NELSON'S

COMMON-SCHOOL ARITHMETIC:

DESIGNED FOR THE USE OF THE LOWEST AS WELL AS THE HIGHEST CLASSES,

AND CONTAINING

THE APPLICATION OF ARITHMETIC TO THE GENERAL PURPOSES OF LIFE,

AND

THE METRIC SYSTEM OF WEIGHTS AND MEASURES,

RECENTLY ADOPTED BY CONGRESS.

BY RICHARD NELSON,

PRESIDENT OF NELSON'S UNION BUSINESS COLLEGE, AND AUTHOR OF NELSON'S MERCANTILE ARITHMETIC.

SECOND EDITION.

CINCINNATI:
R. W. CARROLL & CO., PUBLISHERS,
No. 117 WEST FOURTH STREET.
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PREFACE.

THIS treatise differs so materially from others of its class, that the space of a preface will not suffice to give reasons for the changes made. The author will, therefore, have to content himself with stating that, in the classification and treatment of subjects, he has been guided by his own experience and that of the most distinguished educators of the country; and in the application of the science to the purposes of life, his authorities have been practical men, who were familiar with the defects of the old system, and desirous that the rising generation should be trained with more direct reference to their probable future callings.

The improvements in machinery having almost superseded the necessity for a knowledge of arithmetic in the mechanic arts, little has been done to adapt the treatise to such purpose; hence, it partakes largely of the mercantile character.

This, the author considers, will be its highest recommendation, as the destination of most American youth is business; and, especially, as every man in this great Nation of Commerce is more or less engaged in mercantile pursuits.

The author acknowledges numerous acts of kindness and courtesy received, in the preparation of this work, from officers of the Government, gentlemen of the legal and mercantile professions, teachers in different parts of the country, especially from the Principals of the Public Schools of Cincinnati, most of whom generously and unreservedly tendered advice regarding topics of interest to their schools. To the distinguished educators, A. J. RICKOFF and JOHN HANCOCK—the latter the friend and associate of the author—he is under peculiar obligations. The former gentleman, having been the first to adopt his *Mercantile Arithmetic*, was the most competent to suggest further improvements and hints for the adaptation of arithmetic to common school purposes; while the latter, from his long experience as Principal of the First Intermediate School in this city, and his immediate knowledge of the wants of teachers, proved a valuable companion during the progress of the work, aiding constantly in the revision of the proof-sheets and tendering professional advice.

Though little reliance has been placed on written authority, many works have been called into requisition, principally of a legal, scientific and educational character. Of these, free use has been made, especially in the department of the tables, which, it is hoped, will be found to subserve the present wants of every-day life.

RICHARD NELSON.

CINCINNATI, *August*, 1866.

A VOCABULARY

OF

TECHNICAL TERMS USED IN BUSINESS.

ABATEMENT, or deduction, an amount taken off a bill for prompt payment, damages, etc.
ACCEPTANCE, agreeing to price or terms proposed; a bill with one's name written in such a way as to bind for payment. See page 197.
ACCOMMODATION PAPER, a bill or note used to raise money, and not to pay a debt.
ACCOUNT, detailed statement of goods sold. A statement showing the indebtedness of one person to another.
ACCOUNTANT, a professional calculator; one skilled in book-keeping.
ACCOUNT-BOOK, a ruled book in which accounts are kept.
ACCOUNT CURRENT, a plain statement of a running account between two persons. See page 194.
ACCOUNT SALES, a detailed statement of goods sold, made by an agent to his principal.
ACQUITTANCE, a written discharge; a receipt in full for money due.
AD VALOREM, according to value, an assessment for custom duty.
ADVANCE, a sum of money paid before value is received.
ADVENTURE, a doubtful speculation; a term used in book-keeping for goods shipped to be sold on commission.
ADVICE, mercantile intelligence.
AFFIDAVIT, a declaration in writing, made on oath before a magistrate, etc.
AGENT, one who acts for another.
ANNUITY, a sum of money paid periodically.
ANNUL, to make void; to cancel.
ANTEDATE, to date beforehand.
APOTHECARIES' WEIGHT, the weight used in compounding medicines.
APPRAISER, a valuator.
ARBITRATION, reference of a controversy or dispute to persons chosen by the parties.
ASSESSOR, or Surveyor, one whose duty it is to estimate the value of property for taxation.
ASSETS, the funds and property of a trader or person in business.
ASSIGNEE, one to whom an assignment is made.

ASSIGNMENT a conditional transfer of property, making it over for safe keeping.
ASSIGNOR, one who makes an assignment.
ASSURANCE. See Insurance.
AVERAGE, allowance made for loss at sea; a rule in arithmetic.
AUDIT, to examine books, vouchers, etc.
AUDITOR, one who inspects or examines and certifies accounts.

B.

BAGGAGE, the wearing apparel, trunks, etc., of a traveler.
BALANCE, a well-known instrument for weighing; to find the difference between two sides of an account; also that difference.
BALANCE OF TRADE, the difference of the money value of the produce received and exported. If a country receives more produce and manufactures than it ships, the balance of trade is said to be against it.
BALANCE-SHEET, a paper containing a concise statement of a merchant's accounts.
BALANCING BOOKS, the business of making a balance-sheet from the accounts in the ledger.
BALE, a package of goods or produce.
BANCO, a word used as a prefix to paper money of some parts of Europe.
BANK-BOOK, the pass-book of a bank.
BANKER, a dealer in money.
BANK HOURS, from 9 to 3 o'clock.
BANK NOTE, a bank-bill payable to bearer.
BANKRUPT, one who is not able to pay his debts.
BANKS. See page 154.
BANK STOCK, the shares of a banking company.
BEAR, a term used to designate a person who makes it his business to depress the price of stocks, in order to buy up.
BILL OF ENTRY, a list of goods entered at the custom-house.
BILL OF EXCHANGE. See page 196.
BILL OF LADING, a receipt from a railroad, ship, etc., for goods entered for conveyance from one place to another.
BILL-HEAD, a printed form, with name of business or address.
BILL, or BILL OF PARCELS, a detailed account of goods sold.
BILL OF SALE, a contract under seal for the sale of goods.
BILLS PAYABLE, the name given by a merchant or other person to notes made and issued, or bills, drafts, etc., accepted by him.
BILLS RECEIVABLE, all notes taken or given in payment, except one's own.
BLANK CREDIT, permission given by house or person to draw money on account.
BONA FIDE, in good faith.
BOND, a note or deed given with pecuniary security.
BONDED GOODS, those for which bonds are given for the duties instead of money.
BROKER, an agent or factor. See Broker, page 95.
BROKERAGE, the percentage, commission, etc., paid to a broker for buying or selling.

BULL, a term applied to a broker or stock jobber, who interests himself to raise the price of stocks in the market, in order to command a high sum for those he holds.

BULLION, uncoined gold and silver.

C.

CAPITAL, stock in trade; the amount of assets employed by a person or company in business.

CAPITALIST, a man of large property or means; one who has large sums invested in stocks.

CAPITATION, a poll tax; a tax levied on male adults.

CARGO, a ship's load.

CARRIAGE, the charge made for conveying goods from one place to another.

CARTAGE, the charge for carrying goods on a cart.

CASE, a box for holding goods or merchandise.

CASH, the general name for coin and bank notes; sometimes checks and sight bills of exchange are called cash.

CASH-BOOK, the book in which merchants and others enter the money paid out and taken in.

CASH CREDIT, the privilege of drawing money at a bank, obtained by depositing suitable security.

CASHIER, one who has charge of money.

CELLARAGE, privileged charge of rooms underground.

CERTIFICATE, testimony given in writing; a paper granting some particular privilege.

CHAMBER OF COMMERCE, an association of merchants for the protection of trade.

CHARTER, license from government to pursue certain kinds of business.

CHARTER PARTY, a contract in writing between the owner and freighter of a vessel.

CHATTELS, all goods and real or personal property, except real estate.

CHECK, an order on a bank for payment on demand.

CHECK-BOOK, a printed book of blank checks.

CHEST, a box or package; tea and opium are packed in chests. A chest of opium contains 141½ lbs.; tare allowed, 1½ lbs. A chest of tea is variable.

CIRCULAR, a printed letter of advertisement.

CLOSING AN ACCOUNT, balancing the two sides, by placing the difference on the smaller side, under the name of balance or profit and loss, and drawing lines beneath.

CLERK, an assistant in a store, office, etc. See page 198.

COLLECTOR, one authorized to receive money for another.

COMMERCE, the business of exchanging one commodity for another; buying and selling; mercantile business.

COMMERCIAL, pertaining to commerce.

COMMISSION. See Commission, page 139.

COMPANY, a number of persons associated in business.

COMPENSATION, remuneration or reward for injury or services.

COMPETITION, rivalry, contention for contract or supremacy.

CONSIDERATION, a bonus; a sum given on account for any thing.

CONSIGN, to send goods to an agent or factor for sale.

CONSIGNEE, a person who receives goods on trust, or to be sold on commission.
CONSIGNMENT, goods sent to a distance for sale by an agent.
CONSIGNOR, the person or party who consigns.
CONSOLS, public stocks in England.
CONSUMER, one who uses or expends goods.
CONSUMPTION, a using up; the quantity consumed.
CONTINGENT, a share arising from an adventure; doutbful.
CONTRA, on the other side; *per contra*, a writing on the opposite side.
CONTRABAND GOODS, articles on which there are heavy duties, or those wholly prohibited by government.
CONTRACT, an agreement between parties for a lawful consideration; a bargain.
CONTRACTOR, one who bargains.
CONTRIBUTION, a joint payment of money to an undertaking.
CONVEYANCE. a legal document, transferring land or other property from one person to another; the transport of goods or passengers from one place to another; a vehicle.
COOPERAGE, money paid to a cooper, or charges made for repairs of casks, etc.
COPARTNER a person engaged in partnership.
COPYING INK, adhesive ink, prepared with gum, etc., for transferring writing.
COPYING PRESS, an instrument for taking impressions from writing; copying letters, etc.
CORRESPONDENT, a letter writer; news writer for a periodical.
COUNTER ENTRY, a contrary entry.
COUNTERMAND, a contrary order.
COUNTING-HOUSE, or COUNTING-ROOM, a merchant's office.
COUPON, that part of a bond or other instrument designed to be cut off.
CREDIT, giving trust; goods supplied without present payment.
CURRENCY, paper money and coin established as the circulating medium of a country.
CUSTOM, a tax levied on goods imported or exported.
CUSTOMER, a regular buyer of goods at a stated place.
CUSTOM-HOUSE, the place appointed to receive custom.
CUSTOM-HOUSE ENTRY, a statement made, and fees and expenses paid in clearing a ship.

D.

DAMAGE, injury inflicted or sustained.
DAMAGED GOODS, articles of merchandise or produce which have been injured.
DAY-BOOK, the book in which merchants record daily transactions.
DAYS OF GRACE. See page 142.
DEBIT, to make any thing debtor in one's books; a charge entered
DEBIT SIDE, the left side of a page in the ledger.
DEBT, something due to another.
DEBTOR, one who owes another.
DECIMAL CURRENCY, moneys reckoned by tens, as the United States currency.
DEED, a legal instrument of agreement under seal.

DEFAULT, a failure of payment.
DEFAULTER, one who makes away with the public funds intrusted to his care.
DEFICIT, a deficiency; something wanted
DEPOSIT, a lodgment; a pledge or pawn; money intrusted to the care of others.
DEPOSITOR, one who has money lodged in bank for safe keeping.
DEPRECIATION, lessening in value.
DESPATCH, to transmit, to forward goods, papers, etc. See Dispatch.
DETERIORATION, damage done; wear and tear sustained.
DIRECTOR, a manager, a superintendent selected by a company or board.
DIRECTORY, an alphabetical guide or address book to the inhabitants of a city.
DISCOUNT, a deduction; something thrown off the amount of a bill or note; the sum paid by way of interest for the advance of money at bank.
DISCOUNT BROKER, one who loans money on notes of hand.
DISCOUNT DAY, some banks discount only on stated days, called discount days.
DISPATCH, a letter or message by telegraph.
DISSOLUTION breaking up of a copartnership.
DITTO, the same.
DIVIDEND, interests on stocks; a share of the proceeds of a joint stock speculation.
DOCK, a secure landing for ships; a place for landing cargoes; also a place to build or repair ships.
DOUBLE ENTRY, a method of keeping books, which considers every business transaction contains both a debit and a credit.
DRAFT, an order to pay money; a deduction from the weight of goods; a rough copy of a writing, etc.
DRAW, to write an order on another for money or goods.
DRAWEE, the person on whom the bill is drawn.
DRAWER, the person who draws a bill.
DRAYAGE, the charge made for goods carried on a dray.
DRUGGIST, one who sells drugs, chemicals, paints, etc.
DRY GOODS, a commercial name for cottons, woolens, laces, etc. In England, for grain, coal, etc.
DUPLICATE, a copy; a second article of a kind.
DUTY, a tax on goods or merchandise.

E.

EFFECTS, goods, property on hand, one's possessions.
ENDORSE, to employ to the exclusion of every thing else. See Indorse.
ENTERPRISE, an adventure; a projected scheme.
ENTRY, a record made in a business book; depositing a ship's papers on landing.
ENGROSS, to monopolize.
ESTIMATE, to appraise or value; to judge by inspection.
EXCHANGE, giving one commodity for another; a place of meeting; percentage arising from the sale of bills, etc.
EXECUTOR, a person appointed to carry out the intentions of a testator.
EXHIBIT, a voucher or document produced in a court of law.

EXPENDITURE, a charge or disbursement; outlay for expenses.
EXPORTER, a shipper who sends goods or produce to another country for sale.
EXPORTS, goods sent out of a country.
EXPRESS, a special messenger; a species of conveyance.

F.

FACE, the amount for which a note is drawn; also the side on which the writing is made.
FAC-SIMILE, an exact copy.
FACTOR, an agent or broker.
FAILURE, a term for suspension of payment; breaking up of business.
FANCY GOODS, ribbons, silks, satins, etc.
FEE, a gratuity; the charge of a professional man for services.
FEE SIMPLE, a property acquired by inheritance or that owned without conditions.
FELLOWSHIP, companionship, partnership.
FINANCE, ready money, funds or resources.
FINANCIER, one skilled in money matters.
FIRE INSURANCE, security against loss from fire, obtained by the payment of a small fee.
FIRE POLICY, the document received from an insurance house when goods are insured.
FIRM, a copartnership, a house of business.
FLAT, low, dull, inactive.
FLUSH, full; an abundance of money.
FORESTALL, to buy up goods or produce before the regular time of sale.
FOLIO, page.
FORWARDER, an agent who attends to the conveyance of goods, etc.
FORWARDING HOUSE, merchants who forward goods from one place to another.
FREIGHT, a load; charge made for carrying goods on ship or railroad.
FUNDS, ready money. See Public Funds.
FURS, preserved skins of wild animals, with fine thick hair.
FUR TRADE, the business of dealing in furs.

G.

GAUGE, to measure the contents of vessels, or barrels, casks, etc.; a measure or standard.
GOODS, a general name for movables.
GROCER, a dealer in sugar, spices, dried fruits and articles of food for the table.
GROSS, the whole weight of merchandise and box, barrel, etc.; 12 dozen; a great gross is 12 times 12 dozen.
GUARANTEE, or WARRANTY, indemnity against loss; one who binds him self to see the stipulation of another performed.
GUNNY BAGS, coarse sacking made in India, used for holding coffee, rice, etc.

H.

HAND, a measure of four inches, used for taking the height of horses.
HARDWARE, goods manufactured from iron.
HAWKER, a peddler.
HOGSHEAD, a large cask, formerly a measure of capacity.
HORSE REPOSITORY, a place kept for the sale of horses.

HUNDRED WEIGHT, a hundred pounds. In England, 112 pounds
HONOR, to accept a draft, by paying or promising to pay.
HYPOTHECATE, to pledge as security.

I.

IMMOVABLES, lands, houses, fixtures, etc.
IMMUNITY, freedom from tax, office or obligation.
IMPERATIVE, positive, commanding.
IMPERISHABLE, not subject to decay or waste.
IMPORTED, brought from a foreign country.
IMPORTER, one who brings goods from abroad.
INCOME, receipts, gains from labor, trade, etc.
INCONVERTIBLE, not transmissible; funds that can not be converted stocks.
INDORSE, to write one's name on the back of a note or draft.
INDORSEMENT, a writing on the back of a note or other paper
INDORSER, one who makes an indorsement.
INITIALS, the first or capital letters of a name.
INLAND BILLS. See page 196.
INTEREST, right or share in business. See page 142.
INSOLVENT, want of ability to pay.
INSURANCE. See page 244.
INTELLIGENCE-OFFICE, a registry office for domestics looking for situations.
INVENTORY, a list of goods or effects.
INVESTMENT, capital employed; money at interest.
INVOICE. See page 100.
INVOICE BOOK, a book containing invoices or copies of invoices.

J.

JOINT-STOCK COMPANY, an association of men to carry on heavy undertakings.
JOURNAL, an intermediate book between a day-book and a ledger.

L.

LAND-WARRANT, a title to a lot of public land.
LEASE, a deed; a contract for the use of property.
LEGAL TENDER, the authorized coins or money of a country.
LETTER OF ADVICE, intelligence.
LETTER OF ATTORNEY, legal authority to act for another.
LETTER OF CREDIT, a letter from a mercantile or banking-house given to a traveler, by which he can collect money in a foreign country.
LEVEE, shipping place or landing.
LICENSE, a grant.
LIEN, a legal claim; power to prevent sale by another.
LIGHTERAGE, charges for conveying goods by a lighter.
LIQUIDATION, the act of settling debts.
LIVE STOCK, animals kept on a farm or for sale, as cows, horses, hogs, etc.
LLOYDS, an establishment in London for the classification of ships; a place of assembly for merchants and underwriters to assemble.
LUGGAGE, baggage, the clothing, etc., of a traveler.

M.

MANIFEST, a list or exhibit of a vessel's cargo.
MARINE, belonging or pertaining to the sea.
MARKET, a place of sale; price.
MARKETABLE, what may be readily sold.
MART, a market.
MATERIALS, the substances from which goods and wares are made up.
MATURITY, the time when a bill or note falls due.
MEASUREMENT GOODS, light goods taken on freight by measurement.
MERCHANDISE, trade goods or wares; goods bought to sell.
MINT, an official place for coining money.
MONEY BROKER, a dealer in money, bills of exchange, etc.
MORTGAGE, a pledge of land for the payment of a debt.
MORTGAGEE, the person who holds the pledge.
MORTGAGER, the person who gives the pledge.
MOVABLES, things that can be moved easily, as furniture, etc.

N.

NEST, a set of tubs, buckets, baskets, etc.
NET, the clear amount, the quantity remaining after all deductions.
NET PROCEEDS, the remainder after deducting expenses.
NOTARIAL SEAL, the seal of a notary public.
NOTARY PUBLIC, an officer authorized to attest documents and protest bills of exchange, notes, etc., for non-payment or non-acceptance.
NOTE, a written promise to pay a debt; a memorandum.

O.

OBLIGATION, a bond, a binding agreement.
ORDER, a request to pay; commission given to supply goods.
ORDER-BOOK, a manufacturer's book, in which orders are copied.

P.

PACKAGE, a bundle.
PACKER, a person who receives goods to pack for shipment.
PANIC, a monetary pressure or crisis.
PAPER, an article in common use; the name given by merchants to notes, bills, etc.
PAPER CURRENCY, paper money of a country.
PARCEL, a small package or bundle.
PARTNER, an associate; the member of a copartnership.
PAR OF EXCHANGE, the value of money, both in weight and fineness, when compared with that of other countries.
PASS-BOOK, a small book kept between a bank and its depositors, a merchant and his customers.
PAYEE, the person to whom money is to be paid.
PEDDLE, to carry about goods for sale.
PERSONAL PROPERTY, money and movable goods outside of one's business.
PETTY CASH-BOOK, a memorandum book of small receipts and expenses.
POLICY, a writing of agreement given by insurance companies.
POST-DATE, to date after the real time.
POSTING, transferring from day-book, journal, etc., to the ledger.

POWER OF ATTORNEY, authority to act for another.
PRICE CURRENT, a published list of market prices.
PRIME, superior.
PRINCIPAL, the head of a school or business.
PRO-FORMA, according to form.
PROMISSORY NOTE, an engagement in writing to pay a specified sum at a a stated time.
PROSPECTUS, outline or sketch of an institution, business, book, etc.
PROTEST, an official notice from a notary public of the non-payment of a bill, preparatory to legal proceedings.
PURVEYOR, one who supplies provisions.

Q.

QUARTER, the fourth part of any thing; a measure of weight, 25 lbs.; also a measure of length, 9 inches.
QUOTATIONS, current price for stock and shares, or articles of produce in the market.

R.

REBATE, discount, a reduction.
RECEIPT, an acquittance, acknowledgement of payment.
RECEIVER, a cashier, a person appointed to take charge of property in litigation.
RECEIVING HOUSE, a depot or store.
RESOURCES, funds, assets, that which may be converted into supplies.
RETURNS, profits or receipts in business; accounts of goods sold by an agent.
REMITTANCE, bills or money sent from one house to another.
RENEWAL of a bill or note, giving a new note for a longer time; extension of time on notes, etc.

S.

SALE, an auction; the disposal of goods to a private bidder.
SALVAGE, a reward claimed for saving property from loss at sea.
SAVINGS BANKS, banks of deposit, where interest is allowed on the amount lodged
SCHEDULE, an inventory of goods on parchment or paper.
SCRIP, a receipt or acknowledgment for installments paid on stocks; a partial receipt, to be substituted by a receipt in full when all has been paid.
SECRETARY, a head clerk or writer; the recording officer of a society.
SHIP-LETTER, a letter forwarded by private ship, instead of a packet chartered for that purpose.
SHIPPED, transmitted by sea; goods forwarded by any conveyance.
SHIPPING CLERK, a person who attends to shipping of goods.
SHIPMENT. the goods forwarded by railroad or steamboat; a term in double-entry book-keeping.
SHOP, a work-room; the name given to stores in England.
SIGHT, or AT SIGHT, the time when a bill is presented to a person on whom it is drawn.
SIGNATURE, the name of a person written by himself.
SILENT PARTNER, a partner who puts in capital, but does not take an active part in the business.
SLEEPING PARTNER, the term used in Britain for silent partner.

SMUGGLING, passing goods into a country clandestinely, so as to avoid the duties.

STAPLE, the commodities which always meet with ready sale; the principal articles of produce or manufacture of a country.

STERLING, according to a fixed standard; a term applied to the money of Great Britain.

STOCK, goods kept for sale; materials of manufacture; animals on a farm.

STORAGE, charge for the use of a warehouse.

STORES, supplies laid in for a ship.

SUNDRIES, in book-keeping, more than one; plurality.

SUSPENDED, temporarily removed from employment; alleged inability to pay debts; stoppage of work or business.

T.

TELLER, an officer in a bank who receives or pays money.

TIERCE, a cask containing about 42 gallons.

TRADE, the commerce of a country; to exchange commodities; a bargain.

TRADE ALLOWANCE, trade price; a discount allowed to merchants.

TRADESMAN, a mechanic; in England, a storekeeper or retailer.

TRAFFIC, trade, exchanging commodities.

TRANSCRIPT, a copy.

TRANSFER, a change of property, government funds, etc.

TRANSHIPMENT, the act of removing from one ship to another.

TRANSPORTATION, the conveyance of goods; a name for a forwarding company.

TRANSITU, on passage; on the way from one place to another.

V.

VENDOR, a seller; one who disposes of goods or property.

VENDUE, a public sale; an auction.

VOUCHER, an instrument of writing; a document produced to substantiate a statement of disbursements.

W.

WALL STREET, the street in New York City where the principal bankers are located.

WAREHOUSE, store-room; a place for depositing goods.

WRIT, an official notice from a law court.

TO TEACHERS.—Attention is requested to the arrangement of the answers, which makes the book subserve the purpose of one without answers and a key.

In some places, as on page 44, the sum of the answers to a group of questions is given. Should the pupil construe his work to suit this answer, the teacher can detect the fraud by working any one of the questions of the group.

In other places, as on page 40, the answers are arranged promiscuously, with usually one or more than the number of questions. This prevents copying or working from the answer, and yet encourages the learner to study.

MONEY, WEIGHTS AND MEASURES.

FEDERAL MONEY consists of four kinds: gold, silver, nickel and paper. The smallest gold coin is of the denomination of one dollar. Other gold coins are the quarter-eagle, half-eagle, eagle and double-eagle.

The silver coins are the dollar, half-dollar, quarter-dollar, dime, half-dime and three-cent piece.

These are also represented by paper of the same denomination.

The nickel coins are the one and two-cent pieces.

The established currency of the United States consists of the eagle, dollar, dime and mill; but accounts are kept in dollars and cents only.

TROY WEIGHT is used in the sale of gold and silver and at the mint for coinage: 24 grains=1 pennyweight; 20 pennyweights=1 ounce; 12 ounces=1 pound. The signs are *gr.*, *pwt.*, *oz.*

The caret, when applied to gold, is only a comparative weight, used to indicate the proportions of pure gold and alloy. It is $\frac{1}{24}$ part of the mass of whatever weight. 18 carets fine is $\frac{18}{24}$ gold, or 18 parts gold and 6 alloy.

COMMERCIAL WEIGHT, used in selling groceries, drugs, etc.: 16 ounces=1 pound; 2000 pounds=1 ton. Signs, *oz.*, *lbs.*, T.

NOTE.—The ounce and pound are the principal parts of avoirdupois weight in use in the United States. Iron ore and hemp are weighed by the old standard, 112 pounds to the hundred (*cwt.*) and 20 hundreds, or 2240 pounds, to the ton. See Weight of a Ton page 17.

MEASURES OF CAPACITY.—The units of measurement are the gallon for liquid, and the bushel for dry measure. The gallon contains 58372.2 grains Troy of the standard pound of distilled water at 39° F., weighed in air of the temperature of 62°, and barometer pressure 30 inches. It contains nearly 231 cubic inches.

The bushel contains 543391.89 grains Troy of distilled water, under the above conditions, and is thus the Winchester bushel of 2150.42 cubic inches.

DRY MEASURE, used for measuring grain, fruit, etc.: 2 pints=1 quart; 8 quarts=1 peck; 4 pecks=1 bushel. Signs, *pt.*, *qt.*, *pk.*, *bu.*

NOTES.—1. The U. S. bushel is a cylindrical vessel, 8 inches deep and 18½ diameter, inside, and contains 2150.42 cubic inches.

2. By statute in Ohio, the bushel for stone coal, coke and unslacked lime contains 2688 cubic inches, and the measure should be 24 inches at the top, 20 inches at the bottom and 14.1 deep, and contain *two bushels.*

3. The bushel of New York State contains 80 pounds of pure water, or 2211.84 cubic inches.

LIQUID MEASURE,* for measuring all liquids: 4 gills=1 pint; 2 pints=1 quart; 4 quarts=1 gallon.

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*1. Liquid measure is the old wine measure, and has superseded that of beer and ale measure, both in the United States and Great Britain.

2. The gill is seldom used, while barrels, tierces, etc., are gauged and reckoned by gallons.

3. The gallon contains 231 cubic inches.

4. A pint of water weighs 1 pound.

5. The capacity of cisterns, vats, etc., is usually reckoned in barrels and hogsheads. 31½ gallons=1 barrel; 2 barrels, or 63 gallons=1 hogshead.

WEIGHTS OF PRODUCE PER BUSHEL, according to usage in Cincinnati, and as fixed by statute in Ohio:

	Usage. lbs.	Stat. lbs.
Apples, dried	25	25
Barley	48	48
Barley malt, weight of bags included	34	
Beans	60	60
Bran	20	
Bran shorts	25	
Broom-corn	30	
Buckwheat	52	50
Coal, bituminous	80	
cannel	70	
Charcoal	30	
Coke	32	
Castor beans	46	
Corn, shelled	56	56
in ear....68 and	70	70
Hair, plastering	8	
wet	16	
Hominy	60	60
Lime, slacked	51	
Malt		34
Meal, corn	50	
Middlings	40	
Oats	33	32
Onions	56	
Onion sets	25	
Peaches, dried	33	33
Peas	60	60
green	24	
Plaster and hair	118	
Peanuts, roasted	22	
Potatoes, Irish	60	60
sweet		50
Rye	56	56
Rye malt, wt. of bags included	40	
Salt	50	
Seed, clover	62	60
timothy	45	45
flax	56	56
hemp	44	44
orchard grass	14	
Hungarian grass	50	50
blue grass	14	
millet	50	50
canary	60	
sorghum	45	
Ship stuff	40	
Shorts	30	
Turnips	60	
Wheat	60	60
Water, distilled	77.6274	

WEIGHTS PER TON:

	lbs.
Pig Iron, chill mold	2240
Pig Iron, sand molds	2268
Blooms	2464
Iron ore	2240
Hemp	2240
Hay	2000

THE WEIGHT OF A PINT OF

	ounces.
Flour	14
Meal	18
Butter	15
Crush sugar	17
Brown sugar	18
Loaf sugar	19

WEIGHT OF A CUBIC FOOT OF

	lbs.		lbs.
Cast iron	450.55	Yellow pine	33.81
Wrought iron	486.65	White oak	35.2
Steel	489.8	Live oak	70
Copper	565	Salt water (sea)	64.3
Lead	708.75	Fresh water	62.5
Brass	537.75	Air	.07529
Tin	456	Steam	.03689
White pine	29.56	Clay	135
Loose earth or sand	95	Sand	113
Common soil	124	Cork	15
Strong soil	127	Tallow	59
Clay	135	Brick	119
Coal	45 to 55	Coke	32
Charcoal	18 to 18.5	Ice	58

23 cubic feet of sand, 18 of earth or 17 of clay make a ton.

APOTHECARIES' FLUID MEASURE, used in compounding medicines: 60 minims=1 fluid drachm; 8 fluid drachms=1 fluid ounce; 16 fluid ounces=1 pint; 8 pints=1 gallon. Signs, *M.*, minim; *f*ʒ., fluid drachm; *f*℥., fluid ounce; ⊙, pint; *Cong.*, gallon.

MEASURES OF TIME.—Time is divided into seconds, minutes, hours, days, weeks, months, years and centuries.

60 seconds=1 minute; 60 minutes=1 hour; 24 hours=1 day; 7 days=1 week; 4 weeks=1 lunar month; 12 calendar months=1 year; 365 days=1 common year; 366 days=1 leap-year; 365 days 5 hours 48 minutes 49.7 seconds, or $365\frac{1}{4}$=1 solar year.

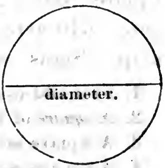
A leap-year is exactly divisible by 4, and has 29 days in February. 1860 and 1864 were leap-years.

The calendar months are

1. January,	31 days.		7. July,	31 days.	
2. February,	28	"	8. August,	31	"
3. March,	31	"	9. September,	30	"
4. April,	30	"	10. October,	31	"
5. May,	31	"	11. November,	30	"
6. June,	30	"	12. December.	31	

Commencing with January, every other month has 31 days to July, inclusive; and commencing with August, every other month has 31 days to December, inclusive.

CIRCULAR MEASURE is divided into seconds, minutes and degrees. 60 seconds=1 minute; 60 minutes=1 degree; 360 degrees=1 circumference.



Signs, ″, seconds; ′, minutes; °, degrees.

25° 31′ 27″, 25 deg., 31 min. 27 sec.

LINEAR MEASURE.—*Long Measure* is used for measuring length, breadth, depth or distance. 12 inches=1 foot; 3 feet=1 yard; $5\frac{1}{2}$ yards=1 rod, perch or pole; 40 rods=1 furlong; 8 furlongs, or 320 rods=1 mile. Signs, *in.*, inches; *ft.*, feet; *yd.*, yard; *rd.*, rod; *fur.*, furlong; *mi.*, mile.

1. In a mile there are 63360 inches, 5280 feet, 1760 yards, 320 rods.
2. The furlong is seldom used.
3. The 12th part of an inch is called a *line.*
4. Cloth is measured by the yard and fractional parts of a yard.

MARINE MEASURE, for measuring distances at sea: 6 feet=1 fathom; 120 fathoms=1 cable length: 880 fathoms=1 mile, called a nautical or geographical mile; 60 geographical or 69.77 statute miles=1 degree.*

The speed of a ship at sea is measured by an instrument called a log-line, the knots in which correspond to the number of miles sailed per hour. *Knot* is therefore synonymous with mile. A ship sailing at 7 knots is moving at the rate of 7 miles an hour.

SQUARE OR SURFACE MEASURE.—Surfaces are measured by taking the length and breadth, by long measure, and multiplying them together. The length and breadth

---

*The depth of the sea is measured by fathoms.

of the surface of a foot are 12 inches each; hence, 12 times 12=144 square inches=1 square foot; 9 square feet=1 square yard.

Land Measure: $30\frac{1}{4}$ square yards=1 square rod; 40 square rods=1 rood; 4 roods, or 10 square chains=1 acre; 640 acres=1 square mile; 36 square miles=1 township. Signs, *sq. yds.*, *sq. rds.*, *R.*, *A.*

1. Feet and even inches may also be used in measuring land.
2. A *square* of flooring or roofing is 100 square feet.
3. A square mile is sometimes called a section.
4. A square rood contains $272\frac{1}{4}$ square feet; an acre, 43560 square feet.

Cubic or Solid Measure includes three dimensions, length, breadth (or width) and thickness (or depth) multiplied together. 1728 cubic inches=1 cube foot; 27 cubic feet=1 cubic yard.

Stone Measure is applied to masonry, which is sometimes paid for by the foot, but usually by the perch. $24\frac{3}{4}$ or 25 cubic feet=1 perch; the former for private, the latter for public contracts, as railroad or government work.

Wood Measure: Wood is sold by the cord, which should measure 128 cubic feet closely piled, or 138 feet if stowed in a boat or barge. A pile of wood 8 feet long, 4 feet wide and 4 feet thick contains a cord.

Bricklayers' Measure.—The common dimensions of a brick are 8 inches long, 4 inches broad and 2 inches thick. There are 21 bricks in a cubic foot of wall, including mortar.

A wall 8 inches or 1 brick in thickness contains 14 bricks to the square foot of surface.

A wall 12 inches or $1\frac{1}{2}$ bricks in thickness contains 21 bricks to the square foot of surface.

A wall 16 inches or 2 bricks in thickness contains 28 bricks to the square foot of surface.

A fall of $\frac{1}{10}$ of an inch in a mile will produce a current in rivers.

Ice 2 inches thick will bear infantry; 4 inches, cavalry or light guns; 6 inches, heavy field pieces.

PAPER.—*For Printers.* Sizes of paper made by machinery:

Double imperial, 32 by 44.
Double super royal, 27 by 42.
Double medium, 23 by 26, 24 by 37½ and 25 by 38.
Royal and half, 25 by 29.
Imperial and half, 26 by 32.
Imperial, 22 by 32.
Super royal, 21 by 27.
Royal, 19 by 24, 20 by 25.
Medium, 18½ by 23½.
Demy, 17 by 22.
Folio post, 16 by 21.
Foolscap, 14 by 17.
Crown, 15 by 20.

A sheet folded in 2 leaves is called a *folio;* in 4 leaves, a *quarto;* in 8 leaves, an *octavo*, or *8vo.;* in 12 leaves, a *duodecimo*, or 12mo.; in 18 leaves, an 18mo.; in 24 leaves, a 24mo.

Stationers.—24 sheets=1 quire; 20 quires=1 ream.

Bookbinders count from 16 to 20 sheets to the quire in binding account books.

Wrapping Papers are sold by the ream and bundle; some reams are *short count;* the long count reams contain full quires.

SUNDRIES:

1 barrel of flour=196 lbs.
1 barrel of pork, etc.=200 lbs.
1 firkin of butter=55 lbs.
12 articles=1 dozen.
12 dozen=1 gross.
144 dozen=1 great gross.
20 articles=1 score.
3 inches=1 palm.
4 inches=1 hand.
9 inches=1 span.
3.28 feet=1 meter.
3.937 or 47 ¼ inches=1 aune.

GAS.—1.43 cubic feet of gas per hour give a light equal that of a candle; 1.96 feet equal 4 candles; 3 cubic feet equal 10 candles.

HORSE POWER, in machinery, is reckoned at 33000 lbs. raised 1 foot in a minute; but the ordinary work of a horse is 22500 lbs. per minute for 8 hours.

STRENGTH OF A MAN.—The mean effect of the power of a man, unaided by a machine, is the raising 70 lbs. 1 foot high in a second for 10 hours a day$=\frac{1}{5}$ of the power of the horse.

NOTE.—Two men working at a windlass, at right angles to each other, can raise 70 lbs. more easily than one man can 30 lbs.

A foot soldier travels 70 yards, making 90 steps, in one minute, common time.

In quick time, 86 yards, making 110 steps.

In double-quick, 109 yards, making 140 steps.

Average weight of men, 150 lbs. each.

Five men can stand in a space of 1 square yard.

A man without a load travels on a level ground $8\frac{1}{2}$ hours a day, at the rate of 3.7 miles an hour, or $31\frac{1}{4}$ miles a day. He can carry 111 lbs. 11 miles in a day.

A porter, going short distances and returning unloaded, can carry 135 lbs. 7 miles a day. He can carry in a wheelbarrow 150 lbs. 10 miles a day.

HAY.—10 cubic yards of meadow hay weigh a ton. When the hay is taken out of old, or the lower part of large stacks, 8 to 9 cubic yards will make a ton.

HILLS IN AN ACRE.—3 feet apart, there are 4840 hills in an acre.

BRITISH MONEY, WEIGHTS AND MEASURES.

In Great Britain, accounts are kept in pounds, shillings, pence and farthings. 4 farthings=1 penny; 12 pence=1 shilling; 20 shillings=1 pound. Signs, (farthings are written as fractions of a penny,) *d.*, pence; s., shillings; £, pounds.

The coins are the *copper* half-penny and penny; *silver*, three-penny, four-penny, six-penny, shilling, half-crown and crown pieces; *gold*, the half-sovereign, sovereign and guinea. The value of the crown is 5 shillings; the sovereign, 20 shillings; the guinea, 21 shillings.

THE COMMERCIAL WEIGHT is the avoirdupois, of which there are in use the ounce, pound, stone, quarter, hundred and ton. 16 drachms=1 ounce; 16 ounces=1 pound; 14 pounds=1 stone; 28 pounds=1 quarter; 4 quarters=1 hundred; 20 hundred=1 ton. Signs *dr.*, drachms; *oz.*, ounces; *lbs.*, pounds; *qrs.*, quarters; *cwt.*, hundreds; *T.*, tons.

IMPERIAL MEASURES OF CAPACITY for all liquids and dry goods, such as grain, potatoes, etc.:

1 gill (gl.)=8.6648 cubic inches=5 oz. water.
4 gills=1 pint=34.65925 cubic inches=1¼ lbs. water.
2 pints (pts.)=1 quart=69.3185 cubic inches=2½ lbs water.
4 quarts (qts.)=1 gallon=277.274 cubic inches=10 lbs water.
2 gallons (gal.)=1 peck=554.548 cubic inches=20 lbs. water.
4 pecks (pk.)=1 bushel=2218.192 cubic inches=80 lbs. water.
8 bushels (bu.)=1 quarter=17745.536 cubic in.=640 lbs. water.
4 quarters (qr.)=1 chaldron.
10 quarters=1 last.

The largest measure for liquids is the gallon; the smallest for grain, etc., the peck.

In London, a chaldron of coal contains 36 bushels.

NOTE.—By act of Parliament, in 1824, wine, ale and dry measures were superseded by the imperial measures of capacity.

Time is divided into quarterly terms not recognized in the United States.

In England.	In Scotland.
Lady day, or 1st term.......Mar. 25	Candlemas, or 1st term....Feb. 2
Midsummer, or 2d term...June 24	Whitsunday, or 2d term...May 15
Michaelmas, or 3d term...Sept. 29	Lammas, or 3d term.......Aug. 1
Christmas, or 4th term ...Dec. 25	Martinmas, or 4th term...Nov. 11

Linear, square or superficial, cubic measures, etc., are the same in both countries.

Articles sold by "tale," or count: 12 articles=1 dozen; 12 dozen=1 gross; 5 score=1 hundred; 6 score=1 long hundred.

MULTIPLICATION TABLE.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

NELSON'S

COMMON-SCHOOL ARITHMETIC.

I. INTRODUCTION.

1. Arithmetic is the art or science of computing numbers.

2. The theory of Arithmetic treats of the properties and relations of numbers.

3. The practice of Arithmetic shows the application of numbers to business, the mechanic arts, etc.

4. ARITHMETICAL SIGNS.

+, called *plus*, is the sign of addition.

= is the sign of equality. 5+4=9, is read, *five plus four equals nine.*

—, called *Minus*, is the sign of subtraction. 5—3=2, is read, *five minus three equals two.*

÷ is the sign of division. 9÷3=3, is read, *nine divided by three equals three.*

. is the decimal sign. Placed at the left of a number, it represents *tenths, hundredths*, etc.

: :: : is the sign of proportion. 3:4::6:8, is read, *three is to four as six is to eight.*

4, 4′, 4″, 4‴, is read, *four, four prime, four second, four third.*

$\sqrt{}$ is called the *radical* sign, or sign of square root. $\sqrt{4}=2$, is read *the square root of four equals two.*

$\sqrt[3]{}$. A figure inserted, as the 3, indicates the root to be taken; 3, the cube root; 4, the fourth root.

3^2. A small figure written as the 2 in the margin, indicates that the number is to be raised to a corresponding power; 2, the second power; 3, the third power. This figure is called the *index* of the power.

II. NOTATION AND NUMERATION.

5. *Notation* is the art of representing numbers by symbols, called *figures* or *digits.* There are ten of these figures:

0	1	2	3	4	5	6	7	8	9
naught,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.

The first is also called zero, or cipher.

6. When a larger number than nine is to be represented, two or more figures are used.

7. *Numeration* is the method of reading these figures when arranged to represent numbers. For this purpose they are usually divided into periods of three from the *right.*

8. The first period on the right contains *units*, *tens* and *hundreds*, thus:

1 2 5
hund., tens, units,

which is read, *one hundred and twenty-five.*

The second period contains units, tens and hundreds of *thousands*, thus:

125 , 000
thous., hund.,

which is read, *one hundred and twenty-five thousand.*

The third period contains units, tens, and hundreds of millions, thus:

125 , 000 , 000.
mills., thous., hund.,

which is read, *one hundred and twenty-five millions.*

The fourth period contains *billions*, the fifth *trillions*, the sixth *quadrillions*, the seventh *quintillions*.

RECAPITULATION.—The first period is hundreds, the second thousands, the third millions, the fourth billions, etc.

To read 12376421097, we point off thus:

12,376,421,097.

Here are four periods—the fourth is billions. The number is 12 billions, 376 millions, 421 thousand and 97.

*WRITE THE FOLLOWING IN WORDS:

1	2	3	4
15	127627	1464780	27100101
25	184194	1700700	198140197
125	710107	4001001	100001009
1125	411001	7119031	600100100
23125	100020	1000010	190999009

5
3214100967831
4191006848219

WRITE THE FOLLOWING IN FIGURES:

1. Ten.
2. Seventeen.
3. One hundred and twenty.
4. Three hundred and twenty-four.
5. One thousand and eighty.
6. One hundred and twenty thousand five hundred.
7. Three hundred and ninety-seven thousand four hundred and forty-four.

* Exercises under articles 10, 11, and 12 may precede these.

8. Twelve millions one hundred and twenty-five thousand one hundred and one.
9. One billion one hundred and one thousand and one.
10. Thirty millions eighty-five thousand one hundred and seven.
11. Seventy-six trillions five hundred and forty billions one hundred and ten millions and sixty-seven.
12. Two hundred millions.
13. One hundred trillions.
14. Seven hundred billions three thousand and seven.
15. One thousand.
16. One million.
17. One trillion one hundred thousand.
18. One billion.
19. Thirteen billions three millions seven thousand.
20. One hundred and ten millions and eighty-seven.
21. Seventy-five millions six thousand and nine.

9. FEDERAL MONEY.

The name usually given to the money of the United States is *Federal Money.* It is reckoned by tens and hundreds. Though there are various kinds of gold, silver and nickel *coins*, money is always reckoned in dollars and cents, or dollars, cents and mills.

$ is the dollar sign.

c, the sign for cents.

m, the sign for mills.

$3456.87,5, is read, *three thousand four hundred and fifty-six dollars, eighty-seven cents, five mills.*

Mills are written one place to the right of cents. In this book mills will sometimes be separated from cents by a comma, as above.

A period is used to separate cents from dollars, the two first figures on the right being cents and tens of cents.

WRITE THE FOLLOWING IN WORDS:

1	2	3
$457.	$31412.875	$507.974
$25.25	$167.476	$1134.643
$364.61	$365.323	$216.132
$112.57	$4767.126	$345.815
$3146.87	$116.254	$341.664
$213.18	$67.141	$416.304
$456.45	.75	.17 c
$1719.97	.353	.413 m
$7.90	$1.273	.674 m
$304.02	$35.144	$100.603
$117.44	$414.124	$210.301
$32.32	.67 c	.164 m

When there are no tens of cents, a cipher is written in the ten's place, as in 304.02, Ex. 1, which is read *three hundred and four dollars and two cents.*

Separate the dollars, cents and mills in the following, observing to write dollars under dollars and cents under cents:

4	5	6	7
307 c	135006 m	3171 c	24136 c
14064 m	2703 m	1463 m	7314 m
32002 m	753 m	12195 m	21364 c
100011 m	35001 m	3697 c	71836 m
1826063 m	61422 c	2468 m	2173 c
116003 m	71922 c	14193 c	1897 m
31463 c	143684 m	21314 c	1367841 m
6897 m	900013 c	2136821 m	146001 c
18987 m	41362 c	21367 m	14367 c
31464 c	13687 m	98076 c	314683 m

8. 3141 c, 1386 c, 19301 c, 71432 c, 7321 c, 906301 c, 2136 c, 4563 c, 10001 c, 30351 c, 9878 c, 45121 c, 31645 c, 13621 c, 21001 c, 60006 c, 213621 c, 113600 c.

9. 21360 m, 31461 m, 14565 m, 24798 m, 46503 m, 106 m, 796543 m, 45631 m, 245 m, 14013 m, 41634 m, 14563 m, 21364 m, 7856 m, 24130 m, 45103 m, 1467 m, 4136 m.

10. 20135 m, 314102 m, 14163 m, 213102 m, 31453 m, 12063 m, 14673 m, 24135 m, 43103 m, 156307 m, 14007 m, 34617 m, 21368 m, 146 m, 178936 m, 213641 m, 14567 m, 36845 m, 213146 m, 145163 m.

11. 12314 m, 2136 m, 100135 m, 21364 m, 146345 m, 213984 m, 136453 m, 1467 m, 13645 m, 14107 m, 30674 m, 1451 m, 147431 m, 14345 m, 41367 m, 1456 m, 21314 m, 16789 m, 13674 m, 14683 m, 1413 m, 31463 m, 16703 m, 4103 m, 163 m.

12. 1456 m, 31463 m, 21456 m, 14637 m, 2136 m, 415 m, 1467 m, 23460 m, 1454 m, 3417 m, 14567 m, 14674 m, 13654 m, 14345 m, 41631 m, 14367 m, 1456 m, 145634 m, 1456 m, 14567 m, 4453 m, 14639 m.

13. 23145 m, 12346 m, 16734 m, 2146 m, 1678 m, 1463 m, 14596 m, 3045 m, 163201 m, 146324 m, 14567 m, 14631 m, 21393 m, 7894 m, 14637 m, 21364 m, 14567 m, 176301 m, 21367 m, 140601 m, 14563 m.

14. Write in columns, as before, the following: Seventy-five dollars, eighty-seven cents; thirty-three dollars, sixty-one cents, five mills; seven hundred and ninety-six dollars and sixty cents; five thousand dollars; five thousand three hundred and eighteen dollars and sixty-three cents; two hundred and fifty-six dollars, fourteen cents, four mills; one hundred dollars, sixty cents, three mills.

15. Thirty-six hundred dollars, seven cents, and five mills; eight hundred thousand dollars, forty cents, seven mills; sixty-seven dollars, eighty cents; nine hundred dollars, seventy-five cents, six mills; three hundred and seventy-six dollars, six cents, four mills; sixty-four dollars, eighteen cents, seven mills; fifty-nine dollars, six cents, three mills; eight hundred dollars, one cent, three mills.

17. One thousand five hundred dollars, sixty cents, eight mills; three hundred and fifty dollars and six mills; seventy-eight dollars, eighty cents, six mills; four hundred fifty-seven dollars, sixty-four cents, seven mills.

10. ODD NUMBERS.*

The numbers 1, 3, 5, 7, and 9 are called *odd* numbers, and every number which has one of these figures in the unit's place, as 11, 13, 15, is also an odd number.

1. Write in columns all the odd numbers from 1 to 151, observing to keep units under units, tens under tens, and hundreds under hundreds.
2. Write in the same way all the odd numbers from 151 to 351.
3. Write in the same way all the odd numbers from 351 to 601.
4. Write in the same way all the odd numbers from 601 to 901.

11. EVEN NUMBERS.

The numbers 2, 4, 6, 8 are called even numbers, and every number which has 0 or one of these as a unit figure, is also an even number.

5. Write in columns, as before, all the even numbers from 2 to 200, inclusive.
6. Write in the same way all even numbers from 200 up to 500, inclusive.
7. Write in the same way all the even numbers from 500 to 800, inclusive.
8. Write in the same way all the even numbers from 800 to 1100, inclusive.

The Teacher should follow these exercises by others, from dictation, until the scholars are taught to write any sum without hesitation.

* With very young learners, these exercises should precede those on pages 26 and 27.

12. ADDING.

In the preceding exercises the learner unconsciously added 2 every time he passed from one number to another. In the following exercises he will be required to add 3, 4, 5, etc., and unite the results in the same way. He should observe to write the figures in straight lines.

9. Commencing at 1, add 3 every time until you reach 97, thus: 1, 4, 7, 10, 13, putting the numbers *under* each other.

10. Commencing at 1, add 4 every time till you reach 121.

11. Commencing at 1, add 5 every time till you reach 161.

12. Commencing at 2, add 6 every time till you reach 200.

13. Commencing at 1, add 7 every time till you reach 232.

14. Commencing at 1, add 8 every time till you reach 265.

15. Commencing at 1, add 9 every time till you reach 307.

16. Add 3 to all the odd numbers up to 101. For this purpose write the odd numbers on the left of the new numbers, thus:

1 4
3 6

17. Add, in the same way, 4, 5 and 6 up to 51.

18. Add, in the same way, 7, 8 and 9 up to 51.

19. Add 3, 4, 5 and 6 to all the even numbers up to 50.

20. Add 7, 8 and 9 to all even numbers up to 80.

21. Add, in the same way, 2 to the following numbers: 19, 29, 39, 49, 59, 69, 79, 89, 99, and add 3, 4, 5, 6, 7, 8 and 9 in the same way.

22. Add 2 to the following numbers, after writing them on the left: 8, 18, 28, 38, 48, 58, 68, 78, 88, 98.

Add 3, 4, 5, 6, 7, 8 and 9 in the same way.

23. Add 3, 4, 5, 6, 7, 8 and 9 to the following numbers: 7, 17, 27, 37, 47, 57, 67, 77, 87 and 97.

24. Add 4, 5, 6, 7, 8 and 9 to 6, 16, 26, 36, 46, 56, 66, 76, 86 and 96.

25. Add 5, 6, 7, 8 and 9 to 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95.

26. Add 6, 7, 8 and 9 to 4, 14, 24, 34, 44, 54, 64, 74, 84 and 94.

27. Add 7, 8 and 9 to 3, 13, 23, 33, 43, 53, 63, 73, 83 and 93.

The Teacher ought to examine his scholars on the terms, signs and principles of each rule. In this subject, on the difference between notation and numeration, how many figures necessary to write one hundred, how many a thousand, etc.

III. ADDITION.*

13. The method of uniting two or more numbers into one is called *Addition.*

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*The Teacher will find exercises for evening study at the end of this chapter.

Beginners should not be allowed to count on their fingers or talk over the process. If drilled in the use of the catch-figure by blackboard exercises, they will not afterward resort to any of the slower methods of computation. The use of the catch-figure is in part taught in exercises, page 32 and 33. For the purpose of drill, it ought to be taught as follows: "7 and 9, the unit figure is what?" "6." "17 and 9?" "26." "27 and 9?" "36." "Observe that when 7 and 9 are added, the unit figure is 6." "47 and 9?" "57 and 9?" "67 and 9?"

Classes should be exercised in this way through all the combinations found in the exercises referred to.

14. The *one* number is called the *sum*, *amount*, *total*, or *footing*.

15. The sign is +, and is called *plus*. When placed between two numbers it indicates that they are to be added together. 3+2=5, is read, *three plus two equals five*.

16. In performing operations in addition, it is necessary to write the units, tens, hundreds, etc., of the one number under the units, tens, hundreds, etc., of the other, which arranges the figures in one straight line.

1. To add together 135, 241 and 323.

```
135
241
323
---
699
```

Explanation.—Here units are placed under units, tens under tens and hundreds under hundreds. After arranging the figures thus, we commence at the right-hand column and add 3 to 1, which makes 4, to which add 5 and we have 9.

Adding the tens' column in the same way, we have 9 tens, which we write in the tens' place.

Adding the hundreds' column in the same way, we have 6, which we write in the hundreds' place.

Find the sum of each of the following groups:*

2	3	4	5	6	7	8
3131	131	211	1534	3143	3141	2131
223	453	765	1232	2102	5432	1036
445	100	23	1002	1413	1426	5812
3799	684	999				

9	10	11	12	13	14	15
211	3145	4512	2131	14132	14413	1613
101	4132	1035	1027	1734	34441	143
65	1712	4241	1720	4113	41104	1233

* Answers arranged promiscuously: 377, 9788, 2989, 19979, 3768, 9999, 6658, 8989, 3989, 8979, 4878, 89958.

To add 325, 42 and 178.

325
42
178
Answer, 545

EXPLANATION.—1. Placing the numbers as directed, we proceed to find the amount of the first column on the right: 8, 2 and 5 are 15; that is, 1 ten and 5 units.

2. Writing the 5 units under the units, we add the 1 ten to the tens' column.

3. This one added to the 7, 4 and 2 makes 14 *tens* or 1 hundred and 4 tens.

4. Writing the 4 under the tens, we add the 1 hundred to the hundreds' column.

5. This 1 added to the 1 and 3 in the hundreds' column makes 5 hundreds, which 5 we write in the hundreds' place and our work is done.

17. To add 4501+3213+1007+302, we write them thus:

4501
3213
1007
302
Ans. 9023

Add together the following numbers:

18. 3478, 3167, 4199, 7854, 3456. *Ans.* 22154
19. 1417, 210, 61907, 216, 3184. *Ans.* 66934
20. 7894, 2176, 7, 109, 7998. *Ans.* 18184
21. 376+100+71+416+709+317. *Ans.* 1989
22. 1006+3009+79999+7098+17. *Ans.* 91129
23. 316+10069+9777+307+198. *Ans.* 20667
24. 789632+4+67+879002+876+970 is how much? *Ans.* 1670551
25. 98632+76398+832+97+10029+97384 is how much? *Ans.* 283372
26. 1324+4354653+12+876+97843+68473 is how much? *Ans.* 4523181

27. 31465+2316532+107+3790+465321+3654563+107653+23650+1007+30672+503102+21063 is how much?

28. 18230+476+41034+9875+65432+5678+12090+9387+8276+565+13654+443=how much?

Ans. Sum of 27 and 28, 7344065.

29. Add together 45679+9837+18708+7967+485+78963+84989+12345+7069+8090+7483+96748.

TAKING TWO AND THREE FIGURES AT A TIME.

To enable scholars to grasp two and three figures at a time, and carry them up as one, they might be exercised on the blackboard in such sums as the following:

1	3	6	3	7	7	4	3	5
9	5	4	1	8	6	9	8	7

1	4	6	7	3	9	2	1	3	6	9	3
7	8	2	1	6	3	8	4	5	6	7	3
2	1	3	4	1	4	1	3	6	2	1	2

Such exercises ought to be of frequent occurrence and scholars encouraged to answer in concert.

The answers should be given instantaneously, *naming only the unit figure*, as shown in the column below:

$$\left.\begin{matrix}3456\\1345\end{matrix}\right\}1 \quad \left.\begin{matrix}3689\\1563\end{matrix}\right\}2 \quad \left.\begin{matrix}9456\\3689\end{matrix}\right\}5 \quad \left.\begin{matrix}8998\\1898\end{matrix}\right\}6 \quad \left.\begin{matrix}9873\\1678\end{matrix}\right\}1 \quad \left.\begin{matrix}1684\\7893\end{matrix}\right\}7 \quad \left.\begin{matrix}1453\\1763\end{matrix}\right\}6 \quad \left.\begin{matrix}2195\\9876\end{matrix}\right\}1 \quad \left.\begin{matrix}7897\\2536\end{matrix}\right\}3 \quad \left.\begin{matrix}3529\\1438\end{matrix}\right\}7$$

After writing on the right of the first column the figures produced by pairing, the teacher may lead the class in adding, thus: 17 and 3? 30 and 1? 41 and 6? 47 and 7? 54 and 1? 65 and 6? 81 and 5? 96 and 2? 108 and 11?

It will be observed that the tens produced in forming the pairs were not named. The same course should be pursued in the class, as the learner is unconscious of making as great an effort as he really does.

When the ten is omitted by mistake, attention should be called to it by giving the full number, as 15 or 11 instead of 5 or 1.

The other columns should be added without the aid of the marginal figures.

After thorough drill in this, the class should be taught to take three figures and even four as rapidly as one.

30. Find the sum of 8934, 16749, 809, 67549, 98697, 746839, 1498, 829555, 9218967, 8347912, 968000, 74685.
Total of the preceding two, 20758557.

Foot up the following columns:

31	32	33	34	35
31645	3454	4213	1565	3654
98760	2136	6314	3657	1095
36875	1364	2316	5437	9014
57893	4633	1369	3457	6789
14567	9897	9306	1234	9687
34564	7879	6039	3421	5764
46387	2164	8109	6789	1567
93178	4163	9876	1746	9139
78163	4569	6789	3456	1456
64518	5496	4567	1378	2345
17514	6428	5679	5932	5432
45678	8297	3263	4567	6542
21364	9287	9457	1657	1395
7198	7928	1459	6574	3642
3165	9872	1455	5638	1365
4124	8729	9375	4932	2315
1345	9314	5976	1397	9365
3146	3162	7639	9765	3510
4165	2136	7938	3765	1096
3216	9364	3959	1456	3765

36. Add together the following numbers: 313, 2109, 6785, 2736, 798, 987, 21363, 316, 4934, 2178, 1009, 396, 298, 2753, 607, 3145, 213, 6709, 6093, 190, 2130, 2160, 716, 213, 9876, 45678, 2137, 2198, 9039, 6789, 3097, 4684, 2136, 2178, 5672, 1987, 6789.

Answers promiscuously arranged: 95368, 77823, 120272, 115098, 667465, 88937, 171411.

The Teacher should not permit his scholars to divide these columns when adding, nor should he allow them to resort to the aid of strokes or practice counting on their fingers.

37	38	39	40	41
3286	2467	34564	46321	3614
6713	109	12345	13632	1364
3654	3178	65435	14567	5436
176	145	87654	53678	7835
3976	6178	34564	86367	4678
6345	4156	13682	85432	8793
9823	7532	75671	36457	701
6023	9890	86317	21836	9804
1367	6821	24328	17354	1306
8965	9854	98713	63542	717
8632	3821	21345	78163	2103
1034	5843	1286	82645	6397
6312	1936	78654	34685	1096
4593	7136	19876	31768	2130
3687	9876	93643	65314	3107
5006	2863	6356	68231	167
7164	123	78397	64037	2109
1763	7436	21602	34685	3678
2139	1567	71346	35962	2176
8236	2563	28653	21363	5432
7860	8432	17648	78636	2137
3613	1345	82351	19854	28639
109	8736	21368	80145	1765
1756	8654	78631	87654	371
6386	1263	17639	12345	71031
9890	1345	82360	78654	1463
8243	3093	45671	12345	3168

42. Find the sum of all the odd numbers under 100.

43. Find the sum of all the even numbers under 100.

44. Find the sum of all the numbers included in Ex. 5, page 31.

45. Find the sum of all in Ex. 6.

Answers promiscuously arranged: 181217, 1300099, 126362, 1325672, 136751, 143267, 52850, 2500, 2450, 10100, 2510.

46 to 49. Find the sum of all in Ex. 9, 10, 11 and 12.*
50 to 53. Find the sum of all in Ex. 13, 14, 15 and 16.
54 to 57. Find the sum of all in Ex. 17, 18, 19 and 20.
58 to 60. Find the sum of all in Ex. 21, 22 and 23.
61 to 64. Find the sum of all in Ex. 24, 25, 26 and 27.

65. Add together all the numbers from 300 to 320, inclusive; from 3120 to 3150, inclusive; from 160 to 200, inclusive; from 1950 to 2000, inclusive.

Answers in direct order: 9615, 19228, 25576, 25512, 19200, 211800.

17. *To add Federal Money*, we place dollars under dollars, cents under cents and mills under mills, and proceed as before.

66. What is the amount of the following sums of money? \$32.74, \$16.73, \$13.09, \$37.40, \$16.74, \$7.07.
Ans. \$123.77.

OPERATION.	
\$32.74	EXPLANATION.—The sum of the first column being 27 cents, we write the 7 and add the 2 tens of cents to the tens' column, making 27 tens, or 2 hundreds and 7 tens. Writing the 7, we add the 2 hundreds to the next, which is the dollar column, and proceed as in the above.
16.73	
13.09	
37.40	
16.74	The second column of cents might be called the *dimes'* column.
7.07	
\$123.77	*Amount.*

67. Find the amount of the following: \$1708.25, \$2076.00, \$709.07, \$109.88, \$999.87, \$370.04, \$695.83, \$797.00, \$87.00, \$400.40, \$198.08, \$109.65, \$364.08, \$217.00, \$364.09, \$785.66, \$699.08, \$776.08.

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* *The Teacher* can work most of these by Arithmetical Progression. As indicated by the numbers, these exercises may be broken into three or four parts, if considered too difficult.

68. \$1670.03+\$1006.01+ \$364.01+\$5432.99
\$2310.00+\$1068.24+\$26107.18+\$2136.18
\$109.79+ \$999.99+ \$666.56+ \$449.99
\$777.00+\$7999.00+ \$6666.00+\$6730.15

69	70	71	72
\$987.67	\$716.27	\$187.20	\$4519.27
873.35	855.60	257.65	7864.20
473.92	219.76	330.17	9510.33
187.87	912.67	700.00	3578.84
119.16	107.30	175.85	4875.60
160.97	87.60	150.50	6115.90
176.01	101.19	37.50	9885.10
634.16	808.08	57.63	1105.75
585.26	981.61	109.87	5760.87
458.39	225.00	987.05	7901.57
385.93	811.29	1285.58	7119.85
589.38	300.92	2327.88	5006.29
107.20	10.15	8900.17	9110.11
70.99	106.30	209.18	4362.17
18.18	547.67	101.01	3210.18
1764.18	336.44	125.00	2133.64
397.27	176.33	117.45	1364.57
444.99	1275.84	361.45	2136.06
222.66	666.57	217.33	1456.27
799.88	1176.22	163.77	376.22

73. Find the amount of the following sums of money: One hundred and twenty-five dollars and twenty-five cents; Sixty-eight dollars and forty-seven cents; Three hundred and ten dollars and eighty-seven cents; Six hundred dollars and seven cents; Four thousand eight hundred and fifty dollars and eighteen cents.

Answers arranged promiscuously, including those to Ex. 67 and 74: \$11467.06, \$64493.12, \$45178062.31, \$5954.84, \$2624.91, \$10422.81, \$16802.24, \$97392.79, \$9457.42, \$96394.79.

74. Add together the following amounts:

Eighteen thousand one hundred and forty-six dollars; Seven thousand one hundred and sixteen dollars and twenty-five cents; Sixty-four thousand one hundred dollars and four cents; Forty-five millions and one thousand dollars; Eighty-seven thousand seven hundred dollars and two cents.

