NEW PRACTICAL ARITHMETIC

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NEW

PRACTICAL

ARITHMETIC:

FOR

GRAMMAR DEPARTMENTS.

By JAMES B. THOMSON, LL. D.,
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PREFACE.

THE New Practical Arithmetic now offered to the public, is the third and last of the works which constitute the author's "New Graded Series."

The old "Practical" was issued in 1845, and revised in 1853. Since that time important changes have taken place in the commercial world. These changes necessarily affect business calculations, and demand corresponding modifications in text books.

To meet this demand, the "New Graded Series" was undertaken. Each part of the series has been *reinvestigated* and *rewritten*;—the whole being readjusted upon the *graded plan*, and brought down to the present wants.

Among the objects aimed at in the present work are the following:

1. To make the definitions *clear*, *concise*, and *comprehensive*.

2. To present the principles of the science in a *series* of *distinct* and *consecutive propositions*.

3. To lead the mind of the pupil, through the analysis of the examples immediately following the respective propositions, to *discover the principles* by which all similar examples are solved, and enable him to sum up the principles thus developed, into a *brief, comprehensive rule*.

4. The "how" and the "why" are fully explained.

5. Great pains have been taken to ascertain the *Standard Weights and Measures* authorized by the Government; and to *discard* from the Tables such denominations as are *obsolete*, or *not used* in this country.*

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* Laws of Congress; Profs. Hassler, Bache and Egleston; Reports of Supts. Harrison and Calkins; also of the Committee on Weights and Measures of the Paris Exposition, 1867.

6. The Metric System is accompanied with brief and appropriate explanations for reducing it to practice. Its *simplicity* and *comprehensiveness* have secured its use in the natural sciences and commerce to such an extent in this and foreign countries, that no student can be said to have a finished education, without a knowledge of it.

7. Particular attention has been paid to the development of *Analysis*, the *grand common sense rule* which business men intuitively adopt as they enter upon practical life.

8. The examples are new and abundant;—being drawn from the various industrial arts, commerce, science, etc.

9. The arrangement of the matter upon the page, and the typography, have also received due attention. Teachers who deal much with figures, will be pleased with the adoption of the Franklin type. The ease with which these figures are read, is sufficiently attested by their use in all recent Mathematical Tables.

Finally, it has been the cardinal object to adapt the science of numbers to the present wants of the farm, the household, the workshop, and the counting-room;—in a word, to incorporate as *much information* pertaining to business forms, and matters of science, as the limits of the book would permit. In this respect it is believed the work is unrivaled. While it puts forth no claim to mathematical paradoxes, it is believed teachers will find that *something worthy* of their attention is gained, in nearly every Article.

In conclusion, the author tenders his most cordial thanks to teachers and the public for the very liberal patronage bestowed upon his former Arithmetics, known as "Day and Thomson's Series." It is hoped the "New Graded Series" will be found worthy of continued favor.

JAMES B. THOMSON.

NEW YORK, *July.*, 1872

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ARITHMETIC.

Art. 1. ***Arithmetic*** is the science of numbers.

Arithmetic is sometimes said to be both a *science* and an *art:* a *science* when it treats of the theory and properties of numbers; an *art* when it treats of their applications.

NOTES.—1. The term *arithmetic,* is from the Greek *arithmētikē,* the *art of reckoning.*

2. The term *science,* from the Latin *scientia,* literally signifies *knowledge.* In a more restricted sense, it denotes an *orderly arrangement* of the facts and principles of a particular branch of knowledge.

2. ***Number*** is a unit, or a collection of units.

A ***Unit*** is any single thing, called *one.* One and one more are called *two;* two and one more are called *three;* three and one more, *four,* etc. The terms one, two, three, four, are properly the *names* of numbers, but are often used for numbers themselves.

NOTE.—The term *unit* is from the Latin *unus,* signifying *one.*

3. The ***Unit One*** is the *standard* by which all numbers are measured. It may also be considered the *base* or *element* of number. For, all *whole* numbers *greater* than *one* are composed of *ones.* Thus, *two* is composed of *one* and *one. Three* is *one* more than *two;* but two, we have seen, is composed of ones; hence, *three* is, and so on.

QUESTIONS.—1. What is arithmetic? What else is it sometimes said to be? When a science? An art? 2. Number? A unit? 3. The standard by which numbers are measured? The base or element of number?

4. Numbers are either *abstract* or *concrete.*

An ***Abstract Number*** is one that is not applied to any object; as, three, five, ten.

A ***Concrete Number*** is one that is applied to some object; as, five peaches, ten books.

NOTATION.

5. ***Notation*** is the art of expressing numbers by *figures, letters,* or other *numeral characters.*

The two principal methods in use are the *Arabic* and the *Roman.*

NOTE.—Numbers are also expressed by *words* or *common language;* but this, strictly speaking, is not Notation.

ARABIC NOTATION.

6. The ***Arabic Notation*** is the method of expressing numbers by certain characters called *figures.*

It is so called, because it was introduced into Europe from *Arabia.*

7. The Arabic figures are the following ten, viz:

1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	naught.

The *first nine* are called *significant figures,* or *digits;* the *last* one, *naught, zero,* or *cipher.*

NOTES.—1. The first nine are called *significant* figures, because each always expresses a number.

2. The term *digit* is from the Latin *digitus,* a *finger,* and was applied to these characters because they were employed as a substi-

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4. What is an abstract number? Concrete? 5. What is notation? The principal methods in use? 6. Arabic notation? Why so called? 7. How many figures does it employ? What are the first nine called? The last one? *Note.* Why called significant figures? Why digits? Meaning of digitus? Why is the last called naught? Meaning of zero? Of cipher?

tute for the *fingers* upon which the ancients used to reckon. The term originally included the *cipher*, but is now generally restricted to the *first* nine.

3. The last one is called *naught;* because, when standing alone, it has no value, and when connected with significant figures, it denotes the *absence* of the *order* in whose place it stands.

4. *Zero* is an Italian word, signifying *nothing.*

The term *cipher* is from the Arabic *sifr* or *sifreen, empty, vacant.* Subsequently the term was applied to all the Arabic figures indiscriminately; hence, calculations by them were called *ciphering.*

8. Each of the *first nine* numbers is expressed by a *single figure,*—each figure denoting the number indicated by its name. These numbers are called *units* of the *first order,* or simply *units.*

8, *a.* *Nine* is the *greatest number* expressed by *one* figure. Numbers *larger* than nine are expressed thus:

Ten (1 more than 9) is expressed by an ingenious device, which groups ten *single things* or *ones* together, and considers the collection a *new* or *second order* of units, called *ten.* Hence, *ten* is expressed by writing the figure 1 in the *second* place with a *cipher* on the right; as, 10.

The numbers from ten to nineteen inclusive are expressed by writing 1 in the *second* place, and the figure denoting the *units* in the *first;* as,

11,	12,	13,	14,	15,	16,	17,	18,	19.
eleven,	twelve,	thirteen,	fourteen,	fifteen,	sixteen,	seventeen,	eighteen,	nineteen.

Twenty (2 tens) is expressed by writing the figure 2 in the *second* place with a *cipher* on the right; as, 20.

Thirty (3 tens) by writing 3 in the *second* place with a cipher on the right; and so on to ninety inclusive; as,

20,	30,	40,	50,	60,	70,	80,	90.
twenty,	thirty,	forty,	fifty,	sixty,	seventy,	eighty,	ninety.

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8. How are the first nine numbers expressed? What are they called? What is the greatest number expressed by one figure? How is ten expressed? Twenty? Thirty, etc.? The numbers between 10 and 20? From 20 to 99 inclusive?

The numbers from twenty to thirty, and so on to *ninety-nine* (99) inclusive, are expressed by writing the *tens* in the *second* place, and the *units* in the first; as,

21,	22,	23,	34,	35,	46,	57,	99.
twenty-one,	twenty-two,	twenty-three,	thirty-four,	thirty-five,	forty-six,	fifty-seven,	ninety-nine.

8, *b*. ***Ninety-nine*** is the *greatest* number that can be expressed by *two* figures.

A *hundred* (1 more than 99) is expressed by grouping ten units of the *second order* together, and forming a *new* or *third order* of *units*, called a *hundred*. Thus, a hundred is expressed by writing 1 in the *third* place with two ciphers on the right; as, 100.

In like manner, the numbers from one hundred to nine hundred and ninety-nine inclusive, are expressed by writing the *hundreds* in the *third* place, the *tens* in the *second*, and the *units* in the *first*. Thus, one hundred and thirty-five (1 hundred, 3 tens, and 5 units,) is expressed by 135.

8, *c*. ***Nine hundred and ninety-nine*** is the *greatest* number that can be expressed by *three* figures.

Thousands, and *larger numbers*, are expressed by forming *other new orders*, called the *fourth*, *fifth*, etc., orders; as, *tens* of thousands, *hundreds* of thousands, *millions*, etc.

NOTES.—1. The *names* of the first ten numbers, one, two, three, etc., are *primitive* words. The terms *eleven* and *twelve* are from the Saxon *endlefen* and *twelif*, meaning *one* and *ten*, *two* and *ten*. *Thirteen* is from *thir* and *teen*, which mean *three* and *ten*, and so on.

2. Twenty is from the Saxon *tweentig*, *tween*, *two*, and *ty*, *tens*; i. e., two tens. Thirty is from *thir* and *ty*, three tens, and so on.

3. The terms *hundred*, *thousand*, and *million* are primitive words, having no perceptible analogy to the numbers they express.

From the foregoing illustrations we derive the following principle:

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8. How is a hundred expressed? Thousands, and larger numbers?

9. The ***Orders of Units*** *increase by the scale of ten* That is, ten *single units* are one *ten;* ten *tens* one *hundred;* ten *hundreds* one *thousand;* and, universally,

Ten of any *lower* order make a *unit* of the *next* higher.

NOTES.—1. If the term *unit* denotes *one*, how, it may be asked, can *ten* things or ones be a *unit*, *ten tens* another unit, etc. And how can the figures 1, 2, 3, etc., sometimes denote *single things* or *ones;* at others, *tens* of ones, and so on.

The answer is, *units* are of two kinds, *simple* and *collective.*

A *simple unit* is a *single thing* or *one.*

A *collective unit* denotes a *group* of ones, regarded as a *whole.*

2. To illustrate these units, suppose a basket of pebbles is before us. Counting them out one by one, each pebble is a *simple unit.*

Again, counting out *ten* single pebbles and putting them together in a group, this *group* forms a *unit* of the *second order*, called *ten.* Counting out *ten* such groups, and putting them together in one pile, this *collection* forms a *unit* of the *third order*, called *hundred.* In like manner, a *group* of *ten hundreds* forms a *unit* of the *fourth order*, called *thousand*, and so on.

These different *groups* of *ten* are called *units* on the same principle that a *group* of ten cents forms a *unit*, called a *dime;* or, a *group* of ten dimes, a *unit*, called a *dollar.* Thus, it will be seen that the figures 1, 2, 3, 4, etc., always mean one, two, three, four *units* as their name indicates; but the *value* of these units depends upon the place the figure occupies.

10. From the preceding illustrations, it will be seen that the *Arabic Notation* is founded upon the following principles:

1st. Numbers are divided into *groups* called units, of the *first, second, third*, etc., *orders.*

2d. To express these different orders of units, a *simple* and a *local* value are assigned to the significant figures, according to the place they occupy.

3d. If any order is *wanting*, its place is supplied by a *cipher.*

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9. How do the orders of units increase? Units make a ten? Tens a hundred?
10. Name the principles upon which the Arabic Notation is founded?

11. The ***Simple Value*** of a figure is the number of units it expresses when it stands *alone*, or in the *right-hand* place.

The ***Local Value*** is the number it expresses when connected with other figures, and is determined by the *place* it occupies, counting from the *right*.

12. It is a *general law* of the Arabic Notation that the *value* of a figure is *increased tenfold* for every place it is moved from the *right* to the *left;* and, conversely,

The *value* of each figure is *diminished tenfold* for every place it is moved from the *left* to the *right*. Thus, 2 in the first place denotes *two* simple units; in the *second* place, *ten times* two, or twenty; in the *third* place, *ten times* as much as in the second place, or two hundreds, and so on. (Art. 8.)

13. The number denoting the *scale* by which the orders of units increase is called the *radix*.

The radix of the Arabic Notation is *ten;* hence it is often called the *decimal* notation.

NOTES.—1. The term *radix*, Latin, signifies *root*, or *base*. The term *decimal* is from the Latin *decem*, *ten*.

2. The *decimal* radix was doubtless suggested by the *number* of fingers (digiti) on both hands. (Art. 7, Note.) Hence,

14. To Express Numbers by *Figures*,

Begin at the left hand and write the figures of the given orders in the successive places toward the right.

If any intermediate orders are omitted, supply their places with ciphers.

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11. The simple value of a figure? Local? 12. What is the law as to moving a figure to the right or left? 13. What is the radix of a system of notation? The radix of the Arabic? What else is the Arabic system called? Why? *Note.* Meaning of radix? Decimal? What suggested the decimal radix? 14. Rule for expressing numbers by figures?

EXERCISES.

Express the following numbers by figures:

1. Three hundred and forty-five.
2. Four hundred and sixty.
3. Eight hundred and four.
4. Two thousand three hundred and ten.
5. Thirty thousand and nineteen.
6. Sixty-three thousand and two hundred.
7. One hundred and ten thousand two hundred and twelve.
8. Four hundred and sixty thousand nine hundred and thirty.
9. Six hundred and five thousand eight hundred and forty-two.
10. Two millions sixty thousand and seventy-five.

ROMAN NOTATION.

15. The ***Roman Notation*** is the method of expressing numbers by certain *letters*. It is so called because it was employed by the *Romans*. The letters used are the following seven, viz.: I, V, X, L, C, D and M. The letter I denotes *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* M, *one thousand.* Intervening and larger numbers are expressed by the repetition and combination of these letters.

16. The Roman system is based upon the following general principles:

1st. It proceeds according to the scale of *ten* as far as a thousand, the *unit* of each order being denoted by

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15. Roman notation? Why so called? Letters employed? The letter I denote? V? X? L? C? D? M? 16. Name the first principle upon which it is based. What is the effect of repeating a letter? Of placing a letter of less value before one of greater value? If placed after? If a line is placed over a letter?

a *single* letter. Thus, I denotes one; X, ten; C, one hundred; M, one thousand.

2d. Repeating a letter, *repeats* its *value*. Thus, I denotes one; II, two; III, three; X, ten; XX, twenty, etc.

3d. Placing a letter of *less* value *before* one of *greater* value, *diminishes* the value of the greater by that of the less; placing the less *after* the greater, *increases* the value of the greater by that of the less. Thus, V denotes five, but IV denotes only four, and VI six.

4th. Placing a *horizontal line* over a letter increases its value a *thousand* times. Thus, $\overline{\mathrm{I}}$ denotes a thousand; $\overline{\mathrm{X}}$, ten thousand; $\overline{\mathrm{C}}$, a hundred thousand; $\overline{\mathrm{M}}$, a million.

TABLE.

I,	denotes	one.	XXX,	denotes	thirty.
II,	"	two.	XL,	"	forty.
III,	"	three.	L,	"	fifty.
IV,	"	four.	LX,	"	sixty.
V,	"	five.	LXX,	"	seventy.
VI,	"	six.	LXXX,	"	eighty.
VII,	"	seven.	XC,	"	ninety.
VIII,	"	eight.	C,	"	one hundred.
IX,	"	nine.	CC,	"	two hundred.
X,	"	ten.	CCC,	"	three hundred.
XI,	"	eleven.	CCCC,	"	four hundred.
XII,	"	twelve.	D,	"	five hundred.
XIII,	"	thirteen.	DC,	"	six hundred.
XIV,	"	fourteen.	DCC,	"	seven hundred.
XV,	"	fifteen.	DCCC,	"	eight hundred.
XVI,	"	sixteen.	DCCCC,	"	nine hundred.
XVII,	"	seventeen.	M,	"	one thousand.
XVIII,	"	eighteen.	MM,	"	two thousand.
XIX,	"	nineteen,	MDCCCLXXI,	denotes	one thousand
XX,	"	twenty.			and eight hundred and seventy-one.

NOTES.—1. *Four* was formerly denoted by IIII; *nine* by VIIII; *forty* by XXXX; *ninety* by LXXXX; *five hundred* by IↃ; and a thousand by CIↃ.

2. Annexing Ↄ to IↃ (five hundred) increases its value *ten* times. Thus, IↃↃ denotes five thousand; IↃↃↃ, fifty thousand.

Prefixing a C and annexing a Ↄ to CIↃ (a thousand) increases its value *ten* times. Thus, CCIↃↃ denotes ten thousand, etc. Hence,

17. To Express Numbers by *Letters*,

Begin at the left hand or highest order, and write the letters denoting the given number of each order in succession.

NOTE.—The Roman Notation is seldom used, except in denoting chapters, sections, heads of discourses, etc.

EXERCISES.

Express the following numbers by letters :

1. Fourteen,	2. Twenty-nine,	3. Thirty-four,
4. Sixty-six,	5. Forty-nine,	6. Seventy-three,
7. Eighty-eight,	8. Ninety-four,	9. Ninety-nine,
10. 107,	11. 212,	12. 498,
13. 613,	14. 507,	15. 608.
16. 724,	17. 829,	18. 928,
19. 1004,	20. 1209,	21. 1363,
22. 1417,	23. 1614,	24. 1671,
25. 1748,	26. 1803,	27. 1876.

NUMERATION.

18. ***Numeration*** is the art of reading numbers expressed by *figures*, *letters*, or other *numeral characters*.

NOTE.—The learner should be careful not to confound *Numeration* with *Notation*. The distinction between them is the same as that between *reading* and *writing*.

19. There are two methods of reading numbers, the *French* and the *English*.

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17. Rule for expressing numbers by letters? 18. Numeration? *Note.* Distinction between Numeration and Notation? 19. How many methods of reading numbers?

FRENCH NUMERATION.

20. The French divide numbers into *periods* of *three figures* each, and then subdivide each period into *units, tens,* and *hundreds,* as in the following

TABLE.

Hundreds of Quadrillions.	Tens of Quadrillions.	*Quadrillions.*	Hundreds of Trillions.	Tens of Trillions.	*Trillions.*	Hundreds of Billions.	Tens of Billions.	*Billions.*	Hundreds of Millions.	Tens of Millions.	*Millions.*	Hundreds of Thousands.	Tens of Thousands.	*Thousands.*	Hundreds.	Tens.	*Units.*
8	2	3	5	6	1	7	2	9	4	5	2	7	8	9	3	8	4
6th period.			5th period.			4th period.			3d period.			2d period.			1st period.		

The first period on the right is called *units* period; the second, *thousands;* the third, *millions,* etc.

The periods in the table are thus read: 823 quadrillions, 561 trillions, 729 billions, 452 millions, 789 thousand, three hundred and eighty-four.

NOTE.—The terms *billion, trillion, quadrillion,* etc., are derived from the Italian *milione* and the Latin *bis, tres, quatuor,* etc. Thus, *bis,* united with *million,* becomes *billion,* etc.

21. To read Numbers according to the French Numeration.

Divide them into periods of three figures each, counting from the right.

Beginning at the left hand, read the periods in succession, and add the name to each, except the last.

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20. The French method? Repeat the Table, beginning at the right. What is the 1st period called? The 2d? 3d? 4th? 5th? 6th? 21. The rule for reading numbers by the French method? *Note.* Why omit the name of the right hand period? The difference between orders and periods?

NOTES.—1. The name of the *right-hand* period is omitted for the sake of *conciseness;* and since this period always denotes *units*, the omission occasions no obscurity.

2. The learner should observe the difference between the *orders* of units and the *periods* into which they are divided. The *former* increase by *tens;* the *latter* by *thousands.*

3. This method of reading numbers is commonly ascribed to the *French*, and is thence called *French numeration.* Others ascribe it to the *Italians*, and thence call it the *Italian method.*

Read the following numbers by the French numeration:

1.	270	11.	840230	21.	3006017
2.	309	12.	4603400	22.	2460317239
3.	1270	13.	35040026	23.	5100024000
4.	2036	14.	80600000	24.	70300510
5.	8605	15.	9001307	25.	203019060
6.	40300	16.	65023009	26.	7800580019
7.	85017	17.	810000	27.	86020005200
8.	160401	18.	75306020	28.	51036040
9.	405869	19.	165380254	29.	621000031
10.	1365406	20.	310400270	30.	93000275320

31.	216327250516	36.	289300210375861
32.	4260300210109	37.	5400000541000600
33.	300073004000	38.	600051000024300
34.	41295000400649	39.	40200000008060704
35.	264300439000200	40.	60004030060723051 6

Express the following numbers by figures

1. Ten millions, five thousand, and two hundred.
2. Sixty-one millions, three hundred and forty.
3. Three hundred and ten millions, and five hundred.
4. Twenty-six billions, seventy millions, three hundred.
5. One hundred billions, four hundred and twenty-five.
6. Sixty-eight trillions, seven hundred and twenty-five.
7. Eight hundred and twenty millions, five hundred and twenty-three.

8. Sixty-seven quadrillions, ninety-seven billions.

9. Four hundred and sixty quadrillions, and eighty-seven millions.

10. Seven hundred and sixty-one quadrillions, seventy-one trillions, two hundred billions, eighteen millions, five thousand, and thirty-six.

ENGLISH NUMERATION.

22. The English divide numbers into periods of six *figures each,* and then subdivide each period into *units, tens, hundreds, thousands, tens of thousands,* and *hundreds of thousands,* as in the following

TABLE.

Hund. of Thou. of Billions.	Tens of Thousands of Billions.	Thousands of Billions.	Hundreds of Billions.	Tens of Billions.	*Billions.*	Hund. of Thou. of Millions.	Tens of Thousands of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	*Millions.*	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	*Units.*
4	0	7	6	9	2	9	5	8	6	0	4	4	1	3	0	5	6
3d period.						2d period.						1st period.					

The periods in the Table are thus read: 407692 billions, 958604 millions, 413 thousand, and fifty-six.

NOTE.—This method is called *English* numeration, because it was invented by the English.

For other numbers to read by this method, the pupil is referred to those in Art. 21.

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22. What is the English method of numeration? *Note.* Why so called?

ADDITION.

23. ***Addition*** is uniting two or more numbers in one. The ***Sum*** or ***Amount*** is the number found by addition. Thus, 5 added to 7 are 12; twelve, the number obtained, is the sum or amount.

NOTES.—1. The *sum* or *amount* contains as *many units* as the numbers added. For, the numbers added are composed of units; and the whole is equal to the *sum* or all its parts. (Art. 3.)

2. When the numbers added are the *same denomination,* the operation is called *Simple Addition.*

SIGNS.

24. ***Signs*** are characters used to indicate the *relation* of numbers, and *operations* to be performed.

25. The ***Sign of Addition*** is a perpendicular cross called *plus* (+), placed before the number to be added. Thus $7+5$, means that 5 is to be added to 7, and is read "7 plus 5."

NOTE.—The term *plus,* Latin, signifies *more,* or *added to.*

26. The ***Sign of Equality*** is two short parallel lines (=), placed between the numbers compared. Thus $7+5=12$, means that 7 and 5 are equal to 12, and is read, "7 plus 5 equal 12," or the sum of 7 plus 5 equals 12.

Read the following numbers:

1. $8+4+2=6+8$	4. $23+7=19+11$
2. $7+3+5=2+1+12$	5. $37+8=30+15$
3. $19+1+0=6+5+9$	6. $58+10=40+28$

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23. What is addition? The result called? *Note.* When the numbers added are the same denomination, what is the operation called? 24. What are signs? 25. The sign of addition? *Note.* The meaning of *plus?* 26. Sign of equality? How is 7+5=12 read?

27. The ***Sign of Dollars*** is a capital S with two perpendicular marks across it ($), prefixed to the number of dollars to be expressed. Thus, $245 means 245 dollars.

NOTE.—The term *prefix*, from the Latin *prefigo*, signifies to place before.

Read the following expressions:

1. $17 + $8 = $15 + $10
2. $13 + $20 = $16 + $17
3. $21 + $7 = $12 + $16
4. $25 + $8 = $10 + $23
5. $25 + $40 = $50 + $15
6. $105 + $36 = $96 + $45

ADDITION TABLE.

1 and			2 and			3 and			4 and			5 and		
1	are	2	1	are	3	1	are	4	1	are	5	1	are	6
2	“	3	2	“	4	2	“	5	2	“	6	2	“	7
3	“	4	3	“	5	3	“	6	3	“	7	3	“	8
4	“	5	4	“	6	4	“	7	4	“	8	4	“	9
5	“	6	5	“	7	5	“	8	5	“	9	5	“	10
6	“	7	6	“	8	6	“	9	6	“	10	6	“	11
7	“	8	7	“	9	7	“	10	7	“	11	7	“	12
8	“	9	8	“	10	8	“	11	8	“	12	8	“	13
9	“	10	9	“	11	9	“	12	9	“	13	9	“	14
10	“	11	10	“	12	10	“	13	10	“	14	10	“	15

6 and			7 and			8 and			9 and			10 and		
1	are	7	1	are	8	1	are	9	1	are	10	1	are	11
2	“	8	2	“	9	2	“	10	2	“	11	2	“	12
3	“	9	3	“	10	3	“	11	3	“	12	3	“	13
4	“	10	4	“	11	4	“	12	4	“	13	4	“	14
5	“	11	5	“	12	5	“	13	5	“	14	5	“	15
6	“	12	6	“	13	6	“	14	6	“	15	6	“	16
7	“	13	7	“	14	7	“	15	7	“	16	7	“	17
8	“	14	8	“	15	8	“	16	8	“	17	8	“	18
9	“	15	9	“	16	9	“	17	9	“	18	9	“	19
10	“	16	10	“	17	10	“	18	10	“	19	10	“	20

☞ More mistakes are made in *adding* than in any other arithmetical operation. The first five digits are easily combined; the results of adding 9, being 1 less than if 10 were added, are also easy. The others, 6, 7, 8, are more difficult, and therefore should receive *special attention.*

27. What is the sign of dollars?

CASE I.

28. To find the *Amount* of two or more numbers, when the Sum of each column is *Less* than 10.

Ex. 1. A man owns 3 farms; one contains 223 acres, another 51 acres, and the other 312 acres: how many acres has he?

ANALYSIS.—Let the numbers be set down as in the margin. Beginning at the right, we proceed thus: 2 units and 1 unit are 3 units, and 3 are 6 units; the sum being less than ten units, we set it under the column of *units*, because it is *units*. Next, 1 ten and 5 tens are 6 tens, and 2 are 8 tens; the sum being less than 10 tens, we set it under the column of tens, because it is *tens*. Finally, 3 hundreds and 2 hundreds are 5 hundreds; the sum being less than 10 hundreds, we set it under the column of hundreds, for the same reason. Therefore, he has 586 acres. All similar examples are solved in like manner.

OPERATION.

	hunds.	tens.	units.
	2	2	3
		5	1
	3	1	2
Ans.	5	8	6

By inspecting the preceding illustration, the learner will discover the following principle:

Units of the same order are added together, and the sum is placed under the column added. (Art. 9.)

NOTES.—1. The *same orders* are placed under each other for the sake of *convenience* and *rapidity* in adding.

2. We add the *same orders* together, units to units, tens to tens, etc., because *different* orders express *units of different values*, and therefore cannot be added to each other. Thus, 5 units and 5 tens neither make 10 units nor 10 tens, any more than 5 cents and 5 dimes will make 10 cents or 10 dimes.

3. We add the columns *separately*, because it is easier to add one order at a time than several.

4. The *sum* of each column is set under the column added, because being less than 10, it is the *same order* as that column.

(2.)	(3.)	(4.)	(5.)	(6.)
2103	3024	1211	2102	21032
4022	1230	2002	1253	52010
1674	4603	5340	4604	24603

7. What is the sum of \$2321 + \$123 + \$3245?

8. What is the sum of 3210 pounds + 2023 pounds + 4601 pounds?

9. What is the sum of 130230 + 201321 + 402126?

10. What is the sum of 2410632 + 1034246 + 320120?

CASE II.

29. To find the *Amount* of two or more numbers, when the Sum of any column is 10, or more.

1. What is the sum of $436, $324, and $645?

OPERATION.
$436
324
645
$1405

ANALYSIS.—Let the numbers be set down as in the margin. Adding as before, the sum of the first column is 15 units, or 1 ten and 5 units. We set the 5 units under the column added, and add the 1 ten to the next column because it is the same order as that column. Now, 1 added to 4 tens makes 5 tens, and 2 are 7 tens, and 3 are 10 tens, or 1 hundred and 0 tens. We set the 0, or right hand figure, under the column added, and add the 1 hundred to the next column, as before. The sum of the next column, with the 1 added, is 14 hundreds; or 1 thousand and 4 hundreds. This being the *last* column, we set down the *whole sum*. The answer is $1405. All similar examples may be solved in like manner.

By inspecting this illustration, it will be seen,

When the sum of a column is 10 *or more, we write the units' figure under the column, and add the tens' figure to the next column.*

NOTES.—1. We set the *units' figure* under the column added, and add the *tens* to the next column, because they are the *same orders* as these columns.

2. We begin to add at the *right hand*, in order to carry the *tens* as we proceed. We set down the *whole sum* of the last column, because there are no figures of the *same order* to which its left hand figure can be added.

30. Adding the *tens* or *left hand figure* to the next column, is called *carrying* the tens. The process of *carrying the tens*, it will be observed, is simply taking a certain number of units from a *lower* order, and adding their *equal* to the next *higher*; therefore, it can neither *increase* nor *diminish* the amount.

NOTE.—We carry for *ten* instead of *seven, nine, eleven,* etc., because in the Arabic notation the *orders* increase by the *scale of ten.* If they increased by the scale of eight, twelve, etc., we should carry for that number. (Art. 13.)

(2.)	(3.)	(4.)	(5.)	(6.)
5689	6898	7585	8456	97504
3792	3365	3748	5078	38786
4358	7987	8667	6904	75979
13839	18250	20000	20438	212269

31. The preceding principles may be summed up in the following

GENERAL RULE.

I. *Place the numbers one under another, units under units, etc.; and beginning at the right, add each column separately.*

II. *If the sum of a column does not exceed* NINE, *write it under the column added.*

If the sum exceeds NINE, *write the units' figure under the column, and add the tens to the next higher order.*

Finally, set down the whole sum of the last column.

NOTES.—1. As soon as the pupil understands the *principle* of adding, he should learn to *abbreviate* the process by *simply pronouncing* the successive results, as he points to each figure added. Thus, instead of saying 7 units and 9 units are 16 units, and 8 are 24 units, and 7 are 31 units, he should say, *nine, sixteen, twenty-four, thirty-one,* etc.

Again, if two or more numbers together make 10, as 6 and 4, 7 and 3; or 2, 3, and 5, etc., it is shorter, and therefore better, to add 10 at once.

---

31. How write numbers to be added? The next step? If the sum of a column does not exceed nine, what do you do with it? If it exceeds nine? The sum of the last column? 28. *Note.* Why write units under units, etc.? Why add the columns separately? Why not add different orders together promiscuously? Is the sum of 3 units and 4 tens, 7 units or 7 tens? When the sum of a column does not exceed 9, why set it under the column? 29. *Note.* If the sum of a column is 10 or more, why set the units' figure under the column added, and carry the tens to the next column? 30. What is meant by carrying the tens? Why does not carrying change the amount? Why carry for 10 instead of 6, 8, 12, etc.

2. Accountants sometimes set the figure carried under the right hand figure in a line below the answer. In this way the sum of each column is preserved, and any part of the work can be reviewed, if desired ; or if interrupted, can be resumed at pleasure.

32. PROOF.—*Begin at the top and add each column downward. If the two results agree, the work is right.*

NOTE.—This proof depends upon the supposition that *reversing* the *order* of the figures, will *detect* any error that may have occurred in the operation.

EXAMPLES.

1. Find the sum of 864, 741, 375, 284, and 542, and prove the operation.

(2.)	(3.)	(4.)	(5.)	(6.)
Dollars.	Pounds.	Yards.	Rods.	Feet.
263	4780	2896	23721	845235
425	7642	8342	70253	476234
846	5036	257	4621	6897
407	7827	3261	342	723468

7. What is the sum of 675 acres + 842 acres + 904 acres + 39 acres ?

8. What is the sum of 8423 + 286 + 7932 + 28 + 6790 ?

9. What is the sum of 82431 + 376 + 19 + 62328 + 4521 + 35787 ?

10. What is the sum of 63428 + 78 + 4236 + 628 + 93 + 8413 ?

(11.)	(12.)	(13.)	(14.)	(15.)
2685	89243	72094	825276	9031253
6543	8284	96308	704394	432567
8479	34567	763	37783	65414
6503	865	4292	1697	9236
1762	3952	23648	349435	843
7395	42678	75965	697678	68

16. If a man pays $2358 for his farm, $1950 for stock, and $360 for tools, how much does he pay for all ?

32. How prove addition ? *Note.* Upon what is this proof based ?

17. A merchant bought 371 yards of silk, 287 yards of calico, 643 yards of muslin, and 75 yards of broadcloth: how many yards did he buy in all?

18. Required the sum of \$2404 + \$100 + \$1965 + \$1863.

19. Required the sum of 968 pounds + 81 pounds + 7 pounds + 639 pounds.

20. Required the sum of 1565 gals. + 870 gals. + 31 gals. + 160 gals. + 42 gals.

21. Add 2368, 1764, 942, 87, 6, and 5271.

22. Add 281, 6240, 37, 9, 1923, 101, and 45.

23. Add 888, 9061, 75, 300, 99, 6, and 243.

24. 243 + 765 + 980 + 759 + 127 = how many?

25. 9423 + 100 + 1600 + 119 + 4004 = how many?

26. 81263 + 16319 + 805 + 2500 + 93 = how many?

27. 236517 + 460075 + 235300 + 275161 = how many?

28. A savings bank loaned to one customer \$1560, to another \$1973, to a third \$2500, and to a fourth \$3160: how much did it loan to all?

29. A young farmer raised 763 bushels of wheat the first year, 849 bushels the second, 1011 bushels the third, and 1375 bushels the fourth: how many bushels did he raise in 4 years?

30. A man bequeathed the Soldiers' Home \$8545; the Blind Asylum \$7538; the Deaf and Dumb Asylum \$6280; the Orphan Asylum \$19260; and to his wife the remainder, which was \$65978. What was the value of his estate?

31. A merchant owns a store valued at \$17265, his goods on hand cost him \$19230, and he has \$1563 in bank: how much is he worth?

32. What is the sum of thirteen hundred and sixty-three, eighty-seven, one thousand and ninety-four, and three hundred?

33. What is the sum of three thousand two hundred and forty, fifteen hundred and sixty, and nine thousand?

34. Add ninety thousand three hundred and two, sixty-five thousand and thirty, forty-four hundred and twenty-three.

35. Add eight hundred thousand and eight hundred, forty thousand and forty, seven thousand and seven, nine hundred and nine.

36. Add 30006, 301, 55000, 2030, 67, and 95000.

37. Add 65139, 100100, 39, 111111, 763002, and 317.

38. Add 81, 907, 311, 685, 9235, 7, and 259.

39. Add 4895, 352, 68, 7, 95, 645, and 3867.

40. Add 631, 17, 1, 45, 9268, 196, and 3562.

41. Add 77777, 33333, 88888, 22222, and 11111.

42. Add 236578, 125, 687256, and 404505.

43. Add 23246, 8200461, 5017, 8264, and 39.

44. Add 317, 21, 9, 4500, 219, 3001, 17036, and 45.

45. Add 10000000, 1000000, 100000, 10000, 1000, 100, and 10.

46. Add 22000000, 22000, 202000, 2200, and 220.

47. If a young man lays up $365 in 1 year, how much will he lay up in 4 years?

48. In what year will a man who was born in 1850, be 75 years old?

49. A man deposits $1365 in a bank per day for 6 days in succession: what amount did he deposit during the week?

50. A planter raised 1739 pounds of cotton on one section of his estate, 703 pounds on another, 2015 pounds on another, and 2530 pounds on another: how many pounds did he raise in all?

51. A man was 29 years old when his eldest son was born; that son died aged 47, and the father died 17 years later: how old was the man at his death?

52. A speculator bought 3 city lots for $21213, and sold so as to gain $375 on each lot: for what sum did he sell them?

53. A's income-tax for 1865 was $4369, B's $3978; C's was $135 more than A's and B's together, and D's was equal to all the others: what was D's tax? What the tax of all?

54. How many strokes does a clock strike in 24 hours?

55. A raised 3245 bushels of corn, and B raised 723 bushels more than A: how many bushels did both raise?

56. In a leap year 7 months have 31 days each, 4 months 30 days each, and 1 month has 29 days: how many days constitute a leap year?

57. The entire property of a bankrupt is $2648, which is only half of what he owes: how much does he owe?

58. What number is 19256 more than 31273?

59. A has 860 acres of land, B 117 acres more than A, and C as many as both: how many acres have all?

DRILL COLUMNS.

(60.)		(61.)		(62.)		(63.)		(64.)		(65.)	
Dols.	cts.	Dols.	cts.	Dols.	cts.	Dols.	cts.	Dols.	cts.	Dols.	cts.
14	65	25	76	37	38	24	91	34	45	49	68
21	43	32	37	3	25	42	73	32	61	26	52
64	61	54	61	46	72	9	61	64	12	38	43
37	28	16	45	28	41	32	44	70	33	27	59
43	24	67	38	7	50	51	62	24	54	12	38
46	03	34	92	63	04	9	28	32	67	45	63
19	41	76	41	16	28	70	36	48	32	50	71
32	34	47	69	7	39	84	52	25	67	26	83
34	32	31	04	82	01	19	24	13	09	34	26
15	73	83	26	7	63	32	41	58	32	72	61
62	64	27	13	24	07	42	35	24	63	23	45
45	76	74	52	52	34	56	72	72	56	67	80

☞ The ability to *add* with *rapidity and correctness* is confessedly a most valuable attainment; yet it is notorious that few pupils ever acquire it in school. The cause is the *neglect* of the Table, and the *want* of drilling. *Practice is the price of skill.*

(66.)		(67.)		(68.)		(69.)		(70.)	
Dols.	cts.	Dols.	cts.	Dols.	cts.	Dols.	cts.	Dols.	cts.
25	63	346	25	273	42	3424	27	4386	48
46	74	405	31	534	97	6213	39	3275	26
82	31	830	62	286	31	5382	13	5327	64
60	46	642	43	347	34	4561	63	4613	04
75	38	761	38	721	35	8324	29	1729	56
22	76	482	71	635	26	5276	34	3537	63
64	28	395	65	453	94	6594	32	8274	31
37	31	762	34	587	63	1723	45	5026	73
25	16	250	25	658	92	2674	56	1586	37
32	10	874	50	894	28	3295	31	2345	67
75	87	328	25	946	25	4463	79	4326	43
62	75	432	12	973	22	5324	28	5437	26
48	12	519	75	564	33	6543	20	6325	41
23	15	438	29	283	12	7035	28	7396	27
18	11	533	25	597	31	8546	37	8325	34
50	25	453	62	296	54	9634	25	9274	52
78	26	374	52	458	73	8346	52	3465	23
39	44	250	37	605	42	6275	35	4289	67
72	51	862	75	486	54	4235	34	5365	72
33	41	953	25	193	12	3271	05	6582	39
58	75	846	62	586	32	6137	94	1205	32
29	31	553	12	101	53	6283	59	4396	84
54	62	228	51	253	72	4346	32	5724	33
27	31	312	52	154	53	7294	58	1065	43
29	50	455	63	276	32	6275	63	4953	27
68	71	729	31	586	34	3284	32	2586	54
97	53	426	76	235	20	1635	34	4234	62
82	43	623	25	463	52	2586	89	1736	44
64	25	321	35	958	76	7434	26	5398	29
18	12	238	17	386	29	5869	73	1234	56
19	50	125	51	315	46	3276	42	7891	01
62	25	436	25	434	57	1635	38	1234	16
64	37	536	63	372	46	5913	84	6843	75
53	63	257	47	657	32	6284	35	7616	24

SUBTRACTION.

33. ***Subtraction*** is taking one number from another.

The ***Subtrahend*** is the number to be subtracted.

The ***Minuend*** is the number from which the subtraction is made.

The ***Difference*** or ***Remainder*** is the number found by subtraction. Thus, when it is said, 5 taken from 12 leaves 7, 12 is the minuend; 5 the subtrahend; and 7 the difference or remainder.

NOTES.—1. The term *subtraction*, is from the Latin *subtraho*, to *draw from under*, or *take away*.

The term *subtrahend*, from the same root, signifies *that which is to be* subtracted.

Minuend, from the Latin *minuo*, to *diminish*, signifies *that which is to be* diminished; the termination *nd* in each case having the force of *to be*.

2. Subtraction is the *opposite* of Addition. One unites, the other separates numbers.

3 When both numbers are the *same denomination*, the operation is called *Simple Subtraction*.

34. The ***Sign of Subtraction*** is a *short horizontal line* called *minus* (−), placed before the number to be subtracted. Thus, 6−4 means that 4 is to be taken from 6, and is read, "6 minus 4."

NOTE.—The term *minus*, Latin, signifies *less*, or *diminished* by.

Read the following expressions:

1. $21-3=2+3+13$.
2. $31-10=6+4+11$.
3. $45-15=8+12+10$.
4. $100-30=55+10+5$.

---

33. What is subtraction? What is the number to be subtracted called? The number from which the subtraction is made? The result? *Note.* Meaning of term Subtraction? Subtrahend? Minuend? Difference between Subtraction and Addition? When the Nos. are the same denomination, what is the operation called? 34. Sign of Subtraction? How read 6−4? Meaning of the term minus?

SUBTRACTION TABLE.

1 from			2 from			3 from			4 from			5 from		
1	leaves	0	2	leaves	0	3	leaves	0	4	leaves	0	5	leaves	0
2	"	1	3	"	1	4	"	1	5	"	1	6	"	1
3	"	2	4	"	2	5	"	2	6	"	2	7	"	2
4	"	3	5	"	3	6	"	3	7	"	3	8	"	3
5	"	4	6	"	4	7	"	4	8	"	4	9	"	4
6	"	5	7	"	5	8	"	5	9	"	5	10	"	5
7	"	6	8	"	6	9	"	6	10	"	6	11	"	6
8	"	7	9	"	7	10	"	7	11	"	7	12	"	7
9	"	8	10	"	8	11	"	8	12	"	8	13	"	8
10	"	9	11	"	9	12	"	9	13	"	9	14	"	9
11	"	10	12	"	10	13	"	10	14	"	10	15	"	10

6 from			7 from			8 from			9 from			10 from		
6	leaves	0	7	leaves	0	8	leaves	0	9	leaves	0	10	leaves	0
7	"	1	8	"	1	9	"	1	10	"	1	11	"	1
8	"	2	9	"	2	10	"	2	11	"	2	12	"	2
9	"	3	10	"	3	11	"	3	12	"	3	13	"	3
10	"	4	11	"	4	12	"	4	13	"	4	14	"	4
11	"	5	12	"	5	13	"	5	14	"	5	15	"	5
12	"	6	13	"	6	14	"	6	15	"	6	16	"	6
13	"	7	14	"	7	15	"	7	16	"	7	17	"	7
14	"	8	15	"	8	16	"	8	17	"	8	18	"	8
15	"	9	16	"	9	17	"	9	18	"	9	19	"	9
16	"	10	17	"	10	18	"	10	19	"	10	20	"	10

☞ It will aid the pupil in learning the Subtraction Table to observe that it is the *reverse* of Addition. Thus, he has learned that 3 and 2 are 5. Reversing this, he will see that 2 from 5 leaves 3, etc. Exercises combining the two tables will be found useful, as a review.

CASE I.

35. To find the *Difference*, when each figure of the Subtrahend is *less* than the corresponding figure of the Minuend.

Ex. 1. From 568 dollars, take 365 dollars.

OPERATION.	
\$568	Min.
365	Subt.
\$203	Rem.

ANALYSIS.—Let the numbers be set down, the *less* under the *greater*, as in the margin. Beginning at the right hand, we proceed thus: 5 units from 8 units leave 3 units; set the 3 in units' place under the figure subtracted, *because it is units.* Next, 6 tens from 6 tens leave 0 tens. Set the 0 in tens' place under the figure subtracted, because there are no *tens.* Finally, 3 hundreds from 5 hundreds leave 2 hundreds. Set the 2 in hundreds' place, *because it is hundreds.* The remainder is $203. All similar examples are solved in like manner.

By inspecting this illustration, the learner will discover the following principle:

Units of the same order are subtracted one from another, and the remainder is placed under the figure subtracted. (Arts. 9, 28.)

NOTES.—1. The *less* number is placed *under* the greater, with units under units, etc., for the sake of *convenience* and *rapidity* in subtracting.

2. Units of the *same order* are *subtracted from* each other, for the same reason that they are *added to* each other. (Art. 28, *n.*)

3. We subtract the figures of the subtrahend *separately,* because it is easier to subtract one figure at a time than several.

4. The remainder is set *under the figure subtracted,* because it is the *same order* as that figure.

	(2.)	(3.)	(4.)	(5.)	(6.)
From	648	726	8652	7230	9621
Take	415	516	3440	4120	8510

	(7.)	(8.)	(9.)	(10.)	(11.)
From	5304	6546	7852	8462	9991
Take	1102	3214	5220	4130	8880

	(12.)	(13.)	(14.)	(15.)
From	$6948	4315 ft.	$5346	9657 yds.
Take	$2416	3212 ft.	$3106	5021 yds.

	(16.)	(17.)	(18.)	(19.)
From	645921	8256072	72567803	965235804
Take	435010	6135052	42103201	604132402

20. What is the difference between 3275 and 2132?

21. What is the difference between 5384 and 3264?

22. A father gave his son $5750 and his daughter $4250: how much more did he give his son than his daughter?

CASE II.

36. To find the *Difference*, when a figure in the Subtrahend is *greater* than the corresponding figure of the Minuend.

Ex. 1. What is the difference between 8074 and 4869?

1st METHOD.	
8074	Min.
4869	Sub.
3205	Rem.

1st ANALYSIS.—Let the numbers be set down as in the margin. Beginning at the right, we proceed thus: 9 units cannot be taken from 4 units; we therefore take 1 ten from the 7 tens in the upper number, and add it to 4 units, making 14. Subtracting 9 units from 14 units leaves 5 units, which we place under the figure subtracted. Since we have taken 1 ten from 7 tens, there are but 6 tens left, and 6 tens from 6 tens leaves 0 tens. Next, 8 hundreds cannot be taken from 0 hundreds; hence, we take 1 thousand from the 8 thousand, and adding it to the 0, we have 10 hundreds. Taking 8 hundreds from 10 hundreds leaves 2 hundreds. Finally, 4 thousand from 7 thousand (8—1) leave 3 thousand. The remainder is 3205.

2d METHOD.	
8074	Min.
4869	Sub.
3205	Rem.

2d ANALYSIS.—Adding 10 to 4, the upper figure, makes 14, and 9 from 14 leaves 5. Now, to balance the ten added to the upper figure, we add 1 to the next higher order in the lower number. Adding 1 to 6 makes 7, and 7 from 7 leaves 0. Next, since we cannot take 8 from 0, we add 10 to the 0, and 8 from 10 leaves 2. Finally, adding 1 to 4 makes 5, and 5 from 8 leaves 3. The remainder is 3205, the same as before.

NOTE.—The chief difference between the two methods is this: In the first, we *subtract* 1 from the next figure in the *upper* number; in the second, we *add* 1 to the next figure in the *lower* number. The former is perhaps the more philosophical; but the latter is more convenient, and therefore generally practiced.

By inspecting this illustration, it will be seen,

If a figure in the lower number is larger than that above it, we add 10 *to the upper figure, then subtract, and add* 1 *to the next figure in the lower number.*

Rem —Instead of adding 10 to the upper figure, many prefer to take the lower figure directly from 10, and to the remainder add the upper figure. Thus, 9 from 10 leaves 1, and 4 make 5, etc.

37. Adding 10 to the upper figure is called *borrowing;* and adding 1 to the next figure in the lower number *pays* it.

NOTES.—1. The first method of borrowing depends upon the obvious principle that the value of a number is not altered by *transferring* a *unit* from a *higher* order to the *next lower*.

2. The reason that the second method of *borrowing* does not affect the difference between the two numbers, is because they *are equally increased*; and when two numbers are equally increased, their difference is not *altered*.

3. The reason for borrowing 10, instead of 5, 8, 12, or any other number, is because *ten* of a *lower order* are equal to *one* of the next *higher*. If numbers increased by the scale of 5, we should add 5 to the upper figure; if by the scale of 8, we should add 8, etc. (Art. 9.)

4. We begin to subtract at the right, because when we *borrow* we must *pay* before subtracting the next figure.

	(2.)	(3.)	(4.)	(5.)	(6.)
From	784	842	6704	8042	9147
Take	438	695	3598	5439	8638

38. The preceding principles may be summed up in the following

GENERAL RULE.

I. *Place the less number under the greater, units under units, etc.*

II. *Begin at the right, and subtract each figure in the lower number from the one above it, setting the remainder under the figure subtracted.* (Art. 35.)

III. *If a figure in the lower number is larger than the one above it, add* 10 *to the upper figure; then subtract, and add* 1 *to the next figure in the lower number.* (Art. 36.)

39. PROOF.—*Add the remainder to the subtrahend; if the sum is equal to the minuend, the work is right.*

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38. How write numbers for subtraction? How proceed when a figure in the lower number is greater than the one above it? 35. *Note.* Why write the less number under the greater, etc.? Why subtract the figures separately? Why set the remainder under the figure subtracted? 37. What is meant by borrowing? What is meant by paying or carrying? *Note.* Upon what principle does the first method of borrowing depend? Why does not the second method of borrowing affect the difference between the two numbers? Why borrow 10 instead of 5, 8, 12, etc.? Why begin to subtract at the right hand? 39. How prove subtraction? *Note.* Upon what does this proof depend?

NOTES.—1. This proof depends upon the Axiom, that the *whole is equal to the sum of all its parts.*

2. As soon as the class understand the rule, they should learn to omit all words but the results. Thus, in Ex. 2, instead of saying 9 from 4 you can't, 9 from 14 leaves 5, etc., say *five, naught, three,* etc.

EXAMPLES.

1. Find the difference between 84065 and 30428.

	(2.)	(3.)	(4.)	(5.)
From	824 rods,	4523 pounds,	6841 years,	7350 acres,
Take	519 rods.	4456 pounds.	3062 years.	5631 acres.

	(6.)	(7.)	(8.)	(9.)
From	23941	46083	52300	483672
Take	12367	23724	25121	216030

	(10.)	(11.)	(12.)	(13.)
From	638024	7423614	8605240	9042849
Take	403015	2414605	3062431	6304120

14. From 85269 pounds, take 33280 pounds.
15. From 412685 tons, take 103068 tons.
16. From 840005 acres, take 630651 acres.
17. What is the difference between 95301 and 60358?
18. What is the difference between 1675236 and 439243?
19. Subtract 2036573 from 5670378.
20. Subtract 35000384 from 68230460.
21. Subtract 250600325 from 600230021.
22. A man bought a house and lot for $36250, and paid $17260 down: how much does he still owe?
23. A man bought a farm for $19200, and sold it for $17285: what was his loss?
24. A merchant bought a cargo of tea for $1265235, and sold it for $1680261: what was his gain?
25. A's income is $645275, and B's $845280: what is the difference in their incomes?

26. A bankrupt's assets are $569257, and his debts $849236: how much will his creditors lose?

27. Subtract 9999999 from 11111110.

28. Subtract 666666666 from 7000000000.

29. Subtract 8888888888 from 10000000000.

30. Take 200350043010 from 490103060756.

31. Take 5310006057 3604 from 8013064500 2120.

32. Take 67500000364906 from 90163800024 1036.

33. From two millions and five, take ten thousand.

34. From one million, take forty-five hundred.

35. From sixty-five thousand and sixty-five, take five hundred and one.

36. From one billion and one thousand, take one million.

37. Our national independence was declared in 1776: how many years since?

38. Our forefathers landed at Plymouth in 1620: how many years since?

39. A father having 3265 acres of land, gave his son 1030 acres: how many acres has he left?

40. The Earth is 95300000* miles from the sun, and Venus 68770000 miles: required the difference.

41. Washington died in 1799, at the age of 67 years: in what year was he born?

42. Dr. Franklin was born in 1706, and died in 1790: at what age did he die?

43. Sir Isaac Newton died in 1727, at the age of 85 years: in what year was he born?

44. Jupiter is 496000000 miles from the sun, and Saturn 909000000 miles: what is the difference?

45. In 1865, the sales of A. T. Stewart & Co., by official returns, were $39391688; those of H. B. Claflin & Co., $42506715: what was the difference in their sales?

46. The population of the United States in 1860 was 31445080; in 1850 it was 23191876: what was the increase?

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* Kiddle's Astronomy.

QUESTIONS FOR REVIEW.

1. A man's gross income was $1565 a month for two successive months, and his outgoes for the same time $965: what was his net profit?

2. A man paid $2635 for his farm, and $758 for stock; he then sold them for $4500: what was his gain?

3. A flour dealer has 3560 barrels of flour; after selling 1380 barrels to one customer, and 985 to another, how many barrels will he have left?

4. If I deposit in bank $6530, and give three checks of $733 each, how much shall I have left?

5. What is the difference between 3658 + 256 + 4236 and 2430 + 1249?

6. What is the difference between 6035 + 560 + 75 and 5003 + 360 + 28?

7. What is the difference between 891 + 306 + 5007 and 40 + 601 + 1703 + 89?

8. What is the difference between 900130 + 23040 and 19004 + 100607?

9. A man having $16250, gained $3245 by speculation, and spent $5203 in traveling: how much had he left?

10. A farmer bought 3 horses, for which he gave $275, $320, and $418 respectively, and paid $50 down: how much does he still owe for them?

11. A young man received three legacies of $3263, $5490, and $7205 respectively; he lost $4795 minus $1360 by gambling: how much was he then worth?

12. What is the difference between 6286 + 850 and 6286 − 850?

13. What is the difference between 11325 − 2361 and 8030 − 3500?

14. What number added to 103256 will make 215378?

15. What number added to 573020 will make 700700?

16. What number subtracted from 230375 will leave 121487?

17. What number subtracted from 317250 will leave 190300?

18. If the greater of two numbers is 59253, and the difference is 21231, what is the less number?

19. If the difference between two numbers is 1363, and the greater is 45261, what is the less number?

20. The sum of two numbers is 63270, and one of them is 29385: what is the other?

21. What number increased by 2343 − 131, will become 3265 − 291?

22. What number increased by 3520 + 1060, will become 6539 − 279?

23. What number subtracted from 5009, will become 2340 + 471?

24. Agreed to pay a carpenter $5260 for building a house; $3520 for the masonry, and $1950 for painting: how much shall I owe him after paying $6000?

25. If a man earns $150 a month, and it costs him $63 a month to support his family, how much will he accumulate in 6 months?

26. A's income-tax is $1165, B's is $163 less than A's; and C's is as much as A and B's together, minus $365: what is C's tax?

27. A is worth $15230, B is worth $1260 less than A; and C is worth as much as both, wanting $1760: what is B worth, and what C?

28. The sum of 4 numbers is 45260; the first is 21000, the second 8200 less than the first, the third 7013 less than the second: what is the fourth?

29. A sailor boastingly said; If I could save $263 more, I should have $1000: how much had he?

30. The difference between A and B's estates is $1525; B, who has the least, is worth $17250: what is A worth?

MULTIPLICATION.

40. ***Multiplication*** is finding the ***amount*** of a number taken or added to itself, a given number of times.

The ***Multiplicand*** is the number to be multiplied.

The ***Multiplier*** is the number by which we multiply.

The ***Product*** is the number found by multiplication. Thus, when it is said that 3 times 6 are 18, 6 is the multiplicand, 3 the multiplier, and 18 the product.

41. The multiplier shows *how many times* the multiplicand is to be taken. Thus,

Multiplying by 1 is taking the multiplicand *once;*

Multiplying by 2 is taking the multiplicand *twice;* and

Multiplying by *any whole number* is taking the multiplicand as *many times* as there are *units* in the *multiplier.*

NOTES.—1. The term *multiplication,* from the Latin *multiplico, multus,* many, and *plico,* to fold, primarily signifies to increase by *regular accessions.*

2. *Multiplicand,* from the same root, signifies *that which is to be* multiplied; the termination *nd,* having the force of *to be.* (Art. 33, *n.*)

42. The multiplier and multiplicand are also called *Factors;* because they *make* or *produce* the *product.* Thus, 2 and 7 are the factors of 14.

NOTES.—1. The term *factor,* is from the Latin *facio,* to *produce.*

2. When the multiplicand contains only *one denomination,* the operation is called *Simple Multiplication.*

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40. What is multiplication? What is the number multiplied called? The number to multiply by? The result? When it is said, 3 times 4 are 12, which is the multiplicand? The multiplier? The product? 41. What does the multiplier show? What is it to multiply by 1? By 2? 42. What else are the numbers to be multiplied together called? Why? *Note.* Meaning of the term factor? What is the operation called when the multiplicand contains only one denomination? 43. The sign of multiplication.

43. The ***Sign of Multiplication*** is an oblique cross (×), placed between the factors. Thus, 7 × 5 means that 7 and 5 are to be multiplied together, and is read "7 times 5," "7 multiplied by 5," or "7 into 5."

44. Numbers subject to the same operation are placed within a *parenthesis* ( ), or under a line called a *vinculum* (——). Thus $(4+5)\times 3$, or $\overline{4+5}\times 3$, shows that the sum of 4 and 5 is to be multiplied by 3.

MULTIPLICATION TABLE.

2 times	3 times	4 times	5 times	6 times	7 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7
2 " 4	2 " 6	2 " 8	2 " 10	2 " 12	2 " 14
3 " 6	3 " 9	3 " 12	3 " 15	3 " 18	3 " 21
4 " 8	4 " 12	4 " 16	4 " 20	4 " 24	4 " 28
5 " 10	5 " 15	5 " 20	5 " 25	5 " 30	5 " 35
6 " 12	6 " 18	6 " 24	6 " 30	6 " 36	6 " 42
7 " 14	7 " 21	7 " 28	7 " 35	7 " 42	7 " 49
8 " 16	8 " 24	8 " 32	8 " 40	8 " 48	8 " 56
9 " 18	9 " 27	9 " 36	9 " 45	9 " 54	9 " 63
10 " 20	10 " 30	10 " 40	10 " 50	10 " 60	10 " 70
11 " 22	11 " 33	11 " 44	11 " 55	11 " 66	11 " 77
12 " 24	12 " 36	12 " 48	12 " 60	12 " 72	12 " 84

8 times	9 times	10 times	11 times	12 times
1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

☞ The pupil will observe that the *products* by 5, terminate in 5 and 0, alternately. Thus, 5 times 1 are 5; 5 times 2 are 10.

The *products* by 10 consist of the *figure multiplied* and a *cipher.* Thus, 10 times 1 are 10; ten times 2 are 20, and so on.

The *first nine* products by 11 are formed by repeating the figure multiplied. Thus, 11 times 1 are 11; 11 times 2 are 22, and so on.

The first figure of the *first nine* products by 9 increases, and the second decreases regularly by 1; while the sum of the digits in each product is 9. Thus, 9 times 2 are 18, 9 times 3 are 27, and so on.

45. When the factors are *abstract* numbers, it is immaterial in what order they are multiplied. Thus, the product of 3 times 4 is equal to 4 times 3. For, taking 4 units 3 times, is the same as taking 3 units 4 times; that is, $4 \times 3 = 3 \times 4$.

NOTES.—1. It is more convenient and therefore customary, to take the *larger* of the two given numbers for the multiplicand. Thus, it is better to multiply 5276 by 8, than 8 by 5276.

2. Multiplication is the same in principle as *Addition;* and is sometimes said to be *a short method of adding a number to itself a given number of times.* Thus: 4 stars + 4 stars + 4 stars = 3 times 4 or 12 stars.

46. The *product* is the *same name* or *kind* as the *multiplicand.* For, *repeating* a number does not *change* its *nature.* Thus, if we repeat dollars, they are still dollars, etc.

47. The *multiplier* must be an *abstract* number, or considered as such for the time being. For, the multiplier shows *how many times* the multiplicand is to be taken; and to say that one quantity is taken as many times as another is *heavy*—is *absurd.*

NOTE.—When it is asked what 25 cts. multiplied by 25 cts., or 2s. 6d. by 2s. 6d., will produce, the questions, if taken literally, are *nonsense.* For, 2s. 6d. cannot be repeated 2s. 6d. times, nor 25 cts. 25 cts. times; but we can multiply 25 cts. by the number 25, and 2s. 6d. by 2½. In like manner we can multiply the *price* of 1 yard by the *number* of yards in an article, and the *product* will be the *cost.*

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45. When the factors are abstract, does it make any difference with the product which is taken for the multiplicand? *Note.* Which is commonly taken? Why? What is Multiplication the same as? 46. What denomination is the product? Why? 47. What must the multiplier be? Why?

CASE I.

48. To find the *Product* of two numbers, when the Multiplier has but one figure.

1. What is the cost of 3 horses, at $286 apiece?

OPERATION.	
$286	Multd.
3	Mult.
$858	Prod.

ANALYSIS.—3 horses will cost 3 times as much as 1 horse. Let the numbers be set down as in the margin. Beginning at the right, we proceed thus: 3 times 6 units are 18 units. We set the 8 in units' place because it is units, and carry the 1 ten to the product of the tens, as in Addition. (Art. 29.) Next, 3 times 8 tens are 24 tens and 1 (to carry) make 25 tens, or 2 hundred and 5 tens. We set the 5 in tens' place because it is tens, and carry the 2 hundred to the product of hundreds. Finally, 3 times 2 hundred are 6 hundred, and 2 (to carry) are 8 hundred. We set the 8 in hundreds' place because it is hundreds. Therefore the 3 horses cost $858. All similar examples are solved in like manner.

By inspecting the preceding analysis, the learner will discover the following principle:

Each figure of the multiplicand is multiplied by the multiplier, beginning at the right, and the result is set down as in Addition. (Art. 29.)

NOTES.—1. The *reason* for setting the multiplier *under* the multiplicand is simply for *convenience* in multiplying.

2. The *reasons* for beginning to multiply at the right hand, as well as for setting down the units and carrying the *tens*, are the same as those in Addition. (Art. 29, *n.*)

3. In reciting the following examples, the pupil should carefully analyze each; then give the results of the operations required.

49. *Units* multiplied by *units*, it should be observed, produce *units*; *tens* by *units*, produce *tens*; *hundreds* by *units*, produce *hundreds*; and, universally,

If the multiplying figure is *units*, the product will be the *same order* as the figure *multiplied*.

Again, if the figure multiplied is *units*, the *product* will

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49. What do units multiplied by units produce? Tens by units? Hundreds by units? When the multiplying figure is units, what is the product?

be the *same order* as the multiplying figure; for the product is the same, whichever factor is taken for the multiplier. (Art. 45.)

	(2.)	(3.)	(4.)	(5.)
Multiply	2563	13278	2648203	48033265
By	4	5	6	7

6. What will 8 carriages cost, at $750 apiece?

7. What cost 9 village lots, at $1375 a lot?

8. At $3500 apiece, what will 10 houses cost?

9. At $865 a hundred, how much will 11 hundredweight of opium come to?

10. If a steamship sail 358 miles per day, how far will she sail in 12 days?

11. Multiply 86504 by 5.

12. Multiply 803127 by 7.

13. Multiply 440210 by 6.

14. Multiply 920032 by 8.

15. Multiply 603050 by 9.

16. Multiply 810305 by 10.

17. Multiply 753825 by 11.

18. Multiply 954635 by 12.

19. What cost 4236 barrels of apples, at $7 a barrel?

20. What cost 5167 melons, at 11 cents apiece?

21. At 6 shillings apiece, what will 1595 arithmetics cost?

22. At $12 a barrel, what will be the cost of 1350 barrels of flour?

23. What will 1735 boxes of soap cost, at $9 a box?

24. When peaches are 12 shillings a basket, what will 2363 baskets cost?

25. If a man travels 8 miles an hour, how far will he travel in 3260 hours?

26. A builder sold 10 houses for $4560 apiece: how much did he receive for them all?

27. What will it cost to construct 11 miles of railroad, at $12250 per mile?

28. What will be the expense of building 12 ferry-boats, at $23250 apiece?

CASE II.

50. To find the *Product* of two numbers, when the Multiplier has two or more figures.

1. A manufacturer bought 204 bales of cotton, at $176 a bale: what did he pay for the cotton?

$176	Multd.
204	Mult.
704	
352	
$35904	*Ans.*

ANALYSIS.—Write the numbers as in the margin. Beginning at the right: 4 times 6 units are 24 units, or 2 tens and 4 units. Set the 4 in units' place, and carry the 2 to the next product. 4 times 7 tens are 28 tens and 2 are 30 tens, or 3 hundred and 0 tens. Set the 0 in tens' place and carry the 3 to the next product. 4 times 1 hun. are 4 hun. and 3 are 7 hun. Set the 7 in hundreds' place. The product by 0 tens is 0; we therefore omit it. Again, 2 hundred times 6 units are 12 hun. units, equal to 1 thousand and 2 hundred. Set the 2 in hundreds' place and carry the 1 to the next product. 2 hundred times 7 tens are 14 hundreds of tens, equal to 14 thousand, and 1 are 15 thousand. Set the 5 in thousands' place, and carry the 1 to the next product, and so on. Adding these results, the sum is the cost.

REMARK.—The results which arise from multiplying the multiplicand by the separate figures of the multiplier, are called *partial products;* because they are *parts* of the whole product.

By inspecting this analysis, the learner will discover,

1. *The multiplicand is multiplied by each figure of the multiplier, beginning at the right, and the result set down as in Addition.*

2. *The first figure of each partial product is placed under the multiplying figure, and the sum of the partial products is the true answer.*

NOTES.—1. When there are *ciphers* between the significant figures of the multiplier, omit them, and multiply by the next significant figure.

2. We multiply by each figure of the multiplier separately, when it exceeds 12, for the obvious reason, that it is not *convenient* to multiply by the whole of a large number at once.

3. The *first* figure of each partial product is placed under the *multiplying figure;* because it is the *same order* as that figure.

4. The several partial products are added together, because the *whole product* is equal to the *sum* of *all its parts.*

	(2.)	(3.)	(4.)	(5.)
Multiply	426	563	1248	2506
By	24	35	52	304

51. The preceding principles may be summed up in the following

GENERAL RULE

I. *Place the multiplier under the multiplicand, units under units, etc.*

II. *When the multiplier has but one figure, beginning at the right, multiply each figure of the multiplicand by it, and set down the result as in Addition.*

III. *If the multiplier has two or more figures, multiply the multiplicand by each figure of the multiplier separately, and set the first figure of each partial product under the multiplying figure.*

Finally, the sum of the partial products will be the answer required.

NOTE.—The pupil should early learn to abbreviate the several steps in multiplying as in Addition. (Art. 31, *n.*)

PROOF.

52. By Multiplication.—*Multiply the multiplier by the multiplicand; if this result agrees with the first, the work is right.*

NOTE.—This proof is based upon the principle that the result will be the same, whichever number is taken as the multiplicand.

53. By excess of 9s.—*Find the excess of 9s in each factor separately; then multiply these excesses together, and reject the 9s from the result;* if this *excess* agrees with the excess of 9s in the answer, the work is right.

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51. How write numbers for multiplication? When the multiplier has but one figure, how proceed? When it has two or more? What is finally done with the partial products? 48. *Note.* Why write the multiplier under the multiplicand, units under units? Why begin at the right hand? 50. What are partial products? *Note.* Why multiply by each figure separately? Why set the first figure of each under the multiplying figure? Why add the several partial products together? 52. How prove multiplication by multiplication? *Note.* Upon what is this proof based? 53. How prove it by excess of 9s?

NOTE.—This method of proof, if deemed advisable, may be omitted till review. It is placed here for convenience of reference. Though depending on a peculiar property of numbers, it is easily applied, and is confessedly the most expediitous method of proving multiplication yet devised.

54. To find the *Excess* of 9s in a number.

Beginning at the left hand, add the figures together, and as soon as the sum is 9 or more, reject 9 and add the remainder to the next figure, and so on.

Let it be required to find the excess of 9s in 7548467.

Adding 7 to 5, the sum is 12. Rejecting 9 from 12, leaves 3; and 3 added to 4 are 7, and 8 are 15. Rejecting 9 from 15, leaves 6; and 6 added to 4 are 10. Rejecting 9 from 10, leaves 1; and 1 added to 6 are 7, and 7 are 14. Finally, rejecting 9 from 14 leaves 5, the excess required.

NOTES.—1. It will be observed that the excess of 9s in any *two* digits is always equal to the sum, or the excess in the sum, of those digits. Thus, in 15 the excess is 6, and $1+5=6$; so in 51 it is 6, and $5+1=6$. In 56 the sum is 11, the excess 2.

2. The operation of finding the excess of 9s in a number is called *casting out the 9s.*

EXAMPLES.

1. What is the product of 746 multiplied by 475?

OPERATION.	*Proof by Excess of 9s.*	*Proof by Mult.*
746	Excess of 9s in multd. is 8	475
475	" 9s in mult. " 7	746
3730	Now............$8\times7=56$	2850
5222	The excess of 9s in 56 is 2	1900
2984	The excess of 9s in prod. is 2	3325
	And................$2=2$	
Ans. 354350		*Ans.* 354350

	(2.)	(3.)	(4.)	(5.)
Multiply	5645	18934	48367	231456
By	43	65	75	87

54. How find the excess of 9s?

	(6.)	(7.)	(8.)	(9.)
Multiply	1421673	2342678	4392460	5230648
By	234	402	347	526

10. Mult. 640231 by 205.
11. Mult. 520608 by 675.
12. Mult. 431220 by 1234.
13. Mult. 623075 by 2650.
14. Mult. 730650 by 2167.
15. Mult. 593287 by 6007.
16. Mult. 843700 by 3465.
17. Mult. 748643 by 2100.
18. Mult. 9000401 by 50001.
19. Mult. 82030405 by 23456.

20. How many pounds in 1375 chests of tea, each chest containing 63 pounds?

21. What cost 738 carts, at $75 apiece?

22. At 43 bushels per acre, how many bushels of wheat will 520 acres produce?

23. At $163 apiece, what will be the cost of 1368 covered buggies?

24. If a man travel 215 miles per day, how far can he travel in 365 days?

25. There are 5280 feet in a mile: how many feet in 256 miles?

26. What cost 2115 revolvers, at $23 apiece?

27. Bought 1978 barrels of pickles, at $17; how much did they come to?

28. If railroad cars are $4735 apiece, what will be the expense of 500?

29. How far will 2163 spools of thread extend, each containing 25 yards?

30. Bought 15265 ambulances, at $117 apiece: what was the amount of the bill?

31. What will 3563 tons of railroad iron cost, at $68 per ton?

32. How far will a man skate in 6 days, allowing he skates 8 hours a day, and goes 7 miles an hour?

CONTRACTIONS.

55. A ***Composite Number*** is the *product* of two or more factors, each of which is *greater* than 1. Thus $15=3\times5$, and $42=2\times3\times7$, are composite numbers.

NOTES.—1. The product of a number multiplied into itself is called a *power*. Thus, $9=3\times3$, is a power.

2. *Composite* is from the Latin *compono*, to *place* together.

3. The terms *factors* and *parts* must not be confounded with each other. The *former* are *multiplied* together to produce a number; the latter are *added*. Thus, 3 and 5 are the *factors* of 15; but 5 and 10, 6 and 9, 7 and 8, etc., are the *parts* of 15.

1. What are the factors of 45? *Ans.* 5 and 9.
2. What are the factors of 24? Of 27? Of 28? Of 30?
3. What are the factors of 32? Of 35? Of 42? Of 48?
4. What are the factors of 54? Of 63? Of 72? Of 84?

CASE I.

56. To multiply by a *Composite* Number.

1. A farmer sold 15 boxes of butter, each weighing 20 pounds: how many pounds did he sell?

ANALYSIS.—15=5 times 3; hence, 15 boxes will weigh 5 times as much as 3 boxes. Now, if 1 box weighs 20 pounds, 3 boxes will weigh 3 times 20, or 60 pounds. Again, if 3 boxes weigh 60 pounds, 5 times 3 boxes will weigh 5 times 60, or 300 pounds. He therefore sold 300 pounds. In the operation, we first multiply by the factor 3, and the product thus arising by the other factor 5. Hence, the

OPERATION.

$$\begin{array}{r} 20 \text{ pounds.} \\ 3 \\ \hline 60 \\ 5 \\ \hline \end{array}$$

Ans. 300 pounds.

RULE.—*Multiply the multiplicand by one of the factors of the multiplier, then this product by another, and so on, till all the factors have been used.*

The last product will be the answer.

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55. A composite number? *Note.* The difference between factors and parts?
56. How multiply by a composite number?

NOTES.—1. This rule is based upon the principle that it is immaterial in what order *two factors* are multiplied. (Art. 45.)

The same illustration may be extended to *three* or *more* numbers. For, the product of two of the factors may be considered as one number, and this may be used before or after a third factor, etc.

2. The process of multiplying *three* or *more factors* together, is called *Continued Multiplication;* the result, the *continued product.*

2. Multiply $568 by 35. *Ans.* $19880.

3. Multiply 2604 by 25. 4. Multiply 6052 by 48.

5. Multiply 8091 by 63. 6. Multiply 45321 by 72.

7. In 1 cubic foot there are 1728 cubic inches: how many cubic inches are there in 84 cubic feet?

8. If a ton of copper ore is worth $5268, what is the worth of 56 tons?

9. What cost 125 houses, at $1580 apiece?

CASE II.

57. To multiply by 10, 100, 1000, etc.

10. What will 100 cows cost, at $31 apiece?

ANALYSIS.—Annexing a cipher to a number moves each figure *one place* to the *left;* but moving a figure one place to the left increases its value *ten times;* therefore annexing a cipher to a number multiplies it by 10. In like manner, annexing *two* ciphers, multiplies it by a hundred, etc. (Art. 12.) Therefore, $31 × 100 = $3100. Hence, the

RULE.—*Annex as many ciphers to the multiplicand as there are ciphers in the multiplier.*

NOTE.—The term *annex,* from the Latin *ad* and *necto,* to join to, signifies to *place after.*

11. What cost 1000 horses, at $356 apiece?

12. Multiply 40530 by 1000.

13. Multiply 9850685 by 10000.

14. Multiply 84050071 by 100000.

15. Multiply 360753429 by 1000000.

---

57. How multiply by 10, 100, 1000, etc.?

CASE III.

58. To multiply, when there are Ciphers on the right of either or both Factors.

20. What is the product of 87000 multiplied by 230?

ANALYSIS.—The factors of the multiplicand are 87 and 1000; the factors of the multiplier are 23 and 10. We first multiply the factors consisting of significant figures; then multiply this product by the other two factors (1000 × 10), or 10000, by annexing 4 ciphers to it. Hence, the

```
     87000
       230
     -----
      261
     174
     -----
Ans. 20010000
```

RULE.—*Multiply the significant figures together; and to the result annex as many ciphers as are found on the right of both factors.*

NOTE.—This rule is based upon the two preceding cases; for, by supposition, one or both the given numbers are *composite*; and one of the factors of this composite number is 10, 100, etc.

	(21.)	(22.)	(23.)
Multiply	2130	64000	83046
By	700	52	2000

24. In 1 barrel of pork there are 200 pounds: how many pounds in 3700 barrels?

25. What will 2300 head of cattle cost, at $80 per head?

26. In $1 there are 100 cts.: how many cts. in $26000?

27. The salary of the President is $50000 a year: how much will it amount to in 21 years?

28. If a clock ticks 86400 times in 1 day, how many times will it tick in 7000 days?

29. If one ship costs $150000, what will 49 cost?

30. 670103700 × 60030040?

31. 800021000 × 80002100?

32. 570305000 × 40000620?

---

58. When one or both factors have ciphers on the right? *Note.* Upon what is this rule based?

33. 467234630 × 27000000?

34. 890000000 × 350741237?

35. 9400000027 × 28000000?

36. Multiply 39 millions and 200 thousand by 530 thousand.

37. Multiply 102 times 700 thousand and 1 hundred by 601 thousand and twenty.

38. Multiply 74 millions and 21 thousand by 5 millions and 5 thousand.

39. Multiply 31 millions 31 thousand and 31 by 21 thousand and twenty-one.

40. Multiply 2 billions, 2 millions, 2 thousand and 2 hundred by 200 thousand and 2 hundred.

CASE IV.

59. To multiply 13, 14, 15, or 1 with a *Significant Figure* annexed.

41. If one city lot costs $3245, what will 17 lots cost?

ANALYSIS.—17 lots will cost 17 times as much as 1 lot. Placing the multiplier on the right, we multiply the multiplicand by the 7 units, set each figure one place to the right of the figure multiplied, and add the partial product to the multiplicand. The result is $55165. Hence, the

```
3245 × 17
 22715
--------
$55165 An.
```

RULE.—I. *Multiply the multiplicand by the units' figure of the multiplier, and set each figure of the partial product one place to the right of the figure multiplied.*

II. *Add this partial product to the multiplicand, and the result will be the true product.*

NOTE.—This contraction depends upon the principle that as the *tens' figure* of the multiplier is 1, the multiplicand is the *second partial product*; hence its *first figure* must stand in *tens'* place.

42. Multiply 1368 by 13. 43. Multiply 2106 by 14.

44. Multiply 3065 by 15. 45. Multiply 6742 by 16.

46. Multiply 25269 by 18. 47. Multiply 83467 by 19.

---

59. How multiply by 13, 14, 15, or 1 with a significant figure annexed? *Note.* Upon what is this contraction based?

DIVISION.

60. ***Division*** is finding how many times one number is contained in another.

The ***Dividend*** is the number to be divided.

The ***Divisor*** is the number by which we divide.

The ***Quotient*** is the number found by division, and shows how *many times* the divisor is contained in the dividend.

The ***Remainder*** is a part of the dividend *left* after division. Thus, when it is said, 5 is contained in 17, 3 times and 2 over, 17 is the dividend, 5 the divisor, 3 the quotient, and 2 the remainder.

NOTES.—1. The term *division*, is from the Latin *divido*, to *part*, or *divide*.

The term *dividend*, from the same root, signifies *that which is to be divided*; the termination *nd*, having the force of *to be*. (Art. 33, *n.*)

Quotient is from the Latin *quoties*, signifying *how often* or *how many times*.

2. The *remainder* is always the *same denomination* as the dividend; for it is an undivided *part* of it.

3. A *proper* remainder is always *less* than the divisor.

61. An obvious way to find how many times the *divisor* is contained in the *dividend*, is to *subtract* the divisor from the dividend *continually*, till the *latter* is exhausted, or till the *remainder* is *less* than the divisor; the *number* of subtractions will be the *quotient*. Thus, taking any two numbers, as 4 and 12, we have $12-4=8$; $8-4=4$; and $4-4=0$. Here are *three* subtractions; therefore, 4 is contained in 12, 3 times. Hence,

Division is sometimes said to be a *short method of continued subtraction*.

---

60. What is division? The number to be divided called? To divide by? The result? The part left? *Note.* Meaning of the term division? Dividend? Quotient? What denomination is the remainder? Why?

But there is a *shorter* and *more direct* way of obtaining the quotient. For we know by the multiplication table, that 3 times 4, or 4 taken 3 times, are 12; hence, 4 is contained in 12, 3 times.

62. Division is the *reverse* of multiplication. In multiplication *both factors* are given, and it is required to find the *product;* in division, *one factor* and the *product* (which answers to the dividend) are given, and it is required to find the other *factor,* which answers to the *quotient.* Hence, division may be said to be *finding a quotient which, multiplied into the divisor, will produce the dividend.*

NOTE.—When the dividend contains *only one denomination,* the operation is called *Simple Division.*

DIVISION TABLE.

1 is in 1, once.		2 is in 2, once.		3 is in 3, once.		4 is in 4, once.		5 is in 5, once.		6 is in 6, once.	
2,	2	4,	2	6,	2	8,	2	10,	2	12,	2
3,	3	6,	3	9,	3	12,	3	15,	3	18,	3
4,	4	8,	4	12,	4	16,	4	20,	4	24,	4
5,	5	10,	5	15,	5	20,	5	25,	5	30,	5
6,	6	12,	6	18,	6	24,	6	30,	6	36,	6
7,	7	14,	7	21,	7	28,	7	35,	7	42,	7
8,	8	16,	8	24,	8	32,	8	40,	8	48,	8
9,	9	18,	9	27,	9	36,	9	45,	9	54,	9
10,	10	20,	10	30,	10	40,	10	50,	10	60,	10

7 is in 7, once.		8 is in 8, once.		9 is in 9, once.		10 is in 10, once.		11 is in 11, once.		12 is in 12 once.	
14,	2	16,	2	18,	2	20,	2	22,	2	24,	2
21,	3	24,	3	27,	3	30,	3	33,	3	36,	3
28,	4	32,	4	36,	4	40,	4	44,	4	48,	4
35,	5	40,	5	45,	5	50,	5	55,	5	60,	5
42,	6	48,	6	54,	6	60,	6	66,	6	72,	6
49,	7	56,	7	63,	7	70,	7	77,	7	84,	7
56,	8	64,	8	72,	8	80,	8	88,	8	96,	8
63,	9	72,	9	81,	9	90,	9	99,	9	108,	9
70,	10	80,	10	90,	10	100,	10	110,	10	120,	10

61. What is an obvious way to find how many times the divisor is contained in the dividend? What is division sometimes called?

OBJECTS OF DIVISION.

63. The ***object*** or ***office*** of Division is twofold:

1st, *To find how many times one number is contained in another.*

2d, *To divide a number into equal parts.*

63, *a*. To find how *many times* one number is contained in another.

1. A man has 15 dollars to lay out in books, which are 3 dollars apiece: how many can he buy?

ANALYSIS.—In this problem the object is to find *how many times* 3 dols. are contained in 15 dols.

Let the 15 dols. be represented by 15 counters, or unit marks. Separating these into groups of 3 each, there are 5 groups. Therefore, he can buy 5 books.

✧ ✧ ✧ | ✧ ✧ ✧ | ✧ ✧ ✧ | ✧ ✧ ✧ | ✧ ✧ ✧

63, *b*. To divide a number into *equal parts*.

2. If a man divides 15 dollars equally among 3 persons, how many dollars will each receive?

ANALYSIS.—The object here is to divide 15 dols. into 3 equal parts. Let the 15 dols. be represented by 15 counters. If we form 3 groups, first putting 1 counter in each, then another, till the counters are exhausted, each group will have 5 counters. Therefore, each person will receive 5 dollars.

✧ ✧ ✧ ✧ ✧ | ✧ ✧ ✧ ✧ ✧ | ✧ ✧ ✧ ✧ ✧

REMARK.—The preceding are *representative* examples of the two classes of problems to which Division is applied. In the *first*, the divisor and dividend are the *same denomination*, and the quotient is *times*, or an *abstract* number.

In the *second*, the divisor and dividend are *different denominations*, and the *quotient* is the same denomination as the *dividend*. Hence,

64. When the divisor and dividend are the *same* denomination, the quotient is always an *abstract* number.

---

62. Of what is division the reverse? What is given in multiplication? What required? What is given in division? What required? *Note*. When the dividend contains but one denomination, what is the operation called? 63. What is the object or office of division? 63, *a*. What is the object in the first problem? 63, *b*. What in the second? *Rem.* What is said of first two problems?

When the divisor and dividend are *different* denominations, the quotient is always the *same denomination* as the *dividend.*

NOTES.—1. The process of separating a number into *equal parts,* as required in the second class of problems, gave rise to the name "Division." It is also the origin of *Fractions.* (Art. 134.)

2. The mode of reasoning in the solution of these two classes of examples is somewhat different; but the practical operation is the same, viz.: to *find how many times* one number is contained in another, which accords with the definition of Division.

65. When a number or thing is divided into *two equal* parts, the parts are called *halves;* into *three,* the parts are called *thirds;* into *four,* they are called *fourths;* etc.

The *number* of parts is indicated by their *name.*

66. A number is divided into two, three, four, five, etc., equal parts by dividing it by 2, 3, 4, 5, etc., respectively.

3. What is a half of 12? A third of 15? A fourth of 20? A fifth of 35? A seventh of 28?

4. What is a sixth of 42? A seventh of 56? A ninth of 63? An eighth of 72? A twelfth of 108?

67. The ***Sign of Division*** is a short horizontal line between two dots ($\div$), placed before the divisor. Thus, the expression $28 \div 7$, shows that 28 is to be divided by 7, and is read, "28 divided by 7."

68. Division is also denoted by *writing the divisor under the dividend,* with a short line between them. Thus, $\frac{28}{7}$ is equivalent to $28 \div 7$. It is read, "28 divided by 7," or "28 *sevenths.*"

69. Division is commonly distinguished as *Short Division* and *Long Division.*

---

64. When the divisor and dividend are the same denomination, what is the quotient? When different denominations, what? 65. When a number is divided into two equal parts, what are the parts called? Into three? Four? 66. How divide a number into 2, 3, 4, etc., equal parts? 67. What is the sign of division? What does the expression $35 \div 7$ show? 68. How else is division denoted?

SHORT DIVISION.

70. ***Short Division*** is the method of dividing, when the *results* of the several steps are carried in the mind, and the *quotient* only is set down.

71. To divide by *Short Division*.

Ex. 1. If apples are $3 a barrel, how many barrels can you buy for $693?

OPERATION.

$$\$3)\underline{\$693}$$

Quot. 231 bar.

ANALYSIS.—Since $3 will buy 1 barrel, $693 will buy as many barrels as $3 are contained times in $693. Let the numbers be set down as in the margin. Beginning at the left, we proceed thus: 3 is contained in 6 hundred, 2 hundred times. Set the 2 in hundreds' place, under the figure divided, *because it is hundreds.* Next, 3 is contained in 9 tens, 3 tens times. Set the 3 in tens' place under the figure divided. Finally, 3 is contained in 3 units, 1 time. Set the 1 in units' place. *Ans.* 231 barrels.

REM.—This problem belongs to the 1st class, the object being to find how many times one number is contained in another. (Art. 63, *a*.)

Solve the following examples in the same manner:

(2.)	(3.)	(4.)	(5.)
$2)\underline{4468}$	$3)\underline{3696}$	$4)\underline{4848}$	$5)\underline{5555}$

6. A man having 27543 pounds of grapes, packed them for market in boxes containing 5 pounds each: how many boxes did he fill, and how many pounds over?

OPERATION.

$5)\underline{27543}$ pounds.

Quot. 5508 boxes,
and 3 p. over.

ANALYSIS.—He used as many boxes as there are times 5 pounds in 27543 pounds. Let the numbers be set down as in the margin. Since the divisor 5 is not contained in the first figure of the dividend, we find how many times it is contained in the first two figures, which is 5 times and 2 over. We set the quotient figure 5 under the right hand figure divided, because it is the same order as that figure, and *prefix* the remainder 2, mentally to the next figure of the dividend, making 25. Now 5 is in 25, 5 times, and no remainder. Again, 5 is not contained in 4, the next figure of the dividend; we therefore place a cipher in the quotient, and prefix the 4 mentally to the next figure of the dividend, as if it were a remainder, making 43. Finally, 5 is in 43, 8 times, and 3 over. He therefore filled 5508 boxes, and had 3 pounds remainder. Hence, the

RULE.—I. *Place the divisor on the left of the dividend, and beginning at the left, divide each figure by it, setting the result under the figure divided.*

II. *If the divisor is not contained in a figure of the dividend, put a cipher in the quotient, and find how many times it is contained in this and the next figure, setting the result under the right hand figure divided.*

III. *If a remainder arise from any figure before the last, prefix it mentally to the next figure, and divide as before.*

If from the last figure, place it over the divisor, and annex it to the quotient.

NOTES.—1. In the operation, the divisor is placed on the *left* of the dividend, and the quotient *under* it, as a matter of convenience. When division is simply represented, the divisor is either placed *under* the dividend, or *on the right,* with the sign (÷) before it.

2 The reason for beginning to divide at the *left hand* is, that in dividing a higher order there may be a *remainder,* which must be prefixed to the next lower order, as we proceed in the operation.

3. We place the quotient figure under the figure divided; because the *former* is the *same order* as the *latter.* (Art. 71.)

4. When the divisor is not contained in a figure of the dividend, we place a *cipher* in the quotient, to show that the quotient has no units corresponding with the order of this figure. It also preserves the *local value* of the subsequent figures of the quotient.

5. The final remainder shows that a part of the dividend is not divided. It is placed over the divisor and annexed to the quotient to complete the division.

---

70. What is Short Division? 71 How write numbers for short division? The next step? When the divisor is not contained in a figure of the dividend, how proceed? When there is a remainder after dividing a figure, how? If there is a remainder after dividing the last figure, what? *Note.* Why place the divisor on the *left* of the dividend? Why begin to divide at the left hand? Why place each quotient figure under the figure divided? Why place a cipher in the quotient, when the divisor is not contained in a figure of the dividend? What does the final remainder show? Why place it over the divisor and annex it to the quotient?

☞ The pupil should early learn to abbreviate the language used in the process of dividing. Thus, in the next example, instead of saying 5 is contained in 7 once, and 2 over, let him pronounce the quotient figures only; as, *one, four, five, seven.*

1. A man divided 7285 acres of land equally among his 5 sons: what part, and how much, did each receive?

SOLUTION.—1 is 1 fifth of 5; hence each had 1 fifth part. (Art. 65.) Again, 7285 A ÷ 5 = 1457 A; hence each received 1457 A. (Art. 66.)

(2.)	(3.)	(4.)	(5.)
2)436784	3)560346	4)689034	5)748239
(6.)	(7.)	(8.)	(9.)
6)3972647	7)4806108	8)7390464	9)8306729

(10.)	(11.)	(12.)
10)57623140	11)667301451	12)8160252397

13. At $2 apiece, how many hats can be bought for $16486? (Art 48, Note 3.)

14. At $4 a head, how many sheep can be bought for $844?

15. How many times are 3 rods contained in 26936 rods?

16. A man having $42684, divided it equally among his 4 children: how much did each receive?

17. If a quantity of muslin containing 366 yards is divided into 3 equal parts, how many yards will each part contain?

18. If 84844 pounds of bread are divided equally among 6 regiments, how many pounds will each regiment receive?

19. Eight men found a purse containing $64968, which they shared equally: how much did each receive?

20. Divide 4268410 by 4.
21. Divide 5601234 by 6.
22. Divide 6403021 by 5.
23. Divide 7008134 by 7.
24. Divide 8210042 by 11.
25. Divide 9603048 by 8.
26. Divide 23468420 by 10.
27. Divide 32064258 by 9.
28. Divide 46785142 by 8.
29. Divide 59130628 by 7.
30. Divide 653000638 by 11.
31. Divide 774230029 by 12

32. In 7 days there is 1 week: how many weeks in 26563 days?

33. If $38472 are divided equally among 6 persons, how much will each receive?

34. How many tons of coal, at $7 a ton, can be purchased for $63456?

35. At $9 a barrel, how much flour can be bought for $47239?

36. In 12 months there is 1 year: how many years in 41260 months?

37. A merchant laid out $45285 in cloths, at $7 a yard: how many yards did he buy?

38. At 8 shillings to a dollar, how many dollars are there in 75240 shillings?

39. In 9 square feet there is 1 square yard: how many square yards are there in 52308 square feet?

40. If a person travel 10 miles an hour, how long will it take him to travel 25000 miles?

41. A market woman having 845280 eggs, wished to pack them in baskets holding 1 dozen each: how many baskets did it take?

42. If a prize of $116248 is divided equally among 8 men, what will be each one's share?

43. If in 7 townships there are 2346281 acres, how many acres are there to a township?

44. At $8 a barrel, how many barrels of sugar can be bought for $111364?

45. At $11 each, how many cows can be had for $88990?

LONG DIVISION.

72. ***Long Division*** is the method of dividing, when the *results* of the several steps and the *quotient* are both set down.

73. To divide by Long Division.

1. A speculator paid $31097 for 15 city lots: what did the lots cost apiece?

ANALYSIS.—Since 15 lots cost $31097, 1 lot will cost as many dollars as 15 is contained times in 31097. Let the divisor and dividend be set down as in the margin.

OPERATION.

```
15)$31097($2073 2/15
    30'''
    ——
     109
     105
     ———
      47
      45
      ——
       2
```

Beginning at the left, the *first* step is to find how many times the divisor 15, is contained in 31 (which is 2 times), and set the 2 on the right of the dividend.

Second, multiply the divisor by the quotient figure, and set the product 30, under the figures divided. *Third*, subtract this product from the figures divided. *Fourth*, bring down the next figure of the dividend, and place it on the right of the remainder, making 10 for the next partial dividend, and proceed as before. But 15 is not contained in 10; we therefore place a cipher in the quotient, and bring down the next figure of the dividend, making 109. Now 15 is in 109, 7 times. Set the quotient figure 7 on the right, multiply the divisor by it, subtract the product from the partial dividend, and to the right of the remainder, bring down the succeeding figure for the next partial dividend, precisely as before. Now 15 is in 47, 3 times. Setting the 3 in the quotient, multiplying, and subtracting, as above, the final remainder is 2. We place this remainder

---

72. What is Long Division? 73. How write the numbers? What is the first step? The second? The third? The fourth? If the divisor is not contained in a partial dividend, how proceed? What is to be done with the last remainder? *Note.* What is the difference between short and long division? Of what order is the quotient figure?

over the divisor and annex it to the quotient. The divisor and dividend being *different* denominations, the quotient is the same as the dividend (Art. 64). Therefore the lots cost $\$2073\frac{2}{15}$ apiece. Hence, the

Rule.—I. *Find how many times the divisor is contained in the fewest figures on the left of the dividend, that will contain it, and set the quotient on the right.*

II. *Multiply the divisor by this quotient figure, and subtract the product from the figures divided.*

III. *To the right of the remainder, bring down the next figure of the dividend, and divide as before.*

IV. *If the divisor is not contained in a partial dividend, place a cipher in the quotient, bring down another figure, and continue the operation till all the figures are divided.*

If there is a remainder after dividing the last figure, set it over the divisor, and annex it to the quotient.

Notes.—1. The *parts* into which the dividend is separated in finding the quotient figure, are called *partial dividends,* because they are *parts* of the *whole* dividend.

2. *Short* and *Long* Division, it will be seen, are the same in principle. The only difference is, that in one the results of the several steps are carried in the *mind,* in the other they are *set down.*

Short Division is the more *expeditious,* and should be employed when the divisor does not exceed 12.

The *reasons* for the arrangement of the parts, and for beginning to divide at the left hand, are the same as in Short Division.

3. The quotient figure in *Long* as well as in *Short* Division, is always of the *same order* as that of the *right hand figure* of the partial dividend.

4. To prevent mistakes, it is customary to place a *mark* under the several figures of the dividend as they are brought down.

5. After the first quotient figure is obtained, for *each succeeding figure of the dividend,* either a *significant figure* or a *cipher* must be put in the *quotient.*

6. If the *product* of the divisor into the figure placed in the quotient is *greater* than the partial dividend, it is plain the quotient figure is *too large,* and therefore must be *diminished.*

If the *remainder* is *equal* to or *greater* than the *divisor,* the quotient figure is *too small,* and must *be increased.*

PROOF.

74. By Multiplication.—*Multiply the divisor and quotient together, and to the product add the remainder. If the result is equal to the dividend, the work is right.*

NOTE.—This proof depends upon the principle, that *Division* is *the reverse of Multiplication*; the *dividend* answering to the *product*, the *divisor* to *one* of the factors, and the *quotient* to the *other*. (Art. 62.)

75. By excess of 9s.—*Multiply the excess of 9s in the divisor by that in the quotient, and to the product add the remainder. If the excess of 9s in this sum is equal to that in the dividend, the work is right.*

2. Divide 181403 by 67, and prove the operation.

Ans. $2707\frac{34}{67}$.

Proof.—By Multiplication.—$2707 \times 67 = 181369$, and $181369 + 34$ the rem. $= 181403$ the dividend.

By excess of 9s.—The excess of 9s in the divisor is 4, and the excess in the quotient is 7. Now $4 \times 7 = 28$, and $28 + 34 = 62$; the excess of 9s in 62 is 8. The excess of 9s in the dividend is also 8.

3. Divide 34685 by 15.
4. Divide 65456 by 16.
5. Divide 41534 by 20.
6. Divide 52663 by 25.
7. Divide 420345 by 39.
8. Divide 506394 by 47.
9. Divide 673406 by 69.
10. Divide 789408 by 77.
11. Divide 4375023 by 86.
12. Divide 5700429 by 93.
13. Divide 6004531 by 59.
14. Divide 8430905 by 78.
15. Divide 7895432 by 89.
16. Divide 9307108 by 98.
17. How many acres of land at $75 per acre, can I buy for $18246?
18. At $83 apiece, how many ambulances can be bought for $37682?

---

73. *Note.* If the product of the divisor into the figure placed in the quotient, is greater than the partial dividend, what does it show? If the remainder is equal to or greater than the divisor, what? How is Division proved?

