NEW YORK UNIVERSITY
COLLEGE OF ENGINEERING
RESEARCH DIVISION
•
Department of Meteorology and Oceanography

NOTES ON THE WIND DRIVEN OCEAN CIRCULATION

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Given in Loving Memory of

Raymond Braislin Montgomery
Scientist, R/V Atlantis maiden voyage
2 July - 26 August, 1931
****

Woods Hole Oceanographic Institution
Physical Oceanographer
1940-1949
Non-Resident Staff
1950-1960
Visiting Committee
1962-1963
Corporation Member
1970-1980
******

Faculty, New York University
1940-1944
Faculty, Brown University
1949-1954
Faculty, Johns Hopkins University
1954-1961
Professor of Oceanography,
Johns Hopkins University
1961-1975

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NOTES ON THE WIND-DRIVEN OCEAN CIRCULATION

By

Gerhard Neumann

New York University

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Preface and Abstract

The main important difference between the dynamics of the atmosphere and of the oceans is concerned with the driving forces. The atmospheric circulation may essentially be compared with a thermodynamic engine driven by the temperature differences between low and high latitudes on the rotating earth, and this circulation is to some degree modified by the land and water distribution and by oceanographical conditions at the sea surface.

In the oceans, the temperature (and salinity) differences between the equator and the poles play only a minor role as long as they are considered as a cause of the significant water movements in the upper layers of the sea. The ocean currents are generated and maintained chiefly by the driving forces of the wind. The "thermodynamic engine" (including the "halinodynamic") is relatively insignificant in the oceans, at least in the upper layers, where currents with measurable velocities occur. Weak thermo-haline circulations, however, will dominate in the deeper strata of the oceans beneath a layer which will be called the "layer of no motion" for the wind induced ocean circulation in this and the following reports.

This report deals only with the wind driven ocean circulation. The temperature and salinity structure is considered only as far as the density stratification is involved in the final development of the wind driven current systems. This density stratification is--beyond the minor influence of "density currents"--very important,

since in a stratified ocean a "zero layer" (layer of no motion) for the wind driven circulation must develop, which may be considered as the lower boundary of the circulation, where the velocity of the wind driven currents vanishes. Therefore, the "zero layer" can also be replaced by a rigid but frictionless boundary, and the necessary resistance in order to balance the driving forces is brought about only by the virtual internal friction.

The wind driven circulation in an ocean with a variable depth, D, of the currents is analyzed, and it is found that the relative change of the depth D with latitude equals the relative change of the Coriolis parameter in the large scale planetary circulation. This suggests that the ocean reacts to the planetary vorticity effect in such a way that it rather tends to adjust its level of no motion for the wind driven circulation than to displace the whole gyre with relatively high current velocities to the west. Since the depth of the layer of no motion for the wind driven circulation is also strongly related to the vertical density stratification, the mechanism of mutual adjustments between the current systems and the mass distribution, as it is finally observed in the oceans, appears rather complicated.

For a quantitative study of the wind driven ocean circulation knowledge of the wind stress at the sea surface, and of the virtual internal friction is necessary. Both of these important forces have been discussed in detail in this report. With regard to the internal friction a simple expression corresponding to Guldberg-Mohn's expression was used, and coefficients of virtual internal friction were determined. It seems that a more general treatment of the internal friction will not essentially change the results.

The opinions of different authors are most controversial in discussion of the wind stress at the sea surface, especially at wind velocities less than 10 m/sec. It is this critical range of wind speeds which comes into consideration when dealing with climatological wind data, over most parts of the sea surface. At present, more and more evidence has been accumulated which shows that the effective wind stress at low wind speeds is larger than it was hitherto assumed by most of the workers on this subject. Therefore, the stress formula as suggested by the author in 1948 was used to calculate the wind stress for given wind velocities.

Some special cases of the wind driven circulation beneath the layer of frictional influence are analyzed, such as the meridional flow along the east coast of the United States, and the zonal flow of the Antarctic Circum Polar Current. The outline of future work briefly discusses the ideas which may lead to a better approach of wind driven circulation in a real ocean, taking the true average wind field and the boundaries into account. This approach is based on a numerical integration of the differential equation for the horizontal mass transport in an ocean where the wind induced current systems have a variable depth.

NOTES ON THE WIND-DRIVEN OCEAN CIRCULATION

I. Introduction

Ocean currents with the greatest velocity are observed in the upper layer of the sea with depths varying between two hundred and two thousand meters. This circulation is generated and maintained mainly by the driving force of the wind. The question is, to what extent can the observed conditions be explained on the basis of the hydrodynamical theories as they have been developed up to the present time?

The problem may be divided into two parts. Part one is concerned with the "gross" or general features of the oceanic circulation, and part two with the details which recently have been revealed in some individual branches of the general circulation by means of refined techniques for oceanographical observations.

There is no doubt that not only the details but also the gross features of the oceanic circulation are the result of a complicated mechanism in which different forces or factors are involved. It is possible that the mechanism which governs some outstanding "gross" features, such as the "intensification" of the Gulf Stream along the east coast of North America and the "intensification" of the Kuro Shio along the coast of eastern Asia, is essentially the same as the mechanism which may account for some "details" as observed in the western North Atlantic and in other individual branches of the general current systems of the oceans. Such details are, for example, the series of overlapping currents "arranged somewhat like the shingles on a roof" (Iselin, 1952), and certain "counter currents." It seems to be the tendency of ocean currents in stratified water to split up into narrow streams of higher velocity rather than to form a broader and weaker band of flow. This "streakiness" of ocean currents is not only present within the swift Gulf Stream and Kuro Shio, but also in other branches of the circulation along the eastern and western boundaries of the oceans as well as in mid-ocean regions, although the individual "streaks" are much weaker. At present, no satisfactory explanation for such details in the flow pattern of ocean currents has been given.

The general circulations in the Atlantic and Pacific Ocean are asymmetrical with respect to the equator, and this asymmetry is related to the asymmetry of the general atmospheric circulation. As a consequence of this a special equatorial current system is observed, which from the dynamical point of view needs some further consideration. Common to the oceans, including the Indian Ocean, is the fact that the equatorial currents are relatively strong compared with the currents of the subpolar gyre. This fact, as well as the east-west asymmetries in the North Atlantic, in the North Pacific and partly in the Indian Ocean cannot be explained in a simple way by the wind field itself. For example, the Gulf Stream and Brazil Current are almost equivalent branches of the subtropical gyre on the Northern and Southern Hemisphere, yet the Gulf Stream transports about ten times as much as the Brazil Current, and the gross features of the equivalent circulation systems on both hemispheres show striking differences.

The present report is concerned with some notes on the general or "gross" features of the wind driven circulation. Although no final answer to the general problem of the horizontal and vertical current stratification can be given, it is hoped that these notes may serve to stimulate further interest in the subject along lines which will result in a resumption of the problem in the way it was initiated by Walfried Ekman.

II. Historical review

Approximately 50 years have passed since Walfried Ekman placed our conception of the wind driven ocean currents on a completely new basis. In his well known theory he showed how the effect of the wind goes far beyond the generation of pure drift currents, and how the gradient currents in the deeper layers are maintained by the indirect action of the wind. His studies were directed toward the calculation of the total current system for the given wind system over the oceans. He tried to solve the problem for the vertical and the horizontal velocity distribution. The difficulties which rendered Ekman's general analysis very difficult have not yet been overcome. Ekman, however, succeeded in solving essential partial problems, and most of his results of the first paper on the subject (1905) have survived half a century; they still belong to our basic knowledge in dynamical oceanography.

Ekman was the first to introduce eddy viscosity (virtual friction) into the theory of ocean currents. In a paper from 1923 he deduced some important laws which, in the case of steady motion, the currents and particularly their curl must obey. The acceleration of the water, originally regarded as effectively eliminated from the equations, was later found to be important as far as the "deep-current" is concerned. In his revised theory of 1932, the dynamical theory of the wind driven ocean circulation and its problems were developed by Ekman to such a degree of completeness, that relatively few additions have been made since that time. Probably, the only point where Ekman's model of steady state ocean currents has been revised essentially, and where in the future essential revisions have to be expected, is concerned with the mechanism and the effect of internal friction and vertical density stratification.

A new epoch of theoretical research in the field of wind induced ocean currents started with the work by H.U. Sverdrup (1947), H. Stommel (1948), K. Hidaka (1949) and W. Munk (1950). These authors essentially gave up trying to evaluate the vertical velocity distribution and confined the analysis to the horizontal mass transport by introducing the vertically integrated mass transport as the dependent variable. This was necessary in order to avoid certain difficulties which rendered Ekman's analysis too complicated. It was possible to examine the more general case of a baroclinic ocean without having to specify the nature of the vertical distributions of density and currents, although even in this case, some simplifying assumptions had to be made. The problem was to derive the gross features of the horizontal mass transport of the oceanic circulation for a given wind system. These stimulating theories can be considered as a remarkable step in dynamic oceanography, and the results obtained by the various models are of great interest.

H. Stommel succeeded in 1948 in showing that due to the effect of the varying Coriolis force with latitude the gyre of the general circulation around the Sargasso Sea must be strongly displaced toward the west in the case of a constant depth of the wind driven circulation. The results are very strong currents along the east coasts of the continents. This "westward intensification" of ocean currents is obvious in the Gulf Stream and the Kuro Shio, and it seems that a satisfactory explanation of this striking gross feature of the oceanic circulation has been given. However, remarkable exceptions are found in the South Atlantic and South Pacific, and Stommel's model needs further consideration.

Using interesting mathematical methods, Munk and collaborators, Hidaka and others extended the theoretical research to more natural wind distributions and more realistic ocean boundaries.

Munk's investigation (1950) has served to emphasize the fundamental importance of the wind stress curl rather than of the wind stress vector in determining the transport of ocean currents in meridionally bounded oceans. In the case of an irrotational wind field, such as a uniform west wind over an ocean of constant depth bounded on its western and eastern sides, it follows that there is no net mass transport, and that the stress is balanced by the pressure distribution resulting from the piling up of water against the lee shore. This result becomes understandable only when keeping the two-dimensional model and its theoretical basis in mind. The theoretical basis of the model is essentially a differential equation representing a vertically integrated version of the vorticity equation in the case of an ocean of constant depth. It expresses a balance between the lateral stress torque, the planetary vorticity, and the wind curl. It is evident that the vertical velocity component must be taken into account in order to explain the steady state circulation in a bounded ocean. Otherwise, the result that the permanent ocean currents are only related to the rotational component of the wind stress field over the ocean would indicate that the currents must vanish were the wind stress field irrotational.

Most of the important assumptions of the previous models are concerned with the lower boundary of the wind-induced circulation system, or the "depth" of the currents, and with the concept of lateral and vertical eddy viscosity, especially with the concept of isotropic lateral mass exchange.

With the attempt to explain some of the observed detail flow patterns in the Gulf Stream north-northwest of the Azores, in the California current and the Norwegian current by taking non-isotropic lateral mass exchange into account, the author's attention was called to some probable relationships between the vertical density gradient, the depth of the current, the variation of latitude while the current flows in meridional direction, and the "age" of the current, that is, the time when a wind of a certain velocity starts to blow over an ocean originally at rest.

<u>The varying depth of the wind generated oceanic circulation</u> is the first fact with which we are concerned, and which has been neglected in the previous models even in the case of a steady oceanic circulation. This depth depends on the density stratification of the ocean, the "age" of the current, and possibly on internal friction (lateral and vertical). At this point some difficulties arose, so that we are facing the problems of the <u>gross</u>

features of the steady oceanic circulation again.

In the present report it will be shown that the depth of the wind induced oceanic circulation plays a significant role in the hydrodynamic analysis of the problem. A more realistic model of the wind driven circulation of the North Atlantic Ocean will be treated by numerical integration of the basic differential equations in a subsequent report.

