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Non Sequitur: Marx (1818-1883)

Hans Brems

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College of Commerce and Business Administration

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BACKGROUND AND ABSTRACT

When in 1867 Marx published volume I of Das Kapital, the best minds of our profession had already discovered the demand function (Cournot), marginal utility (Bernoulli), and marginal productivity (von Thünen). To Marx such discoveries meant nothing. While Mill had freed himself from the straitjacket of the labor theory of value, Marx tightened it further around himself. To Marx, surplus value was generated by the application of variable (wage) capital but never by the application of constant (machine) capital. With this beginning Marx himself had created most of the logical difficulties haunting his system.

The purpose of the present paper is to show that Marx's system suffered from three non sequiturs: first that rates of surplus value should be equalized among industries; second that under technological progress the rate of profit should be falling; and third that the real wage rate should also be falling.

January 7, 1985

NON SEQUITUR: MARX (1818-1883)

By Hans Brems

Why don't people argue about the "meanings" of Wicksell the way they do about those of Ricardo and Marx?

P. A. Samuelson (1974: 64)

1. Marx's Problem

Marx was more than an economic theorist: a philosopher, an historian, a journalist, an agitator, and a remote-control labor organizer. But to Marx economic theory came first, and as a theorist he is our man; we have known him for 119 years and shall not let philosophers, historians, or others tell us what to think of his economic theory.

As an economic theorist Marx wanted to find the laws of motion of relative price, the rate of profit, and the real wage rate in a capitalist competitive economy.

2. Marx's Method

Marx was fond of Quesnay and saw the interdependence of industries. As Ricardo before him, Marx had two industries, a producers' good and a consumers' good industry. But his model was a step forward from Ricardo: to Marx it took producers' goods to produce producers' goods. Unlike Ricardo, Marx ignored land and used fixed input-output coefficients, hence could have no diminishing returns to anything.

Like Cantillon and Ricardo, Marx used words plus numerical examples. But Marx was neither a born nor a trained mathematician. Mathematical training might have saved him from his non sequiturs.

3. Our Own Restatement

Let us try to restate algebraically what parts of Marxian theory are well enough specified to permit such restatement. Samuelson (1957, 1971) has shown the way, and we shall follow him except on one point. We replace his (1957: 884), (1971: 413n.) strong assumption of a one-year useful life and simple interest by our weaker, more Marx-like, and more realistic assumption of a u -year long useful life of producers' goods. In other words, we think of Marx's producers' goods as being as

durable as the Ricardian ones. If so, we should adopt the same compound interest with continuous compounding we used in our chapter 3 on Ricardo.

I. NOTATION

1. Variables

$c \equiv$ constant capital

$H \equiv$ revenue minus operating labor cost

$J \equiv$ present net worth of an investment project

$L \equiv$ labor employed

$P \equiv$ price

$r \equiv$ rate of interest or profit

$S \equiv$ physical capital stock

$s \equiv$ surplus value

$v \equiv$ variable capital

$W \equiv$ wage bill

$w \equiv$ money wage rate

x_{ij} \equiv physical units of i th industry's good demanded by j th industry

X_i \equiv physical output of i th industry's good

Z \equiv profits bill

2. Parameters

a \equiv labor coefficient

b \equiv capital coefficient

u \equiv useful life of producers' goods

The symbol e is Euler's number, the base of natural logarithms.
The symbol t is the time coordinate. All flow variables refer to the instantaneous rate of that variable measured on per annum basis.

II. SIMULTANEOUS EQUALIZATION OF RATES OF SURPLUS VALUE AND PROFIT?

1. Surplus Value

Marx imagined a capitalist-entrepreneur producing commodities from labor hired and a physical capital stock of producers' goods owned.

The value in use of the labor hired was the value in exchange of the commodities produced. The value in exchange of the labor hired was called "variable" capital v and in Marx's words [1867 (1908: 190)] "is the value of the means of subsistence necessary for the maintenance of the labourer."

The value in use of the physical capital stock of producers' goods owned equaled their value in exchange. That, in turn, was called "constant" capital c and equaled the labor necessary to produce them.

"Surplus" value s was defined as the difference between the value in exchange of the commodities produced and the cost of all capital used, variable as well as constant. Notice that the source of surplus value was variable capital only, never constant capital.

