Keyser

THE NEW INFINITE AND THE OLD THEOLOGY

By

CASSIUS J. KEYSER, PH.D., LL.D.

Adrain Professor of Mathematics in Columbia University

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PREFACE

Some years ago, in the course of a lecture dealing with Mathematics regarded as a distinctive type of thought and with its relations to other varieties of philosophic and scientific activity, I ventured to say: "I do not believe that the declined estate of Theology is destined to be permanent. The present is but an interregnum in her reign, and her fallen days will have an end. She has been deposed mainly because she has not seen fit to avail herself promptly and fully of the dispensations of advancing knowledge. The aims, however, of the ancient mistress are as high as ever, and when she shall have made good her present lack of modern education and learned to extend a generous and eager hospitality to modern light, she will reascend and will occupy with dignity, as of yore, an exalted place in the ascending scale of human interests and the esteem of enlightened men. And Mathematics, by the inmost character of her being, is especially qualified, I believe, to assist in the restoration."

The following pages have been written under the stress of that conviction, which the intervening years have but deepened and confirmed. Rational theology is a legitimate and venerable member of the great family of spiritual enterprises of man: natural science, philosophy, jurisprudence, religion, art, mathematics, theology. These are all of them children of one great passion: the imperious craving of the human spirit for an inner ideal adjustment of life to the tragic limitations of life in a flowing world. The distinctive problems of rational theology are regarded as in a special sense originating in what may be called the supernalizing tendence or power of the human mind. This propensity or power, so strange and so familiar in every category of the understanding, ever and everywhere manifesting the presence of a kind of divine energy in the world, is a 'natural' agency, being at once a human faculty and a cosmic force, deeper than will; and so rational theology is conceived to be a species of 'natural' science—that branch of it which has for its special task to study and to appraise the phenomena of Idealization.

The aim has been to set the matter in the increasing light of certain ideas and methods of modern mathematics. But the reader need not be deterred by any fear of technique. All that is required is a fair share of mathematical spirit, which is a pretty common possession, being simply the spirit of right thinking, or logical righteousness.

I have to thank the Editor of the *Hibbert Journal* for permission to employ here, in some instances with only slight change, a few considerations adduced by me in an article published in that journal several years ago under the title, "The Message of Modern Mathematics to Theology."

Columbia University
April 15, 1915

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It is the aim of this essay to show that the modern concept of infinity together with certain kindred ideas that have come into mathematics in the course of the last hundred years have qualified this science to shed new light upon some of the harder problems of rational theology. No demand will be made upon the reader's knowledge of mathematical technique; all that is required is a fair measure of mathematical spirit, which is simply the spirit of logical rectitude.

The reader is entitled to know at the outset that the following words are not those of a professional theologian; they have no official authority, nor any merit beyond what may prove to be their reasonableness; they are offered as the words of a layman who, in his earlier and more expectant years, listened attentively to some hundreds of sermons, who has diligently read some theological works, and has reflected a good deal, not without some temperamental interest in the themes, upon the great questions that attend a poignant sense of the world's mystery and wait upon the leisure hour and the pensive mood.

The problems are many and difficult and old. No one who has seriously reflected upon them or is familiar with their history will expect to find in these pages a universal resolvent for theological difficulties. Problems that triumphed over the keen and sanguine dialectic of the ancient world, problems that baffled the infinitely subtle genius of the middle age, problems that the profoundest meditations of modern philosophy have not been able to solve, present grave difficulties. Many of them not even the adventurous spirit of modern mathematics may confidently assail. My task is limited to showing that some of them may be partly or wholly overcome by mathematical means. That all the rest may be subdued in future by similar means I do not maintain. Who knows? It may be that some of the difficulties are insuperable and so are destined to be everlasting. In that reflection there is nothing to bewail. One need not have "passed on life's highway the stone that marks the highest point" before learning to be content with less than the full measure of intellectual conquest dreamed of in youth. To be happy it is not necessary to conquer the invincible; it is sufficient to advance a little where progress is possible. Indeed it would be a matter for sorrowing if in the course of time all problems were solved and questions ceased to be, for a world without wonder were a dreary place. But of that there is no danger. Wonder increases with knowledge and knowledge with time. "It is no longer true," said Henri Poincaré, "that there are solved problems and others that are not solved; there are only problems *more or less* solved." As with natural science and mathematics, so too with philosophy and theology: not complete solutions of their problems, not final answers to the deepest questionings of the spirit, but ever increasing illumination of them, the acquisition of fresh viewpoints and new perspectives—the advancement, in a word, and multiplication of insight and vision—, these, I take it, are the reasonable expectations, the precious fruits, the ample rewards of serious speculation.

This, then, and not any magical formula

for the solution of riddles, is the kind of service that rational theology may expect from mathematics. I am aware that, owing to the popular misconception of mathematics, the claim is not an easy one to vindicate. To the many who are accustomed to regarding mathematics as merely a useful drudge, the claim will naturally seem to be groundless or visionary. But their conception of the science is far from adequate or just. Mathematics is indeed a humble servant—a drudge, if you please—an unsurpassed drudge—in the sense that nothing else does a larger share of humble and homely work. To imagine, however, that her place in the hierarchy of knowledges is thereby defined is hardly the beginning of wisdom in the matter. It is necessary to look much higher. Her rank in the ascending scale is not that of a useful drudge, immeasurable as is her service in that capacity; it is not merely the rank of a metric and computatory art, invaluable as the latter is as well in science as in the affairs of the workaday world; it is not even that of servant to other sciences in their fields of experimental and observational research, indispensable as mathematics is in that regard; over and above these things, she is charged with a sacred guardianship—in her keeping are certain ideals, the ideal forms of science and the standards of perfect thinking; she is concerned, not with the vagaries, but with the verities, of thought, with select matters independent of opinion, passion, accident, and will; it is thus peculiarly hers to release human faculties from the dominion of sense by winning allegiance to things that abide; her meditations transcend the accidents of time and place; it is their idiosyncrasy to have for subject proper, not the fickle and transitory elements in the stream of a flowing world, but those aspects of being that present themselves under the forms of the infinite and eternal.

It will be a useful preliminary to reflect a little upon the relations of rational theology to religion on the one hand and to science on the other, with a view to ascertaining what the province of theology may be rightly said to be. What, then, are those relations? It is evident that the answer must materially depend upon the relations that science and religion themselves bear to one another. This subject I have discussed elsewhere. In a recent lecture* dealing with science and religion I have undertaken to examine the relations between these two great interests of mankind from what may perhaps be regarded as in some respects a new point of view. I cannot here repeat the considerations adduced in support of the doctrine there sketched. It will be helpful, however, and possibly sufficient, to set down briefly some of its cardinal propositions. Those most intimately related to our present enterprise are these:

A.—In respect of method, structure, and content, science is conceptual and logical. Any branch of science, at any given stage of its development, consists of a certain group of ideas, or concepts, together with the relations that bind them into a logically organic whole. The potential domain of science and the domain of the rational—whatever is open, that is, to conquest by the means of concept and logic—are one and the same. All else—whatever is below or above that domain—is subrational or superrational.

* Science and Religion: the Rational and the Superrational. The Yale University Press.

B.—Religion, on the other hand, is not essentially a body of ideas nor a body of ideas together with their interrelations. Religion is essentially and ultimately a complex of emotions, of emotions as felt in their integrity. It is thus a kind of life not known nor knowable conceptually, logically, rationally, scientifically; it is known or knowable only "emotionally" and is even thus knowable, like love, for example, to none but such as feel or have felt the constituent emotions.

C.—Religion does not belong to the rational domain. There is indeed possible a science of emotions but these can not, as emotions, be constituents or elements of it. For it they do not exist as feelings. It can know only their outward manifestations and can know these only as science may know other objects of the external world. What is called the scientific study of religion does not—as scientific it can not—deal with religion as "emotionally known"; it can not know religion as a felt life, as a life conscious of itself; the most, the best, the last it can do is to know, as objects, as externalities, the exterior manifestations of what is essentially, being emotional, an inner life. Concepts can not feel, logic can not fear nor love, it can not revere, wonder, worship, nor adore. For scientific method religion is not a life, it is an hypothesis.

D.—The doctrine that it is a characteristic mark of religion "essentially to deal with the uncharted region of human experience" is untenable. Ignorance is not the presence of religion—else every body would be profoundly religious—, it is the absence of knowledge. Religion and the spirit of science are not incompatible; being capable of dwelling together harmoniously in a single personality, they are compatible *practically;* and they are compatible *theoretically:* under the influence of advancing science, forms of religion age and pass but new forms succeed them, and the religious emotions change but they do not die; in this respect it is with religion as with knowledge—there is transformation and supersession of form, there is advancement, enlargement, and elevation, but no breach of continuity, no essential extinction, no death.

E.—The rational implies and in a measure reveals the superrational. The rational world—the potential domain of science, the field of concept and logic—is not the whole sphere of our psychic life. It is but a mid-region, the median zone; under it lies a sub-rational zone—the zone of sense, which we share jointly with the beasts; above it, a world superrational, which millions have fancied angels share with us. Though it is above and beyond the dominion of concept and logic, the existence of that world is yet betrayed and its nature in part displayed, by rational means: by a process known in mathematics as the method of limits but elsewhere known as the process of idealization. Operating amid the activities of concept and logic and upon their subject-matter, the great process occurs in every division of the rational understanding; its function is, in every category where the laws of reason reign, to point aloft to an appropriate limit beyond their range, to some ideal form above the laws: in the category of classes, to an ideal universe as the manifold of all; in the realm of propositions or that of relations, to the sum or the product of all propositions or all relations; in that of time, to eternity; in knowledge, to omniscience; in ubiety, to omnipresence; in power, to omnipotence; in order and law, to necessity or fate; in indetermination, to absolute freedom or self-determination; in wisdom or love, to the "beauty absolute" of Plato's dream; and so on and on throughout the circuit and scope of rational thought. And so it is that the realm of superrational reality—the ultimate source of the religious emotions—thus indicated by the supernalizing process of idealization operating in the fields of reason, presents itself as an over-world of ideals.

In view of these considerations respecting the relations of Science and Religion, what shall we say is the place of Theology? What are the essential relations of Theology to Science and to Religion? What and where is the province of the venerable "Queen"? It is evident, I think, that theology *has* a province. Certainly there is in the heart of mankind a perennial craving for a kind of wisdom that the ages have taught us to regard as the peculiar object of theological aspiration; and there is a corresponding realm of truth: a field of enquiry that theology may rightly claim as her own. We are in a position, I believe, to see pretty clearly what her province is and what it is not. I speak, of course, of *rational* theology. No one need be told nowadays that theology is not religion. Religion, we have seen, is essentially and ultimately a certain complex of emotions—of emotions, not as analyzed into their elements, but as felt in their native integrity. Religion, so taken, is not only more immediate and more fundamental than theology but differs from it in kind: theology is not emotion, it is doctrine. No doubt religion is, in a sense, pregnant with theology, containing it, so to speak, in a "state of solution", *in potentia;* theology thus is, in a sense, religion's offspring and is naturally pervaded and tinged by religious reference and feeling. But to confound or to identify the two things would be like confusing a doctrine of æsthetic with the sentiment of beauty, or an ethical theory with the sense of right and wrong, or mathematical science with a feeling for logical implication and intellectual harmony, or science in general with the feeling of wonder, the delight of understanding, the lure of truth, the joy of knowledge and light. All doctrine, all theory, all science results from the reaction of intellect to feeling. "*Gefühl*" may not be "*alles*" but it is back of all and under all. The emotional source, however, or background of a doctrine is not itself doctrine. A dogmatist may feel, but dogmas are not emotions, they are propositions. Rational theology, in order to be rational, must be an affair of intellect, it must be an affair of ideas and their relations, of concepts and logic; it must be scientific—scientific in subject-matter, in method, and in structure; and so must deliver its message in the form, not of poetry or song or ejaculation, but of reasoned propositions concatenated into an intelligible and coherent body of doctrine addressed primarily to the understanding.

If theology is to be thus regarded as a science, what shall we say is its subject-matter? What is theology the science of? The answer is hardly to be found in the etymological meaning of the term. Names are stabler than their meanings. Time is ever pouring new wine into old bottles but the bottles do not always burst. Geometry, as every one knows, is, etymologically, earth-measurement, and the corresponding term in the Chinese language means 'show it by a figure.' Geometricians know, however, that their science is not mainly concerned with measurement of any kind, much less with measurements of Earth, and they know that, far from depending on figures, which are things of sense and imagination, geometry is a purely conceptual architecture, always strictly and for the most part obviously transcending sense and imagination. Theology may indeed, in the future as in the past, discourse about gods or about God; she may do so legitimately, conveniently, often consistently with an immense literature and a vast tradition. But to speak of "a science of God", even if the locution were clear, which it is not, could hardly serve as a felicitous indication of the subject-matter, or the field, of a science. It does not ring right: it sounds extravagant, pretentious, irreverent. In so far as God must be supposed to be superrational, the speech is absurd; and it has, moreover, the fatal disadvantage of seeming to exclude from the circle of theological thought the finest spiritual meditations of many millions of our fellow men. If we insisted upon defining the province of theology etymologically, a devout adherent of so great and noble a religion as Buddhism, for example, however profound his understanding of spiritual things, would have to be denied, and he would disclaim, the interests and the character of theologian. Such a conception is shallow and narrow. The domain of what is to be called theology must be conceived with sufficient depth and catholicity to include the thought of all men and women, whatever their time, place or creed, whose vocation it is to cherish the kind of wisdom that seeks to understand and to interpret rationally the supreme ideals of the human spirit.