III. The differential equation of horizontal mass transport

Rectangular coordinates may be assumed for a first simplified model. Let the y-axis point northward, the x-axis eastward, and the z-axis vertically upwards from the undisturbed sea surface which coincides with a level surface. The boundaries of the ocean model may be considered as vertical walls, which at first approximation would account for the continental slope. Besides these lateral boundaries the lower boundary of the current system has to be fixed.

It is known that in equatorial regions the layer of no motion ("zero layer") for the wind driven circulation is found in about 200-300 m of depth, and it increases with increasing latitude to approximately 1500 m in 45°-50°N. (The depth of this "zero layer" should not be confused with the depth of the layer of frictional influence.) There is much evidence for this fact, and it has been assumed by most oceanographers that there is a continuous change of this "level of no motion" between low and high latitudes. Much support of this point of view has been given by A. Defant (1941) in his extensive analysis of the Atlantic circulation, where interesting details of the layer of no motion have been revealed.

The "zero layer" of the oceanic circulation in the wind affected surface and subsurface layers may be considered as the lower boundary of the current system. It may be replaced by a rigid surface where the velocity vanishes (no friction at this boundary). Therefore, the "zero layer" has to be considered as the depth D of our model of wind driven ocean currents, and D is a function of x and y.

Let $f = 2\omega \sin\varphi$ be the Coriolis parameter, p the pressure, u and v the horizontal components of the current velocity vector $\vec{c}$ in the x and y direction, respectively, $\nu\,[cm^{-1}\ gr\ sec^{-1}]$ the coefficient of eddy viscosity in the lateral, and μ the coefficient of eddy viscosity in the vertical direction, ∇^2 the Laplace operator in the x-y plane and ρ the density. The hydrodynamic equations of steady horizontal motion are

$$\left.\begin{aligned} \rho f v + \mu \frac{\partial^2 u}{\partial z^2} + \nu \nabla^2 u - \frac{\partial p}{\partial x} &= 0 \\ -\rho f u + \mu \frac{\partial^2 v}{\partial z^2} + \nu \nabla^2 v - \frac{\partial p}{\partial y} &= 0, \end{aligned}\right\} \tag{1}$$

the hydrostatic equation

$$\frac{\partial p}{\partial z} = -g\rho, \tag{2}$$

and the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{3}$$

The system of hydrodynamic equations (1) to (3) together with adequate boundary conditions provides the basis for a very general solution, that is, to find the horizontal and the vertical distribution of u and v. Although this solution is of great interest, the mathematical analysis is too complicated. Therefore the question of the horizontal mass transport may be considered only. If the vertically integrated mass transport M, with the components U, V is introduced as the dependent variable,

$$U = \int_{-D}^{0} \rho u \, dz \quad , \qquad V = \int_{-D}^{0} \rho v \, dz \quad .$$

Since the integration is carried out from a depth where the total motion is zero ($u = v = w = 0$), to the surface, ($z = 0$)*, it follows from (3) that

$$\frac{\partial}{\partial x} \int_{-D}^{0} \rho u \, dz + \frac{\partial}{\partial y} \int_{-D}^{0} \rho v \, dz = 0 \ ,$$

and that

$$\text{div } M = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \ . \tag{4}$$

Equation (4) states that the horizontal divergence of the total mass transport equals zero, in contrast to the divergence of the mass transport in individual levels. For example in the layer of frictional influence (wind drift current), the divergence may be zero in exceptional cases only.

The special form of the equation of continuity for the vertically integrated mass transport (4) permits the introduction of a "stream function", ψ , for the mass transport, and

$$\begin{aligned} U &= \partial \psi / \partial y \\ V &= - \partial \psi / \partial x \end{aligned} \tag{5}$$

Integration of equations (1) over the depth between the level of no motion ($z = -D$) and the surface ($z = 0$) gives

$$\left.\begin{aligned} f \frac{\partial \psi}{\partial x} - \mu \int_{-D}^{0} \frac{\partial^2 u}{\partial z^2} dz - \frac{\nu}{\rho} \left(\nabla^2 \frac{\partial \psi}{\partial y}\right) + \frac{\partial \bar{p}}{\partial x} D = 0 \\ f \frac{\partial \psi}{\partial y} - \mu \int_{-D}^{0} \frac{\partial^2 v}{\partial z^2} dz + \frac{\nu}{\rho} \left(\nabla^2 \frac{\partial \psi}{\partial x}\right) + \frac{\partial \bar{p}}{\partial y} D = 0 \end{aligned}\right\} \tag{6}$$

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* Actually $z = \zeta$, where ζ the elevation or depression of the sea surface with reference to the level surface of the undisturbed sea surface. However, since ζ is a very small quantity it can be neglected against D.

where $\partial\bar{p}/\partial x$ and $\partial\bar{p}/\partial y$ represent average values of the horizontal pressure gradients in the whole layer between $z = -D$ and $z = 0$.

The two integrals in equations (6) are equal to the differences of the components of the tangential stress at the surface and at the depth $-D$. The tangential stress at the surface can be evaluated from the wind at "anemometer height" by means of an empirical formula (Neumann, 1948). This question, however, needs some further consideration and will be discussed in more detail in section IX of this report. The tangential stress in the depth $z = -D$ equals zero.

Besides the wind stress at the surface, horizontal and vertical shearing stresses in the water have to be taken into account. The dynamical effect of these internal stresses is evident in the virtual internal friction. Since it is difficult to introduce internal friction in a most general form, and in order to simplify the differential equations for the mathematical analysis, it seems justifiable for the purpose of the present report to assume that the frictional forces are proportional to the total mass transport M. The combined effect of the virtual friction in horizontal and vertical direction will be considered as an "effective" internal friction. For the whole water column between $z = -D$ and $z = 0$ the components of the effective internal frictional forces are assumed to be

$$\left.\begin{aligned} R_x &= - r \frac{\partial \psi}{\partial y} \\ R_y &= + r \frac{\partial \psi}{\partial x} , \end{aligned}\right\} \tag{7}$$

where $r[\sec^{-1}]$ represents an effective "friction"-coefficient. A plausible average value of this coefficient has to be determined later.

With these assumptions equations (6) can be written

$$\left.\begin{aligned} f\frac{\partial\psi}{\partial x} - \tau_x + r\frac{\partial\psi}{\partial y} + \frac{\partial\bar{p}}{\partial x}D = 0 \\ f\frac{\partial\psi}{\partial y} - \tau_y - r\frac{\partial\psi}{\partial x} + \frac{\partial\bar{p}}{\partial y}D = 0 \end{aligned}\right\} \qquad (8)$$

τ_x and τ_y are the components of the wind stress at the sea surface in the direction of the x-axis and y-axis, respectively.

Cross differentiation of (8), and subtraction leads to

$$r\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + \frac{\partial f}{\partial y}\frac{\partial\psi}{\partial x} + \frac{\partial\bar{p}}{\partial x}\frac{\partial D}{\partial y} - \frac{\partial\bar{p}}{\partial y}\frac{\partial D}{\partial x} = \frac{\partial\tau_x}{\partial y} - \frac{\partial\tau_y}{\partial x} \qquad (9)$$

According to equations (8)

$$\frac{\partial\bar{p}}{\partial x}\frac{\partial D}{\partial y} - \frac{\partial\bar{p}}{\partial y}\frac{\partial D}{\partial x} =$$

$$\frac{f}{D}\frac{\partial D}{\partial x}\frac{\partial\psi}{\partial y} - \frac{f}{D}\frac{\partial D}{\partial y}\frac{\partial\psi}{\partial x} + \frac{\tau_x}{D}\frac{\partial D}{\partial y} - \frac{\tau_y}{D}\frac{\partial D}{\partial x} - \frac{r}{D}\left(\frac{\partial D}{\partial y}\frac{\partial\psi}{\partial y} + \frac{\partial D}{\partial x}\frac{\partial\psi}{\partial x}\right) \qquad (10)$$

By substituting this into equation (9), it follows

$$r\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + \left(\frac{\partial f}{\partial y} - \frac{f}{D}\frac{\partial D}{\partial y} - \frac{r}{D}\frac{\partial D}{\partial x}\right)\frac{\partial\psi}{\partial x} + \left(\frac{f}{D}\frac{\partial D}{\partial x} - \frac{r}{D}\frac{\partial D}{\partial y}\right)\frac{\partial\psi}{\partial y}$$

$$= \frac{\partial\tau_x}{\partial y} - \frac{\partial\tau_y}{\partial x} - \frac{\tau_x}{D}\frac{\partial D}{\partial y} + \frac{\tau_x}{D}\frac{\partial D}{\partial x} \quad . \qquad (11)$$

With this differential equation and adequate boundary conditions the horizontal mass transport is uniquely determined in the ocean under consideration. The boundary conditions for the North Atlantic Ocean will be specified later.

In the special case where the depth D is assumed to be constant, it follows from (11)

$$r\nabla^2\psi + \frac{\partial f}{\partial y}\frac{\partial\psi}{\partial x} = -\,\mathrm{curl}_z\,\tau \text{ , for } D = \text{const.}$$

This equation is identical with Munk's differential equation for the horizontal mass transport (1950; equ. 6), if the sign of ψ is changed (Munk defined $V = \partial\psi/\partial x$, $U = -\partial\psi/\partial y$) and the term $r\nabla^2\psi$

is replaced by $A_H \nabla^4 \psi$ in Munk's notation.

Before an evaluation of equation (11) several important factors involved in the model have to be discussed and to be fixed. This is concerned with D as a function of x and y, with the wind stress at the sea surface and with the effective friction coefficient, r, in the water as defined before.

IV. The "zero-layer" in the Atlantic Ocean

A successful attempt to determine the depth of the zero layer (layer of no motion) in the Atlantic Ocean was made by A. Defant (1941). When comparing the differences in the dynamic depth of given pressure surfaces between adjacent oceanographic stations, it was found that in certain levels this difference was almost constant over a rather large depth interval. This would mean that either the whole body of water throughout this depth interval (sometimes with a thickness of 500-800 meters) has a constant velocity in the vertical direction, or that this layer is absolutely at rest.

Defant's analysis of the North and South Atlantic Ocean has shown that it is much more plausible to assume that this layer is absolutely at rest, than that it is in relative motion to some other "reference level" in the body of water. Defant assumed the average depth of this layer to be the "level of no motion" for his dynamical computations. This hypothetical level of no motion was traced throughout the whole North and South Atlantic, and it was found that it composes without any constraint to a closed layer of varying thickness. This fact and others as pointed out by Defant may serve to strengthen our confidence in Defant's analysis of the circulation of the Atlantic. Any other assumption of the level of no motion would lead to unreasonable and contradictory results about the vertical structure of the Atlantic circulation. The same hypothesis was applied to a dynamical analysis of the Black Sea circulation by the author (1942,1943). In this case the results could be checked against other techniques in determining the level of no motion, and the computed dynamical structure of the Black Sea was in good agreement with the observed conditions.

Defant's results for the Atlantic Ocean are shown in figure 1. The depth of the "zero layer" (D) increases with increasing latitude, both, on the Northern and on the Southern Hemisphere. In the North Atlantic, the depth of the zero layer is more irregular than in the South Atlantic, which has its analogy in the more complicated current pattern of the North Atlantic.

