A

NEW, SIMPLE, AND GENERAL METHOD

OF

SOLVING NUMERICAL EQUATIONS OF ALL ORDERS.

BY THOMAS WEDDLE.

Wm. Carter R.N.
Sat: 24 June 1905.

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The method of solving Numerical Equations, developed in the following pages, occurred to the Author early in 1839. He subsequently drew up the Paper which now issues from the press, and sent it as a communication to the Royal Society. It was read before that learned body in June, 1841, and they were pleased to transmit their thanks to its Author. The encouragement which he thus received induces him to lay the result of his enquiries in this important branch of Mathematics before the Public.

Newcastle-upon-Tyne,
March, 1842.

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A

NEW METHOD OF SOLVING

NUMERICAL EQUATIONS.

1. The object of the present paper is to develope a new method of solving numerical equations. It is a process of great simplicity and elegance. And although, in comparison with existing methods, it is not without defects, yet I trust it will be found to possess some advantages of considerable importance.

2. We shall, in the first place, exhibit the methods of transformation which it will be necessary to employ.

$$\text{Let } Ax^n + Bx^{n-1} + Cx^{n-2} \ldots\ldots\ldots\ldots + \gamma x^2 + \beta x + \alpha = 0 \ldots\ldots\ldots\ldots (X)$$

be an equation of the n.th degree. If the roots of this equation be divided by R, the resulting equation or the transformation in Rx', is

$$AR^n x'^n + BR^{n-1} x'^{n-1} + CR^{n-2} x'^{n-2} \ldots\ldots\ldots + \gamma R^2 x'^2 + \beta Rx' + \alpha = 0.$$

When the significant part of R consists of a single digit, the transformation may be effected with great facility as below, each number being found by multiplying that above it by R :

A	B	C	..................γ	β	α
AR	BR	CR	γR	βR	
AR^2	BR^2	CR^2	γR^2		
⋮	⋮	⋮			
AR^{n-2}	BR^{n-2}	CR^{n-2}			
AR^{n-1}	BR^{n-1}				
AR^n					

3. Instead of dividing the roots of (X) by R let them be divided by $(1+r)$, where r is a decimal, then the transformed equation will be

$$A(1+r)^n x'^n + B(1+r)^{n-1} x'^{n-1} + C(1+r)^{n-2} x'^{n-2} \ldots\ldots\ldots + \gamma(1+r)^2 x'^2 + \beta(1+r)x' + \alpha = 0 \ldots\ldots\ldots (Y).$$

If we put $A_1=(1+r)\ A=A+Ar$

$$\begin{aligned}
A_2&=(1+r)^2\ A=(1+r)\ A_1=A_1+A_1r\\
A_3&=(1+r)^3\ A=(1+r)\ A_2=A_2+A_2r\\
&\vdots\\
A_n&=(1+r)^n\ A=(1+r)\ A_{n-1}=A_{n-1}+A_{n-1}r\\
B_1&=(1+r)\ B=B+Br\\
B_2&=(1+r)^2\ B=(1+r)\ B_1=B_1+B_1r\\
B_3&=(1+r)^3\ B=(1+r)\ B_2=B_2+B_2r\\
&\vdots\\
B_{n-1}&=(1+r)^{n-1}\ B=(1+r)\ B_{n-2}=B_{n-2}+B_{n-2}r
\end{aligned}$$

&c., &c.,

then the method of computing the coefficients of the transformed equation (Y) will be understood from the following formula :—

A	B	C	γ	β	α
$+Ar$	$+Br$	$+Cr$	$+\gamma r$	$+\beta r$	
A_1	B_1	C_1	γ_1	β_1	
$+A_1r$	$+B_1r$	$+C_1r$	$+\gamma_1 r$		
A_2	B_2	C_2	γ_2		
$\vdots$	$\vdots$	$\vdots$			
A_{n-2}	B_{n-2}	C_{n-2}			
$+A_{n-2}r$	$+B_{n-2}r$				
A_{n-1}	B_{n-1}				
$+A_{n-1}r$					
A_n					

Whence the transformed equation (Y) becomes

$$A_n\ x'^n+B_{n-1}\ x'^{n-1}+C_{n-2}\ x'^{n-2}\ldots\ldots\ldots\ldots+\gamma_2\ x'^2+\beta_1\ x'+\alpha=0.$$

4. When the significant part of r consists of a single digit, the addends in the preceding transformation need not be written, as they may be incorporated mentally with great ease. For the operation consists merely in multiplying the successive digits of a certain number by a digit, and adding to each product two other digits, viz., that which was carried forward, and that figure of the same number which stands immediately above the place where the unit's figure of the result is to be set down.

5. Since $A_1=(1+r)\ A$

$$A_2=(1+r)\ A_1$$

&c. *(See Art. 3.)*

$$\begin{aligned}
\therefore\ A_1r&=(1+r)\ .\ Ar\\
A_2r&=(1+r)\ .\ A_1r\\
&\vdots\\
A_{n-1}r&=(1+r)\ .\ A_{n-2}r\\
B_1r&=(1+r)\ .\ Br\\
B_2r&=(1+r)\ .\ B_1r\\
&\vdots
\end{aligned}$$

$$\vdots$$

$$B_{n-2}r=(1+r)\,.\,B_{n-3}r$$

&c.

Now Ar, A_1r, A_2r.........$A_{n-1}r$; Br, B_1r, B_2r.........$B_{n-2}r$; &c., which we shall denote by A', A'_1, A'_2 ....A'_{n-1}; B', B'_1, B'_2.........B'_{n-2}; &c., are the addends employed in article 3, and it is plain from the above values that they may be derived from each other by the process in article 4. It is also obvious (article 3) that

$$A_n=A+Ar+A_1r\ldots\ldots\ldots\ldots+A_{n-1}r$$
$$=A+A'+A'_1\ldots\ldots\ldots\ldots+A'_{n-1}$$
$$B_{n-1}=B+Br+B_1r\ldots\ldots\ldots+B_{n-2}r$$
$$=B+B'+B'_1\ldots\ldots\ldots+B'_{n-2}$$

&c.

Hence instead of employing the process in article 4 we may employ the following:

$$\begin{array}{llllll}
A & B & C\ \ldots\ldots\ldots\ldots & \gamma & \beta & \alpha \\
+A' & +B' & +C' & +\gamma' & \beta_1 & \\
+A'_1 & +B'_1 & +C'_1 & \underline{+\gamma'_1} & & \\
\vdots & \vdots & \vdots & \gamma_2 & & \\
+A'_{n-3} & +B'_{n-3} & \underline{+C'_{n-3}} & & & \\
+A'_{n-2} & \underline{+B'_{n-2}} & C_{n-2} & & & \\
\underline{+A'_{n-1}} & B_{n-1} & & & & \\
A_n & & & & &
\end{array}$$

6. We shall now proceed to to develope our method.

Let us represent the given equation by $F(x)$, and the first of the succeeding transformed equations by $F_1(x_1)$, the second by $F_2(x_2)$, the third by $F_3(x_3)$, &c.; also the limiting equations of $F(x)$ by $F'(x)$, $F''(x)$, &c.; those of $F_1(x_1)$ by $F'_1(x_1)$, $F''_1(x_1)$, &c., &c.

Let R be the value of the first significant figure of a root of $F(x)$, transform the equation by dividing its roots by R, or, which amounts to the same thing, by substituting Rx_1 for x, one value of x_1 will obviously be between 1 and 2, put r_1 for the value of the first significant figure of the decimal part of x_1, transform the equation in $(1+r_1)\,x_2$, assume r_2 for the value of the first significant figure of the decimal part of x_2, and transform the equation in $(1+r_2)\,x_3$, &c.

7. In effecting the preceding transformations we shall not be obliged to extend each column to an indefinite number of decimals, but each must be restrained to that number (q) of places, which is requisite to give the root to the extent required. Suppose that in continuing the transformations we arrive at an equation $F_m(x_m)$ (which denote by $Ax_m^n+Bx_m^{n-1}+Cx_m^{n-2}\ldots\ldots\ldots\ldots+\gamma\,x_m^2+\beta\,x_m+\alpha=0$) whose coefficients are so related to each other, that when it is transformed in $\left\{1-\frac{Fm\,(1)}{F'_m\,(1)}\right\}x_{m+1}$ the addends (see article 3), when restricted to q places of decimals, are constant in each of the columns, then the root of $F_m(x_m)$ is $1-\frac{F_m\,(1)}{F'_m\,(1)}$.