2. Rates of Surplus Value Versus Rates of Profit

Marx defined his rate of surplus value as s/v or surplus value divided by its source, variable capital. In volume I [1867 (1908)] he thought that competition would equalize rates of surplus value among industries or, in two industries, that

$$s_1/v_1 = s_2/v_2 \tag{1}$$

Marx defined his rate of profit as $s/(c + v)$ or surplus value divided by all capital, constant as well as variable. In volume III [1894 (1909)] Marx thought that competition would equalize rates of profit among industries:

$$s_1/(c_1 + v_1) = s_2/(c_2 + v_2) \quad (2)$$

In both (1) and (2) multiply across, subtract first result from second, and find

$$c_2 s_1 = c_1 s_2 \quad (3)$$

Multiplied across (2) may be written

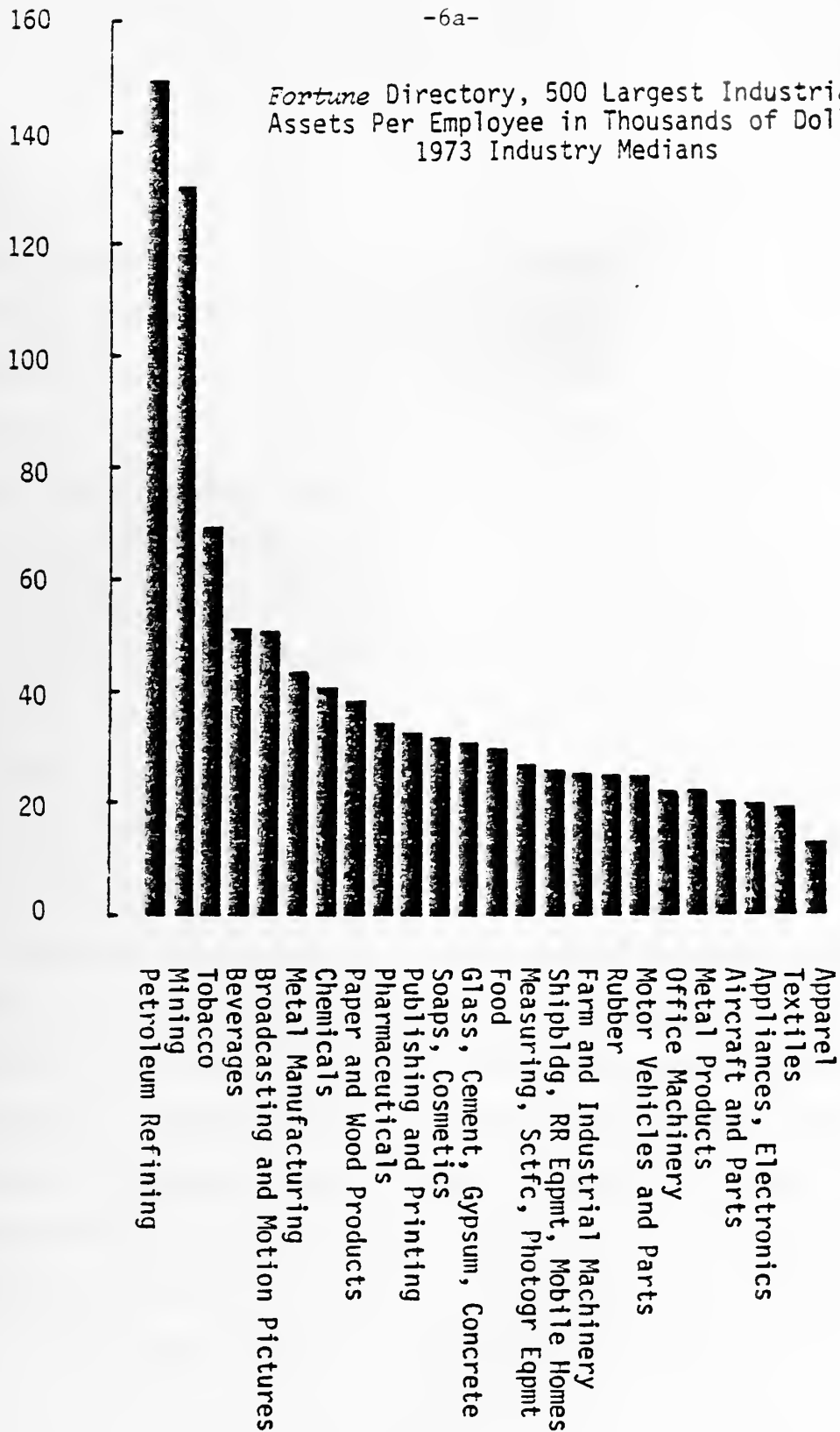
$$s_1 v_2 = s_2 v_1 \quad (4)$$

Divide (3) by (4) and find

$$c_1/v_1 = c_2/v_2 \quad (5)$$

from which we see that for simultaneous equalization of rates of surplus value (1) and of rates of profit (2) the ratio c/v between

Fortune Directory, 500 Largest Industrials,
Assets Per Employee in Thousands of Dollars,
1973 Industry Medians



Data:
Fortune, May 1974, 254. © 1974 Time, Inc.

Figure 4-1. U.S. Capital Intensities By Industry

constant and variable capital must be the same in the two industries. Is it the same? In our own time it certainly isn't: figure 4-1 shows a wide variation of assets per employee among industries. In Marx's day the ratio probably wasn't the same either, and Marx struggled [1894 (1909: 183-186)] to "transform" the "values" of his volume I into the "prices" of his volume III. Let us examine his transformation problem.

III. THE TRANSFORMATION PROBLEM

1. "Values" Versus "Prices"

Marx distinguished between "values" and "prices of production." The values of his volume I [1867 (1908)] resulted from equalization of rates of surplus value among industries. The prices of his volume III [1894 (1909)] resulted from equalization of rates of profit among industries. Does the difference matter? It does. Industry by industry, competitive prices will not normally reflect Marxian values. We must choose between values and prices. The choice is easy once we realize what perfect mobility of labor and capital does and does not do.

