NUMERICAL PROBLEMS IN PLANE GEOMETRY

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WITH

METRIC AND LOGARITHMIC TABLES

BY

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PREFATORY NOTE

When arithmetic was dropped from the requirements for admission to Yale College, in 1894, the following substitute was adopted : "Plane Geometry (b)—Solution of numerical problems involving the metric system and the use of Logarithms, also as much of the theory of Logarithms as is necessary to explain their use in simple arithmetical operations.—Five-figure tables will be used in the examination." (1896–97 Catalogue.)

At the conference on uniform requirements for admission to college, in February, 1896, at Columbia College, representing Harvard, Yale, Princeton, University of Pennsylvania, Columbia, and Cornell, and nearly all the large preparatory schools of the East, the Mathematical Conference voted unanimously to recommend that arithmetic be dropped from the college entrance requirements, and that a knowledge of the metric system and the ability to solve numerical problems in Plane Geometry be required.

These two facts account for the writing of this little book.

The most of the problems have had class-room test. They add interest to the study of formal geometry. They are helpful, too, in making clear, and fastening in the memory, the principles and propositions of formal geometry. They enforce the practical application of truths which boys are apt to think have no application. They furnish a drill that is just as valuable to those who are not preparing for college as for those who are. These problems are not to take the place of other geometries, but are to be used with them. And, therefore, the division into Books is made to correspond pretty closely with that of the geometries in most general use.

The use of the metric system is begun at the very first, simple as that necessarily makes the problems of the first book, for the most part. No other book contains a graded set of problems on the first two books of geometry.

No apology is considered necessary for putting in quite a number of problems which presuppose some knowledge of algebra.

The order of the problems is not the same as the order of the propositions of any geometry; neither are all the problems which illustrate an important principle placed together. The reason for this is obvious. Still, the order of the problems in the different books is approximately the same as the order of the propositions in the most popular text-books. On account of this difference in order it will be best to keep the text-book work somewhat ahead, unless one cares to select the problems beforehand to give out with the text-book lesson. Some may prefer to use the problems only with the review of the geometry.

Boys preparing for college will certainly take a lively interest in the questions, problems, and exercises selected from the college entrance papers.

The entrance papers were selected with great care, with the hope that they may prove helpfully suggestive both to teachers and pupils.

The discussion of logarithms, the explanation of their use, and the use of the table have been made as simple and clear as possible.

Only such symbols are used as are almost universally employed.

Some few proofs are put in because they are not found in all the text-books.

Notice of errors, or any suggestion, will be gratefully received.

J. G. ESTILL.

HOTCHKISS SCHOOL,
LAKEVILLE, CONN., January 8, 1897.

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NUMERICAL PROBLEMS

IN

PLANE GEOMETRY

BOOK I.

1. What is the complement of 43° ? of 75° 15′ ? of 81° 11′ 11″ ? of 14° 18″ ? of $\frac{11}{30}$ ⌊R ? of $m°$ n' ? of 82° 40′ – .4 ⌊R ?

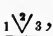
2. What is the supplement of 28° 31′ 18″ ? of 115° 39″ ? of 140° 1.84″ ? of 1.2 ⌊R ? of $\frac{8}{11}$ ⌊R ? of $c° - t°$?

3. Find the supplement of the complement of 50° ; of 85° 13′ 22″ ; of $x°$; of $t° - 31°$ 18′.

4. Find the complement of the supplement of 169° 44′ 42″ ; of 155° 55″ ; of $g° - 15°$; of $c° - 8°$ 5″.

5. How many degrees in the difference between the supplement and the complement of an ∠ ?

6. How many degrees in each of the ∠s made by two intersecting straight lines, when one of the ∠s lacks only 2° of being $\frac{3}{4}$ of $\frac{8}{9}$ of a ⌊R ?

7. In this figure , $\angle_1 = \angle_2 = \angle_3 + 12°$; how many degrees in each ∠ ?

8. Of two supplementary-adjacent $\angle$s, one lacks 7° of being ten times as large as the other; how many degrees in each?

9. Two complementary $\angle$s are such that if 7° be added to one and 8° to the other they will be in the ratio of 3 to 4?

10. If an $\angle$ divided by its supplement gives a quotient of 5 and a remainder of 6°, how many degrees in the $\angle$?

11. How many degrees in each of the five $\angle$s about a point, if each, in a circuit from right to left is 5° greater than its adjacent $\angle$?

12. Three $\angle$s make up all the angular magnitude about a point. The difference between the first and second is 10°; the difference between the second and third is 100°; how many degrees in each?

13. When the $\angle$s formed by one straight line meeting another are in the ratio 7 : 11 how many degrees in each?

14. Find the $\angle$ whose complement and supplement are in the ratio 4 : 13.

15. Find the $\angle$ the sum of whose supplement and complement is 15° less than four times its complement.

16. How many degrees in the $\angle$ whose supplement taken from three times its complement leaves 1° 18′ less than the difference between the $\angle$ and 50°?

