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Monetarism Simulated

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Abstract

Any model admitting inflation as an equilibrating variable will con-
tain a derivative of price with respect to time, hence be intrinsic-
ally dynamic, and will immediately have two additional equilibrating
variables, the nominal and the real rate of interest. Any model deny-
ing that monetary policy can peg the rate of unemployment for more
than very limited periods, must dismiss and go beyond such periods
and become a long-run model.

A long-run, dynamic, and two-interest-rates model is clearly in-
compatible with the short-run, static, one-interest-rate IS-LM frame-
work offered by Friedman (1970) as his theoretical framework.

The paper works its way backwards from Friedman's conclusions
to a model which will deliver them and shows that a neoclassical
growth model will require little modification to do so.

MONETARISM SIMULATED

Hans Brems

...The monetary authority controls nominal quantities...

It cannot use its control over nominal quantities to peg

a real quantity--the real rate of interest, the rate of

unemployment...

Friedman (1968: 11)

1. Friedman's Problem

Friedman wished to include the rate of inflation among his equilib-
rating variables. To do so he unfroze Keynesian price. But merely making
the level of price a variable wouldn't do. It is one thing to tell
how high price would be. It is quite a different thing to tell how ra-
pidly price would be changing--which is what inflation is all about.
Any model admitting inflation will contain a derivative with respect
to time dP/dt, hence be intrinsically dynamic. Any model admitting in-
flation as an equilibrating variable will immediately have two addition-

al ones, i.e., the nominal and the real rate of interest—as perhaps

Turgot (1769-1770) and certainly Fisher (1896) taught us. Consequently

a Friedman model must be a dynamic two-interest-rates model.

According to Friedman (1968: 5), monetary policy cannot peg the rate

of unemployment for more than very limited periods. Consequently

a monetarist model must dismiss and go beyond such limited periods

and become a long-run model.

2. Friedman's Method

Such a long-run, dynamic, and two-interest-rates model is clearly

incompatible with the short-run, static, one-interest-rate IS-LM frame-

work offered by Friedman himself (1970) as his "theoretical framework".

As Thygesen (1977) observed in his Nobel article, Friedman "is clearly

uncomfortable with it".

Crowding-out is a Friedman idea treated equally cavalierly. Crowd-

ing-out refers to the crowding-out of private investment by government

purchase of goods and services. To simulate it one would need a model

including an investment function and a tax rate, yet in his own writing

Friedman offered neither. Indeed, recent monetarist tradition [Brunner

-Meltzer (1976)] does not even distinguish between consumption and in-

vestment demand.

Such silence about behavior hypotheses was common in the days of David Hume and survived in Friedman's reliance (1959, 1966) on single-equation reduced-form econometrics. The idea of his "positive" economics seems to be that if a reduced-form equation will fit the facts then never mind which particular behavior hypotheses it might be derived from. To us, however, such things matter, and we must work our way backwards from Friedman's conclusions to a model which will deliver them. As it happens, a neoclassical growth model requires little modification to do so.[1] We shall now see how.

I. NOTATION

1. Variables

$C \equiv$ physical consumption

$D \equiv$ desired holding of money

$G \equiv$ physical government purchase of goods and services

$g_v \equiv$ proportionate rate of growth of variable v

$I \equiv$ physical investment

k ≡ present gross worth of another physical unit of capital stock

κ ≡ physical marginal productivity of capital stock

L ≡ labor employed

n ≡ present net worth of another physical unit of capital stock

P ≡ price of goods and services

R ≡ tax revenue

r ≡ before-tax nominal rate of interest

ρ ≡ after-tax real rate of interest

S ≡ physical capital stock

w ≡ money wage rate

X ≡ physical output

Y ≡ money national income

y ≡ money disposable income

2. Parameters

a ≡ multiplicative factor of production function

α, β ≡ exponents of a production function

c ≡ propensity to consume

F ≡ available labor force

g_v ≡ proportionate rate of growth of parameter v

λ ≡ proportion employed of available labor force

M ≡ supply of money

m ≡ multiplicative factor of demand-for-money function

μ ≡ exponent of demand-for-money function

T ≡ tax rate

The model will include derivatives with respect to time, hence is dynamic. All parameters are stationary except a, F, and M whose growth rates are stationary.

