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STOCK INDEX FUTURES

ABSTRACT

This paper investigates the validity of the simple OLS model developed by Johnson (1960) and Stein (1961) and used by numerous subsequent studies to estimate the optimal hedge ratio using futures contracts. Focusing on the variance structure of the model, this paper provides some theoretical reasons for possible existence of heteroscedasticity (conditional as well as unconditional). Using data on three index futures we find significant heteroscedasticity and nonnormality in the conventional model. Alternative hedge ratios are obtained using an autoregressive conditional heteroscedastic (ARCH) model. Information provided by empirical results in this paper suggests the importance of taking account of the ARCH effects in estimating the optimal hedge ratio.

A NOTE ON THE ARCH EFFECTS IN HEDGE RATIO ESTIMATION: STOCK INDEX FUTURES

I. INTRODUCTION

One of the important functions of futures contracts is to facilitate hedging, i.e., transferring the risk inherent in spot positions to speculators in the futures market. In this regard, stock index futures contracts are of particular interest to investors since they can provide a means to hedge the market risk exposure.

The key to any hedging strategy using futures contracts is a knowledge of the hedge ratio, i.e., the number of futures contracts to sell short per a long position in the cash market. Following Johnson (1960) and Stein (1961), the predominant method used in previous studies to estimate the optimal hedge ratio is the regression approach (the ordinary least squares regression) relating changes in cash prices to changes in futures prices. Inherent in the regression is the assumption that the optimal combination of cash position with futures is the one whose variance is minimized (see Ederington 1979 for a literature review on the hedging theory).

This paper investigates the validity of the simple regression model to estimate the optimal hedge ratio in stock index futures, focusing on the variance structure of the model. We provide some theoretical reasons for possible existence of heteroscedasticity in the conventional hedge ratio estimation model, and attempt to reestimate the hedge ratio taking heteroscedasticity into account. The most common method to correct for heteroscedasticity is to introduce some exogeneous variables which may predict the variance. However, as pointed out by Engle (1982), this method requires a specification of the causes of the changing variance in an ad-hoc fashion rather than recognizing that both means and variances conditional on the past information available may jointly evolve over time. We obtain alternative hedge ratios based upon the autoregressive conditional heteroscedastic (ARCH) model, introduced by Engle (1982), which is characterized by mean zero, serially uncorrelated processes with non-constant variances conditional on the past but constant unconditional variances.

In Section II, we provide some reasons for possible existence of heteroscedasticity in the conventional hedge ratio model for index futures. In Section III, we discuss econometric methodology. Section IV describes the data and presents empirical results. A brief summay is contained in Section V.

II. HETEROSCEDASTICITY IN THE CONVENTIONAL MODEL

Following Johnson (1960) and Stein (1961), the commonly used ordinary least squares (OLS) technique to estimate the optimal hedge ratio can be written as:

$$\Delta S_t = \alpha + \beta \Delta F_t + \varepsilon_t \tag{1}$$

where ΔS_t and ΔF_t are the random changes in spot and futures prices, respectively, in period t. The slope coefficient β measures the optimal hedge ratio.[1] Apart from assuming that the functional form of model (1) is correct, the success of the OLS procedure heavily relies on at least three assumptions on the distribution of ε_t; (i) homoscedasticity (both conditional and unconditional), (ii) normality and (iii) serial independence. Although we discuss issues (ii) and (iii), this paper mainly deals with the problem of heteroscedasticity, particularly conditional heteroscedasticity.

Following Engle (1982), suppose we could write:

$$\varepsilon_t = \nu_t \, (\gamma_o + \gamma_1 \, \varepsilon_{t-1}{}^2 + \ldots. + \gamma_p \, \varepsilon_{t-p}{}^2)^{1/2}, \tag{2}$$

where the ν_t's are independent N(0,1). Then the conditional first two moments are (conditional on ε_{-t}, the past values of ε_t):

$$E(\varepsilon_t | \varepsilon_{-t}) = 0$$

$$V(\varepsilon_t | \varepsilon_{-t}) = \gamma_o + \gamma_1 \, \varepsilon_{t-1}^2 + \ldots + \gamma_p \varepsilon_{t-p}{}^2 \tag{3}$$

whereas unconditionally $E(\varepsilon_t) = 0$ and $V(\varepsilon_t) = \gamma_0/(1 - \sum_{i=1}^{p} \gamma_i)$. Therefore, the disturbances are conditionally heteroscedastic but unconditionally homoscedastic. We can have both conditional and unconditional heteroscedasticities just by assuming γ_o as a time varying parameter, say γ_{ot}, and there will be only unconditional heteroscedasticity if $\gamma_1 = \gamma_2 = .. = \gamma_p = 0$. However, there are a number of reasons to suspect possible existence of conditional heteroscedasticity.