Assume perfect mobility of labor among industries. If one industry generated a higher rate of surplus value than another, would labor leave the low-rate industry and enter the high-rate industry? The answer is no, and the obvious reason is that a worker can tell what rate of surplus value he is creating if and only if the capitalist-entrepreneurs will open their books to him. What he can tell without the books is what money wage rate he is paid. If one industry offers a higher money wage rate than another, labor will leave the low-wage industry and enter the high-wage industry. That's all.

Next assume perfect mobility of capital among industries. If one industry generated a higher rate of surplus value than another, would capital leave the low-rate industry and enter the high-rate industry? Again the answer is no. From his books the capitalist-entrepreneur can tell what his surplus value s is, divide it by variable capital v , and find his rate of surplus value s/v . But why should he care? What matters to him is his rate of profit $s/(c + v)$ on all his capital, whether constant or variable. He will leave industries offering a low rate of profit and enter industries offering a high rate of profit. We can't say it better than Marx himself did in volume III [1894 (1909: 181)]:

...There is no doubt that, aside from unessential, accidental, and mutually compensating distinctions, a difference in the average rate of profit of the various lines of industry does not exist in reality, and could not exist without abolishing the entire system of capitalist production.

2. Conclusion

We conclude that perfect mobility of neither labor nor capital will equalize rates of surplus value among industries. What perfect mobility of labor will equalize is the money wage rate, and what perfect mobility of capital will equalize is the rate of profit. Marx's equalization of rates of surplus value among industries was a non sequitur.

We can now take our stand on the transformation problem: we choose the prices of volume III rather than the values of volume I. We are in good company. Samuelson (1971: 413-414) has shown that Marx's own struggle with the transformation problem was inconsistent and (1971: 418-422) that Marx's treatment in volume I is redundant and may safely be replaced by his treatment in volume III. Joan Robinson [1942 (1966: 22)] agreed, and that is precisely what we shall do.

IV. RELATIVE PRICE

1. Technology

As a Ricardian model, let a Marxian one have two industries, a producers' good and a consumers' good industry, called $i = 1, 2$, respectively. In either industry let labor input and physical capital stock be in proportion to output:

$$L_i = a_i X_i \tag{6}$$

$$S_i = b_i X_i \tag{7}$$

Both industries use both inputs, so $a_i > 0$ and $b_i > 0$.

