NASA TECHNICAL TRANSLATION

NASA TT F-15,937

MESON PRODUCTION AS A SHOCK WAVE PROBLEM

W. Heisenberg

Translation of "Mesonenerzeugung Als Stosswellenproblem", Zeitschrift für Physik, Vol. 133, 1952, pp. 65 - 79.

(NASA-TT-F-15937) MESON PRODUCTION AS A SHOCK WAVE PROBLEM (Scientific Translation Service) 25 p HC $4.25 CSCL 20H

N74-34191

Unclas

G3/24 50355

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 SEPTEMBER 1974

STANDARD TITLE PAGE

1. Report No. NASA TT F-15,937	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Meson production as a shock wave problem		5. Report Date October 1974
		6. Performing Organization Code
7. Author(s) W. Heisenberg		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address SCITRAN Box 5456 Santa Barbara, CA 93108		11. Contract or Grant No. NASw-2483
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Mesonenerzeugung Als Stosswellenproblem" Zeitschrift für Physik, Vol. 133, 1952, pp. 65 - 79.

16. Abstract

The production of many mesons in the collision of two nucleons is described as a shock wave process represented by a nonlinear wave equation. The quantum-theoretical features of the process can then be considered approximately according to the principle of correspondence, as we are dealing with a "high quantum number process". Statements on the energetic and angular distributions of the various types of mesons arise from the discussion of the solutions of the nonlinear wave equation.

PRECEDING PAGE BLANK NOT FILMED

17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified - Unlimited		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 23	22. Price

MESON PRODUCTION AS A SHOCK WAVE PROBLEM

W. Heisenberg

The experimental information gained in recent years on the origin of the π-mesons makes it seem very probable that many mesons are often produced at once in the collision of two high-energy nucleons. It has for a long time been established that a strong interaction of nucleons with mesons, and particularly between mesons, can lead to such multiplication [4]. For a quantitative estimate, one can compare the energy dissipation in the meson field with turbulence in flow fields [5], or , as Fermi [3] has done, one can think of a temperature equilibrium being attained at the moment of collision, from which the energetic distribution of the mesons can be calculated. / 65 *

The following considerations, however, are intended to take up the problem from the viewpoint which the author presented in 1939 in relation to the Yukawa theory [4]. Meson production will be considered as a shock wave process, described by a nonlinear wave equation, and it will be shown that through such a treatment one can arrive at quantitative results for the spectral and spatial distribution of the mesons.

I. Perceptual Description of the Shock Wave

In the following, meson production is always described in the center-of-mass system. Transformation into the laboratory system can be undertaken without difficulty as a supplement, and has been done in earlier works; it need not be explained

* Numbers in the margin indicate pagination in the original foreign text.

here [5].

a) In the center-of-mass system, both nucleons approach each other from opposite directions (Figure 1) until they overlap in a certain region (shaded in Figure 1). The nucleons are shown as flat discs. Because of the Lorenz contraction, their thickness is less by the factor of $\sqrt{1-\beta^2}$ (β = center of mass velocity) than their diameter, which one can take to be of the order of magnitude of the Compton wavelength of the meson, i. e., on the order of $1.4 \cdot 10^{-13}$ cm. At the moment /66

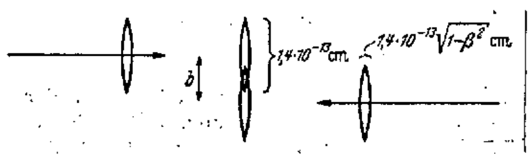

Figure 1

of collision the velocity of the nucleons changes, so that in their total region energy is transferred to the meson field. In the first moment of the shock wave, then, the entire energy of the meson field is concentrated in the thin flat layer which was filled by both nucleons at the moment of collision.

b) If one ignores the interaction of the mesons, it would expand after the first moment according to the wave equation

$$\Box\varphi - \varkappa^2\varphi = 0 \tag{1}$$

(or according to a complex linear wave equation which contains the different meson types). The spectral and angular distribution of the meson wave would then no longer change in the course of the wave expansion. They could, therefore, be determined by a Fourier expansion of the wave at the first moment. We find that the energy contained in the meson wave between the frequencies k_0 and $k_0 + dk_0$ (k_0 corresponds to the energy of a single meson) would be nearly independent of k_0 up to frequencies having their wavelengths of the order of magnitude of the thickness of the layer in which the collision occurs; i. e., on the order of $\frac{\sqrt{1-\beta^2}}{\varkappa}$. ($\varkappa$ is the meson mass). For $k_0 > k_{0m} = \frac{\varkappa}{\sqrt{1-\beta^2}}$ the intensity will decrease rapidly as a function of k_0.

