THE MONIST

A QUARTERLY MAGAZINE.

DEVOTED TO THE PHILOSOPHY OF SCIENCE

VOLUME XXVIII

CHICAGO
THE OPEN COURT PUBLISHING COMPANY
1918

CONTENTS OF VOLUME XXVIII.

ARTICLES AND AUTHORS.

PAGE

Alexander, Hartley B. Plato's Conception of the Cosmos 1
Atomism, The Philosophy of Logical. By Bertrand Russell 495
Bateman, H. The Genesis of an Electro-Magnetic Field 586
Berkeley's Logic of Mathematics. By G. A. Johnston 25
Biologist's Religion, A. By Walter Sonneberg 567
Body and Mind. By C. D. Broad 234
Broad, C. D. Body and Mind 234
Carus, Paul. In Reply to Dualistic Conceptions of Mind, 259.—Suggestions for a New Logic. Dr. Mercier's Logical Work, 302.
Child, J. M. Critical Notes on K. I. Gerhardt's "Leibniz and Pascal" ... 550
Christian Theophagy: An Historical Sketch. By Preserved Smith 161
Conceptions of the History of Philosophy, The. By Victor Delbos 394
Construction of Magic Squares and Cubes with Prime Numbers, General Notes on the. By Harry A. Sayles 141
Cosmos, Plato's Conception of the. By Hartley B. Alexander 1
Delbos, Victor. The Conceptions of the History of Philosophy 394
Drake, Durant. An Empirical View of the Trinity 135
Dualism, Monism and. By Ernst Jonson 624
Dualistic Conceptions of Mind, In Reply to. By Paul Carus 259
Edmunds, Albert J. The Washington Manuscript and the Resurrection in Mark 528
Electro-Magnetic Field, The Genesis of an. By H. Bateman 586
Empirical View of the Trinity, An. By Durant Drake 135
Galileo and Newton. By Philip E. B. Jourdain 629
Genesis of an Electro-Magnetic Field, The. By H. Bateman 586
Gerhardt, Karl Immanuel. Leibniz and Pascal 530
Heaton, Charles. A Philosophical Litterateur 608
Hirshberg, L. K. Things Are not Always what they Seem 456
Hyde, Walter Woodburn. The Two-Hundredth Anniversary of the Birth of Winckelmann 76
Hyslop, James. H. Predicaments in Philosophy 352
Imagination, Servant or Master (Poem). By C. L. Marsh 68
Infinity as Method. By Henry Lanz 46

PAGE
Johnston, G. A. Berkeley's Logic of Mathematics 25
Jonson, Ernst. Monism and Dualism 624
Jourdain, Philip E. B. Galileo and Newton 629
Lane, Charles Alva. Wheeler's Hundredth-Century Philosophy 481
Lanz, Henry. Infinity as Method 46
Lawrence, Edward. Prayer. Its Origin, Meaning, and Ethical Significance. 410
Leibniz and Pascal. By Karl Immanuel Gerhardt 530
Lindsay, James. Rationalism and Voluntarism 433
Logic in Numbers. By Chas. P. R. Macaulay 472
Logic of Mathematics, Berkeley's. By G. A. Johnston 25
Logic, On the Construction of a Non-Aristotelian. By Henry Bradford Smith 465
Logic, Recent Work in Mathematical. By Dorothy Maud Wrinch 620
Logic, Suggestions for a New. Dr. Mercier's Logical Work. By Paul Carus 302
Logical Atomism, The Philosophy of. By Bertrand Russell 495
Lucretius Returns. A Philosophical Poem. By George Seibel 282
Macaulay, Chas. P. R. Logic in Numbers 472
Magic Squares and Cubes with Prime Numbers, General Notes on the Construction of. By Harry A. Sayles 141
Marsh, C. L. Imagination, Servant or Master (Poem) 68
Marsh, C. L. The Super-Soul (Poem) 73
Mathematical Logic, Recent Work in. By Dorothy Maud Wrinch 620
Mathematics, Berkeley's Logic of. By G. A. Johnston 25
Mercier's Logical Work, Dr. Suggestions for a New Logic. By Paul Carus 302
Mills, Lawrence Heyworth. Obituary 314
Mind, Body and. By C. D. Broad 234
Mind, In Reply to Dualistic Conceptions of. By Paul Carus 259
Mind, the Creator of Matter. By L. L. Pimenoff 209
Minkowski, Hermann. Time and Space 288
Monism and Dualism. By Ernst Jonson 624
Mors Mortis. By William Benjamin Smith 321
Muscio, Bernard. The Mechanical Explanation of Religion 123
Newton, Galileo and. By Philip E. B. Jourdain 629
Pascal, Leibniz and. By Karl Immanuel Gerhardt 530
Philosophical Litterateur, A. By Charles Heaton 608
Philosophy? Is There an Intellectual Content in. By James G. Townsend. 597
Philosophy, Predicaments in. By James H. Hyslop 352
Philosophy, The Conceptions of the History of. By Victor Delbos 394
Philosophy of Logical Atomism, The. By Bertrand Russell 495
Pimenoff, L. L. Mind, the Creator of Matter 209
Plato's Conception of the Cosmos. By Hartley B. Alexander 1
Pragmatic Issue, A Psychological View of the. By Theodore Schroeder . 273
Prayer. Its Origin, Meaning, and Ethical Significance . By Edward Lawrence 410
Predicaments in Philosophy. By James H. Hyslop 352
Probability, On the Conception of. By H. M. Westergaard 613
Psychological View of the Pragmatic Issue, A. By Theodore Schroeder . 273
Rationalism and Voluntarism. By James Lindsay 433

PAGE
Religion, A Biologist's. By Walter Sonneberg 567
Religion, The Mechanical Explanation of. By Bernard Muscio 123
Resurrection in Mark, The Washington Manuscript and the. By Albert J. Edmunds 528
Rignano, Eugenio. The School of To-Morrow 379
Russell, Bertrand. The Philosophy of Logical Atomism 495
Sayles, Harry A. General Notes on the Construction of Magic Squares and Cubes with Prime Numbers 141
School of To-Morrow, The. By Eugenio Rignano 379
Schroeder, Theodore. A Psychological View of the Pragmatic Issue 273
Seibel, George. Lucretius Returns. A Philosophical Poem 282
Smith, Henry Bradford. On the Construction of a Non-Aristotelian Logic. 465
Smith, Preserved. Christian Theophagy: An Historical Sketch 161
Smith, William Benjamin. Mors Mortis 321
Sonneberg, Walter. A Biologist's Religion 567
Suggestions for a New Logic. Dr. Mercier's Logical Work. By Paul Carus 302
Super-Soul, The (Poem). By C. L. Marsh 73
Theophagy, Christian: An Historical Sketch. By Preserved Smith 161
Things Are not Always what they Seem. By L. K. Hirshberg 456
Time and Space. By Hermann Minkowski 288
Townsend, James G. Is There an Intellectual Content in Philosophy?... 597
Trinity, An Empirical View of the. By Durant Drake 135
Voluntarism, Rationalism and. By James Lindsay 433
Washington Manuscript and the Resurrection in Mark, The. By Albert J. Edmunds 528
Wells, Wesley Raymond. The Fallacy in H. G. Wells's "New Religion".. 604
Wells's "New Religion," The Fallacy in H. G. By Wesley Raymond Wells 604
Westergaard, H. M. On the Conception of Probability 613
Wheeler's Hundredth-Century Philosophy. By Charles Alva Lane 481
Winckelmann, The Two-Hundredth Anniversary of the Birth of. By Walter Woodburn Hyde 76
Wrinch, Dorothy Maud. Recent Work in Mathematical Logic 620

BOOK REVIEWS AND NOTES.

PAGE
Durkheim, Emile. The Elementary Forms of the Religious Life 158
Elliot, Hugh. Herbert Spencer 638
Harris, Rendel. The Ascent of Olympus 640
Jaini, Jagmanderlal. Outlines of Jainism 320
Lindsay, James. A Philosophical System of Theistic Idealism 639
McQueen, E. Neil. The Distribution of Attention 636
Merz, John Theodore. Religion and Science 319
Science Progress 479, 633
Scientia 480
Wundt, Wilhelm. Elements of Folk-Psychology 159

vi

VOL. XXVIII. JANUARY, 1918 NO. 1

THE MONIST

PLATO'S CONCEPTION OF THE COSMOS.

I.

"PYTHAGORAS was the first," says Plutarch, "who named the compass of the whole a Cosmos, because of the order which is in it."

The notion that all things knowable and all things existent form one orderly and comprehensive system, in which every event is linked with every other by causal necessity while all the elements with mechanical nicety mutually enmesh, is to us of to-day an intellectual commonplace. We make no difficulty in thinking an Everything which is made up of all things, an Entirety or a Totality which is just the commingled sum of the numberless particularities which our lives are always itemizing; and we call this Totality, this All, this Thing of things, the Universe or the World. It rarely occurs to us to question either the unity or the reality of this omnium-gatherum, which, even if it occupies a somewhat concealed position in our thoughts, is yet a well-nigh indispensable convenience; it stands an ever-ready and capacious receptacle for all the perplexities and inconsistencies which the apparent nature of things is constantly presenting, but which, we feel, are in some benign way healed by the alchemical mystery of an all-inclusive World.

Ideas are habits; and when an idea gets so fixed that the habit has become automatic, it is usually good medicine to revive, now and again the habit-forming period, that we may judge with refreshed intelligence the safety and truth of our continued course. This is our purpose in turning to certain Greek conceptions of the world as a cosmos.

For we must remember that the notion, so familiar to us, of what they variously called τὸ πᾶν, the All, or τὸ ὅλον, the Whole, or again ὁ οὐρανός, the Heaven, or ὁ κόσμος, the Order of Things, was to the Greeks a new invention. The idea that all things are somehow one is by no means self-evident, and when it was suggested the wary Hellenic mind approached it with canny suspicion and cautious circumlocution. Is the World limited or unlimited? Is it truly One or is it Many? Does the Whole, or Totality, exhaust the All? Or indeed may not the All indefinitely transcend the Realm of Order, the Cosmos? These were questions which were raised and discussed—questions with a dangerous smack of impiety—by the men who were interested in what Xenophon characterizes as "that which is called by sophists 'the world.'"

Doubtless it was Pythagoras, as Plutarch states, or some Pythagorean, who first daringly pronounced the Whole to be a Cosmos, the realm of reality and the realm of order to be coextensive. For the Pythagoreans were the earliest of men to be entirely enamored of that first principle and foundation of law and order, the idea of number. They devoted themselves to mathematics and music and astronomy, and in the numerical analogies which they discovered in the properties of sound and in the movements of the heavenly bodies there burst upon their minds, with what must have seemed a very blaze of creative intelligence, the great conception of number in nature, which has since been the foundation of all our science. They conceived all nature to be organized according to mathematical proportions, and because they found these proportions to be most emblematically realized in musical strings and pipes they named the principle of it a harmony, and again because they seemed to see it regnantly imaged in the motions of the heavenly spheres they regarded these too as a harmony and a music. It was indeed primarily to the heavens that the name Cosmos was given, and it was only later, when the seasons of Earth were observed to follow the periods of the Sun while the figures of the stars were regarded as prognostics of human events, that the conception of order was extended from celestial to terrestrial phenomena.

The background of Hellenic thought, like the natural thought of mankind everywhere, was pluralistic. To the normal Greek, even in the days of Plato and Aristotle, the obvious facts of life indicated not a consistent and close-locked universal scheme, but a *mêlée* of whim and purpose, blind chance and blinder fancy, while the most reasonless of all the powers he recognized was that to which he gave the name Necessity. To him it seemed evident that the affairs of men and nature are innumerable and unorganized, and while certain of the more stable aspects of existence were regarded as the charge of the Olympian gods, not even such mercurial control as emanated from the hoydenish family of Zeus divine obtained in the generality of experience: the vast majority of events were not to be explained at all; they were simply the manifestation of the hostility, indifference, idiosyncracy and anarchy which appear in the elemental facts of life.

This, I say, was the view of the normal Greek even in his classical hey-dey, as it is the view of the naive and natural man everywhere. But the foundations of our own sophisticated philosophy had been set long before, in two first conditions which, as I see it, go far to account for the whole edifice of reason.

One of these is a psychological condition. It is what is known in Kantian philosophy as the "unity of apperception" and in scientific method as the "law of parcimony," or economy of thought. Essentially it is just our native simple-mindedness, expressed in the maxim, "Attend to one thing at a time." Intellectually we are unable to cope with complex facts; we have to simplify them, analyze them, in order to see them. Hence we regard simplicity as the supreme virtue, not only in reason but also in nature; and hence also our invincible conviction that reason's simplifications are more genuine than nature's empirical complexities. In spite of its multitudinous and multiplying variety the very limitations of our intellectual powers compel us to see Nature as one, as a unity, and thus out of chaos is created an orderly world.

Such is the inner condition, but it is mightily helped outwardly by the natural allegory of Sky and Earth, Day and Night, Summer and Winter. These antitheticals seem to form a great division of Nature into the Intelligible and the Unintelligible: Sky and Day and Summer not only symbolize but embody motion and light and life, which are in turn the image and essence of reason; while Earth and Night and Winter no less surely body forth the inert and void and deathly realm of anti-reason. Thus we have a realm of order, Cosmos, set over against a realm of disorder, a Chaos; and because the orderly Sky images the rulership of reason, and because Day is the revealer and Summer the life-giver, these powers are regarded as friendly to man and in the great contention of Nature as encroaching upon and subduing the dark forces of Chaos.

Such a sense of duality is omnipresent in human thought. Its metaphors are the very breath of life of poetry, and even in philosophies which deny its reality the problems to which it gives rise—problems of the formal and material, spiritual and physical, good and evil,—are the crucial perplexities. Greek thought is no exception to the rule. Already in the epic theogonies Uranus and Gaea, Sky and Earth, appear as ancestral and gigantic forms of creation emerging from primeval chaos....

> "First Chaos was, and then broad-bosomed Earth....
> And Earth bare starry Heaven, thence to be
> The habitation of the blessed gods."

This is the Hesiodic genesis, and the Orphic differs from it only in making Heaven and Earth a coequal and wedded pair, from whose union multitudinous nature was begotten. Euripides preserves it in the utterance of the seeress Melanippe:

> "It is not my word, but my mother's word,
> How Heaven and Earth were once one form; but stirred,
> And strove, and dwelt asunder far away:
> And then, re-wedding, bore unto the day
> And light of life all things that are, the trees,
> Flowers, birds and beasts and them that breathe the seas,
> And mortal man, each in his kind and law."[1]

This dualism of the epic age passed over into the philosophic tradition with little more than a change of names. In place of Heaven and Earth, the antithesis is set between Chaos and Nous, Anarchy and Intelligence, or between Chaos and Cosmos, Void and Order,—though we must remember that the word οὐρανός persisted as a synonym of κόσμος even with Plato and Aristotle, and that κόσμος itself was at first used of the heavenly firmament, and only with advancing insight into the orderliness of the world beneath the spheres was it made to include terrene nature.

The lesson of intelligence was in fact learned first of all from observation of the heavens. No phenomena so vividly impress the natural mind with a sense of their divinity as do the regular and brilliant courses of the heavenly bodies. Repetition is the gateway and light is the outer image of learning, and in the sun and moon and stars we have our permanent exemplars of repetition and light.

> "All mankind thou guidest as a single being;
> Expectantly, with raised head, they look up to thee!"

[1] Gilbert Murray's translation.

says a Babylonian hymn to the sun, for which the nineteenth psalm—

> "The Heavens declare the glory of God,
> And the firmament sheweth his handywork"—

is only a later parallel. Plato, in describing the works of the Demiurge, tells how "of the heavenly and divine, he created the greater part out of fire, that they might be the brightest of all things and fairest to behold, and he fashioned them after the likeness of the universe in the figure of a circle, and made them follow the intelligent motion of the supreme, distributing them over the whole circumference of heaven, which was to be a true cosmos or glorious world spangled with them all over."[2] And in another passage Plato derives from the image of the heavens, as does the psalmist, his conviction of the goodness of God: for if, he says, "we say that the whole path and movement of heaven, and of all that is therein, is by nature akin to the movement and revolution and calculation of mind, and proceeds by kindred laws, then, as is plain, we must say that the best soul takes care of the world and guides it along the good path." Perhaps the sublimest expression of this thought in Greek literature is Aristotle's characterization of Xenophanes: "He cast his eyes upon the expanse of Heaven, and saw that it was one, and that one God."

Thus the heavens were at once the embodiment of reason and divinity, the symbol of divine rulership and the exemplar of divine perfection. But it was the reverse of obvious that either the mathematical regularity of the heavenly reason or the perfection of heavenly form extend to the world beneath the moon. What seems to have been really the first suggestion that such is the case was the Pythagorean discovery that musical intervals vary with the length of the sound-producing strings according to certain simple and regular numerical ratios. This discov-

[2] This and other citations from Plato are in Jowett's translation.

ery burst upon men's minds as a sudden revelation of order where order had hitherto never been suspected, and in their first delirious application of it the Pythagoreans seemed to see number everywhere, in the world of change below as in the world of constancy above, in the conduct of men as in the conduct of gods and stars, and so they proclaimed the Whole to be a One, whose emanating numbers gave coherence and system to all things, and they named this systemic All a Cosmos.

There remained one further step. Xenophanes had seen God in the heavens; Pythagoras had lifted Earth up into the Cosmos; but neither had as yet perceived that the world of sense and of physical numbers is only a symbol and an image of the true realm of law, that the cosmic citadel must be sought inwardly in thought and not outwardly in fact. This had been darkly intimated by the dark Heraclitus. "Better is the hidden harmony than the manifest," he had said; and again, "In one thing is wisdom, to know the reason by which all through all is guided." But it was Socrates who first clearly and explicitly emphasized the inner nature of the cosmic principle. "Socrates was the first," says Cicero, "to call philosophy down from the sky, and to settle it in the city and even introduce it within the house, and compel it to inquire concerning life and death and things good and ill." Probably, in saying this, Cicero, like Xenophon, merely saw Socrates turning from astronomy as from a vain speculation. The truth of Socrates' mission is perhaps better indicated by Aristotle's statement that it was Socrates who invented definition. We know what he strove to define—courage and temperance and justice and wisdom, the principles of conduct and the laws of an orderly life. Socrates was seeking cosmos, reason, not in the physical image, but in the spiritual reality. That Socrates was genuinely interested in physical science there is every reason to believe, but his final attitude is best expressed in the words which Plato puts into his mouth, "Those who elevate astronomy into philosophy appear to me to make us look downward and not upward."

The predecessors of Plato had modelled two great conceptions. The physical and mathematical thinkers had evolved the grandiose notion of a Cosmos, an Order, written upon the face of Chaos. Heraclitus and, far more distinctly, Socrates had proclaimed this order of nature to be only the outward image and reflection of the inner order of reason. Pythagoras and Heraclitus and Socrates, more than all others, were the teachers of Plato, and it was from the inspirations of their insights that he drew his own magnificent vision of the world.

II.

The vivid impression one derives from a reading of Plato is of the intensity of his conviction of the unreality of sensible things. The world of sense, of sight and hearing and taste and touch, in which most men chiefly dwell is for him a shadow world. At its best it is but a symbol obscurely imitating the character of the reality which it veils; in its normal function it is a delusional mirage; and at its worst, when it conveys the deception of knowledge, it is the fount of corruption and the seed of damnation. The Greek argument against our commonsense conviction that what we see and touch is real is about as follows: All objects of sense suffer perpetual change; they never *are* this or that, but are always in a process of becoming or of ceasing to be this or that; hence, we cannot justly describe them as being anything, or indeed as having any true existence of any sort. Heraclitus remarked that one cannot bathe in the same river twice, and Cratylus, the sceptic, after remarking that we cannot in fact bathe in the same river even once, finally, as Aristotle tells us, ceased speech altogether on the ground that it was impossible to say any thing that is true; to inquisitors he would reply merely by a wagging of the finger, his mutely eloquent asseveration of his master's dogma that "All things flow." Plato accepted this doctrine, as he also accepted Socrates's conception that ignorance is essential vice, and combining the two, to the sceptical he added a moral condemnation of the world of sense: not only does it not give us truth, but because, as he says, "ignorance is the aberration of a mind bent on truth," through the intensity of its illusions it betrays the soul's integrity.

The Cratylean denial of the possibility of discourse is thus, for Plato, the proclamation of moral ruin, and at such his sanity revolts. Nor is the way of salvation hard to find. If sense be false, ideas may yet be true, and in its own proper world discourse may be dealing with reality. "Knowledge"—these are Plato's words—"does not consist in impressions of sense, but in reasoning about them; in that only, and not in the mere impression, truth and being can be obtained." And again: "Things of which there is no rational account are not knowable. . . . things which have a reason or explanation are knowable." Plato's "world of Ideas," as it is called, is in fact but the assertion that our speech is significant, and that this significance, not the courses of sense, is what we mean by reality. "The word expresses more than the fact" and "in the nature of things the actual must always fall short of the truth."

Plato's idealism is thus simply a sane and unconquerable conviction that there is a realm of truth, and his whole philosophy is an effort to find out this truth. In the *Phaedrus* he speaks of truth as "the pilot of the soul"; in the *Philebus* he asserts that the soul has "a power or faculty of loving truth and of doing all things for the sake of it"; and in the *Phaedo* he makes Socrates, about to take the hemlock, preface his great argument for the soul's immortality with a wise caution against the bias of desire, "I would ask you to be thinking of the truth and not of Socrates."

Yet Plato has no illusory notion that truth is of easy access. Immersed as we are in a sea of distorting sensation, our knowledge at its best is only a faith. "For there is no light of justice or temperance or any of the higher ideas which are precious to souls in the earthly copies of them: they are seen through a glass darkly." In the famous image of the den, wherein mankind are the chained prisoners, with their eyes fixed upon the shadows of reality, Plato reminds us that even were our eyes opened to the upper world the light of reality would sear our vision. All that we can hope for is such intimations of the truth as we can gather from the allegory of nature.

And with a curious astuteness he emphasizes the affinity of vision—"the clearest aperture of sense"— to the inner perception of truth. "Sight in my opinion," says Timaeus, "is the source of the greatest benefit to us, for had we never seen the stars and the sun and the heavens, none of the words which we have spoken about the universe would ever have been uttered. But now the sight of day and night, and the months and the revolutions of the years, have created number, and have given us a conception of time, and the power of inquiring about the nature of the universe; and from this source we have derived philosophy, than which no greater good ever was or will be given by the gods to mortal men. . . . God invented and gave us sight to the end that we might behold the courses of intelligence in the heavens, and apply them to the courses of our own intelligence which are akin to them, the unperturbed to the perturbed; and that we, learning them and partaking of the natural truth of reason, might imitate the absolutely unerring courses of God and regulate our own vagaries."

In this remarkable passage Plato compresses not only the actual history of science, but its psychological foundations and its metaphysical ends, with a precision truly astonishing. I cannot dwell upon the multitude of analogies that it suggests, but the fundamentals are obvious; for the sense of sight is in fact the pattern of intelligence; perception of the heavens has given us our measures of time, and has created number and the science of the calendar which is the parent of all the sciences and of philosophy as well; and again the constancies of the celestial bodies have ever seemed to men, as Plato says, the regulation and the healing of their own errant ways. The whole life of reason is summarized and prophesied in this natural allegory.

And yet, let us repeat, it remains for Plato throughout an allegory. All science is an allegory and an art. What men call nature, the experiences that in human life stand over against our essential humanity, is after all unreal. It may image reality because it is the product of creative reason, but beyond this power of imaging its only being is scenic and mirage-like.

"The starry heaven which we behold is wrought upon a visible ground, and therefore, although the fairest and most perfect of visible things must necessarily be deemed inferior far to the true motions of absolute swiftness and absolute slowness, which are relative to each other, and carry with them that which is contained in them, in the true number and in every figure. Now, these are to be apprehended by reason and intelligence, but not by sight... The spangled heavens should be used as a pattern and with a view to that higher knowledge; their beauty is like the beauty of figures or pictures excellently wrought by the hand of Daedalus, or some other great artist, which we may chance to behold; any geometrician who saw them would appreciate the exquisiteness of their workmanship, but he would never dream of thinking that in them he could find the true equal or the true double, or the truth of any other proportion....And will not the true astronomer have the same feeling when he looks at the movements of the stars? Will he not think that heaven and the things in heaven are framed by the Creator of them in the most perfect manner? But he will never imagine that the proportions of night and day, or of both to the month, or of the month to the year, or of the stars to these and to one another, and any other things that are material and visible can also be eternal and subject to no deviation—that would be absurd."

Where the ancients said "astronomy" we say "physics," remarks a savant of our own day; and is it not obvious that Plato's words hold with perfect truth of our own science? For we, like Plato, to not look to the visible and sensible world for our realities, but to an ideal world which is only faintly intimated by the riddle of the senses. Whether it be as in our mechanical sciences a world of atoms and molecules or of ether vortices or of electrons and ions, or as in our biological sciences a world of genera and species, in every case we hypothecate a realm of forms, of ideas, as the essential reality of all natural phenomena. We vary no whit from Plato in all this; and indeed, little as they may suspect it, all our scientists are good Platonians.

But where we do vary from Plato is in the kind of value which we set upon our ideas. For we regard our scientific knowledge as ultimate and as a kind of divine possession in itself, whereas Plato held it to be only a means whereby men can dimly approach the being of divinity. In his own phrase we are "thrice removed from the king and the truth": behind the world of sense is the world of mathematical forms which are in turn but the reflection of the divine intelligence. Sense is the allegory of science, but science itself is only our human parable of divinity—a myth whose meaning is the mind of God. Science is thus a purely human instrument, and truth, our human, intellectual truth, is but the device whereby we adumbrate the nature of being. "The Deity," says Plutarch in one of his expositions of Plato, "stands in no need of science, as an instrument to withdraw his intellect from things engendered and to turn it to the realities; for these are all in him, and with him, and about him." It is only the weakness of human insight that makes the world-myth a significant myth.

III.

Plato, his critics are accustomed to say, resorts to allegory, to what he himself calls myth, when he encounters problems with which rational analysis alone is unable to cope. The lordly tales which adorn his dialogues these critics view as imaginative ornaments which Plato himself takes only half seriously. This I believe to be a misunderstanding. It is characteristic of these myths that they are introduced not when Plato is analyzing the nature of being, but when he has passed to a discussion of becoming, that is, when cosmic history rather than metaphysical organization is his theme. Now it is this province of becoming, which we should call the field of empirical science, which is, in Plato's view, itself an allegorical reality. And in resorting to allegory for its description he is but emphasizing the duplex nature of the fact. There is no field of discourse where positive statement is so easy and so dangerous as in the field of science (in our modern sense), and in discussing the problems of change Plato employs myths primarily in order that he may avoid dogmatism. Empirical science is for him a work of human art, just as the empirical universe is God's work of art; and he would not have us forget, what we are so prone to forget, that our constructions of cosmic realities give us at best but a verisimilitude, or as he would say, an "imitation" of the truth. In speaking of the empiric world, he repeats again and again, we can use but the "language of probability," and the language of probability is myth.

When therefore Plato, in the language of probability or of myth, sketches for us the cosmic drama which is the history of the world, it is with no Laplacean confidence in the invulnerability of his representation. Rather he is aware that at the core it cannot be the essential truth of the cosmos: science is given us in order that we may "imitate," as he says, "the absolutely unerring courses of God and regulate our own vagaries": it is not and cannot give dogmatic knowledge. "Law and order," to quote once more, "deliver the soul"; and there is a trenchant difference between this and our modern conception that the soul is but an automatic reflection of external laws and orders.

The motive which animates Plato's cosmological speculations is thus clearly a humanistic motive. He is concerned for truth, but only for such truth as bears directly upon men's conduct, and this he does not expect to find in the sensible world. For him, as for Dante, the world in time and space is but the vesture of man's life, whose essence and reality is to be sought in that divine nature of which apparent nature is the image. Truth, then, must be appraised, and the appraiser is the Good and the Perfect,—for "nothing imperfect is the measure of anything."

The conception of a cosmic drama—a world-play having, as Aristotle would say, a beginning, a middle and an end, a complication and a solution,—is not new with Plato. It appears in the theogonic epics and in the notions of the physical philosophers of the earlier period. But it is with Plato that the proper motive of the plot appears; and this is the striving for the good. With Plato's predecessors the moral problem had been (as it is to our scientists) adventitious; with Plato it is central, and we can understand his science of first and last things only when we see in it, as he saw in nature, a cosmic staging of the search for salvation.

Genesis and eschatology represent respectively the complication and solution of the plot. Genesis, the tale of origins, is treated most completely in the *Timaeus*; cosmic justice and its judgments is the theme of the speculative cosmology of Socrates in the *Phaedo* and of the vision of Er in the *Republic*. In these and in allied passages Plato draws for us his world emblem.

Plato begins his genesis, in the *Timaeus*, with an assertion of dualism. "First," says Timaeus, "we must make a distinction and ask, What is that which always is and has no becoming; and what is that which is always becoming and never is? That which is apprehended by intelligence and reason is always in the same state; but that which is conceived by opinion with the help of sensation and without reason, is always in a process of becoming and perishing and never really is." In its inception this dualism is a logical one, hypostatized into the familiar Platonic antithesis of the World of Sense and the World of Ideas. But very speedily we perceive that the moral antithesis of good and evil is in it also. The kernel of Plato's thought is the old philosophical dualism of Nous and Chaos, and even the older mythic dualism of Heaven and Earth; and, as does the earlier thought, he identifies Mind and Light with Goodness, and Disorder and Darkness with Evil.

"God desired that all things should be good and nothing bad, so far as this was attainable. Wherefore also finding the whole visible sphere not at rest, but moving in an irregular and disorderly fashion, out of disorder he brought order, considering that this was in every way better than the other. Now the deeds of the best could never be or have been other than the fairest; and the creator, reflecting on the things which are by nature visible, found that no unintelligent creature taken as a whole was fairer than the intelligent taken as a whole; and that intelligence could not be present in anything which was devoid of soul. For which reason, when he was framing the universe, he put intelligence in soul, and soul in body, that he might be the creator of a work which was by nature fairest and best. Wherefore, using the language of probability, we may say that the world became a living creature truly endowed with soul and intelligence by the providence of God."

In these words of Timaeus, Plato outlines his conception of creation. God, perceiving the disorder of Chaos, designs to redeem it by imparting to it the image of mind, of Cosmos, order. He creates it, therefore, in the likeness of a perfect animal (παντελές ζῷον), "the very image of that whole of which all other animals both individually and in their tribes are portions." First he created its soul, the *anima mundi,* "to be the ruler and mistress, of whom the body was to be the subject," organized from the categories of thought, from identity and difference and essence, in harmony of number. Afterwards he gave it body, interfusing with the visible body the rational soul, so that the whole universe of being became one animal endowed with soul (ζῷον ἔμψυχον).

"And he gave to the world the figure which was suitable and also natural. Now to the animal which was to comprehend all animals, that figure was suitable which comprehends within itself all other figures. Wherefore he made the world in the form of a globe, round as from a lathe, having its extremes in every direction equidistant from the center, the most perfect and the most like itself of all figures; for he considered that the like is infinitely fairer than the unlike. This he finished off, making the surface smooth all round for many reasons; in the first place because the living being had no need of eyes when there was nothing remaining outside of him to be seen; nor of ears when there was nothing to be heard; and there was no surrounding atmosphere to be breathed; nor would there have been any use of organs by the help of which he might receive his food or get rid of what he had already digested, since there was nothing that went from him or came into him: for there was nothing beside him. . . . And, as he had no need to take anything or defend himself against any one, the Creator did not think it necessary to bestow upon him hands: nor had he any need of feet nor of the whole apparatus of walking; but the movement suited to his spherical form was assigned to him, ... and he made the universe a circle moving within a circle, one and solitary, yet by reason of its excellence able to converse with itself, and needing no other friendship or acquaintance. Having these purposes in view he created the world a blessed god."

"When the father and creator saw the creature which he had made moving and living, the created image of the eternal gods, he rejoiced, and in his joy determined to make the copy still more like the original; and as this was eternal, he sought to make the universe eternal, so far as might be. Now the nature of the ideal being was everlasting, but to bestow this attribute in its fullness upon a creature was impossible. Wherefore he resolved to have a moving image of eternity, and when he set in order the heaven, he made this image eternal but moving according to number, while eternity itself rests in unity; and this image we call time." Time came into being with the heavens which measure it, and will be dissolved with them, says Plato; but space is of another origin. For besides the reason which gives cosmic form there is another cause of being, a principle of limitation which Plato calls necessity. We must conceive, he says, of three natures: first, that which is in process of generation, and this would be the world of nature as we experience it; second, that in which the generation takes place, and this is the recipient or matrix of nature; and third, that of which the generated world is an image, and this is the cosmic reason or form. "We may liken the receiving principle to a mother, and the source or spring to a father, and the intermediate nature to a child," he says, and we think immediately of the mythopoetic union of Earth and Heaven and of the Life of Nature which is its offspring. But for Plato this is a mere trope; he does not rest without being scientifically explicit. There are three kinds of being: that which is uncreated and indestructible, changeless, eternal, imperceptible to any sense, open only to the contemplation of the intelligence, and this is the principle of the Father, the ideal or formal essence of the world; again, that which is sensible and created and always in motion, the Child, the world of change and life; and finally, there is a third nature, the Mother, which, like the Father, is eternal and admits not of destruction, which provides a home for all created things, and is apprehended "without the help of sense, by a kind of spurious reason, and is indeed hardly real." This nature is space, and we "beholding as in a dream, say of all existence that it must of necessity be in some place and occupy a space, but that what is neither in heaven nor in earth has no existence."

This mothering space which is hardly real, yet is the cause of the determinism of nature, Plato identifies as the material element of being. As pure matter it is purely indeterminate, but it is receptive of all determinations. The four elements, earth, air, fire and water, are formed from it, for "the mother substance becomes earth and air, in so far as she receives the impressions of them." Plato's conception of the formation of these elements from the original substance was as purely mathematical as are our modern physical notions. "God fashioned them by form and number," he says; and the forms which he assigned were the forms of the regular solids. Thus the form of the fiery element is the pyramid, of air the octahedron, of water the icosahedron, of earth the cube. The fifth solid, the dodecahedron is the form of the universe as a whole, or perhaps one might say the scaffold upon which the spherical universe is constructed. Further, these elements are themselves compounded of simpler mathematical forms, the pyramid, octahedron and icosahedron of scalene, the cube of equilateral triangles; so that if we regard the elements as molecules, we may view the triangles as atoms of the material substrate.

Doubtless it was this geometrical account of matter which gave rise to the saying ascribed to Plato that "God always geometrizes,"—for God, says Plutarch in his commentary on the saying, made the world in no other way than by setting terms to infinite and chaotic matter. But it is not with the mathematical aspect of Plato's theory that we are here most concerned, but with its moral bearings. For it is in matter that Plato finds the root of evil, and, if we may so put it, the villainy of the world. In framing the inhabitants of the world, according to the account of Timaeus, the Creator made first the race of gods, perfect and immortal; but of the race of men he made only the souls, their bodies were handed over to the created gods to be composed of perishable matter. "The part of them worthy of the name immortal, which is called divine and is the guiding principle of those who are willing to follow justice and you (the gods)—of that divine part I will myself," saith the Creator, "sow the seed, and having made a beginning, I will hand the work over to you. And do ye then interweave the mortal with the immortal, and make and beget living creatures, and give them food, and make them to grow, and receive them again in death."

And having made souls equal in number to the stars, and having assigned each soul to a star, and there placed them as in a chariot, God "showed them the nature of the universe, and declared to them the laws of destiny, according to which their first birth would be one and the same for all,—no one should suffer a disadvantage at his hands,"—and he showed them how "he who lived well during his appointed time was to return and dwell in his native star, and there would have a blessed and congenial existence; but if he failed in attaining this," he would be reborn into some brute who resembled him in evil nature, nor would his toils and transformations cease until the principle of reason had enabled him to overcome "the turbulent and irrational mob of later accretions, made up of fire and air and water and earth" and return to his first and purer state. And "having given all these laws to his creatures, that he might be guiltless of future evil in any of them, the Creator sowed some of them in the earth, and some in the moon, and some in the other instruments of time; and when he had sown them he committed to the younger gods the fashioning of their mortal bodies, and desired them to furnish what was still lacking to the human soul, and to rule over them, and to pilot the mortal animal in the best and wisest manner which they could, and avert from him all but self-inflicted evils."

In these passages we see the *rationale* of the Platonic doctrines of anamnesis and metempsychosis, or recollection and transmigration. The great image in the *Phaedrus* of the soul in its chariot driving the unruly and the ruly steed, and the descriptions of a future-world judgment in the *Phaedo* and *Republic,* in which these doctrines are presented, appear as necessary scenes in the cosmic drama. The motive of that drama is the conflict of form and matter, Nous and Chaos, which on its theological side is the conflict of God and Necessity as the two principles of being, and in its moral aspect is the strife of Good and Evil. In each of these senses Plato is a dualist; and if he describes chaos and matter and evil in negative terms, this is not because he views them as non-existent (as our modern idealists seem to do), but because he regards them as impermanent, and hence as unreal; for Plato defines the real as the permanent, never, however, meaning thereby to deny genuineness of our experience of change and hence of imperfection and evil.

Nevertheless, Good and Evil, God and the Devil, are not in Plato's conception coordinate powers. Their difference is a difference of dramatic position. In the world-conflict we, as human beings, are all enlisted on the side of the good, and if we are traitorous to it this is because of the deceit of the enemy. "For as we acknowledge the world to be full of many goods and also of evils, and of more evils than goods, there is, as we affirm, an immortal conflict going on among us, which requires marvelous watchfulness, and in that conflict the gods and demigods are our allies and we are their property." No Persian has ever stated this fundamental dualism more emphatically nor adhered to it more uncompromisingly. From it Plato deduces the ascetic rule of life which recurs in his writings so repeatedly. "Evils," says Socrates in the *Theaetetus*, "can never pass away; for there must always remain something which is antagonistic to good. Having no place among the gods in heaven, of necessity they hover around the mortal nature and this earthly sphere. Wherefore we ought to fly away from earth to heaven as quickly as we can." And from it, too, comes Plato's clear-eyed perception that the idea of good holds the hegemony over all our interests, scientific and esthetic as well as moral. It is the good—as our pragmatists say—which makes truth true and is indeed the measure of reality. For "that which imparts truth to the known and the power of knowing to the knower is what I would have you term the idea of good, and this you will deem to be the cause of science, and of truth in so far as the latter becomes the subject of knowledge; beautiful too, as are both truth and knowledge, you will be right in esteeming this other nature as more beautiful than either; and as light and sight may be truly said to be like the sun and yet not to be the sun, so in this other sphere, science and truth may be deemed to be like the good, but not the good; the good has a place of honor yet higher."

IV.

Let me briefly recapitulate Plato's view. In the beginning were God and Chaos. And God strove to impress the spirit of order, which is his own divine spirit, upon the face of the Void. And in his own image he created a soul of the World, and the name of this soul is Cosmos, Order. And to this divine soul he united a body, hewn from Chaos, and this soul in this body forms the visible Heaven and all that is therein. And he created inhabitants for the world which he had made, the race of gods and of demigods and the race of mortal men; and these were to be his allies and his helpmates in the redemption of Chaos. For Chaos is ruled by blind Necessity, and the horror of its blindness enters into all being in which it has a share; so that not men nor demigods nor gods are free from the peril of Darkness, which is the peril of their material and temporal being. Wherefore it behooves them, men and gods, to strive nobly after the Good, holding fast to the image of divinity which is in them. And to this strife there is and there can be no end. For Chaos is coequal with God, infinite in change as God is infinite in might; and the conflict of the two is the eternal struggle for the world's salvation which is the world's life.

In conclusion, I would say a word in regard to the wonderful vitality of Plato's thought; for to no other philosopher has it been given to lay such lasting hold at once upon men's reason and upon their affectionate imagination. I think the clue to this will appear if we compare his attitude with that of his great pupil and competitor toward the man from whom both derive their inspiration. For Aristotle, the arch-intellectualist, saw in Socrates but the inventor of definition—"two things may be fairly ascribed to Socrates, inductive arguments and universal definition" —and he made definition the very core of his own metaphysics. But for Plato Socrates is first and last that "midwife of souls" which he would have himself to be. Plato, in other words, had caught what Aristotle missed, the central spirituality of Socrates's teaching.

Plato is a great dialectician and a master of the things of the intellect, but he knew, as Socrates had taught, that reason alone cannot bring us to the truth, and that science is no capable vessel of reality. When "all philosophers proclaim, as with one voice, that mind is the king of heaven and earth—in reality they are but magnifying themselves," he says; for he knows well that beyond the symbols of sense, which are the symbols of our reason, there is a more splendid reality. We can see this other-world truth but as in a glass darkly; we can speak of it only in myth and allegory; we can hope for its realization never save in those aeon-parted moments of the cosmic cycles when the soul, after its hour of agony, has brought its steeds to that outer revolving heaven whence the things that are beyond stand revealed. And "of that heaven which is above the heavens, what earthly poet ever did or ever will sing worthily?. . . . There abides the very being with which true knowledge is concerned; the colorless, formless, intangible essence, visible only to mind, who is the pilot of the soul. The divine intelligence, being nurtured upon mind and pure knowledge, and the intelligence of every soul which is capable of receiving the food proper to it, rejoices at beholding reality, and once more gazing upon truth, is replenished and made glad, until the revolution of the worlds brings her round again to the same place."

Such is the beatific vision, and "how can he who has magnificence of mind and is the spectator of all time and all existence think much of human life?" Surely he will value it only for this spiritual prospect which it promises; "he will look at the city which is within him" whereof the pattern is the heavenly city; and "he will live after the manner of that city, having nothing to do with any other."

Is it not because of this faith in the spiritual reality of the world-life, which is a faith in the spiritual power of mankind, that Plato has brought conviction to the minds of his fellows, generation after generation, the edifice of his thought standing secure amid the rise and decay of competing systems? And is there other measure of truth than this?

HARTLEY B. ALEXANDER.

LINCOLN, NEBRASKA.

BERKELEY'S LOGIC OF MATHEMATICS.*

THAT Berkeley was keenly interested in mathematics is well known. In the *Commonplace Book* a great deal of attention is paid to mathematical questions; it is noticeable, indeed, that in its pages Berkeley refers to mathematicians far more frequently than to philosophers. The extent of his interest in mathematics is indicated also by a group of early writings, *Arithmetica absque Algebra aut Euclide demonstrata,* and *Miscellanea Mathematica* which includes papers "de Radicibus Surdis," "de Cono Aequilatero et Cylindro eidem Sphaerae circumscriptis," "de Ludo Algebraico," and "Paraenetica quaedam ad studium matheseos praesertim Algebrae." Both of these tracts were written in 1705 and first published in 1707. Belonging to the same period is the essay "Of Infinites," which is in part concerned with the infinitesimal calculus. Berkeley deals with mathematical questions also in *The Principles* (1710) and in *De Motu* (1721), and his criti-

* The following article contains, in its treatment of Berkeley's early work which was not published for generations after it was written, a new and important contribution to the history of mathematics. It will also be of interest to our readers to know that editions of the books by Barrow and Wallis mentioned in this article are in preparation. They are edited by Mr. J. M. Child and will appear in the "Open Court Classics of Science and Philosophy." Further, in the same series a small volume by Prof. Florian Cajori on the history of fluxional concepts from the time of Newton is also in preparation. It will contain a detailed account of the *Analyst* controversy. Finally it is to be noticed that Berkeley's doctrine of "compensation of errors" in the calculus was later advocated by the eminent mathematicians Lagrange and Lazare Carnot.—Proofs of this article did not reach the author who was absent on military service.—ED.

cisms of the logical basis of the infinitesimal calculus in *The Analyst* (1734) and *A Defence of Free-Thinking in Mathematics* (1735) are of considerable importance in the history of mathematics.

In this paper I propose to consider the mathematical views stated in Berkeley's *Commonplace Book* and *Analyst.* In both cases he is concerned mainly with the logical basis of mathematics.

Berkeley very clearly perceived that his "new principle" involved difficulties with regard to the nature of mathematics. The "new principle" implied that lines consist of a finite number of points, that surfaces consist of a finite number of lines, and that solids consist of a finite number of surfaces. Thus ultimately all geometrical figures are composed of complexes of points, which are regarded by Berkeley as ultimate individualities. These indivisibles are *minima sensibilia,* the minutest possible objects of sense. It is impossible that the *minimum sensibile* should be divisible, because in that case we should have something of which our senses could not make us aware, and that, Berkeley believes, is simply a contradiction.[1]

Sensation, then, is the test of all geometrical relations. Thus equality depends simply on our inability to distinguish in sense-perception. "I can mean nothing by equal lines but lines which it is indifferent whether of them I take, lines in which I observe by my senses no difference."[2] He explicitly considers the claims of imagination and pure intellect to judge of geometrical relations, and summarily rejects their pretensions. Imagination, he points out, is based on sensation, and has no other authority than that of the senses. It has no means of judging but what it derives from the senses, and, as it is removed by one stage from immediate sense-perception, and has its knowledge

[1] Berkeley's *Works,* Oxford, 1901, Vol. I, p. 86.

[2] *Ibid.,* I, 22.

only at second-hand, it is in fact not so well fitted as sensation to judge and discriminate. Pure intellect, Berkeley continues, has no jurisdiction in mathematics, for it is concerned only with the operations of the mind, and "lines and triangles are not operations of the mind."[3]

Now this view of the nature of geometry is the direct consequence of Berkeley's early metaphysical doctrine, but it is interesting to note that it also connected itself in his mind with the method of indivisibles maintained by the Italian mathematician Cavalieri. "All might be demonstrated," he says, "by a new method of indivisibles, easier perhaps and juster than that of Cavalerius."[4] What precisely Cavalieri meant by his conception of indivisibles is open to doubt, but it is certain that Berkeley's sympathy would be elicited by his demonstration that quantities are composed of indivisible units, a line being made up of points, a surface of lines, and a volume of surfaces. It is possible, though he is very obscure, that he regarded areas as composed of exceedingly small indivisible atoms of area. Berkeley's conception is very similar to this; but whereas Cavalieri maintained that the number of points in a line is infinite, Berkeley was convinced that no line or surface can contain more than a finite number of points, points for him being *minima sensibilia*. This, then, is Berkeley's "new method of indivisibles."

It will follow that geometry must be conceived to be an applied science. The only pure science will be algebra, for it alone deals with signs in abstraction from concrete things. Geometry may be regarded as an application of arithmetic and algebra to points, i. e., the *minima sensibilia* which constitute the whole of concrete reality, Berkeley admits that it is difficult for us "to imagine a minimum,"[5] but the reason is simply that we have not been accustomed to take note of it separately. In reading we

[3] *Ibid.,* I, 22; cf. I, 14. [4] *Ibid.,* I, 87. [5] *Ibid.,* I, 85.

do not usually notice explicitly each particular letter; but the words and pages *can* be analyzed down to these minimal letters. Similarly, though we are not explicitly aware of the *minima sensibilia*, they do exist separately, and may be analyzed as indivisibles in the complex sense-datum presented to us in perception. Geometry, then, is an applied science dealing with finite magnitudes composed of indivisible *minima sensibilia.*

If this conception of geometry be adopted, it immediately follows, as Berkeley very clearly perceived, that most of the traditional Euclidean geometry must be rejected. (1) In the first place, on the new theory, not all lines are capable of bisection.[6] Only those lines which consist of an even number of points can be bisected. If the number of points composing the line be odd, then (supposing bisection to be possible) the line of bisection would have to pass through the central point. But the point is *ex hypothesi* indivisible; hence the line does not admit of bisection. (2) Again, the mathematical doctrine of the incommensurability of the side and diagonal of the square must be rejected.[3] For since both the side and the diagonal of the square consist of a finite number of points, the relation between these lines will always be capable of exact numerical expression. Berkeley even makes the general statement, "I say there are no incommensurables, no surds."[8] (3) It follows that one square can never be double another, for that is possible only on the assumption of incommensurables. And it also follows that the Pythagorean theorem (Euclid, I, 47) is false.[9] (4) Further, it is no longer possible to maintain that a mean proportional may be found between any two given lines. A mean pro-

[6] *Ibid.*, I, 79, 80.

[7] *Ibid.*, I, 60, 78, 79.

[8] *Ibid.*, I, 14.

[9] *Ibid.*, I, 19.

portional will be possible, on Berkeley's theory, only in the special case where the numbers of the points contained in the two lines will, if multiplied together, produce a square number.[10] (5) Finally, the important work that had recently been done on the problem of squaring the circle is, in Berkeley's view, quite useless. Any visible or tangible circle, i. e., any actually constructed circle, may be squared approximately; and it is therefore time thrown away to invent general methods for the quadrature of all circles.

That his new doctrine necessitated such a clean sweep of important mathematical propositions, most of which had been accepted for hundreds of years, might well have given pause to an even more confident man than Berkeley; for (to take only one instance) apart from its startling theoretical aspects, serious practical difficulties would arise if some lines should prove incapable of bisection. Berkeley therefore suggests that for practical purposes small errors may be neglected. Though we cannot bisect a line consisting of 5 points, we can divide it into two parts, one containing 3 points, the other 2; and, as the *minimum sensibile* is so minute, it makes no practical difference that the lines are only approximately equal. Berkeley was influenced to make this suggestion by the method of neglecting differences practised in the calculus.[11] If differentials, which are admitted to be something, are overlooked under certain circumstances in the calculus, are we not justified in the new geometry, Berkeley asks, in neglecting everything less than the *minimum sensibile*?[12] The resulting errors will be so slight that the usefulness of geometry,

[10] *Ibid.*, I, 14.

[11] *Ibid.*, I, 85.

[12] It might seem that in our approximate bisection of the line we have neglected a whole *minimum sensibile*. But from the point of view of the error involved in each of the resulting parts we are not guilty of that. Each of the parts ought to contain 2½ points. Now each of the lines obtained by the approximate method differs from this by only ½ a point. Hence the error to be neglected in each case is less than a *minimum sensibile*. And this is the condition laid down by Berkeley.

which it must be remembered is a practical science, will not seriously be impaired.[13]

It is of peculiar interest to notice that Berkeley was influenced to neglect small errors, and to justify his procedure, by the example of the differential calculus. For in the *Analyst,* written nearly thirty years later, he vigorously attacked the method of ignoring small errors in the calculus. What a triumph for his opponents in the *Analyst* controversy if they could have seen the *Commonplace Book*!

But though Berkeley made use of the illegitimate method suggested by the calculus, his attitude to the calculus itself was from the first exceedingly critical. And his motive for criticism is not far to seek. If the calculus were sound, then his conception of geometry could not be maintained. For the calculus, whether in the form of Newton's theory of fluxions or Leibniz's method of differentials, rested, Berkeley believed, on the assumption of the existence of infinitely small quantities. Now if these infinitesimals were admitted to exist the significance of his *minima sensibilia* would disappear, and indeed the foundations of his philosophy as a whole would be seriously shaken. For if quantities could be proved to exist which were neither sensible nor imaginable he would need to revise his theory of knowledge altogether. He therefore had every reason to look with critical eyes on the conception of infinitely small quantities.

In the *Commonplace Book* he says nothing of importance with regard to the use to which infinitesimals are put in the calculus. Yet even then he was certainly acquainted with a good deal of the work that had been done on fluxions and differentials. His notes contain references, on matters connected with infinitesimals, not only to Newton and Leibniz but also to Barrow, in whose *Lectiones*

[13] *Ibid.,* I, 78.

opticae et geometricae (1669) was given the chief impulse to Newton's theory of fluxions; to Wallis (1616-1703), whose *Arithmetica infinitorum* (1656) paved the way for the invention of the calculus; to Keill (1671-1721), who, in addition to his *Introductio ad veram physicam* (1702), had written of fluxions in the *Philosophical Transactions* of the Royal Society, and took a prominent part in the famous "Priority controversy" in which he accused Leibniz of having derived the fundamental ideas of the calculus from Newton; to Halley (1656-1742) who in addition to his works on astronomy and magnetism wrote on fluxions in the *Philosophical Transactions*; to Cheyne (1671-1743), whose *Fluxionum methodus inversa* (1703) and *Philosophical Principles of Natural Religion* (1705) gained him admission to the Royal Society; to Joseph Raphson, whose *De Spatio reali seu ente infinito* (1697) contained a definition of the infinitely small, and who was afterwards to write a *History of Fluxions*; and also to two more elementary writers, Hayes (1678-1760) who published in 1704 his *Treatise of Fluxions,* and John Harris whose *New Short Treatise of Algebra....Together with a Specimen of the Nature and Algorithm of Fluxions* (1702) was the first elementary book on fluxions to be published in England. And that he had not confined his reading to English works is proved by his reference to the *Analyse des Infiniments Petits,* and to the controversy between Leibniz and Bernhard Nieuwentijt, a Dutch physician and physicist, which took place in 1694-5 in the pages of the Leipsic *Acta Eruditorum.*[14]

It is clear, then, that even when the *Commonplace Book* was written Berkeley was acquainted with much of the work that had been done in the calculus. But at that time he was not in possession of the arguments which he

[14] The last-mentioned references occur, not in the *Commonplace Book,* but in the essay "Of Infinites" (*Works,* III, 411).

afterwards advanced against it in the *Analyst*.[15] In the *Commonplace Book* he does not venture any criticism in detail of the use of infinitesimals *in the calculus*.[16] What he is concerned to do there is to prove that infinitesimals have no real existence at all.

His line of argument is indicated twice over, and is based on his own metaphysic. For the purpose of his proof he posits two axioms: (I) "No word to be used without an idea," and (II) "No reasoning about things whereof we have no idea." Now we have no idea, Berkeley says, of an infinitesimal. By this he means, if his terminology be translated, that infinitesimals cannot be either objects of sense-perception or objects of representation in imagination. Hence, as we have no idea of an infinitesimal, it is simply a word. Further, according to axiom I, it is a word which means nothing; and, according to axiom II, we have no right to use it in our calculations.

We have now considered in outline Berkeley's attitude, as revealed in the *Commonplace Book*, to contemporary mathematical problems. His willingness to throw overboard the solid achievements of the established geometry simply because they did not accord with an *aperçu* of his own does not encourage us to rate his mathematical ability very highly. Or perhaps it would be truer to say that when he wrote the *Commonplace Book* he had not had time to steady his outlook upon science and the world; and allowance may fairly be made for his youthful dreams of a new idea which was destined to revolutionize the sciences, when we remember that it was only about seventy-three years since Galileo expounded the Copernican theory and thus changed entirely the orientation of astronomy and indeed of science as a whole. Another Copernican change,

15 Some of his remarks show that he was at that time, far from understanding its principles and methods (Cf. *Commonplace Book*, I, 84f).

16 But there is some criticism of the calculus itself in the essay "Of Infinites" (*Works*, III, 411). And cf. *Commonplace Book*, I, 83-86.

he believed, was not impossible; and in any case he was inclined to think that the wonderful mathematical renaissance of the previous few decades had, among all its triumphs, grown not a few excrescences which it would do no harm but much good to pare off. What he really wished to do was to examine the logical basis of mathematics. He did not advance very far in the *Commonplace Book,* but it was part of what he attempted, and with greater success, in the *Analyst.* To the argument of the *Analyst* we now turn.

The *Analyst* was published in 1734. It is a curious work, and though its purpose is ultimately theological rather than mathematical, it gave rise to a mathematical controversy which lasted for several years and produced more than thirty controversial pamphlets and articles. We have no concern with the theological argument of the *Analyst,* but before passing to consider its mathematical importance, it may be well to mention that the essay is primarily intended as a defense of Christianity, and that Berkeley, acting on the principle that the best defense is in attack, criticizes the foundations of mathematics on the same lines as those on which Christianity had been opposed by "mathematical infidels." In reply to the criticism that the dogmas of Christianity are mysterious and incomprehensible, Berkeley maintains that mathematics, universally admitted to be the most demonstrable department of human knowledge, is, in that regard, in precisely the same position as Christianity. For it also makes use of mysterious and incomprehensible conceptions, e. g., fluxions and infinitesimals. If mathematicians accept mystery and incomprehensibility in mathematics they have no right to object to it in Christianity. This is the kernel of Berkeley's argument.

Berkeley is often regarded, but quite unjustly, as an enemy of the infinitesimal calculus. In reality he had no objection in the world to the calculus as such. What he did was to submit its logical basis to a searching examination. He criticized the conception of infinitely small quantities, which were at that time vaguely conceived as neither zero nor finite, but somehow in an intermediate state. They were said to be "nascent" and "evanescent" quantities, not quite nothing and not quite anything. It was against this "vague, mysterious and incomprehensible notion" that all Berkeley's attacks were directed; and as soon as it was clearly pointed out by one of the parties to the controversy, Benjamin Robins,[17] that the calculus did not necessarily involve this conception of infinitely small quantities, but might be demonstrated by the methods of limits, the controversy was abandoned by Berkeley. He had replied to his other critics, such as Jurin of Cambridge ("Philalethes Cantabrigiensis") and Walton of Dublin, because these mathematicians persisted in trying to defend the conception of infinitely small quantities. But as soon as it became clear, and Robins was the first to make it so, that that conception was not essential to the calculus, the controversy lost interest for Berkeley. For the method of limits, as he seems to have realized, is not incomprehensible; and therefore an attack on it would not have enabled him to use his *tu quoque* argument, and would thus no longer serve his purpose, which, it must be remembered, was primarily theological.[18]

[17] Robins's contributions to the controversy were contained in his *Discourse concerning the Nature and Certainty of Sir Isaac Newton's Methods of Fluxions, and of Prime and Ultimate Ratios* (1735), and in a series of articles in the *Republic of Letters* in 1736 and in the *Works of the Learned* in 1737.

[18] The course of the *Analyst* controversy, so far as Berkeley was concerned, was as follows. In 1734 the *Analyst* appeared. It was almost immediately attacked by Jurin in an anonymous tract entitled *Geometry no Friend to Infidelity; or a Defence of Sir Isaac Newton and the British Mathematicians.* To this Berkeley replied in *A Defence of Free-Thinking in Mathematics,* published in March, 1735. To this reply Jurin wrote a rejoinder which was published in July of the same year. Berkeley took no notice of it.

Berkeley had another critic. This was Walton of Dublin, who produced in 1735 a *Vindication of Sir Isaac Newton's Fluxions.* It was replied to in an appendix to the second edition of *A Defence of Free-Thinking in Mathematics.* Walton replied, and Berkeley then published his *Reasons for not replying to*

But though his motive in writing the *Analyst* was theological, the chief importance of the book, as we must now try to show, is mathematical. It is, indeed, an able treatise on the logic of mathematics. Berkeley saw that the brilliance of the rapidly accumulating results attained by means of the calculus had tended to put into the background the question of its logical basis and the validity of the methods employed by it. And he did good service to mathematics by the publication of the *Analyst,* for he forced upon mathematicians the investigation of the logical basis of the new mathematics. "I have no controversy," he says, "about your conclusions, but only about your logic and method. . . . I beg leave to repeat and insist that I consider the geometrical analyst as a logician, i. e., so far forth as he reasons and argues; and his mathematical conclusions, not in themselves, but in their premises, not as true or false, useful or insignificant, but as derived from such principles and by such inferences."[19] As a direct result of this investigation originated by Berkeley two highly important principles were firmly established, (1) that the calculus must be grounded on the method of limits, and (2) that the then current conception of infinitely small quantities must be abandoned.

These points will become clear if we examine Berkeley's criticism of Newton's theory of fluxions. In our investigation there are three main questions which we must ask. (1) Is Berkeley's criticism of Newton valid? (2) Is his criticism of current conceptions of infinitesimals

Mr. Walton's Full Answer. All this took place in 1735. Walton issued a rejoinder, but Berkeley took no further part in the controversy.

It is noticeable that Berkeley participated in the controversy vigorously until Robins's book appeared. After that he said not a word. The reason is, as we have suggested, that Robins showed that infinitesimals are not essential to the calculus. Berkeley must have been convinced by his arguments, and therefore realized that it was no longer possible, from his point of view, to take part in the controversy.

[19] *The Analyst,* § 20.

sound? (3) Did he really expose any fallacies in the calculus?

1. First, then, we must consider whether Berkeley is successful in his criticism of Newton. There is one special point in Newton's theory which must be examined with some care, for upon it depends the applicability of Berkeley's criticism. The question is this. Did Newton really use the conception of infinitely small quantity (in which case he would be exposed to the full force of Berkeley's arguments), or was his method really that of limiting ratios (in which case Berkeley's criticisms would be directed, so far as Newton is concerned, against a man of straw)?

It is often held that Newton never used the conception of infinitely small quantity, but it was conclusively established by De Morgan that this conception does appear in some of his works. De Morgan maintains that until the year 1704 when his *Opticks* was published Newton did believe in infinitely small quantities. "In Newton's earliest papers," writes De Morgan, "the velocities are only differential coefficients: when A changes from x to $x + o$, B changes from y to $y + oq/p$, the velocities being p and q. Those terms in which o remains are 'infinitely less' than those in which it is not, and are therefore 'blotted out.' And 'those terms also vanish in which o still remains, because they are infinitely little.'"[20] Again, in the first edition of the *Principia,* published in 1687, fluxions are founded on infinitesimals, moments being regarded as infinitely small quantities. De Morgan confirms this by relevant quotations from Newton's *Method of Fluxions,* written in the period 1671-1676) and his *Quadratura Curvarum,* which was originally written about the same time, though

[20] "On the Early History of Infinitesimals in England," *Philosophical Magazine,* 1852, IV, 322-3.

it was not published till later. So far, Newton certainly made use of the conception of infinitely small quantity.

But in 1704 the *Quadratura Curvarum* was issued in an appendix to the *Opticks*. It contained a new preface with some important statements regarding infinitesimals. "I here consider mathematical quantities," Newton says, "not as consisting of minimal parts, but as described by continuous motion."[21] "I was anxious to show that in the method of fluxions there is no need to introduce into geometry figures infinitely small."[22] Now Berkeley was well aware that the conception of infinitesimals had been thus disclaimed by Newton. In the early essay "Of Infinites" he says, "Sir Isaac Newton, in a late treatise,[23] informs us his method of fluxions can be made out *a priori* without the supposition of quantities infinitely small."[24]

But in 1713, when the second edition of the *Principia* was published, Newton again admitted, though very obscurely, infinitely small quantities.[25] From all this we may conclude that while Newton did not give exclusive adhesion to the method of infinitesimals, yet the conception of infinitely small quantity does occur in his writings prior to 1704, and though it was renounced in that year it reappears in the second edition of the *Principia* in 1713. It therefore follows that Berkeley's criticism is pertinent. Newton, we have decided, did maintain the existence of

[21] "Quantitates mathematicas non ut ex partibus quam minimis constantes, sed ut motu continuo descriptas hic considero."

[22] "Volui astendere quod in methodo fluxionum non opus sit figuras infinite parvas in geometriam introducere."

[23] This refers to the *Quadratura Curvarum*. Berkeley's "Of Infinites" was written about 1706-7.

[24] Berkeley's *Works*, III, 412.

[25] This point has been regarded as open to doubt. It depends on Newton's definition of moment. The definition is stated somewhat differently, but very obscurely in both cases, in the first and second editions of the *Principia*, in Book II, lemma II. But Edleston cites a letter from Newton to Keill written in May, 1714, in which he says explicitly, "Moments are infinitely little parts" (J. Edleston, *Correspondence of Sir Isaac Newton and Professor Cotes*, p. 176). This seems to be conclusive evidence that Newton still clung to infinitesimals.

infinitely small quantities, and it is against these that Berkeley argues.

Berkeley points out a serious inconsistency in Newton's conception. He shows that at one time Newton admits that infinitely small moments may under certain circumstances be altogether omitted in calculation. Against this admission he arrays Newton's declaration that in mathematical operations even the smallest errors must not be overlooked. Now the former of these statements is made by Newton in the *Principia* and the latter in the *Quadratura Curvarum.* The two are obviously inconsistent. Berkeley's critics in the controversy tried to defend Newton in various ways, but neither of them dared to admit, even if they perceived it, that the inconsistency was due to a change in Newton's system. In the *Principia*, holding a conception of infinitesimals, he is forced, precisely as the continental exponents of the differential calculus were forced, to admit that infinitely small quantities are negligible in calculation in comparison with those of finite magnitude. On the other hand, in the *Quadratura Curvarum,* having renounced infinitesimals, he is free to assert that even the smallest errors cannot be permitted. Benjamin Robins was the first of Newton's defenders to see clearly that the systems are different, and that if Newton's position is to be seriously defended it will be necessary to admit frankly the change of system, and to maintain that for Newton the really fundamental method is the method of limits.[26]

[26] Berkeley has been accused of bad faith in advancing this criticism. He must have seen, it is argued, that the Newton of the *Principia* was in a different position from the Newton of the *Quadratura Curvarum,* and therefore he was not justified in arraying the statements of these two periods against one another as evidence of present inconsistency (cf. A. De Morgan, *op. cit.,* p. 329). But such an argument overlooks two or three very material facts. The first is that Newton himself nowhere explicitly admits a change of system; in fact he seems anxious to conceal that such a change had taken place. Further, with the exception of Robins, Newton's followers were far from clear whether or not a change had taken place; and, in any case, as we have seen, Newton seems to have returned to the conception of infinitely small

This is what Robins did, and it has come to be realized that the conception of limits forms the true logical basis of the calculus. Berkeley's general criticism of Newton is perfectly valid, and it was largely owing to his objections that the difference between the two methods came to be fully appreciated, and that eventually a method of limits akin to that of Newton was established as the foundation of the calculus.

But in two respects Berkeley was unfair to Newton.

a. He never lets his reader know that Newton used the method of limits at all, and always speaks as if Newton had always held that the method of infinitesimals was essential to his theory of the calculus. Now the truth is, as Robins pointed out, that everything of fundamental importance in Newton's work is perfectly consistent with the method of limits.

b. He gives Newton no credit for his doctrine of continuity. Newton's infinitesimals are, after all, never so self-contradictory as those of Leibniz or even of his own followers. His infinitely small quantities are not, like Leibniz's differentials, discrete particulars. The Leibnizians hold that the "difference" of a line is an infinitely little line, the "difference" of a plane an infinitely little plane, and so on. And Newton's own followers used the conception of infinity in an equally rash way. Thus De Moivre regards the fluxion of an area as an infinitely small rectangle; and Halley, to whom Berkeley refers in the *Commonplace Book,* speaks of infinitely small *ratiunculae* and *differentiolae* in much the same way as the Leibnizians. Hayes, again, another follower of Newton to whom Ber-

quantity in 1713. Now, the *Analyst* was not published till 1734, and at that distance of time Berkeley may quite well have regarded Newton's renunciation of infinitesimals in 1704 as a temporary aberration. In that case he would be perfectly justified in his criticism.

[27] For an appreciation of Benjamin Robins, see Prof. G. A. Gibson's review of Cantor's *Geschichte der Mathematik* in *Proc. Edin. Math. Soc.,* 1899, pp. 20ff.

keley also refers, maintains the conception of infinitely small quantity with much frankness. "Magnitude," he says, "is divisible *in infinitum*. Now those infinitely little parts, being extended, are again infinitely divisible; and those infinitely little parts of an infinitely little part of a given quantity are by geometers called *infinitesimae infinitesimarum* or *fluxions of fluxions*."[28] But Newton himself does not speak in that way. He never forgets that his whole system is based on the continuity of motion. Lines are generated by the motion of points, planes by the motion of lines, and solids by the motion of planes. Fluxions are strictly the velocities of the generating motions. And the continuity of motion, generating lines, surfaces, and the like with varying velocities involves the conception of prime and ultimate ratios. To this aspect of Newton's theory Berkeley seems to be blind.

2. Having considered the respects in which Berkeley's criticism of Newton is valid, we may now proceed to ask whether his criticism of infinitesimals in general will bear examination. The general criticism of infinitesimals consists of two arguments, only one of which seems to be sound.

a. Berkeley argues—to take first the contention that seems unsound—that infinitesimals are impossible because imperceptible. An infinitely small quantity cannot be the object either of sense-perception or of imagination, and in accordance with the formula *esse est percipi* it can therefore have no existence. "As our sense is strained and puzzled with the perception of objects extremely minute, even so the Imagination, which faculty derives from Sense, is very much strained and puzzled to frame clear ideas of the least particles of time, or the least increments generated therein; and much more so to comprehend the

[28] *A Treatise of Fluxions*, 1704. Quoted by A. De Morgan in *Essays on the Life and Work of Newton*, edited by P. E. B. Jourdain, Chicago and London, 1914, p. 91.

moments, or those increments of the flowing quantities *in statu nascenti,* in their very first origin or beginning to exist, before they become finite particles.[29]

Now, this argument is simply at the level of picture-thinking. It does not follow that what we are unable to perceive in sense-perception or to represent in imagination is non-existent. At one time Berkeley's new principle would have necessitated this argument, but when the *Analyst* was written he had outgrown the cruder form of his early theory, and in his doctrine of notions he admitted that we can have knowledge which comes neither through sense nor imagination. He was thus prepared to allow that we might have real knowledge not sensuous in its origin. His retention of the argument here is a sign that he was not completely emancipated from his early sensationalism.

b. Berkeley's second general argument against infinitesimals is perfectly sound. He points out that the conception of the infinitely small, whether in the form in which it appears in Newton and his followers or as maintained by Leibniz, is impossible. It is impossible because it is self-contradictory. Whether we regard infinitesimals with Leibniz as differences, i. e., as infinitely small increments or decrements, or with Newton as fluxions, i. e., velocities of nascent or evanescent increments, they involve in their nature an ultimate contradiction. On the one hand, an infinitesimal seems to be something, for otherwise it would not be used in mathematics; but on the other it seems to be nothing, for mathematicians say it may be neglected in calculations without affecting the accuracy of their results. Sometimes it is called a nascent quantity, i. e., one which has left being nothing, but has not yet quite become anything; at other times it is called evanescent, i. e., a quantity which is still something but is tending to be almost

[29] *The Analyst,* § 4.

(though not quite) nothing. This conception, Berkeley insists, is ultimately incomprehensible and contradictory. His argument here is, of course, perfectly sound. Infinitesimals, conceived in this vague and loose way, have now, very largely owing to the process of criticism initiated by Berkeley, been entirely extruded from the calculus.

3. The last problem which we set before ourselves is this. Did Berkeley, apart from stimulating the investigation of the logical basis of the calculus, expose any real errors in it? From his arguments in the *Analyst* it would seem that two main errors affect the calculus. Berkeley maintains that (*a*) any attempt to demonstrate the value of a fluxion involves the violation of ultimate logical principles, and (*b*) the maxim that infinitely small errors compensate one another is vicious. A word or two must be said on each of these points.

a. In order to prove the illogicality of the methods of determining the value of fluxions, Berkeley examines in some detail the two independent demonstrations given by Newton. In the *Principia* Newton gives a geometrical proof, in the *Quadratura Curvarum* an algebraic one. In each case, Berkeley seeks to show, a closely analogous error is committed.

Take first Newton's geometrical demonstration. We wish to find the fluxion of the rectangle AB generated by the continuous motion of one side upon the other. Let the moments or momentaneous increments of A and B be a and b respectively.

When the sides of the rectangles are each diminished by half their moments, the rectangle becomes

$$(A - \tfrac{1}{2}a)(B - \tfrac{1}{2}b)$$

$$\text{i. e., } AB - \tfrac{1}{2}aB - \tfrac{1}{2}bA + \tfrac{1}{4}ab.$$

Similarly, when the two sides are increased by half their moments, the rectangle becomes

$$(\mathrm{A} + \tfrac{1}{2}a)(B + \tfrac{1}{2}b)$$
$$\text{i.e., } \mathrm{AB} + \tfrac{1}{2}a\mathrm{B} + \tfrac{1}{2}b\mathrm{A} + \tfrac{1}{4}ab.$$

Subtract now the former rectangle from the latter, and the remainder is $a\mathrm{B} + b\mathrm{A}$. This remainder is the moment of the rectangle generated by the moments a, b of the sides. Such is Newton's proof.

In criticism of it Berkeley maintains that the natural and direct method of obtaining the moment of the rectangle AB, when the moments of its sides are a and b, is to multiply into one another the sides increased respectively by their *whole* moments.[30] The moment of the rectangle is therefore

$$(\mathrm{A} + a)(\mathrm{B} + b) - \mathrm{AB},$$
$$\text{i. e., } \mathrm{AB} + a\mathrm{B} + b\mathrm{A} + ab - \mathrm{AB},$$
$$\text{i. e., } \quad a\mathrm{B} + b\mathrm{A} + ab.$$

This, Berkeley says, is the true moment or increment. It differs from that obtained by Newton's proof by the quantity ab. Now, as it was essential for the method of fluxions to eliminate the term ab, Newton and his followers said that it was so infinitely small that it could simply be neglected. But against this defense Berkeley quotes Newton's own words, "In rebus mathematicis errores quam minimi non sunt contemnendi."[31]

Berkeley shows that Newton's algebraic proof also rests on illegitimate assumptions.[32] In this demonstration we are given the uniformly flowing quantity x, and it is required to find the fluxion of x^n.

Suppose that x, in process of constant flux, becomes $x + o$, then x^n becomes $(x + o)^n$. Expanding this by the method of infinite series we get

$$x^n + nox^{n-1} + \tfrac{1}{2}n(n - 1)o^2x^{n-2} + \ldots.$$

(i. e., the increment of x^n is $nox^{n-1} + \tfrac{1}{2}n(n-1)o^2x^{n-2} + \ldots.$).

[30] *The Analyst,* §§ 9ff.

[31] *Opticks,* Introduction to *Quadratura Curvarum.*

[32] *The Analyst,* §§ 13ff.

It follows that the increments of x and x^n are to each other as o to $nox^{n-1} + \frac{1}{2}n(n-1)o^2x^{n-2} + \ldots\ldots$
or, dividing by the common quantity o,

$$\text{as } 1 \text{ to } nx^{n-1} + \tfrac{1}{2}n(n-1)ox^{n-2} + \ldots\ldots$$

Now, "let the increments vanish," and the last or limiting proportion is $1 : nx^{n-1}$. The ratio of the fluxion of x to that of x^n is as 1 is to nx^{n-1}.

Berkeley points out that this reasoning is illogical. If we say, "Let the increments vanish," we imply that the increments are really nothing, seeing that they are negligible. But we are enabled to arrive at the proportion between the fluxions only by assuming that the increments are something. Berkeley accordingly maintains that it is illogical to reject the increments, and still retain an expression, i. e., the proportion of the fluxions, obtained by means of them. If we let the increments vanish, we must also in consistency let everything derived from the supposition of their existence vanish with them.

This criticism Berkeley supports with a lemma, which he states as follows, "If, with a view to demonstrate any proposition, a certain point is supposed, by virtue of which certain other points are attained; and such supposed point be itself afterward destroyed or rejected by a contrary supposition; in that case, all the other points attained threby, and consequent thereupon, must also be destroyed and rejected, so as from thenceforward to be no more supposed or applied in the demonstration."[33]

b. Berkeley goes on to urge that, even though correct results are attained by the application of the method of fluxions, that does not validate the method as method. That the conclusion of a syllogism is true does not necessarily imply that the process of reasoning is correct. The conclusion may be true, and yet logical errors may have been

[33] *The Analyst,* § 12.

committed in the process of proof. It is possible to reach a true conclusion from false premises by erroneous reasoning. One error compensates the other, and thus, though the conclusion is true, the logic is faulty. Precisely similar is the case of the calculus. True conclusions may be attained by it, and results of great practical value may be achieved, but its method is unsound because it is based upon the illogical principle of the compensation of errors.

These are the main arguments advanced by Berkeley in the *Analyst*. In the controversy which ensued all the points that he raised were traversed and re-traversed, with the result that (1) the vague notion of infinitely small quantity was abandoned, (2) the method of limiting ratios was firmly established, and (3) the principle of the compensation of errors was seen to be inconsistent with the logical foundation of the calculus.

G. A. Johnston.

University of Glasgow.

INFINITY AS METHOD.

AFTER the mathematical theory of assemblages had been developed, the logic of infinity entered upon a new stage. If we cannot as yet determine the final form of the doctrine, we can, at least, see at the present time in what direction it is tending. After the discovery of the calculus of infinite aggregates and after the establishment of different exactly distinguished kinds of infinity, the perpetual problem as to the potential or actual character of mathematical infinity seems to incline toward a solution in terms of actuality.[1] But this actuality seems to belong rather to the methodological than to the quantitative character of infinite aggregates; *it is a property of methods, not of quantities,* and expreses merely a peculiar system of laws and principles logically working in and upon finite magnitudes. From this point of view infinity cannot be regarded as a kind of quantity, but rather as the *type of structure* of certain quantities;—it does not pass the "limits of our possible experience" as if it were an expression for something "we never could find on sea or land." It is one of the constructive laws of our normal

[1] G. Cantor, "Mitteilungen zur Lehre vom Transfiniten" (*Zeitschrift für Philos. und philosoph. Kritik,* Vol. 91, pp. 81 ff. Comp. Vol. 88, pp. 224 f). Couturat, *De l'infinie mathématique,* pp. x, 213-256; 488-563. Royce, *The World and the Individual,* Vol. II, pp. 554 ff. Spaulding, *Defense of Analysis* (New Realism). H. Lanz, "The Problems of Immortality" (in Russian language, *Logos,* 1913). Gavronsky, *Das Urtheil der Realität* (Dissert., Marburg). Cohen, *Logik der reinen Erkenntniss,* pp. 102 ff. The actual character of infinitely small elements has been mathematically established in Veronese's *Grundlagen der Geometrie von mehreren Dimensionen* (transl. from the Italian). Methodological character of infinity without acceptance of its actuality is emphasized by Brown, *Intelligence and Mathematics* (Creative Intelligence).

experience, perfectly incorporated within every finite object. If two parallel lines do not cross each other in some infinitely distant point, the geometrical structure of our triangles, circles and the like will be different. This mysterious "point" has an *influence* upon finite geometrical forms which is somehow manifested in their inner relations. But is it the "point" that makes the relations what they are? Why not invert our presumption? Why not suppose the structure of finite relations to be prior, infinity being a result of them?

If we can discover all the methodological principles that have to do with infinity, and investigate the reflection of "endlessness" into the world of finite magnitudes, we shall be led to the conclusion that infinity is only an aspect of our normal experience, a property of finite things; that the infiniteness of "space" is nothing but a character of "single spaces," the infinity of series only a property of certain magnitudes. Mathematically however every "property" is an expression of some constructive method. Accordingly the old question as to whether infinity is a category of the qualitative or the quantitative sort, appears unexpectedly in a new light: it seems to be prior to both sorts, being a complex of methods rather than a *quale* or even *quantum.* Those peculiar "qualities" which belong as much to the spatial point as to the instant of time, as much to the sum total of algebraic numbers as to every general concept in its infinite integrating function, lose thus their metaphysical mystery. A point without determined "position" has no peculiar "quale"; taken abstractly it is pure nonsense. This mystical "quale" properly belongs to a point only in so far as it is a *point of a curve. It is a moment in the continuous change of direction,* expressed in the methodical exactitude of a derivative function, not a mysterious "part" of space without extension. For mathematically infinity is always connected with exactly determined methods of operation, in principle different from addition or division, even in their indefiniteness. This difference in the methods of operation makes every instance of mathematical infinity actual in spite of the impossibility of its production by any arbitrary long process of addition or division. The method of integrals, for instance, perfectly achieves a task which the ordinary methods of addition and division can never fulfil: it leads us to apparent "qualitative" new results,—sometimes to new kinds and unexpected generalizations of number.

The matter in question has often been discussed in mathematical and philosophical literature; but it may be of interest to follow the development of the methodical meaning of infinity as well in the deepest metaphysical speculations of antiquity as in the exactest mathematical achievements of our own epoch. We shall find, then, that this aspect, consciously or unconsciously, is predominant in every "case" and every "kind" of infinity. Thus will become clearer and clearer that positive moment, implied in the earliest unmethodical negations of finiteness, and the "dark" quality as well as the unintelligible "endlessness" of the infinite will be seen to be but concentrated expressions for certain methods of operation involved. This point of view may, possibly, remove the usual distrust and disgust of mathematicians for the "actual infinite."

Qualitative infinity. The infinite is not "not-finite." From the first historical use of the term by Anaximander, the ἄπειρον purports to be a *positive* principle for the explanation of all single and separate "quales," being far from an equivalent for pure nothingness or endlessness. As an ἀρχή or first principle of being, τὸ ἄπειρον is "infinite in a positive sense because it expresses a belief in and a demand for a "different nature" (ἑτέραν τινὰ φύσιν) required for the genesis and derivation of "beings" (ὄντα) —a kind of primitive generating ἕτερον (ἕτερον τι τῶν στοιχείων). To be principle is essential for the ἄπειρον; it is nothing but principle or basis (ὑποκείμενον) and has no meaning apart from its "consequences," innate or implicit in it.[2] The *infinitum* is possible and intelligible only in its relation to finite objects, in its logical activity as constructive and productive principle of finite results. It is prior to everything, because τὸ ἀόριστον πρὶν ὁρισθῆναι,[3] and it cannot be constructed by the successive addition or condensation of all things (μεταβολὴ τῆς ὕλης) because its nature is positive, simple and indivisible.[4]

These vague speculations of the earliest Greek philosophy do little more than mark out the field for later analysis. Nevertheless they clearly indicate the primary phase of the problem, in which the finite being begins to look for its origin beyond itself—*in infinitum.* The non-finite, that which is to explain the finites, defines itself as the *problem* of "in-finite." The dim historical previsions, concealed in this definition, soon reveal their positive purport. Aristotle—that scholastic lover of subtle and sterile distinctions—in our present problem brought out a discrimination of great importance. He set up two different concepts of infinity which might have been of a great historical influence and systematic fruitfulness. In his Metaphysics we meet with the clear distinction between the potential (τὸ δυνάμει ἄπειρον) and the actual infinity (ἐνεργείᾳ ἄπειρον). The potential infinite persistently remains within the confines of finite processes and means nothing but the possibility of continuing these processes indefinitely. It remains in the methodical power of the measure, in the

[2] Ἁμαρτάνει οὖν, τὴν μὲν ὕλην ἀποφαινόμενος, τὸ δὲ ποιοῦν αἴτιον ἀναιρῶν· τὸ γὰρ ἄπειρον οὐδὲν ἄλλο. . . . ἔστιν. *Anaximander Milesius* (Neuhaeuser, p. 6). Comp. Simplicius, *Phys.*, p. 32, β: ἐνούσας γὰρ τὰς ἐναντιότητας ἐν τῷ ὑποκειμένῳ ἀπείρῳ. Therefor the ἄπειρον being a different nature, it is not apart from reality. These sentences of Anaximander may be the first indication of the latter metaphysical speculations of Fichte and Schelling, according to which doctrine the "principle" without appearance is nothing; it appears necessarily and exists only in its manifestation in the world of finites; comp. the author's "Fichte and his Doctrine of the Absolute" in Russian).

[3] Aristotle, Met. I, 8, p. 989. [4] Neuhaeuser, *Anaximander Milesius*, p. 44.

field of its logical activity; it is adjusted for measurement and adapted to its "being gone through," but cannot be "gone through" because of the absence of the end.[5] This kind of infinity is only possible as a finite *variable* quantity in process of augmentation or diminution, something remaining always "beyond."[6] This purely negative concept of endlessness, unintelligible and contradictory as it is, has been generally acknowledged by mathematicians and is still current in that branch of science. The other logical type or variant of infinity given by Aristotle is without mathematical import or value and was meant to satisfy the purely logical interest in the notion of οὐσία. This ἐνεργείᾳ ἄπειρον (*infinitum actu*) is apparently the historical source of the "qualitative infinite"—an infinite transcending the problem, the function and the methods of measurement, and as remote from any implication of process as melody is inaccessible for sight. This kind of infinity *being beyond the concept of measure* is thus without "extension"; it has no "middle," no "above," no "below";[7] it does not consist of "parts" and is in the strict sense indivisible.[8] We must give over enumeration if we want to grasp infinity *in actu*; it is impossible to understand or to construct it in terms of continued recurrence of finite elements; in a word actuality marks in this primitive stage the creation of a new quality, the elevation of the mind to an entirely different level expressible only in terms of "ideality." Historically the meaning of ideality is connected inseparably with infinity[9] because to consider any fragment of reality under the aspect of ideality means to consider it as an instance of universal conformity to law. "Ideality" is the explanation of infinity *in actu,* or the resulting "quale" of infinity.[10]

This new qualification has been for many centuries the

[5] Aristotle, Met., κ, 10. [6] Aristotle, Phys., γ, 6. [7] Aristotle, Phys., γ, 5.
[8] Aristotle, Met., κ, 10. [9] Comp. Schelling, *Bruno, S. W.*, I, 4, pp. 342 ff.
[10] Hegel, *Logik, S. W.*, III, pp. 165, 171 ff.

main subject of philosophical reflection. In terms of infinity Plotinus defines his overtemporal realm of creative intelligence, where every "part" possesses the same "power" as the whole.[11] In the same terms Spinoza constructs his concept of substance, by which the must have meant to express neither more nor less than the logical inevitability of all the laws of nature.[12] In medieval literature also we meet with a very instructive instance of the qualitative infinity. I mean the doctrine of eternity. Thus Anselm constructs his concept of God in terms of "eternal truth" by the method of time-negation;[13] and this eternity is no potential or repetitive infinity of time. It transcends all lapse of time; it is an "indivisible unity" beyond time,[14]—*tota sibi praesens.*[15] This is the original source from which the modern concept of *Geltung* or Ideality has been derived. And what is of more importance, Anselm in his explanation of "over-temporality" goes further perhaps than Bolzano, Husserl, or even Bradley. According to his doctrine, the irrelevance to time limits not only produces a peculiar quality but is caused by negation and suspension of all the methodical means used for the explanation of temporal reality. *The positive ground for this new quality is discovered in the conformity to a new law.*[16] which has found its positive expression and justification in the laws of Logic.

Thus the definition of infinity as "quale" reveals its positive value when applied to the problems of pure logic.

[11] Plotinus, Enneades, III, 8, 8; VI, 9, 6. Comp. Henry Lanz, "Speculative Transcendentalism in Plotinus" (printed in Russian in the Journal of Ministry of National Education, 1914, I, 2.

[12] Comp. Wenzel, *Die Weltanschauung Spinozas.*

[13] Sancti Anselmi opera omnia (*Patrologiae cursus completus* T., 159), Monologium, pp. 160, 198 ff; Proslogion, pp. 235, 237, 239; Dialogus de veritate, p. 479.

[14] Anselm, Proslogion, p. 237.

[15] *Ibid.*, p. 238.

[16] Anselm, *Monologium*, p. 175: "Procul dubio summa substantia, quae nulla loci vel temporis continentia cingitur, *nulla eorum lege constringitur.*" Comp. *ibid.*, p. 166: "Ita uno modo, *una consideratione* est, quicquid est essentialiter." Comp. *ibid.*, 184-185. Thus according to Anselm's conception the essentia or idea is not apart from reality but a certain "consideration" of it, the result of the methodical application of certain laws.

Every logical content, every proposition in its value and relative truth can be regarded as an *instance of infinity,* because it marks the abandonment of the primitive attitude of enumerating the single cases—a turning from the sheer pluralism of sense-perception to the universality of method established first in Plato's "idea." The result was a different kind of logical operation, absorbing in the *process of deduction* all the possible cases *in infinitum,* instead of their enumeration *in indefinitum. The infinity implied in every logical concept is "actual" not because all its single cases have been enumerated, but because enumeration is no longer significant or serviceable.* In this sense "actual" infinity is simply the expression of a generality implied and employed in all use of "general concepts." It is no inherent or peculiar quality of "ideas" enabling us to apply to them our deductive, dialectical or transcendental methods; on the contrary by our methods we fashion our "ideas" into a logical form and adaptability in which they have for us the semblance of "transcendent entities."

Our modern logic is a positive system of methods, laws and categories which has grown out of these metaphysical speculations concerning eternal ideas, substances, God, *absoluta* and the like. It is an historical outcome of the simple resolve to consider the separate cases not in their plurality but in their systematic totality. So considered, they inevitably stand revealed as infinite logical complexes. Their being instances of a qualitative infinity is nothing but the expression of what they are as instances of a methodical (in a large sense deductive) thinking, and the peculiar quality of logical concepts, expressed in their eternality, overtemporality and so on, is nothing but a peculiar kind of operation with such complexes which justifies them and gives them a definite meaning. They have no existence for us until our methods of dealing with our world have made them seem to exist. Whereupon we say metaphysically, "they exist in no space or time," "they have but an intentional being." The peculiar property or power of logical complexes to embrace an infinity of single cases compels no recognition of a mysterious realm of truths or ideas, beyond reality. It is a natural consequence of our methodological emphasis upon the significant proportions and relations of the finite cases ("The essence is immanent to the appearance,"[17] "objectivity is created by consciousness," "the mind prescribes its law to nature").

Critics of the potential infinity. The usual explanation of infinity in mathematics consists in the assertion that the true infinite is nothing but a symbol expressing the possibility of continued counting or measurement—in a word "potential" infinity.

We may urge against this "subjective" principle of explanation the general objection, that our process of counting does not belong to the objective value or logical purport of any mathematical relations,—it can explain nothing, it can prove nothing with regard to them: it is absolutely irrelevant to their logical constitution or value, and can play no part in their mathematical establishment. The relations which govern in this operation, as executed by our mind, are psychological or epistemological relations which can have nothing to do with arithmetic or with the theory of aggregates. The constructive principle or method of arithmetic excludes by its essential purport all influence of consciousness so that what is impossible for consciousness may be quite possible in principle. This *elimination*

[17] The "essence" by Spinoza—perhaps the most important instance of the qualitative infinity—is based on the same methodological ground; to regard anything in the essence, as a modus of substance, means to regard it ***sub specie aeternitatis, as a moment in the deductive development of the system;*** everything *is* "substance" in so far as it is an instance of "method," i. e., in so far as it has truth (***sub specie veritatis***) and can be proved. The "essence" does not point out a different thing provided with special qualities, side by side with its real appearance, but the essential relation *within* the appearance itself, its conformity to the immanent law, ***its ability of being proved, its position in the system of deduction.***

of consciousness belongs in fact to the logical intention of all theory as such and is of the essence of all reasoning.[18]

This general proposition has an application to our present discussion. To base the concept of infinity on the ground of mental possibilities or processes means the same as to construct it without any ground, because the recurrence to the process as such has no logical value; the whole reasoning represents a very simple example of *quaternio terminorum.* What properly plays a part in the constructive definition of infinity is not the process as such (as executed in a finite time), but the inner methodical principle of it;[19] the process itself is going on according to this principle, changing no element or item of its logical content or axiomatical formulation. It would be absolutely meaningless to say: the principle does not define the totality of a certain series, because we are unable to stop in our process (*quaternio terminorum*). On the contrary: we cannot rest in our process because the principle does not permit us to rest, because it gives no guidance or indication as to a particular point of absolute rest. The relation must be reversed: it is the logical nature of infiniteness which gives our processes of counting or measuring indefinitely large or small, not contrariwise. Thus the *infinitum potentiale* may be called groundless infinity; the process of its construction is based on a well-known logical fallacy.

[18] The general position here indicated has been elsewhere worked out by the author. Comp. "Das Problem der Gegenständlichkeit in der modernen Logik" (Ergänzungsheft d. Kantstudien, No. 26, 1912); "Fichte und der transcendentale Wahrheitsbegriff." It has also been developed in certain writings of the author in the Russian language (in "Logos" and "Problems of Philosophy and Psychology").

[19] From this point of view may be profitably discussed a very old doubt of the sceptics, that for the purpose to know the infinite "we" must have an infinite capacity, and since we do not have any, we are unable to know anything about infinity. Pascal says, for example: "....il ne faut pas moins de capacité pour aller jusqu'au néant que jusqu'au tout; il la faut infinie pour l'un et l'autre." (*Pensées,* I, p. 82. Nouvelle éd. par Brunschvicg). But as we have seen, any of our "capacity" does not belong and does not have any logical influence upon the content known; therefore we don't need to have an infinite

On the other hand, the mathematical possibility of setting up a limitation in counting ("to construct the concept of number") is dependent upon a long series of special, and only for that purpose inevitable, pre-suppositions, which shall axiomatically define the meaning of the end. After the infinite aggregates had been mathematically defined by means of equivalence between part and whole, a long series of presuppositions was required to construct the finite numbers. The principles of enumeration, established in Dedekind's system of arithmetical axioms by means of "similar representation" and the concept of "chain," are similarly limitations added to Cantor's "axiomatic." Every aggregate of elements may be a "number" not by itself, but only in so far as a certain principle of representation is used; the same aggregate might be ordered by some other principle in a different way, which does not define the fundamental laws of finite arithmetic.[20] Thus the system of axioms which define the transfinite aggregates is prior to the system which defines the finite numbers (in metaphysical language: "finite things are limitations of infinite," ἀόριστον πρὶν ὁρισθῆναι); that is to say: the actual infinite is presupposed by the potential infinity.

Infinity of Series. If the subjective ability to continue a certain kind of operation *in indefinitum* does not belong to the constructive value of infinity, then what is the meaning of this peculiar term? It has to be determined without any reference to subjectivity—otherwise it would have no meaning at all. Let us start with the consideration of infinite series.

We may express the approximate value of π in decimal notation.

$$\pi = 3.14159\cdots$$

mind for the purpose to grasp the infinity *in actu*. The same might be said against Kant's doctrine.

[20] Dedekind, *Was sind und was sollen die Zahlen?* p. 37.

It is plain there is no sense in speaking forthwith about the whole decimal series in this expression, simply because the terms of the series are not represented by it. The equation gives no enunciation of what numbers are to be united into a whole; however long we may continue the enumeration of decimal terms, the value of π still remains under the curse of the undeveloped "potentiality." Nevertheless, π has an exact geometrical value, being a symbolic expression of a certain type of relation which cannot be expressed by any other value.[21] Otherwise, *every* decimal in the above series not only can be but objectively *is* determined—whether we compute it or not—by a certain method of operation, *each* of them *is* a result of exactly defined proportions and relations between the finite numbers. The theorem of Taylor supplied the foundation upon which have been based different methods for estimation of the value of π. Suppose we have carried out the calculation of π until we have reached the 707th decimal; we ask: are the 708th or 1000th decimals objectively undetermined? Would they be "created" in the process of our further calculation and begin to be only in that moment when we *know* their value? However it may be with the question of the dependence or independence of being upon consciousness, it is evident enough that the logical value of a certain mathematical proposition does not begin to "exist" with our temporal act of knowing it; all the roots of a certain algebraic equation of *n*th degree, for example, have their mathematical "existence," i. e., they are perfectly determined, in spite of the fact that we are forever unable to know them. How much more right then we have to conclude that *every* decimal in the objective value of π is determined by a certain type of preserial relations (theo-

[21] Comp. Couturat, *De l'infinie mathématique*, pp. 216-217: "En admettant que le symbole $m/0$ n'ait pas de sens numérique, il ne laisse pas d'en avoir un parfaitement intelligible en géométrie: car ce qui est absurde ou illégitime au point de vue du nombre pur ne l'est plus au point de vue géométrique." Comp. pp. 257; 213-256; 488-503.

rem of "middle worth"); and we are able to continue our calculation indefinitely only *because* every term—whether we actually know it or not—is objectively determined. Not in the process, but in the method of this determination our series becomes infinite. We should have no right to speak of the infinity (not even the potential infinity) of our series were there no *law* generating all the terms of the series. Therefore the working principle and real essence of infiniteness in our present case is represented by this generating law which renders it actual. Mathematically the infinite series may be regarded as given in its totality ("the series defines an infinite number")[22] only when its "general term" is given, because the constructive law is then expressed immediately in the form of series:

$$\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots\cdots\pm\frac{1}{2n-1}\pm\cdots.$$

Of course the infinity in this case would be meaningless if we tried to construct it by the successive addition of the terms; because this addition can never be fulfilled and the series does not mean anything else but the virtual determination of "every" term by a certain law; and the indefiniteness of series is nothing but a reflex of the actual infinity of law, being a negative expression of its positive character. Thus the infinity, being an instrument or methodical concept rather than one of process, is free of every continuation; it cannot be constructed by the methodical means of continuation, because the process as such is irrelevant and does not belong to the purport of infinity; it is rather an expression of the structure of certain processes than a concrete process by itself.

The reproach that we are unable to accomplish the measurement of a circle whose diameter is equal to one, does not prove the impossibility of such a circle; its circum-

[22] This terminus: infinite number is introduced by Dingler to denominate every mathematical expression, containing infinite chain of operations and united by a certain and expressible law.

ference possesses its exactly determined value *in the constructive law* of π; this transcendent value is *not approximately but exactly* determined by this law, expressed by its "general term" or more deeply by Taylor's general formula. We don't need to estimate all the terms for the purpose of determination of series, which is sufficiently determined independently upon the estimation of all approximative value in the series.[23] It is absolutely wrong to suppose that the mathematical determination is possible only by the arithmetical calculus; it can be afforded also by the simple indication of the constructive law. The arithmetical determination represents only a particular case of this general rule, every "number" is but a symbolic expression of a certain "type of order"); it is not a self-contained entity separated from the concrete reality but rather a peculiar way of bringing order into the concrete world of experience, a method of operation. The mathematical "existence" of the various kinds of numbers signifies only the possibility of determining magnitudes by their constructive law; "existence" means nothing but determinateness of whatever sort. The number 2 is not less determined when not defined by the constructive serial law (general term)

$$2 = \frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\cdots + \frac{1}{2^n} + \cdots$$

than by its arithmetical definition:

$$2 + 1 - 1$$

Therefore the number 2 may be an "infinite number" as well as π; the difference is only a formal one, the determination of irrational number by infinite series may be used as their definition. This is impossible in reference to the rationals only, because it would be an apparent circle

[23] Comp. Euler, *Vollständige Anleitung zur Algebra,* Ges. Werke, Vol. I, pp. 50-51. He says: "One cannot say, that $\sqrt{12}$ is in itself undetermined, but from what has been just said it follows only that $\sqrt{12}$ cannot be expressed *by fractions,* nevertheless it necessarily has a determined value."

in definition as may be seen from the equation given above. Returning to our previous example, π is sufficiently distinguished from every other value by the simple character of its constructive law and can be used as a well-defined complex of numerical relations;[24] its value is an actual one, and we have the right to regard it as actual only because it can be expressed in the terms of series having on infinite law. Thus *infinity is rather a property of law than of series as such.*

As to the character of the constructive law in general, another limitation has to be made. It is evident that our reasoning has no application to the divergent series. Of course, the law of a divergent series may be actual too, but it is meaningless to call it constructive, because it does not construct anything. The infiniteness in this case loses its purpose and that may be so important as to lose ground. For infiniteness is completely expressed only in and by the fulfilment of a certain task; where no determinate task is set, no fulfilment is possible. This might have made the concept of "limit" of such a great logical importance. But, from the logical point of view the limit is a statement of problem rather than a solution of it; the "existence" of limit is proved by the constructive law of series, not contrariwise, and the peculiar "jump" made by our mind in transition to the limit remains still unexplained and unintelligible if the methodical sense of actual infinity is disregarded. The infinite constructive law leads us to a determinate, mathematically positive result, i. e., it justifies the establishment of limit only when the series satisfies the conditions of convergence; that is to say: "the *infinitum* is possible and intelligible only in its relation to finite magnitudes, in its logical activity as constructing and pro-

[24] Every "infinite number," like π, may be regarded as an instance of such a "whole" which contains more than *all* its "parts"; because every "part" or "cut" of the series is a rational number (according to the definition) and the "whole" leaves the limits of rationality in principle, breaking the methodical law of it.

ducing the finite result" (metaphysically: "to be principle is essential for the ἄπειρον," "*das Prinzip erscheint nothwendig*").

The following important conclusions may be derived from the analysis of series given above: (1) Infinity has its logical basis in the constructive law (positive) which *makes* the series endless (negative). (2) The possibility of continuing a certain process indefinitely is nothing but a reflex of the objective activity of certain laws (not every methodical law gives this possibility). (3) The purport of infinity is justified only if it gives and implies a method for the genetic creation of the finite (conditions of convergence). (4) The qualitative moment in the logical explication of infinity turns out to be the methodical efficiency and defines itself more precisely as the logical activity of a certain type of laws.

The infinitely large (transfinite) numbers.[25] We are brought to the same conclusions by the consideration of transfinite numbers. What did Cantor mean by his "actual infinity"?

If we compare the whole lot of algebraic numbers with the series of integers, no conclusions can be immediately derived as to the quantitative difference between the two assemblages; there is no sense in speaking of the totality of an indefinite fraction, in default of a method for its construction. To Cantor chiefly is due the credit of preparing the way for the methodical comparison and mathematical treatment of such indefinitely large multitudes. Instead of classifying "numbers" by themselves he classifies the roots of *all* the algebraic equations, and in doing so he makes it possible to insert them in a proper order in which every equation finds its "enumerable" place in accordance

[25] This paragraph I suppose to be in agreement with Pr. Brown in his remarkable article: "Intelligence and Mathematics" (*Creative Intelligence*) in which is emphasized the methodical conception of transfinite numbers. But I cannot agree with the author concerning the potential character of this type of "numbers."

with its "height"; the classification of real roots follows immeditely and of itself because every equation has a finite number of roots which may be disposed in a proper order in accordance with their magnitudes. By this simple method of disposition he was able to coordinate all roots and consequently all algebraic numbers to the series of integers; obviously in the process of this coordination, not one root remains without a "number."[26] This method of "univocal coordination" first opens to us the logical possibility of applying the concept of "whole" to such indefinite aggregates, "wholeness" (infinity) signifying nothing but the unrestricted action of a certain law of coordination within certain limits. The aggregate of all the algebraic numbers can be regarded as a "whole" (transfinite number) only because *it can be arranged in the same way as the series of integers is arranged*; this prior constructive principle makes both classes of numbers equivalent, in spite of their indefiniteness rendering them of the same "class" and endowing them with the same "power." Every different "power," i. e., every *aleph,* means a different way or arrangement rather than a new (larger) quantity; by the general method of "covering" (*Belegung*) we are able to produce new and newer kinds of infinity, i. e., ever new ways of arrangement. Thus here also infinity is perfectly imprisoned in the finite magnitudes and expresses nothing but a peculiar method of organization of our usual experience.

The mysterious equivalence between "whole" and "part" loses its paradoxical character if we consider the situation from this methodical point of view. *It does not express equivalence in* quantis, *but only an equivalence in* methodis. The whole lot of algebraic numbers quantitatively is the same aggregate as the whole lot of integers, but differ-

[26] Cantor, "Ueber eine Eigenschaft aller realen algebraischen Zahlen (*Journal für reine und angew. Mathematik,* Vol. 77, pp. 258-268).

ently arranged; the "addition" of new terms does not change anything in this arrangement, which *methodically* remains the same, i. e., does not increase it at all, just as the designation of new officials does not increase the population. Cantor categorically distinguishes the "logical function" by which the transfinite numbers are established and proved, from the method of successive addition of terms.[27] The transfinite number, consequently, does not "consist" of its parts, because it is not constructed by regular addition. To take away a proper part from a certain transfinite aggregate does not change anything in it, *because the part has never been added.*

The infinitely small. The usual protest of mathematicians against the actual character of infinitely small elements has its ground in the tendency to understand it in terms of extensive magnitudes, i. e., infinitely small *parts.* Of course under such an aspect the concept of the differential becomes impossible and even contradictory. But still the infinitely small may be regarded as actual from the standpoint of such categories as lie beyond the competency of the primitive opposition between part and whole; this opposition may be irrelevant in the process of its logical construction. For differentiation is not division, and the mathematical procedure of this operation has no resemblance to the process of division. It is logically impossible to establish the concept of an indivisible part.[28]

The task of differentiation according to Leibniz is not to find infinitely small parts; the differentiation has to do with laws, i. e., functions, instead of with extensive magnitudes. If the constructive law of a certain line is given, we determine by differentiation, not a "point," but the direction of the tangent in this point; if a certain law

[27] Cantor: *Grundlage einer allgemeinen Mannigfaltigkeitslehre,* No. 11.

[28] Leibniz, *Mat.* III, 524: "Etsi enim concedam, nullam esse portionem materiae quae non actu sit secta, non tamen ideo devenitur ad elementa insectabilia aut ad minores portiones, imo nec ad infinite parvas."

of movement is given, differentiation determines the velocity at every moment of this process. Thus differentiation is not an algebraic action with extensive magnitudes but a way of dealing with laws or functions as such: when a certain law is given, differentiation permits us to determine the action of this law at every moment and for every element, i. e., to find the derivative function. On the contrary, if the derivative function is given, i. e., the action of a certain law at every moment is known, we are able by the inverse operation of integration to determine the law itself (the equation of a certain curve for instance), i. e., to find the original function (*data lege declivitatum curvae, posse describere curvam*). Within this operation the curve is regarded not as an aggregate of points, not as what it "consists of," but as a continuous change of direction; in the same way we are to conceive movement not as a product of "rests" but as a continuous change of velocity. The well-known paradox of Zeno is a simple case of fallacy; from the fact that in the 0 of time a point traverses no space, nothing follows as to the point's velocity. The expression 0/0 is arithmetically undetermined and undeterminable; but Zeno fallaciously ascribes to this point a determined velocity—rest being a peculiar case of velocity, where $v = 0$.

The differential does not exist as a part of the integral; under the aspect of part and whole (arithmetically) it is a pure nothingness; it has a meaning only as an instrument for certain operations with magnitudes, not with numbers. From this numerical point of view 0 is an expression for the simple negation, or absence of being: 0/0 here has no sense and must be regarded as an absolutely undetermined form, precluding all mathematical treatment. In the series of magnitudes the zeroes do not exist at all; here the concept of zero must be supplanted by the concept of "moment" which always conserves its specific qualitative character. A point in space, an instant of time and a moment in a movement are qualitatively distinguished in spite of the absence of any quantitative element in them. But the concept of this peculiar quality again is a statement of the problem rather than a solution of it; the quality has no mathematical expression and cannot be used in the process of calculation. What may be regarded as conserved is not the quality but an exactly determinable relation which is active in every moment of the process. In this sense the point of a circle is different from the point on a parabola, because each keeps the direction continually produced by all other points of its "class"; every element of a curve implies the law of its curvature; every moment of the movement continuously keeps the law of its velocity. Therefore the point, in so far as it is an element in the continuous change of direction, is not a simple null, but such a null as logically to imply the law of the whole line, i. e., the "infinitely small element" of "differential."[29] This infinitely small element has a meaning only in reference to a corresponding magnitude, determined in its whole character by a determinate formula or law (*interventu infiniti finitum determinatur*).[30] Another account of the infinitely small is possible. According to Leibniz's principles the differential has no positive meaning in itself; it seems to be only a symbolical expression of what is logically tctive *behind* it in the methodical meaning of the derivative function (dx:dy).[31] Is it possible to ascribe any positive value to dx independently of dy? Veronese tries to solve this problem geometrically. He defines the infinitely small "segment" of the order m as follows: "If a number in relation to another number is infinitely large, then this second number in relation to the first may be called infinitely

[29] Leibniz, *Opera Mathem.*, IV, 218: "Interea infinite parva concipimus non ut nihila simpliciter, sed ut nihila respectiva....id est ut evanescentia quidem in nihilum, retinentia tamen characterem ejus quod evanescet."

[30] *Ibid.*, VII, 53.

[31] Comp. also Euler, *De calculo integrali*, I, 2.

small."[32] In a special theorem he emphasizes the actual character of elements so defined, exactly distinguishing them from the "indefinitely small" elements: the latter still remain in the process of diminution and belong to the series of finite magnitudes; but it would be meaningless to seek the infinitely small segments in the series.[33] Bernoulli already believed the actual existence of infinitely small elements: "Sic omnes termini hujus progressionis, ½, ¼, ⅛, 1/16,.... actu existunt, ergo existit infinitesimus....si decem sunt termini existit utique decimus, si centum sunt termini, existit utique centesimus...ergo si numero infinito sunt termini, existit infinitesimus." But Bernoulli still seems to believe that the *terminus infinitesimus* is given in the series itself; of course, were this true, Veronese says, the concept of the infinitely small would involve a contradiction, because all the terms of this series according to the definition are finite. But it is logically possible to regard the series as defining a certain element beyond itself, which does not belong to the class of numbers given in the series, being always smaller than every term of it; and nevertheless this element may have some exactly determined and definable properties. In a certain circle of problems the assumption of such element may even be inevitable. Suppose we hypothetically accept the assumption of such a system, where "every finite segment, variable as to length and becoming indefinitely small, contains an element, which is different from its terminal points"; in the first place this presupposition is logically possible and implies the definition of infinitely small elements given above; in the second place it is obvious that certain systems (for instance the system of the spatial points) satisfy this hypothetically accepted condition.

From what has just been said it seems to follow that

[32] Veronese, *Grundzüge der Geometrie von mehreren Dimensionen*, p. 116.
[33] *Ibid.*, p. 141.

the infinitely small may have mathematical existence by itself. But still it remains true that all the determinations ascribed to this element are, properly speaking, of a derivative nature. What Veronese might have had in mind is only a convenient way of expressing certain properties and proportions in a certain class of systems called continuous. He explains himself clearly in this way.[34] If the distance between the two foci of the ellipse remains only indefinitely small, without any suggestion of an element of a different order transcending the potential series of such indefinitely small distances, the circle might never be considered as a particular case of ellipse. The "actual existence" of an infinitely small distance in this case means nothing but the possibility of passing from the formula given for the ellipse to that for the circle. The "actual existence" has here no meaning beyond the methodical value of certain operations; and this methodical value of the "infinitely small" element consists in what it is doing in the system, rather than in what it is. The correlation between the finite distance and the infinitely small element defined by the process of its continuous determination, is no relation between quantities, but in our present case, the relation of affinity between two different laws (circle—ellipse). Their "truth" consists in what stands behind their formal definition, in the methodological back-ground of their "existence." Still in terms of our present example, we may say that the point (as infinitely small distance) can be regarded as a "part" of the line only because a certain class of analytic forms (circles) can be regarded as a "part" of another more general class of forms (ellipses). It is obvious that the problem here again harks back to the problem of qualitative infinity.

Résumé. From what has been said it follows that infinity in all the cases of its application has a purely method-

[34] *Ibid.*, p. 144.

ical value. It is not a "thing in itself," not something ready given and self-existent independent from science; it is not a "thing" of whatever sort; it is a method, rather a methodical aspect of reality than reality itself. I don't want to allege that it is a pure product of our mind, unless we understand this term "mind" in a purely logical sense, as a system of methodical presuppositions of science, action or art. Then and only then, in this exactly restricted sense of "our mind," it may be logically created by it, i. e., every instance of infinity may be and, as a matter of fact, is a result of certain presuppositions. The reproach of artificiality does not affect our position in any way; in this broad and vague sense everything may be called artificial; I don't see any reason why any finite magnitude or any limited field of experience is less artificial than a transfinite number. We are too much inclined to forget, that a long period first of biological adaptation and then of logical and mathematical reasoning were required to perceive the limits of the real objects and to conceive the meaning of the "end." It is an old truth that all the boundaries in this world are artificial, i. e., they are based upon a long system of presuppositions. But since these presuppositions are not artificial at all, since they have their meaning and purpose, the result of their logical activity loses its artificial character also. Thus to persist in the thesis that everything in our world of experience is limited, is in itself a logical limitedness: It must be considered as a modern positivistic extreme, as a reaction against the metaphysical exaggeration of the value of infinity. The opposition of finite and infinite is not a contraposition of the different realms or worlds separated from each other; it is only a cooperation of two different methods one of which is quite as justified as the other.

Henry Lanz.

Palo Alto, California.

IMAGINATION.

SERVANT OR MASTER.

REASON'S eye is calm and steady,
Gazing ever straight ahead,
Seeing clearly every object
In its level vision spread.
But Imagination cries: "Look upward!
Here are wondrous things to see!
Leave your sober, steady plodding,
Trust my wings and fly with me."
Reason answers: "I will follow
Throughout all your fairy land,
But forget not, pretty maiden,
I shall always hold your hand."
Then the sprite Imagination
Guides him to the Ivory Door,
Lets him see the deeper meaning
Of his slowly gathered lore.
Never master had a servant
Who could give him such delight,
But 'tis well that Reason watch her,
See her safely home at night.

The scholar struggles slowly
Through the records of the past,
Sifting, balancing, rejecting,

Pondering o'er their meaning vast.
Suddenly Imagination
Breaks from Reason's curbing rein
As the lightning leaps from heaven,
Flashing through the startled brain
Swiftly vivid pictures, blending
In *one truth* the scattered train
Of the facts which toil unending
Strove to reconcile in vain.

He who walks beside the river
Hears its vexed and sullen roar,
Sees it sweeping swiftly onward,
Sees—a fact—and nothing more.
He who views it from the mountain
Sees a gleaming silver rod,
Silent, motionless, completed,
Like the changeless truth of God.

There's a pathway up the mountain,
Steep, laborious, and slow,
Lighted only by the witch-fire
Of Imagination's glow.
That lone path which thought has traveled
Since the Reason's earliest youth,
Struggling upward toward the cloud-cap
That still veils the Greater Truth.
Not for fame and not for riches
The explorer scales these heights,
But for the exhilaration
Of revealing hidden lights.
There's no joy for human nature
Like the mind's exultant thrill
When the new-born thought leaps living,
Bringing that ecstatic chill

Which has in it more than nature,
Holds the heart and brain in thrall,
Makes us wonder, spite of reason
If we're not immortals all.

When Galileo saw the lamp
Swing slowly to and fro,
A light leaped up within him,
'Twas Imagination's glow.
His reason fed and fanned it
Till its radiance burned away
A dozen dogmas of that church
In which he came to pray.
When Newton saw the apple fall
Imagination gleamed.
With all his hoarded learning
He never yet had dreamed
Of what that searchlight showed him
Which his reason gripped and steered
Through vast sidereal spaces
Where worlds on worlds are veered.

When the thinker meets the barrier
Of the "Ultimate First Cause,"
Reason fails him, for the problem
Seems transcending Reason's laws.
Then Imagination murmurs,
"Set me free and I will tell
All that Reason cannot show you,
All the truths of heaven and hell."
When the seeker, worn and weary,
Meets no answer to his quest,
Finds his Reason baffled, beaten,
Helpless at his great behest,
Yearns to know—what mortal knows not—

That which follows after death,
Then Imagination whispers,
"Lean on me, for I am Faith."

But if once Imagination
Is set free from Reason's hand
She assumes a thousand figures,
For they're all at her command.
Now an angel in the brothel,
Now a devil at the shrine,
She endows each human error
With an origin divine.
She has led Utopian dreamers
Into many a grave mistake,
And inspired the grim fanatic
To burn Reason at the stake.
Like the "Genius of the Bottle"
In the oriental tale,
She's a servant true and mighty
Till the magic word shall fail,
Then she becomes the Master,—
Oft a tyrant and a curse,
Leading blinded Reason captive,
Speeding on from bad to worse,
Till at last the frenzied dreamer
Thinks he hears the voice of God
In his wild Imagination,
Uncontrolled by Reason's rod.

All the palsying superstitions
That in ignorant minds find place,
All the cruel, false "religions"
That have cursed the human race,—
All the torments and the furies
That have harried every land—

Are Imagination's children
When released from Reason's Hand.

Yet the greatest truths discovered
Own Imagination's sway,
She the seeress, she the wakener,
Lights the torch for Reason's way.
All of poetry and music,—
All the beauty and the grace
Of the arts that help to sweeten
And uplift the human race—
Are Imagination's children,
Owning Reason as their Sire,
Sane and splendid, looking upward
With the soul's divine desire,
Yet they are the true half-brothers
Of that deadly bastard spawn
Which has kept the shadow lingering
O'er the promise of the dawn.

C. L. Marsh.

THE SUPER-SOUL.

FROM Sun-begotten sires of earliest life,
We draw our being, stronger through their strife.
They fought their way from formless germ to man,
With "climbing instinct," following nature's plan—
The plan, however planned—that life shall rise.
The "Right" of Nature lives, the "Wrong" still dies,
Not individual, but of the race.
The greatest man leaves no enduring trace
Save in the building, strengthening of the power
From which the coming man derives a dower
Of added strength and upward striving will,
That cumulates through generations still.
Not power of wealth or rank, but that of thought,
Which lives and grows forever and has wrought
All that has lifted life from brute to man
Since first the gleam of reason's light began.
Thought moves forever on in peace or wrath
Though blood of retribution stain its path.

Science, with searchlight of unbiased will,
Through dust of dream and dogma peering still,
Strikes to the solid rock of Nature's youth
And says with confidence: "Here rests the truth;
This earth was once impossible of life,
Now with life's varied forms the earth is rife."
The necessary sequence checks the breath.

By unknown forces *Life was born from Death*;
By natural law, to chemist still unknown,
Man has evolved from inorganic stone.
Strange confirmation of primeval thought—
From dust of earth the Gods mankind have wrought.

But whence is this that lifts us far above
The dawn of life, this power of thought and love?
This upward striving, this "divine desire,"
This restless star-ideal, ever higher?
The mind that through the tiny cells of brain
Can weigh the universe, and call again
The wisdom of all ages to its side,—
The heart that still with patience can abide
The scorn of *men* for higher love of *man*,—
The souls creative, lifting us to scan
The half-veiled beauty which we dimly see
In those rare hours that seem from earth set free—
Was this potential infinite alone
In that first germ that lived from lifeless stone?
Or was there influx of a mighty soul,
In-forming, lifting toward a rising goal—
Through each new form infusing greater strength
And greater self-dependence, till at length
The soul of conqueror Man, still incomplete,
Sees all the powers of nature at its feet?

Yet life evolved from inorganic stone,
How—matters little, so the fact be known.
Then was the power to live before the life,
'Twas in the stone itself through all the strife
Of world-formations; in the glowing mold
Of lifeless suns, in sunless realms of cold.
From unbeginning past we reach to-day;
To endless future lies our upward way.

In that eternity from whence we came
Lives still the potence of the higher aim,
The concepts *Good* and *Beautiful* and *True*
Forever widen to man's widening view.
There is no absolute, no perfect whole.
Perfection means stagnation, *growth is soul.*
The mountain tops that genius may attain
Rise ever higher from the grovelling plain,
But o'er their loftiest crags still loom afar
Unending peaks beyond the utmost star.
The immortal past is parcel of our life,
It breathes into our souls the upward strife.
The scientists, the poets and the seers,
The best and wisest of the bygone years,
Point ever upward from the heights they won,
And cry: "Beyond! Above! 'Tis but begun!"

Perhaps correlative to high desire
The Universal Soul's undying fire,
The *Growth-Law which is God,* informs us still.
Unrecognized, dream-felt, it molds the will;
It beckons ever to remoter goal,
The all-embracing Mind, the *Super-Soul.*

C. L. Marsh.

THE TWO-HUNDREDTH ANNIVERSARY OF THE BIRTH OF WINCKELMANN.

THE year 1917 is not only the four-hundredth anniversary of the birth of the Reformation, but also the two-hundredth of the birth of Winckelmann, the founder of scientific archeology and the father of modern art criticism. There is more of similarity in the work of Luther and Winckelmann, if both are judged by the influence which they wrought on posterity, than appears at first sight. While the one brought a complete change into the attitude of men's minds toward religion, teaching that an independent judgment is the inalienable right of every religious man, the other effected no less complete a change in the world of esthetics, by overthrowing the false taste in art and wrong conception of classical learning which obtained throughout Europe in his day, and by laying the foundations of a wholly new science.

In reading the biography of Winckelmann by Karl Justi[1] one feels that he is in the presence not only of one of the greatest scholars, but one of the greatest of men. His greatness as a scholar is indubitably attested by the scientific work which he left behind him, as well as by the influence which he exerted not only over his immediate contemporaries, but over the whole world of learning and culture since; his greatness as a man is no less clearly discernible in the infinite capacity which he possessed for

[1] *Winckelmann, sein Leben, seine Werke und seine Zeitgenossen,* 3 vols. Leipsic, 1865-72. 2d ed. 1898.

overcoming the almost insuperable difficulties of his early career until he reached his life's ambition. Nor was he only concerned with books and monuments, but with men, constantly seeking the help and inspiration of true friends, since he believed that friendship was the greatest of human virtues. For one born and schooled in adversity in an age and in an environment whose ideals were out of harmony with his very nature; for one who not only lacked the means to properly prosecute his studies, but the inspiration of contemporary science and art; for one who had never seen a genuine monument of ancient art until he had passed his thirtieth year; for such a one to have raised himself by sheer ability and industry to the highest place in European scholarship and to have been the means of completely reversing the attitude of his day toward art—all this discloses greatness of a rare order. For Winckelmann was not one of those fortunate mortals who are born in the lap of luxury, whose genius is slowly but easily unfolded throughout a long life and at the end crowned with great rewards; on the contrary he was of lowly birth and only with incredible difficulty accomplished his life work, and then was suddenly cut off by an appalling calamity after having barely passed his fiftieth year. His brief life was one of great contrasts in which the shadows and lights were about equally balanced—his journey to Rome in his thirty-eighth year dividing it into two distinct parts. It was the contrast of want and competence, of removal from the rudest environment to association with the world's best collections of art and intercourse with the greatest personalities of Italy and Europe, of being the teacher of recalcitrant village school children to becoming the preceptor of Europe and posterity. It is surely a life story well worthy our study and emulation.

To understand the character and significance of the change in the esthetic view-point wrought by Winckelmann's influence, we must understand how it was that Italian taste with its prejudice in favor of Latin studies over Greek and indifference to the latter had dominated Europe for two hundred years before his time.

The study of Greek, which had been so enthusiastically begun by the Greek immigrants and Italian humanists in the fourteenth and fifteenth centuries, as well as the great period of Italian art beginning with 1300 and so intimately connected with the commercial prosperity of the free states of central and northern Italy, began to languish after the first quarter of the sixteenth century. This decline was primarily due to the loss of political independence in these states during the disasters which befell them in the time of Michelangelo. Italy, the fairest and richest of countries, then became the prey of foreign armies and could no longer under the leadership of the popes present a united front against invasion. An army of Charles V sacked the eternal city in 1527 and took Pope Clement prisoner; two years later Florence was besieged by another imperial army and by its surrender in 1530 lost its liberty, and by the reestablishment of the Medici in 1532 as hereditary dukes of the capital and later of all Tuscany, Italian freedom was doomed. From then on until 1796—over two hundred and fifty years—Italy had no political history of its own: its annals were filled with records of dynastic changes and redistributions of territories, and it became the theater of desolating wars fought for the most part by the armies of contending foreign princes and for ambitions in which the Italian people had no share. The brilliant aristocracies which had long cultivated humanistic studies were ruined and the predominant influence of the reformed Catholic Church looked with no friendly eye on the worship of pagan ideals, an attitude which was bound to divert Italy from classical learning. The Greek elements and influences in Roman art and letters had been so thoroughly assimilated at the end of antiquity by the Imperial Age of Rome that there were few Italians in the seventeenth and eighteenth centuries who were aware of their independent Greek origin. The Mohammedans were holding Greek lands in thraldom, and no one visited them to bring back a truer knowledge to counteract the growing tendency to treat Roman studies as superior to Greek and to look upon them as original. Patriotism, moreover, naturally led Italian scholars to exalt their own country as the center of the old Empire of Rome. They knew Italy's debt to the Romans in both literature and art and enthusiastically imitated them without any critical idea that the Romans had largely copied the Greeks. The fact that Italian was descended from the language of Rome made it easy for them to unlock the treasures of Latin literature. Thus, generally speaking, it had come to be customary in Italy to ignore Greek studies and to prefer everything Roman, and this way of looking at things spread over all Europe until finally, in Winckelmann's century, Italian taste, founded upon a wholly mistaken historical conception, ruled all cultivated nations. The great Italian humanist, Julius Cæsar Scaliger, long before had declaimed against Greek in favor of Latin, and his book of Latin verses — the *Poetice* — which appeared posthumously in 1561, remained a standard of taste down into the eighteenth century. Though the French historian de Thou exalted him above all scholars ancient and modern for his learning and talent, we know that he only looked upon classical studies as an agreable relaxation from the severer pursuits of life. In fact his chief amusement in later years was the composition of Latin verses. Thus within a century of Byzantium's fall, the Renaissance had already begun to take on in Italy its characteristic Roman bias.

In France the sixteenth-century Greek tradition inaugurated by Stephanus and Turnebus soon began to wane.

The French schools were deserted by Joseph Scaliger in 1593, by Casaubon in 1610 and by Salmasius in 1631. By the end of the seventeenth century classical enthusiasm had yielded to a taste which found pleasure in ridiculing Greek studies with characteristic Gallic wit. The age of Louis XIV,—the founder in 1663 of the Academy of Inscriptions—was marked in the years 1687-92 by the literary quarrel between Perrault and Boileau over the respective literary merits of the ancients and moderns. In his *Parallèle des anciens et des modernes,* Perrault, after a superficial survey of ancient and modern literature, gave the palm to the moderns. He declaimed not so much against the genius of the ancients as against their technique, the impersonal and objective character of their art. He compared Homer's immortal lays with the ballads of the Parisian street singers and looked upon his heroes as of lower stature than the dandies of Versailles, more like their landed thralls. His book was the signal to a controversy which passed over to England and again, in the days of Antoine Houdart de la Motte and Fénelon, returned to the land of its origin. La Motte, like his master, was an enemy of Greek and measured Homer by the rules of romantic French poetry. Voltaire, the dates of whose long life included those of Winckelmann's, expressed his sorrow that "the most beautiful language of the world" was neglected in France in his day. While praising the truth to nature and the descriptive power of Homer, he nevertheless found many unhewn stones in his marble palace and was content to set the second, fourth and sixth books of the Aeneid above not only the Iliad but all Greek poetry. He thought that the *Jerusalem Delivered* was at least the equal of the Iliad. He admired the dignity of Demosthenes, but looked upon the immortal Aristophanes as a mere *farçeur.* Plato did not please him because he made virtue too attractive and vice too repulsive; in his opinion Cicero was the equal of any Greek thinker. Out of respect for his judgment of the Iliad we must remember that Voltaire also essayed to write an Epic, which, I fear, but few even of the professors of French to-day read with any pleasure.

In England humanism had not yet recovered from the effects of the Civil War. There was no great name in classical scholarship until that of Richard Bentley, who was destined to become the greatest figure in the learned world of Europe during the first half of the eighteenth century. Sir William Temple, who knew no Greek, nevertheless entered into the controversy begun across the channel and championed the ancients with his *Essay on Ancient and Modern Learning.* A challenge to prolong the conflict was given by his statement that the best examples of Greek literature were the fables of Æsop and the letters of Phalaris, which he looked upon as nearly contemporary. The challenge was first accepted by Wotton, who, in his *Reflections upon Ancient and Modern Learning* (1694), calmly examined Sir William's reasoning. His friend Bentley told him that the fables of Æsop were not the work of Æsop at all and that the letters of Phalaris were a late forgery, the work, perhaps, of a sophist of the second century A. D. Temple's advertisement made a great demand for these worthless letters, and a young Oxford scholar named Boyle published an edition in 1695. A second edition of Wotton's essay was followed in 1697 by Bentley's famous *Dissertation on Æsop and Phalaris.* Nothing can better show the real state of Greek studies in England at his time than the fact that for some time public opinion favored the enemies of Bentley; however, the second edition of his *Dissertation* in 1699 marked a new epoch in English scholarship by heralding a new era of criticism. We have interesting hints of how Greek was neglected at Oxford at this time in Macaulay's *Essay on Addison.* While Addison had an intimate knowledge of the Latin poets and could write an excellent Latin style, his knowledge of Greek, though such as was deemed respectable at Oxford in his day, was evidently less than that which is carried away by many high school boys of to-day. An account of his Italian journey also shows how preponderating was his interest in things Roman.