76. To find the *Quotient Figure*, when the Divisor is *large*.

Take the *first figure* of the divisor for a *trial divisor*, and find how many times it is contained in the *first* or *first two* figures of the dividend, making due allowance for carrying the tens of the product of the *second* figure of the divisor into the quotient figure.

19. Divide 18046 by 673.

ANALYSIS.—Taking 6 for a trial divisor, it is contained in 18, 3 times. But in multiplying 7 by 3, there are 2 to carry, and 2 added to 3 times 6, make 20. But 20 is larger than the partial dividend 18; therefore, 3 is too large for the quotient figure. Hence, we place 2 in the quotient, and proceed as before. (Art. 73, *n.*)

```
673)18046(26
    1346
    ----
     4586
     4038
     ----
      548
```

20. Divide 3784123 by 127. 21. Divide 4361729 by 219.

22. Divide 8953046 by 378. 23. Divide 9073219 by 738.

24. How many shawls at $95, can be bought for $42750?

25. In 144 square inches there is 1 square foot: how many square feet are there in 59264 square inches?

26. A quartermaster paid $29328 for 312 cavalry horses: how much was that apiece?

27. If 128 cubic feet of wood make 1 cord, how many cords are in 69240 cubic feet?

28. If a purse of $150648 is divided equally among 250 sailors, how much will each receive?

29. In 1728 cubic inches there is 1 cubic foot: how many cubic feet are there in 250342 cubic inches?

30. If 560245 pounds of bread are divided equally among 11200 soldiers, how much will each receive?

31. Div. 36942536 by 4204. 32. Div. 57300652 by 5129.

33. Div. 629348206 by 52312. 34. Div. 730500429 by 61073.

35. Divide 7300400029 by 236421.

36. Divide 8230124037 by 463205.

37. Divide 843000329058 by 203963428.

38. A stock company having \$5000000, was divided into 1250 shares: what was the value of each share?

39. A railroad 478 miles in length cost \$18120000: what was the cost per mile?

40. A company of 942 men purchased a tract of land containing 272090 acres, which they shared equally: what was each man's share?

41. A tax of \$42368200 was assessed equally upon 5263 towns: what sum did each town pay?

42. The government distributed \$9900000 bounty equally among 36000 volunteers: how much did each receive?

43. Since there is 1 year in 525600 minutes, how many years are there in 105192000 minutes.

CONTRACTIONS.

CASE I.

77. To divide by a *Composite Number*.

Ex. 1. A farmer having 300 pounds of butter, packed it in boxes of 15 pounds each: it is required to find how many boxes he had, using the factors of the divisor.

OPERATION.

5)300 lbs.

3)60 5 lb. boxes.

Ans. 20 boxes.

ANALYSIS.—15=5 times 3. Now if he puts 5 pounds into a box, it is plain he would use as many boxes as there are 5s in 300 or 60 *five-pound* boxes. But 3 *five-pound* boxes make 1 *fifteen-pound* box; hence, 60 *five-pound* boxes will make as many 15 *pound boxes*, as 3 is contained times in 60, which is 20. In the operation, we first divide by one of the factors of 15, and the quotient by the other. Hence, the

RULE.—*Divide the dividend by one of the factors of the divisor, and the quotient thence arising by another factor, and so on, until all the factors have been used. The last quotient will be the one required.*

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77. When the divisor is a composite number, how proceed?

NOTES.—1. This contraction is the *reverse* of *multiplying* by a composite number. Hence, dividing the dividend (which answers to the product) by the several factors of one of the numbers which produced it, will evidently give the quotient, the other factor of which the dividend is composed. (Art. 56.)

2. When the divisor can be resolved into different sets of factors, the result will be the same, whichever set is taken, and whatever the order in which they are employed. The pupil is therefore at liberty to select the set and the order most convenient.

2. Divide 357 by 21, using the factors.

3. If 532 oranges are divided equally among 28 boys, what part, and how many will each receive?

4. A dairyman packed 805 pounds of butter in 35 jars: how many pounds did he put in a jar?

5. How many companies in a regiment containing 756 soldiers, allowing 63 soldiers to a company?

6. Divide 204 by 12, using different sets of factors.

7. Divide 368 by 16, using different sets of factors.

8. Divide 780 by 30, using different sets of factors.

78. To find the *True Remainder*, when Factors of the divisor are used.

Ex. 9.—A lad picked 2425 pints of chestnuts, which he wished to put into bags containing 64 pints each: it is required to find the number of bags he could fill, and the number of pints over, or the true remainder.

OPERATION.

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2)2425
8)1212 — 1                      1 pt. 1st r.
4)151 — 4; 4 × 2 =              8 pt. 2d r.
   37 — 3; 3 × 8 × 2 = 48 pt. 3d r.
                       ——
         Ans. 37 bags, and 57 pt. over.
```

ANALYSIS. — The divisor 64 equals the factors $2 \times 8 \times 4$. Dividing 2425 pints by 2, the quotient is 1212 and 1 remainder. But the *units* of the quotient 1212 are 2 times as large as pints, which, for the sake of distinction, we will call *quarts*; and the remainder 1, is a *pint*, the same as the dividend. (Art. 60, *n.*)

Next, dividing 1212 quarts by 8, the quotient is 151, and 4 remainder. But the *units* of the

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78. How is the true remainder found? *Note.* To what is it equal? What should be done with it? The object in multiplying each partial remainder by all the preceding divisors except its own?

quotient 151 are 8 times as large as quarts; call them *pecks;* the remainder 4, which denotes quarts, must be multiplied by the *preceding* divisor 2, to reduce it back to pints. Finally, dividing 151 pecks by 4, the quotient is 37, and 3 remainder. But the *units* of the quotient 37 have four times the value of pecks; call them *bags;* and the remainder 3, which denotes pecks, must be multiplied by the last divisor 8, to reduce them back to quarts, and be multiplied by 2 to reduce the quarts to pints. Having filled 37 bags, we have three partial remainders, 1 pint, 4 quarts, and 3 pecks.

The next step is to find the *true remainder.* We have seen that a unit of the 2d remainder is twice as large as those of the given dividend, which are pints; hence, 4 quarts = 8 pints. Again, each unit of the 3d remainder is 8 times as large as those of the preceding dividend, which are quarts, therefore, 3 pecks = 3 × 8 or 24 quarts; and 24 qts. = 24 × 2 or 48 pts. The sum of these partial remainders, 1 pt. + 8 pt. + 48 pt. = 57 pts., is the true remainder. Hence, the

RULE.—*Multiply each partial remainder by all the divisors preceding its own; the sum of these results added to the first, will be the true remainder.*

NOTES.—1. Multiplying each remainder by all the preceding divisors except its own, reduces them to *units* of the same denomination as the given dividend. Hence, the *true* remainder is equal to the *sum* of the *partial remainders* reduced to the same denomination as the dividend.

2. When found, it should be placed over the given divisor, and be annexed to the quotient.

10. Divide 43271 by 45.
11. Divide 502378 by 63.
12. Divide 710302 by 72.
13. Divide 3005263 by 84.
14. Divide 63400511 by 96.
15. Divide 216300265 by 144.

CASE II.

79. To divide by 10, 100, 1000, etc.

Ex. 16. Divide 2615 by 100.

OPERATION.

1|00)26|15

Ans. 26|15 Rem.

ANALYSIS.—*Annexing* a cipher to a figure, we have seen, multiplies it by 10; conversely, *removing* a *cipher* from the right of a number must *diminish* its *value* 10 times, or *divide* it by 10; for, each figure in the number is removed *one place* to the right. (Art. 12.) In like manner, cutting off *two* figures from the right, divides it by 100; cutting off *three*, divides it by 1000, etc.

In the operation, as the divisor is 100, we simply cut off two figures on the right of the dividend; the number left, viz., 26, is the *quotient;* and the 15 cut off, the *remainder.* Hence, the

RULE.—*From the right of the dividend, cut off as many figures as there are ciphers in the divisor. The figures left will be the quotient; those cut off, the remainder.*

17. Divide 75236 by 100.
18. Divide 245065 by 1000.
19. Divide 805211 by 10000.
20. 9820341 by 100000.
21. 9526401 by 1000000.
22. 80043264 by 10000000.

CASE III.

80. To divide, when the Divisor has *Ciphers* on the *right.*

23. At $30 a barrel, how many barrels of beef can be bought for $4273?

OPERATION.

$3|0)427|3$

Quot. 142, 1 Rem.

Ans. $142\frac{13}{30}$ bls.

ANALYSIS.—The divisor 30 is composed of the factors 3 and 10. Hence, in the operation, we first divide by 10, by cutting off the right-hand figure of the dividend; then dividing the remaining figures of the dividend by 3, the quotient is 142, and 1 remainder. Prefixing the 1 remainder to the 3 which was cut off, we have 13 for the true remainder, which being placed over the given divisor 30, and annexed to the quotient, gives $142\frac{13}{30}$ bls. for the answer. Hence, the

RULE.—I. *Cut off the ciphers on the right of the divisor and as many figures on the right of the dividend.*

II. *Divide the remaining part of the dividend by the remaining part of the divisor for the quotient.*

III. *Annex the figures cut off to the remainder, and the result will be the true remainder.* (Art. 78.)

NOTE.—This contraction is based upon the last two Cases. The *true* remainder should be placed over the *whole* divisor, and be annexed to the quotient.

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79. How proceed when the divisor is 10, 100, etc.? 80. How when there are ciphers on the right of the divisor? *Note.* Upon what is this contraction based?

24. Divide 45678 by 20.
25. 81386 by 200.
26. Divide 603245 by 3400.
27. 740321 by 6500.
28. Divide 7341264 by 87000.
29. 8004367 by 93000.
30. Divide 61273203 by 125000.
31. 416043271 by 670000.

32. In 100 cents there is 1 dollar: how many dollars in 37300 cents?

33. At $200 apiece, how many horses will $45800 buy?

34. If $75360 were equally distributed among 1000 men, how much would each receive?

35. At $4800 a lot, how many can be had for $25200?

36. How many bales, weighing 450 pounds each, can be made of 27000 pounds of cotton?

QUESTIONS FOR REVIEW, INVOLVING THE PRECEDING RULES.

1. William, who has 219 marbles, has 73 more than James: how many has James? How many have both?

2. A farmer having 368 sheep, wishes to increase his flock to 775: how many must he buy?

3. The difference of two persons' ages is 19 years, and the younger is 57 years: what is the age of the elder?

4. What number must be added to 1368 to make 3147?

5. What number subtracted from 4118 leaves 1025?

6. What number multiplied by 95 will produce 7905?

7. The product of the length into the breadth of a field is 2967 rods, and the length is 69 rods: what is the breadth?

8. A man having 5263 bushels of grain, sold all but 145 bushels: how much did he sell?

9. What number must be divided by 87, that the quotient may be 99?

10. If the quotient is 217, and the dividend 7595, what must be the divisor?

11. If the divisor is 341 and the quotient 589, what must be the dividend?

12. A merchant bought 516 barrels of flour at $9 a barrel, and sold it for $5275: how much did he gain or lose?

13. How long can 250 men subsist on a quantity of food sufficient to last 1 man 7550 days?

14. How many pounds of sugar, at 11 cents, must be given for 629 pounds of coffee, at 17 cents?

15. At $13 a barrel, how many barrels of flour must be given for 530 barrels of potatoes worth $3 a barrel?

16. A man having $15260, deducted $4500 for personal use, and divided the balance equally among his 7 sons: how much did each son receive?

17. A man earns 12 shillings a day, and his son 8 shillings: how long will it take both to earn 1200 shillings?

18. If a man earns $19 a week, and pays $2 a week for boarding each of his 3 sons at school, how much will he lay up in 12 weeks?

19. A farmer sold 6 cows at $23, 150 bushels of wheat at $2, and 75 barrels of apples at $4, and laid out his money in cloth at $7 a yard: how many yards did he have?

20. If I buy 1361 barrels of flour at $7, and sell the whole for $12249, how much shall I make per barrel?

21. The earnings of a man and his two sons amount to $3560 a year; their expenses are $754. If the balance is divided equally, what will each have?

22. A man having $23268, owed $1733, and divided the rest among four charities: how much did each receive?

23. How many sheep at $5 a head, must be given for 30 cows at $42 apiece?

24. A father bought a suit of clothes for each of his 3 sons, at $123 a suit, and agreed to pay 17 tons of hay at $12 a ton, and the rest in potatoes at $4 a barrel: how many barrels of potatoes did it take?

25. If you add $435 to $567, divide the sum by $334, multiply the quotient by 217, and divide the product by 59, what will be the result?

26. If from 1530 you take 319, add 793 to the remainder, multiply the sum by 44, and divide the product by 37, what will be the result?

27. A man's annual income is $4250; if he spends $1365 for house rent, $1439 for other expenses, and the balance in books, at $3 apiece, how many books can he buy?

28. A farmer having $3038, bought 15 tons of hay at $11, 3 yoke of oxen at $155, 375 sheep at $5, and spent the rest for cows at $41 a head: how many cows did he buy?

GENERAL PRINCIPLES OF DIVISION.

81. From the nature of Division, the *absolute value* of the *quotient* depends both upon the *divisor* and the *dividend.*

The *relative value* of the quotient; that is, its value compared with the dividend, depends upon the *divisor.* Thus,

1. If the divisor is *equal* to the dividend, the quotient is 1.
2. If the divisor is *greater* than the dividend, the quotient is *less* than 1.
3. If the divisor is *less* than the dividend, the quotient is *greater* than 1.
4. If the divisor is 1, the quotient is *equal* to the dividend.
5. If the divisor is *greater* than 1, the quotient is *less* than the dividend.
6. If the divisor is *less* than 1, the quotient is *greater* than the dividend.

82. The relation of the *divisor, dividend,* and *quotient* is such that the divisor remaining the same,

Multiplying the *dividend* by any number, *multiplies* the *quotient* by that number. Thus, $12 \div 2 = 6$; and $(12 \times 2) \div 2 = 6 \times 2$. Conversely,

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81. Upon what does the absolute value of the quotient depend? Its relative value? If the divisor is equal to the dividend, what is true of the quotient? If greater? If less? If the divisor is 1, what is the quotient? If greater than 1? If less than 1? 82. What is the effect of multiplying the dividend? 83. Of dividing it?

83. *Dividing* the dividend by any number, *divides* the *quotient* by that number. Thus, $(12 \div 2) \div 2 = 6 \div 2$.

84. *Multiplying* the *divisor* by any number, *divides* the *quotient* by that number. Thus, $24 \div 6 = 4$; and $24 \div (6 \times 2) = 4 \div 2$. Conversely,

85. *Dividing* the *divisor* by any number, *multiplies* the *quotient* by that number. Thus, $24 \div (6 \div 2) = 4 \times 2$.

86. *Multiplying* or *dividing* both the *divisor* and *dividend* by the same number, does not *alter* the quotient. Thus, $48 \div 8 = 6$; so $(48 \times 2) \div (8 \times 2) = 6$; and $(48 \div 2) \div (8 \div 2) = 6$.

NOTE.—Multiplying or dividing the dividend, produces a *like* effect on the quotient; but multiplying or dividing the divisor, produces the *opposite* effect on the quotient.

86, *a*. If a number is both *multiplied* and *divided* by the *same number*, its *value* is not altered. Thus, $(7 \times 6) \div 6$ is equal to 7.

PROBLEMS AND FORMULAS PERTAINING TO THE FUNDAMENTAL RULES.

87. A ***Problem*** is something *to be done*, or a *question* to be *solved.*

88. A ***Formula*** is a *specific rule* by which problems are *solved,* and may be expressed by *common language,* or by *signs.*

89. The *four great* Problems of Arithmetic have already been illustrated. They are—

1st. When *two or more* numbers are given, to find their *sum,* or *amount.* (Art. 29.)

2d. When *two numbers* are given, to find their *difference.*

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84. What is the effect of multiplying the divisor? Of dividing it? 86. What is the effect of multiplying or dividing both the divisor and dividend? 87. What is a problem? 88. A formula? 89. The four great problems of arithmetic?

3d. When *two factors* are given, to find their *product.* (Art. 50.)

4th. When *two numbers* are given, to find *how many times* one is contained in the other. (Art. 73.)

90. These problems constitute the *four fundamental rules* of *Arithmetic,* called *Addition, Subtraction, Multiplication,* and *Division.*

NOTES.—1. These rules are called *fundamental,* because upon them are based all arithmetical operations.

2. As *multiplication* is an abbrevated form of *addition,* and *division* of *subtraction,* it follows that every *change* made upon the *value* of a number, must *increase* or *diminish* it. Hence, strictly speaking, there are but *two* fundamental operations, viz.: *aggregation* and *diminution,* or *increase* and *decrease.*

3. The following problems, though subordinate, are so closely connected with the preceding, that a passing notice of them may not be improper, in this connection.

91. To find the *greater* of two numbers, the *less* and their *difference* being given.

1. A planter raised two successive crops of cotton, the smaller of which amounted to 4168 bales, and the difference between them was 1123 bales: what was the greater crop?

ANALYSIS.—If the *difference* between two numbers be added to the *less,* it is obvious the *sum* must be equal to the *greater.* Therefore, 4168 bales + 1123 bales or 5291 bales must be the greater crop. Hence, the

RULE.—*To the less add the difference, and the sum will be the greater.* (Art. 39.)

2. One of the two candidates at a certain election, received 746 votes, and was defeated by a majority of 411: how many votes did the successful candidate receive?

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90. What do these constitute? *Note.* Why so called? What is given and what required in addition? In subtraction? In multiplication? In division? Where begin the operation in addition, subtraction, and multiplication? Where in division? What is the difference between addition and subtraction? Between addition and multiplication? Between subtraction and division? Between multiplication and division?

92. To find the *less* of two numbers, the *greater* and their *difference* being given.

3. The greater of two cargoes of flour is 5267 barrels, and their difference is 1348 barrels: how many barrels does the smaller contain?

ANALYSIS.—The *difference* added to the *less* number equals the *greater;* therefore, the *greater* diminished by the *difference,* must be equal to the *less;* and 5267 barrels minus 1348 barrels leaves 3919 barrels, the smaller cargo. Hence, the

RULE.—*From the greater subtract the difference, and the result will be the less.* (Art. 38.)

4. At a certain election one of the two candidates received 1366 votes, and was elected by a majority of 219 votes: how many votes did the other candidate receive?

93. The Product and one Factor being given, to find the other Factor.

5. A drover being asked how many animals he had, replied that he had 67 oxen, and if his oxen were multiplied by his number of sheep, the product would be 37520: how many sheep had he?

ANALYSIS.—Since 37520 is a *product,* and 67 *one* of its factors, the *other* factor must be as many as there are 67s in 37520; and $37520 \div 67 = 560$. (Art. 93.) Therefore, he had 560 sheep. Hence, the

RULE.—*Divide the product by the given factor, and the quotient will be the factor required.* (Art. 62.)

6. The length of a certain park is 320 rods, and the product of its length and breadth is 51200 rods: what is its breadth?

94. The Product of three or more Factors and all the Factors but *one* being given, to find that Factor.

7. The product of the length, breadth, and height of a certain mound is 62730 feet; its length is 45 feet, and its breadth 41 feet: what is its height?

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91. How find the greater of two numbers, the less and difference being given?
92. How find the less, the greater and difference being given?

ANALYSIS.—The contents of solid bodies are found by multiplying their *length, breadth,* and *thickness* together. Now as the length is 45 feet, and the breadth 41 feet, the product of which is 1845, the height must be 62730 feet divided by 1845, or 34 feet. Hence, the

RULE.—*Divide the given product by the product of the given factors, and the quotient will be the factor required.* (Art. 93.)

8. The continued product of the distances wh' ;h 4 men traveled is 1944630 miles; one traveled 45, .nother 41, and another 34 miles: how far did the fourth travel?

95. To find the *Dividend*, the Divisor and Quotient being given.

9. If the quotient is 7071, and the divisor 556, what is the dividend?

ANALYSIS.—Since the *quotient* shows how many times the *divisor* is contained in the *dividend,* it follows, that the product of the *divisor* and *quotient* must be equal to the dividend. Now 7071 × 556 =3931476 the dividend. Hence, the

RULE.—*Multiply the divisor by the quotient, and the result will be the dividend.* (Art. 74.)

10. What number of dollars divided among 135 persons will give them $168 apiece?

96. To find the *Divisor*, the *Dividend* and *Quotient* being given.

11. What must 8640 be divided by that the quotient may be 144?

ANALYSIS.—Since the quotient shows how many times the divisor is contained in the dividend, it follows that if the dividend is divided by the quotient the result must be the divisor, and 8640÷144=60. Therefore, the divisor is 60. Hence, the

RULE.—*Divide the dividend by th. quotient, and the result will be the divisor.* (Arts. 62, 93.)

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93. The product and one factor being given, how find the other factor?

12. If the dividend is 7620, and the quotient 127, what must be the divisor?

97. The *Sum* and *Difference* of two numbers being given, to find the *Numbers*.

13. The sum of two numbers is 65, and their difference 15: what are the numbers?

ANALYSIS.—The sum 65 is equal to the greater number increased by the less; and the greater diminished by the difference 15, is equal to the less (Art. 38). Hence, if 15 is taken from 65, the remainder 50, must be twice the less. But $50 \div 2 = 25$ the less number; and $25 + 15 = 40$ the greater number.

Proof.—$40 + 25 = 65$ the given sum. Hence, the

RULE.—*From the sum take the difference, and half the remainder will be the less number.*

To the less add the difference, and the result will be the greater. (Art. 39.)

14. A merchant made $5368 in two years, and the difference in his annual gain was $976: what was his profit each year?

15. The whole number of votes cast for the two candidates at a certain election was 5564, and the successful candidate was elected by a majority 708: how many votes did each receive?

16. A lady paid $250 for her watch and chain; the former being valued $42 higher than the latter: what was the price of each?

17. Two pupils A and B, solved 75 examples; B solving 15 less than A: how many did each solve?

18. A and B found a pocket-book, and returning it to the owner, received a reward of $500, of which A took $38 more than B: what was the share of each?

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94. When the product of three or more factors, and all but one are given, how find that one? 95. How find the dividend, the divisor and quotient being given? 96. How find the divisor, the dividend and quotient being given?

ANALYSIS.

98. ***Analysis*** primarily denotes the *separation* of an object into its elements.

99. ***Analysis,*** in Arithmetic, is the *process of tracing the relation* of the conditions of a problem to each other, and thence *deducing the successive steps* necessary for its solution.

NOTES.—1. The application of Analysis to arithmetic is of recent date, and to this source the late improvements in the mode of teaching the subject are chiefly due. Previous to this, Arithmetic to most pupils, was a *hidden mystery,* regarded as beyond the reach of all but the favored few.

2. The pupil has already learned to *analyze* particular examples, and from them to *deduce specific rules* by which similar examples may be solved; but *Arithmetical Analysis* has a much wider range. It is applied with advantage to those classes of examples commonly placed under the heads of *Barter, Percentage, Profit and Loss, Simple* and *Compound Proportion, Partnership,* etc. In a word, it is the *grand Common-Sense Rule* by which business men perform the great majority of commercial calculations.

100. No *specific directions* can be prescribed for *analytical solutions.* The following suggestions may, however, be serviceable to beginners:

1st. In general, we reason from the *given value of one,* to the *value* of *two* or *more* of the same kind. Or,

2d. From the *given value of two* or *more,* to that of *one.*

In the first instance we reason from a *part* to the *whole;* in the second, from the *whole* to a *part.*

3d. Sometimes the *result* of certain combinations is given, *to find the original number* or *base.*

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98. What is the primary meaning of analysis? 99. What is arithmetical analysis? *Note.* What is said as to its utility in arithmetical and business calculations? 100. Can specific rules be given for analytical solutions? What general directions can you mention?

In such cases, it is generally best to begin with the *result*, and *reverse* each *operation* in succession, till the original number is reached. That is, to reason from the *result* to its *origin*, or from *effect* to *cause*.

1. A farmer bought 150 sheep at $2 a head, and paid for them in cows at $20 a head: how many cows did it take to pay for the sheep?

2. How much tea, at 85 cents a pound, must be given for 425 pounds of rice, at 10 cents a pound?

3. How much sugar, at 12 cents a pound, must be given for 288 pounds of raisins, at 18 cents a pound?

4. How much corn, at 80 cents a bushel, must be given for 160 pounds of tobacco, at 30 cents a pound?

5. How much butter, at 40 cents a pound, must be given for 62 yards of calico, at 20 cents a yard?

6. Bought 189 yards of linen, at 84 cents a yard, and paid for it in oats, at 42 cents a bushel: how many bushels did it take?

7. Paid 18 barrels of flour for 30 yards of cloth, worth $6 a yard: what was the flour a barrel?

8. A farmer gave 15 loads of hay for 45 tons of coal, worth $6 a ton: what did he receive a load for his hay?

9. A sold B 35 hundred pounds of hops, at $27 a hundred, and took 45 sacks of coffee, at $14 a sack, and the balance in money: how much money did he receive?

10. James bought 96 apples, at the rate of 4 for 3 cents, and exchanged them for pears at 4 cents apiece: how many pears did he receive?

11. A farmer being asked how many acres he had, replied, if you subtract 20 from the number, divide the remainder by 8, add 15 to the quotient, and multiply by 5, the product will be 125: how many had he?

ANALYSIS.—Taking 125 as the base, and reversing the several operations, beginning with the last, we have $125 \div 5 = 25$, the number before the multiplication. Again, subtracting 15 from 25, we have $25 - 15 = 10$, the number before the addition.

Next, multiplying 10 by 8, we have $10 \times 8 = 80$, the number before the division. Finally, adding 20 to 80, we have $80 + 20 = 100$, the number required.

12. What number is that, to which, if 25 be added, and the sum multiplied by 9, the product will be 504?

13. What number, if diminished by 40, and the remainder divided by 8, the quotient will be 58?

14. A man being asked how many children he had, answered, if you multiply the number by 11, add 23 to the product, and divide the sum by 9, the quotient will be 16: how many children had he?

15. The greater of two numbers is 3 times the less, and the sum of the numbers is 36: what are the numbers?

ANALYSIS.—The smaller number is 1 part, and the larger 3 parts; hence, the sum of the two is 4 parts, which by the conditions is 36. Now, if 4 parts of a number are 36, 1 part is equal to as many units as there are 4s in 36, or 9. Therefore 9 is the smaller number, and 3 times 9 or 27, the greater.

16. The sum of two numbers is 72, and the greater is 5 times the less: what are the numbers?

17. Divide 472 into three such parts, that the second shall be twice the first, and the third 3 times the second plus 13.

ANALYSIS.—Calling the first 1 part, the second will be 2 parts, and the third 6 parts plus 13; hence the sum of the three, in the terms of the first, is 9 parts plus 13, which by the conditions is 472. Taking 13 from 472 leaves 459, and we have 9 parts equal to 459. Now if 9 parts equal 459, 1 part is equal to as many units as 9 is contained times in 459, or 51. Therefore the first is 51, the second 2 times 51 or 102, the third 3 times 102 plus 13 or 319.

18. A and B counting their money, found that both had $473, and that A had 3 times as much as B plus $25: how much had each?

19. The sum of two numbers is 243, the second is three times the first minus 25: what are the numbers?

20. What number is that to which if 315 be added, the sum will be 250 less than 2683?

(For further applications of Analysis, see subsequent pages.)

CLASSIFICATION

AND PROPERTIES OF NUMBERS.

101. Numbers are divided into *abstract* and *concrete*, *simple* and *compound*, *prime* and *composite*, *odd* and *even*, *integral*, *fractional*, and *mixed*, *known* and *unknown*, *similar* and *dissimilar*, *commensurable* and *incommensurable*, *rational* and *irrational* or *surds*.

DEF.—1. *Abstract Numbers* are those which are not applied to things; as, one, two, three.

2. *Concrete Numbers* are those which are applied to things; as, two caps, three pencils, six yards.

3. *Simple Numbers* are those which contain only *one denomination*, and may be either *abstract* or *concrete;* as, 13, 11 pounds.

4. A *Compound Number* is one containing *two* or *more denominations*, which have the same base or nature; as, 3 shillings and 6 pence; 4 yards 2 feet and 6 inches.

5. A *Prime Number* is one which *cannot* be produced by multiplication of any two or more numbers, except a *unit* and *itself*.

All prime numbers except 2 and 5, end in 1, 3, 7, or 9.

6. A *Composite Number* is the *product* of two or more *factors*, each of which is greater than 1; as, 15 (5 × 3), 24 (2 × 3 × 4), etc. (Art. 5[illegible].)

A *prime* number differs from a *composite* number in two respects: *First*, In its *origin; Second*, In its *divisibility;* the former being divisible only by a *unit* and *itself;* the *latter* by each of the factors which produce it.

7. Two numbers are *prime to each other* or *relatively prime*, when the only number by which both can be divided without a *remainder*, is a unit or 1; as, 5 and 6.

8. An *Even Number* is one which can be divided by 2 without a *remainder;* as, 4, 6, 10.

9. An *Odd Number* is one which *cannot* be divided by 2 without a remainder; as, 3, 5, 7, 9, 11.

Name the kind of each of the following numbers, and why: 3, 10, 17, 21, 28, 31, 56, 63, 72, 81, 44, 39, 91, 67, 51, 84, 99, 100.

10. An *Integer* is a *number* which contains one or more *entire units only;* as, 1, 3, 7, 10, 50, 100.

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101. How are numbers divided? An abstract number? Concrete? Simple? Compound? Prime? Composite? When are two numbers prime to each other? Even? Odd? An integer? A fraction? A mixed number?

11. A *Fraction* is *one* or *more* of the *equal* parts into which a *unit* is divided; as, 1-half, 2-thirds, 3-fourths, etc.

12. A *Mixed Number* is an *integer* and a *fraction* expressed together; as, $5\frac{1}{2}$, $11\frac{3}{8}$, etc.

13. *Like* or *similar numbers* are those which express units of the same kind or denomination; as, 3 shillings and 5 shillings, four and seven, etc.

14. *Unlike Numbers* are those which express units of *different* kinds or denominations; as, 2 apples and 3 oranges.

15. *Commensurable Numbers* are those which can be divided by the *same* number, without a remainder; as, 9 and 12, each of which can be divided by 3.

16. *Incommensurable Numbers* are those which *cannot* be divided by the same number without a remainder. Thus, 3 and 7 are incommensurable.

17. A *given number* is one whose *value* is *expressed.*

A number is also said to be given when it can be *easily inferred* from something else which is given. Thus, if two numbers are given, their *sum* and *difference* are given.

18. An *Unknown Number* is one whose value is not given.

102. ***A Factor*** of a number is one of the numbers, which multiplied together, produce that number.

103. An ***Exact Divisor*** of a number is one which will divide it without a *remainder*. Thus 2 is an exact divisor of 6, 3 of 15, etc.

NOTES.—1. An *exact divisor* of a number is always a *factor* of that number; and, conversely, a *factor* of a number is always an *exact divisor* of it. For, the dividend is the *product* of the *divisor* and *quotient,* and therefore is *divisible* by each of the numbers that produce it. (Art. 62.)

2. The terms *divisor* and *factor* are here restricted to *integral* numbers.

104. A ***Measure*** of a *number* is an exact divisor of that number. It is so called because the *comparative magnitude* of the number divided, is determined by this standard.

Like numbers? Unlike? Commensurable? Incommensurable? What is a given number? An unknown? A factor? An exact divisor? A measure?

105. An ***aliquot part*** of a number is a *factor* or *an exact divisor* of that number.

NOTE.—The terms *factors, measures, exact divisors,* and *aliquot parts,* are different names of the same thing, and are often used as synonymous. Thus, 3 and 5 are respectively the factors, measures, exact divisors, and aliquot parts of 15.

106. The ***reciprocal*** of a *number* is 1 divided by that number. Thus, the reciprocal of 4 is $1 \div 4$, or $\frac{1}{4}$.

COMPLEMENT OF NUMBERS.

107. The ***Complement*** of a *Number* is the difference between the number and a *unit* of the *next higher* order. Thus, the complement of 7 is 3; for $10-7=3$; the complement of 85 is 15; for $100-85=15$.

108. To find the *Complement* of a Number.

Subtract the given number from 1 *with as many ciphers annexed, as there are integral figures in the given number.*

Or, begin at the left hand, and subtract each figure of the given number from 9, *except the last significant figure on the right, which must be taken from* 10.

NOTE.—The *second* method is based upon the principle that when we *borrow,* the next upper figure must be considered 1 less than it is. (Art. 37, *n.*)

Find the complement of the following numbers:

1. 328.	6. 6072.	11. 56239.	16. 73245.
2. 567.	7. 8256.	12. 64123.	17. 1234567.
3. 604.	8. 9061.	13. 102345.	18. 2301206.
4. 891.	9. 13926.	14. 261436.	19. 3021238.
5. 4638.	10. 23184.	15. 40061.	20. 7830426.

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105. An aliquot part? *Note.* What is said as to the use of these four terms? 106. The reciprocal of a number? 107. The complement? 108. How find the complement of a number?

DIVISIBILITY OF NUMBERS.

109. One number is said to be *divisible* by another, when there is no *remainder*. The division is then *complete.*

When there *is a remainder,* the division is *incomplete;* and the dividend is said to be *indivisible* by the divisor.

NOTES.—1. In treating of the *divisibility* of numbers, the term *divisor* is commonly used for *exact divisor.*

2. Every *integral* number is divisible by the *unit* 1, and by *itself.* It is not customary, however, to consider the unit 1, or the number itself, as a *factor.* (Art. 102.)

110. In determining the divisibility of numbers the following *properties* or *facts* are useful:

PROP. 1. Any number is divisible by 2, which ends with 0, 2, 4, 6, or 8.

2. Any number is divisible by 3, if the *sum* of its *digits* is divisible by 3. Thus, 147 the sum of whose digits is $1+4+7=12$, is divisible by 3.

3. Any number is divisible by 4, if its *two right hand* figures are divisible by 4; as, 2256, 15368, 19384.

4. Any number is divisible by 5, which *ends* with 0 or 5; as, 130, 675.

5. Any *even* number is divisible by 6, which is divisible by 3. Thus, $1344 \div 3 = 448$; and $1344 \div 6 = 224$.

6. Any number is divisible by 8, if its *three right hand* figures are divisible by 8; as, 1840, 1688, 25320.

7. Any *number* is divisible by 10, which ends with 0; by 100, if it ends with 00; by 1000, if it ends with 000, etc.

8. Any *number* is divisible by 12 which is divisible by 3 and 4.

9. Any number divided by 9 will leave the *same remainder* as the *sum* of *its digits* divided by 9. Hence,

10. Any number is *divisible* by 9 if the *sum* of its *digits* is divisible by 9. Thus, the sum of the digits of 54378 is $5+4+3+7+8$, or 27. Now, as 27 is divisible by 9, we may infer that 54378 is divisible by 9.

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109. When is one number divisible by another? When indivisible? 110. When is a number divisible by 2? By 3? By 4? By 5? By 6? By 8? By 10?

11. As 9 is a multiple of 3, any number divisible by 9, is also divisible by 3.

12. Any *number* divided by 11 will leave the *same remainder* as the *sum* of its digits in the *even* places, taken from the *sum* of those in the *odd* places, counting from the right, the latter being increased or diminished by 11 or a multiple of 11. Thus, the sum of the digits in the even places of 314567, viz., 6+4+3, or 13, is equal to the sum of the digits in the odd places, viz., 7+5+1, or 13; therefore, 314567 is divisible by 11. Hence,

13. Any number is *divisible* by 11 when the sum of its digits in the *even* places is *equal* to the *sum* of those in the *odd* places, or when their *difference* is divisible by 11. (For a demonstration of the properties of 9 and 11, see Higher Arithmetic.)

14. If one number is a *divisor* of another, the former is also a divisor of any *multiple* of the latter. (Art. 103, *n.*) Thus, 2 is a divisor of 6; it is also a divisor of the product of 3 times 6, of 5 × 6, and of any whole number of times 6.

15. If a number is an *exact divisor* of each of two numbers, it is also an exact divisor of their *sum*, their *difference*, and their *product*. Thus, 3 is a divisor of 9 and 15 respectively; it is also a divisor of 9+15, or 24; of 15−9, or 6; of 15 × 9, or 135.

16. A *composite* number is *divisible* by each of its *prime factors*, by the *product* of any two or more of them, and by *no other* number. Thus, the prime factors of 30 are 2 × 3 × 5. Now 30 is divisible by by 2, 3 and 5, by 2 × 3, by 2 × 5, by 3 × 5, and by 2 × 3 × 5, and by no other number. Hence,

17. The *least* divisor of a composite number, is a *prime* number.

18. An *odd* number cannot be divided by an *even* number without a *remainder*.

19. If an *odd* number divides an *even* number, it will also divide *half* of it.

20. If an *even* number is *divisible* by an *odd* number, it will also be divisible by *double* that number.

If a number is divided by 9, to what is the remainder equal? When is a number divisible by 9? If a number is divided by 11, to what is the remainder equal? When is a number divisible by 11? If a number is a divisor of another, what is true of it in regard to any multiple of that number? If a number is a divisor of each of two numbers, what is true of it in regard to their sum, difference, and product? By what is a composite number divisible? *Note.* What is true of the least divisor of every composite number? Is an odd number divisible by an even? If an odd divides an even number, what is true of half of it? If an even number is divisible by an odd, what is true of it in regard to double that number?

FACTORING.

111. ***Factors,*** we have seen, are numbers which multiplied together, produce a *product.* (Art. 42.) Hence,

112. ***Factoring*** a number is finding *two* or *more factors,* which multiplied together, *produce* the number. Thus, the factors of 15 are 3 and 5; for, $3 \times 5 = 15$. (Art. 56, *n.*)

NOTE.—Every number is divisible by *itself* and by 1; hence, if multiplied by 1, the *product* will be the number itself. (Art. 41.) But it cannot properly be said that 1 is a *factor,* nor that a *number* is a *factor* of itself. If so, all numbers are *composite.* (Art. 109, *n.*)

113. To resolve a Composite Number into *two Factors.*

Divide the number by any exact divisor; the divisor and quotient will be factors. (Art. 62.)

NOTE.—Every *composite* number must have *two factors* at least; some have *three* or *more;* and others may be resolved into *several different pairs* of factors. (Art. 101, Def. 6.)

1. What are the two factors of 35? Of 49? Of 121?
2. Name two factors of 45? Of 56? Of 72? Of 108?

114. To find the *Different Pairs* of factors of a Composite Number.

3. What are the different pairs of factors of 36?

ANALYSIS.—Dividing 36 by 2, we have $36 \div 2 = 18$. Again, $36 \div 3 = 12$; $36 \div 4 = 9$; and $36 \div 6 = 6$. That is, the different pairs of factors of 36 are 2×18; 3×12; 4×9; and 6×6. Hence, the

$$36 \div 2 = 18$$
$$36 \div 3 = 12$$
$$36 \div 4 = 9$$
$$36 \div 6 = 6$$

RULE.—*Divide the given number continually by each of its exact divisors, beginning with the least, until the quotient obtained is less than the divisor employed.*

The divisors and corresponding quotients will be the different pairs of factors required.

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111. What are factors? Is 1 a factor? Is a number a factor of itself? Why not? What is meant by Factoring? *Note.* How resolve a number into two factors? 112. How find the different pairs of factors?

NOTE.—The division is stopped as soon as the *quotient* is *less* than the *divisor*; for, the *subsequent factors* will be *similar* to those already found.

Find the different pairs of factors of the following numbers:

4.	20,	8.	38,	12.	96,	16.	256,
5.	27,	9.	45,	13.	110,	17.	475,
6.	30,	10.	56,	14.	144,	18.	600,
7.	32,	11.	75,	15.	225,	19.	1240.

PRIME FACTORS.

115. The ***Prime Factors*** of a number are the *Prime Numbers* which, multiplied together, produce the number.

116. Every ***Composite Number*** can be resolved into *prime factors*. For, the factors of a composite number are divisors of it, and these divisors are either *prime* or *composite*. If *prime*, they accord with the proposition. If *composite*, they can be resolved into other factors, and so on, until all the factors are *prime*. (Art. 114.)

117. To find the *Prime Factors* of a Composite Number.

Ex. 1. What are the prime factors of 210?

OPERATION.

1st divisor, 2	210, given.
2d " 3	105, 1st quot.
3d " 5	35, 2d "
4th " 7	7, 3d "
	1, 4th "

Hence, $210 = 2 \times 3 \times 5 \times 7$.

ANALYSIS.—Dividing 210 by any prime number that will exactly divide it, as 2, we resolve it into the factors 2 and 105. Again, dividing the 1st quotient 105 by any prime number, as 3, we resolve it into the factors 3 and 35. In like manner, dividing the 2d and 3d quotients by the prime numbers 5 and 7, the 4th quotient is 1. The divisors 2, 3, 5 and 7 are the *prime factors* required. For, each of these divisors is a *prime* number, and the division is continued until the *quotient* is a

115. What are prime factors? 116. What is said of composite numbers? How find the prime factors of a number?

unit; therefore, the several divisors must be the *prime factors* required. Hence, the

RULE.—*Divide the given number by any prime number that will divide it without a remainder. Again, divide this quotient by a prime number, and so on, until the quotient obtained is* 1. *The several divisors are the prime factors required.*

NOTE.—As the *least divisor* of every number is *prime,* beginners may be less liable to mistakes by taking for the divisor the *smallest number* that will divide the several dividends *without a remainder*

Find the prime factors of the following numbers:

2.	72.	7.	184.	12.	1000.	17.	2348.
3.	96.	8.	215.	13.	1208.	18.	10376.
4.	121.	9.	320.	14.	1560.	19.	25600.
5.	132.	10.	468.	15.	1776.	20.	64384.
6.	144.	11.	576.	16.	1868.	21.	98816.

118. To find the *Prime Factors* common to two or more Numbers.

22. What are the prime factors common to 42, 168, and 210?

ANALYSIS.—Dividing the given numbers by the prime factor 2, the quotients are 21, 84, and 105. Again, dividing these quotients by the prime factor 3, the quotients are 7, 28, and 35. Finally, dividing by 7, the quotients are 1, 4, and 5, no two of which can be divided by any number greater than 1. Therefore, the divisors 2, 3, and 7 are the *common* prime factors required. Hence, the

OPERATION.

```
2)42, 168, 210
3)21,  84, 105
7) 7,  28,  35
   1,   4,   5
```

RULE.—I. *Write the given numbers in a horizontal line, and divide them by any prime number which will divide each of them without a remainder.*

II. *Divide the quotients thence arising in the same manner, as long as the quotients have a common factor; the divisors will be the prime factors required.*

118. How find the prime factors common to two or more numbers?

Find the prime factors common to the following numbers:

23. 12, 18, 30.
24. 48, 66, 72.
25. 64, 108, 132.
26. 71, 113, 149.
27. 48, 72, 88, 120.
28. 60, 84, 108, 144.
29. 75, 90, 135, 150.
30. 132, 144, 196, 240.
31. 168, 256, 320, 500.
32. 200, 325, 540, 625.
33. 316, 396, 484, 936.
34. 462, 786, 924, 858, 972.

119. To find the Prime Numbers as far as 250.

I. *Write the odd numbers in a series, including* 2.

II. *After* 3, *erase all that are divisible by* 3; *after* 5, *erase all that are divisible by* 5; *after* 7, *all that are divisible by* 7; *after* 11 *and* 13, *all that are divisible by them: those left are primes.*

35. Find what numbers less than 100 are prime.
36. Find what numbers less than 200 are prime.

CANCELLATION.

120. ***Cancellation*** is the method of *abbreviating operations* by *rejecting equal factors* from the divisor and dividend.

The ***Sign of Cancellation*** is an oblique mark drawn across the face of a figure; as, 3̸, 5̸, 7̸, etc.

NOTE.—The term *cancellation* is from the Latin *cancello,* to *erase.*

121. ***Cancelling a Factor*** of a number *divides* the number by that factor. For, multiplying and dividing a number by the *same factor* does not alter its value. (Art. 86, *a.*) Thus, let 7×5 be a dividend. Now cancelling the 7, divides the dividend by 7, and leaves 5 for the quotient; cancelling the 5, divides it by 5, and leaves 7 for the quotient.

120. What is cancellation? 121. Effect of cancelling a factor? Why?

122. To divide one Composite Number by another.

1. What is the quotient of 72 divided by 18?

ANALYSIS.—The dividend $72=9\times4\times2$, and the divisor $18=9\times2$. Cancelling the common factors 9 and 2, we have 4 for the quotient.

OPERATION.

$$\frac{72}{18}=\frac{9\times4\times2}{9\times2}=4$$

2. Divide the product of 45×17, by the product of 9×5.

ANALYSIS.—In this example the product of the two factors composing the divisor, viz.: 9×5, equals one factor of the dividend. We therefore cancel these equals at once, and the factor 17 is the quotient.

9	45
5	17
Ans.	17

Remark.—In this operation we place the numbers composing the divisor on the *left* of a *perpendicular* line, and those composing the dividend on the *right*. The result is the same as if the divisor were placed *under* the dividend as before. Each method has its advantages, and may be used at pleasure. Hence, the

RULE.—*Cancel all the factors common to the divisor and dividend, and divide the product of those remaining in the dividend by the product of those remaining in the divisor.* (Art. 86.)

NOTES.—1. When either the *divisor, dividend,* or *both,* consist of two or more factors connected by the sign $\times$, they may be regarded as *composite* numbers, and the common factors be cancelled before the multiplication is performed.

2. This rule is founded upon the principle that, if the divisor and dividend are both divided by the same number, the quotient is not altered. (Art. 86.) Its utility is most apparent in those operations which involve both *multiplication* and *division;* as Fractions, Proportion, etc.

3. When the *factor* or *factors* cancelled *equal* the number itself, the unit 1, is always left; for, dividing a number by itself, the quotient is 1. When the 1 stands in the dividend, it must be *retained.* But when in the divisor, it is *disregarded.* (Art. 81.)

3. Divide the product $5\times6\times8$ by 30×48.

SOLUTION.—Cancelling the common factors, we have 1 in the dividend and 6 in the divisor. (Art. 112, *n.*)

$$\frac{5\times6\times8}{30\times48}=\frac{5\times6\times8}{30\times48},\ \frac{1}{6}=\frac{1}{6}$$

---

122. Rule for cancelling in division? *Note.* On what is the rule founded?

4. Divide 24×6 by 12×7.

5. Divide 42×8 by 21×2.

6. $18 \times 3 \times 4$ by 12×6.

7. Divide 18×15 by 5×6.

8. Divide 56×16 by 8×4.

9. $28 \times 6 \times 12$ by 14×8.

10. Divide $21 \times 5 \times 6$ by $7 \times 3 \times 2$.

11. Divide $32 \times 7 \times 9$ by $8 \times 5 \times 4$.

12. Divide $27 \times 35 \times 14$ by $9 \times 7 \times 21$.

13. Divide $33 \times 42 \times 25$ by $7 \times 5 \times 11$.

14. Divide $36 \times 48 \times 56 \times 5$ by $96 \times 8 \times 4 \times 2$.

15. Divide $63 \times 24 \times 33 \times 2$ by $9 \times 12 \times 11$.

16. Divide $175 \times 28 \times 72$ by $25 \times 14 \times 12$.

17. Divide $220 \times 60 \times 48 \times 69$ by $13 \times 110 \times 12 \times 8$.

18. Divide $350 \times 63 \times 144$ by $50 \times 72 \times 24$.

19. Divide $500 \times 128 \times 42 \times 108$ by $256 \times 250 \times 12$.

20. How many barrels of flour, at $12 a barrel, must be given for 40 tons of coal, at $9 per ton?

21. How many tubs of butter of 56 pounds each, at 28 cents a pound, must be given for 8 pieces of shirting, containing 45 yards each, at 26 cents a yard?

22. How many bags of coffee 42 pounds each, worth 4 shillings a pound, must be given for 18 chests of tea, each containing 72 pounds, at 6 shillings a pound?

23. Bought 24 barrels of sugar, each containing 168 pounds, at 20 cents a pound, and paid for it in cheeses, each weighing 28 pounds, worth 16 cents a pound: how many cheeses did it take?

24. A grocer sold 10 hogheads of molasses, each containing 63 gallons, at 7 shillings per gallon, and took his pay in wheat, worth 14 shillings a bushel: how much wheat did he receive?

25. A young lady being asked her age, replied: If you divide the product of 64 into 14, by the product of 8 into 4, you will have my age: what was her age?

26. A has 60 acres, worth $15 an acre, and B 100 acres, worth $75 an acre: B's property is how many times the value of A's?

COMMON DIVISORS.

123. A ***Common Divisor*** is any number that will divide *two* or *more* numbers without a *remainder.*

124. To find a *Common Divisor* of two or more Numbers.

1. Required a common divisor of 10, 12, and 14.

ANALYSIS.—By inspection, we perceive that $10=2\times5$; $12=2\times6$, and $14=2\times7$. The factor 2, is common to each of the given numbers, and is therefore a *common* divisor of them. (Art. 123.) Hence, the

$$10=2\times5$$
$$12=2\times6$$
$$14=2\times7$$
Ans. 2.

RULE.—*Resolve each of the given numbers into two factors, one of which is common to them all.*

Find common divisors of the following numbers:

2. 8, 16, 20, and 24. *Ans.* 2 and 4.
3. 12, 15, 18, and 30.
4. 36, 48, 96, and 108.
5. 42, 54, 66, and 132.
6. 21, 28, 35, 49, 63.
7. 20, 30, 70, and 100.
8. 60, 75, 120, and 240.
9. 16, 24, 40, 64, 116, 120, 144, 168, 264, 1728.

GREATEST COMMON DIVISOR.

125. The ***Greatest Common Divisor*** of two or more numbers, is the *greatest number* that will divide *each* of them without a remainder. Thus, 6 is the greatest common divisor of 18, 24, and 30.

NOTES.—1. A *common divisor* of two or more numbers is the same as a *common factor* of those numbers; and the *greatest* common divisor of them is their *greatest common* factor. Hence,

2. If any two numbers have not a *common factor,* they cannot have a *common divisor greater* than 1. (Art. 112, *n.*)

3. A *common* divisor is often called a *common measure,* and the *greatest common* divisor, the *greatest common measure.*

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123. A common divisor? 125. Greatest common divisor? *Note.* A common divisor is the same as what? Often called what?

FIRST METHOD.

126. To find the *Greatest Common Divisor* by continued Divisions.

1. What is the greatest common divisor of 28 and 40?

ANALYSIS.—If we divide the *greater* number by the *less*, the quotient is 1, and 12 remainder. Next, dividing the first divisor 28, by the first remainder 12, the quotient is 2, and 4 remainder. Again, dividing the second divisor by the second remainder 4, the quotient is 3, and 0 rem. The last divisor 4, is the *greatest common divisor*.

OPERATION.

