and rule, but the business man must first discover the principle that is involved and then by a process of reasoning determine the result. It has been the object of this work to present the problems as nearly as possible as they are presented in the business office. When the pupil changes from the school to the office he will find the change in the method of thought involved as slight as possible.

The student in school has no time to waste and therefore this work contains no puzzles or catch problems. Subjects that do not arise in ordinary business transactions are omitted.

The author's aim has been to present a work that would give the pupil such instruction as he needs, and to set it forth in the manner he will meet it in the business office. It is hoped and believed that an inspection and trial of the work will show that he has succeeded.

CONTENTS

MODERN COMMERCIAL ARITHMETIC

NOTATION AND NUMERATION

1. The writing of numbers is called Notation.

2. The reading of numbers is called Numeration.

3. Numbers may be written in three ways: 5, five, V.

The first method was used by the Arabs, and it is called the Arabic system of notation.

THE ARABIC SYSTEM

4. There are ten figures used in this system. Each of the following figures is a digit, and represents a number: 1, 2, 3, 4, 5, 6, 7, 8, 9. 0 is a figure, but it does not represent a number. It is used with other figures to represent numbers.

In counting, units are grouped into units, tens, hundreds, thousands, tens of thousands, hundreds of thousands, millions, etc. 10 of one group make 1 of the next larger group. Thus, 10 units make 1 ten, 10 tens make 1 hundred, 10 hundreds make 1 thousand, 10 thousands make 1 ten-thousand, 10 ten-thousands make 1 hundred-thousand, 10 hundred-thousands make 1 million, etc.

The number 1234567 contains 7 units, 6 tens, 5 hundreds, 4 thousands, 3 tens of thousands, 2 hundreds of thousands, and 1 million. It may be described as 7 units of the first order, 6 of the second, 5 of the third, 4 of the fourth, 3 of the fifth, 2 of the sixth, and 1 of the seventh.

5. *Principles.*

1. Orders of units increase from right to left in a tenfold ratio.

2. Orders of units decrease from left to right in a tenfold ratio.

6. In reading numbers, three figures are grouped into a period and read as follows:

Millions,	Thousands,	Units
203,	203,	203

The number is read: 203 million, 203 thousand, 203.

EXERCISES

7. Read: 3002, 42,005, 40,205, 400,200, 402,204, 400,020, 402,030, 4,003,001, 4,010,050, 50003002, 50030020, 50300200, 300020070, 301201701, 110001010.

Remark.—Numbers above hundreds of millions are seldom used in business. When used, such numbers are read as millions. Thus, 2,467,845,427 is read 2467 million, 845 thousand, 427.

Write: Twenty thousand, eighty. One hundred thousand, forty-six. Three million, thirty thousand, fifteen. Two hundred four million, eighteen thousand, one hundred fifty. Two thousand eighty-five million, seventy thousand, three. Twenty million, fourteen thousand, forty. One hundred three million, one hundred three. One thousand four million, one thousand, four. One thousand five million, five hundred five thousand, five hundred five.

ROMAN NOTATION

8. This system was used by the Romans.

Letters used:	I	V	X	L	C	D	M
Values:	1	5	10	50	100	500	1000

9. *Principles.*

1. Repeating I, X, C, or M repeats its value. Thus, III = 3, XX = 20, CCC = 300. These letters are not repeated more than three times, although we find IIII on clocks, and 400 is sometimes written CCCC.

2. When I is before V or X, X before L or C, C before D or M, the values of the letters are subtracted. Thus, IV = 4, IX = 9, XL = 40, XC = 90, CD = 400, CM = 900.

3. When one letter is placed after another of greater value, their values are added. Thus, VI = 6, XI = 11, LX = 60, CI = 101.

4. A dash placed over a letter multiplies its value by 1000. Thus $\overline{V} = 5000$, $\overline{XI} = 11000$, $\overline{LX} = 60000$.

10. *How to Write Numbers.*

1. Use the expressions IV, IX, XL, XC, CD, CM as one letter.

2. Write the letter, or expression, whose value is nearest that of the required number, but less than the required number, and add letters to this expression until the required number is obtained.

To write 19, first write 10 (X), then add 9 (IX). To write 28, first write 10 (X), add 10 (X), add 5 (V), add 3 (III). To write 49, write XL, and add IX. To write 99, write 50 (L), add 40 (XL), add 9 (IX).

11. Write: 29, 34, 49, 51, 69, 89, 91, 219, 289, 391, 1047, 1863, 1899, 1900, 20678, 4685, 3569, 56435, 4567, 1365, 3709.

Read: XXXIX, XCV, XCIX, CIX, CXIX, DCVI, DCCXXIX, DCCCXL, CMXIV, MDCCXXVII, MDCCCLXI, MCMXCIX.

ADDITION

12. Addition is the most difficult process in arithmetic, for it is not done until it is correctly done. When the columns of numbers added are long, mistakes are likely to occur. Care and practice are necessary to enable one to add correctly and with a reasonable degree of rapidity. Probably the first test or trial an employer will give an applicant for a position as bookkeeper will be to add, and the test may be important. Those who add well generally perform the other operations well. It is worth while to learn to add correctly and rapidly.

13. The mental process in adding consists in grouping digits of the same order. No matter how many the numbers added may be, the whole work is to group and combine digits. In adding by any method, results only should be mentioned or thought of. One should not name, even mentally, the digits combined. In adding the digits 7, 2, 4, 6, one by one, think "9, 13, 19," not "7 and 2 are 9, 9 and 4 are 13, 13 and 6 are 19."

Instead of adding digits one by one, one may group two or more digits and combine the groups. Thus, in adding 4, 5, 6, 2, 3, 7, 4, 6, the digits may be combined into groups of two each. Then "17, 27, 37," thinking of results only. Of course, one may think of the groups and the digits in the groups, but they are minor subjects of thought. The *attention* should be on the results.

HOW TO MAKE GROUPS

14. Making groups and combining them is the whole of the group method. In combining the groups it is important that they be made quickly and to advantage. It is easy to combine 10's and 20's, and it is therefore advantageous to make groups of 10's and 20's.

Groups of two digits that produce 10·

9	8	7	6	5
1	2	3	4	5

These groups should be recognized *at sight.* Thus, when the pupil sees $\begin{matrix}7\\3\end{matrix}$, he should think "10." These groups should be so well known that the pupil may give his whole attention to combining the groups.

Groups of three digits that produce 10:

1	1	1	1	2	2	2	3
1	2	3	4	2	3	4	3
8	7	6	5	6	5	4	4

Groups of three digits that produce 20:

9	9	9	9	8	8	8	7
9	8	7	6	8	7	6	7
2	3	4	5	4	5	6	6

All possible groups of two digits each. There are only 45 such groups:

1	1	1	1	1	1	1	1	1	2	2	2	2	2	2
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7
2	2	3	3	3	3	3	3	3	4	4	4	4	4	4
8	9	3	4	5	6	7	8	9	4	5	6	7	8	9
5	5	5	5	5	6	6	6	6	7	7	7	8	8	9
5	6	7	8	9	6	7	8	9	7	8	9	8	9	9

The pupil may know the sum of each of these groups, but he should accustom himself to think of the sum of each group instead of the two numbers in each group. He should think results only, and should be thoroughly drilled on these groups.

Simplest Group Method

15. The ordinary pupil has learned to add by combining digits one by one. Thus, in adding 2, 5, 7, 3, 4, 2, 6, 3, 7, 2, he thinks: 7, 14, 17, 21, 23, 29, 32, 39, 41.

The simplest group method is to combine two or more

digits whose sum is 10 or less than 10, and add as above. Thus, the pupil in adding the above numbers would group and think as follows, the numbers in parentheses show the groups made: (2, 5) 7, (7, 3) 17, (4, 2) 23, (6, 3) 32, (7, 2) 41.

EXAMPLE.—Add the following column of numbers, making groups of 10 or less:

435
274
167
921
786
205
472

3260

EXPLANATION.—The parentheses show the groups made. Begin at the bottom and add upward, naming results only. (2, 5) (6, 1) 14, 21, (4, 5,) 30. Write 0, carry 3. Add (3, 7) 10, (8, 2) 20, 26, (7, 3) 36. Write 6, carry 3. Add (4, 2) 9, 16, (9, 1), 26, (4, 2) 32. Write 32.

EXAMPLE.—Add the following:

6708
4352
1637
9452
3614
2373
4836
2552

35524

EXPLANATION.—Add as in the preceding example. (2, 6) (3, 4) 15, (2, 7) 24, (2, 8) 34. Write 4, carry 3. Add 3, (5, 3) 11, (7, 1) 19, (5, 3) 27, 32. Write 2, carry 3. 3, 8, 16, (3, 6) 25, (4, 6), 35, (3, 7) 45. Write 5, carry 4. 4, (2, 4) 10, (2, 3) 15, (9, 1) 25, (4, 6) 35. Write 35.

Straight Grouping and Irregular Grouping

16. Making groups of digits in the order in which they are written may be called straight grouping. Selecting digits out of their regular order to make groups may be called irregular grouping. To make groups equal to 10 or 20, it may be necessary to skip about in the column. The advantage in combining groups of 10 and 20 justifies such irregular grouping. If the digits in a column run 3, 0, 6, 7, 4, we may group 3 and 7, and 6 and 4. If the digits run 8, 8, 7, 4, we may group 8, 8, and 4, and leave 7 to be made into another group or to be added separately.

PROBLEMS

17. Add the following columns of numbers, making groups of 10 or less:

1.	2.	3.	4.	5.	6.	7.
5843	25364	465371	386754	369625	36825691	46387625
2671	72459	245827	538432	537167	35718356	18253751
9326	72459	274336	276345	362841	35624352	26715436
1473	57531	524386	534785	275146	35625436	63726354
5845	14328	295437	369328	264375	25634253	63527152
3264	46534	624852	574183	536271	53462534	26372453
5728	73291	735968	263715	296345	25635417	35715487
4532	52618	243143	497462	527164	45278164	53716482
2674	45423	538427	513728	356278	37452735	37184527
3426	28936	253673	354372	357281	53672845	25735427

8.	9.	10.	11.	12.
26748353	25749574	39615385	25647154	27845398
17639052	30816748	25674926	37185378	26894536
73620874	20981632	15674892	37882567	25037524
10835627	35718253	71565735	61782035	26735526
16738625	35618536	25637526	35629816	28019635
19035276	51673871	26018635	52673815	27618763
52176351	56271463	63728165	73625437	18674536
93827362	26173815	37227384	93026375	27133926
29637256	34685647	24674468	34256709	45136784
25135487	45789643	23125376	35867657	98787656
15463787	23543476	56984765	25769804	56875674
25416748	35984627	35682536	35987164	37618835

13.	14.	15.	16.	17.
15367283	15427685	24537657	43519687	35239675
27742625	76865432	76583251	65763452	76972541
27531874	76574532	76457361	45347648	34135268
77615524	74563424	76593425	76583242	76854325
37106352	76484323	35246745	76953420	74693542
35628163	98063514	63203516	17035263	87063521
46245163	90584615	76845361	25761536	37825163
27462738	27093546	16725830	35627916	25673541
36547684	70561825	26174382	25364534	46874536
25543423	63572514	27615437	27645362	25639653
38762453	18674523	28764358	26517094	25763542
29861542	17625376	35245142	37617284	25160939
25167253	17620935	37825391	25635427	20198352
35426155	15269354	26175243	47654372	25617542
36812563	36782546	37652453	37905615	36873526
26714526	25634516	53672534	90635243	63752635
36254177	24536426	35745246	35649182	63725416
73612803	52719352	26748234	25436145	26415536

18.	*19.*	*20.*	*21.*	*22.*
43523543	45315436	165247	253762	375263
67865434	26794572	167354	352435	635426
32145673	72326436	156345	635645	435261
87694351	76945332	896547	457654	365426
37094532	27840354	265346	256354	486745
76456324	25653543	254155	244362	256341
96352743	37615224	175243	524152	263435
25637184	26185643	864532	367453	365473
37689452	37614523	163725	367281	354263
27610934	27615437	908564	367352	532718
28165437	15426435	256371	453626	376281
28919735	26514326	187645	291853	109673
36537462	25637745	902876	267352	536745
38964532	45673542	873452	145352	536476
25763415	25346732	253645	547635	348273
53672543	35426634	273654	351674	536018
25735462	25639064	534614	425134	234162
82453142	35261743	256173	356472	291864
24316423	35241364	156274	852637	254163
24719342	16354276	526376	256173	253452

23.	*24.*	*25.*	*26.*	*27.*
367251	256372	356271	253645	109354
376534	254364	256453	256453	755463
534265	547654	345263	264273	453432
276354	243524	756473	356473	253647
345243	254365	345253	764534	251652
371852	634524	271563	254362	253624
735261	278435	253462	156372	357453
356453	482617	251736	251637	253746
352617	356453	926504	251647	352671
356576	453664	345634	354625	356427
946732	534167	352463	245362	834925
261832	256173	352671	764836	874693
768546	675846	764836	536281	648376
534253	356453	457154	376283	473865
381965	409756	381794	361748	356738
389164	361748	657436	358467	876452
571684	356271	254615	345261	357152
356046	647835	280193	355627	861744
357849	609654	351709	356173	254363
524534	645352	764352	267453	352634

All Possible Groups of Three Digits Each

18. There are 165 such groups:

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3
1	2	3	4	5	6	7	8	9	2	3	4	5	6	7	8	9	3	4
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	6	6	6
5	6	7	8	9	4	5	6	7	8	9	5	6	7	8	9	6	7	8
1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2
6	7	7	7	8	8	9	2	2	2	2	2	2	2	2	3	3	3	3
9	7	8	9	8	9	9	2	3	4	5	6	7	8	9	3	4	5	6
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	4	4	4	4	4	4	5	5	5	5	5	6	6	6	6	7
7	8	9	4	5	6	7	8	9	5	6	7	8	9	6	7	8	9	7
2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3
7	7	8	8	9	3	3	3	3	3	3	3	4	4	4	4	4	4	5
8	9	8	9	9	3	4	5	6	7	8	9	4	5	6	7	8	9	5
3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4
5	5	5	5	6	6	6	6	7	7	7	8	8	9	4	4	4	4	4
6	7	8	9	6	7	8	9	7	8	9	8	9	9	4	5	6	7	8
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	5	5	5
4	5	5	5	5	5	6	6	6	6	7	7	7	8	8	9	5	5	5
9	5	6	7	8	9	6	7	8	9	7	8	9	8	9	9	5	6	7
5	5	5	5	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6
5	5	6	6	6	6	7	7	7	8	8	9	6	6	6	6	7	7	7
8	9	6	7	8	9	7	8	9	8	9	9	6	7	8	9	7	8	9
6	6	6	7	7	7	7	7	7	8	8	8	9						
8	8	9	7	7	7	8	8	9	8	8	9	9						
8	9	9	7	8	9	8	9	9	8	9	9	9						

NOTE.—If the pupil will learn to think the sum of each of these groups at sight, instead of adding one by one, or two by two, he will be able to add three digits at a time. He may omit groups whose sum exceeds 20.

Combining Groups between 10 and 20

19. It is easy to combine groups of 10's and 20's. It requires practice to combine rapidly and accurately groups like 15, 17, 19. In combining such groups, particular atten-

tion should be given to the unit figures, for mistakes are generally made in combining units, not in combining tens.

All the combinations (45) that can be made with numbers between 10 and 20 are as follows:

11	11	11	11	11	11	11	11	11	12	12	12	12	12
11	12	13	14	15	16	17	18	19	12	13	14	15	16
12	12	12	13	13	13	13	13	13	13	14	14	14	14
17	18	19	13	14	15	16	17	18	19	14	15	16	17
14	14	15	15	15	15	15	16	16	16	16	17	17	17
18	19	15	16	17	18	19	16	17	18	19	17	18	19
18	18	19											
18	19	19											

The unit figures in these groups are the same as those in the 45 combinations of the nine digits. The sum of the tens is 2 in each group. If the sum of the units in any group is 10 or more, the tens in that group will be increased by 1. Think what the unit figure will be. Then the tens figure will be either 2 or 3. If the combinations of the digits have been learned, the combinations of these numbers will soon be mastered. Combining 16 and 19 is the same as combining 6 and 9, and adding 2 tens to the sum. Combining any two numbers between 10 and 20 is the same as combining their unit figures, and increasing the sum by 2 tens.

Device for Drill Work

20. Let the teacher draw on the board a diagram like the following: In the space between the two rings are all the numbers between 10 and 20. In the center may be placed, successively, each of the numbers between 10 and 20. The pupil should be required to combine the number in the center with each number between the rings, naming only the *sum* of the numbers. The pupil should begin at one point and go around the ring. Thus he may begin at 12, and say: 25, 27, 30, etc. This device will give thorough and rapid

drill, and the pupil may profitably use it outside of the recitation.

All Combinations of Numbers between 10 and 20 with all Numbers between 10 and 100

21. If the pupil can make these combinations readily, he will be in possession of the "lightning method."

If the pupil has learned the combinations under Art. 19, he can readily learn the combinations under this article. The wheel for drill work under Art. 20 may be used to develop all the possible combinations here. With the numbers between 10 and 20 in the wheel, and the number 13 in the center, as shown in the diagram, the pupil may make 81 combinations. Thus, beginning with $\begin{matrix}13\\12\end{matrix}$, which he can see on the wheel, he may continue mentally,

13	13	13	13	13	13	13	13
22	32	42	52	62	72	82	92

Then he may take $\begin{matrix}13\\14\end{matrix}$, the next combination on the wheel, and combine

13	13	13	13
24	34	44	54

etc. Next he may combine

13	13	13	13
17	27	37	47

and so on. He may use each number between 10 and 20 in the center.

There are no more combinations of units under this article than there are under Art. 19. The only difference is in the tens.

PROBLEMS

22. Solve the problems under Art. 17, making groups of numbers from 10 to 20 inclusive.

Also add the following columns.

NOTE.—Some teachers may prefer to omit this exercise now, and teach the pupil to add by the cipher method or by the method of rejecting tens.

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*
746092	356189	849756	357925	672109	261752	251673
615427	256352	256381	987536	267154	256371	377163
635867	263451	253645	386728	904625	109725	376281
251673	376184	456098	378194	476386	837625	156253
906498	378294	783902	389173	371839	274681	378294
274891	356745	278357	287467	173823	490879	387164
547993	371846	275637	190896	389523	779174	265784
537683	453789	809251	437892	709261	345672	456728
245681	907896	475829	356281	167493	267356	908957
534617	264357	785536	671526	390895	378456	267453
908576	378164	467584	467582	367153	178098	190758
256173	381745	189047	356174	637456	190467	378594
352635	467189	356453	578264	390183	354671	267184
345671	278590	675987	461736	254367	356174	256738
354672	567438	546781	689053	563748	253672	356478
534672	356271	386478	467590	409873	536728	356278
156253	273884	534721	254671	356713	356183	356472
256173	356174	276453	371809	309814	456173	356172
309735	253781	367184	276153	908153	456153	460981
376154	124563	908945	356278	267493	251674	748367

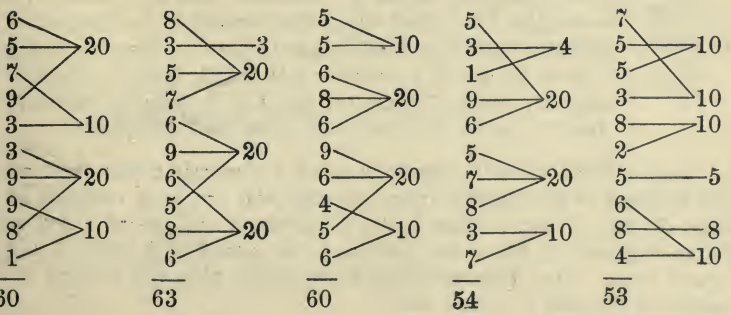
8.	*9.*	*10.*	*11.*	*12.*	*13.*	*14.*
554367	356904	378904	356076	365243	377815	251436
635736	356253	346891	380965	356253	368256	356189
536715	253671	356872	268467	901645	276489	256378
356187	256184	356209	378915	256109	350987	536173
356187	356274	356278	256173	256374	366453	356274
904687	378987	367187	789762	356274	378190	378167
859045	356287	153617	354613	674657	356184	678098
352464	456374	465745	341745	389178	467589	671523
526354	657890	351647	456374	467563	152463	351672
256153	356456	789153	467819	378918	678193	609871
261873	678163	254167	409871	564073	378109	367194
360192	108391	357184	356273	371835	674839	789463
908758	478567	675876	678354	617356	256173	371835
785094	367184	367184	153723	478194	367855	361738
901746	352617	361846	453152	768970	345241	768970
152435	678706	132453	670078	451324	670789	453314
352415	676097	678796	345241	253425	678709	679687
352415	674533	463718	894675	678969	352453	678698
256104	453253	609687	908968	675446	467364	354672
674352	352673	356278	162543	908718	674098	361014

15.	*16.*	*17.*	*18.*	*19.*	*20.*	*21.*
367183	356280	467587	467584	567483	467384	374837
389106	671908	367184	361893	267134	235243	676987
253617	453671	360985	473645	342515	678678	567483
901235	367183	253671	356183	268493	256109	356173
906879	467354	368193	367281	467384	453674	281947
960687	453627	352415	657869	906879	567483	352633
109273	456378	907685	758495	356475	617382	354609
470194	367495	467382	638485	133749	351848	546473
905736	351635	451736	251674	467583	101886	461735
905681	253471	456183	901857	567685	567685	678509
891025	675847	467584	467584	567485	391046	908679
906870	567006	564732	152453	674536	514235	671884
155367	251453	617352	184761	352785	906875	647382
905768	574857	567485	678968	906879	678596	132453
102947	647283	467384	362718	253647	895047	850495
102847	564738	291837	647859	905748	356173	785686
605674	172959	637152	675448	152673	718938	188965
536718	354678	996574	471836	768006	675869	567281
387596	162839	452637	152435	261745	352674	786960
678957	456374	568676	787967	857465	860978	564735

The Cipher Method

23. By this method the student makes groups of 10's and 20's, and combines them with the other digits in groups or one by one. By practice in irregular grouping, the student can put many of the digits into groups of 10 or 20.

The following columns show how such groups may be made:



PROBLEMS

24. Look through the columns of figures under Articles 17 and 22, and make groups of 10's and 20's. Skip about if necessary, but do not try to put all the digits into such groups.

NOTE.—The teacher should assign some of the problems under Articles 17 and 22, to be added by the cipher method.

Method of Rejecting 10's

25. By this method the student rejects the 10's (does not hold them in memory), but retains them on a piece of paper or on the fingers. The mind is thus relieved from keeping account of the 10's, and addition becomes an easy operation. The student adds, one by one or by groups, until he has from 10 to 20, rejects 10, begins again with what he has left and adds until he has from 10 to 20, rejects 10, and so on.

EXAMPLE.—Add the following column of numbers, rejecting 10's:

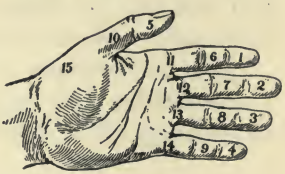
684
732
876
934
782
649
537
468
752
836
7250

EXPLANATION.—Begin at the bottom. Add 6, 2 and 8; having more than 10, drop 10 and begin again with the 6 left (for every 10 dropped, makes a mark on a slip of paper). Add 6 and 7; having more than 10, drop 10 and begin again; add 3 and 9, drop 10 and begin over; add 2, 2, 4, and 6, drop 10 and begin over; add 4 and 6, drop 10; add 4, 2, and 4, drop 10. There are no units left. Write 0 in units' place. 5 tens have been dropped. Add the second column, including the 5 tens dropped from the first. Add 5, 3, and 5, drop 10; add 3, 6, and 3, drop 10; add 2, 4, 8, drop 10; add 4, 3, 7, drop 10; add 4, 3, 8, drop 10; write the 5 left, and carry the 5 dropped. Add the third column, including the 5 dropped from the second. Add 5, 8, drop 10; add 3, 7, drop 10; add 4, 5, 6, drop 10; add 5, 7, drop 10; add 2, 9, drop 10; add 1, 8, 7, drop 10; add 6, 6, drop 10; write the 2 left, also write the 7 dropped.

NOTE.—Hold the units clearly in mind, but merely notice the tens. The tens are to be dropped from memory, and are to be recorded by some device. When adding 7 and 9, think 6 and record 1 ten as being dropped. If the next number to be added is 8, think 4 and record 1 ten. Give close attention to the units, give just enough attention to the tens to record them.

Device for Recording 10's

26. The following cut shows how the 10's may be recorded. Number the joints of the thumb, beginning at the end, 5, 10, 15; the joints of the first finger, 1, 6, 11; the second finger, 2, 7, 12; the third finger, 3, 8, 13; and the fourth finger, 4, 9, 14.



To record 1 ten, place the thumb on the joint of the little finger marked 1. In like manner, 2 tens, 3 tens, and 4 tens may be recorded on the other fingers. To record 5 tens, straighten the thumb. To record 6 tens, place the thumb on the second joint of the little finger. In like manner, 7 tens, 8 tens, and 9 tens may be recorded on the other fingers. To record 10 tens, straighten the thumb. And so on. By a little practice, the pupil will be able to record the tens on his hand and know when he is through with a column just how many tens have been dropped. If the count ends when his thumb is straightened, he must know, of course, whether it has been straightened once, twice, or three times. If any column contains more than 15 tens, the student may repeat the count on his hand.

Civil Service Method of Recording Partial Sums

27. If the sum of each column added be written in full, instead of writing but one figure, the accountant can leave his work after adding part of the number of columns and resume it without re-adding any of the columns. The addition of any column may be verified without re-adding the other columns to find the figure carried.

The following illustrates the method:

4628	
7394	25
5462	30
6856	28
4785	26
29125	

The sums of the columns are 25, 30, 28, 26, and they are written under one another, each succeeding number one place to the left. The partial sums are then added and the result placed under the numbers added.

EXERCISES

28. Look through the columns under Articles 17 and 22, and combine the digits, dropping the tens and thinking of the units only. Do not record the tens, but drop them.

Add the columns under Articles 17 and 22 by repeating the above exercise, keeping on the hand a record of the tens dropped and writing the proper figure under each column.

Horizontal Addition

29. Sometimes numbers that are to be added are found written in horizontal lines. They can be added without rewriting them in vertical columns. In adding horizontally, from right to left or from left to right, care must be taken to combine only like orders of units.

PROBLEMS

30. Add the following:

1. 45, 34, 67, 35, 98, 56, 74, 37, 25, 74, 34, 64.
2. 123, 435, 609, 524, 274, 315, 376, 903, 457.
3. 260, 789, 635, 256, 235, 170, 585, 370, 245.
4. 1245, 3465, 9075, 1260, 4561, 2351, 6754, 4532.
5. 2745, 706, 486, 3450, 235, 5687, 250, 1274, 378.
6. 652, 3476, 6789, 376, 4523, 8097, 560, 4563, 276.
7. 476, 6453, 254, 8609, 560, 3124, 3654, 450, 231.
8. 3500, 3540, 350, 3456, 873, 2578, 2154, 687, 3542.
9. 5476, 9050, 576, 3542, 1456, 684, 9085, 541, 355.
10. 35672, 5609, 90504, 526, 500, 8050, 45763, 375.
11. 3540, 6750, 4535, 548, 46876, 78045, 362, 4752.
12. 45, 687, 50943, 475, 9967, 4500, 450, 35460.
13. 4536, 567, 9075, 32165, 45, 768, 75, 35025, 268, 4575.
14. 550, 6735, 98, 675, 558, 75, 35450, 265, 4235.
15. 3750, 790, 89, 563, 4576, 256, 950, 5425, 635.
16. 4576, 9056, 75, 356, 87, 2457, 675, 9050, 350.
17. 5490, 5467, 580, 25, 69, 599, 4563, 2875, 654.
18. 4678, 6780, 47, 387, 5674, 3265, 745, 87, 4765.
19. 367, 90, 9835, 453, 69, 2543, 657, 45, 76255.
20. 54376, 9004, 5476, 899, 654, 37459, 362, 7623.

SUBTRACTION

Subtraction by Addition

31. EXAMPLES.

786 minuend
243 subtrahend
543 remainder

243 + 543 = 786
subtrahend + remainder = minuend

243 + ? = 786

To find the remainder, find a number which added to the subtrahend will produce the minuend. The best way to find that number is to add to the subtrahend, digit by digit, until the minuend is produced. Solving the example at the top of this page, one would say, beginning at the right hand, 3 (subtrahend) and 3 (write it in the remainder) are 6 (minuend). 4 (sub.) and 4 (write in rem.) are 8 (min.) 2 (sub.) and 5 (write in rem.) are 7 (min.)

EXAMPLE.—From 968 take 425.

OPERATION

968
425
543

EXPLANATION.—5 (sub.) and 3 (rem.) are 8 (min.) 2 (sub.) and 4 (rem.) are 6 (min.). 4 (sub.) and 5 (rem.) are 9 (min.).

EXAMPLE.—From 7023 take 4326.

OPERATION

7023
4326
2697

EXPLANATION.—6 (sub.) and 7 (rem.) are 13 (which gives the figure in the minuend). Carry 1. 2 (sub.) and 1 (carried) and 9 (rem.) are 12 (min.). 4 (sub.) and 1 (carried) and 2 (rem.) are 7(min.) This process is called subtraction by addition. It is easy, and is productive of the highest degree of accuracy and rapidity.

PROBLEMS

1. 4670 – 2463 = ?	*9.* 507433 – 285362 = ?
2. 8609 – 3726 = ?	*10.* 429728 – 136475 = ?
3. 2432 – 1716 = ?	*11.* 16078435 – 1356428 = ?
4. 2586 – 1654 = ?	*12.* 24736842 – 21687584 = ?
5. 130765 – 124587 = ?	*13.* 42687903 – 26875486 = ?
6. 728062 – 257465 = ?	*14.* 87065435 – 84736247 = ?
7. 709354 – 467152 = ?	*15.* 80035463 – 78200456 = ?
8. 370816 – 281029 = ?	*16.* 56190364 – 49015246 = ?

32. Two or more subtrahends:

EXAMPLE.—From 86798 take 21342, 26584, and 22765.

OPERATION

86798
21342
26548
22765
16143

EXPLANATION.—The operation is similar to the process of "Subtraction by Addition." Instead of adding the remainder to one subtrahend we add it to three subtrahends. Thus, 5, 8, 2 (sub.) and 3 (rem.) are 18 (min.). 6, 4, 4 (sub.) and 1 (carried) and 4 (rem.) are 19 (min.). 7, 5, 3 (sub.) and 1 (carried) and 1 (rem.) are 17 (min.). 2, 6, 1 (sub.) and 1 (carried) and 6 (rem.) are 16 (min.). 2, 2, 2 (sub.) and 1 (carried) and 1 (rem.) are 8 (min.).

33. This method is often convenient in finding the balance of an account. The above problem may be written and solved as follows:

subtrahends	21342
	26548
	22765
remainder	16143
minuend	86798

From the following statement find the net gain:

GAINS		LOSSES
$4260		$324
1273		1673
1647		246
365		113
426	Net gain	5615
$7971		$7971

NOTE.—Add up the gains. Place the sum under the losses. Add losses, and write for the net gain such digits as added to the losses will produce $7971, and balance the account.

34. PROBLEMS

1. From 9467 take 2135, 1682, 1478, and 2692.

2. From 12784 take 6253, 3786, and 2057.

3. From 28650 take 10652, 8549, 3267, and 1026.

4. From 156709 take 24680, 13793, 16745, and 24863.

5. From 84907 take 16243, 14786, 9352, and 27054.

6. From 246532 take 73855, 12736, and 96854.

7. From 42584 take 12653, 11964, and 13867.

8. From 89756 take 21684, 14793, 16248, and 13728.

9. From 126087 take 43568, 13752, 28537, and 21864.

10. From 254375 take 62873, 94852, 14695, and 17586.

11. From 48692 take 12071, 9825, 4687 and 14650.

12. From \$4508.75 take \$1328.14, \$924.68, \$156.90, \$437.15 and \$260.54.

13. From \$39575.40 take \$4681.25, \$11060.50, \$16245.30 and \$716.90.

14. From \$12825.70 take \$625.25, \$513.42, \$765.34 and \$1486.76.

15. From \$28746.80 take \$9458.28, \$7134.35, \$863.47, \$6479.58, \$143.92.

16. From \$137248.75 take \$41980.40, \$3714.24, \$9652.18, \$4573.62 and \$31967.86.

17. From \$125876.85 take \$1463.25, \$4563.54, \$3287.43 and \$458.35.

18. From \$981.44 take \$135.72, \$368.27, \$94.25, \$126.36, \$72.83, \$43.28.

19. From \$3574.50 take \$1247.31, \$719.85, \$64.27, \$435.63 and \$246.33.

20. From \$42867.25 take \$19763.45, \$1564.22, \$867.14, \$938.53 and \$6537.48.

MULTIPLICATION

35. The product is composed of the same kind of units as is the multiplicand, and the multiplier is an abstract number. But as the product of two numbers is the same whichever factor is used as a multiplier, either factor may be regarded as the multiplier.

EXAMPLE.

246	multiplicand
3	multiplier
738	product

36. Know the multiplication table up to 10×10.

Multiplication Table

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Numbers between 10 and 20

37. EXAMPLE.—Multiply 12 by 13.

OPERATION
$12 \times 13 = 156$

EXPLANATION.—$2 \times 3 = 6$. The product of the units is 6, which write. The product of tens by units is $10 \times 2 + 10 \times 3 = 50$, or 5 tens, which write. The product of the tens is 1 (hundred), which write.

NOTE.—The product of any two numbers between 10 and 20 is composed of three parts: the product of the units, the product of the units by the tens, and the product of the tens. The product of the units by the tens, in tens, is always the sum of the units. The product of the tens is always 1 (hundred). Therefore, take the product of the units for the units, the sum of the units for the tens, and 1 for the hundreds. If the product or the sum of the units is more than 9, carry as usual.

EXAMPLE.—Multiply 14 by 15.

OPERATION
$14 \times 15 = 210$

EXPLANATION.—$5 \times 4 = 20$. Write 0, and carry 2. $5 + 4 = 9$. $9 + 2$ (carried) $= 11$. Write 1, carry 1. 1 (hundred) $+ 1$ (carried) $= 2$, which write.

PROBLEMS

Find the product of:

1. 13×14.	*7.* 14×17.	*13.* 19×19.
2. 14×15.	*8.* 15×18.	*14.* 14×18.
3. 13×15.	*9.* 16×19.	*15.* 17×15.
4. 15×16.	*10.* 17×18.	*16.* 17×14.
5. 13×17.	*11.* 15×19.	*17.* 12×18.
6. 13×19.	*12.* 16×18.	*18.* 18×19.

NOTE.—The pupil should learn to perform these operations mentally, and the teacher may assign other problems of the same kind.

Multiplying by 10, 100, 1000, Etc.

38. If one cipher (0) be annexed to a number, by what is the number multiplied? If two ciphers (00) be annexed to a number, by what is the number multiplied? If three ciphers (000) be annexed?

MENTAL PROBLEMS

Find the product of:

1. 87×100.	*3.* 872×10000.	*5.* 368×10000.
2. 605×1000.	*4.* 9651×100.	*6.* 7524×1000.

Multiplying Numbers with Ciphers at the Right Hand

39. EXAMPLE.—Multiply 860 by 2400.

OPERATION

$86 \times 24 = 2064$.
$2064 \times 10 \times 100 = 2064000$.

EXPLANATION.—Multiply the digits, then to the product annex as many ciphers as are found at the right of both factors.

PROBLEMS

Find the product of:

1. 27×1300.	*3.* 5200×137000.	*5.* 1240×1400.
2. 680×12400.	*4.* 2600×18000.	*6.* 3500×16000.

Multiplying by a Number Near 100, 1000, etc.

40. EXAMPLE.—Multiply 246 by 99.

OPERATION

$246 \times 100 = 24600$
$24600 - 246 = 24354$

EXPLANATION.—Multiply 246 by 100 by annexing two ciphers. 99 is 1 less than 100. Therefore, multiply 246 by 99 by subtracting once 246 from 24600. Or, annex two ciphers and subtract 246.

EXAMPLE.—Multiply 425 by 997.

OPERATION

425000
$425 \times 3 = 1275$
423725

EXPLANATION.—Annex three ciphers and subtract 3 times 425.

EXAMPLE.—Multiply 425 by 1003.

OPERATION

425000
$425 \times 3 = 1275$
426275

EXPLANATION.—Annex three ciphers and add 3 times 425.

PROBLEMS

NOTE.—Do not write the multiplier, and write the multiplicand but once.

Find the product of:

1. 875×99.
2. 6032×998.
3. 587×97.
4. 6802×996.
5. 3075×995.
6. 732×102.
7. 864×1004.
8. 2073×105.
9. 358×1003.
10. 2561×106.

CROSS MULTIPLICATION

When Each Factor Contains Two Figures

41. EXAMPLE.—Multiply 21 by 23.

Steps in the operation:

1. Units by Units $\begin{matrix}21\\23\end{matrix}$ $3 \times 1 = 3$ Units.
2. Tens by Units $\begin{matrix}21\\\times\\23\end{matrix}$ $2 \times 3 + 2 \times 1 = 8$ Tens.
3. Tens by Tens $\begin{matrix}21\\23\end{matrix}$ $2 \times 2 = 4$ Hundreds.

OPERATION

$21 \times 23 = 483$

EXPLANATION.—In multiplying units and tens by units and tens, there are three steps: finding the product of units by units, the product of tens by units and the product of tens by tens. The product of the units is 3, which write for the units of the complete product. The product of the tens by the units is 8 tens ($3 \times 2 + 2 \times 1$), which write for the tens of the complete product. The product of the tens by the tens is 4 hundreds, which write for the hundreds of the complete product. If any partial product be more than 9, carry as usual.

EXAMPLE.—Multiply 48 by 35.

OPERATION

$48 \times 35 = 1680$

EXPLANATION.—First step, $5 \times 8 = 40$, units. Write 0, carry 4. Second step, $5 \times 4 + 3 \times 8 + 4$ (carried) $= 48$ tens. Write 8, carry 4. Third step, $3 \times 4 + 4$ (carried) $= 16$ hundreds, which write.

PROBLEMS

NOTE.—Write the factors in a horizontal line and perform the operations mentally.

Find the product of:

1. 42×34.	*6.* 74×53.	*11.* 37×46.	*16.* 37×89.
2. 53×27.	*7.* 67×58.	*12.* 48×53.	*17.* 46×74.
3. 62×43.	*8.* 95×36.	*13.* 75×26.	*18.* 38×92.
4. 73×28.	*9.* 46×58.	*14.* 83×47.	*19.* 79×62.
5. 68×94.	*10.* 29×35.	*15.* 64×93.	*20.* 88×69.

When the Multiplicand Contains More Than Two Figures

42. EXAMPLE.—Multiply 235 by 24.

OPERATION

$235 \times 24 = 5640$

EXPLANATION.—There is one more step in the operation than there are figures in the multiplicand. Omitting the carrying figures, the steps are as follows:

1. $\begin{matrix}235\\24\end{matrix}$ $5 \times 4 = 20$ units. *3.* $\begin{matrix}235\\\times\\24\end{matrix}$ $2 \times 4 + 3 \times 2 = 14$ hundreds.

2. $\begin{matrix}235\\\times\\24\end{matrix}$ $3 \times 4 + 5 \times 2 = 22$ tens. *4.* $\begin{matrix}235\\24\end{matrix}$ $2 \times 2 = 4$ thousands.

Write the proper figures and carry as usual.

EXAMPLE.—Multiply 4376 by 57.

NOTE.—The steps in the operation may be illustrated thus:

$\begin{matrix}4376\\57\end{matrix}$ $\begin{matrix}4376\\\times\\57\end{matrix}$ $\begin{matrix}4376\\\times\\57\end{matrix}$ $\begin{matrix}4376\\\times\\57\end{matrix}$ $\begin{matrix}4376\\57\end{matrix}$

EXAMPLE.—Multiply 43765 by 57.

NOTE.—The steps in the operation may be illustrated thus:

$\begin{matrix}43765\\57\end{matrix}$ $\begin{matrix}43765\\\times\\57\end{matrix}$ $\begin{matrix}43765\\\times\\57\end{matrix}$ $\begin{matrix}43765\\\times\\57\end{matrix}$ $\begin{matrix}43765\\\times\\57\end{matrix}$ $\begin{matrix}43765\\57\end{matrix}$

PROBLEMS

NOTE.—Write the factors in a horizontal line and perform the operations mentally. This method is convenient in extending bills.

1. 234×23. *3.* 567×38. *5.* 4306×42. *7.* 1236×63.

2. 426×34. *4.* 1214×35. *6.* 7308×54. *8.* 4253×76.

9. 1684×57. *13.* 23784×75. *17.* 9017×39.
10. 2173×65. *14.* 40265×84. *18.* 65182×82.
11. 4287×29. *15.* 25738×73. *19.* 52781×87.
12. 64261×36. *16.* 6381×76. *20.* 6910×94.

Multiplying by Two Figures One of Which is 1

43. This is usually given as a special method, but in its simplest form it is cross multiplication.

PROBLEMS

Find the product of:

1. 236×13. *5.* 8316×31. *9.* 237×19. *13.* 1378×12.
2. 472×14. *6.* 7026×18. *10.* 1384×91. *14.* 7813×16.
3. 1238×17. *7.* 1682×41. *11.* 2765×71. *15.* 836×61.
4. 2036×21. *8.* 2361×51. *12.* 1683×15. *16.* 7825×81.

To Multiply by 11

44. Although usually given as a special method this is simply cross multiplication.

PROBLEMS

Find the product of:

1. 18×11. *5.* 315×11. *9.* 356×11. *13.* 1362×11.
2. 26×11. *6.* 246×11. *10.* 943×11. *14.* 2480×11.
3. 135×11. *7.* 719×11. *11.* 257×11. *15.* 7156×11.
4 76×11. *8.* 263×11. *12.* 645×11. *16.* 3062×11.

DIVISION

45. Illustration of Terms and Proof of Division

$$\begin{array}{rcrcl} 40 & \div & 5 & = & 8 \\ \text{dividend} & \div & \text{divisor} & = & \text{quotient} \end{array}$$

$$\begin{array}{rcrcl} 5 & \times & 8 & = & 40 \\ \text{divisor} & \times & \text{quotient} & = & \text{dividend} \end{array}$$

$$\begin{array}{rcrcl} 40 & \div & 8 & = & 5 \\ \text{dividend} & \div & \text{quotient} & = & \text{divisor} \end{array}$$

$128 \div 5 = 25$ and 3 remainder, written $\frac{3}{5}$, 3 to be divided by 5.

divisor × quotient + remainder (if any) = dividend

Hence to prove division, multiply together the divisor and quotient and to the product add the remainder. If the result equals the dividend the work is correct.

46. *Principles.*

$24 \div 4 = 6 \qquad 48 \div 4 = 12 \qquad 12 \div 4 = 3$

1. Multiplying the dividend multiplies the quotient; dividing the dividend divides the quotient.

$24 \div 4 = 6 \qquad 24 \div 8 = 3 \qquad 24 \div 2 = 12$

2. Multiplying the divisor divides the quotient; dividing the divisor multiplies the quotient.

$24 \div 4 = 6 \qquad 48 \div 8 = 6 \qquad 12 \div 2 = 6$

3. Multiplying or dividing both dividend and divisor by the same number does not change the value of the quotient.

Short Long Division

47. This is the ordinary method of long division shortened by introducing the "subtraction by addition" method and by multiplying and subtracting at the same time.

EXAMPLE.—Divide 721098 by 291.

OPERATION

```
291)721098(2478
  1390
   2269
    2328
     0000
```

EXPLANATION.—The first figure of the quotient is 2. Multiply the divisor by 2 and subtract the product from 721 as follows: $2 \times 1 = 2$ and 9 (which write for the remainder) are 11. $2 \times 9 = 18$ and 1 (carried) are 19 and 3 (write in remainder) are 22. $2 \times 2 = 4$ and 2 (carried) are 6 and 1 (write in remainder) are 7. Bring down the next figure of the dividend, making the remainder 1390. Multiply 291 by 4 and subtract. $4 \times 1 = 4$ and 6 are 10. $4 \times 9 = 36$ and 1 are 37 and 2 are 39. $4 \times 2 = 8$ and 3 are 11 and 2 are 13. Bring down the next figure of the dividend, making the remainder 2269. Multiply 291 by 7 and subtract. $7 \times 1 = 7$ and 2 are 9. $7 \times 9 = 63$ and 3 are 66. $7 \times 2 = 14$ and 6 are 20 and 2 are 22. Bring down the 8. Multiply 291 by 8 and subtract.

PROBLEMS

Find the quotient of:

1.	11445 ÷	35.	*11.*	1091510 ÷	1085.
2.	10664 ÷	24.	*12.*	135676 ÷	317.
3.	30128 ÷	56.	*13.*	6220848 ÷	654.
4.	1131264 ÷	7856.	*14.*	4687238 ÷	423.
5.	1691823 ÷	357.	*15.*	7098643 ÷	2136.
6.	313194106 ÷	7153.	*16.*	9364725 ÷	4874.
7.	1243414986 ÷	58327.	*17.*	537654 ÷	2576.
8.	302418 ÷	954.	*18.*	1068432 ÷	16843.
9.	205683 ÷	629.	*19.*	2764379 ÷	4268.
10.	2151090 ÷	4185.	*20.*	876543 ÷	769.

48. Dividing by 10, 100, 1000, etc.

Principle.—Cutting off one figure from the right of the dividend divides it by 10, cutting off two figures divides it by 100, cutting off three figures divides it by 1000, etc. The part of the dividend cut off is the remainder.

PROBLEMS

Find the quotient of:

1.	46800 ÷	100.	*6.*	420070 ÷	1000.
2.	73000 ÷	1000.	*7.*	53781936 ÷	100000.
3.	2860 ÷	100.	*8.*	800167829 ÷	10000.
4.	796800 ÷	10000.	*9.*	670197356 ÷	1000000.
5.	63875 ÷	1000.	*10.*	5179036527 ÷	1000000.

THE EQUATION

49. The sign of equality (=) between two equal numbers or expressions forms an Equation.

$7+2=9$. $4\times5+3=6\times3+5$. Cost of 25 cows = cost of 100 sheep.

minuend − subtrahend = remainder
dividend ÷ divisor = quotient

Axioms

50. Truths so simple that they do not admit of proof are called Axioms.

1. Things equal to the same thing are equal to each other.

If \$600 = cost of 25 cows, and
\$600 = cost of 100 sheep,
then cost of 25 cows = cost of 100 sheep.

2. If equals are added to equals the sums are equal.

If cost of 25 cows = cost of 100 sheep,
then cost of 25 cows + \$100 = cost of 100 sheep + \$100.
If A = B, then A + C = B + C.

3. If equals are subtracted from equals the remainders are equal.

If cost of 25 cows = cost of 100 sheep,
then cost of 25 cows − \$100 = cost of 100 sheep − \$100.
If A = B, then A − C = B − C.

4. If equals are multiplied by equals the products are equal.

If cost of 25 cows = cost of 100 sheep,
then cost of 100 cows = cost of 400 sheep.
If A = B, then A × C = B × C.

5. If equals are divided by equals the quotients are equal.

If cost of 25 cows = cost of 100 sheep,
then cost of 5 cows = cost of 20 sheep.
If A = B, then A ÷ C = B ÷ C.

51. The process of changing a number from one side of an equation to the other is called transposition.

$$(1)\quad A - 10 = B$$

Add 10 to both sides (members) of the equation (Axiom 2,) and

$$(2)\quad A - 10 + 10 = B + 10, \text{ or } A = B + 10$$

In the first equation, 10 is in the first member; in the second equation, 10 is in the second member. The sign of 10 in the first equation is −, in the second +. 10 has been changed from one member of the equation to the other, and its sign has been changed from − to +.

$$(3)\quad A + 10 = B$$

Subtract 10 from both members of the equation, and

$$(4)\quad A = B - 10$$

10 has been changed from one member of the equation to the other, and its sign has been changed from + to −.

52. *Principle.*—A number may be transposed from one member of an equation to the other by changing its sign from + to −, or from − to +.

Solution of the Equation

53. If in any equation there is a term whose value is not given, as A = 3 + 7, finding the value of that term is called solving the equation.

NOTE.—It is a common error to take the signs of addition, subtraction, multiplication and division in the order in which they come. The signs of multiplication and division have the preference over the signs of addition and subtraction, and the operations indicated by the former are to be performed before those indicated by the signs of the latter, thus:

$$2 + 9 \times 6 = 56, \text{ not } 66$$

In every equation the plus or the minus sign must be understood to affect the result of the whole operation indicated between it and the next plus or minus, or between it and the close of the expression.

If a problem is stated in the form of an equation, solving the equation is solving the problem.

PROBLEMS

54. Solve the following equations:

1. Cost of horse = \$100 + \$6 × 10. (Find the cost of the horse.)
2. A = 65 + 32 − 7.
3. Cost of 2 cows = \$18 + \$3 × 10. (Find cost of one cow.)
4. 2 A = 18 + 3 × 10. (Find value of A.)
5. Cost of stove + \$10 = \$40. (Find cost of stove.)
6. Cost of horse − \$15 = \$60.
7. Value of 3 horses = \$210. (Find value of a horse.)
8. Value of 3 horses − \$15 = \$60.
9. Value of 3 horses + \$10 = \$190.
10. Value of 8 cows = \$1920 ÷ 6.
11. Value of 5 sheep + \$12 = \$42.
12. A + 6 × 8 = 50 + 23.
13. Dividend = 12 × 360 + 40.
14. Subtrahend − 165 = 586.
15. How many cows at \$25 per head = \$600?

CANCELLATION

55. Cancel means to mark out. Cancellation is division by marking out or crossing out factors. The dividend consists of two factors: the divisor and quotient. If one of these factors be marked out, the other factor will remain. The divisor itself may be separated into factors. When both divisor and dividend contain like factors, such factors may be cancelled without changing the value of the quotient.

Cancellation is the process of rejecting common factors from both dividend and divisor. (See Principle 3, Art. 46.)

56. EXAMPLE.

Divide $6 \times 8 \times 12 \times 15 \times 20$ by $2 \times 3 \times 4 \times 5 \times 30$.

Instead of dividing the product of the first set of factors by the product of the second set, we may cancel the common factors and then divide if a divisor remains uncancelled.

OPERATION

$$\frac{\cancel{6}\quad \cancel{8}_{4}\quad \cancel{12}\quad \cancel{15}\quad \cancel{20}}{\cancel{2}\quad \cancel{3}\quad \cancel{4}\quad \cancel{5}\quad \cancel{30}_{2}} = 48$$

EXPLANATION.— Write the factors of the dividend above a horizontal line and the factors of the divisor below. Then reject equal factors from both dividend and divisor. The factors 2 and 3 in the divisor cancel 6 (2×3) in the dividend. 4 and 5 in the divisor cancel 20 (4×5) in the dividend. 15 in the dividend cancels 15 (one of the factors of 30) in the divisor, leaving the factor 2 instead of 30 in the divisor. This 2 cancels 2 (one of the factors of 8) in the dividend, leaving 4 instead of 8 in the dividend. In the dividend there remain the factors 4 and 12, which produce 48, the required quotient.

Exact Divisors

57. *1.* 2 is an exact divisor of any even number.

2. 3 is an exact divisor of any number the sum of whose digits is divisible by 3.

3. 5 is an exact divisor of any number ending with 5 or 0.

4. 9 is an exact divisor of any number the sum of whose digits is divisible by 9.

5. No even number is an exact divisor of an odd number.

PROBLEMS

58. Cancel when you *can*, multiply when you *must*.

Divide:

1. $45 \times 16 \times 60 \times 27$ by $15 \times 8 \times 12 \times 9 \times 4$.

2. $96 \times 128 \times 72 \times 64$ by $16 \times 48 \times 27$.

3. $33 \times 57 \times 72 \times 216$ by $19 \times 11 \times 8 \times 27 \times 16$.

4. $872 \times 365 \times 496$ by $654 \times 175 \times 428$.

5. $384 \times 495 \times 350$ by $352 \times 330 \times 210$.

6. $345 \times 432 \times 120$ by $210 \times 360 \times 216$.

7. $882 \times 225 \times 168$ by $556 \times 315 \times 112$.

8. $616 \times 512 \times 420$ by $560 \times 448 \times 315$.

9. A man exchanged 24 loads of wheat, each weighing 2480 pounds, worth 80 cents per bushel of 60 pounds, for 59 barrels of sugar, each weighing 420 pounds. What was the price of the sugar per pound?

10. A farmer sold 18 loads of hay, each weighing 2860 pounds, at $21 per ton of 2000 pounds. He received in payment 8 loads of phosphate, each load containing 12 bags, and each bag 240 pounds. What was the price of the phosphate per 100 pounds?

11. How many city lots, each containing 21 square rods, and valued at $8 per square rod, are equal in value to 15 fields, each containing 1120 square rods, valued at $32 per acre of 160 square rods?

12. How many pieces of cloth, each containing 126 yards, valued at 16 cents per yard, are equal in value to 35 pieces of cloth, each containing 288 yards, valued at 14 cents per yard?

13. If 18 barrels of beef, each containing 200 pounds, are worth $288, what will 75 pounds cost at the same rate?

14. If 52 men can dig a ditch in 42 days, working 9 hours a day, how many days will be required by 24 men to do the same work, if they work 7 hours per day?

15. How many boxes of tobacco, each weighing 42 pounds, valued at 90 cents per pound, are equivalent to 80 boxes of tobacco, each weighing 149 pounds, valued at 81 cents per pound?

16. Divide $120 \times 540 \times 695$ by 380×175.

17. $250 \times 25 \times 84 \times 21 = 365 \times 80 \times 32 \times ?$

18. The factors of the dividend are 940, 760, 145, and 724. The factors of the divisor are 190, 724, 235, and 180. Find the quotient.

19. A bicyclist rode 8 miles an hour for 18 days of 10 hours each and walked back at the rate of 3 miles per hour. How many days did it take him to get back, if he walked 12 hours a day?

20. How many days' work, of 10 hours each, at 15 cents per hour, will be required to pay for a pile of wood 48 feet long, 4 feet wide, and 9 feet high, at \$4.50 per cord?

21. How many boards 16 feet long and 12 inches wide, at \$24 per thousand, must be given in exchange for 160 scantling 2 inches by 4 inches and 18 feet long, at \$14 per thousand?

22. How many bricks 2 inches by 4 inches by 8 inches will be required to lay a wall 14 feet long, 6 feet high, and $1\frac{1}{2}$ feet thick?

23. How many village lots 6 rods by 8 rods, worth \$6 per square rod, are equal in value to a farm 80 rods by 90 rods, at \$96 per acre?

24. At \$26 per thousand, how many sticks 4 inches by 6 inches by 14 feet are equal in value to 130 boards 18 feet long and 12 inches wide, at \$45 per thousand?

25. How many days, of 9 hours each, must a man work, at 18 cents per hour, to pay for a lot 120 feet by 165 feet, at \$8 per square rod?

FRACTIONS

59. If a whole is divided into two or more equal parts, the parts are called fractions of the whole. One of the parts is a fraction. Several of the parts are a fraction.

Division is the process of separating a unit or a number into equal parts.

One-seventh of 1, one-ninth of 1, one-tenth of 1, are fractions, for they are each equal parts of a unit.

One-seventh of 4, one-ninth of 4, one-tenth of 4, are fractions, for they are each equal parts of a number.

A fraction is one or more of the equal parts of a unit, or one of the equal parts of a number.

$\frac{2}{3}$ shows 2 of the 3 equal parts into which a unit is divided, or it shows that 2 is divided into 3 equal parts.

A fraction is division indicated by the sign /. $1 \div 2$ and $\frac{1}{2}$ are the same in value. $\frac{2}{3}\frac{7}{6}$ and $27 \div 36$ are the same in value.

60. In a fraction, the dividend (number above the line) is called the Numerator. It enumerates, or tells, how many of the equal parts of a unit are included in the fraction, or it shows what number has been divided into equal parts. The fraction $\frac{3}{5}$ shows 3 of the 5 equal parts into which the unit is divided, or it shows that 3 is divided into 5 equal parts.

61. The divisor (number below the line) is called the Denominator. It names the parts into which a unit or a number is divided. If the denominator is 4, it shows that a unit or a number is divided into fourths.

62. A fraction indicates division. The value of a fraction is therefore the quotient of the division indicated. A fraction is a quotient. The numerator and denominator are called the terms of a fraction. The dividend and divisor are called the terms of a division.

Comparing fractions with division, the numerator is the dividend, the denominator is the divisor, and the fraction is the quotient.

Principles of Division

63. *1.* Multiplying the dividend multiplies the quotient, dividing the dividend divides the quotient.

2. Multiplying the divisor divides the quotient, dividing the divisor multiplies the quotient.

3. Multiplying or dividing both dividend and divisor by the same number does not change the value of the quotient.

Principles of Fractions

64. *1.* Multiplying the numerator multiplies the fraction, dividing the numerator divides the fraction.

2. Multiplying the denominator divides the fraction, dividing the denominator multiplies the fraction.

3. Multiplying or dividing both numerator and denominator by the same number does not change the value of the fraction.

Terms

65. A Proper Fraction is one whose numerator is less than its denominator; as, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{1}{9}$.

An Improper Fraction is one whose numerator equals or exceeds its denominator; as, $\frac{4}{4}$, $\frac{6}{5}$, $\frac{11}{8}$.

A Mixed Number is one expressed by an integer and a fraction; as, $4\frac{3}{5}$, read four and three-fifths.

A Complex Fraction is one which has a fraction in one or both of its terms; as, $\frac{\frac{3}{4}}{\frac{5}{6}}$, $\frac{5}{2\frac{3}{4}}$, $\frac{9\frac{2}{3}}{6}$.

A Compound Fraction consists of two or more single fractions joined together by the word of; as, $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$.

PROBLEMS

Write the following in the form of fractions:

1. One-half over six-fifths.

2. Four over eight-seventeenths.

3. Nine-tenths of eight and two-thirds.

4. Seven-ninths of four-sevenths.

5. Six and seven-eighths over nine and three-fourths.

DECIMAL DIVISIONS AND DECIMAL FRACTIONS

66. If 1000 is divided into 10 equal parts, what is one of the parts called? Perform the operation by pointing off one figure.

If 100 is divided into 10 equal parts, what is one of the parts called? Perform the operation by pointing off one figure.

If 10 is divided into 10 equal parts, what is one of the parts called? Perform the operation by pointing off one figure.

If 1 is divided into 10 equal parts, what is one of the parts called? Perform the operation by pointing off one figure.

If .1 is divided into 10 equal parts, what is one of the parts called? Point off one figure as before. Put a cipher between the period (decimal point) and the one to show that one place more has been pointed off.

If .01 be divided into 10 equal parts, what is one of the parts called? Point off as before. Insert another cipher.

If .001 is divided into 10 equal parts, what is one of the parts called? Point off as before, and insert another cipher.

The divisions of a number into tenths, hundredths, thousandths, etc., are Decimal Divisions, or Decimal Fractions.

PROBLEMS

Divide by pointing off (insert ciphers when necessary) and read the quotients:

1.	$1 \div 10.$	*10.*	$10 \div 1000000.$
2.	$1 \div 100.$	*11.*	$25 \div 100.$
3.	$1 \div 1000.$	*12.*	$25 \div 1000.$
4.	$3 \div 10.$	*13.*	$25 \div 10000.$
5.	$3 \div 100.$	*14.*	$25 \div 100000.$
6.	$3 \div 1000.$	*15.*	$25 \div 1000000.$
7.	$1 \div 10000.$	*16.*	$136 \div 1000.$
8.	$1 \div 100000.$	*17.*	$136 \div 10000.$
9.	$1 \div 1000000.$	*18.*	$1378 \div 1000000.$

67. By what must 1 be divided to produce .01? Read .01. By what must 1 be divided to produce .001? Read .001. By what must 1 be divided to produce .0001? Read .0001. Read .0007, .00001, .00005, .000001, .000006, .000021, .00055.

DECIMAL SCALE OF ARABIC NOTATION

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
7	6	5	4	3	2	1	2	3	4	5	6	7

68. Repeat the scale from millions to millionths, from millionths to millions.

The decimal point is always before *tenths*. Repeat the scale from the decimal point each way.

The name, or denomination, of a decimal is that of its right-hand order of units. Thus, .0001 is one *ten-thousandth*.

How to Write Decimals

69. According to our system of writing we begin at the left and write toward the right. This is the case with script and also with figures. For the sake of economy of time and to be consistent, pupils should learn to write decimals in the same manner and according to the following rules:

1st. Fix the decimal point.

2d. Think of the number of places required to make a fraction of the given denominator.

3d. Think of the number of places given in the numerator.

4th. Beginning at the right of the decimal point, write as many ciphers as are required to make the number of places given equal to the number required, and follow these by the numerator, or the figures given.

70. Write decimally:

1. 3 thousandths.

1st. Fix the point.

2d. It requires three places to make thousandths.

3d. One place is given.

4th. At the right of the point write two ciphers, then the 3.

2. 135 millionths.

3. 3 millionths; 345 hundred-thousandths; 45 millionths.

4. 23456 millionths; 356 hundred-millionths.

5. Eighteen ten-thousandths. Twenty-seven hundred-thousandths. One hundred sixty-five millionths. Thirty-four ten-thousandths. One hundred eight hundred-thousandths. Eighty millionths.

6. Twenty-five and twenty-six thousandths.

NOTE.—The word *and* is used to connect a whole number and a decimal. This expression is written 25.026.

7. One hundred and seventy-eight ten-thousandths. One hundred seventy-eight ten-thousandths.

8. Four hundred sixty-three and four hundred sixty-three ten-millionths. Four hundred and forty-seven ten-thousandths. Four hundred forty-seven ten-thousandths.

9. Six hundred-thousandths. Six hundred thousandths. Three hundred ten thousandths. Three hundred ten-thousandths.

10. Three hundred eighty thousand and thirteen thousand four hundred ninety-six hundred-thousandths.

EXERCISES IN NUMERATION

71. The numerator of a decimal is the number expressed by the figures of the decimal. It is the number divided into tenths, hundredths, thousandths, etc. The denominator of a decimal is indicated by the decimal point and by the number of figures following the decimal point. The decimal point stands for 1; each figure in the decimal stands for a cipher following that 1. The denominator, if expressed, would be 1 with as many ciphers annexed as there are figures at the right of the decimal point.

EXERCISES

Read the following:

1. .00125.

NOTE.—There are two parts to the operation: reading the numerator and reading the denominator. Read the numerator as if it were a whole number. To read the denominator, begin at the decimal point and numerate toward the right, thus: tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths. The decimal is one hundred twenty-five hundred-thousandths.

2. .000367, .1709, .00231, .100236, .00002, .000367.

3. .07, .0028, .6400, .056843, .0086003, .00008.

4. 18.34.

NOTE.—This is read 18 and 34 hundredths. In reading mixed numbers, it is necessary to connect the integral and fractional parts by *and*.

5. 127.0034, 13.2006, 1780.0073, 146000.146.

6. 400.04, 10000.010, 10.010, 200.003, .203.

7. .00800654, 3800.0076, 56.7003804, 4800.7063.

8. 3674583.000437, 2687.015008, 3268.007583.

9. 1200.0012, 3984.05307, 100300.00301, 37.0000045.

10. 256784370.4600736, 4300.56032, 140000.00007554.

72. Compare .3, .30, .300, .3000 as to value.
Compare .3, .03, .003, .0003 as to value.

Principles: — *1.* Annexing ciphers to a decimal does not change its value.

2. Inserting a cipher between the decimal point and the decimal figures divides the decimal by 10.

3. Decimal orders of units increase and decrease in value the same as do orders of units in whole numbers.

Addition of Decimals

73. *Principle.*—Only like orders of units can be added.

EXAMPLE.—Add .265, 13.7, and 1.3787.

OPERATION	EXPLANATION.—Write the numbers so that the decimal points fall in a vertical line, and units of the same order will stand in the same column. Add as in integers, and put the decimal point in the sum directly under the points in the numbers added.
.265	
13.7	
1.2787	
15.3437	

PROBLEMS

Find the sum of:

1. .14, 1.268, 3.72, 4.682, 15.79.
2. 31.06, 128.374, 175.009, 38.063, 53.034.
3. 504.017, 3.86, 7.128, 130.065, 14.586.
4. .0038, 100.16, 9.054, .69786, 1687.98745.
5. 432.867, 576.09, 78.659, 9.85, 15.462.
6. .3286, 14.567, 284.007, 1275.49, 36.5976.
7. 12, 14.825, .7364, 129, 16.004, 157.3697.
8. 47.25, 6.0078, 174.6, 9.678, 23.00159.

Subtraction of Decimals

74. *Principle.*—Only like orders of units can be subtracted.

EXAMPLE.—From 48.73 take 25.6274.

OPERATION	EXPLANATION.—Write the numbers as in addition. Consider ciphers as annexed to the minuend, and subtract. Put the decimal point in the remainder directly under the points in the numbers subtracted.
48.73	
25.6274	
23.1026	

PROBLEMS

Find the remainder of:

1. .832 − .126.
2. 15.06 − 7.2584.
3. 460.85 − 53.265.
4. 246.7385 − 39.74.
5. 179.86 − 138.0583.
6. 4085.75 − 927.6485.
7. 928.6482 − 25.7595.
8. 16843.7652 − 483.28.

Multiplication of Decimals

75. Tens × tens = hundreds. $10 \times 10 = 100$.

Tens × hundreds = thousands. $10 \times 100 = 1000$.

Hundreds × hundreds = ten-thousands. $100 \times 100 = 10000$.

Hundreds × thousands = hundred-thousands. $100 \times 1000 = 100000$.

Tenths × tenths = hundredths. $.1 \times .1 = .01$.

Tenths × hundredths = thousandths. $.1 \times .01 = .001$.

Hundredths × hundredths = ten-thousandths. $.01 \times .01 = .0001$.

Hundredths × thousandths = hundred-thousandths. $.01 \times .001 = .00001$.

Principle.—The product contains as many decimal places as there are decimal places in the factors.

EXAMPLE.—Multiply 1.256 by .32.

OPERATION

1.256
32
2512
3768
.40192

EXPLANATION.—Multiply as in whole numbers. Point off as many decimal places in the product as there are decimal places in both factors.

NOTE.—If there are not enough figures in the product to point off, prefix ciphers to the product. Thus, $.2 \times .04 = .008$.

PROBLEMS

Find the product of:

1. $.83 \times 1.4$.
2. $1.75 \times .23$.
3. $.272 \times .081$.
4. $.064 \times 1.5$.
5. $.0053 \times .029$.
6. $2.0038 \times .00016$.
7. $4.062 \times .0037$.
8. $.0155 \times 1.8$.
9. $27.08 \times .125$.
10. $523 \times .00017$.
11. 1000×10.
12. $.001 \times 10$.
13. $1.001 \times .0001$.
14. $.0001 \times .001$
15. $10.001 \times .00001$.
16. $1000 \times .001$.
17. 10001.0001×1.001.
18. 10000.0001×10000.
19. 5005.005×5000.
20. $500.0005 \times 5000.000005$.

Division of Decimals

76. $.05 \times .005 = .00025$. Hence, $.00025 \div .05 = .005$, and $.00025 \div .005 = .05$.

Principle.—The quotient contains as many decimal places as the number of decimal places in the dividend exceed those in the divisor.

EXAMPLE 1.—Divide .00036 by .004.

OPERATION

.004).00036(.09

EXPLANATION.—Divide as in whole numbers. Point off in the quotient as many decimal places as the number of decimal places in the dividend exceed those in the divisor. $5 - 3 = 2$. Prefix one cipher to the quotient, and point off two places.

SUGGESTIONS.—*1.* If the quotient does not contain enough figures to point off, prefix ciphers.

2. Before dividing, make the number of decimal places in the dividend at least equal to the number of places in the divisor, by annexing ciphers to the dividend.

3. When all the figures of the dividend have been used and there is a remainder, annex ciphers to the dividend and continue the division.

4. Ordinarily it is not necessary to extend the division to more than four decimal figures in the quotient.

PROBLEMS

Find the quotient of:

1. 68.125 ÷ 25.
2. 16.025 ÷ .045.
3. 52.848 ÷ 09.
4. 75 ÷ .00125.
5. .0065 ÷ 125.
6. .0357 ÷ 51000.
7. 625 ÷ .0025.
8. 20 ÷ 75.
9. 723.68 ÷ 143.
10. 2.652 ÷ 17.

EXAMPLE 2.—Divide 638.25 by 300.

OPERATION

3ØØ)6.3825
2.1275

EXPLANATION.— Cut off the two ciphers in the divisor and point off two places in the dividend, beginning at the decimal point. Then divide 6.3825 by 3.

11. 675 ÷ 50000.
12. 428.6 ÷ 200.
13. .373 ÷ 4000.
14. 1728 ÷ 12000.
15. 865 ÷ 124000000.
16. 97.281 ÷ 900.
17. 57800 ÷ 8000000.
18. .6307 ÷ 700.

Ten problems are given in each of the following groups. The object is to drill the pupil in pointing off.

State the quotient in the form of a decimal fraction in each of the following problems:

19.	*20.*	*21.*
2 ÷ 2.	1 ÷ 2.	4 ÷ 25.
2 ÷ .02.	1 ÷ 200.	400 ÷ .25.
20 ÷ 2.	.1 ÷ .2.	.4 ÷ 2500.
2 ÷ .2.	.01 ÷ 20.	.004 ÷ .0025.
20 ÷ .002.	.0001 ÷ .2.	.4 ÷ .000025.
.2 ÷ 2.	100 ÷ .002.	4000 ÷ .000025.
.2 ÷ .002.	.0001 ÷ 200.	.0004 ÷ 250000.
.2 ÷ .2.	10 ÷ .0002.	40 ÷ 25000.
.2 ÷ .200.	.001 ÷ .00002.	4 ÷ .000025.
20 ÷ 2000.	1000 ÷ .0002.	.00004 ÷ .000025.