2. Zero Present Net Worth

Let a capitalist-entrepreneur in the i th industry consider acquiring a capital stock of S_i new physical units of producers' goods whose useful life is u . So he must be planning for u years. Let his annual physical output be X_i of the i th good to be sold at the price P_i . His

revenue will then be $P_i X_i$. Since he is employing L_i men at the money wage rate w , his operating labor cost is $L_i w$. At time t , then, his revenue minus operating labor cost would be

$$H_i \equiv P_i X_i - L_i w \quad (8)$$

and per small fraction dt of a year located at time t in the future his revenue minus operating labor cost would be $H_i dt$.

Let there be a market in which money may be placed or borrowed at the stationary rate of interest r . Let that rate be applied when discounting future cash flow. As seen from the present time τ , then, his revenue minus operating labor cost would be $e^{-r(t - \tau)} H_i dt$. Define the present gross worth of the investment project as the present worth of the sum total of all future revenue minus operating labor cost over the entire useful life u of the new capital stock S_i or

$$k_i(\tau) \equiv \int_{\tau}^{\tau + u} e^{-r(t - \tau)} H_i dt \quad (9)$$

Here H_i as defined by (8) is not a function of t , hence may be taken outside the integral sign. The rate of interest r was said to be stationary, hence the coefficient of t is stationary. As a result find the integral to be

$$k_i = \frac{1 - e^{-ru}}{r} H_i \quad (10)$$

P_1 is the price of a new physical unit of producers' goods. Assume the salvage value of the unit when retired to be zero. The present net worth of the investment project is then defined as the present gross worth (10) minus the cost of acquisition of the new capital stock S_i or

$$J_i \equiv k_i - P_1 S_i \quad (11)$$

Insert (8) and (10) into (11) and write the present net worth of the investment project as

$$J_i = \frac{1 - e^{-ru}}{r} (P_i X_i - L_i w) - P_1 S_i \quad (12)$$

What can we do with our present net worth (12)? Ricardian present net worth had a maximum--which we found. Marxian present net worth has none. In Marx there is no diminishing return to anything--land, labor, or capital. Under a stationary technology (6) and (7), L_i and S_i are

in direct proportion to X_i . For a given competitive price P_i , rate of interest r , useful life u , and money wage rate w , then, present net worth J_i of the investment project is in direct proportion to physical capital stock S_i and has no maximum. All is not lost however.

3. Relative Price of Producers' and Consumers' Goods

Pure competition, freedom of entry, and freedom of exit will make prices P_i adjust until--as in volume III but not volume I--rates of profits have been equalized among industries. We who have distinguished between interest and profits might now say that the equalized rate of profit in equilibrium must equal the rate of interest common to all borrowers. That equality is nothing but zero present net worth in all industries.

So good volume-III Marxists may drop our distinction between a rate of interest and a rate of profit, call both of them r , set present net worth (12) equal to zero, multiply it by $r/(1 - e^{-ru})$, divide it by physical output X_i , use (6) and (7), and write a Marxian price equation

$$P_i - a_i w - P_l b_i r / (1 - e^{-ru}) = 0 \quad (13)$$

Now first write (13) for $i = 1$ and find the price of producers' goods

$$P_1 = \frac{a_1}{1 - b_1 r / (1 - e^{-ru})} w \quad (14)$$

Then write (13) for $i = 2$, insert (14), and write the price of consumers' goods

$$P_2 = \frac{1 + a_1 (b_2/a_2 - b_1/a_1) r / (1 - e^{-ru})}{1 - b_1 r / (1 - e^{-ru})} a_2 w \quad (15)$$

Finally divide (14) by (15) and write the relative price of producers' and consumers' goods

$$\frac{P_1}{P_2} = \frac{1}{1 + a_1 (b_2/a_2 - b_1/a_1) r / (1 - e^{-ru})} \frac{a_1}{a_2} \quad (16)$$

Does (16) have all that Marx called "socially necessary labor" in it, i.e., direct as well as indirect labor? It does. The production of the i th good absorbs direct labor according to the labor coefficient a_i and indirect labor according to the capital coefficient b_i .