```
28)40(1
   28
   12)28(2
      24
       4)12(3
         12
```

Demonstration.—We wish to prove two points: 1st, That 4 is a *common* divisor of the given numbers. 2d, That it is their *greatest* common divisor.

First. We are to prove that 4 is a *common* divisor of 28 and 40. By the last division, 4 is contained in 12, 3 times. Now, as 4 is a divisor of 12; it is also a divisor of the *product* of 12 into 2, or 24. (Art. 110, Prop. 14.) Next, since 4 is a divisor of itself and 24, it must be a divisor of the *sum* of 4+24, or 28, which is the *smaller* number. (Prop. 15.) For the same reason, since 4 is a divisor of 12 and 28, it must also be a divisor of the *sum* of 12+28, or 40, which is the *larger* number. Hence, 4 is a *common* divisor of 28 and 40.

Second. We are now to prove that 4 is the *greatest* common divisor of 28 and 40. If the greatest common divisor of these numbers is not 4, it must be either *greater* or *less* than 4. But we have shown that 4 is a *common* divisor of the given numbers; therefore, no number *less* than 4 can be the *greatest* common divisor of them. The assumed number must therefore be *greater* than 4. By supposition, this assumed number is a divisor of 28 and 40; hence, it must be a divisor of their *difference* 40—28 or 12. And as it is a divisor of 12, it must also divide the *product* of 12 into 2 or 24. Again, since the assumed number is a divisor of 28 and 24, it must also be a divisor of their *difference*, which is 4; that is, a *greater* number will divide a *less* without a *remainder*, which is impossible. (Art. 81, Prin. 2.) Therefore, 4 must be the *greatest common divisor* of 28 and 40. Hence, the

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126. Explain Ex. 1. Prove that 4 is the greatest common divisor of 28 and 40. Rule? *Note.* If there are more than two numbers, how proceed? If the last divisor is 1, then what? What is the greatest common divisor of two or more prime numbers, or numbers prime to each other?

RULE.—*Divide the greater number by the less; then divide the first divisor by the first remainder, the second divisor by the second remainder, and so on, until nothing remains; the last divisor will be the greatest common divisor.*

NOTES.—1. If there are more than *two* numbers, begin with the smaller, and find the *greatest common divisor* of two of them; then of this divisor and a third number, and so on, until all the numbers have been taken.

2. The greatest common divisor of two or more *prime* numbers, or numbers *prime* to *each other*, is 1. (Art. 112, *n.*)

3. If the last divisor is 1, the numbers are *prime*, or *prime* to *each other*; therefore their greatest common divisor is 1. Such numbers are said to be *incommensurable.* (Art. 101, Def. 16.)

2. What is the greatest com. divisor of 48, 72, and 108?

3. What is the greatest common divisor of 72 and 120?

SECOND METHOD.

127. To find the greatest *Common Divisor* by Prime Factors.

4. What is the greatest common divisor of 28 and 40?

ANALYSIS.—Resolving the given numbers into factors common to both, we have $28=2\times2\times7$, and $40=2\times2\times10$. Now the product of these factors, viz., 2 into 2, gives 4 for the greatest common divisor, the same as by the first method.

OPERATION.

$28=2\times2\times7$

$40=2\times2\times10$

Ans. $2\times2=4$.

5. What is the greatest com. div. of 30, 45, and 105?

ANALYSIS.—Setting the numbers in a horizontal line, we divide by any prime number, as 3, that will divide each without a remainder, and set the quotients under the corresponding numbers. Again, dividing each of these quotients by the prime number 5, the new quotients 2, 3, and 7, are prime, and have no common factor. Therefore, the product of the common divisors 3 into 5, or 15, is the greatest common divisor. (Art. 110, Prop. 16.) Hence, the

OPERATION.

3)30, 45, 105
5)10, 15, 35
2, 3, 7

$3\times5=15$, *Ans.*

127. What is the rule for the second method? *Note.* What is the object of placing the numbers in a horizontal line?

RULE.—I. *Write the numbers in a horizontal line, and divide by any prime number that will divide each without a remainder, setting the quotients in a line below.*

II. *Divide these quotients as before, and thus proceed till no number can be found that will divide all the quotients without a remainder. The product of all the divisors will be the greatest common divisor.*

NOTES.—1. This rule is the same as resolving the given numbers into prime factors, and multiplying together all that are common.

2. The advantage of placing the numbers in a *horizontal line,* is that the *prime factors* that are common, may be seen at a glance.

3. When required to find the greatest common divisor of *three* or *more* numbers, the second method is generally more *expeditious,* and therefore preferable.

4. When the given numbers have only *one common* factor, that factor is their *greatest common divisor.*

5. Two or more numbers may have *several common* divisors; but they can have only *one greatest* common divisor.

Find the greatest common divisor of the following:

6. 63 and 42.
7. 135 and 105.
8. 24, 36, 72.
9. 60, 75, 12.
10. 75, 125, 250.
11. 42, 54, 60, 84.
12. 72, 100, 168, 136.
13. 60, 84, 132, 108.
14. 10, 28, 40, 64, 90, 32.
15. 12, 36, 60, 108, 132.
16. 16, 28, 64, 56, 160, 250.
17. 576 and 960.
18. 1225 and 592.
19. 703 and 1369.
20. 1492 and 1866.
21. 2040 and 4080.

22. A merchant had 180 yards of silk, and 234 yards of poplin, which he wished to cut into equal dress patterns, each containing the greatest possible number of yards: how many yards would each contain?

23. Two lads had 42 and 63 apples respectively: how many can they put in a pile, that the piles shall be equal, and each pile have the greatest possible number?

24. A man had farms of 56, 72, and 88 acres respectively, which he fenced into the largest possible fields of the same number of acres: how many acres did he put in each?

MULTIPLES.

128. A ***Multiple*** is a number which *can be divided* by another number, *without a remainder.* Thus, 12 is a multiple of 4.

REMARK.—The *Term Multiple* is also used in the sense of *product:* as when it is said, "if one number is a divisor of another, the former is also a divisor of any multiple (product) of the latter." Thus, 3 is a divisor of 6; it is also a divisor of 7 times 6, or 42.

NOTE.—*Multiple* is from the Latin *multiplex, having many folds,* or *taken many times;* hence, a *product.*

But every product is *divisible* by its *factors;* hence, the term came to denote a *dividend.* The former signification is derived from the *formation* of the number; the *latter,* from its *divisibility.*

129. A ***Common Multiple*** is any number that can be divided by *two* or *more* numbers without a remainder. Thus, 18 is a common multiple of 2, 3, 6, and 9.

130. A *common multiple* of two or more numbers may be found by *multiplying them together.* That is, the product of two numbers, or any *entire number* of *times their product,* is a *common multiple* of them. (Art. 128, *n.*)

NOTES.—1. The *factors* or *divisors* of a multiple are sometimes called *sub-multiples.*

2. A number may have an *unlimited number* of multiples. For, according to the second definition, every number is a multiple of itself; and if multiplied by 2, the *product* will be a *second* multiple; if multiplied by 3, the product will be a *third* multiple; and universally, its product into any *whole* number will be a multiple of that number. (Art. 128.)

Find a common multiple of the following numbers:

1. 2, 3 and 5.	4. 2, 8 and 10.	7. 13, 7 and 22.
2. 3, 7 and 11.	5. 7, 6 and 17.	8. 19, 2 and 40.
3. 5, 7 and 13.	6. 11, 21 and 31.	9. 17, 10 and 34.

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128. Meaning of the term multiple? 129. What is a common multiple? 130. How found? *Note.* What are sub-multiples? How many multiples has a number?

LEAST COMMON MULTIPLE.

131. The ***Least Common Multiple*** of two or more numbers, is the *least* number that can be divided by each of them without a remainder. Thus, 15 is the least common multiple of 3 and 5.

132. A ***Multiple***, according to the first definition, is a *composite* number. But a *composite* number contains all the *prime factors* of each of the numbers which produce it. Hence, we derive the following Principles:

PRIN. 1. That a *multiple* of a number must contain all the *prime factors* of that number.

2. A *common multiple* of two or more numbers must contain all the *prime* factors of each of the given numbers.

3. The *least common multiple* of two or more numbers *is the least number* which contains all their *prime* factors, each factor being taken as many times only, as it occurs in either of the given numbers, and no more.

133. To find the *Least Common Multiple* of two or more numbers..

1. What is the least common multiple of 18, 21 and 66?

1st OPERATION.

2)18, 21, 66
3)9, 21, 33
3, 7, 11

$2 \times 3 \times 3 \times 7 \times 11 = 1386$

1st METHOD.—We write the numbers in a horizontal line, with a *comma* between them. Since 2 is a prime factor of one or more of the given numbers, it must be a factor of the least common multiple. (Art. 132, Prin. 1.) We therefore divide by it, setting the quotients and undivided numbers in a line below. In like manner, we divide these quotients and undivided numbers by the prime number 3, and set the results in another line, as before. Now, as the numbers in the third line are *prime*, we can carry the division no further; for, they have no common divisor greater than 1. (Art. 101.) Hence, the divisors 2 and 3, with the numbers in the last line, 3, 7 and 11, are all the prime factors contained in the given numbers, and each is taken as many times as it occurs in either of them.

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131. What is the least common multiple? 132. What principles are derived?

Therefore, the continued product of these factors, $2 \times 3 \times 3 \times 7 \times 11$, or 1386, is the least common multiple required.

2d METHOD.—Resolving each number into its prime factors, we have $18=2 \times 3 \times 3$, $21=3 \times 7$, and $66=2 \times 3 \times 11$. Now, as the least common multiple must contain all the prime factors of the given number, it must contain those of 66, viz., $2 \times 3 \times 11$; we therefore retain these factors. Again, it must contain the prime factors of 21, which are 3×7, and of 18, which are $2 \times 3 \times 3$, each being taken as many times as it is found in either of the given numbers. But 2 is already retained, and 3 has been taken once; we therefore cancel the 2 and one of the 3s, and retain the other 3 and the 7. The continued product of these factors, viz., $2 \times 3 \times 11 \times 7 \times 3$, is 1386, the same as before. Hence, the

2d OPERATION.

$$18=2 \times 3 \times 3$$
$$21=3 \times 7$$
$$66=2 \times 3 \times 11$$
$$2 \times 3 \times 11 \times 7 \times 3=1386$$

RULE.—I. *Write the numbers in a horizontal line, and divide by any prime number that will divide two or more of them without a remainder, placing the quotients and numbers undivided in a line below.*

II. *Divide this line as before, and thus proceed till no two numbers are divisible by any number greater than* 1. *The continued product of the divisors and numbers in the last line will be the answer.*

Or, resolve the numbers into their prime factors; multiply these factors together, taking each the greatest number of times it occurs in either of the given numbers. The product will be the answer.

NOTES.—1. These two methods are based upon the same principle, viz.: that the *least common multiple* of two or more numbers is the least number which contains *all their prime factors*, each factor being taken as *many times only*, as it occurs in either of the given numbers. (Art. 132, Prin. 3.)

2. The reason we employ *prime* numbers as divisors is because the given numbers are to be resolved into *prime* factors, or factors *prime*

133. How find the least common multiple? *Note.* Upon what principle are these two methods based? Why employ prime numbers for divisors in the first method? Why write the numbers in a horizontal line? What is the second method? Advantage of it? How may the operation be shortened? When the given numbers are prime, or prime to each other, what is to be done?

to each other. If the divisors were *composite* numbers, they would be liable to contain factors common to some of the quotients, or numbers in the last line; and if so, their *continued product* would not be the *least* common multiple.

3. The object of placing the numbers in a horizontal line, is to resolve all the numbers into prime factors at the same time.

4. The chief advantage of the second method lies in the *distinctness* with which the prime factors are presented. The former is the more *expeditious* and *less liable* to mistakes.

5. The operation may often be shortened by *cancelling* any number which is a *factor of another number* in the same line. (Ex. 2.) When the given numbers are *prime*, or *prime to each other*, they have no common factors to be rejected; consequently, their continued product will be the least common multiple. Thus, the least common multiple of 3 and 5 is 15; of 4 and 9 is 36.

2. Find the least common multiple of 5, 6, 9, 10, 21.

ANALYSIS.—In the first line 5 is a factor of 10, and is therefore cancelled; in the second, 3 is a factor of 9, and is also cancelled.

$$\begin{array}{l} 2)\not{5},\ 6,\ 9,\ 10,\ 21 \\ \hline 3)\not{3},\ 9,\ 5,\ 21 \\ \hline \quad 3,\ 5,\ 7 \end{array}$$

The product of $2 \times 3 \times 3 \times 5 \times 7 = 630$, the answer required.

Find the least common multiple of the following:

3. 8, 12, 16, 24.
4. 14, 28, 21, 42.
5. 36, 24, 48, 60.
6. 25, 40, 75, 100.
7. 16, 24, 32, 40.
8. 22, 33, 55, 66.
9. 30, 40, 60, 80.
10. 36, 48, 72, 96.
11. 42, 68, 84, 108.
12. 120, 144, 168, 216.
13. 96, 108, 60, 204.
14. 126, 154, 280, 560.
15. 144, 256, 72, 300.
16. 250, 500, 1000.

17. Investing the same amount in each, what is the smallest sum with which I can buy a whole number of pears at 4 cents, lemons at 6, and oranges at 10 cents?

18. A can hoe 16 rows of corn in a day, B 18, C 20, and D 24 rows: what is the smallest number of rows that will keep each employed an exact number of days?

19. A grocer has a 4 pound, 5 pound, 6 pound, and a 12 pound weight: what is the smallest tub of butter that can be weighed by each without a remainder?

FRACTIONS.

134. A ***Fraction*** is one or more of the *equal parts* into which a *unit* is divided.

The *number* of these parts indicates their *name*. Thus, when a unit is divided into *two* equal parts, the parts are called *halves;* when into *three,* they are called *thirds;* when into *four, fourths,* etc. Hence,

135. A *half* is one of the *two* equal parts of a unit; a *third* is one of the *three* equal parts of a unit; a *fourth,* one of the *four* equal parts, etc.

NOTE.—The term *fraction* is from the Latin *frango,* to *break.* Hence, Fractions are often called *Broken Numbers.*

136. The *value* of these *equal parts* depends,

First. Upon the *magnitude* of the *unit* divided.

Second. The *number* of *parts* into which it is divided.

ILLUSTRATION.—1st. If a *large* and a *small* apple are each divided into *two, three, four,* etc., equal parts, it is plain that the parts of the *former* will be *larger* than the corresponding parts of the *latter.*

2d. If *one* of two equal apples is divided into *two* equal parts, and the *other* into *four,* the parts of the first will be *twice* as *large* as those of the second; if one is divided into *two* equal parts, the other into *six,* one part of the *first* will be equal to *three* of the *second,* etc. Hence,

NOTE.—A *half* is *twice* as large as a *fourth, three times* as large as a *sixth, four times* as large as an *eighth,* etc.; and generally,

The *greater* the *number* of equal parts into which the unit is divided, the *less* will be the *value* of each part. Conversely,

The *less* the *number* of equal parts, the *greater* will be the value of each part.

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134. What is a fraction? What does the number of parts indicate? 135. What is a half? A third? A fourth? A tenth? A hundredth? 136. Upon what does the size of these parts depend?

137. Fractions are divided into two classes, *common* and *decimal.*

A ***Common Fraction*** is one in which the unit is divided into *any number* of equal parts.

138. Common fractions are expressed by *figures* written above and below a line, called the numerator and denominator; as $\frac{1}{2}$, $\frac{4}{5}$, $\frac{5}{12}$.

139. The ***Denominator*** is written *below* the line, and shows into *how many* equal parts the unit is divided. It is so called because it *names* the parts; as halves, thirds, fourths, tenths, etc.

The ***Numerator*** is written above the line, and shows *how many parts* are expressed by the fraction. It is so called because it *numbers* the parts taken. Thus, in the fraction $\frac{3}{4}$, *four* is the denominator, and shows that the unit is divided into *four* equal parts; *three* is the numerator, and shows that *three* of the parts are taken.

140. The ***Terms*** of a fraction are the *numerator* and *denominator.*

NOTE.—Fractions primarily arise from dividing a *single unit* into equal parts. They are also used to express a *part* of a *collection* of units, and a *part* of a *fraction* itself; as 1 half of 6 pears, 1 third of 3 fourths, etc. But that from which they arise, is always regarded as a *whole,* and is called the *Unit* or *Base* of the fraction.

141. Common fractions are usually divided into *proper, improper, simple, compound, complex,* and *mixed* numbers.

1. A ***Proper Fraction*** is one whose numerator is *less* than the denominator; as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$.

2. An ***Improper Fraction*** is one whose numerator *equals* or *exceeds* the denominator; as $\frac{4}{4}$, $\frac{5}{3}$.

3. A ***Simple Fraction*** is one having but *one*

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137. Into how many classes are fractions divided? A common fraction? 138. How expressed? 139. What does the denominator show? Why so called? The numerator? Why so called? 140. What are the terms of a fraction?

numerator and *one* denominator, each of which is a *whole* number, and may be proper or improper; as $\frac{2}{3}$, $\frac{5}{4}$.

4. A ***Compound Fraction*** is a fraction of a fraction; as $\frac{1}{3}$ of $\frac{3}{4}$.

5. A ***Complex Fraction*** is one which has a *fractional* numerator, and an *integral* denominator; as, $\frac{\frac{1}{2}}{4}$, $\frac{2\frac{1}{3}}{5}$.

REMARK.—Those *abnormal expressions*, having *fractional denominators*, commonly called *complex fractions*, do not *strictly* come under the *definition* of a fraction. For, a fraction is *one* or *more* of the *equal parts* into which a unit is divided. But it cannot properly be said, that a unit is divided into $3\frac{1}{2}$ths, $4\frac{2}{3}$dths, etc.; that is, into $3\frac{1}{2}$ equal parts, $4\frac{2}{3}$ equal parts, etc. They are expressions *denoting division*, having a *fractional* divisor, and should be treated as such.

6. A ***Mixed Number*** is a *whole* number and a *fraction* expressed together; as, $5\frac{3}{7}$, $34\frac{1}{23}$.

NOTE.—The primary idea of a fraction is a *part* of a *unit*. Hence, a fraction of *less* value than a unit, is called a *proper* fraction.

All other fractions are called *improper*, because, being *equal* to or *greater* than a unit, they cannot be said to be a *part* of a unit.

Express the following fractions by figures:

1. Two fifths.
2. Four sevenths.
3. Three eighths.
4. Five twelfths.
5. Eleven fifteenths.
6. Thirteen twenty-firsts.
7. Fifteen ninths.
8. Twenty-three tenths.
9. Thirty-one forty-fifths.
10. Sixty-nine hundredths.
11. 112 two hundred and fourths.
12. 256 five hundred and twenty-seconds.

13. Explain the fraction $\frac{1}{4}$.

ANALYSIS.—$\frac{1}{4}$ denotes $\frac{1}{4}$ of 1; that is, one such part as is obtained by dividing a *unit* into 4 equal parts. The denominator names the parts, and the numerator numbers the parts taken. It is *common* because the unit is divided into any number of parts taken at random; *proper*, because its value is less than 1; and *simple*, because it has but one numerator and one denominator, each of which is a whole number.

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141. Into what are common fractions divided? A proper fraction? Improper? Simple? Compound? Complex? A mixed number?

14. Explain the fraction $\frac{3}{4}$.

ANALYSIS.—$\frac{3}{4}$ denotes $\frac{3}{4}$ of 1, or $\frac{1}{4}$ of 3. For, if 3 equal lines are each divided into 4 equal parts, 3 of these parts will be equal to $\frac{3}{4}$ of 1 line, or $\frac{1}{4}$ of the 3 lines. It is *common*, etc. (Ex. 13.)

Read and explain the following fractions:

15. $\frac{3}{5}$.	20. $\frac{45}{7}$.	25. $16\frac{3}{4}$.
16. $\frac{1}{2}$ of $\frac{3}{4}$.	21. $\frac{68}{21}$.	26. $28\frac{11}{12}$.
17. $\frac{11}{11}$.	22. $\frac{85}{32}$.	27. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$.
18. $\frac{4}{5}$ of $\frac{12}{30}$.	23. $\frac{35}{35}$.	28. $\frac{2}{3}$ of $\frac{5}{7}$ of $2\frac{1}{2}$.
19. $\frac{4\frac{1}{3}}{8}$.	24. $\frac{6\frac{1}{2}}{10}$.	29. $\frac{11\frac{1}{4}}{15}$.

NOTE.—The 21st and 22d should be read, "68 twenty-*firsts*," "85 thirty-*seconds*," and not 68 twenty-*ones*, 85 thirty-*twos*.

142. Fractions, we have seen, arise from division, the *numerator* being the *dividend*, and the *denominator* the *divisor*. (Art. 64, *n*.) Hence,

The *value* of a fraction is the *quotient* of the numerator divided by the denominator. Thus, the value of 1 fourth is $1 \div 4$, or $\frac{1}{4}$; of 6 halves is $6 \div 2$, or 3; of 3 thirds is $3 \div 3$, or 1.

143. To find a *Fractional Part* of a number.

1. What is 1 half of 12 dimes?

ANALYSIS.—If 12 dimes are divided into 2 equal parts, 1 of these parts will contain 6 dimes. Therefore, $\frac{1}{2}$ of 12 dimes is 6 dimes.

NOTE.—The solution of this and similar examples is an illustration of the *second object* or *office* of Division. (Art. 63, *b*.)

4. What is $\frac{1}{4}$ of 36? $\frac{1}{5}$ of 45? $\frac{1}{6}$ of 60? $\frac{1}{7}$ of 63?
5. What is $\frac{1}{8}$ of 48? $\frac{1}{9}$ of 63? $\frac{1}{10}$ of 190? $\frac{1}{11}$ of 132?
6. What is $\frac{2}{3}$ of 12 dimes?

ANALYSIS.—2 thirds are 2 times as much as $\frac{1}{3}$. But 1 third of 12 is 4. Therefore, $\frac{2}{3}$ of 12 dimes are 2 times 4, or 8 dimes. Hence, the

142. From what do fractions arise? Which part is the dividend? Which the divisor? What is the value of a fraction?

RULE.—*Divide the given number by the denominator, and multiply the quotient by the numerator*

To find $\frac{1}{2}$, divide the number by 2.
To find $\frac{1}{3}$, divide the number by 3.
To find $\frac{2}{3}$, divide by 3, and multiply by 2, etc.

7. What is $\frac{3}{4}$ of 56? *Ans.* 42.
8. What is $\frac{4}{5}$ of 30 apples? Of 45? Of 60?
9. What is $\frac{2}{6}$ of 42? $\frac{3}{7}$ of 56? $\frac{5}{8}$ of 72?
10. What is $\frac{5}{9}$ of 45? $\frac{7}{10}$ of 50? $\frac{6}{11}$ of 66? $\frac{7}{12}$ of 108?

GENERAL PRINCIPLES OF FRACTIONS.

144. Since the *numerator* and *denominator* have the *same relation* to each other as the dividend and divisor, it follows that the *general principles* established in division are applicable to fractions. (Art. 81.) That is,

1. If the numerator is *equal* to the denominator, the value of the fraction is 1.

2. If the denominator is *greater* than the numerator, the value is less than 1.

3. If the denominator is *less* than the numerator, the value is greater than 1.

4. If the denominator is 1, the value is the numerator.

5. *Multiplying* the *numerator, multiplies* the fraction.

6. *Dividing* the *numerator, divides* the fraction.

7. *Multiplying* the *denominator, divides* the fraction.

8. *Dividing* the *denominator, multiplies* the fraction.

9. *Multiplying or dividing both the numerator and denominator by the same number does not alter the value of the fraction.*

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144. If the numerator is equal to the denominator, what is the value of the fraction? If the denominator is the greater, what? If less, what? If the denominator is 1, what? What is the effect of multiplying the numerator? Dividing the numerator? Multiplying the denominator? Dividing the denominator?

REDUCTION OF FRACTIONS.

145. ***Reduction*** of fractions is *changing* their *terms*, without *altering* the *value* of the fractions. (Art. 142.)

CASE I.

146. To Reduce a Fraction to its *Lowest Terms.*

DEF.—The ***Lowest Terms*** of a fraction are the *smallest numbers* in which its numerator and denominator can be expressed. (Art. 101, Def. 7.)

Ex. 1. Reduce $\frac{24}{32}$ to its lowest terms.

1st METHOD.

$$2)\frac{24}{32}=\frac{12}{16} \text{ and }$$

$$4)\frac{12}{16}=\frac{3}{4} \quad \textit{Ans.}$$

ANALYSIS.—Dividing both terms of a fraction by the same number does not alter its value. (Art. 144, Prin. 9.) Hence, we divide the given terms by 2, and the terms of the new fraction by 4; the result is $\frac{3}{4}$. But the terms of the fraction $\frac{3}{4}$ are prime to each other; therefore, they are the lowest terms in which $\frac{24}{32}$ can be expressed.

2d METHOD.

$$8)\frac{24}{32}=\frac{3}{4} \quad \textit{Ans.}$$

Or, if we divide both terms by their *greatest common* divisor, which is 8, we shall obtain the same result. (Art. 126.) Hence, the

RULE.—*Divide the numerator and denominator continually by any number that will divide both without a remainder, until no number greater than* 1 *will divide them.*

Or, divide both terms of the fraction by their greatest common divisor. (Art. 126.)

NOTES.—1. It follows, conversely, that a fraction is reduced to *higher terms*, by *multiplying the numerator and denominator* by a *common multiplier*. Thus, $\frac{2}{3}=\frac{16}{24}$, both terms being multiplied by 8.

2. These rules depend upon the principle, that dividing both terms of a fraction by the same number does not alter its value. When the terms are large, the second method is preferable.

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145. What is Reduction of Fractions? 146. What are the lowest terms of a fraction? How reduce a fraction to its lowest terms?

Reduce the following fractions to their lowest terms:

2. $\frac{24}{48}$.
3. $\frac{24}{36}$.
4. $\frac{18}{54}$.
5. $\frac{35}{49}$.
6. $\frac{12}{84}$.
7. $\frac{63}{96}$.
8. $\frac{17}{68}$.
9. $\frac{19}{95}$.
10. $\frac{108}{144}$.
11. $\frac{121}{256}$.
12. $\frac{288}{192}$.
13. $\frac{134}{134}$.
14. $\frac{75}{300}$.
15. $\frac{19}{31}$.
16. $\frac{132}{156}$.
17. $\frac{41}{83}$.
18. $\frac{600}{375}$.
19. $\frac{568}{1000}$.
20. $\frac{375}{850}$.
21. $\frac{263}{1052}$.
22. $\frac{764}{1160}$.
23. $\frac{560}{1680}$.
24. $\frac{2104}{3156}$.
25. $\frac{345}{920}$.
26. $\frac{800}{2000}$.
27. $\frac{1215}{2187}$.
28. $\frac{1160}{2340}$.
29. $\frac{59}{413}$.

CASE II.

147. To reduce an Improper Fraction to a *Whole or Mixed Number*.

1. Reduce $\frac{45}{4}$ to a whole or mixed number.

ANALYSIS.—Since 4 fourths make a unit or 1, 45 fourths will make as many units as 4 is contained times in 45, which is $11\frac{1}{4}$. Hence, the

$$4)\underline{45}$$
$$11\frac{1}{4}\ Ans.$$

RULE.—*Divide the numerator by the denominator.*

NOTES.—1. This rule, in effect, divides both terms of the fraction by the same number; for, *removing* the denominator *cancels* it, and *cancelling* the denominator divides it by itself. (Art. 121.)

2. If fractions occur in the answer, they should be reduced to the *lowest terms*.

Reduce the following to whole or mixed numbers:

2. $\frac{111}{3}$.
3. $\frac{145}{9}$.
4. $\frac{152}{8}$.
5. $\frac{184}{12}$.
6. $\frac{108}{9}$.
7. $\frac{400}{25}$.
8. $\frac{113}{9}$.
9. $\frac{443}{12}$.
10. $\frac{587}{13}$.
11. $\frac{684}{108}$.
12. $\frac{183}{55}$.
13. $\frac{825}{160}$.
14. $\frac{5801}{126}$.
15. $\frac{5876}{314}$.
16. $\frac{10000}{111}$.
17. $\frac{25713}{1168}$.
18. $\frac{30256}{144}$.
19. $\frac{27342}{254}$.
20. $\frac{98236}{256}$.
21. $\frac{100000}{4800}$.
22. $\frac{306245}{125}$.
23. $\frac{490230}{45200}$.
24. $\frac{510030}{17144}$.
25. $\frac{978911}{72850}$.
26. In $\frac{25625}{620}$ of a pound, how many pounds?
27. In $\frac{360256}{12254}$ of a dollar, how many dollars?
28. In $\frac{2531520}{720}$ of a year, how many years?

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147. How reduce an improper fraction to a whole or mixed number?

CASE III.

148. To reduce a Mixed Number to an *Improper Fraction*.

1. Reduce $8\frac{3}{7}$ to an improper fraction.

ANALYSIS.—Since there are 7 sevenths in a unit, there must be 7 times as many *sevenths* in a number as there are units, and 7 times 8 are 56 and 3 sevenths make $\frac{59}{7}$. Therefore, $8\frac{3}{7}=\frac{59}{7}$.

$$\begin{array}{l} 8\frac{3}{7} \\ \underline{7} \\ \frac{59}{7} \text{ Ans.} \end{array}$$

Or thus: Since $1=\frac{7}{7}$, $8 = 8$ times $\frac{7}{7}$ or $\frac{56}{7}$, and $\frac{3}{7}$ make $\frac{59}{7}$. In the operation we multiply the integer by the given denominator, and to the product add the numerator. Hence, the

RULE.—*Multiply the whole number by the given denominator; to the product add the numerator, and place the sum over the denominator.*

REMARK.—A *whole* number may be reduced to an *improper* fraction by making 1 its denominator. Thus, $4=\frac{4}{1}$. (Art. 81.)

Reduce the following to improper fractions:

2. $15\frac{3}{7}$.	6. $145\frac{35}{63}$.	10. $1573\frac{3}{5}$.	14. $478\frac{5}{83}$.
3. $18\frac{4}{5}$.	7. $187\frac{41}{72}$.	11. $2561\frac{4}{7}$.	15. $57\frac{25}{250}$.
4. $35\frac{7}{8}$.	8. $295\frac{81}{100}$.	12. $3640\frac{5}{8}$.	16. $8\frac{327}{411}$.
5. $81\frac{11}{20}$.	9. $806\frac{95}{110}$.	13. $8624\frac{7}{10}$.	17. $9\frac{4523}{10000}$.

18. In $263\frac{3}{16}$ pounds how many sixteenths?

19. Change $641\frac{12}{40}$ mile to fortieths of a mile.

CASE IV.

149. To reduce a Compound Fraction to a *Simple one*.

1. Reduce $\frac{3}{4}$ of $\frac{2}{3}$ to a simple fraction.

ANALYSIS.—3 fourths of $\frac{2}{3}$ = 3 times 1 fourth of $\frac{2}{3}$. Now 1 fourth of $\frac{2}{3}$ is $\frac{2}{12}$; for multiplying the denominator divides the fraction; and 3 fourths of $\frac{2}{3}$ = 3 times $\frac{2}{12}$ or $\frac{6}{12}$; for multiplying the numerator multiplies the fraction. (Art. 144.) Reduced to its lowest terms $\frac{6}{12}=\frac{1}{2}$.

1st METHOD.

$$\frac{3}{4}\times\frac{2}{3}=\frac{6}{12}$$

$$\text{Ans. } \frac{6}{12}=\frac{1}{2}$$

148. How reduce a mixed number to an improper fraction? *Rem.* A whole number to an improper fraction?

Or, since numerators are dividends and denominators divisors, the factors 2 and 3, which are common to both, may be cancelled. (Art. 144, Prin. 9.) The result is $\frac{1}{2}$. Hence, the

2d METHOD.

$$\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} \times \frac{\cancel{2}}{\cancel{3}} = \frac{1}{2} \text{ Ans.}$$

RULE.—*Cancel the common factors, and place the product of the factors remaining in the numerators over the product of those remaining in the denominators.*

NOTES.—1. *Whole* and *mixed* numbers must be reduced to *improper* fractions, before multiplying the terms, or cancelling the factors.

2. Cancelling the common factors reduces the result to the *lowest terms*, and therefore should be employed, whenever practicable.

3. The numerators being dividends, may be placed on the right of a perpendicular line, and the denominators on the left, if preferred. (Art. 122, *Rem.*)

4. The *reason* of the rule is this: multiplying the numerator of one fraction by the numerator of another, *multiplies* the value of the former fraction by as many units as are contained in the numerator of the latter; consequently the result is as many times too large as there are units in the denominator of the latter. (Art. 144, Prin. 5.) This error is corrected by multiplying the two denominators together. (Art. 144, Prin. 7.)

2. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $2\frac{1}{2}$ to a simple fraction. *Ans.* $\frac{35}{44}$.

Reduce the following to simple fractions:

3. $\frac{5}{6}$ of $\frac{3}{10}$.	9. $\frac{5}{3}$ of $\frac{7}{25}$ of 60.	15. $\frac{7}{12}$ of $\frac{48}{63}$ of $\frac{27}{32}$.
4. $\frac{3}{5}$ of $\frac{4}{7}$.	10. $\frac{7}{9}$ of $\frac{27}{35}$ of $\frac{13}{19}$.	16. $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{9}$.
5. $\frac{4}{7}$ of $\frac{14}{20}$.	11. $\frac{11}{17}$ of $\frac{21}{45}$ of $\frac{63}{84}$.	17. $\frac{32}{39}$ of $\frac{26}{44}$ of $65\frac{1}{4}$.
6. $\frac{3}{8}$ of $\frac{16}{27}$.	12. $\frac{13}{19}$ of $\frac{1}{2}$ of $\frac{42}{51}$.	18. $\frac{45}{55}$ of $\frac{22}{63}$ of $84\frac{1}{9}$.
7. $\frac{3}{7}$ of $\frac{13}{15}$.	13. $\frac{18}{22}$ of $\frac{4}{57}$ of 45.	19. $\frac{5}{8}$ of $8\frac{2}{5}$ of $\frac{19}{20}$.
8. $\frac{8}{9}$ of $\frac{7}{11}$.	14. $\frac{2}{3}$ of $\frac{5}{7}$ of $\frac{11}{13}$.	20. $\frac{7}{9}$ of $\frac{3}{5}$ of $163\frac{3}{4}$.

21. Reduce $\frac{16}{22}$ of $\frac{44}{70}$ of $\frac{30}{44}$ of $9\frac{5}{8}$ to a simple fraction.

22. To what is $\frac{1}{7}$ of $\frac{21}{32}$ of $3\frac{1}{2}$ bushels equal?

23. To what is $\frac{2}{5}$ of $4\frac{1}{2}$ of $\frac{3}{39}$ of a yard equal?

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149. How reduce a compound fraction to a simple one? *Note.* What must be done with whole and mixed numbers? What is the advantage of cancelling?

CASE V.

150. To reduce a Fraction to any required *Denominator*.

1. Reduce $\frac{6}{8}$ to twenty-fourths.

1ST ANALYSIS.—8 is contained in 24, 3 times; therefore, multiplying both terms by 3, the fraction becomes $\frac{18}{24}$, and its value is not altered. (Art. 144.)

$$24 \div 8 = 3 \qquad \frac{6 \times 3}{8 \times 3} = \frac{18}{24}$$

REM.—If required to reduce $\frac{6}{8}$ to fourths, we should divide both terms by 2, and it becomes $\frac{3}{4}$. (Art. 144.)

2D ANALYSIS.—Multiplying both terms of $\frac{6}{8}$ by 24, the required denominator, we have $\frac{144}{192}$. (Art. 144, Prin. 9.) Again, dividing both terms of $\frac{144}{192}$ by 8, the given denominator, we have $\frac{18}{24}$, the same as before. Hence, the

$$\frac{6 \times 24 = 144}{8 \times 24 = 192} \qquad \frac{144 \div 8 = 18}{192 \div 8 = 24}$$

RULE.—*Multiply or divide both terms of the fraction by such a number as will make the given denominator equal to the required denominator.*

Or, Multiply both terms of the given fraction by the required denominator; then divide both terms of the result by the given denominator. (Art. 144, Prin. 9.)

NOTES.—1. The *multiplier* required by the 1st method is found by *dividing the proposed denominator* by the *given* denominator.

2. When the required denominator is neither a *multiple*, nor *an exact divisor* of the given denominator, the result will be a *complex* fraction. Thus, $\frac{1}{3}$ reduced to tenths $= \frac{3\frac{1}{3}}{10}$; $\frac{5}{8}$ reduced to sevenths $= \frac{4\frac{3}{8}}{7}$.

Change the following to the denominator indicated:

2. $\frac{5}{13}$ to fifty-seconds.
3. $\frac{7}{12}$ to sixtieths.
4. $\frac{4}{11}$ to eighty-eighths.
5. $\frac{15}{24}$ to 144ths.
6. $\frac{19}{34}$ to 204ths.
7. $\frac{29}{41}$ to 246ths.
8. $\frac{41}{72}$ to 288ths.
9. $\frac{64}{90}$ to 360ths.
10. $\frac{65}{100}$ to 1000ths.
11. $\frac{119}{2000}$ to 10000ths.

12. Reduce $\frac{3}{8}$ to fourths. *Ans.* $\frac{1\frac{1}{2}}{4}$.

13. Reduce $\frac{5}{12}$ to sixths.

14. Reduce $\frac{7}{9}$ to thirds.

15. Reduce $\frac{8}{9}$ to twenty-sevenths.

16. Reduce $\frac{11}{16}$ to fourths.

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150. How reduce a fraction to any required denomination?

151. To reduce a Whole Number to a *Fraction* having a *given Denominator.*

1. Reduce 7 to fifths.

ANALYSIS.—1st. Since there are 5 fifths in a unit, any number must contain 5 times as many *fifths* as units; and 5 times 7 are 35. Therefore $7 = \frac{35}{5}$.

Or, 2d. Since $1 = \frac{5}{5}$, 7 must equal 7 times $\frac{5}{5}$, or $\frac{35}{5}$. In the operation 7 is both multiplied and divided by 5; hence, its value is not altered. (Art. 86, *a*.) Hence, the

OPERATION.

$$7 = (7 \times 5) \div 5 = \frac{35}{5}.$$

RULE.—*Multiply the whole number by the given denominator, and place the product over it.*

2. Change 7 to a fraction having 9 for its denominator.
3. Change 63 to 5ths.
4. Reduce 79 to 7ths.
5. Reduce 83 to 9ths.
6. Reduce 105 to 16ths.
7. Reduce 217 to 20ths.
8. Reduce 321 to 49ths.
9. 468 to 76ths.
10. 500 to 87ths.
11. 1560 to hundredths.
12. 2004 to thousandths.
13. 500 to ten-thousandths.
14. 25 to millionths.

CASE VI.

152. To reduce a *Complex Fraction* to a *Simple one.*

1. Reduce the complex fraction $\frac{3\frac{1}{3}}{5}$ to a simple one.

ANALYSIS.—The denominator of a fraction, we have seen, is a divisor. Hence, the given complex fraction is equivalent to $3\frac{1}{3} \div 5$. (Art. 142.) Now $3\frac{1}{3} = \frac{10}{3}$; therefore, $3\frac{1}{3} \div 5 = \frac{10}{3} \div 5$. (Art. 148.) But $\frac{10}{3} \div 5 = \frac{2}{3}$; for, dividing the *numerator* by any number, divides the fraction by that number. (Art. 144, Prin. 6.) Therefore, $\frac{2}{3}$ is the simple fraction required.

OPERATION.

$$\frac{3\frac{1}{3}}{5} = 3\tfrac{1}{3} \div 5;$$

$$3\tfrac{1}{3} = \tfrac{10}{3};$$

$$\tfrac{10}{3} \div 5 = \tfrac{2}{3}.$$

Ans. $\frac{2}{3}$.

Or, multiplying the denominator by 5, divides the fraction, and we have $\frac{10}{15} = \frac{2}{3}$, the same as before. (Prin. 7.)

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150. Upon what principle is this rule founded? 151. How reduce a whole number to a fraction having a given denominator?

2. Reduce the complex fraction $\frac{\frac{3}{4}}{7}$ to a simple one.

OPERATION.

$\frac{\frac{3}{4}}{7}=\frac{3}{4}\div 7$;

$\frac{3}{4}\div 7=\frac{3}{28}$.

Ans. $\frac{3}{28}$.

ANALYSIS.—Reasoning as before, the given fraction is equivalent to $\frac{3}{4}\div 7$. But we cannot divide the numerator of the fraction $\frac{3}{4}$ by 7 without a remainder, we therefore multiply the denominator by it; for, multiplying the *denominator*, divides the fraction; and $\frac{3}{4}\div 7=\frac{3}{28}$, the simple fraction required. (Art. 144, Prin. 7.) Hence, the

RULE.—I. *Reduce the numerator of the complex fraction to a simple one.*

II. *Divide its numerator by the given denominator.*

Or, *Multiply its denominator by the given denominator.*

NOTES.—1. When the numerator can be divided *without a remainder*, the *former* method is preferable. When this cannot be done. the *latter* should be employed.

2. After the numerator of the complex fraction is reduced to a simple one, the factors *common* to it and the given denominator, should be cancelled.

☞ If this case is deemed too difficult for beginners, it may be omitted till review.

(For the method of treating those expressions which have *fractional denominators,* see Division of Fractions, p. 130.)

2. Reduce $\frac{5\frac{5}{8}}{24}$ to a simple fraction.

SOLUTION.—$5\frac{5}{8}=\frac{45}{8}$; and $\frac{45}{8}\div 24=\frac{45}{192}$, or $\frac{15}{64}$, *Ans.*

Reduce the following complex fractions to simple ones.

3. $\frac{3\frac{3}{4}}{4}$.

4. $\frac{2\frac{2}{3}}{25}$.

5. $\frac{9\frac{1}{7}}{4}$.

6. $\frac{\frac{24}{4}}{2}$.

7. $\frac{\frac{108}{18}}{4}$.

8. $\frac{\frac{168}{7}}{3}$.

9. $\frac{2\frac{4}{5}}{7}$.

10. $\frac{3\frac{10}{66}}{2}$.

11. $\frac{5\frac{10}{34}}{5}$.

12. $\frac{\frac{176}{11}}{4}$.

13. $\frac{59\frac{1}{2}}{78}$.

14. $\frac{5\frac{5}{296}}{9}$.

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152. How reduce a complex fraction to a simple one? *Note*. What should be done with common factors?

CASE VII.

153. To reduce Fractions to a *Common Denominator*.

DEF.—A ***Common Denominator*** is one that *belongs equally* to *two or more fractions;* as, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{5}{7}$.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ to a common denominator.

REMARK.—In reducing fractions to a common denominator, it should be observed that the *value* of the fractions is not to be altered. Hence, whatever change is made in any denominator, a corresponding change must be made in its numerator.

ANALYSIS.—If each denominator is multiplied by all the other denominators, the fractions will have a common denominator, and if each numerator is multiplied into all the denominators except its own, the terms of each fraction will be multiplied by the same numbers; consequently their value will not be altered. (Art. 144.) The fractions thus obtained are $\frac{12}{24}$, $\frac{16}{24}$, and $\frac{18}{24}$. Hence, the

OPERATION.

$$\frac{1}{2}=\frac{1\times3\times4}{2\times3\times4}=\frac{12}{24}.$$
$$\frac{2}{3}=\frac{2\times2\times4}{3\times2\times4}=\frac{16}{24}.$$
$$\frac{3}{4}=\frac{3\times2\times3}{4\times2\times3}=\frac{18}{24}.$$

RULE.—*Multiply the terms of each fraction by all the denominators except its own.*

NOTES.—1. *Mixed* numbers must first be reduced to improper fractions, *compound* and *complex* fractions to simple ones.

2. This rule is founded upon the principle, that if *both terms* of a fraction are *multiplied* by the same numbers, its *value* is not altered.

3. A *common* denominator, it will be seen, is the product of all the denominators; hence it is a *common multiple* of them. (Art. 129.)

Reduce the following to a common denominator:

2. $\frac{3}{4}$ and $\frac{2}{5}$.
3. $\frac{1}{2}$, $\frac{4}{5}$, and $\frac{6}{7}$.
4. $\frac{2}{3}$, $\frac{3}{7}$, and $\frac{4}{11}$.
5. $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{9}{13}$.
6. $\frac{5}{7}$, $\frac{6}{11}$, and $\frac{6}{17}$.
7. $\frac{2}{6}$, $\frac{4}{9}$, $\frac{5}{10}$, and $\frac{4}{11}$.
8. $\frac{4}{7}$, $\frac{6}{8}$, $\frac{15}{40}$, $\frac{17}{11}$.
9. $\frac{35}{63}$, $\frac{40}{45}$, $\frac{75}{100}$.
10. $\frac{23}{84}$, $\frac{19}{56}$, $\frac{22}{108}$.

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153. What is a common denominator? How reduce fractions to a common denominator? *Note.* What is this rule based upon? What is to be done with mixed numbers, compound and complex fractions?

11. Find a common denominator of 4, $3\frac{1}{2}$, and $\frac{1}{3}$ of $\frac{3}{4}$

ANALYSIS.—$4=\frac{4}{1}$; $3\frac{1}{2}=\frac{7}{2}$; and $\frac{1}{3}$ of $\frac{3}{4}=\frac{1}{4}$. Reducing $\frac{4}{1}$, $\frac{7}{2}$ $\frac{1}{4}$ to a common denominator, they become $\frac{32}{8}$, $\frac{28}{8}$, and $\frac{2}{8}$.

12. Reduce $3\frac{1}{7}$, $1\frac{1}{3}$, $2\frac{1}{5}$.
13. Reduce $6\frac{1}{2}$, $7\frac{10}{11}$, $\frac{2}{13}$, $\frac{3}{4}$.
14. Reduce $\frac{2}{5}$ of $\frac{1}{7}$, $5\frac{1}{8}$, $\frac{5}{9}$.
15. Reduce $5\frac{1}{2}$, $\frac{1}{7}$ of 8, $\frac{11}{19}$.
16. Reduce $\frac{3}{4}$ of $\frac{1}{2}$ of 9, $11\frac{1}{2}$, $\frac{3}{11}$
17. Reduce $13\frac{1}{4}$, 17, $\frac{5}{7}$ of $\frac{2}{3}$.

18. Find a common denominator of $2\frac{1}{4}$, $5\frac{3}{12}$ and $2\frac{2}{3}$.

19. Reduce $\frac{3}{10}$, $\frac{2}{5}$, and $\frac{11}{20}$, to the common denominator 100.

SOLUTION.—The fraction $\frac{3}{10}=\frac{30}{100}$; $\frac{2}{5}=\frac{40}{100}$; and $\frac{11}{20}=\frac{55}{100}$.

20. Change $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{5}{9}$ to 72ds.

21. Change $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{11}{16}$ to 96ths.

22. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ and $\frac{16}{24}$ to 90ths.

23. Reduce $\frac{1}{6}$ of $10\frac{4}{5}$ and $\frac{8}{20}$, to 75ths.

24. Reduce $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{13}{24}$ to 168ths.

25. Reduce $\frac{7}{20}$, $\frac{45}{60}$, and $\frac{119}{250}$, to 1000ths.

CASE VIII.

154. To Reduce Fractions to the *Least Common Denominator.*

DEF.—A ***Common Denominator*** is a *common multiple* of all the denominators. (Art. 153, *n.*) Hence,

The ***Least Common Denominator*** is the *least common multiple* of all the denominators.

1. Reduce $\frac{2}{3}$, $\frac{5}{12}$, and $\frac{11}{15}$ to the least common denominator.

ANALYSIS.—Here are two steps: 1st. To find the *least common multiple* of the *denominators;* 2d. To *reduce* the given fractions to this *denominator.* The least common multiple of the given denominators is 60. (Art. 133.)

OPERATION.

$$\begin{array}{r} 3\,)\,\underline{3,\ 12,\ 15} \\ 1,\ \ 4,\ \ 5 \end{array}$$
$$3\times4\times5=60$$

154. What is the least common denominator? How reduce fractions to the least common denominator? *Note.* What is to be done with mixed numbers, compound and complex fractions?

To reduce the given fractions to 60ths, we multiply both terms of $\frac{2}{3}$ by 20, and it becomes $\frac{40}{60}$. In like manner, if both terms of $\frac{5}{12}$ are multiplied by 5, it becomes $\frac{25}{60}$; and multiplying both terms of $\frac{11}{15}$ by 4, it becomes $\frac{44}{60}$. Therefore, $\frac{2}{3}$, $\frac{5}{12}$, and $\frac{11}{15}$ are equal to $\frac{40}{60}$, $\frac{25}{60}$ and $\frac{44}{60}$. Hence, the

RULE.—*Find the least common multiple of all the denominators, and multiply both terms of each fraction by such a number as will reduce it to this denominator.* (Art. 150, *n.*)

NOTES.—1. *Mixed* numbers must be reduced to improper fractions; *compound* and *complex* fractions to simple ones; and all fractions to the *lowest terms*, before applying the rule. If not reduced to the lowest terms, the least common multiple of the denominators is liable not to be the least common denominator. (See Ex. 2, 18.)

2. This rule, like the preceding, is based upon the principle that *multiplying both terms* of a fraction by the same number does not *alter* its value. (Art. 144, Prin. 9.)

2. Reduce 15, $2\frac{1}{3}$, $\frac{2}{3}$ of $\frac{3}{5}$, and $\frac{6}{12}$ to the least common denominator.

ANALYSIS.—$15 = \frac{15}{1}$, $2\frac{1}{3} = \frac{7}{3}$, $\frac{2}{3}$ of $\frac{3}{5} = \frac{2}{5}$, and $\frac{6}{12} = \frac{1}{2}$.

Now the least common multiple of the denominators of $\frac{15}{1}, \frac{7}{3}, \frac{2}{5}, \frac{1}{2}$, is 30. *Ans.* $\frac{450}{30}, \frac{70}{30}, \frac{12}{30}, \frac{15}{30}$.

Reduce the following to least common denominator:

3. $\frac{2}{3}, \frac{3}{4}, \frac{5}{7}$.
4. $\frac{1}{4}, \frac{5}{20}, \frac{3}{12}$.
5. $\frac{2}{5}, \frac{4}{6}, \frac{10}{18}$.
6. $\frac{3}{7}, \frac{2}{9}, 4\frac{1}{2}$.
7. $\frac{7}{10}, \frac{9}{12}, \frac{4}{5}$ of $12\frac{1}{2}$.
8. $\frac{5}{12}, \frac{7}{8}, \frac{2}{3}$ of 10.
9. $\frac{5}{4}, \frac{14}{35}, 3\frac{1}{2}, 8\frac{3}{4}$.
10. $\frac{9}{10}$ of $8\frac{2}{5}$, $\frac{7}{8}$ of 40.
11. $\frac{8}{40}, \frac{6}{30}, \frac{14}{56}, 5\frac{2}{8}$.
12. $\frac{3}{7}, \frac{2}{3}, \frac{1}{2}, \frac{4}{5}, \frac{6}{7}$.
13. $9\frac{2}{3}, 11\frac{3}{4}, \frac{5}{7}$ of 40.
14. $\frac{4}{10}$ of 13, $\frac{25}{100}, \frac{42}{120}$.
15. $7\frac{3}{4}, \frac{8}{12}$ of 17, $\frac{44}{60}$.
16. $\frac{49}{294}, \frac{250}{750}, \frac{1260}{5040}$.
17. $\frac{113}{256}, \frac{269}{512}, \frac{1447}{5120}$.
18. $\frac{150}{375}, \frac{265}{400}, \frac{1728}{3456}$.

ADDITION OF FRACTIONS.

155. ***Addition of Fractions*** embraces *two classes* of examples, viz.: those which have a *common denominator* and those which have *different denominators.*

156. When *two* or *more* fractions have a *common denominator,* and refer to the *same kind* of unit or base, their *numerators* are *like parts* of that unit or base, and therefore are *like numbers.* Hence, they may be *added, subtracted,* and *divided* in the same manner as whole numbers. Thus, $\frac{5}{8}$ and $\frac{7}{8}$ are 12 eighths, just as 5 yards and 7 yards are 12 yards. (Art. 101, Def. 13.)

157. When two or more fractions have *different denominators,* their numerators are *unlike* parts, and therefore cannot be *added to* or *subtracted from* each other *directly,* any more than yards and dollars. (Art. 101, Def. 14.)

158. To add Fractions which have a *Common Denominator.*

1. What is the sum of $\frac{3}{8}$ yard, $\frac{5}{8}$ yard, and $\frac{7}{8}$ yard?

ANALYSIS.—3 eighths and 5 eighths yard are 8 eighths, and 7 are 15 eighths, equal to $1\frac{7}{8}$ yard. (Art. 147.) For, since the given fractions have a common denominator, their numerators are *like numbers.* Hence, the

OPERATION.

$$\tfrac{3}{8}+\tfrac{5}{8}+\tfrac{7}{8}=\tfrac{15}{8}, \text{ or } 1\tfrac{7}{8} \text{ y.}$$

RULE.—*Add the numerators, and place the sum over the common denominator.*

NOTE.—The answers should be reduced to the *lowest terms*; and if improper fractions, to *whole* or *mixed* numbers.

Add the following fractions.

2. $\frac{6}{7}$, $\frac{5}{7}$, and $\frac{1}{7}$.
3. $\frac{5}{12}$, $\frac{7}{12}$, $\frac{1}{12}$, and $\frac{11}{12}$.
4. $\frac{7}{20}$, $\frac{3}{20}$, $\frac{9}{20}$, and $\frac{11}{20}$.
5. $\frac{17}{100}$, $\frac{29}{100}$, $\frac{62}{100}$, and $\frac{112}{100}$.
6. $\frac{47}{144}$, $\frac{63}{144}$, $\frac{116}{144}$, and $\frac{265}{144}$.
7. $\frac{164}{256}$, $\frac{117}{256}$, $\frac{210}{256}$, and $\frac{121}{256}$.

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158. How add fractions that have a common denominator?

159. To add Fractions which have *Different Denominators.*

8. Find the sum of $\frac{1}{4}$ of a pound, $\frac{2}{5}$ of a pound, and $\frac{5}{8}$ of a pound.

ANALYSIS.—As these fractions have *different* denominators, their numerators cannot be added in their present form. (Art. 157.) We therefore reduce them to a *common denominator*, which is 160. Then $\frac{1}{4}=\frac{40}{160}$, $\frac{4}{5}=\frac{64}{160}$, and $\frac{5}{8}=\frac{100}{160}$. Adding the numerators, and placing the sum over the common denominator, we have $\frac{204}{160}$, or $1\frac{11}{40}$, the answer required. For, reducing fractions to a common denominator does not alter their value; and when reduced to a common denominator, the numerators are *like numbers.* (Arts. 153, 156.)

OPERATION.

$4\times5\times8=160$, com. d.
$1\times5\times8=\ 40$, 1st nu.
$2\times4\times8=\ 64$, 2d "
$5\times4\times5=100$, 3d "
Sum of nu., 204. Hence,

$$\frac{40}{160}+\frac{64}{160}+\frac{100}{160}=\frac{204}{160}, \text{ or } 1\frac{11}{40}$$

Or, we may reduce them to the *least common denominator*, which is 40, and then add the numerators. (Art. 154.) Hence, the

RULE.—*Reduce the fractions to a common denominator, and place the sum of the numerators over it.*

Or, reduce the fractions to the least common denominator, and over this place the sum of the numerators.

REMARKS.—1. The *fractional* and *integral* parts of mixed numbers should be added separately, and the results be united.

Or, *whole* and *mixed* numbers may be reduced to *improper fractions*, then be added by the rule. (Art. 148.)

2. *Compound* and *complex* fractions must be reduced to *simple* ones; then proceed according to the rule. (Ex. 20, 28.)

(For Addition of Denominate Fractions, see Art. 315.)

9. What is the sum of 5, $2\frac{1}{3}$, and $10\frac{3}{4}$?

ANALYSIS.—Reducing these fractions to a common denominator 12, we have, $5=5$; $2\frac{1}{3}=2\frac{4}{12}$; and $10\frac{3}{4}=10\frac{9}{12}$. Adding the numerators, 4 twelfths and 9 twelfths are $\frac{13}{12}=1\frac{1}{12}$. 1 and 10 are 11 and 2 are 13 and 5 are 18. *Ans.* $18\frac{1}{12}$. Or, $5=\frac{5}{1}$ or $\frac{60}{12}$; $2\frac{1}{3}=\frac{28}{12}$; and $10\frac{3}{4}=\frac{129}{12}$. Now $\frac{60}{12}+\frac{28}{12}+\frac{129}{12}=\frac{217}{12}$, or $18\frac{1}{12}$ *Ans.*

$5=5$
$2\frac{1}{3}=2\frac{4}{12}$
$10\frac{3}{4}=10\frac{9}{12}$
Ans. $18\frac{1}{12}$

159. How when they have not? *Note.* How add whole and mixed numbers? Compound and complex fractions?

Add the following:

(10.)	(11.)	(12.)	(13.)	(14.)
$2\frac{7}{8}$	$4\frac{2}{3}$	$27\frac{4}{10}$	$\frac{19}{20}$	$\frac{23}{50}$
$1\frac{5}{9}$	$5\frac{3}{4}$	$8\frac{3}{24}$	$\frac{47}{60}$	$\frac{37}{75}$
$3\frac{8}{12}$	$11\frac{5}{6}$	$46\frac{5}{8}$	$\frac{53}{84}$	$\frac{87}{100}$

(15.)	(16.)	(17.)	(18.)	(19.)
$19\frac{1}{4}$	$24\frac{5}{7}$	$207\frac{3}{4}$	$175\frac{5}{8}$	450
$47\frac{1}{2}$	$10\frac{31}{35}$	$62\frac{3}{5}$	207	$67\frac{5}{24}$
$68\frac{2}{3}$	$68\frac{5}{14}$	$49\frac{9}{10}$	$368\frac{7}{9}$	$37\frac{47}{60}$

20. What is the sum of $\frac{1}{3}$ of $\frac{3}{4}$, $\frac{5}{6}$ of 7, and $\frac{1}{2}$ of $9\frac{3}{8}$?

21. What is the sum of $\frac{3}{8}$ of $\frac{5}{7}$, $\frac{3}{5}$ of $\frac{5}{9}$, and $\frac{3}{4}$ of 7?

22. What is the sum of $\frac{5}{9}$ of $4\frac{1}{2}$, $\frac{5}{8}$ of 3, and $\frac{7}{8}$ of $10\frac{1}{2}$?

23. A beggar received $\$1\frac{3}{5}$ from one person, $\$2\frac{1}{4}$ from another, and $\$3\frac{3}{4}$ from another: how much did he receive from all?

24. If a man lays up $\$43\frac{5}{8}$ a month, and his son $\$27\frac{3}{4}$, how much will both save?

25. What is the sum of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{11}{20}$, and $\frac{4}{5}$ pound?

26. A shopkeeper sold $17\frac{3}{4}$ yards of muslin to one customer, $8\frac{1}{2}$ yards to another, $25\frac{1}{4}$ to another: how many yards did he sell to all?

27. A farmer paid $\$18\frac{3}{4}$ for hay, $\$45\frac{3}{18}$ for a cow, $\$150\frac{3}{4}$ for a horse, and $\$275$ for a buggy: how much did he give for all?

28. What is the sum of $\frac{4\frac{1}{2}}{5}$, $\frac{5\frac{1}{4}}{3}$, and $\frac{\frac{2}{3}}{4}$.

ANALYSIS.—Reducing the complex fractions to simple ones, we have, $\frac{4\frac{1}{2}}{5}=\frac{9}{2}\div 5=\frac{9}{10}$; $\frac{5\frac{1}{4}}{3}=\frac{21}{4}\div 3=\frac{7}{4}$; and $\frac{\frac{2}{3}}{4}=\frac{2}{3}\div 4=\frac{2}{12}$, or $\frac{1}{6}$. Now, $\frac{9}{10}+\frac{7}{4}+\frac{1}{6}=\frac{54}{60}+\frac{105}{60}+\frac{10}{60}=2\frac{49}{60}$, *Ans.*

29. Add $\frac{\frac{48}{4}}{2}$, $\frac{6\frac{6}{35}}{3}$, and $\frac{7\frac{21}{25}}{7}$. 30. Add $\frac{2\frac{2}{9}}{5}$, $\frac{\frac{140}{5}}{7}$, and $\frac{6\frac{3}{5}}{3}$.

SUBTRACTION OF FRACTIONS.

160. ***Subtraction of Fractions*** embraces *two classes* of examples, viz.: those which have a *common denominator,* and those which have *different denominators.*

161. To subtract Fractions which have a *Common Denominator.*

1. What is the difference between $\frac{15}{16}$ and $\frac{13}{16}$?

ANALYSIS.—13 sixteenths from 15 sixteenths leave 2 sixteenths, the answer required. For, since the given fractions have a common denominator, their numerators, we have seen, are *like numbers.* (Art. 156.) Hence, the

OPERATION.

$$\frac{15}{16} - \frac{13}{16} = \frac{2}{16}.$$

RULE.—*Take the less numerator from the greater, and place the difference over the common denominator.*

2. From $\frac{25}{40}$ take $\frac{19}{40}$.
3. From $\frac{63}{75}$ take $\frac{49}{75}$.
4. From $\frac{121}{263}$ take $\frac{85}{263}$.
5. From $\frac{271}{435}$ take $\frac{169}{435}$
6. From $\frac{568}{804}$ take $\frac{205}{804}$.
7. From $\frac{739}{1000}$ take $\frac{504}{1000}$.

162. To subtract Fractions which have *Different Denominators.*

8. From $\frac{6}{7}$ of a pound, subtract $\frac{3}{4}$ of a pound.

ANALYSIS.—Since these fractions have *different* denominators, their numerators cannot be subtracted in their present form. We therefore reduce them to a *common* denominator, which is 28; then subtracting as above, have $\frac{3}{28}$, the answer required. (Art. 153.) Hence, the

$$\frac{6}{7} = \frac{24}{28}$$
$$\frac{3}{4} = \frac{21}{28}$$
$$Ans.\ \frac{3}{28}$$

RULE.—*Reduce the fractions to a common denominator, and over it place the difference of the numerators.*

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161. How subtract fractions that have a common denominator? 162. How when they have different denominators? *Rem.* How subtract mixed numbers? Compound and complex fractions? A proper fraction from a whole number?

REMARKS.—1. The *fractional* and *integral* parts of mixed numbers should be subtracted separately, and the results be united.

Or, they may be reduced to improper fractions, then apply the rule.

Compound and *complex* fractions should be reduced to *simple* ones, and all fractions to their *lowest terms*.

2. A *proper fraction* may be subtracted from a *whole* number by taking it from a *unit;* then annex the remainder to the whole number *minus* 1.

Or, the *whole number* may be reduced to a fraction of the *same denominator* as that of the given fraction; then subtract according to the rule. (Art. 151.)

3. The operation may often be shortened by finding the *least common denominator* of the given fractions. (Art. 154.)

(For subtraction of Denominate Fractions, see Art. 317.)

9. What is the difference between $12\frac{1}{2}$ pounds and $5\frac{3}{4}$ pounds?

ANALYSIS.—The minuend $12\frac{1}{2}=12\frac{2}{4}$. Now $12\frac{2}{4}-5\frac{3}{4}=6\frac{3}{4}$, *Ans.* Or, $12\frac{1}{2}=\frac{25}{2}$, or $\frac{50}{4}$; and $5\frac{3}{4}=\frac{23}{4}$. Now $\frac{50}{4}-\frac{23}{4}=\frac{27}{4}$, which reduced to a mixed number, equals $6\frac{3}{4}$, the same as before.

	(10.)	(11.)	(12.)	(13.)	(14.)
From	$\frac{7}{8}$	$5\frac{2}{3}$	$7\frac{1}{4}$	$23\frac{4}{5}$	$\frac{43}{50}$
Take	$\frac{4}{9}$	$3\frac{1}{2}$	$5\frac{2}{3}$	$15\frac{3}{7}$	$\frac{37}{75}$

	(15.)	(16.)	(17.)	(18.)	(19.)
From	$5\frac{25}{38}$	$7\frac{31}{49}$	$6\frac{39}{55}$	$8\frac{64}{75}$	$9\frac{68}{96}$
Take	$3\frac{14}{43}$	$4\frac{17}{63}$	$3\frac{23}{46}$	$5\frac{33}{90}$	$7\frac{37}{108}$

20. From a box containing $56\frac{1}{2}$ pounds of sugar, a grocer took out $23\frac{3}{4}$ pounds: how many pounds were left in the box?

21. From a farm containing $165\frac{9}{10}$ acres, the owner sold $78\frac{3}{4}$ acres: how much land had he left?

22. From 13 subtract $\frac{5}{7}$.

ANALYSIS.—1st. Reducing 13 to sevenths, we have $13=\frac{91}{7}$, and $\frac{91}{7}-\frac{5}{7}=\frac{86}{7}$, or $12\frac{2}{7}$ *Ans.*

Or, 2d. Borrowing 1 from 13, and reducing it to 7ths, we have $13=12\frac{7}{7}$, and $12\frac{7}{7}-\frac{5}{7}=12\frac{2}{7}$, *Ans.*

23. From 46 take $7\frac{1}{2}$.
24. From 58 take $20\frac{3}{4}$.
25. From $84\frac{3}{7}$ take 41.
26. From $150\frac{5}{9}$ take 83.
27. From 110 take $7\frac{29}{125}$.
28. From 1000 take $999\frac{45}{60}$.

29. Subtract $\frac{1}{3}$ of $\frac{1}{4}$ of 3 from $\frac{1}{2}$ of $\frac{4}{5}$ of $1\frac{1}{2}$.

ANALYSIS.—Reducing the compound fractions to simple ones, then to a common denominator, the subtrahend becomes $\frac{5}{20}$, the minuend $\frac{12}{20}$. Now $\frac{12}{20}-\frac{5}{20}=\frac{7}{20}$. *Ans.*

$$\frac{1}{2} \text{ of } \frac{4}{5} \text{ of } 1\frac{1}{2} = \frac{1}{2} \text{ of } \frac{4}{5} \text{ of } \frac{3}{2} = \frac{3}{5} = \frac{12}{20}$$

$$\frac{1}{3} \text{ of } \frac{1}{4} \text{ of } 3 = \frac{1}{3} \text{ of } \frac{1}{4} \text{ of } \frac{3}{1} = \frac{1}{4} = \frac{5}{20}$$

$$\textit{Ans.}\ \frac{7}{20}.$$

	(30.)	(31.)	(32.)	(33.)
From	$\frac{2}{3}$ of $\frac{5}{8}$	$\frac{5}{9}$ of $\frac{11}{20}$	$\frac{6}{11}$ of $4\frac{1}{2}$	$\frac{7}{24}$ of 28
Take	$\frac{1}{4}$ of $\frac{2}{3}$	$\frac{2}{7}$ of $\frac{5}{12}$	$\frac{4}{7}$ of 3	$\frac{5}{9}$ of $4\frac{1}{5}$.

34. From $\frac{18}{20}$ of $62\frac{1}{2}$ subtract $\frac{3}{15}$ of $16\frac{1}{4}$.

35. A's farm contains $256\frac{5}{8}$ acres, B's $431\frac{3}{10}$ acres: what is the difference in the size of their farms?

36. If from a vessel containing $230\frac{11}{16}$ tons of coal, $119\frac{1}{2}$ tons are taken, how much will there be left?

37. A man bought a cask of syrup containing $58\frac{1}{2}$ gallons; on reaching home he found it contained only $17\frac{5}{8}$ gallons: how many gallons had leaked out?

38. A lady having \$100, paid $\$8\frac{1}{4}$ for a pocket handkerchief, $\$15\frac{1}{2}$ for a dress hat, $\$46\frac{5}{8}$ for a cloak: how much had she left?

39. From $\dfrac{5\frac{1}{2}}{6}$ subtract $\dfrac{1\frac{1}{3}}{5}$.

ANALYSIS.—Reducing the complex fractions to simple ones, then to a common denominator, the minuend becomes $\frac{55}{60}$, the subtrahend $\frac{16}{60}$. (Arts. 152, 153.) Now, $\frac{55}{60}-\frac{16}{60}=\frac{39}{60}$, or $\frac{13}{20}$, *Ans.*

$$\frac{5\frac{1}{2}}{6}=\frac{11}{2}\div 6=\frac{11}{12}=\frac{55}{60}$$

$$\frac{1\frac{1}{3}}{5}=\frac{4}{3}\div 5=\frac{4}{15}=\frac{16}{60}$$

$$\textit{Ans.}\ \frac{39}{60}=\frac{13}{20}$$

40. From $\dfrac{11\frac{1}{4}}{5}$ take $\dfrac{4\frac{1}{2}}{8}$.

41. From $\dfrac{5\frac{1}{7}}{3}$ take $\dfrac{5\frac{1}{2}}{22}$.

MULTIPLICATION OF FRACTIONS.

CASE I.

163. **To multiply a *Fraction* by a *Whole Number.***

Ex. 1. What will 4 pounds of tea cost, at $\frac{5}{8}$ of a dollar a pound?

1st METHOD.—Since 1 pound costs $\$\frac{5}{8}$, 4 pounds will cost 4 times as much, and 4 times $\$\frac{5}{8}$ are $\frac{20}{8}=\$2\frac{1}{2}$, which is the answer required. For, *multiplying* the *numerator, multiplies* the fraction. (Art. 144, Prin. 5.)

1st OPERATION.

$\$\frac{5}{8}\times 4=\$\frac{20}{8}$
and $\$\frac{20}{8}=\$2\frac{1}{2}$

2d METHOD.—If we divide the given denominator by the given number of pounds, we have $\frac{5}{8}\div 4=\$\frac{5}{2}$, or $\$2\frac{1}{2}$, which is the true answer. For, *dividing* the *denominator, multiplies* the fraction. (Art. 144, Prin. 8.) Hence, the

2d OPERATION.

$\$\frac{5}{8}\div 4=\$\frac{5}{2}$, or $\$2\frac{1}{2}$

RULE.—*Multiply the numerator by the whole number.*

Or, *divide the denominator by it.*

REMARKS.—1. When the *multiplicand* is a *mixed number*, the *fractional* and *integral* parts should be multiplied separately, and the results be united.

Or, the *mixed number* may be reduced to an *improper fraction*, and then be multiplied as above.

2. If a fraction is multiplied by its *denominator*, the *product* will be *equal* to its *numerator*. For, the numerator is both multiplied and divided by the *same* number. (Art. 86, *a*.) Hence,

3. A fraction is *multiplied* by a number *equal* to its *denominator* by *cancelling* the denominator. Thus, $\frac{5}{8}\times 8=5$. (Art. 121.)

4. In like manner, a fraction is *multiplied* by *any factor* of its denominator by *cancelling* that factor.

2. Multiply $27\frac{3}{5}$ by 6.

ANALYSIS.—Multiplying the fraction and integer separately, we have 6 times $\frac{3}{5}=\frac{18}{5}$, or $3\frac{3}{5}$; and 6 times $27=162$. Now $162+3\frac{3}{5}=165\frac{3}{5}$, the answer.

Or, thus: $27\frac{3}{5}=\frac{138}{5}$; and $\frac{138}{5}\times 6=\frac{828}{5}$, or $165\frac{3}{5}$, *Ans.*

OPERATION.

$$\begin{array}{r} 27\frac{3}{5} \\ 6 \\ \hline \textit{Ans. } 165\frac{3}{5} \end{array}$$

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163. How multiply a fraction by a whole number? Upon what does the first method depend? The second? *Rem.* How multiply a mixed number by a whole one? How multiply a fraction by a number equal to its denominator? How by any factor in its denominator?

	(3.)	(4.)	(5.)	(6.)	(7.)
Mult.	$\frac{20}{56}$	$\frac{34}{43}$	$23\frac{2}{3}$	$35\frac{3}{4}$	$48\frac{3}{5}$
By	7	9	8	10	12

	(8.)	(9.)	(10.)	(11.)	(12.)
Mult.	$\frac{63}{84}$	$\frac{97}{35}$	$\frac{423}{685}$	$98\frac{7}{10}$	$85\frac{7}{23}$
By	48	100	78	26	48

13. Multiply $\frac{275}{239}$ by 239.

14. What cost 8 barrels of cider, at $\$7\frac{3}{5}$ a barrel?
15. At $\$25\frac{3}{4}$ each, what will 9 chests of tea cost?
16. What will 25 cows come to, at $\$48\frac{3}{4}$ apiece?
17. What cost 27 tons of hay, at $\$29\frac{4}{5}$ a ton?
18. What cost 35 acres of land, at $\$45\frac{1}{2}$ per acre?
19. At $\$34\frac{3}{4}$ apiece, what cost 50 hogsheads of sugar.
20. At $\$45\frac{3}{8}$ apiece, what will 100 coats come to?
21. What cost 6 dozen muffs, at $\$75\frac{1}{4}$ apiece?

CASE II.

165. **To Multiply a *Whole Number* by a *Fraction*.**

DEF. ***Multiplying by a Fraction*** is taking a *certain part* of the multiplicand as many times as there are *like parts* of a *unit* in the multiplier.

But to find a *given part* of a number, we divide it into as many equal parts as there are *units* in the denominator, and then take as many of these parts as are indicated by the numerator. (Art. 143.) That is,

To multiply a number by $\frac{1}{2}$, *divide* it by 2.

To multiply a number by $\frac{1}{3}$, *divide* it by 3.

To multiply a number by $\frac{3}{4}$, *divide* it by 4 for $\frac{1}{4}$, and multiply this quotient by 3 for $\frac{3}{4}$, etc. Hence,

REMARKS.—1. *Multiplying a whole number* by a *fraction* is the same as *finding a fractional part* of a number, which the pupil should here review with care. (Art. 143.)

Def. What is it to multiply by a fraction? How multiply by $\frac{1}{2}$? By $\frac{1}{3}$? By $\frac{3}{4}$?

2. When the *multiplier* is 1, the *product* is *equal* to the *multiplicand.*

When the multiplier is *greater* than 1, the product is *greater* than the multiplicand.

When the multiplier is *less* than 1, the product is *less* than the multiplicand.

1. What will $\frac{3}{4}$ of a ton of iron cost, at \$55 a ton?

1st METHOD.—Since 1 ton costs \$55, $\frac{3}{4}$ of a ton will cost $\frac{3}{4}$ times \$55, or $\frac{3}{4}$ of \$55. But $\frac{3}{4}$ of \$55 = 3 times $\frac{1}{4}$ of \$55. Now $\frac{1}{4}$ of \$55=\$55÷4, or \$13$\frac{3}{4}$; and 3 times \$13$\frac{3}{4}$=\$41$\frac{1}{4}$, the answer required. For, by definition, multiplying by a *fraction* is taking a *part* of the multiplicand as *many times* as there are *like parts of a unit* in the multiplier. Now dividing the whole number by 4, takes 1 fourth of the multiplicand *once;* and multiplying this result by 3, *repeats* this part 3 times, as indicated by the multiplier. (Art. 143.)

OPERATION.

$$\begin{array}{r} \$4)\underline{55} \\ \$13\frac{3}{4} \\ \underline{3} \\ Ans.\ \$41\frac{1}{4} \end{array}$$

2d METHOD.—$\frac{3}{4}$ of \$55=$\frac{1}{4}$ of 3 times \$55. But \$55 × 3 = \$165, and \$165÷4=\$41$\frac{1}{4}$, the same as before. For, $\frac{1}{4}$ of 3 times a number is the same as 3 times $\frac{1}{4}$ of it. Hence, the

$$\begin{array}{r} \$55 \\ \underline{3} \\ 4)\underline{165} \\ Ans.\ \$41\frac{1}{4} \end{array}$$

RULE.—*Divide the whole number by the denominator of the fraction, and multiply by the numerator.*

Or, *Multiply the whole number by the numerator of the fraction, and divide by the denominator.*

REMARKS.—1. The *fraction* may be taken for the *multiplicand,* and the *whole number* for the *multiplier,* at pleasure, without affecting the result. (Art. 45.)

2. When the multiplier is a *mixed number,* multiply by the fractional and integral parts separately, and unite the results.

Or, reduce it to an *improper fraction;* then proceed according to the rule. (Ex. 2.)

2. What will 8$\frac{3}{5}$ tons of copper ore cost, at \$365 a ton?

ANALYSIS.—8$\frac{3}{5}$ = $\frac{43}{5}$; now $\frac{43}{5}$ tons will come to $\frac{43}{5}$ times \$365; and \$365 × $\frac{43}{5}$=\$3139. *Ans.*

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165. How is a whole number multiplied by a fraction? *Rem.* What is the operation like? When the multiplier is 1, what is the product? When the multiplier is greater than 1, what? When less, what? *Rem.* How multiply a whole by a mixed number?

	(3.)	(4.)	(5.)	(6.)	(7.)
Mult.	65	89	96	87	100
By	$\frac{3}{8}$	$\frac{3}{10}$	$3\frac{1}{2}$	$4\frac{2}{3}$	$5\frac{3}{7}$

8. What cost $\frac{7}{10}$ of an acre of land, at $38 an acre?
9. At $29 a ton, what cost $8\frac{5}{8}$ tons of hay?
10. What cost $28\frac{5}{6}$ tons of lead, at $223 per ton?

11. What is $\frac{41}{63}$ of 845?
12. What is $\frac{75}{100}$ of 1876?
13. What is $\frac{56}{87}$ of 1000?
14. What is $\frac{125}{200}$ of 2010?
15. Multiply 79 by $7\frac{3}{4}$.
16. Multiply 103 by $9\frac{4}{5}$.
17. Multiply 1001 by $21\frac{7}{10}$.
18. Multiply 1864 by $37\frac{5}{9}$.

CASE III.

166. **To multiply a *Fraction* by a *Fraction*.**

1. What will $\frac{5}{6}$ of a pound of tea cost, at $\$\frac{9}{10}$ a pound?

ANALYSIS.—$\frac{1}{6}$ of a pound will cost 1 sixth as much as 1 pound; and $\frac{1}{6}$ of $\$\frac{9}{10}$ is $\$\frac{9}{60}$. Again, $\frac{5}{6}$ of a pound will cost 5 times as much as $\frac{1}{6}$; and 5 times $\$\frac{9}{60}$ are $\$\frac{45}{60}=\$\frac{3}{4}$, the answer required.

OPERATION.

$\frac{9}{10} \times \frac{5}{6} = \frac{45}{60}$, or $\$\frac{3}{4}$

BY CANCELLATION.

$$\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} = \$\frac{3}{4}$$

Or, having indicated the multiplication, *cancel* the *common factors* 3 and 5, and then multiply the *numerators* together and the *denominators*; the result is $\$\frac{3}{4}$, the same as before

The reason is this: Multiplying the denominator of the multiplicand $\frac{9}{10}$ by 6 the denominator of the multiplier, the result, $\$\frac{9}{60}$, is 5 times *too small;* for, we have multiplied by $\frac{1}{6}$ instead of $\frac{5}{6}$ of a unit, the true multiplier. To correct this, we must *multiply this result* by 5, which is done by multiplying its numerator by 5. (Art. 144.)

The object of *cancelling common factors* is twofold: it *shortens* the operation, and gives the *answer* in the *lowest* terms. Hence, the

RULE.—*Cancel the common factors; then multiply the numerators together for the new numerator, and the denominators for the new denominator.* (Art. 144, Prin. 9.)

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166. How multiply a fraction by a fraction? Object of cancelling? *Rem.* How multiply compound fractions? Mixed numbers? Complex fractions?

REMARKS.—1. *Mixed numbers* should be reduced to *improper* fractions, and complex fractions to simple ones; then apply the rule.

2. *Multiplying* by a fraction, *reducing* a *compound* fraction to a *simple* one, and *finding* a *fractional part* of a number, are *identical* operations. (Arts. 143, 149, 166.)

2. Multiply $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ by $\frac{1}{2}$ of $\frac{4}{7}$.

SOLUTION.—It is immaterial as to the result, whether the fractions are arranged in a horizontal or in a perpendicular line, with the numerators on the *right* and the denominators on the *left*. (Art. 122, *Rem.*)

$$\begin{array}{r|l} \not3 & \not2 \\ \not4 & \not3 \\ 6 & 5 \\ \not2 & 1 \\ 7 & \not4 \\ \hline 42 & 5 = \frac{5}{42} \end{array}$$

Perform the following multiplications:

3. $\frac{5}{7} \times \frac{7}{9}$	7. $\frac{13}{24} \times \frac{8}{13}$	11. $\frac{3}{5} \times \frac{2}{3} \times \frac{5}{7}$
4. $\frac{8}{10} \times \frac{5}{16}$	8. $\frac{11}{17} \times \frac{51}{77}$	12. $\frac{5}{8} \times \frac{2}{10} \times \frac{3}{7}$
5. $\frac{10}{12} \times \frac{24}{3}$	9. $\frac{17}{20} \times \frac{7}{11}$	13. $\frac{12}{30} \times \frac{45}{50} \times \frac{10}{42}$
6. $\frac{11}{13} \times \frac{3}{7}$	10. $\frac{21}{45} \times \frac{9}{11}$	14. $\frac{7}{28} \times \frac{13}{17} \times \frac{35}{63}$

15. Multiply $\frac{4}{5}$ of $\frac{3}{7}$ of 25 by $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{3}{4}$.

16. Multiply $\frac{5}{8}$ of $\frac{6}{15}$ of $33\frac{1}{3}$ by $\frac{3}{5}$ of $\frac{10}{12}$ of $18\frac{3}{4}$.

17. If 1 quart of cherries costs $\frac{1}{2}$ of $\frac{2}{3}$ of 45 cents, what will $\frac{3}{4}$ of $\frac{8}{10}$ of a quart cost?

18. What will $5\frac{1}{3}$ barrels of chestnuts cost, at $6\frac{3}{4}$ dollars a barrel? *Ans.* $36.

	(19.)	(20.)	(21.)	(22.)
Mult.	$24\frac{1}{8}$	$37\frac{1}{2}$	$62\frac{1}{2}$	$165\frac{3}{4}$
By	$8\frac{3}{7}$	$18\frac{3}{4}$	$31\frac{1}{4}$	$92\frac{5}{8}$

23. What is product of $\frac{5\frac{1}{4}}{6}$ multiplied by $\frac{2\frac{2}{3}}{3}$.

ANALYSIS.—$\frac{5\frac{1}{4}}{6} = \frac{21}{4} \div 6 = \frac{21}{24}$, and $\frac{2\frac{2}{3}}{3} = \frac{8}{3} \div 3 = \frac{8}{9}$.

Cancelling common factors, we have $\frac{7,\not{21}}{3,\not{24}} \times \frac{\not8}{\not9,3} = \frac{7}{3 \times 3} = \frac{7}{9}$, *Ans.*

24. Multiply $\frac{9\frac{1}{3}}{7}$ by $\frac{2\frac{1}{2}}{10}$.

25. Multiply $\frac{\frac{54}{10}}{9}$ by $\frac{5\frac{1}{3}}{12}$.

167. The preceding principles may be reduced to the following

GENERAL RULE.

I. *Reduce whole and mixed numbers to improper fractions, and complex fractions to simple ones.*

II. *Cancel the common factors, and place the product of the numerators over the product of the denominators.*

EXAMPLES.

1. What cost $\frac{3}{4}$ of a yard of calico, at $\$\frac{1}{6}$ a yard?
2. What cost $\frac{4}{5}$ of a pound of nutmegs, at $\$\frac{5}{8}$ a pound?
3. At $\$\frac{10}{12}$ a gallon, what will $\frac{3}{4}$ of a gallon of molasses cost?
4. Multiply $23\frac{7}{8}$ by $6\frac{1}{4}$.
5. Multiply $42\frac{2}{11}$ by $8\frac{4}{5}$.
6. Multiply $65\frac{5}{8}$ by $9\frac{1}{7}$.
7. Multiply $91\frac{4}{5}$ by $43\frac{3}{7}$.
8. Multiply $164\frac{3}{8}$ by $75\frac{3}{10}$.
9. Multiply $200\frac{5}{9}$ by $86\frac{7}{20}$.
10. What is the product of $\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{8}{9}$ into $\frac{5}{7}$ of $\frac{6}{11}$ of $\frac{4}{10}$?
11. What is the product of $\frac{2}{7}$ of $3\frac{1}{2}$ into $\frac{3}{5}$ of $19\frac{1}{4}$?
12. When freight is $\$1\frac{7}{8}$ per hundred, what will it cost to transport 27 hundredweight of goods from New York to Chicago?
13. What will $16\frac{1}{2}$ quarts of strawberries come to, at $26\frac{1}{4}$ cents per quart?
14. At $\$3\frac{4}{5}$ a barrel, what will be the cost of $47\frac{2}{3}$ barrels of cider?
15. Bells are composed of $\frac{1}{5}$ tin and $\frac{4}{5}$ copper: how many pounds of each metal does a church-bell contain which weighs 3750 pounds?
16. What will 45 men earn in $15\frac{1}{2}$ days, if each earns $\$2\frac{3}{4}$ per day?
17. How many feet of boards will a fence $603\frac{9}{10}$ rods long require, allowing $74\frac{1}{4}$ feet of boards to a rod?

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167. **What is the general rule for multiplying fractions?**

DIVISION OF FRACTIONS.

CASE I.

168. To divide a *Fraction* by a *Whole Number.*

1. If 4 yards of flannel cost $\$\frac{8}{9}$, what will 1 yard cost?

1st Method.—1 yard will cost 1 fourth as much as 4 yards; and dividing the numerator by 4, we have $\$\frac{8}{9} \div 4 = \$\frac{2}{9}$, the answer required. For, dividing the numerator by any number, divides the fraction by that number.

1st method.

$$\frac{8 \div 4}{9} = \$\tfrac{2}{9} \text{ } Ans.$$

2d Method.—Multiplying the denominator by the divisor 4 (the number of yards), the result is $\frac{8}{36} = \frac{2}{9}$, the same as before. For, *multiplying* the *denominator* by any number, *divides* the fraction by that number. (Art. 144, Prin. 7.) Hence, the

2d method.

$$\frac{8}{9 \times 4} = \frac{8}{36}$$

$$Ans. \ \frac{8}{36} = \$\tfrac{2}{9}$$

Rule.—*Divide the numerator by the whole number.* Or, *multiply the denominator by it.*

Remark.—When the *dividend* is a mixed number, it should be reduced to an *improper fraction;* then apply the rule.

2. Divide $\$\frac{5}{7}$ by 9. *Ans.* $\$\frac{5}{7} \div 9 = \$\frac{5}{63}$.

Perform the following divisions:

3. $\frac{16}{29} \div 4$.	7. $\frac{150}{87} \div 25$.	11. $\frac{4060}{1327} \div 20$.
4. $\frac{22}{30} \div 9$.	8. $\frac{268}{121} \div 67$.	12. $\frac{52326}{12345} \div 10$.
5. $\frac{49}{23} \div 21$.	9. $\frac{350}{119} \div 70$.	13. $\frac{60045}{45631} \div 15$.
6. $\frac{88}{30} \div 33$.	10. $\frac{488}{261} \div 81$.	14. $\frac{101920}{111111} \div 91$.

15. A man having $\frac{7}{8}$ of a barrel of flour, divided it equally among 5 persons: what part of a barrel did each receive?

16. If 12 oranges cost $\$\frac{84}{100}$, what will 1 orange cost?

17. If 7 writing-books cost $\$\frac{8}{9}$, what will that be apiece?

18. If 6 barrels of flour cost $\$45\frac{3}{4}$, what will 1 barrel cost?

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168. How divide a fraction by a whole number? Upon what does the first method depend? The second? Which is preferable? *Rem.* How is a mixed number divided by a whole one?

Perform the following divisions:

19. $80\frac{4}{13} \div 12$.
20. $76\frac{7}{29} \div 14$.
21. $28\frac{4}{7} \div 20$.
22. $51\frac{2}{3} \div 27$.
23. $865\frac{3}{15} \div 82$.
24. $490\frac{4}{20} \div 40$.
25. $758\frac{12}{25} \div 48$.
26. $975\frac{13}{40} \div 63$.
27. $1000\frac{25}{100} \div 50$.
28. $4684\frac{4}{5} \div 68$.
29. $7896\frac{2}{3} \div 25$.
30. $9684\frac{4}{5} \div 84$.

31. If 5 yards of cloth cost $\$42\frac{3}{8}$, what will 1 yard cost.
32. If 6 horses cost $\$756\frac{3}{4}$, what will 1 horse cost?
33. If 45 lbs. of wool cost $\$54\frac{3}{4}$, what will 1 lb. cost?

CASE II.

168. *a.* **To divide a *Whole Number* by a *Fraction*.**

1. At $\$\frac{3}{4}$ a yard, how many yards of cashmere can you buy for $20?

ANALYSIS.—At $\$\frac{1}{4}$ a yard, $20 will buy as many yards as there are fourths in $20, and 4 times 20 are 80. But the price $\$\frac{3}{4}$, is 3 times as much as $\$\frac{1}{4}$; therefore, at $\$\frac{3}{4}$, you can buy only $\frac{1}{3}$ as many yards as at $\$\frac{1}{4}$; and $\frac{1}{3}$ of 80 yards is $26\frac{2}{3}$ yards, the answer required. For, multiplying the whole number by the denominator 4, reduces it to *fourths*, which is the same *denominator* as the given divisor. But when fractions have a common denominator, their numerators are *like numbers*; therefore, one may be divided *by the other*, as whole numbers. (Art. 156.) But multiplying the *whole number* 20, by the denominator 4, and *dividing* the *product* by the numerator 3, is the same as *inverting* the fractional divisor, and then *multiplying* the dividend by it. Hence, the

OPERATION.

$20 \times 4 = 80$ y. at $\$\frac{1}{4}$.
$80 \div 3 = 26\frac{2}{3}$ y. at $\$\frac{3}{4}$.
Or, $20 \times \frac{4}{3} = 26\frac{2}{3}$ y. at $\$\frac{3}{4}$.

RULE.—*Multiply the whole number by the fraction inverted.* (Art. 165.)

REMARKS.—1. When the *divisor* is a *mixed number* it should be reduced to an *improper fraction*; then divide by the rule. (Ex. 16.)

2 A fraction is *inverted*, when its terms are made to *exchange places*. Thus, $\frac{3}{5}$ inverted becomes $\frac{5}{3}$.

3 After the denominator is inverted, the common factors should be cancelled. (Art. 149, *n.*)

168, *a.* How divide a whole number by a fraction? How does it appear that this process will give the true answer?

Perform the following divisions:

2. $95 \div \frac{3}{4}$.
3. $168 \div \frac{3}{12}$.
4. $245 \div \frac{7}{10}$.
5. $175 \div \frac{7}{15}$.
6. $261 \div \frac{3}{17}$.
7. $348 \div \frac{7}{9}$.
8. $576 \div \frac{7}{20}$.
9. $1236 \div \frac{2}{25}$.
10. $6240 \div \frac{11}{100}$.

11. How many yards of muslin, at $\$\frac{1}{3}$ a yard, can be bought for \$19?

ANALYSIS.—At $\$\frac{1}{3}$ a yard, \$19 will buy as many yards as there are thirds in 19, and $19 \times \frac{3}{1} = 57$. Therefore \$19 will buy 57 yards.

12. 27 = how many times $\frac{1}{5}$?
13. 53 = how many times $\frac{1}{9}$?
14. 38 = how many times $\frac{1}{7}$?
15. 67 = how many times $\frac{1}{12}$?

16. How many cloaks will 45 yards of cloth make, each containing $4\frac{1}{2}$ yards?

ANALYSIS.—Reducing the divisor to an improper fraction, we have $4\frac{1}{2} = \frac{9}{2}$, and 45 yds. $\times \frac{2}{9} = \frac{90}{9}$, or 10 cloaks., *Ans.*

17. Divide 88 by $10\frac{2}{5}$.
18. Divide 100 by $12\frac{1}{2}$.
19. Divide 785 by $62\frac{1}{2}$.
20. Divide 1000 by $87\frac{1}{2}$.

CASE III.

169. To divide a Fraction by a Fraction, when they have a *Common Denominator*.

REMARK.—This case embraces *two classes* of examples:
First, those in which the fractions have a *common denominator.*
Second, those in which they have *different* denominators.

21. At $\$\frac{3}{8}$ apiece, how many melons can be bought for $\$\frac{7}{8}$?

ANALYSIS.—Since $\$\frac{3}{8}$ will buy 1 melon, $\$\frac{7}{8}$ will buy as many as $\$\frac{3}{8}$ are contained times in $\$\frac{7}{8}$; and 3 eighths are in 7 eighths, 2 times and 1 over, or $2\frac{1}{3}$ times. (Art. 156.) Hence, the

OPERATION.
$\frac{7}{8} \div \frac{3}{8} = 2\frac{1}{3}$
Ans. $2\frac{1}{3}$ m.

RULE.—*Divide the numerator of the dividend by that of the divisor.*

NOTE.—When two fractions have a *common denominator*, their numerators are *like numbers*, and the quotient is the same as if they were *whole numbers.* (Arts. 64, 156.)

22. Divide $\frac{27}{11}$ by $\frac{3}{11}$.
23. Divide $\frac{35}{16}$ by $\frac{11}{16}$.
24. Divide $\frac{61}{23}$ by $\frac{17}{23}$.
25. Divide $\frac{78}{35}$ by $\frac{19}{35}$.

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169. How divide a fraction by a fraction when they have a common denominator?

170. To divide a Fraction by a Fraction, when they have *Different Denominators.*

1. At $\$\frac{2}{3}$ a pound, how much tea can be bought for $\$\frac{6}{8}$?

1ST ANALYSIS.—$\$\frac{6}{8}$ will buy as many pounds as $\$\frac{2}{3}$ are contained times in $\$\frac{6}{8}$. $\$\frac{6}{8}=\$\frac{3}{4}$. Reducing $\frac{3}{4}$ and $\frac{2}{3}$ to a common denominator, they become $\frac{9}{12}$ and $\frac{8}{12}$, and their numerators *like numbers*. Hence, $\frac{9}{12}\div\frac{8}{12}=9\div8$, or $1\frac{1}{8}$. *Ans.* $1\frac{1}{8}$ pound.

$$\frac{6}{8}=\frac{3}{4}=\frac{9}{12}$$
$$\frac{2}{3}=\frac{8}{12}$$
$$\frac{9}{12}\div\frac{8}{12}=9\div8$$
$$9\div8=1\tfrac{1}{8}\text{ lb.}$$

REMARKS.—1. In reducing two fractions to a *common denominator,* we multiply the *numerator* of each into the *denominator* of the other, and the two denominators together. But in dividing, no use is made of the common denominator; hence, in practice, multiplying the denominators together may be omitted. (Art. 153.)

2D ANALYSIS.—$\$\frac{6}{8}=\$\frac{3}{4}$. At $\$\frac{2}{3}$ a pound, $1 will buy as many pounds as $\$\frac{2}{3}$ are contained times in $1. Now $1\div\frac{2}{3}=\frac{3}{3}\div\frac{2}{3}$, and $\frac{3}{3}\div\frac{2}{3}=\frac{3}{2}$, or $\frac{3}{2}$ pounds, the *quotient* being the *divisor inverted.* Again, if $1 will buy $\frac{3}{2}$ pounds, $\$\frac{3}{4}$ will buy $\frac{3}{4}$ of $\frac{3}{2}$ pounds; and $\frac{3}{4}$ of $\frac{3}{2}=\frac{9}{8}$, or $1\frac{1}{8}$ pounds, the same as before.

REMARKS.—2. In this analysis, it will be seen, by inspection, that the *numerator* of each fraction is also multiplied into the *denominator* of the other. Both of these solutions, therefore, bring the *same combinations* of terms, and the *same result,* as *inverting* the divisor, and *multiplying* the *dividend* by it. (Art. 166.) Hence, the

RULE.—*Reduce the fractions to a com. denominator, and divide the numerator of the dividend by that of the divisor.*

Or, multiply the dividend by the divisor inverted.

NOTES.—1. The *first* method is based upon the principle that numerators of fractions having a com. denom. are *like numbers.*

The *second* is evident from the fact that it brings the same combinations as reducing the fractions to a *common denominator.*

The divisor is inverted for convenience in multiplying,

2. After the divisor is inverted, the *common factors* should be *cancelled,* before the multiplication is performed.

3. *Mixed numbers* should be reduced to *improper fractions, compound* and *complex fractions* to *simple* ones.

4. Those *expressions* which have *fractional* denominators, are reduced to *simple fractions* by the above rule. Ex. 14. (Art. 141, Rem.)

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170. How when they have not? *Note.* Upon what principle is the first method based? The second? Show this coincidence. What is done with mixed numbers, compound, and complex fractions.

Perform the following divisions:

2. $\frac{7}{12} \div \frac{3}{14}$.	6. $\frac{12}{63} \div \frac{8}{59}$.	10. $\frac{75}{100} \div \frac{25}{150}$.
3. $\frac{4}{17} \div \frac{3}{17}$.	7. $\frac{15}{35} \div \frac{9}{45}$.	11. $\frac{132}{256} \div \frac{300}{512}$.
4. $\frac{3}{35} \div \frac{4}{49}$.	8. $\frac{27}{42} \div \frac{28}{54}$.	12. $\frac{628}{749} \div \frac{314}{1498}$.
5. $\frac{11}{18} \div \frac{5}{38}$.	9. $\frac{21}{45} \div \frac{42}{63}$.	13. $\frac{573}{1264} \div \frac{824}{1867}$.

14. Reduce $\dfrac{27\frac{5}{9}}{10\frac{1}{3}}$ to a simple fraction.

SOLUTION.—The given expression is equivalent to $27\frac{5}{9} \div 10\frac{1}{3}$, and is reduced to a simple fraction by performing the division indicated. *Ans.* $\frac{8}{3}$ or $2\frac{2}{3}$. (Art. 141, Rem.)

15. Reduce $\dfrac{4\frac{1}{2}}{2\frac{2}{3}}$

16. Reduce $\dfrac{11\frac{1}{4}}{2\frac{1}{17}}$

17. Reduce $\dfrac{15\frac{3}{4}}{5\frac{1}{8}}$

18. Reduce $\dfrac{120\frac{5}{7}}{16\frac{1}{14}}$

19. Divide $777\frac{6}{10}$ dollars by $129\frac{3}{5}$ dollars.

20. How many rods in $2320\frac{1}{4}$ feet, at $16\frac{1}{2}$ feet to a rod?

21. How many times $30\frac{1}{4}$ sq. yards in $320\frac{1}{16}$ sq. yards?

22. What is the quotient of $\frac{2}{3}$ of $\frac{3}{4}$ of $4\frac{1}{2}$ divided by $\frac{2}{7}$ of $\frac{5}{8}$?

SOLUTION.—$\frac{2}{3}$ of $\frac{3}{4}$ of $4\frac{1}{2}$ divided by $\frac{2}{7}$ of $\frac{5}{8}$ =

$$\frac{2}{3} \times \frac{3}{4} \times \frac{9}{2} \times \frac{7}{2} \times \frac{8}{5} = \frac{63}{5} = 12\frac{3}{5}.\ \textit{Ans.}$$

$$\begin{array}{r|l} 3 & 2 \\ 4 & 3 \\ 2 & 9 \\ 2 & 7 \\ 5 & 8 \\ \hline 5 & 63 = 12\frac{3}{5} \end{array}$$

☞ When the perpendicular form is adopted, the *divisor* must be *inverted*, before its terms are arranged.

23. Divide $\frac{4}{5}$ of $\frac{3}{7}$ of $\frac{1}{3}$ by $\frac{7}{9}$ of $\frac{1}{3}$.

24. Divide $\frac{7}{9}$ of $\frac{3}{4}$ of $\frac{5}{7}$ by $\frac{1}{2}$ of $\frac{3}{4}$.

25. Divide $\frac{5}{7}$ of $\frac{9}{9}$ of $4\frac{1}{2}$ by $\frac{1}{2}$ of $\frac{2}{3}$ of $4\frac{1}{4}$.

26. What is the quotient of $\dfrac{\frac{1}{9}}{\frac{2}{3}}$ divided by $\dfrac{8}{3\frac{1}{5}}$?

ANALYSIS.—The dividend $\dfrac{\frac{1}{9}}{\frac{2}{3}} = \frac{1}{9} \times \frac{3}{2}$; the divisor $\dfrac{8}{3\frac{1}{5}} = \dfrac{8}{1} \times \dfrac{5}{16}$.

Inverting the divisor, etc., $\frac{1}{9} \times \frac{3}{2} \times \frac{1}{8} \times \frac{16}{5} = \frac{1}{15}$, (Art. 141, *n.* 2.)

27. What is quotient of $\dfrac{6\frac{1}{4}}{\frac{3}{4}}$ divided by $\dfrac{8\frac{1}{4}}{\frac{4}{11}}$?

171. The preceding principles may be reduced to the following

GENERAL RULE.

Reduce whole and mixed numbers to improper fractions, compound and complex fractions to simple ones, and multiply the dividend by the divisor inverted.

NOTE.—After the divisor is inverted, the common factors should be cancelled.

EXAMPLES.

1. If a young man spends $\$2\frac{1}{2}$ a month for tobacco, in what time will he spend $\$13\frac{1}{3}$? (Art. 48, Note 3.)

2. If a family use $5\frac{1}{4}$ pounds of butter a week, how long will $45\frac{1}{2}$ pounds last them?

3. If $\$\frac{3}{4}$ will buy 1 yard of gingham, how much will $\$\frac{9}{10}$ buy?

4. How many tons of coal, at $\$7\frac{1}{4}$ a ton, can be bought for $\$125\frac{7}{8}$?

5. How many times will a keg containing $13\frac{1}{8}$ gallons of molasses fill a measure that holds $\frac{7}{8}$ of a gallon?

6. What is the quotient of $24\frac{9}{10}$ divided by $8\frac{3}{5}$?

7. What is the quotient of $\frac{45}{78}$ divided by $\frac{15}{23}$?

8. What is the quotient of $\frac{110}{165}$ divided by $\frac{19}{33}$?

9. What is the quotient of $45\frac{7}{8}$ divided by $25\frac{3}{4}$?

10. How much tea, at $\$1\frac{1}{2}$ a pound, can be bought for $\$75\frac{3}{8}$?

11. How many acres can be sowed with $57\frac{3}{4}$ bushels of oats, allowing $1\frac{7}{8}$ bushel to an acre?

12. A man having $57\frac{1}{2}$ acres of land, wished to fence it into lots of $5\frac{3}{4}$ acres: how many lots could he make?

13. How many yards of cloth, at $\$6\frac{3}{8}$, can you buy for $\$268\frac{3}{9}$?

14. What is the quotient of $\frac{5}{8}$ of $\frac{4}{7}$ of $\frac{14}{35} \div \frac{2}{3}$ of $\frac{1}{8}$ of $2\frac{1}{4}$?

15. What is the quotient of $\frac{4}{5}$ of $\frac{3}{4}$ of $8\frac{2}{5} \div 4\frac{12}{13}$?

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171. What is the general rule for dividing fractions?

QUESTIONS FOR REVIEW.

1. A book-keeper adding a column of figures, made the result $\$563\frac{3}{8}$; proving his work, he found the true amount to be $\$607\frac{1}{4}$: how much was the error?

2. A merchant paid two bills, one $\$278\frac{3}{8}$, the other $\$340\frac{2}{3}$, calling the amount $\$638\frac{1}{4}$: what should he have paid? What the error?

3. What is the sum of $35\frac{3}{4}$ and $23\frac{5}{8}$ minus $8\frac{1}{2}$?

4. A speculator bought two lots of land, one containing $47\frac{3}{10}$ acres, the other $63\frac{4}{5}$ acres: after selling $78\frac{1}{2}$ acres, how many had he left?

5. What is the sum of $\frac{1\frac{2}{3}}{3\frac{3}{4}}$ plus $\frac{\frac{5}{6}}{3\frac{2}{6}}$?

6. What is the sum $\frac{6\frac{3}{5}}{4}$ plus $\frac{4\frac{1}{2}}{12}$ plus $\frac{8}{5\frac{1}{3}}$?

7. A man owning $\frac{11}{16}$ of a ship worth $\$48064$, sold $\frac{1}{4}$ of his share. What part of the ship did he sell; what part does he still own, and what is it worth?

8. A farmer owning $75\frac{3}{8}$ acres, sold $31\frac{1}{4}$ acres, and afterward bought $42\frac{2}{5}$ acres: how many acres did he then have?

9. What is the difference between $\frac{\frac{3}{5}}{\frac{2}{3}}$ and $\frac{4\frac{5}{9}}{6}$?

10. What is the difference between $\frac{6}{3\frac{1}{2}}$ and $\frac{5\frac{1}{2}}{6}$?

11. What is the difference between $\frac{12\frac{1}{4}}{6\frac{1}{9}}$ and $\frac{18\frac{1}{3}}{12\frac{1}{2}}$?

12. If it requires $1\frac{3}{4}$ bushels of wheat to sow an acre, how many bushels will be required to sow $28\frac{5}{8}$ acres?

13. How many feet in $148\frac{3}{5}$ rods, allowing $16\frac{1}{2}$ feet to a rod?

14. If a pedestrian can walk $45\frac{7}{8}$ miles in 1 day, how far can he walk in $18\frac{3}{4}$ days?

15. What will $37\frac{1}{2}$ barrels of apples come to, at $\$2\frac{3}{4}$ per barrel?

16. The sum of two numbers is $68\frac{19}{20}$, and the difference between them is $13\frac{2}{3}$: what are the numbers?

17. What is the product of $\frac{4\frac{1}{2}}{3\frac{1}{2}}$ into $\frac{2\frac{1}{3}}{4\frac{1}{4}}$?

18. What is the product of $\frac{4}{6\frac{2}{3}}$ into $\frac{5\frac{1}{3}}{12}$?

19. What is the product of $\frac{18\frac{1}{3}}{12\frac{1}{2}}$ into $\frac{\frac{25}{35}}{2\frac{1}{5}}$?

20. How many days' work will 100 men perform in $\frac{16}{20}$ of a day?

21. A man owning $\frac{9}{10}$ of a section of land, sold $\frac{1}{3}$ of his share for $\$12\frac{3}{8}$: what is the whole section worth, at that rate?

22. How many times is $\frac{3}{10}$ of $\frac{5}{8}$ of $5\frac{1}{4}$ contained in $23\frac{1}{2}$?

23. Divide $\frac{3}{4}$ of $18\frac{3}{4}$ by $\frac{4}{5}$ of $\frac{20}{16}$ of $\frac{5}{9}$ of $31\frac{5}{9}$.

24. If a gang of hands can do $\frac{19}{20}$ of a job in $5\frac{1}{2}$ days, what part of it can they do in 1 day?

25. If $\frac{5}{8}$ of a yard of satin will make 1 vest, how many vests can be made from $31\frac{1}{4}$ yards?

26. How many oil-cans, each containing $1\frac{3}{8}$ gallon, can be filled from a tank of $61\frac{3}{4}$ gallons?

27. If a man walks $3\frac{3}{5}$ miles an hour, how long will it take him to walk $45\frac{3}{10}$ miles?

28. By what must $\frac{19}{20}$ be multiplied to produce $15\frac{5}{6}$?

29. How many bushels of apples, at $\frac{5}{8}$ of a dol., are required to pay for 6 pair of boots, at $\$6\frac{1}{4}$?

30. A farmer sold $330\frac{6}{10}$ pounds of maple sugar, at $16\frac{2}{3}$ cents a pound, and took his pay in muslin, at $22\frac{1}{2}$ cents a yard: how many yards did he receive?

31. Divide the quotient of $12\frac{1}{2}$ divided by $3\frac{1}{8}$ by the quotient of $6\frac{1}{4} \div 3\frac{1}{8}$?

32. What is the quotient of $\frac{6\frac{1}{4}}{\frac{3}{4}} \div \frac{8\frac{1}{4}}{\frac{4}{11}}$?

33. What is the quotient of $12\frac{1}{2}$ times $\frac{3\frac{1}{2}}{4} \div \frac{12}{3\frac{1}{2}}$?

FRACTIONAL RELATION OF NUMBERS.

172. That *Numbers* may be *compared* with each other *fractionally*, they must be so far of the same nature that one may properly be said to be a *part* of the other. Thus, an *inch* may be compared with a *foot*; for *one* is a *twelfth part* of the other. But it cannot be said that a *foot* is any part of an *hour*; therefore the *former* cannot be compared with the *latter*.

173. To find what *part* one number is of another.

1. What part of 4 is 1?

ANALYSIS.—If 4 is divided into 4 equal parts, one of those parts is called 1 fourth. Therefore, 1 is $\frac{1}{4}$ part of 4.

2. What part of 6 is 4?

ANALYSIS.—1 is $\frac{1}{6}$ of 6, and 4 is 4 times $\frac{1}{6}$, or $\frac{4}{6}$ of 6. But $\frac{4}{6}=\frac{2}{3}$ (Art. 146); therefore, 4 is $\frac{2}{3}$ of 6. Hence, the

RULE.—*Make the number denoting the part the numerator, and that with which it is compared the denominator.*

NOTE.—1. This rule embraces *four classes* of questions:
1st. What part one *whole* number is of another.
2d. What part a *fraction* is of a whole number.
3d. What part a *whole* number is of a fraction.
4th. What part one *fraction* is of another.
2. When *complex fractions* occur, they should be reduced to *simple* ones, and all answers to the *lowest* terms. (Art. 146.)

3. What part of 75 is 15? Of 84 is 30?

4. Of 91 is 63? 6. Of 81 is 18? 8. Of 256 is 72?
5. Of 48 is 72? 7. Of 100 is 75? 9. Of 375 is 425?

10. What part of 1 week is 5 days?

11. A man gave a bushel of chestnuts to 17 boys: what part did 5 boys receive?

12. At $13 a ton, how much coal can be bought for $10?

13. A father is 51 years old, and his son's age is 17: what part of the father's age is the son's?

14. If 8 pears cost 35 cents, what will 5 pears cost?

ANALYSIS.—5 pears are $\frac{5}{8}$ of 8 pears; hence, if 8 pears cost 35 cents, 5 pears will cost $\frac{5}{8}$ of 35 cents. Now $\frac{1}{8}$ of 35 cents is $4\frac{3}{8}$ cents, and 5 eighths are 5 times $4\frac{3}{8}$ cents, which are $21\frac{7}{8}$ cents.

15. If 5 bar. of flour cost $45, what will 28 bar. cost?
16. If 50 yds. of cloth cost $175, what will 17 cost?
17. If 25 bu. of apples cost $30, what will 110 bu. cost?
18. What part of 5 is $\frac{3}{4}$?

ANALYSIS.—Making the *fraction* which denotes the *part* the numerator, and the *whole number* the denominator, we have a *fraction* to be divided by a *whole number*. For, all *denominators* may be considered as *divisors*. Thus, $\frac{3}{4}\div5=\frac{3}{20}$, *Ans.* (Art. 142.)

19. What part of 25 is $\frac{7}{8}$?
20. What part of 35 is $\frac{15}{4}$?
21. What part of 30 is $\frac{9}{10}$?
22. What part of 40 is $\frac{24}{3}$?
23. If 5 acres of land cost $100, what will $\frac{3}{4}$ acre cost?

ANALYSIS.—1 acre is $\frac{1}{5}$ of 5 acres, and $\frac{3}{4}$ of an acre is $\frac{3}{4}$ of $\frac{1}{5}$, or $\frac{3}{20}$ of 5 acres. Hence, $\frac{3}{4}$ of an acre will cost $\frac{3}{20}$ of $100. Now $\frac{1}{20}$ of $100 is $5; and 3 twentieths are 3 times 5 or $15.

24. When coal is $95 for 15 tons, what will $\frac{4}{5}$ ton cost?
25. If 19 yards of silk cost $60, what will $\frac{7}{8}$ yard cost?
26. What part of $\frac{3}{5}$ is 2?

ANALYSIS.—Making the *whole number* which denotes the *part*, the numerator, and the *fraction* the denominator, we have a *whole number* to be divided by a *fraction*. Thus, $2\div\frac{3}{5}=2\times\frac{5}{3}=\frac{10}{3}$, *Ans.*

27. What part of $\frac{3}{4}$ is 8?
28. What part of $\frac{5}{8}$ is 12?
29. What part of $\frac{4}{7}$ is 11?
30. What part of $\frac{7}{10}$ is 20?
31. What part of $\frac{4}{5}$ is $\frac{2}{3}$?

ANALYSIS.—Making the fraction denoting the *part* the numerator, and the other the denominator, we have a fraction to be divided by a fraction. Thus, $\frac{2}{3}\div\frac{4}{5}=\frac{2}{3}\times\frac{5}{4}=\frac{5}{6}$, *Ans.* (Art. 170.)

32. What part of $\frac{5}{8}$ is $\frac{2}{7}$?
33. What part of $\frac{28}{35}$ is $\frac{27}{63}$?
34. What part of $\frac{15}{17}$ is $\frac{9}{34}$?
35. What part of $\frac{33}{48}$ is $\frac{38}{63}$?
36. $6\frac{1}{4}$ is what part of 25?

ANALYSIS.—Reducing the mixed number to an improper fraction, we have $6\frac{1}{4}=\frac{25}{4}$, and $\frac{25}{4}\div25=\frac{1}{4}$, *Ans.*

37. What part of 100 is $12\frac{1}{2}$?
38. What part of 100 is $33\frac{1}{3}$?
39. What part of 100 is $16\frac{2}{3}$?
40. Of 100 is $62\frac{1}{2}$?
41. Of 100 is $18\frac{3}{4}$?
42. Of 100 is $87\frac{1}{2}$?
43. $12\frac{1}{2}$ is what part of $18\frac{3}{4}$?

ANALYSIS.—Reducing the mixed numbers to improper fractions we have, $12\frac{1}{2}=\frac{25}{2}$, and $18\frac{3}{4}=\frac{75}{4}$. Now $\frac{25}{2}\div\frac{75}{4}=\frac{2}{3}$, *Ans.*

44. What part of $62\frac{1}{2}$ is $18\frac{3}{4}$?
45. Of $87\frac{1}{2}$ is $31\frac{1}{4}$?
46. At $\$\frac{19}{20}$ a pound, how much tea will $\$\frac{3}{4}$ buy?
47. At $\$\frac{15}{40}$ per foot, how many feet of land can be bought for $\$\frac{12}{16}$?
48. A lad spent $18\frac{3}{4}$ cents for candy, which was $62\frac{1}{2}$ cents a pound: how much did he buy?
49. A can do a certain job in 8 days, and B in 6 days: what part will both do in 1 day?
50. What part of 4 times 20 is 9 times 16?
51. What part of 75×18 is $105 \div 25$?
52. What part of $(68 - 24) \times 14$ is $168 \div 12$?

174. To find a Number, a Fractional Part of it being given.

Ex. 1. 9 is $\frac{1}{3}$ of what number?

ANALYSIS.—Since 9 is 1 third, 3 thirds or the whole number must be 3 times 9 or 27. Therefore, 9 is a third of 27.

Or, thus: 9 is $\frac{1}{3}$ of 3 times 9, and $3 \times 9=27$. Therefore, etc.

2. 21 is $\frac{3}{4}$ of what number?

OPERATION.

$21 \div \frac{3}{4} = 21 \times \frac{4}{3}$

$21 \times \frac{4}{3} = 28$.

ANALYSIS.—Since $\frac{3}{4}$ of a certain number is 21 units, $\frac{4}{4}$ or the whole number must be as many units as $\frac{3}{4}$ are contained times in 21; and $21 \div \frac{3}{4} = 21 \times \frac{4}{3} = 28$, the answer required. For, a whole number is divided by a fraction by multiplying the former by the latter inverted. (Art. 168, *a*.)

Or, thus: Since 21 is $\frac{3}{4}$ of a certain number, 1 fourth of it is 1 third of 21, or 7. Now as 7 is 1 fourth of the number, 4 fourths must be 4 times 7 or 28, the same as before. Hence, the

RULE.—*Divide the number denoting the part by the fraction.*

Or, *Find one part as indicated by the numerator of the fraction, and multiply this by the denominator.*

NOTE.—The learner should observe the difference between finding $\frac{1}{4}$ of a number, when $\frac{4}{4}$ or the whole number is given, and when only $\frac{3}{4}$ or a part of it is given. In the *former*, we divide by the *denominator* of the fraction; in the *latter*, by the *numerator*, as in the *second* analysis. If he is at a loss which to take for the *divisor*, let him substitute the word *parts* for the denominator.

3. 56 is $\frac{2}{3}$ of what?
4. 68 is $\frac{3}{4}$ of what?
5. 85 is $\frac{2}{5}$ of what?
6. 115 is $\frac{5}{6}$ of what?
7. 436 is $\frac{3}{7}$ of what?
8. 456 is $\frac{5}{8}$ of what?
9. 685 is $\frac{3}{10}$ of what?
10. 999 is $\frac{7}{10}$ of what?

11. A market man being asked how many eggs he had, replied that 126 was equal to $\frac{7}{15}$ of them: how many had he?

12. If $\frac{9}{16}$ of a ship is worth $8280, what is the whole worth?

13. A commander lost $\frac{5}{7}$ of his forces in a battle, and had 9500 men left: how many had he at first?

14. $\frac{50}{75}$ is $\frac{1}{4}$ of what number?

ANALYSIS.—$\frac{50}{75}$ is $\frac{1}{4}$ of 4 times $\frac{50}{75}$; and 4 times $\frac{50}{75}=2\frac{2}{3}$.

15. $\frac{21}{28}$ is $\frac{3}{7}$ of what number?

ANALYSIS.—Since $\frac{21}{28}=\frac{3}{7}$ of a certain number, $\frac{1}{7}$ of that number must be $\frac{1}{3}$ of $\frac{21}{28}$, and $\frac{1}{3}$ of $\frac{21}{28}=\frac{7}{28}$, or $\frac{1}{4}$. Now if $\frac{1}{7}$ of the number $=\frac{1}{4}$, $\frac{7}{7}$ must equal 7 times $\frac{1}{4}=\frac{7}{4}$, or $1\frac{3}{4}$, *Ans.*

Or, dividing $\frac{21}{28}$ by $\frac{3}{7}$ we have $\frac{7,\ 21}{4,\ 28}\times\frac{7}{3}=\frac{7}{4}$, or $1\frac{3}{4}$, *Ans.*

16. $\frac{35}{45}$ is $\frac{3}{5}$ of what number?

17. $\frac{72}{96}$ is $\frac{5}{8}$ of what number?

18. $18\frac{3}{4}$ is $\frac{5}{8}$ of what number?

ANALYSIS.—$18\frac{3}{4}=\frac{75}{4}$. Since $\frac{75}{4}=\frac{5}{8}$, $\frac{1}{8}=\frac{15}{4}$, and $\frac{8}{8}=\frac{120}{4}$, or 30, *Ans.* Or, $\frac{75}{4}\div\frac{5}{8}=\frac{75}{4}\times\frac{8}{5}=\frac{600}{20}=30$, the same as before.

19. $37\frac{1}{2}$ is $\frac{3}{5}$ of what number?

20. $66\frac{2}{3}$ is $\frac{4}{5}$ of what number?

21. 48 is $\frac{5}{9}$ of $\frac{4}{5}$ of what number?

ANALYSIS.—$\frac{5}{9}$ of $\frac{4}{5}=\frac{4}{9}$. The question now is, 48 is $\frac{4}{9}$ of what number? *Ans.* $48\times\frac{9}{4}$, or 108.

22. 112 is $\frac{7}{8}$ of $\frac{4}{9}$ of what number?

23. In $\frac{7}{9}$ of 120 how many times 15?

ANALYSIS.—$\frac{1}{9}$ of 120 is $13\frac{1}{3}$; and $\frac{7}{9}$ are 7 times $13\frac{1}{3}$ or $93\frac{1}{3}$. Now $93\frac{1}{3} \div 15 = \frac{280}{3} \div 15 = \frac{280}{45}$ or $6\frac{2}{9}$, *Ans.*

24. How many yards of brocatelle, at $9 a yard, can be bought for $\frac{7}{8}$ of $100?

25. A man paid $\frac{9}{10}$ of $280 for 84 arm-chairs: what was that apiece?

26. 90 is $\frac{6}{7}$ of how many times 17?

ANALYSIS.—As 90 is $\frac{6}{7}$ of a certain number, $\frac{1}{7}$ is $\frac{1}{6}$ of 90, which is 15; and $\frac{7}{7}$ are 7 times 15 or 105. Now 17 is in 105, $6\frac{3}{17}$ times. Therefore, etc.

27. 125 is $\frac{5}{9}$ of how many times 20?

28. A man paid 60 cents for his lunch, which was $\frac{6}{10}$ of his money, and spent the remainder for cigars, which were 5 cents each: how much money had he; and how many cigars did he buy?

29. $\frac{9}{10}$ of 110 is $\frac{9}{7}$ of what number?

ANALYSIS.—$\frac{1}{10}$ of 110 is 11, and $\frac{9}{10}$, 9 times 11 or 99. Now, since 99 is $\frac{9}{7}$ of a number, $\frac{1}{7}$ of it must be $\frac{1}{9}$ of 99, which is 11, and $\frac{7}{7}$ must be 7 times 11 or 77. Therefore, etc.

30. $\frac{7}{9}$ of 126 is $\frac{3}{4}$ of what number?

31. $\frac{5}{9}$ of 90 is $\frac{5}{8}$ of how many times 11?

ANALYSIS.—$\frac{5}{9}$ of 90 is 50. Now as 50 is $\frac{5}{8}$ of a number, $\frac{1}{8}$ is $\frac{1}{5}$ of 50 or 10; $\frac{8}{8}$ is 8 times 10 or 80. Finally, 11 is contained in 80, $7\frac{3}{11}$ times. Therefore $\frac{5}{9}$ of 90 is $\frac{5}{8}$ of $7\frac{3}{11}$ times 11.

32. $\frac{7}{8}$ of 96 is $\frac{6}{10}$ of how many times 20?

33. $\frac{5}{6}$ of 120 is $\frac{4}{9}$ of how many sevenths of 56?

ANALYSIS.—$\frac{5}{6}$ of 120 is 100. If 100 is $\frac{4}{9}$ of a number, $\frac{1}{9}$ is $\frac{1}{4}$ of the number; now $\frac{1}{4}$ of 100 is 25, and $\frac{9}{9}$ is 9 times 25 or 225. Finally, $\frac{1}{7}$ of 56 is 8, and 8 is contained in 225, $28\frac{1}{8}$ times. Therefore, $\frac{5}{6}$ of 120 is $\frac{4}{9}$ of $28\frac{1}{8}$ times $\frac{1}{7}$ of 56.

34. $\frac{5}{7}$ of 35 is $\frac{5}{8}$ of how many tenths of 120?

DECIMAL FRACTIONS.

175. ***Decimal Fractions*** are those in which the *unit* is *divided into tenths, hundredths, thousandths,* etc. They arise from *continued divisions* by 10.

If a unit is divided into *ten equal parts,* the parts are called *tenths.* Now, if one of these *tenths* is subdivided into *ten* other equal parts, each of these parts will be *one-tenth* of a tenth, or a *hundredth.* Thus, $\frac{1}{10} \div 10$ or $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$. Again, if one of these *hundredths* is subdivided into *ten* equal parts, each of these parts will be *one-tenth* of a hundredth, or a *thousandth.* Thus, $\frac{1}{100} \div 10 = \frac{1}{1000}$, etc.

NOTATION OF DECIMALS.

176. If we multiply the *unit* 1 by 10 continually, it produces a series of whole numbers which *increase* regularly by the *scale* of 10; as,

1, 10, 100, 1000, 10000, 100000, 1000000, etc.

Now if we divide the highest term in this series by 10 continually, the several quotients will form an *inverted* series, which *decreases* regularly by *ten,* and extends from the highest term to 1, and from 1 to $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, and so on, indefinitely; as,

100000, 10000, 1000, 100, 10, 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc.

177. By inspecting this series, the learner will perceive that the *fractions* thus obtained, regularly *decrease* toward the right by the *scale* of 10.

If we apply to this class of fractions the great law of Arabic Notation, which assigns *different values* to figures,

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175. What are decimal fractions? How do they arise? Explain this upon the blackboard. 177. By what law do decimals decrease?

according to the place they occupy, it follows that a figure standing in the *first* place on the *right* of units, denotes *tenths*, or 1 *tenth* as much as when it stands in units' place; when standing in the *second* place, it denotes *hundredths*, or 1 *tenth* as much as in the *first* place; when standing in the *third* place, it denotes *thousandths*, etc., each succeeding order below units being *one tenth* the value of the preceding. Hence,

178. *Fractions* which decrease by the *scale of ten*, may be expressed like whole numbers; the *value* of each figure in the decreasing scale being determined by the place it occupies on the right of units. Thus, 3 and 5 tenths may be expressed by 3.5; 3 and 5 hundredths by 3.05; 3 and 5 thousandths by 3.005, etc.

178, *a*. Decimals are distinguished from whole numbers by a *decimal point.*

NOTES.—1. The decimal point commonly used (.), is a *period.*

2. This class of fractions is called *decimals,* from the Latin *decem, ten,* which indicates both their *origin* and the *scale* of decrease.

179. The ***Denominator*** of a decimal fraction is always 10, 100, 1000, etc.; or 1 with as *many ciphers annexed* to it as there are *decimal places* in the given numerator. Conversely,

The ***Numerator*** of a decimal fraction, when written alone, contains as *many figures* as there are *ciphers* in the denominator. Thus $\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{1000}$, expressed decimally, are .5, .05, .005, etc. If the ciphers in .05 and .005 are omitted, each becomes 5-tenths.

What place do tenths occupy? Hundredths, thousandths, etc.? 178. How is the value of decimal figures determined? 178, *a*. How are decimals distinguished from whole numbers? *Note.* What is the decimal point? From what does this class of fractions receive its name? 179. What is the denominator? How do the number of ciphers in the denominator and the decimal places in the numerator compare?

180. The different *orders* of decimals, and their *relative position,* may be seen from the following

TABLE.

Millions, etc.	Hundreds of thousands.	Tens of thousands.	*Thousands.*	Hundreds.	Tens.	*Units.*	(Decimal Point.)	Tenths.	Hundredths.	*Thousandths.*	Ten-thousandths.	Hundred-thousandths.	*Millionths.*	Ten-millionths.	Hundred-millionths.	*Billionths, etc.*
8	4	2	5	6	7	2	.	3	2	6	7	2	5	4	5	8
Integers.								Decimals.								

NOTES.—1. A *Decimal* and an *Integer* written together, are called a *Mixed Number;* as 35.263, etc. (Art. 101, Def. 12.)

2. A *Decimal* and a *Common Fraction* written together, are called a *Mixed Fraction;* as $.6\frac{1}{4}$, $.33\frac{1}{3}$.

181. Since the orders of decimals *decrease* from left to right by the scale of 10, it follows:

First. Prefixing a cipher to a *decimal diminishes* its value 10 times, or *divides* it by 10. Thus $.7=\frac{7}{10}$; $.07=\frac{7}{100}$; $.007=\frac{7}{1000}$.

Annexing a cipher to a decimal does not *alter its* value. Thus .7, .70, .700 are respectively equal to $\frac{7}{10}$, $\frac{70}{100}$, $\frac{700}{1000}$, or $\frac{7}{10}$.

These effects are the *reverse* of those produced by annexing and prefixing ciphers to *whole* numbers. (Arts. 57, 79.)

Second. Each removal of the *decimal point* one figure to the *right, increases* the number 10 times, or *multiplies* it by 10. Each removal of the *decimal point* one figure to the *left, diminishes* the number 10 times, or *divides* it by 10. Hence.

180. Name the orders of decimals toward the right. Name the orders of integers toward the left. 181. How is a decimal figure affected by moving it one place to the right? How, if a cipher is prefixed to it? How if ciphers are annexed?

182. To write *Decimals.*

RULE.—*Write the figures of the numerator in their order, assigning to each its proper place below units, and prefix to them the decimal point.*

If the numerator has not as many figures as required, supply the deficiency by prefixing ciphers.

Write the following fractions decimally:

1. $\frac{7}{10}$.	6. $\frac{7}{100}$.	11. $93\frac{9}{1000}$.
2. $\frac{11}{100}$.	7. $\frac{99}{100}$.	12. $7\frac{45}{1000}$.
3. $\frac{49}{100}$.	8. $4\frac{7}{10}$.	13. $10\frac{508}{100000}$.
4. $\frac{65}{100}$.	9. $21\frac{6}{100}$.	14. $46\frac{7}{10000}$.
5. $\frac{3}{100}$.	10. $84\frac{45}{100}$.	15. $80\frac{364}{1000000}$.

17. Write 6 hundredths; 63 thousandths; 109 ten thousandths.

18. Write 305 thousandths; 21 hundred-thousandths; 95 millionths.

19. Write 4 thousandths; 108 ten-thousandths; 46 hundreths; 65 millionths; 1045 ten-millionths.

20. Write sixty-nine and four thousandths; ten and seventy-five ten-thousands; 160 and 6 millionths.

21. Write 53 ten-thousandths; 63 and 28 hundred-thousandths; 352 ten-millionths.

183. To read *Decimals* expressed by Figures.

RULE.—*Read the decimals as whole numbers, and apply to them the name of the lowest order.*

NOTE.—In case of *mixed numbers,* read the *integral* part as if it stood alone, then the *decimal.*

Or, pronounce the word *decimal,* then read the *decimal* figures as if they were *whole* numbers.

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182. What is the rule for writing decimals? *Note.* If the numerator has not as many figures as there are ciphers in the denominator, what is to be done? 183. How are decimals read? *Note.* In case of a mixed number, how?

Or, having pronounced the word *decimal*, repeat the *names* of the decimal figures in their order. Thus, 275.468 is read, "275 and 468 thousandths;" or "275, decimal four hundred and sixty-eight;" or "275, decimal *four, six, eight.*"

1. Explain the decimal .05.

ANALYSIS.—Since the 5 stands in the second place on the right of the decimal point, it is equivalent to $\frac{5}{100}$, and denotes 5 hundredths of *one*, or 5 such parts as would be obtained by dividing a *unit* into 100 equal parts.

Read the following examples:

(1.)	(2.)	(3.)
.36	2.751	32.862
.479	4.8465	40.0752
.0652	7.25025	57.00624
.00316	8.400452	81.20701

183, *a.* ***Decimals*** differ from *common* fractions in three respects, viz.: in their origin, their notation, and their limitation.

1st. *Common fractions* arise from dividing a unit into *any number* of equal parts, and may have *any number* for a denominator.

Decimals arise from dividing a unit into *ten, one hundred, one thousand,* etc., equal parts; consequently, the denominator is always 10, or some power of 10. (Arts. 55, *n.* 179.)

2d. In the *notation* of common fractions, both the *numerator* and *denominator* are written in full.

In decimals, the *numerator only* is written; the *denominator* is understood.

3d. Common fractions are *universal* in their application, embracing all classes of fractional quantities from a *unit* to an *infinitesimal.*

Decimals are *limited* to that particular class of fractional quantities whose *orders* regularly *decrease* in value from left to right, by the scale of 10.

NOTE.—The question is often asked whether the expressions $\frac{3}{10}$, $\frac{6}{100}$, $\frac{7}{1000}$, etc., are *common* or *decimal* fractions.

All fractions whose *denominator* is *written under* the *numerator, fulfil the conditions* of *common* fractions, and may be treated as such. But fractions which arise from dividing a unit into 10, 100, 1000, etc., equal parts, *answer to the definition of decimals,* whether the denominator is expressed, or understood.

183, *a.* How do decimals differ from common fractions?

REDUCTION OF DECIMALS.

184. To Reduce Decimals to a *Common Denominator*.

1. Reduce .06, 2.3, and .007 to a common denominator.

ANALYSIS.—Decimals containing the same number of figures, have a *common denominator*. (Art. 179.) By annexing ciphers, the number of decimal figures in each may be made the same, without altering their value. (Art. 181.) Hence, the

.06 = 0.060
2.3 = 2.300
.007 = 0.007

RULE.—*Make the number of decimal figures the same in each, by annexing ciphers.* (Art. 181.)

2. Reduce .48 and .0003 to a common denominator.

3. Reduce 2 to tenths; 3 to hundredths; and .5 to thousandths. *Ans.* 2.0 or $\frac{20}{10}$; 3.00 or $\frac{300}{100}$; .500.

185. To Reduce *Decimals* to Common Fractions.

1. Reduce .42 to a common fraction.

ANALYSIS.—The denominator of a decimal is 1, with as many ciphers annexed as there are figures in its numerator; therefore the denominator of .42 is 100. (Art. 179.) *Ans*, $.42 = \frac{42}{100}$. Hence, the

RULE.—*Erase the decimal point, and place the denominator under the numerator.* (Art. 179.)

2. Reduce .65 to a common fraction; then to its lowest terms. *Ans.* $.65 = \frac{65}{100}$, and $\frac{65}{100} = \frac{13}{20}$.

Reduce the following decimals to common fractions:

3. .128	7. .05	11. .0007	15. .200684
4. .256	8. .003	12. .04056	16. .0000008
5. .375	9. .0008	13. .00364	17. .12400625
6. .863	10. .0605	14. .00005	18. .24801264

186. To Reduce *Common Fractions* to Decimals.

1. Reduce $\frac{3}{8}$ to a decimal fraction.

ANALYSIS.—$\frac{3}{8}$ equals $\frac{1}{8}$ of 3. Since 3 cannot be divided by 8; we annex a cipher to reduce it to *tenths*. Now $\frac{1}{8}$ of 30 tenths is 3 tenths, and 6 tenths over. 6 tenths reduced to *hundredths* = 60 hundredths, and $\frac{1}{8}$ of 60 hundredths = 7 hundredths and 4 hundredths over. 4 hundredths reduced to *thousandths* = 40 thousandths, and $\frac{1}{8}$ of 40 thousandths = 5 thousandths. Therefore, $\frac{3}{8} = .375$. Hence, the

OPERATION.
8)3.000
Ans. .375

RULE.—*Annex ciphers to the numerator and divide by the denominator. Finally, point off as many decimal figures in the result as there are ciphers annexed to the numerator.*

NOTE.—If the number of figures in the quotient is *less* than the number of ciphers annexed to the numerator, supply the *deficiency* by *prefixing ciphers.*

Demonstration.—A fraction indicates division, and its value is the numerator divided by the denominator. (Arts. 134, 142.) Now, annexing *one* cipher to the numerator multiplies the fraction by 10; annexing *two* ciphers, by 100, etc. Hence, dividing the numerator with *one, two,* or *more* ciphers annexed, gives a quotient, 10, 100, etc., times *too large.* To *correct* this *error* the *quotient* is divided by 10, 100, 1000, etc. But dividing by 10, 100, etc., is the same as pointing off an *equal number* of decimal figures. (Art. 181.)

Reduce the following fractions to decimals:

2. $\frac{1}{4}$	6. $\frac{5}{8}$	10. $\frac{19}{20}$	14. $\frac{3}{8000}$
3. $\frac{2}{5}$	7. $\frac{3}{12}$	11. $\frac{30}{50}$	15. $\frac{2}{256}$
4. $\frac{3}{4}$	8. $\frac{7}{8}$	12. $\frac{7}{80}$	16. $\frac{7}{800}$
5. $\frac{4}{5}$	9. $\frac{12}{15}$	13. $\frac{3}{150}$	17. $\frac{18}{1600}$

18. Reduce $\frac{2}{3}$ to the form of a decimal.

ANALYSIS.—Annexing ciphers to the numerator and dividing by the denominator, as before, the quotient consists of 6 repeated to infinity, and the remainder is always 2. Therefore $\frac{2}{3}$ cannot be exactly expressed by decimals.

OPERATION.

3)2.000
.6666, etc.

19. Reduce $\frac{5}{37}$ to the form of a decimal.

ANALYSIS.—Having obtained three quotient figures 135, the remainder is 5, the same as the original numerator; consequently, by annexing ciphers to it, and continuing the division, we obtain the same set of figures as before, repeated to infinity. Therefore $\frac{5}{37}$ cannot be exactly expressed by decimals.

OPERATION.

37)5.000000
.135135, etc.

187. When the *numerator*, with ciphers annexed, is *exactly divisible* by the denominator, the decimal is called a *terminate* decimal.

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185. How reduce a decimal to a common fraction? 186. How reduce a common fraction to a decimal? *Note.* If the number of figures in the quotient is less than that in the numerator, what is to be done? Explain the reason for pointing off the quotient.

When it is *not exactly divisible*, and the *same figure* or *set of figures* continually recurs in the quotient, the decimal is called an *interminate* or *circulating* decimal.

The *figure* or *set* of figures repeated is called the *repetend* Thus, the decimals obtained in the last two examples are *interminate*, because the division, if continued forever, will leave a *remainder*. The repetend of the 18th is 6; that of the 19th is 135.

NOTES.—1. After the quotient has been carried as far as desirable the sign (+) is annexed to it to indicate there is still a remainder.

2. If the remainder is such that the next quotient figure would be 5, or more, the *last figure obtained* is sometimes *increased* by 1, and the sign (−) annexed to show that the decimal is too large.

3. Again, the remainder is sometimes placed over the *divisor* and annexed to the quotient, forming a *mixed* fraction. (Art. 180, *n*.) Thus if $\frac{2}{3}$ is reduced to the decimal form, the result may be expressed by .6666+ ; by .6667− ; or by .6666$\frac{2}{3}$.

(For the further consideration of Circulating Decimals, the student is referred to Higher Arithmetic.)

Reduce the following to four decimal places:

20. $\frac{1}{3}$	22. $\frac{2}{7}$	24. $\frac{3}{11}$	26. $\frac{29}{48}$
21. $\frac{5}{6}$	23. $\frac{4}{9}$	25. $\frac{9}{14}$	27. $\frac{41}{75}$

Reduce the following to the decimal form:

28. $75\frac{3}{5}$	30. $261\frac{17}{25}$	32. $465\frac{1}{400}$	34. $740\frac{11}{800}$
29. $136\frac{7}{8}$	31. $346\frac{61}{75}$	33. $523\frac{1}{256}$	35. $956\frac{5}{640}$

ADDITION OF DECIMALS.

188. Since *decimals increase* and *decrease* regularly by the *scale* of 10, it is plain they may be *added, subtracted, multiplied,* and *divided* like *whole* numbers.

Or, they may be reduced to a *common denominator*, then be *added, subtracted,* and *divided* like Common Fractions. (Arts. 156, 184.)

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187. When the numerator with ciphers annexed is exactly divisible by the denominator, what is the decimal called? If not exactly divisible, what? What is the figure or set of figures repeated called?

189. To find the *Amount* of two or more Decimals.

1. Add 360.1252, 1.91, 12.643, and 152.8413.

ANALYSIS.—Since units of the *same order* or *like numbers* only can be *added* to each other, we reduce the decimals to a *common denominator* by annexing ciphers; or, which is the same, by writing the decimals one under another, so that the decimal points shall be in a perpendicular line. (Arts. 28, *n.* 156.) Beginning at the right, we add each column, and set down the result as in whole numbers, and for the same reasons. (Art. 29, *n.*) Finally, we place the decimal point in the amount directly under those in the numbers added. (Art. 178, *a.*) Hence, the

OPERATION.
360.1252
1.9100
12.6430
152.8413
Ans. 527.5195

RULE.—I. *Write the numbers so that the decimal points shall stand one under another, with tenths under tenths, etc.*

II. *Beginning at the right, add as in whole numbers, and place the decimal point in the amount under those in the numbers added.*

NOTE.—Placing tenths under tenths, hundredths under hundredths, etc., in effect, reduces the decimals to a *com. denominator;* hence the ciphers on the right may be omitted. (Arts. 181, 184.)

(2.)	(3.)	(4.)	(5.)
41.3602	416.378	36.81045	4.83907
4.213	85.1	.203	.293
61.46	.4681	5.3078	.40
375.265	4.38	87.69043	5.1067
482.2982 *Ans.*	375.2956	9.25	3.75039

6. What is the sum of 41.371 + 2.29 + 73.402 + 1.729?

7. What is the sum of 823.37 + 7.375 + 61.1 + .843?

8. What is the sum of .3925 + .64 + .462 + .7 + .56781?

9. What is the sum of 86.005 + 4.0003 + 2.00007?

10. What is the sum of 1.713 + 2.30 + 6.400 + 27.004?

11. Add together 7 tenths; 312 thousandths; 46 hundredths; 9 tenths; and 228 ten-thousandths.

12. Add together 23 ten-thousandths; 23 hundred-thousandths; 23 thousandths; 23 hundredths; and 23.

---

189. How are decimals added? How point off the amount?

13. Add together five hundred seventy-five and seven-tenths; two hundred fifty-nine ten-thousandths; five-millionths; three hundred twenty hundred-thousandths.

14. A farmer gathered $17\frac{3}{5}$ bushels of apples from one tree; $8\frac{1}{2}$ bushels from another; $10\frac{1}{4}$ bushels from another; and $16\frac{3}{4}$ bushels from another. Required the number of bushels he had, expressed decimally.

15. A grocer sold $7\frac{1}{2}$ pounds of sugar to one customer; 11.37 pounds to another; $10\frac{3}{5}$ pounds to another; $25\frac{1}{8}$ pounds to another; and 21.375 pounds to another: how many pounds did he sell to all?

SUBTRACTION OF DECIMALS.

190. To find the *Difference* between two Decimals.

1. What is the difference between 2.607 and .7235?

ANALYSIS.—We reduce the decimals to a common denominator, by annexing ciphers, or by writing the same orders one under another. (Art. 189, *n.*) For, units of the *same order* or like numbers *only*, can be subtracted one from the other.

OPERATION.

