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# The "Natural" Rate of Unemployment in Historical Perspective

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FACULTY WORKING PAPER NO. 92-0122


College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

April 1992

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April 6, 1992

THE "NATURAL" RATE OF UNEMPLOYMENT IN HISTORICAL PERSPECTIVE

By Hans Brems

Abstract

*Between the seventeenth and the mid-eighteenth century macroeconomic theory reversed itself from a demand-side to a supply-side equilibrium. The reversal repeated itself in the last half of our own century. The first half of the paper will derive such a reversal as succinctly as possible. Here at a short-run frozen capital stock labor can have a higher real wage rate at a lower natural rate of employment. The second half of the paper will examine what difference it makes if capital stock is unfrozen: at a long-run unfrozen capital stock labor can have a no higher real wage rate at a lower natural rate of employment.*





## I. INTRODUCTION

### 1. Macroeconomics--The Oldest Part of Our Building

Macroeconomics considers an economy producing a single good. Unemployment theory determines the physical output of that good, inflation theory determines its price. Macroeconomics is the oldest part of our building: we have practiced it since the last half of the seventeenth century.

### 2. Petty-Yarranton

The concern of the mercantilists was unemployment. Petty [1662 (1899: 30)] estimated it at ten percent and analyzed it within the framework of a demand-side equilibrium. Here physical output was seen as bounded by demand. Supply was no problem: demand would always create its own supply. There was always excess capacity. The rate of interest could be affected by the money supply. Petty thought that ample money had reduced it to six percent. Yarranton [1677 (1854: 38)] believed that the use of paper money would reduce it to four percent. Thus monetary policy could raise physical output and employment.

Capitalism left to itself might be incapable of utilizing its own resources. Government action was the remedy.<sup>1</sup>

Within less than a century such a demand-side equilibrium was to reverse itself.

### 3. Hume

Hume's (1752) concern was inflation, and he analyzed it within the framework of a supply-side equilibrium. Here physical output was seen as bounded by supply. Demand was no problem: supply would always create its own demand. There was never excess capacity. The rate of interest was determined by saving and investment. Doubling the money supply would not reduce it but, in a fully monetized economy, would merely double prices. Capitalism was entirely capable of utilizing its own resources. Government action was the problem.<sup>2</sup>

### 4. Reversal Repeated

The reversal repeated itself in our own century: this time the demand-side equilibrium of Keynes (1936) reversed itself into the supply-side equilibria of Friedman (1968), Lucas (1972), and Sargent (1973). The purpose of the paper is to derive such a reversal and to

see how much of it will survive if dynamized. We use the following notation.

## 5. Variables

$C$   $\equiv$  physical consumption

$D$   $\equiv$  demand for money

$g$   $\equiv$  proportionate rate of growth

$I$   $\equiv$  physical investment

$k$   $\equiv$  present gross worth of another unit of physical capital stock

$\kappa$   $\equiv$  marginal productivity of physical capital stock

$L$   $\equiv$  labor employed

$P$   $\equiv$  price of good

$r$   $\equiv$  nominal rate of interest

$\rho$   $\equiv$  real rate of interest

$S$   $\equiv$  physical capital stock

$w$   $\equiv$  money wage rate

$X$   $\equiv$  physical output

## 6. Parameters

$a$   $\equiv$  joint factor productivity

$\alpha, \beta$   $\equiv$  exponents of a Cobb-Douglas production function

$b \equiv$  inducement to invest

$c \equiv$  propensity to consume

$F \equiv$  available labor force

$f \equiv$  inducement to hold money

$h \equiv$  elasticity of investment with respect to rate of interest

$i \equiv$  elasticity of real demand for money with respect to rate of interest

$\lambda \equiv$  "natural" employment rate

$M \equiv$  supply of money

All parameters are stationary except  $a$ ,  $F$ , and  $M$  whose growth rates are stationary.

