THE NONSTATIONARITY OF SYSTEMATIC RISK FOR BONDS

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Summary:

Recently a number of researchers have attempted to employ the
market model to estimate systematic risk (i.e., beta) for bonds. In
this study we reviewed theoretical evidence which suggests bond
betas can be expected to be nonstationary. This nonstationarity is
a function of the duration of a bond, the standard deviation of the
change in the yield to maturity of a bond relative to the standard
deviation of the return on the market portfolio, and the correlation
between the change in the yield to maturity of a bond and the return
on the market portfolio. However, all bonds will not necessarily
have nonstationary betas in a given time period since it is possible
that these factors may occasionally counteract one another.
Empirical tests indicated that over 80 percent of the bonds ex-
amined had nonstationary betas. The primary factor differentiating
bonds with nonstationary betas from those with stationary betas was
the substantially higher relative standard deviation in the change in
the yield to maturity for bonds with nonstationary betas. The larger
standard deviation was caused by the higher average coupon rates and
yields to maturity for bonds with nonstationary betas. The theoretical
and empirical results of this study indicate bond betas, in general,
tend to be nonstationary. Hence, further use of them appears to be
of very questionable value.

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I. INTRODUCTION

In the last decade increasing use has been made of the Capital Asset Pricing Model (CAPM) developed by Sharpe [31], Lintner [7] and extended by Black [2]. In this model the only relevant risk of an asset is the systematic risk which is measured by the covariance of the ex-ante return on the asset with the ex-ante return on the market portfolio. Since the ex-ante returns cannot be observed, researchers have used historical data to estimate the systematic risk. The market model, which has been the most common method of estimating the relative systematic risk (β_i) states that:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it},\qquad\qquad(1)$$

where R_{it} and R_{mt} are returns on i^{th} asset and the market portfolio respectively, and β_i is computed as $cov(R_{it}, R_{mt})/\sigma^2(R_{mt})$. The use of historical data to estimate β_i is justified only if the joint distribution of returns on the asset and the market portfolio is stable over time. Under these conditions β_i will be stationary and hence the market model will be an appropriate method of estimating β_i.

Recently a number of researchers have applied the market model to estimate systematic risk for bonds. Percival [27] estimated bond betas and then attempted to explain them as a function of the bond characteristics. Friend and Blume [14] and McCallum [21] also estimated bond systematic risk while Reilly and Joehnk [28] examined the relationship between bond betas and bond ratings. Finally, Warner [34] estimated

bond betas and then examined the risk adjusted performance of bonds for firms in bankruptcy versus the performance of bonds for similar firms not in bankruptcy.

The increasing use of the bond betas appears to be without support since there are theoretical considerations suggesting bond betas are inherently nonstationary. In the presence of this nonstationarity, bond betas appear to be poor estimates of systematic (or any other kind of) risk for bonds. The purposes of this study are threefold: 1) to examine the theoretical considerations indicating nonstationarity of bond betas; 2) to test empirically whether the betas of individual bonds are stationary over the 1969-1975 time period; and 3) to explain the observed stationarity/ nonstationarity in terms of the factors that cause the nonstationarity in bond beta. In Section II theoretical arguments for the nonstationarity of bond betas are reviewed, while Section III contains the methodology employed. The empirical results are presented and discussed in Section IV and the conclusions are contained in Section V.

II. THEORETICAL CONSIDERATIONS

Several recent studies [4,15,18,35] have examined a specific time-risk relationship using a measure of time known as duration. The concept of duration was first introduced by Macaulay [19] in his study of bond yields. Unlike the time to maturity, which looks only at the last payment, duration gives some weight to the time at which each cash payment is received. The weight assigned to each period is the present value of the cash payment for that period divided by the current market price of the security. For a bond, duration at time t_0 is computed as:

$$D_{t_0} = [\sum_{j=1}^{N} (t_j - t_0) A_{t_j}] / \sum_{j=1}^{N} A_{t_j} \qquad (2)$$

where A_{t_j} is the present value measured at time t_0 of cash flows to be received at time t_j and N is the number of years to maturity. From (2) it is apparent that duration is a function of the time to maturity, the size of interim coupon payments, the yield to maturity and the size of the principal payment. For a zero-coupon bond, duration is identical to the time to maturity. The link between the bond price volatility and duration was developed by Fisher [13] and extended by Hopewell and Kaufman [15]. Assuming continous compounding, the percentage change in a bond price is related to duration by:

$$\frac{dP_{it}}{P_{it}} = -D_{it} \, dy_{it}, \qquad (3)$$

where dP_{it} and P_{it} are the price change and initial price of bond i at time t respectively. D_{it} is the duration of the bond at time t and dy_{it} is the change in the yield to maturity. Equation (3) shows that duration is a constant of proportionality relating percentage changes in bond prices to changes in the yield (dy_{it}).