Shall we, then, say that theology is the science of religion? There are, I think, insuperable objections to doing so. For one thing, the words have been gradually appropriated in recent times to another use; they carry a different import; they point to something else. They point, on the one hand, to psychological analysis of the religious emotions and, on the other, to study of their external manifestations—to their sensible embodiments in institutions, customs, ceremonies, and rites. Such analysis and such study are important enterprises; they are intimately related to theology; in a measure, they fall within its scope, but only so as auxiliaries and adjuvants and not as constituting the center or bulk of its concern. Not only do they differ from theology in their attitude towards religion, being less warm, less sympathetic, less constructive, less philosophic in their interest and bearing, less interior and spiritual, but—what is more significant—they differ from it in respect of content and subject-matter. Theology may analyze the religious emotions, or try to do so, and it may study their exterior manifestations in time and place, but these enterprises are not, singly or jointly, its chief concern. Theology is neither a branch of analytic psychology nor a branch of anthropology nor yet a combination of them. What is called the science of religion—the anthropological study of religion—is related to religious life very much as botany would be related to the life of plants if we supposed plants to be conscious of what we call their life and if botanists were fairly representative vegetables. But before botany could develop a branch, or acquire an interest or a function, analogous to that of theology, it would be necessary to endow plant life much more highly than we have just supposed. We should have to suppose it endowed with fear and love, reverence and awe, hope and aspiration, with supernalizing power, with dreams and ideals, with responsive sensibility to the light of a higher world. The relation of theology to religion is, then, not that of a science to its subject-matter. Granted that religion is theology's source, its motive, reference, and goal, its *raison d'etre.* That does not mean that religion is its subject-matter. We have seen that religion is essentially emotion; theology is doctrine; the former feels; the latter thinks; theology is a structure—an edifice of thought; religion is a flow—a stream of sentiment; theology is subject to the governance of ideas, deriving its authority from the *rules* of reason; religion is under the sway of ideals, deriving its authority from reason's *dreams;* the materials of the former are near at hand, they belong to the domain of the rational; the emotions of the latter come from afar, having their ultimate source in a realm superrational; the light of theology is the light of the understanding; that of religion is the mystic radiance of an overworld.

In this triune scheme of distinct but kindred things of the spirit, in this triple combination and interplay of idea, ideal, and feeling—of reason, overworld, and religion—, it is now at length evident, I believe, where we are to find the province and the rôle of theology. It is evident that its part in the great drama is the part of idea and reason, the part of intellect. The subject-matter of theology is not immediately nor primarily the religious emotions nor is it the interior constitution of their superrational ground and source: it is neither the feelings themselves nor the essential inner nature of the overworld that thrills them into being and sustains their life. Its subject-matter proper consists of rational phenomena: it consists of those facts and processes of the rational understanding that serve at once to indicate the existence of an overworld and to manifest its shining upon the things below. It is thus the task of theology to study those implications of logical thought that are hyperlogical, and, in so far as possible, to interpret them in rational terms; it is its function to examine the nature of rational thinking in its various categories, to unfold its hid intent, to clarify the manner in which thought, following endless courses within its own domain, perpetually approximates, forever pursues, and intimates, by the laws of its going, limits that lie beyond. There *is* in Reason a life-process deeper and finer than the mechanical movements of ratiocination, there is a kind of divine energy there, a beholding presence, a faculty within a faculty, a soul, if you please, in Reason, that fills her heart with dreams, points to a shining canopy above the summits of her thought, discerns in the light and atmosphere of her common activities the sheen of ideals—the glory of perfections—above and beyond them all. The nature and significance of that supernalizing power—there, I take it, is theology's problem. Theology is, in a word, the science of Idealization.

It is a natural science; not indeed a laboratory science; not, in ordinary sense, an observational science, for the objects it observes are inner things, things beheld only in psychic light, not things stained with refracted radiance of the sun; neither is it an experimental science save in the sense in which all thinking whatsoever—all logical procedure—is essentially experimental; but it is, none the less, a natural science. Granted that its materials are not things of sense; granted that they are things of reason—familiar shinings there of strange supernal lights; they are not on that account unnatural. The phenomena of idealization are not artificial nor forced; they are spontaneous, springing from foundations deeper than will; they are as natural as the dawn; their credentials are cosmic.

What it is that makes the task of theology so difficult and delicate is clearly to be found in the peculiar character of its subject-matter—in the essential nature of it and especially in the relation it bears to the over-world. We have to do with the phenomena of idealization. There are no special difficulties to be encountered in dealing with the great process as a process; the major fact about it—that of its existence, its ubiquitous presence, its ceaseless operation—is plain; there is nothing insuperable in ascertaining where and how it begins—here, there and yonder, as we have seen—in every category of the rational understanding; nor in ascertaining how it advances, from initial points in the domain of reason along innumerable paths of thought that run endlessly on and on, like an increasing sequence of terms, outward towards the border; nor yet in ascertaining how the process ends, in the presentation, namely, of limits that lie beyond. In all this, in all that pertains to the process as such, there are indeed difficulties, subtleties of thought, delicate considerations, but nothing of a kind to baffle the methods of science. But what shall we say of the *results* of the process, of the limits presented by it, of those great ideals themselves of which it is the function of idealization to make us aware? It must be noted and borne in mind that they are not concepts, they are not ideas, they are ideals. How is theology, how is theology as a science, to deal with *them?* Their similitudes and differences are to be detected; they are to be compared, ordered, and classified; their significance is to be appraised; their authority determined; their claim to supremacy in the ascending scale of values must be examined. How may all this be done scientifically? In such an enterprise the primal instinct to seize and subjugate is of no avail. Ideals are not things to be grasped, they are things to be reached for; they are not subjects for conquest, they are objects for aspiration; they are not properties to be possessed, they are perfections to be pursued; logic can not harness them, it can not reduce them, as it reduces ideas, to the ranks of obedient servants in the fields of reason; they hover aloft; they can not be pounced upon; to realize an ideal is not to possess it; it is to own its authority, to respond to its appeal, to follow its leading, to be drawn to higher elevations by the charm and persuasiveness of its majesty and beauty. It is evident, I believe, what must be the answer to the foregoing question. In dealing with the great ideals, theology must approach them from below, from their ground and source, which are a rational ground and source; she must approach them through an understanding of the infinite sequences that have the ideals, not as final terms, in reason, but as superrational limits; she can know them only as they are revealed in the mode and light of their genesis; she must study them as *results* of a *process*—results that she can not immediately handle or seize—a process with which she is competent so to deal.

An even greater source of theological difficulty and confusion is the subtle and bewildering relation the ideals in question bear to the overworld. Theology is to be rational, scientific. The overworld is superrational. It is obvious that such a world can not be the subject of a science. It is evident that theology can not be a science of the overworld as astronomy, for example, is the science of the heavenly bodies, as physics is the science of matter and motion, as biology is the science of organic life, or as mathematics is the science of logical implication. To speak of explaining superrational being in rational terms is folly. Does it, therefore, follow that theology must remain silent regarding superrational reality? It does not. The overworld has downward-facing aspects; it presents aspects to the upward gaze of reason: these are reason's ideals, superrational limits, as we have seen, of rational thought. Of these theology may speak; she may speak of their origin, of the process and mode of their presentation, of their significance, of their majesty, of the lure of their beauty, of their glory; she may speak of their genuineness and authority, of their relation to hope and aspiration, yearning and love, reverence and awe. But she can not without folly undertake to explore nor pretend to explain the inner constitution, the ultimate nature, of an overworld.

The task of theology, thus conceived, is one of exceeding delicacy. It is little wonder that in her long, long career she has often gone astray, that she has committed innumerable blunders, that she has sometimes despaired, that she has frequently incurred, sometimes deservedly, the disrespect, the antipathy, even the contempt, of scientific men. It is little wonder, too, that she fares ill in a practician age, that she wins but little encouragement or support in comparison with those physical sciences that have the advantage of being able constantly to vindicate their worth in the eyes of a tinkering and huxtering world through "useful" applications, multiplying the conveniences of men, advancing their physical welfare, expanding and subliming their petty pursuits to the proportions and elevation of vast and dazzling commercial and industrial enterprises. There is no domain of thought, no branch of science or speculation, where the subject-matter is quite so subtle, where the facts are so intangible, so elusive, so remote from sound and touch and sight, where the conceptions are so tenuous, where the hypotheses are so generic and broad, so hard to verify, and where it is so difficult to discriminate appearance from reality, separating from out the wildering maze problems that are genuine from those that are not. It is precisely on this account, however, that modern mathematics, as I hope we may see, is qualified by the inmost character of her being to lend a helping hand.

The answer of Laplace to Napoleon's question, why he had not in his *Mécanique Céleste* mentioned the name of God, is known to all: "Sir," the savant replied, "I had no need of that hypothesis." Not so generally known is the instant response of the great author of the *Mécanique Analytique* when the Emperor made prompt report to him of the memorable conversation: "Nevertheless," said Lagrange, "that is an hypothesis that accounts for many things."

Let us not mistake the point of these fine words. Superficially the speeches appear to be mutually antagonistic; they do somewhat resemble the sudden saber-thrust and counter thrust of battle. Yet they are in perfect accord. Their semblance of mutual opposition is illusion, due to the dramatic character of the situation and a certain contrast of sound. It entirely disappears on closer examination. There is neither irreverence in the one speech nor reverence in the other. If Laplace's *mot* indicate a lack of veneration, then that of Lagrange must indicate a lack of *scientific* temper. Scientific temper lacking in Lagrange! It is true that Laplace, at the close of his immortal work, might, like Newton before him, have discharged the mood essential to its production; he might have given himself to another kind of meditation, to leisured contemplation of the cosmic visions gained in years of analytic toil; and thus receptively musing on the mighty mechanism of the stellar universe—its unfathomable deeps, the immeasurable energies of swift-revolving worlds of flame, the all-pervasive order, the silent reign throughout of majestic law—, he might have felt a reverent sense of admiration akin to religious awe, and—again like Newton—have owned in words that such unity and power betoken the dominion of a Supreme Ruler and Lord of all. Had he done so, had he thus chosen to crown his scientific work by some expression of belief in a divine source and ruler of a universe whose profounder beauties he had been enabled to behold and disclose, the testimony could not but seem fitting to everyone; it would be especially grateful to those fortunate folk who see in every great display of power a witness to omnipotence, in every striking manifestation of natural law an evidence of divine decree, in every nobler scene of beauty a token of divine perfection. But—and this is the thing to be noted—such an expression of belief, however creditable to the great astronomer in his character as a man, would not have been in any sense a constituent of the *Mécanique Céleste*—neither a postulate nor a theorem, no proper part whatever of the great *description*, but only an aftereffect, a note of veneration evoked by subsequent recall and contemplation of the celestial scenes *described.* Had some soldier of Euclid's time demanded of the illustrious geometrician why he had not in the Elements made mention of Zeus, no doubt the wit provoked but yesterday by the challenge of Napoleon's question had framed itself in Greek two thousand years before. Or does some one imagine that that least perishable work among the scientific monuments of the ancient world could have been scientifically improved by adding to its underlying postulates the statement, There is a God? If one asks, for example, why planetary paths are elliptic, or why camels have humps, or why the earth is flattened at the poles, and receives for answer that there is a God and that God so wills, the answer may indeed be a statement of fact, and yet as a scientific answer it would be absolutely worthless; it would be silly; and any one who could solemnly offer it as scientific would seem less logical than pathological. The resolute attempt of science to explain the universe in terms of mechanics can not be furthered by the postulation of a God; indeed it would be abandoned thereby; for one thing is certain: God, if God there be, is no machine. Laplace was right; he had "no need of that hypothesis." Nay, his problem being one of mechanics, he could not, without stultifying himself, have even pretended to use it.

"Sir, I had no need of that hypothesis." Laplace was right. "Nevertheless that is an hypothesis that accounts for many things." Lagrange was right. It is evident that the significance of the two speeches lies, not in their seeming discord, but in their real concord: in their common point of view; it consists in what neither one asserts but both of them imply: namely, that *God is an hypothesis.*

Let me say, for what it may be worth, that personally I am far from prepared to contend that God is the name of an hypothesis and *nothing more.* It is perfectly true and perfectly clear that science, viewed as an attempt to explain, in mechanical terms, all phenomena, the attempt itself included, is, thoroughgoingly, an atheistic enterprise. It is a legitimate enterprise; it is carried on under a working hypothesis that men may make—the hypothesis that mechanical principles are sufficient; under it great things have been achieved; there is every reason to expect that even greater things will follow with the years; it is a right, it may be a duty, to pursue it for all it may yield. But, while Science, thus defined, is essentially atheistic, scientific Man is not. Man is greater, infinitely greater, than science, as he is greater than art or philosophy or religion or any mode or form in which his life may manifest itself. Many a scientific man is temperamentally disqualified to regard the mechanistic hypothesis as all-sufficient, and who is qualified to say that temperament has no essential relation to the problem? Many a scientific man, even the hardiest of the kind—unless cut off before the mellowing touch of pensive years can ripen knowledge into wisdom—comes sooner or later to feel that the mechanistic hypothesis, fruitful as it is, can not embrace the whole of life, that it can never give an adequate account of the finer elements of "man's unconquerable mind"—its radiance and joy, its conscience and love, its spiritual yearnings, its holy aspirations; and so, under the chastening influences of time and meditation, more and more awake to the subtler claims of his being, he comes, reluctantly perhaps, slowly it may be and late in life, to reconsider and rectify his earlier estimates, and from the doubt that is "hungry and barren and sharp as the sea," craves and seeks relief, finding it at length in a sense of a sympathizing consciousness not his own, in subtle intimations of the pervasive presence of a living Spirit.