## II. KEYNES

### 1. Essence

Keynes saw two markets, a goods market and a money market, and thought of capital stock and price as frozen. His frozen capital stock did not keep him from thinking of investment as a declining function of the rate of interest, say,

$$I = br^h \quad (1)$$

where  $h < 0$ . Consumption was an increasing function of output, say,

$$C = cX \quad (2)$$

where  $0 < c < 1$ . Real demand for money was an increasing function of output and a declining one of the rate of interest, say,

$$D/P = fXr^i \quad (3)$$

where  $i < 0$ . Two equilibrating variables, output and the rate of interest, would clear the goods and money markets, respectively:

$$X = C + I \quad (4)$$

$$D = M \quad (5)$$

## 2. Solutions

Solve the system (1) through (5) for those equilibrating variables:

$$X = \left( \frac{b}{1-c} \right)^{i/(h+i)} \left( \frac{M/P}{f} \right)^{h/(h+i)} \quad (6)$$

$$r = \left( \frac{1-c}{b} \frac{M/P}{f} \right)^{1/(h+i)} \quad (7)$$

### 3. Properties of Keynesian Solutions

The elasticity of output (6) with respect to the propensity to save  $1 - c$  is  $-i/(h + i) < 0$  and of interest (7) is  $1/(h + i) < 0$ : if the propensity to save is up, output and the rate of interest are both down.

The elasticity of output (6) with respect to the money supply  $M$  is  $h/(h + i) > 0$  and of interest (7) is  $1/(h + i) < 0$ : if the money supply is up, output is up and the rate of interest down--as Petty had said.

Within a third of a century such a demand-side equilibrium was to reverse itself into a supply-side equilibrium. Let us see how.

### III. FRIEDMAN-LUCAS-SARGENT

#### 1. A "Natural" Rate of Employment

New Classicals saw three markets, a goods market, a money market, and a labor market. To reverse a Keynesian into a New Classical equilibrium keep our old Keynesian equations (1) through (5) and let capital stock remain frozen. But add three new equations and three new variables. The new variables are labor employed  $L$ , unfrozen price  $P$ , and a real wage rate  $w/P$ .

The first new equation is a production function displaying diminishing returns to labor, say,

$$X = aL^{\alpha}S^{\beta} \quad (8)$$

where  $0 < \alpha < 1$ ,  $0 < \beta < 1$ ,  $\alpha + \beta = 1$ , and  $a > 0$ . The second new equation says that competitive firms optimize labor employed by equating the real wage rate with the physical marginal productivity of labor:

$$\frac{w}{P} = \frac{\partial X}{\partial L} = a\alpha L^{-\beta}S^{\beta} \quad (9)$$

Facing such a demand curve (9) for labor how do unions respond?

Friedman's (1968) answer was a "natural" rate of employment reflecting union density. Natural rate and union density vary across countries and over time. But they do indeed tend to vary in opposite directions.

Over the thirteen years 1973-1986 Freeman (1988b: 294-295) found actual employment as a fraction of working-age population declining steadily in OECD-Europe as a whole but largely rising in the United States. Over the fifteen years 1970-1985 Freeman (1988a: 69) found a corresponding contrast between Europe and the United States in union density: union density was rising sharply in Denmark, Finland, and Sweden; rising moderately in France, Germany, Ireland, Italy, and Switzerland; rising slightly in Norway and the United Kingdom; declining slightly in Austria and the Netherlands; declining sharply in the United States.

Current labor-market literature adds institutional color:

Lindbeck and Snower (1986) and Blanchard and Summers (1988) distinguish between "insiders," who are employed hence decision-making, and "outsiders," who are unemployed hence disenfranchised.

## 2. Solutions

We solve the New Classical model for its equilibrating variables, old and new. Let insiders accept a "natural" employment rate  $0 < \lambda \leq 1$ .



They accept it in the sense that if  $L > \lambda F$  they will insist on a higher real wage rate. If

$$L = \lambda F \quad (10)$$

they will be happy with the existing one. If  $L < \lambda F$  they will settle for a lower one. The natural employment (10) is our third new equation.

Call the real wage rate insiders will be happy with the natural one. Solve for it by inserting (10) into (9):

$$w/P = a\alpha(\lambda F)^{-\beta}S^{\beta} \quad (11)$$

At the frozen capital stock  $S$ , then, labor can have a  $\beta$  percent higher real wage rate by accepting a one percent lower natural rate  $\lambda$  of employment.