Based on the chart shown in figure 1, zonal averages of the depth of no motion (D) were computed for the North Atlantic, and plotted against the latitude in figure 2. For the South Atlantic, individual values were used as given along the meridian of 20°W, since the variation of D in the east-west direction is small compared with the variation of D in the meridional direction. The values of D, according to A. Defant's chart, are indicated in figure 2 by circles, whereas the curve represents the function $D = -K \sin\varphi$. The constant K is different on the Northern and Southern Hemisphere, but the variation of D with latitude follows this law closely. It is seen that the zonal averages of the depth of the layer of no motion in the North Atlantic, and the corresponding depths along 20°W in the South Atlantic, can be represented very closely by

$$D(\varphi) = -K \sin\varphi, \tag{12}$$

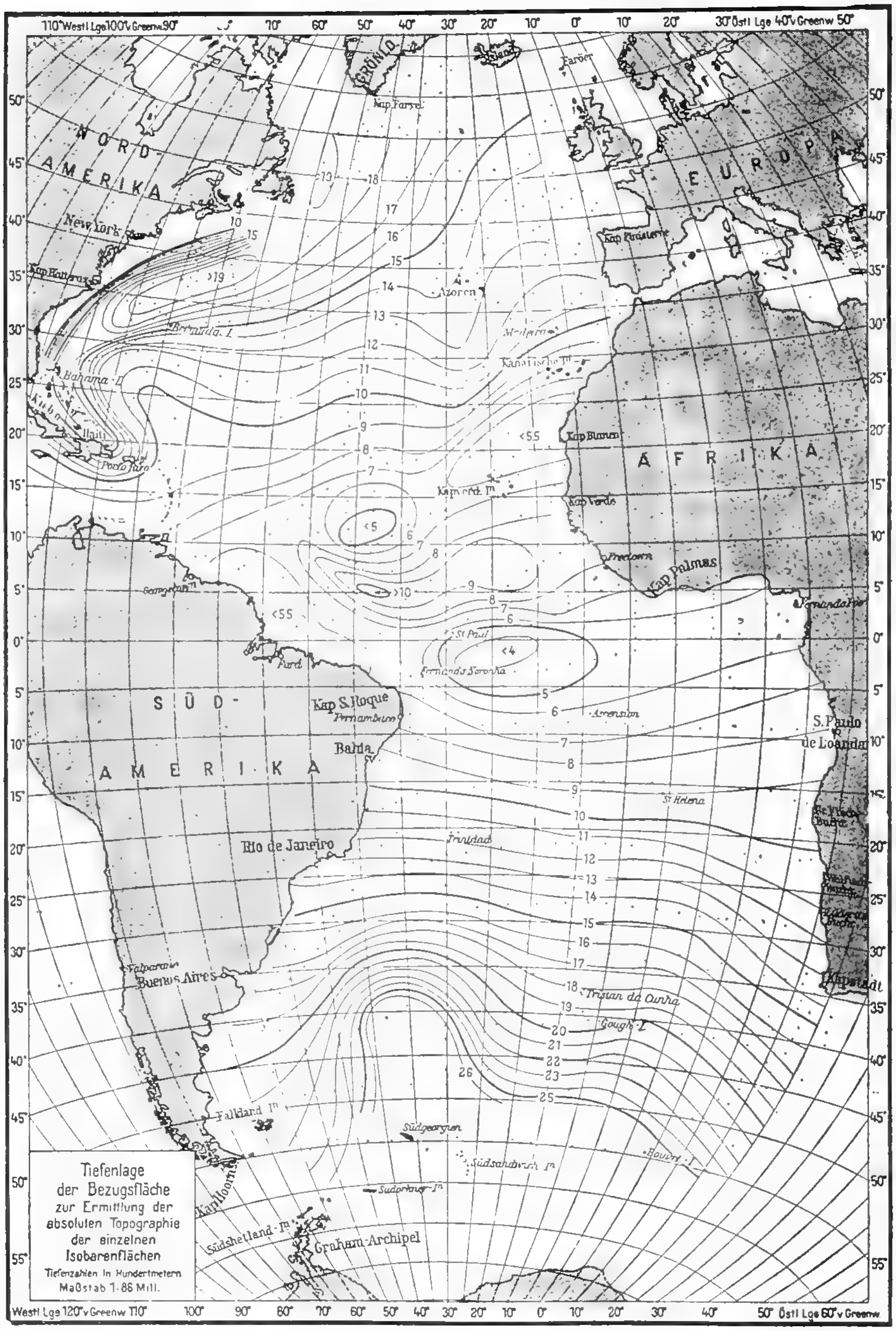

Fig. 1. Depth of the "Zero Layer" in the Atlantic Ocean, according to A. Defant (1941). The figures at the lines are to be multiplied by 100, thus denoting the depth in meters.

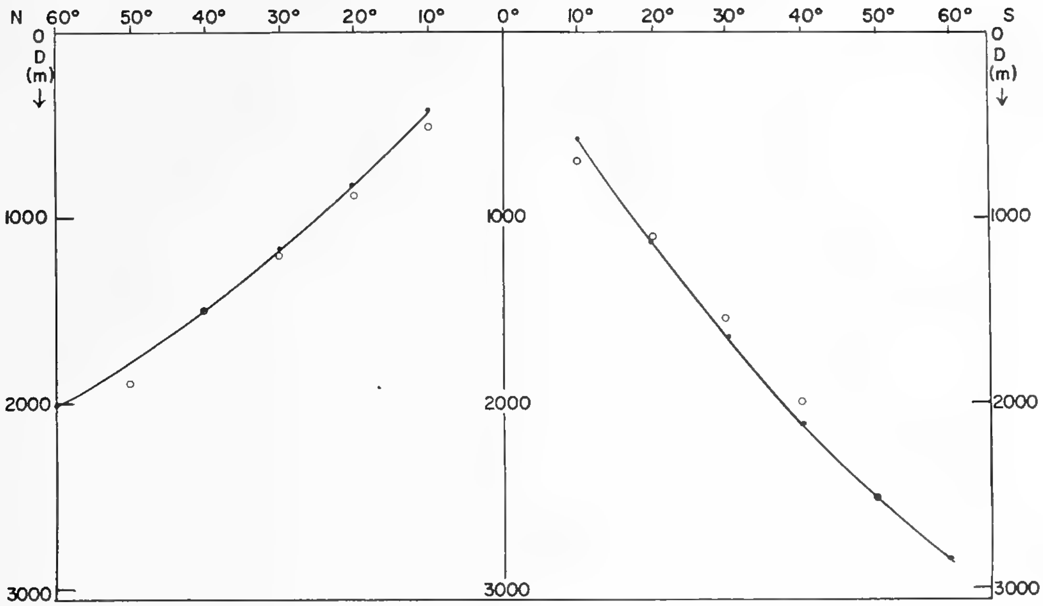

Figure 2. Average depth, D, of the layer of no motion in the Atlantic Ocean. The curves —•—•— represent D (φ) = const. sin φ , and the values according to A. Defant (1941) are indicated by circles (0).

or, the relative variation of D is given by

$$\frac{1}{D}\frac{\partial D}{\partial y} = \frac{1}{R}\,\text{ctn}\,\varphi \;,$$

where R is the radius of the earth. Since

$$\frac{1}{f}\frac{\partial f}{\partial y} = \frac{1}{R}\,\text{ctn}\,\varphi \;,$$

there is much evidence that in the "gross" features of the oceanic circulation

$$\frac{\partial f}{\partial y} = \frac{f}{D}\frac{\partial D}{\partial y} \;, \qquad (13)$$

and the important term

$$\left(\frac{\partial f}{\partial y} - \frac{f}{D}\frac{\partial D}{\partial y}\right)\frac{\partial \psi}{\partial x}$$

in equation (11) drops out in the "gross" features of a model where only zonal conditions are considered.

In the model by Stommel (1948), Hidaka (1949) and Munk (1950)

D was assumed to be constant. Therefore, the term responsible for the "westward intensification", $\frac{\partial f}{\partial y}\frac{\partial \psi}{\partial x}$, which contains the variation of the Coriolis parameter with latitude (planetary vorticity) was not cancelled out by the variation of the depth D with latitude (topographic vorticity, according to Ekman's terminology).

It seems unlikely that the east-west asymmetry of the gross features of the oceanic circulation is caused by the variation of the Coriolis parameter with latitude, but since there is an intensification of ocean currents along the east coasts of North America and Eastern Asia, this particular flow pattern requires another explanation. This may also answer the question why an analogous intensification of the subtropical gyre (Munk, 1950) is not found in the Southern Hemisphere.

It seems more likely that the ocean reacts to the planetary vorticity effect in such a way that it rather tends to adjust its level of no motion for the wind driven circulation than to displace the whole gyre with relatively high current velocities to the west. Whether or not this adjustment is complete is a question which still has to be answered.

It may be added that the relationship as stated in equation (13) does not say anything about "cause and effect". The zero layer may be the result of the current system, and probably it is to some degree in the gross features and even in the details. However, several other external and internal forces and factors, besides those taken into account, are involved in the mutual relationships between the oceanic circulation and the stratification. Some of these factors are the large scale heat gain and loss through the sea surface and evaporation, precipitation, and lateral and vertical mixing processes. The first mentioned climatic factors and mixing processes are especially responsible for some large scale density stratifications. With "large scale density stratification" in this sense is meant, for example, the very weak vertical density gradient in polar regions and the strong, significant vertical density gradient in tropical regions, both taken as an average between the surface and the bottom water. The vertical density stratification is an important factor in the oceanic circulation, which, at least more indirectly, may determine the wind driven circulation. This will be shown by a simple model in the next two sections.

V. A simple model of a stratified ocean

Disregard at first the differences in the salinity distribution and assume the temperature distribution as the main factor of the density stratification. The vertical temperature distribution as observed in the deeper layers of the oceans is to a certain degree usually considered as the result of the oceanic circulation or the "spreading" of water masses (G. Wüst, 1935). This is only partly true, as shown by H.U. Sverdrup (1939). Stationary conditions which appear on the average "gross" features to be rather similar to the observed conditions in the ocean may exist even in the case where the components u, v, w, of the oceanic circulation are zero; that is, in an ocean completely at rest.

Assume in the first step an ocean at rest. This "ocean" may cover the whole earth, and only the sun's radiation (with zero declination) may warm up the equatorial regions. The depth of the ocean may be constant (= h meters). In this case, the resulting temperature distribution will be symmetrical in the meridional direction and equal in the zonal direction. Since $u = v = w = 0$ by assumption,

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\frac{A_x}{\rho}\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{A_y}{\rho}\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{A_z}{\rho}\frac{\partial T}{\partial z}\right) \quad ,$$

where T is the temperature, and A_x, A_y, A_z are exchange coefficients for heat conduction. With an east-west symmetry, as in our model, we need only to consider the resulting temperature distribution in a meridional cross section. Stationary conditions are possible if

$$\frac{\partial}{\partial y}\left(\frac{A_y}{\rho}\frac{\partial T}{\partial y}\right) = -\frac{\partial}{\partial z}\left(\frac{A_z}{\rho}\frac{\partial T}{\partial z}\right) \quad .$$

Due to the sun's radiation there is a heat gain at the equator and a heat loss at the poles. Suppose that a meridional temperature distribution is maintained at the sea surface, and let T(y) at the sea surface be

$$T(y) = T_p + T_o \cos\frac{\pi}{2b}\,y \ , \qquad (14)$$

where T_p is the temperature of the sea surface at the pole, (y = b), $T_p + T_o$ the temperature at the equator (y = 0).

If, for simplicity, we assume A_y and A_z to be constant in space, but $A_y \gg A_z$, a solution of the differential equation

$$A_y\frac{\partial^2 T}{\partial y^2} + A_z\frac{\partial^2 T}{\partial z^2} = 0 \qquad (15)$$

with the boundary condition (14) is

$$T(y,z) = T_p + T_o\frac{\cosh \alpha(h - z)}{\cosh \alpha h}\cdot\cos\frac{\pi}{2b}\,y \ , \qquad (16)$$

where

$$\alpha = \frac{\pi}{2b}\sqrt{\frac{A_y}{A_z}} \ .$$

Assume that the ratio $A_y/A_z \sim 10^7$, and then, with b = 90° ~ 10,000 Km,

$$\alpha = 0.497 \ .$$

Further, let T_o = 25°C, T_m = 0°C, and let h = 6000 m.

With the assumptions made, stationary conditions of the temperature distribution in an ocean at rest would be possible if the temperature in different depths at different latitudes follows equation (16). In the following table this temperature distribution in a meridional cross section between 90° latitude and the equator is computed on the assumption that $A_y/A_z = 10^7$.

