Faculty Working Papers

NOTES ON OPTIMALITY AND FEASIBILITY OF INFORMATIONALLY DECENTRALIZED ALLOCATION MECHANISMS

Andrew Postlewaite, Associate Professor,
Department of Economics
David Schmeidler, Tel-Aviv University
and the University of Illinois

#583

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

May, 1979

NOTES ON OPTIMALITY AND FEASIBILITY OF INFORMATIONALLY DECENTRALIZED ALLOCATION MECHANISMS

Andrew Postlewaite, Associate Professor,
Department of Economics
David Schmeidler, Tel-Aviv University
and the University of Illinois

#583

Summary:

There has been much recent work investigating the possibility of implementing social choice functions in a game theoretic sense. This paper reviews and relates some of the recent work in social choice and neoclassical economic models.

Acknowledgment:

This work was supported by Grant #SOC 77-27403 from the National Science Foundation.

NOTES ON OPTIMALITY AND FEASIBILITY OF INFORMATIONALLY DECENTRALIZED ALLOCATION MECHANISMS

Andrew Postlewaite
University of Illinois

Mailing Address:

Box 34, Commerce West
Department of Economics
University of Illinois
Urbana, Illinois 61801

David Schmeidler
Tel-Aviv University and
University of Illinois

Mailing Address:

P. O. Box 39286
Ramat Aviv, Tel-Aviv
Israel

This work was supported by Grant #SOC 77-27403 from the National Science Foundation.

I. INTRODUCTION TO ALLOCATION MECHANISMS

In 1973 Gerard Debreu reviewed and summarized four major developments in general equilibrium theory [D.2]. The developments to which Debreu referred were the relation of the core and competitive equilibria, existence of equilibria in more general settings (i.e., measure theoretic models), computation of competitive equilibria, and topological properties of competitive equilibria derived by differential topological methods. Looking now, we would add another major line of research which differs from the above in that it alters the basic parameters of the model. Previously the conceptual framework consisted of a description of an economy in terms of agents characteristics (preferences and endowments and in the case of production, the technological possibilities).

The analysis focused on the correspondence between the economies so described and their competitive allocations. However it was never specified how these allocations arose. There was no specification of how the trades were made. Using the language of Hurwicz [H.2], the analysis was of the competitive "performance correspondence" without specifying what the mechanism is which implements this performance correspondence. The first efforts in this attempt to specify how outcomes arose was to cast the problem in a non-cooperative game theoretic framework. A mechanism was to be a precise specification of the set of strategies (or signals) which were available to an agent and the outcome (or allocation) which would arise from any simultaneous choice of strategies by the agents. We will call the function relating outcomes to joint strategies a strategic outcome function.

As an example, Hurwicz considered the set of classical pure exchange economies and let an agent's strategy be the announcement of a (classical) utility function. The outcome would be (assuming uniqueness) the competitive allocation. A problem arises, though, in that for many economies some agents would find that if they announced a utility function other than their true utility function, the outcome would be preferable to the one which would have arisen had they revealed correctly. In game theoretic terms, correct revelation is not a dominant strategy equilibrium; in Hurwicz's terms, this mechanism is not incentive compatible. But what of other mechanisms? Perhaps there is some mechanism which has utility functions as strategies for the agents and is incentive compatible. It is trivial to design such a mechanism--simply let the outcome be no trade regardless of the utility functions announced. But here the correspondence between the economies and the outcomes thus realized (i.e., the performance function) is undesirable from an economic point of view in that the outcomes will not in general be efficient.

The question of interest then is whether or not, when economically interesting restrictions are placed on a performance correspondence, there exists an incentive compatible mechanism which will realize (or implement) such a performance correspondence. Hurwicz [H.2] showed that no Pareto efficient and individually rational performance function can be implemented with an incentive compatible mechanism.

This might seem to leave an economic planner in a quandary. If he designs a strategic outcome function which picks Pareto efficient and individually rational outcomes when people reveal their preferences correctly, then in some economies some agents will find it in their own best interest to misrepresent their preferences. The outlook is not so dismal for the planner however. He can ask what the outcome would be even if agents misrepresented their preferences. The initial concern was that if agents misrepresented their preferences, this would destroy any efficiency properties that the planner might have designed, but this may not necessarily be so. It is at least conceivable that the combination of agents' strategic behavior leads to "nice" outcomes. If agents are playing strategically in their announcement of preferences, we are led to consider as the outcome of the game not dominant strategy equilibria, but rather Nash equilibria. At the same time the planner might ask why the strategies of an agent should be announcement of preferences. Why must the form of the strategies be even related to agents' characteristics? Possibly by choosing some quite abstract set of available strategies for agents, the planner might be able to avoid some of the problems Hurwicz pointed to. So we leave the realm of strategies in which an agent necessarily has a correct or "truthful" strategy. Instead the agents are faced with some arbitrary sets of strategies of which they are to choose whichever they wish, and an outcome will be selected depending upon these strategies. This has been done for models with public goods. In Schmeidler [Sc.1] there is an attempt to introduce voting to a model of an Arrow-Debreu economy with public goods. Schmeidler proposed a mechanism whereby people are taxed proportionately according to the value of their private goods and the tax revenue is used to finance public goods according to each taxpayer specifications. This individual "earmarking" is aggregated to determine the quantity of each public good to be produced. The valuation of the private goods is made at the equilibrium prices. An equilibrium price and Nash equilibrium of individual decisions as to how to allocate the taxes is shown to exist. However, the Nash equilibria of this mechanism are not Pareto efficient.