First, omitted variables in the model may cause heteroscedasticity. As pointed out earlier, the regression model in (1) is based solely on risk-minimization alone. Recall that hedging in the modern portfolio theory should be viewed as an activity that reduces total expected return in exchange for a smaller variance (see Howard and D'Antonio (1984)). Taking account of the risk-return tradeoff, the hedge ratio can be alternatively derived as (see the Appendix for proof):

$$\beta^* = \frac{Cov(\Delta S_t, \Delta F_t)}{Var(\Delta F_t)} - \frac{E(F_\tau)-F_t}{2\phi\ Var(\Delta F_t)} , \quad (4)$$

where ϕ represents risk aversion parameter and $\tau > t$.

For analytical convenience, suppose that an investor holds a futures position until its maturity date, i.e., $E(F_T) = E(S_T)$, where T represents the maturity date. Note that on the maturity date, the futures price should be equal to the spot price to rule out the costless arbitrage (see Cox, Ingersoll and Ross (1981), and Richard and Sundaresan (1981)). Then, it becomes clear that eq. (4) narrows down to the conventional hedge ratio only if the futures price is an unbiased estimate of the expected spot price and/or a hedger is extremely risk averse ($\phi \to \infty$).

Whether or not the futures price is a systematically biased estimate of the expected spot price has been a long-time controversial issue in financial literature, i.e., the issue of normal-backwardation or contango[2] (e.g., Keynes (1930), Hicks (1939), Houthakker (1957), Cootner (1960), Carter, Rausser and Schmitz (1983), Rockwell (1967), Richard and Sundaresan (1981) and Telser (1958)). However, the existence of normal backwardation or contango, if any, and/or changing risk aversion over time will be reflected in the disturbance term, ε_t, in the simple regression model. It is more clear by rewriting (1) using (4) as

$$\Delta S_t = \alpha + [\beta^* + \frac{E(\Delta F_t)}{2\phi Var(\Delta F_t)}]\Delta F_t + \varepsilon_t$$

$$= \alpha + \beta^*\Delta F_t + [\varepsilon_t + \frac{E(\Delta F_t)\Delta F_t}{2\phi Var(\Delta F_t)}] \quad (5)$$

$$= \alpha + \beta^{*}\Delta F_t + [\varepsilon_t + \eta_t],$$

where $E(\Delta F_t) = E(F_\tau) - F_t$.

We can easily see from (5) that as long as the futures price is a systematically biased estimate of the expected spot price over time and thus $E(\Delta F_t)$ depends on t, the variance of $(\varepsilon_t+\eta_t)$ may depend on t even if we assume that ε_t and η_t are independent.

Second, conditional heteroscedasticity may exist because of the basis risk (the changes in the relationship between spot and futures prices). Figlewski (1984) provides evidence that the basis risk is positively related to the market trend and decreases as the time to maturity decreases. To the extent that the basis risk is significant and related to market trend (i.e., the underlying index itself) over time, the heteroscedasticity may exist and be reflected in γ_i.

Intuitively, the variance of ε_t represents the uncertainty about the agents choice of the hedge ratio in each period. As the maturity of a futures contract approaches, investors will have more information about the spot price in the future, so that the heterogeneity of information among investors will be reduced and thus the variance of ε_t will decrease over time. Therefore, the possibility exists that $V(\varepsilon_t)$ depends in part on the past information, ε_{-t}. This error-learning hedging behavior, if any, will be reflected in γ_i and its impact will be smaller as the maturity approaches. The reasoning is quite conceivable in light of the speculative behavior of investors due to heterogeneous information, and the ample evidence of maturity effect (e.g., Figlewski (1984) and Ederington (1979)).

III. ECONOMETRIC METHODOLOGY

Effects of unconditional heteroscedasticity are well known and one of them is the inconsistency of the standard errors if we use the usual formula. However, asymptotically valid influences can be drawn by using the White (1980) heteroscedasticity consistent covariance matrix estimator. This consistent covariance matrix is easily obtained by running an instrumental variable (IV) regression (see Messer and White (1984)). The procedure can be summarized as follows:

Suppose we have the OLS estimates $\hat{\alpha}$ and $\hat{\beta}$ and the residuals $\hat{\varepsilon}_t = \Delta S_t - \hat{\alpha} - \hat{\beta}\Delta F_t$. Then define the variables:

$$s_t^* = \frac{\Delta S_t}{\hat{\varepsilon}_t}, \quad f_t^* = \frac{\Delta F_t}{\hat{\varepsilon}_t} \text{ and } z_t = \Delta F_t \cdot \hat{\varepsilon}_t ,$$

and run an IV regression of s_t^* on f_t^* using z_t as an instrument. The resulting covariance matrix from this regression will provide the correct standard errors. White (1980) also provides a test for the presence of unconditional heteroscedasticity which does not assume any particular form of heteroscedasticity. Assuming that the ε_t's are homokurtic, the White test statistics can be calculated as the product of the sample size and the coefficient of determination of the regression of $\hat{\varepsilon}_t^2$ on ΔF_t and ΔF_t^2. Under homoscedasticity, this test statistic asymptotically follows χ^2 distribution with 2 degrees of freedom. We also test for the normality of the disturbances and use the following test statistic:

$$n \left[\frac{(\sqrt{b_1})^2}{6} + \frac{(b_2-3)^2}{24} \right]$$

where n is the sample size, and $\sqrt{b_1}$ and b_2 are the sample skewness and kurtosis coefficients of the OLS residuals $\hat{\varepsilon}_t$, respectively. Under normality, this statistic also asymptotically follows χ^2 distribution with two degrees of freedom (see Bera and Jarque (1982) for details).

Concerning conditional heteroscedasticity, Engle (1982) has shown that ARCH disturbances are uncorrelated (but dependent) and have a fatter tail than the normal distribution. Therefore, the above normality test may also reveal ARCH effects and vice-versa. For simplicity, we consider only a first order ARCH model, i.e., p = 1 in (3).

$$V(\varepsilon_t | \varepsilon_{t-1}) = \gamma_0 + \gamma_1 \varepsilon_{t-1}^2 .$$

Note that γ_1 is the simple correlation coefficient between ε_t^2 and ε_{t-1}^2. Therefore, the first order ARCH model postulates a non-linear relationship between ε_t and ε_{t-1}.

The log-likelihood function for the first-order ARCH model can be written as

$$\ell = \text{constant} - \frac{1}{2} \sum_{t=2}^{n} \log [\gamma_o + \gamma_1 (\Delta S_{t-1} - \alpha - \beta \Delta F_{t-1})^2]$$
$$- \frac{1}{2} \sum_{t=2}^{n} \frac{(\Delta S_t - \alpha - \beta \Delta F_t)^2}{\gamma_o + \gamma_1 (\Delta S_{t-1} - \alpha - \beta \Delta F_{t-1})^2} \tag{6}$$

Maximum likelihood estimates (MLEs) are obtained by maximizing (6) with respect to $(\alpha, \beta, \gamma_o, \gamma_1)$ using the GRADX Subroutine of the R.E. Quandt's program GOOPT3. The program also gives the asymptotic standard errors of MLEs. A significance test on γ_1 provides evidence of the presence of ARCH effects.

This type of conditional heteroscedastic model has particular appeal for estimating hedge ratios. First, realistic measures of hedge ratios can be estimated when the underlying variance may change over time and is predicted by past forecasting errors rather than making conventional assumptions about the disturbance. Second, it provides more efficient estimators using the maximum likelihood method. Third, it does not employ an arbitrary exogeneous variable to explain heteroscedasticity. Lastly, by the nature of the ARCH process, the effect of possibly omitted variables from the estimated model as discussed earlier might be picked up.

IV. DATA AND EMPIRICAL RESULTS

In February 1982, the Commodity Futures Trading Commission approved the trading of futures contracts on the Value Line Index in the Kansas City Board of Trade. This action was followed by the introduction of futures contracts on the S&P500 Index (Chicago Mercantile Exchange) and the NYSE Index (New York Futures Exchange) in April and May of 1982, respectively. This paper uses daily data on S&P500, NYSE and Kansas City Value Line futures (KCVAL) from the first trading dates of each contract to June 1985. All of the data (spot and futures closing prices) were secured from the MJK associate computer tapes.[3]

The OLS and ARCH model results on three stock indexes: S&P500, NYSE and KCVAL for 36 futures contracts are presented in Table 1. First, for the OLS results, the White test indicates the presence of strong unconditional heteroscedasticity in two thirds of the cases. The normality test statistics are significant for almost half of the contracts. The D.W. test statistics, which are not presented here to save space, do not show any presence of serial correlation. In the presence of non-normality and heteroscedasticity, the estimates of the OLS hedge ratios are inefficient and the t-statistics are unreliable.