Neither do I deny that, far from being a *mere* hypothesis, God may be a real being—an infinite personality—whose reality is, at times, to persons of a certain temperament, an immediate object of a genuine kind of knowledge—the kind that mystics have sometimes claimed to have. That they have been sincere there is no reason to doubt. Have they been mistaken? I do not know. Knowledge of the kind in question is said to be ineffable. If it exists, it is ineffable. That does not mean that it does not exist; it merely means that, if it does exist, it is not *scientific* knowledge, for scientific knowledge is effable: it is communicable knowledge; it rests on a kind of evidence that, if it is for you, is also for me, or, if for me, then also for you—it is not essentially private or personal, it is essentially public and impersonal. But knowledge, we know, is not all of a kind. It would be stupid to maintain that all knowledge must be scientific or else ungenuine. I know how to move my arms, we say, or how to walk, to cast a stone, to wink, to swallow, or to think; a squirrel, we all say, knows how to climb a tree or gnaw a nut, a horse how to find its stall: such knowledge is not scientific. To my wife the full moon appears the size of a dinner plate; to me, the size of a large cart-wheel. How big is the moon? That is not the question; if it were, the right answer would belong to scientific knowledge. How big does it seem? That is meaningless. How big does it seem to you? That you can know. To me? That I know. But your certitude and mine are not common to us—they are not impersonal, they are individual, private, personal certitudes.

In this connection it is worth while to mention another type of evidence—a kind of evidence that is, like the mystic's, ineffable—at all events exceedingly hard to communicate—and yet is, I suspect, available to the normal intellect, provided it will be at the pains to try a certain psychological experiment. The experiment relates to the great question of cosmic *purposefulness.* To deny the universe that quality is so easy to do in words. But to do so in fact—to gain, that is, a poignant sense of the denial's essential meaning—appears to be a matter of exceeding difficulty. May I refer to my own experience? I have tried the experiment many times. Mood is essential, and time and place—springtime or autumn, evening or the still night, rural solitude under the moon and the stars. In the course of thirty years I have won, perhaps a hundred times, what seemed to be a realizing sense of what it is that the denial means. Words fail. To know the sense one must feel it. When it comes, it comes like a sudden apparition, but it does not stay. Its momentary presence seems to involve an instant's failure of its support, like a swooning of mind immediately checked and healed, like the integrity of being itself suddenly recovered from the brink of dissolution. The coming and going are quick as a streak of lightning; only the apparition is dark, like the passing shadow of a flitting bird, like a mid-day moment's dream of dusk at once dissolved in the light, like a cut in consciousness instantly closed as a cleft in a sea: the denial being no sooner achieved in feeling than it has been completely overwhelmed by the inrushing flood of 'What, then, *is* it for?'—as if some suddenly roused instinct, vital to Intelligence, had leaped to the defense of her integrity and life. Such experience leads me to suspect that cosmic purposefulness is something profounder than a doctrinal postulate with which thought may, if it choose, dispense. I suspect it is an essential part of what mind means by mind.

But, after all such claims have been duly allowed, we must not fail to see clearly that, for theology regarded as a purely *scientific* enterprise, God *is* an hypothesis and nothing more. For the rapt vision of the seer, faith's evidence of things not seen, the mystic's immediate sense of divine communion, the above-mentioned evidence of cosmic purposefulness, all these and their kind being essentially personal, private, ineffable, incommunicable experiences, are none of them forms of scientific knowledge: because, as I have said, scientific knowledge always is, potentially at least, impersonal, public, sharply discriminated in kind from other varieties of knowledge by what we may call its *social* character, by its transmissibility from mind to mind. Knowledge of the outward differences between Greek architecture, for example, and Hindu architecture is scientific but your knowledge that one of these pleases you more than the other is not scientific, it is private. The idiosyncrasy of scientific knowledge is that, though perchance there may be at a given time but one individual who has it, yet it does not belong to him in his individual capacity; it is his as a member of society, as a representative of humankind. Of expert logicians, for example, there may be in a given community but few. Yet the science is not theirs. Logic is impersonal. It belongs to Man.

Here, then, we are face to face with a capital theme of theological meditation: the assumption, namely, or hypothesis of a being called God. How shall we frame it in speech? How describe the august Being it seeks to represent? If we appeal to the greatest physical philosopher of all time, the author of the Principia and inventor of the Infinitesimal Calculus returns the terse reply: "A Being eternal, infinite, absolutely perfect." If we listen to him whose genius established the great alliance between the doctrines of Number and Space, thus bringing together the sundered hemispheres of apodictic thought and so creating the world of Analytic Geometry, we hear the resounding words of Descartes: "Infinite, eternal, immutable, independent, all-knowing, all-powerful." If we ask the "God-intoxicated" philosopher of Amsterdam, we receive from the great Spinoza a similar characterization not less impressive: "Absolutely infinite, consisting of infinite attributes, each expressing eternal and infinite essentiality." These familiar citations will serve to remind the reader of like efforts, among the best of human thought, to formulate in adequate terms the hypothesis God. About things that are very familiar it is exceedingly difficult to bring ourselves to think, and the terms of the hypothesis have been familiar for hundreds of years. Were it new and fresh instead of being so old and stale, we should all of us be immediately struck by what is, among its distinctive features, a very obvious mark. The hypotheses that we meet elsewhere, as the nebular, the corpuscular, the ionic, the atomic, the molecular, the hypothesis of a space-pervading ether, of universal gravitation, of Euclidean space, of organic evolution, of conservation of energy or of mass, all such—all the hypotheses we encounter in the literature of ordinary science—have one character in common: each of them is restricted in scope, limited to some fragment of reality, they divide in order to conquer, each is confined to a field that is bounded; not so, however, the hypothesis God: it is distinguished by the fact that, among hypotheses, it alone attempts to span and bind the Whole. That is a very remarkable characteristic. And soon we must note another. But first we must ask, What does the hypothesis mean?

"The light of human minds," says Hobbs, "is perspicuous words, but by definitions first snuffed and purged from ambiguity." Accordingly it is necessary to ask: what, if any, precise meaning, available for the purposes of logical discourse, may be assigned to the terms of the hypothesis? Infinite, Eternal, Omnipotent, Omniscient, Omnipresent, and the rest: what do these mighty terms mean? I do not now ask for their meaning as instruments for energizing life. I do not now ask for their meaning as cries of the spirit—voices from the deeps of feeling. At present I am not concerned with their meaning for reverence, for love, for awe. I do not here seek their relation to the moods of poetry and prayer. I am not enquiring for their *emotional* significance, so like that of mountain scenery, a vast wilderness, the heavens above, or the "solemn anthem of the sea." I enquire for their logical value, for their meaning in Thought. It is essential to note at once a very remarkable thing: the great terms in question are not names of scientific notions, they are not names of concepts, they are not names of ideas; they are names of ideals—super-rational ideals, outlying limits of rational thought. This gives the hypothesis an appearance of being an hypothesis respecting the nature of the overworld. Is it such in fact? So to take it, as many consciously or unconsciously do, is fatal. It removes the question from the jurisdiction of theology regarded as a science: a world of super-rational reality can not be the subject of any science; to suppose the contrary is to ignore the sole restriction that the muses have placed on freedom of thought: thought must be free from internal contradiction, it must be harmonious. It is not necessary, however, to take the hypothesis so. The overworld, we have seen, has downward-facing aspects. These aspects, presented to the upward gaze of reason by the process of idealization operating in reason's fields, are precisely the ideals that the terms of our hypothesis serve to designate. The hypothesis is accordingly to be regarded as an hypothesis respecting, not the essential inner nature of the overworld, but the nature of the downward-facing aspects presented by it through the process of idealization—a supernalizing agency working below. But, if the ideals, the aspects in question, are super-rational, how is it possible for science to say aught about them? Science, we have seen, must approach them from below, in the light of their genesis and manner of presentation. By this method science is enabled to say of them that such and such ideals are of a nature to appear as limits of such and such processes or sequences of rational thought. To be able to say *that*, however, with all it implies, is much: just as it is much—if I may illustrate* great things by small—to be able to say of a curve, for example, that, though outside the domain of broken lines, it is yet the limit of an endless sequence of broken lines; or just as it is much to be able to contemplate a curved surface as an ideal indicated by an endless series of plane-bounded figures approximating it forever, though the ideal itself does not belong to the field of the approximating figures; or just as it is much to be able to view what is called an irrational number as an ideal or limit beyond the domain of rational numbers but indicated and endlessly pursued by series of these; or just as, in general, it is important for the life of understanding, to be able to make out, in whatever field it operates, the endless courses of ever increasing approximation that by the law of their progress at once betray appropriate perfections beyond

* For a fuller explanation of the force and point of such illustrations, see the author's "Science and Religion," herein cited on an earlier page.

and, though never attaining them, yet lead us more and more deeply into their far-shining light.

Such, then, must be the method and such the ways of rational theology. If it is to have a motto, the motto must be: From ideas to ideals. The former indicate, the latter are indicated. These are to be understood scientifically only in so far as their meaning is revealed in the ideas indicating them and in the manner of the indication. Among the ideals with which we are here concerned, among the great ideals assembled in the hypothesis God, it is obvious that there is one which has the distinction of seeming to be at once coordinate with the rest and yet in a sense involved in each of them. I refer, of course, to the ideal denoted by the adjective Infinite. There is a Being, so the hypothesis runs, at once infinite *and* omniscient *and* omnipotent *and*, so on. Omniscience, however, involves Infinitude; so does Omnipotence; so does Eternality; so does every pealing note of the great diapason. Any illumination of what is meant by the term Infinite will, therefore, serve in a measure to illuminate the meaning of the kindred terms.

The reader doubtless knows that the Infinite of theology has never been defined—defined, that is, for logical use—, and he is now in a position, I believe, to see why it has not. It is not because the centuries have not witnessed many ingenious attempts to define it. It is because the term denotes, not an idea, but an ideal, a superrational ideal, and so does not admit of definition. There can be no doubt that a great deal of the confusion found in theological literature has resulted from the fact that theologians, failing in respect of this logical distinction, have gone on discoursing about what they have called the Infinite, as if the term stood for something—a concept or an idea—that had been, or, at all events could be, defined. The remedy for the kind of confusion that thus results is simple: it consists in not ignoring the distinction; it consists in ceasing, once for all, the attempt to treat an ideal as an idea; it consists in refraining from the hopeless endeavor to deal with a superrational limit as if it were a rational term of the endless sequence whose nature it is, not to contain the limit nor to attain it, but to indicate it and approximate it.

There are, however, other difficulties connected with the term—difficulties inherent in the nature of the case, proper difficulties, we may say, because they belong to the ideas constituting what we may call the ideal's rational basis, its basis in reason. Here it is necessary to note an important distinction, to be henceforth kept in mind. Thus far we have been speaking of the *theological* infinite. Fortunately or unfortunately the same term is constantly employed in science and especially in mathematics in another sense—in a sense closely related indeed, as we shall see, to theology's sense of the term but yet quite distinct therefrom. In theology, we have seen, the term denotes an ideal, a super-rational ideal, which can not be defined; in mathematics, as we are going to see, it denotes an idea, a concept that not only is sharply definable but has been in fact sharply defined. Presently we shall begin to see the beautiful relation between the two senses in which the term is employed and why it is and wherein the mathematical sense is an indispensable means for making clear the theological sense. The ideas—the objects or things—that mathematics calls infinite are not all of them of one order. There are countless mathematical infinities, or infinitudes, or infinites, as they are variously called,—countless types of them. Like the stars, they differ in glory. They constitute, as we shall see, an endless sequence of ever increasing terms—an endless series or succession of terms mounting ever higher and higher in respect of order or dignity or rank. Each term in the endless march of terms includes the type of infinitude represented by the preceding term but is itself of higher type. The major relation in the scheme delineated is evident at once: the *limit* of this endless series of infinite *ideas* is an infinite *ideal:* the Infinite of theology is the limit of the endless sequence of more and more embracing Infinitudes presented by science. It is not one of them; it is, so to speak, their envelope, enfolding them all.

It is now time to look into the great relation a little more deeply. We must try to see quite clearly what scientific infinities are; we must endeavor to understand how they are related to the finite things of sense and how they embrace and penetrate the common affairs of men; we must learn something of the law in accordance with which they are disposed, rank above rank, in a hierarchy of orders, without a summit; we must observe how the process of Idealization, operating in and among them, pervades the atmosphere of the grand array, and creates or finds there a subtle radiance that seems to reveal a downward-shining aspect of an overworld. The task is not an easy one; it demands a little patience and a little penetration; a part of the discussion, which is for such as prefer not being entertained to being fooled, must seem to some a little arid—a pretty dry way to a valley of fruits; a sense of its full significance can not be gained at once; it must be won as the fruit of reflection.

