NATTES'S

Practical Geometry

OR

an introduction to Perspective.

Translated from the French of Le Clerc.

with Additions & Alterations.

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PRACTICAL GEOMETRY,

OR

Introduction to Perspective.

TRANSLATED FROM THE FRENCH OF LE CLERC;
WITH ADDITIONS AND ALTERATIONS.

The Explanations rendered so simple, that very young People, by Attention, may soon be enabled to go through the different Problems with perfect Ease.

A WORK

NOT ONLY USEFUL TO THOSE WHO CULTIVATE THE ELEGANT ART OF DRAWING, BUT ALSO RECOMMENDED TO THE STUDENT IN VARIOUS BRANCHES OF THE ARTS AND SCIENCES.

TO WHICH IS ADDED,

AN EASY METHOD OF MAKING AN OVAL
OF ANY GIVEN PROPORTIONS:

ALSO

THE RULE FOR FORMING A GEOMETRICAL PLAN AND ELEVATION;
Being the last Problem previous to the Commencement of the Study of Perspective.

WITH FORTY VIGNETTES,
ETCHED FROM DESIGNS ANALOGOUS TO THE DIFFERENT GEOMETRICAL FIGURES,
BY W. H. PYNE.

THE PROBLEMS ENGRAVED BY T. KING.

LONDON:
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INTRODUCTION.

In an age like the present, when education branches out into so many studies, it is presumed he that can facilitate the acquirement of any useful science may be excused for adding another book to the general stock. The utility of Practical Geometry (at the same time that it lays a foundation for the study of Perspective, so essential to drawing local views) has rendered it a necessary part of the education of a gentleman. It forms, perhaps, the most interesting branch of mathematics, and prepares the mind for that species of knowledge which confers upon the possessor additional consequence as a scholar.

The Problems exhibited in this work are rendered so simple by the written Explanations, that the study of them cannot appear like a task; and the youth who possesses sufficient ingenuity to invent stars for his kite, is likely to be insensibly led to draw these mathematical figures for his evening's amusement.

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BOOK II.

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BOOK IV.

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BOOK V.

OF PROPORTIONAL LINES.

LIST

OF THE

VIGNETTES UNDER EACH PLATE.

THE FRONTISPIECE,

A Mill by the Bridge at Bray, near Dublin.

Plate

1. A Horse-mill, used in a Manufactory for making Whiting.
2. Stonemasons, with the Stone-sawyer's Shed, &c.
3. A Capstan for raising Stones, Timber, and other heavy Bodies, from Vessels on the River.
4. Wheelwrights at Work.
5. Men grinding.
6. Stonemasons laying a Pavement.
7. Copper-plate Printers at Work.
8. Figures at the Invalids Well, Paris.
9. Horses and Sheep at a Crib.
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11. Bricklayers at Work.
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16. A Water-conduit, with a Group of Water-carts.
17. Tile-kilns.
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19. Gipsies cooking.
20. A Well, with a Horse raising Water.
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23. Machine for raising Timber, &c.
24. A Pump, with a Water-cart.
25. Cooperage, with a Brewer's Vat, &c.
26. A Brickmaker's Mill for grinding Clay.
27. Sawyers at Work.
28. Market-cross, with Booths, Figures, &c.
29. An Ice-house in a Garden, with Men rolling a Gravel Walk.
30. Making a Hay-stack, with a Horse-sledge, &c.
31. Loading a Timber-tug under a Triangle.

OF

GEOMETRY IN GENERAL.

THE word Geometry is of Greek origin; in its primary sense it means the art of measuring the earth, or any distances thereon. We nevertheless understand by it the principal part of the mathematics, which is a science whose object is continued quantity.

Continued quantity is that, all of whose parts are conjoint, as are all sorts of extensions, sizes, and dimensions.

These dimensions principally consist either in lines, angles, superficies, or bodies, considered in themselves, and without any regard to matter.

Geometry is divided into speculative and practical. Speculative geometry teaches us to perceive and demonstrate the truth of geometrical propositions. Practical geometry is that which applies the theorems of speculative geometry to practice.

OF ITS ORIGIN.

Geometry had its rise in Egypt, where the inundations of the Nile rendered it necessary to distinguish lands by considering their figures; to be able to measure their respective quantities; and to know how to lay them out in their just dimensions and situations.

By the Egyptian studies and observations this very mechanical exercise was insensibly raised to one of the highest rank among the sciences.

OF ITS UTILITY.

Geometry is not only useful, but it may be said to be absolutely necessary. It is by geometry that the astronomer makes his observations, measures the extent of the heavens, the motions of the stars, the regularity of the seasons, and the duration of time. It is by its means that the geographer can bring into one point of view, the whole earth, the vast extent of the sea, and the division of empires, kingdoms, and provinces. To it the architect is indebted for the knowledge of the just and necessary rules for the structure of edifices both public and private.

By it the engineer is enabled to construct his works; to observe the situation and take the plan of places, even when by their locality they are only accessible to the sight. To persons of the military profession it is absolutely necessary, not only as an introduction to fortification, by which they are taught to build ramparts for the security of places, and to construct works for their destruction; but it likewise gives them a considerable degree of knowledge and facility in the military art, such as drawing up an army in order of battle, marking out encampments, and taking such military plans of a country as shall make them as much esteemed for their professional knowledge as for their courage.

The artist is likewise under the necessity of knowing something of geometry, as without it he cannot have such a perfect knowledge of architecture or perspective as is absolutely necessary for him.

Geometry is established upon three Sorts of Principles, viz. Definitions, Axioms, and Petitions.

Definitions are succinct explanations of names and terms.

Axioms are sentences whose truth is so clear, that they are incontestable.

And the petitions are such clear and intelligible demands, that the execution and practice require no demonstration.

N. B. It is particularly recommended that a pupil should never go to a second proposition before he understands the first well; and in order to practise with precision it is necessary to have good instruments: the compiler never found any maker superior to WELLINGTON, Mathematical Instrument Maker to their Royal Highnesses the Dukes of Gloucester and Cumberland, Crown Court, St. Ann's, Soho, London.

DEFINITIONS.

THE DEFINITION OF A POINT.

A POINT is that which has no parts. By this definition you may easily perceive, that a point has neither length, nor breadth, nor depth; that it is not any thing sensible, but only intellectual; for nothing falls under the notice of our senses that has nothing of quantity, and nothing is quantity that has not parts; so that to say a point is sensible, would be to say it has parts, which would contradict this definition. Notwithstanding, since no operation can be performed without the intervention of something corporeal, we usually represent a mathematical point by a physical point, which is an object of sight the smallest and the least sensible that can be, and which has no geometrical magnitude divisible to our senses, and is made by the prick of a pin, point of a compass, pen, or pencil, as the point marked . . . . A

A central point, or centre, is a point from which a circle, or circumference, is described; or rather it is the middle of a figure, as the point . . B

A secant point, or, as some call it, a point of intersection, is a point where two or more lines cross one another, as the point C

THE DEFINITION OF A LINE.

A line is a length without any breadth.

A line is nothing but the track made by a point passing from one place to another; and would not be perceived, if it were not delineated by a physical point, which by its motion represents a line to us, as . . AB, CD, EF

There are as many sorts of lines as there are different kinds of motions, which a point, the principle of a line, is capable of; though there are but two, which are simple and the principal, viz. a right and curve, and a third, which is called a mixed line, because made up of the two former, that are usually considered in geometry.

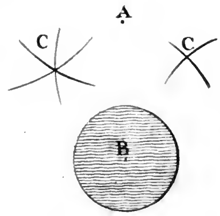
A right line is one that lies equally between its extremities.

Otherwise it is a line that goes from one point to another without any deviation, as . . . . AB

A curve line is that which turns out of its way by one or more deviations, as CD

When such a line as this is described by a pair of compasses, it is called a circular line, as . . . . E

A mixed line is that which is both straight and a curve, as the line V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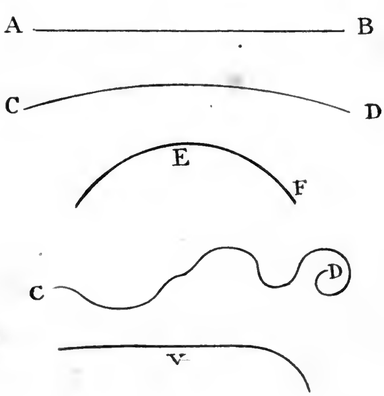
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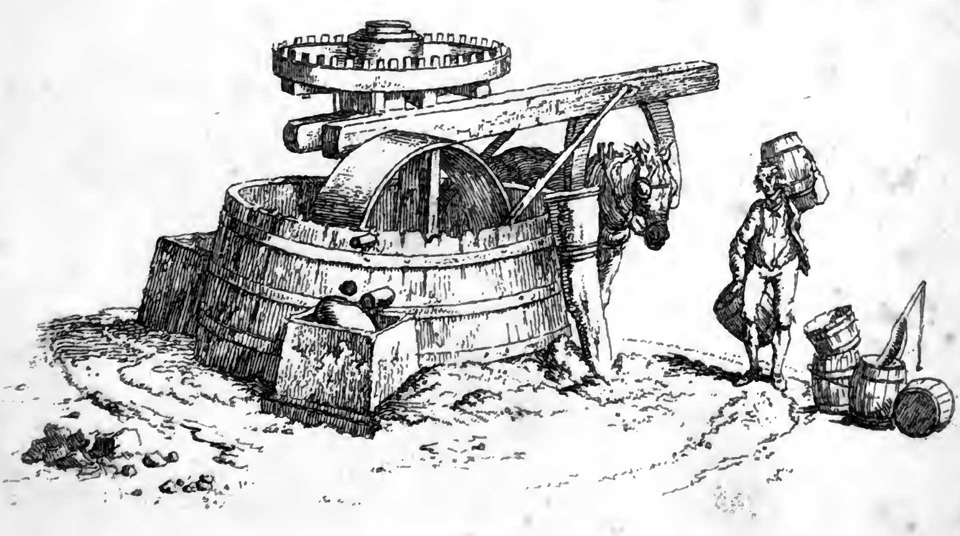

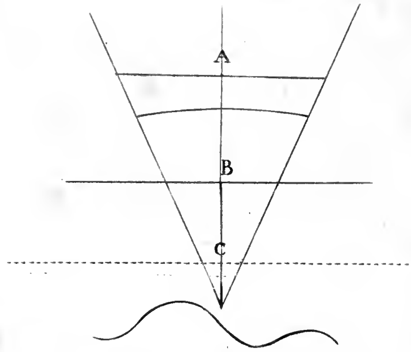
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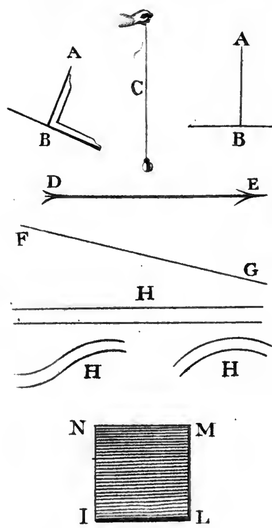
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A Line is distinguished into finite and infinite, into apparent and occult.

A FINITE line is a bounded line, containing or supposing a necessary length, as A

An infinite line is an undetermined line, having no precise length, as B

An apparent line is one described with ink or a pencil, as . . A B

An occult, or white line, is only made with the point of a pair of compasses, or marked by points, and then it is called a pricked line, as . . C

A Line receives also several Denominations, according to its different Positions and Properties.

A perpendicular is a right line that is let fall or erected upon another, making the angles on each side equal, as . . . . . . A B

A plumb line is that which hangs down without inclining to the right or left, and would pass through the centre of the earth, if it were produced infinitely, as C

A horizontal line is a line in equilibrio, equally inclined on both sides D E

Parallel lines are such as follow one another at an equal distance . . H

An oblique line is one that is neither horizontal nor perpendicular . F G

A base is a line upon which the figure rests, as . . . . I L

Sides are the lines that contain a figure, as . . . I N, L M

A diagonal is a right line crossing a figure, and terminated at its two opposite angles . . . . AB

A diameter is a right line passing through the centre of a circle, and terminated at the circumference . . . . CD

A spiral line is a curve line issuing from a centre, and continually going off from it at every turn . . . . EF

A chord or subtense is a right line that joins the two extremities of an arc GH

An arc is any part of a circumference . . . . . . GIH

A tangent is a line that touches a figure without cutting it, nor would it cut or cross the figure, though it were produced, as . . . . LM

A secant is a line that does cross or cut a figure . . . LO, MO.

If two lines meet at their extremities, they meet either directly or indirectly: if directly, they make but one line; if indirectly, they form an angle.

THE DEFINITION OF AN ANGLE.

An angle is the indirect concourse of two lines in the same point; or rather it is the space contained between the indirect concourse of two lines meeting in a point, as . . . . ABC

If the concourse be formed by two right lines, the angle is called a rectilineal; if by two curve lines, a curvilineal; but if by one right and one curve line, a mixtilineal angle.

A denotes a rectilineal angle.

B a curvilineal angle.

C a mixtilineal or compound angle.

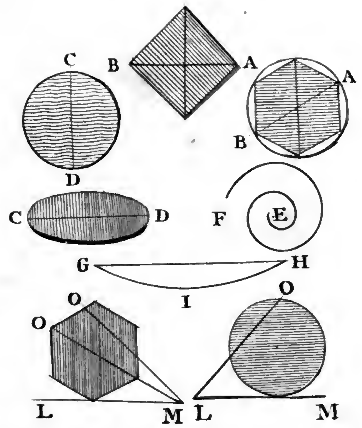
A rectilineal angle receives several particular names according as it has a greater or less aperture, as right, acute, obtuse: thus the terms of rectilineal, curvilineal, and mixed, express the quality of the lines, and those of right, acute, obtuse, the quantity of the space contained between the said lines.

An angle is right, when one of the lines is perpendicular to the other EDF

An angle is acute, when its aperture is less than that of a right angle EDG

An angle is obtuse, when its aperture is greater than that of a right angle FDG

The middle letter D denotes the angle.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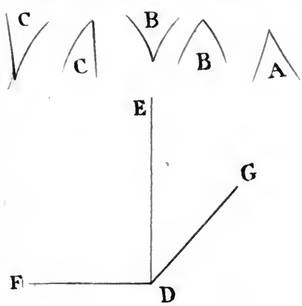
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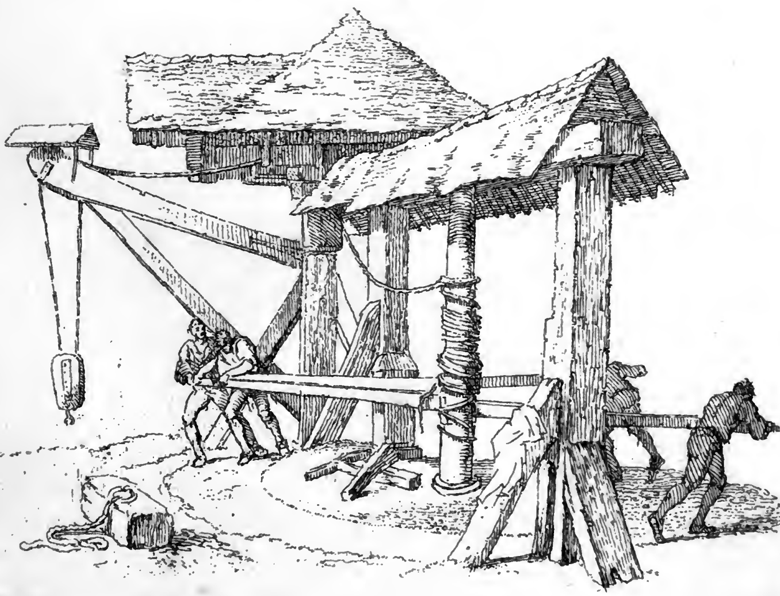

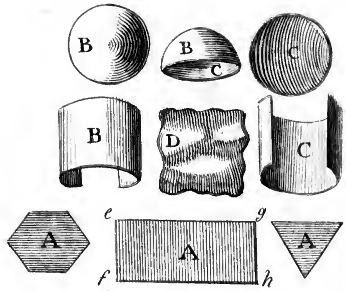
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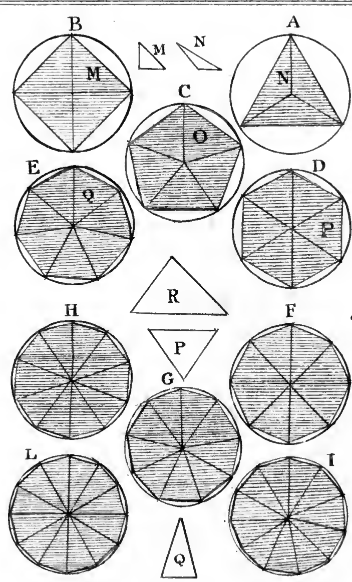
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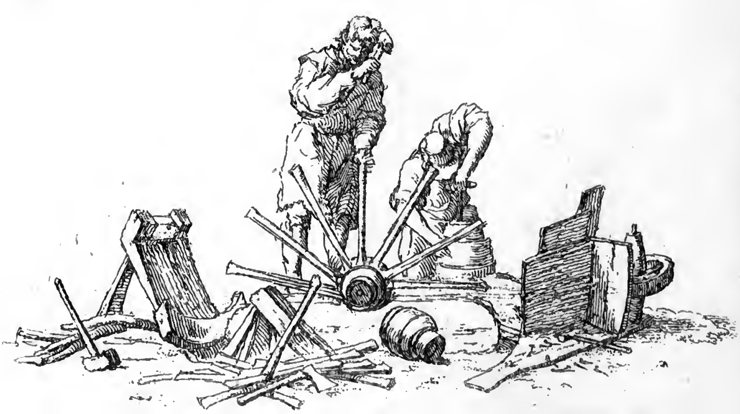


DEFINITION OF A SURFACE.