In order to prove this it will obviously be sufficient to show that the root of the transformed equation $F_{m+1}(x_{m+1})$ is unity. Put $-\frac{F_m\,(1)}{F'_m\,(1)}=a$, then since the addends are constant, and as there are n of them in the first column, $(n-1)$ in the second, $(n-2)$ in the third, &c., we shall have

$$F_{m+1}(x_{m+1}) = \left\{A + nAa\right\} x^n_{m+1} + \left\{B + \overline{n-1} \,.\, Ba\right\} x^{n-1}_{m+1} + \left\{C + \overline{n-2}Ca\right\} x^{n-2}_{m+1} \cdots\cdots\cdots$$

$$\cdots\cdots\cdots + \left\{\gamma + 2\gamma a\right\} x^2_{m+1} + \left\{\beta + \beta a\right\} x_{m+1} + \alpha.$$

$$= Ax^n_{m+1} + Bx^{n-1}_{m+1} + Cx^{n-2}_{m+1} \cdots\cdots\cdots + \gamma x^2_{m+1} + \beta x_{m+1} + \alpha.$$

$$+ ax_{m+1}\left\{nAx^{n-1}_{m+1} + \overline{n-1} \,.\, Bx^{n-2}_{m+1} + \overline{n-2}\, Cx^{n-3}_{m+1} \cdots\cdots\cdots + 2\gamma x_{m+1} + \beta\right\}.$$

$$= F_m(x_{m+1}) + a \,.\, x_{m+1} F'_m(x_{m+1}) = F_m(x_{m+1}) - x_{m+1} \frac{F_m(1)}{F'_m(1)} F'_m(x_{m+1}).$$

$$\text{Let } x_{m+1} = 1, \quad \therefore F_{m+1}(1) = F_m(1) - \frac{F_m(1)}{F'_m(1)} F'_m(1) = F_m(1) - F_m(1) = 0.$$

Hence a root of $F_{m+1}(x_{m+1})$ is unity, and consequently a root of $F_m(x_m)$ is $1 - \frac{F_m(1)}{F'_m(1)}$. We shall, therefore, obviously obtain the following development of the root of the original equation $F(x)$, viz.:

$$x = R\left\{1 + r_1\right\} \,.\, \left\{1 + r_2\right\} \,.\, \left\{1 + r_3\right\} \cdots\cdots\cdots \left\{1 + r_{m-1}\right\} \,.\, \left\{1 - \frac{F_m(1)}{F'_m(1)}\right\}.$$

8. It is obvious that it would be a great acquisition could we discover an easy method by which r_1, r_2, r_3, &c., may be suggested, nor is this desirable object unattainable,

$$\text{For } F_p(x_p) = F_p(1 + \overline{x_p - 1}) = F_p(1) + F'_p(1) \,.\, \overline{x_p - 1} + \frac{F''_p(1)}{2} \overline{x_p - 1}^2 \cdots\cdots\cdots$$

Now whatever be the value of p, $(x_p - 1)$ is a decimal, and after the first or second transformation, a very small one too, it is therefore obvious that $-\frac{F_p(1)}{F'_p(1)}$ will generally be sufficient to suggest at least the first significant figure of $(x_p - 1)$, which figure is the value of r_p.

9. In the preceding investigation, we have supposed the significant part of r_1, r_2, r_3, &c., to consist of only one digit, but this restriction is not necessary as they may evidently consist of any number of digits whatever.

10. If any of the transformed equations, $F_p(x_p)$, be such that were it transformed in $\left\{1 - \frac{F_p(1)}{F'_p(1)}\right\} x_{p+1}$, and each column *supposed* to be restricted to a certain number (k) of places of decimals, the addends would be constant, then it follows from what has been said in article 7, that $x_p = 1 - \frac{F_p(1)}{F'_p(1)}$, where $F_p(1)$, $F'_p(1)$, must be restrained to k places, and the quotient resulting from the (contracted) division of the former, by the latter carried to that number (f) of decimals, which can be found without annexing cyphers to the dividend $F_p(1)$, hence as $1 - \frac{F_p(1)}{F'_p(1)}$ gives the value of x_p true to about f places of decimals, it may sometimes be convenient to make $r_p = -\frac{F_p(1)}{F'_p(1)}$ to f places.

We may here remark that should we find that we have taken any of the numbers r_1, r_2, r_3, &c., as r_p, a little too large, we need not erase the transformation, for it will merely have the effect of making the next r_{p+1} negative.

11. The calculation of $F_1(1)$, $F_2(1)$, $F_3(1)$, &c., and of $F'_1(1)$, $F'_2(1)$, $F'_3(1)$, &c., which must be computed for the purpose of discovering r_1, r_2, r_3, &c., is easily effected, thus if

$$F_p\ (x_p) = Ax_p^{\ n} + Bx_p^{\ n-1} + Cx_p^{\ n-2} \ldots\ldots\ldots\ldots + \gamma x^2_{\ p} + \beta x_p + \alpha.$$

$$\text{Then } F_p\ (1) = A + B + C \ldots\ldots\ldots\ldots\ldots\ldots + \gamma + \beta + \alpha.$$

$$\text{And } F'_p\ (1) = n\,.\,A + \overline{n-1}\,.\,B\,.\ + \overline{n-2}\,.\,C \ldots\ldots\ldots\ldots + 2\gamma + \beta.$$

By referring to the formulas of transformation in Articles 2, 3, and 5, it will be seen that the absolute term remains unaltered, and consequently no column will be formed under it, this vacant space we propose to appropriate to the calculation of $F_1(1)$, $F_2(1)$, &c.; also an additional column a little to the right, to the computation of $F'_1(1)$, $F'_2(1)$, &c.

In order to elucidate our method, we shall now proceed to apply it to the solution of a few equations.

Example 1. Given $x^4 - 30\ x^3 + 700\ x^2 + 15132\ x - 1804827 = 0$, to find x.

Here x 7 30 ∠ 40, hence R=30, and the operation is as follows:

```
1
30
900
2700 0
8100 0 0
1620 0 0
9720 0 0
1944 0 0
11664 0 0
2332 8 0
13996 8 0
2799 3 6
16796 1 6
167 9 6 16
16964 1 2|16
169 6 4|12
17133 7 6|28
171 3 3|76
17305 1 0|04
173 0 5|10
17478 1|5 14
69 9|1 26
17548 0|6 40
70 1|9 23
17618 2|5 63
70 4|7 30
17688 7|2 93
70 7|5 49
17759|4 8 42
10|6 5 57
17770|1 3 99
10|6 6 21
17780|8 0 20
10|6 6 85
17791|4 7 05
10|6 7 49
17802 1 4 54
```

```
— 30
900
27000
8100 0 0
1620 0 0
9720 0 0
1944 0 0
11664 0 0
2332 8 0
13996 8 0
139 9 6·8
14136 7 6|8
141 3 6|77
14278 1 3|57
142 7 8|14
14420 9|1 71
57 6|8 37
14478 6|0 08
57 9|1 44
14536 5|1 52
58 1|4 61
14594|6 6 13
8|7 5 68
14603|4 1 81
8|7 6 20
14612|1 8 01
8|7 6 73
14620 9 4 74
```

```
700
2100 0
6300 0 0
1260 0 0
7560 0 0
1512 0 0
9072 0 0
90 7 2
9162 7 2
91 6 272
9254 3|472
37 0|174
9291 3|646
37 1|655
9328|5 301
5|5 971
9334|1 272
5|6 005
9339 7 277
```

```
1513 2
4539 6 0
907 9 2
5447 5 2
54 4 752
5501 9|952
22 0|080
5524|0 032
3|3 144
5527 3 176
```

```
—180 4827
45 × 1=
63 × 2=
— 81 × 3=
81 × 4=
—72 =F_1(1)
—180|5
54|5
90|7
—140|0
168|0
—7|3=F_2(1)
—180|48
55|02
92|54
—144|21
174|78
—2|35=F_3(1)
—180|483
55|240
93|285
—145|947
177|595
—|310=F_4(1)
—180|4827
55|273176
93|397277
—146|209474
178|021454
```

$$-\frac{F_m(1)}{F'_m(1)}=\cdot00000052)\quad -267=F_m(1)$$

```
258
9
10
```

```
(30=R
45 (1·2=1+r_1
126
—243
324
252=F'_1(1)

55 (1·01=1+r_2
181
—420
672
488=F'_2(1)

55 (1·004=1+r_3
185
—433
699
506=F'_3(1)

55 (1·0006=1+r_4
187
—438
710
514=F'_4(1)

55
187
—439
712
515=F'_m(1)
```