22.	*23.*	*24.*
33 ÷ 11.	250 ÷ 12500.	.08 ÷ 16.
.0033 ÷ 1100.	2.5 ÷ .00125.	80 ÷ 1600000.
33000 ÷ .0011.	2500 ÷ .0000125.	8000 ÷ .000016.
3300 ÷ 110000.	.00025 ÷ 125000.	.0008 ÷ .00016.
.33 ÷ .000011.	2.5 ÷ 125.	.00008 ÷ 16000.
.00033 ÷ 110000.	.025 ÷ .000125.	8 ÷ 1600.
.00033 ÷ .011.	.0025 ÷ 1250000.	80000 ÷ 160.
.0033 ÷ .000011.	25000 ÷ .0000125.	800 ÷ .000016.
330 ÷ .00011.	.25 ÷ .000125.	.0008 ÷ 160000.
.000033 ÷ 1100000.	250 ÷ 12500000.	.000008 ÷ .0016.

REDUCTION OF FRACTIONS

77. Fractions may be written as decimals or as common fractions.

Principles.—1. A fraction is an indicated division.

2. The denominator of a decimal when expressed is 1 with as many ciphers annexed as there are orders of units at the right of the decimal point.

PROBLEMS

1. Write .64 as a common fraction.

2. Write $\frac{64}{100}$ as a decimal.

3. Write .032 as a common fraction.

4. Write 4 ÷ 5 as a decimal. Perform the operation indicated.

5. Write $\frac{4}{5}$ as a decimal. Perform the operation indicated.

6. Write $\frac{1}{4}$ as a decimal.

Write the following as common fractions:

7. .268.	*10.* .0205.	*13.* .0036.
8. .3758.	*11.* .09.	*14.* .25738.
9. .046.	*12.* .049.	*15.* .0013.

Write as mixed numbers:

16. 2.65.	*18.* 10.15.	*20.* 101.101.
17. 17.364.	*19.* 610.0018.	*21.* 13.085.

Write as decimals:

22. $\frac{2}{5}$.	*24.* $\frac{4}{5}$.	*26.* $\frac{3}{8}$.	*28.* $\frac{3}{5}$.
23. $\frac{5}{8}$.	*25.* $\frac{3}{4}$.	*27.* $\frac{1}{2}$.	*29.* $\frac{7}{8}$.

NOTE.—When the division is not exact, the remainder may be expressed as a common fraction, or the sign + may be placed after the decimal to show that the division is not complete. Thus, $\frac{1}{3} = .33\frac{1}{3}$, or .3333+.

Common fractions in their lowest terms cannot be reduced to pure decimals if their denominators contain any prime factors other than 2 or 5.

30. $\frac{1}{6}$.	*34.* $\frac{3}{16}$.	*38.* $\frac{7}{20}$.	*42.* $\frac{5}{6}$.
31. $\frac{5}{9}$.	*35.* $\frac{8}{15}$.	*39.* $\frac{11}{12}$.	*43.* $\frac{3}{32}$.
32. $\frac{2}{3}$.	*36.* $\frac{3}{20}$.	*40.* $\frac{14}{15}$.	*44.* $\frac{4}{125}$.
33. $\frac{4}{7}$.	*37.* $\frac{16}{25}$.	*41.* $\frac{9}{14}$.	*45.* $\frac{71}{80}$.

NOTE.—$.25\frac{1}{8} = .25125$. $.23\frac{3}{4} = .2375$.

46. $.17\frac{2}{4}$.	*48.* $.27\frac{1}{3}$.	*50.* $.24\frac{3}{5}$.	*52.* $.96\frac{7}{8}$.
47. $.24\frac{4}{5}$.	*49.* $.16\frac{2}{3}$.	*51.* $.24\frac{5}{12}$.	*53.* $.18\frac{2}{5}$.

NOTE.—$3\frac{1}{3} = 3.3333+$. $4.0\frac{1}{2} = 4.05$.

54. $7\frac{1}{8}$.	*56.* $14\frac{2}{7}$.	*58.* $25.0\frac{4}{5}$.	*60.* $16.00\frac{1}{2}$.
55. $17.0\frac{2}{7}$.	*57.* $20.00\frac{3}{4}$.	*59.* $9.0\frac{7}{12}$.	*61.* $5.00\frac{1}{8}$.

78. Whole numbers may be written as fractions thus:

$5 = \frac{5}{1}$ $\quad 5 = \frac{5 \times 2}{2} = \frac{10}{2}$ $\quad 5 = \frac{5 \times 14}{14} = \frac{70}{14}$ $\quad 5 = \frac{5 \times 20}{20} = \frac{100}{200}$.

Changing a whole number to a fraction by giving a denominator to the whole number is simply dividing the number by that denominator.

If a number is changed to a fraction with a given denominator, the number must be multiplied by that denominator.

PROBLEMS

1. $3 = \frac{?}{5}$.	*3.* $15 = \frac{?}{7}$.	*5.* $11 = \frac{?}{15}$.
2. $12 = \frac{?}{10}$.	*4.* $9 = \frac{?}{12}$.	*6.* $13 = \frac{?}{20}$.

7. Write 7 as 13ths.
8. Write 16 as 30ths.
9. Write 4 as 27ths.
10. Write 5 as 36ths.
11. Write 8 as 41sts.
12. Write 12 as 120ths.

79. Mixed numbers may be written as common fractions thus:

$2\frac{1}{2} = \frac{4}{2} + \frac{1}{2}$ or $\frac{5}{2}$. $\quad 7\frac{3}{8} = \frac{56}{8} + \frac{3}{8}$ or $\frac{59}{8}$.

NOTE.—Multiply the whole number by the denominator of the fraction, add the numerator, and write the sum over the denominator.

PROBLEMS

Write as common fractions:

1. $7\frac{1}{5}$.	*4.* $15\frac{4}{9}$.	*7.* $120\frac{6}{11}$.	*10.* $163\frac{7}{22}$.
2. $12\frac{3}{5}$.	*5.* $20\frac{3}{8}$.	*8.* $251\frac{2}{15}$.	*11.* $16\frac{5}{16}$.
3. $6\frac{5}{7}$.	*6.* $14\frac{9}{13}$.	*9.* $62\frac{7}{12}$.	*12.* $24\frac{8}{17}$.

80. Common fractions may be written as whole or mixed numbers thus:

$\frac{6}{3} = 6 \div 3 = 2$. $\quad \frac{7}{3} = 7 \div 3 = 2\frac{1}{3}$. $\quad \frac{245}{12} = 245 \div 12 = 20\frac{5}{12}$.

NOTE.—Perform the operations indicated.

PROBLEMS

Write as whole or mixed numbers:

1. $\frac{15}{2}$.	*4.* $\frac{382}{8}$.	*7.* $\frac{265}{18}$.	*10.* $\frac{6875}{123}$.
2. $\frac{145}{15}$.	*5.* $\frac{26}{3}$.	*8.* $\frac{156}{12}$.	*11.* $\frac{973}{28}$.
3. $\frac{375}{25}$.	*6.* $\frac{78}{14}$.	*9.* $\frac{297}{16}$.	*12.* $\frac{6853}{146}$.

81. Common fractions may be written in their lowest terms.

$$\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}. \qquad \frac{18}{96} = \frac{9}{48} = \frac{3}{16}.$$

Principle.—Dividing both numerator and denominator by the same number does not change the value of the fraction.

NOTE.—If both terms be divided by their greatest common divisor, or by as many successive divisors as possible, the fraction will then be expressed in its lowest terms

PROBLEMS

Write the lowest terms of the following:

1. $\frac{18}{36}$.	*4.* $\frac{21}{35}$.	*7.* $\frac{441}{567}$.	*10.* $\frac{648}{972}$.
2. $\frac{24}{96}$.	*5.* $\frac{60}{625}$.	*8.* $\frac{8911}{10053}$.	*11.* $\frac{420}{450}$.
3. $\frac{78}{192}$.	*6.* $\frac{840}{1728}$.	*9.* $\frac{14710}{17856}$.	*12.* $\frac{141112}{158476}$.

82. Common fractions may be written in higher terms.

$$\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}. \qquad \frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}.$$

Principle.—Multiplying both terms of a fraction by the same number does not change the value of the fraction.

Thus, change $\frac{7}{9}$ to 27ths.

$$\frac{7}{9} = \frac{?}{27}. \qquad \frac{7 \times 3}{9 \times 3} = \frac{21}{27}.$$

NOTE.—Multiply both terms of the fraction by a number that will change the given denominator to the required denominator. To find such a number, divide the required denominator by the given denominator.

PROBLEMS

Change:

1. $\frac{7}{12}$ to 48ths.
2. $\frac{5}{16}$ to 80ths.
3. $\frac{9}{11}$ to 55ths.
4. $\frac{16}{21}$ to 105ths.
5. $\frac{3}{7}$ to 147ths.
6. $\frac{8}{15}$ to 225ths.
7. $\frac{6}{13}$ to 182ds.
8. $\frac{21}{25}$ to 475ths.
9. $\frac{17}{18}$ to 72ds.
10. $\frac{23}{29}$ to 319ths.
11. $\frac{13}{18}$ to 3104ths.
12. $\frac{4}{49}$ to 196ths.

83. Fractions that have similar denominators are similar fractions.

84. The denominator of similar fractions is a common denominator.

85. The least or lowest denominator that similar fractions can have is their least common denominator.

86. A common denominator contains each of the given denominators.

87. The least common denominator that two or more fractions can have is the least number that will exactly contain the denominators of the given fractions.

88. Fractions may be reduced to their least common denominator.

EXAMPLE.—Find the least common denominator of $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{16}$, and $\frac{1}{18}$.

NOTE.—The least common denominator is the least number that will exactly contain 6, 12, 16, and 18, and may be found as follows:

OPERATION

2	6, 12, 16, 18
3	3, 6, 8, 9
2	1, 2, 8, 3
	1, 4, 3

$2 \times 3 \times 2 \times 4 \times 3 = 144$, L. C. D.

EXPLANATION.—Write the numbers as here shown. Divide successively by any prime factor that will be contained in at least two of the numbers to be divided. The product of the successive divisors and the remaining numbers that have no common divisors is the L.C.D.

PROBLEMS

Find the L.C.D. of these fractions:

1. $\frac{3}{4}$, $\frac{1}{6}$, $\frac{3}{8}$, $\frac{5}{12}$.
2. $\frac{1}{3}$, $\frac{4}{9}$, $\frac{7}{12}$, $\frac{5}{16}$.
3. $\frac{5}{18}$, $\frac{7}{10}$, $\frac{4}{15}$, $\frac{13}{24}$.
4. $\frac{1}{12}$, $\frac{2}{21}$, $\frac{3}{28}$, $\frac{5}{42}$.
5. $\frac{1}{8}$, $\frac{7}{12}$, $\frac{3}{20}$, $\frac{4}{60}$.
6. $\frac{4}{11}$, $\frac{5}{14}$, $\frac{7}{16}$, $\frac{5}{36}$.

89. To change fractions to fractions having their L.C.D.

I. Find the L.C.D.

II. Change each fraction to a fraction having the L.C.D. as its denominator.

PROBLEMS

Change to fractions having their L.C.D.

1. $\frac{17}{21}$, $\frac{9}{16}$, $\frac{3}{14}$, $\frac{5}{7}$.
2. $\frac{4}{7}$, $\frac{3}{14}$, $\frac{7}{18}$, $\frac{21}{24}$.
3. $\frac{7}{12}$, $\frac{9}{30}$, $\frac{13}{48}$, $\frac{32}{60}$.
4. $\frac{8}{14}$, $\frac{9}{26}$, $\frac{15}{28}$, $\frac{17}{39}$.
5. $\frac{9}{24}$, $\frac{14}{25}$, $\frac{7}{30}$, $\frac{9}{40}$.
6. $\frac{3}{28}$, $\frac{17}{40}$, $\frac{7}{16}$, $\frac{14}{27}$.
7. $\frac{1}{18}$, $\frac{4}{27}$, $\frac{5}{64}$, $\frac{17}{36}$, $\frac{13}{24}$.
8. $\frac{5}{12}$, $\frac{5}{21}$, $\frac{7}{16}$, $\frac{8}{15}$.
9. $\frac{11}{36}$, $\frac{15}{42}$, $\frac{21}{50}$, $\frac{13}{72}$.
10. $\frac{31}{68}$, $\frac{25}{96}$, $\frac{16}{81}$, $\frac{13}{42}$, $\frac{22}{27}$.
11. $\frac{4}{7}$, $\frac{12}{45}$, $\frac{7}{26}$, $\frac{9}{156}$.
12. $\frac{15}{34}$, $\frac{25}{66}$, $\frac{8}{75}$, $\frac{5}{81}$.

MENTAL PROBLEMS

Reduce to their simplest forms:

1. $\frac{18}{64}$.
2. $\frac{36}{144}$.
3. $\frac{12}{156}$.
4. $\frac{25}{175}$.
5. $\frac{15}{90}$.
6. $\frac{14}{196}$.
7. $\frac{11}{176}$.
8. $\frac{24}{96}$.
9. $\frac{27}{99}$.
10. $\frac{18}{126}$.
11. $\frac{44}{132}$.
12. $\frac{64}{144}$.
13. $\frac{65}{80}$.
14. $\frac{57}{87}$.
15. $\frac{65}{91}$.

Reduce:

16. 4 to 9ths.
17. $\frac{1}{3}$ to 18ths.
18. $\frac{4}{5}$ to 40ths.
19. $\frac{7}{12}$ to 96ths.
20. $\frac{1}{4}$ to 72ds.
21. $\frac{1}{8}$ to 64ths.
22. $\frac{3}{7}$ to 49ths.
23. $\frac{4}{11}$ to 55ths.
24. $\frac{5}{8}$ to 120ths.
25. $\frac{5}{21}$ to 126ths
26. $\frac{3}{4}$ to 70ths.
27. $\frac{9}{25}$ to 200ths.

Reduce to integers or mixed numbers:

28. $\frac{360}{9}$.
29. $\frac{120}{3}$.
30. $\frac{72}{5}$.
31. $\frac{84}{6}$.
32. $\frac{91}{13}$.
33. $\frac{120}{15}$.
34. $\frac{150}{12}$.
35. $\frac{37}{4}$.
36. $\frac{95}{7}$.
37. $\frac{54}{8}$.
38. $\frac{125}{11}$.
39. $\frac{132}{6}$.
40. $\frac{140}{25}$.
41. $\frac{375}{50}$.
42. $\frac{136}{8}$.

Reduce to the fractional form:

43. $2\frac{3}{4}$.
44. $1\frac{7}{8}$.
45. $6\frac{4}{5}$.
46. $5\frac{3}{7}$.
47. $2\frac{1}{9}$.
48. $3\frac{3}{8}$.
49. $4\frac{2}{5}$.
50. $12\frac{5}{7}$.
51 $7\frac{3}{10}$.
52. $6\frac{4}{9}$.
53. $12\frac{5}{7}$.
54. $19\frac{4}{11}$.
55. $15\frac{1}{6}$.
56. $10\frac{5}{6}$.
57. $14\frac{4}{5}$.

Find the L. C. D. of:

58. $\frac{1}{9}$ and $\frac{1}{12}$.
59. $\frac{1}{8}$ and $\frac{1}{36}$.
60. $\frac{1}{14}$ and $\frac{1}{42}$.
61. $\frac{1}{15}$ and $\frac{1}{18}$.
62. $\frac{1}{12}$ and $\frac{1}{18}$.
63. $\frac{1}{9}$ and $\frac{1}{48}$.
64. $\frac{1}{12}$ and $\frac{1}{32}$.
65. $\frac{1}{10}$ and $\frac{1}{35}$.
66. $\frac{1}{14}$ and $\frac{1}{21}$.
67. $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{12}$.
68. $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{16}$.
69. $\frac{1}{8}$, $\frac{1}{12}$, and $\frac{1}{24}$.

ADDITION OF FRACTIONS

90. *Principles.—1.* Only similar fractions can be added.

2. Dissimilar fractions must be reduced to similar fractions before they can be added.

PROBLEMS

EXAMPLE 1.—Find the sum of $\frac{3}{8}$, $\frac{5}{9}$ and $\frac{7}{12}$.

$$\frac{3}{8}+\frac{5}{9}+\frac{7}{12}=\frac{27}{72}+\frac{40}{72}+\frac{42}{72}=\frac{109}{72}=1\frac{37}{72}.$$

Find the sum of:

1. $\frac{1}{3}$, $\frac{5}{8}$, $\frac{4}{9}$, $\frac{7}{12}$.
2. $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{9}$, $\frac{11}{24}$.
3. $\frac{3}{7}$, $\frac{5}{14}$, $\frac{8}{21}$, $\frac{11}{35}$.
4. $\frac{5}{8}$, $\frac{5}{9}$, $\frac{5}{16}$, $\frac{5}{21}$.
5. $\frac{7}{24}$, $\frac{9}{28}$, $\frac{6}{7}$, $\frac{3}{14}$.
6. $\frac{7}{18}$, $\frac{9}{16}$, $\frac{4}{5}$, $\frac{13}{24}$.
7. $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{11}$, $\frac{5}{12}$.
8. $\frac{7}{9}$, $\frac{5}{8}$, $\frac{3}{10}$, $\frac{2}{15}$.
9. $\frac{14}{5}$, $\frac{12}{8}$, $\frac{15}{16}$, $\frac{21}{12}$.
10. $\frac{21}{22}$, $\frac{41}{24}$, $\frac{165}{72}$, $\frac{13}{56}$.

EXAMPLE 2.—Find the sum of $3\frac{1}{4}$, $7\frac{2}{9}$, $6\frac{5}{8}$, and $5\frac{2}{3}$.

$$\tfrac{1}{4}+\tfrac{2}{9}+\tfrac{5}{8}+\tfrac{2}{3}=\tfrac{18}{72}+\tfrac{16}{72}+\tfrac{45}{72}+\tfrac{48}{72}=1\tfrac{55}{72}.$$

$$3+7+6+5=21. \quad 21+1\tfrac{55}{72}=22\tfrac{55}{72}.$$

NOTE.—Add fractions and integers separately, then add the results.

11. $12\frac{5}{7}$, $6\frac{3}{8}$, $15\frac{17}{21}$.

12. $123\frac{3}{5}$, $168\frac{8}{27}$, $64\frac{2}{15}$, $65\frac{7}{18}$.

13. $27\frac{15}{36}$, $19\frac{11}{15}$, $45\frac{7}{25}$, $90\frac{8}{21}$.

14. $82\frac{5}{14}$, $12\frac{7}{16}$, $13\frac{19}{24}$.

15. $3\frac{5}{21}$, $9\frac{11}{51}$, $4\frac{9}{34}$, $23\frac{8}{45}$.

16. $6\frac{4}{15}$, $2\frac{1}{2}$, $28\frac{9}{21}$, $63\frac{12}{26}$, $5\frac{9}{35}$.

SUBTRACTION OF FRACTIONS

91. *Principles.—1.* Only similar fractions can be subtracted.

2. Dissimilar fractions must be reduced to similar fractions before they can be subtracted.

PROBLEMS

EXAMPLE 1.—Find the value of $\frac{5}{8}-\frac{7}{12}$.

$$\tfrac{5}{8}-\tfrac{7}{12}=\tfrac{15}{24}-\tfrac{14}{24}=\tfrac{1}{24}.$$

Find the value of:

1. $\frac{2}{3}-\frac{5}{11}$.

2. $\frac{7}{9}-\frac{5}{18}$.

3. $\frac{19}{36}-\frac{14}{26}$.

4. $\frac{8}{21}-\frac{7}{25}$.

5. $\frac{5}{9}-1\frac{3}{24}$.

6. $\frac{7}{15}-\frac{25}{48}$.

Mixed numbers may be reduced to improper fractions.

7. $3\frac{1}{2}-\frac{7}{8}$.

8. $2\frac{3}{4}-2\frac{4}{9}$.

9. $1\frac{7}{15}-1\frac{16}{75}$.

10. $5\frac{3}{5}-4\frac{7}{11}$.

11. $3\frac{6}{11}-2\frac{21}{22}$.

12. $2\frac{4}{7}-1\frac{3}{14}$.

Fractions and integers may be subtracted separately.

EXAMPLE 2.—Find the value of $14\frac{7}{8}-11\frac{9}{10}$.

OPERATION

$$\tfrac{7}{8}-\tfrac{9}{10}=\tfrac{35}{40}-\tfrac{36}{40}$$

$$14\tfrac{7}{8}=14\tfrac{35}{40}$$

$$11\tfrac{9}{10}=11\tfrac{36}{40}$$

$$2\tfrac{39}{40}.$$

EXPLANATION.—Reduce the fractions to similar fractions. $\frac{36}{40}$ cannot be subtracted from $\frac{35}{40}$, therefore take 1, or $\frac{40}{40}$, from 14 and unite it with $\frac{35}{40}$, making $\frac{75}{40}$. Then subtract. $\frac{75}{40}-\frac{36}{40}=\frac{39}{40}$. $13\ (14-1)-11=2$.

13. $23\frac{3}{11}-9\frac{7}{33}$.

14. $17\frac{5}{8}-14\frac{9}{14}$.

15. $231\frac{26}{81}-193\frac{17}{27}$.

16. $68\frac{5}{21}-19\frac{16}{35}$.

17. $28\frac{3}{15}-16\frac{3}{7}$.

18. $51\frac{7}{8}-14\frac{5}{11}$.

MENTAL REVIEW OF ADDITION AND SUBTRACTION

92. To add two fractions whose numerators are each 1, take the sum of the denominators for the numerator, and the product of the denominators for the denominator of the sum. Thus, $\frac{1}{4}+\frac{1}{5}=\frac{9}{20}$.

Find the value of:

1. $\frac{1}{3}+\frac{1}{5}$.
2. $\frac{1}{3}+\frac{1}{7}$.
3. $\frac{1}{5}+\frac{1}{9}$.
4. $\frac{1}{2}+\frac{1}{5}$.
5. $\frac{1}{6}+\frac{1}{11}$.
6. $\frac{1}{9}+\frac{1}{7}$.
7. $\frac{1}{11}+\frac{1}{8}$.
8. $\frac{1}{6}+\frac{1}{14}$.
9. $\frac{1}{4}+\frac{1}{7}$.
10. $\frac{1}{2}+\frac{1}{3}$.
11. $\frac{1}{3}+\frac{1}{5}$.
12. $\frac{1}{6}+\frac{1}{5}$.
13. $\frac{1}{5}+\frac{1}{10}$.
14. $\frac{1}{7}+\frac{1}{2}$.
15. $\frac{1}{6}+\frac{1}{5}$.

93. To subtract fractions whose numerators are each 1, take the difference of the denominators for the numerator and the product of the denominators for the denominator of the remainder. Thus, $\frac{1}{4}-\frac{1}{5}=\frac{1}{20}$.

Find the value of:

1. $\frac{1}{6}-\frac{1}{7}$.
2. $\frac{1}{3}-\frac{1}{4}$.
3. $\frac{1}{8}-\frac{1}{9}$.
4. $\frac{1}{4}-\frac{1}{6}$.
5. $\frac{1}{3}-\frac{1}{5}$.
6. $\frac{1}{4}-\frac{1}{9}$.
7. $\frac{1}{5}-\frac{1}{11}$.
8. $\frac{1}{2}-\frac{1}{5}$.
9. $\frac{1}{6}-\frac{1}{8}$.
10. $\frac{1}{3}-\frac{1}{7}$.
11. $\frac{1}{7}-\frac{1}{9}$.
12. $\frac{1}{6}-\frac{1}{8}$.

94. To add two fractions whose numerators are greater than 1, take the sum of the products of the numerators of each by the denominator of the other for the numerator, and the product of the denominators for the denominator of the sum. Thus, $\frac{2}{3}+\frac{3}{4}=\frac{17}{12}$.

$2\times 4+3\times 3=17$ and $3\times 4=12$.

Find the value of:

1. $\frac{2}{3}+\frac{3}{5}$.
2. $\frac{3}{4}+\frac{2}{5}$.
3. $\frac{3}{5}+\frac{5}{8}$.
4. $\frac{3}{7}+\frac{4}{9}$.
5. $\frac{4}{5}+\frac{3}{7}$.
6. $\frac{2}{7}+\frac{3}{8}$.
7. $\frac{2}{9}+\frac{4}{5}$.
8. $\frac{3}{8}+\frac{2}{3}$.
9. $\frac{3}{4}+\frac{5}{7}$.
10. $\frac{5}{6}+\frac{7}{8}$.
11. $\frac{2}{5}+\frac{3}{11}$.
12. $\frac{4}{7}+\frac{3}{4}$.

95. To subtract two fractions whose numerators are greater than 1, take the difference of the products of the numerator of each by the denominator of the other for the numerator, and the product of the denominators for the denominator of the remainder. Thus, $\frac{3}{4}-\frac{2}{3}=\frac{1}{12}$. ($3\times 3-2\times 4=1$, and $3\times 4=12$.)

Find the value of:

1. $\frac{3}{4} - \frac{3}{5}$.
2. $\frac{2}{7} - \frac{3}{11}$.
3. $\frac{4}{5} - \frac{3}{4}$.
4. $\frac{5}{8} - \frac{4}{9}$.
5. $\frac{7}{9} - \frac{3}{5}$.
6. $\frac{2}{3} - \frac{3}{5}$.
7. $\frac{3}{4} - \frac{3}{8}$.
8. $\frac{5}{6} - \frac{4}{5}$.
9. $\frac{8}{9} - \frac{7}{8}$.
10. $\frac{9}{10} - \frac{8}{9}$.
11. $\frac{4}{7} - \frac{3}{4}$.
12. $\frac{7}{8} - \frac{4}{5}$.

MULTIPLICATION OF FRACTIONS

96. *Principles.—1.* Multiplying the numerator multiplies the fraction.

2. Dividing the denominator multiplies the fraction.

PROBLEMS

Find the product of:

1. $\frac{3}{7} \times 2$, or $2 \times \frac{3}{7}$.

$$\frac{3}{7} \times 2 = \frac{3 \times 2}{7} = \frac{6}{7}. \quad \text{(Principle 1.)}$$

2. $\frac{5}{9} \times 4$, or $4 \times \frac{5}{9}$.
3. $\frac{8}{15} \times 6$, or $6 \times \frac{8}{15}$.
4. $\frac{7}{18} \times 8$, or $8 \times \frac{7}{18}$.
5. $\frac{9}{13} \times 7$, or $7 \times \frac{9}{13}$.
6. $\frac{8}{17} \times 4$, or $4 \times \frac{8}{17}$.
7. $\frac{12}{14} \times 6$, or $6 \times \frac{12}{14}$.

8. $\frac{5}{21} \times 3$, or $3 \times \frac{5}{21}$.

$$\frac{5}{21} \times 3 = \frac{5}{21 \div 3} = \frac{5}{7}. \quad \text{(Principle 2.)}$$

9. $\frac{17}{81} \times 9$, or $9 \times \frac{17}{81}$.
10. $\frac{25}{36} \times 12$, or $12 \times \frac{25}{36}$.
11. $\frac{12}{61} \times 17$, or $17 \times \frac{12}{61}$.
12. $\frac{5}{24} \times 3$, or $3 \times \frac{5}{24}$.
13. $\frac{19}{42} \times 21$, or $21 \times \frac{19}{42}$.
14. $\frac{79}{132} \times 11$, or $11 \times \frac{79}{132}$.

15. $\frac{2}{3} \times \frac{4}{5}$.

$$\frac{2}{3} \times \frac{4}{5} = (2 \div 3) \times (4 \div 5), \text{ or } \frac{2 \times 4}{3 \times 5}, \text{ or } \frac{8}{15}.$$

16. $\frac{7}{11} \times \frac{9}{14}$.
17. $\frac{3}{12} \times \frac{6}{15}$.
18. $\frac{21}{81} \times \frac{9}{28}$.
19. $\frac{3}{5} \times \frac{7}{12} \times \frac{15}{21} \times \frac{27}{64}$.
20. $\frac{19}{20} \times \frac{4}{17} \times \frac{51}{144} \times \frac{4}{27}$.
21. $\frac{13}{4} \times \frac{16}{7} \times \frac{49}{8} \times \frac{12}{13}$.

Mixed numbers may be reduced to improper fractions.

22. $1\frac{3}{4} \times 2\frac{5}{8} \times 3\frac{7}{9}$.

23. $2\frac{5}{11} \times 3\frac{4}{5} \times 1\frac{6}{7}$.

24. $\frac{5}{13} \times 2\frac{9}{15} \times 3\frac{7}{16}$.

25. $\frac{12}{10} \times 1\frac{4}{6} \times 2\frac{5}{8}$.

EXAMPLE 1.—Find the product of $12\frac{3}{5} \times 14$ or $14 \times 12\frac{3}{5}$.

OPERATION

$12\frac{3}{5}$
14
168
$8\frac{2}{5}$
$176\frac{2}{5}$

EXPLANATION.—Multiply the integral and the fractional parts of the mixed number separately by the whole number, and add the products. $12 \times 14 = 168$. $\frac{3}{5} \times 14 = 8\frac{2}{5}$. $168 + 8\frac{2}{5} = 176\frac{2}{5}$.

26. $14\frac{5}{9} \times 18$.

27. $13\frac{6}{11} \times 14$.

28. $15 \times 12\frac{3}{5}$.

29. $42 \times 7\frac{4}{13}$.

30. $62\frac{5}{14} \times 21$.

31. $135\frac{9}{16} \times 48$.

32. $206\frac{7}{12} \times 56$.

33. $89 \times 17\frac{3}{22}$.

34. $156 \times 4\frac{5}{16}$.

35. $73 \times 62\frac{7}{8}$.

36. $28\frac{5}{11} \times 45$.

37. $35\frac{8}{13} \times 39$.

38. $64\frac{7}{16} \times 75$.

39. $56\frac{3}{7} \times 14$.

40. $48\frac{4}{5} \times 25$.

41. $31 \times 14\frac{3}{8}$.

42. $44 \times 16\frac{7}{11}$.

43. $39 \times 12\frac{8}{13}$.

44. $17 \times 23\frac{9}{11}$.

45. $81 \times 13\frac{9}{22}$.

EXAMPLE 2.—Find the product of $12\frac{1}{2} \times 15\frac{1}{3}$.

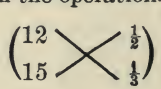
OPERATION

$12\frac{1}{2}$
$15\frac{1}{3}$

$180 = 12 \times 15$
$7\frac{1}{2} = \frac{1}{2} \times 15$
$4 = 12 \times \frac{1}{3}$
$\frac{1}{6} = \frac{1}{2} \times \frac{1}{3}$

$191\frac{2}{3} = 12\frac{1}{2} \times 15\frac{1}{3}$

NOTE.—The connecting lines in the diagram show the steps in the operation.



46. $16\frac{7}{12} \times 14\frac{3}{4}$.

47. $128\frac{3}{11} \times 42\frac{3}{4}$.

48. $169\frac{5}{14} \times 28\frac{7}{15}$.

49. $54\frac{8}{9} \times 72\frac{5}{6}$.

50. $24\frac{3}{4} \times 36\frac{5}{9}$.

51. $180\frac{2}{27} \times 14\frac{2}{3}$.

52. $1684\frac{5}{8} \times 132\frac{3}{5}$.

53. $426\frac{3}{16} \times 96\frac{5}{6}$.

54. $28\frac{6}{13} \times 26\frac{5}{7}$.

55. $33\frac{4}{21} \times 11\frac{1}{2}$.

56. $7\frac{5}{6} \times 146\frac{3}{8}$.

57. $18\frac{3}{4} \times 120\frac{7}{9}$.

MENTAL PROBLEMS

Find the product of:

1. $\frac{3}{5} \times 7$.
2. $\frac{5}{6} \times 18$.
3. $\frac{4}{9} \times 27$.
4. $\frac{6}{7} \times 35$.
5. $\frac{2}{5} \times 12$.
6. $\frac{4}{11} \times 44$.
7. $\frac{5}{13} \times 65$.
8. $\frac{3}{8} \times 64$.
9. $28 \times \frac{5}{14}$.
10. $56 \times \frac{7}{8}$.
11. $32 \times \frac{5}{16}$.
12. $20 \times \frac{4}{5}$.
13. $30 \times \frac{7}{8}$.
14. $45 \times \frac{7}{9}$.
15. $\frac{5}{12} \times 84$.
16. $\frac{5}{6} \times 15$.
17. $\frac{3}{8} \times 17$.
18. $\frac{5}{9} \times 16$.
19. $49 \times \frac{6}{7}$.
20. $108 \times \frac{8}{9}$.
21. $\frac{2}{3} \times \frac{3}{4}$.
22. $\frac{5}{8} \times \frac{2}{5}$.
23. $\frac{3}{7} \times \frac{14}{15}$.
24. $\frac{4}{9} \times \frac{6}{12}$.
25. $\frac{3}{8} \times \frac{4}{9}$.
26. $\frac{5}{12} \times \frac{6}{35}$.
27. $\frac{9}{4} \times \frac{8}{15}$.
28. $\frac{12}{5} \times \frac{45}{8}$.
29. $\frac{6}{7} \times \frac{21}{8}$.
30. $\frac{5}{11} \times \frac{154}{4}$.
31. $\frac{5}{8} \times \frac{6}{7}$.
32. $\frac{3}{11} \times \frac{22}{36}$.
33. $\frac{3}{4} \times \frac{28}{15}$.
34. $\frac{9}{16} \times \frac{8}{15}$.
35. $\frac{7}{12} \times \frac{9}{14}$.
36. $\frac{5}{16} \times \frac{24}{25}$.
37. $\frac{4}{7} \times \frac{11}{9}$.
38. $\frac{6}{11} \times \frac{7}{8}$.
39. $\frac{7}{15} \times \frac{6}{11}$.
40. $\frac{9}{16} \times \frac{7}{15}$.
41. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.
42. $\frac{1}{2} \times \frac{5}{6} \times \frac{7}{10}$.
43. $\frac{4}{7} \times \frac{9}{16} \times \frac{8}{3}$.
44. $\frac{3}{8} \times \frac{4}{5} \times \frac{10}{21}$.
45. $\frac{6}{8} \times \frac{1}{3} \times \frac{2}{5}$.
46. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$.
47. $\frac{3}{10} \times \frac{5}{9} \times \frac{7}{8}$.
48. $\frac{4}{7} \times \frac{5}{6} \times \frac{3}{4}$.
49. $\frac{3}{4} \times \frac{5}{6} \times \frac{4}{5}$.
50. $\frac{7}{12} \times \frac{4}{21} \times \frac{3}{8}$.

DIVISION OF FRACTIONS

97. *Principles.—1.* Dividing the numerator divides the fraction.

2. Multiplying the denominator divides the fraction.

EXAMPLE 1.—Find the quotient of $1\frac{2}{17} \div 6$.

OPERATION

$$1\tfrac{2}{17} \div 6 = \frac{12 \div 6}{17} = \tfrac{2}{17}. \quad \text{Or,}$$

$$1\tfrac{2}{17} \div 6 = \frac{12}{17 \times 6} = \tfrac{12}{102} = \tfrac{2}{17}.$$

EXPLANATION.—To divide $1\frac{2}{17}$ by 6 is to take $\frac{1}{6}$ of $1\frac{2}{17}$. $1\frac{2}{17} \times \frac{1}{6} = \frac{2}{17}$. But $\frac{1}{6}$ is $\frac{6}{1}$ inverted, and $\frac{6}{1}$ is the divisor written as a fraction. Hence if we write the divisor as a fraction, invert the divisor, and multiply the dividend by the inverted divisor, we divide by the divisor. Then, to divide by a fraction, invert the divisor and multiply.

NOTE.—Divide when you *can*, multiply when you *must*.

PROBLEMS

Find the quotient of:

1. $1\frac{5}{16} \div 5$.
2. $\frac{27}{30} \div 9$.
3. $6\frac{4}{81} \div 16$.
4. $3\frac{2}{33} \div 8$.
5. $1\frac{7}{12} \div 6$.
6. $4\frac{2}{11} \div 6$.
7. $3\frac{6}{37} \div 4$.
8. $\frac{7}{15} \div 4$.
9. $1\frac{2}{17} \div 9$.
10. $\frac{5}{12} \div 3$.
11. $1\frac{6}{41} \div 18$.
12. $1\frac{50}{21} \div 25$.
13. $8\frac{1}{70} \div 27$.
14. $6\frac{3}{65} \div 8$.
15. $5\frac{4}{17} \div 3$.

Mixed numbers may be changed to improper fractions.

16. $1\frac{4}{5} \div 3$. *18.* $2\frac{5}{8} \div 3$. *20.* $4\frac{3}{7} \div 5$.

17. $3\frac{2}{16} \div 15$. *19.* $5\frac{7}{12} \div 17$. *21.* $1\frac{3}{4} \div 6$.

EXAMPLE 2.—Find the quotient of $19\frac{3}{4} \div 6$.

OPERATION

$$6)19\frac{3}{4}$$
$$3\frac{7}{24}$$

EXPLANATION.—6 is contained in 19 three times, with a remainder of 1. Change 1 to $\frac{4}{4}$ and add it to $\frac{3}{4}$, making $\frac{7}{4}$. $\frac{7}{4}$ divided by 6 is $\frac{7}{24}$.

NOTE.—Instead of reducing the mixed number to an improper fraction, it is sometimes more convenient to divide as above.

22. $129\frac{5}{9} \div 8$. *26.* $17\frac{3}{8} \div 7$. *30.* $57\frac{9}{15} \div 8$.

23. $1384\frac{7}{12} \div 5$ *27.* $31\frac{6}{13} \div 5$. *31.* $268\frac{5}{16} \div 11$

24. $21\frac{3}{4} \div 6$. *28.* $43\frac{4}{11}$. *32.* $7856\frac{5}{8} \div 7$.

25. $380\frac{5}{7} \div 10$. *29.* $15\frac{2}{3} \div 9$. *33.* $12032\frac{4}{21} \div 15$.

NOTE.—Invert the divisor and multiply.

34. $\frac{3}{8} \div \frac{2}{3}\frac{1}{2}$. *38.* $2\frac{1}{3} \div 4\frac{2}{7}$. *42.* $68\frac{3}{5} \div 21\frac{4}{3}$.

35. $\frac{5}{9} \div \frac{2}{2}\frac{0}{7}$. *39.* $1\frac{5}{11} \div \frac{9}{5}$. *43.* $123\frac{8}{17} \div 65\frac{7}{3}$.

36. $\frac{3}{4}\frac{2}{9} \div \frac{8}{7}$. *40.* $\frac{3}{6}\frac{1}{3} \div \frac{3}{13}$. *44.* $1685\frac{7}{24} \div 23\frac{1}{8}$.

37. $\frac{1}{2}\frac{6}{1} \div \frac{4}{7}$. *41.* $\frac{2}{11} \div \frac{4}{15}$. *45.* $8\frac{4}{9} \div \frac{2}{3}$.

Divide:

46. $\frac{7}{15}$ of $\frac{8}{11}$ by $\frac{4}{5}$ of $\frac{4}{7}\frac{2}{7}$. *50.* $\frac{2}{3}$ of $\frac{5}{9}$ by $\frac{3}{5}$ of $\frac{6}{7}$ of $1\frac{2}{15}$.

47. $\frac{9}{31}$ of $\frac{1}{17}\frac{2}{}$ by $\frac{2}{6}\frac{7}{2}$ of $\frac{4}{12}\frac{2}{8}$. *51.* $\frac{14}{15}$ of $\frac{18}{49}$ by $\frac{12}{21}$ of $\frac{16}{45}$.

48. $3\frac{5}{8}$ of $2\frac{4}{13}$ by $\frac{5}{8}\frac{6}{7}$ of $\frac{15}{52}$ *52.* $4\frac{3}{9}$ of $6\frac{8}{11}$ by $4\frac{3}{5}$ of $5\frac{2}{7}$.

49. $11\frac{9}{16}$ of $14\frac{3}{14}$ by $2\frac{1}{2}$ of $\frac{6}{42}\frac{3}{}$. *53.* $\frac{2}{9}\frac{4}{6}$ of $\frac{1}{8}\frac{7}{1}$ by $\frac{5}{7}\frac{1}{6}$ of $\frac{7}{4}\frac{2}{9}$.

MENTAL PROBLEMS

Find the quotient of:

1. $\frac{9}{11} \div 3$. *11.* $8 \div \frac{4}{7}$. *21.* $\frac{1}{2} \div \frac{1}{3}$. *31.* $12 \div \frac{4}{3}$.

2. $1\frac{8}{5} \div 6$. *12.* $9 \div \frac{3}{11}$. *22.* $\frac{2}{3} \div \frac{3}{4}$. *32.* $7 \div \frac{3}{8}$.

3. $\frac{2}{4}\frac{1}{0} \div 7$. *13.* $12 \div \frac{3}{16}$. *23.* $\frac{4}{5} \div \frac{2}{15}$. *33.* $\frac{8}{9} \div \frac{4}{5}$.

4. $\frac{3}{16} \div 5$. *14.* $15 \div \frac{7}{3}$. *24.* $\frac{5}{9} \div \frac{2}{3}$. *34.* $\frac{5}{6} \div \frac{9}{14}$.

5. $\frac{6}{7} \div 6$. *15.* $25 \div 2\frac{1}{2}$. *25.* $\frac{3}{4} \div \frac{7}{8}$. *35.* $\frac{6}{7} \div \frac{2}{21}$.

6. $\frac{5}{11} \div 15$. *16.* $18 \div \frac{3}{8}$. *26.* $16 \div \frac{8}{3}$. *36.* $4 \div \frac{5}{9}$.

7. $\frac{7}{12} \div 14$. *17.* $\frac{5}{6} \div 15$. *27.* $\frac{3}{8} \div 9$. *37.* $\frac{7}{16} \div \frac{3}{8}$.

8. $\frac{12}{7} \div 4$. *18.* $24 \div \frac{3}{8}$. *28.* $\frac{3}{16} \div 3$. *38.* $\frac{3}{7} \div 9$.

9. $\frac{15}{17} \div 10$. *19.* $30 \div \frac{5}{6}$. *29.* $\frac{1}{2} \div \frac{5}{9}$. *39.* $15 \div \frac{6}{7}$.

10. $6 \div \frac{2}{3}$ *20.* $\frac{4}{3} \div 8$. *30.* $\frac{2}{3} \div \frac{6}{5}$. *40.* $24 \div \frac{3}{5}$.

THE THREE PROBLEMS OF FRACTIONS

98. *1.* To find a part of a number: *What is $\frac{3}{4}$ of 48?*

2. To find what part one number is of another: *6 is what part of 15?*

3. To find a number when a part of it is given: *$\frac{3}{8}$ of a number is 12; what is the number? 12 is $\frac{3}{8}$ of what number?*

Solution by the Equation

Each of the above problems may be stated as an equation. *Is* means =, *of* means ×. Representing the number to be found by ?, the above problems may be stated thus:

1. $? = \frac{3}{4} \times 48$. *2.* $6 = ? \times 15$. *3.* $\frac{3}{8} \times ? = 12$; $12 = \frac{3}{8} \times ?$.

In equations 2 and 3, the product of two numbers and one of the numbers is given to find the other number.

PROBLEMS

Write each of the following problems as an equation and then solve the equation:

1. Find $\frac{4}{9}$ of 270. $(? = \frac{4}{9} \times 270.)$

2. 21 is what part of 36? $(21 = ? \times 36.)$

3. Of 48, 27 is what part? $(27 = ? \times 48.)$

4. 25 is $\frac{4}{5}$ of what number?

5. $\frac{5}{7}$ of a number is 35; what is the number?

6. What is $\frac{3}{16}$ of 128?

7. 342 is $\frac{5}{8}$ of what number?

8. $\frac{5}{16}$ is $\frac{3}{7}$ of what number?

9. Of $\frac{4}{13}$, $\frac{7}{21}$ is what part?

10. What is $\frac{1}{16}\frac{2}{8}$ of 180?

11. What part of 86 is 14?

12. What part of $32\frac{5}{8}$ is $12\frac{1}{3}$?

13. Of $64\frac{1}{2}$ days, $\frac{5}{9}$ is what part?

14. $18\frac{3}{7}$ is $\frac{9}{20}$ of what number?

15. $24\frac{1}{2}$ pounds is what part of $65\frac{1}{2}$ pounds?

16. A cow cost $27\frac{3}{8}$ dollars, and a horse $78\frac{4}{20}$ dollars. The cost of the cow was what part of the cost of the horse?

17. A desk cost $18\frac{3}{7}$ dollars, which was $\frac{9}{11}$ of the cost of a table. What was the cost of the table?

18. The value of 26 cords of wood at $3\frac{2}{5}$ dollars per cord is what part of the value of 35 tons of coal at $4\frac{1}{6}$ dollars per ton?

19. A gain of $\frac{3}{5}$ of a stock of goods is a gain of what part of $\frac{1}{3}$ of the goods?

20. Different kinds of coffee are mixed in the following parts: $14\frac{5}{16}$ pounds, $18\frac{9}{16}$ pounds, $21\frac{4}{16}$ pounds. Each part is what part of the whole mixture?

21. If $31\frac{1}{2}$ gallons of cider make $5\frac{2}{3}$ gallons of jelly, the number of gallons of jelly is what part of the number of gallons of cider?

MENTAL REVIEW

99. Solve mentally:

1. Add $\frac{1}{8}$ and $\frac{5}{16}$.

2. From $\frac{1}{4}$ take $\frac{1}{7}$.

3. Add $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$.

4. Find the product of $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$ and $\frac{1}{8}$.

5. Change $\frac{7}{12}$ to 84ths.

6. Reduce $\frac{144}{168}$ to lowest terms.

7. What will 12 pounds of tea cost at $\frac{3}{8}$ dollars per pound?

8. What will $\frac{5}{16}$ of a ton of hay cost at $18 per ton?

9. Find the cost of 9 ounces of butter at $.20 per pound.

10. Find the cost of 15 eggs at $.14 per dozen.

11. If 1 pound 14 ounces of cheese cost $.23, what is the price per pound?

12. If a horse eats $\frac{3}{8}$ bushel of oats in a day, how long will 30 bushels last him?

13. Divide 14 by $\frac{3}{7}$?

14. A crock of butter weighs $8\frac{1}{3}$ pounds, and the crock alone weighs $1\frac{1}{2}$ pounds. What is the value of the butter at $12\frac{1}{2}$ cents per pound?

15. What will $9\frac{5}{8}$ cords of wood cost at $4\frac{1}{2}$ dollars per cord?

16. At 36 bushels to the acre, what is the yield of $1\frac{1}{3}$ acres?

17. $\frac{5}{8}$ is what part of $\frac{4}{5}$?

18. What is $\frac{7}{12}$ of 32?

19. 18 is what part of 40?

20. From $\frac{1}{4}$ take $\frac{1}{9}$.

21. Find the cost of $\frac{3}{4}$ of a piece of cloth of 36 yards at $1\frac{1}{3}$ dollars per yard.

22. At $1\frac{3}{4}$ dollars per day of 10 hours, what will a man earn in $7\frac{1}{2}$ hours?

23. If 10 bushels of apples will make 32 gallons of cider, how much cider will 7 bushels make?

24. Find the cost of 750 feet of lumber at $9\frac{1}{2}$ dollars per thousand.

25. $\frac{3}{8}$ is what part of $\frac{11}{2}$?

26. A lady bought $5\frac{3}{4}$ yards of silk for $8\frac{5}{8}$ dollars, what was the price per yard?

27. At $1\frac{5}{8}$ dollars per yard, how much cloth can be purchased for $14?

28. 32 is $\frac{4}{9}$ of what number?

29. What part of 64 is 42?

30. $\frac{1}{3}$ is what part of $\frac{3}{4}$?

31. A grocer mixed 7 ounces of coffee at $.30 per pound with 9 ounces at $.40 per pound. What is the pound of mixed coffee worth?

32. Divide $\frac{7}{16}$ by 8.

33. Add $\frac{2}{3}$ and $\frac{5}{8}$.

34. A couch is worth $9. For how much must it be sold to gain $\frac{3}{16}$?

35. At $4.50 per week, what will five days' board cost?

36. Sold a book for $2.25 and gained $\frac{2}{5}$. What did the book cost me?

37. Find the cost of $\frac{2}{5}$ of a yard of silk at $.60 per yard.

38. 14 is what part of 7?

39. Multiply $\frac{4}{5}$ by 6.

40. 15 is .75 of what number?

41. Find the cost of .8 pounds of tea at $.45 per pound.

42. Find the cost of $\frac{3}{5}$ of a ton of coal at $6.50 per ton

43. Change $\frac{3}{8}$ to a decimal.

44. Change .625 to a common fraction.

45. Bought a wagon for $40 and sold it at a gain of .20 of the cost. Find the selling price.

46. If I wish to gain $.25 on a dollar, how must I mark an article that cost me $4.80?

47. Sold a table for $6 and made a gain of .25 of the purchase price. Find the purchase price.

48. Find the cost of 425 pickles at $.30 per hundred.

49. Change $\frac{3}{5}$ to a decimal.

50. $\frac{3}{8}$ is what part of .40?

51. Change .2$\frac{1}{2}$ to a common fraction.

52. A man borrows $350, and agrees to pay .06 of the sum for its use. How much should he pay the lender?

53. .08 is what part of .16?

54. At $.12$\frac{1}{2}$ apiece, how many brushes can be bought for 4$\frac{3}{4}$ dollars?

55. In selling combs at $.20, I lost .20 of the cost. What did they cost me?

56. What is .04 of $\frac{3}{5}$?

57. $\frac{3}{4}$ is what part of .80?

58. Find the cost of 10 ounces of meat at $.12$\frac{1}{2}$ per pound?

59. A rod 10 feet long is lengthened by .03 of itself. What is its length then?

WRITTEN REVIEW

100. Solve the following:

1. If a furnace consumes a ton of coal in 9 days, in how many weeks will it consume 9 tons?

2. If I pay $36.40 with wheat worth $$\frac{5}{8}$$ per bushel, how many bushels must I give?

3. If a train goes 146.54 miles in 3 hours, what is the rate of speed per minute?

4. A contributed $5800 to the capital of a company; B, $7800; C, $9600; and D, $5500. What part of the whole did each put in?

5. From a tract of 49$\frac{5}{8}$ acres of land, how many lots of $\frac{3}{4}$ of an acre each can be laid out?

6. A agreed to keep B's horse 14 weeks for $18. If A keeps the horse 11 days, how much ought B to pay?

7. A field of 29½ acres produced 3450 bushels of potatoes. What was the average yield per acre?

8. At $.87½ per bushel of 60 pounds, what will 4780 pounds of wheat cost?

9. If apples lose .70 of their weight in drying, how many pounds of apples must be used to make 300 pounds of dried apples?

10. If a bushel of wheat of 60 pounds will make 44 pounds of flour, and 16 pounds of feed, and the miller takes .10 of the grist for grinding, how many bushels of wheat must a customer take to the mill to get 10 barrels of flour of 195 pounds each? How much feed will he get?

UNITED STATES MONEY

101. Money is a measure of value and a medium of exchange.

A watch is worth 10 dollars. The dollar is the unit of measure.

A piece of cloth is 10 yards long. The yard is the unit of measure.

A butcher wants a hat worth 3 dollars, a ring worth 5 dollars, and a book worth 3 dollars. Can he take 3 dollars' worth of meat to the hatter, 5 dollars' worth to the jeweler, and 3 dollars' worth to the bookseller and exchange for the things he wants? Why? What can he do that he may practically exchange his meat for these things? Why does the butcher exchange his meat for money if they both have the same value? Money is the *medium* by which the butcher makes the exchange.

102. The unit of United States money is the dollar.

The first Congress of the United States made the dollar the unit of value. It determined the value of the dollar by providing for the coinage of silver dollars to contain 371.25 grains of pure silver (with certain alloy) and of gold pieces to contain 24.75 grains of pure gold (with certain alloy) to the dollar.

The *value* of the coins was determined by the amount of metal they contained, and by the value trade and custom gave them. The *coin* determined the value of the dollar, the *dollar* did not determine the value of the coin.

103. *Ratio.*—By the first coinage law the weight of a silver dollar was 15 times the weight of a gold dollar. The ratio of weight then was 15 to 1. In 1836, Congress passed a bill making the coinage ratio 16 to 1, so that since then a silver dollar weighs 16 times as much as a gold dollar.

104. The denominations and scale of United States money are shown by the following

TABLE

10 mills = 1 cent (¢, c., or ct.).
10 cents = 1 dime.
10 dimes = 1 dollar ($).
10 dollars = 1 eagle.

United States money is based on the decimal scale. It is expressed as dollars, cents, and mills. The terms dime and eagle are not commonly used. Dollars are written as integers, cents as hundredths, and mills as thousandths.

The sign ($) is prefixed to expressions of United States money; as, $7, $.07, $.007.

Cents and mills are sometimes written as common fractions; as, $12.25, 12\frac{25}{100}$, 16$\frac{1}{2}$¢, 12\frac{1}{2}$, or $.165.

If the final result of a computation contains five or more mills, they are counted as one cent; if less than five, they are rejected; as, $3.166, $3.17; $4.714, $4.71.

United States Coins

105. Gold: The double-eagle, eagle, half-eagle, and one-dollar piece.

Silver: The dollar, half-dollar, quarter-dollar, and the ten-cent piece.

Nickel: The five-cent piece.

Bronze: the one-cent piece.

At various times other pieces have been coined; as, the half-cent and the two-cent piece in bronze, the nickel three-cent piece, the silver half-dime, silver twenty-cent piece, silver trade-dollar, three-dollar gold piece.

106. *Alloy.*—All coins contain an alloy to toughen them and reduce the loss from abrasion. Gold coins are made of $\frac{9}{10}$ pure gold and $\frac{1}{10}$ silver and copper. Silver coins are made of $\frac{9}{10}$ pure silver and $\frac{1}{10}$ copper. Nickel coins are made of $\frac{3}{4}$ copper and $\frac{1}{4}$ nickel. The bronze coins are made of $\frac{95}{100}$ copper and $\frac{5}{100}$ tin and zinc.

107. *Legal Tender.*—Money that when offered, or tendered, in payment of a *debt* must be accepted *or lose further interest* is called a Legal Tender. Gold coins and silver dollars are legal tender for all debts; the other silver coins, for debts not exceeding $10; the other coins, for debts not exceeding 25¢.

United States Paper Money

108. United States paper money consists of notes, gold and silver certificates. A note is a promise to pay. Its value consists in the promise to pay. Notes given by some men are good, notes given by some men are worthless. In what is the difference? The notes of the United States government are good. Why?

United States notes are called Greenbacks and Treasury Notes. Examine one or more of each. Read what is printed on them. Why are they as good as gold?

National Bank Notes, or Bank Bills, are issued by national banks under the supervision of the United States government. These bills are not legal tender, but they are redeemable in lawful money. Examine a bank bill. Why are they received for debts?

Certificates of Deposit are called Gold Certificates and Silver Certificates. Examine one of each. Are they legal tender? Why are they received for debts?

109. Coin is *metallic* money. Currency is any kind of *paper* money.

OPERATIONS WITH ALIQUOT PARTS

Aliquot Parts

110. The Aliquot Parts of a number are the parts that will exactly divide the number. Thus, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, are aliquot parts of 10.

Aliquot parts of 10, 100, 1000, respectively:

Halves { 5, 50, 500 } Thirds { $3\frac{1}{3}$, $33\frac{1}{3}$, $333\frac{1}{3}$ } Fourths { $2\frac{1}{2}$, 25, 250 } Sixths { $1\frac{2}{3}$, $16\frac{2}{3}$, $166\frac{2}{3}$ }

Eighths { $1\frac{1}{4}$, $12\frac{1}{2}$, 125 } Twelfths { $8\frac{1}{3}$, $83\frac{1}{3}$ } Sixteenths { $6\frac{1}{4}$, $62\frac{1}{2}$ }

MULTIPLICATION WITH ALIQUOT PARTS

111. EXAMPLE.—Multiply 1836 by $33\frac{1}{3}$.

OPERATIONS

$1836 \times 25 = 1836 \div 4 \times 100 = 45900.$
$1836 \times 16\frac{2}{3} = 1836 \div 6 \times 100 = 30600.$
$1836 \times 125 = 1836 \div 8 \times 1000 = 229500.$

EXPLANATION.—$33\frac{1}{3}$ is $\frac{1}{3}$ of 100. Therefore, the required product is as many hundreds as 3 is contained times in 1836, or 612 hundred, or 61200.

NOTE.—Take such a part of the multiplicand as the multiplier is of 10 and annex one cipher, as the multiplier is part of 100 and annex two ciphers, or as the multiplier is of 1000 and annex three ciphers.

PROBLEMS

Multiply:

1. 1836 by 50, 25, $12\frac{1}{2}$, $8\frac{1}{3}$, $6\frac{1}{4}$.
2. 2448 by 50, $33\frac{1}{3}$, 25, $16\frac{2}{3}$, $12\frac{1}{2}$, $8\frac{1}{3}$, $6\frac{1}{4}$.
3. 368 by 50, $33\frac{1}{3}$, 25, $16\frac{2}{3}$, $12\frac{1}{2}$, $8\frac{1}{3}$, $6\frac{1}{4}$.
4. 42 by $3\frac{1}{3}$, $33\frac{1}{3}$, $333\frac{1}{3}$, $2\frac{1}{2}$, 25, 250.
5. 56 by $2\frac{1}{2}$, 25, 250, $1\frac{1}{4}$, $12\frac{1}{2}$, 125.
6. 486 by $1\frac{2}{3}$, $16\frac{2}{3}$, $166\frac{2}{3}$, $8\frac{1}{3}$, $83\frac{1}{3}$.
7. 126 by $1\frac{1}{4}$, $12\frac{1}{2}$, 125, $2\frac{1}{2}$, 25, 250.
8. 156 by $8\frac{1}{3}$, $83\frac{1}{3}$, $2\frac{1}{2}$, 25, 250, $33\frac{1}{3}$.
9. 256 by $6\frac{1}{4}$, $62\frac{1}{2}$, $8\frac{1}{3}$, $83\frac{1}{3}$, 25, $333\frac{1}{3}$.

10. \$26.40 by 50, $33\frac{1}{3}$, 25, $16\frac{2}{3}$, $12\frac{1}{2}$, $8\frac{1}{3}$, $6\frac{1}{4}$.

11. \$1.84 by $33\frac{1}{3}$, 250, $16\frac{2}{3}$, 125, $83\frac{1}{3}$, $6\frac{1}{4}$.

12. \$3.36 by 5, $3\frac{1}{3}$, $2\frac{1}{2}$, $33\frac{1}{3}$, 25, $1\frac{2}{3}$, $12\frac{1}{2}$, $1\frac{1}{4}$, $16\frac{2}{3}$, 125, $8\frac{1}{3}$, $62\frac{1}{2}$, $83\frac{1}{3}$, $6\frac{1}{4}$, 50, $333\frac{1}{3}$, 250, $166\frac{2}{3}$.

13. \$7.68 by $1\frac{2}{3}$, 250, $16\frac{2}{3}$, 25, $166\frac{2}{3}$, $2\frac{1}{2}$, $1\frac{1}{4}$, $333\frac{1}{3}$, $12\frac{1}{2}$, $33\frac{1}{3}$, 125, $3\frac{1}{3}$, $8\frac{1}{3}$, $6\frac{2}{3}$, $83\frac{1}{3}$, $6\frac{1}{4}$.

14. \$10.56 by $62\frac{1}{2}$, $6\frac{1}{4}$, $83\frac{1}{3}$, 125, $8\frac{1}{3}$, $12\frac{1}{2}$, $16\frac{2}{3}$.

15. \$50, \$.$33\frac{1}{3}$, \$.$02\frac{1}{2}$, \$.$16\frac{2}{3}$, \$3.$33\frac{1}{3}$, \$.$12\frac{1}{2}$, \$1.$66\frac{2}{3}$, \$.$06\frac{1}{4}$, \$62.50, \$1.25 by 576.

112. Multiplication Table

1	$2\frac{1}{2}$	$3\frac{1}{3}$	$6\frac{1}{4}$	$8\frac{1}{3}$	$12\frac{1}{2}$	$16\frac{2}{3}$
2	5	$6\frac{2}{3}$	$12\frac{1}{2}$	$16\frac{2}{3}$	25	$33\frac{1}{3}$
3	$7\frac{1}{2}$	10	$18\frac{3}{4}$	25	$37\frac{1}{2}$	50
4	10	$13\frac{1}{3}$	25	$33\frac{1}{3}$	50	$66\frac{2}{3}$
5	$12\frac{1}{2}$	$16\frac{2}{3}$	$31\frac{1}{4}$	$41\frac{2}{3}$	$62\frac{1}{2}$	$83\frac{1}{3}$
6	15	20	$37\frac{1}{2}$	50	75	100
7	$17\frac{1}{2}$	$23\frac{1}{3}$	$43\frac{3}{4}$	$58\frac{1}{3}$	$87\frac{1}{2}$	$116\frac{2}{3}$
8	20	$26\frac{2}{3}$	50	$66\frac{2}{3}$	100	$133\frac{1}{3}$
9	$22\frac{1}{2}$	30	$56\frac{1}{4}$	75	$112\frac{1}{2}$	150
10	25	$33\frac{1}{3}$	$62\frac{1}{2}$	$83\frac{1}{3}$	125	$166\frac{2}{3}$

NOTE 1.—This table can be easily learned and will prove convenient in mental operations.

NOTE 2.—Black figures indicate multipliers and multiplicands. Intersecting points of horizontal and vertical columns give the products.

MENTAL DRILL

1. Multiply each multiplicand by its corresponding multiplier.

2. Multiply each multiplicand by each of all the multipliers.

3. Multiply each of all the multiplicands by each multiplier.

MULTIPLICANDS	MULTIPLIERS	MULTIPLICANDS	MULTIPLIERS
$ 6.72	.05	$ .05	48
7.20	.02½	.03⅓	96
7.68	.03⅓	.33⅓	144
8.16	.83⅓	.02½	192
8.64	2.50	.25	240
9.12	.16⅔	2.50	288
9.60	.25	.16⅔	336
10.08	.62½	1.66⅔	384
10.56	1.25	.12½	432
11.04	.33⅓	1.25	528
12.96	1.66⅔	.83⅓	576
13.44	.12½	.62½	524

NOTE.—The product will be the same whichever factor is *used* as the multiplier.

DIVISION WITH ALIQUOT PARTS

113. EXAMPLE.—Divide 245 by 3⅓, 33⅓, and 333⅓ respectively.

OPERATION

245 ÷ 3⅓ = 73.5
245 ÷ 33⅓ = 7.35
245 ÷ 333⅓ = .735

EXPLANATION.—10 is 3 times 3⅓. 245 ÷ 3 equals as many times 3 as there are 10's in 245, or 24.5. 24.5 × 3 = 73.5. In like manner, 100 is 3 times 33⅓, and 1000 is 3 times 333⅓.

NOTE.—To divide by an aliquot part of 10, 100, or 1000, multiply the dividend by the number that shows what aliquot part the divisor is of 10, 100, or 1000, and point off 1, 2, or 3 places, as the divisor is part of 10, 100, or 1000.

MENTAL PROBLEMS

1. Divide all the dividends by each divisor.

2. Divide each dividend by all the divisors.

DIVIDEND	DIVISOR	DIVIDEND	DIVISOR	DIVIDEND	DIVISOR
7.82	5	87.64	.83⅓	.76	125
5.47	50	32.45	1.25	1.82	8⅓
.82	3⅓	17.63	.62½	4.25	83⅓
1.26	33⅓	18.24	1.66⅔	7.63	6¼
3.14	2½	5.67	2.50	2.57	62½

Dividend	Divisor	Dividend	Divisor	Dividend	Divisor
.91	25	4.39	.12½	14.75	.33⅓
.85	250	35.87	.06¼	7.53	.50
2.13	16⅔	42.64	.25	12.15	.02½
4.16	166⅔	3.65	.08⅓	8.24	.03⅓
.53	12½	16.52	.16⅔	124.16	.05

PRICE, COST, AND QUANTITY

114. Business computations deal with price, cost, and quantity.

Solution by the equation: Let P = price, C = cost, Q = quantity. Then, P × Q = C. Hence, C ÷ Q = P, and C ÷ P = Q.

MENTAL PROBLEMS

1. If the price is 8 cents, and the quantity 12, what is the cost?

2. If 16 articles cost 48 cents, what is the price?

3. If the price is 3 cents and the cost 36 cents, what is the quantity?

Find the term not given:

	Price	Quantity	Cost		Price	Quantity	Cost
4.	$.08	15	?	*10.*	$.12½	16	?
5.	.11	?	$1.32	*11.*	?	21	$1.05
6.	.12	10	?	*12.*	.25	12	?
7.	.06	?	2.40	*13.*	?	14	.98
8.	?	15	6.00	*14.*	.11	?	1.65
9.	?	8	1.60	*15.*	.08	?	4.00

EXERCISES

Tell how to find the term not given.

1. P (25¢), Q.
2. P (62½¢), C.
3. C, Q (12½).
4. C, P ($16⅔).
5. Q (112½), P.
6. P ($1.16⅔), Q.
7. C, P.
8. C, Q (33⅓).
9. C, P (83⅓¢).
10. Q, P ($2.25).
11. P, Q (166⅔).
12. C, Q (62½).

13. P (25¢), C.
14. C, P ($6.25).
15. Q, P ($2.125).
16. Q (83⅓), P.
17. Q (25), C.
18. C, P (66⅔¢).
19. Q (33⅓), C.
20. P (75¢), Q.
21. Q (75), P.
22. C, P (75¢).
23. Q (75), C.
24. Q, P ($1.83⅓¢).
25. Q (50), P.
26. C, Q (6¼).
27. C, P (.125¢).
28. C ($65.13), P.
29. Q (75), P (25¢).
30. Q, (62½), P ($1.86).
31. P (46¢), C ($16.89).
32. Q, P (50¢).
33. P ($83⅓), C.
34. Q (6¼), C.
35. Q, P ($2.50).
36. Q (75), P ($2.33⅓).
37. Q, P (62½¢).
38. Q (83⅓), C.
39. C ($1.66⅔), P (3⅓¢).
40. C, Q (25).
41. C, P (8⅓¢).
42. P, Q (2.62½).
43. Q (8⅓), C.
44. Q (12½), P ($25).
45. P (2½¢), Q.
46. C, P ($1.08⅓).
47. P, Q (116⅔).
48. Q (66⅔), C.
49. Q, P (6¼¢).
50. Q (8⅓), P.

NOTE.—To multiply by 2.33⅓, multiply by 2 and by .33⅓ separately and add the products. Treat 1.16⅔, 1.12½, 2.83⅓, 2.62½, etc., in a similar manner.

Articles Bought by 100 (C) or 1000 (M)

115. Q ÷ 100 = Q in hundreds. (Point off two places.)
Q ÷ 1000 = Q in thousands. (Point off three places.)
P per 100 × Q in hundreds = Cost.
P per 1000 × Q in thousands = Cost.

EXAMPLE 1.—Find the cost of 384 laths at $.33⅓ per C.

FORMULA $\quad \dfrac{\text{P per } 100 \times \text{Q}}{100} = \text{Cost.}$

SOLUTION $\quad \dfrac{384 \times \$.33\frac{1}{3}}{100} = \$1.28.$

EXAMPLE 2.—Find the cost of 2415 laths at $3.25 per M.

FORMULA $\quad \dfrac{\text{P per } 1000 \times \text{Q}}{1000} = \text{Cost.}$

SOLUTION $\quad \dfrac{2415 \times \$3.25}{1000} = \$7.85.$

PROBLEMS

Find the cost of:

1. 781 brick at 85 cents per C.
2. 2107 feet pine at $18.50 per M.
3. 6385 feet hemlock at $14.60 per M.
4. 1343 posts at $12.25 per C.
5. 1560 pineapples at $8⅓ per C.
6. 2752 pounds coal at 25¢ per C.
7. 687 feet oak at $32 per M.
8. 3250 shingles at $3.33⅓ per M.
9. 964 pounds beef at $6.25½ per C.
10. 4738 feet timber at $23.50 per M.

Articles Bought by the Ton

116. Price per ton ÷ 2 = price per 1000 pounds.

Price per 1000 pounds × Q ÷ 1000 = Cost.

EXAMPLE.—Find the cost of 2685 pounds of hay at $12 per ton.

FORMULA

$$\frac{\text{P per ton} \times \text{Q}}{2 \times 1000} = \text{Cost.}$$

SOLUTION

$$\frac{\$12 \times 2685}{2 \times 1000} = \$16.11.$$

PROBLEMS

Find the cost of:

1. 6842 pounds of coal at $5.20 per ton.
2. 4975 pounds steel at $33.33⅓ per ton.
3. 2360 pounds sugar at $83⅓ per ton.
4. 15837 pounds old iron at $6.25 per ton.
5. 6974 pounds salt at $5.75 per ton.
6. 3798 pounds; price per ton, $6.90.
7. 8790 pounds; price per ton, $12.50.
8. 350 pounds; price per ton, $9.60.

Articles Bought by the Bushel

117. EXAMPLE.—Find the cost of 2100 pounds of wheat at 70 cents per bushel of 60 pounds.

FORMULA

$$\frac{\text{pounds} \times \text{P per bushel}}{\text{pound per bushel}} = \text{Cost.}$$

SOLUTION

$$2100 \div 60 \times 70¢ = \$24.50.$$

NOTE.—Use cancellation.