In Winckelmann's own land classical studies had fared but little better. Their systematic study inaugurated in the fifteenth century by Huysman and continued by the labors of the humanists Reuchlin, Melanchthon and Camerarius had already begun to languish by the close of the sixteenth century. The leaders of the Protestant Reformation, Luther, Melanchthon and Zwingli, were all classically trained men whose minds had been broadened and whose powers of expression had been increased by the study of the Latin and Greek classics. Most of the Latin schools of the sixteenth century were founded under the direction of Melanchthon, and his educational plan was taken over by the universities which he reorganized. In his *Discourse on Reforming the Studies of Youth*, which he, a youth of twenty-one years, delivered as his inaugural as the first professor of Greek at Wittenberg, Melanchthon expressed his determination to plead the cause of the classics against those who found them "more difficult than useful" and who maintained that "Greek was studied only by disordered intellects and that, too, for display." His appointment at Wittenberg marked an epoch in German university education; for under this *praeceptor Germaniae* Wittenberg became the school of the whole nation. In laying aside the old scholastic methods of instruction, he showed that he had caught the real spirit of the Renaissance and was fitted to be one of its greatest leaders. In lecturing on Homer he announced that he, "like Solomon, was seeking Tyrian brass and gems for the adornment of God's temple" and he also asserted that "by going to the sources we are led to Christ." But despite its glorious initial promise the Reformation was bound to react detrimentally on classical learning. Luther, though he began his work at Wittenberg with lectures on Aristotle's *Physics* and *Dialectics,* soon found his influence harmful to the new theology and came to look upon Aristotle as the personification of scholasticism, the great enemy of the Church. He, therefore, wished to banish the *Ethics* and *Metaphysics* entirely from the university curriculum and to retain only the *Rhetoric, Poetic* and *Logic,* because these works might help young men to preach and pray better. The whole Protestant principle in art, isolated by the cleavage from Italian influences, was destined to cut Germany off from the ancient tradition of beauty and culture. The Thirty Years War in the following century had, like the Civil War in England, a disastrous effect upon every form of learning and culture. With the peace of Westphalia in 1648 neither art nor classical learning revived. The age of the giants of humanism had passed. After the death of Camerarius in 1574 there was not a name of importance in German classical scholarship for a hundred and fifty years until that of Johann Albert Fabricius (died 1736) is reached, and he is to be remembered mostly only for his great learning and industry, which won for him the title of the modern Didymus. The Flemish philologist Justus Lipsius had long before heralded the decay of Greek studies by his dictum that Greek was merely an ornament which for a scholar was not an indispensable possession. Latin continued to be taught in Germany and was still largely the medium of university instruction and the language of the learned world. Ancient literature, however, was regarded everywhere as a barren field, quite superfluous to the scholar. In Winckelmann's boyhood Greek was nowhere seriously studied; what Greek was taught was mainly intended for students of divinity for the sake of the New Testament and the early Church Fathers—that is, as the handmaid of theology. No Greek book of importance had been published for nearly a century and a half, from the time of Sylburg toward the end of the sixteenth century down to that of Ernesti, whose *Memorabilia* of Xenophon appeared in 1737. No Plato had appeared anywhere in Europe since 1602. No Greek text-books, except selections, were to be had. Scientific archeology was yet unknown and scientific philology was yet to be created at Halle by Wolf at the end of the eighteenth century. Porson's gibe that "the Germans in Greek were sadly to seek" was not without point. Only the seeds of the coming revival in Greek studies had been sown. Gesner, the older contemporary of Winckelmann, who was professor of eloquence at Göttingen for twenty-seven years until his death in 1761, was the first to re-introduce the best Greek classics into a German university, by publishing his *Chrestomathia graeca* in 1731 when Winckelmann was a boy of fourteen. This event really marked the advent of the new humanism by rekindling the national enthusiasm for ancient learning. It was Gesner's aim no longer merely to imitate the style of the Latin authors, but to understand the content of both Latin and Greek literature. Though himself a Latinist, he was the first to set a high value on Greek and the first to teach it in Germany and, therefore, may rightly be looked upon as the prophet of the Greek revival to be later instituted by Winckelmann, Lessing and Goethe. The revived classical tradition was carried forward by Ernesti, who, as professor of ancient literature at Leipsic from 1742, was the only official exponent of Greek in any German university in Winckelmann's day; by Reiske, who combined a critical knowledge of Greek with an unrivalled acquaintance with Arabic; and by Heyne, who lectured as Gesner's successor at Göttingen for half a century until his death in 1812. Heyne possessed neither the enthusiasm nor the penetration of Winckelmann, nor the philosophical nor critical power of Lessing, but he surpassed them both in accuracy and method. Johann Friedrich Christ, the professor of history and poetry at Leipsic after 1754, urged his students not to confine their attention merely to the ancient languages, but to include ancient art, and consequently he may be regarded as the immediate forerunner of Winckelmann in archeology, as Gesner was of Wolf in philology. It is significant of the condition of classical study in Germany in Winckelmann's day that its leading exponents—with the exception of Reiske—were such men as the uncritical Latinists Gesner, Ernesti and Heyne. Many greater German philologists, like Ruhnken and Wyttenbach, had sought the more congenial atmosphere of the Netherlands for their life-work, while others, like Reiske, had been compelled to go there for instruction. Joseph Scaliger, on leaving France at the end of the sixteenth century, had called Holland "the only corner of Europe"; classical scholarship there, which had extended from Erasmus to Grotius, was again flourishing in Winckelmann's time under the influence of the great Hellenist Hemsterhuis, who had founded the only real school of Greek learning which had existed in Europe since the days of Scaliger and Casaubon.

In the last half of the eighteenth century these prejudices in favor of Latin studies over Greek were destined to be overthrown largely by the work of one man—Johann Joachim Winckelmann. Through his influence the older custom of looking upon the relics of antiquity on Italian soil as those exclusively of Roman civilization had to yield to the true origin of these things in Greece. In his first book, *Thoughts on the Imitation of Greek Works in Painting and Sculpture,*[2] which appeared in 1755 just as

[2] The German title of this work is: *Gedanken über die Nachahmung der griechischen Werke in der Malerey und Bildhauerkunst.* 2d ed., 1756.

he was leaving Dresden for Rome, Winckelmann for the first time clearly disclosed the distinction between a Greek original of sculpture and painting and a Roman copy. In the next thirteen years down to his death his researches were destined to revolutionize the esthetic taste of Europe. His notion that there was an independent Greek art, from which Roman art was derived, was, strange as it may seem to us, a revelation to his contemporaries, who had uncritically accepted the interpretations of art works which had been based on the early enthusiasm for Roman history and literature. He showed that the realistic Italian sculpture of the day, which was more interested in anatomical accuracy than in the expression of the beautiful, copied merely the decadent phase of Greek art and that all such dramatic effects were directly opposed to the simplicity and repose of even Roman imitations of Greek works. With the disclosure that Roman art was derivative there was involved a new conception of the general origin of everything else in Roman civilization; for if Roman sculpture, painting and architecture were Greek, it followed that Roman literature and culture in general largely depended upon Greek. The change in viewpoint was to be fundamental and permanent; an entirely new inspiration was to come to Europe—an inspiration only comparable with that of the Renaissance itself. The taste of the succeeding period became Hellenic rather than Roman. Everything Greek—art, literature, history—began to be studied. The resulting intensity and expansion of interest in things Greek we now call the Greek Revival, whose waning we are unfortunately fated to see in our own time. This revival, beginning even in the lifetime of Winckelmann, came to full fruition after his death in the last quarter of the eighteenth century and was destined to become the most prominent spiritual feature of later European history. Lessing, by the publication of his famous essay

Laocoon in 1766—a work chiefly inspired by Winckelmann's ideas and studies—helped the nascent movement by critically establishing the superiority of Homer and thereby lowering the prevailing literary taste inaugurated by the French critics. Goethe's transcendent genius raised it into the higher realm of poetry. But the foundation of it all is to be sought in Winckelmann. He can rightly be called not only the founder of a science—for the principles which he laid down for antiquarian investigation have been followed since with ever increasing results—but also the greatest connoisseur and teacher of the Beautiful. His influence was be no means confined to the world of scholarship. The manifestations of the revival were manifold and far-reaching. The new inspiration entered not only into the more spiritual structure of culture—into the fine arts—but also into politics and every-day life. Here I can only most briefly and generally indicate a few of the more prominent manifestations which resulted from the stimulus of his work.

I have already spoken of the immediate effect of Winckelmann's influence on Lessing and Goethe. It was no less marked on all the Augustan writers of Germany, who owed their greatness to Winckelmann's disclosure of the Greek spirit. The new humanism soon, however, passed the boundaries of Germany and influenced all European letters. Travel to Greek lands began and a long line of English, German, French, Italian, Dutch and Scandinavian scholars studied the monuments on their native soil and wrote glowing accounts of their experiences, which immeasurably enlarged the horizon of scholarship. The new impulse was phenomenal in its influence on architecture, sculpture and painting. The simplicity of form of Greek porticoes and temples caused them everywhere to be copied; the theatrical and sentimental in sculpture yielded to Greek canons of restraint and dignity; Greek simpicity was taken over into painting. In architecture Schinkel von Klenze and Semper appeared in Germany; Vignon, Hittorff and Chalgrin in France; Soane, Inwood and Wilkins in England, and the architects of many famous Greek buildings in the older cities of the United States. In sculpture the Italian Canova and the Danish Thorwaldsen were followed by the German Dannecker and the English Gibson; in painting the French David, the contemporary of the Revolution and Napoleon, was the best exponent of the new style. Though in all forms of art the imitation of Greek subjects and forms proved ephemeral, the standards of taste taken from Greek art will always remain authoritative. Only after the first quarter of the nineteenth century did the imitation of Greek forms in all the branches of art yield to more independent styles, like the great Gothic revival in architecture, which reached its zenith about 1850, when practically every church built in Europe and America was Gothic. In music the subjects of the operas of Gluck reflected the new spirit. Even in dress and furniture the same spirit was revealed: the short-waisted dress of the Revolutionary period, known as the Directoire in Europe and that of Martha Washington in our country, was merely an effort to recover Greek simplicity: furniture, even clocks, imitated Greek designs. In politics it is hard to overestimate the effect of the revival. The Revolutions in both America and France were certainly largely influenced by the account of republican institutions in *Plutarch's Lives,* the most popular book of the day, while the Greek War of Independence in the last century was due in great part to the sympathy of European scholars and statesmen and men like Byron, who were directly influenced by the sentiments awakened by the second Renaissance of Greek studies.

To have furnished the inspiration and the stimulus for such a change in the spiritual history of the world is indeed an achievement of the highest order. As Walter Pater[8] has said, the highest that can be said of any critical effort is that "it has given a new sense, that it has laid open a new organ"; and this honor he pays to Winckelmann. Hegel, in his *Lectures on the Philosophy of Art,* has also paid a tribute to the humble German scholar in these words: "Winckelmann, by his contemplation of the ideal works of the ancients received a sort of inspiration, through which he opened a new sense for the study of art. He is to be regarded as one of those who, in the sphere of art, have known how to initiate a new organ for the human spirit." Winckelmann was a man to whom art was both religion and fatherland; when he wrote he thought not of Germany alone nor of his own time, but of all Europe and posterity. When one reflects on what he accomplished and the honor which he brought to his native land, one should not be surprised that his memory has been so highly esteemed in the past by his countrymen as to have amounted almost to Winckelmannolatry, a sort of cult in which he was regarded as a spiritual superman, the patron saint of archeology and art criticism. A more reasonable appreciation of his merits is the custom now long obtaining in Rome and in many of the universities of Germany of repeatedly commemorating his natal day—December ninth—by the publication of contributions to the science which he founded.

It is interesting to know something of the personality and life story of the man who wrought so great a change in men's outlook. Voltaire would hear nothing of the biographies of great writers, for he maintained that the life of a quiet scholar lay open in his works. This is largely true of the authors of scientific works, where facts and methods are the paramount interest and the personality of the writers is secondary. But it is certainly not true

[8] See his essay on "Winckelmann," in his *Studies in the Art and Poetry of the Renaissance,* 1873. I have followed his translation of the passage from Hegel.

of poets, essayists nor of literary men in general, whose life work is more concerned with sentiment and emotion. It is for this reason that we are vastly more interested in the romantic lives of a Cellini or a Shelley than in the more prosaic ones of a Laplace or a Darwin. In the case of Winckelmann, the idea which was the soul of his life's activities was the very human one of beauty and it was through this alone that his personality has influenced successive generations of art lovers. In trying to express this idea he had to pass his early manhood in the uncongenial atmosphere of the north, condemned to subsist by teaching rudiments to children; but he spent his nights in reading Homer and Sophocles, which fired his enthusiasm and finally drew him to the south. The fulfilment of his life's work was of such importance that Lessing, on hearing of his untimely death, could say that Winckelmann was the second writer to whom he gladly would have given some years of his own life—meaning thereby that his life had been shortened by that catastrophe.

Winckelmann was of very lowly origin, the only son of a poor cobbler of Stendal, a town in the ancient Prussian province of Brandenburg. The house in which he was born consisted of only one thatch-roofed room, which was used by the family as working, living and sleeping quarters. His father naturally wished his son to follow his trade and only with the greatest difficulty was persuaded to let the boy go to the town Latin school. Here he received his first instruction from the almost blind rector whose *famulus* he became, reading to him, walking with him and looking after his library. His childish imagination was impressed by the medieval appearance of his native town, by its ancient gabled houses, lofty cathedral and massive city walls and gates, all of which aroused in him thus early a love of the historical and monumental. His boyhood was passed amid great poverty and trials which ever after left their mark on his melancholy disposition. Years later while viewing the Roman Forum in full emancipation of spirit he said: "One gets spoiled here; but God owed me this, for I suffered too much in my youth." But he who was destined to interpret the charm and beauty of the spirit of Greece to his age, had first to serve an unhappy apprenticeship in the rude intellectual life of Germany. There is no wonder that, as Pater says, after "passing out of that into the happy light of the antique, he had a sense of exhilaration almost physical."

The old rector, seeing the boy's studious nature, wanted him to enter the Church. Consequently it was necessary for him to go beyond the Latin school to prepare himself for the university. At sixteen we find him at the Cologne Gymnasium in Berlin. This was at that time under the direction of a Greek scholar of note, Christian Tobias Damm, the lexicographer of Homer and Pindar. Winckelmann lived in his home as tutor to his children. He soon found, however, that he could get little instruction at the Gymnasium outside of Latin. The recent reform in German schools which had started in Halle under Francke paid little attention to Greek; everything was Latin, German and the positive sciences. Consequently it is no wonder that Winckelmann was more interested in the lectures given at the Academy of Arts and Sciences than in the work of the school. This naturally aroused the hostility of the rector, who showed his resentment by writing in the student register after Winckelmann's name the opinion that he was a *homo vagus et inconstans,* quite unaware on whose side the irony would eventually fall. However, it was not difficult for him to imbue the mind of his young pupil with the idea that Greek was superior to Latin and that Greek models must be imitated to raise the level of German culture. The imitation of Greek models in Art was destined to be the theme of the first work published by his most famous student.

After three years Winckelmann left Berlin to enter the gymnasium of the Gray Cloister at Salzwedel, from which he entered the University of Halle with the intention of studying theology. The university at this time had about fifteen hundred students and a library of ten thousand volumes and was important in philosophy, theology and law. The Wolfian philosophy was then dominant there as at all German universities. Winckelmann studied philosophy and esthetics under the great Baumgarten, and he also studied Hebrew, mathematics, physics and law. Halle had no professor of Greek, but Schulze, a teacher of medicine and linguistics, admitted Winckelmann to his course on ancient coins. He got but little out of his theological studies except Hebrew; from his legal studies he received valuable lessons on the universality of history, a sense for outlining great epochs and an idea of clearness in exposition, lessons which stood him in good stead in later years. Where Goethe confessed the influence of Kant on his life, Winckelmann's study of philosophy led him to protest against all philosophers except Plato—and Plato was excepted merely because of his redeeming literary style. He never received a degree at Halle nor wrote a dissertation, but contented himself with receiving in February 1740 a certificate of membership in the theological class. For a half year longer he stayed at Halle in charge of the library of the university chancellor, where he spent most of his time reading Greek. He had arrived at the certainty that he was in no wise fitted for a theological career. In after years he spoke disparagingly of his university education, maintaining that he was his own teacher. We would expect such an opinion from a poet, who receives little assistance from a formal education, but from a scholar, such as the historian of ancient art and the votary of the greatest intellectual tradition, we are surprised at such an admission.

After spending a short time as tutor in a family at Osterburg, Winckelmann in 1741 entered Jena to study medicine. Here he soon found that he had as little aptitude for medicine as he had for theology; his private work of tutoring left him little leisure either for his new studies or for his beloved Greek. The next spring he left Jena and became tutor to the sons of a high Prussian official named Lamprecht living near Magdeburg. Here he met von Hansen, a former secretary of the Danish ambassador to Paris, whose library, rich in modern literatures, was hospitably thrown open to him. From these books the young tutor became acquainted with the French sceptical movement, especially with the *Historical and Critical Dictionary* of Pierre Bayle. After a year and a half Winckelmann received a call as con-rector and teacher of Hebrew, logic and geometry at the gymnasium at Seehausen.

The five years which Winckelmann spent at Seehausen were the dreariest of his life. He always looked upon them in after years as a martyrdom. In one of his later letters we read: "I have enacted the schoolmaster with great fidelity and taught children with scabby heads their a b c's, while during this pastime I was ardently longing to attain to a knowledge of the beautiful and was repeating similes from Homer....At that time I was constantly saying to myself what I still say at the present time:

τέτλαϑι δή, κραδίη, καὶ κύντερον ἄλλο ποτ' ἔτλης."[4]

No one in Seehausen could doubt his ability or skill as teacher; but a man whose head was full of such lofty ideas must necessarily have presided over his classes in an indifferent manner. His predecessor Boysen had been a veritable Orbilius and Winckelmann found his pupils de-

[4] From the Odyssey, Bk. 20, l. 18: "Endure, my heart; yea, a baser thing thou once didst bear" (Butcher and Lang).

ficient in taste and far more interested in facts than in sentiment. Boysen, who had become a preacher in Magdeburg, wrote that he could say without self-praise that he "had done incomparably more for literature and the sciences in the year and a half that he had acted as assistant rector than was done in five years by his successor." After his day's work in the schoolroom, Winckelmann had to spend the early evening tutoring Lamprecht's son whom he had brought along with him, and he was only free to do his own reading after ten o'clock. He spent the greater part of the night reading Homer, Sophocles, Herodotus, Xenophon and Plato. He ordinarily retired at midnight but arose at four the next morning to read until six, when his school duties began again. It is said that for a whole year he never undressed to go to bed, but slept in his chair. He literally followed the poet's advice:

> "vos exemplaria graeca
> nocturna versate manu, versate diurna."

Apart from his Greek authors he also read largely in modern literatures. He even found pleasure in Voltaire's artificial classicism; the subtle Frenchman, whose superficial taste Winckelmann was one day to supplant by the clear ring of the genuine ancient spirit, at least gave him a love for French letters, which contrasted with his contempt for German books. We must remember that Goethe was not yet and that there was nothing in German literature which could have anticipated his *Iphigenie*.

In teaching Greek Winckelmann had little in the way of texts. There were in Germany at this time only a few Italian and Dutch texts of the classics and about the only Greek books for class-room use were the selections of Borst and Gesner. Not satisfied with such ἀποσπάσματα Winckelmann made handwritten copies of commentators and scholiasts. Some of these manuscripts, beautifully written, are still in existence. He planned with a fellow teacher to publish a collection of classical authors and actually annotated parts of Sophocles as well as Juvenal and Persius, which were never published. He was mercifully saved for something higher than the editing of text-books.

In addition to his poverty—he received only two hundred and fifty thalers a year—overwork and school duties which he hated, Winckelmann also got into trouble at Seehausen with the rector. As assistant rector he was unable to hold chapel himself and so was obliged to listen to the preaching of his superior. Instead of listening to the service Winckelmann would read his Homer in church and was also untactful in expressing his contempt of his colleague's abilities. His remarks naturally reached the ears of the rector, who in retaliation denounced Winckelmann's knowledge of Latin. However there is a letter in existence whose Latinity at least is above reproach, in which years after Winckelmann expressed his contempt of his superior in language which would have done justice to Martial. Among other things he wrote: "I still remember the looks with which I was insulted by a man lighter than the shadow of a cork-tree, and, of all bipeds, the most worthy to be the wiper of Silenus, the most stupid of the gods."[5] In the year 1747 his unhappiness reached a climax: as the poet says,

> "When sorrows come, they come not single spies
> But in battalions."

His school work oppressed him; he had little time for his own studies; his pupils were stupid; the attitude of the rector had become unbearable; he became lonely and melancholy, and his tiny income scarcely met his simple wants; on top of all his mother, to whom he was passionately attached, died. He longed for a change, but did not know

[5] In a letter to Kleinow: "....Haerent infixi pectore vultus, quibus nobis insultavit homo umbra suberis levior, et omnium bipedum dignissimus, qui Sileno, stupidissimo Deorum, a clunibus sit." (Translation of G. Henry Lodge, *Winckelmann's History of Ancient Art,* I, p. 12.)

where to turn. He knew he was unfitted for the Church, for law or medicine; school teaching had become utterly loathsome to him . Only the literature of art pleased him, and he longed to leave Germany and visit the countries of the classical tradition. As he wrote at this time: "It is my misfortune that I was not born to great place, where I might have enjoyed cultivation and the opportunity of following my instinct and forming myself." But he was thirty years old and had not received as yet a single favor of fortune. In 1748 he wrote Count Bünau of Nötheniz near Dresden—the first German historian and the author of a History of the Holy Roman Empire—"for a corner in his library." In his letter he hinted at his unhappy position "in a metaphysical age by which humane literature is trampled under foot," and continued: "Nowadays little value is set on Greek literature, to which I have devoted myself so far as I could when good books are scarce and expensive." Soon afterward we find him ensconced in Bünau's library of over forty thousand volumes, lodged and paid from fifty to eighty thalers a year. He had finally found congenial work.

During the six years which he spent at Nötheniz he made frequent visits to the collection of antiquities at Dresden nearby. Hitherto he had only known the words of Greek poetry; now for the first time he was in the presence of the visible remains of Greek culture. In Dresden he got acquainted with many artists, especially with Oeser, Goethe's frend and teacher, whose culture and knowledge of art were of great assistance to Winckelmann. Through Oeser's influence he finally moved to Dresden where he spent the year 1754-5—the most important and decisive in his life. Here in the Saxon capital he felt at home: for though born in Prussia, Winckelmann was no Prussian: his gentle nature rebelled against the Spartan military discipline and the police system of that despotic land, and he was fond of boasting that his fatherland was Saxony and that no drop of Prussian blood flowed in his veins.

Dresden at this time was the most cultivated city of Germany. During the reign of the splendor-loving August the Strong (1694-1733) and that of his successor, the art virtuoso August III, the city had become greatly embellished and had reached a prominent place as a cradle of art. August the Strong had made the grand tour and had become captivated by the spirit of the reign of Louis XIV. On his return he had his architect Pöppelmann begin the erection of the Zwinger, the original plan being to make this building the center of a grand architectural display. It recalls the palatial French edifices which had been built as monuments to glorify the reign of the Grand Monarch. The age of Louis had been fond of comparing itself with the Golden Age of Rome; so the Zwinger was intended to embrace the most sumptuous features of Roman palaces, baths and pleasure buildings. The purpose of Rococo art, which we see in part worked out in this building, was to invest even the domestic life of princes with pomp and state, to show to the people the royal cabinet and private office. The Dresden opera and theater were also French; sculpture, however, was here as elsewhere in Europe dominated by the Italian taste of Bernini. The collection of paintings had been founded in 1722, while that of sculpture, mostly formed from the Chigi and Albani collections of Rome, had started with the Brandenburg collection in 1723. These were the only art collections of any importance in Germany. The Sistine Madonna had been brought to Dresden in 1753 just before Winckelmann left for Italy. The art treasures of the city were so rich that the sculptor Cavaceppi, the fellow traveler of Winckelmann on his last journey, could flatteringly say that Dresden might strive for first place with the Capitoline collection at Rome. A colony of foreign artists lived here, as also several native ones of note. In short, as Winckelmann said, "Dresden is becoming ever more the Athens of artists"—a sentiment echoed some years later by Herder, who called Dresden the German Florence. The seven years which Winckelmann spent in and near Dresden were indeed happy years. These were the years from the end of the War of the Austrian Succession to the outbreak of the Seven Years War in 1756 and were the most peaceful which Europe had seen for a long period. During these years, as Voltaire says in his *Le siècle de Louis XIV*: "Industry bloomed from Petersburg to Cadiz; the fine arts were everywhere in honor; all nations had intercourse with one another; Europe was like a big family which had become united after its troubled days." Nowhere were the fruits of peace better to be enjoyed than in Dresden, which at this time had the most illustrious court in Europe.

Winckelmann was already past his thirty-seventh year and the world as yet had seen no public proof of his ability and learning. He had begun to ask himself with Juvenal:

"Semper ego auditor tantum?"

In 1755, the year that he left Dresden forever, he published the first of the three great works by which he is remembered, his *Thoughts on the Imitation of Greek Works in Painting and Sculpture*. This was followed immediately by a pretended attack and then by a defence of its principles. Winckelmann had studied the simplicity and repose of Raphael's great Madonna and found that the same elements were also characteristic of Greek art. In this first book was the kernel of his fundamental view of art, words which were soon to be memorable in later essays and in his great History of Art: "One must imitate the Greeks and not nature only; for the Greeks knew the secret." This secret was that art should be characterized by "noble simplicity" and "calm grandeur." As sculpture was the chief product of Greek art, he discussed it most and showed why it was superior to modern sculpture, which was dominated by Bernini's theatrical taste and characterized by strange and uncustomary poses and treatment. The book, though full of obscurities, reached its purpose—the direct appeal from the artificial classicism of the day to the study of the ancients. It was enthusiastically received; every one was amazed at the author's boldness in assailing the prevailing taste. Lessing got from it the inspiration for his *Laocoon*, the book which Macaulay said "filled him with wonder and despair," in which the author analized the boundaries of poetry and the plastic arts, and enunciated the principle that each art was subject to very definite conditions and could only attain its end by limiting itself to its own function. Winckelmann in a few months was recognized as belonging to the first rank of German writers. It was the turning point in his life. Not only do misfortunes come in battalions, but also, even if more seldom, fortune's favor. By this book he achieved not only celebrity, but, best of all, the opportunity to go to Italy. His Dresden sojourn had filled him with an overwhelming desire to see Rome. For here in Dresden it had become clear to him that art was the main interest of his life and that Italy was the only place in which properly to continue its study. His success was all the sweeter because it was unexpected and in such contrast with his earlier years of struggle. From now on we have a different Winckelmann. His *Lehrjahre* are now over: art has become his religion and now that he has attained his freedom and maturity he appears to us, as Goethe said, "consummate, entire, complete in the ancient sense."

As the Saxon court was Catholic the only road to favor at Dresden was through the Roman ecclesiastics. Back in 1751 Archinto, the Papal Nuncio at Dresden, had visited Nötheniz and had suggested to Winckelmann, who had acted as his guide through the library, that Rome best suited his health and temperament. He had then held out to him the hope of a place in the Papal library and had told him how Cardinal Passionei, an ardent student of Greek, had been pleased with his beautiful Greek handwriting and would be ready to play the Maecenas if only he would accede to the indispensable condition of joining the Roman Church. The bribe was finally accepted and Winckelmann, after a great deal of hesitation, became a Catholic in 1754. Goethe explains this conversion by pleading that Winckelmann was a pagan spirit to whom Christianity was nothing. That Winckelmann had no intention of deception by the disguise is shown by the fact that he had a book by Voltaire in his pocket when searched at the Roman custom house, and that later during his residence in Rome he lived in constant fear of an inquisition. He gives his own version of the affair in a letter to a friend: "It is a love of knowledge, and that alone which can induce me to listen to the proposal made me." In 1760 he had an opportunity of holding a fat office in Vienna if he would only take the tonsure. At that time he answered: "I was born free and I will die free." Doubtless the fact that the Roman Church was in so many ways bound up with pagan grandeur had made this superficial change of heart easy. In any case his religious sentiments were all merged in those of art. As for his embracing Roman Catholicism he would have turned Mohammedan with equal ease if he could have gained thereby a good chance to study antique marbles. On reaching Rome he was mercifully excused from kissing the pontiff's foot, and Benedict XIV assured him of his continued favor. Dresden had proven to be the gateway to Italy. The Elector of Saxony, pleased with his book, promised him a pension of two hundred and fifty thalers, and in September 1755 he started for Italy.

For the next thirteen years of his life Winckelmann devoted himself entirely to the study of art and archeology. It was fortunate that he, like Goethe, had come to Italy in full maturity of mind. The effect of Rome on the poor German scholar was immense. Everything about the Eternal City pleased him—its free artist life, its antiquities, libraries, language, climate and above all the spell of the past. Here there was no bureaucracy, no military, no police. Everything in the congenial atmosphere of the city with its Hellenic affinities made a truly artistic ensemble for him. From a long familiarity with ancient literature his mind had acquired an antique cast; when he reached the Niobe of nations and viewed its ruins and art treasures, he felt as if he belonged not to the present but to the past. He said in the fulness of his rapture: "All is nothing compared with Rome! Formerly I thought I had thoroughly studied everything and behold, when I arrived here, I found I knew nothing." In a letter three years later to a friend in Dresden he says: "In Rome, I believe, is the university for all the world, and I have been purified and tried in it." He also felt that he was in a sense out of place, for he wrote: "I am one of those whom the Greeks called ὀψιμαθεῖς—I have come into the world and into Italy too late." He was pleased with the cordial reception which he received from the Cardinals Passionei and Albani; their democratic ways contrasted strangely with the *hauteur* of Germans of high position, for he was immediately invited to drive and walk with them on terms of the greatest intimacy. His life was one of the utmost simplicity; at first he lived in the artists' quarter; he never went to the theater nor the opera, but went early to bed where he slept undisturbed by the street noises which at this time were worse than in the days of Juvenal. His delicate constitution only allowed him the simplest fare—generally only bread and wine, though he drank the latter neat like a German. After remaining four years in Rome Winckelmann lodged in the palace of Cardinal Albani, living on his pension from the Saxon prince and another of about $120 from the cardinal. Four years later, in 1763, he was appointed to the high-sounding office of *Commissario della Antichità della camera apostolica,* with oversight over all the antiquities in and near Rome, at a salary of about $180, his pension from Dresden by then having been stopped. In the same year he was also given a clerkship in Hebrew in the Papal library at a salary of $50, a position which entailed practically no work, but confined him from eight to twelve hours a day. Thus his total income in Rome was never over $350, though this amount was enough in those days for a quiet scholar.

At this time Rome was the center of classical studies. The collections of the Louvre, the Glyptothek and the British Museum were not yet in existence. There was little of importance in Berlin or in any German city outside of Dresden. The sculptures of the Uffizi in Florence, mostly from the Roman Villa Medici, were not set up until the end of the century when also the Farnese collection was taken to Naples. In the latter city the finds from the buried towns of Pompeii and Herculaneum had not yet been made public. So Rome was the only place in which to properly study ancient art. It was the day before travels, excavations, reviews and books on art. The only cast collection in existence was the small one gathered in Rome by Raphael Mengs. It is doubtful if Lessing ever saw a copy of the Laocoon when he wrote his famous essay. Bonn University was the first to have a collection of casts, which was made in the early part of the nineteenth century; now not only all the German universities, but many of those in the United States have them. Even in Rome there were no public museums. Only three of the five great Roman collections of the present day existed in Winckelmann's time—those of the Villas Albani, Borghese and Ludovisi, and these were all under private ownership. The Medici collection was moved to Florence a little later; the present Capitoline collection was originally in the Villa Albani and after its sale the present Villa Albani collection was begun; the present Vatican museum of antiquities, now the finest in the world, had as its beginning in Winckelmann's day the statues of the Belvedere collection, which had been begun in the sixteenth century by Pope Julius II and was named from the garden house in the Vatican grounds where they were exposed down to the beginning of the eighteenth century. Only a few, however, of the present masterpieces, like the Hercules Torso, the Apollo and the Laocoon, date their appearance in the Vatican from that period. The Museo Pio-Clementino was started at the end of the eighteenth century, while the Chiaramonti and the Braccio Nuovo had their beginnings in the nineteenth.

It was in the year 1758 that Winckelmann made his first visit to Naples to visit the sculptures there and to view the recently opened excavations in the neighborhood. At Resina, on the site of Herculaneum, the theater had already been laid bare and at Pompeii a portion of the amphitheater and the eastern end of the town had been excavated. He stayed in and around Naples for two months, enjoying everything he saw and did, even the eating of enormous cauliflowers and the drinking of Lacrima Christi. He also went on to the site of Paestum, which at that time was merely a malaria-stricken wilderness containing a few shepherds' huts. Here he saw the first Greek temples. Their existence until a short time before had been a secret even to artists and antiquaries. Macaulay, in describing Addison's visit to these ruins at the end of the seventeenth century, graphically writes: "Though situated within a few hours journey of a great capital where Salvator had not long before painted and where Vico was then lecturing, these noble ruins were as little known to Europe as the ruined cities overgrown by the forests of Yucatan." Winckelmann made in all three more visits to Naples in the years 1762, 1764 and 1767. It was in 1760 that the statue of Diana had been discovered inside a little temple at Pompeii, the first example of an ancient sulpture which retained traces of color. As the fruit of his second and third visits Winckelmann gave to the world in two letters the first authentic information about the excavations at Pompeii and Herculaneum.[6] During his last visit, he, like the elder Pliny, was able to witness a great eruption of Vesuvius. Accompanied by von Riedesel, he went to Portici, whence the party walked out over the ancient lava beds to the new and was compelled, in order to reach the crater's mouth, to pass over hot lava which scorched the soles of their shoes. He had also planned during his first journey south to make a tour of Calabria and Sicily. A journey to Southern Italy, however, was no easy task. The conditions of travel were barbarous; the roads were nearly impassable and were beset by theives and cutthroats. In the Kingdom of Naples one could only go on foot or on horseback and had to be accompanied from place to place by a soldier. If one had no servants and no letters of introduction to landed proprietors along the way, he had to put up with the food of an anchorite and to sleep on pallets no strangers to vermin. In a letter Winckelmann recounts how his journey to Paestum in 1758 was filled with a hundred annoyances. Of the danger of brigands he says: "One must go with two pistols in his sack, two in his girdle, and with a good claymore at his side and a gun on his shoulder." Despite the ridiculous figure the poor scholar must have cut in such a panoply, he says he bought all these necessaries in Naples. The ignorance

[6] *Sendschreiben von den herculanischen Entdeckungen* (1762) and *Nachricht von den neuesten herculanischen Entdeckungen* (1764).

which prevailed among educated men of that day about Calabria is shown by Winckelmann's belief that there were ruined temples there. He only gave up his intention of visiting them when he learned from the English noble Brudnell, who had just returned from a journey along the coast as far as Taranto, that, outside the temple of Juno at Croton, there were no ruins to be seen. In his last journey to Naples he also again seriously had in mind a trip to Sicily to visit the Doric architectural ruins there. His enthusiasm had been fired by the descriptive letters written by Riedesel, who was the first scholar to make the island known to lovers of art. Goethe, years later when in Girgenti (1787), spoke of Riedesel's little volume, which he says he carried about with him "in his bosom like a breviary or talisman." At the beginning of 1760 Winckelmann also seriously considered a trip to Greece with Lady Oxford. He then wrote: "Nothing in the world have I so ardently desired as this; willingly would I allow one of my fingers to be cut off, indeed I would make myself a priest of Cybele, could I see that land." Again in 1768, the year of his death, he was invited to accompany von Riedesel to Greece. But he was destined never to see either Sicily or Greece. New vistas of travels and plans for work were constantly being opened up to his mind; the infinity of possibilities made him sadly reflect—

> "Ach, das Leben ist am Ziele
> Und die Kunst noch kaum begonnen."[7]

During his first visit to Naples in 1758 Winckelmann had been recalled to Rome by the last illness of the Pope and immediately after his death went to Florence, which he described as "the most beautiful place I have ever seen and far superior to Naples," and as "the true cradle of the Italian art spirit." While here he studied the art treasures of the city and worked assiduously on a catalogue of the

[7] "Alas that life has reached its goal
And art is scarce begun."

great collection of gems owned by the Prussian Baron von Stosch, who resided there. He says that he never before had worked so hard; for six months he only allowed himself a half-hour's relaxation in the evening. He had to complete the catalogue in Rome the following year, where he could avail himself of the study of the gems in the Museo Kircheriano and of the advice of connoiseurs in that field. The work finally appeared in 1760 under the title *Descriptions des pièrres gravées du feu Baron de Stosch*—his first scientific work. It was while he was in Florence that his old friend Archinto, the secretary of Cardinal Albani, died, and he was summoned to Rome to become the librarian and companion of the aged prelate. It was after this that he wrote many essays on various phases of the subject of art and antiquities; he also carefully studied the descriptions of monuments in Pausanias and the conception of the *Beautiful* in Plato. Many of these minor writings—like those on the *Apollo Belvedere* and on *Grace in Works of Art* and the study of *The Capability of the Beautiful in Sculpture*—are among the most beautiful from his pen. But the results of all his studies and writings finally culminated in his greatest work *Die Geschichte der Kunst des Altertums.* It will be convenient at this point in recounting the chief events in Winckelmann's career to briefly bring together what relates to the origin and fate of this work and also of his last book the *Monumenti antichi inediti.*

Winckelmann had had the plan of writing the *History of Ancient Art* in mind ever since his second year in Rome. He continually visited the treasures of the Belvedere to arouse his spirit, and from these visits grew his desire to write such a book. He looked upon all his preceding reading and essays as merely preparatory to this work, which for years robbed him of most of his time. He was long in doubt in what language such a history should be written. Cardinal Albani suggested Italian on the theory that *dum vivis Romae, Romano vivito more.* But Winckelmann finally decided on his mother tongue. The first draft of the work had been sent to Dresden for publication in 1758, but, in consequence of delay, it had been withdrawn the following year. This proved to be a fortunate circumstance, for it allowed the author to recast it and to produce an almost entirely new work. This revision extended over the years 1758-61, and in 1762 he again looked about for a printer. The work finally appeared in Dresden in two volumes quarto in the year 1764 and was dedicated to Friedrich Christian, the Elector of Saxony, who had succeeded August III, Winckelmann's patron, the year before. The size of the edition made it impossible to bring out a new edition for some time, so that the author had to content himself with collecting emendations and additions for a second work entitled *Anmerkungen über die Geschichte der Kunst,* which appeared in 1767. Just before his death he had begun to recast the material for a second edition which was to appear in French; but fate was against him. On his last journey he carried the manuscript for this edition with him, and the very last words that he penned while in Trieste, where he was murdered, were in reference to it. After his death the manuscript was sent to Vienna, where it was published with great negligence. From this publication came the Italian edition and another French translation.

Winckelmann's *History of Ancient Art* is the earliest work in which the origin and development of sculpture and painting in Egypt, Phœnicia, Persia, Etruria, Greece and Rome is systematically presented in connection with the general progress of culture. Following the custom of French writers on art, he wrote an art history in general, but one of Greece in particular. He recognized that art was but one phase of the history of mankind, though it was the flower of national life and evolution. Not content with merely presenting the beautiful monuments of art, he investigated the sources of beauty in Greece and the reason why Greek art still commands the world's admiration. In unfolding the theory of the *Beautiful* he finds that "the highest purpose and the central point of art" is beauty rather than instruction. This thought was to dominate artists, critics and poets for the next two generations. Ideal beauty can only be attained when individual features are subordinated to the general scheme in the mind of the artist. The artist selects his theme from the natural world and combines it with his imagination, thereby creating an ideal type marked by the two characteristics of "noble simplicity" and "calm greatness" or "repose." All details, like muscles and veins, must not be allowed to impair the harmony and the proportions. He discerned for the first time what is now a commonplace of knowledge that "beauty has been esteemed by no people so highly as by the Greeks." The beauty of the youth of life was so extolled by the Greeks that Aristobulus in Xenophon's *Symposium* is made to say: "I swear by all the gods that I would not choose the power of the Persian king in preference to beauty." This Greek ideal of beauty is nowhere so preeminent as in sculpture, where it is especially associated with youth, for in youth more than in manhood the artist finds the causes of beauty," in unity, variety and harmony"; "the forms of beautiful youth resemble the unity of the surface of the sea, which at a distance appears smooth and still like a mirror, although it is constantly in movement with its heaving swell."

In writing his History Winckelmann used everything—monuments and books both ancient and modern. His own artistic sense, helped on by vast erudition and by a vigorous imagination, enabled him to make remarkably true suggestions about periods of Greek art where little real information then existed. He overthrew many of the older interpretations of monuments, which had been based on the false theory of the Roman origin of ancient art. Thus he found that the portrait busts in Italian collections were far too realistic to be Greek, too much out of harmony with Greek ideality. He was the first to divide Greek art into epochs, indicating the sequence of styles corresponding to changes in society and politics. These divisions are still kept in our histories of art; the archaic style (*älterer Stil*); the grand style (*hoher Stil*) of the age of Phidias, characterized by grandeur, beauty and truth to nature; the beautiful style (*schöner Stil*), beginning with Praxiteles and characterized by elegance and grace; and lastly the style of the imitators, when the old ideals of simplicity were lost and a pretentious and decadent taste came in. At the end of the work he devotes a few pages to Roman art, a period in which all originality had been lost and art was devoted to the repetition of earlier types.

There was really no one in 1764 who was able to criticise adequately this work. The few who knew Greek literature knew nothing of the monuments and those in Rome who were acquainted with the latter knew little of Greek letters or history. The work was nothing short of a revelation to his contemporaries and it profoundly influenced the best minds everywhere. It was praised by learned societies and scholars for its flowing style, its erudition, its sane judgments, its insight and its sense of beauty and proportion. It was soon recognized as a permanent contribution to European science and *belles lettres.* Lessing received a copy while still at work on his *Laocoon,* and was unbounded in his praise; the contemporary French sculptor Falconet said he had "read nothing better on the beautiful in art"; Diderot was more guarded, for while praising the author's enthusiasm, he felt the application of his ideas to sculpture was wrong, since he did not agree with Winckelmann's fundamental notion of art, that it should imitate the antique rather than nature. The Italian architect Visconti and the Frenchman Quatremère de Quincy years later found nothing but praise for it; Madame de Staël, in her *Allemagne,* said that it was Winckelmann who "brought about an entire revolution in the manner of considering the arts" and that he had "banished from the fine arts of Europe the mixture of ancient and modern taste" and that "no one before him had united such exact and profound observation with admiration so animated."[8] Winckelmann's Roman contemporaries, like Raphael Mengs, were unbounded in their approval. Heyne in Göttingen some years later wrote a eulogy of the author, though he, like Diderot, tempered his praise with real criticism. Heyne had written on Pliny's art epochs and was surprised that Winckelmann had made so little use of that author; but the author of the *History of Ancient Art* knew that Pliny was no evangelist in matters of art. Heyne called attention to the weakness of the work—its uncritical statements and inaccuracies, though he was wrong to conclude that the historical part was therefore "practically useless."

The *History of Ancient Art* was a masterpiece of German prose; though primarily a scientific work, it possessed all the grace, rhythm and dignity which we expect in a work of pure literature. With Lessing's Essay it may be said to be the beginning of modern German prose. These two writers brought German literature into line with the world literatures and by opening to the Germans the empire of beauty brought a plastic element into their poetry. Winckelmann confessed that he had followed the dictum of Roscommon that the "greatest masterpiece of everything in which mankind has been distinguished is good writing." The style, always original, is at times grand as when treating of the essence of beauty and in certain descriptions

[8] See her eulogy, *Allemagne,* Part II, Chap. VI (transl. by O. W. Wright).

it actually soars. Of his eloquence in describing the Apollo Belvedere and the Laocoon, Madame de Staël found his style as "calm and majestic as the object of his consideration." But its style and poetic beauty are its least important features. It instituted the historic study of art and indicated the methods by which that subject must be approached; greatest of all, it overthrew the false taste of the day and for the first time scientifically showed the existence of an independent Greek art.

In criticising the contents of this work to-day we must bear in mind that it entered an almost new field of criticism and therefore was influenced but little by anything which had preceded it; furthermore we must remember that it was composed at a time when but few monuments of the great period of Greek art were known. In the preface Winckelmann mentions the now forgotten works of the painter Monier, of Durand and of Turnbull. The best preceding work on ancient painting, that of Franz Junius, which had appeared well over a century before (in 1637) and remained the source for the study of Greek art all that time, he does not mention. This work was, however, more philological and philosophical than historical in character and had been written by a man who had lived most of his life in England and who had never seen Italy. The Frenchman Goguet published in 1758 a work on the *Origin of the Laws of the Arts and Sciences*, "one of the best books of our times," as Winckelmann termed it; but this work was anthropological and historical in character rather then esthetic. The most exact and learned work on Greek sculpture was a part of the recent *Recueil d'antiquités* of the Compte de Caylus,[9] who had traveled extensively in Italy, Greece and the East. As for the monuments of Greek art then available to Winckelmann in Italy,

[9] *Recueil d'antiquités égyptiennes, étrusques, grecques, romaines et gauloises,* 6 vols. Paris, 1752-5.

it may be said that the most important examples now known to us had not yet been discovered. Of the archaic period there were but few significant works and almost none of the time of Phidias and little of the fourth century B. C. Still Winckelmann's treatment of the "grand" and "beautiful" styles of the fifth and fourth centuries are for all times and all peoples.

Many of his historical conclusions about art are, of course, mistaken. Thus his idea that the Greeks first worked in clay and then in wood, ivory, stone and bronze successively must be given up, as well as his idea that the Greek sculptor first used unwrought cornered blocks, which were subsequently rounded and then fashioned into herms by placing heads at one end and later differentiated by sex, followed by sculpturing the upper part of the body and then the lower, until at last Daedalian statues with the legs separated were evolved. He was quite as mistaken in his contention that Greek art was independent in origin, quite uninfluenced by the art of Egypt and the East, the Greeks "not deriving the first seeds from elsewhere," but "appearing to have been original discoverers." We must also remember that Winckelmann had to reach Greek art largely through Roman copies and imitations; consequently many of his conclusions are inadequate in their basis and have been either completely overthrown or largely modified by subsequent discoveries. Thus no one to-day would echo his excessive praise of such monuments of sculpture as the Laocoon, the Hercules Torso or the Apollo of the Belvedere. He assigns the Laocoon to Alexander's time, but concludes that posterity has been "unable to produce anything worthy of being compared with it even remotely." The torso he places in the age of Alexander's immediate successors and looks upon it as the "lofty ideal of a body elevated above nature"; the Apollo belongs to the imperial times after Nero's reign and is "the highest ideal of art among all the works of antiquity which have escaped destruction," and the effect produced on him by its aspect is "indescribable." He even explains the lack of veins in both the Torso and the Apollo as a sign of their "heavenly essence." The excellencies which he saw in these and similar works we can now see in far less contaminated purity in many monuments which were unknown in his day, and consequently we judge them from a very different standard when we compare them with genuine products of Greek art of the great period instead of with those of the decadent epoch. If Winckelmann had seen such beautiful statues at those of the Hermes or the Melian Aphrodite, the pride of the Louvre and by many looked upon as the most beautiful of all sculptures, their "noble simplicity" and "calm greatness" would have called forth the encomium which, in the absence of such noble works, he gave to decadent pieces. In that case he doubtless would have seen in the Belvedere torso not a resting Hercules at all, but perhaps merely a Cyclops—as Sauer maintains— who is holding up his hand to shade his eyes as he looks out over the sea to get a glance of his beloved Galatea, and so this piece, with all its fine modelling, would fall into place among Pergamene works of the Hellenistic period; its lack of veins, then, could not be explained by an attempt to deify a hero or to etherealize his body. Lessing, instead of using the Laocoon as an example of what sculpture should not attempt—for it not only groans as he and Winckelmann said, but shrieks—uses it to illustrate the difference between the principles of poetry and sculpture. If either had seen the masterpieces of sculpture from the Elgin marbles down, they would have judged it very differently and seen that it, like the Laocoon, belongs to Pergamene art, as an extreme example of the tendencies of that art toward dramatic power and exaggerated pathos. In the case of the Apollo, however, we must admire Winckelmann's insight; for many modern critics believe it is a copy of an original bronze dating, perhaps, from the fourth century B. C.; its original has even been assigned to Leochares, who worked with the great Scopas on the Mausoleum.

To one so imbued with the Greek essence of beauty, it is not strange that Winckelmann denounced the fantastic and exaggerated conceits and affectations of modern art and in opposition fearlessly preached his admiration of the purity, naturalness and simplicity of ancient works. But it was just this insistence on Greek ideals that at times led him into wrong appraisals of certain modern artists. Thus his wholesale condemnation of the greatest of the Renaissance sculptors seems to us not only harsh but wholly unjust, even if we try to excuse it by the fact that he doesn't judge him from the point of view of modern artists but from that of the ancients. In a classic simile he says Michelangelo compared with Raphael is what Thucydides is compared with Xenophon. To him the supreme interpreter of the Old Testament, the immortal artists of Sibyls and Prophets, by his striving after the difficult and extraordinary, and his "studious employment of scientific knowledge," is merely the originator and promoter of the corruption of taste, which culminated in the theatrical motives and strained attitudes of Bernini's art. While admitting that he "contemplated lofty beauty," he finds this feature in his poetry rather than in his sculpture and painting. His Christ heads are "mean and vulgar" and "borrowed from the barbarous works of the Middle Ages." The youthful beardless heads of Christ painted by Raphael and Annibale Caracci, as well as the bearded Christ of Leonardo, he found far more noble. Winckelmann's insistence on Greek ideals led him to affirm that subjects drawn from the Christian religion were not favorable to art, and consequently he endeavored to arouse in the artist enthusiasm for classical mythology. Thus he said that artists should copy their Saviours from the models of Greek heroes and their Holy Virgins from Amazon heads, not perceiving that any such slavish tendency would mean the deathknell to all independence and progress. In violent contrast to his disapproval of Michelangelo, he lauds his friend Raphael Mengs as "the most accomplished instructor in his art," and speaks of the "immortal works" of him who had "reached the highest point of excellence to which the genius of men has ever risen." He ends his panegyric of Mengs by calling him "the German Raphael." We are reminded that a German admirer of the author of Paradise Lost called Klopstock "the German Milton," and that Coleridge sneeringly rejoined "a very German Milton, indeed!"

Winckelmann's second great work was written in Italian, the splendidly illustrated *Monumenti*. On his forty-fourth birthday in 1761 he announced this work which finally appeared six years later. This "classic work," as Visconti called it, was chiefly intended for Italian scholars and lovers of art and not, like the *History of Ancient Art,* for the general reader. It was more the fruit of Winckelmann's Italian sojourn than any of his other works, a fitting tribute of the author to his adopted land. Casanova furnished the drawings for the more than two hundred copper plates and vignettes, which were mainly taken from sarcophagi reliefs; the expense of draftsmen, engravers and printing were all borne by the author. The plates, selected from unpublished monuments, were accompanied by explanations of mythology, customs and history. Winckelmann spent much time and energy on this monumental work. He says in a letter: "It is known to God and myself how I have sweated over it. There are pieces in it over each one of which I have sat for five months." In recent years it has been objected that the work was overloaded with unnecessary learning after the Italian fashion, on the assumption that the author wished to make a display of his erudition among his Italian contemporaries. In any case it is an invaluable work and shows the same original and independent style which we see in all his writings.

We now come to the last scene in Winckelmann's life—his untimely end. On leaving Germany for Italy thirteen years before, he had had no intention of remaining there permanently. But soon after he left Dresden the Seven Years War broke out and Saxony, especially the capital city, suffered terribly. He was fortunate to receive his pension at first; but after two years it was cut in two, and in 1763, on the death of his patron August II, it was withdrawn entirely. In the winter of 1767-8 on returning from his last visit to Naples, he was hard at work on the revision of his History, for he intended in the spring, in conformity with a plan which he had long had in mind, to revisit Germany and especially Berlin, where he was to see a French edition of his work through the press. The recent invitation extended to him by Frederick the Great, to come there and take charge of the royal collection of antiquities, was well known in Italy, and every attempt was made to dissuade him from going, as it was felt that he was the only man in Italy with a critical knowledge of Greek literature and art, and it was feared that he might never return if he again visited his native land. It was also just at this time that von Riedesel invited him to accompany him to Greece and the East, a journey which Winckelmann had longed for all his life. It was a far easier thing for him to get permission to go to Greece than to cross the Alps. It was hard for him not only to refuse Riedesel but to break the ties which bound him to his Roman friends, especially to the aged Cardinal Albani. Still the desire to see his old home finally decided him to go north. In his last letter to his old friend Franke at Nötheniz he fondly referred to the *Ruheort* where they were to meet; but he was destined to see neither him nor any of his other early friends.

He started north on April 10, accompanied by his friend the sculptor Cavaceppi, who has left us a description of a part of the journey. They traveled via Bologna, Venice and Verona, and all went well until they reached the Tyrolean Alps. It was here on his journey to Rome years before that the grandeur of the mountain scenery had delighted him so much that later he regarded this part of the journey south as the most agreeable; at that time he had written to his friend Berends: "I should fill my whole letter with things about the Tyrol, if I should attempt to describe the rapture into which I was thrown." Now all was changed; thirteen years in Italy lay between. He now looked upon the same nature with aversion, calling it a "shocking, horrible landscape," and he even found fault with the architecture of the picturesque thatched Alpine chalets. He told Cavaceppi he could not find words to express his feelings of aversion. His companion at first thought he was jesting. In a few days they reached Munich and finally Ratisbon, where Winckelmann came unalterably to the determination not to continue the journey, but to return at once to Italy. Though he recognized that Cavaceppi's remonstrance was just and that he was leaving him in a country whose customs and language he did not understand, he answered that he felt "an overpowering impulse within him which he could not withstand," and immediately wrote Cardinal Albani his intention of returning. Only with the greatest difficulty was he prevailed upon to return by the longer route via Vienna. On reaching the Austrian capital Winckelmann was received with great honors, and Prince Kaunitz tried to persuade him to renounce his determination. His emotion grew so great that he lay sick of a fever for days and finally Cavaceppi gave up hope of dissuading him and left. On his recovery he had an audience with Maria Theresa at Schönbrunn and received from her and Kaunitz several gold and silver medallions as tokens of their regard. A promise was even exacted of him to return the next year to arrange the empress's cabinet. But Winckelmann was counting the days before he could go; he wrote the young Baron von Stosch in Florence that there was no pleasure left for him in this world outside Rome.

To many this sudden determination of Winckelmann to abruptly terminate his long-planned journey has seemed inexplicable. The circumstances of the last few months of his life explain it only in part. He was certainly worn out with his arduous work; two years before he had suffered from fainting fits, and had gone to Anzio for rest, and in March of the present year he had had a recurrence of the same malady, which, as he said, warned him "to bring his house into order." He suffered also from weak eyes and stomach. The fatigue of weeks of post-traveling through scenery which he no longer cared for aggravated the annoyance caused by suddenly breaking into the quiet of his Roman life. The contrast between the joyous Italian primavera and the bleak and lonely Tyrolean and Bavarian mountains brought on a Roman homesickness. Doubtless the memory of the hardships of his youth also came back to him as he approached his old home. But all these things together do not explain his feelings, for they could not have affected to such a degree a strong and healthy nature in the prime of life. However, it is not necessary to see anything mysterious in his decision, a kind of presentiment of evil which came to him in his weakened nervous and physical condition, even if many sentences in the letters of his last few years speak of his expected early death. This Italian homesickness is by no means an uncommon phenomenon. While to most of us the yearnings which draw us to the ancient world remain faint and remote, to

Winckelmann they were strong and insistent. As Madame de Staël says: "He felt himself attracted with ardor toward the South; we still find in German imagination some traces of that love of the sun, that weariness of the North (*cette fatigue du nord*), which formerly drew so many northern nations into the countries of the South. A fine sky awakens sentiments similar to the love we bear our country."[10] Zimmern, in his recent book on *The Greek Commonwealth,* has expressed a similar thought when he says that one must enter deeply into the spirit and life of the south before one realizes the difference in outlook. Even northern poets who have sung of the Southland have done so for the most part as visitors to whom the real spirit of the country has remained largely exotic, even if it arouses their enthusiasm. The gulf in most cases is not bridged by a lifetime: often a northern invader of Greece would finally return home because of homesickness. Many a Frankish baron of medieval Greece left his domain to go home and die by the Loire or Rhine. Thus Otto de la Roche, the first feudal lord of Attica, who "had the Acropolis for his castle and the Parthenon for his minster" left all in his old age and with his son returned to Burgundy to die. Just so Winckelmann swayed between the desire to see the land of his birth and to return to the land of his adoption. His real home was Italy and not the flat steppes of Germany; he was, to quote the words of Goethe, "of an ancient nature reappearing, so far as that is possible, among his contemporaries."

He reached the port of Trieste on June 1, whence he intended to take ship for Venice. The closing scene[11] of his life drama took place in the *Grosser Gasthof* on the Petersplatz. In the next room to his was lodging an Italian

[10] *Allemagne, loc. cit.*, (Wight's translation).

[11] See the little book by von Rosetti entitled *Johann Winckelmann's letzte Lebenswoche* (Dresden, 1815).

adventurer named Arcangeli, who was formerly a cook and who four years before had been sent to prison as a thief. This man had come to Trieste on foot and without luggage and was also awaiting a chance to return to Italy. The two men became companions at table and the Italian volunteered to aid Winckelmann in finding a ship. During the week of waiting the two were constantly thrown together at table, and Winckelmann asked the Italian to visit him in his room, and they also took walks together. It seems strange that such an intimacy could have grown up between scholar and peasant; but Winckelmann wanted to remain incognito and was glad to while away the tedium of the days that passed in talking his beloved Italian, and Arcangeli pressed the acquaintance for his own purpose. With characteristic frankness Winckelmann had shown him the medallions which he had brought from Vienna. The avarice of the Italian was at last aroused by these paltry souvenirs. The last morning while Winckelmann, without coat, cravat or wig, was seated at his table writing a letter, Arcangeli entered his room and the two spent a half hour walking up and down conversing. Winckelmann invited his companion to visit him in Rome and promised he would then disclose to him his identity and show him the palace in which he lived. His mysterious hints as to who he was aroused the suspicions of the Italian, who concluded that he was either a Jew or a Lutheran or perhaps a spy. After returning to his own room, he put a knife into his pocket and again entered Winckelmann's chamber on the plea of recovering his handkerchief. He then asked him again if he would show the medallions at the dinner table and, on Winckelmann's refusing once more, asked him why he was so reticent about his identity. Winckelmann, offended at his impertinence, did not answer, but reseated himself and began to write. Then Arcangeli quickly threw a noose over his head, dragged him to the ground and stabbed him five times in the chest and stomach. A servant, aroused by the uproar, rushed in and found the Italian over the prostrate body of Winckelmann, who was groaning deeply. The murderer forthwith ran hatless out into the street. Winckelmann lived for six hours, during which he dictated his will and received the last offices of the Church. In his traveling chest were found his favorite authors—Homer, Plautus and Martial. He was buried in the plot of a brotherhood in the churchyard of the cathedral of San Giusti. Later, when his remains were crowded by new arrivals, his bones were cast into the common charnel house. It is pleasing to know that the cowardly assassin was soon caught on the Italian frontier and was brought back to Trieste and tried, and six weeks later, on the same day and at the same hour in which he committed the murder and before the window of the hotel where it had occurred, suffered the punishment of Ixion.

Thus Winckelmann departed from life as poor as he had entered it. But behind him lay his brief, though glorious, life of struggle and service. A more fearful end can scarcely be imagined. The gods, however, were kind to him, for they brought him death near the border of the two countries to which he, half German, half Italian belonged. He was only fifty-one years old and therefore still in the prime of vigor. In the beautiful words of Goethe "he had the advantage of figuring in the memory of posterity as one eternally able and strong; for the image in which one leaves the world is that in which one moves among the shadows." Goethe, then a lad of nineteen, just leaving the University of Leipsic for Strasburg, was eagerly awaiting the promised opportunity of meeting the great Hellenist, when he received the tidings of his death. In a letter which he wrote years after in Rome (1786), in speaking of the emotion which he felt on reading some of the correspondence of Winckelmann which had come into his possession, he said: "How bravely and diligently did he not work his way through all difficulties; and what good does it not do me—the remembrance of such a man in such a place." Walter Pater calls it a calamity that the expected meeting of these two never took place, for thereby German literary history lost a famous friendship. Though a bust of Winckelmann was set up in the Roman Pantheon only four years after his death, no monument marked the place of his passing until fifty years had gone by, when a beautiful statue was erected in the square of Trieste. It was almost a century before his native Stendal set up a monument to its greatest citizen. In 1805 Goethe wrote his *Winckelmann und sein Jahrhundert,* the title of which rightly appraises the European position of this famous scholar; in 1865-72, a full century after his death, Karl Justi gave to the world the first accurate account of Winckelmann's short life. In these latter years he has received the full meed of honor which his abilities and influence have merited.

WALTER WOODBURN HYDE.
UNIVERSITY OF PENNSYLVANIA.

CRITICISMS AND DISCUSSIONS.

THE MECHANICAL EXPLANATION OF RELIGION.

There has been a common opinion in the past that what, in a broad sense, is known in philosophy as mechanical explanation—that is, explanation by antecedent cause,—is absolutely opposed to teleological explanation. According to this view, if some theory of a mechanical form is true, any teleological theory (concerning the same explicandum) must be false, and vice versa. This opinion is even still not uncommon, notwithstanding the philosophy of Leibniz and Kant's third critique. To me the contrary view appears to be correct, and the recognition of its truth to be very important, especially in the treatment of religious phenomena. A detailed argument in support of this contrary view has already appeared in this journal from the pen of its editor, Dr. Carus.[1] In the present article I propose to offer an analysis of the situation, from a somewhat different standpoint, in further support of this theory.

Since the time of Fichte, the dominant school of speculative thought has tended toward explanation that is teleological. With regard to mechanical explanation two attitudes have been adopted. It has been said, on the one hand, that mechanical explanation, accurately carried out, is true "so far as it goes," but that it is not the whole truth, and that in particular, it must be supplemented by teleological explanation. On the other hand, the position that mechanical explanation is *no* explanation seems to have been held not infrequently. Advocates of this view would probably admit that to many physical phenomena no teleological explanation can reasonably be assigned; and, in so far as they held that explanation

[1] Vol. XXIII, No. 2.

must be either mechanical or teleological, they would consequently have to admit that of such physical phenomena a mechanical explanation is the alternative to no explanation at all. They would be disinclined, nevertheless, to accept such explanation. And they would resent any attempt to explain mechanically something with regard to which they believed themselves possessed of a teleological explanation. This happens most frequently concerning things that are judged to be valuable, and perhaps most conspicuously in connection with religion.

Even those who admit that mechanical explanation is true "so far as it goes" often appear to share this resentment toward attempts to treat religion from the mechanical point of view. This is not because such attempts have always issued in explanations which were *mechanically* inadequate, even if this be true. Any thinker, even though he were to believe in the universality of mechanical explanation, would object to such explanation in so far as it was inaccurate. He might resent any general acceptance of the proposition that water under normal pressure boils at 211° F. And he might be willing to admit that the mechanical explanation of religion which sees its origin in the lust of priests and the tyranny of kings cannot be accepted as remotely probable. This, however, is not the attitude of the teleologist. He says, in effect, that such a phenomenon as religion, being of vital importance to man, must be explained by its function, not by its cause; and that its significance is destroyed by any theory which is in essence mechanical. He objects to "the Enlightenment" treatment of religion, not because it was inaccurate in detail, but because it was wrong in its *form*. And his position seems at first sight to be borne out by modern philosophical speculation.

What is usually regarded as the idealistic attitude toward mechanical explanation, when this is offered as ultimate philosophical hypothesis, seems now established. We obviously cannot explain the whole of existence by something, as it were, antecedent to it. The alternative is for mechanical theory to explain *part* of the whole by an antecedent and "ultimate" *part*. The objection which then finally emerges is that the explanatory reality,—whatever form it assumes,—remains *mechanically* inexplicable. Explanation has been obtained perhaps even at the cost of creating a new explicandum.

It seems to be sometimes assumed that the only conclusion to be drawn from this position is that philosophical explanation must be teleological; but the considerations which make mechanical explanation finally impossible, make teleology impossible also. We certainly cannot explain the whole of existence by an end that is its consequent,—which is somehow to come after it, and is, at any rate, additional to it. There is nothing after, just as there is nothing antecedent to, the whole of existence. The alternative here also is to explain (teleologically) one *part* of the whole by another *part*. But this is formally no better than the analogous situation with regard to mechanism. At least, this is so if "explaining a thing teleologically" be defined as "showing that it is a means to the realization of something else." For the teleologist's "end" is a final inexplicable just as is "the first cause" of the mechanist.

It of course might be held that we give a teleological explanation of anything if we show that it is *either* means or *end*; and upon such a view we have, in showing that one thing is means to another, given a teleological explanation of both things. But to such a theory the advocate of mechanical explanation could reply that he, in his turn, will hold that both the explicandum and its cause are (mechanically) explained once the latter has been assigned. The teleologist could not reasonably deny the justification for this procedure without renouncing his own formally similar procedure. It must therefore be concluded that if ultimate philosophical explanation cannot be mechanical because it involves the postulating of a reality not mechanically explicable, an analogous reason leads to the abandonment of the idea that such explanation can be teleological.

The teleologist may endeavor to escape from this position by suggesting that everything is to be explained as both means and end, the significance of this suggestion being that everything is teleologically explained in so far as everything is a means. Apart from the difficulty of proving such a theory, the formal difficulty still remains. No doubt everything is explained so far as everything is means; but, on the other hand, nothing is explained so far as everything is end. The explanation still leaves an inexplicable, an inexplicable, indeed, which now permeates the whole.

The result which thus appears to be forced upon us by considering the formal characteristics of the two methods of explanation, is corroborated by an impartial survey of actual theories embodying them. If it must be admitted, in the face of the teleologist's charge, that the mechanical "first cause" alternates between possessing more content than can be justified and possessing no content at all; even if it must be admitted that this explanatory reality always tends to approximate to the empty abstraction, the thing-in-itself; it must be charged to the account of the teleologist that his professedly complete explanations are purely verbal. The theory that all the parts of the universe are means to a certain end, such as the realization of self-consciousness or spirit, is plausible, if at all, only so long as we keep to the most general form of statement. Descend to particular facts; inquire what evidence there is for believing that the realization of self-consciousness or spirit demands, for example, just the existing number of human beings, or of birds, or of trees; and the whole "explanation" is seen to be "on paper" only.

It must therefore be concluded that when regarded as ultimate philosophical explanation, theories of mechanism and theories of teleology are, formally and materially, equally unsatisfactory. The recognition of this fact has been stated in the form that of the universe as a whole no explanation can be given. According to this view, which appears correct, the totality of existence is the one great inexplicable which must simply be accepted; and the most philosophy can do is to illuminate its nature.

So far as philosophy is concerned, the interesting question at this point is what form this illumination of the nature of the universe is to take,—in other words, precisely what is the problem of philosophy. The importance of this question is due to the fact that no one can be expected to solve a problem which is not definitely stated. It is outside the scope of the present article to deal with this point further than to emphasize its importance. For it is in fact partly due to the neglect of it that the conflict between the teleologist and the mechanist breaks out once more, but in a form somewhat different from that already considered. Whether or not the attempt to give an explanation of "the whole" is now definitely abandoned, the actual problems dealt with are of a more modest character. Each side clings to its form of explanation as the vitally important one, and, applying it to one finite phenomenon after another, endeavors to extend its range indefinitely. Attempts are made to show, on the one hand, that the category of means, on the other hand, that the category of antecedent cause, is of universal application, each position being stated as in some sense a methodological principle, while the presumption in favor of each is considered to vary directly with the number of phenomena explicable, and inversely with the number not explicable, by it.

When the two positions are thus opposed, it may seem difficult to decide between them, or to see on what principle any decision is to be reached. In practice the universalization of each type of explanation is fraught with difficulties, upon which its opponents fasten. Mechanical explanation appears to proceed smoothly so long as it keeps to the inorganic sphere. Immediately it leaves this sphere, its task becomes harder, and the admission must be made that of much that is organic no mechanical explanation has yet been given. The universal applicability of this type of explanation cannot therefore be considered more than a methodological ideal. The teleologist appears to occupy an analogous position. He moves with ease in the organic realm, although even here he cannot yet maintain that his task is completed; but when he enters the realm of the inorganic, he is unable to proceed at all unless he adopts some elaborate and unverifiable hypothesis about the nature of matter. Thus, the obvious fact is that phenomena which can readily be given a mechanical explanation are such as afford little apparent ground for a teleological explanation, and vice versa.

Suppose now that all phenomena whatsoever could be divided into three classes as follows: (1) phenomena of which there exists a mechanical but not a teleological explanation, (2) phenomena of which there exists a teleological but not a mechanical explanation, and (3) phenomena of which there exists neither a mechanical nor a teleological explanation. In such a situation, the chance of a conflict between the mechanist and the teleologist would appear to be remote. Of course the mechanist might maintain that present-day teleological explanation is illusory, and that future knowledge will make it clear that the phenomena in classes (2) and (3) can be explained mechanically. And if the teleologist were to adopt an analogous position with regard to present-day mechanical explanations and the phenomena in classes (1) and (3), a conflict certainly would result. Such a conflict, however, would be based upon faith in the universal applicability of the methods concerned, and, in the absence of evidence, would be unreasonable. Hence, the conflict that now arises is not based upon such faith. The fact is that the above supposition is not true, there being a fourth class of phenomena, those, namely, of which explanations of both kinds are offered. This brings the trouble to a definite head. For of these phenomena the teleologist maintains the correctness of his own and the falsity of the mechanical explanation, while the mechanist similarly asserts that his explanation is the only true one.

The coloration of the humming-bird, for instance, has been explained both teleologically and mechanically. Teleology urges that the phenomenon is explained by its function. This function, it says, is sexual attraction, as a result of which the survival of those birds that were colored in special ways was guaranteed. The mechanist points out, however, that the humming-bird is normally in incessant motion, and urges that the relatively large quantities of waste products accumulating in its feathers as a result of its activity is the correct explanation of its coloration.

Now, in so far as the teleologist denies that the mechanical, and the mechanist that the teleological explanation, is true, there is a definite conflict between them; but it is a conflict which investigation, theoretically at least, could remove. A carefully performed investigation might show that the coloration of the humming-bird subserves no biological purpose, or it might prove that the mechanical explanation is incorrect. The important point, however, is that it could, theoretically, prove that both explanations *are true*. For these explanations, so far from being logical contradictories, are not even logical contraries. In themselves they do not conflict at all. The only possible conflict occurs when the advocate of one explanation denies the other. And this is unjustifiable. It might quite well be that the coloration of the humming-bird is caused as the mechanical explanation asserts, or at least in some rather similar way, and that it has had a biological significance. The one explanation states that the phenomenon has a cause; the other, so far from denying this, merely asserts that it has a function. No conflict is possible between two theories one of which states that a certain phenomenon has a cause and the other that it has an effect.

The harmony between the two explanations is rendered clearer by considering what "natural selection," which is the essence of much teleological explanation, really is. Darwin accepted organic variations as one of his ultimates, and he then endeavored to show that, with variations in the environment (in the widest sense of the term), one organic variation survives rather than another. He did not deny that organic variations have causes, although he professed himself entirely ignorant as to what these causes are. Now with regard to such a phenomenon as the coloration of the humming-bird, the teleologist really adopts the Darwinian hypothesis in its original form, while the mechanist has proceeded, not to deny this hypothesis, but rather to enlarge it by assigning a cause for one of the organic variations.[2]

The conflict between the mechanist and the teleologist nevertheless continues, and its most acute stage is reached when both put forward explanations of religious phenomena. Let us suppose that it be asked why a certain man prays. The teleologist, having regard to the function of prayer in the man's life, may say that it is because prayer uplifts and strengthens him. If he answers thus, he is giving what is formally a perfectly good teleological explanation, which may also be true in fact. But the mechanist may say that a man prays because his mother has taught him to pray. Is there then a conflict between the mechanist and the teleologist? Not if "because" be used in an appropriate sense by each. For it seems unquestionable that both explanations may be correct: the man may be uplifted and strengthened by prayer and he may pray now because he has been taught to pray. While this cannot very well be denied, the teleologist usually exhibits a certain hostility toward the mechanical explanation. This hostility increases in proportion as the phenomena concerned become more extensive, reaching a climax when such phenomena become synonymous with religion as a whole. The situation, however, is clear, and there is no ground whatever for the hostility. Just as the coloration of the humming-bird may have both a mechanical and a teleological explanation, so may any phenomenon whatsoever. If therefore the teleologist is interested in the function of religion, in the purpose it plays in social and individual life, he may limit his statements to the making explicit of this aspect of religion, and is quite justified in so doing. But the man who seeks for the causal antecedents of religion is equally justified, and his mechanical explanation is in no way opposed to that of the teleologist. The one type of explanation is without prejudice to the other.

The most that an advocate of one type can say to an advocate of the other is that he is only interested in his own type of explanation. Such an attitude, however, would be onesided. An impartial and complete review of religion will include a consideration both of its function and of its antecedent conditions. Indeed, as

[2] By assigning a particular kind of cause, however, namely, a conscious purpose, it is often thought that a teleological explanation of organic variations is given; but see below, with regard to the explanation of a man's possession of a boat.

we are able to control events only in so far as we know their causes, the mechanical explanation of religion seems to be as valuable as, if not more valuable than, its teleological explanation. And there is a further reason for the importance of mechanical explanations of religious phenomena at the present time. In the past, such explanations have often involved too much hypothesis about the beginnings of mental life to render them either convincing or useful, not to mention that they have usually laid themselves open to the charge of merely reconstructing complex evolved phenomena out of elements that are themselves products of evolution. What is now needed is a treatment of religious phenomena which shall connect them with psychical processes such as we know them. Such a treatment would distinguish between a man's psychical constitution and what tradition gives him. With this distinction in mind, it would explain the life of a present-day religious man by reference to *his* psychology and not by reference to that of some primitive ancestor. And it would endeavor to indicate the *psychical tendencies* which lead the religious man *to accept the religious tradition.*

It is fairly obvious that the easy adoption of this standpoint requires a recognition of the fact that mechanical and teleological explanations, so far from being in opposition, are complementary. In the past, various obstacles have stood in the way of this recognition. It will therefore be of use to consider now what the more important of these obstacles are.

A reference may first of all be made to the influence of rash statement. It is always difficult for the enthusiast to keep his speech within the bounds of logic; but when the mechanist, for example, asserts positively, and in the absence of anything that could be called strict scientific evidence, that the cause of religious experience is matter in motion, he inevitably finds himself in conflict both with the sober teleologist and with the teleologist who is as rash as himself. This, however, is obvious; and since the conflict thus arising is not in itself any reason why the complementary character of the two types of explanation should not be recognized, it is unnecessary to consider it further.

We now come to a more important point. Some confusion seems frequently to have been due to the employment of the term "explanation" without any precise definition of its meaning. It is commonly thought that an explanation is a complete account of a thing, so that if one explanation is true, any other must be false.

This no doubt is only felt in a vague kind of way; and it would, in fact, be an error to say that the philosophical explanation of a thing consists in stating all the propositions that are true of it. All explanation is the establishing of certain kinds of connection between the phenomena to be explained and others; and no explanation includes the proposition that it is a complete explanation. Possibly, it would be an advantage if the term "explanation" were not to be used in the present connection. If it were to be definitely recognized that a mechanical "explanation" consists in assigning causal antecedents, while a teleological "explanation" is the assignment of function, it would be realized that the two types of "explanation" do not conflict at all, but that they are, in fact, a viewing of phenomena from two standpoints.

Apart from this difficulty concerning the term "explanation," there has been a perhaps greater difficulty in connection with the term "teleology." There has as a rule been no clear conception of what was being done in giving a teleological explanation. Perhaps the chief point of confusion here has been in the idea that a teleological explanation of any phenomenon consisted in showing that there existed antecedent to the phenomenon a purpose in some mind, and that this purpose brought the phenomenon into being. Let us suppose, for example, that a man who lives on the bank of a river is asked why he has a boat. He may reply that he has it for pleasure. In giving such a reply, he would be offering a good teleological explanation of his possession of a boat,—he would be explaining this fact by the function which the boat has in his life. But he might conceivably have given a mechanical explanation. He might have replied that he has the boat because a friend one evening suggested he should procure one, that the idea struck his fancy, that he happened to possess the necessary money to buy one, that there already existed a boat which could be bought, and so on. Such an explanation would certainly be as true as any teleological explanation. If the boat did give the man pleasure, his possession of it on the other hand certainly had causal antecedents. That is to say, his possession of the boat had both a cause and an effect. And so far as the mechanist restricts himself to the assignment of a cause and the teleologist restricts himself to the assignment of a function, no conflict is possible.

But now there frequently arises a misapprehension. Among the causal antecedents of the man's possession of the boat, there was a certain state of his mind, which included a purpose. It might be maintained that of all the causal antecedents this was the most important and deserves to be specially emphasized in any explanation of the man's possession of the boat. And it must be admitted that this state of the man's mind *was* important, although whether it was more important than the fact that he possessed a certain amount of money, is not easy to determine. But however important it may have been, it must be noted that it is a part of the mechanical explanation of the fact to be explained: it is one of the fact's causal antecedents.[8] Now the teleologist seems to have frequently held that this factor constitutes the teleological explanation of the fact. He sees that the conscious purpose is important; and he appears to confuse a conscious purpose which is an antecedent cause of a phenomenon, and therefore part of its mechanical explanation, with the "purpose" of the phenomenon in being a means to an end. "Purpose," in the latter sense, however, does not refer to a conscious state, but merely to the manner in which a thing functions,—to what might be called, in a broad sense, an effect of the thing.

That a conflict should have arisen between the mechanist and the teleologist as a result of such a confusion was inevitable. It frequently led to the teleologist selecting *a part* of the mechanical as *the* teleological explanation; and his conflict with the mechanist has consequently often been due to the fact that what is admittedly an important part of the mechanical explanation is asserted by the teleologist to be the whole explanation and to be teleological. The teleologist did not see that his explanation was really mechanical, nor that it was precisely this which caused the conflict between himself and the mechanist. The two kinds of explanation could not be regarded as complementary while the teleologist asserted one antecedent cause and the mechanist another. In fact, the conflict between mechanist and teleologist has often been essentially similar to a dispute which would arise between two scientists were one to say that the *cause* of a certain event is A, and the other that this *cause* is B, or at least A plus C.

Connected with this misapprehension of the true character of teleological explanation, is the more or less popular view of the divine ordination of religion. According to this view, the true explanation of religion, which as a result of the above confusion

[8] The psycho-physical parallelist would deny this; but, then, *he* could not hold that the man's purpose was of the slightest importance, as it, for him, could not affect *any physical* phenomenon.

is regarded as teleological, is that a purpose in God's mind determined its existence. This is, of course, a mechanical explanation. And it conflicts with what is ordinarily called mechanical explanation of religion because this is usually, in its aim, scientific,—in that it attempts to discover *verae causae*. In so far, therefore, as such a "teleological" explanation was implicitly accepted, it would produce hostility toward such mechanical explanation as endeavored to be scientific. The conflict thus arising would really be between a scientific and an unscientific mechanical theory.

The misapprehension just considered leads to the position that any given "teleological" explanation must conflict with any given mechanical explanation. But apart from such *post facto* conflict, the teleologist, as has been noted above, frequently objects to the very idea of mechanical explanation. It appears to me that the reason of this attitude is largely to be found in two ideas, which must be considered briefly.

The first of these concerns the causal relation. Among the various theories with regard to the nature of this relation there has appeared the view that cause and effect are identical, the effect being merely a transformation of the cause. Such a theory is not compatible with certain common usages of causal terminology; but, on the other hand, it appears to be implicit in much popular belief. Now a person who accepted such a theory would be disinclined to admit that something regarded by him as valuable had a cause less valuable than itself. As it is generally considered that superstition in its more primitive forms and the feelings associated with it are less valuable than enlightened religion, a certain hostility arises toward any attempt to show that the former is at least part of the cause of the latter. Since this, however, cannot be very well denied, the teleologist admits it with a somewhat bad grace, maintaining that the assignment of causal antecedents is not *explanation*. Such an assertion has no point once it is seen what explanation is.

It is unnecessary to enter here upon any criticism of this conception of the causal relation, although it could be shown not to be in agreement with recent conclusions on the subject. The important point is to maintain that the value of a thing is independent of its genesis. If an institution, for instance, really is valuable at the present day, it remains valuable whatever its origin may have been. To say that its value is depreciated by supposing that its cause was less valuable than itself is to admit that it is not *it* that is valued but some belief about it. The value then naturally disappears when the belief is proved to be false. It is therefore unreasonable to depreciate the value of religion, for example, because of the undoubted fact that superstition moulded its earlier forms. Indeed, it would be more reasonable for the man who is convinced of the value of religion to place upon earlier superstitions a higher value; for have not they been part of thc cause of this valuable thing? There is, however, a strong disinclination to adopt this point of view. In spite of anthropological evidence, the teleologist will almost deny that religion could have an origin with which evil was intimately associated. It is probably here that we should look for the explanation of the not uncommon attitude of hostility toward such theories of religion as were propounded by Holbach and the Encyclopedists. Of course, objection may be taken to such theories on the ground that they assign causes wrongly; but to him who considers religion valuable, no reasonable objection to them can be made to depend upon the fact that the causes assigned would be considered by us destitute of value, or even positively bad.

Perhaps an even more important factor in the teleologist's hostility toward the mechanical explanation of religion is the idea, often felt more or less dimly rather than clearly cognized, that any account of the genesis of an institution tends to lessen its authority over the individual. Those who believe that the authority exercised by religion is productive of excellent results, naturally desire that the authority of religion shall not be weakened. The teleologist, who often holds this belief, therefore tends unconsciously to object to *any* mechanical explanation of religion. This tendency is perhaps strengthened by a kind of unconscious pragmatism, by the almost unconscious belief that a theory that has bad consequences must be false. There thus arises on the part of the teleologist a hostility to the mere form of mechanical explanation, a hostility which is rendered greater by a great hope and a great fear.

The question involved here must be admitted to be important. But there appears to have been some misapprehension as to what this question is. The situation is not such as to justify hostility toward explanation merely because it is mechanical in form. To suppose that it is, is to miss the point entirely. The question belongs, in fact, to the sphere of practice rather than to that of theory, and may be termed in a general sense educational. From a practical standpoint, it can be inquired whether it is justifiable to propagate a knowledge of the genesis of certain institutions if it seems likely that such knowledge will have bad consequences; but the question, what is the truth, is quite distinct from the question, should the truth be freely given to all. It may be practically best at a given period to prevent the knowledge of the genesis of religion, for example, (supposing that we possess it), from becoming popular; but this is by no means equivalent to admitting that religion has no antecedent cause. If, therefore, the teleologist here makes an objection to the mechanical theory, he must base it on the unwisdom of the mechanist in publishing his theory and not on that theory's form.

Such appear to be the chief factors in the mechanical explanation of the belief that teleological and mechanical theories of religious phenomena are absolutely opposed to each other. Contrary to this belief, our general conclusion must be that the two forms of explanation are complementary and in no sort of conflict. It is perhaps necessary to add that by "mechanical explanation" here is not meant "explanation by matter in motion exclusively" (*vide* first paragraph of article).

Bernard Muscio.

University of Sydney, Australia.

AN EMPIRICAL VIEW OF THE TRINITY.

There are many rationalistically-minded theists to-day who wonder how intelligent persons can continue to use the language of the old Trinitarian dogma. God to them is an unquestioned reality; although we never see or hear Him, and cannot clearly say where or how He exists, we can be sure that He does exist. But that He is One God in Three Persons seems to them utterly unintelligible and a remnant of scholastic metaphysics which modern common sense should repudiate. Surely, when Christianity is thoroughly rationalized, this incomprehensible and self-contradictory doctrine must yield to the clean-cut Unitarian conception.

In answer to this familiar contention, it would not be a paradox to say that the mystical Trinitarian formula, though, to be sure, it is clothed in the creeds with an unwarranted license of language, is based far more firmly upon experience than the more sharply defined theistic conception of the rationalists.

For how do we know, after all, that God exists? The old naive faith in the Biblical legends of a Jehovah who walked in the garden in the cool of the afternoon, who conversed with the saints, and wrote the Decalog with his finger upon tables of stone, is obsolescent. We can no longer believe in the existence of such a person just because the writers of the Bible-documents believed in him, any more than we can believe in Apollo or Artemis because the Greek epic poets believed in them. We cannot even believe in him because Jesus believed in him; a Jew of the first century, steeped in the pious hopes of his countrymen, how could he possibly help believing? His genius was religious, not scientific; he was no analyst, as Phillips Brooks once said; the whole bent of his nature led him to adopt the faith of his fathers, deepening, sweetening, spiritualizing it, but certainly never questioning its essential truth.

Nor can we rely any longer upon the stock arguments of the older theology—arguments from design, first-cause, *et al.* Every one of these has been so riddled with objections, or had its fallacies so exposed, that it needs an unread or obtuse theologian to rely upon them. The younger generation leaves the dust to gather upon all this laborious argumentation, pro and con, and turns to religious experience as the sole source of its faith and hopes. The question that now engages attention is, How is God revealed in human experience? Psychology and biography take the place of metaphysics, introspection and observation of *a priori* reasoning. The conviction is growing that the conception of God does not rest upon inferences from the nature of the universe, and still less upon a supernatural revelation, but upon the concrete facts of the religious life. God-experiences (if we may use the phrase) are primary, God-theories are secondary. So that even if our theorizing remains dubious and confused, these experiences are indisputable and precious; even if we were to discard the term "God" entirely, the Reality which we seek to name thereby would remain, of profound importance in the religious life of man.

And now, from historical and phychological studies this conclusion emerges: our experience of God is of three types—there are three sorts of human experience which, together, give us our conception of God.

Historically, the concept of God came into existence in these three principal ways: it was the crystallization of the awe and reverence and fear and faith felt in the contemplation of nature, felt in the thought of deceased heroes or chiefs believed to be still alive, felt in the response to that inward pressure that we call conscience. We have passed beyond the stage of disputing whether religion had its origin in animism or in the belief in ghosts; it sprang from both sources. And—what is less commonly recognized—it sprang from the inner struggles of those prehistoric ancestors of ours who, millenniums before St. Paul, found two natures battling within them, the one devilish, the other divine.

And what is true of its origin is true of our God-idea still. In its fulness it is formed by the convergence of three great streams of mental tendency—the recognition of the Divine in nature, in our spiritual heroes, and in ourselves. God about and beyond us, in the vastness of the cosmic life; God in whatever religious leader the believer follows, the spiritual power in that other human life upon which he leans for guidance and inspiration; God in his heart, the Holy Spirit in him, to which he must give his entire allegiance if he would find lasting satisfaction and peace,—man's conception of God is naturally Trinitarian. For the Christian, Christ is preeminent among spiritual heroes, epitomizing and typifying that Divinity in other men which is our greatest source of salvation; he is the supreme revealer of God to us, the symbol and concrete incarnation of Godliness in man. And so the Trinitarianism of Christianity, derived as it was by a devious and blind process of intuition and easily refutable dialectic, has not been, after all, a wide departure from man's spontaneus and instinctive reactions to the great and mysterious forces without him and within.

The belief in God is not, then, a mere act of credulity, a venture of faith in the unknown, an "over-belief," sufficient for our personal needs but unverifiable, unprovable to others. Such an adventurous belief might indeed be legitimate; but is it all we can have? No. The concept of God is, in its foundation-sense, empirical; it is not, at the outset, a matter of blindly believing, but of opening our eyes to see. It is one of Matthew Arnold's greatest services to thought that he insisted upon this truth. We may not deem his definition of God, as "the Power not ourselves that makes for righteousness," comprehensive enough; but we must applaud his attempt to point out in the conception of God, so largely being discarded as a mere superstition, a substratum of truth.

For there surely is in the world a great current pushing us into righteousness. In struggling to do right we are not setting up merely arbitrary and conventional standards, we are moving in the direction which the forces of Nature have ordained. No matter how men may rebel and kick against the pricks, morality is bound eventually to win the day. In choosing virtue we choose the winning side; the cosmos is backing us. In this knowledge there is inspiration and assurance.

Moreover, the same cosmic process that has, from the beginning, been moving irresistibly in the direction of producing, in due time, human virtue and valor, has also flowered out into beauty and all other forms of good. Forces making for evil there also are, for ugliness and sorrow and sin; and men have widely differed in their reaction to that truth, some forming from it a devil-concept, some clinging to the faith that it contributes in the end to good, some ignoring it, throwing it overboard, as the mere waste-product of life. But to realize through every fiber of our being the presence of the Power for Good, the God-Power, as enduring and ultimately winning, to pledge our individual efforts on its side, and rejoice in its triumphs, is the essential *differentia* of the religious life.

The great seers and saints have realized more vividly than the average man the presence of this God in nature. From the Psalmists to Wordsworth, Carlyle, Emerson, we find men of vision inspired and consoled by the sight of this tide of Good that sweeps man on to a destiny which he but dimly sees. Much that was superstition and error has been mingled, no doubt, with this vision. This God whose glory the heavens declare was deemed a partisan Jewish deity, with a manlike form and speech, offering a crude extraneous reward and punishment to those who followed or disobeyed his will. But such anthropomorphism is better than an absence of vision. For rewards and punishments for virtue and vice there are, though they are intrinsic, and brought about in natural ways. And to lose the sense of the divineness of nature, to lose the faculty of worship, of reverence, of joy in the beauty and wonder of the world in which we live, is not only to be, in so far, irreligious, but therein to miss one of the essential ingredients in the noblest and happiest life.

But more than in nature do we find God in men—the best men we know; and for us at least of Christendom, in Christ. This need not imply any disloyalty to a truly historic conception of his life and teachings, need not imply anything miraculous or supernatural about him. The divineness in Christ may be as much a natural fact, produced according to natural laws, as the divineness in the outer world may be. For our purpose here we need not debate that question. For certainly it was not the debated fact of supernaturalness (in the scientific sense) that made Christ divine; it was his character. His will was wholly merged with the will of God; there was no selfishness, no self-indulgence, in him. The Christ-life is the divine life for men, the measure of the amount of Godliness that we are capable of. To call his life divine is not in the least to assert that he was born of a virgin, raised the dead, rose himself from the tomb; it is a different sort of judgment, a value-judgement. The facts about his life must be decided by historical methods, as we would sift the records of the life of any other personage of the past; no ardent believer or entrenched ecclesiasticism ought to attempt to bias the impartial judgment of scholars upon them. But the question of the divineness of this life is to be decided by men of spiritual vision. And the verdict of truly religious men is all but unanimous; the great warrior, the great statesman, the great inventor, the great poet, have a veritable spark of God in them; but the life that is most truly divine, that most fully reaches up to God, is the life of purity and charity and self-sacrifice. Preeminent among such lives, dazzling men of all races for the two millenniums since he lived, is the life of the Carpenter of Nazareth.

But if divinity is especially incarnate in the spiritual heroes of mankind, it may also, in some measure live in each of us. We recognize amid the tangle and clutter of selfish and sensual desires a holier spirit within us. Sometimes an uprush of noble feeling or high resolve, a power for good, wells up in us, and we know it to be divine. This fountain of inner holiness springs up at times abruptly, even unexpectedly, and then oftenest subdues our other impulses into hushed obedience. But we need not consider these conversion-experiences as supernatural; the new spirit is holy, not because of its miraculous way of working, but because its influence in our lives is divine. The practically significant fact is that this power is ready for our use. As Emerson wrote, "It is a secret which every intelligent man quickly learns, that beyond the energy of his possessed and conscious intellect, he is capable of a new energy (as of an intellect doubled on itself) by abandonment to the nature of things; that besides his power as an individual man, there is a great public power upon which he can draw, by unlocking at all risks his human doors, and suffering the ethereal tides to roll and circulate through him."

What all this may imply about the ultimate metaphysical nature of God is, no doubt, worth discussing, and conceivably of great import. But men are coming in these latter days to a humbler sense of their intellectual limitations; we are realizing that we know nothing of the inner nature of anything, save of our own conscious life as it passes. What is matter? What is electricity? What is God? Perhaps we cannot know. But what is practically important is to understand and utilize the experiences out of which these concepts have grown. If we can use electricity in our telephones and dynamos and trolley cars, we can be content to confess our ignorance of its inner nature. So if we can comprehend and repeat the religious experiences out of which the concept of God has arisen, it matters less if our knowledge of God is limited to that experience-contact.

Souls of different types and needs will naturally formulate their experience in different terms; there is no need that any one to whom the generalization is not personally useful should express his God-idea in a trinitarian formula. Trinitarianism should never be a dogma. And with the arguments and disputes of the Greek doctors of the third and fourth centuries, through whom that dogma took shape, we may have scant patience. Certainly all that sort of speculation is very alien to our modern scientific world-view. But on the other hand, the arguments of the rationalists of to-day for a God-idea divorced from those experiences in which it has its natural roots, are equally alien to the outlook and spirit of science. To believe in God is a mere act of credulity, except as we see the meaning of the God-idea in human life. When we do thus turn to experience, we find ourselves led to the God-conception from the three sorts of experience mentioned. So, as an embodiment of the profound truth of the threefold basis of our human conception of God, the Trinitarianism of the saints should command our sympathy and respect.

Trinitarianism, Unitarianism—as mutually exclusive dogmas, both are cramping and arrogant. What is important is to keep alive the experience that each term enshrines. The essential oneness of all God-experiences, and of the God-idea which they unite in producing, is important, no doubt. But the bare insistence upon unity has, now that the extravagances of polytheism are forever past, little religious value, and tends to a contentment with less than the full gamut of religious experience. No one of the three forms of God-experience can be dispensed with in a rich and fruitful spiritual life; and it is no wonder that the orthodox have generally felt a merely negative Unitarianism to have an impoverishing tendency. However crude the creedal affirmations of Trinitarianism may be, the fulness of the Christian life has by it been fostered and preserved. So, however loath we may be to seem to accept the description of a quasi-human Being who is somehow Three Persons and yet One, if we take the doctrine (as we must take all religious doctrines) in its inner and spiritual sense, which is its empirical foundation-sense, we shall see it as a more or less blind expression of a great truth—that Christians can attain to the vision of God in three ways, through contemplation of the outer world, through faith in their Master Christ, and through obedience to the Holy Spirit in their hearts.

DURANT DRAKE.

VASSAR COLLEGE.

GENERAL NOTES ON THE CONSTRUCTION OF MAGIC SQUARES AND CUBES WITH PRIME NUMBERS.

The series of numbers generally used in the construction of magic squares are in arithmetical progression. The progression of the prime number series is very irregular, and therefore cannot be used as freely as an arithmetical series. This naturally leads to the investigation of the possible irregularities in groups or series of numbers which may be formed into magic squares. It is also necessary to find means of discovering these groups of numbers in the prime number series.

It is the writer's aim to describe here simple rules for constructing prime number squares, methods of finding the numbers to be used, and to point the way to the solution of a few of the problems not yet mastered.

THE SQUARE OF THE 3d ORDER.

There is only one possible construction of this square and there is only one rule governing the series, and that is, when the series is written in tabular form, as in Fig. 1, the differences between all vertically adjacent cells must be equal, and the differences between all horizontally adjacent cells must be equal, but the vertical and horizontal differences must be unequal to avoid duplicate numbers. These differences are indicated by numbers at the sides of the lattice and it will be by these that we will identify the nature of the series

	42	42	
	5	47	89
12	17	59	101
12	29	71	113

Fig. 1.

	x	x	
a			
a			

Fig. 2.

71	5	101
89	59	29
17	113	47

Fig. 3.

used in the following magic squares. We will represent these differences by letters, using the letters of the fore part of the alphabet for one set of differences and those of the other end of the alphabet for the other set, as is shown in Fig. 2, like letters indicating the necessity of like differences.

Fig. 2 is arranged into the magic by using the middle column and middle line as diagonals, the position of the remaining numbers then being easily found. The resulting magic is shown in Fig. 3.

THE SQUARE OF THE 4th ORDER.

Any series or set of 16 numbers, when written in the tabular form previously mentioned, wihch gives the differences a, b, c and x, y, z, may be formed into a magic square by the Jaina method as follows. Fig. 4 shows a table of prime numbers with irregular differences. Four sets of the upper line of numbers of this table are arranged

	x	y	z	
	1	11	23	37
a	31	41	53	67
b	61	71	83	97
c	127	137	149	163

Fig. 4.

1	11	23	37
23	37	1	11
37	23	11	1
11	1	37	23

Fig. 5.

1	71	149	67
53	163	61	11
97	23	41	127
137	31	37	83

Fig. 6.

in a subsidiary square, as shown in Fig. 5, so that each line, column, and the two diagonals contain each of the four different numbers. Subtracting the initial number of the table (in this case 1) from each of the numbers in the left-hand column of the table, will give the numbers, 0, 30, 60, 126, which are to be arranged in a second subsidiary square with the same arrangement as in Fig. 5, only that the pattern is turned 90 degrees. The two subsidiary squares are then added together, cell to cell, to produce the magic square. A resulting square is shown in Fig. 6.

In selecting numbers from the tables for the subsidiary squares, the column and line containing the lowest numbers should be chosen, but it makes no difference which set the initial number is subtracted from.

A balanced series of numbers whose tabular differences are

$x \quad y \quad x$

$a \quad b \quad a$

17	47	137	167
29	59	149	179
31	61	151	181
43	73	163	193

Fig. 7.

193	47	137	43
29	151	61	179
31	149	59	181
167	73	163	17

Fig. 8.

17	73	149	181
179	151	47	43
61	29	193	137
163	167	31	59

Fig. 9.

a, b, a, and *x, y, x,* may be arranged into an *associated* square, or a *pandiagonal* square. Such a series is shown in Fig. 7. By revolving the two diagonals of Fig. 7, 180 degrees, it will produce the associated magic square shown in Fig. 8. To produce a pandiagonal square, we select, as before mentioned, two subsidiary sets of numbers from which are formed two subsidiary squares of the pattern shown in Fig. 5. The numbers in the upper line should be so arranged that the sum of the left-hand pair equals the sum of the right-hand pair. One of these subsidiary squares is revolved 90 degrees and added to the other to produce the magic. A pandiagonal square resulting from such a construction is shown in Fig. 9.

Another form of subsidiary square which may be used to produce a pandiagonal square from a balanced series is shown in Fig. 10, which is exemplified with arbitrary numbers. The numbers

1	4	1	4
2	3	2	3
4	1	4	1
3	2	3	2

Fig. 10.

167	163	17	73
29	61	179	151
193	137	43	47
31	59	181	149

Fig. 11.

must be so arranged that the pairs indicated by dotted lines will have like summations. One subsidiary square is revolved 90 degrees from the other and the two added together to produce the final square. Fig. 11 is a pandiagonal square produced from the series in Fig. 7, by the method last described.

THE SQUARE OF THE 5th ORDER.

A series of 25 numbers whose tabular differences are *a, b, c, d* and *w, x, y, z* is shown in Fig. 12. Such a series may be formed

w x y z

a b c d

1	7	61	67	127
13	19	73	79	139
23	29	83	89	149
37	43	97	103	163
41	47	101	107	167

Fig. 12.

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

Fig. 13.

1	29	101	79	163
73	103	127	23	47
149	41	19	97	67
43	61	89	167	13
107	139	37	7	83

Fig. 14.

into a *pandiagonal* square as follows. Two sets of subsidiary numbers are selected and arranged in two subsidiary squares according to the pattern shown in Fig. 13; the pattern of one subsidiary square should, however, be in a reversed or reflected order from the other. The two squares are then united to form the final square. Fig. 14 shows one example resulting from the series in Fig. 12.

To produce an *associated pandiagonal* square of the 5th order, it requires a series whose tabular differences are *a, b, b, a* and *x, y, y, x*. The writer, at present, knows of only one series which suits the above requirements. Its initial number is 41 and the tabular differences are 60, 390, 390, 60 and 72, 138, 138, 72 respectively.

1013	251	449	911	881
839	1301	941	113	311
41	173	701	1229	1361
1091	1289	461	101	563
521	491	953	1151	389

Fig. 15.

The subsidiary squares are arranged in associated formation and according to the pattern in Fig. 13. The solution of this difficult problem was accomplished by Mr. Chas. D. Shuldham, and his resulting magic is shown in Fig. 15.

Mr. Shuldham has succeeded by other methods in constructing *associated* squares of the various orders including the 12th.[1] These are not pandiagonal however.

THE SQUARE OF THE 6th ORDER.

A balanced series whose tabular differences are *a, b, c, b, a* and

x y z y x

a b c b a

47	179	199	1301	1321	1453
257	389	409	1511	1531	1663
467	599	619	1721	1741	1873
677	809	829	1931	1951	2083
887	1019	1039	2141	2161	2293
1097	1229	1249	2351	2371	2503

Fig. 16.

1	6	6	6	1	1
5	2	5	2	2	5
3	4	3	3	4	4
4	3	4	4	3	3
2	5	2	5	5	2
6	1	1	1	6	6

Fig. 17.

x, y, z, y, x may, by a common method, be formed into a magic square as follows. Fig. 16 shows a series with the above differences. It will be noted that the values for *a, b* and *c* are each 210, though the method of construction does not necessitate these like differences. Subsidiary sets of numbers are selected as previously explained and formed into subsidiary squares according to the pattern shown in Fig. 17. One of the squares thus formed is revolved 90 degrees and added to the other, cell to cell, to form the final square. One

47	2293	2083	1873	257	1097
1321	389	1951	599	1019	2371
1249	1511	619	829	2141	1301
2351	1039	1721	1931	409	199
1229	1531	809	1741	2161	179
1453	887	467	677	1663	2503

Fig. 18.

v w x y z

a a 2a a a

19	41	47	67	101	173
229	251	257	277	311	383
439	461	467	487	521	593
859	881	887	907	941	1013
1069	1091	1097	1117	1151	1223
1279	1301	1307	1327	1361	1433

Fig. 19.

example is shown in Fig. 18, which is magic only in its lines, columns and two diagonals.

A series of prime numbers to suit the above differences is very difficult to find, but the following form of series is easily found, and it is only recently that the writer has succeeded in discovering a

[1] See "Associated Prime Number Magic Squares," *The Monist*, July, 1914, Vol. XXIV, No. 3.

method of arranging the numbers of such a series in magic square formation.

Fig. 19 shows a series whose tabular differences are *a, a, 2a, a, a* and *v, w, x, y, z*. The subsidiary numbers with differences corresponding to the *a* values are arranged as shown in Fig. 20, each

1260	1260	1260	0	0	0
210	210	210	1050	1050	1050
420	420	420	840	840	840
210	210	210	1050	1050	1050
1260	1260	1260	0	0	0
420	420	420	840	840	840

Fig. 20.

19	41	47	67	101	173
101	67	19	173	47	41
41	47	173	19	67	101
47	173	41	101	19	67
173	101	67	47	41	19
67	19	101	41	173	47

Fig. 21.

line made up of three like pairs. The other set of subsidiary numbers is arranged as shown in Fig. 21, with a full set of the six different numbers in each line, column, the two central diagonals, and in the cells corresponding to the like numbers in Fig. 20. These two subsidiary squares combined, will produce the magic square shown in Fig. 22.

1279	1301	1307	67	101	173
311	277	229	1223	1097	1091
461	467	593	859	907	941
257	383	251	1151	1069	1117
1433	1361	1327	47	41	19
487	439	521	881	1013	887

Fig. 22.

To construct an *associated* or *pandiagonal* square of the 6th order would require a table of numbers with the differences *a, b, c, b, a* and *x, y, z, y, x* and each set of subsidiary numbers would have to be of values permitting thier arrangement in magic 2×3 rectangles.[2] It is doubtful if a series of this nature can be found, but, by methods previously published, the problem may be solved

[2] To construct, see "Notes on the Construction of Magic Squares" by Messrs. Andrews and Frierson, *The Monist*, April 1912, Vol. XXII, No. 2. For an example, try any series with the tabular differences 2, 1, 4, 1, 2 and 9, 11, 18, 11, 9, with the illustrations on p. 306 (of the above issue) as a guide.

by first forming an *associated* square and transforming it into a *pandiagonal* square.[8]

THE SQUARE OF THE 7th ORDER.

This square, like the 5th order square, may be formed into a pandiagonal magic by using a series having tabular differences of

	s	t	v	w	x	y	z
17	41	173	347	613	853	1997	2693
23	47	179	353	619	859	2003	2699
227	251	383	557	823	1063	2207	2903
233	257	389	563	829	1069	2213	2909
443	467	599	773	1039	1279	2423	3119
653	677	809	983	1249	1489	2633	3329
857	881	1013	1187	1453	1693	2837	3533
863	887	1019	1193	1459	1699	2843	3539

Fig. 23.

a, b, c, d, e, g and *t, v, w, x, y, z.* The subsidiary squares have like numbers running in knight paths, the pattern of one may be a reflection of the other, or they may have the same pattern, but 90 degrees apart.

THE SQUARE OF THE 8th ORDER.

This square may be formed with an irregular series having tabular differences of *a, b, c, d, e, g, h* and *s, t, v, w, x, y, z.* One series is shown in Fig. 23, and Fig. 24 shows one pattern of sub-

1	2	5	6	3	4	7	8
4	3	8	7	2	1	6	5
2	1	6	5	4	3	8	7
3	4	7	8	1	2	5	6
5	6	1	2	7	8	3	4
8	7	4	3	6	5	2	1
6	5	2	1	8	7	4	3
7	8	3	4	5	6	1	2

Fig. 24.

17	257	619	1063	599	1193	2633	3533
353	383	2693	2213	677	857	1279	1459
467	863	1489	1453	347	389	2699	2207
809	1187	2423	3539	23	251	613	1069
823	859	233	41	2837	3329	1019	773
2909	1997	557	179	1699	1039	881	653
1693	1249	887	443	2903	2003	563	173
2843	3119	1013	983	829	853	227	47

Fig. 25.

[8] See "Pandiagonal Prime Number Magic Squares," by Mr. Chas. D. Shuldham, *The Monist,* Oct. 1914, Vol. XXIV, No. 4.

sidiary square, other patterns being easily found. The magic square shown in Fig. 25 was constructed from the above series with the subsidiary squares arranged according to Fig. 24, one being revolved 90 degrees from the other.

THE SQUARE OF THE 9th ORDER.

It will be noted in Fig. 24 that each line, column and the two diagonals contain no like numbers. The numbers are also arranged

	r	s	t	v	w	x	y	z	
a	17	41	173	347	613	853	1997	4871	5431
b	23	47	179	353	619	859	2003	4877	5437
c	227	251	383	557	823	1063	2207	5081	5641
d	233	257	389	563	829	1069	2213	5087	5647
e	443	467	599	773	1039	1279	2423	5297	5857
g	653	677	809	983	1249	1489	2633	5507	6067
h	857	881	1013	1187	1453	1693	2837	5711	6271
k	863	887	1019	1193	1459	1699	2843	5717	6277
	1277	1301	1433	1607	1873	2113	3257	6131	6691

Fig. 26.

so that the pattern will set upon itself, by reflection or revolving, and not produce duplicate numbers in the final square. In like manner, the subsidiary squares for the 9th order square are ar-

3	8	4	7	6	2	5	1	9
7	6	2	5	1	9	3	8	4
5	1	9	3	8	4	7	6	2
4	3	8	2	7	6	9	5	1
2	7	6	9	5	1	4	3	8
9	5	1	4	3	8	2	7	6
8	4	3	6	2	7	1	9	5
6	2	7	1	9	5	8	4	3
1	9	5	8	4	3	6	2	7

Fig. 27.

5641	863	829	1187	5507	179	467	853	3257
881	1489	2003	5857	17	1873	557	5717	389
773	4871	1433	251	1699	2213	6271	653	619
233	823	6277	4877	1013	983	2113	2423	41
859	2837	677	1277	1039	5431	5087	383	1193
6131	599	347	1069	2207	887	23	1453	6067
1459	5647	227	809	353	5711	1997	1301	1279
2633	47	1693	613	6691	443	1019	563	5081
173	1607	5297	2843	257	1063	1249	5437	857

Fig. 28.

ranged. Fig. 26 shows an irregular series, and Fig. 27 shows a pattern for the subsidiary squares, one being a reflection of the other. A resulting square is shown in Fig. 28.

It may also be noted that, from Fig. 26, sets of numbers may be chosen for squares of the 4th, 5th, 7th, and 8th orders, and the possible number of distinct squares which may be formed from these depends on the laws of *permutations* and *combinations*.

TWIN SQUARES.

Two magic squares having like summations and having no numbers in common have been termed "twin squares."

1	599	701	337
281	757	421	179
547	71	389	631
809	211	127	491

29	619	677	313
257	733	449	199
523	47	409	659
829	239	103	467

Fig. 29.

To construct twin squares of the 4th order, we select a table of numbers having 8 or more columns and 4 numbers per column. The tabular differences may be irregular. From the top line of numbers, select two groups of four numbers each, that have like summations. Each of these groups will indicate the columns of numbers to be used in the respective squares. Each square is constructed as was explained in reference to Figs. 4, 5 and 6.

Twin squares of the 4th and 5th orders are shown in Figs. 29 and 30 respectively. In both examples, the above method was employed.

The foregoing methods of constructing prime number squares, by the use of tabular series, are obviously of little use for constructing squares of the higher orders, due to the increased difficulty in finding series of the necessary requirements. Large squares have been constructed, however, by other methods.[4]

389	643	6113	5927	1321
5717	2161	179	433	5903
223	5693	5507	1951	1019
1741	809	1063	5483	5297
6323	5087	1531	599	853

2909	3583	653	1097	6151
887	6991	2699	3373	443
3163	233	677	6781	3539
6571	3329	4003	23	467
863	257	6361	3119	3793

Fig. 30.

[4] See squares by Messrs. Shuldham and Muncey in *The Monist*, Oct. 1913, Vol. XXIII, No. 4, pp. 623 to 630.

PRIME NUMBER TABLES.

To facilitate the finding of sets of prime numbers to be used in the construction of prime number magic squares, the prime number series has been arranged in tabular form in various ways.

One form of table is shown in Fig. 31, which is composed of a lattice having five cells in each line, the columnar length of the table depending on the range of numbers to be tabulated. The cells are counted by odd numbers in natural order from left to right, first through the top line, then through the second, and so on down through the table, a dot being placed in each cell where a prime number falls. In some cases, the table is made more convenient by extending it to form extra columns containing numbers which are duplicates of those in the table proper. Fig. 31 shows two supplementary columns which contain duplicates of the numbers in the first two columns. It should also be noted that the numbers in the supplementary columns must be one cell nearer the head of the table than the numbers they duplicate.

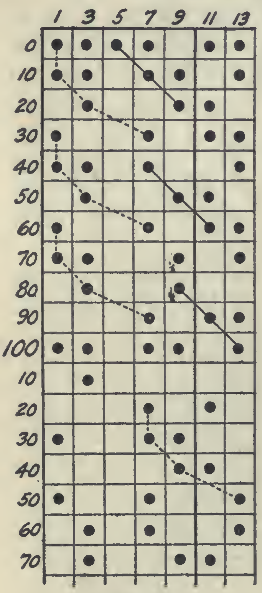


Fig. 31.

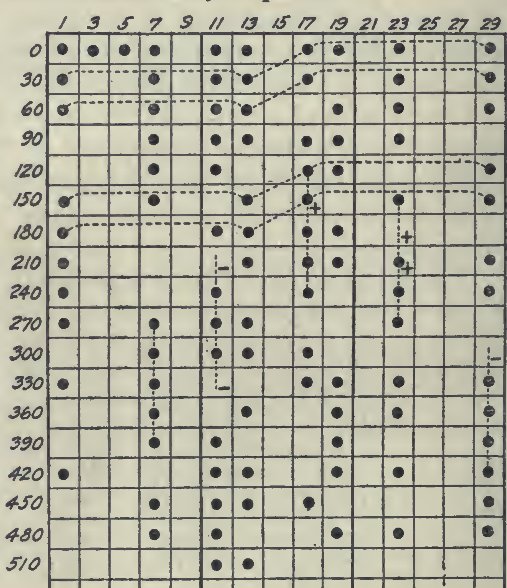


Fig. 32.

At the top of the table, the columns are numbered 1, 3, 5,. . . ., and at the side, the lines are numbered 0, 10, 20, 30,. . . ., the latter numeration depending on the number of cells in a line. The value of a number represented by any dot, may be determined by adding together the index numbers at the end of the line and column in which the dot lies.

Fig. 32 shows another table in which the increments between lines are 30; and a portion of a still larger form of table is shown in Fig. 33, where the increments between lines are 210.

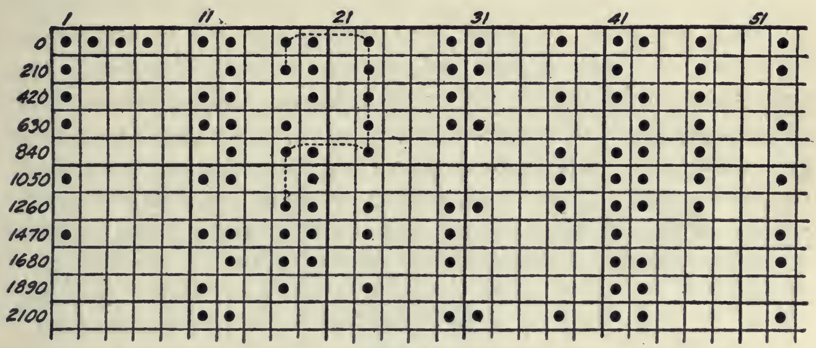


Fig. 33.

To get the best formation of tables, the increments between lines should be a multiple of two or more different small prime factors. In Fig. 31, the increment is $10=2\times5$; in Fig. 32, the increment is $30=2\times3\times5$; and in Fig. 33, the increment is $210=2\times3\times5\times7$. The value of this may be observed by the resulting linear grouping of numbers in the tables.

To illustrate the use of these tables, we will point out a few of the groups of numbers which have been used in the preceding magic squares.

In Fig. 31 will be found three triads, the dots in each triad being indicated by a straight line connecting them. This group of 9 numbers is shown in Fig. 1 and is arranged in magic square formation in Fig. 3. It will be noted that the 9 dots in the table form a symmetrical figure, which indicates an *associated* group of numbers. In these forms of tables, all *associated* groups of numbers will show symmetrical figures, unless the grouping runs over the edge of the table. If the table in Fig. 31 had not been extended, the 9-number group would have appeared irregular if kept within the table.

In the same table will be found four irregular groups having four dots each, as indicated by dotted lines. Each group has the same form, though their relation is unsymmetrical. This group of 16 numbers was used to form the magic square shown in Fig. 6.

A symmetrical group of 16 numbers is pointed out in Fig. 32, which was used to form the associated square in Fig. 8 and the pandiagonal square in Figs. 9 and 11.

In Fig. 33 is shown an irregular group of 9 numbers. Nine groups of this form were used in constructing the square shown in Fig. 28.

The foregoing rules of construction will aid in the simple formation of magic squares with prime numbers, but these rules are apparently inadequate in certain instances, for example, to construct a pandiagonal 9th order square, or to form a magic cube with prime numbers. The writer believes that these and other problems can be mastered if we bring further irregularities into the magic square series. In the following pages will be shown some of these irregularities, with an introduction of the "kink."

THE KINK.

The "kink" was first discovered in analyzing the prime number square, S = 102. Fig. 34 shows this square resolved into two La

3	71	5	23
53	11	37	1
17	13	41	31
29	7	19	47

S = 102

=

2	36	12	18
12	18	2	36
18	12	36	2
36	2	18	12

+

1	35	-7	5
5	-7	35	1
35	1	5	-7
-7	5	1	35

La Hireian Subsidiaries.

+

36			-36
-36			36

Kink.

Fig. 34.

Hireian subsidiaries of Jaina formation and a kink. It will be noticed that this kink involves four cells, rectangular in position, and of equal numeral values, two being plus and two minus. For

			+	−
−		+		
			−	+
+		−		

A

			+	−
			−	+
−	+			
+	−			

B

+	−			
	−			+
	+			−
−	+			

C

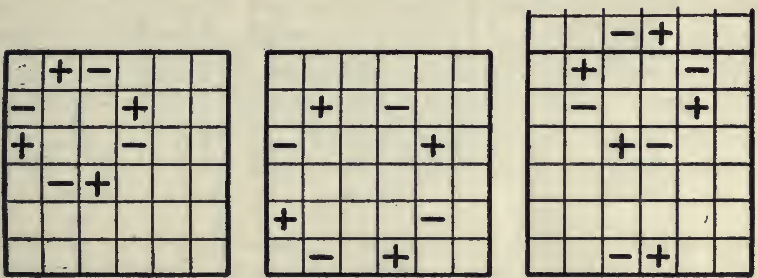
−		+		
+			−	
			−	+

D

Fig. 35.

convenience of designation we will give this form the term ***rectangular kink.*** It will be observed in the rectangular kink, that any two values in a horizontal or vertical line, counteract each other; therefore, a rectangular kink of any plus or minus values, may be added to any magic square without affecting the magic summations, providing any part of the kink does not fall in a magic diagonal. A rectangular kink may affect a diagonal, providing a second kink is added to correct the fault. Fig. 35 illustrates a few double rectangular kinks by which the main diagonals of the square have been corrected. In the central cell of the square "D," two kink values are neutralized, and are therefore not shown.

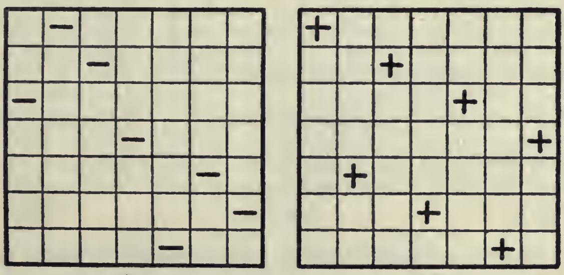
Two like rectangular kinks may be combined so as to form a kink that will not destroy the values of any of the magic summations of a pandiagonal square. This form might be termed *pan-*



A B C

Fig. 36.

diagonal or *octagonal kink.* Fig. 36 shows a few of the latter type, the square "C" showing how the octagonal form may be apparently missing in some cases, due to the kink running over the edge of the square.



A B

Fig. 37.

minus or plus as shown in Fig. 37. In square "A" the same value is taken from each line, column, and the two diagonals, which
There are also other kinks where all the values are either
would mean a lowering of the summations but would not destroy

its principal magic qualities. In square "B" each line, column and all diagonals are affected alike, and may therefore be classed as a *pandiagonal kink.* The values in this latter kink are shown to run in a knight's path, which formation has suggested the term *path kink.*

When applying kinks to squares, it is not necessary that the same numeral values are applied to all the kinks involved. This

	$+n$			$-t$	$+t$	
			$+t$	$+n$		$-t$
$+n$	$-s$		$-t$		$+s$	$+t$
			$+n$	$+t$	$-t$	
	$+s$				$-s$	$+n$
		$+n$				
					$+n$	

Fig. 38.

10	18	1	14	22
11	24	$\underset{+}{7}$	20	$\underset{-}{3}$
17	$\underset{-}{5}$	13	21	$\underset{+}{9}$
23	6	19	2	15
4	$\underset{+}{12}$	$\underset{-}{25}$	8	16

Fig. 39.

1	2	$\underset{-}{3}$	4	$\underset{-}{5}$
6	$\underset{+}{7}$	8	$\underset{+}{9}$	10
11	$\underset{+}{12}$	13	14	15
16	17	18	19	20
21	22	23	24	$\underset{-}{25}$

Fig. 40.

is illustrated in Fig. 38, where are shown a rectangular, an octagonal, and a path kink. It can be realized from this illustration that a magic square series can be greatly distorted by the addition of a few kinks.

The following example will suffice to show the constructive application of the kinks. Fig. 39 is a magic square containing the

		w	*x*	*y*	*z*
a	277	173	$\underset{-}{329}$	137	$\underset{-}{221}$
b	307	$\underset{+}{203}$	359	$\underset{+}{167}$	251
c	337	$\underset{+}{233}$	389	197	281
d	367	263	419	227	311
	397	293	449	257	$\underset{-}{341}$

Fig. 41.

277	173	179	137	71
307	353	359	317	251
337	383	389	197	281
367	263	419	227	311
397	293	449	257	191

Fig. 42.

251	419	277	197	293
337	257	353	311	179
263	71	389	397	317
449	307	227	173	281
137	383	191	359	367

Fig. 43.

series 1, 2, 3,.... n^2, constructed with subsidiaries of the pattern shown in Fig. 13. The numbers of this square are shown in tabular form in Fig. 40. The position of two arbitrary rectangular kinks has been designated in Fig. 39, and these kinks have been transferred to the same respective numbers in Fig. 40. Any set of numbers with tabular differences of a, b, c, d, and w, x, y, z, that is affected by kinks, as shown in Fig. 40, is susceptible to transformation into a magic square. In Fig. 32 are shown by vertical dotted lines, five sets of numbers which have been tabulated in Fig. 41, the strings of the former being arranged as columns in the latter. The selections in Fig. 32 were made with the kinks in mind, which are indicated with small plus or minus signs. Where the minus signs occur, we have substituted the number five cells above in the same column, and where the plus signs occur, the number five cells below has been used; these differences of position being equal to 150, which is the numeral value of the kink used. This kink value has been added to the cells indicated in Fig. 41 which gives the numbers shown in Fig. 42. The numbers in Fig. 42 are now transposed into the magic square Fig. 43, in the same respective order that the numbers of Fig. 40 are shown in Fig. 39.

It can be seen from the foregoing that unusual irregular series, formed by the use of kinks, may be anticipated, and in a great many cases, found and arranged in magic squares. The writer believes that the 3d order cube will be solved in this manner, which will be referred to in the following pages.

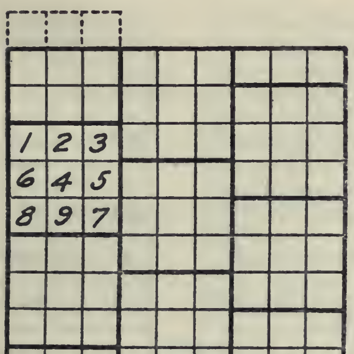


Fig. 44.

677	1069	601	761	269	401	11	577	887
937	313	139	151	487	389	643	1181	1013
307	101	59	467	1289	1399	1117	281	233
239	1351	1371	1523	499	37	17	43	173
1061	811	569	23	83	157	509	669	1371
71	31	103	53	787	1051	1547	883	727
431	773	1283	1151	379	439	443	293	61
1109	191	557	461	541	397	359	977	661
421	613	571	663	919	983	607	349	127

Fig. 45.

Another method of using the kink, is to construct the desired magic using as few composite numbers as possible, and then add various kinks at the places where the composite numbers occur; the process being continued until all the composite numbers are eliminated. The pandiagonal square of the 9th order will undoubtedly be accomplished in this way.

For this square, the preliminary series (containing as few composites as convenient) should be so chosen as to allow each of the two subsidiary groups of numbers to be divided into three triads of equal values. The numbers are arranged in quarrels in the subsidiary squares, as shown in the pattern, Fig. 44, the triads being placed in vertical strings. The subsidiary squares are similar in pattern and are placed 90 degrees apart, or, one subsidiary may be reflected on either of its two diagonals.

Fig. 45 illustrates a pandiagonal square constructed by Chas. D. Shuldham. By the above method he has succeeded in transforming it to contain as low as six composites, which are indicated by circles.

THE CUBE.

To the writer's knowledge, the prime number cube of the 3d order has not yet been constructed. The kink will undoubtedly aid in its construction, and the following example will suffice to show the application of the kink to the cube.

Fig. 46 shows the three respective layers of one of the various magic cubes constructed with the natural series of numbers, and

4 −	26	12 +
17	3	22
21 +	13	8 −
18	1	23
19	14	9
5	27	10
20 +	15	7 −
6	25	11
16 −	2	24 +

Fig. 46.

1	2	3
4 −	5	6
7 −	8 −	9
10	11	12 +
13	14	15
16 −	17	18
19	20 +	21 +
22	23	24 +
25	26	27

Fig. 47.

1831	1747	1663
1597 −	1777	1693
1627 −	1543 −	1723
991	907	1087 +
1021	937	853
787 −	967	883
151	331 +	247 +
181	97	277 +
211	127	43

Fig. 48.

1597	127	1087
967	1663	181
247	1021	1543
883	1831	97
151	937	1723
1777	43	991
331	853	1627
1693	211	907
787	1747	277

Fig. 49.

Fig. 47 shows this series in tabular form. A magic cube of ordinary construction must have a series whose tabular differences are *a, a* and *p, p* and *x, x,* as indicated in Fig. 47, the numbers being arranged in the cube in the same order as any cube of a straight series.

The simple *cubic kink* is shown diagrammatically in Fig. 50 and has the form of a right parallelopiped. A single kink of this form can be added to the 3d order cube in only one way, that is, its eight values must fall in the eight corner cells of the cube, otherwise the summation of the diagonals would be altered.

The position of a simple kink is indicated in Fig. 46, and is transposed to the same respective numbers in Fig. 47.

The series of prime numbers for the cube would be discovered in the same manner as were the numbers in Figs. 42 and 43, that is, find any set of numbers in the prime number table that has tabular differences, as indicated by letters in Fig. 47, disregarding any composite numbers that may occur where the kink is indicated. For the plus values of the kink, a new set of values is discovered in the table which has the same geometric relation to each other as the originals. This is diagrammatically illustrated in Fig. 51. Care should be taken that a new set for the minus values can also be found in a symmetrically opposite direction.

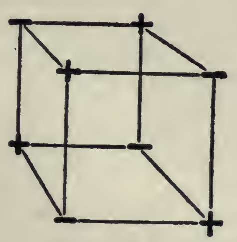


Fig. 50.

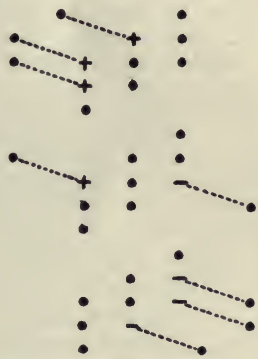


Fig. 51.

According to the above method, the writer has succeeded in finding, in a table of the form shown in Fig. 32, the series shown in Fig. 48, which contains the one composite number indicated with a circle. These numbers being transposed into a cube according to Figs. 46 and 47, produces the magic cube shown in Fig. 49.

Combinations of kinks can be added to cubes in various ways which the reader can easily discover for himself. The variations in tabular differences and in kink formation and combinations would apparently indicate that the discovery of a prime number series for a cube is possible. There is greater freedom in the application of kinks to the 4th order cube, though the writer has not investigated beyond the 3d order.

It also seems possible that the cube of the 4th order may be constructed by an extension of the *method of pseudo-complementaries.*[5]

Patience and perseverance will be found to be the principal requirements in solving these difficult problems in prime number magics. Who will claim the honor of being the first to solve them?

HARRY A. SAYLES.

SCHENECTADY, N. Y.

BOOK REVIEWS.

THE ELEMENTARY FORMS OF THE RELIGIOUS LIFE. A study in religious sociology. By *Emile Durkheim,* professor of the faculty of letters at the University of Paris. Translated by *J. W. Swain, M.A.* London, George Allen & Unwin, 1916. Price 15s. net.

This book must be valued from three different points of view. It contains reinterpretations of the principal social phenomena of primitive peoples; it contains a theory of the genesis of knowledge with doubtful philosophic implications; and it contains what we may assume for the present to be M. Durkheim's definitive pronouncement on the nature and the future of religion. All of these strands of argument are bound together by M. Durkheim's well-known theory of the group-consciousness, but this theory itself must be assigned different values in these three developments.

It is in the more purely anthropological aspect that this book is most successful. Here M. Durkheim's views must be judged in comparison with those of the older interpreters such as Tylor, Müller, Lang, Frazer, Jevons, Robertson Smith, Mannhardt. As in most works of the sort, the author is most convincing when he sticks closest to the facts, when he is least metaphysical, and when he is engaged in refuting his predecessors. In fact, he is most convincing when he is showing us what the phenomena of primitive religion do *not* mean. M. Durkheim confines his observation almost entirely to Australia, and his theory of Australian totemism is distinctly the best that has yet been evolved. Why? Because he is able to show that totemism is not animal worship, that it is not derivative from ancestor worship, or from the "nature cult"; the totem is not a name; the group totem is not, as Frazer holds, a development from the conception totem. M. Durkheim's theory is the best because it is the nearest to being no theory at all. And when he comes

[5] See "Even Order Magic Squares with Prime Numbers," *The Monist,* Jan. 1916, Vol. XXVI, No. 1.

to state it in positive terms, he finds almost as much difficulty as his predecessors in avoiding intellectualization. His "group-consciousness" is a contribution. But is it capable of articulate expression? "The totem," he says, "is the flag of the clan." This is just what some of the earlier theorists have said. It is a "collective representation." It has for the group-consciousness a significance quite different from the significance which that animal or plant has for the individual consciousness. We are not sure that this means anything more than that it is incapable of explanation. Totem is the origin of the idea of force. "Religious force is nothing other than the collective and anonymous force of the clan....If religious force, in so far as it is conceived as incorporated in the totemic emblem, appears to be outside of the individuals and to be endowed with a sort of transcendence over them, it, like the clan of which it is the symbol, can be realized only in and through them; in this sense, it is imminent in them and they necessarily represent it as such." M. Durkheim has given reason to believe that the examination of the individual consciousness is inadequate to explain social phenomena. He does not convince us that his social psychology is anything but an admission of the inexplicable, that the "group-consciousness" and the "collective representation" are more than a definition of the limits of individual psychology.

We should have liked to discuss the theory of the "origin of the categories" at length; although the exposition of this theory is much slighter than its place in the analytical table of contents would lead us to expect. It is open to the same charge of negativity, and leaves epistemology, we think, precisely where it was before. The theory of the nature of religion is stated in the conclusion. We have only space to draw attention to one difficulty. On page 416 we read that "the real function of religion is not to make us think, to enrich our knowledge, nor to add to the conceptions which we owe to science others of another origin and another character, but....to make us act." On page 428 we find that a religion "is not merely a system of practices, but also a system of ideas whose object is to explain the world."

The whole book is intensely interesting. The translation is good, but of less literary finish than the original.

η

ELEMENTS OF FOLK PSYCHOLOGY. Outlines of a psychological history of the development of mankind. By *Wilhelm Wundt.* Authorized translation by *E. L. Schaub, Ph.D.,* Professor of Philosophy in Northwestern University. New York: Macmillan. Price 15s. net.

