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OF THE

STEAM ENGINE,

AND THE

MODE OF CALCULATION BY MEANS OF IT,

OF

THE EFFECTIVE POWER &c. OF

EVERY KIND OF STEAM ENGINE, STATIONARY OR LOCOMOTIVE.

BY

THE CHEVALIER G. DE PAMBOUR.

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In our Treatise on Locomotive Engines, the first edition of which appeared in the beginning of 1835, was published the basis of a new theory of the steam engine. We then limited ourselves to showing its application to locomotives, merely announcing that it was no less indispensable for calculating with exactitude both the effects and the proportions of stationary steam engines of every kind. The memoir of which we now offer an analysis, and which was read by parts at the *Institute Royale* of France, from February till the close of the year, 1837, has for its object to give a farther development of that theory, and to extend it to the various systems of steam engines in use. It consists of three parts, namely:—

Part i.—Proofs of the inexactitude of the ordinary methods of calculation, used to determine the effects or the proportions of steam engines; and a succinct exposition of the method proposed.

Part ii.—General formulæ for the calculation of the effects, &c. of rotative, stationary or locomotive, high or low pressure, expansive or unexpansive, condensing or uncondensing steam engines, according to the proposed theory.

Part iii.—Special application of these formulæ to the divers systems of steam engines in use.

PART I.

PROOFS OF THE INEXACTITUDE OF THE ORDINARY METHODS, AND EXPOSITION OF THE ONE PROPOSED.

§ 1. *Mode of calculation hitherto in use.*—All the problems in the application of steam engines merge into these three—

The velocity of the motion being given, to find the load the engine will move at that velocity.

The load being given, to find the velocity at which the engine will move that load;

And, the load and the velocity being given, to find the vaporization necessary, and consequently the area of heating surface requisite for the boiler, in order that the given load be set in motion at the given velocity.

The problem, which consists in determining the useful effect to be expected from an engine of which the number of strokes of the piston per minute is counted, that is, whose velocity is known, evidently amounts to determining the effective load corresponding to that velocity; for that load being once known, by multiplying it by the velocity we have the useful effect required.

According to the mode of calculation hitherto admitted, when it is wanted to know the useful effect an engine will produce at a given velocity, or, in other words, the effective load that it will set in motion at that velocity, the area of the cylinder is multiplied by the velocity of the piston, and that product by the pressure of steam in the boiler; this gives, in the first place, what is called the theoretical effect of the engine. Then, as experience has shown that steam engines can never completely produce this theoretical effect, it is reduced in a certain proportion, indicated by a constant number, which is the result of a comparison between the theoretical and practical effects of some engines previously put to trial; and thus is obtained the number which is regarded as the practical effect of the engine, or the work it really ought to execute.

A mode perfectly similar is followed, for determining the vaporization which an engine ought to produce in order to produce a desired effect; that is to say, for resolving the third of the problems which we have presented above. As to the second of these problems, that which consists in determining the velocity the engine will assume under a given load, no solution of it has been proposed in this way, and we shall expose, farther on, some fruitless essays that have been made to resolve it in another way.

As in the above-mentioned calculation no account is taken of friction, nor of some other circumstances which appear likely to diminish the power of the engine, the difference observed between the theoretical and the practical result excites no surprise, and is readily attributed to the circumstances neglected in the calculation.

§ 2. *First objection against this mode of calculation.*—This mode of calculation is liable to many objections, but for the sake of brevity we limit ourselves to the following:—

The coefficient adopted to represent the ratio of the practical effects to the theoretical, varies from $\frac{1}{3}$ to $\frac{2}{3}$, according to the various systems of steam engines; that is to say, that from $\frac{2}{3}$ to $\frac{1}{3}$ of the power exerted by the machine is considered to be absorbed by friction and divers losses. Not that this friction and these losses have been measured and found to be so much, but merely because the calculation that had been made, and which might have been inexact in principle, wanted so much of coinciding with experience.

Now it is easy to demonstrate, that the friction and losses which take place in a steam engine can never amount to $\frac{2}{3}$, nor to $\frac{1}{3}$ of the total force it developes. It will suffice to cast an eye on the explanation attempted, on this point, by Tredgold, who follows this method in his Treatise on Steam

Engines.* He says (art. 367,) that, for high pressure engines, a deduction of $\frac{4}{10}$ must be made from the *total* pressure of the steam, which amounts to a deduction of $\frac{5}{10}$ on the ordinary *effective* pressure of such engines; and to justify this deduction, which however is still not enough to harmonize the theoretical and practical results in many circumstances, he is obliged to estimate the friction of the piston, with the losses or waste, at $\frac{2}{10}$ of the power, and the force requisite for opening the valves and overcoming the friction of the parts of the machine, at $\frac{6}{100}$ of that power. Reflecting that these numbers express fractions of the gross power of the engine, we must readily be convinced that they cannot be correct; for, in supposing the engine had a useful effect of 100 horses, which, from the reduction or coefficient employed, supposes a gross effect of 200 horses, 12 would be necessary to move the machinery, 40 to draw the piston, &c.! The exaggeration is evident.

Besides, in applying this evaluation of the friction to a locomotive engine, which is also a high pressure steam engine, and supposing it to have 2 cylinders of 12 inches diameter, and to work at 75 ℔s. total pressure, which amounts to 60 ℔s. effective pressure, per square inch, we find that from the preceding estimate, the force necessary to draw the piston would be 5650 ℔s., whereas our own experiments on the locomotive engine, the *Atlas*, which is of these dimensions, and works at that pressure, demonstrate that the force necessary to move, not only the two pistons, but all the rest of the machinery, including the waste, &c., is but 48 ℔s. applied to the wheel, or 283 ℔s. applied on the piston.

It is then impossible to admit, that in steam engines the friction and losses can absorb the half, nor the third, much

* The author here refers to the first edition of "Tredgold on the Steam Engine:" in the new edition just published the algebraic parts are transformed by the editor into easy practical rules, accompanied by examples familiarly explained for the working engineer.

less the $\frac{2}{3}$ of the total power developed; and yet there do occur cases wherein, to reconcile the practical effects with the theoretical ones thus calculated, it would be necessary to reduce the latter to the fourth part, and even to less; and what is more, it often happens, that the same engine which in one case requires a reduction of $\frac{3}{4}$, will not in other cases need a reduction of more than about $\frac{1}{6}$. This is observed in calculating the effects of locomotive engines at very great velocities, and afterwards at very small ones.

There is no doubt, then, that the difference observed between the theoretical effect of an engine and the work which it really performs, does not arise from so considerable a part of the applied force being absorbed by friction and losses, but rather from the error of calculating in this manner the theoretical effect of the machine. In effect, this calculation supposes that the motive force, that is, the pressure of the steam *against the piston or in the cylinder,* is the same as the pressure of the steam in the boiler; whereas we shall presently see, that the pressure in the cylinder may be sometimes equal to that of the boiler, sometimes not the half nor even the the third of it, and that it depends on the resistance overcome by the engine.

§ 3. *Formulæ proposed by divers authors to determine the velocity of the piston under a given load, and proofs of their inexactitude.*—We have said that this problem was not resolved by the foregoing method. The following are the attempts made to that end by another way. Tredgold, in his Treatise on Steam Engines (art. 127 and following,) undertakes to calculate the velocity of the piston from considerations deduced from the velocity of the flowing of a gas, supposed under a pressure equal to that of the boiler, into a gas supposed at the pressure of the resistance. He concludes from thence, that the velocity of the piston would be expressed by this formula,

$$V = 6{\cdot}5 \sqrt{h},$$

in which V is the velocity in feet per second, and h stands for the difference between the heights of two homogeneous columns of vapour, one representing the pressure in the boiler, the other that of the resistance. But it is easily seen, that this calculation supposes the boiler filled with an inexhaustible quantity of vapour, since the effluent gas is supposed to rush into the other with all the velocity it is susceptible of acquiring, in consequence of the difference of pressure. Now such an effect cannot be produced, unless the boiler be capable of supplying the expenditure, however enormous it might be. This amounts consequently to supposing that the production of steam in the boiler is unlimited. But, in reality, this is far from being the case. It is evident that the velocity of the piston will soon be limited by the quantity of steam producible by the boiler in a minute. If that production suffice to fill the cylinder 200 times in a minute, there will be 200 strokes of the piston per minute; if it suffice to fill it 300 times, there will be 300 strokes. It is then the vaporization of the boiler which must regulate the velocity, and no calculation which shall exclude that element can possibly lead to the true result; consequently the preceding formula cannot be exact.