17. If the bisector of *one* of two supplementary-adjacent $\angle$s makes an $\angle$ equal to one-sixth of the other, how many degrees in each of the $\angle$s?

18. How many degrees in each of the five $\angle$s about a point if they are in the ratio 1 : 2 : 3 : 4 : 5?

19. What answer to 18 if the ratio is 2 : 3 : 7 : 11 : 13 ?

20. If the complement of the ∠ A is three and one-half times as large as A, what part of 7 ∟R is the ∠ A ?

21. Find the ∠ whose supplement increased by 26° will be three times its complement.

22. How many degrees in the ∠ whose supplement and complement added together make 144° ?

23. How many degrees in the ∠ whose supplement, increased by 9°, is to its complement, decreased by 1°, as 7 to 2 ?

24. Find the number of degrees in each of these ∠s, if b is 2° less than $\frac{8}{7}$ of a ; c is $\frac{2}{3}$ $(a + b - 1°)$; d is 13° less than the sum of a, b, and c ; and e is 2° more than the difference between the sum of b and d, and the sum of a and c.

25. How many degrees in the ∠ whose complement is one-fifth its supplement ?

26. How many degrees in the ∠ whose supplement, increased by 20°, divided by its complement, decreased by 5°, gives a quotient 4 and a remainder 25° ?

27. If a ⊥ is 1 foot 10 inches from one end of a line and 55^{cm} from the other, at what point of the line is this ⊥ ?

28. Of two lines from the same point to the same straight line, one is 1 yard 1 foot 4 inches, the other is 130^{cm}, what can you say of them ?

29. Two lines from a point to the extremities of a straight line are 15 feet 4 inches, and 11 feet 11 inches,

respectively. Two similarly drawn are 4^{m} 6^{dm} and 3.2^{m}. Which pair includes the other? Why?

30. Of two oblique lines from a point to a straight line one is 3 feet 10.8 inches, the other, 1^{m} 1^{dm} 7^{cm}; which cuts off the greater distance from the foot of the perpendicular from the point to the straight line.

31. What answer to 30, if the lines are 35 feet and 1^{Dm}, respectively?

32. If the bisector of one of two supplementary-adjacent $\angle$s makes with their common side an $\angle = \frac{3}{5}$ $\llcorner R$ lacking $5°$, how many degrees in the other $\angle$?

33. Of two lines from a point to a straight line, one is 30^{cm} and the other is 11 inches, which is a $\perp$, if either is? Why?

34. Which is the greater of two oblique lines from a point to a straight line, cutting off, the one 20 yards, the other 15^{m}, from the foot of the $\perp$ from the point to the line?

35. Answer the same when the distances cut off are 1^{m} 7^{dm} 5^{cm} and 5 feet 10 inches.

36. In the $\triangle$s A B C and A′ B′ C′, $a = 3$ feet, $b = 7$ feet, $c = 8$ feet, $\angle A = \angle A'$, $b' = 7$ feet, $c' = 8$ feet. Find the length of a' in centimetres.*

37. In the $\triangle$ A B C, $a = 4^{m}$, $b = 5^{m}$, $c = 7^{m}$; find in feet (approximately) the sides of a $\triangle$ equal to the $\triangle$ A B C.

38. One side of a $\triangle$ is 1^{m} 5^{dm}, another 7 feet 5 inches. What is the greatest value the third side can have (1) in metric units, (2) in English units? What is the least?

* a, b, c, represent the sides of a $\triangle$ opposite the $\angle$s A, B, C, respectively.

39. Find the ∠s of the △ A B C, when A is 43° more than $\frac{2}{9}$ of B, which is 18° less than 4 times C.

40. In the two △s A B C and A′ B′ C′, A = 37°, B = 111°, c = 2.5 feet, A′ = 111°, B′ = 37°, c' = 7^{dm} 5^{cm}. What can you say of them? Why?

41. In the △ A B C, a = 13 feet, b = 17.3 feet, and c = 22.4 feet, find in metres (approximately) the sides of a △ = the △ A B C. (Log.*)

42. One of the acute ∠s of a right △ = 37° and the hypotenuse is 1.5 miles, how many kilometres in the hypotenuse of an equal right △ which has an acute ∠ of 37°?

43. In the △ A B C, a = 11^{Km}, b = 32^{Km}, what is the least possible value in miles of the side c?

44. If in two △s A B C and A′ B′ C′, a = 1^{m} 5^{cm}, b = 1^{m} 2^{dm} 5^{cm}, C = 48°, a' = 3 feet 6 inches, b' = 4 feet 2 inches, C′ = 148°, what can you say of c and c'? Show by your work how you reached your conclusion.

What would your answer be if all the given values were the same except C′ = 48°? Why?

45. If in two △s A B C and A′ B′ C′, a = 7 miles, b = 13 miles, c = 15 miles, a' = $11\frac{1}{5}^{Km}$, b' = 21^{Km}, c' = 24^{Km}, what about the ∠s B and B′? If b' = $20\frac{4}{5}^{Km}$, what of these △s?