For each of the contracts, we calculate the White consistent estimate of the variance and report the resulting modified t-statistics in brackets in Table 1. It is interesting to note that these t-statistics are overall lower than those calculated using the standard formula which in most cases underestimates the variance. These alternative estimates of the variances of the hedge ratios have some practical importance. They take account of unconditional heteroscedasticity and provide reliable confidence intervals which are of interest to the investors.

It is difficult to formulate an explicit model for the non-normal disturbances. Non-normality coupled with zero autocorrelation may imply some form of dependency among the disturbances. As we mentioned before, the ARCH model whose distrubances allow a simple form of dependency is a very convenient tool to capture non-normality. The results in the first-order ARCH model are given in the last columns of Table 1. Significance of the test $H_o: \gamma_1 = 0$ indicates the presence of first-order ARCH effects, i.e., conditional heteroscedasticity. Note that the test statistics are significant in all of the contracts of S&P500 and NYSE, and half of the contracts of KCVAL.

Comparing the OLS with ARCH hedge ratios, we can see that the OLS regressions overestimate (underestimate) significantly the optimal hedge ratio for some cases and thus cause the investors to sell short too many (few) futures contracts. Of particular interest is the improved efficiency of the hedge ratio through the ARCH model. Since the ARCH model takes account of conditional heteroscedasticity and in part of dependency and non-normality, they are more efficient than the OLS regression. The results on the tests for non-normality and homoscedasticity are similar to those for the OLS model. The ARCH model incorporates non-normality, but the presence of unconditional heteroscedasticity once again make the t-statistics given in parentheses invalid. A somewhat better estimate of the variance can be obtained by computing the White variance estimate using the ARCH residuals. The resulting t-statistics are in brackets. As before, these t-statistics are much lower, and they are more reliable in the presence of conditional and unconditional heteroscedasticity.

We also broke down the data of each index futures into four categories in terms of time-to-maturity of the futures to examine the impact of the time-to-maturity on the hedge ratio estimates. Four non-overlappig time-to-maturity contracts were chosen for each index futures: contracts maturing within three months, three to six months, six to nine months, and finally over nine months.

Table 2 shows the results of the OLS and ARCH models on each category of the three index futures. Presence of strong heteroscedasticity (conditional as well as unconditional) and non-normality is evident in all the cases. We do not observe much evidence of the relationship between the degree of heteroscedasticity (or non-normality) and the time-to-maturity in both the OLS and ARCH models. Only for KCVAL, the conditional and unconditional heteroscedasticities become stronger with the length of the time horizon. An interesting result to note in Table 2 is a general pattern of decreasing hedge ratios as the time-to-maturity horizon is lengthened. Theoretically, this is what we should expect. For shorter horizon, in equation (4), the numerator of the first part increases because of close correspondence between spot and futures prices and the numerator of the second part becomes smaller.

V. SUMMARY

We have investigated the validity of the conventional OLS model developed by Johnson (1960) and Stein (1961) and used by numerous subsequent studies to estimate the optimal hedge ratio using futures contracts. Focusing on the variance structure of the model, we provide some theoretical reasons for possible existence of heteroscedasticity (conditional as well as unconditional).

Using data on spot and futures prices of three indexes: S&P 500, NYSE and K. C. Value Line indexes from the first trading date of each contract to June 1985, we find significant heteroscedasticity (conditional and unconditional) and non-normality of the disturbance term of the OLS regression. This paper provides alternative hedge ratios based on an ARCH model, introduced by Engle. Comparing the OLS and ARCH hedge ratios, we find that the conventional hedge ratio estimating model causes investors to sell short too many or few futures contracts. Of particular interest is the markedly improved efficiency of the hedge ratio estimates. Information provided by the empirical results in this paper suggests the importance of taking account of the ARCH effects in estimating the optimal hedge ratios for futures contracts.

Footnotes

[1]See Ederington (1979) for a brief derivation of the hedge ratio.

[2]Normal backwardation/contango refers to the process in which futures prices are systematically downward/upward biased estimates of expected spot prices over time.

[3]The MJK Associate is a computer service in California specializing in futures markets. All prices are quoted in their normal trading units as determined by the various exchanges.

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Appendix

An alternative hedge ratio can be derived using the basic principles of the modern portfolio theory. Let us use the following notations:

S_t: Spot price at time t today

S_τ: Spot price at time τ, where $\tau > t$ and $\tau - t$ represents hedging period.

F_t: Futures price at time t maturing at sometime later than τ

F_τ: Futures price at time τ maturing at sometime later than τ

N: The proportion of the portfolio held in futures contracts with $N > 0$ representing a long position and $N < 0$ representing a short position

W_τ: End-of-period return distribution at τ

ϕ: Hedger's risk aversion parameter

U: Hedger's utility which is a function of only expected return and its variance.