```
     2.607
      .7235
Ans. 1.8835
```

Beginning at the right, we perceive that 5 ten-thousandths can not be taken from 0; we therefore borrow ten, and then proceed in all respects as in whole numbers. (Art. 38.) Hence, the

RULE.—I. *Write the less number under the greater, so that the decimal points shall stand one under the other, with tenths under tenths, etc.*

II. *Beginning at the right hand, proceed as in subtracting whole numbers, and place the decimal point in the remainder under that in the subtrahend.*

NOTE.—Writing the same orders one under another, in effect reduces the decimals to a common denominator. (Art. 189, *n.*)

	(2.)	(3.)	(4.)	(5.)
From	13.051	7.0392	20.41	85.3004
Take	5.22	.43671	3.0425	67.35246

190. How are decimals subtracted? How point off the remainder?

Perform the following subtractions:

6. 13.051 minus 5.22.
7. 7.0392 minus 0.43671.
8. 20.41 minus 3.0425.
9. 85.3004 minus 7.35246.
10. 93.38 minus 14.810034.
11. 3 minus 0.103784.
12. 10 minus 9.1030245.
13. 100 minus 99.4503067.
14. 1 minus .123456789.
15. 1 minus .98764321.
16. 0.1 minus .001.
17. 0.01 minus .00001.

18. From 100 take 1 thousandth.

19. From 45 take 45 ten-thousandths.

20. From 1 ten-thousandth take 2 millionths.

21. A man having $673.875, paid $230.05 for a horse: how much had he left?

22. A father having 504.03 acres of land, gave 100.45 acres to one son, 263.75 acres to another: how much had he left?

23. What is the difference between 203.007 and 302.07?

24. Two men starting from the same place, traveled in opposite directions, one going 571.37 miles, the other 501.037 miles: how much further did one travel than the other; and how far apart were they?

MULTIPLICATION OF DECIMALS.

191. **To find the *Product* of two or more Decimals.**

1. What is the product of 45 multiplied by .7?

ANALYSIS.—$.7=\frac{7}{10}$. Now $\frac{7}{10}$ times $45=\frac{315}{10}=315\div10$, or 31.5, *Ans.* In the operation, we multiply by 7 instead of $\frac{7}{10}$; therefore the product is 10 times too large. To correct this, we point off 1 figure on the right, which divides it by 10. (Arts. 79, 143, 165.)

OPERATION.

$$\begin{array}{r} 45 \\ .7 \\ \hline 31.5 \end{array}$$ *Ans.*

2. What is the product of 9.7 multiplied by .9?

ANALYSIS.—$9.7=\frac{97}{10}$, and $.9=\frac{9}{10}$. Now $\frac{9}{10}$ times $\frac{97}{10}=\frac{873}{100}=873\div100$, or 8.73, *Ans.* In this operation we also multiply as in whole numbers, and point off 2 figures on the right of the product for decimals, which divides it by 100. (Art. 79.)

OPERATION.

$$\begin{array}{r} 9.7 \\ .9 \\ \hline 8.73 \end{array}$$ *Ans.*

REM.—By inspecting these operations, we see that each answer contains as many *decimal figures* as there are *decimal places* in both factors. Hence, the

RULE.—*Multiply as in whole numbers, and from the right of the product, point off as many figures for decimals, as there are decimal places in both factors.*

NOTES.—1. *Multiplication* of *Decimals* is based upon the same principles as Multiplication of *Common* Fractions. (Art. 166.)

2. The *reason* for pointing off the product is this: The *product* of any two decimal numerators is as *many times too large* as there are *units* in the product of their denominators, and pointing it off divides it by that product. (Art. 79.) For, the product of the denominators of two decimals is always 1 with as many ciphers annexed as there are decimal places in both numerators. (Arts. 57, 179, 186, Dem.)

3. If the product has not as many figures as there are decimals in both factors, supply the deficiency by prefixing ciphers.

2. Multiply .015 by .03.

SOLUTION.—.015 × .03 = .00045. The product requires 5 decimal places; hence, 3 ciphers must be prefixed to 45.

	(3.)	(4.)	(5.)	(6.)
Multiply	29.06	.07213	.000456	4360.12
By	.005	.0021	.0037	5.000

7. 4.0005 × .00301.
8. 5.0206 × 4.0007.
9. 3.0004 × 106.
10. 7.2136 × 100.
11. 0.0048 × .0091.
12. 15.004 × .10009.
13. 6.0103 × .00012.
14. 20007 × .000001.

15. If 1 box contains 17.25 pounds of butter, how many pounds will 25 boxes contain?

16. What cost 20.5 barrels of flour, at $10.875 a barrel?

17. If one acre produces 750.5 bushels of potatoes, how much will .625 acres produce?

18. What cost 53 horses, at $200.75 apiece?

19. Multiply 28 hundredths by 45 thousandths.

20. What cost 73.25 yards of cloth, at $9.375 per yard?

---

191. How are decimals multiplied? How point off the product? *Note.* Explain the reason for pointing off. If the product does not contain as many figures as there are decimals in the factors, what is to be done? 192. How multiply a decimal by 10, 100, etc.

21. Multiply 5 tenths by 5 thousandths.

22. Multiply two hundredths by two ten-thousandths.

23. Multiply seven hundredths by seven millionths.

24. Multiply two hundred and one thousandths by three millionths.

25. Multiply five hundred-thousandths by six thousandths.

26. Multiply four millionths by sixty-three thousandths.

27. Multiply a hundred by a hundred-thousandth.

28. Multiply one million by one millionth.

29. Multiply one millionth by one billionth.

192. To multiply *Decimals* by 10, 100, 1000, etc.

30. Multiply .43215 by 1000.

ANALYSIS.—Removing a figure *one place* to the *left*, we have seen multiplies it by 10. But moving the decimal point *one place* to the right has the same effect on the position of the figures; therefore, it multiplies them by 10. For the same reason, moving the decimal point two places to the right multiplies the figures by 100, and so on. In the given example the multiplier is 1000; we therefore move the decimal point *three* places to the right, and have 432.15, the answer required. Hence, the

RULE.—*Move the decimal point as many places toward the right as there are ciphers in the multiplier.* (Art. 181.)

31. Multiply 32.0505 by 100.

32. Multiply 8.00356 by 1000.

33. Multiply 0.000243 by 10000.

34. Multiply 0.000058 by 100000.

35. Multiply 0.000005 by 1000000.

36. If a newsboy makes $0.005 on each paper, what is his profit on 10000 papers?

37. What is the profit on 100000 eggs, at $0.006 apiece?

38. If a farmer gives 4.25 bushels of apples for one yard of cloth, how many bushels should he give for 6.5 yards?

39. If a man walks 3.75 miles an hour, how far will he walk in 17.5 hours?

DIVISION OF DECIMALS.

193. **To divide one Decimal by another.**

1. How many times .2 in .8?

ANALYSIS.—These decimals have a com. denom.; hence, we divide as in Common Fractions, and the quotient is a *whole number*. (Art. 169, *n.*)

.2).8
Ans. 4 times.

2. How many times .08 in .7?

ANALYSIS.—Reduced to a common denominator, the given decimals become .08 and .70. Now .08 is in .70, 8 times, and .06 rem. Put the 8 in *units'* place. Reduced to the next lower order, .06=.060, and .08 is in .060, .7 of a time, and .004 rem. Put the 7 in *tenths'* place. Finally, .08 is in .0040, .05 of a time. Write the 5 in *hundredths'* place. *Ans.* 8.75 times.

.08).7000
Ans. 8.75

3. How many times is .5 contained in .025?

ANALYSIS.—Reduced to a common denominator, .5=.500, and .025=.025. Since .500 is not contained in .025, put a cipher in *units'* place, and reduce to the next lower order. But .500 is not contained in .0250. Put a cipher in *tenths'* place, and reducing to the next order, .500 is in .02500, .05 of a time. Write the 5 in *hundredths'* place.

.500).02500
Ans. 0.05 t.

REM.—When two decimals have a *com. denom.* the *quotient* figures thence arising are *whole numbers*, as in common fractions. (Art. 169, *n.*)

If ciphers are annexed to the remainder, the *next* quotient figure will be *tenths*, the second *hundredths*, &c. Hence, the

RULE.—*Reduce the decimals to a common denominator, and divide the numerator of the dividend by that of the divisor, placing a decimal point on the right of the quotient.*

Annex ciphers to the remainder, and divide as before. The figures on the left of the decimal point are whole numbers; those on the right, decimals.

Or, divide as in whole numbers, and from the right of the quotient, point off as many figures for decimals as the decimal places in the dividend exceed those in the divisor.

NOTES.—1. If there are not figures enough in the quotient for the decimals required by the second method, *prefix* ciphers.

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193. How divide decimals? If there is a remainder? *Rem.* When two decimals have a com. denom., what is the quotient? When ciphers are annexed to the remainder, what? When the 2d method is used, how point off?

2. If there is a remainder after the required number of decimals is found, annex the sign + to the quotient.

2. Divide .063 by 9.
3. Divide .856 by .214.
4. Divide 642 by 1.07.
5. Divide 4.57 by 11.

Perform the following divisions:

6. 78.4 ÷ 2.6.
7. 8.45 ÷ 3.5.
8. 1.262 ÷ 9.7.
9. .4625 ÷ .65.
10. 97.68 ÷ 100.
11. 6.75 ÷ 1000.
12. .576 ÷ 10000.
13. 45.30 ÷ 3020.
14. .03753 ÷ .00006.
15. 12 ÷ 1.2.
16. 1.2 ÷ .12.
17. .12 ÷ 12.
18. .00001 ÷ 5.
19. .00005 ÷ .1.
20. .0003 ÷ .000006.
21. .27 ÷ 1000000.

22. If 2.25 yards of cloth make 1 coat, how many coats can be made of 103.5 yards?

23. How many rods in 732.75 feet, at 16.5 feet to a rod?

24. At $18.75 apiece, how many stoves can be bought for $506.25 ?

194. To Divide Decimals by 10, 100, 1000, etc.

25. What is the quotient of 846.25 divided by 100?

Analysis.—Moving the *decimal point one place* to the *left* divides a number by 10. For the same reason, moving the decimal point *two places* to the left, divides it by 100. In the given question, removing the decimal point two places to the left, we have 8.4625, the answer required. Hence, the

Rule.—*Move the decimal point as many places toward the left as there are ciphers in the divisor.* (Art. 181.)

26. Divide 4375.3 by 1000.
27. Divide 638.45 by 10000.
28. Divide 2.53 by 100000.
29. Divide .5 by 1000000.

30. Bought 1000 pins for $.5: what was the cost of each?

31. If a man pays $475 for 10000 yards of muslin, what is that a yard?

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194. How divide decimals by 10, 100, 1000, etc.?

UNITED STATES MONEY.

195. ***United States Money*** is the *national currency* of the United States, and is often called *Federal Money.* It is founded upon the *Decimal Notation,* and is thence called *Decimal Currency.*

Its denominations are *eagles, dollars, dimes, cents,* and *mills.*

TABLE.

10	mills (*m.*)	are 1 cent, - - - - -	*ct.*
10	cents	" 1 dime, - - - - -	*d.*
10	dimes	" 1 dollar, - -	*dol.* or $.
10	dollars	" 1 eagle, - - - -	*E*

NOTATION OF UNITED STATES MONEY.

196. The ***Dollar*** is the *unit;* hence, dollars are whole numbers, and have the sign ($) prefixed to them.

In $1 there are 100 *cents;* therefore *cents* are *hundredths* of a dollar, and occupy *hundredths'* place.

Again, in $1 there are 1000 *mills;* hence, mills are *thousandths* of a dollar, and occupy *thousandths'* place.

NOTES.—1. The *origin* of the sign ($) has been variously explained. Some suppose it an *imitation* of the two pillars of Hercules, connected by a scroll found on the old Spanish coins. Others think it is a modified *figure* 8, stamped upon these coins, denoting 8 reals, or a dollar.

A more plausible explanation is that it is a *monogram* of United States, the curve of the U being dropped, and the S written over it.

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195. What is United States money? Upon what founded? What sometimes called? The denominations? Repeat the Table. 196. What is the unit? What are dimes? Cents? Mills? *Note.* What is the origin of the sign $? The meaning of dime? Cent? Mill? 197. How write United States money? *Note.* How are eagles and dimes expressed? If the number of cents is less than 10, what must be done? Why? If the mills are 5 or more, what considered? If less, what?

2. The term *Dime* is the French *dixième*, a *tenth*; *Cent* from the Latin *centum*, a *hundred*; and *Mill* from the Latin *mille*, a *thousand*.

3. United States money was established by act of Congress, in 1786. Previous to that, pounds, shillings, pence, etc., were in use.

197. To express United States money, decimally.

Ex. 1. Let it be required to write 75 dollars, 37 cents, and 5 mills, decimally.

ANALYSIS.—Dollars are *integers*; we therefore write the 75 dollars as a *whole number*, prefixing the ($), as $75. Again, *cents* are *hundredths* of a dollar; therefore we write the 37 cents in the first two places on the right of the dollars, with a *decimal point* on their left as $75.37. Finally, *mills* are *thousandths* of a dollar; and writing the 5 mills in the first place on the right of cents, we have $75.375, the decimal required. (Art. 179.) Hence, the

RULE.—*Write dollars as whole numbers, cents as hundredths, and mills as thousandths, placing the sign* ($) *before dollars, and a decimal point between dollars and cents.*

NOTES.—1. Eagles and dimes are not used in business calculations; the former are expressed by *tens of dollars*; the latter by *tens of cents*. Thus, 15 eagles are $150, and 6 dimes are 60 cents.

2. As *cents* occupy *two* places, if the number to be expressed is *less* than 10, a *cipher* must be prefixed to the figure denoting them.

3. Cents are often expressed by a common fraction having 100 for its denominator. Thus, $7.38 is written $\$7\frac{38}{100}$, and is read "7 and $\frac{38}{100}$ dollars."

Mills also are sometimes expressed by a *common fraction*. Thus, 12 cts. and 5 mills are written $0.125, or $\$0.12\frac{1}{2}$; 18 cts. and $7\frac{1}{2}$ mills are written $0.1875, or $\$0.18\frac{3}{4}$, etc.

4. In business calculations, if the mills in the *result* are 5 or more, they are considered a *cent*; if less than 5, they are omitted.

1. Write forty dollars and forty cents.

2. Write five dollars, five cents and five mills.

3. Write fifty dollars, sixty cents, and three mills.

4. Write one hundred dollars, seven cents, five mills.

5. Write two thousand and one dollars, eight and a half cents.

6. Write 7 hundred and 5 dollars and one cent.

7. Write 84 dollars and $12\frac{1}{2}$ cents.

8. Write 5 and a half cents; 6 and a fourth cents; 11 and three-fourths cents, decimally.

9. Write 7 dollars, 31 and a fourth cents, decimally.
10. Write 19 dollars, $31\frac{1}{4}$ cents, decimally.
11. Write 14 eagles and 8 dimes, decimally.
12. Write 5 eagles, 5 dollars, 5 cents, and 5 mills.

198. To read United States money, expressed decimally.

RULE.—*Call the figures on the left of the decimal point, dollars; those in the first two places on the right, cents; the next figure, mills; the others, decimals of a mill.*

The expression $37.52748 is read 37 dollars, 52 cts., 7 mills, and 48 hundredths of a mill.

NOTE.—*Cents* and *mills* are sometimes read as *decimals* of a dollar. Thus, $7.225 may be read 7 and 225 thousandths dollars.

Read the following:

1. $204.30	5. $78.104	9. $1100.001
2. $360.05	6. $90.007	10. $7.3615
3. $500.19	7. $1001.10	11. $8.0043
4. $61.035	8. $1010.01	12. $10.00175

REDUCTION OF UNITED STATES MONEY.

CASE I.

199. To reduce *Dollars* to Cents and Mills.

1. In $67 how many cents?

ANALYSIS.—As there are 100 cents in every dollar, there must be 100 times as many cents as dollars in the given sum. But to multiply by 100 we annex two ciphers. (Art. 57.)

OPERATION.
$67 \times 100 = 6700$
Ans. 6700 cts.

2. In $84, how many mills?

ANALYSIS.—For a like reason, there are 1000 times as many mills as dollars, or 10 times as many mills as cents. Hence, the

$84 \times 1000 = 84000$
Ans. 84000 mills.

RULE.—*To reduce dollars to cents, multiply them by* 100.
To reduce dollars to mills, multiply them by 1000.
To reduce cents to mills, multiply them by 10.

---

198. How read United States money?

NOTE.—Dollars and cents are reduced to cents; also dollars, cents, and mills, to mills, by erasing the sign of dollars ($), and the decimal point.

2. Reduce $135 to cents.
3. Reduce $368 to mills.
4. Reduce $100 to mills.
5. Reduce $1680 to cents.
6. Reduce 97 cents to mills.
7. Reduce $356.25 to cents.
8. Reduce $780.375 to mills.
9. Reduce $800.60 to mills.

CASE II.

200. To reduce Cents and Mills to Dollars.

1. In 6837 cents how many dollars?

ANALYSIS.—Since 100 cents make 1 dollar, 6837 cents will make as many dollars as 100 is contained times in 6837, and 6837÷100=68, and 37 cents over. (Art. 79.)

OPERATION.
1)68|37
$68.37 *Ans.*

In like manner any *number of mills* will make as many dollars as 1000 is contained times in that number. Hence, the

RULE.—*To reduce cents to dollars, divide them by* 100.
To reduce mills to dollars, divide them by 1000.
To reduce mills to cents, divide them by 10. (Art. 194.)

NOTE.—The *first two figures* cut off on the right are *cents*, the next one *mills*.

2. Reduce 1625 cts. to dols.
3. Reduce 8126 m's to dols.
4. Reduce 10000 m's to dols.
5. Reduce 9265 m's to cents.
6. Change 89567 cts. to dols.
7. Change 94283 m's to dols.
8. Change 85600 m's to cents.
9. Change 263475 m's to dols.

10. A farmer sold 763 apples, at a cent apiece: how many dollars did they come to?

11. A market woman sold 5 hundred eggs, at 2 cents each: how many dollars did she receive for them?

12. A fruit dealer sold 675 watermelons at 1000 mills apiece: how many dollars did he receive for them?

---

199. How reduce dollars to cents? To mills? How cents to mills? *Note.* How dollars to cents and mills? 200. How reduce cents and mills to dollars? *Note.* What are the figures cut off?

ADDITION OF UNITED STATES MONEY.

201. *United States Money,* we have seen, is founded upon the *decimal notation;* hence, all its operations are precisely the same as the corresponding operations in *Decimal Fractions.*

202. To find the *Amount* of two or more Sums of Money.

1. What is the sum of \$45.625; \$109.07; and \$450.137?

ANALYSIS. — Units of the same order only can be added together. For convenience in adding, we therefore write dollars under dollars, cents under cents, etc., with the decimal points in a perpendicular line. Beginning at the right, we add the columns separately, placing the *decimal point* in the *amount* under the points in the numbers added, to distinguish the dollars from cents and mills. (Art. 197.) Hence, the

OPERATION.

$$\begin{array}{r} \$45.625 \\ 109.07 \\ \underline{450.137} \\ \$604.832 \end{array}$$

RULE.— *Write dollars under dollars, cents under cents, etc., and proceed as in Addition of Decimals.*

NOTE.—If any of the given numbers have *no cents,* their place should be supplied by ciphers.

2. A man paid \$13.62½ for a barrel of flour, \$25.25 for butter, \$9.75 for coal: what did he pay for all?

3. A farmer sold a span of horses for \$457.50, a yoke of oxen for \$235, and a cow for \$87.75: how much did he receive for all?

4. What is the sum of \$97.87½; \$82.09; \$20.12½?

5. What is the sum of \$81.06; \$69.18; \$67.16; \$7.13?

6. What is the sum of \$101.101; \$210.10½; \$450.27¼?

7. Add \$7 and 3 cents; \$10; 6¼ cents; 18¾ cents.

8. Add \$68 and 5 mills; 87½ cents; 31¼ cents.

9. A man paid \$8520.75 for his farm, \$1860.45 for his stock, \$1650.45 for his house, and \$1100.07 for his furniture: what was the cost of the whole?

---

201. What is said of operations in United States Money? 202. How add United States Money? *Note.* If any of the numbers have no cents, how proceed?

10. A lady paid $\$31\frac{1}{2}$ for a dress, $\$15\frac{1}{4}$ for trimmings. and $\$7\frac{3}{4}$ for making: what was the cost of her dress?

11. A grocer sold goods to one customer amounting to $17.50, to another $\$30.18\frac{3}{4}$, to another $\$21.06\frac{1}{4}$, and to another $51.73: what amount did he sell to all?

12. A young man paid $31.58 for a coat; $11.63 for a vest, $14.11 for pants, $10.50 for boots, $\$7\frac{1}{4}$ for a hat, and $\$1\frac{3}{4}$ for gloves: what did his suit cost him?

SUBTRACTION OF UNITED STATES MONEY.

203. To find the *Difference* between two Sums of Money.

1. A man having $\$1343.87\frac{1}{2}$, gave $750.69 to the Patriot Orphan Home: how much had he left?

ANALYSIS.—Since the same orders only can be subtracted one from the other, for convenience we write dollars under dollars, cents under cents, etc. Beginning at the right, we subtract each figure separately, and place the decimal point in the remainder under that in the subtrahend, for the same reason as in subtracting decimals. (Art. 190.) Hence, the

OPERATION.

$$\begin{array}{r} \$1343.875 \\ 750.69 \\ \hline \$593.185 \end{array}$$

RULE.—*Write the less number under the greater, dollars under dollars, cents under cents, etc., and proceed as in Subtraction of Decimals.* (Art. 190.)

NOTE.—If only one of the given numbers has *cents*, their place in the other should be supplied by ciphers.

2. A man having $861.73, lost $328.625 in gambling: how much had he left?

3. If a man's income is $1750, and his expenses $\$1145.37\frac{1}{2}$, how much does he lay up?

4. A gentleman paid $1500 for his horses, and $\$975\frac{1}{2}$ for his carriage: what was the difference in the cost?

5. A merchant paid $\$2573\frac{1}{4}$ for a quantity of tea, and sold it for $\$3158\frac{1}{2}$: how much did he make?

---

203. How subtract United States money? *Note.* If either number has no cents, what is to be done?

6. If I pay $5268 for a farm, and sell it for $4319.67, how much shall I lose by the operation?

7. From 673 dols. 6¼ cents, take 501 dols. and 10 cents.

8. From 1011 dols. 12½ cents, take 600 dols. and 5 cents.

9. From 1 dollar and 1 cent, subtract 5 cents 5 mills.

10. From 7½ dols. subtract 7¼ cents.

11. From 500 dols. subtract 5 dols. 5 cents and 5 mills.

12. A young lady bought a shawl for $35¼, a dress for $23½, a hat for $10¾, a pair of gloves for $1¾, and gave the clerk a hundred dollar bill: how much change ought she to receive?

MULTIPLICATION OF UNITED STATES MONEY.

204. To multiply United States Money.

1. What will 9½ yards of velvet cost, at $18¼ per yard?

ANALYSIS.—9½ yards will cost 9½ times as much as 1 yard. Now 9½ yds.=9.5 yds., and $18¼=$18.25. We multiply in the usual way, and since there are *three decimal figures* in both factors, we point off three in the product for the same reason as in multiplying decimals. (Art. 191.) Hence, the

```
  $18.25
     9.5
  ------
    9125
  16425
  ------
$173.375
```

RULE.—*Multiply, and point off the product, as in Multiplication of Decimals.* (Art. 191.)

NOTES.—1. In *United States Money*, as in simple numbers, the *multiplier* must be considered an *abstract* number.

2. If either of the given factors contains a *common* fraction, it is generally more convenient to change it to a *decimal*.

2. What will 65 barrels of flour cost, at $11.50 a barrel?

3. What will 145.3 pounds of wool cost, at $1.08 a pound?

4. What cost 75 pair of skates, at $3.87½ a pair?

5. What cost 63 gallons of petroleum, at 95½ cents a gallon?

6. At $4.17½ a barrel, what will 110 barrels of apples cost?

---

204. How multiply United States money? *Note.* What must the multiplier be?

Perform the following multiplications:

7. $\$51\frac{3}{4} \times 1.9\frac{1}{2}$.
8. $\$6.07\frac{1}{2} \times 2.3\frac{1}{4}$.
9. $\$10.05 \times 6\frac{3}{4}$.
10. $\$100.031 \times 3.105$.
11. $\$765.40\frac{1}{2} \times 6.05$.
12. $\$.07 \times .008$.
13. $\$.005 \times 1000$.
14. $\$1.011 \times .001$.

15. What cost 35 pounds of raisins, at $18\frac{3}{4}$ cents a pound?

16. What cost 51 pounds of tea, at $\$1.15\frac{1}{2}$ a pound?

17. What cost 150 gallons of milk, at $37\frac{1}{2}$ cts. a gallon?

18. What cost 12 dozen penknives, at $31\frac{1}{4}$ cents apiece?

19. What cost 13 boxes of butter, each containing $16\frac{1}{2}$ pounds, at $37\frac{1}{2}$ cents a pound?

20. A merchant sold 12 pieces of cloth, each containing 35 yards, at $\$4\frac{1}{2}$ a yard: what did it come to?

21. What is the value of 21 bags of coffee, each weighing 55 pounds, at $47\frac{1}{2}$ cents a pound?

22. What cost 55 boxes of lemons, at $\$3\frac{1}{4}$ a box?

23. The proprietor of a livery stable took 37 horses to board, at $32 a month: how much did he receive in 12 months?

DIVISION OF UNITED STATES MONEY.

205. *Division of United States money,* like simple division, embraces *two classes* of problems:

First. Those in which both the divisor and dividend are *money.*

Second. Those in which the dividend is *money* and the divisor is an *abstract number,* or regarded as such.

In the *former,* a given sum is to be divided into *parts,* the *value* of each part being equal to the *divisor;* and the *object* is to ascertain the *number* of parts. Hence, the quotient is *times* or an *abstract* number. (Art. 63, *a.*)

In the *latter,* a given sum is to be divided into a given number of *equal parts* indicated by the divisor; and the *object* is to ascertain the *value* or *number* of dollars in each part. Hence, the quotient is *money,* the same as the dividend. (Art. 63, *b.*)

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205. What does Division of U. S. money embrace? The first class? The second?

206. To Divide Money by Money, or by an Abstract Number

1. How many barrels of flour, at $9.56 a barrel, can be bought for $262.90?

ANALYSIS.—At $9.56 a barrel, $262.90 will buy as many barrels as $9.56 are contained times in $262.90, or 27.5 barrels. As the divisor and dividend both contain *cents*, they are the same denomination; therefore, the quotient 27, is a *whole* number. Annexing a cipher to the remainder, the next quotient figure is *tenths*. (Art. 193, *Rem.*) Hence, the

OPERATION.

```
$9.56)$262.90(27.5
       1912
       ----
        7170
        6692
        ----
         4780
         4780
         ----
```

RULE.—*Divide, and point off the quotient, as in Division of Decimals.* (Arts. 64, 193.)

NOTES.—1. In business matters it is rarely necessary to carry the quotient beyond mills.

2. If there is a remainder after all the figures of the dividend have been divided, *annex ciphers* and continue the division as far as desirable, considering the ciphers annexed as decimals of the dividend.

2. If 63 caps cost $123.75, what will 1 cap cost?

3. If 165 lemons cost $8.25, what will 1 lemon cost?

4. Paid $852.50 for 310 sheep: what was that apiece?

5. If 356 bridles cost $1040, what will 1 bridle cost?

Find the results of the following divisions:

6. $17.50÷$.175	10. $.005÷$.05
7. $365.07÷$1.01	11. $101÷$1.01
8. $1000÷25 cts.	12. $500÷$.05
9. $.25÷$25	13. $1200÷$.002

14. If 410 chairs cost $1216, what will 1 chair cost?

15. At $9¼ a ton, how much coal will $8560 buy?

16. When potash is $120.35 per ton, how much can be bought for $35267.28?

17. If 2516 oranges cost $157.25, what will one cost?

18. Paid $273.58 for 5000 acres of land: what was that per acre?

---

206. How divide money? *Note.* If there is a remainder, how proceed?

COUNTING-ROOM EXERCISES.

207. The ***Ledger*** is the *principal book* of accounts kept by business men. It contains a *brief record* of their monetary transactions. All the items of the *Day Book* are transferred to it in a condensed form, for reference and preservation, the debits (marked Dr.) being placed on the left, and the credits (marked Cr.) on the right side.

208. ***Balancing an Account*** is finding the *difference* between the debits and credits.

Balance the following Ledger Accounts:

(1.) Dr.	(1.) Cr.	(2.) Dr.	(2.) Cr.
$8645.23	$2347.19	$76421.26	$43261.47
160.03	141.07	2406.71	728.23
2731.40	2137.21	724.05	6243.41
4242.25	3401.70	86025.21	75.69
324.31	2217.49	9307.60	53268.75
3313.17	168.03	685.17	3102.84
429.18	329.17	61.21	456.61
4536.20	2334.67	7824.28	32921.70
641.46	4506.41	60708.19	6242.09
182.78	239.06	1764.85	20374.34
2634.29	2067.12	32846.39	290.25
3727.34	675.89	385.72	4536.68
840.68	1431.07	23.64	42937.74
6219.77	4804.31	6072.77	819.35
4727.91	6536.48	50641.39	30769.27
23.45	720.34	2062.40	8506.35
650.80	36.45	301.53	97030.48
3267.03	7050.63	26.47	876.20
680.47	904.38	805.03	89030.50
64.38	78.05	2403.67	384.06
8350.60	307.63	10708.79	70207.48

207. What is the ledger? What does it contain? How balance an account?

MAKING OUT BILLS.

209. A ***Bill*** is a *written statement* of goods sold, services rendered, etc., and should always include the *price* of each item, the *date*, and the *place* of the transaction.

A ***Bill is Receipted,*** when the person to whom it is due, or his agent, writes on it the words "Received payment," and his name.

209, *a*. A ***Statement of Account*** is a copy of the *items* of its *debits* and *credits*.

NOTES.—1. The abbreviation *Dr.* denotes *debit* or *debtor; Cr., credit* or *creditor; per, by;* the character @, stands for *at*. Thus, 5 books, @ 3 shillings, signifies, 5 books the price of which is 3 shillings apiece.

2. The learner should carefully observe the *form* of Bills, the *place* where the *date* and the *names* of the buyer and seller are placed, the *arrangement* of the items, etc.

Copy and find the amount due on the following Bills:

(1.) PHILADELPHIA, *March 1st*, 1872.

HON. HENRY BARNARD,

TO J. B. LIPPINCOTT & CO., *Dr.*

For 6 Webster's Dictionaries, 4to.,	@ $12.50	$75	00		
" 8 reams paper,	@ $3.75	30	00		
" 36 slates,	@ $0.27	9	72	$114	72
CREDIT.					
By 10 School Architecture,	@ $5.45	$54	50		
" 8 Journals of Education,	@ $3.75	30	00	$84	50
	Balance,			$30	22

209. What is a bill? 209, *a*. A statement of account? Where is the date placed? (See form.) The name of buyer? Seller? How is the payment of a bill shown? *Note.* What does Dr. denote? Cr.?

(2.)

NEW YORK, *April 2d*, 1871.

MRS. W. C. GARFIELD,

Bought of A. T. STEWART.

28 yds. silk,	@ $3.50				
35 yds. table linen,	@ $2.12½				
6 pair gloves,	@ $1.75				
43½ yds. muslin,	@ $0.33				
1 doz. pr. cotton hose,	@ $0.80				
Amount,					

Received Payment,

A. T. STEWART.

(3.)

NEW ORLEANS, *May 5th*, 1872.

GEORGE PEABODY, ESQ.,

Bought of JACOB BARKER.

Feb. 1	75 bls. pork,	@ $25.00				
17	160 bls. flour,	@ $8.75				
March 3	500 gals. molasses,	@ $0.93				
25	75 boxes raisins,	@ $5.37½				
April 30	256 gals. kerosene,	@ $0.87½				
	Amount,					

Received Payment,

J. BARKER,
By JOHN HOWARD.

(4.)

BOSTON, *May 23d*, 1871.

MESSRS. FAIRFIELD & WEBSTER,

TO H. W. HALL, *Dr.*

For 319 yds. broadcloth,	@ $5.87½				
" 416 yds. cassimere,	@ $2.10				
" 1110 yds. muslin,	@ $0.28				
" 265 yds. ticking,	@ $0.47				
Amount,					

Received Payment by Note,

GEORGE ANDERSON,
For H. W. HALL.

5. James Brewster bought of Horace Foote & Co., New Haven, May 16th, 1867, the following items: 175 pounds of sugar, at 17 cents; 5 gallons of molasses, at 63 cents; 3 boxes of raisins, at $6\frac{1}{2}$; 15 pounds of tea, at $1\frac{1}{4}$: what was the amount of his bill?

6. George Bliss & Co. bought of James Henry, Cincinnati, June 3d, 1867, 1625 bushels of wheat, at $1.95; 130 barrels of flour, at $11; 265 pounds of tobacco, at 48 cents; and 1730 pounds of cotton, at 27½ cents: what was the amount of their bill?

7. Pinkney & Brother sold to Henry Rutledge, Richmond, July 15th, 1867, 1 shawl, $450; 19 yards of silk, at $3.63; 16 yards point lace, at $11; 6 pair gloves, at $2.05; and 12 pair hose, at 87½ cents: required the amount.

8. Bought 37 Greek Readers, at $1.85; 60 Greek Grammars, at $1.45; 75 Latin Grammars, at $1.38; 25 Virgils, at $3.62; 14 Iliad, at $3.28: required the amount.

BUSINESS METHODS.

210. *Calculations* in United States money are the same as those in *Decimals;* consequently, in ordinary business transactions no additional rules are required. By *Reflection* and *Analysis*, however, the operations may often be greatly abbreviated. (Arts. 100, 211–214.)

1. What will 265 hats cost, at $7 apiece?

ANALYSIS.—265 hats will cost 265 times as much as one hat; and $7 × 265 = $1855.

Or thus: At $1 each, 265 hats will cost $265; hence, at $7, they will cost 7 times $265, and $265 × 7 = $1855.

NOTES.—1. Here the *price* of *one* and the *number* of articles are given, to find their cost; hence, it is a question in Multiplication. (Art. 51.)

2. A mechanic sold 12 ploughs for $114: what was the price of each?

ANALYSIS.—The price of 1 plough is $\frac{1}{12}$ as much as that of 12 ploughs; and $114 ÷ 12 = $9.50.

NOTE.—2. Here the *number* of articles and their *cost* are given, to find the *price* of one, which is simply a question in Division.

3. A farmer paid $921.25 for a number of sheep valued at $2.75 apiece: how many did he buy?

ANALYSIS.—At $2.75 apiece, $921.25 will buy as many sheep as $2.75 is contained times in $921.25; and $921.25 ÷ $2.75 = 335.

NOTE.—3. Here the *whole cost* and the *price* of *one* are given, to find the *number*, which is also a question in Division.

4. What will 287 chairs cost, at $2¾ apiece?

5. What will 75 sofas cost, at $57.50 apiece?

6. What will 350 table books cost, at 12½ cents apiece?

7. What cost 119 tons of coal, at $7½ a ton?

8. Paid $84 for 252 pounds of butter: what was that a pound?

9. If 8¾ cords of wood cost $46.06, what will 1 cord cost?

10. If 46 acres of land are worth $1449, what is 1 acre worth?

11. How many Bibles at $5½ can be bought for $561?

12. How many vests at $8½, can be bought for $1262.25?

13. How much will it cost a man a year for cigars, allowing he smokes 5 a day, averaging 6¼ cents each, and 365 days to a year?

14. A lady bought 18 yards of silk, at $2.12½ a yard; 14 yards of delaine at 63 cents; 12 skeins of silk, at 6¼ cents a skein: what was the amount of her bill?

15. What will it cost to build a railroad 265 miles long, at $11350 per mile?

16. A farmer sold his butter at 34 cents a pound, and received for it $321.67½: how many pounds did he sell?

17. The cheese made of the milk of 53 cows in a season was sold for $1579.40, at 20 cents a pound: how many pounds were sold; and what the average per cow?

18. A merchant sold 3 pieces of muslin, each containing 45 yards, at 40½ cents a yard, and took his pay in wheat at $1½ a bushel: how much wheat did he receive?

19. If a man earns $\$9\frac{1}{2}$ a week, and spends $\$4\frac{3}{4}$, how much will he lay up in 52 weeks?

20. If a man drinks 3 glasses of liquor a day, costing 10 cents a glass, and smokes 5 cigars at 6 cents each, how many acres of land, at $\$1\frac{1}{4}$ an acre, could he buy for the sum he pays for liquor and cigars in 35 years, allowing 365 days to a year?

21. A grocer bought 175 boxes of oranges, at $\$6.37\frac{1}{2}$, and sold the lot for \$637.50: what did he make, or lose?

211. To find the Cost of a number of articles, the Price of one being an *Aliquot part* of \$1.

1. What will 168 melons cost, at $12\frac{1}{2}$ cents apiece?

ANALYSIS.—By inspection the learner will perceive that $12\frac{1}{2}$ cents $= \frac{1}{8}$ dollar. Now, at 1 dollar apiece, 168 melons will cost \$168. But the price is $\frac{1}{8}$ of a dollar apiece; therefore, they will cost $\frac{1}{8}$ of \$168, which is \$21. Hence, the

OPERATION.

$$8\overline{)168}$$
$$\$21 \textit{ Ans.}$$

RULE.—*Take such a part of the given number as indicated by the aliquot part of* \$1, *expressing the price of one; the result will be the cost.* (Arts. 105, 270.)

2. What will 265 pounds of raisins cost, at 25 cents?
3. What cost 195 pounds of butter, at $33\frac{1}{3}$ cents?
4. What cost 352 skeins of silk, at $6\frac{1}{4}$ cents?
5. What cost 819 shad, at 50 cents apiece?
6. At $12\frac{1}{2}$ cents a dozen, what will 100 dozen eggs cost?
7. At 20 cents a yard, what will 750 yards of calico cost?
8. At $16\frac{2}{3}$ cents, what will 1250 pine apples come to?
9. What cost 1745 yards of delaine, at $33\frac{1}{3}$ cents?
10. At $8\frac{1}{3}$ cents apiece, what will 375 pencils cost?
11. At 25 cents apiece, what will 1167 slates cost?
12. How much will 175 dozen eggs cost, at 20 cts. a doz.?
13. What will 219 slates cost, at $12\frac{1}{2}$ cents each?
14. At $16\frac{2}{3}$ cts. apiece, what will 645 melons cost?
15. At $33\frac{1}{3}$ cts. apiece, how much will 347 penknives cost?

212. To find the Cost of a number of articles, the Price of one being $1 plus an Aliquot part of $1.

16. What cost 275 pounds of tea, at $1.25 a pound?

ANALYSIS.—The price of 1 pound $1.25=$1+$¼. At $1 a pound, the cost of 275 pounds would be $275. But the price is $1+$¼; therefore 275 pounds will cost once $275+¼ of $275=$343.75. Hence, the

$$\begin{array}{r} 4)\underline{275} \\ \underline{68.75} \\ \$343.75 \end{array}$$

RULE.—*To the number of articles, add ½, ⅓, ¼ of itself, as the case may be; the sum will be their cost.*

17. At $1.25 per acre, what will 168 acres of land cost?

18. At $1.50 apiece, what will 365 chairs cost?

19. What cost 512 caps, at $1⅛ apiece?

20. At $1.16⅔ apiece, what cost 12 dozen fans?

21. A man sold 200 overcoats at a profit of $1.20 apiece: how much did he make?

213. To find the Number of articles, the Cost being given, and the Price of one an Aliquot part of $1.

22. How many spellers, at 12½ cents apiece, can be bought for $25?

ANALYSIS.—12½ cents=$⅛; therefore $25 will buy as many spellers as there are *eighths* in $25, and 25÷⅛=200. *Ans.* 200 spellers. Hence, the

$$\$.12\tfrac{1}{2}=\$\tfrac{1}{8}$$
$$\$25\div\$\tfrac{1}{8}=200$$

RULE.—*Divide the cost of the whole by the aliquot part of* $1, *expressing the price of one.*

23. How many yards of flannel, at 50 cents, can be bought for $6.83?

24. How many pounds of candy, at 33⅓ cents, can be purchased with $375?

25. Paid $450 for cocoa-nuts which were 25 cents each: how many were bought?

26. At 20 cents each, how many pine-apples can be purchased for $538?

---

211. How find the cost of articles by aliquot parts of $1? 212. How when the price is $1 plus an aliquot part of $1? 213. How find the number of articles, when cost is given, and the price of one is an aliquot part of $1?

8

214. To find the Cost of articles, sold by the 100, or 1000.

1. What will 1765 oranges cost, at $6.125 a hundred?

OPERATION.
$6.125
1765
$108.10625

ANALYSIS.—At $6.125 for each orange, 1765 oranges would cost 1765 times $6 125, and $6.125 × 1765 = $10810.625. But the price is $6.125 for a hundred; therefore this product is 100 times too large. To correct it, we divide by 100, or remove the decimal point *two* places to the left. (Art. 181.)

In like manner if the price is given by the 1000, we multiply the *price* and *number* of articles together, and remove the decimal point in the product *three* places to the left, which divides it by 1000. Hence, the

RULE.—*Multiply the price and number of articles together, and divide the product by* 100 *or* 1000, *as the case may require.* (*Art.* 181.)

NOTE.—In business transactions, the letter C is sometimes pui for *hundred;* and M for *thousand.*

2. What will 4532 bricks cost, at $17.25 per M.?

3. What cost 1925 pounds of maple sugar, at $12.50 per hundred?

4. What cost 25268 feet of boards, at $31.25 per thousand?

5. At $5¾ per hundred, how much will 20345 pounds of flour come to?

6. At $6.25 per hundred, what will 19263 pounds of codfish come to?

7. What cost 10250 envelopes, at $3.95 per thousand?

8. What cost 1275 oysters, at $1.75 per hundred?

9. What cost 13456 shingles, at $7.45 per M.?

10. What cost 82 rails, at $5½ a hundred?

11. What cost 93 pine apples, at $15.25 a hundred?

12. What cost 355 feet of lumber, at $45 per thousand?

---

214. How find the cost of articles sold by the 100 or 1000? *Note.* What does C stand for? What M?

COMPOUND NUMBERS.

215. ***Simple Numbers*** are those which contain units of *one denomination* only; as, three, five, 2 oranges, 4 feet, etc. (Art. 101.)

216. ***Compound Numbers*** are those which contain units of *two* or *more* denominations of the *same nature;* as, 5 pounds and 8 ounces; 3 yards, 2 feet and 4 inches, etc.

But the expression 2 feet and 4 pounds, is not a compound number; for, its units are of *unlike* nature.

NOTES.—1. *Compound Numbers* are restricted to the divisions of *Money, Weights,* and *Measures,* and are often called *Denominate Numbers.*

2. For convenience of reference, the Compound Tables are placed together. If the teacher wishes to give exercises upon them as they are recited, he will find examples arranged in groups corresponding with the order of the Tables in Arts. 276, 279.

MONEY.

217. ***Money*** is the *measure* or *standard* of *value.* It is often called *currency,* or *circulating medium,* and is of two kinds, *metallic* and *paper.*

Metallic Money consists of *stamped pieces* of metal, called *coins.* It is also called *specie,* or *specie currency.*

Paper Money consists of *notes* or *bills* issued by the Government and Banks, redeemable in *coin.* It is often called *paper currency.*

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215. What are simple numbers? 216. Compound? Give an example of each. *Note.* To what are compound numbers restricted? 217. What is money? Metallic money? Paper money?

UNITED STATES MONEY.

218. ***United States Money*** is the *national currency* of the United States, and is often called *Federal Money.* Its denominations are *eagles, dollars, dimes, cents,* and *mills..*

TABLE.

10	mills (*m*)	are	1 cent, - - - - -	*ct.*
10	cents	"	1 dime, - - - - -	*d.*
10	dimes	"	1 dollar, - - - -	*dol.* or $.
10	dollars	"	1 eagle - - - - -	*E.*

219. The ***Metallic Currency*** of the *United States* consists of *gold* and *silver* coins, and the *minor* coins.*

1. The *gold* coins are the *double eagle, eagle, half eagle, quarter eagle, three dollar* piece, and *dollar.*

The dollar, at the standard weight, is the *unit* of *value.*

2. The *silver* coins are the *dollar, "trade" dollar, half dollar, quarter dollar,* and *dime.*

3. The *minor coins* are the 5-*cent* and 3-*cent* pieces, and the *cent.*

220. The ***weight*** and ***purity*** of the coins of the United States are regulated by the laws of Congress.

1. The *standard weight* of the gold dollar is 25.8 gr.; of the quarter eagle, 64.5 gr.; of the 3-dollar piece, 77.4 gr.; of the half eagle, 129 gr.; of the eagle, 258 gr.; of the double eagle, 516 gr.

2. The *weight* of the dollar is 412½ grains, Troy; the "trade" dollar, 420 grains; the half dollar, 12½ grams; the quarter dollar and dime, *one-half* and *one-fifth* the weight of the half dollar.

3. The *weight* of the 5-cent piece is 77.16 grains, or 5 grams; of the 3-cent piece, 30 grains; of the cent, 48 grains.

4. The *standard purity* of the gold and silver coins is *nine-tenths* pure metal, and *one-tenth* alloy. The *alloy* of gold coins is *silver* and *copper;* the silver, by law, is not to exceed *one-tenth* of the whole alloy. The *alloy* of silver coins is *pure copper.*

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219. Of what does the metallic currency of the United States consist? What are the gold coins? The silver? The minor coins? 220. How is the weight and purity of United States coins regulated?

* Act of Congress, March 3d, 1873.

5. The *five-cent* and *three-cent* pieces are composed of *one-fourth* nickel and *three-fourths* copper; the *cent*, of 95 parts copper and 5 parts of tin and zinc. They are known as *nickel* and *bronze* coins.

NOTES.—1. The *Trade dollar* is so called, because of its intended use for *commercial purposes* among the great Eastern nations.

2. The *gold* coins are a legal tender in *all payments;* the *silver* coins, for any *amount* not exceeding $5 in any one payment; the *minor* coins, for any *amount* not exceeding 25 cents in any one payment.

3. The *diameter* of the nickel 5-cent piece is *two centimeters*, and its *weight* 5 grams. These *magnitudes* present a simple relation of the *Metric* weights and measures to our own.

4. The silver 5-cent and 3-cent pieces, the bronze 2-cent piece, the old copper cent and half-cent are no longer issued. Mills were never coined.

221. The ***Paper Currency*** of the United States consists of *Treasury-notes* issued by the Government known as *Greenbacks*, and *Bank-notes* issued by Banks.

NOTE.—Treasury notes less than $1, are called *Fractional Currency.*

ENGLISH MONEY.

222. ***English Money*** is the *national currency* of Great Britain, and is often called *Sterling Money*. The denominations are *pounds, shillings, pence,* and *farthings.*

TABLE.

4 farthings (*qr.* or *far.*)	are	1 penny,	*d.*
12 pence	"	1 shilling,	*s.*
20 shillings	"	1 pound or sovereign,	£
21 shillings	"	1 guinea,	*g.*

NOTES.—1. The *gold coins* are the *sovereign* and *half sovereign.* The *pound sterling* was never coined. It is a bank note, and is represented by the *sovereign.* Its *legal* value as fixed by Congress is $4.8665. This is its *intrinsic* value, as estimated at the U. S. Mint.

What is the alloy of gold coins? Of silver? 221. Of what does U. S. paper currency consist? 222. English money? The denominations? The Table? *Note.* Is the pound a coin? How represented? What is the value of a pound?

2. The *silver* coins are the crown (5s.); the half-crown (2s. 6d.), the *florin* (2s.); the shilling (12d.); the six-penny, four-penny, and three-penny pieces.

3. The *copper* coins are the penny, half-penny, and farthing.

4. *Farthings* are commonly expressed as *fractions* of a penny, as $7\frac{1}{2}$d.

5. The oblique mark (/) sometimes placed between shillings and pence, is a modification of the long *ſ*.

CANADA MONEY.

223. ***Canada Money*** is the *legal currency* of the Dominion of Canada. Its denominations are *dollars, cents,* and *mills,* which have the same value as the corresponding denominations of U. S. money. Hence, all the operations in it are the same as those in U. S. money.

Note.—The present system was established in 1858.

FRENCH MONEY.

224. ***French Money*** is the *national currency* of France. The denominations are the *franc,* the *decĭme,* and *centĭme.*

TABLE.

10 centĭmes	- - - - are	1 decĭme.
10 decĭmes	- - - - "	1 franc.

Notes.—1. The system is founded upon the *decimal* notation; hence, all the operations in it are the same as those in U. S. money.

2. The *franc* is the *unit;* decimes are *tenths* of a franc, and centimes *hundredths.*

3. *Centimes* by contraction are commonly called *cents.*

4. *Decimes,* like our dimes, are not used in business calculations; they are expressed by *tens* of centimes. Thus, 5 decimes are expressed by 50 centimes; 63 francs, 5 decimes, and 4 centimes are written, 63.54 francs.

5. The *legal* value of the *franc* in estimating duties, is 19.3 cents, its *intrinsic* value being the same.

Shilling? Florin? Crown? How are farthings often written? 223. What is Canada money? Its denominations? Their value? 224. French money? Its denominations? The Table? *Note.* The unit? The value of a franc?

WEIGHTS.

225. ***Weight*** is a *measure* of the force called *gravity*, by which all bodies *tend toward* the *center* of the earth.

226. ***Net Weight*** is the weight of goods without the bag, cask, or box which contains them.

Gross Weight is the weight of goods with the bag, cask, or box in which they are contained.

The weights in use are of *three* kinds, viz: *Troy*, *Avoirdupois*, and *Apothecaries'* Weight.

TROY WEIGHT.

227. ***Troy Weight*** is employed in weighing *gold*, *silver*, and *jewels*. The denominations are *pounds*, *ounces*, *pennyweights*, and *grains*.

TABLE.

24 grains (*gr.*)	are	1 pennyweight,	*pwt.*
20 pennyweights	"	1 ounce,	*oz.*
12 ounces	"	1 pound,	*lb.*

NOTE.—The *unit* commonly employed in weighing *diamonds*, *pearls*, and other *jewels*, is the *carat*, which is equal to 4 grains.

228. The ***Standard Unit*** of weight in the United States, is the *Troy pound*, which is equal to 22.794377 cubic inches of distilled water, at its maximum density (39.83° Fahrenheit),* the barometer standing at 30 inches. It is exactly equal to the Imperial Troy pound of England, the former being copied from the latter by Captain Kater.†

NOTE.—The *original element* of weight is a *grain* of *wheat* taken from the middle of the ear or head. Hence the name *grain* as a unit of weight.

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225. What is weight? 226. Troy weight? The denominations? The table? 227. The standard unit of weight? *Note.* The original element of weight? 228. Avoirdupois weight? The denominations? The Table?

* Hassler. † Professor A. D. Bache.

AVOIRDUPOIS WEIGHT.

229. ***Avoirdupois Weight*** is used in weighing *all coarse articles;* as, hay, cotton, meat, groceries, etc., and all metals, except *gold* and *silver.* The denominations are *tons, hundreds, pounds,* and *ounces.*

TABLE.

16 ounces (*oz.*)	are 1 pound,	*lb.*
100 pounds,	" 1 hundredweight,	*cwt.*
20 cwt., or 2000 lbs.,	" 1 ton,	*T.*

The following denominations are sometimes used:

1000 ounces	are	1 cubic foot of water.
100 pounds	"	1 quintal of dry fish.
196 pounds	"	1 barrel of flour.
200 pounds	"	1 barrel of fish, beef, or pork.
280 pounds	"	1 barrel of salt.

NOTES.—1. The *ounce* is often divided into *halves, quarters,* etc.

2. In business transactions, the *dram,* the *quarter* of 25 lbs., and the *firkin* of 56 lbs., are not used as *units* of Avoirdupois Weight.

REM.—In calculating duties, the law allows 112 pounds to a hundredweight, and custom allows the same in weighing a few coarse articles; as, coal at the mines, chalk in ballast, etc. In all departments of trade, however, both *custom* and the *law* of most of the States, call 100 pounds a *hundredweight.*

230. The ***Standard Avoirdupois*** pound is equal to 7000 grains Troy, or the weight of 27.7015 cubic inches of distilled water, at its maximum density (39.83° Fah.), the barometer being at 30 inches. It is equal to the Imperial Avoirdupois pound of England.

231. *Comparison of Avoirdupois and Troy Weight.*

7000	grains	equal	1 lb. Avoirdupois.
5760	"	"	1 lb. Troy.
437½	"	"	1 oz. Avoirdupois.
480	"	"	1 oz. Troy.

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229. To what is the Avoirdupois pound equal? 230. What is net weight? Gross weight?

APOTHECARIES' WEIGHT.

232. ***Apothecaries' Weight*** is used by physicians in prescribing, and apothecaries in mixing, dry medicines.

20 grains (*gr.*)	are	1 scruple, - - - -	*sc.*, or ℈.
3 scruples	"	1 dram, - - - - -	*dr.*, or ʒ.
8 drams	"	1 ounce, - - - -	*oz.*, or ℥.
12 ounces	"	1 pound, - - - -	*lb.*, or ℔.

NOTE.—The only difference between *Troy* and *Apothecaries'* weight is in the *subdivision* of the *ounce*. The *pound*, *ounce*, and *grain* are the same in each.

232, a. ***Apothecaries' Fluid Measure*** is used in mixing liquid medicines.

60 minims, or drops (♏ or *gtt.*)	are	1 fluid drachm, - -	*f*ʒ.
8 fluid drachms	"	1 fluid ounce, - -	*f*℥.
16 fluid ounces	"	1 pint, - - - - - -	*O.*
8 pints	"	1 gallon, - - - - -	*Cong.*

NOTE.—*Gtt.* for *guttae*, Latin, signifying drops; *O*, for *octarius*, Latin for one-eighth; and *Cong*, *congiarium*, Latin for gallon.

MEASURES OF EXTENSION.

233. ***Extension*** is that which has *one* or *more* of the dimensions, *length*, *breadth*, or *thickness*; as, *lines*, *surfaces*, and *solids*.

A ***line*** is that which has *length* without *breadth*.

A ***surface*** is that which has *length* and *breadth* without *thickness*.

A ***solid*** is that which has *length*, *breadth*, and *thickness*.

NOTE.—A *measure* is a *conventional standard* or *unit* by which *values*, *weights*, *lines*, *surfaces*, *solids*, etc., are computed.

234. The ***Standard Unit*** of length is the *yard*, which is determined from the scale of Troughton, at the temperature of 62° Fahrenheit. It is equal to the British Imperial yard.

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232. In what is Apothecaries' Weight used? Table? 233. What is extension? A line? A surface? A solid? *Note.* A measure? 224. What is the standard unit of length?

LINEAR MEASURE.

235. ***Linear Measure*** is used in measuring that which has *length* without *breadth;* as, lines, distances. It is often called *Long Measure.* The denominations are *leagues, miles, furlongs, rods, yards, feet,* and *inches.*

TABLE.

12 inches (*in.*)	are	1 foot, - - - - - - - - -	*ft.*
3 feet	"	1 yard, - - - - - - - - - -	*yd.*
$5\frac{1}{2}$ yds. or $16\frac{1}{2}$ ft.	"	1 rod, perch, or pole, - - -	*r.* or *p.*
40 rods	"	1 furlong, - - - - - - - -	*fur.*
8 fur. or 320 rods	"	1 mile, - - - - - - - - - -	*m.*
3 miles	"	1 league, - - - - - - - -	*l.*

The following denominations are used in certain cases:

4 in. = 1 hand, for measuring the height of horses.
9 in. = 1 span.
18 in. = 1 cubit.
6 ft. = 1 fathom, for measuring depths at sea.
3.3 ft. = 1 pace,* for measuring approximate distances.
5 pa. = 1 rod, " " "
$1\frac{1}{6}$-statute mi. = 1 geographic or nautical mile.
60 geographic, or $69\frac{1}{2}$-statute m. nearly, } = 1 degree on the equator.
360 degrees = 1 circumference of the earth.

A *knot,* used in measuring distances at sea, is equivalent to a nautical mile.

(For the English and French methods of determining the standard of *length,* see Higher Arith.)

NOTES.—1. The *original element* of linear measure is a *grain* or *kernel* of barley. Thus, 3 barley-corns were called an inch. But the barley-corn, as a measure of length, has fallen into disuse.

2. The *inch* is commonly divided into *halves, fourths, eighths,* or *tenths;* sometimes into *twelfths,* called *lines.*

3. The *mile* of the Table is the *common land mile,* and contains 5280 ft. It is called the *statute mile,* because it is recognized by law, both in the United States and England.

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235. For what is linear measure used? The denominations? The Table? *Note.* The original element of linear measure? How is the inch commonly divided?

* A military pace or step is variously estimated $2\frac{1}{2}$ and 3 feet.

236. The ***Linear Unit*** employed by *surveyors* is *Gunter's Chain,* which is 4 rods or 66 ft. long, and is subdivided as follows:

7.92 inches (*in.*)	are	1 link, - - - - - -	*l.*
25 links	"	1 rod or pole - - -	*r.*
4 rods	"	1 chain, - - - - -	*ch.*
80 chains	"	1 mile - - - - -	*m.*

NOTE.—*Gunter's* chain is so called from the name of its inventor. *Engineers* of the present day commonly use a *chain,* or *measuring tape* 100 feet long, each foot being divided into *tenths.*

CLOTH MEASURE.

237. ***Cloth Measure*** is used in measuring those articles of commerce whose *length only* is considered; as, cloths, laces, ribbons, etc. Its *principal unit* is the *linear yard.* This is divided into *halves, quarters, eighths,* and *sixteenths.*

TABLE.

3 ft. or 36 in.	are	1 yard, - - - -	*yd.*
18 in.,	"	1 half yard - - -	$\frac{1}{2}$ *yd.*
9 in.,	"	1 quarter yard - -	$\frac{1}{4}$ *yd.*
$4\frac{1}{2}$ in.,	"	1 eighth " - -	$\frac{1}{8}$ *yd.*
$2\frac{1}{4}$ in.,	"	1 sixteenth " - -	$\frac{1}{16}$ *yd.*

NOTES.—1. The *old Ells Flemish, English,* and *French,* are no longer used in the United States; and the *nail* ($2\frac{1}{4}$ inches), as a unit of measure, is practically obsolete.

2. In calculating duties at the Custom Houses, the yard is divided into *tenths* and *hundredths.*

SQUARE MEASURE.

238. ***Square Measure*** is used in measuring surfaces, or that which has *length* and *breadth* without *thickness;* as, land, flooring, etc. Hence, it is often called *land* or *surface* measure. The denominations are *acres, square rods, square yards, square feet,* and *square inches.*

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236. What is the linear unit commonly employed by surveyors? 237. Cloth measure? Its principal unit? How is the yard divided? The Table? 238. Square measure? The denominations? The Table?

TABLE.

144 square inches (*sq. in.*)	are	1 square foot,	*sq. ft.*
9 square feet	"	1 square yard,	*sq. yd.*
$30\frac{1}{4}$ sq. yards, or $272\frac{1}{4}$ sq. feet,	"	1 sq. rod, perch or pole,	*sq. r.*
160 square rods	"	1 acre,	*A.*
640 acres	"	1 square mile,	*sq. m.*

239. The ***Unit of Land Measure*** is the *Acre*, and is subdivided as follows:

625 sq. links	are	1 pole or sq. rod,	*p.*
16 poles	"	1 square chain.	*sq. c.*
10 sq. chains, or 160 sq. rods	"	1 acre,	*A.*

NOTES.—1. The *Rood* of 40 sq. rods is no longer used as a unit of measure,

2. A *Square*, in Architecture, is 100 square feet.

239, *a.* The public lands of the United States are divided into *Townships, Sections,* and *Quarter-sections.*

A ***Township*** is 6 miles square, and contains 36 sq. miles.

A ***Section*** is 1 mile square, and contains 640 acres.

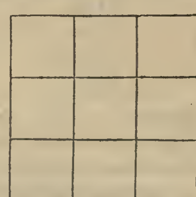
A ***Quarter-section*** is 160 rods square, and contains 160 acres.

240. A ***Square*** is a *rectilinear* figure which has *four equal sides,* and *four right angles.* Thus,

A ***Square Inch*** is a square, each side of which is 1 inch in length.

A ***Square Yard*** is a square, each side of which is 1 yard in length.

This measure is called *Square Measure,* because its measuring unit is a *square.*



9 sq. ft. = 1 sq. yd.

241. A ***Rectangle*** or ***Rectangular Figure*** is one which has *four sides* and *four right angles.* When *all* the sides are equal, it is called a *square;* when the *opposite* sides only are equal, it is called an *oblong* or *parallelogram.*

242. The ***Area*** of a *figure* is the *quantity* of *surface* it contains, and is often called its *superficial contents.*

Note. How are the Government lands divided? How much land in a township? In a section? In a quarter-section? 240. What is a square? A square inch? A square yard? Why is square measure so called? 241. What is a rectangular figure? 242. What is the area of a figure?

243. The *area* of *all rectangular surfaces* is found by *multiplying the length and breadth together.*

244. The *area* and *one side* being given, the *other side* is found y *dividing* the *area* by the given side. (Art. 93.)

CUBIC MEASURE.

245. ***Cubic Measure*** is used in measuring *solids*, or that which has *length, breadth,* and *thickness;* as, timber, boxes of goods, the capacity of rooms, ships, etc. Hence it is often called *Solid Measure.* The denominations are *cords, tons, cubic yards, cubic feet,* and *cubic inches.*

TABLE.

1728 cubic inches (*cu. in.*)	are	1 cubic foot,	*cu. ft.*
27 cubic feet	"	1 cubic yard,	*cu. yd.*
128 cubic feet	"	1 cord of wood,	*C.*

245, ***a.*** A ***Cord*** of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high; for $8 \times 4 \times 4 = 128$.

A ***Cord Foot*** is *one* foot in length of such a pile; hence, 8 cord ft.=1 cord of wood.

245, ***b.*** A ***Register Ton*** is the standard for estimating the capacity or tonnage of vessels, and is 100 cu. ft.

A ***Shipping Ton,*** used in estimating cargoes, in the U. S., is 40 cu. ft.; in England, 42 cu. ft.

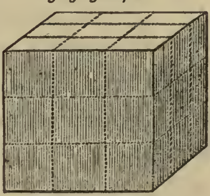
NOTE.—The *ton* of 40 ft. of *round,* or 50 ft. of *hewn* timber is seldom or never used.

246. A ***Cube*** is a regular solid bounded by *six equal squares* called its faces. Hence, its *length, breadth,* and *thickness* are equal to each other. Thus,

A ***Cubic Inch*** is a cube, each side of which is a *square inch;* a ***Cubic Yard*** is a cube, each side of which is a *square yard,* etc.

This Measure is called *Cubic Measure,* because its measuring unit is a *cube.*

$3 \times 3 \times 3 = 27$ cu. ft.



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243. How is the area of rectangular surfaces found? 245. Cubic measure? The denominations? Table? *Note.* Describe a cord of wood? A cord foot? A register ton? Shipping ton? 246. What is a cube? A cubic inch? Yard?

247. *A rectangular body* is one bounded by *six rectangular* sides, each *opposite* pair being *equal* and *parallel;* as, boxes of goods, blocks of hewn stone, etc.

When *all* the sides are equal, it is called a *cube;* when the *opposite* sides only are equal, it is called a *parallelopiped.*

248. The *contents* or *solidity* of a body is the *quantity* of *matter* or *space* it contains.

249 The *contents* of a *rectangular solid* are found, by *multiplying the length, breadth,* and *thickness* together.

MEASURES OF CAPACITY.

250. The ***capacity*** of a vessel is the *quantity* of *space* included within its limits.

Measures of Capacity are divided into two classes, *dry* and *liquid* measures.

DRY MEASURE.

251. ***Dry Measure*** is used in measuring *grain, fruit, salt,* etc. The denominations are *chaldrons, bushels, pecks, quarts,* and *pints.*

TABLE.

2 pints (*pt.*) - -	are	1 quart, - - - - -	*qt.*
8 quarts - . -	"	1 peck, - - - - - -	*pk.*
4 pecks, or 32 qts.,	"	1 bushel, - - - - -	*bu.*
36 bushels - - -	"	1 chaldron, - - - -	*ch.*

252. The ***Standard Unit*** of Dry Measure is the *bushel,* which contains 2150.4 cubic inches, or 77.6274 lbs. avoirdupois of distilled water, at its maximum density.

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247. What is a rectangular body? When all the sides are equal, what called? When the opposite sides only are equal? 248. What are the contents of a solid body? 249. How find the contents of a rectangular solid? 250. The capacity of a vessel? 251. Dry measure? The denominations? The Table? 252. The standard unit of dry measure?

It is a cylinder $18\frac{1}{2}$ in. in diameter, and 8 in. deep, the same as the old Winchester bushel of England.* The British Imperial Bushel contains 2218.192 cu. inches.

NOTES.—1. The *dry quart* is equal to $1\frac{1}{6}$ liquid quart nearly.

2. The *chaldron* is used for measuring coke and bituminous coal.

253. The ***Standard Bushel*** of different kinds of grain, seeds, etc., according to the laws of New York, is equal to the following number of pounds:

32 lbs. = 1 bu. of oats.	58 lbs. = 1 bu. of corn.
44 lbs. = 1 " Timothy seed.	60 lbs. = 1 " { wheat, peas, potatoes, or clover-seed.
48 lbs. = 1 " { buckwheat, or barley.	62 lbs. = 1 " { beans, or blue-grass seed.
55 lbs. = 1 " flax-seed.	100 lbs. = 1 cental of grain.
56 lbs. = 1 " rye.	

NOTES.—1. The *cental* is a standard recently recommended by the Boards of Trade in New York, Cincinnati, Chicago, and other large cities, for *estimating grain, seeds,* etc. Were this standard generally adopted, the discrepancies of the present system of grain dealing would be avoided.

2. *Bushels are changed to centals,* by multiplying them by the *number of pounds* in one bushel, and dividing the product by 100. The remainder will be *hundredths* of a cental.

LIQUID MEASURE.

254. ***Liquid Measure*** is used in measuring *milk, wine, vinegar, molasses,* etc., and is often called *Wine Measure.* The denominations are *hogsheads, barrels, gallons, quarts, pints,* and *gills.*

TABLE.

4 gills (*gi.*)	are	1 pint,	*pt.*
2 pints	"	1 quart,	*qt.*
4 quarts	"	1 gallon.	*gal.*
$31\frac{1}{2}$ gallons	"	1 barrel,	*bar.* or *bbl.*
63 gallons	"	1 hogshead,	*hhd.*

254. Liquid measure? The denominations? Table?

* Professor A. D. Bache.

255. The ***Standard Unit*** of Liquid Measure is the *gallon,* which contains 231 cubic inches, or 8.338 lbs. avoirdupois of distilled water, at its maximum density. The British Imperial Gallon contains 277.274 cu. inches.

NOTES.—1. The *barrel* and *hogshead,* as units of measure, are chiefly used in estimating the contents of cisterns, reservoirs, etc.

2. *Beer Measure* is practically obsolete in this country. The old beer gallon contained 282 cubic inches.

CIRCULAR MEASURE.

256. ***Circular Measure*** is used in *measuring angles, land, latitude* and *longitude,* the *motion* of the heavenly bodies, etc. It is often called *Angular Measure.* The denominations are *signs, degrees, minutes,* and *seconds.*

TABLE.

60 seconds (″)	are	1 minute,	- - - ′
60 minutes	"	1 degree,	- - - °, or *deg.*
30 degrees	"	1 sign,	- - - - *s.*
12 signs, or 360°	"	1 circumference,	- *cir.*

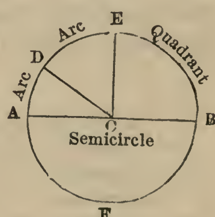
NOTE.—*Signs* are used in Astronomy as a measure of the Zodiac.

257. A ***Circle*** is a plane figure bounded by a curve line, every part of which is *equally distant* from a point within called the *center.*

The ***Circumference*** of a circle is the curve line by which it is bounded.

The ***Diameter*** is a *straight line* drawn through the *center,* terminating at each end in the *circumference.*

The ***Radius*** is a straight line drawn from the center to the *circumference,* and is equal to *half* the diameter.



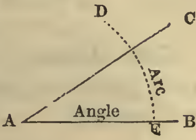
An ***Arc*** is any part of the circumference.

In the adjacent figure, A D E B F is the circumference; C the center; A B the diameter; C A, C D, C E, etc., are radii; A D, D E, etc., are arcs.

255. The standard of Liquid measure? *Note.* What of Beer measure? 256. In what is Circular measure used? The denominations? The Table? 257. What is a circle? The circumference? Diameter? Radius? An arc?

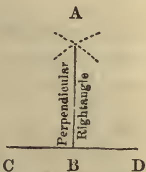
258. A ***Plane Angle*** is the *quantity* of *divergence* of two straight lines starting from the same point.

The ***Lines*** which form the angle are called the *sides*, and the point from which they start, the *vertex*. Thus, A is the vertex of the angle B A C, A B and A C the sides.



259. A ***Perpendicular*** is a straight line which meets another straight line so as to make the two adjacent angles equal to each other, as A B C, A B D.

Each of the two lines thus meeting is *perpendicular* to the other.



260. A ***Right Angle*** is one of the two equal angles formed by the meeting of two straight lines which are *perpendicular* to each other. All other angles are called *oblique*.

260, *a*. The ***Measure*** of an angle is the *arc* of a circle included between its two sides, as the arc D E, in Fig., Art. 258.

261. A ***Degree*** is one 360th part of the *circumference* of a *circle*. It is divided into 60 equal parts, called *minutes*; the *minute* is divided into 60 *seconds*, etc. Hence, the *length* of a *degree, minute*, etc., varies according to the *magnitude* of different circles.

The length of a degree of longitude at the equator, also the *average length* of a degree of latitude, adopted by the U. S. Coast Survey, is 69.16 statute miles. At the latitude of 30° it is 59.81 miles, at 60° it is 34.53 miles, and at 90° it is nothing.*

262. A ***Semi-circumference*** is *half* a circumference, or 180°.

A ***Quadrant*** is *one-fourth* of a circumference, or 90°.

263. If two diameters are drawn *perpendicular* to each other, they will form *four right angles* at the center, and divide the circumference into *four* equal parts. Hence,

A *right angle* contains 90°; for the quadrant, which measures it, is an arc of 90°.

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258. A plane angle? The sides? The vertex? 259. A perpendicular? 260. A right angle? 260, *a*. What is the measure of an angle? 261. What is a degree? Upon what does the length of a degree depend? What its length at the equator? 262. What is a semi-circumference? How many degrees does it contain? A quadrant? 263. If two diameters are drawn perpendicular to each other, what is the result? How many degrees in a right angle?

* Encyclopedia Britannica.

MEASUREMENT OF TIME.

264. ***Time*** is a ***portion*** of duration. It is divided into *centuries, years, months, weeks, days, hours, minutes* and *seconds.*

TABLE.

60 seconds (*sec.*)	are	1 minute	*m.*
60 minutes	"	1 hour	*h.*
24 hours	"	1 day	*d.*
7 days	"	1 week	*w.*
365 days, or 52 w. and 1 d.	"	1 common year	*c. y.*
366 days	"	1 leap year	*l. y.*
12 calendar months (*mo.*)	"	1 civil year	*y.*
100 years	"	1 century	*c.*

NOTE.—In most business transactions 30 days are considered a month. Four weeks are sometimes called a *lunar month.*

265. A ***Civil Year*** is the year adopted by *government* for the computation of time, and includes both *common* and *leap* years as they occur. It is divided into 12 *calendar months,* as follows:

January (Jan.)	1st	mo.,	31 d.	July (July)	7th	mo.,	31 d.
February (Feb.)	2d	"	28 d.	August (Aug.)	8th	"	31 d.
March (Mar.)	3d	"	31 d.	September (Sep.)	9th	"	30 d.
April (Apr.)	4th	"	30 d.	October (Oct.)	10th	"	31 d.
May (May)	5th	"	31 d.	November (Nov.)	11th	"	30 d.
June (June)	6th	"	30 d.	December (Dec.)	12th	"	31 d.

NOTES.—1. The following couplet will aid the learner in remembering the months that have 30 days each:

"Thirty days hath September,
April, June, and November."

All the other months have 31 days, except *February,* which in *common* years has 28 days; in *leap* years, 29.

266. Time is naturally divided into *days* and *years.* The *former* are measured by the revolution of the earth on its axis; the *latter* by its revolution around the sun.

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264. What is time? The denominations? The Table?

267. A ***Solar*** year is the *time* in which the earth, starting from one of the *tropics* or *equinoctial* points, revolves around the sun, and returns to the same point. It is thence called the *tropical* or *equinoctial* year, and is equal to 365d. 5h. 48m. 49.7 sec.*

NOTE.—1. The excess of the *solar* above the *common* year is 6 hours or $\frac{1}{4}$ of a day, nearly; hence, in 4 years it amounts to about 1 day. To provide for this excess, 1 day is added to every 4th year, which is called *Leap* year or *Bissextile.* This additional day is given to *February,* because it is the shortest month.

2. Leap year is caused by the *excess* of a *solar* above a *common* year, and is so called because it *leaps over* the limit, or runs on 1 day more than a common year.

3. Every year that is exactly divisible by 4, except centennial years, is a *leap year;* the others are *common* years. Thus, 1868, '72, etc., are leap years; 1869, '70, '71, are common. Every centennial year exactly divisible by 400 is a *leap* year; the other centennial years are *common.* Thus, 1600 and 2000 are *leap* years; 1700, 1800, and 1900 are *common.*

268. An ***Apparent Solar Day*** is the *time* between the apparent departure of the sun from a given meridian and his return to it, and is shown by *sun dials.*

A ***True*** or ***Mean*** solar day is the average length of apparent solar days, and is divided into 24 equal parts, called *hours,* as shown by a perfect clock.

269. A ***Civil Day*** is the day adopted by governments for business purposes, and corresponds with the mean solar day. In most countries it begins and ends at midnight, and is divided into two parts of 12 hours each; the former being designated A. M.; the latter, P. M.

NOTE. A *meridian* is an imaginary circle on the surface of the earth, passing through the poles, perpendicular to the equator. A. M. is an abbreviation of *ante meridies, before midday;* P. M., of *post meridies, after midday.*

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266. How is time naturally divided? How is the former caused? The latter?

* Laplace, Somerville, Baily's Tables.

MISCELLANEOUS TABLES.

12 things are 1 dozen.	12 gross are 1 great gross.
12 doz. " 1 gross.	20 things " 1 score.
24 sheets are 1 quire of paper.	2 reams are 1 bundle.
20 quires " 1 ream.	5 bundles " 1 bale.
2 leaves are 1 folio.	12 leaves are 1 duodecimo or 12mo.
4 leaves " 1 quarto or 4to.	18 leaves " 1 eighteen mo.
8 leaves " 1 octavo or 8vo.	24 leaves " 1 twenty-four mo.

NOTE.—The terms *folio, quarto, octavo,* etc., denote the *number* of leaves into which a sheet of paper is folded in making books.

270. *Aliquot Parts of a Dollar,* or 100 cents.

50 cents = $\$\frac{1}{2}$	$12\frac{1}{2}$ cents = $\$\frac{1}{8}$
$33\frac{1}{3}$ cents = $\$\frac{1}{3}$	10 cents = $\$\frac{1}{10}$
25 cents = $\$\frac{1}{4}$	$8\frac{1}{3}$ cents = $\$\frac{1}{12}$
20 cents = $\$\frac{1}{5}$	$6\frac{1}{4}$ cents = $\$\frac{1}{16}$
$16\frac{2}{3}$ cents = $\$\frac{1}{6}$	5 cents = $\$\frac{1}{20}$

271. *Aliquot Parts of a Pound Sterling.*

10 shil. = £$\frac{1}{2}$	3s. 4d. = £$\frac{1}{6}$
6s. 8d. = £$\frac{1}{3}$	2s. 6d. = £$\frac{1}{8}$
5 shil. = £$\frac{1}{4}$	2 shil. = £$\frac{1}{10}$
4 shil. = £$\frac{1}{5}$	1s. 8d. = £$\frac{1}{12}$

272. *Aliquot Parts of a Pound Avoirdupois.*

12 ounces = $\frac{3}{4}$ pound.	4 ounces $\frac{1}{4}$ pound.
8 " = $\frac{1}{2}$ "	2 " $\frac{1}{8}$ "
$5\frac{1}{3}$ " = $\frac{1}{3}$ "	1 " $\frac{1}{16}$ "

273. *Aliquot Parts of a Year.*

9 months = $\frac{3}{4}$ year.	3 months = $\frac{1}{4}$ year.
8 " = $\frac{2}{3}$ "	2 " = $\frac{1}{6}$ "
6 " = $\frac{1}{2}$ "	$1\frac{1}{2}$ " = $\frac{1}{8}$ "
4 " = $\frac{1}{3}$ "	1 " = $\frac{1}{12}$ "

274. *Aliquot Parts of a Month.*

20 days = $\frac{2}{3}$ month.	5 days = $\frac{1}{6}$ month.
15 " = $\frac{1}{2}$ "	3 " = $\frac{1}{10}$ "
10 " = $\frac{1}{3}$ "	2 " = $\frac{1}{15}$ "
6 " = $\frac{1}{5}$ "	1 " = $\frac{1}{30}$ "

REDUCTION.

275. ***Reduction*** is changing a number from one denomination to another, without *altering* its *value.* It is either *descending* or *ascending.*

Reduction Descending is changing *higher* denominations to *lower;* as, yards to feet, etc.

Reduction Ascending is changing *lower* denominations to *higher;* as, feet to yards, etc.

276. To reduce *Higher* Denominations to *Lower.*

1. How many farthings are there in £23, 7s. $5\frac{1}{4}$d.?

OPERATION.

```
£23, 7s. 5d. 1 far.
 20
 ---
 467 s.
  12
 ----
5609 d.
   4
 -----
22437 far. Ans.
```

ANALYSIS.—Since there are 20s. in a pound, there must be 20 times as many shillings as pounds, *plus* the given shillings. Now 20 times 23 are 460, and 460s. + 7s. = 467s. Again, since there are 12d. in a shilling, there must be 12 times as many pence as shillings, *plus* the given pence. But 12 times 467 are 5604, and 5604d. + 5d. = 5609d. Finally, since there are 4 farthings in a penny, there must be 4 times as many farthings as pence, *plus* the given farthings. Now 4 times 5609 are 22436, and 22436 far. + 1 far. = 22437 far Therefore, in £23, 7s. $5\frac{1}{4}$d. there are 22437 far. Hence, the

RULE.—*Multiply the highest denomination by the number required of the next lower to make a unit of the higher, and to the product add the lower denomination.*

Proceed in this manner with the successive denominations, till the one required is reached.

2. How many pence in £8, 10s. 7d.? *Ans.* 2047d.
3. How many farthings in 12s. 9d. 2 far.?
4. How many farthings in £41, 5s. $4\frac{1}{2}$d.?

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275. What is Reduction? How many kinds? Descending? Ascending? 276. How are higher denominations reduced to lower? Explain Ex. 1 from the blackboard?

277. To reduce *Lower* Denominations to *Higher*.

5. In 22437 farthings, how many pounds, shillings, pence, and farthings?

OPERATION.

4)22437 far.
12)5609d. 1 far.
20)467s. 5d.
£23, 7s.
Ans. £23, 7s. 5d. 1 far.

ANALYSIS.—Since in 4 farthings there is 1 penny, in 22437 farthings there are as many pence as 4 farthings are contained times in 22437 farthings, or 5609 pence and 1 farthing over. Again, since in 12 pence there is 1 shilling, in 5609 pence there are as many shillings as 12 pence are contained times in 5609 pence, or 467 shillings and 5 pence over. Finally, since in 20 shillings there is 1 pound, in 467 shillings there are as many pounds as 20 shillings are contained times in 467 shillings, or 23 pounds and 7 shillings over. Therefore, in 22437 farthings there are £23, 7s. 5d. 1 far. Hence, the

RULE.—*Divide the given denomination by the number required of this denomination to make a unit of the next higher.*

Proceed in this manner with the successive denominations, till the one required is reached. The last quotient, with the several remainders annexed, will be the answer.

NOTE.—The *remainders*, it should be observed, are the same denomination as the respective *dividends* from which they arise.

278. PROOF.—Reduction *Ascending* and *Descending prove* each other; for, one is the *reverse* of the other.

6. In 2047 pence how many pounds, shillings, and pence?

7. In 614 farthings, how many shillings, pence, etc.?

8. How many pounds, shillings, etc., in 39610 farthings?

9. An importer paid £27, 13s. 8d. duty on a package of English ginghams, which was 4d. a yard: how many yards did the package contain?

10. A railroad company employs 1000 men, paying each 4s. 6d. per day: what is the daily pay-roll of the company?

11. Reduce 18 lbs. 6 ounces troy, to pennyweights.

---

277. How are lower denominations reduced to higher? 278. Proof? Explain Ex. 5 from the blackboard.

12. Reduce 32 lbs. 9 oz. 5 pwt. to pennyweights.

13. How many pounds and ounces in 967 ounces troy?

14. How many pounds, etc., in 41250 grains?

15. How many rings, each weighing 3 pwt., can be made of 1 lb. 10 oz. 9 pwt. of gold?

16. What is the worth of a silver cup weighing 10 oz. 16 pwt., at $12\frac{1}{2}$ cents a pennyweight?

17. Reduce 165 lbs. 13 oz. Avoirdupois to ounces.

18. Reduce 210 tons 121 pounds 8 ounces to ounces.

19. In 4725 lbs. how many tons and pounds?

20. In 370268 ounces, how many tons, etc.?

21. What will 875 lbs. 11 oz. of snuff come to, at 5 cents an ounce?

22. A grocer bought 1 ton of maple sugar, at 10 cents a pound, and sold it at 6 cents a cake, each cake weighing 4 oz.: what was his profit?

23. In 250 lbs. 6 oz. Apothecaries' weight, how many drams?

24. In 2165 scruples, how many pounds?

25. How many feet in 1250 rods, 4 yards, and 2 feet?

For the method of multiplying by $5\frac{1}{2}$ or $16\frac{1}{2}$, see Art. 165.

Ans. 20639 feet.

26. How many feet in 365 miles 78 rods and 8 feet?

27. In 5242 feet, how many rods, yards and feet?

$$
\begin{array}{l}
3)\underline{5242} \text{ ft.} \\
5\tfrac{1}{2})\;1747 \text{ yd. } 1 \text{ ft.} \\
\underline{2}\quad\;\; \underline{\;\;\;2} \\
11)\underline{3494} \text{ half yds.} \\
\textit{Ans. } 317 \text{ r. } 3\tfrac{1}{2} \text{ yd. } 1 \text{ ft.}
\end{array}
$$

REMARK.—In order to divide the second dividend 1747 yards by $5\frac{1}{2}$, the number of yards in a rod, we reduce both the divisor and dividend to *halves;* then divide one by the other. Thus, $5\frac{1}{2}=11$ halves; and $1747=3494$ halves; and 11 is contained in 3494, 317 times and 7 over. But the remainder 7, is *halves;* for the dividend was halves; and 7 half yards$=3\frac{1}{2}$ yards. (Art. 168, *a*, 169.)

28. Reduce 38265 feet to miles, etc.

29. How many rods in 461 leagues, 2 miles, 6 furlongs?

30. Reduce 7 m. 6 fur. 23 r. 5 yds. 8 in. to inches, and prove the operation.

31. Reduce 1 m. 7 fur. 39 r. 5 yds. 1 ft. 6 in. to inches, and prove the operation.

32. If a farm is $2\frac{1}{2}$ miles in circumference, what will it cost to enclose it with a stone wall, at $1.85 a rod?

33. At $6\frac{1}{4}$ cents a mile, what will be the cost of a trip round the world, allowing it to be 8299.2 leagues?

34. How many eighths of a yard in a piece of cloth 57 yards long?

35. How many sixteenths of a yard in 163 yards?

36. In 578 fourths, how many yards?

37. In 1978 sixteenths, how many yards?

38. How many vests will $16\frac{1}{2}$ yards of satin make, allowing $\frac{3}{4}$ yard to a vest?

39. A shopkeeper paid $2.50 for $18\frac{3}{4}$ yards ribbon, and making it into temperance badges of $\frac{1}{8}$ yard each, sold them at $12\frac{1}{2}$ cents apiece: what was his profit?

40. How many square rods in 43816 square feet?

41. How many acres, etc., in 25430 square yards?

42. Reduce 160 acres, 25 sq. rods, 8 sq. ft. to sq. feet.

43. Reduce 100 sq. miles to square rods?

44. A man having 64 acres 8 sq. rods of land, divided it into 6 equal pastures: how much land was there in each pasture?

45. My neighbor bought a tract of land containing $15\frac{1}{4}$ acres, at $500 an acre, and dividing it into building lots of 20 square rods each, sold them at $250 apiece: how much did he make?

46. Reduce 85 cubic yards, 10 cu. feet to cu. inches.

47. Reduce 250 cords of wood to cu. feet.

48. Reduce 18265 cu. inches to cu. feet, etc.

49. Reduce 8278 cu. feet to cords.

50. Reduce 164 bu. 1 pk. 3 qts. to quarts.

51. Reduce 375 bu. 3 pks. 1 qt. 1 pt. to pints.

52. Reduce 184 chaldrons 17 bu. to bushels.

53. In 8147 quarts how many bushels, etc.?

54. A seedsman retailed 75 bu. 3 pk. of clover seed at 17 cents a quart: what did it come to?

55. A lad sold 2 bu. 1 pk. 3 qts. of chestnuts, at $8\frac{1}{2}$ cents a pint: how much did he get for them?

56. In 98 quarts 1 pt. 2 gills, how many gills?

57. In 150 gals. 3 qts., how many quarts?

58. In 45 barrels, 10 gals. 3 qts., how many quarts?

59. How many gills in 17 hhds. 10 gals. 3 qts.?

60. How many gallons, etc., in 86673 pints?

61. How many bottles holding $1\frac{1}{2}$ pt. each are required to hold a barrel of wine?

62. What will a hogshead of alcohol come to, at $6\frac{1}{4}$ cents a gill?

63. Reduce 30 days 5 hours 42 min. 10 sec. to seconds.

64. Reduce 17 weeks 3 days 5 hr. 30 min. to minutes.

65. Reduce a solar year to seconds.

66. Reduce 6034500 sec. to weeks, etc.

67. Reduce 5603045 hours to common years.

68. Reduce 10250300 minutes to leap years.

69. An artist charged me 75 cents an hour for copying a picture; it took him 27 days, working $10\frac{1}{2}$ hours a day: what was his bill?

70. In 48561 seconds, how many degrees?

71. In 65237 minutes, how many signs?

72. In $237°, 40', 31''$, how many seconds?

73. In $360°$, how many seconds?

74. How many dozen in 1965 things?

75. How many eggs in 125 dozen?

76. How many gross in 100000?

77. How many pens in 65 gross?

78. How many pounds in 3 score and 7 pounds?

79. How many sheets of paper in 75 quires?

80. How many quires of paper in 10000 sheets?

APPLICATIONS OF WEIGHTS AND MEASURES.

279. The use of ***Weights*** and ***Measures*** is not confined to ordinary trade. They have other and important applications to the farm, the garden, artificers' work, the household, etc.

THE FARM AND GARDEN.

280. To find the Contents of Rectangular Surfaces.

1. How many square yards in a strawberry bed, 4 yards long, and 3 yards wide?

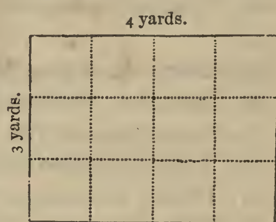

ILLUSTRATION.—Let the bed be represented by the adjoining figure; its length being divided into *four* equal parts, and its breadth into *three*, each denoting *linear* yards. The bed, obviously, contains as many *square yards* as there are *squares* in the figure. Now as there are 4 squares in 1 row, in 3 rows there must be 3 times 4 or 12 squares. Therefore, the bed contains 12 sq. yards. Hence, the

RULE.—*Multiply the length by the breadth.* (Art. 243.)

NOTES.—1. Both *dimensions* should be reduced to the *same denomination* before they are multiplied.

2. The *area* and *one side* of a rectangular surface being given, the *other* side is found by *dividing the area by the given side.*

3. One *line* is said to be multiplied by another, when the *number* of units in the former are taken as many times as there are *like units* in the latter. (Art. 47, *n.*)

2. How many acres in a field 40 rd. long and 31 rd. wide ? wide ?

3. If the width of an asparagus bed is 66 feet, what must be the length to contain $\frac{1}{4}$ of an acre?

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279. What is said of the applications of Weights and Measures? 280. How find the area of a rectangular surface? *Note.* If the dimensions are in different denominations, what is to be done? If the area and one side are given?

4. What is the length of a pasture, containing 15 acres, whose width is $33\frac{1}{3}$ rods?

5. A speculator bought 30 acres of land at $50 per acre, and sold it in villa lots of 5 rods by 4 rods, at $200 a lot: what was his profit?

6. A garden 230 ft. long and 125 ft. wide, has a gravel walk 6 ft. in breadth extending around it: how much land does the walk contain?

7. What is the difference between 2 feet square and 2 square feet?

8. A farmer having 3 acres of potatoes, sold them at 25 cents a bushel in the ground; allowing a yield of $2\frac{1}{2}$ bu. to a sq. rod, how many bushels did the field produce; and what did he receive for them?

281. To find the number of Hills, Vines, etc., in a given field, the Area occupied by each being given.

9. How many hills of corn can a farmer plant on 2 acres of ground, the hills being 4 ft. apart?

ANALYSIS.—Since the hills are 4 ft. apart, each hill will occupy 4×4 or 16 sq. ft. Now 2 acres $= 160$ sq. rods $\times 272.25 \times 2 = 87120$ sq. ft. Therefore, he can plant as many hills as 16 is contained times in 87120; and $87120 \div 16 = 5445$ hills. Hence, the

RULE.—*Divide the area planted by the area occupied by each hill or vine.*

10. How many bulbs will a lady require for a crocus bed, 12 ft. long and 4 ft. wide, if planted 6 inches apart?

11. What number of grape vines will be required for $\frac{3}{4}$ of an acre, if they are set 3 ft. apart? What will it cost to set them at $5.25 per hundred?

12. How large an orchard shall I require for 100 apple trees, allowing them to stand 33 ft. apart?

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281. How find the number of hills, vines, etc., in a given field? 282. How are flooring, plastering, etc., estimated?

ARTIFICERS' WORK.

282. *Flooring, plastering, painting, papering, roofing, paving,* etc., are estimated by the number of *sq. ft.* or *sq. yds.* in the *area,* or by the "*square*" of 100 sq. feet.

13. What will be the cost of flooring a room 18 ft. long and 16 ft. 6 in. wide, at $18\frac{3}{4}$ cts. a sq. foot?

SOLUTION.—18 ft. × $16\frac{1}{2}$=297 sq. ft.; and $18\frac{3}{4}$ cts. × 297=\$$55.68\frac{3}{4}$.

14. If a school house is 60 ft. long and 45 ft. wide, what will be the expense of the flooring, at \$1.08 per sq. yard?

15. What will it cost to plaster a ceiling 18 ft. 6 in. long, and 15 ft. wide, at \$3.20 per square of 100 sq. ft.

16. What will be the cost of flagging a side-walk 206 ft. long and $9\frac{1}{2}$ ft. wide, at \$1.85 per sq. yard?

17. What will be the expense of painting a roof 60 ft. long and 25 feet wide, at \$1.50 per "square"?

18. What cost the cementing of a cellar 65 ft. by 25 ft., at \$.25 per square yard?

MEASUREMENT OF LUMBER.

283. To find the *Contents of Boards,* the length and breadth being given.

DEF.—A ***Standard Board,*** in commerce, is 1 in. thick. Hence,
A ***Board Foot*** is 1 foot long, 1 foot wide, and 1 inch thick.
A ***Cubic Foot*** is 12 board feet. Hence, the

RULE.—*Multiply the length in feet by the width in inches, and divide the product by* 12; *the result will be board feet.*

NOTES.—1. If a board *tapers regularly,* multiply by the *mean width,* which is *half* the *sum* of the two ends.

2. *Shingles* are estimated by the *thousand,* or *bundle.* They are commonly 18 in. long, and average 4 in. wide. It is customary to allow 1000 to a square of 100 feet.

3. The contents of boards, timber, etc., were formerly computed by *duodecimals,* which divides the *foot* into 12 in., the *inch* into 12 sec., etc.; but this method is seldom used at the present day.

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283. The thickness of a standard board? What is a board foot? A cubic foot? How find the contents of a board? If the board is tapering, how?

19. If a board is 10 ft. long, and 1 ft. 3 in. wide, what are its contents, board measure?

SOLUTION.—$10 \times 15 = 150$; and $150 \div 12 = 12\frac{1}{2}$ board ft., *Ans.*

20. What are the contents of a board 13 ft. long, and 1 ft. 5 in. wide?

SOLUTION.—$13 \times 17 = 221$; and $221 \div 12 = 18\frac{5}{12}$ board ft.

21. Required the contents of a board 14 ft. long, 1 ft. 4 in. wide; and its value at $7\frac{1}{2}$ cts. a foot?

22. Required the contents of a tapering board 15 ft long, 14 in. wide at one end, and 6 in. at the other.

23. Required the contents of a stock of 9 boards 14 ft long, and 1 ft. 2 in. wide.

24. The sides of a roof are 45 ft. long and 20 ft. wide; what will it cost to shingle both sides, at $15.45 per M., allowing 1000 shingles to a square?

284. To find the Cubical Contents of Rectangular Bodies.

REM.—*Round Timber*, as masts, etc., is estimated in *cubic feet.*
Hewn Timber, as beams, etc.. either in *board* or *cubic feet.*
Sawed Timber, as planks, joists, etc., in *board* feet.

25. How many cubic feet are there in a rectangular block of marble 4 ft. long, 3 ft. wide, and 2 ft. thick?

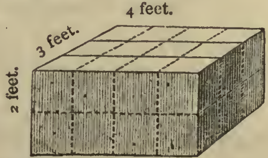

ILLUSTRATION.—Let the block be represented by the adjoining diagram; its length being divided into *four* equal parts, its breadth *three*, and its thickness into *two*, each denoting linear feet.

In the upper face of the block, there are 3 times 4, or 12 sq. ft. Now if the block were 1 foot thick, it would evidently contain 1 time as many *cubic* feet as there are *square* feet in its upper face; and 1 time 4 into 3=12 cubic feet. But the given block is 2 ft. thick; therefore it contains 2 times 4 into 3=24 cu. ft. Hence, the

RULE.—*Multiply the length, breadth, and thickness together.* (Art. 249.)

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284. How are hewn timber, etc., estimated? How joists, studs, etc.? 285. How

285. **To find the *Contents of Joists,* &c., 2, 3, 4, &c., in. thick.**

RULE.—*Multiply the width by such a part of the length, as the thickness is of* 12; *the result will be board feet.*

NOTES.—1. The *approximate* contents of *round timber* or *logs* may be found by *multiplying* $\frac{1}{4}$ of the mean circumference by *itself*, and this *product* by the *length.*

2. *Cubic feet* are reduced to *Board feet* by multiplying them by 12. For, 1 foot board measure is 12 inches square, and 1 in. thick; therefore, 12 such feet make 1 cubic foot. Hence,

If *one* of the dimensions is *inches*, and the other two are *feet*, the *product* will be in *Board feet.*

3. To estimate hay.—

A *ton* of hay upon a scaffold measures about 500 cu. ft.; when in a mow, 400 cu. ft.; and in well settled stacks, 10 cu. yards.

4. To find the *weight* of *coal* in bins.—

A *ton* (2000 lbs.) of Lehigh white ash, egg size, measures $34\frac{1}{2}$ cu. ft.
A *ton* of white ash Schuylkill, " " 35 cu. ft.
A *ton* of pink, gray and red ash, " " 36 cu. ft.

26. How many cu. feet in a stick of hewn timber 20 ft. long, 1 ft. 3 in. wide, 1 ft. 4 in. thick? *Ans.* $33\frac{1}{3}$ cu. ft.

27. How many board feet in a joist 18 ft. long, 5 in. wide, and 4 in. thick? How many cubic feet?

SOLUTION.—4 is $\frac{1}{3}$ of 12; and $\frac{1}{3}$ of 18 ft. is 6 ft. Now $5 \times 6 = 30$ bd. ft.; and $30 \div 12 = 2\frac{1}{2}$ cu. ft.

28. What cost 45 pieces of studding 11 ft. long, 4 in. wide, and 3 in. thick, at 3 cts. a board foot?

29. At 45 cts. a cu. foot, what will be the cost of a beam 32 ft. long, 1 ft. 6 in. wide, and 1 ft. 1 in. thick?

30. What is the worth of a load of wood 8 ft. long, 4 ft. wide, and 5 ft. high, at $\$3\frac{1}{2}$ a cord?

31. How many tons of white ash, e-s Sch. coal, will a bin 10 ft. long, 8 ft. wide, and 8 ft. high, contain. *Ans.* $18\frac{2}{7}$ T

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find the contents of a rectangular body? *Note.* How find the contents of round timber? When the contents and two of the dimensions are given, how find the other dimension? How reduce cubic feet of timber to board feet? Why If one dimension is inches, and the other two ft., what is the product?

32. How many cords of wood in a tree 60 feet high, whose mean circumference is 10 feet?

33. How many tons of hay in a mow 22 ft. by 20 ft., and 15 ft. high?

34. At $18.50 a ton, what is the worth of a scaffold of hay 30 ft. long, 12 ft. wide, and 10 ft. high?

286. *Stone masonry* is commonly estimated by the *perch* of 25 cubic feet.

Excavations and *embankments* are estimated by the cubic yard. In removing earth, a cu. yard is called a *load.*

Brickwork is generally estimated by the 1000, but sometimes by cubic feet.

NOTES.—1. A *perch* strictly speaking=$24\frac{3}{4}$ cu. feet being $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high.

2. The *average size* of bricks is 8 in. long, 4 in. wide, and 2 in. thick.

In estimating the labor of brickwork by cu. feet, it is customary to measure the length of each wall on the outside; no allowance is made for windows and doors or for corners.

In finding the *exact number* of bricks in a building, we should make a deduction for the windows, doors, and corners; also an allowance of $\frac{1}{10}$ of the solid contents for the *space* occupied by the mortar.

35. What will be the cost of digging a cellar 45 feet long, 24 feet wide, and 8 feet deep, at 35 cents a cu. yard?

36. At $5.25 a perch (25 cu. ft.), what cost the labor of building a cellar wall, 1 ft. 6 in. thick, and 8 ft. high, the cellar being 22 by 45 feet?

37. How many bricks of average size will it take to build the walls of a house 50 ft. long, 35 ft. wide, 21 ft. high, and 1 ft. thick, deducting $\frac{1}{10}$ of the contents for the space occupied by the mortar, but making no allowance for windows, doors, or corners?

38. At 45 cents a cubic yard, what will it cost to fill in a street 800 ft. long, 60 ft. wide, and $4\frac{1}{2}$ ft. below grade?

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286. How are stone masonry, excavations, etc., estimated? How brick work? *Note.* The average size of bricks?

THE HOUSEHOLD.

287. To find the Quantity of material of given width required to line or cover a given Surface.

39. How many yards of serge $\frac{5}{8}$ yd. wide are required to line a cloak containing $7\frac{3}{4}$ yds. of camlet $1\frac{1}{4}$ yd. wide?

ANALYSIS.—The surface to be lined$=7\frac{3}{4}$ yds. $\times 1\frac{1}{4}=\frac{31}{4}\times\frac{5}{4}=\frac{155}{16}$ sq. yards; 1 yd. of the lining$=1$ yd. $\times\frac{5}{8}=\frac{5}{8}$ sq. yd. Now as $\frac{5}{8}$ sq. yard of camlet requires 1 yard of lining, the whole cloak will require as many yards of lining as $\frac{5}{8}$ is contained times in $\frac{155}{16}$; and $\frac{155}{16}\div\frac{5}{8}=\frac{155}{16}\times\frac{8}{5}=\frac{31}{2}$ or $15\frac{1}{2}$ yards, the serge required. Hence, the

RULE.—*Divide the number of square yards or feet in the given surface by the number of square yards or feet in a yard of the material used.*

40. How many yards of Brussels carpeting $\frac{3}{4}$ yd. wide, are required for a parlor floor 21 ft. long and 18 ft. wide?

41. How many yards of matting $1\frac{1}{4}$ yd. wide will it take to cover a hall 16 yds. long and $3\frac{1}{2}$ yds. wide?

42. How much cambric $\frac{7}{8}$ yd. wide is required to line a dress containing 15 yds. of silk $\frac{3}{4}$ yd. wide?

43. How many sods, each being 15 in. square, will be required to turf a court yard 25 by 20 feet?

44. How many yards of silk $\frac{5}{8}$ yd. wide are required to line 2 sets of curtains, each set containing 10 yds. of brocatelle $1\frac{1}{2}$ yd. wide?

45. How many marble tiles 9 in. square, will it take to cover a hall floor 48 ft. long and $7\frac{1}{2}$ ft. wide?

46. How many rolls of wall paper 9 yds. long and $1\frac{1}{2}$ ft. wide, are required to cover the 4 sides of a room 18 by 16 ft. and 9 ft. high, deducting 81 sq. ft. for windows and doors?

47. How many gallons of water in a rectangular cistern, whose length is 8 ft., breadth 6 ft., and depth 5 ft.?

NOTE.—*Cubic measure* is changed to *gallons* or *bushels*, by reducing the former to cu. inches, and *dividing* the *result* by 231, or 2150.4, as the case may be. (Arts. 252, 255.)

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287. How find the quantity of material required to line or cover a given surface?

48. A man constructed a cistern containing 20 hogsheads, the base being 6 ft. square: what was its depth?

49. How many bushels of wheat will a bin 6 ft. long, 4 ft. wide, and 3 ft. deep, contain?

50. How many cu. feet in a vat containing 50 hogsheads?

NOTE.—*Gallons* and *bushels* are reduced to *cubic inches* by *multiplying* them by 231 or 2150.4, as the case may be.

51. A man having 10 bu. of grain, wishes to store it in a box 5 ft. long and 3 ft. wide: what must be its height?

52. A man built a cistern in his attic, 5 ft. long, 4 ft. wide, and 3 ft. deep: what weight of water will it contain, allowing 1000 oz. to a cu. ft.?

288. To change Avoirdupois Weight to Troy, and Troy Weight to Avoirdupois.

53. If the internal revenue tax is 5 cents per ounce Troy, on silver plate exceeding 40 ounces, what will be the tax on plate weighing 7 lb. 8 oz. Avoirdupois?

ANALYSIS.—7 lb. 8 oz. Avoirdupois=120 oz.; and 120 oz. × $437\frac{1}{2}$= 52500 grains. Now 52500 gr. ÷ 480 = $109\frac{3}{8}$ oz. Troy. Again, $109\frac{3}{8}$ oz. −40 oz. (exempt)=$69\frac{3}{8}$ oz.; and 5 cts. × $69\frac{3}{8}$=\$$3.46\frac{7}{8}$. *Ans.*

54. How many spoons, each weighing 1 oz. Avoirdupois, can be made from 1 lb. 9 oz. 17 pwt. 12 gr. Troy?

ANALYSIS.—1 lb. 9 oz. 17 pwt. 12 gr.=10500 grains; and 10500 grs. ÷ 437.5 grs. (1 oz. Avoir.)=24 spoons. *Ans.* Hence, the

RULE.—*Reduce the given quantity to grains; then reduce the grains to the denominations required.* (Art. 231.)

55. If a family have 12 lb. 6 oz. Avoir. of silver plate, what will be the tax, exempting 40 oz., at 5 cts. per oz. Troy?

56. A lady bought a silver set weighing 11 lbs. 4 oz. Troy: how many pounds Avoir. should it weigh?

57. How many rings, each weighing $3\frac{1}{3}$ pwt., can be made from a bar of gold weighing 1 pound Avoirdupois?

58. What is the value of a silver pitcher weighing 2 lb. 8 oz. Avoir., at \$2 per ounce Troy?

59. A miner sold a nugget of gold weighing $3\frac{1}{4}$ lbs. Avoir., at \$17 per oz. Troy: what did he get for it?

DENOMINATE FRACTIONS.

289. A ***Denominate Fraction*** is *one or more* of the equal parts into which a *compound* or *denominate* number is divided.

Denominate Fractions are expressed either as *common* fractions, or as *decimals.* The former are usually termed *denominate fractions;* the latter, *denominate decimals.*

REDUCTION OF DENOMINATE FRACTIONS.

CASE I.

290. To reduce a Denominate Fraction from *Higher* denominations to *Lower*.

1. What part of an inch is $\frac{2}{96}$ of a yard?

ANALYSIS.—$\frac{2}{96}$ of 1 yard equals $\frac{1}{96}$ of 2 yards. Reducing the numerator to inches, we have 2 yards $=2\times3\times12$, or 72 inches; and $\frac{1}{96}$ of 72 in. is $\frac{72}{96}$ or $\frac{3}{4}$ inch.

$2\text{ yd.}\times3=6\text{ ft.}$
$6\text{ ft.}\times12=72\text{ in.}$
Ans. $\frac{72}{96}$ or $\frac{3}{4}$ in.

Or, denoting the multiplications, and *cancelling* the factors common to the numerators and denominators, we have

$$\frac{2}{96}\text{ yd.}=\frac{2\times3}{96}\text{ ft.}=\frac{2\times3\times12}{96}\text{ in.}=\frac{\cancel{2}\times3\times\cancel{12}}{\cancel{96},4}=\frac{3}{4}\text{ in.}$$ Hence, the

RULE.—*Reduce the numerator to the denomination required, and place it over the given denominator; cancelling the factors common to both.* (Art. 276.)

NOTE.—The steps in this operation are the same as those in *Reduction Descending.*

2. Reduce $\frac{3}{60}$ of a bushel to the fraction of a quart.
3. Reduce £$\frac{1}{396}$ to the fraction of a penny.
4. Reduce $\frac{5}{1000}$ of a day to the fraction of an hour.
5. What part of an ounce is $\frac{3}{25}$ of a pound?
6. What part of a square inch is $\frac{2}{512}$ of a sq. foot?

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289. What are denominate fractions? How are they expressed? 290. How are denominate fractions reduced from higher denominations to lower? *Note* What are the steps in this operation like? Explain Ex. 1 from the blackboard.

CASE II.

291. To reduce a Denominate Fraction from *Lower* denominations to *Higher*.

7. What part of a yard is $\frac{3}{4}$ of an inch?

ANALYSIS.—Reducing 1 yard to fourths of an inch, we have 1 yd. $=36$ in.; and 36 in. $\times 4=144$ fourths inch. Now 3 fourths are $\frac{3}{144}$ of 144 fourths; therefore $\frac{3}{4}$ of an inch is $\frac{3}{144}$ or $\frac{1}{48}$ of a yard. Hence, the

RULE.—*Reduce a unit of the denomination in which the required fraction is to be expressed, to the same denomination as the given fraction, and place the numerator over it.*

8. What part of a dollar is $\frac{5}{8}$ of a mill?
9. What part of a pound is $\frac{5}{8}$ of a farthing?
10. What part of a Troy ounce is $\frac{5}{12}$ of a grain?
11. Reduce $\frac{3}{5}$ of a gill to the fraction of a gallon.
12. Reduce $\frac{2}{3}$ of a rod to the fraction of a mile.
13. Reduce $\frac{7}{8}$ of a pound to the fraction of a ton.

CASE III.

292. To reduce a *Denominate Fraction* from higher to a *Whole Number* of lower denominations.

14. Reduce $\frac{4}{5}$ of a yard to feet and inches.

ANALYSIS.—Reducing the given fraction to the next lower denomination, we have $\frac{4}{5}$ yard $\times 3=\frac{12}{5}$ ft. or 2 ft. and $\frac{2}{5}$ ft. remainder.

Again, reducing the remainder to the next lower denomination, we have $\frac{2}{5}$ ft. $\times 12=\frac{24}{5}$, or $4\frac{4}{5}$ in. Therefore, $\frac{4}{5}$ yards equals 2 ft. $4\frac{4}{5}$ in. (Arts. 147, 276.) Hence, the

RULE.—*Multiply the given numerator by the number required to reduce the fraction to the next lower denomination, and divide the product by the denominator.* (Art. 276.)

Multiply and divide the successive remainders in the same manner till the lowest denomination is reached. The several quotients will be the answer required.

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291. How are denominate fractions reduced from lower denominations to higher? 292. How are denominate fractions reduced from higher to whole numbers of lower denominations?

15. Reduce $\frac{3}{4}$ of an eagle to dollars and cents. *Ans.* $7.50.

16. Reduce $\frac{2}{3}$ of a pound sterling to shillings and pence.

17. How many pecks and quarts in $\frac{15}{16}$ of a bushel?

18. How many pounds in $\frac{5}{8}$ of a ton?

19. How many ounces, etc., in $\frac{7}{8}$ of a Troy pound?

20. What is the value of $\frac{5}{12}$ of a mile?

21. What is the value of $\frac{11}{12}$ of an acre?

22. Reduce $\frac{7}{8}$ of a cord to cu. feet.

CASE IV.

293. To reduce a *Compound Number* from lower to a *Denominate Fraction* of higher denominations.

23. Reduce 2 ft. 3 in. to the fraction of a yard.

ANALYSIS.—2 ft. 3 in.=27 inches; and 1 yard=36 inches. The question now is, what part of 36 in. is 27 in.? But 27 is $\frac{27}{36}$ or $\frac{3}{4}$ of 36. (Art. 173.) Therefore 2 ft. 3 in. is $\frac{3}{4}$ of a yard. Hence, the

RULE.—I. *Reduce the given number to its lowest denomination for the numerator.*

II. *Reduce to the same denomination, a unit of the required fraction, for the denominator.*

NOTES.—1. If the lowest denomination of the given number contains a fraction, the number must be reduced to the parts indicated by the denominator of the fraction.

2. If it is required to find *what part* one compound number is of another which contains different denominations, *reduce both to the lowest denomination mentioned in either, and proceed as above.*

24. Reduce 2 qt. 1 pt. $3\frac{1}{2}$ gills to the fraction of a gal.?

25. What part of a pound Troy is 5 oz. 2 pwt. 10 gr.?

26. Reduce 3 pk. 2 qt. 1 pt. to the fraction of a bushel.

27. Reduce 6 cwt. 48 lb. to the fraction of a ton.

28. What part of a square yard is 5 sq. ft. $20\frac{4}{5}$ sq. inches?

29. What part of £3, 10s. 2d. is 7s. 9d. 2 far.?

30. What part of a week is 3 days, 5 hrs. 40 min?

31. What part of a cord is $25\frac{3}{5}$ cu. feet of wood?

32. What part of 5 miles 2 fur. 14 rods is 2 m. 1 fur. 2 rods?

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293. How are denominate numbers reduced from lower denominations to fractions of a higher denomination? Explain Ex. 23 from the blackboard.

CASE V.

294. **To reduce a *Denominate Decimal* from a higher to a *Compound Number* of lower denominations.**

33. Reduce .89375 gallon to quarts, pints, and gills?

ANALYSIS.—This example is a case of Reduction Descending. (Art. 276.) We therefore multiply the given decimal of a gallon by 4 to reduce it to the next lower denomination, and pointing off the product as in multiplication of decimals, the result is 3 qts. and .57500 qt. In like manner, we multiply the decimal .57500 qt. by 2 to reduce it to pints, and the result is 1 pt. and .15000 pt. Finally, we multiply the decimal .15000 pt. by 4 to reduce it to gills, and the result is .6 gill. Therefore, .89375 gal. equals 3 qt. 1 pt. 0.6 gi. Hence, the