Durkheim's *Formes élémentaires de la vie religieuse* and Wundt's *Elemente der Völkerpsychologie* appeared in the same year, 1912. In the preface to the present work Professor Wundt states the difference in method between the earlier *Völkerpsychologie* and this shorter book. "Instead of considering successively the main forms of expression of the folk mind, the present work studies the phenomena, so far as possible synchronously, exhibiting their common conditions and their reciprocal relations....The chief purpose of investigations in folk psychology must be found in a synthetic survey." This is in accord with Durkheim's theory that the religious phenomena must not be isolated by the investigator from the rest of the social life of a people. Otherwise the books of the two men are strikingly different. Durkheim's psychology is metaphysics. Wundt's psychology is descriptive anthropology. His method is descriptive and historical. He divides the stages of culture into four: (1) primitive man, including prehistoric man and such existing tribes as the Veddahs, Bushmen and Negritos; (2) the totemic stage, including the Australians and the Iroquois; (3) the "age of gods and heroes," the age of the folk epic; (4) the "development to humanity," which includes a discussion of "world-empires" and "world-religions." In each of these stages he takes up cult, social organization, myth, art, language; except that in the last stage the treatment is vaguer and these divisions are abandoned.

In his account of primitive and savage society Wundt is in general sound, but unsatisfying. When we turn to totemism, for example, he gives the impression of painstaking common sense. He is certainly right in rejecting the "eugenic" theory of exogamy, and in combating the "conceptional" theory of the totem. But it is improbable that the group totem is (as Wundt apparently holds) an outgrowth of the individual totem. Wundt is an animist. "Totemic ideas arise as a result of the diremption of primitive soul ideas into the *corporeal soul* and the *breath-* and *shadow-soul*" (p. 192). The soul is regarded "as a moving form, particularly as an animal, a bird, a rapidly gliding snake, or a lizard." We are inclined to believe that this "breath-soul" which totemism introduces was at first, and in fully developed totemism, much more indefinite and impersonal than Wundt would lead us to suppose. And he does not succeed in showing the relation between totemism as a social organization and totemism as a religious cult.

For the rest, Wundt is less concerned with explaining motive and meaning than with explaining the development of forms. Thus, his account of art is taken up largely with the development of the stringed instrument out of the bow, and kindred problems; he engages in a discussion of the beginnings of domestication of animals. The major part of his subject matter, in short, is not psychological at all; it belongs, in the earlier stages, to descriptive anthropology, and in the later stages, to the philosophy of history. And of the role which the sexual instinct plays in the religion and mythology of primitive peoples (indeed in all religion) Wundt has almost nothing to say. The psychoanalysis of myths, pursued by some of Freud's disciples, is surely capable of throwing considerable light on the primitive mind. It is possible that Wundt is still under the domination of a Hegelian conception of history. Although he criticises Hegel for applying a "logical schematism which is in large measure imposed upon history," his own account is very rationalistic. The book is a sound and valuable handbook, enriched by Wundt's ideas. But we think that any further advance in folk psychology is conditioned by advance in individual psychology. η

VOL. XXVIII. APRIL, 1918 NO. 2

161

THE MONIST

CHRISTIAN THEOPHAGY: AN HISTORICAL SKETCH.

I. PRAEPARATIO EVANGELICA.

THOSE who have attended the celebration of a mass have witnessed the most ancient survival from a hoary antiquity. There, in the often beautiful church, in gorgeous vestments, with incense and chanted liturgy, the priest sacrifices a God to himself and distributes his flesh to be eaten by his worshipers. The Divine Son is offered to the Father as "a pure victim, a spotless victim, a holy victim,"[1] and his holy body and blood become the food of the faithful. The teaching of the Church is explicit on this point. The body eaten is the same as that once born of a virgin and now seated at the right hand of the Father; the sacrifice of the mass is one and the same as that of the cross, and is so grateful and acceptable to God that it is a suitable return for all his benefits, will expiate sin, and turn the wrath of the offended Deity "from the severity of a just vengeance to the exercise of benignant clemency."[2]

All this goes back to the time when man was just emerging from the animal; it is the most striking of the many instances of the conservatism of religion. The further back we go historically the more religious do we find our ancestors; the story of progress has been one of constant secularization. But there was a prehistoric time when there

[1] *The Missal*: Canon of the Mass.

[2] *Catechism of the Council of Trent,* transl. by J. Donovan, 1829, pp. 156ff.

was nothing that we would recognize as religion at all. Behind the savage culture that we know, when religion rules the tribes with a rod of iron, there must have been a period when the grandsons of the ape were accumulating their theological ideas. Their first concept was not, apparently, that of personal gods, but that of a vast mystery; it was the weird or uncanny quality of certain things they did not understand. Along with this was the overmastering power of tribal custom. They had the conservative instinct to the highest degree; as children and savages and certain neurotics[3] to-day, they felt an imperative need, the reason of which they could not explain, that things should be done in the ways to which they were accustomed. The real reasons, of course, lay deep in the laws of habit and imitation; but, because they could not understand this, they gave their acts a mysterious sanction, the taboo. It was in this, and the related idea of "mana," both of them founded in the sacredness, i. e., mysteriousness, wierdness, of certain objects and acts, that the germs of all religions lay. In the earliest stages the ape-men were unable to conceive of anything very personal and definite as god. Not only was the conception of Being "without body, parts or passions" impossible to them, but even an anthropomorphic god was too abstract. Nor was this period so remote as we sometimes think. Just as in Latin the word *sacer*, meaning both "sacred" and "accursed," retains the old connotation of "taboo," so in Greek θεός was used with a far wider significance than we should use the word "god." The fact of success was a "god" and more than a "god"; to recognize a friend after long absence is a "god"; wine is a "god" whose body was poured out in libation to the gods.[4] Nor was this mere poetry or philosophy; it was, to the speakers, literal prose.

[3] S. Freud, *Zwangshandlungen und Religionsübungen. Kleine Schriften zur Neurosenlehre.* 2d ed., 1909, 122ff.

[4] G. Murray, *Four Stages of Greek Religion,* 1912, p. 26.

This earliest stage of theology was totemism, at one time probably universal. The totem was a specially sacred thing connected, by some fancied resemblance, with the tribe—at that period Church and State in one. It was a sort of dreadful mascot; a thing usually an animal, that was felt to be akin to the tribe and that could bring both bad luck and good according to the treatment it received. Ordinarily it was treated with reverence, awe and fear; it could not be killed or annoyed. But at times when things were going badly, or there was urgent need of stimulating the crops on which the existence of the people depended, or the bravery of the men or the fecundity of the women which were no less essential, some more drastic form of government regulation of totems was felt to be desirable. How could the tribe absorb the good qualities of the sacred thing; its "mana," as some of us, or "grace," as others would say?

Compared with the first mystics who brooded over the problem of union with the divine, Caliban was a gentleman and a scholar, the exquisite flower of a long refinement by civilization. Practically the whole content of their experience, as far as it gave them any suggestion of union, was food and sex. The "god" must be either eaten, or united with his worshipers in sexual intercourse.[5] Both ideas have colored the language and thought of all religions, including Christianity.

The eating of the sacred animal, or, later, of the god in the form of an animal, is the one with which we are at

[5] See A. Dietrich. *Eine Mithrasliturgie,* 1910, pages 121 and the following. On sexual intercourse with deity in classical antiquity, see, for instance, *Alcestis,* 839; Josephus, *Antiquities,* Chapter XVIII, 3, 4. The analogy of sex in the union with God, witnessed by a thousand "brides of Christ" (cf. Mark ii. 19; Eph. i. 6; v. 32) is carried out by Staupitz (T. Kolde, *Die Augustiner-Kongregation,* 1879, p. 291) and Luther (*Vorlesung über den Römerbrief, Scholien,* 206). On homosexual ideas in mysticism, cf. Pfarrer O. Pfister, *L. v. Zinzendorf* (Schriften zur angewandten Seelenkunde, VIII, 1910). On pederasty as a "means of grace," analogous to the Christian "laying on of hands," cf. E. Bethe, "Die dorische Knabenliebe," *Rheinisches Museum,* LXII, 3, pp. 438ff, 1897.

present concerned. The classic example of it is that found by Robertson Smith in the works of St. Nilus, a hermit who lived on Sinai in the fourth century of our era.[6] He tells how the Arabs would sacrifice boys to the Morning Star, but, when boys failed, would take a white camel, and after wounding it mortally, would suck its blood and eat its raw and still living flesh. Robertson Smith thought of the camel as a tribal god; but he was partly wrong; it was really only the raw material from which gods are made.[7] The animal was devoured to get its "mana," its strength, swiftness and endurance, and doubtless other more subtle qualities. For the savage thought of all the original character passing over with the flesh and blood. If bread could strengthen man and wine make glad his heart.[8] surely the brave, strong, sacred body of an animal could impart its own excellence..[9]

The eating of an animal or in some cases a human being in the same sacramental way, has been found also in Australia,[10] in Nigeria, and among North American Indians.[11]

But the totem was not the only divine being eaten. In the primitive sacrament of the first-fruits, the spirit of the corn was thus absorbed by his votaries. Thus in Wendland, Sweden, to the present day, "the farmer's wife uses the grain of the last sheaf to bake a loaf in the shape of a little girl; this loaf is divided among the whole household and eaten by them. Here the loaf represents the corn-spirit conceived as a maiden." "The new corn is itself eaten sacramentally, that is, as the body of the corn-spirit."[12] A similar custom is found in Lithuania.[13]

"In one part of Yorkshire it is still customary for the

[6] J. E. Harrison, *Prolegomena to the Study of Greek Religion,* 1903, 486f.

[7] Murray, 35f.

[8] Psalm civ. 15. These words were quoted by Luther as applying to the bread and wine of the eucharist.

[9] J. G. Frazer, *The Golden Bough,* 3d ed., *Spirits,* 1912, II, 138.

[10] Frazer, *Totemism and Exogamy,* 1910, I, 120; II, 590; IV, 230ff.

[11] Frazer, *Spirits,* I, 18ff. [12] *Ibid.,* II, 48. [13] *Ibid.,* 49.

clergyman to cut the first corn; and my informant," says Sir J. G. Frazer, believes that the corn so cut is used to make the communion bread. If the latter part of the custom is correctly reported (and analogy is all in its favor) it shows how the Christian communion has absorbed within itself a sacrament which is doubtless far older than Christianity."[14]

Among the heathen Cheremiss on the Volga, when the first bread from the new crop of wheat is to be eaten, the villagers assemble in the house of the oldest inhabitant, open the eastern door and pray with faces toward it. The sorcerer or priest then gives each a mug of beer to drain; next he cuts and hands to every person a morsel of bread. "The whole ceremony," says the writer who has described it, "looks almost like a caricature of the eucharist."[15] In fact it is its crude prototype.

The Incas of Peru also ate bread and drank liquor in a manner compared by the Spaniard to the eucharist.[16]

The Aino of Japan also regard their cereal offering as an eaten god,[17]and the East Indians, Buru, call their sacramental meal "eating the soul of the rice."[18] "In all such cases," observes Frazer, "we may not improperly describe the eating of the new fruit as a sacrament or communion with a deity, or at all events with a powerful spirit." In many cases the rite was preceded by the administration of a purgative or emetic, the idea being to preserve the sacred food from contact with profane nourishment. Thus the Catholics take the eucharist fasting.[19]

In some cases the sacrament of the first-fruits was combined with a sacrifice or offering of them to the gods or spirits, and at times the latter element of the rite throws the earlier into the shade.[20] Here, too, the analogy with

[14] *Ibid.*, 51. [15] *Ibid.* [16] Prescott, *Conquest of Peru*, Chap. III.
[17] Frazer, *Spirits*, II, 52. [18] *Ibid.*, 54.
[19] *Ibid.*, 83. [20] *Ibid.*, 86.

the mass is striking, as in the connection made by Paul between the feast of unleavened bread, "Christ our passover sacrificed for us," and Christ the "first-fruits of them that slept."[21]

The custom of eating a god sacramentally was practised by the Aztecs before the discovery of Mexico. Twice a year, in May and December, an image of the great god Vitziliputzli was made of dough and then broken in pieces and solemnly consumed. Acosta says that the Aztec virgins made the paste of beets and maize, which they called the flesh and bones of Vitziliputzli, and adored as such. Then, after a holocaust of victims, the priests distributed the dough after the manner of communion. The people said that they ate the flesh and bones of God. A similar mystic communion was held by the Brahmans in India, upon which Frazer remarks: "On the whole it would seem that neither the ancient Hindoos nor the ancient Mexicans had much to learn from the most refined mysteries of Catholic theology."[22]

At the festival of the winter solstice the Aztecs first killed their god Huitzilopochtli in effigy and then ate him. They made their idol in the form of a man, from various seeds, with bones of acacia wood. A priest, who took the name and part of the god Quetzalcoatl pierced the image through and through, which was called killing it. Then they cut out the heart, which was given to the king, and divided the rest among the people. The name of the festival was "god is eaten."[23] As we shall see later on, at one time the Christian host was baked in the form of a man and stabbed by the priest.

When the Mexicans craved a closer union with the living god, they endeavored to attain it by cannibalism; making a man impersonate their deity and then devouring

[21] 1 Cor. v. 7f; xv. 20. [22] Frazer, *Spirits*, II, 89. [23] *Ibid.*, 90.

him.[24] A curious survival of communion with a god by eating his image is found among the Huichol Indians of Mexico, who have an idol carved from lava, bits of which they scrape off with their nails and eat.[25]

The Hindus furnish two further customs which are also found in Christianity. The Malas eat a goddess in effigy at the time of their marriage,[26] just as Catholics commune before wedding.[27] The Veddas of Ceylon make an offering to the spirits of the dead, which they eat sacramentally, believing that it will give them health and good luck. They even extend this inestimable privilege to their dogs, hoping that the heavenly food will make them better hunters.[28] Even so at the "palio," a horse-race held for centuries twice every year at Siena, which I myself have witnessed,[29] before the race the horses and jockeys are taken into a church, where the host is offered to the jockey to kiss and to the horse to smell. This powerful charm did not, however, when I witnessed the race, prevent one of the blessed riders from getting a bad fall.

But not all our examples of god-eating are to be found among "the beastly devices of the heathen." "In Europe the Catholic Church has resorted to similar means for enabling the pious to enjoy the ineffable privilege of eating the persons of the Infant God and his Mother. For this purpose images of the Madonna are printed on some soluble and harmless substance and sold in sheets like postage stamps. The worshiper buys as many of these sacred images as he has occasion for, and, affixing one or more of

[24] *Ibid.*, 92. [25] *Spirits*, II. 93. [26] *Ibid.*

[27] Decree of Council of Trent, C. Mirbt, *Quellen zur Geschichte des Papstthums und des römischen Katholizismus*, 3d ed., 1911, 251.

[28] C. G. Seligman, *The Veddas*, p. 130, quoted W. M. Groton, *The Christian Eucharist and the Pagan Cults*, 1914, 8.

[29] I saw the race, but not the consecration of the horses. This was witnessed by my sister, Dr. Winifred Smith, of Vassar College. So in Spain, I am informed, bullfighters take the sacrament before they enter the arena. As the danger of death is almost nil, it is probably conceived as a charm to strengthen them.

them to his food, swallows the bolus.... In his youth Count Hoensbroech and his devout mother used to consume portions of God and his Mother with their meals." The practice was officially sanctioned by a decree of the Inquisition, in July, 1903.[30]

It is a fact of the highest importance that the sacramental meal attained great prominence in many religions among the peoples of the Mediterranean during the centuries just preceding and just following the rise of Christianity. Such meals were in many cases interpreted by a refined culture in a way less gross than had been the case earlier. They were compared to the banquets given at funerals in memory of the dead; they were likened to the common meals at Sparta and elsewhere;[31] they were communion with the god simply in that he was the host and the worshipers his guests. Thus dinners of a purely social nature were sometimes held in temples in order to enjoy the company of the god.[32] But the fundamental idea, vaguely expressed but always present, was the old one, that the consecrated food was the means of obtaining obsession by a good spirit, of becoming identified with the god of the Mystery.[33] Caution had to be exercised lest bad demons would also enter the body of the communicant. So comparatively enlightened a philosopher as Porphyry[34] assures us that demons delight in impure meats and enter those who use them.

Fanatic Egypt saw nothing incongruous in treating her gods like cattle from whose milk or flesh divinity could be extracted. One of her Pharaohs achieved immortality by sucking the breast of a goddess;[35] another took a more

[30] Frazer, *Spirits,* II. 94.

[31] P. Gardner, *Religious Experience of St. Paul,* 1911, 110.

[32] *Papyri Oxyr.,* I, 110, edited by Milligan, p. 97; cf. Carpenter, *Phases of Early Christianity,* 251ff.

[33] K. Lake, *Earler Epistles of St. Paul,* 196.

[34] Eusebius, *Praeparatio evangelica,* IV, 23.

[35] Dietrich, 101.

drastic method: "His servants," we are told, "have captured the gods with a lasso, they have found them and brought them down, have bound them and cut their throats and taken out their entrails and carved them and cooked them in hot cauldrons. The king consumes their power and eats their souls. The great gods are his breakfast, the middle-sized ones his dinner and the small ones his supper. . . . . The king consumes all that comes to him. Eagerly he swallows all their magic power. He becomes an heir of might, greater than all heirs; he becomes lord of heaven, for he ate all the crowns and bracelets; he ate the wisdom of every god."[36]

The blood of Osiris was a great charm, which, poured in a cup of wine, made Isis drinking it feel love for him in her heart.[37] When the blood could not be procured, its place was taken by simple wine, consecrated by this hocus-pocus said seven times: "Thou art wine and not wine but the head of Athene. Thou art wine and not wine, but the bowels of Osiris."[38]

From Persia marched forth Mithra to dispute the empire of the world with Christ. His warriors told how the hero Saoshyañt would kill a bull and of his fat, mingled with the juice of the white haoma, would prepare a beverage assuring immortality to all who tasted it.[39] That the bull was a divine animal goes without saying, for how otherwise could his flesh be the "drug of immortality"?[40] The sacramental banquet, however, was also a love-feast, done in remembrance of the supper celebrated by the sun before his ascension.[41] It could only be partaken of after long initiation, and was rightly regarded at Rome as "a

[36] *Ibid.*, 100.

[37] Griffith, *Demotic Magical Papyrus*, p. 107. Reitzenstein, *Die hellenistischen Mysterienreligionen und Paulus*, 1910, 204.

[38] Kenyon, *Greek Papyri*, I, 105; Reitzenstein, 205.

[39] Dietrich, 102.

[40] As Ignatius called the eucharist. *Ad Ephesios*, 20.

[41] F. Cumont, *The Mysteries of Mithra*, 1903, pp. 158ff.

magical meal."[42] So similar was it to the Christian Supper that Justin Martyr informs us it was directly imitated from the institution of Christ by evil demons, who, "in the mysteries of Mithra, set forth bread and a cup of water with certain explanations in the ceremonial of initiation."[43] Tertullian also noted the resemblance, so dangerous for simple souls, between Mithraism and Christianity.[44]

Attis, the Phrygian god who was born of a virgin, and who died and rose again at Easter time, also left his followers a sacramental meal.[45] His worshiper could say: "I have eaten from the drum, I have drunk from the cymbal, I have carried the earthen dish." From pictures we know that this latter was carried on the head in exactly the style in which, in the Greek Church, the holy food of the eucharist was carried by the deacons.[46] Another point of similarity between the communions of Attis and Christ was the use in each of fish.[47]

The connection of fish with the eucharist, made as early as the composition of the Gospel of Mark,[48] and witnessed by inscriptions in the catacombs,[49] is another case of the absorption by the conquering cult of the elements of vanquished superstitions. One cannot, indeed, explain it, as has been done,[50] by saying that "Jesus found at Bethsaida . . . a local pagan cult of the widely-spread fish-god, availed himself of it, and spiritualized it by means of an etymolog-

[42] Dietrich, 102. Pliny, *Hist. Nat.*, XXX, 1, 6.

[43] Justin Martyr, *First Apology*, I, 66; Clemen, *Primitive Christianity and its Non-Jewish Sources*, 1912, 261.

[44] Reinach, *Cultes, Mythes et Religions*, 1905ff, II, 227.

[45] Frazer, *Adonis*, I, 272ff, 309f.

[46] Dietrich, 103f.

[47] M. Brückner, "Attis," *Die Religion in Geschichte und Gegenwart*, 5 vols., 1909ff.

[48] Mark vi. 38; Matt. xiv. 17; Luke ix. 13. That this meal was eucharistic will be shown later.

[49] An epitaph at Rome, dating 100-130, represents the eucharist by loaves and fishes. M. Goguel, *L'Eucharistie des origines à Justin Martyr*, 1910, 279.

[50] Eisler, *Transactions of Third International Congress of Religions*, II, 352.

ical coincidence between *lehem*, bread, *luhm*, fish, and *luhm*, breath or spirit." This is too uncritical of the documents, and assumes too much history in them. But of the connection there can be no doubt. Dagon, meaning "fish," was worshiped by the Philistines (Judges xvi. 23), and Lucian tells us of fish kept in sacred fountains from which they were ritually taken and eaten.[51] The designation of Christ as Ἰχθύς was not, as commonly stated, an anagram, but a genuine case of syncretism. He was called the Big Fish and his worshipers little fishes. Thus an ancient Christian inscription of Abercius says: "Faith shows me my way everywhere and furnishes my food: even a fish from a fountain, large and pure, which a chaste virgin captures." An allusion to baptism is often seen in this, though it much better suits the eucharist, or perhaps the ancient custom of administering the eucharist immediately after baptism. In former centuries eating fish was symbolic of eating Christ's flesh, just as now it is eaten by Catholics on fast-days, especially as a preparation for communion.

Rome, too, did not lack her sacramental meals. One of the titles of Jupiter was "dapalis," "he of the feast," and the priest who presided at the sacrifice was called "epulo," "feaster."[52] At ancient Aricia, near Rome, it is believed that loaves were baked in the image of the King of the Wood and eaten sacramentally.[53]

Something has been made of the fact that the students of comparative religion have found the eating of a god in so many and diverse religions. Surely, it is said, one key is too simple to fit so many locks; the day of the vegetation god, killed and eaten and reviving will go the way of the sun-god theory of Max Müller. When one sees the vegetation myth in Australia and Mexico, in Orestes and Ham-

[51] Reinach, C. M. R., III, 46ff.

[52] Dietrich, 229.

[53] Frazer, *Spirits*, II, 95.

let,[54] he must be the victim of a monomania. But it is certain that many other religious ideas, whether true or delusive, the existence of gods, immortality, the power of witchcraft, have until recently been held all but universally: *semper, ubique et ab omnibus.* Communion with a god by eating him is just one of those ideas which arise naturally in a certain stage of culture, and, under myriad forms, survive in a hundred different societies. A similar one is baptism; the idea found in very many cults, that, by washing, a man can cleanse his soul as well as his body.

So in Greece we find the pre-Christian communion in many forms. After the great age of art and philosophy there was a reaction which Gilbert Murray has called "The Failure of Nerve." The hungry generations trod men down as they had never done before; there went up a great cry for respite from this world, for salvation. To supply this neeed arose the Mystery Religions, of which Orphism is a good example, promising rest for the soul and union with God. But they kept the old forms to a great extent, particularly the myth and ritual of the god torn to pieces and devoured by his adorers.

Traces of this belief are found in the ancient Minoan civilization.[55] A god was there sacrificed in the form of a bull, possibly at some earlier period than we know in the form of a child.[56] In many an old Greek legend we see the original sacrifice and devouring of a divine animal. So common were these *motivs* that Greek has special words to designate them: σπαραγμός for the ritual tearing of the animal to pieces and ὠμοφαγία for the feast of raw flesh. Thus Acteon was a sacred stag worshipped at Plataeae

[54] Gilbert Murray, *Hamlet and Orestes,* 1914. "One of my friends has assured me that every one knew it before; another has observed that most learned men, sooner or later, go a little mad." He refers primarily to the Hamlet of Saxo Grammaticus.

[55] Farnell, *Greece and Babylon,* 26.

[56] Harrison, *Prolegomena,* 489. On the omophagia in general, 478ff.

and torn by adorers who called themselves does;[57] Hippolytus was a horse rent by horses;[58] Orpheus was a fox similarly treated by "vixens," as, quite rightly no doubt, his devotees called themselves.[59] In Orpheus the early Church justly saw a prototype of Christ.[60] It is interesting to note that the worshipers frequently, if not always, called themselves by the name of the beast or god they adored. Thus the followers of Bacchus were called Bacchi and Bacchae;[61] thus the worshipers of Jesus "put on Christ." By eating the eucharist they became ἔνθεοι ἐν Χριστῷ just as did the votaries of Dionysus.[62]

Zeus himself was sacrificed at Athens in the form of a bull. At this feast, called the buphonia, near the summer solstice, an ox was killed, eaten and restored to life in pantomime.[63] It is interesting to note that the feast—Δαὶς—became a personified divinity, just as the Roman Church, in instituting the feast of Corpus Christi day, near midsummer, has presented the mystery of the mass as an object to the adoration of the people. At Delphi also a bull, called Hosiater, or the Consecrator, and Isodaitos, "He of the equal feast," was immolated.[65] Plato doubtless had in mind one of these ceremonies when he describes[66] the killing of a bull in Atlantis, and the drinking of his blood mingled with wine. This was accompanied by an oath to deal justly, reminding us of the oath (*sacramentum*) that Pliny says the Christians took at their sacred meal.[67]

In the Eleusinian mysteries animals were immolated

[57] Reinach, C. M. R., III, 24ff.

[58] *Ibid.*, 54ff.

[59] *Ibid.*, II, 85ff.

[60] Harrison, *Prolegomena*, 474; Reinach, C. M. R., II, 83.

[61] Farnell, *Cults*, V, 150ff.

[62] Lake, *Epistles of Paul*, 214; Reinach, C. M. R., II, 105.

[63] Harrison, *Themis*, 141.

[64] *Ibid.*, 146.

[65] *Ibid.*, 155.

[66] *Ibid.*, 163; Plato, Critias, 119.

[67] Pliny, *ep.* 96.

to Demeter and their flesh eaten on the spot;[68] there was also a meal of κικέων, a mixture of grain and water, but there is no evidence that this was regarded as representing the goddess.[69]

But of all the "mysteries" known to us, that of Dionysus bears the closest resemblance to that of Christ. The god of wine died a violent death and was brought to life again; his "passion," as the Greeks called it, and his resurrection were enacted in his sacred rites. According to the common legend the son of Zeus and his daughter Proserpina was given by jealous Hera to the Titans, who tore him to pieces, boiled his body and ate it with herbs. His heart was taken back to Zeus and Semele, from whom he was reborn.[70] As this doctrine was spiritualized his resurrection was represented in a different way and was followed by an ascension to heaven.[71] Thus was inculcated the doctrine of immortality; Plutarch consoles his wife for the death of a daughter by the belief in a future life as taught by tradition and revealed by the mysteries of Dionysus.

All this was enacted ritually in various parts of Greece. As is so often the case, the ritual preceded the legend, which was invented to explain a misunderstood custom, in this case the sacramental eating of a totemic bull,[72] or, in some cases, of a kid,[73] for the god inherited the ritual of both beasts. Thus it was celebrated at Delphi;[74] and thus in Crete. In all cases the animal was torn to pieces and a fragment of his flesh given to each worshiper and eaten raw as a sacrament, in order to impart to each some of the divine life.[75] At first this was doubtless conceived of as purely a physical benefit, but by the fourth century, B. C.,

[68] Foucart, *Les Mystères d'Eleusis*, 1914, 375f.

[69] *Ibid.*, 378ff.

[70] Frazer, *Spirits*, I, 12ff; Reinach, C. M. R., II, 58ff.

[71] Justin Martyr, *First Apology*, 54; *Dialogue with Trypho*, 69.

[72] Reinach, C. M. R., II, 58ff.

[73] *Ibid.*, 96.

[74] Harrison, *Prolegomena*, 440.

[75] Frazer, *Spirits*, II, 16.

the excellent moral effects of the initiatory feast are stressed. Thus, in a fragment of Euripides's Cretans, one speaks of "lengthening out a life of purity from the day when I became an initiate of Idæan Zeus, and a herdsman of night-roaming Zagreus [Dionysus], a celebrant of the meal of raw flesh."[76] At a later stage of Orphic theology, some offence was taken at the idea of killing a god, and the myth was changed to make the deity the sacrificer and communicant. Thus we find a god sacrificed to himself, and eating his own flesh,[77]—a striking parallel to the Last Supper and to the mass. It was not always in the interests of humanity to anthropomorphize the rite too much, for in Chios and Tenedos Dionysus was represented by a human victim who was subjected to the barbarous rite of holy cannibalism.[78]

Now all this seems to us such revolting savagery that it is hard to believe that it became imbedded in a religion of great moral purity and lofty idealism. Such, however, is the case. "The belief in the sacrifice of Dionysus himself and the purification of man by his blood," remained, according to Gilbert Murray, "a curious relic of superstition firmly imbedded in Orphism, a doctrine irrational and unintelligible, and for that reason wrapped in the deepest and most sacred mystery."[79] But the rite continued; for the wild worshipers roamed in the woods and tore to pieces and ate raw whatever animals they could cope with. "It is noteworthy, and throws much light on the spirit of Orphism, that apart from this sacramental tasting of blood, the Orphic worshiper held it an abomination to eat the flesh of animals at all. . . . It fascinated him just because it was so incredibly primitive and uncanny; because it was a mystery which transcended reason."[80] Euripides has trans-

[76] Quoted, Kennedy, *St. Paul and the Mystery Religions,* 1913, 257.

[77] Frazer, *Spirits,* I, 23.

[78] *Ibid.,* 24. [79] *Bacchae,* note on p. 85f. [80] *Ibid.,* p. 86.

muted the beastly rite into immortal poetry. He thus describes the rending of the animals:[81]

> "Great uddered kine then hadst thou seen
> Bellowing in sword-like hands that cleave and tear,
> A live steer riven in sunder, and the air
> Tossed with rent ribs of limbs of cloven tread
> And flesh upon the branches and a red
> Rain from the deep green pines. Yea, bulls of pride,
> Horns swift to rage, were fronted and aside
> Flung stumbling by those multitudinous hands
> Dragged pitilessly."

And through it all the maenads feel the divine presence, and adjure it, "O God, Beast, Mystery, come!" It is Dionysus who is the god and the bull, to whom Pentheus speaks, when he sees him, as follows:[82]

> "Is it a Wild Bull this, that walks and waits
> Before me? There are horns upon thy brow!
> What art thou, man or beast? For surely now
> The Bull is on thee!"

When the new religion was introduced into Italy, it ran a course for a time something like that of Christianity later. In the first place its votaries were accused, like the Christians, of celebrating holy meals followed by sexual debauches.[83] Later they were suppressed by the government.[84] That nothing might be wanting to make the parallel with Christianity, the word "sacrament,"[85] originally a military oath, was applied by the Romans to the initiation. Indeed it is certain that that word had the connotation of consecration long before the rise of the Roman Church or its founder. It was employed, for example, by Apuleius,

[81] *The Bacchae,* line 700ff; *ibid.,* p. 44.

[82] *Ibid.,* line 920ff, p. 55.

[83] Livy, XXXIX, 8, 5, quoted Reitzenstein, 88.

[84] E. Gibbon, *Decline and Fall of the Roman Empire,* Chap. XV. He says that the language of Tacitus in describing the introduction and attempted suppression of the Christian worship, is almost similar to that of Livy about the Bacchanalia.

[85] Livy, XXXIX, 15, 13; Reitzenstein, 66.

for the visible sign of the spiritual grace vouchsafed to the worshipers of Isis.[86]

As men became softer and more fastidious, substitutes were found for the raw flesh and blood which were originally elements of their communion. Thus the sacred ivy, regarded as an impersonation of Dionysus, was substituted for his flesh,[87] and wine for his blood.[88]

The connection of wine and blood was as familiar to antiquity as it is to us through the eucharist. It was often an offering to the gods and a means of communion with them.[89] The blood was the life; who imbibed it absorbed the spirit. A Greek word for soul, θυμός, is etymologically *fumus*, the hot "steam" from the blood.[90] The Romans sealed their oaths by drinking a mixture of wine and blood called *asseratum*.[91] Among the Hebrews, too, wine was called the "blood of the grape,"[92] Offerings of bread and wine were made to Asklepios, the god of healing.[93]

It must be remembered that this tradition of the eaten god was kept up by the mysteries among the lower strata of society only. In the world of art and letters best known to us there prevailed an enlightened skepticism. Not many wise, not many noble, were called to salvation by the blood of Bacchus or of Attis. The expressed opinion of a Roman philosopher as to the Real Presence is very much what the expressed opinion of a modern scientist is now: "When we call corn Ceres and wine Bacchus," says Cicero,[94] "we use a common figure of speech; but do you imagine that any-

[86] Apuleius, XI, 15, quoted *ibid.*

[87] Plutarch, *Quaestiones Rom.*, 112; Clemen, 258; J. Rendel Harris, "Origin of the cult of Dionysus," *Bulletin of J. Rylands Library*, 1915, p. 119ff.

[88] Justin Martyr, *First Apology*, 54; *Dialogue with Trypho*, 69.

[89] Kircher, *Die sakrale Bedeutung des Weines im Altertum*, 1910, 45.

[90] *Ibid.*, 78.

[91] *Ibid.*, 83.

[92] *Ibid.*, 85. They also treated wine as blood, pouring it out at the base of altars. Robertson Smith, *Religion of the Semites*, 1894, p. 230.

[93] Kircher, 92f.

[94] *De Natura deorum*, III, 16, 41. Frazer, *Spirits*, II, 167

body is so insane as to believe that the thing he feeds on is god?" The answer then, as now, was in the affirmative.

II. PAUL AND HIS SYMMYSTAE.

"The most excellent of the sacraments"[1] was borrowed by the Christians from the older mystery religions. That they attributed the institution of their rite to their founder was inevitable. Many of the classic myths originated as explanations of ritual, in the desire to show how Dionysus or Attis or Osiris had once done what their initiates now re-enacted.[2] The account of the Last Supper is but an etiological cult story, analogous to the Greek myths or to the Hebrew fable of the Passover in Exodus xii, designed to authorize a custom otherwise established in the earliest community.[3] "The Christ of Mark," says Loisy, "is like the gods of the mysteries; what he does is the type of what happens to his worshipers and what they must do....The idea and form of this institution were suggested....by Paul, who conceived them in a vision, on the model of the pagan mysteries."[4] In fact, as soon as any institution was established, firmly or otherwise, it was fathered on Christ, or at least on the apostles. Thus the mingling of water with wine was said by Cyprian to have begun by Jesus;[5] thus the self-communion of priests was wrongly said to have descended "as it were from apostolic tradition."[6] On the way the Gnostics attributed all their peculiar institu-

[1] So called by the Council of Trent, Mirbt, 226.

[2] Reinach, C. M. R., II, p. vi, says it is simply a matter of good faith to apply to the Gospels the same process which has been generally acknowledged as the correct solution of the classic myths. Some Christians now admit the likeness of the eucharist and the earlier theophagy. See *Catholic Encyclopædia,* and E. A. James, *Primitive Belief and Ritual,* 1917.

[3] So called by Heitmüller, R. G. G., I, 25, though illogically he tries to extract some history from the ἱερὸς λόγος. Long arguments against his position and that of Reitzenstein and Dietrich in Schweitzer, *Paulinische Forschung,* 152ff, and by G. P. von Wetter in *Z. N. T. W.*, 1913, pp. 202ff.

[4] Loisy, *L'évangile selon Marc,* 1912, 405.

[5] Quoted in *Catechism of Council of Trent.*

[6] Council of Trent, Mirbt, 228.

tions to Jesus a long and instructive essay has been written by C. Schmidt.[7]

But though we see nothing historic in the Last Supper, and are convinced that Paul founded the eucharist, it is worth while asking what analogous conceptions, if any, prevailed in the pre-Pauline community about the sacramental use of food. We shall find that there are two such conceptions plainly discernible; the first that of the Messianic feast, the second that of a spiritual nourishment. Both these are founded in the Old Testament. There, though sacrifice is a covenant with Yaweh, and a communion meal, there is no trace of the eating of a divine animal.[8] The Jews of the historic period had gone beyond this conception, just as had the "Olympian" religion of the Ionians, represented by Homer. But the idea that when the Messiah came he should eat and drink with his elect, is found in many places in the Jewish writings,[9] and doubtless considerably influenced the Christian supper. It is represented in the document known as "Q" by the marriage feast of the king's son.[10] It is also prominent in the Apocalypse,[11] though neither it nor Q nor the Jewish-Christian epistles of James or Jude or 2 Peter, know anything of the eucharist.[12] Thus also Luke makes Jesus say to his disciples: "And I assign unto you, as my Father has assigned unto me, a kingdom, that ye may eat and drink at my table in my kingdom."[13]

[7] *Texte und Untersuchungen,* VIII.

[8] H. P. Smith, *The Religion of Israel,* 1914, pp. 39f.

[9] Isaiah lv. 1ff; lxv. 12ff; xxv. 68; Enoch, xxiv and xxv; Test. Levi, xxiii. 11 and lxii. 14. Schweitzer, *Quest of the Historical Jesus,* 1910.

[10] Matt. xxii. 1-14; Luke xiv. 15-24.

[11] Apoc. ii. 7, 17; iii. 21; vii. 16f; xix.

[12] The idea that Apoc. ii. 17 refers to the eucharist is untenable. *Hibbert,* XI, 140ff. "Q" has nothing even on the Passion. Harnack, *Sayings of Jesus,* 1908, 233. W. Haupt, *Worte Jesu und Gemeinde-Ueberlieferung,* 1913.

[13] Luke xxii. 30. It is uncertain whether the original was in Q. Probably not, as Matt. lacks the verse, and the word διατίθεμαι is eucharistic.

The other idea which amalgamated naturally with the eucharist was that of a spiritual nourishment. "Man cannot live by bread alone," says the Deuteronomist, "but by every word that proceedeth out of the mouth of God."[14] The manna was to the Psalmist "bread from heaven."[15] Isaiah offered bread and wine and milk of a spiritual nature without money and without price.[16] "Those who eat me," says Wisdom in Ecclesiasticus,[17] "will always hunger for me; those who drink me will always thirst for me again." Philo, too, spoke of the Logos as the bread from heaven.[18] Nor do I doubt that this is the meaning of the fourth petition in the Lord's Prayer: "Give us this day our supernatural [i. e., spiritual] bread." The Greek word ἐπιούσιος is translated in the Latin versions *supersubstantialis,*[19] followed by Wyclif with "bread above other substance" and the Douai Bible with "supersubstantial bread." One ancient Latin manuscript in the British Museum reads "Panem verbum Dei celestem da nobis hodie,"[20] evidently a gloss, but a good one. To express so simple an idea as "daily" the author of Q would certainly not choose a word so rare that it is not met with elsewhere, was absolutely unknown to learned Origen,[21] and puzzled early evangelists.[22] Moreover "daily" would be tautological, having just been said.[23] Further, the petition for bread would

[14] Deut. viii. 3. [15] Psalm lxxviii. 24f. [16] Isaiah lv. 1f.

[17] XXIV, 29. Many other references in Stone, *History of the Doctrine of the Holy Eucharist,* 1909, i. 3.

[18] Quoted Pfleiderer, IV, 23ff.

[19] In Matt. vi. 11. The translation of the same word in Luke xi. 3 is *quotidianus,* and this form is adopted in the ritual. Most modern versions follow this second rendering, "daily," which is also supported by F. S. Chase, *The Lord's Prayer,* 1891; F. Blass, *Grammatik des neutestamentlichen Griechisch,* fourth edition, 1913, § 123; Dobschütz, *Harvard Theological Review,* 1914, p. 313.

[20] E. S. Buchanan, ἐπιούσιος, *Expositor,* 1914, p. 423.

[21] *De oratione,* XXVII, 7.

[22] The Gospel of the Hebrews rendered "to-morrow's bread." The Acts of Thomas (Pick, *Apocryphal Acts,* 1909, 144) omitted this petition altogether. Cf. Cyril's *Catechetical Lectures,* quoted by Stone, I, 91.

[23] Matt. vi. 25; Luke xii. 22.

contradict the injunction given a little later, to take no thought for what to eat or to drink, but to seek first the kingdom. All the other petitions in this early Christian prayer are for spiritual blessings, and the intrusion of the mere bodily needs would be strange. Etymologically the word is compared by Liddell and Scott to ἐπηετανός, but it seems better to derive it from ἐπι meaning "super" and οὐσία meaning "substance," and to compare it with ἐπουράνιος, "superheavenly," in other New Testament writings.

The idea of a spiritual nourishment offered directly by God to the believer is also developed in the Johannine writings and in what was one of their principal sources, the Odes of Solomon. Written probably by a Disciple of the Baptist at Ephesus very near the middle of the first century,[24] one of these poems (XIX, 1ff) says: "A cup of milk was offered to me and I drank it in the sweetness of the delight of the Lord. The Son is the cup, and he who was milked is the Father and she who milked him is the Holy Spirit."[25] Elsewhere in these poems, which nowhere have any allusion to the eucharist,[26] milk and honey are spoken of as the mystic food of believers.[27] It is interesting to note in this connection that milk and honey were added to the first communion in the Monophysite churches of Armenia.[28] This would seem to indicate that feeding with milk was actually done as symbolic of the new and spiritual birth of the child. Sallustius[29] speaks of "feeding on milk as though we were being born again," in the ritual

[24] Preserved Smith, "The Disciples of John and the Odes of Solomon," *Monist*, 1915, pp. 161-190.

[25] Reading of Burkitt's manuscript of the Odes, *Journal of Th. Studies*, 1912.

[26] *Monist*, 186.

[27] J. Rendel Harris, *The Odes and Psalms of Solomon*, second edition, 1911, p. 80.

[28] Conybeare, "Eucharist" in *Encyclopædia Britannica*.

[29] "On the Gods," translated by G. Murray, *Greek Religion*, p. 193.

of Attis. Perhaps the same thought lies back of Paul's simile "milk for babes" (1 Cor. vi. 5). But it is plainest in the First Epistle of Peter, so called, in the words translated in our Revised Version:[30] "As newborn babes, long for the spiritual milk which is without guile." The Authorized Version in this case came nearer to the true meaning when it rendered λογικὸν ἄδολον γάλα "sincere milk of the word," provided only we write Word with a capital, and understand it of the Logos.

But neither the celestial bread nor the milk of the Logos constituted a ritual meal. It is practically certain, however, that the first Christian community had such prior to the institution of the eucharist by Paul.[31] Precedent for such could be found in Jewish custom,[32] and among the Essenes[33] and probably also in the custom of the Disciples of John.[34] This meal was known as the "love-feast," and persisted in certain quarters side by side with the eucharist for many years. It is alluded to by Jude[35] and described by Tertullian.[36] Whether any traces of it can be found in the Gospels or in Acts, colored as these are by Pauline theology, is more than doubtful.

If we read the books of the New Testament in the order in which they were written, the first account of the eucharist is found in 1 Corinthians, written from Ephesus at about Easter time, probably in the year 55. There Paul speaks of its institution in words (xi. 23ff) which, to bring

[30] 1 Peter ii. 2. On this Reitzenstein, *Mysterienreligionen,* 156, and on similar thoughts in Egyptian religions, *ibid.,* 157.

[31] Achelis, *Das Christentum in den ersten drei Jahrhunderten,* 1912, I, 172-83; II, 78ff; Carpenter, 251ff.

[32] Josephus, *Ant.,* XIV, 10, 8; S. J. Case, *The Evolution of Early Christianity,* 1914, p. 340.

[33] R. G. G. I., 38.

[34] The Mandaeans or Sabaeans, the spiritual descendants of the Disciples of the Baptists, had a supper consisting of "bites and water." M. Brückner, *Der sterbende und auferstehende Gottheiland,* 1908, p. 47.

[35] Jude, 12.

[36] Tertullian, *Apology,* cap. 39.

out their literal meaning, I translate into unavoidably awkward English: "For *I* received over from the Lord that which also I delivered over to you, how that the Lord Jesus in the night in which he was delivered over, took bread, and having blessed it, broke and said: This is my body which is for you. This do in remembrance of me. In like manner also the cup after supper, saying, This cup is the new covenant in my blood. Do this, as often as you drink it, in remembrance of me. For as often as ye eat this bread and drink this cup, ye proclaim the Lord's death till he come. So that whoever eats the bread and drinks the cup of the Lord unworthily is guilty of the body and blood of the Lord. But let a man try himself and thus eat of the bread and drink of the cup. For who eats and drinks not discerning the body is eating and drinking judgment to himself. For this cause many among you are weak and sickly and not a few sleep."

It is an official dogma of the Catholic Church that these words should be taken as history.[37] The Catholics, less subjective than the Protestants, admit that Paul received a special revelation on the subject, only they say that it revealed to him exactly what really happened.[38] Modern Protestant scholars have felt the intrinsic absurdity of this and have argued that Paul could not have received a special revelation on this point, because it would not be in accordance with "the acknowledged principles of economy in the use of miracles," for Paul to receive by revelation what might have been learned by other means.[39] This old-fashioned point of view will have less weight with impartial scholars than the other argument advanced, that Paul uses the words "received" and "delivered" in his account of the death and resurrection of Jesus, which, it is commonly

[37] Syllabus of Pius X, 1907, Mirbt, p. 409.

[38] Renz, *Geschichte des Messopfer-Begriffs,* 2 vols., 1901f, I, 122.

[39] Lambert, *The Sacraments in the New Testament,* 1903.

believed, he learned from the other apostles. But reasons have been put forward to show that here, too, Paul is really giving the results of his own subjective visions.[40] These very words, "received" and "delivered," were used in the Pirke Aboth, i. 1, of what Moses received directly from Jehovah on Sinai and delivered to the elders.[41] They were also technical terms of the pagan mysteries.[42] If we will only listen to Paul himself we shall learn whence he got his doctrine: "The gospel which was preached by me is not after man. For neither did I receive it from man, nor was I taught it, but it came to me through revelation of Jesus Christ....When it was the good pleasure of God ....to reveal his Son in me,....immediately I conferred not with flesh and blood, neither went I up to Jerusalem to them which were apostles before me: but I went up into Arabia: and again I returned unto Damascus. Then *after three years* I went up to Jerusalem to visit Cephas and tarried with him fifteen days."[43] Later, Paul was kind enough to instruct these Jewish apostles in the gospel he had received, though he dared not to do it publicly.[44] How he obtained these revelations in Paradise he tells elsewhere.[45] As he "received" the story of Christ's death and resurrection thus,[46] he was perfectly consistent in asserting "Christ was raised according to *my* gospel."[47] The whole thing was "God's wisdom in a mystery,"[48] and this mystery itself was Christ: "He who was manifested in the flesh,

[40] Preserved Smith, "A New Light on Peter and Paul," *Hibbert*, July, 1913. The conclusions here advanced have been accepted by Solomon Reinach who translated the article in French and published it in the *Bibliothèque de propagande*, Oct. 15, 1913.

[41] J. Weiss, in *Archiv für Religionswissenschaft*, 1913.

[42] Clemen, 233.

[43] Galatians i. 11ff.

[44] *Ibid.*, ii. 2

[45] 2 Cor. xii. 2ff.

[46] 1 Cor. xv. 4.

[47] 2 Tim. ii. 8. The pericope, according to many scholars, is Paul's, though the whole epistle is not.

[48] 1 Cor. ii. 7.

justified in the spirit, seen of angels, preached among the nations."[49]

The German Wrede has put us under a great debt by at last writing a biography of the Tarsian,[50] showing both how it was possible psychologically for Paul to evolve these myths and possible historically for him to foist them on the Christian Church. But this is not the place to discuss the whole extent of Paul's mythology; all that here concerns us is his derivation of the eucharist. *A priori,* the possibility of his dependence on the Mysteries cannot be denied.[51] It has been proved from linguistic evidence, proved to the hilt, that Paul was saturated in the current conceptions of the Mystery Religions,[52] prominent among which was that of the eaten body of the Saviour God, who, in human form, should live, suffer violent death and rise again. He himself speaks of "the table of demons," i. e., of false gods, and of "communion with demons" as analogous to the communion with Jesus (1 Cor. x. 21). Moreover, in this particular case the evidence of his derivation of his doctrine from a vision is peculiarly strong. Hardly any scholar, not under the double dogmatic prepossession of the historicity of the Last Supper and the improbability of revelations, has denied it. Among a vast number who have admitted the vision are Chrysostom, Osiander, Calvin, Gardner,[53] Conybeare[54] and Reitzenstein.[55]

In fact the force of the language is overwhelming. The

[49] 1 Tim. iii. 16. The letter is not by Paul, but well expresses the primitive Christian idea.

[50] *Paul,* English translation by J. F. Carpenter, 1908. According to Schweitzer the book belongs "not to theology but to world-literature."

[51] Heitmüller in *R. G. G.,* "Abendmahl."

[52] Reitzenstein, *Mysterienreligionen und Paulus, passim.*

[53] Gardner, *Exploratio Evangelica,* second edition, p. 453, gives references for the older scholars. He here withdraws his former theory that Paul derived the Supper from the Eleusinian Mysteries, but says that Paul was influenced by mystery concepts in general.

[54] *Myth, Magic and Morals,* 251ff.

[55] *Mysterienreligionen,* 50f.

emphatic "I," the positive statement that the doctrine was received "from the Lord," ought to be decisive. But this is not all. Note that Paul uses the same word for that which he "delivered over" to the Corinthians, and that which was done on the night in which the Lord was "delivered over." Prof. W. B. Smith has pointed out that this could not mean "betrayed," as it is commonly rendered, but must mean "delivered up" or "surrendered."[56] This explanation has now been adopted by Messrs. A. Robertson and A. Plummer, in their Commentary on 1 Corinthians.[57] They state that the words in question refer "perhaps chiefly to the Father's surrender of the Son, and the Son's self-sacrifice may also be included." Better, possibly, to say that Jesus was himself, as a mystic concept, delivered over to Paul and by him so delivered over to his neophytes.

One more point requires exegesis before we proceed to the consideration of Paul's eucharistic doctrine in general. The words "new covenant," here used first of the cup, were probably borrowed by Paul from the Jewish Messianic sect of the Zadokites,[58] who made a "new covenant" at Damascus, shortly before Paul's sojourn there. The Greek word διαθήκη commonly means "testament," and is so used by the author of the epistle to the Hebrews.[59] But as it is the equivalent of the Hebrew *berith,* and was used to translate this word in the Septuagint,[60] "covenant" is almost certainly the true meaning of the word here.[61]

What is Paul's understanding of the words "This is my body"? It is certain that he took them literally. The "*hoc est corpus meum*" which has been decisive for the

[56] *Ecce Deus,* English edition, 1912, pp. 303ff. German edition, 1911.

[57] *International Critical Commentary,* p. 243.

[58] Fragments of a Zadokite Word, *Apocrypha and Pseudepigrapha,* ed. R. H. Charles, II, 792.

[59] Hebrews, ix. 15ff.

[60] E. g., Job xxxi. 1.

[61] Dibelius, *Das Abendmahl,* 1911, 76ff.

Catholic Church, and which, Luther declared, was "too strong" for him, meant exactly what it said. The reason why many Protestants have maintained the contrary is simply that they believed it impossible themselves. Of course it is impossible—but that does not mean that Paul did not believe it. Kirsopp Lake puts the point aptly: "Much of the controversy between Catholic and Protestant theologians has found its center in the doctrine of the eucharist, and the latter have appealed to primitive Christianity to support their views. From their point of view the appeal fails; the Catholic doctrine is much more nearly primitive than the Protestant. But the Catholic advocate in winning his case has proved still more: the doctrine which he defends is not only primitive but pre-Christian."[62] And again: "It is necessary to insist that the Catholic is much nearer to early Christianity than the Protestant."[63]

The part of the text stressed by those who wish to make the rite merely commemorative is, "Do this in remembrance of me." Let us hear an expert on the subject: "Frankly," says Reitzenstein,[64] "I can never interpret these words of a mere commemorative meal, such as the Greek cult of the dead knows. The whole sacramental teaching which Paul adds immediately, contradicts that interpretation. The words can be better understood in a mystical sense analogous to that of an approximately contemporary narrative in a magic text in which Osiris gives Isis and Horus his blood to drink in a cup of wine, in order that they may not forget his death, but must seek him in yearning plaint, until he again becomes alive and unites with them." This then explains also the words "ye proclaim the Lord's death till he come." If the eucharist be regarded as analogous to the meals held in memory of dead friends by the Greeks,

[62] Lake, *Earlier Epistles of St. Paul,* 215.

[63] *H. T. R.,* 1914, p. 429.

[64] *Mysterienreligionen,* 51.

it must be recognized that these meals, also, were sacrificial."[65]

In the same sense must be read the words that he who eats and drinks unworthily, not discerning the body, eats and drinks judgment (or "damnation") to himself. The meaning is so clear that Mr. Scott is able to say that practically all commentators agree that the phrase refers to the failure on the part of the worshiper to see that the bread represented the body of Christ.[66] "Behind these words," says Bousset quite rightly, "we catch glimpses of definitely sacramental feeling, the belief in the marvelous virtue of sacred food, for weal or woe."[67] How perfectly crude were Paul's ideas of this magical effect is brought out in verse 30, where he attributes the prevalence of sickness and death among his converts to the misuse of the holy food. But the benefits of the Christian mysteries did not go the length of guaranteeing salvation irrespective of conduct. Paul devotes the best part of a chapter to the confutation of this belief which had evidently gained currency among the Corinthians.[68] Indeed some of them turned their eucharists into drunken orgies.[69] Whether the abominable sexual disorders among them[70] originated in these debauches, cannot be told. Somewhat later the accusations were made against the Christians that they united "Thyestean banquets and Oedipean intercourse" at their meetings.[71]

Almost all that Paul says implies his belief that bread and wine were body and blood of Christ. Thus (1 Cor. x. 16): "The cup of blessing which we bless, is it not a sharing

[65] Lake, *Earlier Epistles*, 214.

[66] *Expositor*, August, 1915, 182ff. He himself, however, proposes that the body here means "fellowship," and "failing to discern it" means being unbrotherly.

[67] *Die Schriften des Neuen Testaments*, 1906f, ed. J. Weiss, *ad. loc.*

[68] 1 Cor. x; Lake, *Earlier Epistles*, 200 and 213.

[69] 1 Cor. xi. 21.

[70] 1 Cor. v.

[71] *R. G. G.*, I, 633. "Nachapostolisches Zeitalter" by Knopf.

of the blood of Christ? The bread which be break, is it not a sharing of the body of Christ?"[72] If we ask *how* he conceived this, the answer must be that he never raised the question of mode, but that he appears to have assumed the reality of his contention with a literalness far surpassing that of the Fourth Lateran Council. In classical antiquity symbol and reality were not separated as we separate them.[73] To Greek philosophy words were things, and that was its greatest weakness. So the personification of bread, wine, war and love as Ceres, Bacchus, Mars and Venus seems to us mere figure of speech, but to the ancients implied a good deal more. Even so a child will now say of her doll "This is my baby," and if you insist that it is not her baby, but only the symbol of one, will not be convinced, and will even begin to cry if you press the point. So to the primitive Christian the bread and wine simply *were* the body and blood of his Saviour; words could not make it plainer to him than that. They just *were*.

This belief of Paul implies the other one held by the Catholic Church that the eucharist is a sacrifice. He never states this with equal clearness, but he assumes it. Indeed it could hardly be otherwise. It is probable *a priori* because it was so in the mystery religions he knew. It is probably *a posteriori* because it can be proved that other Christians of the first century, e. g., Clement of Rome, so regarded it. But it is not entirely a matter of inference. Conybeare correctly points out that the germ of the idea, at least, is found in the words, "body, *which is for you,*" and (in the Gospels), "blood, *poured out for you.*"[74] Thus Paul also speaks in one breath of "keeping the feast" and of "Christ

[72] Lake's translation.

[73] Bergh van Eysinga, *Radical Views about the New Testament,* 1912, 104. Ramsay in *Expository Times,* XXI, 516. Harnack makes the same remark. "At that time 'symbol' denoted a thing which, in some way, really is what it signifies." *Dogma,* Eng., II, 144. Cf. also IV, 289, n. 2, and Loofs in Realencyclopädie für protestantische Theologie und Kirche, 3d ed., I, 58.

[74] Conybeare, "Eucharist," E. B.

our passover that hath been sacrificed for us."[75] Thus, further, he compares the holy bread with the sacrifices of Israel, which gave the Jews "communion with the altar,"[76] and with the things which the heathen sacrificed to devils: "Ye cannot," says he, "partake of the cup of the Lord and the cup of devils; ye cannot partake of the table of the Lord and the table of devils."[77] In this verse, which incidentally furnishes invaluable proof that Paul was familiar with the sacrificial meals of the pagan mysteries, the Catholics rightly see a clear support to their doctrine of the sacrifice of the mass.[78] The idea here is the same as that expressed in the Pseudo-Clementine Recognitions, that he who worships pagan gods, or tastes meat sacrificed to them has communion with demons.[79] Further the words "This *do* in remembrance of me" had the connotation in both Greek and Latin (ποιεῖτε, *facite*) of "doing sacrifice."[80]

Indeed it was inevitable that the communions should be regarded as the counterpart of sacrifices, both Jewish and pagan.[81] And in the later developments of both religions, Paul would find prepared for him the idea of "spiritual and bloodless sacrifices," a phrase soon borrowed to denote the eucharist. According to the Testament of the Twelve Patriarchs the angels offer such sacrifices to God.[82] In the Hermetic literature the same phrase λογικὴ θυσία is applied to the offering brought by Tat to his father Hermes.[83] The victim here thought of was the

[75] 1 Cor. v. 7.

[76] 1 Cor. x. 17f.

[77] I Cor. x. 21. Srawley, in *Encyclopædia of Religion and Ethics,* V, 544.

[78] Council of Trent, Mirbt, 242.

[79] II, 71. Kennedy, 273.

[80] Conybeare in E. B., "Eucharist." Renz, I, 152. Cajetan, quoted below; Stone I, 9. The same double meaning is in Hebrew עשה.

[81] Conybeare, *Myths, Morals and Magic,* 252.

[82] Test. Levi, III, 6.

[83] *Corpus Hermeticum,* XIII. 18; Reitzenstein, *Mysterienreligionen,* 35, 88.

Logos,[84] just as in similar words about Isis the victim offered to the goddess was herself.[85] And this victim was represented by the body of the worshiper, a comparison also made by Livy in describing the Bacchanalia.[86] All this serves to illuminate Paul's injunction to the Romans (xii. 1) to present their bodies to God as a spiritual service. The allusion is not directly to the eucharist but is from a circle of ideas closely analogous to that of the sacrifice of the communion. It is expressed more clearly in 1 Peter ii. 5.

Other passages in the Pauline epistles[87] doubtless have the eucharistic doctrine as a background, but they are too vague, apart from one in Colossians, to be discussed presently, to be of importance for our present purpose.

It will be objected that if Paul really introduced a new and pagan rite into Christianity, it would have been withstood violently by the Jewish Christians and especially by the previous apostles.[88] To this the answer is that he really was so opposed and on this very point. Since F. C. Baur,[89] few church historians have realized the tremendous strain that existed between the Jerusalem community and the Apostle of the Gentiles. It became so virulent that when Mark wrote his gospel, entirely along Pauline lines,[90] he could find scarcely anything to say about Peter save that

[84] *Ibid.*

[85] *Ibid.*, p. 91.

[86] Livy, XXXIX, 10, 7; Reitzenstein, p. 88.

[87] 1 Cor. xii. 13; Galatians iii. 6-26; Romans iv. 25 to v. 9; Eph. ii. On these see B. W. Bacon in *Harvard Theological Review*, 1915, 505ff. He finds not only the Pauline epistles but the Gospels "polarized" about the two sacraments of baptism and the supper.

[88] Schweitzer, *Paulinische Forschung, Einleitung.*

[89] *Paul*, English translation, 1876, Introduction and Part I, *passim.* On this, Schweitzer, *Paulinische Forschung*, 10 and 194. Cf. further, *Hibbert*, 1913, 737ff.

[90] On Mark's Paulinism, Loisy, *Les évangiles synoptiques*, I, 25, 116; B. W. Bacon, *The Beginnings of the Gospel Story*, 1909, pp. xxvff. Harnack, *Sayings of Jesus*, 248. The theory, originating with Papias, that Mark represents Peter, has been exploded.

he had denied his Lord and that Christ had called him Satan.[91] When, on the other hand, the Jewish faction expressed itself, it was to brand Paul as "a false apostle and a liar,"[92] and, "Balaam, who taught the children of Israel to eat things sacrificed to idols and to commit fornication."[93] Not only the Jews but the disciples of John at Ephesus and Damascus anathematized him as the perverter of their law, "the man of scoffing."[94] That the great schism in the early Church does not occupy a still more important place in the New Testament is due partly to the fact that Peter and Paul apparently divided the field into two spheres of influence, the Jerusalem apostles agreeing, for the sake of a tribute, to allow Paul to preach what he wished to the Gentiles.[95] It is also due in part to the complete triumph, after the destruction of Jerusalem, of the Pauline faction and to the desire of irenic historians like Luke to smooth everything over and make all appear according to Paul's gospel from the beginning.[96]

As to the eucharist, though there was opposition, its adoption was made easier to the Jewish Christians by the fact that they already had a common meal with which it was soon identified. This "love-feast," as we know from Jude, Tertullian and other sources, continued to the second century at least.[97] The difference of opinion among scholars as to whether it was identical with or different from the eucharist, is doubtless due to the fact that the two, at

[91] Mark viii. 31-34; xiv. 66-72.

[92] Apocalypse ii. 2; the allusion to Paul has been recognized by Renan and many others.

[93] Apocalypse ii. 14. The reference is to the doctrine of 1 Cor. x. Spiritual fornication, or idolatry, is meant.

[94] In the recently discovered *Fragments of a Zadokite Work,* cf. G. Margoliouth in *Expositor,* Dec. 1911 and March 1912.

[95] Galatians ii. 7. Conybeare, *Myth, Magic and Morals,* 11. *Hibbert,* 1913, pp. 748ff.

[96] *Hibbert,* 757. Harnack, *Luke the Physician,* 158f.

[97] Conybeare, "Agape" in *Encyclopædia Brit.*

first distinct, were gradually merged. It is noteworthy that the purely Jewish Christian literature, so far as it has survived in the New Testament—namely Q, James, Jude, 2 Peter and the Apocalypse—says nothing of the great rite of the Gentile Church. Nor—and this is very significant[98] —does the Shepherd of Hermas, one of the earliest Roman Christian writings. Little later the Didache,[99] in giving an account of the eucharist, carefully refrains from speaking of the Last Supper, of the body or blood or of the sacrifice of the cross. Instead of the words of institution, he recommends a simple prayer connecting the cup with the "vine of David."

A somewhat stronger opposition is probably seen in the Epistle to the Hebrews. O. Holtzmann has recently pointed out in this book a polemic against the eucharist.[100] Other scholars[101] have seen reference to the eucharist without polemic, and still others[102] have denied that there are any references at all. The verses which Holtzmann relies on are xiii. 9f: "Be not carried away by diverse and strange teachings: for it is good that the heart be stablished by grace, not by foods wherein they that occupied themselves were not profited. We have an altar of which they have no right to eat which serve the tabernacle." This seems to agree well with the interpretation of Holtzmann, and it is on the whole supported by other verses in the epistle. Thus in vi. 2, the writer speaks of baptism and laying on of hands but omits the eucharist. More striking is ix. 9: "gifts and sacrifices which cannot, as touching the conscience, make the worshiper perfect, being only, with meats and drinks and divers washings, carnal ordinances." The reference is,

[98] Réville, *Révue de l'histoire des religions,* LVI, 26.

[99] IX, 10; Gardner, *Exploratio Evan.,* 458; *Religious Experience of Paul,* 119, etc.

[100] *Z. N. T. W.,* 1909, 251-60, against him, Goguel, 219.

[101] Srawley, *E. R. E.,* V, 543.

[102] Lambert, 391.

of course, to the old dispensation, but through it the author seems to hit at the new ceremonialism. Again, the insistence in x, 12 that Jesus was sacrificed once only for our sins seems to read almost like a Protestant polemic against the repeated sacrifice of the mass. The Paulinists also seem to be scored in the verse against those who have counted the blood of the covenant a common thing (xii. 29). The verse "forget not to do good and to communicate," refers, naturally, not to communion but to giving to the poor, as in Romans xv. 26, 2 Cor. ix. 13.

One other passage in Paul has been left for discussion until now, because it seems to refer to those who oppose his eucharist doctrine. I mean Col. ii. 16f: "Let no man therefore judge you in food or in drink, or in respect to a feast day or a new moon or a sabbath day: which are but a shadow of things to come; but the body is Christ's."

The Synoptic gospels adopt the Pauline view entire. I will spare my reader the exhibition of the texts relating to the Last Supper in parallel columns, and the long comparison of them, with the purpose of discovering what is historic or original in them. All such attempts have definitely failed. Those who favor Mark and those who prefer Luke,[103] cannot show that there is anything but Paul in the lesson of the narratives. The words attributed to Jesus, are, says Loisy, "the doctrine of Paul and are simply incomprehensible as addressed by Jesus to his disciples on the day of his death."[104] Mark did not need to copy them from 1 Corinthians, for the usage had become established at Rome when he wrote. His omission of the Pauline words "Do this in remembrance of me" has no significance, for they seemed to Mark implied, or, as Germans would say, *selbstverständlich*. Schweitzer and others have seen in the verse added by Mark, in which Jesus says that he will

[103] As Heitmüller, and Bacon, *H. T. R.* V, 322ff.

[104] *L'évangile selon Marc,* 403.

no more drink of the fruit of the vine until he shall drink it new in the kingdom of God, a genuine reminiscence. This, however, is untenable; for the idea here is also Pauline, closely similar to that of 1 Cor. xi. 26.

There are at least three other allusions to the eucharist in Mark besides the account of its institution. The first of these of which I shall speak is positive proof that words about the sacrament could be attributed to Jesus, though he could not possibly have spoken them. When the sons of Zebedee ask for the chief places in Christ's kingdom, he replies (x. 38). "Can ye drink of the cup that I drink of and be baptized with the baptism that I am baptized with?" This joining of the cup and baptism is surely a figurative allusion to the two Christian sacraments. But as the content of the pericope is a prophecy of the death of James and John, a *vaticinium ex eventu* certainly not genuine, the allusion to the eucharist placed in Jesus's mouth is certainly later than his time.

From the earliest days it has been recognized that the miraculous feeding of the multitudes is a symbol of the spiritual nourishment of mankind by the communion bread. John, the first commentator on the synoptics, so took it, and joins on to it his version of the sacramental words attributed to Christ.[105] How carefully the symbolism is carried out is shown in one narrative of Mark by the seating of the people in groups, as was done in the early Church, and his other narrative by the instructions to pick up the fragments. This may be compared with the miraculous instructions given by Tertullian,[106] and followed in the Roman Church to-day, to let none of the precious body of the Lord be left on the floor, if dropped.

The use of fish in connection with the eucharist at Rome

[105] Loisy, *L'évangile selon Marc,* 191ff; 225ff, to Mark vi. 32ff and viii. 1ff. Cf. John vi.

[106] *De corona mil.,* 3.

where Mark wrote has been noticed above. The reason for his repetition of substantially the same miracle is probably to be found in his use of sources, though it has been conjectured that he wished to symbolize the callings of the Jews and Gentiles respectively.

Matthew and Luke add nothing on this subject to Q and Mark. In Luke, however, we have an interesting textual problem on which I believe I can throw light. Some manuscripts,[107] headed by D, omit the words (xxii. 19b-20): "given for you. Do this in remembrance of me. And in like manner the cup, after supper, saying, This cup is the covenant in my blood, which is poured out for you." The textual evidence together with "the suspicious resemblance of this passage to 1 Corinthians" led Westcott and Hort to bracket it as an interpolation. The words are evidently taken from Paul, but as it is just as possible that Luke borrowed them as that his copyist did, and as they are present in most of the decisive authorities, they are retained by Von Soden and regarded as genuine by Jülicher, Cremer, Clemen, Schweitzer, Lambert and others.[108] If, then, they were in the original, why does the Codex Bezae (D) omit them? The answer is this: The reviser of D (or rather, probably the scribe of an earlier manuscript he copies), was from Asia Minor,[109] probably from Ephesus, at which place there was the strongest opposition both to Paul and to his eucharistic doctrine. The Disciples of John there, as is proved by the Odes of Solomon[110] and the Johannine writings, presently to be discussed, refused to take the eucharist bread or to recognize it as the flesh of Christ. Even as late as the second cen-

107 Besides D, the old African and Italic Latin versions omit them, and Tatian changes the order of words.

108 Lambert, 245.

109 Ramsay, *Church in the Roman Empire,* 151ff.

110 Preserved Smith, "The Odes of Solomon and the Disciples of John," *Monist,* April 1915, pp. 186f.

tury the Docetae of Asia Minor, probably an offshoot of the Johannites, took the same position.[111] Now the reviser of the manuscript represented by D and the Latins did not dare to omit the story of the institution as a whole, but he did delete the words implying a sacrifice and the command to repeat. Like the Fourth Evangelist later he hoped thus to keep the spiritual lesson and to avoid the ritual repetition.

Acts occasionally mentions the celebration of the Supper (ii. 42; xx. 7), but as it adds nothing to our knowledge, save to show that it and Paul's interpretation of it were thoroughly established in the community and at the late date at which Luke wrote, the book need not be further noticed.

Of the New Testament writings there remain to be discussed only the Gospel and First Epistle of John. On their teaching the most extraordinary diversity of opinion has prevailed. Some scholars have denied that the Gospel refers to the eucharist at all. Others have seen in it only an intensification and emphasis on the sacramental theory of Paul. Many think that John "spiritualizes" Paul's teaching, though without saying definitely how. The data are these: (1) John omits the account of the Last Supper and substiutes for it foot-washing, with a probable allusion to baptism. (2) In the sixth chapter he joins to the narrative of the miraculous feeding a long discourse of Jesus on the necessity of eating his flesh and drinking his blood: "I am the bread of life. He who cometh unto me shall never hunger and he who believeth on me shall never thirst." "I am the living bread coming down from heaven. If any one eat of this bread he shall live forever. For the bread which I shall give him is my flesh which is for the life of the world. Then the Jews contended with one another saying, How can this man

[111] Ignatius *ad Smyrn.*, 6.

give us his flesh to eat? Then said Jesus to them, Verily, verily I say unto you, if ye eat not the flesh of the Son of man and drink not his blood, ye have not life in yourselves. The feeder on my flesh and the drinker of my blood hath life eternal, and I shall raise him up at the last day. For my flesh is true nourishment and my blood is true drink. The feeder on my flesh and the drinker of my blood remaineth in me and I in him."

Knowing the methods of the Fourth Evangelist, his total independence of historical tradition and his custom of writing into the narrative the lessons he thought needed in his own day, it is easy to see in this debate, nowhere recorded in the Synoptics, the controversy actually in process at Ephesus, between the Pauline Christians on one side and the Jewish and Baptist parties in the Church on the other. (3) It is possible that there is some allusion to the eucharist in the story of the wedding at Cana, but, if so, it is vague and not to our purpose.[112] The water and the blood issuing from Jesus's side at the passion have been interpreted as referring to the two sacraments. It is quite possible that the parable of the true vine (John xv. 1ff) situated as it is in Jesus's last discourse to the disciples, is an allusion to the eucharist cup, suggested by Mark xiv. 25. It is noteworthy that the prayer of consecration in the Didache connects the cup with the vine of David.

How shall we interpret these seemingly conflicting data? Why did John refuse to regard the Last Supper as historical, while embodying the doctrine of the flesh and blood of Jesus in such strong language? Did he omit the Last Supper simply as he omitted the baptism of Jesus and as he says that the master baptized not, but his disciples, as though his Christ were superior to sacramental

[112] John ii. 1ff. His sources were Mark ii. 18-22;Matt. xxii. 1-14; Luke xiv. 15-24, and IV Ezra X. Similar tales were told of Dionysus turning water into wine at his epiphany. This pericope was in ancient rituals a lesson for Epiphany. Bacon, *H. T. R.,* 1915, p. 115.

acts?[113] Surely not. His Jesus, who weeps and suffers hunger and washes his disciples' feet, is not above eating with them a ritual meal. Or does he transpose the institution of the eucharist to the earlier account of the feeding of the multitudes to show that Jesus's eating with his disciples was no new thing at his death, but that his every meal with them was consecrated? This view[114] also seems insufficient, and at variance with certain verses in the discourse quoted above (John vi).

The solution of the enigma, I am persuaded, will be found in the situation at Ephesus where the evangelist wrote. There, as we know (Acts xviii. 19ff) was a church founded by Paul, in which, naturally, the eucharist would be celebrated. But there was also a powerful element in the church drawn from the Disciples of John,[115] who had no eucharist, and who would doubtless oppose it, just as the Bohemian Brethren absorbed into Protestantism for long kept their own distinctive tenets. But we have already proved from Hebrews, from Colossians and from the D recension of Luke xxii, that there was opposition to the eucharist, and especially at Ephesus. Now, though the sources of the Fourth Gospel are many—the Synoptics, the Apocalypse, Philo, the Hermetic literature, and of course the Jewish scriptures—the ones from which he drew most heavily for his doctrine were the Pauline epistles and Odes of Solomon,[116] these latter written at Ephesus by the Disciples of John, and consequently full of allusions to baptism, but with none to the eucharist. Unhampered as he was by any trace of independent tradition,[117] he felt

[113] John iv. 2. Schweitzer advances this view, *Paulinische Forschung,* 157ff.

[114] Bacon, 434f, maintains it.

[115] Acts, xix. 1ff. That the Disciples would have no eucharist is obvious and is also proved by the Odes of Solomon. *Monist,* April, 1915, p. 186f.

[116] So Harnack and Rendel Harris. *Monist,* 1915, pp. 171ff.

[117] This fact, still disputed, has been pretty well established by Loisy, Bacon and others.

free to deal with the facts as he liked. As a follower of Paul he wished to preserve and emphasize the great spiritual lesson which he found in the words about eating the flesh and drinking the blood of Jesus. On the other hand he could not ignore the Disciples of John and their heirs, supported as they were by Jewish Christians, who abominated the supper as a heathen rite. Whether the evangelist had once himself been a disciple of the Baptist remains uncertain,[118] but that he did write with them constantly in his eye has long been recognized.[119] He therefore rejected the founding of the eucharist, and substituted for it a washing reminiscent of the one sacrament universally accepted, while at the same time conserving the lesson that Jesus is the bread of life. Not without reason does his language hark back to the Jewish Scriptures, to the Apocrypha and to Philo,[120] in showing that the Logos is the true nourishment of the soul. "Except ye eat the flesh of the Son of man and drink his blood," says he, "ye have no life in you." By this he would not have understood in the old, literal way: "It is the spirit that quickeneth; the flesh profiteth nothing. The words that I speak unto you, they are spirit and they are life" (John vi 63).

How then shall we explain the emphasis on the "water and the blood," i. e., the sacraments of baptism and the eucharist, in John xix. 34 and 1 John v. 6? It has been proposed to regard the "blood" here simply as an allusion to the passion. It is probable that the Docetae,[121] at whom these verses may have been aimed, denied the passion, and it has been shown that it would be most appropriate to connect the blood of martyrdom with the water of baptism,

[118] Gardner, *Ephesian Gospel,* 87f.

[119] Baldensperger, *Der Prolog zum vierten Evangelium,* 1897; Dibelius, *Johannes der Täufer,* 1911; B. W. Bacon, *Fourth Gospel,* 290.

[120] Psalm lxxviii. 4; Ecclesiasticus xxiv. 29; Pfleiderer, *Primitive Christianity,* 1906ff, IV, 231ff. Probably also to the supersubstantial bread of the Lord's prayer.

[121] This explanation offered by Bacon.

for the one might well follow the other.[122] Such an explanation would obviate all difficulties, but I am inclined, nevertheless, to see at least a secondary allusion to the eucharist in the "blood." If this is true, there is certainly a contrast to the teaching of the earlier chapters of the gospel. It can be instantly seen by comparing John iii. 5 with 1 John v. 6. The first passage reads: "Except a man be born by water and the spirit, he cannot enter the kingdom of God." The second: "This is he that cometh by water and blood and spirit, Jesus Christ. . . . Because these three are witnesses, the spirit and the water and the blood." In the first chapter of the gospel, then, the spirit and baptism were all that was necessary, but in the epistle and in the later, probably subsequently added, verse in the gospel, the eucharist is joined with them as one of the means of salvation. Though I am no friend of the hypothesis of interpolation, by which many wild theories have been proved, I have unusually strong reason for claiming that this verse is subsequently added. Bacon,[123] among other authorities, recognizes that the whole of chapter xxi, and that John xix. 35 are added by a later editor. The evidence for the last verse is overwhelming; it reads: "And he that hath seen hath borne witness, and his witness is true, and that man knoweth he speaketh the truth that ye may believe." The introduction without antecedent of "that man," ἐκεῖνος, *ille,* would be simply incomprehensible in the original narrative. The word points to the author of the gospel as seen by some one else. The solemn asseveration, as to a new and disputed fact, also strongly indicates editorial revision. Now it is absurd to regard the asseveration, and that alone, as interpolated. Something else must have been introduced with it, something

[122] So R. Winterbotham in *Expositor,* 1911, 62ff, and J. Denney, *ibid.,* 1908, 416ff. The latter regards the "blood" as referring primarily to the passion and martyrdom, secondarily to the eucharist.

[123] P. 191.

to which the asseveration applies, and this can only be the previous verse about the water and the blood. This, then, was added by the editor, who introduced it from the epistle. If we regard the gospel and epistle as by the same hand, we are then reduced to the necessity of reconciling the omission of the eucharist in one to its recognition in the other document. The true explanation has been suggested by Percy Gardner:[124] "In old age, when he wrote the epistle, the Evangelist seems to have relied, as was natural to a man of failing powers, somewhat more on the visible rites of the Church." It is remarkable that we find exactly such a change in Luther's dogma, and that completed in ten short years. In 1520 he put the essence (*res*) of the sacrament in the Word, and stated that the actual rite was not necessary to salvation; in 1530 he was ready to affirm that the real essence (*res*) of the sacrament was in the elements, and that participation in them was absolutely indispensable to secure their benefits. So with the Evangelist; in his younger years the spiritual lesson was all important; later, as the rite became more firmly established and as he became more ecclesiastical, he accepted the communion as essential.

Most of the Gnostic sects known to us adopted the eucharist, with its ideas of immolation and theophagy.[125] Many of their dogmas were probably founded directly on mystery cults with which they were connected in pre-Christian times. How easily pagan ideas amalgamated with Christian is seen in the eucharistic prayer in the Acts of Thomas:[126] "Come, communion of the male.... Come, thou that disclosest secrets and makest manifest the mysteries. .... Come and communicate with us in thy eucharist."

[124] *Ephesian Gospel*, 213.

[125] A good account of their dogmas in W. M. Groton, pp. 35ff.

[126] Chaps. xlix and l; Pick, *Apocryphal Acts*, 268f.

Here emerge the two primitive conceptions of the mysteries and of communion with the divine after the manner of sex.

Clement of Rome in the first century calls the communion an offering and a sacrifice.[127] By making it the "liturgy" *par excellence* of the Church, he puts it in the place of the highest form of divine worship which it has ever since held in the Roman Church.

Ignatius also thinks of it as a sacrifice, and as charged with a magical quality for keeping both body and soul deathless. "The bread," says he, "is the medicine of immortality, the antidote preserving us that we should not die, but live for ever in Jesus Christ."[128] This is but a literal interpretation of John's teaching by a younger contemporary. Ignatius also states plainly that the body is the same as that which suffered on the cross.[129]

According to Justin Martyr, "God, anticipating all the sacrifices offered in his name by the command of Jesus Christ, namely the eucharist of the bread and the cup, which are offered by Christians in all places throughout the world, testified that they are well-pleasing unto him."[130] He also speaks of the eucharist as becoming the body and blood of Christ through the prayer of the Logos. To him also it is a memorial of the passion and a magical charm for giving men immortality. His comparison of this sacrament with that of Mithra has already been mentioned. In this connection it is interesting to note that with him and quite a number of other early Christians, the elements were not bread and wine but bread and water.[131] Paul speaks only of the "cup," without denoting

[127] Ad Cor. 40, 44; cf. 36. Srawley, *Encyclopædia of Religion and Ethics*, V, 546; *Encyclopædia Britannica*, IX, 868; Goguel, 224; Lambert, 412.

[128] *Ad Eph.*, 20. Srawley, 546.

[129] *Ad. Smyr.*, 6; cf. *Ad Rom.*, 7.

[130] *Dialogue with Trypho*, 117. *First Apology*, 66, 67. Srawley, 547; Lambert 415.

[131] Harnack, *Brot und Wasser. T. & U.*, VII, 2, 1891.

its contents, but both he and the gospels imply that it was wine.[132]

It was the insistence on the element of sacrifice that gave rise to the rumors in the Roman world of "Thyestean banquets." Early in the second century Pliny[133] felt it necessary to inform Trajan that the meal partaken of by the Christians was of harmless and ordinary food, and that he found nothing criminal in it but only a perverse and excessive superstition. In the same letter he uses the word *sacramentum* of the morning service, but does not connect it with the supper which was eaten later in the day. The word, which we have seen was already used of the rites of Bacchus and Isis, became the regular translation of the Greek "mysterium," the initiation into holy secrets and magical practices characteristic of all the "mystery-religions," including Christianity. The word is found in the Septuagint only in the latest books, Daniel and the Apocrypha, when the Hellenization of the Jews was well under way.

Though Clement of Alexandria does not emphasize the sacrificial aspect of the eucharist, he is familiar with the conception of sacrifice as originally a feast upon a victim, and neither the idea of the Real Presence nor that of transubstantiation are foreign to his thought.[134]

Irenaeus call the bread and wine an offering to God the Father of the body and blood of his Son, and says that it is efficacious for the body as well as for the soul. When consecrated, the bread is no longer bread but of two elements, a heavenly and an earthly, and prepares our bodies for the resurrection. He compares it to the sacrifices of the Jews to its advantage, as being offered by children, not servants.[135]

[132] 1 Cor. xi. 21; Mark xiv. 25 etc. [133] Ep., 96.

[134] Tollington, *Clement of Alexandria*, 1914, II, 155.

[135] *Adv. Haer.*, IV. xviii, 4. *De corpore et sanguine*, V, ii, 2. Srawley, 547.

As has been shown, the fundamental idea in eating the God was to become like him. This was carried so far in the pagan religions, that the initiates not only imitated what the god was fabled to have done, but were actually called by his name. The adorer of Bacchus became a Bacchus; the follower of Attis was called Attis.[136] This dogma could not be better expressed than it was by Cyril of Jerusalem, who, in his Fourth Mystagogic Catechism teaches: "By taking the body and blood of Christ, you become one body and one blood with him. For thus we become Christ-bearers (χριστοφόροι) by his body and blood being digested into our members."[137] The language of ritual again became the mother of legend, and the myth of St. Christopher was born.

The "highest" doctrine of the sacrifice of the commuion is found in Cyprian near the middle of the third century. "The priest," says he, "imitates what Christ did, and offers then in the Church to God the Father a true and complete sacrifice,"[138] and again: "The passion of the Lord is the sacrifice we offer."[139]

Cyprian's idea of the effect of the magic food was that of the savage medicine-man. He tells in one place of a little girl who had eaten some meat sacrificed to idols and thus became possessed by devils. When she came to the Lord's table, she accordingly refused the consecrated cup and fell into fits.[140] A similar magical effect is attributed to the host by the Acts of Thomas.[141] A youth who had murdered his mistress partook of the eucharist and immediately had his hand withered. The Apostle forthwith invited him to confess his crime, "for," said he, "the

[136] As in Catullus's famous poem of that name.

[137] Quoted, Dietrich, 107.

[138] Ep. LXVIII, 14. Mirbt, 24b.

[139] *Ibid.*, 17.

[140] *De lapsis*, cap. 25. Dietrich, 107.

[141] Cap. XLVIII.

eucharist of the Lord hath convicted thee." It is well to bear in mind that the magic of the host is not a medieval invention but as primitive as the rite itself.

The Didascalia, in the second half of the third century, speaks of "offering the acceptable eucharist, which is the symbol (ἀντίτυπον) of the royal body of Christ."[142]

In the next age the Apostolic Constitutions call the bread and wine "symbols (ἀντίτυπα) of his precious body and blood" and an "unbloody sacrifice," celebrated to commemorate the Lord's death.[142]

Eusebius of Caesarea says that Christians are "fed with the body of the Saviour," and that Christ delivered to his disciples the symbols of his divine incarnation, charging them to make the image of his own body.[143] (Are we listening to the priest of Aricia and his image of the Wood-King baked in bread?) Here and elsewhere the words for image (εἴκων, *figura*), imply the real presence.

Tertullian's fetishism made him dread any disrespect offered to the magic food. He speaks of "handling the Lord's body" and of "offering violence to it." The bread he also calls the "figure of the body," and "that which represents the body," without, however, implying that the body is absent. Rather than saying that he began to confound the bread with the body, it is truer to see in him the first to distinguish them.[144]

In many writers of the period of Rome's decline and fall the sacrificial idea comes to dominate all others. Strange, this fascination of blood, that *ganz besonderer Saft,* for the savage and religious mind! Only by some horrible cruelty and suffering inflicted, generally against their wills, on others, can man escape from the bogies of his own conscience! Like other Christian doctrines, that of the

[142] Srawley, *E. R. E.,* v. 549.

[143] *De Solemnitate Pasch.,* 7.

[144] Srawley, *E. R. E.,* v. 549.

atonement is rooted in the primeval practice of the savage in cursing some senseless object, or killing some harmless animal or innocent person, in order to get rid of his own sins on vicarious shoulders.[145] Some such idea haunted the mind of Athenagoras when he speaks of "the bloodless sacrifice of the Christians," as the counterpart of the bloody sacrifice of the cross. Thus does Cyril of Jerusalem dilate upon the "holy and most awful sacrifice," "Christ immolated for our sins to propitiate God who loves men," offered in the eucharist. Thus Chrysostom gloats over "the Lord lying slain, and the priest standing over the victim praying, all reddened with that blood."[146]

Before closing this section on the primitive Church, it is pertinent to notice one question which early came up, as to the ministration of women in the eucharist. From the first, women had taken a part in divine service and had prophesied with the men. Such were the daughters of Philip the Evangelist, from whom, according to Harnack,[147] Luke derived much of his peculiar material. But St. Paul, who commonly lent his influence to the worst social oppressions of the age,[148] in this also advocated the subjection of women,[149] thus adding to the burden of that much suffering sex. As, however, the practice continued here and there, we meet with later efforts to deal with it. The most interesting of these is in the Apostolic Church Order.[150] It is but one instance of many to show the inveterate tendency of men to refer back to authority, and, if there is not a command of God covering the subject they desire to deal with, to invent one. Just as Paul

[145] J. G. Frazer, *The Scapegoat.*

[146] *De Sacerdot.*, VI, 4; Srawley, *E. R. E.*, 551f.

[147] *Luke the Physician.*

[148] E. g., passive resistance to tyranny, Romans xiii. 1ff, and slavery, 1 Cor. vii. 20f.

[149] 1 Cor. xiv. 34ff; cf. 1 Tim. ii. 12.

[150] Bauer, *Das Leben Jesu im Zeitalter der neutestamentlichen Apocryphen*, 1909, 165. Pick, *Paralipomena*, 68b.

fabled that Christ had instituted the Supper, so the later author felt free to write history as follows: "The Apostle John said: 'You have forgotten, brethren, that when the master demanded the cup and the bread and consecrated them with the words, That is my body and blood, he did not allow them [*sc.* Mary and Martha] to come to us.' Martha said, 'It was on account of Mary, for he saw her smile.' Mary said: 'I did not laugh; it is rather as he said to us before that weakness should be saved by strength.' "[151]

This obvious invention did not entirely suppress the abuse at which it was aimed, or else the practice cropped up afresh from time to time. The service of women at the altar was condemned by a council of Nimes in 394, but still persisted in certain parts of France. In the sixth century in Brittany women called "conhospites" offered the blood of Christ to the people and carried the elements around on portable altars. This "unheard-of superstition" was denounced and suppressed by the bishops Licinius of Tours and Melaine of Rennes. It is continued elsewhere, however, until the ninth century.[152] It is profitable to compare with this the service of maidens at the grail, an ancient vegetable sacrifice which finally became identified with the eucharist.[153]

Preserved Smith.

Poughkeepsie, N. Y.

[151] I. e., woman by man.

[152] *Monumenta Germ. Hist., Leges,* I, cap. 2, p. 42. I owe this reference to Miss R. J. Peebles. Other examples of women who dispensed the eucharist in the early Church or in heretical sects given in article *"Frauenämter,"* in *R. G. G.*; Lydia Stöcker, *Die Frau in der alten Kirche,* 1907; L. Zscharnack, *Der Dienst der Frau in den ersten Jahrhunderten der christlichen Kirche,* Göttingen, 1902.

[153] Peebles, *The Legend of Longinus,* 1911, 209.

MIND, THE CREATOR OF MATTER.

A NEW era is dawning upon the world—an era in which religion will become the deepest science, and science in its truest sense religion. The deeps of existence, psychical as well as physical, are being sounded, and the recent conquests in the domain of science unfold prospects of a mastery of nature such as never before occurred to man—to modern man, at least—in his wildest dreams. What is the domination to be? Are we to witness merely a fresh following of the old road, with telescope, spectrum analysis and chemist's balance for the instruments of advance; or is it the power of mind over matter, directly exerted, which is to be the new solvent of nature's problems, the new agency for bending nature to man's will, for remoulding it into harmony with his highest needs and aspirations?

The question summons us at once to a brief consideration of what we mean by matter and by mind. Are they totally different things? The whole trend of modern thought is in the direction, if not of identifying them with one another, at least of bringing them closer and closer together. The most salient feature of modern progress is the steady shifting of the emphasis from the material to the mind values in the broadest sense of the word. We cognize objects only in terms of our conscious states; in the last analysis all material values are found to be mind values. The world as we know it presents itself in two aspects—the outside realm of matter, consisting of objects revealed to us through our senses, and the inner realm of feeling and thought which is without anything like location in space.

Take the outer world and see how far we can trace it. I hold an apple in my hand. What do I know of it? I know its color, its form, its hardness, its taste, its odor—in a word, I know what my five senses tell me. Remove eye, hand, tongue, and these qualities will become non-existent, being conditioned by the nature of my organism. Professor James in his transmission theory maintains that our organism, instead of revealing to us the nature of the universe, limits our knowledge of it by our very constitution to what we acquire with our five senses. We cognize only the things for the perception of which we have corresponding organs. To illustrate: a colored pane of glass —say red—transmits only red rays, shutting out many other vari-colored rays, which, although they cannot be transmitted by the red pane, exist nevertheless. "In my Father's house are many mansions."

Matter is not the ultimate. Beneath matter, science tells us, is ether—the medium which pervades all space—the interstellar immensities as well as the infinitesimal interstices between material atoms. Some modern physicists have compared it to a jelly; others describe it as denser than steel; all agree that it is incompressible and is in some way the reservoir of energy for all material phenomena. The existence of ether is not merely speculation, it is a reality; and Sir Oliver Lodge calls it the most important reality with which we are acquainted. The senses tell us nothing of it, but without ether such phenomena as light, electricity, magnetism, radio-activity, would be impossible. It is through this continuous substance that light passes to us from the sun, stars and other luminous objects; without the undulations which it transmits the whole world of objects which we now see in their shapes, colors and distances, would be invisible. Not only in this way is ether the vehicle of energy flowing in to us from without—it is the very source of the things to which we have access through our sense organs. Subject to some sort of stress, it differentiates itself into matter; in all probability the so-called electrons, which unite to form atoms, are knots or rings formed in and of the ether itself.

Such is the outer world. What now of the inner, which we know only as states of consciousness? We know it neither as matter nor as motion, but we find it intimately connected with brain, and as the brain is made up of molecules, these must vibrate during the activities of thought. We here have the link connecting the two worlds which at first sight would seem to be so remote from each other. Matter we know through our senses; matter is evolved from ether; ether we do not know; our thought we know. Our thought is not matter, but it is accompanied by vibrations in the ether. Thought is connected with matter through our bodily frame; it is not less distinctly connected with ether through its *modus operandi*. Meanwhile that which logic asserts is fast becoming the favorite conclusion of science. Not only naturalists like Naegeli and Haeckel maintain that matter is endowed with elementary feeling; the physicists also incline to this view, and Sir Oliver Lodge in his book on *The Ether of Space* writes:

"The universe we are living in is an extraordinary one, and our investigation of it has only just begun. We know that matter has a psychical significance, since it can constitute brain, which links together the physical and the psychical worlds. If any one thinks that ether, with all its massiveness and energy, has probably no psychical significance, I find myself unable to agree with him."

And Camille Flammarion sums up the argument of his book on *Mysterious Psychic Forces* in the words:

"The phenomena of which we are speaking are manifestations of the universal dynamism with which our five senses put us very imperfectly in relation. We live in the midst of an unexplored world in which the psychical forces play a role still very insufficiently investigated. These forces are of a class superior to the forces usually analyzed in mechanics, in physics, in chemistry. They are of the psychical order, have in them something vital and a kind of mentality. They confirm what we know from other sources, that the purely mechanical explanation of nature is insufficient, and that there is in the universe something other than so-called matter. It is not matter that rules the world; it is a dynamic and psychic element. . . . There is in nature, especially in the domain of life, the manifestation of instinct in vegetables and animals, in the general soul of things, in humanity, in the cosmic universe, a psychical element which appears more and more in modern studies, especially in researches in telepathy, and in the observation of the unexplained phenomena which we have been studying in this book."

Science not only shows us how moving matter causes vibrations in the ether, producing motion in other matter at a distance; it also enables us to realize the possibility of action at a distance by means of thought, and this without the instrumentality of speech, telegraph wires, or other physical agencies. For if thought be accompanied by molecular vibrations in the brain, the ether must be moved by these just as it is moved by the vibrations which produce the phenomena of light and electricity. Said Sir William Crookes, the famous English physicist, in his address as president of the British Association for 1898:

"It would be well to begin with telepathy, with the fundamental idea that thoughts and images may be transferred from one mind to another without the agency of the recognized organs of sense; that knowledge may enter the human mind without being communicated by any hitherto known or recognized ways. . . . If telepathy takes place, we have two physical facts—the physical change in the brain of A, the suggestor, and the analogous physical change in the brain of B, the recipient of the suggestion. Between these two physical events there must exist a train of physical causes. Such a sequence can only occur through an intervening medium. All the phenomena in the universe are presumably in some way continuous, and it is unscientific to call in the aid of mysterious agencies when with every fresh advance in knowledge it is shown that ether vibrations have powers and attributes abundantly equal to any demand—even to the transmission of thought. . . . It is known that the action of thought is accompanied by certain molecular movements in the brain, and here we have physical vibrations capable from their extreme minuteness of acting direct on individual molecules, while their rapidity approaches that of the internal and external movements of the atoms themselves. . . . It will be found possible to discover a path by which telegraphing without wires and transferring thought from mind to mind can be found to harmonize."

Examples of this possibility of moving the matter of the human brain at a distance by the putting forth of purely mental power have been gathered in thousands by societies for psychical research on both sides of the water. It is only a step further to show that matter outside the human brain—matter which is inorganic—may also be moved and influenced by the action of mind. In 1871, Sir William Crookes published an account of experiments conducted by him, under a system of rigid scientific tests, which established "the existence of a new force in some unknown manner connected with the human organization, which for convenience may be called the psychic force." In this account Sir William demonstrated that by putting forth of the psychic force it is possible to alter the weight of bodies and play upon musical instruments without direct human intervention.

But there is still a third stage in the power thus exerted by mind upon matter—that of actually creating it. For what does the creation of matter really mean? It does not mean the bringing of matter into existence out of nothing; it simply means the rearrangement of the atoms which already exist. The ultimate parts of all kinds of matter are the same; the different types of matter known to us are due to different combinations and motions of the ultimate units, and these units are simply modifications of the ether itself. Tarde, the French writer, maintains that all spatial likenesses in the universe, and therefore the likenesses of the ultimate parts of matter, are due to likenesses of vibration; and if the mind be capable of giving rise to vibrations in the ether, it should be able to call matter from the ether. It was Sir William Crookes who, alluding to Tyndall's assertion that he saw in matter the promise and potency of all forms of life, said: "I should prefer to reverse the apothegm and say that in life I see the promise and potency of all forms of matter."

Some time ago Professor Ramsey startled the scientific world with the announcement:

"I have found that when electricity is passed through a vacuum tube containing a little hydrogen, two other gases, helium and neon, appear....The chief value of these experiments is that they point the way for a change of one form of matter—an element supposed to be incapable of change—into another, or that it shows that what we have hitherto considered as substance is but a manifestation of forces. In any case a severe blow has been dealt to the present theories concerning the constitution of matter. . . . It means that we must cease to believe in and to speak of 'elements.' We must adopt new phrases. New theories, founded largely on the old ones but with serious modifications, must be advanced. We must experiment further with electricity and its effects upon matter. I imagine that if any experiments are carried on, we shall discover many startling facts concerning electricity and may even discover a new and enormously large source of electrical power."

Where shall we look for this unknown source of power, and what may be its *modus operandi*? Thus far we have seen that logic and scientific research admit of the possibility of synthesizing matter out of ether. The principle that underlies this creation and predetermines the resultant forms and shapes is the one with which human thought has wrestled from time immemorial, whether calling it the "idea" of Plato, the "*élan vital*" of Bergson, or simply the "first cause." In the universe this principle manifests itself as motion along the path of least resistance, thus fulfilling that law of harmony and unity which like a thread of gold runs through the whole fabric of creation, from the orbits of planets and stars to the circling motions of electrons in atoms, from the formation of a snow crystal and the wing of a butterfly to the Greek Parthenon, a Madonna of Raphael, a symphony of Beethoven or a sonnet of Shakespeare. In other words, the intelligent principle is the mode in which the universe works in what is called evolution, and therefore has its outcome in forms and motions characterized by harmony and unity. Ether, in differentiating itself, first produces electrons, which in their turn give rise to atoms. These, following the path of the least resistance and subject to unlike stress, unite to form different kinds of matter, until finally the creative activity exerted by the universe passes over into the organism in a highly complex form.

Man as an organism also creates intelligent forms by virtue of his derivation from the universe. At first he creates by moulding objects with his hands, and later by means of machines. But this is only the initial stage of his creative power. Mind comes to have greater and greater meaning in his development and activities. As the individual mind gets more from and cooperates more with the race mind, human advance is accelerated. A further stage of this development will come when, instead of using hands, tools and machines to supply his needs and shape his environment, man will accomplish these ends by means of brain waves generated by his thought. These waves will be creative, just as the ether which produced and moulded matter is creative, since both are ultimately of the same nature. They will result in forms which are intelligent because the production of such is the end toward which all activities ultimately tend. The link between these effects of thought and the effects produced by the universe is the link of a fundamental process which is common to all existence. But in order to be creative these thought waves must be rhythmic, that is to say, they must move in accordance with the laws of harmony and unity—this is the cardinal condition, the *sine qua non* of the mind's mastery over matter, the *raison d'être* of all religions, the logical deduction of all modern science. The power of rhythm in ether may be brought home to us by the force of rhythmic waves in our material world. It is a well-known fact that a body of soldiers must break step when crossing a bridge to prevent its collapse under their rhythmic tread. The fall of Jericho's walls at the trumpet blasts of the Israelites may be cited in illustration of the force exerted by sound-waves. Religion is in complete accord with science when it inculcates goodness, righteousness, moral perfection in man, for in this direction lies man's oneness with his Creator. "Except ye become as little children ye shall not enter into the kingdom of heaven." By achieving unity and harmony within himself and with his fellow beings man will put himself in tune with the universe, and wield the powers of the cosmos itself—his rhythmic thought-waves acquiring under these conditions the power to liberate what are called the "intra-atomic energies" and shape them at will. This power at present is dormant or dissipated, largely because of the conflicts and cross-purposes which continue to divide mankind.

The newly-discovered phenomena of radio-activity go far toward bridging the gap between matter and mind. They show that matter is continually undergoing disintegration, and that by rearrangements due to this disintegration material substances change one into another—uranium into radium, radium into helium, neon or argon; copper into lithium, thorium into carbon, the series closing, it is believed, with lead. Nature, presenting us these transformations in progress on a large scale, reveals one of the methods by which matter is created, and suggests the possibility of advance, not only to physical means of imitating such processes, but also to the reproducing of them by mental action. Has not such a power of creating matter been already exercised in the past? Does not progress move in cycles, and has not humanity gained again, by infinite struggle that which it lost through some mistake or blunder of its own? This is the view expressed by Professor Soddy, the English physicist, in his *Interpretation of Radium.*

"Some of the beliefs and legends which have come down to us from antiquity are so deep-rooted that we are accustomed to consider them almost as old as the race itself. One is tempted to inquire how far the unsuspected aptness of some of these beliefs and sayings to the point of view so recently disclosed is the result of mere chance or coincidence and how far it may be evidence of a wholly unknown and unsuspected civilization of which all other relic has disappeared. It is curious to reflect, for example, on the remarkable legend of the philosopher's stone, one of the oldest and most universal beliefs, the origin of which, however far back we penetrate into the records of the past, we do not seem able to trace to its source. Let us give the imagination a moment's further free scope in this direction. . . . What if this point of view that has now suggested itself is true, and we may trust ourselves to the slender foundation accorded by the traditions and superstitions which have been handed down to us from prehistoric time? Can we not read into them some justification for the belief that some former forgotten race of man attained, not only to the knowledge we have so recently won, but also to the power which is not yet ours? Science has reconstructed the story of the past as one of the continuous ascent of man to the present-day level of his powers. In face of the circumstantial evidence existing of this steady upward progress of the race, the traditional view of the fall of man from the higher, former state has become more and more difficult to understand. From our new standpoint, the two points of view are by no means as irreconcilable as they appear. A race which could transmute matter would have little need to earn its bread by the sweat of its brow. If we can judge by what our engineers accomplish with their comparatively restricted supplies of energy, such a race could transform a desert continent, thaw the frozen poles, and make the whole world one smiling garden of Eden. Possibly they could explore the outer realm of space, migrating to more favorable worlds, as the superfluous to-day migrate to more favorable continents. One can see also that such dominance may well have been short-lived. By a single mistake, the relative positions of nature and man, as servant and master, would, as now, become reversed, but with infinitely more disastrous consequences, so that even the whole world might be plunged back again under the undisputed sway of nature, to begin once more its upward, toilsome journey through the ages. The legend of the fall of man possibly may indeed be the story of such a past calamity."

Our modern knowledge of psychical phenomena may thus be a merely recovered remnant of knowledge once possessed in vastly greater fulness. There may have been a time when telepathy was as common and as highly developed for the practical purposes of life as telephony and telegraphy are to-day. The occultism of the East and the strange powers of communicating information to a distance possessed by many savage tribes suggest an age in which, by mind action alone, results were achieved that in our time would be called miraculous. And when an exact man of science suggests that our race was once able to disintegrate matter and mould it into new forms at will, it is surely no mere poetic fancy to imagine that this power, existing at all times potentially, may finally be put forth dynamically in acts of mind.

The merely physical aspect of radio-activity is itself a revelation of such startling importance as to be almost incredible.

"Radio-activity," says Professor Soddy, "has raised an issue which it is safe to say will mark an epoch in the progress of thought. With all our mastery over the powers of nature, we have adhered to the view that the struggle for existence was a permanent and necessary condition of life. To-day it appears as though it might well be but a passing phase to be altogether abolished in the future, as it has to some extent been mitigated in the past by the unceasing, and as it now appears, unlimited ascent of man to knowledge, and through knowledge, to physical power and domination over Nature."

The struggle for existence to-day among human beings is largely a struggle for energy, and it is because of the insufficient supply of this that competition is so fierce, in many respects even so unmoral, and that money, which is simply so much energy, is so unequally distributed. Science now opens up the prospect of such a mitigation of the struggle as will lay at least a physical foundation for a new order of society. When it becomes possible to liberate and to utilize the "intra-atomic energies," matter will supply us with the powers needed for life and social progress in such unlimited quantity as literally to change the face of the world.

The process of creating matter out of ether is much simplified by the consideration that atoms, being already in existence, the transmutation of one set of them into another, would be all that is necessary to obtain any desired material results. Says Soddy in *Matter and Energy*:

"The discovery of the relation of the atom to energy within the last decade recalls the strange medieval myth that the Philosopher's Stone, which had the power of transmuting metals, when discovered, would prove also to be the elixir of life. Transmutation, the pulling to pieces and putting together of atoms, would render available the primary sources of energy, which maintain the time-defying processes of cosmical evolution."

Such a philosopher's stone man possesses in his brain with its brain waves. It is generally agreed now that all life is inseparably bound up with electricity, and that our own organism is nothing but a storage electric battery, while our brain waves are disturbances in the ether similar to light or electric waves. The extreme minuteness of these brain waves does not militate in the least against their becoming, while acting rhythmically, the greatest source of power ever at the disposal of man, any more than the extremely small attractive power of amber known to the ancient Romans militated against electricity becoming the mighty agent of power it is at present. Sir Oliver Lodge says:

"If the ether can be set spinning, we may have some hope of making it imitate the properties of matter, or even of constructing matter by its aid. But how are we to spin the ether? Matter alone seems to have no grip of it.... We cannot spin ether mechanically. But we can vibrate it electrically; and every source of radiation does that. An electric charge, in sufficiently rapid vibration, is the only source of ether waves that we know; and if an electrical charge is suddenly stopped, it generates the pulses known as X-rays, as the result of the collision. Not speed, but sudden change of speed, is the necessary condition of generating waves in the ether by electricity. We can also infer some kind of rotary motion in the ether; though we have no such obvious means of detecting the spin as is furnished by vision for detecting some kinds of vibration. Rotation is supposed to exist whenever we put a charge into the neighborhood of a magnetic pole. Round the line joining the two the ether is spinning like a top. I do not say it is spinning fast; that is a question of its density; it is, in fact, spinning with excessive slowness, but it is spinning with a definite moment of momentum.... The fact of such definite rotation was discovered by Faraday.... In whatever way it is regarded, it is an example of the three rectangular vectors. The three vectors at right angles to each other, which may be labelled Current, Magnetism and Motion, respectively, represent the quite fundamental relation between ether and matter, and constitute the link between electricity, magnetism and mechanics. Where any two of them are present, the third is a necessary consequence. This principle is the basis of all dynamos, of electric motors, of light, of telegraphy, and of most other things. Indeed, it is a question whether it does not underlie all that we know in the whole of the physical sciences;

and whether it is not the basis of our conception of the three dimensions of space."

Since mathematics offers us a glimpse into a fourth dimension of space as yet transcending our actual experience, why could we not put the brain waves, of whose force we have already a glimmering, however faint, in the same group with current, magnetism and motion—all being fundamentally but modes and manifestations of one common energy, the ether?

The fact that it is associated with man's organism, and is an emanation of both his physical and spiritual nature, invests with a special significance this hitherto almost wholly unexplored force. Nor were the scientists slow to recognize the portentous value of brain waves and try to reduce their manifestations to strictly scientific data.

A discovery of this kind was a few years ago announced to the French Academy of Sciences in the form of observations showing the existence of a vital emanation from the human body, "analogous in its behavior to that of radium, Crookes' bulbs, X-rays, radio-activity" and like them capable of producing images on the photographic plate. The evidence for these experiments cannot yet be considered conclusive, but it is a fact of enormous significance that such experiments are being made. Science first heard of them thirty years ago; they have been frequently repeated since then; and the French savant's belief that there is some new knowledge to be won in this direction may very well indicate the path along which humanity is yet to travel. If a tiny bit of inorganic matter like radium, extracted with infinite toil and patience from a mass of pitchblende, can display powers which astonish the scientific world and revolutionize scientific conceptions, what might not be expected from "vital rays," and still more from the subtle activities of the human brain?

It is interesting to note that in the past human reverence has persistently surrounded the heads of saints and divine personages with halos. May these halos not only represent power outflowing from the human brain, but also stand for harmony between spirit and matter—a harmony such as was meant by Kant when he spoke of "good-will," such as found complete manifestation in the divine personality of Christ at whose word spirit (or ether, or whatever it may be called) was translated into matter? Are we not justified in assuming that the mistake, the blunder to which Professor Soddy refers—the serpent that crept into the garden of Eden and caused the fall of man—may well have been the evil which, entering the human soul, disrupted the harmony between matter and spirit and plunged the race into the lower state of toil and pain?

Staggering and stupendous as is the contention that rythmic thought waves are capable of synthesizing matter from ether, it is by no means far fetched, but fully borne out by the past achievements in the domain of science. Electricity, as a force, has always existed; as a factor in human progress, it is a mere infant born of yesterday. Could you have told an ancient Roman, as he amused himself by rubbing a piece of amber and watching it attract light bodies, that this selfsame power would one day propel over the seas ships a thousand times heavier than the largest Roman galley, that it would carry the human voice over distances farther than the farthest reach of the Roman aqueducts, and even signal human thought far beyond the pillars of Hercules, the ancient Roman would surely have viewed such a prophecy as arrant nonsense if not downright madness.

Thus far we have dealt with the scientific aspect of the problem. Let us now see how far we can rely upon metaphysics in support of this view. From what we know of the universe, both through personal experience, and by way of pure reasoning and deduction, it is clear that the creative process in nature must have mind behind it. That is, the contrivances we see in nature cannot arise (1) either by chance, through purely mechanical causation, or (2) by voluntary action and manipulation in the manner in which we human beings produce results and carry out our plans. The mechanical process in nature, that is, causation, must therefore be guided to ends, to the realisation of the results which we call teleological, that is, planned with view to ends. If mind is operative in nature it cannot be merely an abstract principle, but must be an impelling force. It must so permeate and influence cause as to give it the character of a working through means to ends. The mind engaged must be omniscient, aware of all forces and of all possible results which may come from the direction and guidance of them in any particular case. Such a mind must act directly as mind, without needing to be embodied in organic form or to work upon its material by way of the voluntary action through which we accomplish our results as human beings. We must assume therefore that creative power is put forth as a direct influence exèrted, first upon ether to differentiate it into matter, and then upon matter to bring into existence the various organic and inorganic forms which make up what we know as the world. Up to the present man's powers of contrivance—of reaching ends by way of means—have been restricted to the physical manipulation of matter by means of his organs, tools and machines, with mind coming in only as a guide to these. In the ultimate stage of our evolutionary progress man's mind must become a direct, and not merely a secondary factor, in his creative activity.

How do the teachings of the foremost metaphysicians of modern times bear out this view? Paul Janet says in *Les causes finales*:

"If each one of the things in the universe taken separately has been produced by another — why, for what end, all taken together, have they been made? Unity of cause presupposes unity of end. If a single cause has done it all, it must have done it for one end, and as the cause is absolute, the end also must be absolute. But as there are no two absolutes, the cause and the end must be identical; and consequently, God must have made this world for Himself. If God, as absolute perfection, could not have created the world for an egotistic end (for then it would have been simpler not to create it at all), if on the other hand we cannot suppose that He created it only by accident or as a plaything, it therefore follows that He created the world only in the interest of the created beings—that is to say, out of goodness. Such is at least the only way in which the human mind can conceive the reason of creation; such is, translated into human language, the only hypothesis which permits us to conceive of the relation between the infinite and the finite, between the imperfect and the perfect, between the Creator and the creature."

According to Paul Janet, therefore: if the first absolute cause is goodness, the absolute end must also be goodness. God, the absolute cause, by creating the world, had goodness in view as His end. Inasmuch as man is derived from God, man's end must be identical with God's, goodness. By achieving this end, since there are no two absolutes, man *ipso facto* becomes an absolute cause, and therefore at one with God, sharing His power, partaking of His wisdom, and striving for that "divine event toward which the whole creation moves"—that is, the setting up of the kingdom of God on this earth.

Not wanting in analogy to this view is the reasoning by which Henri Bergson seeks to guide us, though along a slightly devious path. His primal cause is the vital urge, the *élan vital,* and he regards it as a sort of divine message which is given out to and expressed in all created things. But it gets garbled in transmission. It is only dimly presented in the unconscious world of inorganic matter; it comes out more clearly in the realm of plants and animals; its highest expression is in man. Suppose we accept this vital thrust, this *élan vital,* as a symbol of creative power, through which God acts in and upon all things. It is evident that when transferred, its efficiency must depend on the nature of the instrument through which it is transmitted, and must flow out in all the greater potency according to the degree in which that instrument approaches perfection. Man as he is now is handicapped by the imperfections of his organism, yet even in his present state the divine message is able to manifest itself through him in flashes of intuition. That is why Bergson lays such stress on intuition and extols it above all knowledge acquired under the merely material limitations of man's intellect. It follows from this that the more perfect and good man becomes, the more harmony he brings into his life and thought, the more completely will he be capable of receiving the full efflatus of the vital urge, the *élan vital,* and of applying its creative power to the human environment and to human conditions. Bergson's intuition, separated from knowledge in his system, will then become one with knowledge. In man will be combined the creature and the Creator, the doer and the deed; through him the divine message will pass out to matter as well as to life, and in creative activities will refashion as well as illuminate the world.

The analogy between creative activity in the universe and the creative activity of man finds its full exemplification in the domain of art. Says Janet:

"Nature is no more artist by chance than she is geometer by chance; her esthetics are no more fortuitous than her industry. It is because there is an industry of nature, a geometry and esthetics of nature, that man is capable of industry, of geometry, of esthetics. Nature is all that we are and all that we are is derived from nature. The creative genius the artist feels in himself is the revelation and the symbol of the creative genius of nature."

And Janet further writes:

"What we call in God the ideal creation (the difference of infinite from finite being understood) is an act analogous to that which we call the creative act of human genius."

According to Goethe's conception, life reaches its highest form in artistic creation because in art nature comes to self-consciousness and shows its inmost essence in visible shape. Schiller assigns to art the same place. His reason is that man, who in the rest of life is divided and shattered, wins his unity through art, so that he may act as one power.

Partaking of the divine creative power, all art is religion. Schopenhauer saw in art the embodiment of the universal will manifested in types that are eternal, time-transcending, immortal. Schopenhauer's theory of art has thus been summarized:

"The idea obtains a relatively complete realization in art (and philosophy) in the creations of genius. While ordinary cognition is merely subservient to the ends of mere living, or is purely relative, esthetic and philosophic cognition are ends in themselves, and reveal the pure idea, since they show the immediate essence or nature,— the "what,"—and not merely the mediate nature,—the "why," or relative causes of things.... In the series of arts constituting a series of objectifications of the idea, architecture contains the idea of mere blind force; sculpture and painting, respectively, of organic (human) form and action; poetry, of historical development; music,—the highest of the arts,—of the inner essence of things."

The secret of the powerful sway of music over the human soul lies in the fact that by its very nature of harmony and rhythm, it is the most immediate in its effects, attuning man to the universal soul. For this reason it has from time immemorial accompanied divine worship, to which all the other arts—architecture, sculpture, painting, the inspired word of man—are ancillary.

Finally: how far are the ideas here expressed capable of practical application to every-day life? Will they stand the test of pragmatism? Will they work? Thus far in human history the incentive to moral perfection in man has been a system of rewards and punishments in the hereafter. Be good, says religion, in order that you may enter the kingdom of heaven after death. This system on the whole worked, and the proof is that the world goes on. But it does so laboriously, painfully, haltingly; social readjustments are slow because of imperfect social unification, because of the fact that humanity fails to grasp the fundamental principle, underlying all creation, that goodness is power, that each individual is simply a unit, that his strength comes from the unity and harmony of all. In a word, the system of rewards and punishments as practised to-day, is too remote, too detached from man's life and experience, and for that reason is inefficient and wasteful; it follows, so to speak, a circuitous route, meeting science half way and winking at it from behind hedges. It is out of date, because while in education and even in the treatment of criminals we are gradually seeing light and abolishing the rod, we still lay emphasis on hell in our religion. This antiquated system is bound to be superseded in the course of time by a system of rewards in this present world as offering the highest possible incentive to moral perfection and goodness, with religion and science as a single cult leading humanity harmoniously along a straight path toward the common goal. A constructive view of this sort would naturally leave punishments out of account, except merely as dissuasives.

It may be pertinent here to remark that the great American pragmatist philosopher, Prof. William James, already referred to in this article, saw the need, toward the end of his life, of a new constructive metaphysics which, by fusing science and religion into one cult, would work for the greatest good of mankind.

To set aright a world manifestly out of joint, we must start at the beginning, with the bringing up of the new generation. What makes a man a Mohammedan, a Buddhist, a Hebrew, a Christian? Clearly, it is the way in which he has been brought up from childhood. It is perfectly practicable to educate children in the idea of attaining goodness in their lives and souls as a means of attaining to God and to all power. The possession of the earth and the fulness thereof is a powerful incentive, and there can be no greater than the cry of the Scriptures, "Oh grave, where is thy victory? Oh death, where is thy sting?"

Far more potent than the lure of immortality beyond the grave is the incentive of immortality in man's actual life, for it is easier for the mind to grasp the idea of uninterrupted bodily existence with all it implies than that of one resumed after the dissolution of the material body with only belief to substantiate the beyond. If, as the scientific argument demonstrates, it is possible to synthesize matter from ether, then it is also possible to achieve immortality. The potentialities of ether being infinite, immortality for human beings may thus mean aspects and modes of existence of which in our present state of knowledge we can form no adequate conception. Nor is the practicability of the incentive invalidated by the consideration that life is too short for any one individual to realize perfection and to achieve divinity in his personal existence. For man does not live for himself alone—more fully and more intensely he lives for his children and for generations yet unborn. But the greatest assurance that this system would work lies in the fact that it would be able to perpetuate itself by the very good it would achieve in uniting mankind as one power.

There is still one more aspect to be considered: that of the methods by which man may be able to create matter and refashion not merely social conditions, but the very physical environment itself, in the midst of which his lot is cast. How, then, shall he, ceasing to adapt himself to his inanimate surroundings as something which he cannot change, but must take for granted, begin to adapt them to his needs and plans? This can be accomplished only by intense concentration, involving not merely individual, but also social unification. We get the model and forecast of these in the organism itself, where concentration is essential to all activities and to every putting forth of will. The human individual is able not only to work out plans, but to hold his mind definitely on particular ends which he wishes to see realized. How, then, are the numerous individuals who make up society, with different interests and ideals and with conflicting aims, to unite their multifarious will powers with a view to a common result? It must be noted in the first place that a social community is not a mere multiplicity of individual units, but has its members linked together in a thousand ways. To a large extent, and for things common to it, the community thinks in common. It is now recognized that there is a social as well as an individual consciousness, a social will as well as an individual will, and that these collective powers of society are coming to be exerted in a more intense way and over a much wider field than was formerly the case. Communities are more unified than before; they think more and will more in common than at any previous period of social history. If individual telepathy is possible, due to individual consciousness and will, what is to exclude the possibility of a telepathy, of a thought transference which is social in character, which proceeds from the community thinking and willing as a whole?

If, however, the prospect of concentration on so vast a scale may seem far too remote from the present, history on the other hand furnishes us with ample illustrations of intense individual concentration associated with transcendental powers. Such, for instance, were the mystics of old who, by abstracting themselves from the interests and impressions that constantly bombard our conscious existence, were enabled to unite with the soul of the universe, with God, and thus gain wondrous wisdom and power. The adepts of Yogi in India, the followers of Zen in Japan, all belong in the same class. But perhaps the most shining individual example we see in Buddha who, forsaking the life of pomp and pleasure, retired into himself and meditated upon the miseries of humankind until he became aware of the essence of things and of his own mission. Even so the Son of Man went into the desert and fasted, and prayed, and wrestled with his soul ere he gave himself to the salvation of the world.

Over the bridge that the Past ever throws to the Future we may tentatively trace the route which humanity in ages to come is to travel. With moral perfection as the cornerstone of all individual and universal existence, life must needs become one common prayer, in which human souls, following the path of unity and harmony, as with one accord will become one force, one power. Acting as an impetus in the world of ether, this force will be potent to transform by means of brain waves the pure idea into creative idea, thus completing the cycle as it is now exemplified in the order of creation. Says Janet:

"The principle of good which is in the universe, must be not only conservator, but organizer, creator, promoter."

But after all, the methods and ways and means of concentration are only a matter of detail which we may safely intrust to the future, once the fundamental truth is grasped that by achieving moral perfection man will achieve his unity with God and partake of the divine power of creation.

By inference this power is at present denied to man because of his imperfect morality, his self-seeking, his lack of harmony and unity within himself and with the rest of mankind. Nor can there be human freedom—all the legislation in the world notwithstanding—until humanity grasps the inner meaning of the fundamental laws of man's spiritual development.

To sum up. The pre-condition of the mastery of mind over matter is harmony and unity in the thoughts of men, and by implication in their lives and actions. Thus harmonized and unified, mind action, giving rise to rhythmic ether waves, would liberate and direct the intra-atomic energies, and open up to humanity the new unknown source of power to which Professor Ramsey alludes. The closing words of Bergson's address before the English Society for Psychic Research seem to point in the same direction, for he predicts that "the science of mind will attain results surpassing all our hopes." The new source of power will enable man to reshape or even create his environment at will. As soon as this truth is realized a new era will dawn upon the world. Blending science and religion into one cult, mankind will devote itself to the task of harmonizing and unifying human thought. The history of civilization is the history of great religious movements. Christianity, Buddhism, Mohammedanism, shaped the destiny of the world by inspiring its people with great ethical principles. The goal of man is to become divine, and the way to attain it is the way indicated by every religion—the *via beatifica.* When every one awakens to the idea that he can help or hinder the bringing of heaven on earth, there will be no need of jails or standing armies. Humanity will then have set out on its quest of the Holy Grail. Thus the greatest power in the world is the power of thought, since behind it is that something—the "substance" of Spinoza, the ether of modern science—in a word, the spirit out of which all things arise. Ultimately, the forces at work in the world will not be measured in terms of horse-power, of foot-pounds, of calories, but in terms of mind—unified, harmonious, creative. There is awe-inspiring grandeur in the conception of the potential worlds embodied in ether which, at the call of man, the creator, may arise and be.

L. L. Pimenoff.

Boston, Mass.

BODY AND MIND.

ALL unsophisticated people believe that their minds act on their bodies and their bodies on their minds. If some one sticks a pin into me and I feel a painful sensation, it seems obvious that the entry of the pin into my body is the cause of the sensation in my mind. Similarly if I will to move my arm it seems obvious that the volition in my mind causes the movement of my body. The view that mind acts on body and body on mind may be called "two-sided interactionism."

In spite of the fact that interactionism seems at first sight to be certainly true, we have to notice that it is at the present time rejected by what is probably a majority of scientists and a majority of philosophers. Most people who have studied the subject from the side either of philosophy, or of physics, or of physiology have come to the conclusion that the mind does not act on the body and that the body does not act on the mind. Such a strange conclusion and one so contrary to the belief with which we all start must need powerful arguments to support it; and what I propose to do in this paper is to state and criticize the most important of these as carefully as I can.

Before entering into these arguments in detail, I would like to point out that this is essentially a question which cannot profitably be discussed by mere philosophers or by mere scientists, but only by persons with a competent knowledge both of philosophy and of natural science. The question is: are events of a certain kind causally connected with events of a certain other kind, or are they not? To answer such a question one must have a competent knowledge of the two kinds of events and their laws, and one must understand exactly what is meant by causation. Now mental events and their laws are treated by psychology, and bodily events and their laws are treated by mechanics, physics, chemistry, and physiology. Hence some knowledge of all these sciences is necessary before one can discuss this question. But, though it is necessary, it is not sufficient. All natural sciences make constant use of the notion of causation, but the notion of causation does not form part of the subject matter of any natural science. Causation, its precise limitations, are part of the subject matter of philosophy. Similarly arithmetic makes continual use of reasoning but it is not about the process of reasoning, for this is dealt with by logic.

With these preliminary remarks we may turn to the special arguments which have been used against interactionism. I will begin with two purely philosophical arguments. They seem to me quite worthless and we may as well clear them out of the way at once.

I. One argument is that body and mind are so entirely unlike each other that it is inconceivable that events in one should cause events in the other. How could two events so different as eating a beefsteak and thinking of a poem, or having a volition and making a bodily movement be causally connected? This argument assumes that events can only cause each other if they be sufficiently similar, that if they be sufficiently similar their causal connection is intelligible, but if they be very different it is inconceivable. The answer is (a) that however similar two events may be the fact that one causes the other is never self-evident but has to be learnt by experience. It is not *a priori* self-evident that one billiard ball moving straight on to another will make the second move in the same straight line; we have simply learnt that this is what actually happens. We have exactly the same kind of evidence for the view that sticking a pin into a man's body causes a painful sensation in his mind. In neither case is the connection intelligible, if by intelligible you mean logically deducible from what is otherwise known of the nature of billiard balls or of pins respectively. In both cases it is intelligible, if by this you mean that it is a fact which involves no contradiction and is actually found to be true. (*b*) We are not told in this argument how dissimilar events must be before it becomes unintelligible that one should cause the other. A draught is not particularly like a cold in the head, but no one who habitually changes trains at Clapham Junction will deny that the former may cause the latter. And if the dissimilarity between a draught and a cold in the head does not render their causal connection impossible, I fail to see why the difference between a pinprick and a painful sensation should make *their* connection unintelligible.

II. A more refined form of philosophical argument is the following. It is said that wherever we have a genuine instance of causation the events are connected by a great many other relations as well as the causal one. The two billiard balls have definite spatial relations to each other, and so on. It is argued that there are no such relations between a pinprick and a painful sensation or a volition and a bodily movement. The mental states are not in space and the bodily events are, hence there can be no spatial relations between them. Hence it is argued that mental and bodily events cannot be causally connected. Although this argument has the support of so eminent a philosopher as Professor Stout, I must confess that I can see very little in it. I have four objections to it. (*a*) How do we know that the causal relation can only subsist between two events when other relations subsist between them too? It does not seem self-evident and I know of no attempt to prove it. (*b*) How do we know that there are not other relations between mental and bodily events? It is perfectly conceivable and even probable that bodies have many qualities which we cannot perceive owing to the very limited range of our senses. It is still more likely that states of mind have many properties which we cannot detect by introspection. I see no difficulty whatever in supposing that there may be plenty of relations between states of mind and states of body of which we are unable to become aware. Now, if this possibility be granted, it seems much more reasonable, in view of the strong appearances in favor of interactions and the difficulties which we shall find in all alternative views, to suppose that there really is interaction and that we are unable to become aware of the other relations than that no other relations exist and consequently there is no interaction. (*c*) But, further, in certain cases we can actually see that there are other relations between mental and bodily events. When I will to move my arm I have to think of my arm and of its present and its future positions. Here we have at once a definite relation between volition and bodily movement, viz., the fact that the part of the body to be moved and its movement must be objects of thought to the mind. This is just as good a relation as the spatial relations of the billiard balls. Since mind and body are very different we need not be surprised to find that the relations between mental and bodily events when they interact are considerably different from those between two bodily events when they interact. (*d*) Finally, a man who believes that mind and body interact is not obliged to suppose that a bodily event is ever the total cause of a mental one or conversely. It is quite open to him to think that a painful sensation has a complex cause one part of which is a pinprick and the other some state of his mind. There is much in our experience to favor such a view and nothing against it. E. g., a person who is kicked with the same hardness, once when he is sitting quietly and at another time when he is playing in a football match, will have considerably different sensations in the two cases. This suggests that the sensation felt is a joint product of his body and his mind. If his body had not been kicked he would not have had the painful sensation, if his mind had not been attending intently to the game the sensation would have been much more painful. But, if states of mind are often the joint products of states of body and of other states of mind, and conversely, the objection that there is no other relation between the alleged cause and the alleged effect obviously breaks down; for there will be an intimate relation between the mental factor in the total effect.

For these reasons I think that the purely philosophical arguments against interaction have no tendency to refute the view of common sense, and therefore we may turn to arguments based on the accurate observations and the accepted laws of natural science.

The most important argument of this kind is based on observations on the energy-changes in the human body and on the physical principle of the conservation of energy. But closely connected with and supporting this argument is one based on the fact that all nervous process is physiologically of the reflex type. I will deal with these two arguments in turn. The one about the conservation of energy will occupy us for some time, for we shall have to make clear (*a*) what are the observed facts, (*b*) what is really meant by the conservation of energy and in what sense it is probably true, and (*c*) what bearing the observed facts and the principle really have on the question of interaction.

a. The following are the observed facts. Very careful experiments have been performed on human beings with a view to testing whether any changes of energy occur in human bodies which cannot be accounted for by the chemical energy produced by the oxidation and other changes in the chemical energy of the food which a man eats. When a man moves his arm there is an increase of kinetic energy. But it is found that, within the limits of experimental error, this increase is compensated for by a decrease in the chemical energy of some part of his body. The upshot of the matter is that competent observers after careful experiments seem to be convinced that the system composed of a human body, the air that it breathes, the food that it eats, and the heat that it evolves is energetically a closed system. That is, it is a system whose total energy remains unchanged, an increase in one factor being compensated by a decrease in some other factor. I do not intend to criticize these observations, which seem to have satisfied competent observers, except on one point. It seems to me that such experiments can only tell us what is true on an average over a long space of time. To make them perfectly satisfactory one would need to know the total chemical energy in the man's body at each moment of the experiments. This we naturally cannot do since it would involve killing the man and analyzing his body at each moment; a process which would be both illegal and physically impossible, since it would involve killing him to get one's observation and bringing him to life again to continue the experiments. Remembering these limitations we can say that the net result is that over the period of the experiment the total amount of energy given out by the body in heat and movement balances that lost by the food eaten and the air breathed. This leaves it perfectly open to us to hold that the balance is not maintained at every moment, that sometimes there is more and sometimes less total energy present in the system, but that these differences average out over a long period and are never very great.