A Surface is whatever has Length and Breadth without Depth or Thickness.

According to the sentiments of the geometricians, a surface is a production of a line, just as a line is the production of a point: thus we are to imagine the line EF moving towards GH to constitute the surface EF GH, which is an extension bounded by lines, and has length and breadth without depth or thickness: this is commonly called a surface; but a figure, if it be considered in regard of its extremities, which are the bounding lines.

If the surface be elevated or raised, it is said to be convex; but if depressed, sunk in, or hollow, it is called a concave; and if even and flat, a plane. Thus,

B is a convex surface.
C a concave surface.
A a plane surface.
D a surface that is convex, concave, and plane.

This first part relates only to plane surfaces.

The terminus, term, or boundary, of any thing is its extremity: thus a point is the terminus of a line, a line is the term of a surface, and a surface is the terminus of a body.

OF SURFACES OR FIGURES THAT ARE RECTILINEAL.

Surfaces take their particular Names from the Number of their Sides.

Thus,

A is a trigon or triangle, a figure with three sides.
B a tetragon or square, a figure of four sides.
C a pentagon, or a figure of five sides.
D an hexagon, or figure of six sides.
E an heptagon, or figure of seven sides.
F an octagon, or figure of eight sides.
G an enneagon, or figure of nine sides.
H a decagon, or figure of ten sides.
I an hendecagon, or figure of eleven sides.
L a dodecagon, or figure of twelve sides.

All these figures are also called by the general name of polygons.

OF TRIANGLES.

Triangles are distinguished by the Quality of their Angles, and by the Disposition of their Sides.

Thus,

M is a right-angled triangle; i.e. has one right angle.
N an obtuse-angled triangle; i.e. has one obtuse angle.
O an acute-angled triangle; i.e. has all three angles acute.
P an equilateral triangle; i.e. has its three sides equal.
Q an isosceles triangle; i.e. has only two sides equal.
R a scalene triangle; i.e. has all its three sides unequal.

OF QUADRILATERALS, OR FIGURES THAT HAVE FOUR SIDES.

A is a square, or figure that has its four sides equal and four angles right.

B, a rectangle, by some improperly called a long square, has all its angles right or equal, but its sides unequal.

C, a rhombus, is a quadrilateral that has its four sides equal, but not its four angles.

D, a rhomboid, has the opposite angles and sides equal, without being equiangular or equilateral.

A B C D, a parallelogram, is a quadrilateral whose opposite sides are parallel.

E, a trapezium, has only two opposite sides parallel, and the two others equal.

F, a trapezoid, has its four sides and angles unequal.

G. If a diagonal be drawn in a parallelogram, as also two lines parallel to the sides, through the same point of the diagonal, the parallelogram will be divided into four parallelograms; and three of them, viz. one of those described upon the diameter and the two supplements (i. e. the two parallelograms, which are not described about the diameter), form a figure called a gnomon; thus the three parallelograms H I L make a gnomon, as do also the three parallelograms I K L.

All figures having more than four sides, are called polygonals or multilaterals.

OF CURVES OR CURVILINEAL FIGURES.

A, a circle, is a surface or figure perfectly round, described upon a centre, from which the circumference in all its parts is equally distant.

a b c d, a circumference, is the extremity of a circle, or it is the circular line that bounds it.

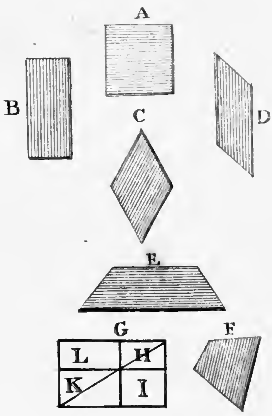
B, an oval, is a curvilineal figure described upon several centres, and divided into two equal parts by all its diameters.

C, an ellipse, is also a curvilineal figure described upon several centres in the shape of an egg, and has but one diameter that divides it into two equal parts.

D, a volute or scroll, is a figure or surface bounded by a spiral line.

E is a cylindric surface.

F is an irregular curvilineal figure, composed of several dissimilar curve lines.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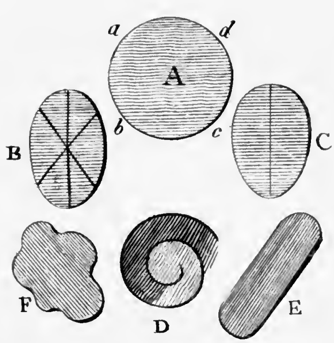
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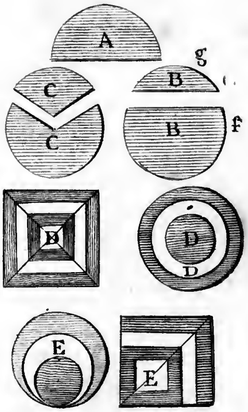
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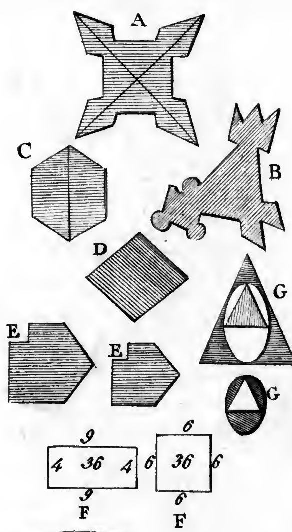
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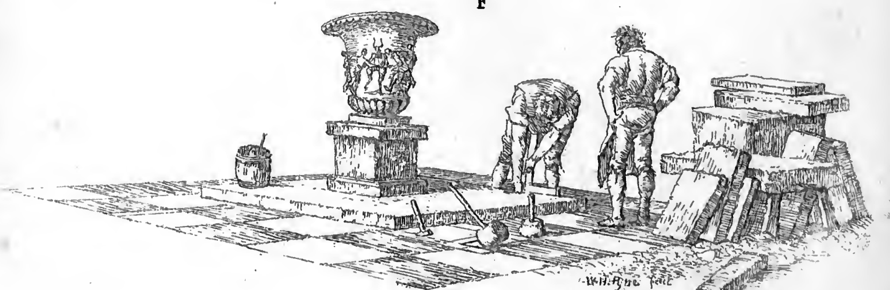


OF COMPOUND FIGURES.

A, A SEMICIRCLE, is a figure contained between half the circumference and the diameter.

B, a portion of a circle, is a figure comprehended within any part of a circle, and a right line.

f, a large portion of a circle, is greater than half the circle.

g, a small portion of a circle, is that which is less than half the circle.

C, a sector, is a figure contained between two semidiameters, and an arc, greater or less than a semicircle.

There is also a large or small sector.

D, concentric figures, are such as have the same centre.

E, eccentric figures, are such as are described upon different centres.

OF REGULAR AND IRREGULAR FIGURES.

A, a regular figure, is that which has its opposite parts similar and equal.

B, an irregular figure, is such a one as is composed of angles and sides that are dissimilar.

E E, similar figures, are such as have all their sides proportional, though one may be greater, equal, or less, than another.

F F, equal figures, are such whose contents are equal, though they may be similar or dissimilar.

C, an equiangular figure, has all its angles equal.

E E, one figure is said to be similar or equiangular to another, when all the respective angles of the one are equal to all the respective angles of the other.

C D, an equilateral figure, is one that has all its sides equal.

G G, similar curvilineal figures, are such as will admit similar polygons to be inscribed in them, or circumscribed about them.

THE AXIOMS.

AXIOMS.

I.

Things equal to the same Third, are equal to one another.

The lines A C, A C, which are equal to A B, are also equal to one another.

II.

If to equal Things, equal Things be added, the whole will be equal.

The lines A C, A C, are equal.
The lines C D, C D, added are equal.
The whole A D, A D, are also equal.

III.

If from equal Things, equal Things be taken away, the Remainders will be equal.

If from the equal lines . .	A D, A D
you take away the equal lines .	A C, A C
the remaining parts . .	C D, C D
will be also equal.	

IV.

If to unequal Things, you add equal Things, the whole will be unequal.

If to the unequal lines . .	D E, D E
you add the equal lines . .	A D, A D
the whole	A E, A E
will be unequal.	

V.

If from unequal Things, equal Things be taken, the Remainder will be unequal.

If from the unequal lines . .	A E, A E
you take away the equals . .	A D, A D
the remainders . . .	D E, D E
will be unequal.	

VI.

Things double the same Third, are also equal to one another.

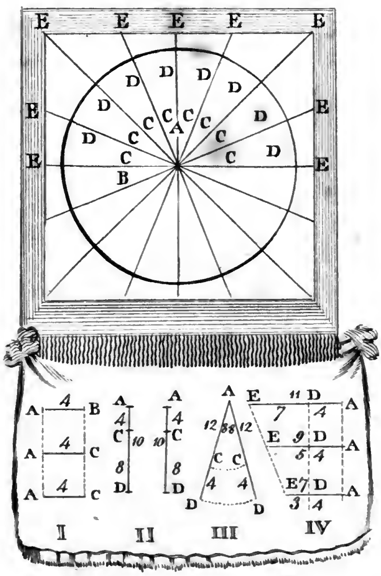
The right lines . . .	D D, D D
that are double the line . .	A D
are equal among themselves.	

VII.

Things, that are Halves of the same, or equal Things, are also equal.

The lines	A D, A D
which are halves of the lines .	D D, D D
are equal to one another.	

What has been said of lines, may also be said of numbers, surfaces, and bodies.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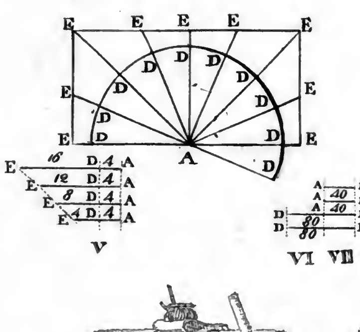
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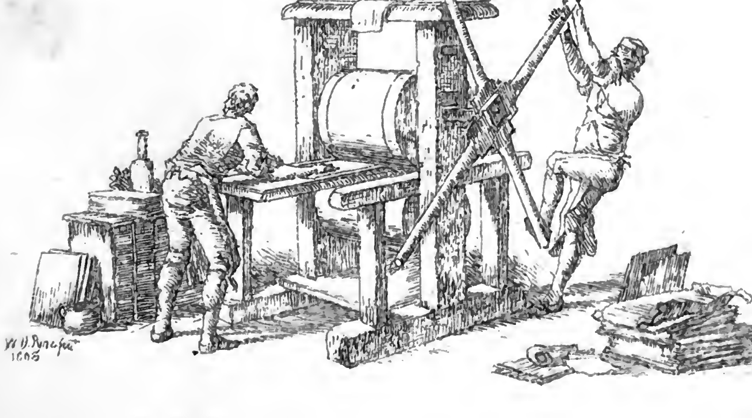
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THE PETITIONS.

THE PETITIONS OR DEMANDS.

PETITION I.

Draw a right line from the point . . . . A
to the point . . . . B

OPERATION.

Apply a ruler to the points . . . A & B
Draw the line demanded . . . AB
by carrying the pencil along the ruler, and
close to it from the point . . A
to the point . . . . . B

PETITION II.

Produce infinitely the line . . . . CD
on the side of the extremity . . . . D

OPERATION.

Join the ruler to the line . . . . CD
Continue infinitely that line . . . CD
on the side of the extremity . . D
by carrying the pen along close to the
ruler towards . . . . . . E

PETITION III.

Describe a circle upon the point . . . . A
and at the distance . . . . AB

OPERATION.

Set one of the points of the compass
upon the given point . . . . . A
Open the other to the given point . . B
Turn the compasses about upon the point . A
and trailing the point . . . . . B
draw the circle demanded . . BCD

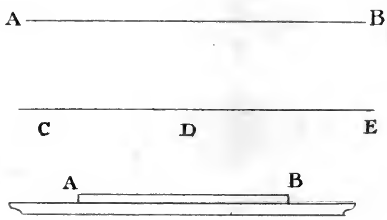
PETITION IV.

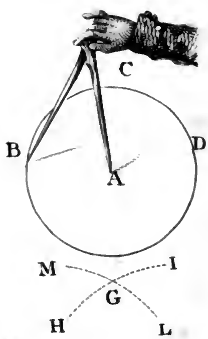
On the points . . . . E &F
make an intersection or section.

OPERATION.

Open the compasses at discretion, but so that the distance of the two points of the compasses may be greater than half the distance of the points proposed E & F

With this distance of the compasses
Upon the point E describe the arc . . . . LM
upon the point F draw the arc . . . . HI
and the intersection required will be . . . . . . G


A
B
C
D
E
A
B


C
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THE

FIRST BOOK

OF THE

DESCRIPTION OF LINES.

BOOK THE FIRST.

PROPOSITION I.

To erect a Perpendicular upon the Middle of a right Line.

POSITION.

Let C be the point proposed in the middle of the line A B, upon which the perpendicular is to be erected.

OPERATION.

Upon the given point . . .	C
describe at pleasure the semicircle	D E
Upon the points	D & E
make the section . . .	I
From the point	C
draw the line demanded . .	C O
through the section . . .	I

This line C O will be perpendicular to the line given A B, and erected upon the point proposed C.

PROPOSITION II.

To erect a Perpendicular upon the Extremity of a right Line proposed.

Let A be the extremity proposed of the line A B, upon which the perpendicular is to be erected.

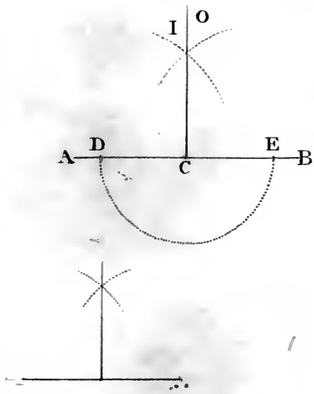
OPERATION.

Take at pleasure the point . . .	C
above the line	A B
from that point	C
with the distance	C A
Describe the portion of the circle .	E A D
Draw the right line . . .	D C E
through the points . .	D & C
Draw the line demanded . .	A E
it will be perpendicular to . .	A B
and at the extremity proposed . .	A

ANOTHER WAY.