In order to explain the scientific or mathematical meaning of the term infinitude, or infinite, let me begin with some simple examples. I will take them from the two hemispheres of rigorous thought, the two great subject-matters of it—Number and Space.

Imagine two concentric spheres, the inner one white and named the silver sphere, the outer (or larger) one yellow and named the golden sphere. (In accordance with the usage of higher geometry I shall mean by sphere a sphere-surface.) Next imagine the sheaf (as it is called) of rays, consisting of all the straight lines that have their beginning at the common center of the two spheres and thence extend outward endlessly in every direction. It is plain that any ray, *R*, of the sheaf pierces the silver sphere in a point, say *S*, and the golden sphere in a point, say *G*. Calling *S* and *G* a pair of points, it is evident that, by considering all the rays of the sheaf, the points of the one sphere are paired with those of the other in a one-to-one, or point-to-point, fashion: in other words, a unique and reciprocal correspondence is thus established between the points of the silver sphere and those of the golden sphere. One silver point corresponds to one and but one golden point; one golden point, to one and but one silver point; and this reciprocal relation holds for every silver point and for every golden one. We see at once that the number of points on the one sphere is exactly the same as the number of points on the other; we see, too, that this number equality subsists no matter how great the difference between the *sizes* of the spheres—one of them may as well be microspically small and the other billions of times larger than the earth or the sun. In other words, the number of points on a surface of given size (given area) is independent of the given area or size, and so will not be changed by changing the area or size. Now imagine a closed curve or ring—red, if you like, for the sake of vividness—to be drawn on the golden sphere and enclosing thereon a portion of it, a region *A*, precisely equal in area to the area of the silver sphere. We need not suppose this latter equality (of areas) but we may as well, for the supposition will reduce a little the shock we are soon to receive. The number of points in the region *A* is, of course, the same as the number on the silver sphere and is, therefore, the same as the number on the golden one. But the collection of points in the region *A* is only a *part* of the *whole* collection on the golden sphere. The shocking thing is this: we have here a *part*—the ensemble of points in the region *A*—and a *whole*—the ensemble of points on the golden sphere—such that the number of points in the *part* is precisely the same as the number of points in the *whole*. It is to be noted carefully and once for all that the astonishing equality subsists, not between the area of the region *A* and that of the golden sphere, but between two *multitudes* (of points), of which one is a *part* and the other the *whole*.

Does some non-mathematical reader, unfamiliar with this kind of thinking, distrust the argument, feeling perhaps that he has been tricked by a juggling use of the notions of surface area and point collection? If so, let him scrutinize the equivalent following argument, in which the notion of area plays no rôle. First, a preliminary word of explanation. If a straight line and a plane are parallel, we say sometimes—in high school, for example,—that they have no common point; but if we continue our study into what is called projective geometry,—never mind the name—, we learn to say that, if a line and a plane be parallel, they *have* a common point—called an "ideal" point to distinguish it from ordinary points—the "ideal" point being so far away that it can not be reached by a step-by-step process of going towards it; any "ordinary" point can be so reached. Such "ideal" points of a plane make up a line, called the "ideal" line of the plane. A plane thus conceived as having such an "ideal" line is called a projective plane, and a line regarded as having an "ideal" point is called a projective line. And now the promised argument. Think of a hemisphere, *H*, and suppose it to rest on a horizontal plane, *Π*, the hollow of *H* being open to the upper sky. The rim of *H* is a circle, *C*. Denote its center by *P*. There is a sheaf of rays running out from *P*. Of this sheaf consider only those rays that lie in the plane containing *C* and those that run below this plane. The imagery is perfectly clear. The rays considered, since each of them pierces *H* in a point and *Π* in a point, plainly establish a one-to-one correspondence between the points on the hemisphere *H* and the points of the plane *Π*; a point on the rim of *H* obviously corresponding to an "ideal" point of *Π*, and conversely. Now imagine a plane above *Π*, parallel to it, and cutting *H* in two. Cast away the upper part of *H* so cut off and keep the lower part—the up-turned cap resting on *Π* as before. The rim of this cap is, again, a circle. Call it C' and its center P'. As before, there is a sheaf of rays running out from P'. Of this sheaf consider, as before, only those rays that lie in the plane of the cap's rim and those that run below this plane. Again the situation is clear: the rays considered, each piercing a point from the cap and a point from Π, set up a one-to-one correspondence between the points of the cap and those of Π. We now see that the number of points of H is the same as the number of points of Π and that the number on the cap is the same as the number on Π; hence, we see, the number on H is the same as the number on the cap. But the collection of points on the cap is a *part* of the *whole* collection on H. The fact is, accordingly, now perfectly evident—whether at first we like it or not—that we are in a world where it is easy to encounter a whole having a part whose elements are precisely as numerous as are the elements of the whole. Every whole of that kind is said to be *infinite*.

Be it understood, then, that the concept of infinity—the scientific or mathematical meaning of the term—is this: namely, a collection, class, set, group, aggregate, ensemble, manifold, or multitude of elements or things—be these points or passions, ions or ideas, relations or terms, quantities or qualities, numbers or instants or colors or sounds, degrees of wisdom or goodness or power or joy, or any other modes, forms, or determinations of being—is said to be *infinite* if and only if the collection, like the ensemble of points on a sphere, contains a part, or subcollection, that is numerically equal to the whole. On the other hand, a collection or multitude is said to be *finite* if and only if, like the collection of trees in yonder forest, like the human population of the globe, like the multitude of sands of the sea or that of the stars within telescopic range, it contains no part or subcollection numerically equal to the whole.

There is here no ground for quibbling, hesitance, or doubt. There stand the two concepts, absolutely clear; and there, too, stand the validating facts, absolutely unmistakable. The latter indeed may be multiplied at will. Examples of collections illustrating the concept of *finitude* are of course familiar to every one, being forced upon the attention by the vulgar necessities of life; indeed they are so familiar that but few persons have so much as dreamed that there are collections or manifolds of another type. We have seen, however, that there are, and the gain is one—if we really make it our own—to work a profound transformation in our view of the world. Of examples illustrating the concept of *infinitude*, we have thus far instanced but two. Similar examples abound, however, in even greater profusion than the other kind, being found in the great and the small, the remote and the near, in Number, in Space, in Time, in qualitative distinctions, in the realm of pure relation—wherever the intellect may penetrate—if the inner eye be only disciplined to detect their omnipresence. A little patience, I have said, is indispensable in this part of the discussion—quite as needful as a little penetration; and I must request the reader's permission to tarry yet a little in order to point out a few further illustrations of what science means by an infinite multitude. It is of the nature of doctrine to grow aloft, higher and higher, into the limpid atmosphere of pure theory, and that is legitimate—architecture must rise; but, however high its head, a doctrine, if it is to stand, must plant its feet upon the solid earth of fact. The facts with which we are here concerned are not facts of sense; they are facts of thought; they do not belong to the domain that we, as animals, share jointly with the beasts; they are the prerogatives of man as man—in his capacity, that is, for "discourse of reason." Let us return for a moment to our image of the sheaf and the concentric spheres. Consider those rays of the sheaf that pierce the points of the region *A* on the golden sphere. Let us call the bunch of these rays a bundle. It is evident that the number of rays of the bundle is the same as the number of points in the region *A*, one ray through each point of *A*, one point of *A* on each ray of the bundle: this number, we have seen, is the same as the number of points on the sphere; and this, again, the same as the number of rays of the entire sheaf; whence it is seen that the bundle, though but a *part* of the sheaf, has the same number of rays as the number of rays in the *whole*. And so the sheaf and the bundle serve to exemplify again the concept of infinite manifolds.

Let me now take a very simple example from the inexhaustible resources of another field. Consider the little equation, $y = 2x$, which every one understands. If we assign a value, say 1, to x, then, as we see, the value of y is thereby also determined: it is just twice as much—in this case 2; if we let x be $\frac{1}{2}$, y must be 1; if x be $\sqrt{2}$, y is $2\sqrt{2}$; and so on: to any value of the variable x, there corresponds one and but one value of the variable y; and conversely, for we could just as well give values to y and so find for each of them its half, or the corresponding value of x. Let us now agree to let x vary in value from *zero* to 1, taking, one at a time, the value *zero*, the value 1, and each of the innumerable host of values between; then y will take, one at a time, each of the values in the range from *zero* to 2, including, of course, *zero* and 2. Thus is set up a one-to-one correspondence between the multitude of values in the x-range from *zero* to 1 and the multitude in the y-range from *zero* to 2. The number of values or numbers in the one range is, therefore, the same as the number of values or numbers in the other. But the collection of numbers in the range from *zero* to 1 is but a *part* of the *whole* multitude in the range from *zero* to 2. Accordingly each of these multitudes is an infinite multitude of things.

As a final example here, let me invite careful attention to an infinite collection that would be the easiest of all to grasp were it not so very simple and if action upon it of the higher understanding were not almost inhibited by our fixed habit of regarding the elements of the collection as having no significance beyond their familiar vulgar uses in the counting-house and the market place. For the elements in question are nothing more romantic than the numbers with which we count. How very prosaic the prospect, you naturally say. I quite agree, and yet I venture to say that, if we will but rise above the stale levels of sense and imagination, we shall not fail to detect here a species of genuine poesy—the poesy of pure thought in touch with the infinite and eternal. Consider the two sequences or series of integers:

$$(W) \quad 1, 2, 3, 4, 5, 6, \quad \ldots\ldots, n,\ n+1, \ldots\ldots\ldots$$
$$(P) \quad 2, 4, 6, 8, 10, 12, \ldots\ldots, 2n, 2(n+1) \ldots\ldots$$

By the series (W) of symbols I wish to call attention, not to that uncompleted row of marks itself, but to a certain definite invisible *whole* that the row suggests and serves to bring as an object before the mind, namely: the *totality* of the positive integers. On being confronted with the notion of this fundamental totality, at once so clear to thought and so baffling to imagination, many persons, especially the uninitiated, become restive for a time. A little reflection, however, will dissipate any reasonable scepticism, and show that our footing here is solid rock. It is true indeed that, however many integers we may singly specify or imagine, there always remain more and more. It is also true that the hand cannot actually write nor the physical eye behold a set of symbols matching one-to-one all the integers composing the asserted totality, if such a thing there be. What of it? Consider, for a moment, a familiar totality so obvious that none may question it—the totality, I mean, of the points of a circle. As in the case of the integers, so here, too, it is impossible to think all the points singly or singly to specify or symbolize them all. Yet there they are—not one now and then another—but all of them at once, a totality persisting as such and unescapable. What is the secret? The secret is that the totality is a conceptual thing, a thing for thought and not for sense or imagination, a thing carved out by a *law* transcending the powers of step-by-step perception or depiction, a law of definition that selects out of the universe of thinkable things a set of them unambiguously—the law, namely, that the things shall be points of a plane and be all of them equally distant from a point therein. So it is precisely with the totality of positive integers. If you say that the totality does not exist, what you mean is that the integers of such a totality can not be written down for sight to look at or that no one can depict them all on the canvas of imagination. Permit me to remind you that I am not here addressing your sense nor your imagination. I am addressing your conception, your thought. The asserted totality does not exist for sense, it does not exist for imagination; it exists for thought. It derives its completeness and one-ness from the completeness and one-ness of the selective law defining it—the law, namely, that *after any definite integer there is another greater than that by one.* Note that the law includes and excludes and that the inclusion and exclusion are both of them precise, decisive, complete, and instantaneous. It is pathetic if one can not see clearly that it is precisely such sense-transcending and imagination-transcending totalities that constitute the essential subject-matter of rigorous thought. For to deny their validity is to evacuate the Reason of its proper content and to bar even the possibility of Science. Science, properly speaking, does not deal with a set of things that we might fancy arranged in a row, like a row of blocks, beginning here and ending there. Science is interested only when the row, if it begins, never ends. Consider a curve. You can not exhaust its points by naming one after another of them. That is just why science is interested. It deals with the *totality* of the points by dealing with the curve, that is with the law—a definite thing—defining the totality, which is, therefore, also definite. Do you think geometry would exist if the points of space could be counted like a heap of marbles? If it did, it would be trivial. So much by way of reassuring those timid persons who, primarily children of sense and imagination, are filled with doubt and trepidation when asked to pass upward from their accustomed atmosphere into the ether of pure thought.

Let us now resume the advance. Compare the totality (W) of integers with the totality (P) of even integers. Let us agree to pair each integer of (W) with the one below it in (P). In this way a one-to-one correspondence is set up between the integers in (W) and those in (P), a result that we may indicate by the following sequence of pairs:

$$(T) \quad 1, 2; 2, 4; 3, 6; \ldots ; n, 2n; \ldots$$

Observe that the pairing is no creeping performance that never gets performed—ever going on and never finishing; neither is it a lightning-swift process, for this were as helpless before the task of pairing the totalities step by step as would be the pace of a snail: an endless course can not be run through by merely going fast. No, the pairing is an instantaneous deed of law, wrought without lapse of time. The law is: *each number shall go with its double.* To choose the law is to say: Let the pairing be done; and—it is done. To contemplate the deed requires time; but the doing of it, none.

