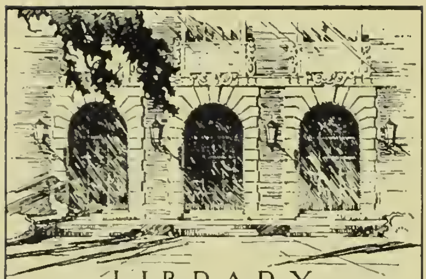

ANTENNA LABORATORY

Technical Report No. 45

NUMERICAL ANALYSIS OF THE EIGENVALUE PROBLEM OF WAVES IN CYLINDRICAL WAVEGUIDES

by

C.H. Tang and Y.T. Lo

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CONTENTS

ILLUSTRATIONS

1. INTRODUCTION

The waves in a cylindrical waveguide are found from solutions of the two dimensional differential equation

$$\nabla^2 u + k^2 u = 0 \qquad (1)$$

where u is a function of the coordinates in the transverse plane and k is a constant. The Dirichlet boundary condition, u = 0 on the boundary of the cross-section, corresponds to Transverse Magnetic modes and the Neumann boundary condition, $\partial u/\partial n = 0$ on the boundary, corresponds to Transverse Electric modes. In (1) k is the wave number and the function u, which is independent of the longitudinal coordinate z, represents the component of electric (TM-case) or magnetic (TE-case) field intensity along longitudinal direction. The time dependence is assumed to be of the form $e^{-i\omega t}$ in this formulation.

The permissible values of k are also called eigenvalues of the differential Equation (1) and u is the eigenfunction. Since the wave is confined to a finite region, the spectrum of the eigenvalues $\{k_n\}$ is a discrete set. According to mode theory, these eigenvalues k_n determine the cut off frequency of each mode propagating along the waveguide. It is necessary only to find the resonant frequency of the two dimensional problem defined by the guide boundary since there is no axial variation at cut-off; all energy does propagate back and forth in the transverse plane.

The exact solution of (1) with prescribed boundary condition can be obtained only when the given boundary constitutes a (or a set of) coordinate surface of the separable coordinate system. It has been shown only few regular cross sections can be treated by the method of separation of variables. Waveguides with odd cross section, yet valuable in practical use (such as folded waveguide and ridged waveguide), demand the result for engineering design.

1.1 Equivalent Network Approach

Some of the "non-separable" problems can be treated by using the equivalent circuit representation of the cross-section[1]. For ridged waveguide, the approximate equivalent circuit is obtained by considering region I (Figure 1) as a capacitor of capacity $C_B = \epsilon S/h$ and regions II as inductances of value $L_A = \mu \ell b$

$$f_C = \frac{1}{2\pi \sqrt{C_B L_A/2}} \tag{2}$$

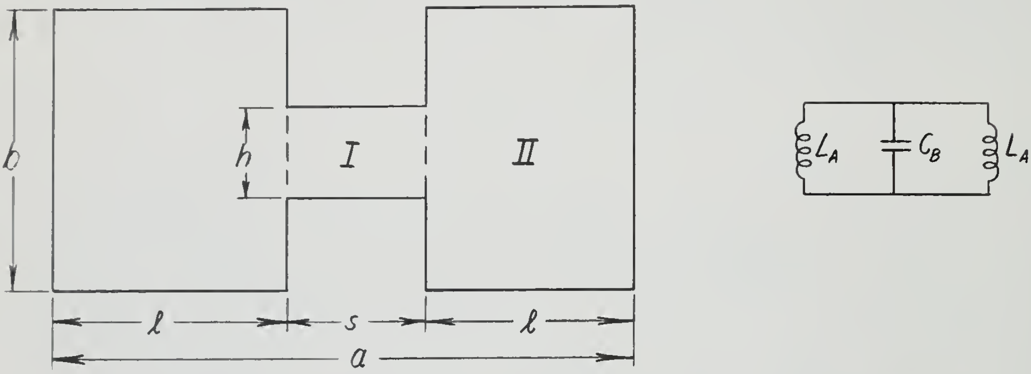


Figure 1. Ridged Waveguide and Its Equivalent Network

It is seen that it applies only for the case of b >> h.

More accurate equivalent representation for cut off calculation has bee obtained by considering the ridges as two step discontinuities[2] (Figure 2) where the step susceptance B is approximated by using that of a capacitive window (Figure 3).

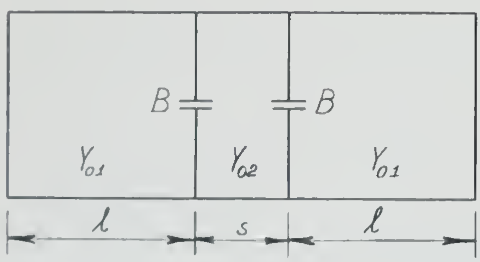


Figure 2. Equivalent Circuit in the Transverse Plane of a Ridged Waveguide

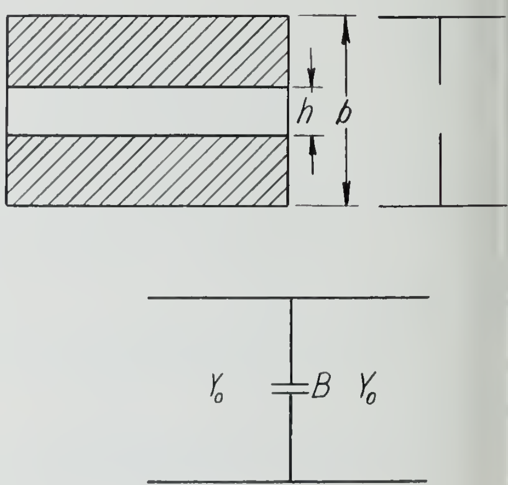


Figure 3. Window in a Rectangul Guide and Its Equival Circuit

Then by symmetry, the resonant input function at the center is either zero or infinity. Thus we have

$$\cot \beta \ell - \frac{b}{d} \tan \frac{\beta s}{2} - \frac{B}{Y_{01}} = 0 \qquad \text{For } TE_{no} \quad n = \text{odd} \tag{3}$$

$$\cot \beta \ell + \frac{b}{d} \cot \frac{\beta s}{2} - \frac{B}{Y_{01}} = 0 \qquad \text{For } TE_{mo} \quad m = \text{even} \tag{4}$$

where $$Y_{01} = \frac{\beta}{\omega\mu} \frac{1}{b} \qquad Y_{02} = \frac{\beta}{\omega\mu} \frac{1}{d}$$

Numerical solutions of above transcendental Equations (3) and (4) can be obtained by tabulation. It is again restricted to the wavelength range of $2b/\lambda < 1$ for the single ridge and $b/\lambda < 1$ for the double ridges.

1.2 Numerical Methods

The approximate solution of (1) with "non-separable" boundary can also be obtained from numerical analysis; such as the variational and finite difference methods or the analogue method of a network analyzer[4].

1.2.1 Variational Method

The approximate eigenvalue of the wave equation can be obtained by using the approximated Rayleigh-Ritz formula

$$k_1^{(o)^2} \approx - \frac{\Sigma\, u^{(o)}\, \nabla^2\, u^{(o)}}{\Sigma\, u^{(o)^2}} \tag{5}$$

We start with the unperturbed eigenfunction $u^{(o)}$ in the given cross-section (which corresponds to the eigenfunction of the rectangular guide of same aspect ratio, but without ridge). After obtaining the first approximation value of $k_1^{(o)}$, the wave equation in finite difference form can be used as a formula to obtain the higher order value of the eigenfunction $u^{(1)}$ (the formula will be derived later). Hence the iterative process consists of successive corrections between the value $k^{(i)}$ and eigenfunction $u^{(i)}$.

It has been shown that the formula (5) always converges to the lowest eigenvalue. For higher order eigenvalue, say the second, the first term of the orthogonal expansion $\Sigma\, C_n\, u_n$ must be eliminated from the assumed function

$u^{(o)}$. By using the orthogonality properties of these normal functions, C_1 can be determined, and $(u^{(o)} - C_1 u_1)$ is then used for the computation of th second eigenvalue.

A combination of variation and relaxation methods has been used by Blacl and von Rohr[3] to calculate the cut off wavelength of semicircular ridges in rectangular waveguide. A typical sequence of values for k^2 has been given as $11.1/a^2$, $8.3/a^2$ and $7.7/a^2$ for nets containing respectively 19, 97 and 42 pts. (where a is the broad face demension of the guide), with the error clai to be less than 2 percent.

2. FINITE DIFFERENCE METHOD

Another way of solving this particular boundary value problem is by the use of finite differences*. The differential operator is first approximated by a finite difference formula. Then by setting up a finite number of mesh points, we transform, approximately, the wave equation and the boundary condition into a matrix eigenvalue problem. From the matrix we get a set of approximate eigenvalues corresponding to a set of cut-off frequencies of the particular waveguide structure.

The study reported here was initiated to investigate the practicality of using a general purpose, high speed digital computer to perform the calculation of cut-off frequencies for cylindrical waveguides with irregular cross-section.

The Laplacian ∇^2 operating on a function u as in the scalar Helmholtz equation, can be replaced by a set of finite difference approximations relating the values of the function at the nodes of a mesh pattern such as shown in Figure 4.

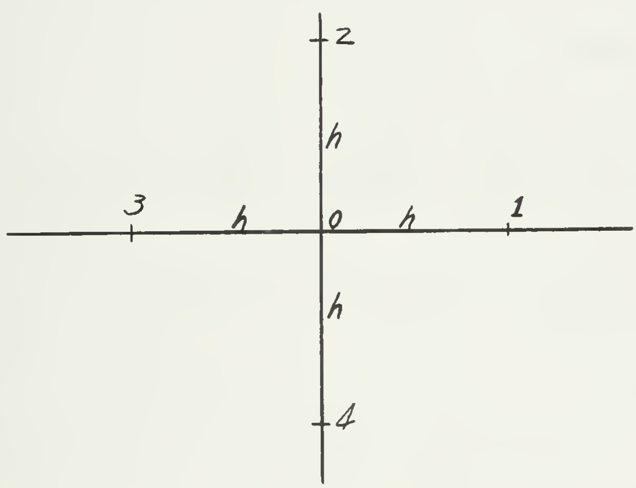


Figure 4. Mesh Pattern for Finite Difference

* The application of the finite difference method to ridge waveguide problems was initiated in this laboratory by Professor E. J. Scott.