Upon the point A describe the arc . .	*g h m*
Upon the point *g* describe the arc . .	A *h*
Upon the point *h* describe the arc . .	*A m n*
Upon the point *m* describe the arc . .	*h n*
Draw the line required . . .	A *n*

I



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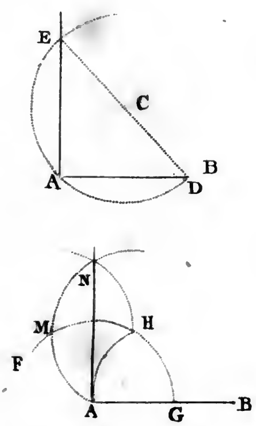

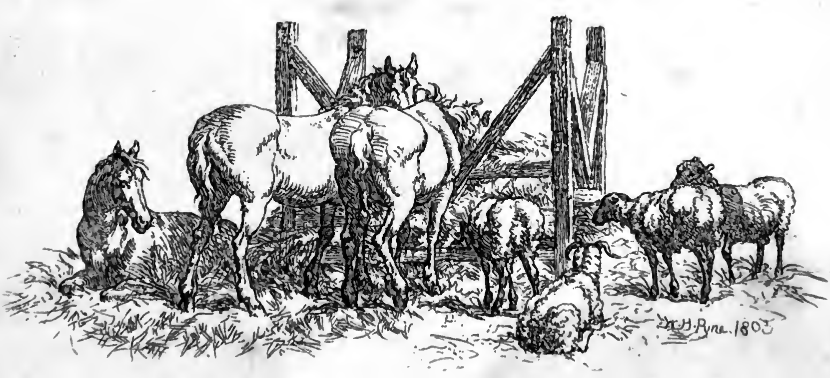

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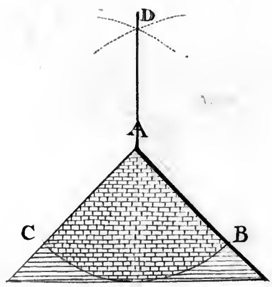

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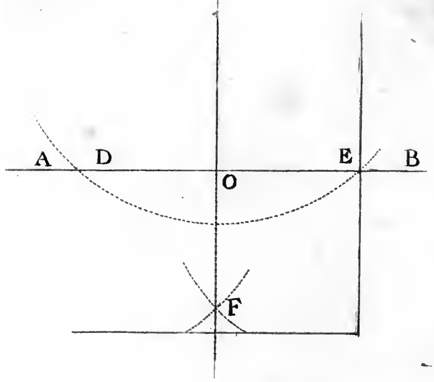


PROPOSITION III.

Upon an Angle given to erect a right Line that inclines neither to the right Hand nor to the left.

Let B A C be the angle upon which the right line is to be raised, that inclines neither to the right hand nor to the left.

OPERATION.

Upon the angle given . . . A
describe at pleasure the arc . . B C
upon the extremities . . . B & C
make the section . . . D
from the point of the angle given . A
draw the line required . . . A D
through the section . . . D
This right line . . . . A D
shall be erected upon the angle . B A C
without inclining either to the right or left.

PROPOSITION IV.

To let fall a Perpendicular upon a given Line, from a Point without the Line.

Let C be the point from which a line is to be let fall perpendicular to A B.

OPERATION.

Upon the given point . . . C
describe at pleasure the arc . . D E
cutting the line . . . . A B
in the points . . . . D & E
upon those points . . . D & E
As centres make the section . . F
draw the line . . . . C F
and the line . . . . C O
will be the line required.

PROPOSITION V.

Through a given Point to draw a Line parallel to a given right Line.

Let A be the given point through which a line is to be drawn parallel to the line B C.

OPERATION.

Draw at pleasure the oblique line	.	A D
upon the point	. . .	A
Describe the arc	. . .	D E
upon the point	. . .	D
Describe the arc	. . .	A F
make the arc	. . .	D G
equal to the arc	. . .	A F
Draw the line required	. .	M N
through the points	. . .	A & G

OTHERWISE.

Upon the centre A describe the arc	.	E F G
touching the line	. .	B C

Without altering the Legs of the Compasses.

Upon the point H describe the arc	.	L R I
The point H *is taken at pleasure in the line*		B C
Draw the line demanded	. .	O P
through the point	. . .	A
and touching the arc	. .	L R I

PROPOSITION VI.

To bisect a given finite right Line.

POSITION.

Let A B be the right line proposed to be divided into two equal parts.

OPERATION.

Upon the extremity	. . .	A
as a centre, describe the arc	. .	C D

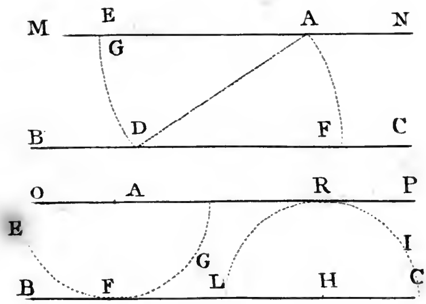
Without altering the Distance of the Legs of the Compasses.

Upon the other extremity	. .	B
as a centre describe the arc	. .	E F

These Arcs are to be made so as to intersect each other.

Draw the right line	. . .	G H
through the intersections	. .	G & H
A B then will be bisected at the point	.	O

V



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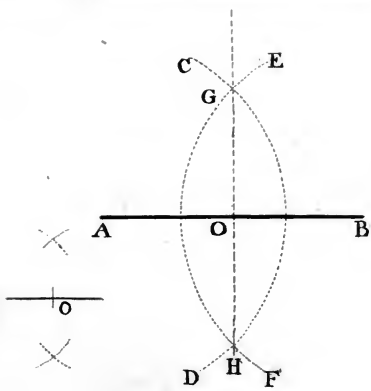

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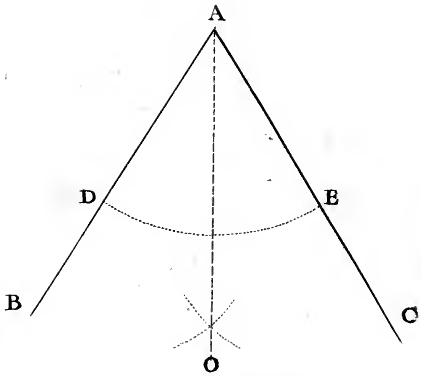

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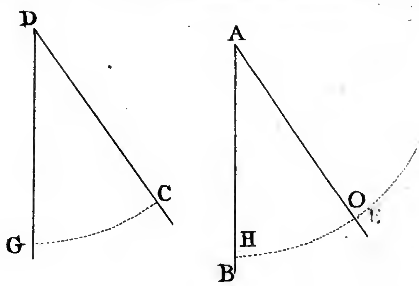
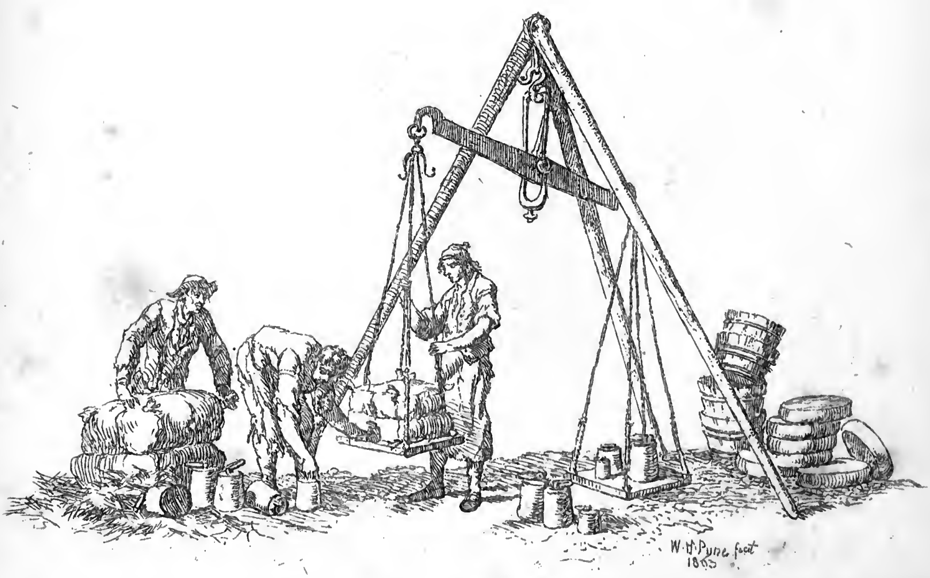




PROPOSITION VII.

To bisect a given rectilineal Angle.

Let B A C be the angle proposed to be bisected.

OPERATION.

Upon the angular point . . A
describe at pleasure the arc . . D E
upon the points . . . D & E
As centres make the section . . O
draw the line . . . A O
This line . . . . A O
will divide the given angle . . B A C
into two equal parts.

PROPOSITION VIII.

At the End of a given right Line to make a rectilineal Angle equal to a given rectilineal Angle.

Let A be the end of the line A B, at which an angle is to be made equal to a given rectilineal angle . . . . . C D G

OPERATION.

Upon the angular point . . D
describe at pleasure the arc . . C G
Without altering the Opening of the Compasses.
Upon the extremity . . . A
describe the arc . . . H O
Make the arc . . . H E
equal to the arc . . . C G
draw the line . . . A E
The angle . . . . B A E
will be equal to the angle . . C D G
which was the thing proposed.

PROPOSITION IX.

To divide a given right Line into any Number of equal Parts required.

Let A B be the line proposed to be divided into six equal parts.

OPERATION.

From the point . . .	A
draw at pleasure the line . .	A C
through the extremity . .	B
Draw the line . . .	B D
parallel to the line . . .	A C
from the points . . .	A & B
and along the lines . .	A C, B D
Carry any six equal parts, viz. .	e f g h I L
along the line . . .	A C
R q p o n m along the line . .	B D
draw the lines, e n, f o, g p, h q, .	I R
then the line . . .	A B
will be divided into six equal parts at the	
Sections . . .	S, T, V, X, Y

PROPOSITION X.

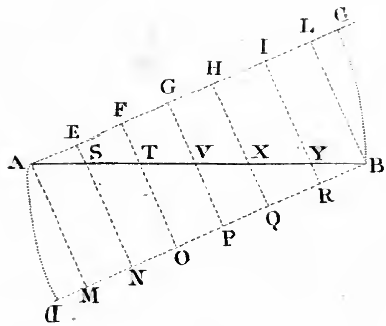
To draw a Tangent to a Circle proposed through a given Point.

Let A be the point through which the tangent to the circle D O P is to be drawn.

OPERATION.

From the centre of the circle . .	B
draw the secant . . .	B A
divide the line . . .	B A
into two equal parts in . .	C
upon the point . . .	C
with the radius . . .	C A
Describe the semicircle . .	A D B
cutting the circle in . . .	D
from the given point . .	A
Draw the right line . . .	A B
through the point . . .	D
This right line . . .	A B
will be the tangent required.	

IX



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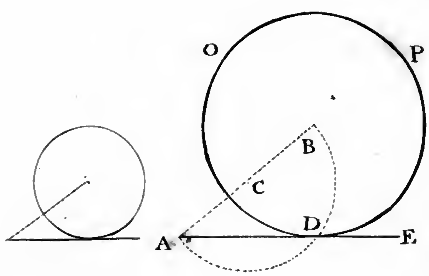
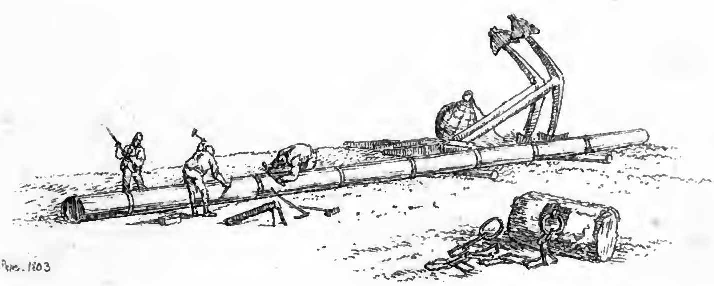
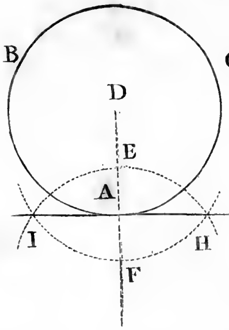




W.H.Pyne. 1803

XI

34 Pl 14



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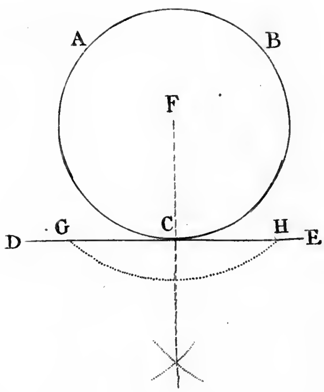
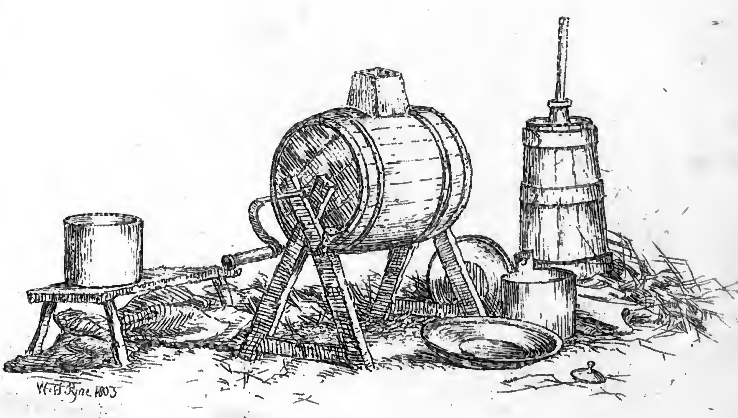




PROPOSITION XI.

To draw a right Line that shall be a Tangent to a Circle at a given Point.

Let A B C be the given circle, and the point of contact in its circumferenee A.

OPERATION.

From the point or centre . .	D
draw the line . . .	D F
through the point proposed . .	A
Through the point proposed . .	A
and to the line . . .	D F
draw the perpendicular . .	A H
continued towards . .	I
This tangent	H I
will touch the circle at the point .	A
which was the thing required.	

PROPOSITION XII.

A Circle and a right Line that touches it, being given, to find the Point of Contact.

Let A B C be the circle to which the line G H is a tangent.

OPERATION.

From the centre of the circle . .	F
let fall the perpendicular . .	F C
upon the tangent . . .	D E
The section	C
will be the point of contact sought.	

PROPOSITION XIII.

To draw a spiral Line about a given right Line.

Let I L be the line about which the spiral line is to be described.

OPERATION.

Divide half the right line . . I L
into as many equal parts as there are to be revolutions.

EXAMPLE.

To make one of four Revolutions.

Divide the half . . .	B I
into four equal parts .	B C E G I
Divide also	B C
into two equal parts in . .	A
upon the point . . .	A
Describe the semicircles	B C, D E, F G, H I
upon the point . . .	B
Describe the semicircles	C D, E F, G H, I L

and you will have the spiral line sought.

PROPOSITION XIV.

Between two given Points to find two others directly interposed.

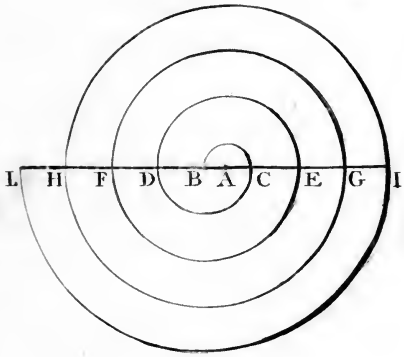
Let A & B be the points given, between which two others are to be found directly interposed, by the help of which a right line may be drawn from the point A to the point B, with a short ruler.

OPERATION.

Upon the points	A & B
as centres, make the intersections .	C & D
upon the points . . .	C & D
As centres make the intersections .	G & H
These points . . .	G & H

are the points required, by the assistance of which a right line may be drawn from the point A to the point B, which could not be done at once with a rule less than the length between . . . . . A B

XIII



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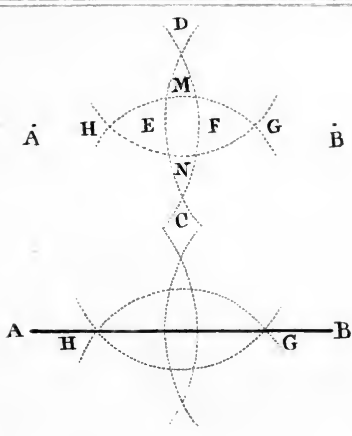

THE

SECOND BOOK.

OF THE

CONSTRUCTION OF PLANE FIGURES.

BOOK THE SECOND.

PROPOSITION I.

To make an equilateral Triangle upon a given Line.

Let A B be the given line upon which the equilateral triangle is to be constructed.

OPERATION.

Upon the extreme point	.	.	A
with the radius	.	. .	A B
Describe the arc	.	. .	B D
upon the extremity	.	.	B
with the radius	.	. .	B A
Describe the arc	.	. .	A E
from the intersection	.	.	C
Draw the lines	.	.	C A, C B

A B C will be the equilateral triangle required.

