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Some Tests and Comparisons

George M. Frankfurter
Thomas J. Frecka

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Naive Diversification and Efficient Portfolios --
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George M. Frankfurter
Syracuse University

Thomas Frecka, Assistant Professor
Department of Accountancy

Abstract

This study reports the results of an experiment which compares and contrasts the empirical properties of naive portfolios and Markowitz-Sharpe efficient portfolios. On an ex post basis, it is shown that the M-S investment strategy resoundingly outperforms a strategy of investing in naive portfolios of the same size as the efficient portfolio.

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## 1. Introduction and Overview

The purpose of this study is to compare and contrast the ex-post,
as well as the ex-ante characteristics of naive and Markowitz-Sharpe
(M-S) efficient portfolios. We offer the term "naive portfolio" to de-
scribe a strategy that, based on the normative implications of the
broadly accepted capital asset pricing model [12, 17], calls for invest-
ment in equal proportions in a predetermined number of randomly selected
securities. In contrast, we define "M-S efficient portfolios" as those
members of the convex set of portfolios that can not be dominated by
any other portfolio in risk-return space.

While the ex-ante characteristics of M-S efficiency were the kernel
of Markowitz's [13] early work, the theoretical underpinnings of naive
diversification were not explored until 1976, 1977. Vertes [19] in 1976,
and independently, Elton and Gruber [4] in 1977, derived the risk-return
tradeoffs of a naive portfolio cast in terms of simple random sampling
without replacement (hereafter, SRS WO/R). Prior to the theoretical
derivation of Vertes and Elton and Gruber, in several papers [5, 6, 10,
11, 18] the empirical ramifications of a naive strategy were broadly ex-
plored. It has been shown in these works, that risk reduction through
increased diversification at the margin is very close to nil after a
relatively small portfolio size. Empirically, after 10-20 securities,
while the expected return is invariant to size, risk as measured by the
variance of the portfolios, is being reduced only fractionally.

By contrast, Vertes and Elton and Gruber derive the analytical
properties of naive diversification as a function of size. In the

following, we adopt the notation and derivation of Vertes because of its simplicity and because of its generality.

In all the works that we have mentioned thus far as well as other works, to the best of our knowledge there were no comprehensive comparisons between the M-S strategy and the naive strategy on an ex-post basis. While it has been shown that ex-ante the M-S strategy is superior to any naive strategy regardless of portfolio size [19] it also has been shown that it is subject to very serious biases in terms of overstatement of expectations and understatement of risk [2, 7, 8]. However, the ex-post relative performance of the two strategies has yet to be explored.

In this paper we report the results of an experiment which compares and contrasts the empirical properties of the two models. The paper is organized in the following fashion: In Section 2 the ex-ante attributes of a naive strategy are considered and contrasted to the same properties of the M-S efficient portfolio. In Section 3 we describe the logic for the selection and construction of an index which is later used in obtaining M-S efficient portfolios and for ex-post testing. Section 4 is devoted to an experiment which culminates in an Analysis of Variance Bi-Factorial design, comparing naive portfolios with M-S efficient portfolios. Conclusions and a summary are offered in the last section of this paper.

2. Ex-Ante Attributes of Two Strategies

According to Vertes [19], and to Elton and Gruber [4], a naive strategy (equal investment in n randomly selected securities), can be

looked upon as a two stage sampling strategy from a universe of N securities for all $n \leq N$, by making the following assumptions:

(a) Security returns are generated by a multivariate process with finite first and second moments such that

$$r_{it} = \frac{P_{i,t+1} - P_{i,t} + D_{i,t+1}}{P_{i,t}} \times 100 \quad \begin{array}{l} i = 1,2,\ldots N \\ t = 1,2,\ldots T \end{array} \tag{1}$$

where:

  $P_i$ is the price of security i
     and
  $D_i$ is the dividend on security i.

(b) The parameters of this process are known ex-ante.