II. THE MODEL

Define the proportionate rate of growth of variable v as

$$g_v \equiv \frac{dv}{dt}\frac{1}{v} \tag{1}$$

Define investment as the derivative of capital stock with respect to time:

$$I \equiv \frac{dS}{dt} \tag{2}$$

Monetarists have shown no interest in specifying a production function but may not object, we hope, to a Cobb-Douglas form

$$X = aL^{\alpha}S^{\beta} \tag{3}$$

where $0 < \alpha < 1$; $0 < \beta < 1$; $\alpha + \beta = 1$; and $a > 0$.

Let profit maximization under pure competition equalize real wage rate and physical marginal productivity of labor:

$$\frac{w}{P} = \frac{\partial X}{\partial L} = \alpha \frac{X}{L} \tag{4}$$

Rearrange (4) and write the neoclassical mark-up-pricing equation

$$P = \frac{wL}{\alpha X} \tag{5}$$

saying that neoclassical price P equals per-unit labor cost wL/X marked
up in the proportion $1/\alpha$.

Define physical marginal productivity of capital stock as

$$\kappa \equiv \frac{\partial X}{\partial S} = \beta \frac{X}{S} \tag{6}$$

Let entrepreneurs be purely competitive ones, then price P of out-
put is beyond their control. At time t, then, after-tax marginal value
productivity of capital stock is $(1 - T)\kappa(t)P(t)$.

Let there be a market in which money may be placed or borrowed at
a stationary before-tax nominal rate of interest r. Let interest earnings be
taxed and interest expense be tax-deductible. Then money may be placed

or borrowed at the after-tax rate $(1 - T)r$. Let that rate be applied when discounting future cash flows. As seen from the present time τ, then, after-tax marginal value productivity of capital stock is $(1 - T)\kappa(t)P(t)e^{-(1 - T)r(t - \tau)}$. Define present gross worth of another physical unit of capital stock as the present worth of all future after-tax marginal value productivities over its entire useful life:

$$k(\tau) \equiv \int_{\tau}^{\infty} (1 - T)\kappa(t)P(t)e^{-(1 - T)r(t - \tau)} dt \qquad (7)$$

Let entrepreneurs expect physical marginal productivity of capital stock to be growing at the stationary rate g_κ:

$$\kappa(t) = \kappa(\tau)e^{g_\kappa(t - \tau)}$$

and price of output to be growing at the stationary rate g_P:

$$P(t) = P(\tau)e^{g_P(t - \tau)}$$

Insert these into (7), define

$$\rho \equiv (1 - T)r - (g_\kappa + g_p) \tag{8}$$

and write the integral (7) as

$$k(\tau) = \int_\tau^\infty (1 - T)\kappa(\tau)P(\tau)e^{-\rho(t - \tau)}dt$$

Neither $(1 - T)$, $\kappa(\tau)$, nor $P(\tau)$ is a function of t, hence may be taken outside the integral sign. Our g_κ, g_p, and r were all said to be stationary, hence the coefficient ρ of t is stationary, too. Assume $\rho > 0$. As a result find the integral to be

$$k = (1 - T)\kappa P/\rho \tag{9}$$

Find present net worth of another physical unit of capital stock as its gross worth minus its price:

$$n \equiv k - P = [(1 - T)\kappa/\rho - 1]P$$

Desired capital stock is the size of stock for which the present net worth of another physical unit of capital stock equals zero, or

$$(1 - T)\kappa = \rho \tag{10}$$

Finally take equations (6) and (10) together and find desired capital stock

$$S = (1 - T)\beta X/\rho \tag{11}$$

In accordance with the definition (2), differentiate desired capital stock (11) with respect to time, and write desired investment

$$I \equiv \frac{dS}{dt} = (1 - T)\beta g_X X/\rho \tag{12}$$

If we think of (11) and (12) as being derived for an individual entrepreneur, then everything except X on their right-hand sides is common to all entrepreneurs. Factor out all common factors, sum over all entrepreneurs, then X becomes national physical output, and (11) and (12) become national desired capital stock and investment, respectively.

For all their attention to crowding-out, monetarists have shown

little interest in deriving investment functions like (12). But let us take a closer look at (11) and (12) just the same. Both are in inverse proportion to ρ. What is ρ? In the definition (8) of ρ let it be correctly foreseen that $g_\kappa = 0$—our steady-state growth and inflation model will indeed have the solutions (26) and (32), and historically the physical marginal productivity κ has displayed no secular trend, see ch. 7 of Brems (1983). In that case ρ collapses into the after-tax real rate of interest.