Boquist, Racette and Schlarbaum [4] developed a theoretical model which links the beta of a default free bond to duration:

$$\beta_{it} = -D_{it} \frac{cov(dy_{it}, R_{mt})}{\sigma^2(R_{mt})} = -D_{it} \, \rho(dy_{it}, R_{mt}) \frac{\sigma(dy_{it})}{\sigma(R_{mt})}, \qquad (4)$$

where $\sigma(dy_{it})$ is the standard deviation of dy_{it}, $\sigma(R_{mt})$ is the standard deviation of the return on the market portfolio, and $\rho(dy_{it}, R_{mt})$ is the correlation coefficient between changes in the yield to maturity and the return on the market portfolio. (As argued by Boquist et al., the correlation coefficient is expected to be negative for most bonds.) From

equation (4) it is apparent that β_{it} is dependent upon the duration of
the bond, the correlation coefficient between changes in the yield to
maturity of the bond and the return on the market, and the standard devia-
tion of the changes in the yield to maturity for the bond relative to the
standard deviation of the return on the market portfolio.[1] Therefore,
depending upon the interaction of changes over time in the following three
factors: 1) D_{it}; 2) $-\rho(dy_{it}, R_{mt})$; and 3) $\sigma(dy_{it})/\sigma(R_{mt})$ the bond beta
may be stationary or nonstantionary. As a bond progresses toward
maturity the duration, D_{it}, will shorten which, ceteris paribus, should
cause β_{it} to decrease. The second factor, $-\rho(dy_{it}, R_{mt})$, may also cause
β_{it} to decrease over time. Through the passage of time the maturity of
a bond becomes shorter. In general short-term yields tend to be less cor-
related with the return on the market portfolio than the long-term yields.
Therefore as time passes the second factor will cause β_{it} to decrease.[2]
Finally, the third factor, $\sigma(dy_{it})/\sigma(R_{mt})$, should cause β_{it} to increase
because, as Malkiel [20] has shown, short-term yields tend to be more
volatile than long-term yields. Unless these factors exactly offset each
other bond betas estimated from historical time series data will be non-
stationary.

III. METHODOLOGY

A. SAMPLE

In order to empirically test for the nonstationarity of bond
systematic risk a homogeneous group of bonds was required. The selection
criteria employed resulted in bonds being selected if they were public
utility or industrial bonds continuously rated (without any change) in
the top four bond rating categories by both Moody's and Standard & Poor's

between May 31, 1969 and May 31, 1975, were issued between January 1,
1966 and March 1, 1969, had an original maturity of at least 20 years
and an original issue size of at least $10 million.[3] In addition, the
bonds could not be subordinated or convertible, nor could they be issued
with warrants attached.[4] In cases where there were more than one bond
per company that met the selection criteria, the most recent issue was
selected. Application of these criteria resulted in 84 bonds being
selected of which 42 were public utility bonds and 42 were industrial
bonds.[5]

B. VARIABLES

1. Holding Period Return

Monthly holding period returns for bonds were computed as:

$$R_t = \frac{\frac{1}{m} I + \Delta P_t}{P_{t-1} + \frac{n-1}{m} I} , \tag{5}$$

where I is the periodic interest payment per $100 of par value;
m is the number of holding periods between interest payments (for most
bonds m = 6 months); n is the number of periods accrued toward the next
interest payment at the end of period t; and P_{t-1} is the market price
of the bond at the end of period t-1.

Some authors have used different methods to measure the return on bonds.
Yawitz and Marshall [36] used purchase yield as a measure of return on
U.S. Government bonds. They reasoned that it is a better measure of the
expected return because over the life of the bond, price changes must
sum to zero. This argument is valid only if the investors' holding period
is equal to the life of the bond. Yield to maturity has also been used
as a measure of the returns on bonds by Duvall and Cheney [10]. They

argued that yield to maturity is a more reliable estimate of the expected return than the ex-post measure as formulated in equation (5). Reilly and Joehnk [28] employed the percentage change in the yield to maturity as a measure of the bond return. Again these authors are implicitly assuming that investors have a holding period equal to the life of the bond, the bond is default free, and investors can reinvest the intermediate interest payments at a rate equal to the yield to maturity. Because of the above mentioned problems with these return measures, we prefer to use equation (5) to measure the holding period return.