But what is the rate of interest or profit r doing in (16)? Well, direct and indirect labor are both absorbed by the i th good but not at the same time. The direct labor absorbed in the n th year of useful life of producers' goods is n years apart from the indirect labor originally absorbed when the producers' goods were being built. Direct and indirect labor n years apart are not additive until synchronized. So one or both of them must be moved through time until they meet. But time is money, and the rate of interest or profit r is its price. That rate is inherent in synchronization, must appear in (16), and does in the transcendental form $r/(1 - e^{-ru})$. A table of powers of e will show that the transcendental form is a rising function of r .

The ratios b_i/a_i between capital and labor coefficients are capital-labor ratios or, as we could call them nowadays, capital intensities. Three possibilities immediately suggest themselves.

First, if the capital intensities of producers' and consumers' goods are the same, i.e.,

$$b_1/a_1 = b_2/a_2 \tag{17}$$

then according to (16) $P_1/P_2 = a_1/a_2$, a plausible result: if producers' goods have the same capital intensity as consumers' goods they also absorb the same indirect man-years per direct man-year and carry the same interest charge inherent in the synchronization of indirect and direct labor. For both reasons their relative price may be expressed, by proxy so to speak, by their relative direct labor coefficient.

Second, if producers' goods have a higher capital intensity than do consumers' goods, i.e.,

$$b_1/a_1 > b_2/a_2 \quad (18)$$

then according to (16) $P_1/P_2 > a_1/a_2$, a plausible result: if producers' goods have the higher capital intensity they also absorb more indirect man-years per direct man-year and carry the higher interest charge inherent in the synchronization of indirect and direct labor. For both reasons their relative price must be higher than indicated merely by their relative direct labor coefficient.

Third, if producers' goods have a lower capital intensity than do consumers' goods, i.e.,

$$b_1/a_1 < b_2/a_2 \quad (19)$$

then according to (16) $P_1/P_2 < a_1/a_2$, a plausible result: if producers' goods have the lower capital intensity they also absorb less indirect man-years per direct man-year and carry a lower interest charge inherent in the synchronization of indirect and direct labor. For both reasons their relative price must be lower than indicated merely by their relative direct labor coefficient.

4. Conclusion

All this was a model of competitive market prices. Its essence was set out in volume III [1894 (1909)], yet there is nothing peculiarly Marxian about it. It does have technologically fixed input-output coefficients, i.e., no diminishing return to anything. But so had Cantillon (and even Walras in his first edition until Barone taught him marginal productivity). Specifically the model set out is not a labor theory of value. If relative price were expressed in nothing but relative labor embodiment, then the rate of interest or profit r should not have appeared in (16) but did--except for the trivial case (17), devoid of practical interest as Gordon (1961) showed. Precisely because it is not a labor theory of value, (16) is rich enough to capture practically interesting cases.

V. THE REAL WAGE RATE

1. The Factor-Price Frontier Under Stationary Technology

The real wage rate is just another relative price, i.e., the relative price of labor and consumers' goods, and is fully contained in our result (15). Rearrange the latter and write it as the real wage rate

$$\frac{w}{P_2} = \frac{1 - b_1 r / (1 - e^{-ru})}{[1 + a_1 (b_2/a_2 - b_1/a_1) r / (1 - e^{-ru})] a_2} \quad (20)$$

Under stationary technology a_1 , a_2 , b_1 , and b_2 , how are the real wage rate w/P_2 and the rate of interest or profit r related? Let us first find how the real wage rate w/P_2 and the expression $r/(1 - e^{-ru})$ are related in (20), so take the derivative of the former with respect to the latter, let lots of things cancel, and find

$$\frac{\partial(w/P_2)}{\partial[r/(1 - e^{-ru})]} = - \frac{a_1 b_2}{\{[1 + a_1 (b_2/a_2 - b_1/a_1) r / (1 - e^{-ru})] a_2\}^2} \quad (21)$$

which is unequivocally negative. Now the expression $r/(1 - e^{-ru})$ is a rising function of r . So if according to (21) the real wage rate (20) and the expression $r/(1 - e^{-ru})$ are negatively related, so are the real wage rate (20) and the rate of interest or profit r . Under stationary technology, then, if the rate of interest or profit r is down the real wage rate w/P_2 is up.