Once priced according to (5), physical output becomes national income: Let capital stock be immortal, so we may ignore capital consumption allowances and define national income as the money value of physical output

$$Y \equiv PX \tag{13}$$

Define money disposable income before capital gains as national income minus government net receipts:

$$y \equiv Y - R \tag{14}$$

Let real wealth in the neoclassical model consist of real money stock M/P and the physical capital stock S. Real capital gains on money stock are $-g_p M/P$ and on physical capital stock zero. Consequently real disposable income after capital gains is $(Y - R - g_p M)/P$, and let consumption be the fraction c of that:

$$C = c(Y - R - g_p M)/P \qquad (15)$$

Let labor employed be the proportion λ of available labor force:

$$L = \lambda F \qquad (16)$$

where $0 < \lambda < 1$. The difference $1 - \lambda$ is the "natural" rate of unemployment below which, according to Friedman (1968: 8) excess demand for labor will push the real wage rate up, and above which excess supply will push it down. Friedman sees the employment fraction λ, then, as a long-run Walrasian equilibrium determined by the intersection of supply and demand curves invulnerable to monetary policy. Could they be vulnerable to

policy other than monetary policy? They could well be. A minimum-wage
statute can cut off the demand curve at the point at which the real
minimum wage rate equals the physical marginal productivity of labor.
As a result the employment fraction λ is down. Or an unemployment-
compensation statute can shift the supply curve to the left by raising
the rate of unemployment compensation or lengthen the maximum period
of entitlement. As a result, the hardship of unemployment is reduced,
the incentive to look for jobs is reduced, and again the employment
fraction λ is down. Even a Walrasian long-run equilibrium thus amended
by modern institutional arrangements would still generate an employment
fraction λ to be considered a parameter by _monetary_ policy.

Let tax revenue be in proportion to money national income:

$$R = TY \qquad\qquad (17)$$

and let the government finance its deficit, if any, by issuing noninterest-
bearing claims upon itself called money. The government budget constraint
is then

$$GP - R = \frac{dM}{dt} \tag{18}$$

Let the demand for money[2] be a function of money national income and of the after-tax nominal rate of interest:

$$D = mY[(1 - T)r]^{\mu} \tag{19}$$

where $\mu < 0$ and $m > 0$.

Goods-market equilibrium requires the supply of goods to equal the demand for them:

$$X = C + I + G \tag{20}$$

Money-market equilibrium requires the supply of money to equal the demand for it:

$$M = D \tag{21}$$

III. STEADY-STATE EQUILIBRIUM GROWTH SOLUTIONS

1. Growth-Rate Solutions

By differentiating equations (1) through (21) with respect to time the reader may convince himself that they are satisfied by the following steady-state growth solutions

$$g_C = g_X \tag{22}$$

$$g_D = g_M \tag{23}$$

$$g_G = g_X \tag{24}$$

$$g_I = g_X \tag{25}$$

$$g_K = g_X - g_S \tag{26}$$

$$g_L = g_F \tag{27}$$

$$g_M = g_Y \tag{28}$$

$$g_R = g_Y \tag{29}$$

$$g_r = 0 \tag{30}$$

$$g_\rho = 0 \tag{31}$$

$$g_S = g_X \tag{32}$$

$$g_{w/P} = g_a/\alpha \tag{33}$$

$$g_X = g_a/\alpha + g_F \tag{34}$$

$$g_Y = g_P + g_X \tag{35}$$

$$g_y = g_Y \tag{36}$$

Our growth was steady-state growth, for no right-hand side of our solutions (22) through (36) was a function of time.

2.   Properties of Growth-Rate Solutions

Our growth-rate solutions neatly deliver Friedman's conclusions.

First, no growth-rate solution for the nine real variables C, G, I, κ, L, ρ, S, w/P, and X has the rate of growth g_M of the money supply in it, directly or indirectly.

Second, the growth-rate solutions for the five nominal variables D, P, R, Y, and y have the rate of growth g_M of the money supply in them, directly or indirectly. The growth-rate solution for the sixth nominal variable, r, does not, but its level may.