75. A merchant has 29 pieces of silk in 1 package, 35 in another, 79 in a third. In the first there are 1497 yards; in the second, 2173 yards; in the third, 4130 yards. How many pieces, how many yards?

SOLUTION.	29		1497
	35		2173
	79		4130
Whole number of pieces	143	*Whole number of yds.*	7800

*76. A coal dealer sells 1254 bushels every day of the week, Sunday excepted; how many does he sell in all?

TO THE TEACHER.—Columns of fifteen or twenty numbers may now be dictated to classes, the teacher observing to increase the speed of the scholars at every effort. The results may be called off as produced, and written by the teacher on the blackboard, or the learners may exchange slates for examination and correction.

In this, as in all competitive exercises, the teacher should not wait until every member of the class has finished the work; but the tardy ones must not be overlooked, nevertheless. Means should be adopted to stimulate them to greater effort. They must be taught that they can not be allowed to fall behind without the risk of being returned to a lower class or grade.

The teacher probably knows that to make boys or girls reckon rapidly he *must lead;* and to this end, it would be to the advantage of both teacher and pupil if such exercises as these were always impromptu.

*In giving the answers, the learner should state whether it is bushels, pounds, etc. The abbreviation *lbs.* stands for pounds.

77. A merchant bought 9 bags of coffee, each bag weighing 215 lbs., at an average cost of $21.20; what weight of coffee did he buy, and how much did he pay for it?

78. A farmer has 118 sheep, 518 hogs, 210 pair of chickens, 5 plows, 6 wagons, 1 dozen hoes, 7 horses, 10 spades and 12 pitchforks; how many animals has he, and how many agricultural implements?

79. How many pupils in a school in which there are 5 classes, the first containing 19, the second 28, the third 32, the fourth 35 and the fifth 29 pupils?

80. Bought three boxes of oranges, in one of which there were 450, in another 469, in the last 510 oranges; how many did I buy?

81. A man walked twenty-five miles on the 20th day of the month, twenty-three on the 22d, twenty-nine on the 23d, thirty-three on the 24th day; how many miles did he walk altogether?

82. How many days in the first nine calendar months of the year?

83. Sir Isaac Newton lived 85 years and was born in 1642; in what year did he die?

Answers: 143, 1429, 273, 110, 274, 1727, 7524, 1935, 19080, 1063, 45.

The Teacher should give numerous exercises besides these, and have his scholars work them on the blackboard before the class.

HOME EXERCISES.

1. Add 1, 2, 3, 4, 5, 6, 7, 8 and 9 to 2, and take them from the result again, writing them out as below:

2 and 1 are 3; 2 from 3 leaves 1.

2 and 2 are 4; 2 from 4 leaves 2.

2 and 3 are 5; 2 from 5 leaves 3.

2. Add and subtract (take from) 3 in the same way.
3 Add and subtract 4 in the same way.
4. Add and subtract 5 in the same way.
5. Add and subtract 6 in the same way.
6. Add and subtract 7 in the same way.
7. Add and subtract 8 in the same way.
8. Add and subtract 9 in the same way.

IV. SUBTRACTION.

18. The process of finding the difference between two numbers is called *Subtraction.*

19. This difference is called the *remainder* or excess.

20. The sign is —, and is called *minus.* When placed between two numbers, it shows that the one on the right is to be taken from the one on the left: 7—5=2, reads, *seven minus five equals two.*

1. To find the difference between 375 and 263.

Solution.—1. Writing the small number under the large one, units and tens of the one under those of the other, we proceed to subtract 3 from 5, which leaves 2; this we write under the 3.

$$\begin{array}{r} 375 \\ 263 \\ \hline 112 \end{array}$$

2. 6 from 7 leaves 1; write it under the 6.
3. 2 from 3 leaves 1. The remainder is 112.

2. From 186436 take 165213 *Ans.* 21223
3. From 786900 take 654300 132600

EXERCISES FOR THE BLACKBOARD.

When subtracting one figure from another, the learner should be taught to *see* the result, rather than to reckon it or talk over the process. This can be done by such exercises as the following:

Taking a row of figures as 4903781236542, point to 9, requiring the class to give the difference between it and 4; then to 0, requiring the difference between 10 and 9, etc.

4. From 49368282 take 15012 *Ans.* 49353270

5. 247896—136785	8. 66145397— 134286
6. 716035— 15012	9. 15176482—4164271
7. 371150— 70000	10. 37898643— 276321
Total, 1113284	Total, 114645644

11. 13176356487645— 13145363435
12. 984960997610899—771900986010098
13. 19899799994896— 7445199821886

Total, 238677822898021.

When some of the figures of the smaller number are greater than those above them, we add ten to both figures.

14. From 342 take 267.

To subtract the 67 from the 42 above it, we add 10 to both numbers, as indicated by the small figures; but instead of adding to both numbers in the same place, we add 10 to the unit 2 of the upper number, and 1 (ten)to the 6 tens of the lower number.

$$\begin{array}{r} {}^{10} \\ 342 \\ {}^{1} \\ 267 \\ \hline 75 \end{array}$$

PROCESS.—1. We can not take 7 from the 2; add 10 which makes 12; 7 from 12 leaves 5, which write.

2. Adding 1 (ten) to the 6 we have 7, which taken from 14, after adding another ten (or one hundred) leaves 7.

3. Adding 1 (hundred) to the 2 we have 3, which taken from 3 leaves nothing. The remainder is 75.

21. PROOF.—By adding the remainder to the smaller number, we should get a sum equal to the larger.

In the above example the remainder was 75, the smaller number 267, which added together equal the larger, 342.

$$\begin{array}{r} 267 \\ 75 \\ \hline 342 \end{array}$$

$$\begin{array}{r} {*}15.\ 1603845732164000000 \\ 98123456789798768 \\ \hline 1505722275374201232\ \textit{Rem.} \end{array}$$

* The Teacher will require the class to read off both question and answer.

16. From 31642789600 take 1278899765.

Rem. 30363889835.

17. From 16782466987 take 123469978.

Rem. 16658997009.

18. 21683673217849—1637642178
19. 221682178368001—1000999999
20. 100000000000000— 1
21. 3681068213682— 11194680

Total, 347044269962674.

22. From $1670 take $389.27.

OPERATION.	$1670.00
	389.27
	$1280.73 *Ans.*

When writing the following questions, be particular to

TO THE TEACHER.—Mental exercises in adding will be found a good means of cultivating the retentive faculty for business purposes. Such exercises should not consist of single digits nor of fractions with terms made up of single digits, but of numbers such as the following:

27 and 64 are how much? To add these numbers the tens should be taken first: 27 and 60=87 and 4=91.

$3.27 and $1.25 are how much? 327 and 120=447 and 5=452.

COMPETITIVE EXERCISES

In addition might come in here, and be introduced at intervals throughout the course. A problem being written on the blackboard, or dictated to the class, scholars should be required to hand in their slates in the order in which they obtain the results, when the teacher would number them, "1, 2, 3, wrong, 4, 5," etc., calling the name of the competitor in each case and returning the slate. Such exercises should be conducted by the teacher with celerity, so that at a single glance he can tell whether the learner has the correct result. Fifteen minutes will be found sufficient time to devote to such exercises at once.

arrange dollars under dollars and cents under cents. When there are no cents, write two ciphers in their place.

23. $10067.89—$2141.98	27. $60000.00—$4670.87
24. $15070.14— 6160.47	28. $23678.45— 4101.00
25. $1001000. — 1.86	29. $1006812.00— 3178.59
26. $6743147. — .97	30. $678997.00— 210.99
Total, $7760979.75.	Total, $1757326.00.

31. $710356.87—$14683.29	35. $68750.37—$1416.44
32. $370968. — 17987.77	36. $71000.90— 87.50
33. $478979. — 14780.99	37. $100000. — .87
34. $100000. — 374.66	38. $61987.15— .99
Total dif., $1612477.16.	Total dif., $300232.62.

*39. What is the sum and difference, when added together, of $36748.94 and $10968.75.

40. Borrowed of A, at different times, $146.87, $6740.18, $310.75, and have paid him $10.00, $450.18 and $61.14; how much do I owe?

41. Out of 5 hogsheads of sugar containing 5761 lbs., I sold 3, containing 1114 lbs., 1311 lbs. and 1001 lbs.; how much was left?

42. After selling 1347 lbs. of sugar from 3 hogsheads, each containing 1000 lbs., how much was left?

EXERCISES IN TAKING THE COMPLEMENT OR "MAKING CHANGE."

To THE TEACHER.—Taking $1 as the complete number, require the complement of 25 c, 27 c, 30 c, 35 c, etc. It will be found that the complement of the teens is in the 80s, of the 20s in the 70s, of the 30s in the 60s, of the 40s in the 50s, of the 60s in the 30s, of the 70s in the 20s, etc. "54?" Ans. 46. 35? Ans. 65.

Taking $2, $3 or $5, the exercise may be practiced in the same way.

*Give the denomination of the answer, whether it be dollars, pounds, etc.

43. A merchant owns goods to the amount of $3147, and lands to the amount of $2107, and is indebted $1400: to A $200, to B $340 and to C $860; what is the amount of his net capital?

44. A merchant sells goods for another to the amount of $4374.23, and is to receive $43.75 for his trouble, besides the expenses of freight, etc., which was $125.15; how much should he return to his principal?

45. What is the difference between 1856 and 1798? When was the individual born who died in 1857 at the age of 45 years? When will the work be completed which was commenced in 1855 and was to take eighteen years?

Answers to the last seven: 420533, 58, 1812, 2454, 1873, 1653, 7349788, 2335, 667648, 410533.*

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*Give the denomination of the answer, whether it be dollars, pounds, etc.

HOME EXERCISES.†

1. $30+25=?$	14. $78+65=?$	27. $94+27=?$
2. $27+40=?$	15. $59+63=?$	28. $79+86=?$
3. $26+30=?$	16. $72+48=?$	29. $56+65=?$
4. $43+27=?$	17. $37+75=?$	30. $33+63=?$
5. $29+12=?$	18. $59+76=?$	31. $98+63=?$
6. $54+16=?$	19. $85+64=?$	32. $27+79=?$
7. $33+17=?$	20. $78+77=?$	33. $56+65=?$
8. $55+75=?$	21. $99+37=?$	34. $73+64=?$
9. $47+43=?$	22. $55+66=?$	35. $55+63=?$
10. $82+12=?$	23. $37+74=?$	36. $97+63=?$
11. $95+15=?$	24. $65+37=?$	37. $64+67=?$
12. $34+45=?$	25. $59+63=?$	38. $73+67=?$
13. $29+64=?$	26. $37+65=?$	39. $45+63=?$

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†To be written on the slate at home, and the operations recited in school.

40. $27+33=?$
41. $95+64=?$
42. $84+33=?$
43. $27+69=?$
44. $55+69=?$
45. $78+73=?$
46. $65+63=?$
47. $73+39=?$
48. $55+97=?$
49. $77+67=?$
50. $59+63=?$
51. $78+64=?$
52. $59+33=?$
53. $98+97=?$
54. $65+36=?$
55. $21+17=?$
56. $76+93=?$
57. $54+64=?$
58. $79+98=?$
59. $63+64=?$
60. $59+16=?$
61. $27+17=?$
62. $86+59=?$
63. $75+38=?$
64. $64+55=?$
65. $29+33=?$
66. $36+93=?$
67. $74+67=?$
68. $36+56=?$
69. $75+27=?$
70. $33+13=?$
71. $29+19=?$
72. $98+29=?$
73. $29+35=?$
74. $18+62=?$
75. $79+84=?$
76. $56+33=?$
77. $49+54=?$
78. $59+39=?$
79. $28+97=?$
80. $97+63=?$
81. $84+37=?$
82. $29+18=?$
83. $63+97=?$
84. $25+88=?$
85. $36+54=?$
86. $97+79=?$
87. $88+69=?$
88. $53+76=?$
89. $95+84=?$
90. $67+96=?$
91. $44+39=?$
92. $29+88=?$
93. $74+86=?$
94. $49+57=?$
95. $64+88=?$
96. $98+77=?$
97. $55+76=?$
98. $74+84=?$
99. $85+99=?$

Write the multiplication table as follows:

1. 2 times 1 or once 2 is 2.
   2 times 2 are 4.
   2 times 3 or 3 times 2 are 6.
   2 times 4 or 4 times 2 are 8.

Continue this to 12.

2. Write the 3 times table to 12 in the same way.
3. Write the 4 times table to 12 in the same way.
4. Write the 5 times table to 12 in the same way.
5. Write the 6 times table to 12 in the same way.
6. Write the 7 times table to 12 in the same way.
7. Write the 8 times table to 12 in the same way.
8. Write the 9 times table to 12 in the same way.
9. Write the 10 times table to 12 in the same way.
10. Write the 11 times table to 12 in the same way.
11. Write the 12 times table to 12 in the same way.

IV. MULTIPLICATION.

22. Multiplication is a short method of adding, when the same number has to be repeated any number of times. $\times$ is the sign. $3\times6=18$, reads, *three times six equals eighteen.*

1. To find the sum of $123+123+123$, by additions we would enter the three amounts as before, and *add* for the result.

In *multiplication*, we multiply each figure of the number to be increased by the number which indicates how often the repetition is to be made, thus:

$$\begin{array}{r} 123 \\ 3 \\ \hline 369 \end{array}$$

3 times 3 are 9; put 9 in the unit's place.
3 times 2 are 6; put 6 in the ten's place.
3 times 1 are 3, which put in the hundred's place. The result is 369, as it would have been by addition.

TERMS.

23. The number to be multiplied is called the *multiplicand*, the number by which it is multiplied, the *multiplier*, and the number produced by multiplying, the *product*. The multiplicand and multiplier are also called *factors*.

$$\text{Factors}\begin{cases} 123 & \textit{Multiplicand.} \\ 3 & \textit{Multiplier.} \end{cases}$$
$$\overline{369}\ \textit{Product.}$$

2. To find the product of 1496 by 7.

Here we say 7 times 6 are 42; write 2 under the 7. Then 7 times 9 are 63, and the 4 we carried make 67; write 7 and carry 6. 7 times 4 are 28 and 6 are 34; write 4 and carry 3. 7 times 1 are 7 and 3 are 10.

$$\begin{array}{r} 1496 \\ 7 \\ \hline 10472 \end{array}$$

Ans. 10472

3. $2146 \times 2 = 4292$
4. $3178 \times 3 = 9534$
5. $4167 \times 4 = 16668$
6. $21007 \times 5 = ?$
7. $31497 \times 6 = ?$
8. $17843 \times 7 = ?$

Sum, 418918.

9. $5189 \times 5 = ?$
10. $7864 \times 6 = ?$
11. $2875 \times 7 = ?$

Sum, 93254.

12. $41679 \times 8 = ?$
13. $98765 \times 9 = ?$
14. $73149 \times 12 = ?$

Sum, 2100105.

Observe to point off the cents in the products of the following:

15. $\$21.37 \times 7 = ?$
16. $\$117.49 = 8 = ?$
17. $\$317.00 \times 9 = ?$

Amount, $3942.51.

18. $\$10.73 \times 9 = ?$
19. $\$117.07 \times 6 = ?$
20. $\$307.49 \times 7 = ?$

Amount, $2951.42.

21. $\$678.39 \times 11 = ?$
22. $\$467.28 \times 12 = ?$
23. $\$999.99 \times 9 = ?$

Amount, $22069.56.

24. $\$671.49 \times 10 = ?$
25. $\$857.37 \times 11 = ?$
26. $\$1096.49 \times 12 = ?$

Amount, $29303.85.

27. $\$47.531 \times 9 = ?$
28. $\$716.145 \times 11 = ?$
29. $\$9871.321 \times 12 = ?$

Amount, $126761.226.

30. $\$1670.053 \times 7 = ?$
31. $\$2199.989 \times 9 = ?$
32. $\$7186.739 \times 8 = ?$

Amount, 88984.184.

33. $\$9057.179 \times 12 = ?$
34. $\$7898.796 \times 9 = ?$
35. $\$5970.463 \times 11 = ?$

Amount, $245450.405.

36. $\$793.179 \times 9 = ?$
37. $\$987.970 \times 8 = ?$
38. $\$213.219 \times 7 = ?$

Amount, $165349.04.

39. $\$7161.213 \times 11 = ?$
40. $\$1409.796 \times 12 = ?$
41. $\$9393.678 \times 9 = ?$
42. $\$4131.196 \times 10 = ?$

Amount. $221545.957.

43. $\$3714.291 \times 9 = ?$
44. $\$7965.379 \times 8 = ?$
45. $\$3768.219 \times 7 = ?$
46. $\$ 419.367 \times 6 = ?$

Amount, $126045.386.

47. 2785×357.

We have here three multipliers—seven, fifty and three hundred.

2785×7=	19495	19495
2785×5 tens=	13925 tens, or	139250
2785×3 hundreds=	8355 hundreds, or	835500
	Total products,	994245

This operation is contracted by arranging the figures as in the margin, and writing the *first* figure of the products of the units in the unit's place and the others to the left of it; the first figure of the product of the tens in the ten's place, or under its own multiplier, 5; and the first figure of the product of the hundreds in the hundred's place.

```
  2785
   357
------
 19495
13925
8355
------
994245
```

Either factor may be used as a multiplier in the following exercises?

48. 3170× 178=?	51. 2896×6789=?
49. 6184×1794=?	52. 7109×9998=?
50. 3867×3784=?	53. 2345×3979=?
Total, 26291084.	Total, 100067481.

TO THE TEACHER.—Blackboard exercises in concert may be given in the following manner:

Writing a line of figures, 379463875426, the teacher would lead by multiplying the second by the first, the third by the second, the fourth by the third, etc., without *speaking the process*. Pointing to 7, he would say 21; to 9, 63; to 4, 36, etc.

To instruct in "carrying," the same line may be used by pointing to the third figure and performing the following operation mentally: $3\times7+9=30$. Pointing to 4, he would say 67; to 6, 42; to 3, 27.

To produce rapidity of thought and action, exercises of this kind ought to be frequent, and *the teacher should lead*, taking care that the *whole* class follows.

Such exercises as these may be profitably continued throughout the entire course of study.

54. 6789×2164=?
55. 1578× 753=?
56. 9409×6781=?
57. 2783×4679=?
Total, 92703716.

58. 8976×7659=?
59. 3968×6483=?
60. 7689×2197=?
61. 6784×7898=?
Total, 1649444493.

```
62.  420001000
        109608
  ------------
    3360008000
    2520006
   3780009
  420001
  ------------
Product, 46035469608000
```

```
         109608
      420001000
   ------------
         109608
       219216
      438432
   ------------
Product, 46035469608000
```

The multiplier of the ten's place in the first operation being 0, we passed it, and multiplied by the 6 hundreds. In the second operation we passed the ten's, hundred's and thousand's places for the same reason.*

63. 12346×30010=?
64. 7684×10900=?
65. 6787× 3009=?
Total, 474681143.

66. 4967× 6007=?
67. 5896×900707=?
68. 7649× 66080=?
Total, 5845851161.

69. 2000× 7010=?
70. 3160×10096=?
71. 2178×90909=?
Total, 243923162.

72. 1009×90910=?
73. 21678×21006=?
74. 31784× 7009=?
Total, 769870314.

24. *To multiply by* 10, 100, 1000, etc., we have simply to annex as many ciphers to the multiplicand as there are in the multiplier.

---

* If the learner will simply observe to write the first figure of each product under its own multiplier, he will have no difficulty in multiplying where there are ciphers. For instance, the first figure of the product by 2, in the second example, is immediately *under* the 2.

$35 \times 10 = 350.$

Proof 35

10 — Ten times 5 are 50, and 10 times 3 are 30 and 5 are 35, making 350.

350

75. $165 \times 10 = 1650$	78. $413 \times 10 = ?$
76. $165 \times 100 = 16500$	79. $1716 \times 100 = ?$
77. $165 \times 1000 = 165000$	80. $9417 \times 1000 = ?$
	Total, 9592730.
81. $374 \times 100 = ?$	84. $9361 \times 10 = ?$
82. $268 \times 1000 = ?$	85. $7342 \times 100 = ?$
83. $189 \times 10000 = ?$	86. $8654 \times 1000 = ?$
Total, 2195400.	Total, 9481810.

25. To multiply dollars, cents and mills, we remove the decimal point to the right.

87. What is the product of \$279.373 by 10?

Ans. \$2793.73.

Explanation.—By multiplying the mills by 10 we make them cents, by multiplying the cents by 10 we make them dimes, and by multiplying the dimes by 10 we make them dollars.

88. \$145.373×100=\$14537.3, or 14537 dollars and 3 dimes or 30 cents.

89. \$356.14,5 × 10=?	92. \$317.98,7 × 100=?
90. \$178.91,3×100=?	93. \$679.97,6 × 10=?
91. \$463.97,8×100=?	94. \$7193.44,5×1000=?
Amount, \$67850.55.	Amount, \$7232043.46.
95. \$713.71,4 × 100=?	99. \$131.71,2×1000=?
96. \$165.79,3×1000=?	100. \$724.26,8 × 100=?
97. \$786.47,5 × 10=?	101. \$413.16,4 × 10=?
98. \$130.14 × 100=?	102. \$236.21 × 100=?
Amount, \$258043.15.	Amount, \$231891.44.

To the Teacher.—Mental exercises on this subject should succeed these written ones.

HOME EXERCISES.

1. Commencing at 13, write the 2 times table to 19 in this way:

2 times 13 are 26; 13 times 2 are 26.
2 times 14 are 28; 14 times 2 are 28.

2. Write the 3 times table in the same way.
3. Write the 4 times table in the same way.
4. Write the 5 times table in the same way.
5. Write the 6 times table in the same way.
6. Write the 7 times table in the same way.
7. Write the 8 times table in the same way.
8. Write the 9 times table in the same way.
9. Write the 13 times table to 9 as follows:

13 times 2 are 26; 2 times 13 are 26.
13 times 3 are 39; 3 times 13 are 39.

10. Write the 14 times table in the same way.
11. Write the 15 times table in the same way.
12. Write the 16 times table in the same way.
13. Write the 17 times table in the same way.
14. Write the 18 times table in the same way.
15. Write the 19 times table in the same way.

1. Write the 2 times table to 19 as follows:

2 times 3 are 6; 2 times 13 are 26.
2 times 4 are 8; 2 times 14 are 28.
2 times 5 are 10; 2 times 15 are 30.

2. Write the 3 times table in the same way.
3. Write the 4 times table in the same way.
4. Write the 5 times table in the same way.
5. Write the 6 times table in the same way.
6. Write the 7 times table in the same way.
7. Write the 8 times table in the same way.
8. Write the 9 times table in the same way.

EXERCISES IN MULTIPLYING THE "TEENS."*

100. 2357×13 and 14?
101. 5398×15 and 16?
102. 6532×17 and 18?
103. 7654×17 and 19?
Total products, 735141.

104. 7890×19 and 18?
105. 2164×17 and 16?
106. 3165×16 and 15?
107. 2137×15 and 14?
Total products, 523430.

108. $45.67×16 and 15?
109. $14.59×15 and 14?
110. $23.08×13 and 14?
111. $21.87×17 and 19?
Total products, $3249.36.

112. $76.54 ×16 and 17?
113. $57.352×19 and 15?
114. $67.185×18 and 17?
115. $45.375×16 and 18?
Total products, $8370.013.

116. $137.67 ×17 and 18?
117. $216.031×15 and 18?
118. $131.75 ×16 and 16?
119. $231.35 ×17 and 19?
Total prod., $84595.96.

120. $43.165×19 and 16?
121. $933 ×18 and 15?
122. $61.751×16 and 17?
123. $311.155×14 and 13?
Total prod., $42738.743.

PRINCIPLES OF MULTIPLICATION.

26. When two numbers are to be multiplied together, we use for the multiplier that which will produce least figures in the operation. This will be accomplished by selecting the smaller number, except where there are many ciphers, as in Ex. 62.

27. If a *number* of articles and the price of *one* article be multiplied together, the product will be the price of *all* at the same rate.

3 yards of muslin at 20 cents.
$$20\times3=60 \text{ cents.}$$

If the price of *one* be in *cents*, the price of *all* will be in cents. If in *dollars*, the price of all will be in *dollars*.

---

* Every boy designed for business pursuits ought to commit to memory the multiplication table up to 19 times, inclusive.

28. The number of articles contained in any box, bale, package, etc., multiplied by the number of boxes, bales, etc., each containing a like number, will give the number of articles in all.

In a box there are 30 articles; how many in 20 such boxes?

$30 \times 20 = 600$ articles.

29. A number multiplied by itself is *squared*, or raised to the *second power*, and the second power multiplied by the same number is *cubed*, or *raised to the third power*. The sign is a small figure on the right of the number, thus, 5^4, which indicates that 5 is to be raised to the fourth power, and is equal to $5 \times 5 \times 5 \times 5$, or 625.

30. Feet multiplied by feet and yards multiplied by yards produce *square feet* and *square yards*.

12 feet $\times$ 12 feet $=$ 144 square feet.

31. Any number of feet multiplied by the number of inches in one foot will give the number of inches in all the feet. Pounds multiplied by the number of ounces in one pound will give the number of ounces in all the pounds, and so with numbers of any other denomination.

How many inches in 37 feet?

$37 \times 12 = 444$ inches.

1. What is the price of 37 bushels of corn at 37 cents per bushel?
2. What should I pay for 357 yards of broadcloth at $2.75 per yard?
3. Find the cost of 325 acres of land at $57 per acre.
4. In 320 bales of cotton there are 460 lbs. each; how many in all?
5. In 557 pieces of muslin there are 35 yards each; how many in all?

6. A ship laden with flour has 7950 barrels on board, and in each barrel there are 196 lbs.; how many pounds in all?

7. In a bushel of dried apples there are 25 pounds; how many are there in 37 bushels?

8. A barrel of flour weighs 196 pounds; what is the weight of 325 barrels?

9. What will be the weight of 134 bushels of wheat when 60 lbs. are allowed to the bushel?

10. Find the cost of 379 boxes of cheese, each of which weighs 22 pounds, at 25 cents a pound.

11. A box of buttons contains a gross; how many buttons are there in 59 boxes?

12. A merchant sold 135 barrels of flour at $6.75 apiece, and with part of the money bought 369 bushels of coal at 25 cents; how much money had he left?

13. In 236 yards how many inches?

14. How many quarts are there in 27 bushels?

15. At 23 cents a quart, how much money can be realized on 18 bushels of strawberries, allowing one quart for loss in measuring?

16. A huckster bought two barrels of apples, each containing 3 bushels, at $6 a bushel, and sold them at 87 cents a half peck; allowing half a peck for loss in measuring, did he gain or lose? and how much?

17. There are 12 inches in a foot and 3 feet in a yard; how many inches are there in 357 yards?

18. How many quarts of wine are there in 6 barrels, each of which contains 42 galls.? how many in 35 barrels? how many in 163?

Answers without their denominations: 13225, 864, 819, 8496, 208450, 8496, 8040, 925, 98175, 21397, 18525, 147200, 63700, 19495, 1558200, 489, 1369, 12852, 34272.

19. Find the cost of 117 bushels of apples at 35 cents a peck.

20. Find the cost of 237 bushels of potatoes at 42 cents a peck.

21. Bought 46 horses at $125 apiece, and sold them for $5900; did I lose or gain, and how much?

22. At 5 cents apiece, what will 22 gross of eggs cost?

23. In 15 acres how many square yards?

24. Find the cost of 35 barrels of molasses at 33 cents a gallon, each barrel containing, on an average, 41 galls.

25. In 257 cords of wood how many solid feet?

26. How many bushels of coal in 17 wagons, each carrying 50? and what will be the cost at $17 a load?

27. How many yards are in a box of silk containing 35 pieces, each piece measuring 52 yards?

28. In a case of muslin there are 45 pieces, each 31 yards; what will be the cost of it at 55 cents a yard?

29. A bushel of hemp-seed weighs 44 pounds; what will be the weight of 137 bushels?

30. How many leaves are in 67 reams of paper?

31. How many half-pence are in 527 pounds sterling?

Answers promiscuously arranged: 72600, 16380, 5175, 150, 15840, 39816, 47355, 64320, 76725, 14450, 1820, 32896, 6028, 252960.

HOME EXERCISES.

1. $3+4-2=?$	7. $7+3\times6=?$	13. $16+12-5=?$
2. $5-3+7=?$	8. $9+8+3=?$	14. $23+20-1=?$
3. $9\times4-3=?$	9. $9\times7-4=?$	15. $19-15+7=?$
4. $7\times6+8=?$	10. $5+5-3=?$	16. $27-12+8=?$
5. $6+9\times6=?$	11. $8\times4+2=?$	17. $29+2-7=?$
6. $9-8-1=?$	12. $9-3-2=?$	18. $40+54-3=?$

19. $35-15\times 6=?$
20. $64+30-2=?$
21. $97-13+7=?$
22. $55+43-8=?$
23. $63+55+2=?$
24. $19+6-5=?$
25. $29+6-5=?$
26. $35+25+15=?$
27. $18+22-10=?$
28. $54+16+10=?$
29. $93+17-20=?$
30. $44+16+32=?$
31. $12+18+33=?$
32. $27+33+54=?$
33. $44+16-15=?$
34. $19-12+28=?$
35. $55+44+21=?$
36. $23+60+47=?$
37. $64+15+36=?$
38. $24+46+33=?$
39. $56+44+71=?$
40. $19+33+21=?$
41. $77+44+23=?$
42. $45+31+55=?$
43. $37+13+34=?$
44. $66+14-12=?$
45. $44+76+33=?$
46. $92+18-10=?$
47. $73+10+67=?$
48. $21+39+22=?$
49. $17+63+14=?$
50. $29+33+19=?$
51. $3\times 15+15=?$
52. $7+16\times 3=?$
53. $57-17\times 7=?$
54. $19+11\times 6=?$
55. $45+15\times 9=?$
56. $84+26\times 12=?$
57. $21+18+15=?$
58. $45+16-11=?$
59. $73+13-16=?$
60. $55+45+35=?$
61. $79+60+30=?$
62. $24+71+5=?$
63. $39+11-10=?$
64. $99-19+12=?$
65. $77+79-14=?$
66. $85+96-11=?$
67. $77+63-20=?$
68. $45+65-22=?$
69. $33+73-20=?$
70. $97+93+50=?$
71. $79+91+93=?$
72. $56+64+49=?$
73. $73+65+37=?$
74. $93+65+35=?$
75. $97+63+47=?$
76. $45\times 3+45=?$
77. $35\times 4+30=?$
78. $29\times 6-14=?$
79. $99\times 3-27=?$
80. $84-14\times 7=?$
81. $75\times 3-6=?$
82. $36\times 4-14=?$
83. $56\times 3+50=?$
84. $75-15\times 6=?$
85. $35\times 12-20=?$
86. $42\times 10+22=?$
87. $57\times 12-14=?$
88. $45-15\times 30=?$
89. $64\times 5-20=?$
90. $18\times 6-28=?$
91. $36\times 12+72=?$
92. $42\times 12+12=?$
93. $35\times 20+12=?$
94. $64\times 32-14=?$
95. $33\times 21-12=?$
96. $14\times 12-13=?$
97. $12+33-6=?$
98. $75\times 16-4=?$
99. $39\times 12+4=?$
100. $56-16+6=?$
101. $35\times 15\times 6=?$
102. $35-22\times 12=?$
103. $64+22-10=?$
104. $75\times 97-37=?$
105. $64\times 14-4=?$
106. $36-16\times 96=?$
107. $55-23\times 20=?$
108. $49-19\times 84=?$
109. $64\times 45-15=?$
110. $97-67-17=?$
111. $37+16-6=?$

1. Write the 2 times table to 9 as follows:
   2 times 2 are 4; 2 is contained in 4, 2 times.
   2 times 3 are 6; 2 is contained in 6, 3 times.
   2 times 4 are 8; 2 is contained in 8, 4 times.
2. Write 3 times table to 9 as above.
3. Write 4 times table to 9 in the same way.
4. Write 5 times table to 9 in the same way.
5. Write 6 times table to 9 in the same way.

6. Write 7 times table to 9 in the same way.
7. Write 8 times table to 9 in the same way.
8. Write 9 times table to 9 in the same way.

1. Write the division table, as below, to 2 in 19:
   2 in 2, 1 time.
   2 in 3, 1 time and 1 left.
   2 in 4, 2 times.
   2 in 5, 2 times and 1 left.
2. Find how often 3 is contained in numbers from 3 to 29.
3. Find how often 4 is contained in numbers from 4 to 39.
4. Find how often 5 is contained in numbers from 5 to 49.
5. Find how often 6 is contained in numbers from 6 to 59.
6. Find how often 7 is contained in numbers from 7 to 69.
7. Find how often 8 is contained in numbers from 8 to 79.
8. Find how often 9 is contained in numbers from 9 to 89.

V. DIVISION.

32. Division is the method of calculation used to separate numbers into equal parts.

33. Division may be *short* or *long*. It is *short* when the process of finding the product and remainder is performed mentally, and *long* when the process is written.

34. The sign is ÷, which placed between two numbers indicates that the one on the left is to be divided by the one on the right.

6÷3, reads, *six divided by three.*

Division is also indicated by a curved line between the numbers, thus: 3)6; and by a straight line, with one number above and the other below, as $\frac{6}{3}$, called the *fractional* form. The period is also used to indicate division. .5 shows that 5 is divided by 10.

35. TERMS.

The number divided or to be divided is the *dividend.*

The number by which the division is performed or to be performed is the *divisor.*

The number which shows how many times the divisor is contained in the dividend is the *quotient.*

The number left after dividing, the *remainder.*

Divisor, 3)16784 *Dividend.*

Quotient, 5594—2 *Remainder.*

SHORT DIVISION.

1. To divide 738 by 3.

$$\begin{array}{r} 3)\underline{738} \\ 246 \end{array}$$

EXPLANATION.—1. Commencing at the left, we find how often 3 is contained in 7 hundred, which is 2 hundred times, with a remainder of 1 hundred. The 2 we write in the hundreds' place.

2. This 1 hundred, with the 38, gives a remainder of 138 to be divided. To divide this, we consider the 13 as tens. In 13 tens 3 is contained 4 times, and 1 ten left as a remainder; so we write the 4 in the tens' place.

3. This 1 ten, with the remaining 8, gives 18 to be divided. In 18, 3 is contained 6 times, which, being written under the 8, gives the result, 246.

Until he becomes familiar with the process, the learner may write the remainders in small figures, as in the following example.

2. $$\begin{array}{l} 8)\underline{1\ \ 3\ \ {}^{5}0\ \ {}^{2}7\ \ {}^{3}6\ \ {}^{4}8\ \ 3\ \ {}^{3}7\ \ {}^{5}6\ \ 1\ \ {}^{1}7} \\ \quad\ \ \ 1\ \ 6\ \ 3\ \ 4\ \ 6\ \ 0\ \ 4\ \ 7\ \ 0\ \ 2\text{—}1 \end{array}$$

EXPLANATION.—1. Commencing at the left we find how many times 8 is contained in 13. The answer being 1 time, with a remainder of 5, we write the 1 under the 13 and the 5 before the 0.

2. This 5 taken with the 0 makes 50, in which 8 is contained 6 times, with a remainder of 2.

3. This 2, with the 7, makes 27, in which 8 is contained 3 times, with a remainder of 3.

4. When 3 in the dividend is reached, it is found that 8 is not

contained in it, so a cipher is placed under it and 3 considered a remainder.*

Divide the following:

		Quotients.	Rem.
3.	134615379÷2	*Ans.* 67307689	—1
4.	21637298452÷3	*Ans.* 7212432817	—1
5.	59368217755÷4	*Ans.* 14842054438	—3
6.	1416823687949÷5	*Ans.* 283364737589	—4

	Rem		Rem.
7. 13645217÷6	5	11. 361745731÷10	1
8. 23176841÷7	2	12. 213764952÷11	5
9. 47896739÷8	3	13. 178961521÷12	1
10. 89765432÷9	8	14. 345678900÷12	0

Total quotients, 21546207 and 99327785.

15. 8092375176÷3=?	19. 33376231825÷ 7=?
16. 7316305416÷4=?	20. 72143250072÷ 8=?
17. 35192364975÷5=?	21. 30608807751÷ 9=?
18. 13138539444÷6=?	22. 81726954390÷10=?

Total quotients, 13754764315 and 25359613454.

23. 65420506756÷11=?	27. 578312908÷3=?
24. 96451487844÷12=?	28. 483796459÷4=?
25. 31463215726÷ 7=?	29. 761147355÷5=?
26. 21347096543÷ 8=?	30. 123450678÷6=?
Total, 21148074953—12.	Total, 486524667—3.

31 to 34. Divide 874109630175 by 2, 3, 4 and 5.

Sum, 1121774025390—4.

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*PROOF OF DIVISION.—Division may be proved by multiplying the quotient by the divisor and adding the remainder. Take the last example:

$$\begin{array}{r} 1634604702-1 \\ 8 \\ \hline 13076837617 \end{array}$$

35 to 38. Divide the same number by 6, 7, 8 and 9.
Sum, 476944738684—21.

39 to 41. Divide the same number by 10, 11 and 12.
Sum, 239717944032—9.

42. $710084973 \div 7 = ?$
43. $394789006 \div 8 = ?$
44. $361007839 \div 9 = ?$
45. $909738697 \div 10 = ?$
Sum, 281875186—17.

46. $31463574213 \div 7 = ?$
47. $91678543210 \div 8 = ?$
48. $76303074368 \div 9 = ?$
49. $21356703648 \div 6 = ?$
Sum, 27992184199—5.

36. The quotient of a number divided by 2 is the $\frac{1}{2}$ (one-half) of it; divided by 3 it is the $\frac{1}{3}$ (one-third); by 4, the $\frac{1}{4}$ (one-fourth); hence, to find the $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ of a number, we have simply to divide by 2, 3 or 4.*

Let it be required to find the cost of $2\frac{1}{2}$ yards of cloth at \$3 a yard.

$$\begin{array}{ll} \$3 & = \textit{cost of } 1 \textit{ yard.} \\ \underline{\ \ 2} & \\ \ \ 6 & = \textit{cost of } 2 \textit{ yards.} \\ \underline{\ \ 1\frac{1}{2}} & = \textit{cost of } \frac{1}{2} \textit{ yard.} \\ \$7\frac{1}{2} & = \textit{cost of } 2\frac{1}{2} \textit{ yards.} \end{array}$$

Here, to multiply by $2\frac{1}{2}$, the 3 had to be divided by 2.

50. $\frac{1}{3}$ of $3716 = 1238\frac{2}{3}$ †
51. $\frac{1}{4}$ of $1367 =$ —— $\frac{3}{4}$
52. $\frac{1}{5}$ of $7854 =$ —— $\frac{4}{5}$
53. $\frac{1}{6}$ of $879 =$ —— $\frac{5}{6}$
54. $\frac{1}{7}$ of $34161143764 =$ —— $\frac{6}{7}$
55. $\frac{1}{8}$ of $37897181237 =$ —— $\frac{5}{8}$
56. $\frac{1}{9}$ of $16872352168 =$ —— $\frac{4}{9}$
57. $\frac{1}{12}$ of $34564185432 =$ —— $\frac{9}{12}$

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*The learner will be particular to observe that finding the $\frac{1}{2}$ or $\frac{1}{3}$ of a number is not dividing by *one-half* or *one-third*, but simply finding one part of something divided into two or three parts which is *multiplying* by one-half or one-third.

†The remainder in this example being 2, we write it as $\frac{2}{3}$, which indicates that 2 is divided by 3, or that it is 2 parts of something divided into 3 parts. The remainders only of the questions which follow will be given.

58. $14567.85 × $9\frac{1}{3}$ is how much?

$$\begin{array}{r} 1456785 \ \textit{cents.} \\ 9\frac{1}{3} \\ \hline 13111065 \\ 485595 \\ \hline 13596660 \end{array}$$

13596660 *or* $135966.60

The decimal point was removed before dividing and replaced after the operation was performed.

59. $\frac{1}{3}$ of $21372 =?
60. $\frac{1}{8}$ of $13744 =?
61. $\frac{1}{5}$ of $73176.35=?
62. $\frac{1}{9}$ of $14537.07=?
Total, $25092.50.

63. $\frac{1}{8}$ of $67849132.87=?
64. $\frac{1}{7}$ of $16493178.00=?
65. $\frac{1}{9}$ of $23610934.10=?
66. $\frac{1}{11}$ of $12310985.47=?
Total, $14579927.51.

67. $345.78 × $37\frac{1}{2}$=how much?

$$\begin{array}{rl} 34578 & \\ 37\frac{1}{2} & \\ \hline 17289 & = \frac{1}{2} \textit{ of multiplicand.} \\ 242046 & = 7 \textit{ times do.} \\ 103734 & = 30 \textit{ times do.} \\ \hline 1296675 & = 37\frac{1}{2} \textit{ times,} \end{array}$$

or $12966.75.

68 and 69. $146.82 × $8\frac{1}{3}$=? $1713.14 × $6\frac{1}{4}$=?
Sum, 11930.62\frac{1}{2}$.

70, 71 and 72. $4563.28 × $45\frac{1}{6}$, $16\frac{1}{3}$ and $18\frac{1}{2}$.
Sum of products, $365062.39,9$\frac{4}{6}+\frac{1}{3}$.

73, 74 and 75. $21763 × $14\frac{1}{7}$, $15\frac{1}{6}$, $29\frac{1}{4}$.
Total, 1274430.916\frac{2}{3}$.

76, 77 and 78. $7649.14 × $76\frac{1}{9}$, $96\frac{1}{9}$, $86\frac{1}{4}$.
Total, 1977090.20\frac{1}{4}+\frac{8}{9}$.

79 to 81. $3146 × $2\frac{1}{2}$, $6\frac{1}{2}$, $12\frac{1}{2}$. Total, $67639.00.

82 to 84. $1567 × $3\frac{1}{2}$, 5, $16\frac{1}{2}$. $39175.00.

85 to 87. $7864 × $6\frac{1}{4}$, $7\frac{1}{5}$, $37\frac{1}{2}$. $400670.80.

88 to 90. $71684.25 \times 8\frac{1}{3}, 7\frac{1}{2}, 5\frac{1}{2}$.
91 to 93. $89647.86 \times 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}$.
94 to 96. $79943.52 \times \frac{1}{4}, 6\frac{1}{3}, 7\frac{1}{2}$.

Amount, $3327494.19.

97 to 100. If a steamboat is worth $3456, what will $\frac{1}{5}$ be worth? What $\frac{2}{5}$? What $\frac{3}{5}$? What $\frac{4}{5}$? Total, $6912.

REMARK.—2 fifths will cost 2 times as much as 1 fifth; 3 fifths, 3 times as much as 1 fifth, etc.

101 to 105. $\frac{1}{12}$ of $155367 is how much? $\frac{1}{9}$=? $\frac{1}{8}$=? $\frac{1}{7}$=? $\frac{1}{6}$=? Total, 97720.910\frac{5}{7}$.

37. *To divide by* 10, *we point off one figure on the right,* by 100, two figures, by 1000, three figures. Those on the left will be the quotient, those on the right the remainder. $500 divided by 10=$50.0. $500 divided by 100=$5.00.

REASON.—By pointing off one figure we remove all the figures one place further to the right, so that the tens stand where the units were, and *are* units, the hundreds where the tens were, and are tens.

2. It will be observed that the number pointed off corresponds with the number of ciphers in the divisor. For 10 we point off *one* figure; for 100, *two;* for 1000, *three.*

$31456 \div 100 = 314.56$, *or* $314\frac{56}{100}$

3. Observe, also, that a decimal fraction, as .65, is changed to a common fraction by removing the point and writing the figure 1, with as many ciphers annexed as there are figures in the decimal: $.753 = \frac{753}{1000}$.

106. 134567 ÷ 10=?	112. 9876 ÷ 10=?
107. 34578 ÷ 100=?	113. 3453 ÷ 100=?
108. 71364 ÷ 1000=?	114. 6543 ÷ 10=?
109. 21789 ÷ 100=?	115. 2197 ÷ 100=?
110. 2163 ÷ 100=?	116. 1763 ÷ 10=?
111. 7865 ÷ 1000=?	117. 7974 ÷ 100=?

Total quotients, 14121.229 and 1954.44.

38. *To divide dollars and cents,* the decimal point is removed to the left, which is the same as pointing off.

To divide by 10, it is removed one figure; by 100, two figures. \$55.10 divided by 10=\$5.510. \$167.56 divided by 100=\$1.67,56.*

Divide the following:

118. \$457.87÷ 10.	122. \$473.04÷1000 and 100.
119. \$1677.45÷ 100.	123. \$15.17÷ 10 and 100.
120. \$6109.88÷1000.	124. \$16.57÷ 100 and 10.
121. \$14999.99÷ 100.	125. \$106.07÷ 100 and 1000.
Total, \$218.66.	Total, \$9.85,9

126. Divide the following sums of money by 100: \$645, \$1678.25, \$87493.57, \$16453.27, \$1998.38, \$643.24, \$2168, \$4137.54. Total answer, \$1152.16,9.

39. It often happens that there are not as many figures to cut off as there are ciphers in the divisor. In such cases we prefix ciphers to the dividend to make up the number.

Divide \$5. by 100. *Ans.* .05.

EXPLANATION.—This is the same as removing the decimal point two places to the left, as above. The \$5 had the decimal point on the right of the 5; it is now two places further to the left, and therefore is divided by 100. The cipher, in this case, as elsewhere, possesses no value.

127. \$5÷ 10=.5	132. \$0.03÷ 10=?
128. \$3÷ 100=.03	133. \$0.02÷100=?
129. \$4÷1000=.004	134. \$0.14÷100=?
130. \$50÷1000=.05	135. \$3.16÷100=?
131. \$457÷1000=.457	136. \$21.30÷ 10=?
Total, 1.041.	Total, 2.1662.

*NOTE.—The value of each and all of the figures decreases tenfold for every figure the decimal point is removed to the left.

The \$5 in the first example became 50 cents and the 10 cents became 10 mills or 1 cent, making the answer 5 dollars, 51 cents, not 5 dollars, 510 cents. The second answer is 7 dollars, 67 cents, 5 mills and $\frac{6}{10}$ of a mill, or \$1.67,5$\frac{6}{10}$.

137. Divide the following sums by 100: 3 cents, 33 cents, \$3.33, \$33.33, \$333.33, \$3333.33.

Total, \$37.03,68.

40. To divide by 20, 300, 5000, etc., we point off as many figures in the dividend as there are ciphers in the divisor, and divide by the 2, 3, 5, etc. The figures pointed off will form part of the remainder.*

138. Divide 317745 by 500.

$$5|00)\underline{3177|45}$$
$$635\text{–}245$$
$$635\tfrac{245}{500}$$

EXPLANATION—Pointing off two figures, we divide by 100; what is left we divide by 5.

139. $467831 \div 20 = 23391\frac{11}{20}$

140. $716893 \div 300 = 2389\frac{293}{300}$

141. $417368 \div 500 = 834\frac{368}{500}$

142. $716849 \div 700 = ?$

143. $897653 \div 900 = ?$

144. $49673 \div 80 = ?$

Total quotients, 2641. Total rem., 1573.

The answers to the following are required in dollars, cents and mills, omitting the remainders:

$\$2131.51 \div 500 = ?$ Reduced to mills, it is 2141510.

$$5|00)\underline{21315|10}$$
$$4263\tfrac{10}{500} = \$4.26,3\tfrac{1}{50}.$$

145. $\$13764.75 \div 50 = ?$

146. $\$73968.23 \div 60 = ?$

147. $\$37437.18 \div 90 = ?$

148. $\$18964.20 \div 80 = ?$

Total, \$2161.11,8.

149. $\$16789.37 \div 80 = ?$

150. $\$67859.67 \div 900 = ?$

151. $\$54168.23 \div 700 = ?$

152. $\$78910.00 \div 600 = ?$

Total, \$494.16,5.

153 to 157. Divide the following sums by 20, and give the answers as above: \$1367.25, \$3143.57, \$2345.87, \$34.57, \$45679.44. Total, 2628.53,3.

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* When dividing dollars and cents, reduce them by erasing the decimal point and annexing ciphers if necessary.

$\$357.27 \div 500 = ?$

Reduced to cents the dividend is 35727. $5 \mid 00)\underline{357 \mid 27}$

$$71\tfrac{227}{500}$$

158 to 165. Divide \$34567.25 by 10, 12, 20, 100, 30, 50, 70 and 90. Total, 11132.84,6.

166 to 176. Divide \$367897.87 by 100, and the quotient by 10, 20, 30, 40, 50, 60, 70, 80, 90. Total, \$4719.74,4.

177 to 187. Divide \$17654.37 by 100, and the quotient by 3, 10, 7, 40, 30, 50, 70, 90 and 80. Total, \$298.78.

188 to 196. Divide \$314937 by 100, and multiply the quotient by 7; then divide the quotient by 30, 60, 40, 12, 9, 80, 90. Total, \$22968.52,7.

VI. PERCENTAGE.*

41. *Percentage* is the method of reckoning by *hundredths.* 1 per cent. is the *one hundredth* part, 2 per cent. twice that amount, 3 per cent. three times that amount.

42. The sign is %. 25% signifies 25 per cent.

43. To compute percentage, or, in other words, to find any rate per cent., we first find one per cent., and multiply it by the given rate.

1. To find 5% of 350.

One per cent. is the number divided by 100 or 3.50. 5 per cent is $3.50 \times 5 = 17.50$ or $17\frac{50}{100}$ or $17\frac{1}{2}$.

2. Find 6% of 3572.

OPERATION.
$$\begin{array}{r} 35.72 = 1 \text{ per cent.} \\ 6 \\ \hline 214.32 \text{ or } 214\frac{32}{100} = 6\%. \end{array}$$

3. 6% of 3146 is how much? *Ans.* 188.76 or $188\frac{76}{100}$.

4. 5% of 1937 is how much? *Ans.* $96\frac{85}{100}$.

*This rule is of such general utility, is so simple in its application, and so strictly belongs to the subject of division, that I can not refrain from introducing it in this place.

5. 7% of 3176 is how much?
6. 9% of 7854 is how much?
7. 8% of 396 is how much?
8. 4% of 243 is how much?

Amount, $970\frac{58}{100}$.

9. 4% of 1300=?
10. 6% of 367=?
11. 9% of 463=?
12. 8% of 6735=?

Amount, 654.49.

13. 20% of 3161=?
14. 33% of 798=?
15. 55% of 654=?
16. 19% of 321=?

Amount, $1316\frac{23}{100}$.

17. 30 % of 4541=?
18. 23 % of 147=?
19. 60 % of 7163=?
20. $33\frac{1}{3}$% of 4371=?

Amount, $7150\frac{91}{100}$.

21. $12\frac{1}{2}$% of 167=?
22. $15\frac{1}{2}$% of 364=?
23. $37\frac{1}{2}$% of 910=?
24. $50\frac{1}{4}$% of 693=?

Amount, $766\frac{7775}{10000}$.

25. 2% of \$320=?
26. 3% of \$976=?
27. 5% of \$8900=?
28. 7% of \$6540=?

Amount, \$938.48.

29. 7% of \$327=?
30. 9% of \$100=?
31. 12% of \$978=?
32. 25% of \$179=?

Amount, \$194.00.

33. 20 % of \$1361=?
34. $45\frac{1}{2}$% of \$316=?
35. 17 % of \$2163=?
36. $19\frac{1}{2}$% of \$1723=?

Amount, 1119.67,5.

37. 25% of \$264.50 is how much?

```
 2.6450=1%
     25
 ------
 132250
 52900
 ------
66.1250 or $66.12,5=25%.
```

38. 3% of \$674.75=?
39. 5% of \$198.45=?
40. 6% of \$786.70=?
41. 7% of \$14.13=?
42. 12% of \$1.19=?

Amount, $\$78.49,8\frac{9}{10}$.

43. 6% of \$397.25=?
44. 12% of \$187.17=?
45. 11% of \$710.00=?
46. 40% of \$1678.00=?
47. 50% of \$7764.82=?

Amount, $4678.00,5\frac{4}{10}$.

48. $37\frac{1}{2}\%$ of \$461.75=?
49. $51\frac{1}{4}\%$ of \$198.18=?
50. $62\frac{1}{2}\%$ of \$213.07=?
51. $18\frac{1}{2}\%$ of \$610.18=?
52. $16\frac{1}{2}\%$ of \$114.14=?

*Amount, 539.60,$8\frac{65}{100}$.

GENERAL EXERCISES.

1. Divide \$367.22 equally among 7 persons. Each will have $\frac{1}{7}$; divide by 7.

2. 12 horses, of equal value, cost \$3456.24; what was the cost of each?

3. If 5 men accomplish a piece of work in 320 days, how long will it take one man to do it?

4. In a year there are 365 days; how many weeks will that make?

5. A ship sails 84 miles in 12 hours; what is her average speed per hour?

6. How many shillings in 13456 pence? how many pounds?

7. If 30 bricklayers can erect the walls of a house in 120 days, how long will it take 12 to do it?

8. How long will it take a writer to copy a speech of 22340 words, if he writes 40 words a minute?

9. If \$345.72 be divided among 12 persons, how much will each receive?

10. In a pound there are 20 shillings; how many pounds are there in 3456 shillings?

11. How many shillings are there in 21345 pence? how many pounds?

12. How many bushels of wheat in 134563 pounds, reckoning 60 pounds to the bushel?

Answers: 1600, \$52.46, \$288.02, $52\frac{1}{7}$, $2242\frac{43}{60}$, 7, 16, £88 18 9, 9, £172 16, 300, $558\frac{1}{2}$, \$28.81, $1121\frac{4}{12}$, 1778 9.

**The Teacher* should give numerous oral exercises on this rule, else scholars will be apt to err in pointing the products.

13. A and B are in partnership, in which A invests \$30 and B \$20. They make \$14; how much should each receive?

A's share is 30 *parts.*
B's share is 20 *parts.*

Together they have 50 *parts.*

1 part of \$14 divided into 50 parts=28 cents; and 20 parts= \$5.60, and 30 parts=\$8.40, which, added together=\$14.

14. 2 men trading horses put in each \$1200 and \$800, and gained \$1250; what was each man's share?

15. If 20 men do a piece of work in 30 days, how long will it take 1 man to do it? how long 15?

16. In 3683 oranges how many dozen?

17. If \$3678.21 be divided between 7 persons, how much will each receive?

18. The profits of a speculation in which was invested, by 3 persons, \$300, \$900 and \$800, are \$919.18; how much should each receive?

19. The cost of muslin is 30 cents per yard; how many yards can be bought for \$397 at that rate?

20. Find the cost of the following articles: 125 lbs. sugar at 27 cents a pound; 37 lbs. of butter at $37\frac{1}{2}$ c; 2 hams, each weighing 13 and 14 lbs., at 21c; 115 lbs. of cheese at $15\frac{1}{2}$ c.

21. How much money will buy $37\frac{1}{2}$ lbs. of tea at \$2 a pound; 150 lbs. of fish at $8\frac{1}{2}$ c; 57 lbs. of sugar at $31\frac{1}{4}$ c; $56\frac{1}{2}$ lbs. of lard at 16 c; 45 lbs. of soap at $7\frac{1}{2}$ c; 31 lbs. of candles at 16 c?

22. A lady bought the following goods and paid for them out of \$100; how much had she left? $8\frac{1}{3}$ yds. of French merino @ \$2.25; 2 pieces brown muslin, 33 and $50\frac{1}{4}$ yds., at 35 c; 1 bonnet at \$7.50; 1 shawl at \$11.50.

Answers: 750, 500, 600, 40, $306\frac{11}{12}$, \$23.73,$7\frac{1}{2}$, \$33.$11\frac{1}{4}$, \$525.$45\frac{6}{7}$, $1323\frac{1}{3}$, \$71.12, \$367.67,2, \$137.87,7 \$413.63,1.

VII. LONG DIVISION.*

THE previous operations in division were performed almost mentally, the learner writing only the quotients. That method is preferable when the divisor is found in the tables, or can be reduced to a number contained in them, as 500, 1200; otherwise the operation would be too difficult and tedious to perform mentally.

44. *Long Division* has then to be used, which consists in writing the products and remainders as well as the quotients. The better to illustrate this method, an example which can be solved by short division is selected.

1. Divide 3147 by 6.

```
Divr. Divd. Quot.
 6)3147(524 3/6
   30
   --
    14
    12
    --
     27
     24
     --
     3/6
```

EXPLANATION.—1. To perform this operation, we try, as before, how often the divisor is contained in part of the dividend. 6 is contained in 31, 5 times, and writing 5 in the quotient, we multiply the divisor by it and write the product (30) under 31.

2. We now subtract the 30 from the 31 as we would perform an operation in subtraction. The remainder is 1. Instead of supposing this 1 to stand before the 4 in the dividend, we bring down the 4 to it, making 14.

3. 6 in 14 is contained 2 times, which 2 we write in the quotient, multiply it upon the divisor (6) and write the product underneath.

4. Subtracting this 12 from the 14, we have a remainder of 2, to which we annex 7 from the dividend, making 27.

5. 6 in 27, 4 times. Writing the 4 in the quotient, we multiply it upon 6, making 24, which we write underneath.

6. Subtracting, as before, we find a remainder of 3, and as there

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*This, considered the most difficult rule in arithmetic, may be deferred until the learner has passed *Easy Fractions*.

are no more figures in the dividend to bring down, we consider this the final remainder and express it in fractional form, as in short division, $\frac{3}{6}$.

To enable the learner to comprehend this part of division more clearly, another example is introduced.

3. Divide 8317517 by 723.

```
723)8317517(11504 125/723
    [1] 723*
    [2] 1087
    [3]  723
    [4]  3645
    [5]  3615
    [6]   3017
    [7]   2892
    [8]    125
```

[1] The product of the divisor (723) and 1 of the quotient.

[2] The remainder of 831—723, with 7 annexed.

[3] The product of 723 and the second figure of the quotient.

[4] The remainder of 1087—723, with 5 annexed.

[5] The product of 723×5, the third figure of the quotient.

[6] The remainder, with two figures (1 and 7) annexed. 723 was not contained in 301, so another figure was annexed.

[7] The product of 723×4, the fifth figure of the quotient.

[8] The remainder. This is represented in fractional form in the quotient.

Remarks.—1. Instead of using the whole divisor in finding a quotient figure, it will generally do to use only the first one or two figures. In the preceding exercise, the first figure alone (7) was used, and in this way: "7 is contained in 8 how many times?"

2. The products should never exceed the numbers above them. Number 3 should not exceed number 2. If on trial, it is found they do so, then a smaller number should be put in the quotient.

3. For every figure brought down from the dividend, there should be one in the quotient. When the divisor is not contained in the new dividend, a cipher should be placed in the quotient and another figure annexed.

4. The divisor can not be contained more than 9 times in the smaller dividends, as 1087, 3645.

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* The learner should put a mark under each figure brought down to prevent its being taken twice.

4. $71036 \div 21 = 3382\frac{14}{21}$
5. $31978 \div 43 = 743\frac{29}{43}$
6. $167864 \div 54 = 3108\frac{32}{54}$
7. $9765837 \div 65 = 150243\frac{42}{65}$
8. $1763 \div 76 = ?$
9. $7964 \div 87 = ?$
10. $89737 \div 98 = ?$
11. $77168 \div 19 = ?$
12. $3167 \div 119 = ?$
13. $3167 \div 129 = ?$
14. $71438 \div 320 = ?$
15. $67898 \div 764 = ?$
16. $78637 \div 892 = ?$
17. $10000 \div 7109 = ?$
18. $7185 \div 1990 = ?$
19. $67416 \div 144 = ?$
20. $3784 \div 642 = ?$
21. $14098 \div 671 = ?$

22. *$730.45 ÷ 126 = ?
23. $164.87 ÷ 144 = ?
24. $1710.14 ÷ 166 = ?
25. $89289.61 ÷ 295 = ?
26. $21008.97 ÷ 444 = ?
27. $10000.00 ÷ 180 = ?

Answers: 8 to 12, inclusive, quotients, 5116; remainders, 211. 13 to 16, quotients, 423 and 956. 17 to 21, 498 and 19246. 22 to 24, $39.54; remainder, 6. 25 to 27, $405.53; remainder, 629.

When there are ciphers in the divisor, they may be pointed off with a corresponding number of figures in the dividend.

28. 67314968 ÷ 163000 is how much?

$$
\begin{array}{r}
163|000)67314|968(412\frac{158968}{163000} \\
\underline{652} \\
211 \\
\underline{163} \\
484 \\
\underline{326} \\
158
\end{array}
$$

The figures pointed off in the dividend were annexed to the remainder, forming the fraction, $\frac{158968}{163000}$.

29. $12986745 \div 7300 = ?$
30. $81098670 \div 18000 = ?$
31. $513643 \div 2500 = ?$

Quot's, 6489; Rem., 9858.

32. $7613412 \div 37100 = ?$
33. $4567800 \div 20900 = ?$
34. $5632710 \div 171000 = ?$

Quot's, 455; Rem., 180222.

---

*Express the dollars in cents before dividing.

35. Find the sum, difference, product and quotient of 128097 and 8070; of 1736009 and 4761; and 4070391 and 71068, omitting the remainders.

Totals, 289282688427, 1033998999, 8268611231 and 29993710999.

PRINCIPLES OF DIVISION.

45. If we divide the price of a number of things of equal value by the number, we obtain the price of one.

46. The quotient will usually be in the same name with the dividend or number to be divided. If the dividend be dollars, the quotient will be dollars; if it be rods, the quotient will be rods.

36. If 75 barrels of flour cost $450, what was the price per barrel?

37. If 125 horses cost $25000, what was the cost of each?

38. If $167809 be divided among 7614 persons, how much should each receive?

39. How much tax should each of 16785 persons pay of a levy of $71683?

40. In 303,656,837 lbs. of cotton, how many bales, supposing each bale to weigh 320 lbs.?

41. If $79640 be divided among 274 persons, how much will each get?

42. The earth moves round the sun at the rate of 66600 miles an hour; at what rate does it move per minute?

43. If 357 yards of broadcloth cost $1035.30, what was the cost per yard? and what would be the cost of 50 yards at the same rate?

Answers arranged promiscuously: 1110, 948, 927$\frac{197}{320}$, $290.65, $6, $22.30,9, $145, $4.27,7, $200, $2.90.

EXERCISES IN MULTIPLICATION AND DIVISION.*

1. If 23 yds. of muslin cost \$3.45, what will one yard cost?

2. If 117 men can do a piece of work in 48 days, how long will it take 3 times that number to do it?

3. How many men can do a piece of work in 5 days, that took 10 men 25 days?

4. If a case holds 29 pieces of muslin, how many will it take to hold 7250 pieces?

5. If 15 men can do a certain piece of work in 75 days, how long will it take 1 man to do it?

6. If 7 dozen silver spoons cost \$35.35, what will 3 dozen cost?

Find the cost of one dozen, then the cost of 3.

7. If $\frac{2}{7}$ of a ship cost \$14602, what will the $\frac{7}{7}$, or the whole ship cost?

8. If $\frac{7}{8}$ of a piece of property cost \$6377, what will $\frac{1}{2}$ of it cost?

9. In a cord of wood there are 128 feet, how many cords are in a pile measuring 4 feet wide, 8 feet deep and 100 feet long?

10. In an acre there are 4840 square yards: how many are there in $\frac{1}{8}$ of an acre?

11. A field contains 12 acres, and is 660 yards long; how many yards is it in breadth?

12. A tract of 5 acres is 220 yards long; how much should be cut off the *breadth* to leave 1 acre?

13. $\frac{1}{8}$ of a dozen books cost \$7.50; what was the cost per dozen?

14. $\frac{2}{3}$ of a dozen cost 22 cents; what was the cost per dozen?

---

*For answers, see next page.

15. $1\frac{1}{2}$ dozen cost $1.20; what was the cost per dozen?

16. $\frac{7}{25}$ of a hundred cost $28; what was the cost per hundred?

17. $\frac{3}{20}$ of a hundred cost $15.75; what was the cost per hundred?

18. $\frac{21}{28}$ of a hundred cost $3.15; what was the price per hundred?

Answers: 51107, 3644, 25, 16, 50, 250, 22, 15.15, 605, 60, 88, 80, 33, 100, 4.20, 105, 1125, 15, 88.