It is doubtless true that we should always find that when less energy was being given out in heat and movement than was being taken in in food and air the weight of the man's body increased. We could thus conclude that chemical products were being stored up in the man's body and might suspect that their chemical energy would make the balance right. But we cannot be sure of this because we cannot kill the man and discover just what these storage products are and hence what their chemical energy is. We cannot therefore be perfectly sure that the total energy of the system never decreases, though we may very strongly suspect this. We are on safer ground in concluding that the total energy of the system never increases. When more energy is given out in heat, movement, and waste products than is being taken in in food we shall find a decrease in weight in the man's body. This will lead us to ascribe the balance to the oxidation of stored products. An analysis of the waste products may then tell us what these stored materials must have been and from this knowledge we can deduce the chemical energy which will be liberated by their oxidation.

The upshot of the matter seems to be that (1) we can be pretty certain that in the long run and on the average the energy given out by the body balances that taken in. (2) That we can be pretty sure that at no moment does the total energy of the system increase. (3) That we may strongly suspect, but can never be quite so certain, that at no moment does the total energy of the system decrease.

b. We have so far spoken of energy as if every one knew what it was, and of the conservation of energy as if this were an unambiguous principle which was certainly true. We must now try to become clear on these two points. The only perfectly clear meaning of energy and its conservation is found in kinetic energy in mechanics and in the collision of perfectly elastic bodies. All other forms of energy and all statements about their conservation are not matters of pure observation but are a mixture of observation and convention. This I will now try to show.

The kinetic energy of a body of mass *m* moving with a velocity v is defined as the product $\frac{1}{2}mv^2$. Since mass and velocity can be measured kinetic energy can also be measured. If two perfectly elastic bodies (*e. g.*, two billiard balls) collide it is found that the sum of their kinetic energies before and after impact is practically the same, though the distribution of it between the two may be greatly changed by the collision. Here everything is measurable, the meaning of the law is perfectly clear and there is no element of convention in it. The next stage is the introduction of the notion of potential energy in mechanics. Suppose that a body with kinetic energy $\frac{1}{2}mv^2$ moves up against a perfectly elastic spring and presses it inward. The velocity of the body and hence its kinetic energy will gradually be reduced to nothing. But subsequently the spring will expand again and impart velocity to the body in the opposite direction. And it is found that when the body once more leaves the spring its kinetic energy will again be approximately $\frac{1}{2}mv^2$. These are the actually observable facts. It is clear that, if we confine ourselves to kinetic energy, this has not been conserved. It has in fact passed through all the values between 0 and $\frac{1}{2}mv^2$, and so at all intermediate stages of the transaction the kinetic energy has been less than at the beginning and end. Now the conservation of energy is only maintained by postulating a new kind of energy *ad hoc* and giving such a measure to it as will preserve the principle intact. It is said that as the body loses kinetic energy the spring gains potential energy and conversely. Now potential energy, unlike kinetic energy, cannot be directly measured; we merely ascribe to it such values at any moment as shall keep the principle true. There is therefore an element of "cooking" or convention in the principle even as applied to such abstract cases as purely mechanical transactions between perfectly elastic bodies. All that we can say is that the assumption of potential energy and the ascription of this value to it are compatible with the observable facts, not that they are necessitated by them.

If now we leave purely mechanical events and purely elastic bodies a further dose of convention is needed to preserve the principle, though there are also further observed facts to take into account. If we used billiard balls of lead or putty we should find that the kinetic energy was nothing like the same after a collision as before. Nor could we put this right by assuming potential energy and giving an appropriate measure to it, for we should find that the bodies, unlike the spring in the last example, had been permanently deformed. And, so long as we keep to mechanics, we must simply say that the principle has broken down beyond hope of further "cooking." But, by extending our observations beyond mechanics, we can discern a further important law of motion; and, by a liberal dose of convention, we can state this law in such a way that the conservation of energy can be retained. We shall find that when bodies are permanently deformed other physical phenomena occur. Their temperature rises, they may give out sound waves, or they may produce electrical phenomena. We can directly measure quantity of heat in its own units. And it has been abundantly proved that when a certain amount of kinetic energy disappears from a system and no other change takes place except a rise in temperature the amount of kinetic energy lost measured in mechanical units and the amount of heat gained measured in thermal units bear a constant relation. The same is true when heat disappears and kinetic energy is the only result. Note that, strictly speaking, there can be no question of equality. Kinetic energy is one thing, heat is another; a unit of kinetic energy is different from a unit of heat, and it is really meaningless to talk about equality between the two. All the observed facts tell us is that the number which measures one in its units bears a constant relation to the number that measures the other in *its* units. The same is found to hold for other physical phenomena like light, sound, and electricity. Now these observed facts can be stated in the form that quantity of heat, electric potential, etc., are forms of energy and that when ever one disappears from a system an equal quantity of the other takes its place. Quite strictly speaking this is nonsense, because you can no more talk of a quantity of heat being equal or unequal to a quantity of electric potential than of an archdeacon being equal or unequal to a quadratic equation. Equality and inequality, in the strict sense, can only hold between two quantities of the same kind; and a quantity of heat is not of the same kind as a quantity of electric potential. But this way of talking is convenient in practice, and, by adopting it, the form of the conservation of energy can be preserved when it would otherwise break down. We may sum up then as follows: Strictly taken the conservation of energy is a meaningless and nonsensical proposition. But, interpreted liberally, it is a statement of the observed fact that in mechanical, physical, and chemical phenomena, when n units of any one kind disappear from a system there will be an increase in the number of units of some of the other kinds in the system, and the numerical values of these increases will bear a constant ratio to n. It must be added that this will only be true if the system is isolated; otherwise, as when heat leaves a system by radiation, the compensating change may happen in some other system. The law will then hold of the two systems taken together, but not of either taken separately.

c. Now this principle, together with the experimental facts about the energy-changes in the human body described above, is taken to prove that the mind does not act on the body and that the body does not act on the mind. The question for us is: Does it prove anything of the sort? I take the argument to be this. Experiment proves that the body, its food, air, etc., form an isolated energetic system. Any change in the energy of the body is completely balanced, in the sense given above, by other changes in the energy of this system. If the mind acted on the body this system could not be isolated, energy would appear in it when we made a voluntary movement, and this energy would not be balanced by the disappearance of energy from any other part of the system. Similarly if the body acted on the mind energy would disappear from the body when the mind had a new sensation, and this energy would not be balanced by an increase somewhere also in the system. As this balance actually does take place mind cannot act on body and body cannot act on mind.

This argument, which has convinced a great many eminent persons of the impossibility of interaction, seems to me to have no weight at all against the evidence from constant experience in favor of interaction. I will now state why it appears to me to be worthless. It assumes that if body and mind interacted with each other we should have to assume a new kind of energy—mental energy—in order to preserve the conservation of energy. We should find energy unaccountably appearing in the body when we made a volition to move and unaccountably disappearing from it when a pin entering our bodies was followed by a sensation in our minds. Since we do not need to assume mental energy it is concluded that there can be no interaction. But this would only follow if it were certain that two things cannot interact without changes of energy in each. Now this is not asserted by the conservation of energy at all. What is asserted is that *if* things interact and *if* their interaction be accompanied by change of energy, then these changes will obey the conservation of energy. The conservation of energy then by itself has no bearing on the question of interaction. It is true however that when physical systems interact with each other there are changes of energy in both; though this could not have been foretold from the conservation of energy. But this does not in the least prove that *all* interaction must be accompanied by changes of energy; in particular it leaves it a perfectly open question whether, when a mind interacts with a body, such changes take place. The experimental facts strongly suggest, though they do not prove, that the interaction of mind and body is not accompanied by changes of energy; they have not the faintest tendency to show that no interaction takes place. And the conservation of energy, which is apparently supposed to be the bulwark of this argument, turns out to have as little to do with the case as "the flowers that bloom in the spring."

On the same experiments and the same physical principle another argument is often based. It is said that the experiments prove that the body and its surroundings obey the conservation of energy and that it follows from this fact that everything would proceed in exactly the same way in the body if it had no mind and in the mind if it were not connected with a body. The results of this suggestion are so startling that it may be worth while to consider them for a moment before dealing with the validity of the argument. The L. N. W. Railway was ultimately built entirely by the bodily movements of human beings, and the trains run at stated times from the same causes. If these bodily movements were to take place just the same apart from minds we should have to believe that, although there had never been the faintest glimmer of intelligence on the earth, the L. N. W. Railway would still have been built and that trains would still run into and out of Euston driven by mindless engine drivers and containing mindless passengers reading newspapers printed by mindless printers. Now it really seems incredible that all these things should go on as before if there had been no minds; we should surely expect to find an immense and noticeable difference in everything (except possibly the newspapers). Similarly if the body never acts on the mind we must believe that all our mental states are caused by other mental states. There could be no question of getting a new idea from reading a book or a new sensation from sitting on a tintack, for books and tintacks are alike physical objects. And if we resolutely reject the obvious physical causes of such new sensations and ideas we can find no trace of any mental cause in our past history for them. Any argument which leads to such extraordinary conclusions as this will need to be very strong indeed before it can be reasonable to accept it. In actual fact the argument is extremely weak. Since *every* physical system obeys the conservation of energy the mere knowledge that some particular system such as the human body obeys it will not tell us what that system in particular will do. The system composed of a gun, a bullet, and an explosive obeys the conservation of energy; when it is not discharged the bullet and gun have no kinetic energy and the explosive has great chemical energy, when it is discharged the kinetic energy gained by the bullet and gun is balanced by the chemical energy lost by the explosive. But this knowledge does not suffice to tell us either that the gun will be discharged, or, if so, when it will be discharged. It does not even tell us in what proportion the kinetic energy will be divided between the gun, the bullet, and the gases evolved when the gun is discharged. Similarly the mere knowledge that the human body obeys the conservation of energy does not tell us that it will do anything at all, nor does it tell us what it will do and when it will do it if it does anything. Once again then an argument against interaction which professes to be based on the conservation of energy and on the experiments that have been made on the energy-changes in human bodies is found to rest on neither. What does this argument really involve then? We find in all purely physical and chemical systems, i. e., non-living material systems, that, although the conservation of energy does not determine whether or when one kind of energy will disappear and another kind appear, yet these transformations do obey definite laws. Thus the gun goes off when the temperature is sufficiently and suddenly raised or when a shock is administered to the explosive. We may then define a purely physico-chemical system as one which obeys the conservation of energy, and in which, further, the transformations of energy which take place and the times when they take place are determined by purely material causes according to the special laws of physics and chemistry. Now if the human body were such a material system as this it would follow that the mind could not act on the body, though it would not follow that the body could not act on the mind. A purely physico-chemical system is defined as one where the only causes of change are material ones acting in accordance with physico-chemical laws. If the only causes be material it is clear that none of them could be mental, and that the mind could not act on the body. On the other hand, even if all the transformations of energy in the human body were determined physically or chemically it would not follow that they might not also cause changes in the mind. It is true that physical and chemical changes do not cause sensations when they occur in non-living bodies, but that may perfectly well be because such bodies do not have any minds attached to them in which sensations could be caused. It may quite well be a law of nature as invariable as any of the laws of physics and chemistry that all material systems of the form and complexity of living bodies are accompanied by minds; and that, although the changes in these systems take place entirely in accordance with the laws of physics and chemistry, yet certain of them also cause changes in the minds which, by an invariable law of nature, are attached to such material systems. Nothing that we know about the experimental facts or the laws of physics and chemistry precludes this possibility, and our knowledge that certain bodily changes are always followed by certain sensations and that no other cause for these sensations can be plausibly suggested makes the possibility highly likely. We may call the view that body acts on mind but mind does not act on body "one-sided interactionism." We see then that if it can be proved that all bodily changes take place entirely through chemical and physical causes the most reasonable view to take of the relation between mind and body will be that of one-sided interactionism.

For some reason one-sided interactionism is always stated in a peculiarly absurd form by philosophers and scientists, and is then easily refuted. It is nearly always identified with what is called "epiphenomenalism." This is the doctrine that mental states have no effect either on the body or on each other, that each is produced separately by some bodily change and makes no further difference to anything either mental or bodily. Now if this were the only form that one-sided interaction could take it might fairly be regarded as a preposterous theory. But there is not the least reason either in logic or in any known facts why one-sided interactionism should take the form of epiphenomenalism. It is perfectly open to us to hold that the mind does not act on the body but that mental states are a joint product of certain bodily processes and of past mental states. And there is no reason whatever why certain mental states should not have purely mental causes.

We have now seen what are the consequences of the hypothesis that all changes in the human body take place in accordance with purely physico-chemical laws and have purely material causes. We must now ask whether there is any reason to suppose that this hypothesis is true. First we must notice that, since this conclusion does not follow from the conservation of energy, the evidence for the truth of that law in general, and the experiments which tend to show that the human body and its surroundings form a closed energetic system, have no bearing whatever on the question whether the human body is a purely physico-chemical system. Secondly we must notice that it might be true that the human body is not a purely physico-chemical system, and yet that the vast majority of the processes in it proceed in accordance with purely physico-chemical laws.

If the mind acts on the body at all it is pretty certain that it does not as a rule act directly on most parts of the body. If it acts on the body at all it acts presumably on certain parts of the brain and determines when and to what extent a transformation of energy shall occur there. All the subsequent consequences of this transformation in all the other parts of the body might proceed in accordance with purely physico-chemical laws, and of course all the bodily changes whether started mentally or materially might obey the conservation of energy. It follows that even if all physiologists were agreed (as I understand they are not) in holding that every bodily process that they had investigated took place in accordance with physico-chemical laws it would not in the least follow that none of these processes are started in the brain by the action of the mind.

When we remember the extreme difficulty of proving a negative about any thing, the extreme complexity of the human body, and the impossibility of accurately determining the details of minute processes in the brain of living beings, we may fairly assert that there is no prospect whatever of a direct experimental proof that every process in a living human body proceeds from beginning to end from purely material causes and in accordance with purely physico-chemical laws. Now when a hypothesis cannot be proved or refuted by direct experiment our only course is to consider what will follow if it is true. No hypothesis can be more probable than its logical consequences; hence, if the logical consequences of a hypothesis be wildly improbable we must conclude that the hypothesis is itself wildly improbable. Now the logical consequence of the hypothesis that the body is a purely physico-chemical system is that all its actions would be precisely the same whether it were accompanied by a mind or not. We have already seen that, when this suggestion is considered in detail, it is so wildly improbable as to be ludicrous. Hence I conclude that the view that the human body is a purely physico-chemical system is preposterous, and therefore that there is no reason to suppose that the mind does not act from time to time on the body.

I cannot however leave this point without saying something about the "enlightened parallelist" who figures in Chapter III, § 6, of Professor Stout's *Manual of Psychology* (third edition). Stout, who himself inclines to accept the arguments against interaction, admits that if the denial of interaction led to such absurd results as we have indicated, he would be forced to reject parallelism. But he thinks that they need not lead to any such absurdities. I will quote his example of the enlightened parallelist's treatment of the writing of *Hamlet*. "The manuscript may be regarded from two points of view, each taking account of only one aspect of its nature. In the first place, it may be regarded merely as one portion of matter among others.... From this point of view its existence can be accounted for through merely material conditions including especially certain occurrences in.... Shakespeare's brain. But the manuscript is not merely a material thing; it is also the manuscript of a play to be read, acted, and criticized. From this point of view explanation in terms of material conditions certainly breaks down. What is essential here is the mind, not the brain, of Shakespeare; what is essential is Shakespeare as a subject, thinking, feeling, willing and adapting means to ends. . . . Whether we adhere to. . . . parallelism or to. . . . interaction, this teleological point of view remains unaffected."

The weakness of this passage is that it starts by professing to tell us how the enlightened parallelist will "account for the production of the manuscript of *Hamlet.*" But it actually tells us nothing of the kind. It tells us what any enlightened person must recognize as the distinctive peculiarity of such material objects as manuscripts (viz., that they have a meaning and design). It does not in the least tell us how the enlightened parallelist can account, *qua* parallelist, for what he has to admit, *qua* enlightened.

But we may go further than this. Does Professor Stout mean that Shakespeare's brain and other material causes brought about the particular collection of marks on paper which constitute the manuscript of Hamlet, and that Shakespeare's mind caused the meaning of this collection of marks without affecting his body? Let us consider in what sense you can be said to cause the meaning of a set of marks. Unless a man is making up for himself a new language or symbolism there seems to be only one sense in which he can cause the meaning of a collection of marks. And the sense is this. Certain collections have, independently of him, a meaning for those who see them; and others do not. Of the former, some have, independently again of him, one meaning; and some have another. The only way in which he can cause a meaning is by causing the particular collection of marks that have that meaning. The only way in which he can do this is by the appropriate use of his body. And the only way in which he can appropriately use his body for this purpose is through his mind thinking of the meaning and causing his body to make the movements which cause the collection of marks that express this meaning. Unless the thoughts and desires of the mind can affect the movements of the body I fail altogether to see how an intentional meaning can be expressed by any material object which is produced by the movements of the body.

So far as I can see the least that an enlightened parallelist could hold would be somewhat as follows: (1) All material systems and their changes have purely material causes. (2) Of material systems some are marked off from the rest by showing traces of meaning or design. (3) Somewhere among the material causes of such peculiar material systems will be a state or states of some one's brain. (4) With this state or these states will always be correlated in some one's mind a thought of the meaning and a desire for its expression.

Such a view seems possible, even if not plausible. But it would still leave parallelism powerless to explain the causes of our sensations. I think therefore that one-sided interaction of body or mind would always be in a stronger position than parallelism. For (*a*) it can give the usual explanation of the causes of our sensations. (*b*) It is, as we have seen, perfectly compatible even with the view that the body is a purely physico-chemical system. (*c*) With regard to the causation of material objects which show traces of meaning or design it could take practically the same view as I have ascribed to a really enlightened parallelist. The only modification would be that for (4) in the enlightened parallelist's position it would substitute the proposition: This state or these states of brain *cause* in the mind connected with this brain a thought of the meaning and a desire for its expression.

Mr. Russell argues in his *Lowell Lectures* that when we once understand that causation is nothing but functional correlation we can see that the quarrel between an interactionist and an enlightened parallelist is largely a matter of words. On this assumption as to the meaning of causation it will at any rate follow that if parallelism be true so is interactionism. If we hold that there is a one-to-one correlation between the states of our brain and the states of our minds, and a one-to-one correlation between the states of our brains and the changes in the physical world which we say that these produce, then there will be a one-to-one correlation between our states of mind and the changes in the physical world. And if causation means nothing but such correlation then we have as much right to say that our states of mind cause the changes in the physical world as that our states of brain do so, or that our states of mind cause our states of brain and that these cause the changes in the physical world.

But, in the first place, I am very doubtful whether functional correlation be the whole of what we mean by causation. This, however, is not the place to embark on this wide inquiry. Secondly, even on Russell's theory of causation, interaction would not imply parallelism. E. g., there might be two bodily states which, as such, were indistinguishable in their qualities. To one there might be correlated a state of mind and to the other no state of mind. Now if we found that the first was correlated with a different kind of change in physical objects from that which is correlated with the second we could say that the state of mind is an essential part of the cause of changes of the first kind. Hence the question at issue between parallelists and interactionists will still be a real one.

It remains to notice a second scientific argument, drawn from the constitution of the nervous system, which is supposed to prove or render it probable that all bodily processes are purely physico-chemical, and hence that mind and body do not interact. If you take a purely reflex action, which may go on without consciousness, the arrangement of the part of the nervous system involved is that the afferent nerves convey the stimulus from the surface of the body and are connected with efferent nerves which convey a corresponding stimulus to the muscles. The two nerves join, or at least come into very close contact, at some place called a *synapsis*; and it looks as if the whole process consisted in some physical or chemical change being started by the external stimulus, pushing along the afferent nerve, affecting the efferent nerve through the synapsis, and producing in it a physical or chemical change which travels along this to a muscle and causes it to contract. There is no stage in such a process when it is necessary or reasonable to invoke anything but physical or chemical causes and laws. Now, it is said, all the nervous mechanism of the body, whether it be associated with mere reflex action or with apparent control of acts by consciousness is of the same type as the reflex arc. It simply consists of an enormous complication of such arcs, so that when a process of change once starts to travel along an afferent nerve there is an immense variety of different possible efferent nerves along which it may travel back to the surface of the body. Hence a single stimulus may be followed by an immense variety of external actions on different occasions. But, it is argued, we do not here have anything qualitatively different from the simple reflex arc, the only difference is one of complication. Hence if we did not need to assume anything but physico-chemical causes at any stage in a simple reflex action there can be no need to assume anything else in the most complex voluntary action. The different actions that follow at different times from the same stimulus will depend on the different resistance at different times of the various *synapses*; but there is no reason to suppose that these variations in resistance are due to aught but physico-chemical causes. If mind and body really interacted, it is said, we should expect to find that certain afferent nerves ended in a kind of blank space in the brain and certain efferent nerves started from the same space. Then we might suppose that a stimulus reaching one end of an afferent nerve would affect the mind and that the mind by its voluntary decision would affect the end of an efferent nerve and thus start a nervous current down it which would finally cause a voluntary movement. Now we do not find any such arrangement as this in the nervous system; hence, it is argued, we may conclude that the mind does not intervene at any stage of the process.

It seems to me that, of these two arguments, which generally appear together, the second is quite worthless, while the first does indeed prove something, though not what its employers suppose it to prove. I call the second worthless because it practically assumes that, if at any point there is a gap in a process of purely physical causation, then must there be a spatial discontinuity, and the mind, in order to act, must somehow be in this gap as a wire has to fill up the gap between a bell-handle and a bell if the former is to ring the latter. Now this assumption simply rests on lack of imagination and abuse of spatial metaphors. When we say that somewhere in a process there is a gap in purely physico-chemical causation we simply mean that at some stage of the process an event occurs which cannot be explained by purely physico-chemical laws. It is obviously unnecessary to suppose that at this stage there must also be a gap or breach of spatial continuity in the process. So far the argument consists in confusing two senses of gap (i) a gap in an explanation, (ii) a gap in space. You must just as well argue that only persons over six feet in height can have high moral characters.

The other confusion consists in supposing that if a mind acts on things in space it must itself occupy a particular portion of space. That is simply due to lack of imagination. We are most accustomed to deal with the actions of things which have definite shapes, sizes, and positions; hence we are inclined to think that all things that act must have these characteristics. The inhabitants of Central Africa had just as good reasons for supposing that all men are black.

The first argument, on the other hand, does, I think, strongly suggest what kind of action the mind has on the body, but does not suggest that it has none at all. It strongly suggests that when the mind acts on the body what it does is to raise the resistance of some *synapses* and lower the resistance of others. It is probable that the resistance of *synapses* has causes which are partly physico-chemical and partly mental, that they may get into a state in which the mind cannot affect them, and that very often the mind does not affect them even though it could. In purely reflex actions it is possible that the mind has no control; in habitual actions which we can control but do not as a rule trouble to control, the non-physical cause is in abeyance; in habitual actions which have got beyond the control of the will the mind has lost its power of interfering with the chemico-physical process. This much the facts about the nervous system do render highly probable. That they do not render it probable that the mind has no control in any case seems to me to result from the following considerations.

The argument that the whole of our nervous processes are of the same type as those which accompany purely reflex actions cuts both ways. Whatever be the similarity in the nervous mechanism it cannot be denied that there is a clear introspective difference between the experience of a purely reflex act, like blinking when something approaches our eye or sneezing when we smell pepper, and a voluntary act, like deciding with difficulty to get out of a warm bath on a cold day. This is a real difference open to any one's inspection. Moreover it is a *qualitative* difference and not a merely quantitative one; the experience of voluntary decision is not simply a mass of experiences of reflex action. Now this qualitative distinction has to be explained somehow; and the more you insist that the whole nervous system differs only quantitatively by its greater complexity from the simple reflex arc the more difficult it becomes to explain the admitted qualitative difference in the two experiences. If then it be certain that the structure of all parts of the nervous system differs only quantitatively from that of the part which is associated with reflex action we seem forced to suppose that there must be some difference, not of structure but of process, in the part associated with voluntary action. And in view of the evidence from daily life that the mind does act on the body in volition it seems reasonable to suppose that this difference consists in the fact that certain processes in the higher nervous system are not entirely physico-chemical. The facts, then, so far from proving that the body is a purely physico-chemical system and that the mind cannot act on it, rather tend in the opposite direction.

We may now sum up our results. (1) The most probable theory is that the mind sometimes acts on the body and the body sometimes acts on the mind. We have evidence for this of the same kind and the same amount as for any other case of causation. None of the objections to it are anything like conclusive, and all alternative theories lead to wildly improbable conclusions. (2) It is probable that in acting on the body the mind does not alter the total energy of the body but only determines in certain cases when and to what extent it shall be transformed. (3) It is probable that in voluntary action the mind affects the body by modifying the resistance of certain synapses. (4) The view that the body is a purely physico-chemical system does not follow from the conservation of energy, and can neither be proved nor disproved by direct experiment. If it were true it would still be possible and reasonable to hold that the body can act on the mind. The reason for thinking that it is not true is that it leads to the conclusion that the body would behave in precisely the same way if it had no mind connected with it, and that this seems most improbable. (5) The arguments based on the structure of the nervous system are partly mere confusions and prejudices. They have no tendency to show that the mind cannot act on the body; but, when all the facts are taken into account, they tend to make it probable that the mind does act on the body. (6) The most foolish of all theories as to the relation of body and minds seems to be epiphenomenalism; next to it comes parallelism, the doctrine that all which goes on in the body is determined by purely bodily causes, that all that goes on in the mind is determined by purely mental causes, and yet that there is a mysterious correlation between events in one series and events in the other.

C. D. Broad.

The University, St. Andrews, Scotland.

IN REPLY TO DUALISTIC CONCEPTIONS OF MIND.

HOW attractive is the idea of "Mind the Creator of Matter"! In a certain sense the theory is old, as old as religion, as old as mankind, as old as the first dawn of civilization, for mind has been considered the creator of the whole world; God is the creator and God has been assumed to be mind in the narrowest sense of the word.

The present number of *The Monist* contains an article under this caption by L. L. Pimenoff, who here presents the proposition of "mind the creator of matter" in a still more specific sense. It is not only the old idea that God created the world in the Biblical sense, "And God said 'Let there be light' and there was light," but the statement is meant in a new sense based upon the latest theories of psychical research. According to these mind is a kind of cerebral battery which sends out electric waves, and these waves have the faculty of creating matter in the sense, not that matter is made out of nothing, but that ether is transformed into tangible and gravitating mass. The author corroborates the proposition by quoting a number of authorities, some of them of scientific repute such as Oliver Lodge and Crookes, but I doubt very much whether their depositions will find credit among scientists of the normal and average stamp who are not affected by psychic theories and by a belief in extraordinary experiences of psychically abnormal people.

The subject presented is one of great interest, and if it contains a mere inkling of truth it would certainly be of enormous importance to the human race, for in that case matter of all kinds, including the most necessary nourishment, could be produced by pure thought. A person in need would have simply to concentrate his mind on the materials he wanted and could thus easily appease his hunger or thirst in a most satisfactory manner. There would no longer be any attempts made to starve whole nations into submission, but the psychical men could produce without great effort the things needed for the sustenance of their comrades and families.

The theory of "mind the creator of matter" as we find it in the Bible is extremely old. All heathen mythologies contain stories in which the gods produce the world, or certain parts of the world, with great ease and by the mere power either of the word or of mental faculties. The word plays an important part in Egyptian mythology, and it almost seems as if the theory of the Logos as proposed first in neo-Platonism and then in the Gospel according to St. John was ultimately derived from Egyptian sources, but even the crudest mythologies make the gods or some god, or if they have already developed into a monotheistic belief, the one sole God, shape the world in one way or another, and so it is natural that a thinking being starts his theories with the idea that mind is the primary factor in the theory of existence. Other religions, those of ancient Babylon, India, Assyria, Persia and China, developed on parallel lines.

The theory of mind as the creator of the world received its first shock when science originated, and wherever we can watch that process we find that a more materialistic theory is substituted. We see mind develop in children. We see first the material bodily existence, and the mind develops gradually, first as mere sentient life endowed with feeling and desire, and then from sentiency mentality is gradually developed until a state of maturity is reached in which thought becomes dominant, and in that phase we speak of mind. Thus mind is the final product of a process which we can observe in every growing being.

It is a new-fangled theory to look upon mind as a kind of dynamo or an electric motor which sends out waves that can be utilized for a physical purpose. The new theory originated with people who start with an exaggerated notion of the significance of spiritual factors, but after all it seems to us that they propose theories that are extremely materialistic. They misunderstand the nature of mind and intellectual functions and render them physical like the activities of mechanical machinery.

Whatever mind may be—whether a mechanical machine that attends to the process of thinking, or some mysterious agency of a spiritual character—it is certainly the most important fact that we meet with in our experience, for it is mind that dominates all our affairs and makes man a rational and thinking being. It is the scepter of man's dominion on earth, and it alone is the quality which endows him with his superiority among other creatures by giving him the faculty of foreseeing coming events, anticipating dangers and adjusting himself to his surroundings.

Mind has risen into existence in living organisms, and we are sure that it did not exist when the earth was still in its primitive condition, uninhabited and uninhabitable, before its crust had cooled down into a state that made plant and animal life possible. Nothing is more certain than this: First the earth was in a fiery state like that of our sun; gradually the planet cooled down and formed a crust on which the watery element covered the greater part of it, and the *terra firma* constituted the place on which life could develop in a regular evolution, reaching higher and higher planes of being. The characteristic feature of higher and lower is determined by the mental stage which has been reached and attains its highest development in man. So there was a time when there existed no mind on earth, and now the earth is peopled with intelligent beings that have evolved in a gradual and regular course of improvement. To the scientist the question is not how mind can produce matter, but how matter can develop from a crude state of mindless existence into a better and higher condition governed by mind.

Thus not the origin of matter from mind is a problem of science, but the origin of mind from matter; and to state briefly the outcome of it, we must insist that both mind and feeling have been declared not to be matter, or of matter, nor possibly to have been derived from matter as one of its qualities, but mind must be something *sui generis.* Mental phenomena are subjective, while motions and actions of matter are objective, and the solution of this problem has been briefly the statement that all subjectivity, including mental actions, constitute one side of existence while material existence is the other; or in other words existence is possessed of an inside and an outside. It presents itself as an objective existence by being matter in motion, but in itself it is neither matter nor motion but feeling, and this theory first clearly formulated by Fechner is commonly called the theory of parallelism.

According to the theory of parallelism, matter and feeling are different. Feeling does not originate from matter nor from energy, but is radically different. It is assumed to accompany, according to form, the different motions of a living body, and different motions in the nervous system of living beings are accompanied by different forms of feeling. These two sides of existence, the mechanical or objective and the sentient or subjective, are as different from each other as a concave or inside curve and the convex or outside curve of a circle. They have a definite correspondence but are different in their very characteristic qualities. The totality of subjective phenomena in its continuous existence is called soul.

If the soul cannot have been produced by matter it must have evolved together with matter as its inside existence, and this inside existence is trying to gain the superiority and take the lead as a central dominance of the whole. Assuming that certain motions are in themselves feelings we will understand how these feelings develop into broader and deeper consciousness. The subjective feature of unorganized nature, often called dead or inert matter, is not such that it can be characterized as being possessed of actual feeling, but assuming that the subjective side of matter exists throughout the world everything existent is possessed of subjectivity or the potential conditions from which feelings develop, and if we ask how such a change of potential feelings into actual feelings may come about, my answer would be, by organization. It is not sufficient for a feeling to be actual consciousness. The feeling must gain clearness to be a real feeling, and that is possible only by organization. An isolated feeling is not actual feeling, it is merely potential feeling. A feeling to become an actual feeling must be interrelated with other feelings. It must feel and be felt. It must be so interlinked with other feelings that one feeling feels the other feeling, and can gain clearness by a contrast with other feelings. Such a process would be called organization, and at any rate it is a fact that sentient life originates only in organisms which are living beings in which sentient parts are interrelated and organized.

Such we may fairly well claim to be the established facts of the origin of feeling from a world endowed throughout with subjectivity, which involves a possibility of developing feeling and may be considered as the inside nature of all existence. The next question is how feeling as it has developed in sentient matter will develop mind, and we may briefly answer this problem with the following considerations. A sentient organism is exposed to all kinds of impressions, such as touch, light, sound waves, etc. These impressions are of the same kind and have continued to affect organisms from the beginning of their birth, and the results of these impressions have affected the organism in such a way that their repeated occurrence has created organs for their reception. The impressions of touch have affected the outside of the body in an outer membrane called the skin, and the skin is so arranged as to receive impressions of touch in a way that they are felt to represent something outside. Air waves come upon the body in a similar way as the impressions of touch, but in a special place an organ is produced which we call the ear, ready to receive these air waves so that they may be incorporated into highly specialized feelings called sounds, and every such feeling of sound is so highly specialized that sentient organisms have different feelings for each different sound, and these feelings so differentiated begin to represent the different sounds so as to become identified with them. The same is true of the impressions which ether waves make. A special organ is formed which we call the eye, and the highly complicated process of seeing has finally made the eye as it is to-day in living beings, animals as well as man.

The eye is so differentiated that a living creature receives through the ether waves impressions which produce definite pictures, and these pictures represent the bodies from which they come. The process of seeing has become so natural to all living creatures that they do not reflect about its nature and origin but simply take the result as a fact of their existence. We see things and animals and all kinds of objects in our surroundings and adapt them to represent the things themselves as given data of our experience. While the light emanating from objects of our surroundings impresses pictures on our retina we take the pictures as facts and say that we see those things as if our pictures were the realities themselves. Considering all in all we find that sense-impressions are made upon organisms and that these sense-impressions by constant repetition become representative, and we may boldly say that representativeness is the character of mind. We have sensations, and these sensations picture the world of our surroundings and by showing them with analogous descriptive details they become symbols and furnish us with the material out of which we construct our views of the world.

The next question is the perfection of man, or the origin of human reason, and that coincides with the origin and introduction of speech. By speaking an animal learns to think in abstract terms which puts life on a higher plane. Reason enables us to think, foresee and adapt ourselves to conditions and to understand better the significance of life; in other words, to think scientifically and to raise consciousness to a higher plane, commonly called self-consciousness. How does this come about? How is it possible that mere animal life can develop into rational or human thought? The answer is this, that it comes about in the normal course of events by repetition and by a continued and higher organization. The same impression follows the same nervous tracts by which it is carried to the same central place in the brain. There it is impressed into a structure which has been formed by the same kind of impressions made by the same kind of object in former experiences. The whole structure thus forms a kind of composite picture, and this composite picture melts into one and is accompanied by an oral expression which denotes the whole. The origin of reason is the origin of language. Man thinks because he speaks. He has learned to think by self-observation through an analysis of his way of thinking.

I will not enter into the origin of language, which has been treated by Ludwig Noiré and also by Max Müller, but I will state here that the speaking animal develops a certain sound to accompany the definite picture of a certain object. Seeing a cat or a horse or a dog we denote all the recollections of cats or dogs or horses with the words, and thus the names instigate and stimulate our recollections of these several animals. They become a kind of label and, as all our mental impressions are registered according to our notions of them in a systematic way, our new sense-impressions run along the tracks of former nervous impressions of the same kind. The brain originates like a store house where different sense-impressions are regularly stored according to their nature, and we thus see that in the develepment of mental arrangements a logical system originates in which species become subdivisions of genera. In this way of systematically registering our sense-impressions according to the principle that the same impression goes to the same place prepared for it by former impressions, we develope a logical arrangement of mentality that prepares us to think clearly and helps us to find ourselves prepared for a logical consideration of our own experience when we reach the scientific method of self-observation.

One mind can exchange thoughts with other minds by using the same kind of symbols and speaking the same language. We understand each other because the same words denote the same objects and the interconnection of words expressed in endings and conjunctions will explain to us the relation in which the words stand to each other. All is grown by nature through the impressions of the surrounding world and thought and observation of their interrelationship. It is the symbolical nature of thought which makes mind useful, and if there is any telepathy such as exists in telegraphy it is in sending out by the quickest possible means (among which electric currents are the most efficient) certain shocks transmitted and so charged as to have definite meanings, and these meanings are understood by the recipient party in the same sense as they are given out by the sender of the telegram. Here again we find that the nature of mind remains representative. We must know that certain impressions, be they dots or dashes or any kind of shocks sent out, represent definite thought and that both parties possess the key to understand them. If mind produces anything it produces definite impressions by any kind of means, sound-waves or electric waves or what not, but always a definite form of a wave must possess a definite meaning. Thus mind is not any mysterious quality of unknown psychical or mental or spiritual waves, but it is produced by the transmission of physical impressions by means of the spoken word or otherwise, and we have not the slightest notion in spite of all the learned believers in the mystic ability of the mind that mind produces any other effects, such as the consolidation of ether into matter, or the change of one chemical element into another, or that there are waves going out from the brain of man possessed with any supernatural or unnatural or hyperphysical faculties.

Considering what science knows about the soul of man, I should say *a priori* that such inventions as are mentioned in the article "Mind the Creator of Matter" are highly improbable, and I would therefore naturally refuse to believe them until they are proved beyond doubt. The strange facts mentioned are interesting enough in so far as they are accepted and considered believable by the author who presents them, and also by the men of science to whom they are attributed. Let us wait until they are verified and hold ourselves open to conviction either way, to accept them if they unqualifiedly can be proved, or to reject them if they remain doubtful or can be proved to be untrue by having been due to misrepresentation or misconceptions of some kind.

It is a nice picture of the potentialities of mind to think that it possesses qualities which would make it divine and a real child of God, the creator. Yet even in this we should say that such a conception is not without a deeper meaning, for mind being the product of organization may truly be said to be the creator of matter if we think of matter as being the product of organization. If this principle may be considered as the prototype on which mind has been formed we may consider it a kind of original mind or proto-mentality, and such a condition is exactly the faculty of making something by combinations. This would be the divinity that pervades the world and its creative faculty, and in so far we could again justify the old proposition that God has made the world; or, in other words, the theory would be justified that mind—not the human mind but the superhuman or divine mind, the principle of organization—has shaped matter from the aboriginal material of ether into the different elements as we see them develop according to their masses on a definite grade of creation according to their weight and complexity. In this case, however, we would find our conception of God justified, which may be called in one word nomotheism, or the principle that natural law is the divine order according to which a chaos is impossible, that all nature develops according to law in a definite orderly way as it is realized in the course of evolution.

The proposition of our author, L. L. Pimenoff, can fairly be regarded as unacceptable to scientific thinkers, and we present it mainly as an interesting vagary of a fantastic theorist who—in the judgment of most scientists—will scarcely expect a serious indorsement of his proposition. In this same number we present an article by Mr. C. D. Broad whose expositions on the subject of "Body and Mind" are of a very different character. He treats the problem of the interrelation of body and mind—but suffers from the misconception that body and mind are separate entities without explaining their character or their mode of intercommunication. But he criticises the current theory of parallelism.

Psychologists since Fechner's day have indeed assumed that feeling is not motion and motion is not feeling. Feeling cannot act as a link in causation, and causation must be a chain of events in which cause and effect are uninterrupted. The question therefore is, how does the mental activity enter in the chain of events? If feeling does not form a part of the chain it plays no part in causation and the mind cannot *ex principio* act on the body. This would be a simple conclusion from the abstract considerations that by feeling we do not understand matter or motion, and by matter or motion we do not understand feeling, otherwise we might follow Mr. Broad in thinking that the theory of parallelism is absurd.

We will therefore make a few remarks on the theory of parallelism which we hold to be true in spite of misrepresentation. In a series of events which act as causes and effects in a mental process it is necessary that step by step brain motions are followed by other brain motions, but some of the brain motions are accompanied by phases of feeling, representing mental acts of thought. Definite thoughts are the inside accompaniment of definite brain motions and the nature of thought depends on definite forms of brain structures. And this definite structure gives them the faculty of acting. The meaning of words or the mental aspect is not endowed with energy, but definite brain structures which are endowed with energy are possessed of meaning, and when their feeling is stirred thought originates and assumes in the mind a definite meaning accompanied by the commotion of its corresponding definite structure. It is this brain motion which forms a chain in the causation and here is the point at which mind actually acts on body.

It is not the mind itself or the feeling which is present in our mind that forms a link in the chain of causation, but it is the energized nerve which stirs the brain and acts as the causal link. It is not impossible that by some diseased condition the nerve fails to act, and in that case there may be a state of will without the ability to execute it—a disease described by Ribot under the name *aboulia.*

Thus a critique of the theory of parallelism may become a verbal quibble. If we understand by mind merely the subjective side we could speak of the inability of mind or of feeling to act on the body, but if we understand by mind not only the subjective aspect of a mental process but also the bodily commotion of the brain which it ensouls, we would have to say that there is no question but that the mind influences the body. We must not lose sight of the fact that feeling is a mere abstraction, and if by this abstraction we mean only the subjective side of a process, only the mere actual feeling to the exclusion of its physical condition or accompaniment, it would naturally be illogical to make it the efficient cause in the chain of causation. But if we include in feeling its bodily condition we naturally include the physiological activity which is freighted with energy and forms a link in the chain of cerebral causation.

Mr. Broad certainly does not present a theory of his own which would be acceptable, or give us a satisfactory explanation as to the nature of mind. No! He leaves us in the dark as to what the mind really is or can be, and for all I can see in his proposition, the mind is a mysterious creation of a dualistic conception which is endowed with several mysterious qualities, acting on bodily forces in an unaccountable way.

According to Fechner feeling does not act on mind, because motions only can be the causes in a chain of causation; what is not mechanical cannot produce an effect, for causation is mechanical. Feeling is different from mechanical action, but it is inefficient not because it is different, but because it is not motion. In order to be a cause, or a link in the chain of causation, it must move or push in order to produce a change of any kind. If the feeling in its narrowest meaning cannot stir motions in the brain, the accompanying brain motion may or probably will do it. In bearing this in mind we find no contradiction in the theory of parallelism.

Mr. Broad favors a "two-sided interactionism" in which "the mind sometimes acts on the body and the body sometimes acts on the mind." He condemns epiphenomenalism, according to which feeling is an epiphenomenon or superadded feature standing outside the regular normal causation of physico-chemical activity. Next in foolishness to this theory he regards parallelism. He claims with great insistence that the body is not a purely physical and chemical system, and in this latter point we can agree fully and without any reservation, for in the scale of natural phenomena we have a domain of purely physical and chemical phenomena and while some scientists assume that vital processes are purely physical and chemical we cannot deny that psychical transactions possess a feature that cannot be regarded as physical or chemical, but possesses something that is absolutely new.

If rightly understood there can be no quarrel on this point, and we fully agree that the influence of psychical items does make a difference in the chain of causation. If it is not the feeling portion of a telegram which makes a man jump from his seat and rush into action, it is the meaning of it which meets with an understanding of a threatening danger or whatever it may be, and this meaning is conveyed by the form of letters, which according to former education possess a definite meaning. The forms of certain words together with the meaning with which they are endowed constitute the factor which causes the reaction and sets energy free, just as a key unlocks the bolts through the arrangement of its wards and it opens the lock on account of the shape of its indentations which fit into the corresponding shape of the lock. It is this correspondence of the meaning of words or of symbols which makes the psychical portion of interrelated events efficient, and it is this fittingness, this correspondence, not exactly the pressure and the energy, which constitutes the significance of spirituality. Thus we might very well say that it is not the energy or pressure of the key that opens the lock, but it is the very form, the singular complexity of its wards which in the Yale lock is reduced to a curve on the stem of the key. The mechanical pressure of the key as well as of the nerve is the moving power that is indispensable in the chain of causation, but the correspondence of the meaning of words determines an action in the same way as the proper key opens the lock into which it fits.

Thus it is seen that in judging of the theory of parallelism we must first of all understand its meaning and not confuse its issues. It is to be feared that Mr. Broad construes a parallelism of his own and condemns it on the ground of a misrepresentation which is either misconstrued or possibly a wrong presentation. At any rate it seems to me that Mr. Broad's criticism does not upset or invalidate the theory of parallelism, which so far as can be seen is the only one on which a monistic theory of the interaction of body and mind can be constructed.

EDITOR.

A PSYCHOLOGICAL VIEW OF THE PRAGMATIC ISSUE.

PROFESSIONAL philosophers have published many volumes trying to solve the problem, whether our ideas of things are true because they work or whether they work because they are true. To my mind the realities of life, and of acquiring knowledge, present no such issue, and the discussion of it has had its chief utility as a necessary step toward the discovery of its futility, and so contributing one factor to our understanding of intellectual evolution. To exhibit my justification for this belief is the reason for this essay. Incidentally it may appear that by adopting the first of these formulas to the exclusion of the second one some pragmatists are guilty of that same absolutism for which they so generously criticise others.

Here as everywhere we must seek the solution of our problem on the basis of a higher intellectual level than that on which it arose. Thus the desire for more efficient observation and a more inclusive synthesis of the factors of the problem will lead us to re-examine the seemingly conflicting formulas with the view of translating them into concepts of behavioristic psychology. From this new viewpoint perhaps we will see the old formulas as presenting mere incomplete and dissociated aspects of the same cognitive process. From this psychologic aspect we may also achieve such an integration as will rid us of our seeming conflict.

The statement of a common premise will be followed by a brief analytic restatement of some of the behavioristic aspects of acquiring knowledge in so far as these seem material to the succeeding discussion. After that will come a statement of the synthetic view, and a psycho-analytic suggestion as to the probable cause, conducing to the past dissociated consideration of these intimately related formulas.

As a common premise for my discussion I assume that the limitations of our thinking faculties are such that we cannot know things in themselves, but only some incomplete and imperfect aspects of things. For the future discussion it is important to bear in mind that the belief in the impossibility of knowing things in themselves rests in part upon the fact that to know things in themselves seems to involve what is supposedly an impossible identity of the arbitrarily distinguished knowing mind and the things known.

This being true, we acquire only some incomplete views of some aspects of objective reality, by becoming conscious of an affect-producing relationship therewith. It then seems that all that any of us think we know, must embody some imperfect aspect of things, and must always hold, at least, some tiny resemblance to truth. From this viewpoint of a contrast between the thing known and the knowing mind, perhaps our conception of a thing can never be an exact transcript of objective reality, because to be that it must also become identical with it, yet being derived from a relation with realities, or being some crude awareness of such related existence, it cannot be absolutely false. Since we cannot yet see intelligence imminent in things, nor in the relation among things, it follows that our conception of things, which is an imperfect understanding of their behavior in and during changing relationships, attains varying degrees of approach to identity with, or to an exact transcript of such relation with and between "objectives." The evolutionary rating of this growing intelligence depends upon the diversity, number and complexity of the aspects of things and their relationships which have entered into our affect-objects, that is, of which we have become aware and which we have coordinated in our awareness, and synthetized in the shaping of our concepts. This is only a behavioristic and descriptive way of saying that we grow toward a perfect accuracy of concepts as transcripts of things and of the relationships of these, according to the efficiency and extent of our observations and the relative completeness of their coordinations.

From this we conclude that no concept is wholly false, and that the only judgments which should be passed upon concepts are those which express an evolutionary classification according to relative degrees of approach toward the completeness of our awareness of things (behavioristically identified) in the first place, a relation mong things in the next place, and our relationship to things and to inter-objective relations. All are essential to an exact transcript of reality. These also measure roughly the degrees to which our intelligence is removed away from the mere intellectualization of desire.

The progressing refinement and completeness of our concepts depends in the first place upon our efficiency as observers. This efficiency in turn depends upon the kind of sense-organs which we possess; the degree to which they are educated and developed toward and by means of extraverted interests; the extent to which our sense-organs are supplemented by mechanical contrivances, the quantity and complexity of previously related material which is available and is coordinated in each present observation and judgment; and lastly the number of behavioristic relationships, and the degree of complexity and remoteness of these, which are coordinated within each last concept. It is this stored material of past experience which determines the multiplicity and complexity of the conditions which we can and do prescribe for testing the workability of each present theory of things. So then, we judge relative degrees in the perfection of our conception of things and their relationships, by the relative degrees of multiplicity, variety and complexity of the conditions under which the test of workability is applied. (For further discussion of this evolutionary classification see my article on "Intellectual Evolution and Pragmatism," in *The Monist,* January, 1916. There I also describe, yet too briefly, my concept of the affect-object.)

From this brief description of evolution in the knowing process it seems to me clear that two, purely hypothetical, extreme propositions have become apparent. For the first, we may assume a person without any previous acquaintance with the workability of any idea. In such a hypothetical individual, if thought were possisble, it would be a pure intellectualization of desire. In this hypothetical case, there would be no possible way for such a person to decide whether his first claim of truth contained any portion of the true aspects of objectives, except by the present or future test of workability. From such a view-point, manifestly *such a being,* bereft of all experience, can *only* say that the "concepts" (desires) contain a measure of truth, in so far as thereafter they can be made to work. Having no other experience or prior concept, nor any general concept capable of a deductive application, it cannot be otherwise than that for such a person the first concept would seem true only *because* it works, and later *so far as* it works.

The second proposition which seems to me equally self-evident is this: Let us assume that our conception of things had attained the impossible perfection; that it has become an exact transcript of objective reality, which is only another way of saying that the knowing intelligence is found imminent in, and so far identical with, the things and relations known. In this situation, there could be no possible failure of the test of workability. One having attained such a state of perfection could say "my ideas about things work *only because* they are true."

Of course no mere human is wholly in either of the hypothetical extremes which are involved in the foregoing propositions, and that is why the controversy under consideration is barren. According to our intellectual development we will see only one, or will emphasize one or the other, of these formulas born of an inefficient observation inducing believe in an impossible extreme situation.

Even in earliest infancy it cannot be said that we are wholly void of experiencial knowledge. Likewise we never attain such intellectual maturity that our concepts are accurate and complete transcripts of any objective realities. Intellectual life, in the sense of continuous change, is in a constant flux between these two extremes. In relative immaturity our concepts are mainly the intellectualization of desires, with a minimum of the check of past or present tests of workability. Toward the other extreme, we approach to a situation where there is a maximum of the checking influence of past conscious and unconscious formations of affect-objects, which tend to be automatically and unconsciously applied.

So then, it is inevitably true that each of us is in an ever changing state of development between the infantile condition, where our concepts are entitled to presumptions of extremely slight correspondence with any objective, until after the conscious subjection to at least some simple test of workability; and that other stage of development, where our concepts may be so largely the products of the experiential checks upon infantile desires as tend toward the presumption that they will work because they approach relatively near to an exact transcript of the realities.

So then from this point of view, of a changing and growing mind, in an ever changing relationship with equally changing objectives which are undergoing changes in interobjective relationships, we see that each of the seemingly conflicting formulas contains an imperfect aspect of relationships such as are at present incapable of even exact definitions, or complete separation. If studied with the desire to understand the behavior of human energy in the process of achieving knowledge, then the controversy under consideration looks like a mere war of words, because it does not describe a real conflict of forces, and so is not adequately related to any behavioristic study of such forces.

There are no concepts which are not somehow ultimately founded upon experiential relations with objectives, no matter how utterly void of that experience our conscious memory may be, nor how small the resemblance between the concept and the realities. However unconscious we are of the influence of that experience, the fact that we acquired the concept shows that at least a feeling confidence in its workability has been impressed upon us unawares, by the experiences themselves. Hence the existence of a corresponding assurance that the concept transcribes the reality with substantial accuracy even though we know nothing of the mechanism by which that feeling-conviction was engendered. We simply know because we feel and are firmly convinced in proportion as we are strongly agitated by desire.

So far we believe our concepts work because the experience upon which they are founded has left an impress like unto that of their having "worked"; that is to say, we have a desire that it shall be so and an accompanying feeling—a feeling-conviction—that it is so, and that conviction, were its source and mechanism to be verbally formulated, would be expressed as being warranted by pragmatic tests which had already been unconsciously applied. Before we can consciously apply tests of workability we must have done a little generalizing, because the conscious application of pragmatic tests is partly a deductive process, even though at the moment we are unconscious of that aspect of it. To the extent that we *consiously* apply further tests of workability we find that the concepts work just so far as they are true. When we attain to a consciousness that we are making deductive application of general ideas, we tend to say our ideas work *because* they are true. As we see this, and coordinate it with our earlier exposition of the inductive aspect of this behavioristic psychology, we again come to the conclusion that there is no such conflict as is postulated in the pragmatic issue.

In the realities of acquiring knowledge both formulas are always actually and practically implicit. Persons who affirm that our ideas are true *only because* they work, are for the moment seeing only the inductive part of the process which perhaps is the first that we become conscious of in our individual development. That person who affirms that our ideas of things work *only because* they are true is seeing the complex intellectual behavior only in its deductive aspects, and is forgetting that sometimes our ideas seem to work because they are relatively false and the conditions of the applied pragmatic test are too simple to expose the error. When we acquire a synthetic view of the behavior of the human energy operative in the knowing process, we see the inductive and deductive methods proceed interdependently even in the unconscious activities. In the more highly developed states of conscious supervision over our intellectual processes, we carefully provide for the interaction and check of both methods. So we tend to become aware of the interaction of the modes of conduct presented by both the formulas in the pragmatic issue at the beginning of this essay. Each represents an incomplete aspect of the realities actually involved in the process of consciously developing intelligence.

If that is so, then why was there ever a philosophic issue made of it? I suspect it is because even philosophers have their unsolved personal problems—their subjective conflicts. In philosophers, as in children and hystericals, these conflicts, induced by a past thwarted integration, conduce to negativism, that is to the future dissociation of different aspects of the inspected realities, and so tend to inhibit and limit the larger synthetic understanding of their problems, including the philosophic ones.

If in infancy a future philosopher was habitually compelled to subordinate the method of his expenditure of energy to the authority of a parent instead of the arbitrament of "facts," he may easily grow to maturity of years with an emotional aversion to accepting things as they are, or their interpretation according to the accepted authorities, even in philosophy. If his own intellect is sufficiently fertile, he will see some aspects of things and of their relations, which his fellow philosophers have overlooked. If his aversion to "authority" is sufficiently strong, that aversion will preclude the coordinating of his new aspect with what is reconcilable to it, in that which is already accepted, and will tend to see the new only in dissociation, that is in its negating aspects, as a conflict with that which is already accepted. Thus probably grew the pragmatic controversy depicted in the first paragraph. The integrating process represents a relatively higher evolutionary level.

In this case the synthetic aspect of the mechanism of growing intelligence is missed by pragmatists who make a philosophic cult of one of these related formulas to the exclusion of the others. In consequence of this limitation upon their powers of coordination, they are impelled to frame up verbal defenses for their unintegrated aspects of things, and such formulas as present the seeming conflict in the first paragraph of this essay are the result.

If now we re-read these formulas and coordinate them as different aspects of the behavior of human forces, it is easy to see that the apparent conflict of theory is due to inaccuracy of observation and statement, probably induced by the necessities of repressed emotional conflicts of the past, which have hitherto precluded, on the part of particular philosophers, efficient effort toward the coordination of the two formulas into a more complete understanding of these related psychologic factors.

THEODORE SCHROEDER.

NEW YORK CITY.

LUCRETIUS RETURNS.

A PHILOSOPHICAL POEM.

> "Now Philosophy is like unto a Garden, wherein upspring all manner of flowers and herbage, sweet of scent and potent to heal. And the Soul is like unto a Moth, leaving the cocoon of the Unconscious to flit through the twilight, seeking the Nectar of Life. Now within the Garden hovers the Soul amid the herbage and flowers, darkly swaying in the dim starlight and the shadow; hither and thither, drawn or repelled, lured by a remembered fragrance or driven back by an unfamiliar form that is but half revealed—thus the Soul wanders through the mysterious dusk of the Garden."—From *The Golden Scroll of Krotona.*

ETERNAL stars that heaven's hill bedew,
Ye looked upon the manger where Mankind
Lay wrapped in rags; ye heard the angels sing,
When royal Magi spread their gifts of myrrh
And frankincense upon a shabrack coarse;
Ye gleamed above the boy at merry play
In Nazareth, and wept at Golgotha—
Smiled on the resurrection, and, at last,
World-wounded, he ascended unto you.

Eternal stars that heaven's hill bedew,
Ye heard Creation's grand exordium—
The moan of seas in the azoic age,
The din of wood and jungle; then, anon,
The war-song of the savage fierce and free,
Grim troglodyte and fleet lacustrian;

At last the pulse of forges and the roar
Of teeming cities—an aubade of joy,
Thrush-throated like the chant of cherubim.

Attend! Mine is no plaint of selfish woe:
Weeping o'er Niobe and Tantalus—
O'er Truth, o'er Justice, and o'er Liberty.

Tears shed for butchered Innocence, the blood
Clotted upon the lacerated back
Of Helotry, the virus from the fangs
Dript of dread hydras preying upon Man,
The sighs of sunless centuries, each curse
That livid lips of trodden Truth have framed,
Pour into Hate's alembic and distill
Into revenge—into a cup of gall
For Tyranny, in stupor gluttonous
Huddled on filthy couch. What of a world
Where Wrong is fattened, Folly wears the crown,
While Justice spreads her ermine over straw,
And Learning feeds upon the hedgerow haws?
Shame on humanity! I have known wights
Who daubed their cheeks as silly damsels do,
And strutted round with rings upon their hands,
Yet sat in senates where to counsel met
The sceptered wisdom of a mighty age.
I have seen gypsum hawked about the streets,
Figures of poets, by a man whose soul
Soared far in song above the paltry souls
Of those he modeled as the unpitying sun
Above his fevered head.

Is there a God
To mete our merits and adjudge our fails,
And could he be thus blind?

The peopled sea
Hath coral castles through whose lurid aisles
White mermaids flash and hippocamps disport
Amid the sunken argosies of Time,
Patined with gold and crusted o'er with gems—
All these are thine, thou unborn Œdipus,
Who canst unveil the riddle of this Sphinx!

Yet, shall we say the rich, ripe fruit of Time
Fell from the womb of Chance? The dreams, fresh-drawn
From lustral fountains by a naiad troop,
And borne like dew in lily chalices
To my lone couch, are witnesses that still
A spark survives of what the race hath been.
See yonder shadow cross the sun; observe
The stoop of Atlas; note the line of care
Which mars the brow of kings; or hear the shriek
Of the mad mænad War—can Glory be
Attained save by the thorny track of Woe?

The universe proclaims there is a God!
From tongueless chaos, lutulent and foul,
He culled the vying wonders that we view;
Scatters the violets upon the heath,
And paints the silken petals of the rose;
He bids the planets sing—aye, and he feeds
The adder's tooth with venom; strews the rocks
Upon the pathway of the mariner;
His whirlwinds filch what his beneficence
Hath lavished on the orchard and the farm;
Anguished we cry for light, and see the forked
Tongue of the tempest lick the midnight's brow.

Lo, now, what festering horrors feed our grief:

Beauty decays, Nobility is slain,
Sin sheds his larve, and curst Hypocrisy
Barters his whine and rheum for place and gold.
'T may be, if there's a Providence all-wise,
The far untraversed forest tracts were grown
That on the Judgment Day there might not be
Dearth of good gallows-timber on this globe.

Tiptoe, the twilight muses on the hill;—
Should it descend into the slumbering town
To gild the misery of the driven mob,
Or, fleeing to primeval solitudes,
Hold discourse with the laughing deities
Of hort and vale, caress the airy fern,
And court the sylvan calm upon the sward's
Pied flocculence, where, mean solicitude
Being banished, all the elder gods again
Resume their interrupted reign, to bid
Sorrow and Sin and Shame begone from earth.

Man must approach to God by purity,
E'en as the highest mountains, undefiled
By human footfall, where the virgin snow
Lies chaste and spotless 'neath the amorous sun,
Are nearest heaven.

Let us, too, be brave!
Canst thou bribe thunderstorms with honeyed words,
Or curb with sandhills the choleric sea?
Where Sorrow strikes, let honorable scars
Remain, the blazon of our fortitude.
Be not of such slight, puny courage as
To drink nepenthes, or that slumbrous juice
Of dream-compelling poppy; can there be
Virtue more potent in a wayside weed

Than in the trodden heart's tear-wine of hope?
What laurel for the soul if Vice be shorn
Of her allurement, or the sword of Sin
Be dulled by sorcery? Could a coward hear
The prating trump of Triumph, and become
Not brave? 'T may be there is no God; whence then
Proceed the whisperings that abjure ourselves?
Could the chill alchemy of atheism
Transmute one earthy atom of this race
To the rich metal of divinity?

Yea, there is One that walks in human hearts,
Sandaled with rose-leaves, and with gentle touch
Weeds out all malice; balm upon her lips,
She stoops to kiss the humblest flowering thing;
The sculptured lily she awakes to life,
Paints irised poems on the sterile rock,
And strows the sod with immortality.
She leads the orchestra of brook and breeze,
Of bird and bee, in symphonies that swell,
Dulcet adagios of a seraph choir,
Across the sobbing solitude. She smiles,
And Sorrow is no more—sojourning Grief
Shoulders his wallet and forsakes thy roof.
Her robes of gossamer in cirrous twills
Bear health and happiness upon their seam;
Touch this, and thou art whole! In her tranced eyes
The gorgeous gonfalon of Day unfolds;
And Peace, sweet child of life's lorn Enna, sleeps
Upon her bosom.

Have ye heard her name;
Knelt in her temples; walked among her groves?
Love that doth thrill the molecule to dance,
And string the cosmic lyre with golden stars—

Love that doth kiss the lids of Death ajar,
Enchantress of the demiurgic word,
Mother of men and genetrix of time—
At once the Law, the King, the Throne; at once
Doer and Deed, the Singer and the Song;
Breath of the gods that fills the lungs of space,
In which the suns are sparks of dust immerst;
The earliest Element and latest Form,
Orbit and Orb alike—immortal Love,
The promise and the potency of Life;
The gladness and the glory of the world!

George Seibel.

Pittsburgh, Pa.

CRITICISMS AND DISCUSSIONS.

TIME AND SPACE.[1]

The conceptions of time and space which I wish to develop here have arisen on the basis of experimental physics. Therein lies their strength. Their tendency is radical. From now on space-in-itself and time-in-itself are destined to be reduced to shadows, and only a sort of union of the two will retain an independent existence.

I.

I wish first to show how from the mechanics now generally accepted we might arrive by purely mathematical considerations at a change in our ideas of space and time. The equations of Newton's mechanics show a double invariance. Their form is maintained, first, if we subject our system of original coordinates in space to any *change of position*; second, if we change its state of motion, that is to say, impart to it any uniform translation; neither does the zero-point of time play any part. We are accustomed to considering the axioms of geometry as settled before we approach the axioms of mechanics, and therefore these two invariances are seldom mentioned together. Each of them represents a certain group of transformations, which transform the differential equations of mechanics back into themselves. The existence of the first group is regarded as a fundamental property of space. It is usually preferred to treat the second group with contempt in order to

[1] Lecture delivered at the eightieth Congress of Naturalists at Cologne, September 21, 1908. Published in *Physikalische Zeitschrift,* X (1909, pp. 104-111, and *Jahresbericht der deutschen Mathematiker-Vereinigung,* Vol. XVIII, pp. 75-88; *Gesammelte Abhandlungen,* edited by D. Hilbert, pp. 431-444; also separately, Leipsic, B. G. Teubner, 1909. Translated from the German by Edward H. Carus who herewith expresses his gratitude to Prof. W. B. Smith of Tulane University for many suggestive criticisms.

pass lightly over the fact that we can never decide from physical phenomena whether the space we have assumed to be at rest is not after all in a state of uniform translation. Thus these two groups have an entirely separate existence, side by side. Their quite heterogeneous character may have discouraged their combination; but precisely this combination into one group gives us food for thought. We shall try to illustrate these relations graphically. Let x, y, z be rectangular coordinates of space and let t represent time. As they occur in our experience places and times are always combined. No one has ever observed a place except at a time, nor a time except in a place. But here I am still respecting the dogma that space and time have each an independent significance. I shall call a point in space at a definite time, that is, a system of values, x, y, z, t, a "world-point (*Weltpunkt*). The multiplicity of all possible systems of values x, y, z, t I shall call the world. I might boldly sketch four world-axes on the blackboard. Even *one* such axis consists merely of vibrating molecules and travels with the earth in space, thus alone furnishing us with sufficient food for abstract thought; the somewhat greater abstraction involved in the number four does not disturb the mathematician. In order not to have an empty void anywhere we shall assume that there is something perceptible everywhere and at all times. To avoid the terms matter or electricity we shall call this something substance. Let us direct our attention to the substance-point (*substantiellen Punkt*) at the world-point x, y, z, t, and imagine that we are able to recognize this substance-point at every other time. Let the changes dx, dy, dz, of the space coordinates of this substance-point correspond to an element of time dt. We thus obtain as a representation so to speak of the eternal course of the substance-point a curve in the world, a world-line whose points can be determined uniquely in terms of a parameter t from $-\infty$ to $+\infty$. The whole world stands resolved into such world-lines, and I wish at once to make the fundamental assertion that according to my opinion physical laws may find their most complete expression as mutual relations among these world-lines.

By the concepts space and time, the x, y, z-manifold $t=0$ and its two sides $t>0$ and $t<0$ become separated. If for simplicity we keep the zero point of time and space fixed, then the first mentioned group of mechanics means that we can give any rotation around the origin to the x, y, z-axes in $t=0$ corresponding to the homogeneous linear transformations of the expression $x^2+y^2+z^2$ into itself.

But the second group or invariance means that without changing the expressions of the laws of mechanics, we can replace x, y, z, t by $x-\alpha t$, $y-\beta t$, $z-\gamma t$, t, α, β, γ being any constants whatever. The time-axis can accordingly be given any direction whatever toward the upper half-world $t>0$. Now what connection has the condition of orthogonality in space with this complete upward freedom of the time-axis?

To exhibit the connection we take a positive parameter c and consider the locus

$$c^2t^2-x^2-y^2-z^2=1.$$

It consists of two sheets separated by $t=0$ analogous to a hyperboloid of two sheets. Considering the sheet in the region $t>0$ we now conceive those homogeneous linear transformations of x, y, z, t into four new variables x', y', z', t', in which the expression for this sheet of the hyperboloid in the new variables corresponds to the original expression. Evidently the rotations of space about the origin belong to these transformations. We shall next obtain a full understanding of the remaining transformations by considering one in which y and z remain unchanged. Let us draw (Fig. 1) the intersection of this sheet with the plane of the x- and t-axes, the

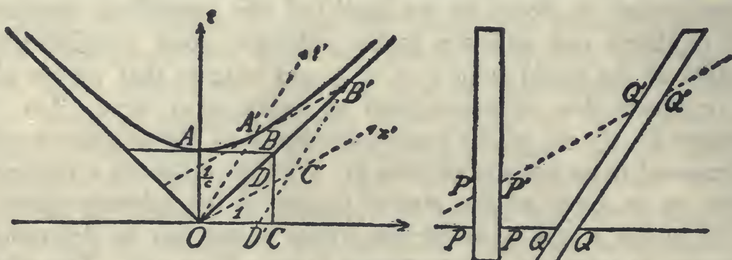


Fig. 1.

upper branch of the hyperbola $c^2t^2-x^2=1$ with its asymptotes. Then let any radius vector OA′ of this branch of the hyperbola be constructed from the origin O, let the tangent to the hyperbola at A′ be extended to the right until it intersects the asymptote at B′, let OA′B′ be completed to form the parallelogram OA′B′C′, and finally for later developments let B′C′ be continued to D′, its intersection with the *x-axis*. If we then take OC′ and OA′ as axes for parallel coordinates x' and t' with units OC′ $=1$, OA′ $=1/c$, then this branch of the hyperbola again has the equation $c^2t'^2-x'^2=1$, $t>0$, and the transition from x, y, z, t to x', y, z, t' is of the type under consideration. We now add to these transformations all arbitrary shiftings of the space and time origin, and in this way construct a group of transformations obviously still dependent on the parameter c, which I designate by G_c.

If we now let c increase to infinity, $1/c$ thus converging to zero, we see from the figure described that the branch of the hyperbola always approaches closer to the x-axis and the angle between the asymptotes widens into a straight angle. At the limit the special transformation changes into one in which the t'-axis can have any upward direction and x' steadily approaches nearer to x. In consequence of this it is clear that the group G_c, in the limit for $c = \infty$, thus as the group G_∞, becomes the complete group of Newton's mechanics. Under these circumstances and since G_c is mathematically more intelligible than G_∞, a mathematician in the free play of his imagination might well have had the idea that, after all, the phenomena of nature do not actually remain invariant for the group G_∞, but rather for a group G_c with a c that is definite and finite but *very large* if taken in the ordinary units. Such an idea would have been an extraordinary triumph of pure mathematics. Now, although mathematics has here been caught napping she still has the satisfaction that, owing to her happy antecedents, through senses made keen by their exercise in broad vistas, she is capable of grasping at once the far-reaching consequences of such a transformation of our conception of nature.

I shall now indicate what value of c will finally come into consideration. For c we shall substitute the *velocity of light in a vacuum.* In order to avoid the terms "space" and "void" we can define this magnitude as the ratio between the electromagnetic and the electrostatic units of electric quantity.

The existence of the invariance of natural laws for the group G_c under consideration would now be expressed as follows:

From the totality of natural phenomena we can derive with ever increasing exactitude by successively closer and closer approximations, a system of reference $x, y, z,$ and t, space and time, in terms of which these phenomena are then represented according to definite laws. But this system of reference is by no means uniquely determined thereby. *It is still possible to change this system of reference at will corresponding to the transformations of the above mentioned group* G_c, without changing thereby the expression of natural laws.

For example, according to the described figure we can also call t the time, but then in connection with it we must necessarily define space by the manifold of the three parameters, x', y, z, in which case physical laws would be expressed in terms of x', y, z, t', exactly the same as in terms of x, y, z, t. According to this there would be in the world not that particular space but an infinite number of spaces, just as there is an infinite number of planes in three-dimensional space. Three-dimensional geometry becomes a chapter of four-dimensional physics. You now understand why I said at the outset that space and time are to fade away into mere shadows and that only a world-in-itself will exist.

II.

Now the question is, what circumstances force the changed conception of space and time on us? Does it never, as a matter of fact, contradict phenomena? And finally, has it advantages for the description of phenomena?

Before we enter into these questions, let us first make an important observation. When we individualize space and time in any manner, then a straight line parallel to the t-axis corresponds as world-line to a substance point at rest, a straight line inclined to the t-axis corresponds to a uniformly moving substance-point, and a world-line curved at will corresponds to a not-uniformly moving substance point. If we consider the world-line passing through any world-point x, y, z, t, and if we there find it parallel to any radius vector OA′ of the above-mentioned hyperboloid sheet, we may introduce OA′ as the new time-axis, and in the new conception of space and time thus obtained substance appears at rest at the world-point in question. Let us now introduce this fundamental axiom:

By a suitable determination of time and space, the substance present at any world-point whatever may always be conceived of as at rest.

This axiom means that in every world-point the expression

$$c^2\,dt^2 - dx^2 - dy^2 - dz^2$$

is always positive or, what amounts to the same thing, every velocity v is always less than c. According to this, c would exist as upper limit for all substance velocities and in this fact would lie the deeper significance of the magnitude c. In this other form the axiom has in it something which at first sight is unsatisfactory. But we must consider that now a modified mechanics will supersede the old—one into which will enter the square root of the above combination of differentials of the second degree, so that cases involving velocities exceeding that of light will only play some such part as figures with imaginary coordinates play in geometry.

The *impulse* and actual motive *for the assumption of the group* G_c originated through the fact[2] that the differential equation for the transmission of light-waves in empty space is actually characterized by the Group G_c. On the other hand the concept of rigid bodies has a meaning only in a mechanics with the group G_∞. If we have an optics with G_c and if on the other hand rigid bodies existed, it is easy to perceive that by the two hyperboloid sheets belonging to G and to G_∞ a definite t direction would be determined, and this would have the further result that we must be able to detect by means of suitable rigid optical instruments in the laboratory, a change in the phenomena at different orientations with reference to the direction of the earth's motion. All attempts, however, at this detection, especially a famous interference experiment of Michelson, had a negative result. To find an explanation for this, H. A. Lorentz constructed a hypothesis the value of which depends on the invariance of optics for the group G_c. According to Lorentz, every body in motion suffers a contraction in the direction of the motion, and for the velocity v this contraction is in the ratio

$$1 : \sqrt{1-(v^2/c^2)}.$$

This hypothesis sounds very fantastic, for the contraction is not to be regarded as a consequence of resistance in the ether but entirely as a gift from above, a phenomenon accompanying the state of motion.

I shall now show by our figure that the Lorentz hypothesis is entirely equivalent to the new conception of space and time through which it may much more readily be understood. If, for simplicity's sake we ignore y and z and consider a world of one space dimension, then parallel strips, an upright one like the t-axis, and one inclined to it (see Fig. 1) represent the path respectively of a stationary and a uniformly moving body which in both cases maintain a constant spatial extent. If OA′ is parallel to the second strip, we can introduce t' as time and x' as the space coordinate, and the

[2] What is practically an application of this fact is to be found as early as 1887 in a contribution by W. Voight in *Nachrichten der K. Gesellschaft der Wissenschaften zu Göttingen,* mathematisch-physikalische Klasse, page 41.

second body then appears at rest and the first in uniform motion. We now assume that the first body conceived as at rest has the length l, that is, the cross-section PP of the first strip on the x-axis $=l.\mathrm{OC}$ where OC denotes the unit on the x-axis; and on the other hand that the second body *conceived as at rest* has the same length l, that is, the cross-section of the secondstrip, measured parallel to the x'-axis gives the equation $\mathrm{Q'Q'}=l.\mathrm{OC'}$. We now have in these two bodies constructions of two *equal* Lorentz electrons, one at rest and one in uniform motion. If we keep the original coordinates x, t, fixed, then the section QQ of the respective strip *parallel to the x-axis,* must be regarded as an extension of the second electron. Now it is clear since $\mathrm{Q'Q'}=l.\mathrm{OC'}$ that $\mathrm{QQ}=l.\mathrm{OD'}$. A simple calculation shows that if $(dx/dt)=v$ for the second strip,

$$\mathrm{OD'}=\mathrm{OC}\sqrt{1-(v^2/c^2)},$$

and therefore also $\mathrm{PP}:\mathrm{QQ}=1:\sqrt{1-(v^2/c^2)}$. But this is the meaning of the hypothesis of Lorentz on the contraction of electrons in motion. If, on the other hand, adopting the system of reference $x'\,t'$, we regard the second electron as at rest, then the length of the first will be denoted by the cross section P′ P′ of its strip parallel to OC′, and we would find the first electron shortened in exactly the same proportion with reference to the second. For it is according to the figure:

$$\mathrm{P'P'}:\mathrm{Q'Q'}=\mathrm{OD}:\mathrm{OC'}=\mathrm{OD'}:\mathrm{OC}=\mathrm{QQ}:\mathrm{PP}.$$

Lorentz called the combination t' of x and t the *place-time* of the uniformly moving electron and used a physical construction of this conception for the better understanding of the contraction hypothesis. But it remained for A. Einstein[3] to recognize clearly that the time of one electron was just as good as that of the other, that is, that t and t' are to be treated alike. Thus time was the first to be discarded as a concept determined uniquely by phenomena.

Neither Einstein nor Lorentz disturbed the conception of space, perhaps for the reason that in the special transformation where the x', t' plane coincides with the x, t plane it is possible to interpret the x-axis of space as remaining fixed in its position. To loftily ignore the conception of space in similar wise is doubtless due to the boldness of mathematical discipline. After this further step which however is indispensable for a true understanding of the group G_c, the expression *postulate of relativity* for the demand for an invariance

[3] A. Einstein, *Annalen der Physik,* XVII, 1905, p. 891; *Jahrbuch der Radioaktivität und Electronik,* IV, 1907, p. 411.

in the group G_c, seems to me very weak. Since the postulate comes to mean that phenomena occur only in the four-dimensional world of space and time but the projection into space and into time can still be assumed with a certain degree of freedom, I would rather call this proposition the *postulate of the absolute world* (or for short, *world-postulate*).

III.

Through the world-postulate a similar kind of treatment of the four determining elements x, y, z, t, becomes possible. Through it, as I shall now show, we gain an insight into the forms under which physical laws operate. Above all, the conception of *acceleration* becomes sharply defined.

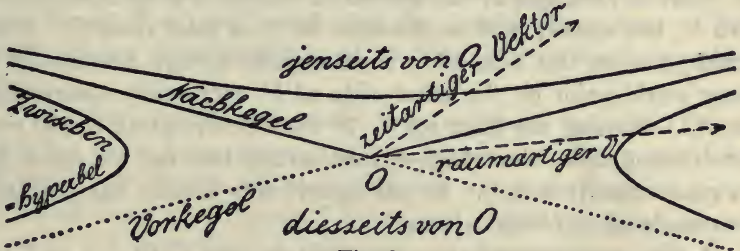


Fig. 2.

I shall use a geometrical mode of expression which at once suggests itself, at the same time tacitly ignoring z in the triplet x, y, z. I take any world-point, O, as the space-time origin. The *cone* $c^2t^2-x^2-y^2-z^2=0$ with O as vertex (Fig. 2) consists of two parts, one with the values of $f<0$, another with the values of $t>0$. The first, the *"past" cone* (*Nachkegel*) *of* O consists, let us say, of all world-points which "send light to O"; the second, the *"future" cone* (*Vorkegel*) *of* O, consists of all points which "receive light from O." The region bounded only by the future cone of O may be designated *this side of* O (*diesseits von* O), and that bounded only by the past cone, *the other side of* O (*jenseits von* O). The hyperboloid sheet considered above,

$F=c^2t^2-x^2-y^2-z^2=1$, $t>0$, falls to the other side of O.

The region between the cones is filled with the hyperboloidic forms of one sheet

$$-F=x^2+y^2+z^2-c^2t^2=k^2$$

for all constant positive values of k^2. Of importance for us are the hyperbolas with O as center which lie on the latter loci. The single branches may be called briefly *interhyperbolas* (*Zwischenhyperbeln*) *with center* O. Such a branch of a hyperbola, considered as the world-line of a substance-point, would represent a motion which, for $t=-\infty$ and $t=+\infty$ aproaches asymptotically the velocity of light, c.

If now in analogy to the concept of a vector in space, we call a directed tract (*gerichtete Strecke*) in the manifold x, y, z, t, a *vector*, then we must differentiate between time vectors (*zeitartigen Vektoren*) with a direction from O to the sheet $+F=1$, $t>0$, and the space-vectors (*raumartigen Vektoren*) with a direction from O to $-F=1$. The time-axis can be parallel to any vector of the first kind. Every world-point between the past cone and future cone of O can be arranged by the system of reference to be *simultaneous* with O, but equally well as *previous to* O or *later than* O. Every world-point on this side of O is necessarily always previous to O, every world-point on the other side of O necessarily always later than O. Passing the limit for $c=\infty$ would correspond to the complete closing up of the wedge-shaped section between the cones into the plane manifold $t=0$. In our figures this section has purposely been made of different widths.

Let us resolve any vector whatever as from O to x, y, z, t, into the four *components*, x, y, z, t. If the directions of two vectors are respectively those of a radius vector OR from O to one of the surfaces $\mp F=1$ and of a tangent RS at the point R of the surface concerned, then the vectors shall be called *normal* to each other. Accordingly

$$c^2 tt_1 - xx_1 - yy_1 - zz_1 = 0$$

is the condition that the vectors with the components x, y, z, t, and x_1, y_1, z_1, t_1 are normal to each other.

The *unit measures* for the scalars of vectors of different directions are to be so determined that the scalar 1 shall always be given to a space-vector from O to $-F=1$, and $1/c$ to a time-vector from

$$\text{O to } F=1,\ t>0.$$

If we now consider the world-line of a substance point passing through a world-point P (x, y, z, t), the scalar

$$d\tau = (1/c)\sqrt{c^2\,dt^2 - dx^2 - dy^2 - dz^2}$$

accordingly then corresponds to the differential time-vector dx, dy, dz, dt in passing along the line.

The integral $\int d\tau = \tau$ of this quantity on the world-line measured from any fixed initial point P_0 to a variable terminal point P, we call the characteristic time (*Eigenzeit*) of the substance-point at P. On the world-line we consider x, y, z, t (the components of the vector OP) as functions of the characteristic time τ and designate their first derivatives with respect to t by $\dot{x}, \dot{y}, \dot{z}, \dot{t}$, the second derivatives with respect to t by $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{t}$, and call the vectors formed from these, the derivative of the vector OP with respect to τ the *velocity--vector at* P and the derivative of this velocity-vector with respect to τ the *acceleration-vector at* P. Then the relations hold:

$$c^2\dot{t}^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2 = c^2$$
$$c^2\dot{t}\ddot{t} - \dot{x}\ddot{x} - \dot{y}\ddot{y} - \dot{z}\ddot{z} - 0,$$

that is, the velocity-vector is the time-vector in the direction of world-line at P of unit length and the acceleration-vector at P is normal to the velocity-vector of P, therefore certainly a space-vector.

Now there exists, as is easily seen, a definite hyperbola branch which has three consecutive points in common with the world-line (*Weltlinie*) at P and whose asymptotes are generators of a past and future cone (see Figure 3 below). Let this hyperbola branch be called the *hyperbola of curvature* (*Krümmungshperbel*) *at* P. If M is the center of this hyperbola we are here concerned with an interhyperbola with its center at M. Let ρ be the length of the vector MP, *then we find the acceleration-vector at* P *to be the vector in the direction* MP *of length* c^2/ρ.

If $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{t}$, are all zero, then the hyperbola of curvature reduces to the straight line touching the world-line at P, and ρ is to be put equal to ∞.

IV.

To show that the assumption of the group G_c as holding in the laws of physics does not lead to a contradiction, it is indispensable to undertake a revision of the whole of physics on the basis of this assumption. This revision has already been successfully carried out within a certain region for questions of thermo-dynamics and radiation of heat,[4] for electromagnetic processes and finally for mechanics with retention of the concept of mass.[5]

[4] M. Planck, "Zur Dynamik bewegter Systeme," *Sitzungsberichte der k. preussischen Akademie der Wissenschaften zu Berlin,* 1907, p. 542; also *Annalen der Physik,* Vol. XXVI, 1908, p. 1.

[5] H. Minkowski, "Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern," *Nachrichten der k. Gesellschaft der Wissenschaften*

In the last-named field the first question that arises is: If a force with components X, Y, Z, along the space-axes is applied at a world-point P (x, y, z, t) where the velocity-vector is $\dot{x}, \dot{y}, \dot{z}, \dot{t}$, as what force is this to be conceived under any possible change of the system of reference? Now there exist tested lemmas about ponderomotive force in the electromagnetic field in the cases where the group G_c is certainly to be allowed. These lemmas lead to the simple rule: *On changing the system of reference the said force is to be applied in the new space coordinates, so that the vector pertaining thereto with the components.*

$$\dot{t}X, \dot{t}Y, \dot{t}Z, \dot{t}T$$

where $$T = 1/c^2 \ (\dot{x}/\dot{t}X + \dot{y}/\dot{t}Y + \dot{z}/\dot{t}Z)$$

is the work that the force divided by c^2 performs at the world-point, all remain unchanged. This vector is always normal to the velocity-vector at P. Such a vector belonging to a force at P shall be called a *moving force-vector at* P.

Now let the world-line running through P be described by a substance-point with a constant *mechanical mass m.* Let m times the velocity-vector at P be called the *impulse-vector at* P, and the m times the acceleration-vector at P be called the *force-vector of the motion at* P. According to these definitions the law describing the motion of a mass point with a given moving force-vector reads:[6] *The force-vector of the motion is equal to the moving force-vector.*

This statement summarizes four equations for the components along the four axes, of which the fourth (because both of the described vectors were *a priori* normal to the velocity-vector) can be regarded as a consequence of the first three. According to the above meaning of T the fourth equation undoubtedly expresses the law of energy. The *kinetic energy* of point-mass is therefore to be defined as c^2 *times the component of the impulse-vector along the t-axis.* The expression for this is

$$mc^2(dt/d\tau) = mc^2/\sqrt{1-(v^2/c^2},$$

which, after subtracting the additive constant mc^2 and neglecting quantities of the order $1/c^2$ is the expression of kinetic energy in

zu Göttingen (mathematisch-physikalische Klasse) 1908, p. 53, and *Mathematische Annalen,* Vol. LXVIII, 1910, p. 527; H. Minkowski, *Gesammelte Abhandlungen,* Vol. II, p. 352.

[6] H. Minkowski, *Gesammelte Abhandlungen,* Vol. II, p. 400. Compare also M. Planck, *Verhandlungen der Physikalischen Gesellschaft,* Vol. IV, 1906, p. 136.

Newtonian mechanics $\frac{1}{2}mv^2$. In this the dependence of energy on the system of reference appears obvious. But since the t-axis can now be taken in the direction of any time-vector, the law of energy, on the other hand, formulated for every possible system of reference, contains the entire system of equations of motion. This fact retains its significance in the above-mentioned limiting case for $c=\infty$, also for the deductive development of the Newtonian mechanics, and in this sense it has already been noted by J. R. Schütz.[7]

We can from the start so determine the relation of unit length to unit time, that the natural limit of velocity becomes $c=1$. If we then introduce $\sqrt{-1}.t\ s$ in place of t the quadratic differential expression becomes

$$d\tau^2 = -dx^2 - dy^2 - dz^2 - ds^2$$

thus completely symmetrical in x, y, z, s, and this symmetry now enters into every law which does not contradict the world-postulate. Accordingly we can express the essence of this postulate very significantly in the mystical formula:

300,000 kilometers $\sqrt{-1}$ second.

V.

Perhaps the advantages secured by the world-postulate are nowhere show more impressively than in stating the effect according to the Maxwell-Lorentz theory of a point-charge moving at will. Let us consider the world-line of such a paint-electron with the charge e and introduce the characteristic time τ from any initial point. To obtain the field determined by the electron at any world-point P_1 we construct the past cone P_1 (Fig. 4). This meets the infinite world-line of the electron at a single point P because its directions are everywhere those of a time vector. We construct the tangent at P to the world-line and through P_1 the normal P_1Q to this tangent. Let the scalar of P_1Q be r. Then, according to the definition of a past cone we must take the scalar value of PQ as r/c.

Now the vector in the direction PQ of length e/r represents in its components along the x-, y-, z-axes the vector potential multiplied by c, and in the component along the t-axis the scalar potential

[7] J. R. Schütz, "Das Prinzip der absoluten Erhaltung der Energie" in *Nachrichten der k. Gesellschaft der Wissenschaften zu Göttingen* (mathematisch-physikalische Klasse), 1897, p. 110.

of the field produced by e for the world-point P_1. This is the basis of the fundamental laws established by A. Liénard and E. Wiechert.[8]

In the description of the field itself produced by the electron it is clearly seen that the separation of the field into electric and magnetic forces is a relative one depending on the time axis of reference. Both forces can be described together most luminously after the analogy, however imperfect, of a force screw in mechanics.

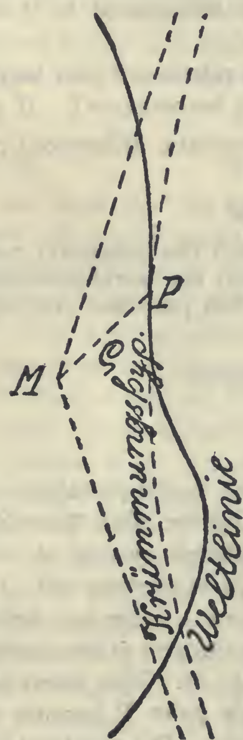


Fig. 3.

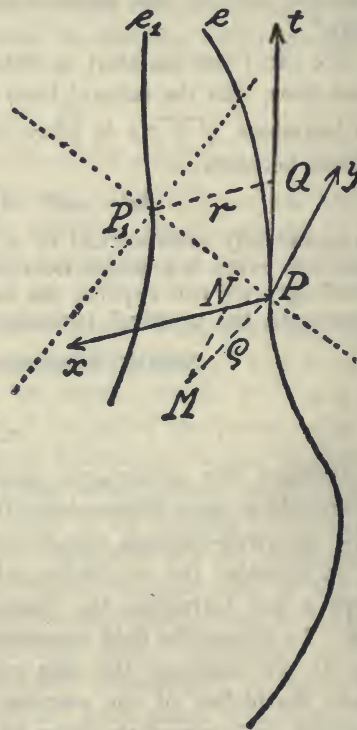


Fig. 4.

I shall now describe the *ponderomotive effect of one point-charge moving at will on another point-charge moving at will.* Let us take the world-line of the second point-electron of charge e_1, passing through the world-point P_1. Let us determine P, Q, r as before, then (Fig. 4) construct the center M of the hyperbola of

[8] A. Liénard, "Champ électrique et magnétique produit par une charge concentrée en un point et animée d'un mouvement quelconque," in *L'éclairage électrique,* Vol. XVI, 1898, pp. 5, 53, 106; E. Wiechert, "Elektrodynamische Elementargesetze" in *Archives néerlandaises des sciences exactes et naturelles* (2), Vol. V, 1900, p. 549.

curvature at P, and finally the normal MN from M to a straight line through P parallel to QP_1. Let us next determine with P as origin a system of reference with the t-axis in the direction of PQ, the x-axis in the direction of QP_1, the y-axis in the direction of MN, so that finally the direction of the z-axis is determined as normal to the t-, x-, y-axes. Let the acceleration vector at P be $\ddot{x}$, $\ddot{y}$, $\ddot{z}$, $\ddot{t}$, and the velocity-vector at P_1 be $\dot{x}_1$, $\dot{y}_1$, $\dot{z}_1$, $\dot{t}_1$. Now the action of the moving force-vector of the first electron e moving at will on the second electron e_1 moving at will at P_1 is formulated thus:

$$-ee_1(\dot{t}_1 - \dot{x}_1/c)\mathfrak{K},$$

in which the three relations between the components $\mathfrak{K}x, \mathfrak{K}y, \mathfrak{K}z, \mathfrak{K}t$, of the vector $\mathfrak{K}$ are: $c\mathfrak{K}t - \mathfrak{K}x = 1/r^2$, $\mathfrak{K}y = \ddot{y}/c^2r$, $\mathfrak{K}z = 0$ and lastly, this vector $\mathfrak{K}$ is normal to the velocity-vector at P_1 and through this circumstance alone is dependent on the latter velocity-vector.

If we compare this statement with the previous formulation[9] of the same fundamental law of the ponderomotive effect of moving point-charges on each other, we cannot but grant that the relations here coming under observation do not manifest their intrinsic character of utter simplicity except in four dimensions, but throw a very complicated projection upon a tri-dimensional space preimposed upon them.

In mechanics reformed according to the world-postulate the disagreements which have caused friction between the Newtonian mechanics and modern electrodynamics disappear of their own accord. I shall touch upon the relation of the *Newtonian law of attraction* to this postulate. I shall assume that when two point masses m and m_1 describe their world-lines a moving force-vector acts from m on m_1 just as in the above expression in the case of electrons, except that now mm_1 is to be substituted for $-ee_1$.

We shall now consider especially the particular case where the acceleration-vector of m is constantly zero, in which case we can so introduce t that m is conceived of as at rest, and the motion of m_1 depends only on the moving force-vector proceeding from m. If we modify this vector first by the factor

$$t^{-1} = \sqrt{1 - v^2/c^2},$$

[9] K. Schwarzschild, *Nachrichten der k. Gesellschaft der Wissenschaften zu Göttingen* (mathematisch-physikalische Klasse), 1903. p. 132. H. A. Lorentz, *Enzyklopädie der mathematischen Wissenschaften,* Vol. V, Art. 14, p. 199.

which, up to quantities of the order $1/c^2$ is equal to 1, then it follows[10] that for positions x_1, y_1, z_1 of m_1 and their corresponding time-positions, Kepler's laws would again obtain, except that in place of the times t_1 the characteristic time τ_1 of m would be substituted.

On the basis of this simple observation we can see that the proposed law of attraction in conjunction with the new mechanics would be no less suitable for explaining astronomical observations than Newton's law of attraction in conjunction with the Newtonian mechanics.

The fundamental equations for electromagnetic processes in ponderable bodies are likewise in complete harmony with the world-postulate. Even the derivation of these equations, as taught by Lorentz, on the basis of conceptions of the electron theory need not for this end by any means be abandoned, as I shall show elsewhere.[11]

The universal validity of the world-postulate is, I should believe, the true core of an electromagnetic world-picture; first discovered by Lorentz, then further developed by Einstein, it is now clearly discernible. In the future development of its mathematical consequences enough indications will be found for experimental verification of the postulate to reconcile by the idea of a pre-established harmony between pure mathematics and physics even those to whom a surrender of old accustomed view-points is uncongenial or painful.

HERMANN MINKOWSKI.

SUGGESTIONS FOR A NEW LOGIC.

The world of logic is in a state of disturbance. A new logic is wanted and anxiously sought after. The logisticians are active and non-Aristotelian thinkers are presenting solutions. Among those dissatisfied with both the traditional and modern logic there is one man of particular originality and distinction. It is Dr. Charles Mercier of Charing Cross Hospital, London, and we take pleasure in presenting a review of his work.

DR. MERCIER'S LOGICAL WORK.

Dr. Charles A. Mercier is a physician whose specialty is mental

[10] H. Minkowski, *Ges. Abhandlungen,* II, p. 403.

[11] This idea is developed in the paper: "Eine Ableitung der Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern vom Standpunkte der Elektronentheorie. Aus dem Nachlass von Hermann Minkowski bearbeitet von Max Born in Göttingen. *Mathematische Annalen,* Vol. LXVIII, 1910, p. 526; *Ges. Abhandlungen,* Vol. II, p. 405.

pathology. Believing that in mental as in bodily disorder the study of order is indispensable to the study of disorder, and that in mental disease the power of logical reasoning is often impaired, he thinks the knowledge of an adequate and correct logic very important and necessary. For this purpose he has found the traditional logic defective in every particular. Nor does he agree with Mill and the modern logicians who follow him, although they too found fault with the traditional school. He has frankly confessed that he does not understand their logic, and the symbolic logic of the logicians is even less satisfactory to him for this he considers no logic at all but "mathematics gone mad." Therefore he has written *A New Logic* of his own in a volume of 422 pages[1] which, though not complete and of course not wholly new, yet is different enough from all previous expositions to warrant the name. He regards his system as an organized and coherent body of doctrine, covering the whole field of reasoning, growing naturally from a single root and forming a harmonious and interdependent whole. And he hopes it will prove of great practical value in clarifying the ideas of the thoughtful and intelligent public with regard to the laws of reasoning. It is impossible to give any comprehensive summary of his method here, for he takes up the defects of the time-honored system one by one, and then in each case works out his own corresponding theory. But we can include here his sweeping indictment of traditional logic, which will give some idea of the scope of his contemplated reform. He says: "In my opinion, its concepts of the composition of the proposition, and of the constituent parts of the proposition, are erroneous; its doctrines of quantity and quality are wrong; its immediate inferences are but a poor few out of multitudes that may be obtained by an adequate logic; the few immediate inferences it does obtain are faulty; its doctrine of the syllogism is artificial and mistaken; the rules of the syllogism are all wrong; there are multitudes of mediate inferences that cannot be reached by the syllogism;....in short, its whole system is insufficient, defective, and erroneous from beginning to end."

Dr. Mercier takes issue with Mill and subsequent writers when they devote a chapter of their logic to the subject of causation and insist that causation lies at the root of induction, for he believes that causation no more belongs to the subject matter of logic than rotation or imitation; that it is neither a principle nor method, but

[1] London: Longmans Green and Co.; Chicago: Open Court Publishing Company, 1912.

rather an example or application of reasoning. It is for this reason that his own treatment of causation is not included in *A New Logic* but is given in a volume by itself.[2] Dr. Mercier writes a direct and trenchant style. He is not only a critic of logic but proves to be a keenly logical critic, and his fearlessness of spirit and acuteness of mind are shown most delightfully in the first chapter of *Causation* which is devoted to the theories of Hume, Mill, Mr. Welton, Professor Pearson, Mr. Bertrand Russell and Dr. McTaggart. We will quote some of his more scathing and fun-provoking passages on the three last named, though for the full line of the author's critical arguments we must refer our readers to the book itself. On pages 18 to 31 he says:

"Much of the authority that Prof. Pearson's *Grammar of Science* has unquestionably achieved is due to his habit of attributing his own opinions to a personified science, a trick that enables him to pose as infallible, while adroitly avoiding the appearance of arrogance that such a pose carries with it. When he says that for science cause is meaningless, he means that Professor Pearson does not understand the meaning of it; when he says that science can in no case demonstrate this or that, he means that Professor Pearson cannot demonstrate it; when he says that science can find no element of enforcement in causation, he means that Professor Pearson is too blind to see the element of enforcement; and so on. This is an adroit method of imposing on the gullibility of his readers, for who, in these 'scientific' days, would have the temerity to question the pronouncements of science? But I must confess to some surprise that it has been so successful. I should have thought that it might have occurred to some one that science in this sense is only a name for a body of opinion; a body of fluctuating opinion, varying from time to time and from person to person, so that what is science to-day was heresy yesterday, and will be superstition to-morrow; what is science to one is stupidity to another, and falsehood to a third. What is science to Professor Pearson, for instance, is nonsense to me.

"Professor Pearson belongs to the school of Hume and Mill, and with them denies that there is any 'enforcement' of an effect by its cause, or any necessary connection between them. The cause is merely the antecedent, the effect merely the subsequent. The one happens to follow the other, but there is no reason or necessity why

[2] *On Causation with a Chapter on Belief.* London and New York: Longmans, Green and Co., 1916.

it should be so: they are in no way connected; but when we see repeated instances of the same succession of events, we deludedly jump to the conclusion that the predecessor is the cause of the successor. Almost as soon as it was stated, Reid blew this doctrine sky high by adducing the instance of night and day. Day always precedes night, and night always follows day, but no one supposes that day is the cause of night or that night is the effect of day. And why not? Manifestly because they are merely antecedent and subsequent; because there is no power in day to produce night; because there is no enforcement of night by day....By cause we do not mean mere antecedence, nor by effect do we mean mere succession. If we did, we should accept day as the cause of night, and night as the effect of day. If we did, the old and notorious fallacy, *post hoc, ergo propter hoc,* would be no fallacy: it would be an unassailable truth; yet the same logicians who declare in their chapters on occasion and induction that causation is nothing but sequence, declare in their chapter on fallacies that it is fallacious to argue from *post hoc* to *propter hoc.* But no inconsistency or self-contradiction in a doctrine ever yet deterred logicians from teaching it; and no doubt they will continue to teach this self-contradiction along with the rest, until the whole silly pseudo-science is swept away, and goes to join judicial astrology, phrenology, and humoral pathology upon the rubbish heap.

"Professor Pearson goes with the crowd, and quotes as from Mill the definition that causation is uniform antecedence; and this definition, says Professor Pearson, is perfectly in accord with scientific concept—that is, with Professor Pearson's concept. It may be a good definition, but when Professor Pearson says it is John Stuart Mill's definition, he is mistaken. Among all of Mill's many definitions of cause and causation this one is not to be found. In this instance 'science' is at fault....

"The most popular doctrine of Professor Pearson's is his distinction between *how* and *why,* a distinction which is either the cause, or the chief effect, of his theory of causation. He denies that we can ever discover *why* a thing happens, or explain it; and limits our powers to saying *how* it happens, or describing it. In this he is demonstrably wrong. It is often as impossible to describe how things happen as to explain why they happen: it is often as easy to explain why they happen as to describe how they happen. The fact is that both *how* and *why* are equivocal words, having more than one meaning; but whichever meaning we take, what I have said is true....

"A good example of the manner in which Professor Pearson poses as a superior being is the advice he gives to his readers, to analyze what is meant by such statements as that the law of gravitation *causes* bodies to fall to the earth. The law, he says, really describes how bodies do fall. Of course it does; but before Professor Pearson gave this advice to his readers, he should have shown some evidence that some one besides himself had ever said such a silly thing. As far as I know, no one has ever pretended that the *law* of gravitation causes bodies to fall to the earth; but if any one should say the *fact* of gravitation—the fact that they attract each other—causes bodies to fall to the earth, he would say what is exactly and punctually true. The law of gravitation describes *how* bodies fall: the fact of gravitation explains *why* they fall; and the explanation is as good and as valid as the description. As far as I know, Professor Pearson never answers the actual arguments of real antagonists; and if he prefers the easier task of answering silly arguments that he puts into the mouth of an imaginary antagonist, then, whatever we may think of his courage and sincerity, we cannot question his wisdom.

"Mr. Bertrand Russell follows Professor Pearson in denying the existence of causes. He say there are no such things. He wants the word abolished, and regards the law of causation, or, as he calls it, of causality, as a relic of a bygone age. To prove this contention he selects from Baldwin's *dictionary* the definitions given therein of Causality, of the notion of Cause and Effect, and so forth; he takes one of Mill's definitions of Causation, and an expression of Bergson's, and analyzes them all destructively.

"All these expressions assume, and Mr. Russell repeatedly in his own expressions assumes, that repetition of instances is necessary before we can identify causation, and I think it is not too much to say that he regards recurrence or repetition as a necessary element, either in causation itself, or in our idea of causation.... He confutes the succession in time of cause and effect, or that antecedence and consequence on which Mill and his school lay so much stress: 'No two instants are contiguous, since the time series is compact.' I cannot see that the conclusion follows from the premise. It seems to me that the more compact the time series, the more closely contiguous must be its instants. If Mr. Russell means that time is continuous, and not made up of instants separated from one another by intervals that are not time, or in which there is no time, I should agree with him; but it is only in such an interrupted time series that the instants would not be contiguous. An instant, like an hour or a day, is a portion of time arbitrarily divided by an imaginary limit from that which precedes and that which follows, with both of which it is continuous or contiguous. But if Mr. Russell is right, and no two instants are contiguous, and if serial contiguity in time between cause and effect is necessary to causation, then this settles the question: then causation is impossible, and Mr. Russell's further argument is redundant, supererogatory, and unnecessary. But he does not think so.....

"He goes on to show that if cause and effect are not contiguous in time, then there must be an interval between them; and 'since there are no infinitesimal time intervals' this lapse of time must be finite. But if there is a finite interval of time between cause and effect, something may happen in that interval to prevent the effect following the cause. It is all very pretty word spinning, and for all I know it may apply to the kind of 'causality' that occurs in the moon, or in a universe of one dimension, but it has no relation whatever to causation as it is known on this earth. Mr. Russell assumes that effect follows cause in the sense of what carpenters call a butt joint, in the sense that the effect does not begin until the cause has ceased to act. That may be what happens in some other universe, but it is not what happens here. What happens here is quite different, as Mr. Russell might have known if he had considered an actual case of causation instead of speculating with $e_1, e_2, \ldots e_n$, and $t_1, t_2, \ldots t_n$, and τ. When, for instance, a man pushes a trolley, he causes it to move. The pushing is the cause, the movement is the effect. But the effect is not postponed until the cause has ceased to act. The effect does not come into existence at an instant contiguous to the cessation of the cause. The effect begins as soon, or almost as soon, as the cause begins; thereafter, cause and effect, the pushing and the movement, accompany one another, and proceed contemporaneously for a certain time; and at length, when the cause ceases, the effect ceases. Cause is contiguous to effect in this case, not end to end, but side by side for the greater portion of their duration. The joint is not a butt joint but a fish joint; and all Mr. Bertrand Russell's pretty word spinning goes for nothing.

"His own statement of 'causality' cannot, he says, be put accurately in non-mathematical language; the nearest approach would be as follows: 'There is a constant relation between the state of the universe at any instant, and the rate of change at which any part of the universe is changing at that instant, such that the rate of change in the rate of change is determinate when the state of the universe is given.' It is with diffidence that I comment on this mysterious formula, but it seems to me clear that if anything can be discovered by its means, it is not the cause of a change, but the rate at which a change takes place, or rather the rate of change in a rate of change; which may be a desirable thing to know, but by no perversity of ingenuity can be twisted or tortured into a cause. But suppose the impossible to be true, and suppose that no cause of any thing can be discovered or assigned unless and until the state of the whole universe is known; then it is clear that no cause of anything ever has been discovered or ever can be discovered, for we can never know the state of the whole universe. But in fact many causes of many things are known, and more are being discovered every day. I know, for instance, that pushing a trolley is a cause of the movement of that trolley. I know that reading such disquisitions as Mr. Welton's, Professor Pearson's, and Mr. Bertrand Russell's, are among the causes of the estimate I have formed of philosophers. Mr. Bertrand Russell may be a great mathematician, Professor Pearson a great statistician, and Mr. Welton a great authority on education; but there is a certain proverb about the cobbler and his last that I would commend to the notice of all three. It may be that I must determine the state of this earth, and of everything upon it, in it, and around it; of all its continents, seas, rivers, lakes, and islands; of all its minerals, from the coal to the diamond; of all its vegetables, from the bacillus to the oak and the orchid; of all its animals, from the spirochæte to the whale; of all its human inhabitants, from the Bushman to Mr. Russell himself; and beyond this, of all the solar system, with its planets, planetary streams, satellites, and comets; of all the stars which we call fixed, with their temperatures, positions, sizes, movements, and chemical composition—it may be that I must know all these things with accuracy before I can discover what it is that is tickling my nose; but for my own part I don't believe it. In fact, I do not know all these things, I know only some of them, and I have already discovered the cause. No doubt

Mr. Bertrand Russell knows best, but my own private belief is that though mathematics cannot err, mathematicians can....

"For thorough mystification, and for the most extreme departure from plain meaning and common sense, Dr. McTaggart runs Mr. Bertrand Russell very hard. According to Dr. McTaggart, 'causation is a relation of implication between existent realities—or to put it more precisely, between existent substances.' This does not on the face of it afford us much help in understanding what causation is, but unlike most philosophers, Dr. McTaggart defines his terms, and for this one cannot be sufficiently grateful to him, not only on general grounds, but also for the surprising meanings that he shows lurk unsuspected in the most ordinary terms. A substance, for instance, according to Dr. McTaggart, is anything that can have qualities and relations; so that, for instance, the battle of Waterloo and a flash of lightning are substances in the McTaggartian sense. This is a bit startling, but definitions are so rare in philosophy that we must be thankful for any we can get, even if they leave us more mystified than before. The battle of Waterloo is presumably not only a substance but also an existing substance in the McTaggartian world, though to the rest of us it ceased to exist a hundred years ago. Causation, then, is a relation of implication between such existing substances as the battle of Waterloo and a flash of lightning; but what is a relation of implication? Here again Dr. McTaggart comes to the rescue with a definition. A relation of implication is a relation between two propositions, P and Q, such that P implies Q, when, if I know P to be true, I am justified by that alone in asserting that Q is true, and if I know Q to be false, I am justified by that alone in asserting P to be false.

"So far, so good, but still we are a long way from attaining a clear idea of causation; but Dr. McTaggart is not done yet. 'Strictly speaking,' he says, 'implication is a relation between propositions or truths [is a proposition, then, necessarily true?] and not between events . But it is convenient to extend our use of it, so as to say that if one proposition implies another, then the event asserted in the first implies the event asserted in the second [but how if neither of them asserts an event?]. It is in this sense that the cause implies the effect'—causes it, in fact. The jump from propositions to events is a bit startling to those who are not accustomed to the proper meaning of realities and substances, but interpreting these expressions to the best of my ability, I gather that when we say the cause implies the effect, we mean that if the cause is true the effect is true, and if the effect is false the cause is false. But what on earth is the meaning of a cause or an effect being true or false? It does not appear that by a true cause Dr. McTaggart means the *causa vera* of the Schools, but what he does mean I cannot conjecture; and supposing this difficulty to be cleared up, what is the meaning of a false effect? Is it an effect that never happens? or is it an effect that is wrongly attributed to a certain cause? or is it something else? It is to be regretted that Dr. McTaggart has not supplemented his definitions with others, explaining the meaning of these terms. In this difficulty the only practicable expedient is to clothe the expression in circumstances—to apply the general rule to an individual case.

"I take, therefore, two propositions, 'Brutus killed Cæsar,' and 'Brutus and Cæsar were contemporaries,' which stand in relation of implication; for if P, or Brutus killed Cæsar, is true, then we are justified by that alone in asserting the truth of Q, that they were contemporaries; and if Q, or Brutus and Cæsar were contemporaries, is false, then we are justified by that alone in asserting the falsity of P, that Brutus killed Cæsar. This specimen fulfils all Dr. McTaggart's conditions. The relation is undoubtedly a relation of implication; and the killing of Cæsar by Brutus is a substance, for it can have qualities, such as treachery, unexpectedness, rapidity, and so forth. It does not seem to me to be an existing substance, it is true, but it is as much an existing substance as the battle of Waterloo. The contemporaneousness of Brutus and Cæsar is a relation, and therefore this also is a substance, and to the same extent the other is an existing substance. All the conditions being satisfied, we may therefore predicate a relation of causation between these two existing substances; but now our difficulties begin, for I cannot understand whether the fact that Brutus killed Cæsar caused them to live at the same time, or whether the fact that they were contemporaries caused Brutus to kill Cæsar, If the latter, why did not all his other contemporaries kill Cæsar? and why did not Cæsar kill Brutus? If the former, what caused Brutus and Cæsar to have so many other contemporaries? I have puzzled over these problems till my brain is almost turned, and I am no nearer a solution, and am obliged to give them up. I doubt whether any one but Dr. McTaggart could solve them; and a method which is useless in the hands of every one but its inventor is never likely to become popular.

"Dr. McTaggart arrives at certain other conclusions that are interesting. He decides that there is no reason to believe 'that a cause exerts an activity or an effect.' What is meant by a cause exerting an effect I do not know, and another definition would be useful here; but if Dr. McTaggart means that a cause does not produce an effect, then I respectfully submit that it is not a cause. Moreover, if a cause does not exert an activity, it is only because it *is* an activity, or more properly an action. Cause and activity can no more be divorced than heat and motion, or solidity and resistance. Dr. McTaggart decides that cause and effect are not identical, a discovery that will not, I think, astonish any one but Mr. Welton; that the effect is not necessarily subsequent to the cause, and, indeed, he is not quite sure that the effect may not sometimes come first, and the cause follow after it; and at last he declares, in despair it seems to me, that though cause and effect are not identical, yet there is no means of knowing which is which, or at any rate, there is no clear distinction between them; and therefore, though we may speak of causal relations as existing between two terms, yet we ought not to speak of one of those terms as cause, and of the other as effect. I think we may legitimately complain that Dr. Taggart does not tell us what we ought to call them. Ought we to call them both X, or the one X and the other Y? Ought we to call the one beef, and the other Yorkshire pudding? Or ought we to call the one petticoat and the other trousers? Dr. McTaggart gives us no guidance, and the reader must choose for himself.

"The lecture in which Dr. McTaggart expounded these views was delivered at Newnham College, presumably to an audience of young women, and I trust he developed to them his views of the impropriety of naming the related terms when describing relations. He convinced them, I trust, that it is convenient to speak of the relation of marriage, but inconvenient (and perhaps improper), to speak of bride and bridegroom, or of husband and wife; that it is convenient to speak of parentage, but not of parents or of children; that it is convenient to speak of the relation of cousinhood, but that they should never allow themselves to use such expressions as Harry or Mary."

The second chapter defines effect, reason, result, cause, and the third is devoted to condition. The fourth deals with causation itself and works up to the author's definition of the term (page 75) as "the necessary connection between an action and the sequent change or accompanying unchange in the thing acted on." Then follows a chapter on "Subsidiary Problems," the last of which is the uniformity of nature. In his first chapter Dr. Mercier had quaintly observed that "no two philosophers agree on what is to be meant by the uniformity of nature; the only thing on which they agree, and when they do agree their unanimity is wonderful, is that nature is not uniform." Chapters six and seven treat various methods of ascertaining causes, and errors in attributing causation and finally one chapter is given to the practical subject of causes of death and insanity. It is an attempt to guide the physician in determining the primary and secondary causes of death as required in England by the Registrar General, and to avoid the confusion of a certain complex "table of causes" of insanity issued by the British Board of Control.

The chapter on belief was added as an afterthought at the request of a friend who was puzzled as to what to believe and disbelieve. In this the author makes no claim to philosophical profundity, but endeavors to furnish a basis by which the ordinary thoughtful man may avoid believing things that are irrational, baseless, absurd or self-contradictory. We quote the author's summary of this chapter (pages 227-228):

"The different meanings of 'believe' are defined, and the meanings of various cognate expressions explained. An assertion of any degree of belief or disbelief expresses an attitude of mind either directly toward a fact, or, while directly toward a statement, indirectly toward the fact stated.

"A fact means anything existing or happening, in the past, present, or future.

"Belief ought to conform to fact, but cannot be directly related to fact, for we have no direct knowledge of fact. Between belief and fact there is always the intermediary of evidence. It is evidence and not fact that impresses our minds, and when we have brought our belief, or the want of it, into accordance with the evidence, we have done all we can, and can do no more.

"Evidence is of three kinds: Evidence of sense, evidence of reason, evidence of hearsay.

"Evidence of sense is certain as to the sensation only; but sensation is of little value until it is interpreted, that is, until its source or cause is arrived at by the elementary process of reasoning called perception. This process may be faulty, and the percept false, or erroneous.

"Evidence of reason gives us two criteria of certainty. That which cannot be conceived is certainly false, and its contradictory is certainly true, and constitutes an axiomatic truth or certainty. It is necessary, in using this test, to be careful not to confuse, as Mill and Spencer did, inconceivability with incredibility.

"Empirical certainty rests upon constancy in experience. That relation which has been found constant (i. e., never contradicted) in experiences diverse and incalculably numerous, is true for us, and cannot be believed to be false, although its contradictory may be conceivable.

"If the relation is not constant in experience, then the degree of belief ought to correspond with the proportion that the positive instances in experience of the relation bear to the negative instances, in which the terms of the relation occur apart. The more nearly constant in experience the relation, the more carefully should apparent exceptions be scrutinized.

"Evidence of hearsay may be maximally trustworthy or may be worthless. The following are the criteria to be depended on:

"1. The statement must be understood in the same sense by the receiver as by the assertor.

"2. The witness must be a witness of truth so far as he knows the truth.

"3. The witness must have means of knowing the truth.

"4. The hearsay evidence must not be inconsistent, or even incongruous with experience.

"Whoso makes an assertion, on him lies the burden of proof. No attention should be paid to bare assertion unsupported by evidence.

"Evidence is anything germane to the issue, and consistent with the assertion.

"Proof is evidence inconsistent with any alternative to the assertion.

"Disproof is evidence inconsistent with the assertion.

"The evidence of a single witness may be received in proportion to his previous record for truthfulness, and in proportion to his responsibility, that is to say to the ill-consequences that would accrue to him if he were found to have given false testimony; also to his freedom from interest and bias in making his assertion.

"The evidence of a plurality of witnesses is valuable in proportion to their independence of one another. Evidence of many independent witnesses goes to prove an assertion if they have means of knowing the truth, and if the assertion is consistent with experience. Otherwise, the evidence of witnesses, however many and however unanimous, has no value."

Though we hesitate to draw inferences with regard to an author who has such a ready eye for fallacies, it seems to us that the logical consequent of this chapter ought to be an essay on New Testament criticism or at least on that phase which deals with the doctrine of the resurrection of Jesus.

EDITOR.

LAWRENCE HEYWORTH MILLS.

1837-1918.

On February first despatches from London announced to American newspapers the death of one of those great Oriental scholars whose researches in difficult fields have been the glory of the nineteenth century. Lawrence Heyworth Mills, professor of Zend philology at Oxford since 1898, died at the ripe age of eighty-one years. In him science and literature lose another great figure, one of those who faithfully and courageously pursued the missionary labor of revealing the religious lore of the great, dead civilizations of the East to a West, which in its selfsufficiency always reacted but slowly and unwillingly to messages deemed by its utilitarian spirit to be of little, or at least, of questionable worth.

Though Professor Mills left for a permanent residence in Europe in 1872, he was of American birth, and we select from Dr. Carus's warm tribute to an esteemed contributor to *Open Court* and *Monist,* as found on pages 505 to 509 of volume XIII, the following salient facts.

He was born in New York in 1837 of German and Irish ancestry, long resident in colonial America. Educated for the ministry and ordained, he was active in ecclesiastical duties from 1861 to 1872 in our country and then in the Protestant Episcopal church of the American colony in Florence, Italy, where he resided until 1877 while continuing the prosecution of those studies on gnosticism that had been begun in America. Removing to Germany in that year on the advice of his physician, he began the publication of his first and tentative edition of the Gāthas with four texts, of which three were translated, between 1879 and 1881. Thus favorably introduced to scholars, he was in 1883 urged by the great Orientalists Max Müller and James Darmesteter, and strongly encouraged to undertake the edition of what was probably the most difficult book of the *Sacred Books of the East,* the XXXIst, including as it did, the translation of the Gāthas. This was his most distinguished service to science, and it brought him to England in 1886 in pursuance of a request of Max Müller to see the work through the press. Henceforth his destinies centered about the great English university, Oxford, for the library of which he was largely instrumental in procuring in 1888 what has been called one of the "most precious gifts ever given it," the oldest manuscript of the Yasna, a present from a distinguished High Priest of the Parsis, a scholar renowned for his five-volume dictionary of the Pahlavi tongue. The Clarendon Press expressed its appreciation in a *de luxe* edition of the manuscript, which is the equal of any specimen of bookmaking produced in the nineteenth century. Spurred on by his first success, Dr. Mills was indefatigable in his efforts to obtain by purchase or gift the valuable manuscripts, or to obtain "diplomatically exact" copies of those he could not acquire, all for the ambitious end of seeing the Bodleian in possession of the finest collection of Parsi manuscripts in Europe.

Dr. Mills's mastery of languages was astounding, nor was he satisfied with a superb mastery of Iranic dialects, but to demonstrate the near relationship of Parsi to Sanscrit he deliberately and successfully translated a large portion of the Parsi sacred books into that difficult and ancient tongue. He was ever busy in learned societies and their publications with tongue and pen furthering the knowledge of his beloved science, and the Open Court Publishing Company has had the esteemed pleasure of printing a number of his volumes, such as the second and enlarged edition in 1900 of the *Gāthas,* originally published in 1892-4, as far as completed; *Zarathushtra, Philo, the Achaemenids and Israel* in 1906, being the two volumes in one of his university lectures, which were published as Vol. I in 1904 and Vol. II in 1905; a further collection of university lectures in 1908 under the title *Avesta Eschatology, Compared with* *the Books of Daniel and Revelations*; and finally, in 1913, *Our Own Religion in Ancient Persia.*

Another fruit of Dr. Mills's professional labors at Oxford, with which he was connected from 1898 on, is the *Dictionary of the Language of the Gāthas,* of which the first volume appeared in 1902 and the last in 1914, the fitting and final labor of a great and useful life.

CURRENT PERIODICALS.

Edward V. Huntington ("On Setting up a Definite Integral without the Use of Duhamel's Theorem," *American Mathematical Monthly,* Vol. XIV, 1917, pp. 271-275) makes a contribution of importance in the principles of the integral calculus. Consider the usual process of setting up an integral in the problem, say, of finding the total attraction P due to a thin rod of length $b-a$ at a point O in line with the rod and at distance a from the nearer end. Suppose the linear density of the rod to be any function, $f(x)$, which is known for all values of x from a to b. Also, suppose the attraction due to a particle to be proportional to F (x) times the mass of the particle. We actually proceed somewhat as follows. First, we think of the rod as divided into small elements, dx, where $dx=(b-a)/n$, and proceed to write down the attraction due to a typical element, say, from $x=x$ to $x=x+dx$. Thus, the mass of the element is seen to be $f(x)dx$, at least approximately—and the formula would be exact if the density throughout the element were the same as at its nearer end. Hence, the attraction at the point O due to the element $kF(x)f(x)dx$, at least approximately—and the formula would be exact if all the attracting material in the element were concentrated at its nearer end. In this k is a factor of proportionality. Having thus found the attraction due to a typical single element, at least approximately, we get the total attraction, P, due to all the elements, by integrating the last expression from a to b, "and in spite of the approximation used in setting up the integral, we feel assured that this final expression for P is exact."

Now, in many text-books, notably W. F. Osgood's *Calculus* of 1907 (revised edition 1909), the process of setting up an integral as the limit of a sum is held to require, for complete rigor, the use of "Duhamel's theorem." This theorem is as follows. If $a_1, a_2, \ldots a_n$ is a set of positive infinitesimals such that

$$\lim[\alpha_1+\alpha_2+\ldots+\alpha_n]=\mathrm{A},$$

and if β_i, $\beta_2, \ldots \beta_n$ is a second set of positive infinitesimals such that each β differs from the corresponding α by an infinitesimal of higher order, so that $\lim[\beta_i/\alpha_i]=1$; then

$$\lim[\beta_1+\beta_2+\ldots+\beta_n]=\mathrm{A}.$$

In these the limits are taken for n going to infinity. This theorem has exceptions, and examples of this falsity of the theorem are given, and it is to be noticed that although Osgood recognized this in a paper in 1903, he retained the incorrect form of Duhamel's theorem, without comment, in his text-book. Osgood gave his reasons for so doing in his article of 1903. If Duhamel's theorem is to be used at all, it must be taken in a modified form; and modified forms have been proposed by Osgood (1903), R. L. Moore (1912), and G. A. Bliss (1914). However in this article Huntington shows that the simple and uncritical process of integration regarded as a method of summation can be counted on to yield the correct result in the case, at least, when the functions $f(x)$ and $\mathrm{F}(x)$ are continuous. "It is not necessary to consider any questions of 'infinitesimal of higher order,' or any questions of 'uniformity'; the simple continuity of the two functions is sufficient." This theorem is stated and proved at some length.

* * *

Louis C. Karpinski ("Algebraical Developments among the Egyptians and Babylonians," *American Mathematical Monthly,* Vol. XXIV, 1917, pp. 257-265) tries to show that "much of the material of our elementary algebra was long ago anticipated, to some extent, in the Orient. Similar anticipations of algebraical reasoning are indicated in the material, such as we have, which shows the progress of mathematics in ancient India and Greece.... In interpreting historical evidence one is constantly in danger of reading modern ideas into the text; on the other hand some writers in discussing Egyptian mathematics have been at great pains to discount the material which we have.... The Egyptians, even as early as 2000 B. C., attained a relatively high development in mathematics along analytical lines. This advance was made by the Egyptian priests who enjoyed that adequate leisure which is a primary essential for scientific advance. The assumption has frequently been made that the mathematics of the Egyptians was the product of their practical needs, this view being the result of a too serious regard for the statement of Herodotus that the Egyptians developed geometry in order to redistribute the lands after the periodic overflow of the Nile. The assumption is absolutely refuted by a study of their mathematical achievements.... A just view of the mathematics involved must regard these points [practical application of mathematics] as applications and not at all as sources of the Egyptian mathematics. Fundamentally and universally mathematics is the achievement of thinking beings, occasioned by the mind and not by the body." Speaking of the Rhind papyrus, the author says that "the manual includes a number of problems in linear equations. The solution while essentially by the method of 'false position' is a definite and scientific procedure, leading to the correct value of the root of the equation. One of these first-degree equations is the following: '*Ahau* (heap, mass, unknown) and its seventh, it makes 19.' An arbitrary value, 7, is assumed as the root and the sum is found to be 8, instead of 19 as required; to obtain 19 from 8 the latter is doubled and multiplied by $\frac{1}{4}$ and $\frac{1}{8}$; the trial root 7 is also multiplied by 2, $\frac{1}{4}$, $\frac{1}{8}$, giving 16, $\frac{1}{2}$, $\frac{1}{8}$ as the value of the unknown; substitution of this value in the original equation follows, as a check, in accordance with the common procedure in Egyptian mathematics." After mentioning that symbols for the unknown, addition, and subtraction, and that simultaneous equations in two unknowns, leading to pure quadratics involving the Pythagorean triad $3^2+4^2=5^2$ are found in this and other ancient papyri, the author notices "that the Egyptian system of unit fractions, which persisted in Europe three thousand years after the times of the Ahmes manual, frequently gives a convenient method for actual computation." Also "the discussion in the Egyptian manual of arithmetical and geometrical progressions reveals an unexpected familiarity with rules which we now express by algebraical formulas, a familiarity which has not received adequate appreciation." There is mention of the weak point, which is apparently universal in Egyptian mathematics, in the discussion of the areas of triangles and trapezoids. "It is difficult to reconcile these crude approximations with the precision of measurements found in the construction of the pyramids and with the use of a method for drawing similar figures corresponding to the use of cross-section paper. The authorities are not in full agreement concerning the interpretation of the texts in question." In the section on algebraical ideas in Babylon, the author mentions the astronomical work of the Babylonians, their number symbols and decimal and sexagesimal systems (the sexagesimal place system of recording numbers appears as early as 3000 B. C.), and the interest on the part of the Babylonians in arithmetical and geometrical series as early as 700 B. C. and in square and cubic numbers. "This brief survey of algebraical developments among the Egyptians and Babylonians shows that much of the material which was developed and extended by Greek mathematicians originated, both in methods and substance, with the scientists of the Orient." Φ

BOOK REVIEWS.

Religion and Science: a philosophical essay. By *John Theodore Merz.* Edinburgh and London, Blackwood and Sons, 1915. Pages, xi, 192. Price, 5s. net.

Clearly, this book belongs to a type. To be in love with emotion has been our affliction since Rousseau; to believe in belief is a form of the same malady. Mr. Merz knows Schleiermacher; he may or may not have read Maeterlinck, or Bergson, or Jean Jacques; but he cannot have escaped Goethe. As for romanticism in theology, we find one fundamental assumption: there is something called religion, independent of articulate creeds; there is the conviction that religion is so valuable that it must be "true"; and there is the prejudice that science is hostile to religion. Strong passions do not need explanation; but just as a man who is not very much in love excuses the follies which he has committed for the purpose of appearing passionate, so the philosophical Christian apologizes for the religion in which he would like to believe, and interprets the weakness of his opponents as evidence of his own strength. Maeterlinck exulted in the "*banqueroute de la science*" because it made religion again possible.

In this book the learned historian of European thought expounds three ideas: (1) Science deals only with an "external" world, which is a development of the world of common sense "with a still greater restriction of fundamental data" (p. 107) out of an earlier and larger reality. (2) Science describes and explains, its terms consist of "spatial data and their connections." Interpretation, i. e., the assignment (or the discovery?) of value and meaning, is reserved for religion. (3) Personality is that which is most real. The highest experience which we can have is the feeling of absolute dependence (Schleiermacher) which we trace to the influence of a higher power.

Mr. Merz decides, first, that the external world is a construction, that conceptual thought abstracts and selects. The products of this selection are subject and object, "an altered and fuller conception of reality," space, time, causality. These entities are carved out of a "primordial stream of thought" which apparently antedates thinking, which is a reality wider (though it is said to be less "full") than the external world. This internal possession is the earlier and truer aspect of our personality—a period (as well as an aspect) when we looked upon everything merely as "internal happenings." We entertained this hypothesis in our infancy, and our age sees the belief justified.

Although this is the earlier and truer aspect of our personality, yet contact with other personality leads us out of it. The first external object that the baby apprehends is its mother, not perhaps in her earlier and truer aspect, but as an influence, a spiritual pressure. Nothing else that we experience is so real as personality. The awareness of a group gives us law and morality. The awareness of a supreme spiritual pressure gives us religion.

Mr. Merz holds that mind is as much an abstraction as is matter. "The totality of experience....is of more importance, being more real, than the particles into which we dissect it" (p. 72). Whether personality is equivalent to this total experience, or is one of the particles, is not made clear.

The phrases "stream of thought" and "firmament of consciousness" recur many times. The account of description, explanation, and interpretation is the best part of the book (pp. 110-120). η

OUTLINES OF JAINISM. By *Jagmanderlal Jaini, M.A.* Edited with preliminary note by *F. W. Thomas.* Cambridge: University Press, 1916. Pp. xl, 156. 4s. net.

A compact little treatise by a distinguished Jain. The author divides his exposition into Theology, Metaphysics, Ethics, and Ritual, and appends a number of Jain texts. The book is a compendium, not an interpretation into terms of western philosophy—which is to its credit. It will appeal chiefly to the student of Sanskrit and Pali who has some acquaintance with Indian and Buddhist philosophy, and perhaps is ignorant in this less explored field; but it should interest others as well. We regret that the author did not find space for a comparative account; we learn nothing of borrowings, analogies, or common sources. There is an historical narrative of the teachings of Jainism, but none of the development of its philosophy. Jainism is dualistic, and one would like to know what relation it bears to the dualism of early Sankhya. From Mr. Jaini's statement of the three cardinal principles (karma, relativism, and ahimsa or non-injury of living beings) we do not discern any fundamental difference from some forms of Buddhism.