2. Market Portfolio

Traditionally a portfolio of common stocks has been employed as a proxy for the market portfolio. According to the CAPM, the market portfolio should contain all risky assets such as common stocks, bonds, preferred stocks, real estate, human capital, etc. Construction of such a portfolio is very difficult, if not impossible, because the data on these assets are not readily available.

A review of the literature on bonds reveals that different proxies for the market portfolio have been employed. Percival [27] and McCallum [21] used an equally weighted portfolio of their bonds, Friend and Blume [14] and Warner [34] utilized a common stock portfolio, while Reilly and Joehnk [28] used three different bond portfolios and two different common stock portfolios. As demonstrated by Roll [29] the choice of the market portfolio greatly affects the estimated beta. In this study a value weighted market portfolio is constructed which includes common stocks, corporate bonds and government bonds each weighted by their corresponding

market value.[6] We believe this is a more reasonable proxy for the
market portfolio and clearly superior to the proxies employed in other
studies.

C. STATISTICAL TECHNIQUES

Since the stationarity of beta is a time related phenomenon, the
traditional method of testing for stationarity using correlation coeffi-
cients is inappropriate. There are basically two problems with the use
of the correlation coefficient as a measure of stationarity. First,
when using equation (1) to estimate β, it is implicitly assumed that β
is stationary during the estimation period. Second, the correlation
coefficient cannot be used to determine the stationary of the individual
securities. It is, in essence, an aggregate measure of stationarity of
the betas for a group of securities or portfolios. An ideal test for
stationarity should detect the constancy of the security beta over time
by examining whether or not the regression coefficients in the market
model vary over time.

Since we were primarily interested in the stability of β_i (not
α_i and β_i simultaneously) we also estimated β_i by:

$$r_{it} = \beta_i r_{mt} + \epsilon_{it}, \tag{6}$$

where $r_{it} = R_{it} - R_{ft}$, $r_{mt} = R_{mt} - R_{ft}$, R_{ft} is the risk free rate of
interest and the intercept (α_i) was supressed.[7] To correctly examine
the behavior β_i over time, equation (6) is rewritten as:

$$y_t = \beta_t x_t + \epsilon_t \tag{7}$$

where subscript t on β indicates that it may vary over time, y_t is the vector of returns on a bond, x_t is the vector of returns on the market portfolio, and ε_t is the vector of disturbances. The null hypothesis for stationarity is formulated as:

$$H_0: \quad \beta_1 = \beta_2 = \ldots = \beta_T. \tag{8}$$

In words, the null hypothesis states that β is stable over time. The alternate hypothesis is that not all β's for an individual bond are equal.

The stationarity of β problem is a special case of the general class of problems concerned with detection of changes in the regression model structures over time. Early work on detecting changes in a model structure employed the ordinary least square (OLS) residuals or the cumulative sum of the OLS residuals. The difficulty with these approaches, however, is that there is no known method of assessing the significance of the nonstationarity in the regression coefficients (cf., Mehr and McFadden [22]). To avoid problems associated with the OLS residuals, Brown and Durbin [6], and Brown, Durbin, and Evans (BDE) [7] proposed using recursive residuals. BDE have shown that under the null hypothesis of stationarity the recursive residuals have the desirable properties of being uncorrelated, with zero mean and constant variance, and therefore are independent of each other under the normality assumption. Recursive residuals are also preferrable to OLS residuals for detecting nonstationarity in β because until a change takes place the recursive residuals behave exactly as specified in the null hypothesis. Recursive residuals are defined as:

$$w_r = (y_r - x_r' b_{r-1})/[1 + x_r' (X'_{r-1} X_{r-1})^{-1} x_r]^{\frac{1}{2}} \qquad (9)$$

$$r = k+1, \ldots, T$$

where k is the number of regression coefficients (1 in equation (7)),
$X'_{r-1} = [x_1, \ldots, x_{r-1}]$, $b_r = (X_r' X_r)^{-1} X_r' Y_r$, and $Y_r' = (y_1, \ldots, y_r)$.
For each value of r, which in our study takes a value between 2 and 72,
the recursive residual was computed using equation (9).[8]