2. Technological Progress: Marx's Own View

What interested Marx, however, was not stationary technology but the effect of technological progress upon the rate of profit and the real wage rate. Let us go back to his definition of the rate of profit $s/(c + v)$, divide numerator and denominator alike by v , and write it

$$\frac{s/v}{1 + c/v}$$

from which we see that if the rate of surplus value s/v stayed the same and if technological progress raised the constant-to-variable capital ratio c/v , then the rate of profit would fall. But would the rate of surplus value s/v stay the same? In a labored numerical example in

volume III [1894 (1909: 247)] Marx assumed it to do so. In other volumes he differed. In volume I [1867 (1908: 422)] he said that "modern industry raises the productiveness of labour to an extraordinary degree." In volume II [1894 (1915: 267)] he exemplified:

Thus machinery shortens the building time of houses, bridges, etc.; a mowing and threshing machine, etc., shorten the working period required to transform the ripe grain into a finished product. Improved ship-building reduces by increased speed the time of turnover of capital invested in navigation.

Raised "productiveness of labour" must mean that either the same number of men produce more commodity value or fewer men are needed to produce the same commodity value. To be sure, constant capital c is up in the first place. Even so, nothing keeps the surplus value s , let alone the rate of surplus value s/v , from going up. But if both s/v and c/v are up, Marx cannot tell what would happen to his rate of profit. Yet, as Gottheil (1966: 99) reports, "in all the examples cited by Marx which deal with increasing organic compositions of capital the assigned

increases in the productiveness of labor never suffice to maintain the rate of profit." We agree with Gottheil (1966: 100) that "this is convenience, not necessity." Marx's falling rate of profit was a non sequitur.

In conclusion, then, Marx's falling rate of profit was no necessity but certainly a possibility--one possibility out of three. We may as well begin our discussion of the three possibilities with the Marxian one.

3. First Possibility: Falling Rate of Profit

Suppose that the capitalist-entrepreneur feels somehow forced to adopt a new technology, although it offers him a lower rate of profit than he was earning before the new technology came along. Capitalist-entrepreneurs have been heard lamenting such misfortune.

What is forcing him? Whatever his competitors are doing, a capitalist-entrepreneur may always remain in the old technology. If he fails to exercise that option the reason can only be that under the old technology his rate of profit would have been lower than it is under the new technology. Here, in our first possibility, the rate of profit r is down. Consequently, under the old technology the rate of profit would have been even more down--or our capitalist-entrepreneur would

have exercised his option of remaining in the old technology! But if under the old technology the rate of profit were down, it follows from the negativity of (21) that a higher real wage rate must be the reason. The new technology adopted by all the competitors has depressed the price of consumers goods P_2 relative to the money wage rate w . We can certainly imagine a two-input production function in which technological progress will raise the real wage rate and reduce the rate of profit-- although we know of no long periods or countries in which it has done so.

4. Second Possibility: Stationary Rate of Profit

Unbelievable to Marx, a stationary rate of profit r is a possibility to us--our second one. Here, under the new technology adopted by the competitors, the rate of profit is still the same as before. Consequently under the old technology the rate of profit would have been down--or our capitalist-entrepreneur would have exercised his option of remaining in the old technology! But once more, if under the old technology the rate of profit were down, it follows from the negativity of (21) that a higher real wage rate must be the reason. The new technology adopted by all the competitors has depressed the price of consumers goods P_2 relative to the money wage rate w . We can certainly imagine a two-input production function in which technological progress will raise the

real wage rate and leave the rate of profit unaffected and we know of long periods in countries like the United States and Britain in which it has actually done so, cf. Phelps Brown (1973) and summary in Brems (1980: 38-42).

5. Third Possibility: Rising Rate of Profit

Equally unbelievable to Marx, a rising rate of profit r is a possibility to us--our third one. Here, under the new technology adopted by the competitors, the rate of profit is up. Consequently under the old technology the rate of profit could have been down, the same, or up--but less up than under the new technology. All three cases would keep our capitalist-entrepreneur from exercising his option of remaining in the old technology. From our three cases and the negativity of (21) it follows that a higher, the same, or a lower real wage rate, respectively, must be the reason. In our third possibility, then, anything may happen to the real wage rate. Strange? Not at all. We can certainly imagine a two-input production function in which technological progress will raise, leave unaffected, or lower the real wage rate and raise the rate of profit--although we know of no long periods or countries in which it has done so.