From a Taylor's series expansion we have

$$u_1\Big|_o = u_o + \left(\frac{\partial u}{\partial x}\right)_o h + \frac{1}{2!}\left(\frac{\partial^2 u}{\partial x^2}\right)_o h^2 + \frac{1}{3!}\left(\frac{\partial^3 u}{\partial x^2}\right) h^3 + 0(h^4)$$

$$u_3\Big|_o = u_o - \left(\frac{\partial u}{\partial x}\right)_o h + \frac{1}{2!}\left(\frac{\partial^2 u}{\partial x^2}\right)_o h^2 - \frac{1}{3!}\left(\frac{\partial^3 u}{\partial x^2}\right) h^3 + 0(h^4)$$

adding these two equations and neglecting the terms after the third gives:

$$h^2 \left[\frac{\partial^2 u}{\partial x^2}\right]_o = u_1 + u_3 - 2u_o + 0(h^4)$$

Similarly $$h^2 \left[\frac{\partial^2 u}{\partial y^2}\right]_o = u_2 + u_4 - 2u_o + 0(h^4)$$

Hence $$h^2 [\nabla^2 u]_o = u_1 + u_2 + u_3 + u_4 - 4u_o + 0(h^4) \qquad (6)$$

For a curved boundary we have

$$\nabla^2 u \approx \frac{1}{h^2}\left[\frac{2\,u_B}{\xi(1+\xi)} + \frac{2\,u_C}{\eta(1+\eta)} + \frac{2\,u_3}{(1+\xi)} + \frac{2\,u_4}{(1+\eta)} - \left(\frac{2}{\xi} + \frac{2}{\eta}\right) u_o\right] \qquad (7)$$

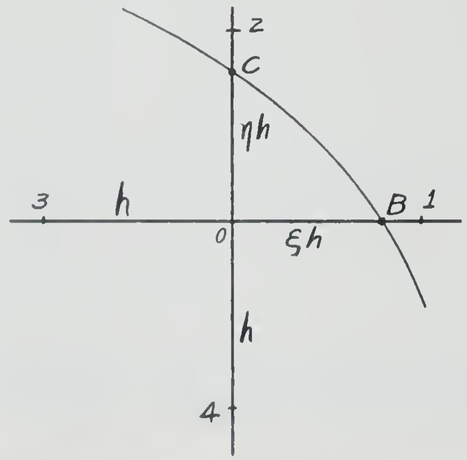


Figure 5. Mesh Pattern with a Curved Boundary

Thus for an ordinary point the wave equation becomes

$$u_1 + u_2 + u_3 + u_4 + (\alpha - 4)\, u_o = 0 \tag{8}$$

In terms of mesh pattern, ∇^2 can be expressed as

$$\nabla^2 = \frac{1}{h^2} \begin{array}{|c|c|c|} \hline & 1 & \\ \hline 1 & -4 & 1 \\ \hline & 1 & \\ \hline \end{array} \tag{9}$$

and

$$\alpha = h^2 k^2 \tag{10}$$

Setting up a suitable number of meshes for a given cross section, and applying the above procedure for each point, we would have as many simultaneous equations as the number of points in the cross section:

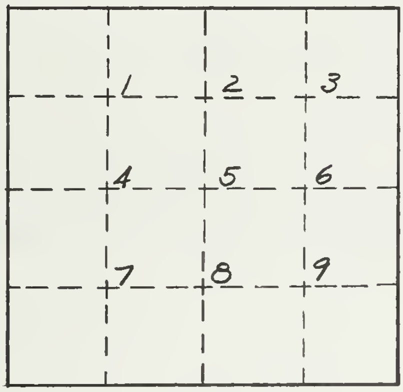


Figure 6. Mesh Points in a Given Cross-Section

The difference equations corresponding to points 1, 2, and 5 with Dirichlet boundary condition are respectively

$$
\begin{array}{llllll}
(\alpha-4)\,u_1 & +\ u_2 & +\ u_4 & & & = 0 \\
u_1 & +\ (\alpha-4)\,u_2 & +\ u_3 & +\ u_5 & & = 0 \\
- & - & - & - & & - \\
u_2 & +\ u_4 & +\ (\alpha-4)\,u_5 & +\ u_6 & +\ u_8 & = 0
\end{array}
\tag{11}
$$

Point 1 is a corner point, point 2 is an ordinary boundary point and point 5 is an interior point.

Writing in matrix form, we get the general formula

$$\bar{\bar{A}}\ \bar{\bar{u}} = \alpha\ \bar{\bar{u}} \tag{12}$$

where $\bar{\bar{A}}$ is the matrix with its element a_{ij} corresponding to the coefficient of u_j at ith equation, $\bar{\bar{u}}$ is the eigenvector $(u_1\ u_2\ \ldots\ u_n)$. In order to get the non-zero eigenvector $\bar{\bar{U}}$, we set

$$\det\,(\bar{\bar{A}} - \alpha\ \bar{\bar{I}}) = 0 \tag{13}$$

where $\bar{\bar{I}}$ is the identity matrix. This equation leads to the set of eigenvalues, $\left\{ \alpha_n \right\}$, corresponding to the roots of the nth order polynomial derived from (13

The relative cutoff wavelength λ_i/a, of the ith mode, in terms of the eigenvalues will be

$$\frac{\lambda_i}{a} = \frac{2\pi\ h}{a\sqrt{\alpha_i}} \tag{14}$$

2.1 Approximations

The approximations involved in the finite difference method are

(1) Finite mesh size, h

(2) Truncation error in Taylor series expansion

(3) Approximation in Neumann boundary condition which will be discussed in 2.3.

2.2 Higher Order Formula

In order to improve the accuracy of the result, especially in the TE case, more terms in the Taylor series expansion may be taken into account. The point pattern representation (see Figure 4) of the Laplacian when neglecting the terms after the third is

$$\nabla^2 = \frac{1}{h^2}\;\begin{array}{|c|c|c|}\hline & 1 & \\ \hline 1 & -4 & 1 \\ \hline & 1 & \\ \hline\end{array} \qquad (9)$$

that for neglecting the terms after the fifth is[5]

$$\nabla^2 = \frac{1}{840\ h^2}\;\begin{array}{|c|c|c|c|c|}\hline -3 & -16 & -32 & -16 & -3 \\ \hline -16 & 176 & 800 & 176 & -16 \\ \hline -32 & 800 & -3636 & 800 & -32 \\ \hline -16 & 176 & 800 & 176 & -16 \\ \hline -3 & -16 & -32 & -16 & -3 \\ \hline\end{array} \qquad (15)$$

Due to the complexity of (15) and the fact that it results in an unsymmetrical matrix in the Neumann problem, the derivation of an alternative formula is desired.

Define the following operators:

$$E\ f(x) = f(x + h)$$

$$D\ f(x) = f'(x)$$

$$\delta\ f(x) = f(x + \frac{h}{2}) - f(x - \frac{h}{2})$$

hence

$$E^n\ f(x) = f(x + nh)$$

$$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$$

$$E^{\frac{1}{2}} = (1 + \frac{1}{4}\ \delta^2)^{\frac{1}{2}} + \delta$$

By Taylor series expansion

$$E\ f(x) = f(x+h) = \left[1 + \frac{hD}{1!} + \frac{h^2D^2}{2!} + \frac{h^3D^3}{3!} + \dots\right] f(x)$$

we obtain

$$E = e^{hD}$$

$$hD = \log E = 2 \log\left[(1 + \frac{1}{4}\delta^2)^{\frac{1}{2}} + \frac{1}{2}\delta\right] = 2 \sin h^{-1} \frac{\delta}{2}$$

$$= \left(\delta - \frac{1^2}{2^2 \cdot 3!}\delta^3 + \frac{1^2 \cdot 3^2}{2^4 \cdot 5!}\delta^5 - \frac{1^2 \cdot 3^2 \cdot 5^2}{2^6 \cdot 7!}\delta^7 + \dots\right)$$

hence $$D^2\, u = u'' = \frac{1}{h^2}\left[\delta^2 - \frac{1}{12}\delta^4 + \frac{1}{90}\delta^6 - \frac{1}{560}\delta^8 + \dots\right] u$$

$$\approx \frac{1}{h^2}\delta^2\left[1 - \frac{1}{12}\delta^2\right] u \approx \frac{1}{h^2}\delta^2 \frac{1}{1 + \frac{1}{12}\delta^2} u \tag{16}$$

The approximate formula is thus obtained

$$h^2\left[1 + \frac{1}{12}\delta^2\right] u'' = \delta^2 u \tag{17}$$

For the one dimensional wave equation

$$u'' + k^2 u = 0 \tag{18}$$

we get

$$\delta^2 u = -h^2 k^2 \left[1 + \frac{\delta^2}{12}\right] u = -\alpha\left[1 + \frac{\delta^2}{12}\right] u$$

hence $$u_{n+1} - 2u_n + u_{n-1} = -\alpha u_n - \frac{\alpha}{12}\left[u_{n+1} - 2u_n + u_{n-1}\right]$$

or $$\left(1 + \frac{\alpha}{12}\right) u_{n+1} - \left(2 - \frac{5}{6}\alpha\right) u_n + \left(1 + \frac{\alpha}{12}\right) u_{n-1} = 0 \tag{19}$$

Similarly, for a two dimensional wave equation, we get

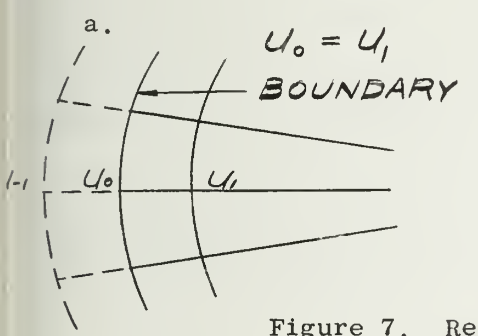
$$24 \begin{array}{|c|c|c|} \hline 1 & 4 & 1 \\ \hline 4 & -20 & 4 \\ \hline 1 & 4 & 1 \\ \hline \end{array} \; u = -\alpha \begin{array}{|c|c|c|} \hline 1 & 10 & 1 \\ \hline 10 & 100 & 10 \\ \hline 1 & 10 & 1 \\ \hline \end{array} \; u \qquad (20)$$

The order of approximation of (20) corresponds to that of (15).

2.3 Approximation in Neumann Boundary Condition

In the Neumann boundary condition, several possible approximations are considered. Let u_o be the value of u at a boundary point adjacent to an interior mesh point with value u_1 and an exterior point with value u_{-1}

a.



$$u_1 = u_o + h\,u_o' + \frac{h^2}{2!}\,u_o'' + \ldots$$

$$u_o' = 0$$

$$u_1 = u_o + O(h^2) \qquad (21)$$

Figure 7. Relation of Interior, Exterior and Boundary Point.

b.

$$u_{-1} = u_1$$

$$u_1 = u_o + h\,u_o' + \frac{h^2}{2!}\,u_o'' + \frac{h^3}{3!}\,u_o''' + O(h^4)$$

$$u_{-1} = u_o - h\,u_o' + \frac{h^2}{2!}\,u_o'' - \frac{h^3}{3!}\,u_o''' + O(h^4)$$

$$u_o' = 0$$

$$u_1 = u_{-1} + O(h^3) \qquad (22)$$

c. In this case, we relate the wave equation to the boundary approximation as following:

$$u_1 - u_o = (E-1)\,u_o = E\,u_o - u_o = e^{hD}\,u_o - u_o$$

$$= [1 + hD + \frac{h^2D^2}{2!} + \ldots]\,u_o - u_o$$

$h^2D^2 \quad h^3D^3$

hence with the wave equation relation $u_o'' = -k^2 u_o$ we have

$$u_1 - u_o = h\,u_o' + \frac{h^2}{2!}\,u_o'' + \frac{h^3}{3!}\,u_o''' + \ldots$$

$$= h\,u_o' - \frac{h^2 k^2}{2!}\,u_o - \frac{h^3}{3!}\,k^2\,u_o' + \ldots$$

$$= h\,u_o' - \frac{\alpha}{2}\,u_o - \frac{\alpha h}{6}\,u_o' + \ldots$$

Since $$u_o' = 0$$

$$u_1 = \left[1 - \frac{\alpha}{2} + \frac{\alpha^2}{24}\ \ldots\right] u_o$$

$$u_1 = \left(1 - \frac{\alpha}{2}\right) u_o + 0(h^3) \qquad (23)$$

2.4 Computational Procedure

The computational procedure used in this finite difference method is as follows

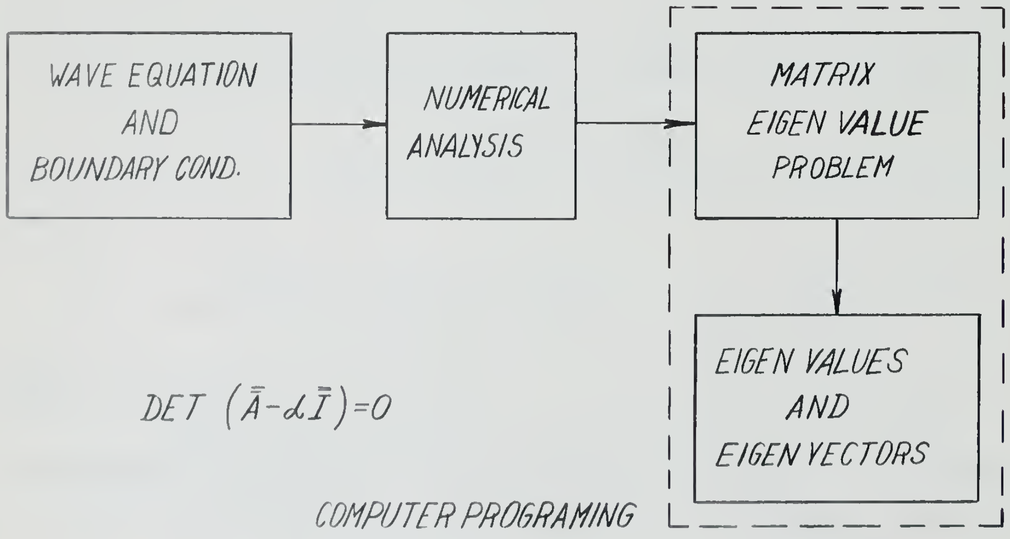


Figure 8. Computation Procedure Diagram

The matrix $\bar{\bar{A}}$ is first scaled so that its norm is less than 1; i.e.,