BDE derived a statistical test for stationarity using the cumu-
lative sum of the squared recursive residuals. This test, the cusum
of squares test, detects both systematic and random changes in the β
and is based on the following formulation:

$$s_r = (\sum_{j=k+1}^{r} w_j^2)/(\sum_{j=k+1}^{T} w_j^2), \quad r=k+1, \ldots, T. \qquad (10)$$

Under the null hypothesis s_r has a beta distribution with mean
$(r-k)/(T-k)$. BDE suggested constructing a confidence internal for s_r as
$[(r-k)/(T-k)] \pm C$ where C is chosen from Table 1 of Durbin [9]. The
stationarity hypothesis will be rejected if $|s_r - ((r-k)/(T-k))| > C$ for
any r included in $[k+1, T]$.

If β is expected to change systematically over time another test
can be used to detect such changes [7]. This type of nonstationarity can
be tested using an F-test. Under the null hypothesis of stationarity,
equation (7) can be rewritten as:

$$y_t = x_t' \beta_0 + \epsilon_t, \qquad (11)$$

where β_0 denotes that the beta coefficient is stationary. Equation (11)
is the reduced model; under the alternate hypothesis β_t is assumed to
change linearly with time, or

$$y_t = x'_t \beta_t + \varepsilon_t, \tag{12}$$

where $\quad \beta_t = \beta_0 + \delta \cdot t, \text{ and} \tag{13}$

where δ is the coefficient of time. Substitution of equation (13) into (12) yields:

$$y_t = x'_t(\beta_0 + \delta \cdot t) + \varepsilon_t, \tag{14}$$

which is the full model. The null hypothesis of stationarity is tested by a comparison of the mean-square increase in the explained variation with the error variance. This F-test is:

$$F = \frac{SSE(R) - SSE(F)}{df(R) - df(F)} \div \frac{SSE(F)}{df(F)} \tag{15}$$

where $SSE(R)$ and $SSE(F)$ are the error sum of squares of the reduced and full models, respectively. Likewise, $df(R)$ and $df(F)$ are the degrees of freedom associated with the $SSE(R)$ and $SSE(F)$. It should be noted that this F-test detects only systematic changes in β, whereas the cusum of squares test detects both systematic and random changes in β. In this study the nonstationarity detected by the F-test is called "systematic nonstationarity", while the nonstationarity detected by the cusum of squares test but not with the F-test is called "random nonstationarity".

IV. EMPIRICAL RESULTS

A. SAMPLE CHARACTERISTICS

In Table 1 the sample characteristics are reported broken down by industrial versus public utility bonds. In general, the coupon rates are lower for the industrial bonds as are the years to maturity while

TABLE 1

Characteristics of the Sampled Bonds

	Number	Coupon Rate	Issue[b] Size	Years to Maturity	β[c]	R^2[c]	β[d]
TOTAL	84	6.452 (.638)[a]	60.071 (48.646)	27.274 (3.671)	.410 (.187)	.181 (.126)	.423 (.178)
INDUSTRIAL	42	6.129 (.564)	81.905 (65.490)	25.714 (2.361)	.412 (.245)	.177 (.154)	.428 (.232)
PUBLIC UTILITY	42	6.775 (.541)	38.238 (41.194)	28.833 (4.090)	.407 (.104)	.185 (.092)	.419 (.102)

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[a]Standard deviation in parethesis.

[b]In millions of dollars.

[c]From the market model given by equation (1).

[d]From the market model given by equation (6).

the issue sizes are larger for the industrial bonds than for public utility bonds. These findings are consistent with the typical characteristics of public utility bonds which have higher coupon rates and longer maturities. The average β of 84 bonds obtained from equation (1) is 0.410 with an average R^2 of 0.181. The average β of the bonds obtained by employing equation (6) is .423 (where α_1 was suppressed) which is virtually the same as that obtained by using equation (1). There are no significant differences in bond betas between the public utility and industrial groups.[9] In the rest of the study β_1 as estimated by equation (6) is employed.