47. Method of Proof.—Division and Multiplication being *converse* operations, the one is proved by the other.

DIVISION.	PROOF.
38)3715(97	97=*quotient.*
342	38=*divisor.*
295	776
266	291
29 *rem.*	3686+*the rem.*, (29)=3715=*dividend.*

MULTIPLICATION.	PROOF. multiplier. product. multiplicand.
465	25)11625(465
25	100
2325	162
930	150
11625	125
	125

VIII. PROPERTIES OF NUMBERS.

48. An *Integer* is any number considered as a whole, as 3, 7, 58, 129.

49. A *Fraction* is a *part* of any thing or number of things considered as a whole; $\frac{1}{2}$, $\frac{7}{8}$, $\frac{25}{100}$.

50. Numbers are divided into *Odd* and *Even*.

An *Odd* number can not be divided into two equal parts without a remainder, as 1, 5, 57.

An *Even* number can be divided into two equal parts without a remainder, as 4, 10, 68.

51. Numbers are either *Prime* or *Composite, Abstract* or *Concrete.*

52. A *Prime* number is an original number, or one which can not be produced by multiplying two other numbers together, as 1, 7, 31.

53. A *Composite* number is one which may be composed of two other numbers multiplied together, as 8, which is composed of 2 and 4 multiplied together; and 27, which is composed of 9 and 3 multiplied together.*

Exercise.—Write out 50 prime and 50 composite numbers.

54. An *Abstract* number is an unapplied number, or one which conveys the idea of number exclusively, as 4, 15, 47.

55. A *Concrete* number is an applied number, or one which conveys the idea of something else besides number. The above numbers become concrete when applied as follows: 4 mills, 15 dollars, 47 pounds, and the names, mills, dollars and pounds, are called *denominations.*

56. A *Multiple* is a number which contains another number a certain number of times without a remainder. 12 is a multiple of 3 as well as of 2, 4 and 6.

57. A *Common Multiple* is one which contains two or more numbers a certain number of times without a remainder. 12 is a common multiple of 2, 3, and 4.

58. The *Least Common Multiple* is the least number

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* NOTE.—Since all even numbers are divisible by 2, an even number can not be a prime number, nor can any number ending with 5; it follows, therefore, that every prime number, except 2 and 5, ends with 1. 3, 7 or 9.

which will contain two or more numbers without a remainder. 6 is the least common multiple of 3 and 2, and 18 of 3 and 6, and 24 of 2, 3 and 8.

59. An *Aliquot* is a number which will divide another without a remainder. The parts of which a multiple is composed are called *aliquot parts* of that number. 1, 2, 3, 4 and 6 are aliquot parts of 12.

60. *Complement:* The number required to be added to another to make it equal to a larger. It is usually applied to 100, 1000 or some other power of 10. Taking 87 as a part of 100, the complement is 13, or 50 as a part of 60, the complement is 10.

Exercise.—Taking 35 as a part of 60, 70 as a part of 100, 18 as a part of 20, 73 as a part of 80, required the complements.

61. Even numbers are divisible by 2 without a remainder.

62. If the *two* right-hand figures of a number are divisible by 4 without a remainder, the whole number is also divisible by 4.

63. Numbers ending with 5 or 0 are divisible by 5 without a remainder.

64. If the *three* right-hand figures of a number are divisible by 8 without a remainder, the whole number is divisible by 8.

65. If the *sum* of the figures of any number is divisible by 3 or 9 without a remainder, the whole number will be divisible by 3 or 9.

MULTIPLICATION BY ALIQUOTS.

66. To multiply by $2\frac{1}{2}$, it will shorten the operation if we multiply by 10, which is 4 times too much, and then divide by 4. In the same way we can multiply by any other aliquot of 10, or by aliquots of 100, 1000, etc.

To multiply 176 by $12\frac{1}{2}$.

$8)\underline{17600}$
2200 *Ans.*

EXPLANATION:—176 being multiplied by 100 is 8 times more than the sum required, so we divide by 8.

To multiply 379 by 250.

$4)\underline{379000}$
94750 *Ans.*

379 being multiplied by 1000 is 4 times too much, so we divide by 4.

To multiply \$49.75 by 125.

$8)\underline{4975\,000}$
$621\,875$ *cents.*
or \$6218.75 *Ans.*

The \$49.75 are considered as cents and multiplied as the preceding.

REMARK.—It will lessen the work still more to simply assume the number to be multiplied by 10, 100 or 1000.

ALIQUOT PARTS OF 10, 100, 1000.

To be committed to memory.

ALIQUOTS OF 10.	ALIQUOTS OF 100.		ALIQUOTS OF 1000.
$5 = \frac{1}{2}$	$50 = \frac{1}{2}$	$14\frac{2}{7} = \frac{1}{7}$	$333\frac{1}{3} = \frac{1}{3}$
$3\frac{1}{3} = \frac{1}{3}$	$33\frac{1}{3} = \frac{1}{3}$	$12\frac{1}{2} = \frac{1}{8}$	$250 = \frac{1}{4}$
$2\frac{1}{2} = \frac{1}{4}$	$25 = \frac{1}{4}$	$10 = \frac{1}{10}$	$166\frac{2}{3} = \frac{1}{6}$
$2 = \frac{1}{5}$	$20 = \frac{1}{5}$	$8\frac{1}{3} = \frac{1}{12}$	$125 = \frac{1}{8}$
$1\frac{2}{3} = \frac{1}{6}$	$16\frac{2}{3} = \frac{1}{6}$	$6\frac{1}{4} = \frac{1}{16}$	$83\frac{1}{3} = \frac{1}{12}$

The pupil can prove the accuracy of his calculations by multiplying in the ordinary way.

4. $140 \times 12\frac{1}{2} = ?$
5. $3767 \times 8\frac{1}{3} = ?$
6. $9987 \times 25 = ?$
7. $9174 \times 125 = ?$
8. $3689 \times 33\frac{1}{3} = ?$
9. $9210 \times 16\frac{2}{3} = ?$
10. $7897 \times 166\frac{2}{3} = ?$
11. $8997 \times 50 = ?$
12. $786 \times 14\frac{2}{7} = ?$
13. $949 \times 333\frac{1}{3} = ?$
14. $179 \times 2\frac{1}{2} = ?$
15. $769 \times 3\frac{1}{3} = ?$
16. $12\frac{1}{2} \times 19 = ?$
17. $125 \times 787 = ?$
18. $250 \times 125 = ?$
19. $16\frac{2}{3} \times 48 = ?$
20. $83\frac{1}{3} \times 756 = ?$
21. $197 \times 12\frac{1}{2} = ?$

23. 675 yards @ $37\frac{1}{2}$ cents.

OPERATION.	675 at a dollar=	\$675.00
	at 25 c=$\frac{1}{4}$	168.75
	at $12\frac{1}{2}$ c=$\frac{1}{2}$	$84.37\frac{1}{2}$
	Ans.	$253.12\frac{1}{2}$

24. $715\times62\frac{1}{2}$ cents.
25. $947\times87\frac{1}{2}$
26. $194\times18\frac{3}{4}$
27. $567\times31\frac{1}{4}$
28. $619\times37\frac{1}{2}$
29. $1060\times32\frac{1}{2}$
30. 197×75
31. $9876\times\$2.18\frac{3}{4}$
32. $719\times\$3.62\frac{1}{2}$
33. $965\times\$4.37\frac{1}{2}$
34. $758\times\$1.25$
35. $197\times\$2.87\frac{1}{2}$
36. $879\times\$3.95$
37. $179\times\$4.32\frac{1}{2}$

38. To find the cost where there are fractions in both factors: $18\frac{3}{4}$ lbs. @ $12\frac{1}{2}$ cents.

OPERATION.	$18\frac{3}{4}$ lbs. @ \$1=	\$18.75
	at $12\frac{1}{2}=\frac{1}{8}$ or	$\$2.34\frac{3}{4}$ *Ans.*

39. $37\frac{1}{2}$ lbs. @ $18\frac{3}{4}$ cents.

OPERATION.	$37\frac{1}{2}$ @ \$1=	\$37.50
	at $12\frac{1}{2}=\frac{1}{8}$	4.687
	at $6\frac{1}{4}=\frac{1}{2}$	2.343
	Ans.	\$7.030

40. $176\frac{1}{2}$ doz. @ $18\frac{3}{4}$ cents.
$731\frac{1}{4}$ doz. @ $12\frac{1}{2}$
$193\frac{1}{2}$ doz. @ $37\frac{1}{2}$
Amount, \$197.06.

41. $164\frac{1}{2}$ lbs. @ $\$1.62\frac{1}{2}$
$374\frac{1}{4}$ lbs. @ 2.25
$693\frac{1}{3}$ lbs. @ $0.16\frac{2}{3}$
Amount, \$1224.93.

NOTE.—The multiplier in the 25th Ex. wants only $12\frac{1}{2}$ cents, or $\frac{1}{8}$, of being a dollar; so we find the cost of 947 at a dollar and take off $\frac{1}{8}$ of it. For $32\frac{1}{2}$ take 30, and $2\frac{1}{2}$ as $\frac{1}{4}$ of 10.

IX. EASY FRACTIONS.*

67. A FRACTION is a part or number of parts of any thing considered as a whole. Fractions are of two kinds, *common* and *decimal*. A common fraction is written with two numbers, called *terms*, having a line between them, as $\frac{1}{2}$; a decimal fraction with one number, having a period at the left, as .5 (five-tenths).

68. A common fraction indicates division, the upper number being the dividend and the lower the divisor. In treating of fractions, the dividend is called the *numerator* and the divisor the *denominator*.

The denominator indicates the number of parts into which the whole is divided, and the numerator the number of such parts under consideration.

69. VALUE OF A FRACTION.—The lowest value of a fraction is expressed by the figure 1 for a numerator, and the highest value a number as great as the denominator less 1.† $\frac{1}{9}$ represents the lowest value of fractions of the denomination of ninths, while $\frac{8}{9}$ represents the highest value of that denomination.‡

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* This chapter is introduced for the benefit of that large class of scholars who leave school before completing the study of Arithmetic. The subject of fractions is treated of at length in the latter part of this book.

† This does not apply to *improper* fractions, which, as the name indicates, are not strictly fractions.

‡ 1. Since this is the case, it is evident that fractions decrease in value as their denominators increase, the numerators remaining the same. $\frac{1}{3}$ is less than $\frac{1}{2}$, $\frac{1}{4}$ than $\frac{1}{3}$, $\frac{1}{5}$ than $\frac{1}{4}$.

2. It is also evident that the value of a fraction depends on the

When a number is divided into two parts, each part is called a *half;* into 3 parts, each part is called a *third;* into 4 parts, each part is called a *fourth;* into 5, a *fifth;* into 12, a *twelfth;* into 18, an *eighteenth;* into 25, a *twenty-fifth;* into 100, a *hundredth;* into 476, a *four hundred and seventy-sixth part.*

ORAL EXERCISES.

1. When a number is divided into 10 parts, what is each part called? Into 11? Into 20? Into 33? Into 45? Into 97? Into 62?

2. When divided into 31, what? Into 69? Into 103? Into 364? Into 155? Into 1000? Into 3144?

3. Which is the greater fraction, $\frac{1}{2}$ or $\frac{1}{3}$? $\frac{1}{4}$ or $\frac{1}{6}$? $\frac{1}{5}$ or $\frac{1}{4}$? $\frac{1}{11}$ or $\frac{1}{10}$? $\frac{1}{20}$ or $\frac{1}{19}$? $\frac{1}{75}$ or $\frac{1}{65}$?

4. Which is greater, $\frac{2}{3}$ or $\frac{3}{4}$? *Ans.* $\frac{3}{4}$.

REASON.—Because it will take less to make it a whole number. The first fraction requires $\frac{1}{3}$ to make it a whole number, while this one requires only $\frac{1}{4}$.

5. Which is the greater, $\frac{7}{8}$ or $\frac{6}{7}$? $\frac{5}{6}$ or $\frac{5}{5}$? $\frac{3}{4}$ or $\frac{4}{5}$? $\frac{8}{9}$ or $\frac{7}{8}$? $\frac{2}{3}$ or $\frac{7}{8}$? $\frac{7}{9}$ or $\frac{8}{10}$? $\frac{12}{13}$ or $\frac{11}{12}$? $\frac{7}{8}$ or $\frac{7}{9}$?

6. Which is the greater, $\frac{15}{16}$ or $\frac{13}{14}$? $\frac{19}{21}$ or $\frac{17}{19}$? $\frac{12}{14}$ or $\frac{13}{15}$? $\frac{21}{24}$ or $\frac{18}{21}$? $\frac{19}{24}$ or $\frac{16}{21}$? $\frac{14}{17}$ or $\frac{16}{19}$?

Since the value of a fraction depends upon the relation of the numerator to the denominator, [note 2, page 82,] both terms may be multiplied or divided by the same number without altering its value.

$$\frac{2\times2}{4\times2}=\frac{4}{8} \quad \text{and} \quad \frac{2\div2}{4\div2}=\frac{1}{2}$$

---

relation of the numerator to the denominator, or, in other words, the number of times the numerator is contained in the denominator. $\frac{3}{6}$ is equal to $\frac{4}{8}$, because the numerator 3 is contained in its denominator, 6, the same number of times that the numerator 4 is contained in the denominator 8.

Now, $\frac{4}{8}$ and $\frac{1}{2}$ possess the same value as $\frac{2}{4}$, because their respective numerators are contained the same number of times in their denominators.

EXERCISES FOR THE SLATE.

5. Change $\frac{3}{4}$ to twentieths.

$\frac{3\times5}{4\times5}=\frac{15}{20}$ EXPLANATION.—By multiplying the 4 by 5, we change the denominator to twentieths; and by multiplying the numerator by the same number we preserve the same value.

1. Change $\frac{3}{4}$ to 8ths; $\frac{1}{2}$ to 12ths; $\frac{1}{5}$ to 20ths; $\frac{2}{7}$ to 14ths; $\frac{3}{4}$ to 12ths; $\frac{5}{6}$ to 18ths; $\frac{4}{5}$ to 30ths.

2. Change $\frac{9}{10}$ to 20ths; $\frac{7}{8}$ to 16ths; $\frac{8}{9}$ to 27ths; $\frac{12}{13}$ to 52ds; $\frac{4}{5}$ to 25ths; $\frac{35}{50}$ to 150ths.

3. Change $\frac{7}{8}$ to 32ds; $\frac{15}{20}$ to 40ths; $\frac{12}{24}$ to 72ds; $\frac{50}{52}$ to 104ths.

4. Change $\frac{12}{20}$ to 80ths; $\frac{13}{26}$ to 52ds; $\frac{29}{32}$ to 128ths; $\frac{4}{10}$ to 50ths.

5. Change $\frac{6}{12}$ to 6ths; $\frac{4}{20}$ to 5ths; $\frac{8}{12}$ to 4ths; $\frac{14}{16}$ to 8ths; $\frac{12}{14}$ to 7ths; $\frac{10}{16}$ to 8ths.

6. Change $\frac{4}{8}$ to halves; $\frac{6}{10}$ to 5ths; $\frac{4}{16}$ to 4ths; $\frac{21}{24}$ to 8ths; $\frac{20}{30}$ to 3ds.

The fractions in last exercise, (6th,) when changed as required, would be *reduced to their lowest terms;* that is, expressed in their simplest form.

70. To *reduce a fraction* to its lowest terms is, therefore, to divide the numerator and denominator by such a number or numbers as will do so without a remainder. When the terms can not be divided exactly by any number greater than 1, the fraction is in its simplest form.

Reduce $\frac{6}{12}$ to its lowest terms.

$$6)\tfrac{6}{12}(\tfrac{1}{2}$$

1. Reduce $\frac{50}{100}$, $\frac{7}{147}$, $\frac{3}{27}$, $\frac{9}{189}$, $\frac{2}{22}$, $\frac{6}{42}$ to their lowest terms. Answers: $\frac{1}{2}$, $\frac{1}{21}$, $\frac{1}{11}$, $\frac{1}{20}$, $\frac{1}{9}$, $\frac{1}{7}$, $\frac{1}{12}$.

When a single number will not reduce the fraction, other numbers may be used, as below.

2. Reduce $\frac{55}{9900}$ to its lowest terms.

$5)\frac{55}{9900}(11)\frac{11}{1980}(\frac{1}{180}$.

3. Reduce to their lowest terms, $\frac{48}{272}$, $\frac{46}{116}$, $\frac{176}{484}$, and $\frac{48}{160}$.

4. Reduce to their lowest terms, $\frac{825}{1920}$, $\frac{644}{1728}$, $\frac{764}{5240}$, and $\frac{1344}{1536}$.

5. Reduce to their lowest terms, $\frac{740}{8675}$, $\frac{1245}{2205}$, and $\frac{346}{1990}$.

Answers: $\frac{105}{384}$, $\frac{55}{128}$, $\frac{3}{10}$, $\frac{23}{58}$, $\frac{3}{17}$, $\frac{4}{11}$, $\frac{161}{432}$, $\frac{173}{995}$, $\frac{148}{1735}$, $\frac{83}{147}$, $\frac{191}{1310}$, $\frac{9}{10}$, $\frac{7}{8}$.

Fractions may be *Proper*, *Improper*, *Simple*, *Compound* or *Complex*. We shall treat of only the three former at present.

A *proper* fraction is one whose numerator is less than its denominator, as $\frac{1}{2}$. An *improper* fraction is one whose numerator is equal to or greater than its denominator, as $\frac{5}{5}$, $\frac{9}{7}$.

71. A *simple* fraction is a single fraction, and may be proper or improper, as $\frac{3}{7}$, $\frac{9}{8}$.

72. When a whole number and fraction appear together, they are called a *mixed number*, as $5\frac{3}{4}$.

73. *Improper fractions may be changed to whole or mixed numbers by dividing the numerator by the denominator.**

To change $\frac{13}{5}$ to a mixed number.

$$\begin{array}{r} 5)\underline{13} \\ 2\frac{3}{5} \end{array}$$

EXPLANATION.—There are 5 fifths in one whole number; in 13 fifths there are as many 1s as the number of times 5 is contained in 13, which is two times, with 3 fifths over, making $2\frac{3}{5}$.

---

* This is simply acting on the principle that the numerator is the dividend and the denominator the divisor.

1. Change $\frac{3}{2}$, $\frac{9}{6}$, $\frac{5}{3}$, $\frac{18}{6}$, $\frac{271}{9}$, $\frac{834}{12}$ to whole or mixed numbers.

2. Change the following: $\frac{21}{5}$, $\frac{37}{4}$, $\frac{56}{3}$, $\frac{22}{7}$, $\frac{66}{8}$, $\frac{55}{4}$, $\frac{120}{9}$.

3. Change $\frac{130}{7}$, $\frac{56}{8}$, $\frac{171}{20}$, $\frac{98}{12}$, $\frac{641}{11}$, $\frac{564}{10}$, $\frac{318}{6}$.

Answers: $1\frac{1}{2}$, $1\frac{2}{3}$, 3, $1\frac{1}{2}$, $30\frac{1}{9}$, $69\frac{1}{2}$, 53, $8\frac{1}{6}$, $8\frac{11}{20}$, 7, $58\frac{3}{11}$, $56\frac{2}{5}$, $18\frac{4}{7}$, $13\frac{1}{3}$, $3\frac{1}{7}$, $8\frac{1}{4}$, $11\frac{1}{4}$, $4\frac{1}{5}$, $9\frac{1}{4}$, $18\frac{2}{3}$.

74. *To change whole or mixed numbers to improper fractions* is an operation the reverse of the last, which scarcely needs explanation.

4. Change $9\frac{4}{5}$ to an improper fraction.

$$\begin{array}{c} 9\frac{4}{5} \\ 5 \\ \hline \frac{49}{5} \end{array}$$

Explanation.—In 1 whole number there are 5 fifths; in 9 there are 9 times 5 or 5 times 9 fifths, to which we add 4 fifths, and we have $\frac{49}{5}$.

5. *Change the following mixed numbers to improper fractions: $3\frac{5}{6}$, $9\frac{7}{8}$, $8\frac{6}{7}$, $5\frac{4}{5}$, $41\frac{3}{4}$, $97\frac{1}{8}$, $16\frac{2}{3}$.

6. Change the following: $7\frac{3}{4}$, $9\frac{11}{12}$, $4\frac{5}{6}$, $7\frac{13}{14}$, $18\frac{8}{9}$, $16\frac{2}{3}$, $12\frac{4}{5}$.

7. Change the following: 21 to fifths; $16\frac{3}{4}$ to eighths; $12\frac{1}{2}$ to fourths; $16\frac{2}{3}$ to twelfths; $8\frac{1}{3}$ to twelfths; $13\frac{1}{8}$ to sixteenths.

75. *To multiply a fraction by a whole number* is simply to multiply the numerator without altering the denominator, or to divide the denominator without altering the numerator.

To multiply $\frac{7}{12}$ by 6.

$$\frac{7\times6}{12}=\frac{42}{12}=3\frac{6}{12} \textit{ or } 3\frac{1}{2}$$

Reason.—Assuming that 7 is a whole number, multiplying it by 6 gives 42; but since it is *not* a whole number, but *twelfths*, the 42 is $\frac{42}{12}=3\frac{6}{12}$ or $3\frac{1}{2}$.

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* The learner should prove the accuracy of his work by last article.

2. $6)\frac{7}{12}(\frac{7}{2}=3\frac{1}{2}$.

By decreasing the denominator, the fraction is increased (as it takes fewer of the small parts to make a whole number); hence, the 7 represents *halves* instead of twelfths. $\frac{7}{2}=3\frac{1}{2}$.

1. $\frac{3}{4}\times7=?$	5. $\frac{9}{10}\times12=?$	9. $\frac{13}{17}\times11=?$
2. $\frac{5}{8}\times9=?$	6. $\frac{15}{19}\times\ 6=?$	10. $\frac{21}{23}\times12=?$
3. $\frac{7}{9}\times8=?$	7. $\frac{21}{25}\times\ 5=?$	11. $\frac{18}{24}\times\ 8=?$
4. $\frac{5}{16}\times4=?$	8. $\frac{18}{33}\times11=?$	12. $\frac{12}{20}\times21=?$

Answers: $1\frac{1}{4}$, $6\frac{2}{9}$, $5\frac{1}{4}$, $5\frac{5}{8}$, $10\frac{4}{5}$, $9\frac{8}{15}$, $7\frac{1}{7}$, $4\frac{14}{19}$, $4\frac{1}{5}$, 6, $1\frac{22}{23}$, $12\frac{3}{5}$, 6.

76. *To multiply a whole number by a fraction*, we multiply the numerator without altering the denominator.

13. Multiply 25 by $\frac{3}{4}$.

25×3 fourths$=75$ fourths, or $\frac{75}{4}$, which, changed to a mixed number, [Art. 73]$=18\frac{3}{4}$.

14. $35\times\frac{4}{5}=?$	18. $134\times\frac{1}{20}=?$	22. $16\times\frac{3}{10}=?$
15. $21\times\frac{6}{7}=?$	19. $215\times\frac{4}{9}=?$	23. $21\times\frac{5}{6}=?$
16. $18\times\frac{1}{5}=?$	20. $112\times\frac{5}{6}=?$	24. $14\times\frac{6}{7}=?$
17. $116\times\frac{3}{10}=?$	21. $36\times\frac{5}{9}=?$	25. $12\times\frac{1}{25}=?$

Answers: 28, $6\frac{7}{10}$, $4\frac{4}{5}$, $17\frac{1}{2}$, $95\frac{5}{9}$, 18, $3\frac{3}{5}$, $93\frac{1}{3}$, 12, $\frac{12}{25}$, 20, $34\frac{4}{5}$, $30\frac{4}{5}$.

77. *To multiply a mixed number by a whole number.*

Multiply $7\frac{3}{4}$ by 9.

$$\begin{array}{r} 7\frac{3}{4} \\ 9 \\ \hline 69\frac{3}{4} \end{array}$$

EXPLANATION.—3 fourths multiplied by $9=27$ fourths, or $6\frac{3}{4}$; and the 7 multiplied by nine$=63$, plus the $6=69$, making the product $69\frac{3}{4}$.

Or thus:

$$\begin{array}{c} 7\frac{3}{4} \\ 4 \\ \hline \frac{31}{4}\times9=\frac{279}{4}=69\frac{3}{4} \end{array}$$

The Teacher will find it important to require the learner to preserve the process, as he will be apt to adopt clumsy methods of solution.

26. $18\frac{3}{4}\times 5=?$	30. $29\frac{3}{4}\times 8=?$	34. $83\frac{1}{3}\times 7=?$
27. $37\frac{1}{2}\times 8=?$	31. $16\frac{2}{3}\times 9=?$	35. $12\frac{1}{16}\times 8=?$
28. $12\frac{1}{2}\times 12=?$	32. $87\frac{1}{2}\times 12=?$	36. $5\frac{2}{26}\times 9=?$
29. $37\frac{1}{2}\times 9=?$	33. $62\frac{1}{2}\times 9=?$	37. $187\frac{1}{2}\times 11=?$

Answers: $3\frac{1}{5}$, $93\frac{3}{4}$, $337\frac{1}{2}$, $562\frac{1}{2}$, $2062\frac{1}{2}$, $45\frac{9}{13}$, $96\frac{1}{2}$, $583\frac{1}{3}$, 238, 150, 1050, 150, 300.

78. *To multiply a whole number by a mixed number.*

38. Multiply 29 by $8\frac{2}{3}$.

$$\begin{array}{r} 29 \\ 8\frac{2}{3} \\ \hline 19\frac{1}{3} \\ 232 \\ \hline 251\frac{1}{3} \end{array}$$

Explanation.—Multiplying 29 by 2 thirds, we have 58 thirds, or $19\frac{1}{3}$, which we write in the first line. Then $29\times 8=232$, which, added to $19\frac{1}{3}=251\frac{1}{3}$.

Or thus: $29\times\frac{26}{3}=\frac{754}{3}=251\frac{1}{3}$

39. $15\times 3\frac{1}{3}=?$	42. $12\times 12\frac{1}{2}=?$	45. $14\times 17\frac{3}{5}=?$
40. $27\times 6\frac{3}{4}=?$	43. $47\times 37\frac{1}{2}=?$	46. $29\times 18\frac{3}{4}=?$
41. $19\times 27\frac{1}{2}=?$	44. $93\times 16\frac{2}{3}=?$	47. $83\times 6\frac{9}{10}=?$

Answers: $572\frac{7}{10}$, 1550, 50, $35\frac{1}{2}$, $246\frac{2}{5}$, $157\frac{1}{4}$, $182\frac{1}{4}$, $543\frac{3}{4}$, $522\frac{1}{2}$, 150, $1762\frac{1}{2}$.

To multiply a fraction by a fraction.

48. Multiply $\frac{3}{4}$ by $\frac{5}{6}$.

Assuming the numerator 5 to be a whole number, $\frac{3}{4}\times 5=\frac{15}{4}$; but 5 is not a whole number, but 5 *sixths;* hence $\frac{15}{4}$ is 6 *times too much.* $\frac{15}{4}$ divided by $6=\frac{15}{24}$, or $\frac{5}{8}$ [Note 1, page 82.]

79. Hence, to multiply a fraction by a fraction, *we multiply the numerators together for a new numerator, and the denominators for a new denominator.*

$\frac{3}{4}\times\frac{5}{6}=\frac{15}{24}$, which, reduced to its lowest terms $=\frac{5}{8}$.*

* It will be observed that to multiply by a fraction does not increase the multiplicand, as in whole numbers; but, on the contrary, *decreases* it, the $\frac{5}{8}$ being less than $\frac{3}{4}$.

To account for this, it is only necessary to remember that a whole

49. $\frac{3}{4} \times \frac{7}{8} = ?$	52. $\frac{7}{8} \times \frac{8}{9} = ?$	55. $\frac{1}{2} \times \frac{21}{23} = ?$
50. $\frac{9}{10} \times \frac{10}{12} = ?$	53. $\frac{6}{9} \times \frac{7}{8} = ?$	56. $\frac{7}{8} \times \frac{6}{21} = ?$
51. $\frac{15}{16} \times \frac{4}{5} = ?$	54. $\frac{3}{4} \times \frac{4}{5} = ?$	57. $\frac{5}{6} \times \frac{6}{7} = ?$

Answers: $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{7}{12}$, $\frac{21}{32}$, $\frac{7}{9}$, $\frac{15}{28}$, $\frac{3}{5}$, $\frac{21}{46}$, $\frac{5}{7}$.

80. *To multiply a mixed number by a fraction or a mixed number.*

58. Multiply $15\frac{3}{4}$ by $\frac{9}{10}$.

$15\frac{3}{4} = \frac{63}{4}$, which, multiplied by $\frac{9}{10} = \frac{567}{40}$ or $14\frac{7}{40}$.

59. Multiply $8\frac{3}{4}$ by $16\frac{2}{3}$.

$8\frac{3}{4} = \frac{35}{4}$ and $16\frac{2}{3} = \frac{50}{3}$. $\frac{35}{4} \times \frac{50}{3} = \frac{1750}{12} = 145\frac{10}{12} = 145\frac{5}{6}$.

60. $12\frac{1}{2} \times 16\frac{2}{3} = ?$	63. $14\frac{2}{7} \times \frac{9}{10} = ?$	66. $18\frac{1}{2} \times 12\frac{1}{2} = ?$
61. $8\frac{1}{3} \times 29\frac{1}{2} = ?$	64. $23\frac{1}{2} \times \frac{5}{4} = ?$	67. $47\frac{2}{3} \times \frac{3}{4} = ?$
62. $37\frac{1}{2} \times 52\frac{3}{4} = ?$	65. $\frac{3}{7} \times 14\frac{1}{2} = ?$	68. $19\frac{1}{2} \times 6\frac{1}{2} = ?$

Answers: $126\frac{3}{4}$, $35\frac{3}{4}$, $231\frac{1}{4}$, $6\frac{3}{14}$, $\frac{9}{14}$, $208\frac{1}{3}$, $245\frac{5}{6}$, $29\frac{3}{8}$, $12\frac{6}{7}$, $1978\frac{1}{8}$.

81. *To divide a whole number by a fraction or a mixed number.*

1. Divide 315 by $\frac{3}{4}$, or, in other words, find how often $\frac{3}{4}$ is contained in 315.

SOLUTION.—Before we can measure 315 by fourths, we must change it to fourths. In 1 there are 4 fourths; in 315 there are 315 times 4 or 1260 fourths, which, divided by 3=420. Hence, $\frac{3}{4}$ is contained in 315 420 times.

OPERATION.
$$\begin{array}{r} 315 \\ 4 \\ \hline 3)1260 \\ \hline 420 \end{array} \quad \textit{or} \quad \frac{315}{1} \times \frac{4}{3} = \frac{1260}{3} = 420$$

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number is reduced to the denomination of a fraction by being multiplied by it. $6 \times \frac{3}{5} = 18$ fifths or $3\frac{3}{5}$. Much more is a fraction reduced in value if multiplied by a fraction. From this we readily infer,

2. That to divide by a fraction increases the dividend.

2. $320 \div \frac{7}{8} = ?$	4. $541 \div \frac{2}{9} = ?$	6. $987 \div \frac{1}{20} = ?$
3. $27 \div \frac{5}{6} = ?$	5. $684 \div \frac{3}{10} = ?$	7. $136 \div \frac{6}{7} = ?$

Answers: $32\frac{2}{5}$, $365\frac{5}{7}$, $2434\frac{1}{2}$, 19740, 2280, $158\frac{2}{3}$, $24\frac{2}{3}$.

8. Divide 25 by $5\frac{1}{2}$.

Operation. 25×2 halves $= \frac{50}{2}$ and $5\frac{1}{2} \times 2 = \frac{11}{2}$. $50 \div 11 = 4\frac{6}{11}$, *or* $\frac{50}{2} \times \frac{2}{11} = \frac{100}{22} = 4\frac{12}{22} = 4\frac{6}{11}$.

82. Hence, to *divide* by a fraction, we multiply by the denominator and divide by the numerator, or invert the divisor and proceed as in multiplication.

9. $157 \div 3\frac{1}{2} = ?$	12. $345 \div 6\frac{3}{4} = ?$	15. $195 \div 16\frac{2}{3} = ?$
10. $22 \div 12\frac{1}{2} = ?$	13. $39 \div 15\frac{1}{3} = ?$	16. $39 \div 12\frac{4}{5} = ?$
11. $16 \div 16\frac{3}{4} = ?$	14. $79 \div 37\frac{1}{2} = ?$	17. $87 \div 31\frac{1}{4} = ?$

To divide one fraction by another.

18. Divide $\frac{3}{4}$ by $\frac{5}{6}$. Operation. $\frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$

Explanation.—By inverting the divisor, we obtain $\frac{18}{20}$, the terms of which, being divided by 2, give $\frac{9}{10}$.

19. $\frac{9}{10} \div \frac{3}{4} = ?$	22. $31\frac{1}{2} \div \frac{6}{7} = ?$	25. $\frac{3}{7} \div 1\frac{1}{2} = ?$
20. $\frac{5}{13} \div \frac{7}{8} = ?$	23. $31\frac{3}{4} \div \frac{9}{11} = ?$	26. $\frac{9}{10} \div 3\frac{1}{2} = ?$
21. $\frac{21}{25} \div \frac{6}{7} = ?$	24. $13\frac{1}{2} \div \frac{5}{6} = ?$	27. $\frac{5}{6} \div 6\frac{3}{4} = ?$

Answers: $1\frac{1}{5}$, $\frac{40}{91}$, $\frac{49}{50}$, $38\frac{29}{36}$, $36\frac{3}{4}$, $16\frac{1}{5}$, $\frac{10}{81}$, $\frac{9}{35}$, $\frac{2}{7}$, $\frac{1}{9}$.

83. *To divide when either divisor or dividend is a mixed number and the other term a whole number*, both terms may be reduced to the same denomination. [Art. 81.]

28. Divide $3457\frac{1}{4}$ by 13.

$$\begin{array}{r} 3457\frac{1}{4} \\ 4 \\ \hline 52)13829(265 \\ 104 \\ \hline 342 \\ 312 \\ \hline 309 \\ 260 \\ \hline 49 \end{array}$$

Explanation.—The dividend containing the fraction of $\frac{1}{4}$, both terms are reduced to fourths, and division performed as in whole numbers. The result shows that the divisor is contained in the dividend 265 times, with a remainder of 49 *fourths* [Art. 46], or $265\frac{49}{52}$ times.

The same by short division.

$$13\overline{)3457\tfrac{1}{4}}$$
$$265\tfrac{49}{52}$$

EXPLANATION.—13 is contained in 3457, 265 times, with a remainder of 12, which, reduced to fourths, including the $\frac{1}{4}$ in the dividend, is 49 fourths. 13 not being contained in this an even number of times, the denominator is increased 13 times, (which is the same as to decrease the numerator,) which gives the same fraction as by long division, $\frac{49}{52}$.

29. $1398\frac{1}{3} \div 56 = ?$	35. $1255\frac{5}{6} \div 350 = ?$
30. $256\frac{1}{2} \div 7 = ?$	36. $796\frac{1}{3} \div 421 = ?$
31. $1939 \div 8\frac{1}{3} = ?$	37. $467\frac{2}{5} \div 12 = ?$
32. $7961\frac{7}{8} \div 300 = ?$	38. $214\frac{1}{4} \div 9 = ?$
33. $9219 \div 6\frac{1}{4} = ?$	39. $713\frac{3}{25} \div 8 = ?$
34. $1391 \div 56\frac{1}{2} = ?$	40. $391\frac{3}{15} \div 6 = ?$

Answers: $24\frac{163}{168}$, $24\frac{70}{113}$, $1475\frac{1}{25}$, $36\frac{9}{14}$, $232\frac{17}{25}$, $26\frac{259}{480}$, $65\frac{1}{5}$, $3\frac{247}{420}$, $38\frac{19}{20}$, $1\frac{1126}{1263}$, $23\frac{29}{36}$, $65\frac{8}{15}$, $89\frac{7}{50}$.

84. *To subtract a fraction from another of the same denomination* is simply to subtract the less numerator from the greater.

1. From $\frac{7}{10}$ take $\frac{3}{10}$.

$$\frac{7}{10}-\frac{3}{10}=\frac{4}{10} \text{ or } \frac{2}{5}$$

2. $\frac{13}{15}-\frac{11}{15}=?$	5. $\frac{9}{10}-\frac{3}{10}=?$	8. $\frac{9}{13}-\frac{4}{13}=?$
3. $\frac{37}{42}-\frac{25}{42}=?$	6. $\frac{12}{21}-\frac{9}{21}=?$	9. $\frac{16}{39}-\frac{3}{39}=?$
4. $\frac{16}{22}-\frac{5}{22}=?$	7. $\frac{16}{45}-\frac{7}{45}=?$	10. $\frac{11}{17}-\frac{6}{17}=?$

Answers: $\frac{2}{7}$, $\frac{1}{7}$, $\frac{1}{3}$, $\frac{5}{17}$, $\frac{1}{5}$, $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{5}{13}$, $\frac{2}{15}$.

85. *To subtract a fraction or mixed nnmber from a whole number.*

11. From 9 take $3\frac{3}{4}$.

The following formula will render the operation simple:

Whole number.	Fourths.
9	0
3	3
5	1
or $5\frac{1}{4}$	

EXPLANATION.—Arranging the less under the greater, we find we can not take 3 fourths from 0 fourths; so a whole number or 1 is added to both terms. In 1 there are 4 fourths, from which take 3 fourths, and we have a remainder of 1 fourth. To the 3 add 1 and we have 4, which, subtracted from 9, leaves 5, giving for the answer $5\frac{1}{4}$.

12. $13 - 4\frac{1}{4} = ?$	15. $11 - 2\frac{1}{3} = ?$	18. $52 - 27\frac{1}{7} = ?$
13. $15 - 5\frac{1}{6} = ?$	16. $7 - \frac{7}{8} = ?$	19. $13 - 12\frac{1}{6} = ?$
14. $29 - 12\frac{1}{2} = ?$	17. $14 - 1\frac{3}{12} = ?$	20. $89 - 75\frac{1}{4} = ?$

Answers: $\frac{5}{6}$, $6\frac{1}{8}$, $9\frac{5}{6}$, $4\frac{1}{2}$, $24\frac{6}{7}$, $8\frac{2}{3}$, $8\frac{3}{4}$, $16\frac{1}{2}$, $13\frac{3}{4}$, $13\frac{3}{4}$.

86. *To subtract one fraction from another of a different denomination*, it will be necessary first to reduce both to the same or a common denominator.

21. From $\frac{7}{8}$ take $\frac{6}{7}$.

By Art. 69, $\frac{7}{8}$ can be changed to 56ths by multiplying both terms by 7, and $\frac{6}{7}$ can be changed to 56ths by multiplying both terms by 8, giving $\frac{49}{56}$ and $\frac{48}{56}$, the difference between which is $\frac{1}{56}$, the answer.

It will be observed that the multipliers used in this case were the two denominators, 7 and 8, which, multiplied together, give a *common denominator*, and multiplied into the numerators of each other give the new numerators.

OPERATION. $\frac{7}{8} - \frac{6}{7} = \frac{49}{56} - \frac{48}{56} = \frac{1}{56}$

22. From $\frac{6}{7}$ take $\frac{1}{5}$.	25. $\frac{3}{4} - \frac{4}{7} = ?$	28. $\frac{7}{8} - \frac{5}{9} = ?$
23. From $\frac{3}{8}$ take $\frac{1}{6}$.	26. $6\frac{1}{2} - \frac{3}{4} = ?$	29. $\frac{4}{12} - \frac{1}{6} = ?$
24. From $\frac{2}{3}$ take $\frac{5}{12}$.	27. $1\frac{1}{2} - \frac{7}{8} = ?$	30. $\frac{3}{13} - \frac{1}{15} = ?$

Answers: $\frac{5}{24}$, $5\frac{3}{4}$, $\frac{23}{35}$, $\frac{5}{8}$, $\frac{32}{195}$, $\frac{1}{6}$, $\frac{23}{72}$, $\frac{1}{4}$, $\frac{5}{28}$, $\frac{3}{5}$.

87. *To add fractions of the same denomination*, the numerators only are added, and the sum reduced to a mixed number or its lowest terms.

1. Add $\frac{3}{8} + \frac{6}{8} + \frac{5}{8} + \frac{7}{8}$.

$$\begin{array}{l} 3 \\ 6 \\ 5 \\ \underline{7} \\ \frac{21}{8} = 2\frac{5}{8} \end{array}$$

EXPLANATION.—Here the four numerators are added together, making 21 eighths, which, reduced to a mixed number, are equal to $2\frac{5}{8}$.

2. $\frac{1}{12}+\frac{5}{12}+\frac{3}{12}+\frac{6}{12}+\frac{4}{12}+\frac{7}{12}+\frac{8}{12}=?$
3. $\frac{3}{9}+\frac{4}{9}+\frac{7}{9}+\frac{8}{9}+\frac{6}{9}+\frac{5}{9}+\frac{1}{9}=?$
4. $\frac{3}{15}+\frac{5}{15}+\frac{7}{15}+\frac{12}{15}+\frac{13}{15}+\frac{2}{15}+\frac{4}{15}=?$
5. $\frac{2}{25}+\frac{5}{25}+\frac{7}{25}+\frac{6}{25}+\frac{9}{25}+\frac{2}{25}+\frac{7}{25}=?$
6. $\frac{3}{19}+\frac{5}{19}+\frac{4}{19}+\frac{2}{19}+\frac{1}{19}+\frac{3}{19}+\frac{1}{19}=?$

Answers: $3\frac{7}{9}$, $2\frac{5}{6}$, 1, $3\frac{1}{15}$, $1\frac{13}{25}$, $\frac{18}{19}$.

88. *To add fractions of different denominations,* they should first be reduced to a common denominator, as in subtraction.

7. $\frac{1}{2}+\frac{3}{4}=?$ $\frac{1}{2}+\frac{3}{4}=\frac{4}{8}+\frac{6}{8}=\frac{10}{8}=1\frac{2}{8}$ *or* $1\frac{1}{4}$.

When three or more fractions of different denominations are to be added together, they may be reduced to a common denominator by multiplying all the denominators together, as above, and then by multiplying each numerator by all the denominators except its own.*

8. Find the sum of $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}$.

$2\times4\times6=\ 48=$ *Common denominator.*
$1\times4\times6=24=$ *First numerator.*
$3\times2\times6=36=$ *Second numerator.*
$5\times2\times4=40=$ *Third numerator.*
$100=$ *Sum of numerators.*

Hence, $\frac{100}{48}=2\frac{4}{48}=2\frac{1}{12}$.

The $\frac{1}{2}$ in the example was multiplied by 24, giving $\frac{24}{48}$; the $\frac{3}{4}$ by 12, giving $\frac{36}{48}$; and the $\frac{5}{6}$ by 8, giving $\frac{40}{48}$.

9. $\frac{3}{5}+\frac{5}{6}=?$	14. $\frac{3}{4}+\frac{4}{8}+\frac{5}{6}=?$	19. $2\frac{3}{4}+\frac{3}{4}+\frac{5}{6}=?$
10. $\frac{1}{5}+\frac{7}{8}=?$	15. $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=?$	20. $5\frac{1}{2}+6\frac{1}{4}+\frac{2}{8}=?$
11. $\frac{3}{12}+\frac{2}{6}=?$	16. $\frac{1}{12}+\frac{1}{6}+\frac{3}{12}=?$	21. $\frac{7}{8}+2\frac{3}{4}+\frac{3}{12}=?$
12. $\frac{7}{8}+\frac{3}{4}=?$	17. $\frac{2}{5}+\frac{1}{12}+\frac{1}{2}=?$	22. $\frac{4}{9}+\frac{3}{6}+\frac{1}{2}=?$
13. $\frac{1}{9}+\frac{1}{12}=?$	18. $\frac{3}{6}+\frac{2}{4}+\frac{5}{12}=?$	23. $\frac{1}{4}+\frac{1}{12}+\frac{1}{6}=?$

Answers: $2\frac{1}{12}$, $1\frac{13}{30}$, $1\frac{3}{40}$, $\frac{11}{12}$, $4\frac{1}{3}$, $\frac{4}{5}$, 12, $3\frac{2}{3}$, $1\frac{4}{9}$, $\frac{1}{2}$, $1\frac{5}{12}$, $\frac{1}{2}$, $\frac{7}{36}$, $1\frac{5}{8}$, $\frac{7}{12}$.

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*This is simply multiplying both terms by the same number. [Art. 69.]

X. THE MERCANTILE PROFESSION.

THE mercantile community may be divided into various classes: Importers, Jobbers, Wholesale Dealers, Commission, Forwarding and Retail Merchants, Brokers, etc.

89. *Importers* purchase goods and produce in foreign countries, and sell them in the home market to jobbers and wholesale dealers. They also receive goods from abroad to sell on commission.

90. *Jobbers.* This term was first applied to persons dealing in *stocks* to a limited extent, but it now includes nearly all classes of wholesale dealers. We speak of *dry goods jobbers*, *produce jobbers*, *cattle jobbers*, etc.

91. *Wholesale Merchants* or dealers buy from importers, jobbers, manufacturers and producers, and sell to retail dealers.

92. *Commission Merchants** act as agents for other persons in buying and selling goods, collecting debts, etc., for which they charge a *percentage* on the whole amount of sale, purchase or collection. Merchants of this class usually keep a wholesale department in their warehouses, where they sell their own goods as well as those of others, and even ship merchandise to distant places for sale on commission, thus acting in the capacity of principals as well as agents.

The person who sends goods to another to be sold on commission is called the *shipper* or *consignor;* the person who receives them, *agent*, *correspondent*, *consignee* or *factor;* and the goods or merchandise sent, *shipment* or *consignment.*

93. *Forwarding Commission Merchants* and *Express Companies* are intrusted with the care of conveying goods

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* Auctioneers belong to this class.

from one city or country to another, for which they charge a percentage, called *forwarding commission.* This class of merchants usually have warehouses, and on rivers, wharf-boats, for the storage of merchandise. A separate charge is made for all goods lodged in these places, according to the time they remain. The fee charged is called storage.

94. *Retail Merchants* dispose of their goods in small quantities suited to the wants of consumers.

95. *Brokers* form a numerous class. They assist commission merchants and dealers generally in finding buyers for their wares, and trade or speculate in stocks, lands, etc. By confining their attention to one line of business, they acquire a more intimate knowledge of its details and of the credit of persons engaged in it, and are thus prepared to render valuable services to both buyer and seller, between whom they act as middle-men.

The business of the broker does not require the investment of much capital, as, unlike commission merchants, they are not under the necessity of keeping stores or warehouses. For the same reason, their fees are smaller than those of the latter.

There are Money, Exchange and Bill, Stock, Customhouse, Real Estate and other Brokers.

MONEY, EXCHANGE AND BILL BROKERS.

96. They buy, sell and exchange specie, bills of exchange, notes, etc.

The entire business of this class is often performed by one individual or company. Being considered a branch of the banking business, many of them adopt the name of bankers.

Real Estate and *Stock Brokers* buy and sell for others, lands, houses, stocks in public funds and joint stock companies, etc.

Custom-house Brokers find employment in maritime cities, by assisting masters of ships in obtaining the necessary papers at the custom-house, and paying duties or taxes incident to the navigation of the ocean.

Ship and *Insurance Brokers* procure freights and cargoes for ships, adjust the terms of *charter parties*, settle accounts between owners and masters of ships, effect insurance on ships and cargoes, etc.

Produce Brokers buy and sell for others various kinds of farm produce, as corn, wheat, cheese, etc. They stand between the producer and dealer or shipper.

The business of other brokers will be sufficiently indicated by their names.

MERCANTILE AND COMMERCIAL COLLEGES

are institutions of learning, having for their ostensible object to prepare young men for entering the mercantile profession. They are got up by private enterprise, some of them being chartered and some not. The chartered institutions possess no advantages over the others, as none of them have the power of conferring degrees.

The course of study in this class of colleges usually comprises instruction in book-keeping, with its application to the various branches of trade, manufactures, etc.; mercantile arithmetic, penmanship and business correspondence; lectures on the usages of trade, negotiation of business paper and the most useful branches of commercial law.

When conducted with ability and integrity, commercial educational establishments rank among the most useful institutions of learning of the day. Though of comparatively recent origin, they are to be found in most of the larger cities of the Union, and receive a liberal patronage by all classes of the community. Professional men, mechanics, farmers, and, in large cities, ladies, are to be found among the number who consider their education unfinished until they have passed a commercial course in one of these, and merited a diploma.

It may be safely asserted that in no institution of learning is there as much useful information obtained in so short a time, and at such trifling expense, as in commercial colleges.

Business men may obtain a knowledge of book-keeping, as applied to their own business, in a few weeks, while most youths might be profitably engaged in a college at least one year.

PERSONS EMPLOYED IN BUSINESS HOUSES.

The persons employed in mercantile houses are: Book-keepers, Correspondents, Solicitors, Salesmen, Travelers; Entry, Bill, Shipping and Engrossing Clerks, Junior Clerks or Boys, Porters, Coopers and Draymen.

DUTIES OF THE VARIOUS OFFICES.

Book-keeper.—The book-keeper's place of business is in the counting-room. His duty is to keep the accounts of the establishment, in a variety of books for that purpose, to receive and pay out all moneys, and deposit money in the banks for safe keeping, to make out bills, render accounts, statements, etc., from the ledger and sometimes to conduct the business correspondence.

Correspondent.—The business of the correspondent is to reply to letters of inquiry, and to write all letters of business connected with the house, etc. In extensive importing houses, the correspondent is usually a person who understands some three or four of the modern languages.

The *Second Book-keeper* assists the first book-keeper. He copies or transfers to a journal or day-book the items found in the sales-book, journalizes and posts to the ledger. It devolves upon him to make out bills, accounts, and to assist in the counting-room generally.

The *Solicitor—traveling agent*—belongs to the broker's class. His business is to solicit orders and secure buyers for houses with which he has made previous arrangements. Accordingly he is found in the hotels, on the steamboats or at the railroad depots. When he finds a buyer, he conducts him to the store for which he is operating, and, if not under salary, receives a commission for his services.

Clerk is a general term applied to all employés, the porter, drayman and cooper excepted.

Salesman.—The duties of the salesman consist in unpacking, marking and arranging goods for sale, receiving customers and selling to them.

In some houses, the salesman receives a commission for the amount of trade he influences, and in all places the amount of his salary very much depends upon this circumstance.

These statements apply principally to wholesale business.

The *Shipping Clerk* receives and examines goods to see if they agree with the conditions of the bill of lading, and attends to the shipping of goods from the establishment. These he enters in a book for that purpose, called the shipping-book.

The *Entry Clerk* records the sales made by the salesman in a book called a blotter or sales-book.

The *Bill Clerk* makes out the bills or outward invoices from the sales-book.

Entry and bill clerks should be rapid penmen and expert in figures, if they would command liberal salaries.

The *Engrossing Clerk* assists generally, sometimes in the counting-room, but more generally as entry or bill clerk. He is simply a copyist.*

Junior Clerks are usually boys from 12 to 17 years of age, whose duty it is to run on errands, pack and unpack goods, mark boxes and packages, and keep sale-rooms in order.

After acting in the capacity of junior clerk for two or three years, they are promoted to more lucrative and responsible offices.

The youth who would aspire to a high degree of usefulness in his profession, should not rest content with the opportunities for improvement afforded by the store or sales-room. His evenings should be devoted to useful study, until he acquires proficiency in all the various branches of mercantile business.

The study of Freedley's excellent "Treatise on Business" is highly recommended, especially chap. iii, page 46.

Reading-rooms, libraries, mercantile associations, mercantile and commercial colleges are to be found in most of the larger cities of the Union, and should form the places of resort for young men of this class in preference to questionable places of amusement, too much frequented by them.

Porter.—The business of the porter is to open and close the store, keep the store and counting-rooms clean and in order, pack and unpack goods, assist in handling and weighing heavy goods, marking packages, etc.

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* In many houses, the whole business of clerking is performed by one person, while in others many more offices are called into requisition than are noticed here.

The office of porter is a more responsible one than most people imagine. By the faithful discharge of its duties, hundreds of men in this country have been placed in possession of respectable retail houses.

In some large establishments, there are two or more porters engaged, between whom the duties of the above are divided.

Cooper.—In liquor and heavy sugar establishments, packing-houses, etc., the services of a cooper are required, whose duty it is to open and close hogsheads, barrels, etc., and to repair damages to which such articles are subject in carrying.

Drayman.—The drayman acts as "carrier" between the store and depot, landing or wharf. He usually keeps a book called a dray-book, in which are entered the contents of each load. This is signed by the clerk at the place of delivery, and when the entire shipment is made the amount is entered in the *Bill of Lading.*

XI. BILLS—INVOICES.

97. WHEN goods are sold, it is the duty of the merchant, or one of his clerks, to make out a statement of the quantity, kind and price of each article, for the satisfaction of the purchaser, and to enter at the foot of such statement the whole amount of the purchase, with the payment received, if any, or the terms of settlement. If the goods are bought to sell again, this statement is commonly called an *Invoice;* otherwise it is called a *Bill*, especially by the purchaser.

A bill or invoice is sometimes delivered to the buyer at the time of purchase; but it is usually sent with the goods, or, if the buyer resides at a distance, by mail.

An invoice should specify the place and date of sale, the name of buyer and seller, a description of the goods, the price of boxes, etc., used for packing, charges for insurance, and, when payment is not made, the terms of sale.

When goods are received, the quality and quantity are compared with the invoice, and the selling prices made out from it, after which it is filed away or pasted in a book for future reference.

98. It is the custom of merchants to have their bill heads (heading of bills) printed, with names of city and street, number of house, and such other matter as will facilitate the labor of clerks, or otherwise advance the interests of the business. A specimen form of such heading will be given in the bills that follow.

99. *Filing Bills.* When there are many bills on hand designed for collection, they should be folded neatly of the same length and breadth, and have the names, addresses and amounts written on the outside at the top. A gum band will then keep them firmly in their places, and permit their being delivered without the trouble of opening for examination.

100. Retail bills are rendered periodically, by the month, quarter, half-year or year, according to agreement or the usage of the house. When a pass-book is not kept, it is well to have a memorandum of each purchase, so that in rendering his final bill the merchant need not insert the items.

101. *Account or Statement.* The final bill of a merchant now goes by the name of account or statement. The head contains the date upon which it is drawn, and the word "To" substituted for "Bought of." In the margin, on the left, are the dates of the several purchases, with the words, "For amt. rendered," or "Amt. pr. bill rendered."

102. In making out bills, the three requisites are rapidity, legibility and accuracy. The principal is accuracy.

In business, it is not enough to be right after one or two, or perhaps repeated, attempts. The clerk should be correct the first attempt, and generally is so. Boys designed for business pursuits ought to spend much time at bill-making, until they acquire familiarity with the numerous abbreviations, and can make out a bill from dictation al-almost as rapidly as the items can be called off.

103. Finding the cost of a number of articles at a certain price, and placing the amount opposite, is called, in bill-making, *extending;* adding the columns, *footing up.*

104. *Receipt on a Bill.* A clerk or agent may write the name of his employer to a receipt and it will be good, if he write his own initials or last name underneath.

BOOKSELLERS AND STATIONERS' BILLS.

CINCINNATI, *June* 16, 1866.

MR. HORATIO NELSON:

Bought of R. W. CARROLL & CO.,

PUBLISHERS, BOOKSELLERS & STATIONERS, WHOLESALE & RETAIL,
117 WEST FOURTH STREET.

TERMS: *In making orders, be particular to avoid mistakes.*
All claims for Errors or Damages to be made within five days of receipt of goods.

1 Gro. Pen-holders, ...		2	25	
1 " " "			75	
3 Doz. Paper-Cutters, ...	2.00	6	00	
2 " Ebony Rulers, ...	3.75	7	50	
1 Rm. Bill Cap, ...		7	00	
2 " Letter Cap, No. 1400, . .	8.25	16	50	
1 2-Qr. Blank Book, ...	40		80	
3 Qr. Blotting Paper, ...	2.25	6	75	
				$47.55

Rec'd Paym't.

R. W. CARROLL & Co.
DAVIS.

Gro., Gross; *Doz.*, Dozen; *Qr.*, Quire; *Rm.*, Ream.

BOOKS—STATIONERY.

105. Books, stationery, etc., at wholesale, are usually sold by the dozen. Paper is sold by the ream, bundle or pound. Writing paper is put up in half-reams; printing paper in bundles of two reams.

In the exercises which follow, the teacher will find it to the advantage of scholars to have them write from dictation. The bills may be made out in favor of the learner or otherwise.

3. $\frac{1}{4}$ dz. Hooker's Nat. Philos., \$14.40; 1 rm. cap. \$6.50; 1 do. bill, \$7.00; 2 dz. 12-in. ebony rulers, \$3.75; 2 dz. paper-cutters, \$2.00; 1 bx. crayons, 35 c; 12 dz. pass-books, 40 c

4. 2 dz. mucilage, \$2.75; 1 dz. carmine, \$2.00; 2 dz. tin cutters, \$2.00; 2 dz. rulers, \$4.50; $\frac{1}{2}$ rm. natl. cap, \$5.50; 3 dz. No. 4 pass-bks 44 c $\frac{1}{2}$ rm. treas'y cap, \$8.40; 1 qr. blotting, \$2.25.

5. 1 bx. 4560 $5\frac{1}{2}$ envelopes, \$2.25; 1 do. $8\frac{1}{2}$ Manilla, \$2.75; 1 dz. Lincoln portraits, \$3.00; 2 dz. check-books, \$12.24; 1000 penman's blanks, \$20.50; $21\frac{9}{12}$ dz. bill-books, \$2.16; 4 dz. do. 2 qr. ea., \$2.50; 20 dz. check, \$2.16; 20 dz. inv., \$1.00; 20 dz. day-books, \$1.80; 60 dz. journals, \$1.80; 20 dz. ledgers, \$1.80; sunds., \$7.00. Credit—cash, \$210.

6. 1 copy'g press, \$10.00; $\frac{1}{4}$ dz. Rec. of a Country Parson, \$20; $\frac{3}{12}$ dz. Wordsworth's Poems, \$25.00; $\frac{7}{12}$ dz. Beginning Life, \$12.50; $\frac{5}{12}$ dz. Heads and Hands in the Wrld of Labor, \$12.00; $\frac{11}{12}$ Essay on Woman's Work, 12.50; $2\frac{3}{12}$ do. Nat'n'l Lyrics, \$4.80.

Answers: \$55.80, \$30.21, \$33.75, \$31.02, \$150.16.

SHOE BUSINESS.

106. Shoes are usually sold at wholesale by the dozen, boxes furnished gratuitously.

Pairs.		
7. 12	Wos. Goat Tips Bals.	1.45
6	Miss " " "	1.75
12	Childs " " "	1.05
12	" Calf "	1.05
12	Wos. Goat Tips "	2.50
10	" Last. Lace Cong.	2.00
6	" " But. Gait.	2.90
6	Miss Kid Cong. "	1.90

Numbers on boxes.	Pairs.		
8. S	12	Child Serge Lace Heel'd Gaiters.	65
474	13	Ladies Kid ch. nl. Balmorals	1.30
533	12	" Goat D. S. "	1.28
2386	11	Wos. Kid Cong. D. S. Boots	$1.37\frac{1}{2}$
2449	12	" Serge Cong. Gaiters	1.20
2593	12	" " " "	$1.12\frac{1}{2}$
2475	24	Child Gai. Peg-Heel'd J. L. Boots	45
2586	12	" End. " " cop. tip.	45
2580	24	" Buff " Lace " "	40
2575	12	Miss Gai. " J. L.	$67\frac{1}{2}$
2413	15	" Goat " Cong Gait...	95
2517	12	" " " " ...	60
2591	9	Ladies Goat, tip. ch. nl. Bals	2.00
2431	12	Youths Buff Peg-H'l'd "	$72\frac{1}{2}$
2461	12	Ladies Calf " tip Bals..	1.25
2588	12	" Kid ch. H'l'd " ..	1.50
2589	12	" Peb. Calf ch. H'l'd tip Bals	1.50
2590	12	Miss Kid Sd. Welt "	1.05

ABBREVIATIONS.— *Wos.*, Womans; *D. S.*, Double soled; *Cong.*, Congress; *Grd.*, Grained; *End.*, Enameled; *J. L.*, Jenny Lind; *Cop. Tip.*, Copper tipped; *Ch. Nl.*, Channel nailed; *Hld.*, Heeled; *Sd. Wlt.*, Sewed Welt; *Peb. Cf.*, Pebbled calf; *Bals.*, Balmorals; *But. Gait.*, Button Gaiters.

MILLINERY.

9.	60	12 15-Braid Bonnets...........	@	$0.62½
	68	6 Swiss Straw Bonnets........	"	1.25
	70	4 7-Braid "	"	1.50
	80	2 7 " "	"	3.00
	86	2 7 " "	"	3.75
		6 Pcs. No. 1 Tafft. Ribbon....	"	15
		5 " " 2 " "	"	28
		3 " " 4 " "	"	48
		2 " " 6 " "	"	75
		1 " " 12 " "	"	1.10
		3 " Bonnet Ribbon.........	"	2.00
		2 " " "	"	2.50
	3	1 Box Ruches................	"	1.50
	415	1 " "	"	2.25
	210	⅓ Doz. Bunches Flowers.......	"	18.00
		½ " " Feathers......	"	36.00
		1 Pc. Black Silk, 20 yds......	"	87½

GROCERY BUSINESS.

107. Tares.—*In Cincinnati:* New Orleans sugar, in hogsheads, 10%; Cuba and Porto Rico, 12%; sugar in boxes, 15%. Rice in tierces, 10%; indigo in ceroons, 11%; in boxes, actual tare; salt in barrels, each 30 lbs.; coffee, cotton, spices, feathers and salt, in bags and bales, no tare; manufactured tobacco in kegs and boxes, (not enumerated,) actual tare; madder in casks, actual tare; lard and bacon in packages, actual tare. Lard kegs tared after being emptied.*

* Butter in firkins is subject to an allowance of 2 lbs. for soakage; in rolls packed in half barrels, 1 lb.; barrels, 2 lbs.; tubs—50 to 70 lbs.—1 lb.

In New York: The tares differ slightly from the above; but the list being a long one, it is reserved for the revised edition of "Nelson's Mercantile Arithmetic."

Extra Charges may be made for drayage, insurance, cooperage, storage, boxes, bags, etc., in which goods are packed in the store. No extra charge is made for the original package. In the bills which follow, some of these charges are introduced.

10.	1 Hhd. N. O. Sugar,	$\frac{*1200}{\dagger 120}$	**1080 lbs..	@$.07
	4 Brls. N. O. Molasses,	$\begin{matrix}\ddagger 44\\42\\42\\41\end{matrix}$	169 gals....	" 35
	1 Trs. Rice,	$\frac{700}{70}$	630 lbs.............	" 4
	20 Bags Rio Coffee,		3200 lbs..........	" 11
	2 Half Chests Black Tea,	$\frac{\begin{matrix}*50-14\dagger\\50-14\end{matrix}}{100-28}$	72 lbs.	" 25
	3 " " Yng. Hyson do.		150 lbs..	" 50
	1 " " Imperial do.		60 lbs..	" 40
	2 " " Gunpowder do.		110 lbs..	" 60
	1 " " Oolong Blk. Tea,		45 lbs.	" 40
	6 Doz. ground Cinnamon...........			" 40
	1 Box 5 lump Tobacco,	$\frac{132}{24}$	108 lbs..	" 25
	1 " pound lump "	$\frac{147}{23}$	124 lbs..	" 20
	1 " Va. pound "	$\frac{140}{20}$	120 lbs..	" 35
	1 " 8 lump "	$\frac{150}{25}$	125 lbs..	" 22
	20 Brls Rect. Whisky		800 gals....	" 17
	4 " Ginger Wine,		160 gals....	" 60
	¼ Cask French Brandy,		40 gals....	" 4.00
	¼ " Port Wine,		45 gals....	" 2·00
	10 Brls. Bourbon Whisky,		405 gals....	" 1.00
	½ Brl. Holland Gin		20 gals....	" 1.50
	5% off..............................			

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* *Gross Weight.* ‡ Gallons in each barrel.

† *Tare,* or weight of bag, box, etc. ** *Net Weight.*

The small figures on the left indicate the prices of boxes, barrels, etc.