This is why, in applying this formula to the case of an ordinary locomotive engine of the Liverpool Railway with a train of 100 tons, the velocity the engine ought to assume is found to be 734 feet per second, instead of twenty miles an hour, or five feet per second, which is its real velocity.

Again, in his Treatise on Railways (page 83,) Tredgold proposes the following formula, without in any way founding it on reasoning or on fact:

$$V = 240 \sqrt{\frac{l\,P}{W}},$$

in which V is the velocity of the piston in feet per minute, l the stroke of the piston, P the effective pressure of the steam in the boiler, and W the resistance of the load. But as this formula makes no mention either of the diameter of the cylinder, or of the quantity of steam supplied by the boiler in a minute, it clearly cannot give the velocity sought; for if it could, the velocity of an engine would be the same with a cylinder of one foot diameter as with a cylinder of four feet, which expends sixteen times as much steam. The area of heating surface, or the vaporization of the boiler, would be equally indifferent: an engine would not move quicker with a boiler vaporizing a cubic foot of water per minute, than with one that should vaporize but $\frac{1}{4}$ or $\frac{1}{20}$. Hence this formula is without basis.

Wood, in his Treatise on Railways (page 351,) proposes the following formula also, without discussion,

$$V = 4\sqrt{\frac{lP}{W}},$$

where V is the velocity of the piston in feet per minute, l the length of stroke of the piston, W the resistance of the load, and P the surplus of the pressure in the boiler, over and above what is necessary to balance the load W. This formula being liable to the same objections as the preceding, is also demonstrated inadmissible *à priori.*

Consequently, of the three fundamental problems of the calculation of steam engines, two have received inaccurate solutions by means of the coefficients, and the third, as we have just seen, has received no solution at all.

§ 4. *Succinct exposition of the proposed theory.*—After having made known the present state of science, with regard to the theory and estimation of the effective power of steam engines, it remains to exhibit the theory we apply to them ourselves.

It is well known, that in every machine, when the effort of the motive power becomes superior to the resistance, a slow motion is created, which quickens by degrees till the machine has attained a certain velocity, beyond which it does not go, the motive power being incapable of producing greater velocity with the mass it has to move. Once this point attained, which requires but a very short space of time, the velocity continues the same, and the motion remains uniform as long as the effort lasts. It is from this point only that the effects of engines begin to be reckoned, because they are never employed but in that state of uniform motion; and it is with reason that the few minutes, during which the velocity regulates itself, and the transitory effects which take place before the uniform velocity is acquired, are neglected.

Now, in an engine arrived at uniform motion, the force applied by the motive power forms strictly an equilibrium with the resistance; for if that force were greater or less, the motion would be accelerated or retarded, which is contrary to the hypothesis. In a steam engine the force applied by the motive agent is nothing more than the pressure of the steam *against the piston* or *in the cylinder.* The pressure therefore in the cylinder is strictly equal to the resistance of the load against the piston.

Consequently the steam, in passing from the boiler to the cylinder, may change its pressure, and assume that which is represented by the resistance of the piston. This fact alone exposes all the theory of the steam engine, and in a manner lays its play open.

From what has been said, the force applied on the piston, or the pressure of the steam in the cylinder, is therefore strictly regulated by the resistance of the load against the piston. Consequently calling P' the pressure of the steam in the cylinder, and R the resistance of the load against the piston, we have as a first analogy,

$$P' = R.$$

To obtain a second relation between the data and the quæsita of the problem, we shall observe that there is a necessary equality between the quantity of steam produced, and the quantity expended by the machine; the proposition is self-evident. Now if we express by S the volume of water vaporized in the boiler per minute, and effectively transmitted to the cylinder, and by m the ratio of the volume of the steam generated under the pressure P of the boiler, to the volume of water which produced it, it is clear that

$$mS$$

will be the volume of steam formed per minute in the boiler.

This steam passes into the cylinder, and there assumes the pressure P′; but if we suppose that, in this motion, the steam preserves its temperature in passing from the boiler to the cylinder, or from the pressure P to the pressure P′, its volume increases in the inverse ratio of the pressures. Thus the volume *m* S of steam furnished per minute by the boiler will, when transmitted to the cylinder, become

$$m\,S.\,\frac{P}{P'}.$$

On another hand, *v* being the velocity of the piston, and *a* the area of the cylinder, *a v* will be the volume of steam expended by the cylinder in a minute. Wherefore, by reason of the equality which necessarily exists between the production of the steam and the expenditure, we shall have the analogy of

$$a\,v = m\,S.\,\frac{P}{P'};$$

which is the second relation sought.

Consequently, by exterminating P′ from the two equations, we shall have as a definitive analytic relation among the different data of the problem:

$$v = \frac{m\,S}{a}.\,\frac{P}{R},$$

This relation is very simple, and suffices for the solution of all questions regarding the determination of the effects or the proportions of steam engines. As we shall develope its terms hereafter, in taking it up in a more general manner, we content ourselves to leave it for the present under this form, which will render the discussion of it easier and clearer.

The preceding equation gives us the velocity assumed by the piston of an engine under a given resistance R. If, on the contrary, the velocity of the motion be known, and it be required to calculate what resistance the engine will move at that velocity, it will suffice to resolve the same equation with reference to R, which will give

$$R = \frac{m S P}{a v}.$$

Finally, supposing the velocity and the load to be given beforehand, and that it be desired to know what vaporization the boiler should have to set the given load in motion at the prescribed velocity, it will still suffice to draw from that analogy the value of S, which will be

$$S = \frac{a v R}{m P}.$$

On these three determinations we rest for the moment, because, as will soon appear, they form the basis of all the problems that can be proposed on steam engines.

§ 5. *New proofs of the exactitude of this theory, and of the inaccuracy of the ordinary mode of calculation.*—The theory just developed demonstrates that the steam may be generated in the boiler at a certain pressure P, but that in passing to the cylinder it necessarily assumes the pressure R, strictly determined by the resistance to the piston, whatever the pressure in the boiler may be. Consequently, according to the intensity of that resistance, the pressure in the cylinder, far from being equal to that in the boiler, or from differing from it in a certain constant ratio, may at times be equal to it, and at other times very considerably different. Hence those who, in performing the ordinary calculation, consider the force applied on this piston as indicated by the pressure in the boiler, begin by introducing into their calculation an error altogether independent of the real losses to which the engine is liable. To this cause, then, and not to the friction and losses, which can form but the smallest part of it, must be attributed the enormous difference which, in this mode of calculation, is found between the theoretical effect of the engine, and the work which it really executes.

We have already proved the mode of action of the steam in the cylinder by the consideration of uniform motion; but in examining what passes in the engine, we shall immediately find many other proofs.

1st. The steam, in effect, being produced at a certain degree of pressure in the boiler, passes into the tube of communication, and thence into the cylinder. It first dilates, because the area of the cylinder is from ten to twenty-five times that of the tube; but it would promptly rise to the same degree as in the boiler, were the piston immoveable. But as the piston, on the contrary, opposes only a certain resistance, determined by the load sustained by the engine, it will yield as soon as the elastic force of the steam in the cylinder shall have attained that point. The piston, in consequence, will be a valve to the cylinder. Hence the pressure in the cylinder can never exceed the resistance of the piston, for that would be supposing a vessel full of steam, in which the pressure of the steam would be greater than that of the safety valve.

2nd. Were it true, that the steam flowed into the cylinder, either at the pressure of the boiler, or at any other pressure which were to that of the boiler in any fixed ratio, as the quantity of steam generated per minute in the boiler would then flow at an identical pressure in all cases, and would consequently fill the cylinder an identical number of times per minute; it would follow, that as long as the engine should work with the same pressure in the boiler, it would assume the same velocity with all loads. Now we know that precisely the contrary takes place, the velocity increasing when the load diminishes; and the reason of it is, that when the load is half, the steam flowing also at a half pressure into the cylinder, and consequently acquiring a volume double what it had before, will serve for double the number of strokes of the piston.

3rd. Applying the same reasoning inversely, we perceive that were the pressure in the cylinder really bearing a constant ratio to that in the boiler, or if it be preferred, constant so long as that in the boiler did not vary, we should, in calculating the effort of which the engine would be capable, always find it the same, whatever be the velocity of the piston. Thus, at any velocity whatever, the engine would always be capable of drawing the same load;

which experience again contradicts, for the greater the velocity of the piston, the lower the pressure of the steam in the cylinder, whence results, that the load of the engine lessens at the same time.