Groves and Ledyard [GL] suggested a different outcome function. They considered an economy with both public and private goods and designed a mechanism, i.e., strategy sets for the agents and a strategic outcome function such that the Nash equilibria are always efficient. One might think that the work of Groves and Ledyard solves the planner's problem of designing a mechanism to implement a desirable performance function. There are several problems however. First, they do not allow full strategic behavior on the part of agents. Rather, they assume that the agents take as given the prices of both public and private goods (but not their taxes).

While this may be appropriate for addressing the free rider problem, as they do, it leaves unanswered the question of what will happen when agents take into account their effect on these prices. More importantly however, there may not exist an equilibrium in their model. There are in fact large families of economies in which equilibria fail to exist.

II. SOCIAL CHOICE MODELS

A parallel development has occurred in the field of social choice. In [A.1] Arrow introduced the modern approach to social choice theory. In our language he asked whether or not there existed performance correspondences which satisfied a priori desirable characteristics such as Pareto efficiency, independence of irrelevant alternatives, and non-dictatorship. Whereas in general equilibrium theory the existence of "desirable" performance correspondences (e.g., competitive equilibria) was a cornerstone of the early work, Arrow obtained an impossibility theorem. Although Arrow was aware of the possibility of strategic behavior on the part of agents, he specifically avoids consideration of it. The very structure of the problem, i.e., considering performance correspondences which are maps from preference profiles into outcomes, preempts the consideration of agents' behavior. In [Fa] Farquharson altered the basic framework of social choice to allow stratetic behavior. He considered a system of sequential majority voting in subsets of the alternatives to determine a final outcome. Every voter now takes into account how his voting interacts with the votes cast by other agents to determine this final outcome. Farquharson suggested various notions of stability which would define an equilibrium. In this way he has presented a model with well-defined strategies sets available to the agents, a strategic outcome function, and an equilibrium concept.

The next decisive step in the line of research was by Gibbard [G] and Satterthwaite [Sa]. Gibbard generalized the structure of Farquharson, removing the restrictions on the specific form of the strategy sets and allowing arbitrary strategic outcome functions. Within this framework he showed that the only outcome functions which have dominant strategy equilibria for all profiles of preferences are dictatorial (under an assumption that the minimum number of outcomes is three). In other words there are no non-dictatorial mechanisms which have dominant strategy equilibria for all profiles of preferences. In the social choice framework this theorem is equivalent to the following: If the strategy sets are taken to be the set of preferences then there exists no non-dictatorial strategic outcome function (with at least three outcomes in the range) for which the truth is a dominant strategy equilibrium for all profiles of preferences. The same result was obtained independently by Satterthwaite. This result is analogous to the result of Hurwicz on economic mechanisms. Hurwicz has an assumption of individual rationality (non-coerciveness) which essentially played the role of non-dictatorship in the Gibbard-Satterthwaite results.

One of the most important conclusions for us arising from the work of Gibbard and Satterthwaite is that within the framework of strategic outcome functions, the insistence on dominant strategy equilibria rules out all reasonable performance functions. Later work by others (e.g., Pattanaik [Pa] and Sengupta [Se] points out that one can weaken somewhat the concept of dominant strategy equilibria and one still obtains impossibility results.

These impossibility theorems led Hurwicz and Schmeidler [HSc] to the Nash solution concept for the strategic outcome function. Precisely, they asked that for any profile of preferences, there exist a Nash equilibrium and that all Nash equilibria be Pareto efficient. If one is considering dominant strategy equilibria, Pareto efficiency relative to the codomaine of the outcome function is a straightforward consequence. Of course, when changing to the solution concept of Nash equilibria one no longer has Pareto efficiency automatically, hence this restriction was added as a desirable criterion for the performance function. Non-dictatorship, or the stronger property, symmetry across people, would also be a desirable characteristic. In this framework existence of a class of mechanisms, i.e., description of strategy sets and stratetic outcome functions, which had these desired characteristics was proved when there are at least three agents. Maskin [Ma] independently attained some of these results and extended others.

III. NASH EQUILIBRIA AND ALLOCATION MECHANISMS

In attempting to apply these results to the other problem of designing economic mechanisms, one confronts several problems. The framework in social choice in general considers only a finite number of outcomes, whereas the economic problem we are trying to analyze generally has an infinite number of outcomes, e.g., allocations. While this problem is not particularly difficult to surmount, there is a more basic difficulty. In the economic model we have the agents' endowments as a parameter of the economy in addition to the preferences. One would like to have the performance function be individually rational (or non-coercive) with respect to these endowments. There is no natural way to embed this concept into the social choice framework without destroying the existence results. Further since the endowments are a parameter of the economic model, the set of feasible outcomes in this economic model are not independent of the agents' characteristics. In the social choice model the set of feasible alternatives is the same regardless of the agent's characteristics. Nevertheless these results provide us with valuable insights into the design of economic mechanisms with desirable features.

Within this framework there are a number of economic models in addition to those mentioned before which can be analyzed using this concept of economic mechanisms. Unless otherwise mentioned, these models deal only with pure exchange private goods economies. Shubik [Sh] introduced a market clearing rule which Shapley and Shubik [SS] used in a general equilibrium model in strategic outcome function form. In this model one commodity is used as money. The strategies of agents consist of bids of the commodity money to purchase other goods, and offers to sell quantities of these other goods. Using the market clearing rule Shubik proposed together with a bankruptcy rule, a Nash equilibrium is shown to exist. Regardless of the agent's strategies the outcome is feasible. This is accomplished by making the strategies available to an agent depend on his initial endowment. The Nash outcomes in this model are individually rational. This is guaranteed since an agent has as a strategy the possibility of not participating in the market. While the Nash outcomes are in general not Pareto efficient, in a modified version of this model [PoS] it was shown by Postlewaite and Schmeidler that the Nash outcome is asymptotically efficient. That is, if there are sufficiently many agents, none of them "too large", then the percentage of resources of the economy which is wasted due to the inefficiency is small.