Hence we have 30 × 1·2 × 1·01 × 1·004 × 1·0006 × 1·00000052 for the development of the root, which may be computed as under:

```
1·0000|0 0 5 2|·0006
6 0 0 0 0
1·0006 0|0 5 2|·004
40 0 2 4 0
1·0046 0 2|9 2|·01
100 4 6 0 3
1·0146 4 8 9|5|·2
2029 2 9 7 9
1·2175 7 8 7 4
3 0
36·527 3 6 2 =x
```

In this solution the process unfolded in Article 3 has been employed; if we suppress the addends, as suggested in Art. 4, the work will take the following form:

```
1
30
900
2700 0
8100 0 0
9720 0 0
11664 0 0
13996 8 0
16796 1 6
16964 1 2|16
17133 7 6|28
17305 1 0|04
17478 1|5|14
17548 0|6 40
17618 2|5 63
17688 7|2 93
17759|4|8 42
17770|1 3 99
17780|8 0 20
17791|4 7 05
17802|1 4 54
```

```
— 30
900
2700 0
8100 0 0
9720 0 0
11664 0 0
13996 8 0
14136 7 6|8
14278 1 3|57
14420 9|1|71
14478 6|0 08
14536 5|1 52
14594|6|6 13
14603|4 1 81
14612|1 8 01
14620|9 4 74
```

```
700
2100 0
6300 0 0
7560 0 0
9072 0 0
9162 7 2
9254 3|472
9291 3|646
9328|5|301
9334|1 272
9339|7 277
```

```
1·0000|0 0 5 2|·0006
1·0006 0|0 5 2|·004
1·0046 0 2|9 2|·01
1·0146 4 8 9|5|·2
1·2175 7 8 7 4|
             3 0
36·527 3 6 2=x
```

```
1513 2
4539 6 0
5447 5 2
5501 9|952
5524|0|032
5527|3 176
```

```
—180 4827
  45
  63
— 81
  81
 —72
—180|5
  54|5
  90|7
—140|0
 168|0
  —7|3
—180|48
  55|02
  92|54
—144|21
 174|78
  —2|35
—180|483
  55|240
  93|285
—145|947
 177|595
   —|310
—180|4827
  55|273176
  93|397277
—146|209474
 178|021454
·00000052)| —267
             258
               9
              10
```

```
(30
45(1·2
126
—243
324
252

55(1·01
181
—420
672
488

55(1·004
185
—433
699
506

55(1·0006
187
—438
710
514

55
187
—439
712
515
||
```

But perhaps the best way of conducting the solution is to perform the first transformation according to Article 2, the second and third according to Article 4, and the remaining two according to Article 5, as follows :

```
1
30
900
2700 0
8100 0 0
9720 0 0
11664 0 0
13996 8 0
16796 1 6
16964 1 2|16
17133 7 6|28
17305 1 0|04
17478 1|5 14
69 9|1 26
70 1|9 23
70 4|7 31
70 7|5 50
17759|4 8 44
10|6 5 57
10|6 6 21
10|6 6 85
10|6 7 49
17802 1 4 56

−30
900
2700 0
8100 0 0
9720 0 0
11664 0 0
13996 8 0
14136 7 6|8
14278 1 3|57
14420 9|1 71
57 6|8 37
57 9|1 44
58 1|4 60
14594|6 6 12
8|7 5 68
8|7 6 20
8|7 6 73
14620 9 4 73

700
2100 0
6300 0 0
7560 0 0
9072 0 0
9162 7 2
9254 3|472
37 0|174
37 1|655
9328|5 301
5|5 971
5|6 005
9339 7 277

1·0000|0 0 5 1|·0006
1·0006 0|0 5 1|·004
1·0046 0 2|9 1|·01
1·0146 4 8 9|4|·2
1·2175 7 8 7 3|
3 0
36·527 3 6 2 = x

1513 2
4539 6 0
5447 5 2
5501 9|952
5524|0|032
5527|3 176

−180 4827
45
63
−81
81
−72
−180|5
54|5
90|7
−140|0
168|0
−7|3
−180|48
55|02
92|54
−144|21
174|78
−2|35
−180|483
55|240
93|285
−145|947
177|595
—|310
−180|4827
55|273176
93|397277
−146|209473
178|021456
·00000051)|−264
258
6
5

(30
45(1·2
126
−243
324
252
55(1·01
181
−420
672
488
55(1·004
185
−433
699
506
55(1·0006
187
−438
710
514
55
187
−439
712
515
||
```

Example 2. Given $x^5+3x^4+2x^3-3x^2-2x-2=0$ to find x.

As R = 1 the transformation in Rx_1 is dispensed with, hence

1
05
1·05
525
1·1025
5512 5
1·15762 5
5788 1 2 5
1·21550 6 2 5
6077 5 3 125
1·27628 1 5|625
1021 0 2|525
1 28649 1 8|150
1029 1 9|345
1 29678 3 7|495
1037 4 2|700
1 30715 8 0|195
1045 7 2|642
1 31761 5 2|837
1054 0 9|222
1 32815|6|2 059
79|6|8 937
9|2|9 709
1 32904|6|0 705
79|7|4 276
9|3|0 332
1 32993|6|5 313
79|7|9 619
9|3|0 955
1 33082|7|5 887
79|8|4 966
9|3|1 579
1 33171|9|2 432
79|9|0 315
9|3|2 203
1 33261 1 4 950

3
15
3·15
1575
3·3075
16537 5
3·47287 5
17364 3 7 5
3·64651 8 7|5
2917 2 1|5
3 67569 0 9|0
2940 5 5|272
3 70509 6 4|272
2964 0 7|714
3 73473 7 1|986
2987 7 8|976
3 76461|5|0 962
225|8|7 690
26|3|5 231
3 76713|7|3 883
226|0|2 824
26|3|6 996
3 76966|1|3 703
226|1|7 968
26|3|8 763
3 77218|7|0 434
226|3|3 122
26|4|0 531
3 77471 4 4 087

2
·1
2·1
105
2·205
11025
2·31525
1852 2
2·33377 2
1867 0 1 76
2·35244 2 1|76
1881 9 5|374
2 37126|1|7 134
142|2|7 570
16|5|9 883
2 37285|0|4 587
142|3|7 103
16|6|0 995
2 37444|0|2 685
142|4|6 642
16|6|2 108
2 37603 1 1 435

−3
15
3·15
1575
3·3075
2646
3·33396
2667 1 68
3·36063|1|68
201|6|3790
23|5|2442
3 36288|3|3032
201|7|7300
23|5|4018
3 36513 6 4350

−2
·1
2·1
168
2·1168
127008
148176
2·118218256

1
−·000000 1 1 7 56|
·999999|8|8 2 44|·00067
599 9 9 9 93
69 9 9 9 99
1·000669 8 8|2 36|·008
8005 3 5 9 06
1·008675 2 4 1|42|·05
50433 7 6 2 07
1·059109 0 0 3 5 = x.