11.	100 Boxes Cheese, $^{4}{}^{100}_{410}$ 3690.......	@ $ .08	
	30 Firkins Butter, $^{3}{}^{360}_{540}$ 2820.......	" 15	
	100 Boxes $^{\$20}$ Starch, 4810..........	" 5	
	100 " $^{\$25}$ Star Candles, 4000.....	" 20	
	20 Bbls. $^{\$25}$ Lard Oil, 810 gals.....	" 85	
	50 " Mess Pork................	" 16.00	
	10 Tierces S. C. C. Hams, $^{3}{}^{350}_{350}$ 3000.	" 11	
	30 Kegs Lard, $^{1}{}^{710}_{376}$ 1334..........	" $12\frac{1}{2}$	
	15 Bbls. Mess Beef................	" 15.00	
	Com. for purchasing, $1521.75.....	" $2\frac{1}{2}\%$	
	Drayages..............................		16.00
	Insurance on $5000....................		59.88

12.	1 bag Pepper, 103..........	$10\frac{1}{2}$		
	1 " Allspice, 128.........	$10\frac{1}{2}$		
	4 dz. Shakers' Brooms......	2.40		
	1 Rm. Cap Paper...........	4.00	10% off.	
	1 " Med "	6.00	" "	
	1 " D. C "	8.00	" "	
	5 bxs. ^{10} Ger. Ex. Soap, 297.	7		
	1 Keg Soda, 112 lbs.........	$5\frac{1}{4}$		
	1 bx. ^{20} Saleratus, 61 lbs....	$5\frac{1}{2}$		
	1 " ^{20} Saltpeter, 47 "	$9\frac{1}{2}$		
	Drayage................................			.79

13. 10 Bbls. $^{2.50}$ Sugar,

246	23	245	20
233	18	246	17
250	21	275	21
227	22	232	19
239	21	266	25
		2459	207=2252 lbs.@$12\frac{1}{2}$c, $

Drayage.................................. 1.60

14.	1 Bbl. ^{125} Lard Oil, 41	85	
	1 Hf. bbl. ^{75} 98% Alcohol, $21\frac{1}{2}$	54	
	1 Dz. Washboards	2.25	
	1 ^{10} Sk. Bar Lead, 96 lbs	$6\frac{1}{2}$	
	1 Pkg. Yarn, 28 lbs	26	
	2 Doz. No. 2 Brooms	2.40	
	1 Trc. Ex. S. C. Drd. Beef $^{206}_{32}$	25	
	1 Keg 10d Nails	3.00	
	$\frac{1}{2}$ Bbl. ^{50} Lard Oil, 23	95	
	Drayage		75

15.	2 Cases H. Cheese, 46 lbs	28
	4 " S. W. R. Cheese, $^{124}_{12}$ 112 lbs..	22
	3 " E. D. " $^{87}_{9}$ 78 " ..	18
	2 " N. W. R. " $^{23}_{2}$ 21 " ..	17
	1 Bbl. Cuba sugar, $^{223}_{21}$ 202 lbs	$13\frac{1}{2}$
	4 Boxes MR. Raisins	4.40
	$2\ \frac{1}{2}$ " " "	2.40
	$2\ \frac{1}{4}$ " " "	1.45
	3 Jars Prunes	45
	5 Drums Figs, $37\frac{1}{2}$ lbs	24
	1 Case 100 half boxes Sardines	42
	1 Box American Castile Soap, 28 lbs..	16
	1 " French " " 35 " ..	16
	1 Doz. Cox Sparkling Gelatine	6.00
	2 Boxes ^{20} Saleratus, 128 lbs	$7\frac{1}{2}$
	5 Half boxes Star Candles, 100 lbs	24
	1 Bbl. ^{25} Com. Smok. Tobacco, 100 lbs	12
	1 " ^{25} S. F. " " 83 "	25
	2 Boxes ^{40} Starch, 96 lbs	7
	2 " Soda, 92 "	13

Sk., sack; Pkg., package; W. R., Western Reserve. $2\ \frac{1}{2}$ boxes raisins, 2 *half boxes.*

16.	1	Bbl. ^{25} Soft Refined Sugar, 240 lbs..	@	16	
	1	" ^{25} Hard " " 220 " ..	"	17	
	1	" ^{25} Granulated " 212 " ..	"	25	
	1	Sack Java Coffee, 136 lbs...........	"	41	
	1	" Rio " 156 "	"	28	
	1	" Laguayra, 111 lbs.............	"	30	
	$\frac{1}{2}$	Chest Y. H. Tea, 82—14=68 lbs...	"	1.25	
	$\frac{1}{2}$	" Black " 53—12=41 lbs...	"	1.35	
	$\frac{1}{2}$	" Y. H. Canton Tea, $^{85}_{17}$ 68 lbs...	"	90	
	$\frac{1}{2}$	" " Fine, $^{80}_{14}$ 66 lbs........	"	1.60	
	$\frac{1}{2}$	" Oolong Common Tea, $^{55}_{14}$ 41 lbs.	"	90	
	1	Sack African Peanuts, 2 bushels....		2.40	
	1	" Roasted do., 3 bushels........		3.10	
	1	Bbl. ^{25} Dried Peaches, $^{114}_{19}$ 95 lbs...	"	21	
	1	" ^{25} " Apples, $^{105}_{20}$ 85 lbs....	"	15	
	1	Box ^{20} German Soap, 64 lbs.......	"	$12\frac{1}{2}$	
	1	" ^{20} Common " 64 lbs.......	"	12	
	1	" ^{20} Cincinnati Extra do., 61 lbs..	"	14	
	1	Bbl. ^{25} Prm. Wh. Beans $^{235}_{19}$ 216 lbs.	"	$2\frac{3}{4}$	
	2	Boxes 2 doz. Baking Powder.......	"	4.80	
17.	2	Sks. Rio Coffee, 323...................		.25	
	1	Bbl. Molasses, 45— $\frac{1}{2}$..............		.60	
	1	" ^{37} Rice, 235—19...............		.10	
	2	" ^{75} Sugar, $^{247—20}_{248—18}$..............		.12	
		Drayage..............................			50

Discount.—An abatement entitled discount is often made on the bill for cash or when goods have fallen in price. When making such abatements, the clerk should remember that he is discounting the profits as well as the first cost. For instance, I buy goods at \$100, and sell them at a profit of 50 per cent., which makes the price \$150. Now, if this is discounted at 40 per cent., it does not follow that a gain of 10 per cent. is made. 40 per cent. of \$150= \$60, which, taken from the selling price, leaves \$90, making, instead of a gain of 10 per cent., a loss to that amount.

18. 20 Bbls. Molasses,

$43\frac{1}{2}$	44	43
$44\frac{1}{2}$	$42\frac{1}{2}$	$42\frac{1}{2}$
$44\frac{1}{2}$	41—1	$44\frac{1}{2}$
$43\frac{1}{2}$	44	$44\frac{1}{2}$
44	$42\frac{1}{2}$	$42\frac{1}{2}$
43	$43\frac{1}{2}$	41—1
44	43	

$865\frac{1}{2}$—*2@75c

Drayage.............................. 2.00

19. 10 Tcs. Shaffer's Hams $^{3215}_{357}$.......... $0.21

5 " A. Spence's S. C. do. $^{1772}_{199}$...... 25

1 Bbl. ^{25} 20 dz. Tongues............ 1.00

Com. on $1013.78............... $2\frac{1}{2}$%

Exch......................... $\frac{1}{2}$%

Drayage.......................... 2.00

Answers: $86.24, $262.26, $1046,23, $138.73, $285.60, $185.51, $4152.87, $1665.97, $649.63, $688.75, $131.90, $228.74, $130, $97.09.

NOTE.—When merchants can not fill the orders from their store, they sometimes buy elsewhere, and charge their customer cost and a commission, as in the last bill.

The Teacher should not confine himself to the few exercises in bill-making given in this work, as he will find it both interesting and profitable to the learner to represent other kinds of business in the same way. Any number of bills can be dictated from price lists or prices current, to be had in any large city.

*Leakage.

DRY GOODS.*

			Yds.	Price	
20.	1 Pc.	H. A. $\frac{7}{8}$ Blea. Muslin	45	25	
	1 "	A $\frac{4}{4}$ Light "	43	15	
	1 "	D. Q. $\frac{7}{8}$ Fine Bro. Muslin	43	22	
	2 "	P. $\frac{4}{4}$ Muslin	70^{2}	24	
	1 "	L. Check	68^{1}	26	
	1 "	W. Jeans	46^{2}	30	
A	3 "	Fcy Prints	126^{1}	15	
B	1 "	" "	37^{2}	16	
C	9 "	" "	355^{2}	17	
	1 "	Buff Chambray	20^{2}	35	
	1 "	Dom Gingham	35	25	
	1 "	L "	60^{2}	25	
	2 Doz.	Coats' Spools		1.10	
	1 "	$\frac{1}{2}$ Hose			3.00
	1 "	Ladies' L. W. Hose			6.75
No. 4	1 Pk.	Pins			70
3	1 "	"			75
2	1 "	"			90
	1 Doz.	Lin. Hdkfs			6.00
	1 "	H. S. "			9.50
†	1 Pc.	Sheetgs to fill	35^{2}	24	
		2 Bxs ^{50} and Strapping ^{50}			2.00

*Though practically correct, some of the answers to these bills will be found mathematically wrong. Accuracy in cents has been sacrificed in conformity with business usage, which often rejects fractions in extensions, alternately adding a cent and rejecting a half or fourth. The letters "H. A." etc., indicate the grade of muslins, or are the initials of the maker or factory; the figures and letters in the margin are marked in the wholesale house to distinguish lots of nearly the same kind from one another.

†Not in the order; but will not be objected to by the buyer, as it prevents goods from shifting in the case.

21.	4 Pcs.	Prints........................	185^3	17	
	1 "	Jeans	45^1	30	
	1 "	Cottonade....................	38^1	37^2	
	5 "	Chamb. Gingham............	154^2	35	
	2 "	Luster	101^2	30	
	2 Doz.	Skirt Braid.................		1.00	
	10 "	Coats' Spools		1.10	
No. 10	1 "	Miss's Hoops...............			5.00
18	1 "	" "			9.00
20	1 "	" "			10.00
30	½ "	2 Gore Miss's Hoops......		26.00	

22.	1 Pc.	Fancy Cassimere.........	$27\frac{3}{4}$	2.20	
	1 "	French "	$28\frac{3}{4}$	2.85	
	1 "	Mixed "	31	2.90	
	1 "	Amoskeag A Ticks.......	60	47	
	1 "	Glassgow Gingham.......	$40\frac{1}{2}$	27	
	1 "	Victoria "	37	42	
	1 "	Shawmut Can. Flannel....	$35\frac{1}{4}$	34^2	
	3 "	I. X. L. Wl. do. 41, 64, 34..	139	73	
	1 "	Roan. Striped Shirting....	$37\frac{1}{2}$	31	
	2 "	Fcy Slk Dress Goods, $^{40}_{31}$..	71	1.10	
	1 "	Village Green Checks.....	$37\frac{3}{4}$	23	
	4 "	Lonsdale Checks $^{34}_{31}$ $^{38}_{40}$...	143	21	
	1 "	Blk French Broadcloth...	$29\frac{3}{4}$	4.25	
	1 "	English " ...	$37\frac{1}{2}$	5.00	
	2 "	Bolivar Denims $^{50}_{48}$.......	98	20^2	
	1 "	Charter Oak Denims......	60	22	
	1 "	Wamsutta Prints.........	$38\frac{3}{4}$	17^2	
	2 "	Duchess Delaines, 31^1 36^1..	$67\frac{1}{2}$	30	
	4 "	Semper Idem, 40 38 27^1 36.	$115\frac{1}{4}$	32^2	
	2 "	Barre Brown Sheeting $^{40}_{36}$..	76	34	
	1 "	Great Falls do. do.........	$42\frac{1}{4}$	28	
		2 Cases and Strapping..........			2.50

23.	75	48	Doz. Gent's Shakspeare Coll.....	2.00
	6	60	" Ladies Plea "	1.26
	1	48	" " Byron Emb. "	1.50
	1	45	" " Gar. " "	1.50
	1	48	" " Per. " "	1.50
	1	120	" " Sq. Gar. C. & C. E. Coll.	1.00
	1	29	" " Mull. Edge Gar. " .	1.50
	1	20	" " Vic. Cuffs, 2 C......	2.50
	1	2	" " Octagon C. & C. E. Sets	4.00
	1	5	" " Sqr. Emb. "	5.50
	1	8	" " Point Wht. Trim. "	4.50
	16	47	" Shirt Fronts..............	3.50

24.	11	10	Pcs. $\frac{4}{4}$ Shirting Linen......	294	$0.37
	12	10	" $\frac{4}{4}$ " "	297	40^2
	13	10	" $\frac{4}{4}$ " "	300	44
	14	15	" $\frac{4}{4}$ " "	433	49
	15	15	" $\frac{4}{4}$ " "	447	55
	16	10	" $\frac{4}{4}$ " "	285	60
	17	10	" $\frac{4}{4}$ " "	287	85

25.	1342	2	Doz. Napkins..................		2.75
	1343	2	" "		3.40
	1356	2	" Huck Towels.............		4.50
	1270	2	" " "		4.00
	1138	4	Felt Table Covers............		2.85
	1416	1	Pc. Irish Linen, No. 2.....	26^2	57^2
	1417	1	" " " " 3.....	26	62^2
	1418	1	" " " " 4.....	26	70
	1419	1	" " " " 5.....	27	75
	1421	1	" " " " 7.....	26	87^2
	1422	1	" " " " 8.....	27	1.00

Emb., Embroidered; *Gar.*, Garote; *Per.*, Persigney; *C. & 2 C.*, Cord and 2 Cord; *Vic.*, Victoria.

QUEENSWARE.

26.

3	Doz. Edged Plates....................	$0.40
10	" " "	50
5	" *CC "	50
½	" " Dishes, ea. $1.75, 2.25, 2.75.	
1	" " Bakers, ea. $1.50, 2.00, 2.50.	
1	" " Beaded Nappies, ea. 1.75, 2.25	
½	" " Tureens	3.50
2	" " Bowls	80
2½	" " "	60
3	" " "	50
½	" " Pitchers	3.50
1	" Colored " ea. $2.50, $4.00....	
4	" " Bowls....................	87^{2}
5	" " "	65
6	" " "	55
9	Sets CC Teas......................	20
36	" Painted Teas....................	20
	Crate..............................	1.00

HAT BUSINESS.

27.

½	Doz. Men's Black Cass. Hungarian.....	21.00
⅓	" " " " "	27.00
5½	" " " "	33.00
¼	" " " Broad Brim Wool...	14.00
1	" " " Wove Senate.......	12.00
¼	" " " Cashmerette........	15.00
½	" Boys' " Hungarian.........	7.00
½	" " Caps, assorted............	12.00
½	" " " "	9.75
¼	" " Cloth Caps...............	9.00
¼	" Children's Fancy Caps...........	8.00

*CC, cream colored.

HARDWARE.

28.

25 Bars $1\frac{1}{2}\times\frac{1}{2}$ Bar Iron	750	lbs.	$0.03	
6 Bund. $\frac{5}{8}$ Round "	625	"	4	
4 " $1\frac{1}{4}\times\frac{3}{8}$ Dandy Tire Iron..	500	"	3^{3}	
3 " $1\times\frac{3}{8}$ Horseshoe Iron				
4 " $1\times\frac{7}{16}$ " "	975	"	4	
2 " $1\times\frac{1}{2}$ " "				
2 Sets $1\frac{1}{2}\times4$ L. Pitts Springs				
3 " $1\frac{3}{4}\times5$ " " "	500	"	10^{2}	
2 " 2×5 " " "				
1 " 2×8 " " "				
4 Slabs $14\times\frac{3}{8}$ Steel	600	"	7^{2}	
3 Doz. Amos No. 2 Shovels				10.50
2 " Rowland's No. 2 Shovels				8.50
1 " " No. 2 Spades				8.50
6 " No. 5684 Pocket Knives				7.50
3 " " 4215 " "				4.50

Answers: $238.88, $160.39, $56.73, $194.92, $1234.04 $832.60, $232.53, $981.51, $318.25, $735.63.

MERCHANT TAILOR'S BILL.

29. New York, *Ap.* 5, 1867.

Mr. A.,

To B.

Jan. 3. For 1 Blue Beav. Overcoat......	70.00	
Feb. 9 " 1 Pr. Blk Doeskn Pants.....	23.50	
		$93.50
Mar. 2. Cr.—By Cash		50.00
		$43.50

To the Teacher.—The learner ought to be taught that bills are seldom punctuated, and that even the dollar or cent signs or @ are rarely used. It is proper, however, to insert the dollar sign at footings.

FARMER'S BILL WITH CREDITS.

RICHMOND, *Nov.* 1, 1869.

MESS. TRADER & Co.,

To JONATHAN FARMER.

Jan. 3.	For 3 Durham Heifers..........	25.00	75.00
" "	" 1 2-yr. old Bay Colt........		50.00
Ap. 11.	" 140^2 Bu. Corn..............	50	70.25
" "	" 5^2 Dz. Chickens............	*1.37^2	7.56^1
9.	" 123^2 lbs. Butter............	18^3	23.15^2
	Cr.		$225.97
Ap. 4.	By 25 Hogs, 3147 lbs 3^2........	110.14^2	
9.	" 1 Plow......................	7.62^2	
	" Cash.......................	108.20	$225.97

JONATHAN FARMER.

MECHANIC'S BILL.

ST. LOUIS, *July* 3, 1867.

MESS. BUREAU & Co.,

To JOHN WORKMAN.

Jan. 1.	For Cab't W'k on 3 dz. hf-Fr. Bed's.	20.00	60.00
Feb. 9.	" " " " 4 " 2-pn'l do...	30.00	120.00
	Cr.		180.00
Jan. 1.	By Cash on Acc.................	10.00	
" 8.	" Order on W. Shoemaker†.....	15.00	
" 15.	" Cash	20.00	45.00
	Settled by note at 90 ds...............		$135.00

JOHN WORKMAN.

When the signature of the merchant is required to a part payment, the phrase *Balance due* should be written opposite the amount unpaid.

* Express the 137^2 in mills before multiplying.

† For form of order and note see index.

Hf. Fr. Bd's., half French bedsteads; *pnl.*, panel.

DRESS-MAKER'S BILL.

Boston, *Sep.* 9, 1866.

Mrs. Affluent,

To M. E. Fashion.

For Making 1 Moire Antique Dress.............	25.00
" Trimming do..................................	75.00
	$100.00

XII. COMPOUND NUMBERS.

Besides the distinction made between numbers in chapter viii, they may be divided into *simple* and *compound.*

108. A *simple number* may be abstract or concrete, of one denomination, as 27 men, 35 dollars.

109. A *compound number* is always concrete and composed of more than one denomination, as 157 dollars 50 cents, 29 pounds 14 shillings and 6 pence—two numbers, each expressing *one sum* of money.

110. *Reduction of compound numbers.*

111. Reduction is the process of changing concrete numbers of one denomination to those of equal value in another. If I multiply 5 bushels by the number of pecks in a bushel, I *reduce them* to pecks, and thus change both the number and denomination, while I preserve the value.*

112. Changing the denomination from a higher to a lower, as bushels to pecks, is called *reduction descending;* while the reverse process, as changing pecks to bushels, is called *reduction ascending.*

A few examples will suffice to teach all that is necessary to be known on this subject.

*On page 70 are several exercises which properly belong to this subject.

1. To reduce 35 acres to, or represent them, in square feet.

35	*Acres*
4	
140	*Roods*
40	
5600	*Sq. Rods*
$30\frac{1}{4}$	
168000	
1400	
169400	*Sq. yards*
9	
1524600	

EXPLANATION.—In 1 acre there are 4 roods, in 35 acres there are 35 times 4 roods (35×4) or 140 roods; in 1 rood there are 40 square rods, in 140 roods there are 140×40 or 5600 sq. rods; in 1 sq. rod there are $30\frac{1}{4}$ sq. yards, in 5600 sq. rods there are $5600\times30\frac{1}{4}$ or 169400 sq. yards; in 1 sq. yard there are 9 sq. feet, in 169400 sq. yards there are 169400×9 or 1524600 sq. feet.

REMARK.—It will be observed that the same result would have been produced by multiplying the 35 by 43560, the number of feet in an acre.

When there are items between the highest and the lowest denominations, they should be added, as shown in the following example:

2. In £75, 13s, $6\frac{1}{2}$d how many farthings?

75	13 $6\frac{1}{2}$
20	
1500	*Shillings in £75.*
13	*Shillings added.*
1513	*Whole number of shillings.*
12	
18156	*Pence in 1513 shillings.*
6	*Pence added.*
18162	*Sum of pence.*
4	*Farthings in 1 penny.*
72648	
2	
72650	*Farthings in $18162\frac{1}{2}$ pence.*

To add the items mentally, the 3 shillings would occupy the place of the first 0; then, multiplying the 5 by the 2, we would obtain 10 to which might be added the 1, making 11; then, multiplying the 7 by 2, we get 14 and the one carried, making 15 or 1513 at once.

3. In 3 miles 21 rods how many yards?

4. In 145 tons 25 lbs. of hemp how many pounds?

5. How many pence in £197, 17s, 9d?

6. How many farthings in £57, 13s, $6\frac{1}{2}$?

7. In 93 barrels of apples how many pecks, each barrel containing 2 bushels 3 pecks?

Answers: 324825, 55370, 47493, 1023, $5395\frac{1}{2}$.

113. To reduce concrete numbers of a lower denomination to those of a higher, the process will be the reverse of the last.

8. Reduce 72650 farthings to pounds.

4)72650 *Farthings.*	In farthings there will be one fourth as many pence; in pence one twelfth as many shillings; and in shillings one twentieth as many pounds. The remainders are 13 shillings, 6 pence and 2 farthings or $\frac{1}{2}$ penny.
12)18162—2 *Pence.*	
2|0)151|3—6 *Shill.*	
£75 13 $6\frac{1}{2}$	

9. Reduce 4163 linear inches to yards.

10. Express 31456739 minutes in years, months and days, allowing 365 days 6 hours to the year.*

11. Reduce 456372 farthings to pounds.

12. At 1 mile in $4\frac{1}{2}$ minutes, how many miles would a locomotive run in 5 hours?

13. A ship sailed 3000 miles in 16 days; what was her average speed per hour?

14. In 35 cubic yards how many cubic inches?

15. In $5\frac{1}{2}$ square rods how many square feet?

16. Divide a log 55 feet in length into 15 equal parts, and express the result in feet and inches.

---

*Reduce the year to minutes and divide them into the minutes in question, which will give the number of years. The remainder being minutes, may be reduced to hours, etc., as in reduction.

17. What will be the cost of $35\frac{3}{4}$ bushels of strawberries at 15 cents a quart?

18. At 5 cents for 3 sheets of paper, how much money can be obtained for $35\frac{1}{2}$ reams, allowing half price for the outside half quires ($35\frac{1}{2}$ qrs.) of each ream?

19. A merchant buys 15 barrels of potatoes, containing 2 bushels $2\frac{3}{4}$ pecks each, at $1.25 per bushel, and sells them at 25 cents a half peck; how much does he make, allowing $\frac{1}{2}$ a peck per barrel for loss in measuring?

20. In 316754 lbs. of hemp, how many tons, cwt., etc.?

Answers: 141, 8, 18; 17160; 3, 8; $66\frac{2}{3}$; 945; 27690; 59, 9, 25, 4, 59; $49\frac{1}{2}$; 475, 7, 9; $7\frac{13}{16}$; $26.48\frac{7}{16}$; $1497\frac{3}{8}$; 1632960.

114. *To add compound numbers.*

1. What is the amount of the following sums of British money?

£	s.	d.
18	17	$4\frac{1}{2}$
19	6	$7\frac{1}{4}$
17	7	$8\frac{3}{4}$
55	11	$8\frac{1}{2}$

SOLUTION.—1. We first add the fractions, calling them *farthings*, which makes 6 farthings; these we reduce to pence by dividing them by 4. $\frac{6}{4}=1\frac{2}{4}$ or $1\frac{1}{2}$. Write $\frac{1}{2}$ and add the 1 penny to the column of pence, which makes 20 pence; this number divided by 12 (the number of pence in a shilling)=1 shilling and 8 pence. Write the 8 under the pence, and add 1 to the units of the shillings' place, which makes 21; write 1 and add the 2 to the ten's column=3 or 31 shillings, which, divided by 20=£1 and 11 shillings left. Write the latter under the shillings and add the 1 pound to the pound's column=£55. *Ans.* £55, 11s, $8\frac{1}{2}$.

Add the following:

2. £17 18 $11\frac{3}{4}$+ £14 17 $2\frac{1}{2}$+ £16 14 8 =?
3. £17 19 $0\frac{1}{4}$+ £45 0 $11\frac{3}{4}$+£111 10 $2\frac{1}{2}$=?
4. £116 16 6 +£320 14 $5\frac{1}{4}$+ £38 18 8 =?

5. £24 18 6 + £180 10 $0\frac{3}{4}$+ £66 19 $11\frac{3}{4}$=?
6. £175 19 $7\frac{1}{4}$+ £90 8 $8\frac{3}{4}$+£575 12 $6\frac{1}{2}$=?
7. £201 17 $6\frac{3}{4}$+£1010 10 $10\frac{1}{4}$+£970 19 $11\frac{3}{4}$=?

Totals, £3297 17 $9\frac{3}{4}$ £700, 10s, 8d.

115. *To subtract compound numbers.*

1. From £19, 4s, 4d take £14, 7s, $6\frac{1}{4}$d.

£	*s.*	*d.*
19	4	3
14	7	$6\frac{1}{4}$
4	16	$8\frac{3}{4}$

Explanation.—We can not take $\frac{1}{4}$ from nothing, so we add a penny to both terms; subtracting $\frac{1}{4}$ from the 1 penny, or 4 fourths, we have $\frac{3}{4}$ left. Adding 1d to the 6d we have 7d, which we can not subtract from the 3d above, and accordingly add 1s to both numbers. 7 from 1s 3d or 15d, leaves 8d. Adding 1s to the shillings, we have 8s, which can not be taken from 4s without adding £1 to both numbers; £1 to 4s=24s; 8s from 24s=16s. Then adding £1 to the 14, we have £15, which, taken from £19=£4, making the answer £4, 16s. $8\frac{3}{4}$d.

Subtract the following:

	£	*s.*	*d.*		£	*s.*	*d.*	
2.	17	10	$8\frac{1}{4}$	—	14	5	3	=?
3.	119	7	6	—	17	19	$5\frac{1}{2}$	=?
4.	500	0	0	—	20	18	8	=?
5.	176	14	$7\frac{1}{4}$	—	129	15	$7\frac{1}{2}$	=?

Total, £630, 13s, $9\frac{1}{2}$d

116. *To multiply compound numbers.**

6. Multiply £17, 4s, $9\frac{1}{4}$ by 8.

Operation.	£17	4	$9\frac{1}{4}$
			8
	£137	18	2

After performing operations in addition, the learner will readily see how this is done.

7. £ 17 18 $8\frac{1}{2}\times$ 7=?
8. £120 16 $6\frac{1}{4}\times$12=?
9. £365 0 $7\frac{3}{4}\times$ 9=?

Total, £4860, 15s, $0\frac{1}{4}$d.

10. £ 48 9 $6\frac{1}{2}\times$2 and 3=?
11. £145 8 $7\frac{1}{2}\times$8=?
12. £705 13 $9\frac{3}{4}\times$6=?

Total, £5639, 19, 6.

---

* Multiplication may likewise be performed by reducing the compound number to one denomination. (See Reduction.)

117. *When the multiplier exceeds* 12 *and is a composite number*, or otherwise.

13. £48, 9, $6\frac{1}{2}$d$\times$24=?

£	s.	d.
48	9	$6\frac{1}{2}$
		3
145	8	$7\frac{1}{2}$=3 *times the amount.*
		8
1163	9	0 =8 *times* 3 *or* 24 *times the amount.*

EXPLANATION.—Here we multiply by the two factors of which the 24 is composed.

14. Multiply £705, 13s, $9\frac{3}{4}$d by 38.

£705	13	$9\frac{3}{4}\times 2$
		6
4234	2	$10\frac{1}{2}$=6 *times the amount.*
		6
25404	17	3 =6 *times* 6 *or* 36 *times the amount.*
1411	7	$7\frac{1}{2}$=2 *times the amount.*
26816	4	$10\frac{1}{2}$=*Sum of* 2+36 *times or* 38$\times$*the amount.*

15. £19 6s 7 d$\times$ 84=?
16. £91 18s $5\frac{1}{2}$d$\times$ 89=?
17. 4s $7\frac{1}{2}$d$\times$129=?
18. £27 8s 8 d$\times$ 87=?
19. £77 17s $7\frac{3}{4}$d$\times$ 95=?
20. £89 17s 6 d$\times$150=?

21. £1 7 6 $\times$17=?
22. £3 4 9 $\times$19=?
23. £4 5 $7\frac{1}{2}\times$23=?
24. £5 7 $6\frac{1}{4}\times$26=?
25. £6 3 8 $\times$29=?
26. £8 4 $7\frac{3}{4}\times$30=?

Answers: £23266, 15, $4\frac{1}{4}$; £9834, 12, 5; £566, 1, 3; £183, 7, $1\frac{1}{2}$.

118. *To divide compound numbers.*

British money being almost the only thing in business to which compound numbers is applied exercises in it have received most attention; and especially as direct importation gives the clerk more to do with it than heretofore. See index for shorter methods of computing this kind of money

27. Divide £157, 13, $6\frac{1}{2}$, by 5.

£	s.	d.
5)157	13	$6\frac{1}{2}$
31	10	$8\frac{1}{2}$

EXPLANATION.—5 is contained in £157 31 times and £2 over. These £2 reduced to shillings, and added to the 13s. of the dividend, make 53s., in which 5 is contained 10 times and 3s. left. In 3s. there are 36 pence, which, added to the 6d. of the dividend, make 42d., in which 5 is contained 8 times and 2d. over. In 2d. and $\frac{1}{2}$d. there are 10 farthings, in which 5 is contained 2 times, making $\frac{2}{4}$ or $\frac{1}{2}$d.

28. Divide £157, 13s, $6\frac{1}{2}$d equally between 25 persons.

25) £157, 13s, $6\frac{1}{2}$d(£6, 6s, $1\frac{1}{2}$d, or £6, 6s, $1\frac{3}{4}$d, nearly.
150
£7=*remainder.*
20
153=*shillings in* £7, *with* 13s *of the dividend added.*
150
3=*remainder in shillings.*
12
42=*pence in* 3 *shillings and* 6 *pence from the div'd.*
25
17=*remainder in pence.*
4
70=*farthings in* 17 *pence and* $\frac{1}{2}$.
50
20=*remainder, or* $\frac{20}{25}=\frac{4}{5}$ *farthings.*

	£	s.	d.			£	s.	d.	
29.	487	13	0	÷ 9=?	32.	167	18	$6\frac{3}{4}$	÷ 25=?
30.	356	7	10	÷36=?	33.	768	14	$3\frac{1}{4}$	÷125=?
*31.	419	15	$6\frac{1}{2}$	÷14=?	34.	17	11	$3\frac{1}{2}$	÷875=?

Answers: £12, 17, $8\frac{3}{4}$, and £94, 1, $4\frac{1}{4}$.

* When the remainder from farthings is $\frac{1}{2}$ or over, add a farthing, otherwise omit it.

GENERAL EXERCISES IN COMPOUND NUMBERS.

28. In 4 yards 2 feet seven inches, linear measure, how many inches?

29. In 100 inches how many yards?

30. Reduce 3520 yards to miles.

31. In 100,000 inches how many miles?

32. How many times will a carriage-wheel turn in a distance of 17 miles, the wheel measuring 2 yards 2 feet in circumference?

33. From a plank 17 yards long was cut 10 yards 2 feet 3 inches; how much of it was left?

34. Reduce 3 acres 140 rods to square yards.

35. Divide 200 acres 100 rods into 10 equal parts.

36. From 406 acres 17 rods take 68 acres 148 rods 15 yards.

37. Divide 64 acres 134 rods 8 yards into 5 equal parts.

38. Find the price of $9\frac{1}{4}$ ounces of gold at £3 17s 8d per lb. Troy.*

39. Find how often £2 4s 6d is contained in £41 10s $7\frac{1}{2}$d.

40. Reduce 1 lb. 3 oz. 5 pwt. to pennyweights.

Answers: 175; 11220; 1, 4, 24, 5, 2, 4; 2; 2, 2, 4; 6, 9; 337, 28, $15\frac{1}{4}$; 2, 19, $10\frac{5}{12}$; 12, 3, 34, $25\frac{4}{5}$; $18\frac{237}{356}$; 18755; 20, 10; 305.

The Teacher should require his scholars to give the denomination of each item in the answers.

* Find the price of 1 ounce; then the price of $9\frac{1}{4}$.

XIII. SHORT METHODS OF MULTIPLYING.

119. Besides the contractions by aliquots, under Art. 66, the expert accountant and arithmetician can find abbreviated methods adapted to almost every calculation. A few will be given in this place, to admit of the learner using them, when opportunity occurs, in the subsequent exercises.

120. *To multiply by* 11, write the first figure of the multiplicand as the first of the product, and add each figure on the left to the one on the right, as below.

1. 38897×11=427867.

Prove the following by multiplying in the ordinary way:

2. 379×11=?	7. $219.168×11=?
3. 1487×11=?	8. $716.573×11=?
4. $37.486×11=?	9. $316.144×11=?
5. $9314.20 ×11=?	0. $137.211×11=?
6. $167.473×11=?	11. $710.22 ×11=?

121. *To multiply by the teens when the tables are not known, and by such numbers of two figures as end with* 1, *as* 21, 31, *etc.*, multiplication by the figure 1 ought to be omitted.

12. Multiply 3174 by 17.

```
3174
 22218
-----
53958
```

Explanation.—The product of 7 is written one place to the right to allow the first line to stand in the tens' place, by which it is multiplied by 10.

13. $3163 ×15=?	18. 435.16\frac{2}{3}$×16=?
14. $216.37 ×19=?	19. $213.14 ×18=?
15. $1139.24 ×13=?	20. 1137.37\frac{1}{2}$×19=?
16. $413.22 ×18=?	21. 713.11\frac{1}{2}$×16=?
17. *131.18\frac{1}{4}$×14=?	22. $4302.87 ×17=?

* Call the $\frac{1}{4}$ of a cent 25 hundredths, making 1311825 for the multiplicand, and point off four figures. (See note, page 66.)

23. $316.27×51=?

31627 158135 1612977	EXPLANATION.—The 5 of the multiplier being in the tens' place, the first figure of the product is written under the tens of the multiplicand.

24. $137.50 ×31=?
25. $298.67 ×51=?
26. 783.37\frac{1}{2}$×61=?
27. $313.17 ×81=?
28. 1987.87\frac{1}{2}$×91=?
29. $2136.22 × 71=?
30. 1394.31\frac{1}{4}$× 41=?
31. 653.18\frac{3}{4}$× 21=?
32. 291.16\frac{2}{3}$×121=?
33. 312.18\frac{3}{4}$× 21=?

122. *When the multiplier wants from* 1 *to* 12 *of being* 100, 200, 3000, *etc.*, the work may be contracted by multiplying by one of these, and subtracting as many times the multiplicand as the multiplier is short of it.

34. To multiply 424 by 97.

OPERATION.
$$\begin{array}{r} 424\times100=42400 \\ 424\times\ \ \ 3=\ \ 1272 \\ \hline 41128 \end{array}$$

35. 765×192=?
36. 1789×398=?
37. 6784×188=?
38. 9876×191=?
39. 671× 39=?
40. 59×689=?
41. $89 ×784=?
42. $167 × 29=?
43. $37.98 ×489=?
44. $478.96 ×499=?
45. 674.82\frac{1}{2}$×992=?
46. 7164.37\frac{1}{2}$× 87=?

123. *When the multiplier is* 29, 39, 49, *etc.*, we multiply by the next higher number and subtract the multiplicand.

47. To multiply 176 by 59.

OPERATION.
$$\begin{array}{r} 176\times60=10560 \\ 176 \\ \hline 10384 \end{array}$$

124. *To multiply by* 601, 1003, 90001, *etc.*

The case differs from Art. 8 only in the intervening figures; so the product is written one place further to the right or left for every cipher.

48. Multiply 317 by 601. OPERATION.

```
       317
      1902
 ---------
Ans. 190517
```

49. Multiply 15704 by 10007 OPERATION.

```
     15704....
        109928
 -------------
Ans. 157149928
```

125. *When one part of the multiplier contains the other without a remainder, as* 248. Here 24 contains 3 times the 8 or first figure, so by multiplying the product of 8 times the multiplicand by 3, one line is saved.

50. Multiply 76439 by 248.

OPERATION.

```
      76439
     ------
     611512=8 times 76439
    1834536 =3  "  611512
   --------
   18956872 Ans.
```

REMARK.—This operation might be shortened by multiplying the product by 8 mentally, and adding that line for the whole product.

51. Multiply 25938 by 936.

OPERATION.

```
 25938×936        or    25938
 ---------              ---------
 233442..               233442..
   933768               ---------
 ---------              24277968
 24277968
```

52. 11457×324=?	57. 7832×64256=?
53. 672×189=?	58. 7498×16144=?
54. 783×357=?	59. 9739× 3972=?
55. 924×218=?	60. 6487× 8109=?
56. 596×426=?	61. 74675× 7206=?

126. *To multiply by* 375, 625, 750 *or* 875, we first multiply by 125 (Art. 66), and that product by 3, 5, 6 or 7, these numbers, 375, etc., being multiples of that number.

62. $1649\times625=\frac{1649000\times5}{8}=\frac{8245000}{8}=1030625$

63. $3156\times375=?$ $\$1703.20\times750=?$ $\$1456\times875=?$

127. *To multiply by a composite number composed of factors under* 13, the latter may be used as multipliers instead of the former.

64. 314×72.

OPERATION. $314\times12=3768\times6=22608$.

65. $932\times64=?$

66. $738\times48=?$

67. $426\times96=?$

68. $\$913.27\times54=?$

69. $\$293.75\times72=?$

70. $\$6318.37\frac{1}{2}\times63=?$

128. *To multiply by any number of* 9*s*, we multiply by the next highest number and subtract the multiplicand.

71. 3145×999.

$$
\begin{array}{rr}
 & 3145\times1000=3145000 \\
\textit{From which take} & 3145 \\
\hline
 & 3141855
\end{array}
$$

129. *To square, mentally, numbers under* 39 *that end with* 9.

72. What is the square of 29?

$$
\begin{array}{r}
29 \\
29 \\
\hline
841
\end{array}
$$

EXPLANATION.—Writing 1 for the first figure of the product, we add 1 to the tens' place of the multiplier, and multiply the sum on the multiplicand less 1: $3\times28=84$, with the 1 annexed$=841$.

73. Find the square of the following numbers mentally: 99, 59, 119, 79, 19, 69, 129, 89.

130. *To square any number of* 9s *instantaneously, and without multiplying.*

Commencing at the *left*, we write as many 9s, less one, as the number to be squared, an 8, as many 0s as 9s and a 1.

74. The square of 9999999 is 99999980000001.

The square of any number of 3s will be one-ninth of the square of the 9s.

131. *To square numbers under* 135 *ending with* 5.

The first two figures on the right of the product will always be 25; and to find the others, we add 1 to the tens' place and multiply it on the tens' and hundreds' places above.

75. To square 115. OPERATION.

$$\begin{array}{r} 11 \\ 12 \\ \hline 13225 \end{array}$$

The reason of this will be apparent by multiplying in the usual way.

132. *To square a number containing a half, as* $12\frac{1}{2}$, we multiply the *whole* number by the next higher number and add a fourth. $8\frac{1}{2}$ squared$=8\times9+\frac{1}{4}=72\frac{1}{4}$.

76. Find the square of the following numbers: 99999, 33333, 75, 45, 65, $62\frac{1}{2}$ $16\frac{1}{2}$, $19\frac{1}{2}$.

XIV. SHORT METHODS OF DIVIDING.

133. *Division may often be contracted by cancellation* when the terms are written in fractional form.*

1. Divide 1463 by 28.

$$\frac{\overset{209}{\cancel{1463}}}{\underset{4}{\cancel{28}}}=52\frac{1}{4}$$

EXPLANATION.—The terms 1463 and 28 were first divided by 7, leaving 209 fourths, and 209 divided by 4 gives $52\frac{1}{4}$.

TO THE TEACHER.—The author does not offer all these contractions as rules of general utility; still, he is of opinion that a familiar knowledge of them will be advantageous to the student of arithmetic in disciplining his mind and showing him the relation of numbers. Where the instructor thinks otherwise, he can omit them.

* To cancel signifies to blot out or make void.

Prove the answers obtained, by dividing in the usual way.

2. $3465 \div 35 = ?$ $2763 \div 81 = ?$ $\$65.45 \div 243 = ?$

3. $1962 \div 22 = ?$ $6876 \div 152 = ?$ $\$54.36 \div 144 = ?$

134. *To divide by aliquots of* 100, 1000, *etc.*

This process is the reverse of that under Art. 66.

135. *To divide by a composite number, as* 96, which is composed of the factors 12 and 8, or 648, which is composed of $9 \times 8 \times 9$. This operation is performed by using the factors instead of the whole number.

4. Divide \$78.54 by 32. OPERATION.

$$\begin{array}{r} 4\underline{)7854} \\ 8\underline{)1963}—2 \\ 245—3 \end{array}$$

How the true remainder is found:

The first remainder is 2 *cents*, because it is left from the *cents* that were divided. The second remainder is *four times* as great as if it were from the first line, because every figure of the second line is four times as great as if it stood in the first line. Four times $3 = 12$ and the 2 of the first remainder equal 14, the *true remainder*.

Ans. $245\frac{14}{32}$.

5. Divide 6371 by 336. OPERATION.

$$\begin{array}{r} 6\underline{)6371} \\ 7\underline{)1061}—5 \\ 8\underline{)\ 151}—4 \\ 18—7 \end{array}$$

REMARK.—The true remainder of this example is found by multiplying the last remainder by 7, to make it of the same value as if it were from the line above, and that by 6, to make it of the same value as if it were from the upper line: $7 \times 7 \times 6 = 294$, to which add $6 \times 4 + 5$ or 29. The true remainder is 323. *Ans.* $18\frac{323}{336}$.

6. Divide	1463	by	28	10. $4571 \div 441 = ?$
7. "	7614	"	72	11. $1987 \div 379 = ?$
8. "	1943	"	49	12. $9843 \div 720 = ?$
9. "	8765	"	343	13. $1456 \div 729 = ?$

14. Divide 7654 by 25.

$$\begin{array}{l} 76.54 = 100th\ of\ 7654. \\ \underline{\quad\quad 4} \\ 306.16 = 4\ times\ as\ much,\ or\ \tfrac{1}{25}\ of\ 7654. \end{array}$$

or $306\frac{16}{100}=306\frac{4}{25}$.

15. \$3675÷125 =? \$213.67÷16⅔=? \$1174÷12½=?

16. \$2153÷ 33⅓=? \$319.25÷ 8⅓=? 316÷25 =?

136. *When the divisor is* 15, 35, 45, 55 *or* 65, it will abbreviate the work to multiply the dividend by 2 and divide by 30, 70, etc.

17. 345÷35=? $345\times2=690\div7=98\frac{4}{7}$.

18. 2756÷15=? \$1324.25÷35=? \$365.75÷45=?

137. *To divide by* 75, 175, 225 *or* 275, the dividend may be multiplied by 4 and the product divided by 300, 700, 900 or 1100.

19. 2136÷75=?

$$\begin{array}{r} 2136 \\ \underline{4} \\ 3{,}00)\underline{85{,}44} \\ 28\frac{144}{300} \end{array}$$

EXPLANATION.—4 three-hundredths being equal to 1 seventy-fifth, it abbreviates the work to divide by 300 and multiply by 4.

20. 3678÷175=? 6317÷175=? \$19.32÷275=?

138. *Long division may be abbreviated by performing a part of the process mentally, and writing only the result.*

21. 76354÷34=?

```
34)76354(2245
    83
    155
     194
      24
```

EXPLANATION.—The products are omitted; only the remainder and the figures brought down are written.

22. 3167÷184=? 1679÷21=? 67831÷498=?

139. *To divide by any number of* 9*s*, the dividend may be pointed off, *from the left*, in periods corresponding with the number of figures in the divisor. If the divisor have two 9s, it may be pointed off in periods of two; if three 9s, in periods of three, and the operation performed as in the formula.

23. 316328÷99=? 7312961÷999=?

31,63,28	731,296,1
31,63	731,2
31	7
3195.22	7320.280
1	1
3195.23	7320.281

REMARK.—It will be observed that each line in the formula is diminished by one period, corresponding with the number of 9s, and the sum of the whole taken. As many figures as 9s should be pointed off for a remainder, plus the carrying figure of the first line.

24. Divide 167389 by 9999 and 7654321 by 99.*

XV. MARKING GOODS—GAIN AND LOSS.

140. ON receiving goods for sale, the merchant or some of his clerks examine them by the bill or invoice, which is usually received in advance of them by mail, after which they proceed to mark them.

141. *Marking Goods.*—This is done by selecting *samples* of each kind or quality of goods, and putting on them a private mark, indicating the cost price, the selling price, or both.

Every house has its own private mark, which usually consists of a word or phrase to represent the ten digits, as the word *importance*, which has ten letters. Commencing at 1, the letters are arranged as follows:

i	*m*	*p*	*o*	*r*	*t*	*a*	*n*	*c*	*e*
1	2	3	4	5	6	7	8	9	0

In addition to these, another letter not contained in the above is selected for a *repeater*, so that when a figure oc-

* It would be no abbreviation to divide by 9 in this way.

curs twice it may be inserted to prevent detection. The letter *g* will do in this case.

To mark $6.55, the letters *trg* would be used, with perhaps a line or period to separate the dollars from the cents.

To fix on a selling price, various circumstances have to be taken into consideration—the cost of transportation, the probable length of time required to sell, cost of rent, wages of clerks, depreciation of stock, etc.

A certain rate is put on for the first item, say 5 to 15 per cent.; then an amount estimated to cover the balance and leave a profit. When adding the entire rate of advance to the cost price, the clerk is not required to be exact, as simplicity of calculation is a greater object than uniformity of profit. Thus, in marking goods to sell by the dozen, a multiple of 12 would be preferred, whether a little above or below the fixed rate. For the same reason, aliquots, or numbers formed of aliquots, of 100 would be selected.

When working the following exercises, the learner will remember that fractions, unless occurring with aliquots of 100, are usually omitted. When under $\frac{1}{2}$ they are rejected; otherwise, a cent is added to the cents.*

1. Add 25% to $3.50, $5.75, $1.82, .75, $2.
Answers: $4.37², 7.18\frac{3}{4}$, $2.28, .94, $2.50.
2. Add 16$\frac{2}{3}$% to $6.20, $3.12², 27c, $3.87.
3. Add 37$\frac{1}{2}$% to 20c, 12²c, $1.15, 6c, 15c.
4. Add 15% to $1.20, 75c, $1.22, $3.57, 16²c, 27c.
5. Add 33$\frac{1}{3}$% to 5c, 15c, 21c, $6.75, 18c, $27.50.
Answers: $15.72, $8.24, $46.46, $2.32.

---

*To indicate the degree of exactness required, a few answers will be given to the examples which follow.

One object in giving these examples being to exercise the judgment of the pupil, he will not be required to obtain precisely the same figures as given in the other answers.

6. $1.75+14%=?
7. .87+12%=?
8. $1.67+53%=?
9. .04+60%=?
10. $2.75+35%=?
11. .16+15%=?
12. .05+25%=?
13. $3.14+18%=?
14. $6.75+26%=?
15. .07+40%=?
16. $3.60+12%=?
17. $4.80+30%=?
18. $1.20+35%=?
19. $7.20+40%=?
20. $6.00+14%=?
21. $5.10+25%=?

Answers: $13.25, $43.79.

22. To .30 add 20 % profit.
23. To $1.20 add 5 % charges and 20% profit.
24. To $1.75 add 2½% freight and 10% profit.
25. To .08 add 16⅔% profit.
26. To .36 add 33⅓% profit.

Answer: $4.41.

27. $0.10+10%=?
28. .05+50%=?
29. .25+20%=?
30. $1.00+16⅔%=?
31. $3.25+33⅓%=?
32. $1.40+12½%=?

Total, $7.57.

The following may be worked by aliquots of 100:

33. Add 5 % to $12.50
34. " 6¼% " $7.80
35. " 8⅓% " $4.85
36. " 9 % " $3.50
37. " 12½% " $0.87
38. " 20 % " $0.45
39. " 18¾% " $0.15
40. " 25 % " $0.12½
41. " 7 % " $6.20
42. " 3⅓% " $1.00
43. " 12½% " $1.35
44. To $3.50 add 4 %
45. " $9.50 " 16⅔%
46. " $0.10 " 12½%
47. " $0.18¾ " 33⅓%
48. " $0.06¼ " 40 %
49. " $0.87½ " 50 %
50. " $0.08⅓ " 75 %
51. " $0.11¼ " 20 %
52. " $0.80 " 15½%
53. " $0.16 " 16⅓%
54. " $0.47 " 14⅕%

Answers: 41.53, $18.56.

ORAL EXERCISES.*

1. $\$0.10 + 25\% = ?$
2. $.25 + 10\% = ?$
3. $.20 + 6\% = ?$
4. $.33 + 33\frac{1}{3}\% = ?$
5. $.16 + 25\% = ?$
6. $.45 + 5\% = ?$
7. $.87 + 30\% = ?$
8. $.16 + 12\frac{1}{2}\% = ?$
9. $.10 + 22\% = ?$
10. $.05 + 30\% = ?$
11. $\$2.20 + 50\% = ?$
12. $\$5.50 + 100\% = ?$
13. $.75 + 60\% = ?$
14. $\$1.25 + 30\% = ?$
15. $\$1.12 + 25\% = ?$
16. $.75 + 20\% = ?$
17. $1.50 + 8\% = ?$
18. $\$0.50 + 20\% = ?$
19. $.25 + 50\% = ?$
20. $.11 + 15\% = ?$
21. $.12\frac{1}{2} + 20\% = ?$
22. $.45 + 9\% = ?$
23. $.66 + 33\frac{1}{3}\% = ?$
24. $.14 + 5\% = ?$
25. $.35 + 20\% = ?$
26. $.16 + 50\% = ?$
27. $.67 + 12\% = ?$
28. $\$1.20 + 16\frac{2}{3}\% = ?$
29. $.27 + 50\% = ?$
30. $\$6.50 + 16\% = ?$
31. $\$2.15 + 8\% = ?$
32. $\$1.87 + 200\% = ?$
33. $.31 + 18\% = ?$
34. $.19 + 12\frac{1}{2}\% = ?$
35. 30% of $\$20.00 = ?$
36. 50% of $\$1000.00 = ?$
37. 25% of $\$700.00 = ?$
38. 16% of $\$500.00 = ?$
39. 10% of $\$350.25 = ?$
40. 15% of $\$200.00 = ?$
41. 80% of $\$500.00 = ?$
42. 20% of $\$20.00 = ?$
43. $\$0.37\frac{1}{2} + 30\% = ?$
44. $.62\frac{1}{2} + 5\% = ?$
45. $\$1.25 + 12\frac{1}{2}\% = ?$
46. $.16 + 5\% = ?$
47. $.54 + 40\% = ?$
48. $\$30.00 + 60\% = ?$
49. $\$90.00 + 9\% = ?$
50. $\$70.00 + 30\% = ?$

---

* *The Teacher* may require the fractions in these exercises.

GAIN AND LOSS.

142. Merchants distinguish between real gain or loss and gain or loss per cent., calling the former the *actual gain or loss*, and the latter the *gain or loss per cent.*

143. *To find the actual gain*, it is simply necessary to subtract the cost price from the selling price.

1. Bought a house and lot for \$4367 and sold them for \$5000; how much did I gain?

$$\begin{array}{rl} \$5000 = & \textit{Selling price.} \\ 4367 = & \textit{Cost or buying price.} \\ \hline \$633 = & \textit{Actual gain.} \end{array}$$

	Cost price.	Selling price.		Cost price.	Selling price.
2.	\$2.75	\$4.87	7.	\$316.17	\$215.25
3.	\$97.35	\$120.10	8.	\$112.14	\$120.48
4.	\$6.87	\$6.98	9.	\$317.18$\frac{3}{4}$	\$219.12$\frac{1}{2}$
5.	\$5.40	\$9.80	10.	\$67.21	\$86.28
6.	\$3.20	\$6.40	11.	\$54.12$\frac{1}{2}$	\$21.18$\frac{3}{4}$

Total gain, \$32.58. Net loss, or loss with gains deducted, \$204.51.

144. *To find the gain per cent.*, is simply to find the gain on every hundred dollars or cents.

Required the gain per cent. on goods which sold at \$1.35 and cost \$1.20.

135—120=15 cents, the actual gain on 120. $15 \div 120 = \frac{15}{120}$=gain on 1 cent. $\frac{15}{120} \times 100 = \frac{1500}{120} = 12\frac{1}{2}$ per cent., or gain, on 100 cents.

OPERATION.

$$\begin{array}{r} 135 \\ \underline{120} \\ 12\cancel{0})\overline{150\cancel{0}} \\ 12\frac{6}{12} \textit{ or } 12\frac{1}{2}\% \end{array}$$

EXPLANATION.—The actual gain is first found; then the gain per cent., by dividing the actual gain (when multiplied by 100) by the first cost.

12. Goods which cost $2.00 were sold at $3.00; required the gain per cent.

13. The cost price was $1.25; the selling price, $1.50; what was the gain per cent.?

14. Goods bought at 75 cents sold at $1.00; what was the gain per cent.?

15. 10 cents was the cost; $12\frac{1}{2}$ the selling price; required the gain per cent.

Total rates of gain of the four, $128\frac{1}{3}\%$.

	First cost.	Selling price.		First cost.	Selling price.
16.	$12.50	$10.00	20.	$3167.00	$3000.00
17.	.18	.20	21.	$1000.00	$1500.00
18.	.05	.06	22.	$27.80	$20.00
19.	$127.52	$111.58	23.	$12.17	$11.50

Net loss, on 16 to 19, $1\frac{7}{18}\%$. Answers to second group, $28\frac{18}{139}\%$, $5\frac{865}{3167}\%$, 50%, $5\frac{615}{1217}\%$.

24. Bought a bbl. of apples for $1.75 and sold it for $2.25; what did I gain per cent.?

25. Sold 25 bbls. of potatoes for $39.00; how much did I gain per cent., if they cost me $1.25 per barrel?

26. Bought 150 bbls. of flour @ $5.25, paid for drayage $7.50 and porterage $1.00; at what per barrel should I sell it to gain 15 per cent.?

27. Bought 15 horses at $125 each, and sold the lot for $3500; what was my gain per cent., after paying $25 for their feed?

28. Sold a safe which cost me $80 for $75; what was my loss per cent.?

29. Bought a bill of goods for $350, paid freight $15.20, insurance $5, drayage $3, and sold them for $425; what was my *actual gain*, and what my gain per cent.?

30. Sold A's note for $750 at a discount of 15%; what did I pay for it?

31. Sold B's note of \$320 for \$300; what was the rate of discount?

Answers: \$637.50, \$51.80, 14% nearly, $84\frac{1}{4}$% nearly, $6\frac{1}{4}$%, $6\frac{2}{3}$%, $28\frac{4}{7}$%, $24\frac{4}{5}$%, \$6.10.

145. *When the selling price and the rate per cent. are known, to find the first cost.*

32. What was the first cost of goods marked \$2.65, the rate per cent. of profit being 25?

```
1.25)2.65(212
     2 50      or $2.12
     ----
     1500
     1500
     ----
```

EXPLANATION.—Every dollar invested in the goods has increased 25 cents, and is worth 125 cents. Hence, there are as many invested dollars in \$2.65 as 1.25 is contained times in it. The remainder, 15 dollars, we reduced to cents, which, divided by 1.25, gives 12 cents.

Another way: Since 25% is $\frac{1}{4}$ of 100, the \$2.65 must be $\frac{1}{4}$ more than the first cost. Let the first cost be $\frac{4}{4}$, then $\frac{4}{4}+\frac{1}{4}=\frac{5}{4}$. Therefore, $\$2.65=\frac{5}{4}$ of the first cost.

$\frac{265}{5}=53$=one-fourth of the first cost.

$53\times4=2.12$, *first cost.*

What was the first cost of the goods marked as follows? The learner can prove his calculations by reversing the process.

33.	\$2.25 @ 10 % gain	38.	\$2.87 @ 10 % loss
34.	\$3.70 " 5 % "	39.	\$1.54 " 6 % "
35.	\$115.87 " $12\frac{1}{2}$% "	40.	\$3.75 " 25 % "
36.	\$14.54 " $33\frac{1}{3}$% "	41.	.$87\frac{1}{2}$ " $12\frac{1}{2}$% "
37.	.87 " $16\frac{2}{3}$% "	42.	.$12\frac{1}{2}$ " 50 % "

43.	\$9.50 @ 50 % gain	47.	\$90.00 @ 20 % gain
44.	\$7.87 " 25 % "	48.	\$75.30 " 15 % "
45.	\$6.50 " $16\frac{2}{3}$% "	49.	\$82.50 " $16\frac{2}{3}$% "
46.	\$8.75 " $18\frac{3}{4}$% "	50.	\$60.00 " 20 % "

XVI. COMMISSION AND BROKERAGE.*

146. COMMISSION, or brokerage, is the percentage charged by a *commission merchant*, *factor*, *agent* or *broker* for transacting business for another.

147. Commission is usually reckoned on the whole amount of *sale*, *purchase* or *collection.*

1. At $2\frac{1}{2}$ per cent., what is the commission on \$17640?

Ans. \$441.

2. A merchant sells goods for another to the amount of \$4371.81; what is his commission at 5 per cent.?

3. A broker receives $\frac{1}{4}$ per cent. for selling \$2500 worth of merchandise for a commission merchant; what is the amount of his brokerage?

4. A of New Orleans buys sugar for B of Cincinnati to the amount of \$7100; what is the amount of his commission at $1\frac{1}{2}$ per cent.?

5. A commission merchant sells goods for his principal to the amount of \$3000, and charges $2\frac{1}{2}$ per cent. commission; what does he make by the operation, after paying a broker $\frac{1}{4}$ per cent. for his services in effecting sales?

6. After receiving 5 per cent. commission on sales amounting to \$520.75, how much should I return to my principal?

7. Gave a lawyer a note of \$50 to collect, at 8 per cent.; how much should I receive?

Omit fractions of a cent in the answer:

Answers: \$106.50, \$6.25, \$218.59, \$67.50, \$494.71, \$46, \$510.71.

---

* See Commission Merchant, page 94; Brokers, page 95.

What is the commission on the following amounts?

8. \$364.15 @ $3\frac{1}{2}\%=?$	12. \$36.21 @ $1\frac{1}{2}\%=?$
9. \$78.54 " $6\frac{1}{4}\%=?$	13. \$174.09 " $3\frac{1}{3}\%=?$
10. \$710.06 " $8\frac{1}{2}\%=?$	14. \$2167.90 " $6\frac{1}{4}\%=?$
11. \$876.75 " $9\frac{1}{2}\%=?$	15. \$78.21 " $5\frac{1}{2}\%=?$
Total, \$161.31.	Total \$146.13.

16. A commission merchant charges $2\frac{1}{2}\%$ com. and $2\frac{1}{2}\%$ *guarantee* on the sale of goods amounting to \$3100; how much should he return to his principal?

17. A merchant receives a consignment of goods valued at \$3000, and sells them for \$4500; how much should he remit to the consignor, after reserving $4\frac{1}{2}\%$ for com. and guar.?

Answers: \$2945, \$4297.50, \$5297.30.

18. A merchant sells a note of \$100 to a money broker, at a discount of 6 per cent.; how much money does he receive?

19. A New York merchant buys a bill of exchange worth \$400, on a Cincinnati banking house, at $\frac{1}{4}$ per cent. discount; how much does he pay for it?

20. Bought a bill of exchange on New York for \$7691, at $1\frac{1}{2}$ premium; what did I pay for it?

21. Purchased 20 shares of railroad stock, worth \$20 per share, @ $12\frac{1}{2}\%$ discount, and sold it at par; what was the amount paid? and how much did I gain?

Answers: \$94, \$399, \$7806.37, \$350, \$50.

22. What is the premium on the following? \$31.46@ $5\frac{1}{2}\%$; \$1760@$6\frac{1}{4}\%$; \$4617@$9\frac{1}{2}\%$. Total, \$550.35.

148. *To find the commission on investments.*

The merchant often has moneys in his hands or remitted to him for investment in goods or stocks, upon which he is allowed commission on the amount invested only.

23. At $2\frac{1}{2}\%$ commission, what amount of money shall I retain of \$2000 in my hands for investment?

This \$2000 contains 100% of the amount to be invested, plus 2.5%, my commission making 102.5% of it.

Reducing both to tenths, we have 2000.0 to be divided by 102.5.

102.5)2000.0(19.5121 *or* \$19.5122=1%,

```
1025
----
 9750
 9225
 ----
  5250000
  5125
  ----
   1250
   1025
   ----
    2250
    2050
    ----
     2000
     1025
     ----
      975
```

Which, multiplied by $2\frac{1}{2}$ gives the commission, or by 100, gives 100 per cent., or the amount to be invested. Com \$48.78.

REMARK.—The 525 dollars remainder were reduced to tenths of mills.

RECAPITULATION.—To find 1 per cent., we divide 2000 by 102.5, which, multiplied by 100, gives 100 per cent., or the amount. The difference is my commission.

24. What will be my com. on \$1300, to be invested at 2%?

25. At 5% commission, how much sugar can I buy for \$3475.25, when it sells at 20 cents a pound?

26. Out of \$987.50 how many pounds of tobacco should I purchase for my principal at 35 cents a pound, commission $2\frac{1}{2}\%$?

27. At $2\frac{1}{2}\%$ commission, how much shall I reserve of \$2136.67, after investing in cotton at 25 cents a pound?

28. To pay $12\frac{1}{2}$ cents a pound for rice, and $\frac{1}{10}\%$ for insurance, and myself 3% commission, how many pounds can I buy for \$345.15?

The learner will prove his work by reversing the process.

29. Invest \$1367.37, $\frac{1}{4}$ in gloves at \$30 a dozen, $\frac{1}{2}$ in sheeting at 35 cents a yard and the balance in muslin at 20 cents a yard; commission $2\frac{1}{2}\%$.

INTEREST.

149. *Interest* is a percentage allowed for the use of capital, which may consist of money, merchandise or debts due. It is regulated by the year or month.

150. The sum upon which interest is reckoned is called the *principal;* the percentage allowed per month or year, the *rate;* and the sum of the principal and interest, the *amount.*

151. Interest is divided into *simple* and *compound* *Simple interest* is percentage on the principal alone; *compound interest* is interest reckoned on both principal and interest.

The legal rates of interest in the several States is variable.

152. Money loaned on interest is usually secured by promissory notes, expressed as follows:

STAMP.

Cincinnati, September 1, 1866.

Three months after date, I promise to pay to the order of H. A. Faber, Five thousand dollars, value received, with legal interest from date.

\$5000.00. *John Hancock.**

153. This note would be due on the 1st day of December but for a law which allows *three days* longer for its payment, called *days of grace.* It will therefore be legally due on December 4th, and interest will be reckoned to that date.

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* Other forms of notes will be found under Banking.

XVII. SIMPLE INTEREST.

154. Interest is usually calculated on the basis of 360 days to the year.* When notes are drawn by the month, calendar months are understood. Thus, a note drawn on the 1st of September, as the preceding, falls due on the 1st of December, plus the days of grace.

155. The simplest method of computing interest is to do it at the rate of 6 per cent. per annum, and add or subtract when it is higher or lower than that.

156. The interest for 60 days at 6% per annum is equal to as many cents as there are dollars, or, in other words, is 1 per cent. of the principal.

The reason of this is obvious. 6 per cent. per annum is $\frac{1}{2}$ per cent. per month, or 1 per cent. for two months or 60 days.

ORAL EXERCISES.