$$\sum_{ij} a_{ij}^2 < 1$$

Then by using the ILLIAC, the University of Illinois digital computer, we can determine the eigenvalues α up to a maximum matrix size of 128 × 128 (Library Routine M20-234, Digital Computer Laboratory, University of Illinois) and we can determine both the eigenvalue α and eigenvector U up to a maximum matrix size of 40 × 40 (Library Routine M18-213). However, in this procedure, we are restricted by the present technique of computer programming to the symmetric matrix, since the possibility of complex eigenvalue for an unsymmetrical matrix makes the programming much more involved.

In some cases, the results thus obtained can be very simply improved by using the Richardson's extrapolation[6].

It is reasonable to suppose that the error in the approximation is a function of mesh size h. If this function is expanded into a Taylor series we can write

$$\lambda_o - \lambda(h) = B\,h + c\,h^2 + \ldots \tag{24}$$

where

$$\lambda_o = \lim_{h \to o} \lambda(h)$$

we assume that the term B h is the major part of the error. If we calculate two approximate solutions λ_1 and λ_2 with different mesh size h_1 and h_2 respectively then

$$h_2(\lambda_o - \lambda_1) - h_1(\lambda_o - \lambda_2) = h_2(ch_1^2 + \ldots) - h_1(ch_2^2 + \ldots)$$

solving for λ_o, we obtain the extrapolation formula

$$\lambda_o = \frac{1}{h_2 - h_1}\left[h_2\,\lambda_1 - h_1\,\lambda_2\right] - c\,h_1\,h_2 + \ldots \tag{25}$$

If the error function can be expressed as an even function of h

$$\lambda_o - \lambda(h) = c\,h^2 + E\,h^4 + \ldots \tag{26}$$

which is the case for our wave equation approximation, we have

$$\lambda_o = \frac{h_2^{\,2} \lambda_1 - h_1^{\,2} \lambda_2}{h_2^{\,2} - h_1^{\,2}} + O(h_1^{\,2}\, h_1^{\,2}) \tag{27}$$

For a set of data which increases monotonically

$$\lambda_2 > \lambda_1 \qquad \text{if } h_2 > h_1$$

the extrapolated value λ_e obtained by dropping the higher order terms in (27), becomes

$$\lambda_e = \lambda_2 + \frac{\lambda_2 - \lambda_1}{h_1^{\,2} - h_2^{\,2}}\, h_2^{\,2} > \lambda_2 \tag{28}$$

If $h_2 \longrightarrow o$ then λ_e approaches λ_2 which in turn approaches the exact value λ_o as seen from (27).

On the other hand, for a set of data decreasing monotonically

$$\lambda_1 > \lambda_2 \qquad \text{if } h_2 < h_1$$

$$\lambda_e = \left(\lambda_2 - \frac{\lambda_1 - \lambda_2}{h_1^{\,2} - h_2^{\,2}}\, h_2^{\,2}\right) < \lambda_2 \tag{29}$$

As $h_2 \longrightarrow o$, the same conclusion as before is reached. In many cases, this method shows a great improvement with practically no further labor added in the overall computation.

By using two data λ_1, λ_2, we can eliminate the necessary knowledge of coefficient c; therefore, the result is accurate to $O(h^4)$. By generalizing this idea with a set of a data $\lambda_1, \lambda_2, \ldots, \lambda_n$, it seems that n coefficients can be eliminated and the result will be accurate to $O(h^{2n})$. This is probably better than repeatedly using the same formula for just a pair of data at a time as has been done later. This may also explain why the extrapolated value for small h (such as h = 1/12, and 1/14) is better than that from all the h's as shown later.

2.5 Study of Convergence

The convergence of the above methods has been tested by the application to a rectangular guide where the exact solution is available for comparison. When the first approximation formula (9) is applied to the wave Equation (1), it shows a better result for the TM case than that for the TE case (Figures 9, 10). It is believed that this is a result of poor approximation in the boundary condition for the TE case. In fact, in the TM case there is no approximation for the boundary condition.

Table I and Figure 9 and 10 show the results (from ILLIAC) for a particular rectangular waveguide (b/a = 0.5) as compared with exact solution.

Table II shows results obtained by repeatedly using the extrapolation Formula (27). They show a remarkable improvement over the results of Table I.

Figures 11 and 12 show the difference between exact relative cut off wavelength and the calculated value where λ_c/a are obtained from the finite set of eigenvalue $\left\{\alpha_h\right\}$.

2.6 Ridge Waveguide

Many authors[7,8] have shown that the insertion of rectangular ridges have the following effects

1. A decrease in the lowest cut off frequency.
2. An increase in mode separation.
3. An increase in attenuation.
4. A concentration of electric field intensity at corners of the ridges.

The results obtained in our calculation have shown good agreement with the data in existing literature.

A typical example shows, for

$$\frac{b}{a} = 0.625 \qquad \frac{s}{a} = 0.375 \qquad \frac{2d}{b} = 0.4 \qquad \text{with net point} = 22$$

mode	TE_{10}	TE_{01}	TE_{11}	TE_{20}	TM_{11}	TM_{21}	TM_{12}	TM_{22}
λ_c/a	2.541	1.059	1.051	1.038	0.675	0.638	0.468	0.466

where the ridge guide modes are given the same designation as the corresponding modes in the rectangular guide. Figures 13 and 14 show the method of mode designations for ridge guide where the boundary condition are shown for only 1/4 of the cross section. The eigenfunctions for this particular ridge guide have also been obtained. Figure 15, 16, and 17 indicate that the largest cut-off wavelength (i.e., dominant mode which corresponds to smallest eigenvalue) are increased due to the distortion of the field distribution.

Sets of higher order mode cut-off wavelength are given in Table III.

Figure 18 shows the difference between two sets of cut-off wavelength obtained by using different numbers of mesh points, namely 9 points and 57 points.

Figure 19 shows the convergence curve for a particular ridge size.