Depth (Km) \ Latitude φ°	90	80	70	60	50	40	30	20	10	0
0	0	0.40	1.50	3.4	5.8	8.9	12.5	16.4	20.7	25.0
1	0	0.25	0.93	2.1	3.6	5.5	7.7	10.2	12.8	15.5
2	0	0.15	0.56	1.27	2.2	3.3	4.7	6.1	7.7	9.2
3	0	0.09	0.35	0.79	1.35	2.1	2.9	3.8	4.8	5.8
4	0	0.06	0.23	0.52	0.89	1.36	1.9	2.5	3.17	3.85
5	0	0.045	0.17	0.38	0.65	1.00	1.4	1.84	2.32	2.8
6	0	0.040	0.15	0.34	0.58	0.88	1.24	1.62	2.05	2.48

This temperature stratification (figure 3a) shows rough similarities with the observed conditions in its very gross features. These similarities are the strong vertical stratification in tropical and subtropical regions with decreasing vertical gradients towards the bottom and the homogeneity at the poles. The distribution of the isotherms in this model (figure 3a) may lead to the impression that the cold water from polar regions spreads out along the ocean bottom towards equatorial regions. Even a more realistic picture could easily be derived with the assumption of an asymmetrical temperature distribution at the surface (Northern Hemisphere warmer than the Southern).

VI. Notes on the influence of the wind

In the second step, suppose that the wind blows over this ocean.

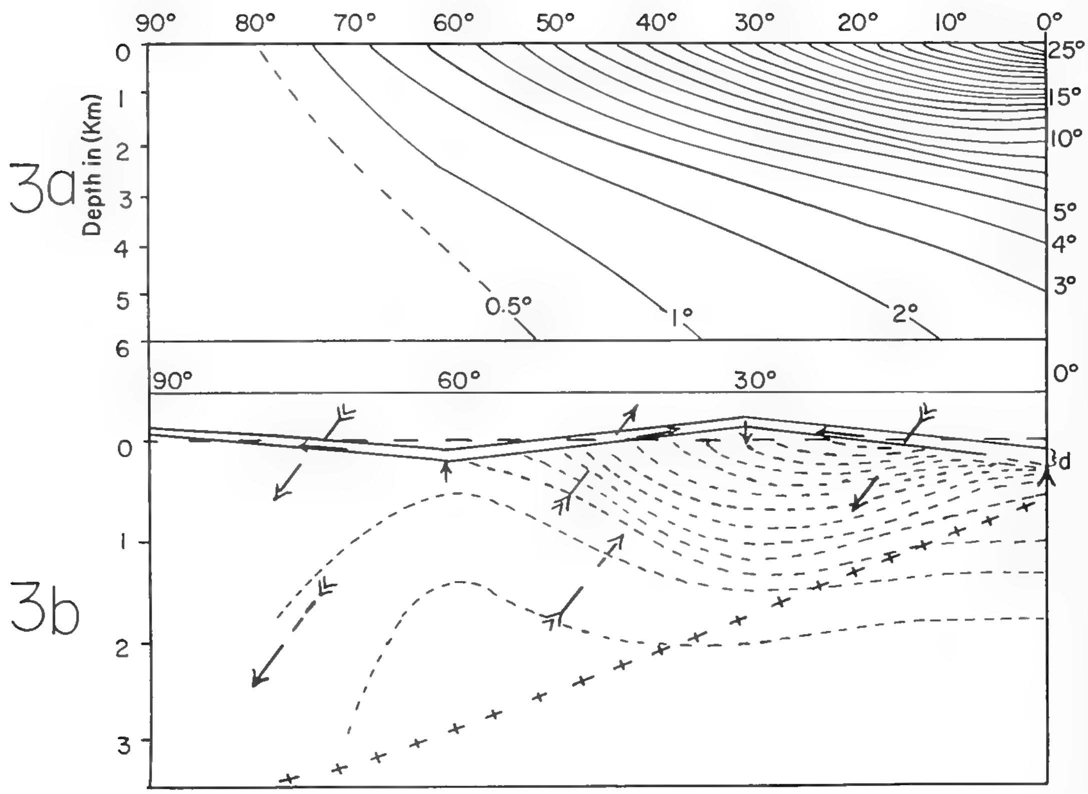

Fig. 3a. Computed temperature distribution in an ocean at rest (equilibrium of heat conduction with given sea surface conditions $A_y/A_z = 10^7$).

3b. Adjustment of mass distribution (temperature) to wind induced circulation with the additional assumption of geostrophic equilibrium (schematic). The line + + + indicates the layer of no motion for the wind induced circulation.

The prevailing winds are east winds between the equator and 30°, west winds between 30° and 60°, and weaker east winds between 60° and the pole. The primary effect of the winds acting at the sea surface is to produce drift currents with velocities which decrease with depth and which are essentially limited to the upper 50 m or 100 m of depth (layer of frictional influence). In our model, figure 3b, which is considered to represent the Northern Hemisphere, the total mass transport of water in the layer of frictional influence (d) is directed toward the north between the equator and 30°N, and between 60°N and the pole, whereas the total mass transport is directed towards the south between 60° and 30°. The consequences are a divergent flow in equatorial regions and around 60° and a convergent flow in the latitudes around 30° and near the pole. The divergence (or convergence) of this mass transport is proportional to the curl of the wind stress, and it can easily be computed.

The unequal divergence at different localities, which strongly depends on details of the wind field, sets up a slope of the sea surface, in our simplified model, which drops from 30° toward the equator and towards 60° and rises again from 60° toward the pole as indicated schematically in figure 3b. From this, hydrostatic pressure gradients result in the water, and, as an indirect consequence of the wind effect, gradient currents will develop in the deeper layers (in addition to the "density gradient currents", simply called "density currents", due to the original horizontal temperature differences). The depth of the wind induced gradient currents depends on the vertical stratification of the ocean water.

In what follows, we may neglect the "density currents" as they would result from the original temperature distribution in the model.

To examine qualitatively the wind effect on the stratified ocean, as shown in figure 3a, let us assume that the velocity of the wind induced gradient currents decreases with increasing depth. In the steady state the sea surface slopes must have reached a certain stationary inclination, and the mass distribution (temperature field) must have changed in such a way that beneath the layer of frictional influence from point to point the geostrophic equilibrium condition

$$\frac{\partial u}{\partial z} = \frac{g}{f} \tan\gamma \frac{1}{\varrho} \frac{\partial \rho}{\partial z} ,$$

is nearly fulfilled, or that, approximately,

$$\frac{\partial u}{\partial z} = \frac{g}{f} \frac{1}{\rho} \frac{\partial \rho}{\partial y}$$

holds. Here, u is the zonal velocity of the gradient current in the x-direction (indicated in figure 3b by feathered arrows), ρ the density (which in our model is only a function of the temperature at constant pressure), and γ the inclination angle between the lines of equal density and the (horizontal) level surfaces. Qualitatively, the distribution of isotherms in the meridional cross-section under these conditions should look like the dashed lines, as shown in figure 3b.

When looking in the direction of the current between 60°N and 30°N the isotherms must slope down from the left hand side to the right hand side, that is, from north to south, provided the current decreases with increasing depth, and they must slope down from the equator towards 30°N, since the wind induced gradient current in this region flows from east to west. This leads to a thick warm water accumulation around 30°N, and to a relative cooling in corresponding depths in equatorial regions. Furthermore, the vertical temperature gradient in certain subsurface levels between 30° and the equator must <u>in</u>crease, and beneath this region <u>de</u>crease, when compared with the original conditions for an ocean at rest, since the geostrophic equilibrium condition requires a tilting up of the isotherms toward the equator in the layer of the equatorial (gradient) current. The depth of this layer depends on the stratification, and it is indicated by the line + + + in figure 3b.

VII. The hydrostatic mass compensation

The condition of a steady state requires that for each column of water, the divergence of the total mass transport M from the surface to the bottom or to the level of no motion equals zero. Let M_1 be the mass transport of the drift current (in the layer of frictional influence), and let M_2 be the mass transport of the gradient current (from the surface to a depth where the current vanishes); then div $(M_1 + M_2) = 0$, or div $M_1 = -$ div M_2 for each column. Only the total mass transport, integrated from the surface to the layer of no motion, is non-divergent. But the layer of frictional influence is a divergent layer, and this divergence must be balanced by divergent gradient currents in deeper layers.

In the case of an ocean with constant, or almost constant potential density from the surface to the bottom, the hydrostatic pressure gradient, as caused by the "wind set up", should practically extend down to the bottom (as in figure 3b near 90° of latitude). Here, a "bottom current" as assumed in Ekman's "elementary current system" would provide the necessary anisobaric mass transport in order to balance the anisobaric mass transport in the layer of frictional influence. However, in a strongly stratified ocean, (as in fig. 3b between 30° and 0°), a layer of no motion may develop (with steady state conditions) in a certain depth above the bottom by "mass-compensation" of the hydrostatic pressure differences resulting from sea surface slopes. In this case, the necessary balance for the mass transport in the layer of frictional influence has to be provided for completely by the wind induced gradient current.

If the hydrostatic equation holds in vertical direction, the pressure p at a depth, h, is given by

$$p = - g \int_{z=-h}^{z=0} \rho \, dz + g \, \rho_o \zeta_o$$

where ζ_o is the elevation of the sea surface above the level surface z = 0, and ρ_o is the density of the surface.

Assume two oceanographic stations, A and B, fairly close together, for which the vertical density stratification is known, and suppose $\Delta \zeta$ is the elevation of the sea surface at station B relative to station A where $\zeta_A = 0$. Then, the depth of the level of no motion for the current perpendicular to the line $\overline{AB}$, between these stations is given at an average (constant) depth, z = -D, where $(p_z)_A = (p_z)_B$, or

$$\int_{-D}^{0} \rho_A \, dz = \int_{-D}^{0} \rho_B \, dz - \rho_o \Delta \zeta \ ,$$

and

$$\Delta \zeta = - \frac{1}{\rho_o} \int_{-D}^{0} (\rho_A - \rho_B) \, dz \ . \qquad (17)$$

Let the coordinate system be oriented with the x-axis in the (zonal) wind direction. Then, in the layer of frictional influence

$$(M_1)_y = -\frac{\tau_x}{f}, \quad (M_1)_x = 0 \tag{18}$$

according to Ekman.

The wind induced gradient current is a non-divergent current only as a first approximation. As an effect of internal friction there must be a slight deviation from the direction parallel to the isobars. The average angle of deviation (from high to low pressure) in the gradient layer may be denoted by α. Let u_g be the zonal component of the geostrophic current in the case of no internal friction, then

$$\rho u_g = -\frac{1}{f}\frac{\partial p}{\partial y},$$

and

$$U_g = \int_{-D}^{0} \rho u_g \, dz = -\frac{1}{f}\int_{-D}^{0} \frac{\partial p}{\partial y} dz$$

Suppose that the actual total horizontal mass transport, U, in the gradient current is deflected slightly from the direction of U_g by the angle α, as schematically indicated in figure 4. The magnitude of α was estimated as 1°-2° in higher latitudes and 5°-6° in lower latitudes (Neumann, 1952).

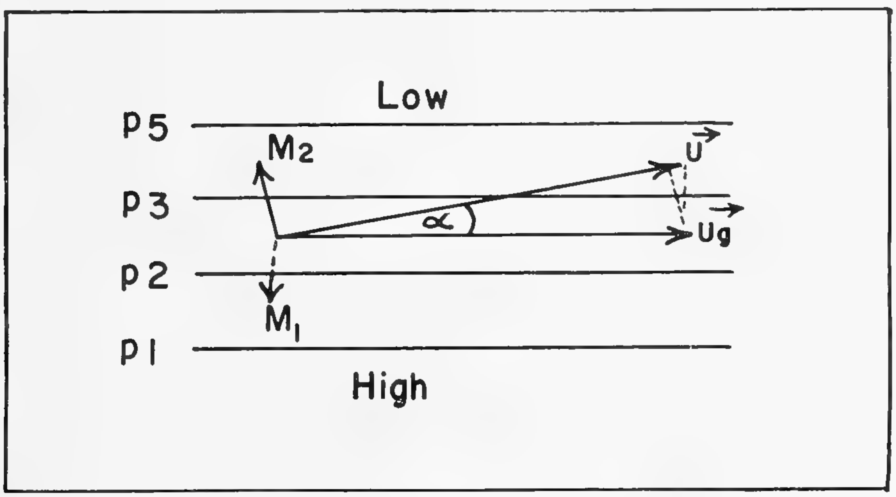

Fig. 4. The difference vector $\vec{U} - \vec{U}_g$ represents approximately the anisobaric mass transport to balance the mass transport M_1 in the layer of frictional influence.