1.	The	interest	for 60	days,	at 6%,	on	$20	=?
2.	"	"	" 60	"	" 6%,	"	$50	=?
3.	"	"	" 60	"	" 6%,	"	$100	=?
4.	"	"	" 60	"	" 6%,	"	$75	=?
5.	"	"	" 60	"	" 6%,	"	$125	=?
6.	"	"	" 60	"	" 6%,	"	$175	=?
7.	"	"	" 60	"	" 6%,	"	$200	=?
8.	"	"	" 60	"	" 6%,	"	$316.50	=?
9.	"	"	" 60	"	" 6%,	"	$715	=?
10.	"	"	" 60	"	" 6%,	"	$50.50	=?
11.	"	"	" 60	"	" 6%,	"	$215.15	=?
12.	"	"	" 60	"	" 6%,	"	$1000	=?

*In New York interest is usually reckoned for the full year.

13.	The	interest	for	60	days,	at 6%,	on	\$976.14=?
14.	"	"	"	60	"	" 6%,	"	\$715.15=?
15.	"	"	"	60	"	" 6%,	"	\$5000 =?
16.	"	"	"	60	"	" 6%,	"	\$5 =?
17.	"	"	"	60	"	" 6%,	"	\$23.13=?
18.	"	"	"	60	"	" 6%,	"	\$67.15=?

19. On	\$7.50=?	28. On	\$269.14=?
20. "	\$8.75=?	29. "	\$198.97=?
21. "	\$118.67=?	30. "	\$267.18=?
22. "	\$368.56=?	31. "	\$1365.50=?
23. "	\$210.33=?	32. "	\$316.18=?
24. "	\$67.67=?	33. "	\$215.16=?
25. "	\$39.37=?	34. "	\$716.16=?
26. "	\$21.37=?	35. "	\$317.60=?
27. "	\$116.16=?	36. "	\$167.37=?

157. Having the interest for 60 days, the interest for any shorter time may be found by

ALIQUOTS OF 60.

30 days $=\frac{1}{2}$	12 days $=\frac{1}{5}$	5 days $=\frac{1}{12}$	2 days $=\frac{1}{30}$
20 " $=\frac{1}{3}$	10 " $=\frac{1}{6}$	4 " $=\frac{1}{15}$	1 day $=\frac{1}{60}$
15 " $=\frac{1}{4}$	6 " $=\frac{1}{10}$	3 " $=\frac{1}{20}$	

When the number is not an aliquot of 60,

For	7	take	6 and	1	For	29	take	1 off 30
"	8	"	6 "	2	"	35	"	30 and 5
"	14	"	12 "	2	"	38	"	30, 6 and 2
"	19	"	15 "	4	"	43	"	30, 12 " 1
"	26	"	20 "	6	"	45	"	15 off 60
"	27	"	15 "	12	"	85	add	20 and 5*

* *The Teacher* should continue these exercises till his scholars are familiar with all the aliquots, and the method of resolving other numbers into aliquots.

1. Find the interest on \$500, for 30 days, at 6% per annum.

$$30 = \tfrac{1}{2})\underline{5.00} = \textit{Interest for } 60 \textit{ days.}$$
$$2.50 = \textit{Interest for } 30 \textit{ days.}$$

2 to 8. Find the interest on \$200 for 12 days, 15 days, 20 days, 10 days, 6 days, 3 days, 1 day, at 6% per annum. *Ans.* \$2.23.3.

9 to 13. Find the interest on \$160 for 1, 2, 3, 4 and 5 days, at 6% per annum. *Ans.* \$0.39.9+.

14 to 18. Find the interest on \$240 for 6, 10, 12, 20 and 30 days, at 6% per annum. *Ans.* \$3.12.

19 to 22. Find the interest on \$1000 for 1, 10, 12 and 6 days, at 6% per annum. *Ans.* \$4.83.3+.

23. Find the interest on \$675, for 27 days, at 6% per annum.

$$\begin{array}{ll} & \underline{\$6.75} = \textit{Interest for } 60 \textit{ days.} \\ 15 = \frac{1}{4} = & 1.687 \\ 12 = \frac{1}{5} = & \underline{1.35} \\ & 3.037 \textit{ or } \$3.04. \end{array}$$

	Principal.	Time.		Principal.	Time.
24.	\$250	for 20 ds=?	30.	\$650	for 35 ds=?
25.	\$567	for 14 ds=?	31.	\$980	for 80 ds=?
26.	\$968	for 25 ds=?	32.	\$216	for 93 ds=?
27.	\$846	for 33 ds=?	33.	\$800	for 67 ds=?
28.	\$610	for 18 ds=?	34.	\$915	for 44 ds=?
29.	\$918	for 27 ds=?	35.	\$1200	for 93 ds=?

Answers: \$16.803+ and \$54.448+.

158. Merchants or bankers seldom reckon interest on cents. When under 50 they are rejected; otherwise, a dollar is added to the dollars.

It should also be observed that business men express their results in dollars and cents, to which usage the learner ought to conform. For practice or review, the teacher may require the exact answers instead of those given.

36. $1000.00 for 27 ds=?	40. $1799.14 for 93 ds=?
37. $71.97 for 47 ds=?	41. $387.66 for 67 ds=?
38. $61.80 for 45 ds=?	42. $199.44 for 41 ds=?
39. $190.27 for 16 ds=?	43. $450.22 for 29 ds=?
Total, $6.04.	Total, $35.75.
44. $719.99 for 11 ds=?	48. $1997.00 for 13 ds=?
45. $55.18 for 9 ds=?	49. $7.88 for 54 ds=?
46. $88.17 for 69 ds=?	50. $17.97 for 35 ds=?
47. $466.00 for 78 ds=?	51. $10.00 for 120 ds=?
Total, $8.47.	Total, $4.71.
52. $1000.00 for 97 ds=?	58. $1999.20 for 23 ds=?
53. $650.00 for 67 ds=?	59. $361.74 for 18 ds=?
54. $10.70 for 13 ds=?	60. $78.93 for 23 ds=?
55. $127.57 for 51 ds=?	61. $1467.20 for 34 ds=?
56. $368.17 for 118 ds=?	62. $7100.18 for 77 ds=?
57. $718.57 for 125 ds=?	63. $29.00 for 99 ds=?
Total, $46.76.	Total, $108.96.

159. *To compute 6% interest for any number of months.* When used in drawing notes or drafts, the month is always calendar; but when computing interest, 30 days is considered a month; hence, the note on page 142 would draw interest for one day more than the three months, exclusive of the days of grace, there being 94 days between the date and the maturity.*

This is a very important distinction, as will be seen by reference to Bank Discount.

160. Since there are half as many 60 days as months, we multiply the interest for 60 days by half the number of months.

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*In practice, when the note remains unpaid till maturity, interest would be charged for only the three months, as if it were 90 days.

64. Find the interest on \$620, for 4 months, at 6% per annum.

6.20=*Interest for* 60 *days.*
2=*Number of* 60 *days in* 4 *months.*
12.40=*Interest for* 4 *months.*

Find the interest, at 6% per annum, on the following:

65. \$750.25 for 6 mos=?	69. \$910.70 for 5 mos=?
66. \$218.87½ for 8 mos=?	70. \$876.33⅓ for 4 mos=?
67. \$1000.00 for 7 mos=?	71. \$937.79 for 3 mos=?
68. \$560.37½ for 9 mos=?	72. \$168.00 for 1 mo =?

Answers: \$91.46 and \$55.21.

73. Required the interest on \$350.25 for 7 mos. 15 ds.

\$3.50 =*Interest for* 60 *days.*
3½=*Number of* 60 *days in* 7 *months.*
10.50
1.75
875=*Interest for* 15 *days or* ¼ *of* 60.
\$13.125 *or* \$13.12².

Another way:

\$3.50 =*Interest for* 60 *days.*
4
14.00 =*Interest for* 8 *months.*
875=*Interest for* 15 *days off.*
13.125 *or* \$13.12².

Compute the interest at 6% pr. an. on the following:

74. *\$36.57 for 3 mos 20 ds.	78. \$1673 for 8 mos 8 ds.
75. \$2977 " 6 " 16 "	79. \$936 " 4 " 19 "
76. \$9856 " 4 " 15 "	80. \$281 " 3 " 27 "
77. \$2836 " 9 " 27 "	81. \$166.27 " 8 " 16 "

Answers: \$460.06 and \$103.40.

* When the principal is small and the time long, interest may be computed on cents.

Find the amounts of the following, and be particular to add the cents of the principal:

		Mos.	Days.				Mos.	Days.
82.	$250.15 for	6	2	86.	$501.19	for	6	6
83.	$380.67 "	10	10	87.	$219.12½	"	4	27
84.	$900.90 "	19	3	88.	$369.16⅔	"	8	12
85.	$216.67 "	8	20	89.	$1220.00	"	10	10

Answers: $1871.11 and $2408.91.

161. *To find the interest for years at 6% per annum.*

Business usage allows only 360 days to the year, which is 6 sixty days; hence, the interest for 60 days, multiplied by 6 times the number of years, gives the result.

90. Find the interest on $120 for 1 year, 4 months and 20 days, at 6% per annum.

```
  1.20
     8
  ----
  9.60
    40
  ----
$10.00 Ans.
```

EXPLANATION.—The interest for 60 days is 120 cents; for 1 year and 4 months it is 8 times 120 or 960 cents; and for 20 days it is ⅓ of 120, or 40 cents, making the sum $10.00—the interest required.

91. Find the interest of $240 for 3 years, 4 months and 10 days. *Ans.* $48.40

92. What is the interest of $1467.45 for 2 years, 6 months and 17 days? *Ans.* $224.21.

Find the interest of the following:

93. $321.00 for 2 years 3 months 15 days.*

94. $1767.00 for 7 years 4 months 21 days.

95. $897.25 for 3 years 6 months 27 days.

96. $898.57 for 2 years 7 months 25 days.†

97. $716.27 for 2 years 1 month 9 days.

Answers: $90.57, $44.14, $783.66, $192.41, $143.09.

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*Find the interest for 2 years 4 months, and deduct the interest for 15 days.

†Call this 2 years 8 months, and deduct the interest for 5 days.

Find the interest on the following:

98. $810.98 for 1 year 6 months 7 days.
99. $50.00 for 9 years 7 months 18 days.
100. $8.00 for 9 years 3 months 27 days.

Answers: $90.58, $73.94, $10.90, $4.48.

101. $3140.79 for 1 year 7 months 7 days=?
102. $795.17 for 2 years 1 month 1 day =?
103. $3.90 for 3 years 5 months 15 days=?
104. $1057.57 for 1 year 11 months 11 days=?

Total, $526.01.

105. $2674.57 for 1 year 8 months 21 days=?
106. $7143.45 for 2 years 1 month 18 days=?
107. $1742.67 for 1 year 9 months 13 days=?
108. $2100.00 for 2 years 1 month 1 day =?
109. $4109.85 for 1 year 6 months 17 days=?

Total, $2022.35.

110. $7856.00 for 1 year 1 month 29 days=?
111. $677.19 for 3 years 3 months 3 days=?
112. $287.17 for 1 year 7 months 16 days=?
113. $97.19 for 5 years 10 months 14 days=?
114. $10.10 for 1 year 3 months 19 days=?

Total, $743.95.

115. $57.87 for 2 years 6 months 14 days=?
116. $120.14 for 7 years 7 months 7 days=?
117. $340.00 for 9 years 1 month 24 days=?
118. $1657.00 for 1 year 3 months 24 days=?
119. $769.75 for 2 years 3 months 18 days=?

Total, $487.40.

The Teacher, when reviewing these exercises, may require his class to compute interest on cents, as is usually done in courts of justice.

162. *Having the interest at 6% per annum to find it at any other rate*, aliquots of 6 may be used.

At 1% it will be $\frac{1}{6}$ of that at 6%.
At 5% it will be $\frac{1}{6}$ less than at 6%.
At 7% it will be $\frac{1}{6}$ more than at 6%.
At 2% it will be $\frac{1}{3}$ of that at 6%.
At 4% it will be $\frac{1}{3}$ less than that at 6%.
At 8% it will be $\frac{1}{3}$ more than that at 6%.
At 9% it will be $\frac{1}{2}$ more than that at 6%.
At 10% it will be $\frac{10}{6}$ of that at 6%.

120. Find the interest on $250 for 1 year, 3 months and 20 days at 7% pr. annum.

$2.50 =*Interest for* 60 *days at* 6%.
$7\frac{1}{2}$=*Number of* 60 *days.*
125
1750
833=*Interest for* 20 *days.*
19.583=*Total interest at* 6%.
3.263=*Interest at* 1%.
$22.846=*Interest at* 7%.
or $22.85.

121. $798.18 for 6 yrs 1 mo 6 ds @ 9% pr. an.=?
122. $1000.00 for 4 yrs 2 mos 4 ds @ 7% pr. an.=?
Answers: $438.10, $292.44.

123. $340 for 2 yrs 2 mos 20 ds @ $2\frac{1}{2}$% pr. an.=?
124. $600 for 3 yrs 4 mos 15 ds @ $6\frac{1}{2}$% pr. an.=?
125. $850 for 1 yr 2 mos 12 ds at $8\frac{1}{2}$% pr. an.=?
Total, $237.22.

Find the interest of

126. $617.18 for 3 mos 18 ds @ 15% pr. an.
127. $460.74 for 2 mos 5 ds @ 18% pr. an.
128. $765.12 for 8 mos 16 ds @ 20% pr. an.
Total, $151.55.

Find the interest on the following at 10% per annum:

129. $710 for 92 days.	133. $496 for 91 days.
130. $1978 for 27 days.	134. $671 for 86 days.
131. $8889 for 128 days.	135. $100 for 104 days.
132. $75 for 117 days.	136. $269 for 73 days.
Total, $351.47.	Total, $36.91.

163. It is customary for bankers to lend money, and discount by the *month* instead of the *year*. This percentage is easily converted into 6% interest, and the work performed with as much ease as before.

1 % per month is 12% per year, or 2 times 6%.
$1\frac{1}{2}$% per month is 18% per year, or 3 times 6%.
2 % per month is 24% per year, or 4 times 6%.

Find the interest on the following:

137. $65 for 80 days @ 2 % per month.
138. $40 for 33 days @ $1\frac{1}{2}$% per month.
139. $190 for 63 days @ 2 % per month.
140. $700 for 93 days @ 3 % per month.

Total, $77.20.

Find the *amount* of the following

141. $710 for 36 days @ $1\frac{1}{2}$% per month.
142. $216 for 45 days @ 2 % per month.
143. $1800 for 57 days @ $1\frac{1}{3}$% per month.
144. $560 for 14 days @ $6\frac{1}{2}$% per month.

Total, $3367.89.

When computing interest, the ingenious student will contrive many ways for abbreviating his work. Sometimes he will take advantage of the aliquots of 100; at other times he will transpose the terms, and consider the days as dollars and the dollars as days, or he will reduce the rate mentally to 6%, if it is some other rate, and thus simplify as well as abbreviate. For instance, in the question 138, he might consider the \$40, \$120, and transposing the terms, which could be done instantly, he would simply have to multiply the 33 by 2, making 66c, the answer.

It will materially abridge the operation and expedite the labor, if the learner will observe to avoid the use of all lines, figures or marks that are not absolutely necessary. As, for instance, when using aliquot parts, to write only the results of division, as shown in the following example:

145. Interest of \$321 for 2 years, 1 month and 22 days at 10% per annum.

EXPLANATION.—Mentally it is found that there are $12\frac{1}{2}$ 60 days in 2 years and 1 month, to multiply by which we divide by 8. The division by 6 and the multiplication by 10 were performed simultaneously, giving \$68.836 or \$68.84 as the answer.

$$\begin{array}{r} 3.21 \\ \hline 40.125 \\ 1.07 \\ 107 \\ \hline 41.302 \\ \hline 68.836 \end{array}$$

In Bank Discount, the learner will find numerous questions upon which to exercise his ingenuity, and the judicious teacher will encourage him.

The method of finding True Discount, and other more difficult calculations, will be found at the end of Banking.

PARTIAL PAYMENTS, OR PAYMENTS BY INSTALLMENTS.

164. Notes, bonds, etc., drawing interest, are sometimes paid by installments, and the amounts thus paid indorsed on them. The legal rule for computing interest on installments may be expressed thus:

Apply the payment to the discharge of the interest, and if there is a remainder, subtract it from the debt. When the payment is less than the interest due, it is not applied to the discharge of the interest or debt, but is indorsed on the note until the installments exceed the interest. The sum of the installments is then taken from the amount due, and interest computed on the remainder as before.

1. $576. CINCINNATI, *Oct.* 9, 1857.

On demand, I promise to pay Robert Ingles, or order, Five hundred and seventy-six dollars, with interest. Value received. SAMUEL DUNNING.

On the note are the following indorsements:

Rec'd, Dec. 16, 1857, $100.
" *Feb.* 28, 1858, 3.
" *July* 27, 1858, 150.

Required the amount due September 3, 1858.

	Yrs.	Mos.	Ds.
From	1857	12	16
Take	1857	10	9
Difference,		2	7 *or* 67 *days.*

$576.00=*Amount of note.*
6.43=*Interest on* $576 *for* 67 *days.*
$582.43=*Total amount due.*
100.00=*Installment to be subtracted.*
$482.43=*Balance due.*

The second payment is less than the interest due and no calculation is required.

From December 16, 1857, to July 27, 1858, is 7 months 11 days.

$482.43=*Balance due.*
17.75=*Interest for* 7 *mos.* 11 *days.*
$500.18=*Amount due.*
153.00=*Amount of payments.*
$347.18=*Balance due.*

From July 27 to September 3, is 38 days.

$347.18=*Balance.*
2.19=*Interest for* 38 *days.*
$349.37=*Amount due September* 3, 1858.

2. $650. BOSTON, *June* 3, 1848.

For value received, I promise to pay, on demand, to H. Crooks, or order, Six hundred and fifty dollars, with interest at 6% per annum. J. F. DAVIS.

Indorsements.

Jan. 6, 1850, $95.
Oct. 13, 1850, 350.
June 3, 1855, 12.

Sept. 7, 1857, paid the balance; how much was it?

Ans. $405.92.

3. On a note drawn September 23, 1857, for $650, with legal interest, there are the following indorsements:

Oct. 4, $100.
Nov. 3, 2.
Dec. 19, 210.

April 3, 1858, the balance; how much was it?

Ans. $354.32.

4. On a note drawn October 3, 1856, for $1000, with 10% interest, are the following indorsements:

Nov. 30, $50.
Jan. 6, 1857, 100.
Feb. 9, " 5.
June 6, " 190.

Feb. 3, 1859, the balance; what was the amount?

XVIII. BANKING.

164. Banking is the business of dealing in money. Banking houses borrow and lend money, receive money for safe keeping, and exchange the money of one country for that of another. There are *National Banks*, *Public* and *Private Banks*, *Banks of Deposit*, *Banks of Issue* or *Circulation* and *Banks of Discount*.

A *National Bank* is one which issues notes secured by bonds of the United States, deposited with the United States Treasurer, and is doing business under the authority of the General Government.

A *Public Bank* is one that is owned by a joint stock company, who commit its management to some of their number chosen for that purpose. These persons are called *President and Directors.*

A *Private Bank* is one which is owned by one or more individuals, who attend to its business personally.

Banks of Deposit receive the ready money of merchants and others for safe keeping. Banks of deposit also loan money on interest and some pay interest on deposits.

Banks of Issue or Circulation manufacture and issue paper money, called *bank-notes* or *bills.* Many of these banks also receive money on deposit and do a discount business.

Banks of Discount lend money on interest, when suitable security is given.

The security required by banking institutions when loaning money, is a note or notes from the borrower, with the names of one or two responsible persons written on the back, in such a manner as to bind them for payment, should the drawer of the note fail to pay it at the proper time. The persons who sign their names thus are called *indorsers*, the writing the *indorsement*, and the person who writes the note and signs his name, the *maker* or *drawer.*

The notes given by borrowers may be their own* or

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* Notes are always known by the names of the makers. We speak of A B's note, though it is in our possession.

To hold an indorser, he must be notified of the maturity of the note at furthest on the next business day, if he resides in the same town or city; otherwise, a notice should be mailed to him within the same time.

those of other persons. If those of others, the borrower's indorsement alone is all that is usually asked.

The person to whom the note is made payable is called the *payee*, and the person possessing it, the *holder*. By the *face* of the note is meant the side containing the promise; also the amount for which it is drawn.

A negotiable note is one that can be transferred. To be negotiable, it should be drawn "to the order" of the payee, or to him "or order" or "to bearer." In the first two cases, to make it negotiable, the payee would have to write his name on the back. To the third class belong bank-notes, which may be passed by any holder without indorsement.

The phrase "value received" is not considered essential to a note, though it is well to insert it.

Days of Grace are three days allowed to the maker of a note beyond the stated time of payment. A note that is drawn payable 90 days after date, is not legally due until the 93d day.

165. MATURITY OF NOTES.

Notes falling due on Sunday or a national holiday are payable the day before.

A note dated the 28th, 29th, 30th or 31st of January, at one month, falls due on the last day of February without, or on the 3d day of March with grace; and a note dated the last day of February, at one month, falls due on the 28th of March, if not a leap-year; otherwise, on the 29th of March, plus the days of grace. Hence, a note drawn by the month, and dated on the last day of the month, falls due on the same day of the month, if the latter month have a corresponding day; otherwise, it falls due on the last day of the month. Thus, a note dated

November 30th, at two months, without grace, would fall due on the 30th of January; at three months, on the 28th or last day of February.

166. REMARKS ON NOTES.

1. A note need not be dated at the place where it is drawn, and can be made payable at any particular place the parties may agree upon; but to hold an indorser, demand must be made at the particular place specified. A note is also good if dated on Sunday.

2. When giving a note, the maker ought to fix the place of payment—say his bank or place of deposit—if the payee be agreeable.

3. In New York and other States, notes draw seven per cent. interest. Notes made payable in those States, accordingly, draw that rate of interest after maturity.

4. If made by more than one person, it is called a *joint* note, or *joint* and *several*. It is a joint note only, unless words are used to indicate individual responsibility, as "We jointly and severally," etc.

5. It is customary for merchants to deposit their notes in bank for collection, the bank, by this means, undertaking to see them paid, or using means to hold the indorsers.

6. A note drawn payable to the maker, and by him indorsed in blank, can be transferred—negotiated—without liability to the holder.

7. When taking up a note—paying it—the drawer ought to require the indorsement of the holder, deface his own signature and file the note away, as it may subserve the purpose of a receipt at some future time.

8. A note drawn under the seal of the maker is called a bond.

Indorsements are of various kinds, and, like the note itself, they require no stated form of words.

There are *Blank*, *Full*, *Special* and *Restrictive* indorsements.

A *Blank Indorsement* is made by the payee or holder writing his name on the back.

A *Full Indorsement*, or, as it is called, an *indorsement in full*, is made by writing such a transfer as "Pay to the order of A," or words to that effect, and signing the name.

A *Special Indorsement* is one made to suit a particular case, as when the indorser wishes to free himself of responsibility should the maker fail to pay the note. "Pay the the contents to B, or order, without recourse on me." "Payee."

A *Restrictive Indorsement* restrains the payment of the note to the party to whom it is indorsed, as "Pay the contents to C only." This kind of indorsement does not limit the payment when made by another person than the payee of the note.

The payee is called the *first indorser*, if he transfers the note, and the party to whom he transfers it, the *indorsee*. Should the latter again transfer it, he would be called the *second indorser* and the party to whom it was transferred, the *second indorsee*.

Indorsements ought to be written across the back, with the left end up, as shown on page 160.

167. Remarks on Indorsing.—1. A holder or indorsee can write over a blank signature a full or restrictive indorsement.

2. A holder may cancel all indorsements (signatures) but the first, and may cancel a full indorsement, except the signature of the first.

3. The object of a full indorsement is to prevent its transfer without the signature of the holder.

4. A note may be transferred after it is due, but the indorser is not then liable.

5. The payee or his agent must make the first transfer.

6. Any partner of a firm may indorse for all. After maturity, it ought to be done severally.

7. Agents may indorse for principals thus:

A B, *Principal*,
By C D, *Agent*.

8. An indorsement cancelled by mistake does not discharge the indorser.

FORMS OF PROMISSORY NOTES.

The following are given as examples for the learner to to copy. The printed blanks differ from these, and are easily filled.

A NEGOTIABLE NOTE.

CLEVELAND, *Sept.* 30, 1866.

Thirty days after date, I promise to pay to the order of J. C. Hutsinpiller Five hundred sixty $\frac{35}{100}$ dollars, at the First National Bank, this city, value received.

\$560$\frac{35}{100}$. E. R. FELTON.

A NON-NEGOTIABLE NOTE.

MILWAUKEE, *Apr.* 4, 1867.

Ninety days after date, I promise to pay to G. W. Nelson One thousand dollars, value received.

\$1000. R. C. SPENCER.

A JOINT AND SEVERAL NOTE WITH INTEREST.

CINCINNATI, *Aug.* 9, 1867.

On demand, six months after date, we severally and jointly promise to pay to the order of William Otte Six thousand dollars, value received, with interest at six per cent. per annum. GEORGE F. SANDS,
W. J. BREED.

FORM OF INDORSEMENT.

Pay to the order of
Henry Holbrook.
John Godar.

When writing the following notes, the learner ought to use cap paper, so that the notes be long enough for filing, and leave a margin above and below of not less than one and a half lines.

HOME EXERCISES IN DRAWING NOTES.

Draw notes from the following data:

1.	Date,	Aug. 18, 1868.	
	Place,	Louisville.	
	Maker,	Yourself.	
	Time,	On demand.	
	Face,	$678.14.	
	Payee,	S. E. Peyton.	

2.	Date,	Nov. 6, 1869.	
	Place,	Cincinnati.	
	Drawer,	Yourself.	
	Payee,	James Moore.	
	Face,	$375.50.	
	Time,	90 days.	

3.	Date,	April 9, 1870.	
	Place,	Pittsburgh.	
	Drawer,	S. S. Sargent.	
	Payee,	Yourself.	
	Face,	$168.75.	
	Time,	3 months.	

4.	Date,	Dec. 3, 1869.	
	Place,	Louisville.	
	Time,	On demand.	
	Drawer,	E. P. Bowers.	
	Payee,	....	
	Indorser,	R. Johnson.	
	Face,	$6745.15.	

5.

Date,	Oct. 19, 1860.
Place,	Wheeling, Va.
Drawer,	Archibald Warren.
Payee,	Yourself.
Indorser,	....
Face,	$1364.
Time,	6 months.
Indorsee,	Saml. Adams.

6.

Date,	Nov. 29, 1867.
Place,	New Orleans.
Drawer,	E. M. Small.
Payee,	H. C. Piner.
1st Indorser,	....
1st Indorsee,	Edward Epply.
2d Indorser,	....
Face,	$138.

7.

Date,	April 3, 1869.
Place,	New York.
Time,	4 months.
Face,	$1468.75.
Drawer,	James Todd.
Payee,	Robt. Emmet.
1st Indorser,	....
Indorsee,	Yourself.

8.

Date,	Jan. 1, 1867.
Place,	Chicago.
Time,	60 days.
Drawer,	R. E. Baird.
Payee,	J. E. Piner.
Indorser,	....
Indorsee,	J. J. Wood.
Face,	$1637.00.

9.

Place,	Mobile.
Date,	April 3, 1867.
Time,	One day after date.
Maker,	S. O. Miner.
Payee,	....
1st Indorser,	James Moore.
2d Indorser,	Yourself.
Face,	$200.

10.

Place,	San Francisco.
Date,	....
Time,	4 months.
Maker,	Henry Adams.
Indorsee,	Thomas Orr.
Payee,	Henry Adams.
Indorser,	....
Face,	$450.

11.

Place,	Boston, Mass.	Maker,	S. Poor.
Date,	July 10, 1870.	Payee,	....
Time,	30 days.	2d Indorser,	....
1st Indorser,	Robt. Penn,	2d Indorsee,	Wm. A. Miller.
1st Indorsee,	James Moon.	Face,	$396.57.

Draw the notes necessary for the following transactions:

12. Take H. P. Spike's obligation, at 1 day after date, for price of a house worth $1695.*

---

* To make this a bond, insert the word "seal" after the name.

13. Pay S. O. Mooney $167.87 with your note at 30 days, bearing interest.

14. Lend Henry Shotwell $1687, on his note, in such a way as to enable you to collect the money at any time.

15. Jno. Emerson's note, our favor, indorsed to G. Young, for $675 at 90 days, was protested for non-payment on January 3, 1866. What was its form?

16. Renew your note of $190, and take 3 months' time, allowing interest for the time the new note has to run.

17. Pay Jno. Sallust $367, on acct., with a note at 90 days.

18. Indorse C D's note of Sept. 3, 1860, for $600 at 90 days (your favor), in such a way that you will not be held responsible for payment.

19. Give W. A. Moore your note at 3 months, and draw it in such a way as will prevent him from negotiating it.

20. Exchange notes with your neighbor, each drawing at 90 days, and indorse.

21. Draw a copy of your note for $675, favor of H. Corncob, at 3 months, upon which you had to pay $2.47 interest on Sept 19, at 6%.

22. Pay Saml. Saul Pickwick the balance of $678.50, you owe him, by giving note in favor of Sam. Weller at 60 ds.

23. Take the note of J. Morgan Henry, Madison, Ind., for $360, at 3 months, and secure yourself against the statute which allows "relief," etc.

(After the words "value received," insert "without any relief whatever from valuation or appraisement laws." For form of note, see page 202.)

24. Indorse the receipt of $167.87, amount paid you on E. C. Johnston's note, dated Aug. 9, 1866, and drawn for $974.35 at 6 months, your favor.

CHECKS—DRAFTS—BILLS OF EXCHANGE.

A *Check* is a written order on a bank or its cashier for the payment of money in its possession belonging to the party making it. A *Draft** is similar to a check, but is written more formally and may be drawn upon any other than a banker; and a *Bill of Exchange* is a check or draft used for transmitting money to distant places.

A check, like a promissory note, may be drawn to bearer, to order, or be made payable to a particular person. When given to strangers, or for large sums of money, they should be drawn to order, so that those receiving them would have to indorse them before payment. In this way, they may also be used to subserve the purposes of receipts.

Checks may be antedated or post-dated. In the former case, they are payable on presentation, if there are funds of the drawer to meet them; in the latter, they will not be payable till the date arrives. Should a bank refuse to pay a check, the party to be sued is the maker, or transferrer, if it has been transferred.

If a bank pay a forged check, the bank, and not the person whose signature is forged, has to sustain the loss, though the forgery may be so well executed that it can not be detected by ordinary inspection.

When a check is so carelessly drawn that an alteration may be easily made, the loss arising must be borne by the drawer. For instance, the amount to be paid should commence at the extreme left of the line, and the part left unfilled be written with a curved line or other mark to prevent additions.

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*For forms of draft, see Exchange.

FORM OF CHECK.

STAMP.

No. 250. Cincinnati, Oct. 21, 1866.

Espy, Heidelbach & Co.,

Pay to HENRY MACK, *or Bearer.*

Five Hundred . . . $\frac{\times}{100}$ Dollars.

\$500.$\frac{00}{100}$ *H. J. Estcourt.*

ORDERS—DUE-BILLS.

A mercantile *Order* is a request for the delivery of goods or money. It differs from a draft in being more simple in its form, and its being usually drawn for the payment of goods instead of money. The following is the common form:

CINCINNATI, *Jan'y* 1, 1867.

MESSRS. JOHN SHILLITO & CO.

Will please let Samuel Steele, or bearer, have goods to the amount of One hundred dollars, and charge to my account.

\$100$\frac{00}{100}$. GEORGE KIDD.

To be more specific, the kind of goods and the price, wholesale or retail, might be inserted.

A *Due-bill* is simply an acknowledgement of a debt. It is usually drawn for a small sum and to settle an account.

Due-bills are considered to draw interest from date, though it is seldom exacted.

FORM OF A DUE-BILL.

INDIANAPOLIS, *Aug.* 3, 1868.

Due Henry Ellsworth, or order, Fifteen dollars. Value received.

$\$15\frac{00}{100}$. HENRY SPENCER.

RECEIPTS.

It may not be considered improper here to introduce a few forms of receipts, as they are essentially connected with the preceding subjects.

A *Receipt* should specify what it was given for, whether money, goods, note, etc., the amount for which it was given and the date, and the amount should be in writing. Receipts for sums over twenty dollars should have a stamp.

Receipts may be made on bills, notes, etc., or given separately, and need not be of any particular form.

RECEIPT ON ACCOUNT.

CINCINNATI, *Jan.* 1, 1868.

Received of Mr. John Cummins, One hundred and twenty-five dollars and 23 cents on account.

$\$125\frac{23}{100}$. JAMES MORGAN.

RECEIPT FOR MONEY ON NOTE.

PITTSBURG, *Apr.* 3, 1867.

Received of Alex. Cowley, One thousand dollars, to be credited on his note, my favor, dated Jan. 3, 1867, for six thousand dollars.

$1000. W. ALLAN MILLER.*

* A payment on a note should be receipted (indorsed) on the back of the same, and a statement made that a receipt was given.

RECEIPT IN FULL.

Received, Cincinnati, June 3, 1868, of Timothy Hay, Thirty-five $\frac{25}{100}$ dollars, in full to date.
$35.25. THOMAS E. YOUNGMAN.

RECEIPT FOR RENT ON ACCOUNT.

BOSTON, *Feb.* 27, 1867.

Received of Mr. Henry G. Judkins Sixty-eight dollars, on account of rent for No. 6 Long St.
$68.00. EDW. FABER.

RECEIPT FOR A NOTE.

PHILADELPHIA, *Jan'y* 30, 1869.

Received of Mr. James Thompson, his note, my favor, at ninety days, for Five hundred dollars to balance acct.
$500. JNO. F. GREEN, JR.

RECEIPT FOR MONEY IN ADVANCE.

COLUMBUS, *Apr.* 3, 1869.

Received of J. Q. A. Miller, Forty dollars, in advance, for live pork, to be delivered to him on or before October 1 1869, at 7 cents a pound.
$40. JAS. HOLLANDER, SEN.

RECEIPT FOR MONEY RECEIVED ON ACCOUNT OF ANOTHER.

CINCINNATI, *Oct.* 10, 1866.

Received from H. D. Brown, on the account H. W. Southey, Sixty-nine $\frac{87}{100}$ dollars, in full of acct to 1st inst.
69\frac{87}{100}$. Y. W. COOK, JR.

RECEIPT FOR RENT IN FULL.

NEW YORK, *Sept.* 12, 1866.

Received from R. Quinn, Twenty-five dollars, in full for rent of house, No. 96 Chestnut St., to 9th inst.
25\frac{00}{100}$. JAS. M. HALL.

RECEIPT FOR MONEY RECEIVED BY A CLERK.

NEW YORK, *Aug.* 3, 1870.

Received of H. Simon One hundred dollars on acct.

$\$100\frac{00}{100}$. JAMES MOORE.

Per JNO. WOOD, *Clk.*

HOME EXERCISES IN DRAWING RECEIPTS.

1. Receipt to John Roberts for \$100 on account.

2. Give H. C. Parker a receipt for \$50 in full of account.

3. Draw a receipt for $\$250\frac{57}{100}$ in favor of C. C. Martin, for his note of this date in settlement of account, and draw a copy of the note.

4. Receipt to Mrs. T. H. Henshaw for \$365, in part for rent of house, 1968 Vine Street.

5. Give your teacher a receipted bill for 257 barrels of flour at \$12.67 per barrel; drayage, \$8.50.

6. Let William Edmundson have your receipt for \$67, on his note, of the 3d of last month, at 6 months, your favor; and draw a copy of the note, showing a receipt to be made on it. Face of note, \$936.

7. Make out to your teacher, this date, the bill on page 111, and take a note in settlement.

8. Receipt for \$150 cash on the first bill, page 112, making it to William Nelson.

9. As clerk of H. J. Estcourt, make out the first bill on page 113 to your teacher, allowing him a discount of 5% for cash.

10. Give your due-bill and an order on your grocer, to teacher, for tuition one session of 5 months, amounting to \$85. Make the order for a balance of \$15.75 in grocer's hands, and draw all the papers.

TIME TABLE

FOR COMPUTING INTEREST AND AVERAGE.

Number of days from 1st of January to any other day of the year. In leap-years, add 1 to the days after 28th of February.

Day of Mo....	January	February....	March........	April........	May	June	Day of Mo....	July..........	August........	September ...	October	November ...	December....	Day of Mo....
1	0	31	59	90	120	151	1	181	212	243	273	304	334	1
2	1	32	60	91	121	152	2	182	213	244	274	305	335	2
3	2	33	61	92	122	153	3	183	214	245	275	306	336	3
4	3	34	62	93	123	154	4	184	215	246	276	307	337	4
5	4	35	63	94	124	155	5	185	216	247	277	308	338	5
6	5	36	64	95	125	156	6	186	217	248	278	309	339	6
7	6	37	65	96	126	157	7	187	218	249	279	310	340	7
8	7	38	66	97	127	158	8	188	219	250	280	311	341	8
9	8	39	67	98	128	159	9	189	220	251	281	312	342	9
10	9	40	68	99	129	160	10	190	221	252	282	313	343	10
11	10	41	69	100	130	161	11	191	222	253	283	314	344	11
12	11	42	70	101	131	162	12	192	223	254	284	315	345	12
13	12	43	71	102	132	163	13	193	224	255	285	316	346	13
14	13	44	72	103	133	164	14	194	225	256	286	317	347	14
15	14	45	73	104	134	165	15	195	226	257	287	318	348	15
16	15	46	74	105	135	166	16	196	227	258	288	319	349	16
17	16	47	75	106	136	167	17	197	228	259	289	320	350	17
18	17	48	76	107	137	168	18	198	229	260	290	321	351	18
19	18	49	77	108	138	169	19	199	230	261	291	322	352	19
20	19	50	78	109	139	170	20	200	231	262	292	323	353	20
21	20	51	79	110	140	171	21	201	232	263	293	324	354	21
22	21	52	80	111	141	172	22	202	233	264	294	325	355	22
23	22	53	81	112	142	173	23	203	234	265	295	326	356	23
24	23	54	82	113	143	174	24	204	235	266	296	327	357	24
25	24	55	83	114	144	175	25	205	236	267	297	328	358	25
26	25	56	84	115	145	176	26	206	237	268	298	329	359	26
27	26	57	85	116	146	177	27	207	238	269	299	330	360	27
28	27	58	86	117	147	178	28	208	239	270	300	331	361	28
29	28		87	118	148	179	29	209	240	271	301	332	362	29
30	29		88	119	149	180	30	210	241	272	302	333	363	30
31	30		89		150		31	211	242		303		364	31

TIME TABLE

FOR COMPUTING INTEREST AND AVERAGE.

Number of days from 1st of July to any other day of the year. In leap-years, add 1 to the days after 28th of February.

Day of Mo...	July......	August......	September ...	October	November ...	December...	Day of Mo...	January	February.....	March.......	April.........	May..........	June	Day of Mo...
1	0	31	62	92	123	153	1	184	215	243	274	304	335	1
2	1	32	63	93	124	154	2	185	216	244	275	305	336	2
3	2	33	64	94	125	155	3	186	217	245	276	306	337	3
4	3	34	65	95	126	156	4	187	218	246	277	307	338	4
5	4	35	66	96	127	157	5	188	219	247	278	308	339	5
6	5	36	67	97	128	158	6	189	220	248	279	309	340	6
7	6	37	68	98	129	159	7	190	221	249	280	310	341	7
8	7	38	69	99	130	160	8	191	222	250	281	311	342	8
9	8	39	70	100	131	161	9	192	223	251	282	312	343	9
10	9	40	71	101	132	162	10	193	224	252	283	313	344	10
11	10	41	72	102	133	163	11	194	225	253	284	314	345	11
12	11	42	73	103	134	164	12	195	226	254	285	315	346	12
13	12	43	74	104	135	165	13	196	227	255	286	316	347	13
14	13	44	75	105	136	166	14	197	228	256	287	317	348	14
15	14	45	76	106	137	167	15	198	229	257	288	318	349	15
16	15	46	77	107	138	168	16	199	230	258	289	319	350	16
17	16	47	78	108	139	169	17	200	231	259	290	320	351	17
18	17	48	79	109	140	170	18	201	232	260	291	321	352	18
19	18	49	80	110	141	171	19	202	233	261	292	322	353	19
20	19	50	81	111	142	172	20	203	234	262	293	323	354	20
21	20	51	82	112	143	173	21	204	235	263	294	324	355	21
22	21	52	83	113	144	174	22	205	236	264	295	325	356	22
23	22	53	84	114	145	175	23	206	237	265	296	326	357	23
24	23	54	85	115	146	176	24	207	238	266	297	327	358	24
25	24	55	86	116	147	177	25	208	239	267	298	328	359	25
26	25	56	87	117	148	178	26	209	240	268	299	329	360	26
27	26	57	88	118	149	179	27	210	241	269	300	330	361	27
28	27	58	89	119	150	180	28	211	242	270	301	331	362	28
29	28	59	90	120	151	181	29	212		271	302	332	363	29
30	29	60	91	121	152	182	30	213		272	303	333	364	30
31	30	61		122		183	31	214		273		334		31

USE OF THE PRECEDING TABLES.

1. *From 1st of July to 9th of June, how many days?*

Opposite 9, in the last column, is 342, the answer.

2. *From 2d of October to 17th November, how many days?*

Opposite 2, in the middle column, is 93, in October column, and opposite 17 is 139, in the November column. The difference is 46.

These tables are specially adapted for averaging accounts, taking 1st of January and 1st of July as dates from which to reckon.

DISCOUNTING NOTES.

168. Discounting notes consists in buying them at less than their nominal value, or the amount for which they are drawn. The difference between the nominal value and the price paid is called discount.

169. Bankers prefer lending money on short time, and by the day, instead of by the month. Notes are usually drawn for 30, 60 or 90 days; and interest is always charged on the days of grace.

170. There are two kinds of discount: *True Discount*, which is the interest paid in advance on the *present value* of a note, and *Bank Discount*, which is interest paid in advance on the *face* of the note. The latter resembles compound interest, as it is interest on both interest and principal.*

When a note is discounted in bank, the interest of the note for the time it has to run, and at the banker's rates, is deducted from the sum called for by the note. The bal-

*The present worth of a note drawn for $100, payable in a year at 6 per cent., is $94.34, and the interest is $5.56; that is, the principal and interest together are equal to $100, or the face of the note; so when a banker discounts from the face of a note, he discounts on both principal and interest.

ance is called the *proceeds*. This species of discount is therefore reckoned in the same way as interest. Bankers reckon interest on every day intervening between the day of discount and that of maturity, including the latter.

171. When a note is drawn by days, subtract the expired term from the number of days for which it is drawn, plus the days of grace; but when drawn by the month, first find the day of maturity and reckon the whole number of days from the day of discount to that date.

1. Discounted on the day of date, how much discount should be deducted from a note of $500 at 90 days?*

	$5.00=	*Interest for*	60	*days.*	
	2.50=	" "	30	"	
	.25=	" "	3	"	(*grace.*)
Ans.	$7.75				

2. $1500. COLUMBUS, *Jan.* 8, 1859.

Sixty days after date, I promise to pay Messrs. M'Ewen and Banfill One thousand five hundred dollars, value received. GEO. K. TENNEY.

Required the discount at 6% per an. *Ans.* $15.75.

3. $3500. WHEELING, *Oct.* 3, 1858.

Ninety days after date, I promise to pay John M'Culloch, or order, at First National Bank, three thousand five hundred dollars, value received.

MILO G. DODDS.

Required the proceeds at 6% per an. *Ans.* $3445.75

4. Find the proceeds of a note for $120 at 60 days at $\frac{1}{2}$% per month.

5. Required the proceeds of a note dated Jan. 1, 1866, and drawn for $575.75 at 90 days.

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*When the rate is not named, six per cent. per annum is understood.

6. What is the bank discount on a note of \$450 for 60 days at 2% per month?*

7 to 12. Find the proceeds of the following:

On \$350 at 30 days at $1\frac{1}{2}$% per month; on \$1678.25 at 90 days and $1\frac{1}{4}$% per month; on \$670 at 60 days and 2% per month; on \$1749.57 at 90 days, $1\frac{1}{2}$% per month; on \$688 at 90 days, $2\frac{1}{4}$% per month; on \$6784 at 60 days, $1\frac{3}{4}$% per month.

Answers: \$118.74, \$566.82, \$344.22, \$18.90, \$1668.19, \$6534.69, 640.01, \$1613.22, \$6594.69, \$641.86.

Find the discount on the following:

13. \$1310.00 for 60 days @ 2 % per month.
14. \$746.87 for 90 days @ $1\frac{1}{2}$% per month.
15. \$219.56 for 30 days @ 1 % per month.
16. \$1867.25 for 20 days @ $2\frac{1}{2}$% per month.
17. \$1367.00 for 15 days @ 3 % per month.

Total, \$152.57.

18. A note drawn on February 13, 1866, for \$900, at 90 days, was discounted on March 23, at 2% per month; how much was paid to the borrower? *Ans.* \$867.

19. What proceeds should be paid on a note of \$346 at 90 days, drawn on November 3, and discounted on December 7, at $1\frac{1}{2}$% per month? *Ans.* \$335.79.

20. A note for \$689, made September 9, payable in 60 days, was discounted on October 5, at 2% per month; what was the discount? *Ans.* \$16.99.

While the cents in the principal are rejected in computing interest and discount, they are always reckoned when finding the amount or proceeds.

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*Such questions as these may be abbreviated by mentally increasing the days or principal in the ratio that the rate is to 6 per cent. In this case, reckon interest on \$450×4 or \$1800 at 6 per cent. per annum.

Required the discount on the following:

	Face of note.	Date.	Time.	When disc'd.	Rate of discount.
21.	\$167.50	Jan. 3, 1869,	60 ds,	Feb. 7,	2 % per mo.
22.	\$9876.00	Feb. 7, 1869,	90 ds,	Mar. 12,	$2\frac{1}{2}$% per mo.
23.	\$789.00	Jun. 18, 1869,	30 ds,	July 3,	$1\frac{1}{2}$% per mo.
24.	\$1897.00	Feb. 21, 1869,	90 ds,	Apr. 1,	$1\frac{1}{2}$% per mo.

Total, \$555.24.

Find the proceeds of the following:

25.	\$676.37	Apr. 3, 1869,	90 ds,	May 9,	2 % per mo.
26.	\$679.39	Mar. 9, 1869,	30 ds,	Apr. 3,	$2\frac{1}{2}$% per mo.
27.	\$7168.00	June 13, 1869,	60 ds,	July 9,	$1\frac{1}{2}$% per mo.
28.	\$816.37	Aug. 12, 1869,	30 ds,	Sep. 6,	$2\frac{1}{4}$% per mo.

Total, \$9172.40.

29. A note for \$4378.35, dated February 1, 1867, at 4 months, was discounted on May 16 at 10% per annum; required the proceeds.

Solution.—This note falls due June 4. From May 16 to that date is 19 days, giving for discount on \$4378, \$23.11. Proceeds, \$4355.24.

30. Required the proceeds of a note dated September 3, 1867, for \$396.87, at 3 months, and discounted on October 5, at $1\frac{1}{2}$% per month.

31. A note dated September 30, at 3 months, and drawn for \$1367.56, was discounted on October 1, at 2% per month; what were the proceeds?

32. Required the proceeds of a note dated December 31, 1867, drawn for \$5363.75, at 2 months, and discounted February 1, at $1\frac{1}{4}$% per month?

33. A note for \$1000, dated Februray 28, 1866, at 2 months, was discounted on March 20, at 1% per month; required the proceeds.

Answers: \$986.00, \$384.56, \$1282.75, \$5296.70 \$529.50, Find the proceeds of the following:

	Face of note.	Date.	Time.	When disc'd.	Rate of discount.
34.	\$2676.00,	Jan. 9, 1869,	90 days,	Feb. 1,	$1\frac{1}{4}\%$ pr mo.
35.	\$7187.00,	Feb. 3, 1869,	60 days,	Mar. 13,	$1\frac{1}{2}\%$ pr mo.
36.	\$768.21,	Mar. 6, 1869,	30 days,	Apr. 3,	2 % pr mo.
37.	\$314.00,	Apr. 7, 1869,	90 days,	May 15,	2 % pr mo.

Total discount, \$181.95.

	Am't of note.	Date.	Time.	When discounted.	Rate of disc't.
38.	\$6785,	Dec. 6, 1868,	6 mo.	Dec. 29,	2 % pr mo.
39.	\$3748,	Jan. 3, 1868,	5 mo.	Feb. 3, 1868,	$2\frac{1}{2}\%$ pr mo.
40.	\$6983,	Mar. 9, 1868,	4 mo.	June 8, 1868,	$1\frac{1}{2}\%$ pr mo.

Total proceeds, \$16272.70.

	Amount.	Date.	Time.	When disc'd.	Rate of discount.
41.	\$3784,	May 6,	2 mo.	July 3,	2 % per mo.
42.	\$6987,	Jun. 8,	3 mo.	Aug. 27,	$1\frac{1}{2}\%$ per mo.
43.	\$7854,	July 24,	4 mo.	Sept. 17,	1 % per mo.

Total proceeds, \$18371.58.

XIX. TRUE DISCOUNT.

172. True Discount is the difference between the *present worth* of a note and the amount for which it is drawn.

The *present worth* of a note or bill due at a future time without interest, is such a sum as would, if put at interest for the same time and rate, amount to the debt; and the difference between this sum and the debt is the *discount.*

Except in courts of justice, this kind of discount is seldom used, business men preferring bank discount for its simplicity.

Discount on Interest-bearing Notes.—Bankers discount off the value of the note, including interest, at maturity; while Real Estate Brokers discount off the face of the note, plus the accrued interest, at the time of discount.

1. What is the true discount on a note of $700 for 90 days at 6%?

The amount of a dollar for 93 days is $1.0155, by which, if we divide $700, we will find the present worth.

OPERATION.
```
$1.0155)700.0000(689.315
        609 30
        ------
         90700
         81240
         -----
          94600
          91395
          -----
           32050
           30465
           -----
            15850
            10155
            -----
             56950
             50775
```

NOTE.—The interest on $1 for 90 days is .0155. The present value of $1.0155, for 93 days is, therefore, $1, and, accordingly, the present value of $700 for 93 days is $700 divided by $1.0155 or $689.31, and the discount $700—$689.31, or $10.69.

PROOF.—The interest on $689.315, 93 days, is $10.683, which, if added to the principal, will give $699.999 or $700.

The pupil can prove his calculations by interest.

2. What is the true discount on a note of $575 for 90 days at 6%?

3. What is the true discount on a note of $137.09 for 90 days at 6%?

4. What will be the proceeds of a note of $1878.67 at 90 days, true discount?

5. A note for $485.44, payable in 30 days after date, is worth how much, true discount?

173. *To find the* FACE OF A NOTE, *when the proceeds, time and rate are given.*

6. Required the principal when the proceeds are $275.23, the time 93 days and the rate 2% per month.

Interest on $1 for 93 days at 2% per month=.062.

Proceeds of $1=$1.000—.062=.938.

Since there are as many dollars in the principal as the proceeds of $1 is contained times in the proceeds given, $275.230÷.938 will give the principal required, $293.42+.

PROOF.—Interest on \$293.42 for 93 days at 2% per month=18.19+, which, subtracted from \$293.42, leaves \$275.23, the proceeds.

7. The proceeds are \$212.60, time 63 days, rate $1\frac{1}{2}$% per month; required the principal.*

8. What principal will realize \$120 proceeds in 6 months at 10% per annum?

9. The time is three months, rate 10% per an., proceeds \$168.97; what is the principal?

10. The rate is 12% per annum, proceeds \$693.75, time 4 months; required the principal.

174. *To find the* RATE PER CENT., *when the principal, interest and time are given.*

11. The principal is \$300, time 60 days, interest \$5; required the rate.

Interest on \$300 for 60 days at 6%=\$3. At 1%=.50. It is obvious that the rate will be as great as the number of times 1% is contained in the interest given. Hence, \$5.00÷50=the rate, 10%.

PROOF.—Interest on \$300 for 60 days at 10%=\$5.

12. The principal is \$396.15, time 13 months 9 days, interest \$26.34,3; required the rate.

13. What is the rate per cent. on \$144 for 5 days, when the interest is 24 cents?

14. Required the rate on \$250 for 60 days, when the interest is \$3.50.

175. *To find the* TIME, *when the principal, rate per cent. and interest are given.*

Grace being allowed only on notes and drafts, where neither is named, it is not reckoned.

---

*The learner can prove his work by computing interest on the principal found.

15. The principal is \$1440, rate 10% per annum, interest \$37.50; required the time.

Interest on \$1440 for 1 day at 10%=40 cents.

Since there are as many days as the interest for 1 is contained times in the interest given, \$37.50÷40=93¾ or 94 days.

PROOF.—Interest on \$1440 for 94 days at 10% per annum=\$37.60.*

16. The principal is \$1674, rate 2% per month, interest \$59.87; required the time.

17. In what time will a note for \$600, at 6% per annum, draw \$27.50 interest?

18. A note for \$375 drew \$21 interest at 6% per annum; how long did it require to do it?

19. A merchant wishes to know the time it will take a balance of \$917.50 to make \$60.80, with interest at 10%.

GENERAL EXERCISES.

1. What is the bank discount on a note of \$375, drawn at 90 days, at 1% per month?

2. What amount of proceeds should I receive from a note of \$796, drawn at 60 days, 2% per month?

3. What is the present value of a note drawn for \$600 at 30 days?

4. What amount of money should I receive on a note of \$675, discounted at 35 days (having 35 days to run), 1½% per month?

The Teacher can increase these exercises to any extent, as the pupils have to furnish proof of their work; hence, only a few have been given under each article.

* Interest is never reckoned on the fraction of a day, hence the difference.

5. June 3, discounted my note of $350 at 10%, having 30 days to run; required the discount.

6. February 6, 1858, had A. Seers's note of $500, dated 20th December, 1857, discounted at $1\frac{1}{2}\%$ per month, time to run, 33 days; what were the proceeds?

7. June 3, 1858, discounted my note of December 9, 1857, drawn for $1000, at 12 months, 10%; what had I to pay?

8. October 3, lifted my note of 17th February, drawn for $1000, at 60 days; what amount of money had I to pay?

9. July 30, had M. Norton's note of $360, dated 23d July, at 90 days, discounted at 2% per month, what sum of money did I receive?

Answers: $596.72, $11.63, $762.57, $663.19, $1027.50, $2.92, $339.36, $946.67, $491.75.

XX. COMPOUND INTEREST.

176. In Compound Interest, the interest is converted into principal, every quarter, half year or year. Capital is thus more rapidly increased than by simple interest.

Any person acquainted with the principles of simple interest will readily understand how to compute this.

When there is a settlement of accounts between the parties, after interest has become due and interest is charged in the settlement, interest may be allowed upon the balance found due by the settlement. So an agreement, after interest is due, to turn it into principal is valid. Where there is a contract between the parties for the payment of interest annually, if not paid, simple interest may be allowed upon the interest from the time it is due.*

* Swan.

1. What is the compound interest on $1000 for $2\frac{1}{2}$ years at 6%, payable semi-annually (half-yearly)?

The interest of $1000 *for* 6 *mos.*,	$30.00
Add the principal,	1000.00
Amount for 6 *mos.*,	1030.00
Interest on $1030 *for* 6 *mos.*,	30.90
Amount for 1 *year*,	$1060.90
Interest on $1060.90 *for* 6 *mos.*,	31.827
Amount for 18 *mos.*,	$1092.727
Interest on $1092.727 *for* 6 *mos*,	32.78181
Amount for 2 *years*,	$1125.50881
Interest on $1125.50881 *for* 6 *mos.*,	33.76526
Amount for 2 *years* 6 *mos.*,	$1159.27407
By deducting the principal,	1000.00
We have the comp'd int. for $2\frac{1}{2}$ *yrs.*,	$159.27

2. What is the compound interest and amount of $672 for 4 years, at 6% per annum?

By computing interest on $1 for a number of years, it will be found that the second amount is equal to the square of the first; the third amount to the cube of the first; the fourth amount to the fourth power of the first; each power corresponding to the number of years. Hence, to find the amount of any principal for any number of years, it is only necessary to multiply the principal by the amount of $1 for the time and rate. Taking the last example, 1.06^4=the amount for 4 years, which, multiplied by 672= amount required.

3. Find the amount of $375 for 20 years, at 6% compound interest, reckoned annually.

At 6 per cent., money will double itself in 11 years, 10 months and 21 days; at 5 per cent., in 14 years, 2 months and 15 days; at 3 per cent., in 23 years, 5 months and $10\frac{1}{4}$ days.

4. Required the compound interest on $600 for 5 years at 7%.

Answers: $241.53, $827.68, $848.38, $176.38, $137.50.

XXI. AVERAGE.

177. AVERAGE signifies mean or medium. An average number or quantity is one which has an intermediate value between two or more numbers or quantities. The average between 3, 4 and 5 is 4.

178. The average may be found by dividing the sum of the quantities by their number.

1. Find the average between 3, 4, 5 and 8.

$$\begin{array}{r} 3 \\ 4 \\ 5 \\ \underline{8} \\ 4)\overline{20} \\ \overline{5} \end{array}$$

EXPLANATION.—In these *four* numbers there are 20 parts; in each of 4 number there are as many equal parts as 4 is contained in 20, viz., 5.

2. What is the average price of 5 cows, which cost, respectively, $50, $60, $70, $80 and $90?

3. The wages of 9 hands in a factory are $5, $6, $7, $8, $10, $11, $12, $15 and $16, respectively; what is the average?

4. Required the average length of the following pieces of calico: 37 yds., 35 yds., 31 yds., 30 yds., 32 yds., $27\frac{1}{2}$ yds., $29\frac{1}{2}$ yds., $30\frac{1}{4}$ yds., $29\frac{3}{4}$ yds.

5. Traveling 5 days, at the rate of 18 miles the first day, 20 the next, $20\frac{1}{2}$ the next, $22\frac{1}{2}$ the next and 25 the next, what was the average speed per day?

Answers: 70, $33\frac{1}{3}$, 10, $21\frac{1}{5}$, $21\frac{1}{15}$.

EQUATION OF PAYMENTS.

179. When average is applied to the settlement of accounts, the process is called *Equation of Payments* or Equation of Time.

Merchants and manufacturers sometimes sell their goods on credit, the time varying from three to nine months, and their customers making numerous purchases before settlement. The object of equation of payments is to obtain an average date of payment for these purchases.

6. I owe \$3 payable in 2 months, \$4 payable in 3 months and 5 payable in 4 months, what will be the average time of payment for the whole amount?

The use of \$3 for 2 mos=that on \$1 for 3×2= 6 mos.
The use of \$4 for 3 mos=that on \$1 for 3×4=12 mos.
The use of \$5 for 4 mos=that on \$1 for 4×5=20 mos.

\$12 38 mos.

Hence, the use of \$12 for those several months is equal to that of \$1 for 38 months; for 12, it will be $\frac{1}{12}$ as much, or $3\frac{1}{6}$ months or 3 months 5 days.

7. A merchant sells a bill of goods amounting to \$4000, to be paid as follows: \$400 in 30 days, \$600 in 60 days, \$1000 in 90 days and the balance in 4 months, or 120 days; what would be a mean or average time of payment for the whole?

A credit of	\$400	for	30 ds.	is the same as	a credit	on	\$1	for	12000	ds.
"	600	"	60 "	"	"	"	1	"	36000	"
"	1000	"	90 "	"	"	"	1	"	90000	"
"	2000	"	120 "	"	"	"	1	"	240000	"
	4000								378000	

On \$1 there is a credit for 378000 days.

On \$4000, there is a credit for 378000 days divided by 4000=$94\frac{1}{2}$ days.

That is, the \$4000 might be paid in $94\frac{1}{2}$ days, or on the 95th day, without either party sustaining loss by interest.

8. A merchant sells goods to the amount of \$1700, \$500 payable in 60 days, \$300 payable in 90 days and \$900 payable in 30 days; what is the average time of payment of the whole?

9. Sold a bill of goods amounting to \$700, $\frac{1}{4}$ of which is payable in 90 days, $\frac{1}{4}$ in 4 months and $\frac{1}{2}$ in 6 months; required the average time of payment.*

180. *To find the average date of purchase.*

10. Purchased goods as follows; what was the average date of purchase?

December 31, a bill of \$300; January 3, a bill of \$100; January 9, a bill of \$200, January 18, a bill of \$800; January 23, a bill of \$500.

$$\begin{array}{rr}
300\times\ 0= & \\
100\times\ 3= & 300 \\
200\times\ 9= & 1800 \\
800\times 18= & 14400 \\
500\times 23= & 11500 \\
\hline
1900 &)\overline{28000} \\
 & 14\frac{14}{19}
\end{array}$$

EXPLANATION.—The first was due at the time of purchase; the second, three days after; the third, nine days after, etc.

REMARK.—If the amounts above were equal, and the intervals also equal, the average date of purchase would be on Jan. 9, because it is midway between the first and last dates.

Or 15 days after December 31, the date of the first purchase, which brings the time up to January 15.

If these debts had been contracted on a credit of three months, a note dated January 15 would be given to settle the bill.

The time tables (pp. 168, 169) are admirably adapted for averaging, prepared as they are for the two common periods of settlement, January and July. For account sales, the book-keeper should prepare similar ones for each month.

* For answers, see end of chapter.

11. The following goods were sold on a credit of 90 days:

Required the average date of purchase, or date of note.

Jan.	1,	Invoice of Coffee	....	$1000.00
Jan.	6,	" " Sugar	....	3500.00
Mar.	9,	" " Sunds	....	9734.00
Mar.	13,	" " "	....	976.50
Apr.	3,	" " "	....	1037.00
				$16247.50

Required the *date of maturity* of a 3 months' note, grace included.

12. Sept.	3,	Invoice of Calicoes	....	$3150.00
"	19,	" " Muslins	....	1174.00
"	20,	" " Silks	....	3500.00
Oct.	19,	" " Sundries	....	1743.00
				$9567.00

Find the equated time of payment for the following, or date of a sixty-days' note:

13. Apr.	3,	$167.25 *	14. May	7,	$674.40
"	9,	374.00	Jun.	7,	168.37
"	19,	176.00	"	10,	370.20
"	20,	371.00	"	15,	167.00
"	25,	197.87	"	19,	679.60
"	30,	300.00	July	23,	679.45
May	9,	150.57	Aug.	18,	993.18
"	23,	720.18	"	19,	875.57

181. *When goods are purchased at different dates and on different lengths of credit.*

15. Purchased the following bills of merchandise; re-

* When the cents are under 50, reject them; otherwise, add a dollar to the dollars.

quired the average date of maturity, or the equated time of payment for all:

Apr. 3,	a bill of	$250	on 3	months' credit.
Apr. 9,	" "	157	" 6	" "
May 7,	" "	250	" 4	" "
Jun. 9,	" "	320	" 2	" "

If we substitute the *date of maturity* of each of these bills for the *date of purchase*, and arrange them in the order of time, we shall have a problem in all respects similar to those under last Art.

The first bill falls due July 3;* the second, October 9; the third, September 7; and the fourth, August 9. Arranged in the order of time, they appear thus:

July 3,	$250 ×	2 =	500
Aug. 9,	$320 ×	39 =	12480
Sep. 7,	$250 ×	68 =	17000
Oct. 9,	$157 ×	100 =	15700
	$977		)45680(46
			3908
			6600
			5862
			738

By the July table we find the difference of time to be 2, 39, 68 and 100 from the *first*, and the average time, 46 and over a half, or 47 days; 47 days after July 1 is August 17.

When time tables are used, especially such as are adopted for periodical settlements, as those in banking, the purchases need not be arranged in the order of date, as above.

16. Find the average date of payment for the following:

Feb. 3,	Mdse,	3 mos,	$678.59	July 27,	Mdse,	30 ds,	$1500.00
Mar. 9,	"	2 "	243.75	Aug. 9,	"	90 "	175.50
Apr. 13,	"	4 "	1000.00	Oct. 3,	"	2 mos,	1673.13
Jun. 17,	"	30 ds,	976.54	Nov. 18,	"	3 "	987.65
19,	"	3 mos,	786.15	Dec. 13,	"	3 "	685.18

*Days of grace are not allowed on invoices.

