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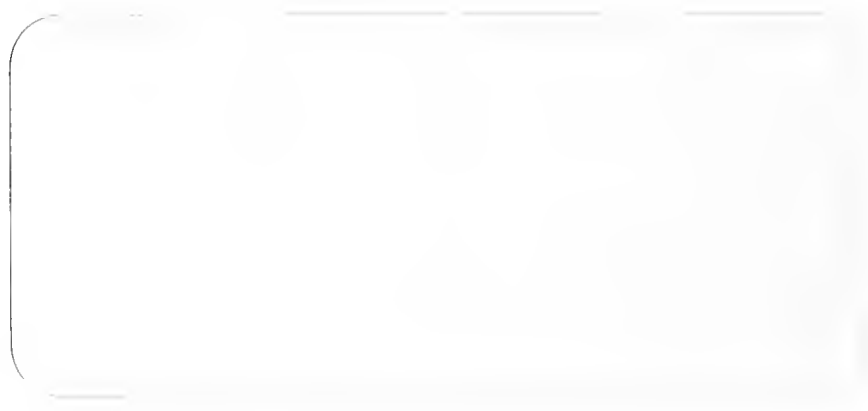
Faculty Working Papers

Nature and Neoclassical Growth

Hans Brems
University of Illinois

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College of Commerce and Business Administration
University of Illinois at Urbana-Champaign



FACULTY WORKING PAPERS

College of Commerce and Business Administration

May 10, 1972

Nature and Neoclassical Growth

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REPORT OF THE COMMISSION

ON THE PROGRESS OF THE WORK DURING THE YEAR 1900

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REPORT OF THE COMMISSION ON THE PROGRESS OF THE WORK DURING THE YEAR 1900

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SUMMARY

111 Economic growth theory and models
720 Natural resources

Brems, H.—Nature and neoclassical growth

In the one-output, two-input neoclassical growth model output has been shown to be converging to steady-state growth defined as a stationary proportionate rate of growth. The article includes the services of irreproducible and stationary nature as a third input in the production function and shows that steady-state growth of capital stock, output, physical marginal productivity of capital, and the real wage rate still results. The article also shows that even a highly developed economy with plenty of capital accumulation could generate a decaying real wage rate.

1. The first part of the document is a letter from the author to the editor, dated 10/10/1964. The letter discusses the author's interest in the subject of the journal and the author's previous work in the field. The author mentions that he has been working on this subject for some time and that he has found some interesting results. He also mentions that he has been thinking about writing a paper on this subject for some time and that he is now ready to do so. The author concludes the letter by expressing his hope that the editor will find the paper interesting and that it will be published in the journal.

2. The second part of the document is the paper itself, which is titled "The Role of the State in the Development of the Economy". The paper is written in a formal, academic style and is divided into several sections. The first section is an introduction, which discusses the importance of the state in the development of the economy. The second section is a discussion of the role of the state in the development of the economy, which is divided into two parts: the role of the state in the development of the private sector and the role of the state in the development of the public sector. The third section is a conclusion, which summarizes the main points of the paper and discusses the implications of the findings.

May 10, 1972

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N A T U R E A N D N E O C L A S S I C A L G R O W T H

By HANS BREMS*

In the one-output, two-input neoclassical growth model [2], [3] output has been shown to be converging to steady-state growth defined [1] as a stationary proportionate rate of growth. Would such steady-state growth be in jeopardy if the production function were to include as a third input the services of irreproducible and stationary nature? Even if not, could the real wage rate in a highly developed economy be displaying steady-state decay?

I. NOTATION

C \equiv consumption

g_v \equiv proportionate rate of growth of the variable v where $v \equiv \kappa, P, S,$ and X

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g_{gv} \equiv proportionate rate of acceleration of the variable v where
 $v \equiv S$

I \equiv investment

κ \equiv physical marginal productivity of capital stock

L \equiv labor employed

P \equiv price of good

S \equiv physical capital stock

X \equiv physical output

Parameters

α, β, γ \equiv exponents of production function

c \equiv propensity to consume

F \equiv available labor force

g_p \equiv proportionate rate of growth of parameter p where $p \equiv F, M,$
and w

M \equiv multiplicative factor of production function

N \equiv nature

w \equiv money wage rate

The parameters listed are stationary except $F, M,$ and $w,$ whose growth rates $g_F, g_M,$ and g_w are stationary.