(c) The actual return on a naive strategy depends on the outcome of two sampling procedures: (1) the specific universe and (2) that set of n securities which is selected from N.

(d) As a result of (c), the statistic which results at the second stage is conditional upon the first stage.

Therefore the return on a naive portfolio $r_{pt}$ equals $\overline{r}_t$ which is the sample mean of a vector $[r_{1t}, r_{2t}, \ldots r_{Nt}]$ taken as SRS WO/R.

Also:

$$\overline{R}_t = \frac{1}{N} \sum_{i=1}^{N} r_{it} \text{ and } \sigma\overline{R}_t^2 = \frac{\sum_{i=1}^{N} (r_{it} - \overline{r}_t)^2}{N} \tag{2}$$

which are the statistics of the first stage.

The outcome at the second stage that is conditional on the first stage is:

$$E[\overline{r}_t | (r_{1t}, r_{2t}, \cdots r_{Nt})] = \overline{R}_t \tag{3}$$

$$\sigma^2[\overline{r}_t | (r_{1t}, r_{2t}, \cdots r_{Nt})] = \frac{N-n}{N-1} \frac{\sigma^2 R}{n} \tag{4}$$

Our interest is, however, in $E(\bar{r}_t)$ and $\sigma^2(\bar{r}_t)$.

It has been shown by Vertes that

$$E(\bar{r}_t) = E(\bar{R}_t) = \frac{E_1+E_2+\ldots E_N}{N} = \bar{E} \tag{5}$$

That is, the expectation of the sum is the weighted sum of the expecta-tions, and hence, the expected return on a sample of n securities is in-variant to n, given N. [16, p. 44].

It has been also shown [ibid, p. 47], that

$$\sigma^2(\bar{r}_t) = \frac{\bar{V}}{n} + \frac{n-1}{n} \cdot \bar{C} + \frac{N}{N-1} \cdot \frac{1-f}{n} \cdot \sigma_E^2 \tag{6}$$

where

$\bar{V}$ is the average variance $= \dfrac{1}{N} \sum\limits_{i=1}^{N} \sigma_i^2$

$\bar{C}$ is the average covariance $= \dfrac{1}{N(N-1)} \sum\limits_{i \neq j}^{N} \sum^{N} \sigma_{ij}$

f is the sampling ratio $= \dfrac{n}{N}$

and $\sigma_E^2$ is the variance of the individual means vis-a-vis the grand mean and is equal to:

$$\sum_{i=1}^{N} \left( \frac{E_i^2}{N} - \frac{\left(\sum\limits_{i=1}^{N} E_i\right)^2}{N^2} \right) .$$

That is, the variance of the strategy is <u>not</u> independent of the port-folio size, but is a decreasing function of n. At the limit, it is simply

$$\frac{\bar{V}}{N} + \frac{N-1}{N} \cdot \bar{C}, \tag{7}$$

when all securities are included in the sample.

The ex-ante mean-variance properties of M-S efficient portfolios are well known. They simply establish the convex set of combinations of securities which for a given level of return have minimum variances. The level of diversification is increasing from the highest return portfolio (which obviously includes one security only) to some point on the efficient frontier, after which it is declining again.

Vertes gives some numerical comparisons between the two strategies for universes of size 400 and 600 respectively (where the 400 member universe is a subset of the 600 member universe).

Keeping the expected returns constant Vertes shows [ibid, p. 33] that the total variance of naive portfolios of size 1 is 107.98% while the portfolio that includes all 600 securities has a variance of 28.44%. At the extremes the composition of the total variance is also interesting. For portfolios of size one, the covariance affect is zero, most of the variance is due to the individual security variances, and the mean effect is very small. For the total universe, the variance is almost exclusively due to the covariance effect. The mean effect is zero, and individual security variances effect very little total portfolio variance. Also, it confirms several empirical studies' results regarding the marginal contribution of diversification. After portfolios of size 16, the reduction of total variance is minimal. By contrast, the variance of the M-S efficient portfolio for the same expected return is 4.90% and it includes 34 securities (in unique proportions) from the universe of 600 securities.