−2
−2
−3
2
3
1
−1
−2
−2·1
−3·308
2 315
3·647
1·276
−·170
−2
−2·1168
−3·36063
2·37126
3·76462
1·32816
−1339
1198
141
140
−2
−2·118218256
−3·3651364350
2·3760311435
3·7747144087
1·3326114950
−·00000011756)
23562
20042
3520
2004
1516
1403
113
100
13
12

− 2(1·05
− 6
6
12
5
15

− 2·1(1·008
− 6·6
6·9
14·6
6·4
19·2

− 2·12(1·00067
− 6·72
7·11
15·06
6·64
19·97

− 2·118
− 6·730
7·128
15·099
6·663
20·042

Or, if we bear in mind that 1·00067 nearly = 1·0006 × 1·00007, the operation may be performed thus:

```
1
1·05
1·1025
1·15762 5
1·21550 6 2 5
1 27628 1 5|625
   1021 0 2|525
   1029 1 9|345
   1037 4 2|700
   1045 7 2|642
   1054 0 9|223
1 32815 6|2 060
     79 6|8 937
     79 7|3 718
     79 7|8 502
     79 8|3 289
     79 8|8 079
1 33214|5 4 585
       9|3 2 502
       9|3 2 567
       9|3 2 632
       9|3 2 697
       9|3 2 762
1 33261 1 7 745
```

```
3
3·15
3·3075
3·47287 5
3·64651 8 7|5
   2917 2 1|500
   2940 5 5|272
   2964 0 7|714
   2987 7 8|976
3 76461 5|0 962
    225 8|7 691
    226 0|1 243
    226 1|4 804
    226 2|8 373
3 77365|8 3 073
     26|4 1 561
     26|4 1 746
     26|4 1 931
     26|4 2 116
3 77471 5 0 427
```

```
2
2·1
2·205
2·31525
   1852 2
   1867 0 1|76
   1881 9 5|374
2 37126 1|7 134
    142 2|7 570
    142 3|6 107
    142 4|4 649
2 37553|2 5 460
     16|6 2 873
     16|6 2 989
     16|6 3 105
2 37603 1 4 427
```

```
−3
3·15
3·3075
   2646
   2667 1 68
3 36063 1|68
    201 6|3790
    201 7|5888
3 36466|5 6478
     23|5 5266
     23|5 5431
3 36513 6 7175
```

```
−2
2·1
2·1168
2·11807|008
2·11821 83449
```

```
  1
−·000000 1 5 9 54
  ·999999|8 4 0 46|·00007
 1·000069 8|4 0 45|·0006
 1·000669 8 8|2 35|·008
 1.008675 2 4 1|41|·05
 1·059109 0 0 3 5 = x.
```

```
−2
−2            − 2(1·05
−3            − 6
 2              6
 3             12
 1              5
−1             15
−2
−2·1          − 2·1(1·008
−3·308        − 6·6
 2·315·         6·9
 3·647         14·6
 1·276          6·4
−·170          19·2
−2
−2·1168       − 2·12(1·0006
−3·36063      − 6·72(1·00007
 2·37126        7·11
 3·76462       15·06
 1·32816        6·64
 −1339         19·97
  1198          | |
   141
   140
−2
−2·1182183449     − 2·118
−3·3651367175     − 6·730
 2·3760314427       7·128
 3·7747150427      15·099
 1·3326117745       6·663
−·00000015954)  31975    20 042
                20042    | | | |
                11933
                10021
                 1912
                 1804
                  108
                  100
                    8
                    8
```

Since the root of the preceding equation is so very much nearer 1·06 than 1·05, if we take r_1=·06 instead of=·05, the solution will become considerably shorter.

```
1                 3                 2                 —3                 —2            —2               — 2(1·06
 06                ·18               12                 18                ·12          —2               — 6
1·06              3·18              2·12              3·18              2·12           —3                 6
  636               1908              1272              1908              — 1696        2                 12
1·1236            3·3708            2·2472            3·3708              —   848       3                  5
  6741 6            20224 8           13483 2           —269 6 64       2·1182192       1                 15
1·19101 6         3·57304 8         2·38203|2           — 13 4 832                    —1
  7146 0 96         21438 2 88        —190|5|6256     3·36796|8|528                   —2
1·26247 6 96      3·78743|0|88        —  9|5|2813       —269|4|3748                   —2·12           — 2·12(1—·00084
  7574 8 6176       —302|9|9447     2 38003|1|0931      — 13|4|7187                   —3·3708         — 6·74
1·33822|5|5776      — 15|1|4972       —190|4|0249     3 36513 9 4345                    2·38203          7·15
  —107|0|5804     3 78424|9|4381      —  9|5|2012                                       3·78743         15·15
  —  5|3|5290       —302|7|3995     2 37803|1|8670                                      1·33823          6·69
1 33710|1|4682      — 15|1|3700       —190|2|4255                                          1689         20·13
  —106|9|6812     3 78107|0|6686      —  9|5|1213                                          1610           | |
  —  5|3|4841       —302|4|8565     2 37603 4 3202                                           79
1 33597|8|3029      — 15|1|2428                                                              80
  —106|8|7826     3 77789|4|5693                                                  —2
  —  5|3|4391       —302|2|3156                                                   —2·1182192         — 2·1182
1 33485|6|0812      — 15|1|1158                                                   —3·3651394345      — 6·7303
  —106|7|8849     3 77472 1 1379                                                    2·3760343202       7·1281
  —  5|3|3942                                                                       3·7747211379      15·0989
1 33373|4|8021                                                                      1·3326144649       6·6631
  —106|6|9878         1                                           —·00000056325)  112885          20·0416
  —  5|3|3494     —·000000 5 63 25                                                 100208          | | | | |
1 33261 4 4649                                                                      12677
                   ·999999|4|36 75|—·00084                                           12025
                  —    799 9 99 55|                                                     652
                  —     39 9 99 98|                                                     601
                   ·999159 4 37|22|·06                                                   51
                     59949 5 66 23|                                                      40
                  1·059109 0 035=x.                                                      11
                                                                                         10
```

Or, if we bear in mind that 1 — ·00084 very nearly = (1 — ·0008) × (1 — ·00004), the work may be performed thus :

```
1
1·06
1·1236
1·19101 6
1·26247 6 96
1 33822 5|5776
  — 107 0|5805
  — 106 9|7240
  — 106 8|8682
  — 106 8|0131
  — 106 7|1587
1 33288|1 2331
  —    5|3 3152
  —    5|3 3131
  —    5|3 3110
  —    5|3 3089
  —    5|3 3067
1 33261 4 6782
```

```
3
3·18
3·3708
3·57304 8
3·78743 0|88
  — 302 9|9447
  — 302 7|5207
  — 302 5|0987
  — 302 2|6786
3 77532|5 6373
  —  15|1 0130
  —  15|1 0070
  —  15|1 0010
  —  15|0 9950
3 77472 1 6213
```

```
2
2·12
2 2472
2·38203 2
  — 190 5|6256
  — 190 4|1011
  — 190 2|5778
2 37631|9 6955
  —    9|5 0528
  —    9|5 0490
  —    9|5 0452
2 37603 4 5485
```

```
—3
  3·18
  3·3708
  — 269 6|64
  — 269 4|4827
3 36540|8 8773
  —  13|4 6164
  —  13|4 6110
3 36513 9 6499
```

```
—2
  2·12
  2·11830|4
  2·11821|92678
```

```
—2
—2              — 2(1·06
—3              — 6
 2                6
 3               12
 1                5
—1               15
—2
—2·12           — 2·12(1—·0008
—3·3708         — 6·74(1—·00004
 2·38203          7·15
 3·78743         15·15
 1·33823          6·69
    1689         20·13
    1610          | |
      79
      80
```

```
 1
—·000000 5 95 28
  ·999999|4 04 72|—·00004
  ·999959 4|04 74|—·0008
  ·999159 4 37|22|·06
 1·059109 0 03 5=x.
```

```
                 —2
                 —2·1182192678     — 2·1182
                 —3·3651396499     — 6·7303
                  2·3760345485       7·1281
                  3·7747216213      15·0989
                  1·3326146782       6·6631
—·00000059528)        119303        20 0416
                      100208        | | | | |
                       19095
                       18037
                        1058
                        1002
                          56
                          40
                          16
                          16
```