```
  .89375 gal.
       4
  3.57500 qt.
       2
  1.15000 pt.
       4
   .60000 gi.
```

Ans. 3 qt. 1 pt. 0.6 gi.

RULE.—*Multiply the denominate decimal by the number required of the next lower denomination to make one of the given denomination, and point off the product as in multiplication of decimals.* (Arts. 191, 276.)

Proceed in this manner with the decimal part of the successive products, as far as required. The integral part of the several products will be the answer.

34. What is the value of £.125445 in shillings, etc.?
35. What is the value of .91225 of a Troy pound?
36. Reduce .35 mile to rods, etc.
37. How many quarts and pints in .625 of a gallon?
38. How many minutes and seconds in .651 degree?
39. How many days, hours, etc., in .241 week?
40. What is the value of .25256 ton?
41. What is the value of .003 of a Troy pound?
42. What is the value of £5.62542?

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294. How are denominate decimals reduced from higher denominations to whole numbers of a lower denomination? Explain Ex. 33 upon the blackboard.

CASE VI.

295. **To reduce a *Compound Number* from lower to a *Denominate Decimal* of a higher denomination.**

43. Reduce 3 pk. 2 qt. 1 pt. to the decimal of a bushel.

OPERATION.

2	1 pt.
8	2.5 qt.
4	3.3125 pk.
	Ans. .828125 bu.

ANALYSIS.—2 pints make 1 quart; hence there is $\frac{1}{2}$ as many quarts as pints, and $\frac{1}{2}$ of 1 qt. is .5 qt., which we write as a decimal on the right of the given quarts. In like manner there is $\frac{1}{8}$ as many pecks as quarts, and $\frac{1}{8}$ of 2.5 is .3125 pk., which we place on the right of the given pecks. Finally, there is $\frac{1}{4}$ as many bushels as pecks, and $\frac{1}{4}$ of 3.3125 is .828125 bu. Therefore, 3 pk. 2 qt. 1 pt. equals .828125 bushel. Hence, the

RULE.—*Write the numbers in a column, placing the lowest denomination at the top.*

Beginning with the lowest, divide it by the number required of this denomination to make a unit of the next higher, and annex the quotient to the next denomination.

Proceed in this manner with the successive denominations, till the one required is reached.

44. Reduce 6 oz. 10 pwt. 4 gr. to the decimal of a pound.
45. Reduce 10 lb. to the decimal of a ton.
46. Reduce 3 fur. 25 r. 4 yd. to the decimal of a mile.
47. Reduce 9.6 pwt. to the decimal of a Troy pound.
48. What decimal part of a barrel are 15 gal. 3 qt.
49. What decimal part of 2 rods are $2\frac{1}{4}$ fathoms?
50. What decimal part of £3, 3s. 6d. are 15s. 10.5d.?
51. What part of a barrel are 94.08 lbs. of flour?
52. Reduce 7.92 yds. to the decimal of a rod.
53. Reduce 1 day 4 hr. 10 sec. to the decimal of a w
54. Reduce 45 sq. rods 25 sq. ft. to the decimal of an a
55. Reduce $53\frac{1}{3}$ cu. feet to the decimal of a cord.

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295. How are compound numbers reduced from lower denominations to denominate decimals of a higher? Explain Ex. 43 upon the blackboard?

METRIC WEIGHTS AND MEASURES.

296. ***Metric Weights and Measures*** are founded upon the *decimal notation,* and are so called because their *primary unit* or *base* is the *Meter.**

297. The ***Meter*** is the *unit of length,* and is equal to *one ten-millionth* part of the distance on the earth's surface from the equator to the pole, or 39.37 inches nearly.

NOTES.—1. The term *meter* is from the Greek *metron,* a *measure.*

2. The *standard meter* is a *bar* of *platinum* deposited in the archives of Paris.

298. The Metric System employs *five* different units or denominations and *seven* prefixes.

The units are the *me'ter, li'ter, gram, ar,* and *ster.*†

299. The names of the *higher* denominations are formed by prefixing to the *unit* the *Greek* numerals, *dek'a* 10, *hek'to* 100, *kil'o* 1000, and *myr'ia* 10000; as, *dek'a-me'ter,* 10 meters; *hek'to-me'ter,* 100 meters, etc.

The names of the *lower* denominations, or *divisions* of the unit, are formed by prefixing to the *unit* the *Latin* numerals, *dec'i* (des'-ee) .1, *cen'ti* (cent'-ee) .01, and *mil'li* (mil'-lee) .001; as, *dec'i-me'ter,* $\frac{1}{10}$ meter; *cen'ti-me'ter,* $\frac{1}{100}$ meter; *mil'li-me'ter,* $\frac{1}{1000}$ meter.

NOTE.—The *numeral prefixes* are the *key* to the whole system; their meaning therefore should be *thoroughly* understood.

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296. Upon what are the Metric Weights and Measures founded? Why so called? 297. What is the meter? 299. How are the names of the lower denominations formed? The higher?

* This system had its origin in France, near the close of the last century. Its simplicity and comprehensiveness have secured its adoption in nearly all the countries of Europe and South America. Its use was legalized in Great Britain in 1864; and in the United States, by Act of Congress, 1866.

† The spelling, pronunciation, and abbreviation of metric terms in this work, are the same as adopted by the American Metric Bureau, Boston, and the Metrological Society, New York.

LINEAR MEASURE.

300. The ***Unit of Length*** is the METER, which is equal to 39.37 inches.*

The denominations are the *mil'li-me'ter, cen'ti-me'ter, dec'i-me'ter, me'ter, dek'a-me'ter, hek'to-me'ter, kil'o-me'ter,* and *myr'ia-me'ter.*

TABLE.

10 *mil'li-me'ters* (*mm.*)	make	1 cen'ti-me'ter - -	*cm.*
10 cen'ti-me'ters	"	1 dec'i-me'ter - -	*dm.*
10 dec'i-me'ters	"	1 ***ME'TER*** - -	*m.*
10 me'ters	"	1 dek'a-me'ter - -	*Dm.*
10 dek'a-me'ters	"	1 hek'to-me'ter - -	*Hm.*
10 hek'to-me'ters	"	1 *kil'o-me'ter* - -	*Km.*
10 kil'o-me'ters	"	1 myr'ia-me'ter -	*Mm.*

NOTES.—1. The *Accent* of each *unit* and *prefix* is on the *first* syllable, and remains so in the compound words.

To *abbreviate* the compounds, pronounce only the prefix and the first letter of the unit; as, centĭm, millĭm, centĭl, decĭg, hektōg, etc.

2. The *meter*, like our *yard*, is used in measuring cloths, laces, moderate distances, etc.

For long distances the *kilometer* (3280 ft. 10 in.) is used; and for minute measurements, the *centimeter* or *millimeter*.

3. *Decimeters, dekameters, hektometers,* like our dimes and eagles, are seldom used.

SQUARE MEASURE.

301. The ***Unit*** for measuring *surfaces* is the *Square Meter,* which is equal to 1550 sq. in.

The denominations are the *sq. cen'ti-me'ter, sq. dec'i-me'ter,* and *sq. me'ter.*

100 sq. cen'tim.	make	1 sq. dec'i-me'ter - -	*sq. dm.*
100 sq. dec'im.	"	1 ***Sq. Me'ter*** - -	*sq. m.*

NOTE.—The *square meter* is used in measuring floorings, ceilings, etc.; *square deci-meters* and *centi-meters,* for minute surfaces.

300. What is the unit of Linear Measure? Its denominations? Recite the Table. 301. What is the unit for measuring surfaces? Recite the table.

* Authorized by Act of Congress, 1866.

302. The ***Unit*** for measuring *land* is the *Ar*, which is equal to a *square dekameter*, or 119.6 sq. yards.

The only *subdivision* of the *Ar* is the *cen'tar;* and the only *multiple* is the *hek'tar.* Thus,

100 cent'ars (*ca.*)	make	1 ***Ar*** - - -	*a.*
100 ars	"	1 hek'tar - -	*Ha.*

NOTES.—1. The term *ar* is from the Latin *area*, a *surface.*

2. In Square Measure, it takes 100 units of a *lower* denomination to make *one* in the *next higher;* hence each denomination must have *two places* of figures. In this respect *centars* correspond to *cents.*

CUBIC MEASURE.

303. The ***Unit*** for measuring *solids* is the *Cubic Meter*, which is equal to 35.316 cu. ft.

TABLE.

1000 cu. mil'lim.	make	1 cu. cen'ti me'ter	- *cu. cm.*
1000 cu. cen'tim.	"	1 cu. dec'i-me'ter	- *cu. dm.*
1000 cu. dec'im.	"	1 ***Cu. Me'ter***	- *cu. m.*

NOTES.—1. The *cubic meter* is used in measuring embankments, excavations, etc.; *cubic centimeters* and *millimeters*, for minute bodies.

2. Since it takes 1000 units of a lower denomination in cubic measure to make *one* of the next higher, it is plain that, like mills, each denomination requires *three places* of figures.

304. In *measuring wood*, the ***Ster***, which is equal to a cubic meter, is sometimes used.

The only *subdivision* of the *ster* is the *dec'i-ster;* and the only *multiple*, the *dek'a-ster.* Thus,

10 dec'i-sters	make	1 ***Ster*** - - -	*s.*
10 sters	"	1 dek'a-ster -	*Ds.*

NOTES.—1. The term *ster* is from the Greek *stereos*, a *solid.*

2. In France, *firewood* is commonly measured in a *cubical box*, whose *length, breadth, and height* are each 1 meter.

3. As the *ster* is applied only to wood, and probably will never come into general use, its divisions and multiples may be omitted.

Note. How many places of figures does each denomination occupy? Why? 302. What is the unit for measuring land? Its divisions? Its multiples? 303. What are the units for measuring solids? Recite the Table. How many places does each denomination in cubic measure occupy? Why?

DRY AND LIQUID MEASURE.

305. The ***Unit of Dry and Liquid Measure*** is the *Li'ter* (*lee'ter*), which is equal to a *cubic decim*, or 1.0567 liquid quart or 0.908 dry quart.

The denominations are the *mil'li-li'ter*, *cen'ti-li'ter*, *dec'i-li'ter*, *li'ter*, *dek'a-li'ter*, *hek'to-li'ter*, *kil'o-li'ter*.

10 mil'li-li'ters (*ml.*)	make	1 cen'ti-li'ter - - -	*cl.*
10 cen'ti-li'ters	"	1 dec'i-li'ter - - -	*dl.*
10 dec'i-li'ters	"	1 ***Li'ter*** - - - -	*l.*
10 li'ters	"	1 dek'a-liter - - -	*Dl.*
10 dek'a-li'ters	"	1 hek'to-li'ter - -	*Hl.*
10 hek'to-li'ter	"	1 kil'o-li'ter - - -	*Kl.*
10 kil'o-li'ters	"	1 myr'ia-li'ter - -	*Ml.*

NOTES.—1. The *liter* is used in measuring milk, wine, etc. For small quantities, the *centiliter* and *milliliter* are employed; and for large quantities, the *dekaliter*.

2. For measuring grain, etc., the *hektoliter*, which is equal to 2.8375 bushels, is commonly used.

3. The term *liter* is from the Greek *litra*, a *pound*.

WEIGHT.

306. The ***Unit of Weight*** is the *Gram*, which is equal to 15.432 grains.

The denominations are the *mil'li-gram*, *cen'ti-gram*, *dec'i-gram*, *gram*, *dek'a-gram*, *hek'to-gram*, *kil'o-gram*, *myr'ia-gram*, and *ton'neau* or *ton*.

10 mil'li-grams (*mg.*)	make	1 cen'ti-gram - -	*cg.*
10 cen'ti-grams	"	1 dec'i-gram - -	*dg.*
10 dec'i-grams	"	1 ***Gram*** - - -	*g.*
10 grams	"	1 dek'a-gram - -	*Dg.*
10 dek'a-grams	"	1 hek'to-gram - -	*Hg.*
10 hek'to-grams	"	1 ***Kil'o-gram*** -	*Kg.*
10 kil'o-grams	"	1 myr'ia-gram - -	*Mg.*
100 myr'ia-grams	"	1 TONNEAU or Ton	*T.*

305. What is the unit of Dry and Liquid Measure? Its denominations? Repeat the Table. 306. What is the unit of Weight? Its denominations?

REMARK.—As the quintal (10 Mg.) is seldom used, and millier means the same as *tonneau* or *ton*, both these terms may be dropped from the table. The metric *ton* is equal to 2204.6 lbs.

NOTES.—1. This Table is used in computing the weight of all objects, from the minutest atom to the largest heavenly body.

2. The *gram* is derived from the Greek *gramma*, a *rule* or *standard*, and is equal to a *cubic centimeter* of distilled water at its greatest density, viz., at the temperature of 4° centigrade thermometer, or 39.83° Fahrenheit, weighed in a vacuum.

3. The *common unit* for weighing groceries and coarse articles is the *kilogram*, which is equal to 2.2046 pounds Avoirdupois.

4. *Kilogram* is often contracted into *kilo*, and *tonneau* into *ton*.

307. To express Metric Denominations decimally in terms of a given Unit.

1. Write 7 Hm. 0 Dm. 9 m. 3 dm. 5 cm. decimally in terms of a meter.

OPERATION.

709.35 m., *Ans.*

ANALYSIS.—Metric denominations increase and decrease by the scale of 10, and correspond to the orders of the Arabic Notation. We therefore write the given meters as units, the dekameters as tens, and the hektometers as hundreds, with the decimeters and centimeters on the right as decimals. Hence, the

RULE.—*Write the denominations above the given unit in their order, on the left of a decimal point, and those below the unit, on the right as decimals.*

NOTES.—1. If any intervening denominations are omitted in the given number, their places must be supplied by *ciphers*.

2. As each denomination in square measure occupies *two places* of figures, in writing *square* decimeters, etc., as *decimals*, if the number is *less* than 10, a cipher must be prefixed to the figure denoting them.

Thus, 17 sq. m., and 2 sq. dm. = 17.02 sq. meters. (Art. 302, *n.*)

3. In like manner, in writing *cubic* decimeters, etc., as decimals, if the number is *less* than 10, two ciphers must be prefixed to it.

Thus, 63 cu. m. and 5 cu dm. = 63.005 cu. meters.

Repeat the Table. *Note.* The common unit for weighing groceries, etc.? 307. How write metric quantities in terms of a given unit? *Note.* If any denomination is omitted, what is to be done?

308. Metric denominations expressed *decimally* are read like numbers in the Arabic notation.

Thus, the Answer to Ex. 1, (709.35), is read, "seven hundred and nine, and thirty-five hundredths meters, or seven hundred nine meters, thirty-five," as we read $709.35, "seven hundred nine dollars, thirty-five," omitting the name cents.

NOTE.—The number of figures following the name of the unit, shows the *denomination* of the last decimal figure.

If the *name* is required, it is better to use abbreviations of the metric terms, as *centims, millims,* etc., than the English words *hundredths,* or *thousandths* of a meter, etc. The former are not only significant of the *kind* of unit and the *value* of the *decimal,* but are understood by *all nations.*

Write decimally, and read the following:

1. 5 Mm., 3 Km., 7 Hm., 0 Dm., 9 m., 8 dm., 4 cm., 5 mm. in terms of a meter, hektometer, kilometer, and decimeter.

2. 4 Kg., 5 Hg., 0 Dg., 5 g., 1 dc., 0 cg., 8 mg. in terms of a dekagram, centigram, hektogram, and kilogram.

3. Write and read 5 Kl., 0 Hl., 6 Dl., 3 l., 0 dl. 8 cl. in terms of a hektoliter. *Ans.* 50.6308 hektoliters.

309. To reduce higher Metric Denominations to lower.

Ex. 1. Reduce 46.3275 kilometers to meters.

ANALYSIS.—From kilometers to meters there are *three* denominations, and it takes 10 of a *lower* denomination to make a *unit* of the next higher. We therefore multiply by 1000, or remove the decimal point *three places* to the right. (Art. 181.) Hence, the

OPERATION.

46.3275 Km.
1000
Ans. 46327.5000 m.

RULE.—*Multiply the given denomination by* 10, 100, 1000, *etc., as the case may require; and point off the product as in multiplication of decimals.* (Art. 191.)

NOTE.—It should be remembered that in the Metric System each denomination of *square* measure requires *two* figures; and each denomination of *cubic* measure, *three* figures.

2. In 43.75 hectares, how many square meters?
3. Reduce 867 kilograms to grams.
4. Reduce 264.42 hectoliters to liters.
5. In 2561 ares, how many square meters?
6. In 8652 cubic meters, how many cubic decimeters?
7. Reduce 4256.25 kilograms to grams.

310. To reduce lower Metric Denominations to higher.

8. Reduce 84526.3 meters to kilometers.

OPERATION.

$$1000\overline{)84526.3} \text{ m.}$$
$$84.5263 \text{ km.}$$

ANALYSIS.—Since it takes 10 linear units to make *one* of the next higher denomination, it follows that to reduce a number from a lower to the next higher denomination, it must be divided by 10; to reduce it to the next higher still, it must be again divided by 10, and so on. From meters to kilometers there are three denominations; we therefore divide by 1000, or remove the decimal point *three* places to the left. The answer is 84.5263 km. Hence, the

RULE.—*Divide the given denomination by* 10, 100, 1000, *etc., as the case may require, and point off the quotient as in division of decimals.* (Art. 194.)

9. In 652254 square meters, how many hectares?
10. Reduce 87 meters to kilometers.
11. In 1482.35 grams, how many kilograms?
12. In 39267.5 liters, how many kiloliters?

APPROXIMATE VALUES.

310, ***a.*** In comparing Metric Weights and Measures with those now in use, the *approximate values* are often convenient. Thus, when no great accuracy is required, we may consider

1 decimeter as	4 in.		1 cu. meter or stere as		$\frac{1}{4}$ cord.
1 meter	"	40 in.	1 liter	"	1 quart.
5 meters	"	1 rod.	1 hectoliter	"	$2\frac{1}{2}$ bushels.
1 kilometer	"	$\frac{5}{8}$ mile.	1 gram	"	$15\frac{1}{2}$ grains.
1 sq. meter	"	$10\frac{3}{4}$ sq. feet.	1 kilogram	"	$2\frac{1}{5}$ pounds.
1 hectare	"	$2\frac{1}{2}$ acres.	1 metric ton	"	2200 pounds.

309. How reduce higher metric denominations to lower? 310. Lower to higher?

APPLICATIONS OF METRIC WEIGHTS AND MEASURES.

311. To add, subtract, multiply, and divide Metric Weights and Measures.

RULE.—*Express the numbers decimally, and proceed as in the corresponding operations of whole numbers and decimals.*

1. What is the sum of 7358.356 meters, 8.614 hectometers, and 95 millimeters?

SOLUTION.—7358.356 m. + 861.4 m., + .095 m. = 8219.851 m. *Ans.*

2. What is the difference between 8.5 kilograms and 976 grams?

SOLUTION.—8.5 kilos — .976 kilos = 7.524 kilos. *Ans.*

3. How much silk is there in $12\frac{1}{2}$ pieces, each containing 48.75 meters?

SOLUTION.—48.75 m. × 12.5 = 609.375 m. *Ans.*

4. How many cloaks, each containing 5.68 meters, can be made from 426 meters of cloth?

SOLUTION.—426 m. ÷ 5.68 m. = 75 cloaks. *Ans.*

312. To reduce Metric to Common Weights and Measures.

1. Reduce 5.6 meters to inches.

ANALYSIS.—Since in 1 meter there are 39.37 inches, in 5.6 meters there are 5.6 times 39.37 in.; and 39.37 × 5.6 = 220.472 in. Therefore, in 5.6 m. there are 220.472 in. Hence, the

$$\begin{array}{r} 39.37 \text{ in.} \\ 5.6 \text{ m.} \\ \hline 23622 \\ 19685 \\ \hline 220.472 \text{ in.} \end{array}$$

Ans. 220.472 in.

RULE.—*Multiply the value of the principal unit of the Table by the given metric number.*

NOTE.—Before multiplying, the *metric number* should be reduced to the same denomination as the *principal unit*, whose value is taken for the multiplicand.

---

311. How are Metric Weights and Measures added, subtracted, multiplied, and divided? 312. How reduce metric to common weights and measures?

2. In 45 kilos, how many pounds? *Ans.* 99.207 lbs.
3. In 63 kilometers, how many miles?
4. Reduce 75 liters to gallons.
5. Reduce 56 dekaliters to bushels.
6. Reduce 120 grams to ounces.
7. Reduce 137.75 kilos to pounds.
8. In 36 ares, how many square rods?

ANALYSIS.—In 1 are there are 119.6 sq. yds.; hence in 36 ares there are 36 times as many. Now $119.6 \times 36 = 4305.6$ sq. yds., and 4305.6 sq. yds. $\div 30\frac{1}{4} = 142.33$ sq. rods. *Ans.*

9. In 60.25 hektars, how many acres?
10. In 120 cu. meters, how many cu. feet?

313. To reduce Common to Metric Weights and Measures.

11. Reduce 213 feet 4 inches to meters.

ANALYSIS.—213 ft. 4 in. = 2560 in. Now, in 39.37 in. there is 1 meter; therefore, in 2560 in. there are as many meters as 39.37 is contained times in 2560; and $2560 \div 39.37 = 65.02+$ meters. Hence, the

OPERATION.

39.37)2560.00 in.
Ans. 65.02 + m.

RULE.—*Divide the given number by the value of the principal metric unit of the Table.*

NOTE.—Before dividing, the given number should be reduced to the *lowest* denomination it contains; then to the denomination in which the *value* of the *principal unit* is expressed.

12. In 63 yds. 3 qrs., how many meters?
13. Reduce 13750 pounds to kilograms.
14. Reduce 250 quarts to liters.
15. Reduce 2056 bu. 3 pecks to kiloliters.
16. In 3 cwt. 15 lbs. 12 oz., how many kilos?
17. In 7176 sq. yards, how many sq. meters?
18. In 40.471 acres, how many hektars?
19. In 14506 cu. feet, how many cu. meters?
20. In 36570 cu. yards, how many cu. meters?

313. How reduce common to metric weights and measures?

COMPOUND ADDITION.

314. To add two or more Compound Numbers.

1. What is the sum of 13 lb. 7 oz. 3 pwt. 18 gr.; 9 oz. 5 pwt. 6 gr.; and 2 lb. 8 oz. 8 pwt. 3 gr.?

OPERATION.

	lb.	oz.	pwt.	gr.
	13	7	3	18
		9	5	6
	2	8	8	3
Ans.	17	0	17	3

ANALYSIS.—Writing the same denominations one under another, and beginning at the right, 3 gr. + 6 gr. + 18 gr. are 27 gr. The next higher denomination is pwt., and since 24 gr. = 1 pwt., 27 gr. must equal 1 pwt. and 3 grains. We set the 3 gr. under the column added, because they are grains, and carry the 1 pwt. to the column of penny weights, because it is a *unit* of that column. (Art. 30, *n.*) Next, 1 pwt. + 8 pwt. + 5 pwt. + 3 pwt. are 17 pwt. As 17 pwt. is less than an *ounce* or a *unit* of the next higher denomination, we set it under the column added. Again, 8 oz. + 9 oz. + 7 oz. are 24 oz. Now as 12 oz. = 1 lb., 24 oz. must equal 2 lb. and 0 oz. We set the 0 under the ounces, and carry the 2 lb. to the next column. Finally, 2 lb. + 2 lb. + 13 lb. are 17 lb., which we set down in full. Hence, the

RULE.—I. *Write the same denominations one under another; and, beginning at the right, add each column separately.*

II. *If the sum of a column is less than a unit of the next higher denomination, write it under the column added.*

If equal to one or more units of the next higher denomination, carry these units to that denomination, and write the excess under the column added, as in Simple Addition.

NOTES.—1. Addition, Subtraction, etc., of Compound Numbers are the *same in principle* as Simple Numbers. The apparent difference arises from their *scales of increase.* The orders of the *latter* increase by the *constant scale* of 10; the denominations of the *former* by a *variable scale.* In both we carry for the *number* which it takes of a lower order or denomination to make *a unit* in the next higher.

2. If the answer contains a *fraction* in any of its denominations, except the lowest, it should be reduced to *whole numbers* of lower denominations, and be added to those of the same name.

---

314. How are compound numbers added? *Note.* The difference between Compound and Simple Addition? If the answer contains a fraction?

3 If they occur in the given numbers, it is generally best to reduce them to lower denominations before the operation is commenced.

2. What is the sum of 6 gal. 3 qt. 1 pt. 2 gi.; 4 gal. 2 qt. 0 pt. 1 gi., and 7 gal. 1 qt. 1 pt. 3 gi.?

Ans. 18 gal. 3 qt. 1 pt. 2 gi.

(3.)

£	s.	d.	far.
7	16	6	1
1	5	8	2
2	7	9	3

(4.)

T.	cwt.	lb.	oz.
3	6	42	4
7	0	26	7
15	17	14	8

(5.)

bu.	pk.	qt.
14	2	5
21	3	6
8	1	7

6. A merchant sold $19\frac{3}{4}$ yd. of silk to one lady; $16\frac{7}{8}$ yd. to a second, $20\frac{1}{2}$ yd. to a third, and $17\frac{5}{8}$ to a fourth: how much silk did he sell to all?

7. If a dairy woman makes 315 lb. 5 oz. of butter in June, 275 lb. 10 oz. in July, 238 lb. 8 oz. in August, and 263 lb. 14 oz. in September, how much will she make in 4 months?

8. A ship's company drew from a cask of water 10 gals. 2 qts. 1 pt. one day, 12 gal. 3 qt. the second, 15 gal. 1 qt. 1 pt. the third, and 16 gal. 3 qt. the fourth: how much water was drawn from the cask in 4 days?

9. A farmer sent 4 loads of wheat to market; the 1st contained 45 bu. 3 pk. 2 qt.; the 2d, 50 bu. 1 pk. 3 qt.; the 3d, 48 bu. 2 pk. 5 qt.; and the 4th, 51 bu. 3 pk. 5 qt.: what was the amount sent?

10. How much wood in 5 loads which contain respectively 1 c. $28\frac{1}{2}$ cu. ft.; 1 c. $47\frac{1}{4}$ cu. ft.; 1 c. 11 cu. ft. 1 c. 110 cu. ft.; and 1 c. $11\frac{1}{4}$ cu. ft.?

11. A miller bought 3 loads of wheat, containing 28 bu. 3 pks.; 36 bu. 1 pk. 7 qts.; 33 bu. 2 pks. 3 qts. respectively: how many bushels did he buy?

If any of the given numbers contain a fraction, how proceed? 315. How add denominate fractions?

12. What is the sum of 13 m. 65 r. 3 yd. 1 ft.; 12 m. 40 r. 2 yd. 1 ft.; and 21 m. 19 r. 1 yd. 2 ft.?

SOLUTION.—In dividing the yards by $5\frac{1}{2}$ (the number required to make a rod), the remainder is $1\frac{1}{2}$ yard. But $1\frac{1}{2}$ yd. equals 1 ft. 6 in.; we therefore add 1 ft. 6 in. to the feet and inches in the result. The *Ans.* is 46 m. 125 r. 1 yd. 2 ft. 6 in.

	m.	r.	yd.	ft.	
	13	65	3	1	
	12	40	2	1	
	21	19	1	2	
	46	125	$1\frac{1}{2}$	1	
$\frac{1}{2}$ yd. =			0	1	6 in.
Ans.	46	125	1	2	6

13. What is the sum of 8 w. $2\frac{1}{2}$ d. 7 h. 40 m.; 4 w. $5\frac{1}{4}$ d. 0 h. 15 m.; and 10 w. 0 d. 16 h. 3 m.?

14. What is the sum of 15 A. 110 sq. r. 30 sq. yds.; 6 A. 45 sq. r. 16 sq. yds; 42 A. 10 sq. r. 25 sq. yds.?

315. To add two or more Denominate Fractions.

15. What is the sum of $\frac{4}{5}$ bu. $\frac{1}{2}$ pk. and $\frac{3}{4}$ quart?

ANALYSIS.—Reducing the denominate fractions to whole numbers, then adding them, the amount is 3 pk. 6 qt. $0\frac{7}{10}$ pt., *Ans.* (Art. 292.)

	pk.	qt.	pt.
$\frac{4}{5}$ bu. =	3 pk.	1 qt.	$1\frac{1}{5}$ pt.
$\frac{1}{2}$ pk. =	0	4	0
$\frac{3}{4}$ qt. =	0	0	$1\frac{1}{2}$
Ans.	3	6	$0\frac{7}{10}$

16. What is the sum of £.125, .65s. and .75d.?

ANALYSIS.—Reducing the denominate decimals to whole numbers of lower denominations, then adding, the result is 3s. 2d. 2.2 far. Hence, the

	s.	d.	far.
£.125 =	2s.	6d.	0 far.
.65s. =	0	7	3.2
.75d. =	0	0	3
Ans.	3	2	2.2

RULE.—*Reduce the denominate fractions, whether common or decimal, to whole numbers of lower denominations: then add them like other compound numbers.*

17. Add .75 T. .5 cwt. .25 lb.

18. Add .25 bu. $\frac{1}{2}$ of 3 pk.

19. Add $\frac{3}{4}$ lb. $\frac{1}{20}$ oz. $\frac{5}{12}$ pwt.

20. Add £.15, .5s. .8d.

21. Bought 4 town lots, containing $\frac{1}{4}$ acre; $\frac{3}{8}$ acre; $121\frac{1}{2}$ sq. rods; and 150.5 sq. rods respectively: how much land was there in the 4 lots?

22. Bought 3 loads of wood; one containing $\frac{4}{5}$ cord, another 75.3 cu. feet; the other $1\frac{1}{4}$ cord: how much did they all contain?

COMPOUND SUBTRACTION.

316. To find the Difference between two Compound Numbers.

1. From 5 mi. 3 fur. 11 r. 4 ft. take 2 mi. 1 fur. 4 r. 9 ft.

	m.	fur.	r.	ft.
	5	3	11	4
	2	1	4	9
Ans.	3	2	6	$11\frac{1}{2}$

ANALYSIS.—We write the less number under the greater, feet under feet, etc. Beginning at the right, 9 ft. cannot be taken from 4 feet; we therefore borrow 1 rod, a *unit* of the next higher denomination. Now 1 rod, or $16\frac{1}{2}$ ft., added to 4 ft. are $20\frac{1}{2}$ ft., and 9 ft. from $20\frac{1}{2}$ ft. leave $11\frac{1}{2}$ ft. Set the remainder under the term subtracted, and carry 1 to the next term in the lower number. 1 rod and 4 r. are 5 r., and 5 r. from 11 r. leave 6 rods. 1 furlong from 3 fur. leaves 2 fur.; and 2 m. from 5 m. leave 3 m. Therefore, etc. Hence, the

RULE.—I. *Write the less number under the greater, placing the same denominations one under another.*

II. *Beginning at the right, subtract each term in the lower number from that above it, and set the remainder under the term subtracted.*

III. *If any term in the lower number is larger than that above it, borrow a unit of the next higher denomination and add it to the upper term; then subtract, and carry* 1 *to the next term in the lower number, as in Simple Subtraction.*

NOTES.—1. *Borrowing* in Compound Subtraction is based on the same principle as in Simple numbers. That is, we add as *many units* to the term in the upper number as are required of that denomination to make a *unit* of the next higher.

2. If the answer contains a *fraction* in any of its denominations except the *lowest*, it should be reduced to *lower denominations*, and be united to those of the same name, as in Compound Addition.

2. From 1 mile, take 240 rods 3 yd. 2 ft.

3. From 14 lb. 5 oz. 3 pwt. 16 gr., take 7 lb. 6 pwt. 7 gr.

4. From 12 T. 9 cwt. 41 lb., take 5 T. 1 cwt. 15 lb.

5. Take 20 gal. 3 qt. 1 pt. 2 gi. from 29 gal. 1 qt. 1 pt.

---

316. How subtract compound numbers? *Note.* What is said of borrowing If the answer contains a fraction, how proceed?

6. From 250 A. 45 sq. r. 150 sq. ft., take 91 A. 32 sq. r 200 sq. ft.

7. A miller has two rectangular bins, one containing 324 cu. ft. 110 cu. in.; the other, 277 cu. ft. 149 cu. in.: what is the difference in their capacity?

8. A man having 320 A. 50 sq. r. of land, gave his eldest son 175 A. 29 sq. r., and the balance to his youngest son: what was the portion of the younger?

9. A merchant bought two pieces of silk; the longer contained 57 yd. 3 qr.; the difference between them was $11\frac{5}{8}$ yards: what was the length of the shorter?

10. What is the difference between 10 m. 2 fur. 27 r. 1 ft. 7 in. and 6 m. 4 fur. 28 r. 1 yd.?

317. To find the Difference between two Denominate Fractions.

1. From $\frac{5}{6}$ of a yard, take $\frac{11}{12}$ of a foot.

$\frac{5}{6}$ yd.	=	2 ft.	6 in.
$\frac{11}{12}$ ft.	=	0	11 in.
Ans.		1 ft.	7 in.

ANALYSIS.—Reducing the denominate fractions to whole numbers, we have $\frac{5}{6}$ yard = 2 ft. 6 in. and $\frac{11}{12}$ foot = 11 in. *Ans.* 1 ft. 7 in. Hence, the

RULE.—*Reduce the denominate fractions, whether common or decimal, to whole numbers of lower denominations; then proceed according to the rule above.*

2. From £.525 take .75 of a shilling.

3. From $\frac{7}{8}$ bu. take $\frac{4}{5}$ of a peck.

4. From $\frac{3}{4}$ of a gal., take $1\frac{4}{5}$ of a pint.

5. A goldsmith having a bar of gold weighing 1.25 lbs., cut off sufficient to make 6 rings, each weighing .15 ounce; how much was left?

6. If a man has .875 acre of land, and sells off two building lots, each containing 12.8 sq. rods, how much land will be left?

7. From $\frac{3}{4}$ of $\frac{5}{6}$ cwt. of sugar, take 31.25 lbs

317. How subtract denominate fractions?

318. To find the Difference of Time between two Dates, in years, months, and days.

1. What is the difference of time between July 4th, 1776, and April 3d, 1870?

	y.	m.	d.
	1870	4	3
	1776	7	4
Ans.	93	8	29

ANALYSIS.—We place the earlier date under the later, with the year on the left, the number of the month next, and the day of the month on the right. Since we cannot take 4 from 3, we borrow 30 days, and 4 from 33 leaves 29. Carrying 1 to the next denomination, we proceed as above. (Art. 264, *n.*) Hence, the

RULE.—*Set the earlier date under the later, the years on the left, the month next, and the day on the right, and proceed as in subtracting other compound numbers.*

NOTE.—*Centuries* are numbered in dates, from the *beginning* of the Christian era; *months*, from the beginning of the year; and *days*, from the beginning of the month.

3. Washington was born Feb. 22d, 1732, and died Dec. 14th, 1799: at what age did he die?

319. To find the difference between two Dates in *Days*, the time being less than a year.

1. A note dated Oct. 20th, 1869, was paid Feb. 10th, 1870: how many days did it run?

Oct. 31 − 20	= 11 d.
Nov.	= 30 d.
Dec.	= 31 d.
Jan.	= 31 d.
Feb.	= 10 d.
Ans.	113 d.

ANALYSIS.—In Oct. it ran 31 − 20 = 11 days, omitting the day of the date; in Nov., 30 days; in Dec., 31; in Jan., 31; and in Feb., 10, counting the day it was paid. Now 11 + 30 + 31 + 31 + 10 = 113 days, the number required. Hence, the

RULE.—*Set down the number of days in each month and part of a month between the two dates, and the sum will be the number of days required.*

NOTE.—The day on which a note or draft is *dated*, and that on which it becomes *due*, must *not both* be reckoned. It is customary to omit the *former*, and count the *latter*.

318. How find the difference between two dates in years? 319. In days?

2. What is the number of days between Nov. 10th, 1869, and March 3d, 1870?

3. A person started on a journey Aug. 19th, 1869, and returned Nov. 1st, 1869: how long was he absent?

4. A note dated Jan. 31st, 1870, was paid June 30th, 1870: how many days did it run?

5. How many days from May 21st, 1868, to Dec. 31st, following?

6. The building of a school-house was commenced April 1st, and completed on the 10th of July following: how long was it in building?

320. To find the difference of Latitude or Longitude.

DEF. 1.—***Latitude*** is *distance* from the Equator. It is reckoned in *degrees, minutes*, etc., and is called *North* or *South* latitude, according as it is north or south of the equator.

2. ***Longitude*** is the *distance* on the *equator* between a *conventional or fixed meridian* and the *meridian* of a given place. It is reckoned in *degrees, minutes*, etc., and is called *East* and *West* longitude, according as the place is *east* or *west* of the fixed meridian, until 180° or half the circumference of the earth is reached.

7. The latitude of New York is 40° 42′ 43″ N.; that of New Orleans, 29° 57′ 30″ N.: how much further north is New York than New Orleans? *

ANALYSIS.—Placing the lower latitude under the higher, and subtracting as in the preceding rule, the *Ans.* is 10° 45′ 13″.

N. Y.	40°	42′	43″
N. O.	29°	57′	30″
Ans.	10°	45′	13″

8. The latitude of Cape Horn is 55° 59′ S., and that of Cape Cod 42° 1′ 57.1″ N.: what is the difference of latitude between them?

ANALYSIS.—As one of these capes is *north* of the equator and the other *south*, it is plain that the *difference* of their latitude is the *sum* of the two distances from the equator. We therefore add the two latitudes together, and the result is 98° 0′ 57.1″.

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320. *Def.* What is latitude? Longitude? How find the difference of latitude or longitude between two places?

* The latitude and longitude of the places in the United States here given are taken from the American Almanac, 1854; those of places in foreign lands, mostly from Bowditch's Navigator.

9. The longitude of Paris is 2° 20′ East from Greenwich; that of Dublin is 6° 20′ 30″ West: what is the difference in their longitude?

ANALYSIS.—As the longitude of one of these places is *East* from Greenwich the *standard* meridian, and that of the other *West*, the *difference* of their longitude must be the *sum* of the two distances from the standard meridian. *Ans.* 8° 40′ 30″. Hence, the

$$\begin{array}{r} 2^\circ\ 20'\ 00'' \\ 6^\circ\ 20'\ 30'' \\ \hline 8^\circ\ 40'\ 30'' \end{array}$$

RULE.—I. *If both places are the same side of the equator, or the standard meridian, subtract the less latitude or longitude from the greater.*

II. *If the places are on different sides of the equator, or the standard meridian, add the two latitudes or longitudes together, and the sum will be the answer.*

10. The longitude of Berlin is 13° 24′ E.;* that of New Haven, Ct., 72° 55′ 24″ W.: what is the difference?

11. The longitude of Cambridge, Mass., is 71° 7′ 22″; that of Charlottesville, Va., 78° 31′ 29″: what is the difference?

12. The latitude of St. Petersburg is 59° 56′ north, that of Rome 41° 54′ north: what is their difference?

13. The latitude of Albany is 42° 39′ 3″, that of Richmond 37° 32′ 17″: what is the difference?

14. The latitude of St. Augustine, Flor., is 29° 48′ 30″; that of St. Paul, Min., is 44° 52′ 46″: what is the difference?

15. The longitude of Edinburgh is 3° 12′ W.; that of Vienna 16° 23′ E.†: required their difference?

16. The latitude of Valparaiso is 33° 2′ S.; that of Havana is 23° 9′ N.: what is the difference?

17. The latitude of Cape of Good Hope is 34° 22′ S.; that of Gibraltar, 36° 7′ N.: what is their difference?

18. The longitude of St. Louis is 90° 15′ 16″; that of Charleston, S. C., 79° 55′ 38″: what is the difference?

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* Encyc. Brit. † Encyc. Amer.

COMPOUND MULTIPLICATION.

321. To multiply Compound Numbers.

1. A farmer raised 6 acres of wheat, which yielded 15 bu. 3 pk. 1 qt. per acre: how much wheat had he?

	bu.	pk.	qt.
	15	3	1
			6
Ans.	94	2	6

OPERATION.

ANALYSIS.—6 acres will produce 6 times as much as 1 acre. Beginning at the right, 6 times 1 qt. are 6 qts. As 6 quarts are less than a *peck*, the next higher denomination, we set the 6 under the term multiplied. 6 times 3 pk. are 18 pk. Since 4 pk. = 1 bu., 18 pecks = 4 bu. and 2 pk. over. Setting the remainder 2 under the term multiplied, and carrying the 4 bu. to the next product, we have 6 times 15 bu. = 90 bu., and 4 bu. make 94 bu. Therefore, etc. Hence, the

RULE.—I. *Write the multiplier under the lowest denomination of the multiplicand, and, beginning at the right, multiply each term in succession.*

II. *If the product of any term is less than a unit of the next higher denomination, set it under the term multiplied.*

III. *If equal to one or more units of the next higher denomination, carry these units to that denomination, and write the excess under the term multiplied.*

NOTES.—1. If the multiplier is a *composite* number, multiply by one of the factors, then this partial product by another, and so on.

2. If a fraction occurs in the product of any denomination except the lowest, it should be reduced to *lower denominations*, and be united to those of the same name as in Compound Addition. (Art. 314.)

	(2.)	(3.)
Mult.	12 T. 7 cwt. 16 lb.	£21, 13s. $8\frac{1}{4}$d.
By	8	7

4. What is the weight of 10 silver spoons, each weighing 3 oz. 7 pwt. 13 gr.?

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321. How are compound numbers multiplied? *Note.* If the multiplier is a composite number, how proceed? When a fraction occurs in any denomination except the last, how?

(5.)

Mult. 9 oz. 13 pwt. 7 gr. by 18.

	9 oz.	13 pwt.	7 gr.
			3
2 lb.	4	19	21
			6
14 lb.	5 oz.	19 pwt.	6 gr. *Ans.*

(6.)

Mult. 10 r. 1 yd. 1 ft. by 7.

	10 r.	1 yd.	1 ft.	0 in.
			7	
	71	$3\frac{1}{2}$	1	
$\frac{1}{2}$ yd.=			1	6
Ans.	71 r.	3 yd.	2 ft.	6 in.

7. If a family use 27 gal. 2 qt. 1 pt. of milk in a month, how much will they use in a year?

8. If a man chops 2 cords 67 cu. feet of wood per day, how much will he chop in 9 days?

9. What cost 27 yards of silk, at 17s. $7\frac{1}{4}$d. sterling per yard?

10. If a railroad train goes at the rate of 23 m. 3 fur. 21 r. an hour, how far will it go in 24 hours?

11. How much corn will 63 acres of land produce, at 30 bu. 3 pk. per acre?

12. How many cords of wood in 17 loads, each containing 1 cord 41 cu. ft.?

13. How much hay in 12 stacks of 5 tons, 237 lbs. each?

14. How much paper is required to print 20 editions of a book, requiring 65 reams, 7 quires, and 10 sheets each?

15. If a meteor moves through $5° 23' 15''$ in a second, how far will it move in 30 seconds?

16. If the daily session of a school is 5 h. 45 min., how many school hours in a term of 15 weeks of 5 days each?

17. A man has 11 village lots, each containing 12 sq. r. 4 sq. yd. 6 sq. ft.: how much do all contain?

18. If 1 load of coal weighs 1 T. $48\frac{1}{2}$ lb., what will 72 loads weigh?

19. How many bushels of corn in 12 bins, each containing 130 bu. 3 pk. and 7 qt.?

20. A grocer bought 35 casks of molasses, each containing 55 gal. 2 qt. 1 pt.: how much did they all contain?

COMPOUND DIVISION.

322. ***Division of Compound Numbers,*** like Simple Division, embraces *two classes of problems :*

First.—Those in which the dividend is a *compound* number, and the divisor is an *abstract* number.

Second.—Those in which both the *divisor* and *dividend* are *compound numbers.* In the former the quotient is a *compound number.* In the latter, it is *times,* or an *abstract* number. (Art. 64.)

323. To divide one Compound Number by another, or by an Abstract number.

Ex. 1. A dairy-woman packed 94 lb. 2 oz. of butter in 6 equal jars: how much did each jar contain?

OPERATION.

6)94 lb. 2 oz.

Ans. 15 lb. 11 oz.

ANALYSIS.—The number of parts is given, to find the *value* of each part. Since 6 jars contain 94 lbs. 2 oz., 1 jar must contain $\frac{1}{6}$ of 94 lbs. 2 oz. Now $\frac{1}{6}$ of 94 lbs. is 15 lbs. and 4 lbs. remainder. Reducing the remainder to oz., and adding the 2 oz., we have 66 oz. Now $\frac{1}{6}$ of 66 oz. is 11 oz. (Art. 63, *b.*)

2. A dairy-woman packed 94 lbs. 2 oz. of butter in jars of 15 lbs. 11 oz. each: how many jars did she have?

94 lb. 2 oz. = 1506 oz.
15 lb. 11 oz. = 251 oz.

251 oz.)1506 oz.

Ans. 6 jars.

ANALYSIS.—Here the *size* of each part is given, to find the *number* of parts in 94 lb. 2 oz. Reduce both numbers to oz., and divide as in simple numbers. (Art. 63, *a.*) Hence, the

RULE.—I. When the divisor is an *abstract* number,

Beginning at the left, divide each denomination in succession, and set the quotient under the term divided.

If there is a remainder, reduce it to the next lower denomination, and, adding it to the given units of this denomination, divide as before.

II. When the divisor is a *compound* number,

Reduce the divisor and dividend to the lowest denomination contained in either, and divide as in simple numbers.

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322. How many classes of examples does division of compound numbers embrace? The first? The second? 323. What is the rule?

NOTE.—If the divisor is a *composite* number, we may divide by its factors, as in simple numbers. (Art. 77.)

3. Divide 29 fur. 19 r. 2 yd. 1 ft. by 7.

4. Divide 54 gal. 3 qt. 1 pt. 3 gi. by 8.

5. A miller stored 450 bu. 3 pks. of grain in 18 equal bins: how much did he put in a bin?

6. A farm of 360 A. 42 sq. r. is divided into 23 equal pastures: how much land does each contain?

7. How many spoons, each weighing 2 oz. 10 pwt., can be made out of 5 lb. 6 oz. of silver?

8. How many iron rails, 18 ft. long, are required for a railroad track 15 miles in length?

9. How many times does a car-wheel 15 ft. 6 in. in circumference turn round in 3 m. 25 r. 10 ft.?

10. How many books, at 4s. 6¼d. apiece, can you buy for £2, 14s. 3d.?

11. If 6 men mow 86 A. 64 sq. rods in 6 days, how much will 1 man mow in 1 day?

12. A farmer gathered 150 bu. 3 pk. of apples from 24 trees: what was the average per tree?

COMPARISON OF TIME AND LONGITUDE.

324. The ***Earth*** makes a *revolution* on its axis *once* in 24 hours; hence $\frac{1}{24}$ part of its circumference must pass under the sun in 1 hour. But the circumference of every circle is divided into 360°, and $\frac{1}{24}$ of 360° is 15°. It follows, therefore, that 15° of longitude make a difference of 1 hour in time.

Again, since 15° of longitude make a difference of 1 hour in time, 15′ of longitude ($\frac{1}{60}$ of 15°) will make a difference of 1 minute ($\frac{1}{60}$ of an hour) in time.

In like manner, 15″ of longitude ($\frac{1}{60}$ of 15′), will make a difference of 1 second in time. Hence, the following

TABLE.

15°	of longitude	are	equivalent to	1 hour	of time.
15′	"	"	"	1 minute	"
15″	"	"	"	1 second	"

CASE I.

325. To find the *Difference of Time* between two places, the difference of Longitude being given.

Ex. 1. The difference of longitude between New York and London is 73° 54′ 3″: what is the difference of time?

OPERATION.

15) 73° 54′ 3″

Ans. 4 h. 55 m. 36.2 s.

ANALYSIS.—Since 15° of lon. are equivalent to 1 hour of time, the difference of time must be $\frac{1}{15}$ part as many hours, minutes, and seconds as there are degrees, etc., in the dif. of lon.; and 73° 54′ 3″ ÷ 15 = 4 h. 55′ 36.2″ Hence, the

RULE.—*Divide the difference of longitude by* 15, *and the degrees, minutes, and seconds of the quotient will be the difference of time in hours, minutes, and seconds.* (Art. 323.)

2. The difference of longitude between Savannah, Ga., and Portland, Me., is 10° 53′ 2″: what is the difference in time?

3. The longitude of Boston is 71° 3′ 30″ W., that of Detroit is 83° 2′ 30″ W.: when it is noon in Boston what is the time at Detroit?

4. The longitude of Philadelphia is 75° 9′ 54″, that of Cincinnati 84° 27′: when it is noon at Cincinnati what is the time at Philadelphia?

5. The lon. of Louisville, Ky., is 85° 30′, that of Burlington, Vt., 73° 10′: what is the difference in time?

6. When it is noon at Washington, what is the time of day at all places 22° 30′ east of it? What, at all places 22° 30′ west of it?

7. How much earlier does the sun rise in New York, lon. 74° 3″, than at Chicago, lon. 87° 35′?

8. How much later does the sun set at St. Louis, whose longitude is 90° 15′ 16″ W., than at Nashville, Tenn., whose longitude is 86° 49′ 3″?

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325. How find the difference of time between two places, the difference of longitude being given?

CASE II.

326. To find the *Difference of Longitude* between two places, the difference of Time being given.

9. A whaleman wrecked on an Island in the Pacific, found that the difference of time between the Island and San Francisco was 2 hr. 27 min. $54\frac{3}{5}$ sec.: how many degrees of longitude was he from San Francisco?

ANALYSIS.—Since 15° of lon. are equivalent to 1 hour of time, 15′ of lon. to 1 min. of time, and 15″ to 1 sec. of time, there must be 15 times as many degrees, minutes, and seconds in the difference of longitude as there are hours, minutes, and seconds in the difference of time; and 2 h. 27 m. 54.6 s. × 15 = 36° 58′ 39″. Hence, the

OPERATION.

$$\begin{array}{r} 2\text{ h. } 27\text{ m. } 54.6\text{ sec.} \\ 15 \\ \hline \textit{Ans. } 36^\circ\ 58'\ 39'' \end{array}$$

RULE.—*Multiply the difference of time by* 15, *and the hours, minutes, and seconds of the product will be the difference of longitude in degrees, minutes, and seconds.*

10. The difference of time between Richmond, Va., and Newport, R. I., is 24 min. 36 sec.: what is the difference of longitude?

11. The difference of time between Mobile and Galveston is 27 min. $\frac{1}{3}$ sec.: what is the difference of longitude?

12. The difference of time between Washington and San Francisco is 3 hr. 1 min. 39 sec.: what is the difference in longitude?

13. The distance from Albany, N. Y., to Milwaukee, is nearly 625 miles, and a degree of longitude at these places is about 44 miles: how much faster is the time at Albany than at Milwaukee?

14. The distance from Trenton, N. J., to Columbus, O., is nearly 400 miles, and a degree of longitude at these places is about 46 miles: when it is noon at Columbus what is the time at Trenton?

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326. How find the difference of longitude, when the difference of time is given?

PERCENTAGE.

327. ***Per Cent*** and ***Rate Per Cent*** denote *hundredths.* Thus, 1 per cent of a number is $\frac{1}{100}$ part of that number; 3 per cent, $\frac{3}{100}$, &c.

328. ***Percentage*** is the *result* obtained by finding a certain *per cent* of a number.

NOTE.—The term *per cent,* is from the Latin *per, by* and *centum, hundred.*

NOTATION OF PER CENT.

329. The ***Sign of Per Cent*** is an *oblique line* between two ciphers (%); as 3%, 15%.

NOTE.—The *sign* (%), is a modification of the sign division (÷), the denominator 100 being understood. Thus 5% $=\frac{5}{100}=5\div 100$.

330. Since per cent denotes a certain *part* of a hundred, it may obviously be expressed either by a *common fraction,* whose denominator is 100, or by *decimals,* as seen in the following

TABLE.

3 per cent is written		.03	$\frac{1}{2}$ per cent is written		.005
7 per cent	"	.07	$\frac{1}{4}$ per cent	"	.0025
10 per cent	"	.10	$\frac{3}{4}$ per cent	"	.0075
25 per cent	"	.25	$\frac{3}{5}$ per cent	"	.006
50 per cent	"	.50	$2\frac{1}{2}$ per cent	"	.025
100 per cent	"	1.00	$7\frac{2}{5}$ per cent	"	.074
125 per cent	"	1.25	$31\frac{1}{4}$ per cent	"	.3125
300 per cent	"	3.00	$112\frac{1}{2}$ per cent	"	1.125

NOTES.—1. Since *hundredths* occupy two decimal places, it follows that every *per cent* requires, at least, two decimal figures. Hence,

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327. What do the terms per cent and rate per cent denote? 328. What is percentage? *Note.* From what are the terms per cent and percentage derived? 329. What is the sign of per cent? 330. How may per cent be expressed? *Note.* How many decimal places does it require? Why? If the given per cent is less than 10, what is to be done? What is 100 per cent of a number?

If the given per cent is less than 10, a *cipher* must be prefixed to the figure denoting it. Thus, 2 % is written .02; 6 %, .06, etc.

2. A *hundred* per cent of a number is equal to the *number itself*; for $\frac{100}{100}$ is equal to 1. Hence, 100 per cent is commonly written 1.00.

If the given per cent is 100 or over, it may be expressed by an integer, a mixed number, or an improper fraction. Thus, 125 per cent is written 125 %, 1.25, or $\frac{125}{100}$. Hence,

331. To express Per Cent, Decimally,

Write the figures denoting the per cent in the first two places on the right of the decimal point; and those denoting parts of 1 per cent, in the succeeding places toward the right.

NOTES.—1. When a given part of 1 per cent cannot be exactly expressed by *one* or *two decimal* figures, it is generally written as a *common* fraction, and annexed to the figures expressing the integral per cent. Thus, $4\frac{1}{3}$% is written $.04\frac{1}{3}$, instead of .043333+.

2. In expressing per cent, when the *decimal point* is used, the words *per cent* and the *sign* (%) must be omitted, and *vice versa.* Thus. .05 denotes 5 per cent, and is equal to $\frac{5}{100}$ or $\frac{1}{20}$; but .05 per cent or .05% denotes $\frac{5}{100}$ of $\frac{1}{100}$, and is equal to $\frac{5}{10000}$ or $\frac{1}{2000}$.

Express the following per cents, decimally:

1. 2%, 6%, 8%, 14%, 20%, 35%, 60%, 72%.
2. 80%, 101%, 104%, 150%, 210%, 300%.
3. $1\frac{1}{2}$%, $4\frac{1}{3}$%, $6\frac{1}{5}$%, $8\frac{1}{4}$%, $10\frac{3}{4}$%.

332. To read any given Per Cent, expressed Decimally.

Read the first two decimal figures as per cent; and those on the right as decimal parts of 1 per cent.

NOTE.—Parts of 1 per cent, when easily reduced to a common fraction, are often read as such. Thus .105 is read 10 and a half per cent; .0125 is read one and a quarter per cent.

Read the following as rates per cent:

4. .05; .07; .09; .045; .0625; .1875; .125; .165; .27.
5. .10; .17; .0825; .05125; $.33\frac{1}{3}$; $.16\frac{2}{3}$; .75375.
6. 1.00; 1.06; 2.50; 3.00; 1.125; 1.0725; $1.83\frac{1}{3}$.

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331. How express per cent, decimally? *Note.* When a part of 1 per cent, cannot be exactly expressed by one or two decimal figures, how is it commonly written? 332. How read a given per cent, expressed decimally?

333. To change a given Per Cent from a Decimal to a Common Fraction.

7. Change 5% to a common fraction. *Ans.* $\frac{5}{100}=\frac{1}{20}$.

8. Change .045 to a common fraction. *Ans.* $\frac{9}{200}$.

RULE.—*Erase the decimal point or sign of per cent* (%), *and supply the required denominator.* (Art. 179.)

NOTE.—When a *decimal per cent* is reduced to a common fraction, then to its *lowest* terms, this fraction, it should be observed, will express an *equivalent rate*, but not the *rate per cent.*

Change the following per cents to common fractions:

9. 5 per cent; 10%; 4%; 20%; 25%; 50%; 75%.

10. $6\frac{1}{4}$ per cent; $12\frac{1}{2}$%; $8\frac{1}{3}$%; $33\frac{1}{3}$%; $62\frac{1}{2}$%.

11. $\frac{1}{2}$ per cent; $\frac{2}{5}$%; $\frac{3}{4}$%; $\frac{1}{3}$%; $\frac{2}{3}$%; $\frac{1}{9}$%; $\frac{8}{25}$%; 25%.

334. To change a common Fraction to an equivalent Per Cent.

12. To what per cent is $\frac{1}{3}$ of a number equal?

$\frac{1}{3}=1.00\div3$

$1.00\div3=.33\frac{1}{3}$

ANALYSIS.—*Per cent* denotes *hundredths.* The question then is, how is $\frac{1}{3}$ reduced to *hundredths?* Annexing ciphers to the numerator, and dividing by the denominator, we have $\frac{1}{3}=1.00\div3$ or $.33\frac{1}{3}$. Hence, the

RULE.—*Annex two ciphers to the numerator, and divide by the denominator.* (Art. 186.)

13. To what % is $\frac{1}{2}$ equal? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{1}{5}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$?

14. To what % is $\frac{1}{10}$ equal? $\frac{7}{10}$? $\frac{9}{10}$? $\frac{1}{20}$? $\frac{7}{20}$? $\frac{3}{25}$? $\frac{3}{50}$?

15. To what % is $\frac{2}{3}$ equal? $\frac{1}{6}$? $\frac{1}{8}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{7}{12}$? $\frac{11}{12}$?

335. In calculations of Percentage, *four elements* or *parts* are to be considered, viz.: the *base*, the *rate per cent*, the *percentage*, and the *amount.*

1. The *base* is the *number* on which the *percentage* is calculated.

2. The *rate per cent* is the *number* which shows *how many hundredths* of the *base* are to be taken.

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333. How change a given per cent from a decimal to a common fraction? 334. How change a common fraction to an equal per cent? 335. How many parts are to be considered in calculations by percentage?

3. The *percentage* is the *number obtained* by taking that portion of the *base* indicated by the rate *per cent.*

4. The *amount* is the *base plus,* or *minus* the *percentage.*

The *relation* between these parts is such, that if any *two* of them are *given,* the *other two* may be found.

NOTES.—1. The term *amount,* it will be observed, is here employed in a modified or enlarged sense, as in algebra and other departments of mathematics. This avoids the necessity of an extra rule to meet the cases in which the final result is *less* than the base.

2. The conditions of the question show whether the *percentage* is to be *added* to, or *subtracted* from the *base* to form the *amount.*

3. The learner should be careful to observe the distinction between *percentage* and *per cent,* or *rate per cent.*

Percentage is properly a *product,* of which the *given per cent* or *rate* per cent, is *one* of the *factors,* and the *base* the other. This care is the more necessary as these terms are often used indiscriminately.

4. The terms *per cent, rate per cent,* and *rate,* are commonly used as *synonymous,* unless otherwise mentioned.

PROBLEM I.

336. To find the *Percentage*, the Base and Rate being given.

Ex. 1. What is 5 per cent of $600?

ANALYSIS.—5 per cent is .05; therefore 5 per cent of a number is the same as .05 times that number. Multiplying the base, $600, by the rate .05, and pointing off the product as in multiplication of decimals, the result is $30. (Art. 191.) Hence, the

	$600 B.
	.05 R.
Ans.	$30.00 P.

RULE.—*Multiply the base by the rate, expressed decimally.*

FORMULA. *Percentage* = *Base* × *Rate.*

NOTES.—1. When the *rate* is an *aliquot* part of 100, the *percentage* may be found by taking a *like part* of the base. Thus, for 20%, take $\frac{1}{5}$; for 25%, take $\frac{1}{4}$, etc. (Arts. 105, 270.)

2. When the *base* is a *compound number,* the *lower* denominations should be reduced to a *decimal* of the *highest;* or the higher to the *lowest* denomination mentioned; then apply the rule.

3. Finding a *per cent* of a number is the same as finding a *fractional part* of it, etc. The pupil is recommended to review with care, Arts. 143, 165, 191.

Explain them. What is the relation of these parts? The difference between percentage and per cent?

3. 3% of $807?
4. 5% of 216 bushels?
5. 8% of 282.5 yds.?
6. 4% of 216 oxen?
7. $5\frac{1}{2}$% of 150 yards?
8. 16% of $72.40?
9. 12% of 840 lbs.?
10. 14% of 451 tons?
11. $5\frac{1}{4}$% of 1000 men?
12. $10\frac{1}{5}$% of 1428 meters?
13. 50% of $1715.57?
14. $\frac{1}{2}$% of £21.2?
15. $\frac{1}{4}$% of 500 liters?
16. $\frac{3}{5}$% of 230 kilograms?
17. 100% of 840 pounds?
18. 200% of $500?

19. A farmer raised 875 bu. of corn, and sold 9% of it: how many bushels did he sell?

20. The gold used for coinage contains 10% of alloy: how much alloy is there in $3\frac{1}{2}$ pounds of standard gold?

21. A man having a hogshead of cider, lost $15\frac{1}{2}$% of it by leakage: how many gallons did he lose?

22. A garrison containing 4000 soldiers lost 21% of them by sickness and desertion: what was the number lost?

23. A grocer having 1925 pounds of sugar, sold $12\frac{1}{2}$ per cent of it: how many pounds did he sell?

ANALYSIS.—$12\frac{1}{2}$% is $\frac{1}{8}$ of 100%, and 100% of a number is equal to the number itself; therefore $12\frac{1}{2}$ per cent of a number is equal to $\frac{1}{8}$ of that number, and $\frac{1}{8}$ of 1925 lbs. is $240\frac{5}{8}$ lbs. In the operation we take $\frac{1}{8}$ of the base.

OPERATION.

8)1925 lbs.

Ans. 240.625

Solve the next 9 examples by aliquot parts:

24. Find 25% of $860.
25. 10% of 1572 pounds.
26. $12\frac{1}{2}$% of 258 meters.
27. 20% of 580 liters.

28. A drover taking 2320 sheep to market, lost 25% of them by a railroad accident: how many did he lose?

29. A farmer raised 468 bu. of corn, and $33\frac{1}{3}$% as many oats as corn: how many bushels of oats did he raise?

30. A young man having a salary of $1850 a year, spent 50 per cent of it: what were his annual expenses?

31. What is $33\frac{1}{3}$% of 1728 cu. feet of wood?

32. What is $12\frac{1}{2}$ per cent of £16, 8s.?

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336. How find the percentage when the base and rate are given? When the rate is an aliquot part of 100, how proceed? When a compound number?

PROBLEM II.

337. To find the *Amount*, the Base and Rate being given.

1. A commenced business with \$1500 capital, and laid up 8% the first year: what amount was he then worth?

ANALYSIS.—Since he laid up 8%, he was worth his capital, \$1500, *plus* 8% of itself. But his capital is 100% or 1 time itself; and 100% + 8% = 108% or 1.08; therefore he was worth 1.08 times \$1500. Now \$1500 × 1.08 = \$1620. We multiply the base by 1 *plus* the given rate, expressed decimally. (Art. 191.)

OPERATION.

```
  $1500    B.
   1.08,   1 + R.
  ------
 120.00
1500
--------
$1620.00   Am't.
```

2. B commenced business with \$1800 capital, and squandered 6% the first year: what was he then worth?

ANALYSIS.—As B squandered 6%, he was worth his capital \$1800, *minus* 6% of itself. But his capital is 100% or 1 time itself; and 100% − 6% = 94% or .94. Therefore he had .94 times \$1800; and \$1800 × .94 = \$1692. Here we multiply the base by 1 *minus* the given rate, expressed decimally. (Art. 191.) Hence, the