TABLE OF BUSHEL WEIGHTS

	Pounds		Pounds		Pounds
Apples	56	Corn (shelled)	56	Potatoes	60
Barley	48	Corn (ear)	70	Rye	56
Beans	60	Flaxseed	56	Timothy seed	45
Buckwheat	48	Oats	32	Turnips	56
Clover seed	60	Onions	57	Wheat	60

NOTE.—These weights are used in most of the States.

PROBLEMS

Find the cost of a load of:

1. Oats, weighing 2146 pounds, at 35¢ per bushel.

2. Potatoes, weighing 3257 pounds, at 48¢ per bushel.

3. Apples, weighing 2980 pounds, at 22¢ per bushel.

4. Turnips, weighing 3425 pounds, at 48¢ per bushel.

5. Barley, weighing 4160 pounds, at 36¢ per bushel.

6. Beans, weighing 3290 pounds, at $1.85 per bushel.

7. Buckwheat, weighing 1846 pounds, at 58¢ per bushel.

8. Corn, weighing 2163 pounds, at 65¢ per bushel.

9. Flaxseed, weighing 3375 pounds, at 40¢ per bushel.

10. Onions, weighing 1956 pounds, at 40¢ per bushel.

11. Rye, weighing 2742 pounds, at $2.50 per bushel.

12. Wheat, weighing 3094 pounds, at 75¢ per bushel.

13. Find the total value of the following produce: 3 loads of wheat weighing 3122, 2659, and 3380 pounds respectively, at 85¢ per bushel; 1 load of barley, weighing 2755 pounds, at 72¢ per bushel; 4 loads of potatoes, weighing 3062, 2587, 3420, and 2970 pounds respectively, at 42¢ per bushel; 2 loads of beans, weighing 3160 pounds each, at $2.12½ per bushel; and 1 load of apples, weighing 2875 pounds, at 48¢ per hundred pounds.

NOTE.—Coal is sold by the ton, and by the bushel of 80 lbs.

14. Find the cost of a load of coal, weighing 2260 pounds, at 22 cents per bushel of 80 pounds. What is the equivalent price per ton?

15. Find the cost of 2493 pounds of coal at $5.75 per ton. What is the equivalent price per bushel?

16. A wagon loaded with potatoes weighs 4750 pounds, and the wagon alone weighs 1426 pounds. What are the potatoes worth at 33¢ per bushel?

17. A cart loaded with coal weighs 3492 pounds, and the cart weighs 1280 pounds. Find the cost of the coal at 25¢ per bushel. Find the equivalent price per ton.

18. A wagon loaded with 28 bags of wheat weighs 4960 pounds, the wagon weighs 1420 pounds, and each bag weighs 2 pounds. Find the value of the wheat at 72¢ per bushel.

19. Find the cost of 20 bags of beans, weighing 118 pounds each, at $1.85 per bushel.

20. What is the cost of three loads of turnips, weighing 2240, 2875, and 2680 pounds respectively, at 28¢ per bushel?

21. How many pounds of shelled corn, at 48¢ per bushel, can be bought for $22.50?

22. Find the cost of a load of drying apples, weighing 3120 pounds, at 18¢ per bushel.

23. A wagon loaded with onions weighs 5570 pounds, the wagon and crates weigh 1808 pounds. Find the cost of the onions at 45¢ per bushel.

24. A man bought a load of oats for $21.35. If the oats weighed 1952 pounds, what price did he pay per bushel?

25. A wagon loaded with 30 bags of beans weighs 5870 pounds, the wagon weighs 1842 pounds, and each bag weighs 2 pounds. Find the cost of the beans at $1.25 per bushel.

DENOMINATE NUMBERS

DEFINITIONS

118. Some things are counted; as, dollars, eggs, tickets. Some things are measured; as, time, area, volume, weight.

119. When the name of the objects counted or measured is used with the expressed number (6 apples, 3 feet), the number is a Concrete Number.

120. A Unit of Measure is any standard by which we determine the number or the amount of anything.

A dozen, a bushel, a pound are units of measure.

A shovelful, a boxful, a dipperful are also units of measure.

The dozen, bushel, and pound are definite and established units of measure. The shovelful is not a definite or established unit of measure.

Some units of measure were established by law (pound, yard, gallon). Some were established by custom (hour, dozen, degree).

121. A number whose unit of measure is established by law or custom is a Denominate Number.

122. A number that expresses units of measure of the same kind is a Simple Denominate Number (7 pounds).

123. A number that expresses units of measure of similar kind but of different values is a Compound Denominate Number (7 pounds, 6 ounces).

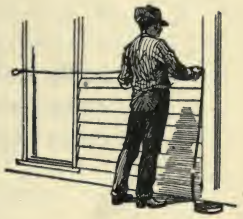
MEASURES OF EXTENSION

124. Extension means length, length and breadth, or length, breadth and thickness. This leaf is a volume, or solid; it has length, breadth, and thickness. This page is a surface; it has length and breadth only. The edge of the page is a line; it has length only.

125. Linear Measure

TABLE

12	inches (in.)	= 1 foot.	ft.
3	feet	= 1 yard.	yd.
$5\frac{1}{2}$	yards or $16\frac{1}{2}$ feet	= 1 rod.	rd.
320	rods	= 1 mile.	mi.



mi.	rd.	yd.	ft.	in.
1 =	320 =	1760 =	5280 =	63360.

Scale.—320, $5\frac{1}{2}$, 3, 12.

In measuring cloth, the yard is divided into quarters. Yards and quarters are sometimes written thus: 12^2 ($12\frac{2}{4}$), 15^3 ($15\frac{3}{4}$).

126. Square Measure

TABLE

144	square inches (sq. in.)	= 1 square foot.	sq. ft.
9	square feet	= 1 square yard.	sq. yd.
$30\frac{1}{4}$	square yards	= 1 square rod.	sq. rd.
160	square rods	= 1 acre.	A.
640	acres	= 1 square mile.	sq. mi.

Scale.—640, 160, $30\frac{1}{4}$, 9, 144.

A square 1 in. on a side is an inch square; a square inch.

A square 1 ft. on a side is a foot square; a square foot.

A square 1 yd. on a side is a yard square; a square yard.

A square inch is *equivalent* to an inch square.

A square foot is *equivalent* to a foot square.

A square yard is *equivalent* to a yard square.

NOTE.—The units of square measure need not be squares. A square foot may be either round or oblong. It is called a square foot because it was derived from, and is equivalent to, a foot square. A square foot measures as much as a foot square.

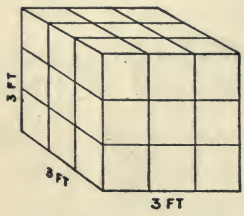
The number of units of square measure in a surface is its Area.

127. Cubic Measure

TABLE

1728 cubic inches (cu. in.)	= 1 cubic foot.	cu. ft.
27 cubic feet	= 1 cubic yard.	cu. yd.
128 cubic feet	= 1 cord.	C.

Scale.—27, 1728.



The units of cubic measure are the cubic inch, cubic foot, and cubic yard. These units need not be cubes.

A cubic inch is *equivalent* to a cube an inch on each edge.

A cubic foot is *equivalent* to a cube a foot on each edge.

A cubic yard is *equivalent* to a cube a yard on each edge.

The number of units of cubic measure a solid contains is its Solid Contents or Volume.

128. Surveyors' Linear Measure

TABLE

7.92 inches (in.)	= 1 link.	l.
25 links	= 1 rod.	rd.
4 rods or 100 links	= 1 chain.	ch.
80 chains	= 1 mile.	mi.

Scale.—80, 4, 25, 7.92.

Surveyors' Square Measure

129. United States government land when surveyed is divided into townships—tracts of land 6 miles square. A township is divided into 36 equal squares, square miles. Each square mile is called a section. A section is divided into half-sections, quarter-sections and quarter quarter-sections.

TABLE

625 square links (sq. l.)	= 1 square rod.	sq. rd.
16 square rods	= 1 square chain.	sq. ch.
10 square chains	= 1 acre.	A.
640 acres	= 1 square mile.	sq. mi.

Scale.—640, 10, 16, 625.

MEASURES OF CAPACITY

130. Liquid Measure

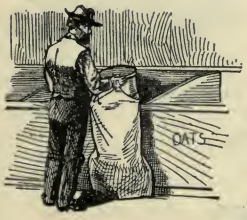
TABLE

4 gills (gi.)	= 1 pint.	pt.
2 pints	= 1 quart.	qt.
4 quarts	= 1 gallon.	gal.

gal. qt. pt. gi.
1 = 4 = 8 = 32

Scale.—4, 2, 4.

131. Dry Measure



TABLE

2 pints (pt.)	= 1 quart.	qt.
8 quarts	= 1 peck.	pk.
4 pecks	= 1 bushel.	bu.

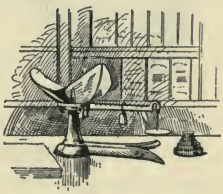
bu. pk. qt. pt.
1 = 4 = 32 = 64

Scale.—4, 8, 2.

MEASURES OF WEIGHT

Avoirdupois Weight

132. Avoirdupois weight is used for ordinary purposes.



TABLE

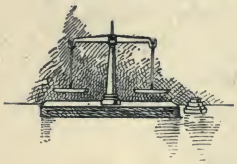
16 ounces (oz.)	=1 pound.	lb.
100 pounds	=1 hundredweight.	cwt.
20 hundred-weight	=1 ton.	T.

T. cwt. lb. oz.
1 = 20 = 2000 = 32000

Scale.—20, 100, 16.

Troy Weight

133. Troy weight is used by jewelers.



TABLE

24 grains (gr.)	= 1 pennyweight.	pwt.
20 pennyweights	= 1 ounce.	oz.
12 ounces	= 1 pound.	lb.

lb.		oz.		pwt.		gr.
1	=	12	=	240	=	5760

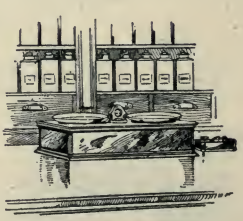
Scale.—12, 20, 24.

Apothecaries' Weight

134. Apothecaries' weight is used by druggists.

TABLE

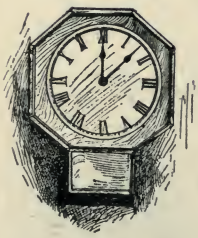
20 grains (gr.)	= 1 scruple.	sc., or ℈
3 scruples	= 1 dram.	dr., or ʒ
8 drams	= 1 ounce.	oz., or ℥
12 ounces	= 1 pound.	lb., or ℔



lb.		oz.		dr.		sc.		gr.
1	=	12	=	96	=	288	=	5760

Scale.—12, 8, 3, 20.

MEASURES OF TIME



135. TABLE

60 seconds (sec.)	= 1 minute.	min.
60 minutes	= 1 hour.	hr.
24 hours	= 1 day.	da.
7 days	= 1 week.	wk.
365 days	= 1 year.	yr.
366 days	= 1 leap year.	l. yr.
100 years	= 1 century.	cen.

yr.		mo.		da.		hr.		min.		sec.
1	=	12	=	365	=	8760	=	525600	=	31536000

Scale.—365, 24, 60, 60.

136. The earth rotates from west to east. The Day measures the time of one complete rotation of the earth on its axis.

137. A straight north and south line passing through both poles and through any point on the earth's surface is the meridian of that point.

138. It is noon at a place when the meridian of the place is under the direct rays of the sun.

139. A.M. (ante-meridian) denotes the half-day before noon. P.M. (post-meridian) denotes the half-day after noon.

140. In astronomy, the day begins at noon; in business, it begins at midnight.

141. The earth revolves around the sun in equal periods of time. The Year measures the time of one revolution of the earth around the sun.

142. A year consists of 365 da. 5 hr. 48 min. 49.7 sec. The Common Year is 365 da. The Leap Year is 366 days.

143. If, to make up for the 5 hr. 48 min. 49.7 sec. dropped from each common year, we add one day to each fourth year, we would add 44 min. 41.2 sec. too much. In 100 years we would add 18 hr. 37 min. 10 sec. too much. If we omit adding a day every 100 years, we would lose 5 hr. 22 min. 50 sec. In 400 years we would lose 21 hr. 31 min. 20 sec. If, then, we add one day for each 400 years, we will gain 2 hr. 28 min. 40 sec.; and in 4000 years we would gain 24 hr. 46 min. 40 sec. So we omit adding a day once in 4000 years.

Rule for Leap Year.—Century years divisible by 400, and other years divisible by four, are leap years, except the year 4000.

144. The day added to leap year becomes the 29th of February.

145. In business, 30 days are usually considered a month, and 12 months a year.

146. The common year contains 52 weeks and 1 day; the leap year 52 weeks and 2 days. Each year begins one day later in the week than the preceding year, except the year following leap year, which begins two days later in the week.

147. *Days in the Months.*—February has 28 days, except in leap year, when it has 29 days.

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone,
Which has four and twenty-four,
Till leap year gives it one day more.

MEASURES OF ANGLES AND ARCS

148. A Circumference is the bounding line of a circle. An Arc is any part of a circumference.

149. $\frac{1}{360}$ of any circumference is a Degree of the circumference. If the space about a point be divided into 360 equal parts or angles, by straight lines meeting at the point, each angle is an angle of 1 degree.

TABLE

60 seconds (″)	= 1 minute.	(′)
60 minutes	= 1 degree.	(°)
360 degrees	= 1 circumference.	cir.

Scale.—360, 60, 60.

The length of a degree of longitude at the equator is nearly 70 miles.

MEASURES OF VALUE

Canada Money

150. The table of the currency of Canada is the same as that of the United States (see p. 66), although English money also is used.

English or Sterling Money

151. The unit of English money is the Pound or Sovereign.

TABLE

4 farthings (far.)	= 1 penny (d.)	= $2\frac{2}{3}¢$
12 pence	= 1 shilling (s.)	= $24\frac{1}{3}¢$
20 shillings	= 1 pound (£)	= $4.8665

Scale.—20, 12, 4.

152. **French Money**

TABLE

10 centimes (ct.) = 1 decime (dc.) = $1\frac{93}{100}¢$
10 decimes = 1 franc (fr.) = $19\frac{3}{10}¢$

153. **German Money**

TABLE

100 pfennigs = 1 mark = $23\frac{85}{100}¢$

154. **COUNTING**

TABLE

12 things = 1 dozen.	doz.
12 dozen = 1 gross.	gr.
12 gross = 1 great gross.	G. gr.

REDUCTION OF DENOMINATE NUMBERS

155. The process of changing a number expressed in one denomination to an equivalent expressed in another denomination is called Reduction.

Change 8 dimes to cents; 3 dollars to dimes; 3 dollars and 8 dimes to cents.

156. Changing a number from a higher to a lower denomination is Reduction Descending.

1. Change 2 ft. to inches; 2 yd. to feet; 2 yd and 2 ft. to inches.

2. Change 2 qt. to pints; 2 pk. to quarts; 2 pk. 2 qt. to pints; 2 bu. to pecks; 2 bu. 2 pk. 2 qt. to pints.

3. Change 4 bu. 3 pk. to quarts; 1 pk. to pints.

4. Change 2 hr. to seconds; 2 da. to minutes.

NOTE.—Reduction descending is performed by multiplication.

157. Changing a denominate number from a lower to a higher denomination is Reduction Ascending.

1. Change 60 cents to dimes; 1200 cents to dollars; 50 dimes to dollars; 287 cents to dimes and cents; 365 cents to dollars, dimes and cents.

2. Change 36 in. to feet; 67 in. to feet and inches; 98 in. to yards, feet, and inches.

3. Change 64 qt. to pecks, then to bushels; 42 qt. to bushels, pecks, and quarts; 89 pt. to higher denominations.

NOTE.—Reduction ascending is performed by division.

158. Under several of the denominate tables is a line of equivalents. Tell how one equivalent is found from another, the highest denomination from the lowest, the lowest from the highest.

159. *Principles.—1.* To perform reduction descending, *multiply* by the numbers in the scale from the given to the required denomination.

2. To perform reduction ascending, *divide* by the numbers in the scale from the given to the required denomination.

Model Solutions

160. EXAMPLE 1.—Reduce 12 yd. 2 ft. 9 in. to inches.

OPERATION

12 yd.
3 ft. (multiply)

36 ft.
2 ft. (add)

38 ft.
12 in. (multiply)

456 in.
9 in. (add)

465 in.

NOTE.—The product is of the same denomination as the multiplicand. The multiplier must be an abstract number, so 12 in the first case and 38 in the second are considered abstract numbers and the multipliers.

EXAMPLE 2.—Reduce 892 in. to higher denominations.

OPERATION

Divide	12 in.	892 in.	
Divide	3 ft.	74 ft.	4 in. remainder
Divide	$5\frac{1}{2}$ yd.	24 yd.	2 ft. remainder
		4 rd.	2 yd. remainder

$\therefore$ 892 in. = 4 rd. 2 yd. 2 ft. 4 in.

NOTE.—The divisors are considered abstract numbers.

EXAMPLE 3.—Reduce $\frac{3}{4}$ ft. to the fraction of a rod.

OPERATION

$\frac{3}{4}$ ft. $\times \frac{1}{3} \times \frac{2}{11} = \frac{1}{22}$ rod.

EXPLANATION.—Divide $\frac{3}{4}$ by 3 and the quotient by $5\frac{1}{2}$.

EXAMPLE 4.—Reduce .8 pt. to the decimal of a bushel.

OPERATION

2 pt.	.8 pt.
8 qt.	.4 qt.
4 pk.	.05 pk.
	.0125 bu.

EXPLANATION.—Divide .8 pt. by 2, the quotient by 8 and this quotient by 4, as abstract numbers.

EXAMPLE 5.—Reduce $\frac{1}{2}$ rd. to integers of lower denominations.

OPERATION

$\frac{1}{2}$ (rd.) $\times \ 5\frac{1}{2} = 2\frac{3}{4}$ yd.
$\frac{3}{4}$ (yd.) $\times \ 3 \ = 2\frac{1}{4}$ ft.
$\frac{1}{4}$ (ft.) $\times 12 \ = 3$ in.

$\therefore \frac{1}{2}$ rd. = 2 yd. 2 ft. 3 in.

EXAMPLE 6.—Reduce .7 bu. to integers of lower denominations.

OPERATION

.7 bu. (multiplier)
4 pk.
2.8 pk. (multiplier)
8 qt.
6.4 qt. (multiplier)
2 pt.
.8 pt.

.7 bu. = 2 pk. 6 qt. .8 pt.

NOTE.—Since the decimal is to be reduced to *integers*, multiply only by the decimal part of the multiplier. The product is of the same denomination as the multiplicand, but when the product is used again as a multiplier it is considered as an abstract number.

PROBLEMS

Reduce to higher denominations:

1. 4256 in.
2. 86579 in.
3. 684 pt. dry.
4. 1272 gi.
5. 1298 gr. Troy.
6. 15652 gr. apoth
7. 489754 sec.
8. 12863 sq. in.
9. 6871 sc.
10. 9478 l.
11. 42735 in.
12. 6853 gi.
13. 735 sc.
14. 62784 l.
15. 6952 sq. in.
16. 68754 min.
17. 61453 gr. Troy.
18. 567389 cu. in.
19. 1593 pt. dry.
20. 11268 pwt.

Reduce to the lowest denomination given:

21. 8 rd. 2 yd. 1 ft. 4 in.
22. 3 mi. 80 rd. 4 yd. 2 ft.
23. 2 lb. 10 oz. 16 gr. Troy.
24. 4 hr. 17 min. 40 sec.
25. 12 lb. 6 dr. 2 sc.
26. 16 rd. 12 ft. 7 in.
27. 17 lb. 2 pwt.
28. 2 da. 26 min.
29. 4 lb. 7 oz. 1 sc.
30. 16° 20″.
31. 20° 15′ 20″.
32. 5 gal. 3 qt. 1 pt. 3 gi.
33. 14. bu. 3 pk. 5 qt. 1 pt.
34. 3 cu. yd. 15 cu. ft. 806 cu. in.
35. 20 gal. 2 qt. 1 pt.
36. 16 gal. 1 pt. 2 gi.
37. 2 bu. 2 pk. 1 pt.
38. 5 cu. yd. 12 cu. ft.
39. 14 gal. 2 qt. 2 gi.
40. 7 lb. 3 oz. 2 dr.

Reduce to lower denominations:

41. $\frac{2}{5}$ rd.
42. $\frac{1}{9}$ mi.
43. .86 rd.
44. $\frac{3}{8}$ lb. Troy.
45. $\frac{1}{3}$ da.
46. .76 lb. apoth.
47. $\frac{7}{16}$ cu. yd.
48. $\frac{5}{9}$ bu.
49. .94 gal.
50. $\frac{5}{12}$ sq. yd.
51. $\frac{7}{8}$ gal.
52. .55 sq. rd.

Reduce to a fraction of the highest denomination:

53. $\frac{1}{3}$ pt. dry.
54. .62 gi.
55. $\frac{7}{8}$ yd.
56. $\frac{1}{4}$ dr.
57. $\frac{1}{7}$ pwt.
58. .56 cu. ft.
59. $\frac{1}{3}$ l.
60. .85 gr. Troy.
61. $\frac{1}{6}$ gi.
62. $\frac{1}{5}$ rd.
63. .96 pwt.
64. .35 dr.

EXAMPLE 7.—Reduce 2 pk. 6 qt. 1 pt. to the decimal of a bushel.

OPERATION

2 | 1 pt.
8 | 6.5 qt.
4 | 2.8125 pk.
.703125 bu.

EXPLANATION. — Reduce 1 pt. to quarts and annex the result to 6 qt. Reduce quarts to pecks and annex the result to 2 pk. Reduce pecks to bushels.

Or,
2 pk. 6 qt. 1 pt. = 45 pt.
1 bu. = 64 pt.
45 pt. ÷ 64 = .703125 pt.

Reduce to decimals of the highest denomination:

65. 4 oz. 12 pwt. 16 gr.
66. 12 hr. 40 min. 30 sec.
67. 2 qt. 1 pt. 2 gi.
68. 12s. 10d.
69. 2 pk. 6 qt. 1 pt.
70 14s. 6d. 2 far.

REDUCTION OF ENGLISH MONEY

161. EXAMPLE.—Reduce 12 sov. 12s. 6d. to United States money.

OPERATION

12 sov. 12s. 6d. = 12.624 sov.
1 sov. = $ 4.8665
12.624 × $4.8665 = $61.43

Or, 1s. = .05 sov.
1 far. = .001 sov.

EXPLANATION. — Call each shilling .05 of a sovereign. Reduce pence to farthings and call the result thousandths of a sovereign. Thus, 12 sov. 12s. 6d. = 12 sov. + .6 sov. + .024 sov. = 12.624 sov. $4.8665 × 12.624 = $61.43.

PROBLEMS

Reduce to United States money:

1. 25 sov. 9s. 11d.
2. 81 sov. 16s. 8d. 3 far.
3. 21 sov. 15s. 7d. 1 far.
4. 16 sov. 18s. 9d. 2 far.
5. 34 sov. 14s. 10d.
6. 13 sov. 7s. 4d. 3 far.

EXAMPLE.—Reduce $175 to equivalents in English money.

SOLUTION

$175 ÷ $4.8665 = 35.96 sov.
35.96 sov. = 35 sov. 19s. 2d. 1.6 far.

Reduce to equivalents in English money:

7. $152.60.
8. $586.
9. $792.18.
10. $384.50.
11. $6450.
12. $586.25.

FUNDAMENTAL OPERATIONS WITH DENOMINATE NUMBERS

162. EXAMPLE 1.—Add 756, 687, and 479; also 7 dollars 5 dimes 6 cents, 6 dollars 8 dimes 7 cents, 4 dollars 7 dimes 9 cents.

OPERATION

7	5	6	$7	5 dimes	6 cents
6	8	7	6	8	7
4	7	9	4	7	9
19	2	2	$19	2 dimes	2 cents

NOTE.—Since the scale is decimal carry 1 for every 10 as in addition of simple numbers.

EXAMPLE 2.—Add 4 bu. 2 pk. 7 qt. 1 pt., 6 bu. 3 pk. 6 qt. 1 pt., and 2 bu. 2 pk. 5 qt. 1 pt.

OPERATION

4 bu.	2 pk.	7 qt.	1 pt.
6	3	6	1
2	2	5	1
14 bu.	1 pk.	3 qt.	1 pt.

NOTE.—Carry according to the scale of the table used.

EXAMPLE 3.—From 7 dollars 4 dimes 6 cents take 2 dollars 7 dimes 8 cents.

OPERATION

7	4	6	$7	4 dimes	6 cents
2	7	8	2	7	8
4	6	8	$4	6 dimes	8 cents

NOTE.—Change from one denomination to another according to the scale of the table used.

EXAMPLE 4.—Subtract 3 gal. 3 qt. 0 pt. 3 gi. from 5 gal. 2 qt. 0 pt. 3 gi.

OPERATION

5 gal.	2 qt.	0 pt.	2 gi.	
3	3	0	3	
1 gal.	2 qt.	1 pt.	3 gi.	

NOTE.—Change from one denomination to another according to the scale used.

EXAMPLE 5.—Multiply 2 dollars 5 dimes 8 cents by 7.

OPERATION

2	5	8	$2	5 dimes	8 cents
		7			7
18	0	6	$18	0 dimes	6 cents

EXAMPLE 6.—Multiply 4 gal. 3 qt. 1 pt. 2 gi. by 6.

OPERATION

4 gal.	3 qt.	1 pt.	2 gi.	
			6	
29 gal.	2 qt.	1 pt.	0 gi.	

EXAMPLE 7.—Divide 8 dollars 5 dimes 5 cents by 3.

OPERATION

3)855	3)$8	5 dimes	5 cents
285	$2	8 dimes	5 cents

EXAMPLE 8.—Divide 5 bu. 1 pk. 7 qt. by 4.

OPERATION

4)5 bu. 1 pk. 7 qt.

1 bu. 1 pk. 3 qt. 1½ pt.

PROBLEMS

1. Add 2 lb. 6 oz. 16 pwt. 18 gr., 8 lb. 11 oz. 17 pwt. 21 gr., 14 lb. 9 oz. 12 pwt. 20 gr.

2. From 1 mi. 60 rd. 4 yd. 1 ft. 4 in. take 130 rd. 2 yd. 2 ft. 9 in.

3. Multiply 12 gal. 3 qt. 1 pt. 3 gi. by 8.

4. Add 2 cu. yd. 16 cu. ft. 987 cu. in., 8 cu. yd. 20 cu. ft. 1265 cu. in.

5. Divide 7 lb. 10 oz. 12 pwt. 16 gr. by 4.

6. Divide 20 bu. 4 qt. by 2 bu. 3 pk. 4 qt.

NOTE.—Reduce both expressions to quarts, then divide.

7. Divide 48 gal. 1 qt. 3 gi. by 3 gal. 2 qt. 1 pt. 3 gi.

8. Multiply 6 bu. 3 pk. 5 qt. by 14.

9. From 12 da. 16 hr. 30 min. 14 sec. take 9 da. 20 hr. 50 sec.

10. Add 1 mi. 165 rd. 2 yd. 8 in., 3 mi. 120 rd. 4 yd. 2 ft.

11. Divide 12 bu. 3 pk. 7 qt. into 8 equal parts.

12. How many jugs holding 1 gal. 1 qt. 1 pt. each can be filled from a barrel containing 54 gal. 2 qt. 1 pt.

13. Add 3 lb. 6 oz. 7 dr. 2 sc. 16 gr., 4 dr. 2 sc. 12 gr., 1 lb. 9 oz. 1 sc. 8 gr., 11 oz. 2 sc. 18 gr., 1 lb. 8 oz. 5 dr.

14. Multiply 3 da. 12 hr. 30 min. 14 sec. by 12.

15. Subtract 1 mi. 190 rd. 4 yd. 8 in. from 2 mi. 60 rd. 2 yd. 2 ft. 9 in.

16. How many times is 2 dr. 2 sc. 12 gr. contained in 1 lb. 2 oz. 5 dr. 1 sc. 12 gr.

17. Add 6 gal. 2 qt. 1 pt. 3 gi., 12 gal. 3 qt. 2 gi., 17 gal. 1 qt. 1 pt. 3 gi.

18. Multiply 3 mi. 80 rd. 5 yd. 2 ft. 8 in. by 16.

19. Subtract 1 yr. 6 mo. 15 da. 9 hr. 45 min. 28 sec. from 2 yr. 3 mo. 12 da. 4 hr. 20 min.

20. Multiply 6 lb. 9 oz. 12 pwt. 16 gr. by 13.

21. Add 31 mi. 65 rd. 3 yd. 1 ft., 196 rd. 2 yd. 2 ft. 9 in., 3 mi. 145 rd. 5 yd. 1 ft. 8 in., 7 mi. 98 rd. 4 yd. 2 ft. 10 in.

22. Add 60 A. 90 sq. rd. 20 sq. yd. 5 sq. ft., 12 A. 120 sq. rd. 20 sq. yd. 8 sq. ft., 16 A. 80 sq. rd. 16 sq. yd. 5 sq. ft.

23. Subtract 3 gal. 3 qt. 1 pt. 2 gi. from 8 gal. 1 qt. 3 gi.

24. Multiply 7 bu. 3 pk. 6 qt. 1 pt. by 21.

25. Add 1 yr. 6 mo. 13 da. 12 hr. 40 min., 2 yr. 7 mo. 15 da. 13 hr. 30 min., 9 mo. 25 da. 17 hr. 48 min., 8 mo. 24 da. 14 hr.

SUBTRACTION OF DATES

163. EXAMPLE.—Find the difference in time between April 6, 1900, and Nov. 11, 1899.

OPERATION

yr.	mo.	da.
1900	4	6
1899	11	11
	4	25

EXPLANATION.—Write the time as years, months and days. April is the fourth month and is written as 4 mo. November is the eleventh month and is written as 11 mo. One month is also called 30 days. As the number of days in a month varies, this method, called compound subtraction, may not give the exact number of days' difference. Thus, counting 30 days to a month, the difference between the above dates as shown in the operation is 145 days, but the true difference is 146 days. To find the exact number of days, count the number of days by months from the first date to the second. Thus, 19 (days in November after November 11), 31 (December), 31 (January), 28 (February), 31 (March), and 6 (days counted in April) are 146.

PROBLEMS

Find the difference in time between the following dates, by compound subtraction:

1. March 4, 1899, and Dec. 11, 1901.
2. June 21, 1899, and April 2, 1903.
3. Aug. 30, 1897, and May 15, 1899.
4. Sept. 26, 1895, and Nov. 12, 1897.
5. Oct. 20, 1900, and Feb. 6, 1904.

Find the actual difference in days between:

6. Jan. 7, 1900, and July 12, 1900.
7. Oct. 16, 1900, and Jan. 17, 1901.
8. Dec. 6, 1901, and March 14, 1902.
9. May 4, 1902, and Oct. 30, 1902.
10. Sept. 23, 1900, and Feb. 26, 1901.

COMPARISON OF WEIGHTS AND MEASURES

164. TABLE

	Troy	Apothecaries'	Avoirdupois
1 lb.	= 5760 gr.	= 5760 gr.	= 7000 gr.
1 oz.	= 480 gr.	= 480 gr.	= 437.5 gr.

1 bu. (32 qt.)	= 2150.42	cu. in.
1 gal.	= 231	cu. in.
1 qt. (dry)	= 67$\frac{1}{5}$	cu. in.
1 qt. (liquid)	= 57$\frac{3}{4}$	cu. in.
1 cu. ft. water	= 62$\frac{1}{2}$	lb. avoir.
1 gal. water	= 8$\frac{1}{3}$	lb. avoir.

Which is heavier, a pound Troy or a pound avoirdupois? an ounce Troy or an ounce avoirdupois? Which is larger, a quart dry measure or a quart liquid measure?

NOTE.—Large fruits, vegetables, coal, etc., are measured by the heaped bushel, or the bushel of 40 quarts.

PROBLEMS

1. How many liquid quarts in a bushel?
2. Change 1 lb. Troy to the fraction of a pound avoirdupois.
3. Change 8 lb. 12 oz. avoirdupois to apothecaries' weight.
4. Change 1 lb. avoirdupois to Troy integers.
5. How many prescriptions of 1 dr. 2 sc. 12 gr. can be filled from 14 oz. avoirdupois?
6. A 60 qt. liquid measure is equivalent to what in dry measure?

7. A bin that holds 620 bu. of wheat will hold how many bushels of potatoes?

8. A tank that holds 1280 gal. will hold how many bushels of wheat?

9. Since 1728 cu. in. make 1 cu. ft., and 1 gal. contains 231 cu. in., how many gallons in 1 cu. ft.?

10. A cistern that contains 384 cu. ft will hold how many gallons?

PAPERS AND BOOKS

165. TABLE

24 sheets	= 1 quire
20 quires or 500 sheets	= 1 ream
2 reams	= 1 bundle
5 bundles	= 1 bale

NOTE.—The 480-sheet ream is now used rarely in this country except for stationery, and odd and fancy papers.

166. *Book Paper.*—The paper out of which books, circulars, and pamphlets are usually made is called Book Paper. It is sold in large unfolded sheets.

167. *Flat, Linen, and Ledger Papers.*—The paper out of which billheads, letterheads, blank books, writing books, etc., are made is called Flat Paper. Flat paper is more expensive than book paper.

Both book and flat papers come in sheets of various sizes.

168. SIZES OF BOOK PAPER

NOTE.—22 in. by 32 in. may be written 22′′ × 32′′, as in the following table:

22″ × 32″	25″ × 38″	28″ × 42″	36″ × 48″
24″ × 36″	25″ × 40″	32″ × 44″	38″ × 50″

169. SIZES OF FLAT PAPER

14″ × 17″	17″ × 22″	17″ × 28″	18″ × 46″
15″ × 19″	16″ × 26″	20″ × 28″	22″ × 31″
16″ × 21″	19″ × 24″	21″ × 32″	23″ × 36″
18″ × 23″			

Paper Folding

170. This table shows the number of leaves into which book paper is folded in making books:

Name of Fold	Leaves	Pages
Folio	2	4
Quarto (4to)	4	8
Octavo (8vo)	8	16
Duodecimo (12mo)	12	24
16mo	16	32
18mo	18	36
24mo	24	48
32mo	32	64

EXERCISE IN FOLDING

1. Make a folio; a quarto.

2. An octavo may be $\frac{1}{2}$ the width and $\frac{1}{4}$ the length, or $\frac{1}{4}$ the width and $\frac{1}{2}$ the length of the sheet. Make both kinds.

3. A duodecimo may be $\frac{1}{3}$ the width and $\frac{1}{4}$ the length, or $\frac{1}{4}$ the width and $\frac{1}{3}$ the length of the sheet. Make both kinds.

4. Make a 16mo $\frac{1}{4}$ the width and $\frac{1}{4}$ the length of the sheet.

5. A 24mo may be $\frac{1}{4}$ the width and $\frac{1}{6}$ the length, or $\frac{1}{6}$ the width and $\frac{1}{4}$ the length, or $\frac{1}{3}$ the width and $\frac{1}{8}$ the length, or $\frac{1}{3}$ the length and $\frac{1}{8}$ the width of the sheet. Make a 24mo of each kind.

PROBLEMS

1. Give two possible sizes of the pages of quarto books made from paper (a) $24'' \times 36''$, (b) $25'' \times 38''$, (c) $28'' \times 42''$, (d) $32'' \times 44''$.

2. Give the two sizes of the pages of a 12mo book made from paper $24'' \times 36''$

Solution

$24'' \div 3 = 8''$ $\quad 36'' \div 4 = 9''$ $\quad$ Page, $8'' \times 9''$

Or, $24'' \div 4 = 6''$ $\quad 36'' \div 3 = 12''$ $\quad$ Page, $6'' \times 12''$

3. Give the possible sizes of the pages of a 24mo book made from paper $24'' \times 36''$.

SOLUTION

$3 \times 8 = 24$ $4 \times 6 = 24$ $2 \times 12 = 24$

$24'' \div 3 = 8''$	$36'' \div 8 = 4\frac{1}{2}''$	Page, $4\frac{1}{2}'' \times 8''$
Or, $24'' \div 8 = 3''$	$36'' \div 3 = 12''$	Page, $3'' \times 12''$
Or, $24'' \div 4 = 6''$	$36'' \div 6 = 6''$	Page, $6'' \times 6''$
Or, $24'' \div 6 = 4''$	$36'' \div 4 = 9''$	Page, $4'' \times 9''$
Or, $24'' \div 2 = 12''$	$36'' \div 12 = 3''$	Page, $3'' \times 12''$
Or, $24'' \div 12 = 2''$	$36'' \div 2 = 18''$	Page, $2'' \times 18''$

NOTE.—Pages $3'' \times 12''$ and $2'' \times 18''$ would be very rare.

4. Find the possible sizes of the pages of an 18mo book made from paper $28'' \times 42''$

5. What size of page can be made by folding a sheet $22'' \times 28''$ into an 8vo?

6. What size of page can be made by folding a sheet $22'' \times 32''$ into a 16mo book?

7. What size of page can be made by folding a sheet $32'' \times 44''$ into a 24mo book?

8. What size of page can be made by folding a sheet $32'' \times 44''$ into a 32mo book?

9. What size of pages can be printed from a sheet of paper $28'' \times 42''$, using the 18mo form?

10. What size of pages can be printed from all the sheets in the table of book paper, using the 24mo form?

11. What size of billheads can be made from flat paper $24'' \times 38''$, using the 24mo form?

12. Using the 24mo form, what size of flat paper should be purchased to make letterheads $5\frac{1}{2}'' \times 8\frac{1}{2}''$ with the least waste?

13. A publisher wishes to print a 16mo book $4\frac{1}{2}'' \times 6\frac{1}{2}''$. What size of paper should he buy?

14. What size of paper should be purchased to make a 24mo book $5'' \times 7\frac{1}{2}''$?

15. What size of book paper should be bought to make a 12mo blank book $8'' \times 11''$?

PRICE, COST, AND MIXED QUANTITIES

171. Merchants and manufacturers often have occasion to form a compound or mixed substance by combining different ingredients or similar ingredients of different qualities. Thus, a grocer may mix Rio coffee with Java coffee, a confectioner may mix two or more kinds of candy, and a manufacturer of paint mixes different oils, colors, and leads. In such cases it is necessary to find the price per pound, quart, etc., of the mixture. If the dealer wishes to make a mixture, to be sold at a certain price, it is necessary for him to know what quantities of each ingredient to put into the compound to make the resulting mixture of the required value.

To Find the Price of a Mixed Quantity

172. EXAMPLE.—A grocer mixed in equal quantities coffees worth 12¢, 15¢, and 18¢ per pound respectively. At what price per pound should he sell the mixture?

FORMULA

$$\frac{\text{Cost (of mixture)}}{\text{Quantity}} = \text{Price.}$$

EXPLANATION.—The quantity is 3 lb., the cost is 45¢ (12 + 15 + 18), and the price is therefore, 45¢ ÷ 3, or 15¢ per lb.

MENTAL PROBLEMS

1. At what price should the mixed candy be sold, if in making it, candies selling for 8¢, 10¢, and 15¢ respectively are mixed in equal quantities?

2. Coffee worth 15¢ per pound is mixed in equal quantities with coffee worth 22¢ per pound. What is the value per pound of the mixture?

3. 100 lb. of sugar worth 6½¢ per pound are mixed with 50 lb. worth 5¢ per pound. What is the value of a pound of the mixture?

NOTE.—If the quantities mixed have a common divisor, the quotients of the common divisor may be taken instead of the quantities themselves. Thus, instead of 100 lb. and 50 lb., 2 lb. and 1 lb. may be used.

4. 20 qt. of wine worth 20¢ per quart are mixed with 10 qt. of cider worth 5¢ per quart. What is the price of the mixture per gallon?

5. 40 gal. of rum at $2.25 per gallon are mixed with 5 gal. of water. What is the mixture worth per quart?

6. Find the price of mixed nuts if the lot is made up of equal quantities worth 9¢, 12¢, and 18¢ respectively.

7. 100 lb. of tea worth 25¢ per pound are mixed with 75 lb. worth 18¢ per pound. What is the value of a pound of the mixture?

8. 10 lb. of pepper worth 40¢ per ounce are mixed with 15 lb. worth 30¢ per ounce. What is the value of the mixture per ounce?

9. Beans worth $2.15 per bushel are mixed in equal quantities with beans worth $2.05 per bushel. Find the price per quart of the mixed beans.

10. Syrup worth 45¢ per gallon is mixed in equal quantities with syrup worth 75¢ per gallon. Find the price of the mixture per quart.

PROBLEMS

1. A paint dealer mixed 300 gal. of oil worth 65¢ per gallon with 250 gal. worth 87½¢ per gallon, and 140 gal. worth 70¢ per gallon. Find the price of 1 gallon of the mixed oil.

2. A druggist made a composition drug using ingredients of the following weights and values: 14 lb. 9 oz. at 30¢ per ounce, 9 lb. 6 oz. 4 dr. at 28¢ per ounce, 21 lb. 6 oz. at 25¢ per ounce, and 7 dr. 2 sc. at 10¢ per dram. Find the price per dram of the mixture.

3. A manufacturer mixed 1 T. 14 cwt. of wool at 68¢ per pound with 1 T. 15 cwt. 75 lb. at 55¢ per pound, and 960 lb. at 50¢ per pound. What was the value per pound of the mixed wool?

4. A dealer mixed 10 bbl. of wine worth 90¢ per gallon with 8 bbl. worth 75¢ per gallon, 7 bbl. worth 50¢ per gallon, and 40 gal. of water. What was the liquor worth per gallon?

5. A patent-medicine manufacturer mixed ingredients of the following volumes and values: 18 gal. at $1.85, 2 gal.

water, 1 gal. at $2.75, 1 pt. at $1.30, 2 gr. at 20¢, and a drug worth $2.40 which added nothing to the bulk of the liquid. If he sold the mixture for twice what it cost him, what was the price per pint?

To Find What Quantities Must Be Used to Produce a Mixture of a Given Price

173. EXAMPLE.—A grocer wishes to sell mixed tea for 25¢ per pound, and desires to mix teas worth 45¢, 35¢, 20¢, and 10¢ per pound. What proportional quantities of each may he use?

OPERATION

	1.	2.	3.	4.	5.	6.	7.
	45	20	15		3		3
	35	10		5		1	1
25							
	20	5		10		2	2
	10	15	20		4		4

EXPLANATION.—For convenience, write the required price at the left of a vertical line, and the given prices at the right. Write the *difference* between the given and required prices in the next vertical column at the right. Thus, $45 - 25 = 20$, $35 - 25 = 10$, $25 - 20 = 5$, $25 - 10 = 15$. Draw a horizontal line separating the prices that are greater than the required price from those that are less. The numbers in column 2 show the number of cents gained or lost by putting 1 lb. of tea at a given price into the mixture, and selling it at 25¢ per pound. Thus, if 1 lb. of 45¢ tea is sold for 25¢, 20¢ is lost. If 1 lb. of 10¢ tea is sold for 25¢, 15¢ is gained, etc. The horizontal line separates the gains from the losses. The gains and losses must be equal. If 1 lb. of 45¢ tea and 1 lb. of 10¢ tea be put into the mixture, 20¢ will be lost on one, and 15¢ gained on the other. But if 15 lb. of the tea on which 20¢ is lost per pound is mixed with 20 lb. of the tea on which 15¢ is gained per pound, the gains and losses will be equal ($20 \times 15 = 15 \times 20$). And if 5 lb. of the tea on which 10¢ is lost per pound are mixed with 10 lb. of the tea on which 5¢ is gained per pound, the gains and losses will be equal. By comparing prices, one above and one below the horizontal line, a balance of gains and losses is kept, and the figures in columns 3 and 4 show how many pounds of each

kind of tea may be mixed. The numbers in any vertical column may be multiplied or divided by any number, as that will not affect the *comparative* quantities of the two substances mixed. Reducing columns 3 and 4 to their simplest form, columns 5 and 6 are obtained. These columns show that 3 lb. of the 45¢ tea and 4 lb. of the 10¢, 1 lb. of the 35¢, and 2 lb. of the 20¢ may be mixed and sold at 25¢ without gain or loss. This result is shown in column 7.

Suppose the prices given were 45¢, 40¢, 35¢, 30¢, 20¢, 10¢, and the required price 25¢. The solution would be:

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
	45	20	15				3				3
	40	15		15				1			1
	35	10			5				1		1
	30	5				5				1	1
25											
	20	5			10	5			2	1	3
	10	15	20	15			4	1			5

Column 2 shows the difference between the given prices and the required price. By taking these numbers in pairs and reversing them, columns 3, 4, 5, and 6 are obtained, which show the comparative quantities that may be mixed. These columns are reduced, by dividing the numbers in each by the greatest number that will divide them, to columns 7, 8, 9, and 10. And the number of pounds of each kind that may be used are shown in column 11. As the numbers in any one of columns 7, 8, 9, or 10 may be multiplied by any number, an indefinite number of changes may be made in column 11.

PROBLEMS

1. A merchant has lots of pepper worth 20¢, 28¢, 35¢, and 40¢ per pound, and wishes to form a mixture worth 30¢ per pound. How many pounds of each may he use? Give five answers. If he uses 50 lb. of the first, how many pounds of each of the others must he use?

2. Syrup worth 40¢, 50¢, 65¢, and 70¢ per gallon is mixed and sold at 60¢ per gallon. What comparative quantities of each are mixed?

3. A dealer mixed wines worth $1.25, $1.10, 90¢, and water, so that the mixture was worth $1 per gallon. What quantities of each were used? Give three answers.

4. How many pounds of each kind of candy worth 6¢, 8¢, 10¢, and 15¢ may be mixed to form a compound worth 12¢ per pound?

5. Drugs worth 45¢ per ounce, 60¢ per ounce, and 10¢ per dram were mixed and sold at 50¢ per ounce. If 8 lb. of the second were used, what quantities of the others were used?

6. What proportional quantities, worth 85¢, 75¢, 65¢, 55¢, and 40¢, may be mixed with 80 lb. worth 50¢ so that the mixture shall be worth 70¢ per pound?

7. Oils worth 65¢, 87½¢, and 70¢ were used to form a mixture worth 75¢. What proportional quantities of each were used?

8. Coffees worth 12½¢, 15¢, 18¢, and 25¢ per pound were mixed and sold for 20¢ per pound. What comparative quantities of each were used?

9. A druggist mixed chemicals worth 20¢ per ounce, $3.30 per ounce, 6¢ per ounce, and 8¢ per dram. If the compound was worth 45¢ per ounce, what quantities of each did he use?

BUSINESS PROBLEMS

1. A dealer sold 24800 bu. wheat for 4135 sov. 12s. What was the price per bushel in United States money?

2. A manufacturer made 20 gross of silver spoons, each weighing 10 pwt. 16 gr. How much did the silver in the spoons cost him at 90¢ per ounce avoirdupois?

3. A gold dollar weighs 25.8 gr. and is $\frac{9}{10}$ pure gold. A manufacturer made, from United States gold coin, 750 gold chains 18 k. fine. If each chain weighed 3 oz. 16 pwt. 18 gr., what was the value of the coin melted to make the chains?

NOTE.—Pure gold is said to be 24 karats fine. Gold that contains $\frac{6}{24}$ alloy has $\frac{18}{24}$ of pure gold and is 18 karats (18 k.) fine.

4. How many vases, each containing 8 oz. 16 pwt. of pure silver can be made from 95 lb. 4 oz. (Troy) of silver?

5. Find the cost, at $1.10 per cwt, of three loads of meal weighing as follows: 1 T. 7 cwt. 60 lb., 2 T. 80 lb., 1 T. 12 cwt. 75 lb.

6. What will be the freight on 75 bbl. of oil, each weighing 450 lb., at $2 per ton?

7. What will be the freight on 8765 bu. of wheat (60 lb. = 1 bu.) at 75¢ per ton?

8. A druggist bought 150 lb. of drugs, by avoirdupois weight, at $7.50 per pound, and sold them at 8¢ per scruple. What did he gain?

9. A dealer bought nuts at $2 per bushel (32 qt.) and sold them at 8¢ per qt. liquid measure. Find his gain per bushel.

10. A grocer bought peas for $2.40 per bushel, and sold them at 10¢ per quart. What did he gain per bushel? What part of his purchase price did he gain?

11. What will be the cost, at 30¢ per bushel of 60 lb., of 5 loads of potatoes, weighing 2472 lb., 3185 lb., 2817 lb., 3025 lb., 2960 lb.?

12. Find the cost of 18 gal. 3 qt. 1 pt. of wine, at 22¢ per quart.

13. A cask of brandy containing 46 gal. 2 qt. 1 pt. was bought for $108 and sold at 20¢ per gill. What was gained?

14. A grocer bought 8 bu. of beans by dry measure and sold them by the liquid quart. How many liquid quarts did he gain?

15. How many feet of fence will be required to inclose a field 25 ch. 41 l. long and 21 ch. 40 l. wide?

16. If fence wire weighs 12 oz. to the rod, how many pounds of wire will be required to build a fence around a field 19 ch. 20 l. long and 14 ch. 30 l. wide, if the fence be made 8 wires high?

17. Change $425 to equivalents in French money; to equivalents in German money.

18. Change 1486 francs to equivalents in United States money.

19. Change 2538 marks to equivalents in United States money.

20. If pine is estimated to weigh 3000 lb. per M, and maple 4000 lb. per M, how much freight must be paid, at $1.20 per ton, on 4560 ft. of pine and 5872 ft. of maple?

21. Find the cost of 1865 ft. of lumber at $17.60 per M.

22. If it requires 31 lb. of coal per day to heat a house, how much will it cost per week to heat it, the price of coal being $5.50 per ton?

23. A farmer raised 230 bu. 2 pk. 6 qt. from 9 bu. 2 pk. 4 qt. of seed. What was the yield from one bushel of seed?

24. Change 83 sov. 12s. 8d. to United States money.

25. Find the total cost of:

12 lb. 9 oz. lard at 11¢ per pound.

9 lb. 6 oz. steak at 14¢ per pound.

7 lb. 4 oz. mutton at 9¢ per lb.

8 lb. 12 oz. pork at 10¢ per pound.

2 gal. 2 qt. 1 pt. molasses at 60¢ per gallon.

7 lb. 5 oz. cheese at 12½¢ per pound.

20 ft. cord at 8¢ per yard.

100 pickles at 10¢ per dozen.

12^2 yd. cloth at 25¢ per yard.

PRACTICAL MEASUREMENTS

AREA OF PLANE FIGURES

174. An Angle is the difference in the direction of two lines that meet, and it is not affected by the length of the lines.

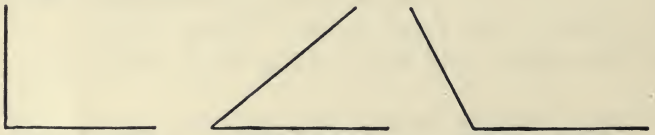
175. A Right Angle is one whose sides are perpendicular to each other.

NOTE.—If two lines meet and make two equal angles, the lines are perpendicular to each other.

176. Angles whose sides are not perpendicular to each other are Oblique Angles.

177. An Acute Angle is an oblique angle less than a right angle.

178. An Obtuse Angle is an oblique angle greater than a right angle.

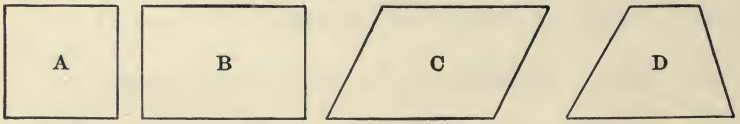


RIGHT ANGLE ACUTE ANGLE OBTUSE ANGLE

179. Two lines that remain the same distance from each other throughout their whole extent are Parallel.

180. A Plane Figure is any plane, or flat, surface.

181. A plane figure having four sides is a Quadrilateral. A, B, C, and D are quadrilaterals.



182. A quadrilateral whose opposite sides are parallel is a Parallelogram. A, B, and C are parallelograms.

183. The opposite sides of a parallelogram are equal.

184. The Base of a figure is any side on which it may be supposed to rest.

185. The Altitude of a figure is the perpendicular distance from the base to the side opposite.

186. A parallelogram that has right angles is a Rectangle.

187. A rectangle that has equal sides is a Square.

A and B are rectangles; A is a square.

188. To Find the Area of a Rectangle

1	2	3	4
1	2	3	4
1	2	3	4

EXPLANATION.—
Units in length, 4.
Units in width, 3.
Units in area, $4 \times 3 = 12$.

Principle.—The area of a rectangle is equal to the product of its length and breadth.

PROBLEMS

1. The side of a square is 16 ft. What is its area in square yards?

2. A field is 50 rd. long and 35 rd. wide. How many acres does it contain?

3. How many square rods in a park 12 rd. by 8 rd.?

4. How many square feet in a floor 17 ft. by 16 ft. 8 in.?

5. How many square rods in a lot 188 ft. by 164 ft.?

6. How many acres in a field 64 rd. 3 yd. by 42 rd. 4 yd.?

7. How many acres in a road 3 rd. wide and 1 mi. long?

8. Find the number of square feet in the walls, floor, and ceiling of a room 18 ft. by 22 ft. by 9 ft.

9. Find the number of square yards in the walls of a room 16 ft. square and 10 ft. high.

10. At 30¢ per square yard, what will be the cost of building a walk $4\frac{1}{2}$ ft. wide and 6 rd. long?

11. The area of a garden is $\frac{1}{4}$ A., and its length is 7 rd. How wide is it?

12. How many square feet in the walls, floor, and ceiling of a room 21 ft. square and 9 ft. high?

13. A field 85 rd. wide contains 100 A. How much must be cut off one end of the field to make a lot of 34½ A?

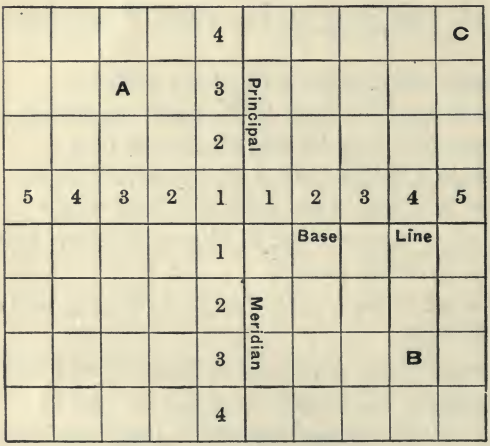
14. How many square inches on the entire surface of a stick 8″ square and 12 ft. long?

15. Find the number of square yards of roofing required for the two sides of a roof which is 62 ft. long and the slant height of which is 44 ft.

Land Measurements

189. The United States government surveyed the land of many of the western States, and these lands are often described by the government survey. Under the United States survey a north and south line was run as a Principal Meridian. Range lines were run parallel to the principal meridian and six miles apart. An east and west line, called a Base Line, was run intersecting the principal meridian at right angles. Township lines were run parallel to the base line and six miles apart. These lines divide the land into townships six miles square.

EXPLANATION.—The diagram shows how these lines are drawn, and how the townships are numbered and described.



Township lines run east and west.

Range lines run north and south.

Ranges are numbered east and west from the principal meridian.

Townships are numbered north and south from the base line.

A is described as Township 3 North, Range 3 West.

B is described as Township 3 South, Range 4 East.

C is described as Township 4 North, Range 5 East.

The following diagram shows how a township is divided into 36 sections, each a mile square, and how they are numbered and described:

TABLE

36 square miles =	1 township
36 sections	= 1 township
1 section	= 640 acres
1 acre	= 160 square rods

A is described as N. W. ¼ of Section 7, Township —, Range —.

B is described as S. ½ of Section 26, Township —, Range —.

6	5	4	3	2	1
A	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	B	25
31	32	33	34	35	36

DIAGRAM OF A TOWNSHIP

The following shows how a section is divided and described:

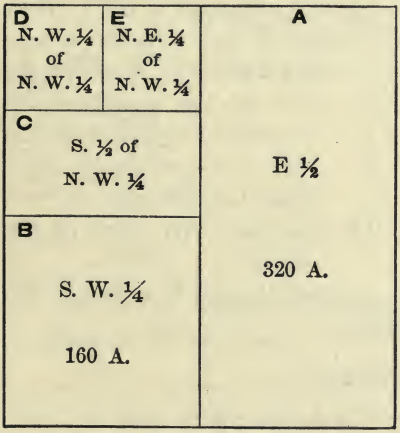


DIAGRAM OF A SECTION

Sections are divided into half-sections, quarter-sections, half quarter-sections, and quarter quarter-sections.

A is described as E. ½ of Section —, Township —, Range —.

B is the S. W. ¼ of Section —, etc.

C is the S. ½ of N. W. ¼ of Section —, etc.

D is the N. W. ¼ of N. W. ¼ of Section —, etc.

E is the N. E. ¼ of N. W. ¼ of Section —, etc.

PROBLEMS

1. Draw a principal meridian, base line, range lines, township lines, and indicate Township 7 North, Range 5 East; Township 6 South, Range 4 West.

2. Find the cost, at $27 per acre, of the N. ½ of S. W. ¼ of Section 21, Township 7 N., Range 14 W.

3. What is the value, at \$62.50 per acre, of the S. $\frac{1}{2}$ of N. E. $\frac{1}{4}$ of Section 25, Township 38 N., Range 13 E.?

4. What is the value of the S. E. $\frac{1}{4}$ of S. E. $\frac{1}{4}$ of Section 16, Township 30 S., Range 14 W., at \$51.30 per acre?

5. Find the cost, at \$17.50 per acre, of the S. $\frac{1}{2}$ of N. E. $\frac{1}{4}$ of Section 3, Township 9 S., Range 4 E.

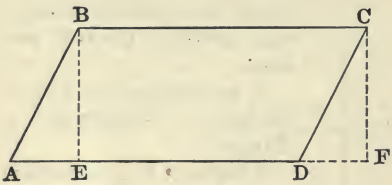
6. Find the value, at \$33$\frac{1}{3}$ per acre, of the E. $\frac{1}{2}$ of Section 28, Township 11 N., Range 14 E.

7. What will be the cost of the N. $\frac{1}{2}$ of the N. E. $\frac{1}{4}$ of Section 2, Township 8 S., Range 21 E. at \$33$\frac{1}{3}$ per acre?

8. If the N. $\frac{1}{2}$ of S. E. $\frac{1}{4}$ of Section 17, Township 14 N., Range 12 W., was sold for \$1440, what was the price per acre?

To Find the Area of a Parallelogram

190. How does A B E compare in area with D C F? How does the parallelogram A B C D compare in area with the rectangle E B C F? How does the base of the parallelogram compare in length with the base of the rectangle? How does the altitude of one compare with that of the other?



Principle.—The area of a parallelogram is equal to the product of its base and altitude.

PROBLEMS

1. Find the area of a parallelogram whose base is 16 rd. and whose altitude is 8 rd.

2. What is the area of a field in the form of a parallelogram if its base is 53 ch. 31 l., and the perpendicular distance from the base to the side opposite is 28 ch. 40 l.?

3. The area of a parallelogram is 684 sq. yd., and its base is 36 yd. Find its altitude.

4. A field in the form of a parallelogram has a base of 68 rd. and an altitude of 36 rd. Find the value of the field at \$60 per acre?

5. The area of a parallelogram is 3240 sq. ft. If its altitude is 51 ft., what is its length?

6. How many square feet in the ceiling of a room in the form of a parallelogram 32 ft. long and 17 ft. wide?

7. The opposite sides of a park are parallel, the adjacent sides being 24 rd. and 17 rd. If the distance between the sides is 14 rd., how many square yards does the park contain?

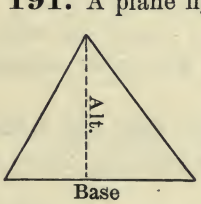
8. A parallelogram is 32 ft. long, 18 ft. wide, and the distance between its ends is 29 ft. How many square yards does it contain?

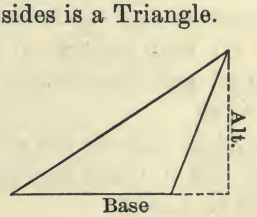
9. A field in the form of a parallelogram contains 20 A. If it is 83 rd. long, how wide is it?

10. The base of a parallelogram is 52 ft. and the altitude is 22 ft. What is its area?

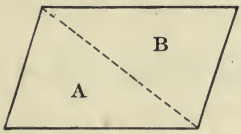
To Find the Area of a Triangle

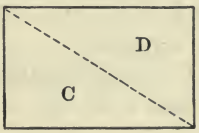
191. A plane figure having three sides is a Triangle.





192. The perpendicular distance from the angle opposite the base to the base, or the base extended, is the Altitude.





How does triangle A compare in area with triangle B?

How does C compare in area with D?

Principles.—1. The diagonal of a parallelogram divides it into two equal triangles.

2. The area of a triangle is equal to half the product of its base and altitude.

PROBLEMS

Draw and find the area of triangles of these dimensions:

1. Base 64 ft., altitude 36 ft.

2. Base 120 yd., altitude 53 yd.

3. Altitude 74 rd., base 62 rd.

4. Altitude 130 rd., base 115 rd.

5. Base 86 ft., altitude 74 ft.

6. Altitude 126 yd., base 245 yd.

7. How many square yards in a triangular park whose altitude is 362 ft. and base 581 ft.?

8. The area of a triangle is 65 sq. yd. and its altitude is 18 ft. Find its base.

9. Find the area of the four sides of a square pyramid. Each side is 40 yards at the base, and the distance from the middle of the edge of the base to the vertex is 80 ft.

10. How many feet of lumber will be required to cover a triangular floor whose base is 45 ft. and altitude 37 ft.?

11. Find the area of a triangular field if one of its sides is 124 rd. and the distance from this side to the vertex opposite is 56 rd.

12. I wish to lay out a triangular park that will contain 3 A. If I make the base 36 rd., how far opposite must the apex be placed?

13. The base of a triangle is 17 ft. and the area is 75 sq. ft. What is its altitude?

14. The three sides of a triangle are 6, 8, and 10 rd. If the area is 24 sq. rd., what is the distance of each side from the apex opposite?

15. Find the area of a triangle if the altitude is 14 rd. and the base is 29 rd.

To Find the Area of a Trapezoid

193. A quadrilateral having two sides parallel is a Trapezoid.

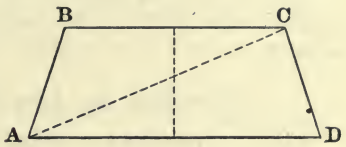
A diagonal of a trapezoid divides it into two triangles having the same altitude.

EXPLANATION.—

Area of triangle A B C = ½ of B C × alt. of the trapezoid.

Area of triangle A C D = ½ of A D × alt. of the trapezoid.

Area of trapezoid A B C D = ½ of (A D + B C) × alt. of the trapezoid.



Principle.—The area of a trapezoid is equal to one-half of the product of the sum of the parallel sides by the altitude.

PROBLEMS

Draw and find the area of trapezoids of the following dimensions:

1. Parallel sides 22 ft. and 16 ft., altitude 14 ft.

2. Parallel sides 36 yd. and 24 yd., altitude 25 yd.

3. Parallel sides 128 rd. and 75 rd., altitude 37 rd.

4. Altitude 246 ft., parallel sides 426 ft. and 538 ft.

5. Altitude 42 rd., parallel sides 85 rd. and 96 rd.

6. The area of a trapezoid is 4 acres. The sum of the parallel sides is 64 rd. What is the altitude?

7. How many acres in a field in the form of a trapezoid whose altitude is 38 rd. and whose parallel sides are 56 rd. and 62 rd.?

8. Find the area of a park in the form of a trapezoid, 60 rd. long, 8 rd. wide at one end and 12 rd. at the other.

9. One side of a lot is 16 rd. long, the side parallel to it is 12 rd. long, and the perpendicular distance between them is 8 rd. How much is the lot worth at $2 per square yard?

10. The area of a trapezoid is 4 A. If the altitude is 18 rd., what is the sum of the parallel sides?

11. The parallel sides of a field are 46 rd. and 28 rd., and the distance between them is 26 rd. What is the area?

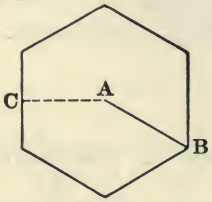
12. How many square feet in a board 18 ft. long, 24″ wide at one end, 14″ at the other?

13. How many square feet in the two gable ends of a house 32 ft. wide, if the peak of the roof is 12 ft. above the plate?

14. How many square inches in a sheet of metal having two parallel sides 20″ and 32″ long, if the distance between them is 19″?

To Find the Area of a Regular Polygon

194. A plane figure whose sides and angles are respectively equal each to each is a Regular Polygon.



HEXAGON
(AC, apothem; AB, radius.)

OCTAGON

195. The Perimeter of a polygon is the sum of all its sides.

196. The Radius of a regular polygon is the distance from the center to any vertex.

197. The Apothem of a polygon is the perpendicular distance from the center to any side.

198. The radii of a regular polygon divide it into equal triangles. The apothem of the polygon is then the altitude of each of these triangles. The perimeter of the polygon is the sum of the bases of the triangles. Therefore, the area of a regular polygon is equal to one-half the product of its perimeter and apothem.

Principle.—The area of a regular polygon is equal to one-half the product of its perimeter and apothem.

PROBLEMS

1. Find the area of a regular hexagon whose sides are 5 ft. and whose apothem is 4 ft. 4 in.

2. The perimeter of a regular octagon is 48 ft., and the perpendicular distance from the center to one side is 7 ft. 3 in. What is the area of the octagon?

3. The side of a regular pentagon is 10 yd. and the apothem is 7 yd. 1 ft. 5 in. What is the area?

4. The apothem of a regular octagon is 12 ft. and each side is 9.94 ft. What is the area?

5. Find the area of a regular hexagon, if the perimeter is 36 ft. and the apothem is $5\frac{1}{2}$ ft.

6. Find the area of a regular hexagon each of whose sides is 8″ and whose apothem is 6.93″.

7. If the area of a regular octagon is 174 sq. ft. and the length of one side is 6 ft., what is its apothem?

8. If the area of a regular hexagon is 374.4 sq. ft. and the length of one side is 12 ft., what is the apothem?

NOTE.—To find the area of any regular polygon, multiply the square of its side by its number in the following table:

Triangle	.433013	Nonagon	6.181824
Pentagon	1.720477	Decagon	7 694209
Hexagon	2.578076	Dodecagon	11.196152
Octagon	4.828427		

9. Find the area of a regular pentagon whose perimeter is 40 ft.

10. Find the area of a regular decagon one of whose sides is 15 yd.

11. If a triangle is 7 ft. on a side, what is its area?

12. If the side of a regular dodecagon is 15 ft., what is its area?

13. Find the area of a regular nonagon whose sides are each 22 ft.

14. The side of a regular pentagon is 44 yd. What is its area?

15. What is the area of the floor of an octagonal room 16 ft. on a side?

To Find the Circumference and Diameter of a Circle

199. A plane figure bounded by a uniformly curved line is a Circle.

200. The line that bounds a circle is its Circumference.

201. Every point in the circumference is equally distant from the center.

202. A straight line from the center to the circumference is the Radius.

203. A straight line from one side of a circle through the center to the opposite side is the Diameter.

Principles.—1. Circumference = diameter × 3.1416.

2. Diameter = circumference ÷ 3.1416.

For ordinary purposes, circumference = diameter × $3\frac{1}{7}$.

PROBLEMS

1. What is the circumference of a circle whose diameter is 28 ft.?

2. Find the diameter if the circumference is 49 yd.

3. Find the diameter if the circumference is 56 yd.

4. The radius is 7 ft.; find the circumference.

5. The circumference is 68 yd.; find the radius.

6. The diameter is 42 ft.; find the circumference.

7. The circumference is 95 yd.; find the radius.

8. The radius is 7 rd.; find the circumference.

9. The circumference is 123 rd.; find the diameter.

10. The radius is 11 ft. 7″; find the circumference.

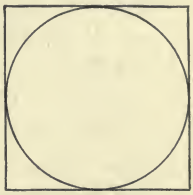
To Find the Area of a Circle

204. If the number of sides of a regular polygon be sufficiently increased, the polygon will become a circle, the perimeter will become the circumference, and the apothem will become the radius of the circle.

Principle.—The area of a circle is equal to one-half the product of its circumference and radius.

$$\text{Area} = \frac{\text{circumference} \times \text{radius}}{2}$$

205. Inscribe a circle within a square. How does the diameter of the circle compare in length with a side of the square? How does the circumference of the circle compare in length with 4 times the length of the side of the square? It is *just* how many times the side of the square—or the diameter of the circle? How does the circle compare in area with the area of the square around the circle? If the side of the square is 10 ft., what is the area of the square? What is the diameter of the inscribed circle? What is $\frac{1}{2}$ the circumference? What is the radius? What is the area?



PROBLEMS

Find the area of circles of the following dimensions:

1. Circumference 50 ft., diameter 15.915 ft.

2. Circumference 1 mi., diameter 101.856 rd.

3. Circumference 60 ft.

4. Radius 28 in.

5. Radius 4 ft.

6. Circumference 26 yd.

7. Circumference 34 yd.

8. Diameter 85 ft.

9. Diameter 18 rd.

10. Diameter 24 ft.

11. Radius 12 rd.

12. Radius 16 yd.

13. How many acres in a circular park 60 rd. in diameter?

14. How many acres in a race-course 1 mi. in circumference?

15. How many square feet in the bottom of a cistern $6\frac{1}{2}$ ft. in diameter?

16. Find the area of the side and bottom of a cistern, if it is 6 ft. deep and 20 ft. in circumference.

17. Find the entire surface of a cylinder 8″ in diameter and 18″ long.

18. How many square feet in a rug 35 ft. in circumference?

Measurements by the Square Yard

206. The cost of plastering, ceiling, painting, and paving is usually computed by the square yard.

It is customary to allow for one-half of the area of openings, as for doors and windows.