Holding N futures contracts short per spot position, the hedger's return distribution at time τ will be

$$W_\tau = S_\tau - S_t - N(F_\tau - F_t) \tag{A1}$$

Then, the hedger's expected utility can be written as

$$EU_t = E(W_\tau) - \phi \, Var(W_\tau) = E(S_\tau) - S_t - N(E(F_\tau) - F_t) \tag{A2}$$

$$- \phi \, [Var\,(S_\tau - S_t) + N^2 \, Var(F_\tau - F_t)$$

$$- 2N \, Cov(S_\tau - S_t,\, F_\tau - F_t)]$$

The first order condition of (A2) yields the optimal hedge ratio as:

$$N = \frac{Cov(S_\tau - S_t,\, F_\tau - F_t)}{Var(F_\tau - F_t)} - \frac{E(F_\tau) - F_t}{2\phi \, Var(F_\tau - F_t)},$$

which is eq. (4).

Table 1

Results Based on the OLS and ARCH Models along with the Normality and Homoscedasticity Test Statistics
(t-statistics are in parentheses and White modified t-statistics in brackets)

	OLS Results			ARCH Results				
Index/Month/Year	$\hat{\alpha}$	$\hat{\beta}$	Normality Homoscedasticity Test Statistics	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}_0$	$\hat{\gamma}_1$	Normality Homoscedasticity Test Statistics
S&P500/June/82 (n=40)	-.036 (-0.52)	.677 (11.00)[a] [13.37][a]	1.26 1.832	-.078 (-1.69)[b]	.654 (15.50)[a] [13.01][a]	.126 (4.87)[a]	.206 (1.79)[b]	0.53 1.84
S&P/500/Sept/82 (n=103)	.047 (0.78)	.610 (17.11)[a] [13.12][a]	3.28 18.365[a]	.030 (0.72)	.595 (24.17)[a] [12.54][a]	.299 (7.36)[a]	.172 (1.64)[c]	4.90 19.57[a]
S&P500/Dec/82 (n=167)	.055 (0.96)	.651 (23.46)[a] [18.30][a]	0.77 17.318[a]	.050 (1.26)	.637 (33.18)[a] [17.58][a]	.450 (9.96)[a]	.165 (2.32)[a]	1.17 19.21[a]
S&P500/Mar/83 (n=230)	.064 (1.34)[c]	.655 (28.35)[a] [23.79][a]	5.62 9.729[a]	.045 (1.39)[c]	.652 (40.23)[a] [23.53][a]	.445 (12.03)[a]	.141 (2.52)[a]	5.86 10.35[a]
S&P500/June/83 (n=247)	.086 (1.91)[b]	.668 (30.97)[a] [26.18][a]	12.72[a] 10.300[a]	.064 (2.15)[b]	.667 (44.43)[a] [26.03][a]	.420 (12.60)[a]	.138 (2.55)[a]	12.82[a] 10.87[a]
S&P500/Sept/83 (n-251)	.053 (1.25)	.719 (33.46)[a] [27.47][a]	6.23[b] 23.025[a]	.041 (1.51)[c]	.728 (49.16)[a] [27.69][a]	.355 (12.08)[a]	.192 (3.26)[a]	5.31 24.35[a]
S&P500/Dec/83 (n=251)	.042 (1.04)	.715 (29.24)[a] [26.04][a]	65.37[a] 6.275[b]	.053 (2.06)[b]	.742 (48.43)[a] [26.45][a]	.259 (11.60)[a]	.307 (4.57)[a]	73.49[a] 6.53[b]
S&P500/Mar/84 (n=249)	.018 (0.29)	.572 (13.26)[a] [10.25][a]	25.94[a] 13.247[a]	.037 (1.44)[c]	.723 (31.42)[a] [12.09][a]	.314 (8.67)[a]	.830 (6.55)[a]	70.74[a] 15.69[a]
S&P500/June/84 (n=251)	-.036 (-0.50)	.387 (7.04)[a] [5.42][a]	1.05 23.700[a]	-.035 (-0.72)	.426 (11.60)[a] [5.95][a]	.962 (11.17)[a]	.256 (3.52)[a]	1.25 23.59[a]
S&P500/Sept/84 (n=319)	.004 (0.06)	.298 (6.05)[a] [4.81][a]	16.34[a] 12.409[a]	-.003 (-0.08)	.367 (8.97)[a] [5.99][a]	1.173 (11.57)[a]	.229 (2.96)[a]	16.66[a] 70.53[a]
S&P500/Dec/84 (n=383)	-.001 (-0.01)	.224 (5.01)[a] [4.04][a]	42.26[a] 11.835[a]	.001 (0.03)	.256 (6.33)[a] [4.65][a]	1.361 (13.18)[a]	.114 (1.79)[a]	42.53[a] 11.11[a]
S&P500/Mar/85 (n=305)	.037 (1.33)	.799 (42.83)[a] [40.24][a]	5.27 0.976	.035 (1.82)[b]	.799 (60.84)[a] [40.21]a	.202 (13.26)[a]	.121 (2.35)[a]	5.26 0.92