```
 $1800    B.
   .94    R.
 -----
  7200
16200
--------
$1692.00  Am't.
```

RULE.—*Multiply the base by* 1 *plus or minus the rate, as the case may require. The result will be the amount.*

FORMULA. *Amount* = *Base* × (1 ± *Rate*).

NOTE.—1. The character (±) is called the *double* or *ambiguous sign.* Thus, the expression \$5 ± \$3 signifies that \$3 is to be added to or subtracted from \$5, as the case may require, and is read, "\$5 plus or minus \$3."

2. The rule is based upon the axiom that the *whole* is equal to the *sum* of all its parts.

3. When, by the conditions of the question, the amount is to be *greater* than the base, the multiplier is 1 *plus* the rate; when the amount is to be *less* than the base, the multiplier is 1 *minus* the rate.

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337. How find the amount when the base and rate are given? 338. How else is the amount found, when the base and rate are given?

338. When the *base* and *rate* are given, the *amount* may also be obtained by first *finding the percentage, then adding it to or subtracting it from the base.* (Art. 336.)

3. C and D have 1000 sheep apiece; if C adds 15% to his flock, and D sells 12% of his, how many sheep will each have?

4. A merchant having $2150.38 in bank, deposited 7% more: what amount had he then in bank?

5. If you have $3000 in railroad stock, and sell 5% of it, what amount of stock will you then have?

6. The cotton crop of a planter last year was 450 bales; this year it is 12 per cent more: what is his present crop?

7. An oil well producing 2375 gallons a day, loses 15% of it by leakage: what amount per day is saved?

8. A gardener having 1640 melons in his field, lost 20% of them in a single night: what number did he have left?

9. A man paid $420 for his horses, and 12% more for his carriage: what was the amount paid for the carriage?

10. A man being asked how many geese and turkeys he had, replied that he had 150 geese; and the number of turkeys was 14% less: how many turkeys had he?

11. A fruit grower having sent 2500 baskets of peaches to New York, found 9% of them had decayed, and sold the balance for 62 cts. a basket: what did he receive for his peaches?

12. A Floridian having 4560 oranges, bought 25% more, and sold the whole at 4 cts. each: what did he receive for them?

13. If a man's income is $7235 a year, and he spends $33\frac{1}{3}$% of it, what amount will he lay up?

14. A man bought a house for $8500, and sold it for 20% more than he gave: what did he receive for it?

15. A merchant bought a bill of goods for $10000, and sold them at a loss of $2\frac{1}{2}$%: what did he receive?

PROBLEM III.

339. To find the *Rate*, the Base and Percentage being given; Or, to find what Per Cent one number is of another.

1. A clerk's salary, being $1500 a year, was raised $250: what rate was the increase?

ANALYSIS.—In this example $1500 is the base, and $250 the percentage. The question then is this: $250 is what per cent of $1500? Now $250 is $\frac{250}{1500}$ of $1500; and $250÷$1500 =.16666, etc., or $16\frac{2}{3}\%$. The first two decimal figures denote the per cent; the others, parts of 1%. (Arts. 331, 2.) Hence, the

OPERATION.
1500)$250.00 P.
Ans. $16\frac{2}{3}\%$ R.

RULE.—*Divide the percentage by the base.*

FORMULA. *Rate = Percentage ÷ Base.*

NOTES.—1. This prob. is the same as finding *what part* one number is of another, then changing the common fraction to *hundredths.* (Arts. 173, 186, 334.)

It is based upon the principle that *percentage* is a *product* of which the *base* is a factor, and that dividing a product by *one* of its *factors* will give the *other factor.* (Art. 93.)

2. The number denoting the *base* is always preceded by the word *of*, which distinguishes it from the *percentage.*

3. The given numbers must be reduced to the same denomination, and if there is a *remainder* after two decimal figures are obtained, place it over the divisor and *annex* it to the quotient.

2. What % of 15 is 2?
3. What % of $20 are $5?
4. What % of 48 is 16?
5. What % of $5 are 75 cts.?
6. What % of £3 are 15s.?
7. What % of 56 gals. are 7 qts.?
8. What % are 5 dimes of $5?
9. What % of $\frac{5}{8}$ ton is $\frac{1}{2}$ ton?

10. The standard for gold and silver coin in the U. S. is 9 parts pure metal and 1 part alloy: what % is the alloy?

11. From a hogshead of molasses 15 gals. leaked out: what per cent was the leakage?

12. A grocer having 560 bbls. of flour, sold $\frac{3}{7}$ of it. what per cent of his flour did he sell?

13. A horse and buggy are worth $475; the buggy is worth $110; what % is that of the value of the horse?

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339. How find the rate, when the base and percentage are given? To what is this problem equivalent? *Note.* Upon what is it based?

PROBLEM IV.

340. To find the *Base,* the Percentage and Rate being given.

1. A father gave his son $30 as a birthday present, which was 6% of the sum he gave his daughter: how much did he give his daughter?

ANALYSIS.—The percentage $30 is the *product* of the base into .06 the rate; therefore $30÷.06 is the *other factor* or *base;* and $30÷.06=$500, the sum he gave his daughter. (Art. 193, *n.*)

OPERATION.
.06)$30.00
Ans. $500

Or, since $30 is 6% of a number, 1% of that number must be $\frac{1}{6}$ of $30, which is $5; and 100% is 100 times $5 or $500.

It is more concise, and therefore preferable, to divide the percentage by the rate expressed decimally; then point off the quotient as in division of decimals. (Art. 193.) Hence, the

RULE.—*Divide the percentage by the rate, expressed decimally.*

FORMULA. *Base = Percentage ÷ Rate.*

NOTES.—1. This problem is the same as finding a number when a given per cent or a fractional part of it is given. (Arts. 174, 334.)

2. The rule, like the preceding, is based upon the principle that *percentage* is a *product,* and the *rate* one of its *factors.* (Art. 335, *n.*)

3. Since the percentage is the same part of the base as the rate is of 100, when the *rate* is an *aliquot* part of 100, the operation will be shortened by using this aliquot part as the divisor.

2. 40 is $12\frac{1}{2}$% of what number?

SOLUTION.—$12\frac{1}{2}\% = \frac{1}{8}$ and $40 \div \frac{1}{8} = 40 \times 8 = 320$. *Ans.*

3. 20=5% of what number? *Ans.* 400.

4. 15 bushels=6% of what number?

5. $29=8% of what?

6. 45 tons=25% of what?

7. £150=$33\frac{1}{3}$% of what?

8. 37.5=$6\frac{1}{4}$% of what?

9. 45 francs=$12\frac{1}{2}$% of what?

10. 40=$\frac{1}{2}$% of what?

11. 50 cts.=$\frac{1}{4}$% of what?

12. $100=$\frac{3}{5}$% of what?

13. $35.20=$\frac{1}{5}$% of what?

14. 68 yds.=125% of what?

340. How find the base, when the percentage and rate are given? *Note.* Upon what does the rule depend? When the rate is an aliquot part of 100, how proceed?

15. 2% of $150 is 6% of what sum?

16. 12% of 500 is 60% of what number?

17. A paid a school tax of $50, which was 1% on the valuation of his property: what was the valuation?

18. B saves $31\frac{1}{4}$% of his income, and lays up $600: what is his income?

19. A general lost $16\frac{1}{2}$% of his army, 315 killed, 110 prisoners, and 70 deserted: how many men had he?

20. According to the bills of mortality, a city loses 450 persons a month, and the number of deaths a year is $1\frac{1}{2}$% of its population: what is its population?

PROBLEM V.

341. To find the *Base*, the Amount and Rate being given.

1. A manufacturer sold a carriage for $633, which was $5\frac{1}{2}$% more than it cost him: what was the cost?

OPERATION.

$1.055)\$633.000$

Ans. $600

ANALYSIS.—The amount received $633, is equal to the cost or base *plus* $5\frac{1}{2}$% of itself. Now the cost is 100% or 1 time itself, and $100\% + 5\frac{1}{2}\% = 1.05\frac{1}{2}$; hence $633 equals $1.05\frac{1}{2}$ times the cost of the carriage. The question now is: 633 is $105\frac{1}{2}$% or $1.05\frac{1}{2}$ times what number? If 633 is $1.05\frac{1}{2}$ times a certain number, *once* that number is equal to as many units as $1.05\frac{1}{2}$ is contained times in 633; and $633 \div 1.055 = 600$. Therefore the cost was $600.

2. A lady sold her piano for $628.25, which was $12\frac{1}{2}$%. less than it cost her: what was the cost?

OPERATION.

$.875)\$628.250$

Ans. $718

ANALYSIS.—There being a loss in this case, the amount received, $628.25, equals the cost or base *minus* $12\frac{1}{2}$% of itself. But the cost is 100% or 1 time itself, and $100\% - 12\frac{1}{2}\% = .87\frac{1}{2}$; hence $628.25 equals $.87\frac{1}{2}$ times the cost. Now if $628.25 equals $.87\frac{1}{2}$ times the cost, once the cost must be as many dollars as $.87\frac{1}{2}$ is contained times in $628.25, or $718. Hence, the

RULE.—*Divide the amount by* 1 *plus or minus the rate, as the case may require.*

FORMULA. *Base* = *Amount* ÷ (1 ± *Rate*).

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341. How find the base, the amount and rate being given? *Note.* Upon what

NOTES.—1. This problem is the same as finding a number which is a *given per cent greater* or *less* than a given number.

2. The rule depends upon the principle that the *amount* is a *product* of which the *base* is one of the factors, and 1 *plus* or *minus* the *rate*, the other.

3. The nature of the question shows whether 1 is to be *increased* or *diminished* by the *rate*, to form the *divisor*.

4. When the *rate* is an *aliquot* part of 100, the operation is often shortened by expressing it as a *common* fraction. Thus $25\% = \frac{1}{4}$; and 1 or $\frac{4}{4} + \frac{1}{4} = \frac{5}{4}$, etc.

3. What number is 8% of itself less than 351? *A.* 325.

4. What number is $5\frac{1}{2}\%$ of itself more than 378? *A.* 400.

5. What number diminished $33\frac{1}{3}\%$ of itself will equal $539\frac{1}{3}$?

6. 2275 is 25% more than what number?

7. $\frac{5}{8}$ is $12\frac{1}{2}\%$ more than what number?

ANALYSIS.—$12\frac{1}{2}\% = \frac{1}{8}$; and $\frac{5}{8} \div 1\frac{1}{8} = \frac{5}{8} \div \frac{9}{8} = \frac{40}{72}$ or $\frac{5}{9}$. *Ans.*

8. $\frac{4}{5}$ is 10% less than what number?

9. A owns $\frac{1}{4}$ of a ship, which is $16\frac{2}{3}\%$ less than B's part: what part does B own?

10. A garrison which had lost 28% of its men, had 3726 left: how many had it at first?

11. A merchant drew a check for $4560, which was 25% more than he had in bank: how much had he on deposit?

12. The population of a certain place is 8250, which is 20% more than it was 5 years ago: how much was it then?

13. A man lays up $2010, which is 40% less than his income: what is his income?

14. A drover lost 10% of his sheep by disease, 15% were stolen, and he had 171 left: how many had he at first?

15. The attendance of a certain school is 370, and $7\frac{1}{2}\%$ of the pupils are absent: what is the number on register?

16. An army having lost 10% in battle, now contains 5220 men: what was its original force?

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does this rule depend? How determine whether 1 is to be increased or diminished by the rate? When the rate is an aliquot part, how proceed? To what is this problem equivalent?

APPLICATIONS OF PERCENTAGE.

342. The ***Principles of Percentage*** are applied to two important classes of problems:

First. Those in which *time* is one of the elements of calculation; as, Interest, Discount, etc.

Second. Those which are *independent* of time; as, Commission, Brokerage, and Profit or Loss.

COMMISSION AND BROKERAGE.

343. ***Commission*** is an *allowance* made to agents, collectors, brokers, etc., for the transaction of business.

Brokerage is Commission paid a broker.

NOTES.—1. An *Agent* is one who transacts business for another, and is often called a *Commission Merchant, Factor*, or *Correspondent.*

2. A *Collector* is one who collects debts, taxes, duties, etc.

3. A *Broker* is one who buys and sells gold, stocks, bills of exchange, etc. Brokers are commonly designated by the department of business in which they are engaged; as, Stock-brokers, Exchange-brokers, Note-brokers, Merchandise-brokers, Real-estate-brokers, etc.

4. Goods sent to an agent to sell, are called a *consignment;* the person to whom they are sent, the *consignee;* and the person sending them the *consignor.*

344. *Commission* and *Brokerage* are computed at a *certain per cent of the amount of business transacted.* Hence, the operations are precisely the same as those in Percentage. That is,

The *sales* of an agent, the *sum* collected or invested by him, are the *base.*

The *per cent for services,* the *rate.*

The *commission,* the *percentage.*

The *sales,* etc., *plus* or *minus* the *commission,* the *amount.*

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342. To what two classes of problems are the principles of percentage applied? 343. What is commission? Brokerage? *Note.* An agent? What called? A collector? Broker? 344. How are commission and brokerage computed? What is the base? The rate? The percentage? The amount? *Note.* The net proceeds?

NOTES.—1. The *rate* of commission and brokerage varies. Commission merchants usually charge about 2½ per cent for *selling* goods, and 2½ per cent additional for *guaranteeing* the payment. Stock-brokers usually charge ¼ per cent on the *par value* of stocks, without regard to their *market value.*

2. The *net proceeds* of a business transaction, are the *gross amount* of sales, etc., minus the commission and other charges.

345. To find the *Commission,* the Sales and the Rate being given.

Multiply the sales by the rate. (Problem I, Percentage.)

NOTES.—1. When the *amount* of sales, etc., and the *commission* are known, the *net proceeds* are found by *subtracting the commission* from the *amount* of sales. Conversely,

2. When the *net proceeds* and *commission* are known, the *amount* of sales, etc., is found by *adding the commission* to the *net proceeds.*

3. When both the amount of sales, etc., and the *net* proceeds are known, the *commission* is found by subtracting the net proceeds from the amount of sales.

4. In the examples relating to *stocks,* a *share* is considered $100, unless otherwise mentioned. (Ex. 2.)

(For methods of analysis and of deducing the rules in Commission, Profit or Loss, etc., the learner is referred to the corresponding Problems in Percentage.)

1. A merchant sold a consignment of cloths for $358: what was his commission at 2½ per cent?

SOLUTION.—Commission=$358 (sales) × .025 (rate)=$8.95. *Ans.*

2. A broker sold 39 shares of bank stock: what was his brokerage, at ½ per cent?

SOLUTION.—39 shares=$3900; and $3900 × .005=$19.500. *Ans.*

3. Sold a consignment of tobacco for $958.25: what was my commission at 3¼%?

4. A man collected bills amounting to $11268.45, and charged 3¼%: what was his commission; and how much did he pay his employer?

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345. How find the commission or brokerage, when the sales and the rate are given? *Note.* How find the net proceeds?

5. A commission merchant sold a consignment of goods for $4561, and charged $2\frac{1}{2}\%$ commission, and 3% for guaranteeing the payment: what were the net proceeds?

6. An agent sold 1530 lbs. of maple sugar at $16\frac{2}{3}$ cts., for which he received $2\frac{1}{4}\%$ commission: what were the net proceeds, allowing $7.50 for freight, and $3.10 for storage?

346. To find the *Rate*, the Sales and the Commission being given.

Divide the commission by the sales. (Prob. III. Percentage.)

7. An auctioneer sold goods amounting to $2240, for which he charged $53.20 commission: what per cent was that?

SOLUTION.—Per cent = $53.20 (com.) ÷ $2240 (sales) = .02375, or $2\frac{3}{8}$ per cent. (Art. 339, 331, *n.*)

8. A broker charged $19 for selling $3800 railroad stock: what per cent was the brokerage?

9. Received $350 for selling a consignment of hops amounting to $7000: what per cent was my commission?

10. An administrator received $118.05 for settling an estate of $19675: what per cent was his commission?

347. To find the *Sales*, the Commission and the Rate being given.

Divide the commission by the rate. (Prob. IV, Per ct.)

11. An agent charged 2% for selling a quantity of muslins, and received $93.50 commission: what was the amount of his sales?

SOLUTION.—Sales = $93.50 (com.) ÷ .02 (rate) = $4675. *Ans.*

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346. How find the per cent commission, when the sales and the commission are given? 347. How find the amount of sales, when the commission and the rate are given?

12. Received $45 brokerage for selling stocks, which was $\frac{1}{2}\%$ of what was sold: what was the amount of stocks sold?

13. A commission merchant charging $2\frac{1}{2}\%$ commission, and $2\frac{1}{2}\%$ for guaranteeing the payment, received $210.60 for selling a cargo of grain: what were the amount of sales, and the net proceeds?

14. A district collector received $67.50 for collecting a school tax, which was $4\frac{1}{2}\%$ commission: how much did he collect, and how much pay the treasurer?

15. An auctioneer received $135 for selling a house, which was $1\frac{1}{2}\%$: for what did the house sell; and how much did the owner receive?

348. **To find the *Sales*, the net proceeds and per cent commission being given.**

Divide the net proceeds by 1 *minus the rate.* (Prob. V.)

16. An agent sold a consignment of goods at $2\frac{1}{2}\%$ commission, and the net proceeds remitted the owner were $3381.30: what was the amount of sales?

SOLUTION.—Sales=$3381.30 (net p.)÷.975 (1−rate)=$3468. *Ans.*

17. A tax receiver charged 5% commission, and paid $4845 net proceeds into the town treasury: what was the amount collected?

NOTE.—In this and similar examples, the pupil should observe that the *base* or *sum* on which commission is to be computed is the *sum collected*, and not the *sum paid over.* If it were the latter, the agent would have to collect his own commission, at his own expense, and his rate of commission would not be $\frac{5}{100}$, but $\frac{5}{105}$. In the collection of $100,000, this would cause an error of more than $350.

18. After retaining $2\frac{1}{2}\%$ for selling a consignment of flour, my agent paid me $6664: required the amount of sales, and his commission.

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348. How find the sales, etc., the net proceeds and the per cent commission being given?

19. After deducting $1\frac{1}{4}\%$ for brokerage, and \$45.28 for advertising a house, a broker sent the owner \$15250: for what did the house sell?

20. An administrator of an estate paid the heirs \$25686, charging $2\frac{1}{4}\%$ commission, and \$350 for other expenses: what was the gross amount collected?

349. To find the *Sum Invested*, the sum remitted and the per cent commission being given.

21. A manufacturer sent his agent \$3502 to invest in wool, after deducting his commission of 3%: what sum did he invest?

ANALYSIS.—The sum remitted \$3502, includes both the sum invested and the commission. But the sum invested is 100% of itself, and 100% + 3% (the commission) = 103%. The question now is: \$3502 is 103% of what number? $\$3502 \div 1.03 = \$3400$, the sum invested. (Art. 340, *n.*) Hence, the

RULE.—*Divide the sum remitted by* 1 *plus the per cent commission.* (Prob. V, Per ct.)

NOTE.—The learner will observe that the *base* in this and similar examples is the *sum invested*, and not the sum remitted. If it were the *latter*, the agent would receive commission on his commission, which is manifestly unjust.

22. A teacher remitted to an agent \$3131.18 to be laid out in philosophical apparatus, after deducting 4% commission: how much did the agent lay out in apparatus?

23. If I remit my agent \$2516 to purchase books, after deducting 4% commission, how much does he lay out in books?

24. Remitted \$50000 to a broker to be invested in city property, after deducting $1\frac{1}{2}\%$ for his services: how much did he invest, and what was his commission?

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349. How find the sum invested, the sum remitted and the per cent commission being given?

ACCOUNT OF SALES.

350. An ***Account of Sales*** is a written statement, made by a commission merchant to a consignor, containing the prices of the goods sold, the expenses, and the net proceeds. The usual form is the following:

Sales of Grain on acc't of E. D. BARKER, ESQ., *Chicago.*

DATE.	BUYER.	DESCRIPTION.	BUSHELS.	PRICE.	EXTENSION.
1871.					
April 3	J. Hoyt,	Winter wheat,	565	@ $2.10	$1186.50
	A. Woodruff,	Spring "	870	@ 1.95	1696.50
	Hecker & Co.,	Corn,	1610	@ 1.05	1690.50
	Gross amount,				$4573.50
		Charges.			
Freight on 3045 bu., at 20 cts.,				$609.00	
Cartage " " $15.30,				15.30	
Storage " 38.75,				38.75	
Commission, 2½%,				114.34	
					$777.39
		Net proceeds,			$3796.11

J. HENDERSON & Co.

NEW YORK, *July 5th*, 1871.

Ex. 25. Make out an Account of Sales of the following:

James Penfield, of Philadelphia, sold on account of J. Hamilton, of Cincinnati, 300 bbls. of pork to W. Gerard & Co., at $27; 1150 hams, at $1.75, to J. Ramsey; 875 kegs of lard, each containing 50 lb., at 8 cts., to Henry Parker, and 750 lb. of cheese, at 10 cts., to Thomas Young.

Paid freight, $65.30; cartage, $15.25; insurance, $6.45; commission, at 2%. What were the net proceeds?

26. Samuel Barret, of New Orleans, sold on account of James Field, of St. Louis, 85 bales cotton, at $96.50; 63 barrels of sugar, at $48.25; 37 bls. molasses, at $35.

Paid freight, $45.50; insurance, $15; storage, $35.50; commission, 2½%. What were the net proceeds?

350. What is an account of sales?

PROFIT AND LOSS.

351. ***Profit*** and ***Loss*** are the sums *gained* or *lost* in business transactions. They are computed at a certain *per cent* of the *cost* or *sum invested,* and the operations are the same as those in Percentage and Commission.

The *Cost* or *sum invested* is the *Base;*

The *Per cent* profit or loss, the *Rate;*

The *Profit or Loss,* the *Percentage;*

The *Selling Price,* that is, the cost plus or minus the profit or loss, the *Amount.*

352. To find the *Profit or Loss,* the Cost and the Per Cent Profit or Loss being given.

Multiply the cost by the rate. (Problem I, Per ct.)

NOTE.—When the *per cent* is an *aliquot part* of 100, it is generally *shorter,* and therefore *preferable* to use the *fraction.* (Art. 336, *n.*)

1. A man paid $250 for a horse, and sold it at 15% profit: how much did he gain? *Ans.* $37.50.

2. A man paid $450 for a building lot, and sold at a loss of 11%: how much did he lose? *Ans.* $49.50.

3. Paid $185 for a buggy, and sold it 12% less than cost: what was the loss?

4. Paid $110 for a pair of oxen, and sold them at 20% advance: what was the profit?

5. A lad gave $87\frac{1}{2}$ cts. for a knife, and sold it at 10% below cost: how much did he lose?

6. Bought a watch for $\$83\frac{1}{4}$, and sold it at a loss of 20%: what was the loss?

7. Bought a pair of skates for $4.20, and sold them at $33\frac{1}{3}$% advance: required the gain?

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351. What are profit and loss? How reckoned? To what does the cost or sum invested answer? The per cent profit or loss? The profit or loss? The selling price? 352. How find the profit or loss, when the cost and per cent are given? *Note.* When the per cent is an aliquot part of 100, how proceed?

353. To find the *Selling Price,* the Cost and Per Cent Profit or Loss being given.

Multiply the cost by 1 *plus or minus the per cent.* (Prob. II. Percentage.)

NOTE.—When the *cost* and *per cent* profit or loss are given, the *selling price* may also be found by first finding the profit or loss; then *add* it to or *subtract* it from the cost. (Art. 338.)

8. A man paid $300 for a house lot: for what must he sell it to gain 20%?

ANALYSIS.—To gain 20%, he must sell it for the cost plus 20%. That is, selling pr.=$300 (cost) × 1.20 (1 + 20%)=$360. *Ans.*

9. A farmer paid $250 for a pair of oxen: for how much must he sell them to lose 15%?

Selling pr.=$250 (cost) × .85 (1 − 15%)=$212.50. *Ans.*

10. A and B commenced business with $2500 apiece. A adds 17% to his capital during the first six months, and B loses 17% of his: what amount is each then worth?

11. A merchant paid $378 for a lot of silks, and sold them at 20% profit: what did he get for the goods?

12. If a man pays $2750 for a house, for how much must he sell it to gain 7%?

13. If a man starts in business with a capital of $8000, and makes 19% clear, how much will he have at the close of the year?

14. If a merchant pays 15 cts. a yard for muslin, how must he sell it to lose 25%?

15. Bought gloves at $15 a dozen: how must I sell them a pair, to lose 20%?

16. Paid $25 per dozen for pocket handkerchiefs: for what must I sell them apiece to make $33\frac{1}{3}$ per cent?

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353. How find the selling price, when the cost and per cent profit or loss are given. *Note.* How else may the selling price be found?

17. Paid \$196 for a piece of silk containing 50 yds.: how must I sell it per yard to gain 25%?

18. Bought a house for \$3850: how must I sell it to make $12\frac{1}{2}$%?

19. A speculator invested \$14000 in flour, and sold at a loss of $8\frac{1}{2}$%: what did he receive for his flour?

354. To find the *Per Cent*, the Cost and the Profit or Loss being given.

Divide the profit or loss by the cost. (Prob. III, Per ct.)

NOTE.—When the cost and *selling price* are given, first find the *profit* or *loss*, then the *per cent*. (Art. 339.)

20. If I buy an acre of land for \$320, and sell it for \$80 more than it cost me, what is the per cent profit?

SOLUTION.—Per cent=\$80 (gain)÷\$320 (cost)=.25 or 25%. *Ans.*

21. A jockey paid \$875 for a fast horse, and sold it so as to lose \$250: what per cent was his loss?

22. If I pay $22\frac{1}{2}$ cts. a pound for lard, and sell it at $2\frac{1}{2}$ cts. advance, what per cent is the profit?

23. If a newsboy pays $2\frac{1}{2}$ cts. for papers, and sells them at $1\frac{1}{2}$ cent advance, what per cent is his profit?

24. If a speculator buys apples at \$2.12$\frac{1}{2}$ a barrel, and sells them at \$2.87$\frac{1}{2}$, what is his per cent profit?

ANALYSIS.—\$2.87$\frac{1}{2}$−\$2.12$\frac{1}{2}$=\$.75 profit per bl. Therefore, \$.75 (gain)÷\$2.12$\frac{1}{2}$ (cost)=.35$\frac{5}{17}$ or 35$\frac{5}{17}$%. (Art. 339, *n.*)

25. If I sell an article at double the cost, what per cent is my gain?

26. If I sell an article at half the cost, what per cent is my loss?

27. If I buy hats at \$3, and sell at \$5, what is the per cent profit?

28. If I buy hats at \$5, and sell at \$3, what is the per cent loss?

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354. How find the per cent profit or loss, when the cost and profit or loss are given?

29. If a man's debts are \$3560, and he pays only \$1780, what per cent is the loss of his creditors?

30. If $\frac{3}{4}$ of an article be sold for $\frac{1}{2}$ its cost, what is the per cent loss?

ANALYSIS.—If $\frac{3}{4}$ are sold for $\frac{1}{2}$ its cost, $\frac{1}{4}$ must be sold for $\frac{1}{3}$ of $\frac{1}{2}$, or $\frac{1}{6}$ the cost, and $\frac{4}{4}$ for $\frac{4}{6}$ or $\frac{2}{3}$ the cost. Hence, the loss is $\frac{1}{3}$ the cost; and $\frac{1}{3} \div \frac{3}{3} = .33\frac{1}{3}$ or $33\frac{1}{3}\%$.

31. If you sell $\frac{1}{2}$ of an article for $\frac{3}{4}$ the cost of the whole, what is the gain per cent?

32. If I sell $\frac{2}{3}$ of a barrel of flour for the cost of a barrel, what is the per cent profit?

33. If a milkman sells 3 quarts of milk for the price he pays for a gallon, what per cent does he make?

34. Bought 3 hhd. of molasses at 85 cts. per gallon, and sold one hhd. at 75 cts., the other two at \$1 a gallon: required the whole profit and the per cent profit?

355. To find the *Cost*, the Profit or Loss and the Per Cent Profit or Loss being given.

Divide the profit or loss by the given rate. (Problem IV, Percentage.)

NOTE.—When the per cent profit or loss is an *aliquot part* of 100, the operation may often be abbreviated by using this *aliquot part* as the *divisor*. (Art. 341, *n.*)

35. A grocer sold a chest of tea at 25% profit, by which he made \$$22\frac{1}{2}$: what was the cost?

SOLUTION.—Cost = \$22.50 (profit) ÷ .25 (rate) = \$90. *Ans.*
Or, cost = \$22.50 ÷ $\frac{1}{4}$ = \$90. *Ans.* (Art. 340, *n.*)

36. A speculator lost \$1950 on a lot of flour, which was 20% of the cost: required the cost?

37. Lost 65 cents a yard on cloths, which was 13% of the cost: required the cost and selling price?

ANALYSIS.—The cost = 65 cts. ÷ .13 = \$5; and \$5 − \$.65 = \$4.35 the selling price. (Art. 340, *n.*)

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355. How find the cost, when the profit or loss and the per cent profit or loss are given.

38. Gained $\$2\frac{1}{2}$ per barrel on a cargo of flour, which was 20%: required the cost and selling price per barrel?

ANALYSIS.—$20\% = \frac{1}{5}$, and $\$2.50 \div \frac{1}{5} = \$12.50$ cost, and $\$12.50 + \$2.50 = \$15$, selling price.

39. If I sell coffee at 10% profit I make 10 cts. a pound: what was the cost?

40. A man sold a house at a profit of $33\frac{1}{3}\%$, and thereby gained $7500: required the cost, and selling price?

41. If I make 20% profit on goods, what sum must I lay out to clear $3500, and what will my sales amount to?

42. A and B each gained $1500, which was $12\frac{1}{2}\%$ of A's and 16% of B's stock: what was the investment of each?

43. If a merchant sells goods at 10% profit, what must be the amount of his sales to clear $25000?

44. A market man makes $\frac{1}{2}$ a cent on every egg he sells, which is 25% profit: what do they cost him, and how sell them?

356. To find the *Cost*, the Selling Price and the Per Cent Profit or Loss being given.

Divide the selling price by 1 *plus or minus the rate of profit or loss, as the case may require.* (Prob. V, Per ct.)

45. A jockey sold two horses for $168.75 each; on one he made $12\frac{1}{2}\%$, on the other lost $12\frac{1}{2}\%$: what did each horse cost him?

SOLUTION.—Cost of one = $168.75 (sel. pr.) ÷ 1.125 (1 + rate) = $150. Cost of other = $168.75 (sel. pr.) ÷ .875 (1 − rate) = $192.86. *Ans.*

46. By selling 568 bls. of beef at $\$15\frac{1}{2}$ a barrel, a grocer lost $12\frac{1}{2}\%$: what was the cost? *Ans.* $\$10061.71\frac{3}{7}$.

47. Sold 5000 acres of land at $\$3\frac{1}{4}$ an acre, and thereby gained 22%: what was the cost?

48. Sold a case of linens for £27, 10s., making a profit of 25%: what was the cost?

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356. How find the cost, when the selling price and the per cent profit are given?

49. If a newsboy sells papers at 4 cts. apiece, he makes $33\frac{1}{3}\%$: what do they cost him?

357. To *Mark Goods* so that a given Per Cent may be deducted, and yet make a given Per Cent profit or loss.

50. Bought shoes, at $2.55 a pair: at what price must they be marked that 15% may be deducted, and yet be sold at 20% profit?

ANALYSIS.—The selling price is 120% of \$2.55 (the cost); $\$2.55 \times 1.20 = \$3.06$, the price at which they are to be sold. But the marked price is 100% of itself; and $100\% - 15\% = 85\%$. The question now is, \$3.06 are 85% of what sum? If $\$3.06 = \frac{85}{100}$, $\frac{1}{100} = \$3.06 \div 85$, or \$.036; and $\frac{100}{100} = \$3.60$. *Ans.* (Art. 340.) Hence, the

RULE.—*Find the selling price, and divide it by* 1 *minus the given per cent to be deducted; the quotient will be the marked price.*

51. Paid 56 cts. apiece for arithmetics: what must they be marked in order to abate 5%, and yet make 25%?

52. If mantillas cost \$24 apiece, at what price must they be marked that, deducting 8%, the merchant may realize $33\frac{1}{3}\%$ profit?

53. When apples cost \$3.60 a barrel, what must be the asking price that, if an abatement of $12\frac{1}{2}\%$ is made, there will still be a profit of $16\frac{2}{3}\%$?

54. A merchant paid $87\frac{1}{2}$ cts. a yard for a case of linen, which proved to be slightly damaged: how must he mark it that he may fall 25%, and yet sell at cost?

55. A goldsmith bought a case of watches at \$60: how must he mark them that, abating 4%, he may make 20%?

56. If cloths cost a tailor \$4.50 a yard, at what price must he mark them, that deducting 10% he will make 15 per cent profit?

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357. When the selling price and the per cent profit or loss are given, how find the per cent profit or loss at any proposed price? How find what to mark goods. that a given per cent may be deducted, and yet a given per cent profit or loss be realized?

QUESTIONS FOR REVIEW.

1. A grocer paid 23 cts. a lb. for a tub of butter weighing 65 lbs., and sold it at 18% profit: how much did he make?

2. Bought a house for $3865, and paid $1583.62 for repairs: how much must I sell it for to make $16\frac{2}{3}$%?

3. Paid $2847 for a case of shawls, and $956 for a case of ginghams; sold the former at $22\frac{1}{2}$% advance, and the latter at 11% loss: what was received for both?

4. A jockey bought a horse for $125, which he traded for another, receiving $37 to boot; he sold the latter at 25% less than it cost: what sum did he lose?

5. What will 750 shares of bank stock cost, if the brokerage is $\frac{3}{4}$% and the stock 3% above par?

6. Sold 3 tons of iron, at $15\frac{1}{2}$ cts. a pound, and charged $3\frac{1}{2}$% commission: what were the net proceeds?

7. Bought wood at $$4\frac{1}{2}$ a cord, and sold it for $$6\frac{1}{4}$: required the per cent profit?

8. A shopkeeper buys thread at 4 cts. a spool, and sells at $6\frac{1}{4}$ cts.: what per cent is his profit, and how much would he make on 1000 gross?

9. Bought 1650 tons of ice, at $12; one half melted, and sold the rest at $1 a hund.: what per cent was the loss?

10. The gross proceeds of a consignment of apples was $1863.75, and the agent deducted $$96.91\frac{1}{2}$ for selling: required the per cent commission and the net proceeds?

11. A lady sold her piano for $\frac{3}{4}$ of its cost: what per cent was the loss?

12. If I pay $2 for 3 lbs. of tea, and sell 2 lbs. for $3, what is the per cent profit?

13. A man sold his house at 20% above cost, and thereby made $1860: required the cost and the selling price?

14. A miller sells flour at 15% more than cost, and makes $1.05 a bar.: what is the cost and selling price?

15. Lost 25 cts. a pound on indigo, which was $12\frac{1}{2}$% of the cost: required the cost and the selling price.

16. A commission merchant received \$260 for selling a quantity of provisions, which was 5%: required the amount of sales and the net proceeds.

17. A broker who charges $\frac{1}{4}$%, received \$60 for selling a quantity of uncurrent money: how much did he sell?

18. A grocer makes \$1.25 a pound on nutmegs, which is 100% profit: what does he pay for them?

19. A merchant sold a bill of white goods for \$7000, and made $33\frac{1}{3}$%: required the cost and the sum gained.

20. Sold a hogshead of oil at $93\frac{3}{4}$ cts. a gallon, and made $18\frac{3}{4}$%: required the cost and the profit.

21. A man, by selling flour at \$$12\frac{1}{2}$ a barrel, makes 25%: what does the flour cost him?

22. Made $12\frac{1}{2}$% on dry goods, and the amount of sales was \$57725: required the cost and the sum made.

23. Sold a quantity of metals, and retaining $3\frac{1}{2}$% commission, sent the consignor \$15246: required the amount of the sale and the commission.

24. Sold 250 tons of coal at \$$6\frac{1}{2}$, and made $12\frac{1}{2}$%: what per cent would have been my profit had I sold it for \$8 a ton?

25. A merchant sold out for \$18560, and made 15% on his goods: what per cent would he have gained or lost by selling for \$15225?

26. Paid \$40 apiece for stoves: what must I ask that I may take off 20%, and yet make 20% on the cost?

27. If a bookseller marks his goods at 25% above cost, and then abates 25%, what per cent does he make or lose?

28. A grocer sells sugar at $2\frac{1}{2}$ cts. a pound more than cost, and makes 20% profit: required the selling price.

29. A man bought 2500 bu. wheat, at \$$1\frac{3}{4}$; 3200 bu. corn, at $87\frac{1}{2}$ cts.; 4000 bu. oats, at 25 cts.; and paid \$450 freight: he sold the wheat at 5% profit, the corn at 11% loss, and the oats at cost; the commission on sales was 5%: what was his per cent profit or loss?

INTEREST.

358. ***Interest*** is a compensation for the use of money. It is *computed* at a certain *per cent, per annum*, and embraces five elements or parts: the *Principal*, the *Rate*, the *Interest*, the *Time*, and the *Amount*.

359. The ***Principal*** is the money lent.

The ***Rate*** is the per cent *per annum*.

The ***Interest*** is the percentage.

The ***Time*** is the period for which the principal draws interest.

The ***Amount*** is the sum of the principal and interest.

NOTES.—1. The term *per annum*, from the Latin *per* and *annus*, signifies *by the year*.

2. Interest differs from the preceding applications of Percentage only by introducing *time* as an *element* in connection with *per cent*. The terms *rate* and *rate per cent* always mean a certain number of hundredths *yearly*, and *pro rata* for longer or shorter periods.

360. Interest is distinguished as *Simple* and *Compound*.

Simple Interest is that which arises from the *principal* only.

Compound Interest is that which arises both from the *principal* and the *interest* itself, after it becomes due.

361. The ***Legal Rate*** of interest is the rate allowed by law. Rates *higher* than the legal rate, are called *usury*.

NOTES.—1. In Louisiana the legal rate is - - - 5%.
In the N. E. States, N. C., Penn., Del., Md., Va., W. Va., Tenn., Ky., O., Mo., Miss., Ark., Flor., Ia., Ill., Ind., the Dist. of Columbia, and debts due the United States, - 6%.
In N. Y., N. J., S. C., Ga., Mich., Min., and Wis., - 7%.
In Alabama and Texas, - - - - - - - 8%.
In Col., Kan., Neb., Nev., Or., Cal. and Washington Ter., - 10%.

2. In some States the law allows higher rates by special agreement.

3. When no rate is specified, it is understood to be the *legal rate*.

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358. What is interest? 359. The principal? The rate? The int.? The time? The amt.? 360. What is simple interest? Compound?

THE SIX PER CENT METHOD.

362. Since the interest of \$1 at 6 per cent for 12 months or 1 year, is 6 cents, for 1 month it is 1-*twelfth* of 6 cents or $\frac{1}{2}$ cent; for 2 months it is 2 halves or 1 cent; for 3 mos. $1\frac{1}{2}$ cent, for 4 mos. 2 cents, etc. That is,

The interest of \$1, *at* 6 *per cent, for any number of months, is half as many cents as months.*

363. Since the interest of \$1 at 6 per cent for 30 days or 1 mo., is 5 mills or $\frac{1}{2}$ cent, for 1 day it is $\frac{1}{30}$ of 5 mills or $\frac{1}{6}$ mill; for 6 days it is 6 times $\frac{1}{6}$ or 1 mill; for 12 days, 2 mills; for 15 days, $2\frac{1}{2}$ mills, etc. That is,

The interest of \$1, *at* 6 *per cent, for any number of days, is* 1-*sixth as many mills as days. Hence,*

364. To find the Interest of \$1, at 6 per cent, for any given time.

Take half the number of months for cents, and one sixth of the days for mills. Their sum will be the interest required.

NOTES.—1. When the rate is *greater* or *less* than 6 per cent, the interest of \$1 for the time is equal to the interest at 6 per cent *increased* or *diminished* by *such a part* of itself as the given rate *exceeds* or *falls short* of 6 per cent. Thus, if the rate is 7%, add $\frac{1}{6}$ to the int. at 6%; if the rate is 5%, subtract $\frac{1}{6}$, etc.

2. In finding 1-sixth of the days, it is sufficient for ordinary purposes to carry the quotient to *tenths* or *hundredths* of a mill.

3. When entire accuracy is required, the remainder should be placed over the 6, and annexed to the multiplier.

1. Int. of \$1, at 7% for 9 m. 18 d.? *Ans.* \$.056.
2. Int. of \$1, at 5% for 11 m. 21 d.? *Ans.* \$.04875.
3. What is the int. of \$1, at 6% for 7 m. 3 d.?
4. What is the int. of \$1, at 6% for 11 m. 13 d.?

Note. Wherein does interest differ from the preceding applications of percentage? 361. The legal rate? What is usury? 362. What is the interest of \$1 at 6 per cent for months? 363. For days? 364. How find the int. of \$1 for any given time? *Note.* When the rate is greater or less than 6 per cent, to what is the interest of \$1 equal?

REM.—The relation between the *principal*, the *interest*, the *rate*, the *time*, and the *amount*, is such, that when any *three* of them are given, the others can be found. The most important of these problems are the following:

PROBLEM I.

365. To find the *Interest*, the Principal, the Rate, and Time being given.

1. What is the interest of \$150.25 for 1 year 3 months and 18 days, at 6%?

ANALYSIS.—The interest of \$1 for 15 m.=.075
" " 18 d. =.003
" 1 y. 3 m. 18 d. =.078

Now as the int. of \$1 for the given time and rate is \$.078 or .078 times the principal, the int. of \$150.25 must be .078 times that sum; and \$150.25 × .078=\$11.71950. Hence, the

OPERATION.
Prin. \$150.25
.078
120200
105175
\$11.71950

RULE.—*Multiply the principal by the interest of* \$1 *for the time, expressed decimally.* (Art. 336.)

For the amount, add the interest to the principal.

REM.—1. The amount may also be found by multiplying the principal by 1 plus the interest of \$1 for the time. (Art. 337.)

2. When the rate is *greater* or *less* than 6%, it is generally best to find the interest of the principal at 6% for the given time; then *add* to or *subtract* from it *such a part* of itself, as the given rate *exceeds* or *falls* short of 6 per cent. (Art. 364.)

3. In finding the *time*, first determine the number of *entire calendar* months; then the *number* of days left.

4. In computing interest, if the *mills* are 5 or more, it is customary to add 1 to the cents; if less than 5, they are disregarded.

Only *three decimals* are retained in the following Answers, and each is found by the rule under which the Ex. is placed.

2. What is the amt. of \$150.60 for 1 y. 5 m. 15 d. at 6%?

ANALYSIS.—Int. of \$1 for 1 y. 5 mos. or 17 mos., equals .085; 15 d. it equals .0025; and .085 + .0025=.0875 the multiplier. Now \$150.60 × .0875=\$13 1775, int. Finally, \$150.60+\$13.1775=\$163.7775, *Ans.*

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365. How find the interest, when the principal, rate, and time are given? How find the amount, when the principal and interest are given? *Rem.* How else is the amount found?

3. Find the interest of $31.75 for 1 yr. 4 mos. at 6%.
4. What is the int. of $49.30 for 6 mos. 24 d. at 6%?
5. What is the int. of $51.19 for 4 mos. 3 d. at 7%?
6. What is the int. of $142.83 for 7 mos. 18 d. at 5%?
7. What is the int. of $741.13 for 11 mos. 21 d. at 6%?
8. What is the int. of $968.84 for 1 yr. 10 mos. 26d. at 6%?
9. What is the int. of $639 for 8 mos. 29 d. at 7%?
10. What is the int. of $741.13 for 7 mos. 17 d. at 5%?
11. What is the int. of $1237.63 for 3 mos. 3d. at 8%?
12. What is the int. of $2046¼ for 13 mos. 25 d. at 4%?
13. What is the int. of $3256.07 for 1 m. and 3 d. at 6%?
14. Find the amount of $630.37½ for 9 mos. 15 d. at 10%?
15. Find the amount of $75.45 for 13 mos. 19 d. at 7%?
16. Find the amount of $2831.20 for 2 mos. 3 d. at 9%?
17. Find the amount of $356.81 for 3 m. 11 d. at 5½%?
18. Find the amount of $2700 for 4 mos. 3 d. at 6½%?
19. Required the amount of $5000 for 33 days at 7%.
20. Required the int. of $12720 for 2 mos. 17 d. at 4½%.
21. What is the amt. of $221.42 for 4 mos. 23 d. at 6%?
22. What is the int. of $563.16 for 4 mos. at 2% a month?

SUGGESTION.—At 2% a month, the int. of $1 for 4 mos. is $.08.

23. What is the int. of $7216.31 for 3 mos. at 1% a month?
24. Find the int. of $9864 for 2 mos. at 2½% a month?
25. Find the amt. of $3540 for 17 mos. 10 d. at 7½%?
26. What is the interest on $650 from April 17th, 1870, to Feb. 8th, 1871, at 6%?

ANALYSIS.—1871 y. 2 m. 8 d.	Int. $1 for 9 m.=.045		$650
1870 y. 4 m. 17 d.	" " 21 d. =.0035		.0485
Time, 0 y. 9 m. 21 d.	Multiplier,	.0485	$31.525

27. What is the interest on $1145 from July 4th, 1867, to Oct. 3d, 1868, at 7%.

28. What is the interest on a note of $568.45 from May 21st, 1861, to March 25th, 1862, at 5%?

29. Required the amount of $2576.81 from Jan. 21st, 1871, to Dec. 18th, 1871, at 7%.

METHOD BY ALIQUOT PARTS.

366. To find the Interest by Aliquot Parts, the Principal, Rate, and Time being given.

1. What is the interest of $137 for 3 y. 1 m. 6 d. at 5%?

ANALYSIS.—As the rate is 5%, the interest for 1 year is .05 ($\frac{5}{100}$) of the principal, - - -	$137 prin.
	.05 rate.
Now $137 × .05 = $6.85, - - - - -	12)$6.85 int. 1 y.
Again, the int. for 3 y. is 3 times as much as for 1 y.,	3 y.
And $6.85 (int. 1 y.) × 3 = $20.55, - - -	20.55 int. 3 y.
The int. for 1 m. is $\frac{1}{12}$ of int. for 1 yr.; and $6.85 ÷ 12 = .57, - - - - -	5) .57 " 1 m.
The int. for 6 d. is $\frac{1}{5}$ of int. for 1 m.; and $.57 ÷ 5 = .114, - - - - - -	.114 " 6 d.
Adding these partial interests together, we have the answer required. Hence, the	$21.234, *Ans.*

RULE.—For *one* year.—*Multiply the principal by the rate, expressed decimally.*

For *two* or *more* years.—*Multiply the interest for* 1 *year by the number of years.*

For *months.—Take the aliquot part of* 1 *year's interest.*

For *days.—Take the aliquot part of* 1 *month's interest.*

That is, for 1 m., take $\frac{1}{12}$ of the int. for 1 y.; for 2 m., $\frac{1}{6}$; for 3 m., $\frac{1}{4}$, etc.

For 1 d., take $\frac{1}{30}$ of the int. for 1 m.; for 2 d., $\frac{1}{15}$; for 6 d., $\frac{1}{5}$; for 10 d., $\frac{1}{3}$, etc.

NOTE.—This method has the advantage of directness for different rates; but in practice, the preceding method is generally shorter and more *expeditious.*

2. What is the int. of $143.21 for 2 y. 5 m. 8 d. at 7%?
3. What is the int. of $76.10 for 1 y. 3 m. 5 d. at $6\frac{1}{2}$%?

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366. How find the interest for 1 year, at any given rate, by aliquot parts? How for 2 or more years? For months? For days? What part for 1 month? For 3 mos.? For 6 mos.? For 1 day? For 5 d.? For 10 d.? For 15 d.? For 20 d.?

4. What is the int. of \$95.31 for 8 m. 20 d. at 7%?
5. What is the int. of \$110.43 for 1 yr. 6 m. 10 d. at 4%?
6. What is the int. of \$258 for 3 yrs. 7 m at 8%?
7. What is the interest of \$205.38 for 5 yrs. at $6\frac{1}{4}$%?
8. Find the interest of \$361.17 for 11 months at 8%?
9. Find the interest of \$416.84 for 19 days at $7\frac{1}{2}$%?
10. At $7\frac{1}{2}$%, what is the int. of \$385.20 for 1 yr. 13 d.
11. At $5\frac{1}{2}$%, what is the int. of \$1000 for 1 y. 1 m. 3 d.?
12. At 8%, what is the int. of \$1525.75 for 3 months?
13. Required the interest of \$12254 for $2\frac{1}{2}$ years at 8%?
14. What is the amount of \$20165 for 5 m. 17 d. at 7%?

METHOD BY DAYS.

367. To find the *Interest* by *Days,* the Principal, Rate, and Time being given.

1. What is the interest of \$350 for 78 days at 6%?

ANALYSIS.—The interest of \$1 at 6% for 1 day is $\frac{1}{6}$ of a mill or $\frac{1}{6000}$ dollar. (Art. 363.) Therefore for 78 days it must be 78 times $\frac{1}{6000}=\frac{78}{6000}$ dol., or $\frac{78}{6000}$ times the principal. Again, since the int. of \$1 for 78 days is $\frac{78}{6000}$ times the principal, the interest of \$350 for the same time and rate must be $\frac{78}{6000}$ times that sum. In the operation, we multiply the principal by 78, the numerator, and divide by the *denominator* 6000. Hence, the

```
        $350
          78
        2800
        2450
6|000)$27|300
  Ans. $4.55
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RULE.—*Multiply the principal by the number of days, and divide by* 6000. *The quotient will be the interest at* 6%.

For *other rates, add to or subtract from the interest at* 6 *per cent, such a part of itself as the required rate is greater or less than* 6%. (Art. 364.)

NOTE.—This rule, though not strictly accurate, is generally used by private bankers and money-dealers. It is based upon the supposition that 360 days are a year, which is an error of five days or $\frac{1}{73}$ of a year. Hence, the result is $\frac{1}{73}$ too large. When *entire accuracy* is required, the result must be *diminished* by $\frac{1}{73}$ part of itself.

367. How compute interest by days? *Note.* Upon what is this method founded?

2. A note for \$720 was dated April 17th, 1871: what was the interest on it the 16th of the following July at 7%?

Omitting the day of the first date. (Art. 339, *n.*)		\$720 prin.
April has 30 days—17 d.=13 d.		90 days.
May "	31 d.	6|000)64|800
June "	30 d.	6)10.80 int. 6%.
July "	16 d.	1.80 " 1%.
Time "	=90 d.	*Ans.* \$12.60 int. 7%.

3. What is the interest of \$517 for 33 days at 6%?
4. What is the interest of \$208.75 for 63 days at 6%?
5. What is the interest of \$631.15 for 93 days at 7%?
6. Find the interest of \$1000 for 100 days at 5%?
7. Find the amount of \$1260.13 for 120 days at 6%?
8. Required the interest of \$3568.17 for 20 days at 7%?
9. Required the amount of \$4360.50 for 3 days at 7%?
10. Find the interest of \$5000 from May 21st to the 5th of Oct. following at 6%?
11. Find the interest of \$6523 from Aug. 12th to the 5th of Jan. following at 7%?
12. Find the interest of \$7510 from Jan. 5th to the 10th of the following March, being leap year, at 6%?

PROBLEM II.

368. To find the *Rate*, the Principal, the Interest, and the Time being given.

1. A man lent his neighbor \$360 for 2 yrs. 3 mos., and received \$48.60 interest: what was the rate of interest?

$360 × .01 = \$3.60 int. 1 y.
3.60 × $2\frac{1}{4}$ = \$8.10 " $2\frac{1}{4}$ y.
\$8.10)\$48.60
Ans. 6 per ct.

ANALYSIS.—The int. of \$360 for 1 year at 1% is \$3.60; and for $2\frac{1}{4}$ y. \$8.10. Now, if \$8.10 is 1% of the principal, \$48.60 must be as *many per cent* as \$8.10 are contained times in \$48.60, which is 6. Therefore the rate was 6%. Hence, the

RULE.—*Divide the given interest by the interest of the principal for the time, at* 1 *per cent.*

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368. How find the rate, when the principal, rate, and time are given?

NOTE.—Sometimes the *amount* is mentioned instead of the *principal*, or the *interest*. In either case, the *principal* and *interest* may be said to be *given*. For, the amt.=the prin.+int.; hence, amt.−int.=the prin.; and amt.−prin.=the int. (Arts. 365, 101, Def. 17.)

2. If $600 yield $10.50 interest in 3 months, what is the rate per cent?

3. At what rate will $1500 pay me $52.50 interest semi-annually?

4. At what rate will $1000 amount to $1200 in 3 y. 4 m.?

5. A lad at the age of 14 received a legacy of $5000, which at 21 amounted to $7800: what was the rate of int.?

6. A man paid $9600 for a house, and rented it for $870 a year: what rate of interest did he receive for his money?

7. At what rate of int. will $500 double itself in 12 years?

8. At what rate will $1000 double itself in 20 y.? In 10 y.?

9. At what rate will $1250 double itself in $14\frac{2}{7}$ years?

10. At what rate must $3000 be put to double itself in $16\frac{2}{3}$ years?

PROBLEM III.

369. To find the *Time*, the Principal, the Interest, and the Rate being given.

1. Loaned a friend $250 at 6%, and received $45 interest: how long did he have the money?

$250 × .06 = $15 int. for 1 y.
$15)$45 int.
Ans. 3 years.

ANALYSIS.—The int. of $250 at 6% for 1 yr. is $15. Now if $15 int. require the given principal 1 yr. at 6%, to earn $45 int., the same principal will be required as many years as $15 are contained times in $45; and $45÷$15=3. He therefore had the money 3 years. Hence, the

RULE.—*Divide the given interest by the interest of the principal for 1 year, at the given rate.*

Note. How find the principal, when the amount and interest are given? How find the interest, when the amount and principal are given? 369. How find the time, when the principal, interest, and rate are given?

NOTES.—1. If the quotient contains *decimals*, reduce them to months and days. (Art. 294.)

2. If the *amount* is given instead of the principal or the interest, find the part omitted, and proceed as above. (Art. 368. *n.*)

2. In what time will $860 amount to $989 at 6%?

ANALYSIS.—The amount $989—$860=$129.00 the int., and the int. of $860 for 1 year at 6% is $51.60. Now $129÷$51.60=2.5, or $2\frac{1}{2}$ years.

3. In what time will $1250 yield $500, at 7%?
4. In what time will $2200 yield $100, at 6%?
5. In what time will $10000 yield $200, at 8%?
6. In what time will $700 double itself, at 6%?

SOLUTION.—The int. of $700 for 1 year at 6%, is $42; and $700÷$42=$16\frac{2}{3}$ years. *Ans.*

7. How long must $1200 be loaned at 7% to double itself?
8. In what time will $7500 amount to $15000, at 6%?
9. In what time will $10000 amount to $25000, at 8%?

PROBLEM IV.

370. To find the *Principal*, the Interest, the Rate, and the Time being given.

1. What principal, at 6%, will produce $60 int. in $2\frac{1}{2}$ yrs.?

ANALYSIS.—$2\frac{1}{2}$ y.=30 mos.; therefore the int. of $1 for the given time at 6% is 15 cents. Now as $.15 is the int. of $1 for the given time and rate, $60 must be the int. of as many dollars as $.15 are contained times in $60; and $60÷$.15=$400, the principal required. Hence, the

$2\frac{1}{2}$ y.=30 m.
int. of $1=$.15.
.15)$60.00
Ans. $400 prin.

RULE.—*Divide the given interest by the interest of* $1 *for the given time and rate, expressed decimally.*

2. At 6%, what principal will yield $100 in 1 year?
3. At 7%, what principal will yield $105 in 6 months?

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370. How find the principal, when the interest, rate, and time are given?

4. What sum, at 5%, will gain $175 in 1 year 6 months?

5. What sum must be invested at 6% to pay the ground rent of a house which is $150 per annum?

6. A gentleman wished to found a professorship with an annual income of $2800: what sum at 7% will produce it?

7. A man invested his money in 6% Government stocks, and received $300 semi-annually: what was the sum invested?

8. A man bequeathed his wife $1500 a year: what sum must he invest in 6% stocks, to produce this annuity?

PROBLEM V.

371. To find the *Principal*, the Amount, the Rate, and the Time being given.

1. What principal will amount to $508.20 in 3 y. at 6%?

ANALYSIS.—3 y.=36 m.; therefore the int. of $1 for the given time at 6% is 18 cts., and the amt.=$1.18. (Art. 365.) Now as $1.18 is the amt. of $1 principal for the given time and rate, $508.20 must be the amount of as many dollars principal as $1.18 are contained times in $508.20; and $508.20÷$1.18 =$430.68, the principal required. Hence, the

3 years=36 m.
int. $1= $.18
amt. $1=$1.18
$1.18)$508.20
Ans. $430.68

RULE.—*Divide the given amount by the amount of* $1 *for the given time and rate, expressed decimally.*

2. What principal, at 6%, will amount to $250 in 1 year?

3. What principal, at 7%, will amount to $356 in 1 year 3 months?

4. What sum must a father invest at 6%, for a son 19 years old, that he may have $10000 when he is 21?

5. What sum loaned at 1% a month will amount to $1000 in 1 year?

6. What sum loaned at 2% a month will amount to $6252 in 6 mos.?

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371. How find the principal, when the amount, rate, and time are given?

PROMISSORY NOTES.

372. A ***Promissory Note*** is a written promise to pay a *certain sum* at a specified time, or on demand.

NOTE.—A Note should always contain the words "value received;" otherwise, the holder may be obliged to prove it was given for a *consideration*, in order to collect it.

373. The person who *signs* a note is called the *maker* or *drawer;* the person to whom it is made payable, the *payee;* and the person who has possession of it, the *holder*.

374. A ***Joint Note*** is one signed by two or more persons.

375. The ***Face of a Note*** is the sum whose payment is promised. This sum should be written *in words* in the body of the note, and in *figures* at the top or bottom.

376. When a note is to draw interest from its *date*, it should contain the words "with interest;" otherwise no interest can be collected. For the same reason, when it is to draw interest from a particular time *after date*, that fact should be specified in the note.

All notes are entitled to legal interest *after* they *become due*, whether they draw it before, or not.

377. Promissory Notes are of two kinds; *negotiable*, and *non-negotiable*.

378. A ***Negotiable Note*** is a note drawn for the payment of *money* to "order or bearer," without any conditions.

A ***Non-Negotiable Note*** is one which is not made payable to "order or bearer," or is not payable in *money*.

NOTES.—1. A note payable to A. B., or "order," is transferable by *indorsement;* if to A. B., or "bearer," it is transferable by *delivery*. Treasury notes and bank bills belong to this class.

2. If the words "order" and "bearer" are both omitted, the note can be collected only by the *party* named in it.

379. An ***Indorser*** is a person who writes his name upon the back of a note, as security for its payment.

380. The ***Maturity of a Note*** is the day it becomes legally due. In most of the United States a note does not become legally due

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372. What is a promissory note? What particular words should a note contain? 373. What is the one who signs it called? The one to whom it is payable? The one who has it? 374. A joint note? 375. The face of a note? 377. Of how many kinds are notes? 378. A negotiable note? A non-negotiable note? *Note.* How is the former transferable? Why is the latter not negotiable? 379. What is an indorser? 380. The maturity of a note?

until *three days* after the time specified. These three days are called *days of grace*. Hence, a note matures on the *last day* of grace.

381. When a note is given for any number of months, *calendar months* are always to be understood.

382. If a note is payable on *demand*, it is *legally due* as soon as presented. If no time is specified for the payment, it is understood to be on demand.

383. A ***Protest*** is a written declaration made by a notary public, that a note has been duly presented to the maker, and has not been paid.

NOTE.—A protest must be made out the day the note or draft matures, and sent to the indorser immediately, to *hold him responsible*.

ANNUAL INTEREST.

384. When notes are made "with interest payable annually," some States allow *simple legal interest* on each year's interest from the time it becomes due to the time of final settlement.

385. To compute *Annual Interest*, the Principal, Rate, and Time being given.

1. What is the amount due on a note of $500, at 6%, in 3 years with interest payable annually?

SOLUTION.—Principal,	$500.00
Int. for 1 y. is $30: for 3 y. it is $30 × 3, or	90.00
Int. on 1st annual int. for 2 y. is	3.60
" 2d " " " 1 y. is	1.80
The Amt. is $595.40. Hence, the	*Ans.* $595.40

RULE.—*Find the interest on the principal for the given time and rate; also find the simple legal interest on each year's interest for the time it has remained unpaid.*

The sum of the principal and its interest, with the interest on the unpaid interests, will be the amount.

2. What is the amount of a note of $1000 payable in 4 years, with interest annually, at 7%? *Ans.* $1309.40

382. When is a note on demand due? 383. What is a protest? 385. How compute annual interest?

PARTIAL PAYMENTS.

386. ***Partial Payments*** are parts of a debt paid at different times. The sums paid, with the date, are usually written on the back of the note or other obligation, and are thence called *indorsements.*

UNITED STATES RULE.

387. To compute Interest on Notes and Bonds, when Partial Payments have been made.

I. *Find the amount of the principal to the time of the first payment. If the payment equals or exceeds the interest, subtract it from this amount, and, considering the remainder a new principal, proceed as before.*

II. *If the payment is less than the interest, find the amount of the same principal to the next payment, or to the period when the sum of the payments equals or exceeds the interest then due, and subtract the sum of the payments from this amount.*

Proceed in this manner with the balance to the time of settlement.

NOTES.—1. This method was early inaugurated by the Supreme Court of the United States, and is adopted by New York, Massachusetts, and most of the States of the Union.—*Chancellor Kent.*

2. The following examples show the common forms of promissory notes. The *first* is *negotiable* by *indorsement;* the second by *transfer;* the third is a *joint* note, but *not negotiable.*

$750.

WASHINGTON, D. C., *Jan. 7th,* 1870.

1. Four months after date, I promise to pay to the order of George Green, seven hundred and fifty dollars, with interest at 6%, for value received.

HENRY BROWN.

386. What are partial payments? 387. What is the United States rule?

On this note the following payments were indorsed. June 10th, 1870, $43. Feb. 17th, 1871, $15.45. Nov. 23d, 1871, $78.60. What was due Aug. 25th, 1872?

OPERATION.

Principal, dated Jan. 7th, 1870,		$750.00
Int. to 1st payt. June 10th, 1870 (5 m. 3 d.) (Art. 365),		19.13
Amount,		= 769.13
1st payment June 10th, 1870,		43.00
Remainder or new principal,		= 726.13
Int. from 1st payt. to Feb. 17th, 1871 (8 m. 7 d.),		29.89
2d payt. less than int. due,	$15.45	
Int. on same prin. to Nov. 23d, 1871 (9 m. 6 d.),		33.40
Amount,		= 789.42
3d payt. to be added to 2d,	$78.60	= 94.05
Remainder or new principal,		= 695.37
Int. to Aug. 25th, 1872 (9 m. 2 d.),		31.52
Balance due Aug. 25th, 1872,		= $726.89

$1500.

NEW ORLEANS, *July 1st*, 1869.

2. Two years after date, we promise to pay to James Underhill or bearer, fifteen hundred dollars, with interest at 7%, value received.

G. H. DENNIS & Co.

Indorsements:—Received, Jan. 5th, 1870, $68.50. Aug. 8th, 1870, $20.10. Feb. 11th, 1871, $100. How much was due at its maturity?

$930.

ST. LOUIS, *March 5th*, 1860.

3. On demand, we jointly and severally promise to pay J. C. Williams, nine hundred and thirty dollars, with interest at 8%, value received. THOMAS BENTON.
HENRY VALENTINE.

Indorsements:—Received Oct. 10th, 1860, $20. Nov. 16th, 1861, $250.13. June 20th, 1862, $310: what was due Jan. 30th, 1863?

MERCANTILE METHOD.

388. When *notes* and *interest accounts* payable within a year, receive partial payments, business men commonly employ the following method:

Find the amount of the whole debt to the time of settlement; also find the amount of each payment from the time it was made to the time of settlement.

Subtract the amount of the payments from the amount of the debt; the remainder will be the balance due.

4. A debt of $720.75 was due March 15th, 1870, on which the following payments were made: April 3d, $170; May 20th, $245.30; June 17th, $87.50. How much was due at 6%, Sept. 5th, 1870?

Principal dated March 15th, 1870,		$720.75
Int. to settlement (174 d.)=$720.75 × .029, (Art. 367.)		20.90
Amount, Sept. 5, '70,		= 741.65
1st payt., $170. Time, 155 d. Amt.,	= $174.39	
2d payt., $245.30. Time, 108 d. Amt.,	= 249.72	
3d payt., $87.50. Time, 80 d. Amt.,	= 88.67	
Amt. of the Payts.,		= 512.78
Balance due Sept. 5th, 1870,		$228.87

5. Sold goods amounting to $650, to be paid Jan. 1st, 1868. June 10th, received $125; Sept. 13th, $75.50; Oct. 3d, $210: what was due Dec. 31st, 1868, at 6% interest?

6. A note for $820, dated July 5th, 1865, payable in 1 year, at 7% interest, bore the following indorsements: Jan. 10th, 1866, received $150; March 20th, received $73.10; May 5th, received $116; June 15th, received $141.50: what was due at its maturity?

7. An account of $1100 due March 3d, received the following payments: June 1st, $310; Aug. 7th, $119; Oct. 17th, $200: what was due on the 27th of the next Dec., allowing 7% interest?

CONNECTICUT METHOD.

389. I. When the first payment is a year or more from the time the interest commenced.

Find the amount of the principal to that time. If the payment equals or exceeds the interest due, subtract it from the amount thus found, and considering the remainder a new principal, proceed thus till all the payments are absorbed.

II. When a payment is made before a year's interest has accrued.

Find the amount of the principal for 1 year; also if the payment equals or exceeds the interest due, find its amount from the time it was made to the end of the year, and subtract this amount from the amount of the principal; and treat the remainder as a new principal.

But if the payment be less than the interest, subtract the payment only, from the amount of the principal thus found, and proceed as before.

NOTE.—If the settlement is made in less than a year, find the amount of the principal to the time of settlement; also find the amount of the payments made during this period to the same date, and subtracting this amount from that of the principal, the remainder will be the balance due.—*Kirby's Reports.*

$650.

NEW HAVEN, *April 12th*, 1860.

8. On demand, I promise to pay to the order of George Selden, six hundred and fifty dollars, with interest, value received.

THOMAS SAWYER.

Indorsements:—May 1st, 1861, received $116.20. Feb. 10th, 1862, received $61.50. Dec. 12th, 1862, received $12.10. June 20th, 1863, received $110: what was due Oct. 21st, 1863?

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388. What is the mercantile method? 389. What is the Connecticut method?

Principal, dated April 12th, 1860,	$650.00
Int. to 1st payt. May 1st, 1861 (1 y. 19 d.),	41.06
Amount, May 1, '61,	= 691.06
1st payt. May 1st, 1861,	116.20
Remainder or New Prin., May 1, '61,	= 574.86
Int. to May 1, '62, or 1 y. (2d payt. being short of 1 y.),	34.49
Amount, May 1, '62,	= 609.35
Amt. of 2d payt. to May 1, '62 (2 m. 19 d.),	62.31
Rem. or New Prin., May 1, '62,	= 547.04
Amt., May 1, '63 (1 y.),	= 579.86
3d payt. (being less than int. due), draws no int.,	12.10
Rem. or New Prin., May 1, '63,	= 567.76
Amt., Oct. 21, '63 (5 m. 20 d.),	= 583.85
Amt. of last payt. to settlement (4 m. 1 d.),	= 112.22
Balance due Oct. 21, 63,	= $471.63

NOTE.—For additional exercises in the Connecticut rule, the student is referred to Art. 387.

VERMONT RULE.

390. I. "When payments are made on notes, bills, or similar obligations, whether payable on demand or at a specified time, 'with interest,' such payments shall be applied; *First*, to *liquidate the interest* that has accrued at the time of such payments; and, *secondly*, to the *extinguishment* of the *principal*."

II. "The annual interests that shall remain unpaid on notes, bills, or similar obligations, whether payable on demand at a specified time, "with interest *annually*," shall be *subject* to *simple interest* from the time they become due to the time of final settlement."

III. "If payments have been made in any year, reckoning from the time such annual interest began to accrue, the amount of such payments at the end of such year, with interest thereon from the time of payment, shall be applied; *First*, to *liquidate* the *simple interest* that has accrued from the *unpaid annual* interests.

"*Secondly*, To liquidate the *annual interests* that have become due

"*Thirdly*, To the *extinguishment* of the *principal*."

391. The Rule of New Hampshire, when partial payments are made on notes "with interest annually," is essentially the same as the preceding. But "where payments are made expressly on account of interest accruing, but not then due, they are applied when the interest falls due, *without interest* on such payments."

$1500.

BURLINGTON, *Feb.* 1st, 1865.

9. On demand, I promise to pay to the order of Jared Sparks, fifteen hundred dollars, with interest annually, value received. AUGUSTUS WARREN.

Indorsements:—Aug. 1st, 1865, received $160; July 12th. 1866, $125; June 18th, 1867, $50. Required the amount due Feb. 1st, 1868.

Principal,		$1500.00
Int. to Feb. 1, '66 (1 yr. at 6%),		90.00
Amount,		= 1590.00
1st payment Aug. 1, '65,	$160.00	
Int. on same to Feb. 1, '66 (6 mos.),	4.80	164.80
Remainder or new principal,		1425.20
Int. on same to Feb. 1, '67 (1 yr.),		85.51
Amount,		= 1510.71
2d payment July 12, '66,	$125.00	
Int. on same to Feb. 1, '67 (6 m. 20 d.),	4.16	129.16
Remainder or new principal,		= 1381.55
Int. on same to Feb. 1, '68 (1 yr.),		82.89
Amount,		= 1464.44
3d payment June 18, '67,	$50.00	
Int. on same to Feb. 1, '68 (7 m. 14 d.),	1.87	51.87
Bal. Feb. 1st, 1868,		= $1412.57

$2000.

CONCORD, *Jan.* 15*th*, 1869.

10. Two years after date, I promise to pay to the order of Lewis Hunt, two thousand dollars, "with interest:" the payee, Jan. 15th, 1870, received, by agreement, $200 on account of interest then accruing. What was due on this note Jan. 15th, 1871, by the Vt. and N. H. rules?

By the Vt. rule, the bal.=$2240−$212=$2028 }
" N. H. rule, " =$2240−$200=$2040 } *Ans.*

In the former, interest is allowed on the payment from its date to the settlement; in the latter, it is not

COMPOUND INTEREST.

392. ***Interest*** may be compounded *annually, semi-annually, quarterly,* or for any other period at which the interest is made payable.

393. To compute *Compound Interest,* the Principal, the Rate, and Period of compounding being given.

1. What is the compound interest of $600 for 3 years, at 6%?

Principal,	$600.00
Int. for 1st year, $600 × .06,	36.00
Amt. for 1 y., or 2d prin.,	= 636.00
Int. for 2d year, $636 × .06,	38.16
Amt. for 2 yrs., or 3d prin.,	= 674.16
Int. for 3d year, $674.16 × .06,	40.45
Amt. for 3 years,	= 714.61
Original principal to be subtracted,	600.00
Compound int. for 3 years,	= $114.61

Hence, the

RULE.—I. *Find the amount of the principal for the first period. Treat this amount as a new principal, and find the amount due on it for another period, and so on through every period of the given time.*

II. *Subtract the given principal from the last amount, and the remainder will be the compound interest.*

NOTES.—1. If there are *months* or *days* after the last regular period at which the interest is compounded, find the interest on the amount last obtained for them, and add it to the same, before subtracting the principal.

2. Compound interest cannot be collected by *law;* but a creditor may receive it, without incurring the penalty of *usury.* Savings Banks pay it to all depositors who do not draw their interest when due.

2. What is the compound int. of $500 for 3 yrs. at 7%?

3. What is the compound int. of $750 for 4 yrs. at 5%?

393. How find compound interest, when the principal, rate, and time are given?

4. What is the com. int. of $1000 for 2 y. 7 m. 9 d. at 6%?

5. What is the interest of $1360 for 2 years at 7%, compounded semi-annually?

6. What is the amount of $2000 for 2 years at 4%, compounded quarterly?

COMPOUND INTEREST TABLE.

Showing the amount of $1, at 3, 4, 5, 6, and 7% *compound interest*, for any number of years from 1 to 25.

Yrs.	3%.	4%.	5%.	6%.	7%.
1.	1.030 000	1.040 000	1.050 000	1.060 000	1.07 000
2.	1.060 900	1.081 600	1.102 500	1.123 600	1.14 490
3.	1.092 727	1.124 864	1.157 625	1.191 016	1.22 504
4.	1.125 509	1.169 859	1.215 506	1.262 477	1.31 079
5.	1.159 274	1.216 653	1.276 282	1.338 226	1.40 255
6.	1.194 052	1.265 319	1.340 096	1.418 519	1.50 073
7.	1.229 874	1.315 932	1.407 100	1.503 630	1.60 578
8.	1.266 770	1.368 569	1.477 455	1.593 848	1.71 818
9.	1.304 773	1.423 312	1.551 328	1.689 479	1.83 845
10.	1.343 916	1.480 244	1.628 895	1.790 848	1.96 715
11.	1.384 234	1.539 451	1.710 339	1.898 299	2.10 485
12.	1.425 761	1.601 032	1.795 856	2.012 196	2.25 219
13.	1.468 534	1.665 074	1.885 649	2.132 928	2.40 984
14.	1.512 590	1.731 676	1.979 932	2.260 904	2.57 853
15.	1.557 967	1.800 944	2.078 928	2.396 558	2.75 903
16.	1.604 706	1.872 981	2.182 875	2.540 352	2.95 216
17.	1.652 848	1.947 900	2.292 018	2.692 773	3.15 881
18.	1.702 433	2.025 817	2.406 619	2.854 339	3.37 293
19.	1.753 506	2.106 849	2.526 950	3.025 600	3.61 652
20.	1.806 111	2.191 123	2.653 298	3.207 135	3.86 968
21.	1.860 295	2.278 768	2.785 963	3.399 564	4.14 056
22.	1.916 103	2.369 919	2.925 261	3.603 537	4.43 040
23.	1.973 587	2.464 716	3.071 524	3.819 750	4.74 052
24.	2.032 794	2.563 304	3.225 100	4.048 935	5.07 236
25.	2.093 778	2.665 836	3.386 355	4.291 871	5.42 743

394. To find *Compound Interest* by the Table.

7. What is the amount of $900 for 6 yrs. at 7%, the int. compounded annually? What is the compound interest?

SOLUTION—Tabular amt. of $1 for 6 yrs. at 7%,	$1.50073
The principal,	900
Amt. for 6 yrs.,	= $1350.65700
The principal to be subtracted from amt.,	900
Compound Int. for 6 yrs.,	= $450.657

Hence, the

RULE.—I. *Multiply the tabular amount of* $1 *for the given time and rate by the principal; the product will be the amount.*

II. *From the amount subtract the principal, and the remainder will be the compound interest.*

NOTE.—If the given number of years exceed that in the Table, find the *amount* for any *convenient period,* as half the given years; then on this amount for the remaining period.

8. What is the interest of $800 for 9 years at 6%, compounded annually?

9. What is the int. of $1100 for 12 years at 7%, compounded annually?

10. What is the int. of $1305 for 16 years at 5%, compounded annually?

11. What is the amount of $4500 for 15 years at 4%, compounded annually?

12. What is the amount of $6000 for 25 years at 7%, compounded annually?

13. What is the amount of $3800 for 30 years at 6%, compound interest?

14. What is the compound interest of $4240 at 5% for 40 years?

15. What is the amount of $1280 for 50 years at 7% compound interest?

16. What will $100 amount to in 60 years at 6% compound interest?

DISCOUNT.

395. ***Commercial Discount*** is a deduction of a certain per cent from the *price-list* of goods, the *face* of bills, &c., without regard to time.

396. ***True Discount*** is the *difference* between a *debt* bearing no interest and its *present worth.*

397. The ***Present Worth*** of a debt payable at a future time without interest, is the *sum,* which, put at legal interest for the given time, will amount to the debt.

397, *a*. To find the Net Proceeds of Commercial Discount.

1. Sold goods marked $1560, at 20% discount, on 4 m., then deducted 5% for cash. Required the net proceeds?

ANALYSIS.—$1560 × .20 = $312.00, and $1560 − $312 = $1248. $1248 × .05 = $62.40, and $1248 − $62.40 = $1185.60, net. Hence, the

RULE.—*Deduct the commercial discount from the list price, and from the remainder take the cash discount.*

2. What is the net value of a bill of $3500, at 15% discount, and 5% additional for cash? *Ans.* $2826.25.

3. What is the net value of a bill amounting to $5280, at 12½% discount? *Ans.* $4620.

398. To find the *Present Worth* of a debt, the Rate and Time being given.

1. What is the present worth of $250.51, payable in 8 months without interest, money being worth 6%?

ANALYSIS.—The amount of $1 for 8 mos., at 6%, is $1.04; therefore $1 is the present worth of $1.04, due in 8 mos., and $250.51 ÷ $1.04 = $240.875. Hence, the

$1.04 = amt. $2
$1.04)$250.51 debt
$240.875 *Ans.*

RULE.—*Divide the debt by the amount of $1 for the given time and rate; the quotient will be the present worth.*

2. Find the present worth of $300, due in 10 m., when interest is 7%.

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395. What is discount? 396. Commercial? True? 397. Present worth? 397, *a*. How find net proceeds of commercial discount? 398. Present worth?

3. Find the present worth of $500, due in 1 year, when interest is 8%.

4. What is the present worth of a note for $1250, payable in 6 mos., interest being 6%?

5. A man sold his farm for $2500 on 1 year without interest: what is the present worth of the debt, money being 7%?

6. What is the present worth of a legacy of $5000, payable in 2 years, when interest is 6%?

399. To find the *True Discount*, the Rate and Time being given.

7. What is the true discount at 6% on a note of $474.03 due in 6 months and 3 days?

$$\$1.0305)\underline{\$474.03}$$
$$\$460$$
$$\$474.03 - 460 = \$14.03$$

ANALYSIS.—The amount of $1 for 6 mos. and 3 days is $1.0305. (Art. 365.) Therefore, the present worth of the note is $474.03÷1.0305, or $460. (Art. 398.) But, by definition, the debt, minus the present worth, is the true discount; and $474.03−$460 (present worth)=$14.03, the *Ans.* Hence, the

RULE.—*First find the present worth; then subtract it from the debt.*

8. Find the discount on $2560, due in 7 months, at 6%.

9. Find the discount on $2819, due in 9 months, at 5%.

10. Bought $2375 worth of goods on 6 months: what is the present worth of the bill, at 8%? The discount?

11. At 6% what is the present worth of a debt of $3860, half of which is due in 3 months and half in 6 months?

12. What is the difference between the int. of $6000 for 1 y. at 6%, and the discount for 1 y. at 6%?

13. Bought a house for $5560 on 1½ year without interest: what would be the discount at 7%, if paid down?

\# 14. If money is worth 7%, which is preferable, $15000 cash, or $16000 payable in a year without interest?

399. How find the true discount, when the rate and time are given?

Dis. Proceeds, Present Worth

BANKS AND BANK DISCOUNT.

400. ***Banks*** are incorporated institutions which deal in money. They are of four kinds: banks of *Deposit*, *Discount*, *Circulation*, and *Savings*.

401. A ***Bank of Deposit*** is one that receives money for safe keeping, subject to the order of the depositor.

A ***Bank of Discount*** is one that loans money, discounts notes, drafts, etc.

A ***Bank of Circulation*** is one that makes and issues *bills*, which it promises to pay, on demand.

A ***Savings Bank*** is one that receives small sums on deposit, and puts them at interest, for the benefit of depositors.

402. ***Bank Discount*** is simple interest paid in advance.

NOTES.—1. A *note* or *draft* is said to be *discounted* when the *interest* for the given time and rate is deducted from the face of it, and the *balance* paid to the holder.

2. The *part* paid to the holder is called the *proceeds* or *avails* of the note; the part deducted, the *discount*.

3. The *time* from the date when a note is discounted to its maturity, is often called the *Term of Discount*.

403. To find the *Bank Discount*, the Face of a Note, the Time, and the Rate being given.

1. What is the bank discount on \$450 for 4 m. at 6%?

ANALYSIS.—The int. of \$1 for 4 m. 3 d. is \$.0205; and \$450 × .0205 = \$9.225, the bank discount required. (Art. 380.) Hence, the

RULE.—*Compute the interest on the face of the note at the given rate, for 3 days more than the given time.*

To find the proceeds:—*Subtract the discount from the face of the note.*

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400. What is a bank? 401. A bank of deposit? Discount? Circulation? Savings bank? 402. What is bank discount? 403. How find the bank discount, when the face of a note, the rate and time are given? How find the proceeds?

NOTE.—If a note is on interest, find its *amount* at maturity, and taking this as the *face* of the note, cast the interest on it as above.

2. Find the term of discount and proceeds of a note for \$500, on 90 d., dated June 5th, 1873, and discounted July 3d, 1873, at 7%. *Ans.* 65 d.; Pro., \$493.68.

3. Find the proceeds of a note of \$730 due in 3m. at 6%.

4. Find the proceeds of a draft for \$1000 on 60 d. at 6%.

5. Find the maturity, term of discount, and proceeds of a note of \$1740, on 6m., dated May 1, '73, and dis. Aug. 21st, '73, at 5%. *Ans.* Nov. 4th, '73; Time, 75 d.; Proceeds, —.

6. What is the difference between the true and bank discount on \$5000 due in 1 year at 6%?

7. A jobber buying \$7500 worth of goods for cash, sold them on 4 mos. at $12\frac{1}{2}$% advance, and got the note discounted at 7% to pay the bill: how much did he make?

404. To find the *Face of a Note,* that the proceeds at Bank Discount shall be a specified sum, the R. and T. being given.

8. For what sum must a note be drawn on 6 mos. that at 6% bank discount, the proceeds shall be \$500?

ANALYSIS.—The bank discount of \$1 for 6 mos. 3 d. at 6% is \$.0305; consequently the proceeds are \$.9695. Now as \$.9695 are the proceeds of \$1, \$500 must be the proceeds of as many dollars as \$.9695 is contained times in \$500; and \$500÷\$.9695=\$515.729, the face of the note. Hence, the

RULE.—*Divide the given proceeds by the proceeds of* \$1 *for the given time and rate.*

9. What must be the face of a note on 4 mos. that when discounted at 7% the proceeds may be \$750?

10. What was the face of a note on 60 days, the proceeds of which being discounted at 5%, were \$1565?

11. If the avails are \$2165.45, the time 4 mos., and the rate of bank discount 8%, what must be the face of the note?

12. Bought a house for \$7350 cash: how large a note on 4 mos. must I have discounted at bank 6% to pay this sum?

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404. How find how large to make a note to raise a specified sum, when the rate and time are given?

STOCK INVESTMENTS.

405. ***Stocks*** are the funds or capital of incorporated companies.

An ***Incorporated Company*** is an association authorized by law, to transact business.

NOTE.—Stocks are divided into equal *parts* called *shares,* and the owners of the shares are called *stockholders.* These shares vary from $25 to $500 or $1000. They are commonly $100 each, and will be so considered in the following exercises, unless otherwise stated.

406. ***Certificates of Stock*** are written statements, specifying the number of shares to which holders are entitled. They are often called *scrip.*

407. The ***Par value*** of stock is the sum named on the face of the scrip, and is thence called its *nominal* value.

The ***Market value*** is the sum for which it sells.

NOTES.—1. When shares sell for their *nominal* value, they are at *par;* when they sell for *more,* they are *above par,* or at a *premium;* when they sell for *less,* they are *below par,* or at a *discount.*

2. When stocks sell at *par,* they are often quoted at 100; when 8% *above* par, at 108; when 8% *below* par, at 92. The term *par,* Latin, signifies *equal;* hence, to be *at par,* is to be on an *equality.*

408. The ***Gross Earnings of a Company*** are its entire receipts.

The ***Net Earnings*** are the sums left after deducting all expenses.

409. ***Instalments*** are portions of the capital paid by the stockholders at different times.

410. ***Dividends*** are portions of the earnings distributed among the stockholders. They are usually made at stated periods; as, annually; semi-annually, etc.

411. A ***Bond*** is a writing under seal, by which a party binds himself to pay the holder a certain sum, at or before a specified time.

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405. What are stocks? An incorporated company? *Note.* Into what are stocks divided? What are stockholders? 406. What are certificates of stock? 407. What is the par value of stock? The market value? *Note.* When are stocks at par? Above par? Below par? The meaning of the term par? 408. What are the gross earnings of a company? The net earnings? 409. What are instalments? 410. Dividends? 411. What is a bond?

UNITED STATES BONDS.

412. ***United States Bonds*** are those issued by Government, and are divided into two classes: those payable at a *given date*, and those payable within the *limits* of two given dates, at the option of the Government.

NOTE.—The *former* are designated by a combination of the numerals, which express the *rate of interest* they bear, and the *year* they become due; as "6s of '81."

The *latter* are designated by combining the numerals expressing the *two dates* between which they are to be paid; as "5-20s."

413. A ***Coupon*** is a certificate of interest attached to a bond, which, on the payment of the interest, is cut off and delivered to the payor.

414. The principal U. S. bonds are the following:

1. "6s of '81," bearing 6% interest, and payable in 1881.

2. "5-20s," bearing 6% interest, and payable in not less than 5 or more than 20 years from their date, at the pleasure of the Government. Interest paid semi-annually in gold.

3. "10-40s,' bearing 5% interest, redeemable after 10 years from their date, interest semi-annually in gold.

4. "5s of '81," bearing 5% interest, redeemable after 1881, interest paid quarterly in gold.

5. "4½s of '86," bearing 4½% interest, redeemable after 1886, the interest paid quarterly in gold.

6. "4s of 1901," bearing 4% interest, the principal payable after 1901, the interest paid quarterly in gold.

415. State, City, Railroad Bonds, etc., are payable at a specified time, and are designated by annexing the numeral denoting the rate of interest they bear, to the name of the State, etc., by which they are issued; as, New York 6s; Georgia 7s.

416. Computations in stocks and bonds are founded upon the principles of percentage; the *par value* being the base, the *per cent* premium the rate, the *premium*, etc., the percentage, and the *market value* the amount.

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412. What are U. S. bonds? How many classes? *Note.* How are the former designated? The latter? 413. What is a coupon? 414. What is meant by U. S. 6s of '81? By U. S. 5-20s? By U. S. 10-40s? By new U. S. 5s of '81?

NOTE.—The *comparative profit* of investments in U. S. Bonds depends upon their *market value*, the *rate* of interest they bear, and the *premium* on gold. That of Railroad and other Stocks upon their *market value*, and the *per cent* of their dividends.

417. To find the *Premium, Discount, Instalment,* or *Dividend,* the Par Value and the Rate being given.

1. What is the premium on 20 shares of the New Orleans National Bank, at 8%?

ANALYSIS.—20s.=$2000; and $2000 × .08=$160, *Ans.* Hence, the

RULE.—*Multiply the par value by the rate, expressed decimally.* (Art. 336.)

2. What is the discount on 27 shares of Michigan Central Railroad at 9%?

3. What is the premium on three $1000 U. S. 6s of '81, when they stand at 17½% above par?

4. The Maryland Coal Company called for an instalment of 15%: what did a man pay who owned 35 shares?

5. What is the discount on $4000 Tennessee 6s, at 10%?

6. The Virginia Manufacturing Company declared a dividend of 17%: to what sum were 28 shares entitled?

418. To find the *Market Value* of Stocks and Bonds, the Rate and Nominal Value being given.

7. What is the market value of 45 shares of New York Central, at 4% premium?

ANALYSIS.—45s.=$4500; and $4500 × 1.04 (1 + the rate)=$4680, the value required. Hence, the

RULE.—*Multiply the nominal value by* 1 *plus or minus the rate.*

Or, multiply the market value of 1 *share by the number of shares.*

NOTE.—In finding the *net value* of stocks, the *brokerage, postage stamps,* and *other expenses* must be deducted from the market value.

8. What is the worth of $14000 of Kentucky 6s, at 92?

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417. How find the premium, etc., when the par value and rate are given?

9. What is the worth of an investment of $9500 in U. S. 5-20s, at 10½% premium?

10. What is the net value of 58 shares of New Jersey Central, at 110; deducting the express and other charges $1.89, and brokerage at ¼%?

11. What will be realized from $7500 Texas 7s, at 15% discount, deducting the brokerage at ½%, and $1.39 for postage and other charges.

12. What will $10000 U. S. 5-20s cost at 8¼% premium, adding brokerage at ¼%, and other expenses $2.37½?

13. What is the value in currency of $15750 gold coin, the premium being 22½%?

14. A person has $10000 U. S. 5-20s: what will he receive annually in currency, when gold is 12% premium?

419. To find the *Rate,* the Par Value, and the Dividend, Premium, or Discount being given.

15. The capital stock of a company is $100000; its gross earnings for the year are $34500, and its expenses $13500: deducting from its net earnings $1000 as surplus, what per cent dividend can the company make?

ANALYSIS.—$13500 + $1000 = $14500, and $34500 − $14500 = $20000, the net earnings. Now 20000 ÷ $100000 = .20 or 20%, the rate required. Hence, the

RULE.—*Divide the premium or discount, as the case may be, by the par value.* (Art. 339.)

16. Paid $750 premium for 50 shares of bank stock: what was the per cent?

17. The discount on 100 shares of the Pacific Railroad is $625: what is the per cent below par?

18. The net earnings of a company with a capital of $480000 are $35000; reserving $3000 as surplus, what per cent dividend can they declare?

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418. How find the market value when the rate and nominal value are given?
419. How find the rate, when the premium, discount, or dividend are given?

420. To find how much Stock can be bought for a specified sum, the Rate of Premium or Discount being given.

19. How much of Kansas 6s, at 20% discount, can be bought for $5200?

ANALYSIS.—$5200÷.80 (val. of $1 stock)=$6500, *Ans.* Hence, the

RULE.—*Divide the sum to be invested by* 1 *plus or minus the rate, as the case may be.* (Art. 349.)

Or, divide the given sum by the market value of $1 *of stock.*

20. How many $100 U. S. 10-40s, at 5% premium, can be bought for $4200?

21. What amount of Virginia 6s, at 90, can be purchased for $10800?

421. To find what Sum must be invested in Bonds to realize a given income, the Cost and Rate of Interest being given.

22. What sum must be invested in Missouri 6s at 90, to realize an income of $1800 annually?

ANALYSIS.—At 6% the income of $1 is 6 cts.; and $1800÷.06=$30000, the nominal value of the bonds. Again, at .90, $30000 of bonds will cost .90 times $30000=$27000, the sum required. Hence, the

RULE.—I. *Divide the given income by the annual interest of* $1 *of bonds; the quotient will be the nominal value of the bonds.*

II. *Multiply the nominal value by the market value of* $1 *of bonds; the product will be the sum required.*

23. What sum must be invested in U. S. 5-20s, at 106, to yield an annual income of $2500 in gold?

24. How much must one invest in Wisconsin 8s, at 95, to receive an annual income of $3000? *Ans.* $35625.

25. How much must be invested in Mississippi 6s, at 80, to yield an annual interest of $4200?

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420. How find the quantity of stock that can be bought for a specified sum, when the rate is given?

EXCHANGE.

422. ***Exchange*** is a method of making payments between distant places by *Bills of Exchange.*

423. A ***Bill of Exchange*** is an order or draft directing one person to pay another a certain sum at a specified time.

NOTE.—The person who signs the bill is called the *drawer* or *maker;* the one to whom it is addressed, the *drawee;* the one to whom it is to be paid, the *payee;* the one who sends it, the *remitter.*

424. Bills of Exchange are *Domestic* or *Foreign.*

Domestic Bills are those payable in the country where they are drawn, and are commonly called *Drafts.*

Foreign Bills are those drawn in one country and payable in another.

425. ***A Sight Bill*** is one payable on its *presentation.*

A Time Bill is one payable at a specified time *after* its date, or presentation.

426. The ***Par of Exchange*** is the *standard* by which the value of the currency of different countries is *compared,* and is either *intrinsic* or *nominal.*

An ***Intrinsic Par*** is a standard having a *real* and *fixed value* represented by *gold* or *silver coin.*

A ***Nominal Par*** is a *conventional* standard, having any assumed value which convenience may suggest.

427. When the *market price* of bills is the same as the *face,* they are *at par;* when it *exceeds* the face, they are *above* par, or at a *premium;* when it is *less* than the face, they are *below* par, or at a *discount.* (Art. 407, *n.*)

NOTES.—1. The *fluctuation* in the *price* of bills from their par value, is called the *Course of Exchange.*

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422. What is exchange? 423. A bill of exchange? 424. What are domestic bills? Foreign bills? 425. What are sight bills? Time bills? 426. What is the par of exchange? An intrinsic par? A nominal par?

2. The *rate* of Exchange between two places or countries depends upon the circumstances of trade. If the trade between New York and New Orleans is equal, exchange is *at par*. If the former owes the *latter*, the demand for drafts on New Orleans is *greater* than the *supply*; hence they are *above* par in New York. If the latter owes the former, the demand for drafts is *less* than the supply, consequently drafts on New Orleans are *below* par.

428. An ***acceptance*** of a bill or draft is an *engagement* to pay it according to its conditions. To show this, it is customary for the drawee to write the word *accepted* across the face of the bill, with the date and his name.

NOTE.—Bills of Exchange are negotiable like promissory notes, and the laws respecting their indorsement, collection, protest, etc., are essentially the same.

DOMESTIC OR INLAND EXCHANGE.

429. To find the *Cost* of a Draft, its Face and the Rate of Exchange being given.

1. What is the cost of a sight draft on New York, for $4500, at $2\frac{1}{2}\%$ premium?

ANALYSIS.—At $2\frac{1}{2}\%$ premium, a draft of $1 will cost $1 + $2\frac{1}{2}$ cts. = $1.025, or 1.025 times the draft. Hence, the cost of $4500 draft will be 1.025 times its face; and $4500 × 1.025 = $4612.50, *Ans.*

$$\begin{array}{r} \$4500 \\ 1.025 \\ \hline \textit{Ans.}\ \$4612.50 \end{array}$$

2. What is the cost of a sight draft on St. Louis of $5740, at 4% discount?

ANALYSIS.—At 4% discount, a draft of $1 will cost $1 − 4 cents = $0.96, or .96 times the draft. Therefore, the cost of $5740 draft will be .96 times its face; and $5740 × .96 = $5510.40, Hence, the

$$\begin{array}{r} \$5740 \\ .96 \\ \hline \$5510.40\ \textit{Ans.} \end{array}$$

RULE.—*Multiply the face of the draft by the cost of* $1 *of draft, expressed decimally.*

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427. When are bills at par? When above par? When below? *Note.* What is the fluctuation in the price of exchange called? Upon what does the rate of exchange depend? Explain? 428. The acceptance of a bill? 429. How find the cost of a draft, when the face and rate are given?

NOTES.—1. When payable at *sight*, the worth of \$1 of draft is \$1 *plus* or *minus* the *rate* of exchange.

2. On *time* drafts, both the exchange and the bank discount are computed on their face. Dealers in exchange, however, make but one computation; the *rate* for *time* drafts being enough *less* than *sight* drafts to allow for the bank discount.

2. A merchant in Galveston bought a draft of \$2000 on Philadelphia at 60 days sight: what was the draft worth, the premium being $3\frac{1}{2}\%$, and the bank discount 6%.

3. What cost a sight draft of \$3560, at 2% discount?

1— 4. Required the worth of a draft for \$4250 on Chicago, at 90 days, sight drafts being 1% discount and interest 7%.

2— 5. The Bank of New York having declared a dividend of 4%, a stockholder living in Savannah drew on the bank for his dividend on 50 shares, and sold the draft at $1\frac{1}{2}\%$ premium: how much did he realize from the dividend?

430. To find the *Face* of a Draft, the Cost and Rate of Exchange being given.

7. Bought a draft in Omaha on New York, payable in 90 days, for \$3043.50, exchange being 3% premium, and interest 6%: what was the face of the draft?

$$\begin{aligned} \$1 + 3\% &= \$1.03 \\ \text{Dis. on } \$1 \text{ for } 93 \text{ d.} &= 0.0155 \\ \text{Cost of } \$1 \text{ dft.} &= \$1.0145 \\ 1.0145)\$3043.50 & \\ \text{Face of dft.} &= \$3000. \end{aligned}$$

ANALYSIS.—The cost of \$1 of draft is \$1 plus the rate, minus the bank discount for the time. Now \$1 + 3% = \$1.03; the bank discount on \$1 for 93 d. is \$0.0155, and \$1.03 − \$0.0155 = \$1.0145. Now if \$1.0145 will buy \$1 of draft, \$3043.50 will buy a draft of as many dollars as 1.0145 is contained times in \$3043.50. \$3043.50 ÷ 1.0145 = \$3000, the draft required. Hence, the

RULE.—*Divide the cost of the draft by the cost of* \$1 *of draft, expressed decimally.*

8. What is the face of a sight draft purchased for \$1250, the premium being $2\frac{1}{2}\%$?

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430. How find the face of a draft, the cost and rate being given?

9. What is the face of a draft at 60 days sight, purchased for $1500, when interest is 8%, and the premium 2%?

10. What is the face of a sight draft, purchased for $2500, the discount being $4\frac{1}{2}$%?

11. What is the face of a draft on 4 m., bought for $3600, the int. being 6%, and exchange 2% discount?

12. A merchant of Natchez sold a draft on Boston at $1\frac{1}{2}$% prem., for $3806.25: what was the face of the draft?

FOREIGN EXCHANGE.

431. A ***Foreign Bill of Exchange*** is a Bill drawn in one country and payable in another.

A Set of Exchange consists of three bills of the *same date* and *tenor*, distinguished as the *First*, *Second*, and *Third* of exchange. They are sent by different mails, in order to save time in case of miscarriage. When one is *paid*, the others are *void*.

432. ***The Legal Par of Exchange*** between Great Britain and the United States, is $4.8665 gold to the *pound sterling*.*

433. To find the *Cost* of a Bill on England, the Face and the Rate of Exchange being given.

1. What is the cost of the following bill on London, at $4.8665 to the £ sterling?

£354, 12s. NEW YORK, *July 4th*, 1874.

At sight of this first of exchange (the second and third of the same date and tenor unpaid), pay to the order of Henry Crosby, three hundred and fifty-four pounds, twelve shillings sterling, value received, and charge the same to the account of O. J. KING & Co.

To GEORGE PEABODY, Esq., London.

431. What is a foreign bill of exchange? What is the legal par of exchange with Great Britain?

* Act of Congress, March 3d, 1873—To take effect Jan. 1st, 1874.

ANALYSIS.—Reducing the given shillings to the decimal of a pound, £354, 12s. = £354.6. (Art. 295.) Now if £1 is worth \$4.8665, £354.6 are worth 354.6 times as much; and \$4.8665 × 354.6=\$1725.661, the cost of the bill.	OPERATION. 4.8665 354.6 *Ans.* \$1725.661

2. What is the value of a bill on England for £436, 5s. 6d., at \$4.85¼ to the £ sterling?

ANALYSIS.—£436, 5s. 6d.=£436.275, and the market value of exchange is \$4.8525 to the £. Now \$4.8525 × 436.275=\$2117.02, the cost of the bill. Hence, the	OPERATION. \$4.8525 436.275 *Ans.* \$2117.02

RULE.—*Multiply the market value of £1 sterling by the face of the bill; the product will be its value in dollars and cents.*

NOTES.—1. If there are shillings and pence in the given bill, they should be reduced to the decimal of a pound. (Art. 295.)

2. Bills on Great Britain are drawn in Sterling money.

3. The *New Par* of Sterling Exchange \$4.8665, is the intrinsic value of the Sovereign or pound sterling, as estimated at the United States Mint, and is 9½% greater than the old par.

4. The *Old Par*, which assumed the value of the £ sterling to be \$4.44⁴⁄₉, is abolished by law, and all contracts based upon it after January 1st, 1874, are *null* and *void*.

3. What is the cost of a bill on Dublin for £381, at \$4.87⅞ to the £ sterling?

4. What is the cost in currency of a bill on England for £750, exchange being at par, and gold 33⅓% premium?

5. B owes a merchant in Liverpool £1500; exchange is \$4.93⅜; to transmit coin will cost 2% insurance and freight; and when delivered its commercial value is \$4.80 to a pound: which is the cheaper, to buy a bill, or send the gold? How much?

6. A New Orleans merchant consigned 568 bales of cotton, weighing 450 lbs. apiece, to his agent in Liverpool; the agent paid 1d. a pound freight, and sold it at 12d. a

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433. How find the cost of a bill on England, when the face and rate are given?

pound; he charged $2\frac{1}{4}$% commission, and £8, 6s. for storage: what did the merchant realize for his cotton, exchange on the net proceeds being 4.91\frac{1}{2}$ to the £?

434. To find the *Face* of a Bill on England, the Cost and the Rate of Exchange being given.

7. What is the face of a bill on England which cost $1725.72, exchange being at par, or $4.8665 to the £?

ANALYSIS.—Since $4.8665 buys £1 of bill, $1725.72 will buy as many pounds as $4.8665 are times in $1725.72, or £354.612 =£354, 12s. 2$\frac{3}{4}$d. Hence, the

$4.8665)$1725.72
£354.612
Ans. £354, 12s. 2$\frac{3}{4}$d.

RULE.—*Divide the cost of the bill by the market value of £1 sterling; the quotient will be the face of the bill.*

8. What is the face of a bill on London which cost $2500, exchange being 4.88\frac{1}{2}$ to the £?

9. What amount of exchange on Dublin can be obtained for $3750, at 4.84\frac{1}{4}$ to the £ sterling?

10. A merchant in Charleston paid $5000 for a bill on London, at 4.87\frac{1}{8}$ to the £: what was the face of the bill?

11. Paid $7500 for a bill on Manchester: what was the face of it, exchange being 4.86\frac{1}{5}$ to the £?

434, *a*. In quoting Exchange on Foreign Countries, the general rule is to quote the money of one country against the money of other countries. Thus, exchange is quoted

On Austria, in cents to the Florin (silver)=$0.476.
On Frankfort, in cents to the Florin (gold)=$0.4165.
On the German Empire, in cents to the Mark (gold)=$0.2382.
On North Germany, in cents to the new Thaler (silver)=$0.714.
On Russia, in cents to the Rouble (silver)=$0.7717.

NOTE.—Bills of Exchange between the United States and foreign countries are generally drawn on some of the great commercial centers; as London, Paris, Frankfort, Amsterdam, etc.

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434. How find the face of a bill on England, when the cost and rate are given?

435. ***Bills on France*** are drawn in French currency, and are calculated at so many *francs* and *centimes* to a *dollar.*

NOTE.—*Centimes* are commonly written as *decimals* of a franc Thus 5 francs and 23 centimes are written 5.23 francs. (Art. 224, *n.*)

436. To find the *Cost* of a Bill on France, the Face and Rate of Exchange being given.

12. What is the cost of a bill on Paris of 1500 francs, exchange being 5.25f. to a dollar?

ANALYSIS.—Since 5.25 francs will buy a bill of $1, 1500 francs will buy a bill of as many dollars as 5.25 is contained times in 1500; and 1500÷5.25=$285.71, the cost required. Hence, the

5.25 fr.)1500.00 fr.
Ans. $285.71+

RULE.—*Divide the face of the bill by the number of francs to* $1 *exchange.*

13. What is the cost of a bill of 3500 francs on Havre, exchange being 5.18 francs to a dollar?

437. To find the *Face* of a Bill on France, the Cost and the Rate of Exchange being given.

14. A traveler paid $300 for a bill on Paris; exchange being 5.16 francs to $1: what was the face of the bill?

ANALYSIS.—If $1 will buy 5.16 francs, $300 will buy 300 times as many; and 5.16 fr. × 300=1548 francs, the face of the bill required. Hence, the

RULE.—*Multiply the number of francs to* $1 *exchange by the cost of the bill.*

15. Paid $2500 for a bill on Lyons, exchange being 5.22 francs to $1: what was the face of the bill?

16. A merchant paid $3150 for a bill on Paris, exchange being 5.23 francs to $1: what was the face of the bill?

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435. How is exchange on France calculated? 436. How find the cost of a bill on France, when the face and rate are given? 437. How find the face, when the cost and rate are given?

INSURANCE.

438. ***Insurance*** is *indemnity* for loss. It is distinguished by different names, according to the *cause* of the loss, or the *object* insured.

439. ***Fire Insurance,*** is indemnity for loss by fire.

Marine Insurance, for loss by sea.

Life Insurance, for the loss of life.

Accident Insurance, for personal casualties.

Health Insurance, for personal sickness.

Stock Insurance, for the loss of cattle, horses, etc.

NOTES.—1. The party who undertakes the risk is called the *Insurer* or *Underwriter.*

2. The party protected by the insurance is called the *Insured.*

440. A ***Policy*** is a writing containing the evidence and terms of insurance.

The ***Premium*** is the sum paid for *insurance.*

NOTE.—The business of insurance is carried on chiefly by *Incorporated Companies.* Sometimes, however, it is undertaken by individuals, and is then called *out-door-insurance.*

441. Insurance Companies are of two kinds: *Stock Companies,* and *Mutual Companies.*

A ***Stock Insurance Company*** is one which has a paid up capital, and divides the profit and loss among its stockholders.

A ***Mutual Insurance Company*** is one in which the losses are shared by the parties insured.

442. Premiums are computed at a *certain per cent* of the sum insured, and the operations are similar to those in Percentage.

NOTE.—Policies are renewed annually, or at stated periods, and the premium is paid in advance. In this respect insurance differs from commission, etc., which have no reference to time.

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438. What is insurance? 439. Fire insurance? Marine? Life? Accident? Health? Stock? 440. What is a policy? The premium? 441. How many kinds of insurance companies? A stock insurance company? A mutual? 442. How are premiums computed?

FIRE AND MARINE INSURANCE.

443. To find the *Premium,* the Sum insured and the Rate for the period being given.

1. What premium must I pay per annum for insuring \$1750 on my house and furniture, at $\frac{1}{2}\%$?

ANALYSIS.—\$1750 × .005 (rate) = \$8.75, *Ans.* Hence, the

RULE.—*Multiply the sum insured by the rate.* (Art. 336.)

NOTE.—The rate of insurance is sometimes stated at a certain number of cents on \$100. In such cases the rate should be reduced to the decimal of \$1 before multiplying. Thus, if the rate is 25 cents on \$100, the multiplier is written .0025. (Art. 295.)

2. At 35 cents on \$100, what is the insurance of \$1900?
3. At $1\frac{1}{2}\%$, what is the premium on \$2560?
4. At $2\frac{1}{4}\%$, what is the premium on \$3750?
5. At 25 cents on \$100, what is the premium on \$4280?
6. At 50 cents on \$100, what is the premium on \$5000?
7. At $3\frac{1}{5}\%$, what is the cost of insuring \$6175?
8. What is the premium for insuring a ship and cargo valued at \$35000, at $2\frac{1}{2}\%$?
9. What is the cost of insuring a factory and its contents, valued at \$48250, at $3\frac{1}{4}\%$, including \$$1\frac{1}{2}$ for policy?

444. To find what Sum must be insured to cover both the Property and Premium, the Rate being given.

10. For what must a factory worth \$20709 be insured, to cover the property and the premium of $2\frac{1}{2}\%$?

ANALYSIS.—The sum to be insured includes the property *plus* the premium. But the property is 100% of itself, and the premium is $2\frac{1}{2}\%$ of that sum; therefore \$20709 = 100% − $2\frac{1}{2}\%$, or $.97\frac{1}{2}$ times the sum to be insured. Now, if \$20709 is $.97\frac{1}{2}$ times the required sum, once that sum is \$20709 ÷ $.97\frac{1}{2}$, or \$21240, *Ans.* (Art. 337.) Hence, the

RULE.—*Divide the value of the property by* 1 *minus the rate.*

NOTE.—This and the preceding problem cover the ordinary cases of Fire and Marine Insurance. Should other problems occur, they may be solved like the corresponding problems in Percentage.

How find the premium, when the sum insured and the rate are given?

11. A merchant sent a cargo of goods worth $15275 to Canton: what sum must he get insured at 3%, that he may suffer no loss, if the ship is wrecked?

12. A house and furniture are worth $27250: what sum must be insured at 2% to cover the property and premium?

13. What sum must be insured at 5% to cover the premium, with a vessel and cargo worth $35250?

LIFE INSURANCE.

445. Life Insurance Policies are of *different kinds*, and the premium *varies* according to the *expectation* of life.

1st. ***Life Policies***, which are payable at the death of the party named in the policy, the annual premium continuing through life.

2d. ***Life Policies***, payable at the death of the insured, the annual premium ceasing at a given age.

3d. ***Term Policies***, payable at the death of the insured, if he dies during a given term of years, the annual premium continuing till the policy expires.

4th. ***Endowment Policies***, payable to the insured at a given age, or to his heirs if he dies before that age, the annual premium continuing till the policy expires.

NOTE.—The *expectation of life* is the average duration of the life of individuals after any specified age.

1. What premium must a man, at the age of 25, pay annually for a life policy of $5000, at $4\frac{1}{2}\%$?

ANALYSIS.—$4\frac{1}{2}\% = .045$, and $\$5000 \times .045 = \$225$, *Ans.* (Art. 443.)

2. What is the annual premium for a life policy of $2500, at 5%?

3. A man at the age of 35 years effected a life insurance of $7500 for 10 yrs., at $3\frac{1}{4}\%$: what was the amt. of premium?

4. A man 65 years old negotiated a life insurance of $8000 for 5 years, at $12\frac{1}{2}\%$: what was the amt. of premium?

5. At the age of 30 years, a man got his life insured for $75000, at 4% per annum, and lived to the age of 70: which was the greater, the sum he paid, or the sum received?

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444. How find what sum must be insured to cover the property and premium?

TAXES.

446. A ***Tax*** is a sum assessed upon the person or property of a citizen, for public purposes.

A ***Property Tax*** is one assessed upon *property*.

A ***Personal Tax*** is one assessed upon the *person*, and is often called a *poll or capitation tax.*

NOTE.—The term *poll* is from the German *polle,* the head; *capitation,* from the Latin *caput,* the *head.*

447. ***Property*** is of two kinds, real and personal.

Real Property is that which is *fixed;* as, lands, houses, etc. It is often called *real estate.*

Personal Property is that which is *movable;* as, money, stocks, etc.

448. An ***Assessor*** is a person appointed to appraise property, for the purpose of taxation.

449. An ***Inventory*** is a list of taxable property, with its estimated value.

450. *Property taxes* are computed at a certain *per cent* on the valuation of the property to be assessed. That is,

The *valuation* of the property is the base; the *sum* to be raised, the percentage; the *per cent* or tax on $1, the rate; the *sum collected* minus the commission, the net proceeds.

Poll taxes are *specific* sums upon those not exempt by law, without regard to property.

451. To assess a *Property Tax,* the Valuation and the Sum to be raised being given.

1. A tax of $6250 was levied upon a corporation of 6 persons; A's property was appraised at $30000; B's, $37850; C's, $40150; D's, $50000; E's, $55000; F's, $37000. What was the rate of the tax, and what each man's share?

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446. What is a tax? A property tax? A personal tax? *Note.* What is a personal tax called? 447. Of how many kinds is property? What is real property? Personal? 448. What is an assessor? 449. An inventory? 450. How are property taxes computed? How are poll taxes levied? 451. How is a property tax assessed, when the valuation and the sum to be raised are given?

ANALYSIS.—The valuation=\$30000+\$37850+\$40150+\$50000+\$55000+\$37000=\$250000. The valuation \$250000, is the base; and the tax \$6250, the percentage. Therefore \$6250÷\$250000=.025, or $2\frac{1}{2}\%$, the rate required.

Again, since A's valuation was \$30000, and the rate $2\frac{1}{2}\%$, his tax must have been $2\frac{1}{2}\%$ of \$30000; and \$30000×.025=\$750.00. The tax of the others may be found in like manner. Hence, the

RULE.—I. *Make an inventory of all the taxable property.*

II. *Divide the sum to be raised by the amount of the inventory, and the quotient will be the rate.*

III. *Multiply the valuation of each man's property by the rate, and the product will be his tax.*

NOTES.—1. If a poll tax is included, the *sum* arising from the polls must be *subtracted* from the *sum* to be raised, before it is divided by the inventory.

2. If the tax is assessed on a large number of individuals, the operation will be shortened by first finding the tax on \$1, \$2, \$3, etc., to \$9; then on \$10, \$20, etc., to \$90; then on \$100, \$200, etc., to \$900, etc., arranging the results as in the following

ASSESSORS' TABLE.

\$1	pays	\$.025	\$10	pay	\$.25	\$100	pay	\$2.50
2	"	.050	20	"	.50	200	"	5.00
3	"	.075	30	"	.75	300	"	7.50
4	"	.100	40	"	1.00	400	"	10.00
5	"	.125	50	"	1.25	500	"	12.50
6	"	.150	60	"	1.50	1000	"	25.00
7	"	.175	70	"	1.75	2000	"	50.00
8	"	.200	80	"	2.00	3000	"	75.00
9	"	.225	90	"	2.25	4000	"	100.00

2. B's valuation=\$37850=\$30000+\$7000+\$800+\$50.

By the table the tax on \$30000=\$750.00
" " 7000= 175.00
" " 800= 20.00
" " 50= 1.25
Therefore, we have B's tax, =\$946.25

Note. If a poll tax is included, how proceed?

3. Required C, D, E, and F's taxes, both by the *rule* and the *table*.

4. A tax levied on a certain township was $16020; the valuation of its taxable property was $784750, and the number of polls assessed at $1.25 was 260. What was the rate of tax; and what was A's tax, who paid for 3 polls, the valuation of his property being $7800?

5. The State levied a tax of $165945 upon a certain city which contained 1260 polls assessed 75 cents each, an inventory of $5427600 real, and $72400 personal property. What was G's tax, whose property was assessed at $15000?

6. What was H's tax, whose inventory was $10250, and 3 polls?

7. A district school-house cost $2500, and the valuation of the property of the district is $50000: what is the rate, and what A's tax, whose property is valued at $3400?

452. To find the *Amount* to be assessed, to raise a net sum, and pay the Commission for collecting it.

8. A certain city required $47500 to pay expenses: what amount must be assessed in order to cover the expenses, and the commission of 5% for collecting the tax?

ANALYSIS.—At 5% for collection, $1 assessment yields $.95; therefore, to obtain $47500 net, requires as many dollars assessment as $.95 are contained times in $47500; and $47500÷.95 (1−5%) = $50000, the sum required to be assessed. (Art. 341.) Hence, the

RULE.—*Divide the net sum by* 1 *minus the rate; the quotient will be the amount to be assessed.* (Art. 348.)

9. What sum must be assessed to raise a net amount of $3500, and pay the commission for collecting, at 4%?

10. What sum must be assessed to raise a net sum of $5260, and pay for the collection, at $4\frac{1}{2}$% commission?

11. What sum must be assessed to raise a net amount of $10500, and pay the commission for collecting, at 5%?

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452. How find the amount to be assessed, to cover the sum to be raised and the commission for collection?

DUTIES.

453. ***Duties*** are sums paid on imported goods, and are often called *customs.*

454. A ***Custom House*** is a building where duties are received, ships entered, cleared, etc.

455. A ***Port of Entry*** is one where there is a *Custom House.*

The ***Collector of a Port*** is an officer who receives the duties, has the charge of the Custom House, etc.

456. A ***Tariff*** is a list of articles subject to duty, stating the *rate*, or the *sum* to be collected on each.

457. An ***Invoice*** is a list of merchandise, with the cost of the several articles in the country from which they are imported.

458. Duties are of two kinds, specific and ad valorem.

459. A ***Specific Duty*** is a *fixed sum* imposed on each article, ton, yard, etc., without regard to its cost.

An ***Ad Valorem Duty*** is a certain *per cent* on the value of goods in the country from which they are imported.

NOTE.—The term *ad valorem* is from the Latin *ad* and *valorem, according to value.*

460. Before calculating duties, certain allowances are made, called *tare, tret* or *draft, leakage,* and *breakage.*

Tare is an allowance for the weight of the box, bag, cask, etc., containing the goods.

Tret is an allowance in the weight or measure of goods for waste or refuse matter, and is often called *draft.*

Leakage is an allowance on liquors in casks.

Breakage is an allowance on liquors in bottles.

NOTES.—1. *Tare* is calculated either at the rate specified in the invoice, or at rates established by Act of Congress.

2. *Leakage* is commonly determined by gauging the casks, and *Breakage* by counting.

3. In making these allowances, if the fraction is less than ½ it is rejected, if ½ or more, 1 is added.

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453. What are duties? 454. A custom house? 455. What is a port of entry? A collector? 456. A tariff? 457. What is an invoice? 458. Of how many kinds are duties? 459. A specific duty? An ad valorem? 460. What is tare? Tret? Leakage? Breakage?

PROBLEM I.

461. To calculate *Specific Duties*, the quantity of goods, and the Sum levied on each article being given.

1. What is the specific duty on 12 casks of brandy, each containing 40 gal., at $1\frac{1}{2}$ per gal., allowing 2% for leakage?

ANALYSIS.—12 casks of 40 gallons each,	=480.0 gal.
The leakage at 2% on 480 gallons,	= 9.6 gal.
The remainder, or quantity taxed,	=470.4 gal.

Now $1.50 × 470.4 = $705.60, the duty required. Hence, the

RULE.—*Deduct the legal allowance for tare, tret, etc., from the goods, and multiply the remainder by the sum levied on each article.*

2. What is the duty, at $1.25 a yard, on 65 pieces of brocade silk, each containing 50 yards?

3. What is the duty on 87 hhds. of molasses, at 20 cts. per gallon, the leakage being 3%?

4. What is the duty, at 6 cts. a pound, on 500 bags of coffee, each weighing 68 lbs., the tare being 2%?

PROBLEM II.

