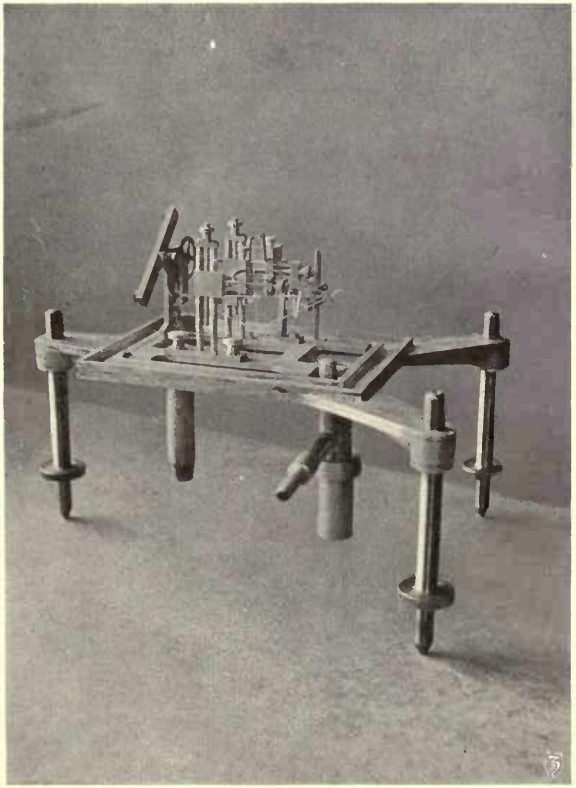
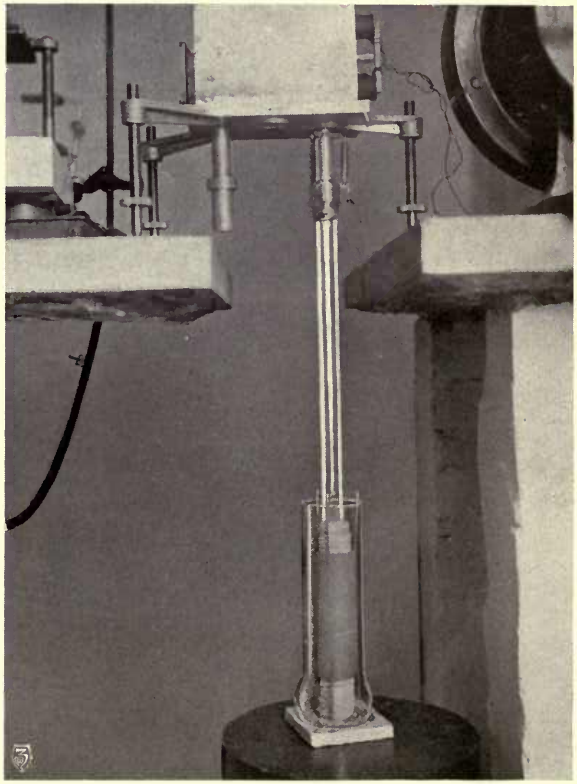
A NEW MICRO-BALANCE AND ITS USE.

BY

HANS PETTERSSON



The new case without its cover.



Weighing at 1000° abs.

A NEW MICRO-BALANCE AND ITS USE.

BY

HANS PETTERSSON

LIC. PHIL.

INAUGURAL DISSERTATION

BY DUE PERMISSION OF

THE PHILOSOPHICAL FACULTY OF THE UNIVERSITY OF STOCKHOLM

TO BE PUBLICLY DISCUSSED IN LECTURE ROOM APRIL TH 1914
AT O'CLOCK A. M.

FOR THE DEGREE OF DOCTOR OF PHILOSOPHY.

GÖTEBORG 1914
WALD. ZACHRISSONS BOKTRYCKERI A.-B.

Ur: Göteborgs Kungl. Vetenskaps- och Vitterhetssamhälles
Handlingar. Fjärde följden. XVI.

CONTENTS.

I. Introductory.

Most weighing balances can be classed either as elastic balances or as lever balances. On instruments of the first type the load is suspended from an elastic structure of some kind, generally a spring coil, which becomes deformed until the elastic forces caused by the deformation counteract the downward pull of the load. The vertical displacement of the latter is read on a scale, and the instrument is standardized by producing different scale-deflections with a set of weights.

On instruments of the second type the load is suspended from one end of a rigid structure, the beam, which can rotate round a horizontal axis, generally a knife-edge. The deflection of the beam from its normal position is read on a scale. In order to keep that deflection within certain limits the load is in most cases counterbalanced by suspending from the other end of the beam a counterpoise, which is either made up from weights or is afterwards, in its turn, counterbalanced by exchanging the load for an equivalent number of weights.

On instruments of the first type an automatic compensation is produced by the elastic forces, whereas the construction of most lever balances aims at reducing all such compensatory effects to a minimum.

The absolute sensibility, S, of a balance we define as the additional load, dW, which produces an observable deflection, da, of the beam. $S = dW$. According to this definition a high sensibility corresponds to a low numerical value of S and *vice versa*.

The sensibility may evidently be increased by reading the deflections more accurately on the scale used for that purpose. It is, however, useless to carry this increase beyond a certain limit, the «instrumental limit», which is reached when da becomes of the same

magnitude as the irregular and uncontrollable variations of the position of the beam that are due to constructional defects.

The absolute sensibility divided by the load is called the relative sensibility; $S_r = S : W$

While the technique of most other physical measurements has been brought to a high degree of perfection during the last decades, it is only in the last few years that any considerable progress has been made with regard to the absolute accuracy of weighings. Still, a considerable number of attempts to that end were made by renowned men of science. A short description of the more important balances which have thus been constructed will be given in the following pages.

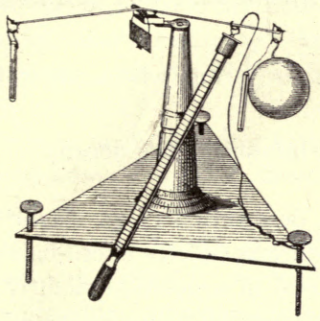
In 1886 Warburg and Ihmori [4] made a sensitive lever balance, figure 1 a, which they used to study the formation of thin films of condensed water-vapour on the surface of solids. The beam, a thin-walled glass tube, slightly bent at the middle, was balanced on a minute knife-edge made from a fragment of a razor. The scale pans were suspended from similar knife-edges, one at each end of the beam. The balance was mounted under the receiver of an ordinary air-pump and was worked in a vacuum. Read with mirror and scale (at 272 cm) its sensibility was $\frac{1}{300}$ mg. with a load of 600 mg.

Ihmori [5] afterwards improved the instrument in several respects and brought its sensibility up to $\frac{1}{2000}$ mg. with a load of 500 mg. Balances of this type are difficult to construct and to work, and they have therefore not been used by later experimenters.

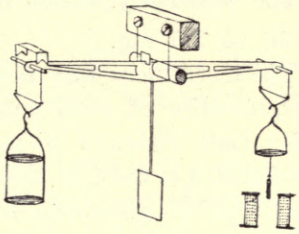
K. Ångström [6] made a balance in 1895 which gave quite good results in spite of its simple construction. The very light beam, figure 1 b, was made from wood, carefully varnished. Instead of being balanced on a knife-edge it was suspended by two silk fibres wound half round a central cylindrical axis, which rolled on the fibres when the beam was swinging. The weight-pans were suspended in the same way. With a load of 1 gr. the sensibility was of the order $\frac{1}{1000}$ mg. Ångström used the instrument for comparing and standardizing light weights.

A balance suspended in the same way was described by G. E. Weber in 1841 [1], a fact of which Ångström was not aware until after his own balance had been completed.

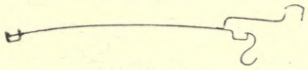
Fig. 1.



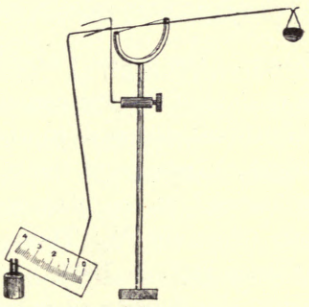
a (1 : 2)
Warburg & Ihmori.



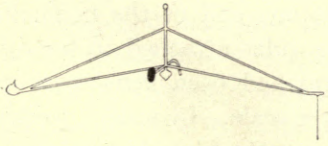
b (1 : 3)
Ångström.



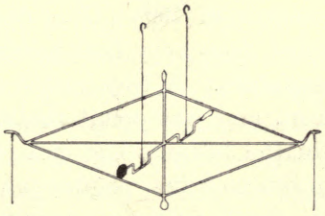
c (3 : 8)
Salvioni.



d (2 : 9)
Nernst.



e (2 : 5)
Steele & Grant.



f (2 : 5)
Pettersson.

Different types of micro-balances.
(a—d reproduced from papers previously cited.)

Guglielmo [7] in 1901 adopted an ingenious device for reducing the weight of the balance beam by immersing it completely in water. It was made from glass tubes filled with air and was balanced on two fine metal points.

The sensibility varied from $\frac{1}{100}$ mg. to $\frac{1}{500}$ mg. with loads weighing 236 to 2.5 mg.

The instrument was used chiefly for measuring the density of minute quantities of solid substances.

In 1901 Salvioni [8] published the description of an elastic balance, figure 1 c, the first instrument which was called a micro-balance. A thin rod of glass was fixed at one end in a position slightly inclined to the horizontal. The load was suspended from the free end of the rod, the downward displacement of which was observed with a reading microscope, a piece of cob-web attached to the rod serving as index. According to the laws of elasticity the displacement should be in direct proportion to the weight of the load, which was varied from 1 mg. to 200 mg. The highest absolute sensibility was of the order $\frac{1}{1000}$ mg. With the aid of this instrument Salvioni [9] found it possible to measure the continual decrease in the weight of a strongly odorous substance like musk. A balance of the same type, but with an elastic strip of silver plate instead of the glass rod, has been described (in 1886) by Lord Kelvin (then Sir W. Thomson) [3].

Giesen [10], in 1903, used a slightly modified Salvioni balance to measure the density of gases and their absorption by charcoal, and also for a repetition of Warburg's and Ihmori's experiments on surface sheets of moisture.

The well-known Nernst balance, figure 1 d, was described by its inventor in 1903 [11]. A thin glass rod, serving as beam, was cemented to a very fine quartz fibre stretched horizontally between two supports. A tiny metal capsule at one end of the beam held the load, and was counterbalanced by a rider attached to the other arm of the beam. The end of that arm had been bent twice and drawn out to a fine pointer moving over a scale. In order to have a very small restoring torque of the quartz fibre it must be extremely thin. For that reason the balance could not be charged with loads heavier than a couple of mg., the sensibility being then about $\frac{1}{500}$ mg. The simple construction of the Nernst balance has caused

it to be widely used in spite of its very limited range. Nernst himself has employed it for a determination of the atomic weights of several rare earths. Later O. Brill [15] has carried out a similar research and has also investigated the dissociation of carbonates at high temperatures with the Nernst balance, after having improved it in certain respects.[1])

It is an interesting fact that an instrument almost identical in construction to the Nernst balance was used for a hygrometer by Hertz [2] in 1882.

So far, the results attained in the course of twenty years had not been very brilliant. It must be admitted, however, that there were considerable technical difficulties to overcome before any further progress could be made.

To find a suitable material for the beam and the weights was the first problem to be solved. As the dimensions of the instrument are reduced, the relative influence of all surface effects, such as the condensation and adsorption of vapours and gases, and the corrosion by the oxygen and the moisture of the air, increases. An ideal substance should therefore be non-hygroscopic and non-corrosive, and must in addition to that have a low density and a high tensile strength.

How to make sufficiently light weights was another most difficult problem. The balances described above were all read by deflection, that is, the exact value of the weight of the load was calculated from the observed scale-reading by interpolation according to a previous standardization of the scale with very light weights. Such an interpolation cannot be made much more accurate than to 0,1 % of the lightest weight employed. Now a weight of less than 0,1 mg. is most troublesome to handle, which gives 0,0001 mg. as the practical limit of accuracy. Again, if such a weight is used as a rider, its position cannot very well be more accurately defined than to a thousandth part of the length of the beam, which gives the same limit as the other method. It would therefore seem to be useless to try to construct balances of a still higher sensibility. This view has been expressed as recently as in 1904 by Scheel [12] in his criticism of Giesen's work with the Salvioni balance. It appears indeed most likely that, but for

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[1]) I have recently found that Brill has since found it possible to increase the sensibility of the balance to $\frac{1}{10000}$ mg. [O. Brill and Miss Evans. Journ. chem. soc. London 93 p. 1442; 1908.]

the inventive genius of two Australian men of science, the aforesaid limit must have remained unsurpassed for many years.

In the Proceedings of the Royal Society of London for 1909 Steele and Grant [16] published a paper on *"Sensitive micro-balances and a new method of weighing minute quantities"*, which opened a new era in the history of weighing instruments.

They found the best possible solution for the problem of the material by making the whole of their balance from fused silica. This substance possesses in a high degree all the desired qualities before mentioned, and in addition to this it presents the advantage of becoming fusible and even slightly volatile at the temperature of the oxy-coalgas flame.

Thin cylindrical silica rods were fused together by means of the blowpipe to a plane structure of considerable rigidity. This beam was balanced on a knife-edge, ground at the end of a short silica rod. A fibre provided with a hook for the load was drawn out from one end of the beam, a lump of silica at the other end serving as counterpoise. A quartz mirror and another counterpoise were fused to opposite sides of the beam near its central axis. The centre of gravity of the beam was adjusted so as to give the balance the desired sensibility simply by drawing off or volatilizing away minute quantities of silica from the lower end of the mirror-counterpoise.

Still more ingenious is the way in which the authors have overcome the difficulty of making sufficiently light weights. The balance was mounted inside a vacuum-tight case and the weighing done by varying the pressure of the enclosed air. This would of course have no effect on the equilibrium of the system if its density were uniform, that is, if the load consisted only of solid silica. There was, however, also a sealed silica bulb of known capacity, v, suspended from the beam. Therefore, when the pressure in the case was reduced from p_1 to p_2, there would be an uncompensated decrease in the buoyancy of the air acting on the bulb, equivalent to an apparent increase, dW, in its weight. It is easy to see that approximately $dW = v \frac{p_1 - p_2}{760} \cdot \delta_t$, where δ_t is the density of the air at normal pressure and at the temperature of the experiment. One might also say that dW is the weight of the air which would have escaped from the bulb, if it had been open when the pressure was reduced.

If the capacity of the bulb is very small, its effective weight can evidently be varied by extremely small amounts, the exact values of which are found by reading the change in pressure on a manometer in communication with the case.

The most delicate balance made in this may, fig. 1 e, was sensitive to $\frac{1}{250000}$ mg. with a load which was probably about 20 mg. The total weight of the balance with mirror, counterpoise, and a bulb of 8,65 mm³., was 177 mg.

With the aid of such balances the authors [17] afterwards made attempts to weigh the active deposit from a small quantity of niton (radium emanation).

The reliability of instruments of this type and their usefulness both for chemical and physical work has received a striking illustration through two investigations by Sir William Ramsay and Dr. Whytlaw-Gray. Operating on aquantity of only 0,1 mm³. of niton they succeeded in measuring its density [18] with a mean error of about one per cent, the result giving a conclusive proof in favour of Rutherford's Disintegration theory. Afterwards they redetermined the atomic weight of radium [20] on a minute quantity (less than 3 mg) of pure Ra Br_2.

The balances used in these investigations were made according to instructions from Steele and Grant, but the technique of their construction and of the weighings was modified in certain respects.

On the plane surface of a block of gas-coal fine grooves were ruled to a figure representing the shape of the beam. Thin silica rods were put into these and their ends fused together with the blowpipe. In this way a perfectly plane structure was secured, free from all after effects due to inner strain, which had sometimes troubled Steele and Grant.

It was also found more convenient to seal a piece cut off from a previously made silica fibre to the end of the beam rather than to draw out the fibre from a *T*-piece of silica fused to the tip of the beam.

The weighings were carried out according to the zero principle, that is, the pressure was adjusted so as to bring the beam back to almost identically the same position.

The most delicate instrument made by Ramsay and Gray had a

sensibility of 2 μmg. ($= 2 \cdot 10^{-6}$ mg) with a load of about 50 mg. The atomic weight of radium was determined on a «rougher» instrument, symmetrical in shape, on which loads of some 70 mg. could be weighed to $\frac{1}{25000}$ mg.

Though balances of this type work perfectly under normal conditions, they have certain drawbacks. The end-fibres, which have to be very thin in order not to offer too great resistance when the beam swings, seem to get very brittle at the points where they are sealed to the beam. For this reason these balances cannot safely be worked with loads heavier than about 0,1 gr.

Fine dust particles will now and then get under the knife-edge and put the balance out of working order until the case is opened and the particle removed.

When I started working at the University College of London in November 1911, Sir William Ramsay suggested to me that I should try to make a new kind of micro-balance, suspended by fine platinum wires instead of the knife-edge, and with the same suspension for the loads. By this arrangement the carrying strength of the instrument would probably be much increased.

In the course of their work with niton [18; 19] Ramsay and Gray had observed that when a tiny gold capsule was heated and suspended from the balance, its weight on cooling continually increased for several hours and even for days. Sir William therefore advised me to use the balance which I was to construct for a closer study of this phenomenon.

It will be seen from the following that the first of the two problems set before me has been solved. A new type of micro-balance suspended by fibres, though not of platinum, has been constructed, which is superior to the knife-edge balances in several respects. The technique of the new instrument has been developed and studied, and its usefulness for certain physical measurements has been demonstrated.

As regards the second problem only preliminary experiments have hitherto been made. This is largely due to the fact that the phenomenon proved to be of too complicated a nature to be studied by the simple methods originally devised.

I wish to express here my sincerest gratidude to Sir William Ramsay for having suggested this research, and for the kind and encouraging interest which he has constantly taken in it.

After working for about ten months at University College, it became necessary for me to go back to Sweden, where I resumed my work at Fysiska Institutet, Stockholms Högskola. I have much pleasure in acknowledging my indebtedness to its director, Professor Carl Benedicks, for having placed the necessary instruments at my disposal, and for much help and advice.

II. The new micro-balance.

§ 1. Construction of the beam.

The shape chosen for the beam (see fig. 1 f) is that of a rhombus with its two intersecting diagonals, a structure which is at the same time light and fairly rigid. The longer, horizontal, diagonal is about 9 cm. in length, the shorter, vertical, diagonal is 4 cm. The diagonals are made to protrude by half a cm. outside the corners of the figure so that each arm of the beam is about 5 cm. long.

The beam was made flat on a piece of gas-coal from silica rods, 0,5 to 1 mm. thick, according to the method of Dr. Whytlaw-Gray, to whom I am much indebted for valuable advice on the technique of the silica work and of the weighings. Short pieces of silica rod were fused to the centre of the beam at right angles to its plane. To these crosspieces the central fibres of suspension were attached.

I first attempted to suspend the beam and its loads by platinum wires of which three different dimensions were tried, viz. wire 0,025 mm. thick, drawn in the ordinary way, and Wollaston wires of 0,012 mm. and of 0,008 mm. The ends of the wires were soldered to the tips of the central crosspieces and of the horizontal diagonal, where the silica had previously been platinized with colloid platinum. The soldering was

carried out under the microscope with the aid of a minute electric soldering apparatus.

Unfortunately the wires got very brittle at the junction, so that they almost invariably snapped when any strain was put on. A considerable number of unsuccessful attempts were made before a beam could be suspended and loaded with all four fibres (0,008 mm) intact. It was, however, not possible to make it more sensitive than to $\frac{1}{1500}$ mg. (scale-distance 0,8 m, loads about 1 gr.), and even at that low sensibility the zero of the instrument was not constant.

This poor result was undoubtedly due to the considerable elastic after-effects which metal wires are known to show.

Nearly three months had been spent on these attempts, and it appeared that I would have to fall back on knife-edge balances of the older type for my investigation. However, before definitely giving up suspension by fibres I decided to try to suspend a beam by fibres of *silica* instead of platinum.

The general shape of the beam, figure 1 *f*, was the same as before. The fibres were drawn out from four short silica rods bent to a hook at one end and fused by the other end to the central crosspiece and to the ends of the beam. These had been bent to the shape shown by the figure in order to make the starting points of all the four fibres fall nearly at the same level as the centre of the beam. Only the first mm's of the fibres next to these points were drawn very thin, some μ in diameter. The theory of the balance, which will be given further on, proves that the thickness of the rest of the fibre is of no importance.

Finally a ground and polished silica mirror, covered with a reflecting layer of palladium by the « spray » method, was sealed to one of the central crosspieces, and a counterpoise of the same weight to the other crosspiece. The balance was then ready to be mounted in its case, loaded, and adjusted to the desired sensibility.

The first instrument of this type which I made possessed a sensibility of $\frac{1}{5000}$ mg. when charged with loads of about 2 grammes each. The period of a complete swing was then some 40 seconds, and the scale, which was at a distance of 1 m., was read to millimetres. After that, a highly sensitive balance was made of the same construction but with thinner fibres. A detailed account of the properties of this instrument will

be given in a succeeding paragraph. After my first two balances had been completed, I was made aware of the fact that an instrument of similar construction, but with elastic strips of metal instead of the fibres, has been described in 1841 by W. Weber in his "*De tribus novis librarum construendarum methodis*" [1]. Quite recently a balance of the Weber type, hanging by two strips of steel, has been used by Piccard [22] for magnetic investigations.