Example 3. Given $x^6+17\,x^3-8\,x-630=0$, to find x.
Here $x \angle -3 \;7\; -4$, hence $R=-3$.

```
1                17                 −8            − 63|0          (−3
−3               −51                +24              2|4          2(1·08
9                153                25·9 2        − 45|9          −138
−27              −459               26·0|2368        72|9          437
81               495·7 2            26 0|4450     − 33|6          301
−243             535·3 7 7|6                      − 63|0
+729             578·2 0|7 81                        2|6          3(1·004
787·3 2            2 3 1|2 83                     − 57|8          −173
850·3 0 5|6        2 3 2|2 08                      115|7           694
918·3 3 0|05       2 3 3|1 37                       −2|5           524
991 7 9 6|45     585 1|7 4 09
1071 1 4 0|17        4|6 8 14                     − 63|0
1156 8 3|1|38        4|6 8 51                        2|6·02       3(1·0008
   4 6 2|7 33        4|6 8 89                     − 58|5·17       −176
   4 6 4|5 84    586 5 7 9 63                      118|4·87        711
   4 6 6|4 42                                        − |4·28       538
   4 6 8|3 08
   4 7 0|1 81
   4 7 2|0 61    1                                − 630
1184 8|7 4 47    −·0000 0 7 04                      26·04450        26
     9|4 7 90     ·9999|9 2 96|·0008              − 586·57963     −1760
     9|4 8 66    1·0007 9|2 95|·004                1190·57324       7143
     9|4 9 42    1·0047 9 6|12|·08        −·00000704) 3811          5409
     9|5 0 17    1·0851 7 9 81                        3786          |||
     9|5 0 93              −3                           25
     9|5 1 69    −3·2555 3 9 4=x.                       22
1190 5 7 3 24
```

12. The preceding belongs to a class of equations, to which it appears to me, that the method of solution here proposed applies very happily. I allude to those which are incomplete, or which want several of their terms. For if any term be absent in the given equation, the corresponding term will also be absent in each of the transformed equations, consequently if an equation have *m* terms less than a complete equation of the same dimensions, we shall get rid of *m* columns.

The preceding example with those following, will, it is thought, demonstrate the superiority of our method in the solution of equations of this kind.

Example 4. Given $7x^{16}+170x^{8}+652x^{4}+1342x^{2}+5326x-3918500=0$, to find x.

```
7
14
28
56
112
224
448
896
1792
3584
7168
14336
28672
57344
114688
229376
458752
----------------
504627·2
555089·92
610598·91 2
671658·80 3|2
738824 68 3|5
812707 15 1|9
893977 86 7|1
983375 65 3|8
1081713 21 9|2
1189884 54 1|1
1308872 99 5|2
1439760 29 4|7
1583736 32 4|2
1742109 95 6|6
1916320 95 2|3
2107953 04|7|5
----------------
2171191 63|8 9
2236327 38|8 1
2303417 20|9 7
2372519 72|6 0
2443695 31|7 8
2517006 17|7 3
2592516 36|2 6
2670291 85|3 5
2750400 60|9 1
2832912 62|7 4
2917900 00|6 2
3005437 00|6 4
3095600 11|6 6
3188468 12|0 1
3284122 16|3 7
3382645 82|8 6
----------------
```

```
170
340
680
1360
2720
5440
1088 0
2176 0
4352 0
----------------
4787 2
5265 9·2
5792 5·1 2
6371 7·6 3 2
7008 9·3 9 5|2
7709 8 3 3 4|7
8480 8 1 6 8|2
9328 8 9 8|5|0
----------------
9608 7 6 5|4 6
9897 0 2 8|4 2
10193 9 3 9|2 7
10499 7 5 7|4 5
10814 7 5 0|1 7
11139 1 9 2|6 8
11473 3 6 8|4 6
11817 5 6|9 5 1
   70 9 0|5 4 2
   71 3 3|0 8 5
   71 7 5|8 8 3
   72 1 8|9 3 8
   72 6 2|2 5 2
   73 0 5|8 2 6
   73 4 9|6 6 1
   73 9 3|7 5 9
----------------
12396 8|6 8 9 7
      6 1|9 8 4 3
      6 2|0 1 5 3
      6 2|0 4 6 3
      6 2|0 7 7 3
      6 2|1 0 8 3
      6 2|1 3 9 4
      6 2|1 7 0 5
      6 2|2 0 1 6
----------------
12446|5 4 3 2 7
     |9 9 5 7 2
     |9 9 5 8 0
     |9 9 5 8 8
     |9 9 5 9 6
     |9 9 6 0 4
     |9 9 6 1 2
     |9 9 6 2 0
     |9 9 6 2 8
----------------
12454 5 1 1 2 7
```

```
652
1304
2608
5216
1043 2
----------------
1147 5·2
1262 2·7 2
1388 4·9 9 2
1527 3·4 9|12
----------------
1573 1 6 9|59
1620 3 6 4|68
1668 9 7 5|62
1719 0 4|4 89
  10 3 1|4 27
  10 3 7|6 16
  10 4 3|8 42
  10 5 0|1 05
----------------
1760 6|7 4 79
     8|8 0 34
     8|8 0 78
     8|8 1 22
     8|8 1 66
----------------
1764|1 9 8 79
    |1 4 1 14
    |1 4 1 15
    |1 4 1 16
    |1 4 1 17
----------------
1764 7 6 3 41
```

```
134 2
268 4
536 8
----------------
590 4·8
649 5·2 8
----------------
669 0·1 3|84
689 0 8|4 26
  4 1 3|4 51
  4 1 5|9 31
----------------
697 3|7 8 08
    3|4 8 69
    3|4 8 86
----------------
698|0 7 5 63
   |  5 5 85
   |  5 5 85
----------------
698 1 8 7 33
```

```
5326
1065 2
----------------
1171 7·2
----------------
1206 8·7|16
----------------
1214 1|1 283
----------------
1214|7 1 989
----------------
1214 8 1 707
----------------
```

```
1·00000|1 1 4 4 5|·00008
1·00008 1|1 4 4 6|·0005
1·00058 1 1|8 5 2|·006
1·00658 4 6 7|2 3|·03
1·03678 2 2 1 2|5|·1
1·14046 0 4 3 3 8|
                 2|
----------------
2·28092 0 8 6 8=x.
```

```
—39|18500              (2
   |1                 0(1·1
   |1                  0
   |1                  0
   |4                  3
  4|6 × { 10=         46
—33 9  {  6=          28
                      77
```

```
—39|2
   |1                 0(1·03
   |1                 0
   |2                 1
   |9                 7
 21|1                 211
—16|8                 127
                      346
```

```
—39|19
   |12                0(1·006
   | 7                0
   |17                1
  1|18                9
 33|83                338
 —3|82                203
                      551
```

```
—39|185
   |121               0(1·0005
   | 70               0
   |176               1
  1|240               10
 37|224               372
 — |354               223
                      606
```

```
—39| 1850
   | 1215             0(1·00008
   |  698             0
   | 1764             1
  1| 2447             10
 37| 5230             375
   | —496             225
                      611
```

```
       —39185|00
          121|481707       121
           69|818733       140
          176|476341       706
         1245|451127       9964
        37571|071579       375711
·0000011445)—|700513       225426
              612068       612068
               88445       ||||
               61207
               27238
               24483
                2755
                2448
                 307
                 306
```

Continuation of the first column.

