NOTES ON LOGIC.

[Printed for Private Circulation.]

PART I.

FORMAL LOGIC.

[illegible] (Aberyscon [illegible])

Plants - [illegible]

Protozoa [illegible]

Coelenterata [illegible]

Echinodermata [illegible]

[illegible] [illegible]

Molluscs [illegible] Cephalopods

Worms [illegible]

Arthropods [illegible]

Vertebrates Fishes

Carboniferous Age.

Plants Vascular Cryp. Conif. & Some Monocotyledons
Protozoa Foraminifera
Coelenterata Corals
Echinodermata end of Crinoids sea urchin abund.
Brachiopoda less numerous
Mollusca very abundant
Arthropods Trilobites extinct. Spiders appear
Vertebrates Fishes & sharks

Lecture I.

OUTLINE.

I. Relation of Logic { 1. To Metaphysics.
2. To Psychology.

{ 1. Cognitive Powers.
2. Reproductive Powers.
3. Comparative Powers.

II. The Logical Power.—Understanding.

III. The Function of the Understanding.—Judgment.

IV. Nature of Judgment,—assertion or denial between 2 terms.

V. Intuitive and Inferential Judgments.

VI. Definition of Logic.

VII. Double Character of Judgments.

VIII. Subdivisions of Logic { 1. Formal.
2. Material.

Lecture Proper.

Rel. to Metaphysics.—Knowledge may be divided into two parts. (1) Immediate or Intuitive. (2) Mediate or Indirect. Logic is the science of mediate knowledge.

Rel. to Psychology.—Psychology seeks to unfold the faculties of the mind. It is inductive. It looks into the mind. It is a science of discovery. Psychology uses two instruments, viz: Sense-perception and Self-consciousness. It seeks to discover these relations which form the basis of Metaphysical science. It is the function of Metaphysics to determine the limits of their validity and application to the problem of knowledge.

Mental Powers, are divided into four groups. (1) Cognitive Powers.—Sense-perception and Self-consciousness. (2) Reproductive Powers.—Memory and Imagination. These

two Powers enable us to obtain fact and reproduce them at pleasure. (3) Comparative Powers.—The faculty of relations. Relation of Logic to these three powers. It derives the materials from which it starts, from these.

II. *Logical Faculty.*—The mind also has the power to *Judge* and draw inferences. This power Kant calls the *Understanding*, meaning judgment and inference. "Logic is the science of the proper use of the Understanding." Logic has an indirect relation to the first three powers and a direct relation to the last.

III. *Function of the Understanding.*—Each mental power has its specific function. "Senses" perceive facts, "Memory" recalls; "Imagination" represents, &c.

IV. *What is the nature of Judgment?*—There has been a long discussion as to whether Logic is a science of discovery. We must determine the nature of Judgment. "Judgment" in its essence is the assertion or denial of a relation between two terms, e. g., If we say, Fire produces heat, 1st term is Fire; 2d term, Heat. Or, A is to the right of B, this gives us the relation of space. Or, Man is a rational being, or animal. This asserts the relation of idenity. It is the function of Logic to assert or deny relations, and the assertion of relation is Judgment.

V. *Kinds of Judgment.*—Two, viz: (1) Intuitive. (2) Inferential.

(a) Intuitive is the one which asserts a direct relation. We always have both terms before the mind.

(b) Judgments which assert an inferred relation, i. e., When A and B are equal and we know that C is equal to A, we pronounce the inferred judgment tnat B and C are equal, e. g., All men are mortal. Peter is a man and still alive ∴ Peter is mortal. This latter judgment is called an *inference*, because it is not pronounced directly.

VI. *Def. of Logic.*—Logic deals directly with the inferential judgment. Not, What can I observe directly, but, What can I deduce from what I observe. Logic is the science which deals with these discursive thoughts.

VII. *Double Character of Judgments.*—In general it corresponds to the definition, which we have given, but there is a subdivision founded on character of Judgments with which we deal. Form of Judgment, i. e. either affirmative or negative.

(1) The "form of Judgment" is the mental affirmation —of some relation. This is called the *form* of the judgment.

(2) "Matter of the Judgment," is the fact or relation which is asserted, as Hannibal was a Carthagenian general.

VIII. *From this are formed two divisions of Logical Science, viz.:*—1. *Formal.* 2. *Material.*

(1) Formal Logic is that science which treats of the laws of discursive thoughts, as such without regard to the truth or the falsity. Assuming the truths of certain judgments, Formal Logic seeks to determine what other judgments can be deduced from these and the laws by which they may be deduced. In these the *form* may be correct, but the premises are false.

(2) We have department of Material Logic. It is that part of the science which treats of discursive thought as an instrument in discovering truth. Logic is therefore the science of discursive thought. It has two forms, (1) Formal. (2) Material.

Lecture II.

OUTLINE.

I. Kinds of Inferential Judgments { 1. Immediate, 2. Mediate.

II. Relation of Judgment in general to the divisions of Logic.

III. Processes involved in the logical use of the Understanding.

(a) Abstraction,

(b) Comparison,

(c) Generalization.

IV. Fundamental laws of Logic { Identity—*a* is *a*. Contradiction—*a* is not *a*. Excluded Middle or Third—a thing is *a* or not *a*.

V. Nature and application of these laws.

VI. Steps in development of Formal Logic.

VII. Analytic and Synthetic Judgments and their relation to Formal Logic.

VIII. Deduction and Induction, Formal Logic, Deductive.

I. *Kinds of Inferential Judgments.*—(1) Immediate, is a judgment which is derived directly from another judgment. e. g., All poets are men ∴ Some men are poets. Given a single judgment, and you can draw a number of other judgments. (2) The Mediate judgment cannot be drawn directly from a single judgment.

II. *Relation of Judgment to different parts of Logic—Notion, Syllogism, &c.*—We cannot form a notion without a judgment. Take the notion man. It involves several predicates, i. e., Man is a *rational, self-conscious animal.* We must exclude several things from him, as height, color of hair, nationality, &c. Judgment is the universal function which operates in all Logical processes.

III. *Processes involved.*—(a) *Abstraction.* What is it? It is simply directing the attention to a particular part of a perception. In looking at a book, I confine my attention to the color, or shape, or some one thing.

(b) Comparison is simply viewing facts in relation, e.g., I bring two books together. I abstract all qualities but those which are alike.

(c) Generalization. e. g., I examine a number of books, and find that they possess a number of qualities in common. This is *abstraction* and *comparison.* When you form a class then you have *generalization* and the three are called *notions.*

IV. *Fundamental Laws of Logic.*—(1) Identity. (2) Contradiction. (3) Excluded Middle or Third.

Identity, *a* is *a*, Macaulay ridiculed it. Properly understood law of Identity is the basis of all positive judgments, e. g., All men are mortal. All judgments which are the same or a part of this judgment are true.

Contradiction, or "*a* is not *a*." This has also been criticised. It lies at the base of all *negative* judgments.

Excluded Middle, or a thing is either *a* or not *a*. It means that a thing must be either *true* or *false*. There is no middle possibility.—Man is mortal or not mortal. They enable us to decide whether a thing is true without proving it. They are fundamental laws of Inferences.

V. *Nature and Application of these Laws.*—These laws do not apply to a relation between subject and predicate of a proposition. Macaulay thought that these laws were to be applied this way. The relation which is asserted between the subject and predicate is not necessarily always the same in all propositions.

(1) Identity. If a judgment be true then all other judgments which agree with it are true.

(2) Contradiction. If a given judgment be true, then all judgments which disagree with it are false.

(3) Excluded Middle. If a given judgment be true, then all judgments which contradict it are false, and of two contradictory judgments about the same thing, one must be true and the other false.

VI. *Steps in the Development of Formal Logic.*—Take up now Formal Logic, discuss the nature of Inference itself.

1st step. Doctrine of the *Notions*, which, expressed in language gives us *Term*. These are the elements with which Logic has to deal.

(1) Critique of the Notion. (2) Discussion of the Logical Judgment. This logical judgment expressed in language is the Proposition. (3) Will be the discussion of Immediate Inference, e. g., All metals are simple elements.—Then we may deduce, A number of elements are metals. (4) Discussion of chains of Inference. We string a number of syllogisms together, to reach a conclusion. This is called *Proof*. In logical argument we have a systematic development.

VII. *Analytic and Synthetic Judgments and their Relations to Formal Logic.*—Analytic Judgment is a judgment which deduces its judgment. The syllogism is a splendid example. A Synthetic Judgment is one which reaches out and grasps a fact, not implied before, or from the outside, e. g., I reach out and grasp a hot stove and say "the stove is hot."

VIII. *Dedu lion and Induction.*—These are simply the names of methods. In the first we proceed from given premises. In Induction we proceed by synthesis. We reach a general law from particulars. Formal Logic is a *Deductive Science.* We have nothing to do with Induction. Material Logic is Inductive.

Lecture III.

OUTLINE.

A—Doctrine of the Notion—embraces
- 1. Generalization.
- 2. Denomination.
- 3. Division.
- 4. Definition.

I. Generalization
- Conditions.
- Processes.
- Result.

(a) Nature of Abstract Notion.

II. *Denomination*—Language.

(a) General Function of Language.

(b) Scientific and Popular.

(c) Division of Scientific Language
- Terminology.
- Nomenclature.

(d) Result of Denomination.

III. Logical Character of Terms, or Terminology.

IV. Division.

(1) Problem of Division.

(2) Categories.

(3) Basis.

A—Doctrine of the Notion.

There is a difference of opinion among scholars, as to whether the discussion of the Notion belongs to Logic. Some knowledge of it is necessary. The Notion contains some element of the Logical Doctrine. The doctrine of the Notion gives rise to (1) The formation of the concept; (2) Its relation to Language; (3) The coördination of Notion: (4) Definition. This gives rise to four topics.

I. Generalization. (a) The *condition* of Generalization. There must be first, classifiable things in Nature. (b) Then there must be a generalizing power in the mind. There are two extreme views in regard to Generalization. (1) That there are ready made classes in Nature. This an extreme view, because it leaves the mind nothing to do. (2) That the mind generalizes *arbitrarily*. That it supplies the relations themselves. These are extreme views on either side. Nature and the mind each supplies a part.

The process involved in Generalization. Generalization is a species of Judgment. (1) Abstraction. (2) Comparison. (3) Comprehension. After we have observed a number of objects and notice that they have several things in common, we then classify. Our notion of man includes everything that men possess in common. (4) Subsumption. Take class notion—mineral for example. It is bringing the multitudes of objects under our observation. The result of Generalization is the formation of general concepts, from percepts. This accounts for two kinds of Notions, viz.:—(1) The Particular. (2) The Abstract.