PROPOSITION II.

To make a Triangle whose three Sides are equal to three given right Lines.

Let A B C be the three given lines; a triangle is to be made whose three sides are equal to them.

OPERATION.

Draw the right line	.	. .	D E
equal to the line	.	. .	A A
upon the point	.	. .	D
with the radius	.	. .	B B
Describe the arc	.	. .	G F
upon the point	.	. .	E
with the radius	.	. .	C C
Describe the arc	.	. .	H I
from the intersection	.	.	O
Draw the lines	. .	.	O E, O D

The triangle	. .	. .	D E O
will be composed of three sides equal to			
the three lines given		.	A A, B B, C C

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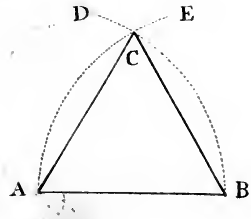
Pl. 16

I



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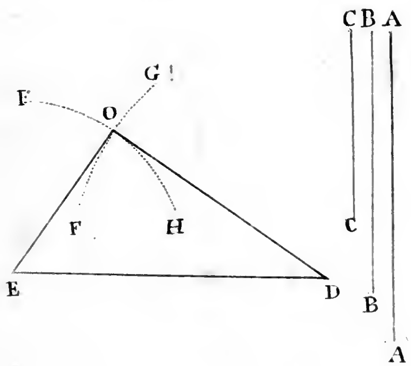

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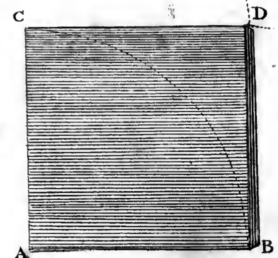

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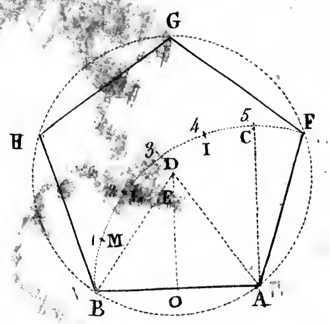


PROPOSITION III.

To make a Square upon a given right Line.

Let A B be the given right line, upon which the square is to be made.

OPERATION.

Erect the perpendicular . .	A C
upon the point . . .	A
As a centre, describe the arc . .	B C
upon the points . . .	B & C
with the radius . . .	A B
Make the section . . .	D
from the point . . .	D
Draw the lines . .	D C, D B

A B C D will be the square required to be constructed upon the given right line . . . . A B

PROPOSITION IV.

To make a regular Pentagon upon a given right Line.

Let A B be the given line, upon which the pentagon is to be constructed.

OPERATION.

Upon the extremity . . .	A
and with the radius . . .	A B
Describe the arc . . .	B D F
Erect the perpendicular . .	A C
Divide the arc . . .	B C
into five equal parts .	I D, L M
Draw the right line . . .	A D
Divide the base . . .	A B
into two equal parts in . .	O
Erect the perpendicular . .	O E
upon the intersection . .	E
with the radius . . .	E A
Describe the circle . .	A B F G H
Carry round five times, the line .	A B

in the circumference of the circle, and a regular equiangular equilateral pentagon will be completed.

PROPOSITION V.

To make a regular Hexagon upon a given right Line.

Let A B be a right line, upon which a regular hexagon is to be made.

OPERATION.

Upon the extremities . . .	A & B
and with the radius . . .	A B
Describe the arcs . .	A C, B C
upon the section . . .	C
Describe the circle . .	A B E F G
carry six times the line given .	A B
in the circumference, and you will have a	
regular hexagon . .	A B E F G D
upon the given line . . .	A B

PROPOSITION VI.

Upon a given right Line to describe any Polygon from an Hexagon to a Dodecagon.

Let A B be a line upon which an hexagon, heptagon, or octagon, &c. is to be made.

OPERATION.

Bisect the line A B in the point .	O
Erect the perpendicular . .	O I
upon the point B describe the arc .	A C
Divide A C in six equal parts	M, N, P, Q, R

This is to be done, if an Heptagon is to be made.

Upon the point C with the interval	
of one part . . .	C M
describe the arc . . .	M D

D will be the centre for describing a circle capable of containing seven times the line given.

For an Octagon.

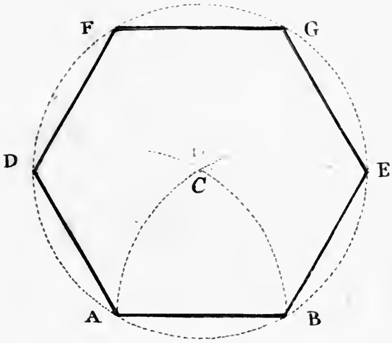
Upon the centre C, with the interval	
of two parts . . .	C N
Describe the arc . . .	N E
E will be the centre of a circle capable of containing eight times the given line .	A B

For an Heptagon.

Take three parts . . .	C P

And so for the rest, adding one part.

V



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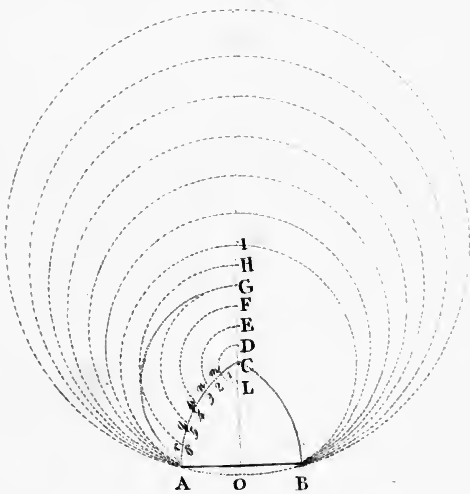
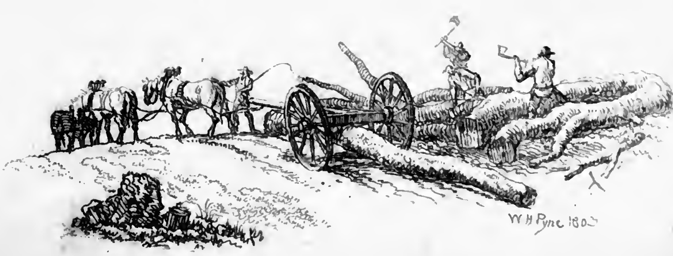
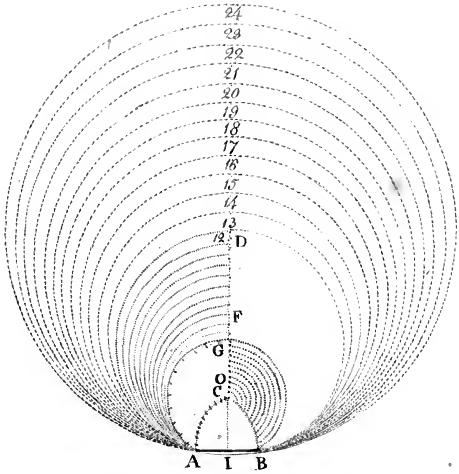




VII

45 Pl 19



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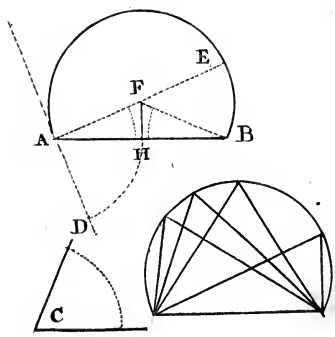

PROPOSITION VII.

To make a Polygon of any Number of Sides from twelve to twenty-four, upon a given right Line.

Let A B be the line, upon which the polygon is to be made.

OPERATION.

Divide the arc . . . A C
into twelve equal parts from the point C
Take as many of the parts of . . C A
as the number of the sides of the polygon is above twelve.

EXAMPLE.

If you would describe a Polygon of fifteen Sides.

Upon the point . . . C
with the radius of three of these parts C E
describe the arc . . . E O
A C of twelve, C O of three together, make fifteen.
Upon the point O with the radius . O B
describe the arc . . . B F
Upon the point F with the radius . F A
describe a circumference, and it will contain the
line given . . . . A B
fifteen times.
And so also for any other Polygon.

PROPOSITION VIII.

To describe a Portion of a Circle capable of containing an Angle equal to an Angle given upon a given right Line.

Let A B be the right line, upon which a portion of a circle capable of containing an angle equal to the given angle, is to be described, C.

OPERATION.

Make the angle . . . B A D
equal to the angle . . . C
Erect upon . . . . A D
the perpendicular . . . A E
Bisect the line . . . A B
in the point . . . H
Erect the perpendicular . . H F
upon the section . . . F
with the radius . . . F A
Describe the portion of the circle . A E B
All the angles you make in this segment of the
circle, and upon the given line . A B
will be equal to the angle . . C

PROPOSITION IX.

To find the Centre of a given Circle.

Let A B C be the circle proposed, whose centre is to be found.

OPERATION.

Draw at pleasure the right line . .	A B
terminating in the circumference .	A B C
Bisect the right line . . .	A B
by the line	D C
Bisect also the right line . .	C D
in the point . . .	F
The point F will be the centre of the circle required	A B C

PROPOSITION X.

To complete the Circumference of a Circle whose Centre is lost.

Let A B C be the part of the circumference given, whose centre is to be found, in order to the finishing the circle.

OPERATION.

Take at pleasure the three points .	A B C
in the circumference begun upon the points	A & B
Make the sections . . .	E & F
Draw the right line . . .	E F
upon the points . . .	B & C
Make the sections . . .	G & H
draw the right line . . .	G H
upon the intersection and centre .	I
and with the interval . .	I A
complete the circumference begun.	

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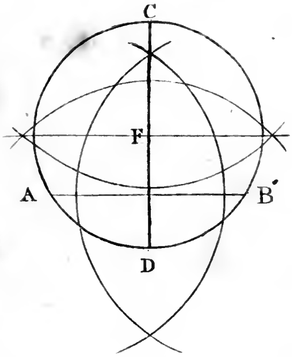
Pl 20

IX



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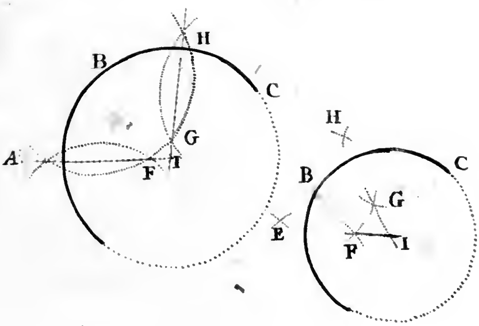
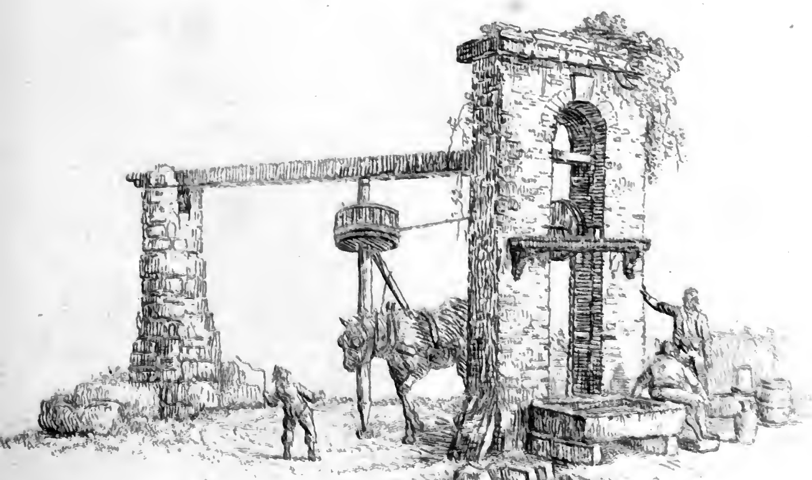
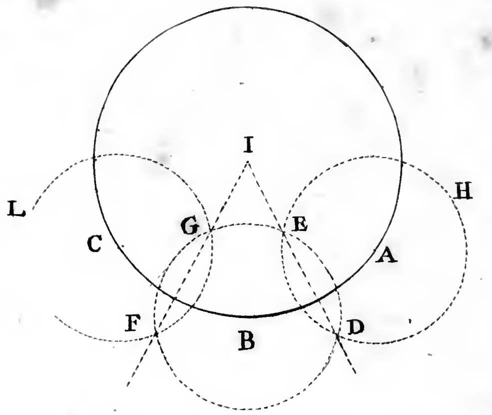




XI



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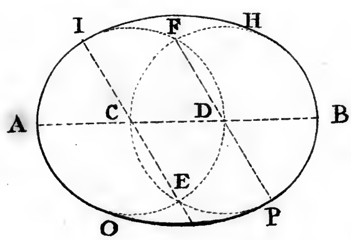
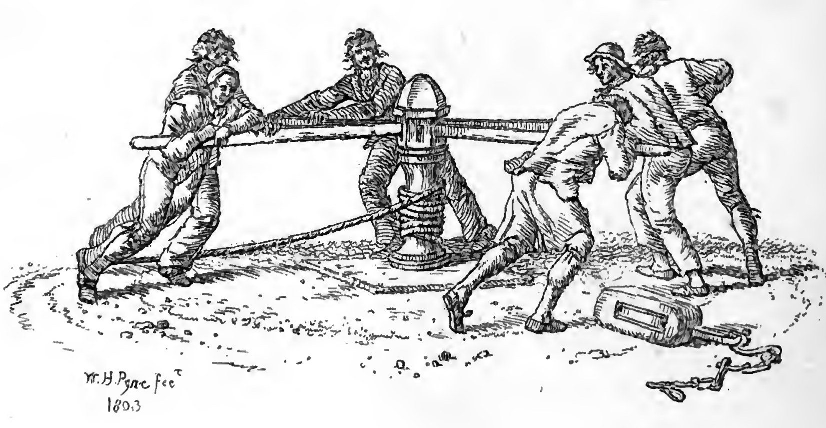




W. H. Pyne fec.t
1803

PROPOSITION XI.

To describe a Circle that shall pass through three given Points.

Let A, B, C, be the three points through which the circle is to pass.

OPERATION.

Upon the points given	A, B, C
describe three circles	D E H, D E F, F G L
with the same radius, and intersecting at	
the points	D & E, F & G
Draw the right lines	D E, F G
till they meet in	I
upon the point	I
with the radius	I A
Describe the circle required.	

This operation is similar to the preceding.

PROPOSITION XII.

To describe an Oval upon a given Length.

Let A B be the length upon which the oval is to be made

OPERATION.

Divide the length given	A B
into three equal parts	A C D B
upon the points	C & D
with the radius	C A
Describe the circles	A E F, B E F
upon the intersections	E & F
and with the diameter	E H
As a radius describe the arcs	I H, O P

A I H B P O will be the oval required.

PROPOSITION XIII.

To describe an Oval upon two given Diameters.

Let A B, C D, be the diameters upon which the oval is to be constructed.

OPERATION.

Make the ruler . . .	M O
equal to the great semidiameter .	A E
upon which mark the length .	M N
equal to the lesser semidiameter .	C E
This Ruler being thus disposed,	
Place it after such a manner upon the diameters	A B, C D
that the point . . .	N
sliding along the line . .	A B
the extremity . . .	O
may always be in the line . .	C D
carrying along thus the rule . .	M O
Describe the oval with the extremity .	M

PROPOSITION XIV.

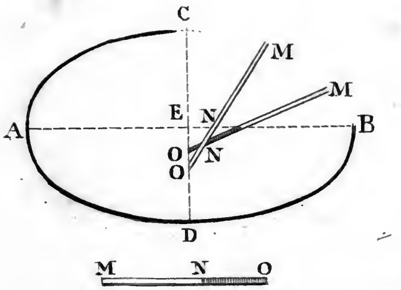
To find the Centre and the two Diameters of an Oval.

Let A B C D be the oval proposed, whose centre and diameters are to be found.

OPERATION.

In the oval proposed . .	A B C D
draw at pleasure . .	
the two parallel lines .	A N H I
Bisect the lines . .	A N H I
in the points . . .	L & M
Draw the line . .	P L M O
Bisect it in	E
and the point E will be the centre	
upon the point . . .	E
Describe at pleasure the circle . .	F G Q
cutting the oval in . . .	F & G
through the intersections . .	F & G
Draw the right line . . .	F G
Bisect it in	R
Draw the greatest diameter . .	B D
through the points . . .	E R
through the centre . . .	E
Draw the least diameter . .	A E C
parallel to the line . .	F G
and what was proposed will be effected.	