These ex-ante attributes say very little about ex-post performance. While, given the assumptions, it is safe to assert that the naive

strategy provides unbiased estimates of ex-post results, the M-S strategy is biased upward; its expected return is overstated while the variance is understated.

For investment decision making the ex-post verification of a given strategy is of utmost importance. Merton [14] has shown that if the assumptions of the CAP model are correct, the market portfolio will coincide with the M-S efficient portfolio that is tangent to the ray originating at the point of the risk free rate on the expectation axis. This theoretical equivalence is of no concern to the portfolio manager. His interest is strictly in the ex-ante promise and in how well this promise is being kept. Therefore, without entering the much debated and often controversial issue of ex-post descriptive validity of the CAP model, it is our intention in the following to compare the ex-post performance of the two strategies.

To facilitate parsimony, and in order to keep this work in reasonable bounds regarding computational requirements, we opt to select the Sharpe single index simplified model of portfolio analysis [16]. Since the "common underlying factor," often referred to as the index of this model is of paramount importance, the next Section is devoted to the description of the index we use in Section 4 in our experiment.

3. The Index of the Market Model

Our data consists of the CRSP file of monthly returns of the New York Stock Exchange for 625 months starting with 12/31/1925 prices. The file includes indexes which are specifically built for the universe. One index is market value weighted while the other is

equally weighted. Both are calculated with and without dividends. The
Table below shows the expectations, the variance-covariance matrix and
the correlation matrix of these 4 indexes.

Table 1
Key Values of the CRSP Indexes

| | Market Value Weighted Dividends | | Equal Value Weighted Dividends | |
| | Included | Excluded | Included | Excluded |
|---|---|---|---|---|
| monthly means | .0085 | .0048 | .0124 | .0090 |
| variances | .0034 | .0035 | .0045 | .0045 |
| covariances | -- | .0035 | .0045 | .0045 |
| | -- | -- | .0067 | .0067 |
| | -- | -- | -- | .0068 |
| correlations | 1.0000 | .9992 | .9326 | .9313 |
| | -- | 1.0000 | .9331 | .9326 |
| | -- | -- | 1.0000 | .9997 |
| | -- | -- | -- | 1.0000 |

A major problem with these indexes is that they have variable mem-
bership. That is, while for the first data point the index consists of
501 securities, for the last it consists of 1546 securities. In between,
the number varies within these two extremes. As indexes of comparison,
they serve a useful role. As a component of the "market model" however,
they are not the best choice for "the common underlying factor" [16].
Since the composition of these indexes is constantly changing, they can
not proxy in a statistical sense for an intratemporal non-stochastic
regressor.

There are, however, two important pieces of information which may
be deduced from Table 1:  (a) the almost near perfect correlation be-
tween dividend included and dividend excluded indexes and (b) the under-
statement of variability of the market value weighted index.

It can be shown [19, pp. 60-65], that a market value weighted index is a ratio estimator, and unless certain assumptions are met (no correlation between market values and periodic returns) this ratio estimator is a biased estimator of the common underlying factor.

For these reasons and for the reason of reducing other sources of bias (such as correlation between the dependent and independent variables in the market model) we opted to build our own index to serve as the proxy for the "common underlying factor."

Using the CRSP monthly returns file, we selected all companies for which a complete history of returns was available for the period January, 1938 through December, 1977 (480 monthly observations). A total of 287 securities met this criterion. From the 287 securities, we randomly selected 37 securities in constructing an equally weighted index. The remaining 250 securities were reserved for constructing portfolios. Some basic characteristics of this index are summarized in Table 2.