(a) *Abstract Notion.* It is necessary to distinguish between *real* and *apparent* abstractions. Take a book. It is blue or red. You abstract that color and you call it *blueness*, then you get a true abstraction, that is, the adjective qualifier is not an abstract quality. It involves two processes *Abstraction* and *Comparison.* When do we form our abstract terms? It is probable that we form them later in life. We get blue or red, before we get *blueness* or *redness.*

II. Denomination. Next step logically considered is denomination, or giving them names. This involves language.

(a) General Function of Language. Language has entered very largely into Logic since the days of Locke. General function of language may be expressed in the single term—*Symbolism*. It is necessary to have language to give things names as well as to talk about them. The mind by generalization, forms "class notions." Symbol to meaning, or "thing symbolised." Language may be classified as *general* and *special*. General language embraces all forms—outer expressions, &c., as cries of animals, &c.

(b) Special language is that of man as expressed in words. Special language may be divided into, (1) Popular. (2) Scientific. In Popular language we can often give no meaning to words which we use. Popular language is deficient in both *Denotation* and *Connotation*. By denotation we mean the denoting of objects. By connotation, qualities or attributes of objects. It is more defective in *Connotation*. The Scientific language is simply critical language. Its relation to popular language is expressed in two points. (1) The Scientific investigator uses popular language, or terms, and makes them definite. (2) Science invents terms of its own. Every science has its technical terms. We deal with Scientific language in Logic. The function is denomination.

(c) Scientific Language has two divisions, viz. :—(1) Terminology. (2) Nomenclature. Terminology embraces the descriptive terms of a science. All those terms which are applied to attributes of objects. For instance class of Mammalia. It is comprised of sub-divisions or species. Terminology takes up these species and discusses them.

(1) Rules of Terminology. (1) There should be a name for every important meaning. (2) Every name should have its *connotation* strictly fixed and precisely determined. A Terminology should be short and *precise*.

(2) Nomenclature. It embraces the names of classes and divisions, e. g., class Radiata. Subdivide it and each subdivision has a name. The nomenclature of a science embraces all the sub-divisions Rule 1st. There should be a distinct name for every kind or distinct species. These two, Terminology and Nomenclature exhaust the sphere of language.

Room for new expanded meaning

III. Log character of Terms. By the process of Generalization we form the Notion, from the Notion we get *Term*. A *term* is a more convenient word and embraces all the distinctions which we give to the Notion. The description of the logical character, embraces the following distinctions according as they are,

Categorematic or Syncategorematic.
Adjective or Substantive.
Concrete or Abstract.
General or Singular.
Collective or Distributive.
Positive or Negative.

{ Denotative and Connotative } or Simply Connotative.

A Categorematic term is one which stands alone. A Syncategorematic term is one which depends on something else. Adjective—blue; Substantive—blueness. Collective Term is one which applies to a whole class. Distributive Term is a general term which applies to each individual member of that term. Denotative and Connotative, are class terms in general.

IV. Division. Problem of Logical Division.

(1) Given a highest class or *genus*, to separate it into its constituent species, and these in like manner until its lowest species has been reached, i. e. the one next above the individual.

(2) Categories. The mind makes use of certain categories, viz.:—(1) Genus. (2) Species. (3) Differentia. (4) Property. (5) Accident. The *genus* stands for the highest class, that which is to be divided. *Species* are the natural divisions of the class or the next lower divisions. *Differentia* is the attribute which is used in division. *Property* is an inseparable attribute. *Accident* is an attribute which may belong to an individual or it may not, as color *red* in horses.

(3) Basis.—This is always some mark or marks which stands for attributes. This must be a *specific* mark: one which represents the special nature of the class of objects to which it is applied.

Lecture IV.

OUTLINE.

I. Rules of Division. 1, 2, 3, 4.

Logical Definition.

(a) Origin and Necessity of Logical Definition.
(b) Nature and scope of Definition.
(c) Relation of Description to Definition.
(d) Rules of Definition. { 1. Main Rule. 2. Subordinate Rules.
(e) Ground of the Main Rule in Definition.
(f) What the Definition represents { 1. Nominalism. 2. Realism. 3. Conceptionalism.

Second Part of Logical Doctrine.

I. The Judgment.
II. The Proposition.

(a) Classification of Proposition { 1. Attributive. 2. Class Inclusive. 3. Class Exclusive.
(b) Nature of the Relation asserted in the Proposition.
(c) Properties of the Proposition { 1. Quality. 2. Quantity. 3. Distribution.
(d) Classification with reference to Quantity, Quality by symbols.
(e) Distribution of terms in Propositions.
(f) Qualification of Proposition { 1st step. 2nd step.
(g) Logical Character of Propositions.

Lecture Proper.

I. *Rules of Division.*—(1) Basis of division must be a specific work. (2) The genus must be divided into approximate species. (3) The division must be mutually exclusive. (4) The sum of the species must make up the genus. To sum up, *Generalization* gives us our notions of classes. *Denomination* names them and *Division* separates them into species and arranges them.

Logical Definition.

(a) Origin and necessity of Logical division. Popular notions are indefinite. Scientific definitions must be distinct.

(b) Nature and Scope of Definition. If we distinguish between the common attributes and the special attribute, we may say that Definition is confined to the general, and must exclude individual attributes. Again, in Definition we must distinguish between essential and non-essential attributes. The Definition, scholars say, must give the essence of the thing, its genus and species.

(c) Relation of Description to Definition. They are frequently confounded. Description has a wider latitude than Scientific Definition. It goes more into properties. It aims to give a complete picture of the the thing described. Definition is confined to essential features of classes; it gives the relation simply.

(d) Rules of Definition.—General Rule.—The definition of a class must include its generic and specific attributes and must exclude all others. Logicians give *subordinate* rules, viz.: (1) Definition must include all the objects of the class defined and no more. (2) The definition must not be negative unless the term to be defined is negative. (3) The definition must not be tautological. (4) The definition must not be ambiguous. It would violate the scientific demands if it was.

(e) Ground of Main Rule in Definition. All scientific knowledge is made up of general statements regarding classes. It groups things into classes and then studies them.

(f) What does the Logical Definition represent? (1) Nominalism holds Definition as a name merely. The name is definite and confined to a multitude of objects. (2) Realism—(Plato) That there is a general being in Nature. And that this general being rests at the basis of Realism. (3) Conceptionalism—(Abbelard) Definition represents primarily, the *Notion*. This Notion has no corresponding object in nature. The Notion equals the thing which the name represents.

Discussion of These Theories.

The notion is derived from singulars by generalization; but the class notion does not represent relations apart from the singulars. With conclusion of doctrine of the notion, have developed the four parts—Generalization, Denomination, Division and Definition.

Second Part Logical Doctrine.

(1) Judgment has been discussed previously, i. e., it asserts a relation. We assume the truth of the Judgment and proceed to discover what formal or correct truth can be founded upon it. (2) The Proposition is simply the verbal affirmation between two objects. A Logical proposition is a grammatical sentence. They are Indicative, Imperative, Interrogative, Declarative, Optative and Exclamatory. Logical Proposition is Indicative. Propositions may be divided into (1) Hypothetical [Conjunctive and Disjunctive], (2) Categorical. A categorical proposition is a simple assertion. Copula is some tense of the verb "*to be.*" In Logic it must be put in this form.

(a) Nature of Relation asserted by the Copula. It may be (1) Attributive. (2) Class Inclusive. Take (1) "Man is mortal." (2) Whales are mammals. Here the class whale is included in the class mammal. That predicate is primarily attribution. That predicate is primarily class exclusion. Important whether this be true. All judgments can be stated in a quantitative form, i. e., man is mortal, it puts it, all men are mortals. This relation is one of class inclusion or exclusion. This is the more common form of Logic. It is possible to put all propositions in class exclusion or class inclusion.

(c) Properties of the Proposition. (1) Quantity. (2) Quality. (3) Distribution. (1) Qualitative propositions are either affirmative or negative. (2) Quantitative propositions are either universal or particular. They are universal when the whole subject class is included or excluded from the predicate.

Inductive Logic.

[illegible]

[illegible]

[illegible]

Legitimate [illegible]

from the known

Legitimate inference of the

[illegible] from particular.

Species of [illegible]

I. from [illegible] to particular;

II. from particular to general laws.

(II) is the [illegible] from

Incomplete induction

Incomplete induction.

[illegible] induction is the

scientific induction.

Enables us to go from

Known to unknown.

(d) Classification with reference to Quality, Quantity with symbols. Fourfold, viz.: (a) Universal Affirmative. (e) Universal Negative. (i) Particular Affirmative. (o) Particular Nagative. (a) Represents total inclusion of subject in predicate. (e) Represents total exclusion of subject from predicate. (i) Represents partial inclusion of subject in predicate. (o) Represents partial exclusion of subject from predicate.

Rules of Distinction.

(1) A Term is said to be distinct when it is wholly included in or excluded from another term.

(2) A Term is said to be undistributive when it is only partly included in or excluded from another term.

Prop. (a) Subject is distributive . ∴ Predicate is undistributive.

Prop. (e) Subject and predicate are both distributive.

Prop. (i) Subject and predicate are both uudistributive.

Prop. (o) Subject is undistributive, Predicate is distributive. What is involved?

(1) Classification of the propositions are *Categorical* or *Hypothetical*. (2) The giving of the parts, subject, predicate and copula. (3) Quality, Quantity, and Symbols. (4) The distribution in the terms of the proposition.

Lecture 5.

OUTLINE.

Quantifications of Proposition.

I. Quantity of Predicate, giving rise to eight propositions.
II. Substitution of class relations for Attribution.
III. Idea of a universe of Discourse and its division into Positive and Negative classes.
IV. Possible Subjects and Predicates and Forms of Propositions—resulting.

V. The Seven (7) Simple Forms, $\begin{cases} x=ry=\text{all } x\text{'s are } y\text{'s} \\ x=r\bar{y}=\text{no } x\text{'s are } y\text{'s} \\ rx=ry=\text{Some } x\text{'s are } y\text{'s} \\ rx=r\bar{y}=\text{“ } x\text{'s are non } y\text{'s} \\ \bar{x}=ry=\text{all non } x\text{'s are } y\text{'s} \\ \bar{x}=r\bar{y}=\text{no non } x\text{'s are } y\text{'s} \\ r\bar{x}=r\bar{y}=\text{Some non } x\text{'s are } y\text{'s.} \end{cases}$

VI. Application (practical) of these to Propositions.

Immediate Inference.

(1) Nature, (2) Methods, $\begin{cases} \text{Opposition.} \\ \text{Conversion.} \\ \text{Contraposition.} \\ \text{Obversion.} \end{cases}$

Lecture Proper.

The idea of Quantification originated with Sir. Wm. Hamilton, with idea of quantifying predicate of Propositions. The utility of such a process, has been much disputed by logicians. (1) The quantification of a predicate of a Proposition, gives rise to eight forms, viz :

(1) All A is all $B=A^2$.
(2) “ “ “ some $B=A$.
(3) No “ “ any $B=E^2$.
(4) “ “ “ some $B=E$.
(5) Some “ “ all $B=I^2$.
(6) “ “ “ some $B=I$.
(7) “ “ “ not any $B=O$.
(8) “ “ “ some $B=O^2$.