338264	5 8	286
2029	5 8	750
2041	7 6	502
2054	0 1	561
2066	3 3	970
2078	7 3	774
2091	2 1	017
2103	7 5	743
2116	3 7	997
2129	0 7	825
2141	8 5	272
2154	7 0	384
2167	6 3	206
2180	6 3	785
2193	7 2	168
2206	8 8	401
2220	1 2	531
372241	0	1 172
186	1	2 051
186	2	1 357
186	3	0 668
186	3	9 983
186	4	9 303
186	5	8 628
186	6	7 957
186	7	7 291
186	8	6 630
186	9	5 973
187	0	5 321
187	1	4 674
187	2	4 031
187	3	3 393
187	4	2 760
187	5	2 131
375230	1	3 323
30	0	1 841
30	0	2 081
30	0	2 321
30	0	2 561
30	0	2 801
30	0	3 041
30	0	3 281
30	0	3 521
30	0	3 761
30	0	4 001
30	0	4 241
30	0	4 481
30	0	4 721
30	0	4 961
30	0	5 201
30	0	5 441
375710	7	1 579

Example 5. Given $x^{11}=100000$, to find x.

```
1                  —100 000            (2
2                      2             22(1·4
4                   —98
8                   —10|0
16                    8|3            91(1·01
32                   —1|7
64                  —10|00
128                   9|25           102(1·007
256                  —|75
512                 —10|0000
1024                  9|9907         110(1·00008
2048                   |—93
2867·2
4014·08
5619·71 2
7867·59 6 8         —100000
11014·63 5 5|2        99995·34794       1099949
15420·48 9 7|3     42293)—4·65206        |||||
21588 68 5 6|2           4 39980
30224 15 9 8|7            25226
42313 82 3 8|2            21999
59239 35 3 3|5             3227
82935 09 4|6|9             2200
83764 44 5|6 4             1027
84602 09 0|1 0              990
85448 11 1|0 0               37
86302 59 2|1 1               33
87165 61 8|0 3
88037 27 4|2 1
88917 64 6|9 5
89806 82 3|4 2
90704 89 1|6 5
91611 94 0|5 7     1·00000|42 2 9 3|·00008
92528 05|9 9 8     1·00008 42|2 9 6|·007
   647 69|6 4 2    1·00708 48 1|9 2|·01
   652 23|0 2 9    1·01715 56 6 7|4|·4
   656 79|5 9 0    1·42401 79 3 4 4|
   661 39|3 4 7                   2|
   666 02|3 2 2    2·84803 58 6 9=x.
   670 68|5 3 8
   675 38|0 1 8
   680 10|7 8 4
   684 86|8 5 9
   689 66|2 6 7
   694 49|0 3 1
 99907|39 4 2 5
     7|99 2 5 9
     7|99 3 2 3
     7|99 3 8 7
     7|99 4 5 1
     7|99 5 1 5
     7|99 5 7 9
     7|99 6 4 3
     7|99 7 0 7
     7|99 7 7 1
     7|99 8 3 5
     7|99 8 9 9
 99995 34 7 9 4
```

13. It is to be observed that there is another method of obtaining the transformed equations, which may often be employed with great advantage, particularly in the solution of very high equations.

If the roots of the equation $Ax^n + Bx^{n-1} + Cx^{n-2} \ldots\ldots\ldots\ldots + \gamma x^2 + \beta x + \alpha = 0$ be divided by any number P, the resulting equation is

$$AP^n x'^n + BP^{n-1} x'^{n-1} + C\cdot P^{n-2} x'^{n-2} \ldots\ldots\ldots\ldots + \gamma P^2 x'^2 + \beta P\cdot x' + \alpha = 0, \ldots\ldots\ldots\ldots (Z).$$

Now if we compute P^2, P^3............P^n in succession, or if the equation be incomplete, such of these powers as we have occasion for, and then take the product of A and P^n, of B and P^{n-1}, of C and P^{n-2}............of γ and P^2, and of β and P, we shall obviously obtain the coefficients of the transformed equation (Z).

The two examples which follow, will, it is hoped, fully elucidate this method.

Example 6. Let the equation $102\ x^{20} - 111\ x^{19} + 72\ x^{18} - 85\ x^{17} - 31\ x^{16} + 67\ x^{15} + 201\ x^{14} + 6\ x^{13} - 83\ x^{12} - 5\ x^{11} + 123\ x^{10} - 234\ x^9 - 22\ x^8 + 93\ x^7 - 7{\cdot}6\ x^6 + {\cdot}932\ x^5 + 82{\cdot}2\ x^4 - 73{\cdot}86\ x^3 - 94\ x^2 - 2{\cdot}638\ x - 290565 = 0$, be proposed for solution.

The solution of this equation extends over the six succeeding pages, and the three last columns of the work are devoted to the computation of $1{\cdot}5^2$, $1{\cdot}5^3$............15^{20}, of $1{\cdot}02^2$, $1{\cdot}02^3$............$1{\cdot}02^{20}$, &c.

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1·218994419	99	1·0040072	0767	1·00002	000018
1·243374308	39	1·0044088	1055	1·00002	200022
1·268241794	56	1·0048105	7407	1·00002	400026
1·293606630	45	1·0052124	9830	1·00002	600031
1·319478763	06	1·0056145	8330	1·00002	800036
1·345868338	32	1·0060168	2913	1·00003	000042
1·372785705	09	1·0064192	3586	1·00003	200048
1·400241419	19	1·0068218	0355	1·00003	400054
1·428246247	57	1·0072245	3227	1·00003	600061
1·456811172	52	1·0076274	2208	1·00003	800068
1·485947395	97	1·0080304	7305	1·00004	000076

Example 7. Given $1379664\,x^{622} + 2686034\cdot10^{432}\,x^{153} - 17290224\cdot10^{518}\,x^{60} + 2524156\cdot10^{574} = 0$, to find x.

The solution of this equation also extends over the six following pages, of which four are occupied by the computation of 8^{622}, 8^{153}, and 8^{60}. of $1{\cdot}04^{622}$, $1{\cdot}04^{153}$, and $1{\cdot}04^{60}$, &c.; these powers are all calculated in the same manner, except that $1{\cdot}0000024^{15}$ is obtained by multiplying $1{\cdot}0000024^{7}$ and $1{\cdot}0000024^{8}$ together.

We have not thought it necessary to compute $F_1(1)$, $F'_1(1)$, $F_2(1)$, and $F'_2(1)$, because it is evident at a *glance* that $-\dfrac{F_1(1)}{F'_1(1)}$ will give no approximation to the value of r_1, nor $-\dfrac{F_2(1)}{F'_2(1)}$ to that of r_2.

13 79664
52 7195760274 $\times 10^{550}$
68 98320
2 759328
9657648
1379664
12416976
689832
96576
8278
28
9
1
72 735301140 $\times 10^{557}$
39 331186129·9
21 820590342
6 546177103
218205903
21820590
727353
72735
58188
4364
73
15
7
1
28 607656674 $\times 10^{568}$
22·2479711429
57 215313348
5 721531335
572153133
114430627
20025360
2574689
200254
2861
286
114
6
3
63 646232016 $\times 10^{569}$
⋮

2 68 6034 $\times 10^{432}$
1 48 856570736 $\times 10^{127}$
2 68 6034
1 07 44136
21 488272
2 1488272
13430170
1611620
134302
18802
188
8
2
3 99 83381012 $\times 10^{566}$
4 03·739194953
1 59 93352405
1 19950143
27988367
1199501
359850
3998
3599
160
36
2
1 61 42858061 $\times 10^{569}$
2·14 490197319
3 22 85716122
16 14285806
6 45714322
64571432
14528572
16143
14529
1130
48
2
1
3 46 24848107 $\times 10^{569}$
⋮

—1 7 290224 $\times 10^{518}$
1 5 3249554087 $\times 10^{43}$
1 7 290224
8 6451120
51870672
34580448
6916090
1556120
86451
8645
692
14
1
2 6 497191181 $\times 10^{569}$
1 0·5196274081
2 6 497191181
1 324859559
26497191
23847472
1589831
52994
18548
1060
21
2 7 874057857 $\times 10^{570}$
1·3 4885015255
2 7 874057857
8 362217357
1 114962314
222992463
22299246
1393703
2787
1394
56
14
1
3 7 597927192 $\times 10^{570}$
⋮