Table 1
(continued)

Index/Month/Year	OLS Results $\hat{\alpha}$	$\hat{\beta}$	Normality Homoscedasticity Test Statistics	ARCH Results $\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}_o$	$\hat{\gamma}_1$	Normality Homoscedasticity Test Statistics
NYSE/Sept/82 (n=101)	.011 (0.30)	0.516 (14.90)^{a} [11.24]^{a}	1.92 15.625^{a}	-.005 (-0.19)	0.506 (21.18)^{a} [10.79]^{a}	.107 (7.68)^{a}	.194 (1.94)^{b}	2.53 17.27^{a}
NYSE/Dec/82 (n=164)	.046 (0.95)	0.731 (11.58)^{a} [7.30]^{a}	114.13^{a} 25.141^{a}	.015 (0.65)	0.503 (26.00)^{a} [8.32]^{a}	.155 (8.17)^{a}	.682 (5.18)^{a}	190.32^{a} 21.32^{a}
NYSE/Mar/83 (n=228)	.061 (1.20)	0.294 (7.52)^{a} [5.39]^{a}	16.65^{a} 28.454^{a}	.040 (1.34)	0.350 (14.72)^{a} [6.34]^{a}	.272 (8.47)^{a}	.643 (5.22)^{a}	22.56^{a} 24.62^{a}
NYSE/June/83 (n=291)	.078 (1.78)^{c}	0.265 (7.46)^{a} [5.42]^{a}	11.52^{a} 33.290^{a}	.069 (2.43)^{b}	0.297 (12.75)^{a} [15.97]^{a}	.362 (10.81)^{a}	.370 (4.23)^{a}	13.37^{a} 30.26^{a}
NYSE/Sept/83 (n=355)	.069 (1.70)^{c}	0.228 (6.72)^{a} [4.99]^{a}	7.11^{b} 33.370^{a}	.069 (2.66)^{a}	0.295 (10.91)^{a} [6.44]^{a}	.426 (11.71)^{a}	.282 (3.59)^{a}	8.91^{b} 27.34^{a}
NYSE/Dec/83 (n=265)	.045 (1.00)	0.196 (4.25) [3.78]^{a}	2.39 5.009^{c}	.052 (1.70)^{b}	0.186 (5.52)^{a} [3.57]^{a}	.465 (12.86)^{a}	.108 (2.10)^{b}	2.39 5.30
NYSE/March/84 (n=328)	.026 (0.65)	0.173 (4.15)^{a} [3.69]^{a}	1.27 5.215^{c}	.029 (1.05)	0.161 (5.35)^{a} [3.43]^{a}	.461 (14.28)^{a}	.105 (2.28)^{b}	1.30 5.58
NYSE/June/84 (n=376)	.017 (0.47)	0.124 (3.10)^{a} [2.77]^{a}	1.38 4.512	.019 (0.75)	0.115 (4.10)^{a} [2.57]^{a}	.457 (16.06)^{a}	.076 (2.04)^{b}	1.47 4.51
NYSE/Sept/84 (n=376)	.020 (0.57)	0.117 (2.80)^{a} [2.42]^{b}	8.66^{b} 6.354^{b}	.015 (0.59)	0.110 (3.46)^{a} [2.28]^{b}	.435 (15.51)^{a}	.079 (1.89)^{b}	8.77^{b} 6.77^{b}
NYSE/Dec/84 (n=370)	.007 (0.52)	0.748 (45.07)^{a} [38.86]^{a}	7.71^{b} 5.254^{c}	.008 (0.87)	0.746 (61.49)^{a} [38.82]^{a}	.066 (15.29)^{a}	.075 (1.76)^{b}	7.86^{b} 5.18
NYSE/Mar/85 (n=338)	.019 (1.23)	0.704 (39.74)^{a} [37.78]^{a}	6.50^{b} 0.507	.018 (1.81)^{b}	0.702 (54.23)^{a} [37.64]^{a}	.065 (14.08)^{a}	.161 (3.12)^{a}	6.62^{b} 0.68

Table 1
(continued)