De Morgan rejected E^2 and O^2 as useless on the ground that we rarely employ them in reasoning. This is the *first* step. The second step in the Quantification of the Proposition, is the substitution of the quantitative relation of inclusion and exclusion for the relation of Attribution.

Third Step—(Most Important introduced by De Morgan, contemporary with Hamilton) as follows, viz : It is founded on the conception of a universe of Discourse, e. g., If the discourse is about blue, then the universe of discourse is not all things existing, but merely all colored things, e. g., Let

x=blue color, the rest of class=non x, and x and non x will exhaust the discourse. Take "Blue is agreeable to the eyes." Here the predicate or universe of discourse is the class of divisible things, Let y=things agreeable to the eyes, and *non* y will stand for things not agreeable.

Fourth Step, Possible,

Four Symbolic Subjects, viz: $\begin{cases} x; \text{ and non } x. \\ \text{some } x; \text{ some non } x. \end{cases}$

Four Symbolic Predicates, viz: $\begin{cases} y; \text{ and non } y. \\ \text{some } y; \text{ some non } y. \end{cases}$

Sixteen Propositions resulting. Let v=some; $\bar{x}$=non x; vx=some x, we have

(1) $x=vy$=all x's are y's.
(2) $x=v\bar{y}$=no x's are y's.
(3) $vx=vy$=some x's are y's.
(4) $vx=v\bar{y}$= " " " non y's.
(5) $\bar{x}=vy$=all non x's " y's.
(6) $\bar{x}=v\bar{y}$=no non x's " y's.
(7) $v\bar{x}=v\bar{y}$=some non x's are non y's.
(8) $v\bar{x}=vy$= " " " " y's.

Thus we have sixteen forms instead of the old notation which seems to be perplexing, but they are not all simple propositions. The "simple propositions" are the first seven.

{Six or seven may be formed by converting subj. for pred and *vice versa*. Three are called *universal substitutive judgments.*}

(6) *Practical Application.*

(1) Example. All crystals are solid. (1) head, $x=vy$.

(2) Ex. Some solids are not crystals, $vx=v\bar{y}$.

(3) Ex. Some non crystals are not solids, $v\bar{x}=v\bar{y}$.

(4) Ex. No crystals are solids, $x=v\bar{y}$.

(5) Ex. Some solids are crystals, $vvx=vy$.

(6) Ex, All gold mines cannot be worked with profit, $vx=v\bar{y}$.

(7) Ex. It is wrong to put an innocent man to death, $x=vy$.

(8) Ex. There was no possibility of substantiating this, $x=v\bar{y}$.

All simple propositions may be symbolised by one of these forms. This is the *new notation.*

Immediate Inference, or

Third Part of Logical Doctrine.

The next problem that arises is the employment of the proposition in drawing inferences, i. e. from given relations between terms to deduce relations not given. This resolves itself into two questions, viz.: (1) From given relations to deduce others *immediately.* (2) From given relations to deduce others *mediately.*

(1) *Immediate Inferences.*—Taking up the four simple propositions, A, E, I, O, there are several methods by means of which others may be deduced, viz.: (1) Opposition. Is said to exist when there is difference in quantity and quality both. How do we deduce relation by Opposition? Let us assume the Prop. A.—"Grass is green." If this proposition is true, then its contrary, (E) "no grass is green" must be false. Then we deduce inference from the first. But if A is false no inference can be drawn. Again if A is true, its Contrary in (O) must be false, and if A is false then (O) must be true.

(2) *Conversion.*—Transposition of Subject and Predicate. The rule in this method is,—In converting a proposition the quality and distribution of the terms must not be changed. e. g., "Some minerals are rare."—Prop. in (I) both distributive, Converting it,="Some rare things are minerals."

(3) *Contraposition.*—The contrapositive of a Proposition is found by negating the predicate of a proposition, and then affirming of it the universal negation of a subject, e. g., "All men are mortal," negated predicate = non mortal, affirming the universal of the subject="all non mortals are non men."

(4) *Obversion.*—The obverse of a Proposition is its implied negative or affirmative. The process of Obversion rests on the fact that every affirmation implies a negation also. e. g., "All men are mortal." Obverse. No men are non mortals. The obverse of "No men are quadrupeds, is "All

men are non quadrupeds." Rule. If the original proposition be true, its obverse, converse, and contrapositive must be true also. We cannot generally infer the truth of the reciprocal of a Proposition.

Lecture VI.

OUTLINE.

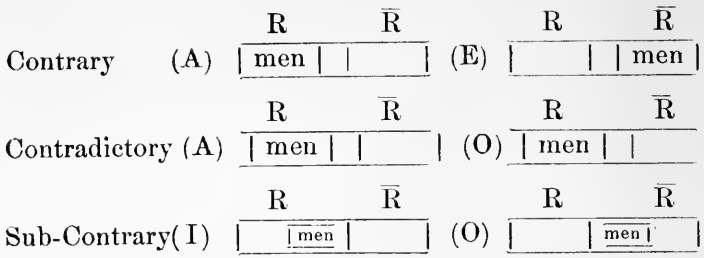
I. Resumé of Immediate Inference.
II. Symbolic representation of the relation of Immediate Inference.
III. Mediate Inference, 4th Part Logical Doctrine.
  (a) Relations of Mediate and Immediate Inference.
  (b) Nature of Mediate Inference.
  (c) Forms of Mediate Inference 

IV. Syllogism
  1. Number of Terms in Syllogism.
  2. Elements of Syllogism.
  3. Relation of the Syllogism to Actual Reasoning.
  4. Does the Syllogism beg the question?
V. Canons for testing the Syllogism.
  (a) The two Canons of De Morgan.
VI. Scope of the Syllogism—embraces Plurative and Numerically Definite Premises.
VII. How this modifies the rule of Distribution.
VIII. Subordinate Rules of Syllogism—1, 2, 3, 4, 5, 6, 7, 8.

Lecture Proper.

1. *Resumé of Immediate Inference.*—All inference is the unfolding of implications of *given data*. Immediate Inference of a single *datum*. Various relations give rise to general relations of 1st, Identity, 2nd, Non-Identity. Identity of relation is the basis of Affirmative Inference. Non-Identity of relations is the basis of Negative Inference.

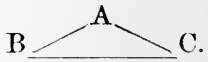
2. *Symbolic Representation of Immediate Inference.*—"All men are mortal."



3. *Mediate Inference or Reasoning.*—The discursive steps by Im. Inference are few; but it is nevertheless important, for all inferences are deduced from given data, e. g., If we could not say that if A is true, E is false, we would have to reason about I.

(a) Relation of Immediate to Mediate Inference. We cannot start with a single proposition and derive others from it, without Mediate Inference. We must bring in propositions from the outside.

(b) Nature of Mediate Inference. It differs from Immediate Inference in being: (1) Indirect. We cannot draw out inference directly, but by comparison, e. g., "All men are mortal." The Greeks are men ∴ the Greeks are mortal. We must bring the two together. (2) It also involves three (3) terms instead of two. It must have three terms and may have more.

(c) Forms of Mediate Inference. 

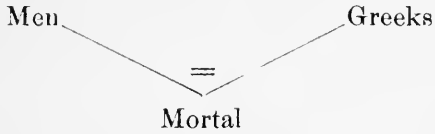
What can be said about relation of B and C? B is related to A, and C is related to A, ∴ B and C are in some way related to each other.

4th. *The Syllogism.*—Explicit form of "Mediate Inference" is the syllogism, (of Greek origin). It is a synthetic process in which we combine two propositions and deduce from the relation of the two a third relation. The parts are 1st—Two Premises and 2nd, one Conclusion.

(1) Nature of terms involved. We have three propositions, and each proposition has two (2) terms, e. g., "All men are mortal." "The Greeks are men," ∴ "The Greeks are mortal." Spenser says the Syllogism has four (4) terms

viz.: $\left.\begin{matrix}\text{Men}\\ \text{Mortal}\end{matrix}\right\} = \left\{\begin{matrix}\text{Greeks}\\ \text{Mortal.}\end{matrix}\right.$

Mortal is repeated. The syllogism only involves three distinct terms.



(2) Elements involve three propositions and three terms, (Premises,—major and minor), and *Middle Term. Major* Premise is that in which the major term is compared with the middle. Minor is that in which the middle is compared with the minor term. The major term is always predicate of the conclusion. The minor term the subject of the conclusion.

(3) Relation of Syllogism to Reasoning. Two objections are made: (1) That it is not the form in which we ordinarily reason. We admit this to be true. We reason straight ahead through the major to the minor premise. It does not profess to embody this common method of reasoning. It is only the test of reasoning. (2) The second objection is, that it begs the question, i. e., That the conclusion only states what is contained in the major premise. This is not true. The major premise contains the ground of the conclusion but does not state the conclusion, e. g., "All men are mortal"—major premise; Greeks are mortal—minor premise. This does not state that the Greeks are men. The minor premise subsumes and settles that point. We would beg the question if we would state the conclusion in one of our premises.

5. *Canons for Testing the Syllogism.*—We bring forward two canons of De Morgan. 1st. If the minor and major terms, each being compared with same third term, agree with it, they agree with each other. This is the canon of affirmative relation. 2nd. If the minor and major terms, both being compared with the same third term, and one agrees with it, and the other disagrees with it, they disagree with each other. These two canons, by considering

them, we would discover that they are but statements of the laws of Identity and Difference and Contradiction. We first state laws of identity as applicable to reasoning in three terms.

6. *Scope of Syllogism.*—In order to reach a conclusion the middle term, must be distributed once in the premise, or the conclusion would involve an "illicit process" of the middle term;—so it was held by older logicians, e. g., "Some men are poets" and "Some men are Americans," ∴ Some Americans are poets. When it is known that *some* means more than half the class, the law of distribution does not hold good. In the same way we can derive the numerically definite premises. Plurative judgments are inconsistent with the rule, that the middle term must be distributed once, but they fall in with the general (canon) law of identity.

7. The law of distribution holds good in all judgments except in Plurative judgments.

8. *Rules of Syllogism.*—1st. Every syllogism must have three terms only. 2nd. Every syllogism contains three propositions only. 3rd. The middle term must be distributed once at least, except in the case of Plurative judgments, and it must not be ambiguous. 4th. No term must be distributed in the conclusion, which was not distributed in one of the premises. 5th. From negative premise nothing can be inferred. 6th. If one premise be negative the conclusion must be negative, and *vice versa.* To prove a negative conclusion, one of the premises must be negative. 7th. From two particular premises (except in the Plurative judgments), no conclusion can be drawn. 8th. If one premise be particular, the conclusion must be particular.