462. To calculate *Ad Valorem Duties*, the Cost of the goods and the Rate being given.

5. What is the ad valorem duty, at 25%, on a quantity of silks invoiced at $3500?

ANALYSIS.—Since the duty is 25% ad valorem,	$3500
it is .25 times the invoice; and $3500 × .25 = $875.	.25
Hence, the	*Ans.* $875.00

RULE.—*Multiply the cost of the goods by the given rate, expressed decimally.*

6. What is the duty, at $33\frac{1}{3}$% ad valorem, on 1575 yds. of carpeting invoiced at $1.80 per yard?

7. What is the ad valorem duty, at 40%, on 110 chests of tea, each containing 67 lbs., and invoiced at 90 cts. a pound, the tare being 9 lbs. a chest?

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461. How calculate specific duties when the quantity of goods and the sum levied on each article are given? 462. How ad valorem?

INTERNAL REVENUE.

463. ***Internal Revenue*** is the income of the Government from Excise Duties, Stamp Duties, Licenses, Special Taxes, Income Taxes, etc.

464. ***Excise Duties*** are taxes upon certain home productions, and are computed at a given *per cent* on their value.

465. ***Stamp Duties*** are taxes upon written instruments; as, notes, drafts, contracts, legal documents, patent medicines, etc.

466. A ***License Tax*** is the sum paid for permission to pursue certain avocations.

467. ***Special Taxes*** are fixed sums assessed upon certain articles of luxury; as, carriages, billiard tables, gold watches, etc.

Income Taxes are those levied upon annual incomes.

NOTE.—In determining income taxes, certain deductions are made for house rent, National and State taxes, losses, etc.

468. To compute *Income Taxes*, the Rate being given.

1. What is a man's tax whose income is $5675; the rate being 5%, the deductions $1000 for house rent, $350 national tax, and $1100 for losses?

ANALYSIS.—Total deductions are $1000+$350+$1100=$2450; and $5675−$2450=$3225 taxable income. Now 5% of $3225= 3225×.05=$161.25, the tax required. Hence, the

RULE.—*From the income subtract the total deductions, and multiply the remainder by the rate.*

NOTE.—If there are special taxes on articles of luxury, as carriages, etc., they must be added to the tax on the income.

2. What was A's revenue tax for 1869, at 5%; his income being $4750; his losses $1185, and exemptions $1200?

3. In 1870, A had $10500 income; 35 oz. taxable plate at 5 cts., 1 watch $2, and 1 carriage $2: what was his tax at 5%, allowing $2100 exemption?

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463. What is internal revenue? 464. Excise duties? 465. Stamp duties? 466. A license tax? 467. Special taxes? Income taxes? 468. How compute income taxes, when the rate is given?

EQUATION OF PAYMENTS.

469. ***Equation of Payments*** is finding the *average time* for payment of two or more sums due at *different* times.

The *average time* sought is often called the *mean*, or *equated* time.

470. Equation of Payments embraces two *classes* of examples.

1st. Those in which the items or bills have the *same date*, but *different lengths* of credit. 2d. Those in which they have *different dates*, and the *same* or *different* lengths of credit.

PROBLEM I.

471. To find the *Average Time,* when the items have the same date, but different lengths of credit.

1. Bought Oct. 3d, 1870, goods amounting to the following sums: $50 payable in 4 m., $70 in 6 m., and $80 in 8 m.: what is the average time at which the whole may be paid, without loss to either party?

$$\begin{array}{r} \$50 \times 4 = 200 \\ 70 \times 6 = 420 \\ 80 \times 8 = 640 \\ \hline 200\ \overline{)1260} \\ \hline \textit{Ans. } 6\tfrac{3}{10}\text{ m.} \end{array}$$

ANALYSIS.—The int. on $50 for 4 m.=the int. on $1 for 50 times 4 m. or 200 m. Again the int. on $70 for 6 m.=the int. on $1 for 70 times 6 m., or 420 m. Finally, the int. on $80 for 8 m.=the int. of $1 for 80 times 8 m., or 640 m. Now 200 m.+420 m.+640 m.=1260 m.; therefore I am entitled to the use of $1 for 1260 m. But the sum of the debts is $50+$70+$80=$200. Now as I am entitled to the use of $1 for 1260 m., I must be entitled to the use of $200 for $\frac{1}{200}$ part of 1260 m., and 1260÷200=$6\frac{3}{10}$ m., or 6 m. 9 d., the average time required. Hence, the

RULE.—*Multiply each item by its length of credit, and divide the sum of the products by the sum of the items. The quotient will be the average time.*

NOTES.—1. When the *date* of the payment is required, add the average *time* to the date of the transaction. Thus, in the preceding Ex., the date of payt. is Oct. 3d+6 m. 9 d., or April 12th, 1871.

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469. What is equation of payments? *Note.* What is the average time called? 470. How many classes of examples in Equation of Payments? 471. How find the average time, when the items have the same date, but different credits

2. This rule is applicable to notes as well as accounts. It is founded upon the supposition that *bank discount* is the same as *simple interest.* Though not strictly accurate, it is in general use.

3. If one item is *cash,* it has no *time,* and no *product;* but in finding the *sum* of items, this must be added with the others.

4. In the answer, a fraction less than $\frac{1}{2}$ day, is rejected; if $\frac{1}{2}$ day or more, 1 day is added.

2. Bought a house June 20th, 1870, for $3000, and agreed to pay $\frac{1}{2}$ down; $\frac{1}{3}$ in 6 m., the balance in 12 m.: at what date may the whole be equitably paid?

3. A owes B $700, payable in 4 mos.; $500, in 6 mos.; $800, in 10 mos.; and $1000 in 12 mos.: in what time may the whole be justly paid?

4. Bought a bill of goods March 10th, 1868, amounting to $2500; and agreed to pay $500 cash, $750 in 10 days, $600 in 20 days, $400 in 30 days, and $250 in 40 days. At what date may I equitably pay the whole?

5. A jobber sold me on the 1st of March $12000 worth of goods, to be paid for as follows: $\frac{1}{4}$ cash, $\frac{1}{4}$ in 2 mos., $\frac{1}{4}$ in 4 mos., and the remaining $\frac{1}{4}$ in 6 mos. When may I pay the whole in equity?

PROBLEM II.

472. To find the *Average Time,* when the items have different dates, and the same or different lengths of credit.

6. Bought the following bills of goods: March 10th, 1870, $500 on 2 m.; April 4th, $800 on 4 m.; June 15th, $1000 on 3 m. What is the average time?

May 10,	00 ×	$500 =	0000 d.	
Aug. 4,	86 ×	$800 =	68800 d.	
Sept. 15,	128 ×	$1000 =	128000 d.	
		$2300	) 196800 d.	
		Average time,	$85\frac{13}{23}$ d.	

ANALYSIS.—We first find when the items are due by adding the time of credit to the date of each, and for convenience place these dates in a column. Taking the earliest date on which either item matures as a standard, we find the number of days from this standard date to the maturity of the other items, and place them on the right, with the sign (×) and the items opposite.

Multiplying each item and its number of days together, the sum of the products shows that the interest on the several items is equal to the interest of $1 for 196800 days.

Now if it takes 196800 days for $1 to gain a certain sum, it will take $2300, $\frac{1}{2300}$ of 196800 d. to gain this sum; and 196800÷2300= $85\frac{13}{23}$, or 86 days from the assumed date. Now May 10+86 d.=Aug. 4th, the date when the amt. is equitably due. Hence, the

RULE.—I. *Find the date when the several items become due, and set them in a column.*

Take the earliest of these dates as a standard, and set the number of days from this date to the maturity of the other items in another column on the right, with the items opposite.

II. *Multiply each item by its number of days, and divide the sum of the products by the sum of the items. The quotient will be the average time of credit.*

NOTES.—1. Add the *average time* to the *standard date*, and the result will be the *equitable date* of payment.

2. The *latest* date on which either item falls due may also be taken as the standard; and having found the *average time*, subtract it from this date; the result will be the date of payment.

3. The date at which each item becomes due, is readily found by adding its time of credit to the date of the transaction.

7. A bought goods as follows: Apr. 10th, $310 on 6 m.; May 21st, $468 on 2 m.; June 1st, $520 on 4 m.; July 8th, $750 on 3 m.: what is the average time, and at what date may the whole debt be equitably discharged?

SUGGESTION.—The *second* item matures earliest; this date is therefore the standard. *Ans.* Av. time 59 d.; date Sept. 18th.

8. Sold the following bills on 6 months credit: Jan. 15th, $210; Feb. 11th, $167; March 7th, $320.25; April 2d, $500.10: when may the whole be paid at one time?

9. Bought June 5th, on 4 m., groceries for $125; June 21st, $230.45; July 12th, $267; Aug. 2d, $860.80: what is the amt. and time of a note to cover the whole?

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472. How find the average time, when the items have different dates, and the same or different credits? *Note.* How find the date of the payment? How find the date at which each item becomes due?

AVERAGING ACCOUNTS.

473. An ***Account*** is a record of the items of debit and credit in business transactions.

NOTE.—The term *debit* is from the Latin *debitus*, owed.

474. A ***Merchandise Balance*** is the difference between the debits and credits of an account.

A ***Cash Balance*** is the difference between the debits and credits, with the interest due on each.

NOTES.—1. Bills of goods sold on time are entitled to interest after they become due; and payments made before they are due are also entitled to interest.

475. To find the *Average Time* for paying the balance of an Account which has both debits and credits.

1. Find the cash Bal. of the following Acct., and when due.

Dr. WM. GORDON in Acct. with JOHN RANDOLPH. *Cr.*

1870.			1870.		
Feb. 10.	For Mdse., 4 m.	$450.00	Mar. 20.	By Sundries, 3 m.	$325.00
May 11.	" " 3 "	500.00	July 9.	" Draft, 60 d.	150.00
July 26.	" " 2 "	360.00	Sept. 15.	" Cash,	400.00

ANALYSIS.—Setting down the date when each item is due, take June 10th, the earliest of these dates, as the *standard*, find the number of days from this standard to the maturity of each item on both sides, and place it on the right of its date with the sign (×), then the items, adding 3 days grace to the time of the draft.

Debits.

June 10, 00 d. × $450 = 00000 d.
Aug. 11, 62 d. × 500 = 31000 d.
Sept. 26, 108 d. × 360 = 38880 d.
Amt. debits, $1310, Int. 69880 d.

Credits.

June 20, 10 d. × $325 = 3250 d.
Sept. 10, 92 d. × 150 = 13800 d.
" 15, 97 d. × 400 = 38800 d.
Amt. credits, $875, Int. 55850 d.
Cash bal. $435, " 14030 d.
$435)14030 d. = 32 d. +.

Ans. Bal. $435, due July 12, '70.

473. What is an account? *Note.* What is the meaning of the term debit?
474. What is a merchandise balance? A cash balance?

Multiplying each item on both sides by its number of days, the int. on the *debits* is equal to the int. of $1 for 69880 days, and the int. on the *credits* is equal to the int. of $1 for 55850 days. (Art. 474.) The balance of int. on the Dr. side is 69880 d.—55850 d. =14030 d.; that is to the int. of $1 for 14030 d. The balance of *items* on the Dr. side is $1310—$875=$435.

Now if it takes 14030 days for $1 to gain a certain sum, it will take $435, $\frac{1}{435}$ of 14030 d. and 14030÷435=$32\frac{22}{87}$, or 32 d. But the assumed standard June 10th+32 d.=July 12th. Therefore the balance due Randolph the creditor is $435, payable July 12th, 1870.

If the greater sum of items and the greater sum of products were on *opposite* sides, it would be necessary to *subtract* the average time from the assumed date. Hence, the

RULE.—I. *Set down the date when each item of debit and credit is due; and assuming the earliest of these dates as a standard, write the number of days from this standard date to the maturity of the respective items, on the right, with the sign* (×) *and the items themselves opposite.*

II. *Multiply each item by its number of days, and divide the difference between the sums of products by that between the sums of items; the quotient will be the average time.*

III. *If the greater sum of items and the greater sum of products are both on the same side, add the average time to the assumed date; if on opposite sides, subtract it; and the result will be the date when the balance of the account is equitably due.*

NOTES.—1. The average time may be such as to extend to a date either *earlier* or *later* than that of any of the items. (Ex. 2.)

2. In finding the maturity of notes and drafts, 3 days grace should be added to the specified time of payment.

3. In finding the *extension* to which the balance of a debt is entitled, when partial payments are made before it is due,

Multiply each payment by the *time* from its date to the maturity of the debt, and *divide* the *sum* of the products by the *balance* remaining unpaid.

4. When no time of credit is mentioned, the transaction is understood to be for cash, and its payment due at once.

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475. How find the average time for paying the balance, when there are both debits and credits?

2. Find the balance of the following Acct., and when due.

Dr. A. B. in account with C. D. *Cr.*

1860.			1860.		
Aug. 11.	For Mdse.,	$160.00	Sept. 2.	By Sundries,	$75.00
Sept. 5.	" "	240.00	Oct. 10.	" your Note on 30 d.,	100.00
Oct. 20.	" 1 Horse,	175.00	Nov. 1.	" Cash	110.00

Dr. OPERATION. *Cr.*

Due.	*Days.*	*Items.*	*Products.*	*Due.*	*Days.*	*Items.*	*Products.*
Aug. 11.	0	$160	0000	Sept. 2.	22	$75	1650
Sept. 5.	25	240	6000	Nov. 9.	93	100	9300
Oct. 20.	70	175	12250	Nov. 1.	82	110	9020
		575	18250			285	19970
		285	1720÷290=6 d. (nearly), av. time.				18250
	Bal=$290		Aug. 11—6 d.=Aug. 5, bal. is due.				1720

SUGGESTION.—In this example, the greater sum of items and the greater sum of products are on opposite sides; hence, the average time must be subtracted from the standard date.

3. A man bought a cottage for $2750, payable in 1 year; in 3 mos. he paid $500, and 3 mos. later $750: to what extension is he entitled on the balance?

SOLUTION.—The sum of the product is 9000; and the balance of the debt $1500. Now 9000÷1500=6. *Ans.* 6 mos. after the maturity of the debt. (Note 3.)

4. A merchant sold a bill of $4220 worth of goods on 8 mos.; 2 months after the customer paid him $720, one month later $850, and 2 months later $1000: how long should the balance in equity remain unpaid?

5. What is the balance of the following account, and when is it due?

Dr. HENRY SWIFT in Acct. with HOMER MORGAN. *Cr.*

1865.			1865.		
March 10.	For Sundries,	$250	April 1.	By Bal. of Acct.,	$110
April 15.	" Flour on 60 d.,	420	May 21.	" Dft. on 30 d.,	300
June 20.	" Mdse. on 30 d.,	600	July 1.	" Cash,	560

RATIO.

476. ***Ratio*** is the *relation* which one number has to another with respect to *magnitude*.

The ***Terms of a ratio*** are the numbers compared. They are often called a *couplet*.

477. Ratio is commonly expressed by a *colon* (:) placed between the two numbers compared. Thus, the ratio of 6 to 3 is written 6 : 3.

478. The *first term* is called the *antecedent*, the second the *consequent*.

The comparison is made by considering what *multiple* or *part* the antecedent is of the consequent.

NOTES.—1. The *sign* of ratio (:) is derived from the sign of division (÷), the horizontal line being dropped.

2. The terms are so called from the order of their position. They must be of the *same kind* or *denomination;* otherwise they cannot be compared.

479. Ratio is measured by a *fraction*, the *numerator* of which is the *antecedent*, and the *denominator* the *consequent;* or what is the same thing, by *dividing* the antecedent by the consequent.

480. The *value* of a ratio is the *value* of the *fraction* by which it is measured. Thus, comparing 6 with 2, we say the ratio of 6 : 2 is $\frac{6}{2}$, or 3. That is, the *former* has a *magnitude* which contains the *latter* 3 times; therefore the value of the ratio 6 : 2 is 3.

481. A ***Simple Ratio*** is one which has but *two* terms; as, 8 : 4.

A ***Compound Ratio*** is the product of *two or more simple* ratios.

Thus: $\left.\begin{matrix}4:2\\9:3\end{matrix}\right\}$ are simple ratios. But $4 \times 9 : 2 \times 3$ is a compound ratio.

NOTE.—The *nature* of compound ratios is the same as that of simple ratios. They are so called to denote their origin, and are usually expressed by writing the corresponding terms of the simple ratios one under another, as above.

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476. What is ratio? The numbers compared called? 477. How is ratio commonly expressed? The first term called? The second? *Note.* Why? 479. How is ratio measured? The value of a ratio? 481. Simple ratio? Compound?

482. Ratio is also distinguished as *direct* and *inverse* or *reciprocal.*

A *direct ratio* is one which arises from dividing the antecedent by the consequent.

An *inverse ratio* is one which arises from dividing the consequent by the antecedent, and is the same as the ratio of the *reciprocals* of the two numbers compared. (Art. 106.) Thus, the direct ratio of 4 to $12=\frac{4}{12}$, or $\frac{1}{3}$; the inverse ratio of 4 to $12=\frac{12}{4}$, or 3. It is the same as the ratio of their reciprocals, $\frac{1}{4}$ to $\frac{1}{12}$.

483. The ratio between two fractions having a common denominator is the same as the *ratio* of their *numerators.* Thus, the ratio of $\frac{6}{9}:\frac{3}{9}$ is the same as 6 : 3.

In finding the ratio of two fractions which have *different denominators,* they should be reduced to a *common* denominator; then *take the ratio* of their numerators. (Art. 153.)

484. In finding the ratio between two *compound* numbers, they must be reduced to the *same denomination.*

NOTE.—Finding the ratio between two numbers is the same in principle as finding *what part* one is of the other, the number denoting the *part* being the antecedent. Thus, 2 is $\frac{2}{4}$ of $4=\frac{1}{2}$; and the ratio of 2 to 4 is $\frac{2}{4}=\frac{1}{2}$. (Art. 173.)

485. Since ratios are measured by fractions whose *numerators* are the *antecedents,* and *denominators* the *consequents,* it follows that *operations* have the same effect upon the *terms* of a ratio as upon the *terms* of a fraction. (Art. 144.) That is,

1. *Multiplying the antecedent multiplies the ratio; and dividing the antecedent divides the ratio.*

2. *Multiplying the consequent divides the ratio; and dividing the consequent multiplies the ratio.*

3. *Multiplying or dividing both the antecedent and consequent by the same number, does not alter the value of the ratio.*

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482. Direct? Inverse? 483. The ratio of two fractions having a common denominator? 484. In finding the ratio of two compound numbers, what must be done? 485. What effect do operations on the terms of a ratio have? Multiplying the antecedent? Dividing it? Multiplying the consequent? Dividing it? What effect has multiplying or dividing both the antecedent and consequent by the same number?

What are the ratios of the following couplets:

1. 12 : 4.	4. 6 : 24.	7. £5 : 10s. 6d.
2. 28 : 7.	5. 8 : 40.	8. 10 y. : 6 ft. 3 in.
3. 36 : 12.	6. 9 : 51.	9. 25 g. : 2 qt. 1 pt.

10. Reduce the ratio of 14 to 35 to the lowest terms?

SOLUTION.—14 : 35 equals $\frac{14}{35}$; and $\frac{14}{35}=\frac{2}{5}$, or 2 : 5. (Art. 146.)

Reduce the following ratios to the lowest terms?

11. 154 : 28.	13. 73 : 511.	15. 238 : 1428.
12. 39 : 165.	14. 113 : 1017.	16. 576 : 1728.

17. Reduce the ratio $\frac{3}{6}:\frac{5}{8}$ to the lowest integral terms?

$\frac{3}{6}=\frac{12}{24}$. and $\frac{5}{8}=\frac{15}{24}$. Now $\frac{12}{24}:\frac{15}{24}$ is the same as 4 : 5. (Art. 483.)

18. Multiply the ratio of 21 : 7 by 4 : 8. *Ans.* 84 : 56, or $1\frac{1}{2}$.

19. What is the value of $5 : 8 \times 4 : 10 \times 7 : 9$?

PROPORTION.

486. ***Proportion*** is an *equality* of ratios.

487. Every proportion must have at least *four* terms; for, the equality is between *two* or *more* ratios, and each ratio has *two* terms, an antecedent and a consequent.

A proportion may, however, be formed from *three numbers;* for, one of the numbers may be repeated, so as to form two terms.

488. Proportion is denoted in two ways; by a double colon (: :), and by the sign of equality (=), placed between the ratios. Thus, each of the expressions 4 : 2 : : 6 : 3, and 4 : 2 = 6 : 3 indicates a proportion; for, $\frac{4}{2}=\frac{6}{3}$.

The former is read, "4 is to 2 as 6 to 3," or "4 is the same part of 2, that 6 is of 3." The latter is read, "the ratio of 4 to 2 equals the ratio of 6 to 3."

NOTE.—The sign (: :) is derived from the sign (=), the *points* being the *extremities* of the parallel lines.

486. What is proportion? 487. How many terms has every proportion? Can three numbers form a proportion? How?

489. The four numbers which form a proportion, are called *proportionals*. The *first* and *last* are the *extremes*, the other two the *means*.

When a proportion has but *three* numbers, the second term is called a *mean proportional* between the other two.

490. If four numbers are *proportional*, the *product* of the *extremes* is equal to the *product* of the *means*. Hence,

491. The relation of the four terms of a proportion to each other is such, that if *any three* of them are given, the *other* or *missing* term may be found.

492. To find the *Missing Term* of a Proportion, the other three Terms being given.

1. Let 6 be the first term of a proportion, 3 and 10 the two means; the other extreme equals $3 \times 10 \div 6 = 5$; for, the product of the means=the product of the extremes; and the product of two factors divided by one of them, gives the other. (Art. 93.)

2. Let 3, 10 and 5 be the last three terms of a proportion, the first term equals $3 \times 10 \div 5 = 30 \div 5$ or 6.

3. Let 6 and 5 be the extremes of a proportion, and 3 one of the means; the other mean equals $6 \times 5 \div 3 = 30 \div 3$ or 10.

4. If 6 and 5 are the extremes, and 10 one of the means, the other mean equals $6 \times 5 \div 10 = 30 \div 10$ or 3. Hence, the

RULE.—I. If *one* of the *extremes* and the *two means* are given, *divide the product of the means by the given extreme.*

II. If *one* of the *means* and the *two extremes* are given, *divide the product of the extremes by the given mean.*

Find the missing term in the following proportions:

1. 52 : 13 : : 62 : —.
2. 15 : 90 : : — : 72.
3. 60 : — : : 100 : $33\frac{1}{3}$.
4. — : 25 : : $\frac{6}{8}$: $\frac{1}{4}$.
5. 4 rods : 11 ft. : : 18 men : —.
6. 24 yd. : 3 yd. : : — : $12.
7. 20 gal. : — : : $40 : $8.
8. — : 40 lb. : : £2 : 8s.

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489. What are the four numbers forming a proportion called? 491. If three terms of a proportion are given, what is true of the fourth? If the two means and one extreme are given, how find the other extreme? If the two extremes and one mean are given, how find the other mean?

SIMPLE PROPORTION.

493. ***Simple Proportion*** is an equality of two *simple ratios.*

Simple Proportion is applied chiefly to the solution of problems having *three terms* given to find a *fourth,* of which the *third* shall be the same *multiple* or *part,* as the *first* is of the *second.*

494. To solve Problems by Simple Proportion.

1. If 5 baskets of peaches cost $10, what will 3 baskets cost?

STATEMENT.

5 bas. : 3 bas. : : $10 : *Ans.*

$$\begin{array}{r} 3 \\ \hline 5)\$30 \\ \hline \$6\ \textit{Ans.} \end{array}$$

ANALYSIS.—The question assumes that 3 baskets can be bought at the same rate as 5 baskets; therefore 5 bas. has the same ratio to 3 bas. as the *cost* of 5 bas. has to the *cost* of 3 bas. That is, 5 bas. : 3 bas. : : $10 : cost of 3 bas. We have then the *two means* and *one extreme* of a proportion to find the *fourth* term, or other extreme. (Art. 492.) Now the product of the means $10 × 3 = $30; and $30 ÷ 5 (the other extreme) = $6, the cost of 3 baskets. Hence, the

RULE.—I. *Take that number for the third term, which is the same kind as the answer.*

II. *When the answer is to be larger than the third term, place the larger of the other two numbers for the second term; but when less, place the smaller for the second term, and the other for the first.*

III. *Multiply the second and third terms together, and divide the product by the first; the quotient will be the fourth term or answer.* (Arts. 490, 491.)

PROOF.—If the *product* of the *first* and *fourth* terms equals that of the *second* and *third,* the answer is right.

NOTES.—1 The arrangement of the given terms in the form of a proportion is called "Stating the question."

2. After stating the question, *the factors* common to the *first* and *second,* or to the *first* and *third* terms, should be cancelled.

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493. Simple Proportion? 494. How solve problems by Simple Proportion?

3. If the *first* and *second* terms contain *different denominations*, they must be reduced to the *same*. If the *third* term is a compound number, it must be reduced to the *lowest denomination* it contains.

495. REASONS.—1. The *reason* for placing that number for the *third term*, which is the same kind as the *answer*, and the other two numbers for the *first* and *second*, is because *money has* a *ratio to money*, but *not* to the other two numbers; and the *other two numbers* have a *ratio to each other*, but *not* to money.

2. Of the two like numbers, the *smaller* is taken for the *second* term, and the *larger* for the *first*, because 3 baskets being less than 5 baskets, will cost less; consequently, the *answer* or *fourth* term must be *less* than the *third*, the cost of 5 baskets.

3. If it were required to find the cost of a quantity *greater* than that whose cost is given, the *answer* would be *greater* than the *third* term; consequently the *greater* of the two similar numbers must then be taken for the *second* term, and the *less* for the *first*.

4. The reason for multiplying the *second* and *third* terms together and dividing the product by the *first*, is because the product of the means divided by one of the extremes, gives the other *extreme* or *answer*. (Arts. 93, 492.)

2. If 9 yards of cloth cost $54, what will 23 yards cost?

Statement.—9 yd. : 23 yd. : : $54 : *Ans.*

And ($\$54 \times 23) \div 9 = \$138$, the *Ans.*

By Cancellation.—Since 9 yds. cost $54, 1 yd. will cost $\frac{1}{9}$ of $54, or $\$\frac{54}{9}$; and 23 yds. will cost $\$\frac{54}{9} \times 23 = \frac{54 \times 23}{9} = \$138$, *Ans.*

OPERATION.

$\$\frac{54}{9} \times 23 = Ans.$

$$\frac{\overset{6}{\cancel{54}} \times 23}{\cancel{9}} = \$138, \textit{Ans.}$$

Proof.—9 yd. : 23 yd. : : $54 : $138; for $9 \times 138 = 23 \times 54$.

3. If 7 barrels of flour cost $56, what will 20 barrels cost?

4. What cost 75 bushels of wheat, if 15 bushels cost $33?

5. What cost 150 sheep, if 17 sheep cost $51?

6. If 5 lb. 8 oz. of honey cost $1.65, what will 20 lb. cost?

7. Paid £1, 15s. 6d. for 6 pounds of tea: what must be paid for a chest containing 65 lb. 8 oz.?

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495. Why take the number which is of the same kind as the answer for the third term, and the other two for the first and second? When place the larger of the other two numbers for the second? Why? When the smaller? Why? Why does the product of the second and third terms divided by the first give the answer? What is the arrangement of the terms in the form of a proportion called? If the first and second terms contain different denominations, how proceed? If the third is a compound number, how?

SIMPLE PROPORTION BY ANALYSIS.

496. The chief difficulty experienced by the pupil in Simple Proportion, lies in "stating the question." This difficulty arises from a *want of familiarity* with the *relation* of numbers. He will be assisted by *analyzing the examples* before attempting to state them.

8. If 7 hats cost $42, how much will 12 hats cost?

ANALYSIS.—1 hat is 1 seventh of 7 hats; therefore 1 hat will cost 1 seventh as much as 7 hats; and $\frac{1}{7}$ of $42 is $6. Again, 12 hats will cost 12 times as much as 1 hat, and 12 times $6 are $72. Therefore, 12 hats will cost $72.

Or, 7 hats are $\frac{7}{12}$ of 12 hats; therefore the cost of 7 hats is $\frac{7}{12}$ the cost of 12 hats. But 7 hats cost $42; hence $42 are $\frac{7}{12}$ the cost of 12 hats. Now, if $\frac{7}{12}$ of a number are $42, $\frac{1}{12}$ of that number is $\frac{1}{7}$ of $42, which is $6; and $\frac{12}{12}$ are 12 times $6, or $72.

By Proportion.—7 h. : 12 h. : : $42 : *Ans.* That is,
7 h. are the same part of 12 h. as $42 are of the cost of 12 hats.

497. Solve the following examples both by *analysis and proportion.*

9. If 11 men can cradle 33 acres of grain in 1 day, how many acres can 45 men cradle in the same time?

10. When mackerel are $150 for 12 barrels, what must I pay for 75 barrels?

11. How far will a railroad car go in 12 hours, if it goes at the rate of 15 miles in 40 minutes?

12. A bankrupt owes $3500, his assets are $1800: how much will a creditor receive whose claim is $560?

13. At the rate of 18 barrels for $63, what will 235 barrels of apples cost?

14. If $250 earns $17½ interest in 1 year, how much will $1900 earn in the same time?

15. If a car wheel turns round 6 times in 33 yards, how many times will it turn round in going 7 miles?

16. If a clerk can lay up $1500 in 1½ year, how long will it take him to lay up $5000?

17. If 6 men can hoe a field of corn in 20 hours, how long will it take 15 men to hoe it?

18. An engineer found it would take 75 men 220 days to build a fort; the general commanding required it to be built in 15 days: how many men must the engineer employ to complete it in the required time?

19. If 5 oz. of silk can be spun into a thread 100 rods long, what weight of silk is required to spin a thread that will reach the moon, 240000 miles distant?

20. How many horses will it take to consume a scaffold of hay in 40 days, if 12 horses can consume it in 90 days?

21. If $\frac{5}{8}$ acre of land costs \$15, what will $25\frac{1}{2}$ acres cost?

22. If $\frac{3}{8}$ of a ton of iron costs £$\frac{4}{5}$, what will $\frac{9}{16}$ of a ton cost?

23. If $10\frac{1}{2}$ lb. sugar cost \$$1\frac{3}{4}$, what will $30\frac{3}{8}$ lb. cost?

24. If $\frac{5}{8}$ of a chest of tea costs \$35.50, what will $15\frac{1}{2}$ chests cost?

25. What will $48\frac{3}{8}$ tons of hay cost, if $12\frac{1}{2}$ tons cost \$$126\frac{3}{4}$?

26. If $\frac{7}{12}$ of a ship is worth \$$16250\frac{5}{8}$, what is $\frac{3}{16}$ of it worth?

27. What will it cost me for a saddle horse to go 100 miles if I pay at the rate of $37\frac{1}{2}$ cts. for 3 miles?

28. What must be the length of a slate that is 10 in. wide, to contain a square foot?

29. How many yards of carpeting $\frac{3}{4}$ yard wide will it take to cover a floor 15 ft. long and 12 feet wide?

30. If a man's pulse beats 68 times a minute, how many times will it beat in 24 hours?

31. If Halley's comet moves 2° 45′ in 11 hours, how far will it move in 30 days?

32. If a pole 10 ft. high cast a shadow $7\frac{1}{2}$ ft. long, how high is a flag-staff whose shadow is 60 ft. long?

33. At the rate of 3 oranges for 7 apples, how many oranges can be bought with 150 apples?

34. If an ocean steamer runs 1250 miles in 3 days 8 h., how far will she run in 8 days?

35. If 12 men can harvest a field of wheat in 11 days, how many men are required to harvest it in 4 days?

36. The length of a croquet-ground is 45 feet; and its width is to its length as 2 to 3: what is its width?

37. A man's annual income from U. S. 6s is $1350 when gold is $112\frac{1}{2}$: what was it when gold was 160?

38. George has 10 minutes start in a foot-race; and runs 20 rods a minute: how long will it take Henry, who runs 28 rods a minute, to overtake him?

39. If the driving wheel of a locomotive makes 227 revolutions in going 206 rods 6 ft., how many revolutions will it make in running 18 miles 240 rods?

40. If 3 lbs. of coffee cost $1.20, and 10 lbs. of coffee are worth 6 lbs. of tea, what will 60 lbs. of tea cost?

41. A can chop a cord of wood in 4 hours, and B in 6 hours: how long will it take both to chop a cord?

42. A reservoir has 3 hydrants; the first will empty it in 8 hours, the second in 10; the third in 12 hours: if all run together, how long will it take to empty it?

43. A man and wife drank a keg of ale in 18 d.; it would last the man 30 d.: how long would it last the woman?

44. A fox is 100 rods before a hound, but the hound runs 20 rods while the fox runs 18 rods: how far must the hound run before he catches the fox?

45. A cistern holding 3600 gallons has a supply and a discharge pipe; the former runs 45 gallons an hour, the latter 33 gallons: how long will it take to fill the cistern, when both are running?

46. A clerk who engaged to work for $500 a year, commenced at 12 o'clock Jan. 1st, 1869, and left at noon, the 21st of May following: how much ought he to receive?

47. A church clock is set at 12 o'clk. Saturday night; Tuesday noon it had gained 3 min.: what will be the true time, when it strikes 9 the following Sunday morning?

48. A market-woman bought 100 eggs at 2 for a cent, and another 100 at 3 for a cent; if she sells them at the rate of 5 for 2 cents, what will she make or lose?

49. Two persons being 336 miles apart, start at the same time, and meet in 6 days, one traveling 6 miles a day faster than the other: how far did each travel?

COMPOUND PROPORTION.

498. ***Compound Proportion*** is an equality of a *compound* and a *simple* ratio. Thus,

$\left.\begin{matrix}4:2\\9:3\end{matrix}\right\}::12:2$, is a compound proportion.

It is read, "The ratio of 4 into 9 is to 2 into 3 as 12 to 2."

1. If 4 men saw 20 cords of wood in 5 days, how many cords can 12 men saw in 3 days?

STATEMENT.

$\left.\begin{matrix}4\text{ m.}:12\text{ m.}\\5\text{ d.}:3\text{ d.}\end{matrix}\right\}::20\text{ c.}:Ans.$

$20\text{ c.}\times12\times3=720\text{ c.}$

$4\times5=20$

$720\text{ c.}\div20=36\text{ c. } Ans.$

ANALYSIS.—The answer is to be in cords; we therefore make 20 c. the third term. The other given numbers occur in *pairs*, two of a kind; as, 4 men and 12 men, 5 days and 3 days. We arrange these pairs in *ratios*, as we should in simple proportion, if the answer depended on each pair alone. That is, since 12 m. will saw *more* than 4 men, we take 12 for the second term and 4 for the first. Again, since 12 men will saw *less* in 3 days than in 5 days, we take 3 for the second term and 5 for the first. Finally, dividing the product of the second and third terms 20 c. × 12 × 3 = 720 c. by the product of the first terms 4 × 5 = 20, we have 36 cords for the answer. Hence, the

RULE.—I. *Make that number the third term which is of the same kind as the answer.*

II. *Take the other numbers in pairs of the same kind, and arrange them as if the answer depended on each couplet, as in simple proportion.* (Art. 494.)

III. *Multiply the second and third terms together, and divide the product by the product of the first terms, cancelling the factors common to the first and second, or to the first and third terms. The quotient will be the answer.*

PROOF.—*If the product of the first and fourth terms equals that of the second and third terms, the work is right.*

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498. What is Compound Proportion? Explain the first example. The rule. Proof. *Note.* How proceed when the first and second terms contain different denominations? When the third does? How else may questions in Compound Proportion be solved?

NOTES.—1. The terms of each couplet in the compound ratio must be reduced to the same denomination, and the third term to the lowest denomination contained in it, as in Simple Proportion.

2. In Compound Proportion, all the terms are given in *couplets* or *pairs* of the *same* kind, except *one*. This is called the *odd term*, or *demand*, and is always the same kind as the *answer*.

3. It should be observed that it is not the ratio of 4 to 2, nor of 9 to 3 alone that equals the ratio of 12 to 2; for, $4 \div 2 = 2$ and $9 \div 3 = 3$, while $12 \div 2 = 6$. But it is the ratio compounded of 4×9 to 2×3, which equals the ratio of 12 to 2. Thus, $(4 \times 9) \div (2 \times 3) = 6$; and $12 \div 2 = 6$. (Art. 498.)

4. Compound Proportion was formerly called "Double Rule of Three."

499. Problems in Compound Proportion may also be solved by *Analysis* and *Simple Proportion*. Take the preceding example:

By Analysis.—If 4 men saw 20 cords in 5 d., 1 m. will saw $\frac{1}{4}$ of 20 c., which is 5 c., and 12 m. will saw 12 times 5 c., or 60 cords, in the same time. Again, if 12 men saw 60 c. in 5 days, in 1 d. they will saw $\frac{1}{5}$ of 60 c., or 12 cords, and in 3 d. 3 times 12 c., or 36 cords, the answer required.

By Simple Proportion.—4 m. : 12 m. : : 20 c. : the cords 12 m. will saw in 5 d.; and 20 c. $\times 12 \div 4 = 60$ c. in 5 days. Again, 5 d. : 3 d. : : 60 c. : the cords 12 men will saw in 3 d. And 60 c. $\times 3 \div 5 = 36$ cords, the same as before.

2. If 4 men earn \$219 in 30 days, working 10 hours a day, how much can 9 men earn in 40 days, working 8 hours a day?

STATEMENT.

$$\left.\begin{array}{l}4\text{ m.} : 9\text{ m.}\\ 30\text{ d.} : 40\text{ d.}\\ 10\text{ h.} : 8\text{ h.}\end{array}\right\} : : \$219 : Ans.$$

$(\$219 \times 9 \times 40 \times 8) \div (4 \times 30 \times 10) = \$525\frac{3}{5}$, *Ans.*

By Cancellation.

~~4~~ m.	~~9~~ m. 3
5, ~~15~~, ~~30~~ d.	~~40~~ d.
~~10~~ h.	~~8~~ h. 4
	: : \$219 : *Ans.*
5	$\$219 \times 4 \times 3 = \$525\frac{3}{5}$, *Ans.*

3. If 6 men can mow 28 acres in 2 days, how long will it take 7 men to mow 42 acres?

4. If 8 horses can plow 32 acres in 6 days, how many horses will it take to plow 24 acres in 4 days?

5. If the board of a family of 8 persons amounts to \$300 in 15 weeks, how long will \$1000 board 12 persons?

6. What will be the cost of 28 boxes of candles containing 20 pounds apiece, if 7 boxes containing 15 pounds apiece can be bought for $23.75?

7. If the interest of $300 for 10 months is $20, what will be the interest of $1000 for 15 months?

8. If a man walks 180 miles in 6 days, at 10 h. each, how many miles can he walk in 15 days, at 8 h. each?

9. If it costs $160 to pave a sidewalk 4 ft. wide and 40 ft. long, what will it cost to pave one 6 ft. wide and 125 ft. long?

10. If it requires 800 yards of cloth $\frac{3}{4}$ yd. wide to supply 100 men, how many yards that is $\frac{7}{8}$ wide will it require to clothe 1500 men?

11. If 75 men can build a wall 50 ft. long, 8 ft. high, and 3 ft. thick, in 10 days, how long will it take 100 men to build a wall 150 ft. long, 10 ft. high, and 4 ft. thick?

12. If it costs $56 to transport 7 tons of goods 110 miles, how much will it cost to transport 40 tons 500 miles?

13. If 30 lb. of cotton will make 3 pieces of muslin 42 yds. long and $\frac{5}{8}$ yd. wide, how many pounds will it take to make 50 pieces, each containing 35 yards $1\frac{1}{8}$ yd. wide?

14. If the interest of $600, at 7%, is $35 for 10 months, what will be the int. of $2500, at 6%, for 5 months?

15. If 9 men, working 10 hours per day, can make 18 sofas in 30 days, how many sofas can 50 men make in 90 days, working 8 hours per day?

16. If it takes 9000 bricks 8 in. long and 4 in. wide to pave a court-yard 50 ft. long by 40 ft. wide, how many tiles 10 in. square will be required to lay a hall-floor 75 ft. long by 8 ft. wide?

17. If in 8 days 15 sugar maples, each running 12 quarts of sap per day, make 10 boxes of sugar, each weighing 6 lb., how many boxes weighing 10 lb. apiece, will a maple grove containing 300 trees, make in 36 days, each tree running 16 quarts per day? *Ans.* 720 boxes.

PARTITIVE PROPORTION.

500. ***Partitive Proportion*** is dividing a number into two or more parts having a *given ratio* to each other.

501. To divide a Number into two or more parts which shall be proportional to given numbers.

1. A and B found a purse of money containing $35, which they agree to divide between them in the ratio of 2 to 3: how many dollars will each have?

By Proportion.—The sum of the proportional parts is to each separate part as the number to be divided is to each man's share. That is, 5 (2+3) is to 2 as $35 to A's share. Again, 5 is to 3 as $35 to B's share. Hence, the

STATEMENT.

5 : 2 : : $35 : A's s.
5 : 3 : : $35 : B's s.
($35 × 2) ÷ 5 = $14 A's s.
($35 × 3) ÷ 5 = $21 B's s.

RULE.—I. *Take the number to be divided for the third term; each proportional part successively for the second term; and their sum for the first.*

II. *The product of the second and third terms of each proportion, divided by the first, will be the corresponding part required.*

By Analysis.—Since A had 2 parts and B 3, both had 2 + 3, or 5 parts. Hence, A will have $\frac{2}{5}$ and B $\frac{3}{5}$ of the money. Now $\frac{2}{5}$ of $35 = $14, and $\frac{3}{5}$ of $35 = $21. Hence, the

RULE.—*Divide the given number by the sum of the proportional numbers, and multiply the quotient by each one's proportional part.*

2. It is required to divide 78 into three parts which shall be to each other as 3, 4, and 6. *Ans.* 18, 24, 36.

3. A man having 200 sheep, wished to divide them into three flocks which should be to each other as 2, 3, and 5: how many will each flock contain?

4. A miller had 250 bushels of provender composed of oats, peas, and corn in the proportion of 3, 4, and $5\frac{1}{2}$: how many bushels were there of each kind?

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500. How divide a number into parts having a given ratio?

5. A father divided 497 acres of land among his four sons in proportion to their ages, which were as 2, 3, 4, and 5: how many acres did each receive?

502. The principles of Partitive Proportion are applicable to those classes of problems commonly arranged under the heads of *Partnership, Bankruptcy, General Average,* etc., in which a given number is to be divided into *parts* having a *given ratio* to each other.

PARTNERSHIP.

503. ***Partnership*** is the association of two or more persons in business for their *common profit.*

It is of two kinds; *Simple* and *Compound.*

504. ***Simple Partnership*** is that in which the capital of each partner is employed for the *same time.*

505. ***Compound Partnership*** is that in which the capital is employed for *unequal* times.

NOTE.—The association is called a *firm, house,* or *company;* and the persons associated are termed *partners.*

506. The ***Capital*** is the money or property employed in the business.

The ***Profits*** are the gains shared among the partners, and are called *dividends.*

NOTES.—1. The *profits* are divided as the partners may agree. Other things being equal, when the capital is employed for the *same time,* it is customary to divide the profits according to the *amount of capital* each one furnishes.

2. When the capital is employed for *unequal times,* the profits are usually divided according to the *amount of capital* each furnishes, and the *time* it is employed.

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502. To what are the principles of Partitive Proportion applicable? 503. What is Partnership? Of how many kinds? 504. Simple Partnership? 505. Compound? *Note.* What is the association called? 506. What is the capital? The profits? What called?

PROBLEM I.

507. To find each Partner's Share of the Profit or Loss, when divided according to their capital.

1. A and B entered into partnership; the former furnishing $648, the latter $1080, and agreed to divide the profit according to their capital. They made $432: what was each one's share of the profit?

ANALYSIS.—The capital equals $648 + $1080 = $1728.

$1728 (capital): $648 (A's cap.) : : $432 (profit) : A's share, or $162.
$1728 " : $1080 (B's cap.) : : $432 " : B's share, or $270.

Or thus: The profit $432 ÷ $1728 (the cap.) = .25; that is, the profit is 25% of the capital. Therefore each man's share of the profit is 25% of his capital. Now $648 × .25 = $162, A's share; and $1080 × .25 = $270, B's share. Hence, the

RULE.—*The whole capital is to each partner's capital, as the whole profit or loss to each partner's share of the profit or loss.*

Or, *Find what per cent the profit or loss is of the whole capital, and multiply each man's capital by it.* (Art. 339.)

2. A and B form a partnership, A furnishing $1200, and B $1500; they lose $500: what is each one's share of the loss?

3. A and B buy a saw-mill, A advancing $3000, and B $4500; they rent it for $850 a year: what should each receive?

4. The net gains of A, B, and C for a year are $12500; A furnishes $15000, B $12000, and C $10000: how should the profit be divided?

5. Three persons entering into a speculation, made $15300, which they divided in the ratio of 2, 3, and 4: how much did each receive?

6. A, B, and C hire a pasture for $320 a year; A put in 80, B 120, and C 200 sheep: how much ought each man to pay?

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507. How find each partner's profit or loss, when their capital is employed the same time?

PROBLEM II.

508. To find each Partner's Share of the Profit or Loss, when divided according to capital and time.

7. A and B enter into partnership; A furnishes $400 for 8 months, and B $600 for 4 months; they gain $350: what is each one's share of the profit?

ANALYSIS.—In this case the profit of the partners depends on two conditions, viz.: the *amount* of capital each furnishes, and the *time* it is employed.

But the use of $400 for 8 months equals that of 8 times $400, or $3200, for 1 m.; and $600 for 4 m. equals 4 times $600, or $2400, for 1 m. The respective capitals, then, are equivalent to $2400 and $3200, each employed for 1 m. Now, as A furnished $3200, and B $2400, the whole capital equals $3200 + $2400 = $5600. Therefore,

$5600 : $3200 : : $350 (profit) : A's share, or $200.
$5600 : $2400 : : $350 " : B's share, or $150.

Or thus: The gain $350 ÷ $5600 (the cap.) = .06¼, or 6¼%. Therefore, $3200 × .06¼ = $200, A's share; and $2400 × .06¼ = $150, B's share. Hence, the

RULE.—*Multiply each partner's capital by the time it is employed. Consider these products as their respective capitals, and proceed as in the last problem.*

Or, *Find what per cent the profit or loss is of the whole capital, and multiply each man's capital by it.* (Art. 339.)

NOTE.—The object of multiplying each partner's capital by the *time* it is employed is, to reduce their respective capitals to *equivalents* for the same time.

8. A, B, and C form a partnership; A furnishing $500 for 9 m., B $700 for 1 year, and C $400 for 15 months; they lose $600: what is each man's share of the loss?

9. Two men hire a pasture for $50; one put in 20 horses for 12 weeks, the other 25 horses for 10 weeks: how much should each pay?

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508. How find each one's share, when their capital is employed for unequal times? *Note.* Why multiply each one's capital by the time it is employed?

10. A, B, C, and D commenced business Jan. 1st, 1870, when A furnished $1000; March 1st, B put in $1200; July 1st, C put in $1500; and Sept. 1st, D put in $2000; during the year they made $1450: how much should each receive?

11. A store insured at the Howard Insurance Co. for $3000; in the Continental, $4500; in the American, $6000, was damaged by fire to the amount of $6750: what share of the loss should each company pay?

REMARK.—The loss should be averaged in *proportion* to the *risk* assumed by each company.

12. A quartermaster paid $2500 for the transportation of provisions; A carried 150 barrels 40 miles, B 170 barrels 60 miles, C 210 barrels 75 miles, and D 250 barrels 100 miles: how much did he pay each?

13. A, B, and C, formed a partnership, and cleared $12000; A put in $8000 for 4 m., and then added $2000 for 6 m.; B put in $16000 for 3 m., and then withdrawing half his capital, continued the remainder 5 m. longer; C put in $13500 for 7 m.: how divide the profit.

BANKRUPTCY.

509. ***Bankruptcy*** is inability to pay indebtedness.

NOTE.—A person unable to pay his debts is said to be *insolvent*, and is called a *bankrupt*.

510. The ***Assets*** of a bankrupt are the property in his possession.

The ***Liabilities*** are his debts.

511. The ***Net Proceeds*** are the assets *less* the expense of settlement. They are divided among the creditors according to *their claims*.

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509. What is bankruptcy? 510. What are assets? Liabilities? 511. Net proceeds? 512. How find each creditor's dividend, when the liabilities and the net proceeds are given?

512. To find each Creditor's Dividend, the Liabilities and Net Proceeds being given.

1. A merchant failed, owing B \$1260, C \$1800, and D \$1940; his assets were \$1735, and the expenses of settling \$435: how much did each creditor receive?

ANALYSIS.—The liabilities are \$1260 + \$1800 + \$1940 = \$5000; and the net proceeds \$1735 − \$435 = \$1300. Now,

\$5000 : \$1260 : : \$1300 : B's dividend, or \$327.60.
\$5000 : \$1800 : : \$1300 : C's " or \$468.00.
\$5000 : \$1940 : : \$1300 : D's " or \$504.40.

Or thus: The net proceeds \$1300 ÷ \$5000 = .26, or 26%, the rate he is able to pay. (Art. 339.) Now \$1260 × .26 = \$327.60 B's; \$1800 × .26 = \$468 C's; \$1940 × .26 = \$504.40 D's. Hence, the

RULE.—*The whole liabilities are to each creditor's claim, as the net proceeds to each creditor's dividend.*

Or, *Find what per cent the net proceeds are of the liabilities, and multiply each creditor's claim by it.*

2. A bankrupt owes A \$6300, B \$4500, and D \$3200; his assets are \$5250, and the expenses of settling \$1500: how much will each creditor receive?

3. A. B. & Co. went into bankruptcy, owing \$48400, and having \$13200 assets; the expense of settling was \$1100. What did D receive on \$8240?

ALLIGATION.

513. ***Alligation*** is of two kinds, *Medial* and *Alternate*.

Alligation Medial is the method of finding the *mean value* of mixtures.

Alligation Alternate is the method of finding the proportional parts of mixtures having a given value.

NOTES.—1. *Alligation,* from the Latin *alligo,* to *tie,* or *bind together,* is so called from the manner of connecting the ingredients by curve lines in some of the operations.

2. *Alternate,* Latin *alternatus,* by *turns,* refers to the manner of connecting the prices *above* the mean price with those *below.*

3. The term *medial* is from the Latin *medius, middle* or *average.*

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513. What is alligation medial? Alternate?

ALLIGATION MEDIAL.

514. To find the *Mean Value* of a mixture, the Price and Quantity of each ingredient being given.

1. Mixed 50 lbs. of tea at 90 cts., 60 lbs. at $1.10, and 80 lbs. at $1.25: what is the mean value of the mixture?

$.90 × 50 =	$45.00
$1.10 × 60 =	$66.00
$1.25 × 80 =	$100.00
190)	$211.00
Ans.	$1.11 +

ANALYSIS.—The total value of the first kind .90 × 50 = $45.00, the second $1.10 × 60 = $66.00, the third $1.25 × 80 = $100.00; therefore the total value of the mixture = $211.00. But the quantity mixed = 190 lbs. Now, if 190 lbs. are worth $211, 1 lb. is worth $\frac{1}{190}$ of $211, or $1.11+. Therefore the mean value of the mixture is $1.11+ per pound. Hence, the

RULE.—*Divide the value of the whole mixture by the sum of the ingredients mixed.*

NOTE.—If an ingredient costs nothing, as water, sand, etc., its value is 0; but the quantity itself must be added to the other ingredients.

2. If I mix 3 kinds of sugar worth 12, 15, and 20 cts. a pound, what is a pound of the mixture worth?

3. A farmer mixed 30 bu. of corn, at $1.25, with 25 bu. of oats, at 60 cts., and 10 bu. of peas, at 95 cts.: what was the average value of the mixture?

4. A grocer mixed 5 gal. molasses, worth 80 cts. a gal., and 107 gallons of water, with a hogshead of cider, at 20 cts.: what was the average worth of the mixture?

5. A goldsmith mixed 12 oz. of gold 22 carats fine with 8 oz. 20 carats, and 7 oz. 18 carats fine: what was the average fineness of the composition?

6. A milkman bought 40 gallons of new milk, at 4 cts. a quart, and 60 gallons of skimmed milk at 2 cts. a quart, which he mixed with 12 gallons of water, and sold the whole at 6 cts. a quart: required his profit?

Note. Why is this rule called alligation? Why alternate? What is the import of medial? 514. How find the mean value of a mixture, when the price and quantity are given?

ALLIGATION ALTERNATE.

PROBLEM I.

515. To find the *Proportional Parts* of a Mixture, the Mean Price and the Price of each ingredient being given.

7. A grocer desired to mix 4 kinds of tea worth 3s., 8s., 11s., and 12s. a pound, so that the mixture should be worth 9s. a pound: in what proportion must they be taken?

OPERATION.

9	3	3 lb.
	8	2 "
	11	1 "
	12	6 "

ANALYSIS.—To equalize the gain and loss, we compare the prices in pairs, one being *above* and the other *below* the mean price, and, for convenience, connect them by curve lines. Taking the *first and fourth;* on 1 lb. at 3s. the gain is 6s.; on 1 lb. at 12s. the loss is 3s., which we place opposite the 12 and 3. Therefore, it takes 1 lb. at 3s. to balance the loss on 2 lb. at 12s., and the proportional parts of this couplet are as 1 to 2, or as 3 to 6. But 3 and 6 are the differences between the mean price and that of the teas compared, taken inversely.

Again, 1 lb. at 8s. gains 1s., and 1 lb. at 11s. loses 2s., which we place opposite the 11 and 8. Therefore, it takes 2 lb. at 8s. to balance the loss on 1 lb. at 11s., and the proportional parts of this couplet are as 2 to 1. But 2 and 1 are the differences between the mean price and that of the teas compared, taken inversely. The parts are 3 lbs. at 3s., 2 lbs. at 8s., 1 lb. at 11s., 6 lbs. at 12s.

If we compare the *first* and *third*, the *second* and *fourth*, the proportional parts will be 2 lbs. at 3s., 3 lbs. at 8s., 6 lbs. at 11s., and 1 lb. at 12s. Hence, the

RULE.—I. *Write the prices of the ingredients in a column, with the mean price on the left, and taking them in pairs, one less and the other greater than the mean price, connect them by a curve line.*

II. *Place the difference between the mean price and that of each ingredient opposite the price with which it is compared. The sum of the differences standing opposite each price is the proportional part of that ingredient.*

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515. How find the proportional parts of a mixture, when the mean price and the price of each ingredient are given?

REM.—Since the results show the proportional parts to be taken, it follows if each is *multiplied* or *divided* by the same number, an endless variety of answers may be obtained.

2D METHOD.—Since the *mean price* is 9s. a pound,

9s. {
1 lb. at 3s. gains 6s.; hence, to gain 1s. takes $\frac{1}{6}$ lb.$=\frac{1}{6}$. lb.
1 lb. at 8s. gains 1s.; hence, to gain 1s. takes 1 lb.$=\frac{6}{6}$ lb.
1 lb. at 11s. loses 2s.; hence, to lose 1s. takes $\frac{1}{2}$ lb.$=\frac{3}{6}$ lb.
1 lb. at 12s. loses 3s.; hence, to lose 1s. takes $\frac{1}{3}$ lb.$=\frac{2}{6}$ lb.

Reducing these results to a common denominator, and using the numerators, the proportional parts are 1 lb. at 3s., 6 lbs. at 8s., 3 lbs. at 11s., and 2 lbs. at 12s. Hence, the

RULE.—*Take the given prices in pairs, one greater, the other less than the mean price, and find how much of each article is required to gain or lose a unit of the mean price, setting the result on the right of the corresponding price.*

Reduce the results to a common denominator, and the numerators will be the proportional parts required.

NOTES.—1. If there are *three* ingredients, compare the *price* of the one which is *greater* or *less* than the *mean* price with each of the others, and take the sum of the two numbers opposite this price.

2. The *reason* for considering the ingredients in *pairs*, one *above*, and the other *below* the mean price, is that the *loss* on one may be counterbalanced by the *gain* on another.

[For Canfield's Method, see Key to New Practical.]

8. A miller bought wheat at \$1.60, \$2.10, and \$2.25 per bushel respectively, and made a mixture worth \$2 a bushel: how much of each did he buy?

9. A refiner wished to mix 4 parcels of gold 15, 18, 21, and 22 carats fine, so that the mixture might be 20 carats fine: what quantity of each must he take?

10. A grocer has three kinds of spices worth 32, 40, and 45 cts. a pound: in what proportion must they be mixed, that the mixture may be worth 38 cts. a pound?

11. A grocer mixed 4 kinds of butter worth 20 cts., 27 cts., 35 cts., and 40 cts. a pound respectively, and sold the mixture at 42 cts. a pound, whereby he made 10 cts. a pound: how much of a kind did he mix?

PROBLEM II.

516. When one ingredient, the Price of each, and the Mean Price of the Mixture are given, to find the other ingredients.

12. How much sugar worth 12, 14, and 21 cts. a pound must be mixed with 12 lbs. at 23 cts. that the mixture may be worth 18 cts. a pound?

18	12	3 lb.
	14	5 "
	21	6 "
	23	4 "

ANALYSIS.—If neither ingredient were limited, the proportional parts would be 3 lbs. at 12 cts., 5 lbs. at 14 cts., 6 lbs. at 21 cts., and 4 lbs. at 23 cts.

But the quantity at 23 cts. is limited to 12 lbs., which is 3 times its difference 4 lbs. Now the ratio of 12 lbs. to 4 lbs. is 3. Multiplying each of the proportional parts found by 3 the result will be 9 lbs., 15 lbs., 18 lbs., and 12 lbs. Hence, the

RULE.—*Find the proportional parts as if the quantity of neither ingredient were limited.* (Art. 515.)

Multiply the parts thus found by the ratio of the given ingredient to its proportional part, and the products will be the corresponding ingredients required.

NOTE.—When the quantities of *two* or *more* ingredients are given, find the *average* value of them, and considering their *sum* as one quantity, proceed as above. (Art. 515.)

13. How much barley at 40 cts., and corn at 80 cts., must be mixed with 10 bu. of oats at 30 cts. and 20 bu. of rye at 60 cts., that the mixture may be 55 cts. a bu.?

SUGGESTION.—The mean value of 10 bu. of oats at 30 cts. and 20 bu. of rye at 60, is 50 cts. a bu. *Ans.* 30 bu. barley, and 24 bu. corn.

14. How much butter at 40, 45, and 50 cts. a pound respectively, must I mix with 30 lbs. at 65 cts. that the mixture may be worth 60 cts. a pound?

15. How many quarts of milk, worth 4 and 6 cts. a quart respectively, must be mixed with 50 quarts of water so that the mixture may be worth 5 cts. a quart?

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516. When one ingredient, the price of each and mean price are given, how find the other ingredients?

PROBLEM III.

517. To find the *Ingredients,* the Price of each, the Quantity mixed, and the Mean Price being given.

16. How much water must be mixed with two kinds of Bourbon costing \$4 and \$6 a gal., to make a mixture of 150 gal. the mean price of which shall be \$3 a gal.?

ANALYSIS.—The price of the water is 0. Disregarding the quantity to be mixed, and proceeding as in Problem I., the proportional parts are 4 g. water, 3 g. at \$4, and 3 g. at \$6, the sum of which is 4 g. + 3 g. + 3 g. = 10 gallons.

But the whole mixture is to be 150 gallons. Now the ratio of 150 g. to 10 g. equals $\frac{150}{10}$, or 15.

Multiplying each of the parts previously obtained by 15, we have 4 gal. × 15 = 60 gal. water; 3 gal. × 15 = 45 gal. at \$4; and 3 gal. × 15 = 45 gal. at \$6. Hence, the

RULE.—*Find the proportional parts without regard to the quantity to be mixed, as in Problem I.*

Multiply each of the proportional parts thus found by the ratio of the given mixture to the sum of these parts, and the several products will be the corresponding ingredients required.

17. A grocer mixed 100 lb. of lard worth 6, 8, and 12 cts. a pound, the mean value of the mixture being 10 cts.: how many pounds of each kind did he take?

18. Having coffees worth 28, 30, 38, and 42 cts. a pound respectively, I wish to mix 200 lbs. in such proportions that the mean value of the mixture shall be 36 cts. a pound: how many pounds of each kind must I take?

19. A grocer wished to mix 4 kinds of petroleum worth 40, 45, 50, and 60 cts. a gal. respectively: how much of each kind must he take to make a mixture of 300 gallons, worth 52 cts. a gallon?

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517. How find the ingredients, when the price of each, the quantity mixed, and the mean price are given?

INVOLUTION.

518. ***Involution*** is finding a *power* of a number.

A ***Power*** is the *product* of a number multiplied into itself. Thus, $2 \times 2 = 4$; $3 \times 3 = 9$, etc., 4 and 9 are powers.

519. Powers are divided into *different degrees;* as, *first, second, third, fourth,* etc. The *name* shows *how many times* the number is taken as a factor to produce the power.

520. The ***First Power*** is the root or number itself.

The ***Second Power*** is the product of a number taken *twice* as a factor, and is called a *square.*

The ***Third Power*** is the product of a number taken *three times* as a factor, and is called a *cube,* etc.

NOTES.—1. The *second* power is called a *square,* because the process of raising a number to the *second* power is similar to that of finding the *area* of a square. (Art. 243.)

2. The *third* power in like manner is called a *cube,* because the process of raising a number to the *third* power is similar to that of finding the contents of a cube. (Art. 249.)

521. Powers are denoted by a small *figure* placed above the number on the right, called the *index* or *exponent;* because it shows *how many times* the number is taken as a factor, to produce the power.

NOTE.—The term *index* (plural *indices*), Latin *indicere,* to *proclaim. Exponent* is from the Latin *exponere,* to *represent.* Thus,

$2^1 = 2$, the first power, which is the number itself.
$2^2 = 2 \times 2$, the *second* power, or *square.*
$2^3 = 2 \times 2 \times 2$, the *third* power, or *cube.*
$2^4 = 2 \times 2 \times 2 \times 2$, the *fourth* power, etc.

522. The expression 2^4 is read, "2 raised to the fourth power, or the fourth power of 2."

1. Read the following: 9^5, 12^7, 25^8, 245^6, 381^{10}, 465^{15}, 1000^{24}.
2. $6^3 \times 7^4$, $25^6 \times 48^5$, $140^8 - 75^3$, $256^{10} \div 97^5$.

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518. What is involution? A power? 519. How are powers divided? 520. What is the first power? The second? Third? *Note.* Why is the second power called a square? Why the third a cube? 521. How are powers denoted?

3. Express the 4th power of 85.
4. Express the 5th power of 348.
5. The 7th power of 340.
6. The 8th power of 561.

523. To raise a Number to any required Power.

7. What is the 4th power of 3?

ANALYSIS.—The *fourth* power is the product of a number into itself taken four times as a factor, and $3 \times 3 \times 3 \times 3 = 81$, the answer required. Hence, the

RULE.—*Multiply the number into itself, till it is taken as many times as a factor as there are units in the index of the required power.*

NOTES.—1. In raising a number to a power, it should be observed that the *number* of *multiplications* is always *one less* than the *number of times* it is taken as a factor; and therefore *one less* than the number of the index. Thus, $4^3 = 4 \times 4 \times 4$, the 4 is taken three times as a factor, but there are only two multiplications.

2. A *decimal* fraction is raised to a power by multiplying it into itself, and pointing off as *many decimals* in each power as there are decimals in the factors employed. Thus, $.1^2 = .01$, $.2^3 = .008$, etc.

3. A *common* fraction is raised to a power by multiplying each term into itself. Thus, $(\frac{3}{4})^2 = \frac{9}{16}$.

4. A *mixed* number should be reduced to an *improper* fraction, or the fractional part to a *decimal;* then proceed as above. Thus, $(2\frac{1}{2})^2 = (\frac{5}{2})^2 = \frac{25}{4}$; or $2\frac{1}{2} = 2.5$ and $(2.5)^2 = 6.25$.

5. All powers of 1 are 1; for $1 \times 1 \times 1$, etc. $= 1$.

Compare the square of the following integers and that of their corresponding decimals:

8. 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90.

9. .5, .6, .7, .8, .9, .01, .02, .03, .04, .05, .06, .07, .08, .09.

Raise the following numbers to the powers indicated:

10. 5^3.
11. 2^6.
12. 132^3.
13. 4^5.
14. 8^4.
15. 25^3.
16. 2.03^3.
17. 4.0003^3.
18. 400.05^3.
19. $\frac{3}{5}^4$.
20. $\frac{7}{8}^3$.
21. $2\frac{1}{4}^4$.

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523. How raise a number to a power? *Note.* In raising a number to a power, how many multiplications are there? How is a decimal raised to a power? A common fraction? A mixed number? 524. How find the product of two or more powers of the same number?

524. To find the Product of two or more Powers of the same Number.

22. What is the product of 4^3 multiplied by 4^2?

ANALYSIS.—$4^3=4\times4\times4$, and $4^2=4\times4$; therefore in the product of $4^3\times4^2$, 4 is taken $3+2$, or 5 times as a factor. But 3 and 2 are the given indices; therefore 4 is taken as many times as a factor as there are *units* in the *indices*. *Ans.* 4^5. Hence, the

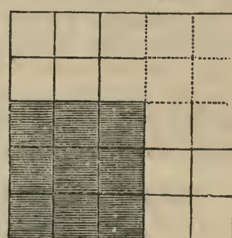
RULE.—*Add the indices, and the sum will be the index of the product.*

23. Mult. 2^3 by 2^2.
24. Mult. 3^4 by 3^3.
25. Mult. 4^3 by 4^4.
26. Mult. 5^4 by 5^2.

FORMATION OF SQUARES.

525. To find the *Square* of a Number in the Terms of its Parts.

1. Find the square of 5 in the terms of the parts 3 and 2.



ANALYSIS.—Let the shaded part of the diagram represent the square of 3;—each side being divided into 3 inches, its contents are equal to 3×3, or 9 sq. in.

The question now is, *what additions* must be made and *how* made, to preserve the *form* of this square, and make it equal to the square of 5.

1st. To preserve the form of the square it is plain equal additions must be made to *two adjacent* sides; for, if made on *one side*, or on *opposite* sides, the figure will no longer be a *square*.

2d. Since 5 is 2 more than 3, it follows that *two rows* of 3 squares each must be added at the top, and 2 rows on one of the adjacent sides, to make its *length* and *breadth* each equal to 5. Now 2 into 3 plus 2 into 3 are 12 squares, or *twice* the product of the two parts 2 and 3.

But the diagram wants 2 times 2 small squares, as represented by the dotted lines, to fill the corner on the right, and 2 times 2 or 4 is the square of the second part. We have then 9 (the sq. of the 1st part), 12 (twice the prod. of the two parts 3 and 2), and 4 (the square of the 2d part). But $9+12+4=25$, the square required.

Again, if 5 is divided into 4 and 1, the square of 4 is 16, twice the prod. of 4 into 1 is 8, and the square of 1 is 1. But $16+8+1=25$.

2. Required the square of 25 in the terms of 20 and 5.

ANALYSIS.—Multiplying 20 by 20 gives 400 (the square of the 1st part); 20×5 plus 20×5 gives 200 (twice the prod. of the two parts); and 5 into 5 gives 25 (the square of the 2d part). Now $400+200+25=625$, or 25^2. Hence, universally,

$$\begin{array}{rr} 25 = & 20+5 \\ 25 & 20+5 \\ \hline 125 & 400+100 \\ 50 & +100+25 \\ \hline 625 = & 400+200+25 \end{array}$$

The *square* of any number expressed in the terms of its parts, *is equal to the square of the first part, plus twice the product of the two parts, plus the square of the second part.*

3. What is the square of 23 in the parts 20 and 3?

4. What is the square of $2\frac{1}{2}$, or $2+\frac{1}{2}$? *Ans.* $6\frac{1}{4}$.

5. What is the square of $\frac{4}{2}$ or $\frac{3}{2}+\frac{1}{2}$? *Ans.* 4.

EVOLUTION.

526. ***Evolution*** is finding a *root* of a number.

A ***Root*** is one of the *equal factors* of a number.

527. *Roots,* like powers, are divided into *degrees;* as, the square, or second root; the cube, or third root; the fourth root, etc.

528. The ***Square Root*** is one of the *two equal factors* of a number. Thus, $5\times5=25$; therefore, 5 is the square root of 25.

529. The ***Cube Root*** is one of the three equal factors of a number. Thus, $3\times3\times3=27$; therefore, 3 is the cube root of 27, etc.

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525. To what is the square of a number equal in the terms of its parts? 526. What is evolution? A root? 528. Square root? 529. Cube root?

530. *Roots* are denoted in *two ways:* 1st. By prefixing to the number the character ($\sqrt{}$), called the *radical sign,* with a figure over it; as $\sqrt[2]{4}$, $\sqrt[3]{8}$.

2d. By a *fractional exponent* placed above the number on the right. Thus, $\sqrt[2]{9}$, or $9^{\frac{1}{2}}$, denotes the sq. root of 9.

NOTES.—1. The figure over the radical sign ($\sqrt{}$) and the denominator of the exponent respectively, denote the *name* of the root.

2. In expressing the *square* root, it is customary to use simply the radical sign ($\sqrt{}$), the 2 being understood. Thus, the expression $\sqrt{25}=5$, is read, "the square root of 25=5."

3. The term *radical* is from the Latin *radix,* root. The sign ($\sqrt{}$) is a corruption of the letter *r,* the initial of *radix.*

531. A ***Perfect Power*** is a number whose exact root can be found.

An ***Imperfect Power*** is a number whose exact root can not be found.

532. A ***Surd*** is the root of an imperfect power. Thus, 5 is an imperfect power, and its square root 2.23+ is a surd.

NOTE.—All *roots* as well as *powers* of 1, are 1.

Read the following expressions:

1. $\sqrt{40}$.	3. $119^{\frac{1}{2}}$.	5. $1.5^{\frac{1}{4}}$.	7. $\sqrt[8]{256}$.	9. $\sqrt[4]{\frac{45}{58}}$.
2. $\sqrt[4]{15}$.	4. $243^{\frac{1}{3}}$.	6. $\sqrt[4]{29}$.	8. $\sqrt[15]{45.7}$.	10. $\sqrt[5]{\frac{125}{278}}$.

11. Express the cube root of 64 both ways; the 4th root of 25; the 7th root of 81; the 10th root of 100.

533. To find how many figures the *Square* of a Number contains.

1st. Take 1 and 9, the *least* and *greatest integer* that can be expressed by *one* figure; also 10 and 99, the least and greatest that can be expressed by *two integral* figures, etc. Squaring these numbers, we have for

The Roots:	1,	9,	10,	99,	100,	999,	etc.
The Squares:	1,	81,	100,	9801,	10000,	998001,	etc.

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530. How are roots denoted? 531. A perfect power? Imperfect? 532. A surd? 533. How many figures has the square of a number?

2d. Take .1 and .9, the *least* and *greatest* decimals that can be expressed by *one* figure; also .01 and .99, the least and greatest that can be expressed by *two* decimal figures, etc. Squaring these, we have for

The Roots:	.1,	.9,	.01,	.99,	.001,	.999,	etc.
The Squares:	.01,	.81,	.0001,	.9801,	.000001,	.998001,	etc.

By inspecting these *roots* and *squares*, we discover that

The square of the number contains twice as many figures as its root, or twice as many less one.

534. To find how many figures the *Square Root* of a Number contains.

Divide the number into periods of two figures each, placing a dot over units' place, another over hundreds, etc. The root will have as many figures as there are periods.

REMARK.—Since the *square* of a number consisting of *tens* and *units*, is equal to the *square* of the *tens*, etc., when a number has *two periods*, it follows that the left hand period must contain the *square* of the *tens* or *first figure* of the root. (Art. 525.)

EXTRACTION OF THE SQUARE ROOT.

535. To extract the *Square Root* of a Number.

1. A man wishes to lay out a garden in the form of a square, which shall contain 625 sq. yards: what will be the length of one side?

OPERATION.

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    6̇25̇(25
    4
45)225
   225
```

ANALYSIS.—Since 625 contains two periods, its root will have *two figures*, and the left hand period contains the *square* of the tens' figure. (Art. 534, *Rem.*)

But the greatest square of 6 is 4, and the root of 4 is 2, which we place on the right for the tens' figure of the root. Now the square of 2 tens or 20 is 400, and 625−400=225. Hence, 225 is twice the product of the tens' figure of the root into the units, plus the square of the units. (Art. 525.)

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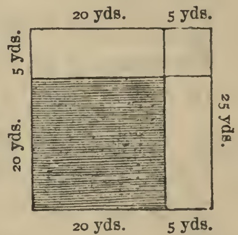
534. How many figures has the square root of a number?

But one of the factors of this product is 2 times 20 or 40; therefore the other factor must be 225 divided by 40; and 225÷40=5, the factor required. (Art. 93.) Taking 2 times 20 into 5=200 (i. e., twice the product of the tens into the units' figure of the root) from 225, leaves 25 for the *square* of the units' figure, the square root of which is 5 Hence 25 is the square root of 625, and is therefore the length of one side of the garden.



2d Analysis.—Let the shaded part of the diagram be the square of 2 tens, the first figure of the root; then 20 × 20, or 400 sq. yds., will be its contents. Subtracting the contents from the given area, we have 625−400=225 sq. yds. to be added to it. To preserve its form, the addition must be made equally to two adjacent sides. The question now is, what is the width of the addition.

Since the length of the plot is 20 yds., adding a strip 1 yard wide to two sides will take 20+20 or 40 sq. yds. Now if 40 sq. yds. will add a strip 1 yard wide to the plot, 225 sq. yds. will add a strip as many yds. wide as 40 is contained times in 225; and 40 is contained in 225, 5 times and 25 over.

That is, since the addition is to be made on two sides, we double the root or length of one side for a trial divisor, and find it is contained in 225, 5 times, which shows the width of the addition to be 5 yards.

Now the length of each side addition being 20 yds., and the width 5 yds., the area of both equals 20 × 5 + 20 × 5, or 40 × 5=200 sq. yards. But there is a vacancy at the upper corner on the right, whose length and breadth are 5 yds. each; hence its area=5 × 5, or 25 sq. yards; and 200 sq. yd. + 25 sq. yd.=225 sq. yd. For the sake of finding the area of the two side additions and that of the corner at the same time, we place the quotient 5 on the right of the root already found, and also on the right of the trial divisor to complete it. Multiplying the divisor thus completed by 5, the figure last placed in the root, we have 45 × 5=225 sq. yds. Subtracting this product from the dividend, nothing remains. Therefore, etc. Hence, the

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535. What is the first step in extracting the square root of a number? The second? Third? Fourth? How proved? *Note.* If the trial divisor is not contained in the dividend, what must be done? If there is a remainder after the root of the last period is found, what? How many decimals does the root of a decimal fraction have?

RULE.—I. *Divide the number into periods of two figures each, putting a dot over units, then over every second figure towards the left in whole numbers, and towards the right in decimals.*

II. *Find the greatest square in the left hand period, and place its root on the right. Subtract this square from the period, and to the right of the remainder bring down the next period for a dividend.*

III. *Double the part of the root thus found for a trial divisor; and finding how many times it is contained in the dividend, excepting the right hand figure, annex the quotient both to the root and to the divisor.*

IV. *Multiply the divisor thus increased by this last figure placed in the root, subtract the product, and bring down the next period.*

V. *Double the right hand figure of the last divisor, and proceed as before, till the root of all the periods is found.*

PROOF.—*Multiply the root into itself.* (Art. 528.)

NOTES.—1. If the trial divisor is not contained in the dividend, *annex a cipher* both to the root and to the divisor, and bring down the next period.

2. Since the product of the trial divisor into the quotient figure cannot *exceed* the dividend, allowance must be made for carrying, if the product of this figure into itself exceeds 9.

3. It sometimes happens that the remainder is *larger* than the divisor; but it does not necessarily follow from this that the *figure* in the root is too *small*, as in simple division.

4. If there is a *remainder* after the root of the last period is found, annex *periods of ciphers*, and proceed as before. The figures of the root thus obtained will be *decimals*.

5. The square root of a *decimal fraction* is found in the same way as that of a whole number; and the root will have as many *decimal figures* as there are *periods* of decimals in the given number.

6. The *left hand* period in *whole numbers* may have but *one* figure; but in *decimals*, each period must have *two* figures. (Art. 533.) Hence, if the number has but *one decimal* figure, or an *odd number* of decimals, a *cipher* must be annexed to complete the period.

536. REASONS.—1. Dividing the number into periods of two figures each, shows *how many figures* the root will contain, and enables us to find its *first figure*. For, the *left hand period* contains the *square* of this figure, and from the *square* the *root* is easily found. (Art. 534, *Rem.*)

2. Subtracting the *square* of the first figure of the root from the *left hand period*, shows what is left for the other figures of the root.

3. The object of *doubling* the first figure of the root, and dividing the remainder by it as a trial divisor, is to find the next figure of the root. The remainder contains *twice* the *product* of the *tens* into the *units;* consequently, dividing this product by double the tens' factor, the quotient will be the *other factor* or *units' figure* of the root.

4. Or, referring to the diagram, it is *doubled* because the *remainder* must be added to *two sides*, to preserve the *form* of the square.

5. The right hand figure of the dividend is *excepted*, to counterbalance the omission of the *cipher*, which properly belongs on the *right* of the trial divisor.

6. The quotient figure is placed in the root; it is also annexed to the trial divisor to *complete* it. The divisor thus completed is multiplied by the second figure of the root to *find the contents* of the additions thus made.

The reasons for the steps in obtaining other figures of the root may be shown in a similar manner.

2. What is the square root of 381.0304? *Ans.* 19.52.

SUGGESTION.—Since the number contains decimals, we begin at the *units' place*, and counting both ways, have four periods; as, 381.0304. The root will therefore have 4 figures. But there are *two periods of decimals;* hence we point off two decimals in the root.

3. What is the square root of 1012036? *Ans.* 1006.

4. What is the square root of 2? *Ans.* 1.41421+.

Extract the square root of the following numbers:

5. 182329.	11. .1681.	17. 5.	23. 19.5364.
6. 516961.	12. .725.	18. 7.	24. 3283.29.
7. 595984.	13. .1261.	19. 8.	25. 87.65.
8. 3.580.	14. 2.6752.	20. 10.	26. 123456789.
9. .4096.	15. 4826.75.	21. 11.	27. 61723020.96.
10. .120409.	16. 452.634.	22. 12.	28. 9754.60423716.

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536. Why divide the number into periods of two figures each? Why subtract the square of the first figure from the period? Why double the first figure of the root for a trial divisor? Why omit the right hand figure of the dividend? Why place the quotient figure on the right of the trial divisor? Why multiply the trial divisor thus completed by the figure last placed in the root?

537. To find the *Square Root* of a Common Fraction.

I. *When the numerator and denominator are both perfect squares, or can be reduced to such, extract the square root of each term separately.*

II. *When they are imperfect squares, reduce them to decimals, and proceed as above.*

NOTE.—To find the square root of a *mixed number*, reduce it to an improper fraction, and proceed as before.

29. What is the square root of $\frac{8}{18}$?

ANALYSIS.—$\frac{8}{18}=\frac{4}{9}$, and $\sqrt{\frac{4}{9}}=\frac{2}{3}$, *Ans.*

30. What is the square root of $\frac{3}{5}$? *Ans.* .7745 +.

31. What is the square root of $1\frac{3}{8}$? *Ans.* 1.1726 +.

Find the square root of the following fractions:

32. $\frac{144}{256}$.	35. $6\frac{2}{5}$.	38. $\frac{256}{625}$.	41. $1\frac{3}{80}$.
33. $\frac{5}{9}$.	36. $13\frac{1}{5}$.	39. $\frac{576}{900}$.	42. $27\frac{9}{16}$.
34. $\frac{9}{12}$.	37. $17\frac{3}{8}$.	40. $\frac{341}{467}$.	43. $51\frac{21}{25}$.

APPLICATIONS.

538. To find the *Side of a Square* equal in area to a given Surface.

1. Find the side of a square farm containing 40 acres.

ANALYSIS.—In 1 acre there are 160 sq. rods, and in 40 acres, 40 times 160, or 6400 sq. rods. The $\sqrt{6400}=$ 80 r. Therefore the side of the farm is 80 linear rods. Hence, the

OPERATION.

40 A. × 160 = 6400 sq. r.

$\sqrt{6400}$ = 80 r. *Ans.*

RULE.—I. *Extract the square root of the given surface.*

NOTE.—The root is in *linear units* of the same name as the given surface.

2. What is the side of a square tract of land containing 1102 acres 80 sq. rods?

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537. How find the square root of a common fraction? *Note.* Of a mixed number? 538. How find the side of a square equal to a given surface?

3. How many rods of fencing does it require to inclose a square farm which contains 122 acres 30 sq. rods?

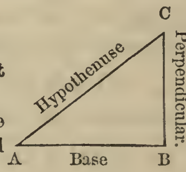
4. A bought 14161 fruit trees, which he planted so as to form a square: how many trees did he put in a row?

5. A general has an army of 56644 men: how many must he place in rank and file to form them into a square?

539. A ***Triangle*** is a figure having *three sides* and *three angles*.

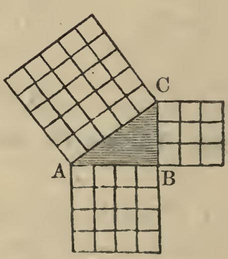
A ***Right-angled Triangle*** is one that contains a *right angle*. (Art. 260.)

The side *opposite* the right angle is called the *hypothenuse;* the other two sides the *base* and *perpendicular*.



540. *The square described on the hypothenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides.**

541. The truth of this principle may be illustrated thus: Take any *right-angled* triangle A B C; let the hypothenuse *h*, be 5 in., the base *b*, 4 in., and the perpendicular *p*, 3 in. It will be seen that the square of *h* contains 25 sq. in., the square of *b* 16 sq. in., and the square of *p* 9 sq. in. Now $25=16+9$, which accords with the proposition. In like manner it may be shown that the principle is true of all right-angled triangles. Hence,



542. To find the *Hypothenuse*, the Base and Perpendicular being given.

To the square of the base add the square of the perpendicular, and extract the square root of their sum.

539. What is a triangle? A right-angled triangle? Draw a right-angled triangle? The side opposite the right-angle called? The other two sides? 540. To what is the square of the hypothenuse equal? 541. Illustrate this principle by a figure? 542. How find the hypothenuse when the base and perpendicular are given?

* Thomson's Geometry, IV, 11; Euclid, I, 47.

543. To find the *Base*, the Hypothenuse and Perpendicular being given.

From the square of the hypothenuse take the square of the perpendicular, and extract the square root of the remainder.

544. To find the *Perpendicular*, the Hypothenuse and Base being given.

From the square of the hypothenuse take the square of the base, and extract the square root of the remainder.

NOTE.—The pupil should draw figures corresponding with the conditions of the following problems, and indicate the parts *given:*

6. The perpendicular height of a flag-staff is 36 ft.: what length of line is required to reach from its top to a point in a level surface 48 ft. from its base?

SOLUTION.—The square of the base . . $=48\times48=2304$
" " perpendicular $=36\times36=1296$
The square root of their sum $=\sqrt{3600}=60$ ft. *Ans.*

7. The hypothenuse of a right-angled triangle is 135 yds., the perpendicular 81 yds.: what is the base?

8. One side of a rectangular field is 40 rods, and the distance between its opposite corners 50 rods: what is the length of the other side?

9. Two vessels sail from the same point, one going due south 360 miles, the other due east 250 miles: how far apart were they then?

10. The height of a tree on the bank of a river is 100 ft., and a line stretching from its top to the opposite side is 144 ft.: what is the width of the river?

11. The side of a square room is 40 feet: what is the distance between its opposite corners on the floor?

12. A tree was broken 35 feet from its root, and struck the ground 21 ft. from its base: what was the height of the tree?

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543. How find the base when the other two sides are given? 544. How find the perpendicular when the other two sides are given?

SIMILAR FIGURES.

545. ***Similar Figures*** are those which have the *same form*, and their *like dimensions proportional.*

NOTES.—1. All *circles*, of whatever magnitude, are similar.

2. All *squares, equilateral triangles,* and *regular polygons* are similar. And, universally,

All *rectilinear figures* are similar, when their several angles are *equal* each to each, and their *like dimensions proportional.*

3. The like dimensions of circles are their *diameters, radii,* and *circumferences.*

546. The *areas* of *similar figures* are to each other as the *squares* of their like dimensions. And,

Conversely, the *like dimensions* of *similar figures* are to each other as the *square roots* of their areas.

13. If one side of a triangle is 12 rods, and its area 72 sq. rods, what is the area of a similar triangle, the corresponding side of which is 8 rods?

SOLUTION.—$(12)^2 : (8)^2 :: 72$: *Ans.*, or 32 sq. rods.

14. If one side of a triangle containing 36 sq. rods is 8 rods, what is the length of a corresponding side of a similar triangle which contains 81 sq. rods?

SOLUTION.—$\sqrt{36} : \sqrt{81} :: 8$: *Ans.*, or 12 rods.

15. If a pipe 2 inches in diameter will fill a cistern in 42 min., in what time will a pipe 7 in. in diameter fill it?

547. To find a *Mean Proportional* between two Numbers.

16. What is the mean proportional between 4 and 16?

ANALYSIS.—When three numbers are proportional, the product of the extremes is equal to the *square* of the *mean.* (Arts. 487, 490.) But $4 \times 16 = 64$; and $\sqrt{64} = 8$. *Ans.* Hence, the

RULE.—*Extract the square root of their product.*

Find the mean proportional between the following:

17. 4 and 36.	19. 56 and 72.	21. $\frac{16}{49}$ and $\frac{4}{36}$.
18. 36 and 81.	20. .49 and 6.25.	22. $\frac{81}{100}$ and $\frac{64}{144}$.

545. What are similar figures? 546. How are their areas to each other?

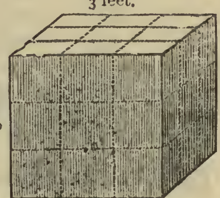
FORMATION OF CUBES.

548. To find the *Cube* of a Number in the terms of its parts.

1. Find the cube of 32 in the terms of the parts 30 and 2.

ANALYSIS.—$32=30+2$, and $32^3=(30+2)\times(30+2)\times(30+2.)$

$$\begin{array}{rl} 32 = & 30+2 \\ \hline 96 = & 30^2+(30\times 2) \\ 64 = & \quad (30\times 2)+2^2 \\ \hline 1024 = & 30^2+2\times(30\times 2)+2^2 \\ 32 = & 30+2 \\ \hline 3072 = & 30^3+2\times(30^2\times 2)+(30\times 2^2) \\ 2048 = & \quad (30^2\times 2)+2\times(30\times 2^2)+2^3 \\ \hline 32768 = & 30^3+3\times(30^2\times 2)+3\times(30\times 2^2)+2^3. \end{array}$$



$3\times 3\times 3=27$ ft.

Or thus: Let the diagram represent a cube whose side is 30 ft.; its contents= the cube of 3 tens, or 2700 cu. ft.

What additions must be made to this cube, and *how* made, to preserve its *form*, and make it equal to the cube of 32.

1st. To preserve its cubical form, the additions must be equally made on *three adjacent* sides; as the top, front, and right.

2d. Since 32 is 2 more than 30, it follows that this cube must be made 2 ft. longer, 2 ft. wider, and 2 ft. higher, that its length, breadth, and thickness may each be 32 ft.

But as the side of this cube is 30 ft., the *contents* of each of these additions must be equal to the square of the tens (30^2) into 2, the units, and their *sum* must be 3 times $(30^2\times 2)=3\times 900\times 2=5400$ cu. ft.

But there are three vacancies along the edges of the cube adjacent to the additions. Each of these vacancies is 30 ft. long, 2 ft. wide, and 2 ft. thick; hence, the contents of each equals 30×2^2, and the sum of their contents equals 3 times the tens into the square of the units $=3$ times $(30\times 2^2)=3\times 30\times 4=360$ cu. ft. But there is still another vacancy at the junction of the corner additions, whose length, breadth, and thickness are each 2 ft., and whose contents are equal to $2^3=2\times 2\times 2=8$. The cube is now complete. Therefore $32^3=$ 27000 $(30^3)+5400$ (3 times $30^2\times 2)+360$ (3 times $30\times 2^2)+8$ $(2^3)=$ 32768 cu. ft. In like manner it may be shown that,

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548. To what is the cube of a number consisting of tens and units equal?

The cube of any number consisting of tens and units is equal to the cube of the tens, plus 3 times the square of the tens into the units, plus 3 times the tens into the square of the units, plus the cube of the units.

549. To find how many figures the *Cube* of a Number contains.

1st. Take 1 and 9, also 10 and 99, 100 and 999, etc., the least and greatest integers that can be expressed by *one, two, three*, etc., figures.

2d. In like manner take .1 and .9, also .01 and .99, etc., the least and greatest decimals that can be expressed by *one, two*, etc., *decimal figures*. Cubing these, we have

The root	1 and	$1^3=1$,	- - - -	.1 and	$.1^3=.1$
"	9 "	$9^3=729$,	- - -	.9 "	$.9^3=.729$
"	10 "	$10^3=1000$,	- - -	.01 "	$.01^3=.000001$
"	99 "	$99^3=970299$,	- -	.99 "	$.99^3=.970299$
"	100 "	$100^3=1000000$,	- -	.001 "	$.001^3=.000000001$
"	999 "	$999^3=997002999$,	-	.999 "	$.999^3=.997002999$.

By comparing these roots and their cubes, we discover that

The cube of a number cannot have more than three times as many figures as its root, nor but two less. Hence,

550. To find how many figures the *Cube Root* contains.

Divide the number into periods of three figures each, putting a dot over units, then over every third figure towards the left in whole numbers, and towards the right in decimals.

REMARKS.—1. Since the *cube* of a number consisting of *tens* and *units* is equal to the *cube of the tens*, plus 3 times the *square* of the tens into the units, etc., when a number has *two periods*, it follows that the left hand period must contain the *cube* of the *tens*, or *first figure* of the root.

2. The left hand period in *whole* numbers may be *incomplete*, having only *one* or *two* figures; but in *decimals* each period *must* always have *three* figures. Hence, if the decimal figures in a given number are *less* than three, *annex ciphers* to complete the period.

How many figures in the cube root of the following:

2. 340566.	4. 576.453.	6. 32.7561.
3. 1467.	5. 5.7321.	7. .456785.

549. How many figures has the cube of a number?

EXTRACTION OF THE CUBE ROOT.

551. To extract the *Cube Root* of a Number.

1. A man having 32768 marble blocks, each being a cubic foot, wishes to arrange them into a single cube: what must be its side?

OPERATION.