Wilson [W] constructed an example of a mechanism in which the strategy sets are feasible net trades (i.e., the strategy sets depend on initial endowments) for all but one agent. This central agent has only one strategy which is determined by his true preferences. The non-central agents propose feasible net trades and the central agent accepts the utility maximal (with respect to his preferences) subset of these trades. The strategic outcome function is such that the outcome is feasible for all choices of strategies and the Nash outcome is in the core, a fortiori it is efficient and individually rational. All the examples considered so far, considered only strategy sets which did not depend on agents' preferences. To the extent that the available strategies differed for different agents, the dependence was on endowments alone. If the preferences of agents are known or observable by others, we need only a computational scheme to achieve a particular performance function. We want to deal with mechanisms which preserve informational decentralization with respect to preferences. That is we do not allow any dependence of strategy sets or strategic outcome correspondences on preferences, which we take to be unobservable. If the central agent in Wilson's example is allowed to pick as a strategy any strategy appropriate for some preference relation, the efficiency of the Nash allocations disappears.

A mechanism discovered by Schmeidler [Sc.2] has the strategy sets the same for all agents and therefore independent of their characteristics, both preferences and endowments. A strategy for an agent consists of a pair, a price and compatible net trade, that is a net trade with value 0 at this price. The agents who have announced the same price trade, and to the extent that their aggregate net trade is not 0, they are rationed proportionately. Given the strategies chosen by the other agents, an agent has a strategy which will give him as an outcome his Walrasian demand for any price announced by other agents. The Nash outcomes of this game (when there are at least three agents) are precisely the Walrasian allocations. This mechanism thus implements the competitive performance correspondence when Nash equilibrium is the solution concept. The price paid for this achievement is nonfeasibility for some (non-equilibrium) strategy choices. That is, for some non-equilibrium strategy profiles, the net trades some agents are to carry out are not feasible given their initial endowments.

Hurwicz [H.-] also constructed a mechanism with this property that the Nash outcomes are precisely the Walrasian outcomes and strategy sets identical for all agents. The strategic outcome function is different from that used by Schmeidler however. Here an agent trades at the average price announced by others, and the price he announces affects other agents only. More importantly he also constructed a similar mechanism with these properties for economies with public goods. Here the Nash outcomes coincide with Lindahl equilibria. Both examples however have the same characteristic as Schmeidler's: non-equilibrium strategies may lead to non-feasible outcomes.

It is worthwhile to add a comment at this point. The non-feasibility of these mechanisms is not an oddity which can be rectified by a simple ad hoc change in the mechanism, such as making a rule which states that no trade will take place in the event of non-feasibility. This type of change fundamentally alters this mechanism and destroys either the existence or optimality of its Nash equilibria.

More generally, Hurwicz, Maskin and Postlewaite [HMP] show that the Walrasian performance function cannot be implemented by a feasible outcome function. A proof of this result can be found in Appendix C.

In a general vein Hurwicz [H.5] has explored the relationship between individually rational and Pareto efficient performance correspondences implemented by Nash equilibria of strategic outcome functions and the Walrasian (competitive) performance function (Lindahl performance correspondence in the case of public goods). He showed that if the Nash equilibrium correspondence is upper semi-continuous, then for any economy the Walrasian allocations must be in the performance correspondence, i.e., Walrasian allocations must be Nash allocations. Hurwicz also shows that with somewhat more restrictive assumptions, all Nash allocations must be Walrasian. To get this result Hurwicz assumes that for any strategies chosen by the other agents, the set of outcomes available to the particular agent is convex (assuming free disposal). The upper semi-continuity required for the first half of the theorem can be justified on the grounds that small changes in the parameters of an economy should cause an equilibrium to change slightly. The convexity assumption used for the other direction seem less compelling. Thus while in general for mechanisms which have individually rational, Pareto efficient Nash equilibria we expect that Walrasian allocations will be Nash, Nash allocations may not be Walrasian if the convexity assumption fails.

IV. FEASIBILITY OF ALLOCATION MECHANISMS

We are interested in the design of an economic mechanism which yields feasible allocations for any joint set of strategies chosen. This is of particular importance since as was mentioned in the introduction, it is impossible to design a strategic outcome function for which dominant strategy equilibria will be Pareto efficient and individually rational. Thus our attention will be focused on Nash equilibria instead. But here an agent's optimal strategy will depend upon the strategies chosen by other agents. Non-equilibrium strategy choices would seem more likely, then, when Nash equilibrium is the solution concept.

It is quite clear that if feasibility is to be obtained, then either agents' available strategies or the outcome function [illegible] will have to depend on agents' endowments. [illegible] endowments, then for some set of agents' strategy choices which result in a non-zero net trade for some agent, we could replace that agent with another whose initial endowment was so small as to make the net trade infeasible. Thus the only hope we have is to introduce dependence of some sort on initial endowments. We will focus on dependence of the strategy sets on initial endowments maintaining an assumption that the stratetic outcome function depends on the initial endowments only through the strategy set dependence. This seems most consistent with the notion of decentralization.

We will still ask that the strategy sets do <u>not</u> depend on preferences which we take to be unobservable. In the spirit of decentralization, we will also ask that the strategy set of an agent depend only on his <u>own</u> endowment, not the endowment of others. Thus we have a mapping from endowments into an abstract space which associates with any initial endowment the permissible strategies for any agent with that endowment in any economy.