Index/Month/Year	OLS Results $\hat{\alpha}$	$\hat{\beta}$	Normality Homoscedasticity Test Statistics	ARCH Results $\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}_o$	$\hat{\gamma}_1$	Normality Homoscedasticity Test Statistics
KCVAL/Mar/82 (n=25)	-.024 (-0.24)	0.633 (7.58)[a] [9.80][a]	1.41 2.570	.006 (0.10)	0.627 (11.65)[a] [9.39][a]	.161 (2.58)[a]	.319 (1.19)	1.41 2.65
KCVAL/June/82 (n=88)	-.028 (-0.49)	0.480 (11.32)[a] [12.31][a]	0.22 0.414	-.031 (-0.78)	0.484 (15.82)[a] [12.39][a]	.259 (7.42)[a]	.043 (0.53)	0.20 0.44
KCVAL/Sept/82 (n=152)	.030 (0.53)	0.406 (12.35)[a] [12.14][a]	0.33 2.310	-.019 (-0.47)	0.380 (16.89)[a] [10.75][a]	.378 (8.48)[a]	.215 (2.31)[b]	0.34 3.19
KCVAL/Dec/82 (n=216)	.092 (1.68)[b]	0.448 (16.96)[a] [15.42][a]	4.63 2.160	.067 (1.74)[b]	0.432 (21.13)[a] [14.66][a]	.540 (10.80)[a]	.155 (2.18)[b]	5.35 3.02
KCVAL/Mar/83 (n=279)	.115 (2.41)[b]	0.460 (20.30)[a] [17.56][a]	17.67[a] 8.258[b]	.106 (3.19)[a]	0.452 (28.55)[a] [17.25][a]	.567 (13.58)[a]	.094 (1.96)[b]	19.21[a] 8.65[b]
KCVAL/June/83 (n=342)	.133 (3.15)[a]	0.484 (23.62)[a] [20.51][a]	15.45[a] 12.586[a]	.126 (4.26)[a]	0.480 (32.12)[a] [20.29][a]	.559 (14.98)[a]	.067 (1.57)[c]	16.21[a] 12.65[a]
KCVAL/Sept/83 (n=375)	.095 (2.38)[a]	0.515 (26.63)[a] [23.34][a]	10.21[a] 13.763[a]	.090 (3.25)[a]	0.514 (36.50)[a] [23.30][a]	.551 (15.77)[a]	.063 (1.59)[c]	10.37[a] 13.88[a]
KCVAL/Dec/83 (n=372)	.116 (2.51)[a]	0.442 (21.45)[a] [10.30][a]	484.41[a] 106.578[a]	.128 (4.07)[a]	0.474 (30.23)[a] [10.21][a]	.640 (16.15)[a]	.149 (3.44)[a]	715.59[a] 120.90[a]
KCVAL/Mar/84 (n=312)	.033 (0.85)	0.574 (27.06)[a] [24.72][a]	2.00 10.546[a]	.034 (1.23)	0.574 (38.18)[a] [24.70][a]	.468 (14.07)[a]	.010 (0.23)	2.03 10.61[a]
KCVAL/June/84 (n=246)	-.039 (-0.95)	0.589 (23.78)[a] [21.37][a]	0.04 2.731	-.040 (-1.37)[c]	0.586 (32.38)[a] [21.25][a]	.392 (12.17)[a]	.059 (1.09)	0.03 2.71
KCVAL/Sept/84 (n=257)	-.022 (-0.52)	0.539 (22.86)[a] [19.19][a]	3.21 5.269[c]	-.030 (-1.00)	0.523 (26.44)[a] [18.28][a]	.401 (12.19)[a]	.102 (1.77)[b]	3.46 6.68[b]
KCVAL/Dec/84 (n=252)	-.014 (-0.33)	.525 (22.56)[a] [18.81][a]	2.78 6.124[c]	-.015 (-0.52)	.519 (31.41)[a] [18.50][a]	.426 (12.81)[a]	.066 (1.34)[c]	2.92 6.55[b]
KCVAL/Mar/85 (n=203)	.073 (1.61)	.458 (19.37)[a] [15.38][a]	2.94 10.901[a]	.063 (1.95)[b]	.446 (24.19)[a] [14.73][a]	.381 (12.18)[a]	.076 (1.65)[b]	3.67 13.80[a]

[a] Significant at the 1% level. The critical values for t-statistics and x2 statistics are respectively 2.57 and 9.21.
[b] Significant at the 5% level. The critical values for t-statistics and x2 statistics are respectively 1.66 and 5.99.