$$d\varepsilon = \text{const} \cdot dk_0 \quad \text{for} \quad k_0 \leq k_{0m}. \tag{2}$$

Correspondingly, for the number of mesons in the interval dk_0 one obtains

$$dn = \text{const} \frac{dk_0}{k_0} \quad \text{for} \quad k_0 \leq k_{0m}. \tag{3}$$

Figure 2 shows the course of φ on the axis perpendicular to the plane of emission (shortly after the act of emission). It also shows $d\varepsilon/dk_0$ and dn/dk_0, under the assumption (1). / 67

Spectrum (3) corresponds to the well-known x-ray braking spectrum of the electrons. Even if a considerable part of the nucleon energy is transferred to the meson field, it never leads to a large number of emitted mesons, because the energy of an individual meson would average $\gtrsim \frac{1}{2} k_{0m}$.

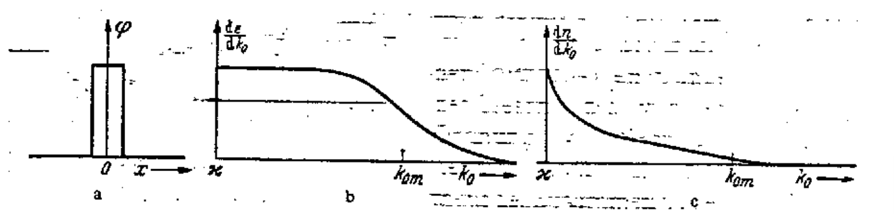

Figure 2 a-c.

c) In reality, though, we cannot ignore the interaction of the mesons. The wave expands according to a nonlinear wave equation. Only in the limiting case of low intensity does it transform approximately into the linear form. The

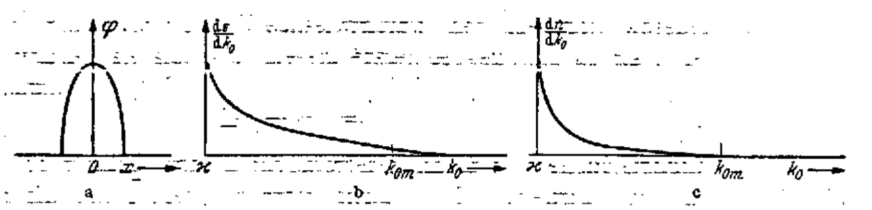

Figure 3 a-c.

nonlinearity has the result, which we shall recalculate later, that the singularity at the head of the wave is somewhat rounded off. As a result, energy is transferred from the short to the

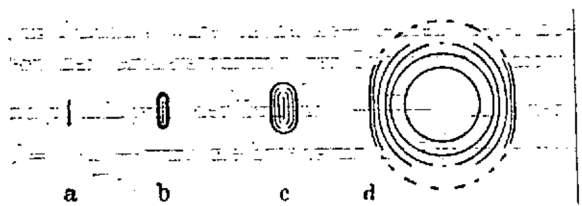

Figure 4 a-d.

long waves during the expansion process, and the spectral distribution at the end of the expansion process falls off more rapidly than if (1) were valid. Qualitatively, one obtains the relations shown in Figure 3.

The spatial expansion is shown in Figure 4 a-d.

At the moment of impact the entire energy is concentrated in the layer of the two nucleons (a). Then two shock fronts

move out to the right and left. The major portion of the energy /68
still resides in the two shock fronts, but there is also wave excitation in the space between them, which contains the rest of the energy (b). Now the shock fronts proceed farther. The excitation in their wake spreads over a wider space, and that near the starting point becomes a new wave expansion. The energy in the shock fronts has become smaller. It has shifted into the remaining wave region [and, therefore, to greater wavelengths (c)]. On continued advance, the excitation at the center decreases. A true wave forms, propagating faster in the direction of the shock fronts than perpendicular to it, because waves of short wavelength have a higher propagation velocity (group velocity). Only at very slight intensity will the excitation spread to all sides, even though with the velocity of light. The energy in the shock fronts has by now become so small that here, too, the nonlinearities play no important role. Continued progress is according to the usual linear wave equation (d).

In this perceptual description we have so far completely ignored the quantum theoretical aspects of the problem. That is a quite useful approximation, as it deals with the production of many mesons; that it, with a process having high quantum numbers. The work mentioned above [4] describes in detail how to undertake the corresponding transformation into quantum theory. Here it is sufficient to take the following qualitatively from Figure 4d: A large part of the energy is radiated out in all directions in the form of mesons having wavelengths comparable with the diameter of the disc; i. e., with $1/\varkappa$. In the direction perpendicular to the axis the momentum will only rarely be able to be greater than $\varkappa$ because the Fourier coefficients of such waves become very small. But the momentum in the direction of the axis can be greater because the shorter wavelengths appear in the shock wave front proper. Therefore, mesons with the energy k_o are generally emitted only in an angular region of the order of magnitude $\varkappa/k_0$ about the two primary directions. The heavier mesons are also emitted principally only in the shock wave front.