B. STATIONARITY OF SYSTEMATIC RISK FOR BONDS

The cusum of squares test (for random nonstationarity) and the F-test (for systematic nonstationarity) were applied to each bond to determine whether the individual bond betas were stable or not over the period examined. The results of these tests (employing a 5 percent significance level) are reported in Table 2. Examination of this table indicates that 69.05 percent [(24+5)/42] of the industrial bonds had nonstationary betas, while 95.24 percent [(24+16)/42] of the public utility bonds had nonstationary bond betas. Overall, 82.14 percent [(48+21)/84] of the bonds examined had nonstationary betas with 25 percent (21/84) of the bonds exhibiting systematic nonstationarity and 57.14 percent (48/84) indicating random nonstationarity. Not only were more of the public utility bond betas unstable, but they also exhibited more systematic nonstationarity than did the industrial bonds. These results, for a very homogeneous set of bonds, provide strong empirical support for the

TABLE 2

Number of Bonds With Nonstationary Beta
Based on the Cusum of Squares and F Tests
(5 percent significance level)

| | Stationary | Nonstationary | | Total |
		Random	Systematic	
INDUSTRIAL	13	24	5	42
PUBLIC UTILITY	2	24	16	42
TOTAL	15	48	21	84

theoretical considerations presented in section II indicating that bond betas are inherently nonstationary.

Since our concern is not with the nature of the nonstationarity, per se, the rest of the analysis will focus on two groups of bonds—those with stationary betas and those with nonstationary betas (encompassing both random and systematic nonstationarity). In Table 3 the salient characteristics of these two groups of bonds are presented. As expected (based on Table 1 and the knowledge that more public utility bonds are included in the nonstationary group), the nonstationary bonds had a significantly higher average coupon rate, significantly smaller average size and significantly lower betas than bonds with stationary betas. While not statistically significant (at the 5 percent level), the bonds with nonstationary betas tend to have slightly lower bond ratings, while there is virtually no difference in the average years to maturity. The higher average coupon rate for bonds with nonstationary betas can also be seen by examining Table 4. Almost 50 percent (34/69) of the bonds with nonstationary betas have coupon rates greater than 6.5 percent while only 13 percent (2/15) of the bonds with stationary betas have coupon rates greater than 6.5 percent.

As presented in Section II, theoretical considerations indicate bond betas should be inherently unstable and this instability is related to: 1) the duration of the bond, D_{it}; 2) the correlation between the change in the yield to maturity of the bond and the return on the market, $-\rho(dy_{it}, R_{mt})$; and 3) the standard deviation of the change in the yield to maturity of the bond relative to the standard deviation of the return on the market, $\sigma(dy_{it})/\sigma(R_{mt})$. (As indicated in equation (4) the

TABLE 3

Statistics on Bonds with Stationary
and Nonstationary Bond Betas

Number	Stationary 15	Nonstationary 69	F Ratio --	Probability --
Coupon Rate	6.005 (.577)[a]	6.550 (.612)	9.93[b]	.0023
Years to Maturity[c]	19.267 (4.399)	20.043 (3.771)	.91	.3439
Issue Size[d]	110.000 (73.969)	49.217 (49.001)	16.61	.0001
Bond Rating[e]	2.333 (.900)	2.696 (.845)	2.21	.1407
Bond Beta	.535 (.166)	.399 (.173)	7.83	.0064

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[a]Standard deviation in parenthesis.

[b]With 1 and 82 degrees of freedom.

[c]As of January 1, 1975.

[d]In millions of dollars.

[e]1 = Aaa/AAA, 2 = Aa/AA, 3 = A/A and 4 = Baa/BBB.

TABLE 4

Coupon Rates for Bonds with
Stationary and Nonstationary Bond Betas

| | COUPON RATE | | | | | | |
	< 5.5	5.51 to 6.00	6.01 to 6.5	6.51 to 7.00	7.01 to 7.50	> 7.50	TOTAL
Stationary	3	6	4	1	1	—	15
Nonstationary	4	14	17	16	17	1	69

relationship between β_{it} and these factors carries a negative sign—for convenience we have appended the negative sign to the correlation.) In order to examine the relative impact of these factors on the observed stability/instability of the bond betas, we arbitrarily divided the study period into three 24 month periods. Then we calculated the average duration, D_{it},[10] the average correlation, $\rho(dy_{it}, R_{mt})$ and the average standard deviation, $\sigma(dy_{it})$ for the first and last 24 month periods and the relative change in these variables from the first to the last period. (Since the standard deviation of the market, $\sigma(R_{mt})$ is the same for all bonds, we ignore it and focus solely on $\sigma(dy_{it})$.