§ 2. The balance cases.[1]

The first case used with the new micro-balance is drawn in figure 2. The floor of the case is a brass plate, 10 mm. thick, which has been carefully tinned over its lower surface. It is carried by three screw legs of brass, of which only one is seen in the figure. The cover is a pneumatic glass trough (9 by 10 by 15 cm.), the edges of which have been ground on the brass plate. When readings are taken the light from a Nernst lamp is sent through a positive lens and then through a kind of window, W, made by cementing two pieces of plate-glass with Canada balsam to the wall of the cover. After a reflection against the balance mirror an image of the incandescent filament is formed across a vertical scale, 4 metres away, which is read to millimetres.

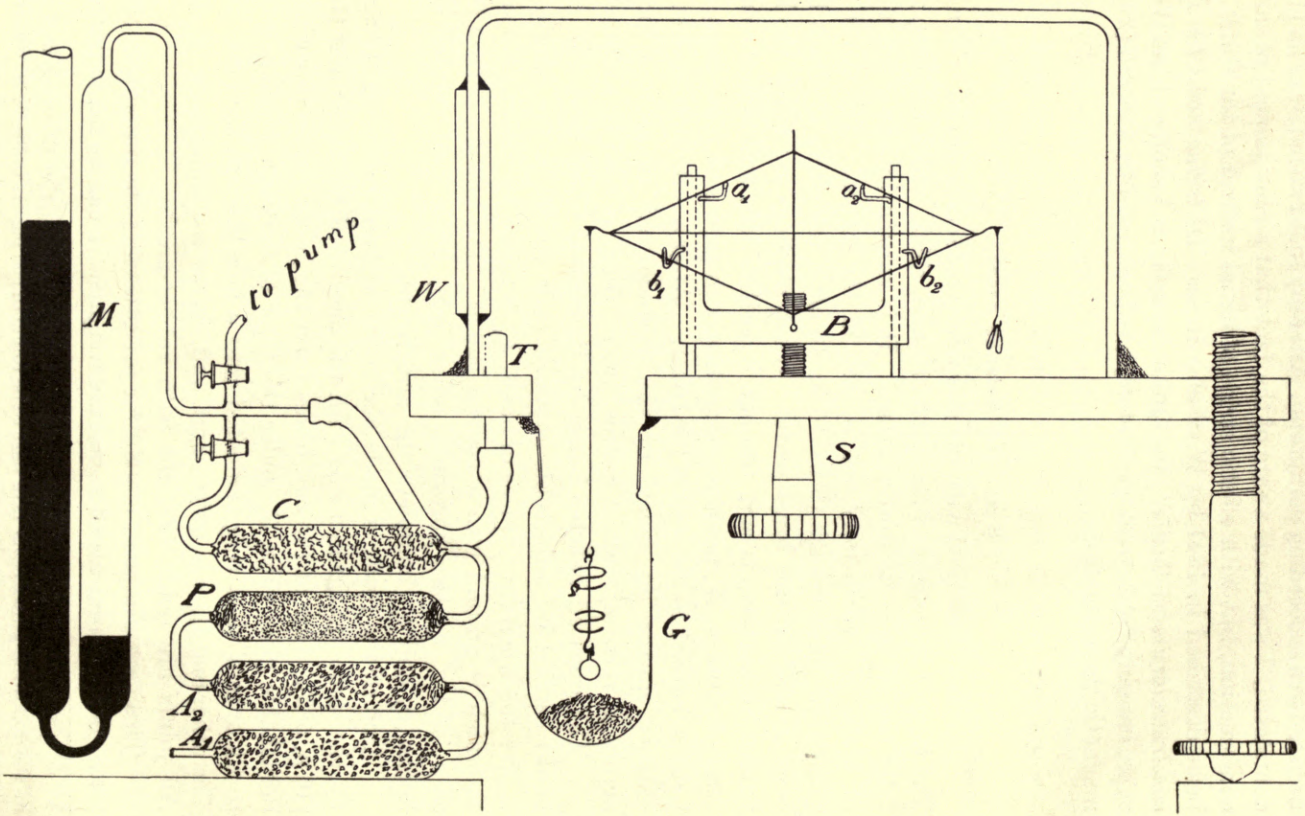
The released balance hangs by its central fibres from two brass hooks carried by adjustable supports (not visible in the figure). It is arrested when the U-piece of brass, B, is raised by turning the screw S, which is ground into a conical jacket. The balance is then lifted at a_1 and a_2 by two horizontal silica rods, while two vertical silica hooks, b_1 and b_2, complete the arrestment.

One of the loads is made accessible without removing the cover, by the same arrangement as with the case used by Steele and Grant [16 p. 582]. One of the end-fibres from the beam comes down into a detachable glass tube, G, under the floor of the case, which is ground to fit over a glass ring cemented into a hole through the brass plate. The other end-fibre is quite short and carries a counterpoise of silica. All joints are made air-tight with rubber-grease.

This case was used chiefly for investigating the properties of the micro-balance and also for making weights. It can be made perfectly

[1]) The description of the technique of the balance and of the weighings given in the following paragraphs is in part to be found in the papers of Steele & Grant and of Ramsay & Gray.

Fig. 2.



The first case.

air-tight, it is comparatively light, and also presents the advantage that the movements of the released balance may be closely observed through the transparent walls of the cover. On the other hand the low thermal conductivity of the cover is a disadvantage, as it is important for the accuracy of the weighings that the air in the case shall be surrounded by an isothermal surface. The case also appears to be rather sensitive to tremors, probably in consequence of its light weight and of the unelastic properties of its supports.

Certain experiments, which will be related further on, had indicated to me, that a considerable number of investigations can only be made with a balance case where it is possible:

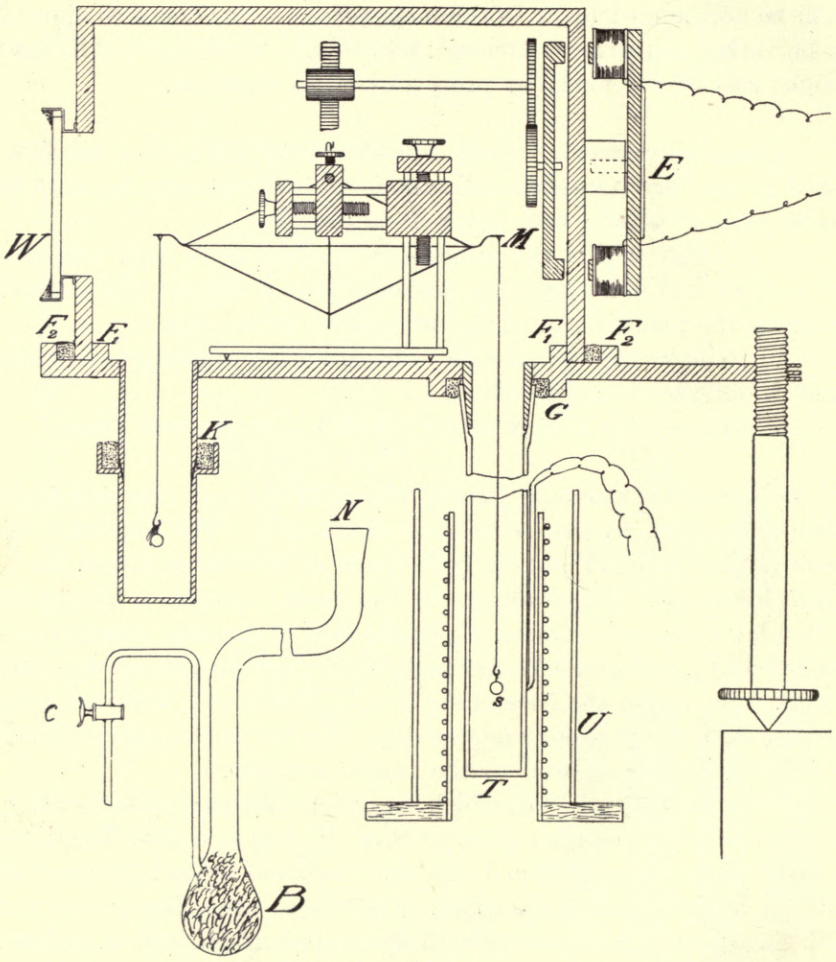
1. to heat one of the weight-tubes suddenly to high temperatures or to cool it in liquid air without risks of leakage and without affecting the temperature of the other parts of the case,
2. to make the case air-tight without having rubber-grease or any kind of volatile cement inside it,
3. to maintain a high vacuum within it for several hours.

In figure 3 is drawn a vertical section of the new case which has been constructed to answer to these demands.[1]) The floor and the cover (10 by 10 by 14 cm) are both made of cast brass, 5 mm. thick, which is carefully tinned all over. The cover is made to fit exactly over a low rectangular frame, F_1, on the floor of the case. Another, wider frame, F_2, runs parallel to the first, leaving a groove, 5 mm. deep and 7 mm. wide, round the base of the cover. The joint is made air-tight by pouring into the groove molten alloy of a low melting point (about $+80°$) which adheres well to the tinned surfaces.

A similar kind of alloy, which has a higher melting point ($+140°$), is used to cement the plate-glass window, W, into its frame. The latter is made from strips of thin brass plate soldered round a rectangular hole cut into the wall of the cover. The edges of the glass are ground with silica powder and covered with layers of platinum, copper, and tin, so as to make the alloy adhere better. This alloy has nearly the same coefficient of thermal dilatation as glass. Its composition was revealed to me (4 p. lead, 4 bismuth, 3 tin, 1 cadmium) by Lic. O. Tenow, to whom my thanks are also due for many other useful hints on practical points.

[1]) I am much indebted to the mechanic of Stockholms Högskola, Mr. C. Andersson, for the excellent way in which he has carried out my designs.

Fig. 3.



The new case.

with charcoal bulb, *B*, and electric furnace, *U*.

One of the weight-tubes, T, is of transparent silica of 15 mm. bore and 50 cm. long. Its open end is conically widened and ground to fit over a tap of «Invar» soldered to the bottom of the case. The outside of the silica cone is platinized, and the tube is cemented to its place with alloy poured into a circular groove, G, round the edge. The other weight-tube is of tinned brass; round the edge of its lower, detachable, part there is an annular cup, K, into which alloy can also be poured.

The support hooks from which the balance is suspended are adjustable in two dimensions by screw motion. The silica rods of the arrestment have the same shape as in the first case and are attached to an H-shaped piece of brass tubing. The latter is moved up or down along two guides by a cog-wheel carried by a horizontal axis. The motion is transmitted to that axis from an anchor-piece of soft iron, M, which can revolve close to the wall of the cover. It is moved by rotating a co-axial electromagnetic key, E, which is mounted outside the cover. The balance is thus arrested and released without any material transmission by the magnetic forces acting through the 5 mm. of brass.

The whole of the arrestment as well as the supports are mounted on a loose framework cut out from a brass plate. Thanks to this arrangement the mounted balance may be lifted out of the case if the latter has to be turned upside down, as is sometimes necessary.

Besides the usual air-inlet for work at ordinary pressures there is a second inlet (not in the plane of the drawing) through which the highest vacua are made A large charcoal bulb of glass provided with a tube, 40 cm. long and 12 mm. wide, was attached in the same way as the silica tube to a second «Invar» tap soldered round a hole through the balance floor. In order to remove the bulb as far as possible from the lower end of the silica tube, the wide glass tube was bent twice, and its neck, N, was ground with silica and covered externally with metals like the edges of the window. The bulb, which is drawn separately in B, also carried another, narrow, tube provided with a mercury stop-cock, C, through which it could be put in communication with an air-pump.

While the bulb was being placed in position it happened to break close to the ground neck. To remove the broken piece, make a new neck, and attach it once more, would have caused considerable delay and involved certain risks to the silica tube, which had already been

fixed in its place. I therefore tried to mend the tube by pressing the pieces firmly together, while alloy was poured into a brass jacket round the broken part. The joint secured in this way was quite firm and considerably increased the strength of the structure, but it could not be expected to keep air-tight. The case was also found to leak badly the first time the air was pumped out of it. After several futile attempts I managed to bring down the leakage to less than 0,1 mm. an hour by covering all the free surfaces of the alloy with a kind of low-volatile vacuum cement.

The means which I could use for evacuating the case were highly effective, thanks to the large capacity of the bulb and the width of the tube leading to it. It therefore appeared probable that, in spite of the leakage, a comparatively high vacuum might be maintained within the case, at least for a few hours while the charcoal was still fresh. I therefore decided to make experiments with the balance case in this slightly imperfect state, and to postpone investigations demanding the highest degree of evacuation until after a reconstruction of the case.

§ 3. Mounting and adjusting the balance.

After the beam has been constructed and the four fibres drawn out from it, it is put on the silica rods of the arrestment, and the silica hooks carried by the central fibres are slipped over the brass hooks of the support. These are adjusted so that the released balance hangs in a vertical position with its end-fibres coming down into the centre of the weight-tubes. If the beam has been correctly made, the strain on each central fibre will be about the same, and the main plane of the beam will keep vertical while the balance swings.

The end-fibres are then charged with the counterpoise and the load, the latter made up from a number of small pieces of solid silica and a bulb of suitable capacity In order to give room for a larger number of weights, the end-fibre should be provided with a kind of double T-piece, the arms of which are bent to a semi-circular shape so as to decrease the danger of touching them with the greased neck of the weight-tube when the latter is put on or taken off. (see fig. 4 a on page 24).

When the beam is at equilibrium with the loads in position, it is adjusted to the desired sensibility, as indicated by the length of the

period of a complete swing. This is done according to the method before described of directing the blowpipe either against the top or the lowest part of the beam and drawing off or sealing on small lumps of silica. The very finest adjustment is attained by volatilizing away a minute quantity of silica with a small and very hot flame.

This operation requires some experience and patience, as it has to be done quite at random. It is easy enough to adjust a balance to a period of 20 to 30 seconds, which corresponds to a moderate sensibility, but the difficulties increase rapidly with the length of the desired period. To raise the period to 80 or 100 seconds and more, is an extremely delicate operation.

§ 4. Weighing by pressure.

Theoretical.

The method of weighing by pressure or by aerostatic compensation seems to have been invented independently by Gwyer [18 p. 538] and by Steele and Grant. The theory of the method is very simple.

Let the balance be at equilibrium at a pressure p', when charged with a load which, like the counterpoise, consists only of solid silica.

The whole suspended system being of uniform density, its equilibrium will obviously not be affected if the pressure of the surrounding air is varied.

Suppose now, that part of the load, weighing w', is exchanged for a sealed silica bulb of the capacity v. Let the silica of the bulb weigh w'', and take the density of the gas it contains to be η, and that of silica d, the density of the air within the case being δ'.

If the equilibrium shall remain unaffected by the exchange we have the condition:

1. $$w' - \frac{w'}{d}\delta' - \left(w'' - \frac{w''}{d}\delta'\right) - v(\eta - \delta') = 0 \quad \text{or}$$

2. $$\frac{w' - w''}{d} = v\frac{\eta - \delta'}{d - \delta'}$$

The change in the total volume of the load produced by the exchange is:

3. $$dV = v - \frac{w' - w''}{d}$$ or, according to equation 2:

4. $$dV = v\left(1 - \frac{\eta - \delta'}{d - \delta'}\right)$$

Now δ' is quite small compared to d, and so is the second term within the brackets compared to unity. We may therefore neglect δ' in its denominator and have:

5. $$dV = v\left(1 - \frac{\eta - \delta'}{d}\right)$$

We now suppose that the pressure is reduced from p' to p'', and the density of the air from δ' to δ''. The change in the buoyancy of the air which surrounds the balance can affect the equilibrium, only if $dV \lessgtr 0$. The change, dW, in the « effective weight » of the bulb must therefore be:

6. $$dW = dV\,(\delta' - \delta'')$$

Assuming Boyle's law to hold exactly for air we have:

$\delta' = \frac{p'}{760}\,\delta_t$; $\delta'' = \frac{p''}{760}\,\delta_t$, where δ_t it the density of air at normal pressure.

From equations 5 and 6 it follows:

7. $$dW = \delta_t \frac{p' - p''}{760}\, v\left(1 - \frac{\eta - \delta'}{d}\right)$$

So far the temperature of the air has been supposed to remain constant. If, on the contrary, it has varied from T' ° abs. in the first case to T'' ° abs. in the second, we find instead of equation 7:

8. $$dW = \delta_o \frac{273}{760}\left(\frac{p'}{T'} - \frac{p''}{T''}\right) \cdot v\left(1 - \frac{\eta - \delta'}{d}\right)$$

where δ_o is the density of air at 760 mm. and at 0°C = 273° abs. Here we have assumed also Gay-Lussacs law to hold exactly for air.

If the gas in the bulb is air of normal density the value of $\frac{\eta}{d}$ is about

$\frac{1}{1800}$; $\frac{\eta - \delta'}{d}$ is, of course, still smaller. In many cases that term may therefore be neglected, so that equation 8 takes on the simple form given by Steele and Grant [16 p. 588]:

9. $$dW = \delta_0 \frac{273}{760} \left(\frac{p'}{T'} - \frac{p''}{T''} \right) \cdot v$$

§ 5. Practical.

The method of weighing by pressure involves the use of an «air-weight» of the same volume as the capacity of the bulb. It is therefore necessary that the air within the case is of known density, i. e. free from carbonic acid and moisture,and also that its pressure and temperature can be accurately measured when a reading is taken. The arrangement I use for that purpose is identical with that of Ramsay and Gray. *M* in fig. 2 on page 12 is a manometer (drawn on a small scale) of large bore, 23 mm., so that no corrections are necessary for the capillarity of the mercury. One of its branches is in communication with the case, and the other branch with an Antropoff air-pump by which a high vacuum is maintained over the mercury in *M*. The pressure is read to $\frac{1}{50}$ mm. with a reading cathetometer.

The air is admitted into the case after filtering slowly through two tubes, A_1 and A_2, charged with soda-lime, then through a $P_2 O_5$ tube, *P*, and finally through a fourth tube, *C*, which is filled with dry cotton-wool tightly packed. In the latter tube the fine dust particles are retained which may be carried with the air from the absorbants.

The inlet to the case consists of a brass tube, *T*, which comes up through the balance floor near one of its corners. The upper end of the tube is closed and its wall perforated by a number of fine holes over the sides which are turned away from the balance.

In order to remove also the moisture and the CO_2 which can come into the case while the weight tube is removed, a small heap of barium monoxide is put into that tube, and a crucible filled with the same substance is placed under the beam.

The accumulation of electric charges on the silica, as well as on the glass walls of the cover and of the weight-tube, is prevented by placing under the beam a small crucible filled with uranous oxide, which keeps the air in the case ionized.

A weighing by pressure is carried out in the following manner. The balance is loaded with pieces of solid silica and a bulb, and the load is adjusted until the lightspot comes to rest near the scale-centre, preferably at a low pressure. An extra cover of bright tin with a thermometer inside is put over the balance case, and the released instrument is left alone for a certain time until the temperature under the cover may be assumed to be uniform. A set of readings is then taken of the scale-deflection, a',[1]) the pressure, p', and the temperature, T'. In order to avoid the heating effect of the strong light on the air within the case, the Nernstlamp is kept screened up to the moment when the scale-reading is taken. The balance is then arrested, and air is slowly admitted until there is atmospheric pressure in the balance case. The weight-tube can then be removed, and the body to be weighed is suspended from the balance. After the tube has been put back in its place the pressure in the case is reduced until the balance, when released, comes back to its original position. Before that is done, the case may, however, as an extra safeguard, be pumped empty, and new air slowly filtered into it until the desired pressure is attained. The released instrument is again left under its extra cover for a certain interval of time, rather longer than before as the pressure is now higher. Then a set of readings giving a'', p'', and T'', is taken.

If $a' = a''$ the load must be the same at both readings[2]), and the apparent weight of the body in air at p'' and T'' is equal to the change in the effective weight of the bulb, that is, to dW in equation 8 or 9. It is, however, not practical to bring the balance back to exactly the same position; a' is therefore generally slightly different from a'', and a corresponding correction, $+ (a''-a') \cdot S_1$, must be introduced in the equation, S_1 being the average sensibility of the balance within the scale-interval a' to a''.

Here we have assumed the scale to be read in such a manner that an increase in a corresponds to an increase in the effective weight of the bulb.

It is therefore necessary to measure the sensibility of the balance. This can be done when it is charged only with solid silica and the bulb

[1]) At ordinary pressures the damping of the released balance is so high that only the rest position of the lightspot is taken.

[2]) With this method of weighing by substitution, i e. with constant loads, it is obviously of no importance that the arms of the beam may be slightly different in length, and that the sensibility of the instrument varies with the weight of the loads.

by varying the pressure by small amounts, each time taking readings of a, p, and T. The change, dW, in the effective weight of the bulb from one reading to another is found from equation 9, and the sensibility within each interval is given by: 9 a. $S_n = \frac{dW_n}{a_{n+1} - a_n}$.

The correction is, however, generally so small and the sensibility so uniform that in most cases only its value at the scale-centre, S_o, need be used.

If the load consists also of one or more bodies of densities different from that of silica, each of them will have the same effect as a bulb, positive or negative, depending on whether its density is smaller or greater than that of silica. Correction terms of the type

$$v_x = V_x \left[1 - \frac{d_x - \delta'}{d}\right]$$

must then be added to the second factor in equations 8 and 9, V_x denoting the volume of the body and d_x its density. If these quantities are not known, the value of the correction term can be found experimentally by taking two sets of readings at different pressures, when the balance is loaded only with the body in question and with solid silica. From these observations and from the known sensibility of the balance the «effective volume», v_x, of the body can be calculated, that is, the volume, positive or negative, of the bulb which would produce the same change in the deflection.