II. Solution of the Shock Wave Equation

a) The expansion of the shock wave depends on the form of the nonlinear wave equation based on the mesons. But it can be shown that there is a limiting case for "strong" interaction in which the spectral distribution of the mesons can be stated independently of the particular form of the wave equation.

If we consider first only the spectral distribution, and not the directional distribution, the solution of the nonlinear wave equation can be eased by some simplifications: Consider the plane in which the emission occurs to be extended to infinity, and the layer infinitely thin. Then, because of the Lorentz invariance of the wave equation, φ can depend only upon $s=t^2-x^2$. The partial differential equation thus transforms into an ordinary differential equation, the solution of which can be discussed more easily. / 69

Two nonlinear wave theories will be considered as examples:

1. The equation discussed by Schiff [10] and Thirring [12] in relation to the nuclear forces:

$$\Box\varphi-\varkappa^2\varphi-\eta\varphi^3=0. \tag{4}$$

2. A wave equation which arises from the Lagrange function

$$L=l^{-4}\sqrt{1+l^4\left[\sum\left(\frac{\partial\varphi}{\partial x_\nu}\right)^2+\varkappa^2\varphi^2\right]} \tag{5}$$

following the pattern of the earlier work by Born [1]. Some time ago, Born commented that nonlinear theories of this type have singular solutions in a smaller degree than the linear theories. That was used at the time for the self-energy of electrons, but it also applies to meson production. Previous studies on meson production have already been based on the Lagrange function (5) [4].

On 1: For $\varphi=\varphi(s)$ the first of these two equations transforms into

$$4\frac{d}{ds}\left(s\frac{d\varphi}{ds}\right)+\varkappa^2\varphi+\eta\varphi^3=0. \tag{4a}$$

For $\eta=0$ one returns to the linear wave equation (1) and the solution is then

$$\left.\begin{array}{ll} \varphi=a J_0(\varkappa\sqrt{s}) & \text{for } s>0 \\ \varphi=0 & \text{for } s<0, \end{array}\right\} \tag{6a}$$

Here a is a constant of integration; see also Figure 2. For $\eta \neq 0$ one can give an exponential series expansion with s = 0:

$$\left.\begin{array}{ll} \varphi=a\left[1-(\varkappa^2+\eta a^2)s+\frac{1}{4}(\varkappa^2+3\eta a^2)(\varkappa^2+\eta a^2)s^2-+\cdots\right] & \text{for } s>0 \\ =0 & \text{for } s<0. \end{array}\right\} \tag{6b}$$

We see immediately that (4) deals with a "weak" interaction which changes nothing with respect to the discontinuity of the wave function at the shock wave front. This is related to the fact that the theory characterized by (4) is one of the group / 70
of renormalizable theories. The coupling parameter, η, has the dimension of a pure number. It has already been established in various ways that the renormalizable theories contain only

"weak" interactions which do not in general give rise to multiple production of mesons.

On 2: The situation is different, however, for the wave equation characterized by (5). For $\varphi=\varphi(s)$ it reads:

$$4\frac{d}{ds}(s\varphi')+\varkappa^2\varphi=8l^4 s\varphi'^2\frac{\varphi'+\varkappa^2\varphi}{1+l^4\varkappa^2\varphi^2}. \tag{7}$$

If we assume that $\varkappa=0$ (vanishing rest mass of the mesons), then the solution can be written immediately:

$$\left.\begin{aligned}\varphi&=\frac{1}{a}\lg\left(1+\frac{a^2}{2l^4}s+\frac{a}{2l^4}\sqrt{4l^4s+a^2s^2}\right) &&\text{for } s\geq 0\\ &=0 &&\text{for } s\leq 0.\end{aligned}\right\} \tag{8}$$

In the general case $(\varkappa\neq 0)$ we can again state series expansions. We set

$$\varphi=\frac{1}{\varkappa l^2}f(\zeta);\qquad \zeta=s\varkappa^2 \tag{9}$$

and obtain

$$\left.\begin{aligned}f(\zeta)&=\sqrt{\zeta}\left(1+a\zeta+\frac{27a^2+2a-1}{10}\zeta^2+-\cdots\right) &&\text{for } \zeta\ll 1\\ &\approx\gamma\zeta^{-\frac{1}{4}}\cos(\sqrt{\zeta}+\delta) &&\text{for } \zeta\gg 1\\ &=0 &&\text{for } \zeta\leq 0.\end{aligned}\right\} \tag{10}$$

The constants γ and δ are unambiguously determined by the integration constant, a, but their values have not been calculated.