XIII



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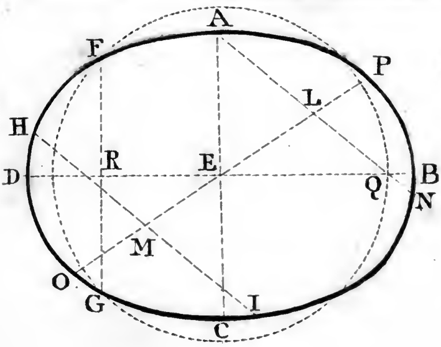

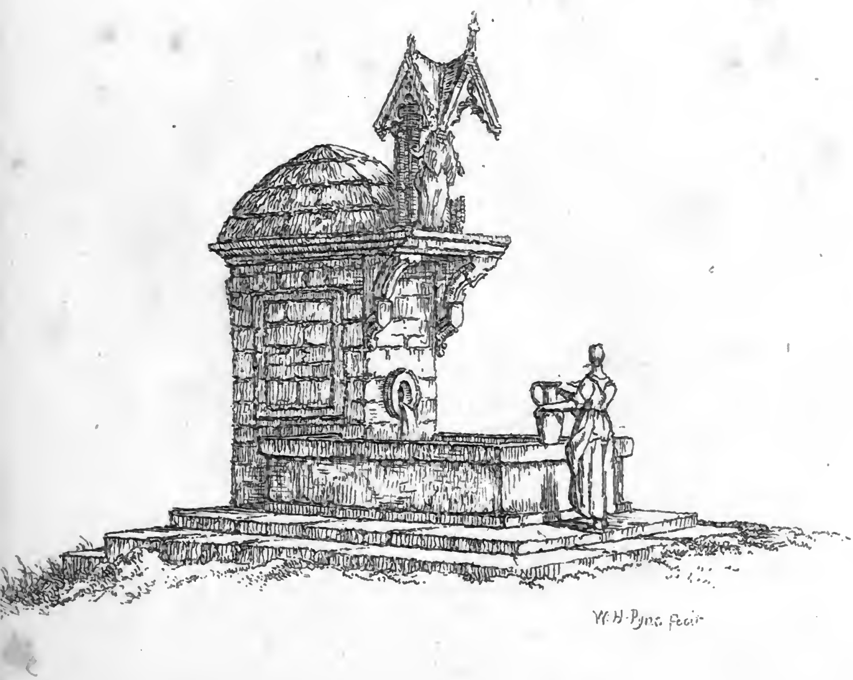

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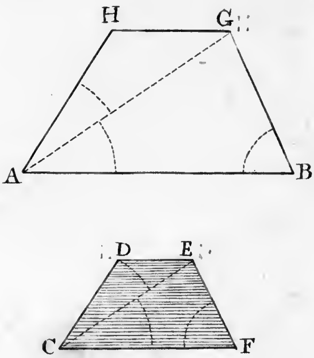
H
G
A
B
D
E
C
F


W H Pyne 1803

PROPOSITION XV.

To make a rectilineal Figure upon a given right Line similar to a given rectilineal Figure.

Let A B be the line upon which a figure similar to the figure C D E F is to be drawn.

OPERATION.

Draw the diagonal . . .	C E
make the angle . . .	A B G
equal to the angle . . .	C F E
Make the angle . . .	B A G
equal to the angle . . .	F C E
the triangle . . .	A B G
will be similar to the triangle . .	C F E

After the same Manner.

Make the triangle . . .	A G H
similar to the triangle . .	C E D
The whole figure . .	A B G H
will be similar to the whole figure	C D E F

THE

THIRD BOOK.

OF THE

INSCRIBING OF FIGURES.

BOOK THE THIRD.

PROPOSITION I.

To inscribe in a given Circle, an equilateral Triangle, Hexagon, or Dodecagon.

Let A C D be the circle in which an equilateral triangle, &c. is to be inscribed.

OPERATION.

For an equilateral Triangle.

Upon a point as . . .	A
with the interval of the semidiameter	A B
Describe an arc . . .	C B D
Draw the right line . . .	D C
Carry that distance . . .	C D
from the point . . .	C
to the point . . .	F
Draw the lines . .	F C, F D
the triangular required will be .	C D F

For an Hexagon.

Carry round six times the semidiameter in the given circumference.	A B

For a Dodecagon.

Bisect the arc of the hexagon . .	A C
in the point . . .	O
the side of the dodecagon will be .	A O

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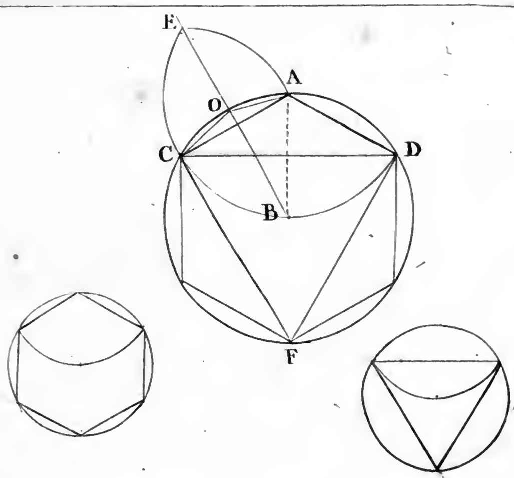

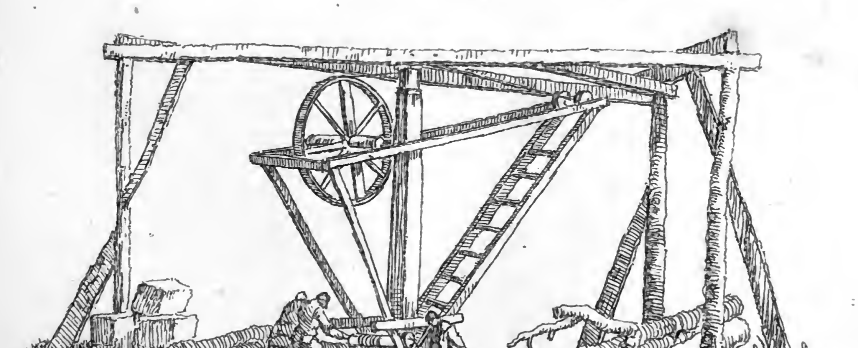

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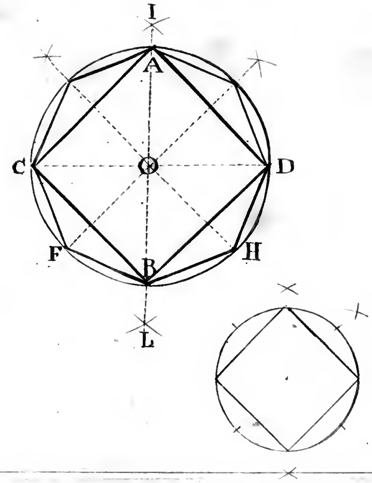
Pl 24

II



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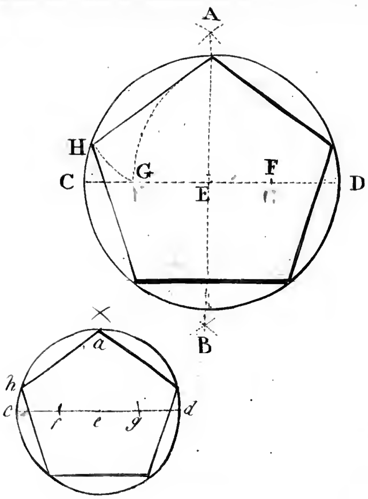
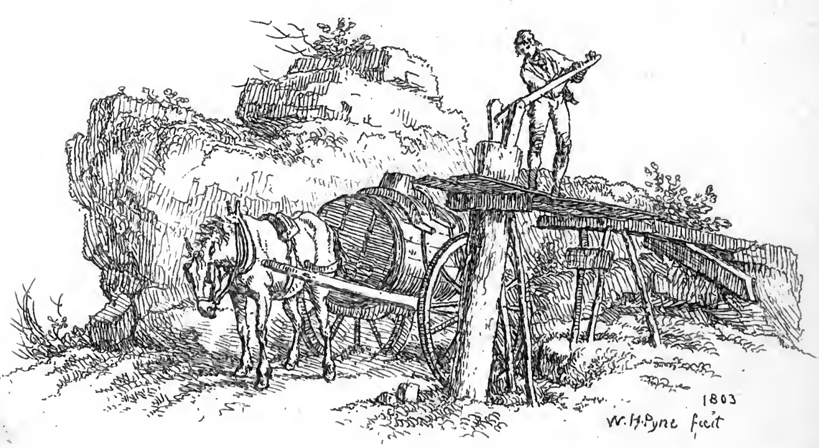




PROPOSITION II.

To inscribe a Square or Octagon in a given Circle.

Let A B C D be the circle in which the square or octagon is to be inscribed.

OPERATION.

For a Square.

Draw the diameters . .	A B C D
intersecting each other at right angles,	
that is, draw the right line . .	C D
through the centre of the circle .	O
Upon the points or extremities .	C & D
Make the intersections . .	I & L
Draw the right line . . .	I L
passing through the centre . .	O
These lines or diameters . .	A B, C D
will intersect at right angles.	

Draw the lines A C, A D, B C, B D, and A C B D will be the square required.

For an Octagon.

Subdivide each quarter of the circle into two equal parts, and you will have an octagon.

PROPOSITION III.

To inscribe a Pentagon or Decagon in a given Circle.

Let A B C D be the circle proposed.

OPERATION.

For a Pentagon.

Draw the two diameters .	A B, C D
intersecting each other at right angles in	E
Bisect the semidiameter . .	C E
in the point . . .	F
Upon the point . . .	F
as a centre, with the radius . .	F A
Describe the arc . . .	A G
upon the point . . .	A
with the radius . . .	A G
Describe the arc . . .	G H
The right line . . .	A H
will divide the circle into five equal parts.	

For the Decagon.

Subdivide each part of the circle into two equal parts.

PROPOSITION IV.

To inscribe an Heptagon in a given Circle.

Let A B C be the circle proposed in which the heptagon is to be inscribed.

OPERATION.

Draw the radius . . .	I A
upon the extremity . . .	A
with the radius . . .	A I
Describe the arc . . .	C I C
Draw the right line . . .	C C
Carry the half . . .	C O

seven times in the circumference of the circle, and the heptagon required will be inscribed.

PROPOSITION V.

To inscribe an Enneagon in a given Circle.

Let B C D be the circle in which the enneagon is to be inscribed.

OPERATION.

Draw the radius . . .	A B
upon the extremity . . .	B
with the distance . . .	B A
Describe the arc . . .	C A D
Draw the right line . . .	C D
produced towards . . .	F
Make the line . . .	E F
equal to the line . . .	A B
upon the point . . .	E
Describe the arc . . .	F G
upon the point . . .	F
Describe the arc . . .	E G
Draw the line . . .	A G
Then the ninth part of the circumference will be	D H

PROPOSITION VI.

To inscribe an Hendecagon in a given Circle.

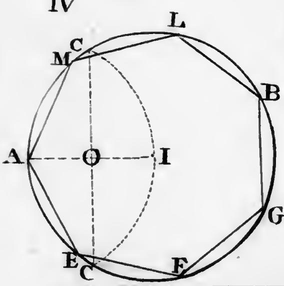
Let A E F be the given circle in which the hendecagon is to be inscribed.

OPERATION.

Draw the radius . . .	A B
bisect the radius . . .	A B
in the point . . .	C
upon the points . . .	A & C
with the distance . . .	A C
Describe the arcs . .	C D I, A D
upon the point . . .	I
with the distance . . .	I D
Describe the arc . . .	D O
the distance . . .	C O

will be the side of the hendecagon exact enough for practice.

IV



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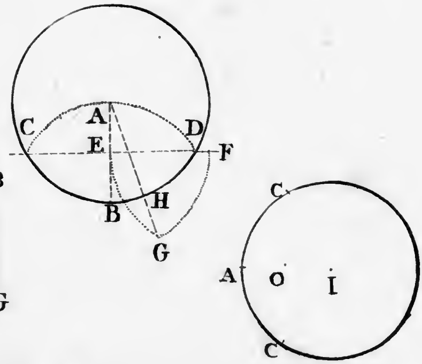

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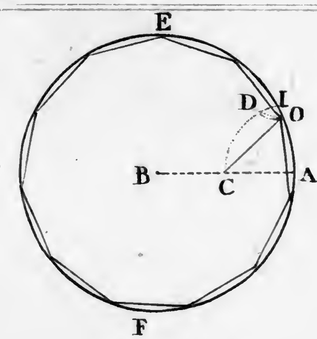

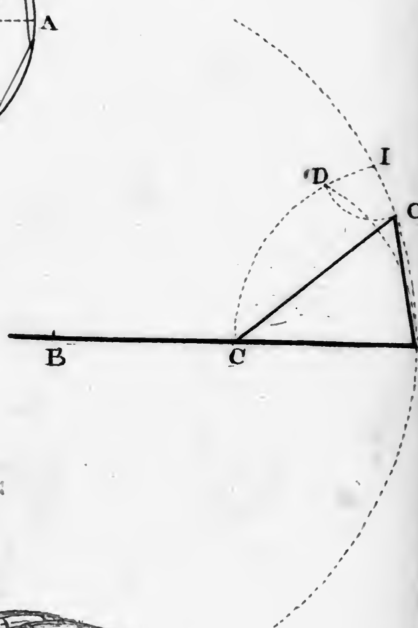

VII

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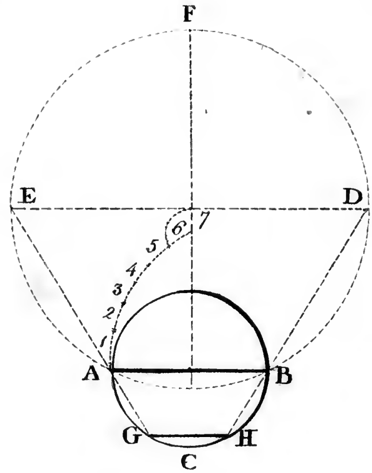
Pl. 26



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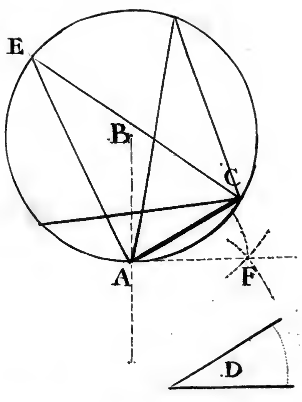
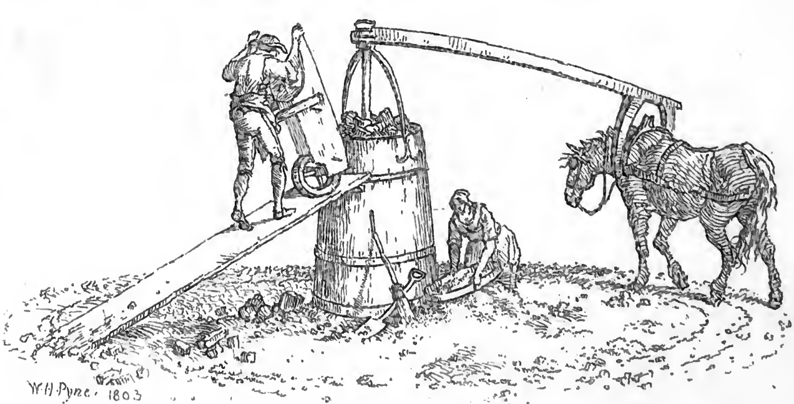




PROPOSITION VII.

To inscribe in a given Circle any Polygon you please.

Let B A C be the circle in which you would have an heptagon inscribed.

OPERATION.

Draw the diameter . . .	A B
describe the circle . . .	A B F
capable of containing seven times .	A B
After the same way as if you would make upon	A B
a polygon similar to that which is to be inscribed in the given circle . .	A B C
Draw the diameter . . .	D E
parallel to the diameter . .	A B
Draw the right lines . .	D A G, E B H
through the extremities .	D A, E B
G H, will divide the circle given	A B C
into seven equal parts.	
So in every other polygon.	