II. THE EQUATIONS OF THE MODEL

Let capitalist-entrepreneurs produce a single good from labor, an immortal capital stock of that good, and nature, hence investment is the act of setting aside part of output for installation as capital stock. Write the equations of such a system as follows. To the four variable growth rates listed in Section I apply the definition

$$(1) \text{ through } (4) \quad g_v \equiv \frac{dv}{dt} \frac{1}{v}$$

Define investment as the derivative of capital stock with respect to time

$$(5) \quad I \equiv \frac{dS}{dt}$$

Let the capitalist-entrepreneurs apply the Cobb-Douglas production function

1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ (Probability of getting two heads)

2. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ (Probability of getting two tails)

3. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ (Probability of getting one head and one tail)

4. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ (Probability of getting one tail and one head)

$$P(\text{Two heads}) = \frac{1}{4}$$

$$P(\text{Two tails}) = \frac{1}{4}$$

5.

$$P(\text{One head and one tail}) = \frac{1}{2}$$

$$(6) \quad X = ML^{\alpha}S^{\beta}N^{\gamma}$$

where $0 < \alpha < 1$; $0 < \beta < 1$; $0 < \gamma < 1$; $\alpha + \beta + \gamma = 1$; $M > 0$; and N is stationary. Let profit maximization under pure competition equalize real wage rate and physical marginal productivity of labor:

$$(7) \quad \frac{w}{P} = \frac{\partial X}{\partial L} = \alpha \frac{X}{L}$$

Physical marginal productivity of capital is

$$(8) \quad \kappa \equiv \frac{\partial X}{\partial S} = \beta \frac{X}{S}$$

Under full employment, available labor force must equal labor employed:

$$(9) \quad F = L$$

Let consumption be a fixed proportion of output:

$$(10) \quad C = cX$$

where $0 < c < 1$. Output equilibrium requires output to equal the sum of consumption and investment demand for it, or inventory would either accumulate or be depleted:

$$(11) \quad X = C + I$$

III. SOLUTIONS FOR PROPORTIONATE RATES OF GROWTH

Despite the diminishing returns imposed by stationary nature, our system (1) through (11) can be shown to possess a set of steady-state solutions for the proportionate rates of growth of

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capital stock, output, physical marginal productivity of capital, and the real wage rate. To find that set, let us begin by inserting (9) into (6) and differentiate the latter with respect to time:

$$(12) \quad g_X = g_M + \alpha g_F + \beta g_S$$

From (10), (11), and (1) through (5) find

$$(13) \quad g_S = (1 - c)X/S$$

Differentiate (13) with respect to time, use (1) through (4), and find the proportionate rate of acceleration

$$(14) \quad g_{gS} \equiv \frac{dg_S}{dt} \frac{1}{g_S} = (\alpha + \gamma)[(g_M + \alpha g_F)/(\alpha + \gamma) - g_S]$$

In (14) there are only three possibilities: If $g_S > (g_M + \alpha g_F)/(\alpha + \gamma)$, then $g_{gS} < 0$. If

$$(15) \quad g_S = (g_M + \alpha g_F) / (\alpha + \gamma)$$

then $g_{g_S} = 0$. Finally, if $g_S < (g_M + \alpha g_F) / (\alpha + \gamma)$, then $g_{g_S} > 0$. Consequently, if greater than (15) g_S is falling; if equal to (15) g_S is stationary; and if less than (15) g_S is rising. Now g_S cannot converge toward anything else than (15), for if it did, then by letting enough time elapse we could make the left-hand side of (14) less than any arbitrarily assignable positive constant ϵ , however small, without the same being possible for the right-hand side. So g_S converges toward (15). Insert (15) into (12) and find

$$(16) \quad g_X = (g_M + \alpha g_F) / (\alpha + \gamma)$$

Output and capital stock, then, will eventually be growing at the same steady-state proportionate rate (15) and (16). That may be lower than when nature was ignored, but it is still a steady-state rate!