One can see that here the nonlinearity has extensively changed the nature of the solution. The discontinuity of φ at $s = 0$ has disappeared. Only φ' behaves discontinuously. In the vicinity of $s = 0$, φ behaves like $\sqrt{s}$.

If one expands $\varphi(s)=\varphi(x,t)$ at a given time into a Fourier integral according to the wave number, k, then, except for constant factors, one obtains an expression of the form

$$\varphi(k,t) \sim k^{-\frac{3}{2}} t^{\frac{1}{2}} e^{\pm i k_0 t}. \tag{11}$$

for the coefficients $\varphi(k,t)$ for large values of k $(k \sim k_0 \gg \varkappa)$. The factor $t^{\frac{1}{2}}$ clearly arises because of the fact that during the entire expansion process, energy flows continuously from the head of the shock wave into the other parts of the wave, and, therefore, into the lower frequencies. Actually the supply of energy in the head of the shock wave is infinite here. This is a necessary consequence of the assumption that the shock wave begins in an infinitely thin plane layer, because from this assumption we concluded that the solution of $\varphi(x,t)$ depends only upon t^2-x^2, and is therefore invariant with respect to the Lorentz transformation in x,t space. But a finite energy momentum vector would indicate a direction in this space, and thus could not be part of an invariant solution. / 71

Actually, of course, the shock wave starts in a layer of finite thickness $\sim\frac{\sqrt{1-\beta^2}}{\varkappa}$. The energy-momentum vector is finite and the rise of the Fourier amplitudes in (11) comes to a stop after a certain time when the energy supply of the wave front is exhausted. Then the Fourier coefficients for large values of t fall off more strongly than as $k^{-3/2}$ as a function of k for $k > k_{0m} = \frac{\varkappa}{\sqrt{1-\beta^2}}$. Thus, one obtains for the intensity distribution

$$\frac{d\varepsilon}{dk_0} = \text{const}\,\frac{dk_0}{k_0} \quad \text{for} \quad \varkappa \lesssim k_0 \lesssim k_{0m} = \frac{\varkappa}{\sqrt{1-\beta^2}} \tag{12}$$

and

$$\frac{dn}{dk_0} = \text{const}\,\frac{dk_0}{k_0^2}, \tag{13}$$

for the same region.

This is the form of the spectrum which was discussed previously in relation to multiple generation [4, 5] and is also presented in Figure 3.

The wave equation (7) taken from Born's theory [1] represents a typical case of a "strong" interaction and leads to multiple production of mesons. The coupling parameter has the dimension of the fourth power of a distance.

b) Now it will be shown that the spectrum (12) and (13) quite generally corresponds to the limiting case of strong interaction, independent of the particular form of the Lagrange function and independent of the special properties of the particles involved.

We begin with an arbitrary Lagrange function for a scalar wave function φ and its first derivative $\partial\varphi/\partial x_\nu$. Because of the Lorentz invariance, L can depend only on φ and $\sum_\nu \left(\frac{\partial\varphi}{\partial x_\nu}\right)^2$. For very small values of φ and $\partial\varphi/\partial x_\nu$, L must transform into the Lagrange function of the ordinary wave equation (1). Now we inquire about the value of $\sum_\nu \left(\frac{\partial\varphi}{\partial x_\nu}\right)^2$ in the vicinity of $s=0\ (s>0)$. For $s\to 0$, $\sum \left(\frac{\partial\varphi}{\partial x_\nu}\right)^2$ can either become infinitely large, take on a finite value, or approach zero. Next, we can exclude the last of these three possibilities, / 72 because then the nonlinearity would play no part just at the critical point, s = 0. But that is impossible because for the usual wave equation (1), (1) $\sum \left(\frac{\partial\varphi}{\partial x_\nu}\right)^2$ at the critical point is by no means zero, but infinite.