PROPOSITION VIII.

To cut off a Segment of a given Circle, capable of an Angle equal to any rectilineal Angle proposed.

Let A C E be the given circle, a portion of which is to be cut off, capable of an angle equal to the angle . . . . . D

OPERATION.

Draw the semidiameter . .	A B
draw the tangent . . .	A F
Make the angle . . .	F A C
equal to the given angle . .	D
All the angles made upon . .	A C
in the segment . . .	A E C
will be equal to the given angle .	D
Therefore the portion . .	A E C
is the segment required.	

PROPOSITION IX.

To inscribe in a Circle a Triangle similar to a given Triangle.

Let A B C be the circle in which a triangle similar to the triangle D E F is to be inscribed.

OPERATION.

Draw the tangent . . .	G H
upon the point of contact . .	A
Make the angle . . .	H A C
equal to the angle . . .	E
Make also the angle . . .	G A B
equal to the angle . . .	D
Draw the line . . .	B C

A B C will be the triangle required to be similar to the triangle given D E F

PROPOSITION X.

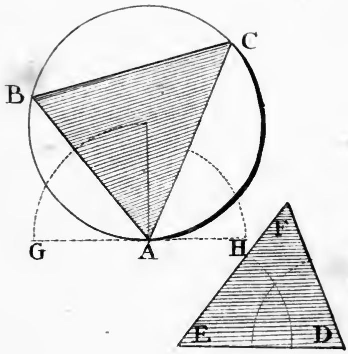
To inscribe a Circle in a given Triangle.

Let A B C be the triangle in which the circle is to be inscribed.

OPERATION.

Bisect the angles . . .	B & C
by the right lines . .	B D & C D
from the intersection . .	D
let fall the perpendicular . .	D F
Upon the centre . . .	D
with the distance . . .	D F
Describe the circle required . .	E F G

IX



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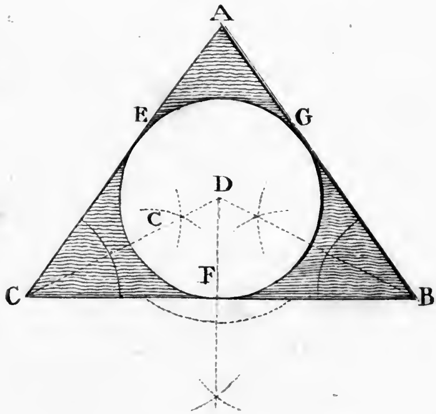

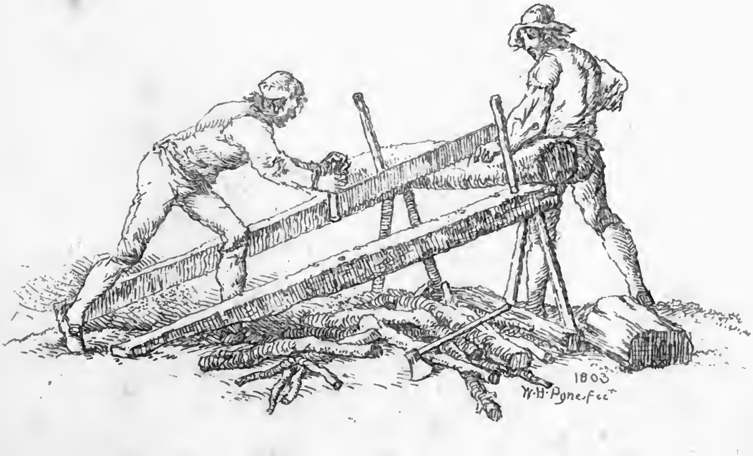

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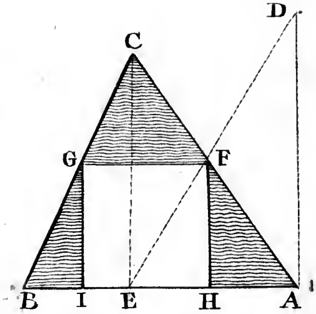
67 Pl 28



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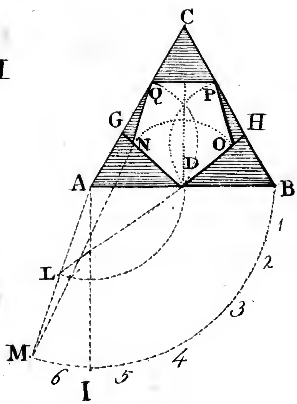


PROPOSITION XI.

To inscribe a Square in a given Triangle.

Let A B C be the triangle in which the square required is to be inscribed.

OPERATION.

Erect the perpendicular . .	A D
upon the extremity of the base .	A B
Make this perpendicular . .	A D
equal to the base . . .	A B
from the angle . . .	C
Draw the line . . .	C E
parallel to the line . . .	A D
Draw the oblique line . . .	D E
through the section . .	F
Draw the line . . .	F G
parallel to the base . .	A B
Draw the lines . .	F H, G I
parallel to the line . . .	C E
And the square required will be	F G H I

PROPOSITION XII.

To inscribe a regular Pentagon in an equilateral Triangle.

Let A B C be the triangle in which the pentagon is to be inscribed.

OPERATION.

Let fall the perpendicular . .	A I
upon the centre . . .	A
Describe the arc . . .	B I M
Divide into five equal parts the arc .	B I
Carry on the sixth . . .	I M
Draw the line . . .	A M
Divide	A M
into two equal parts . . .	L
Upon the point . . .	A
describe the arc . . .	L D
Draw the right line . .	L D to H
Make the part . . .	A G
equal to the part . . .	B H
Draw the right lines . .	D G, M C
upon the centre . . .	D
with the distance of the section .	N
Describe the arc . . .	N O
upon the points . . .	N O
Describe the arcs . .	D Q, D P
Draw the lines . .	O P, P Q, N Q
And then the pentagon demanded will be	D O P, Q N

PROPOSITION XIII.

To inscribe an equilateral Triangle in a Square.

Let A B C D be the square in which the equilateral triangle is to be inscribed.

OPERATION.

Draw the diagonals . .	A C, B D
upon the centre . .	. E
and with the distance . .	E A
Describe the circle . .	A B C D
upon the point . .	. C
with the distance . .	. C E
Describe the arc . .	G E F
Draw the right lines . .	A F, A G
Draw the right line . .	. H I
The equilateral triangle required is	A H I

PROPOSITION XIV.

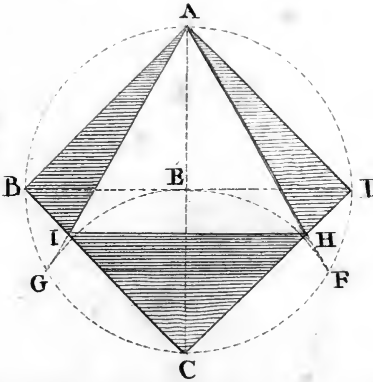
To inscribe an equilateral Triangle in a Pentagon.

Let A B C D E be the pentagon in which an equilateral triangle is to be inscribed.

OPERATION.

Circumscribe the circle .	A B C D E
upon the point . . .	A
and with the distance of the radius	A F
Describe the arc . . .	F L
Cut that arc . . .	F L
into two equal parts in . .	N
Draw the line . . .	F N I
upon the point . . .	A
with the distance . . .	A I
Describe the arc . . .	H O I
draw the lines . .	A H, H I
The triangle demanded will be .	A H I

XIII



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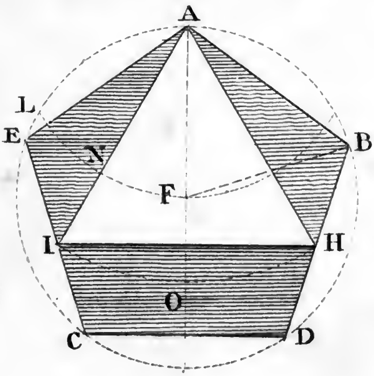

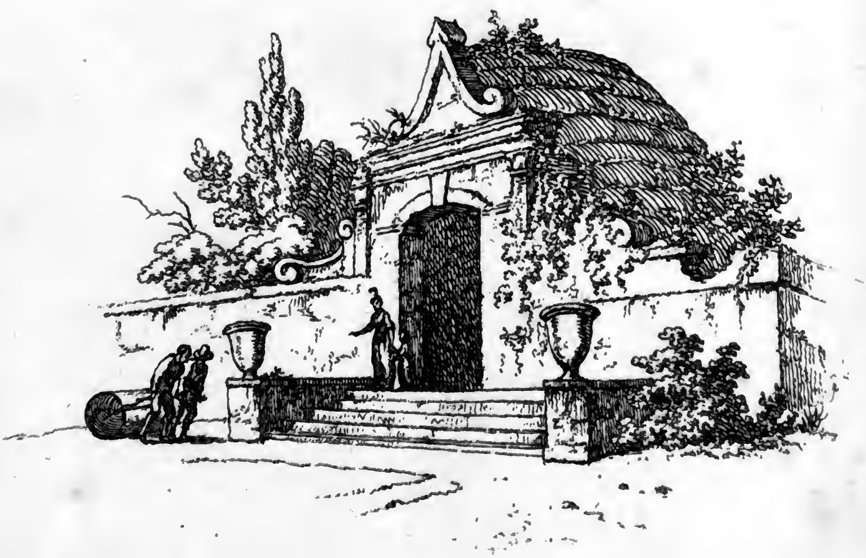

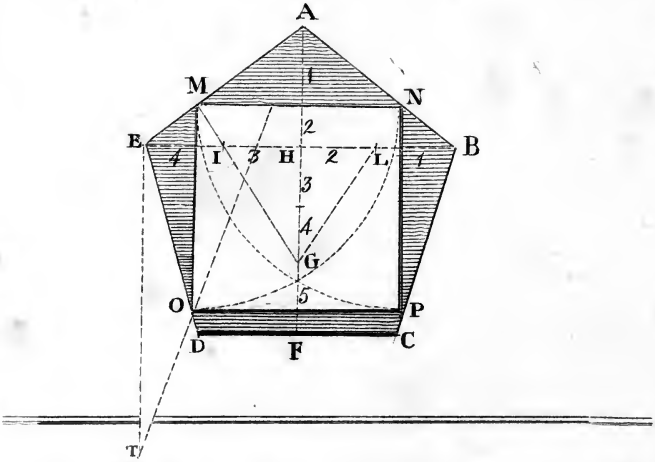

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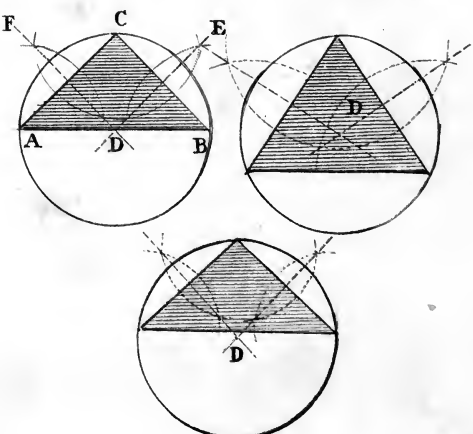

PROPOSITION XV.

To inscribe a Square in a Pentagon.

Let A B C D E be the pentagon in which a square is to be inscribed.

OPERATION.

Draw the line . . .	B E
let fall the perpendicular . .	E T
from the extremity of . .	B E
Make this perpendicular . .	E T
equal to the line . . .	B E
Draw the line . . .	A T
through the section . .	O
Draw the line . . .	O P
parallel to the side . .	C D
On the extremities . .	O & P
erect the perpendiculars . .	O M, P N
Draw the line . . .	N M
The square required will be . .	N M O P

BOOK THE FOURTH.

OF THE CONSCRIPTION OF FIGURES.

PROPOSITION I.

To circumscribe a Circle about a given Triangle.

Let A B C be the triangle about which the circle is to be circumscribed.

OPERATION.

Describe the circumference . .	A B C
through the three points . .	A, B, C

and the thing required will be done.

PROPOSITION II.

To circumscribe a Circle about a Square.

Let A B C D be the square about which the circle is to be circumscribed.

OPERATION.

Draw the two diagonals .	A B, C D
upon the intersection or centre .	G
with the distance . . .	G A
Describe the circle demanded .	A B C D

PROPOSITION III.

To circumscribe a Triangle similar to a given Triangle, about a given Circle.

Let D E V be the circle, about which a triangle, similar to the triangle F G H, is to be described.

OPERATION.

Draw the diameter . . .	A B
through the centre . . .	C
Make the angle . . .	A C E
equal to the angle . . .	H
Make the angle . . .	B C D
equal to the angle . . .	G
Produce the lines . .	E C, D C
towards	R & S
Draw the tangent . . .	N O
parallel to the line . . .	D R
Draw the tangent . . .	O I
parallel to the line . . .	E S
Draw also the tangent . . .	N I
parallel to the diameter . .	A B

I N O will be the triangle required similar to the triangle F G H, and circumscribed about the circle D E V.

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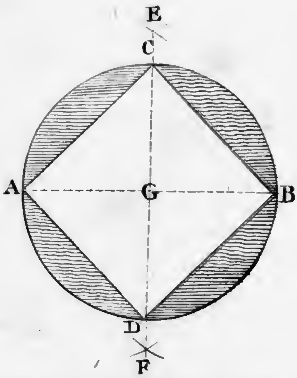
Pl. 31

II



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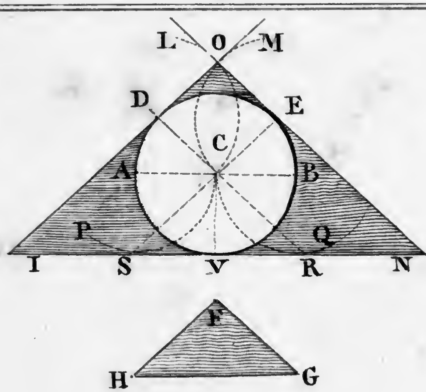

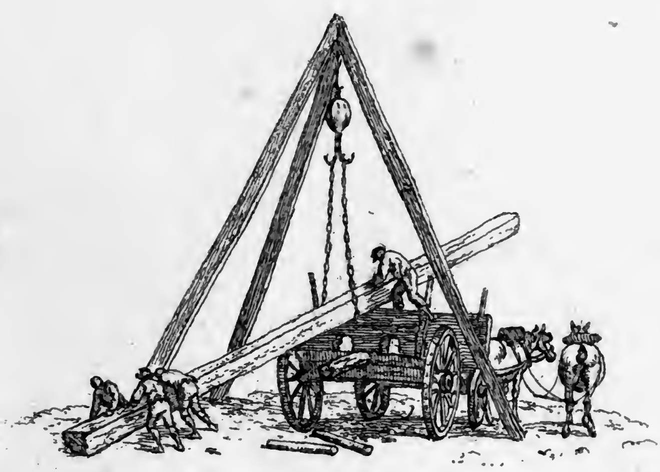

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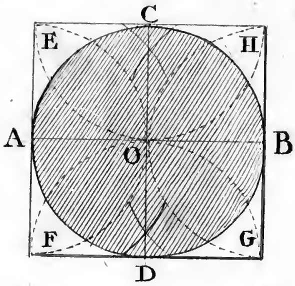

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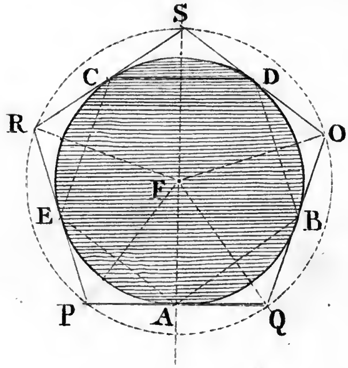
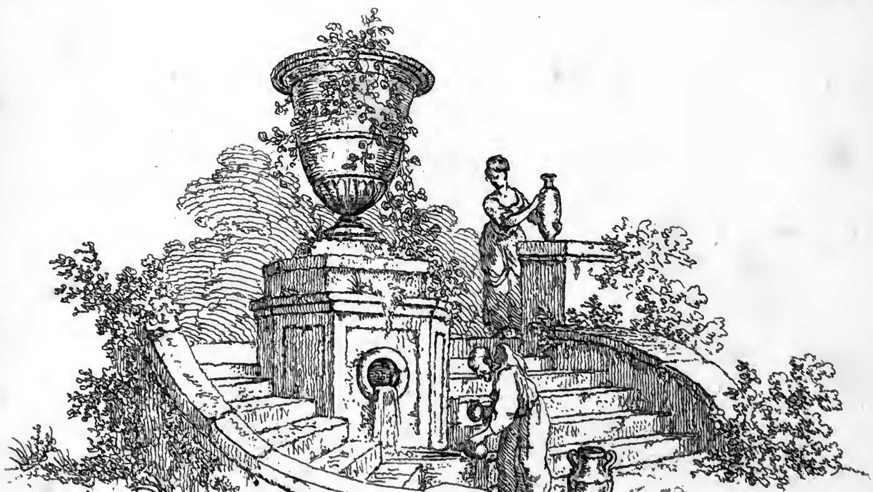




PROPOSITION IV.