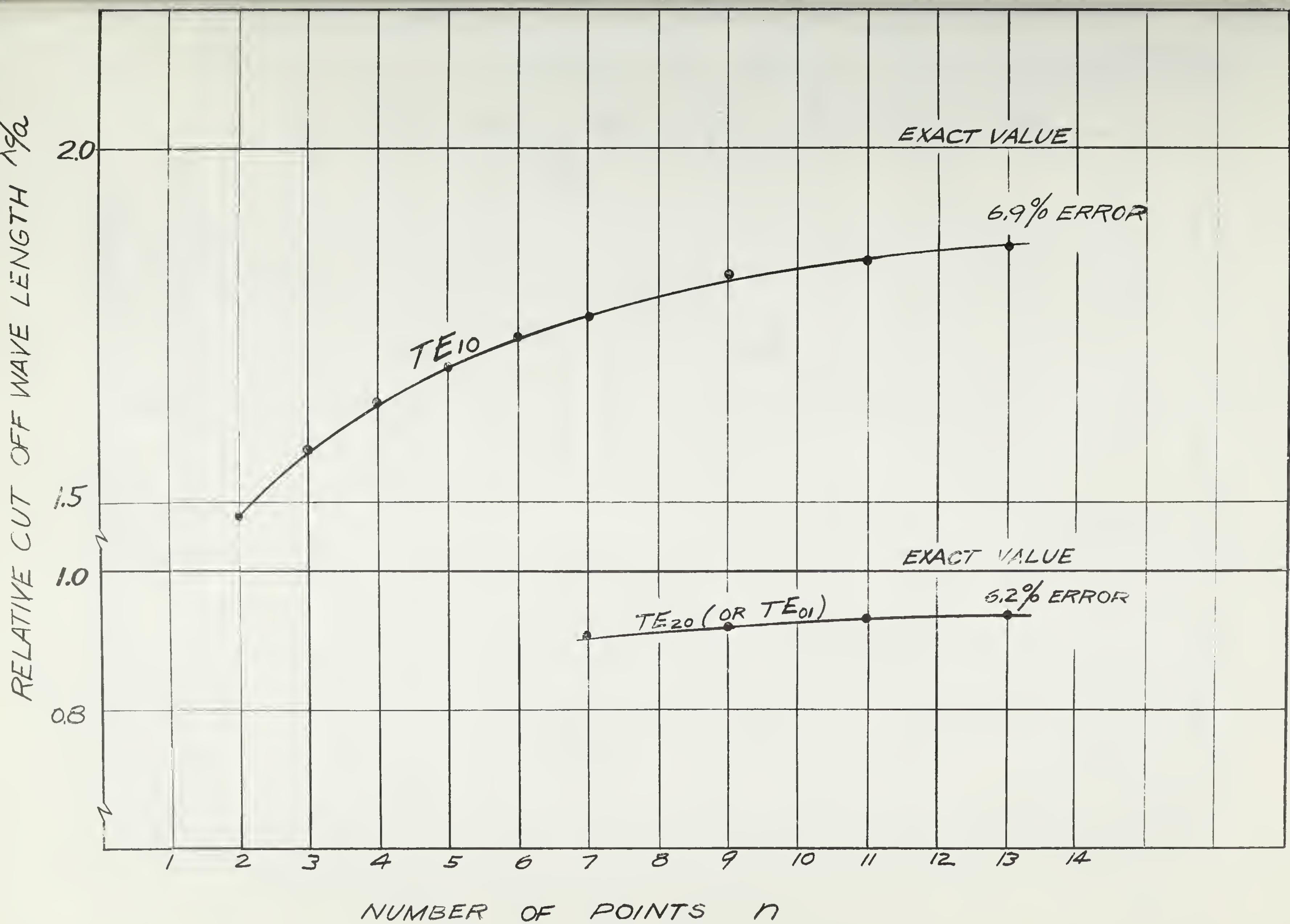


Figure 9. Convergence Curves for TE_{10} and TE_{20}

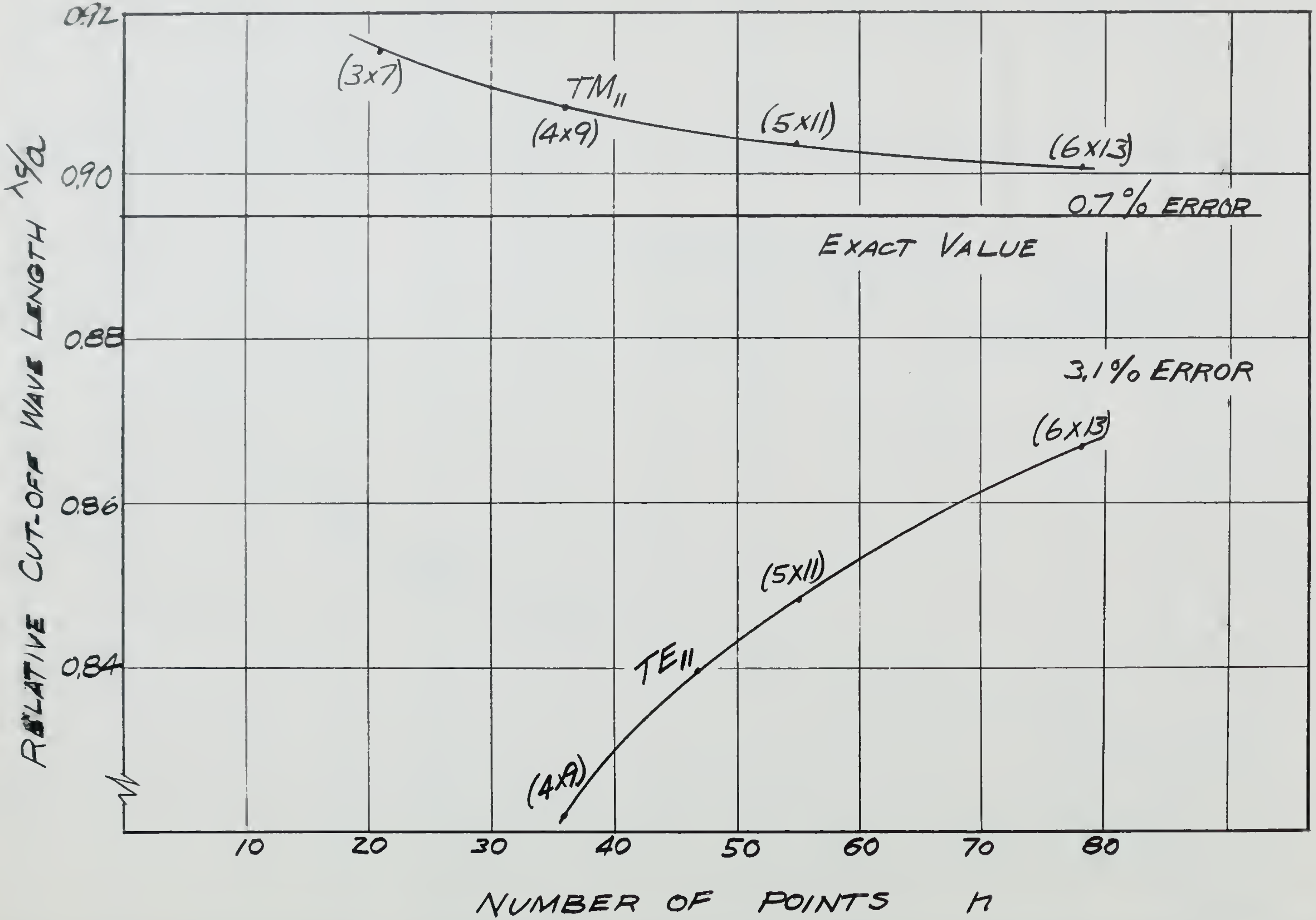


Figure 10. Convergence Curves For TE_{11} and TM_{11}

TABLE I

Set of Approximated Cut Off Wavelengths

For a Rectangular Waveguide

(p×q)	(3×7)		(4×9)		(5×11)		(6×13)		Exact Solution		$\frac{\lambda_c}{a} = \frac{2}{\sqrt{m^2+4n^2}}$	
λ_c/a	TE	TM	TE	TM	TE	TM	TE	TM	TE	TM	m, n	
	∞		∞		∞		∞		∞		0,0*	
	1.765		1.82		1.84		1.861		2		1,0	
	0.905		0.919		0.931		0.938		1		2,0	0,1
	0.785	0.915	0.822	0.908	0.848	0.904	0.867	0.901	0.895		1,1	
	0.718	0.726	0.748	0.72	0.77	0.716	0.787	0.714	0.708		2,1	
	0.63		0.628		0.631		0.637		0.667		3,0	
	0.594	0.583	0.613	0.573	0.626	0.567	0.633	0.564	0.555		3,1	
	0.503		0.499		0.507		0.511		0.5		4,0	0,2
	0.491	0.535	0.49	0.517	0.485	0.507	0.483	0.501	0.488		1,2	
	0.454	0.488	0.445	0.473	0.446	0.465	0.449	0.46	0.448		2,2	4,1
	0.44	0.437	0.433	0.423	0.433	0.416	0.437	0.412	0.4		3,2	5,0
	0.436	0.429		0.407		0.396		0.389	0.372		5,1	
	0.424	0.416		0.382		0.371		0.366	0.354		4,2	
	0.405								0.333		6,0	0,3
	0.403	0.393		0.378		0.365		0.355	0.329		1,3	
	0.381	0.373		0.363		0.349		0.339	0.316		2,3	6,1
	0.368	0.365							0.312		5,2	
	0.358	0.36							0.298		3,3	
	0.346								0.285		7,0	
	0.314	0.338							0.278		4,3	6,2
	0.301	0.325							0.274		7,1	
	(21)		(36)		(55)		(78)					

* There is no field corresponding to this particular eigenvalue.

TABLE II

Results Obtained by Extrapolation

1. TE_{10} Exact value 2

h	α	λ_c/a								
1/3	2.0	1.48								
			1.84							
1/4	1.0	1.57		1.88						
			1.925		1.91					
1/5	0.585	1.641		1.945		1.918				
			1.965		1.942		1.936			
1/6	0.381	1.695		1.953		1.96		1.96		
			1.94		1.97		1.984		1.947	
1/7	0.269	1.73		1.975		1.999		1.975		1.966
			2.01		2.008		1.98		1.977	
1/8	0.198	1.765		2.03		1.995		1.983		
			2.04		1.994		1.987			
1/10	0.1206	1.82		1.99		1.992				
			1.94		1.989					
1/12	0.081	1.84		1.961						
			1.987							
1/14	0.0581	1.861								

2. TE_{20} Exact value 1

h	α	λ_c/a			
1/8	0.753	0.905			
			0.977		
1/10	0.468	0.919		0.983	
			0.991		0.982
1/12	0.317	0.931		0.967	
			0.98		
1/14	0.229	0.938			

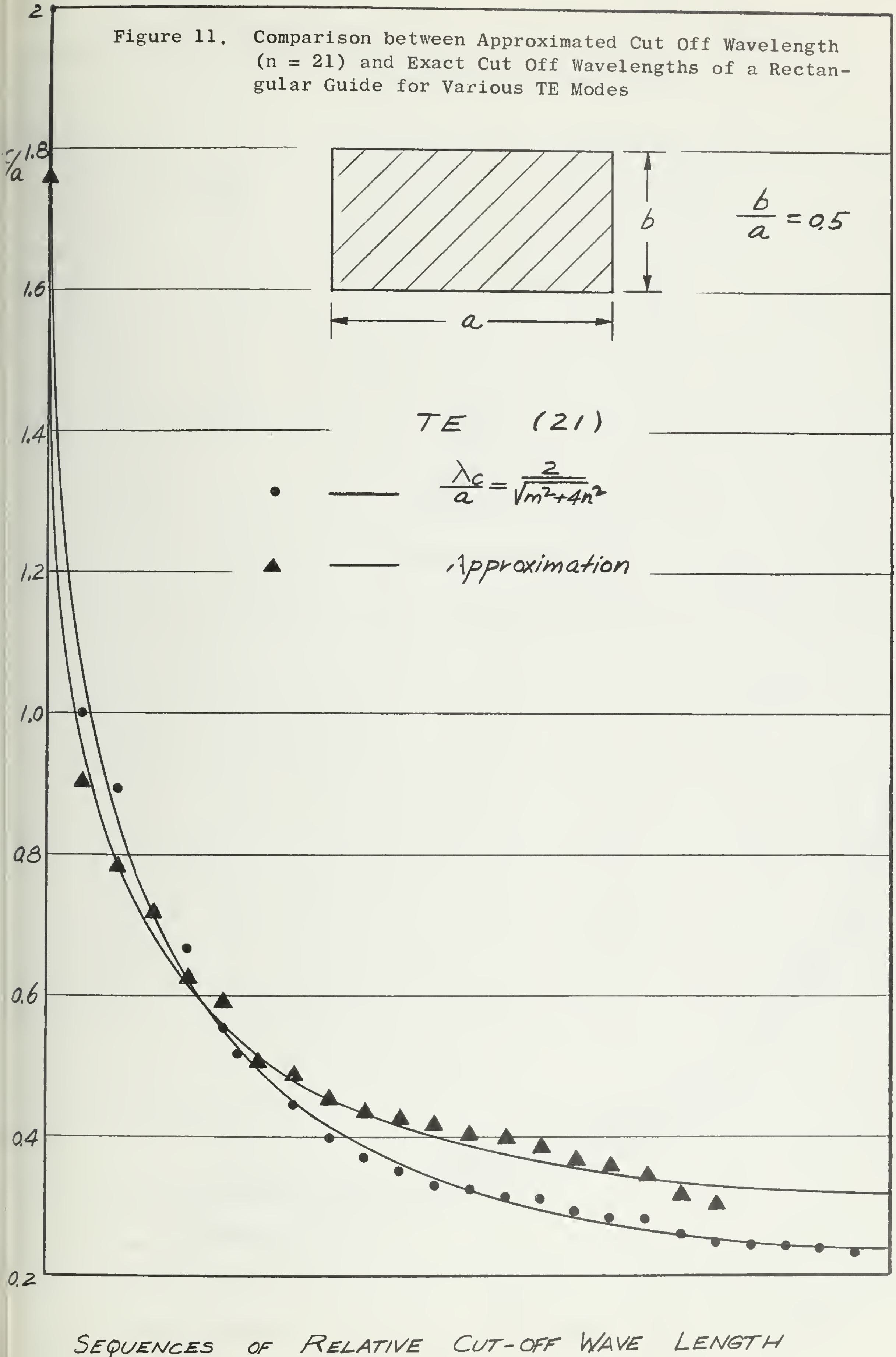


Figure 11. Comparison between Approximated Cut Off Wavelength (n = 21) and Exact Cut Off Wavelengths of a Rectangular Guide for Various TE Modes