There is possible a yet deeper view of the matter. I mean the static view. We may say, that is,—and this is correct—, that the integers as elements of the existing world of ideas already and always stand at once in all sorts of interrelations of which it is the nature of integers to admit, among such relations being that indicated by (T). In this view, the pairing is not a *process* of associating an integer with its double, then another with *its* double, and so on, thus establishing progressively, so to speak, the relation (T). It is not that; it is simply a single act of will choosing out a certain eternal relation from among hosts of relations also eternal. Whichever view of the matter be taken—and either is admissible—it is clear that a one-to-one relation does subsist between the elements in (W) and the elements in (P). The two totalities are therefore equally rich in elements: the number of integers in the one is the same as the number of those in the other. But every integer in (P) is an integer in (W), while (W) has integers that are not in (P). Hence (P) is a *part* and (W) the *whole;* and so (W) contains an *infinitude* of integers; and the like is true of (P), for whatever matches an infinite, in the way now repeatedly exemplified, is, of course, itself infinite—indeed, infinite of the same rank.

It is needless, I trust, to cite here further examples. "These slight footprints suffice to enable a keen-searching mind to find out"—not "*all* the rest", as the maddened poet sang—but more and more. For, to eyes once opened, the brood of the infinite is everywhere. The light of the great concept shines in every aspect of being. The reader is now aware that this our world is a world that presents two great types of wholes, or manifolds, of thinkable realities—manifolds that are finite and manifolds that are infinite. He is now aware that each of the latter is characterized by the marvelous fact that it is a whole containing a part (countless parts indeed) matching the whole perfectly, as we have seen, in elemental wealth, in richness of content, in dignity of structure. The principle of discrimination is very simple—so simple indeed as to have eluded the eye of thought for thousands of years—for the doctrine is very modern, a faint first glimmer of it appearing in a work of Galileo and a little later in a hint of Pascal but not again, it seems, for two centuries. By it the universe of thinkable reality, as we now see, is riven asunder, not spatially indeed but logically. The two grand divisions—the realm of the finite and the realm of the infinite—, which are wonderfully interlocked, together constitute a dual world answering to our dual life, the life of action and the life of thought. The realm of finite things is the domain of action, of Practical Life: it contains no multitudes but man may count them—the coins in the coffer, the cattle in the field, the deeds of a hero, the years of an empire; any series in it *begins* and *ends;* no totality or whole found there is matched by one of its parts: the world of finite things is an island-world suspent in a sea. And what is the immersing sea? It is the realm of infinite things—an ocean without bottom or surface or shore. It contains no totalities but such as are law-defined, never a whole of any kind that has not countless parts each matching it perfectly in respect of number of elements, coequal with it in *Mächtigkeit* as it is called, in potence or power, in complexity of structure, in dignity and wealth of reality. This is not the domain of Practical Life, though it penetrates the latter domain, intersects it in numberless ways, surrounds it, contains it in a sense: for a series that terminates is but part of one that does not; every ensemble that admits of tabulation is a fragment of one than can not be fully represented by tabulation but only by a law; every whole that is an overmatch for its every part belongs to some vaster whole owning parts with respect to which it is not an overmatch; every finite manifold is a sub-collection of an infinite one. No, the realm of infinite totalities, though embracing, in the sense explained, the domain of Practical Life, is not that domain; it is the domain of Reason, the province of Thought, the realm of Science; for, as Poincaré has acutely pointed out, there can be no science, properly speaking, of a finite subject-matter.

Very well, one may wish to say, I grant what you have said, but what of it? Where, pray, is Deity? I ask for bread; you give me a stone. I ask for a vision of God; you invite me to thread endless mazes of mathematics; you invite me to contemplate vast and dazzling splendors of Number and Space. What does it all avail?

> "I heap up numbers enormous,
> Mountains of millions extend,
> Piling time upon time,
> World on world without end,
> But when from the awful height
> I would a vision of Thee behold:
> The total sum of number's Might,
> Tho' multiplied a millionfold,
> Is yet no part of Thee."

The protest is easy to understand, it is temperamental. May I reply, by way of a reminder, that I have promised no "vision" of God? I am dealing with the hypothesis God. My aim is to throw some light on the meaning of its mighty terms. Chief among these is the theological Infinite. In pursuance of the aim I have been here trying to clarify the meaning of *scientific* infinitudes, of which the Infinite of theology is the supernal ideal or limit. None but the infinite, it is said, can comprehend the infinite. How familiar are the words! How often have they been solemnly pronounced in courts of philosophy and sunken in the soul like a leaden decree of fate! But are they not true? "Comprehend" here means, of course, comprehend rationally, it signifies to understand as ideas are understood. Said of the infinites of science, the words are, then, true. Said, however, of the theological Infinite, they are neither true nor false; they are meaningless, for the theological Infinite is, as already said and as I hope we are beginning to see, a superrational ideal, and to talk of comprehending or not comprehending *such* an ideal as we talk of understanding ideas is not to utter what is true or false but what is void of meaning. If, however, 'comprehend the Infinite' be—contrary to usage—taken to mean, not comprehend it as an idea, which it is not, but to signify having or gaining that kind of sense of what it means which comes from regarding it as the ideal or limit of infinites that we *can* comprehend as *ideas*, then the old maxim, applied to the theological Infinite, is false, for we *can* win the mentioned sense in the mentioned way, and we do not, I believe, regard ourselves as superrational ideals. But have we not involved ourselves in contradiction? For how can we gain the mentioned sense in the mentioned way, seeing that this way requires ordinary, or logical, comprehension of infinites, and seeing that, regarding these infinites of science, we have admitted that none but the infinite can comprehend the infinite? The answer is no, there is no contradiction, for we are ourselves infinite in the scientific meaning of the term, where by "we" I mean the commonwealth of ideas over which your mind or mine can range. That this is true we know at length, thanks to mathesis, and we know it, not merely as an intimation or intuitive apprehension, but as a proved proposition of science. We know, that is, as Richard Dedekind has rigorously demonstrated, that the world of man's ideas *as ideas*—the human *Gedankenwelt* as the author calls it—is an infinite manifold. Shorn of contest and other non-essentials, the proof may be rendered in a line. Some readers will not require it. A friend tells me that he does not understand the proof and does not need it. I may add that he is a man of extraordinary spiritual sensibility. Intuition, however, precious as it is, is often wrong, and I give the proof for the comfort of those who think it important to submit their intuitions, when it is possible, to the rigors of logical demonstration.

Denote by G the *Gedankenwelt*—the world of ideas; by I any idea therein, as that of a meal, a song, a deed of charity, a bargain, a moon beam, a diamond, a birth, a death; by I_1 the idea of I, for plainly the idea that a given idea is an idea is another idea; by I_2 the idea of I_1; and, generally, by I_{n+1} the idea of I_n. As any thought may itself be object of another thought, as this one may be object of a third different from the former two, and so on forever, it is seen that I_{n+1} can never fail, however large n may come to be, and so we have the two totalities:

$$(T) \quad I, I_1, I_2, \ldots, I_n, I_{n+1}, \ldots;$$
$$(T') \quad I_1, I_2, I_3, \ldots, I_{n+1}, I_{n+2}, \ldots;$$

the latter is a part of the former; each of them is a part of G. Now pair the two totalities as in the following scheme:

$$I, I_1; I_1, I_2; I_2, I_3; \ldots; I_n, I_{n+1}; I_{n+1}, I_{n+2}; \ldots;$$

each thing in (T) being thus associated with the thing below it in (T'). At once it is seen that the *whole* totality (T) is completely matched in one-to-one fashion by its *part* (T'); whence it follows that (T) is infinite; that (T') is infinite; and, *a fortiori*, that their common container, *G*, the *Gedankenwelt*, is infinite.

In this simple demonstration, so free from pomp, and in its conclusion, so significant for a right conception of man, there is large gain for rational theology, if indeed we may hope that professional theologians will one day be moved to avail themselves of such considerations. It is no small gain to vindicate by logic a great intuition of the soul: it is no small thing to *know*, not merely at times to feel, that our faculties are framed to comprehend, scientifically, infinite elements in the architecture of the world. For in the presence of such knowledge, the terrors of Naturalism dwindle and vanish. Kant's exclamation that "modern astronomy has annihilated my own importance" ceases to have significance when once we know that with countless infinitudes encountered in time and space our faculties are competent to deal,

"Times unending
Comprehending,
Space and worlds of worlds transcending."

We desire no instauration of the shallow and timid humanism that derived its estimate of man from a geocentric theory of the universe, cried alarm at the crumbling of a Mosaic cosmogony and shudders still at the shrinking of the earth to a pebble in the cosmic perspectives opened to the view by modern science. Bigness does not daunt Mathesis; she seeks it; vastness is the æther that sustains her wing. In her modern doctrine of infinite manifolds, of which I am here trying to give a rather slight indeed but hintful sketch, she has extended the dominion of logic far beyond the utmost borders of finite things out into the realm of transfinite reality. And when, if ever, theology learns to follow thither, when if ever, she acquaints herself with the procedures of science there and learns to contemplate the innumerable infinitudes that science can understand, she will find that the hierarchy they constitute is a ladder for her, an endless ladder by which she may ascend higher and higher into a better and better sense of what she ought to mean by her own Infinitude, which at once o'ertops and includes them all.

At this point the reader may naturally

desire to enter the discussion and have a share in shaping its course. I have, he may wish to say, now acquired a pretty clear conception of what science means by an infinite manifold; I have grasped the abstract idea and have seen it realized and illustrated in a variety of concrete examples; I am now prepared to find other examples for myself, for I am beginning to see that such multiplicities compose the intelligible portion of the embracing world, that they are literally omnipresent, that even in the surface of common life and common thought they gleam here, there and yonder like shining bassets of gold. But I do not see, he may say, that they are not of a single type; I have not glimpsed the ladder; I am far from seeing that, in respect of dignity, they dispose themselves rank above rank in a hierarchy without a rank supreme.

These words of the reader imply a legitimate demand, which must now be met—met, that is, in so far as circumstances will allow, for the matter is pretty subtle, involving some technical considerations known only to mathematicians, and does not admit of public presentation in a line. It is possible, however, without too many words, so to delineate the matter as to give what is sufficient—a realizing sense of its truth. For this purpose I must ask the reader to look again at the infinite manifold denoted above by (W). It is a homely affair. How dreary it looks and commonplace. But let us not be disheartened. We are going to see that it has beautiful aspects not yet disclosed, a dignity and character to quicken the pulse of our thought and win our admiration; for we are going to see that it belongs to an immense family of similar manifolds, many of them of singular beauty,—a countless host of them—, and that this homely one has the distinction and honor to represent the type. Of this family many members differ so much in appearance as to have concealed for thousands of years the deep similitude that makes them kin. Why is it, I wonder, that things are not what they seem? Is it because, if they were, life would be void of its finest interest—the zest of research, the joy of discovery, the surprise and delight of detecting the hid? Perhaps so, but let the question pass, and attend very sharply to what is now to be said. Any ensemble or manifold of elements such that a one-to-one correspondence can be set up between them and the numbers composing the manifold (*W*) is said to belong to the type of infinite manifolds represented by (*W*). This type has a beautiful designation—it is called the Denumerable Type: the manifolds belonging to it are called denumerable infinities or denumerably infinite manifolds or classes. Their name is legion. One of them, as we have seen, is the manifold of even integers, above denoted by (*P*); obviously another is the ensemble of odd integers; another may be got by taking from (*W*) for elements, say the number *one*, then a *million*, then a *million million*, and so on; it being thus evident that (*W*) contains countless denumerably infinite *parts*. But let us go outside of (*W*). Consider any straight line *L* running endlessly up and down—zenith-ward and nadir-ward—piercing or passing the stars, like a thread stretched through the universe of space. Consider the ensemble of miles it contains. On it choose a starting point *O*. Conceive as marked by 1 the end of the first upward mile, by 2 the end of the first downward mile, by 3 the end of the next upward mile, by 4 the end of the next downward mile, and so on and on. Will the integers fail? No, you say. Will thread length fail? Again you say no. So, then, the law of correlation holds, the required correspondence is established between the mile posts of *L* and the integers of (*W*). Obviously the same would be the case if we chose any other unit of length. Accordingly, we see that the ensemble of unit lengths composing a thread or course that traverses endlessly the abysses of Space is an infinite manifold of the denumerable type. To the same type of infinitude plainly belongs the aggregate of minutes or hours or centuries in the stretch or course of Time conceived as running eternally backward and eternally forward. Well might the apostle exclaim that one day is with the Lord as a thousand years, and a thousand years as a day. No wonder Lucretius could say that, however many years you may prolong your life, you can not diminish by a single jot the length of time you will be dead. Without knowing it, these men were thinking in terms of denumerable infinitudes. They did not indeed understand the matter scientifically, but they felt it, and in their utterance is the throb of its mighty power. I wish now to present what is—if one will but ponder it till it is clearly seen in the light of meditation—perhaps the most impressive, certainly the most astonishing, known example of the type of infinity here in question. Consider the totality of rational fractions, of those fractions, that is, whose terms (numerators and denominators) are integers, or whole numbers. Take any two integers, say 3 and 4, and reflect a little upon the multitude of fractions that lie *between*, being greater, that is, than 3 and less than 4. Take any two of these; between them there is another and another; between these, another and another; and so on *forever*. How thickly they are crowded together! More numerous than the sands of the sea, than the drops in the ocean, for these sands or drops, if arranged in a row, would not go on forever. In the interval between *any* two consecutive integers stands, then, a countless crowd of fractions. Do but reflect and reflect again upon the amazing multitude: an infinite host in each interval of an infinite host of intervals. Surely we have here—have we not?—