Table 2

Summary Characteristics of Index

|  | Mean | Variance |
|---|---|---|
| 480 observations | .0118 | .0031 |
| 1st 240 observations | .0136 | .0039 |
| last 240 observations | .0110 | .0023 |

Several interesting items surface in Table 2. First, our index is remarkably similar to the CRSP equally weighted (with dividends) index in terms of its mean return and variability, even though it covers a somewhat different time period. Second, and more important, the index

is relatively stationary over time. After splitting the data in half,
a test for differences in means yields an F statistic of .46 which is
only significant at levels of $\alpha > .50$.

## 4. Ex-Ante and Ex-Post Efficiency of Naive vs. M-S Portfolios

### a. Stationarity

In the experiment which follows we are applying the Sharpe simpli-
fied model of portfolio selection because of its many advantages as a
statistically tractable model and because of simplicity.

The model requires $2 + 3N$ estimates, where N is the number of secu-
rities in the population. Estimates are required for the mean and vari-
ance of the "common factor" (index) and intercept, slope, and residual
variance from a regression of security returns on the index as follows:

$$r_{it} = \hat{a}_i + \hat{b}_i r_{It} + e_{it} \tag{8}$$

where:

$r_{it}$ is the rate of return of security i in period t,

$r_{It}$ is the rate of return of the index in period t,

$\hat{a}_i$ and $\hat{b}_i$ are parameter estimates, and

$e_{it}$ is a random error term, $\sim N(\theta, \sigma^2)$.

In the context of this model a great deal of attention has been di-
rected toward the question of stationarity of the process that generates
rates of return [1, 2, 7]. The only point at issue, however, is the
stationarity of the market model in sample space. For if the process
is stationary, portfolios can be created over the time domain of obser-
vation by randomly selecting a subset of the universe and estimating

each time the parameters of interest and the attributes of efficient portfolios [9]. These attributes then can be compared with the naive strategy. Thus, the number of repetitions is basically controlled by the constraints imposed on academic computing time, resulting in a meaningful number of degrees of freedom for statistical testing.

A test of stationarity of the coefficients of (8) is fashioned by Chow in [3]. The logic of the test is as follows. Suppose we split our t = 1 ... 480 observations in half and estimate equation (8) separately using each half of the data. If

$$a_i^1 = a_i^2 \text{ and } b_i^1 = b_i^2 \tag{9}$$

where the superscripts refer to estimates obtained using each sub-period, then we can test the restriction implied in (9) by using an F test. The test statistic is

$$Q = \frac{(RRSS - URSS)/p}{URSS/(n_1 + n_2 - 2p)} \tag{10}$$

where:

RRSS = the restricted residual sum of squares, obtained by pooling the data and estimating a single equation,

URSS = the unrestricted residual sum of squares, obtained by estimating separate equations for each sub-period and adding the residual sums of squares,

p = 2 = the number of parameters estimated, and

$n_1 = n_2 = 240$ = the number of observations in each sub-period, and $Q \sim F(P, n_1 + n_2)$.

In Table 3 we present the test statistics calculated according to (10) and the frequency (at different levels of significance) at which the null hypothesis of stationarity could not be rejected.

Table 3

Summary Results of Chow's Test

| F(2,∞) | α level | Not Significantly Different # | % Total |
|--------|---------|------|---------|
| 3.00 | .05 | 146 | 51% |
| 4.61 | .01 | 185 | 64% |
| 6.91 | .001 | 219 | 76% |

The results indicate that the hypothesis of stationarity cannot be rejected for the majority of our population. These results are surprisingly good given the fact that observations were taken over a forty-year period. We exploit this property of stationarity for power and precision in our experimental design that is discussed in part c of this section.

b. Ex Ante Portfolio Selection

Our objective is to compare and contrast the performance of Sharpe efficient portfolios on both an ex ante and ex post basis. Our analysis is based on 50 Sharpe and 50 naive portfolios formed as follows.

Using the remaining 250 securities and the index based on 37 securities, we randomly select 100 monthly returns over the period January, 1938 through November, 1977. December, 1977 data is reserved for ex post testing. Using the 100 observations, estimates of the index mean and variance as well as the market model of (8) are computed. The six-month Treasury bill rate, a proxy for the risk free rate of interest, is obtained each time for the last randomly selected time period. The

Sharpe algorithm is then run and the tangent portfolio is selected for analysis.[1] This procedure is repeated 50 times.