4th. Another no less evident proof of this is easily adduced. Were it true that the pressure in the cylinder were to that in the boiler in any fixed proportion, since the same locomotive engine always requires the same number of revolutions of the wheel, or the same number of strokes of the piston to traverse the same distance, it would follow that, as long as those engines worked at the same pressure, they would consume in all cases the same quantity of water for the same distance. Now the quantity of water, far from remaining constant, decreases on the contrary with the load, as may be seen by the experiments we have published on this subject. Here therefore again it is proved, that, notwithstanding the equality of pressure in the boiler, the density of the steam expended follows the intensity of the resistance, that is to say, the pressure in the cylinder is regulated by that resistance.

5th. Similarly, the consumption of fuel being in proportion to the vaporization effected, it would follow, if the ordinary theory were exact, that the quantity of fuel consumed by a given locomotive, for the same distance, would always be the same, with whatever load. Now we again find by experience that the quantity of fuel diminishes with the load, conformably to the explanation we have given of the effects of the steam in the engine.

6th. It is again clear that if the pressure in the cylinder were, as it is believed, constant for a given pressure in the boiler, that so soon as it was recognised that an engine could draw a certain load with a certain pressure, and communicate to it a uniform motion, it would follow, that the same engine could never draw a less load with the same pressure, without communicating to it a velocity indefinitely accelerated; since the power, having been found equal to the resistance of the first load, would necessarily be superior to that of the second. Now experience proves, that in the second case the velocity is greater, but that the motion is no less uniform than in the first; and the reason of this is, that though the steam may indeed be produced in the boiler at a greater or less pressure, and that it matters little, yet on passing into the cylinder, it always assumes the pressure of the resistance, whence results that the motion must remain uniform as before.

7th. Finally, in looking over our experiments on locomotives, it will be seen that the same engine will sometimes draw a light load with a very high pressure in the boiler, and sometimes a heavy load with a very low pressure. It is then impossible to admit, as the ordinary calculation supposes, that any fixed ratio *whatever* has existed between the two pressures. Moreover, the effect just cited is easy to explain, for it depends simply on this, that in both cases the pressure in the boiler was superior to the resistance on the piston; and it needed no more for the steam, generated at that pressure or at any other, satisfying merely that condition, to pass into the cylinder and assume the pressure of the resistance.

It is then visible, from these various proofs, that the pressure in the cylinder is strictly regulated by the resistance on the piston, and by nothing else; and that any method like that of the coefficients in the ordinary calculation, which tends to establish a fixed ratio between the pressure in the cylinder and that of the boiler, must necessarily be inexact.

§ 6. *Verification of the two modes of calculation by particular examples.*—We have sufficiently demonstrated the want of basis of the ordinary calculation; but as the inaccuracy we have just exposed in that method might by some be supposed to be of slight importance, and they might conceive that, in practical examples, it amounted to the obtaining of results, which if not quite exact, were at least very near the truth, we will now attempt to apply it to some particular cases.

The coefficient of reduction for high pressure engines, working without expansion and without condensation, not being given by the authors who have treated on these subjects, we propose, in order to determine it, the two following facts which took place before our eyes:—

I. The *Leeds* locomotive engine, which has two cylinders eleven inches in diameter, stroke of the piston sixteen inches, wheel five feet in diameter, drew a load of 88·34 tons, in ascending a plane inclined 1 in 1300, at the velocity of 20·34 miles an hour; the effective pressure in the boiler being 54 ℔s. per square inch, or the total pressure 68·71 ℔s. per square inch.

II. The same day, the same engine drew a load of 38·52 tons in descending a plane inclined 1 in 1094, at the velocity of 29·09; the pressure in the boiler being precisely the same as in the preceding trial, and the regulator open to the same degree. These experiments may be seen in pages 201 and 202 of our Treatise on Locomotives.

If on one hand be reckoned, according to the ordinary method, the theoretic effort applied to the piston, and on the other hand the effect really produced, viz., the resistance opposed by the load *plus* that of the air against the train, we find, on referring the pressure and the area of the pistons to the foot square:—

1st case.—Theoretic effort applied on the piston, according to the ordinary calculation $1{\cdot}32 \times (68{\cdot}71 \times 144)$ . .	13,060 ℔s.
Real effect	8,846
Coefficient of correction . .	0·68
2nd case.—Theoretic effort, the same as above	13,060
Real effect	6,473
Coefficient of correction . .	0·50

The *mean* coefficient, to apply to the total pressure, to convert the theoretic effects to the practical, is then ·59.

We find, then, three very different coefficients: choose the first case, then an error occurs in the second; choose the second, and an error must arise in the first; by taking the third, you will only divide the error between the two. In every way an error is inevitable, and that alone suffices to prove that every method, like the ordinary one, which consists in the use of a *constant* coefficient, is necessarily inexact, whatever be the coefficient chosen, and to whatever engine the application be made; for it is evident that the same fact would occur in every kind of steam engine. Only that it might be less marked, if the velocities at which the engine were taken were less different; and this is what has hitherto prevented the error of this method from being perceived, for all the engines of the same system being imitated from each other, and moving nearly at the same velocity, the same coefficient of correction seems tolerably to suit them, from the factitious limit that had been laid down for the speed of the piston.

Besides, in stationary engines, one cannot, for want of precise determinations of the friction, disengage in the result the part which is really attributable to it from that which constitutes a positive error. But here we may easily be convinced that neither of these coefficients of correction represents, as the ordinary theory would have it, the friction, losses, and various resistances of the machine; for direct experiments made on the engine under consideration, and noted in our Treatise on Locomotives, enable us to estimate separately all these frictions, losses, and resistances. Reckoning, then, the friction of the engine at 82 ℔s. taking account besides of its additional friction per ton of load, and adding for each case the pressure subsisting on the opposite side of the piston by the effect of the blast pipe, we find, as the sum of the friction and indirect resistances—

1st case.—Friction	1,257 ℔s.
or ·10 of the theoretic result.	
2nd case.—Friction	873 ℔s.
or ·07 of the theoretic result.	

Thus we see that in each of the two cases, the friction and indirect resistances, omitted in the calculation, do not in reality amount to more than 10 or 7 hundredths of the theoretic result; and if we should be disposed to add to that $\frac{1}{20}$ or ·05, for the filling of the vacant spaces of the cylinder, which we could not estimate in ℔s., it will be ·15 and ·12; whereas the coefficients of correction would raise them to ·32 in one case, and ·50 in the other; that is, to 2 and 4 times what they really are. If, then, from these coefficients, be deducted the true value of the friction and losses, it will appear that the theoretic error, introduced into the calculation under the denomination of friction, is 17 per cent. of the *total power of the engine* in the one case, and 38 per cent. in the other.

But it is to be remarked, that, from the preceding evaluations, viz., of the direct resistances first, and then of the friction and indirect resistances, we have, for each of the two cases in question, the sum of the total effects really produced by the machine, as follows:—

1st case.—Direct resistances . . .	8,846 ℔s.
Friction	1,257
	10,103
2nd case.—Direct resistances . . .	5,473
Friction	873
	6,346

We are therefore enabled now to compare these effects produced with the results either of the ordinary calculation or of our theory.

1°. In applying the ordinary calculation with the mean coefficient ·56 determined above, and comparing its result with the real effect, we find—

1st case.—Effort applied on the piston, according to the ordinary calculation, 1·32 × (68·71 × 144) × ·59 .	7,705 ℔s.
Effect produced, including friction and every resistance . . .	10,103
Error over and above the friction and resistances	2,398
2nd case.—Effort applied on the piston, according to the ordinary calculation, the same as above . . .	7,705 ℔s.
Effect produced, including friction and every resistance . . .	7,346
Error over and above the friction and resistances	359
Mean error of the two cases . .	1,378

It is then evident what error would have been committed in calculating the effects of this engine from the coefficient ·59; but it is equally evident, that in applying any other coefficient *whatever*, the error would only transfer itself from one case to the other, without ever disappearing; and thus it is that the coefficient ·59 has almost annulled the error of the second case, by transferring it to the first.