Of the two remaining possibilities, the second obviously gives the smoother curve for φ at the singular point. Thus, it corresponds to the stronger interaction. Here, in the vicinity of s = 0, we get

$$\sum\left(\frac{\partial\varphi}{\partial x_\nu}\right)^2 = -4s\left(\frac{d\varphi}{ds}\right)^2 = \text{const}\ (\neq 0 \text{ and } \neq \infty), \tag{14}$$

from which

$$\varphi(s) \sim \text{const}\sqrt{s} \tag{15}$$

so that behavior as in (7) and (10) follows.

c) But one can give a still more general proof for (12) and (13), which also applies for arbitrary particles of high spin value. It has already been mentioned under IIa that in the limiting case in which the shock wave begins in an infinitely thin layer its total energy content must be infinite, because the wave function is then invariant to rotations in x,t space. Now the energy spectrum of the mesons falls off more steeply the greater the energy dissipation due to the interaction is. Thus, to the extent that the spectrum has the form of a power law at all (and that could apply for most of the simple wave equations), it cannot fall off more sharply than in (12) and (13), because here the total energy still diverges for $k_{0m} \to \infty$ (namely, logarithmically). The spectrum (12) and (13) therefore just corresponds to the limiting case of strong interaction. Thus, as has already been said, the Lagrange function (5) taken from Born's theory gives only a special example of a theory with strong interaction. But the spectrum, (12) and (13), remains correct also for very much more complex Lagrange functions which contain various types of mesons as a solution for the limiting case of small interaction, if we deal with a theory with strong interaction.

III. Application to Meson Production

The multiple production of mesons will now be treated quantitatively, with the assumption of strong interaction.

a) One of the most important quantities for characterization of a meson shower is the average energy of the mesons in the center-of-mass system. To a very crude approximation, one can consider the spectrum (12, 13) as exactly valid between $k_0 = \varkappa_i$ (rest mass of the type of mesons concerned) and $k_0 = k_{0m}$. /73

Then we have

$$\left.\begin{aligned} \varepsilon_i &= A_i \int_{\varkappa_i}^{k_{0m}} \frac{dk_0}{k_0} = A_i \lg \frac{k_{0m}}{\varkappa_i} \\ n_i &= A_i \int_{\varkappa_i}^{k_{0m}} \frac{dk_0}{k_0^2} = \frac{A_i}{\varkappa_i}\left(1 - \frac{\varkappa_i}{k_{0m}}\right) \end{aligned}\right\} \tag{16}$$

and it follows that

$$\overline{k_{0i}} = \frac{\varepsilon_i}{n_i} = \varkappa_i \frac{\lg \frac{k_{0m}}{\varkappa_i}}{1 - \frac{\varkappa_i}{k_{0m}}} \quad \text{for} \quad k_{0m} > \varkappa_i. \tag{17}$$

For $k_{0m} \leq \varkappa_i$ the type of mesons concerned would not occur at all.

In reality, the spectrum will have to contain the factor kdk_0 just because of the phase space volume, and will not have the form of (12, 13) at all for small k. Furthermore, it will not disappear completely for $k_0 > k_{0m}$, but only diminish more strongly than in (12) and (13). One can try

$$d\varepsilon_i = A_i \frac{k\,dk_0}{k_0^2\left(1 + \frac{k_0^2}{k_{0m}^2}\right)} \tag{18}$$

as probably a somewhat better solution. Then we obtain

$$\left.\begin{aligned} \varepsilon_i &= A_i\left(-1+\sqrt{1+\alpha^2}\,\lg\frac{1+\sqrt{1+\alpha^2}}{\alpha}\right) \\ n_i &= \frac{A_i}{\varkappa_i}\frac{\pi}{4}\left(1+2\alpha^2-2\alpha\sqrt{1+\alpha^2}\right), \end{aligned}\right\} \tag{19}$$

$$\overline{k_{0\,i}} = \varkappa_i\frac{4}{\pi}\,\frac{-1+\sqrt{1+\alpha^2}\,\lg\frac{1+\sqrt{1+\alpha^2}}{\alpha}}{1+2\alpha^2-2\alpha\sqrt{1+\alpha^2}}, \tag{20}$$

where we set $\varkappa_i/k_{0\,m} = \alpha$.

Both approximations, (17) and (20), are plotted as functions of $\lg(1/\alpha)$ in Figure 5. The difference between the two curves gives a measure for the inaccuracy of the entire estimate.

It appears from these calculations that in the limiting case of strong interaction, the average meson energy increases only logarithmically and that, therefore, the number of mesons increases almost in proportion to the energy transferred into the meson field in the center-of-mass system.

b) To be sure, the relations are complicated more by the occurrence of new types of mesons at higher energies. We can assume that for sufficiently high values of $k_0\,(k_0 \gg \varkappa_i)$ the relative proportion g_i of the meson species $\varkappa_i$ is independent of k_o, and depends only on the form of the shock wave equation. In this region, then, the various species of mesons generally occur in comparable frequency, but the g_i need not be simply proportional to the statistical weight of the species concerned. We normalize /74