in this infinity of infinities of rational fractions an overmatch for the mere ensemble (W) of integers. Undoubtedly it seems so. But it is seeming only: the appearance deceives. The imposing array of all the fractions, as soon we shall see, belongs to the denumerable type of infinitude. Nay, we may even throw all the integers and all the rational fractions together, and then show that the new multitude is, in respect of multiplicity, perfectly matched by the array of integers alone, notwithstanding these seem in comparison so few and scarce. Let us prove this astounding fact, for it is but a sample—a model, if you please, or pattern—of surprising relationships literally saturating the subject-matter of theology and there awaiting disclosure for the enlightenment and edification of man. The argument is easy to follow. Take a fraction at random, say $2/3$; note that the sum of its terms—its term-sum—is an integer, in this case 5; note that there are other fractions having the same term-sum; arranged in the order of increasing numerators, they are: $1/4$, $2/3$, $3/2$, $4/1$. Any other integer will similarly give rise to a set of fractions, which we may arrange in similar order. Thus the term-sum 2 gives (a): $1/1$. The term-sum 3 gives (b): $1/2$, $2/1$. The term-sum 4 gives (c): $1/3$, $2/2$, $3/1$. The term-sum 5 yields (d): $1/4$, $2/3$, $3/2$, $4/1$. The term-sum 6 furnishes (e): $1/5$, $2/4$, $3/3$, $4/2$, $5/1$. And so on forever. Observe that this procedure is one that sooner or later presents us with any fraction whatever that we may designate. Whole numbers appear among the fractions, as $4/2$ or $4/1$, for example, and a same integer is repeated, as $2/1$ and $4/2$, for example; and a same fraction appears repeatedly, as $1/2$, $2/4$, for example. We agree, however, to take each but once in the matching process now to follow. Bear in mind what we are to show: it is that the integers of (W), taken alone, perfectly match, in one-to-one fashion, all the rational fractions and all the integers taken together. The correlation proceeds as follows: pair 1 of (W) with $1/1$ of (a); next pair 2 and 3 of (W) respectively with $1/2$ and $2/1$ of (b); next pair 4 and 5 of (W) respectively with $1/3$ and $3/1$ of (c), omitting $2/2$ as a repetition of $1/1$ already paired; and so on and on. We thus get the following scheme of one-to-one association:

1, $1/1$; 2, $1/2$, 3, $2/1$; 4, $1/3$, 5, $3/1$; 6, $1/4$, 7, $2/3$, 8, $3/2$, 9, $4/1$; . . . ; . . . . . .

Observe that the law of procedure matches each integer of (W) with some definite fraction or whole number, and each fraction or whole number in the grand totality of fractions and whole numbers with a definite integer of (W). The result, then, is this: that grand totality, embracing, as we saw, an infinitude of infinitudes of things, so far surpassing, it seemed, in elemental wealth the manifold of integers, is nevertheless perfectly matched by it in that regard, owns precisely the same *Mächtigkeit*, and is thus only an exceptionally impressive member of the great family or type of Denumerable Infinity.

On discovering results so astonishing as the one I have just now presented, it is little wonder that mathematical students of the subject suspected for a time that possibly all thinkable or discoverable infinitudes would be found upon examination to be of one family, of one and the same type—the denumerable type—, however much one infinity might *seem* to surpass another in wealth of elements. But the suspicion was short-lived: it was soon discovered that everywhere round about us there are innumerable infinitudes of higher type—infinitudes, that is, such that you can no more exhaust the wealth of one of them by removing from it a denumerable infinity of its elements than you can exhaust a denumerable infinitude by taking from it a finite collection however large. Indeed it is now well known that the denumerable type is the *lowest* type of infinite manifolds and that above it, as I have said, there rise in the world of thought an endless scale of types. I regret the necessity in this connection of having to request the reader, if he be not a mathematician, to accept a few mathematical facts or propositions on the authority of my report, for to prove all of them here would both expand this volume beyond desirable limits and include in it argumentation of a kind too technical for the general reader, whatever his abilities or attainments in other fields. The reader knows that besides the rational numbers, above considered, there exist what we call, somewhat unhappily but for good historical reasons, *irrational* numbers. He knows that these are such as $\sqrt{2}$, the number denoted by π—ratio of the circumference to the diameter of a circle—, the number denoted by e—base of the Naperian system of logarithms—and many others equally familiar. He probably does not know—what is nevertheless true—that the irrationals are, unlike the rationals, not denumerable; they are too numerous for that—a fact that mathematicians have rigorously demonstrated; the irrationals constitute an infinite manifold of higher rank or type than the denumerable type; if from the totality of irrationals we take away a denumerable infinitude of them there will always remain infinitely more irrationals than we have taken away. Nay, this will be true if we take away, not merely one denumerable infinitude of them, but another such, then another, and so on endlessly, thus removing a denumerable infinitude of denumerable infinitudes! The original ensemble remains absolutely undiminished—its wealth of elements, its *Mächtigkeit*, or "power", its dignity, its rank or type, is the same as before the great removal or decimation. This wonderful type of infinitude has, like the other type considered, a fine name: it is called the Type of the Continuum; it is so called because the totality of points in a continuous line of any length, however short, is a familiar example of an infinite manifold belonging to the type in question. "What *is* it that you say?" may interject a reader. "Do you mean to tell me the ensemble of points in a little short line or the ensemble of instants in a little stretch of time would not be exhausted if we could take away from it a denumerable infinitude of points in the one case or of instants in the other?" The answer is, I do: such a taking away would not only not *exhaust* the ensemble but it would not even *diminish* its wealth of elements—no, not if the great subtraction were repeated a billion times in each second in an endless succession of seconds! Assuming that this is mathematically sound, which it is, is it unreasonable to say, as I do say, that our amiable theological friends and guides, in preparing to instruct regarding the theological Infinite, which is over and above all other infinitudes, would do well to gain some insight into the wonders of those that are below? Is it unreasonable to contend that a course of lectures in this matter ought to be regularly provided in our theological seminaries? Personally I have no doubt at all that a competent student of theology could be, not only much informed, but thrilled, joyed, and inspired, by the marvels of insight and perspective that such a course could open to his gaze. That, however, by the way. Members of the great family of the Continuum type of infinity are omnipresent in our world. One of them is the manifold of rational and irrational numbers taken together; another is the collection of instants in a second or a thousand years; another is the ensemble of points in a sphere or in the universe of space; another is the ensemble of angles among the lines of a sheaf or the ensemble of the lines themselves; another is the pencil of planes having a line in common, or the collection of spheres centered at a point, or the totality of relations between points of time and states of the world, or the aggregate of possible motions, or the group of possible poses of a tiger or a statue, or the multitude of simple equations showing how two variables may change together, or the multitude of spaces like ours that coexist in a space of higher dimensionality; and so on and on endlessly and forever. Is the Continuum type the *next* above the Denumerable type? Probably so but no one knows. Whoever answers the question will thereby immortalize his name.

Shall we proceed to infinite types that are superior to that of the Continuum in respect of dignity or rank? It were possible so to do but the way is steep. I fear to weary the reader by too much demonstration of what he is now, I venture to hope, prepared to assume. For as we go up from level to level in the ever ascending scale of more and more embracing infinitudes, the thought becomes abstracter and abstracter, the heights dizzier and dizzier. At least for the present we have perhaps climbed enough. At another time, when our lungs have become accustomed to the tenuous air, the ascent may be resumed. Here and now it is sufficient to know that the hierarchy exists, that each rank includes all ranks below it, and that, taken together, these ranks above ranks of Infinitude, amenable to the ways of Reason, constitute, as we have said, an endless ladder, an ever rising scale, along which the subtle process of Idealization, with the velocity of spirit, proceeds upward forever, attaining never a summit, for there is no summit, but intimating, indicating, and ever approximating, an outlying Limit, supernal, above all ranks, embracing all, reflecting the glory of all—the Infinite of theology.

The foregoing sketch has, I trust, made it fairly clear in a *general* way that in the study of the infinites of science, which are infinite ideas, and not elsewhere, there is a scientific way to the meaning of theology's Infinite, which is, not an infinite idea, but something more supreme—an infinite ideal. In "a general way," I have said, for the considerations adduced have been, in the main, pretty broad and general. I wish now in approaching the end—for this writing must terminate—to descend to particulars and to show by some concrete examples how the study in question can render the service claimed.

As every one knows, the indictment that men of rationalistic temper, including for the most part scientific men, have brought against theology, is not the same as their objection to religion. It is very far from the same. Their indictment of theology charges that theology is not coherent, that it is replete with internal contradictions, that it thus fails to meet the rightful demand of intellect for harmony, and so fails to meet the standard essential alike to science and to art. The indictment is fatal unless the alleged contradictions, familiar to all, can be purged away or else transcended. Broadly speaking they are of two kinds—*foreign* and *domestic*—contradictions, that is, arising from theology's use of assumptions or postulates that, however available elsewhere, are entirely outside theology's proper domain, and contradictions that do not arise from imported postulates but present themselves properly in theology's subject-matter as seen from interior but inadequate or fragmentary points of view. Contradictions of the foreign variety may, I think, be gradually purged away by ridding theology of imported postulates; contradictions of the domestic kind may, I am equally confident, be transcended more and more by seeking viewpoints more and more commanding.

I wish to indicate what appears to me to be the right manner of dealing with these two comprehensive varieties of theological contradiction or difficulty. And first a word respecting the foreign kind. These are like the contradictions that would defeat the ends of justice if, in the trial of a case at law, it were *assumed* and *held throughout* that all witnesses are honest or that none can be mistaken; or like the hopeless confusion that would result to the science of hydraulics, did the student adhere to the postulate, as *universally* valid, that water runs down hill; or like the confusion that would arise in chemistry if the chemist assumed that the rate of chemical reaction depends solely upon the kind, and never upon the amount, of the substances involved; or like the contradictions that would confound the theory of functions if it laid down as a postulate that every continuous function possesses a derivative; or like the contradictions that would stay the progress of geometry did this science assume that all geometric constructions are feasible with ruler and compasses; or, in general, like the entanglements that must always ensue whenever, in any field of thought, we consciously or unconsciously employ one or more postulates that, though valid elsewhere, in a more restricted field, are not valid in the field of our actual operation. What is the remedy? It obviously is to reject the postulates whence the entanglements arise. Theology is, then, confronted with the task of weeding her garden of *alien* postulates. The task is difficult. Theology must ascertain what her postulates are—what assumptions she actually makes—and this is not easy to do, for assumptions are sly—they do not, as a rule, loudly proclaim their arrival or their presence. Moreover, when once they are ascertained, there remains the difficult problem of discrimination: which of them are legitimate, or domestic, and which are illegitimate, or foreign?

Perhaps the most noxious, certainly the most flagrant, of theology's foreign postulates—one that has engendered endless confusion within and brought from without no end of ridicule—is the hoary assumption that, in every subject-matter or field of thought, *the whole is greater than the part.* It is not perhaps strange that this so-called axiom became an article of universal belief in the early stages of human development, when the interests of men were of necessity confined to the concrete things of sense, but it is strange, very strange, that the belief persisted as universal throughout the history of thought despite the fact that the subject-matter of thought is everywhere and continuously vocal with its denial. Except in the case of mathematicians and some philosophers, the proposition is even today universally held to be universally valid. The fact is, however, as we have abundantly seen, that the proposition, instead of being universally true, is generally false. The discovery of this fact—the discovery, about fifty years ago, that, instead of being an essential principle of reason, the proposition merely serves as a principle of classification, as a logical blade, we may say, sundering the universe of thinkable things into two components; the discovery that one of these—the world of *finite* things—is composed of wholes to which the proposition does indeed apply without exception, but that the other component—the world of infinites—is composed of wholes for which, without exception, the proposition is false; the discovery that the latter world, the world of infinite wholes, is *par excellence* the domain of reason, and that, in respect of content, it is immeasurably richer than the world of finite wholes: that discovery I judge to be second in importance, for the future of thought, to no event in the history of mankind. And auspicious for theology will be the day when she really discovers that Discovery, when she really learns that her subject-matter belongs strictly to the world of *infinite* wholes, and accordingly relinquishes, as then she will, the ancient dogma of whole and part as alien to her field.