```
      32768(32
      27
2700|5768
 180|
   4|
2884|5768
```

ANALYSIS.—Since 32768 contains two periods, we know its root will have two figures; also that the left hand period contains the *cube* of the *tens* or *first* figure of the root. (Art. 550, *Rem.*)

The greatest cube in 32 is 27, and its root is 3, which we place on the right for the tens or first figure of the root. Subtracting its cube from the first period, and bringing down the next period to the right of the remainder, we have 5768, which by the formation of the cube is equal to 3 times the square of the tens' figure into the units, plus 3 times the tens into the square of the units, plus the cube of the units. We therefore place 3 times the square of the tens or 2700, on the left of the dividend for a trial divisor; and dividing, place the quotient 2 on the right for the units' figure of the root.

To complete the divisor, we add to it 3 times 30 the tens into 2 units=180; also 4, the square of the units, making it 2884. Multiplying the divisor thus completed by the units' figure 2, we have $2884 \times 2 = 5768$, the same as the dividend. *Ans.* 32 ft.

Or thus: Let the cube of 30, the tens of the root, be represented by the large cube in the set of cubical blocks.* The remainder 5768, is to be added equally to three adjacent sides of this cube.

To ascertain the thickness of these side additions, we form a trial divisor by squaring 3, the first figure of the root, with a cipher annexed, for the area of one side of this cube, and multiply this square by 3 for the three side additions. Now $30^2 = 30 \times 30 = 900$; and $900 \times 3 = 2700$, the trial divisor. Dividing 5768 by 2700, the quotient is 2, which shows that the side additions are to be 2 ft. thick, and is placed on the right for the units' figure of the root.

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551. The first step in extracting the cube root? The second? Third? Fourth? Fifth? *Note.* If the trial divisor is not contained in the dividend, how proceed? If there is a remainder after the root of the last period is found, how?

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* Every school in which the cube root is taught is presumed to be furnished with a set of Cubical Blocks.

To represent these additions, place the corresponding layers on the top, front, and right of the large cube. But we discover three vacancies along the edges of the large cube, each of which is 30 ft. long, 2 ft. wide, and 2 ft. thick. Filling these vacancies with the corresponding rectangular blocks, we discover another vacancy at the junction of the corners just filled, whose length, breadth, and thickness are each 2 ft. This is filled by the small cube.

To complete the trial divisor, we add the area of one side of each of the corner additions, viz., $30 \times 2 \times 3$, or 180 sq. ft., also the area of one side of the small cube$=2 \times 2$, or 4 sq. ft. Now $2700+180+4=2884$. The divisor is now composed of the area of 3 sides of the large cube, plus the area of one side of each of the corner additions, plus the area of one side of the small cube, and is complete.

Finally, to ascertain the contents of the several additions, we multiply the divisor thus completed by 2, the last figure of the root; and $2884 \times 2=5768$. (Art. 249.) Subtracting the product from the dividend, nothing remains. Hence, the

RULE.—I. *Divide the number into periods of three figures each, putting a dot over units, then over every third figure towards the left in whole numbers, and towards the right in decimals.*

II. *Find the greatest cube in the left hand period, and place its root on the right. Subtract its cube from the period, and to the right of the remainder bring down the next period for a dividend.*

III. *Multiply the square of the root thus found with a cipher annexed, by three, for a trial divisor; and finding how many times it is contained in the dividend, write the quotient for the second figure of the root.*

IV. *To complete the trial divisor, add to it three times the product of the root previously found with a cipher annexed, into the second root figure, also the square of the second root figure.*

V. *Multiply the divisor thus completed by the last figure placed in the root. Subtract the product from the dividend; and to the right of the remainder bring down the next period for a new dividend. Find a new trial divisor, as before, and thus proceed till the root of the last period is found.*

NOTES.—1. If a trial divisor is *not contained* in the dividend, put a *cipher* in the root, *two ciphers* on the right of the divisor, and bring down the next period.

2. If the product of the divisor completed into the figure last placed in the root *exceeds* the dividend, the root figure is too large. Sometimes the remainder is *larger* than the divisor completed; but it does not necessarily follow that the root figure is *too small.*

3. When there are three or more periods in the given number, the first, second, and subsequent trial divisors are found in the same manner as when there are only two. That is, disregarding its true local value, we simply multiply the square of the root already found with a cipher annexed, by 3, etc.

4. If there is a *remainder* after the root of the last period is found, annex periods of ciphers, and proceed as before. The root figures thus obtained will be *decimals.*

5. The cube root of a *decimal* fraction is found in the same way as that of a whole number; and will have as many decimal figures as there are periods of decimals in the number. (Art. 549.)

552. REASONS.—1. Dividing the number into *periods* shows how many figures the root contains, and enables us to find the *first figure* of the root. For, the left hand period contains the *cube of the first figure* of the root. (Art. 550.)

2. The object of the trial divisor is to find the *next figure* of the root, or the *thickness* of the side additions. The root is *squared* to find the *area of one side* of the cube whose root is found, the cipher being annexed because the first figure is *tens.* This square is multiplied by 3, because the additions are to be made to *three sides.*

3. The root previously found is multiplied by this second figure to find the area of a side of one of the vacancies along the edges of the cube already found. This product is multiplied by 3, because there are three of these vacancies; and the product is placed under the trial divisor as a correction. The object of squaring the *second figure* of the root is to find the *area of one side* of the cubical vacancy at the junction of the corner vacancies, and with the other correction this is added to the trial divisor to complete it.

4. The divisor thus completed is multiplied by the second figure of the root to find the *contents of the several additions* now made.

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552. Why divide the number into periods of three figures? What is the object of a trial divisor? Why square the root already found? Why annex a cipher to it? Why multiply this square by 3? Why is the root previously found multiplied by the second figure of the root? Why multiply this product by 3? Why square the second figure of the root? Why multiply the divisor when completed by the second figure of the root?

2. What is the cube root of 130241.3?

ANALYSIS.—Having completed the period of decimals by annexing two ciphers, we find the first figure of the root as above. Bringing down the next period, the dividend is 5241. The trial divisor 7500 is not contained in the dividend; therefore placing a cipher in the root and two on the right of the divisor, we bring down the next period, and proceed as before.

OPERATION.

	130241.300(50.6 +
	125
750000	5241.300
9000	
36	
759036	4554216
	687084 Rem.

3. Cube root of 614125?
4. Cube root of 84.604?
5. Cube root of 373248?
6. Cube root of 3?
7. Cube root of 21.024576?
8. Cube root of 17?

9. What is the cube root of 705919947264?

10. What is the cube root of .253395799?

11. What is the side of a cube which contains 628568 cu. yards?

NOTE.—The root is in *linear units* of the same name as the given contents.

12. What is the side of a cube equal to a pile of wood 40 ft. long, 15 ft. wide, and 6 ft. high?

13. What is the side of a cubical bin which will hold 1000 bu. of corn; allowing 2150.4 cu. in. to a bushel?

553. To find the Cube Root of a *Common Fraction.*

If the numerator and denominator are perfect cubes, or can be reduced to such, extract the cube root of each.

Or, reduce the fraction to a decimal, and proceed as before.

NOTE.—Reduce *mixed numbers* to *improper fractions,* etc.

14. What is the cube root of $\frac{54}{128}$? *Ans.* $\frac{3}{4}$.
15. Cube root of $\frac{3}{7}$? *Ans.* .75 +.
16. Cube root of $\frac{729}{1750}$?
17. Cube root of $\frac{12167}{15625}$?
18. Cube root of $\frac{729}{1728}$?
19. Cube root of $81\frac{5}{8}$?

553. How find the cube root of a common fraction? *Note.* Of a mixed number?

APPLICATIONS.

554. ***Similar Solids*** are those which have the *same form*, and *their like dimensions proportional.*

NOTES.—1. The like dimensions of *spheres* are their *diameters, radii*, and *circumferences;* those of cubes are *their* sides.

2. The like dimensions of *cylinders* and *cones* are their *altitudes*, and the *diameters* or the *circumferences* of their bases.

3. *Pyramids* are similar, when their *bases* are similar polygons, and their *altitudes* proportional.

4. *Polyhedrons* (i. e., solids included by any number of plane faces) are similar, when they are contained by the *same number* of similar *polygons*, and all their *solid angles* are *equal* each to each.

555. The *contents* of *similar solids* are to each other as the *cubes* of their *like dimensions;* and,

Conversely, the *like dimensions* of *similar solids* are as the *cube roots* of their contents.

1. If the side of a cubical cistern containing 1728 cu. in. is 12 in., what are the contents of a similar cistern whose side is 2 ft. *Ans.* $1^3 : 2^3 :: 1728$ cu. in. : con., or 13824 cu. in.

2. If the side of a certain mound containing 74088 cu. ft. is 84 ft., what is the side of a similar mound which contains 17576 cu. ft.?

3. If a globe 4 in. in diameter weighs 9 lbs, what is the weight of a globe 8 in. in diameter? *Ans.* 72 lbs

4. If 8 cubic piles of wood, the side of each being 8 ft., were consolidated into one cubic pile, what would be the length of its side?

5. If a pyramid 60 ft. high contains 12500 cu. ft., what are the contents of a similar pyramid whose height is 20 ft.?

6. If a conical stack of hay 15 ft. high contains 6 tons, what is the weight of a similar stack whose height is 12 ft.?

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554. What are similar solids? *Note.* What are like dimensions of spheres? Of cylinders and cones? Of pyramids? 555. What relation have similar solids?

ARITHMETICAL PROGRESSION.

556. An ***Arithmetical Progression*** is a series of numbers which increase or decrease by a *common difference.*

NOTE.—The numbers forming the series are called the *Terms.* The *first* and *last* terms are the *Extremes;* the *intermediate* terms the *Means.* (Arts. 476, 489.)

557. The ***Common Difference*** is the difference between the successive terms.

558. An ***Ascending Series*** is one in which the successive terms increase; as, 2, 4, 6, 8, 10, etc., the common difference being 2.

A ***Descending Series*** is one in which the successive terms decrease; as, 15, 12, 9, 6, etc., the common difference being 3.

559. In arithmetical progression there are *five parts* or *elements* to be considered, viz.: the *first term,* the *last term,* the *number of terms,* the *common difference,* and the *sum* of all the terms. These parts are so related to each other, that if any three of them are given, the other two may be found.

560. To find the *Last Term,* the First Term, the Number of Terms, and the Common Difference being given.

1. Required the last term of the ascending series having 7 terms, its first term being 3, and its common difference 2.

ANALYSIS.—From the definition, each succeeding term is found by *adding the common difference* to the preceding. The series is:

$3, \ 3+2, \ 3+(2+2), \ 3+(2+2+2), \ 3+(2+2+2+2)$, etc. Or
$3, \ 3+2, \ 3+(2\times 2), \ 3+(2\times 3), \ 3+(2\times 4)$, etc. That is,

561. The *last term* is equal to the *first* term, *increased* by the *product* of the common difference into the number of terms *less* 1. Hence, the

RULE.—*Multiply the common difference by the number of terms less* 1, *and add the product to the first term.*

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556. What is an arithmetical progression? *Note.* The first and last terms called? The intervening? 557. The common difference? 558. An ascending series? Descending? 560. How find the last term, when the first term, the number of terms, and the common difference are given?

NOTES.—1. In a *descending* series the product must be *subtracted* from the first term; for, in this case each succeeding term is found by subtracting the common difference from the preceding terms.

2. Any term in a series may be found by the preceding rule. For, the series may be supposed to stop at any term, and that may be considered, for the time, as the last.

3. If the *last term* is given, and the first term required, *invert the order of the terms, and proceed as above.*

2. If the first term of an ascending series is 5, the common difference 3, and the number of terms 11, what is the last term? *Ans.* 35.

3. The first term of a descending series is 35, the common difference 3, and the number of terms 10: what is the last?

4. The last term of an ascending series is 77, the number of terms 19, and the common difference 3: what is the first term? *Ans.* 23.

5. What is the amount of $150, at 7% simple interest, for 20 years?

562. To find the *Number of Terms,* the Extremes, and the Common Difference being given.

6. The extremes of an arithmetical series are 4 and 37, and the common difference 3: what is the number of terms?

ANALYSIS.—The last term of a series is equal to the first term *increased* or *diminished* by the product of the common difference into the number of terms *less* 1. (Art. 561.) Therefore 37−4, or 33, is the product of the common difference 3, into the number of terms less 1. Consequently 33÷3 or 11, must be the number of terms less 1; and 11+1, or 12, is the answer required. (Art. 93.) Hence, the

RULE.—*Divide the difference of the extremes by the common difference, and add* 1 *to the quotient.*

7. The youngest child of a family is 1 year, the oldest 21, and the common difference of their ages 2 y.: how many children in the family?

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562. How find the number of terms, when the extremes and common difference are given?

563. To find the *Common Difference,* the Extremes and the Number of Terms being given.

8. The extremes of a series are 3 and 21, and the number of terms is 10: what is the common difference?

ANALYSIS.—The difference of the extremes $21-3=18$, is the product of the number of terms *less* 1 into the common difference, and $10-1$, or 9, is the number of terms less 1; therefore $18\div9$, or 2, is the common difference required. (Art. 93.) Hence, the

RULE.—*Divide the difference of the extremes by the number of terms less* 1.

9. The ages of 7 sons form an arithmetical series, the youngest being 2, and the eldest 20 years: what is the difference of their ages?

564. To find the *Sum* of all the Terms, the Extremes and the Number of Terms being given.

10. Required the sum of the series having 7 terms, the extremes being 3 and 15.

ANALYSIS.—(1.) The series is 3, 5, 7, 9, 11, 13, 15.
(2.) Inverting the same, 15, 13, 11, 9, 7, 5, 3.
(3.) Adding (1.) and (2.), $18+18+18+18+18+18+18=$ twice the sum.
(4.) Dividing (3.) by 2, $9+9+9+9+9+9+9=63$, the sum.

By inspecting these series, we discover that half the sum of the extremes is equal to the average value of the terms. Hence, the

RULE.—*Multiply half the sum of the extremes by the number of terms.*

REMARK.—From the preceding illustration we also see that,
The *sum* of the *extremes* is equal to the sum of any *two terms equidistant* from them; or, to *twice* the *sum* of the *middle* term, if the number of terms *be odd.*

11. How many strokes does a common clock strike in 12 hours?

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563. How find the common difference when the extremes and number of terms are given? 564. How find the sum of all the terms, when the extremes and number of terms are given?

GEOMETRICAL PROGRESSION.

565. A ***Geometrical Progression*** is a series of numbers which increase or decrease by a *common ratio.*

NOTE.—The series is called *Ascending* or *Descending,* according as the terms *increase* or *decrease.* (Art. 558.)

566. In Geometrical Progression there are also *five parts* or *elements* to be considered, viz.: the *first* term, the *last* term, the *number* of terms, the *ratio,* and the *sum* of all the terms.

567. To find the *Last Term,* the First Term, the Ratio, and the Number of Terms being given.

1. Required the last term of an ascending series having 6 terms, the first term being 3, and the ratio 2.

ANALYSIS.—From the definition, the series is

$3,\ 3\times 2,\ 3\times(2\times 2),\ 3\times(2\times 2\times 2),\ 3\times(2\times 2\times 2\times 2)$, etc. Or
$3,\ 3\times 2,\ 3\times 2^2,\ 3\times 2^3,\ 3\times 2^4$, etc. That is,

Each successive term is equal to the first term multiplied by the ratio raised to a power whose index is *one* less than the *number* of the term. Hence, the

RULE.—*Multiply the first term by that power of the ratio whose index is 1 less than the number of terms.*

NOTES.—1. Any term in a series may be found by the preceding rule. For, the series may be supposed to stop at that term.

2. If the last term is given and the first required, *invert* the *order* of *the terms,* and *proceed as above.*

3. The preceding rule is applicable to *Compound Interest;* the *principal* being the first term of the series; the *amount* of $1 for 1 year the ratio; and the *number of years* plus 1, the number of terms.

2. A father promised his son 2 cts. for the first example he solved, 4 cts. for the second, 8 cts. for the third, etc.: what would the son receive for the tenth example?

3. What is the amount of $1500 for 5 years, at 6% compound interest? Of $2000 for 6 years, at 7%?

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565. What is a geometrical progression? 567. How find the last term, when the first term, the ratio, and number of terms are given?

568. To find the *Sum of all the Terms,* the Extremes and Ratio being given.

4. Required the sum of the series whose first and last terms are 2 and 162, and the ratio 3.

ANALYSIS.—Since each succeeding term is found by multiplying the preceding term by the ratio, the series is 2, 6, 18, 54, 162.

(1.) The sum of the series, $=2+6+18+54+162$.
(2.) Multiplying by 3, $= \quad 6+18+54+162+486$.
(3.) Subt. (1.) from (2.), $486-2=484$, or twice the sum.

Therefore $484 \div 2 = 242$, the sum required. But 486, the last term of the second series, is the product of 162 (the last term of the given series) into the ratio 3; the difference between this product and the first term is $486-2$ or 484, and the divisor 2 is the ratio $3-1$. Hence, the

RULE.—*Multiply the last term by the ratio, and divide the difference between this product and the first term by the ratio less* 1.

5. The first term is 4, the ratio 3, and the last term 972: what is the sum of the terms?

6. What sum can be paid by 12 instalments; the first being $1, the second $2, etc., in a geometrical series?

569. To find the *Sum* of a Descending Infinite Series, the First Term and Ratio being given.

REMARK.—In a *descending* infinite series the last term being *infinitely small,* is regarded as 0. Hence, the

RULE.—*Divide the first term by the difference between the ratio and* 1, *and the quotient will be the sum required.*

7. What is the sum of the series $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, continued to infinity, the ratio being $\frac{1}{2}$? *Ans.* $1\frac{1}{3}$.

NOTE.—The preceding problems in Arithmetical and Geometrical Progressions embrace their ordinary applications. The others involve principles with which the pupil is not supposed to be acquainted.

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568. How find the sum of the terms, when the extremes and ratio are given?
569. How find the sum of an infinite descending series, when the first term and ratio are given?

MENSURATION.

570. ***Mensuration*** is the measurement of magnitude.

571. ***Magnitude*** is that which has one or more of the three dimensions, *length, breadth,* or *thickness;* as, lines, surfaces, and solids.

A ***Line*** is that which has *length* without *breadth.*

A ***Surface*** is that which has *length* and *breadth,* without *thickness.*

A ***Solid*** is that which has *length, breadth,* and *thickness.*

572. A ***Quadrilateral Figure*** is one which has *four sides* and *four angles.*

NOTES.—1. If all its sides are straight lines it is *rectilinear.*

2. If all its angles are *right angles* it is *rectangular.*

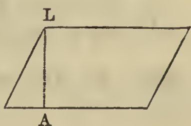
3. Figures which have more than *four* sides are called *Polygons.*

573. Quadrilateral figures are commonly divided into the rectangle, square, parallelogram, rhombus, rhomboid, trapezium, and trapezoid.

☞ For the definition of rectangular figures, the square, etc., see Arts. 240, 241.

574. A ***Rhombus*** is a quadrilateral which has all its sides *equal,* and its angles *oblique.*

575. A ***Rhomboid*** is a quadrilateral in which the *opposite* sides only are *equal,* and all its angles are *oblique.*



576. The *altitude* of a quadrilateral figure having two parallel sides is the perpendicular distance between these sides; as, A, L.

577. A ***Trapezium*** is a quadrilateral which has only two of its sides parallel.* (See next Fig.)

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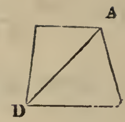
570. What is Mensuration? 571. Magnitude? Line? Surface? Solid? 574. A rhombus? 575. Rhomboid? 576. The altitude? 577. A trapezium?

* Legendre, Dr. Brewster, Young, De Morgan, etc.

578. A ***Diagonal*** is a straight line joining the vertices of two angles, not adjacent to each other; as, A D.



579. The *common measuring unit* of surfaces is a *square,* whose side is a *linear* unit of the same name. (Thomson's Geometry, IV. Sch.)

☞ To find the area of *rectangular surfaces,* see Art. 280.

The *rules* or *formulas* of mensuration are derived from Geometry to which their demonstration properly belongs.

580. To find the *Area* of an Oblique Parallelogram, Rhombus, or Rhomboid, the Length and Altitude being given.

Multiply the length by the altitude.

NOTE.—If the area and altitude, or one side are given, the other factor is found by dividing the area by the given factor. (Art. 244.)

1. What is the area of a rhombus, its length being 60 rods, and its altitude 53 rods? *Ans.* 3180 sq. r.

2. A rhomboidal garden is 75 yds. long, the perpendicular distance between its sides 48 yds.: what is its area?

3. The area of a square field whose side is 120 rods?

4. If one side of a rectangular grove containing 80 acres is 160 rods, what is the length of the other side?

581. To find the *Area of a Trapezium*, the Altitude and Parallel Sides being given.

Multiply half the sum of the parallel sides by the altitude.

5. The altitude of a trapezium is 11 ft., and its parallel sides are 16 and 27 ft.: what is its area? *Ans.* 236.5 sq. ft.

582. To find the *Area of a Triangle,* the Base and Altitude being given.

Multiply the base by half the altitude. (Art. 539.)

NOTE.—Dividing the area of a triangle by the altitude gives the *base.* Dividing the *area* by *half the base* gives the *altitude.*

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578. A diagonal? 579. The common measuring unit of surfaces? 580. How find the area of an oblique parallelogram or rhombus? 581. How find the area of a trapezium? 582. Of a triangle?

6. What is the area of a triangle whose base is 37 ft., and its altitude 19 ft.? *Ans.* 351.5 sq. ft.

7. Sold a triangular garden whose base is 50 yds., and altitude 40 yds., at $2.75 a sq. rod: what did it come to?

583. To find the *Circumference of a Circle,* the Diameter being given.

Multiply the diameter by 3.14159.

☞ For definition of the *circle* and its *parts,* see Art. 257.

8. The diameter of a cistern is 12 ft.: what is its circumference? *Ans.* 37.69908 ft.

9. The diameter of a circular pond is 65 rods: what is its circumference?

584. To find the *Diameter of a Circle,* the Circumference being given.

Divide the circumference by 3.14159.

10. What is the diameter of a circle whose circumference is 150 ft.?

11. The diameter of a circle 100 rods in circumference?

585. To find the *Area of a Circle,* the Diameter and Circumference being given.

Multiply half the circumference by half the diameter.

12. Required the area of a circle whose diameter is 75 ft.

13. What is the area of a circle 200 r. in circumference?

586. The *common measuring unit* of solids is a *cube,* whose sides are *squares* of the same name. The sides of a cubic inch are square inches, etc.

☞ For definition of rectangular solids, and the method of finding their contents, see Arts. 246, 247, and 284.

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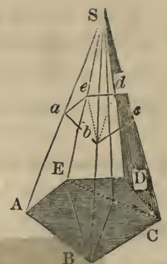
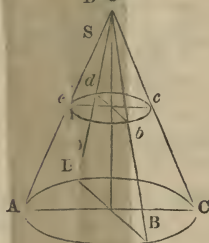
583. How find the circumference of a circle when the diameter is given? 584. How find the diameter of a circle, when the circumference is given? 585. How find the area of a circle? 586. What is the measuring unit of solids?



587. A ***Pyramid*** is a solid whose base is a *triangle, square,* or *polygon,* and whose sides terminate in a point.

NOTE.—This point is called the *vertex* of the pyramid, and the sides which meet in it are *triangles.*



588. A ***Cone*** is a solid which has a circle for its base, and terminates in a point called the vertex.

589. A ***Frustum*** is the part which is left of a *pyramid* or *cone,* after the *top* is cut off by a plane parallel to the base; as, *a, b, c, d, e.*

590. To find the *Contents of a Pyramid or Cone,* the Base and Altitude being given.

Multiply the area of the base by $\frac{1}{3}$ of the altitude.

NOTE.—The contents of a frustum of a pyramid or cone are found *by adding the areas of the two ends to the square root of the product of those areas, and multiplying the sum by $\frac{1}{3}$ of the altitude.*

14. What are the contents of a pyramid whose base is 22 ft. square, and its altitude 30 ft. *Ans.* 4840 cu. ft.

15. Of a cone 45 ft. high, whose base is 18 ft. diameter?

16. The altitude of a frustum of a pyramid is 32 ft., the ends are 5 ft. and 3 ft. square: what is its solidity?

591. A ***Cylinder*** is a roller-shaped solid of uniform diameter, whose ends are *equal* and *parallel circles.*

592. To find the *Contents of a Cylinder,* the Area of the Base and the Length being given.

Multiply the area of one end by the length.

17. What is the solidity of a cylinder 20 ft. long and 4 ft. in diameter? *Ans.* 251.3272 cu. ft.

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587. What is a pyramid? 588. A cone? 590. How find the contents of each?

593. **To find the *Convex Surface of a Cylinder,* the Circumference and length being given.**

Multiply the circumference by the length.

18. Required the convex surface of a cylindrical log whose circumference is 18 ft., and length 42 ft.?

594. A ***Sphere*** or ***Globe*** is a solid terminated by a *curve surface,* every part of which is *equally distant* from a point within, called the *center.*

595. **To find the *Surface of a Sphere,* the Circumference and Diameter being given.**

Multiply the circumference by the diameter.

19. Required the surface of a 15 inch globe. *Ans.* 4.91 sq.ft.

20. Required the surface of the moon, its diameter being 2162 miles.

596. **To find the *Solidity of a Sphere,* the Surface and Diameter being given.**

Multiply the surface by $\frac{1}{6}$ of the diameter.

21. What is the solidity of a 10 inch globe? *A.* 523.6 cu. in.

22. What is the solidity of the earth, its surface being 197663000 sq. miles, and its mean diameter 7912 miles?

597. **To find the *Contents of a Cask,* its length and head diameter being given.**

Multiply the square of the mean diameter by the length, and this product by .0034. *The result is wine gallons.*

NOTES.—1. The dimensions must be expressed in inches.

2. If the staves are *much curved,* for the *mean* diameter add to the head diameter .7 of the *difference* of the head and bung diameters; if *little curved,* add .5 of this difference; if a *medium curve,* add .65.

23. Required the contents of a cask but little curved, whose length is 48 in., its bung diameter 36 in., and its head diameter 34 inches. *Ans.* 199.92 gal.

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591. What is a cylinder? 592. How find its contents? 594. What is a sphere or globe? 595. How find its surface? 596. Its contents? 597. Contents of a cask?

MISCELLANEOUS EXAMPLES.

1. A square piano costs $650, which is 3-fifths the price of a grand piano: what is the price of the latter?

2. A man sold his watch for $75, which was 5-eighths of its cost: what was lost by the transaction?

3. Two candidates received 2126 votes, and the victor had 742 majority: how many votes had each?

4. A man owning 3 lots of 154, 242 and 374 ft. front respectively, erected houses of equal width, and of the greatest possible number of feet: what was the width?

5. Three ships start from New York at the same time to go to the West Indies; one can make a trip in 10 days, another in 12 days, and the other in 16 days: how soon will they all meet in New York?

6. Two men start from the same point and travel in opposite directions—one goes $33\frac{1}{4}$ m. in 7 h.; the other $27\frac{1}{2}$ m. in 5 h.: how far apart will they be in 14 h.?

7. Divide $200 among A, B and C, giving B twice as much as A, and C $3\frac{1}{2}$ times as much as B.

8. How many bushels of oats are required to sow $35\frac{3}{8}$ acres, allowing $11\frac{1}{4}$ bushels to 5 acres?

9. Bought $4\frac{1}{2}$ bbls. of apples at $$3\frac{4}{5}$, and paid in wood at $$3\frac{1}{4}$ a cord: how many cords did it take?

10. A mason worked $11\frac{2}{3}$ days, and spending $\frac{3}{7}$ of his earnings, had $20 left: what were his daily wages?

11. A fruit dealer bought 5250 oranges at $31.25 per M., and retailed them at 4 cents each: what did he make or lose?

12. Cincinnati is $7° 50' 4''$ west of Baltimore: when noon at the former place, what time is it at the latter?

13. A grocer bought 1000 doz. eggs at 12 cts., and sold them at the rate of 20 for 25 cts.: what was his profit?

14. Bangor, Me., is $21° 13'$ east of New Orleans: when 9 A. M. at Bangor, what is the hour at New Orleans?

15. What is the cost of a stock of 12 boards 15 ft. long and 10 in. wide, at 16 cts. a foot?

16. A farmer being asked how many cows he had, replied that he and his neighbor had 27; and that $\frac{2}{3}$ of his number equaled $\frac{1}{12}$ of his neighbors: how many had each?

17. How many sheets of tin 14 by 20 in. are required to cover a roof, each side of which is 25 ft. long and 21 ft. wide?

18. A and B counting their money, found they had \$100; and that $\frac{3}{4}$ of A's plus \$6 equaled $\frac{3}{5}$ of B's: how much had each?

19. How many pickets 4 in. wide, placed 3 in. apart, are required to fence a garden 21 rods long and 14 rods wide?

20. Bought a quantity of tea for \$768, and sold it for \$883.20: what per cent was the profit?

21. What must be the length of a farm which is 80 rods wide to contain 75 acres?

22. What must be the height of a pile of wood 36 ft. long and 12 feet wide to contain 27 cords?

23. Sold 60 bales of cotton, averaging 425 lb., at $22\frac{1}{2}$ cts., on 9 m., at 7% int.: what shall I receive for the cotton?

24. When it is noon at San Francisco, it is 3 h. 1 m. 39 sec. past noon at Washington: what is the difference in the longitude?

25. A man bought a city lot 104 by $31\frac{1}{4}$ ft., at the rate of \$$22\frac{1}{2}$ for 9 sq. ft.: what did the lot cost him?

26. If $\frac{17}{20}$ of a ton of hay cost £$3\frac{1}{2}$, what will $\frac{37}{40}$ ton cost?

27. What sum must be insured on a vessel worth \$16500, to recover its value if wrecked, and the premium at 2%?

28. How many persons can stand in a park 20 rods long and 8 rods wide; allowing each to occupy 3 sq. ft.?

29. A builder erected 4 houses, at the cost of \$$4284\frac{1}{4}$ each, and sold them so as to make 16% by the operation: what did he get for all?

30. A man planted a vineyard containing 16 acres, the vines being 8 ft. apart: what did it cost him, allowing he paid $6\frac{1}{4}$ cts. for each vine?

31. If a perpendicular pole 10 ft. high casts a shadow of 7 ft., what is the height of a tree whose shadow is 54 ft.?

32. A miller sold a cargo of flour at 20% profit, by which he made $2500: what did he pay for the flour?

33. Paid $1.25 each for geographies: at what must I mark them to abate 6%, and yet make 20%?

34. Sold goods amounting to $1500, $\frac{1}{2}$ on 4 m., the other on 8 m.; and got the note discounted at 7%: what were the net proceeds?

35. A line drawn from the top of a pole 36 ft. high to the opposite side of a river, is 60 ft. long: what is the width of the river?

36. A school-room is 48 ft. long, 36 ft. wide, and 11 ft. high: what is the length of a line drawn from one corner of the floor to the opposite diagonal corner of the ceiling?

37. The debt of a certain city is $212624.70: allowing 6% for collection, what amount must be raised to cover the debt and commission?

38. A publisher sells a book for $62\frac{1}{2}$ cents, and makes 20%: what per cent would he make if he sold it at 75 cents?

39. What will a bill of exchange for £534, 10s. cost in dollars and cents, at $\$4.87\frac{4}{5}$ to the £ sterling?

40. Three men took a prize worth $27000, and divided it in the ratio of 2, 3, and 5: what was the share of each?

41. An agent charges 5% for selling goods, and receives $135.50 commission: what are the net proceeds?

42. A, B, and C agreed to harvest a field of corn for $230; A furnished 5 men 4 days, B 6 men 5 days, and C 7 men 6 days: what did each contractor receive?

43. A and B have the same salary; A saves $\frac{1}{4}$ of his, but B spending $40 a year more than A, in 5 years was $50 in debt: what was their income, and what did each spend a year?

44. If a pipe 6 inches in diameter drain a reservoir in 80 hrs., in what time would one 2 ft. in diameter drain it?

45. A young man starting in life without money, saved $1 the first year, $3 the second, $9 the third, and so on, for 12 years: how much was he then worth?

ANSWERS.

ADDITION.

Ex.	Ans.
Page 25.	
2.	13839
3.	18250
4.	20000
5.	20438
6.	212269
Page 26.	
1.	2806
2.	$1941
3.	25285 lbs.
4.	14756 yds.
5.	98937 r.
6.	2051834 ft.
7.	2460 A.
8.	23459
9.	185462
10.	76876
11.	33367
12.	179589
13.	273070
14.	2616263
15.	9539381
16.	$4668
Page 27.	
17.	1376 yds.
18.	$6332
19.	1695 lbs.
20.	2668 g.
21.	10438
22.	8636
23.	10672
24.	2874
25.	15246
26.	100980
27.	1207053
28.	$9193
29.	3998 bu.
30.	$107601
31.	$38058
32.	2844
33.	13800
Page 28.	
34.	159755
35.	848756
36.	182404
37.	1039708
38.	11485
39.	9929
40.	13720
41.	233331
42.	1328464
43.	8237027
44.	25148
45.	11111110
46.	22226420
47.	$1460
48.	1925 y.
49.	$8190
50.	6987 lbs.
51.	93 yrs.
52.	$22338
Page 29.	
53.	$16829, D's; $33658, all.
54.	156 str.
55.	7213 bu.
56.	366 d.
57.	$5296
58.	50529
59.	3674 A.
60.	$437.44
61.	$571.54
62.	$376.02
63.	$476.19
64.	$501.31
65.	$475.89
Page 30.	
66.	$1704.28
67.	$16988.71
68.	$16580.34
69.	$179403.71
70.	$157011.73

SUBTRACTION.

Ex.	Ans.
Page 35.	
2.	346
3.	147
4.	3106
5.	2603
6.	509
Page 36.	
1.	53637
2.	305 r.
3.	67 lbs.
4.	3779 y.
5.	1719 A.
6.	11574
7.	22359
8.	27179
9.	267642
10.	235009
11.	5009009
12.	5542809
13.	2738729
14.	51989 lbs.
15.	309617 T.
16.	209354 A.
17.	34943
18.	1235993
19.	3633805
20.	33230076
21.	349629696
22.	$18990
23.	$1915
24.	$415026
25.	$200005
Page 37.	
26.	$279979
27.	1111111
28.	6333333334
29.	1111111112
30.	289753017746.
31.	27030584428516.
32.	226637999876130.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
33. 1990005	36. 999001000	41. 1732 yrs.	44. 413000000
34. 995500	39. 2235 A.	42. 84 yrs.	45. $3115027
35. 64564	40. 26530000	43. 1642 y.	46. 8253204

QUESTIONS FOR REVIEW.

Page 38.

1. $1200
2. $1107
3. 2365 sold, 1195 left.
4. $4331
5. 4471
6. 1279
7. 3771
8. 803559
9. $14292
10. $963
11. $12523
12. 1700
13. 4434
14. 112122
15. 127680

Page 39.

16. 108888
17. 126950
18. 38022
19. 43898
20. 33885
21. 762
22. 1680
23. 2198
24. $4730
25. $522.
26. $1802
27. $13970 B's $27440 C's
28. 5673
29. $737
30. $18775

MULTIPLICATION.

Page 46.

2. 10224
3. 19705
4. 64896
5. 761824

Page 47.

2. 242735
3. 1230710
4. 3627525
5. 20136672

Page 48.

6. 332671482
7. 941756556
8. 1524183620
9. 2751320848
10. 131247355
11. 351410400
12. 532125480
13. 1651148750
14. 1583318550

Page 48.

15. 3563875009
16. 2923420500
17. 1572150300
18. 450029050401
19. 1924105179680
20. 86625 lbs.
21. $55350
22. 22360 bu.
23. $222984
24. 78475 m.
25. 1351680 ft.
26. $48645
27. $33626
28. $2367500
29. 54075 yds.
30. $1786005
31. $242284
32. 432 m.

Page 50.

3. 65100
4. 290496
5. 509733
6. 3263112
7. 145152 cu. in.
8. $295008
9. $197500

Page 51.

21. 1491000
22. 3328000
23. 166092000
24. 740000 lbs.
25. $184000
26. 2600000 cts.
27. $1050000
28. 604800000 t.
29. $7350000
30. 402263519151480000
31. 640033600441000000
32. 228125535891000000

Page 52.

33. 12615335010000000
34. 312159700930000000
35. 263200000756000000
36. 2077600000000
37. 429189584040000
38. 370475105000000
39. 652303302651
40. 400800840440000
42. 17784
43. 29484
44. 45975
45. 107872
46. 454842
47. 1585873

SHORT DIVISION.

Ex. Ans.

Page 59.

2. 218392
3. 186782
4. $172258\frac{2}{4}$
5. $149647\frac{4}{5}$
6. $662107\frac{5}{6}$
7. $686586\frac{6}{7}$
8. 923808
9. $922969\frac{8}{9}$
10. 5762314
11. $60663768\frac{3}{11}$
12. $680021033\frac{1}{12}$
13. 8243 h.
14. 211 s.
15. $8978\frac{2}{3}$ r.
16. $10671
17. 122 yds.
18. $14140\frac{4}{6}$ lbs
19. $8121

Page 60.

20. $1067102\frac{2}{4}$
21. 933539
22. $1280604\frac{1}{5}$
23. 1001162
24. $746367\frac{5}{11}$
25. 1200381
26. 2346842
27. $3562695\frac{3}{9}$
28. $5848142\frac{6}{8}$
29. $8447232\frac{4}{7}$
30. $59363694\frac{4}{11}$
31. $64519169\frac{1}{12}$
32. $3794\frac{5}{7}$ w.
33. $6412
34. $9065\frac{1}{7}$ t.
35. $5248\frac{7}{9}$ bar.
36. $3438\frac{4}{12}$ yr.
37. $6469\frac{2}{7}$ yds.
38. $9405
39. 5812 sq. yd.
40. 2500 hrs.
41. 70440 b.
42. $14531
43. 335183 A.
44. $13920\frac{4}{8}$ bar.
45. 8090 cows

LONG DIVISION.

Page 63.

3. $2312\frac{5}{15}$
4. 4091
5. $2076\frac{14}{20}$
6. $2106\frac{13}{25}$
7. $10778\frac{3}{39}$
8. $10774\frac{16}{47}$
9. $9759\frac{35}{69}$
10. $10252\frac{4}{77}$
11. $50872\frac{31}{86}$
12. $61294\frac{87}{93}$
13. $101771\frac{42}{59}$
14. $108088\frac{41}{78}$
15. $88712\frac{64}{89}$
16. $94970\frac{48}{98}$
17. $243\frac{21}{75}$
18. 454 am.

Page 64.

20. $29796\frac{31}{127}$
21. $19916\frac{125}{219}$
22. $23685\frac{116}{378}$
23. $12294\frac{247}{738}$
24. 450 sh.
25. $411\frac{80}{144}$ s. ft.
26. $94
27. $540\frac{120}{128}$ C.
28. \$$602\frac{148}{250}$
29. $144\frac{1510}{1728}$ cu. ft.
30. $50\frac{245}{11200}$ lbs.
31. $8787\frac{1988}{4204}$
32. $11171\frac{4593}{5129}$
33. $12030\frac{34846}{52312}$
34. $11961\frac{6276}{61073}$
35. $30878\frac{192391}{236421}$
36. $17767\frac{360802}{463205}$
37. $4133\frac{19481134}{20396342 8}$

Page 65.

38. $4000
39. \$$37907\frac{454}{478}$
40. $288\frac{794}{942}$ A.
41. \$$8050\frac{1050}{5263}$
42. $275
43. $200\frac{72000}{525600}$

Page 66.

2. 17.
3. $\frac{1}{28}$; 19 or.
4. 23 pounds.
5. 12 com.
6. 17
7. 23
8. 26

Page 67.

9. Given
10. $961\frac{26}{45}$
11. $7974\frac{16}{63}$
12. $9865\frac{22}{72}$
13. $35776\frac{79}{84}$
14. $660421\frac{95}{96}$
15. $1502085\frac{25}{144}$

Page 69.

24. $2283\frac{18}{20}$
25. $406\frac{186}{200}$
26. $177\frac{1445}{3400}$
27. $113\frac{5821}{6500}$
28. $84\frac{33264}{87000}$
29. $86\frac{6367}{93000}$
30. $490\frac{23203}{125000}$
31. $620\frac{643271}{670000}$
32. $373
33. 229 horses
34. \$$75\frac{360}{1000}$
35. $5\frac{1200}{4800}$ l.
36. 60 bales

QUESTIONS FOR REVIEW.

Ex.	Ans.
	Page 69.
1.	146 J's; 365 both
2.	407 sheep
3.	76 years
4.	1779
5.	3093
6.	$83\frac{20}{95}$
7.	43 rods
8.	5118 bu.
9.	8613
10.	35
11.	200849
	Page 70.
12.	\$631 gain
13.	$30\frac{50}{250}$ days
14.	$972\frac{1}{11}$ lbs.
15.	$122\frac{4}{13}$ bar.
16.	$\$1537\frac{1}{7}$
17.	60 days
18.	\$156
19.	$105\frac{3}{7}$ yds.
20.	\$2
21.	$\$935\frac{1}{3}$
22.	$\$5383\frac{3}{4}$
23.	252 sheep
24.	$41\frac{1}{4}$ bar.
25.	$11\frac{2}{59}$
26.	$2383\frac{5}{37}$
27.	482 books
28.	13 cows

PROBLEMS AND FORMULAS.

Page 73-6.

2. 1157 votes
4. 1147 votes
6. 160 rods
8. 31 miles
10. \$22680
12. 60
14. \$2196, 1st. \$3172, 2d.
15. 2428, 1st. 3136 2d.
16. \$104 ch. \$146 w.
17. 45, A's. 30, B's.
18. \$269, A's. \$231, B's.

ANALYSIS.

Page 78.

1. 15 cows
2. 50 lbs.
3. 432 lbs.
4. 60 bu.
5. 31 lbs.
6. 378 bu.
7. \$180 cloth; \$10
8. \$18
9. \$315
10. 18 pears

Page 79.

12. 31
13. 504
14. 11 chil.
16. 12 less, 60 greater
17. Given
18. \$112 B's. \$361 A's.
19. 67, 1st. 176, 2d.
20. 2118

FACTORING.

Page 88.

23. 2 and 3
24. 2 and 3
25. 2 and 2
26. None.
27. 2, 2, and 2
28. 2, 3, and 2
29. 5 and 3
30. 2 and 2
31. 2 and 2
32. 5
33. 2 and 2
34. 2 and 3

35. 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

36. From 100–200 are 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199.

CANCELLATION.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
Page 90.	8. 28	13. 90	18. $36\frac{3}{4}$	23. 180 ch.
4. $1\frac{5}{7}$	9. 18	14. $78\frac{3}{4}$	19. 378	24. 315 bu.
5. 8	10. 15	15. 84	20. 30 bar.	25. 28 yrs.
6. 3	11. $12\frac{3}{5}$	16. 84	21. $5\frac{95}{98}$ t.	26. $8\frac{1}{3}$ t.
7. 9	12. 10	17. $318\frac{6}{13}$	22. $46\frac{2}{7}$ bgs	

COMMON DIVISORS.

Page 91.	8. 5 and 3	*Page 94.*	11. 6	18. one
	9. 2 and 4		12. 4	19. 37
3. 3		6. 21	13. 12	20. 2
4. 4, 3, 6	*Page 93.*	7. 15	14. 2	21. 2040
5. 6		8. 12	15. 12	22. 18 yd.
6. 7	2. 12	9. 3	16. 2	23. 21 each
7. 10	3. 24	10. 25	17. 192	24. 8 A.

MULTIPLES.

Page 98.	6. 600	10. 288	14. 55440	17. 60 cts.
3. 48	7. 480	11. 12852	15. 57600	18. 720 ro's
4. 84	8. 330	12. 15120	16. 1000	19. 60 lbs.
5. 720	9. 240	13. 73440		

FRACTIONS.

Page 105.	15. $\frac{19}{31}$	29. $\frac{1}{7}$	13. $5\frac{5}{32}$	27. $\$29\frac{2445}{6127}$
2. $\frac{1}{2}$	16. $\frac{11}{13}$		14. $46\frac{5}{126}$	28. 3516 yr.
3. $\frac{2}{3}$	17. $\frac{41}{83}$	*Page 105.*	15. $18\frac{112}{157}$	
4. $\frac{1}{3}$	18. $\frac{8}{5}$	2. 37	16. $90\frac{10}{111}$	*Page 106.*
5. $\frac{5}{7}$	19. $\frac{71}{125}$	3. $16\frac{1}{9}$	17. $22\frac{17}{1168}$	2. $\frac{108}{7}$
6. $\frac{1}{7}$	20. $\frac{15}{34}$	4. 19	18. $210\frac{1}{9}$	3. $\frac{94}{5}$
7. $\frac{21}{32}$	21. $\frac{1}{4}$	5. $15\frac{1}{3}$	19. $107\frac{82}{127}$	4. $\frac{287}{8}$
8. $\frac{1}{4}$	22. $\frac{191}{290}$	6. 12	20. $383\frac{47}{64}$	5. $\frac{1631}{20}$
9. $\frac{1}{5}$	23. $\frac{1}{3}$	7. 16	21. $20\frac{5}{6}$	6. $\frac{9170}{63}$
10. $\frac{3}{4}$	24. $\frac{2}{3}$	8. $12\frac{5}{9}$	22. $2449\frac{24}{25}$	7. $\frac{13505}{72}$
11. $\frac{121}{256}$	25. $\frac{3}{8}$	9. $36\frac{11}{12}$	23. $10\frac{3823}{4520}$	8. $\frac{29581}{100}$
12. $\frac{3}{2}$	26. $\frac{2}{5}$	10. $45\frac{2}{13}$	24. $29\frac{3215}{4286}$	9. $\frac{88755}{110}$
13. 1	27. $\frac{5}{9}$	11. $6\frac{1}{3}$	25. $13\frac{31861}{72850}$	10. $\frac{7868}{5}$
14. $\frac{1}{4}$	28. $\frac{58}{117}$	12. $3\frac{18}{55}$	26. $41\frac{41}{124}$ lb	11. $\frac{17931}{7}$

Ex.	Ans.
12.	$\frac{29125}{8}$
13.	$\frac{86247}{10}$
14.	$\frac{39679}{83}$
15.	$\frac{14275}{250}$
16.	$\frac{3615}{411}$
17.	$\frac{94523}{10000}$
18.	$\frac{4211}{16}$ lbs.
19.	$\frac{25652}{40}$ m

Page 107.

3. $\frac{1}{4}$
4. $\frac{12}{35}$
5. $\frac{2}{5}$
6. $\frac{2}{9}$
7. $\frac{13}{35}$

Ex.	Ans.
8.	$\frac{56}{99}$
9.	28
10.	$\frac{39}{95}$
11.	$\frac{77}{340}$
12.	$\frac{273}{969}$
13.	$\frac{540}{209}$
14.	$\frac{110}{273}$
15.	$\frac{3}{8}$
16.	$\frac{2}{9}$
17.	$\frac{348}{11}$
18.	$\frac{1514}{63}$
19.	$\frac{399}{80}$
20.	$\frac{917}{12}$
21.	$\frac{3}{1}$
22.	$\frac{21}{64}$ bu.
23.	$\frac{9}{65}$ yds.

Page 108.

2. $\frac{20}{52}$
3. $\frac{35}{60}$
4. $\frac{32}{88}$
5. $\frac{90}{144}$
6. $\frac{114}{204}$
7. $\frac{174}{246}$
8. $\frac{164}{288}$
9. $\frac{256}{360}$
10. $\frac{650}{1000}$
11. $\frac{595}{10000}$
13. $\frac{2\frac{1}{2}}{6}$
14. $\frac{2\frac{1}{3}}{3}$
15. $\frac{24}{27}$
16. $\frac{2\frac{3}{4}}{4}$

Page 109.

2. $\frac{63}{9}$
3. $\frac{315}{5}$
4. $\frac{553}{7}$
5. $\frac{747}{9}$
6. $\frac{1680}{16}$
7. $\frac{4340}{20}$
8. $\frac{15729}{49}$
9. $\frac{35568}{76}$
10. $\frac{43500}{87}$
11. $\frac{156000}{100}$
12. $\frac{2004000}{1000}$
13. $\frac{5000000}{10000}$
14. $\frac{25000003}{1000000}$

Page 110.

3. $\frac{15}{16}$
4. $\frac{8}{75}$
5. $\frac{16}{7}$
6. $\frac{3}{1}$
7. $\frac{3}{2}$
8. $\frac{8}{1}$
9. $\frac{2}{5}$
10. $\frac{52}{33}$
11. $\frac{18}{17}$
12. $\frac{4}{1}$
13. $\frac{119}{156}$
14. $\frac{165}{296}$

Page 111

2. $\frac{15}{20}, \frac{8}{20}$
3. $\frac{35}{70}, \frac{56}{70}, \frac{60}{70}$
4. $\frac{154}{231}, \frac{99}{231}, \frac{84}{231}$
5. $\frac{65}{130}, \frac{78}{130}, \frac{90}{130}$
6. $\frac{935}{1309}, \frac{714}{1309}, \frac{462}{1309}$
7. $\frac{1980}{5940}, \frac{2640}{5940}, \frac{2970}{5940}, \frac{2160}{5940}$
8. $\frac{14080}{24640}, \frac{18480}{24640}, \frac{9240}{24640}, \frac{38080}{24640}$
9. $\frac{157500}{283500}, \frac{252000}{283500}, \frac{212625}{283500}$
10. $\frac{139104}{508032}, \frac{172368}{508032}, \frac{103488}{508032}$

Page 112.

12. $\frac{330}{105}, \frac{140}{105}, \frac{231}{105}$
13. $\frac{7436}{1144}, \frac{9048}{1144}, \frac{176}{1144}, \frac{858}{1144}$
14. $\frac{144}{2520}, \frac{12915}{2520}, \frac{1400}{2520}$
15. $\frac{1463}{266}, \frac{304}{266}, \frac{154}{266}$
16. $\frac{594}{176}, \frac{2024}{176}, \frac{48}{176}$
17. $\frac{1113}{84}, \frac{1428}{84}, \frac{40}{84}$
18. $\frac{324}{144}, \frac{756}{144}, \frac{384}{144}$
20. $\frac{18}{72}, \frac{27}{72}, \frac{40}{72}$
21. $\frac{60}{96}, \frac{56}{96}, \frac{66}{96}$
22. $\frac{36}{90}, \frac{60}{90}$
23. $\frac{135}{75}, \frac{30}{75}$
24. $\frac{105}{168}, \frac{98}{168}, \frac{91}{168}$
25. $\frac{350}{1000}, \frac{750}{1000}, \frac{476}{1000}$

Page 113.

3. $\frac{56}{84}, \frac{63}{84}, \frac{60}{84}$
4. $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$
5. $\frac{18}{45}, \frac{30}{45}, \frac{25}{45}$
6. $\frac{54}{126}, \frac{28}{126}, \frac{567}{126}$
7. $\frac{14}{20}, \frac{15}{20}, \frac{200}{20}$
8. $\frac{120}{264}, \frac{231}{264}, \frac{1760}{264}$
9. $\frac{25}{20}, \frac{8}{20}, \frac{70}{20}, \frac{175}{20}$
10. $\frac{189}{25}, \frac{875}{25}$
11. $\frac{4}{20}, \frac{4}{20}, \frac{5}{20}, \frac{105}{20}$
12. $\frac{90}{210}, \frac{140}{210}, \frac{105}{210}, \frac{168}{210}, \frac{180}{210}$
13. $\frac{812}{84}, \frac{987}{84}, \frac{2400}{84}$
14. $\frac{104}{20}, \frac{5}{20}, \frac{7}{20}$
15. $\frac{465}{60}, \frac{680}{60}, \frac{44}{60}$
16. $\frac{2}{12}, \frac{4}{12}, \frac{3}{12}$
17. $\frac{2260}{5120}, \frac{2690}{5120}, \frac{1447}{5120}$
18. $\frac{32}{80}, \frac{53}{80}, \frac{40}{80}$

ADDITION OF FRACTIONS.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
Page 116.	12. $82\frac{3}{20}$	17. $320\frac{1}{4}$	22. $13\frac{9}{16}$	26. $51\frac{1}{2}$ y.
	13. $2\frac{51}{140}$	18. $751\frac{29}{72}$	23. $\$7\frac{3}{5}$	27. $\$489\frac{2}{3}$
8, 9. Given	14. $1\frac{247}{300}$	19. $554\frac{119}{120}$	24. $\$71\frac{3}{8}$	29. $9\frac{31}{175}$
10. $8\frac{7}{72}$	15. $135\frac{5}{12}$	20. $10\frac{37}{48}$	25. $2\frac{39}{40}$ lbs.	30. $6\frac{29}{45}$
11. $22\frac{1}{4}$	16. $103\frac{67}{70}$	21. $5\frac{143}{168}$		

SUBTRACTION OF FRACTIONS.

Page 118.	15. $2\frac{543}{1634}$	*Page 119.*	27. $102\frac{96}{125}$	34. 53
	16. $3\frac{160}{441}$		28. $\frac{1}{4}$	35. $174\frac{27}{40}$
10. $\frac{31}{72}$	17. $3\frac{23}{110}$	22. Given	29. Given	36. $111\frac{3}{16}$
11. $2\frac{1}{6}$	18. $3\frac{73}{150}$	23. $38\frac{1}{2}$	30. $\frac{1}{4}$	37. $40\frac{7}{8}$ gal
12. $1\frac{7}{12}$	19. $2\frac{79}{216}$	24. $37\frac{1}{4}$	31. $\frac{47}{252}$	38. $\$29\frac{5}{8}$
13. $8\frac{13}{35}$	20. $32\frac{3}{4}$ lbs.	25. $43\frac{3}{7}$	32. $\frac{57}{77}$	40. $1\frac{11}{16}$
14. $\frac{11}{30}$	21. $87\frac{3}{20}$ A.	26. $67\frac{5}{9}$	33. $5\frac{5}{6}$	41. $1\frac{13}{28}$

MULTIPLICATION OF FRACTIONS.

Page 121.	18. $\$1592\frac{1}{2}$	13. $643\frac{59}{87}$	11. $\frac{2}{7}$	2. $\$\frac{1}{2}$
	19. $\$1737\frac{1}{2}$	14. $1256\frac{1}{4}$	12. $\frac{3}{56}$	3. $\$\frac{5}{8}$
3. $2\frac{1}{2}$	20. $\$4537\frac{1}{2}$	15. $612\frac{1}{4}$	13. $\frac{3}{35}$	4. $149\frac{7}{32}$
4. $7\frac{5}{43}$	21. $\$5418$	16. $1009\frac{2}{5}$	14. $\frac{65}{612}$	5. $371\frac{1}{5}$
5. $189\frac{1}{3}$		17. $21721\frac{7}{10}$	15. $3\frac{33}{49}$	6. 600
6. $357\frac{1}{2}$		18. $70003\frac{5}{9}$	16. $78\frac{1}{8}$	7. $3986\frac{26}{35}$
7. $583\frac{1}{5}$	*Page 123.*		17. 9 cts.	8. $12377\frac{7}{16}$
8. 36	3. $24\frac{3}{8}$	*Page 124.*	18. $\$36$	9. $17317\frac{35}{36}$
9. $277\frac{1}{7}$	4. $26\frac{7}{10}$		19. $203\frac{19}{56}$	10. $\frac{4}{55}$
10. $48\frac{114}{685}$	5. 336	3. $\frac{5}{9}$	20. $703\frac{1}{8}$	11. $11\frac{11}{20}$
11. $2566\frac{1}{5}$	6. 406	4. $\frac{1}{4}$	21. $1953\frac{1}{8}$	12. $\$50\frac{5}{8}$
12. $4094\frac{14}{23}$	7. $542\frac{6}{7}$	5. $6\frac{2}{3}$	22. $15352\frac{19}{32}$	13. $433\frac{1}{8}$ cts.
13. 275	8. $\$26\frac{3}{5}$	6. $\frac{33}{91}$	24. $\frac{1}{3}$	14. $\$181\frac{2}{15}$
14. $\$60\frac{4}{5}$	9. $\$250\frac{1}{8}$	7. $\frac{1}{3}$	25. $\frac{4}{15}$	15. 750 tin, 3000 cop.
15. $\$231\frac{3}{4}$	10. $\$6429\frac{5}{6}$	8. $\frac{3}{7}$	*Page 125.*	16. $\$1918\frac{1}{8}$
16. $\$1218\frac{3}{4}$	11. $549\frac{58}{63}$	9. $\frac{119}{220}$		
17. $\$804\frac{3}{5}$	12. 1407	10. $\frac{21}{55}$	1. $\$\frac{1}{8}$	17. $44839\frac{23}{40}$

DIVISION OF FRACTIONS.

Page 126.

3. $\frac{4}{29}$
4. $\frac{11}{135}$
5. $\frac{7}{69}$
6. $\frac{4}{45}$
7. $\frac{2}{29}$
8. $\frac{4}{121}$
9. $\frac{5}{119}$
10. $\frac{488}{21141}$
11. $\frac{203}{1327}$
12. $\frac{26163}{61725}$
13. $\frac{4003}{45631}$
14. $\frac{160}{15873}$
15. $\frac{7}{40}$ bar.
16. $\$\frac{7}{100}$
17. $\$\frac{8}{63}$
18. $\$7\frac{5}{8}$

Page 127.

19. $6\frac{9}{13}$
20. $5\frac{181}{406}$
21. $1\frac{3}{7}$
22. $1\frac{74}{81}$
23. $10\frac{113}{205}$
24. $12\frac{51}{200}$
25. $15\frac{481}{500}$
26. $15\frac{1213}{2520}$
27. $20\frac{1}{200}$
28. $68\frac{76}{85}$
29. $315\frac{13}{15}$
30. $115\frac{31}{105}$
31. $\$8\frac{19}{40}$
32. $\$126\frac{1}{8}$
33. $\$1\frac{13}{60}$

Page 128.

2. $126\frac{2}{3}$
3. 672
4. 350
5. 375
6. 1479
7. $447\frac{3}{7}$
8. $1645\frac{5}{7}$
9. 21630
10. $56727\frac{3}{11}$
12. 135
13. 477
14. 266
15. 804
17. $8\frac{6}{13}$
18. 8
19. $12\frac{14}{25}$
20. $11\frac{3}{7}$
22. 9
23. $3\frac{2}{11}$
24. $3\frac{10}{17}$
25. $4\frac{2}{19}$

Page 130.

2. $2\frac{13}{18}$
3. $1\frac{1}{3}$
4. $1\frac{1}{20}$
5. $4\frac{29}{45}$
6. $1\frac{51}{126}$
7. $2\frac{1}{7}$
8. $1\frac{47}{196}$
9. $\frac{7}{10}$
10. $4\frac{1}{2}$
11. $\frac{22}{25}$
12. 4
13. $1\frac{28255}{1041536}$
15. $1\frac{11}{16}$
16. $5\frac{13}{28}$
17. $3\frac{3}{41}$
18. $7\frac{23}{45}$
19. 6
20. $140\frac{41}{66}$ r.
21. $10\frac{281}{484}$ s. r.
23. $\frac{108}{245}$
24. $1\frac{1}{9}$
25. $2\frac{32}{119}$
27. $\frac{400}{1089}$

Page 131.

1. $5\frac{1}{3}$ mo.
2. $8\frac{2}{3}$ lbs.
3. $\$1\frac{1}{5}$
4. $17\frac{21}{58}$
5. 15 times
6. $2\frac{77}{86}$
7. $\frac{23}{26}$
8. $1\frac{3}{19}$
9. $1\frac{161}{206}$
10. $50\frac{1}{4}$ lbs.
11. $30\frac{4}{5}$ A.
12. 10 lots
13. $42\frac{14}{153}$ y.
14. $\frac{16}{21}$
15. $\frac{273}{400}$

Page 132.

1. $\$43\frac{7}{8}$ err.
2. $\$619\frac{1}{24}$ c.
   $\$19\frac{5}{24}$ er.
3. $50\frac{7}{8}$
4. $32\frac{3}{5}$ A.
5. $\frac{25}{36}$
6. $3\frac{21}{40}$
7. $\frac{11}{64}$ sold; $\frac{33}{64}$ own; \$24783
8. $86\frac{19}{24}$ A.
9. $\frac{19}{135}$
10. $\frac{67}{84}$
11. $\frac{71}{132}$
12. $50\frac{3}{32}$ bu.
13. $2451\frac{9}{10}$ f.
14. $860\frac{5}{32}$ m.
15. $\$103\frac{1}{8}$

Page 133.

16. $41\frac{37}{120}$ l.
    $27\frac{77}{120}$ s.
17. $\frac{12}{17}$
18. $\frac{4}{15}$
19. $\frac{10}{21}$
20. 80 days
21. $\$41\frac{1}{4}$
22. $23\frac{55}{63}$
23. $\frac{3645}{4544}$
24. $\frac{19}{110}$
25. 50 vests
26. $44\frac{10}{11}$ c.
27. $12\frac{7}{12}$ hr.
28. $16\frac{2}{3}$
29. 60 bu.
30. $244\frac{8}{9}$ yd.
31. 2
32. $\frac{400}{1089}$
33. $3\frac{73}{384}$

FRACTIONAL RELATIONS OF NUMBERS.

Page 134.

3. $\frac{1}{5}$, $\frac{5}{14}$
4. $\frac{9}{13}$
5. $\frac{3}{2}$
6. $\frac{2}{9}$
7. $\frac{3}{4}$
8. $\frac{9}{32}$
9. $\frac{17}{15}$
10. $\frac{5}{7}$ wk.
11. $\frac{5}{17}$ bu.
12. $\frac{10}{13}$ ton
13. $\frac{1}{3}$

Page 135.

15. \$252
16. $\$59\frac{1}{2}$

Ex.	Ans.
17.	\$132
19.	$\frac{7}{200}$
20.	$\frac{3}{28}$
21.	$\frac{3}{100}$
22.	$\frac{1}{5}$
24.	\$$5\frac{1}{15}$
25.	\$$2\frac{29}{38}$
27.	$\frac{32}{3}$
28.	$\frac{96}{5}$
29.	$\frac{77}{4}$
30.	$\frac{200}{7}$
32.	$\frac{16}{35}$
33.	$\frac{15}{28}$
34.	$\frac{3}{10}$
35.	$\frac{608}{693}$
Page 136.	
37.	$\frac{1}{8}$
38.	$\frac{1}{3}$
39.	$\frac{1}{6}$
40.	$\frac{5}{8}$
41.	$\frac{3}{16}$
42.	$\frac{7}{8}$
44.	$\frac{3}{10}$
45.	$\frac{5}{14}$
46.	$\frac{15}{19}$ lb.
47.	2 ft.
48.	$\frac{3}{10}$ lb.
49.	$\frac{7}{24}$
50.	$\frac{9}{5}$
51.	$\frac{7}{2250}$
52.	$\frac{1}{44}$
Page 137.	
3.	84
4.	$90\frac{2}{3}$
5.	$212\frac{1}{2}$
6.	138
7.	$1017\frac{1}{3}$
8.	$729\frac{3}{5}$
9.	$2283\frac{1}{3}$
10.	$2283\frac{3}{7}$
11.	270
12.	\$14720
13.	33250
16.	$1\frac{8}{27}$
17.	$1\frac{1}{5}$
19.	$62\frac{1}{2}$
20.	$83\frac{1}{3}$
Page 138.	
22.	288
24.	$9\frac{13}{18}$ yds.
25.	\$3 each
27.	$11\frac{1}{4}$ t.
28.	100 cts. 8 cigars
30.	$130\frac{2}{3}$
32.	7 times
34.	$3\frac{1}{3}$

REDUCTION OF DECIMALS.

Page 142.

8. 4.7
9. 21.06
10. 84.45
11. 93.009
12. 7.045
13. 10.00508
14. 46.0007
15. 80.000364
17. .06; .063; .0109
18. .305; .00021; .000095
19. .004; .0108; .46; .000065; .0001045
20. 69.004; 10.0075; 160.000006

Page 144.

3. $\frac{16}{125}$
4. $\frac{32}{125}$
5. $\frac{3}{8}$
6. $\frac{863}{1000}$
7. $\frac{1}{20}$
8. $\frac{3}{1000}$
9. $\frac{1}{1250}$
10. $\frac{121}{2000}$
11. $\frac{7}{10000}$
12. $\frac{507}{12500}$
13. $\frac{91}{25000}$
14. $\frac{1}{20000}$
15. $\frac{50171}{250000}$
16. $\frac{1}{1250000}$
17. $\frac{19841}{160000}$
18. $\frac{1550079}{6250000}$

Page 145.

1. Given
2. .25
3. .4
4. .75
5. .8
6. .625
7. .25
8. .875
9. .8
10. .95
11. .6
12. .0875
13. .02
14. .000375
15. .0078125
16. .00875
17. .01125

Page 146.

19. Given
20. .3333+
21. .8333+
22. .2857+
23. .4444+
24. .2727+
25. .6428+
26. .604166+
27. .5466+
28. 75.6
29. 136.875
30. 261.68
31. 346.8133+
32. 465.0025
33. 523.00390625
34. 740.01375
35. 956.0078125

ADDITION OF DECIMALS.

Page 147.

1, 2. Given
3. 881.6217
4. 139.26168
5. 14.38916
6. 118.792
7. 892.688
8. 2.76231
9. 92.00537
10. 37.417
11. 2.3948
12. 23.25553

Page 148.

13. 575.729105
14. 53.1 bu.
15. 75.97 lbs.

SUBTRACTION OF DECIMALS.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
Page 148.		*Page 149.*		12.	.8969755	19.	41.9955
1.	Given	6.	7.831	13.	.5496933	20.	.000098
2.	7.831	7.	6.60249	14.	.876543211	21.	$443.825
3.	6.60249	8.	17.3675	15.	.01235679	22.	139.83 A.
4.	17.3675	9.	77.94794	16.	.099	23.	99.063
5.	17.94794	10.	78.569966	17.	.00999	24.	70.333 m.
		11.	2.896216	18.	99.999		1072.407 m

MULTIPLICATION OF DECIMALS.

Page 150.

3. .1453
4. .000151473
5. .0000016872
6. 21800.6
7. .012041505
8. 20.08591442
9. 318.0424
10. 721.36
11. .00004368
12. 1.50175036
13. .000721236
14. .020007
15. 431.25 lbs.
16. $222.9375
17. 469.0625 bu.
18. $10639.75
19. .0126
20. $686.71875

Page 151.

21. .0025
22. .000004
23. .00000049
24. .000000603
25. .0000003
26. .000000252
27. .001
28. 1 [00001
29. .00000000000
31. 3205.05
32. 8003.56
33. 2.43
34. 5.8
35. 5
36. $50
37. $600
38. 27.625 bu.
39. 65.625 m.

DIVISION OF DECIMALS.

Page 153.

2. .007
3. 4
4. 600
5. .4154+
6. 30.153+
7. 2.4142+
8. .13+
9. .7115+
10. .9768
11. .00675
12. .0000576
13. .015
14. 625.5
15. 10
16. 10
17. .01
18. .000002
19. .0005
20. 50
21. .00000027.
22. 46 coats
23. 44.409+ r.
24. 27 stoves
26. 4.3753
27. .063845
28. .0000253
29. .0000005
30. $.0005
31. $.0475

ADDITION OF U. S. MONEY.

Page 158.

1. Given
2. $48.625
3. $780.25
4. $200.09
5. $224.53
6. $761.4785
7. $17.28
8. $69.1925
9. $13131.72

Page 159.

10. $54.50
11. $120.48
12. $76.82

SUBTRACTION OF U. S. MONEY.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
Page 159.	5. $585.25	7. $171.9625	10. $7.4275
2. $533.105		8. $411.075	11. $494.945
3. $604.625	*Page 160.*	9. $.955	12. $28.75
4. $524.50	6. $948.33		

MULTIPLICATION OF U. S. MONEY.

Page 160.	*Page 161.*	12. $.00056	18. $45
2. $747.50	7. $100.9125	13. $5	19. $80.4375
3. $1569.24	8. $14.124375	14. $.001011	20. $1890
4. $290.625	9. $67.8375	15. $6.5625	21. $548.625
5. $60.165	10. $310.596255	16. $58.905	22. $178.75
6. $459.25	11. $4630.70025	17. $56.25	23. $14208

DIVISION OF U. S. MONEY.

Page 162.	4. $2.75	9. .01	14. $2.965 +
	5. $2.921	10. .1	15. 925.405 +
1. Given	6. 100	11. 100	16. 293.039 +
2. $1.964	7. 361.455 +	12. 10000	17. $.0625
3. $.05	8. 4000	13. 600000	18. $.0547 +

COUNTING-ROOM EXERCISES.

Page 163.	*Page 165.*	4. $3183.07½	6. $5201.70
1. $13958.38 bal.	2. $206.83	*Page 166.*	7. $717.77
2. $159857.16 bal.	3. $4367.125	5. $71.15	8. $395.37

ANALYSIS.

Page 167.	13. $114.06¼	20. 6132 A.	10. $31¼
4. $789.25	14. $47.82 amt.	21. $478.125 l.	11. $291¾
5. $4312.50	15. $3007750	2. $66¼	12. $35
6. $43¾	16. 946.1+lb.	3. $65	13. $27⅜
7. $892½	17. 7897 lb. s'd;	4. $22	14. $107½
8. $0.33⅓	149 lb. av.	5. $409½	15. $115⅔
9. $5.264	18. 36.45 bu.	6. $12½	
10. $31.50		7. $150	*Page 169.*
11. 102 bibles	*Page 168.*	8. $208⅓	17. $210
12. 148½ vests	19. $247	9. $581⅔	18. $547½

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
19.	$576	25.	1800 coc's	3.	$240.625	8.	22.31\frac{1}{4}$
20.	$168	26.	2690 pine's	4.	$789.625	9.	$100.2472
21.	$240			5.	1169.83\frac{3}{4}$	10.	$4.51
23.	13.66 yds.		*Page 170.*	6.	1203.93\frac{3}{4}$	11.	14.18\frac{1}{4}$
24.	1125 lbs.	2.	$78.177	7.	$40.4875	12.	$15.975

REDUCTION.

Page 190.

3. 614 far.
4. 39618 far.
6. £8, 10s. 7d.
7. 12s. 9d. 2 far.
8. £41, 5s. 2d. 2 far.
9. 1661 yds.
10. £225
11. 4440 pwts.

Page 191.

12. 7865 pwts.
13. 80 lbs. 7 oz.
14. 7 lbs. 1 oz. 18 pwt. 18 grs.
15. 149$\frac{2}{3}$ rings
16. $27
17. 2653 oz.
18. 6721944 oz.
19. 2 t. 725 lb.
20. 11 t. 1141 lb. 12 oz
21. $700.55
22. $280 profit
23. 24048 drams
24. 7 lb. 6 oz. 1 dr. 2 sc.
26. 1928495 ft.
28. 7 m. 79 r. 1$\frac{1}{2}$ ft.
29. 443440 rods

Page 192.

30. 495782 in.
31. 126720 in.
32. $1480
33. $1556.10
34. 456 eighths
35. 2608 sixteenths
36. 144$\frac{1}{2}$ yds.
37. 123$\frac{5}{8}$ yds.
38. 22 vests
39. $16.25 profit
40. 160 r. 256 sq. ft.
41. 5 A. 40 s. r. 20 s. y.
42. 6976414$\frac{1}{4}$ sq. ft.
43. 10240000 sq. r.
44. 10 A. 108 sq. r.
45. $22875 profit
46. 3983040 cu. in.
47. 32000 cu. ft.
48. 10 c. ft. 985 c. in.
49. 64 C. 86 cu. ft.
50. 5259 qts.
51. 24051 pts.

Page 193.

52. 6641 bu.
53. 254 bu. 2 p. 3 q.
54. $412.08
55. $12.75
56. 790 gi.
57. 603 qts.
58. 5713 qts.
59. 34616 gi.
60. 10834 gal. 1 pt.
61. 168 bottles
62. $126.00
63. 2612530 sec.
64. 176010 min.
65. 31556929.7 sec.
66. 9w. 6 d. 20h. 15m.
67. 639 y. 225 d. 5 h.
68. 19 y. 164 d. 6 h. 20 min.
69. $212.625
70. $13° 29' 21''$
71. 36 s. $7° 17'$
72. 855631''
73. 1296000''
74. 163$\frac{3}{4}$ dozen
75. 1500 eggs
76. 694$\frac{4}{9}$ gross
77. 9360 pens
78. 67 lbs.
79. 1800 sheets
80. 416$\frac{2}{3}$ quires

APPLICATIONS OF WEIGHTS AND MEASURES.

Page 194.

1. Given
2. 7$\frac{3}{4}$ A.
3. 165 ft.

Page 195.

4. 72 rods
5. $46500 pr.
6. 4116 s. ft.
7. 2 sq. ft.
8. 1200 bu. $300
9. Given
10. 192 bulbs
11. 3630 gr. v. $190.575 s.
12. 2$\frac{1}{2}$ A.

Ex.	Ans.
	Page 196.
14.	\$324
15.	\$8.88
16.	\$402.27$\frac{2}{9}$
17.	\$22.50
18.	\$45.13$\frac{8}{9}$
	Page 197.
21.	18$\frac{2}{3}$ b. ft.; \$1.40 val.
22.	12$\frac{1}{2}$ b. ft.
23.	147 b. ft.
24.	\$278.10
	Page 198.
28.	\$14.85
29.	\$23.40
30.	\$4$\frac{3}{8}$
	Page 199.
32.	2$\frac{119}{128}$ cords
33.	16$\frac{1}{2}$ tons
34.	\$133.20
35.	\$112
36.	\$337.68
37.	86751 br.
38.	\$3600
	Page 200.
40.	56 yds.
41.	44$\frac{4}{5}$ yds.
42.	12$\frac{6}{7}$ yds.
43.	320 sods
44.	48 yds.
45.	640 tiles
46.	13$\frac{1}{9}$ rolls
47.	1795$\frac{25}{77}$ g.
	Page 201.
48.	56$\frac{7}{48}$ in.
49.	57.857 + b.
50.	421$\frac{3}{32}$ c. ft.
51.	9.955$\frac{5}{9}$ in.
52.	3750 lbs.
55.	\$7$\frac{3}{128}$
56.	9$\frac{57}{175}$ lbs.
57.	87$\frac{1}{2}$ rings
58.	\$72$\frac{11}{12}$
59.	\$805$\frac{35}{48}$

DENOMINATE FRACTIONS.

Page 202.

2. $\frac{8}{5}$ qt.
3. $\frac{20}{33}$ d.
4. $\frac{3}{25}$ hr.
5. $\frac{48}{25}$ oz.
6. $\frac{9}{16}$ s. in.

Page 203.

8. \$$\frac{1}{1600}$
9. £$\frac{1}{1536}$
10. $\frac{1}{1152}$ oz.
11. $\frac{3}{160}$ gal.
12. $\frac{1}{480}$ m.
13. $\frac{7}{16000}$ t.

Page 204.

16. 13s. 4d.
17. 3 pk. 6 qt.
18. 1250 lbs.
19. 10 oz. 10 p.
20. 3 fur. 13 r. 1 yd. 2 ft. 6 in.
21. 146 s. r. 20 s. y. 1 s. ft. 72 s. in.
22. 112 cu. ft.
24. $\frac{47}{64}$ gal.
25. $\frac{1229}{2880}$ lb.
26. $\frac{53}{64}$ bu.
27. $\frac{81}{250}$ ton
28. $\frac{463}{810}$ sq. yd.
29. $\frac{187}{1684}$
30. $\frac{233}{504}$ wk.
31. $\frac{1}{5}$ C.
32. $\frac{31}{77}$

Page 205.

34. 2s. 6d. 0.4 + far.
35. 10 oz. 18 p. 22.56 grs.
36. 2 furlongs 32 rods.
37. 2 qt. 1 pt.
38. 39′ 3 6″
39. 1 d. 16 h. 29 m. 16.8 s.
40. 505 lb. 1.92 oz.
41. 17.28 grs.
42. £5, 12s. 6d. 0.4032 far.

Page 206.

44. .5423 + lb.
45. .005 ton
46. .45539 + m.
47. .04 lb.
48. .5 bl.
49. .409 + r.
50. .25
51. .48 bl.
52. 1.44 r.
53. .166 + wk.
54. .28182 +
55. .41666 + C.

METRIC NOTATION AND NUMERATION.

Page 212.

1. 5370.9845 dekameters; 537.09845 hektometers; 53.709845 kilometers; 537098.45 decimeters.
2. 450.5108 dekagrams; 450510.8 centigrams; 45.05108 hektograms; 4.505108 kilograms.

REDUCTION OF METRIC WEIGHTS AND MEASURES.

Ex. Ans.

Page 213.

1. Given
2. 437500 sq. m.
3. 867000 grams
4. 26442 liters
5. 256100 sq. m.
6. 8652000 cu. dm.
7. 4256250 grams
8. Given.
9. 65.2254 hektars.
10. 0.087 kilos.
11. 1.48235 kg.
12. 39.2675 kl.

APPLICATIONS OF METRIC WEIGHTS AND MEASURES.

Page 215.

3. 39.14631 m.
4. $19.8131\frac{1}{4}$ gals.
5. 15.89 bu.
6. 4.2324 oz.
7. 303.68365 lbs.
8. Given
9. 148.87775 A.
10. 4237.92 cu. ft.
12. 58.293 + m.
13. 6236.959 + kg.
14. 236.585 + liters.
15. 72.492 + kl.
16. 143.223 + kg.
17. 6000.06 + s. m.
18. 16.378 + hect.
19. 410.748 + c. m.
20. 27958.715 + c.m

COMPOUND ADDITION.

Page 217.

3. £11, 10s. 0d. 2 f.
4. 26 T. 3 cwt. 83 lb. 3 oz.
5. 45 bu. 0 pk. 2 qt.
6. $74\frac{3}{4}$ yds.
7. 1093 lb. 5 oz.
8. 55 gal. 2 qt.
9. 196 bu. 2 pk. 7 q.
10. 6 C. 80 cu. ft.
11. 98 bu. 3 pk. 2 q.

Page 218.

12. Given
13. 23 wk. 1 d. 17 h. 58 m.
14. 64 A. 7 s.r. $10\frac{1}{2}$ s.y
17. 15 cwt. 50 lb. 4 oz
18. 2 pk. 4 qt.
19. 9 oz. 1 pwt. 10 gr.
20. 3 s. 6 d. 3.2 far.
21. 2 A. 52 sq r.
22. 2 C. 81 cu ft. $1209\frac{3}{5}$ cu. in

COMPOUND SUBTRACTION.

Page 219.

2. 1 fur. 39 r. 1 yd. $2\frac{1}{2}$ ft.
3. 7 lb. 4 oz. 17 p. 9 grs.
4. 7 T. 8 cwt. 26 lb.
5. 8 gal. 1 qt. 1 pt. 2 gi.

Page 220.

6. 159 A. 12 sq. r. $222\frac{1}{4}$ sq. ft.
7. 46 cu. ft. 1689 cu. in.
8. 145 A. 21 sq. r.
9. $46\frac{1}{8}$ yd.
10. 3 m. 5 fur. 38 r. 5 yd. 0 ft. 1 in.

2. 9 s. 9 d.
3. 2 pk. 5 qt. 1.2 p.
4. $4\frac{1}{5}$ pt.
5. 1 lb. 2 oz. 2 p.
6. 114.4 sq. r.
7. 31.25 lbs.

Page 221-2.

3. 67 y. 9 m. 22 d.

2. 113 days
3. 74 days
4. 150 days
5. 224 days
6. 101 days

Page 223.

10. $86^\circ\ 19'\ 24''$
11. $7^\circ\ 24'\ 7''$
12. $18^\circ\ 2'$
13. $5^\circ\ 6'\ 46''$
14. $15^\circ\ 4'\ 16''$
15. $19^\circ\ 35'$
16. $56^\circ\ 11'$
17. $70^\circ\ 29'$
18. $10^\circ\ 19'\ 38''$

COMPOUND MULTIPLICATION.

Ex. Ans.	Ex. Ans.	Ex. Ans.
Page 224.	8. 22 C. 91 cu. ft.	15. 161° 37′ 30″
2. 98 T. 17 cwt. 28 lb.	9. £23, 15s. $3\frac{3}{4}$d.	16. 431 h. 15 m.
3. £151, 15s. $9\frac{3}{4}$d.	10. 562 m. 4 fu. 24 r.	17. 133 sq. r. 21 sq. yd. $\frac{3}{4}$ sq. ft.
4. 33 oz. 15 pw. 10 g.	11. 1937 bu. 1 pk.	
	12. 22 C. 57 cu. ft.	18. 73 T. 1492 lbs.
Page 225.	13. 61 T. 844 lbs.	19. 1571 bu. 2 p. 4 q.
7. 331 gal. 2 qt.	14. 1307 r. 8 qr. 8 s.	20. 1946 gal. 3 q. 1 p.

COMPOUND DIVISION.

Page 227.

3. 4 fur. 8 r. 2 yd. $2\frac{1}{14}$ ft.
4. 6 gal. 3 qt. 0 pt. $3\frac{7}{8}$ gi.
5. 25 bu. 0 pk. 1 qt. $\frac{2}{3}$ pt.
6. 15 A. 106 sq. r. 5 sq. yd. $2\frac{8}{23}$ sq. ft.
7. $26\frac{2}{5}$ spoons
8. 8800 rails
9. $1049\frac{6}{31}$ times
10. 12 books
11. 2 A. 64 sq. r.
12. 6 bu. 1 pk. 1 qt.

COMPARISON OF TIME AND LONGITUDE.

Page 228.

1. Given.
2. 43 m. 32.13+ s.
3. 11 o'c. 12 m. 4 s.
4. 12 o'c. 37 m. 8.4 s.
5. 49 m. 20 s.
6. 1 o'c. 30 m. E.; 10 o'c. 30 m. W.
7. 54 m. 19.8 sec.
8. 13 m. 44.86+ s.

Page 229.

9. Given
10. 6° 9′
11. 6° 45′ 5″
12. 45° 24′ 45″
13. 56 m. 49+ sec.
14. 12 o'clk. 34 m. 47 sec.

PERCENTAGE.

Page 232.

13. .50; .25; .75; .20; .40; .60; .80
14. .10; .70; .90; .05; .35; .12; .06
15. $.66\frac{2}{3}$; $.16\frac{2}{3}$; .125; .625; .875; $.58\frac{1}{3}$; $.91\frac{2}{3}$

Page 234.

3. $24.21
4. 10.8 bu.
5. 22.6 yds.
6. 8.64 oxen
7. 8.25 yds.
8. $11.584
9. 100.8 lbs.
10. 63.14 T.
11. 52.5 men
12. 145.656 m.
13. $857.785
14. £.106
15. 1.25 l.
16. 1.38 k.
17. 840 lbs.
18. $1000
19. 78.75 bu.
20. .35 lbs.
21. 9.765 gals.
22. 840 men
24. $215
25. 157.2 lbs.
26. 32.25 m.
27. 116 l.
28. 580 sheep
29. 156 bu.
30. $925

Ex. Ans.

31. 576 cu. ft.
32. £2, 1s.

Page 236.

3. 1150 s. C.; 880 s. D.
4. $2300.91
5. $2850
6. 504 bales
7. 2018.75 gal.
8. 1312 m.
9. $470.40
10. 129 t.
11. $1410.50
12. $228
13. $4823.33⅓

Ex. Ans.

14. $10200
15. $9750

Page 237.

2. $13\frac{1}{3}\%$
3. 25%
4. $33\frac{1}{3}\%$
5. 15%
6. $9\frac{3}{8}\%$
7. $3\frac{1}{8}\%$
8. 10%
9. 80%
10. 10%
11. $23\frac{17}{21}\%$
12. $42\frac{6}{7}\%$
13. $30\frac{10}{73}\%$

Ex. Ans.

Page 238.

4. 250 bu.
5. $362.50
6. 180 tons
7. £450
8. 600
9. 360 fr.
10. 8000
11. $200
12. $16666⅔
13. $17600
14. 54.4 yds.
15. $50
16. 100
17. $5000
18. $1920

Ex. Ans.

19. 3000 m.
20. 360000 p.

Page 240.

5. 809
6. 1820
7. Given
8. $\frac{8}{9}$
9. $\frac{3}{10}$ B's
10. 5175 m.
11. $3648
12. 6875 p.
13. $3350
14. 228 sheep
15. 400 pupils
16. 5800 m.

COMMISSION AND BROKERAGE.

Page 242.

3. $31.1431¼
4. $366.225 com.; $10902.225 paid

Page 243.

5. $4310.145
6. $238.66¼
8. $\frac{1}{2}\%$
9. 5%
10. $\frac{3}{8}\%$

Page 244.

12. $9000
13. $4212 sales; $4001.40 net p.
14. $1500 col.; $1432.50 paid
15. $9000 sale; $8865 rec'd
17. $5100
18. $6834.872 sales; $170.872 com.

Page 245, 6.

19. $15488.89+
20. $26635.294+
22. $3010.75
23. $2419.23+
24. $49261.083+in.; $738.916+ com.
25. $13687.50 g. a.; $360.75 ch.; $13326.75 net p.
26. $12127.82 net p.

PROFIT AND LOSS.

Page 247-9.

3. $22.20
4. $22
5. $.0875
6. $16.65
7. $1.40
10. $2925 A.; $2075 B.
11. $453.60
12. $2942.50
13. $9520
14. 11¼ cts.
15. $1 a pair
16. 2.77\frac{7}{9}$ each
17. $4.90 a yd.
18. $4331.25
19. $12810
21. $28\frac{4}{7}\%$
22. $11\frac{1}{9}\%$
23. 60%
25. 100%
26. 50%
27. $66\frac{2}{3}\%$
28. 40%

Ex.	Ans.
	Page 250.
29.	50%
30.	Given
31.	50%
32.	50%
33.	$33\frac{1}{3}\%$
34.	$12.60 pr.;
	$7\frac{43}{51}\%$
36.	$9750
	Page 251.
39.	$1 cost per lb.
40.	$22500 cost;
	$30000 sell'g pr.
41.	$17500 spend;
	$21000 amt. sales
42.	$12000 A's inv.;
	$9375 B's inv.
43.	$250000 cost;
	$275000 amt. s.
44.	$.02 cost;
	$.025 selling pr.
45.	Given
46.	$\$10061.71\frac{3}{7}$
47.	$13319.672
48.	£22 cost
49.	3 cents

Ex.	Ans.
	Page 252.
51.	$.736 marked p.
52.	$34.78 + m. pr.
53.	$4.80 m. pr.
54.	$\$1.16\frac{2}{3}$ m. pr.
55.	$75 each
56.	$5.75 per yd.
	Page 253.
1.	$2.691 prof.
2.	$\$6356.72\frac{1}{3}$
3.	$3487.575 sh.;
	$850.84 ging.;
	$4338.415 both
4.	$22 loss
5.	$77812.50 cost
6.	$897.45 net pr.
7.	$38\frac{8}{9}\%$
8.	$56\frac{1}{4}\%$ prof.;
	$3240 prof.
9.	$16\frac{2}{3}\%$ loss
10.	.052 or $5\frac{2}{10}\%$ c.;
	$1766.835 net p.
11.	25% loss
12.	125%
13.	$9300 cost;
	$11160 sell'g pr.

Ex.	Ans.
14.	$7 cost;
	$8.05 sell'g pr.
15.	$2 cost;
	$1.75 sell'g pr.
	Page 254.
16.	$5200 sales;
	$4940 net
17.	$24000 sales
18.	$1.25 cost
19.	$5250 cost;
	$1750 gain
20.	$.7894\frac{14}{19}$ per gal.;
	.1481 gain per g.;
	$49.73 wh. cost;
	$9.33 wh. gain
21.	$10 a barrel
22.	$\$51311\frac{1}{9}$ cost;
	$\$6413\frac{8}{9}$ gain
23.	$15798.963 sale;
	$552.963 com.
24.	$38\frac{6}{13}\%$
25.	$5\frac{66}{100}\%$ loss
26.	$60 m. p.
27.	$6\frac{1}{4}\%$ loss
28.	15 cts.
29.	$10\frac{9}{10}\%$ loss

INTEREST.

	Page 258.
3.	$2.54
4.	$1.676
5.	$1.224
6.	$4.523
7.	$43.356
8.	$110.77
9.	$33.42
10.	$23.364
11.	$25.577
12.	$94.35
13.	$17.91
14.	$680.28
15.	$81.44 +
16.	$2875.792
17.	$362.315
18.	$2759.962
19.	$5032.083
20.	$122.40
21.	$226.69
22.	$45.053
23.	$216.489
24.	$493.20
25.	$3923.47
27.	$99.96
28.	$24.00
29.	$163.842;
	$2740.652
	Page 259.
2.	$24.44
3.	$6.252
	Page 260.
4.	$4.82
5.	$6.75
6.	$73.96
7.	$64.18
8.	$26.49
9.	$1.65
10.	$29.933
11.	$60.04
12.	$30.52
13.	$2450.80
14.	$20819.80
	Page 261.
3.	$2.8435

Ex.	Ans.
4.	\$2.192
5.	\$11.413
6.	\$13.89
7.	\$1285.33
8.	\$13.876
9.	\$4363.044
10.	\$114.16$\frac{2}{3}$
11.	\$185.18
12.	\$81.358

Page 262.

2. 7%
3. 7%

4. 6%
5. 8%
6. 9$\frac{1}{16}$%
7. 8$\frac{1}{3}$%
8. 5%; 10%
9. 7%
10. 6%

Page 263.

3. 5$\frac{5}{7}$ y., or 5 y. 8 m. 17 d.
4. 9 m. 2 d.
5. $\frac{1}{4}$ y., or 3 m.

7. 14$\frac{2}{7}$ yrs.
8. 16$\frac{2}{3}$ yrs.
9. 18$\frac{3}{4}$ yrs.

2. \$1666$\frac{2}{3}$
3. \$3000

Page 264.

4. \$2333$\frac{1}{3}$
5. \$2500
6. \$40000
7. \$10000
8. \$25000

2. \$235.85
3. \$327.36
4. \$8928.57
5. \$892.86
6. \$5582.142

Page 268, 9.

1. Given
2. \$1519.71
3. \$538.63
5. \$270.19
6. \$388.23
7. \$516.32

COMPOUND INTEREST.

Page 273, 4.

1. Given
2. \$112.52
3. \$161.63
4. \$164.61
5. \$200.63
6. \$2165.713

Page 275.

7. Given
8. \$551.58
9. \$1377.41
10. \$1543.65
11. \$8104.25
12. \$32564.58
13. \$21825.26
14. \$29849.56
15. \$37704.95
16. \$3298.77

DISCOUNT.

Page 277.

1. Given
2. \$283.47+
3. \$462.96
4. \$1213.59
5. \$2336.45
6. \$4464.28
7. Given
8. \$86.57
9. \$101.892
10. \$91.35 dis; \$2283.65 p.
11. \$3775.264
12. \$20.377
13. \$528.33
14. \$46.73 for.

Page 279.

3. \$718.685
4. \$989.50
5. \$1721.875
6. \$17.25
7. \$735.70
8. Given
9. \$768.44
10. \$1578.81
11. \$2226.23
12. \$7503.83

STOCKS AND BONDS.

Page 282.

2. \$243
3. \$525
4. \$525
5. \$400
6. \$476
8. \$12880

Page 283.

9. \$10497.50
10. \$6363.61
11. \$6336.11
12. \$10852.37$\frac{1}{2}$
13. \$19293.75
14. \$672 cur.
16. 15%
17. 6$\frac{1}{4}$%
18. 6$\frac{2}{3}$%

Page 284.

20. 40 bonds.
21. \$12000
23. \$44166$\frac{2}{3}$
24. \$35625
25. \$56000

EXCHANGE.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
Page 287.	10. $2617.801	*Page 290.*	11. £1542, 11 s. 6d.
2. $2049	11. $3751.95	6. $56125.12+	
3. $3488.80	12. $3750	7. Given.	*Page 291.*
4. $4130.647		8. £511, 15 s. 4 d. 3.2 far.	12. Given
5. $203	*Page 289.*	9. £774, 7 s. 10 d. 1.28 far.	13. $675.68
8. $1219.51	1, 2. Given.	10. £1026, 8 s. 7.2 d.	14. Given
	3. $1858.80⅜		15. 13050.00 f.
Page 288.	4. $4866.50		16. 16474.50 f.
9. $1491.053	5. $56.62½ g.		

INSURANCE.

Page 293.
1. Given
2. $6.65
3. $38.40
4. $84.375
5. $10.70
6. $25.00
7. $197.60
8. $875
9. $1569.625

Page 294.
10. Given
11. $15747.423
12. $27806.122+
13. $37105.263+

1. Given
2. $125.00
3. $2437.50
4. $5000
5. $45000 gr.

TAXES.

Page 297.
3. $1003.75, C's tax
$1250, D's "
$1375, E's "
$925, F's "
4. 2% rate;
$159.75, A's tax
5. 3% rate;
$450, G's tax
6. $309.75 H's tax
7. 5% rate;
$170, A's tax
9. $3645.83⅓
10. $5507.853+
11. $11052.63+

DUTIES.

Page 299.
2. $4062.50
3. $1063.314
4. $1999.20
6. $945
7. $2296.80

Page 300.
2. $118.25
3. $425.75

EQUATION OF PAYMENTS.

Page 302.
2. 4 m. from J. 20, or Oct. 20
3. 8 m. 18 d.
4. Mch. 10+17 d.=Mch. 27
5. June 1, or in 3 m.

Page 303.
6. Given
8. July 15+51 d.=Sept. 4
9. $1483.25 amt.;
42.9 d. av. time; due Nov. 17

AVERAGING ACCOUNTS.

Page 306.
1–3. Given
4. 7 m. extension;
Bal., $1650
5. $300 bal. debts;
22320 bal. prod.;
74 d. av. time
Due May 23.

SIMPLE PROPORTION.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
Page 312.	14. $133	26. $\$5223\frac{93}{224}$	38. 25 min.
3. $160	15. 2240 times	27. $12.50	39. 6600 rev.
4. $165	16. 5 years	28. 14.4 in.	40. $40
5. $450	17. 8 hours	29. $26\frac{2}{3}$ yds.	41. $2\frac{2}{5}$ hrs.
6. $6		30. 97920 t.	42. $3\frac{9}{37}$ hrs.
7. £19, 7s. $6\frac{1}{2}$d.	***Page 314.***	31. 180°	43. 45 days
	18. 1100 men	32. 80 ft.	44. 1000 rods
Page 313.	19. 240000 lb.	33. $64\frac{2}{7}$ orang.	45. 300 hrs.
9. 135 A.	20. 27 horses	34. 3000 mi.	46. $191.78
10. $\$937\frac{1}{2}$	21. $612	35. 33 men	47. 8 m. 51 s. before 9
11. 270 miles	22. £$1\frac{1}{5}$	36. 30 ft.	48. $3\frac{1}{3}$ cts. loss
12. $288	23. $\$5.06\frac{1}{4}$	***Page 315.***	49. 150 m. less; 186 m. gr.
13. $\$822\frac{1}{2}$	24. $880.40	37. $1920	
	25. $\$490.52\frac{1}{4}$		

COMPOUND PROPORTION.

Page 317.
3. $2\frac{4}{7}$ days
4. 9 horses
5. $33\frac{1}{3}$ weeks

Page 318.
6. $\$126.66\frac{2}{3}$
7. $100
8. 360 miles.
9. $750
10. $102857\frac{5}{7}$ yd.
11. $37\frac{1}{2}$ days
12. $\$1454\frac{6}{11}$
13. 750 lbs.
14. $62.50
15. 240 sofas
16. 864 tiles

PARTITIVE PROPORTION.

Page 319.
3. 40 s.; 60 s.; 100 s.
4. 60 bu. oats; 80 b. p.; 110 b. c.
5. 71; $106\frac{1}{2}$; 142; $177\frac{1}{2}$ A.

PARTNERSHIP.—*Page 321-3.*

2. $\$222\frac{2}{9}$, A's loss; $\$277\frac{7}{9}$, B's "
3. $340, A's share; $510, B's "
4. $\$5067\frac{21}{37}$, A's g'n; $\$4054\frac{2}{37}$, B's " $\$3378\frac{14}{37}$, C's "
5. $3400; $5100; and $6800
6. $64, A's share; $96 B's; $160 C's
8. $\$142.85\frac{5}{7}$, A's sh. $\$266.66\frac{2}{3}$, B's " $\$190.47\frac{13}{21}$, C's "
9. $\$24.48\frac{48}{49}$, one; $\$25.51\frac{1}{49}$, other
10. $\$424\frac{16}{41}$, A's sh.; $\$424\frac{16}{41}$, B's " $\$318\frac{12}{41}$, C's " $\$282\frac{38}{41}$, D's "
11. $1500, H'd.; $2250, Cont.; $3000, Am.
12. $\$263.38\frac{1018}{1139}$, A's; $\$447.76\frac{136}{1139}$ B's; $\$691.39\frac{679}{1139}$, C's; $\$1097.45\frac{445}{1139}$, D.
13. $\$4021\frac{157}{183}$, A's; $\$3846\frac{182}{183}$, B's; $\$4131\frac{27}{183}$, C's

BANKRUPTCY.

Page 324.
2. $1687.50, A's sh.; $\$1205.35\frac{5}{7}$, B's sh.; $\$857.14\frac{2}{7}$, D's "
3. $2060, D rec'd; $12100 net proc.

ALLIGATION.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
Page 325.	9. 2 p. at 15;	***Page 328.***	18. $54\frac{6}{11}$ lb. 28 c;
2. $15\frac{2}{3}$ cts.	1 p. at 18;	14. $3\frac{1}{3}$ lbs. ea.	$18\frac{2}{11}$ lb. 30;
3. $95\frac{5}{13}$ cts.	2 p. at 21;	15. 50 q. at 4 c.;	$54\frac{6}{11}$ lb. 38;
4. $9\frac{17}{35}$ cts.	5 p. at 22	300 q. at 6	$72\frac{8}{11}$ lb. 42
5. $20\frac{10}{27}$ carats	10. 9 lb. at 32 c;		For other ans.
6. $15.68	6 lb. at 40;		see Key.
Page 327.	6 lb. at 45	***Page 329.***	19. $53\frac{1}{3}$ g. 40 c.;
8. 35 bu. 1st;	11. 8 lb. at 20 c;	17. 20 lb. at 6 c.;	$53\frac{1}{3}$ g. 45;
40 bu. 2d;	3 lb. at 27;	20 lb. at 8;	$53\frac{1}{3}$ g. 50;
40 bu. 3d	5 lb. at 35;	60 lb. at 12	140 g. 60
	12 lb. at 40		

INVOLUTION.—*Page 331.*

8. 25; 36; 49; 64; 81; 100; 400; 900; 1600; 2500; 3600, 4900; 6400; 8100.

9. .25; .36; .49; .64; .81; .0001; .0004; .0009; .0016; .0025; .0036; .0049; .0064; .0081.

10. 125	13. 1024	16. 8.365427	19. $\frac{81}{625}$
11. 64	14. 4096	17. 64.014401080027	20. $\frac{343}{512}$
12. 2299968	15. 15625	18. 64024003.000125	21. $\frac{6561}{256}$

EXTRACTION OF THE SQUARE ROOT.

Page 338.	12. .8514+	20. 3.16+	28. 98.7654	37. 4.1683
5. 427	13. .355+	21. 3.316+		38. $\frac{16}{25}$
6. 719	14. 1.635+	22. 3.46+	***Page 339***	39. $\frac{24}{30}$
7. 772	15. 69.47+	23. 4.42	32. $\frac{12}{16}$	40. .8545+
8. 1.892+	16. 21.275+	24. 57.3	33. .745+	41. 1.018+
9. .64	17. 2.236+	25. 9.36+	34. .866+	42. $\frac{21}{4}$
10. .347	18. 2.64+	26. 11111.11+	35. 2.529+	43. $\frac{36}{5}$
11. .41	19. 2.828+	27. 7856.4	36. 3.63+	

APPLICATIONS.

Page 339-41.	9. 438.29+ m.	16. Given
2. 420 rods.	10. 103.61+ ft.	17. 12
3. 559.28 r.	11. 56.56+ ft.	18. 54
4. 119 trees	12. 75.81+ ft.	19. 63.49+
5. 238 men		20. 1.75
7. 108 yds.	***Page 342.***	21. $\frac{8}{42}$
8. 30 rods	15. $3\frac{3}{7}$ min.	22. $\frac{72}{120}$

EXTRACTION OF THE CUBE ROOT.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
Page 348.	7. 2.76	13. 129.07+in	***Page 349.***
1-2. Given	8. 2.57+	14. Given.	1. Given
3. 85	9. 8904	16. .746+	2. 52 ft.
4. 4.38+	10. .632+	17. $\frac{23}{25}$	4. 16 ft.
5. 72	11. 85.6+ yds.	18. $\frac{9}{12}$	5. 462.96+c f
6. 1.44+	12. 15.3+ ft.	19. 4.3+	6. 3.072 tons

ARITHMETICAL PROGRESSION.—*Page 351.*

1. 15	5. $360	***Page 352.***	10. Given
3. 8	7. 11 children	9. 3 yrs.	11. 78 strokes

GEOMETRICAL PROGRESSION.

Page 353.	3. $2007.3383664;	***Page 354.***
1. 96	$3001.460703698	5. 1456
2. $10.24	4. 242	6. $4095

MENSURATION.—*Page 356-9.*

1. Given	9. 204.20335 r.	15. 3817.03185 cu. ft.
2. 3600 s. y.	10. $47.746\frac{164386}{314159}$ ft.	16. $522\frac{2}{3}$ cu. ft.
3. 14400 s.r.	11. $31.831\frac{4871}{314159}$ r.	18. 756 sq. ft.
4. 80 rods	12. 4417.8609375 s. ft.	20. 14684558.20796 s. m.
7. $$90.90\frac{10}{11}$	13. 3183.1 s. r.	22. $260651609333\frac{1}{3}$ cu. m.

MISCELLANEOUS EXAMPLES.—*Page 360-2.*

1. $$1083\frac{1}{3}$	$$45.93\frac{3}{4}$ pr.	21. 150 rods	36. 61 ft.
2. $120 cost;	12. 12 o'cl. 31	22. 8 ft.	37. $226196.489
$45 loss	m. 20 [illegible]/15 s.	23. $6038.72	38. 44%
3. 692 less;	13. $120 cost;	24. $45^\circ\ 24'\ 45''$	39. $2607.291
1434 great.	$30 profit	25. $8125	40. $5400, 1st.;
4. 22 ft.	14. 35 m. 8 s.	26. £$3\frac{55}{68}$	$8100, 2d.;
5. 240 days	past 7 A.M.	27. $16836.734	$13500, 3d.
6. $143\frac{1}{2}$ miles	15. $24	28. 14520 per.	41. $2574.50
7. $20, A's;	16. 3 farmer's;	29. $19878.92	42. $50, A;
$40, B's;	24 neighb's	30. $680.625	$75, B;
$140, C's	17. 540 sheets	31. $77\frac{1}{7}$ ft.	$105, C
8. $79.59\frac{3}{8}$ bu.	18. $40, A's;	32. $12500	43. $120 each
9. $5.26\frac{2}{13}$ C.	$60, B's	33. $1.59	$130, B
10. $3	19. 1980 pick.	34. $1446.625	44. 5 hrs.
11. $$164.06\frac{1}{4}$ c.	20. 15%	35. 48 ft.	45. $265720

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