Call the physical output corresponding to the natural rate of employment the natural supply of goods. Solve for it by inserting (10) into (8):

$$X = a(\lambda F)^{\alpha}S^{\beta} \quad (12)$$

To solve for the rate of interest  $r$  insert (1), (2), and (12) into (4):

$$r = \left[ a (\lambda F)^\alpha S^\beta \frac{1 - c}{b} \right]^{1/h} \quad (13)$$

To solve for price P insert (5), (12) and (13) into (3):

$$P = [a (\lambda F)^\alpha S^\beta]^{-(h+i)/h} \left( \frac{1 - c}{b} \right)^{-i/h} M/f \quad (14)$$

### 3. Properties of New Classical Solutions

No propensity to save  $1 - c$  occurs in employment (10), natural real wage rate (11), or supply (12). But the elasticity of the rate of interest (13) with respect to  $1 - c$  is  $1/h < 0$  and of price (14) is  $-i/h < 0$ : if the propensity to save is up, employment, real wage rate, and output are unaffected but the rate of interest and price are both down.

The natural rate  $\lambda$  of employment occurs in all five solutions. Their elasticities with respect to it are  $1$ ,  $-\beta$ ,  $\alpha$ ,  $\alpha/h < 0$ , and  $-\alpha(h+i)/h < 0$ , respectively: if the natural rate  $\lambda$  is down, employment is down, real wage rate is up, output down, and price and rate of interest are up.

No money supply  $M$  occurs in employment (10), natural real wage rate (11), supply (12), or rate of interest (13), but the elasticity of price (14) with respect to money supply  $M$  is 1: if the money supply is up, nothing is affected except price, which is up in proportion--as Hume<sup>3</sup> had said.

May actual differ from natural employment? According to New Classical agents act as if they knew the structure of the model as well as any systematic monetary policy applied to it. If so, only random hence unanticipated variations of the money supply can generate deviations of actual from natural.

For example, let employers anticipate an expanded money supply and its effect upon price sooner than does labor. In other words, labor suffers from a money illusion. Seeing a real wage rate lower than the natural one (11), employers will expand employment beyond its natural rate (10). But eventually labor has no money illusion: new rounds of collective bargaining will restore the equality between actual and natural employment.

The New Classical equilibrium was as static as the Keynesian one had been: capital stock remained frozen. Now we must unfreeze it.

## IV. CAPITAL STOCK UNFROZEN

1. Desired Capital Stock

Keep our old Keynesian equations (2) through (5) as well as our New Classical equations (8), (9), and (10). But unfreeze capital stock, drop the Keynesian investment function (1), and look for something better.

Desired capital stock is a long-run commitment to be found by maximizing present net worth. We begin by defining physical marginal productivity of capital stock as

$$\kappa \equiv \frac{\partial X}{\partial S} = \beta \frac{X}{S} \quad (15)$$

At time  $t$  marginal value productivity of capital stock is  $\kappa(t)P(t)$ . Let there be a market in which money may be borrowed at the stationary nominal rate of interest  $r$ . Let that rate be applied when discounting future cash flows. As seen from the present time  $\tau$ , then, marginal value productivity of capital stock is  $\kappa(t)P(t)e^{-r(t-\tau)}$ . Define present gross worth of another physical unit of capital stock as

the present worth of all future marginal value productivities over its entire useful life:

$$k(\tau) \equiv \int_{\tau}^{\infty} \kappa(t) P(t) e^{-r(t-\tau)} dt \quad (16)$$