The same method can evidently also be used in order to find the vacuum correction for the weighed body, so that its true weight at the pressure $p'' = 0$ can be calculated.

Finally it must be observed that commercial silica of the ordinary quality is not perfectly homogeneous, but always contains a number of small air-bubbles. Each of them will act as a minute bulb when the pressure is varied. If the bubbles are uniformly distributed in the silica of the suspended system, their total effect will of course be nil. If not, the capacity, v_b, of a bulb which is equivalent to all these bubbles together can be found by the method just described, that is, by varying the pressure when the balance is charged only with solid silica,[1]) and noting the change produced in the deflection.

The corresponding correction is always quite small and need only be considered at very accurate weighings when the pressure is varied by large amounts.

[1]) which ought to be as free from air-bubbles as possible, preferably of the « optical » quality.

On introducing these corrections in equation 9, the complete equation for weighings by pressure is found to be:

$$10. \quad dW = \delta_0 \frac{273}{760}\left[\frac{p'}{T'} - \frac{p''}{T''}\right] \cdot \left[v\left(1 - \frac{\eta - \delta'}{d}\right) + \right.$$
$$\left. + \Sigma V_x \left(1 - \frac{d_x - \delta'}{d}\right) + v_b \right] + [a'' - a'] S_0$$

§ 6. Accuracy of a weighing by pressure.

The absolute accuracy, A, of a weighing by pressure depends on how accurately the variable quantities which enter into equations 9 or 10 are measured. Taking A equal to the maximal error from the two readings required for a weighing by pressure, we have approximately (T' and T'' being generally not much higher than 273):

$$11. \quad A = \frac{v\delta_0}{760}\left[\Delta p' + \Delta p'' + \frac{\Delta T'}{T'} p' + \frac{\Delta T''}{T''} p''\right]$$

where Δp and ΔT denote the accuracy with which the pressure and the temperature are observed. If the weighing is carried out at low pressures the last two terms within the brackets may be neglected. Taking $v = 1$ mm^3 and $\Delta p' = \Delta p'' = 0{,}1$ mm., which is a very moderate precision, we find $A = 0{,}35$ µmg. This proves that it is possible to adjust and to measure the effective weight of such a bulb with an accuracy corresponding to the highest sensibility hitherto attained with any micro-balance.

The relative accuracy is given by $A_r = A : \Delta W$, where ΔW is the change in the effective weight of the bulb produced at the weighing. Theoretically the relative accuracy should be as high as possible (numerical minimum of A_r) when ΔW is a maximum, that is, when the pressure is varied from $p' = 760$ mm. to $p'' = 0$ mm.

Taking $T' = 273$, we then find from equations 9 and 11:

$$\Delta W = v\delta_0 \text{ and } A_r = \frac{\Delta p' + \Delta p''}{760} + \frac{\Delta T'}{273}$$

If $\Delta T' = 0°{,}1$ and $\Delta p' = \Delta p'' = 0{,}1$ mm. as before, we find

$$A_r = 6 \cdot 10^{-4}.$$

In practice, experimental errors, which are chiefly due to convection currents in the air, make it impossible to reach the same ab-

solute accuracy when the pressure is nearly atmospheric as when it is quite low. The highest relative accuracy which is actually attained is, therefore, generally lower than the theoretical maximum. The difference depends mainly on how carefully an unequal distribution of the temperature in the case is avoided by the experimental arrangement.

If the result of a weighing is to be given in absolute units, in grammes, its relative accuracy will be slightly lower, owing to the uncertainty with which the absolute values of v and δ_o in equation 11 are known. For bulbs of larger capacity v can easily be measured to $1 : 10^4$; and the error[1]) in δ_o is probably less than $5 : 10^5$. In the given example these additional errors are much smaller than A_r.

A weighing by pressure can of course be carried out quite as well with another gas of known density, δ_f, in the case instead of air. In equations 8 to 12 we must then substitute δ_f for δ_o. With hydrogen the absolute accuracy of a weighing is about 14 times higher than with air; the relative accuracy is, of course, theoretically the same with both gases.

§ 7. The bulbs and the weights.

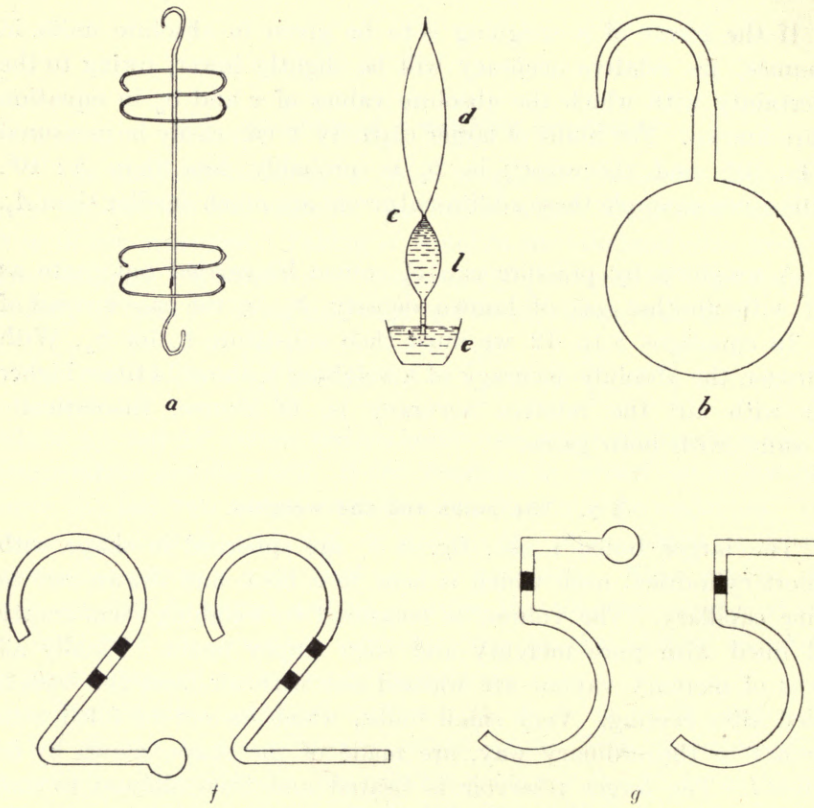
The larger bulbs I use, fig. 4 *b*, are spherical in shape with a short cylindrical neck which is bent to a hook and drawn out to a fine capillary. The volume is measured by weighing them empty and filled with pure mercury and then empty again. Finally all traces of mercury vapour are washed out with air, and the bulb is sealed after cooling. Very small bulbs, which cannot be filled with mercury in the ordinary way, are made of the shape shown by figure *4 l*. The larger reservoir is heated and then allowed to cool while the capillary opening is dipped in mercury until *l* is completely filled. Finally the communication is broken at *c*, and both ends of *l* are sealed. In this way bulbs may be made of the smallest capacity required for work with any micro-balance.

I generally made two independent determinations of the capacity of each bulb, which always agreed to within a few hundredths of a mm^3. The constants for the bulbs I have most frequently used are given here:

[1]) Landolt—Börnstein—Roth. Phys-chem. Tabellen. 4:th Ed. p. 16, 1912.

Bulb. no.	weight	capacity	range
1	14,5 mg	8,16 mm.^{3}	0,01056 mg.
2	43,1 »	39,74 »	0,05144 »
3	94,2 »	554,53 »	0,7178 »

Fig. 4.



The « range » of a bulb denotes the greatest possible change in its effective weight, which is produced when the pressure in the case is varied from 0 to 1 atm. At 0° it is equal to $v\delta_0$ (last column of the table) where the density of the air or $\delta_0 = 0{,}0012944$.

The limited range over which the air-weight can be used makes it necessary to supplement it with weights, and to use it only for the finest adjustment like the rider on an ordinary balance. Very light weights, down to 0,1 mg., can be made from capillary silica tubing open at one end. Such small objects are troublesome to handle,

and there is a certain danger of altering their weight by breaking off minute fragments from the edge of the tubing. All my weights are therefore made of solid silica rod and the very lightest according to the following method. Two pieces of nearly the same weight, a couple of mg., are cut from a silica rod and are bent to the shape of the same figure, one of them, the «zero piece», with an extra blob or bend at one end. The pair is then adjusted through weighings on the balance together with a bulb, until the difference in weight, ε, between the pieces has nearly the value, in tenths or hundredths of one mg, which is indicated by their mutual shape. When the weights are used the zero piece is normally suspended from the balance. By exchanging it for the other piece of the pair the load is evidently increased or reduced with ε.

In order to distinguish between the different decimals of a set, the pieces may be marked with narrow strips of platinum leaf (one for the tenths and two for the hundredths of a mg.) which are heated white hot until they stick permanently to the silica. I have made a set of «difference-weights», of which one pair is reproduced in fig 4 *f*, ε = 0,0197 mg., and another pair in 4 *g*, ε = 0,4990 mg.

Such difference-weights do not break and are easy to handle. They can obviously be made with much smaller values of ε. On the other hand they will also make it possible to carry out less accurate weighings, say to 0,001 mg., in air at atmospheric pressure without the aid of any bulb. A certain disadvantage is that the necessity of always having one piece of the pair suspended from the balance reduces the useful part of the load. With a set of two decimals, 8 pairs, of which each piece weighs 1 to 2 mg. this reduction amounts to 8—16 mg.

The largest difference-weight of the set may be used as a unit from which a set of direct weights, from 1 mg. and upwards, can be built up.

Before the bulb or the weights are suspended from the balance they should be cleansed by heating them for a moment to red heat in a small flame which must not contain any trace of oxygen.

§ 8. Errors in the weighings.

Besides the errors due to mechanical disturbances from the outside, tremors, there are mainly three different causes which may produce errors at weighings with the micro-balance.

Unequal distribution of the temperature in the case will give rise to convection currents and inequalities of the density of the air which may seriously affect the results of a weighing. Such disturbances are more pronounced the denser the air in the case. It is therefore advisable, as Steele and Grant observe, to do more accurate weighings at a reduced pressure, if possible at less than 50 mm., when these troubles are minimized. If the use of the air-weight makes it necessary to take readings at higher pressures, the time during which the released balance must be left under the extra cover to take on a uniform temperature has to be increased accordingly.

At very low pressures radiometer action will set in and may then produce considerable displacements from the true equilibrium. Already at pressures of a couple of mm. this effect begins to make itself felt. In ordinary work with the balance the pressure is generally so high that all troubles of this kind are practically excluded.

Surface effects on the silica. As silica is only slightly hygroscopic no errors can be caused by the traces of moisture which may be present in the case, in spite of the absorbants used to dry the air. Nor is there any risk of vapours from the rubber-grease used to make the case air-tight becoming condensed on the silica, as is borne out by experiments related in the following. Other experiments prove that the normal constituents of the air are not adsorbed or occluded by the silica even after it has been kept at red heat for hours in a high vacuum. Finally there is experimental evidence to show that although silica is volatile in a high vacuum already at + 600°, its evaporation is almost unobservable in air of 5 mm. even after a prolonged heating to nearly + 800°. See this paper part III, § 1, and 4.

Inconstancy of the weights. A too intense heating of the standardized weights, say with a flame which contains traces of oxygen, as well as their contamination by metallic or alcaline dust, subsequently burnt into the silica, must be strictly avoided. Provided that these precautions are observed, the facts just mentioned make it appear extremely probable that weights made of silica should keep constant for any length of time, even to the high degree required for work with the most sensitive micro-balance.

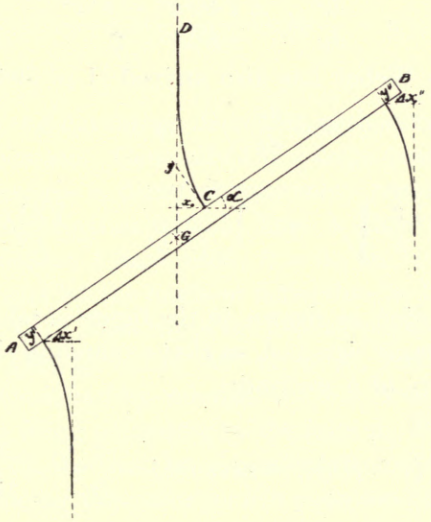
§ 9. Theory of the balance with fibre suspension.

The first part of the following deduction is almost the same as that given by Weber in his «*De tribus novis librarum construendarum me-*

thodis» [1], where the elements of a theory for the balance with elastic suspension are set out.

Let a rigid bar, AB (figure 5), be suspended at C by an elastic fibre, CD, of negligible weight and thickness. When AB is horizontal the whole length of the fibre is supposed to be perfectly straight; when AB is deflected through an angle α, as in the figure, CD will become curved. The system will be at equilibrium provided that its centre of gravity, G, falls on the vertical asymptote to the fibre, and that the momentum of the gravitational and of the elastic forces are equal at all points on CD.

Fig. 5.



Taking the vertical through G as y-axis, and the horizontal line through C as x-axis we must have:

12. $Px = \frac{k}{\rho}$; P being the total weight of the suspended system, ρ the radius of curvature of the fibre, and k a constant which represents the elastic resistance of the fibre against the bending forces. According to the elementary theory for the bending of cylindrical rods:

13. $k = \frac{\pi}{4} ER^4$, where R is the radius of the cylinder and E Young's modulus for the substance.

Now:

14. $$\frac{1}{\rho} = \frac{\frac{d^2x}{dy^2}}{\left[1+\left(\frac{dx}{dy}\right)^2\right]^{\frac{3}{2}}}$$

Calling $\frac{P}{k} = h^2$, and $\frac{dx}{dy} = z$, we have $\frac{d^2x}{dy^2} = z\frac{dz}{dx}$. For $x = 0$ $z = 0$.

Equation 12 gives: $h^2x = \frac{z\frac{dz}{dx}}{[1+z^2]^{\frac{3}{2}}}$; $\frac{h^2x^2}{2} = \int_0^z \frac{zdz}{[1+z^2]^{\frac{3}{2}}} = 1 - \frac{1}{\sqrt{1+z^2}}$

15. $$z = \frac{dx}{dy} = \frac{\pm\sqrt{4h^2x^2 - h^4x^4}}{h^2x^2 - 2}$$

a result at which Weber has also arrived [1 p. 508.]

For $y = 0$ $x = x_0$ and $\frac{dx}{dy} = -tg\,\alpha = -b$. Consequently:

16. $$x_0 = \pm\frac{\sqrt{2}}{h}\sqrt{1-\sqrt{\frac{1}{1+b^2}}} = \pm\frac{\sqrt{2}}{h}\sqrt{1-\cos\alpha}$$

If the deflection as drawn in the figure is taken to be positive, x_0 will be of the same sign as b, so that expanding the last expression with rising powers of b we find:

17. $$x_0 = \frac{b}{h}[1-f(b)]; \text{ where } f(b) = \frac{3}{8}b^2 - \frac{31}{128}b^4 \ldots .$$

For small values of b the series converges so rapidly that all its terms except the first may be neglected.

The axis of rotation of the system must pass through the point of intersection between the tangent through C and the y-axis. The y-coordinate of that point is (see fig. 5!):

18. $$\bar{y} = \frac{x_0}{b} = \frac{1}{h}[1-f(b)]$$

A system suspended in this way will therefore behave as if it were balanced on a knife-edge, displaced from the point of suspension, C, by the horizontal distance x_0 and by the vertical distance $\bar{y}$.

For small deflections, according to equations 17 and 18, x_0 is proportional to b, and $\bar{y}$ is independent of the deflection.

According to equation 15 we have for small deflections:

19. $$\frac{dx}{dy} = -h\,x \qquad x = x_0 e^{-h\,y}$$

20. $$x = \frac{b}{h} e^{-h\,y}$$

Equations 12 and 20 give approximately:

21. $$\frac{1}{\rho} = b\,h\,e^{-hy}$$

which shows how the curvature decreases with increasing height over the point of suspension. Generally h is a large quantity, and the decrease in the curvature is therefore extremely rapid. This is best illustrated by a numerical example. Take a silica fibre of the same radius as the thickest I have used, $R = 3{,}5$ cm. 10^{-4}, charged with a load of 200 mg., and let the angle of deflection be 10°. Taking for the torzional rigidity, Q, of silica fibres the result from the careful measurements of Horton [14], $Q = 3{,}00° \; 10^{11}$ G. G. S., and for Poissons coefficient, $\varkappa$, the value found by Schulze [13], $\varkappa = 0{,}260$, we find Young's modulus for silica fibres according to the formula $E = 2\,Q\,[1+\varkappa] = 7{,}56 \cdot 10^{11}$ C. G. S. We then have in absolute units $k = 8{,}9 \cdot 10^{-3}$ and $h = 149$. Introducing this value for h in equation 21, we find that the radius of curvature at the point of suspension should be $\rho = 0{,}038$ cm.; at $y = +\,0{,}01$ cm. $\rho = 0{,}17$ cm.; and at $y = +\,0{,}1$ cm. $\rho = 1{,}1 \cdot - 10^5$ cm. Thus, already at a height of 1 mm. over the point of suspension the curvature may be regarded as infinitely small. This fact is of considerable importance for the drawing of the fibres, as it proves that it is unnecessary to make them very thin with the exception of the parts which are quite close to the points of suspension.

The horizontal and the vertical displacement of the axis of rotation from the point of suspension may also be given for the case we have considered: $x_0 = 0{,}0012$ cm.; $\bar{y} = 0{,}0067$ cm.

We further assume that AB (fig. 5) is provided with a fibre at each end by which loads, weighing W' and W'', are suspended. Charged

with these loads AB is supposed to be at equilibrium in a horizontal position with all three fibres quite straight. The coordinates for the points of suspension of the end-fibres may then be $(x'\ y')$ for the left and $(x''\ y'')$ for the right fibre.

When AB is deflected from the horizontal by adding a small weight, $\Delta W'$, to left load, W', the fibres will be bent as in the figure. The end-fibres will then have their axes of rotation displaced in the horizontal direction by distances which are given by equations analogous to 17:

22. $\Delta x' = b\sqrt{\frac{k'}{W'}}[1-f(b)]$ 23. $\Delta x'' = b\sqrt{\frac{k''}{W''}}[1-f(b)]$

where k' and k'' represent the elastic resistance of each fibre according to equation 13.

The conditions of equilibrium for the two positions are (compare fig. 5): deflection $= 0$ 24. $W'x' + W''x'' + px_c = 0$

deflection $= \alpha$ 25. $[W' + \Delta W']\ [x' \cos\alpha - y' \sin\alpha + x_o + \Delta x'] +$

$+ W''[x'' \cos\alpha - y'' \sin\alpha + x_o + \Delta x''] + p[x_c \cos\alpha - y_c \sin\alpha + x_o] = 0$

where p is the weight of the beam, and $(x_c\ y_c)$ the coordinates for its centre of gravity at $\alpha = 0$.

In equation 25 y', x_o, and $\Delta x'$, are all very small compared to x'. Their products with $\Delta W'$, which is also very small compared to W', may be therefore neglected. Multiplying equation 24 by $\cos\alpha$ and subtracting it from 25 we have:

$$\Delta W'x' \cos\alpha = [W'y' + W''y'' + py_c] \sin\alpha - Px_o - W'\Delta x' - W''\Delta x''$$

where $P = p + W' + W''$.

In this expression the lenght, L, of the left arm of the beam may be substituted for $-x'$ whereas for x_o, $\Delta x'$, and $\Delta x''$, we substitute their values according to equations 16, 22, and 23.

Dividing by $\cos\alpha = \frac{1}{\sqrt{1+b^2}}$ we then find:

$$\Delta W' = \frac{1}{L}\{-b[W'y' + W''y'' + py_c] + b\sqrt{1+b^2}[1-f(b)] \cdot$$

$$\cdot [\sqrt{Pk} + \sqrt{W'k'} + \sqrt{W''k''}]\}$$

In order to find the absolute sensibility, s, of the balance, referred to a linear scale at a distance of D scale-units, we must differentiate the last expression with regard to b and divide it by D.

26. $$s = \frac{1}{D}\frac{d\Delta W'}{db} = \frac{1}{LD}\{-K + H[1 + \varphi(b)]\}$$

where:

27. $$K = W'y' + W''y'' + py_c$$

28. $$H = \sqrt{Pk} + \sqrt{W'k'} + \sqrt{W''k''}$$

and $\varphi(b) = \frac{3}{8}b^2 - \frac{45}{128}b^4 \ldots\ldots$ For small values of b $\varphi(b) = \frac{3}{8}b^2$

Equation 26 may be written:

29. $$s = s_0 + \frac{H}{LD} \cdot \varphi(b)$$ where 30. $$s_0 = \frac{1}{LD}[H - K]$$

is the minimum value of s, corresponding to a maximum of the sensibility, at the deflection $\alpha = 0$

It has hitherto been assumed that the balance hangs by a single fibre, whereas there are really two, one at each side of the beam. Suppose the starting points of these fibres to be at the same distance, $\pm z$, from the point C (figure 5) on the perpendicular to the figure which passes through C. The strain on each fibre will then be the same $= \frac{P}{2}$. Each of them will tend to have a separate axis of rotation; and when the beam swings these axes can be assumed to undergo horizontal displacements, x_1' and x_2', given by equations analogous to 16. As the displacements are quite small compared to the distance, $2z$, between the fibres, the axis round which the whole system rotates will keep very nearly perpendicular to the plane of the figure, and the horizontal displacement, X, of its intersection with that plane will be at small deflections:

$$X = \frac{1}{2}[x_1' + x_2'] = \frac{b}{\sqrt{2P}} \cdot [\sqrt{k_1} + \sqrt{k_2}]$$

where k_1 and k_2 are given by equations analogous to 13.
In the 25th and the following equations X must be substituted for x_0, so that equation 28 will read:

28 a. $$H = \sqrt{\frac{P}{2}}[\sqrt{k_1} + \sqrt{k_2}] + \sqrt{W'k'} + \sqrt{W''k''}$$

Discussion of the results.