One is glad to see that honor is paid to the labors of that greatest of orientalists, Jacobi. The book is published under the auspices of the Jain Literature Society. We hope that it will spread the interest in a noble religion and ethics and an important philosophy. η

VOL. XXVIII JULY, 1918 NO. 3

321

THE MONIST

MORS MORTIS.*

ἔσχατος ἐχθρὸς καταργεῖται ὁ θάνατος.
—1 Cor. xv. 26

WE read in Ivanhoe, if any one now ever reads Ivanhoe, that in the single combat between De Bois Guilbert and the Disinherited Knight, the latter, as their steeds rushed together, first leveled his lance at the corslet of the champion, but almost at the very moment of collision he changed his aim to the visor, a mark much more difficult to attain but where the shock would be irresistible. Slightly similar has been the procedure of your speaker. It was his purpose long cherished to address you under some sufficiently cryptic title on the general mission of philosophy as the guide of life and the guardian of the higher ideas and ideals that dignify humanity and vindicate the claim of man to be the head of creation. However, regarding the subject more and more nearly, he grew appalled at its magnitude and convinced of the impossibility of any adequate discussion within the limits of your patience. Then it was that the choice of the narrower mark was finally made, a mark most difficult to attain, but yet most certainly well worth attaining. Even now he fears that the barrel is too big for the hoop, that it will be impossible to compress any half-way sufficient presentation within the time allowed. Hence it may be that the necessary directness of statement

* Address before the Phi Beta Kappa Society, Tulane University, June, 5, 1916.

will often take on the appearance of dogmatism. Time fails for establishing in detail every position assumed, some things will have to be taken for granted, but only such as it seems certain can be proved beyond reasonable doubt. Now as the hour contracts, and the way, though broad and smooth, is also exceeding long and exceeding steep, let us without further preliminaries go straight for the heart of the matter.

The basis of all that follows is a strictly spiritual, psychical, or idealistic conception of the universe. When you look round you upon the stars, the sky, the sun, the moon, the earth, the sea, the land, the walls of the house, the bodies of animals and plants, the bodies of your fellows, yea, your own body, the impulse is almost irresistible to declare that these things are the world, or at least its main elements, that they are precisely what they are quite independently of you and your thought or your existence, that you do not make them at all or in any sense, but that your own every-day experience is shaped and determined by them in all its details. You rise in the morning because the sun has arisen and poured its light upon you and dispelled the dark and revealed the smiling countenance of creation.

> "Awake! for morning in the bowl of night
> Has flung the stone that puts the stars to flight,
> And lo! the Hunter of the East has caught
> The Sultan's turret in a noose of light."

But the stars and the great stone of the sun and the bowl of heaven and the light itself all seem to be just what they are, no matter what you are, objects independent of you, existing before you and after you, and moulding your own activity at every step, and all in apparent indifference to you as to a puny pygmy. You seem to be but the veriest mote in the sunbeam, dancing there for a moment and then shaken out and falling asunder forever, re-swallowed by the infinite ocean swifter and surer than even Goethe dreamed of when he wrote:

> "We by a billow
> Are lifted, a billow
> Engulfs us, we sink
> And are heard of no more."

Such, apparently, is the tremendous pronouncement of common sense, and it receives daily more and more solemnly the sanction of science, particularly of the grand science of life, with all of her handmaids, zoology, and botany, and physiology, and chemistry, and mechanics, the chiefest of them all.

Against this awful oracle of science and of common sense it is in vain that authority and tradition in any and all of their forms raise an empty protest and appeal to creeds outworn and to dogmas whose origin is only too well understood. What Coleridge declared a century ago of the fair humanities of old religion may now be declared with added emphasis concerning the *whole* body of extra-rational doctrines that for millenniums have swayed the minds and inspired the hearts of the European. All these have vanished, they live no longer in the faith of reason. The common-sense and quasi-scientific view of man and the universe moves on daily with firmer and surer and haughtier tread, reminding us of Homer's description of Discord:

> "Small indeed when at first her front she uplifteth, but later
> Holding her head up in heaven, the while on earth she is treading."

There is only one name given under heaven whose magic may arrest the march of this conception, which now rushes over the earth like the shadow of a dim eclipse shedding disastrous twilight over the soul. And that name is philosophy,—not any visionary and unreal speculation, but philosophy more scientific than science herself, philosophy that is the equator and Venus-girdle of the whole sphere of the sciences, philosophy that neglects no element of experience but submits all the data of all the sciences to the severest analysis of which the human mind is capable. It is only scientific philosophy, philosophy that is the science of science, or science in the second degree, that can transfigure and glorify science herself and weave the harsh words of her oracles into rhythmic verse and set them to heavenly music.

Ah! you smile incredulous, you say these are lofty pretensions, but what semblance of justification can be offered? Well, let us see. The domain of science is the objective world about us, sun, moon, and stars, earth, sea, and air, plants, animals, and minerals, blood and bone and nerve and cells, ether and atoms and sub-atoms and electrons and ions and protions, in a word, the whole universe of mass and motion. All of these science struggles with ever finer and finer subdivision to arrange and order and describe harmoniously and consistently in regular forms called laws. A prodigious, an infinite task, which can never be perfectly performed, but which may be advanced on its way further and further without end; a great and a glorious task, which it is the honor and the dignity, the necessity and the blessedness of the human soul to set itself and to work at forever.

But what are all these objects, this whole sensible world around us? Are they the ultimates of the universe? Are they its finalities? Are they all? Is everything derived from them? and beside them is there no other? To give the answer Yes! as so often is hastily done, even where we might expect something better, is to make the greatest mistake of which human nature is capable. That the universe, the sum total of being, consists of atoms or anything like atoms, or of masses in motion, is the greatest error possible to our understanding, and also the most dangerous; for however noble may be the spirit that strikes into this path, it must be led thereby ever downward deeper and deeper into the shade, to the City of Dreadful Night. The conception of the universe as a mere dance of atoms is indeed an appalling, a paralyzing conception; nothing to me sounds more piteous than the cry of a mighty soul, of some strong swimmer in his agony, as of Bertrand Russell or Matthew Arnold, while this tremendous quasi-scientific conception enswathes it with impenetrable gloom. Hear the poet in his famous lines on "Dover Beach."

> "Ah, love, let us be true
> To one another! For the world which seems
> To lie before us like a land of dreams,
> So various, so beautiful, so new,
> Hath really neither joy, nor love, nor light,
> Nor certitude, nor peace, nor help for pain;
> And we are here as on a darkling plain
> Swept with confused alarms of struggle and flight,
> Where ignorant armies clash by night."

The temptation is strong to dwell upon the deep and widespread and all-pervasive working of this materialistic conception, to show how it moulds, and how it tinges where it does not mould, all our modes of thought and feeling, all the activities of our life, our politics, our society, our amusements, our literature, and even our art. Consider the Omar Khayyam craze that swept over us some years ago, consider the beautiful illustrations by Elihu Vedder, with their ever recurrent swirl expressing the alternate collection and dissipation of the life-elements, now gathered up into a person, now scattered to the winds. But the minutes will not wait, we must hurry on. The popular, the current, the quasi-scientific answer "yes," is then an utterly hopeless answer; but that is not the whole of it; the answer is not only hopeless, it is also false. Precisely here philosophy must and does administer its great corrective, not by way of any abridgement but by way of enlargement and supplementation. What science maintains about the physical world is just and true, and immensely important. But such is not the whole story. The physical world of moving masses is not the final, not the ultimate world. On the contrary, it is a construct, a world of images, of symbols that are not at all like the things they symbolize. Such is the central and fundamental proposition of philosophy, the pivot on which all our thinking turns.

As already hinted, no complete proof can be given at present, though certain clear indications should suffice. Hold up a pencil before your face and look with both eyes open straight at the moon,—you will see two pencils; now look straight at the pencil,—you will see two moons. The two pencils and two moons are clearly constructs which you make in the act of seeing; your seeing consists in the making of these constructs. What is said of the pencils holds equally of the world around you: it is made by you in the act of seeing, it is made double, to every point P there corresponds a point P′; only in a certain region, a line or surface called the horopter, do the corresponding points P and P′ fall together into one. Moreover, this horopter changes immensely from instant to instant, it flutters like a flag in a September gale. Since this world of sight is thus built up and changed from instant to instant and is in general always double, it is idle to talk of this visible world as being an ultimate or final thing; it is demonstrably a vision, a construct of your own spirit activity.

Similarly, if you press gently on your eye-ball you will see the page before you divide and another page swim out just like the first, and you will also see a bright ring appear above the other eye. These visions are also constructs, or spatial interpretations of your unspatial mental states. So, too, if you fall and strike the back of your head while roller-skating on the sidewalk, you will perhaps see stars. If you stand before a mirror, you will construct a world behind the mirror and call it a reflection, but it is a contruct none the less. If you look at the smooth pictures in a stereoscope, you will construct what has no depth into endless depths of space. If disturbed in sleep or treated with hashish or opium, you may have amazing dreams, in which you see or construct titanic scenes and enact a long-drawn-out history. All these are constructs, all of the same general nature, all the creatures of your ever active soul.

But you say, all these are unreal imaginations, whereas the tree, the house, your friend's body, and your own are all real objects permanent and sensible to all. However, we have just seen that the apparently permanent things, like pencil and moon, are not permanent and unchanging, they are swiftly changing every moment. But is there no difference between the real and the unreal? Certainly, an immense difference. The real is what we *all* construct alike, or so nearly alike that it may and does pass as exactly alike; hence we speak of it as the same. A and B looking at the sky construct each his own moon, but since A and B are *nearly identical,* the two constructs or moons are also nearly identical. The Real then is the common and constant element in the constructs of individual spirits; hence it appears permanent, unchanging, the same for all men.[1]

What has been said about seeing may be said about all other forms of sensing, as hearing, tasting, smelling, touching etc.: all are modes of constructing, of forming space-and-time symbols of spiritual activities that are not in space or time.

It is curious to note what seemingly strange forms these constructions take. You have an experience of strain, of muscle contraction, and you construct a certain sight as near; you have an experience of relaxation, and you construct the object as far away. You have a certain feeling of rotation of the eyes, and you construct the object as *tall* or *high*; you have another different *feeling* of rotation

[1] For an interesting though unsatisfactory discussion of this point see F. Enriques, *Problemi della Scienza,* Chap. II, pp. 58-107.

and you construct the object as *long* or horizontally extended. You have a certain *experience* that rises and falls and returns to its first stage, and you construct the sight as of a circle or other closed curve. You have another *experience* of even, gentle relaxation, and you construct a level moon-beam or perhaps a straight railroad track.

Well, then, your visible, tangible, audible, sensible world is a world of constructs, the products of your own spirit-activity, and it is *real* just so far as your spirit-activity agrees with itself and with the communal spirit-activity of your spirit-fellows. But what, you ask, is a spirit, a soul, a mind, anyway? That is a question each of you must answer for yourself, no one can answer it for you. Your inner experience is known to you and to you only. I can only guess whether you are interested or bored. You may be in accord, or you may be spurning my words as nonsense. But you know, each one of you, though your knowledge is strictly incommunicable, whatever signs or gestures you may make; for a word is a sign, it is a gesture of the vocal organs.

What then do you know? I can never tell. But I may be allowed to make a bold and momentous hypothesis. I guess that you are like me. Observing that my own body, as a sense-construct, corresponds to my own spirit-nature, to my own soul-experiences, and observing that your body, as also a sense-construct, resembles my own in general plan and countless details, I apply the familiar Rule of Three and form the proportion: As my body is to my spirit, so is your body to—your spirit.

Such is my reason, not a strictly logical, but an extremely probable analogical, reason for supposing there are other spirits than my own, and that I am not now talking to a congregation of vapors and automata. Correct or incorrect, we proceed on this hypothesis. I suppose then that there are as many inner experiences as there are faces before me, inner experiences much like my own. If so, then you feel, you think, you will, you hope, you fear, you love, you hate, and do a million other such things that you know about and that no other can or at least does know about. All these incommunicables are elements or contents of your experience.

But who are *you*? Well, these contents of experience, these hopes, fears, pains, pleasures, thoughts, feelings, wills, and the rest, known and knowable to you and you only, are not a mere bundle; they are all tight interwoven at each and every instant, each essential to every other, and all interlocked in a definite way not quite the same for any two of you. No one of you thinks or feels precisely the same as any other at this instant. This is not all, however. No one of you is quite the same at any two instants, that is, the total complex of your experience varies from instant to instant, like an iridescent garment gleaming in the sun. Such a total complex (at any instant) of your thoughts, feelings, desires, and the like we may call a cross-section of your being. These cross-sections vary from instant to instant as your life runs along. But they do not vary wildly and at random from moment to moment, from hour to hour, from day to day, from year to year. On the contrary, they change in a very definite way as you move on in life, a way that is very much alike in us all, but not exactly alike in any two, though extremely alike in unioval twins. So then your total soul-experience, the sum of your psychic experiences (both conscious and subconscious), hangs together in a definite unity at every instant and also in a definite series of such unities from instant to instant. Now this whole definite way in which your experience hangs together not only at every moment but all the time from moment to moment, this entire *connectivity*, is your Self, your Ego, your Personality, which is thus seen to be a *Law of Psychic Form.*

Now that there actually is such a thing as psychic experience, such a thing as thought, feeling, volition, as hope and fear, pleasure and pain, purpose and the like, is the one thing that I know and you know, each for himself as a fact, and we each *assume* it for all of our fellows. Not merely, however, for all our fellow men, but also for all our fellow beasts: we assume that each of these is really a spirit, that it has psychic experience similar to our own, though of far lower order, and all our daily life proceeds on this assumption. We ascribe feelings, such as fear and desire and pain and pleasure and the like, to dogs and cats and horses and birds. These latter, indeed, Aristophanes seems to have regarded with awe and wonder as an airy antemundane thing, as being

> "Born the first of things
> Before the sun, before the wind,
> Before the gods, before mankind
> . . . .
> Wishes there and feelings strong,
> Incommunicable throng."

Note, however, very carefully. We must not think of the body of any animal as the dwelling of its soul, as a place or region where the soul lives and has all these psychic experiences. By no means; the soul does not dwell in any body, it is vain to hunt for it there or anywhere else. It dwells in no place at all, it is placeless. All the bodies that you see are your own constructs, the creatures of your own soul-activity, and not one of these bodies has any soul, not even your own body. But you are a complex of well-ordered soul-experiences which correspond to your body and to which your body corresponds. And since as a matter of fact your soul-experience corresponds to your body, you *assume* that there is a soul-experience corresponding to your neighbor's body; and also to your pet parrot's body, and also to the fierce tiger's body, and to the body of the oyster and the earth-worm, and of all the rest. But these assumed soul-experiences corresponding to these bodies by no means dwell *in* these bodies. If then we speak of the soul of any body it is only an elliptical expression; we mean the complex of soul-experiences not dwelling in that body but only corresponding to that body. Pardon me for insisting so much on this point, but it is all important and extremely likely to be misunderstood.

Well, then, with this made clear once for all, we can see at once that it is quite impossible to stop anywhere in the descent upon the Jacob's ladder of spirits. It is a question of degree and not of kind. If we assume a soul-life corresponding to the body of A, and we must do the like for B and C and D and so on clear down to Z, through the whole alphabet of bodies; there must be a soul-life corresponding to every animal and as well to every plant. Nay, we cannot stop there, for the biologist can find no clear dividing line between the organic and the inorganic, as Shaefer so recently declared in his famous Dundee address. We must assume a soul-life, though of inexpressibly low degree, as corresponding to the colloids, to the crystals, to the molecules, to the atoms, to the sub-atoms, the electrons, and to whatever other finer pulverizations may be discovered in the constitution of matter. In other words, we must assume soul-life, psychic experience, of order however infinitely low, as *corresponding* to every phase, however elemental, in the vast complex of constructs that each soul builds up around it and calls the physical world.

If such be the case, then each one of us is a soul, a spirit, and the universe is a republic of spirits, a city of souls. And each one of us builds up around him at every instant a vast world of constructs, of *symbols* that represent to him the unbounded spiritual realm of which he is a citizen. It is only through the medium of these constructs, of this amazing system of symbols, this consummate social device, that any one spirit can or does enter into communication with any other. You can not tell what your friend or your foe is feeling or thinking or willing except through his words and deeds, but these words and deeds are phenomena of mass and motion; they are not spirit, they are only the signs, the symbols of spirit. When your friend smiles, when your foe scowls, you do not see his love or his hate, you see only certain motions of his features, certain changes in the configuration of masses. You interpret these changes to signify love or hate. But the whole body of your friend or foe was your own construct, your own mental creation, which you made involuntarily as the sign or symbol of your own mental state, and your own mental state not in itself but in relation to another assumed mental or spiritual being called your friend or your foe.

Let us then grasp firmly and hold tenaciously this important notion, that each of us is a spirit in the midst of spirits; that we are acting and interacting with each other continually, and that the vast image of this system of mutual interactions is the boundless physical world of sights and sounds and masses and motions with which each one, each spirit, engirdles himself at every moment, spinning the universe of space and time all round him as the silk worm spins its costly cocoon.

This is not yet all, however. Not only is every spirit compassed about by an infinite engirdling cloud of spirits symbolized by earth and heaven and all that in them is, but the *union of these spirits is complete and perfect.* There is not only a Many, there is also a One. The universe is a unit. It is a Whole. All the exactest science proceeds and must proceed on this supposition. The law of Newton declares that every two particles attract each other directly as the product of their masses and inversely as the squared distance between them. Newton indeed was thinking solely of our solar system, but his successors do not hesitate to extend his law to the remotest stars. If in the depths of space there should be found any exception, that would only be an occasion to seek for some still higher law of mutual interaction; the physicist does not admit the notion of any particle in the universe out of harness with the rest, —if there were any such, it would be of itself another Universe. He thinks of each atom as the center of a web whose fibers shoot thence in every direction to every other atom in the world. Thus, with its radiant lines of force, every atom fills the whole physical world. But none excludes any other, they are all interpenetrative.

The most modern physicist, who thinks of the atom or ion as a phase of strain in the universal ether, illustrates the same necessity of viewing the world as a whole; for his ether is universal, and each phase at every point is determined by the total stress and strain of the one all-comprehending whole. We do indeed roughly and inaccurately imagine the universe as granular, as like an immense swarm of bees broken up into a countless host of subordinate swarms, and these seem to us to be separate and very distinct. Thus you say the desk is here, the door is there, the tree is yonder. But this segregation is artificial, for convenience only. The physicist, the astronomer, the man of science cannot endure it. His thinking restores and forces him to restore the shattered unity of the world. Similarly a sentence is granulated into words, and these into letters; but it is the sentence that is relatively primitive and unital.

Now this physical frame of things is only a construct, a symbol of spirit interacting amid spirit. The merely seeming separation of the elements of this material universe is a defect, or at least a peculiarity, in this symbolism, which we have just seen it is the self-imposed task of scientific thought to overcome. Since thought cannot rest satisfied with a granulated or subdivided world, but insists on thinking the physical world as one, it would seem that we cannot hesitate, but must regard the spirit world, the original of the physical picture, as also a unit, as an entirety, as a whole. This unital spirit is what a Hegel might call the Absolute, but we do not need the term. There are many other indications that point clearly toward this spiritual oneness of the world, many other paths of thought that lead to the same goal. But time fails us, we can not pursue them now. Mark well, however, that this unity is noway inconsistent with infinite multiplicity. The individual spirit may be one with the universal spirit and yet by no means cease to be individual. This individual spirit is perhaps best conceived not as a part, but as a phase, of the universal spirit, even as the modern physicist may think of his electron or protion as a phase of strain or displacement in his universal ether. In fact, this conception, though certainly difficult and at first puzzling, admits of the most various illustrations. Even if none of these be quite satisfactory by itself, yet the general convergence of their indications may content us. When we find the meridians all coming together towards a pole, we feel sure there is something of the kind there somewhere, though it is unlikely that any of Dr. Cook's tracks are to be found in its vicinity. Since this conception of the unity of all spirits in one spirit is essential for what follows, it may be well to pause and resort to some of these illustrations.

Imagine a sphere of water, like the earth before dry land appeared, with its surface swaying in gentle waves, and consider one of those waves. Look at it closely, and you see it made up of countless crinkles and wavelets. Suppose you would define one of these wavelets precisely, would tell exactly what it is. To do this you would have to consider the adjacent wavelets and tell what they were; for the wavelet is what it is only by virtue of the bordering wavelets being each exactly what they are; any change in the next lying wavelet would induce a corresponding change in the central wavelet. But the wavelets of this first ring are similarly determined by the second ring, and these by the third, and so on throughout. It is plain that the central wavelet is thus determined by the whole sphere. The being of the tiniest dimple on the face of the ocean thus extends itself throughout the whole. In this sense then we may say that the wavelet is identical with the sphere, but every other wavelet is similarly identical, they differ only in degree, not in kind, and the whole sphere is the perfect unity of all the wavelets.

Consider also the case of a vibrating chord, as of a violin, or of an ether-beam, a ray of light. The physicist will tell you that either of these is or may be vibrating in millions of ways all at the same time. The unital sensation in question and corresponding to this physical construct called vibrating ether-thread is (we may say) that of white light; this white-light sensation (or may be purple-light sensation) is felt as just as simple as the purest blue, or the purest yellow of the line D, yet it is resoluble into indefinitely many frequencies of vibration and may be spread out in a long rainbow spectrum (not to mention higher and lower frequencies). In case of the vibrating chord, one form of vibration (of the chord as a whole) corresponds to the fundamental tone, while the other so-called over-tones or upper harmonics correspond to the vibrations of the chord in parts. These overtones coexist with the fundamental ground tone, the chord vibrates at the same time as a whole and as subdivided in countless ways into parts. These vibrations coexist and in no way interfere with each other. The corresponding over-tones coexist and in no way interfere with each other or with the ground-tone, but all melt into one tone which is rich in its timbre because of the over-tones, whereas without them the ground-tone would be thin and poor. But the tone is felt as one, though it may be thus mathematically and even experimentally analyzed into many.

But we need not go to light nor to sound for an example of this coexistence of unity with multiplicity. Your daily life is full of it. You get on the Samson for a river trip and steam up against the current. This current is bearing you downstream four miles an hour, but the wheel drives on the vessel upstream much faster. Meantime you are spinning round the earth's axis from west to east say 800 miles per hour; and with the earth you are racing round the sun nearly 19 miles per second; and with the sun and all the planets and a motley crowd of eccentric comets and meteors you are driving through the sky toward the constellation of Hercules. All the while you are moving every way on deck and perhaps throwing a ball with accuracy; for the movement of the throw melts together with all these other motions into perfect unity. They all coexist and mutually determine but nowise interfere.

Nor is there any limit whatever to this composition or resolution, as there seems to be no limit to the refinement of the physicist in his dissipation of masses into molecules, and molecules into atoms, and atoms into sub-atoms, and so on without end. There is a wonderful curve known as the curve of Weierstrass, that prince of mathematical exactness. At first sight it would look like an ordinary smooth curve of sines, such as you see when you shake a line that is fastened at one end, or as when you snap a whip-cracker. But on scanning it closely you would see that it was not smooth but undulatory up and down like a sea-surface or the asphalt pavement of a New Orleans street. On looking at it still closer with a microscope you would see that each little undulation was wrinkled with a host of other still smaller undulations of the same kind; and each of these in turn under a still more powerful microscope would shiver into still smaller undulations, and so on forever. But the curve is meanwhile one, precisely given by its definition.

So too the indefinitely fine subdivision of the physical world by the physicist does not militate against its unity, which he is compelled to reconstruct in thought. Accordingly we may hold confidently that the spiritual universe is a coexistence of many in one, and you who like mathematics may find a much clearer, more beautiful and more convincing analogy in an algebraic equation connecting $a, b, c, d \ldots . x, y, z \ldots$, holding them all clasped together in a mental unity, while respecting the individuality of each one. Nay, more, you know that you can often solve such an equation, that is, you can express one symbol, one of the magnitudes, in terms of all the others, and even when you cannot solve the equation, that is due only to the inadequacy of your mathematics, you may still think of and deal with the equation as if it were solved. When it is thus solved, the original equational relation is not changed, it is the same as before, but it now consists in declaring that x (for instance) equals some expression involving all the other symbols in some definite combination. Thus the one symbol x, so expressed, through the other symbols, is the equivalent of the whole original equation, certainly a striking illustration of the identity of one with all.

So, then, by this long and toilsome path we reach this conception of the universe, of the spiritual universe, the original whereof the physical universe is each man's construct or picture, constructed or painted *according to each man's ability* as an artist. This spiritual world we think of as *one*, as a garment of life and thought and feeling and will, a garment woven without seam from top to bottom. *Woven* did I say? Nay, not so. Goethe does indeed put these noble lines into the mouth of the Earth-Spirit:

> "Through Time's whirring loom so the shuttle I drive
> And weave of the Godhead the garment alive."

But the living vestment of Deity is not woven, the image is imperfect. As the shuttle flies back and forth it lays the threads side by side, and no matter how close, they are still distinct, like the lines of a diffraction grating. But the living vesture of the Deity is not thus woven, there are no threads, however close, side by side. The garment of the Godhead is a *continuum.* It is like a line, which is not made up of points no matter how dense you may crowd the points together.

It is very tempting to enlarge upon this beautiful and wonderful notion of a continuum, but the time is short and concise subtlety might repel you. It is enough for the present to know that the straight line between two points A and B is a *continuum,* as containing not very many points compacted, but *all* positions that a point would need to take in passing straight from A to B without making any jumps whatever. Such is the continuum, one of the most important of all exact human concepts. As some such continuum we conceive spirit to be, not of course as a line nor as a surface, nor the like; these space- and time-continua are only the constructs that image or symbolize the activities of the spirit-continuum.

As it is once for all our nature to think all things in symbols, especially the deepest things, even as Goethe has said: "The deepest can be said in symbols only" (*Das Tiefste lässt sich nur symbolisch sagen*), it may be well to have some sense-image of the spirit as thus conceived. The sea-surface or a vast spherical flag may partially serve the purpose, a sea-surface heaving now in the light of consciousness, now in the dark of subconsciousness, a flag sunlit here and there in its swells and elsewhere shaded in endless degrees, iridescent as the rainbow, and gleaming and glooming beyond the day and the night. But the flag and the sea-surface are both continuous and unrent, one and indivisible. You have doubtless seen a very ordinary flag floating lazily from the mast of an anchored ship, while the smooth face of the water swayed in a thousand oscillating mirrors below; and you have noticed how the flag was reflected in a thousand distorted and fragmentary images in the waters beneath; the fragments were distinct and a great multitude, but the flag was one. So in the world-image of the spirit we behold millions and decillions of separate forms, the stars and skies and earth and ocean and stones and trees and men; and again, though the images are countless, the spirit that is imaged is one.

It is this unity of the spirit that lies at the basis of all history, of all life, of all science, of all morality. It is because all thought is ultimately one, that we can have a doctrine of logic; because life is one, we can classify and develop a biology; because all soul is one we can have an ethics, both a theory and a practice of morality. In fact, all morality rests upon sympathy, as Adam Smith so deeply divined, and as Sutherland has so clearly illustrated. But sympathy and love, which are the regnant facts of social life, are only forms and specializations of unity, of oneness with our fellows. Behold then the reconciliation of egoism and altruism, of selfishness and unselfishness. The great logical advantage of the egoist has long been felt and was set forth by Plato with tremendous energy in the first books of the Republic. The young logician excites the utmost admiration of Socrates, who feels that it is impossible to confute him without going back exceeding far into ultimate questions. Indeed, he is irrefutable so long as we retain the ordinary notion of self. It is only by an immense expansion of this concept that we gain a coign of logical vantage. Altruism can overcome egoism only by ingurgitation, by swallowing it alive. By this process alone the antagonism is removed. Yourself is in truth your only object of interest or obligation, but only yourself in its largest and only proper sense. But this largest sense extends your self throughout the world, even as the complete definition of the wavelet must extend the wavelet over the whole sphere. You cannot wrong your neighbor without wronging yourself, for behold your neighbour is an aspect of your own universal self.

Now the logic of the situation admits of no escape from these conclusions, but it is one thing to know and it is quite another to *feel*. Logical conviction may be attained and yet leave us cold and lifeless. The head may be converted and the heart remain unmoved. It is for the *feeling* of Universal Unity, the consciousness of the cosmic Self, the enkindling, ennobling, enlightening, inspiring sense of the world-soul, of pan-psychic selfhood that I plead to-night. To be sure, the development, the birth, the growth of any such sense is not the affair of a day, of a year, of a century, or even of a millennium. It is the growth of myriads of years, it is the child of everlasting time. But this need in no way surprise us. How long has any and every sense, by which we construct the world and depict the spirit, been in growing? Did all your remote ancestors have such glorious orbs of light as those wherewith you build up about you the wide roof of the heaven, and the steadfast footstool of the earth? Could the Ninth Symphony have been heard by your forest-ranging forebears or their own progenitors that huddled in the lap of the sea? Nay, your ancestors, that is you yourself at that early dawn, had no specific organs of sight or hearing; you had only a more or less sensitive surface with perhaps here and there a spot of especial tenderness. Neither had you any definite sense of beauty or duty or truth or right. All of these you had then only as infinitesimal germs, now they adorn you as the diadem of your being. So too the world-sense, the consciousness of your universal selfhood, exists with you as only the feeblest spark, but the breath of time shall fan it into heaven-ascending flame.

Some, though, may question whether there is any such sense at all, however nascent. None the less, the proof of the fact is overwhelming, the indications are numberless and unequivocal. No one can look far back upon the vista of the vanished years and doubt that the moral sense, the feeling of obligation, has been growing steadily through all that undistinguished lapse of ages. We need not go back to the amœba in this exploration. We may stop at our ancestors of only a few thousand or even hundred years ago, and we shall find there only the feeblest sense of brotherhood, extending only to the family or at most to the tribe. Within that narrow circle there was a sense of duty, of right, but not beyond; the stranger was the enemy, to whom nothing was due. But now we recognize not only our duties to all men but also our obligations to the dumb brutes of the field. We organize societies for the prevention of cruelty to animals, and there are at least some in whom buds the feeling of obligation to the plants. Meanwhile we continue not only to extensify but also to intensify the feeling of obligation, which is a budding sense of our life as not merely narrowly individual. No matter how much your views may vary in the present war, you must unreservedly admire the immeasurable spontaneity with which the attacked countries have leaped to the defense of the national life in danger; even the English, that most insulated and individualistic of the great peoples, have at length roused themselves to intense national consciousness, and now rallying throughout the length and breadth of their earth-wide empire, they present a seamless and continuous front to the foe. Contrast herewith the state of the world ten thousand years ago, when the largest people would hardly measure up now to our smallest, when the bulk of the population consisted of vagrant groups of a few hundred or perhaps thousands, and the immense strides of human consciousness toward solidarity must become evident. Consider also the great international movements that meet us on every hand, the universal congresses that gather more and more frequently in our great cosmopolitan cities, above all consider Social Democracy, beyond doubt the most impressive of recent political phenomena, and it seems impossible to mistake the indications that we may now behold the faint purpling over all the tree of human life, which betokens the putting forth of a new and glorious foliage, the faint streakings of the dawn of a broader and brighter day.

Some one may say all this is but the progress of civilization. Perhaps; but what is civilization? May we not now perceive it in a clearer light as the history of the birth and growth of the world-consciousness, the progressive reconciliation of the Many and the One? There are many other aspects of this matter that deserve presentation, but I have chosen only a few and these perhaps not the most impressive. Hastening on now we must not fail to note that this bourgeoning sense of worldhood has already come to premonitory recognition in the consciousness of many of the noblest sons of earth. Naught else indeed inspired the great Stoic idea of universal humanity, of the world as one living being, of our citizenship in heaven. The same high note is heard as an overtone all through the dissertations of Epictetus and the meditations of the noble Emperor Aurelius (after whom our own city of New Orleans is named). It is the same great thought that inspired Giordano Bruno and upheld his spirit unbowed even at the stake. It is the same that animated the illustrious Spinoza, the God-intoxicated Jew of Amsterdam, of whom alone among men Schleiermacher could use these words: "Offer with me reverentially a lock of hair to the manes of the holy but proscribed Spinoza. The Divine Spirit transfused him, the Infinite was his beginning and his end, the Universe his only and everlasting love. Into this eternal world he mirrored himself and saw how he was its noblest mirror. Full of religion was he and full of a holy spirit, and therefore he stands alone and unrivaled, master in his art, but exalted above profane society, without disciples, without even citizenship."

Yet, though without disciples, it was the spark of his spirit that enkindled the greatest minds of Germany, such as Lessing and Herder and Schiller, and chief of all Goethe, in whom we find the sense of oneness with the world the liveliest of all. It would be easy to quote by the hour in proof hereof, but the time is nigh out. Consider only a few of the Xenions of Goethe and Schiller (so beautifully translated and published of late by Dr. Paul Carus), such as,

> "Strive on much as thou mayest, thou standest alone there forever
> Until Nature the Strong knitteth thee unto the whole."

And again:

> "Let none equal another, yet every one equal the Highest!
> How can that be? Let each one be complete in himself."

or this from Faust:

> "How each to All its being gives!
> One in the other works and lives."

If now we pass on in haste to Wordsworth, the poet-child of Spinoza, we shall find that this thought of the oneness of man with the world has transfused all his writings and often uplifted an otherwise unsoaring nature to the highest pinnacles of poesy, as when he declares,

> "And I have felt
> A presence that disturbs me with the joy
> Of elevated thoughts; a sense sublime
> Of something far more deeply interfused,
> Whose dwelling is the light of setting suns,
> And the round ocean, and the living air,
> And the blue sky, and in the mind of man:

> A motion and a spirit, that impels
> All thinking things, all objects of all thoughts,
> And rolls through all things."

Such illustrations might be multiplied almost without end, and they show clearly enough that we are here dealing with a profound reality. It is the same nascent consciousness, the quickening and awakening sense of world-oneness, of the divine eternal unity of the All, that not only informs our science, and grounds our morality, and directs our world-politics and all the collective processes of our civilization, but also inspires the oracles of our most philosophic and deep-thoughted poets.

And does any one believe that such a process as we have thus detected can stop now and here or anywhere short of its far distant, its ever unattainable, but yet ever more and more nearly approachable goal? Surely not. It must go on and on forever; the faint purple flush must deepen into richer and richer bloom. Nothing is more absurd than to imagine that the dawning consciousness of the world has more than begun to open its eyes; it is yet but a babe in arms, peeping out upon the world in inarticulate wonder. We cannot indeed foretell the course of its growth, we cannot trace out its way beforehand, it may rush out into the most unexpected paths. But one may be sure it will grow and perhaps at an astounding rate. No one beholding some ascidian ancestor of man ten million years ago could have foretold its descendant with eyes and ears that organize universes of light and color and of melody and harmony, and with still more refined senses of the true and the beautiful and the good that build up unending palaces of exact thought, and colossal fabrics of social and political polity, and far-shining temples of plastic art, and star-y-pointing pyramids of song. Verily the step seems longer by far from such remote ancestry to Goethe or Wordsworth or the average man of to-day than from him to the over-man of myriad years to come, who will clasp the universe to his heart in the nuptial rapture of a consciousness divine.

And now finally we may touch the inmost nerve of the whole matter. In the minds of every one of you perhaps has arisen the question, "But what has all this to do with death?"—the all-important matter, death, which, Seneca says, is the fairest invention of nature? Much every way, as we shall now see. It was August Weismann, the greatest continuator of Darwin, who in his essays on heredity called emphatic attention to a native immortality of the elementary life-form, the cell. When the single-celled organism grows to a certain size it splits in two, and each of the cells goes on living and growing as before; and so on, just as long as the outer conditions of life are present. If the cell dies, it is from some form of accident, and not because it has run its life-course.* The reason of the splitting in two, the so-called spontaneous fission, is to gain greater nourishing surface with the same volume, for two cells of a given shape and containing together a certain volume have a greater surface than one cell of the same shape and the same total volume—an extremely important principle on which we cannot dwell. In the interest of better nutrition cells have kept on dividing and gradually have become specialized in their functions. These specialized cells constitute the body and by becoming specialists have lost their inborn immortality. Meantime the continuous germ-plasm, as Weismann calls it, lives on and grows unceasing through the ages.

Such *very* briefly is the great biologist's doctrine. He, of course, is speaking and very properly speaking of the physical organism solely. We have learned not to disparage this organism in the least, rather to revere it, but at the same time to understand it, as not a thing in itself,

* More recent observations would seem to amend the contention of Weismann.

but as a construct of spirit, as a sign, a symbol, a spatial image of a long series of soul-experience. Well, then, for us the physiologic process called the death of the body is a process taking place not in the world of spirit, of soul-experience, but in the world of the *symbols of that experience*. When the body B dies it does not mean that the corresponding spirit S dies, for there is no meaning in the words "a spirit dies"; neither does it mean that a spirit S has forsaken a body B in which it has been dwelling. The notion that a spirit dwells in a body is a very ancient, very venerable notion, to be treated with great respect; but it is not correct, it is an old-world form outworn. No spirit dwells in any body. Your own body and all the world you see is the construct or outward symbol, which you form at every instant, of your own experience; the bodies of your friends are the signs or images of other spirits with which you are at every instant related. If then your friend dies, the meaning is not that the corresponding symbolized spirit dies, by no means, but only that a certain aspect of your own experience is no longer representable under the image or symbol of your friend's body. For mind you, that friend's body was a construct of your own experience, it was a way of representing another spirit with which you were in the intimate relation called friendship.

But you ask, if this spirit-friend is no longer constructible by me under the form of a body, does it not mean that *some* profound change has taken place in that spirit or in my relation with it? Yes, so much seems to be indicated, but not more. That spirit has changed profoundly its relation to you and its other fellows, but it has not died, for death is a term that has meaning only as applied to physical constructs formed by spirits and corresponding to spirits, but not as applied to spirits themselves.

This fact comes out clearly only when we bear constantly in mind the nature of spirit as a continuum and as a unit. The death, the dissolution, the ceasing to be of such a continuous unit seem quite unthinkable, it would be nothing more nor less than the extinction, the annihilation of the universe, of all that is.

And now at last we come to the final question of the individual conscious existence. We cannot argue but must merely assume that consciousness is the highest stage, yet known or developed, of spirit activity, and that self-consciousness is the highest stage of consciousness. It may sound strange, yet it seems to be the greatest general achievement of the human spirit, that which marks it off most distinctly from all other spirits mounting upward through the spires of form, to be able to say, "It is I." Toward this self-consciousness we may behold the soul struggling through all the ages of the past. But now that this pinnacle is attained, is the onward and upward march to stop? By no means! The path still leads on higher and higher. "Hills peep o'er hills, and Alps on Alps arise." As men we have reached the consciousness of ourselves as individuals, but only as individuals, only in apparent isolation and insulation, as of things in the physical world. In such insulation and isolation we are finite and bounded in time as things are finite and bounded being separated in space.

And precisely herein lies the key and significance of our mortality. It is the symbol of the insulation and isolation of the individual spirit that has attained or is attaining a consciousness and even a selfconsciousness, but has not yet attained a universal consciousness. It is the mark of a spirit that can say "It is I" and "I am Some," but not yet "I am All." Such a spirit that has not yet risen to World-self consciousness, but feels itself as only one among many and not yet as one that permeates, transfuses, unifies, and comprehends all the Many, such a spirit must objectify, externalize, and construct both itself and all its fellows as finite, separate, individualized images, which we call bodies bounded in space and bounded in time, and death is the sign or symbol of this latter definition. But the spirit that rises inexpressibly higher, soaring as on eagle wings above and beyond self-consciousness, mounting aloft to the glittering peaks of World-consciousness divine, that spirit leaves death behind.

A mystic or religionist might say, that soul pillows itself upon the breast of God, but we use not here the language of mysticism or religion. We shape our words to fit the soberer doctrines of development, of the gradual unfolding of the higher forms of life, of the continuous exaltation of psychic experience, through all the endless grades of soul-activity, ever upward and upward to the highest self-consciousness of man. And here not only do we find it logically impossible to stop, but we have found that the general direction of spirit growth as it now shows itself among men is steadily set along the whole front of progress toward the enlargement and, we might say, the solidarification of the individual into a general consciousness. We have seen that under this sign the great historical movements, whether of science or art or politics or of social, industrial or commercial enterprise, take on new meaning and are stamped with the signet of cosmic significance. We have seen also that the choicest spirits both of ancient and of modern times have foreboded the movement of which we speak, have foreseen its goal, and have flung themselves gladly into its current, as it were into the drift of the stars.

Yea, too, they have felt, though unable to justify the feeling, that on this path alone was it possible to seek for triumph over the last enemy, death. Says Goethe:

> "Art thou affrighted at death? and yearnest for life everlasting?
> Live in the whole! When thou long hast departed, it stays."

Similarly the deepest-thoughted of recent poets, George Meredith:

"Our life is but a little holding, lent
To do a mighty labor; we are one
With heaven and the stars when it is spent
To serve God's aim; else die we with the sun."

But you will readily recognize the oracles of both these seers as dubious and at best only half correct; for neither has any inkling of the scientific and philosophic truth that his words darkly adumbrate. Similarly Tennyson tells us, "the individual withers, and the world is more and more."* But the truth they miss is that cosmic history is the process of unfolding, of growing, a psychic experience that passes on up to consciousness and to self-consciousness and does not stop there but expands and ascends ever wider and higher to universal self-consciousness, to the realization of the world-selfhood, the identity of the individual with the universal, a consciousness that transcends death, because it removes the bonds and the bars of which death is the sign. There is nothing Utopian, nothing visionary in the prospect here set forth; it is in line, as we have seen, with all the surest teachings of the austerest science. A hundred illustrations lie at hand, but only one have you patience to hear. When a one-celled organism splits in two, we must suppose the physical fact images some psychic process of too low an order for us to name, something *most distantly* akin to a feeling, to the mother-instinct of a bird or a dam that flutters in agony about her brood or defends her offspring with her own life. Perhaps it is thence a still farther cry to the intense love of the human mother, who loses her very being in her child and finds herself again therein and hardly less in her grand-children and even in remoter descendants.

Now as this lofty triumphant feeling of love is an absolutely uninterrupted outgrowth from the nameless sub-sub-feeling in the single cell, unless we make the impossible

* Especially notable in this connection is the allegory of Mr. Herbert Trench, *Apollo and the Seaman.*

supposition that history is to call a halt in its forward march and henceforth retire or spin round in a circle, it must be that this feeling will grow as the ages roll on, into higher and higher super-feelings that shall identify the life of its descendants, that shall expand and intensify the parent consciousness and the parent love unendingly through all generations to come. Such is only one of a million paths along which the enlarging consciousness pursues its steady and unceasing march toward the infinite and immortal world-consciousness which is its heavenly goal. Even as a wave of the sea issuing from a pebble thrown into it spreads wider and wider till it compasses the whole sphere and gathers itself up in the opposite pole.

"Reflection," says the Dhammapada, "is the path of immortality; thoughtlessness is the path of death." We must amend the wisdom of the Indian sage. It is consciousness that is the path not so much of immortality as of eternality; not mere narrow self-consciousness, but the consciousness of the larger Self that eradiates over the Whole and sees and feels that it is itself the world and that its fellows are each of equal right the world. Herein lies no contradiction, for the modern doctrine of the infinite, grounded by Bolzano and developed by Cantor, Dedekind, Keyser and others, shows clearly how the parts of an infinite may each equal the whole. Such then is the path to immortality, the way to eternal life. Not indeed a narrow path, but the wide-expanding sweep of advancing consciousness, which flashes upon us here as science and there as art and yonder as democracy and liberty and equality and justice and culture and morality and self-sacrifice and virtue and truth and love and everywhere as philosophy, the guide of life. All of these, by no means excluding the lower but no less essential aspects of trade and commerce and industry and wealth and amusement and social enjoyment, all are but manifold phases of the brightening, expanding, ascending individual consciousness that more and more will burst all bounds, above, below, and uplift itself to the Universal and Eternal Whole.

Of course there are many objections you could urge, not many perhaps that have not already been pondered. But these would require the introduction of a new order of notions, for which there is now no time. Enough that a rational interpretation of cosmic history opens before our eyes an increasing prospect for humanity, a vista that broadens and brightens unto perfect day.

William Benjamin Smith.
Tulane University, New Orleans, La.

PREDICAMENTS IN PHILOSOPHY.

PROFESSOR Lovejoy's address before the Philosophical Association last year expressed the suspicion that "something was the matter with philosophy" and returned to criticism and discussion as the way out of the difficulty. It offered nothing constructive in the solution of the problem. "Criticism" is only a euphemism for scepticism, and while scepticism is a necessary weapon in that field, it is not the method of making philosophy. Philosophy began under the discovery of illusions and scepticism was the means of discovering and exposing them, but it was not the method employed by such men as Plato and Aristotle in their constructive work.

There are three functions which philosophy can perform, two of them not being adequately distinguished from each other and not occupying as much attention since Kant and Hume as the first one. They are (1) *Criticism*, (2) the *Acquisition*, and (3) the *Communication* of knowledge. Criticism is the means of breaking up dogmatism and stagnant ideas in our thinking. Acquisition explains itself, while we too often forget the difference between it and the conditions for communicating what we have acquired. Criticism adds nothing in content to knowledge. It only demands clarification and perhaps certitude, though it does not supply it. Communication adds nothing, but transmits what has been acquired, while acquisition is the means of discovery and addition.

I cannot enter into the analysis of the problem of "knowledge" at any length. That would take us far into epistemology and it is only a part of the general problem with which we are concerned here. But I must call a brief attention to the equivocal import of that term, a fact which neither Kant nor Hamilton seemed to have noticed, or to have sufficiently allowed for, if they did notice it. The term "knowledge" has two very different conceptions for which it does duty. The first is *unity* and the second is *certainty*. Or the first is unification, classification, relation, and the second is certification, certitude, assurance. If we can only keep these apart in our discussions, we would quickly come to agreement in our problem. But we are perpetually confusing them and committing fallacies as evident as in the paradoxes of Zeno about motion. Hamilton defined knowledge as *relation* and Herbert Spencer followed him. It was easy to see in this conception why he denied any "knowledge" of the Absolute. It was not comprehensible in terms of a higher genus. It was not classifiable, or unified with a more general concept. It was the *summum genus* itself. But Hamilton sought certitude for the fact of the Absolute in *Faith*, and this was opposed to "knowledge," an opposition quite clear on his definition, but absurd on the definition that "knowledge" implied certitude. Hamilton, however, while correct as to the scholastic use of the term "faith" did not see that it, too, was equivocal. It did duty for the most certain thing in consciousness and also for the most uncertain things, namely, dogmas that required proof or some means of certification. Hence the attack of Mill upon him without discovering exactly what Hamilton was after. If Hamilton's doctrine had not been invoked in the defence of theology it might have been either disregarded or admitted as harmless. It was at least perfectly logical and irrefutable as reasoning on his premises. The point of criticism should have been against his definitions or assumptions and not his reasoning. It was Descartes that suggested the definition of *certitude* for the term, as his doubt was convertible with uncertainty, and whatever other conceptions may have lurked in his employment of the term, that of certitude was reflected in his position and became permanently embodied in its use. There and then the conflict began between "knowledge" as *relation* and knowledge as *certitude.* We shall see the importance of this later.

But what is the problem of philosophy? Most people, perhaps all, would answer, the "knowledge" of Reality. But what is reality? One answers *phenomena* and the other *noumena.* One says sensory data and the other supersensory data. But at this point the problem divides further into the *process* of "knowing" them and the *object* of "knowledge." This gives rise to epistemology along with metaphysics. Epistemology is concerned primarily with the process and metaphysics with the object of "knowledge." But in fact the two cannot be separated except logically, so to speak, and we have always to have reference to both in the philosophic problem as a whole. At one time it comprised the whole field of things known, but became limited by the development of the special sciences and in that way was left the dowry of the insoluble problems of the universe. The consequence was that, in leaving the determination of facts, the acquisition of "knowledge," considered in terms of its objects, to science, it was confined to the criticism and analysis of these data and to the exposition and communication of ideas while the discovery and acquisition of them was made subordinate. In criticism and analysis scepticism either served as the basis or was concealed behind an effort to clarify concepts. The constructive function of philosophy was lost in the effort to find its elements. But the problem is complicated and requires preliminary analysis of its aspects.

Let me state, therefore, some important facts and distinctions with which I undertake the analysis of our problem. (1) There are the separate and yet connected problems of the *acquisition* and of the *communication* of "knowledge." Psychological question and processes are involved, but these two problems are mainly occupied with the *content* or matter of "knowledge." (2) There is the problem of *certitude,* as complicated with both acquisition and communication of "knowledge." This, too, involves psychological processes, but puts the stress of thought on the *modality of judgment,* or the degree of assurance connected with the state of mind involved. (3) There is the problem of the personal equation in acquiring and communicating "knowledge." This concerns the question whether the subject is a visuel, an audile, or a motile, or the problem of the center of reference for the connections and assimilation of experience. (4) There is the problem of the formulation of "knowledge," or the embodiment of it in language which will convey it most intelligently.

Now if "knowledge" and certitude are made convertible in meaning, the first and second problems are the same, but the distinction between acquisition and communication will remain. The third problem will concern the psychological peculiarities that affect the representative ideas of the subject and will determine some, if not all, the differences of opinion that arise in the discussion of fundamental problems. The fourth is a problem for communication, not for acquisition.

Now the modern philosopher, perhaps the ancient philosopher also, is less an inquirer than he is an expositor or teacher. When he was the discoverer and depository of all the "knowledge" men possessed, he had no competitors. He was the wise man in general and had no special problem. But the off-shoots of his general information in the sciences have deprived him of the monopoly of "knowledge" and left him a purveyor rather than an investigator. As a communicator of "knowledge" he labors under limitations which the discoverer does not. He must adapt himself to the experience and limitations of his auditor or reader. He must impress his ideas in the mould of another intelligence, even though he has to modify or abandon the terminology of his natural habits in thinking. He must employ *ad hominem* methods. Communication involves social categories affected by the personal equation of the receiver. Acquisition is not a social affair. It is individual and may employ methods that are difficult to convert into transmissive agencies. This will be apparent as we proceed.

The wide general problem of knowledge is the unification and the explanation of facts. Perhaps we could say the unification or explanation of facts, according as we accept the wider or the narrower meaning of explanation. But the problem is to make the world of experience intelligible and the question is how this is done. What are the conditions on which the mind proceeds in doing this?

In answer to this question, preliminary to the discussion of the difficulties of the philosopher in his appropriate work, I may reduce all these fundamental principles to one general root: namely, that of *causality* with allowance for its divisions and for nomology. I use the term causality in its widest sense for the moment and shall notice its divisions presently. I must mention nomology as concerned with the laws of things and as not entering into final explanations, whatever relation it may have to practical questions. It is *par excellence* the function of science, whatever else may be conceded to that department of intellectual activity. But causality is the fundamental conception on which all philosophy is built and it is divided into two branches. I shall call them *efficient* and *material* causes. The former is the usual conception of the term outside technical philosophical problems, but there are reasons for philosophy wanting the wider use of the term for certain special matters, especially as efficient and material causes may be so closely associated in the same facts. An efficient cause is one which is active and produces events. It is originative, or creative in some sense of the term. A material cause is one which is constitutive of the nature of a thing and may not be active or creative at all. Efficient causes I divide into *Internal* or subjective and *External* or objective causes. Material causes I divide into conferential and differential, or identity and difference. A tabular view of them with appropriate characterization will make this clearer, and will enable readers to understand better what follows.

Causality.	Causa efficiens. Ratio fiendi. (Aetiological)	Internal. Subjective. Free.	
		External. Objective. Determined.	
	Causa materialis. Ratio essendi. (Ontological)	Identity.	Numero eadem. Unity.
			Arte eadem. Similarity.
		Difference.	Numero diversa.
			Arte diversa.

It will be apparent in this tabular scheme that the general idea of "causality" is ambiguous, and in our use of it in this discussion we shall have occasion to employ it in the narrower sense as convertible with the notion of efficient or aetiological agency while the discussion of certain problems will limit the material or ontological causality to the principle of identity, one branch of it, with the principle of difference playing a minor role in the present question. But the main point is that philosophical or metaphysical problems are occupied with *aetiological* and *ontological* principles of explanation, the former concerned with the origin and the latter with the nature of phenomena. I shall not pursue these into their detailed forms. The reader may do this from the table.

In the pursuit of "knowledge" we may not try to go beyond the phenomenal or nomological aspect of things and so may content ourselves with the uniformities of coexistence and sequence. Practical life may not demand more than this. But this depends on the question whether metaphysics may or may not involve "higher" practical questions than mere nomology. Whether it does or not, it is certain that there are mental interests transcending the mere laws of events and "knowledge" seeks realization in both aetiological and ontological facts. But as we have shown there are two separate problems here. The first is the acquisition of "knowledge" and the second is the communication of it.

Now how do we acquire "knowledge"? The brief answer to this is that, in so far as it is systematization of experience, we acquire it by the application of the principles of causality in their wider sense. When we see a fact or phenomenon, we either relate it or explain it, or both relate and explain it, assuming that "explain" here is convertible with assigning its efficient cause. In frequent use, "explanation" may be or is reference to a class, or even showing its law. But here I am using the term, at least for the moment, as the equivalent of assigning the cause. I am never satisfied with the mere event by itself. I must connect it with something else to explain it, if I am to understand it. I relate it to its kind, its material cause, or I refer it to that which *produces* it, its active or efficient cause. Classification explains unity; causation explains occurrence.

In ascertaining how we acquire "knowledge," we come to the question as to what it is. This can be answered in two ways. (1) We may name and analyze the processes of it. This is epistemology and psychology. (2) We may examine the deposit in language which is the result of the process. We may have briefly to speak of both of these. For my purposes, sensation and judgments may constitute the psychological sources of "knowledge," one of them representing it as *having* a mental state and the other as *asserting* a fact or truth. Usually "knowledge" is convertible with certitude of conviction, whatever its source. Sensation and judgment represent the distinction between the given and the asserted. Sensation is experience; judgment is connecting experiences. Both may be "knowledge," but sensation is *having* a state of consciousness as the result of stimulus; judgment an act of relating a fact of experience, and represents usually the conception of "knowledge" which the philosopher has in mind. In acquiring knowledge you use both sensation and judgment; in communicating it, you can use judgment alone, and only one type of that. The sequel will show us this. "Knowledge" in sensation is presentative and has certitude of the immediate sort. But "knowledge" in judgment will have degrees of certitude to be determined by criteria which we do not need to discuss here.

Let us, then, take up the problem of judgment and study it in the forms which it takes in language which represents the petrified forms of thought and may be made to reveal the processes implied.

Sigwart has eight forms of judgment and for some purposes this or any other classification of judgments may be legitimate. But I reduce all of them to two types, which I call *intensive* and *extensive* judgments. Intensive judgments embody the connection between *substance* and *attribute*; extensive judgments the relation between *genus* and *species*. "Snow is white" and "John struck James" are intensive judgments, the one static and the other dynamic. "Iron is a metal" is an extensive judgment. Every possible form of judgment can be reduced to one or the other of these two types, and indeed each of the two is convertible into the other. For instance, "Iron is metallic" is the intensive form of the extensive judgment, "Iron is a metal." Extensive judgments embody the idea of *causa materialis*, of identity in affirmative and of difference in negative propositions. Intensive judgments embody the idea of *causa efficiens,* objective and mechanical when phenomenal, and subjective or free when noumenal, the latter with some qualification in the use of the term "free." The principles here involved show how we think in the presence of a fact of experience, and illustrate how we explain the origin and the nature of facts. They are the basis of all acquisition, whatever may be the basis of communication.

"Knowledge" begins with sensation and perception, if we mean by it *having* a mental state, and if we give it no other meaning it stops there. Judgment is relating and assertory "knowledge." It unifies or classifies and explains or causifies facts of experience. We unify or classify by extensive judgments and explain or causify by intensive judgments. We acquire "knowledge" of fact by immediate perception or having it in consciousness, but we acquire relative "knowledge" by the two types of judgment, while we *communicate* it by only one of them, the extensive. Let us further examine the process of acquisition.

A complex concept, that is, a synthesis of attributes, is the result of judgment and hence acts of judgment precede the use of terms in propositions. As the extensive judgment involves comparison of two or more facts or things, it is the later form to develop. The intensive judgment is the most primitive, though in its later form it involves complex concepts for the subject. It is based upon the aetiological principle. Being the most primitive form of mental action after sensation, the simplest illustration of it is the *impersonal* judgment. "It is warm," "It is cold," "It rains," "It is fine," etc., show the intensive judgment in its first and ultimate form. What we have is sensation, and we apply the category of causality, efficient or aetiological causality, to it in the indefinite form. The term "It" is merely the indication of a subject which we do not name or imply by any special property other than the one concerned in the present experience. The subject is the most general possible, and so far as knowledge is concerned may not have any property but the one in presentation. When we have found a synthesis of qualities we employ a name for them, and the property expressed in the predicate is a new one, or not necessarily implied by the name, at least until the additional property becomes an essential attribute of it. When we have found that the "It" is a complexus of other attributes than the one in immediate perception, our concept denotes that synthesis. For instance, "Apple," "Iron," "Tree" etc. Intensive judgments are involved in forming them and any future reference of a quality to this same subject or synthesis involves another intensive judgment. We are not comparing attributes or things in this process. We are referring events, phenomena, attributes, qualities etc., whether static or dynamic, to a subject in which they inhere. The principle of causality, aetiological causality, is used to make the facts intelligible. We are superposing a category on a fact. In the impersonal judgment, this cause or ground is not named in terms of any other properties than the one in present experience. In other forms of intensive judgment, the subject represents a given synthesis already formed and the predicate is a quality on which we wish to lay stress.

All this means that aetiological conceptions are prior to ontological ones in the process of "knowledge," *ratio fiendi* to *ratio essendi*. The result is that the acquisition of "knowledge" involves contact with facts of experience and offers the way to constructive processes, while communication involves nothing constructive for the mind that is imparting "knowledge." It is analytic and construction is synthetic.

I would not object to expressing the facts in terms of phenomenal syntheses. That is, I am willing to put myself on the basis of pure empiricism, so far as the present contention is concerned. The synthesis may be merely a connection between phenomena, if you like, though I might reserve the right to raise the question whether subject and predicate involve the connection between phenomena at all. But conceding the empirical point of view, we should seem to dispense with the idea of causality or ground, or to make it convertible with coexistences and sequences of events. This, however, would not alter the problem of acquiring "knowledge." It only evades or postpones the question whether there is causality or not. One thing, however, it does correctly enough. Construing "knowledge" as *having* a mental state, it evades the scepticism which attaches to the discussion of the validity of causality. But it does not alter the relation between subject and predicate in intensive judgments, which express ideas in terms of inherence.

The main point, however, is that intensive judgment is the first in the order of "knowledge," as embodying the connection between substance and attribute, ground and property, or the primary idea of causality. The extensive judgment comes second. It absolutely requires two facts for the formation of an assertion. These facts must resemble or differ in order to have the judgment formed. In the intensive judgment comparison does not enter, or is not a necessary part of it. In the extensive judgment this comparison is an absolutely essential condition. The synthesis of intensive judgments is that of the organic unity either of an attribute in a subject, definite or indefinite, or of several attributes in the same subject, unity in time and space, if phenomenal, and unity in time and space plus causality, if noumenal. Sameness of subject depends on the synthesis of qualities in the same time and space; differences of subjects depend on synthesis in different times and spaces. But the synthesis of extensive judgments depends on the unity of *kind,* identity or similarity, regardless of time and space, and causality or ground may be disregarded, though actually present. Thus we establish greater unity of nature in the cosmos by the extensive judgment, and hence it simplifies the *use* of "knowledge."

In the acquisition of "knowledge" by these processes we are in contact with facts of experience. The methods of observation, experiment, classification and explanation are employed and we may not be communicating truth at all. We are simply having sensations and perceptions of facts and superposing categories on them, or seeing them under these principles of "knowledge." In the intensive judgment we are superposing the idea of efficient causes on the facts and in extensive judgment superposing the idea of material causes on them. We are simply exercising aetiological and ontological categories in the processes of explaining and unifying experience.

But when it comes to the *communication* of "knowledge," we can employ only material causes in the act of transmitting it. We may use intensive judgments as well as extensive ones, but we are social beings when we do it and are transmitting rather than acquiring information, and in spite of employing intensive judgments we must rely upon the identity of experience in others with our own to "communicate" at all. The individual can acquire "knowledge" by both processes, as indicated, but he can transfer it only by one of them and that is the principle of identity and difference or material causes. This is the reason that definition and ratiocination are so necessary. If we cannot reproduce identical experiences in the party to whom we wish to convey information, we must press our ideas into the mould of his experience. Without the facts of experience or the power to imagine them, the other party would not use the category of aetiological causes, but must rely on his experience to make communication intelligible. The slightest difference between them will frustrate the transfer. *Causa efficiens,* and perhaps the second branch of the *causa materialis,* cannot be used in communication. The individual in that case must have his own experience. Communication is only an economic device to save time and experience in education, and it does not wholly divest the subject of responsibility for his own experience and thinking. It is successful in proportion to the amount of personal experience. In fact this is the case in all instances, as experience is the primary condition of intelligent receptivity, and communication can occur only in the realm of abstract ideas, not in those of the concrete. The receipt of concrete "knowledge" is a matter of individual experience and it cannot be transferred. This fact puts communication under greater limitations than acquisition. Communication is limited to the *causa materialis* of things.

The best proof of this is the fact that *no syllogism can be constructed out of intensive judgments.* There must be at least one extensive judgment in every syllogism, in order to secure a middle term, or identity of middle terms. The syllogism is to impart conviction or certitude and it can be done only by means of the principle of ontological causes, identity for affirmative judgments and difference for negative judgments. No principle of aetiological causes can be employed in imparting this conviction. Only the individual can apply them to the facts of experience. We cannot make him see this. But by the principle of identity and difference, we may force him to see a conclusion, as it is expressed in the mould of his previous experience. The conclusion is but an instance of the belief he has in general and the certitude transmitted is in direct proportion to the certitude of his premises. The existence of causality, aetiological causality, cannot be imparted to him either by judgment or ratiocination. He must be able to see and apply this for himself.

Now for the application of these general truths to the practical situation.

The philosopher in most cases is not a scientific investigator. He is so generally a teacher, or transmitter, that he gets into the necessary habit of communicating "knowledge." He is not always in contact with concrete facts. He is always trying to make things intelligible to those of less information than himself, and even when he is a scientific inquirer, he is condemned to the use of material causes in his discussion and communication of truth. He has to make his information fit into the experience of others. He has always to employ *ad hominem* methods. He cannot always, if ever, use *ad rem* means in imparting truth. He must embody all his information in the principle of identity to transmit it, as is clearly proved by the instrument of language and the syllogism. If no language embodying this principle of identity exists, no communication is possible. Even mimic art conforms to this and depends on the principle of identity for its effectiveness. But the habit and necessity of employing this principle of identity, whether in judgment or ratiocination—and ratiocination is only a complexus of judgments—*create the tendency to interpret the world by this principle alone.* The condition of communication is made the condition of "knowledge" throughout, though the fact is that causality, or *causa efficiens* is far more fundamental than this and is prior to *causa materialis* in the problem of "knowledge." It insists on the presence of a correlate of phenomena because the fact of experience is this or an event, and implies this correlate. The mind may not be able to name this correlate in terms of experience, or sensation, though it does so in "phenomenal causation," which is merely coexistence and sequence, but it as inevitably thinks of this correlate or causal agent as it thinks of a fact of experience as an effect. Hence when at a loss for a term to express this cause in conceptions of phenomenal antecedent, it resorts to the indefinite or impersonal form of subject or substance, such as "It rains" or "This is sweet." It simplifies its conception of the situation by choosing the most skeletonized form of causality conceivable, not implying any other datum of experience or sensation than the one present. As this concept does not represent a datum of experience, sensory experience at least, it is not communicable, but must be realized in the mental action of the person asked to recognize the facts. That is why aetiological causality is always transcendental. It is not a communicable datum, while anything expressible in sense terms can be transmitted, because the principle of identity can be employed to express it. We may think in intensive judgments, but we must communicate in extensive ones. True, we also think in extensive judgments, but we cannot communicate in any other, and as the philosopher, in the function of a teacher, tends always to communicate information, his habit of mind, determined by the practice of definition and ratiocination, tends to make him try to solve the problem of knowledge by the *causa materialis* without the *causa efficiens,* by ontological without aetiological causes. When he finds himself blocked or frustrated by the defects of definition and ratiocination, he imagines that there is no other principle involved in "knowledge" than that of identity. He becomes sceptical of causality and assumes that acquisition is not different from communication. But when he cannot transmit information, the whole problem has to be left to the perceptions of the recipient. If the recipient lacks in the power of perception, the "knowledge" is not transmitted. We cannot prove the *pons asinorum* to an idiot. If the recipient has the mental experience or power of using his own judgment, we may facilitate his perception of truth, but otherwise we are powerless. As already remarked, communication is but an economic device for saving the expense of time and direct experience with concrete facts. It suggests what this experience would be by indicating its identity in some particular with the existing experience of the recipient.

Now let us apply this result to the main problem of philosophy; namely, the controversy between realism and idealism. Outside of this dispute there is perhaps little to engage controversy among philosophers, but at this fundamental point they are always at odds and we seem to have made little or no progress since Plato.

Naive realism is based upon—or is usually represented as based upon—the conception of some sort of identity between experience and reality, between sensation and the external world. I say "some sort" of identity, because there are the rudiments of discussion and scepticism in the most naive realism. We generally express the situation by saying that the naive realist, who is the unsophisticated layman, assumes that he perceives things as they are, and that the idealist assumes that we do not perceive reality as it is or *per se*. The naive realist does not think of the antithesis between sensation and reality as the idealist does. To him things are as they appear. We see or perceive them. We do not create them. Cause and effect are like each other, or if that is debatable and not the correct way of stating the fact, the cause is identical in kind, more or less, with the appearance. That is, we naturally interpret reality by the principle of identity, because we have to disregard causality in communication of ideas about reality. But the moment that we discover any illusions in perception, we are perplexed. We find that the principle of identity as we are accustomed to employ it fails us, or fails to express the full meaning of things. We discover some sort of antithesis or difference between the subjective and objective. We can no longer communicate our "knowledge." The principle of difference has come into play and as that abstracts all that was assumed to represent the real, we are left without any criterion of "reality" as previously conceived and have to fall back upon efficient or aetiological causes for an explanation of the situation or positing the real, and this is incommunicable. This principle is not convertible with the facts which it explains. When classification will not tell what a thing *is,* we are either lost or fall back upon telling what it *does,* and this is an appeal to causality to determine the nature of things, but that is not communicable.

The whole problem is seen in all its complexity in illusions. Whatever will solve them will remove the perplexities of the realist and the idealist. The philosopher is always looking for universal propositions or judgments that will be true without qualification, but illusions seem to disturb this ideal. They show variation from the normal. He wants to discover identity, whether differences exist or not, and he often finds it difficult to discover this identity where the differences are extremely marked. But the layman goes along without comparing judgments about the straight and crooked stick in the water, or those of normal vision and the image in the mirror, though he may feel as puzzled as the philosopher may be, because the layman is governed by pragmatic considerations. The layman is content with the knowledge of the cause of the abnormality, and makes no attempt to reconcile the different appearances. For practical purposes he is correct, and these in the end may lead also to the philosophical explanation. But the philosopher wants to find the unity between two apparently contradictory phenomena. He discards the question of causality in the case and tries to solve the problem of illusion by that of identity alone, and this is not the correct standard, though it is the only means of communicating his ideas.

For instance, the illusion about the image in the mirror is not about the *existence* of the object, but about its *locus* in space. Its existence is as fully guaranteed by the image in the mirror as if no mirror were there. The illusion concerns space, not objectivity. Causality enters into the explanation and the illusion is due to the attempt to apply identity where it is not applicable. It is much the same about the crooked stick in the water. Its objectivity is protected by causality and not by sensation. The mechanical conditions affect the specific sensation, but not the application of causality. Besides, we assume that "straightness" is a percept or concept of vision alone when it is not. Permanent "straightness" is a concept produced by the abstractions of several senses or at least two of them, and this abstraction may not involve any identity between the two percepts except the fact of permanence in normal conditions, and then, between the normal and abnormal conditions, the permanence of causality for like effects. The illusion is caused by the attempt to apply the principle of identity to the phenomena that are alike in all characters except the causal situation.

It is the principle of causality, *causa efficiens,* that solves the problem. It does not require identity of any kind between subject and object, between appearance and reality, between antecedent and consequent, in order to satisfy the terms of the case, though that identity may actually be there, whether it be *numero eadem* or *arte eadem.* We too hastily assume that illusion implies non-reality in the object of consciousness, when the situation is complicated with inferences and abnormal conditions. The stimulus is there, but it does not require to be what the naive realist assumes, though he may be nearer right than the idealist. The idealist assumes a difference between cause and effect which the realist may not do. At any rate the philosopher is influenced by naive realism long after he has given it up, because its point of view is necessary for the communication of "knowledge," though not for the possession of it. Hallucinations are the best illustration of what I mean. They are always represented as indicating an apparent reality, whose "real" existence we deny. But there are two things to be noted here. Hallucinations have stimuli just as well as normal sensations have. This is a universally recognized fact, but the stimuli are not normal ones. They are secondary, not primary, but they illustrate the law of *causa efficiens*, but not *causa materialis,* as applied by the naive realist to normal sense-perception.

In the second place, it is impossible to affirm the existence of illusions and hallucinations unless we assume a reality as the criterion of them. An illusion has no meaning apart from our "knowledge" of the truth. Lotze well expresses this in the following language. "Die psychologische Entstehungsweise eines Irrthums schliesst den Beweis, dass er ein Irrthum sei, immer erst dann ein, wenn man die Wahrheit schon kennt, von der die Bedingungen seiner Entstehung nothwendig ablenken mussten."

Hegel, I believe, it was who said that we cannot criticize the faculty of knowledge and this was synonymous with the dictum of Lotze. Error implies knowledge of the truth as a condition of discovering the error. Illusion exists only because we insist upon applying the principle of identity where it is not applicable as we conceive it. We make the conditions of communication convertible with those of acquisition, when they are only partly so. Causality holds good after identity has been disqualified. This is unmistakably true in the case of supersensible causes, even though we regard them as hypothetical and though we may later discover elements of identity in them with the sensible. The man who sets up atoms, molecules, ions, electrons, ether, corpuscles, etc., as conditions of phenomena is not appealing to the law of identity as revealed in sense-perception for his explanations, but to some supersensible reality beyond sense, and he must either abandon his hypothesis of such things or accept the law of causality as primary and as not always convertible with that of identity as exemplified in sense-perception, which is the condition of communication, but not the only condition of "knowledge."

But the philosopher, as a teacher, is always trying to communicate "knowledge" to facilitate the student's learning, to save time in his contact with experience, and in this process he comes to regard as untenable all that will not subscribe to the law of identity. This may be true for *proof,* but not for perception or acquisition. A little reflection will show that no "knowledge" is really transmitted, but that this idea of "communication" is a euphemism for economy in the employment of observation and experiment. But we may retain the term for that conception while the actual fact is that no man can acquire knowledge except by his own activity. The communication of "knowledge" is but the pressing of our ideas into the moulds of another's experience and shortens or saves effort to acquire by personal experience and contact with the facts. In this transmission we can use only the barest outline of the facts and the individual receiver must supply the full contents himself. Only the *abstract* can be transmitted. The *concrete* must be experienced.

Now causality of the aetiological type is always transcendental; identity or *causa materialis,* ontological cause, is not. Cause is other than the fact to be explained by it, whether *numero diversa* or *arte diversa,* or independent and transcendental in time and space when phenomenal, and different or immanental when noumenal. Its ultimate conception is immanental and so coexistent with phenomena, as is shown by the fact that substance is the primary criterion of it, and the ordinary representation of it in terms of antecedence and consequence, is only the evidence, the *ratio cognoscendi,* of causality, not its *ratio essendi.* You can transmit "knowledge" about causality only when it expresses itself in antecedence and sequence, and this can be done only in sensory data. Hence it functions only as the *ratio cognoscendi* of cause, not its *ratio essendi.* This is precisely the reason that true causality cannot be communicated by the facts which make it necessary. The individual must supply this "knowledge" by his own insight or ability to *see* it, or to posit it, if "see" is equivocal. This broad principle holds good of all appreciation of truth, but in matters of causality the insight cannot be transmitted or supplied when the abstraction of the facts can be transmitted, and this because the abstract can be expressed in the forms of identity. Only when the actual cause is "phenomenal" can it be communicated and then only as a phenomenon, not as a cause. The causal factor is concealed from sense and must be realized by the perceptive insight of the subject obtaining the "knowledge." The scientific man never looks for the cause in the phenomenon or event itself. He goes "outside" of this, even though he does not transcend time and space for it. The cause may be like the effect in kind, but it is other than the event. It may differ in kind, even if it does not differ in time and space. But being transcendent, causality, the aetiological type, is never an object of sense-perception. Time and space are the principles of individuation, but not of causality in its aetiological aspects. This is the reason that we cannot make causality and identity convertible, though in the final solution of our problem we may always find them associated. But being transcendent the *causa efficiens* is never an object of sense-perception; identity may be such an object and certainly is such in most instances. Hence the communication of "knowledge" will always depend on the ability to appeal to sensory experience. Causal "knowledge," aetiological "knowledge," will not take that form and so must depend on the insight of the subject of experience.

Now if we apply the principle to the perception of reality we shall dsicover the illusions of many thinkers and perhaps we shall run upon the close relation between aetiological and ontological influences in "knowledge," and at the same time the difficulties between acquisition and the communication of it.

When Democritus began a theory of the perception of objects by his doctrine of idola he did not think of idealism as the outcome and assumed both the principle of identity as his means of explanation and the sensation of touch as the standard. He said we perceived objects by the idola, or simulacra of the reality seen, thrown off from the objects. He could not conceive of perception without the idea of contact and the principle of identity between cause and effect. But later thinkers substituted motion, and then luminous undulations when it was found that light was undulatory, to account for the phenomena. But here the principle of identity was abandoned and idealism began its career. Most people still assumed that touch or contact was necessary for perception of objects, whether tactual or visual, and may have squinted toward the same idea in hearing. But here the puzzle for naive realism began. Undulations were not the object and yet a necessary intermediary in perceiving it. When Berkeley came to the problem he too assumed that contact was the condition of perception as well as of sensation and also some sort of identity between sensation and the real. He could not conceive that an object could be perceived at a distance when distance or the third dimension was not in the sensation. He was consciously or unconsciously governed by the principle of *causa materialis* in his conception and explanation of perception. Hamilton came nearer a solution, but did not live to clear up completely his analysis which he based upon the principle of identity, though he was dimly aware that it was not the fundamental criterion of reality. In any case he did not solve it. He too did not see that perception might defy the doctrine of identity and yet be valid and that causality, aetiological causality, might satisfy the problem while we waited for further investigation to adjust ontological causality to it. As long as identity is assumed to be the prior criterion of reality, it will give trouble in the problem of perception. If illusions had not occurred, the problem might never have arisen. But whatever illusion did to create perplexity, the discovery of mechanical and physical conditions affecting the perception of objectivity greatly complicated it. We have gotten away from the naive view of Democritus, but we have not wholly divested ourselves of the assumptions that governed him and subsequent thinkers. The moment that we got rid of idola to explain it, we simply set up a more perplexing intervention in the undulations of light. This perplexity, of course, arose from our failure to emphasize aetiological principles as a satisfactory solution of the problem and that perception might not require contact to determine its validity. The undulations of light were supposed to be different from the object and yet to condition the perception of it. Only idealism cut the Gordian knot here and thought of the object as subjective in its nature. That is, it was sensation which we perceived and not the object *per se,* if there was an object *per se.* It still clung to the assumption that to be seen must be contact with the sensorium. That is, in Berkeleyan parlance, *esse* is *percipi,* whatever that may mean. But the illusion came from supposing that sensation and perception were the same thing. They are simultaneous, but are functionally different, and this cannot be made clear by the law of identity. But if we once see that contact may not be necessary for perception, we shall not be so much influenced by the law of *causa materialis* in our explanation of perception.

The idealistic theory depends on two assumptions. (1) That contact and therefore some kind of identity between sensation and object is necessary for perception. (2) That undulations are the cause of the sensation and are themselves different from the object and the sensation. In the first place the undulatory theory is hypothetical and with it the difference assumed between "physical light" and "psychological light." The corpuscular theory may modify this. But we have to proceed with the undulatory hypothesis. The idealistic theory assumes that the sensation can be called light because there must be some identity between the sensation and the thing "known." This enables it to eliminate the object as non-existent or as "unknown." The assumed difference between the undulations of light and the assumed object of naive realism helps it in this view. But it never satisfies us with its assumption that we can "know" these undulations and yet that we cannot "know" the object. The whole problem of perception and "knowledge" is involved in the doctrine of undulations quite as much as in that of external reality or matter. If you cannot trust perception in the one, you cannot in the other. The fact is that, viewed from the analogies of touch, vision gives no sensation at all. The very existence of visual sensation is an inference, when adjudged by the principle of contact. It is the object we "know" or perceive, and neither the sensation nor the undulations of light. The only common element between touch and vision as sensations is the reaction against stimulus and that relation is no part of the "sensation" as such. The object is no part of the sensation and the perception of the object is not dependent upon any identity between what is in the sensation and what is in the object, though some identity may be found by further analysis of the problem. Let us see if this can be done.

I have said that the puzzle for most people lies in the fact that we are supposed to perceive objects in spite of the fact that the immediate stimulus is either no part of the sensation or has no resemblance to either the sensation or the object, the mental state or the cause. Let us see, however, just what the facts of nature are.

In ordinary photography we have undulations, according to the hypothesis, assumed to be wholly different from the object from which they emanate, passing to the plate of the camera and forming or producing an image there. The result is to produce an image so exactly similar to the object in certain essential characteristics as to be perfectly recognizable in comparison with the reality. A man can be recognized from his picture, though he had never been seen before. The undulations are not like the object and are not like the image, and yet the image is like the object. This is more true in color photography where the actual colors of the object are transferred to the image on the plate. On a larger scale the law of color adaptation in nature illustrates the same law. The cause transfers its characteristics to the animal it affects. Cause and effect have certain identical characteristics in all these phenomena.

Now if nature establishes a law of similarity between subject and object, between cause and effect, between object and image by which we perceive the object, may not perception bridge the chasm as easily as nature does that between object and image in the camera? Why may not nature provide a means of adjusting perception to the situation as well as the identity between object and image in spite of a causal intermediary unlike both of them? Why should I interpret perception after mechanical analogies?

If I trust perception or hypothesis in asserting the nature of undulations, why may I not trust it when it affirms reality in spite of the real or apparent antithesis between sensation and the object, or the difference between undulations and both of them?

That is to say that perception does not depend on identity between object and sensation and may be correct when they are antithetic to each other. The identity may be there, but it is not the identity that determines the perception and its validity. Its judgment about the nature of reality or the object may easily be as valid as that about the undulations and their relation to both object and image. In this, too, we may find a way to recognize a place for *causa materialis* in the problem of perception, though not allowing it to take the place of *causa efficiens*. It is manifest in the phenomena of photography and color adaptation, so that the analogy of these with the phenomena of visual perception may suggest conceptions that will help to solve the problem at this point and to resolve the illusions that center about the acquisition and communication of "knowledge," on the one hand, and about logical and descriptive definitions, on the other. We try too hard to communicate "knowledge" instead of making the recipient do his own thinking by coming into direct contact with facts. We abstract from conditions under which abnormal phenomena occur and then seek a unity where there is none and where we need none. In other words, we substitute ratiocination for perception and assume too readily that "knowledge" can be transmitted without the employment of the functions of acquisition. The latter require the individual to do his own work while communication can only instigate, not produce. Perception is an individual function, ratiocination a social one. Scepticism and criticism, important as they are, may easily develop into intellectual paralysis. The individual must exercise his own power of insight.

His perplexities in the face of illusions may be respected, but contact with facts will dispel them. They are largely of his own creation, as were the paradoxes of Zeno and the puzzles of the Sophists and the New Academy. A little more than superficial analysis and criticism will find the way out of the labyrinth. It was the hopeless entanglement of formal logic, important in its place, that led Professor James into pragmatism. He, like Herbert Spencer, found the solution in contact with facts, or the priority of science. It was Spencer's absurd juggling with the Unknowable that fascinated logic choppers who never discovered the illusions and equivocations that perplexed the case while his knowable was a perpetual source of charm and interest. It is the concrete, and not the abstract that solves problems. If philosophy, then, can do its thinking in the processes of acquisition and confine its critical methods to the communication of "knowledge" it may hope to escape the "ego-centric predicament," reduce abstractions to their place, and find that it can have as much confidence in perception as in ratiocination.

James H. Hyslop.

New York.

THE SCHOOL OF TO-MORROW.

WE are taught by social and by organic evolution alike that the development both of species and of societies does not always take place at the same rate, but is effected rather by an alternation of periods of stagnation or semi-stagnation during which the evolutive process is very slowly unfolded, with other periods in which the rhythm receives almost unprecedented acceleration. This occurs when the gradual accomplishment of events brings about such a contrast between the being which is evolved and the environment in which it has to live that a new and very rapid adaptation is necessary if an inevitable catastrophe is to be avoided. The nations of Europe, and particularly those of the *Entente,* are passing through such an experience, for, even if they emerge completely victorious from the armed conflict with Germany, they none the less run the danger of collapse in the world-wide economic struggle in the after-war period. if they are not re-organized so as to adapt themselves to that profound and radical change in the environment which has been gradually taking place, and which has arisen from the existence of such a competitor as the German Empire, dominated by its ideal of a hegemony, and in possession of all the psychical, economical and technical elements that are necessary for the accomplishment of its aims.

Renovation, in the case of a nation, does not so much imply a change in the aspect of its external institutions, as a moral and intellectual re-modeling upon new lines of all those members upon which depend its institutions, its economic life, and its social progress.

This has been instinctively realized by all the nations of the *Entente,* and they have set to work, anxiously, if one may say so,—as if they felt their very existence threatened—to examine their educational systems, and to study those introduced by Germany, in order to discover where their own are defective, and where those of their rival are worthy of imitation.

This examination has merely confirmed the suspicion that no mysterious secret, no wonderful pedagogic discovery is to be found in the German systems, with perhaps a single exception, that they succeed better than ours in providing the community at large (and not a small minority belonging to the higher classes, but the mass of the people) with that valuable body of concrete knowledge, that elasticity of adaptation to the environment, that capacity for transforming the latter into a shape appropriate to its own ends, which in the struggle for existence have always been considered the very certainty of success.

Let us then examine in the first place whether our own systems are the best suited to effect that continual contact with the greatest possible number of different objects or facts in the external world, and to develop the spirit of observation which alone can furnish the child with that vast aggregate of knowledge of its environment which constitutes the basis indispensable both to its adaptation to that environment, and to its ability to effect a further transformation of it in accordance with needs.

For that purpose we have from the earliest awakening of the child a valuable auxiliary in its innate curiosity. The observation of everything that comes before its eyes should not give rise to fatigue, especially if it is made a matter of play by the wise use of its toys. The Germans in their toys have done wonders in the faithful reproduction in miniature of all that can be reproduced of the external world. They have been no less successful in dealing with the side of that world spontaneously presented to us by nature, and with the technical side gradually brought into being by the industry of man. In every other country this magnificent opportunity has been neglected. We have, indeed, often allowed our toys to give us a false idea of reality. For instance, the little tin engines which delight our children are set going by the winding up of a spring. But the German locomotive has its little boiler, and its little spirit-lamp, and thus the child itself makes the steam, and it is the steam which moves the piston in toy and real machine alike. Thus the child, by that spontaneous curiosity which leads it to endeavor to understand the working of the little mechanism, acquires without an effort something of that mental habit, that instinct of the engineer, which will later stand him in good stead when he enters the technical school or the polytechnic, into which too many of our children are pitchforked without ever having been near a machine. I am not referring to all those wonderful toys which, because they are so cheap, are more and more within the reach not only of the wealthy but of all classes of the community: railway stations, factories, stables, farms, etc., completely fitted up and suitable for giving an exact idea of the agricultural and industrial environment in which the man of the future at a later period will have to exercise his activity, whatever his condition in life may be; kitchens and rooms, all complete and presenting to the child every object required in a well-managed household; Noah's Arks, with faithful reproductions of the various types of animals; miniature botanical gardens with their trees and plants; and so on. Unfortunately we are still very far from this ideal in which the toy is a faithful reproduction in miniature of the external environment, both natural and technical,—the ideal by which the environment which the child will be called upon some day to dominate and to transform is made part and parcel of its mental furniture.

Our infant and elementary schools are not successful in this exercise of the spirit of observation, and in the bringing of the mind into contact with reality. With those rare exceptions in which the Montessori system has been applied with success, these schools seem to place every imaginable stumbling-block in the way of furnishing the child with the slightest experience of the world and of life. The school itself is too often a bare and empty room, containing nothing but forms and desks. It should be first and foremost a rich and varied museum. The teaching, instead of consisting of lessons on things, is purely verbal. Reading and writing, instead of being taught as a means of acquiring the experience of others, and of communicating to others our own experiences, becomes an end in itself. At far too early a period grammar is made to exercise a wicked strain on the infant intelligence, and checks at its very birth the vital impulse of the child mind—a mind that is eager to know everything. As far as life is concerned, the essential utility of the memory consists in the power it gives of storing up in the mind the recollection of the experiences we have lived through, or the experiences of our fellows. Thus the memory of the child should be exercised by encouraging him to recall and to relate to accurate terms what he has seen and noticed during the past few days. Instead of this, he is wearied out by oral repetition of passages of insipid poetry, exercises in mechanical recitation, which are all the more irksome to the pupil because, wiser than his master, he sees no object in them. And to crown all, there are the essays, in which the poor child has to make bricks without straw. Surely the mere written description, *carefully drawn up in consecutive* *order,* of concrete objects which have interested him, or may have been placed before him with the purpose of interesting him, would have the twofold effect of exercising his powers of observation, and of training him in that *clear, accurate* and *systematic* expression which is all that should be expected in compositions from children in either elementary or secondary schools.

Drawing from nature and geometrical drawing are either completely neglected or are taught by old and defective methods, in spite of the fact that again and again it has been insisted that they are useful, in the one case as giving a knowledge of the fundamental geometrical properties of objects, and in the other as cultivating the power of observation. The same may be said of manual work, which has rightly been claimed as invaluable in developing the faculty of observation, in bringing to light the fundamental physical properties of matter, and in giving to man that sense of power over matter and the forces of nature which raises him morally and strengthens his will and energy in action.

In all cases the mere knowledge of facts, the mere experience that comes from ourselves or our fellows, is not in itself enough to produce an adaptation to the environment, or, to put it better, to give us the power of adapting the environment to ourselves, our needs and our ends. What really makes us masters of nature is *reason,* because it is only by means of reason that we are able to determine what results will follow this act or that; reason points out to us the path by which the desired result will be achieved; in a word, it is reason that gives us the power to foresee and serves as a guide to all our actions. Now in our schools, and especially in our secondary schools where this faculty should be more particularly cultivated, an infinite number of opportunities of developing it are neglected, and in certain cases one might even assert that the object of instruction seems rather to destroy than to develop the precious faculty that Mother Nature, wiser than the school, has given us.

It is true that mathematics are excellent as a gymnastic for this faculty of reasoning, but mathematics are not enough. This subject degenerates, especially after the intuitional period of instruction has passed, into a purely mechanical exercise, especially for those pupils who have no genuine aptitude for the subject. Take for example the case of the schoolboy who in his final examination did all his calculations correctly, but was at a loss to explain the tiresome π which came into nearly every formula he used! In any case, as mathematics are usually taught, they develop but one side of the reasoning faculty, the deductive, while they tend rather to dry up the synthetic or intuitive side, by means of which we are able to see analogies between certain phenomena which at the first glance may seem to be quite dissimilar, and thereby to extend to quite a new category of phenomena what we already know from another category which is more familiar to us. Besides, mathematics, either because they are too mechanical, or because of the over-development they produce on the deductive side, tend rather to atrophy what Pascal called *l'esprit de finesse,* which is so necessary to men of business and to men of action in general, and which, thanks to the synthetic view it gives us of a complicated aggregate of circumstances, consists in the faculty of forming for oneself an accurate idea of the relative importance of the different factors or phenomena which combine to produce a whole. Charles Darwin, who himself confessed his aversion from mathematics, shows us nevertheless in his masterly works that he possesses this synthetic faculty, and that in vigor of thought he is inferior to none of the most eminent mathematicians.

The natural sciences could lend themselves wonderfully to the development of this reasoning faculty, and to its development on the deductive or analytical side as well as on the intuitive or synthetic side. But we know only too well how, with the rarest exceptions, they are taught in most schools. In the first place the greatest care seems to be taken to keep out of sight of the student the objects with which he should be closely familiar. Instead of the objects themselves, he is given long and minute verbal descriptions which cannot give him the least idea of what the objects are. He is compelled to learn by heart that a stork has a long bill and long legs, although he has never seen even a stuffed specimen of that fowl. Time is wasted over classifications and sub-classifications, and woe to the unfortunate examinee who cannot repeat like a parrot the species and the genus of birds to which the stork belongs! This is no exercise for the reasoning powers or for the spirit of observation. And yet, the doctrine of evolution, set forth as the nucleus of all the natural sciences, accompanied by concrete presentation, or by very clear images of the different species and of their environment, would explain the genesis of the most fundamental peculiarities of the structure of animal and vegetable organisms, and would thus keep the reasoning faculty constantly at work. Instead of allowing the instinctive mental inertia of the child full play while he is receiving and storing up in his memory the master's verbal statements as to the morphological characteristics of the different species, the pupil should be steadily induced to find out for himself the why and the wherefore of certain characteristics presented by certain organisms compelled to live and move in a stated environment. He would thus acquire a synthetic vision combined with an intimate knowledge of the organic world about him, and at the same time he would find in his hand the precious thread of Ariadne which will in the future guide him in all the transformations which he may find it useful or necessary to effect in his zoological or botanical environment.

The branch of the natural sciences which comprises notions of the structure, the functioning, and the physiological and physical hygiene of our organism must in future have a much more important place in our system than it holds at present. Of themselves these ideas would constitute a solid basis for individual positive morality, and from the social point of view would eventually secure to the nation the maximum return from its potential energies, and would in particular prevent the early decadence or premature destruction of those energies.

Geography based on the naming of capes and bays, of latitudes and longitudes, also fails in its object, which should be that of giving to man a knowledge of the physical, economical, and social environment in which it is his lot to live. Nor does it assist the development of the reasoning faculty either on the deductive or on the intuitive side. And yet no other subject can equip the future *homo oeconomicus,* the worker in the fields or in the factories, the clerk or the emigrant, with information more indispensable to the different activities which some day he may be called upon to exercise. Nor is there any other subject of study which can more effectively induce him to compare the civilizations and institutions of other lands with those of his own country, and so give him in his political duties as citizen both inspiration and impulse to the reform and betterment of the social environment of which he forms a part. And finally there is no other science which lends itself more to the development of his reasoning powers. But if this is to be secured the teaching must not be purely informative in character. As Irving Elgard Miller, the well-known American teacher and psychologist, maintains, we must proceed by continual questions, e. g.: Why is the climate of England warmer than that of Labrador? Why are the countries to the east of the Rocky Mountains arid? Why have the United States spent so much money and energy in cutting through the Isthmus of Panama? What are the conditions which have made New York, Chicago, and St. Louis such important towns?, and so on.

The same may be said of history based on dates, names of kings and battles, and isolated events, all of which teach us nothing of the present moment in history, which alone is of interest to us in completing our knowledge of the environment in which we live. From any one single historical fact of the past, pure and simple, we can draw no conclusion that will throw light on the facts of the present. It has been said that man, with reference to his historical environment, is like a traveler who has lost his way in the forest, and who, while he can see the individual trees, is nevertheless incapable of forming such a general and synthetic view of the forest itself, as alone will enable him to find out unaided the direction he must take. Now history, if taught so as to illustrate in its general lines, and at the same time in its deeply-rooted causes, the complicated development of historical facts, and thereby making possible a comparison of general historical situations in the past with those of a similar generality in the present, would then really fulfil the highly important task of facilitating the adequate and complex comprehension of *our* historical environment which, I again assert, is the only one which concerns us. At the same time such teaching would lead to a better comprehension of the resisting power of certain traditions and the prestige of certain institutions, even after the object of their existence has passed away, and the direction of certain evolutive tendencies, which in their aggregate are so many important factors in the complex play of the social forces which make history. And finally, such teaching no less than the teaching of geography, would lead to the continual exercise of practical reasoning, and would develop the political sense of the future citizen. Questions and problems such as the following would suffice from this point of view: Why did Richelieu in these circumstances or those act in this way or that? In consequence of what conflict between parties or interests did this or that legislative or constituent assembly arrive at this or that decision? What complex historical situation made Napoleon's *coup-d'état* successful?, and so on.

To geography and history must be added with even wider developments the teaching of economic, juridical and administrative science. Not only will this give information that is essential as to the environment in which man must work, earn his livelihood, assert his rights and develop his activity as a citizen, but it will also, by the very questions that are raised and by his efforts at their solution, lead the student to reflect, and will form in him the habit of that accurate evaluation and appreciation of things which is so important a factor of success in life. The mere setting forth of these subjects, and of law in particular, by showing the student the conditions that are necessary for the maintenance and progress of society, would at the same time be a training in what we may call social hygiene, and therefore in that positive social morality which would be the natural complement of the positive individual morality already based on the hygiene of the organism.

But the development of the reasoning faculty in its twofold aspect of the analytical and the synthetic is not sufficient. The student must in the first place be supplied with the direct and tangible proof of the great domination over matter and the forces of nature which is furnished by the concrete knowledge of external facts, and by reasoning based on them; and he must further be trained in the unceasing application of that concrete knowledge and in the use of his reasoning faculty in such a way as to become accustomed to making them the infallible guide and criterion of all his actions. This lofty function of education is fulfilled by nothing more effectively than by the teaching of chemistry and physics, throughout accompanied by that work in the laboratory which should be possible in every secondary school. By its direct action on matter and the forces of nature, by the constant overcoming of the difficulties which beset the path of all experiment, and which are overcome by reflection alone, by investigation under the impulse of the eagerness to discover why this or that experimental result is not what was expected—by all these means will the adolescent find that his powers of observation and his reasoning faculty are being refined. At the same time the will and the resolution to attain the desired end will be strengthened, and the result will be to realize in one and the same individual the happy union of the man of action and the man of thought.

If the subjects we have mentioned aim at the intellectual cognitive development properly so called, the teaching of literature must not only develop and enrich the creative fancy of the student, an inestimable possession in all the really new contingencies of life, but it must also have a highly educative end, the endowment of the youthful mind with lofty moral sentiments, sentiments which are as necessary for the well-being as for the progress of the community.

If the objects to be attained by the teaching of literature are those I have indicated, here then is the unquestionable opportunity of banishing the dead languages from our secondary schools, except of course in the case of students who are destined for literature and for the law.

The old question of the utility of the dead languages is not an absolute but a relative question. The question is: Shall they usurp the place of other and more useful subjects? In this form it admits of but one answer. It is idle to assert that Latin and Greek afford an incomparable intellectual gymnastic, for the modern languages and the subjects already dealt with are even better fitted to achieve that end. Nor can it be claimed that the dead languages furnish the young with ideas which are useful in modern life. On the contrary, it has with as much reason been asserted that the study of the classics unfits men for practical life, and detaches them from the prosaic occupations to which they must some day devote themselves. Nor can it fairly be said, since they speak to us from a distant past, that they can inspire us with sentiments in harmony with the tendencies and aspirations of modern times. And finally, a knowledge of the classics can no longer be claimed as the sole means of knowing the masterpieces of antiquity, for as every one knows, most schoolboys never acquire such a knowledge of Latin and Greek as will enable them to taste the beauties of those masterpieces; and if they know them at all, it is by the means of good translations.

If Latin is absolutely essential to the future students of law, and if Latin and Greek are essential, as they undoubtedly are, to the future students of literature (we do not agree that they are necessary to the students of medicine and the natural sciences, in spite of the few Latin and Greek roots in their technical terminology)—they can always be taught in a special section. They must be taken in extra hours, without encroaching on the time required for the other subjects (and if this supplementary work were to prevent a few young folk from taking up the legal profession, there are few who will question the advantage to society). Or again, the time allotted to the practical work of the laboratory may be omitted by the future students of literature and the law, and given to instruction in the classics.

As for instruction in literature, properly so called, i. e., the knowledge and study of the principal literary masterpieces, ancient and modern, of each country in turn, in the original text or in good translations—its principal object, I repeat, should be the development of the creative faculty of the fancy, bold and unfettered, without which even the most powerful intellect is but a machine, and at the same time to give every young student an ethical preparation for the exigencies of civic life and social progress, to inspire him with lofty civic sentiments and to make him an upright, noble and generous soul. It is precisely with this object in view that we can and should count on the profoundly emotional and irresistibly suggestive influence that no really classic work in literature ever fails to exercise. Thus the time devoted to literature would be for the pupil a period of rest and gracious respite from the continual strain of the powers of observation and reflection he would be compelled to exercise in the other subjects of a scientific character. Literature, and if required, the history of art, would thus really transport the young mind into an atmosphere full of life, full of fancy, of free inspiration, of noble and lofty sentiments; and his impulse toward the pure skies of the ideal would be spontaneous and vigorous in proportion to the mental constraint of the hours devoted to the other subjects.

As for the teaching of philosophy, the present course must be recast completely and with the utmost care. I would go so far as to say with the most anxious care, for, unfortunately, philosophy as it is taught in our schools, with an insidious metaphysics for its basis, a metaphysics more dangerous than if it were openly declared, seems to have the Mephistophelian function of disturbing and obscuring that lucidity of ideas, that reasoning based on sound sense, that upright and healthy judgment which are innate in the normal man. Teaching of the subject could be given, on a reduced scale, in the literature hours, as the history of philosophy, and then only if it is considered good for the development of the fancy of the student to know something of those nebulous poems in which the great metaphysical constructions of the past consist. As a discipline in itself, the course should be transformed, partly into one of scientific synthesis, and partly into one of the analysis of the human mind and the history of science, so that the student may acquire that wide and general view which makes him conscious of the illimitable power of which the human intellect is capable, provided that it continues to exercise his activity in the direction imposed upon it by its very nature.

But, the benevolent reader will say, all this has been discussed over and over again. That is perfectly true. But many of these questions must be opened up anew, and not only these, but also those of professional training and of higher education upon which I have not here touched. They are questions which must be re-examined with a fresh mind, and in the light of the harvest of facts revealed to us by the great war. Questions once regarded as of merely academic interest, have now become problems of vital importance. Action is necessary on the part of those who realize the terrible dilemma by which we are faced: There must be reform, or we perish. Safety lies alone in continuous, unwearying effort; no detail in the teaching of to-day must be neglected, no fact in the life of the school must escape examination. Every question in connection with the training of the new generations must be re-opened and thoroughly discussed. The real aims of education must be subjected to the closest scrutiny; the courses of the schools must be overhauled from top to bottom. Every change and improvement must be enforced with implacable tenacity and with every ounce of our energy. Not for one moment must we allow ourselves to be checked in the work of reformation by the inertia of institutions that are now out of date, or by the culpable indolence of legislators or bureaucrats.

Only thus shall we achieve our supreme aim: the equipment of the democracies for the bitter life and death struggle, for the task of opening up the road to the complete attainment of their glorious destiny.

Eugenio Rignano (Editor of *Scientia*).
Milan, Italy.

THE CONCEPTIONS OF THE HISTORY OF PHILOSOPHY.[1]

IT is less easy than one would think to form an exact idea of *the history of philosophy,* of its function in the order of human disciplines, and the way in which it must endeavor to carry out this function effectively. Like all history, naturally, its task is to find out and reconstitute, and as far as possible to explain, realities which have previously come to pass; but how far does the nature of these realities agree with the labor of reconstruction, and in any case is it not of such a nature as to require special methods or special mental attitudes for accomplishing the task? It is not enough to say or to presume that the methods of investigation proper to history have simply to be applied in the present case; for limits must be assigned to the particular object to which these methods are applied, and the meaning of the questions we must ask ourselves in order to understand it should be determined: now it is from the nature of the object that the enunciation of these questions, to a considerable extent at all events, is evidently deduced. Consequently, we must inquire as to the precise way in which philosophy lends itself to historical study.

At the outset, we must note that philosophy is not a thing that exists objectively, at least in an objective mate-

[1] The following article is a lecture given by the late Victor Delbos and is entitled "Les conceptions de l'histoire de la philosophie." It is printed in the *Revue de métaphysique et de morale* for March, 1917 (Vol. XXIV, pp. 135-147), and the translation is by Fred Rothwell.

rial existence conformable with the unity and simplicity of the word. *Philosophy* does not exist, but *philosophies* do, philosophical doctrines or conceptions which have appeared either successively or simultaneously, and many of which, strange to say, have claimed to be the vehicles of complete and certain truth, to supply the formulas which conclude investigation, and hence, in a way, as regards knowledge of their object, to arrest the course of history. These philosophies are diverse and frequently opposed to one another, not only in the solutions they reach but also in the problems from which they originate and still more in the faculties they bring into play and the methods they employ, and even in the representation of their ideal, which is strictly systematic in some and in the others more divided and parceled out, less engaged in the quest for unity: so that we have actually some difficulty in indicating those characteristics whereby philosophic doctrines or conceptions are really distinguished from other forms of intellectual production.

Nevertheless, an attempt must be made to indicate these characteristics. Without either prejudging or excluding anything, we may lay it down that the various philosophic doctrines or conceptions have come about, in part or in whole, *with reference to* this end: from the resources of the human mind alone to supply an all-embracing explanation of reality and also such an idea of the destiny of man as will enable us to determine his essential task in this world. I state that the various philosophies have appeared *with reference to* this end, by which term I mean that while some of them have had this end directly in view and believed they could actually realize it more or less completely, others have wished to examine and dispute this very claim, in such a way at times as to consider it more or less illegitimate, more or less capable of realization. Some philosophies are doctrinal and dogmatic, others are critical and sceptical; others again combine or link together these two tendencies in varying proportions. From another point of view, the importance allotted to any particular part of the philosophic problem varies according to the philosophers. All the same, speaking generally, all philosophy is qualified by its relation, whether total or partial, affirmative or critical, or even consciously negative, to the end we have just set forth.

Hence, while we can set approximate limits to the object of the history of philosophy, it would all the same appear that this object scarcely lends itself to historical research without being misrepresented. Indeed, is it not characteristic of many philosophic doctrines that they claim completely to reconstruct all previous work and hence to suppress their dependence on the past? To some extent, history imposes on them like dependence. Is it not also the characteristic of many of them that they claim to express the whole truth, i. e., an adequation of mind and matter, freed from the conditions of time? Now, history subordinates them, however slightly, to conditions of this kind. Cartesianism offers us a striking and decisive instance of the conflict between the historical and the philosophic mind. Descartes, when he philosophizes, does not want to know if men existed before him or not; moreover, he asserts the truth he discovers through the content and the concatenation of clear, distinct ideas, the meaning and scope of which are eternal and immutable. The philosopher contemplates or explains things *sub specie aeternitatis.* The historian can only consider them under the form of time. Malebranche, like a good Cartesian, is ever telling us that the knowledge of the philosophic opinions of other men is quite useless and may be extremely prejudicial to the search after truth. It cannot be denied, on the other hand, that the history of philosophy has frequently set forth the contrast between systems and the perpetual and useless going over the same ground again and again on the part of philosophers themselves. Still, we must discover if such a view has not been added on to history, both by the prejudices of a sceptical mind which likes to find an irremediable contradiction everywhere, and by the excesses of the dogmatic mind which, presupposing that philosophic truth is absolutely realized somewhere in a given system, delights in proving the impotence or the obscurity of everything outside this system. To do this, however, we must endeavor to form a clear idea of the history of philosophy, for this idea is itself of varying nature and does not readily reveal itself in its exact form. Let us examine a few modern works which, directly or indirectly, have either contributed to this revelation or have claimed to do so.

Strictly speaking, it is impossible to count among the works dealing with the history of philosophy such a work as Bayle's *Dictionnaire historique et critique* (Rotterdam, 1695-1697, 2 vols). Many of his articles, however, which treat of doctrines of the past, bear witness to far-reaching sources of information that is less well arranged than curious in its nature. Moreover, the restoration or the evocation of these doctrines is calculated to confound human reason and that along the lines of scepticism. Here we are rather dealing with a philosophical criticism of the doctrines handed down to us than with a historical criticism of the methods in which their transmission has come about. Directed along these lines, Bayle's intelligence does not always penetrate to the real essence of a system, far from it. None the less has this exposition of various doctrines resulted in a certain notion of them being spread abroad and popularized, while a stimulus has been given to the spirit of research in the domain of the history of philosophy.

Before assuming a form capable of combining the philosophical interest of the object with the historical requirements of research, the history of philosophy has again in modern times acquired characteristics which have kept it more or less aloof from the one or the other of these two conditions. *The History of Philosophy*, by Thomas Stanley, published in London in 1655, 2d ed. 1687,—translated into Latin, Leipsic, 1711,—is no more than a pragmatic and narrative history; it follows to a considerable extent the same lines as the work of Diogenes Laërtius; moreover, it is concerned only with the philosophy previous to Christianity, on the ground that, "Christian theology being the receptacle of truth, there is no longer any reason why philosophy should seek it."—Brucker's works certainly indicate an advance on this method of procedure; his principal work, *Historia critica philosophiae, a mundo incunabulis ad nostram usque aetatem deducta,* 5 vols. (Leipsic, 1742-1744), is not only very erudite and unambiguous, it is also largely critical. The doctrines are stated faithfully, though the idea of their concatenation and their relative importance is lacking. Convinced that truth has its home in Protestant orthodoxy and in the philosophy of Leibniz, Brucker judges doctrines by this standard, and occasionally almost misjudges the causes that have produced discordant systems—causes that have a deeper origin than human perversity. Truth being single, error is multiple,—and the history of philosophy, by disclosing the multiplicity of systems, shows *infinita falsae philosophiae exempla.*—Brucker confuses the history of philosophy in its origin with that of religions, mythologies and poetry. Here, no doubt, he was right in thinking that the origins of philosophic thought raised the problem of its relation to those forms of belief which involved ideas on the world; but in those days he had no means of stating clearly and attacking effectively—let alone solving—this kind of problem. At all events, even his formal statement of purely philosophical doctrines still resembles too closely the pragmatic statement of Diogenes Laërtius.

There is more coherence and organization in Tiedemann's work: *Geist der speculativen Philosophie* (7 vols., Marburg, 1791-1797). This is an exposé of the doctrines of theoretic philosophy from Thales down to Berkeley, an exposé which aims at stricter impartiality, and frequently, if not always, succeeds in entering profoundly into the meaning of the doctrines. Tiedemann possesses a relatively objective criterion for the examination of systems. He believes that it would be arbitrary to gauge their importance by the truth of any particular one regarded as absolutely certain; above all, he would try to find out if a philosopher has contributed something new, if he bases his affirmations on solid reasons, if he is able to connect his thoughts mentally and ensure their mutual agreement, and what difficulties could be brought up against him. Tiedemann is one of the opponents of the new philosophy, at that time the Kantian; for his part, he holds to conceptions which combine the philosophy of Leibniz and Wolff with that of Locke.

It is not surprising however that the appearance and the predominance of the Kantian philosophy tended to make Kantism a guide alike in the exposition and the examination of the doctrines,—especially since Kantism claims to solve by critical idealism the conflicts of reason, the manifest origin of the conflicts between systems. This tendency we find in Buhle, a Kantian along the lines of Jacobi,—though he is somewhat cautious and not too manifestly prejudiced in favor of historic truth. His *Lehrbuch der Geschichte der Philosophie und einer kritischen Litteratur derselben* (8 vols., Göttingen, 1796-1804); and his *Geschichte der neueren Philosophie* (6 vols., Göttingen, 1800-1805), are mainly valuable by reason of the bibliographies they contain. The *Geschichte der neueren Philosophie* also contains important extracts from rare works.

Faith in the truth of Kantism is more pronounced in the work of Tennemann.

In the years 1798 to 1819 Wilhelm Gottlieb Tennemann published his great history of philosophy: *Geschichte der Philosophie,* in eleven volumes; it was meant to be completed in thirteen volumes. This work has certain good points: a careful and occasionally critical investigation into origins, great clarity of exposition, considerable wealth of information and numerous references. Its defect is that it judges doctrines too much in their relation to Kantism; all the same, its conception of the evolution of philosophic doctrines is one that removes from them the contingent character of being successive and disconnected opinions. Its object is to set forth the efforts of reason to realize the idea of the science of the ultimate laws and principles of nature and liberty. This conception of a progressive development of reason in its strivings toward science was also held by Kant, and, in passing, it is curious to note that Kant had the idea of a rational history of philosophy; one that differed from empiric history in the fact that, instead of noting the succession of the doctrines purely and simply, it must explain their sequence by the progressive evolution of reason itself.—This quasi-Hegelian conception of the history of philosophy was not developed by Kant in his works: traces of it are found among his notes (Reicke, *Lose Blätter,* II, p. 277 etc.; 285 etc.) The main points of Tennemann's great work are included in his manual: *Grundriss der Geschichte der Philosophie für den akademischen Unterricht* (1st edition, Leipsic, 1812, 5th edition, Leipsic, 1829). This manual, of which Cousin published a French translation (2d edition, 1839) supplies important bibliographical information.

Though in Germany the authority gained by the philosophy of Leibniz and Wolff, and later on by that of Kant, might render somewhat partial the study of the doctrines of the past, still, the speculative character of these two philosophies predisposed one to feelings of sympathy for the various historical manifestations of philosophic thought; —whereas in France, the mind, less inclined to speculation, evoked the doctrines of a more or less distant past only to bring out their inadequacy or vanity. In France, the spirit of the eighteenth century is a struggle against the philosophic structures of the preceding century, against Cartesianism and its offshoots. The historical element of philosophy in the works of that time affects a polemical character. In his *Traité des systèmes* Condillac deals thus with Malebranche, Spinoza and Leibniz, though he does it mainly to prove that their systems, based on abstract principles, set forth as certain, propositions that are arbitrary, vague and unintelligible—a testimony to the error which consists in thinking that abstract formulas are capable of affording determinate knowledge.—Nevertheless, it is to Degérando, a writer belonging to the ideological school, that the merit of attempting a general history of philosophy in the beginning of the nineteenth century is due, though Degérando had been influenced in this direction less by the tendencies of the school to which he belonged than by an acquaintance with the German works of this class which he had acquired.

The comparative history of the systems of philosophy, first published in three volumes by Degérando (1804), re-edited in four volumes in 1822-1823, and translated into German by Tennemann (1806-1807), attempted to add to the historical exposition of the systems a critical analysis of the cause from which these systems are derived. This historical exposition sets forth, as a center for all historical ventures, the problem of the universal principles of all knowledge, interpreted largely after the ideologists; critical analysis adopts as its final object, by comparing the essential data of systems with their consequences, an inquiry into the system which is the best in itself. This system, regarded by Degérando as the philosophy of experience,—experience that is complete, both interior and exterior, and interpreted by the mind which only refuses to supply *a priori* knowledge, of itself,—in turn enables us to recognize the relative truth of other systems: prior to Cousin, Degérando would indeed appear to have admitted the necessity and importance of eclecticism.—Nevertheless, insight and vigor are too frequently wanting in this history, which deliberately gives the doctrines a certain meaning; the very concatenation of the doctrines is here but partially grasped.

By reintroducing as a law of the mind an idea which Wolffianism had rather unfortunately neglected in Leibniz, the idea of development, of *Entwickelung,* post-Kantian German speculation supplied a concept capable of giving a meaning to the sequence of systems. Schleiermacher is one of those who approached the history of philosophy under the influence of this concept, more or less strictly applied; but we are mainly indebted to Schleiermacher not only for a profound sense of that which, in the history of ideas, links past to present and gives it a renewal of life, but also for original and suggestive views on ancient philosophy, principally set forth in various articles and in the commentaries that accompany his translation of Plato. Schleiermacher greatly influenced H. Ritter, the author of a history of philosophy, *Geschichte der Philosophie,* in twelve volumes, ranging from the most ancient times to the period of Kant (1829-1853). Ritter looks upon philosophy as a whole which continues to develop; he refuses, however, to consider preceding doctrines as moments of the doctrine which replaces them in time; he expressly opposes all methods of dealing with the history of philosophy by *a priori* construction; hence he would withdraw from the influence of the man who, and the doctrine which, in spite of the criticisms which may be launched against them, have contributed most to attract men to the history of philosophy and to enable them to see how interesting it is: Hegel and the Hegelian doctrine.

To Hegel undoubtedly is due the credit for introducing a conception of the history of philosophy which, while subject to reserve and criticism (for the spirit in which the history of philosophy becomes with him a philosophy of history, and, more than that, a philosophy of becoming—a philosophy set up once for all as an absolute and itself setting up becoming as an absolute—might easily corrupt or do away with the historical sense itself), has at least claimed to show forth the compatibility or even the profound agreement of philosophy with its history and has reconciled the historic with the philosophic spirit. In a general way, it is known that Hegel regarded philosophy as the science of the absolute in the form of a dialectic development of thought proceeding by way of thesis, antithesis and synthesis from the most indeterminate and abstract to the richest and most concrete concepts: the dialectic method reproduces in the consciousness of the thinking subject the sense of reality itself. In these conditions, philosophy is amenable to historical understanding, provided that history itself is not simply a description of unconnected events but an intelligent and regular concatenation. The usual idea of the history of philosophy, however, is far removed from such a conception: what is offered under this name, or rather, the idea we form, is a disorderly succession of opinions that are often strange, a veritable museum of extravagant ideas: and what could be more futile than the mere knowledge of a series of opinions? What curiosity is it capable of satisfying, apart from that pedantic curiosity which indeed clings to the futile? Let me add that this succession of conflicting opinions only strengthens all the prejudices which the superficial mind so readily welcomes against the possibility of philosophy: every effort to introduce philosophic truth into the world is met by Pilate's ironical question: What is truth?

It is the idea of development that enables us to reject this superficial view of the history of philosophy. All development is the realization of a potentiality. That which is potential in a subject passes into action as the result of development. Thus, one and the same subject passes through a number of states and degrees: this does not prevent it from being essentially one and the same subject. In the present instance, the subject that is the one and the same is philosophy; whereas the various historical philosophies are states and degrees in its realization. The fundamental theme is the knowledge of pure thought for itself; the successive and progressive variations of this theme—the various philosophies—correspond each of them to a determination of thought which, *per se,* is necessary, immutable and eternal; each of these determinations appears in process of time as the principle of a doctrine; indeed, the doctrine is born and dies in time, like everything that obeys the law of time; the motion, however, which serves it as a principle, is immutable and indestructible; it is a necessary moment in the dialectic development of thought. In other words, the many succeeding symptoms are the chronological manifestations of a dialectic order of development which is eternal in itself; it is the temporal forms that the categories of thought assume. And just as the concepts by means of which human thought attains to higher levels do not altogether abolish the logically anterior concepts, but include as well as transcend them, so do the systems which replace the other systems retain of these latter, dominating and explaining it the while, the essential element which had been their *raison d'être.* Consequently, all philosophy exists necessarily; no philosophy has wholly died; they all actually exist in genuine philosophy, as moments of a whole (see Hegel, *Vorlesungen über die Geschichte der Philosophie*, 1st vol., 13th vol. of the complete works, pp. 19-64).

The chief interest of this conception of Hegel's is that instead of contrasting philosophy with its history in such a way as to make this latter inoperative or even of a paralyzing effect on present philosophic thought, it makes philosophy—the philosophy which tends most to pursue the absolute—one with its history. It also makes of the sequence of the doctrines, not a contingent succession of episodes and opinions, but rather the expression of a continuous and regular effort to reach truth, through all its contradictory forms. When we agree that the knowledge of truth is subject to a law of evolution, we cannot set up as an argument against it the evolutionary development it has had to carry out, any more than we can regard this development as meaningless. In the sequence of doctrines, then, we find a reasonableness which enables us largely to recognize the reason of to-day. We may imagine that this conception of the history of philosophy, as set forth by the genius of Hegel, has attracted many a mind to this class of study; indeed, many of the great historians of philosophy produced by nineteenth-century Germany, such as Ed. Erdmann, Ed. Zeller, Kuno Fischer, while more or less repudiating, along the lines of research, the too constructive and too unanalytical methods of Hegel, all the same retained a great deal of his spirit.

The thing of course that is most arbitrary to us in Hegel's conception, is that it connects too closely the meaning of the history of philosophy as a whole with the triumph of the Hegelian philosophy. It must however be noted that a certain interpretation of this philosophy, while not altogether indisputable, at all events fairly natural, would do away with all future evolution of the philosophy that would supply it with a history. Being the whole of truth and completely expressed, Hegel's doctrine would appear to leave to the dialectic order of concepts no other manifestations to produce in time.—Even if we remove from the Hegelian doctrine its most dogmatic content, it would still appear very arbitrary to interpret the march of systems in accordance with the law of a dialectic progress whose moments are predetermined. While it sometimes happens that we can bring some particular succession of doctrines within the scope of this very general scheme, it is only on condition we give this scheme no more importance than that of being a frame whose relation to the picture it encircles is merely one of symmetry or external proportion.

The main question, however, is to discover whether the sequence and the filiation of doctrines in time can be determined for us essentially by logical necessities. Now, if we admit that, throughout the successive doctrines, there is a certain regular development of philosophic thought and human intellect, and if we also admit that a new doctrine is linked on to those that have preceded it by relations which may be represented logically,—relations from principle to consequence, relations of opposition and of conciliation,—it does not therefore result that the transition from the prior doctrines to the following ones comes about in historic reality that can be apprehended by a law which imposes these relations *a priori*. In the way in which a new doctrine is built up, there are many factors supplied by the personality of the philosopher,—his own distinctive methods of formation, reflection and spiritual invention,—as well as by social traditions and renewals, sentimental aspirations and scientific requirements. Even if a kind of universal mind, advancing gradually in the direction of truth, were operating in all these particularities and contingencies; it would none the less follow that it is in these particularities and contingencies, which are offered us at the outset, that we are able to understand something of them. At all events, we have no right to infer in what way the doctrines succeed one another; we ought mainly to attempt to determine a host of circumstances, irreducible to concepts, which guide this succession. The method of *a priori* construction in the history of philosophy must be rejected, from the historical point of view at least. Usually, when more or less consciously practised, this method is but the expression of a philosopher's thought projected into history, in order to direct its course.

We must apply the same reserve as regards attempts which, although mitigating the Hegelian method or even opposing it, make use of certain general determinations in planning the history of philosophy or distributing the doctrines. Having received in it the thought of Hegel,—without fully understanding it,—at all events accepting it only in order to modify and pervert it, Victor Cousin tried to prove that the multitude of systems can be quickly resolved into a few principal systems which, through their relations and combinations, are the essential and lasting factors of the entire historic development of philosophy. These systems, each of which is connected with no more than a part of the total reality to exalt it into a whole, both in type and in principle, are sensualism, idealism, scepticism and mysticism. Sensualism firmly believes in the authority of the senses and in the existence of matter; but it believes in nothing else. Idealism firmly believes in the existence of the mind and in the authority of the ideas belonging to it; but it believes in nothing else. While the inadequacy of sensualism brings about as a natural reaction the appearance of idealism, these two dogmatisms, by opposing each other, cast reflection into a state of uncertainty and cause it to proclaim the vanity of all scientific investigation: hence scepticism. Scepticism in its turn, unable to satisfy the need to believe, awakens in the mind confidence in spontaneous and irrational inspiration: hence mysticism. These are the four great systems to be found at the root of all the historical developments of philosophy: naturally they combine and blend together; still, these are the true factors, alike vouched for by an investigation into the progress of reflection and by the study of history. And Cousin, with certain reservations, is inclined to think that they succeed one another in this order, for the mind investigates things of the senses before it investigates ideas; the contrast of the two dogmatisms is needed to lead to scepticism, just as lassitude regarding scepticism is needed to make mysticism into a doctrine. (*Histoire générale de la philosophie,* 1st lesson.)

In these considerations there is much that is vague and arbitrary: it would not be possible to include the history of philosophy in this law of the generation of the four systems except by very indeterminate definitions and artificial suppressions. Above all, at the origin of these remarks there is a very incorrect conception, the belief that the systems proceed from a kind of general element; we may call Epicurus, Locke and Condillac sensualists, and this may be regarded as true enough, though it overlooks the effort by which this element has been specified: now, it is specification that causes the interest, originality and potency of the doctrine.—There is nothing more vague than the term idealism, it may be used to include very different and even opposite doctrines.—On the other hand, while it may be said that the development of a certain intellectual tendency, a development carried more or less to extremes, causes the appearance of a contrary tendency, this is but a very simple scheme which affords us no information whatsoever as to the manner in which it assumes a concrete form.

In contrast with the Hegelian and eclectic spirit, Ch.

Renouvier in his *Esquisse d'une classification* and his *Dilemmes de la métaphysique pure,* has set forth a general view of the history of philosophy in the form of dilemmas dealing with various subjects: the dilemma of the unconditioned or of the conditioned, of substance or of law, of the infinite or of the finite, of determinism or of freedom, of things or of persons. These dilemmas call for an exclusive option, in contradistinction from the Hegelian antinomies which call for a reconciliation, and the series of the terms which are on one and the same side,—the first in the present instance,—must be rejected to afford room for the acceptance of the other series. This method of subjecting the whole of the systems to a dichotomic method may be interesting philosophically; still, it gives us no idea of historic truth. A doctrine like that of Leibniz, for instance, includes arguments which depend on the contrasted parts of the dilemmas: and while this is a striking instance, it is far from being the only one. The eclectic method, by preparing us to understand the comparing of ideas and their fusion, ideas that are at the outset heterogeneous or incongruous, is probably more favorable than this dichotomic method for studies in the history of philosophy.

By setting forth and criticizing some of the principal attempts by means of which we have tried to fix the objects and methods of the history of philosophy, it has been our sole object to show that the practice of the history of philosophy may not be so easy a matter, since an exact and definite idea of it is so slow and difficult to reach.

VICTOR DELBOS.

PARIS, FRANCE.

PRAYER.

ITS ORIGIN, MEANING AND ETHICAL SIGNIFICANCE.

IT MAY be said that the time has passed when the study of religion and of that religious feeling which is the "essential basis of conduct"[1] could be claimed as the exclusive product of a single body of men. With the growth of the science of comparative religion, and with the great importance now attached to the study of religious phenomena by psychologists and ethnologists, it is to anthropology that one must turn if religious values are to be fully understood. What is most remarkable is the fact that while on the one hand we have many Christian churches deploring the falling off in numbers of their communicants together with the universal apathy displayed by the laity at large to all matters of a religious character, we should have on the other hand, and as a result of recent scientific investigation, a value and a significance attached to the religious instinct which promises to be pregnant with future possibilities. If it were necessary to indicate, by one fact more than another, how great this interest is, one might point to that valuable and monumental work, now in course of publication, which deals with all the main factors of religious life and culture—with its mythology and its history, its superstitions and its ethics, its philosophy and psychology,[2] for "it is safe to say that there is no

[1] Thomas Henry Huxley.

[2] *The Encyclopædia of Religion and Ethics,* Edinburgh, edited by Dr. Hastings, M.A., F.R.A.I., and Dr. Selbie, M.A.

subject of modern research which concerns all classes as nearly as the study of religions."[3]

Until recent years it was held for the most part that barbaric and uncivilized man possessed little of the sentiment and feeling which we associate with the term "religion." He was given credit for the practice of hideous superstitions and of rites of the most abominable kind, but it was explicitly denied that he possessed religious feeling in any higher form.[4] Even authorities like the late Lord Avebury held that prayer itself, being to us a necessary part of religion, was quite independent of the lower forms of religion.[5] We know now that, not only is religion a matter of vital importance in the every-day life of the savage, being interwoven with all his habits, customs and mode of thought,[6] but that the practice of prayer itself is found to exist among some of the most savage races known to us. Even certain savage customs, barbarous and cruel as we may deem them, when traced to their fountain head are found to have arisen from the most pious motives and are carried into effect through the most earnest conditions.[7] What adds a deep significance to the value of the religious impulse is the undoubted fact that wherever and whenever a religion has been brought into ridicule and contempt, physical and moral decrepitude have followed as a fixed and a natural consequence. Having for my part paid no inconsiderable attention for some years past, to the effect of outside or alien influences upon the character of civilized and uncivilized man in various parts of the globe, it would be a most difficult task for me to name any race or tribe whose morale has not undergone serious degeneration

[3] See Committee on Publication, Brinton's *Lectures on the Religions of Primitive Peoples*, New York, 1897.

[4] Dr. Brinton, *ibid.*, pp. 30-31, referring to Lubbock and H. Spencer.

[5] *Origin of Civilization*, 6th ed., 1911, p. 402.

[6] See Ellis, *Tshi-Speaking Peoples of the Gold Coast*, 1887, p. 9.

[7] Ellis, *ibid.*, p. 9.

when once its ancient ritual and its religion have been brought into contumely. This being granted, the paramount importance of religion may be considered to be almost beyond discussion.

Writing some years ago, the late Auguste Sabatier, formerly Dean of the faculty of Protestant theology, Paris, declared that nothing better reveals the worth and moral dignity of a religion than the kind of prayer it puts into the lips of its adherents;[8] a truism which we shall find to be as applicable to the most primitive, as it is to the highest forms of religious development.

Many prayers have been recorded in recent years from savage races. An examination of these petitions shows that, in the great majority of cases, it is for material prosperity and gain that the savage prays. He asks that his crops may prosper, that he himself may be freed of danger, that no disease may befall his cattle or that they may not die.

Thus the Egbos, a tribe living in the depths of the bush in Southern Nigeria, pray to the sun and say:

"Sun of morning, sun of evening, let me be freed from danger to-day."[9] In another instance the prayer is to Obassi—a kind of ancestor god—"Obassi, everything was made by you; you made earth and heaven; without you nothing was made, everything comes from you."[10]

The natives of Brass, in the Niger Delta, before eating and drinking, present a little food and liquid to the household deity, and then offer the following prayer:

"Preserve our lives, O Spirit Father who has gone before and make thy house fruitful, so that we thy children shall increase, multiply, and so grow rich and powerful."[11]

[8] *Philosophy of Religion Based on Psychology and History,* 1897, p. 109.

[9] P. Amaury Talbot, *In the Shadow of the Bush,* 1912, p. 21.

[10] Talbot, *ibid.,* p. 66.

[11] A. G. Leonard, *The Lower Niger and its Tribes,* 1906, p. 292.

Writing of the New Caledonians, Dr. J. G. Frazer says: "If only wrestling in prayer could satisfy the wants of man, few people should be better provided with all the necessities and comforts of life than the New Caledonians."[12]

The Todas, a pastoral tribe inhabiting the Nilgiri plateau, offer prayer continually in their daily life. Dr. W. H. R. Rivers, tells us that these prayers are in the form of supplications to invoke the aid of the gods to protect their buffaloes. "May it be well with the buffaloes, may they not suffer from disease or die, may they be kept from poisonous plants, and from wild beasts, and from injury by flood or fire, may there be water and grass in plenty."[13]

To take another example from the Dark Continent, we find that the Bawenda, a Bantu tribe living in the northeastern portion of the Transvaal, offer the following appeal during their annual sacrifices at the graves of their ancestors:

"O Modzimo, Thou art our father; we, Thy children, have congregated here, we humbly beg to inform Thee that a new year has commenced. Thou art our God; Thou art our Creator; Thou art our Keeper; we pray Thee give us food for us and our children; give us cattle; give us happiness, preserve us from illness, pestilence and war."[14]

While this feature, the desire for material gain, is a predominant one in all primitive ritual, it is hardly necessary for us to be reminded that it is also a dominant characteristic of all the higher religions. The great difference between the creed of the savage and the creed of the higher races is this, that while among the former it is material gain that is chiefly sought, among the latter the material

[12] *The Belief in Immortality,* Vol. I, p. 332, 1913.

[13] *The Todas,* 1906, p. 216.

[14] Rev. E. Gottschling in *Journal of the Anthropological Institute,* 1905, Vol. 35, p. 380.

factor has become, as it were, spiritualized, as we shall see when we come to examine the liturgy of the higher races.

Nevertheless, an ethical element is present in many prayers offered by races whom we, in common parlance, classify as "savage." Thus the Sioux of North America say:

"O my Grandfather the Earth, I ask that Thou givest me a long life and strength of body. When I go to war let me capture many horses and kill many enemies, *but in peace let not anger enter my heart.*"[15]

It will scarcely be denied that in the portion of this prayer italicized we have the appearance of an ethical element which is absent from the supplications taken from a lower stage of culture. Indeed, it may be said that, with a few verbal alterations this Sioux prayer might well stand side by side with many of those which still find utterance in the congregations of Christendom. And if it be thought that the ethical element in this prayer be an exception, surely the following incident would serve to dispel it.

At Fort Yates, overlooking the Missouri River, there exists at this moment a remarkable petrification in the shape of a woman with a child on her back, very life-like in appearance and which is venerated by the red Indians as a sacred relic. This figure was brought to the Indian Agency and set up in its present position at the suggestion of Mr. James McLaughlin, formerly Indian agent to the Sioux. A great council of Indians was held, at which it was agreed that the unveiling of the image should be performed by some Indian who could truly claim possession of all the Indian virtues. A warrior named Fire Cloud was selected. On the day of the ceremony, Fire Cloud, addressing the Great Spirit, prayed for peace, hoping that the erection of the monument would establish a lasting peace in all the land, not only between the Indians and

[15] Capt. Clark quoted by Dr. Brinton. *Ibid.*, p. 106.

the white men, but between the Indians themselves. He prayed that the Great Spirit would bless the rock and the place, so that they might be regarded as a pledge of the eternal cessation of warfare. Then, turning to his brother Indians assembled, he charged them that it was *their* duty to observe the laws of the Great Spirit, and that those among them who had not clean hearts and hands should stand abashed and humiliated in the presence of the woman of the Standing Rock and the Great Spirit. He then and there called upon them to repent and devote themselves to lead clean and pure lives in the future.[16]

During one of their ceremonies for initiation into the mysteries of manhood, the youth of the Omaha (a Sioux tribe) prays to Wako—the great permeating life of visible nature, itself invisible, but which reaches everything and everywhere. Standing alone in a solitary place, with clay upon his head and tears falling from his eyes, he, with hands uplifted, supplicates the Great Spirit to aid him in his need.[17]

These instances in themselves may perhaps suffice to show how important a place prayer does occupy in the mind of so-called savage and uncivilized man.