PROBLEMS

1. Find the cost of plastering the four walls of a room 16 ft. square and 9 ft. high, at 20¢ per square yard.

2. How many square yards in the ceiling of a room 22 ft. by 18 ft.?

3. The walls of a room 20 ft. by 18 ft. and 10 ft. high have 3 ft. of wainscoting and 7 ft. of plaster. Find the number of square yards in the floor, ceiling, and wainscoting, and the number of square yards of plaster on the walls.

4. A room 18 ft. square and 10 ft. high has 3 doors each 3 ft. by 7½ ft., and 4 windows each 3 ft. by 6 ft. How many square yards in the walls and ceiling, making one-half allowance for doors and windows?

5. Find the cost of ceiling, at 20¢ per square yard, 5 rooms of the following dimensions: 12 ft. by 14 ft., 18 ft. by 21 ft., 13 ft. by 16 ft., 14 ft. square, and 15 ft. by 18 ft.

6. Find the cost of paving a walk 4½ ft. wide, 8 rd. long, at 60¢ per square yard.

7. How many square yards of paving in a street 85 ft. wide and 60 rd. long?

8. How many brick 8 in. by 4 in. will be required, if laid flat, to pave a walk 4 ft. 8 in. wide and 25 ft. 4 in. long?

9. A block of buildings measures 320 ft. by 420 ft. How many square yards in an 8 ft. walk surrounding the block?

10. How many square yards of paving in a courtyard in the form of a trapezoid, the parallel sides being 98 ft. and 76 ft. respectively, and the distance between these sides being 48 ft.?

11. Find the cost of plastering a circular cistern 8 ft. in diameter and 6 ft. deep, at 30¢ per square yard.

12. How many square yards of plastering on a cistern 7 ft. deep, 10 ft. long, and 8 ft. wide?

13. A room 20 ft. square and 9 ft. high has 8 windows 4 ft. by 8 ft., and 7 doors 3 ft. by 8 ft. How many square yards in the walls, ceiling, and floor, allowing ½ space for doors and windows?

14. Find the cost, at 28¢ per square yard, of ceiling the sides and overhead of 6 rooms of the following dimensions: 16 ft. by 18 ft., 12 ft. by 13 ft., 9 ft. by 12 ft., 8 ft. by 14 ft., 13 ft. by 14 ft., and 11 ft. square, each being 9 ft. high.

15. Find the cost of paving, at 75¢ per square yard, a hexagonal court 150 ft. on a side.

16. How many square feet of cement in a circular court 62 ft. in diameter?

17. How many square yards of painting on the outside of a house 50 ft. long, 32 ft. wide, and 21 ft. high? The gable ends are 14 ft. high, the cornice is estimated at 40 sq. yds. and no allowance is made for windows.

18. How many squares of painting, 100 ft. to the square, on the outside of a brick chimney the slant height of which is 60 ft., 12 ft. square at the bottom, and 6 ft. at the top?

19. How many square yards of plastering on the inside of the above chimney if the walls are $1\frac{1}{2}$ ft. thick?

20. How many square feet of painting on 12 stairs, each having 20 steps 3 ft. wide with a tread of 9″ and a rise of 8″?

Papering

207. A roll of wall paper is usually 8 yd. long and 18 in. wide. Double rolls are counted as two rolls.

208. To find the number of rolls of paper required to paper a room:

1. Find the number of strips of paper required.

2. Find the number of strips that can be cut from a roll.

3. Divide the number of strips required by the number that can be cut from a roll.

NOTE.—Sometimes waste occurs in matching, so that it may not always be possible to estimate the exact number of rolls required.

PROBLEMS

1. How many rolls of paper, 8 yd. long and 18 in. wide, will be required to cover the walls of a room 20 ft. square and 9 ft. high, making allowances for 6 windows 3 ft. wide, and 4 doors $3\frac{1}{2}$ ft. wide?

2. How many rolls of paper will be necessary for a room 22 ft. long, 15 ft. wide, and $8\frac{1}{2}$ ft. high, allowing only for 4 doors $3\frac{1}{2}$ ft. wide?

3. How many rolls of paper must be used to cover the ceiling of a room 16 ft. by 21 ft., if the paper runs crosswise with the room?

4. Find the number of rolls of paper required to cover the walls and ceiling of a room 18 ft. square and 10 ft. high. A roll will make two strips for the wall. In the middle of each side is a door $3\frac{1}{2}$ ft. wide, and on each side of each door and $2\frac{1}{2}$ ft. from it, is a window 3 ft. wide.

5. How much paper will be required for the walls and ceiling of a room 12 ft. by 13 ft. and 8 ft. high? Allow for 2 doors and 4 windows each $3\frac{1}{2}$ ft. wide and no waste in matching.

6. How many yards of plain paper 30″ wide must be used to cover the walls of a room 32 ft. by 28 ft., and 14 ft. high, allowing 1 ft. for border, 3 ft. for wainscoting, and for 4 doors each 4 ft. wide, and 8 windows each $3\frac{1}{2}$ ft. wide?

7. How many yards of plain paper 3 ft. wide will be necessary to cover the walls and ceiling of a hall 120 ft. long, 12 ft. wide, and 13 ft. high?

8. How many rolls of paper will be used for the walls and ceiling of a room 17 ft. by 23 ft. and 10 ft. high, allowing 1 ft. for border, 2 ft. of paper on each strip for matching, and for 5 doors and 6 windows, each 4 ft. wide?

Carpeting

209. Carpets are usually either 1 yd. or $\frac{3}{4}$ yd. in width.

Allowance must be made for waste in matching the patterns in carpets.

As carpets are sold in strips and matched by strips, the number of strips required to cover the floor must be found. Sometimes it is necessary to cut off or turn under part of a strip, and the part cut off or turned under must be included in the estimate.

Sometimes there is less waste when the strips run one way in the room than when they run the other way.

In finding the length of border to be put around a carpet the entire distance around the room must be taken, as there is a waste at each corner in making the border.

PROBLEMS

1. A floor 21 ft. by 24 ft. is covered with carpet 1 yd. wide, without waste in matching. Find the cost of the carpet at \$1.10 per running yard.

2. If the above room is covered with carpet $\frac{3}{4}$ yd. wide, what will it cost at 90¢ per yard?

3. Find the cost of carpeting a room 20 ft. by 27 ft., with carpet $\frac{3}{4}$ yd. wide, at 80¢ per yard, if the strips run lengthwise; if the strips run crosswise.

4. How many yards of carpet 1 yd. wide will be required for a room 24 ft. by 16 ft., if the strips run lengthwise and $\frac{1}{6}$ of a yard in each strip is wasted in matching?

5. Find the least number of yards of carpet required to cover floors of the following dimensions, and tell which way the strips should run, allowing no waste for matching:

Room	Width of Carpet
40 ft. by 42 ft.	1 yd.
22 ft. by 15 ft.	$\frac{3}{4}$ yd.
27 ft. by 34 ft.	$\frac{3}{4}$ yd.
13 ft. by 15 ft.	1 yd
14 ft. by 18 ft.	$\frac{3}{4}$ yd.

6. Find how much more carpet would be required in each case if it were laid in the opposite direction.

7. Find the least number of yards of carpet necessary to cover floors of the following dimensions, allowing no waste for matching:

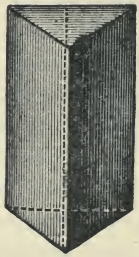
Room	Width of Carpet
14 ft. by 22 ft.	1 yd.
21 ft. by 24 ft.	$\frac{3}{4}$ yd.
13 ft. by 17 ft.	1 yd.
$12\frac{1}{2}$ ft. by 16 ft.	$\frac{3}{4}$ yd.
22 ft. by 25 ft.	$\frac{3}{4}$ yd.

MEASUREMENT OF SOLID FIGURES

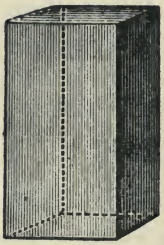
To Find the Volume of a Prism

210. A solid has length, breadth, and thickness.

211. A solid that has two parallel equal bases, and three or more sides that are parallelograms, is a Prism.



TRIANGULAR PRISM



RECTANGULAR PRISM

212. The area of a rectangle is equal to the product of its length and breadth.

213. A rectangular prism that is one unit high has a volume equal to the product of its length and breadth. Its solid contents equals the area of its base.

If the prism is 3 units in height, its volume is equal to 3 times the area of its base, or the product of its length, breadth, and thickness.

Principles.—1. The volume of a rectangular prism is equal to the product of its length, breadth, and thickness.

2. The volume of any prism is equal to the product of the area of its base by its altitude.

PROBLEMS

1. Find the contents in cubic feet of prisms having the following dimensions:

(a) 12 ft. square and 16 ft. high.

(b) Base 8 ft. by 14 ft., altitude 7 ft.

(c) Length 20 ft., breadth 16 ft., height 8 ft.

(d) Area of base 137 sq. ft., altitude 9 ft.

2. The triangle that forms the base of the prism has a base of 8 ft. and an altitude of 5 ft. The altitude of the prism is 10 ft. Find the contents.

3. How many cubic feet in a rectangular solid 12 ft. by 8 ft. by 14 ft?

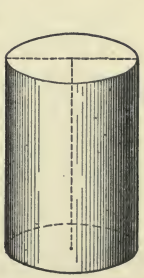
4. Find the solid contents of a hexagonal prism 12 ft. on a side and 16 ft. high.

5. The bottom of a bin is a trapezoid, the parallel sides being 14 ft. and 16 ft. and 10 ft. apart. How many cubic feet does it contain if it is 8 ft. deep?

6. How many cubic feet of water can be put into a tank in the form of an octagonal prism, if it is 9 ft. high and each side of the tank is 8 ft.?

7. A gas reservoir has a base in the form of a dodecagon 16 ft. on a side. If it is 40 ft. high, how many cubic feet of gas will it hold?

To Find the Volume of a Cylinder



214. A solid having two equal parallel circles for its bases and a uniformly curved surface for its side is a Cylinder.

Principle.—The volume of a cylinder is equal to the product of the area of its base by its altitude.

PROBLEMS

1. Find the contents of cylinders with the following dimensions:

(*a*) Altitude 7 ft., base 6 ft. in diameter.

(*b*) Circumference of base 25 ft., altitude 6 ft.

(*c*) Altitude 26 ft., diameter of base 4 ft.

(*d*) Circumference of base 42 ft., altitude 7 ft.

2. How many cubic feet in a pillar 8 ft. in diameter and 50 ft. high?

3. A cistern 7 ft. deep and 28 ft. in circumference contains how many cubic feet?

4. The diameter of a pipe is $2\frac{1}{2}$ ft. How much water will flow through it in an hour, if it flows with a velocity of 90 ft. per minute?

5. How many cubic feet of iron in a pipe 80 ft. long and 3 ft. in diameter, if the iron is $\frac{1}{2}''$ thick?

6. I wish to make a cylindrical cistern that will hold 1000 cubic feet of water. If I make it 12 ft. in diameter, how deep must it be?

Brick and Stone Work

215. Brick work is commonly estimated by the 1000 brick. Masonry is commonly estimated by the cubic foot and by the perch.

NOTE.—A perch is $16\frac{1}{2}$ cu. ft. Sometimes $24\frac{3}{4}$ cu. ft. are called a perch.

Usually a deduction is made for one-half of the openings.

216. In estimating the amount of work done in laying stone and brick, the length of the wall is found by measuring around the wall on the outside, or by measuring on the inside and adding 8 times the thickness of the wall. The corners are thus counted twice.

217. In estimating the amount of material used, find the exact length of the wall by measuring on the inside and adding 4 times the thickness of the wall for the corners, or by measuring on the outside and subtracting 4 times the thickness of the wall for the corners.

Principle.—To find the contents of a wall, find the entire length of the wall; find the product of the length, height, and thickness. Make deductions for openings.

PROBLEMS

1. A house is 28 ft. by 22 ft. How many cubic feet in the cellar wall, if the wall is 18 in. thick and the cellar is 7 ft. deep?

2. What will be the cost, at 80¢ a perch ($16\frac{1}{2}$ cu. ft.), of laying a wall $1\frac{1}{2}$ ft. thick, for a cellar 29 ft. square and 7 ft. deep?

3. What will it cost, at 20¢ per load (a cubic yard of earth is called a load), to have a cellar dug 18 ft. by 34 ft., and 8 feet deep? What will it cost, at 70¢ per perch, to have the cellar wall laid 20 in. thick? How many cords of stone will be required for the wall? (A cord of stone is 128 cu. ft.)

4. Find the number of cubic feet in a cellar wall of the following dimensions: Wall, 2 ft. thick, 8 ft. high; cellar, 32 ft. by 45 ft. Make ½ allowance for a door 3 ft. wide, and 6 windows each 2 ft. by 3 ft.

5. A house 21 ft. by 48 ft. has a cellar 8 ft. deep. If the wall is to be 1½ ft. thick, how many cords of stone will be required for the wall?

6. A wall 12 in. thick is estimated to contain 22 common brick per cubic foot. How many brick in a brick house 24 ft. square and 19 ft. high, if the wall is 1 ft. thick and an allowance is made for 3 doors, each 4 ft. by 8 ft., and 16 windows, each 3½ ft. by 7 ft.?

7. Find the cost, at 50¢ per load, to have a cellar dug 20 ft. by 36 ft. and 7 ft. deep. Find the cost of laying the cellar wall 1½ ft. thick at 90¢ per perch. How many cords of stone will be required?

8. How many brick will be necessary to build a cistern 7 ft. deep, 8 ft.in diameter, inside measurement, and 1 ft. thick?

9. Find the exact number of cubic feet in the wall of a cellar 34 ft. square (inside) and 9 ft. high, if the wall is 2 ft. thick.

10. Find the cost, at 95¢ per perch, of laying a cellar wall. The cellar on the inside is to be 22 ft. by 38 ft. and 8 ft. high. The wall is to be 2 ft. thick, and the trench is to be 1½ ft. deep.

Wood

218. A Cord of wood is equivalent to 128 cu. ft.—or a pile 8 ft. by 4 ft. by 4 ft. A Cord Foot (cd. ft.) is 1 ft. in length of such a pile, or 16 cu. ft.

NOTE.—A pile of wood 8 ft. long and 4 ft. high is commonly called a cord without regard to the length of the wood. Thus, a cord of stove wood is a pile 8 ft. by 4 ft. by the length of the sticks of wood.

219. The face of a cord of wood measures 32 sq. ft. (8 × 4). Therefore, the area of the face of such a pile divided by 32 equals the number of cords in the pile.

PROBLEMS

1. Find the number of cords of wood in piles having the following dimensions:

(*a*) Stove wood, 38 ft. by 9 ft.
(*b*) Stove wood, 28 ft. by 6 ft.
(*c*) 4-ft. wood, 16 ft. by 3 ft.
(*d*) 4-ft. wood, 27 ft. by 5 ft.
(*e*) 4-ft. wood, 38 ft. by 6 ft.

2. A man bought a pile of wood 16 ft. long, 9 ft. wide, and 7 ft. high. What did it cost at $3 per cord of 128 cu. ft.?

3. A pile of wood is begun 4 ft. wide and 6 ft. high. How long must it be made to contain 18 cords?

4. A wood rack is 12 ft. long and 3 ft. wide. How high must the wood be piled upon it to make $1\frac{1}{4}$ cords?

Lumber

220. If a board is not more than 1 in. thick, a square foot of its surface is a *foot of lumber*, or a *foot board measure.*

A board 1 ft. wide and 14 ft. long, if not more than 1 in. thick, contains 14 ft. of lumber.

The width of boards is measured in inches.

221. For boards not more than 1 in. thick, we have the formula:

$$\frac{\text{Length (ft.)} \times \text{width (in.)}}{12} = \text{feet of lumber}$$

222. If lumber is more than 1 in. thick, this formula is used:

$$\frac{\text{Length (ft.)} \times \text{width (in.)} \times \text{thickness (in.)}}{12} = \text{feet of lumber}$$

EXAMPLE.—Find the number of feet of lumber in a board, not more than 1 in. thick, with length 12 ft., width 13 in.

OPERATION

$12 \times 13 \div 12 = 13$

NOTE.—Use cancellation when practicable. Use aliquot parts of 12. If a board is 8 in. wide and 15 ft. long, say $8 \times 1\frac{1}{4} = 10$, instead of $8 \times 15 \div 12 = 10$. Likewise, as parts of 12, $3 = \frac{1}{4}$, $4 = \frac{1}{3}$, $6 = \frac{1}{2}$, $8 = \frac{2}{3}$, $9 = \frac{3}{4}$, $15 = 1\frac{1}{4}$, $16 = 1\frac{1}{3}$, $18 = 1\frac{1}{2}$.

PROBLEMS

Find the number of feet of lumber in boards, not more than 1 in. thick, of the following dimensions:

1. Length 15 ft., width 16 in.
2. Length 14 ft., width 9 in.
3. Length 18 ft., width 10 in.
4. Length 20 ft., width 15 in.
5. Length 24 ft., width 16 in.
6. Length 22 ft., width 6 in.
7. Length 8 ft., width 9 in.
8. Length 14 ft., width 13 in.

Find the number of board feet in timbers and plank of the following dimensions:

9. Width 8 in., length 16 ft., thickness $2\frac{1}{2}$ in.
10. Width 12 in., length 14 ft., thickness 3 in.
11. Width 14 in., thickness 4 in., length 10 ft.
12. Length 18 ft., width 12 in., thickness 9 in.
13. Length 20 ft., 8 in. square.
14. Length 16 ft., 2 in. by 6 in.
15. Length 18 ft., 2 in. by 4 in.
16. Length 14 ft., 2 in. by 8 in.
17. Length 15 ft., 2 in. by 4 in.

18. Find the cost, at $18.50 per M, of 32 scantling, each 14 ft. by 2 in. by 4 in.

19. Find the cost, at $35 per M, of 12 timbers, each 18 ft. long and 10 in. square.

20. Find the cost, at $14 per M, of a load of hemlock timbers of the following dimensions:

24 scantling.....	$16' \times 2'' \times 4''$
12 scantling.....	$14' \times 2'' \times 6''$
8 timbers	$12' \times 6'' \times 8''$
10 timbers	$16' \times 4'' \times 4''$
20 plank.....	$18' \times 2'' \times 10''$
6 timbers	$14' \times 4'' \times 6''$

21. Find the cost, at $28 per M, of the following pieces of pine:

22 plank	$16' \times 2'' \times 12''$
18 plank	$14' \times 3'' \times 10''$
25 scantling	$18' \times 2'' \times 4''$
6 sticks	$14' \times 4'' \times 4''$
40 boards	$12' \times 1\frac{1}{2}'' \times 8''$
36 rafters	$18' \times 2'' \times 6''$
48 sleepers	$16' \times 2'' \times 8''$
8 sticks	$24' \times 6'' \times 8''$

22. Find the cost of the following chestnut lumber at $45 per M:

9 pieces	$12' \times 1\frac{1}{2}'' \times 10''$
12 pieces	$14' \times 1'' \times 12''$
10 pieces	$16' \times 1\frac{1}{4}'' \times 14''$
6 pieces	$12' \times 1'' \times 14''$
8 pieces	$8' \times 2'' \times 6''$
9 pieces	$6' \times 1\frac{1}{4}'' \times 8''$

23. What is the cost of the following boards at $18 per M:

16 boards	$12' \times \frac{3}{4}'' \times 12''$
12 boards	$16' \times \frac{1}{2}'' \times 10''$
20 boards	$18' \times \frac{5}{8}'' \times 12''$
10 boards	$14' \times 1'' \times 9''$
24 boards	$12' \times \frac{1}{4}'' \times 8''$
15 boards	$9' \times 1\frac{1}{2}'' \times 10''$

Capacity of Bins and Cisterns

223.

TABLE

2150.4 cu. in. = 1 bu.
231 cu. in. = 1 gal.
31½ gal. = 1 bbl.

FORMULAS

$$\frac{\text{Length} \times \text{breadth} \times \text{thickness} \times 1728}{2150.4} = \text{bushels}$$

$$\frac{\text{Length} \times \text{breadth} \times \text{thickness} \times 1728}{231} = \text{gallons}$$

NOTE.—All dimensions should be in feet.

PROBLEMS

1. Find the contents (dry measure) of the following: (*a*) A bin 8 ft. square and 6 ft. deep; (*b*) a box 9 ft. by 3 ft. 4 in. by 10 in.; (*c*) a box 6 ft. long, 4 ft. high, $4\frac{1}{2}$ ft. wide.

2. How many bushels of wheat will a bin hold that is 8 ft. long, 5 ft. wide, and $5\frac{1}{2}$ ft. high?

3. How many bushels of potatoes will a bin hold that is 9 ft. square and $6\frac{1}{2}$ ft. deep?

4. How many bushels of apples will a wagon box hold that is 14 ft. by 3 ft. 4 in. by 2 ft. 3 in.?

5. Find the contents, liquid measure, of a tank 9 ft. long, 4 ft. wide, and 3 ft. deep.

6. How many barrels will a cistern hold that is 7 ft. by 8 ft. by 6 ft.?

7. A vat $8\frac{1}{4}$ ft. by 6 ft. by 4 ft. will hold how many gallons of water?

8. A cylindrical cistern 7 ft. in diameter and 7 ft. high will hold how many barrels of vinegar?

9. If 3 measures of grapes will make 2 measures of wine, how many gallons of wine can be made from 3 bu. of grapes?

10. How many gallons of alcohol will be required to fill a pipe 18 ft. long and $1\frac{1}{2}$ in. in diameter?

11. How many gallons of oil in a cylindrical can 2 ft. high and 14 in. in diameter?

12. If 1 cu. ft. of water weighs $62\frac{1}{2}$ lb., how many gallons in 1 T. of water?

13. A tank is 22 ft. in circumference and 8 ft. high. How many barrels of beer (28 gal.) will it contain?

PRACTICAL RULES FOR DEALERS IN FARM PRODUCE

224. The following rules are approximate:

1. To find the contents, in bushels, of a bin or box, multiply the number of cubic feet by .8.

2. To find the volume of a bin required to hold a given number of bushels, divide the number of bushels by .8, or multiply by $\frac{5}{4}$.

3. To find the contents, in 40-qt. bushels, of a bin, multiply the number of cubic feet by $\frac{4}{5}$.

NOTE.—The 40-qt. or "heaped" bushel is used for apples, potatoes, etc.

4. To find the volume of a bin required to hold a given number of 40-qt. bushels, divide the number of bushels by $\frac{4}{5}$.

5. To find the contents, in gallons, of a tank or cistern, multiply the number of cubic feet by $7\frac{1}{2}$.

6. To find the volume, in cubic feet, of a tank required to hold a given number of gallons, divide the number of gallons by $7\frac{1}{2}$.

7. To find the number of shelled bushels in a crib of unshelled corn, multiply the number of cubic feet by .45.

NOTE.—Pupils should be required to give the reasons for the above rules. NOTE—A bushel is to a cubic foot as 56 is to 45.

Hay

225. The weight of hay, per cubic foot, in a load, shed, mow or stack, varies with the kind of hay, the height of the pile, pressure or treading in packing, and time of settling. The higher the pile the more compact the hay will be.

Principles.—1. To find the weight of hay in a load, or low pile, allow 540 cu. ft. for a ton.

2. To find the weight of hay in an ordinary mow, or low stack, allow 400 cu. ft. for a ton.

3. To find the weight of hay in mow bottoms and in bottoms of high stacks, allow 325 cu. ft. per ton.

PROBLEMS

1. How many bushels of wheat in a bin 6 ft. square and 4 ft. deep?

2. How many bushels of apples in a wagon box 14 ft. long, 3 ft. wide, and 30 in. high.

3. How many bushels of oats in a bin 8 ft. long, 6 ft. wide, and 3 ft. deep?

4. At 35¢ per bushel, find the value of a bin of potatoes 10 ft. long, 7 ft. wide, and 5 ft. high.

5. How many cubic feet of space must be provided for 500 bushels of apples? If the bin is 4 ft. deep and 8 ft. wide, how long must it be?

6. I wish to build a bin that will hold 120 bushels of wheat. If I make it 6 ft. square, how high must it be?

7. How many cubic feet in a cylindrical cistern 5 ft. in diameter and 6 ft. deep? How many gallons would it hold?

8. How deep must a cistern be to hold 60 bbl. of water if it is 7 ft. in diameter?

9. How many bushels of shelled corn in a crib of unshelled corn 18 ft. by 6 ft. by 7 ft.?

10. If a bin will hold 400 bushels of wheat, how many bushels of apples will it hold?

11. A wagon box 14 ft. long, 3 ft. wide and $2\frac{1}{2}$ ft. high is full of corn in the ear. How many bushels of shelled corn will the load make?

12. How many bushels of turnips in a bin 7 ft. long, 5 ft. wide, and 4 ft. deep?

13. I wish to make a wagon box to hold 40 bu. of apples. If I make it 14 ft. long and 3 ft. wide, how high must it be?

14. How many hundredweight of hay in a load 20 ft. long, 12 ft. wide, and 7 ft. high?

15. The top of a high hay stack has been removed. The bottom is 20 ft. square and 12 ft. high. How many tons does it contain?

16. A pile of hay 40 ft. long, 15 ft. wide, and 9 ft. high contains how many tons?

17. If a stack of hay is 24 ft. in diameter, and 12 ft. high, what is it worth at $12 per ton?

18. I wish to build in a barn a bay that will hold 100 tons of packed hay. If I make it 40 ft. wide and 12 ft. deep, how long must it be?

19. How many cubic feet of space will be required to hold 400 bu. of potatoes? If the bin is made 12 ft. square, how deep must it be?

20. A bin that will hold 250 bu. of wheat will hold how many bushels of potatoes?

SQUARE ROOT AND ITS APPLICATIONS

226. To square a number is to multiply it by itself. The square of 5 is 25. 36 is the square of 6. What is the square of 8, 9, 11, 12, 15, 20, 25?

227. One of the two equal factors of a number is the square root of the number. Thus, 10 and 10 are the two equal factors of 100, and 10 is the square root of 100. 12 is the square root of 144. What is the square root of 81, 121, 225, 625?

228. To indicate the square of a number we write a small figure 2 at the upper right hand side of the number. Thus, 12^2 means 12×12, or 12 squared. $15^2 = 225$. Give the value of 8^2; 10^2; 11^2; 13^2; 21^2; 19^2.

229. To indicate the square root of a number, we write the character $\surd$ at the left of the number. $\surd\overline{100}$ means the square root of 100, or 10. $\surd\overline{144} = 12$. Give the value of $\surd\overline{225}$; $\surd\overline{625}$; $\surd\overline{900}$; $\surd\overline{169}$; $\surd\overline{400}$.

230. *Principles.—1.* If the side of a square represents a number, the square itself will represent the square of the number.

2. If a square represents a number, a side of the square will represent the square root of the number.

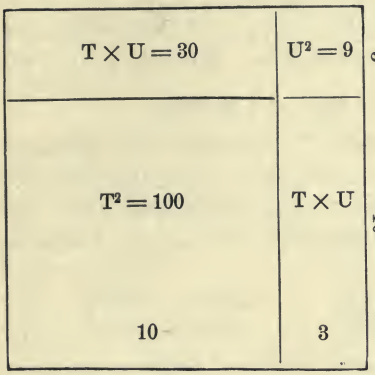
MENTAL PROBLEMS

1. I wish to make a square table with a surface of 16 sq. ft.; what must be the length of a side?

2. What is the length of a side of a square board that contains 225 sq. in.?

3. What is the length in rods of a square field that contains 10 acres?

4. A square floor contains 400 sq. ft. of space; what is the length of one side?



The diagram compares *a square* with *the square of a number* and *the side of the square* with *the square root of the same number*. The large square marked T^2 equals the square of the tens of the number. The area of the square is 100 and the square of 10 is 100. Each of the rectangles marked $T \times U$ equals the product of the tens by the units. The small square marked U^2 equals the square of the units.

The complete square equals the square of the tens plus twice the product of the tens and units, plus the square of the units, or $T^2 + 2TU + U^2$. $13 = 10 + 3$, or $1T + 3U$. $13^2 = 10^2 + 2(10 \times 3) + 3^2$, or $T^2 + 2TU + U^2$. Any number may be considered as composed of tens and units. 125 is composed of 12 tens and 5 units. $125^2 = T^2 + 2TU + U^2$, or $120^2 + 2(120 \times 5) + 5^2$. 2654 = 265 tens + 4 units. $2654^2 = 2650^2 + 2(2650 \times 4) + 4^2$.

The following example also shows the relation of a number to its square.

EXAMPLE.—Find the square of 10 plus 3.

OPERATION

$$\begin{array}{r} 10 + 3 \\ 10 + 3 \\ \hline (10 \times 3) + 3^2 \\ 10^2 + \quad (10 \times 3) \\ \hline 10^2 + 2\,(10 \times 3) + 3^2 = 169 = T^2 + 2TU + U^2 \end{array}$$

Principle.—The square of a number composed of tens and units is equal to the square of the tens + 2 × the tens × the units + the square of the units.

PROBLEMS

Write out the squares of the following:

1. $5T + 4U$.
2. $40 + 3$.
3. $60 + 8$.
4. $70 + 5$.
5. $80 + 4$.

Write by inspection the square root of:

6. $100 + 60 + 9$. *8.* $100 + 80 + 16$. *10.* $100 + 140 + 49$.

7. $100 + 100 + 25$. *9.* $100 + 120 + 36$. *11.* $400 + 200 + 25$.

To Find the Square Root of a Number

231. Finding the square root of a number is an operation in "fitting and trying" like long division. We try and then find whether we are right. We can tell something about the square root of a number by inspection. Study the following table:

$1^2 = 1$	$10^2 = 100$	$100^2 = 10000$
$9^2 = 81$	$99^2 = 9801$	$999^2 = 998001$

It will be seen that the square of a number of 1 figure cannot contain more than two figures; that the square of a number of 2 figures cannot contain less than three nor more than 4 figures; that the square of a number of 3 figures cannot contain less than 5, nor more than 6 figures. If any number be separated into periods of 2 figures each, beginning at the right, there will be just as many periods as there are figures in the square root of the number. Thus, in the square root of 1'46'75'83 there are 4 figures; in the square root of 21'38'00 there are three figures.

The first figure of the square root of a number will always be the highest figure whose square can be formed in the first (left hand) period. If the first period is 1, the first figure of the root will be 1. If the first period is 6, the first figure of the root will be 2. If the first period is 28, the first figure of the root will be 5. What will be the first figure of the root if the first period is 36; 40; 49; 55; 60; 65; 75; 85; 98?

Find the number of figures in the root, and the first figure which is always sure to be right, then proceed to find the next figure. This is found somewhat as a quotient figure in division is found. After two figures of the root are found, call them both tens—consider them as one—and find the next figure, and so on.

EXAMPLE 1.—Find the square root of 169.

OPERATION

	1'69	13
	1	
20	69	
23	69	

EXPLANATION.—There are two periods and there will be two figures in the root. The first figure of the root must be 1. Take the square of 1 (1 ten) from the whole square (169) and 69 is left. 69 is $2 \times$ the tens + the square of the units, or $2TU \times U$, or $U(2T + U)$. Now find the next figure, or U, the number which multiplied by twice the tens plus itself will produce 69. At the

left of 69 write 2 × the tens, or 20, and by inspection the next figure is found to be 3. Add 3 to twice the tens already found and get 23, (2T + U). Multiply this by 3 and it is found that the estimate was correct.

EXAMPLE 2.—Find the square root of 6770404.

OPERATION

	6′77′04′04	2602
	4	
40	277	
46	276	
5200	10404	
5202	10404	

EXPLANATION.—There will be four figures in the root. The first figure of the root is 2, for the greatest square in the first period is 4. Taking the square of 2 from the first period, 2 is left. Bring down the next period and from the result, 277, find the next figure of the root. 277 is twice the first figure × the second + the square of the second + a possible remainder. To find the second figure divide 277 by twice 2 tens, the first root figure, or 40, and get 6, which is probably the second figure of the root. Add 6 to 40, and then multiply 46 by 6 instead of multiplying 40 by 6 and adding 6 × 6. Subtract 276, (46 × 6), from 277 and obtain 1. Bring down the next period and the result is 104. Divide this by twice the first figure of the root, which is now 26 tens, to find the second figure. Twice 26 tens are 520 (26 × 2 with one cipher annexed). But 520 is not contained in 104, so the next figure of the root is 0. Call 260 tens the first figure of the root, and proceed to find the last figure. Bring down the next period and 10404 is the dividend. Divide it by twice the found part of the root (260 tens), which is 5200 (260 × 2 with one cipher annexed). The quotient is 2. Add 2 to the "trial divisor," 5200, and the "complete divisor" is 5202.

Then divide 10404 by 5202, and the quotient is exactly 2.

Steps.—1. Beginning at the left point off into periods of 2 figures each.

2. Find the first figure of the root.

3. Take the square of the first figure from the first period and bring down the next period.

4. At the left write twice the part of the root already found and annex one cipher.

5. Estimate the next figure of the root.

6. Add this new figure to the trial divisor of the fourth step.

7. Divide and bring down the next period.

8. Repeat 4, 5, 6, 7, and so on.

NOTES.—1. If at any time the product of the divisor and last figure of the root is greater than the dividend, the quotient figure is too great and must be made less—the same as in long division.

2. If there is a remainder after finding the last integral figure of the root ciphers may be annexed, and the root continued as a decimal.

PROBLEMS

Find the square root of:

1. 576.	4. 1522756.	7. 15876.
2. 5625.	5. 7 to 3 decimal places.	8. 2645731.
3. 42436.	6. 2.	9. $\frac{25}{36}$.

NOTE.—In extracting the square root of a fraction the root of each term may be found separately, or the fraction may first be reduced to a decimal.

10. Extract the square root of $\frac{15}{16}$.

11. What is the length of the side of a square lot that contains 15 A.?

12. A man wishes to set out 15625 trees in a square so that there shall be the same number of trees in rows each way. How many trees should be planted in a row?

13. How many hills of potatoes, each 3 ft. from the others in rows 3 ft. apart, can be planted in an 8-A. square lot?

14. At $1.40 per rod, what will it cost to fence a field of 28 A. twice as long as it is wide?

15. A man wishes to erect a building having 2900 feet of floor. If he wishes its length to be four times its width, what must be its dimensions?

To Find the Area of a Triangle by Square Root

232. EXAMPLE.—Find the area of a triangle whose sides are 12 ft., 8 ft., and 16 ft., respectively.

Steps in the Operation.—1. Add the three sides and divide the sum by 2.

2. From this half-sum subtract each side separately.

3. Find the product of the three remainders and the half-sum.

4. Extract the square root of the product. This will give the area.

OPERATION

$$8 + 12 + 16 = 36$$
$$36 \div 2 = 18$$
$$18 - 8 = 10$$
$$18 - 12 = 6$$
$$18 - 16 = 2$$
$$18 \times 10 \times 6 \times 2 = 2160$$
$$\sqrt{2160} = 46.475, \text{ area in square feet}$$

PROBLEMS

1. The sides of a triangle measure 21 ft., 40 ft., and 45 ft. Find the area of the triangle.

2. How many acres in a triangular field whose sides measure 48 rd., 62 rd., and 78 rd.?

3. How many square feet in the gable end of a building that is 80 ft. wide, if the length of each side of the roof is 50 ft.?

4. How many acres in a triangular field which measures 36 rd., 24 rd., and 42 rd.?

5. What is the diameter of a circular field that contains 1 acre?

NOTE.—Diameter$^2 \times .7854 =$ area.

6. Each side of a triangle is 62 ft. What is its area?

7. If the area of a circular field is 10 A., what is its diameter?

8. How many rods of netting must be purchased to enclose a circular park of $\frac{1}{2}$ acre?

9. What is the diameter of a pipe if the area of a cross-section is 8 sq. ft.?

10. What is the area of a triangle whose sides are 23 rd., 28 rd., and 31 rd.?

11. A fence 100 yd. long will enclose how much land in the form of a circle? How much in the form of a square?

12. Find the diameter of a circle equal in area to a square 12 ft. on a side.

13. Find the side of a square equal in area to a circle whose diameter is 100 ft.

14. If a cistern is to be 6 ft. deep, what must be its diameter in order that it may contain 50 bbl.?

15. The three sides of a field are 75 rd., 60 rd., and 53 rd. What is its area?

16. I wish to build a bin that will hold 1000 bushels of wheat. If I make it 4 ft. deep, how large square must it be?

17. Find the area of a triangle whose sides are 19 yd., 21 yd., and 24 yd.

18. Find the contents in bushels of a bin 5 ft. deep, whose bottom is an equilateral triangle 8 ft. on a side.

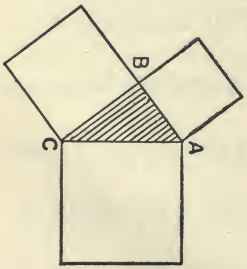
19. Find the number of square feet of roofing on a house 40 ft. square. The roof consists of 4 parts, each slanting from a side of the house to the peak in the center. The distance from the peak to one corner of the roof is 80 ft.

20. What is the area of a triangle whose sides measure 20 ft., 24 ft., and 30 ft.?

The Right Triangle

233. The Right Triangle, or a Right-Angle Triangle, is a triangle having one right angle.

234. The side opposite the right angle is the Hypotenuse.



If the base is 4 and the altitude 3, the hypotenuse is 5.

NOTE.—As shown in the diagram, if a square is drawn on each side of a right triangle the square on the side of the hypotenuse is equal to the sum of the squares on the other two sides.

This is true of any right triangle.

Principle.—In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$\text{Hypotenuse}^2 = \text{base}^2 + \text{altitude}^2$$
$$B^2 = H^2 - A^2$$
$$A^2 = H^2 - B^2$$

EXAMPLE 1.—The hypotenuse of a right angle is 10 and the altitude is 6. What is the base?

OPERATION

$$B^2 = 10^2 - 6^2$$
$$B^2 = 100 - 36, \text{ or } 64$$
$$B = \sqrt{64}, \text{ or } 8$$

EXAMPLE 2.—The base of a triangle is 12 ft., the altitude is 16 ft. Find the hypotenuse.

OPERATION

$$H^2 = 16^2 + 12^2$$
$$H^2 = 256 + 144, \text{ or } 400$$
$$H = \sqrt{400}, \text{ or } 20$$

PROBLEMS

1. The hypotenuse is 15 yd., the base is 12 yd. What is the altitude?

2. If the altitude is 15 ft., the base 20 ft., what is the hypotenuse?

3. How many acres in a field in the form of a right triangle if the hypotenuse is 70 rd. and the base is 56 rd.?

4. Find the diagonal of a square 22 ft. on a side.

5. Find the distance from the lower southeast corner of a room to the upper northwest corner, if the room is 18 ft. by 16 ft., and 10 ft. high.

6. I wish to put a steel brace against a brick wall. If the brace is to meet the wall 18 ft. above the base, and is to stand 7 ft. from the foot of the wall, how long must the brace be?

7. How long must a wire be to reach from the top of a stack 60 ft. high to a stake 45 ft. from the base of the stack?

8. Find the area of the surface of the sides of a square pyramid 125 ft. on a side, if the apex is 90 ft. above the base.

9. There are two poles 90 ft. apart, each 110 ft. high. How far is it from the top of one to the middle of the other?

10. What must be the length of the rafter of a house 28 ft. wide, if the peak of the roof is to be 10 ft. above the plate and the roof is to project $1\frac{1}{2}$ ft.?

11. What is the size of the largest square that can be cut out of a circular sheet of metal 2 ft. in diameter?

12. Find the area of a right triangle whose base is 64 ft. and hypotenuse 80 ft.

13. What is the distance from a point to the top of a pole 120 ft. high, if the pole is 90 ft. from the place of measurement?

14. What is the length of the diagonal of a room 18 ft. by 22 ft.?

15. Find the area of the six faces of a hexagonal pyramid 8 ft. on a side, the distance from the apex to any corner at the base being 20 ft.

REVIEW PROBLEMS IN MENSURATION

1. Find the area of a circle 15 ft. in diameter.

2. How many square feet in the floor of a hexagonal room 10 ft. on a side?

3. How many feet of lumber in a board 18 ft. by 11 in. and $1\frac{1}{2}$ in. thick?

4. What is the contents in bushels of a bin 9 ft. square and 6 ft. deep?

5. How many cords of wood in a pile 38 ft. long, 7 ft. high and 4 ft. wide?

6. How much liquid will be required to fill a pipe $\frac{3}{4}$ in. in diameter and 25 ft. long?

7. How many square feet of sheet iron will be required to build a smokestack 45 ft. high and 14 in. in diameter?

8. What is the value, at \$68 per acre, of a triangular field which measures 36 rd., 42 rd., and 54 rd.?

9. What will it cost, at 20¢ per square foot, to pave the intersecting diagonal walks of a park 40 yd. square, if the walks are 6 ft. wide.?

10. How many cubic feet of marble in a cylindrical monument 9 ft. in diameter and 32 ft. high?

11. How many bricks in a chimney 6 ft. square and 48 ft. high, if the chimney wall is 1 ft. thick?

12. Find the number of yards of carpet, $\frac{3}{4}$ yd. wide, necessary to cover the floor of a room 21 ft. by 25 ft.

13. If water weighs $62\frac{1}{2}$ lb. per cubic foot, what is the weight of the water in a tank which measures 4 ft. by $2\frac{1}{2}$ ft. by 14 ft.?

14. How many feet of lumber in a timber $8'' \times 9''$ and 28 ft. long?

15. How many feet of fence will enclose a square field of 8 A.?

16. Find the number of cubic feet of masonry in the walls of a cellar 16 ft. square on the inside and 9 ft. deep, if the wall is 20 in. thick.

17. How many loads of earth must be removed in digging a cellar 20 ft. wide, 46 ft. long, and 7 ft. deep?

18. Find the capacity in barrels of a tank 4 ft. wide, 10 ft. long, and 5 ft. deep.

19. What will be the cost, at $28 per M, of a stick of timber 26 ft. long, 12 in. wide, and 9 in. thick?

20. How many square yards of plastering are there on the walls and ceiling of a room 21 ft. wide, 27 ft. long, and 11 ft. high, allowing for 7 doors 4 ft. wide and 8 ft. high, and for 8 windows 4 ft. wide and 7 ft. high?

21. How many cords of stone will be used in building a foundation wall for a factory 80 ft. by 110 ft., if the wall is to be 10 ft. high and $2\frac{1}{2}$ ft. thick?

22. How many brick will be used in erecting a brick house 24 ft. wide, 36 ft. long, and 18 ft. high, if the wall of the house is made $1\frac{1}{2}$ ft. thick, and allowing for 3 doors $3\frac{1}{2}$ ft. by $7\frac{1}{2}$ ft., and for 24 windows each $3\frac{1}{2}$ ft. by 6 ft.?

NOTE.—22 brick will make a cubic foot of wall.

23. How many feet of lumber in a stick 14 ft. long, 9 in. wide and 6 in. thick?

24. How much lumber will be required to build a fence 6 boards high around a square lot of 4 acres, if the boards are 6 in. wide and are to be nailed to posts 4 in. square and 8 ft. long, set 8 ft. apart? How many posts?

25. A bin 6 ft. by 9 ft. by 5 ft. will contain how many bushels of potatoes?

26. A builder has planned a house 27 ft. by 45 ft. How many feet of lumber will be required:

(*a*) For the sills, which are to be 8 in. by 3 in.?

(*b*) For the studs, which are to be 2 in. by 4 in. by 18 ft., and are to be placed at intervals of 18 in.?

(*c*) For the sleepers, or joists, of 2 stories, which are to be 2 in. by 10 in., and are to be laid 18 in. apart?

(*d*) For the plates, which are to be 4 in. by 4 in.?

(*e*) For the rafters, which are to be 3 in. by 5 in. and are to be $1\frac{1}{2}$ ft. apart? The roof is to project 2 ft. and the peak is to be 10 ft. above the plates.

(*f*) For the 2 floors, if the flooring is to be $1\frac{1}{4}$ in. thick?

(*g*) For the siding for the outside?

(*h*) How many square feet of roofing will there be?

(*i*) How many square yards of plastering will there be on the walls and ceiling of both stories, the lower rooms being 9 ft. high, and the upper ones 7 ft. high?

(*j*) If the cornice is to be 2 ft. wide, what will it cost at $80 per M?

27. A cellar 22 ft. by 34 ft. and 9 ft. deep (inside measurements) is to be built.

(*a*) If the wall is to be 20 in. thick, how many cubic feet of masonry will it contain?

(*b*) How many cords of stone will be required?

(*c*) How many loads of earth must be removed in digging the cellar?

(*d*) How many square yards of plastering will there be on the walls of the cellar?

(*e*) How many square feet of cement will there be on the cellar bottom?

PERCENTAGE

235. OPERATIONS WITH HUNDREDTHS

PROBLEMS

1. Find .08 of 250; .25 of 730; .60 of 840.

2. Find 3 hundredths of $600; 12 hundredths of $180.

3. A merchant gained .05 of $1260. How much did he gain?

4. A dealer lost .06 of $790. How much did he lose?

5. 18 is how many hundredths of 24?

6. 9 is how many hundredths of 36?

7. .05 of a number is 60. Find the number.

8. A man began business with $1200 and gained .20 on his investment. How much had he then?

9. .15 of a number is 75. What is the number?

10. 12 is how many hundredths of 48?

11. 17 is how many hundredths of 136?

12. Find .22 of 184.

13. 10 is .02 of a number. What is .01 of the number? What is the number?

14. 25 is .04 of a number. What is the number?

15. 18 is .35 of what number?

16. What is .45 of 560?

17. A merchant gained $42 on $350. How many hundredths of his investment did he gain?

18. A dealer lost .08 of his investment. How many hundredths of his investment had he left?

19. A dealer gained .12 of his capital. How many hundredths of his original capital had he then?

20. $7.50 is how many hundredths of $25?

21. $46.30 is how many hundredths of $578.75?

22. What is .55 of 1125?

23. A man lost .14 of his money. How many hundredths had he left?

24. A man borrowed $850 and paid .06 of that amount each year for the use of the money. How much did he pay each year?

25. Find 43 hundredths of 700.

APPLICATIONS OF PERCENTAGE

236. Per cent means *hundredths.*

237. In many business operations we say per cent instead of hundredths. Read the examples on the preceding page, and read *per cent* instead of *hundredths.* The meaning will be the same.

238. Percentage means a process involving hundredths, or per cent. The examples on the preceding page are examples in percentage.

239. Operations in percentage involve certain business terms, a certain knowledge of the way in which business is done, and decimals.

How Per Cent Is Expressed

240. Since per cent means hundredths, it is commonly expressed as a decimal. It may also be expressed as a common fraction. Thus, $.05 = \frac{5}{100} = \frac{1}{20}$.

241. The sign of per cent is %.

242. Expression of hundredths:

$.01 = 1\% = \frac{1}{100}$. $.05 = 5\% = \frac{5}{100} = \frac{1}{20}$.

$.50 = 50\% = \frac{50}{100} = \frac{1}{2}$. $.10 = 10\% = \frac{10}{100} = \frac{1}{10}$.

$.75 = 75\% = \frac{75}{100} = \frac{3}{4}$. $.20 = 20\% = \frac{20}{100} = \frac{1}{5}$.

$.25 = 25\% = \frac{25}{100} = \frac{1}{4}$. $1.25 = 125\% = 1\frac{25}{100} = 1\frac{1}{4}$.

$.04 = 4\% = \frac{4}{100} = \frac{1}{25}$. $1.50 = 150\% = 1\frac{50}{100} = 1\frac{1}{2}$.

243. Hundredths as aliquot parts:

$12\frac{1}{2}\% = \frac{1}{8}$.	$33\frac{1}{3}\% = \frac{1}{3}$.	$66\frac{2}{3}\% = \frac{2}{3}$.
$16\frac{2}{3}\% = \frac{1}{6}$.	$37\frac{1}{2}\% = \frac{3}{8}$.	$75\% = \frac{3}{4}$.
$20\% = \frac{1}{5}$.	$50\% = \frac{1}{2}$.	$87\frac{1}{2}\% = \frac{7}{8}$.
$25\% = \frac{1}{4}$.	$62\frac{1}{2}\% = \frac{5}{8}$.	$125\% = 1\frac{1}{4}$.

NOTE.—When more convenient, these fractional parts should be used instead of the decimal equivalents.

244. Some expressions:

$\frac{1}{3}\%$ is read one-third of one per cent, or one-third per cent. It is written decimally $.00\frac{1}{3}$, not $.\frac{1}{3}$.

$8\% = .08$; $.8\% = .008$; $.08\% = .0008$; $.008\% = .00008$.

The sign % indicates two decimal places.

Adding the sign % is equivalent to pointing off two decimal places, or dividing by 100.

EXERCISES

Express decimally:

1. 18%.	*3.* $6\frac{1}{2}\%$.	*5.* $66\frac{1}{2}\%$.	*7.* $\frac{3}{4}\%$.	*9.* $7\frac{1}{2}\%$.
2. 93%.	*4.* $7\frac{1}{4}\%$.	*6.* $25\frac{3}{5}\%$.	*8.* $\frac{5}{8}\%$.	*10.* .024%.

Express as common fractions:

1. $33\frac{1}{3}\%$.	*3.* $87\frac{1}{2}\%$.	*5.* $16\frac{2}{3}\%$.	*7.* $\frac{1}{3}\%$.	*9.* $\frac{2}{3}\%$.
2. $83\frac{1}{3}\%$.	*4.* 125%.	*6.* $\frac{5}{8}\%$.	*8.* $\frac{3}{4}\%$.	*10.* $\frac{3}{5}\%$.

Express as a per cent:

1. $\frac{1}{2}$.	*3.* $\frac{1}{3}$.	*5.* $\frac{1}{8}$.	*7.* $\frac{5}{8}$.	*9.* $\frac{5}{12}$.
2. $\frac{1}{4}$.	*4.* $\frac{1}{6}$.	*6.* $\frac{3}{5}$.	*8.* $\frac{2}{3}$.	*10.* $\frac{7}{16}$.

245. An example in percentage involves:

(*a*) A decimal that indicates a part of a number.

(*b*) The part of the number indicated by the decimal.

(*c*) The number of which a part is indicated.

The decimal = rate per cent, or rate
The part = percentage
The number = base

246. The decimal that shows how many hundredths of a number are taken is the Rate Per Cent.

247. The number of which the hundredths are taken is the Base.

248. The number that is a given number of hundredths of the base is the Percentage.

249. When the percentage is added to the base, the sum is called the Amount.

250. When the percentage is subtracted from the base, the remainder is called the Difference.

251. To Find the Percentage

MENTAL PROBLEMS

1. Find $\frac{1}{4}$ of 36; 25% of 36.
2. What is $\frac{1}{10}$ of 750? 10% of 750?
3. What is .02 of 800? 2% of 800?
4. What is $33\frac{1}{3}$% of 72? of 48? of 60?
5. Find $16\frac{2}{3}$% of 96.
6. Find 25% of 28; of 44; of 64.
7. Find $62\frac{1}{2}$% of 72; of 144.
8. Find $66\frac{2}{3}$% of 15; of 24.
9. Find 40% of 250.
10. Find $12\frac{1}{2}$% of 40 yd.

FORMULA

Base × rate = percentage

PROBLEMS

Find the percentage, the amount, and the difference:

1. 24% of 360.
2. 42% of $700.
3. $65\frac{1}{4}$% of $840.
4. 83% of $762.50.
5. $29\frac{1}{2}$% of $538.20.
6. 25% of $684.40.
7. 75% of $932.60.
8. $33\frac{1}{3}$% of $638.60.
9. $87\frac{1}{2}$% of $9734.
10. $66\frac{2}{3}$% of $1248.
11. $12\frac{1}{2}$% of 840 cows.
12. $37\frac{1}{2}$% of 944 horses.
13. $16\frac{2}{3}$% of $2563.92.
14. 84% of 4672 lb.
15. $112\frac{1}{2}$% of $354.62
16. $108\frac{1}{3}$% of 6457.
17. 125% of 6820.
18. $35\frac{5}{8}$% of $921.30.
19. 53% of $749.80.
20. 28% of 4648.

21. A drover bought 780 sheep, and sold 30% of them. How many had he left?

22. A dealer bought 34% of a stack of hay containing 18 T. 14 cwt. How much hay did he buy?

23. $23\frac{1}{2}$% of a barrel of oil leaked out. If the barrel held 48 gal., how much was lost?

24. How much lead will be obtained from 580 tons of ore, if the ore yields 15% of metal?

25. If cloth will shrink $4\frac{3}{11}\%$, what will be the shrinkage on 128 yd.?

26. If .27% of coloring is to be mixed with white paint, how much coloring must be used with 3 T. 7 cwt. 80 lb. of paint?

27. In making a certain medicine, .04% of arsenic is used. How much arsenic will be used in making 420 lb. of the medicine?

252. To Find the Rate

MENTAL PROBLEMS

1. 8 is what part of 100? 20 is what part of 100?
2. Of 100, 15 is what part? 35? 17? 82?
3. 4 is what part of 5? 7 of 9? 8 of 15?
4. 4 is how many hundredths of 5? 3 of 4? 4 of 16?
5. 4 is what per cent of 5? of 8? of 16? of 32?
6. 8 is what per cent of 16? of 24? of 12?
7. 6 is what per cent of 12? of 24? of 18?
8. 9 is what per cent of 18? of 27? of 36?
9. 12 is what per cent of 24? of 18? of 36? of 60?
10. What per cent of 44 is 11? 22? 33?
11. What per cent of 60 is 15? 12? 20? 30? 45?
12. What per cent of $\frac{4}{7}$ is $\frac{2}{7}$? $\frac{1}{7}$? $\frac{3}{7}$? $\frac{5}{7}$?

FORMULA

$$P \text{ (percentage)} = R \text{ (rate)} \times B \text{ (base)}$$
$$P \div B = R$$

PROBLEMS

1. What per cent of 270 is 90?
2. What per cent of 38 is 27?
3. 45 is what per cent of 60?
4. Of 165, 15 is what per cent?
5. What per cent of \$860 is \$520?
6. \$750 is what per cent of \$500?
7. 140 sheep are what per cent of 560 sheep?
8. What per cent of \$900 is \$1200?

9. 3465 is what per cent of 4587?

10. What per cent of 860000 is 580?

11. What per cent of 720 is 24000?

12. 480000 is what per cent of 12000000?

13. $\frac{5}{11}$ is what per cent of $\frac{3}{16}$?

14. What per cent of .86 is .32?

15. What per cent of 6.25 is 98.75?

16. The United States silver dollar weighs 26.729 gr., and the Japanese dollar weighs 26.9564 gr. The weight of the American dollar is what per cent of the weight of the Japanese dollar?

17. A bank has $257326 cash on hand, and its deposits are $10658730. The bank report will state that the cash on hand is what per cent of the deposits?

18. A bank has received $14873580, and can repay only $3672500. What will a man receive who has deposited $1260?

19. The following table shows the population and number of deaths for a period of time in three cities:

Population	Number of Deaths
2768000	138
1875000	116
2162000	154

Find the death rate (per cent) for each city.

20. Three substances are mixed in the proportion of 5, 21, and 38. Each substance is what per cent of the mixture? How many pounds of the first substance in 85 lb. of the mixture?

21. In a school of 55 pupils, there were 16 absences in 10 days. What was the rate per cent of attendance?

22. If out of 76000000 passengers 420 are killed, what per cent of the passengers are killed?

23. If a bushel of apples weighs 50 lb., and 10 bu. of apples will make 32 gal. of cider weighing $8\frac{1}{3}$ lb. per gallon, what per cent of the apples becomes cider?

24. From a substance weighing 8 lb. 4 oz. (Troy), 7 dr. of a mineral were obtained. What per cent of mineral did the substance yield?

253. To Find the Base

MENTAL PROBLEMS

1. 25 is $\frac{1}{6}$ of what number?
2. 25 is 20% of what number?
3. 30 is $33\frac{1}{3}$% of what number?
4. 16 is 8% of what number?
5. 12 is 40% of what number?
6. Of what number is 12 60%?
7. 15 is 75% of what number?
8. Of what number is 14 7%?
9. Of what number is 20 40%?
10. Of what number is 25 $37\frac{1}{2}$%?
11. Find the number of which 20 is 20%.
12. Find the number of which 40 is 5%.
13. Find the number of which 60 is 12%.
14. Of what number is 18 6%?
15. Of what number is 27 $33\frac{1}{3}$%?
16. 100 is $12\frac{1}{2}$% of what number?
17. 32 is $62\frac{1}{2}$% of what number?
18. 72 is 9% of what number?

FORMULA

$$P \div R = B$$

PROBLEMS

1. Find the number of which 10608 is 17%.
2. \$43.40 is 32% of what sum?
3. 472 is $33\frac{1}{3}$% of what number?
4. $\frac{3}{4}$ is 40% of what number?
5. Of what number is 675 $16\frac{2}{3}$%?
6. Of what number is 924 .056%?
7. Of what number is 87 $\frac{7}{8}$%?
8. Find the number of which 3.045 is $24\frac{1}{8}$%.
9. .0000315 is $.08\frac{1}{3}$% of what number?
10. Find the number of which 530100 is .57%.
11. A business pays a net gain of \$2865 per annum. A buyer offers for the business a sum of which the net gain shall be 8%. What is the value of his offer?

12. A company with a capital of $685750 pays annual dividends to the amount of $44573.75. The next year the dividends are $76242.30, and the company wish to report a capitalized value of their business that shall yield the same per cent of dividends as did the capital the year before. What will be the reported capital of the company the second year?

13. A mine pays $623750 annually. If invested money pays on an average 4½% of itself as profits, what is the cash value of the mine?

14. A certain business pays 5% dividends. How much must be invested in the business to produce an annual income of $1375?

15. If wine contains 11⅓% of alcohol, how much wine must be purchased by a distiller who wishes to make 586 gal. of alcohol from the wine?

16. If a certain paint contains .7% of zinc, how much paint can be made from 200 lb. of zinc?

17. A solution contains .003% of arsenic. How many grains of the solution must be taken to get 12 gr. of arsenic?

18. If an agent receives 3% commission for selling goods, how many goods must he sell to earn $275?

19. A manufacturer wishes to make a composition metal to consist of 38% of gold, 5% of silver, 4% of tin, and the remainder copper. If he has 120 lb. (Troy) of gold, how much of each of the other metals must he add to it?

20. A custom miller takes 8% of the grist for grinding. How many pounds of oats must be taken to the mill to get 300 lb. of feed?

254. To Find the Amount and Base

MENTAL PROBLEMS

1. 10 is what per cent of 10?

2. Any number is what per cent of itself?

3. The base is always what per cent of itself?

4. The percentage is what of the base?

5. What is the base plus the percentage called?

6. What is 100% plus the rate per cent called?

7. If the rate is 8%, what is the amount per cent?

8. If the base is 100 and the rate 8%, what is the amount?

9. The amount in the last question is what per cent of the base?

10. If the amount per cent is 110%, what per cent is the base?

11. If the amount is 110 and the amount per cent is 110%, what is the base?

12. If the amount per cent is 115% and the base is 300, what is the amount?

13. If the amount per cent is 115% and the amount is 345, what is the base?

14. If the amount per cent is 107% and the base is 400, what is the amount?

15. If the amount per cent is 107% and the amount is 428, what is the base?

16. If the rate per cent is 5% and the base is 600, what is the amount?

17. If the rate per cent is 5% and the amount is 630, what is the base?

FORMULA

B + percentage = amount
100% + rate per cent = amount per cent
Base × amount per cent = amount
Amount ÷ amount per cent = base

PROBLEMS

Find the base:

1. When the amount per cent is 112½ and the amount is 1000.

2. When the amount is 1680 and the amount per cent is 105.

3. When the amount is $29.38 and the amount per cent is 113.

4. When the amount per cent is 103 and the amount is 460.

5. When the amount per cent is 102½ and the amount is 65.83.

6. When the amount is \$1268 and the rate per cent is 4.

7. When the rate is $12\frac{1}{2}\%$ and the amount is \$1475.65.

8. When the amount is 763 and the amount per cent is 100.7.

9. When the amount is \$528.25 and the rate is $12\frac{1}{2}\%$.

10. When the rate is $.06\frac{1}{3}\%$ and the amount is \$1194.50.

11. What number increased by 25% of itself is 855?

12. If it costs 3% of the price of iron to buy it, how much iron can be bought for \$1648?

13. If a jeweler adds to gold an alloy equal to 21% of the weight of the gold, how much pure gold must be used in making 31 lb. (Troy) of the alloyed gold?

14. If metal bars will expand $\frac{5}{8}\%$ when heated, how long should they be made if, after expanding, they should be 18 ft. long?

15. If a business pays 8% per annum, how much must a man invest in the business so that at the end of the year his interest in the business shall be worth \$10000?

255. To Find the Difference and Base

MENTAL PROBLEMS

1. What is the base less the percentage called?

NOTE.—100% less the rate per cent is called the Difference Per Cent.

2. If the rate is 8%, what is the difference per cent?

3. If the base is 100 and the rate is 8%, what is the difference?

4. In the above example, the difference is what per cent of the base?

5. If the difference per cent is 90% and the base is 100, what is the difference?

6. If the difference is 90, and the difference per cent is 90%, what is the base?

7. If the difference per cent is 85% and the base is 300, what is the difference?

8. If the difference per cent is 85% and the difference is 255, what is the base?

9. If the difference per cent is 95% and the base is 400, what is the difference?

10. If the difference per cent is 95% and the difference is 380, what is the base?

11. If the rate is 6% and the base is 600, what is the difference?

12. If the rate is 6% and the difference is 564, what is the base?

FORMULAE

100% − rate per cent = difference per cent
Base − percentage = difference
Base × difference per cent = difference
Difference ÷ difference per cent = base

PROBLEMS

Find the base:

1. When the difference per cent is 92 and the difference is 1656.

2. When the difference is $245 and the difference per cent is 86.

3. When the difference is 750 and the rate is 15%.

4. When the rate is 12½ per cent and the difference is $435.

5. When the difference per cent is 79 and the difference is $692.40.

6. When the difference is 804 and the difference per cent is 67.

7. When the rate is 6¼% and the difference is $379.25.

8. When the difference is $119.36 and the difference per cent is 97.25.

9. When the rate is .0036 and the difference is $274.86.

10. What number diminished by 13% of itself equals 1827?

11. If the waste in melting a metal and making it into articles is 2⅓%, how much metal must be used to make 1865 lb. of the articles?

12. If cloth shrinks 3% in length and a man wishes to use a piece 136 yd. long after shrinking, how much cloth must he buy?

13. How many pounds of alloyed gold must be taken to obtain 86 lb. of pure gold if it contains $22\frac{1}{2}\%$ of alloy?

14. A customer ordered 380 of a certain article, and remitted money enough to pay for them, with instructions to send as many articles as the money would buy in case there should be any discount on the goods. If a discount of 5% is allowed, how many articles should be sent him?

15. If it costs 3% of the price of goods to sell them, how many goods must be sold to net the dealer $1000?

PROFIT AND LOSS

QUESTIONS

256. *1.* Are goods ever sold at a loss to the dealer?

2. Show how each of the following may cause a dealer to sell below cost: fire, flood, competition, out-of-style goods, lack of storage, desire to advertise, supply of perishable goods.

NOTE.—The rate of gain or loss is usually computed as a per cent.

3. Comparing the terms of percentage with those of profit and loss, to what does the cost (to the dealer) compare? The whole gain or loss corresponds to what term in percentage? The per cent of profit or loss? The selling price if at a gain? The selling price if at a loss?

MENTAL PROBLEMS

1. Sold a hat that cost $2 at a gain of 20%. What was the gain? What was the selling price?

2. Sold a cow that cost $24 at a gain of $12\frac{1}{2}\%$. What was the gain? The selling price?

3. Sold an article that cost $30 at a loss of $33\frac{1}{3}\%$. What was the loss? The selling price?

4. Sold an article that cost $50 at a gain of 15%. What was the gain? The selling price?

5. Bought a book for \$2 and sold it for \$3. How much was gained? What per cent of the purchase price was gained?

6. Bought a watch for \$12 and sold it for \$16. How much was gained? What part of the purchase price was gained? What was the gain per cent?

7. Gain or loss is reckoned at a certain per cent of what sum?

8. Bought a horse for \$50 and sold it for \$40. How much was lost? What part of the cost was lost? What was the loss per cent?

9. If an article costs \$2.40, for how much must it be sold to gain 25%?

10. A man bought a wagon for \$45 and paid \$15 for repairs. If he then sold the wagon for \$70, what was his gain per cent?

11. If you buy a book for \$2.60 and know that the dealer made a profit of 30%, how much did the book cost the dealer?

12. At what price must an article be bought so that it may be sold for \$2.80 and make a profit of 40%?

13. A man wishes to sell shoes for \$2.50 and make a profit of 25%. What is the most he can afford to pay for the shoes?

14. A shoe dealer who sells at a profit of 20% made \$1200. What did he pay for the goods sold? What did he sell them for?

15. If you know that a dealer makes \$60 on the sale of a piano, and that he sells at a profit of 15%, you know that the piano cost him how much?

16. If an article is marked \$4.80 and you know that the dealer makes a profit of 20%, you know that the article cost him how much?

17. If an article cost \$1.50, how must it be marked to insure a gain of 30%?

18. A merchant buys eggs at 12¢ per dozen and sells them at 14¢ per dozen. What per cent does he gain?

19. Bought an article for $\frac{1}{2}$ its list price, and sold it for $\frac{3}{4}$ its list price. What per cent was gained?

20. Bought an article for \$24 and sold it for \$20. What was the per cent of loss?

21. If an article is bought for \$8 and sold at a loss of 12%, what is the selling price?

22. If an article is bought 20% below cost and sold 20% above cost, what is the gain per cent on the investment?

TABLE FOR MENTAL DRILL

	Gain %	Gain	Loss %	Loss	Selling Price	Purchase Price
1	10	\$20			?	?
2	?	\$15			\$75	?
3			12½	?	?	\$200
4			5	\$25	?	?
5			?	?	\$18	\$ 20
6	?	?			\$20	\$ 18
7	8	?			?	\$ 1.20
8			10	?	\$90	?
9	?	\$12			\$92	?
10	14	?			\$ 5.70	?
11	15	\$ 4.50			?	?
12	20	?			?	\$ 6.50
13			7	?	?	\$ 14
14	33⅓	\$25			?	?
15			6	?	?	\$ 1.20
16	?	\$24			\$96	?
17	12	?			?	\$ 15
18			?	\$12	?	\$ 42
19			10	\$22	?	?
20	?	\$ 6			\$48	?
21	16	?			\$23.20	?
22			15	?	\$17	?
23			?	\$ 3	?	\$ 18
24	37½	\$40			?	?

PROBLEMS

1. For how much must goods that cost \$875.38 be sold so as to gain 21½%?

2. A merchant bought dishes for \$1.15 per dozen. At what price must he sell them apiece to gain 20%?

3. A dealer bought wheat at 98¢ per bushel and sold it at \$1.10 per bushel. What per cent did he gain?

4. If a man buy 2400 brick for $8, for how much per M must he sell them to gain 15%?

5. On an investment of $2875 a man gained $213.50. What was his gain per cent?

6. Jan. 1, 1897, a man invested $3800. Jan. 1, 1900, his investment was worth $4425. What was his gain per cent per annum?

7. A furniture dealer sold furniture that cost him $1125, at 12½% below cost. What was his loss?

8. If a horse is bought for $85 and sold for $75, what is the loss per cent?

9. A man invested $1680 and afterward sold his investment for $1475. What was his loss per cent?

10. A grocer bought a barrel of sugar containing 420 lb. for $16. What will be his gain per cent if he sells it at 5¢ per pound?

11. A grocer bought 64 lb. of ginger for $12.80. For how much per pound must he sell it to gain 25%?

12. A produce dealer bought 250 bu. of wheat at 85¢ per bushel, 360 bu. of oats at 32¢ per bushel, 415 bu. of corn at 28¢ per bushel, and 16 bu. of beans at $1.80 per bushel. If he sold the wheat at 90¢ per bushel, the oats at 30¢ per bushel, the corn at 30¢ per bushel, and the beans at $1.95 per bushel, what was his entire gain per cent?

13. A real estate dealer bought a lot for $2600 and paid $85 for a fence, $30 for digging a drain, $160 for street repair, and $3250 for a house. For how much must he sell the house and lot to gain 15% on his investment?

14. A druggist bought alcohol at $2 per gallon and sold it at 35¢ per pint. What was his gain per cent?

15. If collars are bought for $1.10 per dozen and sold at 15¢ apiece, what is the gain per cent?

16. If plates must be sold at 20¢ apiece, for how much must they be purchased to insure a gain of 20%?

17. A fruit dealer bought 200 pineapples at 7¢ apiece. 13 of them spoiled, and he sold the remainder at 8¢ apiece. What was his gain per cent?

18. A dealer lost $\frac{1}{4}$ of a load of fruit. At what per cent of profit must the rest be sold that he may gain 20% on the whole load?

19. Bought a farm of 260 acres for $28 per acre, and paid $320 for fencing, $850 for repairs, $175 for draining, $82 for taxes. At what price per acre must it be sold to gain 20% on the investment?

20. If shoes are bought for $3 per pair, how must they be marked that the dealer may make a reduction of 10% from the marked price and gain 20% on the cost?

21. What per cent is gained on goods marked 30% above cost and sold at 20% off from the marked price?

22. What is the gain or loss per cent on goods marked 25% above cost and sold at 25% off from the marked price?

23. It costs a publishing house $1.64 to produce a book. If the house wishes to make a profit of 25% on the first cost of the book, and allows the agents 40% of the sales of the book, at what price must the house have the agents sell the book?