Our objective is to compare Sharpe and naive portfolios based on the same degree of diversification. To form the naive portfolios, we randomly select n securities from the 250 security population, where n is the number of securities in the ith Sharpe portfolio (i = 1, ... 50). The ex ante characteristics of the naive portfolios are then computed based on the same 100 randomly selected returns used to compute the Sharpe inputs.

The summary ex-ante statistics of the portfolios are shown on the left side of Table 4 below.

Table 4

Summary Characteristics of the Portfolios
(Monthly Percentages)

| Average: | Ex Ante Characteristics | | Ex Post Characteristics | |
|---|---|---|---|---|
| | Sharpe | Naive | Sharpe | Naive |
| Return | 1.54% | 1.12% | 0.52% | 0.19% |
| Variance | 8.30% | 28.39% | 18.93% | 36.36% |
| Sharpe Performance Measure[2] | .381 | .120 | .012 | -.010 |

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[1]The tangent portfolio is defined as that portfolio of the convex set to which a ray drawn from the risk free rate on the expectation axis is tangent. We wish to bring to the reader's attention that this portfolio is neither a proxy nor is it equivalent to the "market portfolio" implied by the CAP model, as the assumptions on which (8) is based are quite different from the assumptions that are required for models of asset pricing.

[2]The Sharpe Performance Measure was computed as (portfolio return-risk free proxy)/portfolio variance.

In terms of expectations, the Sharpe portfolios indicate a superior risk-return relationship. The expected returns are on average higher and the variances lower, thus resulting in higher Sharpe performance measures than for the naive portfolios. However, while naively selected portfolios provide unbiased estimates of both return and risk, the bias associated with Sharpe portfolios is well known [7, 8]. In addition to the information in Table -4, we also note that the portfolios are well-diversified, containing a range of 12 to 37 securities, with an average portfolio size of 23 securities. The composition of the naive portfolio variances also confirm the earlier results of Vertes. Referring to the three terms in equation (6), averages for the fifty naive portfolios where: $\overline{V}$, 74.02 and $\overline{V}/n$, 3.22; $\overline{C}$, 25.16; and the $\sigma_E^2$ effect was close to zero.

c. Ex Post Performance Analysis

In this section we compare the ex post performance of the two portfolio selection strategies. We analyze the performance of each portfolio in the period following the last randomly selected period used in determining portfolio ex ante characteristics. The ex post results are detailed on the right side of Table 4.

For ex-post performance analysis consider the following linear model:

$$Y_{ijk} = U + T_i + P_j + TP_{ij} + e_{ijk} \tag{11}$$

$$i = 1, 2, \ldots 11$$
$$j = 1, 2$$
$$k = 1, 2, \ldots 50$$

where:

$Y_{ijk}$ = Normalized ex-post return

$T_i$ = Time effect

$P_j$ = Portfolio effect

TP = Interaction

$e_{ijk}$ = a random error, $\sim N(0, \sigma_E^2)$

The random selection process which was not limited to any time period resulted in 11 actual ex-post periods for testing. However, all except two of test months were in 1977. It should be noted that the frequencies per cell for these 11 months are unequal, and proper adjustment has been taken for the inequality of the number of observations per cell across the time factor [15]. We formulate three null hypotheses on model (11):

$H_0^1$: $T_1 = T_2 = \ldots = T_{11}$;  there is no time effect

$H_0^2$: $P_1 = P_2$  there is no portfolio effect, and

$H_0^3$: $TP_{ij} = TP$ for all i, j; there is no interaction effect between time and portfolio.

The alternate hypotheses are obviously that there are two main effects and one interaction effect.