The suspended system is evidently stable as long as s in equation 26 remains positive. If $s = 0$ the balance is top-heavy and a stable equilibrium impossible.

With an ideal suspension, that is, with infinitely flexible fibres, the factor H vanishes from equation 26 which then becomes identical with the formula given by the elementary theory for knife-edge balances. In that case the equilibrium would be stable only if K in the equation had a negative value. This means that of the three points $(x' \; y')$, $(x'' \; y'')$, and $(x_c \; y_c)$, viz., the points of suspension of the end-fibres, and the centre of gravity of the beam, all three *may*, and at least one of them *must* fall *under* the level of the central point of suspension, C, at $\alpha = 0$.

The sensibility of such an instrument would not vary with the deflection, but would be the same at all parts of the scale.

With fibres of a finite thickness H is always a positive quantity, and so is $\varphi\,(b)$. In order to increase the sensibility as far as possible K must therefore be made positive, which means that of the three points just mentioned all three *may*, and one at least *must* be raised *over* the level of C when $\alpha = 0$. (see adjustment of the balance on page 17.

The system is then actually top-heavy with regard to the central point of suspension and would become overturned except for the elastic resistance of the four fibres which tend to keep it upright.

If the three points are raised too high it can happen that $H - K < 0$, while $H \; [1 + \varphi \; (b_g)] - K = 0$ where $b'_g = tg \; \alpha_g$. Then the equilibrium is unstable «near the scale-centre», i. e. at small deflections, but becomes stable at the sides of the scale, as soon as the deflection to either side exceeds α_g.

The sensibility is not independent of the deflection, but has a maximum at the centre of the scale, $\alpha = 0$ (minimum of $s = s_o$), and decreases symmetrically to both sides (*increase* of s). From eq. 29 it follows that this decrease is independent of K. Therefore, if the thickness of the fibres and the weight of the beam and of the loads are given, the decrease (and $s - s_o$) shall at every point on the scale have a certain value, which remains constant also when the sensibility at the scale-centre (s_o) is varied.

The *relative* decrease in the sensibility with growing deflection, and the flatness of the curve which represents the sensibility as a

function of the deflection, must therefore vary both with K and with H.

If both members of equation 29 are divided by s_0 it becomes

31. $$\frac{s - s_0}{s_0} = \frac{H}{s_0 LD} \varphi(b)$$ which proves that the *relative* decrease of the sensibility (rel. *increase* of s) is greater, and the sensibility curve less flat, the *higher* the sensibility is at the scale-centre.

Multiplying the right member of equation 31 by $\frac{W'}{W'}$ we have:

32. $$\frac{s - s_0}{s_0} = \frac{H}{W' LD} \varphi(b) : \frac{s_0}{W'}$$

Equation 28 *a* shows that only the square roots from W' and W'' and from $P = W' + W'' + p$ enter into H. If therefore the sensibility at the scale-centre together with the weight of the beam or of the loads are varied, while the relative sensibility at the scale-centre, $\frac{s_0}{W'}$, is kept constant, then the *relative* decrease of the sensibility with increasing deflection is smaller, and the sensibility curve more flat, the heavier the loads are and the lighter the beam is.

The preceding conclusions may be summarized thus:

The fibre suspended balance works more perfectly, i. e. its sensibility is less dependent on the deflection, *the thinner the fibres are, the lower the sensiblity is, the heavier the loads are, and the lighter the beam is.*

It is evident from the foregoing equations that the sensibility must vary with the weight of the loads. Suppose that these are equally heavy, as is very nearly the case with my balances, $W' = W'' = q$, and that all the three terms in K, equation 28 *a*, are positive. Equation 30 may then be written:

$$s_0 = \frac{1}{LD} [- py_c - Aq + B\sqrt{q} + C\sqrt{2q + p}]$$

For $q = 0$ $s_0 = M = \frac{1}{LD} [- py_c + C\sqrt{p}]$

If M is kept constant and of negative sign, as it generally is on highly sensitive balances, it is then easy to see that s_0 will vanish for two different values of q and will have a maximum of positive sign (*minimum* of sensibility) for an intermediate value, whereas it will be negative (unstable equilibrium) for all values of q outside the range so defined.

This peculiarity of the balance with elastic suspension, to remain stable only as long as the loads are kept within certain values, has already been pointed out by Weber.

In the preceding theory the beam has been supposed to be of a very simple shape, from which the instruments actually made differ in various respects. Most of these differences are insignificant, so that they can safely be assumed not to affect the validity of the conclusions we have drawn. There are, however, two points which must be considered as more important.

The beam is not a perfectly rigid structure, but must become slightly deformed when the loads are suspended from it. As long as these are kept constant the deformation can be disregarded, but with varying loads it will also vary. The y-coordinates for the points of suspension of the end-fibres will in the first hand be affected by these variations, which may therefore also have other effects on the sensibility than those just mentioned.

The fibres were supposed to be of a perfectly cylindrical shape, i. e. to have a constant radius which suddenly becomes infinite at the points of suspension. As the fibres are drawn from rods of much greater diameter they are actually conical in shape, tapering more or less slowly to a minimum thickness which remains nearly constant over a comperatively long stretch.[1]) I have not considered it worth while to attempt to develop a theory for the bending of such fibres, which would be rather complicated, as it would have to consider the individual shape of each fibre. It appears most probable, however, that a complete theory would lead to equations closely resembling those we have found, but in which the term H would be of a less simple form than according to equation 28 a.

It will be seen from the following pages in how far the results found by this theoretical study are in accordance with the experimental observations.

[1]) See Table I on page 36.

§ 10. Investigation of a micro-balance.

The case used in this investigation, figure 2 on page 12, is not particularly well adapted for weighings of the highest precision (for reasons already mentioned). Also the tremors in the building of Stockholms Högskola are so considerable that I found it impossible to take accurate readings during day-time on a balance suspended inside this case.

I therefore had to work at night, but even then scale-readings could only be taken during the comparatively rare intervals when no vehicles happened to be driving near the Högskola[1]).

For this reason I found it useless to attempt to reach an extremely high accuracy of the weighings. Instead I have tried to investigate whether the variations in the sensibility with varying loads and period would be of the general character indicated by the preceding theory.

The case was mounted with two of its legs on one and the third leg on the other of two solid pillars of brickwork and concrete built up from the floor of a subterranean room. A vertical scale was attached to the opposite wall of the room, 412 cm. away from the balance mirror which was at the same level as the scale-division 1000. The beam of the balance had the following dimensions:

length of vertical diagonal = 3,8 cm.; length of left arm = 5,0 cm.
» » horizontal » = 8,6 » ; » » right » = 4,9 »
distance between the central fibres = 1,6 cm.
total weight of the beam = 377 mg.
min. diam. in μ of the central fibres: $R_1 = 3$; $R_2 = 1{,}5$
» » » » » end » $R' = 3{,}5$; $R'' = 1{,}5$

[1]) My sincerest thanks are due to the Superintendent of Police in Stockholm, W. A. Tamm Esq., by whose courtesy a police guard was ordered to divert the traffic from the byestreets at the back of the Högskola during certain nights when I had particularly delicate observations to take.

The error in the measurement by means of a microscope of the radii of the fibres may have amounted to 0,5 μ Table I shows how the radius, R, in μ (= 10^{-4} cm) of one of the central fibres varied with the distance, l, (in mm.) from a certain point on the conical part of the fibre.

Table I.

l mm.	R_2 μ	l mm.	R_2 μ
0,00	35	1,20	2
0,20	24	1,40	1,5
0,40	13	1,60	1,5
0,60	8	1,80	1,5
0,80	4	2,15	1,5
1,00	2,5	5,20	2

The vacuum correction of the beam itself, due to the non-homogeneity of the silica rods, proved to be very small. When the beam was charged with loads of solid silica weighing 20 mg. I found for the quantity v_b in equation 10 on page 22: $v_b = -0{,}02$ mm^3, a correction which can be neglected in the following.

A point of special interest was to study how the sensibility varied with the deflection when the loads are very light and the period high. The balance was therefore charged with a load which, inclusive of bulb no. 1[1]), weighed only 20 mg. and with a corresponding counterpoise. The period at the scale-centre was raised to about 58 seconds, and the sensibility measured according to the method described on page 21. The results are recorded in table II. The first three columns give the scale-reading, a, in mm., the pressure, p, and the temperature, *temp*. The fourth column gives S, in μmg., calculated from the equations 9 and 9a, where I have taken $\delta_0 = 0{,}001294$. We can assume that S, which is the average sensibility within each scale-interval, is equal to the true sensibility, $\frac{d\,\Delta W}{da}$ at the centre of the interval, a'. The fifth column of the table gives $a'_n = {}^1/_2\,(a_n + a_{n+1})$. In figure 6 the values of S are plotted

[1]) see page 24.

against those of a' and a smooth curve drawn through the points. It will be seen from the curve that the sensibility has a maximum somewhere near the scale-division *1260*, and that the minimum value of S at that point is about $S_0 = 0{,}60$ μmg.

Fig. 6.

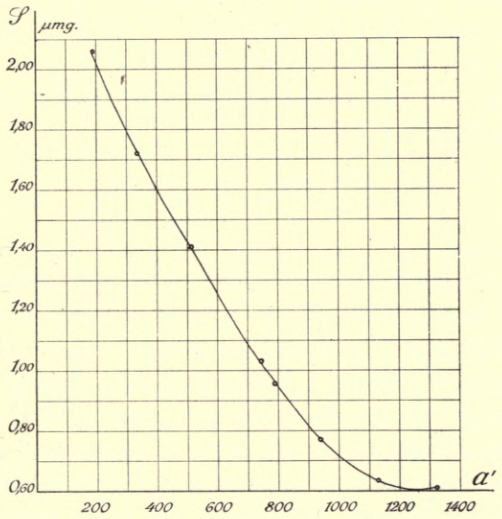


Table II.

a mm.	p mm. Hg.	*temp.*	S μmg.	a' mm.	H erg.
1407	58,60	16°,60	—	—	—
1235	50,68	16°,70	0,605	1321	(9,8)
1025	40,52	16°,75	0,635	1130	(15,2)
854	30,50	16°,80	0,770	940	12,2
727	21,24	16°,90	0,955	791	11,8
880[1])	93,02	16°,50	—	—	—
610	71,77	16°,30	1,03	745	11,9
417	50,99	16°,65	1,41	514	10,7
254	29,52	16°,50	1,72	336	9,6
128	9,73	16°,50	2,06	191	9,4
				mean value of H =	11,0

[1]) The last five observations were taken after the load had been slightly adjusted so as to bring the lightspot to the other parts of the scale.

In order to apply the foregoing theory to these results we must count the deflection of the balance from its position of maximum sensibility. Now, the angular deflection of the lightspot is twice as large as that of the beam. The angles being quite small we have very nearly for the quantity b in equations 16 and following;

$$b = b' = {}^1/_2\,(1260 - a') : D$$

where D is the scale-distance = 4120 mm. By the same approximation we have in equations 29 and 30: $s = 2\,S$; $s_\circ = 2\,S_\circ$. Equation 29 may also be written: $H = L\,D[s - s_\circ] : \varphi\,(b)$ where we can put $\varphi\,(b) = {}^3/_8\,b^2$.

Introducing in this expression $b = b'$; and s and $s_\circ$ from the preceding equations we find:

33. $$H = \frac{64}{3}\,LD^3\,\frac{S - S_\circ}{[1260 - a']^2}$$

In order to have H in absolute units we have to express all the quantities in equation 33 in C. G. S., except D which must be in mm. (scale-units) We therefore take: $L = 5$; $D = 4120$; 1 μmg. = $9{,}82 \cdot 10^{-7}$ dynes. With the aid of equation 33 I have calculated the values for H which are given in the last column of table II. The variations in H from its average value[1]), given at the end of the table, are not greater than the possible errors of experiment. At the highest deflections the table shows an apparent decrease in the values for H. It must, however, be observed, that between the 5th and the 6th measurement in the table the Nernst lamp happened to become slightly displaced. The corresponding shift of the lightspot, which was not directly measured, was estimated to be about 100 scale-units; and the subsequent scale-readings have been corrected accordingly. However, there remains, therefore, a slightly increased uncertainty in the four last values of H.

Very many attempts were made to raise the sensibility of the balance still further. Finally the period at the scale-centre (which was then very difficult to measure) was brought up to about 120 seconds. The sensibility was measured and the results are set out in table III and in figure 7.

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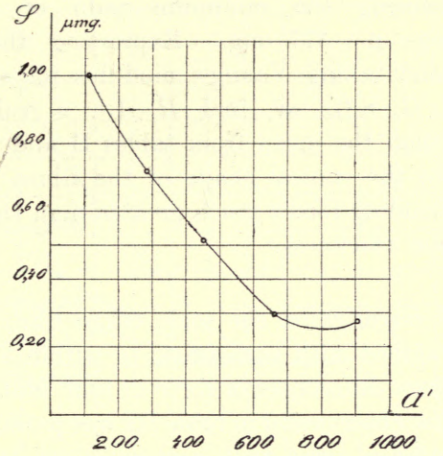
[1]) The first two values of H, which are most uncertain, are not included in the average.

Fig. 7.



Before this was done the Nernst-lamp had been displaced so as to make the lightspot fall nearer to the centre of the scale at the maximum of the sensibility. The shift amounted to about 440 scale-units. The maximum of the sensibility is also seen to be displaced by the same amount, from $a' = 1260$ in figure 6 to $a' = 820$ in figure 7, where the value of S_0 is seen to be about:

$S_0 = 0{,}255$ µmg., that is. a sensibility of nearly *one four-millionth part of a milligram*. The last column of table III gives H as calculated from equation 33, but with $(820 - a')^2$ in its denominator instead of $(1260 - a')^2$. The variations in H are rather larger than before, but the mean calculated from these values is seen to agree fairly well with the mean from table II.

These results are evidently in accordance with the theory we have developed.

Table III.

a mm.	p mm. Hg.	*temp.*	S µmg.	a' mm.	H erg.
1026	69,36	16°,10	—	—	—
785	64,34	16°,15	0,275	905	(20)
538	58,70	16°,10	0,300	662	13,2
364	51,86	16°,00	0,515	451	14,0
203	43,03	15°,90	0,720	234	9,9
20	29,09	15°,90	1,00	111	10.9
				mean value of H =	12,0

A theoretical lower limit for H may be calculated from equation 28 *a* by introducing the minimum radii of the fibres[1]) and $W' = W'' = 20$ mg.; $p = 377$ mg. Expressing these quantities in absolute units and taking Young's modulus for silica fibres to be $E = 7{,}56 \cdot 10^{11}$ C. G. S.[2]), we find $H = 1{,}7$, a value which is considerably lower than the mean from tables II and III. This is undoubtedly due to the conical shape of the fibres making their resistance to the bending forces much greater than that corresponding to their *minimum* thickness.

The control readings on the scale taken during these observations generally agreed with each other to within a couple of scale-units; in a few cases differences of three or four units were observed. Only with the first observation in table III was it impossible to make the different readings agree to less than about 10 units. It must be emphasized that these discrepancies cannot be taken as a sign that the sensibility had been pushed beyond the limit set by the *instrumental* errors, i. e. the errors due to possible defects of the beam and the fibres. They were most probably caused by *experimental* errors, of which those due to temperature variations in the case would be particularly prominent, as the pressure was rather high at some of the measurements.

Shortly after the experiments recorded in table III one of the end-fibres was accidentally broken. A new fibre was drawn out instead, with a minimum radius of only $R = 0{,}75$ μ. Some measurements made afterwards at different periods with loads of 45 mg. are given graphically in figure 8 together with the sensibility curves already mentioned. The curves are not strictly comparable for the reason just mentioned, but the general character of their shape is seen to be in accordance with the theoretical conclusions on page 29.

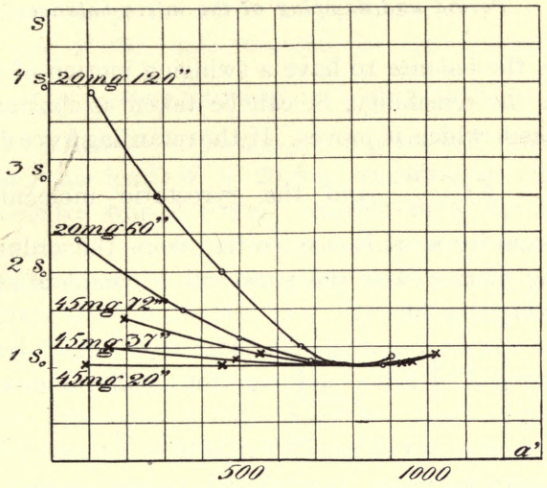
Finally I have given in table IV values found for the sensibility, both absolute and relative, at the scale-centre for different periods, τ, and with different loads, W.

The tendency of the relative sensibility to increase with increasing loads is well borne out by the results. In the last observation

[1]) See page 35.

[2]) See page 29.

Fig. 8.



given in the table, the balance was charged with loads weighing over a quarter of a gramme, (one of which was carried by a fibre 1,5 μ thick). The result proves that the relative sensibility which can be reached with the new balance is of the same order as that attained with the most perfect instruments used to standardize the prototypes of weight.

With these heavy loads the balance also appeared to be much easier to work, and less disturbed by tremors than with lighter loads. Two scale-readings taken in succession after the last observation in the table at a pressure of 1.30 mm. gave: after 30′ $a = 1027$; after 20′ $a = 1030$. The balance was then very roughly arrested, again released, and after 30′ gave: $a = 1029$.

Table IV.

W mg.	τ seconds	S_0 μmg.	$S_r \cdot 10^6$	W mg.	τ seconds	S_0 μmg.	$S_r \cdot 10^6$
20	40″	1,45	1 : 14	108	20″	18,1	1 : 6,0
20	60″	0,60	1 : 33	108	40″	4,50	1 : 24
20	~120″	0,255	1 : 80	108	~85″	1,00	1 : 108
45	20″	9,80	1 : 4,6	255	40″	7,70	1 : 33
45	37″	2,90	1 : 14,5	255	100″	1,40	1 : 180
45	~72″	0,90	1 : 50				

Period and damping of the micro-balance.

Suppose the balance to have a swinging motion of so small amplitude that the sensibility, S, can be taken as constant within the interval through which it moves. If the retarding force due to friction is given by $2\,C\,\frac{da}{dt}$, and the gyrostatic momentum of the swinging system by m, it is easy to find from the ordinary equation of a swinging motion that the period of a complete swing will be given approximately by:

$$\tau = \pi \sqrt{\frac{m}{s}} \left[1 + \frac{C^2}{2\,sm} \right] \quad \text{or for small values of } C\text{:}$$

$$s\tau^2 = \pi^2\, m = \text{const.}$$

These equations prove, as might have been foreseen *a priori*, that the period varies with the sensibility, so that a long period indicates a high sensibility and *vice versa*.

Therefore, as the sensibility of this balance varies with the deflection, the period must also vary, its variations being more pronounced the higher the sensibility. In accordance with this it was found in the observations given in table III that the period near the scale-centre ($a \sim 820$) was about $\tau = 120''$; whereas at $a \sim 440$ it had fallen to $\tau = 75''$; and at $a \sim 250$ to $\tau = 60''$.

It is also easy to prove that with a given period the relative sensibility will be higher the heavier the loads (see last column of table IV).

At ordinary pressures the period is practically independent of the pressure, but when a high vacuum is made within the case the period decreases slightly. In some experiments, which will be described later in this paper, the period at the scale-centre was $46''$ when $p = 5$ mm., and fell to $42''$ when a charcoal vacuum was established within the case.

The damping of the balance depends on several different factors, such as the length of the period, the weight and the shape of the loads, to a certain extent on the pressure, and generally also on the direction to which the balance swings. With the highest sensibility, table III, the motion of the balance at the scale-centre was nearly

aperiodic at ordinary pressures, so that I had to reduce p to about 0,05 mm. before the period could be measured.

At the side of the scale ($a \sim 450$) the natural logarithmic decrement, $\Lambda = \frac{C}{m}\tau$, was then found to be: $\Lambda = 1{,}4$ when $p = 45$ mm., and $\Lambda = 1{,}3$ when $p = 0{,}95$ mm., i. e. practically the same value.

With the same load, $W = 20$ mg., and $\tau = 40''$ the decrement at the scale-centre was $\Lambda = 0{,}85$; whereas with the heaviest loads $W = 255$ mg., and $\tau = 40''$ it had a still smaller value: $\Lambda = 0{,}22$.

The greater part of the damping is obviously caused by the friction against the air in the case, but another and not inconsiderable part might reasonably be attributed to the loss of energy by friction at the points of suspension. This is most probably true for knife-edge balances. I have, however, made experiments with the suspended balance in a high vacuum of the order $p = 10^{-4}$ mm., where the friction against the residual air must have been almost nil. When the balance was left released it kept up perfectly regular vibrations of a small amplitude ($\pm$ 5 mm.) which remained sensibly constant for over half-an-hour. To some extent the swinging motion must have been maintained by occasional faint tremors. Anyhow its persistence proves that the damping due to inner friction in the fibres is very small, a fact which illustrates the perfect properties of the fibre suspension. Evidently a suspension must be regarded as the more ideal in proportion as the energy of the swinging motion is the less dissipated through it.