Lecture VII.

OUTLINE.

I. Statements of the Canons in terms of genus and species.
   (a) Affirmative.
   (b) Negative.

In a syl. there should be only 3 terms:
only one middle term.
One premise must be aff.
If either is neg. conclusion
must be neg.
To prove neg. conclusion one
of premises must be neg.

II. Mood and Figure.
III. Special function of each Figure.
IV. Reduction of the Syllogism $\begin{cases} 1) \text{ Nature of Reduction.} \\ 2) \text{ Relation of other Figures to the first.} \\ 3) \text{ Method of Reduction.} \end{cases}$
V. Reduction under New Notation,
(a) The elementary propositions.
(b) Reduction of these to two forms $\begin{cases} xy > 0. \\ xy = 0. \end{cases}$
VI. Possible combinations of Premises from these.
VII. Valid Forms.
VIII. Reduction of various Syllogisms to these forms.
IX. Logical character of syllogism includes,
$\begin{cases} 1. \text{ Terms.} \\ 2. \text{ Propositions.} \\ 3. \text{ Symbolic Propositions.} \\ 4. \text{ Mood and Figure.} \end{cases}$

Lecture Proper.

I. *Statement of Canons, etc.* — The Canons of De Morgan are somewhat indefinite. There are certain kinds of agreement and disagreement. Hence a more specific statement.

(a) Affirmative.) What is included in a species is also included in the containing genus. Principle of Affirmative Inf. in 1st and 4th Figures. When two classes overlap, the common part belongs to both classes. This belongs in Fig. 3d, and also in Plurative Judgments.

(b) What is excluded from a genus is also excluded from the contained species. Principle of Neg. Inf. in Fig's. 1 and 2. Conversely what is excluded from a species, is excluded from part of the containing genus. Principle of Neg. Inf. in Fig. 3, also negative Plurative Judgments.

II. *Mood and Figure.*—The Mood of the syllogism may be defined as the symbolic expression of the quantity and quality of the propositions of a syllogism.

The Figure of the syllogism is determined by the position which the middle term holds with reference to the extreme terms—*i. e.*, major and minor terms.

III. *Special function of each Figure.*—Lambert has given the best statement of each Figure. The 1st Figure is suited to the discovery or proof of the properties of a thing. 2d Figure to the discovery or proof of the distinction between things. 3d Figure to the discovery or proof of the instances and exceptions. 4th Figure to the discovery or exclusion of the different species of a genus. These distinctions possess some value.

IV. *Reduction of the Syllogism.*—(1) It is a movement towards simplification or generalization. What are the fundamental forms to which all syllogisms can be deduced? This is the question in Logic. (2) Aristotle characterized the 1st Fig. as "perfect," the 2d and 3d as "imperfect." Figure 1st gives conclusion corresponding to the four fundamental judgments, viz., (A), (E), (I), (O). Other figures are special cases, able to be reduced to 1st Fig. We can reduce all possible forms to the four forms of the 1st Figure.

IN TEXT-BOOK.

(3) Method of Reduction. Mnemonic lines—
Artifice of old logicians, *e. g.*,
Syllogism in 1st Figure,
"All men are rational,"
No cats are rational.
Therefore, No cats are men.

To change this to the 1st Figure, transpose the premises, putting 2d for 1st, and then convert this simply.

Ex. II. Special case in 2d and 3d Figures,
"All men are rational."
Some animals are not rational.
Therefore, Some animals are not men.

Syllogism in 1st Fig. Process is "*Reductio per Impossibile.*" Take contrapositive of major term, change the minor into its corresponding affirmative and the conclusion follows in Fig. I, as follows:

"No non rationals are men."
Some animals are non rationals.
Therefore, Some animals are non men.

V. *Reduction under New Notation.*—(a) Begins with more complicated conditions. It has 16 propositions which admit of 256 combinations of premises. This may be reduced from 16 to 7, therefore to 49 combinations. We can further reduce them to 2 under the "new," instead of 4 under the "old," *e. g.*, $xy > 0$, $xy = 0$.

We may state the seven (7) propositions in the following forms, viz.:

1st Prop.	$x = vy$	becomes	$xy > 0$.
2d "	$x = v\bar{y}$	"	$xy = 0$.
3d "	$vx = vy$	"	$xy > 0$.
4th "	$vx = v\bar{y}$	"	$x\bar{y} > 0$.
5th "	$\bar{x} = vy$	"	$\bar{x}y > 0$.
6th "	$\bar{x} = v\bar{y}$	"	$xy = 0$.
7th "	$v\bar{x} = v\bar{y}$	"	$xy > 0$.

(b) From this it appears that 5 out of the 7 Props. reduce to the form $xy > 0$, and the remaining two to $xy = 0$.

All propositions that represent a relation of total exclusion, fall under forms $xy=0$. While all judgments which represent partial inclusion fall under first (1st) form, $xy > 0$.

VI. *Possible Combinations of these two forms.*—In these two forms, x may be taken to represent x or $\bar{x}$ (non x) and y may equal y or $\bar{y}$ (non y). Bearing this in mind and taking y and z to represent extreme terms, and x the middle term, we have the following six combinations:

(1) $xy = 0. : zx = 0.$
(2) $xy = 0. : zx > 0.$
(3) $xy > 0. : zx > 0.$
(4) $xy = 0. : z\bar{x} = 0.$
(5) $xy = 0. : z\bar{x} > 0.$
(6) $xy > 0. : z\bar{x} > 0.$

VII. *Valid Forms.*—Take an example under the first, and determine empyrically the validity: Take $xy = 0$ and $zx = 0$, when that is transformed it means no x's are y's and no z's are x's, therefore false.

(2) $xy = 0$ and $zx > 0$ means no x's are y's, some z's are x's, therefore, z's are not y's. This gives us a conclusion and is a valid combination.

(3) $xy > 0 : zx > 0$. Some x's are y's, some z's are x's. Fallacy of two particular premises, therefore invalid.

(4) $xy = 0$, $z\bar{x} = 0$. No x's are y's; all z's are x's, *valid.*

(5) $xy = 0$, $z\bar{x} > 0$. No x's are y's; some z's are not x's. Fallacy of negative premises.

(6) $xy > 0$, $z\bar{x} > 0$. Some x's are y's; some z's are not y's. Not good.

Only two out of the six give rise to conclusions, viz.:

(1) $xy = 0 : zx > 0$, therefore, $\bar{y}z > 0$,
(2) $xy = 0 : z\bar{x} = 0$, " $yz > 0$.

1st. By substituting *non y* for y, gives a conclusion in (I), which was (O). 2d. From (E) we get a conclusion in (A) by substituting $\bar{y}$ for y in first part.

VIII. *Reduction of various Syllogisms to these forms.*

a. (1) "All men are rational."
(2) Greeks are men.
(3) Therefore, Greeks are rational.

$$(1) = x\bar{y} = 0.$$
$$(2) = z\bar{x} = 0.$$
$$(3) = z\bar{y} = 0.$$

b. (1) "No Americans are slaves.
(2) All New Englanders are Americans.
(3) Therefore, No New Englanders are slaves.

$$(1) = xy = 0.$$
$$(2) = z\bar{x} = 0.$$
$$(3) = zy = 0.$$

c (1) "All men are mortal."
(2) No angels are mortal.
(3) Therefore, No angels are men.

$$(1) = \bar{x}y = 0.$$
$$(2) = xz = 0.$$
$$(3) = yz = 0.$$

d. (1) "All men are mortal."
(2) All men are rational.
(3) Therefore, Some rationals are mortal.

$$(1) = x\bar{y} = 0.$$
$$(2) = xz > 0.$$
$$(3) = zy > 0.$$

IX. *Logical character of Syllogism includes,*

1. *Terms.*
2. *Propositions.*
3. *Symbolic Propositions.*
4. *Mood and Fig.*

Definitions, etc., given in connection with Text-book.

Meaning of Conditional Reasoning, and its relations to Categorical Reasoning.

1st. In categorical reasoning, we assume the truth of both propositions to draw our conclusion. The truth of the one does not depend on the truth of the other. They are both true. In conditional propositions the truth or alternative of one depends on the truth or falsehood or alternative of another.

2d. We substitute term conditional for hypothetical. Hypothetical is a term applied to this kind of reasoning in old Logics. In science, hypothesis has a different meaning, since it is a supposition which is to be tested. Taking conditional to represent this kind of reasoning, we may say that in conditional reasoning we have two propositions associated together in relation of ground and consequent. There are two cases. Fowler has: 1st. If two propositions or sets of propositions be associated together, so that the truth of the one depends on the truth of the other, it is *conjunctive* reasoning. 2d. If they be dissociated so that the truth of the one depends on the falsity of the other, and the falsity of the one on the truth of the other, it is *disjunctive* reasoning.

Hence we have the subdivisions of Conditional Reasoning into 1. Conjunctive, 2. Disjunctive.

Law of Conditional Reasoning, as a whole, may be called *Sufficient Reasoning.*

Every conclusion or inference must have an adequate ground. This law points to the fact that all inference depends on some data for its justification. Hence all Mediate Reasoning may be put in both forms. The *conditional* form makes explicit the dependence of the conclusion on the data which give rise to it. The *categorical* form keeps this dependence in the back ground and emphasizes the data which lead to the conclusion.

Law of Disjunctives.

The general law is that of *excluded middle.*

Special Laws.

1. The members of the disjunctive must exclude one another. It must not be possible for two or more to be true at the same time. 2. The disjunctive must be complete. If any alternative is left out it spoils the reasoning. The Constructive Disjunctive Syllogism is "*modus tollendo ponens.*" The Destructive Disjunctive Syllogism is "*modus ponendo tollens.*"

Definition of Dilemma.

A dilemma is a syllogism having a dilemmatic judgment for its major premise, with a minor premise so affirming or denying some member or members of the major premise, as to lay the foundation for an inference. A dilemmatic judgment is one which has a conjunctive antecedent and a disjunctive consequent.

Ways of Meeting a Dilemma.

1. You may show that the major premise or the conjunction is not valid. 2. You may show that the disjunction is not complete or exclusive. 3. You may oppose the dilemma by another dilemma, which will neutralize it.

Form of Dilemma.

1. If A is B, C is D, and E is F, but either C is not D, or E is not F; therefore A is not B. 2. If A is B or E is F, C is D; but either A is B or E is F; therefore, C is D.

3. If A is B, C is D, and if E is F, G is H; but either A is B, or E is F; therefore, either C is D or G is H. 4. If A is B, C is D, and if E is F, G is H; but either C is not D or G is not H; therefore, either A is not B, or E is not F.

Sorites.