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1. The first part of the document is a letter from the President of the United States to the Secretary of State, dated August 14, 1914. The letter discusses the situation in Mexico and the United States, and the need for a strong and efficient government in Mexico. The President expresses his confidence in the Secretary's ability to handle the situation and his hope that the Secretary will be able to bring about a peaceful and stable government in Mexico.

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What is happening to the physical marginal productivity of capital? Take the derivative of (8) with respect to time, insert (1) through (4), and find

$$(17) \quad g_K = g_X - g_S$$

Insert (12) into (17) and find

$$(18) \quad g_K = (\alpha + \gamma)[(g_M + \alpha g_F)/(\alpha + \gamma) - g_S]$$

Comparing (18) to (14) we find that the rate of growth of physical marginal productivity equals the rate of acceleration of capital stock. Since the latter converges to zero, so must the former: Physical marginal productivity of capital eventually grows at the steady-state rate of zero, i. e., becomes stationary.

What is happening to the real wage rate? Take the derivative of (7) with respect to time, insert (1) through (4) and (9), and find

$$(19) \quad g_W - g_P = g_X - g_F$$

THEOREM 1. Let \mathcal{L} be a linear operator on \mathbb{R}^n .

Suppose that \mathcal{L} is symmetric, i.e., $\langle \mathcal{L}x, y \rangle = \langle x, \mathcal{L}y \rangle$ for all $x, y \in \mathbb{R}^n$. Then \mathcal{L} has n real eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$ and a corresponding set of orthonormal eigenvectors v_1, v_2, \dots, v_n .

$$\mathcal{L}v_i = \lambda_i v_i$$

where $\langle v_i, v_j \rangle = \delta_{ij}$ (Kronecker delta).

Proof. Let $\mathcal{L}x = \lambda x$ be an eigenvalue equation.

Take the inner product of both sides with x :

$\langle \mathcal{L}x, x \rangle = \langle \lambda x, x \rangle = \lambda \langle x, x \rangle = \lambda \|x\|^2$.
On the other hand, since \mathcal{L} is symmetric, $\langle \mathcal{L}x, x \rangle = \langle x, \mathcal{L}x \rangle = \langle x, \lambda x \rangle = \lambda \langle x, x \rangle = \lambda \|x\|^2$.
Thus, $\lambda \|x\|^2 = \lambda \|x\|^2$, which is always true. This shows that λ is real.
Next, let $\mathcal{L}x = \lambda x$ and $\mathcal{L}y = \mu y$ be two eigenvalue equations with $\lambda \neq \mu$.
Take the inner product of the first equation with y : $\langle \mathcal{L}x, y \rangle = \langle \lambda x, y \rangle = \lambda \langle x, y \rangle$.
Take the inner product of the second equation with x : $\langle \mathcal{L}y, x \rangle = \langle \mu y, x \rangle = \mu \langle y, x \rangle$.
Since \mathcal{L} is symmetric, $\langle \mathcal{L}x, y \rangle = \langle x, \mathcal{L}y \rangle = \langle x, \mu y \rangle = \mu \langle x, y \rangle$.
Thus, $\lambda \langle x, y \rangle = \mu \langle x, y \rangle$. Since $\lambda \neq \mu$, we must have $\langle x, y \rangle = 0$.
Therefore, eigenvectors corresponding to distinct eigenvalues are orthogonal.
By the Gram-Schmidt process, we can find an orthonormal basis of eigenvectors.

Insert (12) into (19) and find

$$(20) \quad g_w - g_p = g_M - (1 - \alpha)g_F + \beta g_S$$

But we have just seen that g_S will converge to (15). Insert (15) into (20), write $1 - \alpha = \beta + \gamma$, and find the rate of growth of the real wage rate converging to

$$(21) \quad g_w - g_p = (g_M - \gamma g_F) / (\alpha + \gamma)$$

We conclude that if the natural-resource elasticity of output γ is high enough, if labor-force growth g_F is high enough, or if technological progress g_M is low enough, then even a highly developed economy with plenty of capital accumulation could generate a decaying real wage rate.

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F O O T N O T E

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R E F E R E N C E S

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- [3] J. Tinbergen, "Zur Theorie der langfristigen Wirtschaftsentwicklung," Weltw. Archiv, May 1942, 55, 511-549.





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