The model in (11) is an Analyses of Variance model [15]. Ex-post observations on the 100 portfolios [50 Sharpe and 50 naive] are obtained where the ex-post observation is for the month immediately following the month of the last observation upon which the portfolio had been created and the ex-ante characteristics had been calculated. These ex-post returns were normalized through the division by the ex-post standard deviations. Then model (11) has been fit to the normalized ex-post returns. The analyses of variance table is presented below.

Table 5
Analyses of Variance of
Ex-Post Normalized Returns

| Source of Variation | P.F. | Sum of Squares | Mean Square | F | Level of Significance |
|---|---|---|---|---|---|
| Time Effect | 10 | 23.991 | 2.399 | 31.637 | .001 |
| Portfolio Effect | 1 | .508 | .508 | 6.696 | .012 |
| Total Main Effects | 11 | 24.999 | 2.227 | 29.370 | .001 |
| Interaction Effect | 10 | 1.903 | .190 | 2.509 | .011 |
| Explained | 21 | 26.402 | | 16.579 | .001 |
| Unexplained | 18 | 5.915 | | | |
| Total | 99 | 32.316 | | | |

For the interest of the reader we also present the means of the

main effects and the grand total in Table 6.

Table 6

Means

| Date: | 2/76 | 11/76 | 3/77 | 5/77 | 6/77 | 7/77 | 8/77 | 9/77 | 10/77 | 11/77 | 12/7 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | .46 | .62 | -.24 | -.17 | .91 | -.12 | -.32 | .06 | -.58 | .80 | -.03 |
| (observations) | (2) | (2) | (2) | (6) | (6) | (8) | (14) | (8) | (14) | (18) | (20 |
| Portfolio: | | | | | | | | | | | |
| Sharpe | .61 | 1.05 | -.23 | .04 | .97 | .24 | -.16 | .14 | -.71 | .82 | -.02 |
| Naive | .32 | .18 | -.25 | -.37 | .86 | -.48 | -.49 | -.02 | -.45 | .79 | -.03 |
| Difference | .29 | .87 | .02 | .41 | .11 | .72 | .33 | .16 | -.26 | .03 | .01 |

| | Sharpe | Naive |
|---|---|---|
| Portfolio | .14 | .00 |

Grand Mean        .07

The two tables (5 and 6) jointly show that the _normative_ model of port-

folio selection formulated by Markowitz and simplified later by Sharpe

resoundingly outperforms the naive diversification model.[3]  This was

true in ten out of the eleven ex-post periods examined.

It is hard to explain why the normative model has not proliferated

in practice and why it was so quickly abandoned, even in academe, for a

positive model of capital asset pricing upon which the logic of naive

diversification is based.  As we can see from Table 6 even for an ultra

conservative portfolio manager an error type I of .012 (the highest in

the Table) for the portfolio effect should suffice.  A more desperate

chap would probably settle for an even bet of doing not worse than the

market.  Applying the Sharpe model for any universe of interest could

not be a computational problem today with the large scale utilization of

computer data bases and the latest generation of mini and micro computers.

Also, the new generation of MBA's from leading schools in which

portfolio theory is taught in addition to the traditional courses of in-

vestment and security analysis could provide the cadre that is needed

by the many institutional investors who are not aware of the technical

aspects of modern portfolio selection.

## Summary and Conclusions

In this paper we tried to statistically test the hypothesis of su-

periority of efficient portfolio selection over naive portfolio selec-

tion of dart throwing.  We have shown the ex-ante and ex-post attributes

of both of these procedures and based on our data and experimental de-

sign we have concluded that the Markowitz-Sharpe model is convincingly

superior to the method of naive diversification.