This dependence of strategy sets on this part of the agent's characteristics is reasonable in that the endowments are at least potentially observable. An audit of an agent can objectively determine whether or not he has a stated endowment as opposed to the general impossibility of determining his preferences.

One question which remains however is to what extent an agent's endowment can be precisely known. A demand to exhibit endowment would prevent an agent from overstating endowment. But whether or not an agent can successfully <u>hide</u> or withhold endowment is a more difficult problem. If agents have complete property rights over their own endowment, a further possibility available to agents is the destruction or elimination of some part of this endowment.

If only a ceiling can be put on an agent's endowment, another avenue of strategic behavior is opened for an agent. By "announcing" an endowment less than his actual endowment, he changes the set of strategies available to him. This could ultimately leave him in a better position than if he reported his endowment correctly.

In [Po], Postlewaite considered mechanisms in which the strategy sets were announcements of endowments; preferences were assumed to be known. It was shown that for any mechanism which yields Pareto efficient, individually rational outcomes, correct announcement cannot be a dominant strategy equilibrium. This is analogous to Hurwicz's result with preferences as the strategies.

As we have seen above, we must give up some of the informational decentralization if we are to design a feasible mechanism whose Nash equilibria are Pareto-optimal. Hurwicz, Maskin and Postlewaite [HMP] introduced such a mechanism whose Nash equilibria are individually rational as well. More specifically, the Nash equilibria of their mechanism coincides with the constrained Walrasian equilibria (see Appendix D). This last term means that each consumer maximizes his utility over the budget set constrained to feasible bundles. The information that must be known by the mechanism is vector of endowments.

There is another variant of the mechanism in which the endowments are not known. Here the individuals as part of their strategy state their endowments. It is assumed that it can be verified that they don't claim to have more endowment than they actually have, but may state that they have less. Again in this variant the mechanism is feasible and its Nash equilibria coincide with the constrained Walrasian equilibria. A similar informational constraint leads to the feasibility of the mechanism in [PaS., PoS, SS].

V. INFORMATIONAL REQUIREMENTS FOR COMPUTING NASH EQUILIBRIA

In our introduction we presented the development of the literature on mechanisms as being necessary to describe how allocations were to arise through the interactions of agents' actions or choices of strategies. As we stated, the first efforts in this area utilized the dominant strategy equilibrium concept. After the impossibility results of Hurwicz, attention was shifted to the concept of Nash equilibrium. It was with this notion of solution that the positive results of Groves and Ledyard, Hurwicz and Schmeidler, Schmeidler, Wilson, Maskin, and others were derived.

There is an essential difference between the two solution concepts however. If there is a dominant strategy equilibrium, each agent has a best response independent of the other agents' choices of strategies. Regardless of what strategies they choose, he can uniformly pick this strategy. He does not need any information of what strategies they choose to calculate his optimal behavior. Viewed in this light, it is quite plausible that if there is a dominant strategy equilibrium, the agents in an economy will arrive at it, at least if they know the strategic outcome function.

The use of Nash equilibrium presents the agents with a much more complex problem. Now an agent trying to choose his "best" strategy finds that in general this best strategy will change as the other agents change their strategies. Thus in addition to knowing the strategic outcome function, an agent may also need to know the strategies of all other agents in order to determine his optimal strategy.

This presents something of a problem. In the previous section we stated that one very desirable characteristic of a mechanism is that it be feasible. Regardless of the strategies chosen by agents, we would like there to be a feasible outcome. But now suppose that the agents in an economy choose some non-Nash profile of strategies resulting in an outcome. Will the agents in fact realize that this is a non-equilibrium position? Clearly they will if they know the messages or strategies of all other agents. But in some environments, particularly in large economies, this is an heroic assumption. If an agent is supposed to know the messages of all other agents, he obviously has to be in possession of "arbitrarily large" amounts of information if we consider economies with arbitrarily many agents.

This leads to a question of the amount or size of the information an agent needs to determine his optimal strategy in response to those of other agents. Ideally, an agent might see only the outcome to him from a joint strategy choice and be able to determine whether or not his choice is in fact optimal and if not, what his optimal strategy is. Failing this one would hope that at least an aggregate "summary" of other agents' messages or strategies would suffice. An aggregate summarization would be a function from the strategy spaces of the other agents into a euclidean space of the same dimension as the individual strategy space. Examples of summarizations would be summations of the agents' strategies, averages of them, etc. Then as the number of agents grows, the "size" of the information needed by an agent would not change as we consider larger economies. Is there then a mechanism which yields individually rational and Pareto efficient Nash outcomes along with an aggregate summarization procedure which guarantees agents' will know their optimal strategy given the aggregate summarization? A somewhat weaker criterion would be that there is a finite dimensional space which would contain enough information for agents to be capable of knowing their optimal response, where the dimension of the space is independent of the number of agents in the economy. The mechanism introduced by Hurwicz [H.4] whose Nash equilibria coincide with Walrasian equilibria satisfies the latter weaker condition. The dimension of the space which contains the summarized information is four times the number of commodities.