To circumscribe a Square about a Circle.

Let A B C D be the circle about which a square is to be circumscribed.

OPERATION.

Draw the diameters . .	A B, C D
intersecting each other at right angles in	O
upon the points . .	A, C, B, D
with the distance . .	A O
Describe the semicircles .	H O G, H O E
	E O F, F O G
Draw the right lines	E F, F G, G H, H E
through the intersections	E, F, G, H
The square demanded will be	E F G H

PROPOSITION V.

To circumscribe a Pentagon about a given Circle.

Let A B C D E be the given circle about which a pentagon is to be circumscribed.

OPERATION.

Inscribe the pentagon . .	A B C D E
upon the centre . . .	F
and through the middle of each side	
Draw the lines .	F O, F P, F Q, F R, F S
Draw the line	F A
Draw the tangent . . .	P Q
through the point . . .	A
Upon the centre . . .	F
with the radius . . .	F P
Describe the circle . .	O P Q R S
Draw the sides of the pentagon demanded through	
the sections .	O, P, Q, R, S

PROPOSITION VI.

To circumscribe a regular Polygon about another of the same Sort.

Let B C D E F G be the polygon given, about which another similar polygon is to be circumscribed.

OPERATION.

Produce two sides as . .	B G, E F
until they meet in . . .	H
Draw the line . . .	A H
Draw the line . . .	E I
bisecting the angle . .	G F H
upon the centre . . .	A
with the distance . .	A I
Describe the arc . . .	I M O
Draw the radius's	A L A M, A N, A O
through the middle of each side.	
Draw the sides of the exterior polygon demanded,	
through the sections	I, L, M, N, O, P

PROPOSITION VII.

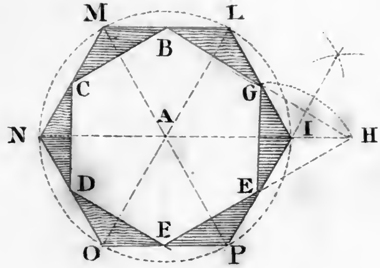
To circumscribe a Square about a given equilateral Triangle.

Let A B C be the equilateral triangle, about which a square is to be circumscribed.

OPERATION.

Bisect the base . . .	B C
in the point . . .	E
Produce the base . . .	B C
both ways towards . .	D & D
Make the lines . .	E D & E D
equal to the line . . .	E A
Upon the point . . .	E
with the distance . .	E C
Describe the semicircle . .	B F C
draw the line . . .	A E F
From the point . . .	F
draw the lines . .	F C G & F B G
and the square required will be	A G F G

VI



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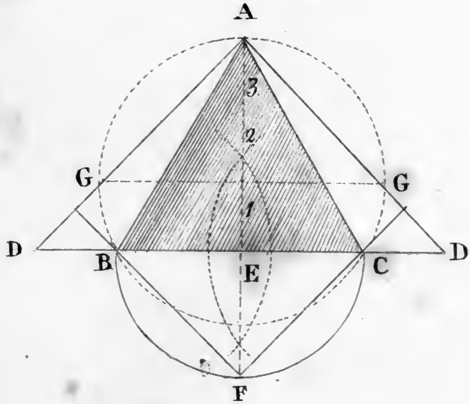

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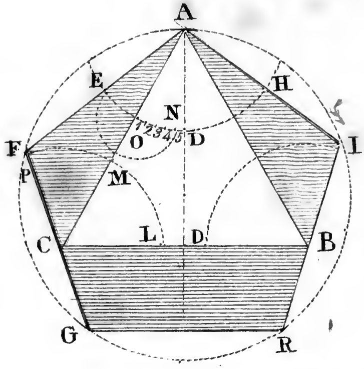
Pl. 34

VIII



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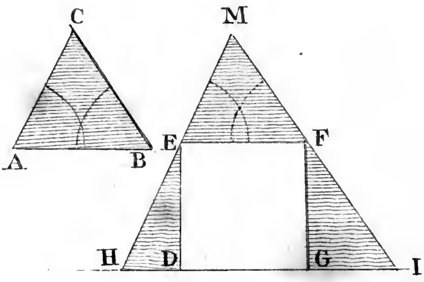
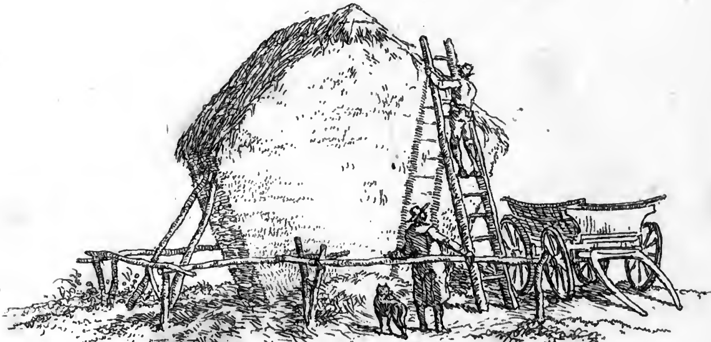




PROPOSITION VIII.

To circumscribe a Pentagon about an equilateral Triangle.

Let A B C be the triangle given, about which a pentagon is to be circumscribed.

OPERATION.

Upon the points or angles . . A, B, C
and with the same opening of the compasses, describe at pleasure the arcs . D E, L P
Divide the arc . . . D O
into five equal parts . 1, 2, 3, 4, 5
upon the centre or section . O
And with the distance of four parts . O N
describe the arc . . . N M E
Draw the right line . . . A E F
Cut off the arc . . . M P
equal to the arc . . . E N
Draw the right line . . . f P c g
equal to the line . . . f A
Make the arc . . . D H
equal to the arc . . . D E
Draw the sides . . . A I, I R
equal to the sides . . . A f, f G
The side . . . . I R
will complete the pentagon demanded.

PROPOSITION IX.

To circumscribe a Triangle similar to a given Triangle, about a Square.

Let D E F G be the square about which a triangle is to be circumscribed similar to the triangle A B C.

OPERATION.

Make the angle . . . E F M
equal to the angle . . . A
Make the angle . . . M E F
equal to the angle . . . B
Produce the lines . . M E, M F, D G
towards . . . . I & H
H I M will be the triangle required, similar to the triangle . . . . A B C
and circumscribed about the square D E F G

PROPOSITION X.

To circumscribe a Pentagon about a Square.

Let A B C D be the square about which a pentagon is to be circumscribed.

OPERATION.

Produce the side . . .	C B
towards . . .	N
Bisect the side . . .	A B
in the point . . .	R
Erect the perpendicular . .	R V
upon the points . . .	B, D, C
with the distance . . .	B R
Describe the arcs . .	R N, S T, S T
Divide the arc . . .	R N
into five equal parts .	R H, G F, E N
Make the angle . . .	R B V
with the distance of two parts .	R G
Make the angles . .	S C T, S D T
with the distance of one part .	R H
Produce the lines . .	V B, C T, to O
Make the line . . .	O Q
equal to the line . . .	O V

Draw the other sides after the same manner, and you will have the thing required.

BOOK THE FIFTH.

OF PROPORTIONAL LINES.

PROPOSITION I.

To find a Mean proportional between two given Lines.

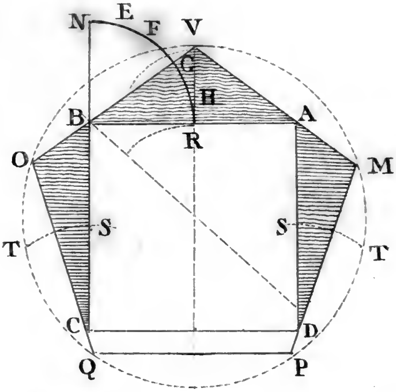
Let A & B be the two lines between which a mean proportional is to be found.

OPERATION.

Draw an indetermined line . .	G H
Make	C E
equal to the line . . .	A
Make	E D
equal to the line . . .	B
Bisect	C D
in the point . . .	I
upon the point . . .	I
and with the distance . .	I C
Describe the semicircle . .	C F D
Erect the perpendicular . .	E F
This line	E F
will be a mean proportional between	A & B

X

Pl = 35



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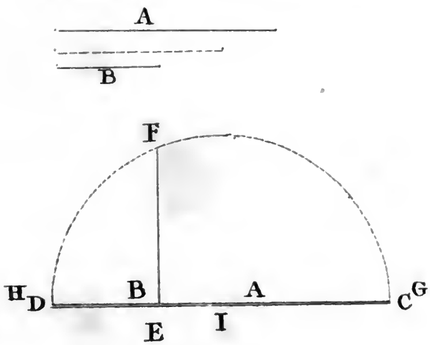

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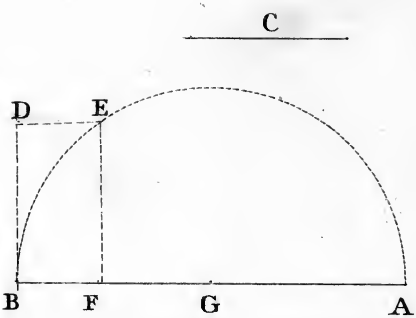

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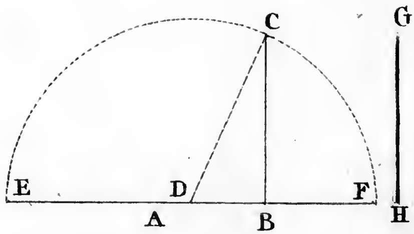


W. H. Pyne 1803

PROPOSITION II.

Given the Sum of the Extremes and the mean Proportional, to distinguish the Means.

Let A B be the sum of the extremes (i. e. the two magnitudes connected without any distinction), and C the mean proportional, by whose assistance the point, where the extremes join, is to be distinguished.

OPERATION.

Bisect the line . . .	A B
in the point . . .	G
upon the point . . .	G
with the interval . . .	G A
Describe the semicircle . .	A E B
Erect the perpendicular . .	B D
equal to the mean proportional .	C
Draw the line . . .	D E
parallel to the line . .	A B
from the section . . .	E
Draw the line . . .	E F
parallel to the line . .	B D
Then will the point where the extremes join be	F
so that C or its equal . .	E F
shall be a mean proportional between	A F & B F

PROPOSITION III.

Given the Mean of three Proportionals and the Difference of the Extremes, to find the Extremes.

Let G H be the mean proportional, and A B the difference of the extremes, required the length of the extremes.

OPERATION.

Erect the perpendicular . .	B C
at the extremity of the difference .	A B
and equal to the mean . .	G H
Bisect the difference . . .	A B
in the point . . .	D
Produce both ways towards . .	E & F
upon the point . . .	D
with the distance . . .	D C
Describe the semicircle . .	E C F
The extremes required will be .	B E, B F

PROPOSITION IV.

Two right Lines being given, to find a third Proportional.

A B, A C, are the two given right lines, to which a third proportional is to be found.

OPERATION.

Make at pleasure the angle . .	D N E
cut off the part . . .	N H
equal to the line . . .	A B
Cut off the part . . .	N O
equal to the line . . .	A C
Draw the line . . .	H O
Draw the line . . .	D E
parallel to the line . . .	H O

E O will be the third proportional required.

PROPOSITION V.

To cut off from a given Line, a Part that shall be a mean Proportional between what remains and another given right Line.

Let A A be the line, of which a part is to be cut off, that shall be a mean proportional between what remains and the line proposed B B.

OPERATION.

Draw the indefinite line . .	C D
cut off the lines . .	D E, E C
equal to the lines . .	A A & B B
Describe the semicircle . . .	C F D
Erect the perpendicular . .	E F
Bisect the lines . . .	C E
in the point . . .	B
upon the point . . .	B
with the distance . . .	B F
Describe the arc . . .	F G
Cut off the part demanded . .	A H
equal to the part . . .	E G

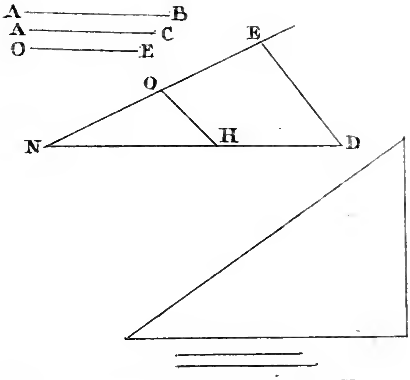
A H will be the mean proportional between the

remainder	H I
and the other line proposed . .	B E

IV

85

Pl. 37



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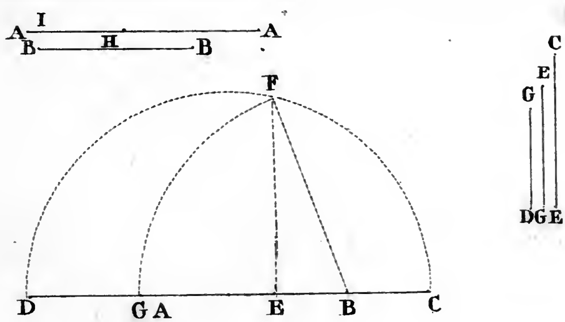

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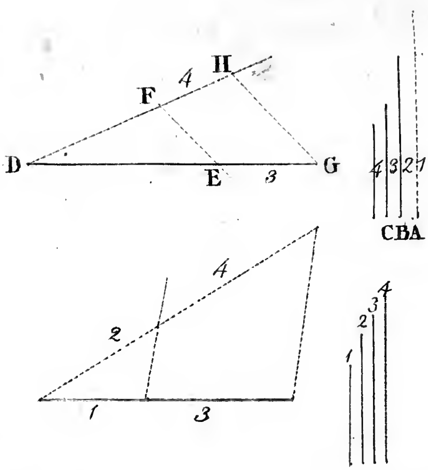

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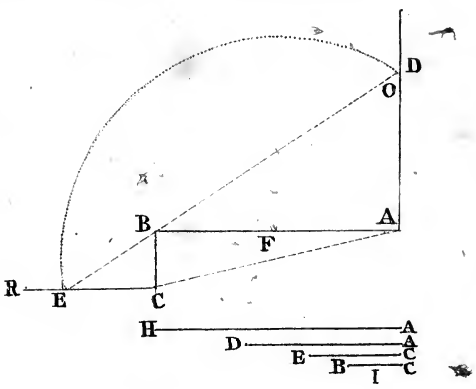
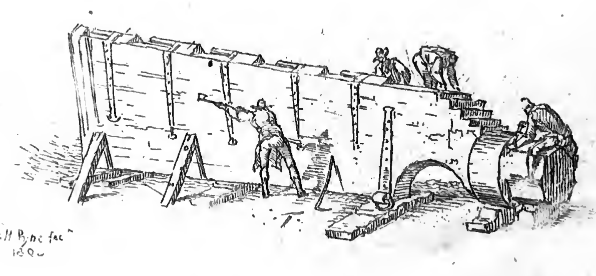




PROPOSITION VI.

To find a fourth Proportional.

A, B, C, are three equal lines proposed; a fourth is to be found, which will be to the third just as the second is to the first.

OPERATION.

Make at pleasure the angle . . .	G D H
cut off the part . . .	D E
equal to the line . . .	A
Cut off the part . . .	D F
equal to the line . . .	B
Cut off the part . . .	E G
equal to the line . . .	C
Draw the line . . .	E F
Draw the line . . .	G H
parallel to the line . . .	E F

F H will be the fourth proportional demanded.

PROPOSITION VII.

To find two mean Proportionals between two given Lines.

Let I & H be the lines proposed, between which two mean proportionals are to be found.

OPERATION.