182. *When cash goods are sold with others on credit.*

Goods are sometimes classed as *cash* or *time.* Those designated cash do not always realize present payment, any time within a month being considered cash; and even at the end of a month cash has not been exacted, the understanding being that a debt contracted on cash terms draws interest from date. When computing average, cash bills are considered due on the day of purchase.

17 JOHN MICKLEBOROUGH.

Jan.	1,	To Mdse on 2 months	$900.00
Feb.	6,	" " " 3 "	800.00
Mar.	8,	" " " cash	700.00
Apr.	17,	" " " "	600.00

$900 \times 59 = 53100$
$800 \times 125 = 100000$
$700 \times 66 = 46200$
$600 \times 106 = 63600$
$300\cancel{0}\ )\ 26290\cancel{0}$
$87\frac{19}{30}$ 88 days after Jan. 1 or Mar. 30.

EXPLANATION.—2 months from January 1 is March 1, or 59 days; 3 months from February 6 is May 6, or 125 days from January 1; March 8 is 66 days from January 1, and April 17 is 106 days from January 1. Hence, March 30 is the average date of maturity or payment.

Find the average date of maturity of the following:

18 D. R. SOMERSET.

Jan. 1,	On 3 mos,	$600.00
Feb. 3,	For cash,	670.00
Mar. 3,	On 6 mos,	950.00
May 3,	For cash,	550.00

19 A. E. NELSON.

July 1,	On 3 mos,	$675.00
13,	On 2 mos,	619.54
19,	For cash,	147.67
23,	On 5 mos,	678.44

20

Sept.	3,	At 30 days,	$937.15
	9,	At 90 days,	897.78
	17,	Cash,	619.18
Oct.	3,	At 60 days,	777.00

Required the amount due on each of the following on July 1:

21. Jan. 9, $678.44 at 60 days; 20th, $419.88 at cash price; 29th, $789.14 at 3 mos.

22. April 9, $1678 on 3 mos; June 18, $1000 at cash price; 21st, $879.55 on 60 days; 23d, $371.19 cash; 29th, $785.25 cash.

183. *Average applied to statements when there are credits.*

Dr.	C. A. Walworth.		*Cr.*
1867.		1867.	
Jan. 1, To Mdse,	$300.00	Feb. 9, By Mdse,	$200.00
Mar. 3, " "	500.00	23, " "	100.00

$$
\begin{array}{llll}
300\times 0 & & & 200\times 39=7800 \\
500\times 61= & 30500 & \textit{Dr. products.} & 100\times 53=5300 \\
\overline{800} & \underline{13100} & \textit{Cr. products.} & \overline{300} \quad \overline{13100} \\
300 & 50\cancel{0})1740\cancel{0} & & \\
\hline
\textit{Bal. due } 500 & 34\tfrac{4}{5} \textit{ or } 35 \textit{ days from Jan. } 1 \textit{ or Feb. } 5. & &
\end{array}
$$

Explanation.—Assuming both purchases and sales to be due on January 1, W. would be entitled to a discount on his purchases equal to that on $1 for 30500 days, and I would be entitled to a discount on my purchases equal to that on $1 for 13100 days, making a difference of 17400 days in W.'s favor on the balance, $500. The discount on $1 for 17400 days is equal to the discount on $500 for 17400 days÷500=34⅘ days from January 1. Whether to be reckoned backward or forward is easily determined. To be in his favor, the time for payment must be reckoned forward; otherwise, it would be reckoned backward.

Required the equated time of the following:

24

	William Sillets.		
Feb. 3, To Mdse,	$250.00	Mar. 2, By Cash,	$300.00
Mar. 9, " "	300.00	Apr. 3, " "	200.00
Apr. 18, " "	500.00		

Dr.			J. C. Hintz.		*Cr.*
1869.			1869.		
July 3,	To Mdse,	\$1000.00	Aug. 1,	By Cash,	\$500.00
7,	" "	500.00	13,	" "	500.00
Au. 18,	" "	250.00			

Assuming the day of settlement to be July 1, we have

1000 × 2 =	2000 *Days.*	500 × 31 =	15500 *Days.*
500 × 6 =	3000 "	500 × 43 =	21500 "
250 × 48 =	12000 "	1000	37000 *Cr. products.*
1750	17000 "		17000 *Dr. products.*
1000			20000 *Difference of do.*
\$750 *Balance due.*			

$$
\begin{array}{l}
75\cancel{0})2000\cancel{0}(26 \textit{ ds.} \\
\quad 150 \\
\quad \overline{\ \ 500} \\
\quad\ \ 450 \\
\quad \overline{\quad\ 50}
\end{array}
$$

Explanation.—The sum of the credit products being greater than that of the debit products, shows that the discount is in my favor. Hence, in order to settle on the assumed date, his payments would be at a discount of 26 days for the balance; but as it would be impossible to settle on a past date, we will have to charge him interest from 26 days *prior to July* 1 (June 5) to the real day of settlement, whatever that may be, or else take his note, dated June 5, bearing interest from date. Say the date of the settlement is September 1. Interest on \$750 from June 5 to September 1 is \$11, which, added to \$750=\$761, balance due.

This balance is 9 cents less than what would be obtained by computing interest on both sides of the account to September 1, and is caused by ignoring the $\frac{1}{15}$ of a day—26 days being taken instead of $26\frac{1}{15}$.

Note.—Book-keepers sometimes omit the tens of dollars when averaging. In the above example the hundreds might have been omitted without serious error.

Dr.	WILLIAM P. WALLACE.		*Cr.*
1869.		1869.	
Feb. 22, To Mdse.	$500.00	Apr. 3, By Cash,	$620.00
Mar. 28, " "	700.00	July 6, " "	520.00
Apr. 30, " "	900.00	Sep. 10, " "	900.00
Jun. 8, " "	600.00	Dec. 1, " "	650.00

Assuming January 1 as the day of settlement, we have the following formula:

$$\begin{array}{rl} 500\times 52= & 26000 \\ 700\times 86= & 60200 \\ 900\times 119= & 107100 \\ \underline{600}\times 158= & \underline{94800} \\ 2700 & 288100 \\ \underline{2690} & \\ 10 & \end{array} \qquad \begin{array}{rl} 620\times 92= & 57040 \\ 520\times 186= & 96720 \\ 900\times 252= & 226800 \\ \underline{650}\times 334= & \underline{217100} \\ 2690 & 597660 \\ & \underline{288100} \\ & 10)\underline{309560} \\ & 30956 \textit{ days}, \end{array}$$

Or 84 years, 9 months and 5 days (allowing for leap-years) *to be counted backward*, because the discount is in my favor.

184. Hence, when the balance of the account and the balance of the discount are both in my favor, I count *backward;* when the former only, I count *forward*, and *vice versa.*

29. Goods bought on January 9, at 60 days, $1376; on 13th, $780; on May 3, $3400. Payments made May 3, $1200; June 8, $3500; equated time required.

30. Balance of last account brought down, including interest to July 1; goods bought August 9, at 3 months, $2300; September 3, $1500; December 9, $500; and payments, October 3, $3000; November 9, $2500; average date of payment required.

31 to 33. Find the equated time of the following:

H. H. Schultz.

1869.		1869.	
July 3, To Mdse, 6 mos,	560.87	July 1, By Balance,	127.15
15, " " 3 "	149.50	30, " Accept. 60 ds,	300.00
Au. 21, " " 3 "	2000.00	Aug. 29, " Cash,	460.00
Sep. 18, " " Cash,	396.40	Oct. 20, " Note, 3 mos,	1000.00
Oct. 15, " " "	175.20	31, " Cash	100.00
21, " " "	425.16	31, " Mdse Ret.,	250.00
27, " Cash,	100.00	Nov. 30, " Cash,	450.00
31, " Mdse, 3 mos,	506.18	30, " Balance,	2144.77
Nov. 28, " " 4 "	197.45		
30, " " 4 "	321.16		
	4831.92		4831.92
Dec. 1, To Bal.,	2144.77		

Theodore Lilienthal.

1867.		1867.	
Jan. 1, To Balance,	650.00	Jan. 8, By Mdse, 3 mos,	160.00
Feb. 3, " Cash,	245.00	15, " " 6 "	710.87
15, " Note, 60 ds	416.87	Feb. 14, " " 2 "	910.14
Mar. 18, " Accept. 30 ds,	1000.00	Apr. 16, " " Cash,	1000.00
Jun. 4, " " 60 "	750.14	June 8, " " 4 mos,	900.00
16, " Note, 3 mos,	987.64	15, " " 4 "	2500.00
30, " Cash,	500.00	17, " " 6 "	1215.00
30, " Mdse, abate.	200.00	30, " Sunds,	700.00

Julius Wise.

1869.		1869.	
July 3, To Balance,	1500.00	Aug. 3, By Cash,	1000.00
18, " Mdse, 4 mos,	750.40	Sept. 7, " Acctc. 60 ds,	500.00
Aug. 29, " " 4 "	128.80	Nov. 5, " " 60 "	750.00
Sep. 30, " " 4 "	916.84	Dec. 14, " Cash,	2000.00
Oct. 10, " " 3 "	500.00		
30, " Cash,	675.14		
Nov. 18, " Sunds,	564.18		

These exercises may be omitted until the learner has studied Exchange.

186. *When payments are made before a note or bill is due, to find how long after maturity it should run to balance the interest on the advanced payments.*

34. A merchant holds a note of \$500 at 6 months. Three months before it is due, he receives \$100, and one month before it is due, he receives \$300; how long should he allow the balance to run to equal the interest on the advance?

The int. on \$100 for 3 mos=int. on \$1 for 300 mos.
The int. on 300 for 1 mo =int. on 1 for 300 mos.
600 mos.

Hence, the interest on the advance payments is equal to the interest of \$1 for 600 months; that is, a balance of \$1 should have run 600 months, but the balance due on the note is \$100; therefore, it should run $\frac{600}{100}$ months=6 months.

PROOF.—*The int. of the* \$100 *for* 6 *mos*=\$3.

The int. of \$100 (the first pay't) for the 3 mos=\$1.50
The int. of 300 (the sec. pay't) for the 1 mo = 1.50
Total interest on advance=\$3.00

35. A note of \$600 was given January 3, 1868, payable in 6 months. 4 months before it was due, \$100 was paid on it, and 3 months before it was due, \$200 was paid; how long in equity should the balance run?

36. A merchant owes \$700, due 8 months from the time he contracted the bill; 5 months afterward, he pays \$200, and two months after that, \$300; how long should the balance remain unpaid?

37. If I borrow \$600 from A at one time, and \$500 at another, each for 4 months, how long should I lend him \$1000 to return the favor?

38. I owe \$400, payable in 10 months; at the end of 4 months I pay \$100; 3 months after that, \$50; how long after the expiration of 10 months may the balance remain unpaid?

39. A owes $1000 due in 6 months; 5 months before it is due he pays $200, and 3 months before it is due he pays $300; how long after the expiration of the 6 months may the balance remain unpaid?

187. *Average applied to account sales.*

An account sales is a detailed statement of goods received by a commission merchant, and sold on account of another. The person who sends the goods is called the shipper or *consignor;* the person to whom they are sent, the *consignee*, and the goods, the *consignment.*

The duty of an agent or commission merchant is to procure the best intelligence of the state of trade at the place where he does business, including the quality and quantity of goods in the market, their present prices, and the probability of their rising or falling; to pay exact obedience to the orders of his employers; to consult their advantage in matters left to his discretion; to execute their business with all the dispatch circumstances will admit; to be early in his intelligence, distinct and correct in his accounts, and punctual in his correspondence.

188. An account sales should state from whom the goods were received, or on whose account and risk they were sold, the dates and terms of sales, the cash paid for freight, drayage, etc., and the various charges, such as insurance, commission, cooperage, etc. The total amount received will appear on the right of the account, and the charges, etc., on the left, or they may be arranged as in the example.

The difference between the two sides is called the *proceeds.*

Advances made on goods are charged to the shipper's account, not in the account sales. Where goods are sold promptly for cash, or on short time, the account sales is not averaged.

COMMISSION HOUSE OF STRAIGHT, DEMING & CO.

Shipment 18. No. 7828.

Sales for account of Messrs. Gaff & Baldwin.

By sundries,

June 4, T. B. Colgan & Co. @ 60 days, 8 hhds. Sugar.						
1095	1020					
1100	1120					
1080	1240	8965				
1200	1110	896	8069	@ $7\frac{7}{16}$		$600.13
June 6, G. Newton & Co., @ 60 days, 10 hhds. Sugar.						
1080	1040					
1090	1340					
1120	1020					
1240	1100	11440				
1200	1210	1144	10296	@ $6\frac{7}{8}$		$707.85
June 10, B. Vilgers & Co., @ 60 days, 20 hhds Sugar.						
1060	1240					
1210	1110					
1180	1005					
1055	1285					
1240	1100					
1185	1210					
1300	1325					
1010	1140					
1120	1205	23185				
1205	1000	2318	20867	@ $6\frac{7}{16}$		1343.31
						$2651.29

CHARGES.

June 1, P'd cash st'r Landis for freight,	$87.18	
" 10, Dray. 9^{50}, ins. 4^{63}, and stor. 9^{50},	23.63	
" Commission and guarantee,	132.56	243.37
Net proceeds due by equation, Aug. 13,		$2407.92

E. O. E.

CINCINNATI, *June* 14, 1858.

STRAIGHT, DEMING & CO.,

Per F. JELKE.

Aug. 3,	600×63=	37800	June 1,	87
" 5,	708×65=	46020	" 10,	156×9=1404
" 9,	1343×69=	92667		243 1404
	2651	)176487		
	243	1404		
	2408	175083(72.6 or 73 days from June 1,		
		16856 which gives August 13.		
		6523		
		4816		
		1707		

2. Find the equated time of payment of the following: Account sales of merchandise sold on account and risk of Morris J. Parry, New York:

Mar. 1,	To Cash for freight,	$50.00	Mar. 15,	By Cash,	$450.00
1,	" Drayage,	10.00	Apr. 3,	" S. Miner,	75.00
10,	" Insurance,	8.50	May 1,	" Cash,	318.75
June 20,	" Storage & Advertising,	10.00	7,	" J. Clark.	92.25
	" Com. on $1224 @ 2½,	30.60	Jun. 19,	" Cash,	288.00
	M. J. Parry's net pro.,	1114.90			
		$1224.00			$1224.00

3. Sales of 100 bbls. of molasses for acct. of C. H. Crane.

July 3,	F. M. Peale, on acct., 20 bbls., 860 gals...	@ 1.25	1075.00
July 9,	Saml. A. Butts, Jr., cash, 15 bbls., 635 gals.	@ 1.20	762.00
July 18,	Geo. T. Ladd, cash, 12 bbls., 495 gals......	@ 1.20	594.00
Sep. 6,	F. M. Peale, on acct., 12 bbls., 495 gals...	@ 1.20	594.00
Oct. 3,	J. J. Marvin, on acct., 31 bbls., 1285 gals.	@ 1.00	1265.00
Dec. 18,	J. J. Marvin, on acct., 10 bbls., 400 gals..	@ .90	360.00
			$4650.00

CHARGES.

July 1,	Cash paid freight..	58.00
	Drayage..	18.00
Dec. 30,	Cooperage	3.20
	Com. and guar. 4 per cent..................................	186.00
	C. H. Crane's net proceeds..................................	4384.80
		$4650.00

17

SALES FOR ACCT. OF R. H. LANGDALE.

Sept. 3, G. F. Sands, 90 days,				
18 bbls. Clo. S. 3120 lbs net=52 bus.	@ $6.50	338.00		
Oct. 9, C. F. Rowe, on 60 ds. note,				
5 bbls. Clo. S. 930 lbs., 15½ bus...	@ 10.00	155.00		
Dec. 5, G. A. Voige, 30 ds. note,				
10 bbls, — 2220 — 37..............	@ 8.00	296.00	789.00	

CHARGES.

Aug. 15, Cash pd. fr't..................................	12.00	
Drayage on acct..................................	6.00	
Sept. 1, Advertising..................................	8.55	
Dec. 30, Storage..................................	10.00	
Com. and guar., 5 per cent....................	39.45	
R. H. Langdale's net proceeds..................	713.00	799.00

E. E.

CINCINNATI, *Jan.* 1, 1867. NELSON, NEPHEW & CO.

Answers: 3 mos. 24 ds.; 3 mos. 10 ds.; 3 mos.; Dec. 31, 1869; 4 mos. 15 ds.; July 15, 1869; Dec. 15, 1867; $1919.85; 4 mos. 12 ds.; Oct. 17; March 25; $4707.34; Nov. 2; 49 ds.; Feb. 21; July 11; April 30; Dec. 23; 143 ds.; April 27; Dec. 6; Oct. 8; May 17; Dec. 21; Aug. 24; June 2; Aug. 16, 1867.

ACCOUNTS CURRENT.

An Account Current is a statement of the entire transactions between two parties, generally for three or six months. It exhibits the whole sums given and received, the interest due on each at the date of the account, and to whom the balance of interest and principal is due.

Without a knowledge of book-keeping, an account current or account sales can not be fully understood.

The name of the party against whom the account current is made out is always written first and on the left, while the maker's name appears on the right. When any sums fall due after the date of the account, the interest is entered on the opposite side, making it discount.

AN ACCOUNT CURRENT WITH INTEREST COMPUTED.

MESSRS. GRAFF & BALDWIN,
In Account Current and Interest Account with

Dr. STRAIGHT, DEMING & Co.

DATE.		DESCRIPTION.	TIME.	IN'T. $	IN'T. c.	AMOUNT. $	AMOUNT. c.
1867.			Days	$	c.	$	c.
Jan.	4	To Mdse, as per bill rendered..........	161	11	40	425	00
"	10	" " " " " "	155	20	93	810	00
Feb.	8	" Your draft on us at 60 days sight, in favor of H. King & Co.........	63	5	25	500	00
"	20	" Your sight draft on us................	114	19	00	1000	00
Mar.	2	" Mdse, as per bill rendered..........	104	2	17	125	00
"	10	" Exchange on $500 at 1 per cent....	96	0	08	5	00
Apr.	18	" Mdse, as per bill rendered..........	57	4	12	434	00
"	20	" " " " " "	55	4	62	504	00
"	25	" Our draft on G. Wright, your favor.	44	1	10	150	00
"	26	" Mdse, as per bill rendered..........	49	8	99	1101	00
May	4	" N. Davis & Co.'s note at 30 days, in our favor	8	0	67	500	00
"	8	" Discount on uncurrent money at 1⅓ per cent............................	37	0	03	5	00
"	25	" Mdse, as per bill rendered..........	20	0	92	275	00
June	5	" Net proceeds, sales 100 bbls. flour.	9	0	74	495	00
"	8	" Mdse, as per bill rendered..........	6	4	15	4152	00
"	14	" Cash paid for your telegram.........				5	00
		" Interest in our favor..................				30	03
				84	17	10515	03
		CR.					
Jan.	20	By draft on New York, ½ p. c. prem.	145	24	29	1005	00
"	28	" Cash..	137	2	28	100	00
Mar.	18	" Your sight draft on Moore & Adams, our favor..................	88	11	74	800	00
"	29	" Draft on New York, ¾ per cent. premium..............................	77	5	17	403	00
Apr.	10	" Net proceeds, sales 50 bbls. molasses..................................	65	6	76	624	00
May	8	" Cash..	37	1	23	200	00
"	29	" "	16	2	67	1000	00
June	14	" Net proceeds, sales 38 hhds. sugar.				2407	00
		Balance of interest....................		30	03		
		Balance in our favor...................				3976	03
		E. & O. E.		84	17	10515	03

CINCINNATI, June 14, 1867.
STRAIGHT, DEMING & Co.,
per HILL.

XXII. EXCHANGE.

189. If A of Liverpool is indebted to B of New York in the sum of $1000 dollars, and B is indebted to C of Liverpool to the same amount, it is evident that the two debts might be paid without either party sending real money to the other. A settlement could be effected by B sending the following order to C:

$1000. New York, *Jan.* 3, 1868.

At sight, pay to the order of C One thousand dollars, value received, and charge to my account.

To A, B

Liverpool, England.

This method of settling accounts between persons in distant places is called *Exchange.* If between persons in the same country, it is called *Home*, *Domestic* or *Inland Exchange;* otherwise, it is called *Foreign Exchange.*

190. A *Bill of Exchange* is an order written like the one above; it is also called a *check* or *draft.* The term draft is usually applied to inland bills. There are *Sight Bills* and *Time Bills.*

191. A *Sight Bill* is one payable at sight or on demand, as the one above.

A *Time Bill* is one that requires payment so many days after sight, or after date.

192. A *Set of Exchange* consists of three copies of the same bill drawn to insure safety of transmission, one of which being paid, the others are void.

The person whose signature is attached to a bill or draft is called the *drawer* or *maker;* the person addressed, the *drawee;* the party to whom payment is to be made, the *payee*, and the one who has possession of it, the *owner* or *holder*.

REMARKS ON BILLS AND DRAFTS.

1. Bills of exchange, unless payable at sight or on demand, require to be "accepted," and should be presented promptly to the drawee for this purpose. The drawer then becomes the *acceptor*, and the bill is said to be *accepted* or *honored*, and is called an *acceptance*.

2. *Accepting* a bill consists in the drawee writing his name across the face, by which act he becomes responsible for its payment. The following is the form:

$5000.00 Cincinnati, Dec. 26, 1866.

Accepted.
Jan'y 1, 1867.
Edward Mason.

To Edw. Mason,
Boston, Mass.

Wm. Massey.

Business men prefer a draft to a promissory note, because there are three parties to it, while the note, ordinarily, has only two. It often happens, however, that notes and drafts are made payable to the drawers, which leaves only one party to the former, and two to the latter, though technically they are considered the same as if drawn in favor of others.

When bankers receive unaccepted bills, they send them out for acceptance or notify the drawees.

Bills of exchange, like promissory notes, may be made negotiable by the insertion of the words, "to the order of," or "bearer," and are subject to protest for non-payment, and the indorsement may be *special* or in *blank.*

3. Bills of exchange can be had at the banking houses or offices of exchange brokers, and may be drawn in favor of the persons buying them, indorsed and mailed to their creditors; or they may be drawn in favor of some other person, indorsed by him, and afterward indorsed by the buyer, and mailed to his creditors, as before.

4. When a bill or draft costs neither more nor less than the amount of its face, it is said to be at *par;* if less than that amount, it is at a *discount;* if more, it is said to command a *premium*, and the rate of discount or premium is called the *rate of exchange.*

5. The phrase, "apply to my account," or "your account," "as advised," etc., are not essential to a bill, but rather indicate the relations of the parties as debtors or creditors. When the drawer is indebted to the drawee, he would say, "apply to my account," but when the drawee is indebted to the drawer, the phrase would be, "apply to your account," or "put it to your account." Should the bill be drawn on account of a third person, he would say, "put it to the account of A."

6. When the words "as per advice," or "as advised," are used, it is presumed that a letter of instructions has preceded the draft. In such case, the drawee honors at his risk in the absence of such advice.

7. Bills or drafts for acceptance must be presented within a reasonable time. If the drawee destroy a bill for acceptance, or refuse to return it in twenty-four hours, he will be deemed to have accepted it.

8. Sight bills for collection should not be mailed to the drawee, as their possession is presumptive evidence of payment.

9. The phrase "value received" is properly omitted when the bill is drawn against funds of the drawer in the hands of the drawee, as is usually the case with banking houses when selling exchange.

10. The place of payment, separate from where it is drawn, is not usually inserted in a draft, unless an understanding to the contrary exists between the parties.

11. Drafts are often drawn by merchants upon each other to raise money or settle accounts. A merchant shipping a large quantity of goods to another to sell on commission, usually draws a draft for a part of the cost on the party and sells it at bank, or passes it to another merchant in the course of business. This kind of paper is called a *mercantile draft*, to distinguish it from one issued by a bank, which is called *exchange*, or a bank check or draft, and is not so available for transmission as the bank draft or exchange. It is a part of the business of a banking house or exchange office to buy this mercantile paper, send it home for collection, and in the mean time sell exchange on the banks to which they transmit it, for such sums as may be demanded.

DRAFT OF A MERCHANT UPON ANOTHER TO WHOM HE HAS SHIPPED GOODS.

NEW YORK, *May* 17, 1867.

At ten days' sight, pay to our order One thousand dollars, value received, and charge to our account.

To HENRY L. WEHMER. A. J. RICKOFF & CO.
Cincinnati, Ohio.

To obtain money on this, Mr. Rickoff would indorse it to a banking house, which would pay him the current rate for mercantile paper, and discount for the time to expire before collection—say 4 days to reach its destination, and 13 days for maturity.

BILLS IN DUPLICATE AND SETS OF EXCHANGE.

To prevent delay and guard against loss, bills of exchange are often drawn in duplicate or in sets.*

A *duplicate* bill is, as the name indicates, a correct copy of the original, with the addition of the word "Duplicate" written or printed across the face.†

A *set of exchange* properly belongs to Foreign Exchange, under which subject the learner will find the form and description.

HOME EXERCISES IN DRAWING DRAFTS.

1. Draw on your teacher, (locating him in San Francisco, California,) at 10 days' sight, for $2136.50, and make the draft payable to yourself.

2. At 60 days after date, draw on O. I. Mitchell for $2000, favor of yourself, and prepare draft for negotiation.

Draw bills and notes from the following data:

3. Draw on M. Garaghan, St. Louis, for $3600, at sight, and prepare the draft for collection by the Central National Bank, Cincinnati.

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* Should a bill be lost in transmission, the amount can be recovered of the bank from which it was bought, unless it can be proved that payment was made by the drawee.

† On bank drafts will often be found writing on both back and face, which can not be represented in type, such as the names of bank officers, through whose hands they pass before issue, the amount written on the back or face a second time to prevent alteration. They are also drawn payable in "gold" or "currency," as occasion requires.

4.		5.	
Drawer,	Yourself.	Drawer,	Yourself.
Drawee,	Your teacher, at N. O.	Drawee,	W. A. Fillmore, N.Y.
Payee,	G. A. Carnahan.	Date,	April 1, 1867.
Date,	The present.	Time,	10 days' sight.
Where,	Cincinnati, Ohio.	Payee,	Yourself.
Time,	At sight.	Amount,	$907.84.
Amount,	$367.25.		

6. Face, $3167.85.
   Date, August 9, 1866.
   Payee, B. O. M. De Beck.
   Indorsee, Luther W. Strafer.
   Drawer, Noble K. Royse, Cincinnati, Ohio.
   Drawee, Herman H. Raschig, New Orleans.
   Time, 10 days' sight.

7. ACCEPTANCE OF F. M. PEALE AT NEW YORK.
   Drawer, Wm. H. Morgan, San Francisco.
   1st Indorser, J. M. Allen.
   Time, 60 days after date.
   Face, $967.18.
   Date, 24th May, 1866.
   Indorsee, Geo. F. Sands.

8. Drawer, John J. Marvin, Cincinnati, Ohio.
   Drawee, John S. Highlands, Columbus, Ohio.
   Payee, E. H. Prichard, Boston, Mass.
   1st Indorser, B. B. Stewart, Face, $2127.
   Time, 30 days after sight.
   Date, January 1, 1867.
   Accepted two days afterward.

9. Indorsee, G. W. Harper.
   Drawee, C. R. Stuntz, Washington, D. C.
   Drawer, G. W. Smith, New York.
   Indorser, G. A. Schmitt.
   Time, 10 days' sight, Face, $5000.
   Date, January 1, 1867.
   Accepted four days after date.

AN INDIANA NOTE.

$500. INDIANAPOLIS, *May* 25, 1867.

Six months after date, I, the subscriber, of Indianapolis, County of Marion, State of Indiana, promise to pay to the order of Geo. W. Runyan, Five hundred dollars, without any relief whatever from valuation or appraisement laws. Value received. Payable at First National Bank.

JAMES W. WILSON.

No. 59. Due November 28. 1867

XXIII. FRACTIONS.

193. A FRACTION is a part of one or more things considered as a whole, and is therefore the result of division.

Whole numbers are sometimes expressed in fractional form. (See Improper Fractions.)

194. The subject of fractions is the method of treating fractional numbers, or showing how they may be added, subtracted, multiplied and divided.

195. Fractions are divided into *common* and *decimal*, according to the way in which they are written. A common fraction requires two numbers to express it, as $\frac{1}{2}$, while a decimal requires only one, with a period at the left, as .5.

The following fractions would be read as shown opposite:

$\frac{1}{12}$ *One Twelfth.*

$\frac{3}{17}$ *Three Seventeenths.*

$\frac{9}{145}$ *Nine One hundred and forty-fifths.*

$\frac{57}{3396}$ *Fifty seven Three thousand three hundred and ninety-sixths*, or *Thirty-three hundred*, etc.

196. The two numbers forming a fraction are called *terms;* the upper term the *numerator*, and the lower term the *denominator*. The line between the terms is the sign of division, and indicates that the upper term is divided by the lower. (Art. 34.) $\frac{3}{12}$ represents the twelfth part of 3, or 3 parts of something divided into 12 parts.

A fraction also expresses the ratio between the two terms. $\frac{3}{12}=3:12$.

PRINCIPLES OF FRACTIONS.

197. If both terms of a fraction be multiplied by the same number, the value of the fraction will remain unaltered.

Let the 1 and 4 of the fraction $\frac{1}{4}$ be multiplied by 2, and we have $\frac{2}{8}$, a fraction of the same value as $\frac{1}{4}$.

198. If both terms of a fraction be divided by the same number, the value of the fraction will remain unaltered.

Let 3 and 6 of the fraction $\frac{3}{6}$ be divided by 3, and we have $\frac{1}{2}$, a fraction of the same value as $\frac{3}{6}$.

199. If the numerator only be multiplied, the value of the fraction will be increased and the whole fraction multiplied.

Let 2 of the fraction $\frac{2}{3}$ be multiplied by 4, and we have $\frac{8}{3}$; that is, *eight* thirds instead of *two* thirds.

200. If the denominator only be multiplied, the value of the fraction will be decreased, and the whole fraction divided.

Let the 3 of the fraction $\frac{2}{3}$ be multiplied by 2, and we have $\frac{2}{6}$; that is, two *sixths* instead of two *thirds*.

201. If the numerator only be divided, the value of the fraction will be decreased, and the whole fraction divided.

Let 4 of the fraction $\frac{4}{8}$ be divided by 2, and we have $\frac{2}{8}$; that is, *two* eighths instead of *four* eighths.

202. If the denominator only be divided, the value of the fraction will be increased, and the whole fraction multiplied.

Let 8 of the fraction $\frac{2}{8}$ be divided by 2, and we have $\frac{2}{4}$; that is, two *fourths* instead of two *eighths.*

Common fractions are divided into *simple*, *proper*, *improper*, *compound*, *complex* fractions and *mixed numbers.*

A *simple fraction* is a single fraction, as $\frac{2}{3}$.

A *proper fraction* is a single fraction whose numerator is less than the denominator, as $\frac{1}{6}$.

An *improper fraction* is a single fraction whose numerator is equal to or greater than the denominator, as $\frac{3}{3}$, $\frac{9}{6}$, which indicates not a *part*, but the whole or more than the whole; hence, the term *improper.*

203. A *compound fraction* is a fraction of a fraction or fractions, as $\frac{1}{2}$ of $\frac{2}{3}$ or $\frac{4}{5}$ of $\frac{11}{13}$ of $\frac{45}{60}$.

204. A *complex fraction* is one having a fraction in the numerator or denominator, or in both, as

$$\frac{\frac{2}{3}}{7} \quad \textit{or} \quad \frac{1\frac{1}{2}}{\frac{7}{8}}$$

205. A *mixed number* is composed of a fraction and whole number together, as $7\frac{3}{4}$.

REDUCTION OF FRACTIONS

206. Fractions are often expressed in terms too large for convenient use, or to estimate their value at sight. The fraction $\frac{9}{27}$ possesses the same value as $\frac{1}{3}$, and for convenience in operating ought to be reduced to that degree of simplicity.

The process of changing the form of a fraction in this manner is called *reducing it.*

207. *To reduce a fraction to its lowest terms.*

We divide both terms by any number or numbers which will do so without a remainder. (Art. 198.)

1. 9 and 27 of the fraction $\frac{9}{27}$, divided by 9, give 1 and 3, or the fraction $\frac{1}{3}$.

When a single number will not reduce a fraction to its lowest terms, other numbers are used and the process continued.*

2. To reduce $\frac{84}{126}$ to its lowest terms.

$$3)\tfrac{84}{126}(\;7\;)\tfrac{28}{42}(\;2\;)\tfrac{4}{6}(\tfrac{2}{3}$$

3 to 13. Reduce the following fractions to their lowest terms:

$\frac{24}{60}$, $\frac{184}{728}$, $\frac{21}{196}$, $\frac{125}{1000}$, $\frac{6874}{24682}$, $\frac{7216}{36080}$, $\frac{234}{6188}$, $\frac{126}{6666}$, $\frac{2168}{4682}$, $\frac{936}{2808}$, $\frac{9876}{79008}$.

Answers: $\frac{1}{8}$, $\frac{1}{3}$, $\frac{21}{1111}$, $\frac{1084}{2341}$, $\frac{2}{5}$, $\frac{3}{28}$, $\frac{1}{8}$, $\frac{23}{91}$, $\frac{491}{1763}$, $\frac{1}{5}$, $\frac{9}{238}$.

208. *To raise a fraction to a higher denomination*, we multiply both terms by the same number—a process the reverse of the last.

209. *When the higher denomination is given*, the multiplier may be obtained by dividing the new denominator by the old.

---

*THE GREATEST COMMON DIVISOR

Is the greatest number which will divide any two or more numbers. It may be found by the following process, but the operation is so long that it is seldom used in practice.

To find the greatest common divisor of 540 and 612.

```
540)612(1
    540
    ---
     72)540(7
        504
        ---
         36)72(2
            72
```

EXPLANATION.—The smaller number is divided into the larger, and the remainder (72) into the first divisor; then the next remainder (36) into the last divisor, etc.

The last divisor is the greatest common divisor, viz., 36. That is, no number higher than 36 will divide both without a remainder.

14. To raise $\frac{5}{6}$ to 24ths, we divide 24 by 6, which gives 4 as a multiplier. $5 \times 4 = 20$ and $6 \times 4 = 24$, making $\frac{20}{24}$.

15 to 21. Raise $\frac{2}{3}$ to 12ths, $\frac{7}{8}$ to 16ths, $\frac{1}{7}$ to 49ths, $\frac{1}{4}$ to 28ths, $\frac{5}{6}$ to 120ths, $\frac{1}{12}$ to 180ths, $\frac{23}{25}$ to 500ths.

210. *To reduce a mixed or whole number to an improper fraction.*

22. Reduce $5\frac{7}{8}$ to an improper fraction; that is, in $5\frac{7}{8}$ how many eighths?

$$\begin{array}{l} 5\frac{7}{8} \\ \underline{8} \\ \frac{47}{8} \end{array}$$

EXPLANATION.—In every whole number there are 8 eighths, and in 5 whole numbers there are 8 times 5, or 40 eighths, to which add 7 eighths and the result is 47 eighths.

23 to 30. Reduce the following numbers to improper fractions: $15\frac{3}{5}$, $7\frac{3}{7}$, $6\frac{4}{5}$, $51\frac{2}{3}$, $17\frac{7}{34}$, $113\frac{16}{80}$, $16\frac{21}{50}$, $21\frac{3}{7}$.

Answers: $\frac{566}{5}$, $\frac{52}{7}$, $\frac{34}{5}$, $\frac{155}{3}$, $\frac{585}{34}$, $\frac{566}{5}$, $\frac{821}{50}$, $\frac{150}{7}$, $\frac{78}{5}$

211. *To reduce improper fractions to whole or mixed numbers,* is an operation the reverse of the last.

31. Reduce $\frac{47}{8}$ to a mixed number.

$$\begin{array}{r} 8\underline{)47} \\ 5\frac{7}{8} \end{array}$$

32 to 41. Reduce the following to whole or mixed numbers, and the remaining fractions to their lowest terms: $\frac{146}{6}$, $\frac{456}{9}$, $\frac{364}{2}$, $\frac{161}{15}$, $\frac{1196}{25}$, $\frac{100}{8}$, $\frac{4160}{6}$, $\frac{3179}{185}$, $\frac{7854}{864}$, $\frac{11000}{85}$.

Answers: 182, $10\frac{11}{15}$, $24\frac{1}{3}$, $50\frac{2}{3}$, $12\frac{1}{2}$ $693\frac{1}{3}$, $17\frac{34}{185}$, $129\frac{7}{17}$, $9\frac{13}{144}$, $47\frac{21}{25}$.

XXIV. DECIMALS.

212. A DECIMAL FRACTION expresses its value in one term, and is known from a whole number by its having a period, called a *decimal point*, at the left. .5 is a decimal.

The value of a decimal is more easily ascertained than that of a common fraction, while operations in decimals are performed with nearly the same ease as those in whole numbers.

213. Figures increase in a tenfold ratio as they are removed one place to the left, and decrease in the same ratio as they are removed one place to the right.

In the number .5 the figure is one place to the right of the unit figure, and therefore possesses only one-tenth of the value it would in that place. In other words, it represents *tenths* instead of units. One place further it would represent *hundredths*, as .05, and one place further, *thousandths*, as .005. As common fractions these would appear thus: $\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{1000}$; hence,

214. *To reduce a decimal to a common fraction*, we erase the decimal point, and write for the denominator as many ciphers as there are figures in the numerator, and prefix the figure 1. .075 would be written $\frac{75}{1000}$ and .0047, $\frac{47}{10000}$.

Ciphers on the extreme right of a decimal possess no value. .500 expresses the same value as .5, the first being $\frac{500}{1000}$, which, reduced to tenths, is $\frac{5}{10}$.

1 to 21. Find the fractional value of the following and read them: .23, .007, .013, .760, .00019, .3401, .67800, .0907, .0076, .3467, .1093, .0770, .3657, .2136, .09876, .000001, .13607, .06789, .03146, .000016, .016037.

215. As in Federal money, the removal of the decimal point one place to the right multiplies a number by 10, and its removal one place to the left divides it by 10.

Removed to the right in 1673.27, we have 16732.7, and removed to the left we have 167.327.

The notation and numeration of decimals being so similar to that of whole numbers, little trouble will be experienced in reading the following:

22. .01	28. .3013	34. .45689	40. .00001
23. .57	29. .0031	35. .18654	41. .0010906
24. .709	30. .2160	36. .36109	42. .3016031
25. .856	31. .1061	37. .00009	43. .0016039
26. .2913	32. .4064	38. .01002	44. .0000067
27. .0016	33. .5067	39. .168002	45. .00001001

Express in figures the following:

1. One tenth.
2. Three hundredths.
3. Five thousandths.
4. Sixty-five ten thousandths.
5. Three hundred seventy-six thousandths.
6. Four hundred ten thousandths.
7. Eighteen hundred and twenty thousandths.
8. Sixty-one and eighteen hundredths.
9. Forty-five hundredths.
10. Eighty-seven thousand and sixty hundredths.
11. Five hundred thousand and seven tenths.
12. One hundred and one thousand and seven.
13. Sixty-four thousand and eight tenths.
14. Nine millions and seventy-nine thousandths.
15. Eighty-six hundredths.
16. Seven thousand and six hundredths.
17. One hundred and ten and sixty-five hundredths.
18. Eighteen hundred and sixty-seven and seventy-five hundredths.
19. Twenty-four hundred and five hundred and one thousandths.

ADDITION OF DECIMALS.

216. When arranged, tenths under tenths, hundredths under hundredths, etc., decimals are added and subtracted precisely as whole numbers. The operations in Federal money, with which the learner is already familiar, properly belong to this subject. In those, the decimal points were placed directly under each other. The same rule should be observed in adding or subtracting decimals generally.

1. To add 1.07+.001+37.045+10.06+.0007.

```
 1.07
  .001
37.045
10.06
  .0007
-------
48.1767
```

Explanation.—Here the decimal points are arranged directly under each other and addition performed as in whole numbers.

2.	2.13	+ .426	+ 21.2	+ 7.63	+ 640.072	=?
3.	43.27	+ 9.042	+ 712.417	+ 41.007	+ .962	=?
4.	820.71	+ 2.006	+ 84.243	+ 217.072	+ 9.341	=?
5.	107.67	+ 1.301	+ 20.0163	+ 684.6	+ 10.06	=?
6.	719.86	+ .2103	+ .1610	+ 310.6	+ 2134.	=?
7.	9.8784	+ 29.8	+ 67.19	+ 7.916	+ 379.	=?
8.	643.72	+ .109	+ 360.06	+ .0006	+ .216	=?

Answers: 2748.3678, 671.458, 1133.372, 823.6473, 806.698, 1004.1056, 493.7844, 2748.3678, 2708.3768, 493.7844, 3164.8313.

9.	.007	+ 31.06	+ .1009	+ 100.07	=?
10.	710.34	+ 2.406	+ 67.709	+ .0006	=?
11.	314.60	+ .0006	+ .0027	+ .001	=?
12.	714.06	+ .003	+ 8.007	+ 800	=?

Answers: 314.6043, 1522.07, 131.2379, 780.4556, 1522.3074.

SUBTRACTION OF DECIMALS.

When the larger number has fewer places of decimals than the smaller, the blanks may be filled with ciphers. (Art. 214.)

1. To find the difference between 107.06 and .213.

OPERATION.
$$\begin{array}{r} 107.060 \\ .213 \\ \hline 106.847 \end{array}$$

2. 617.07—41.7106=?	6. 341. —.213 =?
3. 10.06— .9092=?	7. .97—.0376=?
4. 36.84— 6.672 =?	8. 4.15—.1999=?
5. 118.09— 7.009 =?	9. 7.96—.9789=?

Totals, 351.6056, 725.7592, 352.6506.

MULTIPLICATION OF DECIMALS.

217. In this rule we multiply as in whole numbers, and mark off as many places of decimals in the product as there are in the two factors.

1. To multiply 6.107 by 5.5.

$$\begin{array}{r} 6.107 \\ 5.5 \\ \hline 30535 \\ 30535 \\ \hline 33.5885 \end{array}$$

There are *three* places of decimals in this factor, and one place in this; so we point off four in the product.

PROOF.
$$\begin{array}{r} 6.107\times 5 = 30.535 \\ 6.107\times .5 = 3.0535 \\ \hline 33.5885 \end{array}$$

EXPLANATION.—The 6 and 107 thousandths multiplied by 5=30 and 535 thousandths, but multiplied by .5 or 5 tenths, it is only one tenth as much, or 3.0535 (Art. 213), which, added to the first product, gives 33.5885, as above.

2. .3507× 10.09 =?	7. 2300.7 × 48.003 =?
3. 17.07 ×200.6 =?	8. 704.23 × .0007=?
4. 785.4 × 36.70 =?	9. .786×100 =?
5. .279 ×160.7 =?	10. 4.862× .75 =?
6. 876.5 × .780=?	11. 200.03 × .002 =?
Total, 32980.4659.	Total, 110523.641621.

218. *When the product contains fewer figures than there are decimals in the factors*, the number is made up by prefixing ciphers.

12. $100 \times .0005 = ?$

100
.0005

500, to which prefix one cipher and we have .0500, or .05, the answer.

PROOF. $.0005 \times 100 = .05$.

13. $.107 \times .05 = ?$
14. $61.04 \times .0007 = ?$
15. $.7103 \times .004 = ?$
16. $.3045 \times .00061 = ?$
17. $.27 \times .27 = ?$
18. $.4102 \times .1004 = ?$

Totals, .0509192 and .114269825.

DIVISION OF DECIMALS.

219. When dividing decimals, the quotient and divisor must contain as many places of decimals as the dividend.

1. $33.5885 \div 6.107 = ?$

```
6.107)33.5885(5.5
      30 535
      ------
       30535
       30535
```

PROOF.—This is the converse of Ex. 1 in multiplication, the multiplier and multiplicand being 6.107 and 5.5 and the product 33.5885.

A further proof is obtained by estimate, if we divide the whole number (33) of the dividend by the whole number (6) of the divisor, which will give one place for whole number.

220. When the dividend does not contain as many decimals as the divisor, ciphers may be annexed to make up the number. The quotient will then be a whole number, as it simply shows the number of times the latter is contained in the former.*

---

*In practice, decimals are seldom carried to more than four places.

2. $3066 \div .1783 = ?$

```
.1783)3066.0000(17195.73752
      1783
      -----
      12830
      12481
      ------
        3490
        1783
        -----
        17070
        16047
        ------
         10230
          8915
         ------
         13150
         12481
         ------
           6690
           5349
           -----
           13410
           12481
           ------
             9290
             8915
             -----
              3750
              3566
              -----
               184
```

In this example four ciphers have been annexed to the dividend, to correspond with the number of decimals in the divisor. From this resulted 17195, the quotient. These ciphers being exhausted, five more were annexed to the remainders to give the five decimals in the quotient.

Another method is to annex ciphers at will, observing to place a mark in the dividend to limit the whole numbers in the quotient, as 3066.0000'000.

221. *When there are not figures enough in the quotient to make up the number of decimals in the dividend*, ciphers should be prefixed to the former.

3. Divide 10.70067 by 370.4.

```
370.4)10.70067(.0288
       7 408
       -----
       32926
       29632
       ------
        32947
        29632
        ------
```

Here the quotient produced only three figures (288), which, with the one in the divisor, makes only four decimals, so to make up the number a cipher is prefixed.

Carry out the following to only four places of decimals:

4. 314.06 ÷ 10.73 =?	8. 6.74÷2.34 =?
5. 17600 ÷785.4 =?	9. 496 ÷ .278=?
6. 3170.09 ÷ 2.4014=?	10. 7.6 ÷ .734=?
7. 417.456÷ 31.145 =?	11. 7.23÷4.06 =?

Totals, $1385.1824 and 1799.1878.

12. 30.640 ÷493.67 =?	16. 724.1 ÷38.07 =?
13. 10.8739÷117.406=?	17. 82.03 ÷ 9.0002 =?
14. 6.342 ÷ 22.973=?	18. 7.624 ÷ 2.001 =?
15. 1467.06 ÷196.04 =?	19. .5213÷ .24121=?

Totals, 7.9142 and 34.10573.

REDUCTION OF DECIMALS.

222. *To reduce a Common Fraction to a Decimal.*

1. Reduce $\frac{1}{2}$ to a decimal.

$$2)\underline{1.0}$$
$$.5$$

By annexing a decimal point and a cipher, the number is properly reduced to tenths or 10 tenths, in which 2 is contained 5 times. This 5, being of the same denomination of the dividend, is tenths, or .5.

2. Reduce $\frac{1}{3}$ to a decimal.

$$3)\underline{1.00000}$$
$$33333$$

This quotient may be carried out indefinitely, and is called a *repeating* decimal. To save writing, a point is usually placed over the repeater thus $.\dot{3}$.

223. The fractional value of a repeating decimal may be restored by using 9 instead of 10 as the denominator, as $\frac{3}{9}=\frac{1}{3}$.

3. Reduce $\frac{1}{7}$ to a decimal.

$$7)\underline{1.000000000000}$$
$$142857142857+$$

This is called a *circulating* decimal and is marked thus: $.\dot{1}4285\dot{7}=\frac{142857}{999999}=\frac{1}{7}$.

Its fractional value is restored in the same manner as that of the $.\dot{3}$ in the preceding example.

Express the following decimally:*

4. $\frac{1}{4}$	8. $\frac{5}{26}$	12. $\frac{11}{16}$	16. $\frac{9}{10}$	20. $\frac{75}{125}$
5. $\frac{1}{5}$	9. $\frac{55}{91}$	13. $\frac{5}{12}$	17. $\frac{3}{64}$	21. $\frac{11}{60}$
6. $\frac{3}{4}$	10. $\frac{9}{20}$	14. $\frac{7}{32}$	18. $\frac{43}{80}$	22. $\frac{112}{240}$
7. $\frac{3}{8}$	11. $\frac{1}{6}$	15. $\frac{19}{40}$	19. $\frac{1}{50}$	23. $\frac{7}{3125}$

Answers: .75, .4, .25, .375, .1923+, .6043, .45, $.1\dot{6}$, .6875, $.41\dot{6}$, .2187+, .046875, .9, .475, .00224, $.4\dot{6}$, $.18\dot{3}$, .6, .02, .5375, .2.

224. To find the value of the decimal part of a compound number, as £0.75 or \$0.33$\frac{1}{3}$.

24. What is the value in shillings, pence and farthings of £0.345?

$$\begin{array}{rl} .345 & \\ 20 & \\ \hline 6.900 & \textit{Shillings} \\ 12 & \\ \hline 10.800 & \textit{Pence} \\ 4 & \\ \hline 3.200 & \textit{Farthings} \end{array}$$

or 6*s* 10*d* 3$\frac{1}{5}$ *far.*

Explanation.—Multiplying .345, that is, $\frac{345}{1000}$ by 20, the number of shillings in a pound, we have $\frac{6900}{1000}$, or 6.900 shillings, [Art. 112,] and multiplying the .900 shillings by 12, the number of pence in a shilling, we have $\frac{10800}{1000}$ or 10.800 pence. Multiplying the .800 by 4, the number of farthings in a penny, we have $\frac{3200}{1000}$ or 3.200 or 3.2 farthings.

The operation might have been abbreviated by dropping the ciphers on the right.

Hence, we have for the result 6 shillings 10 pence, 3 farthings and $\frac{2}{10}$ or $\frac{1}{5}$.

Find the value of

25. .625 of a gallon.
26. .1425 of a year.
27. .8323 of a £.
28. .1374 of a ton.
29. .0037 of a lb. Troy.

Answers: 21$\frac{3}{10}$; 2, 1; 274, 12$\frac{8}{10}$; 16, 7$\frac{3}{4}$; 1, 21$\frac{3}{10}$.

225. *To reduce denominate values to decimals.*

---

* The plus sign will be used when the decimal can be carried out further.

30. Reduce 6 shillings 10 pence $3\frac{1}{5}$ farthings to the decimal of a pound sterling.

$$\begin{array}{rll} 5) & \underline{1.0} & \\ 4) & \underline{3.2} & \text{or } 3\tfrac{2}{10}\ \textit{far.} \\ 12) & \underline{10.8} & \text{or } 10\tfrac{8}{10}\ \textit{pence.} \\ 20) & \underline{6.9} & \text{or } 6\tfrac{9}{10}\ \textit{shill.} \\ & .345 & \text{or } £0.\tfrac{345}{1000}. \end{array}$$

The first step in this operation is to reduce $\frac{1}{5}$ to a decimal, which gives .2. Prefixing 3 farthings, we divide by 4, the number of farthings in a penny, and obtain .8 of a penny, to which we prefix 10 and divide by 12, the number of pence in a shilling and obtain. 9. Prefixing 6 shillings, we divide by 20, the number of shillings in a pound, and obtain .345 of a pound, the answer. The reverse of this process is found in example 24.

The pupil can prove his calculations by last Art.

31. Reduce 3 quarters to the decimal of a yard.

32. Reduce 6 lbs. 3 oz. to the decimal of a cwt

33. Reduce 12s. $6\frac{3}{4}$d. to the decimal of a £.

34. Reduce 12 lbs. to the decimal of a tun.

35. Reduce 1 ft. $3\frac{1}{6}$ in. to the decimal of a yard.

36. Reduce 16 oz. to the decimal of a ton.

37. At 56 cents a pound, what will 127 lbs. 6 oz. of tea come to?

$$\begin{array}{r} 16)\underline{60} \\ .375 \end{array} \quad \textit{decimal part of a pound.}$$

$$\begin{array}{r} 127.375 \\ 56 \\ \hline 764\,250 \\ 6368\,75 \\ \hline 7133\,000 \end{array} \quad \textit{cents, or } \$7133.$$

REMARK.—This is not strictly a practical question, nor the shortest method of computing the above, the object being merely to show the application of decimals.

38. At $5 for a pound sterling, what will be the value of £16 8s. 10d.?

39 to 41. What will be the value of the following sums of money at the same rate? £167 10s. $3\frac{1}{4}$d., £19 2s. 6d., £10 10s. $10\frac{1}{2}$d.

Answers: $95.63, $837.57, $52.72, $82.21.

XXV. COMMON FRACTIONS.

MULTIPLICATION.

226. A fraction is multiplied by a whole number by simply multiplying the numerator without altering the denominator. [Art. 199.]

$\frac{3}{4}\times7=7\times3$, or $\frac{21}{4}$, which, reduced to a mixed number, equals $5\frac{1}{4}$.

227. Fractions can also be multiplied by dividing the denominator, without altering the numerator. [Art. 202.]

$$\frac{4}{15}\times5=\frac{20}{15}=\frac{4}{3}, \text{ or } 1\frac{1}{3}.$$

Multiply the following fractions:

1. $\frac{2}{9}\times\ 5=1\frac{1}{9}$
2. $\frac{7}{8}\times\ 4=3\frac{1}{2}$
3. $\frac{2}{3}\times12=8$
4. $\frac{3}{14}\times11=2{\cdot}357$
5. $\frac{7}{19}\times\ 9=3.316$
6. $\frac{8}{17}\times\ 6=2.824$

228. Mixed numbers may be multiplied like compound numbers.

7. Let it be required to multiply $4\frac{5}{8}$ by 7.

	Whole Nos.	Eighths.
Illustration.	4	5
		7
	32	3
	or $32\frac{3}{8}$	

EXPLANATION.—Seven times 5 eighths equals 35 eighths, or 4 whole numbers and 3 eighths. Seven times 4=28 and 4 make 32. *Ans.* $32\frac{3}{8}$.

It will not be necessary for the pupil to write his work in so formal a manner as in this illustration.

8. Multiply $6\frac{7}{9}$ by 12.

$$\begin{array}{r} 6\frac{7}{9} \\ 12 \\ \hline 81\frac{3}{9} \\ or\ 81\frac{1}{3} \end{array}$$

Multiplying 7 ninths by 12, we have 84 ninths or 9 and 3 ninths. Then 12 times 6 are 72 and 9 make 81, giving for the answer $81\frac{1}{3}$.

9. $6\frac{3}{4} \times 8=?$*	13. $35\frac{1}{9} \times 9=?$	17. $914\frac{2}{3} \times 120=?$
10. $7\frac{1}{5} \times 7=?$	14. $60\frac{7}{8} \times 12=?$	18. $63\frac{1}{2} \times 15=?$
11. $8\frac{7}{8} \times 6=?$	15. $45\frac{2}{3} \times 8=?$	19. $127\frac{2}{11} \times 20=?$
12. $1\frac{12}{113} \times 12=?$	16. $64\frac{7}{13} \times 6=?$	20. $110\frac{1}{6} \times 14=?$

Answers: 54, $50\frac{2}{5}$, $13\frac{31}{113}$, $53\frac{1}{4}$, 109760, 952.5, 387.29, $2543.\dot{6}3\dot{6}$, $1542.\dot{3}$, 316, 730.5, $387\frac{3}{13}$, $365.\dot{3}$.

229. *To multiply one fraction by another*, the numerators may be multiplied together for a new numerator and the denominators for a new denominator.

21. $\frac{2}{3} \times \frac{3}{7} = \frac{6}{21}$ *or* $\frac{2}{7}$.

Here $\frac{2}{3}$ is multiplied by 3, giving 6 thirds; but this 3 being sevenths, 6 thirds is 7 times too much. Multiplying the denominator [Art. 200] by 7 gives $\frac{6}{21}$ or $\frac{2}{7}$.

22. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} = \frac{120}{360} = \frac{1}{3}$.

These operations might have been abbreviated by what is called *cancellation*. In the first example, for instance, $\frac{2}{3}$ is to be multiplied by $\frac{3}{7}$; that is, the numerator 2 is to be *multiplied* by the numerator 3; but the 2 is also *to be divided* by 3, for $\frac{2}{3}$ signifies that 2 is to be divided by 3; therefore, since the 2 is to be multiplied by 3 and divided by 3, it remains exactly the same, and the 3 of the denominator is said to cancel or make void the 3 of the numerator. In the following operations the canceled figures will be known by having a line drawn through them.

---

*Many answers in this chapter are given in decimals carried to two, three, or four places, as in practice. Occasionally the plus sign is used to show that the decimals may be continued; at other times the last figure will be found increased.

$$\frac{2}{\cancel{3}}\times\frac{\cancel{3}}{7}=\frac{2}{7}$$

$$\frac{2}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{5}}\times\frac{\cancel{5}}{6}$$

$$\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{5}}\times\frac{\cancel{5}}{\underset{3}{\cancel{6}}}$$

In the second operation, the 2 of the numerator and the 6 of the denominator are uncanceled, making $\frac{2}{6}$, which, reduced by dividing both by 2, equals $\frac{1}{3}$.

The 2 and 6 might have been canceled also, by dividing both where they stood in the question, as in the third example, placing only 3 as a denominator, and 1 as a numerator. 1 is always to be understood where a number has been canceled.

23. $\frac{1}{2}\times\frac{18}{24}\times7\frac{1}{2}\times9\frac{2}{8}=?$

$$\begin{array}{r|l} \cancel{2} & 1 \\ 8\ \cancel{24} & \cancel{18}\ \cancel{9}\ 3 \\ \cancel{2} & 15 \\ 8 & \cancel{74}\ 37 \end{array}$$

Some prefer arranging the terms of canceling fractions as in the margin, with the denominator or divisor on the left, and the numerator on the right.

EXPLANATION.—The first 2 was canceled in the 18, leaving 9; the 24 and 9 were canceled by dividing both by 3, leaving 8 and 3; the 74 was canceled by the second 2.

$$\frac{3}{8}\times\frac{15}{1}\times\frac{37}{8}=\frac{1665}{64}=26\frac{1}{64}.$$

24. $\frac{1}{5}\times\frac{7}{8}\times\frac{5}{7}=?$
25. $\frac{2}{3}\times\frac{4}{7}\times\frac{14}{15}\times\frac{5}{16}=?$
26. $\frac{1}{9}\times\frac{8}{15}\times\frac{5}{24}\times\frac{7}{21}=?$
27. $\frac{1}{2}\times1\frac{1}{9}\times\frac{2}{7}\times\frac{1}{6}=?$
28. $1\frac{1}{2}\times2\frac{1}{2}\times\frac{9}{10}=?$
29. $6\frac{1}{2}\times\frac{4}{5}\times\frac{12}{13}=?$
30. $87\frac{1}{2}\times\frac{2}{175}\times\frac{1}{20}=?$
31. $\frac{161}{211}\times52\frac{3}{4}\times\frac{1}{7}=?$

Answers: 3.375, 4.8, $\frac{1}{8}$, $\frac{1}{9}$, .05, 5.75, .0041, .0264+.

230. *Compound fractions are reduced to simple ones by multiplication.* Let it be required to reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to a simple fraction. We know, by inspection, that one-half of $\frac{2}{3}$ is $\frac{1}{3}$, and that $\frac{1}{3}$ of $\frac{3}{4}$ is $\frac{1}{4}$, the answer.

By multiplication: $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{6}{24}=\frac{1}{4}$.

By cancellation: $\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}=\frac{1}{4}$.

32. $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{9}{21}$=?
33. $\frac{1}{7}$ of $\frac{14}{21}$ of $3\frac{1}{2}$=?
34. $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{8}{16}$=?
35. $\frac{1}{2}$ of $1\frac{1}{2}$ of $\frac{7}{8}$=?
36. $\frac{7}{8}$ of $\frac{8}{9}$ of $9\frac{1}{2}$ =?
37. $\frac{2}{7}$ of $\frac{1}{6}$ of $\frac{5}{35}$ =?
38. $4\frac{1}{2}$ of $1\frac{1}{3}$ of $\frac{1}{4}$ =?
39. $\frac{1}{2}$ of $\frac{2}{3}\times\frac{6}{7}\times1\frac{1}{2}$=?

Answers: 7.389, 1.75, .0068, .286, 1.5, $\frac{1}{4}$, .4286, $.\dot{3}$.

231. *To multiply one mixed number by another.*

The mixed numbers are reduced to improper fractions and multiplied as in Art. 229.

40. $3\frac{7}{8}$ by $5\frac{4}{5}$=?

$$\frac{31}{8}\times\frac{29}{5}=\frac{899}{40}=22\frac{19}{40}.$$

41. $4\frac{1}{5}\times7\frac{1}{8}$=?
42. $2\frac{3}{4}\times1\frac{7}{9}$=?
43. $3\frac{3}{7}\times2\frac{5}{11}$=?
44. $4\frac{5}{11}\times3\frac{6}{7}$=?
45. $2\frac{1}{4}\times\frac{5}{7}$=?
46. $6\frac{7}{8}\times2\frac{1}{2}$=?
47. $1\frac{3}{4}\times8\frac{1}{3}$=?
48. $72\frac{3}{5}\times62\frac{1}{2}$=?
49. $87\frac{1}{2}\times16\frac{2}{3}$=?

Answers: $29\frac{37}{40}$, $17\frac{2}{11}$, $1\frac{17}{28}$, $8\frac{32}{77}$, 2.7, $1\frac{17}{28}$, $17\frac{3}{16}$, 4537.5, $1458.\dot{3}$, $4.\dot{8}$, $14.58\dot{3}$.

50. At $11\frac{1}{4}$ cents a pound, what will $147\frac{1}{2}$ lbs. of coffee cost?

51. What will $7\frac{1}{2}$ lbs. of cheese cost, at $9\frac{1}{2}$ cents per pound?

52. At $12\frac{1}{3}$ cents a pound, what will 120 lbs. of sugar cost?

53. What will $14\frac{1}{2}$ lbs. of beef cost, at $6\frac{3}{4}$ cents a pound?

54. Fifteen and a half yards of muslin, at $9\frac{1}{4}$ cents, will cost how much?

Answers: 71c, \$14.80, \$16.59, 98 c, \$1.43, \$16.50.

DIVISION.

232. Division being the reverse of multiplication, *to divide a fraction by a whole number*, we divide the numerator or multiply the denominator. [Art. 200–1.]

1. $\frac{6}{21}\div3$=?

$$3)\frac{6}{21}=\frac{2}{21} \textit{ or } \frac{6}{21\times3}=\frac{6}{63}=\frac{2}{21}.$$

2. $\frac{16}{18} \div 4 = ?$	5. $\frac{37}{28} \div 7 = ?$	8. $\frac{9}{27} \div 9 = ?$
3. $\frac{14}{16} \div 7 = ?$	6. $\frac{16}{21} \div 10 = ?$	9. $\frac{4}{54} \div 6 = ?$
4. $\frac{18}{36} \div 3 = ?$	7. $\frac{18}{27} \div 4 = ?$	10. $\frac{7}{81} \div 9 = ?$

Answers: $\frac{1}{27}$, $\frac{1}{81}$, $\frac{7}{729}$, .1887, .076, $.1\dot{6}$, $.1\dot{6}$, .125, $\frac{2}{9}$.

233. *To divide mixed numbers.*

11. $21\frac{3}{5} \div 6 = ?$

Whole Nos.	Fifths.
6)21	3
3	3
or $3\frac{3}{5}$	

EXPLANATION.—6 is contained in 21, 3 times and 3 left. In the 3 of the remainder there are 15 fifths, which, added to the 3 fifths in the question, make 18 fifths. 6 in 18, 3 times. *Ans.* $3\frac{3}{5}$.

12. $12\frac{1}{7} \div 8 = ?$

$$8)\underline{12\frac{1}{7}}$$
$$1\frac{29}{56}$$

EXPLANATION.—In this example we had 4 remainder, in which were 28 sevenths, and the one in the question made 29. Then, as 8 would not divide 29 without a remainder, we multiplied it into the denominator, which made 56. *Ans.* $1\frac{29}{56}$.

13. $67\frac{7}{8} \div 7 = ?$	17. $167\frac{1}{9} \div 25 = ?$	21. $72\frac{8}{9} \div 9 = ?$
14. $44\frac{5}{6} \div 3 = ?$	18. $21\frac{1}{3} \div 14 = ?$	22. $148\frac{1}{2} \div 27 = ?$
15. $119\frac{2}{3} \div 6 = ?$	19. $16\frac{1}{7} \div 7 = ?$	23. $276\frac{3}{5} \div 12 = ?$
16. $118\frac{1}{2} \div 12 = ?$	20. $22\frac{1}{3} \div 12 = ?$	24. $175\frac{5}{7} \div 15 = ?$

Answers: $11\frac{6}{7}$, $8\frac{8}{81}$, 5.5, $9\frac{39}{56}$, $14\frac{17}{18}$, 6.684, 1.523, 2.306, 23.05, 11.7142+, $19\frac{17}{18}$, $9\frac{7}{8}$, 1.861.