Table 2

Results Based on the OLS and ARCH Models for Different Maturity Periods
(t-statistics are in parentheses and White modified t-statistics are in brackets)

Index/Maturity Period (t: month)	OLS Results $\hat{\alpha}$	OLS Results $\hat{\beta}$	Normality Homoscedasticity Test Statistics	ARCH Results $\hat{\alpha}$	ARCH Results $\hat{\beta}$	$\hat{\gamma}_0$	$\hat{\gamma}_1$	Normality Homoscedasticity Test Statistics
S&P500/t ≤ 3 (n=748)	.040 (2.46)[a]	.784 (64.06)[a] [40.74][a]	77.65[a] 38.30[a]	.037 (2.64)[a]	.752 (81.97)[a] [41.83][a]	.281 (22.53)[a]	.126 (4.48)[a]	94.95[a] 55.35[a]
S&P500/3 < t ≤ 6 (n=748)	.038 (2.26)[b]	.768 (62.77)[a] [42.12][a]	60.60[a] 37.85[a]	.034 (2.42)[a]	.736 (80.51)[a] [43.31][a]	.290 (22.35)[a]	.126 (4.40)[a]	67.93[a] 38.90[a]
S&P500/6 < t ≤ 9 (n=748)	.040 (1.88)[b]	.720 (57.00)[a] [40.35][a]	117.64[a] 39.87[a]	.034 (2.38)[a]	.724 (80.33)[a] [41.65][a]	.290 (21.65)[a]	.141 (4.49)[a]	134.88[a] 43.38[a]
S&P500/t > 9 (n=647)	.037 (2.04)[b]	.772 (58.16)[a] [39.93][a]	35.63[a] 31.19[a]	.035 (2.26)[a]	.733 (74.99)[a] [40.46][a]	.296 (20.07)[a]	.138 (4.01)[a]	36.07[a] 21.35[a]
NYSE/t ≤ 3 (n=745)	.027 (2.15)[b]	.660 (53.00)[a] [37.83][a]	68.00[a] 29.43[a]	.023 (2.87)[a]	.663 (75.29)[a] [38.65][a]	.090 (22.01)[a]	.183 (5.81)[a]	79.61[a] 43.96[a]
NYSE/3 < t ≤ 6 (n=745)	.026 (2.07)[b]	.644 (52.72)[a] [39.36][a]	47.85[a] 29.13[a]	.023 (2.81)[a]	.645 (73.95)[a] [39.80][a]	.092 (22.14)[a]	.177 (5.73)[a]	52.47[a] 32.04[a]
NYSE/6 < t ≤ 9 (n=745)	.025 (1.98)[b]	.635 (52.59)[a] [39.66][a]	41.91[a] 27.34[a]	.023 (2.80)	.635 (73.54) [40.04][a]	.092 (22.19)	.177 (5.79)	45.28[a] 30.55[a]
NYSE/t > 9 (n=707)	.023 (1.83)[b]	.632 (51.45)[a] [40.52][a]	26.42[a] 19.30[a]	.023 (2.72)	.626 (69.99) [40.64][a]	.094 (21.66)	.160 (5.23)	26.64[a] 16.97[a]
KCVAL/t ≤ 3 (n=794)	.055 (2.19)[b]	.509 (38.14)[a] [28.15][a]	19.69[a] 18.52[a]	.051 (2.88)[a]	.504 (50.57)[a] [28.89][a]	.471 (22.81)[a]	.076 (2.74)[a]	16.77[a] 54.06[a]
KCVAL/3 < t ≤ 6 (n=795)	.063 (2.36)[a]	.480 (35.26)[a] [30.25][a]	12.96[a] 23.61[a]	.052 (2.79)[a]	.476 (46.75)[a] [29.68][a]	.498 (22.69)[a]	.112 (3.92)[a]	12.18[a] 27.83[a]
KCVAL/6 < t ≤ 9 (n=767)	.057 (2.13)[b]	.483 (35.12)[a] [29.64][a]	12.46[a] 23.85[a]	.047 (1.73)[b]	.480 (46.60) [29.18][a]	.478 (21.78)	.135 (4.31)	12.15[a] 30.68[a]
KCVAL/t > 9 (n=514)	.066 (1.71)[b]	.433 (24.26)[a] [18.01][a]	85.95[a] 44.79[a]	.047 (1.73)[b]	.420 (31.32)[a] [16.68][a]	.624 (15.36)[a]	.188 (3.35)[a]	12.34[a] 37.84[a]

[a]Significant at the 1 percent level. The critical values for t-statistics and χ^2 statistics are respectively 2.57 and 9.21.

[b]Significant at the 5 percent level. The critical values for t-statistics and χ^2 statistics are respectively 1.66 and 5.99