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[3]We remind the reader that these are the standardized results.
In terms of the raw data, the reader should refer back to the right
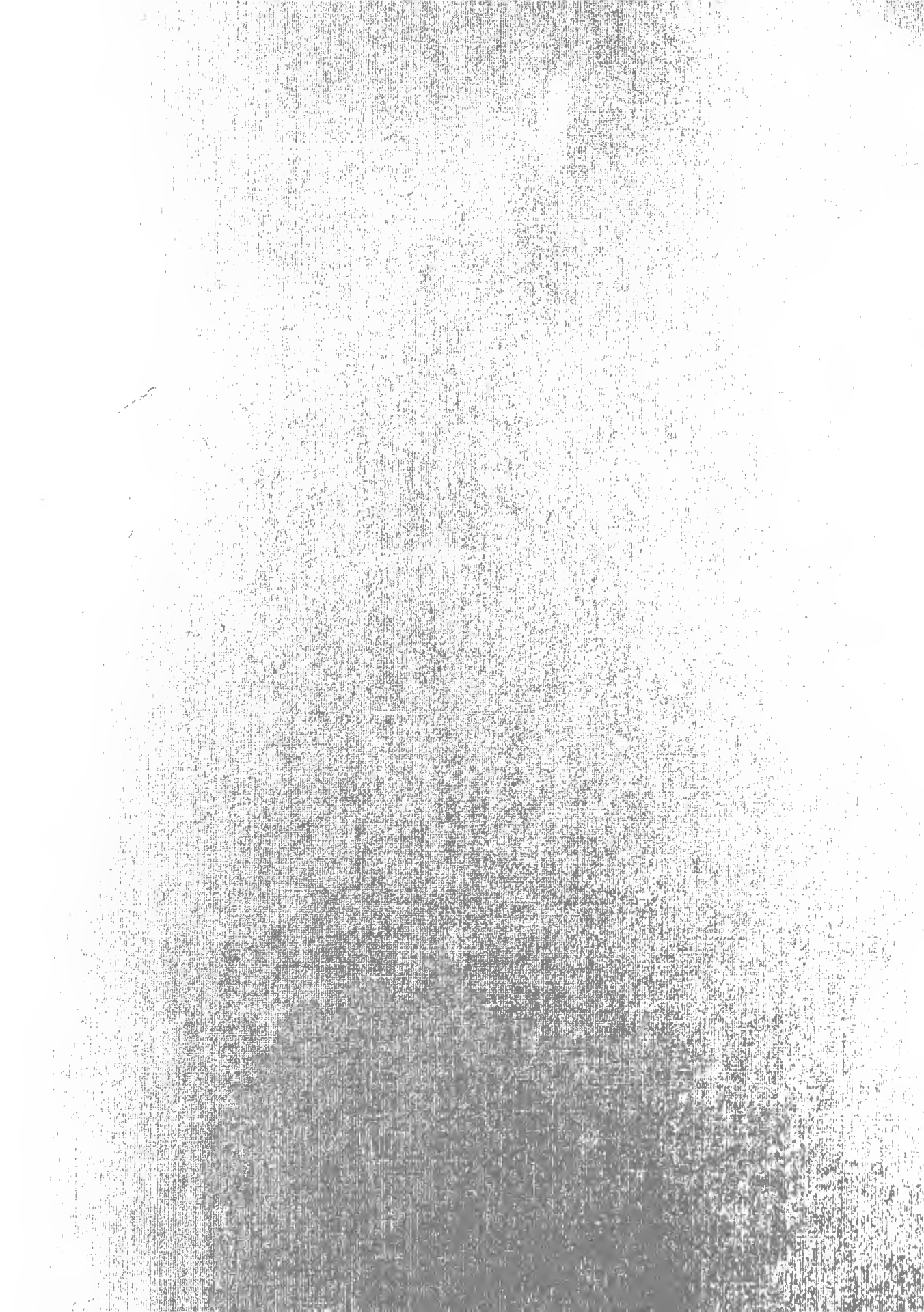side of Table 4.

While we cannot safely argue, that in the long run the M-S model will survive, we are at loss in explaining why it is not widely accepted in practice and why there has not been much academic work done for its improvement and for its proliferation. We hope that our experiment will convince somewhat the skeptics, and encourage further research related to the M-S model. In such case, we hope to see further evidence in support of efficient diversification as opposed to naive "index buying."

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