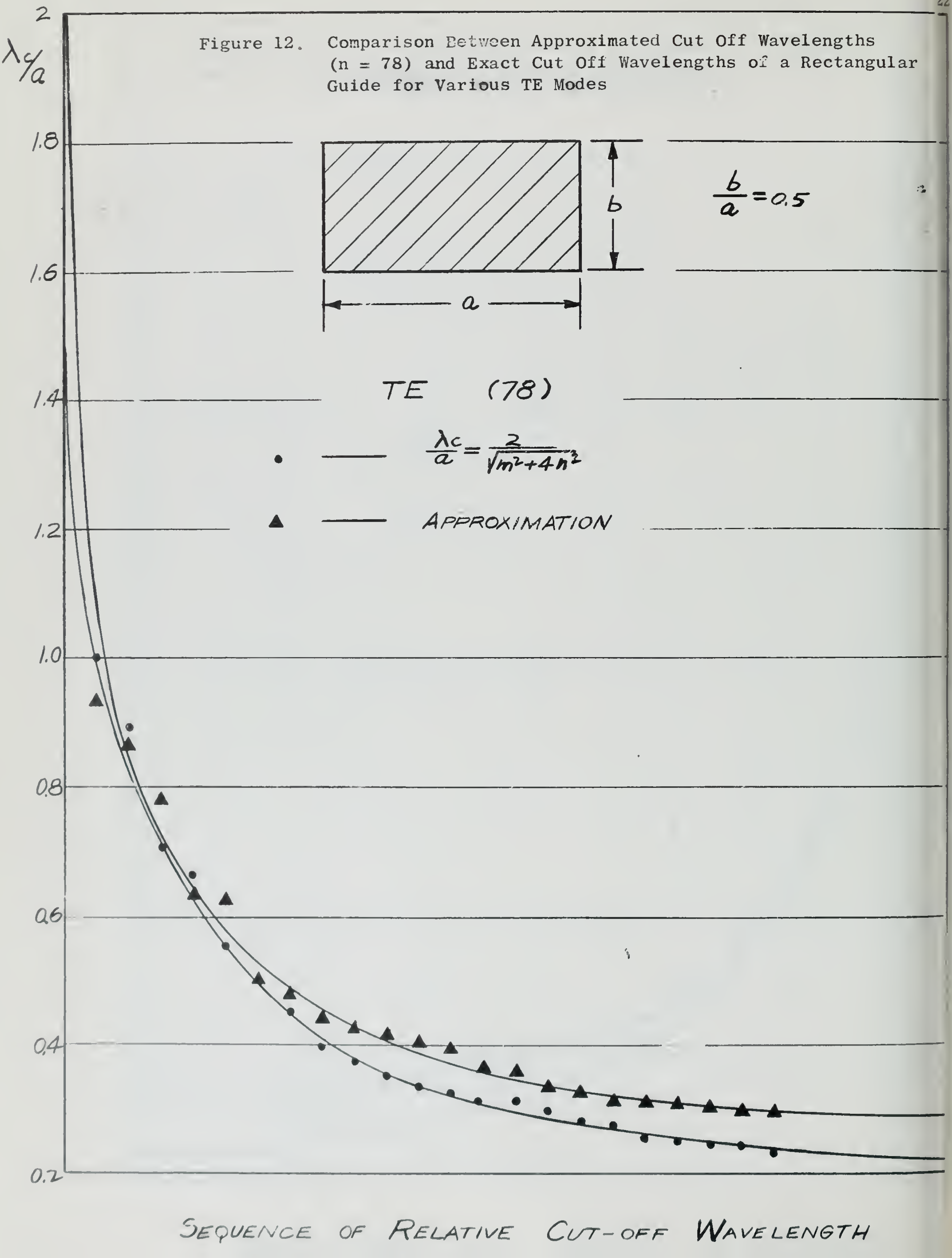


Figure 12. Comparison Between Approximated Cut Off Wavelengths (n = 78) and Exact Cut Off Wavelengths of a Rectangular Guide for Various TE Modes