Third, the rate of growth g_M of the money supply may be thought of as a policy instrument used to control inflation: Take the growth-rate solutions (28) and (35) together, insert (34), and find

$$g_P = g_M - (g_a/\alpha + g_F) \tag{37}$$

or, in English: Knowing the rate of technological progress g_a and the rate of growth of the labor force g_F and knowing the elasticity α of physical output with respect to labor, the monetary authorities may control the rate of inflation g_P by controlling the rate of growth g_M of the money supply.

3. The Level of the After-Tax Real Rate of Interest

In steady-state growth theory it is usually easier to solve for growth rates than for levels. Can we solve for our after-tax real rate of interest ρ? Insert the goods-market equilibrium condition (20) into the left-hand side of the investment function (12) and write the after-tax real rate of interest as

$$\rho = \frac{(1 - T)\beta g_X X}{X - (C + G)}$$

Insert the definition (13) of national income and the tax function (17) into the consumption function (15) and the government budget constraint (18). Divide by P and write physical consumption C and government purchase G. Divide both numerator and denominator by $(1 - T)X$ and use (13) to write PX as Y. Insert the definition (8) and the equilibrium condition (21) into the demand-for-money function (19) and insert the result for the ratio M/Y. Finally insert our newly found equilibrium-growth-rate solutions (26), (28), (32), and (35) and write the solution for the after-tax real rate of interest

$$\rho = \beta g_X / A, \qquad\qquad (38)$$

where

$$A \equiv 1 - c - \frac{[(1 - c)g_M + cg_X]m(\rho + g_M - g_X)^\mu}{1 - T}$$

Here g_X stands for our equilibrium-growth-rate solution (34) expressing g_X in terms of nothing but the parameters α, g_a, and g_F. Thus (38) is a solution for the level of the after-tax real rate of interest ρ in terms of the policy instrument g_M. It is not an explicit solution and cannot be, for it has the sum $\rho + g_M - g_X$ raised to the power μ on its right-hand side. Still, for realistic values of μ--say within the range $-\frac{1}{2} < \mu < 0$--(38) is easy to interpret[3]. If g_M is up the numerator of A is up, and since the numerator is preceded by a minus sign, A is down and ρ up. In other words, comparing a more inflationary steady -state equilibrium growth track having a higher $g_M = g_Y$ to a less inflationary track, we find the former to have the higher after-tax real rate of interest.

IV. CONCLUSIONS

To a neoclassical steady-state growth model we have added money, taxes, and a government budget constraint and derived an investment function. Thus modified, the model delivered most of Friedman conclusions. His growth-<u>rate</u> solutions were delivered impeccably: no growth-rate solution for a real variable had the rate of growth of the money supply in it. The growth-rate solutions for five nominal variables did have the rate of growth of the money supply. But our <u>level</u> (38) of the after-tax real rate of interest did not deliver Friedman's conclusion that "the monetary authority ... cannot ... peg a real quantity--the real rate of interest." For realistic values of μ it was clear that ρ was up if g_M was up.

Such an effect of monetary policy upon the level of the after-tax real rate of interest may be in ill accordance with Friedman's words but in excellent accordance with his views. Our economy is a closed one in which money can come into existence in no other way than by financing a government budget deficit. Vice versa, a government budget deficit can be financed in no other way than by expanding the money supply. The economy on the more inflationary steady-state growth track having a higher

$g_M = g_Y$, then, is also the economy having the higher budget deficit and hence the higher physical government purchase G. In Friedman's view such a higher physical government purchase G will crowd out private physical investment I. But how could it without the very effect upon the after-tax real rate of interest we found? According to our investment function (12), investment was a function of the after-tax real rate of interest and will be down only if that rate is up.

FOOTNOTES

[1]Drud Hansen (1979) used a neoclassical growth model to simulate monetarism but—like Friedman himself—ignored taxation.

[2]On the money-market side, monetarist tradition disaggregates more than we are doing and distinguishes among money, credit, and securities markets. On the goods-market side, monetarist tradition disaggregates less than we are doing and does not even distinguish between consumption and investment demand [Brunner-Meltzer (1976)]. A good recent survey of crowding-out is Svindland (1980).

[3]For correspondance suggesting the form (38) I am grateful to J. Drud Hansen.

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