Let us now turn to the ancient civilized peoples of the Old World. A great number of prayers and invocations have come down to us from ancient Babylonia; many of them being exquisite invocations put into the mouth of worshipers, expressive of their deep sense of moral quiet, yet ending as Dr. Jastrow points out, in a dribble of incantations which had survived from a more archaic period.[18]

The prayers of the ancient Egyptians are familiar to most of us. Wake quotes from Bunsen the following

[16] James McLaughlin, *My Friend the Indian,* 1910, pp. 36-39.

[17] See 27th *Annual Report, Bureau of American Ethnology,* Washington, 1901, by Alice Fletcher and Francis la Flesche (the latter a member of the Omaha tribe).

[18] *The Study of Religion,* 1901, p. 213.

which shows how great has been the growth of the moral element in what had originally been nothing more than a magical formula:

"Oh, thou great God, Lord of Truth, I have come to thee, my Lord, I have brought myself to see thy blessings, I have known thee——I have brought ye truth. Rub ye away my faults. I have not told falsehoods in the Tribunal of Truth. I have had no acquaintance with evil."[19]

Turning to ancient Persia, we find in the Gāthas or Sacred Chants attributed to Zoroaster and forming part of the Yashna, the great liturgical book of the Avesta, many prayers of a high and lofty character. These chants are concerned with the nature and attributes of Ahura-Mazda, the Great Living Lord, the Most Wise. The first chant has been described by one of its translators—Canon Cook—as a perfect example of intercessory prayer, in which Ahura-Mazda is addressed as the Supreme Deity, and before whom Zoroaster stands as his prophet. Too long to quote here, it begins and ends with prayer and praise to the Lord of the Universe, but the following lines will give a faint idea of its import:

"With hands in prayer uplifted
  To Mazda, the quickening Spirit,
I fain would give due honor
  To all who, by good works, win favor
From Him, the Good, the Holy.

"The just, whom thou approvest—
  Righteous and pure in spirit,
Do thou, O mighty Ormuzd
  With thine own mouth instruct from Heaven!
Teach me thy words of power
  By which creation first was fashioned!"[20]

In another chant Zoroaster presents himself—body and soul—intellectual faculties, moral and spiritual—as an ob-

[19] Bunsen, *Egypt*, Vol. IV, pp. 644-5, quoted by Wake, *Evolution of Morality*, 1887, Vol. 2, p. 132.

[20] F. C. Cook, *Origins of Religion and Language*, 1884, pp. 212-216.

lation to the Supreme Being. Canon Cook considers this particular chant to approach more closely than any other Gentile teaching the Christian idea of worship as set forth in the New Testament.[21] We quote the following lines:

"Teach me to know the two laws,
By which I may walk in good conscience,
And worship thee, O Ormuzd,
With hymns of pious adoration."
* * *
"Oh, holy pure Armaiti,
Teach me the true law of purity."
* * *
"This offering Zoroaster,
The vital principle of his whole being,
Presents in pure devotion;
With every action done in holiness;
This above all professing—
Obedience to thy word with all its power."[22]

Zoroaster's noble moral code, epitomized as it has been in three short simple words, "Good thoughts, good words, good deeds,"[23] is well illustrated by this translation of those beautiful psalms.

Modern Persia, through its thirteenth-century poet, may lay claim to have given Christendom one of those great lessons which, as experience has so painfully shown, is so difficult for many of us to learn and to practise—the lesson of toleration. In that poem known as the Mathnavi, which has been described as being perhaps the greatest mystical poem of any age,[24] Jalal-al-Din gives us the following exposition of the doctrine of largemindedness. Moses once heard a shepherd praying: "O Lord, show me where thou art, that I may become thy servant. I will clean thy shoes and comb thy hair, and sew thy clothes, and fetch thee milk." When Moses heard him praying so senselessly

[21] *Ibid.*, p. 256.

[22] *Ibid.*, pp. 247-248.

[23] Art. "Zoroastrianism," *Encycl. Biblica*, 1907, Vol. 4, col. 5435.

[24] *Encycl. Religion and Ethics*, Vol. 7, p. 474.

he rebuked him and said: "O foolish one, though thy father was a Muslem, thou hast become an infidel! God is a Spirit and needs not such gross ministrations as in thy ignorance thou supposest." Abashed at this stern rebuke the shepherd rent his clothes and fled to the desert. Then from heaven a voice was heard saying: "O Moses, why hast thou driven away my servant? Thine office is to reconcile my people with me, not to drive them away, for I have given to men different ways and forms of praising and adoring me. I have no need of their praises, being exalted high above all such needs. I regard not the words which are spoken, but the heart that offers them."[25]

The religion of the Arabian prophet abounds with beautiful prayers and moral teaching of the highest order. Probably the best known prayer is the opening supplication of the Koran: "Praise be to God, the Lord of all creatures, the most merciful. Thee do we worship and of Thee do we beg assistance. Direct us in the right way, in the way of those to whom Thou hast been gracious, not of those against whom Thou art incensed, nor of those who go astray."

In other prayers it is declared that it is not the formal act of praying that justifies, but the doing of that which is held to be right and good. "It is not righteousness that ye turn your faces in prayer toward the east or the west; but righteousness is of him who believeth in God, who giveth money for God's sake unto his kindred, and unto orphans, and the needy, and the stranger....and of those who perform their covenants when they have covenanted, and who behave themselves patiently in hardship and adversity and in times of violence, these are they who are true."[26] In another prayer the petitioner says: "O Lord,

[25] Whinfield's translation, quoted in L. M. J. Garnett's *Mysticism and Magic in Turkey*, 1912, pp. 51-52.

[26] Syed Ameer Ali, *Islam*, 1909, p. 9.

I supplicate Thee for firmness in faith and direction toward rectitude,—I supplicate Thee for an innocent heart, which shall not incline to wickedness; and I supplicate Thee for a true tongue and for that virtue which Thou knowest."[27]

From Mohammedanism it is not unfitting to turn to Buddhism, from that great religious system of Arabia,—with its imageless adoration of Allah, the All-Powerful—to the religion of the Buddha, whose ethical system of philosophy is perhaps one of the the greatest the world has ever received, and whose image may be met with in thousands of shrines and temples in the Far East.

For four hundred years no greater contention has vexed Christendom than that of the use of images in religious worship. Yet, it may be seriously questioned, whether, after all, its true import and significance—its inwardness—has even been realized and understood; certainly not by those who are its chief opponents!

The study of the image ritual of uncultured races throws an unexpected light upon the attitude of those who profess a higher creed, but who still retain their images of wood and of stone. Not even the most barbaric of men believes that the image to which he prays and to which he makes his offering, is of itself a deity. It is to the spirit which enters the idol, as it were, that he makes his supplication. Thus, it can hardly be open to reasonable doubt but that such an attitude has been the precursor and the inaugurator of religion of a greater and a nobler type. Certain it is that not only in its lower manifestations, but in its higher ones as well the presence of an image, to those who believe in it, exerts a most powerful influence over its votaries, but that influence is, in the majority of instances, misunderstood by unsympathetic witnesses who may profess an alien creed.

Near Calcutta, in the little village of Bodh Gaya, there

[27] *Ibid.*, p. 8.

exists the temple of the Mahabodhi—"of the great enlightenment"—a spot sanctified and held to be the most holy on earth by some hundred and forty millions of the human race. That temple, recently repaired by the Indian government, contains a medieval statue of the Buddha.[28] What mystic influence that image must have upon the Buddhist worshiper, may be gathered from Moncure D. Conway's description of his own feelings, when he, the rationalist, paid a visit to that shrine during his "Pilgrimage to the Wise Men of the East." He says: "I feel as if I know something of Zoroaster and of Jesus, and these two are to me the men who knew the true religion. The real Buddha is more dim, but at Gaya the thought of that young prince, burdened with the sorrows and delusions of mankind, reached far down in me and touched some subconscious source of tears and love for the man, and I longed to clasp his knees."[29] Again, the Rev. John Hedley, a Protestant missionary, who visited a few years since the Pagoda of T'ai Ming T'a in Mongolia, tells us in glowing language of the emotions produced in his own mind when he beheld the standing figure of the Buddha erected in that "pagan temple." He says the image affected him strangely and profoundly, so much so that, at the risk of offending his sturdy nonconformist brethren, he calls it but simple truth to state that it would have been a comparatively easy thing for him to have knelt down before that image and pay homage to "One greater" than Buddha, of whose selfless life Buddha himself was so marvelous a forerunner.

"The sweet and gracious expression on that gentle face would have charmed an artist, inspired a poet, and captured the love of a devotee....Had this figure stood in some venerable cathedral of the Catholic faith in Europe, the

[28] Mitra Rajendralala, LL.D., *Buddha Gaya, the Hermitage of Sakya Muni,* Calcutta, 1878. *Encycl. Religion and Ethics,* Vol. 6, pp. 182-185.

[29] Conway, *My Pilgrimage,* 1906, p. 263.

most appropriate word to have written over it would have been the old familiar words of love and blessing, 'Come unto me, all ye that labor and are heavy laden, and I will give you rest.' I do not wonder now that some people find images and icons helpful to their faith. . . . For myself, it is not irreverent to say that though I bowed not my knee nor even momentarily inclined my head as I gazed on what in vulgar parlance we must call an idol, I realized my Lord more distinctly and drew nearer in spirit to Him."[30]

Surely it is time for us to pause, to rub our eyes, to ask ourselves whether we be in the twentieth century—with its coal and its iron, its corn and its pigs—or whether, after all, we are not back again in the old medieval times—with its saints and its sinners, its Madonnas and its suffering Christ? Once more the picture of Savonarola in his cell, with his crucifix before him, rises before us, as he re-pens the lines of that great prayer of his, known as the "Hymn to the Cross":

"Jesus! would my heart were burning
With more fervent love of Thee,
Would my eyes were ever turning
To Thy Cross of agony.

"Would that, on that cross suspended,
I the martyr-pangs might win,
Where the Lord, the Heaven-descended,
Sinless suffered for my sin!"[31]

Santa Teresa tells us, how, losing her mother at the tender age of twelve years, she went in her affliction to the image of Our Lady, and, with many tears, supplicated *her* to be her mother.[32] Upon another occasion, entering her oratory, her eyes by chance fell upon the image of the wounded Christ. "As I gazed on it, my whole being was stirred to see Him in such a state, for all He went through was well set forth; such was the sorrow I felt for having

[30] John Hedley, F.R.G.S., *Tramps in Dark Mongolia,* 1906, pp. 140-142.
[31] See G. S. Godkin, *The Monastery of San Marco,* 1901, pp. 67-68.
[32] *Santa Teresa* by Gabriela C. Graham, 1894, Vol. 1, p. 93.

repaid those wounds so ill, that my heart seemed rent in twain."[33]

Western civilization, with its immense and its intense material prosperity, has almost forgotten what it owes to the past. It may be that in some near future the infinity of that debt will be recognized and acknowledged. For, were one to search for the most beautiful examples of Christian prayer, which form such an essential feature of the Christian faith, it is to pre-Reformation times that one must turn. No greater battle has ever been waged over any book than over the Book of Common Prayer. Abhorred and hated by the early Puritans, denounced by them as being "full of abominations," and branded as "ridiculous and blasphemous,"[34] that book remains still unrivaled and unsurpassed in Christendom as a manual of true devotion. Yet nine-tenths of this book are no recent creation, but belong to the most ancient periods of Christian history. To certain Protestant historians is due the everlasting credit of indicating how vast our debt is. Milner says that the litanies which were collected by Gregory the First, in the sixth century, were not much different from those in use by the Church of England to-day.[35]

Perhaps the greatest eulogy of all has been pronounced by the Congregational historian, Dr. Stoughton. He says that, "as the sources whence the book was compiled are so numerous and so ancient, belonging to Christendom in the remotest times, as there is in it so little that is really original, so little that belongs to the reformed Episcopal Church of England, any more than to other churches constrained by conscience to separate from Rome—the bulk of what the book contains, including all that is most beautiful and noble—like hymns which, by whomsoever written,

[33] *Ibid.*, p. 142.

[34] See Hardwick, *History of the Christian Church*, "The Reformation," 2d ed., 1865, p. 260.

[35] *History of the Church of Christ*, Edinburgh, 1841, p. 414.

are sung in churches of every name—ought to be regarded as the rightful inheritance of any who believe in the essential unity of Christ's Catholic Church, and can sympathize in the devotions of a Chrysostom, a Hilary, and an Ambrose."[36]

In the Bishop's Book—known as the "Institution of a Christen Man" (Instruction of a Christian Man)— issued during the reign of Henry VIII, there is an exceedingly beautiful paraphrastic exposition of the Lord's Prayer, which may be considered a notable instance of that spiritualization of worldly desires to which allusion has already been made. The passage is too long for quotation in full, but we select the following which may prove sufficient to denote its character:

"Oh, our Heavenly Father, we beseech Thee give us this day our daily bread. Give us meat, drink and clothing for our bodies. Send us increase of corn, fruit and cattle. Give us health and strength, rest and peace, that we may lead a peaceful life in all godliness and honesty. . . . Give also Thy grace to us, that we have not too much solicitude and care for these transitory and unstable things, but that our hearts may be fixed in things which be eternal and in Thy Kingdom which is everlasting. . . . Give us grace, that we may be fed and nourished with all the life of Christ, that is to say, both His words and works; and that they may be to us an effectual example and spectacle of all virtues. Grant that all they that preach Thy word may profitably and godly preach Thee and Thy Son Jesus Christ through all the world; and that all we which hear Thy word preached may be so fed therewith that not only may we outwardly receive the same but also digest it within our hearts; and that it may so work and feed every part of us, that it may appear in all the acts and deeds of our life."[37]

[36] *History of Religion in England,* new ed., 1881, Vol. 3, p. 215.

[37] See J. H. Blunt, *The Reformation of the Church of England,* Vol. 1, 1868, pp. 448-449.

A passing reference at least must be made to the prayers contained in the Roman Catholic Service books,—of a church which has perhaps been more misunderstood and misrepresented than any other world-wide faith. From the prayers at mass we select the following, which show the high ethical standard of her creed at its best. "O Lord. . . . Have mercy upon all heretics, infidels, and sinners; bless and preserve all my enemies; and as I freely forgive them the injuries they have done or mean to do to me, so do Thou in Thy mercy forgive me my offenses." Or again, take the prayer where the penitent prays for a spiritual cleansing: "O Lord, who once didst vouchsafe to wash the feet of Thy disciples—wash us also, we beseech Thee, O Lord; and wash us again—not only our feet and hands, but our hearts, our desires and our souls, that we may be wholly innocent and pure."

Can Protestant Christendom present to us anything more touchingly beautiful than the following? At Puenta-del-Inca, between Argentina and Chili, and perched upon the highest pinnacle of the Great Andes, there is to be seen a colossal figure of Christo Redemtor—Christ the Redeemer. Cast from bronze cannon taken from the arsenal at Buenos Ayres, and erected to celebrate the establishment of peace between those two countries, it was bequeathed, not only to Argentina and to Chili, but to the whole world, that from that monument it might learn its lesson of universal peace. On its pedestal one may read: "Sooner shall these mountains crumble to dust than Argentineans and Chilians break the peace which at the feet of Christ they have sworn to maintain."

At the opening ceremony the Archbishop of Argentina, Monsignor Espinosa, offered the following prayer so inexpressibly beautiful that one cannot refrain from quoting it *in extenso*:

"Lord, when my voice is silent, when my eyes cannot behold Thee, and my heart, already changed to dust, disappears with the remembrance of my existence, Thine image, represented in eternal bronze, shall be a perpetual offering on the highest pinnacle of Argentina. When the white snows shall close the path to men, permit that my spirit may keep vigil at the foot of this mountain. Protect, Lord, our country. Ever give us faith and hope. Let our first inheritance be the peace which shall bear fruit and let its fine example be its greatest glory, so that the souls of those who have known Thee shall be able to bring forth from Thee all forms of blessing for the two Americas. Amen."[38]

This noble petition may well form a fitting close to our review of the invocations of civilized and of barbaric man. Having passed under examination the attitude both of civilized and uncultured man toward the Unseen, as illustrated to us by examples of his petitions and prayers, we are now in a position to form an estimate as to their moral value.

As we have said, the study of a religion can no longer be claimed as the exclusive business of the theologian or the divine. A new science has dawned—the science of mankind—and with it, that mantle which formerly rested upon the shoulders of its Elijah, has fallen upon those of the Son of Shaphat. Therefore, it is for science now to estimate religious values, to measure all moral worth, and it is not too much to say that her verdict will be in accordance with nature's laws. Like all her sister sciences, the science of ethnology recognizes law everywhere, no less in the prayers of man than in those starry realms far beyond his unaided ken.

Prof. Max Müller once declared that he who knows but one religion knows none. With equal truth it may be said that he who scorns the religion of others is not himself

[38] Percy F. Martin, F.R.G.S., *Through Five Republics,* 1905, pp. 358-359.

religious. The day of the scoffer, of him who jeered and held to contempt the faith of another, has passed away. Scientific men at least have too great a respect for nature herself to jibe and jeer at those things which, after all, they may not understand. All they do claim is that all knowledge and experience shall be subjected to the same method for investigation, whether it be the study of a piece of granite, or the interpretation of a prayer.

Just as the exposition of certain "spiritual phenomena" at the hands of Christian theologians is not necessarily in accordance with religion itself in its highest aspects, so the explanation of the phenomena of nature by scientific men is not necessarily "science" in itself. For example, some theologians tell us that the answer to prayer is a process of violation of natural law. "The general providence of God acts through what are called the laws of nature. By his particular providence, God interferes with these laws."[39] In opposition to this particular theologic doctrine, the student of nature holds that, so far as human experience is concerned, *all* phenomena—subjective and objective—*must* be interpreted in accordance with natural law. So far as his knowledge reaches, nature never discards her own laws, for if she *could* set them aside she would cease to be natural. Therefore, if the act of prayer possesses any value to man at all, it is from man himself, as part of nature, that one must obtain an answer. The appeal must be to the natural, not to the supernatural; it must be based upon human experience, not upon human supposition.

There is definite reason to believe, outside all supernatural explanation, that the art of prayer and the desires that prayer itself inculcated, is as necessary a part of the psychological evolution of man as any other process of nature.[40] In itself the act is an outcome of an ethical law

[39] See Hook, *Church Dictionary,* 6th ed., 1852, art. "Prayer."

[40] See (Sir) E. W. Brabrooks' "Anniversary Address," *Annual Address Anthropological Institute,* 1898. *J. A. I.,* Vol. 27.

of the highest order, and is only foolish and inconsistent when it becomes a mere jumble of impossible requests.

In its higher manifestations it creates in the mind of the supplicant moral feeling and desire of the highest character, exciting him to attain those spiritual ends of which his feelings are but the expression. As Lecky has so well put it: "The man who offers up his petitions with passionate earnestness, with unfaltering faith, and with a vivid realization of the presence of an Unseen Being, has arisen to a condition of mind which is itself eminently favorable both to his own happiness and to the expansion of his moral qualities."[41]

Man recognizes as a universal law that certain results follow certain acts—be they good or be they bad—as sure as night follows day. The naked savage knows instinctively as it were, that if his actions follow a certain course, certain ills may befall him. While the reason the savage gives in explanation may be a superstitious reason, and therefore no reason or explanation at all, still we cannot fail to discern a natural law which, whatever its origin in the native's mind may be, is nevertheless productive of ethical results. It is for this reason that uncontaminated primitive man is a moral man—as nature herself hath willed. He holds that calamity and disease, fire and flood, are punishments sent in some way or other because of wrong-doing. He believes that nature is angry with him, therefore by his acts he desires and attempts to appease her. While it is true that nature may not show her anger in the way that uncultured man thinks, there is more in this recognition than one might deem.

In a theological work published quite recently, it has been declared that "the scientific student knows nature is not angry and does not require appeasement."[42] As a mat-

[41] *History of European Morals,* 1894, Vol. 1, p. 36.

[42] "Concerning Prayer," art. by the Rev. Harold Anson, M.A., 1916, p. 83.

ter of mere fact, the "scientific student" knows nothing of the kind; rather he has reason to believe that nature *is* angry, angry because certain of her laws have been thrust aside, and that she has replaced them by other laws, not less natural, but which produce disease. "The sins of the fathers" and the results thereof, are no less a process of natural law than is the unconscious act of the falling apple a law of gravitation. Even the savage recognizes this, hence his abstention from doing certain acts which are prohibited to him by ancient custom.

For hundreds of years in Christian lands, it has been considered an incontrovertible truth that suffering and calamity are punishments sent by God. In the work just quoted—a work in which the lack of modern prayer is bewailed,—we are told that religion has contributed much to immorality by speaking of suffering and calamity as a judgment imposed by God upon sin, for God does not impose the consequence of evil.[43] This is a most remarkable pronouncement, a pronouncement which shows the position to which recent theologic thought has been driven. The old Hebrew prophet knew life better when he declared that God created the evil as well as the good.[44] Substitute the word "Nature" for "God" and we have the clearly defined position of the man of science. But while we are content to leave to the theologian the interpretation of the mind and the acts of God, so far as modern science is concerned, there can be no possible doubt but that suffering and calamity *are* in many cases imposed upon man by nature, as a consequence of ill-doing.

When a man prays, he asks to be taken by the hand and led away from destruction, so that he may prosper and the right prevail. Modern psychology has shown that the creation of ideals in the human mind leads by a natural

[43] *Ibid.*, art. by Arthur C. Turner, M.A., p. 428.

[44] Isaiah, xlv. 7.

process to the desire to attain those ideals.[45] Prayer feeds that desire and so leads to their ultimate attainment.

We have pointed out the fundamental difference that exists between the prayer of great religions like Christianity and Islam, and the prayer of some of the lower races of mankind. While the former supplicants pray that they may possess all the great moral qualities, and that their life and character may be moulded so as to produce the noblest and the highest result, the latter ask, in the majority of instances, for those things which add to their material well-being. By examples we have shown that, though the material factor is constantly present in the higher religions, still it is spiritualized in the highest possible way.

Mankind at large has many lessons yet to learn; not the least of these is the serious recognition of that law of nature which goes under the name of "evolution."

Among all "civilized" peoples, there is a growing tendency to forsake that narrow path their forefathers trod, and to divert their course to that broad way which, as we were formerly taught, leadeth to destruction. To-day science can only emphasize this truth our forebears taught us.

Looking around we find man bent upon destruction—everywhere—waging iconoclastic wars of all descriptions. He topples over old idols—some of them foolish ones maybe—and erects in their place idols more hideous than existed before. He destroys that which the past itself held to be bad with that which the past knew to be good. He attempts to substitute the "gospel of hatred" for the "gospel of peace and good will" as a "new way to righteousness."[46] He flings "overboard law, religion and author-

[45] See Ribot, *Psychology of the Emotions,* 2d ed., 1911.

[46] "We preach the Gospel of Hatred, because in the circumstances it seems the only righteous thing we can preach." Leatham quoted by Sir William E. Cooper, C.I.E., *Socialism and its Perils,* 1908, pp. 33-302.

ity,"[47] to give us in place thereof a society where atheism and anarchy are supreme, and where the family exists no more![48] Man is thus attempting to divert nature's course to lead her into paths of his own devising; nevertheless, whatever theologians may now teach, it will be with nature herself that man will have to reckon and whose bill he will have to pay upon her just demand.

The pronounced evils of our day—envy and hatred, malice and greed, no less than war and pestilence—have ever been the result of evil-thinking and evil-speaking; our forefathers were not so far wrong after all when they held that these were punishments, and that war followed in their trail. Were an analysis to be attempted as to the origin of many great wars, it would be found that they were brought about by the greed of man and by the desire to obtain that to which the offender had no right. The story would be that of Naboth's vineyard over and over again. It is from disasters such as these that it is the duty of the Christian to pray, so that his desire may become the father of acts which will frustrate those ends to which his greed would otherwise lead.

There are other great evils beside those of war and of greed. He who manifests ridicule and attempts to bring into contempt those beliefs held sacred by others, has his own lesson to learn. Toleration is the one great virtue which the West may well learn from the East. Even the naked savage never ridicules the religious beliefs of his

[47] Prince Krapotkin, quoted by G. W. Tunzelmann, *The Superstition Called Socialism,* 1911, p. 108.

[48] Congress held in London, July 14-19, 1881. "Resolved—that all revolutionaries be united into an International Revolutionary Association, to affect a social revolution, money to be collected to purchase poison and weapons, ministers of state, the nobility, clergy and capitalists to be annihilated." See E. V. Zenker, *Anarchism,* transl. from the German (1898, p. 231).

"In the new moral world, the irrational names of husband and wife, parent and child, will be heard no more." Robert Owen, quoted by Sir W. E. Cooper, *loc. cit.,* p. 41.

It has been stated that a large number of Labor M.P.'s have been or are local preachers of anarchism. See Peter Latouche's *Methods and Aims of Anarchism,* 1908, p. 14.

fellows; it is a besetting sin, not of savage, but of Christian lands.

To live aright, man must conserve, not destroy. He must once again learn to "leave undone those things which he ought not to have done," and "do those things which he ought to have done," for Nature herself insists.

Were modern science asked for one final word, surely it would be this: If to pray means to create and nourish in our minds those thoughts and aspirations whereby we may live a "righteous and sober life" and not follow the "devices and desires of our own hearts," then she would say—"PRAY WITHOUT CEASING."

Pray that our actions may be so shaped that they conform to Nature's will: that she may be our protector, not our avenger; pray that all erroneous teachings—those superstitions of to-day which arouse the passions of the hustings—MAY CEASE!

To the Christian especially she would say—Pray ye in the spirit and in like manner of that old Catholic saint who told you that,

"YOU were made Christians to this end, that you may always do the works of Christ; that is, that you love chastity, avoid lewdness and drunkenness, maintain humility and detest pride, because our Lord Christ both showed humility by example and taught it by forwards, saying, 'Learn of Me, for I am meek and lowly in heart, and ye shall find rest for your souls.' It is not enough for you to have received the name of Christians if you do not do Christian works, for a Christian is he who does not hate anybody, but loves all men as himself, who does not render evil to his enemies, but rather prays for them; who does not stir up strife, but restores peace to those who are at variance."[49]

To those, whatever their creed may be, who are unable

[49] Homily of Caesarius, Bishop of Arles, attributed to St. Eligius, quoted by Dr. Maitland, *The Dark Ages*, 5th ed., 1890, pp. 134-139.

to share those thoughts which others revere, she would say: Let us not forget how very little our exact knowledge really is and remember that there may still be many more things than we wot of. Pray therefore that you may sympathize where you cannot understand; for what matters it if some tread a devious path, so long as nature wills?

Lastly, she would ask all mankind—with its divers antagonistic creeds, with its love and its hate, its war and its peace, its weal and its woe—to turn to that great figure in bronze, which tops the heights of the Volcanic Andes—that sublime symbol not of the peace that is, but of the peace that ought to be—and in the silence of those now quiescent rocks, say with Shelley:

> "Join then your hands and heart, and let the past
> Be as a grave, which gives not up its dead
> To evil thoughts."[50]

So that all storm and strife, and sobs and tears may cease, and a new era dawn, where Nirvana—that "peace which passeth all understanding"—shall reign, and where, once more,

> "'neath the sky
> All that is beautiful shall abide,
> All that is base shall die!"[51]

EDWARD LAWRENCE, F.R.A.S.

ESSEX, ENGLAND.

[50] *Revolt of Islam.*

[51] R. Buchanan, *Balder the Beautiful.*

RATIONALISM AND VOLUNTARISM.

IT may be of interest to consider some of the relative claims of rationalism and voluntarism, that real and explicit antithesis of recent times, whether we regard either theory in full or extreme form as satisfactory or not. Neither of them is, in fact, satisfactory in any absolute or exclusive sense. Their consideration is the more necessary as extreme forms of voluntarism are by no means rare in the thought of to-day. There is no need in doing so, to forget that, in every psychosis, there will be elements or rudiments of feeling, willing, and thinking, though one of these may have a dominating influence. Rationalism stands for thinking, as the great form or mode of realizing conscious content. That is to say, the essential activity of mental life is for it thought or ideation. Rationalism is concerned with logical priority rather than with the question of genesis, hence it here stands aside from psychology—though I do not mean to leave it untouched—which is concerned with genetic order. The logical priority of thought — thought-activity as the absolute *prius* of the world—is the maintenance of rationalism. For in no other way can you get the world as a world of meaning. Neither blind feeling nor blind will can yield such. But thought, standing by itself, does not suffice to create a world.

Pure thought needs the supplementing of will. That is the defect of rationalism. Will is not moved by reason alone, thought Hume, for he subjects reason to the feelings, as some still do. His stress on passion fails of justice to reason. A further defect or mistake of rationalism has been to undervalue the senses. But experience is too exigent for the tendency to neglect or underrate the senses to be wise. The part played by sense in experience-processes is too important to be overlooked without impoverishment. Thought can come to its own without countenancing this mistaken tendency. Thought, as we know it, never does exist severed or divorced from feeling and will. That is not to say that thought or reason may not have a dominance, a logical priority, a primacy of rank. That is quite another matter from time priority. The time primacy claimed for feeling by some phychologists is denied by rationalism in respect of any feeling-consciousness taken as pure or wholly without rudimentary representation, real however latent. Representation in some sort must be taken to precede feeling—feeling as accompanying sensation.

But, if we distinguish these two, I should take feeling as purely subjective, and sensation as carrying an objective reference or element. This, although certain German philosophers hold all sensations of subjective origin. The unity of sensation, for Rosmini, was intelligence. Not much help is vouchsafed by Höffding's rather vague definition of feeling as "an inner illumination which falls on the stream of sensations and ideas." Feeling is often regarded or treated only as it springs from the stimuli in sensational experience. Thought supervenes on such sensation; and in this usage of feeling, my next remark holds good of it; feeling wholly without presentation or idea must be valueless for action. That is not to deny the dominance of feeling that may exist in certain cases or stages. But that is not the case where reflection is developed, for there the idea or the presentational element is supreme. "In tal modo," says an Italian writer, "l'attività del sentire progredisce dall'interno all'esterno" (N. R. D'Alfonso, *Piccola Psicologia*, Rome, 1917, p. 30). For our knowledge of the external world, sensations are to be followed, not despised. But reason is the organ for the supreme discovery of truth.

Voluntarism stands for the primacy of will or some form or mode of effort-consciousness. It takes will to be the source and the sustaining power of mental life. It may be blind will or impulse, as in Schopenhauer; it may take the form of impulse and idea in synthesis as exemplified in Lotze and in Wundt, although Lotze may be held to recognize too much more than one fundamental mind-function for a real voluntarist; or it may begin with the idea, but hold, as in Royce, that the idea appears in consciousness as an act of will. Touching what has just been said of Lotze, it is he who has said, for example, that all the acts of daily life never demand "a distinct impulse of the will," but are "adequately brought about by the pure flux of thought." Lotze veers, indeed, from a rationalistic mode of thought toward positivist tendency or direction. On genetic grounds, of course, voluntarism will have much to say for itself—hence Paulsen and Wundt have striven to set it upon a psychological basis—since, in the matter of time, early or rudimentary forms of consciousness will be largely blind or impulsive in nature. Paulsen accordingly makes impulse the basal function of the inner life. More generally, I may remark the very unscientific and unwarranted tendency of voluntaristic psychology to found itself on "conation" in ways whereby that term has been stretched far beyond anything consciousness can sanction as processes really volitional in character.

But the weakness of voluntarism lies in the fact that not even the earliest forms of *Trieb,* impulse, or feeling-will, can be admitted to be without germinal representation or rudimentary thought. We must think of some undifferentiated whole, out of which the various mental powers, or characteristics, evolve, instead of assuming will as the base of a gradual intelligence. We must take account of the progressive embodiment of reason to be found in all sentient life. We must hold to internal structure in such wise that the psychosis is not the absolutely simple thing it is sometimes supposed to be. Binet has declared that psychic manifestations are much more complex than is supposed, even in the lowest scales of animal life. Schopenhauer sets his world of feeling-will over against reason or thought, but his *Trieb* or impulse is not really will in any proper or developed sense, and is not exclusive of feeling. In fact, the ground of life, which Schopenhauer chose to call the will in all things, was in reality something psychologically so chaotic, that no world could have come of it that was not irrational and meaningless. Nietzsche made voluntarism the underlying moment of his psychology of religion. For a central experience of will is what he always seeks, as affording a measure in the direction of religious metaphysic. But of the will-theories of Schopenhauer and Nietzsche, it is to be said that the will, properly conceived, never acts blindly or without reason, which latter is, in fact, the determining factor of mental life, since it enlightens and directs the activity of the will.

The world of appetitions, to which, since the time of Leibniz, the term will has, in inexact and even mythological fashion, been applied, does not constitute the realm of will at all. For, obviously, there can be no proper willing without an idea of something that is willed. The qualitative constancy which Wundt has sought in the will —as compared with ideas and with feeling—is too abstract and mythical an affair to be psychologically satisfactory. The fault of a radical voluntarism, like that of Fichte, is that in it pure will regards itself as an end, and wills merely for the sake of willing. It is, for it, not a case of objects, but of willing itself—absolute will, cloaked as a natural impulse to independence. Clumsy and confused is the way in which Fichte tells us that "reason is reason," and in the same breath insists that "the will is the living principle of reason—is itself reason." The truth is, reason or thought is by him subordinated to will or our striving energies, in unwarrantable voluntaristic fashion: will is made antecedent to knowledge. But this idea of absolute will is unsatisfactory, in that it only too easily becomes a detached and unrealized ideal, arbitrarily viewing everything as a mere expression of its will. It is a case of the transcendentalism of will overleaping itself, and vaulting the heavens. This brings us to note the absurdity of voluntarism in taking, as the chief characteristic of life's mental powers, something which is found in complete abeyance as life reaches its highest. For in hours of pure thought, or in seasons of calm esthetic contemplation, it cannot be ignored how disinterested is thought, nor can it be pretended that anything like actual or conscious willing is anything but absent, in both cases. This is all that is then evidenced of Royce's true but irrelevant saying that "our will is always dramatic in its expressions" (*The Problem of Christianity,* Vol. II, p. 297). Yet voluntarism thinks it congruous to make this sleeping partner figure as the most distinctive, and indeed the all-devouring factor in our mental life. It is extreme, and straining experience, to say of such times, like Höffding, that "we must *will* to see, in order to see aright." That, of course, is not meant to imply that will is not present.

What Royce calls his "absolute voluntarism" begins with the idea, but immediately asserts that the idea appears in consciousness as an act of will. This seems a somewhat hasty and violent psychological treatment of the idea, almost reminding one of Condillac's, when he made the idea a sensation representative of something, in spite of their difference being one of kind: my ideas, *as* ideas, *are* ideas, and not just anything else you please. It might surely have sufficed to make ideas also aims and ideals: they are not yet acts of will. But that would not satisfy Royce's mystical pan-egoistic epistemology, and so his rather chaotic voluntaristic psychology declares that "the idea is a will seeking its own determination." "Ideas are thinkable but absolutely unknowable," a writer has said, in the sense of knowledge as ordinarily understood. And "every idea," said Rosmini (in his work *On the Origin of Ideas*), is "universal and necessary." In another connection, Royce has said, less objectionably, that "the motives of an idea are practical, and the constituents of an idea are either the data of perception, or the conceptual processes whereby we characterize or predict or pursue such data" (*The Problem of Christianity,* Vol. II, pp. 181-182). The ideas appear to be really, at most "proposals for volition," as the case has been put, and the idea must be selected, as Bradley says, by something which is not an idea; they may thus become idea-forces, as Fouillée termed them; but the primacy of the idea is not to be obscured or lost sight of, even though its intellectual functioning is not to be disjoined from the volitional and emotional activities which it mediates and determines. The dominance of the idea in consciousness is the primary fact with which we are here concerned, and one which cannot be filched away by voluntaristic violence. This primacy can be maintained without giving the intellectual ideas or terms any too abstract air or character. But let the idea vanish, and what will become of motive-feeling and volitional impulse? The particularity of sensation, and the universality of the idea, need not be forgotten. Even in stages where, psychologists say, sensation dominates, it might be worthy of better remembrance that we become aware of the presence of a sensation only through thinking. Not sensations, but our thought of them, is what differentiates us from the animal creation. Sensation is concrete and particular, while thought always carries an element of universality. Where there are sensations, there, said Rosmini, the primitive synthesis is made by the mind in a spontaneous manner. And (in his work on *Logic*) Rosmini differentiated intelligence from sensation in a meritorious manner. "No other faculty," said he, "except the understanding, has for its term an *object*." This last is intuited, but, to know this, he maintains, there must be an act of reflection upon the intuition. Therein the understanding is different from the feeling. For "the felt is not object but simple *term*, and the faculty of feeling has not the *essential property* of the faculty of understanding." Rosmini thus avoids the confusions as to sense which marked Aristotle, Kant, and others of more recent date. Feeling, as Rosmini insists, is made up of that which feels and of that which is felt, and intellective perception is not to be confounded with feeling, since feeling in this sense must "precede *the act of thought* which observes it." It is not to be forgotten that, as Stout is pleased to put the matter, sensation exists *in,* as well as *for,* the mind (*Manual of Psychology,* p. 209), although this requires some further explication to render it quite satisfactory.

Reason remains a power perceptive, regulative, dynamical—the concrete unity of our organized mental energy. It is by virtue of this dynamic reason that we act in freedom. Freedom is a necessity of the purpose-positing activity of intelligence. That means the freedom of the reasonable will, not the blind voluntaristic will that treats reason as its bond-servant. The reasonable will rules the feeling-life and the impulse-life in the quest of its concrete ideal. Not even the appeal of Rousseau to inward feeling or sentiment was free of considerable elements of ratiocination. True, in his unsystematic way, he could say that ideas came from without, and that sentiments sprang up within the soul. But he did not completely disjoin them, there being, in his view, senses in which "ideas are sentiments, and sentiments are ideas." But he sometimes joined the sentiments to reason, treating them as its necessary completion. For, with all his insistence upon the "heart," he uses sentiment in a way which does not always exclude cognitive elements. Still, there is in Rousseau the tendency to make the sentimental outweigh the rational, although it cannot be said that the sentimental was, in him, void of reference to reason, or always destitute of theoretic thought. The importance of feeling, however, is not to be underestimated, since it reflects the ethical quality of the person or represents the personality in its immediate self-consciousness. Ribot has represented a revolt against intellectualist theory here, freeing feeling from dependence on presentation, and treating it as an original state, and it may be allowed that the intellectualist theory was often unduly pressed. At the same time Höffding is right in holding that cognitive elements are already present, and do not simply arise out of formless and primitive feeling, as is seen in the early calling forth of memory in connection with early pleasure and pain experiences.

Hume had already given high place to feeling or passion, for what was taken to be the determination of the will by reason, Hume regarded as really its determination by calmer or more tranquil feelings. His rejection of the primacy of will was, of course, unsatisfactory, being in favor of a species of impression: reason was by him made subject to the feelings. Dr. Bradley does vastly better in his *Essays on Truth and Reality* in rejecting "in any form the primacy of will" (p. 96). He rightly contends that "bare will is no will," and that "will involves not only perception but also idea," which he finds "hard to reconcile with a secondary position of intelligence." I have myself in a large work, entitled *A Philosophical System of Theistic Idealism* (Blackwood, 1917), not only opposed voluntarism and taken reasonable will to be the only true idea of will, but have shown the straits of voluntarism, and its baleful influences in recent philosophical thought and philosophy of religion. In this I have ranged myself, but on independent grounds, with Meumann and other continental thinkers who stand for the primacy of intelligence. An all-controlling will, at whose demand alone all reason, no less than all value, can have any being, in the manner there shown, can only yield a very bald and unsatisfying psychology, one which is utterly impotent to do any manner of justice to reason. In taking reasonable will—will enlightened by prevenient reason—to be the only true idea of will, I hold, like Bradley, idea to be essential to will. I take, equally with him, the notion of the idea being often the creature of a blind impulse to be quite inconclusive (*Mind,* 1902, p. 462). For impulse without consciousness of end is not will in any proper sense. If there has been no suggestion of idea, there has been no real willing. Dr. Bradley even speaks of the "monarchy" of the idea, and of the "single idea," all other ideas present in the volitional process being, in his view, subordinate or contributory to the "total idea." I should prefer to think more of the primacy of reason than of idea, taking the process to be more concrete, as a unity of reason. Reason views all in the unity of the idea, and it effects the needful fusion of ideas. Bradley's stress on one idea seems to me apt to make the volitional process appear rather thin and bald for all the facts. Even if we take volition to be "the self-realization of an idea with which the self is identified," such self-identification must be taken to imply that the volition is the act of my concrete self, in which the idea reigns. But it might be objected that ideas do not—in the modifying light of evolution—dominate and function in us in the detached and isolated manner which Bradley is apt to represent. They are set in the reason, which is a representative of the world-reason, and it is of a unity or totality of reason we have first of all to think. It is, however, desirable that the idea, as a psychical existent, should be as clear and distinct as possible. But stress on the willing must not be obscured. "In the end," says Bradley, "my union with the idea must remain essentially a felt union" (*Mind,* 1903, p. 152). And again, "volition is the identification of my felt self with the idea" (*ibid.,* 161). But this seems to me rather artificial, and separates the idea too much from the self, for the idea is already my idea; reason in me is a unified force, which goes out from the unity of the idea, and forms the totality of the idea of which Bradley speaks. Reason is the "I" itself indeed, which proves itself reasonable in the process, as the idea is taken up as a willing. Bradley denies that "desire and conation are to be found in all cases of will," and says that to make them the "bridge" in volition would be "absurdly deficient" (*Mind,* 1904, pp. 20-21). On both points I agree with him. Blind conations are not volition; mere desire is not will. He therefore abides by the view that will is not "original or ultimate," since the passage of an idea into existence is, for him, the essence of will. Varisco, too, holds it "essential to will" that it be "enlightened by cognition," and be "altogether one with cognition," but his attitude is less clear-cut and defined. There is, in my view, a lordship of reason in the entire process which leads to harmony, for the resultant whole is the unity of intelligence and will in the human consciousness. The impulse of reason toward unity is not satisfied until such unity is achieved. The content of reason is the ideal, the necessary, the universally valid. But the universality, Rosmini clearly laid down, is of the mind or the intelligence, and not in things or sensations; we may not even speak, *sensu stricto,* of a universal idea, for not in their content, but in their applicability, are ideas universal. Ideas are singulars; the qualities that belong to universality are given them by mind.

Thought has none of the particularization of sensation: to think is to universalize. The idea is all-important to Rosmini, for it is the light of the mind, however impossible that it should be defined. It will be seen that I take reason or intelligence to precede and determine the will, and the psychical activity involved to be fundamentally real; the time relations connected therewith do not prevent or disturb me, for though time in some aspects and relations is real, it is not ultimate, nor regnant in the realm of spirit. Thus I do not regard all inner psychic activity as in the end will-activity, for there are many psychic occurrences outside will-activity. I reject bare will, in all its arbitrariness, as the ultimate source, while not denying, of course, how will-activity sticks fast in all thought. I am, of course, aware how it has been attempted to justify the statement that all psychic activity is will-activity, by seeking to distinguish an empirical-psychological voluntarism from a metaphysical voluntarism, the latter partaking of the universal character of metaphysic. But I am here only incidentally concerned with empiric voluntarism, in which will is made to include or swallow up feelings and sensations, and impulses are taken as lower forms of will, and even made at times to figure as if they were pure will. But even when the distinction just made is observed, it does not follow that the empirical-psychological account of the development is never overweighted in its stress on will, when ideas or representations and feelings are all taken to be developed therefrom. I am myself sceptical of this account of the development, both as to its doing preludial justice to the representation or reason-elements in the process, and still more as to its being a satisfactory account of the relations found to exist between developed intelligence and developed will. It is only by abstraction that we can determine or fix upon the part played by all the individual psychic elements or factors in the process, and though the phenomena of will lend themselves most easily to observation, it does not follow that justice has always been done to the potency of rational and feeling elements or moments likewise. I do not admit will, in its active efficiency, to be anything else than bound, in certain fundamental ways or principles, to representation and thought connections, and the question is, whether this, the more difficult and recondite part of the process, has been satisfactorily performed. I do not think it has. Will, of course, has had its development, just like every other psychic function, and besides will, there is at least always representation, if arbitrariness is to be shunned. For there is no pure activity, but only such as has been qualitatively determined by representation or content. The element of knowledge is an inseparable moment in consciousness, and it is not derivable from will. Not even the representations should be derived from will, when sensations and feelings are also present.

Wundt's theory of "idea-object," as original datum of thought, might surely have led to more satisfactory issue touching the ideating forces. It seems to me not without arbitrariness that Wundt makes the will a standing element in knowledge in the way he has done, and treats the representations as accidental or contingent. His qualitatively constant will is an untenable conception, and the standing thing is the self-identical subject, to whom the will belongs. Activity has no content save as belonging to such a concrete subject, of whom it is a manifestation. Talk of complexes and totalities of psychic elements is vain without this being recognized. Nor do I think it admissible—because arbitrary and not true to experience—to regard the manifoldness of the representations found in experience, as bound into a unity only through will. This seems to me to indicate some failure to appreciate or realize the unifying force or activity in reason, which does not stand idly by will.

If will is never bare will, never mere activity, but always representing activity, there appears to me no adequate ground for blindly quenching or ignoring any rational elements involved—the unifying power or activity of reason—in order to hypostatize will alone. Intellectual elements are already present with the representations; thought begins only with these last, not yet with concepts, which arise out of them; in the original perceptions thought has already found the conditions for its exercise. But I had not meant to do more than make passing reference to empirical-psychological aspects. We must not forget that hypothetical metaphysical conceptions or ground principles must not be applied to, or exchanged with, empirical-psychological abstractions, in the treatment of reality.

But empirical-psychological treatment is not therefore final, or above the need of criticism. Metaphysical voluntarism, however, is my main present concern. Analysis of the concepts of the understanding and inquiry into the transcendent ideas, are a special care of metaphysics, whose fundamental principles are immanent in the impulse of human reason to knowledge. Pure will is to Wundt the end of the psychological regress, but pure will is merely an abstraction of metaphysical value in bringing into clear view the essence of absolute being. To make, in the Wundtian style, the "inner impulses" the source of all need for thought is no satisfactory theory of our mental life or personality; nor do we recognize as will what acts blindly, without reason, or motives, or reflection.

On the other hand, the rationalism which we oppose to one-sided voluntarism is not one in which there is a mere *ens rationis,* but a subject with the characters of concreteness and individuality. The subject must have a content, original and individual, and not independently of external relations, the external world being its necessary correlative; as Wundt says, "a consciousness without objects is an empty abstraction." When the voluntarist tells us the many mighty things wrought by will, he is apt to forget that will essentially implies cooperation of the individual and concrete subject, whereas reason can be conceived without such subjective reference, as capable of being embodied, objectively and universally, in laws or in relational systems standing by themselves (Cf. F. de Sarlo, *Il Concetto dell' Anima nella Psicologia Contemporanea,* Florence, 1900, pp. 33-34). It is not surprising that Mr. A. F. Shand should say that "the profoundest introspection will not show us the universal character of will" (*Mind,* 1897, p. 325). But the varied and different types of will need not keep us, for all that, from saying with Ladd that "willing is of essentially one kind" (*Philosophy of Knowledge*, p. 190).

To treat of synthesis without an individuality, of spontaneity without an individual subject, in Wundt's fashion, can never be satisfactory in result. The psychic elements and functions owe their efficacy and worth to their seat in the real subject, however we may try to abstract them for supposedly scientific purposes. There is no very convincing reason why the treatment should deprive itself of concreteness and lucidity, by trying to dispense with, or ignore, a real subject. Of course, the procedure is intelligible enough, in its desire to avoid older modes of thought in which the soul or subject was viewed too substantially rather than potentially, too much as something given rather than something formed, but the avoidance of wrong ways of regarding the subject does not necessitate vain attempts to eliminate an abiding, self-identical subject as persisting through experience. The facts of unity, coherence, continuity, identity, and evolution, in mental life or personality, are, otherwise, not adequately covered or dealt with. The psychic acts or facts by which we live are not so sufficient unto themselves as Wundt would make it appear, and the reduction of everything to will-activity is far from satisfying.

Dr. Stout has made the significant admission that it is "the cognitive side of our character which gives determinate character to the conative." But what we have already seen of the attempt to set out the psychological origin, nature, and growth, of this cognitive side, has been by no means promising or satisfactory, for it has been mainly in terms of that which is not cognition. In the end we are driven pretty much to let cognition certify itself. Not even Wundt's position that the active mental representation or *Vorstellung* is originally identical with the object can be sustained. Cognition would be defeated by the object being so identified with the representing subject.

Wundt says thinking is willing, and so distinguished a thinker as Ladd remarks that this is "admirably" said. But is it so admirable? If the thinking is not a willing *per se*, it seems to me only a needless confusion. One does not deny the presence of a will-element in thinking, but the thinking is still thinking, and is not, so far as it is thought, to be called willing without a misuse of language. At least I am rationalist enough to think so. I am not unmindful, in saying this, that Bradley—whom I greatly honor in spite of some deep divergences from him—has said, properly enough, that will and thought are implicated the one with the other (*Appearance and Reality*, p. 474); but he has also said, less desirably, that "the same psychical state is indifferently will or thought, according to the side from which you view it" (*ibid.*, 468). Surely the facts can have justice done to them without countenancing so many terminological inexactitudes of this sort in psychology as a "science." In no other "science" are clearness and distinction at such a discount.

The dependence of will on thought or idea, and the dependence of thought on will, can surely be recognized without blindly identifying them. It is only "to a certain extent," says Bradley, they are essentially one, but they are "not two clear functions in unity," which may be granted; but, granting this partial fusion or identity, their divergence is the thing that waits for explanation. This Dr. Bradley does not attempt, but is content to urge that neither thought nor will is primary and ultimate. What he fails to bring out is the unity of human personality, the unity of consciousness, in which feeling, thinking, and willing are three sufficiently fundamental modes of expression. Ideation may be a process given to consciousness, and thinking a more self-conscious and selective affair, but, though there may be a teleology of thinking, and though will may enter as a moment in the thinking process, yet thinking is still distinctively of the nature of thinking, and not willing or anything else.

There need be no failure to appreciate the part played by the will-element in thinking as a discriminating and relating activity, in so maintaining the distinctively rational character of the thinking process, even when it is the "sinewy thought" of stressful life. I reject, in like manner, the position of those who, like Bradley, treat thought as unreal, and make it consist of feeling transformed. Thought is still thought, and not feeling, though they are, of course, inseparably joined in the unity of consciousness or knowledge.

Willing, too, is unique, and not resolvable into thought or feeling. I have declined to run the whole primary consciousness back into pure will-activity, but in that early stage, though presentation or the knowledge-term was present, intelligence may very well have been so far under the dominating influence of will and feeling elements as not to have attained any real independence. The presentative faculty may well have needed growth and development before cognition came to anything like independence and mastery. The process was a complex one, and must not be too abstractly conceived in the cognitive interest, without consideration of feeling and volitional factors. But when the distinctively cognitive supremacy was at length gained, the idea or the presentational element took the place of clear control, which rationalism claims for it, over all else. Will-activity I have not taken to be the ultimate thing, for that activity appears to be only a mode of realizing some condition of consciousness which is not of the nature of will.

In the developed subject it is that knowing and feeling and willing find their deepest point of unity, or the final ground of their hanging together, however one or the other may have at one time been found predominating. This is the *Gesammt-Ich* or total-ego, a personal unity. There is in such a subject an identity of knowing and willing—I mean, in the unity of consciousness or the personality. And it is, as I have already pointed out, not with the genetic point of view we are really concerned, but with the metaphysics of consciousness as here and now developed. In this consciousness relation, the voluntarist cannot be allowed to hypostatize the will-element alone, while the rationalist claims to do so for the knowledge-element also, and the primacy indeed of the idea, the perception, is the contention of the latter. For there is certainly something absurd in the idea of volition without any idea on the part of the willer of the end or thing to be willed.

A voluntary act includes, among other things, a volition or determination to bring about a particular result. Even Münsterberg holds an idea of the result to be brought about an essential factor in voluntary action. In volition there is always an idea seeking realization. Volition is sufficiently complex to require both presentation and feeling. But the transition from idea to realization is not effected so simply as might be supposed, or without extraneous considerations and connections. And, again, in the case of cognition, no combination of ideating-processes and no theory of ideas, will suffice to yield cognition. The processes are, as I have insisted, all bound up, both in the case of thought and in that of will, in the personal unity of individual life or consciousness. But in the complex called consciousness, the primacy of the idea is, to rationalism, to be maintained, for to it belongs the power of initiative, but this primacy of intelligence is not exercised without mediation of the feeling and willing factors. For a purely thinking consciousness would be an utter unreality and abstraction.

The relations of thinking and willing with which I have just been dealing belong to consciousness itself, which latter admits of no explanation that does not presuppose that very consciousness. The inner connection of the various contents of consciousness is indubitable. But the synthesis of elements which goes to form consciousness or personality is one which has never yet been explained. This conception of personality is of central importance for psychology, and calls for more explicit recognition than Bradley has given to it. For what we plainly are called to do is to give more rational character to the relation of the single elements — even the non-intellectual ones — whereof it is composed. And to the thought or knowledge element this task of imparting greater rationality is difficult enough, for it is involved in being itself, which is also in process of becoming.

As Höffding, in dealing with the "Problems of Philosophy," has said, "it is a strange contradiction in the grand rationalistic systems, that, although they may be able to explain everything else, yet they are powerless to explain the striving laboring nature of the thought which produces them." And should it be, as he remarks later, that "the empire of Being may be much vaster than the possibilities of our experience," the limitations to our complete rationality of view come into sight. For all that, it is the business of reason or the speculative activity to follow on to the furthest limits possible, so that thought and being may grow always more approximately one. In doing so, thought must not be regarded as a purely subjective activity, or isolated from its objects and their relations. For, as Riehl has remarked, in these objective relations "there must be something analogous to the activity of thought, something corresponding to the form of this activity, else this activity could not arise" (*Science and Metaphysics,* ed. by A. Fairbanks, p. 306).

I am an ideating self and a willing self, but I am a willing self because, and after, I am an ideating self: the connection, however, may be as swift and intimate as you please. But my ideas are certainly present, as rationalism contends, before they are actualized by will. They do not wait on will demand, as voluntarism contends. Nor is their actualization a pure matter of idea and accordant volition, for being other than the idea or the volition is involved in the actualization, as Ladd has clearly shown in his *Theory of Reality* (pp. 482-483).

In the light of all I have advanced, the view of Wundt —adopted by Külpe—which regards apperception and will as ultimately one and the same function, is not at all satisfying. Needlessly complicated, it is too emotional, the feelings being the spring of action and not the representation, and all the processes which are made up of feelings being taken to arise from volition as fundamental fact. Wundt says it is impossible to find out how a volition proceeds in any other way than by following it exactly as it is presented to us in immediate experience. I entirely agree, and it is on this precise ground that I reject his theory of it.

Is it not surprising that Rehmke should have felt dissatisfied with the uses made of the term *Vorstellung* in voluntaristic discussions. At one time you may find it stand for something given; at another time it means an inner activity or event; in another instance it will serve for an image in us; it does duty for the represented, but again for the representing; now it is superfluously styled conscious, and now it is, in self-contradictory fashion, termed unconscious. And the apparently simple and easy theory of a blind, dull, senseless will which is supposed in voluntarism to have first borne sway, and worked its way in the world up to self-consciousness, is by no means either easy or accountable, for how this unconscious comes to consciousness is never satisfactorily explained, at least in the higher spheres of spirit, even when we allow for unconscious occurrences in nature. It has been vainly attempted to explain consciousness as only the passive product of unconscious actions, without taking any proper account of the reason immanent in the process.

There is no sure footing for our deepest experience in feeling; we need valid ideas—ideas not dissociate from reality. Feeling has need of idea, which, however, must not get divorced from feeling, of which it is meant to be the guide. But reason is not the mere adventitious thing which voluntarists like Schopenhauer would make it, waiting on the bidding of will. Reason is to be regarded as intellectual rather than conative; it is concerned with axiomatic truths or the fundamental ideas, principles, norms, or laws of reason. Reason is utterly underestimated or misconceived when it is reduced by such voluntarism to a merely pragmatist attendance on will and practical needs. Will, when divested by Schopenhauer's voluntarism of the element of knowledge, is utterly abstract and unreal.

But, of course, rationalism by itself does not suffice to give a rounded whole in our view of reality, and, in claiming primacy for intelligence, it is not meant that due consideration is not also to be given to will and feeling factors. Man is not reason alone, however disinterested, any more than he is will alone or feeling alone. But in freeing reason from non-rational factors, we must take an organic conception of man in his truth-seeking capacities and powers, and give will and feeling values their due place. This can be done, without forgetting that these values are stamped with relativity and subjectivity. This will keep us from falling into the modern snare of undervaluing the truth or reality values so dear to reason. Nothing will be exempt from the sway and scrutiny of reason, but truth will be sought with the whole man, feeling and will cooperating toward the vital and concrete results of the quest.

But this reckoning with the non-intellectual factors does not suffice, in our view of the meaning or philosophy of life, for we must go on to a world-view, infinite in its reaches beyond our own world of reason. And if the will and feeling facts and values import pluralistic tendency and direction as against the monistic tendency of reason, justice may yet be done these former elements or factors, in our system of thought, while the constructive power and activity of reason systematically builds up its final or ultimate monistic issue.

It can, of course, be said that under this monistic sway of reason, justice to facts and values may not be done, but it is just the task of infinitely patient constructive reason to see that justice is done. The thing is to see that reason remain living, concrete, and grow not rigid, abstract, and unreal. Such reason will advance the realization of the normative ideals, but not in merely formal fashion, without comprehending the foundations of the empiric world. Facts and values must not be distorted or wrenched but properly articulated in the system, while not allowed, in recalcitrant fashion, to defeat or impede a final unity of reason or of system.

Although not primarily concerned with psychological developments, but rather with the experience of the developed consciousness, I have yet noticed some of the more extreme and insupportable contentions of psychological voluntarism. I shall add yet another example of the somewhat overdone emphasis and over-dogmatic tone of such presentations as exemplified by Prof. J. H. Leuba (in *The American Journal of Religious Psychology and Education,* 1907, p. 309). He says, "Aristotle characterized man as *thinking-desire.*" We are to take this as an epitome of Aristotle! The fine things uttered by Aristotle touching intellect and reason—reason in its rule of desire and passion—apparently do not exist for the voluntarist. "Will without intelligence may be possible," Leuba says; rationalists deny it is anything of the sort. It would not then be will. And the converse is much more conceivable—if that were of any consequence—as Meumann and others have contended.

Leuba takes the usual voluntaristic pleasure in minimizing thought, reason, and intellect. "The function of intelligence" is reduced by Leuba to the "gratifying" of "desires, needs, cravings," a not very exalted role. All spontaneity of thought, all finely disinterested reason, are swept away in this crude subservience to desire. "Thought does not exist for itself; it is the instrument of desire." "We think because we will." It is scarcely to be wondered at that the rationalist finds little satisfaction in these modes of indulging in the humiliation or degradation of reason, the highest, divinest thing in man. But it reacts in lowering the psychological system itself, which seeks to effect such reduction.

I have run intelligence and will back into unity or harmony within the human consciousness—into the unity of personality. And from this, and what we have seen of the impulse of reason for unity, we may say that the constitution of the mind "predisposes man for monism" (Dr. P. Carus, *Fundamental Problems,* p. 21). My own results lead me finally to a spiritual monism, in which spiritual reason is for me the ultimate principle. One finds a correlation of subject and object, of "I" and "not-I," of soul and body, of consciousness and existence, of nature and spirit, of God and the world, but we cannot rest in the end without running these back, under causal points of view where necessary, into some principle or power that embraces them all, and inwardly binds them all together. For though we may have a relative dualism and individualism —which, though relative, does not contemplate anything of the nature of blank absorption—yet is the impulse of reason for unity never satisfied short of an all-unity such as I find in the Absolute and Eternal Reason. For monism is the last word in philosophy, and such a spiritual monistic principle is for me *fons et origo* of the universe, with dualisms and correlations finally grounded in it as fundamental principle. But that World-Reason has effectiveness, for it is also World-Will, and is indeed the unity of the Ideal and the Real.

James Lindsay.

Irvine, Scotland.

CRITICISMS AND DISCUSSIONS.

THINGS ARE NOT ALWAYS WHAT THEY SEEM.

No man or woman is ever so much deceived by another as by himself or herself. The girl deceived by the lover, the rube fooled by the bunco-steerer, the merchant lured by the stock market, the fat gentleman with the bank roll duped by the sweet little maid, the lobster hooked by the salamander, are gulled less by the hocus-pocus, chicanery and deceit of the Salomes and Judases than by the tricks of their own thoughts.

Experimental psychology has contributed a large number of new discoveries which explain all this. Time was when philosophers, beetle browed, knitted and knotted in wrinkles, with ponderous spectacles and professorships, would sit in their garrets or hermitage and evolve some theory or notion to explain whether the world was made of green cheese, a blue fancy, or something real.

For tens and hundreds or thousands of years philosophers have fought French duels of wordy battles as to the existence of anything round about or not. To plain people, who have bumped their heads on door knobs or burned their fingers in a fire, it might seem the Olympus of folly to debate whether a piece of sausage and a dog are the flames from your heated imagination or something actual and real.

But philosophers are not supposed to be either plain or matter of fact. They are apt to pursue words and phrases, no less than thoughts, into all sorts of mazes and devious channels. If they at times run into a blind alley, a *cul-de-sac,* or against a stone wall, the matter is lightly dismissed with "we shall return to that later."

Experimental psychology takes neither philosophies, philosophers, innermost thoughts, or words seriously. This experimental science of the real world as distinct from the image of thought world may be likened to philosophy and the psychology of other years, as a man is to his reflection in a mirror.

The one is active, movable, changeable, up and doing, while the other is merely the reflected ray of light. One is the substance, the other is the shadow. One is a creature that acts upon and is acted on by everything round about. The other is uninfluenced by or uninfluencing the world.

In fine, laboratory, experimental, objective, and the "test" psychology of to-day, takes nothing for granted, admits no "authorities" other than real facts open to, admitted and acknowledged by ninety-nine and nine-tenths percent of sane persons. The older psychology of psychics, spirits, mind reading, telepathy, "seeing-things," "spiritism," and images and thoughts of isolated "professors," "mediums," "experts," "writers on malaria," "descendants of Oliver Wendell Holmes," and the like, are all found wanting by objective psychology.

Recently this refreshing science has undertaken to find out why everybody sees things, not as they actually and truly are, but each in a different way. It has been found for the first time that there is no such thing as a pure, unadulterated, accurate, unmixed sensation.

This will be a blow to physiologists, physicians, and medical men generally, all of whom still teach that when you see a bulldog with his teeth in the seat of a pedestrian's trousers, you really see what you think you see. Nothing else. This is a clean uncomplicated sensation you are falsely taught.

Philosophers of a certain ilk may teach, if they like, that when a saucer of milk is lapped up by a kitten, there never was any real milk there in the first place. They may hold to this superideal world of non-reality. That is not what these experiments of psychology show. What they do prove, however, is the fact that the eye, ear, and other sensation receivers and mouthpieces, as years advance from infancy upward, become moulded and impressed in such a way with repeated happenings of the past in such a wise that they have a real physical power of prophecy.

Coming events cast their shadows before simply because the eye, muscles, tongue and ear are set like a mouse-trap or like the trigger of a gun—to wit, to spring forward far beyond the needed requirement; to foresee and forehear, to forestall what one has seen and heard so often before.

In other words, if you see an automobile, a runaway horse, or a batted ball, although each one is entirely different and describes an absolutely new and distinct kind of motion, yet you will see it exactly as you have seen many others before it.

When you meet a new acquaintance you are prone to think you have met him before or see that he "looks the spitting image of a dear friend, Mr. Blank." I, myself, wear a Van Dyke beard and an imperial mustache. There are a score of men stouter, taller, shorter, darker, lighter, and with hair on their heads—I am well-nigh bald—who do not resemble me in the slightest, yet who are constantly told because they happen to wear beards—also unlike mine—that they look like me or I look like them.

That the ear is never true; that even Caruso, Farrar, Galli-Curci and the best musicians cannot hear sounds as they actually are, is easily discovered experimentally; that even those with a marvelous sense of hearing can never hear exactly what took place or what caused a particular sound, is proved in the laboratory. Little instruments that resemble brass helmets can be made to imitate bees, birds, the sighing of the wind through trees, the breaking of waves on a beach, thunder, roll of drums, violins, oboes, and so on.

Various sounds are made from these "resonators" and real bees, flies, parrots, musical instruments, and noises are also used. Any series of sounds used for some days previously leaves such an impress upon the subject's ears, that subsequent tones or noises are interpreted and heard almost before they are made, in terms of the sounds previously and formerly repeated.

It is a law of nature that light travels faster than sound. You can see a puff of smoke some time before you hear the shot. You can see the batter hit the ball some seconds before you hear the crack of the bat.

Yet you will find on analysis that bits from operas and songs that you hear hundreds of times a day, and other familiar and oft repeated tones are heard as quickly as you see. The experimental psychologist knows this to be another example of hearing things before they happen. This is true, scientific foresight due to habit, past experience, and multitudes of repetitions. The eye and the ear have become linked thus so often that the instant the eye sees a certain thing, the ear hears its necessarily associated sound. This fraction of a moment's anticipation or "prophecy" becomes fused with actual sound, which comes a moment later.

Echoes are often heard double for this reason. The sound is heard from habit and also as a later rebound. People who "see things" such as ghosts, spirits, and departed guests have much the same experience.

Seeing halos around the head; seeing people before you meet them—wrongly explained as coincidences or as something mysterious—are all due to the fact that you see the things which you have seen oftenest.

A patch of color, of light, and of shadow is usually all you see of anything. Yet you instantly recognize that distant blend as Larrie Jones or Goldie Summers; Don Quixote, who in Cervantes's novel charges and takes distant windmills for knights, is not a bit more amusing than the rest of humanity. Knights were in his thoughts as well as among his associates—at least in costume —hence he saw them.

There is but a slight difference between sane persons who see an orange when a yellow colored globe is thrown into the air, and the drunken man who sees rats without cause or the insane one who has the delusion that the veins on his arms are wriggling worms.

Indeed the only way you recognize a friend, a book, a doorstep, a fruit, a tree, or what not, is not altogether because of any sensation you receive at that moment, but from the past experiences, repetitions, and intimate memories of the past.

When you absentmindedly trace your steps home at night you may not be aware that past experiences are responsible for your seemingly rational behavior, but you have not consciously seen a house number, a doorstep, a post, a tree, or any of the landmarks which are needed to guide a stranger.

A dog, a cat, or a horse is no different from you. They find their way home, not because they see any peculiar home signs, but because they perceive a lot of complex, conglomerate things oft associated in their cosmos with that spot. A dog perceives his master, not by smell or sight according to Prof. John B. Watson of Johns Hopkins—as has been taught, but just as the master himself recognizes his children, namely, by a mixture of complex perceptions.

You turn corners, cross roads, avoid lamps as well as people, not because you see them, but because you perceive them. You may be talking to a companion, and at the end of your walk you may find yourself quite unable to recall a single moment when your movements were specially modified to suit an actual need, though you have probably accommodated yourself in this way many times. The frequency of past experiences of the kind has established what you have previously called a psycho-physical disposition which now works itself out on the occasion of the appropriate stimulus with the slightest intervention of consciousness. In like manner, an experienced teacher pursues the course of his lesson without any conscious effort to watch the more mischievous members of his class—yet no irregularity escapes his notice, or fails to produce a suitable though to the casual observer scarcely noticeable, response.

In the young child, all such dispositions are in the making. His mental life is therefore necessarily bound up very closely with his actual environment, as it changes from moment to moment. If he is walking in the road he must attend to the line of the footpath, the gas lamps and the people, or disaster would attend him at every turn. Repeated experience leads him to make the necessary muscular adjustments whenever he is about to step across the line of shadow or of light which marks the change of level from road to footpath, until finally the muscular changes take place with accuracy and precision with the exercise of little, if any, conscious control, whenever the situation demands it. This leaves the mind free to pursue any line of activity without reference to normal changes going on in the immediate surroundings.

You see then, how closely the process of perception is related to that which governs the formation of habits. It is possible only because of that fundamental quality of retentiveness which leads to the formation of psycho-physical dispositions. At the same time, it must not be supposed that the development of the perceptive powers is merely a development toward automatism.

The sensory bases upon which experiences rest are so slight that it is not surprising to find error creeping in, especially when perception takes place under the influence of unsatisfactory proof-readers. The thought and the particular phrases in which it is cast suggest the words before the eye reaches them. You tend to see what you expect to see, and miss the printer's errors. Under emotional influences, like that of fear, for example, such misinterpretations are particularly common. A nervous person walking along a country lane finds a miscreant's footsteps in the fall of every leaf, if you are waiting anxiously for a telegram, how many times do you hear the footsteps of the messenger and the sound of the door-bell!

Every slight sound is the occasion of such erroneous mental construction. It is clear, however, that illusions, which is the name given to misunderstandings of this peculiar kind, are not due to any inaccurate working of the nervous mechanism of sensation. The possibility of mistakes of the kind may perhaps be regarded as the price paid for the power which the accumulated but latent fruits of experience give to you in your perceptual adjustments. The sensory element in perception is often so entirely outweighed by those traces of the past which are involved in the process, that the actual sensory object is enormously modified or even practically replaced by something else which corresponds more closely to existing and very lively dispositions.

In both perception and illusion there is always present some sensory element and even those traces of past experience which are revealed when either process is subjected to analysis are also sensory in origin. Ultimately, then, the knowledge of the physical environment rests upon the evidence of the senses.

Every one knows what Bunyan meant when he wrote of the "five gateways of the soul," but increasing knowledge has taught that the traditional five senses do not exhaust the list. Perhaps the most important of the more recently discovered sensations are those which are due to the movements of muscles, tendons, and joints, which play so large a part in enabling you to gain control of your movements, sensations of heat and cold, other organic sensations from internal parts of the body and sensations of pain, all of which are due to the stimulation of nerve-structures specially adapted to respond to a particular type of stimulus. A visual sensation may be more or less bright, a sound-sensation more or less loud, a sensation of pressure may be more or less light and so on. These are differences in intensity. Again, visual sensations vary in color, sound-sensations in pitch, temperature sensations may be hot or cold and taste sensations may be sweet or salty, sour or bitter. These are typical of what are called qualitative differences, and the student will readily notice how much more delicately these differences are related in the case of light and sound than in the other cases.

It is particularly important that one should realize the difference between the sensation and the stimulus to which it owes its rise. Most people see sufficiently for all practical purposes, without knowing anything about vibrations of the ether or the change which they cause in the minute structures which lie in the sensitive layer of the retina. The psychologist is not directly concerned with either of these things. It is in seeing as you all experience it that he is interested. The physicist or the physiologist tells you that those other things happen and you accept his word for it, but you are not conscious of these events; they do not enter into the experience of the person who sees, in the way that color and brightness and light and shade do. These, then, are the sensory objects the apprehension of which he discusses. A like distinction is also to be drawn between all other sensory objects and the stimuli to which they owe their appearance in consciousness.

Moreover, in actual experience you never merely sense color for instance, but perceive a colored thing. The mental processes which are set up by sensory stimuli are always interpretative and therefore perceptual in character. Whenever you see, you see something. Ordinarily you can name or describe it. So with what you hear or touch or taste. But these interpretations had to be learned, except in so far as precise reflex machinery provided for right response to such stimuli.

In general, the tendency is to shrink from those contacts which produce discomfort, and to seek those which give satisfaction. This shrinking or seeking attitude which the infant learns to adopt toward objects around him is his first interpretation of his sense experience. Conscious purpose is still undeveloped, but when he hears a voice, his head turns, seeking, as it were, the visual sensations which usually accompany that sort of sound. His mental life is at first chiefly of this order. Increase of motor control greatly enriches his sensory experiences and deepens the significance of the things around him. In other words, percepts become fuller: color differences, differences in size and shape, position and distance are all perceived with gradually increasing accuracy; to sensory stimuli his reactions grow increasingly varied and delicate with these growing powers of discrimination. The process is especially rapid in regard to the things which afford him bodily comfort or with which he plays or which he otherwise puts to use. Instincts like fear and curiosity prompt experimental interpretations of new sensory experiences, but his action in these cases, even when most foolish, has its basis in what he has done previously.

In your own perception you will readily distinguish the dominant play of purpose. When you are thirsty, the cup of tea has only one aspect—a thing to take in the hand and carry to your mouth. When thirst is quenched, your china-collecting interest may assert itself, and the shape and design of the particular cup may strike your eye. If you want a certain book from your shelves, to that and that only your eye is directed. You may not even notice that other books surround it. In a casual outward glance, the unfamiliar strikes you and excites a closer examination, but commonly your interests and purposes determine your perceptions. If you are enthusiastic about birds, every twitter catches your ear as you walk through country lanes and a new note instantly arrests your attention, while your friend the botanist sees nothing but the flowers in the hedge bottom.

What you call observation is precisely this purposeful attention to the things which strike your senses. You do at times give yourself over to casual and almost meaningless noting of the things that pass before your eyes, as you sit in a railway train for example. But this is not observation in the right sense of the word. If, on the other hand, by force of habit, or by specific intention you are on the lookout for special features in the changing landscape, geological, historical, or other landmarks, your survey is purposeful, you become observant. Under the influence of a particular interest, your perception becomes remarkably acute. The sailor sees land on the horizon long before the passengers on the ship, and the traditional red Indian can follow a trail through the woods which would defy the ordinary white man. Popular opinion is apt to ascribe the power of the red Indian to special acuteness of vision, but recent researches into the psychology of savage races throw considerable doubt upon this view. It seems more probable that experience, quickened by the necessities of the situation, has taught him just what to look for, and how to interpret what he sees. The same explanation is, in all probability, true of the sailor's quickness to see the coast line which may be fraught with danger, or the first sign of the nearness of home.

At the same time, the capacity for sensory discrimination may be improved by the formal training of graduated exercises. Within certain limits fixed by physiological conditions that vary with every individual, the delicacy of the ear is improved by exercises which necessitate discrimination in the pitch of musical notes. Similarly, you will find that regular practice will prove the power of "seeing" distances, or delicately adjusting your muscles to the handling ot a billiard cue. But improvement in sensory discrimination goes ahead much faster when you feel that something really depends upon it. In the life of the young child, formal training has usually no place. His sensory development is a product of experience, and of his growing sense of power among things which every day acquire new meanings for him. He has no established interests, but the objects about him have for the most part become familiar, in the first instance, as sources of pleasurable sensory activity. He has "played" with them; then he puts them to use on his own initiative and in original ways. Informally he "picks up" a great deal of practical knowledge concerning the physical properties of objects. He finds out that some things will break when they fall and others will not, that some things are hard and others soft, that he cannot carry water or milk as he carries a piece of wood, that his father's chair is heavier than his stool. He is already in the path of learning, but his experiences are disordered, and his actions are almost entirely prompted by momentary circumstances. His development will be marked by an increasing coherence in his behavior. His perceptions will come more and more into the service of purpose, gaining thereby in acuteness as well as in richness of content.

It is important to realize how relatively late the power to look at objects in an impersonal way develops. A child in the Kindergarten is interested in objects because of the part they play in his everyday life—not in their shape or color, or size, or in their relations one to another. The ordinary child of three or four who looks at a picture still sees the persons and objects upon it in isolation. If you ask him to tell you what he saw, you will learn that there was a man, and a girl, and a horse, and so on. The pictured objects are just representations of things that have entered into his own experiences, and nothing more. At five or six he is curious to know what is going on in the picture—he is interested in other people's doings as well as his own. A year or two later he will observe more particularly the relative position of objects and suggest reasons for things—"the man is sitting down on a stool and looks very tired"—"the sun is just peeping behind the hill and the man is going out to his work." "There is a clock by the window on the wall—it says half-past five." Last of all comes the tendency to notice the details of individual objects—what they are made of, their peculiarities of form and position.

The bearing of this upon the so-called observation lessons in school is clear. Internal factors and felt needs are the springs of successful activity on the part of the children, and when you talk of training a child's power of observation, you may profitably keep in mind the possibility of cultivating his powers of purposeful action, success in which will depend upon watchfulness and care in the use of his senses. When mistakes in observation really matter, they become relatively infrequent. Many of the school observation lessons are, psychologically considered, nothing more than a formal attempt to associate names to things or to the specific sensory qualities of things. Whether they are justified or not it is not the business of psychology to say.

At the same time, the psychological qualities of a good observer include something more than interest in and knowledge of the subject under examination. Interest in a subject is not infrequently accompanied by preconceptions which may even be strong enough to vitiate the observations altogether. Until Galileo's time, people believed that a stone of ten pounds weight would fall ten times more quickly than a stone of one pound. That was the current belief, and nobody thought of questioning it. Yet the actual fall of stones must have been watched many times in the interval, but it was only with difficulty that Galileo persuaded his contemporaries to look at facts in freedom from the bias of preconception. In a like way, every teacher of science knows how difficult it is to prevent the quite honest "cooking" of results which comes when a pupil knows beforehand what he ought to find. Hence to train observation implies also a training in intellectual honesty and serves to lay the foundation of a love of truth for its own sake, which enables one to recognize facts whether or not they are in accordance with the preconceived ideas or hopes.

LIEUTENANT LEONARD KEENE HIRSHBERG.
JOHNS HOPKINS UNIVERSITY.

ON THE CONSTRUCTION OF A NON-ARISTOTELIAN LOGIC.

In a paper read at the Christmas meeting of the American Philosophical Association at Princeton University in December 1917,[1] the writer pointed out the existence of a group of logics, in which many of the implications of the traditional science become untrue.

[1] See also the writer's *Primer of Logic*, (B. D. Smith and Brothers, Pulaski, Va., 1917).

The members of this family are each one more general than the common logic, while certain of their underlying axioms stand in contradiction to one another. It is proposed now to construct in some detail that member of the group, whose characteristic postulate asserts the untruth of the proposition, *all a is all a,* for all meanings of *a*. In order to keep the discussion within the narrowest limits consistent with its purpose, we shall confine our attention as far as possible to a single type of implication. Because of the central importance of the syllogism in any system of inference, it will be deemed enough to deduce all the true and all the untrue propositions of that type.

There are four forms which the logician may recognize as necessary and sufficient to express the manner in which any two classes, a and b, may be related categorically. These will be represented by the letters α, β, γ and ϵ. Accordingly let

$$\alpha(ab) = \text{All } a \text{ is all } b,$$
$$\beta(ab) = \text{Some } a \text{ is some } b,$$
$$\gamma(ab) = \text{All } a \text{ is some } b,$$
$$\epsilon(ab) = \text{No } a \text{ is } b,$$

the word *some,* which is explicit in β and γ, being interpreted to mean *some at least, not all.* This meaning of the word is unambiguously established by the properties of the four forms.

By $x(ab)$, $y(ab)$ etc., we shall understand a proposition, which may take on any one of these four meanings, the *terms* being the *subject a,* which is written first, and the *predicate b,* which is written second. When it shall be necessary to indicate that one of these forms is false, we shall place a prime (′) to the right. Thus $x(ab)$ *is false* will be represented by $x'(ab)$, $[x'(ab)]'$ by $x''(ab)$. A comma between the terms will indicate that the term order is not settled. Accordingly $x(a, b)$ will stand either for $x(ab)$ or $x(ba)$.

In addition to the categorical forms the logician distinguishes,

1. the *Hypothetical* relationship,
   $x \angle y = x$ implies y,
   $(x \angle y)' = x$ does not imply y,
2. the *Conjunctive* relationship,
   $xy = x$ and y,
3. the *Disjunctive* relationship,
   $x + y = x$ or y.

By the *null-proposition* he understands an impossibility and by the *null-class,* a class, which contains no objects. By the *one-proposition* he understands a proposition, which is true in any given system of inference for all meanings of the terms and by the *one-class,* a class, which contains all the objects, which are in question. We shall denote the null-class and the null-proposition by the symbol o, the one-class and the one-proposition by the symbol i, and we shall from time to time replace the a and b, the x and y, by these special values. In every case it will be clear from the position of the symbol, whether class or proposition is meant.

The *syllogism* is a form of implication belonging to one of the types:

$$x(ba)y(cb) \angle z(ca),$$
$$x(ab)y(cb) \angle z(ca),$$
$$x(ba)y(bc) \angle z(ca),$$
$$x(ab)y(bc) \angle z(ca).$$

These differences are known as the first, second, third and fourth *figures* of the syllogism respectively. The two forms conjoined to the left of the implication sign are called the *premises* and the form, which stands to the right of the implication sign, is called the *conclusion.* The predicate of the conclusion is called the *major term* and points out the *major premise* and the subject of the conclusion is called the *minor term* and points out the *minor premise.* The term, which is common to the premises and which does not appear in the conclusion, is called the *middle term.* The conjunctive relationship of logic being commutative, the order of the premises is indifferent, but, as a matter of convention, we agree always to write the major premise first. This will always be possible by applying the principle, $(xy \angle z) \angle (yx \angle z)$.

Since x, y and z may take on any one of the four values, α, β, γ, and ϵ, there will be sixty-four modes in each one of the four figures, in which $x(a,b)y(b,c) \angle z(ca)$ can be expressed. Each member of this *array* of syllogistic variations is called a *mood* of the array. The true propositions of the array are called *valid moods* of the array and the other moods are called *invalid moods* of the array.

The principles of deduction, which are given below and which we shall assume as necessity requires, are, of course, not all independent, but no attempt will be made here to point out their interconnection. We shall assume:

I^2 $(xy\angle z)(z\angle w)\angle(xy\angle w)$
$(xy\angle z)(w\angle x)\angle(wy\angle z)$
$(xy\angle z)(w\angle y)\angle(xw\angle z)$

II $(xy\angle z)\angle(xz'\angle y')$
$(xy\angle z)\angle(z'y\angle x')$

III $(xy\angle z)'(w\angle z)\angle(xy\angle w)'$
$(xy\angle z)'(x\angle w)\angle(wy\angle z)'$
$(xy\angle z)'(y\angle w)\angle(xw\angle z)'$

IV $(xy\angle z)'\angle(xz'\angle y')'$
$(xy\angle z)'\angle(z'y\angle x')'$

V $(x\angle y)\angle(y'\angle x')$
$(x\angle y')\angle(y\angle x')$
$(x'\angle y)\angle(y'\angle x)$

VI $(x\angle y)\angle(xy'\angle o)$
$(xy\angle o)\angle(x\angle y')$

VII $(x\angle y)(y\angle z)\angle(x\angle z)$

VIII $(x\angle y)\angle(wx\angle wy)$

IX $(x\angle y)(x\angle y')\angle(x\angle o)$
$(x\angle o)\angle(x\angle y)$

The valid moods of the array $x(a,b)y(b,c)\angle z(ca)$, (x, y and z representing only the unprimed letters), which number twenty-one in the system of inference we are about to construct, are all gotten by the aid of principles I from the postulates given below,[3] viz.

i. $\alpha(ba)\beta(cb)\angle\beta(ca)$ ii. $\alpha(ba)\epsilon(cb)\angle\epsilon(ca)$
iii. $\gamma(ba)\gamma(cb)\angle\gamma(ca)$ iv. $\gamma(ab)\epsilon(bc)\angle\epsilon(ca)$
v. $\beta(ab)\angle\beta(ba)$ vi. $\alpha(ab)\angle\alpha(ba)$

The valid moods of the arrays, $x(a,b)y'(b,c)\angle z'(ca)$ and $x'(a,b)y(b,c)\angle z'(ca)$, of which there are twenty-three and nineteen respectively, may be derived at once from the results now obtained by principles II.

The valid moods of the array, $x(a,b)y(b,c)\angle z'(ca)$, there being one hundred and fourteen of this type, may be obtained from

[2] Most of these principles are well known to logicians. I owe my knowledge of them for the first time—but more especially I owe my knowledge of the method here employed—to certain lectures of Prof. E. A. Singer, Jr., delivered at the University of Pennsylvania about ten years ago.

[3] The operation of *simple conversion* consists in the interchange of subject and predicate. Postulates v and vi express the simple convertibility of α and β. It should be pointed out that this same character may be proven of ϵ by the aid of the characteristic postulate, $i\angle\gamma(aa)$, (see below), as follows:

$\gamma(ab)\epsilon(bc)\angle\epsilon(ca)$ yields $\gamma(aa)\epsilon(ac)\angle\epsilon(ca)$, for $a=b$, and
$[\gamma(aa)\epsilon(ac)\angle\epsilon(ca)]\,[i\angle\gamma(aa)]\angle[\epsilon(ac)\angle\epsilon(ca)]$

by the second member of I.

The non-convertibility of γ, expressed by $[\gamma(ab)\angle\gamma(ba)]'$, may be established at once by making $a=o$ and $b=i$, (see the characteristic postulates xi below).

those of the array, $x(a,b)y(b.c)\angle z(ca)$, by the aid of the additional postulates:[4]

$$\begin{array}{rl} \text{vii.} & \alpha(ba)\beta(cb)\angle\gamma'(ca) \\ \text{viii.} & \beta(ba)\,\alpha(cb)\angle\gamma'(ca) \\ \text{ix.} & \alpha(ba)\epsilon(cb)\angle\gamma'(ca) \\ \text{x.} & \alpha(ab)\angle\gamma'(ab) \\ & \qquad\alpha(ab)\angle\epsilon'(ab) \end{array}$$

The implications,

$$\alpha(ab)\angle\alpha''(ab), \qquad \gamma(ab)\angle\gamma''(ab),$$
$$\beta(ab)\angle\beta''(ab), \qquad \epsilon(ab)\angle\epsilon''(ab),$$

which we should have to use in this connection may be established thus:

iii will yield $\gamma(ba)\gamma(bb)\angle\gamma(ba)$, for $b=c$, and
$[\gamma(ba)\gamma(bb)\angle\gamma(ba)]\ [i\angle\gamma(bb)]\angle[\gamma(ba)\angle\gamma(ba)]$, by I.
Also $[\gamma(ba)\angle\gamma(ba)]\angle[\gamma(ba)\gamma'(ba)\angle o]$, by VI,
and $[\gamma(ba)\gamma'(ba)\angle o]\angle[\gamma(ba)\angle\gamma''(ba)]$, by VI.

No valid implications of syllogistic form exist, other than the ones that have now been enumerated, as will appear later on, when all of the remaining variants shall have been declared untrue. It will be necessary at this point to state the characteristic postulates of the logic, which we have been constructing. It has not been essential to do this up to now, because every form of inference, which is valid here, is also valid in the common logic. They are required in order to establish the invalidity of those forms, which ordinarily taken to be valid, are invalid here. These characteristic postulates, which are, however, evidently not independent,[5] are:

[4] It must be noticed that $\alpha(ab)\angle\beta'(ab)$ and $\alpha(ab)\angle\epsilon'(ab)$ may be derived from vii and ix respectively through the use of the characteristic postulate, $\gamma'(aa)\angle o$, as follows:

ix yields $\alpha(ba)\epsilon(ab)\angle\gamma'(aa)$, for $a=c$, and, by I,
$[\alpha(ba)\epsilon(ab)\angle\gamma'(aa)]\ [\gamma'(aa)\angle o]\angle[\alpha(ba)\epsilon(ab)\angle o]$.
Also $[\alpha(ba)\epsilon(ab)\angle o]\angle[\alpha(ba)\angle\epsilon'(ab)]$, by VI, and
$[\alpha(ab)\angle\alpha(ba)]\ [\alpha(ba)\angle\epsilon'(ab)]\angle[\alpha(ab)\angle\epsilon'(ab)]$, by VII.

A similar derivation will yield $\alpha(ab)\angle\beta'(ab)$.

From these two, together with x, by principle V, we obtain immediately,

$$\epsilon(ab)\angle\alpha'(ab), \qquad \epsilon(ab)\angle\beta'(ab),$$
$$\gamma(ab)\angle\alpha'(ab), \qquad \beta(ab)\angle\alpha'(ab),$$

results which we have constantly to employ in conjunction with the valid moods of the array, $x(a,b)y(b,c)\angle z(ca)$, in order to obtain the valid moods of the array, $x(a,b)y(b,c)\angle z'(ca)$.

[5] The selection of these postulates, while in large measure arbitrary, has been such as not to contradict the definition of the null-class. Then o and $\imath$ in the parentheses refer, of course, to the null- and the one-class; the o to the right of the implication sign refers to the null-proposition.

$$\begin{array}{llll} \text{xi. } \alpha(oo)\angle o & \alpha(oi)\angle o & \alpha(io)\angle o & \alpha(ii)\angle o \\ \beta(oo)\angle o & \beta(oi)\angle o & \beta(io)\angle o & \beta'(ii)\angle o \\ \gamma'(oo)\angle o & \gamma'(oi)\angle o & \gamma(io)\angle o & \gamma'(ii)\angle o \\ \epsilon'(oo)\angle o & \epsilon'(oi)\angle o & \epsilon'(io)\angle o & \epsilon(ii)\angle o \end{array}$$

If we postulate in addition, $\alpha(aa)\angle o$ and $\gamma'(aa)\angle o$, the members of the following sets may be made to depend upon those which have just been written down, i. e.,

$$\begin{array}{ll} \alpha(aa)\angle\alpha'(aa) & [\alpha'(aa)\angle\alpha(aa)]' \\ [\beta(aa)\angle\beta'(aa)]' & [\beta'(aa)\angle\beta(aa)]' \\ [\gamma(aa)\angle\gamma'(aa)]' & \gamma'(aa)\angle\gamma(aa) \\ [\epsilon(aa)\angle\epsilon'(aa)]' & [\epsilon'(aa)\angle\epsilon(aa)]' \end{array}$$

$$\begin{array}{ll} [\beta(ab)\angle\gamma'(ab)]' & [\gamma(ab)\angle\beta'(ab)]' \\ [\gamma(ab)\angle\epsilon'(ab)]' & [\epsilon(ab)\angle\gamma'(ab)]' \\ [\gamma(ab)\angle\gamma'(ba)]' & \end{array}$$

In continuation of our task of deducing the invalid moods of the syllogism, it will be convenient to begin with the array, $x(a,b)y(b,c)\angle z'(ca)$. Ninety-six of the invalid moods of this type may be reduced to simpler invalid forms of inference already established, and so shown to be invalid, (1) either by identifying terms in a γ-premise or a γ- conclusion and suppressing the part $\gamma(aa)$, or (2) by replacing the subject and predicate of a β-premise or a β-conclusion by unity and suppressing the part $\beta(ii)$.[6] The

[6] (1) Suppose $\beta(ba)\gamma(cb)\angle\epsilon'(ca)$ to be a valid mood.
$\beta(ba)\gamma(cb)\angle\epsilon'(ca)$ yields $\beta(ii)\gamma(oi)\angle\epsilon'(oi)$ for $a=b=i, c=o$;
$[\beta(ii)\gamma(oi)\angle\epsilon'(oi)]\,[\epsilon'(oi)\angle o]\angle[\beta(ii)\gamma(oi)\angle o]$, by VII;
$[\beta(ii)\gamma(oi)\angle o]\angle[\beta(ii)\angle\gamma'(oi)]$, by VI.
But the last result is invalid and
$\therefore \beta(ba)\gamma(cb)\angle\epsilon'(ca)$ is invalid.
(2) Suppose $\beta(ba)\beta(bc)\angle\gamma'(ca)$ to be a valid mood.
$\beta(ba)\beta(bc)\angle\gamma'(ca)$ yields $\beta(ba)\beta(ba)\angle\gamma'(aa)$, for $a=c$;
$[\beta(ba)\beta(ba)\angle\gamma'(aa)]\,[\gamma'(aa)\angle o]\angle[\beta(ba)\beta(ba)\angle o]$, by VII;
$[\beta(ba)\beta(ba)\angle o]\angle[\beta(ba)\angle\beta'(ba)]$, by VI.
But the last result is invalid and
$\therefore \beta(ba)\beta(bc)\angle\gamma'(ca)$ is invalid.
The four non-implications,

$$\begin{array}{ll} [\alpha(ab)\angle\alpha'(ab)]', & [\gamma(ab)\angle\gamma'(ab)]', \\ [\beta(ab)\angle\beta'(ab)]', & [\epsilon(ab)\angle\epsilon'(ab)]', \end{array}$$

which we should continually have to employ in applying the method of the last example, may be established as follows:
$[\alpha(ab)\angle\alpha'(ab)]\angle[\alpha(ab)\alpha''(ab)\angle o]$, by VI;
$[\alpha(ab)\alpha''(ab)\angle o]\,[\alpha(ab)\angle\alpha''(ab)]\angle[\alpha(ab)\alpha(ab)\angle o]$, by I;
$[\alpha(ab)\angle\alpha(ab)\alpha(ab)]\,[\alpha(ab)\alpha(ab)\angle o]\,[\angle[\alpha(ab)\angle o]$, by VII;
$[\alpha(ab)\angle o]\angle[\alpha(ab)\alpha(ca)\alpha(cb)\angle o]$, by VIII;
$[\alpha(ab)\alpha(ca)\alpha(cb)\angle o]\angle[\alpha(ab)\alpha(ca)\angle\alpha'(cb)]$, by VI.

remaining forty-six invalid moods of this type may be derived from results already obtained by the aid of principles III and IV and the additional postulates.

xii. $[\alpha(ba)\alpha(cb)\angle\alpha'(ca)]'$	xiii. $[\alpha(ba)\epsilon(cb)\angle\epsilon'(ca)]'$
xiv. $[\alpha(ba)\beta(cb)\angle\beta'(ca)]'$	xv. $[\beta(ba)\beta(cb)\angle\epsilon'(ca)]'$
xvi. $[\alpha(ba)\gamma(cb)\angle\gamma'(ca)]'$	xvii. $[\gamma(ba)\alpha(cb)\angle\gamma'(ca)]'$

All but eight[7] of the two hundred and thirty-five invalid moods of the array, $x(a,b)y(b,c)\angle z(ca)$, may now be obtained, those of the arrays, $x'(a,b)y(b,c)\angle z'(ca)$ and $x(a,b)y'(b,c)\angle z'(ca)$ can be gotten at once by principles III and IV, and it will be easy to show that all of the two hundred and fifty-six moods of each one of the arrays $x'(a,b)y(b,c)\angle z(ca)$, $x(a,b)y'(b,c)\angle z(ca)$, $x'(a,b)y'(b,c)\angle z(ca)$ and $x'(a,b)y'(b,c)\angle z'(ca)$ are invalid, without making any further assumptions.

Henry Bradford Smith.

University of Pennsylvania.

But this last result contradicts postulate xii below.

$\therefore \alpha(ab)\angle\alpha'(ab)$ is invalid.

The second and third forms, $\beta(ab)\angle\beta'(ab)$ and $\gamma(ab)\angle\gamma'(ab)$, will be seen to be established as invalid on making $a=b=i$, and the last, $\epsilon(ab)\angle\epsilon'(ab)$, on making $a=b=o$.

[7] The mood $\beta(ba)\epsilon(cb)\angle\gamma(ca)$, which will have to be set down as a postulate, implies at once its own invalidity in the other three figures. The remaining four follow from $\gamma(ba)\beta(cb)\angle\beta(ca)$.

The two examples which follow will be enough to illustrate the method of deducing the moods of this array.

(1) $\alpha(ba)\beta(cb)\angle\beta'(ca)$ is an invalid mood.

$[\alpha(ba)\beta(cb)\angle\beta'(ca)]'[\alpha(ca)\angle\beta'(ca)]\angle[\alpha(ba)\beta(cb)\angle\alpha(ca)]'$, by III.

(2) Suppose $\alpha(ba)\beta(cb)\angle\gamma(ca)$ to be a valid mood.

$[\alpha(ba)\beta(cb)\angle\gamma(ca)]\,[\alpha(ba)\beta(cb)\angle\gamma'(ca)]\angle[\alpha(ba)\beta(cb)\angle o]$, by IX;

$[\alpha(ba)\beta(cb)\angle o]\angle[\alpha(ba)\beta(cb)\angle\alpha(ca)]$, by IX.

But $\alpha(ba)\beta(cb)\angle\alpha(ca)$ is invalid by the last example.

$\therefore \alpha(ba)\beta(cb)\angle\gamma(ca)$ is invalid.

LOGIC IN NUMBERS.

Logic is the science of consistency. Given a set of propositions; the fundamental problem of logic is to determine whether the propositions can be true together. It is possible to reduce this fundamental problem to a purely mathematical form and to transfer the problem from the domain of philosophy to the domain of mathematics. The sytem of Boole and other systems derived from his, employ mathematical symbols in logical investigations, but meanings are attributed to the symbols that prevent the application of ordinary mathematical processes and it is impossible to proceed beyond cases of extreme simplicity. By the method here outlined the problem of the logician, however intricate, may be expressed as a purely mathematical problem, in the statement of which + means *plus* and – means *minus* and $2+2=4$. The propositions may be expressed as a set of whole numbers and the consistency of the propositions depends upon whether the numbers can be divided into two groups such that the sum of the numbers placed in one group is equal to the sum of the numbers placed in the other group. If the two equal groups can be formed, the propositions are consistent. If it is impossible to form the two equal groups, then the propositions are inconsistent, that is to say, the propositions cannot all be true. Whether the two equal groups can be formed from the numbers arising from the proposition, is a question for the mathematician to answer. It will be necessary to define a few terms.

The sum of the coefficients of a polynomial, all being regarded as positive, is the *weight* of the polynomial. Half the weight is the *semi-weight*. If the polynomial can be made equal to 0 by making each variable either $+1$ or -1, the polynomial is a *balance*. If the variables are written down (without coefficients) and those that are made $+1$ in order to make the polynomial vanish are written with the + sign, and those that are made -1 in order to make the polynomial vanish are written with the – sign, the expression is a *solution* of the balance.

Thus $2a+3b+7c-5d-e$ is a balance, for if a, c, d and e are each $+1$ and b is -1, the polynomial becomes $2-3+7-5-1$, which is equal to 0. $a-b+c+d+e$ is a solution of the balance. The weight of the balance is $2+3+7+5+1$ or 18.

Note that if it is possible to divide the coefficients of a polynomial, all being regarded as positive, into two groups such that the sum of the coefficients placed in either group is the semi-weight of the polynomial, the polynomial is a balance. If the variables belonging to the coefficients placed in one of such groups are written with the signs they have in the balance, and the variables belonging to the coefficients placed in the other group are written with signs contrary to the signs they have in the balance, the variables form a solution.

If letters that do not appear in a balance are added to or are subtracted from a solution, the expression is still a solution. Such additional letters may be regarded as being in the balance with the coefficient 0.

Thus $a-b+c+d+e+f-g$ is a solution of the balance, $2a+3b+7c-5d-e$.

If two or more balances have a common solution they are said to be *consistent* and to form a consistent system, and the common solutions are solutions of the system. But if there is no common solution the balances are *inconsistent* and form an inconsistent system.

Thus a system composed of the balances,

$$\begin{array}{lllllllllll} 13m & +4a & +10b & +3c & +9d & +n & +3p & +9q & & & \\ 4m & +3a & & +c & -2d & & & & +r & +3s & \end{array}$$

is consistent, for $m+a-b-c-d-n-p+q+r-s$ is a solution of both balances. But a system composed of the same two balances and the balance,

$$\begin{array}{lllllllllll} m & & -b & -c & & & & & & & +t \end{array}$$

is inconsistent, for these three balances have no common solution.

If any letter must have the same sign as another letter in every solution of a balance or system of balances, the two letters are said to be *identical,* and two letters that have different signs in every solution are said to be *contradictory*.

Thus, in the system of two consistent balances mentioned in the preceding paragraph, the letters b and c are identical and m and b are contradictory, and so are m and c.

If a polynomial is constructed such that all solutions it may have are solutions of a system of balances, and such that all solutions there may be of the system are solutions of the polynomial, the polynomial is a *summary* of the system.

Thus $4m+a+b+n+3c+3d+3p$ is a summary of the two balances, $m+a+b+n$ and $m+c+d+p$.

The *sum* (and the difference) of any two given consistent balances is a balance consistent with them.

If there are two balances, a summary may be obtained by adding them together (or by subtracting one from the other), after multiplying one of them by any number that is greater than half the weight of the other.

Thus, if there are two balances,

$$\begin{array}{llllll} 2m+2a & +b & +c & +n & +p & \\ m & -b & -c & & & +t \end{array}$$

multiply the first by any number greater than 2 (which is half the weight of the second balance), say 3, and then add the second. We obtain,

$$7m+6a+2b+2c+3n+3p+t,$$

which is a summary of the two balances.

We may obtain a summary of any system of balances by adding the balances together (or by adding some and subtracting others) after multiplying the first of the balances by 1 and each of the others by successive powers of any number greater than half the weight of the balance that has the greatest weight.

Thus, if we have the system,

$$\begin{array}{lll} 4m+a+c+e+g+i+n & & \\ 3m+a+c+e+g+i & & +b+d+f+h \\ 4m \qquad +e & & +b+d+f+h+p, \end{array}$$

a summary may be obtained by adding the balances together after multiplying the first by 1, the second by any number greater than 6 (which is half the weight of the second balance, which has the greatest weight), say 7, and the third by 49 (the square of 7). We thus obtain

$$221m+8a+8c+57e+8g+8i+n+56b+56d+56f+56h+49p.$$

This expression is a summary of the given system of three balances. (This summary is not a balance; hence it may be inferred that the system from which it is derived is inconsistent.)

A summary of a system of balances may be at once obtained thus: Arrange the balances so that the several letters, as they occur in the different balances, are each in a separate column. (When a letter that appears in the system does not appear in any particular balance, it may be supposed to be inserted in that balance with the coefficient 0.) The coefficient of any letter in the summary is a number obtained by writing the coefficients of the letter in the order in which they appear in the column containing that letter, commencing with the coefficient in the first balance as standing in the unit's place, the number so obtained being regarded as expressed in any scale whose radix is greater than half the weight of the balance that has the greatest weight.

Applying this method to the example in the preceding paragraph, the coefficient of m is 434; that of a is 11; that of n is 1; that of b is 110; that of p is 100; etc.; all read in any scale greater than 6. The summary may be written,

$$434m+11a+11c+111e+11g+11i+n+110b+110d+110f+110h+100p.$$

If any of the coefficients of the balances have negative signs the same rule may be applied for obtaining the coefficients of a summary, but in the number expressing the coefficient of such letter in the summary, negative numerals are used to correspond with the negative coefficients of the letter in the balances.

Thus, in the system

$$\begin{array}{l} 3m+2a+n+p+t \\ 3m \qquad\qquad\quad +t+2d+q+r \\ \;m-\;a \qquad\qquad\quad -\;d \qquad\;\; +s \end{array}$$

a summary is

$$133m+(-1)02a+n+p+11t+(-1)20d+10q+10r+100s$$

the numbers being in any scale greater than 4. The summary may be written in the scale of 5 thus,

$$133m-43a+n+p+10t-30d+10q+10r+100s.$$

If a summary is a balance, the system from which it is derived must be consistent; if, however, a summary is not a balance, the system from which it is derived must be an inconsistent system. The consistency, therefore, of a system of balances may be tested by reading off a summary and determining whether the summary is a balance.

Universal propositions may be expressed as balances. A balance represents a universal proposition, if all its solutions represent all cases that are conceivable, if the proposition be true. In the solution of a balance, or system of balances, let m (the first letter of *mundus*) represent something that is conceivable as existing in the universe of discourse; let $m+a$ (or $-m-a$) represent something that is conceivable as existing and as having an attribute denoted by a; let $m-a$ (or $-m+a$) represent something that is conceivable as existing and as not having the attribute denoted by a; similarly, let $m+a-b$ (or $-m-a+b$) represent something that is conceivable as existing and as having the attribute denoted by a and as not having the attribute denoted by b; generally, let any solution represent something that is conceivable as having the attributes denoted by the letters with one sign and as not having the attributes denoted by the letters with the other sign.

The balance $m+a+b+n$ expresses the universal proposition "No a is b," for all its solutions represent all cases that are possible, if the proposition be true. In every solution one of the letters, m, a and b, has a sign different from that of the other two, which is exactly what is required by the proposition.

The balance $m+a-b+p$ expresses the universal affirmative, "All a is b." In every solution, if a has the same sign as m, b has also the same sign as m; but if m and a have different signs, then b may be + or −.

Sometimes it may be convenient to express a proposition by a system of balances instead of by a single balance. Thus "All a is b" may be expressed by the two balances.

$$\begin{array}{l} m+a+q+r \\ \phantom{m+a+{}} q+r+b+s \end{array}$$

Any universal proposition may be stated as a balance or system of balances. The following are given as illustrations:

Whatever is conceivable is a: $m-a$.

Nothing can be a: $m+a$.

a and b are identical: $a-b$.

a and b are contradictory: $a+b$.

a is neither b nor c: $\begin{array}{l} m+a+b+q \\ m+a \phantom{{}+b+q} +c+r. \end{array}$

a is either b or c, or both b and c: $\begin{array}{l} m+a+p+q \\ \phantom{m+a+{}} p \phantom{{}+q} +b+c+r. \end{array}$

a is either b or c, but not both: $\begin{array}{l} m+a-b-c+s+t \\ m+a+b+c \phantom{{}+s+t} +u+v. \end{array}$

Of the three terms, a, b and c, two, at least, are absent: $2m+a+b+c+n$.

Everything has at least two of the attributes denoted by a, b and c: $2m-a-b-c+p$.

Of n things, a_1, $a_2 \ldots a_n$, p at least are present and q at least are absent: $(q-p)m+a_1+a_2 \ldots +a_n+r_1+r_2+\ldots+r_{n-p-q}$.

To test the consistency of universal propositions, therefore, they may be expressed as a system of balances, a summary may be read off, and whether the propositions are or are not consistent depends upon the purely mathematical question whether the summary is a balance.

Universal propositions express rules that must be observed in every solution of a system of balances expressing the propositions. A particular proposition expresses a rule that must be observed in at least one solution of the system expressing universal propositions. If a set of universal propositions and a particular proposition is given, to test their consistency a summary may be obtained of the universal propositions, and then certain variables may be given + or − signs in accordance with the particular proposition; then, if the summary is a balance, the particular proposition is consistent with the universal propositions; otherwise, it is not. Thus, if universal propositions and the particular proposition, "Some *a* is *b*" are given, there must be a solution of the summary of the universal propositions in which *m*, *a* and *b* have the same sign. If in the summary *m*, *a* and *b* are made $+1$, and the summary is still a balance, the propositions are consistent. If there are several particular propositions, the summary should be tested as to each one separately. It is to be observed that there is no implication that any solution of the summary must comply with more than one of the particular propositions.

The method here outlined is a general method of converting logical problems into a mathematical form. It is possible, however, to solve many problems by manipulating balances otherwise and there are a number of important theorems in regard to these expressions, but it would be beyond the purpose of this paper to enter upon a discussion of them. The following problems may serve to illustrate a method of obtaining solutions of balances.

If five chess queens are placed on a board containing 25 cells arranged in the form of a square, so that no two queens attack each other, prove that neither of the diagonals of the square can be without a queen.

Let (x, y) represent a cell which is the xth from the left and the yth from the bottom of the board. Of the cells, (1, 5), (2, 4), (3, 3), (4, 2) and (5, 1), forming a diagonal, four at least are vacant. Hence the balance,

$$4m+(1,5)+(2,4)+(3,3)+(4,2)+(5,1)+p_1 \qquad \text{(I)}$$

Similarly fourteen other balances may be formed, each of which contains $4m$ and p with a different suffix, and also five cells indicated

1 8. 12.	3 9 12.	7.. 12.	3 10. 12.	2 11. 12.
4 8. 13.	1 9. 13.	3 4 6. 13.	2 10. 13.	6 11. 13.
7.. 8.	3 4 5 9.	1 2 7..	3 5 6 10.	7.. 11.
4 8. 14.	2 9. 14.	4 5 6 14.	1 10. 14.	6 11. 14.
2 8. 15.	5 9. 15.	7.. 15.	5 10. 15.	1 11. 15.

in the diagram by a number corresponding with the suffix of p to be used with them. Thus, one of such balances is

$$4m+(2,5)+(4,5)+(3,4)+(2,3)+(4,3)+p_3.$$

If the fifteen balances are added together, after multiplying by 3 the balance that has p_7 and by 2 the balances that have p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, and p_{15}, we obtain,

$$100m+5S+p_1+p_2+p_3+p_4+p_5+p_6+3p_7+2(p_8+p_9+p_{10}+p_{11}+p_{12}+p_{13}+p_{14}+p_{15}) \quad \ldots\ldots\ldots\ldots \text{(II)}$$

where S represents the 25 cells of the board.

Since 5 cells are occupied and 20 are vacant, we have the balance

$$15m+S \quad \ldots\ldots\ldots\ldots \text{(III)}$$

Subtracting 5 times (III) from (II) we get

$$25m+p_1+p_2+p_3+p_4+p_5+p_6+3p_7+2(p_8+p_9+p_{10}+p_{11}+p_{12}\ p_{13}+p_{14}+p_{15}).$$

The weight of this last balance is 50, and the coefficient of m is 25. Hence m is the contradictory of each of the p's. Therefore,

$$m+p_1 \quad \ldots\ldots\ldots\ldots \text{(IV)}$$

is a balance. Subtracting (IV) from (I) we get,

$$3m+(1,5)+(2,4)+(3,3)+(4,2)+(5,1).$$

In every solution of this balance one of the cells has the same sign as m, and the other four cells have the opposite sign. Hence, one of these five cells, forming that diagonal, must be occupied and the other four must be vacant.

Similarly it may be shown that one of the cells forming the other diagonal must be occupied.

Charles P. R. Macaulay.

Chicago.

CURRENT PERIODICALS.

In *Science Progress* for January, 1918, "Recent Advances" occupy about one-third of the number. The subjects dealt with in this number are: Mathematics (10 pages) ; Astronomy (7) ; Physics (5½) ; Physical Chemistry (3) ; Inorganic Chemistry (3) ; Organic Chemistry (4¼) ; Geology (5¾) ; Mineralogy and Crystallography (6½) ; Botany (3) ; Plant Physiology (5½) ; Zoology (6) ; Paleontology (5) ; and Anthropology (2). J. Reilly and W. N. Rae give an account of recent work in the determination of the density of liquids. By Lamb and Lee's "refinement of the hydrostatic method it is possible to obtain results correct to one unit in the seventh decimal place." The pyknometer method is criticised, and various specific gravity bottles are described.—James Small gives an account of the "age and area" law associated with the name of J. C. Willis of the Ceylon Botanic Gardens. The law is thus stated: "The geographical distribution of a species within a fairly uniform country not broken by serious barriers depends upon the age of that species within that country," with certain limitations. He gives an account of the controversy that has arisen, discusses the whole question, and states that his own new work on the evolution and geographical distribution of the *Compositae* has found the "age and area" law "very valuable indeed, confirming in the case of every tribe the phylogenetic conclusions reached in the study of the morphology and physiology of the subdivisions of that large and undoubtedly recent family."—K. M. Parker sums up all that is known up to the present of the structure and development of the pituitary body in all classes of vertebrata.—J. Reid Moir sees no valid reason for accepting the doctrine that Asia witnessed the earliest stages of man's evolution, and sees no cause or causes to preclude England, as far as pure theory is concerned, from having the distinction of being the home of earliest man. A Pliocene Age is indicated by our paleolithic flint implements, the Piltdown treasures, and other finds, which are all sufficiently significative to warrant care in awarding any preeminence in men's pre-paleolithic history to "unknown" Asia.—W. C. McC. Lewis gives under the heading "Popular Science" the first part of a paper "On the Structure of Matter,"—excellent, as far as it goes; but, if it prove to be as "popular" as it is good, we would have much reason to be pleased with the knowledge and taste of the masses in Britain.—Lord Leverhulme deals in an optimistic spirit with the abolition of slums. Being himself a broad-minded and public-spirited man, he sees no serious difficulty ahead in the assault upon vested interests.—Philip E. B. Jourdain reviews in characteristic style the very remarkable collection of papers and addresses published last year by A. N. Whitehead under the title *The Organization of Thought.*—Characteristic also is the notice of Garrison's *History of Medicine,* in which the Editor curtly demolishes certain claims: "I cannot see how F. Schaudinn did anything of any importance whatever in connection with malaria, except to make bad mistakes." Thus are reputations made, and unmade!

* * *

In *Scientia* for February, 1918, Andrew C. D. Crommelin, in his article on "The Galactic Circle as a Plane of Reference for Star Places," offers for criticism the particular plane that he has suggested for adoption, and sets forth a scheme which, as he says, "has been so widely advocated, and appeals to so many minds from its symmetry and simplicity, that I have little doubt that it will sooner or later be realized." He calls for suggestions and amendments, and hopes with their aid to place the scheme in such a form as "to command general assent."—U. Pierantoni develops from his own researches and those of others an argument which tends to throw a flood of light on the long-vexed problem of phosphorescence. He points out that further work along definite lines is still needed, and prophesies that the zoologist, botanist, physiologist, chemist and perhaps the pathologist, will all be called upon to play a part in the discovery of the mystery connected with photogenic bacteria, and the part played by micro-organisms in the phenomena of luminescence. γ

VOL. XXVIII OCTOBER, 1918 NO. 4

THE MONIST

WHEELER'S HUNDREDTH-CENTURY PHILOSOPHY.

OCCASIONALLY a writer, unversed in academic formulas, arises from the rank and file of men to challenge the truth of our most cherished creeds and combat tenets that bear the sanction of armies of sages. Jacob Boehme, the "Shoemaker Philosopher," despite his lack of scholastic training, deemed his inspiration commanding and potent. Flashes of light that to him could have no other source than the *Urgrund* of Eternal Verities broke in upon his soul. Their illuminations must be reported to the world or his charisma would fail, and God's purposes be thwarted.

Such, too, though with a different attitude toward the traditional God-conception, was the propagandic spirit of the late Dr. Charles K. Wheeler, author of the *Hundredth Century Philosophy* series.[1] His earlier works, at least, contain a thoughtful message to the philosophic investigator. The problems presented were mainly fundamental, and their treatment commendably original. His ontology is radically at variance with prevailing systems, and in his psychology many startling propositions appear.

The author's unfortunate idiosyncrasies of expression must prove a barrier to popularity until some patient interpreter recasts the English into forms more conventional,

[1] Published by Mrs. C. A. B. Wheeler, 330 Massachusetts Ave., Lexington, Mass.

and more readily intelligible. The fault lies not in a novel nomenclature, but in the use of unwonted and unwarranted grammatical forms, compelling the suggestion that some type of aphasia worked havoc amid the subtle play and trenchant piercings of an intellect otherwise endowed with analytic powers of a high order. It is to be regretted also that the author indulged so freely in an undignified pasquinade in his final volume: *A Critique of Pure Kant.*

But his work deserves attention for its unique methods of attack upon the Kantian metaphysics, whatever value may be assigned to his interpretation of the tenets of the Königsberg sage. The Ontological Realism which our author submits as a substitute for all academic philosophies based upon idealistic epistemologies as presented by the triumvirate, Kant, Hegel, and Schopenhauer, merits investigation also, as it presents many interesting features.

But despite the iconoclasm and revolutionary boldness of Wheeler's intellectual ventures, his work has, as yet, won but little attention from philosophic inquirers. This neglect results, no doubt, in large measure from what the *Westminster Review* terms "a style excruciatingly tortuous and obscure." Nevertheless the *Review* finds "much that deserves serious attention, and which it would be difficult to refute." "The author," says the reviewer, referring to Wheeler's *Critique of Pure Kant,* "often speaks out—we wish we could say always in plain English—what many students must have thought."

In common with all reformers and idol-breakers, Wheeler regarded his affirmations final and abiding. But finding in his latter days that the world paid little heed to his revelations, he exclaimed, with the pathos of a hope deferred, "Some far day I will be rediscovered as was the revealer of Mendle's Law." But his importunate desire for recognition seems doomed, at least for a time.

Boston papers, recording his death, quoted a line originally written on the death of Bayard Taylor, "Dead he lay amid his books," for in his casket was placed a set of the volumes he had written, and pine-branches were strewn upon his coffin "as fitting emblem of a long and fruitful life," for Wheeler's learning had long been recognized, and his work as a lecturer had often given inspiration to Boston's intellectual circles. Besides, he had for years served the city officially as Assistant Physician. His story is that of many a passionately earnest seeker after Truth, and his confidence in his conclusions is echoed in the somewhat presumptious, or, perhaps, despairing title that identified his speculative volumes: *The Hundredth Century Philosophy.*

But let us learn somewhat of the thinker who assigned to the basis of all academic systems of philosophy no more substantial a value than that of an evolutionary and transient product, and who denied the primacy and transcendency of mind and consciousness, designating them as degenerate forms of a Primordial Mental, even as is matter also. Let us consider the dialectic processes by which our author essays to prove the invalidity of our most intimate and persistent concept, that of our being conscious and self-conscious egos. Let us learn why he refuses to subsume sensibility and memory under the category of mental attributes, and how he argues that all conviction of truth, whether in mathematics or in morals, reaches back into sensuous experience, and even into the mechanicality of that experience, for its source. And let us trace briefly the arguments through which are challenged all ontologies that deny the existence of an outside world absolute and identify all its manifestations with mental experience. For these and other theorems quite as startling constitute the contributions Wheeler has offered to the thinkers of the world.

The most fundamental of his postulates is that mind and consciousness are not ultimate entities, but the aftermath of a Primordial Mental of wholly different character, and of which matter is, perhaps, a basic manifestation. This Primordial Mental, and not its by-product, human reason, is the only "substance" that can properly serve as a basis of knowledge, even though that reason be assumed to find its validity in transcendental realms, as the root of Yggdrasil was fabled to pierce the mystic kingdom of Hela, and draw its sustenance thence.

While products of no state of the physical of which we yet have any knowledge, mind and consciousness will nevertheless be explained eventually in terms interchangeable with definitions of the material; for matter, being of a nature self-existent, must be esteemed of a higher grade of being than are the dependent existents, life, mind, and consciousness, in the media of which thought revels with such assumed superiority.

"Whether consciousness is limited to the brain and nervous tissue of animal life, or whether not, no one knows, or is ever likely to know, with the probabilities, however, that it is not. But consciousness, whether obtaining outside animal life, or only inside, results from an event logically and historically anticipating it, and is, therefore, in any case, itself nothing aboriginal, nothing primary, but is an evolutional product lamentably restricted in function." But the Primordial Mental is both soul and body of the universe, for matter is the mental *in statu quo,* perhaps a permanent, normal manifestation of the Absolute Reality, as indeed was posited millenniums ago by the Vedantists in their creeds of Prana.

Upon the modes of the activity of this substantive entity depends the character of all human mentality. These protean manifestations, however, present no clue to the ultimate nature of the substratum. A ball or cube whirled rapidly yields to the eye the phenomenon of a stable circle from which no inference could be adduced of the revolving object producing the delusion. Matter can be cognized only through, or in the phases of, its activities, and our interpretations of these phenomena are perhaps as erroneous as is the sense of the ring produced by the revolving ball. Nevertheless, these interpretations serve our uses; for the whole nexus of our interrelations with the manifest universe involves the application of these interpretations.

The ultimate is, therefore, to us only the activity of the Absolute. Mind itself is an activity. Introspection reveals nothing more than the acts of perceiving, feeling, conceiving, etc. Of what these acts are the manifestations we possess no faculties for discerning. "Can a thing in motion," asks Wheeler, "be the motion of that in motion? Can the ball itself be what is only the motion of the ball? Is there identity of nature between the clock which goes and the going of the clock?. . . . How then shall that which is active be itself mind, if what is its activity be mind?"

We cannot reason back from the finite to the unconditioned infinite forms of mind. Analogies are conceivable and inviting, but not proofs. "It is absolutely unthinkable," says Wheeler, "that the infinite shrunken to finite should not suffer a jolt in the transition that constitutionally must radically alter it as well as circumscribe it."

Wheeler's demonstration of his postulate that conscious mind is but a by-product is presented in the following quotation: "Modern science informs us that visual light is due to the impact of ether-vibrations on eye and brain. Vibrations are something mechanical. So that visual light is due to the impact of something mechanical on something. But science, again, informs us that to a change, itself something mechanical, of the wave-lengths and of the frequency of their impact (also something mechanical) is due a change of consciousness, that is to say, of its content, from a consciousness, say of red, to a consciousness of blue, green, or other color. But now, if to a change of the mechanical making the impact, is due a change of consciousness, that is, of its content, then, *prima facie,* to the mechanical minus the change, that is, to the mechanical *in statu quo,* is due content of consciousness at all, since we know the latter only as with content as we know reflection in a mirror only as with objects before it for reflection. But now again, consciousness due to the impact of something mechanical on something, is consciousness preceded, of course, by that something making the impact and on which the impact is made, and which, as preceding, is itself the thing of primacy and transcendency, and not consciousness that is such, which itself then obtains only as a development, and as only a development then only as an aftermath."

The most primary movement of the Absolute Reality of which we have any knowledge, says Wheeler, is that centrifugal gyre by reason of which solar and stellar revolutions result. The centripetal, or return-stroke of this movement, with the ensuing impact upon itself, engenders life, mind, and consciousness.[2] These products are in-

[2] "I have sought to make evident that consciousness springs of the impact of the Absolute Reality on *itself*. Indeed, if I might be indulged in a certain distinction between, say, Absolute Being and Existent Being, that I may express an idea, then I would say that the Absolute Reality's most primary move out of Absolute Being into Existent Being, or what is recognized as the manifest universe, is one of projection of itself in the centrifugal and return upon itself in the centripetal, coincident with which movement, worlds revolve around worlds and suns around suns; while synchronous with the return-stroke of that movement and impact upon itself, obtain in ascending order, existence, life, mind, consciousness. That is, everything within our knowledge takes its rise primarily in the *mechanical* consciousness.

"Let me be understood about this. Every physician makes the distinction between the predisposing cause of disease, and the exciting cause. Now, in the impact of the Absolute Reality on itself, it is the mechanical of that Reality that, as exciting cause, making impact on what, as the predisposing cause, is not the mechanical of that Reality, that gives rise to the events of which I spoke, and to the event of consciousness, with the rest. Clearly, if this be so, then consciousness is not the primary thing, but the Absolute Reality, itself consciousless. With it is lodged, latently, both sorts of cause. Water freezes at 32°; but the temperature can be carried lower without freezing if the water is not disturbed. But touch it with a stick, and instantly it congeals. The water, however, has no power to disturb itself; with it is not lodged the exciting cause but only the predisposing cause. But with the Absolute Reality is lodged the power and impulse to be its own exciting cause, that is, power to make impact on itself, which is its exciting cause to consciousness...... Moreover, if any well-nigh universal consciousness as above implied really obtain, it must yet be an aftermath of Existent Being, and no more conscious capable of perceptual cognoscence save through the efficiencies of stimuli imparted by an external world, the intermediary and interpreting medium being neural and cephalic substance in living organisms, and without which intercommunication were impossible. But the Primordial Mental possesses within itself the capacity for consciousness unassisted from without. The mental of Kant and of all academic theorists is wholly of the former type.

The brain, in its functionary capacity, is not the demiurge of consciousness, nor can it properly be said to yield the content of consciousness: it merely determines its content, such as it is. A content cannot be conceived as conscious, much less as self-conscious. Since consciousness can know only its content, that supposititious self back of consciousness, in order to become known, must become content, even though consciousness be reckoned a function or attribute of that assumed entity.[3]

Furthermore, consciousness, by the same token, cannot cognize itself. Some image, evoked by imagination, must pose as a symbol of it. But this symbol is neither conscious nor self-conscious: it is simply consciousness, Dr. Wheeler declares, "holding a lifeless picture of itself upon its lap." To the exploitation of this idea our author has devoted an

subjectively of anything going on within Existent Being than are we conscious of the bones, nerves, blood-corpuscles, and goings-on within our physical bodies."
—From a personal letter.

[3] This idea is presented in the following quotation, which also affords an example of the verbal spirality and indirectness with which Wheeler worms his way through a thought: "There can be no self-consciousness of a thing not, itself, conscious. But, if there can be no self-consciousness of a thing not itself, conscious, or until it is conscious, then, once it is conscious, there can be no self-consciousness but of what was in the consciousness of the thing simply conscious. And, as what was in the consciousness of the latter or the conscious self, the self not consciousness itself, yet, however, that something having consciousness or being conscious, was nothing of consciousness of that self (or it would be already self-conscious) nothing of that self, even, much less of that self as such, than in the any self-consciousness as of such conscious self, there could be nothing either of consciousness of that self as such, nor, even of what was that self merely. That is, the self as something distinct from consciousness, not being in the consciousness of the conscious self, would be utterly beyond the reach of any act of self-consciousness."
—*Autobiography of the I or Ego*, pp. 19-20.

ingenious volume entitled: *The Autobiography of the I or Ego.* This work was editorially reviewed some months ago in *The Open Court.* It is an iconoclastic attack, as stated above, upon our most intimate and persistent concept, that of our being self-conscious egos.

Actors, simulating others, and certain types of insane persons, losing the individuality of their personalities, and, therefore, the identities of their metaphysical egos, assume the identities of other personalities, and are invested apparently with new ego-natures and mentalities. Thus the ego seems to be a mutable and protean thing, attaching itself to any personage fancy may present with sufficient vividness, or disappearing wholly during moments of intense mental concentration or physical exertion. Perhaps in some phases of insanity and in early childhood, the very sense of being an ego-creature is lacking, and the phantasmagoria of mental states becomes an independent and not an individual experience.

Thus consciousness might overleap the bounds of the ego-self and become a free, cosmic, non-personal and uncircumscribed experience. Some such attitude as this toward the All may be the basis of the Vedantic solipsism involved in the formula: *Brahma-atman-aikyam*, the identity or unity of all selfhoods with the infinite and absolute Brahm. What but something of this nature is the Kantian "Universal Consciousness" (*Bewusstsein überhaupt*)?

But Wheeler's Primordial Mental is consciousless, yet thrilled into unrest by an urge toward self-realization, which culminates, so far as we know, in the mind of man, whose furthest reach toward self-consciousness, even yet, is only a delusive concept, a figment of fancy which is conceived by the real thinker as conscious and even self-conscious.[4] For this thinker is not the object seen, as supposed,

[4] "We do not know with absolute knowledge," says Wheeler, "that there is a thinker behind thinking....but that thinking is its own thinker, that thinking is thinker thinking, or that thought is itself thinker thinking, is quite a

in the critical moment of self-consciousness. That projection of the thinker which results in the concept or idea of a self as contradistinguished from the not-self becomes the thought-object of consciousness, subject and object thus delusively merging or identifying.

Wheeler in his attempt to exploit that presumptive entity, that psychic Achilles in his tent, back of the idea or concept of self, there is employed largely the terminology of Buddhistic psychology. It is not an entelechy or unitary, indiscerptible being, but rather "a group of qualities, having no existence independent of the mind," and dependent for the integrity of its composition upon the containing vital organism. The thinker is perhaps only a set of conditions developed in vital organization, and through which some of the fathomless potentialities inherent in matter manifest themselves in mental phenomena, and of which awareness may be reckoned a response to Reality's urge toward self-realization. What and how the thinker thinks is unquestionably referable to the structural and essential character of brain and nerve tissue.

As to the origin of the ego-concept, Wheeler seems to assign it to nothing more substantial than the recognition of the brain as a haunting presence which, in a nondescript manner, develops in consciousness an indefinite image, entertained conceptually as our ego. The brain is, indeed, the only internal organ impinging with sufficient intimacy upon the functional field of consciousness to impart a direct stimulus. For the existence of all other organs of

little too much for our minds to grasp. It is as inconceivable by us as acting without an actor, or motion without a thing in motion." Assuming there is a thinker distinct from thought, what happens in the supposed experience of self-realization or self-consciousness is this: "There is the thought-subject as well as the thought-object; and the thought-subject thought-conscious and self-conscious. In other words, there is the thinker thinking the subject, and then thinking that thought-subject conscious and self-conscious; exactly as McCready, the actor, lost in the idea of himself as King John, thinks first, in logical order, the King of his imagination, then thinks that creature of his thought conscious and self-conscious, as he must be if beside himself thinking himself that creature."

the body indirect testimony must be adduced; that is, they are purely of objective character, while the brain is, in a sense, semi-subjective. Indeed, Wheeler actually attributes to the cephalic substance a mental consistency, for he says: "The brain must be something mental, as it is absolutely unthinkable that that which should be so correlated with mind should not be itself something mental, even if yet nothing of mind or consciousness."

Thus brain-matter seems to partake of two natures. Its marvelous radio-activity, functioning inward, creates the pageantries of mental states; functioning outward with its neural tentacles, it receives and transmits sensorial material in vibratory symbols from an outlying world. As to the transmission and transmutation of these sensorial data from the substratum of mind into their specific forms amid the content of consciousness, Wheeler has much to say. Indeed this matter forms the crux of his attack upon the metaphysics of Kant.