To apply our formula with reference to the same problem, viz.:—

$$a\,R = \frac{m\,S\,P}{a\,v},$$

we have nothing more to do than to substitute for the letters their value, taking care to refer all the measures to the same unit. In making then these substitutions, which give

$P = 68\cdot71 \times 144$ lbs., $m = 411$, $a = 1\cdot32$, and observing that the effective vaporization of the engine has been $S = \cdot77$ cubic foot of water per minute, we find,—

1st case.—Effort applied by the engine at the given velocity, according to our theory, $\frac{411 \times 0\cdot77 \times (68\cdot71 \times 144)}{298}$	10,507 lbs.
Effect produced, including friction and resistances, as above . .	10,103
Difference	404
2nd case.—Effort applied by the engine at the given velocity, according to our theory, $\frac{411 \times 0\cdot77 \times (68\cdot71 \times 144)}{434}$	7,215
Effect produced, including friction, &c.	7,346
Difference	131
Mean difference of the two cases . .	267

It appears, then, that by this method, the useful effect is found with a difference only of 267 lbs., a very inconsiderable difference in experiments of this kind, wherein so much depends on the management of the fire.

2°. To continue the same comparison of the two theories, let it be required to calculate what quantity of water per minute the boiler ought to vaporize, to produce either the first effect or the second. The method followed by the ordinary theory, again consists in previously supposing that the volume described by the piston has been filled with steam at the same pressure as in the boiler, and then in applying to it a fractional coefficient to account for the losses.

Now, in the first case, the volume described by the piston at the given velocity, is $1\cdot32 \times 298 = 393$ cubic feet.

Had this volume been filled with steam at the pressure of the boiler, it would have required a vaporization of $\frac{393}{411} =$ ·96 cubic foot of water per minute. But the real vaporization was but ·77; wherefore, in the first case, the coefficient necessary to lead from the vaporization indicated by the ordinary calculation, to the real vaporization, $\frac{.77}{.96} = .81$.

In the second case, we find in the same manner, that the coefficient should be ·55; whence, in this problem, as in the preceding one, no constant coefficient whatever can suffice.

Performing, however, the calculation with the mean coefficient, ·68, we find,—

1st case.—Vaporization per minute, calculated by the ordinary theory, with the coefficient, $\frac{1.32 \times 298}{411} \times .68$ . . . . . . ·65

Real vaporization . . . . ·77

Error . . . . ·12

2nd case.—Vaporization per minute, calculated by the ordinary theory, with the coefficient, $\frac{1.32 \times 434}{411} \times .68$ . . . . . . ·95

Real vaporization . . . . ·77

Error . . . . ·18

The mean error committed is then $\frac{1}{5}$ of the vaporization, and being, as it is, a mean, it may, in extreme cases, become $\frac{2}{5}$, or amount to half of the whole vaporization.

This is the error committed in seeking a coefficient *expressly* for the vaporization. But when the coefficient, determined in the preceding case, that is, by the comparison of the theoretical and practical effects, is used as a divisor, as by many authors it is, much greater errors are induced, which we will show by an example farther on.

In our theory, on the contrary, the vaporization necessary to set in motion the resistance a R at the velocity v, is given by the formula

$$S = \frac{a R \times v}{m P}.$$

We have then,—

1st case.—Vaporization calculated from our theory

$\frac{10103 \times 298}{411 \times (68{\cdot}71 \times 144)}$ - - - - -	·74
Real vaporization - - - - - -	·77
Difference - - - - - - -	·03

2nd case.—Vaporization calculated from our theory,

$\frac{7346 \times 434}{411 \times (68{\cdot}71 \times 144)}$ - - - -	·78
Real vaporization - - - - - -	·77
Difference - - - - - - -	·01

3°. Lastly, in the case of finding the velocity of the piston, supposing the resistance to be given, any method similar to the ordinary one must inevitably lead to errors; but we must dispense with comparison, since this problem has never been resolved, and we shall therefore in this case merely show the verification of our own theory. The formula relative to this problem is

$$v = \frac{m S P}{a R}.$$

We find then,—

1st case.—Velocity of the piston in feet per minute, calculated from our theory,

$$\frac{411 \times \cdot 77 \times (68 \cdot 71 \times 144)}{10103} \quad - \quad - \quad - \quad 310$$

Real velocity - - - - - -	298
Difference - - - - - - -	12

2nd case.—Velocity of the piston from our theory,

$$\frac{411 \times \cdot 77 \times (68 \cdot 71 \times 144)}{7346} \quad - \quad - \quad 426$$

Real velocity - - - - - -	434
Difference - - - - - - - -	8

It consequently appears, that in each of the three problems in question, our theory leads to the true result; whereas the ordinary theory, besides that it leaves the third problem unresolved, may, in the other two, lead to very serious errors.

Before abandoning this comparison, we request attention to an effect, in calculating by the ordinary theory, which we have already mentioned, but which is here demonstrated, viz., that this calculation gives the same force applied by the engine in both the cases considered, notwithstanding their difference of velocity; and such will always be the result, since the calculation consists merely in multiplying the area of the piston by the pressure in the boiler, and reducing the product in a constant proportion. This theory therefore, maintains, in principle, that the engine can always draw the same load at all imaginable velocities. Again we see, that, in the same calculation of the load or effort applied, the vaporization of the engine does not appear, which would imply that the engine would always draw the same load at all velocities, whatever might be the vaporization of the boiler, which is inadmissible.

We shall also remark; that in calculating by the ordinary theory the vaporization of the engine, no notice is taken of the resistance which the engine is supposed to move; so that the vaporization necessary to draw a given load would be independent of that load—another result equally impossible.

To these omissions, therefore, or rather to these errors in principle, are to be attributed the variations observable in the results given of the ordinary theory in the examples proposed.

PART II.

ANALYTIC THEORY OF THE STEAM-ENGINE.

ARTICLE I.

CASE OF A GIVEN EXPANSION WITH ANY VELOCITY OR LOAD WHATEVER.

§ 1. *Of the change of temperature of the steam during its action in the engine.*—When an engine is at work, the steam is generated in the boiler at a certain pressure; it passes from thence into the cylinder, assuming a different pressure, and, in an expansive engine, the steam, after its separation from the boiler, continues to dilate itself more and more in the cylinder, till the piston is at the end of the stroke. It is generally supposed, that in all the changes of pressure which the steam may undergo, its temperature remains the same; and it is consequently concluded, that during the action of the steam in the engine, the density and volume of that steam follow the law of Mariotte, namely, that its volume varies in the inverse ratio of the pressure. This supposition greatly simplifies the formulæ; but, as reason and experience prove it to be altogether inexact, we are compelled to renounce it, and will substitute in its place another law, deduced from observation of the facts themselves.

We have recognised in a numerous series of experiments, by applying simultaneously a manometer and a thermometer, both to the boiler of a steam-engine, and also to the tube through which the steam, after having terminated its effect, escaped into the atmosphere, that during all its action in the engine the steam remains in the state denoted by the name of saturated steam, that is, at the maximum density for its temperature. The steam in fact was produced in the boiler at a very high pressure, and escaped from the engine at a very low one; but on its issuing forth, as well as at the moment of its formation, the thermometer indicated the temperature corresponding to the pressure marked by the manometer, as if the steam were immediately generated at the pressure it had at that moment.

Thus during its whole action in the engine, the steam remains constantly at the maximum density for its temperature.

Now, in all steams, the volume depends at once on the pressure and the temperature; but in the steam at the maximum density, the temperature itself depends on the pressure. It should then be possible to express the volume of steam of maximum density, in terms of the pressure alone.

The equation which gives the volume of the steam in in any state whatever, in terms of the pressure and temperature, is very simple: it is deduced from Mariotte's law combined with that of M. Gay-Lussac. The equation which gives the temperature in terms of the pressure, for the steam at the maximum density, is also known: it has been deduced from the fine experiments of Messrs. Arago and Dulong on steam at high pressures, and from those of Southern and other experimenters on steam produced under low pressures. By eliminating then the temperature in these two equations, we shall obtain the analogy required, which will give immediately, with regard to steam at the maximum density, for its temperature, the volume in terms of the pressure alone.

But here arises the difficulty. The equation of the temperatures is not invariable; or rather, the same equation does not apply to all points of the scale. To be used with accuracy, it requires to be changed according as the pressure is under that of one atmosphere, or comprised between one and four atmospheres, or again if it be above four atmospheres. Now when the steam is acting in an engine, it may happen, according to the load, or to other conditions of its motion, that the steam generated at first at a very high pressure, may act or be expanded in the engine sometimes at a pressure exceeding four atmospheres, sometimes at a pressure less than four atmospheres, but yet exceeding one, and sometimes at a pressure under that of one atmosphere. It is impossible then to know which of the three formulæ is to be used in the elimination; and consequently it is impossible by this means to attain a general formula representing the effects of the engine in all cases.