$$\sum g_i = 1 \tag{21}$$

and set

$$A_i = g_i A. \tag{22}$$

/75

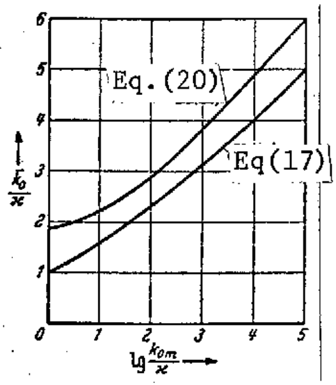

Figure 5

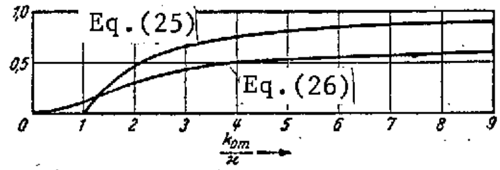

Figure 6

Then, in the rough approximation of (16) and (17) we have

$$\varepsilon = A \sum g_i \lg \frac{k_{0m}}{\varkappa_i}, \tag{23}$$

$$n = A \sum \frac{g_i}{\varkappa_i}\left(1 - \frac{\varkappa_i}{k_{0m}}\right), \tag{24}$$

therefore,

$$\left.\begin{aligned} n_i &= \varepsilon \frac{\frac{g_i}{\varkappa_i}\left(1 - \frac{\varkappa_i}{k_{0m}}\right)}{\sum_l g_l \lg \frac{k_{0m}}{\varkappa_l}} && \text{for} \quad \varkappa_i \le k_{0m} \\ &= 0 && \text{for} \quad \varkappa_i \ge k_{0m}. \end{aligned}\right\} \tag{25}$$

For large k_{0m} ($k_{0m} \gg \varkappa_i$), therefore, the numbers in the various groups of mesons behave as $g_i/\varkappa_i$. As k_{om} decreases, the number of heavy mesons decreases faster than that of the lighter. As soon as k_{om} decreases below the value $\varkappa_i$, the species of mesons in question disappears completely. Instead of (25), then, in the approximation of Equations (18) to (20), we would have

$$n_i = \varepsilon \frac{\frac{g_i}{\varkappa_i}\frac{4}{\pi}\left(1 + 2\alpha_i^2 - 2\alpha_i\sqrt{1+\alpha_i^2}\right)}{\sum g_l\left(-1 + \sqrt{1+\alpha_l^2}\lg\frac{1+\sqrt{1+\alpha_l^2}}{\alpha_l}\right)}. \tag{26}$$

The factor of $g_i/\varkappa_i$ in (25) and (26) which is characteristic for the dependence of n_i on k_{om} is shown graphically in Figure 6. With the second approximation formula, there would still be a small number of mesons of the type $\varkappa_i$ remaining even for $k_{0m} < \varkappa_i$, as is also to be expected physically. /75

c) If one wishes to make statements about the total number of mesons emitted, one must also know the total energy, ε, of the meson field in (25) and (26). For this quantity, we can at first state only a maximum value: ε can be no greater than the kinetic energy of both nucleons in the center of mass system before the collision.

Because in general only a fraction of this energy is actually transmitted to the meson field, it is convenient to introduce this fraction, γ, as the "degree of inelasticity" of the collision. Then we have (M = mass of the nucleons):

$$\varepsilon = \gamma \cdot 2M\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right), \tag{27}$$

where

$$0 \leq \gamma \leq 1.$$

One would expect that for a central collision γ would have a value near 1, while only a small fraction of the kinetic energy will be transferred to the meson field for a grazing collision.

If we call the distance between the centers of the nucleons at the moment of collision b, then we can consider the overlap integral of the π meson fields of the two nucleons as a measure for the strength of the interaction. If one simply sets γ equal to this overlap integral as a very crude estimate of the degree of inelasticity, one gets

$$\gamma = e^{-b\varkappa}, \tag{28}$$

in which $\varkappa$ specifically signifies the mass of the π mesons. It follows from this that the effective cross section for a value of γ between γ and $\gamma + d\gamma$ is

$$d\sigma = 2\pi\, b\, d\, b = \frac{2\pi}{\varkappa^2}\frac{d\gamma}{\gamma}\lg\left(\frac{1}{\gamma}\right). \tag{29}$$

If one wishes to define a total effective cross section, one must define a minimum value of γ. For instance, if one wishes to determine the total effective cross section for multiple production, one must establish as the mimimum value of γ that which will produce at least two mesons.

$$\gamma_{\min} = \frac{\bar{k}_0}{M\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)}. \tag{30}$$

(Here $\bar{k}_0$ refers to the lightest type of mesons, i. e., to the π mesons.)