§ 11. Conclusions.

From the preceding theoretical and experimental study of a fibre-suspended micro-balance we can draw certain conclusions of practical importance.

The fibres by which the beam and the loads are suspended need not be very fine except quite close to their points of suspension.

The sensibility of the balance varies with the length of the period of its swing, with the weight of the loads, and also with the scale-deflection. The balance should therefore be worked with con-

stant loads (weighing by substitution) and preferably at a nearly constant scale-deflection.

With constant loads the sensibility is roughly proportional to the square of the period. The variations of the period and of the sensibility with the scale-deflection are more pronounced the higher the sensibility. Therefore, by adjusting the balance to a long period the sensibility at the scale-centre may be given a very high value, but the relative increase in the sensibility produced at the side of the scale will be much lower.

The relative sensibility of the investigated balance has been raised to the order of $1 : 10^8$ with loads weighing a quarter of a gram, the absolute sensibility with loads of 20 mg. to nearly one four-millionth part of a mg. Owing to unfavourable experimental conditions the greatest *accuracy of a weighing* hitherto attained cannot, however, be given as higher than about one millionth part of a mg.

The balance has more ideal properties, i. e. the sensibility curve is flatter the lighter the beam is, the heavier the loads, and the thinner the fibres are.

As regards the last point it is clear from equations 13 and 28 *a* that the term H, which represents the joint resistance of the fibres to the bending forces, decreases as the section of the fibres, πR^2, provided, of course, that Young's modulus does not vary with the thickness of the fibres. On the other hand the tensile strength of thin fibres is known to decrease, not as R^2, but at a much slower rate. It therefore appears that by reducing the weight of the suspended system as well as the diameter of the fibres, the latter will gain more in flexibility than they lose in carrying strength. Consequently, the more slender the instrument the more perfectly it should work.

With a beam weighing 18 times more than each load I have been able to reach a sensibility of one four-millionth part of a mg. There is no doubt that if the beam be made considerably lighter, say of 50 mg., it would be possible to reach the same relative sensibility with a load of 20 mg. that I have attained with loads of 255 mg. This would correspond to an absolute sensibility of about one ten-millionth part of a mg.

The sensibility may also be increased by reading the scale more accurately, say to 0,1 mm., or by increasing the scale-distance. Whether such an improvement will be practically possible depends on how

effectively all disturbing influences can be excluded by the experimental arrangement.

The greatest possible degree of accuracy would undoubtedly be reached with a very high vacuum in the case. Instead of with the air-weight the balance might then be worked by electromagnetic compensation, which has already been used by Ångström [6] for his balance, and more recently by Urbain [21], and by Piccard [22].

The fibre-suspended micro-balance presents considerable advantages over the knife-edge instruments.

The danger of having the balance disarranged by dust adhering to the suspension is obviously out of the question with the fibres.

A knife-edge must be expected to become blunt gradually by use, whereas the fibres will most probably retain their ideal elastic properties indefinitely, provided they are not accidentally broken or heated by the blowpipe when the balance is adjusted.

The more slender the knife-edge balance is made, the greater will be the difficulties of making a very good knife-edge. On the other hand it is almost as easy to draw out a fibre to 0,5 μ as to ten times that diameter. Moreover the fibre-suspension will have more ideal properties the thinner the fibres are, as has already been proved. The new balance, therefore, appears to offer better facilities than the knife-edge instruments for attaining a still higher degree of accuracy in the weighings than I have yet realized.

At present the physicist has to resort to torsion balances for measuring very small forces. If we assume that the arm of a torsion balance is $l = 5$ cm. long, and that the silica fibre which carries it is $L = 20$ cm., we can find from the following formula[1]) what radius that fibre must have in order to give the instrument the same sensibility as the highest limit reached by me: $R = \sqrt[4]{\frac{2\,l\,L\,P}{\eta\,K\,\alpha}}$. Taking the modulus of torsional rigidity to be $K = 3{,}00 \cdot 10^{11}$ *C. G. S.* and $P = 0{,}255 \cdot 10^{-9} \cdot 982$ dynes, and for the angular deflection $\alpha = \frac{1}{2 \cdot 412}$, we find for the radius: $R = 8 \cdot 10^{-4}$ cm.

The sensibility of the micro-balance can therefore well stand a

[1]) Winkelmann, Handbuch der Physik. 2:d Ed. I: 1 p. 648. Leipzig 1908 (Barth.)

comparison with that of a good torsion balance. The great disadvantage of the latter instrument is that it can only be used to measure forces in the horizontal plane. Another drawback is that its sensibility cannot be directly measured, but has to be found by calculation from the period of the swing and from the gyrostatic momentum of the suspended system. The latter quantity is particularly difficult to measure.

With the aid of a micro-balance and its air-weight, it is possible to measure equally small forces in the vertical plane, and to do that by direct weighing, that is, by comparison with the gravitational force. This instrument can therefore be said to enable us to turn the technique of various physical measurements through 90°, i. e. from the horizontal plane to the vertical.

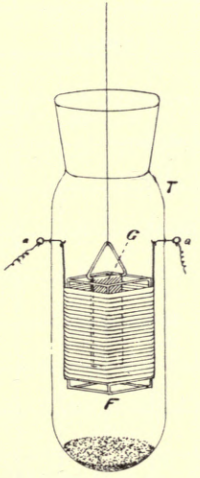
Some examples confirming this statement will be given in the following.

III. Experiments with the micro-balance.

§ 1. The change in the weight of gold caused by heating it.[1])

Most of my experiments on the above mentioned subject were made with the « niton balance » of Ramsay and Gray, which was kindly lent me by its owners. For a detailed description of that instrument I must refer to their paper [18] and to that by Steele and Grant [16]. When I used the instrument it had a sensibility of about 4 μmg.

Fig. 9.

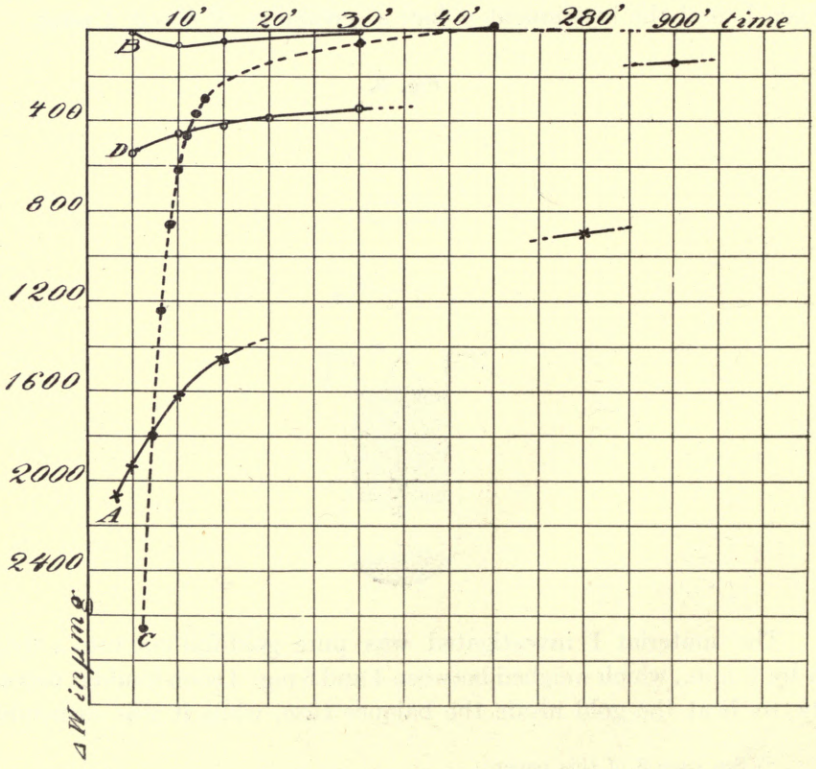


The material I investigated was pure gold-foil cut into a strip, 25 by 8 mm., which weighed between 4 and 5 mg. I soon found it necessary to heat the gold inside the balance case, while it was suspended

[1]) See page 8 of this paper.

from the beam, so that the first stages of the effect could be observed. The arrangement is sketched in figure 9. B is the weight-tube of glass attached to the balance floor; G is the piece of gold-foil carried by a triangle of silica suspended from the end-fibre. It is surrounded by a minute electric furnace, F, made from a structure of silica rods covered with platinum leaf. The temperature of the furnace was raised to bright red heat by sending through it the current from a battery of five storage cells. The heating could only be mantained for ten seconds at a time for fear of over-heating the glass tube. The balance was then immediately released, and scale-readings were taken at short intervals. The experiments were generally carried out at low pressures in order to avoid disturbances by convection currents from the heated walls of the tube.

Fig. 10.



Curve A in figure 10 (table V: A) gives an example of the observed variations, ΔW, in the weight, in μmg., plotted against the time, τ, in minutes, counted from the end of the heating. After 24 hours the apparent weight was still 320 μmg. lower than its original value, and the increase was still observable although its rate had fallen to 1/3 μmg. per minute, as compared to 70 μmg. between the two first observations.

Blank experiments were made with a piece of silica (table V: B) substituted for the gold. The result, curve B in the figure, proves that the silica was practically free from the same effect, the small observed variations being probably due to convection currents in the residual air. This fact, which is of considerable importance for the technique of the balance, has been referred to on page 26.[1])

Evidently matter of some kind is expelled from the gold by the heating and is again accumulated on it after it has grown cold. The simplest explanation seems to be, either that one or more of the gases in the air, or else some vaporous admixture to it, are occluded or adsorbed by the metal.

According to the first assumption, the effect ought to take place at a reduced rate in rarefied air. Experiments were therefore made in

ΔW in μmg. **Table V.**

A		B		C				D	
gold: $p = 31{,}6$ mm.		silica: $p = 27{,}6$ mm.		gold: p = vacuum; afterwards increased.				gold: $p = 26{,}2$ mm. filtered air.	
τ	ΔW	τ	ΔW	τ	ΔW	τ	ΔW	τ	ΔW
3′	—2070	5′	—16	6′	—2650	45′	+ 15	5′	—550
5′	—1930	10′	—72	7′	—1800	150′[2])	+ 25	10′	—460
10′	—1620	15′	—52	8′	—1240	180′[3])	+230	15′	—430
15′	—1460	30′	—16	9′	—860	185′	+265	20′	—390
280′	—900			10′	—620	190′	+290	30′	—350
1160′	—530			11′	—470	1060′	+470	900′	—140
1380′	—420			12′	—370	1180′	+510	(2880′	0)
1680′	—320			13′	—300	1330′	+520		
				30′	—60				

[1]) It must, however, be observed that in the first experiments the total heated surface, gold and silica, was considerably larger, say 10 times, than in the blank experiments.

[2]) At $\tau = 50'$ the pressure was increased to 27,0 mm. by slowly admitting fresh air through filter cooled to —190°.

[3]) A $\tau = 170'$ the pressure was for a few minutes raised to 80 mm. by admitting air which had for some days been left within the manometer. The pressure was then again reduced to 26,7 mm.

a vacuum, produced by putting the case in communication for about an hour with a large charcoal bulb cooled in liquid air. The results, exemplified by the broken curve *C* in the figure (table V: C), prove that the initial weight was practically regained after half an hour. The first large decrease may partly have been due to radiometer action. No further increase was observed when the pressure was raised to 27.0 mm. by admitting air, slowly filtered through a tube cooled in liquid air,

These results speak strongly in favour of the second explanation, as the diffusion of vapours to the gold will go on at a quicker rate the lower the pressure is in the case.

The only vapours which can come into account are those from the rubber-grease used to make the case and the weight-tube air-tight, and mercury vapours which might come into the case from the manometer. The latter alternative was excluded when practically the same effect was observed in air from which all traces of mercury were removed by passing it slowly through gold- and silver-leaves cooled in liquid air.

Finally I repeated the same experiments after the case had been washed out with air filtered in the same manner, while the furnace was being heated repeatedly as intensely as it could be with safety. In this way I hoped to drive off as much as possible of the condensed vapours of the rubber-grease which might adhere to the gold and to the walls of the glass tube. I then found that the first stage of the effect had fallen to about one fourth of the ordinary value, table V:D, curve *D* in the figure. The later stages of the increase went on much in the same way as before, indicating that the vapours were distilling back from other parts of the case.

I also made some experiments with my own balance and the first case (fig. 2 on page 12). The gold was suspended by a long silica fibre inside a small cylindrical brass box at the end of a brass tube, 50 cm. long, which depended from the balance floor as a kind of second weight-tube (not drawn in the figure). The gold was heated in the same way as before, but more intensely. Still the effect was much smaller than before, viz. about 10 % of its ordinary value. This is undoubtedly because the large heat capacity of the brass walls prevents the air inside from becoming so hot and saturated with the vapours as in a glass tube.

The results from these experiments can be summarized:

1. The phenomenon observed by Ramsay and Gray, that a piece of gold apparently gains in weight after it has been heated and suspended from the balance, has been observed and studied.

2. Silica does not show the same effect to an appreciable amount.

3. The experiments seem to prove that the effect is caused by vapours from the rubber-grease becoming condensed on the surface of the metal.

Evidently the method hitherto used has certain disadvantages. The temperature to which the gold is heated cannot be measured, and it may vary considerably from one experiment to another.

The weight-tube cannot be heated to high temperatures or cooled so that the vapours inside are condensed. Finally is it impossible to exclude all traces of rubber-grease from a case of the older type.

With the new case, described on page 13, I hope to be able to repeat these experiments with more definite results, and also to extend the investigation to other metals heated in different gases.

The phenomenon is of some importance for the technique of the balance, as it may cause errors when metallic objects are weighed on it. Another point is, that the supposed tendency of gold, and probably also of other metals, to condense low-volatile vapours on its surface may explain the well-known difficulty of obtaining perfectly pure metallic surfaces, a phenomenon which has been investigated, among others, by Quincke [23], by Lord Rayleigh [24], and quite recently by Seeliger.[1])

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§ 2. The magnetic susceptibility of gases.

The magnetic susceptibility, $\varkappa$, has such low values for all gases except oxygen (air) and nitrous oxide, that an experimental determination of this quantity meets with considerable difficulties. The discrepancies between the results found by different observers are so

[1]) Verh. d. D. Physik. Ges. *15* p. 950; 1913.

considerable that at present even the sign of $\varkappa$ is not definitely known for many gases.

The two methods which have hitherto been most frequently used for measuring $\varkappa$ were given by Faraday [25] and by Quincke [26]. By the first method the mechanical force acting on a certain quantity of the gas in a strongly divergent magnetic field is measured with a torsion balance. Quincke's method, which has been modified by Toepler [27], consists in observing the displacement of a liquid surface in one branch of a U-tube, which occurs when that branch is traversed by an intense magnetic field, the space over the liquid being filled with the gas.

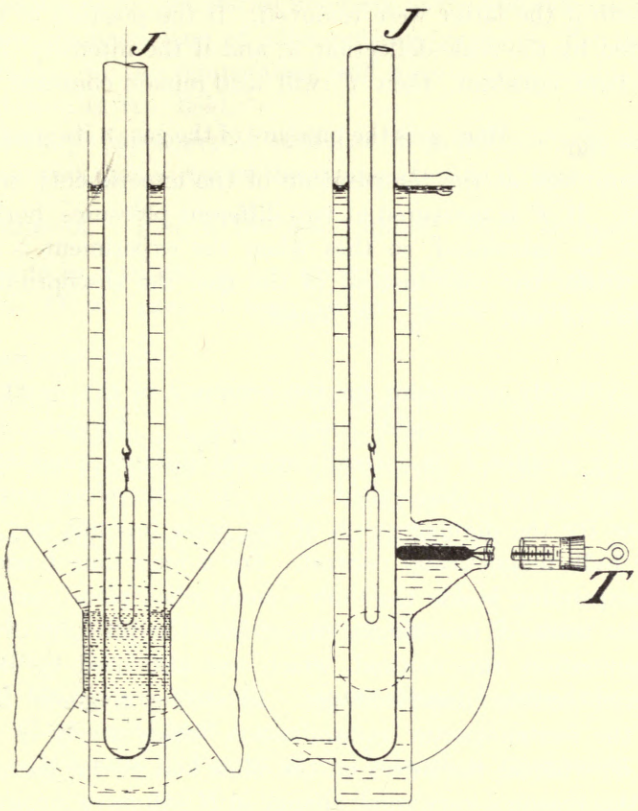
The highest sensibility hitherto attained by the balance method was realized in 1909 by Bernstein [28] who has measured $\varkappa$ for hydrogen, carbonic acid, and chlorine, and also for the vapours of some liquids. I must refer to his paper or to Winkelmann's Handbuch der Physik[1]) for a detailed account of previous experiments on this subject.

The high sensibility of the micro-balance, as well as the low magnetic susceptibility of the silica from which it is made, appear to make it a suitable instrument for magnetic investigations. I have therefore used it for relative measurements of the magnetic susceptibility of hydrogen and nitrogen referred to that of air, which is known with considerable accuracy. The balance and the case used in these experiments were the same as in fig. 3 on page 14, except that the charcoal bulb had been removed and the long silica tube exchanged for a narrower tube of Jena glass, 25 cm. in length. The arrangement given in fig. 2 for measuring the pressure within the case and for admitting air into it were used, the only alteration being that from the tube between the case and the manometer a side-tube led through a three-way tap to a mercury pocket. By means of the latter the investigated gases could be pumped from a gasometer into the case and *vice versa*. The whole system was found to keep perfectly airtight for days.

The lower end of the Jena tube, J, in figure 11, which was surrounded by a jacket-tube with circulating water, depended between the poles of a powerful electromagnet. For the loan of this instrument I

[1]) Vol. V p. 272; 2:d Ed. Leipzig 1908 (Barth.).

Fig. 11.



am indebted to Docent E. Holm [29] who has found it to be practically free from any temperature coefficient at room temperature.

A cylindrical silica bulb was suspended from the balance by a fine fibre, the lower end of the bulb being near the central axis of the field The ratio between the scale-distance (410 cm.) and the arm of the beam was about 82:1, so that a change in the scale-deflection of 1 mm. corresponded to a vertical displacement of the bulb of only 0,006 mm.

When a current was sent through the electromagnet there would be a vertical force[1]) due to the magnetic field acting on the bulb, given by $F = F' - F''$. Here F' denotes the force which would act on the bulb in a vacuum, while F'' is the «magnetic buoyancy» of the

[1]) A deduction of the magnetic force acting on a similar bulb is to be found in Piccards' paper [22].

surrounding gas, i. e. the force which would act on the gas displaced by the bulb if the latter were removed. If the position of the bulb, as indicated by the scale-deflection, a, and if the intensity, H, of the field are kept constant, then F' will also remain constant, whereas $F'' = C \cdot \frac{p}{760} \varkappa$. Here p is the pressure of the gas, $\varkappa$ its susceptibility (at 760 mm. and at the temperature of the experiment), while C is a constant. If F is measured at two different pressures, both F' and $C \cdot \varkappa$ can be calculated, so that when the experiment is repeated with air within the case instead of the gas, the susceptibility relative to that of air, $\varkappa : \varkappa_{\text{air}}$, is found.

It is evidently important for the accuracy of the result that F' should be as small as possible compared to F''. In spite of the low susceptibility of silica F' is normally several hundred times larger than F'', even with a very thin-walled bulb. It is therefore necessary to compensate for the diamagnetism of the silica in some way, so as to make the bulb magnetically neutral. For a long time I tried to do that by suspending beside the bulb a small silica piece contaminated with minute traces of iron filings. Apart from the difficulty of «dosing» the contamination, this method, which was also used by Bernstein, suffers from serious disadvantages. In strong magnetic fields the force on the ferromagnetic contamination varies nearly as the first power of H, whereas the forces on the silica and on the gases vary as H^2. For this reason a very small change in H will have inconveniently great effects on the equilibrium. Owing to the swinging motion of the balance before it comes to rest when the field is on, the hysteresis of the contamination may give different values to F', even though H and a be kept perfectly constant. As I found it impossible to get accurate results by this method I have instead tried to use *paramagnetic* compensation, that is, to make the bulb neutral by filling it with a mixture of oxygen and air. This method is free from the disadvantages just mentioned and also makes it possible to «dose» the contamination. From two measurements in air at different pressures with a known quantity of oxygen enclosed in the bulb, F' and F'' are found. It is then easy to calculate an approximate value for the amount of oxygen that will make F' vanish. In this manner the bulb was made practically neutral already at the second attempt, when it was filled at normal pressure with an atmosphere of about 82 % O_2 + 18 % N_2.

The only disadvantage of this method is due to the fact that the paramagnetism of oxygen varies inversely as the absolute temperature, whereas the diamagnetism of silica remains constant. For this reason a small change in the temperature will have a considerable effect on the equilibrium in the field.

Preliminary experiments, made at 15°,00 and 16°,90 in air of low pressure, proved that a rise in the temperature of 1° gave a change in the deflection of $\frac{da}{dt} = + 135$ scale-units.[1]) More accurate experiments, made in nitrogen at low pressure (table X: D & E) between 16°,00 and 17°,00, gave a temperature coefficient in scale-units of $\frac{da}{dt} = + 140$ scale-units.

The temperature, t, could not be read with certainty nearer than to 0°,05 on the thermometer inserted into the water-jacket (see fig. 11.) Owing to the variations in the temperature of the room, t would sometimes vary by 0°,10 and even more in the course of the 20 to 30 minutes required for a series of 5 observations.