The difference vector $\vec{U} - \vec{U}_g$ represents approximately the anisobaric mass transport M_2 to balance the mass transport M_1 in the steady state, and

$$M_2 \cong - \frac{1}{f} \sin\alpha \int_{-D}^{0} \frac{\partial p}{\partial y} \, dz \; . \tag{19}$$

With the hydrostatic equation

$$\frac{\partial p}{\partial y} = - g \int_{-D}^{0} \frac{\partial \rho}{\partial y} \, dz \; ,$$

and

$$\int_{-D}^{0} \frac{\partial p}{\partial y} \, dz = - g \int_{-D}^{0} \left[\int_{-D}^{0} \frac{\partial \rho}{\partial y} \, dz \right] dz \; ,$$

the mass transport M_2 can be written, approximately,

$$M_2 = \frac{1}{f} g \sin\alpha \int_{-D}^{0} \left[\int_{-D}^{0} \frac{(\rho_A - \rho_B)}{\Delta y} \, dz \right] dz \tag{20}$$

for two closely spaced stations, A and B, with a distance Δy apart.

From the steady state condition, div M = 0, and from (18) and (20) we have with the assumption that between the two closely spaced stations the variation of M_1 and M_2 can be neglected

$$\tau_x = g \sin\alpha \frac{1}{\Delta y} \int_{-D}^{0} \left[\int_{-D}^{0} (\rho_A - \rho_B) dz \right] dz. \tag{21}$$

This equation can be solved numerically for D, the level of no motion. The zonal wind stress, τ_x, can be determined from the wind field, and with regard to α the assumption may be made that α in the region of the equatorial current is approximately 5°-6° and decreases to 1.5° in 60° latitude.

Figure 5 shows the zonal average density distribution in a meridional cross section through the South Atlantic Ocean. The zonal average of the density was computed from the data as given by G. Wüst and A. Defant (1936) in the atlas of the "Meteor" work.

From Schott's (1942) wind charts, zonal average wind velocities in the South Atlantic have been derived, and the wind stress, τ_x, has been computed by means of an empirical formula (Neumann, 1948). More information about the wind stress is given in section IX of this report. The zonal wind velocities W_x, the wind stress τ_x, and the estimated values for α are given in Table 1.

Let us assume equally spaced "stations" with a meridional distance of $\Delta y = 5°$, between 5.5°S and 57.5°S. From (21) with $\Delta y = 555$ km and $\sigma_t = (\rho - 1) \times 10^3$, we have for an estimate

$$\frac{\tau_x}{\sin\alpha} \times 5.65 \times 10^7 = \int_{-D}^{0} \left[\int_{-D}^{0} (\sigma_{t_A} - \sigma_{t_B}) dz \right] dz \qquad (22)$$

The average vertical density distribution as shown in figure 5 is tabulated for the numerical integration in table 2.

Table 1

Latitude	0	5	10	15	20	25	
W_x(m/sec)	-0.5	-3.0	-4.0	-5.3	-3.5	0	
τ_x(dyne/cm^2)	0.05	0.57	0.9	1.38	0.74	0	
$\alpha°$	-	7.7	6.2	4.7	3.9	3.2	
Latitude	**30**	**35**	**40**	**45**	**50**	**55**	**60°S**
W_x(m/sec)	2.7	4.9	7.4	9.4	8.7	7.6	4.2
τ_x(dyne/cm^2)	0.50	1.22	2.27	3.25	2.87	2.36	9.97
$\alpha°$	2.7	2.3	2.1	1.9	1.7	1.6	1.5

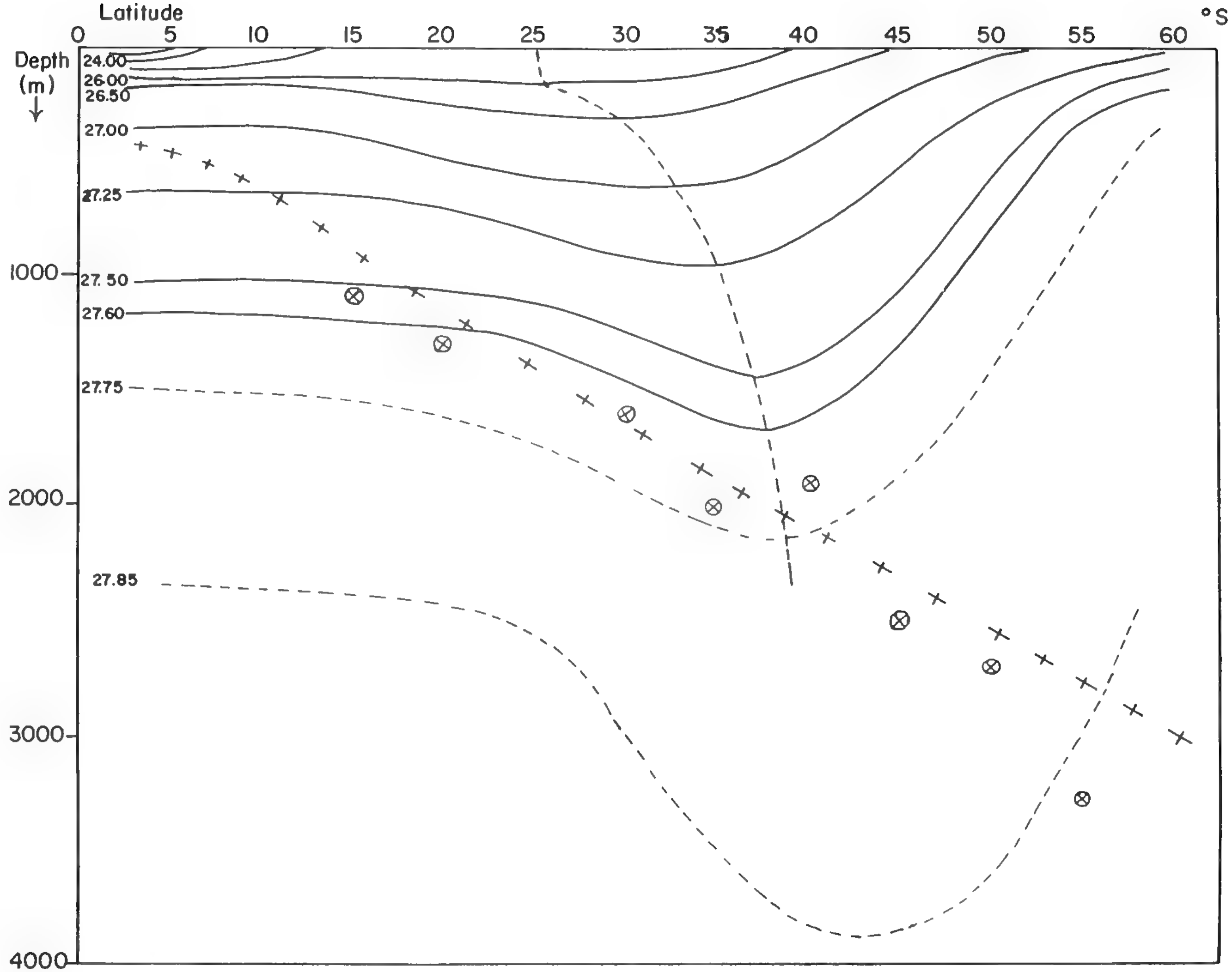

Fig. 5. Average meridional density section (σ_t) through the South Atlantic Ocean, and average depth of the layer of no motion according to A. Defant (—+— —+— —+—) compared with calculations (⊗).

Table 2

Lat. °S / depth (m)	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
0	23.53	24.15	24.44	25.31	25.30	25.34	25.56	25.77	26.34	26.74	27.00	27.16
200	26.66	26.71	26.57	26.39	26.30	26.33	26.47	26.68	26.91	27.15	27.39	27.63
400	27.04	27.07	26.05	26.94	26.79	26.72	26.73	26.90	27.14	27.30	27.56	27.73
600	.23	.23	.24	27.19	27.11	27.03	27.00	27.08	.23	.40	.63	.78
800	.35	.36	.35	.34	.31	.23	.20	.21	.32	.49	.69	.79
1000	.47	.48	.47	.46	.44	.37	.31	.31	.41	.57	.73	.81
1250	.65	.65	.63	.62	.59	.54	.46	.43	.51	.65	.78	.82
1500	.76	.75	.74	.73	.70	.66	.59	.54	.62	.72	.81	.83
1750	.81	.80	.79	.79	.78	.74	.68	.63	.68	.76	.82	.84
2000	.83	.83	.82	.82	.82	.80	.75	.73	.75	.78	.83	.85
2500	.86	.86	.84	.85	.85	.85	.81	.80	.80	.81	.84	.85
3000	.87	.87	.86	.86	.87	.86	.84	.81	.82	.82	.85	.85
3500	.88	.87	.87	.86	.87	.86	.86	.84	.83	.84	.85	.86

For two fictitious "stations" at 42.5°S and 47.5°S, respectively, the following vertical distribution of

$$I = \int_{-D}^{0} \left[\int_{-D}^{0} (\sigma_{t_A} - \sigma_{t_B}) dz \right] dz$$

is derived:

depth (m)	$\Delta\sigma_t = \sigma_{t_A} - \sigma_{t_B}$	$\overline{\Delta\sigma_t} \cdot \Delta z$	$T = \Sigma(\overline{\Delta\sigma_t} \cdot \Delta z)$	$\bar{T}$	$\bar{T}\Delta z$	I
		$\times 10^4$	$\times 10^4$	$\times 10^4$	$\times 10^8$	$\times 10^8$
0	0.40		0			0
		0.64		0.32	0.64	
200	0.24		0.64			0.64
		0.40		0.84	1.68	
400	0.16		1.04			2.32
		0.33		1.205	2.410	
600	0.17		1.37			4.73
		0.34		1.54	3.08	
800	0.17		1.71			7.81
		0.33		1.875	3.75	
1000	0.16		2.04			11.56
		0.375		2.228	5.57	
1250	0.14		2.415			17.13
		0.300		2.565	6.42	
1500	0.10		2.715			23.55
		0.225		2.828	7.08	
1750	0.08		2.940			30.63
		0.137		3.085	7.70	
2000	0.03		3.077			38.33
		0.100		3.127	15.64	
2500	0.01		3.177			53.97
		0.025		3.189	15.95	
3000	0.00		3.202			69.92

With $\tau_x = 3.25$ dyne/cm^2, and $\alpha = 1.9°$, $\frac{\tau_x}{\sin\alpha} \cdot 5.65 \times 10^7 = 53 \times 10^8$,

therefore D = 2500 m for the zonal component. This compares fairly well with the depth of the layer of no motion as shown in figure 2. Similar computations for higher latitudes (55°S) lead to values which are a little too large, whereas the depth D computed for the

South East Trade wind region (10°S through 35°S) agree well with Defant's values. The computed values D are plotted in the density section, figure 5, for comparison with the depth D in the central parts of the South Atlantic according to figure 2.

Probably, the estimated angles α are the most inaccurate. However, the estimate of D by equation (22) should serve only to check the order of magnitude of the factors involved in this possible mechanism of "mass compensation" to determine the level of no motion. Another fact which must be kept in mind is that this calculation only considers the meridional component of the anisobaric mass transport.