Moreover, were either one of these formulæ adopted, the high radical quantities they contain would so complicate the calculations as to render them unfit for practical purposes. And it is to be remarked, that these diverse formulæ, after all, are not the expression of the true mathematical law which connects the temperature and the pressure in saturated steam, but merely empirical relations, which experiment alone has demonstrated to have a greater or less degree of approximation.

A formula of temperatures given by M. Biot is indeed adapted to all points of the scale, and may be useful in a great number of delicate researches relative to the effects of steam; but as it gives only the pressure in terms of the temperature, and is, from its form, incapable of the inverse solution, namely, the general determination of temperatures in terms of the pressure, it is unfit for the elimination proposed.

Under these circumstances the only resourse is to seek a direct relation in terms of the pressure alone, whose results shall represent immediately those of the two preceding formulæ combined; that is, to calculate first by means of those formulæ a table of volumes of the steam, and then to seek a direct and simple relation to represent those results. This we have done.

M. Navier had proposed a formula for this purpose. But that formula, though sufficiently exact in high pressures, differs widely from experience in pressures below that of the atmosphere, which are useful in condensing engines; and it is possible to find one much more exact for non-condensing engines, namely, that we are about to offer. We propose then, for this purpose, the following formulæ, in which p represents the pressure of the steam expressed in pounds per square foot, and μ the ratio of the volume of the steam to that occupied by the same weight of water:

Formula for high or low pressure engines *with condensation*

$$\mu = \frac{10000}{0{\cdot}4227 + 0{\cdot}00258\,p}$$

Formula for high pressure *non-condensing* engines

$$\mu = \frac{10000}{1{\cdot}421 + 0{\cdot}0023\,p}$$

The first formula is equally suitable to pressures above and below that of the atmosphere, at least within the limits likely to be considered in applying it to condensing steam-engines. Those limits are eight or ten atmospheres for the highest pressures; and eight or ten pounds per square inch for the lowest, in consequence of the friction of the engine, the pressure subsisting against the piston after imperfect condensation in the cylinder, and the resistance of the load. Within these limits then the proposed formula will be found to give very approximate results.

This first formula might also be applied, without any error worthy of notice, to non-condensing engines. But as, in these, the steam can scarcely operate with a pressure less than two atmospheres, by reason of the friction of the engine and the resistance of the load, it is needless to require of the formula exact results of volumes for pressures under two atmospheres.

In this case then the second formula will be found to give those results with much greater accuracy, and will consequently be preferred in practice. This will be readily recognised in a table annexed to the work, presenting a comparison of the volume of the steam calculated by the ordinary formulæ in terms of the pressure and temperature, and by the proposed formulæ in terms of the pressure alone.

We state then generally this analogy:

$$\mu = \frac{1}{n + q\,p} \cdots (a)$$

Consequently, if the steam pass in the engine, from a certain volume m' to another known volume m, and thereby abandon its primitive pressure P′, to assume an unknown pressure p, it is easy to recognise that the following relation will exist between those two pressures, and will serve to determine the unknown quantity p, viz.:

$$\frac{p}{P'} = \frac{m'}{\mu} \cdot \frac{1 - n\,\mu}{1 - n\,m'}.$$

This is the relation which we substitute in lieu of that hitherto employed, and according to which the volume appears to vary in the inverse ratio of the pressure. It will be observed that such an hypothesis may be deduced from the analogy we have just offered, by making $n = 0$, and $q = \frac{m\,P}{p}$, m being the volume, and P the pressure of the steam in the boiler; for it is plain that we shall then have,

$$\mu = \frac{m\,P}{p},$$

that is to say, the volumes are inversely as the pressures.

§ 2. *Of the divers problems which present themselves in the calculation of steam engines.*—We distinguish three cases in an engine: that wherein it works with a given rate of expansion of the steam, and with a load or a velocity indefinite; that in which it works with a given rate of expansion, and with the load and velocity proper to produce its maximum of useful effect with that expansion; and lastly that wherein, the engine having been previously regulated for the expansion of the steam most favourable in that engine, it bears, moreover, the load most advantageous for that expansion; which, consequently, produces the absolute maximum of useful effect in the engine.

We have said that the three fundamental problems of the calculation of steam engines consist in finding successively the velocity, the load, and the vaporization of the engine. After the solution of these three problems, that which first presents itself, as a corollary to them, consists in determining the *useful effect* of the engine, which may be expressed under six different forms, viz.: by the work done, or the number of pounds raised one foot high by the engine in a minute; by the horse power of the engine; by the actual duty or useful effect of one pound of coal; by the useful effect of a cubic foot of water converted into steam; and by the number of pounds of coal, or of cubic feet of water, that are necessary to produce one horse power.

Another research, in fine, no less important, is the rate of expansion at which the steam must work in an engine, in order that it may produce given effects. We shall present successively the solution of all these questions.

The various problems will be resolved in each of the three cases above mentioned. In the last two, the question will be to calculate the rate of expansion, the velocity, the load, and the effects which correspond to the maximum of, relative or absolute, useful effect of the engine.

In the ordinary calculations of steam engines, the solution of three questions only had been attempted, viz.,—to find the load, the vaporization, and the useful effect, under its different forms; which solution is, as we have seen, faulty. As to the determining of the velocity for a given load, and that of the rate of expansion for given effects, the calculation of these had not been proposed. Moreover, the very nature of the theory employed in those calculations did not allow of distinguishing, in the machine, the existence of the three cases which are really found in it. The distinction we establish may, therefore, at first appear obscure, expressed, as it is, in general terms, and including relations unusual in the consideration of steam engines; but, on a closer view of the question, these relations will be seen to be of indispensable necessity, in order to calculate with exactitude either the effects or the proportions of steam engines of all systems.

§ 3. *Of the velocity of the piston under a given load.*—To embrace at once the most complete mode of action of the steam, we will suppose an engine working by expansion, by condensation, and with an indefinite pressure in the boiler; and to pass on to unexpansive or uncondensing engines, it will suffice to make the proper suppressions or substitutions in the general equations.

From what has been already shown of our theory, the relations sought between the various data of the problem are necessarily deduced from two general conditions: the first expressing that the engine has attained a uniform motion, and consequently that the quantity of labour impressed by the motive power is equal to the quantity of action developed by the resistance: the second, that there is a necessary equality between the emission of steam through the cylinder and the production by the boiler.

The limits of this extract will not allow us to develop those calculations, simple as they may be; but that the proceeding may be understood, we shall state that, expressing by P the pressure of the steam in the boiler, and by P′ the pressure of the same steam in the cylinder before the expansion, by L the length of stroke of the piston, and by L′ the portion traversed at the moment the expansion begins, by a the area of the piston, and by c the clearance of the cylinder, or the space at each end of the cylinder beyond the portion traversed by the piston, and which necessarily fills with steam at each stroke; lastly, by r the resistance of the load, by p the pressure subsisting on the other side of the piston after imperfect condensation, by f the friction of the engine when not loaded, and by δ the increase of that friction per unit of the load r, these four forces, as well as the pressures, being moreover referred to the unit of surface of the piston; the first of the above conditions produces the following analogy:

$$\frac{P' a\,(L'+c)}{1-n\,a\,(L'+c)}\left\{\frac{L'}{L'+c}+\log.\frac{L+c}{L'+c}-n a\,L\right\}= a\,L\Big((1+\delta)r+p+f\Big)\;.\;.\;.\;.\;.\;(A)$$

This equation expressing that the labour developed by the mover is found entire in the effect produced, be it remarked, that it is not essentially necessary for the motion to be strictly uniform. It may equally be composed of equal oscillations, beginning from no velocity, and returning to no velocity, provided the change of velocity take place by insensible degrees, so as to avoid the loss of *vis viva*, and that the successive oscillations be performed in equal times.

As to the second condition of the motion; if we denote by S the volume of water vaporized by the boiler in a unit of time and transmitted to the cylinder, by m the volume of the steam formed under the pressure P of the boiler, compared with the volume of the same weight of water unvaporized, and by v the velocity of the piston, the equality between the production of the steam and its consumption will be found to furnish the second general analogy:

$$\frac{S}{n + q P'} = \frac{v}{L} a \, (L' + c) \quad . \; . \; . \; . \quad (B)$$

Consequently, by eliminating P′ from these two equations, and writing, for greater simplicity,

$$\frac{\frac{L}{L' + c} - n\,a\,L}{\frac{L'}{L' + c} + \log. \frac{L + c}{L' + c} - n\,a\,L} = \varkappa,$$

we find definitively:

$$v = \frac{L}{L' + c} \cdot \frac{S}{a} \cdot \frac{1}{n + q\,\varkappa \left\{ (1 + \delta)\, r + p + f \right\}} \quad . \; . \quad (1)$$

an equation which gives the velocity of the motion in terms of the load and of the other data of the problem.