6. A Fourth Possibility?

The one and only possibility we cannot imagine is a two-input production function in which technological progress will reduce both the real wage rate and the rate of profit. The fruits of technological progress must accrue somewhere! Yet this possibility is the very one Marx imagined and thought would prevail or, in his own words [1867 (1908: 708-709)]: "It follows therefore that in proportion as capital accumulates, the lot of the labourer, be his payment high or low, must grow worse." For documentation, Marx [1867 (1908: 739)] quotes Ducpétiaux, "inspector-general of Belgian prisons and charitable institutions, and member of the central commission of Belgian statistics," who asked how such immiserization was possible and answered:

...by adopting expedients, the secret of which only the labourer knows; by reducing his daily rations; by substituting rye-bread for wheat; by eating less meat, or even none at all, and the same with butter and condiments; by contenting themselves with one or two rooms where the family is crammed together, where boys and girls sleep side by side, often on the same pallet; by economy of clothing, washing, decency; by

giving up the Sunday diversions; by, in short, resigning themselves to the most painful privations.

What about a three-input production function like Ricardo's? Here technological progress could reduce both the real wage rate and the rate of profit but raise the real rent rate. Marx refused his teacher's help and had no land. His falling real wage rate was a non sequitur.

Enough about prices, wages, and profits. Let us finally turn to interindustry equilibrium.

VI. INTERINDUSTRY EQUILIBRIUM

1. Quesnay-Marx

Marx [1904 (1923: 34)] called Quesnay's table "the most ingenious invention of which political economy has until now been guilty." In his volume II Marx saw the interdependence of his two industries but dimmed in two respects. First, Marx never admitted preferences, consequently his consumers' goods industry never produced anything else

than a single consumers' good. That was a retreat from Quesnay's distinction between farm and city products. Second, written before volume III, volume II assumed [1894 (1915: 454)] "that products are exchanged at their value."

So Marx's interindustry equilibrium had two industries, i.e., his producers' good and his consumers' good industries. In our table 4-I let us write a Leontief transactions table for Marx's two-industry model. But this time let us distinguish between prices and quantities and define transactions x_{ij} as physical units of i th industry's good demanded by j th industry and output X_i as physical output of i th industry's good. Multiplying x_{ij} and X_i by their price P_i will express them in terms of dollars as shown in table 4-I.

Can we solve Marx's model of interindustry equilibrium for the physical outputs of its two industries? Let us write as many equations as Marx permits and begin with investment.

2. Investment

Marx's "simple reproduction" meant a stationary economy. Here there is no net investment. But just like a Ricardian stationary economy with a finite useful life u of producers' goods, a Marxian one must replace retired producers' goods. Let producers' goods have the same

TABLE 4-I. A TWO-SECTOR LEONTIEF TRANSACTIONS TABLE

	Producers' Goods Industry	Consumers' Goods Industry	Row Total
	(1)	(2)	
Producers' Goods Industry (1)	P_1x_{11}	P_1x_{12}	P_1X_1
Consumers' Goods Industry (2)	P_2x_{21}	P_2x_{22}	P_2X_2

TABLE 4-II. MARK'S TRANSACTIONS IN "SIMPLE REPRODUCTION"

	Producers' Goods Industry	Consumers' Goods Industry	Row Total
	(1)	(2)	
Producers' Goods Industry (1)	P_1x_{11}	P_1x_{12}	P_1X_1
Consumers' Goods Industry (2)	$P_1X_1 - P_1x_{11}$	$P_2X_2 - P_1x_{12}$	P_2X_2
Column Total	P_1X_1	P_2X_2	$P_1X_1 + P_2X_2$

useful life whether used to produce producers' or consumers' goods. Let age distribution be even, then each year $1/u$ of the physical capital stock of the i th industry is retired and must be replaced. So investment demand by the i th industry is

$$x_{1i} = S_i/u \tag{22}$$

where $i = 1, 2$.