Let me give an example to illustrate the great emancipation that will ensue. Not long ago in a western city of the United States a great orator, speaking of the dogma that the persons of the Trinity are *each* Almighty and yet together constitute but *one* Almighty, speaking of the doctrine that each of the Persons is equal to the One composed by all of them, evoked general applause from a vast audience by characterizing the venerated creed as "infinitely absurd." Why? Because the speaker and his hearers tacitly assumed that as a matter of course the whole must exceed the part. And why does not theology explain the difficulty? Why does she content herself with avowing that the alleged composition of the Trinity is an "incomprehensible mystery"? Because she, too, makes the same assumption. And yet it is not the dogma but the orator's characterization of it that is "infinitely absurd." Let us see clearly that this is so. It is plain that we have here to do with the structure of infinite manifolds. More than fifty years ago, that profound mathematician, philosopher and theologian, Bernhardt Bolzano, pointed out that "there are points of view from which we perceive in God an infinite multiplicity (*unendliche Vielheit*), and there are no other viewpoints from which we attribute infinity to him." "Ich sage nun," he adds in explanation, "wir nennen Gott unendlich, weil wir ihm Kräfte von mehr als einer Art Zugestehen müssen, die eine unendliche Grösse besitzen. So müssen wir ihm eine Erkenntnisskraft beilegen, die wahre Allwissenschaft ist, also unendliche Menge von Wahrheiten, weil alle ueberhaupt, umfasst, und so weiter." The key-word, as the context shows, is the term *unendliche Menge*, infinite ensemble or multitude or manifold. Now consider, for example, the following infinite manifolds: the totality E of even integers, the totality O of odd ones, the totality F of fractions having integers for their terms. Denote by M the totality of rational numbers. M, you see, is composed of the elements of E and O and F. M contains each of these elements once and only once and contains nothing else. Any child knows that there is an even integer for every odd one, an odd integer for every even one, and so, it is plain, E and O are equally rich in constituents. Recall, as we proved some pages back, that the same is true of E, which we there denoted by (P), and our old friend, the ensemble (W). Hence it is true of O and (W). It is true also of F and (W) for we saw that we could match the integers with the rational fractions. Hence it is true of E and O and F and (W): these are equally rich in elements. But did we not show the like for M and (W)? We did, whence it follows that, in wealth of constituents, E and O and F and M are exactly on a par: they belong to the same type of infinitude. It happens that it is the denumerable type but that fact is not important. What is important is now obvious: it is that we have here *three* infinite manifolds, E, O, F, no two of which have so much as a single element in common, and yet the three together constitute one manifold M exactly equal in wealth of elements to *each* of its infinite components. Have we proved that there is a Trinity composed of three components related to one another and to the Trinity as the dogma asserts? No. We have proved that the *conception* of such a Trinity, instead of being rendered absurd by a so-called axiom having no application to infinite manifolds, is rigorously thinkable, perfectly possible and rational, and that our brilliant orator was indeed in this instance an ass. Far from being absurd, the conception would be rigorously thinkable—as mathematicians know and as the reader of these pages ought now to see—even if the One it contemplates were asserted to be, instead of a trinity of persons, a multiplicity of order four or a million. Nay, an infinite I of even the *lowest* type *always* contains, not merely two or three or a million components each equal to it in plenitude of elements, but an infinity of such components. The like is equally true of the infinites of whatever type in the endless scale of types. Must we suppose the truth to fail in the case of theology's Infinite, the great ideal towards which the others mount forever, ever rising from the level of one sublimity to another yet more sublime? Is the nature of an ideal inferior to that of the ideas it hovers above? Is perfection inferior to approximation?

For another example of the great emancipation that will come to theology the moment she casts off the yoke of the 'whole-part axiom' that has hitherto hampered the proper movement of her thought, witness the possibility of handling anew and radically the question of Omniscience in relation to that of Freedom. I purpose to treat here briefly a single phase of the matter, a central difficulty of it, familiar to every one. If, so the dialectic runs, God is omniscient, he knows what I shall do, he knows my future, he knows, before I make decisions, what they will be, and if he knows that, then to trust the feeling that I am free to choose is "to cheat the eye with blear illusion." On the other hand, if God does not know all future events, he is not omniscient and the supreme *dignity* ascribed to him is thereby impaired. The argument is specious but, as we are going to see, it is false. The problem is to reconcile, not Freedom and Omniscience, but Freedom and the Dignity of omniscience. Let it be granted that, if you are free, God is not omniscient. It does not follow that he is less in respect of dignity than if he were omniscient. Suppose two Beings: one of them capable at once of knowing all and of not knowing all; the other one capable of knowing all but incapable of not knowing all. Are they coequal in respect of dignity? No, you will probably say, the latter one is, in respect of dignity, distinctly inferior to the former. If that be your answer, I shall agree. I do not, however, intend to depend here upon such intuitive estimates of worth. I purpose to prove that a Being of infinite knowledge may have all the Dignity of Omniscience without being omniscient. To do so, we must again have recourse to the nature of infinite manifolds. Instead, however, of employing, as I might, any of those hitherto presented, I shall ask you to consider a more shining one, one that appeals to the imagination like the open sky.

Let Π be an entire plane; it bisects the universe of Space. I must ask the reader to assume—for it is true and might easily be shown did space allow—that a one-to-one correspondence, of the kind with which he is now familiar, can be established between the totality T of points in space, those of Π included, and the totality S of points on either side of Π. Note carefully that, as Π is *any* plane, the correspondence will be equally possible, if Π be moved parallel to itself any finite distance. Now suppose each point of the infinite totality T to *represent* an *element* e of knowable reality, and denote by d the *element* of spiritual *dignity* that attaches to knowledge of e. At once we see that a knowledge K_s extending to all and only the elements e of the *part*-totality S_e of knowable reality represented by the points of S is precisely as rich in elements d of scientific or spiritual *dignity* as is a knowledge K_t extending to all the elements e of the *whole*-totality T_e of knowable reality represented by the points of T. Now suppose that

T_e is the *whole* of knowable reality, then K_t is omniscience. We behold the astounding fact that omniscience does not by even the smallest mite surpass in *dignity* the partial knowledge K_s. But how, one may ask, does this fact advance the solution of our problem? How does it enable us to maintain the doctrine of Freedom and still attribute to God a dignity of knowledge equal to the Dignity of omniscience? For, our interlocutor will say, knowledge is related to Time, it is of things that have been or are or will be; omniscience must cover them all, it must extend at once through Past, Present and Future; whilst Freedom means that you and I are capable of determining what the Past is to be by choosing in the Present, for actualization, from among the possibilities that constantly descend upon us out of Time-to-come like in-rolling waves of an infinite sea. But, our critic will urge, such capability does not exist if omniscience cover the Future and if, accordingly, the destiny of possibilities is determined before they arrive. And what, he will say, is to be said of the Dignity of knowledge that, though covering the Past, does not extend to all events that are yet to be? In answer let me ask the reader to change a little the imagery employed in our previous argument: let us suppose that Π is, not as before an ordinary plane bisecting Space, but what we may call a moving Time-plane—the Present—bounding off the Future from the Past. Behind Π is an eternity of time that has been; before it, an eternity of time that will be. The two eternities, regarded as manifolds of the things they contain, are infinitudes of the same type, and—what is important to note—they are, as infinites, each of the same type as the one Eternity that together they constitute. In respect, then, of Dignity of knowledge, complete knowledge of the eternal Past is not inferior to knowledge extending both backward and forward, covering the composite Eternity of both Future and Past. It is important to observe that the proposition continues to be true as the Time-plane Π—advancing forefront of Universal History—with infinite range and sweep of wing moves continuously forward; for, though the Past, as we say, thus grows continuously longer and longer, and the Future shorter and shorter, yet the two eternities keep forever their common membership in the type of infinity to which they belong. And so it appears that Freedom is entirely compatible with the Dignity of omniscience, though it is not compatible with Omniscience itself. I fancy that many a spiritual-minded defender of the doctrine of Freedom would find it no great hardship to give up that of Omniscience, seeing that the sacrifice does not involve denying to God the Dignity of omniscience. Such a defender could say: 'I maintain that, to the Supreme Intelligence, the Past alone is *completely* known; I maintain that the Future is not completely known; I maintain that, as the Present moves on continuously forward into the realm of potentialities, the eligible gets sifted, becoming in part the chosen, that part of the possible and unknown becomes the actual and known; I maintain that meanwhile the infinite Dignity attaching to knowledge of the growing Past remains forever invariant, equal absolutely to the dignity of omniscience itself; and that Freedom remains.' Many will be glad to know that such a dogma, whether true or not, is at all events, thanks to the nature of infinite manifolds, free from internal contradiction and may, therefore, be held without surrendering reason. Unless I am much mistaken, the distinction, herewith mathematically drawn, between the Dignity of omniscience and Omniscience itself, whereby we may affirm the doctrine of Freedom without imputing to God's knowledge a Dignity less than that of knowing all, is fundamental. I leave it to the reader to see, in the light of his own reflection, that a similar distinction is available, if required, in dealing with other attributes—Omnipotence, for example, or Omnipresence—commonly ascribed to Deity. I purpose to deal here with Omnipresence but from another point of view.

Our task is to vindicate the logical possibility of Omnipresence—not by such inadequate analogies as immortal Bruno, for example, ingeniously employed in comparing it to a voice audible at every point of a room—but by considerations bringing it strictly within the category of doctrines rigorously thinkable. Consider a sphere. Let it be so small that, even if it were a brilliantly colored globe, the most powerful microscope could not reveal its presence.

It is to be carefully noted that the following statements regarding it are absolutely independent of its size, and remain true if it be supposed shrunken to any degree of parvitude, however small, so long as it has not vanished utterly. Denote by *s* the totality of points *within* the tiny sphere, and by *S* the ensemble of all the other points of the whole of Space. In the course of recent years and by means within the grasp of the average student a little disciplined in the ways of rigorous thought, it has been demonstrated that there are precisely as many points in *s*, as in *S*, and that the former are joined to the latter in one-to-one fashion by relational rays of correspondence. As such correlation subsists in countless modes, suppose one of them chosen. This done, to any point of *S*, say the center of the sun, corresponds a definite point of *s;* to any other point of *S*, say the center of the moon or the mass-center of the Milky Way, corresponds another definite point of *s;* and so on and on throughout the range of both totalities: no element of either manifold but it has a match or mate in the other and no two of either manifold have a common mate.

Let no one fail to see clearly that in that tiny sphere, too small, mind you, for even microscopic vision, small indeed at will, there nevertheless exist point *configurations* matching perfectly in detail and every respect of inner constitution each and all of the infinitely infinite hosts of point configurations, minute and vast, simple and complex, here, there, and yonder, everywhere throughout the height and depth and length and breadth of Space. We have now only to reflect that the same scheme of representation obtains universally, being valid at once for *all* infinitesimal spheres, and the truth dawns that the Whole really *is* incarnate in every Part—the Emersonian aphorism that "the universe contrives to integrate itself in every smallest particle" being thus completely justified on scientific ground. But this is yet not all. The universe is dynamic, charged throughout with innumerable modes of motion. Each point, however, of any moving thing—an ion of gas, a vibrating fiber of brain—is represented by a corresponding point in *s*, and so within the tiny sphere—indeed in *every* room however small—the whole dynamics of the universe is depicted completely and coenacted by motion of points and transformation of point configurations. There in miniature proceed at once the countless play and interplay of every kind of motion, small and large, simple and complex, the quivering dance of the molecule, the wave and swing of universal æther.

> "Wie Alles sich zum Ganzen webt!
> Eins in dem andern wirkt und lebt!
> Wie Himmelskräfte auf und nieder steigen
> Und sich die goldnen Eimer reichen!
> Mit segenduftenden Schwingen
> Vom Himmel durch die Erde dringen,
> Harmonisch all' das All durchdringen!"

The immense labor to be performed by theology in eradicating from the proper domain of her study the whole-part dogma with its ubiquitous progeny of confusion; and the light, the freedom, and the power that will more and more accrue to her as the work proceeds: these are not the end but are only the beginning of her emancipation. For the whole-part "axiom" is not the sole postulate of the imported kind that troubles her thought. Once she seriously enters upon the search, she will find that there are others. I have already repeatedly pointed out that the subject-matter of her thought—the realm of transfinite reality—presents infinitudes in a hierarchy without a summit. As she passes upward in her study from level to level, she will find that a postulate available at a given elevation may have to be relinquished on passing to a higher rank. For example, nothing can seem more natural or axiomatic than to suppose that, if we have any manifold of elements, these are capable of being arranged in a row, like marbles, so that after *each* there is a *next*—none, that is, between. Nevertheless, as mathematicians have recently ascertained, that seemingly universal possibility is restricted very narrowly. The possibility—let us call it the postulate of Nextness—does indeed hold for all infinite manifolds of the Denumerable type but it fails utterly for every manifold of the Continuum type or of any higher type. The employment of foreign postulates is equally disastrous whether the importation be, as in case of the whole-part postulate, from the realm of the finite, where it is valid, to the realm of the infinite, where it is not, or, as in case of the nextness postulate, from a type of infinitude, in which it applies, to a higher type, in which it does not. It is now, I believe, sufficiently evident that eternal vigilance against the admission of alien assumptions is part of the price theology must pay for freedom, for freedom, that is, from the fatal presence of internal confusion.