Sorites are of two kinds, viz.: 1. Categorical; 2. Conditional. A categorical sorites is a series of dependent categorical propositions, which may be unfolded into syllogisms. A conditional sorites is a series of conditional statements, so arranged that the affirmation of the first antecedent leads to the affirmation of the last consequent; and the denial of the last consequent leads to the denial of the first antecedent.

Categorical sorites are of two forms—1. Regular sorites; 2. Goclenian sorites.

(For definition of Regular Sorites, see Text-book.)

Rules for Regular Sorites.

1. The unexpressed propositions are conclusions of the next preceding syllogism.

2. All the intermediate expressed propositions between the first and the last are major, the first alone being minor.

3. No premise can be negative except the last.

Definition of Goclenian Sorites.

The Goc. Sorites is a series of propositions in which the subject of each becomes the predicate of the next following, until in the last step, the predicate of the first proposition is affirmed or denied of the subject of the last.

Rules.

1. The unexpressed propositions are conclusions of the next preceding syllogism, and each becomes the major of the following syllogism.

2. All the intermediate expressed propositions are minors, the first alone being major.

3. The last proposition alone may be particular.

4. The first alone may be negative.

Conditional Sorites.

Two Forms—1. Direct; 2. Regressive.

Direct Form—If a is b, c is d; if c is d, e is f; if e is f, g is h; but a is b; therefore, g is h.

Regressive Form—If e is f, g is h; If c is d, e is f; If a is b, c is d; but a is b; therefore, g is h.

PART II.

MATERIAL LOGIC.

Lecture I.

OUTLINE.

I. Different views of the Sphere of Logic.
II. General Logic embraces Formal and Material branches.
III. Subject matter of Material Logic.
IV. Relations of the elements of Material Logic to Reality.
   a. The Notion.
   b. The Judgment.
   c. The Proposition.
V. Relation of the Gen. Proposition to Reality.
VI. Analysis of Reasoning, { 1. The Datum. 2. The Inferential Act. 3. The Result. }
VII. Reasoning and Association—points of resemblance and difference.

Lecture Proper.

1. Aristotle recognized two methods of Reasoning—(1) From particular to general laws, (Induction). 2. From general principles to particulars, (Deduction).

The founder of Logic looked upon it as a deductive science. According to the ancients it is a formal deductive science.

There are different schools.

Kant (1750—1800) makes a distinction between *form* and *matter* of thought. Also Sir Wm. Hamilton seeks to limit the sphere of Logic to the syllogism. Bacon attempted to lay the foundation of a new Logic, and John Stuart Mill laid down the Canons of Induction. Now it is considered that Logic is not merely a Formal Science, but has a Material bearing.

III. *Subject matter of Material Logic.*—General Logic is the science of the laws of Inference. Inference expresses it-

self in Judgment and Propositions. These have a double meaning, *i. e.*, a formal and material meaning. In Formal Logic, we treat of the formal laws of Inference. It assumes the data and seeks to draw the result from these. Material Logic considers reasoning as a means of reaching objective truth. It calls the data of Formal Logic into question. How may our reasoning faculties grasp objective truth? We must test the truth of our data, but also draw correct inference from them.

IV. *Relation of the elements of Material Logic to Reality.* The elements of Material Logic are Notion, Judgment, Proposition.

(a) The Notion.—Material logic regards the notion as an adjective. The noun in Material logic stands for the complete meaning of the thing; while the adjective stands for a partial meaning, *e. g.*, "The book is small." Small expresses only a partial meaning. This is true of all notions, as, 1. of the singular concrete notion; 2. of the general concrete notion; 3. of abstract terms, as "The tiger is rapacious."

(b) Judgment in its essence is the assertion of a conviction. It is always accompanied with a belief. Every judgment has two elements, *i. e.*, subject and predicate. The predicate contains some qualifying matter, not of the notion or subject, but of reality. The judgment asserts a reality and also establishes a relation.

(c) The Proposition is a judgment clothed in language. It may not always express conviction or belief. There are two classes of propositions—1. the Enunciative; 2. the Real. The Enunciative does not always express conviction of the truth, as "Let us suppose that such and such," etc.

V. *Relation of General Proposition to Reality.*—Propositions are also particular and general. Hence it is important to know the relation of every proposition to reality. What do these given propositions stand for? This is a dispute between the nominalists and conceptionalists as well as the realists. John Locke took the position that the given propositions assert nothing but relations among ideas. It

asserts nothing regarding the objective world according to him. Proposition is the assertion or denial of a relation between two ideas (Locke).

There are Static laws, and Dynamic laws. A Dynamic law, as opposed to a Static law, is a statement regarding the relation which exists in the objective world. The law of falling bodies is a dynamic law. The scientific proof is a statement which is conceded to exist in the objective world.

VI. *Analysis of Reasoning* takes up the reasoning process and developes conclusions. The reasoning process is composed of three parts—1. Datum; 2. the Inferential Act; 3. the Result.

1. The Datum is some fact or proposition on which the truth of something else depends. The datum must be of the nature of an antecedent or ground. It is something present to us, which involves the existence of something absent.

2. Inferential act is always a judgment. It does not see the result or what it asserts, but judges it on the strength of what it does see. The essential feature of the Inferential act is that of Judgment.

3. The *Result* of a real act is a "new truth." This is disputed on the ground that it has the truth already in one of the premises. Two facts may depend on each other without either being contained in the other. The truth in regard to the result of reasoning is this. It gives a new truth, but not an *independent* truth. It gives a dependent truth.

VII. *Reasoning and Association*,—resemblance and difference. It has been claimed by some that reasoning is merely a developed form of the association of ideas. They resemble each other in certain points, but—

1. Association, etc., is *subjective*, while Reasoning is *objective*.

2. Association simply recalls what has been already before the mind and gives us nothing new, but reasoning enables us to reach something new.

3. Association does not involve any act of judgment, as to the reality or truth of its ideas, while reasoning ends in an act of judgment. There is a close dependence of reasoning on association. Reasoning must depend in association, for the data or material from which it must proceed. Association gives the mind command over an organized experience. Association gives the mind command over the past, and hence enables us to form expectation for the future.

Lecture II.

OUTLINE.

I. Connecting link between Reasoning and Association.

II. Kinds of Reasoning
- 1. From Typical Instances.
- 2. Induction Proper.
- 3. Deduction.

III. Relation of the laws of thought to Material Logic.
1. Identity. 2. Contradiction. 3. Sufficient Reason.

IV. Modal Categories of Reasoning
- 1. Actuality.
- 2. Possibility.
- 3. Necessity.

Lecture Proper.

I. *Connecting link between Reasoning and Association.* (1) They are not identical, but closely related. That some claim that they are not related, is false. Association is objective and Reasoning subjective. Hence they are different. The connecting link between them is Recognition. Some would make Recognition, merely Reproduction. This cannot be. There may be reproduction without recognition. Recognition is a spontaneous judgment and it is simply the awareness of the mind. While it is not identical with reproduction, it is closely allied to it. We may regard Recognition as the connecting link between Reasoning and Association.

(2) The ordinary doctrine respecting kinds of Reasoning is, that there are two, viz.:

1st Particulars to Generals. (Induction.)
2d Generals to Particulars. (Deduction.)

J. S. Mill says another form is to reason from Particulars to Particulars. Ex. "A child puts its hand in the fire and is burnt, it sees the fire again a few hours afterwards, and withdraws the hand, because it remembers," &c., or, that of a man eating a poisonous herb; which, makes him sick. Seeing it the next day he remembers his former experience, &c. The relation may be one of association in which the refusing to act would be instinctive revulsion. It may be an inference that the eating of the herb will make him sick. The second act involves judgment and reasoning. The simplest act is not reducible to association. The next point is that, the inference is not particular. The starting point is a particular inference, and the real inference is that, what "sickens him once will sicken him again." Inference, whether it starts with an example or a multitude of examples reaches s conclusion which is more or less general. The simplest form of reasoning may be styled reasoning by typical inference. This is the form mostly used in practice.

II. *There are two other kinds of Reasoning.* 1st. Reasoning from particular facts to laws, e. g., "Many diamonds are combustible," &c. 2nd. Inference from general laws to particular facts.

III. Relation of the laws of thought to Material Logic. In Formal Logic we find that Identity and Contradiction govern categorical reasoning, while sufficient reasoning govern Material Logic. In what sense do these laws serve as guarantees of reasoning? Three laws. 1st. Identity. 2nd. Contradiction. 3rd. Sufficient Reason.

1st. Identity.—It is the law of the real subject.

2nd, Contradiction.—Is the law of compatibility in Reasoning. It applies to the relation between the subject and predicate notion as belonging to the same real subject. Ex. "This round body is square." The law of Contradiction shows us that round and square are inconsistent, ∴ false.

3rd. The law of Sufficient Reason. This is the law of sequence. It applies to the relation between the data and

conclusion in reasoning. Contradiction decides only the compatibility of the notion, but sufficient reasoning shows whether they are connected or not. These fundamental laws qualify the same proposition, at the same time.

IV. *Modal Categories of Reasoning.* Actual, possible, necessary, and impossible are the categories. Kant first called attention to the categories of quantity, quality, relation and modality. The necessity of the study of the Modal category is, that thinking and knowing are different things entirely. How should we think about the real in order that our mind may not deceive us?

1st. We assert a thing to be actual when we know all the conditions of its existence to be actual.

2nd. The possible, is one of the most difficult categories. It has so many shades of meaning. There are three. 1. The Improbable. 2. The Barely Possible. 3. The Probable.

1st. A thing is pronounced improbable when it is not known to be impossible. But when the conditions known to be favorable to its existence are greater than those known unfavorable, it is said to be possible.

2nd. A thing is pronounced to be "barely possible" when no conditions are known to exist either for or against it. A thing is pronounced to be possible when the balance of conditions are in favor of its existence.

3rd. A thing is pronounced to be necessary when its non-existence involves the non-existence of something which is known to exist either actually or by hypothesis. Ex. We pronounce "time" to be a necessary thing.

Lecture III.

OUTLINE.

I. Relation of the Modal Categories to the different parts of Logic.
II. Reason and Causality.
III. Nature of Scientific Explanation.

IV. Media of Explanation,
- 1. Similarity and Difference.
- 2. Time Relations,
  - Coexistence and Succession.

V. Time Relations.—(a) Distinction between relations of Coexistence and Succession in Science.

VI. Coexistence
- 1. Species
  - 1.
  - 2.
  - 3.
  - 4.
- 2. Classification
  - 1 and 3 Give Scientific Laws.
  - 2 and 4 Give Empirical Laws.

VII. Succession.

Lecture Proper.

I. *Rel. of the Modal categories to different parts of Logic.*—In Formal Logic all conclusions are necessary. In Formal Logic we take in consideration only the conclusion as regards the premises, and the premises must be such as to satisfy the conditions. There is no sphere for probability in Formal logic. All categories must be categorically or conditionally necessary. In its relation to objective reality, a conclusion may rest in any one of the three modal categories. If we have assurance that a causal relation exists between a and b, and we know b exists, then a must.