As another example we consider the form of the model of Shapley and Shubik [SS] introduced in [PaS] and analyzed in [PoS]. Here the strategy sets of the agents are contained in a euclidean space of dimension 2ℓ where ℓ is the number of commodities. Given a joint choice of strategies, an outcome (allocation) arises. From one agent's share of the allocation, it is not generally possible for an agent to determine whether or not his strategy is optimal. Suppose though, that he is given aggregate information in the form of the sum of the other agents strategies. He then knows the combined quantities of the offers to buy and sell all goods by the other agents. This information is sufficient for the agent to calculate his optimal response to the other agents' strategies. We note that the size (dimension) of this information he needs depends only on the number of commodities, not on the number of agents. Also note that the strategic outcome function of Hurwicz, Maskin and Postlewaite [HMP] for the case that the endowments are known (appendix D) also has a summary function. That is if a person knows a vector of dimension four plus twice the number of commodities he is able to compute his best response or arbitrarily good responses in the case that there is no best response (a situation that may happen with this outcome function).

VI. INFORMATIONAL REQUIREMENTS FOR OUTCOME FUNCTIONS

The question of informational efficiency deals with the amount of information which must be transferred within the system. An obvious aspect of this question, not treated in the previous section, is the size or dimension of the strategy (or message) space. This question has been treated by Hurwicz [H.3] and Mount and Reiter [MR]. The question here is what is the minimal "size" of information agents must send so as to be able to effect individually rational and Pareto efficient allocations. Mount and Reiter present a general framework in which questions of informational efficiency can be investigated. More specifically, in their framework each agent sends a message in some space M. When their messages are consistent (in a well specified sense) an outcome arises from the messages. They present a system of messages consisting essentially of prices and net trade allocations which results in the competitive performance function, and show that no other system of messages can use a message space which is of smaller dimension and still accomplish this. That is, prices and proposed trades are essentially the most efficient messages which can effect competitive equilibria. Hurwicz [H.3] with a somewhat less general framework obtained similar results, though the assumptions could not be reduced by considering different Pareto efficient individually rational performance correspondences than the competitive one.

The results of Mount and Reiter and Hurwicz are not directly applicable to the model of strategic outcome functions. Strategic behavior on the part of agents is not considered in their work on information. Rather, they asked what was the smallest message space which would contain enough information to affect "good" allocations *if* agents followed some prescribed procedure for choosing messages. The fact is that these messages are in general *not* dominant strategies if agents consider "manipulating".

If we turn to strategic outcome functions, the use of prices and net trades, as messages (strategies) still suffices to generate Pareto optimal and individually rational Nash equilibria. For instance this is the case in Schmeidler's outcome function in [Sc.2]. We note that in [PoS] the mechanism is feasible asymptotically efficient as the number of agents gets large, and has message spaces of dimension twice the number of commodities. The mechanism in [HMP] is feasible, achieves Pareto optimal and individually rational Nash equilibria and has strategy spaces of dimension twice the number of commodities.

MATHEMATICAL APPENDICES

APPENDIX A: SOCIAL CHOICE

Let T and A be finite non-empty sets and for each $t \in T$, let S_t be a non-empty set also. Ω is the set of transitive, reflexive and total binary relations on A. Denote $\underline{S} = X_{t \in T} S_t$. A function $f: \underline{S} \to A$ is called a strategic outcome function (SOF). In the case that $S_t = \Omega$ for all $t \in T$, i.e., $f: \Omega^T \to A$, f is called a social choice function (SCF). We refer to elements of T as persons, A as alternatives, S_t as strategies, Ω as preferences. An element $\underline{P} = (P_t)_{t \in T} \in \Omega^T$ is called a profile of preferences, where an element $\underline{s} = (s_t)_{t \in T} \in \underline{S}$ is called a (strategy) selection. Given an SOF f and $\underline{P} \in \Omega^T$, a selection $\underline{s}^*$ is called a dominant strategy equilibrium (DE) if for all $h \in T$ and all $s \in S_h$, $\underline{s} \in \underline{S}$: $f(\underline{s}|h, s^*_h) P_h f(\underline{s})$ where $(\underline{s}|h, s^*_h) = (r_t)_{t \in T}$ with $r_t = s_t$ for all $t \neq h$ and $r_h = s^*_h$. An SOF f is said to be straightforward if it has a DE for every $p \in \Omega^T$. A social choice function is said to be non-manipulable if for every $p \in \Omega^T$, $\underline{p}$ is a DE. An SOF f is said to be dictatorial if there is an $h \in T$ (called the dictator) such that for all $\underline{s}, \underline{s}' \in \underline{S}$, $s_h = s'_h \Rightarrow f(\underline{s}) = f(\underline{s}')$.

<u>Theorem</u> (Gibbard-Sartertuwaite): If the image of a straightforward SOF f contains at least three alternatives, then f is dictatorial.

An SCF f is said to be veto proof if for all $\underline{P} \in \Omega^T$ and for all $x \in A$: $\# \{t \in T \mid \text{for all } y \in A \; xP_t y\} \geq |T|-1 \Rightarrow f(\underline{P}) = x$. An SCF f is said to be monotonic if for all $\underline{P}', \underline{P} \in \Omega^T$ and $x \in A$ if $f(\underline{P}) = x$ and for all $y \in A$ and for all $t \in T$, $xP_t y \Rightarrow xP'_t y$, then $f(\underline{P}') = x$. Given an SOF f and $\underline{P} \in \Omega^T$, a selection $\underline{s}^* \in \underline{S}$ is said to be a Nash equilibrium (NE) if for all $h \in T$ and all $s \in S_h$: $f(\underline{s}^*) P_h f(\underline{s}^*|h, s)$.

Theorem (Maskin): For any veto proof and monotonic SCF f, there exists an SOF f' such that for all P [illegible], $f(P)$ [illegible] $\{f'(s) | s$ is an NE for $P\}$. (In this case we say that f' implements f.)