The results of this analysis of the change in duration, correlation and standard deviation for the two groups of bonds are presented in Table 5. For the bonds with stationary betas, the duration decreased, the standard deviation in the yield to maturity increased, and the correlation between the change in the yield to maturity and the return on the market portfolio decreased from the first to the third 24 month period. The same directional changes occurred for the bonds with nonstationary betas. However, the important difference in the two groups of bonds is the relative change (columns (3) and (6) of Table 5) in these three variables for the two bond groups.

Starting with duration, Table 5 indicates that the relative change in duration between bonds with stationary or nonstationary betas are approximately the same. Hence, differences in average duration are not significant in differrentiating between bonds with stationary versus nonstationary betas (given the relatively homogeneous maturity of the bonds under study).
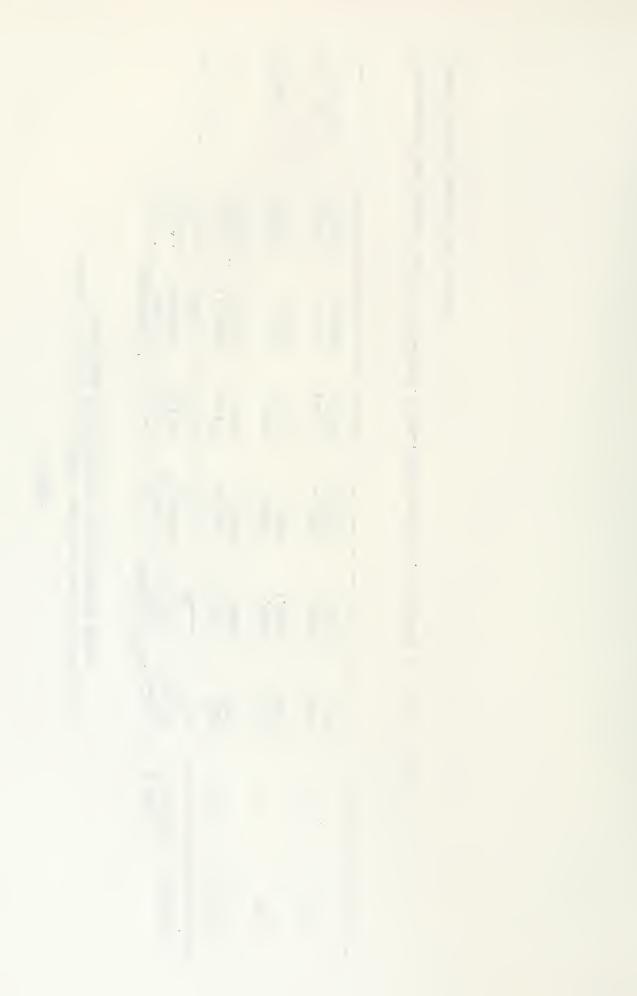
TABLE 5

Changes in Duration, Standard Deviation and Correlation for
Bonds with Stationary and Nonstationary Betas

| | Stationary | | | Nonstationary | | | F Ratio | |
	First 24 months (1)	Last 24 months (2)	Ratio of (2)/(1) (3)	First 24 months (4)	Last 24 months (5)	Ratio of (5)/(4) (6)	(3) to (6)	Probability
D_t	12.4793 (.7877)	11.4880 (.9187)	.9173[a] (.0183)	12.1964 (.7387)	11.3178 (.8865)	.9265[a] (.0235)	2.02[b]	.159
$\sigma(dy_{it})$	.0030 (.0004)	.0031 (.0006)	1.0640 (.1679)	.0027 (.0010)	.0033 (.0011)	1.2910 (.4232)	4.14	.045
$\rho(dy_{it}, R_{mt})$	-.5373 (.1469)	-.4707 (.1809)	.8847 (.3138)	-.5264 (.2283)	-.2232 (.2554)	.6875 (.6442)	1.33	.253

[a] Calculated from figures with more decimal points than contained in Columns (1), (2), (4) and (5).
[b] With 1 and 82 degrees of freedom.