This formula is quite general, and suits every kind of steam engine with continued motion. If the engine be expansive, L′ will be replaced by its value corresponding to the point of the stroke where the steam begins to be intercepted; if the engine be unexpansive, it will suffice to make $L' = L$, which will give at the same time $\varkappa = 1$. If it be a condensing engine p must stand for the pressure of condensation; if it be not a condenser, p will represent the atmospheric pressure. And finally, the quantities n and q will have, according to the case considered, the above-mentioned value.

§ 4. *Of the load and useful effects of the engine.*—If, instead of seeking the velocity in terms of the load it be required, on the contrary, to know the load suitable to a given velocity, the same equation resolved with reference to r becomes,

$$ar = \frac{\frac{L}{L'+c} S - n a v}{(1+\delta) q v \varkappa} - a \frac{p+f}{1+\delta} \quad . \; . \; . \; . \; (2)$$

3°. To find the vaporization of which the engine ought to be capable, in order to put in motion a resistance r with a known velocity v, the value of S must be drawn from the same analogy, thus:

$$S = \frac{L'+c}{L} av \left(n + q \varkappa \{ (1+\delta) r + p + f \}\right) \quad . \; . \; . \; (3)$$

4°. The useful effect produced by the machine, in the unit of time, at the velocity v, is evidently arv. Hence that useful effect will have for its measure,

$$uE. = \frac{\frac{L}{L'+c} S - nav}{(1+\delta) q \varkappa} - av \frac{p+f}{1+\delta} \quad . \; . \; . \; . \; (4)$$

5°. If it be desired to know the useful effect, in horse power, of which the engine is capable at the velocity v, or when loaded with the resistance r, it suffices to observe that what is called one horse power represents an effect of 33,000 ℔s. raised one foot per minute. All consists then in referring the useful effect produced by the engine in a unit of time, to the new unity just chosen, viz, to one horse power; and it will consequently suffice to divide the expression already obtained in the equation (4) by 33,000. Thus the useful effect in horse power will be,

$$uHP. = \frac{uE.}{33000} \quad . \; . \; . \; . \; (5)$$

6°. We have just expressed, in the two preceding questions, the effect of the engine by the work which it is capable of performing. We are now on the contrary about to express that effect by the force which the engine expends to produce a given quantity of work. The useful effect of the equation (4) being that which is due to the volume of water S converted into steam, in the unit of time, if we suppose that in the same unit of time N pounds of fuel be consumed, it is clear that the useful effect produced by each pound of fuel will be the Nth part of the above effect. It will then be,

$$\text{uE. 1 ℔. co.} = \frac{\text{uE.}}{\text{N}} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad (6)$$

To apply this formula, it will suffice to know the quantity of coal consumed in the furnace per minute, that is, during the production of the vaporization S; and this datum may be deduced from a direct experiment on the engine, or from known experiments on boilers of a similar construction.

7°. The useful effect of the equation (4) being that which proceeds from the vaporization of the volume of water S, if it be required to know the useful effect that will be produced by each cubic foot of water, or by each unit of S, it will be sufficient to divide the total effect uE. by the number of units in S. It will then be,

$$\text{uE. 1 ft. wa.} = \frac{\text{uE.}}{\text{S}} \cdot \cdot \cdot \cdot \cdot \quad (7)$$

8°. In the sixth problem we have obtained the useful effect produced by one pound of fuel. We may then, by a simple proportion, deduce from thence the quantity of fuel necessary to produce one horse power, viz.

$$\text{Q. co. for 1 hp.} = \frac{33000\ \text{N}}{\text{uE.}} \cdot \cdot \cdot \cdot \cdot \quad (8)$$

9°. And similarly, the quantity or volume of water necessary to produce one horse power will be,

$$\text{Q. wa. for 1 hp.} = \frac{33000\ \text{S}}{\text{uE.}} \cdot \cdot \cdot \cdot \cdot \quad (9)$$

§ 5. *Of the expansion of steam, to be adopted in an expansive engine, in order to produce wanted effects.*

10° Finally, if it be required to know what rate of expansion the engine must work at, in order to obtain from it determined effects, the value of L′ must be drawn from equation (1.) It will be given by the formula,

$$\frac{L'}{L'+c} + \log. \frac{L+c}{L'+c} = q\,a\,L \left\{ (1+\delta)\,r + p + f \right\}$$

$$\frac{\frac{v}{L} - nav\frac{L' + c}{L}}{S - nav\frac{L' + c}{L}} + naL \quad \ldots . (10)$$

This formula not being of a direct application, we annex to the work a table which gives its solutions for the expansion from hundredth to hundredth, with a very short calculation.

We confine ourselves to these inquiries as being those which may most commonly be wanted; but it is clear that by means of the same general analogies, any one whatever of the other quantities which figure in the problem may be determined, as the case may require. Thus, for instance, may be determined the area of the piston, or the pressure in the boiler, or the pressure in the condenser, corresponding to determined effects of the machine, as has been done for locomotives in our work on that subject.

ART. II.

CASE OF THE MAXIMUM USEFUL EFFECT, WITH A GIVEN RATE OF EXPANSION.

§ 1. *Of the velocity of the maximum useful effect.* We have resolved the above problems in all their generality, that is, supposing the engine to move any load whatever with any velocity whatever, under this single condition, that the load and the velocity be compatible with the capability of the machine. The question is now to find what velocity and what load are most advantageous for the working of the engine, and what are the effects which, in this case, may be expected from it; that is to say, its maxima effects for a given rate of expansion.

1°. In examining the general expression of the useful effect produced by the engine at a given velocity, we perceive that the expression attains its maximum for a given rate of expansion when the velocity is a minimum; now from the equation (B) the smallest value of v will be given by $P' = P$. The velocity corresponding to the maximum useful effect will therefore be,

$$v' = \frac{S}{a(n+qP)} \cdot \frac{L}{L'+c} \ldots (11)$$

Let us however remark, that, mathematically speaking, the pressure P' of the steam in the cylinder can never be quite equal to P, which is the pressure in the boiler; because there exist between the boiler and the cylinder conduits through which the steam has to pass, and the passage of these conduits offers a certain resistance to the motion of the steam: whence results that there must exist, on the side of the boiler, a trifling surplus of pressure equivalent to the overcoming of the obstacle. But as we have proved elsewhere, that, with the usual dimensions of engines, this difference of pressure is not appreciable by the instruments used to measure the pressure in the boiler, the introduction of it into the calculations would render the formulæ more complicated without making them more exact. For this reason we neglect that difference here.

The velocity given by the preceding equation is, then, that at which the engine will produce its maximum effect for a given expansion. This velocity will result from the condition $P' = P$, or reciprocally, when this velocity takes place in the engine, the steam enters the cylinder with full pressure, that is, with the same pressure it has in the boiler. It is necessary to remark that the velocity of full pressure will not be the same for all engines; on the contrary, it will vary in direct ratio with the vaporization, and in the inverse ratio of the area of the cylinder. It may then occur to be, in one engine, the half or the double of what it would be in another; which shows that it is an error to believe that, because the piston of stationary engines does not in general exceed a certain velocity of from 150 to 250 English feet per minute, the steam of the boiler necessarily reaches the cylinder with no change of pressure.

It is easy to be seen that a fixed limit, whatever it may be, cannot in this respect suit all engines; and that the only means of knowing the velocity of the maximum effect, or of full pressure of an engine, is to calculate it directly for that engine. Such is the object of the formula we have just given. This formula, moreover, is of a remarkable simplicity, and requires no other experimental knowledge than that of the production of steam of which the boiler is capable.

§ 2. *Of the load and maximum useful effect of the engine.*—2°. The useful resistance which the machine is capable of putting in motion at its velocity of the maximum effect above, is to be drawn from equation (2,) substituting for v the value just obtained. Calling the load r' we shall find it expressed by

$$ar' = \frac{a\text{P}}{(1+\delta)n} - a\,\frac{p+f}{1+\delta}; \ldots . (12)$$

and it is at the same time visible that this load is the greatest the engine can put in motion with the given expansion L′, for it corresponds to the lowest value of v in equation (2.) Thus the greatest effect of the machine, with a given rate of expansion, is attainable by working the machine at its smallest velocity and with its maximum load.