Draw the line . . .	A B
equal to the line . . .	H
Let fall the perpendicular . .	B C
equal to the line . . .	I
Draw the line . . .	A C
Bisect the line . . .	A C
in the point . . .	F
Erect the perpendiculars .	A O, C R
upon the point or centre . .	F
Describe the arc . . .	D E
so that the cord . . .	D E
may touch the angle . .	B

A D, C E, will be the mean proportionals between the given lines . . . I & H

PROPOSITION VIII.

To cut two given Lines, each into two Parts, so as that the four Segments may be proportional.

A B, A C, are the lines proposed to be cut according to the proposition.

OPERATION.

Make the right angle	B O C
Cut off the line	B O
equal to the line	A B
Cut off the line	O C
equal to the line	A C
Draw the hypothenuse	B C
Describe the semicircle	B D O
from the section	D
Draw the line	D E
parallel to the line	C O
and the line	D F
parallel to the line	E O
A B will be cut in	E
O C also in	F
so that B E will be to	D
as E D to	D F, & E D
to D F, as D F is to	F C

PROPOSITION IX.

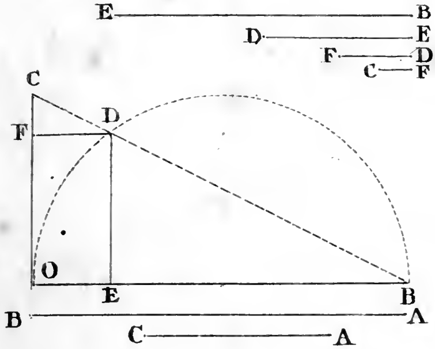
The Excess of the Diagonal of a Square above the Side, being given, to find its Side.

Let A B be the excess of the diagonal of a square above its side, to find its magnitude.

OPERATION.

Erect the perpendicular	B C
equal to the excess	A B
Draw the line	A C
produced towards	D
upon the point	C
and with the distance	C B
Describe the arc	B D
A D will be the side of the square	A
the excess	A B
of whose diagonal	A E
above the said side	A D

VIII



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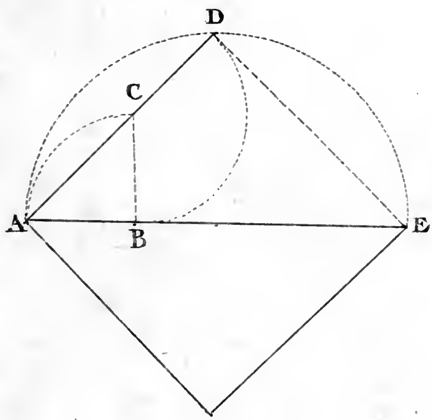

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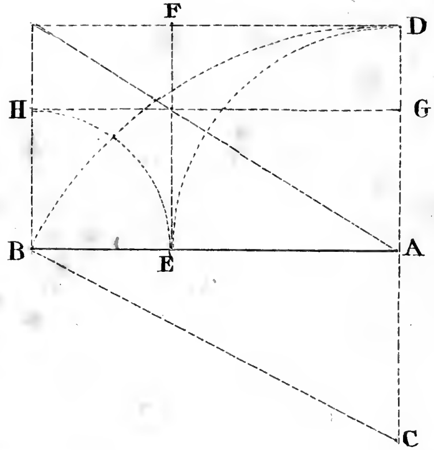

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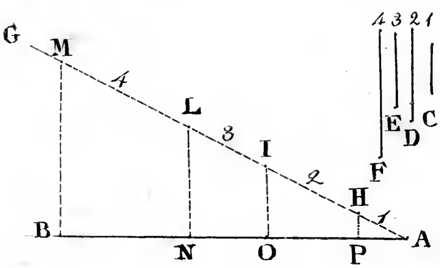
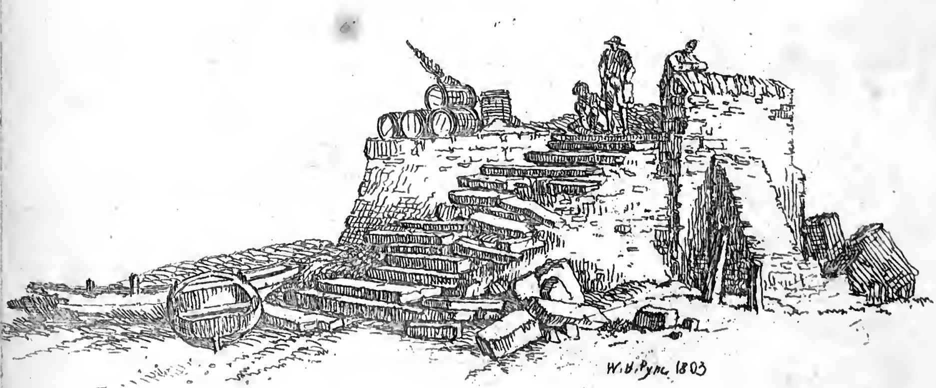




PROPOSITION X.

To cut a given finite Line in extreme and mean Proportion.

Let A B be the line that is to be cut, so that the rectangle of the whole line, and one of the parts, may be equal to the square of the other.

OPERATION.

Erect the perpendicular . .	A D
produce it towards . . .	C
Make	A C
equal to half . . .	A B
Upon the point . . .	C
and with the distance . .	C B
Describe the arc . . .	B D
upon the point . . .	A
with the distance . . .	A D
Describe the arc . . .	D E
The line	A B
will be cut in the point . .	E

in the proportion required: for if you make the rectangle A h of the whole A B and part B E, it will be equal to the square A f made upon the other part . . . . A E

PROPOSITION XI.

To divide a given right Line in any Ratio proposed.

Let A B be the line proposed to be divided according to the ratios of C, D, E, F.

OPERATION.

Upon the point or extremity . .	A
Draw at pleasure the line . .	A G
Make	A H
equal to the line or ratio . .	C
Make	H I
equal to the line . . .	D
Make	I L
equal to the line . . .	E
Make	I M
equal to the line . . .	F
Draw the line . . .	B M
Draw the lines . .	L N, I O, H P
parallel to the line . . .	B M
The line A B will be divided in the points	P, O, N

according to the ratio demanded.

PROPOSITION XII.

To make upon a given right Line two Rectangles, that shall be in any given Ratio to one another.

Let A B be the line upon which two rectangles are to be made, which shall be to one another as C to D.

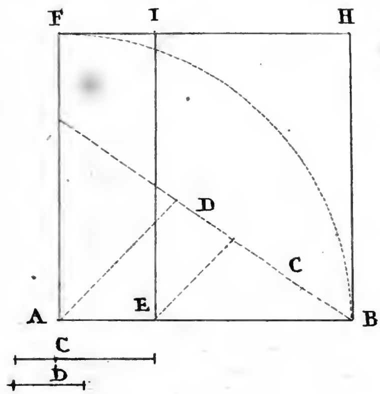
OPERATION.

Divide the right line . . .	A B
at the point	E
in the ratio of . .	C to D
Make the square . .	A B H F
Draw the line . . .	E I
parallel to the line . . .	A F

B E I H, A E I F, will be the rectangle required.

The rectangle . . .	A I
is to the rectangle . . .	E H
as the line	D
is to the line . . .	C

XII



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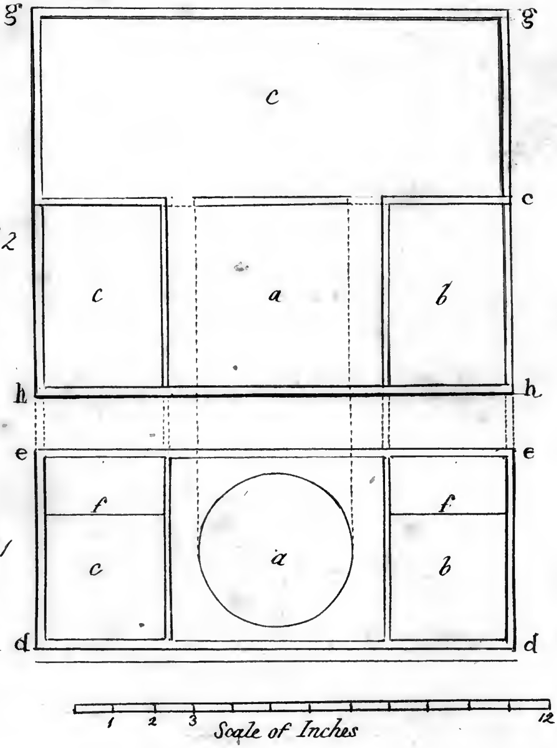

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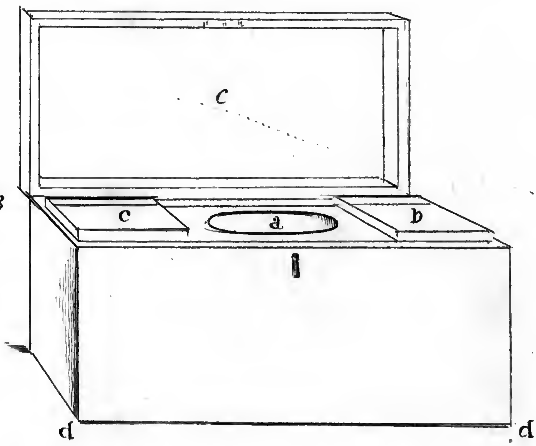


PROPOSITION XIII.

To make a geometrical Plan and Elevation.

The subject fixed on in this plate is a tea-caddy.

No. 1 Shews the geometrical plan.
2 The geometrical elevation.
3 The tea-caddy open in perspective.

OPERATION.

Make a scale of inches the size you wish to introduce on paper, measure the length of the tea-caddy with a common ruler, then take as many parts of your scale as it measures.

Set it on the line	d d
Measure the side of the tea-caddy, and fix it upon the line .	d e, d e
With the number of parts answering to the measurement,	
Measure the canister	b c
Measure the sugar-glass	a
Make the line f f parallel to	e e
Take the height of the tea-caddy open, and make the line .	g g
Raise the perpendicular on each side	a b c
Take the width e d, and mark it upon the line . .	g g, h h

You will then have your geometrical plan, No. 1, your geometrical elevation, No. 2, with the lid open, as seen in No. 3, designed purposely to shew the difference of a geometrical plan and the same subject in perspective.

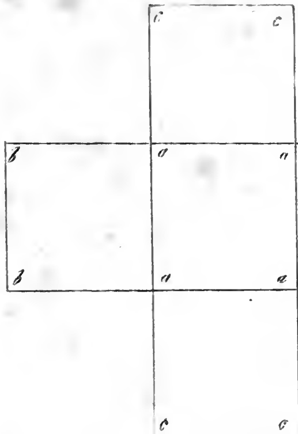
PROPOSITION XIV.

Containing a double Cross.

No. 1 is seen in perspective.
2, The geometrical plan.
3, The geometrical elevation.

No. 3, a a a a represents the centre of the elevation.

No. 2, a a a a the geometrical plan of a a a a.
c c c c the geometrical plan of the elevation c c c c.
b b b b the geometrical plan of the cross beam b b b b.



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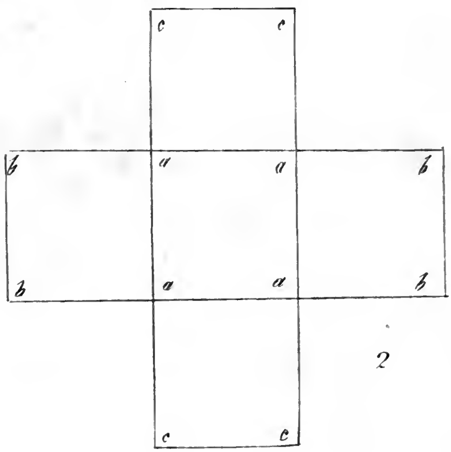

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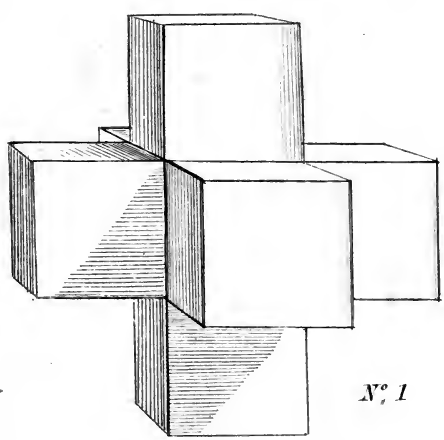

N.º 1

XV

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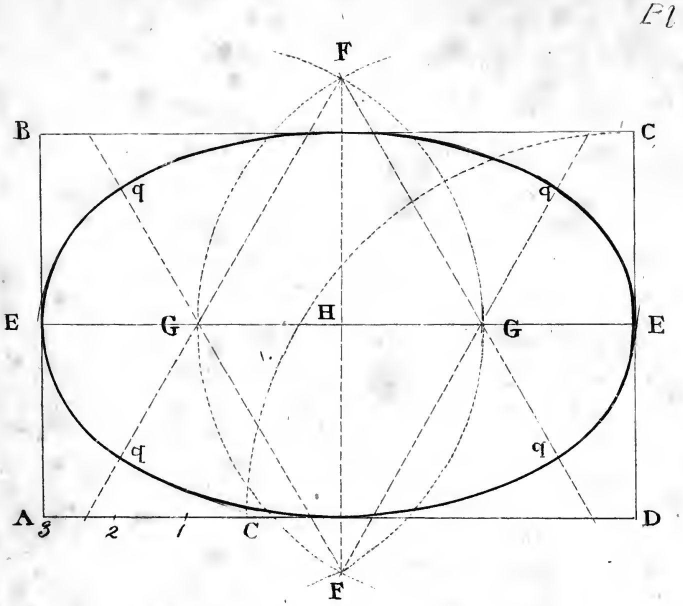
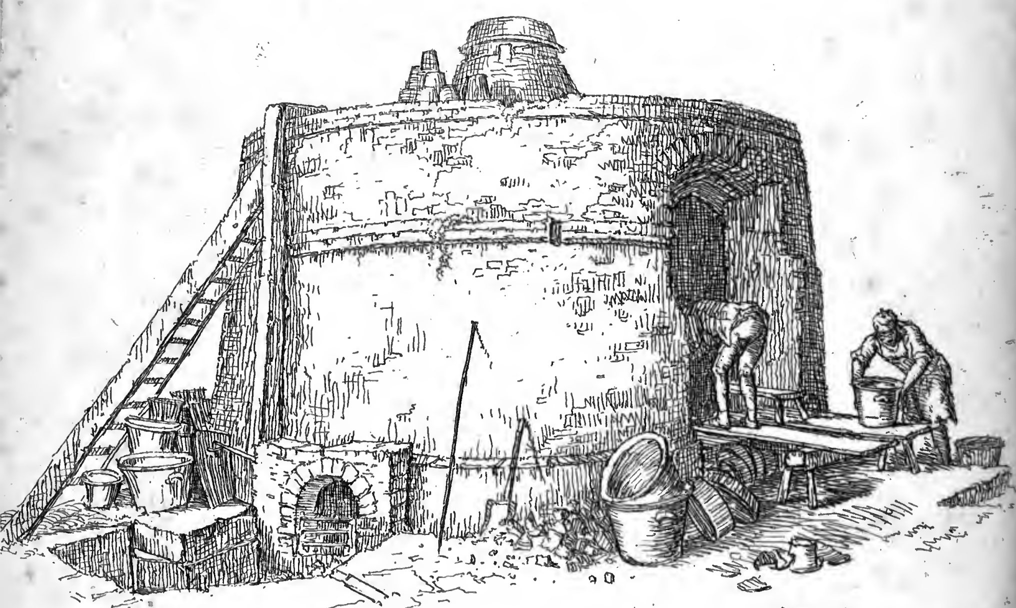
Pl 44





PROPOSITION XV.

To make an Oval of any given Dimension.

Let A B C D be the given size.

Draw the two centres . . E E F F
Draw the semicircle . . . C C
Upon the line . . . A D
Divide the remainder of the line A D in three equal parts, ... 1, 2, 3
Take two of those parts, and mark it on each side of the centre . . . H
to . . . . G G
Open the compass to G G, and make the intersection F F, draw the four radii F G q, F G q.
With the compass open G E, G E, draw part of the circle . . . q E q, q E q
With the compass open to F q, join the circle q q q q

THE END.

S. Gosnell, Printer, Little Queen Street, Holborn.