Moving to the changes in the standard deviation of the change in the yield to maturity, the instability of the change in the yield to maturity increased for both groups of bonds. (This is to be expected because of the shorter average maturity of all bonds in the last period relative to the first period. In addition, the wider dispersion in corporate bond returns[11] in the last period relative to the first period may also contribute to the increase in the observed standard deviations.) However, the important point concerning the standard deviations is that the standard deviation of the nonstationary bonds increased relatively more (1.291 to 1.064) than for bonds with stationary bond betas. We believe the _primary_ reason for the higher relative standard deviation for the bonds with nonstationary betas is due to the higher coupon rates and associated higher yields to maturity for the nonstationary bonds. (Not only do the nonstationary bonds have higher average coupon rates, but they also have lower average bond ratings. It is well known, _ceteris paribus_ that the yield to maturity on lower rated bonds are larger than for higher rated bonds.) As interest rates in general fluctuate, the changes in the yield to maturity is larger for the nonstationary bonds (which have higher average coupon rates and lower bond ratings); hence, they have larger relative standard deviations than bonds with stationary betas.[12] Thus, the most important factor identified in this study which differentiates between bonds with stationary betas versus those with nonstationary betas is the relative standard deviations in the changes in the yield to maturity. Higher coupon rates and yields to maturity (leading to larger standard deviation in the changes in the yield to maturity) are associated with bonds having nonstationary betas.

Finally, it is noted that the correlation between the changes in the yield to maturity and the return on the market decreased for both stationary and nonstationary bonds from the first to the last periods. This is as expected since the sampled bonds in the third period have shorter maturities and hence their yields tend to move less with the returns on the market which are influenced by common stock as well as bond returns.[13] While not significantly different (at the .15 level), the absolute value of $\rho(dy_{it}, R_{mt})$ tended to be lower over time for the non-stationary bonds (.6875 to .8847) than for bonds with stationary betas. Again, this difference appears to be due to the higher coupon rates and yield to maturity carried by the nonstationary group of bonds relative to the stationary bonds.

In order to test the overall ability of the three hypothesized factors to differentiate between bonds with stationary betas and those with nonstationary betas, Hotellings T^2 was employed. It resulted in an F ratio of 2.22 which, with 3 and 80 degrees of freedom, has a probability value of .091. Thus, at the 10 percent significance level the three hypothesized factors (in combination) differentiated between bonds with stationary betas and those with non-stationary betas.

V. SUMMARY AND CONCLUSIONS

Recently a number of researchers have attempted to employ the market model to estimate systematic risk (i.e., beta) for bonds. In this study we reviewed theoretical evidence which suggests bond betas can be expected to be nonstationary. This nonstationarity is a function of the duration of a bond, the standard deviation of the change in the yield to maturity

of a bond relative to the standard deviation of the return on the market portfolio, and the correlation between the change in the yield to maturity of a bond and the return on the market portfolio. However, all bonds will not necessarily have nonstationary betas in a given time period since it is possible that these factors may occasionally counteract one another.

Empirical tests indicated that over 80 percent of the bonds examined had nonstationary betas. The primary factor differentiating bonds with nonstationary betas from those with stationary betas was the substantially higher relative standard deviation in the change in the yield to maturity for bonds with nonstationary betas. The larger standard deviation was caused by the higher average coupon rates and yields to maturity for bonds with nonstationary betas. The substantial presence of nonstationarity in public utility bond betas is caused by the peculiar nature of long term financing in the public utility industry which results in generally higher coupon rates and yields to maturity than in the industrial sector. The theoretical and empirical results of this study indicate bond betas, in general, tend to be nonstationary. Hence, further use of them appears to be of very questionable value.

FOOTNOTES

[1]Livingston [18] extended Boquist et al.'s work by taking into account the duration of both the security and the market portfolio. He shows that:

$$\beta_{1t} = \frac{D_{it}}{D_{mt}} \cdot \frac{\rho(dy_{it}, dR_{mt})\sigma(dy_{it})}{\sigma(dR_{mt})},$$

where D_{mt} is the duration of the market portfolio and dR_{mt} is the change in the return on the market portfolio. Since the duration of the market portfolio (which is dominated by common stocks with infinite maturity) does not change much over time we have chosen to work with equation (4). The notation follows that of Boquist et al. [4] and Livingston [18] except y_{it}, rather than r_{it}, is used for the yield to maturity.