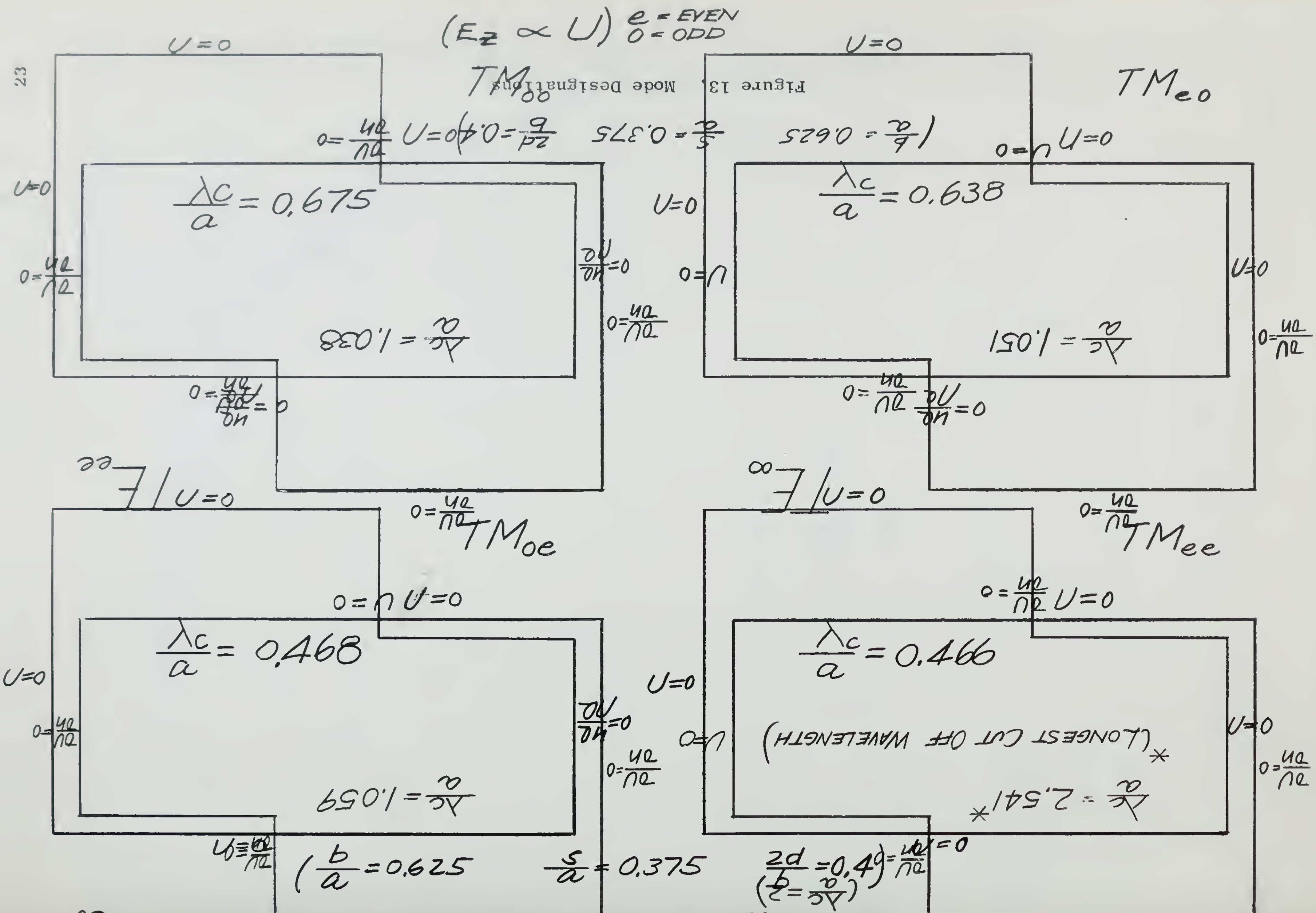


Figure 14. Mode Designations

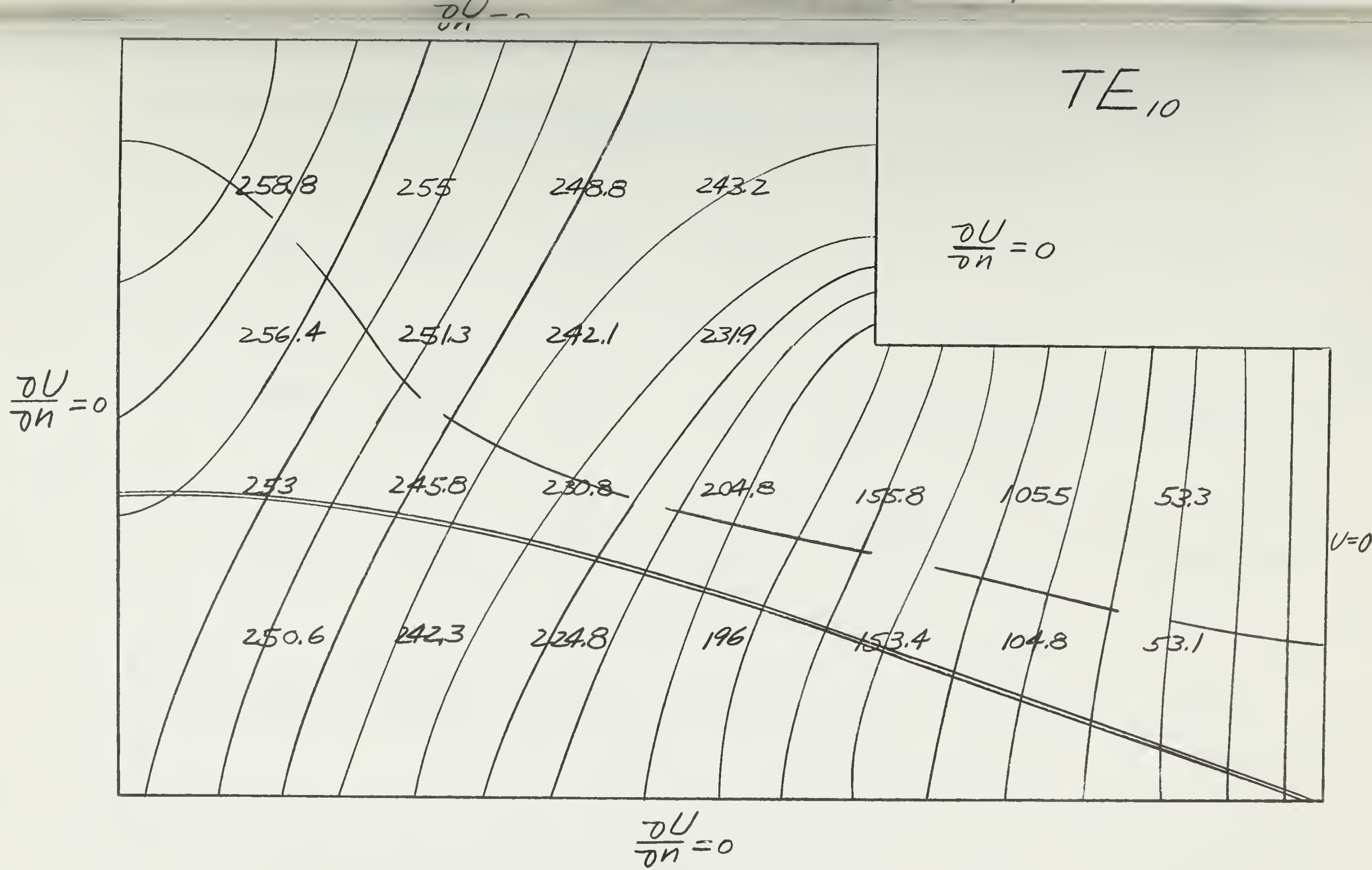


Figure 15. Field Distribution in Ridged Guide

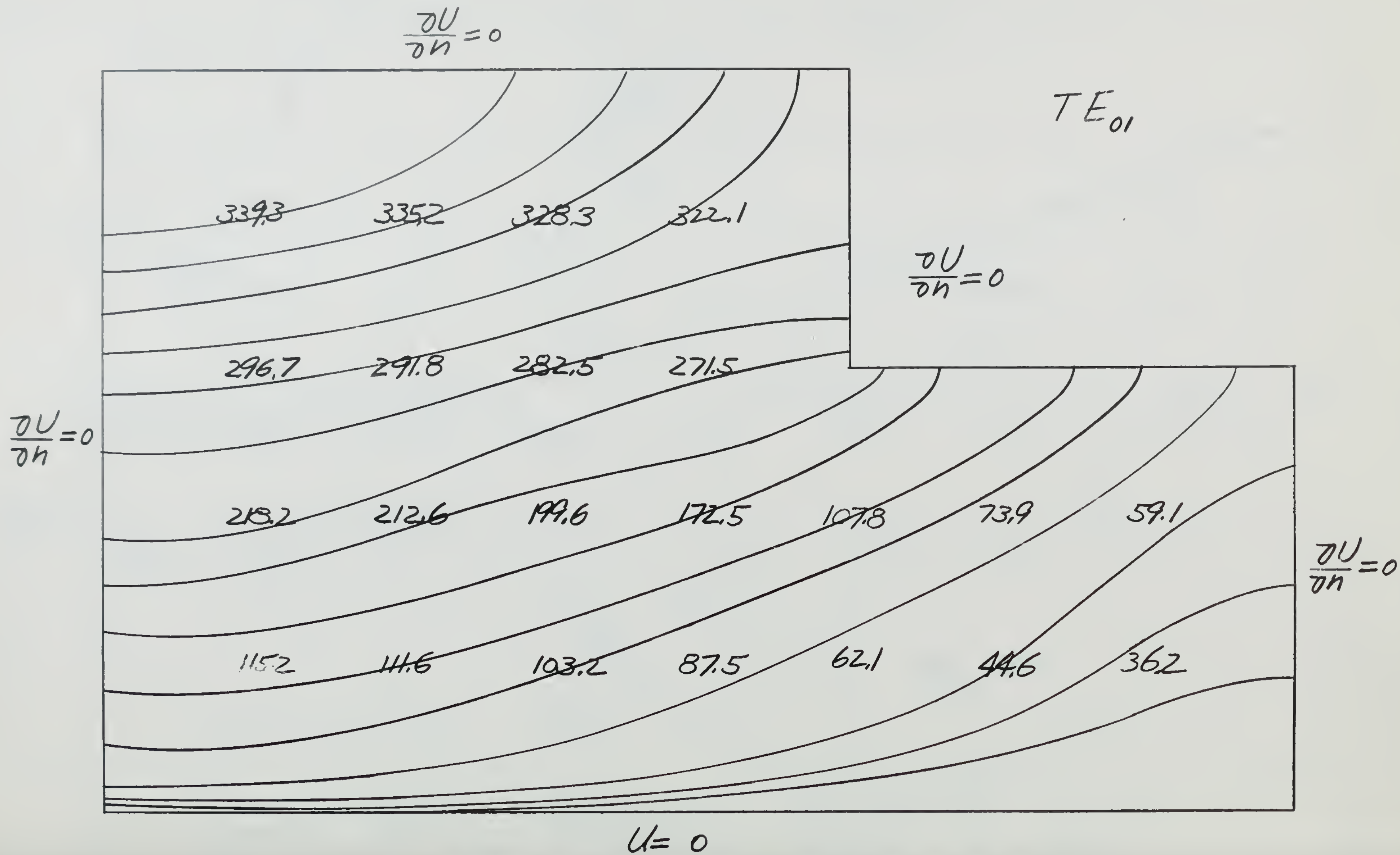


Figure 16. Field Distribution in Ridged Guide

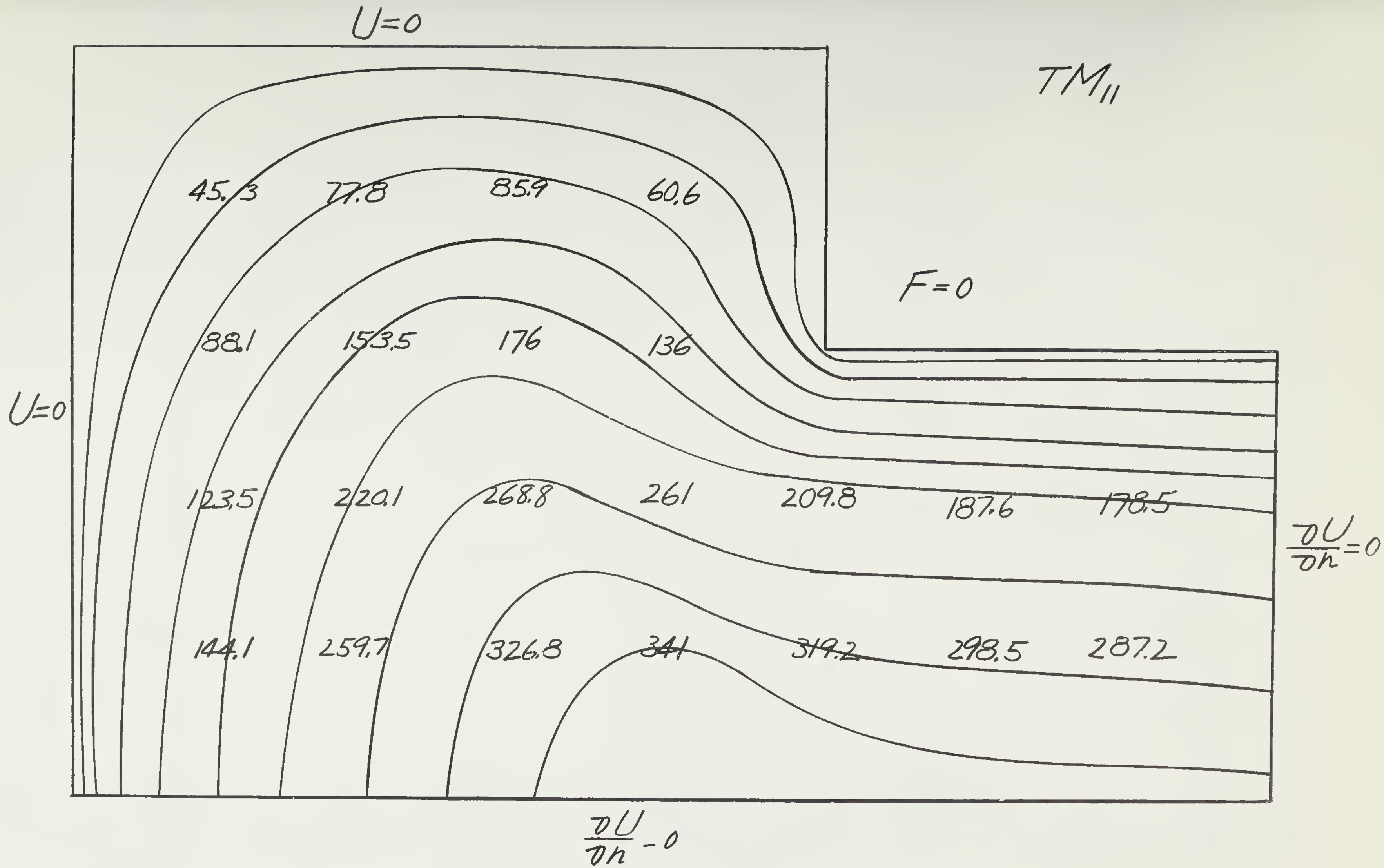


Figure 17. Field Distribution in Ridged Guide

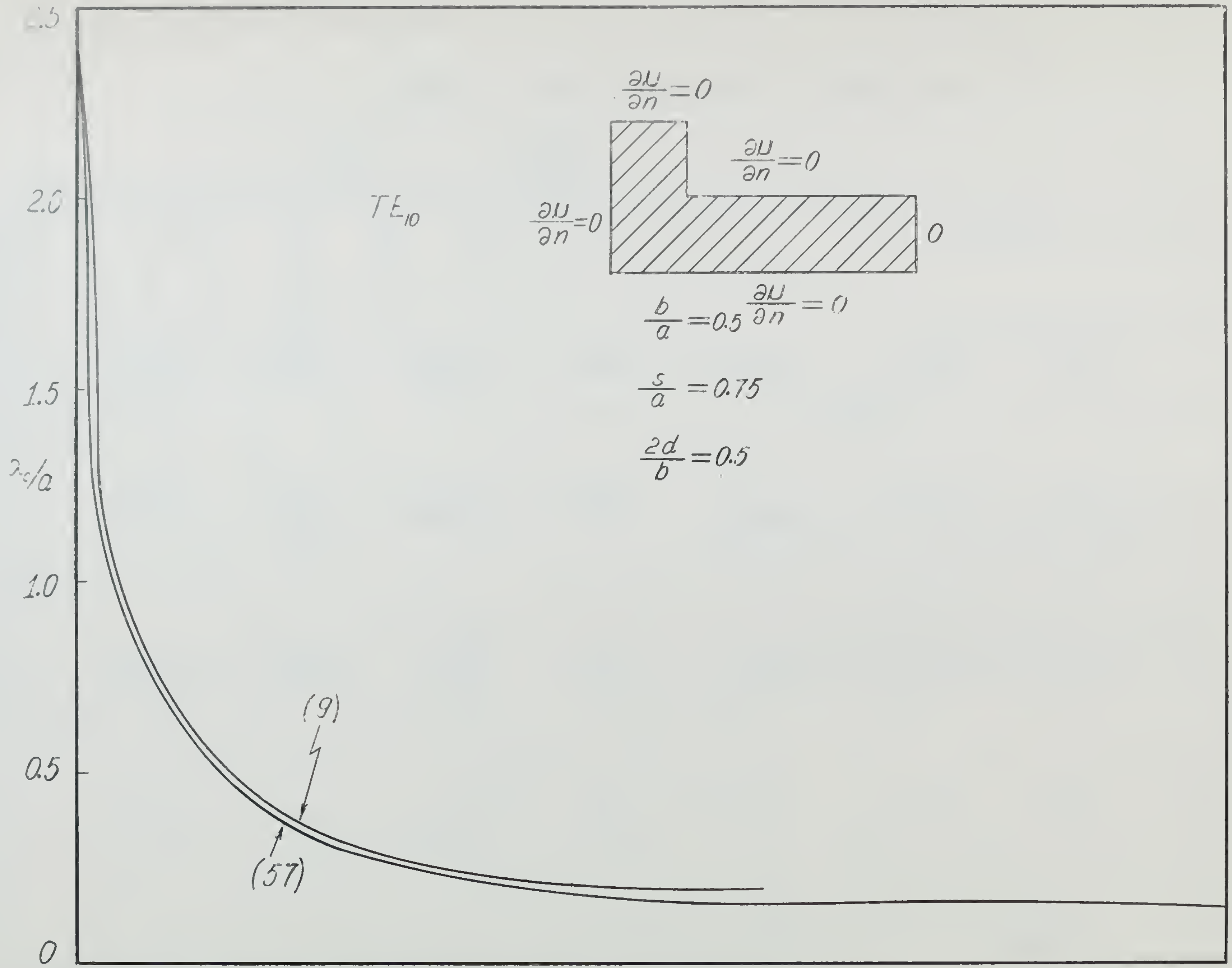


Figure 18. Comparison Between Sets (n = 9 and n = 57) of Approximate Cut Off Wavelengths of a Ridged Guide.