Wheeler's attitude toward that "scandal of the universe," the problem of evil, presents a suggestion of Fatalism, though high ethical ideals characterize all his thought. His metaphysics of the ethical may, perhaps, be formulated as follows: What exists primordially and permanently exists necessarily, with all its involutions and capacity for evolving. The aboriginal Mental is without consciousness of the type known to human thought. If there exists a universal awareness as related to that which is perceived as the physical, this can have no cognoscence of what may be termed the subjective activities of the cosmos, any more than we cognize the functionings of our own vital organs. Neither could such a pervasive consciousness be, in any wise, responsible for what obtained before it in the unconscious Primordial Mental any more than our consciousness is responsible for the structural conformations and interrelations of our internal organs.

To such a consciousness the relationships and adjustments of the cosmic parts would not become a matter ratiocinated as good or evil; rather would all be good, or a subject of indifference. It is man's reading into the nexus of being the fiction of a universal or infinite personality endowed with qualities existent only as evolved human concepts that gives rise to the problem of evil.[5]

The polemic, *A Critique of Pure Kant,* is an elaborate declaration that, despite the imperfections of the mechanicality of perception, "we never come so near the Absolute Reality, certainly never so primarily and directly near, as we do in sensuous experience. No thinking can bring us so near." The interpretation which he adopts of the Kantian system as it relates to the outer world, is summarized as follows: "What does a man see when he looks, or rather what perceive when he perceives? Is it what primarily he aims at, or something short of it? According to Kant we perceive, indeed, in the direction of an outlying world which he dogmatically assumes and affirms to exist," but with which we fail to connect in any direct contiguity. "We perceive in the direction of such world, but perceive only its effects upon the mind, and nothing of that world itself:—or no, perceive not even those effects upon the mind, but only the mind's effects on those effects; which is to perceive, in fact, not quite those effects on the mind, even."

The doctrine of the phenomenality of our world, and not that of the possibility of knowledge independent of sen-

[5] "Every theist acknowledges the principle, if I may so call it, of something's obtaining of necessity, and claims it for his Being of God, who, if not originating Himself, nor anything originating Him, must obtain of necessity. The only difference between the theist and myself is that he maintains that his Being of God obtains of necessity, and I that it is the universe that does.

"But again, if the universe obtains of necessity, then nothing in it obtains of aboriginal *intent*. And now notice that this accords perfectly with the universe as an evolution, which, as making mind and consciousness a development, debars anything like intent in the beginning. And with no goodness or anything else in intent in the universe, there is left only goodness or such *in effect*."—From a personal letter.

suous experience, says Wheeler, constitutes the essence of Kant's transcendental Esthetic and Metaphysic. "In Kant's time, as well as in all time before him, the well-nigh universal conviction was a snap creation. . . . Even to-day," says our bellicose author, "the academic hierarchy at the centers of learning, in their deadly Bourbonism, go right on discoursing metaphysics in terms of a snap, or fiat creation." The primacy and transcendency of conscious mind are arbitrarily assumed, and averred to be interrelated with a universal intelligence. *A priori* intuition implies it, and yet the validity of consciousness is questioned or repudiated. The existence of an outside world was regarded by Kant, especially in his latter years, as probable; but between it and consciousness he interposed the buffer of phenomenality.

If doubt exists as to the validity of perception, as an interpreter of the external, why assume that this conscious principle, glancing inward, interprets with any more exactness the subjective phenomena? An intuitional or *a priori* deliverance may be quite as misleading, albeit quite as sincere, as is the eye's message that the sun moves in the heavens, and that the grass is green at our feet. It has been Wheeler's chief endeavor to discover and dissolve the fundamental illusions of the understanding, which are old as apperception, and as persistent as is the delusion of the sun's motion. Thus he attacks our concept of the ego, our illusion as to self-consciousness, and other errors of intuition and understanding.

No philosophy, says Wheeler, can stand permanently upon any less a foundation than that of the three primary facts of consciousness: recognition of an outlying world absolute, sensuously perceptible; awareness of our own existence; and the additional recognition of the existence of other minds than our own. Yet never in the history of speculation has a philosopher based his system upon all these primary facts.

Physical demonstrations of the existence of an outside world, and our veritable perception thereof, are essayed by Wheeler. Time and space are likewise regarded as entities, and not forms of the sensibility or conception. "These innate *a priori* forms, conceptions or what not are no more necessary conditions of cognition, no more a subjective necessity of it, than they are of a glass mirror that it should cognosce objects in order to reflect them. . . . Not finding time and space 'given' in the sensations, Kant sets himself the task of discovering them elsewhere. . . . He does not, as he expected, find the eggs in the hennery, and so is warranted, he thinks, in assuming them laid in a tree-top by a crow." Not finding them in the sensations, he arbitrarily, and without valid logical process, relegates them to the sensibility.

Of sensibility Wheeler says it is a faculty, not an activity; but "an intuition, *a priori* or any other is an activity, a form of perceiving. And so when Kant says, space, for example, is a form of the sensibility, and in the same breath that it is an intuition, he contradicts himself." Sensibility is but the potentiality of which sensation is the kinetic exponent. If the concepts space and time lay in the sensibility only "as a symphony in an orchestra, how should we have advance notice of their being there? and, indeed, what need of an *a priori* divining of space and time in the sensibility in advance of their obtaining in actuality in the sensations?"

There is implicit in Kant's doctrine of the *a priori* the assumption that as mind is it always has been since its "creation"—a complete, unitary faculty—an entelechy—a thing of primordial perfection—a Minerva sprung from the head of Jove. Let there be Mind! And there was Mind—so complete, so furnished that it can forestall the functional yieldings of all developing faculty, all sensuous deliverances, all products of the toilsomely logical activities of the understanding. But the fruit does not determine the character of the flower and the leaf. "Why then," asks Wheeler, "should we expect the faculty for entertaining things *as such,* or as abstractions, the faculty, namely, of the understanding, before it exists to meddlesomely interfere and determine the nature and functioning of the faculty for sensations, the faculty, namely, of the sensibility?" For here the concept is assumed to exist before the sensuous experience that creates it.

One of Kant's interpreters is quoted by Wheeler as saying, "Space is not an object of sense, but a fundamental *con*ception that makes external *per*ception possible." Then, says Wheeler, "the flea and the fly have no external perception unless they are endowed with one of the highest faculties of the mind, the power to entertain concepts. But they have external perception, and have it in space and time; not as things conceived, but as things primarily realized and perceived, whether subsequently conceived or not."

This in part constitutes Wheeler's account of the absoluteness of time and space, and the identification of these with the conditions logically and physically demanded for the existence of that external world which Kant, in common with all rational minds, believed to exist.

In like manner Wheeler has applied his catalytic Mind-not-aboriginal in efforts to disintegrate and destroy the logical integrity of all idealistic theories.

CHARLES ALVA LANE.

ALLIANCE, OHIO.

THE PHILOSOPHY OF LOGICAL ATOMISM.

(The following articles are the first two lectures of a course of eight lectures delivered in London in the first months of 1918, and are very largely concerned with explaining certain ideas which I learnt from my friend and former pupil Ludwig Wittgenstein. I have had no opportunity of knowing his views since August, 1914, and I do not even know whether he is alive or dead. He has therefore no responsibility for what is said in these lectures beyond that of having originally supplied many of the theories contained in them. The six other lectures will appear in the three following numbers of *The Monist*.—B. R.)

I. FACTS AND PROPOSITIONS.

THIS course of lectures which I am now beginning I have called the Philosophy of Logical Atomism. Perhaps I had better begin by saying a word or two as to what I understand by that title. The kind of philosophy that I wish to advocate, which I call Logical Atomism, is one which has forced itself upon me in the course of thinking about the philosophy of mathematics, although I should find it hard to say exactly how far there is a definite logical connection between the two. The things I am going to say in these lectures are mainly my own personal opinions and I do not claim that they are more than that.

As I have attempted to prove in *The Principles of Mathematics,* when we analyze mathematics we bring it all back to logic. It all comes back to logic in the strictest and most formal sense. In the present lectures, I shall try to set forth in a sort of outline, rather briefly and rather unsatisfactorily, a kind of logical doctrine which seems to me to result from the philosophy of mathematics—not exactly logically, but as what emerges as one reflects: a certain kind of logical doctrine, and on the basis of this a certain kind of metaphysic. The logic which I shall advocate is atomistic, as opposed to the monistic logic of the people who more or less follow Hegel. When I say that my logic is atomistic, I mean that I share the common-sense belief that there are many separate things; I do not regard the apparent multiplicity of the world as consisting merely in phases and unreal divisions of a single indivisible Reality. It results from that that a considerable part of what one would have to do to justify the sort of philosophy I wish to advocate would consist in justifying the process of analysis. One is often told that the process of analysis is falsification, that when you analyze any given concrete whole you falsify it and that the results of analysis are not true. I do not think that is a right view. I do not mean to say, of course, and nobody would maintain, that when you have analyzed you keep everything that you had before you analyzed. If you did, you would never attain anything in analyzing. I do not propose to meet the views that I disagree with by controversy, by arguing against those views, but rather by positively setting forth what I believe to be the truth about the matter, and endeavoring all the way through to make the views that I advocate result inevitably from absolutely undeniable data. When I talk of "undeniable data" that is not to be regarded as synonymous with "true data," because "undeniable" is a psychological term and "true" is not. When I say that something is "undeniable," I mean that it is not the sort of thing that anybody is going to deny; it does not follow from that that it is true, though it does follow that we shall all think it true—and that is as near to truth as we seem able to get. When you are considering any sort of theory of knowledge, you are more or less tied to a certain unavoidable subjectivity, because you are not concerned simply with the question what is true of the world, but "What can I know of the world?" You always have to start any kind of argument from something which appears to you to be true; if it appears to you to be true, there is no more to be done. You cannot go outside yourself and consider abstractly whether the things that appear to you to be true are true; you may do this in a particular case, where one of your beliefs is changed in consequence of others among your beliefs.

The reason that I call my doctrine *logical* atomism is because the atoms that I wish to arrive at as the sort of last residue in analysis are logical atoms and not physical atoms. Some of them will be what I call "particulars,"—such things as little patches of color or sounds, momentary things—and some of them will be predicates or relations and so on. The point is that the atom I wish to arrive at is the atom of logical analysis, not the atom of physical analysis.

It is a rather curious fact in philosophy that the data which are undeniable to start with are always rather vague and ambiguous. You can, for instance, say: "There are a number of people in this room at this moment." That is obviously in some sense undeniable. But when you come to try and define what this room is, and what it is for a person to be in a room, and how you are going to distinguish one person from another, and so forth, you find that what you have said is most fearfully vague and that you really do not know what you meant. That is a rather singular fact, that everything you are really sure of, right off is something that you do not know the meaning of, and the moment you get a precise statement you will not be sure whether it is true or false, at least right off. The process of sound philosophizing, to my mind, consists mainly in passing from those obvious, vague, ambiguous things, that we feel quite sure of, to something precise, clear, definite, which by reflection and analysis we find is involved in the vague thing that we started from, and is, so to speak, the real truth of which that vague thing is a sort of shadow. I should like, if time were longer and if I knew more than I do, to spend a whole lecture on the conception of vagueness. I think vagueness is very much more important in the theory of knowledge than you would judge it to be from the writings of most people. Everything is vague to a degree you do not realize till you have tried to make it precise, and everything precise is so remote from everything that we normally think, that you cannot for a moment suppose that is what we really mean when we say what we think.

When you pass from the vague to the precise by the method of analysis and reflection that I am speaking of, you always run a certain risk of error. If I start with the statement that there are so and so many people in this room, and then set to work to make that statement precise, I shall run a great many risks and it will be extremely likely that any precise statement I make will be something not true at all. So you cannot very easily or simply get from these vague undeniable things to precise things which are going to retain the undeniability of the starting-point. The precise propositions that you arrive at may be *logically* premises to the system that you build up upon the basis of them, but they are not premises for the theory of knowledge. It is important to realize the difference between that from which your knowledge is, in fact, derived, and that from which, if you already had complete knowledge, you would deduce it. Those are quite different things. The sort of premise that a logician will take for a science will not be the sort of thing which is first known or easiest known: it will be a proposition having great deductive power, great cogency and exactitude, quite a different thing from the actual premise that your knowledge started from. When you are talking of the premise for theory of knowledge, you are not talking of anything objective, but of something that will vary from man to man, because the premises of one man's theory of knowledge will not be the same as those of another man's. There is a great tendency among a very large school to suppose that when you are trying to philosophize about what you know, you ought to carry back your premises further and further into the region of the inexact and vague, beyond the point where you yourself are, right back to the child or monkey, and that anything whatsoever that *you* seem to know—but that the psychologist recognizes as being the product of previous thought and analysis and reflection on your part—cannot really be taken as a premise in your own knowledge. That, I say, is a theory which is very widely held and which is used against that kind of analytic outlook which I wish to urge. It seems to me that when your object is, not simply to study the history or development of mind, but to ascertain the nature of the world, you do not want to go any further back than you are already yourself. You do not want to go back to the vagueness of the child or monkey, because you will find that quite sufficient difficulty is raised by your own vagueness. But there one is confronted by one of those difficulties that occur constantly in philosophy, where you have two ultimate prejudices conflicting and where argument ceases. There is the type of mind which considers that what is called primitive experience must be a better guide to wisdom than the experience of reflective persons, and there is the type of mind which takes exactly the opposite view. On that point I cannot see any argument whatsoever. It is quite clear that a highly educated person sees, hears, feels, does everything in a very different way from a young child or animal, and that this whole manner of experiencing the world and of thinking about the world is very much more analytic than that of a more primitive experience. The things we have got to take as premises in any kind of work of analysis are the things which appear to *us* undeniable—to us here and now, as we are—and I think on the whole that the sort of method adopted by Descartes is right: that you should set to work to doubt things and retain only what you cannot doubt because of its clearness and distinctness, not because you are sure not to be induced into error, for there does not exist a method which will safeguard you against the possibility of error. The wish for perfect security is one of those snares we are always falling into, and is just as untenable in the realm of knowledge as in everything else. Nevertheless, granting all this, I still think that Descartes's method is on the whole a sound one for the starting-point.

I propose, therefore, always to begin any argument that I have to make by appealing to data which will be quite ludicrously obvious. Any philosophical skill that is required will consist in the selection of those which are capable of yielding a good deal of reflection and analysis, and in the reflection and analysis themselves.

What I have said so far is by way of introduction.

The first truism to which I wish to draw your attention—and I hope you will agree with me that these things that I call truisms are so obvious that it is almost laughable to mention them—is that the world contains facts, which are what they are whatever we may choose to think about them, and that there are also beliefs, which have reference to facts, and by reference to facts are either true or false. I will try first of all to give you a preliminary explanation of what I mean by a "fact." When I speak of a fact—I do not propose to attempt an exact definition, but an explanation, so that you will know what I am talking about—I mean the kind of thing that makes a proposition true or false. If I say "It is raining," what I say is true in a certain condition of weather and is false in other conditions of weather. The condition of weather that makes my statement true (or false as the case may be), is what I should call a "fact." If I say "Socrates is dead," my statement will be true owing to a certain physiological occurrence which happened in Athens long ago. If I say, "Gravitation varies inversely as the square of the distance," my statement is rendered true by astronomical fact. If I say, "Two and two are four," it is arithmetical fact that makes my statement true. On the other hand, if I say "Socrates is alive," or "Gravitation varies directly as the distance," or "Two and two are five," the very same facts which made my previous statements true show that these new statements are false.

I want you to realize that when I speak of a fact I do not mean a particular existing thing, such as Socrates or the rain or the sun. Socrates himself does not render any statement true or false. You might be inclined to suppose that all by himself he would give truth to the statement "Socrates existed," but as a matter of fact that is a mistake. It is due to a confusion which I shall try to explain in the sixth lecture of this course, when I come to deal with the notion of existence. Socrates[1] himself, or any particular thing just by itself, does not make any proposition true or false. "Socrates is dead" and "Socrates is alive" are both of them statements about Socrates. One is true and the other false. What I call a fact is the sort of thing that is expressed by a whole sentence, not by a single name like "Socrates." When a single word does come to express a fact, like "fire" or "wolf," it is always due to an unexpressed context, and the full expression of a fact will always involve a sentence. We express a fact, for example, when we say that a certain thing has a certain property,

[1] I am here for the moment treating Socrates as a "particular." But we shall see shortly that this view requires modification.

or that it has a certain relation to another thing; but the thing which has the property or the relation is not what I call a "fact."

It is important to observe that facts belong to the objective world. They are not created by our thoughts or beliefs except in special cases. That is one of the sort of things which I should set up as an obvious truism, but, of course, one is aware, the moment one has read any philosophy at all, how very much there is to be said before such a statement as that can become the kind of position that you want. The first thing I want to emphasize is that the outer world—the world, so to speak, which knowledge is aiming at knowing—is not completely described by a lot of "particulars," but that you must also take account of these things that I call facts, which are the sort of things that you express by a sentence, and that these, just as much as particular chairs and tables, are part of the real world. Except in psychology, most of our statements are not intended merely to express our condition of mind, though that is often all that they succeed in doing. They are intended to express facts, which (except when they are psychological facts) will be about the outer world. There are such facts involved, equally when we speak truly and when we speak falsely. When we speak falsely it is an objective fact that makes what we say true when we speak truly.

There are a great many different kinds of facts, and we shall be concerned in later lectures with a certain amount of classification of facts. I will just point out a few kinds of facts to begin with, so that you may not imagine that facts are all very much alike. There are particular facts, such as "This is white"; then there are general facts, such as "All men are mortal." Of course, the distinction between particular and general facts is one of the most important. There again it would be a very great mistake to suppose that you could describe the world completely by means of particular facts alone. Suppose that you had succeeded in chronicling every single particular fact throughout the universe, and that there did not exist a single particular fact of any sort anywhere that you had not chronicled, you still would not have got a complete description of the universe unless you also added: "These that I have chronicled are all the particular facts there are." So you cannot hope to describe the world completely without having general facts as well as particular facts. Another distinction, which is perhaps a little more difficult to make, is between positive facts and negative facts, such as "Socrates was alive"—a positive fact,— and "Socrates is not alive"—you might say a negative fact.[2] But the distinction is difficult to make precise. Then there are facts concerning particular things or particular qualities or relations, and, apart from them, the completely general facts of the sort that you have in logic, where there is no mention of any constituent whatever of the actual world, no mention of any particular thing or particular quality or particular relation, indeed strictly you may say no mention of anything. That is one of the characteristics of logical propositions, that they mention nothing. Such a proposition is: "If one class is part of another, a term which is a member of the one is also a member of the other." All those words that come in the statement of a pure logical proposition are words really belonging to syntax. They are words merely expressing form or connection, not mentioning any particular constituent of the proposition in which they occur. This is, of course, a thing that wants to be proved; I am not laying it down as self-evident. Then there are facts about the properties of single things; and facts about the relations between two things, three things, and so on; and any number of different classifications of some of the facts in the world, which are important for different purposes.

[2] Negative facts are further discussed in a later lecture.

It is obvious that there is not a dualism of true and false facts; there are only just facts. It would be a mistake, of course, to say that all facts are true. That would be a mistake because true and false are correlatives, and you would only say of a thing that it was true if it was the sort of thing that *might* be false. A fact cannot be either true or false. That brings us on to the question of statements or propositions or judgments, all those things that do have the duality of truth and falsehood. For the purposes of logic, though not, I think, for the purposes of theory of knowledge, it is natural to concentrate upon the proposition as the thing which is going to be our typical vehicle on the duality of truth and falsehood. A proposition, one may say, is a sentence in the indicative, a sentence asserting something, not questioning or commanding or wishing. It may also be a sentence of that sort preceded by the word "that." For example, "That Socrates is alive," "That two and two are four," "That two and two are five," anything of that sort will be a proposition.

A proposition is just a symbol. It is a complex symbol in the sense that it has parts which are also symbols: a symbol may be defined as complex when it has parts that are symbols. In a sentence containing several words, the several words are each symbols, and the sentence composing them is therefore a complex symbol in that sense. There is a good deal of importance to philosophy in the theory of symbolism, a good deal more than at one time I thought. I think the importance is almost entirely negative, i. e., the importance lies in the fact that unless you are fairly self-conscious about symbols, unless you are fairly aware of the relation of the symbol to what it symbolizes, you will find yourself attributing to the thing properties which only belong to the symbol. That, of course, is especially likely in very abstract studies such as philosophical logic, because the subject-matter that you are supposed to be thinking of is so exceedingly difficult and elusive that any person who has ever tried to think about it knows you do not think about it except perhaps once in six months for half a minute. The rest of the time you think about the symbols, because they are tangible, but the thing you are supposed to be thinking about is fearfully difficult and one does not often manage to think about it. The really good philosopher is the one who does once in six months think about it for a minute. Bad philosophers never do. That is why the theory of symbolism has a certain importance, because otherwise you are so certain to mistake the properties of the symbolism for the properties of the thing. It has other interesting sides to it too. There are different kinds of symbols, different kinds of relation between symbol and what is symbolized, and very important fallacies arise from not realizing this. The sort of contradictions about which I shall be speaking in connection with types in a later lecture all arise from mistakes in symbolism, from putting one sort of symbol in the place where another sort of symbol ought to be. Some of the notions that have been thought absolutely fundamental in philosophy have arisen, I believe, entirely through mistakes as to symbolism—e. g., the notion of existence, or, if you like, reality. Those two words stand for a great deal that has been discussed in philosophy. There has been the theory about every proposition being really a description of reality as a whole and so on, and altogether these notions of reality and existence have played a very prominent part in philosophy. Now my own belief is that as they have occurred in philosophy, they have been entirely the outcome of a muddle about symbolism, and that when you have cleared up that muddle, you find that practically everything that has been said about existence is sheer and simple mistake, and that is all you can say about it. I shall go into that in a later lecture, but it is an example of the way in which symbolism is important.

Perhaps I ought to say a word or two about what I am understanding by symbolism, because I think some people think you only mean mathematical symbols when you talk about symbolism. I am using it in a sense to include all language of every sort and kind, so that every word is a symbol, and every sentence, and so forth. When I speak of a symbol I simply mean something that "means" something else, and as to what I mean by "meaning" I am not prepared to tell you. I will in the course of time enumerate a strictly infinite number of different things that "meaning" may mean, but I shall not consider that I have exhausted the discussion by doing that. I think that the notion of meaning is always more or less psychological, and that it is not possible to get a pure logical theory of meaning, nor therefore of symbolism. I think that it is of the very essence of the explanation of what you mean by a symbol to take account of such things as knowing, of cognitive relations, and probably also of association. At any rate I am pretty clear that the theory of symbolism and the use of symbolism is not a thing that can be explained in pure logic without taking account of the various cognitive relations that you may have to things.

As to what one means by "meaning," I will give a few illustrations. For instance, the word "Socrates," you will say, means a certain man; the word "mortal" means a certain quality; and the sentence "Socrates is mortal" means a certain fact. But these three sorts of meaning are entirely distinct, and you will get into the most hopeless contradictions if you think the word "meaning" has the same meaning in each of these three cases. It is very important not to suppose that there is just one thing which is meant by "meaning," and that therefore there is just one sort of relation of the symbol to what is symbolized.

A name would be a proper symbol to use for a person; a sentence (or a proposition) is the proper symbol for a fact.

A belief or a statement has duality of truth and falsehood, which the fact does not have. A belief or a statement always involves a proposition. You say that a man believes that so and so is the case. A man believes that Socrates is dead. What he believes is a proposition on the face of it, and for formal purposes it is convenient to take the proposition as the essential thing having the duality of truth and falsehood. It is very important to realize such things, for instance, as that propositions are not names for facts. It is quite obvious as soon as it is pointed out to you, but as a matter of fact I never had realized it until it was pointed out to me by a former pupil of mine, Wittgenstein. It is perfectly evident as soon as you think of it, that a proposition is not a name for a fact, from the mere circumstance that there are *two* propositions corresponding to each fact. Suppose it is a fact that Socrates is dead. You have two propositions: "Socrates is dead" and "Socrates is not dead." And those two propositions corresponding to the same fact, there is one fact in the world which makes one true and one false. That is not accidental, and illustrates how the relation of proposition to fact is a totally different one from the relation of name to the thing named. For each fact there are two propositions, one true and one false, and there is nothing in the nature of the symbol to show us which is the true one and which is the false one. If there were, you could ascertain the truth about the world by examining propositions without looking round you.

There are two different relations, as you see, that a proposition may have to a fact: the one the relation that you may call being true to the fact, and the other being false to the fact. Both are equally essentially logical relations which may subsist between the two, whereas in the case of a name, there is only one relation that it can have to what it names. A name can just name a particular, or, if it does not, it is not a name at all, it is a noise. It cannot be a name without having just that one particular relation of naming a certain thing, whereas a proposition does not cease to be a proposition if it is false. It has these two ways, of being true and being false, which together correspond to the property of being a name. Just as a word may be a name or be not a name but just a meaningless noise, so a phrase which is apparently a proposition may be either true or false, or may be meaningless, but the true and false belong together as against the meaningless. That shows, of course, that the formal logical characteristics of propositions are quite different from those of names, and that the relations they have to facts are quite different, and therefore propositions are not names for facts. You must not run away with the idea that you can name facts in any other way; you cannot. You cannot name them at all. You cannot properly name a fact. The only thing you can do is to assert it, or deny it, or desire it, or will it, or wish it, or question it, but all those are things involving the whole proposition. You can never put the sort of thing that makes a proposition to be true or false in the position of a logical subject. You can only have it there as something to be asserted or denied or something of that sort, but not something to be named.

DISCUSSION.

. . . . Do you take your starting-point "That there are many things" as a postulate which is to be carried along all through, or has to be proved afterward?

Mr. Russell: No, neither the one nor the other. I do not take it as a postulate that "There are many things." I should take it that, in so far as it can be proved, the proof is empirical, and that the disproofs that have been offered are *a priori*. The empirical person would naturally say, there are many things. The monistic philosopher attempts to show that there are not. I should propose to refute his *a priori* arguments.

I do not consider there is any *logical* necessity for there to be many things, nor for there not to be many things.

.... I mean in making a start, whether you start with the empirical or the *a priori* philosophy, do you make your statement just at the beginning and come back to prove it, or do you never come back to the proof of it?

Mr. Russell: No, you never come back. It is like the acorn to the oak. You never get back to the acorn in the oak. I should like a statement which would be rough and vague and have that sort of obviousness that belongs to things of which you never know what they mean, but I should never get back to that statement. I should say, here is a thing. We seem somehow convinced that there is truth buried in this thing somewhere. We will look at it inside and out until we have extracted something and can say, now that is true. It will not really be the same as the thing we started from because it will be so much more analytic and precise.

.... Does it not look as though you could name a fact by a date?

Mr. Russell: You can apparently name facts, but I do not think you can really: you would always find that if you set out the whole thing fully, it was not so. Suppose you say "The death of Socrates." You might say, that is a name for the fact that Socrates died. But it obviously is not. You can see that the moment you take account of truth and falsehood. Supposing he had not died, the phrase would still be just as significant although there could not be then anything you could name. But supposing he had never lived, the sound "Socrates" would not be a name at all. You can see it in another way. You can say "The death of Socrates is a fiction." Suppose you had read in the paper that the Kaiser had been assassinated, and it turned out to be not true. You could then say, "The death of the Kaiser is a fiction." It is clear that there is no such thing in the world as a fiction, and yet that statement is a perfectly sound statement. From this it follows that "The death of the Kaiser" is not a name.

II. PARTICULARS, PREDICATES, AND RELATIONS.

I propose to begin to-day the analysis of facts and propositions, for in a way the chief thesis that I have to maintain is the legitimacy of analysis, because if one goes into what I call Logical Atomism that means that one does believe the world can be analyzed into a number of separate things with relations and so forth, and that the sort of arguments that many philosophers use against analysis are not justifiable.

In a philosophy of logical atomism one might suppose that the first thing to do would be to discover the kinds of atoms out of which logical structures are composed. But I do not think that is quite the first thing; it is one of the early things, but not quite the first. There are two other questions that one has to consider, and one of these at least is prior. You have to consider:

1. Are the things that look like logically complex entities really complex?
2. Are they really entities?

The second question we can put off; in fact, I shall not deal with it fully until my last lecture. The first question, whether they are really complex, is one that you have to consider at the start. Neither of these questions is, as it stands, a very precise question. I do not pretend to start with precise questions. I do not think you can start with anything precise. You have to achieve such precision as you can, as you go along. Each of these two questions, however, is *capable* of a precise meaning, and each is really important.

There is another question which comes still earlier, namely: what shall we take as *prima facie* examples of logically complex entities? That really is the first question of all to start with. What sort of things shall we regard as *prima facie* complex?

Of course, all the ordinary objects of daily life are apparently complex entities: such things as tables and chairs, loaves and fishes, persons and principalities and powers—they are all on the face of it complex entities. All the kinds of things to which we habitually give proper names are on the face of them complex entities: Socrates, Piccadilly, Rumania, Twelfth Night or anything you like to think of, to which you give a proper name, they are all apparently complex entities. They seem to be complex systems bound together into some kind of a unity, that sort of a unity that leads to the bestowal of a single appellation. I think it is the contemplation of this sort of apparent unity which has very largely led to the philosophy of monism, and to the suggestion that the universe as a whole is a single complex entity more or less in the sense in which these things are that I have been talking about.

For my part, I do not believe in complex entities of this kind, and it is not such things as these that I am going to take as the *prima facie* examples of complex entities. My reasons will appear more and more plainly as I go on. I cannot give them all to-day, but I can more or less explain what I mean in a preliminary way. Suppose, for example, that you were to analyze what appears to be a fact about Piccadilly. Suppose you made any statement about Piccadilly, such as: "Piccadilly is a pleasant street." If you analyze a statement of that sort correctly, I believe you will find that the fact corresponding to your statement does not contain any constituent corresponding to the word "Piccadilly." The word "Piccadilly" will form part of many significant propositions, but the facts corresponding to these propositions do not contain any single constituent, whether simple or complex, corresponding to the word "Piccadilly." That is to say, if you take language as a guide in your analysis of the fact expressed, you will be led astray in a statement of that sort. The reasons for that I shall give at length in Lecture VI, and partly also in Lecture VII, but I could say in a preliminary way certain things that would make you understand what I mean. "Piccadilly," on the face of it, is the name for a certain portion of the earth's surface, and I suppose, if you wanted to define it, you would have to define it as a series of classes of material entities, namely those which, at varying times, occupy that portion of the earth's surface. So that you would find that the logical status of Piccadilly is bound up with the logical status of series and classes, and if you are going to hold Piccadilly as real, you must hold that series of classes are real, and whatever sort of metaphysical status you assign to them, you must assign to it. As you know, I believe that series and classes are of the nature of logical fictions: therefore that thesis, if it can be maintained, will dissolve Piccadilly into a fiction. Exactly similar remarks will apply to other instances: Rumania, Twelfth Night, and Socrates. Socrates, perhaps, raises some special questions, because the question what constitutes a person has special difficulties in it. But, for the sake of argument, one might identify Socrates with the series of his experiences. He would be really a series of classes, because one has many experiences simultaneously. Therefore he comes to be very like Piccadilly.

Considerations of that sort seem to take us away from such *prima facie* complex entities as we started with to others as being more stubborn and more deserving of analytic attention, namely facts. I explained last time what I meant by a fact, namely, that sort of thing that makes a proposition true or false, the sort of thing which is the case when your statement is true and is not the case when your statement is false. Facts are, as I said last time, plainly something you have to take account of if you are going to give a complete account of the world. You cannot do that by merely enumerating the particular things that are in it: you must also mention the relations of these things, and their properties, and so forth, all of which are facts, so that facts certainly belong to an account of the objective world, and facts do seem much more clearly complex and much more not capable of being explained away than things like Socrates and Rumania. However you may explain away the meaning of the word "Socrates," you will still be left with the truth that the proposition "Socrates is mortal" expresses a fact. You may not know exactly what Socrates means, but it is quite clear that "Socrates is mortal" does express a fact. There is clearly some valid meaning in saying that the fact expressed by "Socrates is mortal" is *complex*. The things in the world have various properties, and stand in various relations to each other. That they have these properties and relations are *facts*, and the things and their qualities or relations are quite clearly in some sense or other components of the facts that have those qualities or relations. The analysis of apparently complex *things* such as we started with can be reduced by various means, to the analysis of facts which are apparently about those things. Therefore it is with the analysis of *facts* that one's consideration of the problem of complexity must begin, not by the analysis of apparently complex things.

The complexity of a fact is evidenced, to begin with, by the circumstance that the proposition which asserts a fact consists of several words, each of which may occur in other contexts. Of course, sometimes you get a proposition expressed by a single word, but if it is expressed fully it is bound to contain several words. The proposition "Socrates is mortal" may be replaced by "Plato is mortal" or by "Socrates is human"; in the first case we alter the subject, in the second the predicate. It is clear that all the propositions in which the word "Socrates" occurs have something in common, and again all the propositions in which the word "mortal" occurs have something in common, something which they do not have in common with all facts but only to those which are about Socrates or mortality. It is clear, I think, that the facts corresponding to propositions in which the word "Socrates" occurs have something in common corresponding to the common word "Socrates" which occurs in the propositions, so that you have that sense of complexity to begin with, that in a fact you can get something which it may have in common with other facts, just as you may have "Socrates is human" and "Socrates is mortal," both of them facts, and both having to do with Socrates, although Socrates does not constitute the whole of either of these facts. It is quite clear that in that sense there is a possibility of cutting up a fact into component parts, of which one component may be altered without altering the others, and one component may occur in certain other facts though not in all other facts. I want to make it clear, to begin with, that there is a sense in which facts can be analyzed. I am not concerned with all the difficulties of any analysis, but only with meeting the *prima facie* objections of philosophers who think you really cannot analyze at all.

I am trying as far as possible again this time, as I did last time, to start with perfectly plain truisms. My desire and wish is that the things I start with should be so obvious that you wonder why I spend my time stating them. That is what I aim at, because the point of philosophy is to start with something so simple as not to seem worth stating, and to end with something so paradoxical that no one will believe it.

One *prima facie* mark of complexity in propositions is the fact that they are expressed by several words. I come now to another point, which applies primarily to propositions and thence derivatively to facts. You can understand a proposition when you understand the words of which it is composed even though you never heard the proposition before. That seems a very humble property, but it is a property which marks it as complex and distinguishes it from words whose meaning is simple. When you know the vocabulary, grammar, and syntax of a language, you can understand a proposition in that language even though you never saw it before. In reading a newspaper, for example, you become aware of a number of statements which are new to you, and they are intelligible to you immediately, in spite of the fact that they are new, because you understand the words of which they are composed. This characteristic, that you can understand a proposition through the understanding of its component words, is absent from the component words when those words express something simple. Take the word "red," for example, and suppose —as one always has to do—that "red" stands for a particular shade of color. You will pardon that assumption, but one never can get on otherwise. You cannot understand the meaning of the word "red" except through seeing red things. There is no other way in which it can be done. It is no use to learn languages, or to look up dictionaries. None of these things will help you to understand the meaning of the word "red." In that way it is quite different from the meaning of a proposition. Of course, you can give a definition of the word "red," and here it is very important to distinguish between a definition and an analysis. All analysis is only possible in regard to what is complex, and it always depends, in the last analysis, upon direct acquaintance with the objects which are the meanings of certain simple symbols. It is hardly necessary to observe that one does not define a thing but a symbol. (A "simple" symbol is a symbol whose parts are not symbols.) A simple symbol is quite a different thing from a simple thing. Those objects which it is impossible to symbolize otherwise than by simple symbols may be called "simple," while those which can be symbolized by a combination of symbols may be called "complex." This is, of course, a preliminary definition, and perhaps somewhat circular, but that does not much matter at this stage.

I have said that "red" could not be understood except by seeing red things. You might object to that on the ground that you can define red, for example, as "The color with the greatest wave-length." That, you might say, is a definition of "red" and a person could understand that definition even if he had seen nothing red, provided he understood the physical theory of color. But that does not really constitute the meaning of the word "red" in the very slightest. If you take such a proposition as "This is red" and substitute for it "This has the color with the greatest wave-length," you have a different proposition altogether. You can see that at once, because a person who knows nothing of the physical theory of color can understand the proposition "This is red," and can know that it is true, but cannot know that "This has the color which has the greatest wave-length." Conversely, you might have a hypothetical person who could not see red, but who understood the physical theory of color and could apprehend the proposition "This has the color with the greatest wave-length," but who would not be able to understand the proposition "This is red," as understood by the normal uneducated person. Therefore it is clear that if you define "red" as "The color with the greatest wave-length" you are not giving the actual meaning of the word at all; you are simply giving a true description, which is quite a different thing, and the propositions which result are different propositions from those in which the word "red" occurs. In that sense the word "red" cannot be defined, though in the sense in which a correct description constitutes a definition it can be defined. In the sense of analysis you cannot define "red." That is how it is that dictionaries are able to get on, because a dictionary professes to define all the words in the language by means of words in the language, and therefore it is clear that a dictionary must be guilty of a vicious circle somewhere, but it manages it by means of correct descriptions.

I have made it clear, then, in what sense I should say that the word "red" is a simple symbol and the phrase "This is red" a complex symbol. The word "red" can only be understood through acquaintance with the object, whereas the phrase "Roses are red" can be understood if you know what "red" is and what "roses" are, without ever having heard the phrase before. That is a clear mark of what is complex. It is the mark of a complex symbol, and also the mark of the object symbolized by the complex symbol. That is to say, propositions are complex symbols, and the facts they stand for are complex.

The whole question of the meaning of words is very full of complexities and ambiguities in ordinary language. When one person uses a word, he does not mean by it the same thing as another person means by it. I have often heard it said that that is a misfortune. That is a mistake. It would be absolutely fatal if people meant the same things by their words. It would make all intercourse impossible, and language the most hopeless and useless thing imaginable, because the meaning you attach to your words must depend on the nature of the objects you are acquainted with, and since different people are acquainted with different objects, they would not be able to talk to each other unless they attached quite different meanings to their words. We should have to talk only about logic—a not wholly undesirable result. Take, for example, the word "Piccadilly." We, who are acquainted with Piccadilly, attach quite a different meaning to that word from any which could be attached to it by a person who had never been in London: and, supposing that you travel in foreign parts and expatiate on Piccadilly, you will convey to your hearers entirely different propositions from those in your mind. They will know Piccadilly as an important street in London; they may know a lot about it, but they will not know just the things one knows when one is walking along it. If you were to insist on language which was unambiguous, you would be unable to tell people at home what you had seen in foreign parts. It would be altogether incredibly inconvenient to have an unambiguous language, and therefore mercifully we have not got one.

Analysis is not the same thing as definition. You can define a term by means of a correct description, but that does not constitute an analysis. It is analysis, not definition, that we are concerned with at the present moment, so I will come back to the question of analysis.

We may lay down the following provisional definitions:

> That the components of a proposition are the symbols we must understand in order to understand the proposition;
>
> That the components of the fact which makes a proposition true or false, as the case may be, are the *meanings* of the symbols which we must understand in order to understand the proposition.

That is not absolutely correct, but it will enable you to understand my meaning. One reason why it fails of correctness is that it does not apply to words which, like "or" and "not," are parts of propositions without corresponding to any part of the corresponding facts. This is a topic for Lecture III.

I call these definitions *preliminary* because they start from the complexity of the proposition, which they define psychologically, and proceed to the complexity of the fact, whereas it is quite clear that in an orderly, proper procedure it is the complexity of the fact that you would start from. It is also clear that the complexity of the fact cannot be something merely psychological. If in astronomical fact the earth moves round the sun, that is genuinely complex. It is not that you think it complex, it is a sort of genuine objective complexity, and therefore one ought in a proper, orderly procedure to start from the complexity of the world and arrive at the complexity of the proposition. The only reason for going the other way round is that in all abstract matters symbols are easier to grasp. I doubt, however, whether complexity, in that fundamental objective sense in which one starts from complexity of a fact, is definable at all. You cannot analyze what you mean by complexity in that sense. You must just apprehend it—at least so I am inclined to think. There is nothing one could say about it, beyond giving criteria such as I have been giving. Therefore, when you cannot get a real proper analysis of a thing, it is generally best to talk round it without professing that you have given an exact definition.

It might be suggested that complexity is essentially to do with symbols, or that it is essentially psychological. I do not think it would be possible seriously to maintain either of these views, but they are the sort of views that will occur to one, the sort of thing that one would try, to see whether it would work. I do not think they will do at all. When we come to the principles of symbolism which I shall deal with in Lecture VII, I shall try to persuade you that in a logically correct symbolism there will always be a certain fundamental identity of structure between a fact and the symbol for it; and that the complexity of the symbol corresponds very closely with the complexity of the facts symbolized by it. Also, as I said before, it is quite directly evident to inspection that the fact, for example, that two things stand in a certain relation to one another—e. g., that this is to the left of that—is itself objectively complex, and not merely that the apprehension of it is complex. The fact that two things stand in a certain relation to each other, or any statement of that sort, has a complexity all of its own. I shall therefore in future assume that there is an objective complexity in the world, and that it is mirrored by the complexity of propositions.

A moment ago I was speaking about the great advantages that we derive from the logical imperfections of language, from the fact that our words are all ambiguous. I propose now to consider what sort of language a logically perfect language would be. In a logically perfect language the words in a proposition would correspond one by one with the components of the corresponding fact, with the exception of such words as "or," "not," "if," "then," which have a different function. In a logically perfect language, there will be one word and no more for every simple object, and everything that is not simple will be expressed by a combination of words, by a combination derived, of course, from the words for the simple things that enter in, one word for each simple component. A language of that sort will be completely analytic, and will show at a glance the logical structure of the facts asserted or denied. The language which is set forth in *Principia Mathematica* is intended to be a language of that sort. It is a language which has only syntax and no vocabulary whatsoever. Barring the omission of a vocabulary I maintain that it is quite a nice language. It aims at being that sort of a language that, if you add a vocabulary, would be a logically perfect language. Actual languages are not logically perfect in this sense, and they cannot possibly be, if they are to serve the purposes of daily life. A logically perfect language, if it could be constructed, would not only be intolerably prolix, but, as regards its vocabulary, would be very largely private to one speaker. That is to say, all the names that it would use would be private to that speaker and could not enter into the language of another speaker. It could not use proper names for Socrates or Piccadilly or Rumania for the reasons which I went into earlier in the lecture. Altogether you would find that it would be a very inconvenient language indeed. That is one reason why logic is so very backward as a science, because the needs of logic are so extraordinarily different from the needs of daily life. One wants a language in both, and unfortunately it is logic that has to give way, not daily life. I shall, however, assume that we have constructed a logically perfect language, and that we are going on state occasions to use it, and I will now come back to the question which I intended to start with, namely, the analysis of facts.

The simplest imaginable facts are those which consist in the possession of a quality by some particular thing. Such facts, say, as "This is white." They have to be taken in a very sophisticated sense. I do not want you to think about the piece of chalk I am holding, but of what you see when you look at the chalk. If one says, "This is white" it will do for about as simple a fact as you can get hold of. The next simplest would be those in which you have a relation between two facts, such as: "This is to the left of that." Next you come to those where you have a triadic relation between three particulars. (An instance which Royce gives is "A gives B to C.") So you get relations which require as their minimum three terms, those we call triadic relations; and those which require four terms, which we call tetradic, and so on. There you have a whole infinite hierarchy of facts,—facts in which you have a thing and a quality, two things and a relation, three things and a relation, four things and a relation, and so on. That whole hierarchy constitutes what I call *atomic* facts, and they are the simplest sort of fact. You can distinguish among them some simpler than others, because the ones containing a quality are simpler than those in which you have, say, a pentadic relation, and so on. The whole lot of them, taken together, are as facts go very simple, and are what I call atomic facts. The propositions expressing them are what I call atomic propositions.

In every atomic fact there is one component which is naturally expressed by a verb (or, in the case of quality, it may be expressed by a predicate, by an adjective). This one component is a quality or dyadic or triadic or tetradic ... relation. It would be very convenient, for purposes of talking about these matters, to call a quality a "monadic relation" and I shall do so; it saves a great deal of circumlocution.

In that case you can say that all atomic propositions assert relations of varying orders. Atomic facts contain, besides the relation, the terms of the relation—one term if it is a monadic relation, two if it is dyadic, and so on. These "terms" which come into atomic facts I define as "particulars."

Particulars = terms of relations in atomic facts.

Definition.

That is the definition of particulars, and I want to emphasize it because the definition of a particular is something purely logical. The question whether this or that is a particular, is a question to be decided in terms of that logical definition. In order to understand the definition it is not necessary to know beforehand "This is a particular" or "That is a particular". It remains to be investigated what particulars you can find in the world, if any. The whole question of what particulars you actually find in the real world is a purely empirical one which does not interest the logician as such. The logician as such never gives instances, because it is one of the tests of a logical proposition that you need not know anything whatsoever about the real world in order to understand it.

Passing from atomic facts to atomic propositions, the word expressing a monadic relation or quality is called a "predicate," and the word expressing a relation of any higher order would generally be a verb, sometimes a single verb, sometimes a whole phrase. At any rate the verb gives the essential nerve, as it were, of the relation. The other words that occur in the atomic propositions, the words that are not the predicate or verb, may be called the subjects of the proposition. There will be one subject in a monadic proposition, two in a dyadic one, and so on. The subjects in a proposition will be the words expressing the terms of the relation which is expressed by the proposition.

The only kind of word that is theoretically capable of standing for a particular is a *proper name,* and the whole matter of proper names is rather curious.

Proper Names = words for particulars.

Definition.

I have put that down although, as far as common language goes, it is obviously false. It is true that if you try to think how you are to talk about particulars, you will see that you cannot ever talk about a particular particular except by means of a proper name. You cannot use general words except by way of description. How are you to express in words an atomic proposition? An atomic proposition is one which does mention actual particulars, not merely describe them but actually name them, and you can only name them by means of names. You can see at once for yourself, therefore, that every other part of speech except proper names is obviously quite incapable of standing for a particular. Yet it does seem a little odd if, having made a dot on the blackboard, I call it "John." You would be surprised, and yet how are you to know otherwise what it is that I am speaking of. If I say, "The dot that is on the right-hand side is white" that is a proposition. If I say "This is white" that is quite a different proposition. "This" will do very well while we are all here and can see it, but if I wanted to talk about it to-morrow it would be convenient to have christened it and called it "John." There is no other way in which you can mention it. You cannot really mention *it* itself except by means of a name.

What pass for names in language, like "Socrates," "Plato," and so forth, were originally intended to fulfil this function of standing for particulars, and we do accept, in ordinary daily life, as particulars all sorts of things that really are not so. The names that we commonly use, like "Socrates," are really abbreviations for descriptions; not only that, but what they describe are not particulars but complicated systems of classes or series. A name, in the narrow logical sense of a word whose meaning is a particular, can only be applied to a particular with which the speaker is acquainted, because you cannot name anything you are not acquainted with. You remember, when Adam named the beasts, they came before him one by one, and he became acquainted with them and named them. We are not acquainted with Socrates, and therefore cannot name him. When we use the word "Socrates," we are really using a description. Our thought may be rendered by some such phrase as, "The Master of Plato," or "The philosopher who drank the hemlock," or "The person whom logicians assert to be mortal," but we certainly do not use the name as a name in the proper sense of the word.

That makes it very difficult to get any instance of a name at all in the proper strict logical sense of the word. The only words one does use as names in the logical sense are words like "this" or "that." One can use "this" as a name to stand for a particular with which one is acquainted at the moment. We say "This is white." If you agree that "This is white," meaning the "this" that you see, you are using "this" as a proper name. But if you try to apprehend the proposition that I am expressing when I say "This is white," you cannot do it. If you mean this piece of chalk as a physical object, then you are not using a proper name. It is only when you use "this" quite strictly, to stand for an actual object of sense, that it is really a proper name. And in that it has a very odd property for a proper name, namely that it seldom means the same thing two moments running and does not mean the same thing to the speaker and to the hearer. It is an *ambiguous* proper name, but it is really a proper name all the same, and it is almost the only thing I can think of that is used properly and logically in the sense that I was talking of for a proper name. The importance of proper names, in the sense of which I am talking, is in the sense of logic, not of daily life. You can see why it is that in the logical language set forth in *Principia Mathematica* there are not any names, because there we are not interested in particular particulars but only in general particulars, if I may be allowed such a phrase.

Particulars have this peculiarity, among the sort of objects that you have to take account of in an inventory of the world, that each of them stands entirely alone and is completely self-subsistent. It has that sort of self-subsistence that used to belong to substance, except that it usually only persists through a very short time, so far as our experience goes. That is to say, each particular that there is in the world does not in any way logically depend upon any other particular. Each one might happen to be the whole universe; it is a merely empirical fact that this is not the case. There is no reason why you should not have a universe consisting of one particular and nothing else. That is a peculiarity of particulars. In the same way, in order to understand a name for a particular, the only thing necessary is to be acquainted with that particular. When you are acquainted with that particular, you have a full, adequate, and complete understanding of the name, and no further information is required. No further information as to the facts that are true of that particular would enable you to have a fuller understanding of the meaning of the name.

BERTRAND RUSSELL.

LONDON, ENGLAND.

DISCUSSION.

Mr. Carr: You think there are simple facts that are not complex. Are complexes all composed of simples? Are not the simples that go into complexes themselves complex?

Mr. Russell: No facts are simple. As to your second question, that is, of course, a question that might be argued—whether when a thing is complex it is necessary that it should in analysis have constituents that are simple. I think it is perfectly possible to suppose that complex things are capable of analysis *ad infinitum,* and that you never reach the simple. I do not think it is true, but it is a thing that one might argue, certainly. I do myself think that complexes—I do not like to talk of complexes—but that facts are composed of simples, but I admit that that is a difficult argument, and it might be that analysis could go on forever.

Mr. Carr: You do not mean that in calling the thing complex, you have asserted that there really are simples?

Mr. Russell: No, I do not think that is *necessarily* implied.

Mr. Neville: I do not feel clear that the proposition "This is white" is in any case a simpler proposition than the proposition "This and that have the same color."

Mr. Russell: That is one of the things I have not had time for. It may be the same as the proposition "This and that have the same color." It may be that white is defined as the color of "this," or rather that the proposition "This is white" means "This is identical in color with that," the color of "that" being, so to speak, the definition of white. That may be, but there is no special reason to think that it is.

Mr. Neville: Are there any monadic relations which would be better examples?

Mr. Russell: I think not. It is perfectly obvious *a priori* that you can get rid of all monadic relations by that trick. One of the things I was going to say if I had had time was that you can get rid of dyadic and reduce to triadic, and so on. But there is no particular reason to suppose that that is the way the world begins, that it begins with relations of order *n* instead of relations of order 1. You cannot reduce them downward, but you can reduce them upward.

.... If the proper name of a thing, a "this," varies from instant to instant, how is it possible to make any argument?

Mr. Russell: You can keep "this" going for about a minute or two. I made that dot and talked about it for some little time. I mean it varies often. If you argue quickly, you can get some little way before it is finished. I think things last for a finite time, a matter of some seconds or minutes or whatever it may happen to be.

.... You do not think that air is acting on that and changing it?

Mr. Russell: It does not matter about that if it does not alter its appearance enough for you to have a different sense-datum.

THE WASHINGTON MANUSCRIPT AND THE RESURRECTION IN MARK.

THE discovery of the Washington MS. will materially alter the critical apparatus of the next editor of the Greek New Testament. Soden has not observed the fact that this manuscript contains a reading of primary import, in agreement with the lost manuscripts of Eusebius. Westcott and Hort give ἐλθουσαι as a marginal reading at Mark xvi. 5, but they omit its correlative, ἀκουσασαι, at xvi. 8, a reading already known to John Mill of Oxford, in 1707. Westcott and Hort's oversight was due to the fact that no Greek manuscript, used in 1881, contained the reading (except the ungrammatical ἀκουσαντες of Gregory's No. 565). Moreover, they had also failed to observe that lost manuscripts quoted by Eusebius read ἀκουσασαι:

και ἀκουσασαι ἐφυγον, και οὐδενι οὐδεν εἰπον· ἐφοβουντο γαρ. (*Ad Marin. Quaest.* I.)

The critical apparatus of the future will read thus:

Mark xvi. 5.

Corrupted text: εἰσελθουσαι, from the parallel in Luke.

Primitive text: ἐλθουσαι, with Evv. Matth., Johann., Petri; Codd. B, 127.[1]

[1] In my article in *The Monist,* April, 1917, pp. 173f, I adduced the Gothic version here, on the authority of Tischendorf and Massmann; but a study of the Gothic has convinced me that Soden is right in ignoring it.

Mark xvi. 8.

Corrupted text: ἐξελθουσαι, inserted as a correlative to εἰσελθουσαι.

Primitive text: ἀκουσασαι, with lost manuscripts of Sæcc. II-III, apud Euseb. *ad Marin., Quaest.* I; Codex W, Sinai Syriac, Armenian, and Sahidic versions.

The Washington MS. exhibits the first stage of corruption:

και ἀκουσασαι ἐξηλθον και ἐφυγον.

The ἐξελθουσαι or ἐξηλθον was then being added before the deletion of the ἀκουσασαι. The Vatican and Sinaitic MSS. betray the last and complete stage of corruption, wherein the ἀκουσασαι is dropped altogether:

και ἐξελθουσαι ἐφυγον ἀπο του μνημειου.

Contrast with this the simplicity of Eusebius's early manuscripts which say nothing about the women going out of the sepulcher (because they had never been in it). The Armenian version omits the ἐξελθουσαι, and the Sinai Syriac omits the ἀπο του μνημειου. Thus do these two ancient witnesses, based upon lost Greek manuscripts of the same age as those used by Eusebius, furnish complete support to the reading of that Father. The translator of the future will end Mark thus:

And when they heard, they fled, and said nothing to any one, for they were afraid of. . . . .

Here endeth the Gospel according to Mark.

Albert J. Edmunds.

Philadelphia, Pa.

LEIBNIZ AND PASCAL.[1]*

IN the History of Mathematics it is generally stated that the higher analysis took its rise in the method of indivisibles of Cavalieri (1635).[2] This assertion, at least as far as the invention[3] of the algorithm of the higher analysis is concerned, is erroneous. In what follows it will be shown, by argument founded on the work of the French mathematicians of the seventeenth century and on the manuscripts of Leibniz, that Leibniz was led to his invention of the algorithm of the higher analysis by a study of the writings of Pascal, more than by anything else.[4]

With regard to the manuscripts of Leibniz, the first letters of the correspondence between Leibniz and Tschirnhaus are weighty; they contain the further discussion of their joint labor during the time that they lived together in Paris (September, 1675, to November, 1676);[5] it is well known that it was during this time that Leibniz invented the algorithm of the higher analysis. Among these letters, one from Leibniz, not hitherto published, which closes the first part of the correspondence between Leibniz and Tschirnhaus, contains a very detailed statement of the studies of Leibniz during his sojourn in Paris; it is beyond dispute of the utmost importance, since it was written only four years afterward and recalls particulars in a most vivid manner.[6]

Next, we have to consider the works of the French mathematicians about the middle of the seventeenth cen-

* For footnotes of both author and translator see *infra*, pp. 550-560.

tury, especially those of Pascal. We know from the facts of Pascal's life that his father, when he moved to Paris in 1631, joined a circle of mathematicians and physicists,[7] of which the history of science has preserved the names of Mersenne, Roberval, Gassendi, Desargues, de Carcavi, Beaugrand, des Billettes, and others. These were in communication, chiefly through Mersenne, with the mathematicians who did not live in Paris, Descartes, Fermat, and de Sluse; so that about the middle of the seventeenth century all that was best (*die Höhe der*) in the science of mathematics was concentrated in Paris.[8] In this circle Pascal moved, hardly yet out of his boyhood, and excited by his eminent talent astonishment and admiration. As an outstanding characteristic of the works of the mathematicians named above there stood forth the endeavor to abandon the method of Cavalieri as lacking every feature of scientific rigor, and to treat the science according to the methods of the Greek mathematicians.[9] Perhaps the ideas of Kepler, in his *Supplementum Stereometriae Archimedeae*,[10] were of influence, when Roberval and Pascal introduced into geometry the ideas of infinity and the infinitely small.[11]

As for those works of Pascal, which belong to this subject, we must mention in particular the solution of the problems, produced by him in 1658 under the assumed name of Dettonville, on the cycloid. By this, and by the method that he employed, he surpassed all the mathematicians contemporary with him, and he earned for himself the fame of being the greatest geometer of his day.

The investigation of the properties of the cycloid had occupied the attention of the most famous mathematicians of the seventeenth century. It is reported that, earlier than anybody else and indeed before 1599, Galileo had had his attention called to this curve in consequence of his construction of arches for a bridge; he endeavored to find its area in a mechanical way, by weighing a plate of lead of uniform thickness having the shape of a plane bounded by a cycloid; and he found that it was very nearly three times as great as the area of the generating circle. This result he was unable to confirm theoretically. In 1615, Mersenne had his attention called to the cycloid as generated by a rolling wheel; he spent a great deal of time in investigating the nature of the curve, but without success; so that, in 1643, he corresponded with Roberval concerning the difficulties that he had encountered with respect to the curve. Roberval proved, by the help of the method of Cavalieri as improved by himself, that the area of the cycloid is exactly three times that of the generating circle; furthermore, in 1644, he determined the content of the solids formed by the rotation of the cycloid about its base, about its axis, and about the diameter of the generating circle; also he found the centroid of the area of the cycloid. In consequence of a bodily infirmity that robbed him of his rest at night, Pascal, in order to obtain some distraction from his pain, once more took up the investigation of the cycloid after an interval of fourteen years, in the year 1658. His design was to find the area of any chosen segment of the cycloid, the centroid of such a segment, the volumes of the solids described by such a segment by a rotation round either the ordinate or the abscissa, either by a complete, or a half, or a quarter revolution.[12] Inasmuch as the solutions of the problems hitherto investigated had not been done by any general method, but rather by special artificial ways of procedure, the question was that of specially creating a treatment that was applicable in general. Pascal reverted to the method of Archimedes, for determining the quadrature of the parabola by means of the equilibrium of the lever; he generalized the method,[13] by supposing, instead of geometrical figures, unequal weights not merely at the extremities of the lever (which he follows Archimedes in terming *balance*) but also at several different distances from the fulcrum; of these, by means of the Arithmetical Triangle which he had invented,[14] he determined the sum and the center of gravity. On the advice of his friends, Pascal, in June, 1658, under the alias of Dettonville,[15] determined to propose to mathematicians for solution the problems that he had solved. October 1, 1658, was settled as the last day for sending in solutions. Particular cases of the proposed problems were solved by Huygens, de Sluse, and Wren, before the appointed day; but this was not sufficient to meet the requirements of Pascal. At the request of de Carcavi, Pascal made known the abovementioned method for solving such propositions in a long letter, at the beginning of October, 1658,[16] and added thereto three further propositions with respect to the cycloid. In this letter are combined five essays, which prepare the way for the solution of the problems of Pascal.

i. *Traitté des Trilignes et leurs Onglets.*[17]

In this essay, the determination of the content and the centroid of a "triligne" and its "double onglet" is reduced to the sum of the ordinates of the axis or the base in a triligne; also Pascal showed that the determination of the content and the center of gravity of the curved surface of the double onglet could be expressed as the sum of the sines of the axis.[18]

The next essay,

ii. *Propriétés des sommes simples, triangulaires et pyramidales,*

is an appendix to the foregoing. By triangular sum, Pascal meant the sum of a number of magnitudes, each one multiplied in succession by the corresponding number in the natural scale. In the same way, a pyramidal sum denoted the sum of a number of magnitudes, each one in succession multiplied by the corresponding triangular number.[19]

Then comes,

iii. *Traitté des sinus du quart de Cercle.*

In this, Pascal begins by proving the theorem: "The sum of the sines of any arc of a quadrant of a circle is equal to the product of the part of the base, intercepted between the extremities of the outside sines, multiplied by the radius of the circle." By the help of this theorem, he investigated the sum of the sines of a quadrant of a circle, their squares, their cubes, fourth and higher powers,[20] the sum of the rectangles of each sine of the base into its distance from the axis, the triangular and pyramidal sums of the sines of the base, and so on.

The next essay,

iv. *Traitté des sinus et des arcs de Cercle,*

contains the determination of the sum of all the arcs of a circle measured from the vertex of a quadrant to any ordinate of the axis, the sum of their squares, or their cubes, the corresponding triangular and pyramidal sums, the simple and triangular sums of the sectors, the sum of the solids formed from every sector of a quadrant and the distance of its center of gravity from the base, and so on.

v. *Petit Traitté des solides circulaires.*

In this is investigated the position of the center of gravity of such bodies as are formed by the rotation of half a band of a circle about the axis or base, the sum of the fourth powers of the ordinates of the axis, of their cubes, the position of the center of gravity of the semisolid of revotution arising from a rotation about the axis, and so on.

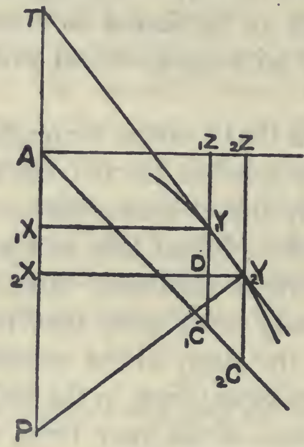
These five essays conclude with:

Un Traitté general de la Roulette, contenant la Solution de tous les Problemes touchant la Roulette qu'il avoit proposez publiquement au mois de Juin 1658.

All these works of Pascal are strictly geometrical in treatment, after the manner of the geometry of the ancients; there is not to be found in them a trace of the method of dealing with geometrical problems introduced by Descartes.[21]

It is well known that Leibniz through his acquaintance with Huygens, who lived in Paris from 1666 to 1681, was encouraged to study higher mathematics. More especially, it was Huygens who advised him to read the writings of Pascal. Upon several occasions later, has Leibniz declared, in conformity with this, that he was led to the higher analysis by the study of the writings of Pascal, and thus made his discoveries; first, in the hitherto unpublished letter to Tschirnhaus, of the year 1679, the part of it that relates to our subject being given later; also in a letter to the Marquis de l'Hospital, in the year 1694; further, in a postscript to a letter to Jacob Bernoulli, in the year 1703; and lastly, in the essay, *Historia et Origo calculi differentialis,* written in the last years of his life.

Up to the present time, among the manuscripts of Leibniz there has been found one of great length, that bears the title: *Ex Dettonvillaeno* (?) *seu Pascalii Geometricis excerpta*: *cum additamentis.* It is not dated; but as it contains work that is in the closest connection with the writings of Pascal to de Carcavi, hence it must be assigned very approximately to the time of his intercourse with Huygens (1673). This cannot be given in its entirety; only the commencement of it follows under the heading III. One special remark has Leibniz made on the five essays which follow Pascal's letter to de Carcavi; he states that the method of Pascal for determining the surface of the sphere,[22] according to which the surface of a solid formed by the rotation round an axis can be reduced to a plane figure proportional to it, was what induced him to make out a general theorem applicable to all plane figures bounded by a curved line.



The coordinates of $_1Y$ and $_2Y$, two points on the curve, are $_1Y_1Z$, $_1Y_1X$ and $_2Y_2Z$, $_2Y_2X$; $_2YT$ is the tangent at $_2Y$, which is supposed to meet the curve again in $_1Y$, and the normal $_2YP$ is drawn. On account of the similarity of the triangles $_1YD_2Y$and $_2Y_2XP$, we have

$$_2XP\,.\,_1YD = {}_2Y_2X\,.\,_2YD;$$

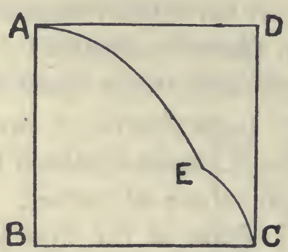
i. e., the subnormal $_2XP$ applied, at right angles to the axis AX, to the element of the axis $_1X_2X(={}_1YD)$, is equal to the ordinate $_2Y_2X$, applied to the element $_2YD$.[23] "But," Leibniz continues, "straight lines which increase from nothing, each multiplied by its corresponding element, form a triangle. For, let AZ be always equal to ZC, and you get the right-angled triangle AZC, which is half the square on AZ, and thus the figure produced by applying the subnormals in order at right angles to the axis is always equal to half the square on the ordinate. Hence, being given a figure to be squared, that figure is sought whose subnormals are equal to the ordinates of the given figure, and the second figure is the quadratrix of the given figure. Thus from this very simple idea, we have the reduction of surfaces produced by rotation to plane quadratures, and also of the rectification of curves;[24] and at the same time, we can reduce these quadratures to problems of inverse tangents." Thus it came about that Leibniz obtained from this a general method for the quadrature of curves.

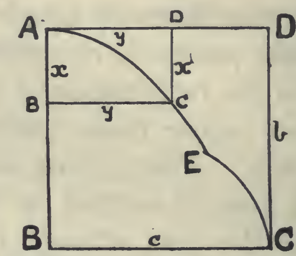
All this was attained to by Leibniz in the first year, 1673/74, of his mathematical studies in regard to the higher analysis. Until this time he had adhered to the rigorous geometrical method, as he found it in the writings of Pascal, in his investigations; acting on the advice of Huygens, he now made himself acquainted with the method of Descartes as being more adapted to computation. The long essay of Leibniz with the title, *Analysis Tetragonistica ex Centrobarycis,* dated Oct. 25, 26, 29, and Nov. 1, 1675, shows clear connection[25] with the above-mentioned method of Pascal; also it shows the improvement that Leibniz had made in consequence of his study of Cartesian geometry, Leibniz commences with Proposition 2 from Pascal's first essay, *Traitté des Trilignes et leurs Onglets,* which he expresses as follows.

"Let any curve AEC be referred to a right angle BAD; let AB $\sqcap$ DC $\sqcap$ a,[26] and let the last $x \sqcap b$; also let BC $\sqcap$ AD $\sqcap$ y, and let the last $y \sqcap c$. Then it is plain that

$$\text{omn. } \overline{yx \text{ to } x} = \frac{b^2c}{2} - \text{omn. } \overline{\frac{x^2}{2} \text{ to } y}\ .$$

For, the moment of the space ABCEA about AD is made up of rectangles contained by BC ($= y$) and AB ($= a$);[26] also the moment about AD of the space ADCEA, the complement of the former, is made up of the sum of the squares on DC halved ($= x^2/2$); and if this moment is taken away from the whole moment of the rectangle ABCD about AD, i. e., from[26] c into omn.x, or from[26] $b^2c/2$, there will remain the moment of the space ABCEA.



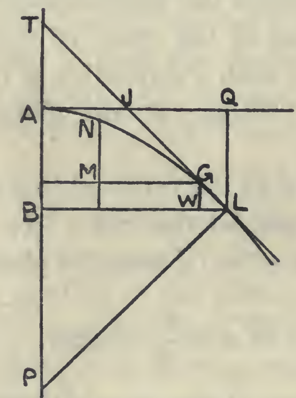


Hence the equation that I gave is obtained; and, by rearranging it, it follows that,

$$\text{omn. } yx \text{ to } x + \text{omn. } x^2/2 \text{ to } y = b^2c/2.$$

In this way we obtain the quadrature of the two joined in one in every case; and this is the fundamental theorem in the center of gravity method."

In the continuation, dated October 29, 1675, in connection with this theorem, Leibniz brings in the characteristic triangle, which has already been mentioned above.



AGL is any curve, BL $= y$, WL $= l$, BP $= p$, AB $= \overline{x}$, GW $= a$, $y =$ omn. l; hence,

$$\frac{l}{a} = \frac{p}{y} = \frac{p}{\text{omn. } l}, \text{ and therefore } p = \frac{\text{omn. } l}{a} . l .$$

Now, by the theorem given above,[27]

$$\text{omn.}\, p = \frac{y^2}{2} = \frac{\overline{\text{omn.}\, l}\,\boxed{2}}{2} = \frac{\overline{\text{omn.}\, l^2}}{2}\ ;$$

hence $$\frac{\overline{\text{omn.}\, l^2}}{2} = \overline{\text{omn.}\ \overline{\text{omn.}\, l}\ .\frac{l}{a}}\ ;$$

"that is," adds Leibniz, "if all the l's are multiplied by their last, and all the other l's again are multiplied by their last, and so on as often as it can be done, the sum of all these will be equal to half the sum of the squares, of which the sides are the sums of these, or all the l's. This is a very fine theorem, and one that is not at all obvious. So is also the theorem,

$$\text{omn.}\, xl \sqcap x\,.\,\text{omn.}\, l - \text{omn.omn.}\, l,$$

where l is supposed to be a term of a progression, and x the number which expresses the position or ordinal that corresponds to the l, i. e., x is the ordinal number and l the ordered quantity.

N.B. In these calculations, a law for all things of the same kind may be observed; for, if 'omn.' is prefixed to a number or ratio, or to something indefinitely small,[28] then a line is produced, also if to a line, then a surface, or if to a surface, then a solid; and so on to infinity for higher dimensions.

It will be useful[29] to write $\int$ for 'omn.,' so that

$$\int l = \text{omn.}\, l, \text{ or the sum of all the } l\text{'s}.$$

Thus, $\int \overline{\frac{l^2}{2}} = \int \overline{\int l}\,\frac{l}{a}$, and $\int \overline{xl} = x\int \overline{l} - \int\int l$."

This was the first time that the algorithm for the higher analysis was introduced. In what then follows, Leibniz obtains the first theorems of the integral calculus:

$$\int x = x^2/2,\ \int x^2 = x^3/3,$$

and adds, "All these theorems are true for series in which the differences of the terms bear to the terms themselves a ratio which is less than any assignable quantity."

Further Leibniz remarks: "These things are new and noteworthy, since they lead to a new kind of calculus. Being given l, and its relation to x, required to find $\int l$. Now this may be obtained by a reverse calculation; thus, if $\int l = ya$, suppose that $l = ya/d$, that is to say, as $\int$ increases the dimensions, so d will diminish them; but $\int$ stands for a sum, and d for a difference.[30] From the given value of y, we can always find y/d or l, or the difference for the y's."

In the investigation that bears the title, *Methodi tangentium inversae exempla,* dated November 11, 1675, Leibniz introduces instead of y/d the notation dy.

Such are the chief points in the story of the introduction of the algorithm of the higher analysis, as far as may be gathered from the extant manuscripts of Leibniz.[31]

In connection with the earlier essay, "Leibniz in London,"[32] I have shown that any influence whatever from external sources upon Leibniz with regard to the introduction of the algorithm of the higher analysis is excluded.

KARL IMMANUEL GERHARDT.

TRANSLATIONS OF THE MANUSCRIPTS

Alluded to by Dr. Gerhardt.

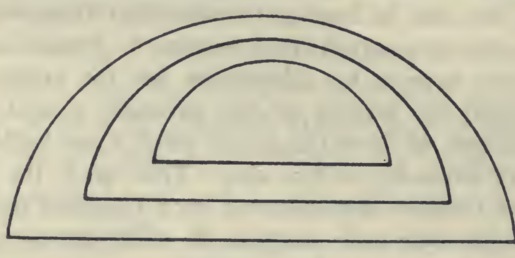
I.

From the letters of Leibniz to Tschirnhaus.

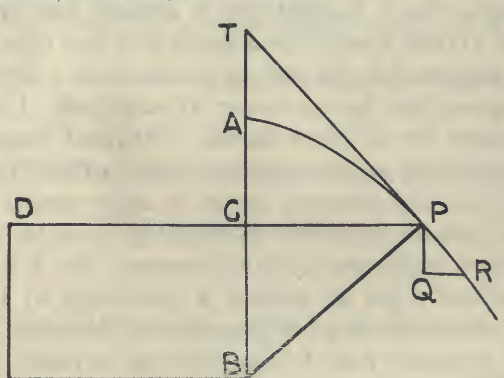
1679.

"You are astonished that Reginaldus[33] should have been able to fall into error over the surface of an elliptic spheroid; but you do not seem to have considered sufficiently how different are the several methods of indivisibles. He certainly understands the Cavalierian method, but that is so circumscribed by narrow limitations that few things of any great importance can be obtained from it. There is no doubt that Cavalieri, Torricelli, Roberval, Fermat, and indeed, as far as I know, all the Italian mathematicians were quite unaware of the utility of tangents for the purpose of finding quadratures, or of that which I have been accustomed to call the infinitely small "characteristic triangle" of the figure; indeed, at the present time also in France, I believe that Huygens is the only man that really understands these matters.[34] Pascal himself could not sufficiently express his admiration for the artifice by which Huygens found the surface of the parabolic conoid. Sluse has given no example of these things, by which I am inclined to think that they are unknown to him also. This too is the reason why Huygens and Gregory demonstrated such theorems by roundabout methods, suppressing their analysis, in order not to divulge their method at once so easy and so fruitful.

"The prime occasion from which arose my discovery of the method of the Characteristic Triangle, and other things of the same sort, happened at a time when I had studied geometry for not more than six months. Huygens, as soon as he had published his book on the pendulum, gave me a copy of it; and at that time I was quite ignorant of Cartesian algebra and also of the method of indivisibles,[35] indeed I did not know the correct definition of the center of gravity. For, when by chance I spoke of it to Huygens, I let him know that I thought that a straight line drawn through the center of gravity always cut a figure into two equal parts; since that clearly happened in the case of a square, or a circle, an ellipse, and other figures that have a center of magnitude, I imagined that it was the same for all other figures. Huygens laughed when he heard this, and told me that nothing was further from the truth. So I, excited by this stimulus, began to apply myself to the study of the more intricate geometry, although as a matter of fact I had not at that time really studied the Elements. But I found in practice that one could get on without a knowledge of the Elements, if only one was master of a few propositions. Huygens, who thought me a better geometer than I was, gave me to read[36] the letters of Pascal, published under the name of Dettonville; and from these I gathered the method of indivisibles and centers of gravity, that is to say the well-known methods of Cavalieri and Guldinus. I immediately committed to paper certain things that occurred to me as I read Pascal, of which I now find that some are absurd, others please me very much even at the present time.[37] Amongst other things, I tried to find a new sort of center. For, I thought that if, to any figure that was given, others that were similar and similarly placed were inscribed, then a "middle point" could be found,[38] at which the figure evanesced, and that being given this point the quadrature could be obtained; later I perceived the difficulty that



made this method ineffective. But to return to the subject, I will tell you how I came to find the method of the Characteristic Triangle. Incidentally Pascal gave a proof of the dimension of the spherical surface proved by Archimedes, that is the moment of a circular curve round the axis,[39] and showed that the radius applied to the axis produced this moment. I, having examined the demonstration with care, observed that, with the aid of the infinitely small characteristic triangle, it was possible to prove the following general proposition for any curve:[40]



"Let AP be any curve and let BP be drawn perpendicular to its tangent AT, to meet the axis in B; then, the ordinate PC being drawn, let the straight line CD be applied to the axis AC, perpendicular to it, and equal to BP. Then if a curve is drawn through all such points as D, we shall have a figure whose area will be the moment of the original curve about the axis, i. e., it will show how to draw a circle equal in area to that of the surface of a curve rotated round the axis. Since in the circle the straight line BP is always of the same length wherever the point P is taken in the curve, hence the figure produced by the perpendiculars[41] applied to the axis is a rectangle, and thus the surface of the sphere is very easily reduced to a plane area. Now, when from this method I had deduced a general method for the dimensions of such surfaces, I at once took it to Huygens; he was surprised and laughingly confessed that he had made use of precisely the same method for obtaining the surface of the parabolic conoid of revolution. For in that case the curve through every D is a parabola, and hence the figure is capable of quadrature. Since I wished to verify the accuracy of my result in the case of the parabola,[42] I began to look for a method of expressing spaces and curves by reckoning, and then for the first time I really understood those matters of which Descartes wrote. For, previously, I used to calculate in my own way, using not letters but the names of lines. Then, for the first time, I read Descartes and Schooten carefully, acting on the advice of Huygens, who told me that the method of reckoning adopted by these authors was very convenient. Meanwhile having once opened the door provided by the characteristic triangle, I very easily discovered innumerable theorems with which at that time I filled innumerable sheets; but later I found that these had also been noted by Huraet, Gregory, and Barrow.[43] Moreover all these things I came upon in the first year of my apprenticeship to geometry. But after that I struggled forward to far greater things, such as I believe that neither Gregory nor Barrow could ever have reached by their methods, far less Cavalieri or Fermat.[44] About the same time, since I perceived that the finding of quadratures could be reduced to the finding of sums of series, and that the finding of tangents could be reduced to the finding of differences, I put together the fundamental principles of my new calculus,[45] which I call the "differential or tetragonistic calculus," by which I can set with a few little lines those things which could be obtained with great difficulty, if indeed at all, by the help of a mighty apparatus of lines. Moreover I considered in general that the finding of the sum of any series was nothing else but the discovering of some other series, the differences of the terms of which gave the given series, and this other series I used to call the summatrix.[46] The occasion for considering infinite series arose from the work of Wallis and Mercator.[47]

When I joined their discoveries to mine, I found out new things with no trouble at all.

"At length, when I considered that problems of quadratures might not be of known degree, and yet might be reduced to equations, in which the exponents of the powers were unknowns, a new light dawned upon me and I began to understand that this was something beyond the ordinary analysis, and I called it transcendent, because it employed equations beyond all degrees; and I see that this method, almost alone of its kind, gives a method of determining whether particular problems of this kind are possible or not. Indeed I can easily prove in other ways, and also by the differential calculus more especially, the impossibility of general quadrature of the circle, or that no algebraical line can be given as its quadratrix. What I call algebraical lines are those that Descartes calls geometrical, and by quadratrixes I mean all curves that, being described, will give the quadrature of any portion of a circle whatever. But the manner of finding the impossibility of any particular quadrature, for instance that of the whole circle, is known to me indeed in two ways, the one by the calculus of transcendent exponents, the other by a certain new kind of calculus, embracing all cases, which has not entered the mind of any one before even in his dreams.[48]

"Here you have the story of some of my meditations...."

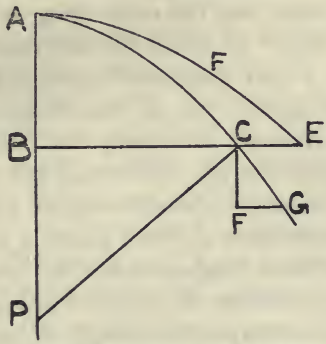
II.

From the correspondence between Leibniz and the Marquis de l'Hospital.

1694.

"I recognize that M. Barrow has advanced considerably, but I can assure you, Sir, that I have derived no assistance for my methods (*pour mes methodes*).[49] At the start I only knew the indivisibles of Cavalieri,[50] and the 'ductions' of Father Gregory St. Vincent, along with the "Synopsis of Geometry" of Father Fabri, and what could be derived from these authors and their like.[51] When M. Huygens lent me the "Letters of Dettonville" (or Pascal), I examined by chance[52] his demonstration of the measurement of the spherical surface, and in it I found an idea that the author had altogether missed; for I remarked that in general, by the same reasoning, the perpendiculars PC, when applied to the axis or set in the position BE, give a line FE, such that the area of the figure

FABEF will furnish a development (*explanation*) of the surface formed by the rotation of AE about AB.



"Huygens was surprised when I told him of this theorem, and confessed to me that it was the very same as he had made use of for the surface of the parabolic conoid. Now, as that made me aware of the use of what I call the "characteristic triangle" CFG, formed from the elements of the coordinates and the curve, I thus found as it were in the twinkling of an eyelid nearly all the theorems that I afterward found in the works of Barrow and Gregory. Up to that time,[53] I was not sufficiently versed in the calculus of Descartes, and as yet did not make use of equations to express the nature of curved lines; but, on the advice of Huygens, I set to work at it, and I was far from sorry that I did so: for it gave me the means almost immediately of finding my differential calculus.[54] This was as follows. I had for some time previously taken a pleasure in finding the sums of series of numbers, and for this I had made use of the well-known theorem, that, in a series decreasing to infinity, the first term is equal to the sum of all the differences. From this I had obtained what I call the "harmonic triangle," as opposed to the "arithmetical triangle" of Pascal; for M. Pascal had shown how one might obtain the sums of the figurate numbers, which arise when finding sums and sums of sums of the natural scale of arithmetical numbers. I on the other hand found that the fractions having figurate numbers for their denominators are the differences and the differences of the differences, etc., of the natural harmonic scale (that is, the fractions 1/1, 1/2, 1/3, 1/4, etc.), and that thus one could give the sums of the series of figurate fractions

$$\tfrac{1}{1}+\tfrac{1}{3}+\tfrac{1}{6}+\tfrac{1}{10}+\text{etc.,} \qquad \tfrac{1}{1}+\tfrac{1}{4}+\tfrac{1}{10}+\tfrac{1}{20}+\text{etc.}$$

Recognizing from this the great utility of differences and seeing that by the calculus of M. Descartes the ordinates of the curve could be expressed numerically, I saw that to find quadratures or the sums of the ordinates was the same thing as to find an ordinate (that of the quadratrix),[55] of which the difference is proportional to the given ordinate. I also recognized almost immediately that to find tangents is nothing else but to find differences (*differentier*), and that to find quadratures is nothing else but to find sums, provided that one supposes that the differences are incomparably small. I saw also that of necessity the differential magnitudes could be freed from (*se trouvent hors de*) the fraction and the root-symbol (*vinculum*), and that thus tangents could be found without getting into difficulties over (*se mettre en peine*) irrationals and fractions.[56] And there you have the story of the origin of my method...."

[At this point Gerhardt quotes his article, *Leibniz in London,* and a long passage from the *Historia,* in corroboration of the foregoing letters. I have omitted them as I have already, in my notes, pointed out the points of resemblance, and the slight differences, between the several accounts that Leibniz gives.—J. M. C.]

III.

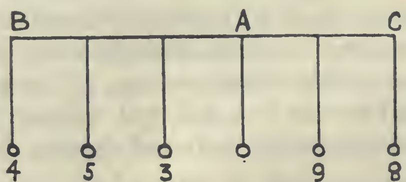
Extracts from the geometry of Dettonville or Pascal; with additions.

Ca. 1673.

1	2	3	4
A	B	C	D
	B	C	D
		C	D
			D

If A, B, C, D, are quantities, their triangular sum, starting with A, is 1A, 2B, 3C, 4D.

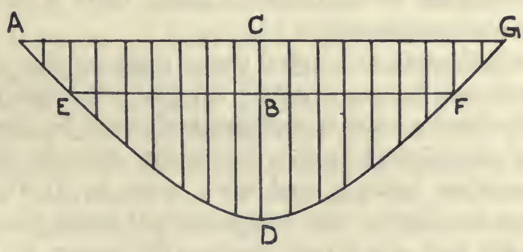
If BC is any straight line divided into any number of equal parts, and any weights, equal or unequal, are suspended at the points of division, and A is supposed to be their point of equilibrium, it is necessary that the triangular sum of the weights on the one arm AB should be equal to the triangular sum of the weights on the other arm AC, where the triangular sum on either side starts from the inner point or from the side A. The reason is that the weights give an effect



that is compounded of the ratio of the weights and their distances from the center. But these distances, on account of the division of the straight line or beam of the balance into equal parts increase as 1, 2, 3, etc.

This is what Pascal says; to which I add the following remarks.

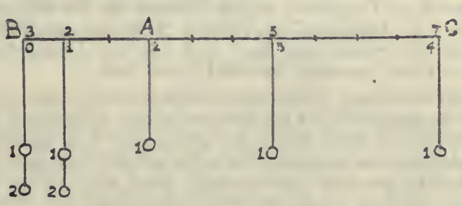
Even if the triangular sums on either side of the point are not the same, that is if the two arms are not in equilibrium, yet the moments will always be to one another as the triangular sums, for the moments are always equal to triangular sums.[57] Hence the far more general rule: If any straight line is divided into any number of equal parts, and weighted with any number of weights suspended at the points of division, and if any point of division is taken to be A, then will the moments of the weights on the one arm BA be to the moments of the weights on the other arm CA as the triangular sums starting from that weight which is nearest to A on each side.[58] Also when any figure, i. e., a line, a surface, or a solid, can be put in such a position that a certain line in it can be taken as parallel to the horizon, that straight line can be taken as a balance, and all the points or all the straight lines or all the planes (where the points in the line are assumed to be placed horizontally, or lying in planes of these points set perpendicular to the horizon), may be considered as weights; and thus, if the quantity or progression of these weights is known, and consequently their triangular sum, then the center of gravity of the figure is known; not indeed its position in the figure, but its position in the straight line that has been taken. The center of equilibrium in the figure itself is of this nature: namely, that a straight line passing through it will cut the figure into two parts, such that on each side the triangular sums of the points, straight lines, or horizontals of the solids are equal to one another. Hence the center of gravity of the whole figure being found, the centers of gravity of arms of this kind supposable without the figure may be obtained; for, let the figure be A, and



let there be taken a line parallel to the horizon in which is the center of gravity B, and suppose that the center of gravity of it is placed above a horizontal style or suspended by a thread: then it is plain that the figure will be in equilibrium. But if it is in equilibrium, then the straight line CD, drawn through the center of gravity, will cut the figure in such a fashion that the triangular sums on each side are equal; and if moreover another straight line perpendicular to CD is supposed to be divided into an infinite number of parts by the infinite parallels to CD, the triangular sums of the infinite rectangles on each side will be equal to one another, for by hypothesis the rectangles can be supposed to be suspended as weights from EF as a balance at the points of division (from which it is clear that the suspended weights need not necessarily be understood to be perpendicular to the horizon, but they may be parallel to it). This being the case, the position of the figure may be changed from the horizontal to the perpendicular, and AG become the balance; in which case it is clear that the point of equilibrium will fall at C, since the triangular sums are by hypothesis equal on each side of it. Hence, given the center of gravity of any figure, and assuming a balance either without or within the figure, to which the figure is supposed to be rigidly attached, the point of equilibrium can be found in it, by merely drawing a perpendicular to it through the center of gravity; for this will cut the balance in the point of equilibrium. On the other hand, if the points of equilibrium of two balances for the same figure are given, the center of gravity for the figure can be found (whether it is within or without the given figure; for sometimes the center will fall within the given figure; and sometimes without, as in the case of annular figures, or curved lines, or other incomplete things); that is to say, at the point of intersection of two perpendiculars drawn from those two balances toward the same parts, in the same plane, if the figure is a plane figure, i. e., if the balances are in one and the same plane; but if the two balances are not in the same plane, there is need for three. This is to be investigated.[59]

But the following is a better way: Suppose that the figure is first affixed to one balance, and let the plane through the common perpendicular be the balance and the horizontal be drawn through the point of equilibrium to cut the figure; then let the figure be affixed to another balance, and once more let another plane be drawn to cut the figure; the intersection of these two planes will give a straight line which will contain the center of equilibrium. If now a third balance is taken in addition, or a third plane, the point of intersection of all the planes, or the point in which the third plane cuts the line already found, will be the center of equilibrium. But if the figures are planes, then two balances and two perpendiculars are sufficient; and also if they are curved lines that lie all in the same plane.

Now it is worth while noting several things in those cases in which the balance is not divided into equal parts; for it may happen that we may know in some way or other the sums of the weights and their progressions, but they are such that, when applied to the balance, they divide it into unequal parts; in that case the progression of the parts into which the balance is divided has to be investigated, as for instance if it is divided into parts that continually increase according to the squares or otherwise. Thus, if we wish to suppose that the weights are equal, while the balance is divided into parts that increase as 1, 2, 3, 4, etc., and yet that this case may come under the rule, we must proceed in this way. Suppose that that point of equilibrium is already found and that it is



2, say; then it is clear that, starting from the point 2 assumed to be the center, the arms should be numbered, and that the point 1 should be marked with the number 2, and the point P with the number 3, and on the other side the point 3 should be marked with the number 3, and the point 4 with the number 7. Now, supposing that the weights are multiplied by the numbers of their own points or arms, it is necessary that the product obtained should be equal;[60] but if it is not, then another point must be sought (or something should be added to, or subtracted from, the weights; for instance, in this case, if the weights are 2, 3 should be supposed to be doubled, or in place of 1,1 we write 2,2 underneath, then there would be an equilibrium on each side, of 10). But to obviate the necessity of going through all the points, a formula should be sought; but if no known progression can be employed for the weights and the parts, a formula will be impossible; but when a known progression can be obtained, then a formula can be found as far as the nature of progression will allow. But the greater part of the difficulty will vanish in those cases in which the weights can be assumed to be equal. What is more, a very simple general rule has been found which is the reciprocal to that of Pascal, namely, that a point may be assumed such that the triangular sums of the numbers on each arm, always starting from the end and going toward the middle, are equal......[61]

NOTES.

A. To Dr. Gerhardt's Article.

[1][Translated by J. M. Child from Dr. K. I. Gerhardt's article, "Leibniz und Pascal," in the *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin,* 1891 (Zweiter Halbband), pp. 1053-1068. All notes added by the translator are put in square brackets.]

[2] "When I speak of the geometry of indivisibles," says Leibniz, "I intend something far more comprehensive than the geometry of Cavalieri, which does not appear to me to be anything but an insignificant (*mediocris*) part of the geometry of Archimedes." [The general statement appears to me to be nearer the truth than that of Gerhardt, who lays unjustifiable stress on the above remark of Leibniz. I have endeavored to show later that there is strong probability that the work of Cavalieri, which Leibniz in the *Historia* acknowledges to have read (see *The Monist,* Oct., 1916, p. 593; also pp. 583, 585 in the same number), was the *Exercitationes Sex,* and not the *Geometria* that was published ten years earlier; perhaps he read them both.]

[3][It seems to me that those who claim merely the symbolism of the Calculus as an "invention" of Leibniz are really detractors from his genius. I have endeavored to show in previous articles in *The Monist* that this symbolism, more especially as regards the sign of differentiation, was a gradual *adaptation and development* of ideas already preconceived for finite differences, until Leibniz had obtained a standardized symbolism for the infinitesimal calculus. This, in my opinion, evidences an immensely greater intellect than that necessary for an "invention"; even if we do take the standpoint that he was helped by the work of his immediate predecessors. Perhaps Gerhardt's word *Erfindung* might be better rendered by "construction" instead of "invention" or "discovery."]

[4][There was absolutely nothing in Pascal to suggest the sign for *differentiation,* and Leibniz might just as easily have obtained his ideas on integration from Galileo or others as from Pascal.]

[5][According to the generally accepted account, Leibniz was in London at the end of the third week of October, 1676, on his way home, via Amsterdam.]

[6][A point therefore to be carefully noticed is that the figure given for the characteristic triangle is totally different from that given in the "Bernoulli postscript" (see *The Monist,* Oct., 1916, p. 585); it is also different from the figure used by him in the manuscript dated Oct. 1674, which is undoubtedly derived from the figure used by Pascal in the opening lemma to the *Traitté des Sinus du quart de Cercle* (see *The Monist,* April, 1917, p. 241, and compare with *The Monist,* Oct., 1916, p. 615); it is different from either of the figures used in the manuscripts of Oct. 29, Nov. 11, 1675 (see *The Monist,* April, 1917, pp. 257, 262, 281), the last of these being like Barrow's Differential Triangle, as used by him throughout his theorems on quadratures. Does this point to a new supposition: namely, that Leibniz originally invented a certain characteristic triangle of his own, essentially different in small detail from that of Pascal, Barrow, or any one else; that then he gradually passed from this to that of Pascal, later to Barrow's form; that he found this the most convenient of all; finally, through lack of memory, he ascribes the earliest form to Pascal, instead

of to himself, making an erroneous apperception of the time at which he had *discovered* this early from? The point is referred to in a later note (40).]

[7] It went by the name of "Compagnie"; out of it grew, in 1666, the "Académie des Sciences."

[8][Gerhardt no doubt here refers to French mathematicians; but the first-mentioned names, of those that lived in Paris, with the exception of Roberval hardly bear comparison with those of the three who did not live there.]

[9] The writings of Roberval and Pascal bearing reference to this have been mentioned in the essay "Leibniz in London." [Unintentionally omitted in the Oct., 1917, *Monist.*]

[10] *Nova Stereometria Doliorum Vinariorum, inprimis Austriaci, figurae omnium aptissimae, et Usus in eo Virgae Cubicae compendiosissimus et plane singularis. Accessit Epitome Stereometriae Archimedeae Supplementum. Lincii an. M DC XV.* See my *Geschichte der Mathematik in Deutschland,* pp. 109ff.

[11] Roberval in a letter to the astronomer Hevelke (Hevelius) in Dantzig, writes: "Concerning analysis, in which I delight, I have far more [theorems]; and no fewer concerning the doctrine of the infinite, which they now call the 'doctrine of indivisibles'...." Published in: *Huygens et Roberval. Documents nouveaux. Par C. Henry*; (Leyden, 1879).

[12][By ordinate and abscissa, Gerhardt means what Pascal calls the axis and base of the segment. Pascal only considered the whole solid of revolution, and the semi-solid, their volumes, their centers of mass, and the centroids of their surfaces; but those for solids generated by a quarter of a revolution could have been deduced quite easily.]

[13][Pascal, in effect, obtained the general formula

$$\bar{x} = \Sigma(mx)/\Sigma(x),$$

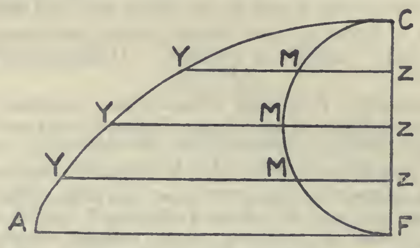
where Σ stands for either a summation of finite quantities, or for the equivalent of integration. If this is to be ascribed to Pascal as an original contribution, then we must assume that he had never seen Cavalieri's *Exercitationes Sex, Exer. quinta,* Theorems 6, 7, 8, and certain others of the fifty propositions that form this section of the book; the section being entirely devoted to centers of gravity, while the method is a direct anticipation of Pascal's.]

[14][What is generally known as the Arithmetical Triangle is not mentioned in the *Lettres de Dettonville;* see Note 19.]

[15][It may be of interest to note that the pseudonym of Amos Dettonville is an anagram on Lovis de Montalte; Lovis, or Louis de Montalte being the pseudonym under which Pascal's *Lettres provinciales* appeared.]

[16] Pascal published what he had written to de Carcavi along with the five essays in the following year, under the title of: *Lettres de A. Dettonville contenant quelques unes de ses Inventions de Geometrie. Scavoir, La Resolution de tous les problemes, touchant la Roulette qu'il avoit proposez publiquement au mois de Juin, 1658. L'Egalité entre les Lignes courbes de toutes sortes de Roulettes et des Lignes Elliptiques. L'Egalité entre les Lignes Spirales et Paraboliques, demonstrée à la maniere des Anciens. La Dimension d'un Solide formé par le moyen d'une Spirale autour d'un Cone. La Dimension et le Centre de Gravité des Triangles Cylindriques. La Dimension et le Centre de Gravité de l'Escalier. Un Traitté des Trilignes et leurs Onglets. Un Traitté des Sinus et des Arcs de Cercle. Un Traitté des Solides Circulaires. A Paris, M DC LIX.* This writing contains the essays of Pascal of the year 1658 together with communications to Huygens, de Sluse, and an unnamed correspondent. From the correspondence of Huygens in the years 1658 and 1659, which is printed in that truly great work: *Oeuvres Complètes de Christiaan Huygens publiées de la Société Hollandaise des Sciences,* we see that a great movement arose among contemporary mathematicians through Pascal's problems, as well as through the printed works that we have mentioned. Leibniz expresses himself thus: "By this time, the controversy [referring to Gregory St. Vincent] had cooled down; when lo! fresh movements in the realm of geometry are stirred up through the whole of France, by Blaise Pascal, a man

of the highest genius, and one who at that time had come nearer to the reputation of Galileo and Descartes than any one else."—This writing of Pascal was recommended for study to Leibniz by Huygens. [As given by Pascal in his letter to de Carcavi, containing the particulars of his method for centers of gravity and the definitions of "trilignes" and "onglets," the problems proposed in June were:

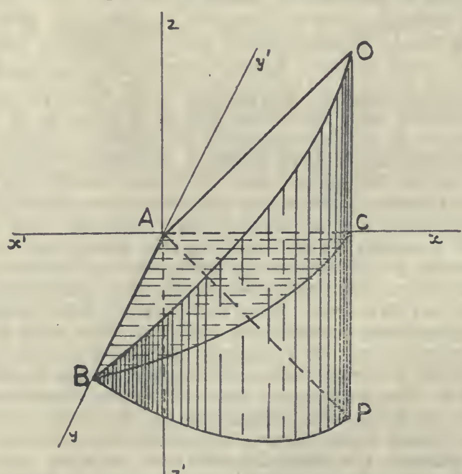


1. To find the dimension and the center of gravity of the space CYZ.
2. To find the dimension and the center of gravity of its semi-solid of rotation about the base ZY, i. e., the solid formed by the triligne CYZ when rotated about the base ZY through half a turn only.
3. To find the dimension and the center of gravity of the solid of revolution about the axis CZ.

To which are added the three proposed in the *Histoire de la Roulette* at the commencement of October:

1. To find the dimension and the center of gravity of the curved line CY.
2. To find the dimension and the center of gravity of the surface of the semi-solid about the base.
3. To find the dimension and the center of gravity of the surface of the semi-solid about the axis.]

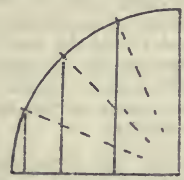
[17] By "Triligne" Pascal intends a plane figure bounded by two straight lines perpendicular to one another and a curved line. One of these perpendicular lines is called the axis and the other the base of the figure. If upon such a figure as a base there is erected a right solid, and this solid is cut by a plane which passes through the axis, or the base, then the portion of the



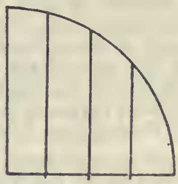
solid that is cut off is called an "onglet." A "double onglet" is obtained if, through the solid formed by production on the other side of the base, there is drawn a plane with the same inclination. [The last sentence does not make it clear that the second cutting plane also passes through the axis, or the base, as the case may be; nor that the plane is anticlinic and not parallel to the first plane; nor that Pascal took in general the inclination of the planes to the plane of the triligne to be 45°. I have therefore tried to represent the onglet and the double onglet in a diagram, see above.

ABC is the triligne, OABC is the onglet of (the axis or base) AB, and OBPCA is the double onglet of AB; the angles OAC, PAC are half right angles.]

[18] By *Sinus* Pascal intends the ordinates multiplied by the indefinitely small portions of the arc. [This is a very misleading statement; for Pascal distinctly distinguishes between *sines* and *ordinates,* and thus makes a considerable advance over his contemporaries. He defines them at the same time for finite section and for infinitesimal section; the distinction is made perfectly obvious in a diagram if we use finite section, say, division into four equal parts, of the quadrant of a circle as a special case of a triligne. Now the sum of the



SINES OF THE BASE.



ORDINATES OF THE BASE.

sines or the ordinates are defined as the sum of the rectangles (for, as with all cases of indivisibles, that is what it comes to), formed by the sines or the ordinates respectively multiplied by the corresponding *equal* sectional parts. Thus, to speak of the sum of the sines as being the ordinates multiplied by the small portions of the arcs is quite wrong. Though only in rare cases is the space drawn, Pascal's idea of the sum of the sines is that of the space formed by *straightening* the arc and erecting at each point of division the corresponding sine. Now, as Pascal says in Prop. 1 of the *Traitté des Trilignes,* the sum of the ordinates, which have to be applied to the base, makes the figure itself; while in Prop. 1 of the *Traitté des Sinus du quart de Cercle,* he shows that the sum of the sines (as a special case of the general theorem quoted in iii by Gerhardt *supra,* p. 534) of a quadrant is equal to the square on the radius. Thus, in modern notation,

$$\text{sum of sines} = \int_0^{\frac{\pi}{2}} r \sin\theta \,.\, d(r\theta) = r^2$$

$$\text{sum of ords.} = \int_0^{\frac{\pi}{2}} r \sin\theta \,.\, d(r\cos\theta) = -\tfrac{1}{4}\pi r^2 \,.$$

The concluding paragraph of the *Traitté des Solides Circulaires* runs thus: "All these results arise from the fact that the straight lines OI are ordinates, that is to say that they are equally distant and proceed from equal divisions of the diameter; this brings it about that the simple sum of the ordinates is the same thing as the space intercepted between the extremes. But this is not true for the sines, since the distances between adjacent ones are not equal to one another, and thus the sum of the sines is not equal to the space intercepted between the extremes; *there must be no mistaken idea on this point.*" We find the same care taken by Barrow; but Tacquet breaks down in determining the surface of a cone through not understanding the necessity of this point, and in consequence condemns the method of indivisibles.]

[19] [The effect is as Gerhardt states, but these sums are differently *defined* by Pascal in his letter to de Carcavi. The triangular sum of the numbers or magnitudes A, B, C, D, starting with A, (which should be stated), is the sum of all of them, plus the sum of all of them except the first, A, plus the sum of all except the first two, A and B, and so on; this is represented by Pascal as in the margin, and he goes on to show that this is equal to the first taken once, the second twice, and so on. Thus defined, the reason why they are named triangular numbers is obvious. The pyramidal sum is similarly defined as the triangular sum of all, plus the triangular sum of all except the first, plus the triangular sum of all except the first two, and so on. As if there were built up a pyramid having the first triangular sum as its bottom layer, the second triangular sum as the next layer, and so on; thus defined, the origin of the name pyramidal is obvious. Pascal then shows that this is the sum of the quantities taken respectively once, three times, six times, and so on, according to the sequence of the triangular numbers. Then using the property that twice a triangular number diminished by its ordinal number is equal to the square of that ordinal (i. e., $n(n+1) - n = n^2$), he also shows that, if two such pyramidal sums of quantities are taken, and from one of them the bottom layer is removed (i. e., the first triangular sum), then the sum of the two is equal to the sum of the quantities respectively multiplied in succession by the squares of the natural numbers. There is no connection between this and what is usually known as the Arithmetical Triangle of Pascal (see *The Monist,* Oct., 1916, p. 603).]

A	B	C	D
	B	C	D
		C	D
			D
A	B	C	D
1	2	3	4

[20] [Pascal simply states the results, as deduced, not from the theorem quoted by Gerhardt, but (together with the theorem quoted) from the preliminary lemma that the radius is to the sine as the hypotenuse of the infinitesimal triangle is to its base: in modern notation, $r : y = ds : dx$, or $r\,dx = y\,ds$, where y is a *sine* and not an *ordinate* in Pascal's sense. All the following theorems are particular cases of the formula $\int y^n\,ds = r . \int y^{n-1} dx$.]

[22] Descartes had spoken disparagingly about Pascal's "Essay on the Conics." Perhaps Pascal's decided opposition to Descartes may be traced back to this. Pascal's niece, Marguerite, writes: "M. Pascal used to speak very little about science; however, when the occasion for doing so occurred, he would state his opinion on those matters about which people were speaking to him. For example, with reference to the philosophy of Descartes, he merely said what he thought. He was of the same opinion as Descartes concerning automatism, but far from being so on the "subtle matter," which he ridiculed. But he could not put up with his (Descartes's) method of explaining the formation of the universe, and he often said: "I cannot pardon Descartes. In the whole of his philosophy, he would have been highly pleased to have dispensed with God; but he could not help making use of him to give a fillip to set the universe in motion. That being done, he had no further use for God." (Fougére, *Lettres, Opuscules et Mémoires de Madame Perier et de Jacqueline, soeurs de Pascal, et de Marguerite Perier, sa nièce.* Paris, 1845, p. 458). [It is more probable that Pascal used geometry, as Barrow did, because he both preferred it and thought it more rigorous than analysis. With regard to the remark on method, Gerhardt does not intend to convey the impression that Pascal abandoned for the more strictly geometrical method of moments the mechanical idea of the *balance,* with which he commences. By the way, to the best of my belief, the word "moment" is never used by Pascal.]

[22] [I have gone carefully through the "Lettres of Dettonville," and I find no mention of Archimedes except in one place, namely, Prop. 1 of the *Traitté des Solides Circulaires*; and the whole of this is devoted to *volumes* of solids and their centers. Nor can I find any place where Pascal determines the surface of a sphere, at least not by reducing it to an equivalent plane figure, I have however shown that Barrow does do this (see *The Monist,* Oct., 1916, pp. 610, 611). Surely Leibniz must be confusing the work of Pascal with that of Barrow on quadratures, the latter being so similar to the former in places that Barrow might easily be suspected of "borrowing" from Pascal; much more

easily indeed than Leibniz could be so suspected with regard to either, in spite of his own assertion with regard to Pascal. See Notes 23, 24.]

[23] [These are far more like Barrow's results than those of Pascal; while the style is entirely Barrovian and quite different from that of Pascal.]

[24] [There is no rectification of curves in Pascal; the whole of this sentence would however serve as a summary of the work of Barrow on rectification.]

[25] [Gerhardt states that the Centrobaryc Method, as considered by Leibniz in the manuscripts dated October 25, 26, 29, and November 1, 1675, shows clear connection with the work of Pascal. He asserts that, from a consideration of Archimedes, Pascal was enabled to extend the method of the ancients; he does not seem to be aware of what Cavalieri had done and published as the fifth section of his *Exercitationes Sex*; or else, knowing all about this, he suppresses that knowledge for fear of discrediting the statements of Leibniz concerning the methods of Cavalieri.

The striking points about the work of Cavalieri in question are as follows. He opens by defining gravity as a property of a body, a descensive force. He then defines a heavy body as one possessing this property, and in a note on the definition, he adds that these must be taken to include surfaces, lines, and points. Then he gives the definition of "moment" in its mechanical sense. "The moment of a weight is its endeavor to descend, no matter at what distance it is hung." This is followed by the note: "Since this moment is different at different distances, as will be seen in what follows, it is to be understood from this that the same weight may have different moments." He then defines uniform and uniformly variable (*difformis*) weights, such as a parallelogram in which the density varies as some power of the distance from one side; also he defines the centers of gravity and equilibrium. In Prop. 6 he shows that the moments of bodies are compounded of the ratio of their weights and the ratio of their distances. In Prop. 8 et seq., he combines the doctrine of indivisibles with that of moments to find the centers of gravity of surfaces, chiefly by means of "analogous figures"; thus, a uniform triangle is analogous to a parallelogram whose "difformity is of the first species," i. e., the density varies as the distance from one edge. He shows that, if the difformity is of the nth species, i. e., if the density varies as the nth power of the distance from the edge, then the medial line is divided by the center of gravity into parts in the ratio of 1 to $n+1$, although it is stated rather differently, and only worked out for the first few values; then, using the idea of moments he proceeds from one degree to another in the case of the triangle, where the axis of moments (*limes*) is a parallel to the base through the vertex, and in the following proposition, the base itself; next the semicircle and the hemisphere are dealt with, whether uniform or varying as the distance from the center. In Prop. 36, he lays down the idea that the axis of moments may be outside the figure under consideration; and then proceeds to consider cylinders, cones, parabolic conoids, and the sphere, and truncated portions of them; and finally he finds the moment of a portion of a hyperbola about the asymptote which is not the base of the portion considered. It is interesting to note that Cavalieri, when speaking of the difformity of weight, uses the phrase "*incrementum* difforme gravitatis," i. e., the word *incrementum* is employed to connote a *gradual* increase that follows a definite law. Also it is worthy of remark that he employs the notation, *o. l., o. p., o. q., o. c.,* etc. for "all the lines," "all the planes," "all the squares," "all the cubes," etc.

From the above it will be seen that Cavalieri has given a fairly comprehensive account of the use of moments for the determination of the center of gravity; thus he not only gives far more than Pascal, but anticipates him. Leibniz's matter is far more like that of Cavalieri than that of Pascal; though he seems to be reading Pascal at the time he wrote the third part of the "Analytical Quadrature," by the method of moments, for the last figure in this manuscript (see *The Monist,* April, 1917, p. 268), with the explanatory diagram that I have added on the right of it, is strongly reminiscent of the idea of the onglet of Pascal; although it may have arisen from Cavalieri's work. The great point about this batch of manuscripts of October and November, 1675, is that nearly every figure has the tangent drawn to the curve; now the tangents are never drawn or used either by Cavalieri or by Pascal. A secondary consideration, but still one of importance, is that the subject-matter of these manuscripts is like nothing in Cavalieri or Pascal, as far as the "center of gravity method" is concerned. As we find Pascal's Infinitesimal Triangle idea in the figure of Leibniz's manuscript of October, 1674, I take it that this was the time at which he *finished reading* his Pascal. Hence, I imagine that in October, 1675, he had got a good knowledge of Descartes's algebraical geometry, and began to study Cavalieri's *Exercitationes Sex*; he did not get very far in this before he appreciated the power given by the method of moments; then, probably wearied by Cavalieri's prolix demonstrations, he laid the book aside, and applied Cartesian analysis to the method of moments, running the idea for all it was worth. If this is the case, these manuscripts represent real *original* research, and are not study notes like some of the others.]

[26] [The misreadings of Gerhardt, as given in his *Geschichte der höheren Analysis* (see *The Monist,* April, 1917, p. 244) are uncorrected even in 1891, the date of this essay, thirty-six years after the publication of the *Geschichte*! We should have "AB $\sqcap$ DC $\sqcap x$" and "AB$(=x)$"—see the figure on the right (p. 538), which is mine while that on the left is the one that Gerhardt gives as that of Leibniz; again Gerhardt's "id est ac in omn.x, sive $a(cb^2/2)$," which makes Leibniz write nonsense, should be "id est a c in omn.x, sive a $cb^2/2$," the "a" being the preposition "away from" and not the length of a line; thus corrected we not only have a sensible reading but the whole paragraph is correct; I have made the correction when translating. Also with regard to Gerhardt's statement that Leibniz starts from an alternative rendering of Prop. 2 of Pascal's *Traitté des Trilignes,* it is worthy of remark that Pascal's figure is altogether different from that of Leibniz; and this is only natural, because there is *no similarity between the theorems, nor is there any relation between the methods of proof.* Pascal's proof is equivalent to the modern method of a change in the independent variable by a conversion to a double integral followed by a change in the order of integration, and is geometrical; that of Leibniz is equivalent to integration by parts, and is merely an example of the theorem of moments.

Thus (Pascal), $\int yx\,dx = \int(\int x\,dx)\,dy = \int \tfrac{1}{2}x^2dy,$

and (Leibniz), $\int yx\,dx = [\tfrac{1}{2}x^2y] - \int \tfrac{1}{2}x^2dy;$

where Pascal's integrals are taken over the *same* area as one another, and those of Leibniz over *complementary* areas. It seems therefore ridiculous to say that "Leibniz commences with Prop. 2.... which he expresses as follows."]

[27] [This means the result obtained geometrically by means of the triangle AZC, in the passage to which Note 23 refers.]

[28] [The connection between number, ratio, and *infinitesimal* is peculiar.]

[29] [Note the word "useful" (*utile*): the "long s" is introduced merely as a convenient abbreviation in accordance with Leibniz's usual idea of obtaining simplification by means of symbols.]

[30] [I have discussed this fully in my translation of Gerhardt's essay, "Leibniz in London" (see *The Monist,* Oct., 1917, p. 545). I have shown there that at least it is highly probable that the d in x/d stands for a certain length, namely the subtangent.]

[31] [Note that, in spite of Gerhardt's opening remarks about the algorithm of the calculus being due to reading Pascal, the symbols of integration and differentiation have not been mentioned in anything quoted by Gerhardt in this essay, except in the paragraph just above.]

[32] [See *The Monist,* Oct., 1917, where a translation has been given. I believe some of those who read what is there given will, while giving Leibniz full credit for the introduction and *development* of the symbols $\int$ and $d,$ that made the calculus of Leibniz the powerful instrument it was, will find it hard

if not impossible to agree with Gerhardt in his assertion that the ideas of Leibniz were not very strongly influenced by the best points of every single author that he studied, and more especially by the *Lectiones Geometricae* of Barrow and the *Exercitationes Sex* of Cavalieri.]

B. Notes to the Manuscripts of Leibniz.

I.

[33][So far I have failed to find any information as to the error into which Reginaldus fell; he does not appear to be mentioned by either Cantor or Zeuthen.]

[34] [The Geometry of Cavalieri is indeed practically all quadratures; but Torricelli himself says (quoted by Tommaso Bonaventura in his preface to an edition of the *Lezione Accademiche,* 1715), in his preface to a *Tract on Proportion,* that he has used indivisibles for tangents as well as for quadratures; Roberval, through his own efforts at concealing his methods, we know comparatively little about; but the germ of Fermat's method is the same as that of Barrow's, namely the Differential Triangle; lastly it is probable that Huygens's knowledge was considerably more than he let anybody know (and so too with Gregory)—cf. Leibniz's words, "suppressing their analysis," a few lines later. It is to be observed that Leibniz deliberately speaks of the mathematicians of France and Italy only; "at the present time," 1679, he must have been aware that Barrow had complete geometrical knowledge, at any rate, of *all* the matters in question.]

[35][The *Horologium* was published in March or April, 1673, and the presentation of a copy to Leibniz was undoubtedly made *after* his return from his first visit to London (Cantor says that the dedication was dated March 25, 1673; see Cantor, III, p. 138). Hence, the date at which Leibniz obtained the Characteristic Triangle can be assigned to some time at least not later than the beginning of October, 1673; and therefore the inclusion of this in the manuscript dated Aug., 1673 (see *The Monist,* April, 1917, p. 238), marks the exact date of its discovery.]

[36][In the "Bernoulli postscript" (see *The Monist,* Oct., 1916, p. 584), Leibniz states that "he obtained a Dettonville from Buotius, a Gregory St. Vincent from the Royal Library, and started to study geometry in earnest." In the *Historia* (see *The Monist,* Oct., 1916, p. 595) Leibniz says that, "in order to obtain an insight into the geometry of quadratures he consulted the *Synopsis Geometriae* of Honoratus Fabri, Gregory St. Vincent, and a little book by Dettonville (Pascal)." In his letter to the Marquis de l'Hospital he says, "At the start I only knew the indivisibles of Cavalieri, and the 'ductions' of Father Gregory St. Vincent, along with the 'Synopsis of Geometry' of Father Fabri" (see *supra,* p. 544). I suggest that the correct explanation of these inconsistencies is that he did get the Dettonville from Huygens as stated here, the St. Vincent from the Royal Library, and the work that he obtained from Buotius was the *Exercitationes Sex* of Cavalieri.]

[37][I think the passage throws considerable light on the character of these manuscripts, besides explaining how it was that Leibniz seems to have taken a very long time to study the works of the authors mentioned. I look on these manuscripts, not as "study notes" merely, nor yet as true "research," but as a mixture of each. I suggest that there is quite enough evidence to make it safe to assert that the characteristic of Leibniz's method of study was to read a very small portion of an author at a time, then to break off and follow out the train of ideas suggested to him by the passage to the furthest limit, before proceeding further with his reading; thus he is led to his own *original* developments. For instance, note in the next sentence how he says he "tried to find a *new* sort of center." This is very characteristic; he is not satisfied with merely acquiring knowledge, even at this early stage, but at once seeks to utilize each point, *as he grasps it,* to obtain something new, something

original previously undiscovered. Cf. the study notes on the work of Pascal, given above under III.]

[38][That is, a "homothetic center."]

[39][As I have been unable to find the word "moment" defined, or even mentioned, in any place except in the *Exercitationes Sex* of Cavalieri, I suggest that this is fairly good circumstantial evidence for the reading of this work by Leibniz before he discovered the theorem in question.]

[40][Observe that this is not the figure used in the manuscript of October, 1674 (see *The Monist,* April, 1917, p. 241), the latter being a diagram that one would naturally expect him to have obtained from the figure in the lemma that commences Pascal's *Traitté des Sinus du quart de Cercle* (cf. Note 6); but is a figure such as one would expect Leibniz to abstract from those given by Barrow, either from Lect. XII, prop. 1, 2, 3, or from Lect. XI, Prop. 1 (see *The Monist,* Oct., 1916, p. 610 and p. 616 respectively. In the latter especially we have the right-angled triangle used by Leibniz on page 596, quoted by Gerhardt in the article translated in the present number). I therefore suggest that Leibniz worked at Barrow and Pascal conjointly, and applied Descartes's analysis to their geometrical theorems. If this is not the case, Leibniz was at fault, for Pascal was discussing *sines* and not *ordinates* (see Note 18); i. e., Pascal was integrating with regard to θ and not with regard to x. Observe also that the figure as given is not correct; the rectangle should be that having AC, CD as adjacent sides.]

[41][Note that the area is taken to be produced by the assemblage of lines applied in order, in the true Cavalierian style.]

[42][Query: urged thereto by a question on the part of Huygens, as to whether Leibniz could now find the properties of the auxiliary curve (see *The Monist,* Oct., 1916, "Bernoulli postscript," p. 585).]

[43][This fits in perfectly with my suggestion that Leibniz attacked Barrow's *Lectiones* at several different times. Having, as I think, taken Barrow's advice given in the preface, he sampled the first few propositions of each lecture, and obtained from those of Lect. XI and XII his Characteristic Triangle. This could I think have been definitely settled if Gerhardt had only given the figure used by Leibniz in the manuscript dated August, 1673. Assuming for the time being that my suggestion is correct and that Leibniz is merely confusing the author that he read at this time, I suggest that characteristically he broke off his reading of Barrow, pursued the idea he had obtained, and made out those theorems on quadratures that he speaks of; this so improved his geometry that later he was able to read Barrow thoroughly and appreciate all that was in it, and to find that his theorems had been anticipated. I also suggest that it was on this second or third reading that he came across the theorem that led to his Arithmetical Tetragonism. A fresh reference to Barrow to find if there were any other ideas that he could develop, considerably later, having already found him a mine of information, would then probably be the occasion on which the marginal notes in his own notation were inserted by Leibniz.]

[44][Leibniz seems to have got these men in true perspective, Cavalieri, Fermat, Gregory, and Barrow, as far as the infinitesimal calculus is concerned. But I doubt whether he, even after he came to his fullest appreciation of Barrow's *geometrical theorems,* or indeed any other person except Bernoulli, ever appreciated the real inwardness of these theorems, or that Barrow's tangent problems could be used, in the manner I have shown in the appendix to my Barrow, to draw a tangent to any curve *given by an equation in either Cartesian or polar coordinates.*]

[45][This I take to mean the principle that differentiation and integration are inverse operations; for it is practically certain that in November, 1675, he could not differentiate a product; otherwise, as previously argued, he would

have verified his solution of the unfortunate equation, $x + y^2/2d = a^2/y$, which he gives as

$$(y^2 + x^2)(a^2 - yx) = 2y^2 \overline{\operatorname{Log} y},$$

by differentiation, as he did with a previous solution that did not contain a product.]

[46][From this probably arose the first germ of the idea of the Quadratrix, in the sense used by Leibniz.]

[47][Substitute *Barrow* and Mercator in conjunction, and we have a feasible suggestion for explaining the first method of proof for the Arithmetical Quadrature of the Circle; the method that Leibniz does not seem ever to have divulged.]

[48][It is impossible for me to conjecture exactly which of his ideas is here referred to by Leibniz; for he calls a mere method by the name of "a calculus," and what we should call a dodge for some particular kind of example by the name of "a method." I think it may be possible that the "transmutation of figures" is referred to.]

II.

[49][Notice that Leibniz says that he has not derived any help from Barrow for his *methods* (*je n'ay tiré aucun secours pour mes methodes*). This is less even than he might have said with perfect truth; for the *methods* of Barrow would have been a veritable hindrance to Leibniz's analytical development. Even when using the Differential Triangle method, and literals for the lengths of his lines, the whole of the working is geometrical in the examples of the method given by Barrow, and not analytical.]

[50][See Notes 35, 36.]

[51][Perhaps this is meant to include Barrow.]

[52][Notice the words "by chance" (*par hasard*); these seem to point to a conclusion that Leibniz read the Pascal in a very desultory manner; this conclusion gets corroborated by the extract given by Gerhardt under the heading III. It is worthy of remark that the "by chance," or "incidentally" (as I have rendered Leibniz's word *forte* in the letter to Tschirnhaus), is made to refer to Pascal. "*Forte Pascalius demonstratbat,*" etc., i. e., "Incidentally Pascal was proving," etc. I think it may be asserted that Pascal missed absolutely nothing that was pertinent to *his purpose*; whereas Barrow certainly missed the opportunity of being the discoverer of the series for the inverse tangent, and thereby the quadrature of the circle, by not applying Mercator's method of division and integration to the result of one of his examples of the Differential Triangle method; as also after giving the method of "transmutation of figures" he missed those things to which it led.]

[53][In a manuscript dated October, 1674 (see *The Monist,* April, 1917, p. 240), Leibniz is using x and y for the variable ordinate and abscissa; while in a manuscript dated August, 1673, he considers "the classification of curves laid down by Descartes." In this manuscript, according to Gerhardt, Leibniz has already constructed the "characteristic triangle," but Gerhardt does not give the particular variant that Leibniz uses in this manuscript. I believe that this will prove to be of the Barrow type, when reference can be made to the original; for the title of the manuscript is strongly suggestive of Barrow, being: *Methodus nova investigandi Tangentes....ex datis applicatis,* etc.; and Pascal's work does not mention tangents.]

[54][That is, as the Characteristic Triangle, leading to integrations, is ascribed to the influence of the work of Pascal, so the Differential Calculus is ascribed to the influence of the work of Descartes. Is this the diplomatic characteristic in Leibniz peeping out? He is writing to a Frenchman, and attributes his work to the respective influences of two Frenchmen. Note that

Leibniz goes on to state that the source of inspiration was summation of series by differences, suggesting the origin of the symbol *dx*.]

[55][In the manuscripts that we have had under consideration, Leibniz does not appear to have made any practical use of the Quadratrix.]

[56][It is precisely this point which formed the really great improvement in the reckoning section of the infinitesimal calculus. It is just this improvement that is due to Leibniz in analysis, and to Barrow in geometry; although Leibniz did not accomplish anything of the kind until 1676 or 1677. Newton's method by means of series for fractions and roots does not bear comparison, let alone the futility of ascribing Leibniz's method to a perusal of Newton's work.]

III.

[57][All that is any good in the following is to be found in Pascal; I think this corroborates the suggestion I have made as to Leibniz's way when studying a book. It looks here as if he had read about twenty pages of Pascal, and about the same number of pages of Cavalieri's section on centers of gravity; moved thereto probably or possibly by Pascal's remark "....the principle of *indivisibles,* which cannot be rejected by any one having pretensions to rank as a geometer." Then he proceeds to work out his own combination of the two ideas, without bothering to see what else either of these authors had to say on the matter.]

[58][Leibniz tacitly assumes that all the points are occupied; this is necessary for the success of the notion of triangular sums.]

[59][Something very like this is indeed investigated fairly thoroughly in a manuscript dated October 25, 1675 (see *The Monist,* April, 1917, p. 245). Hence these extracts from Pascal were certainly made before that time, though probably not long before.]

[60][This is the rendering for "*productum fieri aequale*"; he probably means that what is produced on the one side, i. e., the sum of the moments on one side of A, should be equal to the sum of the moments on the other side. But this endeavor to obtain *something new* seems rather futile.]

[61][It would have been interesting to have seen what this simple rule was. Probably nothing more than the propositions given by Pascal as Prop. 1, 2, 3 of his method of the *balance*; this would corroborate my suggestion that Leibniz did not study Pascal very steadily or thoroughly (cf. Notes 37, 43, 52, 57).]

SUMMING UP.

The notes and criticisms that I have made in these five articles on the manuscripts of Leibniz may give the impression that I am an anti-Leibnizian. This is quite wrong. My prime object was to show, to the best of my power, that the charges of plagiarism brought against Leibniz by partisans of Newton, and indeed by Newton himself in the *Recensio* published in the *Philosophical Transactions,* were unfounded. I considered that the charges in the *Recensio* were perhaps the hardest to be answered, since they were not only direct charges, backed with circumstantial evidence, but they were also set forth very cleverly. Also I thought that the method of defense adopted by Gerhardt and other partisans of Leibniz did as much harm to him as the strongest attack of avowed opponents, such as Sloman. If I am anti-anyone, I am anti-Gerhardt. Never surely did any man have such a glorious opportunity as Gerhardt, in the whole history of scientific controversies; surely there never was an advocate who left himself so open to the attacks of the opponents. Gerhardt starts with the theory that every single word of Leibniz represents gospel truth; and that it is almost blasphemy to doubt it; in consequence he is soon in difficulties when it comes to reconciling the varying statements of the sequences of events that are made by Leibniz at different times. But, once the idea is accepted that Leibniz, while perfectly reliable on the general run of events, is unreliable when it comes to unimportant details, and then all difficulty disappears. I therefore set out with the determination to break down, if I could, the credibility of Leibniz as a witness in his own defense, when it came to unimportant details; then to show that he had opportunities for obtaining everything necessary to the development of the Calculus, that he could not be expected to supply for himself by original work, without having need to know anything of the work of Newton; then to show that these sources of information were set out in a form far more suitable to the requirements of Leibniz than the work of Newton; finally, to clinch the matter, that the analogy of Leibniz's work was so close to these sources, that it was idle to suppose that he made use of any other sources. In other words, (i) the *Analysis per aequationes* was unnecessary to Leibniz, (ii) Newton's method of dodging fractions and roots by means of infinite series was clever, but futile for the needs of Leibniz when developing an operational calculus.

The unreliability of Leibniz with regard to details may be in some measure due to his admittedly bad memory (which is evidenced by his habit of committing everything to writing), and to passage of time. But in a far greater degree it must be ascribed to the circumstances and characteristics of Leibniz. We know that he designed to compile an encyclopedia of *all* science, and for this he considered not at all the nationality or the personality of the discoverer or the author: all he was interested in were the facts or principles discovered.

That he was unreliable with regard to details is proved by the facts I have adduced:

i. the confusion between Mouton and Mercator in the account of the first charge of plagiarism made against him, or rather an assertion that he had been anticipated (see *The Monist,* Oct., 1916, p. 594, and Note 73);

ii. the varied assortment of figures that he gives to illustrate how he found the Characteristic Triangle (see *The Monist,* Oct., 1916, p. 585, and compare them with the figures given in the accounts quoted by Gerhardt in this essay and those published in *The Monist,* Oct., 1917);

iii. the circumstantial detail of the context of the Archimedean measurement of the surface of the sphere being absent from the author he quotes;

iv. the several different accounts of the order in which he obtained his different books for study, and even the persons from whom he obtained them;

v. the error with regard to the time of the presentation of the copy of the *Horologium* (see *The Monist,* Oct., 1916, p. 594, where, in the *Historia,* it is stated that he received it *before* he left for England on his first visit);

vi. the confusion as to the date at which he obtained his Barrow (see *The Monist,* Oct., 1916, p. 586, where, in the Bernoulli postscript, he states that he found the greater part of his theorems anticipated in "Barrow, when his *Lectures* appeared");

and many other things, all unimportant details singly; but, when taken in combination, they show distinctly that we must only take Leibniz's word as accurately describing the *general* course of events.

Another characteristic of Leibniz seems to have been insistent at all times; he burned to distinguish himself as a discoverer of new things. I have suggested that there may have been an ulterior motive to this desire, namely, to get himself taken into the select circle of mathematicians who corresponded with one another. Thus, when he studied an author, and came across some new idea, he would break off his reading to follow that idea to the limit and exhaust all its possibilities, committing his results to writing, whether they were important or not; there is some evidence, too, that while doing this, he would refer to other authors who had discussed the point under consideration, before returning to his reading.

My motive in trying to show that he got everything from Barrow, *except his methods,* was to remove any charge of plagiarism; for, I consider that even if he had merely rewritten Barrow in terms of Descartes, adding his own notation for the sake of convenience, he would still have done a great thing, and would no more have been guilty of plagiarism from either Descartes or Barrow than Stephenson was from Watt, or Parsons from either of these. Leibniz's Calculus was his own, and would have been his own even on the supposition above. Lastly, it was not only more complete than that of Newton, in that it was an operational calculus, though it did perhaps miss the idea of rate; but also from an intellectual standpoint it was greater, in that it was developed, after its first principles were found out, as a practical theory, while Newton's was developed as a mere instrument for his own purposes.

Assuming, then, that Leibniz did not remember, or did not really care, what his text-books were, so long as he was not accused of using somebody else's *methods,* I will try and reconstruct the progress of his reading and his discoveries. His text-books were,

i. Lanzius and Clavius in algebra, and Leotaud for geometry, in his early youth; he also looked through, more or less without understanding them, Descartes and Cavalieri's *Geometria Indivisibilibus.*

ii. On his return from London he brought back with him Barrow, some portions of which he had glanced at in London and on his journey; he obtained Pascal, St. Vincent, and Cavalieri's *Exercitationes Sex*, perhaps a little later than the others; besides these, Wallis and Mercator specially.