Length of the bulb = 40 mm. ; outer diam. = 3,5 mm. ; thickness of walls = 0,1 mm.
Weight of the bulb = 99,4 mg. ; Inner diam. of Jena tube = 9 mm. outer diam. = 11 mm.
Pole gap = 19,6 mm. ; Period of the balance = 38 seconds.

The sensibility of the balance was measured in air at low pressure. Two determinations gave for the change in the deflection caused by a variation of ± 1 mm. in the pressure: + 82,4 and − 83,6 scale-units. Mean value: $\frac{da}{dp} = + 83$.

The capacity of the bulb was calculated from its dimensions to be some 340 mm³. Hence the absolute sensibility of the balance was about 6,5 μmg.

Before being suspended from the balance the bulb had been cleansed in boiling sulphuric acid. In spite of this there were residual traces of ferromagnetic contamination, recognizable from the negative deflections produced at low values of H.

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[1]) A lifting force on the bulb and the corresponding change in the scale-deflection are counted positive.

Method of observation.

The load on the balance was adjusted until there would be equilibrium at a suitable pressure, p, of the gas, which was then admitted into the case. The pressure was adjusted so that when a current of a certain intensity, $i = i_0 = 2{,}800$ Amp., was sent through the electromagnet, the lightspot came to rest very near a certain scale-division, $a_0 = 1065$. The scale-deflection a'' was observed when the current i was on as well as immediately before ,a', and afterwards, a''', Generally a' and a''' differed by a few scale-units. A value in scale-units of the force F is given by $\delta a = a'' - {}^1/_2 [a' + a''']$, which must, however, be reduced to the standard values, i_0 and a_0, $(\delta_t a)$ as well as to a certain temperature, generally $16°{,}00$, $(\delta' a)$. The necessary corrections were generally small, their values being found from separate observations at which a, i, and t, were intentionally varied by relatively large amounts.

Similar sets of observations were made, either with the same gas or with air, after the load had been re-adjusted so as to give equilibrium at a low pressure. From the « magnetic deflection » (reduced to the standard values of a, i, and t) which was then observed, $\delta'' a$, the magnetic deflection in a vacuum $= \delta_0 a$ can be calculated by extrapolation. We then find the magnetic deflection, Δa, which would take place in the gas at *normal* pressure with a perfectly neutral bulb, from the equation:

$$34. \qquad \Delta a = \frac{760}{p} [\delta' a - \delta_0 a].$$

This last quantity is directly proportional to the magnetic susceptibility of the gas: $\Delta a = C_1 \varkappa$.

Errors of experiment.

The current through the electromagnet was taken from a battery of storage cells. Its intensity, i, was read to $0{,}002$ Amp. on an ampèremeter from Siemens and Halske. Care was taken to avoid all errors from hysteresis of the iron core.

The temperature of the bulb was not directly observed, but experiments showed that an abrupt variation in the recorded temperature had an almost instantaneous effect on the magnetic deflection. The «lag»

in the temperature change of the bulb relative to that of the thermometer must therefore have been vanishing. The thermometer could not be read with certainty nearer than to 0°,05, corresponding to a possible error of $\pm$ 7 scale-units, which is about 15 % of the observed magnetic deflection for hydrogen. By far the greatest part of the existing uncertainty in the present results must therefore be put down to temperature variations.

Air.

The air was admitted into the case in the ordinary way (page 19), being thus purified from moisture, CO_2 and dust. At higher pressures the magnetic deflection, which would then have been too large for direct measurements, was compensated in the following manner.

The lightspot was brought to rest near a_0, and the pressure measured and corrected to the value p' which it ought to have for $a = a_0$. It was then reduced so far that the increased effective weight of the bulb was just compensated by the lifting forces produced by the magnetic field when a current of normal intensity, $i = i_0$, was put on. The observed value of the pressure was corrected to p'', corresponding to the standard values a_0 and i_0. Finally the current was broken and the pressure increased until equilibrium was regained at p''', reduced to a_0. The time required for such a set of observations being from 15 to 30 minutes, p' would generally differ from p''' by a small fraction of a mm. owing to temperature variations. The exact time for each scale-reading was therefore noted and an intermediate value, p_m, calculated which would correspond to the moment when the scale-reading at p'' was taken. The difference, $\delta_t p = p_m - p''$, was corrected to $\delta' p$ at normal temperature, 16°,00. This gives the value of F, expressed in «mm. of air», from which its value, $\delta' a$, in scale-units is calculated by multiplication with the sensibility: $\delta' a = 83 \cdot \delta' p$.

Observations made by the other method at low pressure give $\delta'' a$ and, by extrapolation, $\delta_0 a$, the «magnetic vacuum deflection», so that the value at atmospheric pressure, Δa, can be calculated from equation 34.

In the following text and in tables VI to X the magnetic deflections, δa and $\triangle a$, are given in scale-units, the pressures, p

and δp, as usual in mm. Hg. The sign * after a temperature value indicates that the actual temperature was estimated to be about $0°{,}025$ below the recorded value.

Table VI: A gives the results from a series of observations made with air at an average pressure of $p'' = 442{,}48$ mm. From the mean value, $\delta' p = 65{,}03 \pm 0{,}06$ mm., we find $\delta' a = 5397 \pm 5$. The corresponding determinations at low pressure (recorded later on in table IX: A & C) gave a mean value of $\delta_o a = -1$, so that:

$$\Delta a = \frac{760}{442{,}48} \cdot [5398 \pm 5] \quad \text{or} \quad \Delta a = 9272 \pm 9.$$

Another series of observations at $p'' = 514{,}92$ mm., table VI: B, gave a mean value of $\delta' p = 76{,}49 \pm 0{,}08$ mm. or $\delta' a = +6349 \pm 7$. Measurements made afterwards at low pressure (table X: D) gave $\delta a_o = +14 \pm 1$ Hence:

$$\Delta a = \frac{760}{514{,}92} \cdot [6335 \pm 8] \quad \text{or} \quad \Delta a = 9350 \pm 12.$$

Between the two values for Δa there is a small difference of less than 1 %, which is, however, considerably larger than the probable errors added together. The greatest part of this difference is, no doubt, due to a small variation in the partial pressure of the oxygen in the room. For very exact measurements it would evidently be advisable to use pure oxygen as a standard gas instead of air.

Table VI.

Air									
A					B				
pm	p''	$\delta_t p$	t'' [1])	$\delta' p$	pm	p''	$\delta_t p$	t'' [1])	$\delta' p$
507,48	442,65	64,83	15°,95	64,91	591,28	514,91	76,37	16°,00	76,37
507,12	442,55	64,57	15°,80	64,91	591,16	515,06	76,10	16°,00	76,10
507,07	442,36	64,71	15°,75	65,13	591,47	514,95	76,52	15°,90	76,69
506,95	442,58	64,37	15°,70	64,88	591,47	514,91	76,56	16°,00	76,56
506,90	442,26	64,64	15°,60	65,31	591,51	514,77	76,74	16°,00	76,74
mean value:	442,48			65,03		514,92			76,49

The mean from these two values gives $\Delta a = 9310 \pm 50$ as the final value for air. This number also represents the sensibility of the arrangement which is about 18 times greater than that of Bernstein [28 p. 33]: $\Delta a = 53{,}1$.

[1]) t'' is the temperature observed at the pressure p''.

With pure oxygen the value of Δa would be about 45000. Taking the capacity of the bulb to be 90 % of its total volume, we find that the *enclosed* oxygen alone would cause a deflection of about:

$$-0{,}9 \cdot 0{,}82 \cdot 45000 = -33000 \text{ scale-units.}$$

If the temperature coefficient of $\varkappa$ for oxygen is taken to be $\frac{d\varkappa}{dt} = -\frac{1}{273}$, we find that the enclosed oxygen alone would give a temperature coefficient for the magnetic deflection of $\frac{da}{dt} = -\frac{1}{273} \cdot (-33000) =$ $= +120$ scale-units, a value which is not much lower than that found by actual experiments (page 44).

From his careful measurements of the susceptibility of oxygen Piccard [22] has calculated that of air, under the assumption that the susceptibility of the nitrogen in the air can be neglected. He finds (in absolute units):

at $t = 20°$; $p = 760$ mm. $\quad \varkappa_A = +294 \cdot 10^{-10}$ which gives:

at $t = 16°$; $p = 760$ mm. $\quad \varkappa_A = +302 \cdot 10^{-10}$

Experiments related in the following have given for the ratio at $16°{,}00$ between the susceptibility of nitrogen and of air:

$$\varkappa_N : \varkappa_A = -0{,}019.$$

We are therefore now able to apply a corresponding correction to Piccard's values and find the true susceptibility of air to be in absolute units:

at $t = 20°$; $p = 760$ mm. $\quad \varkappa_A = +290 \cdot 10^{-10}$

at $t = 16°$; $p = 760$ mm. $\quad \varkappa_A = +298 \cdot 10^{-10}$

Hydrogen.

The hydrogen was made by electrolysis of dilute sulphuric acid in the apparatus devised by Dr Gaede [33], who has found the gas produced in this way to be perfectly pure hydrogen. Before the electrolysis commenced the apparatus was evacuated and was afterwards repeatedly washed out at low pressure with the evolved gas. The hydrogen was collected over boiled-out water in a gasometer, from which

the water was allowed to escape into a large flask where it was kept under an atmosphere of hydrogen.

As an extra precaution the hydrogen was passed slowly over reduced, red hot copper gauze, and then through $P_2 O_5$ and dry cotton-wool before being admitted into the case. Each time the case and the manometer were filled with the gas, they were first washed out repeatedly at low pressure. The hydrogen used at the experiments can therefore be regarded as sufficiently pure for these measurements.

The arrangement was evidently not well adapted for an accurate measurement of the density, d_H, of the hydrogen. An approximate value for that quantity may, however, be derived from the fact, that the equilibrium pressure (at $a = 1000$ and $i = 0$) with hydrogen, which was measured at one set of experiments, was 579,80 mm. at 17°,5, while with air before and afterwards it had a mean value of 40,19 mm. (reduced to 17°,5). This gives for the ratio between the densities of hydrogen and of air: $d_H : d_A = 1 : 14{,}43$, which agrees fairly well with the value found by Morley: $d_H : d_A = 1 : 14{,}35$.

Table VII.

A			B		
Air at $p = 3{,}31$			Hydrogen at $p = 426{,}14$		
$\delta_t a$	t	$\delta'' a$	$\delta_t a$	t	$\delta' a$
+33	16°,00	+33	—18	16°,20	—46
33,5	16°,00	33,5	15,5	16°,20	43,5
32,5	16°,00	32,5	17	16°,25	52
40	16°,00	40	11	16°,30	53
			10,5	16°,35	59,5
mean value:		+34,8			—50,8
reduced to vacuum $\delta_o a =$		— 5,6			

Table VII: B gives the results from the first series of experiments with hydrogen at $p = 426{,}14$ mm., while table VII: A gives the value of $\delta'' a$ found with air at low pressure. Assuming the magnetic deflection caused by the presence of 1 mm. of air to be $\frac{9310}{760} = +12{,}2$ scale-units, we find $\delta_o a = -5{,}6$ The mean from table VII: B:

$\delta' a = -50{,}8$ gives for the magnetic deflection in hydrogen at atmospheric pressure:

$$\Delta a = \frac{760}{426{,}14} [-50{,}8 + 5{,}6] \quad \text{or} \quad \Delta a = -80{,}6.$$

Another set of experiments which were carried out with hydrogen at $p = 610{,}65$ mm. gave $\delta' a = -54{,}1$, (table VIII: A), while air at $p = 8{,}06$ mm. gave $\delta'' a = +102{,}5$, (table VIII: B) reduced to vacuum $\delta_0 a = +4{,}2$, so that:

$$\Delta a = \frac{760}{610{,}65} \cdot [-54{,}1 - 4{,}2] \quad \text{or} \quad \Delta a = -72{,}6.$$

Finally a series of observations made with hydrogen at $p = 456{,}69$ mm. (table VIII: C) gave $\delta' a = -64{,}1$; with hydrogen at $p = 10{,}99$ mm. $\delta a = -10{,}1$ (table VIII: D). We then find:

$$\Delta a = \frac{760}{456{,}69 - 10{,}99} [-64{,}1 + 10{,}1] \quad \text{or} \quad \Delta a = -92{,}1.$$

Table VIII.

A			B			C			D		
Hydrogen at $p = 610{,}65$			Air at $p = 8{,}0$			Hydrogen at $p = 456{,}69$			Hydrogen at $p = 10{,}99$		
$\delta_t a$	t	$\delta' a$	$\delta_t a$	t	$\delta'' a$	$\delta_t a$	t	$\delta' a$	$\delta_t a$	t	$\delta'' a$
−72,5	15°,90	−58,5	+93	15°,95	+100	+26,5	16°,35*	−61	0	16°,10*	−10,5
55	15°,95	48	99	16°,00	99	22,5	16°,65*	65	+2,5	16°,10*	8
59	15°,90	45	104	16°,00	104	23	16°,65*	64,5	4	16°,10	10
64,5	16°,00*	61	114	16°,05	107	23	16°,65*	64,5	6,5	16°,10	7,5
54,5	16°,05*	58	—	—	—	22	16°,65*	65,5	6,5	16°,15	14,5
mean value: −54,1					+102,5			−64,1			−10,1
reduced to vacum $\delta_0 a = +4{,}2$											

Three independent determinations of Δa have thus given: $-80{,}6$ $-72{,}6$ and $-92{,}1$, the differences being within the possible experimental errors. Taking the mean from these values we get the final result: $\Delta a = -81{,}8$ which gives for the ratio between the susceptibilities of hydrogen and air $\varkappa_H : \varkappa_A = -81{,}8 : 9310 = -0{,}0088$ Assuming the susceptibility of air at 16°,00 to be $\varkappa_A = 298 \cdot 10^{-10}$ we have for the absolute susceptibility of hydrogen at 16°,00 and 760 mm.:

$$\varkappa_H = -2{,}62 \cdot 10^{-10}.$$

The error in this result is probably not larger than 10 %.

This value is much lower than that of Bernstein, $\varkappa_H = -5{,}25 \cdot 10^{-10}$, but it agrees well with that found for liquid hydrogen by Kamerlingh Onnes & Perrier [29], $\varkappa_H = -2{,}4 \cdot 10^{-10}$, and also with that calculated by Pascal [30] from the diamagnetism of hydrogen compounds, $\varkappa_H = -2{,}6 \cdot 10^{-10}$. The present result is therefore a new proof in favour of the theory of M. Langenvin [31] who assumes diamagnetism to be an atomistic quality, which is independent of the physical conditions.

Nitrogen.

The nitrogen was made from an aqueous solution of pure $K\,NO_2$ and NH_4Cl gently heated in a Jena flask on a water-bath, the temperature of which was never raised over +80°. The gas was passed through strong chromic-sulphuric acid, then through dilute ammonium hydrate, and finally over red hot copper. The whole system was washed out several times at low pressure, and the gas was collected in a gasometer in the same way as with hydrogen, except that the gasometer water was slighty acidulated in order to remove traces of NH_3. Before it was introduced into the balance case, the gas happened to become contaminated by a few small air-bubbles coming accidentally into the gasometer. Preliminary measurements made with this slightly impure nitrogen are set out in table IX. A and C give the determinations of $\delta''a$ at low pressure, while B gives $\delta'a$ measured at $p = 530.83$ mm. The mean from A and C being $\delta''a = -6{,}7$ at $p = 29{,}28$ mm. we find:

$$\Delta a = \frac{760}{530{,}83 - 29{,}28}\,[-106{,}5 + 6{,}7] \qquad \text{or} \qquad \Delta a = -151{,}3$$

a result which must evidently be a little too low numerically, owing to the traces of oxygen present in the gas.[1])

[1]) The total volume of the air-bubbles was estimated to be less then 2 cm³, i. e. less than 0,1 % of the total volume of the nitrogen. Hence the error in Δa due to the contamination should have been less than 5 %.

Table IX.

A			B			C		
Nitrogen at p = 29,39			Nitrogen at p = 530,83			Nitrogen at p = 29,26		
$\delta_t a$	t	$\delta'' a$	$\delta_t a$	t	$\delta' a$	$\delta_t a$	t	$\delta'' a$
—12,5	15°,95	—5,5	—110	16°,00*	—106,5	—15	15°,95	—8
14,5	15°,95	7,5	114	15°,95	107	19,5	15°,90	5,5
15	15°,95	8	115,5	15°,95	108,5	21,5	15°,90	7,5
20,5	15°,90	6,5	116	15°,95*	105,5	20,5	15°,90	6,5
19	15°,90	5	119	15°,90	105	—	—	—
mean value:		—6,5			—106,5			—6,9

Next time the nitrogen was admitted into the case it was first purified by being passed slowly over reduced copper at red heat. The first two series (table X: A & B) gave: $\delta'' a = -4{,}3$ at $p = 29{,}34$ mm. and $\delta' a = -151{,}7$ at $p = 614{,}90$ mm., so that:

$$\Delta a = \frac{760}{614{,}90 - 29{,}34}\,[-151{,}7 + 4{,}3] \quad \text{or} \quad \Delta a = -191{,}3$$

Two other series made on the following night, (X: C & D) gave: $\delta' a = -132{,}3$ at $p = 617{,}17$ mm; $\delta'' a = +7{,}4$ at $p = 26{,}89$ mm. Hence

$$\Delta a = \frac{760}{617{,}17 - 28{,}62}\,[-132{,}3 - 7{,}5] \quad \text{or} \quad \Delta a = -180{,}1$$

Table X.

A			B		
Nitrogen at p = 29,34			Nitrogen at p = 614,90		
$\delta_t a$	t	$\delta'' a$	$\delta_t a$	t	$\delta' a$
—3	16,°05*	—6,5	—148	16°,00	—148
+1	16°,05*	2,5	151,5	16°,00	151,5
—1	16°,05*	4,5	151	16,°00	151
—0,5	16°,05*	4	154,5	16°,00	154,5
—0,5	16°,05*	4	153,5	16°,00	153,5
mean value:		—4,3			—151,7

C			D			E		
Nitrogen at p = 617,17			Nitrogen at p = 28,62			Nitrogen at p = 26,89		
$\delta_t a$	t	$\delta' a$	$\delta_t a$	t	$\delta'' a$	$\delta_t a$	t	$\delta'' a$[1])
—128,5	16°,00	—128,5	—1,5	15°,90	+12,5	+158,5	17°,05	+151,5
130,5	16°,00	130,5	5	15°,90	9	151,5	17°,05*	148
131	16°,00	131	7	15°,90	7	150,5	17°,05*	147
135	16°,00	135	13	15°,90	4,5	150	17°,05*	146,5
136,5	16°,00	136,5	16,5	15°,85	4,5	149	17°,05*	145,5
mean value:		—132,3			+7,5			+147,6
					reduced to $p = 28{,}62$: $\delta'' a =$			+147,2

[1]) Reduced to 17°,00, instead of to 16°,00.

The mean from these three values for Δa being $-$ *174,2*, we find for the relative susceptibility of nitrogen: $\varkappa_N : \varkappa_A = -174{,}2 : 9310$, and for its absolute value:

$$\varkappa_N = -5{,}57 \cdot 10^{-10}$$

with an error which is probably lower than 10 %. The corresponding value calculated by Pascal is:

$$\varkappa_N = -5{,}0 \cdot 10^{-10}.$$

The column E in table X gives the results from measurements made at 17°,05 in nitrogen of $p = 26{,}89$ mm. The mean value of $\delta''a$ (which is corrected to 17°,00) together with $\delta''a$ in column D gives the temperature coefficient for the magnetic deflection before mentioned (page 56): $\frac{da}{dt} = +140$ scale-units.

Extension of the investigation.

I hope to be able to repeat these experiments with a more efficient arrangement for keeping the temperature constant. It does not appear impossible that a vacuum vessel may be made sufficiently narrow to be used instead of the water-jacket (fig. 11). It will then probably be possible to keep the temperature constant and to measure it thermo-electrically to within 0°,01. The sensibility of the present arrangement will then allow me to measure $\varkappa$ for the same gases to 1 %. Besides it would be comparatively easy to reach a sensibility ten times higher by increasing the period of the balance and the intensity of the field. It is, however, doubtful whether it will be possible to keep the temperature constant to a corresponding degree (0°,001).

With such an improved arrangement I hope to be able to extend these experiments also to other gases, in the first place to gaseous NH_3, in order to see whether its susceptibility will be equal to that calculated from $\varkappa_N$ and $\varkappa_H$ according to the additive law. It would also be interesting to investigate the susceptibility of ozone, or rather of ozonised oxygen. Finally I intend to try whether an exposure of a gas to intense light will have any effect on $\varkappa$. It does not appear

impossible that a diamagnetic gas like NH_3 may acquire a temporary paramagnetism when exposed to radiation of a wave-length for which it has selective absorption.

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§ 3. The influence of temperature upon weight.

The first attempts to find by exact experiments whether the weight of a body varies with its temperature, or rather whether heat has a weight of its own, were made more than a century ago (1785) by Count Rumford [34]. In one instance he weighed a ball of pure gold placed in a porcelain cup on a sensitive balance, first cold, and then while it was still nearly white hot after having been heated in a furnace. A small diminution was observed which amounted to only one forty-thousandth part of the weight of the gold. Control experiments, in which the cup was covered in different ways, gave so variable values for the decrease as to make Rumford attribute the whole of the effect to convection currents in the air. Assuming the highest temperature at these weighings to have been between $+700°$ and $+800°$, we find his experiments prove that for each degree the temperature of the gold was raised, its weight could not have varied by more than one part in thirty millions; $\frac{dW}{di} > 1 : 3 \cdot 10^7$. This result illustrates the admirable precision with which the great physicist worked.