3. Consumption

As we saw in IV, 2 above, a capitalist-entrepreneur in the i th industry used a physical capital stock S_i to produce the physical output X_i sold at the price P_i . His revenue was then $P_i X_i$. Since he employed L_i men at the money wage rate w , his operating labor cost was $L_i w$, and his revenue minus operating labor cost was $H_i \equiv P_i X_i - L_i w$. That was his gross income before capital consumption allowances. Subtract his capital consumption allowances $P_1 x_{1i}$ and find his profits bill

$$Z_i \equiv P_i X_i - L_i w - P_1 x_{1i} \tag{23}$$

where $i = 1, 2$. In a stationary economy there is no accumulation, so let him consume his entire profits bill (23). Let labor employed in the i th industry consume its entire wage bill

$$W_i \equiv L_i w \quad (24)$$

Total consumption demand by the i th industry is then the sum of (23) and (24):

$$P_2 x_{2i} = W_i + Z_i = P_i X_i - P_1 x_{1i} \quad (25)$$

where $i = 1, 2$.

4. Interindustry Equilibrium

Recall two conditions for interindustry equilibrium in a Leontief transactions table. Here, a row will account for all demand satisfied by a sector's supply. Consequently, in equilibrium the row total must equal the sector's supply. In either industry, goods-market equilibrium will require the supply of goods to equal the demand for them:

$$X_i = \sum_{j=1}^2 x_{ij} \quad (26)$$

A column will account for all supplies satisfying a sector's demand. Consequently the column total must equal the sector's demand. In equilibrium a sector must break even: its revenue must equal its expenditure. In other words, its row total must equal its column total.

Does a Marxian intersector equilibrium satisfy both equilibrium conditions? Multiply (26) by P_i and see that the first condition is satisfied. Insert (25) into the second row of our Leontief transactions table 4-I, find the column totals, and see that the second condition is satisfied. With both conditions satisfied we may write the Leontief transactions table 4-I as the Marx transactions table 4-II.

5. Solution for Physical Outputs?

Define aggregate employment as the sum of labor employed in the two industries:

$$L \equiv \sum_{i=1}^2 L_i \tag{27}$$

Insert (6) into (27) and find

$$L = a_1 X_1 + a_2 X_2 \tag{28}$$

Insert (7) into (22) and the result into (26) and find

$$X_1 = b_1 X_1 / u + b_2 X_2 / u \quad (29)$$

Now if L were a parameter, (28) and (29) would be two linear equations in the two unknowns X_1 and X_2 and could easily be solved:

$$X_1 = \frac{b_2}{a_1 b_2 + a_2 (u - b_1)} L \quad (30)$$

$$X_2 = \frac{u - b_1}{a_1 b_2 + a_2 (u - b_1)} L \quad (31)$$

Was L a parameter to Marx? Marx was enough of an English classicist to think of labor as reproducible at a value in exchange equaling "the value of the means of subsistence necessary for the maintenance of the labourer," as we saw in the quote from Marx [1867 (1908: 190)]. But he was not enough of an English classicist to use this notion to determine sustainable employment, as Ricardo had done. To go that far, he would have needed a fixed quantity of land coupled with either Cantillon's

fixed input-output coefficients or Ricardo's diminishing returns. Marx admired Ricardo but despised Malthus and overcame his dilemma by removing land from his model.

What are we to do with our labor employed L , then?

A simple possibility would be to get rid of it by dividing it away. Divide (30) by (31), let L and the denominators cancel, and find the relative size of the two sectors

$$X_1/X_2 = b_2/(u - b_1) \quad (32)$$

In English: the relative size X_1/X_2 of the two sectors depends upon their capital coefficients b_1 and b_2 and the useful life of capital stock but not on total labor employed L .

Another possibility would be to treat labor employed L as a parameter, give it a growth rate g_L , treat that rate as a parameter, too, assume capitalists always to be willing and able to accumulate the necessary capital, differentiate (6), (7), (30), and (31) with respect to time, thus building a growth model with the solutions

$$g_{Li} = g_{Si} = g_{Xi} = g_L \quad (33)$$

Towards the end of volume II and still assuming products to be exchanged at the "values" of volume I, Marx actually hinted at such a growth model.

VII. CONCLUSIONS

As a theorist measured by Cantillon, Ricardo, or Böhm-Bawerk standards, Marx is disappointing. Perhaps because of its sheer bulk, his system was inconsistent. Its first non sequitur was that rates of surplus value would be equalized among industries. Even if they were, the second non sequitur would still be that under technological progress the rate of profit would be falling. Even if it were, the third non sequitur would still be that the real wage rate would also be falling.

In economic history the three non sequiturs fared no better than they did in economic theory: none of them came true.

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