He read portions of the Barrow afresh, and obtained the Characteristic Triangle, and found his general theorem from this; meanwhile he is also studying Descartes, and we have the materials for the manuscript of August, 1673. Probably he has had a look through Pascal during this time. He remembers the similarity between the complicated diagrams of Barrow and some of those of Pascal, and starts studying the *Traitté des Sinus,* in which he finds the second variant of the differential triangle that appears in the manuscript of October, 1674. Previous to this, however, his attention has been arrested by Barrow's proof of the inverse nature of the operations of finding a tangent and an area, and the analogy between this and sums and differences strikes him. He has also considered the examples on the differential triangle given by Barrow; one of them suggests the method of Mercator to him, he has already got an idea from Wallis of the summation of the several powers of the variable; he applies this to Barrow's expression, equivalent to

$$d(\tan^{-1}x)/dx = 1/(1 + x^2),$$

in modern notation, performs the division as Mercator had done, and obtains the series for the inverse-tangent by a summation according to Wallis, i. e., practically an integration. This answers the charge made by Newton that somehow or other he got this series from him or James Gregory. In the same way, he thought that he could obtain other series, but later found that it was beyond his power. We find in this manuscript of October, 1674, an attempt to get something out of an analogous series, the logarithmic series, showing that it is very probable that he has been studying Mercator during the interval between August, 1673, and October, 1674. And in the *Historia* he definitely states that he came upon the Arithmetical Tetragonism in 1674; so that I think that I have offered a reasonable suggestion as to the course his studies took so far. Also in the meanwhile he has been doing a lot of work on series, and has invented his Harmonic Triangle. I now suppose that he completes his study of Pascal, is led by a remark in it to study the *Exercitationes Sex* of Cavalieri (he has already got some acquaintance with the *Geometria Indivisibilibus*, read as a youth), he does not find much in that to his liking, except the notion of moments. He breaks off his reading and proceeds to work out an application of Descartes's algebra to this new idea of moments, the result being the manuscripts of October and November, 1675; here he is led on to the introduction of the symbols for summation and differentiation, though as yet applied to series, and sums of powers. The consideration of the Quadratrix leads him to make a further study of Barrow; and he is led to x/d, by a consideration of Barrow's propositions on the inverse nature of the operations of integration and differentiation. This, combined with the analogy to the inverse nature of summations and differences, leads him to search for a reason why x/d should represent a difference such as he has considered to be denoted by dx. This at a later date necessitates the discussion of what the result of *operating* with *d* on a product or a quotient will be. Meanwhile the study of Barrow brings him to that proposition which gives the polar differential triangle; in it he perceives at once the method of "transmutation of figures." I now suppose that he appreciates Barrow more fully and begins to apply Cartesian geometry to Barrow's theorems; in a manuscript dated November, 1675, he attacked the problem of tangents, and in connection with it considered the method of Descartes. In the next manuscript that we have, dated June, 1676, he practically obtained the differentiation of the sine and the inverse sine; his figure, if he had given one, would have been the same as that of Barrow for the differentiation of the tangent. In July, 1676, he attacked the inverse-tangent problem, still considering the work of Descartes. I think, however, that his work on Barrow has taken effect, for from now on he includes the differential factor dx under the integral sign. This is the last manuscript before he went to London for the second time.

Thus, I take it that all Leibniz's work is the result of his own original methods on ideas that have been suggested chiefly by two books, those of Barrow and Descartes; at least, everything could have been suggested by these two books alone, except the notion of "moment," which came from Cavalieri. Thus it was unnecessary for him to have known anything about the work of Newton before he went to London for the second time. What he saw there may have had the effect of corroborating his own work; it could have had little other effect. The final polishing of his method I put down to a study of the Differential Triangle method of Barrow, which Leibniz perceived to be powerful, but found distasteful on account of the geometrical nature of the work.

J. M. CHILD.

DERBY, ENGLAND.

A BIOLOGIST'S RELIGION.

"All are but parts of one stupendous whole,
Whose body Nature is, and God the soul."

WHEN I first read Darwin's *Origin of Species* it seemed too plausible to be true. Its conclusiveness appeared to shut out all future speculators from speculating in naturalism. Can it be, I asked myself, that this problem of creation, so perplexing for so many centuries, has been practically solved? Logically, in view of the intellectual assurance of previous generations, the solution, somehow, smacked of prematurity.

Subsequent developments justified the doubt. Investigation uncovered some glaring errors in previously plausible assumptions. What had passed as facts proved to have been purely fiction. The veil of speculation covering many natural phenomena was rent, exposing naked mechanisms whose design and capacities fitted no better to an evolutionary doctrine than to a vitalistic one. A complicated and marvelously correlated animated world revealed itself; each revelation becoming an obstacle to, instead of assisting, further speculation.

Vistas are opening up beyond the temporarily fascinating mechanistic reviews which make manifest their shortcomings. We see in them a kernel of truth surrounded by a mass of chaff.

A pragmatic enlargement of viewpoint suggests itself; a viewpoint which should include the possibilities of special creation, combined with evolution. The present age cannot expect to monopolize the ultimate truth. History declares for the acceptance of a fundamentally mechanistic groundwork for all living things, functioning by grace of some "vitalistic" or external influence.

In other words, every new evidence points to the correctness of science's diagnosis in so far as it is confined to the mere fact of observed evolution or change of form, and as unmistakably indicates a limitation to these changes conditioned by circumstances as yet beyond our understanding. The traditional conception of creation has lost nothing through competition with the mechanistic hypothesis. Rather it has gained. For the mechanistic hypothesis has failed at just those points where failure counts for special creation.

There is special significance in the expressed need by experts in both biology and physics for another Darwin and another Newton to embody newly discovered facts in theories more consistent with observed phenomena than the several diverse hypotheses now tentatively offered.

Selection, under the first general survey of phenomena, had been made the key-note to evolution, but proved to be an inadequate solution. Here we had a wonderful plausible solution of the world riddle all cut out and dried, when facts cropped up to destroy our confidence. A set of investigators, dissatisfied with the offhand explanations of how evolution came about, determined to ascertain the truth for themselves. Unfortunately they discovered too much—for the fair name of evolution. After careful and prolonged examination of the conditions under which selection occurs, observers found that agency did not work out at all in nature as it had been worked out on paper. They tried many cases of selection and found them wanting. The cases investigated offered absolutely no foothold for such a progressive selection as had been pictured.

Nor were the radical evolutionists more happy in another important particular. They originally placed great confidence in the coincidence between phases through which the animal passes in development (as from egg to maturity its form in embryo suggests the fish, bird, etc.), between these phases and the present geographical distribution from old-world primitive types to new-world more complex ones.

Thus it is the eye can pass in review the respective stages through which the primitive animal supposedly gained its monkey-like complexity both by observation of the course of development of higher animals, and by placing the existing animals of the world in a corresponding scale.

The fact that the embryonic form of the highest vertebrate recalls in its earlier stages the first representatives of its type in geological times and its lowest representatives at the present day, speaks only of an ideal relation, existing, not in the things themselves, but in the designing mind. While these transient resemblances of the young among higher animals in one type to the adult condition of the lower animals in the same type, suggest physical continuity, each one of the primary divisions of the animal kingdom is bound to its own form of development, which is absolutely distinct from all others. No type of animal diverges in the slightest degree from its own structural character. The lower animals are never seen to rise a shade beyond the level which is permanent for the group to which it belongs. The higher ones are never seen to stop short of their final aim, either in the mode or the extent of their transformation.

The hopes of the mechanist to read the book of life from the embryonic development of higher animals seems to have come to naught. Wherever practical tests are available the evidence indicates at least its minor inapplicability. Among others Tower found that his modified beetles represented a process of synthesis rather than of accumulation. The modified beetles skipped stages represented by transitions in their ancestors, and altogether physiologically behaved in a manner to suggest the futility of determining relationship and directions of evolution through life histories of species. This plausible recapitulation theory, so important in the earlier phases of evolutionary doctrine, thus lost much of its importance as a measurer of biological movements.

The mechanistic interpretation carries its own penalty. A study of growth and form undertaken with a view to substantiate the mechanistic claims reveals the fact that in simpler organisms, "whose form is due to the direct action of a particular physical force, similarity of form is not necessarily an indication of phylogenetic relationship."

The appeal of biological formula-making is well nigh irresistible. It is so tempting to reduce biological movements to a definite and precise rule consistent with our conception of its activities. The "Age and Area" formula is a case in point. All the more interesting because it brings into consideration the still disputed flora and fauna of New Zealand. The "Age and Area" formula applied to plants presupposes the area occupied by any given species depends upon the age of that species in that country. The older the species the wider its range. Perfectly simple and effective—if true. The problem, however, is a little more complex and less obvious than that. And the formula, which hinges on the distribution of the New Zealand flora, is found faulty in its fundamental assumption. A large and characteristic element of the New Zealand flora, it seems, entered the islands not from Australia on the west, as the author of *Age and Area* supposes, but from the Antarctic regions to the south.

The factors governing the distribution of animals are even more complex than those effecting plants. Food supply, rainfall, humidity, wetness or dryness of soil, altitude, atmospheric density, safety of breeding-places, water (to land species), land (to water species), nature and availability of cover, light-intensity, temperature, interspecific pressure, parasitism, and individual or racial preferences, are some of the factors responsible for the distribution of animals.

The mere catalogue of these influences dooms any simple analysis of their effects. The mechanisms of geographical distribution alone indicate a constant process of adjustment. Frontier individuals, those on the margin of the habitat of the species, may not prosper as readily or reproduce as prolifically, as those in the more favorable center regions of the species, but they certainly do not, as a rule, beat themselves to death individually against their limiting barrier, of whatever nature it may be.

The most important factor for one species is likely to be of minor importance for another species. Always a combination of factors is accountable. Then, too, there are indications of influences at work other than strictly physical ones. Side by side with facts, apparently the direct result of physical laws, are other facts, the nature of which points quite otherwise. The fauna of the Arctic and that of the Alps show a direct relation between climatic conditions and animal life. Yet even there where the shades of specific difference between many animals and plants of the same class are so slight as to elude the keenest, we have representative types among both plants and animals as distinct and peculiar as those of widely removed and strongly contrasted climatic conditions. Shall we attribute the similarities and differences alike to physical causes? On mountain heights of equal altitudes, where not only climate, but other physical conditions would suggest a recurrence of identical animals we do not find the same, but representative types.

It is now admitted that even among unicellular organisms, specific stability is of much wider application than the first loose judgments—under the spell of the evolutionary logic—had persuaded us was the case.

Recognizing the inadequacy of physical explanations, the mechanists by force of their experiments, have switched from these tangible, external influences to internal, intangible ones. They have not quite decided yet whether these internal influences manifest themselves in the form of mutations—large steps, or selection—small steps.

A warm controversy is at present waging between these two schools. The fact that there is a controversy indicates that scientists are still in the dark, still out of harmony with evolutionary causes; and this lack of harmony is characteristic of humanity in its attacks upon all the problems of life. To expect too much of partially applicable principles, to push too far, perfectly legitimate, but limited, formulas is a fatal fault. Evolution has every earmark of being true—up to a certain extent. Scientists can never hope to approximate this extent so long as they are determined not to limit it.

Though the heart can be made to function temporarily outside the organism, this complaisance entails no wholesale organic obligation to the mechanistic dogma with its limitless vistas of restless molecules and chemical affinities. It shadows forth the inevitable precision of the incidental as distinguished from the particular. Respiration and circulation owe their machine-like precision to a conscious inspiration, whose remoteness is a guarantee of individuality. In the cosmic sense respiration is just as consciously performed as though oxygen were hand-pumped into the blood-stream; only the consciousness is mercifully and tactfully activated from a distance. Imagine the labor and concentration necessary for a person to remember breathing and pumping his blood at regular intervals! The very indefiniteness of the manner of approach of this distant consciousness, and the vagueness as to its point of contact, intrigue us into denying its reality.

The accurate and regular working of the mechanical parts argues a designer more eloquently than tons of logic. The further the investigator goes into the details of the marvels of life processes the further he gets from proving these marvels take issue in the chance arrangement of simple chemicals.

Reviewing Dr. J. P. Lotsy's recent extreme advocay of mechanistic doctrine, Professor Jeffrey remarks, "It would apparently be well for the mechanistic biologists, who swarm at the present time, to admit also their indebtedness to the oldest if not the least dogmatic of the sciences, theology. If they had the grace to do so, their debt would doubtless be to Bishop Butler's famous *Analogy of Religion.* Lotsy's comparison of hybrids with metal ores is on all fours with the well-known Butlerian argument that the human worm will enjoy a future winged state because the lowly caterpillar later becomes the resplendent butterfly. Analogies are interesting but they do not constitute scientific argument, however much they may appeal to the socialistic and half-educated mind. Much of the present-day mechanism has a foundation not more substantial than the resemblance between a butterfly and an angel."

Once embarked in the business of making comparisons to substantiate their hypothesis, the imagination will carry far. Not far enough, however, to overcome the facts uncovered by investigation. Here there is another story to tell.

It is difficult to believe that the known world is merely a huge dice-box from which a capricious fate shakes out an occasional fortunate combination of materials in the shape of an oyster or an elephant.

Morgan's wonderfully intricate and detailed work analyzing the movement of factors which decide the make-up of the fruit-fly, Drosophila, has given a valuable insight into the mechanism of heredity. It portrays the distribution, through mating, of already existent factors; but it gives us no inkling whatsoever of the creative agency making these factors possible. It increases rather than diminishes, the mystery of creation, by showing how characters move from the fertilized egg to the mature organism with marvelous regularity, yet fails to enlighten as to the designer back of their regularity.

Numerous experiments are in progress to test out various mechanistic phases of life processes. These experiments usually culminate in the same conclusion. The mechanical processes are mechanical. In experiments carried on by other investigators to discover causes which regulate the duration of life in Drosophila, it was found that the termination of the first stage of metamorphosis is determined by the production in the body of certain chemical constituents not before present. Further investigation by temperature-rises decreased the length of life of these flies. Hence it seems probable that longevity is determined by chemical reaction. Then we come to the endless chain which leaves the chemical reaction in mid-air, minus a known cause. Male insects of some species die immediately after mating; the female of some species die immediately after laying their eggs. Between the two are all grades of longevity with all sorts of chemical reactions, inspired by causes equally unknown.

The imperfections in the mechanistic logic are matched by imperfections in the geological record. These imperfections are glossed over by popular paleontologists. Paleontologists are imaginative students, who are more impressed by the dry bones of the past than by the living facts of the present. While they are usually cautious to confine their claims of actual cases of evolution to relatively brief geological periods, the popular scientist does not scruple to extend the scope through all geological history. That is why the popular conception of prehistoric man has to be revised every so often to keep in line with the discovery of each newly dug-up prehistoric skull.

Calculations based on comparatively short periods in the world's history fail to enlighten as to origins. We cannot hope to approach very close to the truth if we are content to judge the whole from a small part. The Tertiary age presents but a fraction of the world's geological history. Beyond stretch centuries of great biological activity, of whose trend and products we are now afforded only occasional fossiliferous glimpses.

The moral for snap judgments in the matter is furnished by the findings of the seven blind men of Hindustan who went out to investigate the nature of an elephant; one of whom came in contact with the tail and declared the elephant to be like a rope, another bumped into the leg and was convinced of the elephant's likeness to a tree, and so on.

Sudden physical changes in the earth's surface, at widely separated intervals, were accompanied by important alterations in the organic world. Marked and violent changes in the earth-crust caused new elevations, and at the same time terminated the existing animate creation, introducing new populations entirely different from the preceding one.

These cataclysms offer barriers to physical continuity which no amount of persuasive logic can overcome. Of course the fertile mind can conceive of ways of holding to the idea of progressive and continuous evolution as opposed to special creation, even in the face of these cataclysms. But there are few real facts to substantiate their claims.

Prof. R. D. Carmichael is authority for the statement that "In the early years of the present century the world of scientific thought has been unexpectedly confronted with a new situation of a rather astonishing sort. Our unquestioning assumption of the continuity of nature appears not to have been well founded."

He demonstrates the likelihood of our being on the verge of interpreting everything in nature as discontinuous. Certainly the concrete evidence indicates the falsity of the continuity principle upon which complete evolution is founded. Students of science, even of the highest ranks, are apt, when drawing to conclusions, to fail to take proper account of altering rates of changes of temperature or pressure. Verified rates for short distances cannot safely be assumed to continue indefinitely without interruption or variation. Helmholtz, the distinguished physicist, limited the earth's atmosphere to twenty-eight kilometers from the surface on a basis of the gradient as then determined. Soundings carried to twice the limit fixed by Helmholtz reveal an interruption of the aero-thermic gradient and entirely upset his calculations of what should be.

Robert Mallet, with his centrum theory of earthquakes, dominated orthodox doctrine among earthquake specialists for full half a century; "and succeeded in keeping seismology out of its rightful field of geology for that period."

Ferrel's predicted whirls about the earth's geographical poles, were proven, by subsequent exploration, to be nonexistent. "Yet so great has been the success of Ferrel's theory as a whole, that despite its contradiction of the facts, the polar calms and whirls are still treated in the latest text-books of meteorology."

On the other hand the continuity theory as applied to evolution in the cosmic sense has been shown also, by investigation, to have been erroneous. The polar researches of Captain Scott and Sir Ernest Shackleton make manifest the fact of a steady glacial retreat. "The bearing of this conclusion upon the ultimate development of the human race is so far-reaching in its consequences that the great sacrifice attendant upon the prosecution of these researches stands forever as a memorial in the correction of the erroneous and wide-spread conception that the earth is in a period of refrigeration, desiccation, and decay; and establishes the conclusion that it is in the springtime of a new climatic control during which the areas fitted for man's use are being extended and that the moss of polar wastes will be replaced by rye and wheat."

So that either way it is taken the fundamental conception of evolution can find no true basis either in physics or mathematics. The momentum of its original plausibility carried it along past the point where the facts patently discredit it.

We hear echoes of this passing of the critical period by such phrases as "The intellectual bankruptcy of the whole evolutionary theory in the late nineties." The strategetic rescue of the evolutionary theory, and the covering of its defeat from public gaze was most unfortunate for humanity. It gave a false value to the doctrine, "Might makes Right," which many evolutionists are, after the event, in haste to disclaim. Particularly in Germany was the obsession carried over the critical period by the Wallingfordian persuasion of Haeckel. Germany, by a blind adherence and unlimited advocacy of a limited principle, worked itself into a conquesting ecstasy which culminated in the war we are suffering—any one who doubts the connection between the mechanistic conception and the world war has but to read Haeckel and the "Kultur manifestoes" side by side—(see also Northcliffe, *Current Opinion*, Oct., 1917).

The mechanistic conception is a banal attempt to standardize our emotions; but one destined to failure because of the essential falsity of both its premises and conclusions. The philosophy of continuity is the outcome of a misguided hopefulness rather than the result of any positive and convincing inductions.

Deductions from any inadequate basis leave us deep in the mire of metaphysics. True science is never dogmatic and deductive; it is pragmatic and inductive. It is built up slowly from an accretion of tried facts, not suddenly as from the framing of brilliant generalizations. The Darwin theory of mimicry is a case in point. For many years established in the scientific mind by a series of then logical deductions, the whole theory of mimicry and adaptive coloration is now badly in need of revision. Dr. Longley's studies of tropical fishes, and others in similar fields, make clear that the criterion of fitness must derive its sanction from the studied animal's intimate enemies, not from man's conception of what the relation ought to be.

Having been pretty well fed up on the mutual obligations and mechanical fitness existing between bees and flowers, we are somewhat shocked at the introduction of a sordid, stubborn fact into the romance of this interesting relationship. It seems that in the haste of logic-making important considerations were carelessly overlooked—an ant appears in the ointment of perfect argument.

The theory of this scientific-romantic ordered relationship is that the flower, in response to the demand of the bee for nectar, developed the nectar-generating habit; while the bee, reciprocating in response to the demand of the flower for cross-fertilization, a perfecting of the system for insuring its best reproduction, developed features of assistance in carrying ripe pollen from the male organs of one flower to the female organs of another.

This might account for nectar in flowers, but how about the extra-floral nectaries, nectar-sacs on leaf branches and in other discouragingly irregular places? Contrary to the earlier superficial expectation that such extra-floral nectaries might divert the attention of ants from the greater treasures of the flowers, it appears that plants having these sacs, sometimes have their flowers more robbed by ants than would probably be the case if they lacked extra-floral nectaries. On the other hand bees are like to visit these irregular sweets and neglect their duties to flowers. In such cases the flower may fail to be cross-pollenated, indicating a distinct disadvantage. So that, whatever way this romance in mechanistics is viewed, something must be sacrificed. Either we must sacrifice the fundamental principle upon which the extreme evolutionist insisted, that "no structure can survive unless it is of use," or we must sacrifice the picturesque reciprocity of the bee.

Having proceeded beyond the simplicity of the Darwinian formula, the up-to-date evolutionist willingly sacrifices the older, narrow view, which fastens a use to every character, to the newer attitude looking to haphazard internal influences. External tangible physical influences failing in their obligations, it was inevitable that internal, intangible influences should have a trial. But in this repudiation of an old love and taking on of a new no additional light is shed on the marvels of coordination. Quite the contrary. As we cut off environmental pressure from initiating variation we depart from the prospect of arriving at a tangible explanation of creation. Saddling responsibility for variation on the germ-plasm of the race submerges in deeper mystery those problems of relationship and cooperation which are manifestly the important ones.

The germ-plasm, ordinarily so inexorable, is now the accepted seat of all organic changes since the first primeval atom. To its idiocyncrasies we owe all beauty in form, expression, and fitness. It is the chemical experimenter *par excellence*. Hydrogen and oxygen playfully throw aside their obstancies in its magic stream, depart from their strictly ordered existence, and enter into the carefree pastime of concocting complicated essential compounds whose composition cannot be duplicated by the centuries-learned brain of man.

The elusive quality of what we call life is illustrated by the failure of scientists to reproduce the living plasms synthetically though their constituents are known and assembled in proper quantities. What nature accomplishes accidentally, man, with all his accumulated knowledge and resources cannot accomplish at all. Through laborious and long-continued effort he can achieve the lifeless replica: the optical form, such as exists in nature, significantly evades his every effort. The validity of the mechanistic doctrine as an explanation of creation is seriously hampered by his inability to react the drama of his own making.

It is now scientifically admitted that we do not know what protoplasm is. "We have analyzed the substance chemically, we have carefully examined and tried (but without complete success) to describe its structure. We know it is more than merely a chemical compound. It is a historical substance. A watch as such is not." (Dr. C. S. Gager, *The Fundamentals of Botany*.)

There seems to be a fatal shortcoming somewhere in the offhand reasoning relegating the organism to the category of fortuitous chanceling carelessly drawn by the hand of fate from out the immensity of the cosmic reservoir.

That the unpremeditated experiments of a bit of protoplasm in chemical affinities would finally culminate after eons in a scampering monkey is difficult enough of belief; but that this same line of individual experimentation could accomplish all the wonderful collective fitness and coordination of star and sun, water and earth, tree and shrub, insect and man, is more mythological than the mythiest myth, save for the order of mind which has long accustomed itself to spurn other than a mechanistic explanation however strained. There are, undoubtedly, certain orders of mind so firmly locked in the embrace of conventionality that they cannot break loose. For science, conventionality demands adherence to the mechanistic doctrine, and the conventional appetite is easily satisfied. Any formula composed according to its strict rules is acceptable. Thus we find it eagerly espousing the cause of a definite theory of the universe while the fundamental atom is still an enigma. From this easy habit of accepting piecemeal the mental product of recognized authorities we perceive an historic exhibition of favor and disfavor which does small credit to our decree of finality.

To Democritus and his disciples the world appeared to have been the result of a fortuitous concourse of atoms. Plato and his school declared for the orderly course of nature as due to a divine plan. Descartes advocated an earth formed by the aggregation of puny particles of matter which have an inherent whirling motion. Laplace further enlarged upon this view which was received for several generations without reservation. Serious defects later developed, however, and within the last few years astronomers and geologists perceived its coming discardment. The Planetesimal Theory of Chamberlain saves the remnant of Laplace's Nebular Hypothesis, by enlarging upon additional phenomena which required explaining. When these revising explanations—made necessary by hitherto unconsidered phenomena—become too cumbersome and complicated for further logical acceptance we may expect their total breakdown and a return to older views.

It is ever the way of humanity to deal in extremes. Pragmatism is the most difficult form of philosophy for it to adopt. No half-way measures are satisfactory. Either truth must be presented concretely or they will have none of it. Either the mechanism is all or the spirit is all. The possibility of a mechanism divinely planned and ordered, a mechanism which is the highest expression of a designing influence, seems not to have occurred to any number of thinking men.

Yet it is just this possibility which keeps the theorist hovering from pole to pole to discover a principle whose real roots are at the equator.

The unit character conception—whereby the organism was supposed to be a compound or mosaic of characters each one definitely represented in the egg—was most attractive. Evidence of certain characters which do not follow the indicated biologic law, made its modification inevitable. Experimenters soon came to see, particularly in regard to color inheritance, that the matter is most complex. In numerous cases of color inheritance there is little to warrant the assumption that these phenomena are based upon representative particles or individualized entities, and very much to warrant the belief that they are the product of a modified chromogen base, a modified enzym or rather vague capacity for carrying on the process.

All roads lead to Rome. From a simplicity of organization which the mechanistic doctrine demanded, there has gradually developed a recognition of complexity involving a "vitalistic" or external influence, a return to the Platonian viewpoint. Having run the gamut of creatorial guesses and pretty well exhaused the visible supply of possibilities, the theoriest is likely ultimately to double back disappointedly to his starting-point. His intellectual pilgrimage has led into no thoroughfare. Obliged by the obvious shortcomings of his directing instruments to retrace his steps, what more sensible than that he should endeavor to stand once more on the threshold and take stock of the faults and realities of his philosophy? To ascertain the limits of his intellect and to gage its value as opposed to more subtle and settled intuitions?

The crux of the whole matter is here: whether we feel safer in trusting to the pronouncements of biased observers, or in being guided by finer intuitions. Since the intellect is only equipped to grapple with the things of the intellect, we cannot rightly expect it to do justice to itself in matters entirely outside its domain. Those of simple faith, unhampered by intellectual trappings, unaffected by dictation of historically authorized formulas, are best fitted to appreciate a process having nothing in common with a restricted human intellect.

The untutored savage is less handicapped than the most erudite scholar, more thinly insulated from the simplicity of the creatorial influence. The eyes of the Indian and trapper can detect signs and tokens unseen by the educated white man; their ears can hear rumblings to which the latter is deaf; they have advantages of perceptions which the higher civilization dulls. Time has made it manifest that the intellect is a poor gage for the creatorial plan, primarily because the conscious mental processes are unable to cope with the intangible—the tangible alone is their province.

Nor are the intuitive activities confined to an appreciation of spiritual truths; they enter more largely into worldly affairs than intellectuals are inclined to admit. Mr. Harvey O'Higgins, who has made an extended and intimate study of subconscious activity, provides concrete evidence of the working of the subconscious in practical men like Judge Lindsey and Detective Burns (*Saturday Evening Post,* Oct., 1917). Granting, as we must, that such men have been actuated to their best efforts through the instrumentality of the subconscious we can no longer question its judgments. Then also we are obliged to place a higher value on the intuitive findings of a Fabre than on the purely intellectual calculations of a Dr. Jacques Loeb or a Haeckel. "What is true of the subconscious mind in artists is almost equally evident in the achievements and careers of many men of great intellect outside the arts. In their biographies, again and again, you will find that the deep secret of their success, the real heart of their mystery, is a gift, an intuition, an instinct that cannot be explained—that is to say a subconscious faculty."

In proportion as men of intellect have been willing to submit to the still small voice of intuition are they successful in a dual world. For where there are two equally important processes in operation each must receive its due share of recognition. Spirit and flesh are the irreconcilable components of duality. The spirit speaks a language quite different and distinct from that of the flesh—no messages from one to the other are translatable. Denial of duality is the easiest way out, but it is a way which leads into endless philosophic sophistries having as their object the confusion of terms, and involves comparisons more pleasing to the imagination than complimentary to the understanding.

For those who see merit in the pragmatic attitude it is possible to differentiate between the strictly mechanistic doctrine and a modified evolution. It is not necessary for them to be either atheistic or egotistic. Limiting God's province carries neither conviction nor appeal. An orderly plan and a variable organism, if viewed from a sufficiently high plain, lose their apparent antagonism, and become merged in one stupendous harmonious whole. No biologist need now fear betraying a trust in admitting "vitalism" or design in the scheme of nature, because he is merely submitting to the dictates of a saner science.

The fact that increasing thousands of intelligent people are eagerly espousing the Christian Science doctrine of the unsubstantiality of matter looks hopeful. The fact also registers the ordinary course of human reactions. From the extreme of an unsatisfactory materialism to a pure spiritism was an inevitable step. Yet it is clear that life, without matter, would be anchorless, chaotic; matter without life, a superfluous, taskless anchor.

Why should there be any difficulty about granting to Cæsar what belongs to Cæsar, and setting aside for Titania that which is rightfully hers? Only the disinclination of an enlightened people to forego a supposed advantage. Preferring to hasten from one pole of the truth to another, these uneasy persons never pause long enough at the equator to take exact observations. Either their practical minds will accept no compromise, or their idealism will permit no taint.

It is doubtless difficult for some orders of minds to keep separate their business and religious convictions. For the biologist the separation is particularly desirable. Research work is necessarily limited to mechanistic processes of life. But this is not equivalent to denying the existence of a "vitalistic" side. Among the unprejudiced initiated the existence of a ruling intelligence is becoming more and more an admitted possibility. With a further swing of the pendulum we can look forward to a freer, franker, less limited recognition of the power and goodness of God.

> "Up from Earth's Centre through the Seventh Gate
> I rose, and on the Throne of Saturn sate,
> And many a Knot unravel'd by the Road;
> But not the Master-knot of Human Fate."

Walter Sonneberg.

Philadelphia, Pa.

THE GENESIS OF AN ELECTRO-MAGNETIC FIELD.

IN the development of a branch of mathematical physics the first stage often consists of a study of the permanent states of a system, for instance the states of equilibrium, states of steady motion, and states of periodic motion; sometimes the development does not proceed much further than this, but frequently the oscillations about the permanent states are considered in full detail. In later stages of development efforts are made to elucidate the way in which the permanent states are attained, to find the conditions that they should be attained, and so forth. In chemical dynamics and in the theory of the conduction of heat a state of equilibrium is generally approached gradually in one direction without over-reaching the mark, while in ordinary dynamics and in the theory of electricity a permanent state is generally attained as a result of a series of damped oscillations.

In nearly every case in which the approach to a permanent state has been discussed, the system under consideration is supposed to be started with an initial motion, and indeed this seems to be necessary, for instance the transition from one permanent state to another could not otherwise be made, while it frequently happens that a given initial motion would have arisen in the natural (or continuous) order of events from motion of a violent character.

In the theory of electromagnetism the discontinuities which render possible the transition from one permanent state to another are propagated as waves of discontinuity. The theory of such waves has been developed by many writers, and Prof. A. E. H. Love has worked out the details in the case of the transition from the electrostatic field of an electric doublet to the periodic electromagnetic field of a vibrating electric doublet, considering also the case in which the vibrations are damped. In this and in many other investigations in electromagnetic theory an electrostatic field is regarded as the simplest initial field, and this is generally imagined to fill the whole of space and to have existed for ever. A field which fills the whole of space seems, however, to require the existence of an infinite ether or medium to support it, and as the idea of an infinite medium is repugnant to some minds, it may be worth while to consider the question whether an electrostatic field, which does not fill the whole of space, but is bounded by a moving surface of discontinuity, can arise from a state of affairs in which there is initially no electromagnetic field at all.

An answer to this question may be derived from a careful study of the different solutions of Maxwell's equations for the propagation of electric waves. These equations when written in the symmetrical form adopted by Hertz and Heaviside are as follows:

$$c \text{ curl } \mathrm{H} = \delta \mathrm{E}/\delta t, \qquad \text{div } \mathrm{E} = 0,$$
$$c \text{ curl } \mathrm{E} = -\delta \mathrm{H}/\delta t, \quad \text{div } \mathrm{H} = 0,$$

where E and H are the electric and magnetic intensities and c the velocity of light.

In a type of solution which we shall regard as fundamental the complex vector $\mathrm{H} + i\mathrm{E}$ is of the form $\mathrm{M} = mf(\alpha, \beta)$ where the vector m depends on both position and time, while f is an arbitrary scalar function of two quantities α and β which are functions of both position and time. Quantities α and β cannot both be real; they have constant values for certain points, which move along straight lines with the velocity of light, and which may be conveniently called "light-particle." The fact that the vector $mf(\alpha, \beta)$ provides us with a solution of the equations, whatever the arbitrary function f may be, suggests that the elements of disturbance associated with the different light-particles can be regarded as independent of one another and this is just what a further study of the above solution indicates.[1] It appears in fact that the collection of light-particles which lie at any instant within a small volume carry with them a certain amount of energy which remains unaltered during their motion.

The electric and magnetic intensities E and H are, moreover, at right angles to the direction of motion of the light-particles at a point and so the flow of energy, as indicated by Poynting's vector, is in the direction of motion of the light-particle. It should be mentioned that E and H are also perpendicular to one another and equal in magnitude so that the field is a "self-conjugate" or simple radiant field in which there is a simple propagation of energy but no accumulation or expenditure of energy at any ordinary point of space. In such a radiant field the energy may, perhaps, be regarded as energy of motion and as analogous to kinetic energy, although the view is unorthodox.

Now mathematicians have thought for a long time that all energy is really kinetic. The idea that potential energy can be regarded as kinetic energy of concealed cyclic motion was put forward in 1888 by Sir J. J. Thomson in his remarkable book *The Applications of Dynamics to Physics and Chemistry* and was adopted independently by Hertz in his work on the principles of mechanics.

Sir Joseph Thomson says: "This view which regards all potential energy as really kinetic has the advantage of keeping before us the idea that it is one of the objects of

[1] It is indicated to some extent by the general theory of the characteristics and bi-characteristics of linear partial differential equations. Cf. Hadamard, *Propagation des Ondes*.

Physical Science to explain natural phenomena by means of the properties of matter in motion. When we have done this we have got a complete physical explanation of any phenomenon and any further explanation must be rather metaphysical than physical. It is not so, however, when we explain the phenomenon as due to changes in the potential energy of the system; for potential energy cannot be said, in the strict sense of the term, to explain anything. It does little more than embody the results of experiments in a form suitable for mathematical investigations."

Since the energy in an electrostatic field is generally regarded as potential energy, it is clear that an electrostatic field ought not to be regarded as fundamental in electromagnetic theory, and it is now necessary for us to see if the type of field we have chosen as fundamental fulfils the requirements which Thomson considers as characteristic of a type of motion which can be regarded as fundamental in an attempt to eliminate the idea of potential energy.

In a second passage Thomson says: "As all the energy is kinetic its magnitude remains constant by the principle of the Conservation of Energy, and so the principle of Least Action takes the very simple form that with a given quantity of energy any material system will by its unguided motion go along the path which will take it from one configuration to another in the least possible time." The requirements are evidently fulfilled, and so the next step is to choose α and β so that the radiant field under consideration is of a simple character.

Let S be a point which moves in an arbitrary manner with a velocity less than the velocity of light and let the light-particles start from the different positions of S. If P be an arbitrary point in space there is just one position of S, viz., S_0 from which a light-particle can start so as to reach P at time t. The time at which this particle must leave

S_0 is a suitable value of α for the position P at time t and it is clear that α is constant for all the space-time points covered by the light-particle. To obtain a suitable value of β we must find a complex quantity which will specify the direction of motion of the light-particle. Let a sphere of unit radius be described with S_0 as center and let us imagine a steady irrotational two-dimensional motion of an incompressible fluid on the surface or on a portion of the surface,[2] then the complex variable $\varphi + i\psi$ whose real and imaginary parts are constant along the equipotentials and stream-lines respectively is a suitable value of β. The imaginary fluid motion may, of course, vary in an arbitrary manner as S_0 varies; in other words, β depends on both the time of creation and the direction of motion of the light-particle which arrives at P at time t.

Let us now consider the simple case in which the stream-lines are cut out by planes through two points A and A_0 on the sphere. These lines may be regarded either as lines of electric or magnetic force. In the former case the corresponding radiant field possesses the following characteristics:

The field is produced by the creation at the moving point S of pairs of oppositely electrified light-particles and the rectilinear motion of these charged particles in different directions with the velocity of light. A pair of oppositely electrified light-particles may perhaps be supposed to have been derived from a neutral particle traveling initially with the velocity of light.

This creation of electricity may take place continuously for any length of time, and the rate at which electricity of one sign is produced may either vary in an arbitrary manner or may remain constant, while the directions of rectilinear motion of the charged light-particles may also

[2] In the latter case the electromagnetic field may be limited to a certain portion of space.

vary in an arbitrary manner or remain constant. The electromagnetic field comes into existence as soon as the creation of electrified light-particles begins, it has an outer spherical boundary at any later instant and also has an inner spherical boundary at any time after the creation of the electrified light-particles has ceased.

The type of radiant field which has just been described will be called a field of Heaviside's type because in 1901 Oliver Heaviside[3] gave a particular example in which the point S is stationary and the directions of rectilinear motion of the electrified light-particles are invariable and exactly opposite to one another. In this case the electric force at P at time t is directly proportional to the rate of production of positive electricity at S_0 and inversely proportional to the distance of P from the line of motion of the electrified light-particles produced at S_0. If Q and Q_0 are the positions of these light-particles at time t the electric force at P is tangential to the circle Q_0PQ.

The last construction also applies in the more general case, but to obtain an expression for the electric force the inverse distance from QQ_0 must be replaced by the ratio of QQ_0 to $PQ.PQ_0$ and we must divide by the Doppler factor $1 - v_r/c$, where v_r is the component along S_0P of the velocity of S_0.

In the field which has just been described the electricity is moving with the velocity of light. To obtain a field in which the electricity appears to move with a smaller velocity we must superpose two fields of Heaviside's type in such a way that there is a combination of oppositely electrified charges except on a short interval between the sources of the two radiant fields. Let us first of all consider the case in which the two sources are consecutive and stationary, then the electrified light-particles must travel along the line joining the sources and there is "interference" or an anni-

[3] *Electromagnetic Theory,* Vol. III, p. 122.

hilation of electricity in an arrangement of the following type:

$$\begin{array}{c} \infty \quad -\leftarrow- S -\rightarrow+ S' \qquad\qquad\qquad \infty' \\ \hline +\leftarrow- \;\; -\rightarrow- \end{array}$$

In this case if the canceling is complete on the lines ∞S, $S'\infty'$, we are left with an apparently stationary positive charge on S S′. In the more general case when the sources are moving we can obtain a similar canceling if the lines of motion of the electrified light-particles overlap.

To pass to the case of a moving electric pole we must make the distance S S′ tend to zero while the rate of production of positive electricity increases indefinitely in such a way that its product with S S′ remains finite.

Let us now study the genesis of the field of the stationary electric pole a little more fully. Let us suppose that initially a source S fires out two oppositely electrified light-particles A and B and that when B arrives at a point T very close to A, a source there becomes active and fires out oppositely electrified light-particles some of which just annul B. If these sources remain active and adjust their activities and relative positions so that there is a continual annihilation of electricity outside the interval ST we shall have a constant electric charge associated with an interval ST while an uncompensated charge of the opposite sign is carried by the light-particle A and its immediate successors. Now it is undesirable that an electric charge should travel to infinity, and to avoid this we must suppose that when A arrives at some point U a source there becomes active and fires out light-particles one of which just annuls A and its immediate successors. If the other light-particles then produced are annulled by some of the light particles fired out from a neighboring source V and the two sources U and V maintain the charge UV constant, it is easy to see that this charge must be equal and opposite to that on ST if no uncompensated electricity is allowed to proceed to infinity.

The genesis of the electromagnetic field of two slowly moving electric poles carrying charges which are equal in magnitude but opposite in sign, proceeds along lines similar to the above. It should be noticed that if each charge varies slightly, perhaps periodically,[4] and the uncompensated electrified light-particles fired out by one are annulled by the other, the two moving poles are joined by two singular moving curves which form the locus of the uncompensated electrified light-particles. The two curves coincide only when the two electric charges are moving in a special manner in a plane or on a hyperboloid of revolution.

It is thought that the uncompensated electrified light-particles may give rise to the phenomenon of gravitation and that the two curves which sometimes join two electric charges of opposite signs may represent singular Faraday tubes which, as Thomson suggests, may be intimately related to the bonds of the chemist.

A theory of gravitation based on these ideas has not yet been formulated in a sufficiently definite form to enable us to make straightforward progress in the solution of the problem of atomic structure, but it seems worth while at present to combine the above results with one or two simple hypotheses of the type usually employed by physicists.

1. We shall assume the existence of discrete electric charges of magnitudes $-e$, $+e$, $+2e$, where e is the elementary quantum of electricity, and shall assume that an electric charge of magnitude $\pm ne$ has $2n$ singular Faraday tubes attached to it. We shall suppose that n of these tubes are described by electrified light-particles moving toward the charge and the other n by electrified light-particles moving away from the charge.

If an atom consists of discrete charges bound together

[4] This seems quite natural if the permanent state has not yet been attained.

by means of their singular Faraday tubes, it is clear that an atom containing more than two elementary charges must either contain at least one charge whose magnitude is greater than e or it must contain a set of charges whose singular Faraday tubes form a closed polygon. The two Faraday tubes issuing from a charge $\pm e$ need not indeed end at the same charge, for it is easy to see that the condition for the canceling of electrified light-particles can be satisfied when the tubes belonging to a number of charges form a closed polygon. We shall find it convenient to assume the existence of both multiple charges and cyclic arrangements of Faraday tubes.

Calling the elementary charge $-e$ an electron, we shall consider elementary nuclei containing a number of elementary positive charges bound together by a number of electrons, the total positive charge being greater than the negative.

2. We shall assume that in a neutral atom the number of electrons outside the elementary nuclei is equal to the atomic number, while the total number of electrons is the number nearest to the atomic weight.

An attempt has been made to build up atoms using the following elementary nuclei:

positive electron	($1e$)	containing	no	electrons
double charge	α ($2e$)	"	two	"
triple "	ϑ ($3e$)	"	three	"
triple "	φ ($3e$)	"	four	"
quintuple "	ψ ($5e$)	"	nine	"

It should be remarked that a quintuple can be regarded as built up either from two triple charges of type φ joined by an electron or from two α-particles and a ϑ-particle joined by two electrons. The phenomenon of radio-activity may sometimes be due to the breaking up of an elementary nucleus carrying a quintuple charge, and it is interesting to note that there are two distinct ways in which this nucleus can break up. The existence of two types of elementary nuclei carrying a triple charge may perhaps account for the existence of isotopes having the same atomic number but different atomic weights.

3. We shall assume that an electron whose two Faraday tubes end on the same elementary nucleus is a valency electron and the number of such electrons is the valency that the atom can exhibit under given conditions. The other electrons outside the elementary nuclei have their Faraday tubes ending on different nuclei and may be supposed to bind them together. The distance between the elementary nuclei may, however, be quite small compared with the distance between an elementary nucleus and an electron. It is clear that an atom built up from a given set of elementary nuclei and electrons can exhibit different valencies and it is quite an interesting mathematical problem to determine the number of different ways in which the parts of our ideal atom can be bound together.

A few examples of atoms built up in the above manner may perhaps suffice.

A carbon atom consisting of two ϑ-particles and six electrons would have an atomic weight 12 and atomic number 6; it would have valency 4 when the two nuclei are bound together by 2 electrons; an odd valency does not seem to be possible.

A cobalt atom consisting of 4 ϑ-particles, 5 φ-particles and 27 electrons would have atomic weight 59 and atomic number 27; it could have various valencies, the numbers 2 and 3 being given by simple arrangements while the number 9 is given by an arrangement in which the 9 nuclei are arranged in a ring, consecutive particles being joined together by two electrons. This fact may be of some interest because in some compounds cobalt appears to have a valency of 9.

A neon atom consisting of two particles, two nuclei with triple charges and ten electrons would have atomic weight 20 or 22 according as the two triple nuclei were of type ϑ or φ. This is of interest in connection with the recent discovery of two types of neon by J. J. Thomson and F. W. Aston.

H. Bateman.

Pasadena, California.

CRITICISMS AND DISCUSSIONS.

IS THERE AN INTELLECTUAL CONTENT IN PHILOSOPHY?

[It is with a sense of personal loss that we here chronicle the death of the author of this article, the Rev. Dr. James G. Townsend, of Jamestown, New York, on June 27, 1917. He was born in Pittsburgh in 1839, and once said that he had taken care of himself since he was thirteen, when he lost father, brother, sister and uncle within three days in the plague of cholera which swept Buffalo where the family then lived. He was educated at Oberlin and Allegheny College, working his way through. He enlisted in 1862, and in the battle of Perryville had his left arm so badly crushed that he was never again able to lift his left hand. After his discharge he entered the Methodist ministry where he continued for eighteen years, but then went to Jamestown to found the Independent Congregational Church (now the Unitarian) because he no longer felt in harmony with the Methodist theology and philosophy of life. Four years of active ministry there were followed by a severe illness extending over several years, after which Dr. Townsend founded the First Unitarian Church at Pittsburgh. He spent the last seventeen years of his life in Jamestown, where he kept up his intellectual interests through correspondence with literary friends and contributions to liberal periodicals. The "gospel of beauty" as a moral and educational force was the message of his latest years.—Ed.]

Is there an intellectual content in philosophy? Is there a solid, immutable basis for a metaphysic? Can philosophy give us something real, widen the empire of thought? Can philosophy answer any of those questions which are of supreme importance? For example, can philosophy tell us truths about the subsistence of the universe; truths about this mysterious human soul (whether or not the soul, conscious and individual, survives death); truths about God (for if God be a reality there must be truths about Him); and whether or not this universe is "moving to a good end"?

Miss Mary W. Calkins, in her recent *History of Philosophy,* says "that the study of metaphysics holds out no promise of definite results." She constantly iterates that philosophy "gives us nothing." Yet despite this denial she asserts that philosophy can tell us whether the ultimate reality "is one or many, spirit or matter." But one wonders why, if philosophy can go this far, it cannot go farther.

Schiller says: "Philosophy makes a difference." May not, then, that "difference" be expressed? It is well for us to be modest, to cry out "*Ignoramus,*" to admit our knowledge is but a single leaf, plucked from an interminable forest. But must we yield to those agnostics who cry out "*Ignorabimus,*" who aver that we have now gone so far that the last word has been said? Is our passion for the highest truth to be expended in a pursuit that reaches no goal, positive or negative?

Prof. William James, in one of his lectures, said: "In this very university (Harvard) I have heard more than one teacher affirm that 'all the fundamental conceptions of truth have already been discovered by science.'" And Professor James said that to make statements of this kind showed a "lack of imagination," in view of the fact that new conceptions have arisen and new problems have been formulated in our generation. And he intimated that a solution of these problems is not impossible.

But it will be said, if the stupendous labors of the great intellects in the long past, Plato, Aristotle, St. Augustine, Calvin, Spinoza, Kant, Fichte, Hegel, Hume, Mill, Spencer, have brought us nothing, if their strong hands have not been able to lift a single inch that veil of impenetrable mystery which hangs over all, why should we expect to be more successful? There are the ultimate enigmas of existence and there they will remain.

This puts the case very strongly, but is it entirely true? Did not Darwin throw light upon the problem of creation, give us a real advance? And certainly we have, what the thinkers in the past did not have, a wealth of scientific facts and the perfecting of the scientific method.

But it may help us to define, in the beginning of this inquiry, what philosophy is. It is often said that philosophy means to discriminate accurately, to avoid fallacies, collect true premises and deduce just inferences, but I should rather call this logic, not philosophy. Nor would I say it was the province of philosophy to describe phenomena and the facts of experience as exactly, as simply and as completely as possible,—that is science, not philosophy.

In a general way I should say it is the business of philosophy to reveal to us reality, the essential truth, the nature of the universe, the meaning of human life. The precocious young Novalis said that philosophy was "homesickness, a desire to be at home everywhere in the universe." This is certainly fine. At another time he said: "Philosophy bakes no bread, but it has given us God, freedom and immortality." While Novalis had gone too far in assuming that philosophy had solved those great intellectual or metaphysical problems, "God, freedom and immortality," he was not mistaken in the mission of philosophy. It is its province to give us a solution of these riddles of our existence or to throw a great light upon them.

Sir Oliver Lodge, a most cautious thinker, says: "A fair comprehension of the nature of life, and the way it is able to interact with matter, must surely be within our human grasp." But if human intelligence shall pass that mysterious realm which has so long divided life and matter, may not some brave thinker cross the boundary between the visible and invisible worlds and prove that the search for reality is the search for God, is a legitimate search, and that the soul evolved and educated at so much cost, shall not be thrown away at death as so much rubbish? Is it not arrogance, is it not ignorance to say that nothing more is possible to philosophy?

God, freedom and immortality may be possible to a courageous philosophy, with our more exact knowledge of the laws of evolution, and of biology. To set up a fence beyond which thought can never cross is the mark of a commonplace, a timid nature. Has not this fecund universe, which has so much for the hand, for the eye, for the ear, for the heart, something for the mind? Is it not presumption to say that the immense horizons of knowledge, hitherto unknown, can never be opened up?

It will be said in reply to my view of a real intellectual content in philosophy, the assertion of the possibility of arriving at a definite goal in thought, that modern philosophy is distinguished by the emphasis it places upon the "relative" spirit above the "absolute." Modern philosophy says nothing is known or can be rightly known except relatively, or approximately. The sciences of observation, in showing how types of life merge in each other in changes infinitely delicate, have brought about this hesitation. The faculty for truth is recognized as a power of delineating and putting upon canvas the most delicate and ephemeral shades of thought.

There is a new theory of the intimacy, the relationship of mind and matter, good and bad, freedom and necessity. Hard and technical, or churchly, moralities are giving way to simpler, more charitable views, a recognition of those inevitable strands woven by necessity in our complex lives. Man, in body, mind and soul, is swayed by forces of his present environment, also by influences of heredity and by instincts which strike their roots in the soil of an interminable past. Millions of pulses beat in his mysteriously complex nature.

Now I have no quarrel with this "relative" spirit, no desire to "apprehend the absolute," in those realms in which hard and fast lines are impossible. But I maintain, while recognizing the value of the "relative" spirit, that some things may be exactly and absolutely known. For example, that, conditions being the same, there will always be a certain color and curve to the rose-leaf; that every touch of the world of form, color, and feeling brings to us some contribution, if only we are ready to regard it; that it may not be able to tell what beauty is, but that it ever abides for the delight and refreshment of the human spirit.

The new philosophy may not be able to tell us what the body is, or what the soul is, but it may tell us absolutely that the soul abides after the body has crumbled in death.

The new philosophy may not tell us of the mystery of the being of God, but it may tell us that He cares for the things for which we care, that he hears our human call, and is guiding the world toward happiness and goodness, as James said so bravely.

In this attitude of the new philosophy, assuming that in some lines it may be possible to apprehend absolute truth, it follows the old Greek and Roman teachers like Socrates and Zeno and Seneca, Marcus Aurelius and Epictetus. These teachers sought to elevate men, not by conversion, an appeal to the feelings making a change of will ("for the will they thought was good"), but by education, by the impartation of truth, familiarity with lofty ideals. They said "that to know the truth was to do it," and they were right; for in the end, if you elevate men intellectually, you elevate them morally.

The widening of knowledge, constant association with noble and beautiful ideas, affect character. It is said in reply: "Coleridge talked like an angel, and did nothing." And men may, in the midst of many opportunities, remain ignorant; but generally, when knowledge is received, when the mind really moves, there is a moral elevation, a higher civilization is created.

In a thoughtful paper entitled "Civilization in Danger," in the latest *Hibbert Journal,* René L. Gérard seems to contend for views directly opposed to those I have been defending. He says "that to believe that philosophic or religious doctrines create morals or civilization is a seductive and fatal error." He maintains that it is not because a people possess noble beliefs, broad and generous ideas, that it is healthy and happy, but rather that, being healthy and happy, it adopts or invents noble beliefs and generous ideas. And further he affirms that a people, by instinct, unconsciously (here he follows Bergson), will draw upon the vast moral and intellectual acquisitions of the past, the rich experience of all the ages, for the beliefs and ideas they need.

There is a profound truth in these suggestions, but is it the whole truth? Professor Gérard admits that noble beliefs and great thoughts are absolutely necessary, that they assure the survival of a people. And he affirms that a healthy people will adopt these ideas "instinctively, unconsciously." They will draw them from the intellectual and spiritual acquisitions of the past, the accumulated experience of the ages. But are not these acquisitions, the great and universal "acceptances" or beliefs, largely the fruit of the gigantic toil of the mighty thinkers of the past, like Isaiah, Socrates, Jesus, Paul, St. Augustine, Luther?

Professor Gérard contends that when the vital instinct of a people is healthy and vigorous it readily suggests to the people the religious and philosophic doctrines it needs to assure its survival. Let us see if this be so. I will take the same illustration that Professor Gérard uses. The barbarians who destroyed the Roman Empire were a people in whom the vital instinct was pure and strong. They were uneducated, but they adopted Christianity, a new religion. And this new religion, with its ideas of brotherhood and forgiveness, was distinctly opposed to their religious ideas of conquest and cruelty. But in receiving or adopting this new religion they were impressed, educated, elevated, (for the Christian missionaries came with the Bible in one hand "and Virgil in the other").

They ascended to a higher level of morals and civilization.

These were the ideas they needed. But if there had been no Christianity, no new religion, no Virgil, would they have invented these ideas, as Professor Gérard says they would? Ideas and beliefs do not grow on the bushes. Is it not most probable that if these new ideas had not been ready for their adoption the barbarians would have remained at the same low level? Without these ideas and teachers there could have been no progress.

Professor Bergson says that the instinct of the hymenopteræ is superior to the intellect of man. Can such a statement be taken seriously? Compare the work of the ants and bees with the proud achievements of the human reason. If instinct is the appropriate organ for apprehending reality, the discovery of truth, why should it be given in such measure to ants and bees, which care not to exercise it, whose range of freedom is so small? How comes it that man, who has a passion for the discovery of truth, has so little of this divine faculty of instinct, the truth-discovering faculty (I mean of course the hard and fast lines of instinct)? Can Bergson or Gérard explain this paradox?

A healthy, vigorous people will have constantly new material, intellectual and spiritual wants or necessities. There must be, then, new and fertile philosophic ideas crystallizing into religious beliefs and ideals. And the sure proof of vitality in a people is the adapting of these new ideas to its new physical, intellectual and spiritual wants.

But unconscious instinct cannot supply these ideas and ideals. Here surely is the inevitable task of the thinker. Professor Gérard even admits that "the role of the conscious reason is, in spite of all, the higher role." We need the inspiration of instinct, of feeling, at times, but we must not forget that the most perfect thing, the most indispensable evolved on this planet is the human intellect.

And when M. Bergson affirms that the sphere of the intellect is "matter" and Professor James, following him, says that its province is "mere surfaces," it seems to me a discrediting of that great faculty which rose to its fulness in Socrates, Isaiah, Jesus, Paul, Pascal, Newton, Darwin. Is not the intellect a metaphysical faculty? And are not its problems metaphysical as well as physical, depths as well as "surfaces"? Show me great art, great music, great poetry, great sculpture that has not the intellectual, the metaphysical strand. Are not the Apollo Belvedere, the Divine Comedy, the Mona Lisa, the C Minor Sonata as well as Newton's laws, in part at least, the fruit of intellect? Draw out the intellectual threads, and would you not destroy the integrity of the whole garment?

I think it will be conceded that religion will take a great step forward if philosophy shall touch the ground of reality, if it shall find a true answer to some of those questions which ever press upon the human spirit; if it shall rise above a mere rephrasing of its attitude to a consciousness of our mental demands.

I maintain it is not the function of religion to teach any theory of the world, any truth. This does not mean that truth is indifferent in religion, for that would imply that education and science are valueless. Ignorance makes for poverty and vice. But there is no coming to independent truth or knowledge, truths about *real things,* through a revelation from within, through intuition. Truth is attained only by observation, experiment, analysis, search, the most patient and exact generalization. Man's religion is of the imagination and the heart, but from his *intellect* he receives his truth, from the intellect the heart receives its *light.* Truth is ever a matter of discovery, of science, of philosophy; religion a matter of feeling.

There are our feelings, our common reactions, it will be said, the great beliefs, the universal "acceptances." But our feelings and our instincts, in the end, must wait upon our intelligence. Whenever Christianity ceases to hold the people intellectually, it will cease to hold their hearts. Mr. Balfour has shown, with fine scorn, the cowardice of those who would stand for the dogmas of Christianity, not because they believed they were true, but because to retain them would be better for the morals of the people. That is to say, they would retain Christianity for policy's sake. These men forget that when the more intelligent lose their faith, the multitude will surely, in the end, lose their faith also.

It is not the province of philosophy (no one will say) to strengthen the gross materialism so transcendent about us unless it shall have been first shown true by science. But the tendency of the thought of our greatest men of science is away from the material hypothesis. They no longer believe religion to be a cunning fable devised by king and priest to be an instrument with which they might control the people. Religion is a mighty force and has its roots deep down among the primitive instincts and feelings of the human race. It will be a sad day for humanity when religion shall have become superannuated.

There must be, as Professor Gérard says, the healthy and beautiful body. There must be intellectual vigor. But there must be, if a people shall endure, the material for intellectual nutriment. There are to-day new intellectual and spiritual wants. There must be new and fertile ideas which may crystallize into new religious beliefs and ideals. To furnish these ideas and ideals is the inevitable task of the poet, the artist and the philosopher.

While Professor Bergson has said many things derogatory of the human intellect, and a lot of nonsense about the original power of the intuitions which we have now lost, he has said many brave words for philosophy. He has made the vital suggestion (which, so far as I have seen, seems to have been unnoticed) that philosophy pursuing certain lines of facts all converging on the same point "may give an accumulation of probabilities which will gradually approximate scientific certainty."

Well, what greater certainty can we ask? And is not the human intellect, with its dogged slave, Observation, its angel-attendant Imagination, practically infinite? May not that intellect which sweeps over infinite time and space, which holds in its hand, like a flower, the whole stellar universe, solve at least some of those problems which are the very cause of philosophy's existence?

Why should we deny the final intelligibility of the universe? Why may not philosophy pass over the threshold of speculation into the domain of actual knowledge? Why may there not be a definite conquest by philosophy as well as by science? Who can limit what philosophy may do when squarely facing the supreme problems and not frittered away, as Bergson says, "upon a host of special problem in psychology, in morals, in logic."

James G. Townsend.

Jamestown, N. Y.

THE FALLACY IN MR. H. G. WELLS'S "NEW RELIGION."

In his book, *God the Invisible King,* which hails the appearance of a "new religion," Mr. H. G. Wells proclaims himself the spokesman of his age, the "scribe to the spirit of his generation."[1] If he claims to speak for the scientists as well as for the less enlightened portions of society, his conclusions are startling, to say the least,

[1] *God the Invisible King,* p. 171.

in view, not only of the commonly observed lack of religious belief among scientists, but also of the statistical study by which Professor Leuba[2] has shown that the majority of scientists in America, and presumably elsewhere, disclaim any belief in God.

Were it not that he implicates "that very great American, the late William James,"[3] whom he calls his master, Wells's religious views would, perhaps, scarcely merit consideration by philosophers. So far, however, as the views of Wells are due to James's influence, they deserve examination; and the fact that *God the Invisible King* is a book intended primarily for popular consumption need not condemn it in the sight even of professional philosophers, when it is remembered that James (and so why not, perhaps, James's disciple?) could be both popular and profound.

A finite God is proclaimed in Wells's new religion, and at once a point of similarity between Wells and James is noted. James was insistent that the Absolute of the philosophers could not be the God of religion, and Wells is equally insistent upon this point. But, whereas James asserted that personal immortality is the core of religion, and that the chief function of God would be the guaranteeing of immortality,[4] Wells regards this question as an irrelevant issue in religon, interest in which is evidence of egotism.[5] Whether James was not nearer than Wells to a correct interpretation of the religious consciousness regarding the belief in immortality is a question that might appropriately be raised, though I shall omit consideration of it here.

Limiting my discussion to questions of the nature of God, and of the evidence for His existence, in Wells's view, I desire to point out the closeness with which Wells follows James's line of thought, to the extent of committing one of the same fallacies that James commits.

Wells's God is not the Life Force or the Will to Live,[6] neither is He the Collective Mind of the Race.[7] Wells, like James, insists that God must be genuinely personal, existing within a temporal environment, aiding mankind in its upward struggle, and accessible to man through what James calls "prayerful communion". In

[2] J. H. Leuba, *The Belief in God and Immortality,* Boston, 1916.

[3] Wells, *op. cit.,* p. 172.

[4] *The Varieties of Religious Experience,* p. 524; *Human Immortality,* Ingersoll Lecture. This view of James's accounts in part for his interest in psychical research.

[5] *Op. cit.,* Preface, p. xix. [6] *Op. cit.,* p. 17. [7] *Op. cit.,* pp. 61, 62.

Wells's view, as in James's, evidence for God's existence is found in so-called religious experiences, mystical in nature. James expresses it as follows: "There *are* religious experiences of a specific nature ....They point with reasonable probability to the continuity of our consciousness with a wider spiritual environment."[8] "Personal religious experience has its root and center in mystical states of consciousness."[9] And Wells says similarly: "Modern religion bases its knowledge of God and its account of God entirely upon experience."[10] "This cardinal experience is an undoubting, immediate sense of God. It is the attainment of an absolute certainty that one is not alone in oneself.... The moment may come while we are alone in the darkness, under the stars, or while we walk by ourselves or in a crowd, or while we sit and muse. It may come upon the sinking ship or in the tumult of battle.... After it has come our lives are changed, God is with us and there is no more doubt of God. Thereafter one goes about the world like one who was lonely and has found a lover.... One is assured that there is a Power that fights with us against the confusion and evil within us and without."[11] In accepting the mystical experience as the basis of religious belief, Wells agrees completely with James. As Wells himself says,[12] "So far as its psychological phases go the new account of personal salvation....has little to tell that is not already familiar to the reader of William James's *Varieties of Religious Experience*."

When God's existence is argued for upon the basis of the religious experience, a crucial question arises regarding the externality and objectivity of the God that is believed in. Here arises what I have called the fallacy of false attribution, "which consists in the erroneous interpretation of an experience whereby the experience is attributed to an external, divine source in cases where a physiological explanation is adequate to account for it."[13] With Wells as with James there is no doubt regarding the objectivity of the God evidence for whose existence is thought to be found in mystical experiences. James classifies himself as a "piecemeal supernaturalist."[14] Piecemeal supernaturalism "admits miracles and

[8] *A Pluralistic Universe*, pp. 299, 300.

[9] *The Varieties of Religious Experience*, p. 379.

[10] *Op. cit.*, p. 20. [11] *Ibid.*, pp. 23, 24. [12] *Ibid.*, p. 21.

[13] Cf. the author's article "Two Common Fallacies in the Logic of Religion," *Journal of Philosophy, Psychology, and Scientific Methods*, Vol. XIV, pp. 653-660. The above quotation is from page 657.

[14] *The Varieties of Religious Experience*, p. 520.

providential leadings, and finds no intellectual difficulty in mixing the ideal and the real worlds together by interpolating influences from the ideal region among the forces that causally determine the real world's details."[15] Nowhere in Wells's writings do we find quite so frank a statement of supernaturalism; but in denying that God is the Collective Mind of Humanity,[16] in admitting "help from without,"[17] in speaking of an "exterior reference"[18] of the religious experience, and in insisting that "God is an external reality,"[19] Wells commits himself to such a view. And with such a view goes the fallacy of false attribution, which is found in the arguments of both Wells and James[20] for the existence of God.

The fallacy of false attribution is committed by Wells, we must agree, so far as it is possible to explain the religious experience in terms of physiological psychology; and there seems to be little difficulty in accounting for the experience as a form of emotionalism, which is *interpreted, after* the experience, as an experience of communion with God. The mystic *believes* that he experiences an objective God, a reality which is more permanent than the passing experience, and which is the source of the experience. Such belief is essential in connection with the mystical experience in order to make it a religious experience. But, though the *belief* in God is present, God need not be real; and, in fact, it is the belief, and not the object of the belief, God, that does the "work" in religion that James and Wells speak of. Thus Wells says:[21] "Prayer is a power. Here God can indeed work miracles." And in saying this he is illustrating very clearly the fallacy of false attribution. If a physiologically grounded psychology is to be admitted to the circle of the sciences, then we must say here that it is the psycho-physiological activity of belief that does the "miraculous" work—work which is falsely attributed to God.

Whoever claims that evidence for the existence of God, defined concretely enough to be significant, as Wells's God is defined, is to be found in experiences of a mystical sort, must give reasons for asserting that the mystical experience is anything more than a strongly marked emotional state, in which the sentiment of love is prominent, together with a strong conviction regarding the divine

[15] *Ibid.*, pp. 520, 521. [16] Wells, *op. cit.*, p. 61.

[17] *Ibid.*, p. 26. [18] *Ibid.*, p. 78. [19] *Ibid.*, p. 82.

[20] See the author's article, *loc. cit.*, pp. 657-660, for a discussion of the fallacy of false attribution as found in James's views.

[21] *Op. cit.*, p. 155.

source of the emotion and the divine object of the human sentiment of love. The conviction, or belief, moreover, as to the source of the experience is not derived from the experience, but from tradition, education, and social influences in general. Professor Hocking's claim that "the love of God is the one natural instinct of man"[22] is ungrounded. The biologist and the psychologist fail to discover "love of God" among the instincts. There is no religious instinct. Love of God is a form taken by instinctive love when *interpreted* in a religious fashion; and the religious interpretation is not instinctive, but is due to social influences. The experience comes from "below," through the sublimation of a very primitive instinct, and is to be explained in naturalistic terms; but it is interpreted by the mystic as coming from "above," and not to be explained naturalistically. In the mystic's interpretation there inheres the fallacy of false attribution.

The mystical solution of the problem of religion, to which Wells resorts, is inadequate except under one condition—that mysticism be made a thorough-going metaphysical doctrine, involving the complete denial of any reality to the world that the sciences study. Only in Nirvana could such a doctrine be consistently maintained.

Wesley Raymond Wells.

Washington University.

A PHILOSOPHICAL LITTERATEUR.

Prof. S. P. Sherman's work has been for the last few years one of the features of the New York *Nation.* Memorable are the vivid character-portrayal of Professor Kittredge (issue of September 11, 1913), the rollicking zest of "The Gaiety of Socrates" (July 15, 1915), and the mordant logic of his dissection of Mr. Roosevelt (November 29, 1917). But chiefly as the upholder of the conservative tradition in literary criticism has Mr. Sherman attained distinction. Most of his reviews of this character have recently been issued in a revised and enlarged form, together with an introduction expounding fully though somewhat loosely the author's *Weltanschauung.*

The book, entitled *On Contemporary Literature* (Henry Holt & Co.), makes delightful and stimulating reading. Though the accent is at times academic, the style is often vivid and racy. Take

[22] W. E. Hocking, *The Meaning of God in Human Experience,* p. 577.

this penetrating criticism of Dreiser's realism: "If you expect to gain credence for the notion that your hero can have any woman in Chicago or New York that he puts his paw on, you had probably better lead up to it by a detailed account of the street-railway system in those cities." Or this satirical sketch of Alfred Austin: "While we who live in territorial homes have been asking, *Où sont les neiges d'antan*?—'Where are the wives who sit on the nest and never stir?', he has sung on imperturbably, celebrating the Lucile, the Dora, the Maud of the mid-Victorian dream—the fair and lissome English maiden blushing and trembling toward her lover and her lord with the reverence implanted in her unsunned bosom by God and Nature."

When, however, Mr. Sherman proceeds to apply philosophical standards to the subjects of his criticism, as he does very often, he shows himself as remote from the modern trend, as much the victim of traditions and formulas as the laureate object of his satire. He modestly disclaims any complex philosophical apparatus, but the fact is that he seems not to have any philosophical apparatus at all beyond what might be gleaned from a sedulous study of *belles-lettres.* He appeals "to the general reason and experience of mankind against the conclusions of the ratiocinative faculty of the individual." The superstition of a solid and homogeneous "general reason" can only be held by those who coolly ignore the thought of the East and arbitrarily suppress the heterodox opinion of the West as voiced by Euripides, Lucretius, Gottfried von Strassburg, Voltaire, and Heine. Only within the realm of science, which nevertheless regards the individual's judgment as the final court of appeal, has there grown up enough real unanimity of opinion to justify an appeal to the "general reason." Mr. Sherman seems reluctant to admit the authority of science, sneering, for instance, at Anatole France's acceptance of the scientific probability that there is a limit to the evolution of life on our planet; yet we propose to test his philosophy by a comparison with the only real approximation we have to a "general reason", the consensus of scientific opinion.

Mr. Sherman's view of man is summed up in these sentences: "On the lowest level is the natural world, which is the plane....of the animal passions or affections....On the middle level is the human world....working upon the natural world; but governed by reason, the special human faculty....On the third level is the spiritual world, which is the plane of spiritual beings and the home of eternal ideas."

How does this analysis tally with the findings of science? Mr. Sherman seems to say that the passions and affections we share with the animals are not a part of our real humanity. Would he have us regard as unhuman the sex instinct, the creative impulses, tribal loyalty, and individual fidelity? He gravely asserts that "it is of the essence of a man to lay down his life out of reverence for his great-grandfather." Yet what is this essence of a man but the Quixotic outgrowth of a filial devotion observable in many animals? Has Mr. Sherman never felt inclined to regard even our noblest instincts as rather typically animal than human, to exclaim with the cynic, "The more I see of men, the more I like dogs?" Elsewhere he remarks that "the impulse to refrain we can find nowhere in nature. It is part of the pattern design of human society that lies in the heart of man." Really it would seem as if he had never seen a squirrel store away a nut, or a trout resist a baited hook, or a mother bird keep a juicy worm for her nestlings. Either the word "refrain" is used in some esoteric sense, or else Mr. Sherman's words are a specious flattery to our nature which it requires no scientist to refute.

Mr. Sherman, however, is not consistent as to what is the vital differentia between man and brute, and perhaps would prefer to have us judge him by his pronouncement that reason (not the impulse to refrain) is "the special human faculty." I wonder if he has consulted the opinions of psychologists on this point? Holmes says: "If we define reason as the derivation of conclusions through the comparison of concepts, it is not improbable that no animal below man employs this faculty. But this is far from implying that animals cannot perform mental operations which are essentially inferential in their nature." Hobhouse declares that, "If we allow reason to the human species in general, and yet restrict it to that species, it must be by identifying the term reason arbitrarily with a certain grade in the development of analysis." Apparently the Jesuit Father Wasmann is almost alone among comparative psychologists in holding the animal and human minds utterly distinct.

Again we must quarrel with Mr. Sherman when he speaks of reason as a power by which the natural world, including our own desires, is "governed." For him the human ideal is attained "when Ariel, the lawless imagination longing for liberty, and Caliban, the incarnation of the lusts, powers, and instincts of our animal nature ....yield to the wonder-working sway and sovereignty of benignant reason, represented by Prospero." Now reason rules no one: it only points the way to the fulfilment of one's strongest desires. That much neglected essayist, Mr. Bernard Shaw, has put this whole matter with inimitable skill. "The difference between Caliban and Prospero is not that Prospero has killed passion in himself whilst Caliban has yielded to it, but that Prospero is mastered by holier passions than Caliban's." Shaw goes on: "The ingrained habit of thinking of the propensities of which we are ashamed as 'our passions,' and our shame of them and our propensities to noble conduct as a negative and inhibitory department called generally our conscience [Mr. Sherman calls it the reason, Mr. More the inner check], leads us to conclude that to accept the guidance of our passions is to plunge recklessly into the insupportable tedium of what is called a life of pleasure....Reactionists against the almost equally insupportable slavery of what is called a life of duty are nevertheless willing to venture on these terms.... No great harm is done beyond the inevitable and temporary excesses produced by all reactions; for, as I have said, the would-be wicked ones find, when they come to the point, that the indispensable qualification for a wicked life is not freedom but wickedness." Here are words which Mr. Sherman and the philosophers of the humanist school may ponder. At least, they will not retort that Mr. Shaw is a pseudo-scientist, for on this point he has the whole psychological world behind him.

Finally we come to the third story of Mr. Sherman's edifice, the spiritual world, defined as "the plane of spiritual beings and the home of eternal ideas." I fear that science has sadly depopulated this world. As Mr. Sherman knows full well,

> "Al was this land fulfild of fayerye;
> The elf-queen with her joly companye
> Daunced ful ofte in many a grene mede;
> * * *
> I speke of manye hundred yeres ago!
> But now can no man see none elves mo."

And alas, with the elves have gone the incubi and witches, the devils and angels, the cherubim and seraphim, until for most of us God on his sapphire throne is the solitary inhabitant of the spiritual world. And there are statistics to show that the most eminent scientists have dismissed this concept also. Mr. Sherman himself seems to reduce God to that benevolent abstraction, "a power not ourselves that makes for righteousness."

But if the third floor front is almost vacant, there is yet "the home of eternal ideas" in the third floor back. Among these eternal verities, doubtless, are the elementary principles of conduct, which, Mr. Sherman declares in his introduction, "have been adequately tested and are now to be unequivocally accepted." When were the tests completed? In 1791, when Burke announced that no more discoveries were to be made in morality? Or within the last fifty years and one, since the *Nation* has upheld the changeless principles of idealism? Only twenty-five years ago the *Nation's* review of *Tess of the D'Urbervilles* came out flat-footed for a double standard of morality. Is this one of the tested principles of conduct to be unequivocally accepted, or has the perfected idealism been defined only within the last quarter of a century? The humanist critics, Messrs. Babbitt, More, and Sherman, seem as reluctant to ventilate their eternal ideas in categorical form as the Germans are in the matter of their war aims, and doubtless for the same reason: they would be starting a fight behind their own lines. The home of the old eternal verities is being prepared for evacuation, and we may look for the eventual passing of the third floor back.

It has always been a taunt of the humanist critics that the romantic writers lived in an ivory tower, remote from the crowd and bustle of life, and dreamed of man as he never was and never could be. Does not Mr. Sherman's analysis of human nature reveal the humanists themselves dwelling in an ivory tower of academic contemplation, the walls lined with the orthodox classics, the vaults containing the latest authoritative copy of the Decalogue for the benefit of the inmates? When they wish to discover what is the nature of man, they take from the shelf Aristotle or Shakespeare or even Hooker, and without asking awkward questions imbibe reverently the inspired conjectures which once upon a time did duty for a systematic, experimental study of the mind. If we do not look to the ivory tower of romanticism for the best that has been thought and said in the world, why should we listen below the ivory tower of humanism for echoes of the rudimentary psychology and ethics of ancient Greece and Elizabethan England!

CHARLES HEATON.

ON THE CONCEPTION OF PROBABILITY.

The desire for a mathematically and a philosophically sound introduction to the theory of probability is dictated by the importance of the field of applications, by the fact that the theory of probability is mathematics, and by the philosophical interest attached to the term probability. The applications of the theory were originally confined to problems in gambling, but they are now found in statistics, theory of error, statistical mechanics, insurance, etc. This expansion of the field of usefulness would by itself be a sufficient cause for minor changes in the fundamental conceptions and definitions, just as, for instance, in the steel industry the development in methods and use has been followed by changes in the standard definitions and specifications of iron and steel. To this comes that the introduction of new mathematical and logical methods, such as the axiomatic method, has furnished new view-points for the initial steps in the theory of probability. And the inquiry of philosophy into the nature of probability can never be expected to be answered absolutely and finally. In this way all of the relations of the theory to its applications and sources are likely to exert their influence on the fundamental part of this subject. The fact remains that it is extremely difficult to give a satisfactory definition of probability; quoting Poincaré, it is even hardly possible.[1] These conditions taken together explain why the question "what is probability?" in spite of its long history is still alive, and this serves as an apology for the reappearance of the subject in this paper.

The historical methods of introducing probability may be classified according to their relation to four principal methods. The first three of these will, for the purpose of orientation, be briefly mentioned. The discussion in connection with the fourth of the principal methods forms the main part of this paper. It presents a view-point which is thought to throw some new light on the question. The classification follows:

1. Bayes's definition based on the notion of mathematical expectation (*espérance mathématique, mathematische Erwartung*).

[1] H. Poincaré, *Calcul des probabilités*, ed. 1912, p. 24.

2. The method involving the introduction of the notion of equally possible cases as a fundamental notion.
3. The axiomatic method.
4. The method which will here be called the statistical, which has been used by statisticians, and which is based primarily on the law of great numbers.

1. Bayes's ideas of probability are perhaps the oldest historically. At least according to Von Kries's interpretation, they are underlying, though not definitely formulated, in the classical correspondence between Pascal and Fermat.[2] Bayes expressed those ideas in a definition.[3] An example will explain the principle involved. Playing head and tail with two dollars at stake, with even chance of winning and losing, the value of the chance to win is reasonably estimated as one dollar. Then, according to Bayes's definition, the probability of winning would be expressed as one dollar divided by two dollars, that is 1/2. If in general the reasonable value of the chance—the mathematical expectation—of obtaining A is B then B/A is the probability. It seems evident that the modern student of applied mathematics can hardly expect to obtain the clearest notion of probability by way of Bayes's definition unless he has occupied himself extensively with gambling. It seems that only thereby may one develop the notion of the value of a chance, the notion of the mathematical expectation of gain, as a fundamental conception or as an idea of an existing tangible reality. The definition depends on a reference to this reality. Bayes's definition though interesting has now chiefly historical value.

2. The second principal method, found in many classical discussions, makes use of the expressions equally possible or equally probable. These expressions are taken from the ordinary spoken language, and their meaning remains essentially unchanged after they have been introduced and used in the mathematical theory. Illustrative examples are used among which one is predominant, or fundamental as far as it may be said to represent schematically the formation of any probability. It is the example of the bag containing balls of different color but otherwise the same. Assume p white balls in the bag out of a total of q, then there are p equally possible cases out of q in which one draws a white ball by taking one out. Then by the definition the probability is p/q. This gen-

[2] Von Kries, *Die Prinzipien der Wahrscheinlichkeitsrechnung,* 1886, p. 267.

[3] Th. Bayes, "An Essay toward Solving a Problem in the Doctrine of Chances," *Philosoph. Transactions,* 1763, p. 370.

eral method has its disadvantages. What is "equally possible"? To answer this additional explanations of great length have been deemed necessary at the various times. It is sufficient at this place to mention the classical works by Laplace, J. F. Fries, Lexis, Von Kries, and Bertrand, and besides, two more recent discussions by Lourié and Grelling.[4] In spite of the high value of these works, in spite of the increased understanding of probability due to them, their inevitable extensiveness certainly makes them less accessible than is to be desired. And if the notion of the equally possible cases is maintained as the fundamental idea, it is not unlikely that further discussions on the same basis will be found necessary in the future.

3. The *axiomatic method* has been tried by Broggi and Bohlmann.[5] Terms such as "event A," "probability of event A," "event A independent of B or excluding B," etc., are introduced in the axioms stating the two principal laws of combining probability. The axioms can be stated briefly, and logical rigidity as far as the mathematical theory itself is concerned may thus be obtained. When partially independent notions are to be introduced, such as, for instance, that of continuity of probabilities, then the necessary additional axioms are established without difficulty.[6] Nevertheless, as mentioned by the originators of the axioms, the method solves the questions involved only in part. If one does not know what probability is before the axioms are stated the chance remains that one will not know it afterward either. The logical problem left is essentially that to which the main efforts of the previously mentioned extensive works were devoted. The axiomatic method then has its value as supplementary to other solutions. Between the axioms by themselves and reality there is no bridge.

4. The fourth and last principal method is the *statistical method*

[4] Laplace, *Théorie analytique des probabilités,* philosophical introduction from 2d ed. on. J. F. Fries, *Versuch einer Kritik der Prinzipien der Wahrscheinlichkeitsrechnung,* 1842. W. Lexis, *Zur Theorie der Massenerscheinungen,* 1877. Von Kries, *Die Prinzipien der Wahrscheinlichkeitsrechnung,* 1886. Bertrand, *Calcul des probabilités,* 1889 (introduction). S. Lourié, *Die Prinzipien der Wahrscheinlichkeitsrechnung,* 1910. K. Grelling, *Die philosophischen Grundlagen der Wahrscheinlichkeitsrechnung, Abhandlungen der Friesschen Schule,* 1910.

[5] G. Bohlmann, *Encyc. d. math. Wiss.,* Vol. I, Part II, Art. 1D 4b (1900-1904). U. Broggi, *Die Axiome der Wahrscheinlichkeitsrechnung,* Dissertation, Göttingen, 1907. G. Bohlmann, *Die Grundbegriffe der Wahrscheinlichkeitsrechnung, Atti del 4. Congresso Internazionale dei Matematici,* Roma, 6-11 Aprile 1908, Vol. III, 1909, pp. 244 etc.

[6] The point of view of the continuous probabilities is emphasized by L. Bachelier in his *Calcul des probabilités,* 1912.

based principally on the law of great numbers. Montessus by deriving his definition of "equally possible" from the experience expressed in the law of great numbers becomes a representative of this point of view.[7] The writer believes that the statistical method is best suited to the needs in the present main fields of applications, though it is realized that the completest understanding of the problem is reached by a study of all the historical methods. The discussion which follows will propose a method of obtaining logical rigidity in definitions on the basis of the law of great numbers. We shall in the first place consider only the probability which applies in the theory of probability. Other types of probability, subjective and psychological probabilities, which do not necessarily follow the same laws, will be briefly mentioned afterward.

In order to prepare the way for a definition of the probability in the the theory of probability two preliminary notions will first be introduced: that of a great probability and that of a great number.

First let us consider the expression "great probability." This expression shall first be taken in the sense in which it is used in the ordinary scientific language. It shall express the almost safe, or as good as safe expectation that a certain event will occur. Such great probabilities exist. They are derived essentially from experience, but it is realized that they also contain a subjective element expressed in the decision to believe in a certain regularity that makes predictions possible, or in the decision to disregard certain very slight possibilities as immaterial.

The other preliminary notion is that of the great number. The expression "a great number" shall first be taken in the sense of the ordinary scientific language, but in order to adapt the notion for mathematical use we add the following statement: let the functional forms

$$F_1(N_1, N_2 \ldots . N_k),\ F_2(N_1 \ldots . N_k), \ldots .$$

represent certain uses made of the numbers N_1, $N_2 \ldots . N_k$; let $n_1, n_2, \ldots . n_k$ represent any k numbers less than a certain number n; then N_1, $N_2 \ldots . N_k$ are said to be great numbers with respect to the use F_1, $F_2 \ldots .$, and compared with the number n, when—besides $N_1 \ldots . N_k$ being great numbers in the ordinary sense—the differences

$$F(N_1+n_1,\ N_2+n_2 \ldots .) - F(N_1,\ N_2 \ldots .)$$

can be neglected.

[7] Montessus, "La loi des grands nombres," *L'enseignement mathématique,* 1905, pp. 122-138. See also his *Calcul des probabilités,* 1908.

That "great numbers" as just defined exist is a matter of experience. When and whether the differences mentioned can be neglected is not merely a mathematical question, but it depends on the empirical realities to which they apply. The explanation "great with respect to a certain use, compared with a certain number" shall always be understood as added whenever the expression "great number" is used in the theory of probability.

After this preparation it is possible to formulate a definition of probability.

Assume that a group of conditions can be indicated under which a certain event may or may not occur. Assume that the nature of this group of conditions allows their repetition any number of times. Among the conditions will be some which limit the knowledge of what actually happens at the individual reproduction of the conditions. Denote by N_2 the unknown number of times in which the event occurs during N_1 repetitions of the conditions. Assume further that N_1 and N_2 are great numbers with respect to any possible use of the fraction N_2/N_1 as represented by

$$F(N_1, N_2) = \phi(N_2/N_1),$$

N_1 and N_2 thereby being compared with some chosen number n. Assume now that the fraction N_2/N_1 with a great probability can be declared to be equal to some distinct value p, and that this great probability can be made a still greater probability by increasing N_1. Then in the sense of the theory of probability p is the probability of the given event under the given conditions. Briefly expressed, the greatly probable ratio of frequency at a great number of repetitions of the conditions is the probability.

That such "probabilities" exist is a matter of experience. Their existence is identified with the existence of the law of great numbers.

In the philosophical introduction to his Theoretical Physics Volkmann[8] advocates what may be termed a repeated epistemological or knowledge-theoretical cyclus. The present problem allows an application of this method of thought, which, though here it may at first appear so, is not a "vicious circle" but rather a "cyclus of logical convergency."

First note that the conception of probability as defined here depends on the previously defined notion of "great probabilities." But the definition of probability just given allows to consider the great probability as that special case of the general probability in

[8] P. Volkmann, *Einführung in das Studium der theoretischen Physik,* 2d ed., 1913, pp. 349 etc.

which this becomes very nearly equal to one. By adding this consideration a sharpened definition of the term "great probability" is obtained, and again, this improvement in rigidity propagates itself into the definition of the general probability. By re-applying the same method the process of sharpening the definitions may be continued.

It is easy to derive the two fundamental theorems of combining probabilities from the definition given here. The second of the theorems, stating that the probability of the contemporary occurring of two mutually independent events is equal to the product of the probabilities of the single events, requires a special definition of the term independency; such definition can be formulated as follows: the event A is said to be independent of the event B when the ratio of cases in which A occurs at a great number of repetitions of conditions is the same whether the total number of cases or only those cases favorable for the event B are considered.

After these two fundamental theorems Bernoulli's theorem of the great numbers can be derived in the usual way. This theorem throws a new light on the definition of probability and on the notion of great numbers, and thus it opens the way for another application and reapplication of a Volkmann's epistemological cyclus.

We are now ready to discuss briefly other types of probability, namely the psychological and subjective probabilities. These are distinct from the already defined empirical or hypothetically empirical or derived hypothetically empirical probabilities treated in the theory of probability. The psychological probability is the degree of expectation. It expresses itself in certain muscular strains and might be measured through these. Our expectations are not always reasonable or logical, therefore it is evident that they are not subject to the laws of the theory of probability. A type of subjective probability may be defined parallel to the empirical probability as the subjectively expected ratio of frequency at a great number of repetitions of generating conditions. Even the so defined subjective probabilities can only approximately follow the laws of the theory of probability, unless by added axioms they are made dependent on these laws. The understanding of the subjective probability improves the knowledge of the subjective element of the "great probability," one of the terms introduced at the beginning of this development. Here again appears the advantage of the epistemological cyclus.

A final cyclus leads now from the last improved conception of probability to the theory of probability, then to the applications

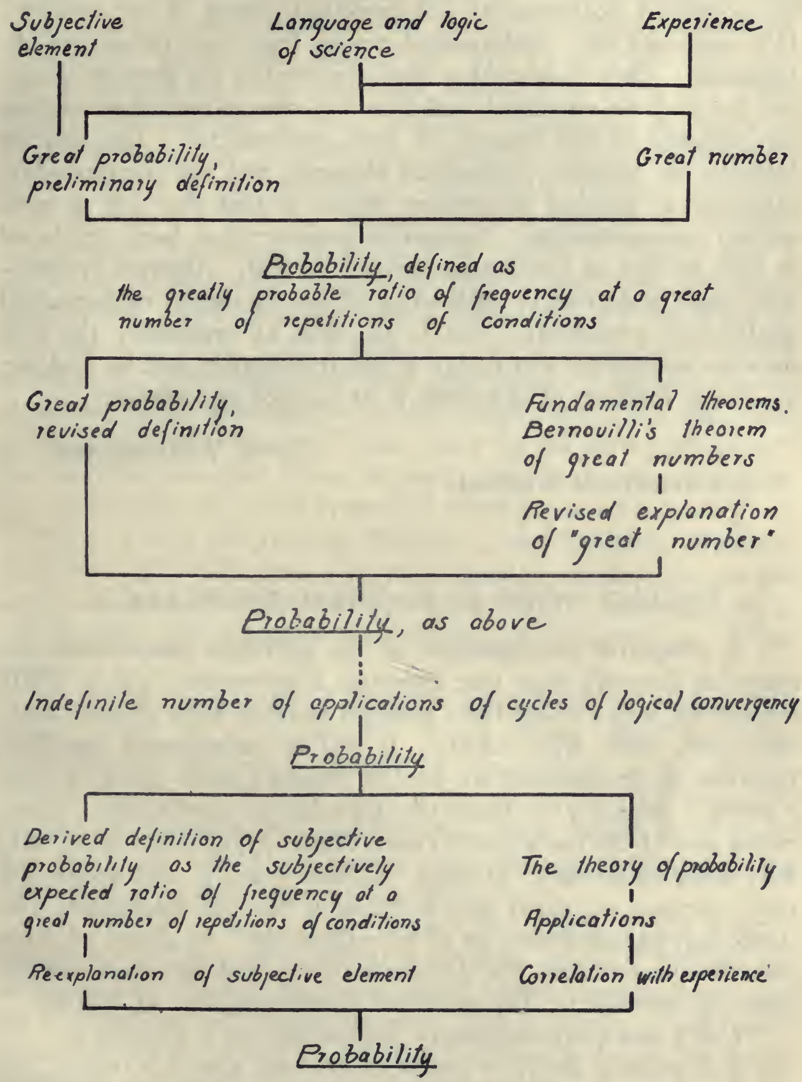


CHART SHOWING PROCESS OF DEFINITION.

of the theory of probability, and therefrom back to the empirical foundation of the definitions.

The chart showing the process of definition reviews the individual steps in this discussion. It should be emphasized that in developing the notion of probability it is necessary to recognize the combined mathematical and empirical nature of the problem. It is conceded that mathematical points, lines, and planes are abstractions. They have neither been observed in the physical world, nor can they be visualized, and thus far, they do not exist outside the paradise of the student of pure mathematics. The relation between the abstract geometrical elements and the corresponding graphical or physical elementary objects is that they are at most mutual approximations. Successful geometry has been developed in spite of that or perhaps on account of that. Abstract probabilities might be derived by eliminating all but the merely mathematical qualities of probability. But in the field of probability the gulf between abstraction and reality is less transparent, and its bridging by proper methods of definition is of decided importance.

H. M. Westergaard.

University of Illinois.

RECENT WORK IN MATHEMATICAL LOGIC.

A delightful simplification of the primitive propositions required in Russell's logic (see *Principia Mathematica,* Vol. I, 1910) is made by J. G. P. Nicod in a graceful piece of work (*Proc. Camb. Phil. Soc.,* 1916, XIX, 32-41). It will be remembered that four functions of propositions are used in Russell's logic—not-*p, p* or *q, p* and *q, p* implies *q*: of these, two are taken as indefinables. Nicod makes use of Sheffer's idea (*Trans. Amer. Math. Soc.,* XIV, 481-488) using "*p stroke q*" to mean "not both *p* and *q*" and defines the four functions ordinarily used in terms of this one indefinable. The stroke may be called the sign of *incompatibility.* By means of *three* primitive propositions the propositions required for mathematical logic are developed. The primitive propositions are as follows:

1. If *p* and *q* are elementary propositions, so is *p stroke q.*

2. If *p* and *p stroke* (*r stroke q*) are true, then *q* is true.

3. P *stroke* (π *stroke* Q), where P stands for *p stroke* (*q stroke r*), Q for [*s stroke q*] *stroke* [(*p stroke s*) *stroke* (*p stroke s*)] and π for *t stroke* (*t stroke t*).

The generalized form of the principle of inference (the second of these propositions) is interesting and deserves notice. The third of these primitive propositions is of course exceedingly complicated and does not commend itself as true so readily as the primitive propositions of *Principia Mathematica,* but it is very interesting to have found it and to have shown how the more familiar propositions can be deduced. In the same issue of the *Proc. Camb. Phil. Soc.* C. E. van Horn uses the same indefinable calling it "*p deltas q*." Then from a certain group of three primitive propositions he claims to be able to deduce the propositions required for logic. These propositions consist of the principle of inference as given in *Principia Mathematica,* a proposition analogous to prop. 1·71 in *Principia Mathematica,* by means of which it is possible to deduce that *p deltas q* is an elementary proposition, when *p* and *q* are elementary propositions; and a third proposition stating that if *p* and *q* are of the *same* truth value, *p* and *p deltas q* are of oppositive truth values, and that if *p* and *q* are of *opposite* truth values, *p deltas q* is true. However, in the development of this system, it is clear that some axiom is required to connect "deltas" with "of the same truth value." It is not difficult to find vicious circles in some of the demonstrations (cf. the criticism of Nicod at the end of his paper, *ibid.,* p. 40).

C. I. Lewis (*Journal of Phil., Psy., and Scientific Methods,* 1917, XIV, 350-355) gives a further statement of his views on the nature of material implication (cf. *Mind,* 1912, number 84; 1914, number 90) and continues his criticism of Russell's use of the notion.

A translation of part of Frege's *Grundgesetze* is given in *The Monist* (1917, XXVII, 114-127). A. E. Heath (*ibid.,* pp. 1-56) contributes an interesting account of Grassmann's work. D. M. Wrinch (*ibid.,* pp. 83-104) gives a sketch of Bernard Bolzano's life and his pioneer work in mathematical logic. Philip E. B. Jourdain (*ibid.,* pp. 142-151) discusses existence and distinguishes the "entity" of a number from its "existence."

N. Wiener (*Trans. Amer. Math. Soc.,* 1917, XVIII, 65-72) attacks a very general problem in Boolean algebras, finding the necessary and sufficient conditions that a relation between any number of elements will remain invariant with reference to all transformations of the algebra into itself which may be expressed in the symbolism and which leave it a Boolean algebra.

An extract from Couturat's unpublished *Manuel de Logistique* is given in the *Rev. de Métaphys.* (1917, XXIV, 15-58). The nature of propositional functions, formal and material implication, and real and apparent variables is examined. The article concludes with a sketch of the more elementary parts of the calculus of classes and the definition of 0 and 1. Part of an unfinished treatise written by Couturat sometime before 1902 is given in the same periodical (*ibid.,* pp. 291-313), which is interesting as an exposition of the *frequency* view of probability originally put forward by Venn and the mathematical writers, as opposed to the *sufficiency* view held by Bosanquet, Lotze, and Sigwart. *Probable* is said to be significantly predicable only of indeterminate judgments (by which is meant, presumably, propositional functions) and not of events or determinate judgments. The probability of a judgment is then defined to be the ratio of the number of cases when it is true to the number of cases when it is true or false. The article ends with a short account of the elementary part of the calculus of probabilities. Tenney L. Davis (*Journal of Phil., Psy., and Scientific Methods,* 1917, XIV, 421-440) gives an interesting study of the theory of probabilities, discussing some of the philosophical problems as well as the calculus.

F. Enriques (*Rev. de Métaphys.,* 1917, XXIV, 149-164), in an article on the mathematical infinite draws a distinction between a *potential* infinite and an *actual* infinite. As an illustration he gives the set

$$\cdot 3,\ \cdot 33,\ \cdot 333, \ldots\ldots\ldots$$

Now, this ordered set can be given in two ways. (1) The general term consists of so many threes and the terms are ordered by the relation "less than." (2) Each term is obtained from the term before, the first term being $\cdot 3$. In the first case we are said to have an *actual* and in the second case a *potential* infinite. The real antithesis, however, seems to be that in the second case we have the terms given as the field of a relation which is one-one, relating only consecutive terms, whereas in the first case the terms are given as the field of a relation which is transitive and symmetrical, and thus relates any early term to any later one. In his brilliant work on the *ancestral* relation (see *Begriffsschrift,* 1879, Part III, pp. 55-87, *Grundgesetze der Arithmetik,* Vol. I, 1893, §§45-46), Frege has shown how to manufacture a relation having the formal characteristics of a series from a one-one-relation: "less than'" is the ancestral relation obtained from the relation between terms in (2). There seems to be no fundamental logical distinction with respect to the nature of the infinite as Enriques suggests. The question of the validity of the assumption of the existence of classes is discussed, but no mention is made of the work of Whitehead and Russell (cf. *Principia Mathematica,* Vol. I) which suggests a method of logical construction by means of which this assumption can be avoided.

A. Padoa (*Rev. de Métaphys.,* 1917, XXIV, 315-325) discusses the general problem of changing the primitive ideas in a deductive system. In an interesting article on the function of symbolism in mathematical logic (*Scientia,* 1917, XXI, 1-12) Philip E. B. Jourdain answers certain charges brought by Rignano against mathematical logic (*ibid.,* 1916, XX). He points out that, though the aim of symbolism in mathematics and logic has been until recently to make the process of reasoning mechanical by pointing out algebraic *analogies,* the aim in modern logic has been to increase the subtlety of our thought by emphasizing *differences.* An account of the work of Peano, Frege, and Russell is given.

In an article, entitled "The Organisation of Thought" in a book of the same title (London, 1917), Prof. A. N. Whitehead investigates the relation between science and logic, and describes some of the features of modern logic.

Prof. L. P. Saunders (*Mind,* 1917, number 101) in a criticism of Mr. Russell's *Lowell Lectures* discusses the nature of logical constructions. The subject is interesting to the mathematical logician, in that, on the validity of this method rests the validity of all but the most elementary parts of mathematical logic, for classes, relations, and numbers are all logical constructions. D. M. Wrinch (*ibid.,* 1917, number 104) attempts to answer some of the points brought up by Professor Saunders in his attack.

Raphael Demos (*Mind,* 1917, number 102) contributes an interesting article on particular negative propositions. He assumes that there are no negative facts and deduces that negative propositions must have reference to the world of positive facts. Particular negative propositions are said to be *ambiguous descriptions of positive propositions* and they are treated as Russell treats descriptive phrases (see *Principia Mathematica,* Vol. I). It seems, however, to the reviewer a little unwise to base a theory on such a disputable point as the non-existence of negative facts.

DOROTHY MAUD WRINCH.

CAMBRIDGE, ENGLAND.

MONISM AND DUALISM.

Are monism and dualism incompatible? or are they complementary aspects of existence? Is human experience such as to be satisfied by either one of these attitudes? or does it demand both?

Our point of view is determined, not immediately by being-in-itself, but by our experience of being. Therefore if man may experience being in two distinct ways he also may see being from two different points of view.

According to Plato's theory of knowledge man may know being either through sense-perception produced by the material manifestations of being, or through immediate intuition of unmanifested being. This thought reconciles monism and dualism. And the fact that it does so would seem to be good reason for accepting it.

Through sense-perception we know the material world. This world is so constituted that it generally leads us to a dualistic point of view. Only those who confine their attention to the physical side of nature come to look on the world as the manifestation of a single principle. Supersensible experience, on the other hand, may contain no duality, and therefore may lead naturally to a monistic point of view. If thus sense-perception and supersensible experience constitute two distinct spheres of experience which never combine in human consciousness, and which are separated by a gulf impassable by thought, it is quite proper that one of these spheres of experience be interpreted dualistically while the other demands a monistic point of view. The student of nature may be right in seeing the world as a manifestation of two principles. The monist, for whom perhaps the world does not exist while he sees being from the supersensible point of view, may also be right in seeing all as one.

The universe is made of forces. All these forces are probably alike. As far as we know the differences between things lie entirely in the difference of arrangement of the forces of which the things are made. The arrangement of the forces results from their local motion. When the local motion of the forces in a piece of wood is increased by throwing it into the fire, these forces cease to make a piece of wood and instead form themselves into smoke and flame and ashes. This local motion, in conjunction with the local arrangement which results directly from the motion, is called energy. Each thing in the universe is a distinct thing by virtue of the energy which it contains.

It is the nature of energy not to stay where it is put, but to dissipate itself. This peculiarity of energy has led to the formulation of the Law of Dissipation of Energy. According to this law energy is never transmitted without being at the same time dissipated. We also know that our sensations result from transmissions of energy, either from things to our bodies, or from our bodies to things. No concentration of energy, however great, could in the least affect our senses so long as the energy is locked up in the thing. Energy produces sensation only when it is transmitted from one thing to another. According to the Law of Dissipation of Energy this transmission results in dissipation. Hence it follows that we are able to perceive the universe only in so far as it is going to pieces. Some things, as the sun for instance, are perceived because they are going to pieces. Other things, as for instance a stone seen by the light of the sun, are perceived because another thing is going to pieces. However, the stone also is going to pieces, although at such a slow rate that that part of its own energy which it liberates produces no sensation. Every part of the universe is unceasingly going to pieces. This is the peculiarity of the universe which enables us to perceive it and to know it, and which, therefore, makes it our universe. This is the reason why we are sometimes tempted to think that the universe never does anything but go to pieces.

But when we reflect we find that the conception of a universe which is just going to pieces throughout infinite time is repugnant to the intellect. The intellect revolts against this impressionistic attitude and boldly infers that there is also creation going on in the universe. Creation consists in concentration of energy. According to the Law of Dissipation of Energy concentration of energy cannot result from transmission of energy. Therefore we are compelled to assume that energy is created at certain times, or possibly at all times.

All mechanical process requires that the elemental forces in it remain unchanged. So long as the forces involved in a process remain unchanged no energy can be created or destroyed in this process. This is the Law of Conservation of Energy. Hence it follows that creation of energy cannot result from the mechanical or physical process, but that there must be in the universe another kind of process in which energy is created, there must be a metaphysical process. We have thus found that while our senses are affected only by the mechanical or physical or material process our intellect demands also a metaphysical process. A mechanistic or materialistic monism can therefore never satisfy the human intellect.

Natural science has shown conclusively that all the immediately observable phenomena of the universe are mechanical. Therefore no genuine idealistic monism can explain the universe. A so-called monism in which the One is merely the owner of two active principles, one mechanical and the other ideal, is not monism. Such a standpoint is dualism in disguise. The actual world evidently arises from the interaction of two opposite principles. One of these principles is mechanical and destructive. The other principle is creative; and therefore it cannot be regarded as mechanical, but it must be regarded as intelligent. Therefore it is impossible to explain the actual world from a monistic standpoint. If our monism be mechanistic we fail to account for the creative process which is implied by the physical process and which is plainly indicated by all vital, social, and intellectual phenomena. If, on the other hand, our monism be idealistic we fail to explain the mechanism and deterioration and strife which we find everywhere about us. It is evident, therefore, that in so far as we study nature or consider any aspect of the world of sense we must adopt a dualistic attitude.

We have seen that the intelligent study of nature requires that we assume not only a mechanism but also a creative principle behind natural phenomena. Through sense-perception this creative principle can be known only in so far as it has manifested itself in nature. It is claimed that through supersensible experience further knowledge of this principle is attained. In order to be freely receptive to supersensible experience it is said the attention must be withdrawn from objects of sense and wholly centered on the supersensible. Hence, the duality of the world of sense has no bearing on our attitude toward supersensible being. In supersensible experience there is found oneness of principle. Such experience therefore leads naturally to a monistic point of view.

In supersensible experience evidently matter or mechanism does not manifest itself. The concept of matter therefore is useless in supersensible experience. Furthermore, if attention is given to the concept of matter this induces sense-experience; and sense-experience excludes supersensible experience. The first step toward supersensible experience therefore is to renounce the concept of matter. And in renouncing the concept of matter we naturally drop into a monistic attitude. Incidentally we also renounce the material world and come to regard the actuality of sense as nothingness. The actuality of sense is then superseded by another actuality —the actuality of the supersensible. Thus monism becomes the point of view of supersensible experience.

Supersensible experience may develop in two different directions. It may follow an intellectual trend and bring insight, or it may consist in practical contact with being. When supersensible experience develops along intellectual lines we call it revelation, or mysticism. Practical contact with supersensible being is practical religion.

The substantial agreement of the sacred books of different peoples and different ages indicates that these books are all to a great extent genuine records of experience. The sacred books practically agree on the fundamental points of supersensible knowledge. These books teach that being is one in principle but infinitely varied in content; that being is neither movement in space nor a process in time, i. e., not material, but eternally perfect Intelligence, Justice, Harmony, Beauty, Love, Bliss; that Being is free from all evil; that it is the absolute Good.

Supersensible experience is described as a peculiarly intimate contact with being, a contact so close that it amounts to practical identity. The Mundaka Upanishad says, "He who knows the highest Brahman, becomes even Brahman." And the Prasna adds, "Yes, O friend, he who knows it, becomes all-knowing, becomes all." Plotinus says, "To see and to have seen that Vision is reason no longer, but more than reason, and before reason and after reason, as also is that Vision which is seen. And perchance we should not speak of sight. For that which is seen—if we must needs speak of the Seer and the Seen as twain and not as one—that which is seen is not discovered by the seer nor conceived by him as a second thing, but, becoming as it were other than himself, he of himself contributeth nought, but as when one layeth center upon center he becometh God's and one with God. Wherefore this vision is hard to tell of, for how can a man tell of that as other than himself, which, when he discerned it, seemed not other but one with himself indeed? And it may be that this was not vision, but some other manner of sight, aye, an ecstasy and a simplicity and a self-surrender, and a still passion of contact and of unison."

This statement of Plotinus calls attention to one of the chief objections raised against the monistic standpoint, namely that the ego seems to go into solution in the One. This is an illusion resulting from a confusion of the subject with the object. The object only is one. The subject remains many. Many subjects may know the object as one and thus be the One in an objective sense while remaining many subjects. The fact is that the genuine monistic attitude intensifies the actuality of the ego. The ego is never more of an actuality than when it succeeds in wiping out the world of sense, including all objective personality, and is maintaining its identity with the One. Absorption in Brahman, or Nirvana, or union with God is not a passive state imposed on the submissive ego by the One. On the contrary, in order to maintain this union the ego must make an exertion far greater than is required in the most strenuous of worldly activities. The extinction of sensible personality results in an awakening to supersensible personality. The change which takes place therefore is essentially a shifting of the scene of consciousness. The subject remains the same subject.

The initial step in all conscious action is the recognition of the idea as a superactual existence, as a form-giving power which may enter the actual world and impress its form upon it. In action we recognize the idea as a creative agent possessing the power to break into the mechanism of matter and add an element to it so that henceforth this mechanism is no longer the same, but a new and different mechanism. This recognition consists essentially in a tacit declaration that the idea is being. This turning from the actual to the ideal is the essence of morality. An act is moral in so far as it springs from a clear recognition of the idea as the creative principle in the world. This recognition implies the complementary concept of matter as the inert medium which the idea moulds into its own form, i. e., as a passive mechanism controlled by an active intelligence. The standpoint of morality, therefore, is dualistic.

When, instead of merely recognizing the idea as revealed in the world of sense, man turns in action altogether away from the world of sense and recognizes only being as revealed in supersensible experience his action becomes practical religion. The viewpoint of practical religion is therefore monistic. Religious action, like mysticism, demands entire renunciation of the world of sense including all sensible personality. This renunciation consists in a withdrawal of attention. This withdrawal of attention is accomplished by means of active denial of the existence of the world of sense. The attention is then centered on the revealed concept of being as found in the sacred books. The results which follow from this turning from the sensible to the supersensible form a body of facts which demonstrate the truth of revelation. These facts are accessible to all who choose to make the experiment of denying the world and recognizing the All-one. Thus through religious action there is opened an unlimited field of practical intercourse with the supersensible. And through this intercourse an intimate practical acquaintance with God may be attained even by those who have never entered the state of mystic experience.

We live in a dualistic world. By rising above the world the problem of the world is solved for us.

Ernst Jonson.

New York City.

GALILEO AND NEWTON.

It is perhaps advisable to consider Newton's debt to Galileo both in mechanics and mathematics rather more fully than I have done in my previous articles on Newton and the principles of mechanics in this magazine. I will try to follow the thread connecting the thoughts of these men in what follows.

One of the most striking results of Galileo's *Discorsi,* published in 1638, is that a motion does not need a force to keep it up. This fact is concealed by the constant presence around us of friction and resistance, and so even Descartes had imagined that each planet is kept in motion by a vortex in a fluid which fills all space. Galileo found, in fact, that a force changes the *velocity* of a body. Thus, without a force which would deflect a planet into a curved orbit, the planet would proceed with uniform velocity along a straight line. Also, when considering the paths of projectiles, Galileo showed that the paths arise from compounding a horizontal uniform velocity and a vertical accelerated motion of falling. The resultant of two motions at right angles to one another is a motion along the diagonal of a parallelogram whose adjacent sides are along the two directions of motion. By considering the various points of arrival for horizontal velocity and vertical acceleration of fixed magnitudes but for various periods of duration, Galileo found that all the points lie on a parabola.

Both these advances made by Galileo were apparently quite familiar to such physicists as Huygens, Wren, Hooke, and Halley. Huygens made a very important application of Galileo's ideas to motion in a circle round a force at the center of the circle. This is, for example, the case of a stone placed on a smooth horizontal table, so as to neutralize the effect of the stone's weight, and then whirled round at the end of a string of which the other end is kept still. In this case the stone is continually deflected from a tangential path by a pull along the string to the center of the circle. We know that Newton discovered quite independently in 1666 the main result of Huygens by a method which was very probably rather different from that of which Huygens published some results in 1673. Huygens noticed that, if a circle of radius r is described with uniform velocity v, any new acceleration must, since there is no change of velocity in the orbit of the body, be at right angles to the circle at the point where the moving body is. There is thus a continual acceleration due to the force acting toward the center of which the "centrifugal force" along the tangent is the accompanying "reaction," as Newton afterward called it, and this acceleration is used up in continually changing the direction of the velocity without bringing the body any nearer to the center of the circle. If no force acted between the center and the body, the body would proceed with uniform velocity v in a straight line PT touching the circle at P, and in a time t would travel a distance vt along this line. Again, if the body at P had no velocity along PT, it would, in the time t, by Galileo's formula, travel a distance $at^2/2$ toward the center, where a is the acceleration. Of course in this case we cannot imagine the force toward the center to be transmitted by a perfectly inextensible string, for unless the string were kept stretched or the time t made infinitesimally short, there would be no force along the string. The actual motion of the body is compounded of both these motions taking place at the same time, and, if we consider t as infinitesimal, the resultant path may be considered as a straight line PQ which coincides with an arc of the circle, so that Q is a point on the circle infinitely near P. The distance TQ, which is parallel to the line joining P to the center (S) represents the distance fallen through toward the center in the short time t. Since P and Q are on a circle, the lengths PT (y), TQ (x) are connected by the relation

$$y^2+(r-x)^2=r^2$$

or, what is the same thing, by the relation between y^2 and $x(2r-x)$ proved in Euclid's *Elements* (III, 36). The above relation gives, when we substitute for y and x in it from what precedes and neglect powers of t higher than the lowest,

$$v^2=ar.$$

From this we get Huygens's formula for the magnitude of the central force,

$$a=v^2/r.$$

It may be mentioned in passing that there is some analogy between this investigation and Galileo's investigation of the path of a projectile. If the projectile starts from P with a velocity v along PT and travels a distance vt in the time t, and at the same time suffers an acceleration in the direction PS by which in the time t it describes a space $gt^2/2$, it arrives, by the end of the time t, at the point Q, where

$$y^2=v^2t^2=2v^2x/g.$$

The difference lies in the fact that in this case t need not be infinitesimal. We may say that, in the former case, the curve *begins* by being a parabola, but that the direction of the force only remains parallel to its original direction at an infinitely small distance along the curve from P, where begins the motion that we are considering.

When the empirical laws of Kepler became generally known, it seems that they were combined with Huygens's theorem by Halley and possibly others as well as by Newton, and this combination gave rise to speculation on the orbits of the planets. Indeed, it seems natural to consider Galileo's parabola of projection as passing over into an ellipse or a circle when the center of attraction is brought from infinity to a finite distance. It is probable that Hooke had no more grounds for his assertions that the force of gravitation varies inversely as the square of the distance and that this law proves that the planets move in ellipses, than this plausible analogy together with the discovery—made also by Newton in 1666 and Halley at a later date—that Kepler's third law and Huygens's theorem between them imply that the force keeping a body revolving in a circle about a larger one, as is approximately the case with the planets and the sun or the moon and the earth, is inversely as the square of the distance.

Another aspect of Galileo's works was of especial importance in influencing Newton probably through Barrow, and this was in a purely mathematical direction. Galileo, in his attempt to find the law according to which a body falls near the surface of the earth, neglecting the resistance of the air, assumed that the velocity acquired by the end of a certain time during which the body falls is proportional to the length of the time of falling. This assumption turned out to be correct, and a previous mistaken assumption that the velocity acquired is proportional to the *space* fallen through will be referred to below. Since it was easier to find out by experiment in what way the *distance* fallen through increased with the time rather than in what way the *velocity* increased with the time, Galileo deduced, from the assumption that v is proportional to t, the relation between s and t. It must be remembered that Galileo was perfectly familiar with the ideas which were expressed in the methods of indivisibles of Kepler and Cavalieri. Galileo considered, unlike his predecessors, velocities varying from point to point, and consequently saw that we could not define "*the* velocity at a point" by the ratio of the space passed over in a finite time to that time, for different lengths of time would give different results. When, however, we consider, round a certain point, a distance which is infinitely small and therefore very nearly a straight line, then for the infinitely small time in which this space is described, we may regard the increase or decrease of space as uniform. This new notion of "velocity" as the (unique) ratio of infinitesimals includes the old one as a particular case; for if the ratio of ds to dt, as we may write these infinitesimal increments in the notation subsequently introduced by Leibniz, is constant, then s is proportional to t.

Galileo represented the lengths of time by lengths on an axis of abscissæ measured from a fixed point on it, and the magnitudes of the corresponding velocities by ordinates. In this diagram, which is like the diagrams introduced into geometry by Descartes soon after Galileo's ideas were formed, except that x and y replaced t and v, Galileo's assumed proportionality of v to t is represented by a straight line through O. That is to say, if P is any point on the t-axis and PQ is the corresponding ordinate at right angles to OP, then, when P varies in position, the ends Q of all such ordinates lie on the above straight line. Now Galileo proved that the triangular area OPQ represents, in units of square measure, the space (s), in units of linear measure, fallen through. Just as the velocity when variable can be measured, at any point, by ds/dt, the slope of the curve at that point, on a diagram in which times are abscissæ and spaces ordinates, so on a diagram in which times are abscissæ and velocities ordinates, the acceleration is measured by dv/dt, the slope of the curve. It can hardly be doubted that Galileo, knowing how areas of curves are found by the method of indivisibles, saw that when the acceleration is *variable,* the area of the figure OPQ, which is no longer a triangle, still represents s. It is also quite possible that Galileo saw in this way the inverse relation of the problems of tangents and quadratures: the ordinate (v) of any point on the st-diagram is given by calculating the corresponding area on the vt-diagram, while the ordinates on the vt-diagram are determined by the slopes or tangents at the corresponding points of the st-curve. Here we may mention that, in Galileo's mistaken assumption referred to above of the proportionality of v to s, triangles like OPQ on a vs-diagram do not represent s. It will be remembered from Mach's *Mechanics* that Galileo rejected this mistaken assumption on grounds which were also mistaken. In fact, if $v=ds/dt$, the integral of $v.dt$ is s, but the integral of $v.ds$ is not s unless v is always unity, so that a vs-diagram does not show that integration is the inverse of differentiation. The notions of time being the independent variable and geometrical curves being generated by motions were used by Isaac Barrow, who was certainly influenced by Galileo and possibly by Roberval. Barrow denoted the areas by "t" and "v," and in this notation and the ideas which it implied he was followed by his pupil Isaac Newton, who, in his "method of fluxions," greatly developed the suggestive ideas of Barrow, especially Barrow's clear perception of the truth that the problems of tangents and of quadratures were inverse problems.

Philip E. B. Jourdain.

Fleet, Hants, England.

CURRENT PERIODICALS.

We hope that the increasing bulk of *Science Progress,* this "Quarterly Review of Scientific Thought, Work and Affairs," is the outward and visible sign of an increasing recognition of its interest and value. The articles in the April 1918 number are of the best and by the best. The "Essay-Reviews," the as-yet-unimitated-and-own-peculiar creation of *Science Progress* (to use the hyphenated form which gave to De Morgan many a chuckle), which combine with effect the brevity of the ordinary review and the majestic *longueurs* of the quarterly, give their writers an opportunity of using the book under discussion as a peg upon which to hang a good deal that is both relevant and urgent. The section entitled "Popular Science" has not perhaps assumed its final form. If it is a matter of popularization, it seems perhaps somewhat out of place in a periodical written by men of science for their brethren and who can manage to assimilate more solid stuff than the name would imply. On the other hand, there is a place for it if it were made to serve the function of a gibbet, so that the world at large might know the kind of nonsense that is dignified in the daily press and the magazines with the honored name of science. The "Notes" have in general a peculiar fragrance of their own, and it is easy to see in them the imprint of the wayward genius—is not all genius wayward, or is the epithet "unscientific"?—who is poet, mathematician, and savant, and who has had every reason to be out in revolt against the malignant stupidity which has characterized the attitude of the governing classes in Britain to those who have been making life possible for the white man in vast territories of the Empire, and who have been doing the work that will count the most in aid of human effort in every field of nature during the next half century. The ordinary reviews are to be counted among the best of their class. All these sections are to the good, most of them are helpful, and if irony does not always bring about the revolution desired, it plays a useful part in titillating the jaded, and in giving voice to those who have been shamefully inarticulate. But of all the features in *Science Progress* the one that strikes us as the most valuable are the forty pages or so entitled "Recent Advances in Science." There is, as far as we can recollect, nothing comparable to this section in scientific literature. Other attempts to keep the student in touch with what is going on in the world of scientific research are rather in the nature of catalogues with, at most, the shortest of summaries. But the summary is really critical, that is not mechanical but selective, that, where need be, reviews past and present and casts a penetrating eye on the future—that is what *Science Progress* gives us, and is as far as we know without a parallel. It is the exceptional value to the writer of this notice of the pages on "Recent Advances" in two of the branches dealt with month by month in this periodical, that leads him to believe that this section is of unique and permanent value.

We have left ourselves no room to deal in detail with the contents of the April number, and must content ourselves with observing that it is brimful of interest from cover to cover, and that "R. R." seems to be here, there, and everywhere, and racier than ever.

BOOK REVIEWS AND NOTES.

The Distribution of Attention. By *E. Neil McQueen. The British Journal of Psychology,* Monograph Supplements, No. 5. Cambridge University Press. Pp. 142. Price 5s. net.

This monograph is a thesis approved for the degree of doctor of science in the University of London, and in it the author sets out to discover whether there is any general power of distributing the attention. In dealing with the study of individual differences, a considerable number of psychological writers have assumed that individual minds may be classified according to types (phlegmatic or choleric, sensorial or muscular in reaction, etc.), the implication being that a given mind has a power of functioning in a constant fashion, without reference to the character of the particular task performed. But this opinion is dying out, and the psychologists are increasingly inclining to the view that the individual's reaction varies in nature according to the type of performance. It is the author's purpose in this monograph to dispose of the assumption in question with reference to the topic of the distribution of attention, and by the citation of experiments of his own, to prove that the distribution of attention in the case of any particular person is a variable factor and that consequently there is no general "span" of attention.

The author begins with a critical review of previous work, making mention of tachitoscopic experiments intended to measure the range of visual attention and other experiments in the field of successive auditory impressions. Reference is made to an interesting experiment by Binet in which the latter instructed his subjects to press a rubber tube according to a certain rhythm which they were to endeavor to maintain while doing something else, as for example, reading aloud, or calculating mentally; Binet believes that the number of impressions which an individual can perform on the tube while doing the other mental work will give some indication of the range of his field of consciousness. The author brings the following criticism to bear on the reasoning usually applied to this kind of experiment. First, that no method is indicated of measuring the two processes simultaneously performed, so that it is impossible to compare the span of the individual who, while recording correctly six impressions on the tube, yet takes ten seconds to perform an addition and then gets it wrong, with that of an individual who performs the addition correctly in five seconds but fails to record correctly more than two impressions on the tube. And, secondly, that no measurement is made of the individual's ability to perform singly each of the two processes performed simultaneously during the experiment, whereas it is clear that an individual who excels in either performance done alone will also do well in the performance when done simultaneously with another. Thus, the experiments are criticised by the author as inconclusive, because they have not been carried out so as to exclude alien factors or in a way to secure precise measurement of results.

The author then relates his own experiments carried out, as he says, to obtain a measure of the ability of the subject for the "undistributed" performances (i. e., of the performances carried on singly), to eliminate the effect of these differences of ability in the undistributed performances, and to see if any evidence were forthcoming of the existence of any general factor of distribution of the attention common to all the "distributed" performances.

In the process of the experiment, the subjects perform a certain pair of tasks, both singly and simultaneously, and in each case their efficiency is measured and recorded respectively; they perform not one pair merely but a number of different pairs of tasks (tapping and adding, card-sorting and counting by 3s, etc.). With these results, the investigator was able to calculate the ability of the subject for the undistributed as well as for the double performance in each case, and, finding that there is an appreciable correlation between the undistributed performances, reasoned—correctly—that the factor which produced this correlation would operate to produce a high correlation between the corresponding distributed performances. But, he argued, if there is a general power of distribution, there must be a further factor owing to the operation of which, the correlation between the distributed performances should be higher than that between the undistributed performances.

Testing this deduction from the hypothesis of a general power of distributing the attention, with the experimental results, the author discovered that the averages for the intercorrelations of the distributed and undistributed performances were nearly equal and hence concluded that there is no evidence of a general power of distribution. From other experiments, which consisted in letting the subject perform repeatedly tasks of the same sort but with varying speed, the author ascertained that the distribution of attention varied as the speed varied, a fact supporting the view of the specific nature of the factors involved in any distribution of the attention.

In succeeding chapters the author deals with a variety of possible deductions from the results of his experimental researches and concludes by considering the vexed question whether it is possible to attend to more than one thing at a time independently. One must remember that what appears as a plurality of objects in practice, may be psychologically a single object; other alternatives in the case of the performance of two tasks simultaneously are an alternation of the attention from the one to the other, an automatization of one of the processes and its consequent relegation to the field of consciousness, and simultaneous attention to the two tasks, both being present independently in the field of attention. Introspective evidence obtained under experimental conditions is brought forward in proof of the occurrence of all these possibilities, including the last. It is rather difficult to understand how one should be able to attend simultaneously to two tasks as separate, but the author takes the sting out of his conclusion by admitting that the conscious effort, at any rate, is directed to only one of the tasks.

It has been possible to give only a brief and very inadequate summary of Mr. McQueen's account of his own experimental researches and of his criticisms of the work of others in the same field. The investigation is strictly scientific and experimental, and the author proves himself resourceful in devising experiments calculated to yield precise results. The exposition is as clear as the subject will allow, and the whole account evidences a happy combination of the empirical-experimental habit of mind with the ability for coherent thinking. The results of Mr. McQueen's investigations should inflict one more blow upon the fast crumbling structure of the old superstition which regards the mind as a substance endowed with general powers as such, instead of as a thing analogous to an organism, whose action is a function in each case of the particular situation in which it finds itself. Δ

HERBERT SPENCER. By *Hugh Elliot.* "Makers of the Nineteenth Century" series, edited by *Basil Williams.* London: Constable and Co., Ltd., 1917. Pp. x, 330. Price, 6s. net.

This volume is a well and clearly written account of all that is of any importance in the life and work of Herbert Spencer (1820-1902). Mr. Elliot is by no means thoroughly in sympathy with either the character or the doctrines of that singularly estimable and unattractive man, who was always admirable except when lack of humor made him ridiculous; although it would seem that at first Mr. Elliot was a very ardent admirer of Spencer's writings. It was certainly much the best plan to give, as Mr. Elliot has given, by far the larger part of the book to a description of the writings of Spencer, which are very numerous for one who was "by nature an exceedingly idle man" p. 61). Mr. Elliot's opinion is that the main interest of Spencer's works at the present time is on the social side. "His scientific and evolutionary writings have already become part of the 'atmosphere' of modern thought, in the sense that they scarcely need to be taught, but constitute the foundation upon which more recent ideas are built. But his social writings have not in the same way become axiomatic...."(p. 2).

There are two fundamental ideas at the root of Spencer's philosophy. The first is that of universal evolution. Early in life Spencer "endeavored to find some law which should describe the tendencies of the constant state of change in Nature,—a law which should be equally applicable to the change of a nebula into a star or stellar system, and of a protozoan animal into a man. This law he called the Law of Evolution. It proposed to describe the various stages characteristic of all progress in all departments of Nature as the universe grows older. He believed that the outlines of such changes were similar throughout all varieties of the changing substance" (p. 79). The second fundamental idea is the guiding principle of his social and political writings, and is that social progress consists in "the admission of every individual to the *maximum* freedom consistent with social order and security" (p. 80).

In the statement of the principles of his philosophy Spencer was very prone to those sonorous definitions which seem to impress some people as evidences of great power and profundity. With their help Spencer brought about an apparent transformation of sciences into a form in which practically everything that seemed, to those people who combine interest in science with lack of scientific ability, to matter, could be logically deduced from them. Examples of such verbiage are to be found in the quotations on pages 244 and 254, for example. However, on a closer logical inspection it can be seen without much trouble that the part played by strict deduction in Spencer's work is usually an illusion. Such a mathematician as Laplace never fell into the Spencerian errors in framing his nebular hypothesis, and consequently Laplace's theory certainly looks, at first sight, less complete than Spencer's. But Spencer only arrived at apparent completeness owing to mathematical ignorance on his own and his readers' parts, and by unhesitatingly taking steps which no trained logician or mathematician would venture to call "deduction." Also no mathematician would do such a ludicrous thing as to "deduce" the motion of more than one body from the principle of the conservation of energy; and yet Spencer does something very much like this when he makes certain conclusions follow from the "Persistence of Force." Thus we cannot agree with Mr. Elliot when he says (p. 57) that Spencer's "philosophy is in many respects Euclidean in form....," unless of course the phrase is meant to be taken in a vague and popular sense, much as some people say of a child who does not cry when he is hurt that he is a "philosopher." The interesting remarks on Spencer's capacity for generalization (p. 62) seem, then, to lay stress on the point that often Spencer was led astray by what he fancied to be analogies.

It is hardly worth while nowadays to insist on the failure of Spencer's attempts to write about metaphysics. Mr. Elliot has said a few words on this subject on pp. 216ff, with most of which it is impossible not to agree. On this subject Spencer's remark, of which he actually seemed proud, that he could not read Kant because he saw he did not agree with him, is rather relevant.

There is a very good portrait of Spencer prefixed to this volume, after the painting in the National Portrait Gallery by J. B. Burgess. ϕ

A Philosophical System of Theistic Idealism. By *James Lindsay, D.D., M.A., B.Sc., F.R.S.E.,* etc. Edinburgh and London: Wm. Blackwood and Sons, 1917. Pp. xi, 530. Price 12s. 6d. net.

Dr. Lindsay claims that his work forms a system of Theistic Idealism, and that the system is intelligible, self-consistent and contravened by no known fact. The book is critical and constructive; its criticism extends to thinkers British, American, German, French, and Italian. Its contents are: Chapter I, "Foundations of Idealism: Laws and Logic of Psychology"; Chapter II, "The God of Theistic Idealism"; III, "The Metaphysics of Creation"; IV, "The Metaphysics of Time and of Eternity"; V, "History and Providence in Theistic Idealism"; VI, "The Philosophy of Nature"; VII, "The Philosophy of Science"; VIII, "The Philosophy of Art"; IX, "Freedom in Theistic Idealism"; X, "The Moral Order, and the Spiritual World, in Theistic Idealism"; XI, "Immortality in Theistic Idealism." Also, a very full Index—twenty-eight columns—of authors and subjects.

The work falls, according to its author, under universal philosophy, and makes appeal to all the philosophical disciplines. It may be of especial importance to note, for the present Journal, that he strongly stands for spritual monism, in which the human consciousness is not simply merged or absorbed in the Divine consciousness, or treated as a mere "part" of it, but is brought into voluntary ethical union with it. The various current philosophical theories are critically dealt with, from the standpoint of Theistic Idealism. The scope and nature of the work cannot be fully described in this place, but must be gathered from study of the work itself. λ

THE ASCENT OF OLYMPUS. By *Rendel Harris.* Manchester: The University Press; New York and London: Longmans, Green & Co., 1917. Pp. vii, 140. Price, 5s. net. [Separately printed: THE ORIGIN OF THE CULT OF DIONYSUS, THE ORIGIN OF THE CULT OF APOLLO, THE ORIGIN OF THE CULT OF ARTEMIS, THE ORIGIN OF THE CULT OF APHRODITE. 1s. net each.]

Dr. Rendel Harris is an expert in that dizzy art of derivation and interpretation to which, in recent studies of comparative religion, we have become accustomed. He has every qualification of scholarship, ingenuity, and plausibility, and he makes his detective work exceedingly interesting. We are familiar with the evolution of the gods out of snakes; Dr. Harris now evolves them out of plants—plants associated, chiefly, with the thunderbolt. The oak is the thunder-tree, being struck more often than any other. Athene is born from the head of Zeus; she is also the owl, i. e., sprung from the hollow tree. From a similar association with riven trees, bees and honey acquire sanctity. "The animistic belief makes everything that thunder touches into thunder." Not only the oak, therefore, but mistletoe and ivy, which cling to the oak, are thunder. And "when the phytomorph becomes the anthropomorph, the name of the new (subordinate) thunder-deity is Dionysus" (p. 5). (The lightning-smitten Semele, his mother, is nothing but the tree.) The association of Dionysus with wine follows naturally: the sacred ivy-leaves were chewed and eaten by the worshipers. Smilax and grape-vines were trained on trees. The goat and fawn, which feed on these plants, became cult-animals. The appearance of an androgynous Dionysus is due to his identification with *both* fire-sticks, the "male" and the "female"; and the fire-sticks were made out of ivy because there is thunder in the ivy. Dr. Harris thinks that the Vedic Soma may have been a surrogate for a more primitive sacred mastication, analogous to the ivy-leaves chewed by Mænads, and suggests a similar source for the custom of drinking ivy beer on Ascension Day at Lincoln College, Oxford.

Apollo is likewise a thunder-god, the laurel being substituted for the oak, and Apollo owes his healing art to his connection with the mistletoe, a plant of supposed medicinal virtues. Artemis is identified with the mugwort, and Aphrodite with the mandrake or love-apple.

If Dr. Harris is ever found to be wrong, it will be because he clings tenaciously to a single (vegetable) line of descent. This line is certainly traceable, but it seems possible that a developed god may have an extremely complex parentage, not to be reduced to a single root. η