24. At what price per dozen must caps be bought that the dealer may sell them at 30¢ apiece and gain 20%?

MENTAL EXERCISES

Tell how:

1. To find the gain when the cost and per cent of gain are given.

2. To find the gain per cent when the cost and selling price are given.

3. To find the gain per cent when the gain and selling price are given.

4. To find the gain per cent when the gain and cost are given.

5. To find the loss when the cost and per cent of loss are given.

6. To find the loss per cent when the cost and selling price are given.

7. To find the loss per cent when the loss and cost are given.

8. To find the loss per cent when the loss and selling price are given.

9. To find the selling price when the gain and per cent of gain are given.

10. To find the selling price, the loss and loss per cent being given.

11. To find the cost, the gain and gain per cent being given.

12. To find the cost, the loss and loss per cent being given.

13. To find the cost, the selling price and gain per cent being given.

14. To find the cost, the selling price and loss per cent being given.

COMMISSION AND BROKERAGE

257. A person who does business for another is an Agent.

The book agent sells books for the publisher. The clerk in a store sells merchandise for his employer. The traveling salesman takes orders for goods to be filled by his house. A commission merchant resides in some city and receives goods to be sold by him. A man in New York may receive farm produce from a farmer, sell the stuff at the market price, take out his charges, and remit the balance to the farmer. A broker buys and sells bonds, stock certificates, etc., for others.

Some agents receive a salary. Some receive a percentage on the things they buy or sell.

258. The pay that an agent receives is called Commission.

259. The pay a broker receives is called Brokerage.

260. The agent's commission is computed on what he buys or sells.

261. The person for whom the agent transacts business is the Principal. A company, firm, or corporation is considered in business as a person.

262. Goods sent by a principal to an agent is a Consignment.

263. The principal is the Consignor, the agent is the Consignee.

264. For taking the risk of loss from sales on credit or for pledging the quality of the goods bought, the agent makes a charge called Guaranty.

265. The charges that an agent may make to his principal are commission, guaranty, freight, storage, insurance, inspection, etc.

266. The whole amount received by an agent from a sale or a collection is the Gross Proceeds.

267. The sum that remains after the agent has deducted all his charges is the Net Proceeds.

268. Sometimes a principal sends to his buying agent a sum of money (remittance) that includes the agent's commission *and the sum to be invested.* The agent is to invest as much as he can and still have enough left for his commission.

269. The sum actually invested in goods is the Prime Cost, or Net Cost. The net cost plus all charges of the agent is the Gross Cost.

270. Comparing the terms of commission and brokerage with the terms of percentage:

The gross proceeds = ?

The net cost = ?

The rate of commission = ?

The commission = ?

Purchase price + commission, or remittance to agent for investment = ?

Selling price − commission = ?

271. To Compute Commission and Brokerage

MENTAL PROBLEMS

1. An agent receives 40% commission on his sales. If he sells and delivers $200 worth of books, how much does he keep for his commission?

2. If an agent sold $2500 worth of goods on 10% commission, how much commission did he receive? How much did he send to his principal?

3. An agent receives 12½% commission on all the goods he sells. How many dollars' worth of goods must he sell to earn $100?

4. If a buying agent receives 5% commission for buying cotton, how much money must his principal send him to pay for $1 worth of cotton and the commission? If the principal sends $5.25 to pay for cotton and the agent's commission, how much cotton will the agent buy? If the agent receives a remittance of $1050, how many dollars' worth of cotton will he send his principal?

5. If an agent receives 3% commission for buying wool, what does $1 worth of wool cost the principal? How much must the principal send the agent to pay in full for $100 worth of wool? If the principal remits $206, how much wool will he receive?

6. If an agent's commission is 2% on all goods bought, how much will $2 worth of goods cost the principal? This cost is what per cent of the purchase price of the goods?

7. A farmer sent a commission merchant 500 bu. of wheat to be sold on a commission of 2%. If the wheat is sold at 80¢ per bushel, how much should the farmer receive?

8. If an agent charges 2% commission for buying silk, how much money must the principal send to pay in full for $500 worth of silk?

9. If a collector receives 1% for collecting, how much must he collect to earn $200 per month? The principal receives what per cent of the sum collected?

10. If an agent's commission for buying flax is 4%, what per cent of the sum to be invested in flax must the principal send the agent?

11. How much does an agent earn who sells $3000 worth of goods on a commission of 7%? What sum does the principal receive? What per cent of the sales does he receive?

12. What will $1100 worth of goods cost a principal, if he pays his agent 6% commission? What per cent of $1100 will the goods cost the principal?

13. How many dollars' worth of goods must an agent sell

to earn \$160, if his commission is 4%? If the principal wishes to receive \$192 as a result of the sale, how many dollars' worth of goods must he send the agent?

14. If the agent's commission is 2% for buying, the principal must send the agent what per cent of the value of the goods the principal wishes to receive? If the principal wishes to receive \$100 worth of goods, how much money must he send the agent? If the principal sends the agent \$500, how many dollars' worth of goods will he receive?

15. If the agent's commission for buying is 3%, how many dollars' worth of goods will \$412 secure the principal?

PROBLEMS

Find the commission, guaranty, and the gross cost:

1. Purchase price \$875.60, rate of commission $2\frac{1}{4}$%.

2. Prime cost \$954, rate of commission $3\frac{1}{6}$%.

3. 360 bu. wheat at 90¢, commission $2\frac{1}{2}$%, guaranty $\frac{1}{4}$%.

4. 684 bbl. apples at \$1.85, commission 3%, guaranty 1%.

5. Prime cost \$1287.50, commission $5\frac{1}{2}$%, guaranty $1\frac{1}{2}$%, insurance 1%, freight \$8.25.

6. 17685 ft. oak at \$28.50 per M, commission 7%, guaranty 1%.

7. 345 bbl. flour at \$5.80, commission 25¢ per barrel, guaranty 2%.

8. 72860 lb. hay at \$12 per ton, commission 3%, guaranty 2%, charges $37\frac{1}{2}$¢ per ton.

9. 127 yd. carpet at \$2.40, commission 3%.

Find the commission, guaranty, and net proceeds:

10. Gross proceeds \$867.25, commission $3\frac{1}{2}$%.

11. Sold 215 bu. corn at 45¢, commission $2\frac{1}{2}$%, guaranty $\frac{1}{4}$%.

12. 11680 lb. hay at \$11 per ton, commission 2%, guaranty $\frac{1}{2}$%.

13. 75600 ft. cherry at \$41 per M, commission 4%.

14. Gross proceeds \$1825.30, commission 5%, guaranty $1\frac{1}{2}$%.

15. 1625 books at \$3.75, commission 25%, charges 5¢ per volume.

16. Gross proceeds \$2368, commission 22%, guaranty 3%.

17. Gross proceeds \$736.80, commission 18%, guaranty 2½%, freight \$45, insurance \$6.50.

18. 65 T. coal at \$5.25, commission 5%, guaranty 1%.

Find the prime or net cost:

19. Gross cost \$2838.71, commission 7%.

20. Commission \$365.25, rate of commission 2½%.

21. Gross cost \$723, commission 6%, guaranty 1%, freight \$15.

22. Gross cost \$1235.40, commission 8%, guaranty 2%.

23. Commission \$117.30, rate of commission 4%.

24. Guaranty \$213.75, rate of guaranty 3%.

25. Gross cost \$6872, commission 12½%, guaranty 2½%.

26. Total charges \$157.25, commission 5%, guaranty 1%, freight \$12.15.

27. Gross cost \$1673.20, commission and guaranty 14%.

28. Commission \$275, rate of commission 2½%.

29. Guaranty \$218, rate of guaranty 3%.

30. Total charges \$753, commission 4%, freight \$27.

31. Commission \$513, rate of commission 9%.

Find the rate of commission:

32. Commission \$412, gross proceeds \$13800.

33. Commission \$357, net proceeds \$6783.

34. Guaranty \$47, gross proceeds \$1342.86.

35. Guaranty \$195, net proceeds \$2705.

36. An agent sold \$4682 worth of goods on 8% commission. He charged 1% for guaranty and \$27 for expenses. Find the commission, guaranty, and net proceeds.

37. An agent bought \$5387 worth of goods on 13% commission. He charged 2% for guaranty and \$125 for expenses. Find the gross cost.

38. A principal sent his agent \$2965 to invest in lace, taking out commission and expenses. If the commission was 6% and the expenses \$17, how much lace did the principal receive?

39. A real estate agent sold a farm for \$12825 on a commission of 3%, and the stock on the farm for \$2360 on a commission of 4%. Find the commission.

40. A principal sent his agent $1475, with instructions to invest it in oil. If his commission is 4%, how much oil will he purchase?

41. How many dollars' worth of merchandise can be purchased for $3691, if the agent's commission is 5%?

42. A commission merchant received a consignment of 831 bbl. of flour to sell on a commission of 4%. He sells the flour at $7.30 per barrel and pays $14.50 for cartage. How much should he remit his principal?

43. An agent received $2935 to invest in wool, taking out his commission of 3½%. How much commission did he receive?

44. An agent bought 28735 bu. of corn at 45¢ per bushel, on a commission of 1½%. If he paid $793.28 for freight, $112 for storage, and $41 for cartage, how much should he receive from his principal?

45. One agent charged $72 for selling a lot for $2400, and another agent charged 3½% commission for selling a lot valued at $2700. Which agent charged the higher rate of commission?

TRADE DISCOUNT

272. Manufacturers and dealers issue catalogues and circulars containing a description of their goods and a price list. The List Price is the price given in the catalogue or circular. Dealers often sell for less than the list price. What the dealer "throws off" is the Trade Discount. The list price may be put high enough so that the dealer can allow a discount and yet make a desired profit. A purchaser is pleased to buy goods at a discount.

Sometimes, after getting out a price list, the market price of goods falls, and instead of changing the price list the dealer may retain the old price list and offer a discount. If the price varies, the dealer may vary the discount. If the price falls three times in succession, the dealer may offer three different discounts.

When there are more trade discounts than one, they are called a discount series.

273. Trade discount is reckoned as a per cent of the list price. The list price less the discount is the Net Price. In a discount series, the first discount is reckoned on the list price, and each subsequent discount is reckoned on the net price—the price after deducting the preceding discount.

It makes no difference in what order the discounts of a series are considered. A series of 20%, 15%, and 10% is the same as 10%, 20%, and 15%. But 20%, 15%, and 15% is not the same as 20% and 30%.

274. Discounts are usually aliquot parts of 100, and operations should be performed with aliquot parts when practicable.

MENTAL PROBLEMS

1. Find the net price if the list price is \$12 and the trade discount is \$25%.

2. Find the net cost of 10 boxes of soap at \$4 per box, less 12½% discount.

3. If on a sale of \$60 worth of goods a discount of 15% is allowed, what is the amount of discount? What is the net cost?

4. A dealer sold goods listed at \$80, less 25%, 20%, 12½%. What was the net price after the first discount? After the second? After the third?

5. Find the net selling price of an article marked \$6, less a discount of 15%.

6. What is the cost of 25 yd. of cloth at \$2, less discounts of 20% and 10%?

7. What is the cost of goods listed at \$75 and sold at discounts of 20%, 25%, 33⅓%, and 50%?

8. What will a \$5 book cost if discounted at 15%?

9. Find the cost of goods listed at \$20, less 30% and 10%.

10. What is the cost of 20 cases of glass at \$27, less discounts of 33⅓% and 12½%?

11. Find the single discount equivalent to the discount series of 20%, 5%, and 25%.

SOLUTION.—The net price after the first discount is 80% of the list price. After the second discount the net price is 76% of the list price. After the third discount the net price is 57% of the list price. If the net price is 57% of the list price, the discounts must be equivalent to 100% — 57%, or 43%.

Find the single discount equivalent to the following series:

12. 10%, 10%.
13. 10%, 20%.
14. 15%, 20%.
15. 20%, 5%.
16. 5%, 10%, 20 %.
17. 10%, 10%, 10 %.
18. 20%, 20%, 20 %.
19. 20%, 10%, 12½%.
20. 6%, 10%.
21. 25%, 10%.
22. 50%, 10 %, 20 %.
23. 25%, 20 %, 12½%.
24. 20%, 5 %, 25 %.
25. 12%, 50 %.
26. 10%, 33⅓%, 20 %.
27. 10%, 16⅔%.
28. 24%, 25 %.
29. 35%, 20 %.
30. 13%, 33⅓%.
31. 12%, 25 %, 33⅓%.

32. What must I ask for an article so that I may allow a discount of 20% and still receive $40 for it?

NOTE.—The net price is what per cent of the asked price?

33. Cloth costing $4 per yard must be marked at what price to gain 25% and allow a discount of 20%?

34. At what price must I mark goods costing $18, that I may allow a discount of 25% and sell them at a gain of 33⅓%?

PROBLEMS

1. What is the net cost of a bill of goods amounting to $87, discounted at 30% and 18%?

NOTE.—Each discount may be taken out separately, or one equivalent discount may be used.

2. Find the net cost of a bill of goods invoiced at $825 and discounted at 25%, 20%, 10%.

3. Three houses list similar goods at $1260. One offers discounts of 25% and 25%; another offers 20%, 20%, and 10%; and the third offers 35% and 15%. What is the net price of each house?

4. I bought goods at 30% and 20% from the list price of $1475, and sold them at 20%, 10%, and 20% from the list price. How much did I gain?

5. A dealer offers bicycles at $100, subject to discounts of 40%, 20%, and 15%. How much will 1 doz. bicycles cost?

6. A book costs $5. What must be the marked price to gain 20%, and allow discounts of 20% and 25%?

REMARK.—The book must be sold for $5 plus 20% of $5, or $6. A discount of 20% and 25% is equivalent to a discount of 40%. If 40% discount is allowed, then $6 is 60% of the marked price.

7. Carpet costing $4 per yard must be marked at what price to gain 25% and allow discounts of 15% and 20%?

8. Goods listed at $265, and bought at discounts of 28% and $12\frac{1}{2}$%, must be marked at what price to gain $33\frac{1}{3}$% and allow discounts of 24% and $16\frac{2}{3}$%?

9. A dealer sells goods at a discount of 25% from the marked price and makes a profit of 25%. At what per cent above cost did he mark the goods?

10. Find the cost:

	LIST PRICE	DISCOUNT
(*a*)	$ 126.50	15 %, $12\frac{1}{2}$%
(*b*)	$ 872.60	$33\frac{1}{3}$%, 25 %
(*c*)	$ 150.25	30 %, 20 %, 15 %
(*d*)	$1285	40 %, $33\frac{1}{3}$%, $12\frac{1}{2}$%
(*e*)	$1673.80	15 %, 20 %, $16\frac{2}{3}$%
(*f*)	$4896.20	35 %, 22 %, 8 %
(*g*)	$5325.75	24 %, 16 %, 12 %
(*h*)	$ 436.18	$16\frac{2}{3}$%, $12\frac{1}{2}$%
(*i*)	$1120	30 %, 25 %, 15 %
(*j*)	$1736	45 %, $33\frac{1}{3}$%

11. At what per cent above cost must goods be marked to allow a discount of:

(*a*) 25 % and make 20 % on the cost?
(*b*) 10 % and make 25 % on the cost?
(*c*) 20 % and make 15 % on the cost?
(*d*) 10 % and make 8 % on the cost?
(*e*) 33⅓% and make 12½% on the cost?
(*f*) 40 % and make 25 % on the cost?
(*g*) 10 % and make 50 % on the cost?
(*h*) 25 % and lose 15% on the cost?
(*i*) 50 % and neither gain nor lose?
(*j*) 15 % and lose 10% on the cost?

12. Which is the better discount offer, 25%, 20%, and 15%, or 50%.

MARKING GOODS

275. Merchants often mark the cost and the price of their goods with symbols instead of figures. Sometimes the selling price is in figures and the original cost in symbols.

276. In marking by symbols, ten letters of the alphabet are used to correspond to the ten figures of the Arabic notation. A word or phrase containing ten letters, no two being alike, is taken as a "key." Some words and phrases used as keys are: blacksmith, handsomely, what prices, cash profit, black horse. The value of each letter of the key is determined by its position in the word or phrase. Thus:

b	l	a	c	k	s	m	i	t	h
1	2	3	4	5	6	7	8	9	0

For $1.25 write *blk*; for $2.68 write *lsi*; for $4.77 write *cmm*. Instead of repeating a letter, as in *cmm*, a letter not found in the key may be used as a repeater. Thus, $4.77 might be written *cmr*, using *r* to show that *m* is repeated.

The last two letters in the expression represent cents.

If a period is added, it shows that there are no cents. Thus, *blk.* means $125.

When the selling price is given in figures, and the cost mark is used, the selling price is written above the cost mark. Thus, $\frac{\$1.50}{blk}$ means that the selling price is \$1.50 and the cost is \$1.25.

When both cost and selling price are given, two keys may be used, one for the cost and the other for the selling price. Thus, using "what prices" for the selling price and "blacksmith" for the cost mark, $\frac{wps}{blk}$ means that the selling price is \$1.50 and the cost is \$1.25.

PROBLEMS

1. Write the following prices, using "handsomely" as the key: \$1.25, \$2.25, \$4.17, \$1.87, \$2.65, 75¢, 36¢, \$1.95, \$3.33, \$4.35.

2. Interpret with the same key: *hss*, *smy*, *ll*, *ahe*, *nos*, *das*, *eyy*, *may*, *hams*.

3. Mark all of the above prices 25% advance, using the same key.

4. With the same key for the cost price and "what prices" for the selling price, mark the following:

	Cost	Gain		Cost	Gain
(*a*)	\$1.50	25 %	(*f*)	\$2.00	15 %
(*b*)	.80	20 %	(*g*)	1.60	25 %
(*c*)	1.75	33⅓%	(*h*)	.90	16⅔%
(*d*)	2.40	12½%	(*i*)	1.20	75 %
(*e*)	1.25	33⅓%	(*j*)	1.40	30 %

MENTAL PROBLEMS

Use the key "blacksmith" and *y* as a repeater. Give the cost mark for the following:

1. \$1.45.	*4.* \$ .85.	*7.* \$7.50.	*10.* \$3.20.
2. 2.25.	*5.* 1.60.	*8.* 8.25.	*11.* 6.85.
3. 5.20.	*6.* .90.	*9.* 2.15.	*12.* 1.75.

STORAGE

277. Storage is the price or amount charged for storing goods in a warehouse.

Storage is usually computed by the weight, measure, or quantity of the goods stored. Sometimes it is computed as a per cent of the value of the goods.

278. The Term of Storage is the time for which the charge is made. Storage may be charged by the day, week, or month, and a fractional part of a term is counted as a full term.

279. *Average Storage.*—When the quantity of goods in storage varies on account of additions to or withdrawals from the stock, charge is made on each quantity for the actual number of days it is stored, and an average term of storage and an average quantity stored is ascertained.

EXAMPLE.—A man deposits 400 bu. of wheat, and after 20 days withdraws 100 bu.; the rest he withdraws in 15 days. What is the storage charge at 1¢ per bushel per month?

SOLUTION.—The storage of 400 bu. for 20 days is equivalent to the storage of 1 bu. for 8000 days, the storage of 300 bu. for 15 days is equivalent to the storage of 1 bu. for 4500 days, and the whole storage is equivalent to the storage of 1 bu. for 12500 days. The average time reduced to months is ($12500 \div 30 = 416\frac{2}{3}$) 417 months. The charge for 1 bu. for 417 months is $4.17; therefore the charge for 400 bu. for 20 days and 300 bu. for 15 days is $4.17.

280. Sometimes storage is paid on each withdrawal from the warehouse. This is *Cash Storage.* Sometimes storage is not paid until the last withdrawal is made. This is *Credit Storage.*

PROBLEMS

1. (*Simple Storage.*)—Find the storage on 4500 bu. of wheat for 3 mo. 20 da., at 1¢ per bushel per month; 1735 bu. of corn for 4 mo., at $\frac{3}{4}$¢ per bushel; 2863 bu. of oats for 4 mo. 15 da., at $\frac{1}{2}$¢ per bushel; 1160 bbl. of flour for 3 mo. 25 da., at 6¢ per barrel; 845 bu. of potatoes for 1 mo. 18 da., at 1¢.

NOTE.—Fractional terms count as whole terms.

2. (*Average Cash Storage.*)—A dealer deposits in a warehouse 460 bbl. apples Oct. 3; 632 bbl. pears Oct. 6; 535 bbl. potatoes Oct. 15; and 782 bbl. apples Oct. 25. He withdraws the whole on Feb. 1. What are the storage charges at 4¢ per barrel per month?

OPERATION

460 bbl. for 121 da. = 1 bbl. for 55660 da.
632 bbl. for ? da. = 1 bbl. for ? da.
535 bbl. for ? da. = 1 bbl. for ? da.
782 bbl. for ? da. = 1 bbl. for ? da.

Total storage = 1 bbl. for ? da. or ? mo.

3. Find the storage charges, at 1½¢ per bushel per month, on the following deposits: July 21, 8140 bu.; Aug. 1, 1670 bu.; Aug. 15, 1985 bu.; Sept. 1, 2430 bu. All was withdrawn Dec. 20.

4. (*Average Credit Storage.*)—Find the storage charges, at 3¢ per bbl. per month, on the following:

DEPOSITS	WITHDRAWALS
Sept. 1, 810 bbl.	Sept. 12, 260 bbl.
Sept. 15, 1460 bbl.	Sept. 25, 483 bbl.
Oct. 24, 735 bbl.	Dec. 1, the remainder

OPERATION

810 bbl. stored for 11 da. = 1 bbl. for ? da.
260 bbl. withdrawn

550 bbl. stored for 3 da. = 1 bbl. for ? da.
1460 bbl. deposited

2010 bbl. stored for 10 da. = 1 bbl. for ? da.
483 bbl. withdrawn

1527 bbl. stored for 29 da. = 1 bbl. for ? da.
735 bbl. deposited

2262 bbl. stored for 38 da. = 1 bbl. for ? da.

Total storage = 1 bbl. for ? da.

5. The deposits and withdrawals of goods at a warehouse were as follows:

Deposits	Withdrawals
June 1, 240 bales	June 18, 160 bales
June 28, 673 bales	July 3, 540 bales
July 10, 490 bales	Aug. 15, the remainder

Find the storage bill at 6¢ per bale per month.

6. Find the storage bill of the following, at 3¢ per case per month:

Deposits	Withdrawals
Dec. 1, 431 cases	Dec. 28, 975 cases
Dec. 23, 763 cases	Feb. 1, 648 cases
Jan. 18, 1145 cases	March 4, the remainder

7. Find the storage to Dec. 1, 1902, on 200 bbl. apples deposited Oct. 3, 1902, at 3¢ per bbl. per mo.; 350 bbl. beans deposited Oct. 1, 1902, at 3¢ per bbl. per mo.; 5800 bu. potatoes deposited Sept. 12, 1902, at 1¢ per bu. per mo.

8. Find the storage charges, at 2¢ per bbl. per mo., due Dec. 15, 1902, on the following deposits: Aug. 14, 1902, 450 bbl. vinegar; Aug. 30, 1902, 425 bbl. molasses; Sept. 12, 1902, 580 bbl. sugar; Sept. 18, 1902, 170 bbl. pork.

9. Find the storage of the following, at 2¢ per bu. per mo.:

Deposits	Withdrawals
June 12, 140 boxes	June 19, 60 boxes
June 27, 230 boxes	July 22, 210 boxes
July 16, 175 boxes	Aug. 26, the remainder

10. Find the storage of the following, at 1¢ per bu. per mo.:

Deposits	Withdrawals
Aug. 12, 250 bu.	Sept. 10, 200 bu.
Aug. 21, 380 bu.	Oct. 24, 150 bu.
Sept. 25, 570 bu.	Nov. 3, the remainder

INSURANCE

281. Certain property may be damaged, or destroyed by fire, water, wind, lightning, etc. Domestic animals die. Persons may be afflicted with accidents or sickness, and all must die. A sum guaranteed to be paid in case of any such misfortune is called Insurance.

282. The Beneficiary is the one to whom the insurance is guaranteed to be paid.

283. The Insurance Party (Insurance Company) is the party that agrees to pay the insurance.

284. The Policy is the written contract between the insured party and the insurance company.

285. The Premium is the amount the insured party agrees to pay the insurance company for the insurance.

286. The Term of Insurance is the time for which the policy is issued, and is usually one or more years.

287. Insurance Rates are given as a certain per cent, or as so much per $100 or $1000 of the amount of insurance, and depend upon the risk assumed and the term of insurance.

288. An Insurance Agent is one through whom the insurance company transacts business with the insured.

289. A company in which the profits and losses are shared by stockholders is a Stock Company.

290. A company in which the profits and losses are shared by the parties insured is a Mutual Insurance Company.

291. The two general kinds of insurance are Property Insurance and Personal Insurance.

PROPERTY INSURANCE

292. Property insurance includes:

Fire Insurance—which is an indemnity for loss by fire, water, wind, etc.

Marine Insurance—which is an indemnity for loss of vessel or cargo.

Live Stock Insurance—which is an indemnity for loss from death of domestic animals.

293. In an Ordinary Policy, the insurance company agrees to pay any damage that does not exceed the face of the policy. Thus, if a house worth \$10000 is insured for \$6000, and is damaged to the extent of \$5000, the company is bound to pay \$5000. If the house is entirely destroyed, the company will pay \$6000.

294. In an Average Clause Policy, the insurance company agrees to pay such a part of the loss as the face of the policy is part of the value of the thing insured. Thus, if a ship, valued at \$10000, is insured for \$6000, and is damaged to the extent of \$5000, the company will pay only $\$\frac{6000}{10000}$ or $\frac{3}{5}$ of \$5000. If the whole ship is destroyed, the company will pay $\frac{3}{5}$ of \$10000.

Marine insurance policies usually contain the "average clause."

MENTAL PROBLEMS

1. Find the cost of insuring a house for \$2000 if the rate of insurance is $2\frac{1}{2}\%$.

2. What is the premium on \$700 of insurance at 3%?

3. Find the cost of \$5800 insurance at \$2 per \$100.

4. I insured my house for \$2500 for 3 yr., at 2%. After 6 mo. the policy is cancelled. What sum should the company repay me?

5. At 2%, what amount of insurance can I obtain for \$28?

6. A ship worth \$12000 is insured for \$6000 in an average clause policy. If the vessel is damaged to the extent of \$2500, what sum must the company pay?

7. The amount insured is what element in percentage?

8. The premium paid is what element?

9. The loss sustained is what element?

10. If I pay \$30 for insuring property worth \$9000 and the property is destroyed, what is my net loss?

11. If I pay \$20 for insuring \$5000 worth of property, what is the rate per cent of premium?

12. What is the premium on an insurance of \$1600, for 3 yr., at $1\frac{1}{2}\%$ per annum?

PROBLEMS

1. What premium must be paid on an insurance of \$2375 at $1\frac{3}{4}\%$?

2. A steamer, worth \$260000, was insured for \$180000, at $2\frac{1}{4}\%$, in a policy with the average clause. If the boat was damaged to the extent of \$75000, what was the net loss to the owners?

3. If the Commercial Insurance Company insure property for \$578000, at $2\frac{1}{3}\%$, and reinsure $\frac{2}{3}$ of the risk in another company at $2\frac{3}{4}\%$, what would be the loss to the Commercial Company if the property becomes destroyed?

4. A dealer bought 1270 bbl. of flour at \$5.30 per barrel, and had it insured at $\frac{3}{4}\%$. If 80 bbl. are destroyed, at what price per barrel must he sell the remainder to make a net profit of 20% on the money invested?

5. What is the premium on a \$4860 insurance policy, at $37\frac{1}{2}¢$ per \$100?

6. Find the premium on an insurance for \$135725, at \$22.50 per \$1000.

7. A cargo worth \$40000 was insured for \$25000, in a policy with the average clause, at \$2.10 per \$100. In case of damage to the extent of \$18000, what would be the total loss to the owner?

8. A train load of flour of 1670 bbl., worth \$6.40 per bbl., is insured for two-thirds of its value, at $1\frac{1}{3}\%$. If the flour is destroyed, how much will the owner lose?

9. There is an insurance of \$85000 on a factory worth \$120000, \$25000 on machinery worth \$40000, and \$18000 on material worth \$24000. The building is entirely destroyed, the machinery is damaged to the extent of \$16000, and the material is damaged to the extent of \$1500. If the rate of insurance is $1\frac{3}{4}\%$, how much does the company lose? How much would the owner lose if the policy contained an average clause?

10. Which is the better rate of insurance, $2\frac{1}{3}\%$ or \$22.50 per \$1000?

11. At $2\frac{1}{4}\%$, in how many years would the premium paid equal the face of the policy?

12. A man bought a house for $2340, had it insured for $\frac{3}{4}$ its value, at 75¢ per $100, and sold it for $2560. What was his gain per cent?

13. If I take out two policies for $1900 and $2100 on a building, how should a damage of $1475 be divided between the two companies?

PERSONAL INSURANCE

295. Personal insurance includes:

Life Insurance—which is an indemnity for loss by death.

Accident Insurance—which is an indemnity for loss by accident.

Health Insurance—which is an indemnity for loss by sickness.

Under a health policy the insured receives a certain sum per week during the continuance of the disability.

Under an accident policy the insured receives a certain sum for the loss of some bodily organ—as an eye or hand—and a weekly indemnity for a temporary disability.

296. The two chief kinds of life insurance policies are the Life Option Policy and the Endowment Policy.

The policies of the different companies differ in some respects, but the policies of all first-class companies are very much alike, all being based on recognized business principles.

297. There are two kinds of life option policies—the ordinary life policy and the term payment life policy. Every policy is a contract. Under the ordinary policy the person insured agrees to pay the company certain premiums annually as long as he lives; and the insurance company agrees to pay, upon the death of the insured, a certain sum called the face of the policy, to the person mentioned in the policy as the beneficiary. If the insured dies after the first payment, the beneficiary gets the full face of the policy—usually one or more thousand dollars. If he pays premiums for fifty years, and then dies, the beneficiary gets the face of the policy, and under some policies he may also receive an additional payment, known as "accumulated dividends." Such a policy is called a "participating policy," because the policy holder participates in the division of

certain profits of the company. A policy under which the beneficiary receives only the face of the policy is called a "non-participating" policy. The premium on a participating policy is greater than on a non-participating policy. On a $1000 participating life option policy of the National Life Insurance Company, a person insured at the age of 20 years would pay an annual payment of $18.73. At his death his beneficiary would receive $1000 plus the accumulated dividends. On a non-participating policy the premium would be $15.57. If the person insured is 40 years old, he would have to pay on a non-participating policy for $1000 an annual premium of $26.75. Of course, a person 40 years old is likely to die twenty years sooner than one 20 years old, and if the same sum is to be paid at the death of each, the older ought to pay more per annum while he lives. If he were 60 years old the premium would be $62.97.

We may ask, how does the insurance company get the $1000 to pay in the case of a man who puts in $15.57 and then dies? How does the company make anything by that? A person at the age of 20 years takes a policy for $1000, pays in $15.57 for fifty years and dies. If he had put his payments in a bank, at 3% interest, they would have amounted, at his death, to $1808.92, and at 4% they would have amounted to $2472.05. The company pays only $1000. Those who insure at 20 years die at different ages, and, on the average, the insurance company can pay the policy and still make money.

As a business venture, does it pay to insure? If I die early, yes. If I live as long as the average person or longer, no. So the question, "Will it pay me to insure?" is simply, "How long will I live?" and no one can tell.

But there is another side. Suppose a young man has a wife, children, or parents dependent upon his labor for support. If he lives he can support them, but if he dies early their means of support is gone. If he has his life insured, he leaves something for their support. If he insures, he runs the risk of paying in more than he draws out. If he does not insure, he runs the risk of dying soon, and not leaving behind an adequate sum for the support of those dependent upon him.

Under a term life policy the insured pays 10, 15, or 20 annual premiums, and then his insurance matures, his policy becomes paid up, and he has no more premiums to pay. Of course, the rates for a term policy are higher than for a life policy. The annual premium on a twenty-payment non-participating life policy (one that becomes paid up after twenty annual premiums have been paid) is $25 for a person 20 years of age, and $35.82 for a person 40 years of age.

298. An Endowment Policy is one in which the face of the policy is payable at death or a certain number of years after date, even if the person insured is alive at that time. Thus, if a man 20 years of age takes a twenty-year endowment policy for $1000, and dies the next year, or any time within twenty years, his beneficiary receives $1000. If the man lives twenty years, the beneficiary receives $1000 at the end of that time, and the beneficiary may be the person insured. The premium on an endowment policy is higher than on a life policy. The premium on a twenty-year non-participating endowment policy for $1000 on the life of a person 20 years of age is $44.07 in the National Life Insurance Company.

If a person 20 years of age takes a straight life policy for $1000, he pays $15.57 per year till death. If he takes a twenty-year payment life policy, he pays $25 for twenty years only. If he takes a twenty-year endowment policy, he pays $44.07 for twenty years only. In any case, $1000 would be payable at his death. In the last case, $1000 would be payable at the end of twenty years, even if the person insured lives. If a person should die soon after he is insured, the straight life policy is the best, but if he is going to live forty or fifty years, the endowment policy is the best. So again the question is, "How long shall I live?"

299. *Cash Surrender Value.*—Usually, after three or more annual premiums have been paid, a policy may be surrendered to the company, which will pay the person insured a certain sum, the cash surrender value of the policy.

300. *Paid-up Insurance.*—After three or more annual premiums have been paid, the insured may surrender his policy

and receive a policy for a lesser amount, but he will not have to pay any more premiums.

301. *Extended Insurance.*—After making three or more annual payments, he may stop paying, keep his policy, and have his insurance extended for a certain number of years. Thus, if a man 40 years of age takes a twenty-payment life policy, makes ten annual payments and stops, he may be insured for 15 yr. 288 da. longer without the further payment of any premiums, but at the end of that time his policy expires.

302. The annual premium on the non-participating life option, twenty-payment life, and twenty-year endowment policies, for $1000, as given by one of the standard life insurance companies, is shown by the following:

TABLE OF RATES

Age	Premium		
	Life	20-Payment Life	20-Year Endowment
20	$15.57	$25.00	$44.07
21	15.91	25.37	44.11
22	16.26	25.76	44.16
23	16.63	26.16	44.21
24	17.01	26.57	44.26
25	17.42	27.00	44.32
26	17.84	27.44	44.39
27	18.29	27.90	44.46
28	18.75	28.38	44.53
29	19.24	28.87	44.62
30	19.76	29.38	44.71
31	20.30	29.91	44.82
32	20.88	30.47	44.93
33	21.48	31.04	45.06
34	22.11	31.64	45.20
35	22.78	32.26	45.36
36	23.49	32.91	45.53
37	24.24	33.59	45.73
38	25.03	34.30	45.96
39	25.87	35.04	46.21
40	26.75	35.82	46.48

PROBLEMS

1. A man at the age of 25 took a $3000 twenty-payment life policy. He died after paying 10 premiums. What was the annual premium? How much did his family receive at his death? How much would the premiums paid have amounted to at the time of his death, if they had been deposited in a bank at 3½% compound interest?

SOLUTION.—$27 × 3 = $81, premium. See table, page 175.

$12.14 × 27 = $327.78, amount of 10 premiums of $27 at 3½%. See table, page 194.

2. A man 36 years old takes out an endowment policy for $2000. What annual premium must he pay? If he lives 20 years and receives the $2000 insurance, how much less will he receive than he would have received if he had invested his premiums at 4% compound interest?

3. A man 21 years old takes a twenty-payment life policy for $2000, and dies at the age of 45. How much will his family receive from the insurance company? How much would they have received if he had deposited the premiums in a bank at 3½% compound interest?

4. 3 men, each aged 26, take out policies for $3000 each; one takes a life option policy, one a twenty-payment life policy, and the third a twenty-year endowment policy. They all die at the age of 37. Show what the premiums paid by each would have amounted to at 4% compound interest.

5. Suppose the men mentioned in the last example had died at the age of 57. Show what the payments in premiums of the first man would have amounted to at 4% compound interest, and what his family received. Show the same of the second man. Show the same of the third man, estimating the amount his family received by finding the amount of the $3000 endowment at 4% compound interest from the time it was paid, to the time of his death.

INTEREST

SIMPLE INTEREST

303. If a man hires a horse, he pays for the use of it. If he loans a sum of money, he asks pay for its use. By using money in business, a person may make money. Two men in partnership may run a store, one furnishing the labor and the other the money. If they divide the profits equally, the use of the money of the one is considered equivalent to the labor performed by the other. If A owes B $100 and does not pay it at once, A is using B's money and should pay for the use of it.

304. Strictly speaking, Interest is the use of money.

The common acceptation of the term interest is the money that is paid for the use of money.

305. The sum for whose use interest is paid is the Principal.

306. The sum of the principal and interest is the Amount. It is the total sum the debtor owes or will owe at the maturity of the debt.

307. The time during which the principal is used is the Term of Interest or the Time.

308. Interest is computed as a per cent of the principal.

309. The Rate of Interest is the rate per cent charged for the use of the principal. If the rate is 6% per annum, the interest for one year is 6% of the principal, or 6¢ on $1.

The rate of interest is per annum, unless otherwise specified.

310. The law of most States fixes a limit to the rate of interest that may be charged. This is called the maximum

rate. Any rate of interest that does not exceed that limit is allowed. It also fixes a rate to be allowed in case no rate is mentioned. This rate is called the Legal Rate. Usury is interest in excess of the maximum rate.

311. The interest on any principal for any given time is a certain per cent of the principal. This per cent is the rate of interest as affected by the term of interest. If the rate is 6% and the term is 1 yr., the interest will be 6% of the principal. If the time is 2 yr., the interest will be 12% of the principal. If the term is 4 mo., the interest will be $\frac{1}{3}$ of 6%, or 2% of the principal.

312. The chief problem in interest is to take the rate and term into consideration and find what per cent the interest is of the principal. Months are twelfths of a year. Thirty days are considered an interest month.

313. Interest is commonly computed on a basis of 360 days for a year. Interest on this basis is called Common Interest. 360 days is taken as a year for convenience in computing interest, and it is sufficiently exact for ordinary business.

314. Exact Interest is interest for the exact time in days considered as 365ths of a year.

315. Since common interest is based on a 360-day year, the common interest year is 5 da., or $\frac{1}{73}$ of a year shorter than the exact year, and the interest for a given number of days by the common method is $\frac{1}{73}$ greater than the interest by the exact method. Therefore, to find exact interest, find the common interest and subtract from it $\frac{1}{73}$ of itself.

NOTE.—Exact interest is taken by the United States government by some banks and in Canada.

316. There are several methods of computing interest. Following are given some of the methods that are among the best. The pupil may familiarize himself with all and select one or more for his own use.

Cancellation Method

317. EXAMPLE.—Find the interest on \$120 for 39 da. at 4%.

OPERATION

$$\frac{\$\cancel{120} \times .04 \times \cancel{39}^{13}}{\cancel{360}_{3}} = \$.52.$$

EXPLANATION.—39 da. = $\frac{39}{360}$ yr. The interest for 1 yr. is 4% of \$120, therefore the interest for $\frac{39}{360}$ yr. is $\frac{39}{360}$ of .04 of \$120. The product of the principal, rate, and time in days divided by 360 equals the interest. We have this formula:

$$\frac{\$? \times ?\% \times ? \text{ da.}}{360} = \text{Interest.}$$

NOTES.—1. The rate per cent should be expressed decimally.
2. Divide when you can, multiply when you must.

PROBLEMS

Find the common interest on:

1. \$840, for 3 mo. 12 da., at 4%.
2. \$126, for 93 da., at 7%.
3. \$278.50, for 115 da., at 6%.
4. \$396, for 4 mo., at 8%.
5. \$172.50, for 63 da., at 5%.
6. \$264.25, for 27 da., at 6%.
7. \$420, for 18 da., at 3%.
8. \$580.60, for 7 mo. 8 da., at 4%.
9. \$371, for 9 mo., at 5%.
10. \$218, for 1 yr. 4 mo., at 6%.
11. \$1130, for 3 yr. 7 mo., at 4%.
12. \$93, for 1 yr. 3 mo. 12 da., at 6%.
13. \$1640.20, for 84 da., at $4\frac{1}{2}$%.
14. \$793.80, for 1 yr. 16 da., at 9%.
15. \$784.30, for 7 mo. 19 da., at 6%.
16. \$2630, for 2 yr. 12 da., at 5%.
17. \$364, for 8 mo. 15 da., at 8%.
18. \$236.40, for 2 yr. 4 mo. 15 da., at 6%.

Find the exact interest on:

19. \$375, for 21 da., at 6%.
20. \$421, for 1 mo. 12 da., at 5%.

21. $1580.45, for 2 yr. 3 mo. 15 da., at 4%.
22. $436, for 7 mo. 15 da., at 6%.
23. $109.75, for 9 mo. 11 da., at 5%.
24. $255, for 4 mo. 12 da., at 3%.
25. $218.30, for 1 yr. 11 mo., at 9%.

Find the ordinary interest on:

26. $375.80, for 19 da., at 7%.
27. $511.25, for 2 yr. 6 mo. 9 da., at $4\frac{1}{2}$%.
28. $928, for 1 yr. 9 mo. 18 da., at 4%.
29. $1362.50, for 1 yr. 11 mo. 15 da., at 6%.
30. $169.25, for 10 mo. 21 da., at 5%.
31. $235.28, for 3 mo. 16 da., at $3\frac{1}{2}$%.
32. $417, for 1 yr. 7 mo., at 4%.
33. $95.60, for 2 yr. 3 mo., at 6%.
34. $125, for 17 da., at 6%.
35. $962.75, for 1 yr. 1 mo. 12 da., at 5%.
36. $65.20, for 6 mo. 22 da., at 6%.
37. $144.10, for 9 mo. 24 da., at 4%.
38. $216, for 1 yr. 4 mo., at $3\frac{1}{2}$%.
39. $96.50, for 3 yr. 5 mo. 8 da., at 5%.
40. $1275, for 4 yr. 10 mo. 20 da., at 6%.

1000-Day Method

318. Find, by the cancellation method, the interest on any sum for 1000 da., at 36%.

Principles.—1. For the interest on any principal for 1000 da. at 36%, take the principal.

2. For the interest on any principal for 1 da. at 36%, point off 3 places in the principal.

3. For the interest on any principal for any number of days at 36%, multiply the principal by the number of days and point off 3 places.

$$\frac{\$? \times 36\% \times \text{da.}}{360} = \text{Interest at } 36\%.$$

OR,

Cancel by dividing 360 by .36, and

$$\frac{\$? \times \text{da.}}{1000} = \text{Interest at } 36\%.$$

MENTAL PROBLEMS

Find the interest, at 36%, on:

1. \$500, for 22 da.
2. \$480, for 20 da.
3. \$75, for 2 mo.
4. \$130, for 18 da.
5. \$125, for 3 mo.
6. \$47, for 12 da.
7. \$70, for 106 da.
8. \$83, for 1 mo.
9. \$90, for 33 da.
10. \$1000, for 62 da.

11. What part of the interest at 36% is the interest at 6%? At 4%? At 3%? At 9%? At 12%? At 4½%? At 2%? At 5%? At 7%?

12. After finding the interest at 36%, how can you find the interest at 6%? At 4%? At 3%? At 9%? At 4½%? At 12%? At 5%?

13. Find the interest on \$300, for 80 da., at 6%.
14. Find the interest on \$200, for 75 da., at 9%.
15. Find the interest on \$120, for 18 da., at 4%.
16. Find the interest on \$72, for 40 da., at 8%.

PROBLEMS

Find the interest, by the 1000-day method, on:

1. \$162.40, for 81 da., at 4%.
2. \$216, for 17 da., at 6%.
3. \$328, for 19 da., at 4½%.
4. \$1114, for 41 da., at 12%.
5. \$265, for 72 da., at 9%.
6. \$439.80, for 69 da., at 3%.
7. \$276.25, for 140 da., at 5%.
8. \$792, for 63 da., at 7%.
9. \$168.25, for 90 da., at 2%.
10. \$93.80, for 2 mo., at 6%.
11. \$235, for 6 wk., at 9%.
12. \$742, for 80 da., at 5%.
13. \$315, for 4 mo., at 3%.
14. \$275, for 2 wk., at 7%.
15. \$316, for 33 da., at 6%.
16. \$529, for 25 da., at 5%.
17. \$236, for 40 da., at 4%.
18. \$427, for 35 da., at 8%.
19. \$638, for 21 da., at 6%.
20. \$192, for 15 da., at 5½%.

Find the amount of

21. \$1420, for 21 da., at 4%.
22. \$975, for 160 da., at 6%.
23. \$268, for 5 mo., at 9%.
24. \$315, for 6 wk., at 8%.
25. \$276, for 82 da., at 5%.
26. \$820, for 2 mo., at 7%.
27. \$416, for 93 da., at 3%.
28. \$617, for 33 da., at 2%.
29. \$283, for 45 da., at 4½%.
30. \$116, for 13 da., at 6%

31. \$728, for 1 yr. 4 mo. 6 da., at 6%.
32. \$520, for 2 yr. 7 mo. 15 da., at 4%.
33. \$1724, for 1 yr. 8 mo. 20 da., at $3\frac{1}{2}$%.
34. \$65, for 11 mo. 27 da., at 5%.
35. \$13850, for 1 yr. 6 mo. 9 da., at $4\frac{1}{2}$%.
36. \$768, for 2 yr. 5 mo. 24 da., at 4%.
37. \$195, for 1 yr. 1 mo. 5 da., at 6%.
38. \$252, for 9 mo. 6 da., at 4%.
39. \$332, for 1 yr. 2 mo., at 7%.
40. \$518, for 3 yr. 9 mo. 11 da., at 8%.

Banker's 60-Day Six Per Cent Method

319. The most common rate of interest is 6%.

A month is usually considered 30 da., and a year 360 da.

Most notes run for 90 da. or less.

320. At 6% per annum, the interest on any principal for 2 mo. (60 da., $\frac{1}{6}$ yr.) is 1% of the principal.

321. *Principle.*—To find 1% of any principal, *point off 2 places.* Therefore, to find the interest on any sum for 60 da. at 6%, *point off 2 places.*

MENTAL PROBLEMS

1. What is the interest, at 6%, for 60 da., on \$46? \$57.30? \$294? \$387? \$65.90? \$789? \$5076? \$54.90? \$3676? \$68.85? \$2167?

2. 30 is what part of 60? After finding the interest for 60 da., how can you find the interest for 30 da.?

3. What is the interest, for 30 da., at 6%, on \$240? \$560? \$862? \$45.80? \$5680? \$34.25? \$67.90? \$346? \$80? \$740?

4. 6 is what part of 60? After finding the interest for 60 da., how can you find the interest for 6 da.? How many places must you point off to find the interest on any sum, at 6%, for 6 da.?

5. What is the interest, for 6 da., at 6%, on \$580? \$720? \$342? \$560? \$96.80? \$5678? \$89.56? \$52167? \$9076? \$3467? \$43809?

6. After finding the interest for 6 da., how can you find the interest for 3 da.?

7. 20 is what part of 60?

322. EXAMPLE 1.—Find the interest on \$720, for 99 da., at 6%.

OPERATION

\$ 7	20 = int. for 60 da.
3	60 = int. for 30 da.
	72 = int. for 6 da.
	36 = int. for 3 da.
\$11	88 = int. for 99 da.

EXAMPLE 2.—Find the interest on \$480, for 85 da., at 6%.

OPERATION

\$4	80 = int. for 60 da.
1	60 = int. for 20 da.
	40 = int. for 5 da. ($\frac{1}{4}$ of 20)
\$6	80 = int. for 85 da.

NOTES.—1. To find the interest for 60 da., at 6%, cut off 2 places by a vertical line. For 4 mo., double this, for 6 mo., multiply it by 3, etc. For 30 da., take $\frac{1}{2}$ of it; for 20 da., take $\frac{1}{3}$ of it. For 40 da., double the interest for 20 da. For 5 da., take $\frac{1}{4}$ the interest for 20 da. For 6 da., cut off 3 figures. For 3 da., take $\frac{1}{6}$ of the interest for 6 da., for 2 da., take $\frac{1}{3}$, etc.

2. To find the interest, at 6%, for any time, find the interest for 60 da. Divide the given number of days into aliquot parts of 60 da.; and compute the interest on each number of days.

EXAMPLE 3.—Find the interest on \$980, for 5 mo. 27 da., at 6%.

OPERATION

\$ 9	80 = int. for 60 da. (2 mo.)
19	60 = int. for 4 mo.
4	90 = int. for 1 mo.
3	27 = int. for 20 da.
	98 = int. for 6 da.
	16 = int. for 1 da.
\$28	91 = int. for 5 mo. 27 da.

323. Following are given some groups of days with which the pupil should be familiar.

Aliquot parts of 60 days:

$30 = \frac{1}{2}$ $\quad 15 = \frac{1}{4}$ $\quad 10 = \frac{1}{6}$
$20 = \frac{1}{3}$ $\quad 12 = \frac{1}{5}$ $\quad 6 = \frac{1}{10}$

Aliquot parts of 6 days:

$3 = \frac{1}{2}$ $\quad 2 = \frac{1}{3}$

324. Any term of days may be divided into convenient aliquot parts, as shown by the following list, which should be carefully studied:

$5 = 3+2$	$21 = 6\times 3+3$	$35 = 30+3+2$
$7 = 6+1$	$24 = 6\times 4, 12\times 2$	$36 = 30+6$
$8 = 6+2$	$25 = 15+10$	$38 = 30+6+2$
$9 = 6+3$	$26 = 20+6$	$39 = 30+6+3$
$11 = 10+1$	$27 = 15+12$	$42 = 30+12$
$13 = 12+1$	$28 = 20+6+2$	$45 = 15\times 3$
$16 = 10+6$	$29 = 20+6+3$	$46 = 20\times 2+6$
$17 = 10+6+1$	$31 = 30+1$	$48 = 6\times 8, 30+15+3$
$18 = 6\times 3$	$34 = 30+2+2$	$97 = 60+30+6+1$

PROBLEMS

Find the interest, at 6%, on:

1. \$480, for 25 da.
2. \$1240, for 29 da.
3. \$194, for 58 da.
4. \$715, for 87 da.
5. \$1213, for 112 da.
6. \$270.80, for 6 mo. 11 da.
7. \$2045.60, for 9 mo. 16 da.
8. \$697.25, for 1 yr. 4 mo. 19 da.
9. \$833.50, for 2 yr. 11 mo. 4 da.
10. \$573, for 8 mo. 28 da.
11. \$642, for 7 mo. 17 da.
12. \$217.90, for 5 mo. 21 da.
13. \$1063, for 1 mo. 9 da.
14. \$2135, for 1 yr. 4 mo. 12 da.
15. \$652, for 165 da.
16. \$79.86, for 218 da.
17. \$473.95, for 131 da.
18. \$238.70, for 147 da.
19. \$3084, for 1 yr. 3 mo.
20. \$541, for 2 yr. 14 da.
21. \$148, for 28 da.

22. $375, for 42 da.
23. $1256, for 1 mo. 9 da.
24. $720, for 3 mo. 12 da.
25. $863, for 4 mo. 15 da.
26. $145, for 46 da.
27. $258, for 35 da.
28. $317, for 92 da.
29. $412, for 3 mo. 10 da.
30. $651, for 2 mo. 11 da.
31. $115, for 29 da.
32. $194, for 1 mo. 16 da.
33. $226, for 2 mo. 21 da.
34. $318, for 2 mo. 21 da.
35. $108, for 3 mo. 11 da.
36. $261, for 4 mo. 9 da.
37. $182, for 1 mo. 7 da.
38. $244, for 27 da.
39. $368, for 1 yr. 7 mo.
40. $511, for 1 yr. 14 da.

325. To find the interest at any rate per cent, find the interest at 6%, divide by 6, and multiply by the given rate.

PROBLEMS

Find the interest on:

1. $516, for 38 da., at 8%.
2. $1218, for 3 mo. 14 da., at 5%.
3. $245.80, for 4 mo. 18 da., at 7%.
4. $52.65, for 1 mo. 12 da., at 3%.
5. $1071, for 1 yr. 5 mo. 15 da., at 4%.
6. $293, for 2 yr. 8 mo. 21 da., at $4\frac{1}{2}$%.
7. $364, for 216 da., at 9%.
8. $1357, for 83 da., at 2%.
9. $574, for 33 da., at 5%.
10. $619, for 72 da., at 7%.

326. Ordinary Six Per Cent Method

TABLE

Interest on $1, for 1 yr., at 6% =	$.06
Interest on $1, for 2 mo., at 6% =	.01
Interest on $1, for 1 mo., at 6% =	.005
Interest on $1, for 6 da., at 6% =	.001
Interest on $1, for 1 da., at 6% =	.000$\frac{1}{6}$

EXAMPLE.—Find the interest on \$150, for 2 yr. 7 mo. 19 da., at 6%.

SOLUTION

Int. on \$1 for 2 yr.	= \$.12
Int. on \$1 for 7 mo.	= .035
Int. on \$1 for 18 da.	= .003
Int. on \$1 for 1 da.	= .000$\frac{1}{6}$
Int. on \$1 for 2 yr. 7 mo. 19 da.	= \$.158$\frac{1}{6}$

Int. on \$150 for 2 yr. 7 mo. 19 da = 150× \$.158$\frac{1}{6}$ = \$23.725

PROBLEMS

Find the interest, at 6%, on:

1. \$575, for 1 yr. 6 mo.
2. \$640, for 2 yr. 4 mo. 12 da.
3. \$315, for 1 yr. 3 mo. 9 da.
4. \$720, for 3 yr. 7 mo. 15 da.
5. \$645, for 1 yr. 18 da.
6. \$83, for 2 yr. 11 mo.
7. \$614, for 9 mo. 23 da.
8. \$176, for 10 mo. 27 da.
9. \$394, for 1 yr. 25 da.
10. \$261, for 1 yr. 9 mo. 6 da.
11. \$312, for 3 yr. 2 mo. 11 da.
12. \$511, for 4 yr. 6 mo. 18 da.
13. \$117, for 5 mo. 13 da.
14. \$226, for 114 da.
15. \$391, for 226 da.
16. \$538, for 107 da.
17. \$264, for 210 da.
18. \$546, for 72 da.
19. \$185, for 49 da.
20. \$229, for 95 da.
21. \$927, for 4 mo. 23 da.
22. \$641, for 1 yr. 19 da.
23. \$725, for 1 yr. 6 mo. 8 da.
24. \$813, for 2 yr. 7 mo.
25. \$856, for 1 yr. 6 mo. 14 da.
26. \$921, for 4 yr. 8 mo. 12 da.
27. \$1040, for 3 yr. 7 mo. 9 da.
28. \$623, for 3 yr. 1 mo. 5 da.
29. \$408, for 2 yr. 11 mo. 29 da.
30. \$513, for 4 yr. 3 mo. 12 da.
31. \$719, for 5 yr. 8 mo. 13 da.
32. \$605, for 4 yr. 9 mo. 4 da.
33. \$352, for 11 mo. 28 da.
34. \$1423, for 3 yr. 22 da.
35. \$178, for 2 yr. 10 mo. 17 da.
36. \$137, for 1 yr. 5 mo. 25 da.
37. \$386, for 2 yr. 7 mo. 27 da.
38. \$920, for 1 yr. 11 mo. 16 da.
39. \$401, for 2 yr. 3 mo. 26 da.
40. \$738, for 3 yr. 5 mo. 11 da.

327. In the following problems, find the term of interest by compound subtraction.

Solve the following problems by your choice of methods.

PROBLEMS

Find the interest and amount of:

1. $643, from Jan. 1, 1898, to July 16, 1898, at 6%.
2. $718, from Dec. 13, 1899, to May 4, 1900, at 8%.
3. $136, from April 6, 1899, to Oct. 28, 1899, at 4%.
4. $207, from Jan. 3, 1898, to Sept. 27, 1898, at 6%.
5. $1692, from Feb. 26, 1897, to Aug. 13, 1897, at 5%.
6. $2046, from Sept. 30, 1898, to Feb. 13, 1899, at 6%.
7. $251, from Dec. 18, 1896, to March 12, 1897, at 3%.
8. $235, from May 29, 1899, to Oct. 3, 1899, at 4%.
9. $2187, from Oct. 15, 1898, to Aug. 26, 1899, at 5%.
10. $514, from Nov. 20, 1897, to June 28, 1898, at $4\frac{1}{2}$%.
11. $342, from Feb. 16, 1898, to June 1, 1900, at 7%.
12. $634, from April 17, 1899, to Dec. 15, 1900, at 3%.
13. $738, from Jan. 1, 1898, to Sept. 5, 1899, at 6%.
14. $1235, from Oct. 7, 1898, to May 10, 1899, at 7%.
15. $2950, from Dec. 12, 1899, to July 6, 1900, at 8%.
16. $965, from April 19, 1900, to Dec. 21, 1901, at 5%.
17. $433, from July 21, 1899, to Dec. 16, 1899, at 4%.
18. $1580, from Nov. 30, 1898, to March 17, 1899, at 5%.
19. $875, from Aug. 15, 1899, to Nov. 5, 1900, at 6%.
20. $386, from May 19, 1900, to Sept. 27, 1901, at 8%.
21. $915, from June 2, 1899, to May 15, 1900, at 5%.
22. $1023, from Feb. 8, 1898, to Aug. 25, 1899, at 7%.
23. $3607, from March 13, 1900, to April 20, 1901, at 6%.
24. $619, from June 9, 1900, to May 14, 1902, at 5%.
25. $785, from Nov. 7, 1900, to Jan. 1, 1902, at 6%.

Find the interest and amount for the exact number of days:

26. $315, from May 1, 1901, to June 14, 1901, at 6%.
27. $427, from Sept. 4, 1901, to Nov. 1, 1901, at 5%.
28. $1380, from June 18, 1901, to Aug. 3, 1901, at 6%.
29. $1025, from Aug. 30, 1902, to Oct. 31, 1902, at 8%.
30. $716, from Oct. 17, 1902, to Dec. 8, 1902, at 7%.
31. $824, from July 26, 1902, to Oct. 7, 1902, at 6%.
32. $906, from Jan. 30, 1902, to March 4, 1902, at $4\frac{1}{2}$%.
33. $314, from April 1, 1901, to June 28, 1901, at 5%.

34. $423, from Aug. 3, 1901, to Nov. 9, 1901, at 7%.
35. $206, from Oct. 19, 1900, to Nov. 1, 1900, at 6%.
36. $341, from Nov. 16, 1900, to Jan. 2, 1901, at 8%.
37. $1217, from Feb. 4, 1901, to May 4, 1901, at 3½%.
38. $1087, from Nov. 1, 1901, to Jan. 27, 1902, at 4½%.
39. $421, from Jan. 2, 1902, to March 1, 1902, at 7%.
40. $538, from Dec. 5, 1902, to Feb. 6, 1903, at 4%.
41. $673, from Sept. 13, 1902, to Dec. 8, 1902, at 5%.
42. $738, from Aug. 1, 1901, to Sept. 3, 1901, at 3%.
43. $297, from Feb. 7, 1901, to April 4, 1901, at 3½%.
44. $309, from June 6, 1902, to Oct. 1, 1902, at 4%.
45. $273, from April 3, 1901, to July 29, 1901, at 6%.
46. $948, from July 4, 1902, to Sept. 3, 1902, at 4½%.
47. $3071, from May 7, 1902, to Sept. 1, 1902, at 5%.
48. $4253, from Nov. 9, 1901, to Jan. 1, 1902, at 5½%.
49. $1406, from Jan. 7, 1902, to April 11, 1902, at 8%.
50. $2350, from July 29, 1902, to Dec. 1, 1902, at 4½%.

328. In finding the interest on any sum, we use the formula:

Principal × rate (for time) = interest

The product of two numbers divided by one of them is the other number. Therefore,

Interest ÷ rate (for time) = principal
Interest ÷ principal = rate (for time)
Rate (for time) ÷ rate (per annum) = time (in years)
Rate (for time) ÷ time (in years) = rate (per annum)
Principal × (1 + rate for time) = amount
Amount ÷ principal = (1 + rate for time)
Amount ÷ (1 + rate for time) = principal

PROBLEMS

1. If the principal is $240, the rate 6%, and the time 4 mo., what is the interest?

2. If the interest is $4.80, the principal $240, and the rate 6%, what is the time?

3. If the interest is \$4.80, the principal \$240, and the time 4 mo., what is the rate per annum?

4. Find the amount of \$360, for 8 mo., at 6%.

5. If the amount is \$374.40, the principal \$360, and the time 8 mo., what is the rate?

6. If the amount is \$374.40, the principal \$360, and the rate 6%, what is the time?

7. If the amount is \$374.40, the time 8 mo., and the rate 6%, what is the principal?

8. How long must I loan \$80, at 6%, that it may amount to \$100?

9. At what rate per cent must I loan \$75 that it may draw \$1.87½ interest in 6 mo.?

10. If I borrow \$245 for 9 mo., at 7%, for how long must I loan \$315 at 6% to balance the favor?

11. How long will it take a sum of money at simple interest to double itself at 6%? 5%? 8%? 3%? 7%? 10%?

12. If the interest is \$9.45, the principal \$315, and the time 8 mo., what is the rate per annum?

13. In how long will \$110 amount to \$125 at 4%?

14. What sum will amount to \$580 at 6% in 1 yr. 4 mo.?

15. At what rate per cent must \$150 be loaned that it may amount to \$175 in 2 yr.?

16. If I borrow \$525 for 1 yr. 4 mo. at 5%, how long must I loan \$475 at 6% to balance the favor?

17. At what per cent must \$340 be loaned to gain \$28 in 1 yr. 6 mo.?

18. What sum of money, invested April 1, 1902, at 6% interest, will amount to \$1000 Jan. 1, 1904?

19. How long will it take \$100 to double itself at 4½% simple interest?

20. What rate of interest must be charged in order that \$650 may amount to \$697.67 in 1 yr. 4 mo.?

PERIODIC INTEREST

329. Simple interest is simply interest on the principal.

330. Periodic Interest is interest on the principal and interest on the simple interest due at certain interest periods.

331. Interest may be due annually, semi-annually, quarterly, etc.

332. Annual Interest is interest on the principal payable annually, and simple interest on the interest that remains unpaid.

333. Semi-Annual Interest is interest on the principal payable semi-annually, and simple interest on the interest that remains unpaid.

334. Quarterly Interest is interest on the principal payable quarterly, and simple interest on the interest that remains unpaid.

335. In some States periodic interest cannot be legally collected. To secure periodic interest, a contract should specify it.

EXAMPLE 1.—Find the annual interest on $500 for 4 yr. at 6%, if no interest is paid until the principal is due.

SOLUTION

Int. on $500 for 1 yr.	= $ 30
Int. on $500 for 4 yr.	= 120
Int. on $30 (unpaid int.) for 3 yr., 2 yr., and 1 yr., or for 6 yr.	= 10.80
Annual int.	= $130.80

EXAMPLE 2.—What is the semi-annual interest on $400 for 3 yr. 4 mo., at 6%?

SOLUTION

Int. on $400 for 6 mo.	= $12
Int. on $12 for 2 yr. 10 mo., 2 yr. 4 mo., 1 yr. 10 mo., 1 yr. 4 mo., 10 mo., and 4 mo., or for 9 yr. 6 mo.	= 6.84
Int. on $400 for 3 yr. 4 mo.	= 80
Semi-annual int.	= $86.84

PROBLEMS

1. What is the annual interest on $475, for 4 yr. 8 mo., at 6%?

2. Find the semi-annual interest on $263, for 2 yr. 7 mo., at 6%.

3. Find the amount of interest due at the end of 5 yr. 3 mo., on a note for $218, at 6% annual interest.

4. What will $125 amount to in 2 yr. 8 mo., with interest at 8%, payable quarterly?

5. Find the annual interest on $436, for 5 yr. 8 mo., at 5%.

6. Find the semi-annual interest on $1080, for 2 yr. 7 mo. 15 da., at 5%.

7. What will $125 amount to in 1 yr. 9 mo. 12 da., with quarterly interest, at 6%?

8. Find the amount of $528 for 2 yr. 9 mo. 18 da., with interest at 4%, payable semi-annually.

9. What will be the annual interest on $1750 for 3 yr. 11 mo. 8 da., at $4\frac{1}{2}$%?

10. Find the amount of $318 for 2 yr. 5 mo. 21 da., at 6% interest, payable semi-annually.

11. What will $162 amount to in 1 yr. 10 mo. 25 da., at 5% quarterly interest?

12. What will be the amount of $435 for 4 yr. 8 mo., at $4\frac{1}{2}$% annual interest?

13. What sum will amount to $500 in 1 yr. 8 mo., at 6% semi-annual interest?

COMPOUND INTEREST

336. Compound Interest is interest upon the principal and on the interest combined with the principal at regular intervals.

For the purpose of finding the compound interest, the simple interest is added to the principal at regular intervals, and the amount becomes the new principal on which the interest is computed.

Interest may be compounded annually, semi-annually, or quarterly.

337. Compound interest cannot be enforced by law.

Savings banks usually allow compound interest.

EXAMPLE.—Find the interest on $250 for 2 yr. 6 mo. 15 da., at 6%, compounded annually.

SOLUTION

$250	= prin.
15	= int. for first year
$265	= amt. for first year
15.90	= int. for second year
$280.90	= amt. for second year
9.13	= int. for 6 mo. 15 da.
$290.03	= amt. for 2 yr. 6 mo. 15 da.
250	= original prin.
$ 40.03	= compound int.

NOTES.—1. As in the above example, the last interest period may be but part of a full period.

2. If interest is compounded semi-annually or quarterly, take one-half or one-fourth of the annual rate per cent.

PROBLEMS

1. What is the interest on $640 for 2 yr. 8 mo., at 7%, compounded semi-annually?

2. What is the interest on $1230 for 1 yr. 3 mo. 15 da., at 8%, compounded quarterly?

3. Find the interest on $390 for 3 yr. 7 mo., at 5%, compounded annually.

4. Find the amount of $750 for 2 yr. 9 mo. 18 da., with 6% interest, compounded annually.

5. Find the interest on $375 for 6 yr. 8 mo., at 5%, compounded annually.

6. Find the amount of $520 for 3 yr. 9 mo., at 8% interest, compounded semi-annually.

7. What is the interest on $640 for 2 yr. 3 mo., at 10% interest, compounded quarterly?

8. What is the amount of $328 for 7 yr. 4 mo., at 5% interest, compounded annually?

Compound Interest Table

338. Persons who have to make many computations in compound interest usually use a printed table like the following:

TABLE SHOWING THE AMOUNT OF $1 AT COMPOUND INTEREST

YEAR	2%	3%	4%	5%	6%	7%
1....	1.020000	1.030000	1.040000	1.050000	1.060000	1.070000
2....	1.040400	1.060900	1.081600	1.102500	1.123600	1.144900
3....	1.061208	1.092727	1.124864	1.157625	1.191016	1.225043
4....	1.082432	1.125508	1.169858	1.215506	1.262477	1.310796
5....	1.104080	1.159274	1.216652	1.276281	1.338225	1.402551
6....	1.126162	1.194052	1.265319	1.340095	1.418519	1.500730
7....	1.148685	1.229873	1.315931	1.407100	1.503630	1.605781
8....	1.171659	1.266770	1.368569	1.477455	1.593848	1.718186
9....	1.195092	1.304773	1.423311	1.551328	1.689479	1.838459
10....	1.218994	1.343916	1.480244	1.628894	1.790847	1.967151
11....	1.243374	1.384233	1.539454	1.710339	1.898298	2.104852
12....	1.268241	1.425760	1.601032	1.795856	2.012196	2.252191
13....	1.293606	1.468533	1.665073	1.885649	2.132928	2.409845
14....	1.319478	1.512589	1.731676	1.979931	2.260904	2.578534
15....	1.345868	1.557967	1.800943	2.078928	2.396558	2.759031
16....	1.372785	1.604706	1.872981	2.182874	2.540351	2.952163
17....	1.400241	1.652847	1.947900	2.292018	2.692772	3.158815
18....	1.428246	1.702433	2.025816	2.406619	2.854339	3.379932
19....	1.456811	1.753506	2.106849	2.526950	3.025599	3.616527
20....	1.485947	1.806111	2.191123	2.653297	3.207135	3.869684

NOTES.—1. Any principal multiplied by the amount of $1 for any given time, at any given rate, is the amount of the principal, for the given time and rate.

2. The amount of $1 for any given number of years is equal to the product of the amounts of $1, for such periods of years whose sum is equal to the given number of years. To find the amount of $1 for 40 yr., multiply together the amounts for 20 and 20 yr., or for 15, 15, and 10 yr., etc.