It will be observed, that this equation may be used to determine the friction of the engine without a load, and its additional friction per unit of the load, upon the same principles that we have employed in our Treatise of Locomotive Engines for similar determinations. This is also the mode we propose for steam-engines of every system.

3°. The vaporization necessary to an engine, in order to exert a certain maximum effort r' at its minimum velocity v' will be given by equation (3,) by substituting in it r' and v', or will be drawn more simply from equation (11,) thus:—

$$\text{S} = (n + q\,\text{P})\, av'. \frac{\text{L}' + c}{\text{L}} \ldots . (13)$$

4°. The maximum of useful effect producible in the unit of time, by an engine working with a given expansion, will be known by formula (4,) by introducing for v the velocity proper to produce that effect. Thus is found,

$$\text{max. uE.} = \frac{L}{L'+c} \cdot \frac{S}{(1+\delta)(n+q\,P)} \left\{ \frac{P}{\varkappa} - (p+f) \right\} \ldots . (14)$$

It will be observed that this maximum useful effect depends particularly on the quantity of water S, evaporated per minute in the boiler. Hence we see plainly the error of those who pretend to calculate the useful effect or the power of engines from the area and the velocity of the piston, which they set in the place of the vaporization produced; this vaporization not only entering not into their calculation, but forming no part of their observations.

5°. The useful effect, in horse power, of the engine will be expressed by

$$\text{uHP.} = \frac{\text{max. uE.}}{33000} \ldots\ldots (15)$$

6°. 7°. 8°. 9°. The various measures of the useful effect will here be deduced from equations similar to those (6,) (7,) (8,) and (9.)

10°. The expansion at which the engine ought to be regulated, in order to draw a given load at the most advantageous velocity, or producing the maximum of useful effect with that load, will be derived from equation (12,) which gives,

$$\frac{L'+c}{L}\left\{\frac{L'}{L'+c} + \log.\frac{L+c}{L'+c}\right\} = \frac{(1+\delta)r+p+f}{P} + na\frac{L'+c}{L}$$
$$\left\{L - L\frac{(1+\delta)r'+p+f}{P}\right\} \ldots\ldots (20)$$

and the solutions of this formula will be found immediately, and without calculation, by means of the table given above, as suggested by equation (10.)

ART III.

CASE OF THE ABSOLUTE MAXIMUM OF USEFUL EFFECT.

The preceding inquiries suffice for engines working without expansion, merely by making $L' = L$; because those engines fall under the case of expansion fixed *à priori.* But it is otherwise with engines in which the rate of expansion may be varied at will. We have seen that, for a given expansion, the most advantageous way of working the engine is to give it the maximum load, which is calculated *à priori* from equation (12.) Hence we know what load is to be preferred for every rate of expansion. But the question now is to determine, among the various rates of expansion of which the engine is susceptible, each accompanied by its corresponding load, which will produce the greatest useful effect.

For this purpose we must recur to equation (14,) which gives the useful effect produced with a maximum load r', and seek among all the values assignable to L', that which will raise the useful effect to a maximum. Now by making the differential coefficient of that expression, taken with reference to L' equal to nothing, we find as the condition of the maximum sought:

$$\frac{L'}{L} = \frac{p+f}{P} + naL \frac{\log.\frac{L+c}{L'+c} - naL\left(1-\frac{L'}{L}\right)}{\left(\frac{L}{L'+c} - naL\right)^2} \ldots\ldots(30)$$

This equation will be resolved in the same manner as the equations (10) and (20,) by means of the table already given; and after having found the value of $\frac{L'}{L}$, it will be introduced in the equations of article II.; and the corresponding velocity, load, and useful effects, will be determined.

However, as the supposition of $n = o$, $q = \frac{1}{mP}$, that is to say, the supposition that the steam preserves its temperature during its action in the engine, will give a sufficient approximation in a great many cases, we present here the corresponding results of all the formulæ. They will show, already to a very near degree, the maximum absolute effects which it is possible to obtain from an engine in adopting simultaneously the most advantageous rate of expansion and the most advantageous load.

$$(21)\ v'' = \frac{m\,S}{a} \cdot \frac{L\,P}{L(p+f)+P\,c}$$ Velocity of the absolute maximum useful effect.

$$(22)\ ar'' = a\,\frac{L\,(p+f)+P\,c}{(1+\delta)\,L}\ \log.\ \frac{(L+c)\,P}{L\,(p+f)+P\,c}$$ Load of the piston corresponding to the absolute maximum useful effect.

$$(23)\ S = \frac{av''}{m} \cdot \frac{L\,(p+f)+P\,c}{L\,P}$$ Vaporization.

$$(24)\ {}^{\text{ab.max.u.}}E = ar''v'' = \frac{m\,S\,P}{1+\delta}\ \log.\ \frac{P\,(L+c)}{L\,(p+f)+P\,c}$$ Absolute maximum of useful effect.

$$(25)\ {}^{\text{u.}}HP = \frac{{}^{\text{ab.max.u.}}E}{33000}$$ Absolute maximum of useful force in horse power.

$$(30)\ L' = \frac{L\,(p+f)}{P}$$ Rate of expansion which produces these effects.

The four determinations of the useful effects of a given quantity of fuel or water will be furnished by equations similar to those (6,) (7,) (8,) and (9.)

The only remark we shall make on the subject of these formulæ is, that the load suitable to the producing of the absolute maximum useful effect is not the maximum load that may be imposed on the engine. In effect, from equation (12,) we know that the maximum load for the engine takes place when $L' = L$, and not when

$$L' = L\,\frac{p+f}{P}.$$

Thus the greatest possible load of the engine is that of the maximum useful effect without expansion; but by applying a lighter load, that of equation (22,) and at the same time the expansion of equation (30,) a still greater useful effect will be obtained.

PART III.

APPLICATION OF THE FORMULÆ TO THE VARIOUS SYSTEMS OF STEAM-ENGINES.

We shall not give here the applications to different systems of steam-engines, which are developed in this part of the work. We shall confine ourselves to what concerns Watt's steam-engines, because they are the most generally employed in the arts.

Watt's rotative double-acting steam-engine.—These engines being without expansion, the proper formulæ for calculating their effects will be deduced from the general formulæ by making $L' = L$, which will give also $\varkappa = 1$, and by replacing the quantity p by the pressure of condensation. We see, moreover, that for these engines, the expansion being susceptible of no variation, since that detent does not exist, the third case, considered as to engines in general, cannot occur. There will be then but two circumstances to consider in their working, viz., the case wherein they operate *with their maximum load, or load of greatest useful effect,* and the case in which they operate *with any load whatever.* The effects therefore of these engines will visibly be determined by the following equations:

General case, or of indefinite load.	*Case of maximum useful effect.*
$v = \frac{L}{L+c} \cdot \frac{S}{a} \cdot \frac{1}{n + q\left\{(1+\delta) r + p + f\right\}}$	$v' = \frac{L}{L+c} \cdot \frac{S}{a(n + q P)}$
$ar = \frac{L}{L+c} \cdot \frac{S}{q(1+\delta) v} - \frac{a}{1+\delta}\left(\frac{n}{q} + p + f\right)$	$ar' = \frac{a}{1+\delta}(P - p - f)$
$S = \frac{L+c}{L} \cdot av\left[n + q\left\{(1+\delta) r + p + f\right\}\right]$	$S = \frac{L+c}{L} \cdot av'(n + q P)$
$_{u}E = arv = \frac{L}{L+c} \cdot \frac{S}{q(1+\delta)} - \frac{av}{1+\delta}\left(\frac{n}{q} + p + f\right)$ . .	$_{\text{max. u.}}E = ar'v' = \frac{L}{L+c} \cdot \frac{S}{(1+\delta)(n + q P)}\left\{P - p - f\right\}$
$_{u}HP = \frac{_{u}E}{33000}$ . . .	$_{u}HP = \frac{_{\text{max. u.}}E}{33000}$,
$_{u}E_{\text{1 lb. co.}} = \frac{_{u}E}{N}$ . . .	$_{u}E_{\text{1 lb. co.}} = \frac{_{\text{max. u.}}E}{N}$,
$_{u}E_{\text{1 ft. wa.}} = \frac{_{u}E}{S}$ . . .	$_{u}E_{\text{1 ft. wa.}} = \frac{_{\text{max. u.}}E}{S}$,
$Q_{\text{co. for 1 hp.}} = \frac{33000\,N}{_{u}E}$ . . .	$Q_{\text{co. for 1 hp.}} = \frac{33000\,N}{_{\text{max. u.}}E}$,
$Q_{\text{wa. for 1 hp.}} = \frac{33000\,S}{_{u}E}$ . . .	$Q_{\text{wa. for 1 hp.}} = \frac{33000\,S}{_{\text{max. u.}}E}$,

Although these formulæ may at first sight appear complicated, they will nevertheless be found very simple in the calculation. It is only necessary to fix attention to refer all the measures to the same unit, as will be seen in the following example. It must be remarked also, that as soon as the velocity and load of the engine are determined, the useful effect will be known immediately, being their produce.