From (30) it follows that / 76

$$\sigma = \frac{\pi}{\varkappa^2}\lg^2\gamma_{\min} \tag{31}$$

and

$$\bar{\gamma} = \frac{2}{\lg^2\gamma_{\min}}\left(1 - \gamma_{\min} + \gamma_{\min}\lg\gamma_{\min}\right). \tag{32}$$

It must be emphasized that the estimation of the frequency distribution of the values of γ in Equations (28) to (32) is independent of the preceding considerations on the expansion of the shock wave, and must be considered as less reliable. So far there is not enough observational material to determine the frequency distribution of γ experimentally.

Table 1, following, gives the total effective cross section, the expected values of γ, n_π and $n_\varkappa$ (number of the π- and $\varkappa$- mesons, respectively), their average energy, and,

TABLE 1*

E	10	10^2	10^3	10^4	BeV
σ	0,18	0,49	0,85	1,3	10^{-24} cm^2
$\bar{\gamma}$	0,34	0,19	0,13	0,09	
$\bar{n}_\pi$	3,6 ± 0,7	4,2 ± 0,8	5,2 ± 0,8	8,0 ± 1	
$\bar{n}_\varkappa$	—	0,9 ± 0,2	2,0 ± 0,4	3,4 ± 0,6	
$\bar{k}_{0\pi}$	0,25 ± 0,04	0,36 ± 0,04	0,50 ± 0,05	0,67 ± 0,06	BeV
$\bar{k}_{0\varkappa}$	—	1,0 ± 0,2	1,4 ± 0,15	2,0 ± 0,18	BeV
$\gamma = 1$ $\bar{n}_\pi$	10,7 ± 2	22,1 ± 4	40,3 ± 6	89 ± 12	
$\gamma = 1$ $\bar{n}_\varkappa$	—	4,7 ± 1	15 ± 6	38 ± 6	

*Translator's note: Commas in numbers represent decimal points.

finally, the number of mesons in the limiting case of $\gamma = 1$ as functions of the primary energy, E (in the laboratory system). Other types of mesons such as π- and $\varkappa$- mesons are not considered. In addition, we arbitrarily set $g_\varkappa = 2g_\pi$, that is, $g_\pi = \frac{1}{3}, g_\varkappa = \frac{2}{3}$, in order to take into account the relatively great frequency of the $\varkappa$- mesons found according to the newer measurements in Bristol. These numbers will have to be revised later on the basis of more accurate measurements. We use 0.61 BeV for the mass of the $\varkappa$- meson. In order to express the inaccuracy of the theoretical estimate, we have taken the average of values obtained from (16, 17) or (18) to (20) (except for the first two columns) and have listed half the difference as the error.

d) The angular distribution of the emitted mesons appears from the perceptual considerations in I. Of course, the details of the angular distribution still depend on the shock wave equation. But, quite generally, the momentum of the mesons perpendicular to the primary direction will only rarely be able to exceed the value $\varkappa$ to any extent. As a rule, mesons with the energy k_0 are emitted in an angular region of the magnitude $\varkappa/k_0$ about the axis. The distribution of the $\varkappa$- mesons /77 is, therefore, always anisotropic, while in the center of mass system the distribution of the slower π mesons will be to some extent isotropic.

IV. Comparison with Experience

So far, only a few meson showers have been observed without gray or black tracks. Only for such showers can one assume with some probability that they were from collisions of only two nucleons, without involvement of a larger atomic nucleus. But if one also includes showers with a few (two to three) thick tracks to test the theory, the perturbation of the shower by the atomic nucleus will generally be small. But because a secondary scattering of the mesons produced could have taken place at the atomic nucleus, the determination of the primary energy from the angular distribution and the evaluation of the angular distribution itself become very unreliable.

TABLE 2*

E	30	40	40	90	130	1000	2000	30 000 BeV
$n_\pi + n_\varkappa$ γ_{emp}	9 0,51	18 0,8	25 1,0	10 0,38	18 0,61	9 0,16	12 0,17	21 0,1

* Translator's note: Commas in numbers indicate decimal points.

Observations of showers suitable for comparison with the theory have been presented so far by Teucher [11], the working group at Bristol [2], by Schein et al. [9], Pickup and Voyvodic [8] and Hopper, Biswas and Derby [6]. If one tries to estimate the primary energies from the angular distribution (which is quite uncertain in some cases) according to the reports in the publications, we obtain the meson numbers for the eight observed showers in the second column of Table 2, if we assume a ratio of 1:2 for neutral to charged mesons. These numbers are already somewhat uncertain because of the neutral mesons. If we assume that the last two columns in Table 1 are correct, we obtain an empirical value of γ for each of these showers, which is shown in the third column of Table 2.