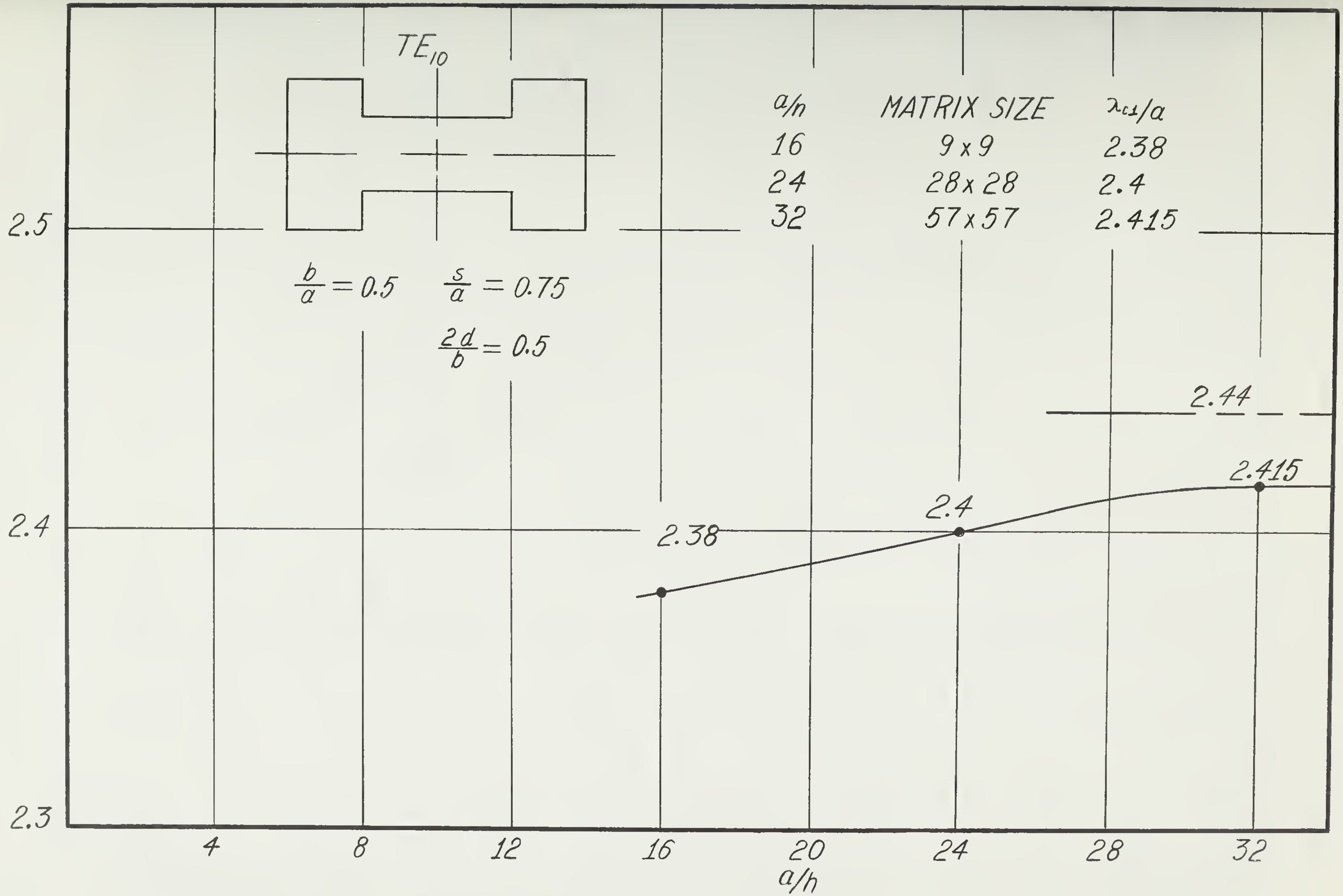


Figure 19. Convergence Curves of Cut Off Wavelength of a Ridged Waveguide

3 COMPUTATIONS BASED UPON VARIOUS BOUNDARY CONDITION APPROXIMATION

3.1 Results Obtained from Improved Approximation for Wave Equation

When Formula (20) was applied to the rectangular guide (with $b/a = 0.5$), considerable improvement was obtained for the Dirichlet problem, but the result for the Neumann problem becomes even worse than the first approximation (Figure 20). It seems to indicate that Formula (20) received more propagation error from approximation in Neumann boundary condition than that of first approximate Formula (9) and it is probably also true that, in our case, the error due to boundary approximation predominates over that of wave equation approximation. In short, we can say that merely improving the wave equation approximation but not the boundary condition does not guarantee better results.

In our first calculation, the wave equation has been approximated with the formula which takes $O(h^3)$ into account with the boundary approximation only up to $O(h)$, while in the second computation a higher order approximation to the wave equation up to $O(h^5)$ is considered but with boundary approximation still to $O(h)$. It happened in our case, that the higher order approximation is only an improvement for the wave equation and is a worse formula when it combines with $O(h)$ Neumann boundary approximation.

The above argument is strengthened by the fact that for Dirichlet boundary condition $u = o$, we do get better results for higher order approximation. It thus seems likely that the approximation for the wave equation and the boundary condition should be of the same order.

3.2 Results Obtained from Improved Approximation for Neumann Boundary Condition

Owing to the above undesirable results, the application of the improved approximation for Neumann boundary condition (22), (23) becomes necessary. The matrix obtained by using the boundary approximation b (see (22)) and wave equation approximation (9) becomes non-symmetric, a case which is difficult to treat. However for a small matrix, computations can be done by a desk calculator. The results are plotted in Figure 21. It is seen that not only is the approximation greatly improved but the direction of convergence is changed from above

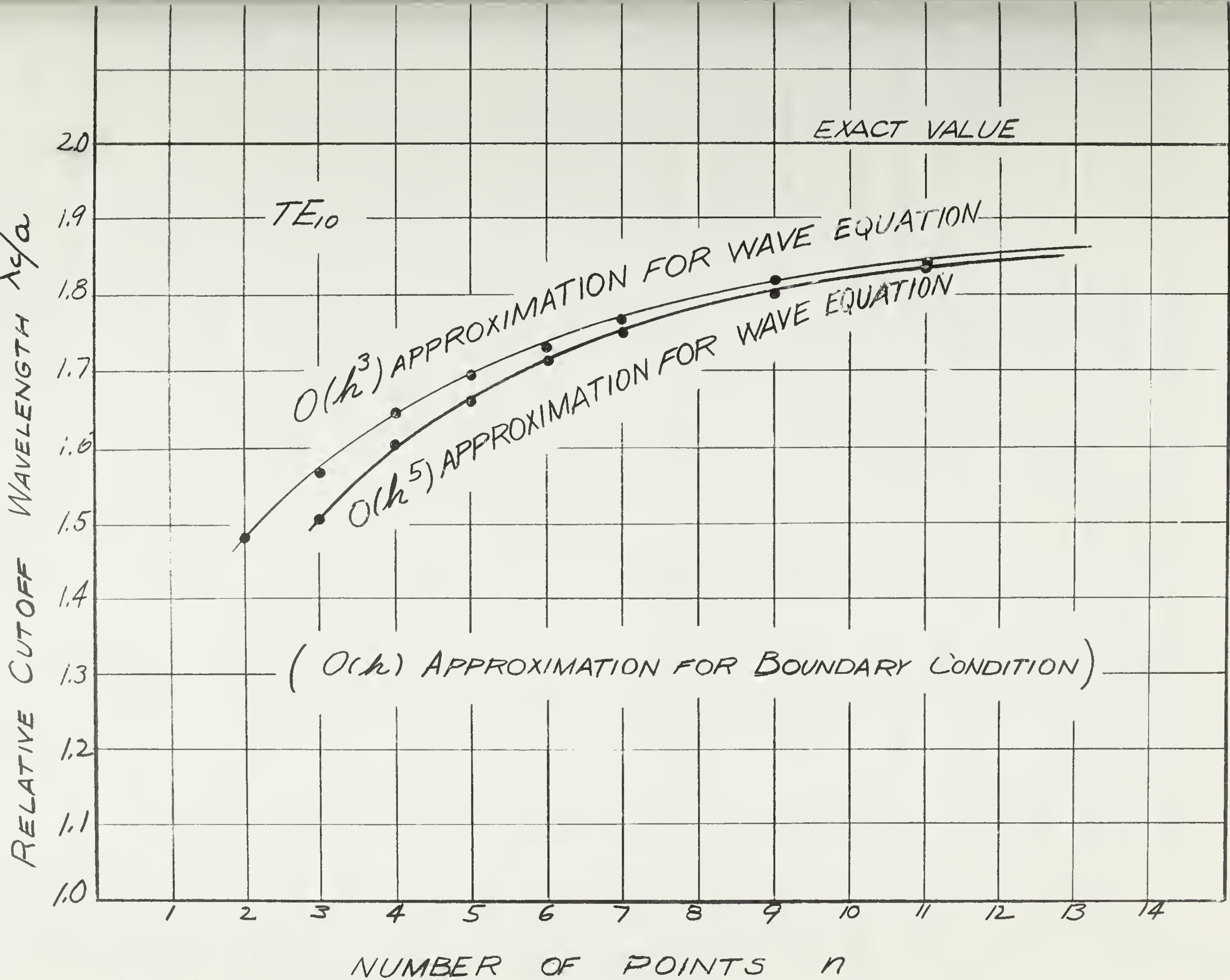


Figure 20. Convergence Curves for Neumann Problem with Different Orders of Approximation on Wave Equation But with Same Order O(h) on Boundary Condition.

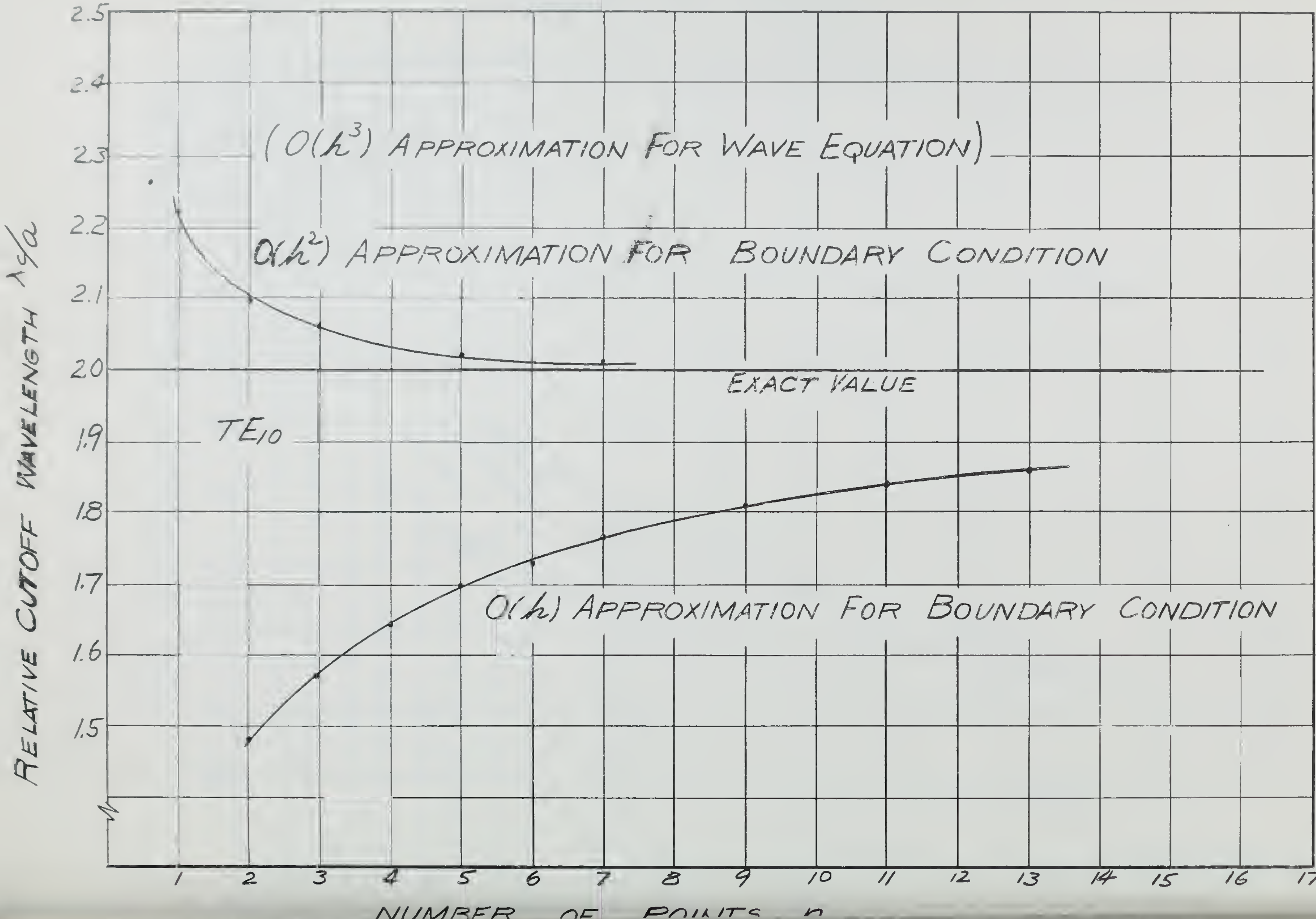


Figure 21. Convergence Curves for Neumann Problem with Different Ord[illegible]

4. CONCLUSION AND DISCUSSION

To connect the differential equation of a boundary value problem to a difference equation one must replace the differential operator by a different operator wherein a truncation of the series representing the operator is involved. Unless the rigorous solution to the problem is known the actual error committed in truncation is unknown, although the upper bound of the truncation error can be determined. A higher order approximation in this process results only in a decreased upper bound of the error, but does not necessarily guarantee lower actual error in a particular computation.