[2]To provide some empirical evidence for the proposition that $-\rho(dy_{it}, R_{mt})$ is smaller for shorter-term bonds than for longer-term bonds we computed $-\rho(dy_{it}, R_{mt})$ using basic yields on corporate bonds with 1, 5, 10, and 15 years to maturity. Over the time period of 1941-1970 the value of $-\rho(dy_{it}, R_{mt})$ are .47, .52, .55 and .56 for bonds with 1, 5, 10, and 15 years to maturity, respectively. Therefore, as expected, $-\rho(dy_{it}, R_{mt})$ becomes smaller the shorter the term to maturity.

[3]The requirement that the bonds be consistently rated (without any change in rating) insures that the relative risk of default (as perceived by the two major rating agencies did not change over the time period employed. Thus, even though the bonds are not default free as required by the Boquist et al.'s model presented in equation (4), the relative probability of default was held constant.

[4]Recent theoretical work by Merton [23], Black and Cox [3] and Brennan and Schwartz [5] suggests that subordination or specific bond indenture provisions influence the value of bonds. Subordination is not a problem since all bonds selected for this study are non-subordinated. In addition, an examination of the call provision indicated that the vast majority of issues required a five year delay if they were to be called for refunding at a rate appreciably lower than the bond's coupon rate. Given the general rise in interest rates during this time period there was no economic incentive to refund. Finally, virtually all of the industrial bonds and a small portion of the public utility bonds are debentures. While some minor differences in the characteristics of the bonds examined exist, there is no reason to believe that any systematic tendencies are present which influence the results.

[5]A list of 84 bonds is available from the authors. The primary source of the monthly price data (for the period May 31, 1969 through May 31, 1975) was the Bank and Quotation Record [1]. Secondary sources included Commercial and Financial Chronicle [8], Moody's Bond Record [24] and Standard and Poor's Bond Guide [32]. The closing bid or sale price was employed; however, it occasionally became necessary to use an opening ask price. The availability of data was less of a problem for the public utility bonds than for the industrials in that closing bid or sale prices were almost uniformly available for the public utility issues examined. Other features of the bonds were determined by reference to Moody's Public Utility [26] and Moody's Industrial [25] manuals.

[6]The common stock returns employed were those from the CRSP value-weighted index while the corporate and government bond returns were those (as updated) provided by Ibbotson and Sinquefield [16]. The common stock weights employed were obtained from the Statistical Bulletin [33] while the corporate and government bond weights were obtained from the Economic Report of the President [11]. It can be shown that the use of a common stock index for R_{mt} will result in lower estimated bond betas. We conducted part of the analysis with the CRSP values-weighted index—there were no significant differences between those results and the reported findings.

[7]We also examined the stationary/nonstationarity of α_i and β_i simultaneously as estimated by equation (1). The subsequent findings are virtually the same whether we focus on the stationarity of both α_i and β_i as estimated by equation (1) or only the stationarity of β_i as estimated by equation (6).

[8]The computer program to test the stationarity of β is provided by BDE [7].

[9]The bond betas did vary by bond rating group with a mean of 0.570 for the Aaa/AAA group, 0.450 for the Aa/AA group, 0.391 for the A/A group and 0.372 for the Baa/BBB group. A one-way analysis of variance yielded an F-ratio of 2.87 which, with 3 and 80 degrees of freedom, was significant at the .041 level. Schwendiman and Pinches [30] reported that mean common stock betas increased as bond ratings decreased; our results indicated that bond betas decrease as bond ratings decrease. While the instability of the bond betas casts serious doubt on the interpretability of bond betas, there appears to be no consistency between bond betas, common stock betas and bond ratings. No other material differences are noted in the sample.

[10]Since duration changes each period, we calculated duration at the middle of the first time period (month 12) and the middle of the third time period (month 60).

[11]The standard deviation of returns on corporate bonds, using the Ibbotson and Sinquefield [16] data, was .0290 for the first time period and .0309 for the last time period. Hence, bond returns in general were more volatile in the last time period.

[12]As an example of the relationship between bond ratings and standard deviation of the change in yield to maturity, weekly yields to maturity were gathered for Standard and Poor's AAA, AA, A and BBB industrial and public utility bonds from July through December 1977. The standard deviations of the change in yield to maturity for the four bond groups over that time period were:

Industrial—AAA - .0393, AA - .0428, A - .0510, BBB - .1962; and Public Utility—AAA - .0415, Aa - .0422, A - .0441, BBB - .0527. In all cases the standard deviation in the changes in the yield to maturity increase as the bond ratings decrease.

[13]See footnote 2.

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