APPENDIX B: ECONOMIC ENVIRONMENTS

For this appendix we will modify somewhat the definitions of the previous appendix. Here the set of alternatives $A = \{x=(x_t)_{t\in T} \in (R^L)^T | \Sigma_{t\in T} x_t = 0\}$ where R^L is the Euclidean space whose coordinates are indexed by elements of the finite non-empty set L (the commodity set). In this section we refer to elements of T also as traders or agents and elements of R^L as net trades, where x_t is the net trade of agent t. The definition of strategic outcome functions, Nash equilibria, dominant strategy equilibria, and straightforwardness carry over from the previous appendix; however, here we are not interested in all profiles. We are interested primarily in modelling Arrow-Debreu pure exchange economies. When an agent compares two alternatives, he considers only his net trades. Furthermore, this agent's preferences over all net trades should satisfy the standard assumptions of convexity, continuity and monotonicity. More formally let us denote by R^L_+ the non-negative orthant of R^L and by R the real numbers and agree that inequalities in R^L hold coordinatewise. Set θ [illegible] $\{P$ [illegible] $| \exists w \in R^L_+, w>0$ and a continuous, quasi-concave, strictly monotonic function u: $R^L_+ \to R$ such that for all [illegible] $\{$[illegible]$^L_+$ or $u(x+w)>u(y+w)$ when $x+w$ and $y+w \in R^L_+\}\}$. The analogue of a social choice function (correspondence) of the previous appendix is a prescription correspondence $g: \underline{\theta} \to A$ where $\underline{\theta} = \theta^T$. Two prescription correspondences which are commonly used are the Walrasian correspondence (W) and PE-IR correspondence (Pareto efficient and individually rational). [illegible] $W(P)$ [illegible] $(x_t)_{t\in T}$ [illegible] $\in R^L_+$ such that for all $t\in T$,

$x \in R^L$: $px_t = pw_t$ and $px \leq pw_t \Rightarrow x_t P_t x$, where w_t is from the definition of Θ above. Given $\underline{P} \in \Theta$, PEIR($\underline{P}$) = {$\underline{x}$=(x_t)$_{t \in T}$ $\in$ A | for all t$\in$T $x_t P_t y_t$} $\cap$ {$\underline{x}$ = $(x_t)_{t \in T}$ $\in$A | for all $\underline{y}$ $\in$A: [for all t$\in$T $y_t P_t x_t$] $\Rightarrow$ [for all t$\in$T $x_t P_t Y_t$]}. Given an SOF f:S → A the Nash performance correspondence NEf is the correspondence from Θ to A that assigns to each $\underline{P}$ in $\underline{\Theta}$ the set NEf($\underline{P}$) ≡ {f($\underline{s}$)$\in$A | $\underline{s}$ is an NE for $\underline{P}$ (and f)} in A.

Given an SOF f and a $\underline{P}$ in Θ a selection $\underline{s}$* in $\underline{S}$ is said to be a strong equilibrium, SE, (for f and $\underline{P}$) if for any subset C of T and any selection s in $\underline{S}$: ([for all t in C, f ($\underline{r}$) P_t f($\underline{s}$*)] ⇒ [for all t in C, f($\underline{s}$*)P_t f($\underline{r}$)], where $r_t = s_t$ for t in C and $r_t = s^*_t$, for t in T∖C).

<u>Theorem:</u> (Schmeidler) 1. If #T≥2 then there is an SOF f such that WE(·)=NEf(·) on $\underline{\Theta}$. 2. If #T>2 then there is an SOF f such that for $\underline{P}$ in $\underline{\Theta}$ induced by a differentiable and strictly quasi-concave (utility) function: WE($\underline{P}$) = NE$_f$($\underline{P}$) = {s$\in$S | s is a SE for $\underline{P}$ and f}.

<u>Theorem</u> (Hurwicz): Let an SOF f be given such that on $\underline{\Theta}$, NEf(·)⊂PEIR(·) and NEf(·) is upper semicontinuous. Then on Θ, WE(·)⊂NEf(·).

If S_t = S for all t in T and an SOF f:S → A is symmetric across persons in T then it is said to be totally (informationally) decentralized. This is the case for the SOFs of this appendix. However in order to get feasibility one has to give up some of the informational decentralization. Indeed the domain of the SOF of the [illegible] appendix includes information on the initial endowments.

Formally we have a function f:S × (R^L_{++})^T → A and a partition of $\underline{\Theta}$ to sets {$\Theta_{\underline{w}}$ | $\underline{w}$ $\in$ (R^L_{++})^T} so that $\underline{P}$ = (P_t)$_{t \in T}$ $\in$ [illegible] for all t in T, P_t [illegible] is induced by w_t. All previous definitions apply under the convention that wherever $\underline{P \in C_w}$ we use the SOF f restricted to $\underline{S}$ $\underline{X(w)}$.

APPENDIX C: IMPOSSIBILITY OF A WALRASIAN AND FEASIBLE MECHANISM

<u>Proposition:</u> No mechanism which has a strategic outcome function which is independent of preferences and yields feasible outcomes for all strategy profiles can have Nash equilibrium allocations coincident with Walrasian allocations for all economies.

<u>Proof:</u> Suppose there is a mechanism which has a strategic outcome function independent of preferences and for all economies has equivalence of the set of Nash equilibrium allocations and Walrasian allocations. We will construct two economies, each with three agents and two commodities, showing that general feasibility is impossible. In the first economy, e, agents 1 and 2 have the same utility function $U^1(x,y) = U^2(x,y) = y - e^{-x}$ and identical endowments $w^1 = w^2 = (1,3)$. Agent 3 has utility function $U^3(x,y) = xy$ and endowment (2,6). It is straightforward to show that price P = (1,1) is competitive resulting in the Walrasian allocation $x^1 = x^2 = (0,4)$ and $x^3 = (4,4)$. By assumption this must be a Nash allocation, i.e., there exists a message triple (m_1, m_2, m_3) which is a Nash equilibrium which results in the Walrasian allocation above.