Our economy is now a growing one. Define the rate of growth of a variable  $v$  as the derivative of its logarithm with respect to time:

$$g_v \equiv \frac{d \log_e v}{dt} \quad (17)$$

and let firms expect physical marginal productivity of capital stock to be growing at the stationary rate  $g_k$ :

$$\kappa(t) = \kappa(\tau) e^{g_k(t-\tau)}$$

and price of output to be growing at the stationary rate  $g_p$ :

$$P(t) = P(\tau) e^{g_p(t-\tau)}$$

Insert these, define

$$\rho \equiv r - (g_k + g_p) \quad (18)$$

and write the integral (16) as

$$\kappa(t) = \int_{\tau}^{\infty} \kappa(\tau) P(\tau) e^{-\rho(t-\tau)} dt$$

Neither  $\kappa(\tau)$  nor  $P(\tau)$  is a function of  $t$  hence may be taken outside the integral sign. Our  $g_k$ ,  $g_p$ , and  $r$  were all said to be stationary, hence the coefficient  $-\rho$  of  $t$  is stationary, too. Assume  $\rho > 0$ . As a result find the integral to be

$$k = \kappa P / \rho$$

Find present net worth of another physical unit of capital stock as its gross worth minus its price:

$$k - P = (\kappa / \rho - 1) P$$

Capital stock desired by the firm is the size of stock at which the present net worth of another physical unit of capital stock would be zero:

$$\kappa = \rho$$

Insert (15) and find desired capital stock and investment

$$S = \beta X / \rho \quad (19)$$

$$I \equiv g_s S = \beta g_s X / \rho \quad (20)$$

What is  $\rho$ ? In its definition (18) let it be correctly foreseen by firms that the economy will have the solution (23), to be found presently. Historically the marginal productivity  $\kappa$  of capital has indeed remained stationary. In that case (18) simply collapses into the real rate of interest.

## 2. Solutions

Our system (2) through (5), (8), (9), (10), (19), and (20) represents a growing economy. We solve first for its growth rates and then for its levels.

As for the growth rates, we begin with Solow's (1956) convergence proof. Insert (2) and the definitional part of (20) into (4):

$$g_s = (1 - c)X/S$$

Differentiate logarithm of  $g_s$  with respect to time and find rate of acceleration of physical capital stock:

$$g_{gS} = g_x - g_s$$

Insert (10) into (8), consider the natural rate  $\lambda$  stationary, and differentiate the logarithm of the result with respect to time:

$$g_x = g_a + \alpha g_f + \beta g_s$$

Insert  $g_x$  into  $g_{gS}$ :

$$g_{gS} = \alpha (g_a/\alpha + g_f - g_s)$$

Three possibilities offer themselves: if  $g_s > g_a/\alpha + g_f$  then  $g_{gS} < 0$ . If

$$g_s = g_a/\alpha + g_f \tag{21}$$

then  $g_{gS} = 0$ . Finally, if  $g_s < g_a/\alpha + g_f$ , then  $g_{gS} > 0$ . Consequently, if greater than (21)  $g_s$  is falling; if equal to (21)  $g_s$  is stationary; and if less than (21)  $g_s$  is rising. We conclude that  $g_s$  must either equal  $g_a/\alpha + g_f$  from the outset or, if it does not, converge to that value.



Once such convergence has been established we may easily find the corresponding values of other growth rates: insert (21) into  $g_x$  and find the growth rate of physical output:

$$g_x = g_s \quad (22)$$

Differentiate logarithm of (15) with respect to time, insert (22), and find the growth rate of physical marginal productivity of capital stock:

$$g_x = 0 \quad (23)$$

Differentiate logarithm of (11) with respect to time, insert (21), and find the growth rate of the natural real wage rate:

$$g_{w/p} = g_a/\alpha \quad (24)$$

Insert (5) into (3), differentiate logarithm of result with respect to time, insert (22) and find the rate of inflation:

$$g_p = g_M - g_s \quad (25)$$

where  $g_s$  stands for (21).

Having solved for growth rates we turn to levels. To solve for the real rate of interest insert (2) and (20) into (4) and divide any nonzero  $X$  away:

$$\rho = \frac{\beta g_s}{1 - c} \quad (26)$$

where  $g_s$  stands for (21).

To solve for the nominal rate of interest insert (23), (25), and (26) into (18):

$$r = [\beta/(1 - c) - 1]g_s + g_M \quad (27)$$

where  $g_s$  stands for (21).

To solve for physical capital stock insert (12) and (26) into (19):

$$S = \left( a \frac{1 - c}{g_s} \right)^{1/\alpha} \lambda F \quad (28)$$

where  $g_s$  stands for (21).