The great bulk of scientific truth rests in probability, therefore the three categories of Reasoning may be called the categories of truth and falsehood.

II. *Reason and Causality.*—The great law of the objective world is Causality. The law of Reasoning is that of Suff. Reason, or ground—of consequent which connects the conclusion with antecedent. These two laws are the objective and subjective aspects of the same principle. The mind seeks for an explanation by asking the ground or "because" of things. The objective world is explained when we have ascertained a sufficient cause for existence. When the mind's asking for things, discovers a real cause, we have discovered

a law of nature. From this it follows that the function of Material logic is *explanation.* How are phenomena which we grasp by explanation to be explained? All experience consists in the discovery of laws. What does the law of a thing involve?

Two extreme views regarding the nature of a scientific law.

First (David Hume).—Law stands simply for a uniform relation, either a coexistence or succession in time, without indicating anything deeper. The character of this is to divorce law from causality and leads to a superficial view of the world.

Second View.—That law is itself an efficient cause. These two views are called extreme views. A law is not a cause, nor is it independent of causality. A law is simply an expression of the mode, in which a cause acts, using cause to include all efficient agency.

III. *Sc. Explanation.*—Scientific explanation refers phenomena to laws, ascertaining the general modes in which causes in nature act. 1. Importance of this view of causality to Philosophy and Science. 2. It increases the simplitude of science. It provides a real connection between science and philosophy. Takes up the problem where science leaves off, &c.

IV. *Media of Explanation.*—The function of Mat. logic being explained leads to Media of Explanation. Scientific explanation is referring of phenomena to their causal laws. But causality cannot be directly distinguished; certain medial relations are therefore necessary. Certain relations may be taken as indices and certain may not. What are Media of Explanation? They are the relation of Similarity and Difference, and secondly are relations of time—Coexistence and Succession. It is not true that we can only know these and no more, but much more. They are simply instruments which the mind must use to discover laws of nature.

V. *Time Relations.*—All phenomena exist in time, which has orders—Coexistence and Succession. (a) Coexistence is the statical relation which exists in a system of things. Suc-

cession is the relation which connects the progressive stages of development. Coexistence is the order of permanence. Succession is the order of progress. Social statics and social dynamics, are statical dynamical laws and are relations of coexistence and succession.

VI. *Coexistence.*—There are various forms,—four distinct species. 1. Coexistence depending on one cause or on interdependent causes. 2. Coexistence which results from coexisting, but independent causes. 3. Coexistence of common attributes in classes or kinds. 4. Accidental coexistence.

The first species of Coexistence is the most reliable and certain, for the phenomena in this case are the effects of one cause, or a group of dependent causes, e. g., As heat rays of the sun and its result in vegetation. When the sun's rays are present, vegetation is present. When sun's rays are absent, vegetation is absent. Here through the medium of coexistence the scientific man discovers underlying causes of explanation.

Second species. Such are of little value in Science,—depend on accidental concurrence of causes. It is possible to derive a hypothetical law, that wherever certain causes coöperate, certain effects will follow. Of little value until effects and individual laws are determined—one of the most fruitful sources of error.

Third species. Rests at the basis of all classification. Classes of kinds depend on attribute of members. It gives rise to a high degree of certitude. Calculations are not accidental. Aristotle expressed it in his attributes.

Fourth species. Partly embraced in second species. Two cases, (1) Coexistences which arise from the coöperation of individual causes. (2) Coexistences which cannot be traced to any causes whatever, and rightly considered are purely accidental.

2. These four classes may be classified into two groups, first and third lead to the discovery of scientific laws; second and fourth lead to generalization—Empirical laws (Mill). Difference between Sc. and Emp. law. Scientific law rests on some principle of causation. Empirical law is based

only upon experiment—what has happened we believe will happen again.

VII. *Succescion.*—Philosophers have recognised the close relation, with time relation and succession. Causation is the unvariable and unconditional sequence. This definition avoids the difficulty by begging the question—means the real cause smuggled in. Time sequences are evidences of causality ∴ media of explanation is divided into 3 classes. 1. Sequences which represent simple causal relation. 2. Sequences which do not represent simple causal relations between terms but point to a common cause or causes of which they are joint effect. 3. Accidental sequences which have no ascertainable relation to causation.

Lecture IV.

OUTLINE.

I. Kinds of rel. of Sequences { 1. 2. 3. 4.

II. Classification { 1 and 2 give scientific laws. 3 and 4 give empirical laws.

III. Summary of Time Relations.

IV. Value of distinction between Scientific and Non-Scientific relations.

V. Similarity and Difference as *media* of explanation.

(a) Similarity—used in *classifying* objects into groups and in *generalizing* phenomena under causes.

(c) Complete similarity and its relation to science.

(b) Gen. kinds of Similarity { 1. Complete. 2. Partial.

(d) Partial Similarity and its relation to science=basis of Analogy.

VI. Nature of Analogical Reasoning.

(a) Cases of Analogy { 1. Analogy in the descriptive sciences. 2. Anal. in dynamic sciences.

(b) Analogy in Des. Sci. { 1. Classifying fixed forms. 2. Classifying transitional forms.

Lecture Proper,

I. *Kinds of rel. of Sequences.*—Succession bears close relation to Causality. Sequences have been regarded of more importance the relation of co-existence. If the order of causality be sequence, it must follow in close order of Time. A fourfold classification—as substitute for the last.

1st. Successions among phenomena which depend on simple causal relations.

2nd. Successions which depend on derivative causal relations.

3rd. Successions which depend on independent causal relations.

4th. Accidental Succession.

Example of 1st Form—succession of lightning and thunder. Example of 2nd Form—succession of day and night. Example of 3rd Form—succession of any two such phenomena, as the establishment of Free Trade in Great Britain and the low condition of laboring classes. Example of 4th Form—succession of high floods in the Ohio river and the Cincinnati riots—accidental.

II. *Classification.*—We may generalize these four kinds into two classes: 1. Including 1st and 2nd species leading to scientific laws. 2. Including 3rd and 4th species which are of little value in science, and lead to empirical laws. All phenomena have time relations.

III. *Summary of Time Relations.*—To sum up the relations —the conclusion of classification of Time Relations. 1. All phenomena have time relations. These time relations are divided into two main classes—Co-existence and Succession. The relation of Co-existence was divided into four species. 1 and 3 give rise to scientific laws; 2 and 4 to empirical laws. Succession is divided into two classes. 1 and 2 give rise to scientific laws; 3 and 4 to empirical laws.

We have used the term empirical in contrast with scientific law. Empirical may be used in two different senses. 1st, Means anything derived from or depending on experience. 2nd, May be used to signify mere uniformity of experience, for which we can give no reason. In this connection it is used in the latter sense.

IV. *Value of Distinction, &c.*—The value of distinction between scientific and non-scientific relations cannot be overestimated. Draws the distinction between science and loose generalization. Shows how many of our laws are unreliable and superficial, e. g., The question of the true industrial policy of this country.

V. *Similarity and Difference, &c.*—How do Similarity and Difference aid the memory in acquiring knowledge? They do not bear the same relation. The relation of Time leads directly to discussion of laws. Similarity and Difference lead indirectly to the same result. (a) Similarity is the relation which the mind uses in Generalization. We generalize by applying the notion to singulars. Similarity rests at the basis of Classification. Certain attributes exist together, we conclude why they coexist.—Given a group of coexisting attributes. If these are laws of nature, then all objects resembling this group belong to a general class. Similarity leads to classification, also to generalisation of phenomena under causes. If we discover a causal relation between two things, similarity leads us to generalize. When through a clue of coexistence or succession, we discover a causal connection, we generalize this by relation of similarity. By using time relation and similarity, we can find a general law. (b) General Kinds of Similarity. There are two cases, 1st. Complete similarity leading to a class. 2nd. Partial similarity which is the basis of Analogy. When we have discovered a relation between two groups of phenomena, we may infer that it will produce similar attributes as an effect.

VI. *Nature of Analogical Reasoning, etc.*—Few principles are more important in Reasoning than Analogy. It is the principle of suggestiveness. The power to conceive analogies is that of genius. The able reasoner is not only the one who

can draw conclusions from premises, but he can also draw analogies. Analogy is also the most dangerous and misleading, (e. g., H. Spenser.) (a) Analogous inference is inference founded on partial similarity. There are two cases: 1. Analogy as employed in the descriptive sciences. 2 Analogy as employed in the dynamic sciences.

1. When the type (in descriptive sciences) of animal or plant is fixed, and a group constitute the class, we get a classification of fixed forms. If characteristics are *a*, *b*, *c*, and *x*, *y*, *z*. If an animal possess *a*, *b*, *c*, and *x*, we conclude by analogy that *y* and *z* belong to it also, and we include it in the class. This does not say that it possesses *y* and *z*, but it is highly probable.

2. When the type is not fixed we have transitional forms. In Zoology it is most difficult to discover transitional forms. Hard to see species of transitional forms. What are the essential characteristics of the several species to which it is most closely allied? We determine the essential characteristics to the organism in question. Lastly, we compare points of resemblance and difference to which it is allied, and the species to which the essential attributes are most closely allied will be the assigned class.

Lecture V.

OUTLINE.

I. Analogy { 1. In the Descriptive Sciences.
2. In the Dynamical Sciences and Math.

II. Forms of Analogy { 1. Scientific.
2. Empirical.

III. Scientific Form { 1. Causal Analogy { 1. Positive.
2. Speculative.
2. Anal. of Coexistence.

IV. Causal Analogy—used in Dynamical Sciences.
(a) Positive form used in discovering Scientific Laws.
(b) Speculative used in Metaphysics.

V. Analogy of Coexistence—used in Classification.

VI. Difference as a medium of Knowledge—sets limits to Analogy.

VII. Resumé of Media of Knowledge.

Lecture Proper.

I. *Analogy.* Two laws dealing. 1st. In dynamical sciences deal with conservation of energy—laws of causation. Cannot limit use of Analogy to descriptive sciences.

Analogy led Descartes in connecting Algebra and Geometry. Analogy thus led to the union of two branches of Mathematics. Astronomy and Chemistry are also fruitful fields for this principle. In Physics we find some of its most remarkable application, e. g., Undulatory theory of light and sound. By a further generalization and use of Analogy, the conclusion has been reached, that in the physical universe, all motion is undulatory or rhythmical.

Analogy has also wide application in the mental and social sciences. Thus, The idea of devolopment, first confined to a few sciences, now tends to the human sphere.

The use of Analogy in the dynamical sciences is also dangerous. It is liable to be carried too far,—so attractive.

The scientific world prevented from the discovery of undulatory theory of light by suppressing the theory of sound and light too closely related.

In social science are also some striking instances of misuse of Analogy, e. g., Spencer seizes upon the obvious relation between the physical and social organism and carries this to the minutest detail.