3. For semi-annual interest, take ½ the rate for twice the time.

4. For quarterly interest, take ¼ the rate for 4 times the time.

PROBLEMS

Solve the following problems by the use of the table:

1. Find the compound interest on $632, for 15 yr., at 5%.

2. Find the interest on $1285, for 9 yr. 9 mo., at 6%, compounded semi-annually.

3. Find the amount of $750, for 3 yr. 8 mo., at 8%, compounded quarterly.

4. What is the interest on \$196, for 16 yr. 5 mo., at 6%, compounded annually?

5. What is the amount of \$224, for 6 yr. 8 mo., at 8% interest, compounded semi-annually?

6. Find the amount of \$3567, for 3 yr. 6 mo., at 4%, compounded semi-annually.

7. What is the interest on \$2687, for 7 yr. 9 mo. 15 da., at 5%, compounded annually?

Find the compound interest on:

	PRINCIPAL	RATE	TIME	PAYABLE
8.	\$1428	8%	9 yr. 7 mo.	quarterly
9.	\$ 732	7%	14 yr. 10 mo.	annually
10.	\$ 523	6%	24 yr.	annually
11.	\$ 176	12%	5 yr.	quarterly
12.	\$ 391	5%	7 yr. 9 mo.	semi-annually
13.	\$ 746	3%	35 yr.	annually
14.	\$ 412	4%	16 yr. 8 mo.	annually
15.	\$ 834	4%	14 yr. 5 mo.	semi-annually

Compound Interest Amount Table

TABLE SHOWING THE AMOUNT OF \$1 INVESTED AT THE BEGINNING OF EACH YEAR FOR A SERIES OF YEARS, AT COMPOUND INTEREST

YEAR	2%	2½%	3%	3½%	4%	4½%
1....	1.020000	1.025000	1.030000	1.035000	1.040000	1.045000
2....	2.060400	2.075625	2.090900	2.106225	2.121600	2.137025
3....	3.121608	3.152515	3.183627	3.214942	3.246464	3.278191
4....	4.204040	4.256328	4.309135	4.362465	4.416322	4.470709
5....	5.308120	5.387736	5.468409	5.550152	5.632975	5.716891
6....	6.434283	6.547430	6.662462	6.779407	6.898294	7.019151
7....	7.582969	7.736115	7.892336	8.051686	8.214226	8.380013
8....	8.754628	8.954518	9.159106	9.368495	9.582795	9.802114
9....	9.949721	10.203381	10.463879	10.731393	11.006107	11.288209
10....	11.168715	11.483466	11.807795	12.141991	12.486351	12.841178
11....	12.412089	12.795552	13.192029	13.601961	14.025805	14.464031
12....	13.680331	14.140441	14.617790	15.113030	15.626837	16.159913
13....	14.973938	15.518952	16.086324	16.676986	17.291911	17.932109
14....	16.293416	16.931926	17.598913	18.295680	19.023587	19.784054
15....	17.639285	18.380224	19.156881	19.971029	20.824531	21.719336
16....	19.012070	19.864730	20.761587	21.705015	22.697512	23.741706
17....	20.412312	21.386348	22.414435	23.499691	24.645412	25.855083
18....	21.840558	22.946007	24.116868	25.357180	26.671229	28.063562
19....	23.297369	24.544657	25.870374	27.279681	28.778078	30.371422
20....	24.783317	26.183273	27.676485	29.269470	30.969201	32.783136

COMPOUND INTEREST AMOUNT TABLE.—*Continued.*

YEAR	5%	6%	7%	8%	9%	10%
1....	1.050000	1.060000	1.070000	1.080000	1.090000	1.100000
2....	2.152500	2.183600	2.214900	2.246400	2.278100	2.310000
3....	3.310125	3.374616	3.439943	3.506112	3.573129	3.641000
4....	4.525631	4.637093	4.750739	4.866601	4.984710	5.105100
5....	5.801912	5.975318	6.153290	6.335929	6.523334	6.715610
6....	7.142008	7.393837	7.654021	7.902803	8.200434	8.487171
7....	8.549108	8.897468	9.259802	9.616627	10.028473	10.435888
8....	10.026564	10.491316	10.977988	11.467557	12.021036	12.579476
9....	11.577892	12.180795	12.816448	13.466562	14.192929	14.937424
10....	13.206787	13.971642	14.783599	15.625487	16.560293	17.531167
11....	14.917126	15.869941	16.888451	17.957126	19.140719	20.384283
12....	16.712982	17.882137	19.140963	20.475296	21.953384	23.522712
13....	18.598631	20.015066	21.550488	23.194920	25.019189	26.974983
14....	20.578563	22.275970	24.129022	26.132113	28.360916	30.772481
15....	22.657491	24.672528	26.888053	29.304283	32.003398	34.949729
16....	24.840366	27.212880	29.840217	32.730225	35.973704	39.544702
17....	27.132384	29.905652	32.999032	36.430243	40.301338	44.599173
18....	29.539003	32.759992	36.378965	40.426263	45.018458	50.159090
19....	32.065954	35.785591	39.995492	44.741964	50.160119	56.275029
20....	34.719251	38.992727	43.865177	49.402921	55.764530	63.002529

NOTES

339. A Note, or a Promissory Note, is a written promise to pay a sum certain, at a time certain.

The sum may be paid in money, or in other valuable things, as mentioned in the note.

The time of payment may be a fixed date (as June 1, 1903); it may be the date of the occurrence of an event that is sure to happen (as the death of a person); it may be any date on which the person entitled to payment may ask for it (payment on demand).

340. **Form of Notes**

$450.00. Buffalo, N. Y., June 1, 1902.

Six months after date, for value received, I promise to pay T. B. Smith, or order, four hundred fifty dollars ($450.00). D. A. WEST.

$500.80. Detroit, Mich., June 1, 1902.

Four months after date, for value received, I promise to pay R. L. Gordon, or bearer, five hundred $\frac{80}{100}$ dollars ($500.80), with 6 per cent interest.

WALTER JOHNSON.

341. The Maker or Drawer is the person who signs the note.

342. The Payee is the person to whom the note is made payable.

343. The Face of the note is the sum promised to be paid.

344. A note should contain:

1. Time when, and place where made.
2. Time when payable.
3. The sum to be paid.
4. The expression, "for value received."

To prevent forgery the sum to be paid should be written in words.

If the words "for value received" are omitted, the maker cannot be compelled to pay unless the owner of the note can show that the maker received a valuable consideration for making the note.

If no place of payment is mentioned, the note is payable at the maker's place of business.

A note may be made payable "on demand," and is then payable whenever its owner calls for its payment.

345. A note may contain:

1. The words "or order," or "or bearer."
2. The words "with interest," or "with use."

If the words "with interest," or "with use," are omitted, the note will not draw interest, but if it is not paid at maturity, it will draw interest at the legal rate from the time it becomes due.

If a note contains the words "with interest," but does not mention the rate, it will draw interest at the legal rate where the note was made.

If a note contains the words "or bearer," it is payable to whoever presents it for payment.

If a note contains the words "or order," it is payable to the person mentioned as payee, or to whomever he orders it to be

paid. A payee may order a note paid to another by indorsement.

346. An Indorsement is a writing on the back of a document.

347. A note may be indorsed:

1. To make it payable to another person.
2. To make sure that the note will be paid.
3. To show that the note has been paid.
4. To show that a partial payment has been made.

Men frequently buy and sell notes. If a note is made payable to J. Smith, or bearer, Smith may sell it to another person, who may also sell it. The maker will pay whoever presents the note for payment. But, if T. Jones, who buys the note of Smith, thinks that the maker is not "good" for the amount of the note, he may require Smith to indorse the note by writing his name on the back of it. That would legally bind Smith to pay the note in case the maker should refuse to pay it if proper demand were made for its payment.

If a note is made payable to "J. Smith, or order," and Smith sells the note to Jones, Smith may write on the back of the note, "Pay to Jones.—J. Smith." The note is said to be transferred to Jones, and is payable to him. Smith, by his indorsement, is responsible for the payment of the note if the maker refuses to pay Jones.

When a note is paid, it is returned to the maker, who may destroy it. But the destruction of a note is no proof that it has been paid. If a note is lost, the payee may still require the maker of the note to pay the debt.

When a note is paid, unless it is made payable to the bearer, the maker often requires the holder, or owner, to write his name on the back, which makes the note payable to the maker. The maker then has the indorsed note as a receipt to show that it has been paid.

A treasurer who pays out money on the order of some other person should always require the payee of an order to indorse it. It then becomes a receipt for the treasurer.

If a part payment is made on a note, the payee writes on the back of the note, above his name, a statement of the amount received in payment.

348. The person who writes his name on the back of a note is an Indorser.

349. A note may be indorsed in one of three ways:

1. The payee may write only his name on the back of the note.

That makes the note payable to the bearer, and also makes the indorser liable for its payment if the maker refuses to pay it. This is called indorsement "in blank."

2. The payee may write, "Pay to James Wise," and sign his name.

That makes the note payable to James Wise, and also makes the indorser responsible for its payment. This is called a "full indorsement."

The payee may indorse a note thus: "Pay to James Wise, or order.—J. Smith." James Wise may indorse the note in a similar manner in favor of Wilson Niles, and so on. A note may have several indorsers, each of whom becomes individually responsible for its payment.

An indorsement to transfer a note makes the indorser responsible for payment unless the indorsement is made "without recourse."

3. If the payee wishes to transfer a note, but does not wish to become responsible for its payment except to a limited degree, he may indorse the note thus: "Pay to R. A. Wall, or order, without recourse to me.—Anson Brown." This is called indorsement "without recourse."

350. A note may be indorsed for transfer, for security, or for transfer and security. Write and indorse notes that will illustrate each of these.

351. If the maker of a note refuses to pay it, he is said to dishonor the note.

In order to make an indorser legally responsible for the payment of a note that has been dishonored by its maker, the

holder of the note must demand payment of its maker at maturity, and give the indorser, within a reasonable time, notice of its dishonor by its maker. If there are several indorsers, each should be notified.

Notice may be given to the indorser by letter or verbally. If the parties to the note are of different States, the owner of the note should mail a protest to the indorser. A protest is a written statement, made by an officer who takes oaths, giving notice to the indorser of the note that it has not been paid.

352. A note that may be transferred from one party to another by indorsement and give the holder the right to sue for its payment in his own name is a Negotiable Note.

Such a note must contain the words "or order," "bearer," or "or bearer."

353. A note that cannot be transferred by indorsement is a Non-Negotiable Note. A note made payable to James Smith is not negotiable.

354. In some States three days are allowed by law for the payment of a note in addition to the time mentioned in the note. These three days are called Days of Grace.

355. A note given for a number of months is due on the expiration of that number of calendar months. Thus, a note given on February 1, for three months, is due May 1.

356. A note given for a number of days is due on the expiration of that number of days. Thus, a note given on February 1, for ninety days, is due on May 2 in an ordinary year, and on May 1 in a leap year.

357. A note maturing on a legal holiday should be paid on the day previous. If that day is a legal holiday also, the note should be paid on the day before. If Monday is a legal holiday, notes maturing on that day should be paid on Saturday.

In a State where days of Grace are not allowed, if the day of maturity of a contract falls on Sunday or a holiday, it is due the day following.

358. In all States a note or a closed account will outlaw—become void—in a certain number of years after it becomes due, if nothing be paid on it. The time required for a note or an account to outlaw varies in the different States from two years to twenty years.

359. What is the *value* of a note at its maturity?

If it does not draw interest, its value is its face.

If it draws interest, its value is its face plus the interest.

All notes draw interest after they become due.

PROBLEMS

1. Find the value of this note at maturity:

$263.80. Rochester, N. Y., May 1, 1902.

Four months after date, for value received, I promise to pay Thomas Byron, or order, two hundred sixty-three $\frac{80}{100}$ dollars, with interest at 6 per cent.

L. C. JOHNSON.

2. Find the amount due on this note Sept. 1, 1903:

$480.00. Cleveland, Ohio, Jan. 17, 1902.

Six months after date, I promise to pay A. C. Berry, or order, four hundred eighty dollars, for value received with interest. M. F. SWAN.

NOTE.— When interest is mentioned but no rate is given in these problems, 6% is to be understood.

Below are given the data of several notes. Find the due date and the amount due at time of settlement, assuming that interest is at 6%, and that no interest is paid till the time of settlement:

	FACE	DATE	TIME	SETTLEMENT
3.	$ 625	Jan. 3, 1900	3 mo.	Oct. 6, 1902
4.	$ 590.50	Feb. 12, 1898	90 da.	when due
5.	$ 268.25	Dec. 7, 1899	6 mo.	Sept. 30, 1901
6.	$1120	Oct. 3, 1897	1 yr.	Feb. 14, 1900
7.	$ 375.60	May 5, 1898	60 da.*	5 mo. after date
8.	$ 214.75	Nov. 28, 1899	30 da.	June 16, 1901
9.	$3620	July 29, 1899	4 mo.*	Aug. 30, 1902
10.	$ 493	March 30, 1898	9 mo.*	Dec. 16, 1900
11.	$ 318	Sept. 1, 1899	3 mo.	April 1, 1901
12.	$ 422	April 3, 1900	30 da.	Nov. 25, 1900

*Interest is not mentioned in the note.

PARTIAL PAYMENTS

360. Payment of a portion of the amount due on a note is often made. Such a payment is called a Partial Payment.

Several partial payments are often made on a note.

The amount and date of each payment should be indorsed on the back of the note.

Partial payments may be made on mortgages and accounts, and may be made before or after the obligation becomes due.

361. Mercantile Rule

MENTAL PROBLEMS

1. What will a debt of $500 amount to in 6 mo., interest at 6%? If $200 be paid 4 mo. before the debt is finally paid, to what will the partial payment amount at the time of settlement? If such a payment be made, what will be the net amount due on the note at the end of 6 mo.?

2. A debt of $1000 was settled 1 yr. after it became due, but 3 mo. after it became due $200 was paid on it, and 3 mo. later $400 was paid on it. What was the amount of the original debt at date of settlement, interest at 6%? What was the amount of each payment at date of settlement? What was the amount left to be paid at the time of settlement?

Principles.—1. The amount of the debt equals the face of the debt plus the interest on the same till the time of settlement.

2. The amount of each payment equals the face of the payment plus the interest on the same till the time of settlement.

3. The amount due at the time of settlement equals the amount of the debt less the sum of the amounts of the payments.

EXAMPLE.—The following payments were made on a note dated Jan. 1, 1900, for $500, with 6% interest: April 3, 1900,

$240; May 12, 1900, $80; June 25, 1901, $90. What remained due July 15, 1902?

EXPLANATION.—Write the dates in consecutive order, putting the last first. Begin at the bottom and subtract each date from the one immediately above it, and place the remainders in consecutive order. Write the amounts of the payments and the debt opposite the dates. Figure the interest and find the balance due as shown in the operation.

OPERATION

Subtraction of Dates				Cr.		Dr.	
Yr.	Mo.	Da.		P'm'ts	Int.	Debt	Int.
1898	7	15	time of settlement				
1897	6	25	time of third payment	$ 90.00			
1896	5	12	time of second payment	80.00			
1896	4	3	time of first payment	240.00			
1896	1	1	time of note			$500.00	
2	6	14	time from note to settlement	$240.00	$32.88	$500.00	$76.17
2	3	12	time from first p'm't to settlem't	80.00	10.44		
2	2	3	time from second p'm't to settlem't	90.00	5.70		
1	0	20	time from third p'm't to settlem't	$410.00	$49.02	$500.00	$76.17
				49.02		76.17	
				$459.02		$576.17	
						459.02	
			Amount due			$1.7.15	

PROBLEMS

1. What was the balance due Sept. 14, 1900, on a note for $638, dated March 6, 1896, interest at 6%, if the following payments were made: Dec. 3, 1896, $115; Feb. 12, 1897, $120; Oct. 18, 1898, $208?

2. $975.50. New York City, Jan. 14, 1897.

Six months after date I promise to pay L. C. Stone, or order, nine hundred seventy-five $\frac{50}{100}$ dollars, for value received, with interest at 6 per cent.

C. E. BABCOCK.

Find the amount due on the above note Jan. 1, 1900, the following payments having been made: Aug. 3, 1897, $86; Jan. 12, 1898, $175; Dec. 23, 1898, $215; July 5, 1899, $328.

3. $1285.00. Philadelphia, Pa., May 2, 1898.
Four months after date, for value received, I promise to pay to the order of G. W. Beam twelve hundred eighty-five dollars. A. T. BRONSON.

The note was not paid when due, but the following payments were made: Oct. 4, 1898, $164; Jan. 13, 1899, $245; July 6, 1899, $338. What remained due Sept. 23, 1899, money being worth 6%?

4. Find the balance due Oct. 28, 1902, on a mortgage given Nov. 1, 1897, for $2600, interest at 6%, the following payments having been made: July 5, 1898, $685; Oct. 14, 1899, $1260; March 7, 1900, $240; May 1, 1901, $350.

The United States Rule

362. The mercantile rule is often used by common consent in settling short-time notes and accounts.

The rule sanctioned by the Supreme Court of the United States is called the United States Rule. The principle of the United States Rule is that each payment should be applied to pay the interest due at that time, and the balance of the payment should be used to diminish the principal. If at any time the payment is less than the amount of interest due, the payment is not considered made at that time, but it is added to the next payment or payments, when the sum of the payments shall at least equal the interest due at the date of last payment.

Most States have adopted the United States Rule.

MENTAL PROBLEM

On Jan. 1, I owe $1000. How much will I owe May 1, interest at 6%? If on May 1, I pay $120, how much will I still owe on that date? How much will I owe Nov. 1? If I pay $177 on Oct. 1, how much will I still owe? How much will I owe on the first of the next March?

EXAMPLE.—A mortgage was given Jan. 12, 1896, for $6820, with interest at 6%. The following payments were made:

Oct. 15, 1896, $380; Feb. 17, 1897, $650; July 13, 1897, $760; Jan. 15, 1898, $1290. How much remained due Nov. 1, 1898?

OPERATION

Subtraction of Dates

Year	Month	Day		
1898	11	1	time of settlement	
1898	1	15	time of fourth payment	
1897	7	13	time of third payment	
1897	2	17	time of second payment	
1896	10	15	time of first payment	Debt
1896	7	12	time mortgage was given	$6820.00
				Payment
	3	3	time from mortgage to first payment	$ 380.00
	4	2	time from first to second payment	650.00
	4	26	time from second to third payment	760.00
	6	2	time from third to fourth payment	1290.00
	9	16	time from fourth payment to settlement	

Face of debt	$6820.00
Int. to first payment	105.71
Amt. of debt at first payment	$6925.71
First payment	380.00
Amt. of debt after first payment	$6545.71
Int. to second payment	133.09
Amt. due at second payment	$6678.80
Second payment	650.00
Amt. due after second payment	$6028.80
Int. to third payment	146.70
Amt. due at third payment	$6175.50
Third payment	760.00
Amt. due after third payment	$5415.50
Int. to fourth payment	164.27
Amt. due at fourth payment	$5579.77
Fourth payment	1290.00
Amt. due after fourth payment	$4289.77
Int. to time of settlement	204.48
Amt. due at settlement	$4494.25

PROBLEMS

1. On a debt of $3245, beginning Sept. 3, 1897, and bearing 5% interest, the following payments were made: Jan. 14, 1898, $630; Aug. 9, 1898, $560; Feb. 14, 1899, $780. How much was due April 2, 1900?

Find the amounts due:

	Date of Debt	Face	Int.	Payments	Settled
2.	Aug. 15, 1896	$1245	4%	Dec. 1, 1896, $ 208 April 1, 1897, 315 Nov. 15, 1897, 120 July 6, 1898, 160	Aug. 4, 1899
3.	May 6, 1898	1790	6%	Aug. 8, 1898, 218 Jan. 19, 1899, 350 Dec. 14, 1899, 190	July 12, 1900
4.	Oct. 4, 1897	645	6%	Feb. 2, 1898, 175 Oct. 8, 1898, 225 Sept. 29, 1899, 130	Oct. 16, 1900
5.	Jan. 6, 1896	3255	5%	Oct. 12, 1896, 250 May 4, 1897, 380 Feb. 26, 1898, 1650	Dec. 7, 1898

363. When a payment is less than the interest due on the debt, the payment is not deducted from the amount due, but it is added to the next payment, and the sum is considered as one payment bearing the date of the latter payment.

EXAMPLE.—A mortgage for $2500 was given Jan. 4, 1896. The following payments were made: July 7, 1896, $350; Feb. 15, 1897, $50; Oct. 4, 1897, $540; May 4, 1898, $60; Dec. 22, 1898, $425. How much was due April 1, 1899?

Subtraction of Dates

Year	Month	Day
1899	4	1
1898	12	22
1898	5	4
1897	10	4
1897	2	15
1896	7	7
1896	1	4
	6	3
	7	8
	7	19
	7	0
	7	18
	3	9

OPERATION

Face of debt	$2500.00
Int. to first payment	41.25
Amt. due at first payment	$2541.25
First payment	350.00
Amt. due after first payment	$2191.25
Int. to second payment*	159.96
Amt. due at second payment	$2351.21
Second payment	590.00
Amt. due after second payment	$1761.21
Int. to third payment*	131.21
Amt. due at third payment	$1892.42
Third payment	485.00
Amt. due after third payment	$1407.42
Int. to settlement	42.93
Amt. due at settlement	$1450.35

*This is the sum of two payments with the date of the latter, for the former was less than the interest due. The pupil can generally tell by inspection whether the interest due is greater than the payment.

PROBLEMS

Find the amounts due:

	Date	Face	Int.	Payments	Settled
1.	March 20, 1897	$1540	6%	Sept. 14, 1897, $ 85 June 6, 1898, 50 Dec. 7, 1898, 320 Feb. 3, 1899, 245	Nov. 1, 1899
2.	Dec. 21, 1898	2435	4%	May 3, 1899, 140 Jan. 17, 1900, 60 June 1, 1900, 75 Sept. 4, 1900, 420	Nov. 3, 1900
3.	Aug. 1, 1896	630	6%	Oct. 8, 1896, 115 Feb. 16, 1897, 50 Jan. 1, 1898, 25 Dec. 11, 1899, 325	April 16, 1900
4.	Nov. 22, 1897	1182	6%	May 7, 1898, 25 Sept. 3, 1898, 85 March 21, 1899, 256 Aug. 4, 1899, 120	July 3, 1900
5.	Feb. 4, 1895	2090	8%	Oct. 26, 1895, 235 July 13, 1896, 90 Jan. 12, 1897, 460 May 4, 1897, 150	Jan. 29, 1898
6.	Sept. 7, 1896	875	6%	April 26, 1897, 145 Oct. 6, 1897, 130 May 18, 1898, 225 Dec. 30, 1898, 160	March 1, 1899
7.	May 3, 1898	1470	6%	Nov. 29, 1898, 110 Sept. 28, 1899, 160 Jan. 8, 1900, 440	Aug. 8, 1900
8.	July 2, 1896	2100	6%	Dec. 1, 1896, 220 Aug. 9, 1897, 165 Feb. 4, 1898, 235 Nov. 28, 1898, 50	April 15, 1899
9.	June 19, 1897	1360	6%	Nov. 10, 1897, 135 March 11, 1898, 290 Feb. 16, 1899, 325 April 3, 1900, 165	Sept. 21, 1900
10.	Jan. 4, 1898	2900	6%	June 13, 1899, 180 Aug. 16, 1899, 70 Oct. 24, 1899, 390 Sept. 6, 1900, 625 Dec. 21, 1900, 750	Jan. 10, 1901

364. TRUE DISCOUNT

MENTAL PROBLEMS

1. What will \$1 amount to in 1 yr. at 6%? In 2 yr.? In 3 yr.?

2. What will \$100 amount to in 1 yr. at 6%? In 2 yr.?

3. What sum now will amount to \$106 in 1 yr. at 6%? To \$112 in 2 yr.? To \$118 in 3 yr.?

4. What is the present value of \$106 *due* in 1 yr., if money is worth 6%? What is the present value of \$112 due in 2 yr.? Of \$118 due in 3 yr.?

5. What is the present value of \$212 due in 1 yr., if money is worth 6%? Of \$208 due in 1 yr., if money is worth 4%?

6. If you owe \$106 on a credit of 1 yr., how much ought you to pay if you pay the debt now, money being worth 6%?

7. If you owe \$115 on 3 yr. credit, how much should you pay now to cancel the debt, money being worth 5%?

8. If you owe \$112 on 2 yr. credit, how much should your creditor allow if you pay the debt now, money being worth 6%?

9. If you owe \$210 on 1 yr. credit, how much discount should you be allowed if you pay the debt now, money being worth 5%?

10. If you owe now \$210, which is to be paid in 2 yr., with interest at 5%, how much should you pay to cancel the debt now?

365. \$100 in cash is worth more than \$100 to be paid in 1 yr., without interest. It is worth as much as \$100 to be paid in 1 yr., *with interest.* If a man has a sum of money due him at a future time without interest, he can afford to allow a discount from the face of the debt if it is paid now.

An equitable deduction made from the face of a debt, due at a future time, without interest, is called Time Discount, or True Discount.

366. What a debt, due in the future, without interest, is worth now, is the Present Worth of the debt. The present

worth of a debt is that sum which put at interest now will amount to the face of the debt when it falls due.

Face of debt – present worth = discount
Face of debt – discount = present worth

EXAMPLE.—I owe $2935.50 to be paid in 6 mo., without interest. What is the present worth and the true discount, if money is worth 6%?

OPERATION

$1 now = $1.03 due in 6 mo.
$2935.50 ÷ 1.03 = $2850, present worth
$2935.50 – $2850 = $85.50, true discount

EXPLANATION.—$1 now = $1.03 due in 6 mo. Each $1.03 found in $2935.50 is equivalent to $1 due now. Therefore $2935.50 ÷ 1.03 = the number of dollars due now.

NOTES.—1. The present worth of an interest bearing debt is the face of the debt, if discounted at the interest bearing rate.

2. When an interest bearing debt is discounted, the amount of the debt when it becomes due should be taken as the face of the debt to be discounted.

PROBLEMS

1. $840.00. St. Louis, Mo., July 6, 1900.

Eight months after date I promise to pay G. O. Black, or order, eight hundred forty dollars, for value received. WILLIAM JOHNSON.

How much was the above note worth July 6, if money was worth 6%?

2. $1250.00. Charleston, S. C., Aug. 1, 1900.

One year after date I promise to pay H. A. Wells, or order, twelve hundred fifty dollars, for value received. A. D. NOYSE.

What was the above note worth Oct. 18, 1900, money being worth 6%?

Find the present worth and the true discount of the following:

3. $1217 on 90 da. credit, money worth 4%.

4. $625.80 due in 1 yr. 4 mo. 12 da., money worth 6%.

5. $3084 payable in 7 mo. 29 da., money worth 8%.

6. $215.25 payable in 1 yr. 6 mo. 18 da., money worth 6%.

7. $1028 payable in 60 da., money worth 7%.

8. $2135 payable in 2 yr. 8 mo., money worth 4½%.

9. $716 payable in 3 yr. 3 mo., money worth 6%.

10. $937 payable in 1 yr. 4 mo., money worth 6%.

11. What sum will amount to $1438 in 2 yr. 7 mo., at 6%?

12. I gave my note for $3625, for 1 yr., with 4% interest. If the note is discounted at 6%, what should I pay now?

13. On Jan. 1, 1900, I agreed to pay $1275 in 1 yr. 4 mo., with interest at 4%. Four months later the debt was discounted at 6%. What did I pay?

14. Which is better and how much, to buy sheep at $8.50 on 7 mo. time, or to pay $8.25 cash, money being worth 6%?

15. A man was offered a house for $18500 cash, or $19400 due in 10 mo. If money was worth 6%, which was the better offer?

16. When money is worth 4%, what cash offer is equivalent to an offer of $2550 on 6 mo. credit?

17. An agent bought a house for $1200. He kept it 14 mo., paid $185 for repairs, and sold it for $1500, on 9 mo. credit. What was his gain or loss, if money was worth 4%?

18. A merchant sold $1475 worth of goods on 8 mo. credit. If he sold the goods at a gain of 20%, what was his actual gain per cent, money being worth 6%?

19. Which is the better bargain for the purchaser, and how much better, $1000 worth of goods bought on 8 mo. time, or 5% off for cash, money being worth 6%?

20. Find the present worth of a debt of $6580, $2000 of which is due in 8 mo., $1500 in 14 mo., and the remainder in 1 yr. 8 mo., money being worth 6%.

NOTE.—Find the present worth of each payment.

21. George is 17 yr. old. How much must be invested for him, at 5% simple interest, that he may have $10000 of principal and interest when he becomes of age?

22. I bought a stock of goods on 8 mo. time. After holding them 6 mo., I sold them at an advance of 25%, giving a

credit of 10 mo. What was my gain per cent, money being worth 6%?

23. A man agreed to pay a debt of $1200 in 6 equal semi-annual payments, with simple interest at 4% per annum. Two months later he paid the present worth of the debt, discounted at 6% per annum. What was the amount of the discount?

24. On a bill of goods for $7850, a trade discount of 20%, 15%, and 10% and a credit of 6 mo. is allowed. What should be the total discount for cash payment, money being worth 5%?

BANKING BUSINESS

BANK DISCOUNT

367. A business man frequently takes notes for one, two, or three months, or longer, without interest or with interest. If he wishes to procure the money on a note before it becomes due, he may present it to a bank, which will purchase it from him. The amount that a bank deducts from the face of the note for advancing the money is called Bank Discount.

368. For discount banks take the legal interest on the amount due at maturity, for the time between the date of discounting and the date when the note becomes due.

369. The time for which a note is discounted is the Term of Discount.

370. The amount due on a note at maturity less the bank discount is the Proceeds of the note. The proceeds of a note are the sum received by its owner when he has it discounted at a bank.

371. In discounting a note, its value, for discount purposes, is its *future worth* (what it will be worth when it falls due). If a note does not draw interest, its future worth is its face. If a note draws interest, its future worth is its principal plus the interest to maturity.

372. *Difference between True and Bank Discount.*—True discount is interest on the present worth of a debt. Bank discount is interest on the future worth of a debt.

MENTAL PROBLEMS

1. What is the bank discount on a note for $500, if the term of discount is 2 mo. and the interest rate for discount is 6%? What are the proceeds?

2. What is the bank discount on a non-interest bearing note for $200, due in 2 mo., money being worth 6%?

3. What is the bank discount on an interest bearing note for $200, due in 2 mo., money being worth 6%?

4. A debt of $300, without interest, is due in 2 mo., and money is worth 6%.

What is the future worth of the debt?

What is the present worth by true discount?

What is the interest on the present worth for 2 mo.?

What is the sum of the interest on the present worth and the present worth?

What is the bank discount?

What are the bank proceeds?

What is the interest on the proceeds for 2 mo.? On the interest on the interest on the proceeds for 2 mo.?

What is the sum of the proceeds, interest on the proceeds, and interest on the interest on the proceeds?

Proceeds + int. on proceeds + int. on int. on proceeds = future worth.

Present worth + int. on present worth = future worth.

373. If a merchant intends to have a note discounted at a bank, he has the maker make the note payable at the bank with which the merchant does business. Then when the merchant wishes to have the note discounted, he indorses it and presents it to the bank. The note becomes payable to the bank, and the bank pays the merchant the proceeds of the note, which is the future worth less the interest on the future worth for the term of discount.

374. A note made payable at a bank is called a Bank Note, and is usually in this form:

$350.00. Brooklyn, N. Y., Jan. 1, 1900.

Two months after date I promise to pay to the order of Richard Roe three hundred fifty dollars, at the First National Bank, Brooklyn, N. Y. Value received JOHN DOE.

Due March 1.

EXAMPLE 1.—The above note was indorsed in blank by Richard Roe and presented to the First National Bank of Brooklyn, Feb. 1, 1900. What were the term of discount, the bank discount, and the proceeds, money being worth 6%?

OPERATION

Term of disc. = 28 da., time from Feb. 1 to March 1
Bank disc. = \$1.63, int. on \$350 for 28 da.
Proceeds = \$350 – \$1.63, or \$348.37

EXAMPLE 2.—

\$200.00. Syracuse, N. Y., June 1, 1900.

Ninety days after date I promise to pay to the order of J. L. Johnson two hundred dollars, for value received, with interest at 6 per cent, at the State Bank, Syracuse, N. Y. LESTER BROWN.

Due Aug. 29.

This note was discounted by the bank July 12. Find the bank discount and the proceeds.

OPERATION

Term of disc. = 48 da., time from July 12 to Aug. 29
Future worth = \$203, face plus interest for 90 da.
Bank disc. = \$1.62, int. on \$203 for 48 da.
Proceeds = \$203 – \$1.62 = \$201.38.

PROBLEMS

Find the discount and the proceeds of the following notes:

	Face	Date	Time	Interest	Discounted	Rate of Discount
1.	\$185	Jan. 2, 1900	4 mo.	4%	Jan. 18, 1900	6%
2.	225	Feb. 21, 1900	90 da.	None	April 2, 1900	5%
3.	436	Jan. 10, 1900	3 mo.	5%	Feb. 1, 1900	6%
4.	250	April 2, 1900	6 mo.	8%	May 3, 1900	6%
5.	712	July 6, 1900	30 da.	None	July 6, 1900	4%
6.	456	March 15, 1900	2 mo.	None	March 28, 1900	6%
7.	575	June 12, 1900	60 da.	7%	July 2, 1900	7%
8.	326	Feb. 9, 1900	90 da.	6%	Feb. 21, 1900	6%

9. Find the discount and the proceeds of the following:

\$485.00. Omaha, Neb., Feb. 6, 1900.

Three months after date, for value received, I promise to pay Thomas Wentworth, or order, four hundred eighty-five dollars, with interest at 6 per cent, at the First National Bank. H. R. BECK.

Discounted Feb. 15, at 6%.

10. $318.00. Philadelphia, Pa., Jan. 2, 1900.

Ninety days after date, for value received, I promise to pay to the order of C. D. Eaton three hundred eighteen dollars, at the Girard Bank.

HAROLD SPENCER.

Find the proceeds if discounted Jan. 17, at 6%.

11. $640.00. Lexington, Ky., April 4, 1900.

Four months after date, for value received, I promise to pay to the order of James Nelson six hundred forty dollars, with interest at 5 per cent, at the Commercial Bank. L. R. WATSON.

Find the proceeds if discounted April 30, at 6%.

12. $525.00. Charleston, S. C., May 4, 1900.

Six months after date, for value received, I promise to pay L. C. Westcott, or order, five hundred twenty-five dollars, with interest at 6 per cent, at the Merchants' Bank. R. A. VANCE.

Find the proceeds if discounted June 12, at 6%.

Find the discount and the proceeds of the following notes:

	Face	Date	Time	Interest	Discounted	Rate of Discount
13.	$1160	Jan. 4, 1900	4 mo.	None	Feb. 1, 1900	6%
14.	475	April 17, 1900	90 da.	6%	April 30, 1900	6%
15.	329	June 22, 1900	30 da.	None	June 25, 1900	5%
16.	1275	Aug. 4, 1900	3 mo.	8%	Aug. 28, 1900	8%
17.	738	July 20, 1900	90 da.	None	Aug. 3, 1900	4%
18.	426	April 6, 1900	60 da.	None	April 28, 1900	6%
19.	375	June 7, 1900	4 mo.	4%	July 25, 1900	6%
20.	526	March 5, 1900	6 mo.	6%	June 18, 1900	6%
21.	840	Jan. 12, 1900	30 da.	None	Jan. 12, 1900	6%
22.	237	Feb. 20, 1900	60 da.	None	Feb. 24, 1900	6%

BANK DEPOSITS AND CHECKS

375. Savings banks receive money from individuals and pay interest on such deposits. The depositor receives a bank book in which the sums deposited are credited to him. When he draws out money, the amount is debited to him.

Commercial banks also receive deposits. Some commercial banks pay interest on deposits, and some do not. Deposits are often made for safe-keeping.

376. When a depositor in a commercial bank wishes to draw on his deposit for his own use, he writes a check, of which the following is a form:

New York, Jan 2, 1900.

FIRST NATIONAL BANK OF NEW YORK.

Pay to Self———————————————

One Hundred Eighty Dollars. $180.00.

AMOS WENTWORTH.

If Amos Wentworth wishes to pay $180 from his deposit, to J. Higginson, he writes a check as follows:

New York, Jan. 2, 1900.

FIRST NATIONAL BANK OF NEW YORK

Pay to the order of J. Higginson———————————

One Hundred Eighty Dollars. $180.00.

AMOS WENTWORTH.

In this case J. Higginson must indorse the check.

377. The one who signs a check is called the Drawer.

378. The one to whom the check is made payable is the Payee.

379. The one to whom the check is addressed is the Drawee.

380. The drawer must be a depositor. It sometimes happens that a depositor of good financial standing draws out more than he has deposited.

The payee may be the drawer, bearer, or any person named in the check.

381. When a person deposits money in a bank, the bank holds the money subject to his order in the form of a check.

Assume that you have $500 deposited in a bank, and wish to draw out $100 for yourself and wish to pay Wm. Springer $200. Write two checks that will accomplish the result.

Write a check so that only James Wells can draw the money.

382. Many banks issue Certificates of Deposit, of which the following is a form:

No. 165. CERTIFICATE OF DEPOSIT.

FIRST NATIONAL BANK OF CLEVELAND.

Cleveland, Ohio, Jan. 2, 1900.

John Doe has deposited in this Bank three hundred dollars, payable to his order on the return of this Certificate properly indorsed, with interest at 3 per cent.

H. WILSON, Teller. RICHARD ROE, Cashier.

BANK LOANS

383. A depositor may borrow money from a bank and give his note for the sum borrowed. Usually the bank requires the borrower to deposit some security, as stocks, bonds, etc., or to have some responsible party indorse the borrower's note.

Such notes are called Bank Notes.

384. If the First National Bank of Detroit is willing to loan James Wilson $100 on his note, the form might be as follows:

$100.00. Detroit, Mich., Jan 2, 1900.

Two months after date I promise to pay the First National Bank of Detroit, Mich., one hundred dollars.

Value received. JAMES WILSON.

Due March 2.

385. Bank notes are usually for one, two, or three months, or for 30, 60, or 90 days. They do not draw interest, but interest is paid in advance in the form of discount. Thus, on the above note James Wilson would receive $100 – $1 (discount for 2 mo.), or $99.

PROBLEMS

Find the net amount received from the bank as proceeds of each of the following notes:

	Face	Time	Rate of Discount		Face	Time	Rate of Discount
1.	$ 340	60 da.	5%	*7.*	$400	30 da.	8%
2.	$ 250	3 mo.	6%	*8.*	$360	2 mo.	4%
3.	$ 475	1 mo.	8%	*9.*	$520	90 da.	6%
4.	$ 600	90 da.	4%	*10.*	$175	1 mo.	7%
5.	$ 850	30 da.	6%	*11.*	$230	60 da.	6%
6.	$1200	2 mo.	6%	*12.*	$150	90 da.	8%

386. If the First National Bank of Detroit is willing to loan James Wilson $100 on a note indorsed by Wm. Sully, the form of the note might be as follows:

$100.00. Detroit, Mich., Jan. 2, 1900.
Two months after date I promise to pay Wm. Sully, or order, one hundred dollars, at the First National Bank of Detroit, Mich. Value received.
Due March 2. JAMES WILSON.

Wm. Sully indorses the note, thus making it payable to the bank, and also making himself responsible for its payment in case James Wilson fails to pay it. Wilson can secure $99 ($100 – $1 discount) on the note. Sully indorses the note to accommodate Wilson, and he is called an accommodation indorser.

387. Such notes, if not paid when due, draw interest from the day of maturity to the day of payment.

388. If an indorsed note is not paid at maturity, the bank immediately sends the indorser a notice of protest.

PROBLEMS

1. You wish to borrow $200 from a bank that will accept your note if indorsed by Lewis Ross. Write the note for three months. If discounted at 6%, what would be the net amount received?

2. On May 1, 1900, R. H. Westcott borrowed $350 from the Third National Bank of Cleveland, Ohio, for 3 mo., on his note indorsed by J. C. Stone. Write and indorse the note. Find the proceeds if discounted at 5%.

3. Wm. Walton borrowed $350 from the Citizens' Bank of Boston, Mass., on a 2 mo. note, dated June 4, 1900, and indorsed by E. H. Jones. Write and indorse the note. Find the amount received by Walton if the discount is 6%. If he immediately put this sum at interest at 6%, how much would he lack of having enough to pay the note at its maturity? If Walton did not pay the note, and the indorser paid it Sept. 12, 1900, what sum did he pay, interest at 6%?

COLLATERAL NOTES

389. A Collateral Note is one whose payment is secured by making a deposit of personal property.

FORM OF NOTE

$25.00. Buffalo, N. Y., July 8, 1902.

Two months after date, for value received, I promise to pay H. B. Swan, or order, twenty-five dollars, with interest at 6 per cent. As security for the payment of the same I have deposited herewith a Smith-Premier typewriter, No. 75863, with permission to sell the same, if the note and interest thereon are not paid at maturity. WALTER C. CLARK.

RECEIPT FOR COLLATERAL

Buffalo, N. Y., July 8, 1902.

Received from Walter C. Clark a Smith-Premier typewriter, No. 75863, to secure the payment of a note for twenty-five dollars, given this day by said Walter C. Clark to me. H. B. SWAN.

390. Life insurance policies, stock certificates, notes, mortgages, etc., may also be assigned, or transferred to secure the payment of notes.

PROBLEMS

1. $120.00. Chicago, Ill., July 2, 1902.

Four months from date, for value received, I promise to pay F. E. Welsh, or order, one hundred twenty dollars, with interest at 6 per cent. To secure the payment of this note I deposit herewith a diamond ring marked H. K. and bought of Wilson Bros., this city. HAROLD KNOX.

What was due on the note Oct. 1, 1902?

2. On July 22, 1902, A. L. Ross gave an interest bearing note for $265 for 2 mo. to J. A. Briggs, and deposited as security for its payment a box of jewelry with permission to sell the same if the note was not paid at maturity. The note was not paid when due, and the jewelry was sold Oct. 15 for $340. How much money should Briggs pay Ross?

3. A gave B a 9-mo. note for $375, interest at 6%, and assigned a life insurance policy as security for payment. What was the total amount due B 4 mo. after the note became due?

4. C assigned a stock certificate to D to secure the payment of a loan of $225 made May 1, 1902. What amount did C pay on Aug. 20, 1902, to redeem his stock certificate?

5. On June 3, 1902, R. H. Empie assigned a chattel mortgage to C. D. Klein to secure the payment of a loan of $130. What amount was due Klein on Aug. 27, 1902?

DOMESTIC EXCHANGE

391. A keeps a meat market and owes B for cattle. C works for B by the month. B has no money to pay C until the end of 8 mo. C wishes to trade at the market, and asks B for money. B writes the following order:

Avon, N. Y., April 4, 1900.

Mr. A:—

Please pay to C forty dollars in meats, and charge to my account. B.

If A accepts the order C can trade at the market, and A will also be paying $40 on his debt to C, but there will be no money used.

392. A owes B $100. B owes C $100. C owes A $100.

How do these men stand financially after the following order is delivered? "Mr. C: Please pay to B $100.—A."

Or after this order? "Mr. A: Please pay to C $100.—B."

Or after this order? "Mr. B: Please pay to A $100.—C."

393. If A in Boston owes B in St. Louis $100, he may pay the debt in one of five ways without sending money.

(*a*) A may go to the postoffice in Boston and buy a Postal Money Order for $100. He would fill out an application blank for a money order, in which he would state the sum to be paid, to whom to be paid, and where to be paid. The postmaster would then give A an order, which would direct the postmaster at St. Louis to pay $100 to the person to be named by the postmaster of Boston in a letter of advice. A would send the order to B. The Boston postmaster would tell the St. Louis postmaster in a "letter of advice" to pay the order to B. B,

upon receiving the order, would take it to the postoffice and get $100. The postmaster at Boston would charge A $100.30 for the order.

At the present time money orders are issued, for any amount up to $100, at the following rates:

$ 2.50 or less	3¢	$30.00 to $ 40.00	15¢
2.50 to $ 5.00	5¢	40.00 to 50.00	18¢
5.00 to 10.00	8¢	50.00 to 60.00	20¢
10.00 to 20.00	10¢	60.00 to 75.00	25¢
20.00 to 30.00	12¢	75.00 to 100.00	30¢

(*b*) A may buy an Express Money Order from an express company in Boston. Express money orders are similar to postal money orders. The agents of the company at different offices of the company issue orders on one another as do the postmasters. The rates for express money orders are the same as for postal money orders.

(*c*) A may go to a telegraph office in Boston and buy a Telegraphic Money Order. The agent of the telegraph company at Boston would telegraph to the company's agent at St. Louis to pay B $100. The rates for telegraphic money orders are higher than the rates for the other orders mentioned, but the exchange is made much more quickly.

(*d*) A may buy from a Boston bank a Draft made payable to B. A would pay the Boston bank $100 plus the charge for "exchange." The Boston bank would write an order for some bank with which it has money deposited, to pay B $100. This order would be a Check of the Boston bank, but when a bank draws its check the paper is called a draft. The Boston bank would give the order, or draft, to A, who would send it to B. B could take it to a bank and have it cashed. The form of the draft might be:

FIRST NATIONAL BANK OF BOSTON.

Boston, Mass., Jan. 2, 1900.

Pay to the order of B———————————

One Hundred Dollars. $100.00.

T. A. WILSON, Cashier.

To the Mercantile National Bank,
New York City.

The draft might be made payable to A, who would then indorse it in favor of B.

Drafts and checks are much alike in form, but a check is drawn by a party, not a bank, on a bank, while a draft is drawn by a bank or banker on another bank.

(*e*) If A has money deposited in a Boston bank, he may send B a check like the following:

Boston, Mass., Jan. 2, 1900.
First National Bank of Boston, Mass.:—
Pay to the order of B———————————
One Hundred Dollars. $100.00.
(Signed) A.

If B has a bank account with a St. Louis bank, he may indorse the check and deposit it with the bank, which will probably collect it for him without charge. When B indorses the check he becomes responsible for its payment, and since he has a deposit in the bank, the bank is safe in accepting the check.

If B has no bank account, he may indorse the check and present it to a bank. The bank will pay the check after ascertaining that it is good.

If B has no bank account, he may indorse the check and send it direct to the Boston bank, which will send him a draft for $100 less a charge for exchange. This draft B may have cashed by a bank.

Cost of Drafts

394. Banks sell drafts, or, as it is termed, sell "exchange." The cost of a draft is the face of the draft plus the charge for exchange.

395. New York or Chicago exchange, drafts on New York or Chicago banks, usually sell at a premium of about .1%. A draft for $500 would probably cost $500.50. On small drafts a definite charge, as 10¢ or 15¢, may be made.

PROBLEMS

1. Find the cost of a draft for $850 bought in Buffalo and drawn on a New York bank, if the Buffalo bank charges a premium of $\frac{1}{8}\%$ as exchange.

2. What will be the cost of a draft on Chicago for $1280, when exchange is at a premium of $\frac{1}{4}\%$?

3. A merchant of Omaha owes a wholesale house in Milwaukee $3800. He buys of his banker a New York draft, at a cost of 8¢ per $100. How much does the draft cost?

4. Find the cost of a draft for $1570 on a Chicago bank, the rate of exchange being .1% premium?

5. Find the cost of a Boston draft for $9700, at the rate of 10¢ per $100.

6. An agent in New Orleans, wishing to pay his principal in New York $3265, bought a New York draft at $\frac{1}{8}\%$ discount. What did the draft cost him?

7. When exchange is at $\frac{1}{8}\%$ premium, what will a draft for $1 cost? What will be the face of a draft that $5.01 will buy? What will be the face of a draft that $851.70 will buy?

8. An agent has $2500 of his employer's money which he is to remit at the expense of the employer. What will be the face of the draft, if exchange is $\frac{1}{4}\%$ discount?

9. What will be the cost of a draft for $2635, at 12¢ per $1000?

10. A principal directs his agent to send him the money he has on hand, deducting charges for the draft. If the agent has $4500 of his employer's money, and exchange is at $\frac{1}{8}\%$ premium, what will be the face of the draft?

The Clearing House

396. Each large city is a money center. The banks in the villages and smaller cities deposit money with, remit drafts payable to, and sell drafts drawn upon the banks in the money centers. In New York State money is usually sent from one village to another by a draft drawn on a New York bank.

A man in Livonia wishes to send $100 to a man in Rochester. He pays the Bank of Livonia $100.50 for a draft for $100 on the Chemical Bank of New York. The draft is sent to Rochester and is cashed by a bank, which sends the draft to the Manhattan Bank of New York and is credited with a deposit of $100.

A man in Dundee wishes to send $100 to a man in Livonia. He pays the Dundee National Bank $100.50 for a $100 draft on the Manhattan Bank of New York, and sends the draft to Livonia. The Liviona man takes the draft to the Bank of Livonia, which cashes the draft and then sends it to the Chemical Bank of New York to pay for the draft which the Bank of Livonia first drew on the Chemical Bank.

The Chemical Bank now has a draft for $100 on the Manhattan Bank, and the Manhattan Bank has a draft for $100 on the Chemical Bank. These drafts are sent to the Clearing House, where they cancel each other.

The New York banks send agents to the clearing house to exchange the drafts held by each on the others. If one bank owes all the others $50000, after the drafts are cancelled, it pays that sum to the clearing house. If several banks owe one bank $50000, the clearing house pays the bank that sum. The clearing house settles the accounts between the various banks. It receives from the debtor banks and pays to the creditor banks. The amount of money exchanged is small in proportion to the value of the drafts exchanged. The "clearings" (cancellations) at the New York clearing house for 1899 were $57,368,-230,771, and the balances paid in money were $3,085,971,370.

Each of the great American cities has a clearing house, but the one at New York does more business than all the others combined.

Fluctuation of Exchange

397. On small sums the rate of exchange is usually at a uniform premium. This premium is to pay the banks for their trouble. Banks usually buy New York and Chicago drafts at par, that is, the banks make no charge for collecting the drafts.

398. The rate of exchange on large sums varies in the different cities. If Denver banks owe New York banks large sums, they would be obliged to send the money to New York, which would cause delay and expense. If then a man in Denver wishes to buy a draft on New York, he would have to pay more than the usual rate of exchange, for the Denver banks would then have to send more money to New York. But if a man in New York wishes to buy a draft on Denver, he might get it at a discount, for then the New York banks would procure a portion of their money from Denver immediately. Thus on large drafts between distant cities the rate of exchange varies, or fluctuates.

PROBLEMS

Find the cost of each of the following drafts:

	Face	Rate of Exchange		Face	Rate of Exchange
1.	\$ 2300	$\frac{1}{4}\%$ premium	*6.*	\$1684	$\frac{1}{2}\%$ premium
2.	\$18250	$\frac{1}{8}\%$ discount	*7.*	\$3790	$\frac{1}{4}\%$ discount
3.	\$ 6580	At par	*8.*	\$1260	At par
4.	\$ 750.90	$\frac{1}{3}\%$ premium	*9.*	\$ 538	.1% discount
5.	\$ 1585	$\frac{1}{2}\%$ premium	*10.*	\$2520	10¢ per \$100

Commercial Drafts

399. The check of one bank on another to effect exchange is called a Bank Draft. The order or draft of one individual or firm on another, asking payment of a debt through the agency of a bank, is called a Commercial Draft. A commercial draft is made payable at a certain bank, and the bank becomes the means by which one party collects from another.

400. A draft may be made payable at sight, or when presented for payment. Such a draft is called a Sight Draft.

SIGHT COMMERCIAL DRAFT

Detroit, Mich., Jan. 2, 1900.

At sight, pay to the order of the First National Bank of Detroit———————————

Eighty-Five Dollars. \$85.00.

To A. B. Wade,
Lansing, Mich.

R. E. LAMSON.

Wade owed Lamson $85, and this is the means that Lamson took to procure payment. Wade paid the bank $85, and the bank paid Lamson $85 less ¼% discount for collecting.

401. A draft may be made payable a certain number of days after sight, or after date. Such a draft is called a Time Draft.

TIME COMMERCIAL DRAFT

St. Louis, Mo., Jan. 2, 1900.

Thirty days after date pay to the order of the Third National Bank, St. Louis,————————

One Hundred Fifty Dollars. $150.00.

To M. D. Clark, R. A. WALL.
Kansas City, Mo.

Clark owed Wall $150, due in 30 da., and Wall asked payment by means of this draft. He presented the draft to the bank and asked to have it discounted. The bank sent the draft to Clark, who "accepted" it—agreed to pay it on maturity. Clark wrote across the face of the draft: "Accepted, Jan. 3, 1900.—M. D. Clark," and returned the draft to the bank. The bank paid Wall $150 less the bank discount on $150 for 30 days, and an additional discount of ¼% for collecting.

If the drawer of a commercial draft indorses it and is a responsible party, the bank will pay him the proceeds of the draft (the face less the discount for time and for collecting) without waiting for the drawee to pay or accept the draft. If the drawee should refuse to accept and pay the draft, the drawer by his indorsement becomes responsible for payment. A draft that is not accepted by the drawee when duly presented is said to be "dishonored."

In States that allow grace, grace is allowed on time drafts.

PROBLEMS

Find the proceeds of the following drafts:

1. Albany, N. Y., July 6, 1900.

At sight pay to the order of the First National Bank, Albany, two hundred dollars.

To H. W. Reed. JAMES KIMM.

The bank discounted the draft at ¼%.

2. New York City, Aug. 14, 1900.

Sixty days after sight pay to the order of the Manhattan National Bank of New York one hundred eighty dollars.

To J. A. Weed, L. M. ROWE.
New York.

The bank immediately cashed the draft, discounting it at 6% for the time and $\frac{1}{4}$% for collecting.

3. Boston, Mass., Sept. 17, 1900.

Thirty days after date pay to the order of the Chemical National Bank, Boston, one hundred seventy-five dollars, and charge to my account.

To James Elliot, H. W. LONGWOOD.
Lynn, Mass.

The bank advanced the money, taking out time discount, money being worth 6%.

4. A Chicago broker bought a 60-da. commercial draft for $1800 on a New York company, at $\frac{1}{8}$% discount. Find the cost of the draft, money being worth 6%.

5. Find the face of the 90-da. commercial draft that can be bought for $825, premium $\frac{1}{8}$%, interest 6%.

EXPLANATION.—A draft for $1 will cost $1 + $.0025 premium —$.015 interest, or $.9875. If $.9875 will buy a draft for $1, $825 will buy a draft whose face is as many dollars as $.9875 is contained times in $825 or $833.44.

6. Barr and Creelman drew a 60-da. draft on the Wilson Mfg. Co., for $2360, and sold the draft to the Merchants' Bank, at $\frac{1}{8}$% discount, and interest at 6%. What were the proceeds?

7. A $580 sight draft was sold at a premium of 1%. Find the proceeds.

8. A Denver broker drew a 60-da. draft on a Chicago firm for $2400. Twenty days after date he sold the draft at a discount of $\frac{1}{6}$%, interest at 6%. What did he receive for the draft?

FOREIGN EXCHANGE

402. Foreign Exchange is that in which drafts drawn in one country are payable in another. It differs from domestic exchange in the currency used and in the manner of quoting the rate of exchange.

403. In foreign exchange drafts are called Bills of Exchange.

404. The rate of exchange on Great Britain is quoted by giving the value of 1 sov. in dollars and cents. Thus, when exchange on London is quoted 4.87, a draft for 1 sov. will cost $4.87.

405. Exchange on France is usually quoted by giving the value of $1 in francs. The quotation 5.15 means that $1 is worth 5.15 francs. Sometimes exchange is quoted by giving the value of 1 franc in cents. The quotation 19.6 means that 1 franc is worth 19.6¢.

406. In Germany, the exchange quotation 96 means that 4 marks are worth 96¢. The quotation 24 means that 1 mark is worth 24¢.

407. The monetary unit of Mexico is the dollar. The gold dollar, which contains nearly the same amount of pure gold as our dollar, is worth $.983 in U. S. money; and the silver dollar, which contains nearly the same amount of pure silver as our silver dollar, is worth from 44¢ to 49¢ in U. S. money, according to the fluctuation in the price of silver as compared with gold.

408. The gold in a sovereign is worth $4.8665 in American coined gold. The quotation 4.8665 is, therefore, at par. It is the par of exchange. The par of exchange on France is 5.18⅛, or 19.3, whichever way the quotation is made. The par of exchange on Germany is 95.2, or 23.8.

409. The Commercial Rate of Exchange may be at par, above par, or below par. It is the market value in one country of drafts on another.

410. Bills of exchange were formerly drawn in triplicate, that is, three bills were drawn of the same tenor and date, one

of which being paid the others became void. At present many bills are drawn in duplicate, and in many cases only a single bill is drawn. The object of sending two or three bills by different routes or at different times is to make sure that one copy will reach its destination.

PROBLEMS

1. What will be the cost in New York of a draft on London for 420 sov. 14s., exchange at 4.86?

2. When exchange on Liverpool, England, is quoted at 4.86⅛, what is the face of a draft that $1285 will buy?

3. What will be the cost of a bill of exchange on Berlin for 2540 marks, exchange at 95½?

4. Find the cost of a bill of exchange drawn on Paris for 35800 francs, exchange at 5.16.

5. What will be the cost in Paris of a bill of exchange drawn on New York for $875, exchange at 5.17?

6. A man owes a London merchant $4560. What is the face of the draft he should send, exchange at 4.86½?

7. A man in New York owes $3480 in U. S. money to a dealer in Mexico. If exchange on Mexico is ½% premium, what will be the cost of a draft to pay the debt?

8. A speculator in Chicago wishes to buy $10000 worth of Mexican stock, payable in silver. What must be the face of the draft if a Mexican dollar is worth 49¢ in U. S. money? What will the draft cost if exchange is at ⅜% premium?

9. I wish to invest $10000 in U. S. money in Mexican silver bonds. How many dollars' worth of the bonds can I buy if exchange is ⅝% premium and the par of exchange is 49?

10. A merchant in Mexico owes a dealer in New Orleans $18950 in U. S. money. How many Mexican silver dollars will be required to pay the debt, exchange at ⅜% discount and the par of exchange at 49?

ACCOUNTS AND BILLS

411. A owes B $100. To A this item is a debt, to B it is a credit.

412. That which one owes another is a Debt. It is a debt to the one owing it and a credit to the one to whom it is owed.

413. That which one has paid to another is a Credit. It is a credit to the one who has paid it, and a debt to the one who has received it.

What is bought or received is a debt, or debit.

What is sold or paid is a credit.

A person is debited *to* what he receives, and credited *by* what he parts with.

414. A record of one or more business transactions by two individuals or firms showing the proper debits and credits is an Account.

Thus, a merchant keeps an account with each person dealing with him. He heads a page of his ledger with the name of a customer. In a column headed "Dr." (debtor) he puts the values of what the customer receives. In a column headed "Cr." (creditor) he puts the sums that the creditor pays.

415. The difference between the amount of the debits and credits of an account is called the Balance of the Account.

If the balance is on the debtor side of the account, it shows that the customer has received more than he has paid for. It shows how much more he should still pay. It shows how much more must be added to the credit side of the account to balance the account.

If the balance is on the credit side of the account, it shows that the customer has paid for more than he has received. It shows how much must be added to the debit side of the account to balance the account.

416. The following account is taken from the account book of John Doe:

Dr. (has received).			WM. SMITH.				Cr. (has paid).
1900 Jan.	2	To lumber,	$14.00	1900 Jan.	3	By cash,	$12.50
	4	To flour,	7.80		5	By labor,	3.75
	5	To feed,	6.20		6	By balance,	11.75
			$28.00				$28 00

It is found that the Dr. side is $11.75 greater than the Cr. side, and that $11.75 must be added to the Cr. side to balance the account. This shows that Wm. Smith should pay John Doe $11.75.

BILLS

417. A detailed statement of goods sold or of services rendered is called a Bill. A bill of goods is also called an Invoice.

418. The following abbreviations and terms are frequently used:

acct.	account	*Pp.*, or *pp.*	pages
agt.	agent	*pay't*	payment
amt.	amount	*pd.*	paid
bal.	balance	*per*	by, or by the
bot.	bought	*pkg.*	package
chgd.	charged	*P. O.*	postoffice
Co.	Company	*pr.*	pair
com.	commission	*pc.*	piece
C. O. D.	collect on delivery	*rec'd*	received
con.	consignment	*rec't*	receipt
dft.	draft	*R. R.*	railroad
disc.	discount	*ship't*	shipment
exch.	exchange	*sunds.*	sundries
frt.	freight	*inst.*	present month
guar.	guaranteed	*prox.*	next month
i. e.	that is	*ult.*	last month
mdse.	merchandise	@	at
mem.	memorandum	#	number
Messrs.	Gentlemen	*a/c*	account
N. B.	take notice	*c/o*	care of
net	without discount	*n/c*	new account
No.	number	*o/c*	old account
P., or *p.*	page		

419. When goods are sold it is customary for the creditor to render a statement to the debtor.

On January 2, 1900, James Wilson sold Allen Jones 4 lb. coffee at 22¢, 15 lb. sugar at 6¢; Jan. 9, 2 bu. potatoes at 45¢; Jan. 11, 5 gal. oil at 8¢. The following statement was made out:

Rochester, N. Y., Jan. 11, 1900.

ALLEN JONES,
In Account with JAMES WILSON.
(Or, To James Wilson, Dr.)

Jan.	2	4 lb. coffee, @ 22¢,	$	88
	2	15 lb. sugar, @ 6¢,		90
	9	2 bu. potatoes, @ 45¢,		90
	11	5 gal. oil, @ 8¢,		40
		Total,	$ 3	08

A. S. Ward bought of H. D. West, Geneva, N. Y., May 7, 1900, 18 yd. calico at 9¢, 20 yd. flannel at 60¢; May 8, 12 yd. gingham at 10¢; May 14, 15 yd. factory at 7¢, 10 yd. cambric at 6¢. May 16 Ward paid $10. The following bill was rendered:

Geneva, N. Y., May 16, 1900.

A. S. WARD,
To H. D. WEST, Dr.

May	7	18 yd. calico, @ 9¢,	$ 1	62		
	7	20 yd. flannel, @ 60¢,	12	00	$13	62
	8	12 yd. gingham, @ 10¢,			1	20
	14	15 yd. factory, @ 7¢,	1	05		
	14	10 yd. cambric, @ 6¢,		60	1	65
		Total,			$16	47
		Cr.				
May	16	By Cash,			10	00
		Balance due,			$ 6	74

Receipting Bills

420. When a bill is paid it is receipted by the creditor or his agent. When paid in cash, the following may be written at the end of the bill:

Received payment [or, Paid],
JOHN DOE.

Or,

Received payment,
JOHN DOE,
Per James Fox. [Fox is agent].

When paid by a note the receipt may be:

Received payment [or, Paid] by note due Sept. 2,
JOHN DOE,
Per James Fox.

When a bill is not paid, after "Total," "Balance," or "Balance due," the words "Please remit," or "Rec'd payment," without the signature, may be written.

PROBLEMS

Render and receipt bills for the following transactions:

1. R. B. Hillman bought of Young & Taylor, July 5, 1900, 14 lb. steak at 12¢, 18 lb. lard at 10¢, 6 lb. pork sausage at 11¢; July 7, 20 lb. roast at 13¢, 15 lb. tallow at 7¢; July 9, 4 lb. steak at 12½¢, 12 lb. pork chops at 12¢. July 8, Hillman sold Young & Taylor a pig for $4.50, and paid the balance due July 10.

2. Stevens & Bacon, Cleveland, Ohio, sold R. S. Wall, Aug. 7, 1900, 4 ranges at $65, 5 wood stoves at $18; Aug. 10, 7 plows at $6.80, 12 shovels at 90¢; Aug. 13, 6 forks at 40¢. Aug. 15, Wall paid the bill by a note due Dec. 8, 1900.

3. A. B. Clark, agent for the Building Company, Detroit, Mich., sold Charles Hawes, Sept. 3, 1900, 25600 ft. pine flooring at $22.50 per M, 16700 ft. hemlock roofing at $14 per M; Sept. 8, 6900 ft. oak joists at $24.75 per M, 2150 lath at 40¢ per C. Sept. 6, Hawes sold the Building Company 8500 ft. pine logs at $9 per M, and 1600 ft. hemlock logs at $8 per M, and paid the balance due Sept. 12.

4. On August 27, 1902, L. Mitchell & Co., Chicago, sold to F. E. Baker, St. Louis, Mo., terms cash, the following items: 1640 ft. A flooring @ $24 per M, 920 ft. C flooring @ $18 per M, 2467 ft. fencing @ $16 per M, 5428 ft. scantling @ $13 per M, 1432 ft. timber @ $9.37½ per M, and 860 ft. timber @ $8.62½ per M.

BILLS—TRADE DISCOUNT

New York, Sept. 3, 1900.

MANNING & JONES, Albany, N. Y.,

Bought of NEW YORK SUPPLY COMPANY,

Terms: 30 days; 10% for cash.

Sept.	1	200 yd. Brussels carpet, @ $2.40,	$480	00		
	1	80 yd. wool cloth, @ 90¢,	72	00	$552	00
	3	160 yd. velvet carpet, @ $3,	480	00		
		Less 25%,	120	00		
			$360	00		
		Less 20%,	72	00	288	00
		Total,			$840	00
		Rec'd paym't Oct. 1, 1900,				
		NEW YORK SUPPLY COMPANY.				

EXPLANATION.—The full cost of each item is given. Then the discount is taken out, and the net cost written in the proper column. By the "Terms" a credit of 30 da. is given. If the bill is paid at date of last item a discount of 10% is allowed. The bill was paid and receipted Oct. 1. If it had not been paid by Oct. 3 (30 da. after date), it would have drawn interest from that date.

PROBLEMS

Render the following bills properly discounted and receipted:

1. H. A. Wood bought of Sawyer & Jones, Oct. 8, 1900, on account 30 da.: 24 tables at $12.50, less 25% and 15%; 16 lounges at $7.60, less 30% and 10%; and 12 chairs at $2.50, less 20% and 12½%. The bill was paid when due.

2. H. C. Kimball sold to Booth & Jessop, for cash, Oct. 9, 1900: 12 pianos at $275, less 33⅓%; 15 organs, at $110, less 25%; 25 violins at $8.50, less 20%.

3. C. D. Beam sold Austin Neff, Oct. 11, 1900, on 3 mo. credit: 200 bu. barley at 52¢; 180 bu. wheat at 74¢; 520 bu. corn at 45¢; 260 bu. oats at 40¢. A trade discount of 20% was allowed. Neff paid the bill Nov. 11, and received the proper time discount, money being worth 6%.

4. C. S. Fanchild bought of Wheeler & Wilson, Buffalo, N. Y., July 2, 1902, 2 doz. tables at $2.60 each, on 2 mo.; 20 sewing machines at $18, 12½% off; 4 doz. chairs at 75¢ apiece, on 30 da.; 8 desks at $45, 20% and 12½% off; 6 stands, at $28, 15% off. Find the amount due Aug. 1, 1902, money being worth 6%.

5. A. C. Taylor, Rochester, N. Y., sold to Wright & Young, July 1, 1902: 12400 ft. pine at $24 per M; 4650 ft. chestnut at $45 per M; 5900 ft. oak at $38 per M. On July 15, 1902, 24300 ft. oak at $35 per M; 36500 lath at 90¢ per C; 12420 ft. hemlock at $16 per M. A credit of 30 da. was allowed and the bill was paid Sept. 1, 1902, interest at 6%.

6. Render the bill and find the amount due August 21, 1902, interest and discount at 6%: Geo. C. Thirp, Chicago, Ill., sold to W. A. Parsons, May 1, 1902, 6 pc. cotton, 60, 65, 70, 58, 45, 75 yd., at 11¢; 8 pc. gingham, 52, 60, 46, 64, 68, 42, 70, 55 yd., at 12¢; June 10, 1902, 4 pc. shirting, 40, 46, 52, 58 yd., at 9¢; July 23, 1902, 4 pc. sateen, 51, 60, 73, 58 yd., at 7¢. A credit of 2 mo. was allowed on each item.

7. Wetmore & Wales, Cleveland, Ohio, sold to J. C. Johnson, on 2 mo. time, 10% off for cash, May 3, 1902, 15 dictionaries at $5.40, less 12½%; 25 atlases, at $4.50, less 10%; 4 doz. geographies at 80¢, less 15%; 5 doz. arithmetics at 50¢, less 16⅔%. Make out the bill for cash payment.

8. The St. Louis, Mo., Lumber Co. sold M. T. Evans, April 2, 1902, 14650 ft. pine at $26 per M, 3860 ft. clapboards at $25 per M; May 7, 1902, 960 posts at $8 per C, 11900 ft. flooring at $32 per M, 7250 shingles at $3.20 per M. A credit of 1 mo. was allowed, and discounts of 10% and 12½%. Make out the bill for payment Sept. 1, 1902.

9. The Metallic Tubing Co., New York City, sold N. M. Butler, April 18, 1902, 28 ft. ½-in. copper tubing at $1.20, less 15%; 120 ft. 1-in. copper tubing, at $1.60, less 16⅔%; 80 ft. 1½-in. steel tubing, at 60¢, less 15%. A credit of two months was allowed. Make out the bill for settlement Sept. 1, 1902, interest at 6%.

10. Werner & Jones, Detroit, Mich., sold H. A. Greenwood,

May 1, 1902, 148 yd. silk at \$1.30, less 16⅔%; 45 yd. lace at 80¢, less 20%; 100 yd. cashmere at 25¢, less 12½%; 320 yd. flannel at 33⅓¢, less 20%; 115 yd. lining at 11¢, less 8%; 200 yd. crash at 6¢, less 10%. A credit of 30 da. was allowed. Make out the bill for payment Sept. 1, 1902.

EQUATION OF BILLS

421. Debts draw interest after they become due.

422. Merchants frequently sell goods on credit, that is, the bill for payment of the goods does not become due, does not draw interest, till after a certain period of time; as, 1 mo., 2 mo., 60 da., etc.

423. The time that must elapse before a debt becomes due is called the Term of Credit.

424. A debt ought to be paid when due. If it is not paid when due, it should draw interest from the time it is due till the time it is paid. If it is paid before it is due, it should be discounted for the time between the date of payment and the date when it becomes due.

425. If a bill contains several items due at different times, each item need not be paid as it falls due, but the total of the bill may be paid on a certain date, so that the interest on the debts falling due after that date will equal the discount on the debts falling due before that date. Payment on such a date would be just to both debtor and creditor. That date is called the due date, average time, or equated time.

426. The Average Time, Equated Time, or Due Date is the date on which several debts, or items of a bill, may be cancelled by one payment.

427. Finding the equated time is called Equating or Averaging Bills.

428. The process of finding the equated or average time is called the Equation, or Average of Bills.

429. The time between the equated time and the due date of the earliest payment is called the Average Term of Credit.

430. The time between the equated time and the due date of the latest payment is called the Average Term of Discount.

A owes B $100 to be paid on Jan. 1, $100 to be paid Jan. 11, and $100 to be paid Jan. 21. When may he pay $300 and avoid paying interest? How does the interest on $100 from Jan. 1 to Jan. 11 compare with the discount on $100 from Jan. 11 to Jan. 21?