The second economy, $\bar{e}$, will be identical to the original in all respects except for the utility function of the third agent. We let $\bar{U}^3(x,y) = x^{1+\varepsilon}y$. The marginal rate of substitution for agent 3 at the point (4,4) is now $1+\varepsilon$, which is different from the price ratio. Hence the Walrasian allocation for the original economy e is <u>not</u> Walrasian in $\bar{e}$. Since we assumed that the set of Nash equilibrium allocations is equivalent to the set of Walrasian allocations (m_1, m_2, m_3) is not a Nash equilibrium in $\bar{e}$. It is clear that if either agent 1 had a message m_1^* such that (m_1^*, m_2, m_3) resulted in an outcome which was strictly preferred to (0,4), this would contradict the fact that (m_1, m_2, m_3) is a Nash equilibrium in e, and similarly for agent 2. Thus if (m_1, m_2, m_3) is not a Nash equilibrium in $\bar{e}$, agent 3 must have a message m_3^* with the outcome resulting from (m_1, m_2, m_3^*) yielding strictly higher utility than (4,4) according to the utility function $\bar{U}^3$. In Figure 1 the indifference curves for both U^3 and $\bar{U}^3$ through the point (4,4) are shown and represented II and $\overline{II}$ respectively. It is straightforward to verify that $\overline{II}$ lies above II to the left of the vertical line through (4,4) and below it to the right.

The outcome to agent 3 from the message triple (m_1, m_2, m_3^*) must lie above $\overline{II}$. If this outcome were on or to the left of the vertical line through (4,4) it would be above II as well as $\overline{II}$. Since this contradicts the fact that (m_1, m_2, m_3) is a Nash equilibrium in e, the outcome to 3 must lie to the right of the vertical line through (4,4). But this means he must receive more than 4 units of good x, while the total quantity is only 4. Thus if (m_1, m_2, m_3) is not a Nash equilibrium in $\bar{e}$, there must be non-feasible outcomes for some strategy profiles.

There are three agents in this example because the examples of Hurwicz and Schmeidler which have equivalence of Walrasian allocations and Nash allocations work for economies in which there are at least three agents. It is straightforward to modify the example so as to provide the same effect with either two agents or more than three. The specific utility functions for agent three are not crucial; the only requirement is that to the [illegible] of the vertical line the new indifference curve should not be below the original indifference curve and that the previously Walrasian allocation not be Walrasian with the new indifference curve. The essential feature is that the Walrasian equilibrium give all of one commodity to this agent. Any preferences for the other agents which permit a Walrasian equilibrium with this feature could have been incorporated instead of those used.

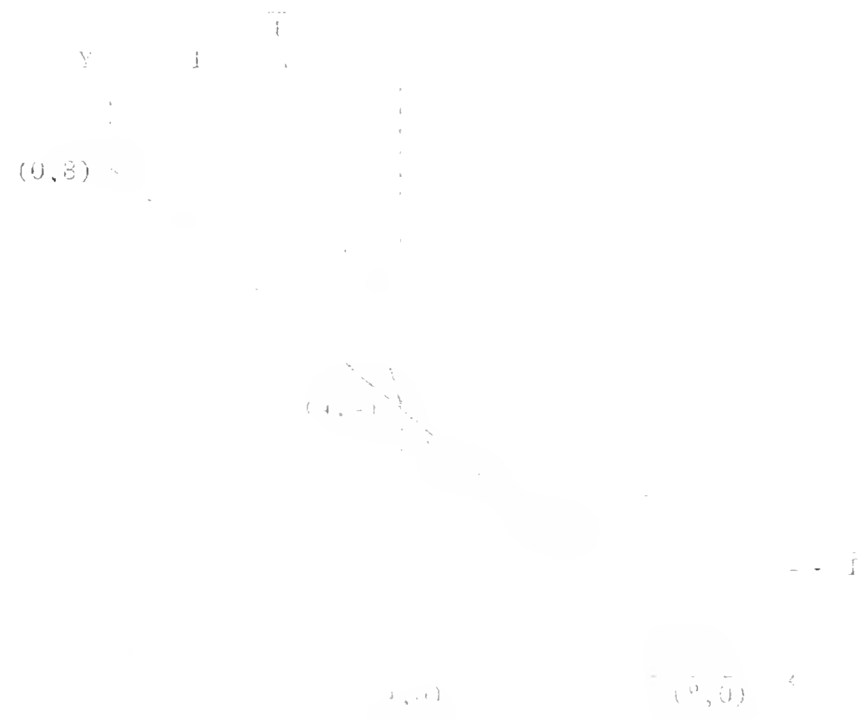

Figure 1

APPENDIX B: [illegible] CONSTRAINED WALRAS EQUILIBRIA

T is the finite set [illegible]

[illegible] set of [illegible]

[illegible] dimensional price simplex.