We note first that the meson numbers are actually not unambiguous functions of the primary energy. The γ values fluctuate strongly, as was to be expected. But they are on the average somewhat greater than would have been conjectured according to Table 1. This could be due to the fact that small showers can be surveyed more easily than large ones; but it could also mean that the estimate in Equation (28) is still too rough *. Also, the empirical values of γ in Table 2 are themselves still quite uncertain because, for /78 instance, the proportion of κ mesons is not accurately known. Perkins [7] also reports relatively high γ values, but one must await still more experimental material.

It has been possible to measure two showers (Teucher, [11], and Hopper, Biswas and Derby [6]) so accurately that the average energy of the mesons in the center of mass system could be reported. In the first case (40 BeV, some 25 mesons) the observed average meson energy is 0.29 BeV, compared with 0.31 BeV according to Table 1. In the second case (1,000 BeV, some 9 mesons) there is some uncertainty because of the possibility

* Comment added in proof. At the Copenhagen Conference in June, 1952, LeCouteur mentioned that the expected value of γ in heavy material (e. g., in the photographic emulsion) must be considerably greater than in hydrogen (Table 1 refers to hydrogen) because "grazing" collisions can occur only with nucleons at the edge of the atomic nucleus. Powell has also reported on new experiments indicating that the particles designated here as κ mesons can be separated into two groups with masses of 0.74 and 0.54 BeV, with quite different properties.

that some of the particles observed could have been $\varkappa$ mesons, not taken into consideration by the authors. (According to Table 1, one should expect some 3 $\varkappa$ mesons among 9 mesons.) If we ignore that, the observed average π meson energy in the center of mass system was 0.44 BeV, compared to 0.50 BeV according to Table 1. Thus, these two measurements confirm the relatively low meson energy of Table 1. On the other hand, Perkins [7] reports the value of 1.5 BeV as the average energy of mesons from a series of showers with a primary energy of 10^2 to 10^3 BeV. This is considerably higher. Here, though, we must consider the uncertainty in the measurement of the primary energy. Any error in the primary energy generally increases the average meson energy, as this has the smallest value just in the center of mass system.

On the frequency of the $\varkappa$ mesons we have as yet only the statement of the Bristol group that it is comparable with that of the π mesons at high energies [7]. For the present, this ratio cannot be determined from the theory. (In Table 1, we arbitrarily set $g_\varkappa/g_\pi=2$.)

With respect to the angular distribution, it is observed that the distribution in the center of mass system is rather isotropic for showers of low energy, while distinct accumulations appear about the primary direction and the opposite direction in showers of high energy. This corresponds exactly to the picture of Ic. In fact, mesons of high energy appear always to be distributed anisotropically, and in particular, quite generally, the $\varkappa$ mesons (Perkins [7]). The degree of the anisotropy also corresponds to the theoretical estimate.

On the whole, then, one has the impression that the formulas derived in III under the assumption of "strong" interaction satisfactorily represent experience; and that, therefore, the interaction of the elementary particles at high energy actually belongs in the group of the "strong" interactions first studied by Born.

REFERENCES

1. Born, M. Proc. Roy. Soc. Lond.,Ser. A, Vol. 143, 1933, p. 410.

   Born, M. and L. Infeld. Proc. Roy. Soc. Lond.,Ser. A, Vol. 144, 1934, p. 425; Vol. 147, 1934, p. 522; Vol. 150, 1935, p. 141.

2. Camerini, U., P. H. Fowler, W. O. Lock and H. Muirhead. Phil. Mag. Vol. 41, No. 7, 1950, p. 413.

3. Fermi, E. Progr. theor. Phys. Vol. 5, 1950, p. 570. Phys. Rev. Vol. 81, 1951, p. 683.

4. Heisenberg, W. Z. Physik, Vol. 113, 1939, p. 61.

5. Heisenberg, W. Z. Physik, Vol. 126, 1949, p. 519.

6. Hopper, V. D., S. Biswas and J. F. Derby. Phys. Rev. Vol. 84, 1951, p. 457.

7. Mr. Perkins has kindly provided us with the results of his next work before publication.

8. Pickup, E. and L Voyvodic. Phys. Rev. Vol. 82, 1951, p. 265. Vol. 84, 1951, p. 1190.

9. Schein, M., J. J. Lord and J. Fainberg. Phys. Rev. Vol. 80, 1950, p. 970; Vol. 81, 1951, p. 313.

10. Schiff, L. J. Phys. Rev. Vol. 84, 1951, p. 1.

11. Teucher, M. Naturwiss. Vol. 37, 1950, p. 260; Vol. 39, 1952, p. 68.

12. Thirring, W. Z. Naturforsch. Vol. 7a, 1952, p. 63.

Translated for National Aeronautics and Space Administration under contract No. NASw 2483, by SCITRAN, P. O. Box 5456, Santa Barbara, California, 93108.