The same problem was attacked in 1905 by Poynting and Phillips [35] with all the resources of modern experimental research. A brass cylinder, weighing 266 grammes, was suspended from one arm of a highly sensitive balance by a fine wire inside a long tube depending from the balance floor. The lowest part of the tube and the cylinder inside could be heated with steam to $+100°$ or cooled in liquid air. Although the experiments were carried out at reduced pressure, the residual air produced relatively large deflections from the equilibrium. These errors the authors tried to evaluate by making control

experiments with hollow brass cylinders of the same surface. Other large corrections were necessary for the shift of the equilibrium caused by the heat spreading to the upper parts of the case. Poynting and Phillips give as final result from their experiments, that between room temperature and + 100° $\frac{dW}{dt} < 1{:}10^9$. Experiments carried out between room temperature and — 190° gave smaller variations and a higher accuracy, $\frac{dW}{dt} < 1{:}10^{10}$.

About the same time Southerns [36] investigated the same subject by the method of «internal heating». The interior of a calorimeter suspended from a balance was heated electrically, and readings of the equilibrium were taken before the heat had had time to spread to the external surface of the calorimeter. I must refer to Southerns' paper for further details of this very ingenious method. He finds that within the rather narrow temperature interval of his experiments (about + 30° from room temperature) no change took place greater than one part in 10^8 per 1°. At the end of his paper Southerns criticizes the experiments of Poynting and Phillips on account of the considerable corrections which they were obliged to make in their results. On the whole he seems justified in asserting that these authors have overrated the accuracy of their method, and that from the existing experimental evidence one could not say with certainty that weight is independent of temperature to a smaller fraction than $1{:}10^8$.

Experiments with heated silica.

It appeared to me that an extension of these experiments to much higher temperatures would be of considerable interest. The arrangement I worked with, which is set out in figure 3 on page 14, was similar to that of Poynting and Phillips, though on a much smaller scale.

A sphere, s, of optical silica, 4,5 mm. in diameter and weighing 103 mg, was suspended from the micro-balance by a very fine silica fibre, 50 cm. long, so that the sphere was hanging a few cm. from the bottom of the long silica tube T. Round the lowest part of the tube a cylindrical electric furnace, U, was placed, the silica sphere hanging about half way down its heated part, which was lined with copper-foil

in order to make the temperature inside more uniform. The temperature of the furnace was read to $\pm$ 5° with a thermo-couple of copper-Konstantan and a Pye millivoltmeter. One of the junctions was kept in contact with the wall of the silica tube at the same level as the centre of the sphere, the other junction being kept at 0° in a vacuum vessel filled with ice. The thermo-couple was standardized by taking the melting points of three pure metals from Kahlbaum (Sn, Zn and Sb). There was no need to measure the temperature very exactly, and there may be errors in the highest of the recorded values amounting to $\pm$ 10°.

The furnace was heated by the current from a battery of storage cells of 140 volts. The temperature could be kept practically constant for hours.

By this arrangement only the temperature of the wall of the silica tube, and not that of the sphere itself, was measured. Evidently the low thermal conductivity of the silica wall and of the vacuum inside must cause a certain « lag » in the temperature curve for the sphere, which will still rise or fall for some time after the furnace temperature has become constant.

Under the assumption that the sphere is only heated by radiation, and that silica radiates and absorbs like a black body under the conditions of the experiments, a maximum value for the « lag » can be calculated from Stefan's law. Taking Kurlbaum's [37] value for the radiation constant ($= 1{,}277 \cdot 10^{-12}$), I have found that if the furnace temperature were abruptly raised from + 17° to + 500°, the temperature of the sphere would be within 1° of the latter value after about a quarter of an hour[1]). As the furnace was heated gradually at the experiments, the actual « lag » in the temperature of the sphere must have been much smaller. The difference between the two temperatures should therefore have been negligible after the former had been kept constant for a quarter of an hour.

No attempts were made in these experiments to measure the pressure which, owing to the leakage[2]), must have increased steadily in the course of each set of experiments. Taking the leakage in-

[1]) In the experiments of Poynting and Phillips the corresponding time was about 3 hours. [l. c. p. 453.]

[2]) See page 16 of this paper.

to the case to amount to 0,1 mm. Hg. an hour, about 0,2 cm^3. of air would in that time come into the case, the volume of the latter being some 1500 cm^3. In order to keep the pressure constant the same amount of air must evidently be carried through the wide glass tube leading to the charcoal bulb. Assuming the pressure over fresh charcoal cooled in liquid air to be vanishing, I have calculated from Knudsen's formula for molecular effusion [38] that the initial equilibrium pressure within the case should have been of the order 10^{-4} mm. Hg. As the charcoal went on absorbing air it must of course get partly saturated, and the pressure in the case would rise accordingly.

Disturbing influences.

Apart from the possibility of any real change in the gravitational force acting on the silica sphere being caused by the heating, there were several disturbing factors which might produce temporary or permanent shifts of the equilibrium position of the balance:

1. The hygroscopic power of the silica
2. Radiometer action
3. Electrostatic charges
4. Occluded gases driven off from the heated sphere
5. Volatilisation of the silica.

I had been warned by the experience of Ramsay and Gray not to have any $P_2 O_5$ present inside the case, nor did I want to use the less effective $Ba\ O$, as I did not know how it would behave at the extremely low pressure I hoped to reach. The air within the case must therefore have held a certain amount of water vapour.

The hygroscopic power of silica has been the subject of careful investigations by Brigg [39] who found that of the water absorbed by silica powder from moist air only a part was given off in air dried with $P_2\ O_5$ (temporary hygroscopic power), while another part was driven off by heating the silica to + 110° (permanent hygroscopic power). In this case one must therefore expect that moisture will be given off:

1 a. from the whole of the beam, the sphere, and its counterpoise, when the high vacuum is established with liquid air,

1 b. from the sphere alone, the first time it is heated after exposure to moist air.

As the total surface of the irregulary shaped silica pieces by which the sphere was counterpoised was considerably larger than that of the sphere, a larger quantity of moisture must be liberated from them than from the latter. The first effect must therefore produce an apparent *increase* in the weight of the sphere, whereas the second effect must naturally be followed by a *decrease*.

By separate tests of the constancy of the zero of the balance, I found that after it had been left for a short time in air at reduced pressure (a few mm. of Hg), the lightspot took up a definite position on the scale, so that perfectly identical scale-readings were taken for several days, although the beam was frequently disturbed.

2. The well known investigations by Knudsen [40] prove that the radiometric force between two surfaces of unequal temperature in a rarefied gas increases with the temperature difference and with the pressure (within certain limits). In accordance with this I found the irregular variations of the equilibrium of the balance to be much greater while the temperature of the furnace was still rising or falling, than after it had become constant. They were also much less pronounced at the beginning of a set of experiments than a few hours later, when the pressure must necessarily have been higher.

In the first two sets of observations I made on the first night, the charcoal was quite fresh and the leakage less than what it appeared to become afterwards. Also the variations in the scale-readings were then very small.

3. Electrostatic charges on the beam were avoided in the same way as before by putting directly below it a silica crucible with $U_3\,O_8$ which had previously been ignited to red heat.

Considering the comparatively low temperature reached at these experiments, it does not appear possible that any spontaneous emission of electrically charged particles could have taken place from the heated sphere.

4 and *5*. Unlike the other disturbing factors the two last will cause a real loss in the weight of the sphere, indicated by a permanent shift of the lightspot on the scale, which will not vanish after normal temperature has been regained, except that in the case of *4* a new quantity of gas may be occluded when air is again admitted into the case.

Method of observation.

Although the balance suspended inside the new case is much less influenced by the tremors in the building than with the old case, I found it preferable to make these experiments also at night, as it was necessary to take very accurate readings on the scale.

At first a Gaede mercury pump in communication with the charcoal bulb and the case was worked for about an hour, while the bulb was being heated to a temperature near the softening point of the glass. After that the mercury stop-cock, C in figure 3, was turned off, and the bulb was allowed to cool before it was immersed in a vacuum vessel with liquid air. The balance was then left for some time, occasional readings on the scale being taken in the meanwhile.

During the whole of the experiments the balance was left released. Its behaviour in the high vacuum was rather interesting and has already been referred to on page 43. It kept up a swinging motion of an amplitude, which for half-an-hour or more remained practically constant at $\pm$ 4 to 5 scale-units. Instead of taking the rest position of the lightspot, as at higher pressures, its extreme positions were read. Each recorded value of the scale-deflection is therefore the mean of three such observations, exactly as in weighings with an ordinary balance. The sensibility was about 4 μmg.

The balance case and the vacuum vessel which held the charcoal bulb were protected by screens of bright tin from becoming heated by radiation and air-currents from the furnace.

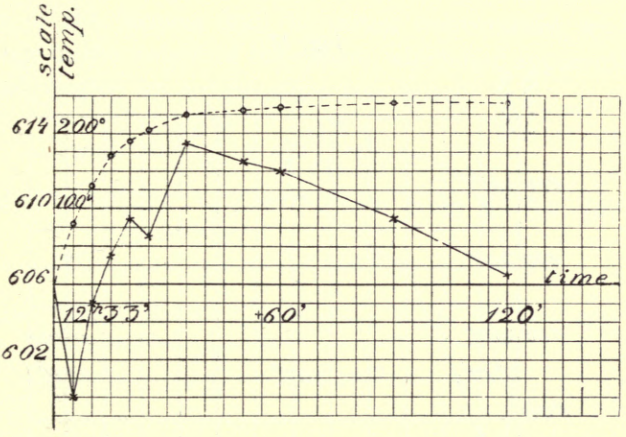
The first sets of experiments were begun in the evening of the 5th of April, 1913, and were continued until 5 o'clock the next morning. The observations are set out in tables XI and XII; the two first columns give the number of each observation and the time (to ½ minute) when it was taken. The third column gives the current through the furnace, the fourth its temperature, while the fifth gives the scale-readings, all taken while the balance was swinging, with the exception of the two first. An increase of one scale-unit denotes a gain in the weight of the sphere of 4 μmg.

Discussion of the observations.

It is necessary to examine the results in detail, as they were slightly complicated by some of the disturbing influences before mentioned.

The balance had for weeks been kept in air at a low pressure. A considerable part of the moisture bound by the temporary hygroscopic power of the silica must therefore have been given off before the beginning of the experiments.

Fig. 12.



The small apparent increase, observed after the liquid air was applied, at $10^h 53'$ p. m., (from No. 1 to No. 3), was evidently caused

Table XI.

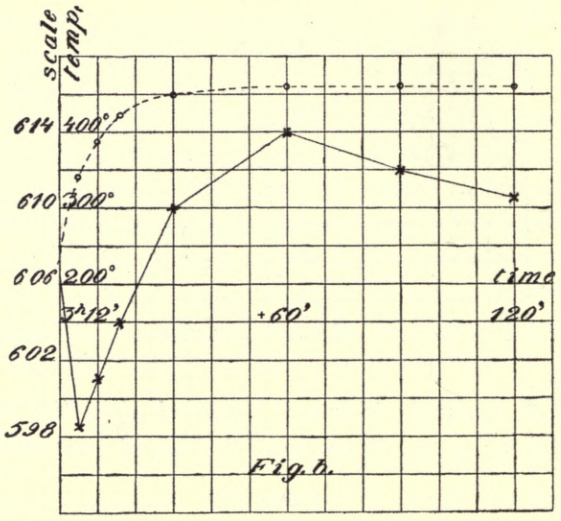
No.	time	current (in Amp.)	temp.	scale a	ΔW[1] (in μmg.)
1	$7^h 15'$ pm.	—	(+18°)	580	—
2	10 53[2])	—	—	—	—
3	11 23	—	—	607	—
4	12 10 am.	—	—	605	—
5	12 33	0,90	—	—	—
6	12 38	0,88	80°	600	—24
7	12 43	0,88	130°	605	—4
8	12 48	0,88	170°	607,5	+6
9	12 53	0,88	190°	609,5	+14
10	12 58	0,88	205°	608,5	+10
11	1 08	0,88	225°	613,5	+30
12	1 23	0,87	230°	612,5	+26
13	1 33	0,87	235°	612	+24
14	2 03	0,87	240°	609,5	+14
15	2 33	0,87	240°	606,5	+2

[1]) counted from the zero = 606.
[2]) liquid air applied.

by traces of moisture being liberated from the beam and the counterpoise (see effect *1 a* on page 68).

A short time before the beginning of the experiments the furnace had been heated red hot for a quarter of an hour so as to make the sphere perfectly dry. The mean value, $a = 606$, from the two first scale-readings in the table can therefore be taken as the true zero of the balance in a high vacuum. The sixth column of tables XI and XII, headed $\triangle W$, gives the apparent variations in μmg. from the normal weight of the sphere as defined by that zero position. The

Fig. 13.



same variations are plotted against time in the figures 12 and 13, full-drawn curves; the broken curves represent the furnace temperature.

Table XII.

No.	time	current (in Amp.)	temp.	scale a	ΔW[1]) (in μmg.)
16	3h12'	1,49	—	—	—
17	3 17	1,49	340°	598,5	—30
18	3 22	1,48	385°	601	—20
19	3 28	1,48	420°	604	—8
20	3 42	1,48	450°	610	+16
21	4 12	1,47	460°	614	+32
22	4 42	1,47	460°	612	+24
23	5 12	1,47	460°	610,5	+18

[1]) counted from the zero = 606.

The variations in these figures and tables are seen to be very small and to be almost identical in character. The latter circumstance indicates that they were mainly due to radiometer forces, produced while the temperature was rising by its local variations within the silica tube.

For reasons already mentioned it was quite sufficient to keep the furnace temperature constant for half an hour before the final scale-reading was taken. Dividing the final value of ΔW by the weight, W, of the sphere multiplied by the difference, t—18, between the temperature of the furnace and the initial temperature, we find the relative change in the weight of the sphere, which had apparently taken place for each degree its temperature was raised:

$$\frac{dW}{dt} = \frac{\Delta W}{W\ (t-18)}$$

Average value from $+18°$ to $+220°$ (from table XI No. 15)

$$\frac{dW}{dt} = +\ 1 : 10^{-10}$$

Average value from $+18°$ to $+460°$ (from table XII No. 23)

$$\frac{dW}{dt} = +\ 4 : 10^{10}$$

Most probably also these minute displacements were only apparent and caused by radiometer forces, so that they would have vanished under more ideal experimental circumstances i. e., in a higher vacuum and with a more uniform furnace temperature.

Considering that the largest temporary variation observed at the second set of experiments (no 21 in table XII) was less than twice the final value, we are justified in concluding from these experiments that:

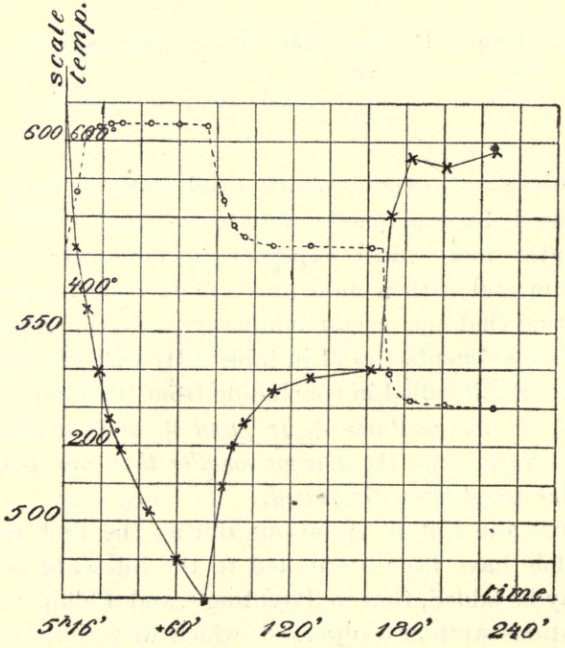
Between the temperatures $+18°$ *and* $+460°$ *weight is constant to a fraction which is on the average smaller than one part in* 10^9 *for each degree the temperature is raised.*

As regards the initial variations during the first period of the heating, which have been attributed to the influence of radiometer action, it may be added, that in Poynting's and Phillip's experiments similar variations were also observed, which were, however, relatively much larger than those dealt with in the present series of experiments. In one of their series a temporary shift of the equilibrium was recorded which was 60 times greater than the final corrected value.

Experiments at higher temperatures.

Immediately after the end of the observations recorded in table XII the current was increased to 1,98 Amp. Already before the first scale-reading could be taken, table XIII, fig. 14, the lightspot had started travelling over the scale, indicating an apparent loss in weight. Instead of taking up an approximately constant position after the temperature had ceased to rise, the lightspot went on moving at a rate which soon became nearly uniform; see last column in the table. When the current was again reduced to 1,48 Amp. the lightspot stopped and began to travel to the opposite direction, but this time it came nearly to rest soon after the furnace had assumed a constant temperature. A similar shift occurred after the current had been brought down to 0,88 Amp.

Fig. 14.



The last two *limited* shifts of the lightspot were undoubtedly caused by radiometer action. That they were so much larger than

the variations in the two first sets of observations indicates that the pressure had risen considerably in the course of the night, the charcoal being evidently partly saturated with absorbed air.

Table XIII.

No.	time	current (in Amp.)	temp.	scale a	$\frac{\delta \Delta W}{\delta \tau}$ (in μmg./min.)
24	$5^h 16'$	1,97	—	610	—
25	5 22	1,97	530°	572	—25
26	5 28	1,97	610°	556	—11
27	5 34	1,97	620°	539,5	—11
28	5 40	1,96	625°	527	—8
29	5 46	1,96	625°	519	—5
30	6 01	1,96	625°	503	—4
31	6 16	1,96	625°	490	—3
32	6 31	1,96	625°	479,5	—3
33	6 35	1,47	—	—	—
34	6 40	1,47	525°	509,5	+26
35	6 45	1,47	490°	520	+8
36	6 51	1,47	475°	526	+4
37	7 06	1,47	460°	534,5	+2
38	7 26	1,47	460°	538	+0,7
39	7 58	1,47	460°	540	+0,3
40	8 03	0,88	—	—	—
41	8 08	0,88	295°	580,5	+32
42	8 19	0,88	260°	596	+6
43	8 37	0,88	255°	593,5	—0,6
44	9 03	0,88	250°	597,5	+0,6

zero before (in vacuum): $a = 606$ (mean from No. 3 & 4)
zero afterwards in vacuum: $a = 511$
permanent loss: $\Delta a = -95$; $\Delta W = -380$ μmg.

The *continued* shift observed at the highest temperature cannot be explained in the same way, but must be attributed to one or to both of the two last disturbing influences which have been mentioned, *4* and *5* on page 69. The true zero of the balance, which was taken the next night in a high vacuum, was also found to have undergone a permanent displacement, from scale-division 606 to 511, indicating a real loss in the weight of the sphere of 380 μmg. This decrease can only be explained:

either by a liberation at the high temperature of gases occluded in the sphere,

or by evaporation of the silica.

The power of occluding gases in measurable quantities does not seem to be in harmony with the other properties of fused optical silica. On the other hand it appears at first strange, considering the high melting point of silica (+1600°), that it should be volatile to a measurable degree already at +625°. Both the liberation of occluded gases and the evaporation would of course be favoured by the high vacuum.

Fig. 15.

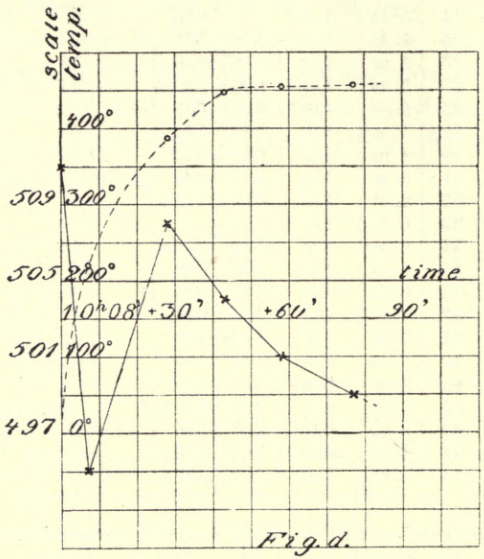


Table XIV.

No.	time	current (in Amp.)	temp.	scale a	ΔW[1] (in μmg.)
1	6h,00' p.m.	—	(+18°)	505	—
2	8 ,45[2]	—	—	—	—
3	10 ,06	—	—	511	—
4	10 ,08	1,48	—	—	—
5	10 ,15	1,43	220°	495	—66
6	10 ,36	1,47	390°	508	—12
7	10 ,51	1,46	450°	504	—28
8	11 ,06	1,47	455°	501	—40
9	11 ,25	1,47	455°	499	—48
10	11 ,53	0	—	—	—

zero before (at low pressure): $a = 505$
» afterwards (at low pressure): $a = 506$

[1]) counted from the zero 511.
[2]) liquid air applied.

Another set of experiments was made the next night in order to find out whether a permanent shift of the zero would be caused merely by heating the sphere to $+450°$, table XIV and figure 15. The charcoal was then no longer fresh, nor was it so carefully heated and exhausted as before the first experiments. The pressure must therefore have been higher, and the apparent variations were consequently also larger than before Still the difference between the initial zero-position in a high vacuum (XIV No. 3) and the last scale-reading does not give a higher value for $\frac{dW}{dt}$ than $-1{,}1 : 10^9$. The zero readings, taken before and after the heating at room temperature and at a pressure of a few mm., only differ by one scale-unit. We may therefore say that *heating the sphere to about* $+450°$ *for nearly an hour produces no permanent change in its weight.*

§ 4. The volatility of silica.