At this point, the dashed curve in figure 5 may at least be mentioned briefly. It connects the maximum depth of the lines of equal density (σ_t), and is in some way related to the tilting of the axis of the subtropical gyre. This tilting in meridional direction toward the poles with increasing depth has already been mentioned by A. Defant (1941), but so far no explanation has been given for this gross feature of the oceanic circulation.

VIII. The coefficient of internal friction (r)

In figure 6, $\vec{M}_g$ is the vector of the geostrophic mass transport, and $\vec{M}$ is the vector of the true mass transport which is deflected by the angle α from the direction parallel to the isobars. The difference vector, $\vec{M}_g - \vec{M}$, is in some way related to the average internal friction in the whole water column from z = 0 to z = -D.

Assume horizontal, non-accelerated motion under the influence of internal friction. Then

$$-\frac{1}{\rho}\,\text{grad}\,\overline{p} + \vec{M} \times \vec{f} + \vec{R} = 0 \,, \tag{23}$$

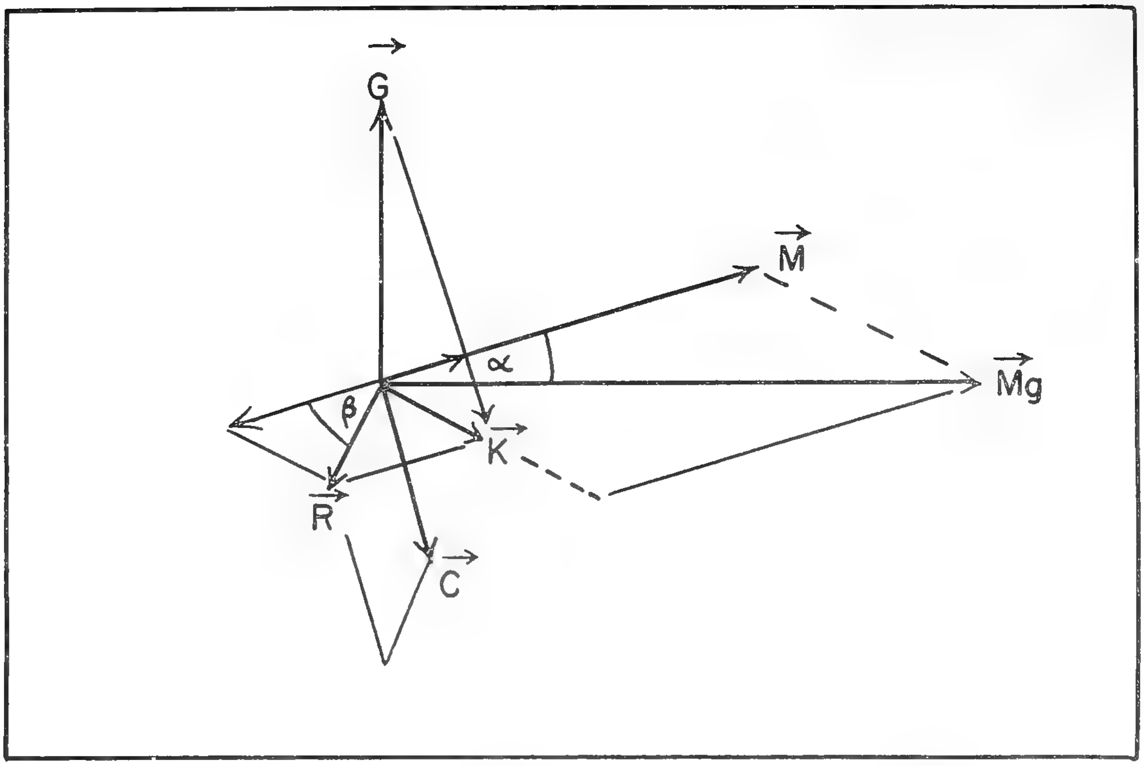

Figure 6. Force diagram of a non-accelerated gradient current with friction.

where $\vec{R}$ is the vector of the internal frictional force. For a geostrophic current without friction,

$$- \frac{1}{\rho} \text{ grad } \bar{p} + \vec{M}_g \times \vec{f} = 0 \qquad (24)$$

On substituting (24) into (23), we have

$$(\vec{M}_g - \vec{M}) \times \vec{f} = \vec{R} . \qquad (25)$$

Thus, the vector of internal friction is perpendicular to the difference vector $\vec{M}_g - \vec{M}$, and only in exceptional cases is β equal to zero.

Suppose α is known, then β and $|\vec{R}| = R$ can be computed. In equation (8), for the components of the effective internal friction, we used for a first approximation Guldberg-Mohn's expression

$$R_x = - rU$$

$$R_y = - rV ,$$

by assuming $R \cong - rM$. As in figure 6, let $\vec{G}$ be the vector of the average pressure gradient force and $\vec{C}$ the vector of the Coriolis force. Then from figure 6,

$$G \sin\alpha = rM \cos\beta \qquad (26)$$

and

$$G \cos\alpha = 2\omega \sin\varphi\, M + rM \sin\beta \ , \qquad (27)$$

since the components of the forces in the direction of M and perpendicular to it must balance. $\vec{K} \propto \vec{M}_g - \vec{M}$ is proportional to the average "dragging force", according to Sprung.

From (26) and (27)

$$\text{ctn}\alpha = \frac{2\omega \sin\varphi}{r \cos\beta} + \tan\beta \qquad (28)$$

$$\tan\beta = \text{ctn}\alpha - \frac{2\omega \sin\varphi\, M}{G \sin\alpha} \qquad (29)$$

In the case where $\beta \to 0$

$$\text{ctn}\alpha \cong \frac{2\omega \sin\varphi}{r} \ . \qquad (30)$$

So far, no direct computations of r have been made in dynamic oceanography, and it seems that the only estimates of α are those used in this report (Neumann, 1952).

Whether (30) can be used instead of (28) to calculate r from α, depends on the order of magnitude of β. However, data which are accurate enough to compute β are not available.

The mass transport of the Antarctic Circum Polar Current between 65°S and 45°S, is given by

$$M \approx 10^{14} \ [\text{gr sec}^{-1}] \ ,$$

approximately, according to Sverdrup et al (1946). (This value may be even a little larger.) The average meridional pressure gradient G can be estimated fairly well from the "Meteor" data published by G. Wüst (1939). Using "Meteor" Stations Mt 130, 124, 60, and 10, the following pressure values, p, in decibar are obtained, relative to the sea surface where p = 0:

p_z in decibars

Depth (m)	Mt. 130 $\varphi = 63.7°S$ $\lambda = 5.3°E$	Mt. 124 $\varphi = 55°S$ $\lambda = 16°W$	Mt. 60 $\varphi = 48.5°S$ $\lambda = 16.9°W$	Mt. 10 $\varphi = 41.2°S$ $\lambda = 17.1°W$
0	0	0	0	0
50	51.368	51.361	51.358	51.336
100	102.772	102.740	102.730	102.682
150	154.192	154.137	154.114	154.043
200	205.626	205.552	205.513	205.425
300	308.529	308.428	308.355	308.237
400	411.481	411.361	411.252	411.110
600	617.534	617.388	617.209	617.021
800	823.783	823.616	823.377	823.137
1000	1030.228	1030.039	1029.758	1029.461
1200	1236.869	1236.660	1236.343	1236.000
1400	1443.703	1443.477	1443.127	1442.745
1600	1650.728	1650.489	1650.109	1649.695
2000	2065.352	2065.092	2064.659	(2064.191)
2500	2584.70	2584.41	2583.93	-
3000	3105.24	3104.92	3104.39	-

From these data $G \sim 1.4 \times 10^{-4}$ dyne/cm^3. With $M \approx 10^{14}$ gr sec^{-1}, $D = 3000$ m and a total width of 2.22×10^3 Km, it follows per unit area of the cross section that

$$M[\text{gr sec}^{-1}\text{cm}^{-2}] = \frac{10^{14}\text{ gr sec}^{-1}}{2.22 \times 3.0 \times 10^{13}\text{ cm}^2} = 1.5 ,$$

and, with $\alpha \sim 1.7°$, according to Table 1, for $\varphi = 55°$,

$$\tan\beta \approx 34 - 43 .$$

This result can only serve to check the order of magnitude of α, G, and M, since $\tan\beta$ is given as a small difference between two larger quantities which are known only approximately. Therefore, β can not be determined from these data. However, since the deviation of the true mass transport from the direction of the geostrophic mass transport is very small, the assumption may be justified that $\beta \sim 0$ for the purpose of estimating r. Then, from (30) and table 1 for $\varphi = 55°$

$$r \sim \frac{2\omega \sin\varphi}{\mathrm{ctn}\alpha} = 3.34 \times 10^{-6} \ [\mathrm{sec}^{-1}] \ .$$

In the lower layers of the atmosphere, r is on the average approximately 100 times larger. H.U. Sverdrup and J. Holtsmark (1917) derived for 10 North American stations average values from 50-100 individual observations. They found r between 7×10^{-5} and 2.7×10^{-4} sec^{-1}, and β between -12° and +48°.

The height of the anemometer above the ground is of great influence. Both r and β decrease with height, as shown by Sverdrup and Holtsmark:

height (meters)	25	25-49	50
$r \times 10^4$	2.20	1.99	1.56 sec^{-1}
β	25°	20°	17°

With increasing height, β and r decrease in the lower layer of the atmosphere, but it seems that no fairly well established information about these friction coefficients is available for the higher troposphere, which is comparable with the layer of wind driven ocean circulation beneath the layer of frictional influence. It may well be, that in the higher troposphere $\beta \rightarrow 0$, magn. $\alpha \approx 2°$, and magn. $r \approx 10^{-5} - 10^{-6}$.

According to equation (30) $\mathrm{ctn}\alpha$ should vary approximately proportionally to $\sin\varphi$, if r is considered to be constant. Since α is given in table 1 by previous results, r in different latitudes is shown in the following table:

Latitude	10°	20°	30°	40°	50°	60°
α	6.2	3.9	2.7	2.1	1.7	1.5
$r \times 10^6$	2.75	3.14	3.43	3.44	3.30	3.30

With the estimated angles α, r is almost constant. This may indicate that in the average for the whole layer of the gradient current, frictional conditions are rather uniform in the different branches of the circulation system, although under special conditions significant local variations of the effective frictional forces may occur. For the numerical integration of the differential equation (11), however, an average value

$$r \approx 3.3 \times 10^{-6} \ [\sec^{-1}]$$

will be used.

IX. The wind stress at the sea surface

Estimates of the total wind stress at the sea surface rest largely on observations of the vertical distribution of mean wind near the sea surface. Such wind profile measurements in the lowest layers over the sea surface have been compared with the velocity distribution in circular pipes, so that the interfacial stress could be deduced from the measured wall stress in pipes. This method simply applies the results of experimental, aerodynamical studies, and is, therefore, based on the assumption that such a comparison is justified.

Regardless of the question as to how far the analogy between the wall stress in pipes and the wind stress at the free sea surface goes, there are still other objections against the method used , especially with regard to the method of observing the average vertical wind profile with cup anemometers at fixed "heights" above the undisturbed sea level (or above the highest wave crests).

Most of the observations seem to indicate that with an adiabatic lapse rate in the lower layers above the sea surface the observed wind profile can be approximated fairly well by a logarithmic law within a certain height interval. However, in the lowest layers, some 50 or 100 cm above the crests of the sea surface roughness elements (waves), the logarithmic curve gradually steepens more and more when approaching the "sea surface". This feature is characteristic for the observed wind profiles over the rough sea surface, and a careful analysis of the observations shows that the logarithmic law for the wind profile just above the waves is a rather questionable approximation.