Before undertaking to deal with the other variety of contradictions—the kind, I mean, that arise properly, because they arise from domestic or native postulates—, I desire to allude briefly to another mathematical idea, one that is destined, I believe, as the eye becomes more and more adjusted to its light, to be of great service in theology, especially enlarging her conception of the Conceivable, and serving to bring not only the attribute of Omnipresence, with which we are here further concerned, but kindred attributes as well, strictly within the category of intelligible ideals. I refer to the radiant concept of Hyperspace. Only a generation ago this concept was regarded even by mathematicians—most adventurous of men—as visionary and vain. Meanwhile it has advanced so rapidly to commanding position that today its instrumental value is—strange to say—recognized even in "natural" science, by Van't Hoff, for example, in chemistry, and by leading physicists in the kinetic theory of gases. The statement made by Poincaré seven years ago before the International Congress of Mathematicians at Rome is well within conservative limits: "Nous sommes aujourd'hui tellement familiarisés avec cetti notion que nous pouvons en parler, même dans un cours d'université, sans provoquer trop d'étonnement." The fact is that the doctrine of hyperspaces already exists in a copious and rapidly growing literature, flourishes in every scientific language of the world, and in its essential principles has become for mathematicians as orthodox as the multiplication table. Indeed, as Professor Klein has shown, the modern physical theory of Relativity is, in point of structure and form, a species of four-dimensional geometry. My aim here is to indicate how the hyperspace concept enables us to show the *conceivability* of an infinite Being being everywhere present in an infinite region without being contained in it. Anyone who will devote a little time to reflecting upon the infinite wealth of points in, say, a straight line L and upon the infinite wealth of detectible combinations and interrelations subsisting among them, will discover to his astonishment that a linear being or intelligence λ inhabiting L and in its experience strictly confined thereto would have, in its own habitation, all the material necessary for constructing mathematical doctrines matching completely, in diversity and in complexity, all branches of geometry and analysis constructible by man, despite the immensely superior resources the latter *seems* to have in inhabiting the three-way spread of Space. Marvelous as it seems, the parity exists. Such a being λ, dwelling in the midst of such magnificence of subject-matter, order, and law, naturally might attempt to construct a rational theology. If so, it would encounter, among other difficulties, that of understanding how the supreme being it hypothetized could be at one and the same time present everywhere in the line-world L. Note that, by hypothesis, λ could have no sense-perception or geometric intuition or image of the fact that the infinite region or line-world L, in which it lives, moves and has its being, is, as we humans happen to know, itself contained or immersed in another infinite region of higher order, namely, a plane *Π;* hence λ could not *per*ceive, though it might *feel*, and it might indeed *con*ceive, the fact that the infinite, *Π*, is actually omnipresent to *L*, every part of this line-world being, as *we* know, completely immersed in *Π;* and so λ could not *per*ceive, yet after some centuries of theologizing it might *con*ceive, how the same attribute—omnipresence in the line-world *L*—could belong to a being whose reality, whatever its nature in other respects, was at least co-extensive with the higher world *Π*. Who can fail to see that precisely like reflections would be equally pertinent, if we replaced the line-world *L* by the plane-world *Π* and the latter by Space itself? *We* live in Space—a three-way spread—and encounter precisely the same difficulties encountered by our linear friend λ, and they are surmountable in the same way, namely, by the concept of Hyperspace. For this world-creating idea, at once exquisite and vast, presents us in the first place with a four-way spread, a four-dimensional space, S_4, completely immersing our ordinary space, being in contact with all its points and present at all of them, just as our ordinary space is omnipresent to all the elements of the plane-world *H*, and this, in turn, to all those of *L;* next, similarly related to S_4, comes a yet higher world S_5; then follow, in order of ascending dimensionality, S_6, S_7, . . ., S_n, . . . and so on *endlessly:* affording thus conceptual provision for the presence everywhere in our dwelling-place of a Being whose reality, if you please, not only pervades but infinitely transcends any assignable space, however high its rank in the summitless scale of hyperspatial grandeur. Is it a small service to show that theology's supreme *ideals* conform to patterns woven of scientific *ideas?* Is it a little thing to demonstrate the reasonableness of reason's dreams?

Finally, I come now to the keeping of my promise regarding theological difficulties of the domestic kind. These are not due to the lurking presence of alien postulates, and are not to be overcome by the process of casting out. They are due to the peculiarly vast and complicate character of theology's subject-matter, to the great diversity of aspects presented by it, and the consequent necessity of examining or beholding them from a corresponding variety of partial or fragmentary points of view. Such native difficulties are to be conquered, progressively of course, not by elimination, but by the method of surmounting, by the process of transcending. What does this method consist in? What does such transcending mean? It does not mean, as commonly supposed, the finding of a point of view from which the difference of two aspects of a matter shall, as this is seen from other points of view, seem to disappear, for that would be, not to clarify, but to obscure, to disguise fact, to hide truth. Transcending does not mean that. It means—and the answer is very important—recognition of the fact that two differing aspects of a matter are indeed, not one, but *two*, and that the matter is, *in truth*, such as to present them both. It thus means submission of the understanding to facts, not facts to the understanding, and, in discourse, to speak of a matter as it is and not as we may wish it to be. Doubtless the aim of science is art but the beauty it seeks does not lie in the way of disguisings or mutilations, for it is the beauty of truth.

Before presenting concrete illustrations may I outline the matter briefly in abstract? Denote by *B* some being, some complex and multi-phased entity, the subject or object of thought. In view of some aspect of *B* we construct a theory T_1, which, as we are not aware of other aspects, we call a theory, not of a phase of *B*, but of *B* itself. Some other aspect of *B*, seen at another time by us or at the same time by some one else, gives rise to another theory T_2, which, like T_1 and owing to the same circumstance, claims to be a theory of *B*; and so on, for other phases of *B*. Let us suppose that the theories have been soundly made after the manner of autonomous doctrines. T_1, then, consists of a definite basal system of compatible postulates together with a superstructure of rigorously deduced implications. Of T_2, we must say the same. Each of the theories is, accordingly, thoroughly coherent, absolutely void of inconsistency among its component elements. They do not, however, coincide: though having perhaps many propositions in common, yet either *T* contains at least one proposition that contradicts some proposition of the other. Let us suppose, moreover, that each theory is true to the aspect that gave it birth: that is, seen from one point of view, *B* appears exactly as T_1 describes it; from another, exactly as T_2 describes it; and so on, of course, if there be other theories. What happens? This: sooner or later, in one or another of the ways familiar to students, T_1 and T_2 get compared; it is noted that each of them claims to be a true theory or account of *B;* it is observed also that in one or more respects they are mutually contradictory. What follows? It follows that the claim must be disallowed in the case of at least one of them: regarded as accounts of one object or subject, two discordant doctrines may be both of them false but they can not both be true. But we have seen that each of them is true to the *B-aspect* that gave it birth; yet they contradict one another. What is to be done? Reject them both? No. The remedy is: Keep both and transcend them. Keep them both, for the contradiction arises from supposing them to be speaking of *B* as a whole, which they are really not; it disappears when we suppose them to be speaking respectively of different phases of *B*, which they really are. The act or process of surmounting consists—not in constructing one theory to cover at once both the aspect covered by T_1 and that covered by T_2, for that is impossible—the nearest possible approach to it would be to construct a theory covering the *common part* (if any) of the two aspects, and plainly such a theory would consist of the common part, or intersection, of T_1 and T_2: no, the surmounting or transcending of T_1 and T_2 consists in recognizing once for all that the object B does in fact present the *two* aspects in question and thereby validates at once *both* of the theories in question. Do you ask what is thus gained? I answer that the proposition stating the recognition is new; it is not in T_1 nor in T_2; it is not a fact about either of the phases dealt with by T_1 and T_2; we have mounted higher—the new truth is a truth about B itself.

Is the matter so clear in the abstract as not to be impressive? I sincerely hope that it is, for in that case it will not require many concrete illustrations and of these, moreover, the simplest will suffice. I will begin with one so simple as to seem trivial. Yet its illustrative value is, I believe, very considerable, unless our familiarity with the phenomena involved, blinds us to their worth. On my table lies a slender rod. As seen there, it appears to be *straight.* I place it at a slant in a vessel of water. As seen there, it appears to be *bent.* Is the rod straight or bent? That is not the question. If it were, we should have to invoke the testimony of at least another sense, which, however, for the purpose of the illustration, I exclude. I am admitting vision only. To vision, then, the rod presents two contradictory aspects—now straight, now bent. Are they, as aspects, false? Is either, as an aspect, false? Neither, as an aspect, is false: as aspects, both are true, both are genuine, both actual. How surmount them? The answer is by recognizing that the rod is *such a* thing in our world that it does, in truth, present to vision both aspects—and that recognition is a valuable event because it tells a truth about the rod, about our world, and about our vision.

For another lean but helpful illustration, consider the quadratic expression, $x^2 — 4$. If you write, $x^2 — 4 = 0$, then I can affirm that $x = 2$ *or* that $x = -2$ but you are right in not allowing me to say that $x =$ both 2 and -2 at once. Everyone knows, however, how to transcend the seeming necessity of the *alternation:* $x = 2$ *or* $x = -2$. We do it, that is, by saying that the given equation is a thing of which 2 *and* -2 are, at the same time, roots. Is such surmounting merely a trick? On the contrary it is a legitimate procedure of thought: the taking of *both* of two things when *either* is allowed, or taking *all* of many when any is allowed: it is the familiar bound of the spirit from alternation to conjunction or more often from the level of partial dissonance to the bridge of an overarching harmony.

A much more impressive example of such surmounting is found in the manner in which geometricians deal with the infinitely distant region of space. There are, as the reader may know, various kinds of geometry of space. In one of these the infinite region of space presents one aspect; in a second, a second aspect; in a third, a third; and so on indefinitely. These various aspects differ among themselves immensely—they are even inconsistent with one another or mutually contradictory and exclusive. Thus, in what is called projective geometry, alluded to previously herein, the aspect presented by the infinite region of space is that of a *plane* all of whose points are infinitely far away; in what is called inversion geometry, which I need not here explain, the same infinite region appears to be a *point* where, curiously enough, all lines of space seem to meet and pass. What do geometricians do in the matter of such conflicts, at first so shocking? Do they reject the aspects as false because they are mutually incompatible? Far from rejecting *any* of them, they keep them all, use them all, rejoice in them all, and—transcend them all. But how transcend? Again the answer is—and how replete with significance for theology!—the answer is that geometricians simply recognize that the infinitely distant portion of space is, whether one likes it or not (and geometricians do like it), in its own nature just such a thing as to present in fact all the diverse aspects in question, and so to validate—*at once*, mind you—all the geometries in question.

Similar matter presses from every side; but enough has been said, I trust, to indicate the method that mathematics would recommend for dealing with theological difficulties of the domestic or native kind. As theology proceeds with her great enterprise of advancing the science of Idealization, as in particular she continues to clarify and estimate the significance of the supernal ideals that she ascribes as attributes to Deity, she is destined to discover that those attributes, however indubitable or undeniable they may be when regarded singly, yet, taken together, involve essential and ineradicable incompatibilities of thought, and, therefore, must finally defeat every possible effort to combine them in *one* self-consistent body of doctrine. The question is, What is to be done in that event? Answering out of the fullness of her own experience in such cases, Mathesis will venture to offer her sister the following counsel. "My years and station," she will say to Theology, "and the character of my occupation entitle me to believe that I am not without some insight into the nature of your gravest difficulty and not without some knowledge of the means available for overcoming it. *Usus, magister egregius, hoc me docuit.* I, too, in the course of my long career have expended, I do not say have wasted, much time and energy in attempting to combine the non-combinable, in attempting, that is, to erect a solid and unitary doctrine respecting some object of my thought upon a basis of postulates that were indeed individually sound and eligible, but that, taken collectively as a system, were subsequently found to involve logical incompatibility and so not to allow any superstructure not doomed to quick decay by the presence within it of fatal contradictions. Fortunately, I have not besought or trusted any hyperlogical providence to preserve such architecture against external criticism or the destructive agency of its own defects, but have had the grace to tear it down myself and prepare to build anew. My practice has been to examine again and patiently to reexamine the basal postulates, to form from them by trial and experiment as many subgroups as possible, subject to the condition that each of these be entirely free of interior inconsistence, and then, upon the *sub*groups as distinct though related foundations, to construct as many distinct but kindred doctrines, each of strength to mock at time and endure for aye. And my practice, as you and all the world may know, has been justified of its fruits. Examples abound in every division of my commonwealth, and some have come to fame. To cite but three of these—behold the noble structures of Euclid, of Bolyai and Lobatschevski, and of Riemann. There stand the great geometries, each upon its own foundation of compatible postulates, and there, flawless within, unassailable from without, they will stand for ever, eternal witnesses of the fact that, contrary to many a venerated but shallow creed, *one* object of thought may, by virtue of its kind and not of limitations of the human mind, transcend the bounds of any one constructible theory, and in its own ultimate nature allow and validate at once, without annulling their differences, a *class* of dissonant doctrines. Thus you perceive, for example, that my Geometry is one, though my geometries are many—just as Music is one, though its forms be as varied as the moods of the sea. And I, Mathesis, am one, as Poetry is one, though my theories, my doctrines, are legion; for these but differ among themselves, as the myriad forms of Art: each is assertable, each being valid, of one great Form common to them all. My meaning, I trust, is clear. Conquest of your gravest difficulty demands division. By the method of trial and experiment, the fundamental attributes that you hypothetise of Deity must be assorted into sets each composed of harmonious elements. Implicit in each such group is a coherent and sacred doctrine. As these doctrines unfold, your conception of yourself will change: you, Theology, will indeed be one; but many your theologies. And thenceforth the Object of all your thought will appear to you and will be shown by you to the world, not in the light of a solitary sun, but in that of a constellation."