[illegible] (p, x) [illegible]

[illegible]

1. If $\exists$ i, j, k$\in$T such that p_i, p_j, p_k are distinct

$$h_t = \frac{\|x_t\|}{\sum \|x_t\|} w \quad \forall t$$

where $w = \Sigma w_t$

2. If $\exists$ only two prices p and p' announced and at least two people announce each p and p'

$$h_t = w_t \quad \forall t$$

3. If $\exists$ $\bar{p}$ such that $p_t = \bar{p}$ for all t (3.1) and $\sum_{t \in T} x_t \neq w$ then $h_t = w_t$

(3.2) and $\sum_{t \in T} x_t = w$ then $h_t = x_t$ $\forall t$

4. If there exist $\bar{p}$ and m s.t. $p_m \neq \bar{p}$, $p_t = \bar{p}$ for all $t \neq m$ then

$$(4.1) \quad \begin{aligned} h_m &= (\bar{p} \cdot w_m / \bar{p} \cdot x_m) x_m \\ h_t &= [(1/\#T-1)](w-h_m) \quad t \neq m \quad \text{if} \quad h_m \leq w \end{aligned}$$

$$(4.2) \quad h_t = w_t \quad \text{if} \; \sim(h_m \leq w)$$

<u>Definition</u>: An allocation $(y_t)_{t \in T}$ and a price p are a <u>constrained Walrasian equilibrium</u> if

i) $\forall t$, $p \cdot y_t = p \cdot w_t$

ii) $\forall t$, $y_t \succsim_t y$ $\forall$ y [illegible]

iii) $\sum_{t \in T} \tilde{y}_t = w$

<u>Theorem</u>: (Hurwicz, [illegible]) [illegible] set of Nash equilibrium allocations (N.E.) [illegible] the set of constrained Walrasian equilibrium allocations (C.W.E.).

REFERENCES

[A.1] Arrow, K., Social Choice and Individual Values, Yale University Press, New Haven, Connecticut, 1951.

[D.1] Debreu, G., Theory of Value. New York: John Wiley. 1959, Chapter 7.

[D.2] Debreu, G., "Four Aspects of General Equilibrium Theory," Proceedings of the International Congress of Mathematicians, Vancouver, 197[illegible].

[Fa] Farquharson, R., Theory of Voting, Yale University Press, New Haven, 1969.

[G] Gibbard, A., "Manipulation of Voting Schemes: A General Result," Econometrica 41 (1973) 587-602.

[GL] Groves, T. and J. Ledyard, "Optimal Allocation of Public Goods: A Solution to 'Free Rider' Problem," Econometrica 45 (1977) 783-810.

[H.1] Hurwicz, L., "On the Interaction Between Information and Incentives in Organizations," Communication and Control, (1977), Ed., Klaus Krippendorf, Gordon and Breach, New York.

[H.2] Hurwicz, L., "On Informationally Decentralized Systems," Ch. 14, Decisions and Organization, C. B. McGuire and Roy Radner (Eds), Amsterdam (1972).

[H.3] Hurwicz, L., "On the Dimensional Requirements of Informationally Decentralized Pareto-Satisfactory Processes," (Manuscript), presented at the Conference Seminar on Decentralization, Northwestern University, Evanston, Illinois, February 1972.

[H.4] Hurwicz, L., "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points," mimeo, November 1976.

[H.5] Hurwicz, L., "On Allocations Attainable Through Nash Equilibria," mimeo, April 1977.

[HMP] Hurwicz, L., E. Maskin, A. Postlewaite, "Constrained Walrasian Equilibria," mimeo, 197[illegible].

[HSC] Hurwicz, L. and D. Schmeidler, "Construction of Outcome Functions Guaranteeing Existence and Pareto Optimality of Nash Equilibria," mimeo, 197[illegible].

[HS] Hurwicz, L. and M. Shapiro, "Incentive Structures Maximizing Residual Gain Under Incomplete Information," Discussion Paper No. [illegible], Center for Economic Research, University of Minnesota, Minneapolis.

Ma] Maskin, E., "[illegible] [illegible] Nash Equilibria," mimeo, Presented at Stanford University, [illegible] 1977.

MR] Mount, K. and S. Reiter, "The Informational Size of Message Spaces," Journal of Economic Theory 8 (197[illegible]) [illegible].

PA] Pattanaik, P., "Counter-Threats and Strategic Manipulation under Voting Schemes," Review of Economic Studies 43 (1976) 11-18.

PaS] Pazner, E. and D. Schmeidler, "Existence of Non-Walrasian Nash-Equilibria,"

Po] Postlewaite, A., "Manipulation via Endowments," to appear in Review of Economic Studies.

PoS] Postlewaite, A. and D. Schmeidler, "Approximate Efficiency of Non-Walrasian Nash Equilibria," Econometrica 46 (1978) 127-136.

Sa] Satterthwaite, M., "Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions," Journal of Economic Theory 10 (1975) 187-217.

Sc1] Schmeidler, D., "An [illegible] [illegible] Rational [illegible] for [illegible] the Provision [illegible] Public Goods," Paper presented to NSF-SSRC Conference on "Individual Rationality: Preference Revelation and Computation Cost in Models of Economic Equilibrium," University of Massachusetts, Amherst ([illegible] 1973) mimeo.

Sc2] Schmeidler, D., "A Remark on Microeconomic Models of an Economy and a Game-Theoretic Interpretation of Walras Equilibria," mimeo, Minneapolis, [illegible] 1976 (to appear in Econometrica).

Se] Sengupta, [illegible], "On a [illegible] in the Analysis of Strategic Voting," to appear in Econometrica.

SS] Shapley, L. and M. Shubik, "Trade Using One Commodity as a Means of Payment," Discussion Paper [illegible]-NSF April 1976, Rand Corporation, Santa Monica, Ca.

Sh] Shubik, M., "A Theory of Money and Financial Institutions: Commodity Money, [illegible] [illegible] and [illegible] in a General Equilibrium Model," Western Economic Journal, [illegible] (1973) 24-38.

W] Wilson, R., "A Competitive Model of Exchange," mimeo (1976)