The value and accuracy of wind profile measurements with cup anemometers for evaluating the wind stress at the sea surface have been discussed in some detail by the author (1949, 1951). There are mainly two facts which make this method unfit for stress computations:

First of all, the questions remains open as to what level has to be chosen as the "height of the sea surface", or, in other words, what is the "sea surface" in the case of an air-sea interface with traveling waves? Any attempt to derive a vertical wind profile from observed wind speeds at fixed heights above the crests, requires a clarification of this question.

Secondly, wind measurements with cup anemometers at fixed heights above the wave crests are unfit to derive true <u>average</u> wind speeds over the rough sea surface. The wind at a fixed height over the trough of a "wave" may have only 40% - 50% or less of its speed over the crest. With steep waves, even a "lee-eddy" may develop somewhere near the trough, and the wind direction may be reversed. Since wind profile measurements near the surface have been taken only at wind speeds of, approximately, less than 10 m/sec, the rather short period wave motion passing by underneath the cup anemometers "speeds up" the anemometers every time a crest passes. The inertia of the anemometers does not allow slowing down over the troughs, and therefore, almost <u>maximum</u> wind speeds are observed instead of average values. When used for stress computations, too small values of the stress are obviously derived by this procedure. The "critical wind speed" at 7 m/sec as discussed by W. Munk (1947) can be explained as a result of the inaccuracy of the data obtained by wind profile measurements (Neumann, 1951).

Another more direct method of deriving the wind stress at the sea surface is based on observations of the steady state sea surface slope in which the water is piled up due to wind ("wind set up"). This method is the oldest, and it was first used by W. Ekman in 1905. Since that time much more data on the "wind set up" in different seas and lakes have accumulated, and with these data, the method was reexamined and critically applied by the author in 1948.

The stress, τ, may be expressed in terms of a resistance coefficient, γ^2, which is defined by

$$\tau = \rho' \gamma^2 W^2 , \qquad (31)$$

where ρ' is the density of the air, and W is the mean wind speed at a given distance above the sea surface. In practice, this distance is given by the "anemometer height", about 10 m. The question is whether γ^2 is a constant, or whether it depends on the wind speed itself. In the case of a flow over a solid surface with well defined and constant roughness conditions, such as grass land or snow,

γ^2 may be a constant for different wind speeds. However, in the case of the sea surface, the "roughness" is given by the composite wave motion. Since the wave motion is a function of the wind speed, W, (and of the state of wave development), γ^2 will probably also be a function of the wind speed. (In the case of stable stratified air, γ also depends on the stability of the air stratification.)

The results obtained by the author in 1948 indicate that under normal conditions (with an adiabatic lapse rate in the lowest layers) γ^2 varies inversely proportional to the square root of W, and

$$\gamma^2 = 9.0 \times 10^{-3} \left(\frac{1 \text{ [cm/sec]}}{W \text{ [cm/sec]}} \right)^{1/2} \tag{32}$$

for winds from 1 to 40 m/sec. Thus, the wind stress at the sea surface (for a fully arisen sea) can be represented by

$$\tau = \rho' \times 0.009 \times W^{3/2} \text{ [dyne/cm}^2\text{]} , \tag{33}$$

where W is to be inserted in centimeters per second.

A comparison of (32) with the corresponding resistance coefficients obtained from wind profile measurements was made in 1951, and the results are reproduced in figure 7, where more recent cup anemometer measurements by U. Roll (1948) show again the characteristic discrepancy between the γ^2 values as derived from "slope" observations and from wind profile observations. By disregarding the results based on "slope" observations at wind speeds of less than 10 m/sec, and replacing them by the results of the inadequate cup anemometer measurements, a "jump" of γ^2 from low to high values appears erroneously at 10 m/sec wind velocity.

A critical review of the accuracy and a summary of the results obtained by different methods, including wind slope observations in wind tunnels has been recently made by R. B. Montgomery (1952).

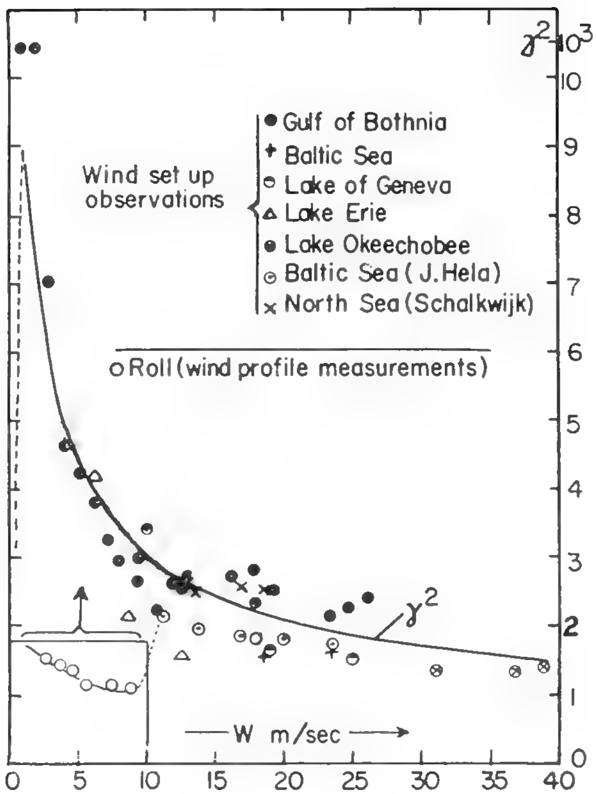

Fig. 7. Resistance coefficient ($\gamma^2 = \frac{\tau}{\rho' v^2}$) at the sea surface as a function of wind velocity (W m/sec), according to slope-observations (wind set up) and wind profile measurements. (After G. Neumann, 1951).

He finally comes to the rather disappointing conclusion that at no wind speed is the stress (or the resistance coefficient) known confidently to within half its value.

At present, a new, direct approach for measuring the interfacial stress at the sea surface is being undertaken by L.A.E. Doe as part of this project. It is hoped to derive the surface stress from direct measurements of the eddy stress, $-\rho[w'u']$, at a short distance from the interface, and from wind profile measurements at constant heights above the true wavy sea surface with a special type of practically inertialess anemometers.

For a quantitative study of the wind driven oceanic circulation, the exact knowledge of τ (or γ^2) is very important. Since the average climatological wind data used in such a study are less than 10 m/sec in most areas over the oceans, the most contraversial region with $W \leqslant 10$ m/sec comes into consideration in a critical way. W. Munk (1950) in his study of the wind driven oceanic circulation came to the conclusion that the stress values as derived from wind profile measurements are much too small to supply the necessary driving force of the wind at low wind speeds. The author agrees completely with W. Munk's statement that the transports of the large oceanic currents are probably as good an indicator of the overall stress exerted by the winds on the ocean as any of the measurements on which the value of γ^2 is now based. Similar conclusions have been drawn with regard to the wave generating forces by wind (Neumann, 1950).

Recently, in a careful analysis of storm tide data on Lake Erie, G. H. Keulegan (1952) also derived wind stresses for larger wind speeds which are in substantial agreement with the values suggested by the author (1948).

There is, at present, no evidence which really shows disagreement with the use of the empirical stress formula as proposed by equation (31) with the variable resistance coefficient of (32). Therefore, after a critical reexamination of the papers on this subject since 1948, this formula has been applied for computing the wind stress in table 1 of this report, and it is also being used for a numerical analysis of wind driven circulation in the North Atlantic Ocean.

X. Conclusions, and some remarks on the model of the wind driven ocean circulation

a) The differential equation (11) for the horizontal mass transport of the wind driven oceanic circulation seems to promise a more realistic approach to a quantitative analysis of the horizontal mass transport in the upper layers of the sea. It comes a little closer to Ekman's original formulation of the problem, since the "topographic curl effect" was shown to be of significant importance throughout all of his papers.

By writing equation (9) in terms of the vertically integrated mass transport, M, with the components, U and V, it follows that

$$\operatorname{curl}_z M = \frac{V}{r}\frac{\partial f}{\partial y} - \frac{1}{r}\left(\frac{\partial \bar{p}}{\partial x}\frac{\partial D}{\partial y} - \frac{\partial \bar{p}}{\partial y}\frac{\partial D}{\partial x}\right) - \frac{1}{r}\operatorname{curl}_z \tau \quad , \qquad (33)$$

where $\partial\bar{p}/\partial x$ and $\partial\bar{p}/\partial y$ again represent average values of the horizontal pressure gradients in the whole layer between z = -D and z = 0. With $\Delta \bar{p}\Big|_{-D}^{\zeta} = g\Delta(\bar{\rho}D)\Big|_{-D}^{\zeta} + g\Delta(\bar{\rho}\zeta)\Big|_{-D}^{\zeta}$ and

$$(D + \zeta)\frac{\partial \bar{\rho}}{\partial x} \approx D\frac{\partial \bar{\rho}}{\partial x}$$

$$\left.\begin{aligned} \frac{\partial \bar{p}}{\partial x}\Big|_{-D}^{\zeta} &= g\bar{\rho}\frac{\partial D}{\partial x} + gD\frac{\partial \bar{\rho}}{\partial x} + g\bar{\rho}\frac{\partial \zeta}{\partial x} \\ \frac{\partial \bar{p}}{\partial y}\Big|_{-D}^{\zeta} &= g\bar{\rho}\frac{\partial D}{\partial y} + gD\frac{\partial \bar{\rho}}{\partial y} + g\bar{\rho}\frac{\partial \zeta}{\partial y} \end{aligned}\right\} \qquad (34)$$

On substituting (34) into (33) we have

$$\text{curl}_z\ M = \frac{V}{r}\frac{\partial f}{\partial y} - \frac{gD}{r}\left(\frac{\partial \bar{\rho}}{\partial x}\frac{\partial D}{\partial y} - \frac{\partial \bar{\rho}}{\partial y}\frac{\partial D}{\partial x}\right)$$
$$- \frac{g\bar{\rho}}{r}\left(\frac{\partial \zeta}{\partial x}\frac{\partial D}{\partial y} - \frac{\partial \zeta}{\partial y}\frac{\partial D}{\partial x}\right) - \frac{1}{r}\,\text{curl}_z\,\tau\ ,$$

and with $M = -\frac{R}{r}$,

$$- \text{curl}_z\ R = V\frac{\partial f}{\partial y} - \text{curl}_z\tau - gD\left(\frac{\partial \bar{\rho}}{\partial x}\frac{\partial D}{\partial y} - \frac{\partial \bar{\rho}}{\partial y}\frac{\partial D}{\partial x}\right)$$
$$- g\bar{\rho}\left(\frac{\partial \zeta}{\partial x}\frac{\partial D}{\partial y} - \frac{\partial \zeta}{\partial y}\frac{\partial D}{\partial x}\right)\ . \tag{35}$$

Only, if D = constant

$$+ \text{curl}_z\ R = - V\frac{\partial f}{\partial y} + \text{curl}_z\tau\ , \tag{36}$$

and the equation expresses a balance between the curl of internal stresses (friction), and the planetary vorticity and the curl of the wind stress. The differential equation was used in this form in the preceding papers on the wind driven circulation.

b) The last term on the right hand side of (35) can be interpreted as the z-component of a vector product of two vectors, grad $D = \vec{a}$ and grad $\zeta = \vec{b}$. If δ is the angle between the vectors $\vec{a}$ and $\vec{b}$, then

$$\frac{\partial \zeta}{\partial x}\frac{\partial D}{\partial y} - \frac{\partial \zeta}{\partial y}\frac{\partial D}{\partial x} = ab\ \sin\delta = 0$$

would mean that $\vec{a} \parallel \vec{b}$, and the lines of equal dynamic height of the sea surface (relative to a level surface) are parallel to the lines of equal depth D.