2524156 $\times 10^{574}$

(8
(1·04

(1·005

2|524
—3|760 × 60 - —226(1·0007
|346 × { 100 = 35
{ 50 = 17
{ 3 = 1
|636 × { 600 = 382
— |254 { 20 = 13
{ 2 . 1
·223

```
        ⋮
6 3646232016 × 10^569
1·545345764
6 36462320
3 18231160
  25458493
   3182312
    190939
     25458
      3182
       446
        38
         3
9 83554351 × 10^571
1·038023974
9 83554351
  29506631
   7868435
     19671
      2951
       885
        69
         4
1 020952997
1·001493913
1 020952997
   1020953
    408381
     91886
      3063
       919
        10
         3
1 022478212
```

```
        ⋮
3 4624848107 × 10^569
1·113003851
3 46248481
  34624848
   3462485
   1038745
      1039
       277
        17
3 85375892 × 10^571
1·009221988
3 85375892
   3468383
     77075
      7708
       385
       347
        31
         3
3 88929824
1·000367267
3 88929824
    116679
     23336
      2723
        78
        23
         3
3 89072666
```

```
        ⋮
3 7597927192 × 10^570
1·0428791555
3 759792719
  150391709
    7519585
    3007834
     263185
      33838
        376
        188
         19
          2
3 921009455 × 10^571
1·0036063794
3 921009455
   11763028
    2352606
      23526
       1176
        274
         35
          2
3 935150102
1·0001440102
3 935150102
     393515
     157406
      15741
         39
          1
3 935716804
```

```
   2|524
  —3|921        —235(1·0006
    |385          39
    |             19
    |              1
    |984         590
  — |              20
  — | 28            2
                  436

   2|52416
  —3|93515      —236(1·0000024
    |38893        39
    |             19
    |              1
   1|02095       613
  — |—111         20
       92          2
       19        458
       18        | |

  252|4156000
 —393|5716804       —23614
   38|9072666         3891
     |                1945
     |                 117
  102|2478212        61349
2161)| —9926          2045
         9187          204
          739        45937
          459        | | | |
          280
          275
            5
            5
```

```
1·0000|0|0̈70 2 1 61|·0000024
1·0000 0 2|4 2 1 61|·00006
1·0000 6 2 4|2 1 76|·0007
1·0007 6 2 4 6|5 45|·005
1·0057 6 6 2 7 7|78|·04
1·0459 9 6 9 2 8 89|
                  8|
8·3679 7 5 4 3 1 = x.
```

8
64
512
4096
32768
262144
2097152
16777216
134217728
1073741824
8589934592
68719476736
549755813888
4398046511104
$8^{15} =$ 35184372088832
35184372088832

105553116266496
17592186044416
351843720888
281474976711
14073748836
1055531163
246290605
7036874
281475
28147
2815
106
7

$8^{30} =$ 123794003928539 × 10^{13}
123794003928539

123794003928539
24758800785708
3713820117856
866558027500
111414603536
4951760157
3713820
1114146
24759
9904
619
37
11

$8^{60} =$ 153249554086592 × 10^{40}
$8^{15} =$ 35184372088832

459748662259776
76624777043296
1532495540866
1225996432693
61299821635
4597486623
1072746879
30649911
1225996
122600
12260
460
31

$8^{75} =$ 539198933343026 × 10^{53}

$8^{75} =$ 539198933343026 × 10^{53}
$8^{76} =$ 431359146674421 × 10^{54}
$8^{77} =$ 345087317339537 × 10^{55}

129407744002326
17254365866977
2156795733372
34508731734
3019514027
129407744
4313591
3019514
129408
12941
3882
216
13
3

$8^{153} =$ 148856570735748 × 10^{124}
$8^{154} =$ 119085256588598 × 10^{125}
$8^{155} =$ 95268205270878 × 10^{126}
$8^{156} =$ 76214564216702 × 10^{127}

6668774368961
571609231625
19053641054
952682053
381072821
47634103
5716092
381073
19054
953
572
67

$8^{311} =$ 7260824748428 × 10^{268}
7260824748428

508257732390
14521649497
4356494849
58086598
1452165
290433
50826
2904
581
29
1
1

$8^{622} =$ 527195760274 × 10^{550}

1·0 4
1·0 816
1·1 24864
1·1 6985856
1·2 166529024
1·2 65319018496
1·3 15931779235 | 84
1·3 68569050405 | 27
1·4 23311812421 | 48
1·4 80244284918 | 34
1·5 39454056315 | 07
1·6 01032218567 | 67
1·6 65073507310 | 38
1·7 31676447602 | 80
1·8 00943505506 91
1·8 00943505506 91

1 8 00943505506 91
1 4 40754804405 53
1620849154 96
72037740 22
5402830 52
900471 75
9004 72
900 47
10 80
1 62
2

3·2 43397510027 52
3·2 43397510027 52

9 7 30192530082 56
6 48679502005 50
1 29735900401 10
9730192530 08
973019253 01
291905775 90
22703782 57
1621698 76
32433 98
64 87
22 70
1 62
6

1 0·51962740805 271
1·8 00943505506 91

1 0 51962740805 27
8 41570192644 22
946766466 72
42078509 63
3155888 22
525981 37
5259 81
525 98
6 31
95
1

1 8·94525466088 49

18·9 452 54660 8 8|49
19·7 030 64847 3 2|03
20·4 911 87441 2 1 31

39 4 061 29694 6 4 1
7 881 22593 8 9 3
1 773 27583 6 2 6
19 70306 4 8 5
1 97030 6 4 8
1 57624 5 1 9
13792 1 4 5
788 1 2 3
78 8 1 2
1 9 7 0
3 9 4
2 0
6

40 3·739 19495 2|8 2
41 9·888 76275 0|9 3
43 6·684 31326 0|9 7
45 4·151 68579 1 4 1

17 4 673 72530 4 4
2 1 834 21566 3 0
1 746 73725 3 0
43 66843 1 3
21 83421 5 7
43668 4 3
26201 0 6
3493 4 7
218 3 4
30 5 7
3 9 3
4
2

19 8 320·91702 6 0
19 8 320·91702 6 0

19 8 320 91702 6
17 8 488 82532 3
1 5 865 67336 2
594 96275 1
39 66418 3
1 78488 8
1983 2
1388 2
4 0
1 2

39 3 311 86129·9

1·005
1·010025
1·015075125
1·02015050062|5
1·02525125312|813
1·03037750939|377
1·03552939694|074
1·04070704392|544
1·04591057914|507
1·05114013204|080
1·05639583270|100
1·06167781186|450
1·06698620092|382
1·07232113192|844
1·07768273758 808
1·07768273758 808

1 07768273758 808
7543779163 117
754377916 312
64660964 255
8621461 901
215536 548
75437 792
3233 048
754 378
53 884
8 621
862
9

1·16140008289 535
1·16140008289 535

1 16140008289 535
11614000828 954
6968400497 372
116140008 290
46456003 316
9291 200
232 280
92 912
10 453
581
35
6

1·34885015254 934
1·07768273758 808

1 34885015254 934
9441951067 845
944195106 784
80931009 153
10790801 220
269770 031
94419 511
4046 550
944 195
67 443
10 791
1 079
11

1·45363252499 547

1·4 536325249 9|547
1·4 609006876 2|045
1·4 682051910 5 855

1 4 609006876 2 05
5 843602750 4 82
876540412 5 72
116872055 0 10
2921801 3 75
73045 0 34
1460 9 01
1314 8 11
14 6 09
7 30
1 17
7
1

2·1 449019731|8 54
2·1 556264830|5 13
2·1 664046154|6 66
2·1 772366385 4 39

4 3 328092309 3 3
2 166404615 4 7
1 516483230 8 3
151648323 0 8
4332809 2 3
649921 3 8
129984 2 8
12998 4 3
649 9 2
173 3 1
10 8 3
8 7
6
2

4·7 167755027 0 4
4·7 167755027 0 4

1 8 867102010 8
3 301742851 9
47167755 0
28300653 0
3301742 9
330174 3
23583 9
2358 4
9 4
3 3

2 2·247971142 9

1·0007
1·00140049
1·002101470|343
1·002802941|3722
1·003504903|4312
1·004207356|8636
1·004910302|0134
1·005613739|2248
1·006317668|8423
1·007022091|2105
1·007727006|6743
1·008432415|5790
1·009138318|2699
1·009844715|0927
1·010551606 3933
1·010551606 3933

1·010551606 3933
10105516 0639
505275 8032
50527 5803
1010 5516
606 3310
6 0633
3032
909
30
3

1·021214549 1840
1·021214549 1840

1 021214549 1840
20424290 9837
1021214 5492
204242 9098
10212 1455
4084 8582
510 6073
40 8486
9 1909
1021
817
41