234. *To divide one fraction by another.*

25. $\frac{1}{2} \div \frac{3}{4} = ?$

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}.$$

Multiplying the denominator of the dividend by 3, we have $\frac{1}{6}$ [Art. 200–1], but as the divisor is not 3 but 3 *fourths*, we multiply the result by 4, giving $\frac{4}{6}$, or $1\frac{1}{2}$; hence, we divide one fraction by another by *inverting the terms of the divisor*, and multiplying as in multiplication.*

235. *To divide by a mixed number.*

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* CAUTION.—The pupil will observe not to invert the terms of the number to be divided.

26. $369 \div 97\frac{1}{3} = ?$

$$\begin{array}{ll} 97\frac{1}{3} & 369 \\ \underline{\quad 3} & \underline{\quad 3} \\ 292\,) & 1107\,(3\frac{231}{292} \\ & \underline{876} \\ & 231 \end{array}$$

Here both terms are reduced to the same denomination (thirds), and division performed as in whole numbers. $97\frac{1}{3}$ is contained in 369, $3\frac{231}{292}$ times.

27. $76 \div 2\frac{5}{8} = ?$ 30. $349 \div 4\frac{1}{2} = ?$ 33. $73\frac{5}{12} \div 8\frac{4}{7} = ?$
28. $84 \div 3\frac{1}{3} = ?$ 31. $106\frac{3}{5} \div 5\frac{4}{5} = ?$ 34. $73\frac{1}{4} \div \frac{1}{4} = ?$
29. $82 \div 7\frac{5}{8} = ?$ 32. $276\frac{3}{5} \div 12 = ?$ 35. $191\frac{7}{8} \div \frac{4}{15} = ?$

Ans. $28\frac{20}{21}$, 23.05, 29.37, $10\frac{46}{61}$, $77.\dot{5}$, $18\frac{11}{29}$, 8.565+, $719\frac{17}{32}$, 293, $25\frac{1}{5}$.

236. *Complex fractions are unsolved questions in division.*

36. $\dfrac{2\frac{1}{2}}{\frac{5}{7}} = 2\frac{1}{2} \div \frac{5}{7} = \frac{5}{2} \times \frac{7}{5}$, which canceled $= \dfrac{\cancel{5}}{2} \times \dfrac{7}{\cancel{5}} = \frac{7}{2} = 3\frac{1}{2}$.

37. $\frac{2}{3} \div \frac{3}{4} = ?$
38. $\frac{7}{9} \div \frac{1}{7} = ?$
39. $\frac{6}{7} \div \frac{1}{3} = ?$
40. $\frac{1}{3} \div \frac{1}{12} = ?$
41. $\frac{1}{2}$ of $\frac{2}{3} \div \frac{5}{8} = ?$
42. $1\frac{1}{2} \times \frac{1}{6} \div \frac{3}{4} = ?$
43. $2\frac{1}{3} \times \frac{1}{7} \div \frac{2}{3}$ of $\frac{3}{4} = ?$
44. $\dfrac{3\frac{1}{9}}{5\frac{1}{4}} \times \dfrac{\frac{1}{2}}{\frac{1}{3}} \div \frac{6}{7}$ of $\frac{7}{8} = ?$

45. If $120\frac{1}{2}$ lbs. of cheese cost \$14.80, what will 1 lb. cost?

46. Find the cost of 1 lb. of coffee, when $15\frac{1}{2}$ lbs. cost \$1.43.

47. If $11\frac{1}{4}$ yards of cassimere cost \$16.59, what will one yard cost?

48. If $9\frac{1}{2}$ yards of muslin cost 71 cents, what will 1 yard cost?

Answers: $7\frac{9}{19}$, $9\frac{7}{31}$, $147\frac{7}{15}$, $12\frac{68}{241}$, 1.185, $.\dot{6}$, $.\dot{3}$, $.5\dot{3}$, .36, $\frac{8}{9}$, $5\frac{4}{9}$, 2.571.

SUBTRACTION.

237. *To subtract fractions or mixed numbers from whole numbers.*

1. From 87 take $25\frac{3}{7}$.

Whole Nos.	Sevenths.
87	0
25	3
61	4

To subtract 3 sevenths, one whole number is added to both terms. In 1 there are 7 sevenths, from which subtract 3 and the remainder is 4. 1 to the 25 makes 26, which take from 87 and 61 is left, giving for the answer $61\frac{4}{7}$.

The above formula being used for illustration only, the learner will be expected to write his operation as follows:

$$\begin{array}{r} 87 \\ 25\frac{3}{7} \\ \hline 61\frac{4}{7} \end{array}$$

2. $210-37\frac{1}{2}=?$ 6. $1063-819\frac{1}{9}=?$ 10. $29-25\frac{1}{4}=?$
3. $119-82\frac{1}{8}=?$ 7. $3785-10\frac{6}{7}=?$ 11. $167-89\frac{7}{8}=?$
4. $61-4\frac{8}{9}=?$ 8. $2168-14\frac{3}{7}=?$ 12. $36-21\frac{2}{19}=?$
5. $54-5\frac{1}{8}=?$ 9. $1765-777\frac{15}{25}=?$ 13. $218-36\frac{3}{8}=?$

Answers: 987.4, 2153.571, 3774.14, 172.5, 36.875, 56.$\dot{1}$, 48.875, 243.$\dot{8}$, $172\frac{1}{2}$, 3.25, 77.125, 181.625, $14\frac{17}{19}$, 3.75.

238. *To subtract a fraction from another of the same denomination.*

14. From $\frac{6}{7}$ take $\frac{4}{7}$.

$$6-4=2 \text{ or } \tfrac{2}{7}$$

6 sevenths less 4 sevenths leaves 2 sevenths, the answer.

15. $\frac{3}{4}-\frac{1}{4}=?$ 18. $\frac{45}{46}-\frac{23}{46}=?$ 21. $\frac{17}{25}-\frac{12}{25}=?$
16. $\frac{7}{8}-\frac{5}{8}=?$ 19. $\frac{8}{9}-\frac{4}{9}=?$ 22. $\frac{18}{21}-\frac{15}{21}=?$
17. $\frac{12}{13}-\frac{1}{13}=?$ 20. $\frac{12}{21}-\frac{9}{21}=?$ 23. $\frac{24}{27}-\frac{21}{27}=?$

Answers: $\frac{1}{2}$, $\frac{11}{23}$, .2, .25, .$\dot{4}$, .14285+, $\frac{11}{13}$, $\frac{1}{7}$, .$\dot{1}$, $\frac{1}{9}$.

239. *To subtract one fraction from another*, both terms must be of the same denomination, or reduced to the same denomination. $\frac{1}{4}$ can be subtracted from $\frac{3}{4}$, but $\frac{1}{3}$ can not be conveniently subtracted from $\frac{3}{4}$ without first changing the denomination of one or both.

24. Subtract $\frac{1}{3}$ from $\frac{1}{2}$.

$$\tfrac{1}{2}\times\tfrac{3}{3}=\tfrac{3}{6} \textit{ and } \tfrac{1}{3}\times\tfrac{2}{2}=\tfrac{2}{6}.\quad \tfrac{3}{6}-\tfrac{2}{6}=\tfrac{1}{6}.$$

EXPLANATION.—The 6, being common to both fractions, is called the *common denominator*. To raise two fractions to a common denominator, both terms of each fraction may be multiplied by the denominator of the other. [Art. 197.]

25. $\frac{3}{7}-\frac{2}{13}=?$	28. $\frac{4}{13}-\frac{1}{20}=?$	31. $\frac{3}{7}-\frac{1}{8}=?$
26. $\frac{4}{6}-\frac{1}{3}=?$	29. $\frac{2}{3}-\frac{5}{11}=?$	32. $\frac{5}{18}-\frac{1}{5}=?$
27. $\frac{7}{8}-\frac{1}{2}=?$	30. $\frac{7}{8}-\frac{4}{5}=?$	33. $\frac{2}{21}-\frac{1}{18}=?$

Answers: $\frac{25}{91}$, .2576, .3035+, $.\dot{3}$, $.0\dot{7}$, $\frac{7}{33}$, .375, .0396+, .075, .3096.

ADDITION.

240. *Fractions of the same denomination are added together by finding the sum of the numerators.*

1. $\frac{2}{3}+\frac{5}{3}+\frac{7}{3}=2+5+7$ thirds, or $4\frac{2}{3}$.

2. $\frac{6}{7}+\frac{7}{7}+\frac{8}{7}+\frac{9}{7}=?$	5. $\frac{4}{160}+\frac{15}{160}+\frac{98}{160}=?$
3. $\frac{5}{11}+\frac{6}{11}+\frac{7}{11}+\frac{8}{11}=?$	6. $\frac{51}{1000}+\frac{82}{1000}+\frac{718}{1000}=?$
4. $\frac{3}{12}+\frac{2}{12}+\frac{13}{12}+\frac{1}{12}=?$	7. $\frac{16}{119}+\frac{18}{119}+\frac{125}{119}=?$

Answers: 1.336, .851, $\frac{117}{160}$, $\frac{1}{3}$, $1\frac{7}{12}$, $2\frac{4}{11}$, $4\frac{1}{14}$, $4\frac{2}{7}$.

241. Fractions of different denominations are added together by finding a common denominator, as in subtraction, and proceeding as in last Art.

8. Find the sum of $\frac{2}{3}+\frac{3}{5}+\frac{5}{6}$.

$3\times5\times6=90$, *Common denominator.*

$$\begin{aligned} 2\times5\times6&=\ 60\\ 3\times3\times6&=\ 54\\ 5\times5\times3&=\ \underline{75}\\ &\quad 189 \end{aligned}$$

That is $\frac{189}{90}=2\frac{1}{10}$.

Here the numerators 2, 3 and 5 are multiplied successively by all the denominators but their own. 2, for instance, is multiplied by 5×6, the product of which is also the multiplier of the denominator 3, giving for the first fraction $\frac{60}{90}$ instead of $\frac{2}{3}$. In the same way $\frac{3}{5}$ becomes $\frac{54}{90}$, and $\frac{5}{6}$ $\frac{75}{90}$, making 189 ninetieths, or 2.1.

When there are many fractions to be brought to the same denomination, it will be better to first divide the common denominator by each denominator. This gives the number which will raise both terms to the required denomination.* [Art. 197.]

9. $\frac{3}{7}+\frac{4}{9}+\frac{1}{4}=?$

10. $\frac{1}{6}+\frac{1}{7}+\frac{1}{8}=?$

11. $\frac{2}{3}+\frac{2}{9}+\frac{2}{11}=?$

12. $6\frac{1}{2}+7\frac{2}{3}+8\frac{1}{9}=?$

13. $8\frac{1}{2}+6\frac{1}{4}+12=?$

14. $2\frac{1}{3}+\frac{17}{21}+\frac{16}{19}=?$

15. $\frac{11}{21}+9+\frac{1}{11}=?$

16. $6\frac{4}{7}+1\frac{3}{5}+2\frac{1}{5}=?$

Answers: $7\frac{1}{3}$, 10.3714+, $1\frac{31}{252}$, .4345+, 1.0707, 9.612+, 32.25, 3.995+, 22.277, 26.75.

17. $\frac{3}{7}+\frac{7}{14}+\frac{7}{21}+\frac{8}{28}+\frac{16}{35}+5\frac{3}{4}+6\frac{1}{4}=?$

18. $\frac{5}{6}+\frac{6}{7}+7\frac{1}{2}+\frac{8}{9}+4\frac{1}{5}+\frac{7}{8}+\frac{2}{16}=?$

19. $\frac{3}{10}+\frac{4}{25}+5\frac{1}{5}+8\frac{1}{4}+6+8\frac{1}{3}+6\frac{2}{3}=?$

20. $2\frac{1}{9}+6\frac{1}{7}+5\frac{3}{8}+67+\frac{2}{7}+3\frac{5}{8}+1\frac{6}{16}=?$

21. $\frac{1}{2}$ of $\frac{2}{3}+\frac{3}{4}$ of $\frac{4}{5}+\frac{6}{7}+\frac{2}{9}+1\frac{3}{5}+\frac{4}{10}=?$

22. $\frac{8}{9}\times\frac{9}{12}+$ of $\frac{2}{7}\times\frac{6}{7}$ of $\frac{7}{8}+1+5\frac{3}{4}+\frac{1}{4}=?$

23. $2\frac{1}{2}\times 6\frac{1}{4}+8\frac{7}{8}+\frac{3}{4}$ of $\frac{6}{7}+\frac{1}{7}$ of $\frac{7}{8}+7=?$

24. $1\frac{1}{2}\times 2\frac{1}{3}+\frac{6}{7}$ of $\frac{7}{8}$ of $1+\frac{8}{9}+\frac{2}{36}+\frac{1}{18}=?$

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*THE LEAST COMMON MULTIPLE

Of several denominators is the least number which can be divided by them without a remainder. The following is the process for finding it:

To find the least common multiple of 3, 4, 6, 8, 9, 12, 15.

$$\begin{array}{r|rrrrrrr}
2 & 3 & 4 & 6 & 8 & 9 & 12 & 15 \\ \hline
3 & 3 & 2 & 3 & 4 & 9 & 6 & 15 \\ \hline
2 & 1 & 2 & 1 & 4 & 3 & 2 & 5 \\ \hline
 & 1 & 1 & 1 & 2 & 3 & 1 & 5
\end{array}$$

$2\times 3\times 2\times 2\times 3\times 5=360.$

EXPLANATION.—2 was used as a divisor of 4, 6, 8 and 12, and the quotients set down. The other numbers were brought down and 3 divided into those divisible by it without a remainder; and so the process was continued until no number could be found to divide the others without a remainder. The divisors and remaining numbers then being multiplied together, produce 360 as the least common multiple.

Like the operose method of finding the greatest common divisor, this is seldom used.

25. $4\frac{2}{7} \times 5\frac{1}{6} + 6\frac{1}{2} \times 2\frac{1}{6}$ of $8\frac{1}{2} + 33\frac{1}{3} + \frac{8}{12} = ?$

26. $\frac{2}{3}$ of $\frac{7}{8} + \frac{6}{7}$ of $\frac{7}{9}$ of $\frac{8}{14} + \frac{9}{54} + \frac{5}{6} = ?$

27. $\frac{1}{2} + \frac{3}{4} + \frac{2}{3} + \frac{3}{4} + \frac{5}{8} + \frac{1}{2} + \frac{3}{6} + \frac{3}{9} + \frac{2}{3} + \frac{1}{2} + \frac{3}{18} + \frac{5}{12} + \frac{8}{9} + \frac{5}{36} + \frac{1}{2} + \frac{7}{18} + \frac{9}{12} + \frac{41}{72} + \frac{5}{6} + \frac{3}{9} + \frac{11}{12} = ?$

28. $\frac{17}{20} + \frac{8}{11} + \frac{3}{4} + 4\frac{1}{4} + 8\frac{7}{8} + 3\frac{1}{2} + 10\frac{2}{5} + 16\frac{3}{8} + 24\frac{3}{16} + 6\frac{1}{2} + 72\frac{4}{9} + 8\frac{3}{18} + \frac{1}{4} = ?$

29. $\frac{2}{5}$ of $\frac{6}{7} + \frac{2}{13} + \frac{2}{3}$ of $\frac{7}{8} = ?$

30. $\frac{4}{7} + \frac{9}{13}$ of $\frac{12}{19} + \frac{4}{9}$ of $5\frac{1}{2} = ?$

31. $\frac{13}{18}$ of $7\frac{2}{3}$ of $9 + \frac{2}{7}$ of $14 = ?$

32. $\frac{21}{23} + \frac{11}{15}$ of $2\frac{2}{3} + \frac{2}{9}$ of $6\frac{2}{3} = ?$

33. $\frac{13}{27}$ of $\frac{19}{23}$ of $9\frac{2}{3} + \frac{11}{35}$ of $8\frac{4}{7} = ?$

34. $\frac{21}{42} + \frac{41}{53} + \frac{71}{89} + \frac{22}{23} = ?$

Answers: $14\frac{1}{210}$, $15\frac{88}{315}+$, 34.91+, 85.9146, .01269, 7.14285, 26.1428, 5.25, 141.85, 175.85, 1.9642, $12\frac{1}{4}$, $157\frac{437}{1584}$, $1\frac{437}{5460}$, 3.4531+, 14.183+, 4.350+, $53.8\dot{3}$, 11.168+, 3.0278+, 4.0126+, 32.2678+, 11.694+, $1\frac{21}{52}+$, 82.61.

PRACTICAL QUESTIONS.

1. In an invoice of goods there are the following items; required the amount.

$27\frac{1}{2}$	doz.	@ $9\frac{1}{4}$c	$13\frac{2}{3}$	doz.	@ $5\frac{1}{2}$c.
$18\frac{7}{12}$	"	" 12	$16\frac{5}{6}$	"	" $3\frac{1}{4}$
$16\frac{5}{12}$	"	" $12\frac{1}{4}$	$118\frac{1}{6}$	"	" $2\frac{1}{6}$

Ans. $10.65.

2. $\frac{2}{3}$ of a merchant's goods were destroyed by fire, and what remained was worth $1637.50; what was his loss?

3. A owns $\frac{2}{3}$ of a steamboat, B $\frac{1}{8}$ and C the remainder, which is worth $10000; what is the value of the boat?

4. $\frac{1}{4}$ of a saw-mill belongs to A, $\frac{1}{8}$ to B, $\frac{3}{16}$ to C, the remainder to D, and the profits for the year amounted to $1680; what is each man's share?

5. The par value of the pound sterling is $\$\frac{40}{9}$; required the value of £1674 at 10% premium.

6. A can do a piece of work in 8 days, B in 7 days and C in 6 days; in what time can they do it if all work together?

SOLUTION.—A can do $\frac{1}{8}$, B $\frac{1}{7}$ and C $\frac{1}{6}$ of the work in a day. The sum of these fractions is $\frac{73}{168}$. If $\frac{73}{168}$ can be done in a day, $\frac{168}{168}$ (the whole) can be done in $\frac{168}{73}=2\frac{22}{73}$, or 2 days $3\frac{1}{7}$ hours.

7. There are 3 pumps placed in a coffer dam; one will empty it in 10, another in 15 and a third in 20 hours; in what time can it be emptied by working all three at once?

8. Express $\frac{3}{7}$ of a day in hours, minutes, etc.

SOLUTION.—$\frac{3}{7}$ of a day is the same as $\frac{1}{7}$ of 3 days.

	Days.	Hours.	Min.	Sec.
7)	3	0	0	0
		10	17	$8\frac{4}{7}$

EXPLANATION.—As 7 is not contained in 3 days, we reduce them to hours=72 hours, which, divided by 7=10 hours and 2 left, etc.

9. In $\frac{5}{6}$ of a pound (British money), how many shillings and pence?

10. In $\frac{7}{8}$ of a bushel, how many pecks, quarts, etc.?

11. In $\frac{1}{6}$ of a ton (long weight), how many hundreds, etc.?

12. Find $\frac{3}{5}$ of £167 18*s.* 6*d.*

13. Reduce $\frac{1}{2}$ an inch to the fraction of a foot.

14. Reduce $\frac{1}{2}$ a cent to the fraction of a dollar.

15. What part of a pound Troy is $\frac{1}{2}$ an ounce?

16. What part of a ton is $\frac{3}{4}$ of a pound?

17. $\frac{4}{5}$ of a farthing is what part of a pound?

18. $\frac{1}{3}$ of $\frac{3}{4}$ of \$1600 is what part of \$1000?

Answers: \$420; \$210; \$315; \$735; \$480.00; \$3275; 16*s.* 8*d.*; 3 pks. 4 qts.; 3 cwt. 1 qr. 9 lbs. $5\frac{1}{3}$ oz.; $4\frac{8}{13}$ hours; \$8184; $\frac{1}{1200}$; $\frac{1}{24}$; $\frac{1}{24}$; $\frac{2}{5}$; $\frac{1}{200}$; $\frac{3}{8000}$.

RATIO.

242. *The relation that one number bears to another is called ratio.* The quotient arising from dividing one number by another, of the same denomination, is the ratio between them.

And as two quotients can be obtained from comparing any two numbers, it follows that two ratios can also be obtained. The relation that 1 bears to 2 is $\frac{1}{2}$, and that which 2 bears to 1 is $\frac{2}{1}$.

The sign of ratio is the colon. The above ratios would be expressed thus: 1 : 2 and 2 : 1, and would be read *one is to two* and *two is to one.* Some mathematicians divide the first term by the second; others, the second by the first. The former method is most used:

$$6:3 \text{ will equal } \tfrac{6}{3} \text{ or } 2, \quad \tfrac{3}{4}:\tfrac{1}{2}=\frac{\frac{3}{4}}{\frac{1}{2}}=\tfrac{6}{4}, \text{ or } 1\tfrac{1}{2}.$$

243. Numbers or quantities of different denominations can not have a ratio. We can not compare 3 trees with 5 books. But if the numbers are capable of being reduced to the same denomination, they can be compared; for we can say 3 feet is to 2 inches, as it is the same as to say, 36 inches is to 2 inches.

XXVI. PROPORTION.

244. Two ratios may be equal to each other. 2 : 4 = 4 : 8.

2 bears the same relation to 4 that 4 does to 8.

245. When ratios are equal, the numbers or *terms* which compose them are said to be in *proportion*, and are written thus: 2 : 4 : : 3 : 6, and read 2 *is to* 4 *as* 3 *is to* 6.

The first and last terms, as the 2 and 6, are called *extremes*, and the second and third the *means*.

246. The same ratio may arise by comparing 4 quantities, two of which are different in denomination from the other two.

Tons.		Tons.		\$		\$
3	:	6	: :	6	:	12.

The ratio is $\frac{1}{2}$, and if reversed, as 6 : 3 : : 12 : 6, it would be 2.

247. If the extremes are multiplied together the product will be equal to the product of the means.

$$3 \times 12 = 36$$
$$6 \times 6 = 36$$

Hence, when any 3 terms are given, we can readily find the fourth by dividing the product by the odd term. If we had only the three first terms of the above proportion, that is, 3 (Tons.) : 6 (Tons.) : : 6 (\$), the fourth term would be found by dividing the product of 6×6, or \$36, by 3 = \$12, or the fourth term as above.

To apply this in practice, we have only to suppose the 3 tons and 6 tons to be coal, and the \$6 the price of 3 tons. Then 3 tons is to 6 tons, as the price of 3 tons is to the price of 6 tons.

1. What will 35 lbs. of sugar cost, if 7 lbs. cost 77 c.?

STATEMENT.—7 : 35 : : 77 is to the price of 35.

```
7 : 35 : : 77
            35
          ----
           385
          231
          ----
       7)2695
          ----
           385 cents,
       or $3.85.
```

EXPLANATION.—The means, 35 and 77, being multiplied together, produce 2695, and this divided by the given extreme, 7, gives the required extreme, 385, which must be of the denomination of cents, in order that a ratio exist between it and the third term 77 cents.

The same by cancellation.

$$\frac{\overset{5}{\cancel{35}}\times 77}{\cancel{7}}=385\text{, or \$3.85, } \textit{Ans.}$$

By placing our terms in fractional form, we have 35×77 for a numerator and 7 for a denominator. Then reducing both terms in the same ratio, the 7 cancels the 35, leaving 5×77 for a numerator and 1 for a denominator.

2. If $27\frac{1}{2}$ lbs. of butter cost \$3.75, what will $16\frac{1}{2}$ lbs. cost?

3. Find the price of $12\frac{1}{2}$ dozen of chickens, at 30 cents a pair.

4. The price of 21 tons, 13 cwt., 3 qrs. and 15 lbs. of hemp is \$1680.55; what will 15 cwt. cost?

5. What will 54 lbs. $7\frac{1}{2}$ oz. cost, if $15\frac{1}{4}$ lbs. cost \$8.47?

6. If $\frac{6}{7}$ of a ship cost \$7000, what will $\frac{9}{10}$ cost?

7. If 6 men do a piece of work in 7 days, how long will it take 5 men to do it?

STATEMENT. $5:6::7$

$$\begin{array}{r} 7 \\ \hline 5)\underline{42} \\ 8\frac{2}{5} \end{array}$$

The 7 (days) having no ratio to the other numbers of the proportion, is placed *to the right.* At first sight, it would seem that the proportion would be $6:5::7$; but 6 men do not bear the same ratio to 5 men that the time of the 6 bears to that of the 5. A little reflection will convince the learner that 5 men would require a longer time to do the work than 6 men, which would fail to complete a proportion, as shown by the following

STATEMENT. *The greater* : *the less* : : *the less* : *the greater*, or, the greater $\times$ the greater $=$ the less $\times$ the less!

Hence, to find the second term of a proportion, it will be necessary to inquire whether more or less will be required. If more, put the greater of the two terms in the second place; if less, put the less of the two terms in the second place.

8. If two men plow a field in 3 days, how long will it take 3 men to do it?

9. If 26 yards of linen cost \$13.50, what will 10 yards cost?

10. If 3 coats can be made from $10\frac{1}{2}$ yards of cloth, how many can be made from $31\frac{1}{2}$ yards?

11. If the interest for \$750 for 3 years, 4 months and 10 days be \$151.25 (360 days to the year), what is it for one year?

12. The interest of £100, from 3d of April to 25th of February, is £6 5*s.* $9\frac{5}{7}\frac{1}{3}$*d.*; what is it per year?

13. A, B and C are in partnership, and their gains for the year are \$6757; what is each man's share, suppose A invested \$1567, B 2600 and C 3798?

The sum of their investments is to each man's investment, as the total gains to each man's gain.

14. M invests \$6500, N \$1487, O \$3654; in 3 months it is found that their gains are \$1678; what is each man's share?

15. A lends B \$1000 for 13 months 10 days; how long should B lend A \$8271, to return the favor?

16. If the shadow from a two-foot rule be 6 inches, what is the height of a tree that throws a shadow of 75 feet?

17. If 7 men can build 21 perches of masonry in a day, how long will it take 14 men to build 147 perches?

Answers: \$5.19, 9, $3\frac{1}{2}$, 7, 300, \$1329.34, \$936.89, 49, \$5.90, \$45, \$4.50, \$2.25, \$22.50, \$7350, \$58.09, \$30.25, 3, 2.

Remark.—This rule is of less utility to the business man or mechanic than is generally claimed for it, as most of the problems can be solved in less time, and with fewer figures, by the application of Multiplication and Division. Take, for instance, the 17th. If

21 perches can be built in a day, 147 can be built in $\frac{147}{21}=7$ days; and if 7 men can do it in 7 days, 14 men can do it in one-half of 7, or $3\frac{1}{2}$ days

COMPOUND PROPORTION.

248. A proportion is said to be *compound* when it is composed of more than two ratios or four terms.

1. If 3 men in 5 days, by working 8 hours a day, dig a cellar 15 feet long, 12 feet wide and 7 feet deep, in how many days will 2 men dig one 17 feet long, 14 feet wide and 6 feet deep, by working 10 hours a day.

In this problem there are 11 terms and 5 ratios: the ratio between men and men, that between hours and hours; between feet and feet of the length; feet and feet of the width, and feet and feet of the depth. In arranging these terms, we proceed as in simple proportion, Ex. 7, page 229.

		Days.
Men,	2 : 3	: : 5
Hours,	10 : 8	
Length in ft.,	15 : 17	
Width in ft.,	12 : 14	
Depth in ft.,	7 : 6	

1. Days are wanted; write days as right-hand term.

2. Comparing men with men, we find that it will take 2 men a *longer* time to do the job than it took 3 men, so we write the greater of the two terms (3) in the second place.

3. Comparing hours with hours, we reason that it will take less time to do the job by working 10 than by working 8 hours a day, so we write the smaller number on the right and under the second term.

4. Comparing length with length, we reason that it will take a longer time to dig a cellar 17 feet long than it did to dig one 15 feet long, so we write the greater (17) term under the second term.

5. Comparing breadth with breadth, it will take a longer time to dig a cellar 14 feet wide than it did to dig one 12 feet wide, so we write the greater (14) under the second term.

6. Comparing depth with depth, it will take less time to dig a cellar 6 feet deep than it did to dig one 7 feet deep, so we write the smaller numbei under the second term.

The same by cancellation.

$$\begin{array}{r|l} & \cancel{5} \\ \cancel{2} & \cancel{3} \\ 5\ \cancel{10} & \cancel{8}\ \cancel{4} \\ \cancel{5}\ \cancel{15} & 17 \\ \cancel{3}\ \cancel{12} & \cancel{14}\ 2 \\ \cancel{7} & \cancel{6}\ \cancel{2} \end{array}$$

$$\frac{17\times2}{5}=6\tfrac{4}{5} \text{ or 6 days 8 hours.*}$$

REMARKS.—1. The pupil should observe to have the terms of each ratio of the same donomination.

2. The answer will be of the same denomination as the right-hand term.

2. If 6 men in 15 days dig a trench 18 feet long, 7 feet wide and 5 feet deep, in how many days will 21 men dig a trench 125 feet long, 9 feet wide and 4 feet deep?

3. What is the interest of $6784 for 2 years, 6 months and 15 days, at 6% per annum?

STATEMENT.		
		$6
Days,	365	927
Dollars,	100	6784

Ans. $1033.77.

4. The interest of $1467 for 3 years, 4 months and 12 days is $450.72; what is the rate per cent.?†

5. The interest on $786.55 at 10% is $176.44; what is the time?

6. The interest for a certain sum of money for 4 years, 2 months and 20 days, at 6%, is $100; required the principal?

---

*In forming this proportion, *we reasoned from what was given to what was required.* For instance, in comparing men with men, we inquired if it would take 2 men a longer or a shorter time than the time (5 days) that it took 3 men to do it.

†The pupil can prove his own work by computing the interest by the method taught in the first part of this book.

XXVII. PARTNERSHIP.

249. WHEN two or more persons associate together to carry on a business, they are said to be in partnership, and are called a *firm*, *house* or *company*.

Partnerships may be general or special. *General partnerships* extend to the whole of the mutual dealings of the parties. *Special partnerships* are formed for some specific purpose, a single dealing or adventure.

When more than two persons are engaged in business, it is usual to select the names of one or two of the members, with the term "Co.," for the name of the partnership; as a business conducted by Messrs. Jones, Evans, Henderson and Norton might be called the *firm* or *house* of *Jones & Co.**

250. Each member of a firm becomes responsible for the acts and contracts of his copartners, in the way of sale, purchase, promise, agreement, etc., performed in the course of the usual business of the firm. If a partner draws a note or bill even in his own name, and it can be proven to be on account of the partnership business, he thereby renders the firm liable.

251. An individual becomes a partner by allowing the community to presume that he is such, or by having his name appear on a sign or in a bill, card, etc. A secret partner becomes equally liable when discovered, as if his name appeared in the firm.

252. A creditor of one of the members of a firm can

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*The name of a firm is not always derived from the members having the largest interest. Where precedence is not given to age, the names of the most influential are usually selected.

claim only the interest of the debtor in the partnership property after all claims against it have been settled.

253. All partnership agreements should be written.

The funds, property and merchandise furnished by partners for carrying on business are called stock or capital, and the gains are called *dividends*.

The *liabilities* of a partnership or individual business are the debts, and the *assets* their available means, including the indebtedness of others to them.

An *inventory* is a list or statement of those things which constitute *assets*.

254. In keeping partnership accounts, each member of the firm should be credited with all that he brings into the concern or business, and be charged or debited with all that he takes out, just the same as if he had no interest in it.

255. The calculations peculiar to partnership relate to the division of property and profits.

1. A, B and C have been in business one year, and find they have made a net gain of \$3476, which is to be divided as follows: A is to have $\frac{1}{2}$, B $\frac{1}{4}$ and C $\frac{1}{4}$; required the share of each.

$\$\frac{3476}{2}=\$1738$, A's share; $\$\frac{3476}{4}=\$869$, B's share; and $\$869=$C's share.

2. X, Y and Z purchase a tract of land for \$2000, X giving \$600, Y \$900 and Z the remainder. In one year afterward they sell it for \$5500; required each person's share of the proceeds.

3. A, B and C invest \$2000 each. In three months their gross gains are \$2000; expenses, including \$250 for additional services of C, \$600: what will be each man's share of the gain?

XXVIII. JOINT STOCK COMPANIES.

STOCKS.

256. A *Joint Stock Company* is a body of men associated together in a species of partnership, to carry out some heavy undertaking requiring the investment of more capital than individuals or partnership companies commonly possess. Joint stock companies are usually incorporated by act of legislature, with certain privileges. Railroads, canals, bridges, etc., are generally constructed by this species of combined interest, and many banking and insurance houses, scholastic institutions, etc., are owned and managed by joint stock companies.

When an association of this kind is to be formed, a few leading persons make an estimate of the probable amount of capital required, divide it into equal shares of from \$10 to \$100 or \$500, according to the nature of the undertaking, and issue certificates of ownership for each share. These are called *certificates of stock*, and are transferable. Persons owning certificates are called *stockholders*.

Joint stock companies are usually managed by a *president* and *board of directors*, elected for the purpose, by the stockholders.

When shares sell for the price named in the certificate, the stock is said to be at *par;* if above this value, they are said to be *above par;* if below it, *below par*.

Besides the stocks of companies, there are *government stocks*, which consist of bonds that have been issued by state officers, for the purpose of borrowing money. These draw interest at a specified rate.

In dividing the profits of joint stock companies, it has been found more convenient to declare the dividend by percentage.

1. What is the cost of 10 shares of railroad stock at 5% below par, the original cost being $100 per share?

Find the cost of 10 shares at $100 and deduct 5%.

2. A banking institution declares a dividend of 18% on a capital of $30000; what amount of money should a stockholder receive who holds 5 shares, valued at $200 each?

3. I hold 15 shares (each $100) of stock in gas-works, which have declared a dividend of 20%; how much am I entitled to after my gas bill of $20 is deducted?

4. How many shares of United States stocks at 2% above par can I buy for $1224, the original cost being $100 per share?

5. What amount of stock can I buy for $1683, if I am allowed 2% commission on the amount invested?

The amount I am to receive is to be $\frac{2}{100}$ or $\frac{1}{50}$ of the amount of stock purchased—not $\frac{1}{50}$ of $1683, for that would be commission *on commission* and investment.

Let the amount to be invested be represented by $\frac{50}{50}$, and to this add $\frac{1}{50}=\frac{51}{50}$, then we discover that $1683 is $\frac{51}{50}$ of the amount to be invested. $\frac{1683}{51}=33=\frac{1}{50}$, or my commission, which, if we multiply by 50, will give us the amount to be spent, $1650.

To *prove* this, find the commission on $1650 at 2%.

6. A broker receives 1685, which he is desired to in-invest in State stocks; how much should he invest, and allow himself $2\frac{1}{2}$% on the investment?

7. What amount of stock can a broker buy for $16700, and allow himself $\frac{1}{4}$% on the investment?

Answers: $180, $950, $280, $16658.35, $6619.51, 12.

257. When declaring dividends, it is customary to reserve a part of the gains of business for current expenses. Such sum is carried to an account called a Contingent Fund or Contingent Expense Account. Dividends, in this way, are usually declared for an even rate per cent.

8. A coal oil company, with a capital of \$150000, gains \$31,493, and has concluded to declare a dividend of 15%: how much will be left to apply to contingent fund?

15% of \$150000=\$22500.
\$31493—\$22500=\$8993, *Ans.*

9. An insurance company gains \$53,369.87, in six mos., on a capital of \$500000, but does not consider it safe to declare a full dividend; how much will apply to contingent fund account after declaring a rate of 10% per annum?

10. What is the largest even dividend which can be declared by a company with a capital of \$150000, whose gains are \$13547.65?

11. A stockholder, owning 20 shares, at \$50 each, receives a dividend of \$120; what is the rate per cent.?

12. The first dividend of a company is payable in bonds, by which a stockholder, owning 20 shares, obtains two shares worth \$100 each; what was the rate of dividend declared?

13. What would apply to contingent account where a dividend of 20% was declared on a gain of \$316784.87, and a capital of \$2000000?

Answers: \$116784.87, 10%, 12%, 9%, \$28369.87.

XXIX. BANKRUPTCY—INSOLVENCY.

258. *Bankruptcy* signifies inability to pay. A person becomes bankrupt when he is obliged to give up his business for want of means to pay his debts, and to carry it on. Such an individual is said to have *failed*. *Bankruptcy* and *insolvency* are synonymous terms.

Insolvent debtors usually transfer their property to other parties for the benefit of their creditors. This is called *making an assignment*, and prevents the individual debtors from recovering more than a share of the property apportioned to the amount of their claims. The person to whom an assignment is made is called an *assignee*, the property and claims of the debtor, his *effects* or *assets*, and his indebtedness, his *liabilities*.

1. A person failing in business has the following effects to meet claims to the amount of $13000; how much should his creditors receive on the dollar? Merchandise to the amount of $3500, railroad stock to the amount of $2100 and personal claims to the amount of $1500.

$$\begin{array}{r} 3500 \\ 2100 \\ \underline{1500} \end{array}$$

Amount of assets, 7100, which, reduced to cents, and divided by the amount of the liabilities=$54\frac{8}{13}$ cents, or $54\frac{8}{13}\%$.

$$\begin{array}{l} 13|000)710|000(54\frac{8}{13} \\ \underline{65} \\ 60 \\ \underline{52} \\ 8 \end{array}$$

2. The amount of assets belonging to an insolvent debtor is $4684, and his liabilities $22000; how much can he pay on the dollar?

XXX. IMPORTING.

259. IMPORTING is the business of buying goods in a foreign to sell in a home market. A tax, under the name of *duties* or *customs*, is imposed by government on most imported articles of commerce. Such taxes are levied for the purpose of creating revenue to defray the expenses of government or to protect home manufactures and agricultural interests. Duties are regulated by a scale of prices called a *tariff*, and are altered according to the exigencies of the times or caprice of the administration.

A *high tariff* signifies high rates of duties, and a *low tariff*, low rates of duties.*

The persons appointed to examine imported goods and collect taxes are called *custom-house officers*, and their place of business, the *custom-house*.

260. Duties are of two kinds: *Ad valorem* and *Specific*. *Ad valorem* duties consist of a rate per cent. on the *value* of goods as stated in the invoice; *Specific* duties, of a stated sum of money on the *quantity imported*, without regard to value, as $1 a gallon, $20 a ton.

261. Certain allowances, called *draft*, *tare*, *leakage* and *breakage*, are made on goods charged with specific duties. These allowances sometimes consist of a percentage of the weight or quantity and sometimes of a specific deduction.

Tare is an allowance made for the weight of the box,

* Taxes are often levied upon exports as well as imports.

barrel, bag, crate, etc., which contains the goods, and is usually calculated by percentage, etc., after the deduction for draft is made.

Draft or *tret* is an allowance made for loss by weighing in small quantities, and for impurities to which some goods are subject.

On	112 lbs.,	or less,	it is		1 lb.	
From	112 "	to	224 lbs.,		2 lbs.	
	224 "	"	336	"	3	"
	336 "	"	1120	"	4	"
	1120 "	"	2016	"	7	"
	More than		2016	"	9	"

Leakage is an allowance of 2% on liquids in casks, paying duties by the gallon.

Breakage is an allowance on bottled liquors, usually 5%, but on ale, beer and porter, 10%.

Gross Weight is the total weight of goods and box, barrel, etc.

Net Weight is what remains after all deductions are made.

We shall not trouble the learner to work out any questions in this chapter, as it rarely happens that young people have them to do in business.

XXXI. FOREIGN EXCHANGE.

262. In calculating Foreign Exchange, the money of one country has to be represented in that of another. A bill drawn in New York on a merchant in England will be expressed in *pounds*, *shillings* and *pence*.

The *draft*, though not stated in the question, is to be deducted before other allowances are made.

263. The relative value of moneys of different countries depends on the *par of exchange* and the *course* of *exchange*.

264. The *par of exchange* is the comparative value of the coins of the different countries, and is fixed, while the relative purity of the coins is the same. The par of exchange between the United States and Great Britain is \$4.86 to the pound sterling. Formerly, when the silver of the United States was purer, the par value of the pound sterling was $4\frac{6}{9}$, and exchange is still quoted from this par, or $9\frac{7}{20}$ *premium* on the old currency being the present par value.

265. The *course of exchange* usually depends upon the relative state of indebtedness of the merchants of the different countries, and the supply of gold and silver; accordingly, the course of exchange will sometime be above and sometimes below par.

EXCHANGE WITH GREAT BRITAIN.

FORM OF A FOREIGN BILL.

Exchange for £1567. CINCINNATI, *June* 3, 1867.

Thirty days after sight of this first of Exchange, (second and third of the same tenor and date unpaid,) pay to the order of William Tuechter, the sum of One thousand five hundred and sixty-seven pounds sterling, value received, and place to my account as advised.*

To William Morgan, Esq., J. B. TREVOR.
Liverpool, England.

* Foreign bills are generally drawn in *sets* of two, three or four; that is, copies of the same bill are made out and transmitted by different conveyances to the payer, one of which being received and accepted, or paid, the others to be void. These copies are called *First, Second* or *Third of Exchange.* The above is a copy of the *first.* The others are drawn in a similar manner.

BRITISH OR STERLING MONEY REDUCED TO FEDERAL MONEY.

GIVING the cost of British money is called *quoting* it, and the rate, the *quotation*.

Sometimes the total cost of a pound is given; at other times, only the premium on the old par.

266. *When the Amount is given in the Quotation.*

1. What is the value of £157 9 2 in Federal money @ $4.86 to the pound sterling?

SOLUTION.—Since £1 is equal to $4.86, £157 will be equal to 157 times 486, or $763.02. Then taking aliquot parts of a pound, we have 6 shillings and 8 pence=$\frac{1}{3}$, and 2 shillings and 6 pence=$\frac{1}{8}$ of a £; add $\frac{1}{3}$ and $\frac{1}{8}$ of $4.86 to $763.02=765.25.

TABLE OF ALIQUOT PARTS

Of a Pound. s.	d.			*Of a Pound.* d.			*Of a Shilling* d.		
10	0	is	$\frac{1}{2}$	10	is	$\frac{1}{24}$	6	is	$\frac{1}{2}$
6	8	"	$\frac{1}{3}$	8	"	$\frac{1}{30}$	4	"	$\frac{1}{3}$
5	0	"	$\frac{1}{4}$	$7\frac{1}{2}$	"	$\frac{1}{32}$	3	"	$\frac{1}{4}$
4	0	"	$\frac{1}{5}$	6	"	$\frac{1}{40}$	2	"	$\frac{1}{6}$
3	4	"	$\frac{1}{6}$	5	"	$\frac{1}{48}$	$1\frac{1}{2}$	"	$\frac{1}{8}$
2	6	"	$\frac{1}{8}$	4	"	$\frac{1}{60}$	1	"	$\frac{1}{1}$
2	0	"	$\frac{1}{10}$	3	"	$\frac{1}{80}$	$0\frac{3}{4}$	"	$\frac{2}{16}$
1	8	"	$\frac{1}{12}$	2	"	$\frac{1}{120}$	$0\frac{1}{2}$	"	$\frac{1}{24}$
1	4	"	$\frac{1}{15}$	$1\frac{1}{2}$	"	$\frac{1}{160}$	$0\frac{1}{4}$	"	$\frac{1}{48}$
1	3	"	$\frac{1}{16}$	1	"	$\frac{1}{240}$			
1	0	"	$\frac{1}{20}$						

2 to 4. At $4.84, what will the following sums amount to? £345 14 0, £15 3 4, £365 7 6.

5 to 7. £147 13 4 @ $4.87, £425 17 6 @ $4.44, £652 10 0 @ $5=? Answers: $5872.53, $3515.02

267. *To reduce British to Federal money when the Premium only is given.*

8. What is the value of £221 15 6 @ 8% premium?

The premium is reckoned on the old par value; that is, \$4.44$\frac{4}{9}$ to the pound, or \$40 to £9.

$$\begin{array}{r} 12)\underline{6.0} \\ 20)\underline{15\ 5} \\ .775 \end{array}$$

Reducing 15 shillings and 6 pence to the decimal of a pound, we have .775.

Hence, £221 15 6=£221.775, which, multiplied by 40 and divided by 9=\$985.666, to which, if we add 8%, we shall have for the answer, \$1064.52.

What is the value of the following in Federal money?

9.	£1424 19 9 @ 7$\frac{1}{4}$%		13.	£313 8 4 @ 8$\frac{1}{2}$%	
10.	£3575 18 6 @ 8 %		14.	£505 19 6 @ 9 %	
11.	£1100 12 6 @ 8$\frac{1}{4}$%		15.	£3737 12 3 @ 9$\frac{1}{4}$%	
12.	£111 @ 9$\frac{3}{4}$%		16.	£649 4 6 @ 9$\frac{1}{2}$%	
	Total, \$29793.55.			Total, \$25270.28.	

267. *When gold is at a premium*, the rate may be added after the rate of exchange.

Take the 8th example, assuming gold to be at a premium of 45 cents, or 45%.

Value of £221 15 6 at the old par=\$985.666.

To this add 8% premium on British money and 45% on gold to the sum, and we have \$1543.56.

268. TO REDUCE FEDERAL TO BRITISH OR STERLING MONEY.

Case I.

When the Amount is given.

17. At \$4.87 to the pound sterling, what will be the value of \$37654?

Since \$4.87=£1, \$37654 will be the equal to as many pounds as \$4.87 is contained times in that number.

3765400÷487=7731.827, which reduced=£7731 16 6$\frac{1}{2}$.

Reduce the following to British money:

18. \$3674.87 @ \$4.87. 19. \$67845.18 @ $4.44\frac{4}{9}$.

Total, £16019 15 2.

CASE II.

When the Premium only is given.

20. At 9% premium on sterling money, what will be the value of \$3964?

At 9% premium \$1.09 is worth only \$1; therefore, \$3964 are worth as many dollars as \$1.09 is contained times in it.

$396400 \div 109 = 3636.697$, which, reduced to pounds by multiplying by 9 and dividing by $40 = 818.2568$, or £818 5 $1\frac{1}{2}$.

21 and 22. Reduce the following to British money: \$3165 @ $9\frac{1}{2}$%, \$1678.90 @ $9\frac{1}{4}$%. Total, £996 2 $2\frac{1}{2}$.

XXXII. INSURANCE.

269. INSURANCE is a guarantee against loss. It may be of several kinds, as *Fire*, *Marine* and *Life* insurance.

270. Insurance on fixed property is called *fire insurance;* that on movable property, as goods in course of transportation, ships, etc., is called *marine insurance;* that which guarantees the payment of a sum of money to a survivor at the death of an individual, *life insurance.*

271. The act of insuring is termed *taking a risk;* the amount paid for insuring, the *premium;* and the paper upon which the contract is written, the *policy.*

272. When the risk is heavy, the insurer sometimes re-insures in another company.

273. In time of war, the rates of insurance increase with the danger to which the property is exposed, or else the company secures itself by inserting in the policy exceptional matter called the *war clause.*

The rates of insurance vary according to the exposure of the property and the character of the property itself; the greater the risk, the higher the rate.

Insurance can be obtained from one day to a term of years, giving a range of rates, from a small fraction of one per cent. to three, four and even higher rates per cent.

1. How much should be paid to insure a house valued at \$1674, premium being $1\frac{1}{2}\%$, and policy \$1.50?

2. At $2\frac{1}{2}\%$ premium, what should I pay on \$6710 worth of goods?

3. At 4% premium, what should I pay on machinery and material in a factory, the estimated value of which is \$6600?

4. A company takes a risk of \$35000 in a block of buildings, at $1\frac{1}{2}\%$, and re-insures \$15000 in another company at $1\frac{1}{4}\%$; how much premium does it realize?

5. At $\frac{1}{10}$ of 1% for 10 days, what should I pay on \$30000?

6. I have insured \$16000 for 3 months at $\frac{5}{10}$ of 1%; how much should I pay?

7. What will be the insurance on merchandise worth £675, to be shipped from Liverpool to New York, at 4 guineas per cent.?

Answers: £28 7*s*., \$26.61, \$167.75, £30.5, \$80, \$30, \$264.00, \$337.50.

XXXIII. DUODECIMALS.

274. DUODECIMALS, like *decimals*, is a species of calculation which enables the operator to compute fractional quantities as whole numbers.

12'''' *fourths* make 1 third.
12''' *thirds* make 1 second.
12'' *seconds* make 1 prime, or inch.
12' *primes* or inches make 1 foot.

1 *inch* is the $\frac{1}{12}$ of a foot.
1 *second* is the $\frac{1}{12}$ of an inch, or $\frac{1}{144}$ of a foot.
1 *third* is the $\frac{1}{12}$ of a second, $\frac{1}{1728}$ of a foot.
1 *fourth* is the $\frac{1}{12}$ of a third, or $\frac{1}{20736}$ of a foot.

As applied to mechanical pursuits, duodecimals have seldom to be subtracted or divided; hence, the following exercises will be confined to multiplication and addition exclusively.

1. Multiply 2 ft. 5 in. by 3 ft. 4 in

2	5	
3	4	
7	3	
	9	8
8	0	8

1. Writing the dimensions as in the margin, we commence with the left-hand figure (3 feet), and say, 3 times 5 inches are 15 inches, or 1 foot 3 inches; write 3 in the inches' place.

2. Then 3 times 2 are 6, and the 1 foot carried makes 7 feet, which we write in the place for feet.

3. We next multiply by the 4 inches; that is, we multiply 5 inches or $\frac{5}{12}$ of a foot by 4 inches, or $\frac{4}{12}$ of a foot. The result will be $\frac{20}{144}$, but to avoid fractions, we call the result 20'' or 1 inch 8''. Write 8 inches to the right of and below the 3 inches.

4. Then 4 times 2 are 8', and 1'=9' or inches, which we write in the inches' place.

5. We now proceed to add them. There being nothing to add to 8'', we set it down; then 9' and 3' are 12' or inches=1 foot. Write 0 and add 1 to the 7, makes 8 feet: the product of 2 ft. 5 in.$\times$3 ft. 4 in.=8 0' 8'.

ANOTHER WAY.

2. Multiply the following dimensions together: 10 ft. 7 in. $\times$ 3 ft. 8 in. $\times$ 7 ft. 9 in.

10	7			
3	8			
30	21			
	80	56		
30	101	56	*1st pro.*	
7	9			
210	707	392		
	270	909	504	
300	8	11	0	

Here we commence to multiply by the left-hand figure (3), and write the result without reducing to a higher denomination. 3×10 ft$=30$ ft., and 7 in.$\times3=21$ in. Then multiplying by the 8, we write the first product under itself as the multiplier, and the second product, 56, one place further to the right. Adding these, we have the product of two divisors.

Proceeding in the same way with the 7 and 9 of the third dimension, we add together the products and reduce them to higher denominations, by which we get 300 ft. 8′ 11″, or $300\frac{8}{12}$ ft.$+\frac{11}{144}=300\frac{3}{4}$ ft., nearly.*

	ft. in.	ft. in.			ft. in.	ft. in.	ft. in.
3.	17 1 $\times$	3 4 $=$?		7.	4 8 $\times$	6 4 $\times$	17 2 $=$?
4.	14 6 $\times$	7 8 $=$?		8.	3 9 $\times$	2 6 $\times$	11 0 $=$?
5.	21 9 $\times$	14 11 $=$?		9.	21 11 $\times$	6 7 $\times$	17 8 $=$?
6.	18 8 $\times$	16 7 $=$					

Total answers: 802 1′ 3″ and 3159 6′ 3″ 8‴.

	ft. in.	ft. in.			ft. in.	ft. in.	ft. in.
10.	21 7 $\times$	11 10 $=$?		13.	31 7 $\times$	3 2 $\times$	3 3 $=$?
11.	13 9 $\times$	17 4 $=$?		14.	26 3 $\times$	9 5 $\times$	7 7 $=$?
12.	33 7 $\times$	29 3 $=$?		15.	17 1 $\times$	6 7 $\times$	0 9 $=$?

Total answers: 1476 7″, 2283 10′ 9″ 6‴.

Mechanics preferring common or decimal fractions to duodecimals, seldom use the latter.

The following example is worked by both methods:

* For this very simple method, we are indebted to J. C. Kinney, Esq., of Reading, Ohio, not having seen it before. Its simplicity would suggest it as the best method to teach this otherwise difficult rule.

16. How many squares of flooring in 3 rooms measuring 18 ft. 6 in.×by 15 ft. 8 in., and what is the cost of laying, at 50 cents per square?

$$18\frac{1}{2}\times15\frac{2}{3}\times3=\frac{37}{2}\times\frac{47}{3}\times\frac{3}{1}=869\frac{1}{2}\text{ ft., } or \text{ } 8.695 \textit{ sqs.}$$

18	6
15	8
277	6
12	4
289	10
	3
869	6

869 6, *or* 869½ sq. ft., which, reduced to squares of 100 feet=8.695 squares. 8.695×50 cents=4.347, or $4.35.

17. What is the cost of laying 4 floors of the following dimensions, at 75 cents per square? 18 ft. 9 in.×17 ft. 3 in.

18. What will be the cost of shingling a roof which measures 53 ft. 6 in. long, and 5 ft. 8. in. from the ridge to the outer edge of the wall, at $1.50 per square?

19. The average breadth of a board is 1 ft. 4 in., and the length 23 ft. 9 in.; what number of feet does it contain?

20. How many solid feet in a log measuring as follows? 45 ft. 4 in.×1 ft. 6 in.×1 ft. 3 in.

Answers: $9.09, 85 ft., $9.70, 31⅔ sq. ft.

XXXIV. INVOLUTION—EVOLUTION.

275. The process of multiplying a number by itself a certain number of times is called *Involution*, while that of finding the number thus raised, or the reverse process, is called *Evolution*.

276. A number multiplied upon itself is raised to the *second power;* the second power multiplied by the number is raised to the *third power;* the number of the power being indicated by the number of times the original number has been used.

The second power is also called the *square*, because the number of square feet, inches, etc., is found by multiplying the number contained in one side by itself. For a similar reason the third power is called the *cube.*

277. The power of a number is indicated by a small figure over the right of the number, thus: 5^3, which shows that the third power of 5 is understood. This figure is called the *index* or *exponent.*

278. Decimals are raised to any power in the same way as whole numbers, with the difference of placing the decimal point, while common fractions are involved by multiplying the numerators and denominators separately. The second power of .5 is $.5 \times .5$ or .25, and the second power of $\frac{3}{4}$ is $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$.

279. The product of two numbers can not consist of more figures than there are in the two factors, and can consist of only one less than in the two factors. Take 9, the largest number of a single figure, and multiplied by itself, it produces only two figures, 81; and take 10, the smallest number of two figures, and multiply it by itself, and it produces three figures. This principle applies in finding the roots of numbers.

1. Square the following numbers: 3, 7, 9, 4, 6, 15, 27, 89, 97, 112.

2. Raise the following numbers to the powers indicated: 3^3 9^5, 26^4, 30^5, 87^4, 250^3, 189^3.

280. The number from which any power is raised is called the *root* of that power, and the process of finding that number is called *extracting* the root.

281. The root of a number derives its name from the exponent of the power, the second or *square root* being from the second power, the third or *cube root* from the third power; and is indicated thus: $\sqrt{\ }$, square root; $\sqrt[3]{\ }$, cube or third root; $\sqrt[4]{\ }$, the fourth root.

282. A *complete* power is one which can have its root extracted. A *surd* is one which can not have its root extracted. 4 is a complete power, while 5 is a surd.

TABLE.

THE SQUARE ROOT OF

1=1	36= 6	121=11	256=16
4=2	49= 7	144=12	289=17
9=3	64= 8	169=13	324=18
16=4	81= 9	196=14	361=19
25=5	100=10	225=15	400=20

283. Since the product of any two numbers can not consist of more than four or less than three digits or figures, the number of which a root is composed can be found by separating the squares into periods of two figures each, thus: 256 indicates that two figures formed the root, 2,56 being the periods, the root of which is 16; and in the same way 746496 is pointed, 74,64,96, indicating that three figures composed the root. The square root is 864.

THE EXTRACTION OF THE SQUARE ROOT.

1. The square root of 765625 is how much?

```
       76,56,25(875
       64
167   )1256
  7    1169
1745   )8725
        8725
```

1. Commencing at the right-hand, the power is separated into periods of two.

2. The nearest square root of the last period is then taken, which gives 8, or 800. [Art. 83.] Writing the 8 in the quotient, and squaring the number, we have 64 (6400), which is written under the 76, or 76,00,00

3. Subtracting this 64 from the 76, we have a remainder of 12, to which another period (56) is annexed, making 1256.

4. For a part of the new divisor, the 8 of the quotient is doubled, giving 16 as a *trial* divisor. Finding it is contained 7 times, (the 7 being included in the divisor,) that figure is annexed, making 167, and the product is completed.

5. 1169 being subtracted from 1256, leaves 87.

6. Annexing the last two figures of the dividend, the last figure of the divisor is doubled, as before, making 174. This number, with the last figure of the quotient (5) annexed, is contained in 8725, 5 times without a remainder, making the *square root* 875.

Recapitulation.—Separating the power into periods, we find the highest root of the last period. This we place as the first quotient figure, and subtract its square from the period. To the remainder annex the next period, and for a trial divisor double the last figure of the divisor. To this divisor we annex the next quotient figure and multiply as in long division. To the next remainder is annexed the next period, and to the last divisor is added its last figure, which is the same as to double the quotient, and the operation proceeds as before.

2. $\sqrt{1683129}$ is how much?

1,68,31,29(1297.354

	1
22	68
2	44
249	2431
9	2241
2587	19029
7	18109
25943	92000
3	77829
259465	1417100
5	1297325
2594704	11977500
	10378816

Note 1.—This answer may be carried out to any number of places by annexing ciphers, as has been done to produce the .354 of the quotient. Three figures, however, are sufficiently correct for practical purposes.

2. To find the square root of a decimal quantity, we commence at the left to point off the periods of two figures. 146.739 would be pointed thus: 1,46.73,90.

The pupil can prove the accuracy of his calculations by squaring the root obtained.

3. $\sqrt{14161}$.
4. $\sqrt{625}$.
5. $\sqrt{99980001}$.
6. $\sqrt{99999.8000001}$.
7. $\sqrt{7837619}$.
8. $\sqrt{16820.17}$.
9. $\sqrt{23467.809}$.
10. $\sqrt{167037.82}$.
11. $\sqrt{456789.375}$.
12. $\sqrt{10963.849}$.

284. The square root of a fractional number is found by extracting the root of each term. The square root of $\frac{4}{25}=\sqrt{\frac{4}{25}}=\frac{2}{5}$.

285. Decimals are pointed off in periods from the right. .31671 is pointed thus: .31,67,10.

286. The square root of the product of two numbers gives a mean proportional between them. $\sqrt{5\times20}=\sqrt{100}=10$, the mean proportional between 5 and 20.

287. The square root of the area of a square is equal to the length of the side.*

13 to 17. Find the mean proportional between 7 and 175, 121 and 36, 6 and 24, 42 and 38, 16 and 49.

Answers: 35, 28, 66, 39.949+, 12.

18. A square garden contains 2916 yards; what is the length of a side in feet?

19. A pavement is 112 feet long and 7 feet broad; what will be the length of the side of a square of equal area?

20. How many yards of ground in the side of a square which would be equal to a lot measuring 144×196?

21. What is the length of the side of a square piece of land which contains 25600 acres?

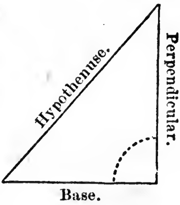
Answers: 28, 168, 162, 160, 2023.85+.

---

*The *area* is the contents of the surface, or the number obtained by squaring the side of a square.

288. The surfaces of circles are to each other as the squares of their diameters or circumferences.

289. A triangle is a figure having three sides and three angles, or corners. If one of these angles is square, it is called right angle and the triangle is called a *right-angled triangle.*



290. Two square figures, one having each of its sides equal to the perpendicular, and the other having each of its sides equal to the base, will, together, be equal to a square, each of whose sides is the same length of the hypothenuse.*

Let the perpendicular be 3, the base 4 and the hypothenuse 5.

Then $3\times3=\ 9$, the area of the first square.
$4\times4=16$, " " " " second "
$\overline{25}$, " " " " third "

But 25 is the hypothenuse squared or multiplied by itself; therefore the square root of the square 25 will be the length of the hypothenuse. The square root of $25=5$.

Hence, the square root of the *sum* of the squares of the base and perpendicular will give the hypothenuse.

And the square root of the *difference* of the squares of the hypothenuse and either of the two sides will give the third side.

22. What length of a ladder will reach across a 15-foot alley to the top of a house 30 feet high?

$15^2=225$
$30^2=900$

1125, *the square root of which is* 33 *ft.* $6\frac{1}{2}$ *in., nearly.*

* The pupil should construct a diagram, with these squares his slate.

23. What is the diagonal of a room 18 feet by 16?

24. A ladder 30 ft. long, placed between two trees, reaches to the height of 27 feet on one of them and 25 on the other; what is the distance between them?

XXXV. EXTRACTION OF THE CUBE ROOT.

291. THE *cube root* of a number is such a number which, if multiplied upon its square, will make that number: $2^2\times2$, or $2^3=8$. The cube root of $8=2$. The sign of the cube root is $\sqrt[3]{}$.

The cube root of any number consisting of three figures will be a number represented by one figure; the cube root of a number containing more than three and less than seven figures, will be one containing two figures; hence, we point off the figures by *threes* instead of *twos*, as in square root.

1. Find the cube root of 262144.

$$\begin{array}{lrl}
 & & 262\overset{\vee}{\,}144(64 \\
6^3= & & 216 \\
 & & \overline{46144} \\
6^2\times300 & =10800 & \\
6\times4\times30= & 720 & \\
4^2 & =\underline{\quad16} & \\
 & 11536 & \underline{46144}
\end{array}$$

EXPLANATION.—The nearest cube root of the first being 6 (60), that number is placed to the left and its cube taken, which is 216 (216000). This subtracted from the dividend, leaves 46, to which is annexed the next period (144), making 46144.

For a trial divisor, 6^3 is then multiplied by 300, giving 10800, which is contained in 46144, 4 times.

The part of the root previously obtained is multiplied by this, and that product by 30, giving 720. The square of the last figure of the root is then taken, and the three results added, making 11536, which, multiplied by the last figure in the quotient, gives 46144.

2. Find the cube root of 17596287801.

$$
\begin{array}{rrl}
 & & 17^{\vee}596^{\vee}287,801(2601 \\
 & 2^3 = & 8 \\
2^2 \times 300 & =1200 &)9596 \\
2 \times 6 \times 30 = & 360 & \\
6^2 & = \ \ 36 & \\
 & 1596 & 9576 \\
 & & 20287801 \\
260^2 \times 300 & =20280000 & \\
260 \times 1 \times 30 = & 7800 & \\
1^2 & = \ \ 1 & \\
 & 20287801 & 20287801
\end{array}
$$

This differs from the last example only in the cipher of the quotient.

Finding the trial divisor ($26^2 \times 300$) was not contained in the new dividend, a cipher was annexed to the quotient, and another to the trial divisor, giving $260^2 \times 300$ or 20280000.

This being contained in the dividend 1 time, the former part of the quotient was multiplied by it and 30, and with the square of the last figure (1) added to the trial divisor, as before, giving 20287801, which, multiplied by 1, completed the extraction. The *cube root* is 2601.

Rule.

Find the greatest root of the left period, place it in the quotient and divisor, and subtract its cube from the dividend.

To the remainder annex the next period, and, for a trial divisor, multiply the square of the root thus obtained by 300.

Divide the new dividend by this divisor, and enter the product of it, with the root already obtained and 30, *under the divisor; under this enter the square of the last quotient figure, and the sum of the three numbers will be the true divisor.*

Multiply this divisor by the last quotient figure, and subtract the product from the dividend; to the remainder annex another period, and proceed as before.

3 to 7. $\sqrt[3]{389017}=?$ $\sqrt[3]{259696072}=?$ $\sqrt[3]{5735339}=?$ $\sqrt[3]{219365327791}=?$ $\sqrt[3]{99252847}=?$

APPLICATION OF CUBE ROOT.

292. A cube is a solid body, having all its sides of equal length. Any two sides of a cube multiplied together will give the superficial contents of one of the faces of the cube, and this multiplied by another side, will give the solid contents; therefore, the cube root of the number of feet, yards, etc., contained in any solid, will give the side of a cube of equal bulk.