If the first of the two suggested explanations had been correct, one would expect the rate of the decrease to show signs of falling off when the heating experiments were repeated, as the amount of gas occluded in the sphere must soon become exhausted. Observations made in the same way during two of the following nights[1]) proved that this was not the case. The experiments from the first night are recorded in tables XV and XVI, figures 16 and 17. Considerable displacement of the lightspot, indicating a comparatively high pressure, occurred while the furnace temperature rose or fell. As before, these displacements were *limited* when the current was kept at about 1,5 Amp (temp. about 440°), while *continued* shifts were again observed when the current was increased to 2,0 Amp. (temp. = 600°). The total shift of the lightspot after these two nights amounted to 110 scale-units, corresponding to a further permanent reduction in the weight of the sphere of some 440 μmg.

In a final set of experiments the furnace was kept at $+785°$ for about an hour. The observations, given in table XVII and figure

[1]) Before these experiments the Nernstlamp had been displaced so as to bring the lightspot back to more central parts of the scale.

18, show that the rate of the decrease was perfectly constant to the end of the heating and nearly five times greater than at + 600°. The displacement of the zero afterwards was 69 scale-units, corresponding to a loss of 280 μmg.

Altogether these heating experiments had caused a total displacement of nearly 300 scale-units, indicating a reduction in the weight of the sphere of some 1100 μmg.

Afterwards air was admitted into the case until the pressure was atmospheric, in order to find out whether the sphere would regain in weight by occluding a fresh supply of air. However, on reducing the pressure to a few mm. after 30 hours, I found that the lightspot had undergone a small shift of 20 scale-units to the *opposite* direction, indicating an apparent *decrease* of 80 μmg, evidently a hygroscopic effect. If any re-occlusion of air really took place, it must therefore have been so small that it was masked by the other effect. Control readings, taken for several days afterwards at the same pressure, proved the final scale-reading to be absolutely constant (see end of table XVII.)

Summing up the experimental evidence hitherto produced we have:

a. Heating the silica sphere for an hour to + 450° did not produce any appreciable change in its weight (table XIV).

b. On heating the sphere to +600°, +625°, and +785°, a continued loss in weight was observed, at first rapid but after some time at a slower and nearly constant rate. (tables XIII, XV, XVI and XVII).

c. Part of this loss was *apparent*, and probably due to radiometer action, as it disappeared after cooling. Part of it was *real* and caused permanent displacements of the balance zero.

d. The rate of the loss rapidly increased with rising temperature. At +785° it was roughly five times its value at +600° (tables XV, XVI and XVII).[1])

e. Altogether matter weighing about 0,001 mg. was given off from the sphere, and still there were apparently no signs of any « exhaustion ».

[1]) It must be observed that the rates of evaporation at the two temperatures are not strictly comparable, as the pressure may have been higher in one set of experiments than in the other.

f. An exposure of the sphere for 30 hours to air at atmospheric pressure was not followed by any appreciable regain in weight.

It is, however, necessary to observe that the permanent loss in weight noticed after the very first intense heating (table XIII) was comparatively *greater* than after the other high-temperature series. It was also much greater than that which would correspond to the final and nearly constant rate; 3 μmg. for about 40 minutes making only 120 μmg. instead of the observed 380 μmg.

The last mentioned fact may be explained by the assumption that traces of occluded gases or of some volatile contamination were driven off from the sphere the *first time* it was heated red hot in a vacuum.

Otherwise the results are decidedly in favour of the second explanation viz., that the loss in weight is caused by an evaporation of the silica favoured by the high vacuum.

An absolutely decisive proof of the correctness of this assumption can only be given by prolonged heating experiments in a permanent high vacuum, where there are no disturbances due to radiometer action. I intend to undertake an investigation to that purpose with my new balance case after its reconstruction.

Judging from the present facts we are, however, brought to the conclusion that:

in a high vacuum silica is so volatile already at +600° *that its evaporation can be observed with a sensitive micro-balance. The rate of the loss from a silicia surface of* 0,6 cm^2 *at that temperature is of the order* 1,5 μ*mg. per minute. At* +785° *the loss is about five times more rapid.*

Finally the sphere was heated in air at a pressure of 5 mm., first to over +600° for five hours, and then to about +750° for more than two hours. The zero positions of the balance, taken at room temperature before and afterwards, gave $a = 477$ and $a = 460$. The small decrease of 17 scale-units or 68 μmg. cannot well have been due to hygroscopic effects, as the zero readings were taken under identically the same conditions. Hence it would appear that silica is really volatile at red heat also in air of 5 mm. pressure, although the rate of evaporation is then only a small fraction of what is it in a high vacuum, where a loss of about 1500 μmg. would have been caused by the same length of heating.

§ 5. Future experiments on the influence of temperature upon weight.

The fact that silica heated to + 600° loses in weight, is of a certain importance for the technique of future research on the constancy of weight. It would, of course, be of particular interest to carry out precision weighings at still higher temperatures. Unfortunately the volatility of silica practically excludes it from use at such an investigation, as the constancy of weight can not very well be tested on a substance which is partly volatilized away at the temperature of the experiments. It will probably not be easy to find a material which can be used for the weight-tube, the heated body, and the fibre by which it is suspended, up to temperatures at which complications due to electric phenomena must make exact weighings impossible.

Within the temperature interval + 18° to + 500° the sensibility of the present arrangement will enable me to test the constancy of weight to a fraction of 10^{-10} per degree, provided that all disturbances due to radiometer action are eliminated. It will, however, be comparatively easy, by increasing the sensibility of the balance and by reading the scale more accurately, to gain another decimal. A somewhat higher accuracy still would be realized, if the weight of the beam rel. to that of the sphere were reduced to one third or to one fourth, so that finer silica fibres could be used for the suspension. Consequently, if a rise of temperature *does* really cause a change in weight, it will be possible with an improved experimental arrangement of the type here described to detect and measure that effect, *if it is not less than of one part in* 10^{-11} *per degree.* If it is smaller still it seems to be inaccessible to our present experimental resources.

Theoretical.

Certain novel speculations which are based on the principle of relativity have led to the conclusion, that a quantity of energy, E, should have a kinetic mass of its own, equal to $\frac{E}{c^2}$, where c is

the velocity of light. Hence the kinetic mass of a body, and according to M. Langevin [41] also its weight, should vary when it is heated or cooled, or generally when is loses or gains energy in some form or other. According to this view "the temperature coefficient of weight" should be given by $\frac{dW}{dt} = \frac{1}{M} \cdot \frac{dM}{dt} = \frac{1}{c^2} \cdot \frac{1}{M} \cdot \frac{dE}{dt}$, where t is the temperature, M the mass, and $\frac{1}{M}\frac{dE}{dt}$ the specific heat of the body. For silica the average value of the last quantity from $+ 20°$ to $+ 500°$ is $0{,}22 \cdot 4{,}12 \cdot 10^7$ C. G. S. Introducing this in the equation we find $\frac{dW}{dt} = + 10^{-14}$. Unfortunately this supposed variation is about a thousand times too small to be detected even with a highly sensitive micro-balance.

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§ 6. Suggested experiments with the micro-balance.

Evidently several important problems in physical science might advantageously be attacked with the aid of the new instrument. Of these the following appear to me to be worthy of special attention.

The hygroscopic power of different materials can of course best be studied with a balance which affords possibilities of varying the pressure and the temperature of the surrounding gas while the body is being weighed. The influence of these two factors on the temporary and the permanent hygroscopic power of silica ought in the first place to be studied.

The adsorption and occlusion of other vapours and of gases is a related phenomenon which is not less important. My own experiments with heated gold were complicated by vapours from a low-volatile substance. With the new case, which can be used without rubber-grease, I intend to investigate whether ponderable quantities of the « permanent » gases are adsorbed on metals and glass. The supposed existence of a « gaseous film » also in a high vacuum is of great importance for photo-electric and other phenomena in rarefied gases[1]).

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[1]) Compare the remarkable investigations by Wiedman & Hallwachs on the photo-electricity of potassium in an ultra-vacuum. Verh. d. D. Physik. Ges. *16* p. 107; 1914.

The density of vapours and gases is particularly easy to study with an instrument worked by aerostatic compensation. As an example we may take a balance sensitive to 3 μmg. carrying a bulb of 0,5 cm^3. If it is at equilibrium in air or another gas of approximately the same density at a pressure of ½ atmosphere, a variation in the density of the gas of one part in a hundred thousand will produce a change in the deflection of one scale-unit. The precision of such a determination will therefore depend mainly on how accurately the temperature and the pressure of the gas are measured.[1])

The vapour tension of metals and of other solids may be measured with the micro-balance also at comparatively low temperatures. Suppose that a body which gives off vapours of the molecular weight 100 is weighed on a balance sensitive to 1 μmg. Allowing the vapours from the body to fill a volume of 30 $cm.^3$ at a certain temperature, then a loss in weight of one scale-unit would indicate a vapour tension of only $5 \cdot 10^{-6}$ mm. Hg.

I owe this suggestion to Professor Benedicks, who intends to have experiments performed on this subject in the near future.

The pressure of light, which has been measured with torsion instruments in the celebrated investigations by Nichols and Hull [42], and by Lebedew [43], might also be studied with the micro-balance. Suppose that light of the intensity employed by Nichols and Hull (1,5 cal. $cm.^{-2}$ $minute^{-1}$), were allowed to impinge from below on a reflecting disc suspended in a high vacuum from a micro-balance. Taking the sensibility of the latter to be 0,5 μmg., the radius of the disc to be 0,5 cm., and its reflection coefficient to be 0,85, we find that the pressure of the radiant energy would cause a deflection of some 100 scale-units.

Theoretically it is therefore possible, even without an extreme sensibility of the balance, to *weigh* the pressure of so intense light to less than one per cent. In practice the chief difficulty will be to avoid errors from radiometer action. Within a short time I hope to be able to make attempts in this direction.

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[1]) The density of neon has recently been measured by Aston with the aid of a micro-balance of the Steele & Grant type. Roy. Soc. Proc. *A 89* p 439; 1914.

IV. Summary.

The following are the main facts set out in this paper.

I. A micro-balance of a new type, suspended by two silica fibres, has been constructed. It is worked by aerostatic compensation, according to the method invented by Gwyer and by Steele & Grant, or also with the aid of «difference weights».

II. The technique of the new instrument has been studied and developed. Its properties have been investigated theoretically and by experiments. It has been found superior in certain respects to micro-balances of older types and also to torsion instruments.

III. A new type of case with magnetic arrestment has been constructed for the micro-balance. With this case an object can be weighed in a charcoal vacuum, both at high and at low temperatures, and also in a strong magnetic field.

IV. Preliminary experiments on the apparent variations in the weight of gold produced by heating it have been made with a micro-balance of the older type and have led to a plausible explanation for the cause of the phenomenon.

V. A balance suspended inside the new case has been used for measuring the magnetic susceptibility of hydrogen and of nitrogen relative to that of air. The sensibility of the arrangement was considerably higher than that attained by any previous experimenter. The results give additional support to the theory of a diamagnetic gas propounded by Langevin.

VI. A sphere of silica has been weighed in a high vacuum at room temperature and at $+460°$. The results prove that within that interval of temperature weight remains constant to less than one part in 10^9 for each degree the temperature is raised.

VII. Similar experiments at higher temperatures have given strong proofs that silica is volatile to a measurable degree already at +600° in a high vacuum. In air at 5 mm. the rate of evaporation is scarcely measurable even at +750°.

VIII. Possible improvements and extensions of the foregoing investigations have been discussed and other subjects for future research suggested.

Fig. 16.

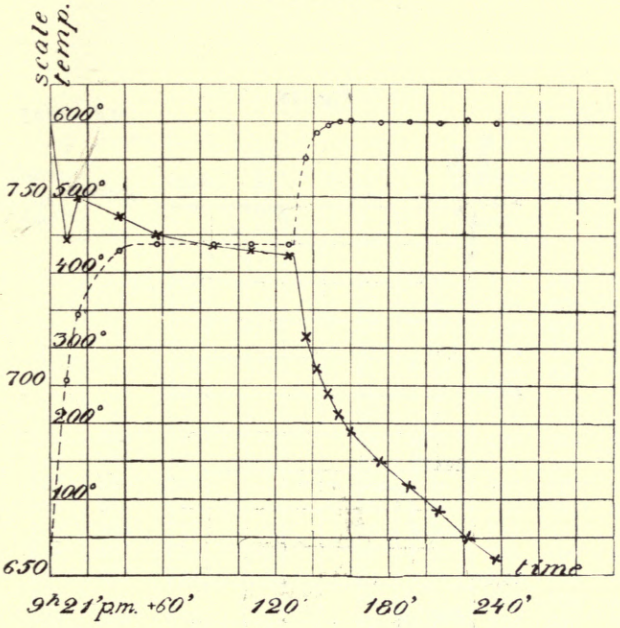
scale temp.
600°
750
500°
400°
300°
700
200°
100°
650
time
9h21'pm. +60'
120'
180'
240'

Fig. 17.

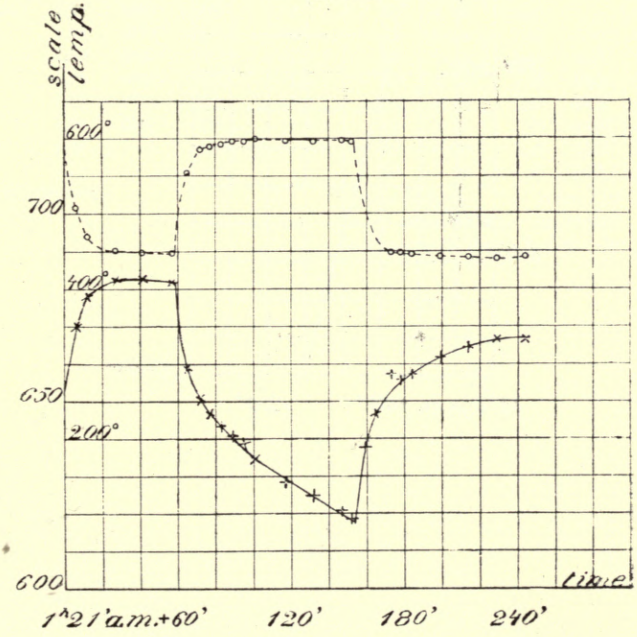
scale temp.
600°
700
400°
650
200°
600
time
1h21'a.m.+60'
120'
180'
240'

Fig. 18.

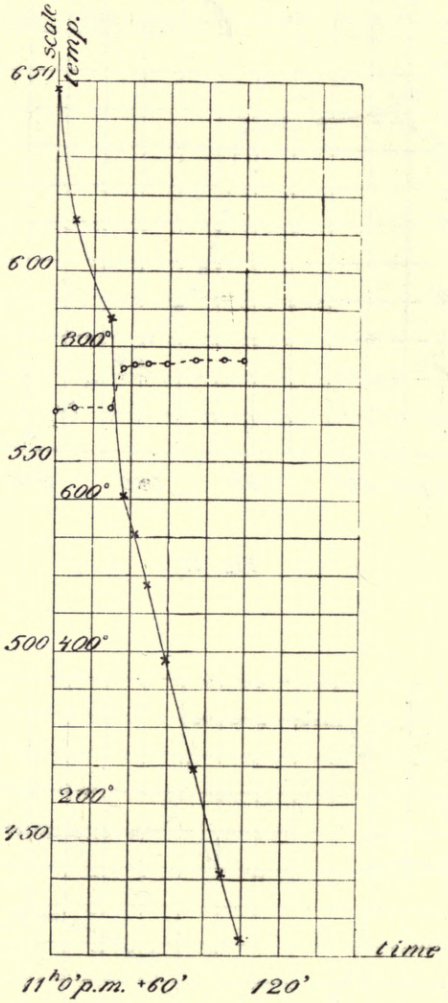
scale temp.
650
600
800°
550
600°
500
400°
200°
450
time
11h0'p.m.
+60'
120'

Table XV.

No.	time	current (in Amp.)	temp.	scale a	$\frac{\delta \Delta W}{\delta \tau}$ (in μmg. pr min.)
1	1^{h}00′ p.m.	—	(+18°)	712	—
2	5 00	—	—	713	—
3	7 30[1])	—	—	—	—
4	8 10	—	—	771	—
5	8 50	—	—	772	—
6	9 15	—	—	770,5	—
7	9 21	1,47	—	—	—
8	9 32	1,39	255°	739	—
9	9 38	1,48	345°	750	+7
10	10 00	1,47	430°	745	—1
11	10 20	1,46	435°	740	—1
12	10 50	1,46	435°	737	—0,4
13	11 10	1,46	435°	736	—0,2
14	11 30	1,45	435°	734,5	—0,3
15	11 33	1,98	—	—	—
16	11 39	1,98	555°	713	—14
17	11 45	1,98	585°	704,5	—6
18	1 51	1,98	595°	698	—4
19	11 57	1,98	600°	692,5	—4
20	12 03 a.m.	1,98	600°	688	—3
21	12 19	1,98	600°	680	—2
22	12 34	1,99	600°	673,5	—1,7
23	12 50	1,98	600°	667	—1,6
24	1 05	1,99	600°	660	—1,9
25	1 20	1,99	600°	654,5	—1,5
26	1 21	1,48	—	—	—

[1]) liquid air applied.

Table XVI.

No.	time	current (in Amp.)	temp.	scale a	$\frac{\delta \Delta W}{\delta \tau}$ (in μmg. pr min.)
27	1^{h}27′	1,48	505°	670	+11
28	1 33	1,48	470°	678	+5
29	1 48	1,48	450°	682,5	+1
30	2 03	1,48	450°	683	+0,1
31	2 18	1,48	450°	682	+0,3
32	2 20	1,99	—	—	—
33	2 26	1,99	555°	659	—15
34	2 33	1,99	585°	650,5	—5
35	2 38	1,99	590°	647	—3
36	2 44	1,99	590°	643,5	—2
37	2 50	2,01	595°	641	—1,6
38	2 56	2,01	595°	639	—1,3
39	3 02	2,03	600°	635	—2,7
40	3 18	2,02	595°	628,5	—1,6
41	3 33	2,03	595°	625	—0,9
42	3 48	2,04	600°	621	—1,1
43	3 53	2,05	595°	618,5	—2
44	3 55	1,48	—	—	—
45	4 00	1,48	—	638	+16
46	4 06	1,48	—	647	+6
47	4 14	1,48	450°	657,5	+5
48	4 19	1,48	450°	656	—1
49	4 25	1,48	445°	657,5	+1
50	4 40	1,48	445°	662	+1
51	4 55	1,48	440°	664,5	+0,7
52	5 10	1,48	440°	665,5	+0,3
53	5 25	1,48	445°	665	—0,1

zero before: $a = 712$
zero afterwards: $a = 662$
permanent loss: $\triangle a = 50$: $\triangle W = 200$ μmg.

Table XVII.

No.	time	current (in Amp.)	temp.	scale a	$\frac{\delta \Delta W}{\delta \tau}$ (in μmg./min.)
1	8^h,30′ p.m.	—	(+18°)	601	—
2	10 ,00[1]	—	—	—	—
3	10 ,36	2,00	—	—	—
4	10 ,50	2,80	—	—	—
5	11 ,01	2,78	715°	648	—
6	11 ,11	2,78	720°	613,5	—14
7	11 ,30	2,78	720°	587,5	—5
8	11 ,31	3,00	—	—	—
9	11 ,37	3,00	770°	541	—30
10	11 ,43	3,00	775°	531	—7
11	11 ,50	3,00	780°	517,5	—8
12	12 ,00	2,99	780°	498	—8
13	12 ,15 a.m.	3,00	785°	469	—8
14	12 ,30	3,00	785°	441,5	—7
15	12 ,40	3,00	785°	424	—7
16	12 ,42	0	—	—	—

zero before: $a = 601$, zero afterwards: $a = 532$ permanent loss: 69 = *280* μmg.
Pressure kept for 30 hours at 760 mm. and again reduced to 5 mm.
Scalereadings: $^{24}/_{4}$ 1^h p.m. $a = 512$
$^{25}/_{4}$ 11^h a.m. $a = 512$
$^{26}/_{4}$ 10^h a.m. $a = 512$

[1]) liquid air applied.

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Note

to table VI on page 58.

In order to reduce the quantity $\delta_t p$ from the observed temperature t'' to 16°,00 the following correction has been applied.

Temp. coeff. due to the bulb: $\frac{dF'}{dt} = +\ 140$ scale-units.

Temp. coeff. due to the magnetic buoyancy of the surrounding air of the pressure p'':

$$\frac{dF''}{dt} = -\frac{1}{273} \cdot 9310 \cdot \frac{p''}{760} = -0{,}045 \cdot p'' \text{ scale-units.}$$

Temp. coeff. for the resultant magnetic force:

$$\frac{dF}{dt} = +\ [140 - 0{,}045 \cdot p''] \text{ scale-units.}$$

Reduction in p'' (= increase in $\delta\, p$) equivalent to this temp. coefficient $= -\frac{1}{83} \cdot \frac{dF}{dt}$ mm. Hg.

Temp. correction per 1° for the thermal expansion of the air =

$$= -\frac{1}{273} \cdot \delta_t\ p$$

Resultant temp. correction $= +\ [t'' - 16{,}00] \cdot \frac{d\ \delta_t p}{dt} =$

$$= +\ [t'' - 16{,}00] \cdot [-\frac{1}{83}\ (140 - 0{,}045\ p'') - \frac{1}{273}\ \delta_t p] \text{ mm. Hg.}$$