II. *Forms of Analogy.*—Analogy in general, with respect to its uses, is divided into two forms, Scientific and Empirical.

III. 1. The Scientific form embraces all those analogies which lead to the discovery of causal laws in the dynamical or of classes in the descriptive sciences. It is divided into *causal analogy* and *analogy of coexistence.* 1. Causal analogy is further divided into two kinds, *Positive* and *Speculative.*

IV. *Causal Analogy.*—Causal analogy is in general of most importance in all reasoning. Its form is intimately related to time relations, which lead to scientific results. We reason that like causes produce like effects and that like effects lead back to like causes, e. g., Let *a* and *b* stand for two phenomena. If we have discovered a causal relation between *a* and *b*, we conclude that like causes produce like effects, &c. (a) The positive form is the form of its use in ordinary scientific reasoning—applies to the relations between phenomena and those causes which appear in antecedents. (b) The speculative form. Here we trace phenomena to their causes and laws. The special form of causal analogy never received due attention from logicians—highly important in Philosophy. Has a wide application to certain branches back of the causes of certain causes. This is a fact, acknowledged by such men as Spencer, when he says, "beneath the relative is the profound." The starting point of our knowledge is *consciousness.*

1st. We infer after the analogy of our own existence, the existence of a ground in nature. This principle is one which is acted on in scientific and philosophical research. 2. We determine concerning the character of this ground of existence. When we are brought in relation to the universe, we refer all things to an invisible ground; and secondly, a cause to which phenomena are referred. It does not differ in essence from positive analogy.

V. *Analogy of Coexistence*—Used in descriptive sciences in classification. Empirical use of analogy extends empirical laws and generalization. These empirical laws while they possess secondary authority are not sufficiently certain to classify them among scientific laws. Such a form widely used in popular reasoning. We conclude here is a certain cause for causality. To sum up. Analogy has two forms—*scientific* and *empirical.* The scientific analogy is divided into *causal* and *analogy of coexistence.* The causal has two forms, *positive* and *speculative*, and is employed in dynamical sciences and philosophy.

VI. *Difference*—In all reasoning we start with certain data and there is a limit where they would be invalid. Dif-

ference is the principle of distinction and limit. Has its principal use as a check to analogy—to limit dangerous tendencies of analogy. How does it enable us to limit analogy?

1. Causal analogy in the positive form—used in physical sciences, e. g., Undulatory theory of light, &c.

2. In the use of speculative analogy where the conclusion cannot be tested by observation, the limit is to be found in the character of phenomenon from which the inference is drawn, e. g., We conclude that our fellowmen have reasoning similar to our own.

In Use of Anal. of Coexistence. The resemblance must exceed the difference before we can conclude, e. g., Of a, b, c, d, and e, which exist in a species, if only one is known to coexist in a species, we cannot conclude. This is modified by a second limitation. We must distinguish between essential and non-essential attributes. Limit, therefore, to be reliable, must be a resemblance of essential attributes. Outside of these limits, analogy of coexistence is unreliable.

VII. *Resumé M. of Knowledge.*—To sum up. The object of scientific investigation is the discovery of laws. In our search we employ media to guide us. These are, 1st. Time Relations—coexistence to discover laws of causation. 2nd. Similarity extends these laws. 3rd. Difference—principle of limit, &c.—we fix limits in which analogical reasoning is valid.

Lecture VI.

OUTLINE.

I. Methods of Reasoning, { 1. Inductive. 2. Deductive.

II. Distinction between Form and Methods of Reasoning.
(a) only one form of Reasoning, but several methods.

III. Relation of Induction to Deduction — science begins with Induction, but gradually becomes deductive.

IV. Analysis of Inductive Method.
(a) Basis of Induction.
(b) Relation of Induction to Facts.
(c) Mental attitude of the Investigation.
(d) Hypothesis in Induction.

Lecture Proper.

I. *The Method of Reasoning.*—There are three main questions in Material Logic—

1. Nature of Reasoning and relation to Reality.
2. Media of Reasoning.
3. Method of Reasoning.

There are three species of Reasoning—

1. From one instance to another. 2. From masses of particular to general laws. 3. From generals to particulars. All logical method falls under induction and deduction, and reasoning process is the same.

We may group the first two of above species = Induction.
" " " " last " " " " = Deduction.

II. In order to proceed it is necessary to consider a preliminary question, which has caused confusion. There must be a distinction between *form* and *method*. In reasoning from a particular instance to a general conclusion, *e. g.*, a countryman eating a poisonous herb, and becoming sick, etc. In the second species of Deductive Reasoning, the form of reasoning is the same. In dealing with masses of facts, we begin reasoning after they have been collected. We do not get this by piecing and patching it together, but an idea passes through the mind. It is universal, not particular.

In deducing conclusions from this hypothesis, we apply tests of the facts to see whether hypothesis is valid. In Deduction the form is the syllogism. Gibbons styles it an inverse method of Induction. In the third species it is valid. So we may lay this down as a proposition. When we speak of method we refer not to different kinds of reasoning, but to different ways of using the same kind of reasoning.

There is no reasoning, if we do not start with data and derive conclusions.

(a) There are two methods of Reasoning, Deductive and Inductive.

Difference between them. In Induction the general proposition is a hypothesis, not an accepted truth,—a hypothesis

assumed to be true. The hypothesis and conclusion are then brought to a test and found valid, or not. In deduction the relation of dependence is just the reverse. The datum is an accepted truth and the conclusion is based on this truth, *e. g.*, Thus if all men are mortal, it follows that Socrates is mortal.

III. *Relation of Induction to Deduction.*—Scientific reasoning begins inductively and gradually becomes deductive, until we have a well attested truth and principle, *e. g.*, In Physics, after preliminary laws of heat, etc., have been established, the investigation goes on and derives other laws. When one thing is a function of another, the laws of one may be deduced from the other. Both methods have a function to perform.

IV. *Analysis of Inductive Method.*—This involves: (a) Basis of Induction; (b) Relation of Induction to Facts; (c) Mental Attitude of the Investigation; (d) Hypothesis in Induction.

(a) *Basis of Induction.*—A profound and interesting discussion regarding true basis. All reasoning which gives real knowledge, is from the known to the unknown. Why does the mind generalize? Mill takes the ground that *basis is a belief in the uniformity of nature.* But, unfortunately, this uniformity is not fixed. Nature is not fixed—full of variety. Our faith in nature is the abiding basis of all our inductive inference. We conclude that it is not reducible to a belief in universal causation. When we refer uniformity of nature back to a uniformity of causality, it rationalizes our opinion.

(b) *Relation of Induction to Facts.*—How do we proceed to discover these laws? The first modern theory was that of Lord Bacon. The main feature of Bacon's conception was the stress which he put on an exhausted collection of facts. Must then classify them and derive laws from observation on them. More recent logicians evince a truer appreciation of facts to inductive reasoning. It is not essential to induction that we have a large mass of facts, but these facts should be true. It is necessary to collect facts enough to convince us that a real law is involved, before we can reason.

(c) *Mental Attribute of the Investigation.*—Bacon devoted all his powers to produce a revolution in mode of investigation. The backwardness of science of the middle ages is due to this cause. "We must approach nature as a little babe, with a child-like spirit," so taught Bacon. Therefore, a close analogy between science and religion,—both require a new heart.

(d) *Hypothesis in Induction.*—What use may we make of hypothesis? A hypothesis is a guess. It is essential as a guide. When we come upon something to be explained, we are in total darkness at first, feeling about for some clue, and it is possible to remain in this baffled state for a long time. Suddenly a light goes up, and the truth is before us. The mind never knows how it comes about by its divine revelation. A hypothesis must be a natural spontaneous suggestion. The human mind in science and religion demands something fresh from nature. Bacon's form is in this respect defective. He lays down a mass of restraint to which the human mind cannot submit.

Lecture VII.

OUTLINE.

I. Tests of Hypotheses, { 1. Before Verification.
2. After "

II. Instruments of Induction, { Observation.
Experiment.

III. Kinds of Scientific Proof.

IV. Methods of Verification.

1. Agreement. Canon and Difficulties.

Method of Observation.

2. Difference. Canon—Method of Experiment.
3. Double Method of Agreement and Difference—Canon.
4. Method of Residues—Canon.
5. Method of Concomitant Variations—Canon.

Lecture Proper.

I. *Tests of Hypotheses.*—A hypothesis is a provisional explanation, and its use a guide to investigation. What are the tests of a valid hypothesis? That discovery is unsatisfactory, which makes no distinction before and after verification. The same rules cannot be applied before and after investigation. (1) The tests before an investigation are two:

1. Must be a reasonable explanation of fact—a certain relation of fitness of the matter to be explained and the conclusion must not be absurd.

2. Must conform to the law of parsimony. This cuts off prodigality. It must be commensurate and adequate by law of parsimony. Must not be more than adequate. The law of parsimony regards two things: 1. That the explanation be no more than adequate. 2. That no unknown causes must be introduced. These are the two main tests of hypothesis.

(2) After investigation. A hypothesis that satisfies the tests may be given as a provisional explanation. It is then submitted to inductive reasoning. To what requisites must it conform?

Dr. Hewell's principal tests of hypothesis are: 1. It must account for all existing or known facts; 2. it must enable us to predict facts not yet observed. These are both useful and perhaps the best. But as Fowler says; "It is possible for a hypothesis to satisfy both these, and yet be untrue," *e. g.*, Corpuscular Theory of Light.

II. *Instruments of Induction.*—In every induction there is something to be explained,—a hypothesis from which conclusions are deduced. We decide so much from *a priori* reasoning. The instruments are *Observation* and *Experiment.* Observation is a critical inspection of phenomena. It involves no control over condition of facts. Thus in the law of tides and planetary motions, observation is the only practical instrument. Experiment is more efficient, becauses it involves command over conditions. As a result, those sciences, in which experiment can be used, are most correct. (Ex.—Astronomy.)

III. *Kinds of Scientific Proof.*—There are two kinds of scientific proof possible,—Direct and Indirect, or Positive and Speculative proof.

The positive consists in establishing a connection between a class of phenomena and certain known causes. The indirect has a wide application in science. The ultimate laws of science rest on indirect proof.

IV. *Method of Verification.* — Various attempts were made by Bacon to devise specific methods of Observation, but it was reserved for Mill to complete them. A large and interesting part of Mill's work is devoted to unfolding laws and canons. The methods he generalizes, he divides into five.

1. Agreement.
2. Difference.
3. Double Method of Agreement and Difference.
4. Method of Residues.
5. Method of Concomitant Variations.

1. The statement of this as by Mill is as follows. If two or more instances of the phenomenon under question have only one circumstance in common, the circumstance, in which alone all instances agree, may be regarded as a cause or effect of the given phenomenon, or as a connection with it through some fact of causation. This, as will appear, is the method of deducing a scientific law from a relation of time.