7. An irregular block of stone contains 15781 cubic feet and 1333 cubic inches, or 27270901 cubic inches; what will be the side of a cube of equal solidity?

8. Required the depth of a cubic cistern that will contain 3375 feet.

9. What will be the side of a cubic bin or box that will contain 20 bushels of wheat?

Answers: 15 ft.; 35.03 in.; 301 in. or 25 ft. 1 in

XXXVI. ARITHMETIC APPLIED TO THE TRADES, FARMING, ETC.

CARPENTRY.

The *Carpenter* proper may be called the *outside* and the *joiner* the *inside* carpenter. The distinction is seldom observed.

Master Carpenters are sometimes called Builders. They will contract for the entire work of an edifice, and superintend its construction. The legitimate business of the carpenter is to prepare and fit all the wood-work used in building houses. His prices depend on the quality of material and style of finish.

Plain work on one side of white pine lumber is taken as the unit of measurement, and is called	1
Plain work on poplar is	$1\frac{1}{2}$
Plain work on ash, oak, etc	$2\frac{1}{2}$
Plain work on maple	3
Segmental or Norman work on white pine	$2\frac{1}{2}$
Gothic work on white pine	3
Serpentine, or the Oriental variety, plain	5
Domes	9

Floors, roofs, partitions and weatherboarding are measured and charged for by the square—100 sq. ft.

The quantity of good lumber required for a square of flooring is 112 feet, $\frac{1}{8}$ or $12\frac{1}{2}\%$ being allowed for waste.

The quantity of good pine shingles required for a square of roofing is 1000.

USEFUL HINTS ON BUILDING.

Persons about to erect buildings in cities, should obtain a permit from the Board of Improvement, else they subject themselves to damages for placing obstructions in the street.

Proprietors of adjoining lots and buildings should be notified of the digging of cellars, or the making of other excavations that would endanger their property. In Cincinnati, cellars may be dug 12 feet deep without the risk of incurring damage. To excavate or build cellars deeper than this, or to build sub-cellars, permission should be obtained from adjoining proprietors.

Where property is valuable, it will be to the advantage of persons who purpose building, to remember that vaults, cellars, cisterns, etc., may be said to occupy no room; and that license may be obtained from the Board of Improvement to extend cellars or vaults under the sidewalk or pavement.

Foundation walls should be made thicker than the main walls of buildings, and the latter, to be secure, should rest on the middle of the foundation, allowing it to project on both sides.

PRICES OF CARPENTER WORK,

WITH GOLD AT PAR, FOR A DWELLING-HOUSE OF TEN ROOMS.

Cellar windows, usual size, with sash	$1.50
Cellar steps, good	2.00
" doors, each	4.00
First floor joists, 1½ in., per square	0.75
Floors, per square	0.65
Trimmers, per foot	0.10
Pinning, " "	0.02
Roofing, per square	1.50
Hip rafters, per foot	0.06
Valley " " "	0.06
Cants	0.01
"	0.03
Trap-doors, each	1.50
Ceiling joists, per square	0.50
Partitions, " "	0.35
Door heads, " "	0.35
Inside door-frames, per ft.	0.04
Outside " " "	0.05
Window frames, plain	0.04
Beads for do., soft wood	0.01
" " " hard "	0.01½
Box. W. frames, soft	0.08
" " " hard	0.11
" " " beads	0.01½
R. sills	0.06
M. rails	0.06
Setting frames	0.07
Pocket and pullies, per pr.	0.35
Hanging sashes, per pair.	0.20
Base, 6 in. wide, plain, per lineal foot	0.05
Base mold	0.08
" " large	0.09
Casings, 6 in	0.05
" 5 "	0.04
Plinths, each	0.10
Caps, 7 in., per foot	0.10
" 8 " " "	0.08
" 10 " " " M.	0.10
" Cornice	0.25
Shelves, common	0.03
Shelves, best	$0.05
Cloak rails	0.03
Trimming doors	0.30
Door sills	0.20
Mantels, each	5.00
" common, each	4.00
Cupboard front, per sq. ft.	0.04
Window stools, per foot	0.06
" " " "	0.18
" " large	0.22

Cornices.

Gutters on eave inverted, per lineal foot	0.13
Do., mold	0.18
Do., 3 members	0.23
Do., 4 "	0.25
Do., mold	0.37
Do., " brackets	0.40
Do., 6 members	0.30
Do., 6 modillions, etc	0.42
Do., truss, each $3 to	5.00
Truss usually referred to measurer.	
Fastening ornaments	0.04
Bracket cornices	0.05
Lining from $0.04 to	0.05
Tubes	0.25

Porticoes.

Square colums, 8 to 9 in., per foot	0.15
Capitals, each	0.75
Best	1.25
Porch, front, per foot	0.15
" panel	0.20
" extra	0.35
" cornice, plain	0.30
" full	0.50
Ornamental left to measurer.	
Floors and roofs, per sq. ft.	0.25
Framework	0.20
Sills	0.14

In carpenter work, the cost of material is about equal to the cost of labor. For calculations, see duodecimals, page 246.

MASONS' WORK.

293. A bill of prices and a standard of measurement are generally fixed upon by mechanics of the various cities of the Union, and contract work is charged for at a certain rate per cent. on the bill, according to agreement.

Master masons, carpenters, etc., usually select one of their number who is expert in figures, and a good judge of work, to attend to the measurement of work done. This person is called a *measurer*, and his fee is paid one-half by the workmen and the other by the employer. The following rules are taken from the Cincinnati Stone-masons' bill of prices:

"RULES FOR MEASURING.

"1. All work is to be measured by the perch of $24\frac{3}{4}$ cubic feet.

"2. All work to be measured from each outside corner, including all openings under eight feet wide.

"3. All openings less than five feet wide to measure solid, and round their jambs, provided there is no frame; but if there is a frame, they measure solid and half round their jambs.

"4. Chimney abutments and common pillars to measure front and both ends for length.

"5. All walls, however thin, to be reckoned eighteen inches thick.

"6. All partition walls to be measured from out to out."*

* "NOTE.—The above rules are for workmanship only. When the

When measuring hewn stone, neatly piled before it is laid in the wall, and in computing masonry for public works, it is customary to reckon 25 feet to the perch. Twenty per cent. is deducted for loose stone in piles.

1. How many perches in a pile of hewn stone measuring 28 feet long, 5 feet high and 4 feet broad?

$$\frac{28\times5\times4}{25}=22\tfrac{10}{25}\text{, or 22 per. 10 ft.}$$

2. What quantity of stone is in a pile $150\times15\times12$ feet?

3. What will it cost to build the foundation of a house, which is 75 feet long, 16 feet wide and 7 feet high, wall 18 inches thick, @ \$2.25 a perch, including materials?

4. What will it cost to build a cellar that is 16 feet square, with walls 2 feet thick and 8 feet high, @ \$2.75 per perch, material included?

Answers: \$173.73. 1080 perches, \$113.77.

BRICKLAYERS' WORK.

294. Bricklayers' work is computed by the thousand bricks. The usual dimensions of a brick are 8 inches long, 4 inches broad and 2 inches thick. There are 21 bricks in a cubic foot of wall, mortar included. The government standard is 1000 bricks to 40 cubic feet. A brick of the above dimensions weighs $4\frac{1}{2}$ lbs.

A bushel of sand weighs 113 pounds; of slacked lime, 51 pounds.

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workman furnishes materials, the value of the materials, both stone and mortar, in the above extra measurements, as well as the value of the materials saved by doors and windows, is to be deducted from the foot of the bill; that is, neither stone nor mortar is to be charged in any instance where they are not used."

RULES OF MEASUREMENT OF THE BRICKLAYERS OF CINCINNATI.

"All lengths shall be exterior or taken on the outside, from corner to corner, and for every return or cross section at openings deducted, nine inches by height for work and materials, or one foot by height for workmanship shall be allowed. Antaes and Pilasters returns allowed in all cases.

"All octagon or circular work of a radius of three feet or less shall measure double, and for larger radius a less but fair allowance shall be made. All walls cut up for or coped with brick, shall measure one foot additional height, and all walls both cut up for and coped with brick, two feet additional height. All work with a batter or material deviation from plumb line, shall measure once and a half.

"Flemish or plumb bond fronts shall measure solid in all cases, and stock or pressed brick with tuck joints shall measure double, with the additional cost of pressed brick if furnished by the contractor.

"Kettles or stills shall measure solid, and those of 50 feet or less, exterior surface, once and a half; and those of 25 feet or less, exterior surface, double. Fire fronts, when cased, double measurement.

"Culverts or sewers, 9 inches thick and two feet or less in diameter, and those of half-brick thick and three feet or less in diameter, shall measure solid. All circular work shall measure exterior girth.

"All openings of sectional area greater than ten feet shall be deducted, except those in bond fronts and ovens, which shall not till exceeding fifty feet; then half of such shall be deducted.

"Twenty-one bricks shall be allowed to the cubic foot for all brick-work, and the same proportion for thicker or thinner walls from one brick in thickness upward; forty-three bricks shall be allowed per yard for brick paving when flat, and eighty for paving on edge in ascertaining the amount of bricks only; but when paving is done and measured as brick-work, the sand shall be allowed for in the measurement, and all materials on edge or cut to, waste allowed for also.

"Cisterns, when measured for brick-work and materials, half brick wall shall count six inches and whole brick wall ten inches, and cistern arches and those over circular vaults shall in all cases measure double.

"The floor joists shall govern the height of stories in all cases till two stories make more than twenty-four feet for carrying materials, and then twenty-four feet shall be allowed for two stories and each ten feet of the additional height a story. When new work is built upon an old building, or one built by another contractor, all such work shall be measured and allowed for an additional story in height for labor of carrying materials.

"Old bricks in piles shall be subject to the usual discount in ascertaining the quantity, and in the absence of special agreement shall, if on the premises and sound, be valued same as new bricks at the kiln; if from a burnt building, or otherwise unsound, shall be valued by the measurer accordingly.

"When hands or materials are furnished for an employer, in the absence of agreement or contract, actual cost, together with fifteen per cent. thereon, shall be charged by the contractor.

"Lumber and materials for scaffolding, mortar beds, etc., and vessels for holding water, shall, in all cases, be furnished by the employer, unless otherwise agreed upon, in which case a reasonable charge shall be made by the contractor.

"For all work not embraced in, or provided for by the foregoing, a fair and reasonable allowance shall be made by the measurer."

1. How many thousands of brick will be required to build a wall 90 feet long, 6 feet high and 20 inches thick?

$90 \times 6 = 540$ square feet of surface.

In one square foot of a 20-inch wall there are 35 bricks; in 540 square feet there are $540 \times 35 = 18900$ bricks, which, divided by $1000 = 18.9$, or $18\frac{9}{10}$.

2. In a house there are 6200 square feet of 20-inch wall and 2000 square feet of 12-inch wall; what will be the cost of building, at 11.50 a thousand, including price of brick and laying?

3. What will it cost to pave a yard 25 by 50 feet, and a walk 75 by 5 feet, @ 50 cents a yard including materials?

Answers: \$90.28, 2978.50.

4. A cistern is 8 feet in diameter and 12 feet deep (average measure); what will be the cost of building at 40 cents a barrel?

$8^2 \times 12 \times .1865 = 143.23$ bbls., $143.23 \times 40c. = \$57.29$.

Instead of multiplying by .7854 (as required by Art. ——), and dividing by 4.211 (the number of feet in a barrel), we merely use the quotient arising from $.7854 \div 4.211 = .1865$, as a multiplier.

The true pitch of a roof is obtained by making the rafters three-fourths of the width of the building.

The Gothic pitch is that produced by making the rafters as long as the building is wide.

For information relating to the department of Building, we are indebted to R. B. Moore, Esq.

STONE CUTTERS.

AVERAGE PRICE AND RULES OF MEASUREMENT FOR COMMON FREESTONE WORK.

295. In measuring plain stone work, all the dressed faces of the stone are taken, and the whole reduced to *superficial measurement.* For instance, a step 4 feet long, 14 inches wide and 7 inches thick would be measured as follows:

Length, 4 ft.+7 in.+7 in.=5 ft. 2 in.
Width, 14 +7 =1 9,

and 5 ft. 2 in.×1 ft. 9 in.=9 ft. 1″, or 9 ft.

Remark.—It will be observed that the ends have been measured twice; this is in accordance with custom.

Window-sills are measured by the running foot, including the projections of the ends. Prices, for 7-inch wide, and 4 to 5 thick, per foot, 18 to 25 cents.

Water-table.—The stone in front of a house and on a level with the door-step. Measured as above, 37½ cents per foot.

Ashler or *slab front.*—Face measure, adding all worked ends. Price, 40 cents per superficial foot.*

Flagging.—Superficial measure, 2½ inches thick, 25 cents; 3 inches, 30 cents; 4 inches, 40 cents; 6 inches, 50 cents; 8 inches, 60 cents.

Fire-wall coping, running measure, 11 by 2 inches, per foot, 25 cents.

Chimney coping, 2½ to 3 inches thick, per foot, 30 cents.

Coping caps, common size, each, $2.

Hearths, common thickness, per superficial foot, 40 cents.

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* Stone-cutters set (build) their own work, the charge for which is included in these prices.

Edge curbings, for walks, 2 to 3 inches thick, per linear foot, 25 cents.

Door and *window cornice*, not exceeding 6 inches thick and 6 inches projection, per foot, $37\frac{1}{2}$ cents. Measured length and returns 3 girts from wall to wall.

Piers for open fronts, face measure, taking the girt.

A common door-piece, comprising 2 each, plinths, piers and caps, with lintel, cornice and blocking, will cost from $50 to $75.

All cut stone should be laid in cement.

Foundations and excavations for steps should be sunk at least three feet below the surface, else the action of the frost on the earth will be liable to displace the stone.

Mortar should not be exposed to the action of frost until it is set.

During the heats of summer, mortar is injured by a too rapid drying in the wall; to prevent this, the other materials, stone or brick, should be thoroughly moistened before being laid; and afterward, if the weather is very hot, the masonry should be kept wet until the mortar gives indications of setting. In very warm weather, the top course should always be well moistened by the workmen on quitting their work for any short period.

PLASTERING.

296. The business of the plasterer is to cover brick and stone work, ceilings and partitions, with plaster, and prepare them for paper, paint, etc.; also, to form cornices and such other decorative portions of walls and ceilings as may be executed in plaster or cement.

To lay off a square corner or a right angle, with a carpenter's rule: Measure 3 feet from the corner in one direction, 4 feet in another direction and separate the points 5 feet apart.

"CINCINNATI PLASTERERS' RULES OF MEASUREMENT.

"1. All work shall be measured superficially, including openings. All heights shall be taken from the floor to the ceiling.

"2. All staircases eight feet wide and under shall be measured double; all over eight feet, once and a half.

"3. All passages four feet wide and under shall be measured once and a half; all over four feet, once and a fourth.

"4. All inclined ceilings to measure once and a half.

"5. All dormor windows, closets and privies to be measured double.

"6. All octagon and circular work, except ceilings of rooms, to be measured double. All arched ceilings of rooms to be measured once and a half.

"7. The deductions for openings occasioned by doors and windows, when the workman furnishes materials, shall be, for lathwork, one-eighth; for brick walls, one-fourth.

"8. The materials for scaffolding and mortar-beds and vessels for holding water, are, in all cases, to be furnished by the employer."

STUCCO WORK.

Moldings, cornices not over 12 inches girth, are measured by the running or linear foot. Flowers, sometimes singly; in moldings, per superficial foot.

Eighteen laths will cover a yard; 500 laths, a square of 100 feet.

Estimate work is made from a bill of prices, the carpenter agreeing to work for a certain percentage on the bill.

1. What will be the cost of plastering a room 18 feet long and 16 feet wide, with a ceiling 10 feet high, at 18 cents?

18
16
$34 \times 2 = 68$, *length round the room.*
$68 \times 10 = 680$ *square feet in walls.*
$18 \times 16 = 288$ " " " *ceiling.*
9)968
107.55 *yds. at* 18c.=\$19.359, *or* \$19.36.

2. How many square yards of plastering in a hall 84 feet long and 40 feet wide, with a ceiling 18 feet high, having a space of 600 square feet occupied by windows?
Ans. $802\frac{2}{3}$ yards.

PAINTING, PAPER-HANGING AND GLAZING.

297. Painters' Work is measured by the square yard, and charged for according to the number of coats, the quality of the paint and the description of the work. Sash frames are charged for singly or by the piece, and sashes by the number of squares. Lettering is charged for by the lineal foot. Common lettering, 25 cents; gilding, 75 cents.

Painting is sometimes charged for by the quantity of paint used, and the time spent in putting it on.

The calculations being so simple, it is considered unnecessary to give any examples.

298. *Glazing* is sometimes charged by the square foot, and sometimes so much per light. When estimated by the foot, it is usual to include the sash in the measurement.

299. *Paper-hanging* is charged for by the piece. The commoner qualities measure about $7\frac{3}{4}$ yards by 19 inches, or about 35 square feet; the better qualities, say from 50 cents a piece, are 9 yards by 21 inches, or 47 square feet. Border paper is made in rolls of the same dimensions as the wall paper, each roll or piece containing two or more strips, each of which is called a piece, and is sold at about the same price as the paper it is designed to match.

Dealers in paper usually contract for the hanging, and charge from 20 to 25 cents a piece, according to quality.

300. To find the quantity of paper required for a room, compute the number of square feet in the walls; deduct the openings and divide the result by the number of feet in a piece. The space occupied by the base will allow for waste and matching pattern.

1. A room is 16 feet square and has a ceiling 12 feet high, with two doors 7 feet by 4, two windows 7 by 3 feet and a fire-place 4 feet square; how much paper will be required to hang it, and what will be the whole cost, including hanging—paper 25, border 35 cents?

$16\times 4=64$, *length of wall around the room.*

$64\times 12=768$ *square feet of wall.*

Windows,	42 feet.	
Doors,	56 "	@
Fire-place,	16 "	
		114
		654 *square feet to be covered.*

$\frac{654}{35}=19$ *pieces.* 35 *number of feet in a piece.*

19×25 cents=cost of paper	4.75
Length of border, 64 feet, or 3 pieces at 35	1.05
Hanging 22 pieces @ 20 cents,*	4.40
Total cost	$10.20

* The border is included in the 22 pieces.

GAS FITTING AND PLUMBING.

301. *Gas fitting* is charged for per foot of pipe, varying according to size. Fittings and chandeliers, per piece.

302. *Plumbing* is charged for like gas fitting. For ordinary house work, say from 25 to 30 cents per foot of pipe; sheeting by weight.

FARMING.

The young farmer will find it to his interest to be a good arithmetician. For those who have not had the advantages of an early education, we will introduce a few of the simpler and more necessary calculations, suggesting, at the same time, that during his leisure moments the farmer should master the entire science as contained in this little work, which any person of ordinary ability, who can read and write, may accomplish without the aid of a teacher.

303. To find the number of acres in a field or tract of land having four square corners,* we multiply the length by the breadth, and divide the result by 160, if the measure was taken in rods; or by 43560, if taken in feet.†

1. The length of a field is 125 rods and its breadth 112 rods; how many acres are in it?

$112 \times 125 = 14000$, which, divided by 160, gives $87\frac{1}{2}$.

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* A figure having square corners, and all its sides equal, is a *square;* one having its opposite sides equal, a *rectangle* or *parallelogram.*

† In a square rod there are $272\frac{1}{4}$ square feet. When there are feet remaining to be reduced to rods, it will be sufficiently accurate to divide by 272.

2. A lot of land is 400 feet long by 110 feet broad; how many acres does it contain? *Ans.* 1 acre 1.7 rods.

304. *To lay off a given quantity of land.*

3. What should be the length of a strip of land 30 rods broad to contain 6 acres?

In 6 acres there are 960 rods, which, divided by 30= 32 rods.

305. *To find the contents of a field in the shape of a right-angled triangle*, we multiply the two shorter sides together, and take one-half the product.

Reason.—A right-angled triangle is half a square or parallelogram, formed by drawing a line between opposite corners.

4. The shorter sides of a right-angled triangle are 45 and 60; required the contents. *Ans.* 1350.

306. *To find the quantity of grain or coal in a bin or wagon*, we multiply the length, breadth and height together, and for *grain* divide the product by 1.2444,* if the diminsions are given in feet; or by 2150.42,† if given in inches. For *coal*, by 1.555, or 2688.

5. A wagon is 8 feet long, 5 feet broad and 18 inches deep; how many bushels of corn does it contain?

$8 \times 5 \times 1\frac{1}{2} = 60$, *the number of cubic feet.*

1.2444)60.0000(48.21, *or* $48\frac{1}{5}$ *bushels.*

```
1.2444)60.0000(48.21
       49 776
       ------
       102240
        99552
       ------
         26880
         24888
        ------
         19920
```

Note.—Two ciphers were annexed to the dividend to correspond with the decimals of the divisor, and produce the *whole numbers*, 48, and two more ciphers were annexed to produce the decimal .23, or $\frac{21}{100}$ or $\frac{1}{5}$.

6. How many bushels of grain in a bin measuring 4 feet every way? *Ans.* $51\frac{1}{2}$, nearly.

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*Feet in a bushel. †Inches in a bushel.

307. *To find the quantity of wood or bark in a pile*, we multiply the three sides given in feet, as before, and divide by 128, the number of feet in a cord.

7. How many cords of wood in a pile 40 feet long, 7 feet high and 4 feet broad? *Ans.* $8\frac{3}{4}$ cords.

308. *Having two sides and the contents of a box, to find the third side*, we divide the cubical contents by the product of the two sides.

Reason.—Since the product of the three sides equals the contents, the contents divided by two of the sides will give the third side.

8. A box is 2 feet wide and 3 feet high; how long should it be to hold 25 bushels of coal?

In 25 bushels there are 2688×25 or 67200 cubic inches.
In 2 ft. there are 24 inches.
" 3 " " " 36 " $24 \times 36 = 864 =$ area of the end.

$$672 \div 864 = 77\tfrac{672}{864}, \text{ or } 6 \text{ ft. } 5\tfrac{3}{4} \text{ in.}$$

9. What must be the height of a bin that will hold 300 bushels of wheat, if its length is 30 feet and its width 4 feet? *Ans.* 3 ft. $1\frac{1}{3}$ in.

10. What must be the depth of a box 16 inches square to hold a bushel? a box 10 inches square to hold a peck? one 8 inches square to hold half a peck?

To find the side of a cube that will hold a certain quantity. See Cube Root.

309. *To find the quantity of grain when heaped against a wall or partition*, take half the perpendicular height for one side, and multiply it by the length and breadth, as in Art. 306.

310. *To find the number of cubic feet in a round log.* See Art. on the Cylinder. To find the solidity of a cylinder, we multiply the area of the end by the length.

11. How many feet are in a log 12 feet long and 30 inches in diameter?

In 30 inches there are $2\frac{1}{2}$, or 2.5 feet: $2.5\times2.5\times.7854=4.9087$, the area of the end. $4.9087\times12=58.9044$, or $58\frac{9}{10}$ feet, the solid contents.

REMARK.—This method of calculating, though correct, is seldom used for practical purposes. It is customary for lumber merchants to throw off one-third of the diameter, and consider the remainder the side of a square log. A log of the dimensions named in the preceding question would thus measure only $33\frac{1}{3}$ feet, or one-third of 100 feet; and is thereby taken as the standard of measurement in some of the Western States. See Lumber Business.

311. *Trade, or barter.*

12. How many cords of wood, at $3.75 a cord, should I get for 50 bushels of wheat at $\$1.12\frac{1}{2}$ a bushel?

$50\times1.12\frac{1}{2}=\$56.25$, which, divided by $3.75, will give the number of cords. $5625\div375=15$ cords.

PROOF.—15 cords at $\$3.75=\$56.25$.

13. How many pounds of sugar, at 8 cents a pound, should I get for 127 pounds of butter, at $12\frac{1}{2}$ cents a pound? *Ans.* $198\frac{1}{2}$.

14. How many days' work of a man, at 75 cents a day, will be equal to 45 days' work of a man at $1.25? *Ans.* 75.

15. How many cords of wood, at $2.25, will be equal to 150 cords, at $3.50?

16. How many yards of muslin, at 8 cents a yard, can be bought for 5 dozen chickens, at $1.25, and 15 dozen eggs, at $8\frac{1}{3}$ cents?

LUMBER BUSINESS.

312. Lumber measure comprises solid and superficial measure. Round logs are measured by deducting *one-third* of the diameter for waste, and calling the remainder the side of a square log.

1. To find the contents of a round log 24 inches in diameter and 30 feet in length.

SOLUTION.—Deducting $\frac{1}{3}$ from 24 for waste, we have 16, which, squared=256 inches, and multiplied by the length= 640 feet *board measure.*

In some places only $\frac{1}{4}$ is deducted for pine lumber.*

Planks or joists are sometimes reckoned by *face measure;* that is, the dimensions of one side of the board are taken instead of the solid contents. A 16-foot board 2 inches thick by 12 inches broad would measure 32 feet *board measure*, or 16 feet *face measure.*

In some places, the *saw-log* is taken as a standard of measurement for round timber. A log 12 feet long and 30 inches in diameter is the standard in some parts of the west. In Pennsylvania, a *saw-log* is one that will cut into 200 feet of lumber.

313. *To measure timber partly squared*, it is customary to deduct the "wane" (the length of the corner) from the thickness of the log, and call the remainder one side. A log 18 inches thick, with a "wane" 3 inches, would be called one of 18 by 15 inches.

2. In an octagonal log, 25 feet long 20 inches thick, with a wane 4 inches, how many solid feet are there?

Ans. $55\frac{5}{9}$.

3. There are 150 logs, the average length and breadth of which are 20 feet by 22 inches, wane 3 inches; required the number of solid feet they contain. *Ans.* $8708\frac{1}{3}$.

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* Inch measure is taken as the standard for lumber. If a board is under an inch, it is measured as a full inch; and if over an inch, it is reduced to inch measurement. A plank 2 inches thick would be considered as two boards 1 inch thick.

4. In a raft there are 450 boards 16 feet long and $1\frac{1}{2}$ inches thick, and measuring in the aggregate 757 feet broad; how many feet of lumber (board measure) does it contain? How many *face measure?*

Ans. 18168 board measure, 12112 face measure.

5. How much lumber can be cut from a tree measuring 20 feet long and 14 inches diameter at the smaller end, allowing for waste one-fourth of the diameter?

HOUSEKEEPING.

314. Housekeepers, and ladies generally, ought to be familiar with the operations in arithmetic which apply in computing house rent, servants' wages, board bills, interest, the quantity of carpet to cover a floor or paper for a room, etc.

HOUSE RENT.

Landlords, in renting by the year, usually collect their rents quarterly; but when renting monthly, collect monthly.

By a quarter is meant three calendar months. As, for instance, if a house is rented on the 17th of April, the quarter would expire on the 17th of July.

When a house is rented for a year, the tenant is liable for the rent during the whole of that time unless the landlord accepts another in his stead. A verbal lease for a year is binding. A lease for three or more years should be recorded.

Tenancy begins on obtaining possession. When there is a lease, however, and the time not stated, it is presumed to commence on the date of the instrument.

When the tenant does not remove at the end of the year, or two weeks afterward, he will be regarded as having rented for another year.

Interest can be collected on rent from the day it is due.

A tenant is released when the landlord accepts a substitute.

A married woman can not make a lease or take one in her own name.

A tenant at will is liable for rent as long as he occupies the premises.

For interest calculations, see page 143.

1. A house which rents at $75 a month is occupied from January 3 to February 9; what is the amount of rent?

Rent for 1 *month* =	75
" " 6 *ds or* $\frac{1}{5}$ =	15
	$90

2. Required the rent for a house from April 3 to August 5, at $1000 a year.

3 *mo* = $\frac{1}{4}$	1000
	250
30 = $\frac{1}{3}$	83333
2 = $\frac{1}{15}$	5555
	338.888
	or $338.89

EXPLANATION.—From April 3 to July 3 is 3 months, or 1 fourth of a year; 1 fourth of $1000 gives $250. From July 3 to August 5 is 1 month and 2 days; 30 days is 1 third of 3 months, and the rent for that time is $83.333, and 2 days is 1 fifteenth of a month, giving the rent for that time, $5.555.

3. Required the rent for a house from December 1 to January 12, at $50 a month. *Ans.* $68.33.

4. What will be the rent of a house from January 20 to August 9, at $750 a year, payable quarterly? *Ans.* $416.67.

The Teacher can give more of such questions as he finds it necessary.

SERVANTS' WAGES.

Servants are hired by the week or month of four weeks or calendar month, and are entitled to wages every day, Sunday included.

5. A girl hires on September 3 and leaves on October 9; what will be her wages at $3 a week?

From September 3 to October 9 is 36 days, or 5 weeks 1 day.

$$\begin{array}{lr} \textit{Wages for 5 weeks at } \$3 = & 15.00 \\ \textit{Wages for 1 day} = \tfrac{1}{7} \textit{ of } \$3 = & .428 \\ \hline & 15.428 \\ & \textit{or } \$15.43 \end{array}$$

6. A man is hired on June 9 at $40 a calendar month, and is discharged on September 3; what is the amount of his wages? *Ans.* $113.33.

This is computed in the same way as house rent.

7. What will be the wages of a man for 7 months and 7 days at $33 a month? *Ans.* $238.70.

To find the quantity of carpet for a floor.

Most carpeting is made one yard in breadth. Brussels and velvet carpeting is usually made only $\frac{3}{4}$ of a yard or 27 inches, though sometimes it is made $\frac{4}{4}$ and even $\frac{6}{4}$, or double the usual breadth.

Oil cloths vary in breadth from 3 to 24 feet, as follows: 3 ft. 9 in., 4 ft. 6 in., 7 ft. 6 in., 12 ft., 18 ft. and 24 ft.

Matting is of three kinds: China Cocoa, Manilla and Cane. China matting is made of a kind of rushes. It looks neat but does not wear long. The best kinds are Gowqua and Manning.

Cocoa matting is made of a kind of grass. The best quality is called "diamond A," from the brand found upon it.

Common ingrain carpeting may be matched by cutting through the center of the pattern; but expensive carpets can be matched only by persons experienced in the business. Some of them require two webs, others more, to make a pattern. Carpet dealers usually furnish their carpets made to any dimensions, and even lay them when required.

The quantity of carpet required for a room is found by multiplying the length by the breadth, in feet or inches, and dividing by the number of square feet or inches in a yard. For $\frac{3}{4}$ carpet, divide the square feet by $6\frac{3}{4}$; for yard, divide by 9.*

ANOTHER WAY

Is to find the number of *breadths* required, and multiply it by the length of the room.

8. How much ingrain carpet will be required to cover a room 15 by 20 feet?

Cutting the carpet in its greatest length, there would be 5 breadths, which, multiplied by the length, gives 100 feet, or $33\frac{1}{3}$ yards.

Required the quantity of ingrain carpet to cover three rooms, measuring as follows: one room 12 by 16 feet; one, 16 by 21; one, 15 by 19; and one room, 20 by 25 with velvet carpet.

9. What quantity of velvet carpet will cover a saloon 20 by 40 feet?

The breadth, 20 feet, reduced to inches=240, which, divided by 27 inches=9 breadths, nearly. The length, 40 feet, multiplied by 9=360 feet, or 120 yards.

For calculations pertaining to wall paper, see Paper Hanging, page 267.

For calculations pertaining to shopping, see page 111.

For weights and measures, see Tables.

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*The spaces for fire-places, etc., will allow sufficient for waste. Carpets should be cut a few inches short to allow for stretching.

To find the cost of articles sold by the dozen.

1 article	will	cost	$\frac{1}{12}$	of the cost of a dozen.
2 articles	"	"	$\frac{1}{6}$	" " " " "
3 "	"	"	$\frac{1}{4}$	" " " " "
4 "	"	"	$\frac{1}{3}$	" " " " "
5 "	"	"	5	times the twelfth.
6 "	"	"	$\frac{1}{2}$	of the cost of a dozen.
8 "	"	"	2	times the third.
9 "	"	"	3	" " fourth.
10 "	"	"	$\frac{1}{6}$	off.
11 "	"	"	$\frac{1}{12}$	"

XXXVII. GEOMETRICAL DEFINITIONS.

An *Angle* is the opening between two lines that meet in a point.

A *Right Angle* is made by one straight line standing perpendicular to another.

An *Obtuse Angle* is wider than a right angle.

An *Acute Angle* is less than a right angle.

A *Triangle* is a figure having three sides and three angles.

An *Equilateral Triangle* has all its sides equal.

An *Isosceles Triangle* has two of its sides equal.

A *Scalene Triangle* has all its sides unequal.

A *Right-Angled Triangle* has one right angle.

An *Obtuse-Angled Triangle* has one obtuse angle.

An *Acute-Angled Triangle* has all its angles acute.

A *Quadrangle* or *Quadrilateral* is a four-sided figure, and may be

A *Parallellogram*, having its opposite sides parallel;

A *Rectangle*, having four right angles, sides unequal;

A *Square*, having all its sides equal, and its angles right angles;

A *Rhombus* or *Lozenge*, having its sides equal and no right angle;

A *Rhomboid*, a parallelogram, with no right angles;

A *Trapezium*, having unequal sides;

A *Trapezoid*, having only two sides parallel.

Polygon, a plain figure having more than four sides.

A *Pentagon* has five sides, a *hexagon* six, a *heptagon* seven, an *octagon* eight, a *nonegon* nine, a *decagon* ten, etc.

A *Circle* is a plain figure, bounded by a curved line, all points of which are equidistant from the center.

An *Arc* is any part of a circumference.

A *Chord* is a straight line joining the extremities of an arc.

A *Segment* of a circle is a part of a circle bounded by an arc and its chord.

The *Radius* of a circle is a line extending from the center to the circumference.

A *Quadrant* is a quarter, a *sextant* a sixth of a circle.

A *Zone*, a part of a circle included between two parallel chords.

A *Prism* is a solid, the sides of which are parallelograms. It may have three or more sides.

A *Pyramid* is a solid with regular sides, tapering to a point.

A *Cylinder* is a solid of uniform thickness, having its ends circular.

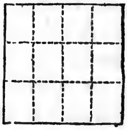
A *Cone* is a round pyramid, having a circle for its base.

The *Circumference* of a circle is the line by which it is bounded. The *Diameter* is a line drawn through the center and terminating at the circumference. The *Radius* is a line drawn from the center to any point in the circumference.

XXXVIII. MENSURATION.

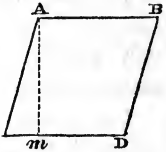
315. *To find the area of a square,* multiply the length by the breadth.

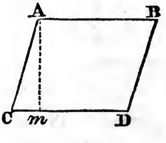
1. Let one side of the annexed parallelogram be 3, and the other 4, then the area will be $3 \times 4 = 12$.



2. What is the area of a square, the side of which is 3 feet 6 inches. *Ans.* $12\frac{1}{4}$.

316. *To find the area of a Rhombus or Rhomboid,* we multiply the length by the perpendicular height. The reason for this will be evident from an inspection of the figures. The triangle A C *m* of the rhombus applied to the side B D, will make a square; and the triangle of the rhomboid applied to the other side will make a parallelogram.

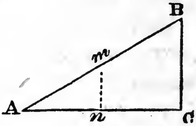




3. Let the perpendicular height (A *m*) be 20, and the length (A B or C D) be 30, then the area will be $20 \times 30 = 600$.

4. What will be the area of a rhomboid, the perpendicular and length being 15 and 25? *Ans.* 375.

317. *To find the area of a right-angled triangle,* we multiply the perpendicular by half the base, or the base by half the perpendicular.

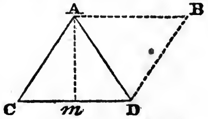


REASON.—The part A *m n*, applied to the part B C *m n*, with the line A *m* applied to *m* B, and the point A on the point B, will make a parallelogram, with a base half of that of the triangle.

5. Let the base be 10 and the perpendicular 8; then $\frac{10}{2}\times 8=40$, *the area.*

6. The perpendicular is 16 and the base 120; what is the area? *Ans.* 960.

318. *When the triangle is not right-angular,* half the base multiplied on the height will give the area.



REASON.—The triangle C A D is half the rhomboid.

7. Let the base be 30 and the height 20, then $20\times\frac{30}{2}=300$, the area.

319. *When the perpendicular is not given,* the area can be found by subtracting each side from half of the sum of the sides; then by multiplying these three remainders and half the sum of the sides together, and extracting the square root of the product

8. Let the sides be 5, 7 and 10; then $\frac{5+7+10}{2}=\frac{22}{2}=11.$

$11-5=6$
$11-7=4$
$11-10=1$ $6\times4\times11=264$, the sq. root of which is 16.3.

9. What is the area of a triangle, the sides of which are 50, 30 and 40? *Ans.* 600.

320. *To find the area of a circle,* multiply the square of the diameter by .7854.

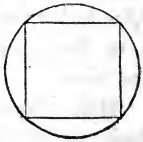
Multiply half the circumference by half the diameter.

321. *To find the side of a square equal in area to a given circle,* multiply the circumference by .2820948, or the diameter by .8862269.

10. The diameter of a circle is 100; required the side of a square having the same area.

$.8862269\times100=88.62269$, side of a square.

822. *To find the side of an inscribed square,* multiply the diameter by .7071068.



11. The diameter of a tree is 2 feet; required the side of a square log that may be cut from it?

.7071068×2=1.4142136, or 1 foot 5 inches nearly.

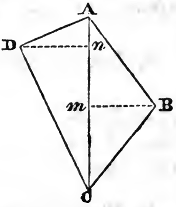
323. *Having the side of a square, to find the diameter of a circumscribed circle,* multiply the side by 1.4142136.

324. *From the side of a square to find the circumference of a circumscribed circle,* multiply the side by 4.4428934.

325. *To find the diameter,* multiply the side by 1.1283791.

326. *From the side of a square, to find the circumference of a circle of equal area,* multiply the side by 3.5449076.

327. *To find the area of a trapezium,* we divide it into two triangles, and the sum of the areas will be the area required.

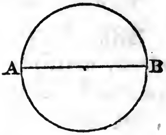


12. Let the diagonal A C be 100, and the perpendiculars B m and D n, 30 and 35; then

$$\frac{(35+30)\times 100}{2}=3250, \textit{ the area.}$$

REMARK.—The areas of irregular polygons are found by dividing the figures into triangles, and taking the sum of their areas.

328. *To find the circumference of a circle,* we multiply the diameter by 3.1416, or $3\frac{1}{7}$, because the circumference of a circle is $3\frac{1}{7}$ times greater than the diameter.



13. Let the diameter be 5; then 3.1416 ×5=15.708, *the circumference.*

The diameter is found by dividing the circumference by 3.1416.

329. *To find the area of a regular polygon,* add all the sides together, and multiply by the perpendicular drawn from the center of the polygon to the middle of one of its sides; *or,*

Multiply the square of the side of the polygon by the number standing opposite to the number of its sides in the following table:

No. of sides.		No. of sides.	
3	0.4333012	8	4.8284271
4	1.	9	6.1818242
5	1.7204774	10	7.6942088
6	2.5980762	11	9.3656404
7	3.6339124	12	11.1961524

14. The side of a pentagon (a five-sided figure) is 20 yds. and its perpendicular 13.76382; required the area.

First method, $\dfrac{20\times5\times13.76382}{2}=688.191$, *Ans.*

Second method, $20^2\times1.720477=688.19$, *Ans.*

15. The side of a nonegon is 50 inches; required its area. *Ans.* 15454.5605.

330. *To measure the heights of objects, the tops of which can not be reached,* the shadow cast by the tree may be used. Measure the length of the shadow cast by the object, and that of some object the length of which is known; then the shadow of the known object will be to that of the first as the length of the known object to the length of the first

16. Let the known object be a man, who, without his hat, measures (in his shoes) 5 feet 8 inches, and whose shadow measures 15 feet, while the shadow of a tree measures 120 feet.

15 : 120 : : 68 inches : the height of the tree in *inches.*

$\dfrac{120\times68}{15}=544$ inches, or $45\frac{1}{3}$ feet.

MEASUREMENT OF SOLIDS.

331. *To find the solidity of a cube*, multiply the side by itself, and the product again by the side.

17. The side of a cubical block of marble is 5 feet 7 inches; what is the solid contents?

$5\frac{7}{12}\times5\frac{7}{12}\times5\frac{7}{12}=174$ feet, nearly.

332. *To find the solidity of a parallelopipedon* (a solid figure with square corners), multiply the length, breadth and thickness together.

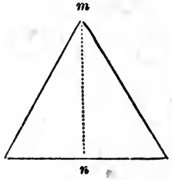
18. A log measures 7 feet in length and 15 by 20 inches in thickness; required the solid contents.

$7\times15\times20=2100$ square inches, or $14\frac{7}{12}$ feet.

333. *To find the solidity of a prism*, multiply the area of the end by the length.

19. What is the solidity of a prism whose ends are equilateral triangles, each side of which is 4 feet and height 8 feet?

An equilateral triangle is made up of two right-angled triangles, the perpendicular of which is found by taking the square root of the difference of the squares of half the base and the other side.



$\sqrt{4^2-2^2}=3.464$, the perpendicular, $m\ n$.

$3.464\times2=6.928$=area of the end.

$6.928\times8=55.424$ feet, the solidity.

334. *To find the solidity of a cone*, multiply the area of the base by one-third the height.

20. A cone is 10 feet in diameter and 10 feet high; required the solidity.

$10^2\times.7854=78.54$, area of the base, which, multiplied by $\frac{1}{3}$ the height, $3\frac{1}{3}=261.8$, *the solidity*.

21. How many cubic feet in a cone whose diameter is 12 feet, and its perpendicular height 100? *Ans.* 3769.92 ft.

335. *To find the solidity of a pyramid*, multiply the area of the base by one-third the height.

22. A square pyramid has a base of 4 feet and height of 12 feet; required the solidity.

$4 \times 4 =$ area of base.

$16 \times \frac{12}{3} = 64$ for *the solidity.*

23. The spire of a church is an octagonal pyramid, each side at the base being 5 feet 10 inches, and its perpendicular height 45 feet; also each side of the cavity or hollow part at the base is 4 feet 11 inches, and its perpendicular height 41 feet; how many solid yards of stone does the spire contain? *Ans.* $32\frac{1}{5}$, nearly.

336. *To find the solidity of the frustrum of a cone or pyramid.**

Find the sum of the areas of the two ends, and of a geometrical mean between them, and multiply by one-third the perpendicular height.

24. What is the solid contents of a frustrum of a square pyramid, whose sides are 5 and 3, and perpendicular height 12?

$$\begin{array}{rl}
5^2 \times .7854 = 19.6350 & \textit{area base.} \\
3^2 \times .7854 = 7.0686 & \textit{area upper end.} \\
\sqrt{19.635 \times 7.0686} = \underline{11.7810} & \textit{geometrical mean.} \\
38.4846 & \\
\underline{4} & \textit{one-third the height.} \\
153.9384 &
\end{array}$$

25. What is the solidity of a squared piece of timber, its length being 18 feet, and sides of the bases 18 and 12 inches? *Ans.* 28.5 ft.

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* A *segment* is a piece cut off by a plane, parallel to the base; a *frustrum* is what remains at the base.

26. How many cubic feet of timber in a tapering log 14.25 ft. long, diameters 9 and 18 in.? *Ans.* 14.689 ft.

Comparison between the globe, cylinder and cone, the diameter and heights being 100:

Solid contents of the cylinder, 785.4
" " " sphere, 523.6=$\frac{2}{3}$ of the cylinder.
" " " cone, 261.8=$\frac{1}{3}$ " "

REMARKS.—1. The cone cut out of a solid cylinder, whose diameter and height are equal, will leave a part equal to the solidity of a sphere of the same diameter.

2. A square pyramid, whose height and side are equal to the side of a cube, if cut out of the latter, will leave $\frac{2}{3}$ of the cube.

GAUGING.

The process of finding the capacity of barrels, etc., is called *gauging.*

337. *Having the head and bung diameter and the diameter between them, to find the capacity of a barrel or cask in gallons,* we add together the square of the head and bung diameters and twice the middle diameter, and multiply the sum by the length, and that by .0004721 for imperial gallon.

27. A cask, having for head, bung and middle diameter 30, 36 and 33, and length 40 inches, holds how many imperial gallons?

$30^2+36^2+(33\times2)\times40\times.0004721=42.72$ *galls.*

Practical method for measuring small cylindrical vessels, is to multiply the square of the diameter by 34, and that by the height in inches, and point off four figures. The result will be the capacity in gallons.

28. An oil-can measures 12 inches in diameter and 2 feet in height; required the contents in gallons.

$12^2\times34\times24=117504=11.75$ *or* $11\frac{3}{4}$ *galls.*

XXXIX. THE METRIC SYSTEM.

338. The *Metric System* is a decimal system of weights and measures, of French origin, deriving its name from *Meter*, the unit of measure upon which the system is based.

Since 1840 it has been adopted by most European governments, including that of Great Britain in 1864, and has been in use by men of science every-where. During its last session (39th), Congress authorized its use in this country, and made provision for its immediate introduction into post-offices.*

339. The units of measure are the *meter*, *are*, *liter*, and *stere;* and the unit of weight the *gram*. Other denominations are formed from these by prefixing Greek or Latin numerals; the former for denominations above the unit, and the latter for denominations below the unit. The Greek prefixes are *deka*, 10; *hecto*, 100; *kilo*, 1000; and *myria*, 10000. A dekameter is ten times and a hectometer one hundred times the length of a meter. The Latin

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**A Bill to authorize the use of the Metric System of Weights and Measures.*

Be it enacted by the Senate and House of Representatives of the United States of America, in Congress assembled, That from and after the passage of this act, it shall be lawful throughout the United States of America to employ the weights and measures of the Metric System; and no contract, or dealing, or pleading in any court, shall be deemed invalid or liable to objection because the weights or measures expressed or referred to therein are weights or measures of the Metric System.

prefixes are *deci*, $\frac{1}{10}$th; *centi*, $\frac{1}{100}$th; *milli*, $\frac{1}{1000}$th. A decimeter is one-tenth and a centimeter one-hundredth of the length of a meter.

LIST OF NAMES AND THEIR PRONUNCIATION.

Name.	*Pronunciation.*	*Abbreviation.*
Meter,	Me′ter,	M.
Kilometer,	Kil′o-meter,	K. M.
Hectometer,	Hec′to-meter,	H. M.
Dekameter,	Dek′a-meter,	D. M.
Decimeter,	Des′i-meter,	d. m.
Centimeter,	Cent′i-meter,	c. m.
Are,	Aer,	A.
Hectare,	Hect′är,	H. A.
Centare,	Sent′är,	c. a.
Stere,	Stere,	S.
Dekastere,	Deck′ĭ-stere,	D. S.
Decistere,	Des′ĭ-stere,	d. s.
Gram,	Grăm,	G.
Dekagram,	Dek′a-gram,	D. G.
Kilogram,	Kil′o-gram,	K. G.
Hectogram,	Hect′o-gram,	H. G.
Decigram,	Des′ĭ-gram,	d. g.
Centigram,	Sent′ĭ-gram,	c. g.
Milligram,	Mil′lĭ-gram,	m. g.
Tonneau,	Ton′no,	Ton.
Millier,	Mil′li-er,	Mil.
Liter,	Le′ter,	L.
Kiloliter,	Kil′o-le-ter,	K. L.
Hectoliter,	Hect′o-le-ter,	H. L.
Dekaliter,	Dek′a-le-ter,	D. L.
Deciliter,	Des′i-le-ter,	d. l.
Centiliter,	Sent′i-le-ter,	c. l.
Milliliter,	Mil′li-le-ter,	m. l.

NOTE.—Denominations below units are abbreviated with small letters.

MEASURES OF LENGTH.

340. Besides its being the base of the new system, the *Meter* is the unit of measure for lengths, and is one *ten-millionth part of the distance from the Equator to the poles*, and is equivalent to 39.37 inches ordinary measure.

TABLE.

Myriameter	=	10,000	Meters	or 393685	inches.
Kilometer	=	1,000	"	or 39368.5	"
Hectometer	=	100	"	or 3936.85	"
Dekameter	=	10	"	or 393.685	"
Meter	=	1	"	or 39.3685	"
Decimeter	=	$\frac{1}{10}$th	"	or 3.9368	"
Centimeter	=	$\frac{1}{100}$th	"	or .39368	"
Millimeter	=	$\frac{1}{1000}$th	"	or .03936	"

REMARK.—Meters, when combined with lower denominations, are treated as whole numbers, and the latter as decimals.

The denominations most used are the Kilometer, Centimeter, and Millimeter. 7 Myriameters, 8 Kilometers, and 5 Hectometers would be written 78.5 K. M.

SQUARE OR SURFACE MEASURE.

341. The unit of measure for large surfaces is the *Are*, from which are derived the *Hectare* and *Centare*. For smaller surfaces the denominations are the same as for measures of length, with the addition of the word square.

Centare = 1 sq. meter, or 1550 sq. inches.
Are = 100 sq. meters, or 1 sq. dekameter, or 119.6 sq. yds.
Hectare = 10,000 sq. meters, or 1 sq. hectometer, or 2.471 acres.
1 sq. decimeter, *d. m.*,^{2} = $\frac{1}{100}$ sq. meter
1 sq. centimeter, *c. m.*,^{2} = $\frac{1}{100}$ sq. decimeter, or $\frac{1}{10000}$ sq. meter.
1 sq. millimeter, *m. m.*,^{2} = $\frac{1}{100}$ sq. centimeter, or $\frac{1}{1000000}$ sq. c. m.
27.354 M.^{2} = 27 sq. meters, 35 sq. decimeters, 40 sq. centimeters.

CUBIC OR SOLID MEASURE.

342. The *Stere* may be called the unit for cubic measure. It is equal to a cubic meter or 1.308 yards.

1 dekastere, *D. S.*, $=10$ steres, or 13.08 yards.
1 decistere, *d. s.*, $=\frac{1}{10}$ stere, or 0.1308 yards.
1 cubic decimeter, *d. m.*,$^3=\frac{1}{1000}$ cubic meter.
1 cubic centimeter, *c. m.*,$^3=\frac{1}{1000}$ cubic decimeter, or $\frac{1}{1000000}$ M.
1 cubic millimeter, *m. m.*, $=\frac{1}{1000}$ cubic centimeter, or $\frac{1}{1000000000}$ M.

The Stere, etc., is used for measuring fire-wood and lumber, and, for computing large numbers, is preferred to the other denominations.

MEASURES OF CAPACITY.

343. The *Liter* is the unit of measure for capacity, and is equal to a cubic decimeter or 1.0567 quarts of United States liquid measure.

TABLE.

Kiloliter $=$ 1,000 liters, or 1 cubic meter of water.
Hectoliter $=$ 100 " or $\frac{1}{10}$ cubic meter.
Dekaliter $=$ 10 " or 10 cubic decimeters.
Liter of water weighs 1 kilogram.
Deciliter $=\frac{1}{10}$ liter, or $\frac{1}{10}$ cubic decimeter.
Centiliter $=\frac{1}{100}$ liter, or 10 cubic centimeters.

A milliliter of water weighs a gram. The liter and hectoliter are most in use.

WEIGHTS.

344. The *Gram* is the unit of weight, and is equal to 15.432 grains Troy, which is the weight of a cubic centimeter of pure water at its greatest density.

TABLE.

Kilogram	= 1,000 grams.	Centigram	= $\frac{1}{100}$ gram.
Hectogram	= 100 "	Quintal	= 100 kilograms.
Dekagram	= 10 "	Tonneau	=1,000 kilograms, or 2,204 lbs.
Gram	= 1 gram.		
Decigram	= $\frac{1}{10}$ "		

REMARK.—The gram and its subdivisions are used in compounding medicines, and wherever great accuracy is required.

The kilogram is the denomination most used, and weighs a little over $2\frac{1}{5}$ pounds. The quintal and tonneau are used for heavy weights, but may be expressed in kilograms.

COMPARISON OF METRIC DENOMINATIONS WITH THOSE IN PRESENT USE.

MEASURES OF LENGTH.

NAMES.	VALUES.	EQUIVALENTS IN USE.
Myriameter	10,000 meters	6.2137 miles.
Kilometer	1,000 "	0.62137 "
Hectometer	100 "	328 1-12th feet.
Dekameter	10 "	393.7 inches.
Meter	1 "	39.37 "
Decimeter	1-10th of a meter	3.937 "
Centimeter	1-100th of a meter	0.3937 "
Millimeter	1-1000th of a meter	0.0394 "

MEASURES OF SURFACE.

NAMES.	VALUES.	EQUIVALENTS IN USE.
Hectare	10,000 square meters	2.471 acres.
Are	100 " "	119.6 square yards.
Centare	1 " meter	1550 square inches.

MEASURES OF CAPACITY.

NAMES.	No. of Liters.	CUBIC MEASURE.	DRY MEASURE.	LIQUID OR WINE MEAS.
Kiloliter or Stere.	1,000	1 cubic meter	1.308 cubic yards	264.17 galls.
Hectoliter	100	1-10th cubic meter	2 bus. & 3.35 pecks.	26.417 galls.
Dekaliter	10	10 cubic decimeters	9.08 quarts	2.6417 galls.
Liter	1	1 cubic decimeter	0.908 quart	1.0567 quarts.
Deciliter	1-10	1-10 cubic decimeter.	6.1022 cubic inches.	0.845 gill.
Centiliter	1-100	10 cubic centimeters	0.6102 cubic inch	0.338 fluid oz.
Milliliter	1-1000	1 cubic centimeter	0.06102 cubic inch	0.27 fluid drm.

WEIGHTS.

NAMES.	NO. OF GRAMS.	WEIGHT IN QUANTITY OF WATER AT MAXIMUM DENSITY.	EQUIVALENTS IN USE.
Millier or Tonneau...	1,000,000	1 cubic meter	2204.6 pounds Av.
Quintal	100,000	1 hectoliter	220.46 " "
Myriagram.............	10,000	10 liters...	22.046 " "
Kilogram	1,000	1 liter	2.2046 " "
Hectogram	100	1 deciliter	3.5274 ounces "
Dekagram	10	10 cubic centimeters	0.3527 ounce "
Gram	1	1 cubic centimeter	15.432 grains Troy.
Decigram	1-10	1-10 cubic centimeter..........	1.5432 " "
Centigram..............	1-100	10 cubic millimeters............	0.1543 grain "
Milligram	1-1000	1 cubic millimeter	0.0154 " "

CALCULATIONS PERTAINING TO THE METRIC SYSTEM.

The principal recommendation of the new system is the simplicity of its arithmetic. Under it the long and intricate calculations of the present system are unknown. Denominations are changed from one to another by the use of the decimal point, with sometimes one or more ciphers, while addition and other operations are performed precisely as in whole numbers and decimals.

1. In 35.2 D. M. how many M? Ans. 352 M.

Solution.—In 1 D. M. there are 10 M., in 35.2 D. M. there are 35.2 times 10 or 352 M.

2. In 35142 millimeters how many meters?

Ans. 35.142 M.

Solution.—1000 m. m. make 1 M., hence we divide by 1000, which gives 35.142 M.

3. In 31475 M. how many kilometers?

Ans. 31.475 K. M.

4. Reduce 371.2 D. M. to meters. Ans. 3712 M.

5. Reduce 45.67 M. to decimeters. Ans. 456.7 d. c.

6. Reduce 13456 decimeters to meters. Ans. 1345.6 M.

7. In 213.21 M. M. how many dekameters?

Ans. 213210 D. M.

8. Reduce 3157.2 c. m. to meters. Ans. 31.572 M.

9. A man traveled 30.5 K. M. in a day, how far would he travel in 5 days at the same rate? Ans. 152.5 K. M.

10. A ship steams 15 K. M. in an hour, how long would it take her to go a distance of 50 myriameters?

Ans. $33\frac{1}{3}$ hours.

11. At 2 cents per kilometer, what would be the cost of traveling 351.27 M. M.? Ans. $70.25.

12. A man walked a distance of 351.27 K. M. in 9 days, how many was that per day? Ans. 39.03 K. M.

13. In 253 d. m., 7 c. m., 5 m. m., how many meters?

Ans. 25.375 M.

14. In 157 d. m^2., 35 c. m^2., how many square meters?

Ans. 1.5735 M^2.

15. At 15 cents per M^2., what would 25 H. M^2. of lumber cost? Ans. $375.

16. How many square meters in a floor which measures 6 M. long and 5.25 M. broad? Ans. 31.5 M^2.

17. A pavement measures 39.2 M. long and 4.15 M. broad, how many square M. does it contain?

Ans. 162.68 $M.^2$

18. A strip of land 120 M. long contains 342 ares, what is its breadth? Ans. 285 M.

19. What will 545.37 centares cost at $4.15 per are?

Ans. $22.63.

20. From 1374.27 M^2. take 7.14 ares.

Ans. 6.6027 M^2.

21. At $50.50 per H. A., what will 1371.154 ares cost?

Ans. $692.43.

22. In 37.27 M^3. how many decisteres? How many dekasteres? Ans. 372.7 d. s., 3.727 D. S.

23. At $1.50 per M^3., what will 157.28 dekasteres cost?

Ans. $2359.2.

24. What will 153 S. of wood cost at $1.75 per S.?
Ans. $267.75.

25. $875.55 was paid for 413.5 S. of wood, what was the cost per stere? Ans. $2.17.

26. A cistern is 3 M. long, 2 M. wide, and 1 M. deep, what weight of water will it contain?
Ans. 13227.6 lbs., or 6 tonneau.

27. What will a decistere of wood cost, at $3.57 per stere? Ans. 36 cents.

28. Find the price of 53 liters of wine, at $3.50 per dekaliter. Ans. $18.55.

29. A vessel measures 53.5 liters, what weight of water does it contain? Ans. 53.5 kilograms.

30. In .3514 H. L. how many liters? Ans. 35.14 L.

31. In 145.37 M^3. how many liters? Ans. 145370 L.

32. Find the price of 5.3 liters of wine, at $1.75.
Ans. $92.75.

33. At 57 cents a meter, what will 317.5 meters cost?
Ans. $180.975.

34. An are of land cost $53, what was the price per meter? Ans. 53 cents.

35. A kilogram costs $37.50, what will a dekagram cost? Ans. $37\frac{1}{2}$ cents.

36. How many ares in a square piece of land which measures 150 meters in length and 500 in breadth?
Ans. 750.

37. At $25 a kilogram, what will a dekagram cost?
Ans. 25 cents.

38. At $7.25 a stere, what will 320 steres of wood cost?
Ans. $2320.

343. *To reduce the denominations of the ordinary to those of the Metric System.*

1. In 3 miles 35 rods, how many meters?

3 miles 35 rods=197010 inches, which, divided by 39.37, (the number of inches in a meter,) gives 5004.06 meters.

2. In $5\frac{1}{3}$ yards how many meters? Ans. 4.877.

3. Reduce $37\frac{1}{2}$ feet to meters. Ans. 11.43.

4. In 29 inches how many centimeters? Ans. 73.66.

5. In 6 inches how many millimeters? Ans. 152.4.

6. How many square meters in 57 rods? Ans. 1441.68.

7. In 5 acres how many ares? Ans. 202.34.

8. How many liters of wine in 37 gallons? Ans. 140.06.

9. In 57 yards of carpet how many meters? Ans. 52.12.

10. How many liters in $5\frac{1}{2}$ bushels? Ans. 193.82.

11. How many cubic meters in 3759 cubic feet? Ans. 106.44

12. Reduce 3 tons to tonneaus. Ans. 2.72.

13. At $2.50 a liter, what will 20 gallons cost? Ans. $189.27

14. At 75 cents a meter, what will 135 yards cost? Ans. $92.58.

344. *To reduce denominations of the Metric to those of the ordinary system.*

1. In 175 meters how many yards? Ans. 191 yds., 1 ft., $1\frac{3}{4}$ in.

In 1 meter there are 39.37 inches, in 175 meters there are 175 times as many. $39.37\times175=6889.75$ inches, which, reduced to yards, =191 yards, 1 foot, $1\frac{14}{100}$ inches.

2. In 379.53 meters how many yards? Ans. 415.058.

3. Reduce 743.5 K. M. to miles. Ans. 461.988

4. In 2435 millimeters how many inches? Ans. 95.866.

5. Reduce 50 pounds to kilograms. Ans. 22.68.

6. At $2.75 per kilogram, what will 375 lbs. cost? Ans. $467.77.

7. At 35 cents per lb., what is it per kilogram? Ans. 77 cents.

8. At $10.50 per cord, what is it per stere? Ans. $2.90.

9. At $1.25 a bushel, what should wheat be sold at per liter? Ans. $3\frac{1}{2}$ cents.

10. A bushel of oats weighs 33 pounds, what should it weigh per liter? Ans. 1.14 lbs.

11. How long will it take a man to travel a distance of 57 K. M. who walks at the rate of $3\frac{1}{2}$ miles per hour? Ans. 10.119 hours.

The following multipliers, taken from a pamphlet written by Prof. H. A. Newton, of Yale College, will facilitate the labor of converting the denominations of one system into those of the other. They will be found sufficiently correct for ordinary purposes:

Meters × 39.3685 = inches. Inches × .0254 = meters.
Meters × 3.2807 = feet. Feet × .30481 = meters.
Meters × 1.09357 = yards. Yards × .91444 = meters.
Meters × .19883 = rods. Rods × 5.0294 = meters.
Kilometers × .62135 = miles. Miles × 1.6094 = kilometers.
Sq. meters × 1550. = sq. inches. Sq. inches × .0006452 = sq. M.
Sq. meters × 10.763 = sq. feet. Sq. feet × .09291 = sq. meters.
Sq. meters × 1.196 = sq. yards. Sq. yards × .8362 = sq. meters.
Ares × 3.953 = sq. rods. Sq. rods × .2529 = ares.
Hectares × 2.4709 = acres. Acres × .4047 = hectares.

Hectares$\times$.003861=sq. miles. Sq. miles$\times$259.=hectares.
Liters$\times$33.81=fluid ounces. Fluid ounces$\times$.02958=liters.
Liters$\times$1.05656=quarts. Quarts$\times$.9465=liters.
Liters$\times$.26414=gallons. Gallons$\times$3.786=liters.
Hectoliters$\times$2.837=bushels. Bushels$\times$.3524=hectoliters.
Liters$\times$61.012=cubic inches. Cubic inches$\times$.01639=liters.
Hectoliters$\times$3.531=cubic feet. Cubic feet$\times$.2832=hectoliters.
Steres$\times$1.3078=cubic yards. Cubic yards$\times$.7646=steres.
Steres$\times$.2759=cords. Cords$\times$3.625=Steres.
Grams$\times$15.44=grains. Grains$\times$.0648=grams.
Kilograms$\times$32.147=troy ounces. Troy oz.$\times$.03108=K. G.
Kilograms$\times$35.30=avoirdupois oz. Av. oz.$\times$.02833=K. G.
Kilograms$\times$2.681=Troy pounds. Troy pounds$\times$.373=K. G.
Kilograms$\times$2.206=Av. pounds. Av. pounds$\times$4536=K. G.
Tonneaus$\times$.985=long tons. Long tons$\times$1.015=tonneaus.
Tonneaus$\times$1.103=short tons. Short tons$\times$.9066=tonneaus.

1. In 37 meters how many yards?

Ans. 37$\times$1.09357=40.462 yds.

2. In 40.462 yards how many meters?

Ans. 40.462$\times$.91444=37 meters.

3. In 120 kilograms how many pounds Avoirdupois?

Ans. 120$\times$2.206=264.72 pounds.

4. In 26.472 pounds how many kilograms?

Ans. 264.72$\times$.4536=120. kilograms.

The meter is nearly 3 feet 3 inches and 3 eighths, or 3 feet $3\frac{3}{8}$ inches.

REMARKS.—The word *Meter* means measure, as in gas-meter, or gas-measure.

Stere means solid, as in stereotype, solid type, or rather a solid mass of type. Stereoscope, an instrument to make two pictures look like a solid.

Are signifies area, a surface included within given lines.

Quintal is from the Latin root *centum*, a hundred; formerly a hundred weight.

Contents

CONTENTS.

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Miss Juliet E. Hogan
[illegible] Ohio
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20, 1870

227 = occurs in the [illegible]
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246 Duodecimals not
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If duodecimals were
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But carry things to the
concrete
Metric System.

Good book ~~for~~ considering
that the author belongs to
a Business College