Similarly, the term

$$\frac{\partial \bar{\rho}}{\partial x}\frac{\partial D}{\partial y} - \frac{\partial \bar{\rho}}{\partial y}\frac{\partial D}{\partial x} = 0$$

would express the requirement that the lines of equal D and the isopycnals $\bar{\rho}$ are parallel. Approximately 35 years ago, Bj. Helland-

Hansen and W. Ekman (1923) noticed that over large areas in the Norwegian Sea the large scale spatial distribution of temperature and salinity (density), followed closely the contour lines of the bottom topography. Furthermore, when studying large scale conditions in the oceans, they found a remarkable parallel between the isotherms and isohalines and the contour lines of the isobaric surfaces. This important law became known as the "law of the parallel solenoids." Although this law seems very well verified in the gross features of the large scale conditions, there are slight deviations which are nevertheless very important.

Precise leveling along the American East Coast shows that the mean sea level increases towards the north from Florida to Nova Scotia (G. Dietrich, 1937). The magnitude of this slope is 10^{-7}, and thus it is of the same order as the sea surface slopes derived from data in the Caribbean Sea (H.U. Sverdrup, et al., 1946). In the Caribbean Sea, as in many other parts of the ocean, the slope of the sea surface is practically compensated for by the distribution of mass. However, this is not the case in the region of the Gulf Stream along the east coast of North America. G. Dietrich (1937) has shown that along the continental slope the distribution of density does not indicate any rise of the sea surface, and this is true also in the case where the sea surface topography is referred to the 2000 dbar-surface instead of the oxygen minimum layer, which was used by Dietrich (H.U. Sverdrup et al., 1946).

With the assumption that within the layer of the wind induced gradient current $\mathrm{curl}_z R \approx \mathrm{curl}_z \tau$, then from (35)

$$V \frac{\partial f}{\partial y} - gD\left(\frac{\partial \bar{\rho}}{\partial x}\frac{\partial D}{\partial y} - \frac{\partial \bar{\rho}}{\partial y}\frac{\partial D}{\partial x}\right) - g\bar{\rho}\left(\frac{\partial \zeta}{\partial x}\frac{\partial D}{\partial y} - \frac{\partial \zeta}{\partial y}\frac{\partial D}{\partial x}\right) = 0 ,$$

or

$$V \frac{\partial f}{\partial y} - g \left[\left(D \frac{\partial \bar{\rho}}{\partial x} + \bar{\rho} \frac{\partial \zeta}{\partial x} \right) \frac{\partial D}{\partial y} - \left(D \frac{\partial \bar{\rho}}{\partial y} + \bar{\rho} \frac{\partial \zeta}{\partial y} \right) \frac{\partial D}{\partial x} \right] = 0 \qquad (37)$$

The x,y-coordinate system can be oriented in such a way that the y axis points in a direction η of the flow of the Gulf Stream along the east coast of the United States. Then, the ξ -axis, perpendicular to η, points toward the southeast, approximately. In this case, it is found from figure 1, that $\partial D/\partial \xi \approx -2.5 \times 10^{-3}$ between Cape Hatteras and Nova Scotia. With the assumption that $\partial D/\partial \eta \approx 0$, or at least $\partial D/\partial \eta \ll \partial D/\partial \xi$, it follows from (37) that

$$V \frac{\partial f}{\partial y} \cos 45° + g \left(D \frac{\partial \bar{\rho}}{\partial \eta} + \bar{\rho} \frac{\partial \zeta}{\partial \eta} \right) \frac{\partial D}{\partial \xi} = 0 \ . \qquad (38)$$

This equation relates the (average) density distribution and the slope of the sea surface in the direction of the flow with the slope of the layer of no motion perpendicular to the flow and the vertically integrated mass transport of the Gulf Stream while moving northeastward.

Suppose that there is no compensation of $\partial \zeta / \partial \eta$ by the density distribution $\rho(\eta)$, and, therefore, in the direction of flow $\partial \bar{\rho}/\partial \eta \approx 0$. Then $\partial \zeta / \partial \eta$ must be positive, since $V(\partial f/\partial y)$ is a positive quantity, and $\partial D/\partial \xi < 0$. The order of magnitude of $\partial \zeta / \partial \eta$ can be estimated with the assumptions made before, and

$$\frac{\partial \zeta}{\partial \eta} \approx \frac{V \frac{\partial f}{\partial y} \cdot \cos 45°}{g\bar{\rho} \cdot 2.5 \cdot 10^{-3}} = V \frac{f}{R} \operatorname{ctn} \bar{\varphi} \ \frac{\cos 45°}{g\bar{\rho} \cdot 2.5 \cdot 10^{-3}} \qquad (39)$$

between Cape Hatteras and Nova Scotia where $\bar{\varphi} = 39°$, and R = 6370 km is the radius of the earth. According to H.U. Sverdrup et al.(1946) the total mass transport of the Gulf Stream towards the NE equals 55 Mill. m^3 per sec. With a total width of the current of, approximately, 250 km,

$$V \approx 2.2 \times 10^6 \; [\mathrm{gr\ cm^{-1}\ sec^{-1}}],$$

and from (39)

$$\partial\zeta / \partial\eta \approx 1.15 \times 10^{-7} \; .$$

This compares well with the observed magnitude of the slope between Cape Hatteras and Nova Scotia (Sverdrup et al., 1946). If this slope did not exist, and the assumptions on which (38) is based hold, ρ would have to increase toward the NE in the axis of the flow.

If the term $V(\partial f/\partial y)$ would be disregarded, it is seen from (38) that an upward slope in the direction of flow must be compensated for by a decrease of the average density of the Gulf Stream water off the continental shelf.

It should be kept in mind that equation (38) holds only in the case of vanishing by small friction. It certainly does not apply to the surface layers of frictional influence, and, if present, to a bottom layer of frictional influence (Ekman's bottom current). Similarly, the law of the parallel solenoids does not apply to layers where frictional influences dominate.

c) With a discussion of equation (11) for the horizontal mass transport, the case of a simple zonal model may be considered, where (see section IV)

$$\frac{\partial f}{\partial y} = \frac{f}{D}\frac{\partial D}{\partial y} ,$$

and any effects of meridional boundaries are disregarded. Further, suppose that $\tau_x = \tau_x(y)$, $\tau_y = 0$, $\partial D/\partial x = 0$, $V = 0$. With these assumptions, equation (11) reduces to

$$\frac{\partial^2\psi}{\partial y^2} - \frac{1}{D}\frac{\partial D}{\partial y}\frac{\partial\psi}{\partial y} = \frac{1}{r}\left(\frac{\partial \tau_x}{\partial y} - \frac{\tau_x}{D}\frac{\partial D}{\partial y}\right) ,$$

or

$$\frac{d}{dy}\left(\frac{1}{D}\frac{d\psi}{dy}\right) = \frac{1}{r}\frac{d}{dy}\left(-\frac{\tau_x}{D}\right) \quad .$$

By integration, it follows that

$$U = \frac{d\psi}{dy} = -\frac{\tau_x}{r} + C \cdot D \ , \qquad (40)$$

where C is a constant, which becomes zero if U = 0 for $\tau_x = 0$. This result shows that the zonal mass transport, U, is proportional to the zonal component of the wind stress, τ_x. This is different from Munk's general result where in a confined ocean of constant depth it was found that the horizontal mass transport of permanent ocean currents depends only on the rotational component of the wind stress field over the ocean. Of course, in an ocean bounded by continents, the assumptions which led to (40) are in general not justified any longer. Even in the case where τ_y may be assumed to be zero, $\partial D/\partial x$ is not necessarily zero, and $V \neq 0$, at least for some distance along the boundaries. The whole system of horizontal mass transport will be modified by the boundary conditions, but besides a dependence of the mass transport on the curl of τ, there will also be a dependence on τ.

Equation (40), however, will hold approximately, in the central parts of the oceans, far away from the boundaries. Probably, it can best be applied to the Antarctic Circum Polar Current, just where W. Munk and E. Palmén (1951) found discrepancies with the results of Munk's theory.

In the Antarctic Circum Polar Current between 65°S and 45°S, according to table 1, $\tau_x \approx 2.0$ dyne/cm^2, and $r \approx 3.34 \times 10^{-6}$ sec^{-1} (see page 30). Thus according to (40)

$$U = 6 \times 10^5 \text{ gr cm}^{-1} \text{ sec}^{-1},$$

and the average total mass transport through a section between 65° and 45°S is, therefore,

$$6 \times 10^5 \times 2.22 \times 10^8 = 1.33 \times 10^{14} \text{ [gr sec}^{-1}\text{]},$$

which agrees well with the observed mass transport. According to A.J. Clowes' (1933) computations the mass transport through the Drake Passage is about 1.1×10^{14} gr sec^{-1}, and it is very probable that the transport in the central parts of the South Atlantic Ocean is approximately 20% greater.

XI. Outline of future work

Equation (11) is being used for determining the horizontal mass transport of the wind driven circulation in the North Atlantic Ocean between the equator and about 55°N. The east and west boundaries of the ocean are the continents. They are approximated as closely as possible by broken straight lines following the continental shelf along the 200 m isobath.

The boundary condition along the continental slopes, which may be identified with the continents themselves, requires that the mass transport be parallel to the boundary curves. Since an additive constant with the stream function ψ makes no difference, the boundary condition is given by $\psi = 0$ along the continents.

However, since this model is not completely enclosed by land, more complicated conditions have to be introduced along the northern and southern boundaries, and between the Antilles Arc and Florida. The Guiana Current along the northeast coast of South America transports an appreciable amount of water across the equator which enters the model ocean in the southwest corner. This fact cannot be disregarded. It has to be taken into account as a "source" by the boundary conditions along the equator. Another "source", which is probably very important, is the flow of water through the Straits of Florida.

Along the northern boundary a "source" has to be considered northeast of the Grand Banks due to the inflow of the Labrador Current. An important "sink" between Great Britain and Iceland is given by the transport of the North Atlantic Current which accounts for a net balance of the water in the region covered by the model. The total net intake of water between the equator and the northern boundary, that is, the sum of the "sources" and "sinks", must be zero.

For a numerical integration of the differential equation (11) the surface of the ocean is thought to be covered by a series of lattice points as given by the intersections of a set of rectangular lines in the north-south and the east-west direction. The lattice of this net consists of "interior" points and "boundary-points". Interior points are called such points which are surrounded by adjacent points belonging to the lattice of the covered ocean area. An interior point, therefore, is always surrounded by four adjacent points, whereas a "boundary point" has less than four adjacent points. The unit mesh width of the lattice was chosen as d = 222 km; however, in the western North Atlantic it may be necessary to use a smaller unit (55.5 km) in order to get some of the details of the Gulf Stream flow pattern. This section will be considered by a special evaluation of the basic differential equation.

The differential equation (11) for the horizontal mass transport can be written in the form

$$r\nabla^2\psi + F(x,y)\frac{\partial\psi}{\partial x} + G(x,y)\frac{\partial\psi}{\partial y} - H(x,y) = 0 ,$$

where the coefficients F, G and H have been plotted as functions of x and y on charts. The wind stress was computed for the month of February by means of formula (33), using the wind charts put out by the Marine Branch of the Meteorological Office, Air Ministry, London (1948).

If the subscripts j,k indicate an interior point, and the adjacent points of the lattice are denoted by subscripts (j,k-1), (j,k+1), (j-1,k), (j+1,k), respectively, the difference equation for the horizontal mass transport becomes

$$r\left(\frac{\psi_{j,k-1} + \psi_{j,k+1} + \psi_{j+1,k} + \psi_{j-1,k} - 4\psi_{j,k}}{d^2}\right) +$$

$$F_{j,k}\left(\frac{\psi_{j,k+1} - \psi_{j,k-1}}{2d}\right) + G_{j,k}\left(\frac{\psi_{j+1,k} - \psi_{j-1,k}}{2d}\right) - H_{j,k} = 0 .$$

Such a difference equation can be established for each interior point. There are as many equations as interior points, and since the number of interior points equals the number of unknowns, with the boundary conditions, ψ is uniquely determined in the interior of the area, provided that the determinant of the system of equations is unequal to zero.

Basic data for the determination of the coefficients F, G and H as functions of x and y, and the results of the numerical integration will be published in the next report.

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