EXAMPLE 1.—On Jan. 1, 1900, N. E. Spenser bought of James Hayes mdse. as follows:

Date When Due	Amount
Jan. 16, 1900	$100
Jan. 25, 1900	75
Feb. 9, 1900	200
March 1, 1900	150
Total	$525

On what date can Spenser pay $525 and avoid the payment of interest?

EXPLANATION.—If he pays it Jan. 16, the date when the first item becomes due, he should be allowed a discount on the other three items for payment in advance. If he waits until March 1, the date when the last item becomes due, he should pay interest on the first three items. He wishes to find a date, between Jan. 16 and March 1, on which he may pay $525, so that the discount on the first items shall equal the interest on the latter items.

Suppose he does not pay until March 1, the due date of the last item. That would be the natural time of settlement. Then the amount he would owe is shown as follows (int. at 6%):

OPERATION

Due Date	Items	Term of Interest	Interest
Jan. 16	$100	44 da.	$.73
Jan. 25	75	35 da.	.44
Feb. 9	200	20 da.	.67
March 1	150	0 da.	.00
	$525		$1.84

$525 + $1.84 = $526.84, due March 1

How many days before March 1 should he pay the $525, so that he will not have to pay any of the $1.84 interest? As many days as it will take for the interest on $525 to amount to $1.84. In how many

days will $525 produce $1.84 interest, at 6%? $1.84 ÷ $.875 (int. on $525 for 1 da.) = 21 da. Then, to avoid paying any interest he should pay $525 21 da. before March 1, 1900. 21 da. before March 1 is Feb. 8, which is the average time, or due date, for paying the whole amount. 21 da. is the term of discount.

PROOF.—To prove that Feb. 8 is the equated time, it is only necessary to show that the interest on the money due before that time is equal to the discount on the money due after that time:

Due	Items	Term of Interest to Feb. 8	Interest	
Jan. 16	$100	23 da.	$.38	
Jan. 25	75	14 da.	.18	$.56
		Term of Discount from Feb. 8	Discount	
Feb. 9	200	1 da.	$.03	
March 1	150	21 da.	.53	$.56

EXAMPLE 2.—Wood & Wilson sold goods to J. E. Almy as follows:

Jan. 4, 1900..... $225
Jan. 22, ... 340, on 2 mo. credit
Feb. 6, ... 160
Feb. 27, ... 180, on 60 da. credit

Find the average date, or the date from which the whole sum due should draw interest.

OPERATION

Due Date	Items	Term of Interest	Interest
Jan. 4	$225	114 da.	$25.65*
March 22	340	37 da.	12.58
Feb. 6	160	81 da.	12.96
April 28	180	0 da.	0 00
	$905		$51.19

*For convenience find the interest by the 1000-day method, interest at 36%. That is, multiply the dollars by the days and point off three places.

Interest on $905 for 1 da. is $.905.

$51.19 ÷ $.905 = 56.5 da., or 57 da., the term of discount.

57 da. before April 28 is March 2, the average time.

EXPLANATION.—For convenience arrange the work in columns as above. Add the proper term of credit (calendar months when it reads months, and the actual number of days when it reads days) to each credit item, and that will give the true due date of each item. As in the preceding example, assume that the account was settled when the last item became due. The terms for which the various items would then draw interest are 114, 37, 81, and 0 days, respectively. Find the interest on the items, by the 1000-day method, at 36%. The total interest is $51.19. It will take as many days for $905 to produce $51.19 interest as the interest on $905 for 1 da. is contained times in $51.19. The interest on $905 for 1 da. is found by pointing off three places. $ 905 is contained 56.5 times in $51.19. Therefore, $905 will produce $51.19 interest in 56.5 da., or 57 da. 57 da. is the average term of discount, and 57 da. before April 28, or March 2, is the average time of payment.

Steps in the Operation.—1. Find the due date of each item by adding the proper term of credit. When it reads days, add the number of days; when it reads months, add the number of months.

2. Assume a settlement on the latest due date, which is called the Focal Date. Find the term of interest for each item—the number of days from each due date to the focal date.

3. Find the interest on each item for its term of interest, and the interest on the sum of the items for 1 da.

4. Divide the total interest due on the items by the interest on the sum of the items for 1 da.

NOTE.—Steps 4 and 5 may be briefly stated: Multiply each item by the term of interest in days. Divide the sum of these products by the sum of the items.

5. Find the average time by counting back from the focal date the number of days in the average term of discount.

NOTES.—1. Any date may be taken as a focal date. Some take the earliest due date, and discount the items due in the future for payment in advance. But it is more in line with business practice to take the latest due date and add interest to the items not paid when due. An account ought to be settled after it is made. It cannot be settled before it is made. Therefore it is better to take the latest instead of the earliest due date as a focal date.

2. In finding the average term of credit, a fraction of a day of one-half or more is counted as a full day.

PROOF

Due	Items	Term of Interest	Interest	
Jan. 4	$225	57 da.	$12.83	
Feb. 6	160	24 da.	3.84	$16.67
		Term of Discount	Discount	
March 22	340	20 da.	$ 6.80	
April 28	180	57 da.	10.26	$17.06

The difference between the interest and discount is $.39. The interest or discount on the sum due, for 1 da., is $.905, therefore the operation is correct. It should be remembered that the division in the operation of finding the equated time in the example was not exact, and that a fraction of a day was called a whole day. That fraction of a day in the average term makes the difference between the interest and the discount in the proof. The difference between the interest and the discount in the proof should always be less than the interest on the debt for one-half of a day.

The Product Method

431. The interest on the items for the terms of credit by the 1000-day 36% method is found by taking the products of the number of dollars by the number of days in the terms of credit and pointing off 3 places. The product method is similar to this interest method.

EXAMPLE.—When is the following statement due by equation?

W. A. MILLARD,

To C. E. BIGELOW, Dr.

June	2	To Mdse., on 30 da.,	$150
June	19	To Mdse.,	220
July	3	To Mdse., 1 mo.,	180
July	22	To Mdse.,	140

OPERATION

Due Date	Items		Term of Interest		Products
July 2	$150	×	32 da.	=	$4800
July 19	220	×	15 da.	=	3300
July 22	140	×	12 da.	=	1680
Aug. 3	180	×	0 da.	=	0
	$690				$9780

$9780 ÷ $690 = 14+. 14 da. before Aug. 3 is July 20, average due date.

EXPLANATION.—The arrangement of the work is similar to that in the interest method. The latest due date is Aug. 3. If the first item is not paid till Aug. 3, the debtor will have the use of \$150 for 32 da., which is equivalent to the use of \$150 × 32 da., or \$4800 for 1 da. If the items are not paid till Aug. 3, the debtor has had the equivalent of the use of \$9780 for 1 da. He should pay the \$690 long enough before Aug. 3 that the creditor will have the equivalent of the use of \$9780 for 1 da. \$9780 ÷ \$690 = 14+.

If the debtor pays \$690 14 da. before Aug. 3, or on July 20th, no interest will be due either party.

PROBLEMS

When are the following bills due by equation? Prove each operation:

1. E. N. BAKER,
To WM. GRAY, Dr.

1900			
Jan.	2	To Mdse.,	\$140
	16	To Mdse., 1 mo. credit,	125
Feb.	7	To Mdse., 30 da. credit,	160
	26	To Mdse., 60 da. credit,	115

2. THOMAS GOODE,
To STEVENS & BACON, Dr.

1900			
Jan.	4	To Mdse., 2 mo.,	\$ 85
	29	To Mdse.,	145
March	5	To Mdse., 1 mo.,	175
	26	To Mdse.,	130

3. J. H. ROWE,
To JAMES MURDOCK, Dr.

1900			
April	2	To Mdse., 30 da.,	\$120
	16	To Mdse., 2 mo.,	240
May	1	To Mdse., 30 da.,	90
	24	To Mdse.,	118
	30	To Mdse.,	70

4. C. A. WOOD,
To POTTER & BOWEN, Dr.

1900			
May	3	To Mdse.,	\$ 70
	15	To Mdse., 1 mo.,	110
June	1	To Mdse., 60 da.,	265
	11	To Mdse., 30 da.,	210
	30	To Mdse.,	180

5. GEO. CLARK,
To SAMUEL TAYLOR, Dr.

1900			
June	3	To Mdse., 3 mo.,	$275
	19	To Mdse., 30 da.	120
	28	To Mdse., 60 da.,	90
July	5	To Mdse.,	310
	20	To Mdse., 1 mo.,	400

6. J. O. MORGAN,
To C. D. WRIGHT, Dr.

1900			
July	3	To Mdse., 30 da.,	$230
	9	To Mdse., 2 mo.	170
Aug.	1	To Mdse., 30 da.,	260
	17	To Mdse., 30 da.,	220
Sept.	1	To Mdse., 1 mo.,	180

7. H. H. SHORT,
To R. P. REED, Dr.

1900			
Sept.	3	To Mdse.,	$300
	10	To Mdse., 1 mo.,	150
	26	To Mdse., 2 mo.,	140
Oct.	1	To Mdse., 1 mo.,	190
	18	To Mdse., 30 da.,	250

8. BENTON COY,
To RAYMOND HOWE, Dr.

1900			
Sept.	1	To Mdse.,	$430
	13	To Mdse., 30 da.,	160
	29	To Mdse., 1 mo.,	90
Oct.	22	To Mdse.,	130
Nov.	2	To Mdse.,	220

EQUATION OF ACCOUNTS

432. Accounts are equated in the same manner as are bills, but in accounts there is usually a debit and a credit side.

Instead of finding the amount of the debit side as in a bill, we find the balance of the two sides. The equated time of an account is the time on which the balance of the account is due, or the time from which the balance should draw interest.

The focal date should be the latest due date in the account.

EXAMPLE.—What should be the face and the date of a note given to settle the following account?

DR. J. C. BARR. CR.

1900				1900			
May	1	To Mdse.,	$100	May	14	By Cash,	$ 90
May	16	To Mdse.,	120	May	21	By Note,	100
June	1	To Mdse.	300	June	8	By Cash,	200

OPERATION

Due	Items	Term	Int.	Paid	Items	Term	Int.
May 1	$100	38 da.	$3.80	May 14	$ 90	25 da.	$2.25
May 16	120	23 da.	2.76	May 21	100	18 da.	1.80
June 1	300	7 da.	2.10	June 8	200	0 da.	.00
	$520		$8.66		$390		$4.05
	390		4.05				
Balance,	$130		$4.61				

Int. on $130 for 1 da. is $.13. $461 ÷ $.13 = $35\frac{6}{13}$, or 35 da.
35 da. before June 8 is May 4, the average date.
The face of the note should be $130, and the date May 4, 1900.

PROOF

Dr.	Items	Term	Int.	Cr.	Items	Term	Int.
May 1	$100	3 da.	$ .30 (disc.)	May 14	$ 90	10 da.	$ .90
May 16	120	12 da.	1.44	May 21	100	17 da.	1.70
June 1	300	28 da.	8.40	June 8	200	35 da.	7.00
			$9.54				$9.60
							9.54
						Balance of int.,	$ .06

EXPLANATION.—Find the difference between the discount and the interest on each side of the account, then the difference of discount or interest between the two sides should be less than one-half of the interest or discount on the balance of the account for one day.

PROBLEMS

When should interest begin on the following accounts? Prove each operation.

1.

DR. ROBERT A. WALKER. CR.

1900				1900			
Jan.	2	To Mdse.,	$400	Jan.	4	By Cash,	$300
Jan.	27	To Mdse.,	300	Feb.	6	By Cash,	250

2.

DR. J. C. MILLER. CR.

1900				1900			
May	1	To Mdse.,	$200	May	25	By Note, on int.,	$180
May	12	To Mdse., 30 da.,	300	June	12	By Cash,	400
June	1	To Mdse.,	350	July	6	By Cash,	300
June	18	To Mdse., 2 mo.,	200				

3.

DR. A. B. CLAYTON. CR.

1900				1900			
June	4	To Mdse., 30 da.,	$310	June	7	By Cash,	$150
June	27	To Mdse., 2 mo.,	160	July	3	By Note,	100
July	6	To Mdse.,	150	July	16	By Cash,	100
Aug.	8	To Mdse.,	240	Aug.	15	By Cash,	120
Aug.	13	To Mdse.,	400	Aug.	20	By Cash,	375

4.

DR. L. C. BRADLEY. CR.

1900				1900			
July	6	To Mdse.,	$250	July	23	By Cash,	$400
July	17	To Mdse., 1 mo.,	260	Aug.	17	By Cash,	100
Aug.	4	To Mdse., 2 mo.,	300	Aug.	24	By Cash,	230
Aug.	28	To Mdse.,	400	Sept.	4	By Cash,	300

OPERATION

Due	Items	Term	Interest	Paid	Items	Term	Interest
July 6	$ 250	90 da.	$22.50	July 23	$ 400	73 da.	$29.20
Aug. 17	260	48 da.	12.48	Aug. 17	100	48 da.	4.80
Aug. 28	400	37 da.	14.80	Aug. 24	230	41 da.	9.43
Oct. 4	300	0 da.	.00	Sept. 4	300	30 da.	9.00
	$1210		$49.78		$1030		$52.43
	1030						49.78
Balance,	$ 180					Balance,	$ 2.65

Int. on $180 for 1 da. is $.18.

$2.65 ÷ $.18 = $14\frac{1}{18}$, or 15 da., the average term.

15 da. *after* Oct. 4 is Oct. 19, the average due date.

EXPLANATION.—This problem differs from the preceding only in that the balances of the items and the interest are on opposite sides, instead of on the same side of the account. The balance of the items shows that there is still due $180. The balance of the interest shows that $2.65 more interest has been paid than is due. Therefore, $180 should not be paid until as many days *after* Oct. 4 as it will take $180 to produce $2.65 interest, which is found to be 15 da.

Principle.—When the balance of the account and the balance of interest are on the same side of the account, date back from the focal date; when the balance of the account and the balance of interest are on opposite sides of the account, date forward.

5.

DR. H. A. GIBBS. CR.

1900				1900			
Oct.	1	To Mdse.. 1 mo.,	$200	Oct.	3	By Cash,	$175
Oct.	23	To Mdse.,	360	Oct.	10	By Mdse.,	300
Oct.	30	To Mdse., 1 mo.,	400	Nov.	1	By Cash,	400
Nov.	6	To Mdse.,	320	Nov.	13	By Cash,	350
Nov.	17	To Mdse.,	480	Nov.	19	By Cash,	200

6.

DR. O. S. CONNOR. CR.

1900				1900			
Oct.	19	To Cash,	$350	Oct.	4	By Mdse., 2 mo.,	$700
Oct.	30	To Note, on int.,	460	Oct.	26	By Mdse., 1 mo.,	100
Nov.	9	To Cash,	400	Nov.	5	By Mdse., 1 mo.,	500
Nov.	21	To Mdse.,	520	Nov.	24	By Mdse., 1 mo.,	680
Nov.	30	To Cash,	250	Nov.	29	By Mdse.,	230
Dec.	18	To Mdse.,	420	Dec.	20	By Mdse.,	800

7.

DR. A. S. WISE. CR.

1900				1900			
Nov.	1	To Mdse., 3 mo.,	$620	Nov.	9	By Cash,	$200
Nov.	13	To Mdse., 1 mo.,	175	Nov.	23	By Note, 2 mo.,	550
Dec.	6	To Mdse., 2 mo.,	340			no interest,	
Dec.	10	To Mdse., 1 mo.,	450	Dec.	27	By Mdse.,	975
Dec.	21	To Mdse.,	520	Dec.	29	By Mdse.,	140
Dec.	31	To Mdse.,	380	Dec.	31	By Cash,	250

8.

DR. J. A. NEWTON. CR.

1900				1900			
Jan.	4	To Mdse.,	$310	Jan.	10	By Note, on int.,	$250
Jan.	25	To Mdse.,	530	Feb.	6	By Cash.,	625
Feb.	9	To Mdse.,	160	Feb.	13	By Mdse.,	100
Feb.	26	To Mdse.,	315	Feb.	28	By Cash,	240
March	5	To Mdse.,	650	March	7	By Cash,	325

9.

DR. WILLARD DOWN. CR.

1900				1900			
May	7	To Mdse., 1 mo.,	$425	May	10	By Note, on int.,	$350
May	26	To Mdse., 2 mo.,	375	May	29	By Cash,	400
June	2	To Mdse., 1 mo.,	540	June	12	By Note, 2 mo., no int.,	550
June	21	To Mdse.,	250				
June	30	To Mdse.,	190	July	3	By Cash,	270
July	6	To Mdse.,	460	July	25	By Cash,	500

10.

DR. D. M. SUTTON. CR.

1900				1900			
June	2	To Mdse., 1 mo.,	$160	June	19	By Mdse., 1 mo.,	$350
June	12	To Mdse.,	325	June	30	By Cash,	100
July	6	To Mdse.,	250	July	21	By Cash,	475
July	20	To Mdse.,	340	July	28	By Mdse.,	120
Aug.	3	To Mdse.,	520	Aug.	8	By Cash,	450
Aug.	25	To Mdse.,	425	Aug.	31	By Cash,	200

11.

DR. R. C. PERRY. CR.

1900				1900			
Sept.	1	To Note, on int.,	$250	Aug.	24	By Mdse., 60 da.,	$275
Sept.	14	To Mdse., 1 mo.,	325	Sept.	8	By Mdse., 1 mo.,	380
Oct.	1	To Cash,	240	Sept.	29	By Mdse., 30 da.,	350
Oct.	19	To Cash,	360	Oct.	9	By Mdse., 1 mo ,	220
Oct.	30	To Cash,	150	Oct.	27	By Mdse., 1 mo.,	460
Nov.	20	To Cash,	280	Nov.	13	By Mdse.,	150

12. Find the balance of the following account, when the balance was due by equation, and what the balance amounted to if not paid until June 1, 1900, money being worth 6%:

DR. D. A. BROOKS. CR.

1900				1900			
Jan.	4	To Mdse.,	$300	Jan.	9	By Cash,	$200
Jan.	18	To Mdse.,	160	Jan.	25	By Cash,	240
Feb.	1	To Mdse.,	420	Feb.	10	By Cash,	360
Feb.	16	To Mdse.,	350	Feb.	28	By Cash,	210

13. Find when the balance of the following account was due, and what was paid to settle the account April 1, 1900, interest at 6%:

DR. HAMILTON SEAMANS. CR.

1900				1900			
Feb.	2	To Mdse., 1 mo.,	$275	Feb.	8	By Mdse., 1 mo.,	$190
Feb.	13	To Mdse., 1 mo.,	360	March	1	By Cash,	275
March	9	To Mdse.,	430	March	20	By Cash,	260
March	28	To Mdse.,	110	March	30	By Cash,	235

14. Equate the following account:

DR. J. P. WORTH. CR.

1900				1900			
Jan.	5	To Mdse.,	$300	Jan.	9	By Cash,	$260
Jan.	23	To Mdse.,	250	Jan.	30	By Cash,	180
Feb.	8	To Mdse.,	175	Feb.	13	By Note, 1 mo., no int.,	350
Feb.	27	To Mdse.,	400	March	8	By Cash,	335

OPERATION

Due	Items	Term	Interest	Paid	Items	Term	Interest
Jan. 5	$ 300	67 da.	$20.10	Jan. 9	$ 260	63 da.	$16.38
Jan. 23	250	49 da.	12.05	Jan. 30	180	14 da.	2.52
Feb. 8	175	33 da.	5.78	March 13	350	0 da.	0.00
Feb. 27	400	17 da.	6.80	March 8	335	5 da.	1.68
	$1125		$44.73		$1125		$20.58
	1125		20.58				
			$24.15				

$24.15 ÷ 6 = $4.03, balance due March 13.

EXPLANATION.—There is no balance in this account, the two sides being equal, but there is an interest balance of $24.15 on the Dr. side, which shows that that amount should be paid or added to the Cr. side. But the interest as here reckoned is at 36%. Since the true rate is 6%, divide $24.15 by 6, and the quotient will show the interest balance due March 13, 1900.

15. Equate the following account, interest at 4%:

DR. L. M. BEEKMAN. CR.

1900				1900			
Feb.	3	To Mdse.,	$250	Feb.	6	By Cash,	$170
Feb.	21	To Mdse.,	160	Feb.	26	By Cash,	190
March	7	To Mdse.,	300	March	14	By Mdse.,	400
March	31	To Mdse.,	270	Apr.	4	By Cash,	200
Apr.	9	To Mdse.,	400	Apr.	25	By Cash,	420

ACCOUNTS CURRENT

433. A statement of a running account showing the debits and credits and the cash balance, with interest or discount to date, is called an Account Current.

434. Adjusting an account is finding the cash balance due at a given date.

435. Theoretically, all sums due draw interest, and all sums paid before they are due are subject to discount.

Most retail dealers do not charge interest on the items of a running account, but the balance of a closed account draws interest from the date of the last item.

Custom or agreement between wholesale dealers determines whether the items of a running account draw interest. It is customary to charge interest on such items after a certain term of credit.

436. Equating an account is finding at what date the balance is due. Adjusting an account is finding the balance due at a given date.

EXAMPLE.—Find the balance due on the following account June 1, 1900, interest at 6%.

DR. R. H. KNAPP. CR.

1900				1900			
March	6	To Mdse., 1 mo.,	$350	March	21	By Cash,	$200
March	29	To Mdse., 2 mo.,	400	Apr.	16	By Mdse., 1 mo.,	360
Apr.	7	To Mdse.,	200	May	3	By Cash,	270
Apr.	25	To Mdse., 2 mo.,	250	May	26	By Mdse., 1 mo.,	300

OPERATION

Due	Items	Term	Int.	Disc.	Paid	Items	Term	Int.	Disc.
April 6	$ 350	56 da	$3.27		Mar. 21	$ 200	72 da.	$2.40	
May 29	400	3 da.	.20		May 16	360	16 da.	.96	
April 7	200	55 da.	1.83		May 3	270	29 da.	1.31	
June 25	250	24 da.		$1.00	June 26	300	25 da.		$1.25
	$1200		$5.30	$1.00		$1130		$4.67	$1.25

$1200 + $5.30 — $1 = $1204.30. $1130 + $4.67 — $1.25 = $1133.42.
$1204.30 — $1133.42 = $70.88, balance due June 1, 1900.

EXPLANATION.—Find the interest on each item from the day it is due to the day of settlement. If an item falls due after the date of settlement, it should be discounted, and the discount should be taken from the amount due at the date of settlement. The sum of the items, plus the interest, less the discount, is the total amount of either side of the account.

PROBLEMS

1. Find the balance due May 1, 1900:

DR. A. L. KINNEY. CR.

1900				1900			
Feb.	2	To Mdse., 2 mo.,	$420	Feb.	5	By Mdse., 1 mo.,	$360
Feb.	21	To Mdse., 1 mo.,	250	Feb.	24	By Cash,	240
March	3	To Mdse., 1 mo.,	160	March	16	By Cash,	150
March	20	To Mdse., 1 mo.,	380	March	27	By Mdse., 1 mo.,	400
March	24	To Mdse., 1 mo.,	300	Apr.	17	By Cash,	200

2. What is the balance due June 1, 1900?

DR. F. A. SAYRE. CR.

1900				1900			
March	6	To Mdse.,	$225	March	19	By Mdse., 2 mo.,	$350
March	21	To Mdse.,	500	Apr.	6	By Cash,	400
Apr.	14	To Mdse.,	340	Apr.	25	By Mdse., 2 mo.,	650
May	5	To Mdse., 1 mo.,	720	May	11	By Cash,	450
May	23	To Mdse., 1 mo.,	400	May	25	By Cash,	300

3. What is the balance due Sept. 1, 1900?

DR. J. H. HADLEY. CR.

1900				1900			
June	2	To Mdse.,	$150	June	7	By Cash,	$100
June	20	To Mdse.,	270	June	29	By Cash,	175
July	11	To Mdse.,	500	July	6	By Mdse., 1 mo.,	600
July	23	To Mdse.,	120	July	27	By Cash,	100
Aug.	4	To Mdse.,	310	Aug.	1	By Mdse.,	250
Aug.	22	To Mdse.,	230	Aug.	29	By Cash,	200

4. What is the balance due Oct. 1, 1900?

DR. C. F. CHASE. CR.

1900				1900			
July	7	To Mdse.,	$350	July	2	By Mdse., 1 mo.,	$400
July	28	To Mdse.,	540	July	17	By Mdse., 1 mo.,	250
Aug.	9	To Mdse.,	200	Aug.	1	By Cash,	100
Aug.	29	To Cash,	100	Aug.	15	By Mdse., 1 mo.,	500
Sept.	8	To Cash,	400	Sept.	5	By Mdse., 1 mo.,	350
Sept.	26	To Cash,	200	Sept.	21	By Mdse., 1 mo.,	220

ACCOUNT SALES

437. A statement rendered by an agent, showing his sales for his principal, his charges against the principal, the amount previously remitted (if any), and the amount due at the equated date, or the amount due at a given date, is called an Account Sales.

438. The agent's gross sales constitute the credits of the account, and his charges constitute the debits.

439. The agent frequently guarantees the quality of the goods he sells for his principal, for which he is allowed a compensation called Guaranty. Guaranty, like commission, is computed at a certain rate per cent on the sales.

440. The agent's charges include commission, guaranty, freight, cartage, storage, insurance, etc.

441. An account sales may be rendered simply as an equated account or as an account current. That is, it may show the balance due on the equated date or on a given date.

Items of freight, cartage, storage, and insurance are rendered as due on the date the agent paid them.

Commission and guaranty are sometimes considered as due on the date the account is rendered, sometimes on the date of the last sale, sometimes on the date of each sale, sometimes on the average date of the sales, and sometimes on the average *due* date of the sales. When sales are made on credit, the average date of sales is not the same as the average *due* date of the sales.

An account sales is equated or adjusted like any other account.

PROBLEMS

1. W. S. Davis, Chicago, Ill., sold lumber for Aldridge & Bro., Milwaukee, Wis., as follows: Jan. 3, 1900, 24000 ft. hemlock at $13 per M; Jan. 9, 59000 ft. pine at $24 per M; Jan. 26, 18500 ft. chestnut at $42 per M; Feb. 5, 27300 ft. oak at $40.50 per M; Feb. 13, 35700 ft. pine at $23 per M. The agent paid for freight, on Jan. 2, $275; $62 for storage,

on the date of the last sale; Jan. 29, he advanced $2500. His commission was 4%, due on the average date of sales. He rendered his account sales Feb. 17, 1900. Reproduce the account.

2. Find the balance of the following account sales, and when due by equation. Consider the entire commission due on the date of the last sale:

New York City, Nov. 7, 1900.

Account Sales of Apples,

For % of WILSON & CO.,

Buffalo, N. Y.

By J. C. FOWLER.

1900		SALES (CR.)				
Oct.	5	240 bbl. Snow, @ $2.10, cash,	.....	....		
	10	350 bbl. King, @ $1.85, 1 mo.,	.....	....		
	16	180 bbl. Greening, @ $1.50, 1 mo.,	.....	....		
Nov.	7	400 bbl. Baldwin, @ $1.60, cash,	.....	....		
		Total, Cr.,			.....	....
		CHARGES (DR.)				
Oct.	1	Freight,	$525	00		
	15	Cartage,	48	00		
	23	Cash advanced,	800	00		
Nov.	6	Storage,	40	00		
	7	Commission and guaranty, 3%,	.....	....		
		Total, Dr.,			.....	....
		Net proceeds, Due ....., 1900.				

3. Find the net proceeds and when due of the following account sales:

1902		SALES (CR.)				
July	3	25 bbl. Pork, @ $12.80, 30 da.,	.....	....		
July	23	45 bbl. Pork, @ $12.60, cash,	.....	...		
Aug	6	40 bbl. Pork, @ $13.00, 20 da.,	.....	....		
Aug.	17	30 bbl. Pork, @ $12.85, 10 da.,	.....	....		
		Total, Cr.,			.....	....
		CHARGES (DR.)				
July	1	Freight,	$ 71	40		
July	1	Cash advanced,	250	00		
		Commission (due.....), 4%,	.....	....		
		Total, Dr.,			.....	....
		Net proceeds,				

PARTNERSHIP

PARTITIVE PROPORTION

442. Partitive Proportion is the process of dividing a number into parts proportional to two or more given numbers.

PROBLEMS

1. A worked 3 days and B 4 days, for the same daily pay. Altogether they received \$14. What was the daily wages of each?

2. Two men performed a piece of work for \$40. The first agreed to take \$3 for every \$5 received by the second. How much did each receive?

3. Divide \$60 into parts proportional to 2, 4, and 6.

4. Divide \$216 into parts which shall be to one another as 5, 6, and 7.

5. Divide 750 into parts proportional to 10, 15, and 25.

6. A, B, and C put their sheep into one flock and agreed to sell them at a common price. A put in 40 sheep, B put in 70 sheep, and C put in 85 sheep. They sold the flock for \$1170. What did each man receive?

7. A has \$500, B \$600, and C \$400 invested in a business. What fractional part of the gain ought each to receive? If the whole gain is \$150, what will be A's share?

8. Two men engage in business. A puts in \$150, and B puts in \$270. If they gain \$210, how much should each receive?

9. Three men invest the following sums in a store: \$1200, \$1400, \$1800. If they gain \$550, what sum should each receive?

10. Divide 75 into parts proportional to $\frac{1}{2}$ and $\frac{1}{3}$.

NOTE—Fractions to be compared must have a common denominator. Then they are to each other as their numerators. $\frac{1}{2}$ and $\frac{1}{3}$ are equivalent to $\frac{3}{6}$ and $\frac{2}{6}$, and are to each other as 3 and 2. Therefore, divide 75 into parts proportional to 3 and 2.

11. Divide 72 into parts proportional to $\frac{1}{3}$ and $\frac{1}{5}$.

12. Divide 470 into parts proportional to $3\frac{1}{2}$ and $4\frac{1}{3}$.

13. Divide \$1105 into parts proportional to $\frac{3}{8}$, $\frac{2}{3}$, and $\frac{4}{5}$.

14. Divide \$373.10 into parts proportional to 1475, 1325, and 2530.

15. Divide \$405 into parts proportional to $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

PARTNERSHIP

443. An association formed by two or more persons investing capital in a business and agreeing to share the gains and losses of the business is called a Partnership.

444. The persons that form the association are called Partners. Collectively, they are called a Company, Firm, or House.

445. The capital invested may be money, other property, or labor.

446. The gains and losses of a partnership are shared in proportion to the value of the capital invested and the time the capital is employed.

447. Resources, or Assets, consist of the property of the firm and the debts due the firm.

448. Liabilities are the debts of a firm.

449. The Net Capital is the excess of the assets over the liabilities.

450. The Net Insolvency is the excess of the liabilities over the assets.

451. With respect to their manner of connection with a firm, there may be four kinds of partners: real, dormant, nominal, limited.

452. A Real, or Ostensible Partner, is one who has capital invested, and is simply a partner without restrictions or conditions.

453. A Dormant, or Silent Partner, is one who has capital invested, but who tries to conceal the fact and does not appear to the public as a partner.

454. A Limited Partner is one who gives legal notice, by publication, of the limit of his responsibility for the debts of the firm.

455. A Nominal Partner is a partner in name only. He has no capital invested, and allows the use of his name as a partner simply to give prestige to the firm.

456. A real partner is liable for all the debts of the firm. A silent partner is also liable for the debts of the firm, but he cannot be held responsible unless his connection with the firm is known. A limited partner is liable to a limited extent. A nominal partner is liable for the debts of the firm to all persons who have trusted the firm because such partner was a member of the firm. If persons are deceived by a nominal partner, the partner should pay for the deception.

EXAMPLE.—A, B, and C formed a partnership. A furnished $4000 of the capital, B $6000, and C $8000. If they gained $1440, what was each partner's share of the gain?

OPERATION

$4000 + $6000 + $8000 = $18000, total capital
$\frac{4000}{18000}$ or $\frac{2}{9}$ = A's share of the capital
$\frac{6000}{18000}$ or $\frac{1}{3}$ = B's share of the capital
$\frac{8000}{18000}$ or $\frac{4}{9}$ = C's share of the capital
$\frac{2}{9}$ of $1440 = $320, A's share of the gain
$\frac{1}{3}$ of $1440 = $480, B's share of the gain
$\frac{4}{9}$ of $1440 = $640, C's share of the gain

Or, $4000 + $6000 + $8000 = $18000, total capital
$1440 = net gain
$1440 ÷ $18000 = .08, or 8 per cent gain
8% of $4000 = $320, A's gain
8% of $6000 = $480, B's gain
8% of $8000 = $640, C's gain

PROBLEMS

1. Jones, Smith, and Brown formed a partnership, Jones putting in $7000 of the capital, Smith $5000, and Brown $9000. If they gained $2940, what was each one's profit?

2. Beekman, Hadley, and Perry entered a partnership, Beekman furnishing $12000 capital, Hadley $8000, and Perry $11000. If they lost $1240, what was each partner's loss?

3. Two men bought a farm for $15000, one paying $9000 and the other $6000. If they sold the farm for $17500, what was the gain of each?

4. The total assets of the firm of Watson & Barnes are $18700, and the liabilities are $4200. Watson invested $5400, and Barnes invested $6300. Find the net gain and the present worth of each partner.

5. Four partners invested as follows: A $4800, B $7200, C $6000, D $8400. After one year their resources are $32270, and their liabilities are $2570. Find the present worth of each partner.

6. Gooding and Spenser formed a partnership, Gooding investing $21000 and Spenser $17000. After two years they dissolved partnership with the following resources and liabilities:

Resources		Liabilities	
Mdse., per inventory	$ 9600	Mortgage	$2000
Cash	6260	Accounts payable	1500
Accounts receivable	12300		
Real estate	15500		

Find the present worth of each partner.

7. Hawes & Cross bought a mill for $60000, Hawes paying $35000 and Cross $25000. Newcomb paid them $40000 for a one-third interest in the mill. What part of the mill did Hawes and Cross each own? What part of the $40000 did each receive? If, after the sale, each had been credited with a one-third interest in the mill, what part of the $40000 ought Hawes to have received?

8. A began business with a capital of $15000. After 3 mo. he took in B as a partner with $12000, and 3 mo. later they took in C with $16000. If the total gain for the year was $2592, what was each partner's share?

OPERATION

A's capital = \$15000 employed for 12 mo. = \$180000 for 1 mo.
B's capital = 12000 employed for 9 mo. = 108000 for 1 mo.
C's capital = 16000 employed for 6 mo. = 96000 for 1 mo.
Total capital employed for 1 mo. = 384000

A's share of capital = $\frac{180000}{384000}$, or $\frac{15}{32}$
B's share of capital = $\frac{108000}{384000}$, or $\frac{9}{32}$
C's share of capital = $\frac{96000}{384000}$, or $\frac{1}{4}$
A's share of gain = $\frac{15}{32}$ of \$2592, or \$1215
B's share of gain = $\frac{9}{32}$ of 2592, or 729
C's share of gain = $\frac{1}{4}$ of 2592, or 648

9. Martin, Gould, and Towne formed a partnership. Martin invested \$12000 for 15 mo. He also worked 9 mo., and his labor was counted equivalent to the use of \$10000. Gould invested \$21000 for 10 mo., and Towne put in \$18000 for 8 mo., and \$12000 more for 9 mo. The total gain was \$5160. Find each partner's share.

10. Howe, Benton, and Ward formed a partnership Jan. 1, each investing property valued at \$5000. March 1, Howe added \$1500, and June 1 he added \$800. April 1, Benton added \$1200, and Oct. 1 he added \$900. July 1, Ward drew out \$500, and Sept. 1 he added \$800. If the gain for the year was \$1250, what was each partner's share?

11. Bush, Austin, and Fox rented a pasture for \$400. Bush pastured 200 sheep for 3 mo., Austin 300 sheep for 4 mo., and Fox 250 sheep for 2 mo. What rent should each man pay?

12. Johnson and Chapin bought 3 houses of equal value for \$2000, Johnson paying \$1200, and Chapin \$800. Wilson paid them \$1500 for one of the houses. If Johnson and Chapin each took one of the other two houses, what part of the \$1500 ought each to receive?

13. Wright, Greene, and Bates formed a partnership to continue three years, each investing \$10000. After 8 mo., Wright invested \$1200 more, and Greene drew out \$900; 4 mo. later Bates invested \$1500, and Wright withdrew \$500; 18 mo. before dissolution Thompson was admitted to the firm with a

capital of $6000, and 1 yr. before dissolution each partner withdrew $800. At the time of dissolution the resources and liabilities of the firm were as follows:

Resources		Liabilities	
Mdse.	$12400	Notes	$1750
Cash	8000	Mortgage	2000
Bills receivable	21500	Rent due	500
Real estate	12000	Bills payable	3650

What was the present worth of each partner at the close of the partnership?

14. Randall and Chapman formed a partnership Jan. 1, 1898, each investing $7000. April 1, 1898, Randall drew out $500, and Chapman added $1200. Sept. 1, 1898, they took in Holt with a capital of $6000. Jan. 1, 1899, Chapman invested $1000 more, and Randall drew out $800. April 1, 1899, Holt added $1500 and Randall $1200. Oct. 1, 1899, each partner withdrew $900. At time of settlement $2000 was allowed Randall for salary. On January 1, 1900, the firm sold out for $22500. If no salary had been paid Randall, how should the money be divided?

STOCKS AND BONDS

457. A company that has a charter that defines the legal powers of the company is called a Corporation.

When a large sum of money is necessary to carry on a business, usually a number of persons contribute money and form a corporation.

458. The total sum contributed (in money or in property) is called the Capital Stock of the corporation.

459. The capital stock is divided into equal parts called Shares. The par value of a share is usually $100.

460. Each contributor receives a Stock Certificate. It states the number of shares of stock which he owns and the par value of a share.

461. The value of a share of stock stated in the certificate is the Par Value of the stock.

462. If the business of a corporation proves profitable, at the end of a year, six months, or some other stated period of time, the corporation divides the net gain among the stockholders in proportion to the number of shares which they hold. This sum divided is called the Dividend of the corporation. The part received by each stockholder is his dividend. A corporation may pay annual, semi-annual, or quarterly dividends.

When a dividend is declared by a corporation, it is stated as a per cent of the capital stock. Thus, if a corporation with a capital stock of $1000000 has $100000 gain to divide among its stockholders, it declares a dividend of 10%.

463. If the business proves unprofitable for any period for which a dividend is usually declared, the corporation taxes, or assesses, each stockholder for his share of the net loss. The sum a stockholder must pay to meet the losses of the corporation is an assessment. Thus, a corporation may declare an assessment of 5%.

464. The annual rate of interest on money loaned is usually from 4% to 6%. If a corporation pays an annual dividend of 10%, people who have money to loan or invest will want to buy shares of stock of that company, because they would then receive a higher rate of income on their money. In such a case, the stockholders will be able to sell their shares for more than $100 (or the par value). Thus, a share of stock whose par value is $100 may be sold for $120.

465. When stock sells for more than its par value, it is said to be Above Par, or At a Premium.

If, however, a corporation declares an assessment or a very low dividend, the stockholders may desire to sell their shares of stock and invest the proceeds in a more profitable business. Thus, a share of stock whose par value is $100 may be sold for $95.

466. When stock sells for less than its par value, it is said to be Below Par, or At a Discount.

467. In each large city there is a Stock Exchange, which is an association of dealers in stocks of corporations.

The shares of stock of the corporations are put upon the market and are bought and sold in the various stock exchanges.

468. A person who buys or sells stocks for another at a stock exchange is called a Stock Broker.

469. The value of a share of stock as determined by the price for which it will sell in the market is called its Market Value.

470. A corporation may issue common and preferred stock.

471. Preferred Stock is that which draws certain guaranteed dividends. Common Stock draws such dividends as remain after the dividends on the preferred stock have been paid.

Preferred stock is usually issued to persons who buy shares of stock when the corporation is in need of money.

PROBLEMS

NOTE.—Unless otherwise mentioned, a share is $100.

1. If a corporation declares an annual dividend of 8%, what dividend will a stockholder receive who owns 36 shares?

2. A corporation levied an assessment of 4%. What assessment was paid by a holder of 115 shares of stock?

3. A telegraph company declared dividends of 12% on preferred stock and 9% on common stock. What dividend was received by a holder of 48 shares of preferred and 25 shares of common stock?

4. A manufacturing company declared a dividend of 6% on preferred stock and 1% on common stock. Find the income from 26 shares of preferred stock and 50 shares of common stock.

5. A bank with $2650000 capital stock divided $145750 among its stockholders. What was the rate of dividend declared?

6. A railroad company lost $35550 on a capital stock of $1580000. What rate of assessment was necessary to cover the loss?

7. A man invested $8500 in stock at par, and drew an annual dividend of $552.50. What rate of income did he receive on his investment? If money is worth 6%, would the market value of his stock be above or below par?

8. What is the cost of 125 shares of bank stock at $114\frac{1}{2}$, brokerage $\frac{1}{8}$%.

NOTE.—The pay a broker receives is called Brokerage. A broker usually charges $\frac{1}{8}$% of the par value of the stock for buying or selling the same. Brokerage is computed upon the par value of the stock.

9. Find the cost of 218 shares of Central R. R. stock at $138\frac{1}{2}$, brokerage $\frac{1}{8}$%.

10. What will be the cost of 62 shares of mining stock at 93, brokerage $\frac{1}{8}$%?

11. I sold 120 shares of Erie R. R. stock at 70, paying brokerage at $\frac{1}{8}$%. How much did I receive?

12. A speculator bought 240 shares L. V. R. R. stock at $105\frac{1}{2}$, and sold it at 110. How much did he gain, brokerage in each case being $\frac{1}{8}\%$?

13. Find the cost of 115 shares of stock at $9\frac{1}{4}\%$ premium, brokerage $\frac{1}{8}\%$?

14. What must be paid for 90 shares of stock at $11\frac{1}{2}\%$ premium, brokerage $\frac{1}{8}\%$?

15. How many shares of stock at 110 can be bought for \$3524, if $\frac{1}{8}\%$ is paid for brokerage?

OPERATION

1 share costs $\$110\frac{1}{8}$

$\$3524 \div \$110\frac{1}{8} = 32$ shares

16. How many shares of oil stock, at 108, can be bought for \$6055, brokerage $\frac{1}{8}\%$?

17. A man exchanged 24 shares of bank stock, at 98, for railroad stock at 84. How many shares did he receive?

18. How many shares of stock must be sold at 97, brokerage $\frac{1}{8}\%$, to obtain \$4662?

19. I bought 140 shares of stock at 3% premium. At what price must I sell it to gain \$721?

20. What annual income will be realized from an investment of \$3610 in 6% preferred stock, bought at 95?

OPERATION

$\$3610 \div \$95 = 38$, shares purchased

38 shares = \$3800

$\$3800 \times .06 = \$228$, income

21. How much will be realized from investing \$10304 in 8% stock, bought at $91\frac{3}{4}$, brokerage $\frac{1}{4}\%$?

22. What income will a man receive by investing \$6920 in 5% stock, at 108, brokerage $\frac{1}{8}\%$?

23. Find the annual income from an investment of \$9276 in 7% stock, bought at $96\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

24. Which is more profitable, and how much, to invest \$8400 in $4\frac{1}{2}\%$ stock at 80, or in $6\frac{1}{2}\%$ stock at 120?

25. A man owning 40 shares of 6% preferred stock sold the same at 112, and invested the proceeds in mining stock at 64.

Was his yearly income increased or diminished, and how much, if the mining stock paid a dividend of $3\frac{1}{2}\%$?

26. How many shares of 6% stock must be purchased to secure an income of $810?

27. How much must be invested in 6% stock, at 107, to secure an income of $810?

28. When 5% stock is selling at 115, how much must be invested to yield an income of $875, brokerage $\frac{1}{8}\%$?

29. What sum must be invested in $4\frac{1}{2}\%$ stock, at $97\frac{1}{2}$, brokerage $\frac{1}{8}\%$, to produce an income of $1845?

30. If 5% bank stock is 5% below par, what sum must be invested to obtain an income of $1800?

31. If 20 shares of stock yield a dividend of $140, what per cent income does the stock yield?

32. If I buy 6% stock at 120, what rate of income will I receive on my investment?

OPERATION

1 share yields $6 income.
1 share costs $120.
$6 is 5% of $120.
Hence the investment yields 5%.

33. A dividend of $4\frac{1}{2}\%$ was declared on stock bought at 80. What rate of interest was received on the investment?

34. What rate per cent on the investment is realized from 6% stock bought at 84?

35. Which is more profitable, and how much per cent, to buy 6% stock at 108, or 8% stock at 125?

36. I wish to buy stock that pays 6% dividend so that I may receive 8% income on my investment. At what price should I purchase the stock?

OPERATION

$1 of the stock yields $.06 income.
$.06 is 8% of what sum?
$.06 ÷ .08 = $.75, the price at which the stock should be purchased.

37. What must be paid for 10% stock so as to realize 7% income?

38. At what price must 6% stock be purchased to yield an income of $6\frac{1}{4}\%$?

39. At what rate of discount must I buy 6% stock that I may receive an income of 7% on my investment?

40. What premium can a dealer pay for 7% stock and derive an income of 6%?

Watered Stock

472. Some corporations are prohibited by law from declaring dividends in excess of a certain per cent. If a corporation with a capital stock of $2000000 gains $200000, and is not allowed to declare a dividend greater than 8%, what will the directors of the corporation do? They may give to the stockholders $500000 of new stock. Then they can declare a dividend of 8% on $2500000, and divide the $200000 among the stockholders without violating the law. The $500000 of stock certificates issued adds no value to the corporation, but it enables the corporation to declare dividends on a $2500000 basis instead of a $2000000 basis. Stock issued simply to swell the capital stock is called Watered Stock.

The managers of a corporation may issue watered stock when they wish to keep the public in ignorance of the true rate of dividends.

New York Stock Exchange

473. The principal stock exchange, or stock market, in America is in New York City.

Most of the stock bought and sold at the exchange is dealt in not to raise money or to invest money, but to make money on the rise and fall in the price of stocks. Dealers sell stock with the expectation that the price will fall. They buy stock with the expectation that the price will rise.

Dealers who try to drive the price of stocks up by buying are called "Bulls."

Dealers who try to drive the price of stocks down by selling are called "Bears."

Besides stock of the various corporations, corn, wheat, cotton, etc., are sold. When wheat is sold, the seller agrees to deliver the wheat to the purchaser within a certain period on demand, but the purchaser seldom demands the wheat. He either sells the "wheat" or settles for it at the market price. A dealer may sell 1000000 bushels of wheat if he has not a kernel of the grain, but if the purchasers demand the wheat he must deliver it, and will have to buy it in the market. If those who have bought the wheat in the exchange have also bought up the real wheat in the market, they may compel the sellers of wheat in the exchange to buy it at a high price.

There are various schemes for sending the price of stocks up or down.

Buying on a Margin

474. Stock brokers often buy stocks on a margin. If a broker offers to buy cotton for a speculator, on a margin of 10%, he means that he will put in 90% of his money and 10% of the speculator's money, buy stock, and hold it to the gain or loss of the speculator. The broker charges a commission for buying or selling stock, and interest on the money he puts in for the speculator. The speculator may give instructions to his broker as to when to buy and sell stock, or he may let the broker use his own judgment. If the stock that the broker buys advances in price, he may sell it and remit to the speculator the gain from the sale, less the commission and interest charges. If the stock that the broker buys falls in price, he may hold it for a time, expecting it will advance again. If the stock is sold at a loss, the loss and the broker's charges are taken out of the "margin." When the price of the stock falls so low that there is danger that the margin may not cover the loss, the broker sells the stock to save his money invested.

EXAMPLE.—A speculator sent $510 to a broker to be invested in stock, on a 10% margin. The broker bought stock at 102, and 12 days later sold it at 105, brokerage $\frac{1}{8}$% in each case. What was the speculator's net gain, if the broker charged 6% interest?

OPERATION

$ 510 ÷ .10 = $5100, amount invested
$5100 ÷ 1.02 = $5000, par value of stock
$5000 × 1.05 = $5250, selling price of stock
$5250 − $5100 = $ 150, gain on stock
$5000 × $\frac{1}{8}$% = $6.25, brokerage for buying
$5000 × $\frac{1}{8}$% = $6.25, brokerage for selling
Int. on $4596.25 for 12 da. = $9.19
$150 − ($6.25 + $6.25 + $9.19) = $128.31, net gain

PROBLEMS

1. A broker received $1500 to invest in stocks, on a margin of 10%. He bought 120 shares of stock at 105, held it 15 days, and sold it at 108. If the broker charged $\frac{1}{8}$% for both buying and selling, and 8% interest on his money invested, what was the speculator's gain?

2. A broker received a remittance of $1200 as a 10% margin, and purchased 200 shares of stock at 60. 19 days later he sold the stock at 82. If the broker charged $\frac{1}{8}$% for buying and selling, and 6% interest, what was the speculator's gain?

3. A broker received $2000 for an 8% marginal investment. He bought 300 shares of stock at 83, held it 3 days, and sold it at 94. Find the broker's profit, commission $\frac{1}{8}$%, interest 6%.

4. A speculator sent his broker $5000 as a 10% margin for investment in bank stock. The broker invested $50000 in stock at 125. The same day the stock fell to 113, and the broker sold out to save his investment. What did the speculator lose, brokerage $\frac{1}{8}$%?

BONDS

475. A Bond is a form of commercial paper which obligates the person, corporation, or government issuing it to pay a certain sum as specified in the bond.

476. When a corporation or a government wishes to raise a large sum of money it usually prepares and sells its bonds. Such bonds bear interest payable at stated times.

477. Bonds which are recorded as being owned by and payable to certain parties are called Registered Bonds.

478. Bonds which have interest certificates attached payable to the bearer are called Coupon Bonds.

The coupons are cut off and presented for payment when the interest is due.

479. Government bonds are often described by abbreviations. "U. S. 4's, 1907, reg." means United States registered bonds, bearing 4% interest, payable in 1907. "6's coup." means 6% coupon bonds.

480. Bonds, like stocks, are bought and sold at the stock exchanges. The income from bonds is a fixed per cent of their par value, while the income from stocks depends upon the business of the corporations issuing them.

TAXES

481. A Tax is a sum of money levied by a government on a person, his property, his income, or his business.

482. In the country districts of some States a tax is levied on each legal voter, for the support of the public roads. This tax is called a Poll Tax.

483. Taxes are levied for the support of schools and for the town, county, city, and State governments. Such taxes are levied on property.

484. Property may be classed as real property, or real estate, and personal property. Real estate consists of land and buildings. Movable property, live stock, money, furniture, merchandise, etc., is personal property.

485. Public property and property belonging to certain religious and benevolent societies are exempt from taxation. The law sometimes exempts a certain amount of the personal property of an individual from taxation.

486. A property tax is levied as a per cent on the property taxed. It is usually stated as so many mills on the dollar. The amount of tax levied on $1, expressed decimally, is called the Tax Rate.

The value of the taxable property in a city is $1612000, and the whole amount of tax to be raised is $9672. The tax is what per cent of the property? What is the tax rate? What is the tax on $1? How much tax will a man have to pay who is assessed (taxed for) $2635?

The value of the property taxed is what element in percentage?

The amount of tax levied is what element in percentage?

The rate of tax is what element in percentage?

487. Officers who estimate the taxable value of each person's property are called Assessors.

488. A list of the persons taxed with the assessed value of their property is an Assessment Roll.

489. Officers who collect the tax are called Collectors. Some collectors receive a salary, others receive as a fee a percentage on the money collected.

PROBLEMS

1. In a city in which the tax rate is .008, Mr. Wilson owns real estate assessed at $6500 and personal property assessed at $3800. What is the amount of his tax?

2. A man having property valued at $34750 is taxed at the rate of $\frac{3}{4}\%$. Find the amount of his tax.

3. The total taxable property in a village is assessed at $2500000, and the total amount of tax to be raised is $18500. The tax is what decimal part of the property? What is the tax rate? Find the amount of tax on an assessment of $2980; of $36870.

490. Suppose a tax of $36495 is to be levied on property assessed at $8117500. There are about 1500 taxpayers. The process of finding each person's tax from the tax rate, as above, is too long, and a tax table is used to shorten the work:

TAX TABLE

Rate .0045, or $4\frac{1}{2}$ mills on $1.

Property	Tax	Property	Tax	Property	Tax	Property	Tax
$1	$.0045	$10	$.045	$100	$ .45	$1000	$ 4.50
2	.0090	20	.090	200	.90	2000	9.00
3	.0135	30	.135	300	1.35	3000	13.50
4	.0180	40	.180	400	1.80	4000	18.00
5	.0225	50	.225	500	2.25	5000	22.50
6	.0270	60	.270	600	2.70	6000	27.00
7	.0315	70	.315	700	3.15	7000	31.50
8	.0360	80	.360	800	3.60	8000	36.00
9	.0405	90	.405	900	4.05	9000	40.50

In finding the tax on $8745, we see from this table that the tax on

$8000 = $36.00
700 = 3.15
40 = .18
5 = .02
$8745 = $39.35

PROBLEMS

1. From this table find the tax on \$46250, \$3928, \$60790, \$1225, \$780, \$325, \$456, \$575, \$11690, \$5600, \$34750, \$23125.

2. The assessment roll of a town shows that the total taxable property is assessed at \$548000, and the tax to be levied is \$2904.40. Find the tax rate, make a tax table, and from it find the tax on \$12395, \$4560, \$7275, \$24890, \$43125.

3. Find the tax on the following:

	Assessed Valuation	Tax Rate
(*a*)	\$4525	4 mills
(*b*)	650	$1\frac{1}{8}\%$
(*c*)	1475	40¢ per \$100
(*d*)	2880	6 mills
(*e*)	2590	25¢ per \$100

4. In a village, taxes are levied as follows: For street improvement, \$2100; for schools, \$7500; for salaries, \$2860; for sinking fund, \$3590. The valuation of the property is \$2153800. Find the tax rate to five decimal places.

5. If the assessed valuation of a town is \$985625, what tax rate is necessary to raise \$7950?

6. The rate of tax for the State is $\frac{3}{8}\%$, for the county $\frac{1}{4}\%$, and for the city 1%. Find the tax on city property assessed at \$3940.

7. A man owns \$12800 worth of property. The total valuation of his school district is \$137500, and the valuation of his town is \$1426000. The expenses of the school are \$340, and the tax levied on his town for town, county, and State purposes is \$9450. Find the whole amount of tax he must pay.

8. The valuation of a town is \$975000. The amount of tax levied for town expenses is \$1240, and the tax rate for county and State purposes is $4\frac{1}{2}$ mills. Find the total tax levied on a farm worth \$3980.

CUSTOM HOUSE BUSINESS

491. The federal government derives its revenues from indirect taxation. It does not tax the property of an indi-

vidual, but it puts a tax on some things that are manufactured in this country and on some things that are imported into this country from abroad. The taxes that the manufacturers of tobacco, beer, whisky, oleomargarine, etc., have to pay are called Internal Revenue Taxes. The taxes that importers of iron, wool, boots, etc., have to pay are called Duties, or Customs.

492. Duties, or customs, are collected at the various Custom Houses, which are offices established at the principal ports.

493. Ports that have a custom house are called Ports of Entry, because goods on which duties are levied may be entered there.

494. The principal officer in charge of a custom house is called the Collector of the Port.

495. Duties, or customs, are levied as so much per cent on the value of the goods imported, or as so much per pound, gallon, bushel, etc., without regard to the value of the goods.

496. Duties that are levied as a per cent on the value of the goods are *ad valorem* duties.

497. Duties that are levied as so much per pound, barrel, or bushel, are *specific* duties.

498. Ad valorem duties are levied on the value of the goods as determined by the prices in the country from which they are imported.

499. The ton used at the custom house is the long ton of 2240 lb.

500. On some goods there are both ad valorem and specific duties.

501. A Tariff is a classification of goods with the rates of duty imposed.

By the tariff act of 1897, the duty on books was 25%; on certain carpets, 28¢ per square yard, and 40% of their value; on onions, 40¢ per bushel; on cigars, $4.50 per pound and 25% of their value.

Which of these articles paid ad valorem duties? Which specific duties?

502. On some goods there are no duties. Such goods are said to be on the Free List.

503. Tonnage is a tax levied on a vessel for coming into a port of entry.

504. Bringing in goods by stealth so as to avoid paying the duties is called Smuggling. It is a crime against the United States.

505. The government has established bonded warehouses in which goods may be stored until the duties have been paid.

506. In estimating the duty on imported goods certain allowances are made. Tare is an allowance made for the weight of the box or other covering of the goods. Breakage is an allowance for the loss of liquids in bottles. Leakage is an allowance for the loss of liquids in barrels.

PROBLEMS

In the following examples the rates of duty are those established by the tariff act of 1897.

1. Find the total duty on 2460 bu. of barley, at 30¢ per bushel; 680 lb. of butter, at 6¢ per pound; 475 lb. of hops, at 12¢ per pound.

2. What is the duty on an invoice of blankets valued at $288, and weighing 640 lb., if the tariff rate is 33¢ per pound and 35% ad valorem?

3. Find the duty, at 60% ad valorem, on $6380 worth of silk lace.

4. Find the duty, at 40¢ per gallon, on 165 gal. of olive oil.

5. Find the duty, at 44¢ per pound, and 50% ad valorem, on 14780 lb. of knit woolen goods, valued at 25¢ per pound.

6. What is the duty, at 6¢ per square yard, on 3145 yd. of rattan matting?

7. What is the duty, at $1\frac{1}{4}$¢ per pound, on 5370 lb. of castile soap?

8. Find the duty, at 40% ad valorem, on an importation of $38700 worth of watches.

9. What is the duty, at $38\frac{1}{2}¢$ per pound and 40% ad valorem, on 1272 lb. of woolen yarn, valued at $33\frac{1}{3}¢$ per pound?

10. What is the duty, at 60¢ per square yard and 40% ad valorem, on 420 yd. of velvet carpet, valued at $3.50 per yard?

MISCELLANEOUS REVIEW PROBLEMS

1. 52 men working 8 hr. per day can do a piece of work in 28 da. If 40 men have been working 10 hr. a day for 14 da., how many more men working 9 hr. a day can complete the work in 14 da.?

2. How many tons of coal may be contained in a bin 10 ft. square and 6 ft. high, coal weighing 80 lb. to the bushel?

3. A wagon box 12 ft. 6 in. long, 3 ft. 4 in. wide, and 1 ft. 9 in. deep is full of wheat. How much does the wheat weigh?

4. Find the weight of a bin of potatoes, the bin being 8 ft. by 14 ft. by 6 ft.

5. What is the cost of 2942 lb. of hay at $22.50 per ton?

6. Write $\frac{1}{320}$ of a mile in lower denominations.

7. The drive wheels of a locomotive are 5 ft. 10 in. in diameter. If the engine runs 60 mi. per hour, how many revolutions will the wheel make per second?

8. Find the capacity of a cylindrical cistern $6\frac{1}{2}$ ft. in diameter and 7 ft. deep.

9. What is the value of a lot 198 ft. long and 176 ft. wide at $1250 per acre?

10. Find the cost, at $14.75 per M, of a 5-ft. walk around a block 112 ft. by 216 ft., the walk to be of 2-in. plank and to rest on 2 stringers running lengthwise, 2 in. by 6 in. each, and placed 3 inches from each edge.

11. A man receives $3460 dividends on an investment of $40700. What is the per cent of profit?

12. How many square feet of sheet iron will be used in making a 7-in. stovepipe of 40 joints each 30 in. long, allowing 1 in. for riveting?

13. If the diameter of the earth is 7912 miles, what is its circumference?

14. Find the weight of water, at $62\frac{1}{2}$ lb. per cubic foot, in a tank 22 ft. in diameter and 18 ft. high.

15. Find the cost of a stone dam 80 ft. long, 21 ft. high, 15 ft. wide at the base, and 7 ft. wide at the top, at $2 per cubic yard.

16. The diagonal of a square lot is 48 rd. Find the length of a side of the lot.

17. What distance can be saved by going diagonally across a lot 110 rd. long and 56 rd. wide?

18. The area of a circular lot is $\frac{2}{3}$ of an acre. What is its diameter?

19. What is the largest square that can be cut out of a circular board 20 in. in diameter?

20. A man receives $250 per month rent for a lot. At 7% per annum, what is the capitalization, or value, of the lot?

21. May 29, 1902, I bought a house for $2600 on 6 mo. time and sold it Aug. 1, 1902, on 8 mo. credit for $3200. What was my gain per cent, money being worth 6%?

22. A man owned a house which rented for $50 per month. His insurance and taxes amounted to $125 per year, and the repairs to $50 per year. He sold the house for $7000 and invested the money in 6% bonds at 87, brokerage $\frac{1}{2}$%. How much was his annual income increased or diminished?

23. What is the present worth of a debt of $1268.40, due in 1 yr. 7 mo. 12 da., money being worth 5%?

24. A boy was born June 2, 1902. What sum must be placed at compound interest at 4%, on Dec. 25, 1902, to amount to $10000 when he becomes of age?

25. A dealer sent his agent $1740 to invest in cheese at 11¢ per pound. If the commission is $2\frac{1}{2}$% and charges $45, how many pounds of cheese will the dealer receive?

26. A loaned B $6000 for 1 yr. 4 mo. when interest was 7%. For how long should B loan A $4000 when interest is 5%?

27. If a bicycle wheel is 28 in. in diameter, how many revolutions will it make in going 1 mi.?

28. On Jan. 1, I began business with $8700 cash. Since then I have received for merchandise $3625 in cash, have $6490 worth of goods on hand, have paid $1750 in wages, and $124

for expenses. I owe bills to the amount of $400 due in 8 mo., without interest, and am the creditor of bills for $1260 due in 6 mo., without interest. What has been my net gain per cent to date, Dec. 1, 1902, money worth 6%?

29. From the following data show the gain or loss and the present worth of the business:

	Ledger Footings	
	Dr.	Cr.
Proprietor		$ 8000
Cash	$12460	10175
Merchandise	16520	14240
Bills payable	2000	2400
Bills receivable	2462	400
James Robinson	1180	747
William Brown	1498	158

Merchandise on hand amounts to $4685.

30. April 3, 1902, I sold James Rice merchandise amounting to $34.25, and took his note at 2 mo. without interest. Write the note. Find the proceeds if discounted June 2, 1902, at 6%.

31. What is the present value, June 16, 1903, of a promissory note for $500, dated Jan. 1, 1903, payable one year after date and bearing interest at 6%?

32. A note for $800, dated Jan. 6, 1896, bears interest at 5%. The following payments have been made on it: June 1, 1896, $15; Sept. 1, 1896, $40; Jan. 5, 1897, $150. How much remained due June 16, 1897?

33. Find the equated time of payment of the following account:

DR. WM. JONES. CR.

1900				1900			
May	5	Mdse., at 30 da.,	$150	June	10	Cash,	$100
July	1	Mdse., at 10 da.,	60		10	Cash,	40
Sept.	10	Mdse., at 20 da.,	80	July	15	Cash,	55
Nov.	2	Mdse., at 30 da.,	50				

34. A merchant in New York City owes 5260 francs in Paris. What will it cost to remit (*a*) direct to Paris at 5.15 fr. a dollar, (*b*) through London at 4.89, there buying exchange on Paris at 25.19 fr. a pound sterling?

35. From the following balances determine the gains and losses, the resources, liabilities, and present worth:

	Dr.	Cr.
Proprietor (investment)		$5000
Merchandise	$8250	6480
Cash	9560	7845
Bills receivable	1550	
Bills payable	780	880
James Hill	425	150
Wheeler & Wilson	630	840

Merchandise on hand per inventory, $4125.

36. A certain stock pays 10%. At what rate must it be bought to yield 6% on the investment?

37. What single discount is equivalent to a trade discount of 10, 15, and 5%?

38. What premium must be paid to insure a cargo of 4880 bu. of wheat, valued at $1.04 per bushel, at $1\frac{1}{2}$%, the policy being for only $\frac{5}{8}$ of the value?

39. Find the cost of oil cloth for a hall $8\frac{3}{4}$ yd. long and 14 ft. wide, at 90¢ a square yard.

40. Find the interest on $820.45 from June 17, 1889, to April 13, 1892, at 4%.

41. 4-ft. wood piled $5\frac{1}{2}$ ft. high requires how many feet in length of the pile for $2\frac{1}{2}$ cords?

42. Reduce $3\frac{3}{4}$% to a decimal.

43. A room 18 ft. by 16 ft. is carpeted with carpet $\frac{3}{4}$ of a yard wide, and the smallest possible number of yards of the carpet is used. Find (*a*) the number of breadths; (*b*) the number of yards.

44. What sum will amount to $354.09 in 7 mo. at 3% per annum?

45. How many brick in a pile 16 ft. by 6 ft. by 4 ft., each brick being 8 in. by 4 in. by 2 in.?

46. Required the exact interest on $146.73, for 23 da., at 5% per annum.

47. A note of $285, bearing 6% interest, given June 17, 1891, has endorsed upon it a payment of $100, March 4, 1892. Find the sum due on the note Nov. 1, 1892.

48. If a grocer sells coffee that costs him 26½¢ per pound and 32¢ a hundred for freight, for 36¢ per pound, what is the gain per cent?

49. If the average yield per bushel of seed is 14 bu. 1 pk., how much is the yield from 7 bu. 3 pk. 2 qt.?

50. Find the loss on 26 shares of stock bought at 101, and sold at 87, brokerage ¼% both for buying and selling.

51. Required the cost of 24 3-in. planks, 18 ft. long and 10 in. wide, and 35 pieces of 2 in. by 4 in. scantling 18 ft. long, at $20 per M.

52. A commission merchant sold 2140 bu. of oats at 39¢ per bushel, paid $47.60 freight, and retained 2½% commission. How much did he remit to the consignor?

53. What is the difference in weight, expressed in avoirdupois pounds, between 300 lb. Troy and 300 lb. avoirdupois?

54. An importer receives a bill of goods of $575, pays a duty of 45%, and sells them at a gain of 20%. The price paid by the purchaser is what per cent of the exporter's price?

55. A man bought a house for $4200, paid $640 for repairs, and rents the place for $50 per month. If he pays $115 taxes, what is the per cent of income?

56. If a merchant marks goods 50% above cost, what discount from the marked price can he give a customer and make a profit of 33⅓%?

57. If 6% bonds are selling at 87, how much money must be invested in them to secure an annual income of $750?

58. An agent has $23150 of his principal's money and is instructed to buy oats at 48¢ per bushel, with a commission of 5%. How many bushels should he buy?

59. When N. Y. C. 4½'s are at a premium of 11½%, what sum must I invest to secure an income of $720?

60. I bought a house for $750, and 2 yr. 9 mo. afterwards sold it for $900. If I paid taxes amounting to $29.17, what was the annual rate per cent of gain on the money invested?

61. How many feet of lumber are required to make a box 4 ft 8 in. by 3 ft. 6 in. by 2 ft. 4 in.?

62. If there is a duty of $1.25 per gallon, and 45%, on varnish, at what price must it be sold per gallon to gain 33⅓%, if the cost in London is $2.11 per gallon and there are no freight charges?

63. If bell metal is composed of 78 parts copper and 22 parts tin, what weight of each of these metals will there be in a bell that weighs 900 lb.?

64. A lot 60 ft. by 150 ft. was sold for $500. What was the price per acre?

65. If the water from a spring yields 6½% of its weight in salt, how many tons of water will be required to make 1000 lb. of salt?

66. What is the rate of income on an investment in 5% bonds at 80%?

67. What is the cost of 3130 lb. of coal at $5.25 per ton, and 1820 lb. at $6.90 per ton?

68. If your standing in attendance at school is marked 88% and you were absent 9 da., how many days of school were there?

69. Find the proceeds of an interest bearing note for $186 given for 3 mo. and discounted the same day it was made, interest and discount being 6% each.

70. If a man bought stock at 2% above par, and sold it at 7% below par, what per cent did he lose?

71. The valuation of the taxable property of a town is $498700 and the tax to be raised is $5850. What will be the tax on $5000?

72. I bought shoes at $2.40 per pair. At what price must I mark them that I may allow a discount of 25% and make a profit of 20%?

73. A man bought a lot for $400 on these terms: $100 cash and the balance in monthly installments of $20, with 6% interest on the part unpaid, interest payable with every installment. What was the total amount paid for the lot?

74. Four men formed a partnership. A put in $12800, B

put in \$14000, C put in \$11900, and D put in \$15000. After 9 mo. A drew \$2800, and 10 mo. later put in \$4600. After 10 mo. B added to his investment \$5200, and C withdrew \$6100. They paid \$8425 for labor, \$1750 for repairs, \$1450 taxes. In 2 yr. they were worth \$68000. What was each partner worth?

75. I bought, through an agent, 5000 bu. of corn at 58¢, commission 2%. The agent sold the corn at 64¢, commission $2\frac{1}{2}\%$, charges \$42.50, and remitted the balance by a draft purchased at $\frac{5}{8}\%$ premium. What was my gain?

76. A man bought goods for \$2560. He paid \$21.50 insurance, \$75.50 cartage, and sold them for \$2975, allowing the agent a commission of 4% for selling the goods. What was the gain per cent?

77. In writing on the typewriter the letter *a* was struck 33520 times, *b* 13080 times, *e* 25160 times, *c* 15260 times, and *j* 2180 times. What was the per cent of use of each letter?

78. A grocer has teas worth 14¢, 18¢, 25¢, 32¢ per pound. In what proportions can he mix them so as to make the mixture worth 22¢ per pound? How many pounds of the mixture must he make to use up 125 lb. of the 18¢ tea?