To apply, however, these formulæ, some previous observations are necessary.

In good engines of that system the pressure in the condenser is usually 1·5 ℔. per square inch, but the pressure in the cylinder itself, and under the piston, is in general 2·5 ℔s. more, which gives $p = 4 \times 144$ ℔s. It has been deduced, moreover, from a great number of trials made on Watt's engines, that their friction, when working with a moderate load, varies from 2·5 ℔s. per square inch of the piston, in engines of smaller dimensions; to 1·5 ℔. in the more powerful ones; which includes the friction of the parts of the machinery and the force necessary for the action of the feeding and discharging pumps, &c. By moderate load in these engines is meant about 8 ℔s. per square inch of the piston. Now our experiments on locomotives, showing the *additional* friction of an engine to be $\frac{1}{8}$ of the resistance, give room to think, that the *additional* friction caused in the engine by that load may be about 1 ℔. per square inch. The above information attributes then to Watt's engines, working unloaded, a friction of from 1·5 ℔. to ·5 ℔. per square inch, according to their dimensions, which would give 1 ℔. for engines of a medium size: this information, agreeing with what we have deduced from our inquiries on locomotives, as has been said above, we shall continue to admit, in this place, respecting the friction, the data already indicated in this respect, viz.:—

$$f = 1 \times 144 \text{ ℔s.} \qquad \delta = \cdot 14.$$

As an application of these formulæ, we will submit to calculation an engine constructed by Watt at the *Albion Mills* near London. The following were its dimensions:—

Diameter of the cylinder, 34 inches, or $a = 6{\cdot}287$ square feet;

Stroke of the piston, 8 feet, or $L = 8$ feet;

Clearance of the cylinder, $\frac{1}{20}$ of the stroke, or $c = {\cdot}4$ foot;

Effective vaporization, ·927 cubic foot of water per minute, or $S = {\cdot}927$ cubic foot;

Consumption of coal in the same time, 6·71 ℔s., or $N = 6{\cdot}71$ ℔s.;

Pressure in the boiler, 16·5 ℔s. per square inch, or $P = 16{\cdot}5 \times 144$ ℔s.;

Mean pressure of condensation, 4 ℔s. per square inch, or $p = 4 \times 144$ ℔s.

And finally, the engine being a condensing one, we have $n = {\cdot}4227$ and $q = {\cdot}000000258$.

The engine had been constructed to work at the velocity of 256 feet per minute, which was considered its normal velocity; but when put to trial by Watt himself, shortly after its construction, it assumed, in performing its regular work, esteemed 50 horse-power, the velocity of 286 feet per minute, consuming at the same time the quantity of water and fuel which we have just reported.

If then we seek the effects it was capable of producing at its velocity of maximum effect, and then at those of 256 and 286 feet per minute, we shall find, by the formulæ already exposed:

Maximum useful effect.

v	=	286	256	$v' = 214$	Velocity of the piston in feet per minute;
ar	=	5,621	6,850	9,133	Total load of the piston in ℔s.;
$\frac{r}{144}$	=	6·21	7·57	10·09	Load of the piston in ℔s. per square in.;

S	=	·927	·927	·927	Vaporization in cubic feet of water per minute;
^{a}·E	=	1,607,610	1,753,600	1,957,180	Useful effect in lbs. raised to one foot per minute.
^{a}·HP	=	49	53	59	Useful effect in horse power.
^{a}·E $^{1 \text{ lb. co.}}$	=	239,585	261,340	291,680	Useful effect of 1 lb. of coal, in lbs. raised to one foot per minute.
^{a}·E $^{1 \text{ p. e.}}$	=	1,734,200	1,891,700	2,111,300	Useful effect due to the vaporization of one cubic foot of water, in lbs. raised to one foot per minute.
Q $^{\text{co. for 1 h.}}$	=	·138	·126	·113	Quantity of coal in lbs., producing the effect of one horse power.
Q $^{\text{wa. for 1 h.}}$	=	·019	·017	·016	Quantity of water, in cubic feet, producing the effect of one horse power.

Such are the effects that this engine should produce, and we see, in consequence, that in performing a labour estimated at fifty horses, it was to be expected the engine would acquire the velocity which in fact it did, viz., that of 286 feet per minute.

Let us now see to what results we should have been led, had we applied the ordinary calculations to the experiment of Watt, which we have just reported. In this experiment, the engine vaporizing ·927 cubic foot of water, and exerting the force of fifty horses, assumed a velocity of 286 feet per minute.

We then find that, since the engine had a useful effect of no more than fifty horses, and that the theoretical force, calculated according to that method, from the area of the cylinder, the effective pressure in the boiler, and the velocity of the piston, was,

$$\frac{6{\cdot}287 \times (16{\cdot}5 - 4) \times 144 \times 286}{33000} = 98 \text{ horses.}$$

It resulted that, to pass from the theoretical effects to the practical, it was necessary to use the coefficient ·51. Consequently, by following the reasonings of that theory, the following conclusions were to be drawn:—

1°. The observed velocity being 286 feet per minute, the vaporization calculated on the quantity of water, which reduced to steam at the pressure of the boiler, might occupy the volume described by the piston, and afterwards divided, as is done, by the coefficient, to take the losses into account, would have been:

$$\frac{\frac{1}{1530} \times 6{\cdot}287 \times 286}{{\cdot}51} = 2{\cdot}305 \text{ cubic feet per minute, instead}$$

of ·927.

2°. The engine having vaporized only ·927 cubic foot of water per minute, the velocity calculated on the volume of steam formed, at the pressure of the boiler, and afterwards reduced by the coefficient, not as has been done, since this problem was not resolved, but as must naturally be concluded from the signification attributed to that coefficient, could but be

$$\frac{1530 \times \cdot 927}{6 \cdot 287} \times \cdot 51 = 115 \text{ per minute, instead of } 286.$$

3°. The coefficient found by the comparison of the theoretical effects to the practical being ·51, the various frictions, losses, and resistances of the engine would amount to ·49 of the effective power; whereas these frictions, losses, and resistances, consisting merely of the friction of the engine and the clearance of the cylinder, could be estimated only as follows:—

Total friction (including the additional friction) 2 ℔s. per square inch, or as a fraction of the effectual pressure, $\frac{2}{12}$	·17
Clearance of the cylinder, $\frac{1}{20}$ of the effective force, or	·05
	·22

Some authors also employ constant coefficients, not however using the same to determine the vaporization as to find the useful effect. This manner of calculating has arisen from those authors having recognised from experience, that the steam has in the cylinder a less pressure and density than in the boiler; but as they cannot settle *à priori* what is that pressure in the cylinder, and that they always seek to deduce it from that of the boiler, instead of concluding it directly and in principle, from the resistance on the piston, as we do; the diminution of pressure observed by them could not be defined in its limits, and it remained simply a practical fact which they used to explain the coefficient. This change in the coefficient employed, avoids the first and second of the contradictions we have just indicated; but the third, as well as all the objections we have developed in the first part against the use of any constant coefficient, remain in full force; that is to say, that in this method, the power of the engine is calculated independently of the vaporizing force of the boiler, and the vaporization independently of the resistance to be moved; that the effort exerted by the machine is found always the same at all velocities; that no account can be taken of the opening of the regulator, unless a new series of coefficients be introduced to that end, as well as for all the changes of velocity, &c.

In consequence, we conclude from this comparison, as well as from what precedes, that the theory in general use for calculating the effects or the proportions of steam engines, cannot lead to any sure results; while the one, which we have deducted from the best known principles in mechanics, and from the direct observation of what takes place in the engines, represents their effects with accuracy.