To solve for physical output insert (28) into (12):

$$X = a^{1/\alpha} \left( \frac{1-c}{g_s} \right)^{\beta/\alpha} \lambda F \quad (29)$$

where  $g_s$  stands for (21).

To solve for the real wage rate insert (28) into (11):

$$w/P = \alpha a^{1/\alpha} \left( \frac{1-c}{g_s} \right)^{\beta/\alpha} \quad (30)$$

where  $g_s$  stands for (21).

### 3. Properties of Unfrozen-Capital-Stock Solutions

No propensity to save  $1 - c$  occurs in any growth-rate solution (21) through (25), but it does occur in every level solution (26) through (30). The elasticities of the real rate of interest (26) and the nominal rate of interest (27) with respect to  $1 - c$  are both negative; the last three elasticities are all positive: if the

propensity to save is up, the real and nominal rates of interest are both down, but physical capital stock, physical output, and the real wage rate are all up.

No natural rate  $\lambda$  of employment occurs in any growth-rate solution (21) through (25) or in the level solutions (26), (27), and (30). But the elasticities of physical capital stock (28) and physical output (29) with respect to  $\lambda$  are both 1: if the natural rate  $\lambda$  is down, growth rates, interest rates, and the real wage rate are unaffected, but physical capital stock and output are down in proportion.

Money is said [Blanchard-Fischer (1989: 207)] to be superneutral "if changes in money growth have no effect on the real equilibrium." Our money is indeed superneutral: no rate of growth  $g_M$  occurs in any real variable (21) through (24), (26), or (28) through (30). But  $g_M$  does occur in the nominal variables (25) and (27). Their derivatives with respect to  $g_M$  are both 1: if the rate of growth of the money supply is up, inflation and the nominal rate of interest are both up but no real variable is affected.

## V. SUMMARY AND CONCLUSIONS

By adding three variables and three equations to it we reversed a Keynesian demand-side equilibrium into a New Classical supply-side equilibrium. Some reversal!

To Keynes a higher propensity to save would lower output: there was unemployment because people were too thrifty. To New Classicals a higher propensity to save would leave output unaffected but lower price. A natural unemployment was the price unions were willing to pay for a higher natural real wage rate.

To Keynes a larger money supply would lower the rate of interest and raise output. Petty-Yarranton would have agreed. To New Classicals a larger money supply would affect neither interest nor output. Instead it would raise price. Hume would have agreed.

But a New Classical equilibrium was as static as the Keynesian one had been: capital stock remained frozen. Unfreezing it made a twofold difference.

First, the reader may recall that under a frozen capital stock the real wage rate (11) and output (12) were unaffected by a higher propensity to save. But unfrozen capital stock (28), output (29), and the real wage rate (30) would all be up if the propensity to save were up.

Second, the reader may recall that under a frozen capital stock in (11) labor could have a  $\beta$  percent higher real wage rate by accepting a one percent lower natural rate  $\lambda$  of employment. Once capital stock was unfrozen, labor could expect no such benefit:  $\lambda$  had disappeared from (30). The reason was simple. Labor (10), capital stock (28), and output (29) were all in direct proportion to  $\lambda$ . A lower  $\lambda$  reduced the economy to a lower scale. Less capital would accumulate and less output be produced. But factor proportions and with them the marginal productivity of labor would remain the same.

Under rational expectations labor would realize that nothing could be gained by lowering  $\lambda$  hence would prefer full employment  $\lambda = 1$ . Only shortsighted labor would choose and uphold a natural rate  $0 < \lambda < 1$ . Such "hysteresis" may explain current high unemployment rates in Australia, Canada, France, Italy, Spain, and the United Kingdom.

## FOOTNOTES

<sup>1</sup>Further documentation in Brems (1986: 19-24).

<sup>2</sup>Further documentation in Brems (1986: 33-37).

<sup>3</sup>Hume, to be sure, knew neither unions nor insiders. But if reflecting the equality sign of our  $0 < \lambda \leq 1$ , i.e., full employment, eighteenth-century institutions would still generate a unique natural supply of goods (12).

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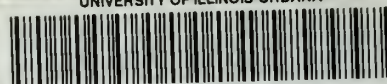
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