Improving the approximate formula for the wave equation does not necessarily give better results; it will depend also on the order of approximation for the boundary condition. In the present investigation, it turns out that the later approximation is of even greater importance.

In our analysis, two different orders of approximations have been considered for the wave equation and two for the boundary condition.

For the wave equation	(I)	$O(h^3)$
	(II)	$O(h^5)$
For the boundary condition	(a)	$O(h)$
	(b)	$O(h^2)$

It turns out that different combinations give results in the following order of accuracy: IIb (best, Ib, Ia, IIa (worst).

Convergence curves show that the Dirichlet boundary problem converges faster than that of the Neumann boundary problem. Furthermore, they converge in different directions, i.e., in Dirichlet boundary problem, the exact value is the lower bound of the set of approximated results $\left\{(\lambda_c/a)_n\right\}$, where n indicates the number of points used in the approximation while in Neumann boundary problem the exact value is the upper bound of the set of approximation results.

This method can be applied to both TE and TM cases. The approximate cut-off wavelength and field distribution have been obtained. However, only the data of a limited range for TE case is available in literature. They are found in good agreement with the present results.

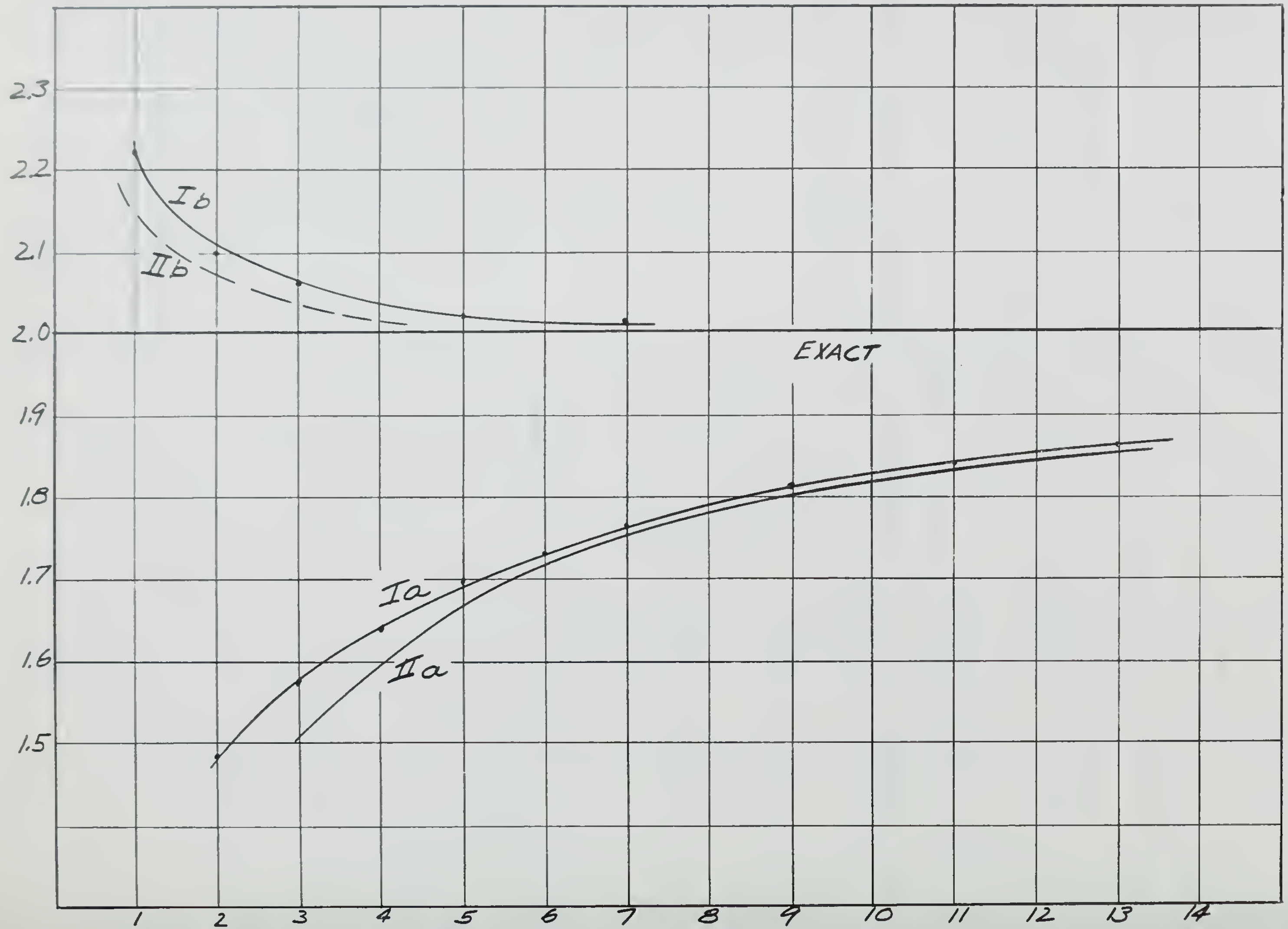


Figure 22. Convergence Curves for Various Approximations for Wave Equation and That for Boundary Condition (Neumann Problem)

This investigation indicates that better results for ridge waveguide might be obtained by the following considerations.

(a) Apply better approximation for boundary condition. It is probably preferable to use the same order of approximation for the boundary condition as for the wave equation. Since these higher order approximation formula always result in non-symmetric matrix, it is thus desirable to investigate the properties of the eigenvalues of a non-symmetric matrix. In general a non-symmetric matrix has complex eigenvalues; however, the computation which has been done for a few small matrices, as indicated in Figure 22, with a desk calculator shows that they possess only real eigenvalues. Therefore, it may infer that in our present problem the non-symmetric matrices could have only real eigenvalues. If this is the case the computation procedure could be simplified somewhat.

(b) An alternative method to solve this ridge guide problem may be suggested as follows: To consider ridged cross section as composed of a few simple regions where their eigenfunction expansion are known, by matching the eigenfunction at the common boundary we may arrive at a set of integral equations. These equations may be solved approximately.

(c) In one-dimension problems, as demonstrated in the Appendix, the fields at interior mesh points can be expressed in terms of those at boundary points only. It is not known whether it is possible to achieve the same goal in a two-dimensional problem. If it can be done, the problem will be solved.

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APPENDIX A

Closed Form for One-Dimensional Case

For the lowest mode of a rectangular guide as discussed previously, the problem is essentially one dimensional. In order to see how the eigenvalue varies with n, the number of mesh points, their difference equations are studied and solved.

Applying (8), we have the approximate wave equation

o	i-1	i	i+1	N
	1	$\alpha-2$	1	

$$u_{i-1} + (\alpha-2)\, u_i + u_{i+1} = 0 \tag{A1}$$

Similarly from (19) we have

o	i-1	i	i+1	N
	$1 + \frac{\alpha}{12}$	$\frac{5}{6}\,\alpha-2$	$1 + \frac{\alpha}{12}$	

$$u_{i-1} + \frac{\frac{5}{6}\,\alpha-2}{1 + \frac{\alpha}{12}}\, u_i + u_{i+1} = 0 \tag{A2}$$

Equations (A1) and (A2) can be generalized as

$$u_{i-1} - 2m\, u_i + u_{i+1} = 0 \tag{A3}$$

where

$$m = 1 - \frac{\alpha}{2} \qquad \text{For (A1)} \tag{A4}$$

$$m' = 1 - \frac{\alpha'}{2 + \frac{\alpha'}{6}}, \qquad \text{For (A2)} \tag{A5}$$

Let $m = m'$, hence

$$\frac{\alpha}{2} = \frac{\alpha'}{2 + \frac{\alpha'}{6}}$$

Therefore $$\alpha < \alpha'$$

Since $$\lambda_c = \frac{2\pi}{k} = \frac{2\pi h}{\sqrt{\alpha}} \tag{A6}$$

we have that λ_c/a obtained from (A1) is always greater than that from (A2) irrespective of boundary approximation, (21) or (22). This also agrees with the numerical results obtained previously. Therefore, for TM case we have better results by applying improved Formula (20), yet the same formula gives worse results for TE case since they converge to their exact value in different directions as shown in Figure 22.

The general solution of Equation (A3) is

$$u_i = a\ z_1^{\ i} + b\ z_2^{\ i} \tag{A7}$$

where z_1 and z_2 are the roots of the quadratic equation

$$z^2 - 2mz + 1 = 0 \tag{A8}$$

Applying the boundary conditions 21, 22, 23 respectively we would have the following cases

(a) $$u_o = u_1 \qquad u_{N-1} = u_N$$

From (7) we obtain

$$a + b = a\ z_1 + b\ z_i$$

$$a\ z_1^{\ N-1} + b\ z_2^{\ N-1} = a\ z_1^{\ N} + b\ z_2^{\ N} \tag{A9}$$

In order to get a non-trivial solution for (29) we set

$$\begin{vmatrix} (1-z_1) & (1-z_2) \\ (1-z_1)\ z_1^{\ N-1} & (1-z_2)\ z_2^{\ N-1} \end{vmatrix} = 0 \tag{A10}$$

Hence $$(1-z_1)\ (1-z_2)\ (z_2^{\ N-1} - z_1^{\ N-1}) = 0$$

The solutions $z_1 = z_2 = 1$ lead to trivial solution $\alpha = 0$ for $(z_z^{N-1} - z_1^{N-1}) = 0$, we get

$$\frac{z_1}{z_2} = \sqrt[N-1]{e^{j2\rho\pi}} = \angle\frac{2\rho\pi}{N-1} \qquad p = 0,\ 1,\ 2,\ \ldots\ N-2 \tag{A11}$$

From (A8) $\qquad z_1 = m + \sqrt{m^2-1} \qquad z_2 = m - \sqrt{m^2-1}$

Substituting in (A11) and solving for m from

$$\tan\left(\frac{\rho\pi}{N-1}\right) = \frac{\sqrt{1-m^2}}{m}$$

we have

$$m = \pm \cos\frac{\rho\pi}{N-1}$$

Following the same routine, we obtain for the other two boundary approximations (22), (23)

(b) $\qquad u_{-1} = u_1 \qquad u_{N-1} = u_{N+1}$

$$m = \pm \cos\frac{\rho\pi}{N}$$

(c) $\qquad u_1 = (1 - \frac{\alpha}{2})\, u_o \qquad u_{N-1} = (1 - \frac{\alpha}{2})\, u_N$

$$m = \pm \cos\frac{\rho\pi}{N}$$

for the lowest normal mode, p = 1, thus we have the lowest cut off wavelength for (21), (22), (23) respectively,

(a) $$\frac{\lambda_c}{a} = \frac{\sqrt{2}\ \pi}{N\sqrt{1 - \cos \pi/N-1}}$$

(b) $$\frac{\lambda_c}{a} = \frac{\sqrt{2}\ \pi}{N\sqrt{1 - \cos \pi/N}}$$

(c) $$\frac{\lambda_c}{a} = \frac{\sqrt{2}\ \pi}{N\sqrt{1 - \cos \pi/N}}$$

It can be shown that the value of cut of wavelength Formula (a) is always less than 2, while those of (22) and (23) are always greater than 2. Again this agrees with the results we obtained previously (Figure 21).

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Mountain View, California

Mr Roger Battie
Supervisor, Technical Liaison
Sylvania Electric Products, Inc.
Electronic Systems Division
P O. Box 188
Mountain View, California

Sylvania Electric Products, Inc.
Electric Systems Division
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