1·042879155 4651
1·010551606 3933

1·042879155 4651
10428791 5547
521439 5777
52143 9578
1042 8792
625 7275
6 2573
3129
939
31
3

1·053883205 8295

1·05388320 5|8295
1·05462092 4|0736
1·05535915 8 7204

1 05462092 4 074
5273104 6 204
527310 4 620
31638 6 277
5273 1 046
949 1 588
10 5 462
5 2 731
8 437
738
21

1·11300385|1 198
1·11378295|3 894
1·11456260|1 962
1·11534279 5 783

1 11456260 1 96
11145626 0 20
1114562 6 02
557281 3 01
33436 8 78
4458 2 50
222 9 13
78 0 19
10 0 31
5 57
78
9

1·24311936 8 54
1·24311936 8 54

1 24311936 8 5
24862387 3 7
4972477 4 7
372935 8 1
12431 1 9
1243 1 2
1118 8 1
37 2 9
7 4 6
9 9
6

1·54534576 4 2

1·00006
1·0001200|036
1·0001800|10800
1·0002400|21600
1·0003000|36001
1·0003600|54003
1·0004200|75606
1·0004801|00811
1·0005401|29617
1·0006001|62025
1·0006601|98035
1·0007202|37647
1·0007802|80861
1·0008403|27678
1·0009003 78098
1·0009003 78098

1 0009003 78098
9008 10340
3 00270
70063
8007
90
8

1·0018015 66876
1·0018015 66876

1 0018015 66876
10018 01567
8014 41254
10 01802
5 00901
60108
6011
801
70
6

1·0036063 79396
1·0009003 78098

1 0036063 79396
9032 45741
3 01082
70252
8029
90
8

1·0045100|04598
1·0045702|75198
1·0046305 49415

1 0045702 7520
40182 8110
6027 4217
301 3711
5 0229
4018
904
40
1
1

1·0092219 8751

1·009221 98751
1·009282 54083
1·009343 09778
1·009403 65837

1 009343 0978
9084 0879
403 7372
3 0280
6056
505
81
3
1

1·018834 6155
1·018834 6155

1 018834 616
10188 346
8150 677
815 068
30 565
4 075
611
10
5
1

1·038023 974

1·00000 2 4
2 000005
400001

1·00000 4 800006
2 000010
400002

1·00000 7 200018
2 000014
400003

1·00000 9 600035
2 000019
400004

1·00001 2 000058
2 000024
400005

1·00001 4 400087
2 000029
400006

1·00001 6 800122
2 000034
400007

1·00001 9 200163
1·00001 6 800122

1 00001 9 200163
1 0 000192
6 000115
800015
122

1·00003 6 000607
1·00003 6 000607

1 00003 6 000607
3 0 001080
6 000216
607

1·00007 2 002510
1·00007 2 002510

1 00007 2 002510
7 0 005040
2 000144
2510

1·00014 4 010204
1·00003 6 000607

1 00014 4 010204
3 0 004320
6 000864
607

1·00018 0 015995
2 000360
400072

1·00018 2 416427
2 000365
400073

1·00018 4 816865

1·0001 8 4816865
1·0001 8 2416427

1 0001 8 481687
1 0 001848
8 001479
200037
40007
1643

1·0003 6 726701
200073
40015

1·0003 6 966789
200074
40015

1·0003 7 206878
200074
40015

1·0003 7 446967
1·0003 7 206878

1 0003 7 44697
3 0 01123
7 00262
20007
688

1·0007 4 66777
1·0007 4 66777

1 0007 4 6678
7 0 0523
4 0030
6004
678

1·0014 9 3913

14. We shall, in conclusion, advert to a method of computing the powers of $1+r_1$, $1+r_2$, &c., although it may not, perhaps, be of much importance.

By the binomial theorem,

$$(1+r_p)^m = 1 + m \,.\, r_p + m . \frac{m-1}{2} r_p{}^2 + \text{ \&c.}$$

Now when r_p is very small a few of the leading terms of this expansion will generally give the value of $(1+r_p)^m$ to that number of decimals which we require, and the computation of these will be very easy, for suppose we can forsee that a certain number of terms, say four, will be sufficient, then we shall have

$$(1+r_p)^m = 1 + m \,.\, r_p + m \,.\, \frac{m-1}{2} r_p{}^2 + m \,.\, \frac{m-1}{2} \,.\, \frac{m-2}{3} \,.\, r_p{}^3,$$

which put $=1 + \text{A}\, r_p + \text{B}\, r_p{}^2 + \text{C}\, r_p{}^3$,

which we compute thus, multiply C by r_p, and add B to the product, multiply this sum by r_p, and add A to the product, multiply once more by r_p, and add unity to the product, we shall thus evidently obtain $1 + \text{A}\, r_p + \text{B}\, r_p{}^2 + \text{C}\, r_p{}^3$ or $(1+r_p)^m$. Should it take a greater number of terms to give the value of $(1+r_p)^m$ to the extent which we require, we must proceed in precisely the same way.

To exemplify this method, let Example V., that is $x^{11} = 100000$, be re-proposed, the root to be found to about twenty places of decimals.

We shall effect the first transformation according to Art. 2, the four next according to Art. 4, and the remaining two according to Art. 13.

In this example we have $(1+r_p)^m = (1+r_p)^{11} = 1 + 11\, r_p + 55 \,.\, r_p{}^2 + 165\, r_p{}^3 + 330\, r_p{}^4 + \text{\&c.}$ We shall hence easily obtain $1{\cdot}000004^{11} = 1 + {\cdot}00011 \times {\cdot}4 + {\cdot}0000000055 \times {\cdot}4^2 + {\cdot}000000000000165 \times {\cdot}4^3 + {\cdot}0000000000000000033 \times {\cdot}4^4$, and $1{\cdot}00000022916^{11} = 1 + {\cdot}000011 \times {\cdot}22916 + {\cdot}000000000055 \times {\cdot}22916^2 + {\cdot}000000000000000165 \times {\cdot}22916^3$.

1		
2		
4		
8		
16		
32		
64		
128		
256		
512		
1024		
2048		
2867·2		
4014·08		
5619·712		
7867·5968		
11014·63552		
15420·489728		
21588·6856192		
30224·15986688		
42313·823813632		
59239·3533390848		
82935·09467471872		
83764·4456214659072		
84602·09007768056627	2	
85448·11097845737193	4	72
86302·59208824194565	4	07
87165·61800912436511	0	61
88037·27418921560876	1	72
88917·64693110776484	9	34
89806·82340041884249	7	83
90704·89163442303092	2	81
91611·94055076726123	2	04
92528·05995627493384	4	36
93175·75637596885838	1	27
93827·98667060064038	9	94
94484·78257729484487	2	67
95146·17605533590878	6	78
95812·19928772326014	8	29
96482·88468273732296	9	33
97158·26487551648423	0	12
97838·37272964509961	9	73
98523·24133875261531	7	07
99212·90402812388362	4	29
99907·39435632075080	9	66

9 9907·394356320750	80966
9 9915·386947869256	46972
9 9923·380178825086	01024
9 9931·374049239392	01712
9 9939·368559163331	16848
9 9947·363708648064	23497
9 9955·359497744756	08011
9 9963·355926504575	66060
9 9971·352994978696	02665
9 9979·350703218294	32233
9 9987·349051274551	78588
9 9995·348039198653	75002
1·0000 440008800105	600845
9 9995 348039198653	75002
3 999813921567	94615
399981392156	79462
79996278	43136
7999627	84314
999	95348
49	99767
5	99972
	800
	40
	5
9 9999·747922509340	72461
1·0000 025207628882	887937
9 9999 747922509340	72461
199999495845	01868
49999873961	25467
1999994958	45019
69999823	54576
5999984	87535
199999	49585
79999	79834
7999	97983
799	99798
19	99995
7	99998
	80000
	7000
	900
	30
	7
9 9999·999998162742	02056

—10 0000 (2
2 22(1·4
— 9 8

—10|0
8|3 91(1·01
— 1|7

—10|00
9|25 102(1·007
— |75

—10|0000
9|9907 110(1·00008
|—93

—10|00000
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