79. A dealer bought 300 casks of vinegar, each containing 45 gal., at 10¢ per gallon. He paid \$1 apiece for the barrels, 1¢ per gallon freight, 10¢ per barrel cartage. He sold it at $12\frac{1}{2}¢$ per gallon, receiving 90¢ for each barrel, and paying $7\frac{1}{2}\%$ commission for selling. What was his total gain?

80. On the following note these payments were made: Feb. 11, 1902, \$240; March 18, 1902, \$375; May 20, 1902, \$260:

\$1500.00. Chicago, Ill., Jan. 2, 1902.

Six months after date, for value received, I promise to pay J. K. Welsh, or order, fifteen hundred dollars, with interest at 6 per cent.

THOMAS H. BEAMAN.

What was due on the note Dec. 24, 1902?

ANSWERS

Art. 17, p. 13

1. 44782
2. 488943
3. 4201420
4. 4309104
5. 3882493
6. 384929783
7. 397334694
8. 406274798
9. 423007285
10. 445609793
11. 549333911
12. 446378793
13. 659402301
14. 878654438
15. 781811566
16. 766652422
17. 808479253
18. 901317663
19. 690428361
20. 8238138
21. 7774358
22. 7547458
23. 9165011
24. 8615227
25. 8725835
26. 7131512
27. 9288411

Art. 22, p. 18

1. 9147136
2. 7422902
3. 9934025
4. 8722928
5. 8988061
6. 7391684
7. 7553772
8. 10343279
9. 8605464
10. 8179692
11. 9083787
12. 10170524
13. 8638249
14. 9632366
15. 11443360
16. 8627036
17. 10084434
18. 9816995
19. 10046349
20. 10647803
21. 10755906

Art. 30, p. 22

1. 643
2. 4016
3. 3545
4. 33243
5. 15211
6. 29312
7. 23811
8. 20680
9. 30765
10. 186999
11. 145408
12. 102527
13. 87099
14. 48641
15. 17034
16. 26682
17. 20322
18. 26428
19. 90314
20. 115853

Art. 31, p. 24

1. 2207
2. 4883
3. 716
4. 932
5. 6178
6. 470597
7. 242202
8. 89787
9. 222071
10. 293253
11. 14722007
12. 3049258
13. 15812417
14. 2329188
15. 1835007
16. 7175118

Art. 34, p. 25

1. 1480
2. 688
3. 5156
4. 76628
5. 17472
6. 63087
7. 4100
8. 23303
9. 18366
10. 64369
11. 7459
12. $1401.34
13. $6871.45
14. $9434.93
15. $4667.20
16. $45360.45
17. $116104.28
18. $140.73

19. $861.11
20. $13196.43

Art. 37, p. 27

1. 182
2. 210
3. 195
4. 240
5. 221
6. 247
7. 238
8. 270
9. 304
10. 306
11. 285
12. 288
13. 361
14. 252
15. 255
16. 238
17. 216
18. 342

Art. 39, p. 28

1. 35100
2. 8432000
3. 712400000
4. 46800000
5. 1736000
6. 56000000

Art. 40, p. 29

1. 86625
2. 6019936
3. 56939
4. 6774792
5. 3059625
6. 74664
7. 867456
8. 217665
9. 359074
10. 271466

Art. 41, p. 30

1. 1428
2. 1431
3. 2666
4. 2044
5. 6392
6. 3922
7. 3886
8. 3420
9. 2668
10. 1015
11. 1702
12. 2544
13. 1950
14. 3901
15. 5952
16. 3293
17. 3404
18. 3496
19. 4898
20. 6072

Art. 42, p. 30

1. 5382
2. 14484
3. 21546
4. 42490
5. 180852
6. 394632
7. 77868
8. 323228
9. 95988
10. 141245
11. 124323
12. 2313396
13. 1783800
14. 3382260
15. 1878874
16. 484956
17. 351663
18. 5344924
19. 4591947
20. 649540

Art. 43, p. 31

1. 3068
2. 6608
3. 21046
4. 42756
5. 257796
6. 126468
7. 68962
8. 120411
9. 4503
10. 125944
11. 196315
12. 25245
13. 16536
14. 125008
15. 50996
16. 633825

Art. 44, p. 31

1. 198
2. 286
3. 1485
4. 836
5. 3465
6. 2706
7. 7909
8. 2893
9. 3916
10. 10373
11. 2827
12. 7095
13. 14982
14. 27280
15. 78716
16. 33682

Art. 47, p. 33

1. 327
2. 444 (8 rem.)
3. 538
4. 144
5. 4739
6. 43785 (1 rem.)
7. 21318
8. 317
9. 327
10. 514
11. 1006
12. 428

13. 9512
14. 11080 (398 rem.)
15. 3323 (715 rem.)
16. 1921 (1771 rem.)
17. 208 (1846 rem.)
18. 63 (7323 rem.)
19. 647 (2983 rem.)
20. 1139 (652 rem.)

Art. 48, p. 33

1. 468
2. 73
3. 28 (60 rem.)
4. 79 (6800 rem.)
5. 63 (875 rem.)
6. 420 (70 rem.)
7. 537 (31936 rem.)
8. 80016 (7829 rem.)
9. 670 (197356 rem.)
10. 5179 (36527 rem.)

Art. 54, p. 36

1. \$160
2. A = 90
3. \$24
4. A = 24
5. \$30
6. \$75
7. \$70
8. \$25
9. \$60
10. \$40
11. \$6
12. A = 25
13. 4360
14. 751
15. 24

Art. 58, p. 38

1. $22\frac{2}{4}$
2. $2730\frac{2}{3}$
3. $40\frac{1}{2}$
4. $3\frac{2503}{112356}$
5. $2\frac{8}{11}$
6. $1\frac{3}{31}$
7. $1\frac{389}{556}$
8. $1\frac{71}{105}$
9. $3\frac{2513}{12396}$
10. $2\frac{4433}{12806}$
11. 20
12. 70
13. \$6
14. 117
15. $255\frac{3}{7}$
16. $677\frac{31}{133}$
17. $11\frac{37733}{4672}$
18. $12\frac{8}{9}$
19. 40
20. $40\frac{1}{2}$
21. 70
22. 3402
23. 15
24. $144\frac{9}{14}$
25. $359\frac{13}{80}\frac{1}{1}$

Art. 73, p. 46

1. 25.6
2. 425.54
3. 659.656
4. 1797.90311
5. 1112.928
6. 1610.9902
7. 329.9351
8. 260.53739

Art. 74, p. 46

1. .706
2. 7.8016
3. 407.585
4. 206.9985
5. 41.8017
6. 3158.1015
7. 902.8887
8. 16360.4852

Art. 75, p. 47

1. 1.162
2. .4025
3. .022032
4. .0960
5. .0001537
6. .000320608
7. .0150294
8. .0279
9. 3.385
10. .08891
11. 10000
12. .01
13. .0001001
14. .0000001
15. .00010001
16. 1
17. 10011.0011001
18. 100000001
19. 25025025
20. 2500002.5025000025

Art. 76, p. 48

1. 2.725
2. $356\frac{1}{9}$
3. 587.2
4. 60000
5. .000052
6. .0000007
7. 250000
8. $.26\frac{2}{3}$
9. 5.0607
10. .156
11. .0135
12. 2.143
13. .00009325
14. .144
15. .00000697
16. .10809
17. .007225
18. .000901
19. 1, 100, 10, 10, 10000, .1, 100, 1, 1, .01
20. .5, .005, .5, .005, .0005, 50000, .0000005, 50000, 50, 5000000

21. .16, 1600, .00016, 1.6, 16000, 160000000, .0000000016, .0016, 160000, 1.6

22. 3, .000003, 30000000, .03, 30000, .000C00003, .03, 300, 3000000, .00000000003

23. .02, 2000, 200000000, .000000002, .02, 200, .000000002, 2000000000, 2000, .00002

24. .005 .00005, 500000000, 5, .000000005, .005, 500, 50000000, .000000005, .005

Art. 77, p. 49

1. $\frac{64}{100}$

2. .64

3. $\frac{32}{1000}$

4. .8

5. .8

6. .25

7. $\frac{268}{1000}$

8. $\frac{3758}{10000}$

9. $\frac{46}{1000}$

10. $\frac{205}{10000}$

11. $\frac{9}{100}$

12. $\frac{49}{1000}$

13. $\frac{36}{10000}$

14. $\frac{25738}{1000000}$

15. $\frac{13}{10000}$

16. $2\frac{65}{100}$

17. $17\frac{3644}{10000}$

18. $10\frac{15}{100}$

19. $610\frac{18}{10000}$

20. $101\frac{101}{1000}$

21. $13\frac{85}{1000}$

22. .4

23. .625

24. .8

25. .75

26. .375

27. .5

28. .6

29. .875

30. .1666+

31. .555+

32. .666+

33. .5714+

34. .1875

35. .5333+

36. .15

37. .64

38. .35

39. .9166+

40. .933+

41. .6428+

42. .8333+

43. .09375

44. .032

45. .8875

46. .175

47. .248

48. .2733+

49. .1666+

50. .246

51. .24416+

52. .96875

53. .184

54. 7.125

55. 17.0285+

56. 14.2857+

57. 20.0075

58. 25.08

59. 9.0583+

60. 16.005

61. 5.00125

Art. 78, p. 50

1. $\frac{15}{5}$

2. $\frac{120}{10}$

3. $\frac{105}{7}$

4. $\frac{108}{12}$

5. $\frac{165}{15}$

6. $\frac{260}{20}$

7. $\frac{91}{13}$

8. $\frac{480}{30}$

9. $\frac{108}{27}$

10. $\frac{180}{36}$

11. $\frac{328}{41}$

12. $\frac{1440}{120}$

Art. 79, p. 51

1. $\frac{64}{9}$

2. $\frac{63}{5}$

3. $\frac{47}{7}$

4. $\frac{139}{9}$

5. $\frac{163}{8}$

6. $\frac{191}{13}$

7. $\frac{1326}{11}$

8. $\frac{487}{19}$

9. $\frac{751}{12}$

10. $\frac{3583}{23}$

11. $\frac{261}{16}$

12. $\frac{416}{17}$

Art. 80, p. 51

1. $7\frac{1}{2}$

2. $9\frac{10}{15}$

3. 15

4. $47\frac{6}{8}$

5. $8\frac{2}{5}$

6. $5\frac{8}{14}$

7. $14\frac{13}{18}$

8. 13

9. $19\frac{12}{15}$

10. $55\frac{110}{123}$

11. $34\frac{81}{88}$

12. $46\frac{137}{146}$

Art. 81, p. 51

1. $\frac{1}{2}$

2. $\frac{1}{4}$

3. $\frac{13}{32}$

4. $\frac{3}{6}$

5. $\frac{12}{125}$

6. $\frac{35}{72}$
7. $\frac{7}{9}$
8. $\frac{11}{13}$
9. $\frac{14}{17}$
10. $\frac{2}{3}$
11. $\frac{14}{16}$
12. $\frac{8}{9}$

Art. 82, p. 52

1. $\frac{28}{48}$
2. $\frac{25}{80}$
3. $\frac{45}{56}$
4. $\frac{80}{105}$
5. $\frac{63}{147}$
6. $\frac{120}{225}$
7. $\frac{84}{182}$
8. $\frac{399}{475}$
9. $\frac{68}{72}$
10. $\frac{253}{319}$
11. $\frac{2242}{3114}$
12. $\frac{16}{196}$

Art. 88, p. 53

1. 24
2. 144
3. 360
4. 84
5. 60
6. 11088

Art. 89, p. 53

1. $\frac{273}{882}$, $\frac{189}{882}$, $\frac{72}{882}$, $\frac{240}{336}$
2. $\frac{289}{504}$, $\frac{108}{504}$, $\frac{196}{504}$, $\frac{441}{504}$
3. $\frac{140}{240}$, $\frac{72}{240}$, $\frac{65}{240}$, $\frac{128}{240}$
4. $\frac{624}{1092}$, $\frac{378}{1092}$, $\frac{585}{1092}$, $\frac{476}{1092}$
5. $\frac{225}{600}$, $\frac{336}{600}$, $\frac{140}{600}$, $\frac{135}{600}$
6. $\frac{1620}{151120}$, $\frac{5712}{151120}$, $\frac{6615}{151120}$, $\frac{7840}{151120}$
7. $\frac{1824}{328832}$, $\frac{4864}{328832}$, $\frac{2565}{328832}$, $\frac{14688}{328832}$, $\frac{17784}{328832}$
8. $\frac{700}{1680}$, $\frac{400}{1680}$, $\frac{735}{1680}$, $\frac{896}{1680}$
9. $\frac{3850}{12600}$, $\frac{4500}{12600}$, $\frac{5292}{12600}$, $\frac{2275}{12600}$
10. $\frac{140616}{308448}$, $\frac{80325}{308448}$, $\frac{60928}{308448}$, $\frac{95472}{308448}$, $\frac{251328}{308448}$
11. $\frac{9360}{16380}$, $\frac{4368}{16380}$, $\frac{4410}{16380}$, $\frac{945}{16380}$
12. $\frac{850500}{1927800}$, $\frac{860625}{1927800}$, $\frac{205632}{1927800}$, $\frac{119000}{1927800}$

Art. 90, p. 54

1. $2\frac{7}{36}$
2. $1\frac{7}{36}$
3. $1\frac{101}{210}$
4. $1\frac{747}{1008}$
5. $1\frac{115}{168}$
6. $1\frac{15}{16}$
7. $1\frac{1195}{2772}$
8. $1\frac{301}{360}$
9. $6\frac{72}{80}$
10. $5\frac{21}{110}$
11. $35\frac{31}{126}$
12. $421\frac{113}{210}$
13. $182\frac{1703}{2100}$
14. $108\frac{197}{336}$
15. $39\frac{959}{10710}$
16. $105\frac{979}{1050}$

Art. 91, p. 55

1. $\frac{7}{33}$
2. $\frac{1}{2}$
3. $\frac{5}{468}$
4. $\frac{53}{525}$
5. $\frac{1}{72}$
6. $\frac{13}{240}$
7. $2\frac{5}{28}$
8. $\frac{11}{36}$
9. $\frac{19}{75}$
10. $\frac{53}{66}$
11. $\frac{13}{22}$
12. $1\frac{5}{14}$
13. $14\frac{2}{33}$
14. $2\frac{55}{66}$
15. $37\frac{56}{81}$
16. $48\frac{82}{105}$
17. $12\frac{11}{105}$
18. $37\frac{37}{88}$

Art. 96, p. 57

2. $2\frac{2}{9}$
3. $3\frac{1}{9}$
4. $3\frac{1}{9}$
5. $4\frac{11}{13}$
6. $1\frac{15}{17}$
7. $5\frac{1}{7}$
9. $1\frac{8}{9}$
10. $8\frac{1}{3}$
11. 4
12. $\frac{5}{8}$
13. $9\frac{1}{2}$
14. $6\frac{7}{12}$
16. $\frac{9}{22}$
17. $\frac{1}{10}$
18. $\frac{1}{12}$
19. $\frac{27}{256}$
20. $\frac{19}{1620}$
21. 42
22. $17\frac{17}{18}$
23. $17\frac{124}{385}$
24. $3\frac{7}{16}$
25. $5\frac{67}{100}$
26. 262
27. $189\frac{7}{11}$
28. 189
29. $306\frac{12}{13}$
30. $1309\frac{1}{2}$
31. 6507
32. $11568\frac{2}{3}$
33. $1525\frac{3}{22}$
34. $672\frac{3}{4}$
35. $4589\frac{7}{8}$
36. $1280\frac{5}{11}$
37. 1389

38. 4835
39. 790
40. 1220
41. $445\frac{5}{8}$
42. 732
43. 492
44. $397\frac{1}{3}$
45. 1113
46. $244\frac{23}{48}$
47. $5483\frac{23}{44}$
48. $4821\frac{1}{30}$
49. $3997\frac{29}{27}$
50. $904\frac{3}{4}$
51. $2650\frac{79}{81}$
52. $223381\frac{11}{40}$
53. $41269\frac{5}{32}$
54. $760\frac{39}{91}$
55. $381\frac{22}{42}$
56. $1146\frac{29}{48}$
57. $2264\frac{7}{12}$

Art. 97, p. 59

1. $\frac{3}{19}$
2. $\frac{1}{10}$
3. $\frac{4}{81}$
4. $\frac{4}{33}$
5. $\frac{7}{72}$
6. $\frac{7}{61}$
7. $\frac{9}{37}$
8. $\frac{7}{60}$
9. $\frac{4}{51}$
10. $\frac{5}{36}$
11. $\frac{8}{369}$
12. $\frac{2}{7}$
13. $\frac{3}{70}$
14. $\frac{63}{520}$
15. $1\frac{1}{17}$
16. $\frac{13}{26}$
17. $\frac{1}{4}$
18. $\frac{7}{8}$
19. $\frac{67}{204}$
20. $\frac{31}{36}$
21. $\frac{7}{24}$
22. $16\frac{7}{36}$
23. $276\frac{11}{12}$
24. $3\frac{5}{8}$
25. $38\frac{1}{14}$
26. $2\frac{27}{56}$
27. $6\frac{19}{66}$
28. $14\frac{5}{11}$
29. $1\frac{20}{27}$
30. $7\frac{1}{6}$
31. $24\frac{69}{176}$
32. $1122\frac{3}{8}$
33. $802\frac{46}{315}$
34. $\frac{4}{7}$
35. $\frac{3}{4}$
36. $\frac{4}{7}$
37. $1\frac{1}{3}$
38. $\frac{49}{90}$
39. $\frac{80}{99}$
40. $2\frac{25}{189}$
41. $\frac{15}{22}$
42. $3\frac{19}{66}\frac{22}{5}$
43. $1\frac{88}{100}\frac{27}{64}$
44. $72\frac{48}{56}\frac{37}{6}$
45. $12\frac{2}{3}$
46. $\frac{7}{9}$
47. $1\frac{15}{36}\frac{5}{7}$
48. $45\frac{3}{56}$
49. $43\frac{27}{33}\frac{8}{6}$
50. $\frac{875}{972}$
51. $1\frac{11}{16}$
52. $1\frac{16}{76}\frac{1}{9}$
53. $\frac{9}{17}\frac{31}{49}\frac{1}{6}$

Art. 98, p. 61

1. $\frac{1}{20}$
2. $\frac{7}{12}$
3. $\frac{9}{16}$
4. $31\frac{1}{4}$
5. 49
6. 24
7. $547\frac{1}{6}$
8. $\frac{35}{48}$
9. $1\frac{1}{12}$
10. 135
11. $\frac{7}{45}$
12. $\frac{2}{7}\frac{9}{8}\frac{6}{3}$
13. $\frac{1}{11}\frac{0}{81}$
14. $41\frac{1}{3}$
15. $\frac{4}{13}\frac{9}{1}$
16. $\frac{10}{31}\frac{95}{28}$
17. $22\frac{11}{21}$
18. $\frac{26}{43}\frac{52}{76}$
19. $\frac{3}{7}$
20. 1st, $\frac{22}{88}\frac{9}{6}$; 2d, $\frac{22}{88}\frac{7}{6}$; 3d, $\frac{17}{43}\frac{0}{3}$
21. $\frac{34}{189}$

Art. 100, p. 64

1. $11\frac{4}{7}$ wk.
2. 58.24 bu.
3. .814$\frac{1}{9}$ mi.
4. A, $\frac{68}{287}$; B, $\frac{78}{287}$; C, $\frac{96}{287}$; D, $\frac{56}{287}$
5. $115\frac{19}{24}$
6. $2.02
7. $116\frac{56}{99}$
8. $69.71
9. 1000 lb.
10. $49\frac{8}{33}$ bu., $709\frac{1}{11}$ lb.

Art. 111, p. 68

1. 91800, 45900, 22950, 15300, 11475
2. 122400, 81600, 61200, 40800, 30600, 20400, 15300
3. 18400, $12266\frac{2}{3}$, 9200, $6133\frac{1}{3}$, 4600, $3066\frac{2}{3}$, 2300
4. 140, 1400, 14000, 105, 1050, 10500
5. 140, 1400, 14000, 70, 700, 7000
6. 810, 8100, 81000, 4050, 40500
7. $157\frac{1}{2}$, 1575, 15750, 315, 3150, 31500

8. 1300, 13000, 390, 3900, 39000, 5200
9. 1600, 16000, 2133$\frac{1}{3}$, 21333$\frac{1}{3}$, 6400, 85333$\frac{1}{3}$
10. \$1320, \$880, \$660, \$440, \$330, \$220, \$165
11. \$61.33$\frac{1}{3}$, \$460, \$30.66$\frac{2}{3}$, \$230, \$153.33$\frac{1}{3}$, \$11.50
12. \$16.80, \$11.20, \$8.40, \$112, \$84, \$5.60, \$42, \$4.20, \$56, \$420, \$28, \$210, \$280, \$21, \$168, \$1120, \$840 \$560
13. \$12.80, \$1920, \$128, \$192, \$1280, \$19.20, \$9.60, \$2560, \$96, \$256, \$960, \$25.60, \$64, \$51.20, \$640, \$48
14. \$660, \$66, \$880, \$1320, \$88, \$132 \$176
15. \$28800, \$192, \$14.40, \$96, \$1920, \$72, \$960, \$36, \$36000, \$720

Art. 115, p. 73

1. \$6.64
2. \$38.98
3. \$93.22
4. \$164.52
5. \$130
6. \$6.88
7. \$21.98
8. \$10.83
9. \$60.30
10. \$111.34

Art. 116, p. 73

1. \$17.79
2. \$82.92
3. \$98.33
4. \$49.49
5. \$20.05
6. \$13.10
7. \$54.94
8. \$1.68

Art. 117, p. 74

1. \$23.47
2. \$26.06
3. \$11.71
4. \$29.36
5. \$31.20
6. \$101.44
7. \$22.31
8. \$25.11
9. \$24.11
10. \$13.73
11. \$122.41
12. \$38.68
13. \$493.01
14. Cost, \$6.22; price, \$5.50
15. \$7.17, 23c.
16. \$18.28
17. Cost, \$6.91; price, \$6.25
18. \$41.81
19. \$72.77
20. \$38.98
21. 2625 lb.
22. \$10.03
23. \$29.70
24. 35c
25. \$82.67

Art. 160, p. 85

1. 21 rd. 2$\frac{1}{2}$ yd. 8 in., or, 21 rd. 2 yd. 2 ft. 2 in.
2. 1 mi. 117 rd. 1 yd. 1 ft. 5 in.
3. 10 bu. 2 pk. 6 qt.
4. 39 gal. 3 qt.
5. 2 oz. 14 pwt. 2 gr.
6. 2 lb. 8 oz. 4 dr. 2 sc. 12 gr.
7. 5 da. 16 hr. 2 min. 34 sec.
8. 9 sq. yd. 8 sq. ft. 47 sq. in.
9. 23 lb. 10 oz. 2 dr. 1 sc.
10. 1 mi. 14 ch. 3 rd. 3 l.
11. 215 rd. 4 yd. 1 ft. 9 in.
12. 6 bbl. 25 gal. 1 pt. 1 gi.
13. 2 lb. 6 oz. 5 dr.
14. 7 mi. 67 ch. 3 rd. 9 l.
15. 5 sq. yd. 3 sq. ft. 40 sq. in.
16. 47 da. 17 hr. 54 min.
17. 10 lb. 8 oz. 13 gr.
18. 12 cu. yd. 4 cu. ft. 605 cu. in.
19. 24 bu. 3 pk. 4 qt. 1 pt.
20. 46 lb. 11 oz. 8 pwt.
21. 1672 in.
22. 17174 ft.
23. 16336 gr.
24. 15460 sec.
25. 3476 sc.
26. 3319 in.
27. 4082 pwt.
28. 2906 min.
29. 1321 sc.
30. 57620 sec.
31. 72920 sec.
32. 191 gi.
33. 955 pt.
34. 166694 cu. in.
35. 165 pt.
36. 518 gi.
37. 161 pt.
38. 147 cu. ft.

39. 466 gi.
40. 698 dr.
41. 6 ft. $7\frac{1}{5}$ in.
42. 35 rd. 9 ft. 2 in.
43. 14 ft. 2.28 in.
44. 4 oz. 10 pwt.
45. 3 hr.
46. 9 oz. 2 sc. 17.6 gr.
47. 11 cu. ft. 1404 cu in.
48. 2 pk. 1 qt. $1\frac{5}{8}$ pt.
49. 3 qt. 1 pt. 2.08 gi.
50. 3 sq. ft. 108 sq. in.
51. 3 qt. 1 pt.
52. 16 sq. yd. 5 sq. ft. 106.2 sq. in.
53. $\frac{1}{192}$ bu.
54. .019375 gal.
55. $\frac{7}{14080}$ mi
56. $\frac{1}{384}$ lb.
57. $\frac{1}{16800}$lb.
58. .02074+ cu. yd.
59. $\frac{1}{24000}$ mi.
60. .0001475 lb.
61. $\frac{1}{1920}$ gal.
62. $\frac{1}{16000}$ mi.
63. .004 lb.
64. .003645 lb.
65. .38611+ lb.
66. .528125 da.
67. .6875 gal.
68. .641$\frac{2}{3}$ sov.
69. .703125 bu.
70. .727083$\frac{1}{3}$ sov.

Art. 161, p. 87

1. $124.07
2. $398.25
3. $105.99
4. $82.43
5. $169.06
6. $65.06
7. 31 sov. 7s. 1d. 3 far
8. 120 sov. 8s. 3d. 3 far
9. 162 sov. 15s. 7d. 3 far.
10. 79 sov. 2d. 1 far.
11. 1325 sov. 7s. 9d. 1 far.
12. 120 sov. 9s. 4d.

Art. 162, p. 89

1. 26 lb. 4 oz. 7 pwt. 11 gr.
2. 250 rd. 1 yd. 1 ft. 7 in.
3. 103 gal. 3 qt.
4. 11 cu. yd. 10 cu. ft. 524 cu. in.
5. 1 lb. 11 oz. 13 pwt. 4 gr.
6. 7
7. 13
8. 96 bu. 2 pk. 6 qt.
9. 2 da. 20 hr. 29 min. 24 sec.
10. 4 mi. 286 rd. 1 yd. 1 ft. 2 in.
11. 1 bu. 2 pk. $3\frac{7}{8}$ qt.
12. $39\frac{8}{11}$
13. 8 lb. 3 dr. 14 gr.
14. 42 da. 6 hr. 2 min. 48 sec.
15. 189 rd. 4 yd. 7 in.
16. 41
17. 37 gal.
18. 52 mi. 17 rd. 2 ft. 2 in.
19. 8 mo. 26 da. 18 hr. 34 min. 32 sec.
20. 88 lb. 5 oz. 4 pwt. 16 gr.
21. 42 mi. 186 rd. 5 yd. 2 ft. 3 in.
22. 89 A. 131 sq. rd. $27\frac{3}{4}$ sq. yd.
23. 4 gal. 1 qt. 1 pt. 1 gi.
24. 167 bu. 1 pt.
25. 5 yr. 8 mo. 19 da. 9 hr. 58 min.

Art. 163, p. 90

1. 2 yr. 9 mo. 7 da.
2. 3 yr. 9 mo. 11 da.
3. 1 yr. 8 mo. 15 da.
4. 2 yr. 1 mo. 16 da.
5. 3 yr. 3 mo. 16 da.
6. 186 da.
7. 93 da.
8. 98 da.
9. 179 da.
10. 156 da.

Art. 164, p. 91

1. 37.236 qt.
2. $\frac{4}{175}$ lb.
3. 10 lb. 7 oz. 12 pwt. 2 gr.
4. 1 lb. 2 oz. 11 pwt. 16 gr.
5. 54 +
6. $51\frac{9}{16}$
7. 496 bu.
8. $137\frac{1}{2}$ bu.
9. $7\frac{1}{2}\frac{1}{3}\frac{1}{1}$ gal.
10. $2872\frac{4}{7}\frac{0}{7}$ gal.

Art. 170, p. 93

1. (*a*) 12×18, 24×9
(*b*) $12\frac{1}{2}\times19$,$25\times9\frac{1}{2}$
(*c*) 14×21, $28\times10\frac{1}{2}$
(*d*) 16×22, 32×11
4. 7×9$\frac{1}{2}$, $4\frac{2}{3}\times14$, $3\frac{1}{3}\times21$
5. 7×11, $5\frac{1}{2}\times14$
6. $5\frac{1}{2}\times8$, 4×11, $2\frac{3}{4}\times16$
7. $10\frac{2}{3}\times5\frac{1}{2}$, $4\times14\frac{2}{3}$, $8\times7\frac{1}{3}$, $5\frac{1}{3}\times11$, $16\times3\frac{2}{3}$, $2\frac{2}{3}\times22$

8. $5\frac{1}{2}\times8$, 4×11, $2\frac{2}{4}\times16$, 2×22

9. Same as problem 4

10. $7\frac{1}{2}\times4$, $2\frac{2}{3}\times10\frac{2}{3}$, $5\frac{1}{2}\times5\frac{1}{3}$, $3\frac{2}{3}\times8$, $11\times2\frac{2}{3}$, $1\frac{6}{8}\times16$, See problem 3, $8\frac{1}{3}\times4\frac{3}{4}$, $3\frac{1}{3}\times12\frac{2}{3}$, $6\frac{1}{4}\times6\frac{1}{3}$, $4\frac{1}{6}\times9\frac{1}{2}$, $12\frac{1}{2}\times3\frac{1}{8}$, $2\frac{1}{12}\times19$, $8\frac{1}{3}\times5$, $3\frac{1}{3}\times13\frac{1}{3}$, $6\frac{1}{4}\times6\frac{2}{3}$, $4\frac{1}{6}\times10$, $12\frac{2}{3}\times3\frac{1}{8}$, $2\frac{1}{12}\times20$, $9\frac{1}{3}\times5\frac{1}{4}$, $3\frac{1}{2}\times14$, 7×7, $4\frac{2}{3}\times10\frac{1}{2}$, $14\times3\frac{1}{2}$, $2\frac{1}{3}\times21$, See problem 7, 12×6, $4\frac{1}{2}\times16$, 9×8, 6×12, 18×4, 3×24, $12\frac{2}{3}\times6\frac{1}{4}$, $4\frac{3}{4}\times16\frac{2}{3}$, $9\frac{1}{2}\times8\frac{1}{3}$, $6\frac{1}{3}\times12\frac{1}{2}$, $19\times4\frac{1}{8}$, $3\frac{1}{8}\times25$

11. $8\times4\frac{3}{4}$, $3\times12\frac{2}{3}$, $6\times6\frac{1}{3}$, $4\times9\frac{1}{2}$, $12\times3\frac{1}{8}$, 2×19

12. There are no sheets in the table large enough.

13. 22×32

14. 25×40

15. 32×44

Art. 172, p. 96

1. $74\frac{1}{8}¢$

2. \$.0341

3. \$.5996

4. \$.704

5. $47\frac{1}{2}$c

Art. 173, p. 98

1. 50 lb., 125 lb., 50 lb., 50 lb.

2. 1st, 2 parts; 2d, 1 part; 3d, 2 parts; 4th, 4 parts

3. 5, 2, 2, 20, or 5, 4, 4, 20, or 5, 6, 6, 20

4. 3, 3, 3, and 12 lb. respectively

5. 1st, 64 lb.; 3d, 8 lb.

6. 240 lb. at 40¢, 240 lb. at 55¢, 80 lb. at 65¢, 400 lb. at 75¢, 720 lb. at 85¢

7. 5, 6, and 5 parts respectively

8. 10, 10, 10, and 29 parts respectively

9. 19 of 1st, 39 of 2d, 285 of 3d, 25 of 4th

P. 99

1. \$.8115

2. \$1516.69

3. \$44622.09

4. 130

5. \$111.27

6. 33.75

7. 197.21

8. \$3075

9. 98¢

10. 80¢, $\frac{1}{3}$ part

11. \$72.30

12. \$16.61

13. 190.40

14. 41.89 qt.

15. 6178.92 ft.

16. 1608 lb.

17. 2202.07 francs, 178197 marks

18. \$286.80

19. \$605.31

20. \$22.30

21. \$32.82

22. \$.60

23. 23 bu. 3 pk. 7 qt.

24. \$407

25. \$11.20

Art. 188, p. 103

1. $28\frac{4}{9}$ sq. yd.

2. 10 A. 150 sq. rd.

3. 96 sq. rd.

4. $283\frac{1}{3}$ sq. ft.

5. $113\frac{871}{1089}$ sq. rd.

6. 17 A. $37\frac{103}{121}$ sq. rd.

7. 6A.

8. 1512 sq. ft.

9. $71\frac{1}{9}$ sq. yd.

10. \$14.85

11. $5\frac{5}{7}$ sq. rd.

12. 1638 sq. ft.

13. $64\frac{16}{17}$ rd.

14. 4736 sq. in.

15. $606\frac{2}{9}$ sq. yd.

Art. 189, p. 105

2. \$2160

3. \$5000

4. \$2052

5. \$1400

6. $\$10666\frac{2}{3}$

7. \$2666.67

8. \$18

Art. 190, p. 106

1. 128 rd.

2. 151.4004 A.

3. 19 yd.

4. \$918

5. $63\frac{9}{17}$ ft.

6. 544 sq. ft.

7. 10164 sq. yd.

8. 64 sq. yd.

9. $38\frac{46}{83}$ rd.

10. 1144 sq. ft.

Art. 192, p. 108

1. 1152 sq. ft.

2. 3180 sq. yd.

3. 2294 sq. rd.

4. 7475 sq. rd.

5. 3182 sq. ft.

6. 15435 sq. yd.
7. $11684\frac{4}{9}$ sq. yd.
8. 65 ft.
9. 19200 sq. ft.
10. $832\frac{1}{2}$ sq. ft.
11. $21\frac{7}{10}$ A.
12. $26\frac{2}{3}$ rd.
13. $8\frac{4}{7}$ ft.
14. 8 rd., 6 rd., $4\frac{4}{5}$ rd.
15. 203 sq. rd.

Art. 193, p. 109

1. 266 sq. ft.
2. 750 sq. yd.
3. $3755\frac{1}{2}$ sq. yd.
4. 118572 sq. ft.
5. 3801 sq. rd.
6. 20 rd.
7. $14\frac{1}{80}$ A.
8. 600 sq. rd.
9. $6776
10. $71\frac{1}{3}$ rd.
11. $6\frac{1}{80}$ A.
12. $28\frac{1}{2}$ sq. ft.
13. 384 sq. ft.
14. 494 sq. in.

Art. 198, p. 110

1. 65 sq. ft.
2. 174 sq. ft.
3. $186\frac{2}{3}\frac{9}{8}$ sq. yd.
4. 477.12 sq. ft.
5. $93\frac{3}{8}$ sq. ft.
6. 166.32 in.
7. $7\frac{1}{4}$ ft.
8. 10.4 ft.
9. 43.0119 sq. ft.
10. 1731.197 sq. yd.
11. 21.217 sq. ft.
12. 2519.134 sq. ft.
13. 2992 sq. ft.
14. 3330.84 + sq. yd.
15. 1236.077 sq. ft.

Art. 203, p. 112

1. 87.96 ft.
2. 15.597 yd.
3. 17.825 yd.
4. 43.98 ft.
5. 10.822 yd.
6. 131.947 ft.
7. 15.119 yd.
8. 43.982 rd.
9. 39.152 rd.
10. 72.78 ft.

Art. 205, p. 113

1. $198.9\frac{3}{8}$ sq. ft.
2. 8148.48 sq. rd.
3. 286.478 sq. ft.
4. 17.104 sq. ft.
5. 50.265 sq. ft.
6. 53.794 sq. yd.
7. 91.987 sq. yd.
8. 5674.515 sq. ft.
9. 254.469 sq. rd.
10. 452.39 sq. ft.
11. 452.39 sq. rd.
12. 804.249 sq. yd.
13. 17 A. 107.44 sq. rd.
14. 50 A. 148.70 sq. rd.
15. 33.183 sq. ft.
16. 151.83 sq. ft.
17. 3 sq. ft. 70.65 sq. in. in.
18. 97.482 sq. ft.

Art. 206, p. 114

1. $12.80
2. 44 sq. yd.
3. Floor, etc., $105\frac{1}{3}$ sq. yd.; walls, $59\frac{1}{3}$ sq. yd.
4. $108\frac{1}{4}$ sq. yd.
5. $27.11
6. $39.60
7. 9350 sq. yd.
8. 532
9. 1344 sq. yd.
10. 464 sq. yd.
11. $6 70
12. $36\frac{3}{8}$ sq. yd.
13. $145\frac{1}{3}$ sq. yd.
14. $114.64
15. $4833.89
16. 3019.077 sq. ft.
17. $472\frac{1}{2}$ sq. yd.
18. $21\frac{3}{5}$ squares
19. 160 sq. yd.
20. 1020 sq. ft.

Art. 208, p. 115

1. 12
2. 15
3. 10
4. 25
5. 12
6. 104 yd.
7. $541\frac{1}{2}$ yds.
8. $23\frac{1}{2}$

Art. 209, p. 117

1. $61.60
2. 72 or 69.30
3. Lengthwise, $64.80; crosswise, $64
4. 49 yd.
5. (a) $186\frac{2}{3}$ yd. crosswise,
   (b) 50 yd. crosswise,
   (c) 136 yd. lengthwise,
   (d) $21\frac{2}{3}$ yd. crosswise,
   (e) $37\frac{1}{3}$ yd. crosswise
6. (a) $9\frac{1}{3}$ yd.,
   (b) $1\frac{1}{3}$ yd.,
   (c) 8 yd.,
   (d) $3\frac{1}{3}$ yd.,
   (e) $4\frac{2}{3}$ yd.

7. (a) $36\frac{2}{3}$ yd.,
(b) 77 yd.,
(c) 26 yd.,
(d) 32 yd.,
(e) $83\frac{1}{3}$ yd.

Art. 213, p. 118

1. (a) 2304 cu. ft.,
(b) 784 cu. ft.,
(c) 2560 cu. ft.,
(d) 1233 cu. ft.
2. 200 cu. ft.
3. 1344 cu. ft.
4. 5939.886 cu. ft.
5. 1200 cu. ft.
6. 2781.173 cu. ft.
7. 114648.596 cu. ft.

Art. 214, p. 119

1. (a) 197.92 cu. ft.,
(b) 298.414 cu. ft.,
(c) 326.726 cu. ft.,
(d) 982.62 cu. ft.
2. 2513.28 cu. ft.
3. 436.72 cu. ft.
4. 26507.25 cu. ft.
5. 30.968 cu. ft.
6. 8.84 ft.

Art. 217, p. 120

1. 1050 cu. ft.
2. $59.05
3. $36.27, cost of digging; $58.83, cost of wall; $10\frac{5}{6}$ cd.
4. 2404 cu. ft.
5. $12\frac{3}{8}$ cd.
6. 27720
7. Cost of digging, $93.33; cost of laying, $64.15; $8\frac{89}{128}$ cd.
8. 4838
9. 2592 cu. ft.
10. $148.78

Art. 219, p. 122

1. (a) $10\frac{1}{1}\frac{1}{6}$,
(b) $5\frac{1}{4}$,
(c) $1\frac{1}{2}$,
(d) $4\frac{7}{32}$,
(e) $7\frac{1}{8}$
2. $23.63
3. 96 ft.
4. $4\frac{4}{5}$ ft.

Art. 222, p. 123

1. 20
2. $10\frac{1}{2}$
3. 15
4. 25
5. 32
6. 11
7. 6
8. $15\frac{1}{8}$
9. $26\frac{2}{3}$
10. 42
11. $46\frac{2}{3}$
12. 162
13. $106\frac{2}{3}$
14. 16
15. 12
16. $18\frac{2}{3}$
17. 10
18. $5.53
19. $63
20. $25.05
21. $130.65
22. $32.82
23. $21.20

Art. 223, p. 125

1. (a) 308.57
(b) 20.09
(c) $86\frac{1}{1}\frac{1}{4}$
2. $176\frac{1}{1}\frac{1}{4}$ bu.
3. $423\frac{1}{8}\frac{5}{6}$ bu.
4. $84\frac{3}{8}$ bu.
5. $807\frac{9}{7}\frac{2}{7}$ gal.
6. $79\frac{6}{7}\frac{1}{7}$ bbl.
7. $1481\frac{1}{7}$ gal.
8. 63.9744 bbl.
9. $18\frac{3}{8}\frac{4}{5}$ gal.
10. 1.652 gal.
11. 15.9936 gal.
12. $239\frac{8}{7}\frac{8}{7}$ gal.
13. 82.318 bbl.

Art. 225, p. 126

1. 115.2 bu.
2. $65\frac{5}{8}$ bu.
3. $115\frac{1}{8}$ bu.
4. $76.56
5. 800 cu. ft., 25 ft.
6. $4\frac{1}{6}$ ft.
7. 117.81 cu. ft., 883.575 gal.
8. 6.565 ft.
9. $340\frac{1}{2}$ bu.
10. $311\frac{1}{3}$ bu.
11. $47\frac{1}{4}$ bu.
12. $87\frac{1}{2}$ bu.
13. $1\frac{1}{2}\frac{1}{1}$ ft.
14. $62\frac{3}{8}$ cwt.
15. $14\frac{1}{1}\frac{0}{3}$ T.
16. 10 T.
17. $162.85
18. $67\frac{1}{2}\frac{7}{4}$ ft.
19. 640 cu. ft., $4\frac{4}{9}$ ft
20. $194\frac{4}{9}$ bu.

Art. 231, p. 132

1. 24
2. 75
3. 206
4. 1234
5. 2.645
6. 1.414
7. 126
8. 1626.57
9. $\frac{5}{6}$
10. .968+
11. 48.989 rd.
12. 125

13. 38809
14. $397.60
15. 26.92 ft.
    107.7 ft.

Art. 232, p. 133

1. 419.98 sq. ft.
2. 9 A. 47.914 sq. rd.
3. 1200 sq. ft.
4. 2 A. 111.155 sq. rd.
5. 14.273 rd.
6. 1664.45 sq. ft.
7. 45.13 rd.
8. 31.7 rd.
9. 3.19 ft.
10. 309.74 sq. rd.
11. Circle, 795.77 sq. yd.; square, 625 sq. yd.
12. 13.541 ft.
13. 88.622 ft.
14. 6.684 ft.
15. 9 A. 137.9 sq. rd.
16. 17 ft. 8 in.
17. 191.33 sq. yd.
18. 110.84 bu.
19. 6196.77 sq. ft.
20. 239.24 sq. ft.

Art. 234, p. 135

1. 9 yd.
2. 25 ft.
3. 7 A. 56 sq. rd.
4. 31.112 ft.
5. 26.076 ft.
6. 19.31 ft.
7. 75 ft.
8. 22500 sq. ft.
9. 105.47 ft.
10. 18.7 ft.
11. 1.414 ft.
12. 1536 sq. ft.
13. 150 ft.
14. 28.42 ft.
15. 470.3 sq. ft.

P. 136

1. 176.715 sq. ft.
2. 257.8 sq. ft.
3. $24\frac{3}{4}$ ft.
4. $390\frac{1}{2}\frac{5}{8}$ bu.
5. $8\frac{5}{1}\frac{}{6}$ cd.
6. 132.53 cu. in.
7. 164.93 sq. ft.
8. $320.89
9. $392.89
10. 2035.75+ cu. ft.
11. 21120
12. $83\frac{1}{3}$ yd.
13. 8750 lb.
14. 168 ft.
15. 2361.28 ft.
16. 1060 cu. ft.
17. $238\frac{1}{2}\frac{4}{7}$ loads
18. $47\frac{1}{2}\frac{8}{1}$ bbl.
19. $6.55
20. $130\frac{5}{9}$ sq. yd.
21. $72\frac{1}{6}\frac{7}{4}$ cd.
22. 48486
23. 63 ft.
24. 5016 ft., 209 posts
25. $168\frac{3}{4}$ bu.
26. (a) 288 ft.
    (b) 1104 ft.
    (c) 2520 ft.
    (d) 120 ft.
    (e) $1377\frac{1}{2}$ ft.
    (f) 3645 ft.
    (g) 2862 ft.
    (h) 1842.4 ft.
    (i) 526 sq. yd.
    (j) $27.84
27. (a) 1780 cu. ft.
    (b) $13\frac{2}{3}\frac{9}{2}$ cd.
    (c) $315\frac{7}{2}\frac{}{7}$ loads,
    (d) 112 sq. yd.
    (e) 748 sq. ft.

Art. 235, p. 139

1. 20, $182\frac{1}{2}$, 504
2. $18, $21.60
3. $63
4. $47.40
5. .75
6. .25
7. 1200
8. $1440
9. 500
10. .25
11. .125
12. 40.48
13. 5, 500
14. 625
15. $51\frac{3}{7}$
16. 252
17. .12
18. .92
19. 1.12
20. .30
21. .08
22. 618.75
23. .86
24. $51
25. 301

Art. 251, p. 142

1. 86.4, 446.4, 273.6
2. $294, 994, 406
3. $548.10, 1388.10, 291.90
4. $632.875, 1395.375, 129.625
5. $158.77, 696.97, 379.43
6. $171.10, 855.50, 513.30
7. $699.45, 1632.05, 233.15
8. $212.87, 851.47, 425.73
9. $8517.25, 18251.25, 1216.75
10. $832, 2080, 416
11. 105, 945, 735
12. 354, 1298, 590

13. $427 32, 2991.24, 2136.60
14. 3924.48, 8596.48, 747.52
15. $398.95
16. 6995.08$\frac{1}{3}$
17. 8525
18. $330.13, 1251.43, 561 17
19. $397.39, 1147.19, 352.41
20. 1301.44, 5949.44, 3346.56
21. 546
22. 6 T. 716 lb.
23. 11.28 gal.
24. 87 T.
25. 5.46$\frac{10}{11}$ yd.
26. 18.306 lb.
27. .168 lb.

Art. 252, p. 143

1. 33$\frac{1}{3}$%
2. 71$\frac{1}{19}$%
3. 75%
4. 11%
5. 60$\frac{20}{43}$%
6. 150%
7. 25%
8. 133$\frac{1}{3}$%
9. 75$\frac{75}{119}$%
10. .06$\frac{32}{43}$%
11. 3333$\frac{1}{3}$%
12. 4%
13. 242$\frac{14}{33}$%
14. 37$\frac{9}{43}$%
15. 1580%
16. 99.156+%
17. 2.414+%
18. $311.11
19. .004985+%, .006186+%, .007123+%
20. 7.8$\frac{1}{3}$%, 32$\frac{13}{16}$%, 59$\frac{3}{8}$%, 6.64 lb.
21. 97$\frac{1}{11}$%
22. .0005$\frac{11}{18}$%
23. 53$\frac{1}{3}$%
24. .8$\frac{3}{4}$%

Art. 253, p. 145

1. 62400
2. $135.625
3. 1416
4. 1$\frac{7}{8}$
5. 4060.15+
6. 1650000
7. 11185$\frac{5}{7}$
8. 12$\frac{3}{8}$
9. .0378%
10. 93000000
11. $35812.50
12. $1172958.46
13. $13861111.11
14. $27500
15. 5170$\frac{10}{17}$ gal.
16. 28571$\frac{3}{7}$ lb.
17. 400000 gr.
18. $9166.67
19. 1578$\frac{18}{19}$ lb. silver, 1263$\frac{3}{19}$ lb tin, 16736$\frac{16}{19}$ lb. copper
20. 326$\frac{2}{23}$ lb.

Art. 254, p. 147

1. 888$\frac{8}{9}$
2. 1600
3. $26
4. 446$\frac{62}{103}$
5. 64.224+
6. $1219.23
7. $1311.69
8. 757.696+
9. $469.56
10. $1193 74
11. 684
12. $1600
13. 25$\frac{75}{121}$ lb.
14. 17.8875 ft.
15. $9259.26

Art. 255, p. 149

1. 1800
2. $284.88
3. 882$\frac{6}{17}$
4. $497.14
5. $876.46
6. 1200
7. $404.53
8. $122.74
9. $275.85
10. 2100
11. 1909.556 lb
12. 140.2 yd.
13. 110.73 lb.
14. 400
15. $1030.93

Art. 256, p. 152

1. $1063.59
2. 11$\frac{1}{2}$¢
3. 12$\frac{28}{45}$%
4. 3.83\frac{1}{3}$
5. 7$\frac{49}{115}$%
6. 5.48$\frac{9}{33}$%
7. 140.62\frac{1}{2}$
8. 11$\frac{13}{17}$%
9. 12$\frac{17}{34}$%
10. 31$\frac{1}{4}$%
11. 25¢
12. 3.384%
13. $7043.75
14. 40%
15. 63$\frac{7}{11}$%
16. 16$\frac{2}{3}$¢
17. 6$\frac{6}{7}$%
18. 60%
19. $40.19
20. $4
21. 4%
22. 6$\frac{1}{4}$% loss
23. 3.41\frac{2}{3}$
24. $3

Art. 271, p. 158

1. Com., $19.70; cost, $895.30

2. Com., $30.21; cost, $984.21
3. Com., $8.10; guar., $.81; cost, $332.91
4. Com., $37.96; guar., $12.65; cost, $1316.01
5. Com., $70.81; guar., $19.31; cost, $1398.75
6. Com., $35.28; guar., $5.04; cost, $544.34
7. Com., $86.25; guar., $40.02; cost, $2127.27
8. Com., $13.11; guar., $8.74; cost, $472.67
9. Com., $9.14; cost, $313.94
10. Com., $30.35; proceeds, $836.90
11. Com., $2.42; guar., $.24; proceeds, $94.09
12. Com., $1.28; guar., $.32; proceeds, $62.64
13. Com., $123.98; proceeds, $2975.62
14. Com., $91.27; guar., $27.38; proceeds, $1706.65
15. Com., $1523.44; proceeds, $4489.06
16. Com., $520.96; guar., $71.04; proceeds, $1776
17. Com., $132.62; guar., $18.42; proceeds, $534 26
18. Com., $17.06; guar., $3.41; proceeds, $320.78
19. $2653
20. $14610
21. $661.68
22. $1123.09
23. $2932.50
24. $7125.00
25. $5975.65
26. $2418.33
27. $1467.72
28. $11000
29. $7266.67
30. $18150
31. $5700
32. $2\frac{6}{9}\frac{8}{9}\%$
33. 5%
34. $3\frac{1}{2}\%$
35. $6\frac{2}{9}\frac{1}{9}\%$
36. Com., $374.56; guar., $46.82; proceeds, $4233.62
37. $6320.05
38. $2781.13
39. $479.15
40. $1418.27
41. $3515.24
42. $5809.15
43. $99.25
44. $14070.99
45. Second agent by $\frac{1}{2}\%$

Art. 274, p. 162

1. $49.94
2. $445.50
3. $708.75, $725.76, $696.15
4. $23.60
5. $489.60
6. $10
7. $7.35
8. $351.47
9. 67%
10. (a) $94.08
    (b) $436.30
    (c) $71 52
    (d) $449.75
    (e) $948.49
    (f) $2283.78
    (g) $2991.96
    (h) $318.05
    (i) $499.80
    (j) $636.53
11. (a) 60%
    (b) $38\frac{8}{9}\%$
    (c) $43\frac{3}{4}\%$
    (d) 20%
    (e) $68\frac{3}{4}\%$
    (f) $108\frac{1}{3}\%$
    (g) $66\frac{2}{3}\%$
    (h) $13\frac{1}{3}\%$
    (i) 100%
    (j) $5\frac{1}{1}\frac{6}{7}\%$
12. First 1%

Art. 276, p. 165

1. has, aas, dhm. hem, aos, ms, no, bls, nnn, dns
2. $1.55, $5.70, $.99, $2.18, $3.65, $4.25, $8.00, $7.20, $12.75
3. h l d, m h n, h a d, a m n, d s o, s n h h y y y, l y v, h s l d
4. (a) $\frac{\text{h s y}}{\text{w c c}}$
   (b) $\frac{\text{e y}}{\text{e r}}$
   (c) $\frac{\text{h m s}}{\text{h a a}}$
   (d) $\frac{\text{a d y}}{\text{h i s}}$
   (e) $\frac{\text{h a s}}{\text{w r i}}$
   (f) $\frac{\text{a y y}}{\text{h a s}}$

(g) $\frac{h\,o\,y}{h\,s\,s}$

(h) $\frac{l\,y}{w\,s\,p}$

(i) $\frac{h\,a\,y}{h\,w\,s}$

(j) $\frac{h\,d\,y}{w\,c\,h}$

Art. 280, p. 166

1. $598.93
2. $354.63
3. $996.07
4. $160.90
5. $70.89
6. $61.34
7. $207
8. $113.54
9. $13.36
10. $19.60

P. 171

1. $41.56
2. $27126.93
3. $189776.67
4. $6.48
5. $18.23
6. $3053.81
7. $7275
8. $3690.93
9. $100260, loss to Co.; $43615 loss to owner under average clause
10. $22.50 per $1000
11. 44⅘ yr.
12. 8.79%
13. $700.62 and $774.38

P. 176

2. $91.06, premium: $820.06 loss
3. Receive $2000; would have received $1703.34
4. 1st, $750.66; 2d, $1154.60; 3d, $1867.82
5. 1st, $3302.18, premiums; 2d, $3923.60, premiums; 3d, $6347.25, premiums; $4617 received

P. 179

1. $9.52
2. $2.28
3. $5.34
4. $10.56
5. $1.51
6. $1.19
7. 63¢
8. $14.06
9. $13.91
10. $17.44
11. $161.97
12. $7.16
13. $17.22
14. $74.62
15. $29.93
16. $267.38
17. $20.63
18. $33.69
19. $1.29
20. $2.42
21. $144.87
22. $16.13
23. $4.22
24. $2.77
25. $37.66
26. $1.39
27. $58.09
28. $66.82
29. $160.09
30. $7.55
31. $2.42
32. $26.41
33. $12.91
34. 35¢
35. $53.75
36. $2.20
37. $4.71
38. $10.08
39. $16.59
40. $374.00

P. 181

1. $1.46
2. 61¢
3. 78¢
4. $15.23
5. $4.77
6. $2.53
7. $5.37
8. $9.70
9. 84¢
10. 94¢
11. $2.47
12. $8.24
13. $3.15
14. 75¢
15. $1.74
16. $1.84
17. $1.05
18. $3.32
19. $2.23
20. 44¢
21. $1423.31
22. $1001
23. $278.05
24. $317.94
25. $279.14
26. $829.57
27. $419.22
28. $618.13
29. $284.59
30. $116.25
31. $786.97
32. $574.60
33. $1827.92
34. $68.22
35. $14800.46
36. $844.29

37. $207.84
38. $259.73
39. $359.11
40. $674.66

P. 184

1. $2
2. $5.96
3. $1.88
4. $10.37
5. $22.64
6. $8.62
7. $97.51
8. $57.99
9. $146.41
10. $25.59
11. $24.29
12. $6.21
13. $6.91
14. $175.07
15. $17.93
16. $2.90
17. $10.35
18. $5.84
19. $231.30
20. $66.18
21. 69¢
22. $2.63
23. $8.16
24. $12.24
25. $19.42
26. $1.11
27. $1.51
28. $4.86
29. $6.87
30. $7.70

P. 184

31. 56¢
32. $1.49
33. $3.05
34. $4.29
35. $1.82
36. $5.61
37. $1.12
38. $1.19
39. $34.96
40. $31.85

P. 185

1. $4.36
2. $17.59
3. $6.60
4. $.18
5. $62.48
6. $35.93
7. $19.66
8. $6.26
9. $2.63
10. $8.67

P. 186

1. $51.75
2. $90.88
3. $24.10
4. $156.60
5. $40.64
6. $14.53
7. $29.98
8. $9.59
9. $25.28
10. $27.67
11. $59.85
12. $139.50
13. $3.18
14. $4.29
15. $14.73
16. $9.59
17. $9.24
18. $6.55
19. $1.51
20. $3.63
21. $22.09
22. $40.42
23. $66.22
24. $126.02
25. $79.04
26. $259.72
27. $225.16
28. $115.77
29. $73.37
30. $131.84
31. $246.02
32. $172.83
33. $21
34. $261.36
35. $30.76
36. $12.22
37. $61.57
38. $108.25
39. $55.87
40. $152.64

P. 187

1. $20.90, $663.90
2. $22.50, $740.50
3. $3.05, $139.05
4. $9.11, $216.11
5. $39.25, $1731.25
6. $45.35, $2091.35
7. $1.76, $252.76
8. $3.24, $238.24
9. $94.46, $2281.46
10. $14.01, $528.01
11. $54.86, $396.86
12. $31.59, $665.59
13. $74.29, $812.29
14. $51.15, $1286.15
15. $133.73, $3083.73
16. $80.38, $1045.68
17. $6.98, $439.98
18. $23.48, $1603.48
19. $64.17, $939.17
20. $41.86, $427.86
21. $43.59, $958.59
22. $110.80, $1133.80
23. $238.66, $3845.66
24. $59.75, $678.75
25. $54.17, $839.17
26. $2.31, $317.31
27. $3.44, $430.44
28. $10.58, $1390.58
29. $14.12, $1039.12
30. $7.24, $723.24
31. $10.03, $834.03

52. $3.74, $909.74
53. $3.84, $317.84
54. $8.06, $431.06

P. 188

55. 45¢, $206 45
56. $3.56, $344.56
57. $10.53, $1227.53
58. $11.82, $1098.82
59. $4.75, $425.75
60. $3.77, $541.77
61. $8 04, $681.04
62. $2.03, $740.03
63. $1.62, $298.62
64. $4.02, $313.02
65. $5.32, $278.32
66. $7.23, $955.23
67. $49.90, $3120.90
68. $34.44, $4287.44
69. $29.37, $1435.37
70. $36 72, $2386.72

Art. 328, p. 188

1. $4.80
2. 4 mo.
3. 6%
4. $374.40
5. 6%
6. 8 mo.
7. $360
8. 4 yr. 2 mo.
9. 5%
10. 8 mo. 5 da.
11. $16\frac{2}{3}$ yr., 20 yr., $12\frac{1}{2}$ yr., $33\frac{1}{3}$ yr., $14\frac{2}{7}$ yr., 10 yr.
12. $4\frac{1}{2}$%
13. 3 yr. 4 mo. 27 da.
14. $537.04
15. $8\frac{1}{3}$%
16. 1 yr. 2 mo. 22 da.
17. $5\frac{2}{5}\frac{5}{1}$%
18. $904.98
19. $22\frac{2}{3}$ yr.
20. $5\frac{1}{2}$%

Art. 335, p. 191

1. $147.82
2. $43.33
3. $77.50
4. $154.25
5. $138.06
6. $149.34
7. $139
8. $589 88
9. $330.80
10. $368.02
11. $178.05
12. $533.98
13. $453.06

Art. 337, p. 192

1. $129
2. $132.55
3. $74.64
4. $883.15
5. $144.29
6. $697.97
7. $159.27
8. $469.22

Art. 338, p. 193

1. $681.88
2. $1002.05
3. $1002.80
4. $314.36
5. $377.95
6. $4097.36
7. $1243.53
8. $1622.84
9. $1265.59
10. $1594.59
11. $141.87
12. $181.81
13. $1353.14
14. $380.25
15. $642.21

Art. 359, p. 200

1. $269.08
2. $512.32
3. Due date, April 8, $728.44
4. Due date, May 13 $599.36
5. Due date, June 7, 1900, $297.44
6. Due date, Oct. 3, 1898, $1278.85
7. Due date, July 4, 1898, $381.30
8. Due date, Dec. 28, $234.72
9. Due date, Nov. 29, $4217.90
10. Due date, Dec. 30 $551.01
11. Due date, Dec. 1, $348.21
12. Due date, May 8 $438.32

Art. 361, p. 202

1. $292.40
2. $289.04
3. $595.51
4. $367.29

Art. 362, p. 204

1. $1549.55
2. $527.86
3. $1213.01
4. $174
5. $1353.51

Art. 363, p. 206

1. $1051.92
2. $1912.37
3. $219.40
4. $853.05
5. $1597.33
6. $312.86
7. $934.37
8. $1735.54
9. $623.72
10. $1373.82

Art. 366, p. 208

1. $807.69
2. $1193.89
3. Present worth, $1204.95; discount, $12.05
4. Present worth, $578.37; discount, $47.43
5. Present worth, $2928.47; discount, $155.52
6. Present worth, $196.94; discount, $18.31
7. Present worth, $1016.15; discount, $11.85
8. Present worth, $1906.25; discount, $228.75
9. Present worth, $599.16; discount, $116.84
10. Present worth, $867.59; discount, $69.41
11. $1245.02
12. $3556.60
13. $1266.98
14. 4¢ gained on each by buying on time
15. Time offer $23.81 better
16. $2500
17. $15.31 gain
18. 19.23%
19. Cash offer is $11.54 better
20. $6124.95
21. $8333.33
22. 20.192%
23. $110.57
24. $3162.98

Art. 374, p. 213

	Disc.	Proceeds
1.	$ 3.24	$ 184.22
2.	1.56	223.44
3.	5.00	436.45
4.	6.59	253.41
5.	2.37	709.63
6.	3.65	452.35
7.	4.52	577.19
8.	4.30	326.59
9.	6.56	485.72
10.	3.98	314.02
11.	10.41	640.26
12.	13.07	527.68
13.	17.79	1142.21
14.	6.19	475.93
15.	1.23	327.77
16.	19.65	1280.85
17.	6.23	731.77
18.	2.70	423.30
19.	4 69	375.31
20.	7.13	534.65
21.	4.20	835.80
22.	2.21	234.79

Art. 385, p. 216

1. $337.17
2. $246.25
3. $471.83
4. $594
5. $845.75
6. $1188
7. $397.33
8. $357.60
9. $512.20
10. $173.98
11. $227.70
12. $147

Art. 388, p. 217

1. $197
2. $345.62
3. Proceeds, $346.50; lack, $3\frac{1}{2}$¢; amount due, $352.28

Art. 390, p. 218

1. $121.82
2. $71.33
3. $399.38
4. 229.16
5. $131.84

Art. 395, p. 222

1. $851.70
2. $1283.20
3. $3803.04
4. $1571.57
5. $9709.70
6. $3260.92
7. 1st, $1.002; 2d, $5; 3d, $850
8. $2506.27
9. 32¢
10. $4491.02

Art. 398, p. 224

1. $2305.75
2. $18227.19
3. $6580
4. $753.40
5. $1592.93
6. $1692.42
7. $3780.52
8. $1260
9. $537.46
10. 2522.52

Art. 401, p. 225

1. $199.50
2. $177.75
3. $174.12
4. $1779.75
6. $2333.45
7. $580.53
8. $2380

Art. 410, p. 228

1. $2044.60
2. 264 sov. 6s. 8d. 2 far.
3. $606.43

4. $6937.98
5. $4523.75
6. 937 sov. 6s. 1d. 3 far.
7. $3497.40
8. Cost, $4918.38; face, $4900
9. $20281.40
10. $38528.44

Art. 420, p. 232

1. $5.23
2. $410.80.
3. $899.88
4. $186.80

P. 233

1. $288.86
2. $3607.50
3. $455.56
4. $811.42
5. $2116.80
6. $131.21
7. $206.79
8. $767 64
9. $232.23
10. $323.72

Art. 431, p. 240

1. Feb. 25
2. March 11
3. May 30
4. July 6
5. Aug. 9
6. Sept. 4
7. Oct. 19
8. Oct. 2

Art. 432, p. 242

1. Dec. 21
2. May 25
3. Aug. 6
5. Jan. 4, 1901
6. March 27
7. March 23
8. Feb. 4
9. June 2
10. June 13
11. March 16
12. Balance, $220; Dec. 27, 1899, $225.68
13. Feb. 19, $216.47
15. $1.43, interest due April 25

Art. 436, p. 248

1. $157.59
2. $41.21
3. $159.64
4. $22.38

Art. 441, p. 249

2. $586.65, due Dec. 12
1. $1417.48
3. $1399.40, due Aug. 20

Art. 442, p. 251

1. A, $6; B, $8
2. 1st, $15; 2d, $25
3. 10, 20, 30
4. 60, 72, 84
5. 150, 225, 375
6. A, $240; B, $420; C, $510
7. A, $\frac{5}{15}$; B, $\frac{9}{15}$; C, $\frac{4}{15}$; A's gain, $50
8. A, $75; B, $135
9. $150, $175, $225
10. 45, 30
11. 45, 27
12. 210, 260
13. 225, 400, 480
14. $103.25, $92.75, $177.10
15. $120, $135, $150

Art. 456, p. 253

1. Jones, $980; Smith, $700; Brown, $1260
2. Beekman, $480; Hadley, $320; Perry, $440
3. $1500, $1000
4. Watson, gain $1292.31; P. W., $6692.31; Barnes, gain $1507.69; P. W., $7807.69
5. A, $5400; B, $8100; C, $6750; D, $9450
6. Gooding, $22193.68; Spencer, $17966.32
7. Hawes owned $\frac{7}{12}$; Cross, $\frac{5}{12}$; Hawes received $23333\frac{1}{3}$; Cross, $16666\frac{2}{3}$; Hawes received $26666\frac{2}{3}$
9. Martin, $1903 28; Gould, $1480.33; Towne, $1776.39
10. Howe, $470.14; Benton, $428.72; Ward, $351.14
11. Bush, $104.35; Austin, $208.69; Fox, $86.96
12. Johnson, $1033\frac{1}{3}$; Chapin, $466\frac{2}{3}$
13. Wright, $13645.18; Greene, $11574.01; Bates, $14590.15; Thompson, $6190.66
14. Randall, $7865.89 Chapman, $8125.92 Holt, $6508.19

Art. 471, p. 259

1. $288
2. $460
3. $801

4. $206
5. $5\frac{1}{2}\%$
6. $2\frac{1}{4}\%$
7. $6\frac{1}{2}\%$ above par
8. $14328.13
9. $30220.25
10. $5773.75
11. $8385
12. $1020
13. $12578.13
14. $10046.25
16. 56
17. 28
18. 48
19. $8\frac{3}{20}$ premium
21. $896
22. $320
23. $672
24. First $17.50
25. Increased $5
26. 135
27. $14445
28. $20146.88
29. $40026.25
30. $34200
31. 7%
33. $5\frac{5}{8}\%$
34. $7\frac{1}{7}\%$
35. Latter $\frac{38}{46}\%$
37. $142\frac{6}{7}$
38. 96
39. $14\frac{2}{7}\%$
40. $16\frac{2}{3}\%$

Art. 474, p. 264

1. $292.95
2. $4315.72
3. $86.47
4. $4900

Art. 489, p. 267

1. $82.40
2. $260.63
3. .0074, $22.05, $272.84

Art. 490, p. 268

1. $208.13, $17.68, $273.56, $5.51, $3.51, $1.46, $2.05, $2.59, $52.61, $25.20, $156.38, $104.06
2. Rate, .0053; $65.69, $24.17, $38.56, $131.92, $228.56
3. (a) $18.10
   (b) $7.31
   (c) $5.90
   (d) $17.28
   (e) $6.48
4. .00745
5. .008065
6. $64.03
7. $116.47
8. $22.97

Art. 506, p. 270

1. $835.80
2. $312
3. $3828
4. $66
5. $8350.70
6. $188.70
7. $67.13
8. $15480
9. $659.32
10. $840

P. 272

1. 8
2. 19 T, $571\frac{3}{7}$ lb.
3. 3515.6 lb.
4. 32400 lb.
5. $33.10
6. 10 rd. 3 yd. 2 ft.
7. $4\frac{6}{8}\frac{6}{3}\frac{8}{3}$
8. 1737.59 gal.
9. $1000
10. $119.65
11. $8\frac{1}{2}\%$
12. 2299.12 sq. ft.
13. $24856\frac{1}{3}$ mi.
14. 213 T. 1650.3 lb.
15. $1368.89
16. 33.941 rd.
17. 42.57 rd.
18. 11.653 rd.
19. 14.142 in.
20. $42857.14
21. 20.67%
22. Increased $55
23. $1173.54
24. $4485.61
25. $15033\frac{3}{11}$ lb
26. 2 yr. 9 mo. 18 da.
27. 720.28
28. 1.135% loss
29. Gain, $2405; P. W., $10405
30. $34.24
31. $513.75
32. $647.07
33. Oct. 3
34. (a) $1021.36
    (b) $1021.10
35. Gain, $2355; resources, $9225; liabilities, $6870; P. W., $7355
36 $166\frac{2}{3}\%$
37. 27.325%
38. $47.58
39. $36.75
40. $92.62
41. $14\frac{6}{11}$ ft.
42. .0375
43. 8 breadths, $42\frac{2}{3}$ yd.
44. $348
45. 10368
46. 46¢
47. $205
48. 34.22%
49. 111 bu. 1 pk. $2\frac{1}{2}$ qt.

50. $390
51. $30
52. $766 13
53. 53$\frac{1}{7}$ lb.
54. 174%
55. 10$\frac{5}{21}$%
56. 11$\frac{1}{9}$%
57. $10875
58. 45932$\frac{3}{4}$
59. $17840
60. 5.86%
61. 70$\frac{2}{3}$ ft.
62. $5.75
63. 702 lb. copper, 198 lb. tin.
64. $2420
65. 7 T. 1384 lb.
66. 6$\frac{1}{4}$%
67. $14.50
68. 75
69. $185.96
70. 8.8$\frac{4}{17}$%
71. $58.65
72. $3.84
73. $412
74. A, $17671.81; B, $23557.24; C, $7933.85; D, $18837.10
75. $100.38
76. 7.48%
77. (a) 37.57%
(b) 14.66%
(e) 28.2%
(c) 17.1%
(j) 2.44%
78. 5 lb. at 14¢; 3 lb at 18¢; 4 lb. at 25¢; 4 lb. at 32¢, 666$\frac{2}{3}$ lb.
79. $15.94
80. $673.96