It is a method of pure observation, employed where experiment is impossible. It does not involve elimination of conditions, but takes them as it finds them.

Difficulty tending to vitiate method of agreement.

1. A plurality of causes. It is possible that many causes produce the same effect, *e. g.*, A gun shot wound and poison, both produce death.

2. A complexity of causes.

3. Complexity of circumstances.

4. Accidental Relation.

2. Difference. Canon.—If an instance, in which phenomenon under investigation occurs and an instance in

which it does not occur, have every circumstance in common, save one, that one occurring only in the former, the circumstance in which alone the two instances differ is the effect or cause or a necessary part of the cause of the phenomenon. This is experimental rather than observational. Presupposes possibility of one in the absence of the other. If a piece of paper is thrown in the stove, we have no hesitation in declaring heat is the cause of its burning. A bullet fired from a gun or a dose of prussic acid administered to an animal, there is no hesitation in ascribing death of the animal to these two causes. The method of difference consists in observing the effect produced by an added circumstance. It has a large application in observation, but its true sphere is experiment. It is a method more applied to industrial elements of a country than that of agreement.

3. Double method of Agreement and Difference. Canon. If two or more instances, in which the phenomenon occurs, have only one other circumstance in common, while two or more instances, from which the phenomenon is absent, have nothing in common, save the absence of that circumstance, the circumstance in which alone the two sets of instances differ, is the effect or cause, or a necessary part of the cause of the phenomenon. This is the combination of the two former (Agreement and Difference) with this difference, that two sets of instances may differ in other respects, provided that difference is constant, except in this, *i. e.*, that in one row of phenomena the antecedent and consequent be present, in the other absent.

Lecture VIII.

OUTLINE.

I. Resumé of Induction.
II. Relation of Similarity and Analogy to the inductive method.
   (a) Relation to five methods.
   (b) Relation to Descriptive Sciences.

III. The Deductive Method.

(a) Requisites of Deduction $\begin{cases} 1. \\ 2. \end{cases}$

IV. The two Methods in Science.

LECTURE PROPER.

I. *Resumé*—The 4th method is the method of residues. The Canon is: Subtract from any phenomenon, such part as is known, by previous induction, to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents, e. g., We have a group of antecedents, A, B, C, D, and a group of consequents, L, M, N, R. Where A, B, C, D is, L, M, N, R is with them. If L, M are effects of A, B, we subtract the known part of either from the two groups. N and R are then the causes of C and D.

5th Method. Is that of concomitant variations. The Canon of this is: Whatever phenomenon varies in any manner, whenever another phenomenon varies in some particular manner, it is a cause or effect, or is connected with it through some fact of causation.

These are Mills' five methods of verification. In our scheme these come in as tests of hypotheses. The mind first devises some hypothetical explanation of facts, which first is a guess and these are the patient tests, applied afterward to test it. We have this painstaking examination, not as previous, to reasoning, but as after tests. This verification of induction also emphasizes the fact that no amount of painstaking observation can serve for human intelligence.

Induction as a method of reasoning begins with particulars and reaches a general law. Begins with facts and phenomena to be explained. The mind's first answer is a provisional hypothesis to be tested. To verify the investigation it first deduces some explanation. Then proceeds by observation and experiment and if it bears the tests then it ceases to be a hypothesis. The tests are the above five methods. By an application of these it may be determined whether the hypothesis is a law or an unfounded truth.

II. *Relation of Similarity and Analogy to Induction*—The full nature of induction can be applied only when its similarity to analogy has been taken into account. It has two phases: (a) Relation to the five methods. (b) Relation to Descriptive Sciences.

(a) The five methods, when properly used, lead to discovery of causal laws. Similarity is used to generalize this causal connection and extend it to adjacent instances. The results of the five methods may be extended indefinitely by analogy.

III. *Deductive Method.*—In Deduction we start with some law, self evident, or verified by previous induction, and draw particular truths. Let it be known that action and reaction are equal and in opposite directions and we gain an important law governing bodies. There are several requisites: 1st. The law from which we proceed must be an established truth. 2nd. Must be certain that the case in question falls under the law from which we reason. Both are equally important.

IV. *The Two Methods in Science.*—Bacon denounced deduction as useless and preached induction. A revolution was necessary He confounded deduction with the bad use of it then in vogue. Since Bacon there has been a reaction. Deduction has now its proper place. Deduction is a method of practical application. Theoretical part obtained by Induction.

Lecture IX.

OUTLINE.

I. Method in the various sciences—
   - (a) Mathematics—Deductive.
   - (b) Nat. Sciences—Inducto-deductive.
   - (c) Science of Man, { 1. Individual — Inducto-deductive. 2. Social—Deducto-inductive. }

II. Application of Deduction to Demonstration or Argument.

III. The Argument.
- 1. Definition.
- 2. Form.
  - 1. Premises.
    - Major.
    - Minor.
  - 2. Prop. to be established

IV. The Proposition to be proved—its requisites.

V. The Major Premise—its laws.
- 1.
- 2.
- 3.

VI. The Minor Premise.

VII. The Proof—a prosyllogism to the Minor Premise.

Lecture Proper.

I. *Method, etc.*—All sciences are inductive in the beginning, but become deductive. Complete scientific knowledge is inducto-deduction. The sciences may be divided into,—1. Mathematics; 2. Natural Sciences; 3. Sciences of Man.

(a) Mathematics is the purest type of Deduction. It begins with axioms and deduces truths. Certain axioms, as two parallel lines never meet, etc., are universal. It would be absurd to apply Induction to Mathematics.

(b) Natural Sciences—primarily inductive, but largely deductive. We approach nature as strangers, must spell out her laws, must approach from the outside and work inward. Method induction, but deduction is growing. Tendency to pass from qualitative to quantitative. Transition from induction to deduction.

(c) Sciences of Man—divided into two parts, 1. Sciences of Individuals; 2. Sciences of the Social Masses. In *Sciences of Individuals*, as Psychology and Ethics, the method is inductive. These sciences deal with facts of consciousness—determine laws of conscious sphere.

2. In Sciences of Social Masses, we deal with man in society. We may approach from outside and by a species of generalization, but this is unsatisfactory, for it gives us empirical laws. The statistical method of studying society is a sort of quasi-induction. Mr. Mill carries deduction to an extreme. In all social investigation, most prominent feature is deduction. When facts demand, we must reconsider our

conclusions. The complete method in Science is not unvaried. It must be both deductive and inductive.

III. *The Argument, etc.*—1. An argument is a chain of reasoning, formed for the conviction of others. Assumed that the reasoner is not in doubt and must bring others to think as he does. 2. Form—Syllogism. First Premises—Major and Minor and conclusion which is to be proved. Second, the proposition to be established is the conclusion of the syllogism. The proof is not direct, but indirect. It takes the form of a prosyllogism to the minor premise.

IV. *The Proposition to be proved.* Its requisites. 1. It must be concisely stated. 2. Must be a truth. 3. Must cut off extraneous matter.

V. *Major Premise.*—The most important part of a syllogism is the major premise, because the validity depends on this. May be an established truth, or undisputed authority. Its laws are: 1. Must be an accepted truth. 2. Must be relevant—embodying the true ground on which the argument rests. 3. Major must be adequate to support the conclusion, else an illicit process.

VI. *Minor Premise.*—In the position which the discussion establishes—Conviction on which the conclusion depends. The form is to be determined by the mood and figure of a syllogism.

For exp. of Art. 7, the Proof, see discussion of Art. 3—Argument.

Lecture X.

OUTLINE.

- I. The Proof in an Argument
  - 1. Rel. between Minor and Proof.
  - 2. Laws of Proof
    - 1. Main.
    - 2.
    - 3.
    - 4.

II. Development of the Proof
- 1. Consists in unfolding a chain of reasoning or sorites.
- 2. Kinds.
  - 1.
  - 2.
  - 3.
    - a.
    - b.

III. *A priori* and *a posteriori proof.*

IV. Limits of various kinds of Argument
- 1. Universally valid.
- 2. Valid within limits.
- 3. Fallacious.

V. The *Onus Probandi.*

LECTURE PROPER.

1. *The Proof.*—The proof is a prosyllogism to the minor premise. 1. The relation of the minor premise to the proof is that of reason and consequent. 2. The Proof has the following laws: 1. the law of sufficiency; 2. the law of explicitness; 3. the law of symmetry; 4. the law of strength or proportion. The law of "sufficiency" is the main law of proof. No proof can be established without it. It must be both formally and materially valid. It is formally valid when it conforms to all laws of the syllogism. To be materially valid, all the propositions brought forward must be directly established, or they must rest on one sufficiently well established already. When a proof is materially sound, it conforms to the law of sufficiency. 2. Law of explicitness, This law requires that the proof be stated in as many propositions as there are distinct steps in the demonstration. 3. Law of symmetry. A proof is symmetrical, when its parts bear the right relation to all other parts. 4. Law of Logical Proportions. This law requires that the elements of proof should be distinguished in accordance with their logical value.

II. *Development of Proof.*—1. It consist of a number of parts, which, according to law of "symmetry," must be arranged in accordance with the law of sequence. 2. Kinds. 1. When the sorites is of the first kind, the first expressed proposition is a minor, and the rest (except conclusion) major of succeeding syllogism. 2. When sorites is of sec-

ond form, two expressed propositions, one being a major, the rest minor. 3. When sorites is of third form, states conclusion or proposition to be established, omits premises altogether.

Under this, third form, we have, (a) 1. Supply premises and prove the first part; 2. use each proposition as a minor to the next and supply the major. (b) When each conclusion becomes the major of the succeeding proposition. Laws of this are same as the other, only change minor in first to major. These three proofs are called Apodictic Proofs.

A fourth kind of Proof is when the Propositions are logically independent of each other.

III. *A Priori and A Posteriori Proof.*—The "*a priori*" —from cause to effect; *a postoriori*—from effect to cause. Also, *a priori*, from necessary and self-evident truths—every event has a cause. *A posteriori*, from facts and phenomena back to their laws and conditions. In Logic, the *a priori* is equivalent to Formal; in Logic, the *a posteriori* is equivalent to Material. The first three kinds of proof are *a priori*, the last *a posteriori*.

IV. *Limits of Aguments, etc.*—1. Universally valid—

"*Argumentum ad rem*" { Ostensive, Apagogical.

The Ostensive is the direct proof; the "apagogical" the indirect proof. 2. Valid within limits—These are such as *Argumentum ad hominem*, etc. 3. Fallacious—All false methods of argument, *e. e.*, "*Argumentum ad ignorantiam*"; "*Petitio principii*," etc.

V. *Onus Probandi.*—The major premise must be an accepted proposition in any proof. The burden of proof rests with him who disputes the major premise. (See further, Text-book.)

FINIS.