46. In the △ A B C, a = 1.3 miles and b = 2^{Km}, what of the ∠s A and B? If a were the same and b = 2.08^{Km}, what could you say of the ∠s A and B?

47. The ∠s A and B in the △ A B C are each 49° 18′

* Certain problems in each book are marked thus for those who care for practice in the use of logarithms.

and $a = 109$ yards 1 foot 1 inch, how many metres in the side b? (Log.)

48. If one of the $\angle$s made by a line cutting two $\parallel$ lines is 3° more than $\frac{7}{18}$ ⌊R, how many degrees in each of the other $\angle$s? (Mark your answers on a figure.)

49. What answer to 48 if one of the $\angle$s is eight times its conjugate $\angle$?

50. If the exterior $\angle$ at A of the $\triangle$ A B C is 115°, and $\angle$ C is three times $\angle$ B, find B and C.

51. The exterior $\angle$s at A and C of the $\triangle$ A B C are 71° and 92° respectively; how many degrees in the $\angle$ B?

52. In the $\triangle$ A B C, A lacks 106° of being equal to the sum of B and C, and C lacks 10° of being equal to the sum of A and B; find A, B, and C.

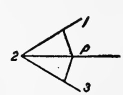
53. Find the $\angle$s of a $\triangle$ which are in the ratio 3 : 4 : 5.

54. Find the $\angle$s of an isosceles $\triangle$ in which the exterior $\angle$ at the vertex is 125°.

55. Find the $\angle$s of an isosceles $\triangle$ in which the exterior $\angle$ at the base is 95°.

56. Find the perimeter of an isosceles $\triangle$, in miles, if a base of 48^{Km} is the longest side of the $\triangle$ by 12^{Km}. (Log.)

57. In the $\triangle$ A B C, $a = 15$ yards and $b = 1^{Dm}\ 2^{m}$, what about the $\angle$s A and B?

58. The point P in the bisector of the angle 

is 5 yards 2 feet from the side *1—2*; how many metres is P from *2—3*?

59. The point P within an ∠ is 6^{dm} 5^{cm} from one side of the ∠ and 2 feet 2 inches from the other side, where does it lie? Show the reason for your answer by your work.

60. The ∠ at the vertex of an isosceles △ is one-third the exterior angle at the vertex, how many degrees in each ∠, exterior and interior, at the base?

61. In the △ A B C, $A = 35°$, $B = 45°$, $a = \frac{1}{8}$ mile; what can you say of the length of b, in metres?

62. Two adjacent sides of a ▱ are respectively 18^{m} and 21^{m}; find the lengths of the other two sides in yards. (Log.)

63. The area of one of the △s made by the diagonal of a ▱ is 5.2^{Ha}. How many acres in the other?

64. If one ∠ of a ▱ $= \frac{3}{4}$ ∟R, how many degrees in each of the other ∠s?

65. If two adjacent ∠s of a ▱ are in the ratio of 17 : 1, how many degrees are there in each ∠ of the ▱?

66. How many degrees in each ∠ of a ▱ where one ∠ exceeds one-third of its adjacent ∠ by two-thirds of a degree?

67. How many degrees in each ∠ of an equiangular icosagon? in each exterior ∠?

68. How many sides has the polygon each of whose exterior ∠s = 12°?

69. How many sides to the polygon each of whose exterior ∠s is only one-eleventh of its adjacent interior ∠?

70. One side of a rhombus is $13.6,^{Km}$ find its perimeter in miles. (Log.)

71. One side of a rhomboid is 4 feet longer than the other, the perimeter is 14^{m}, what are the lengths of the sides in feet and inches? (Log.)

72. Find the number of acres in a rhombus in which one of the four $\triangle$s made by the diagonals contains 5.11^{Ha}. (Log.)

73. Find the $\angle$s of an isosceles $\triangle$ when one of the $\angle$s at the base is equal to one-half the $\angle$ at the vertex.

74. What answer to 73, when the $\angle$ at the vertex is 9° greater than an $\angle$ at the base?

75. What are the $\angle$s of an isosceles $\triangle$ in which the $\angle$ at the vertex is 12° more than one-third the sum of the base $\angle$s?

76. The sides of a quadrilateral taken in order are 6 inches, 18^{cm}, 15^{cm}, $7\frac{1}{5}$ inches, respectively. What is the nature of this quadrilateral?

77. How many sides has the polygon each of whose interior $\angle$s $= 171°$?

78. The line joining the middle points of two sides of a $\triangle$ is 2.5 miles, what is the length of the third side in kilometres?

79. How many sides has the polygon the sum of whose interior $\angle$s exceeds the sum of its exterior $\angle$s by 3240°?

80. One of the diagonals of a rectangle is 40 yards 2 feet 10 inches; find the length of the other in metres. (Log.)

81. One base of a trapezoid is 125^{cm}, the line joining the middle points of the non-parallel sides $.7^{m}$, find the length of the other base.

82. How many sides has the equiangular polygon each of whose interior $\angle$s exceeds its adjacent exterior by 108°?

83. How many sides has the polygon the sum of whose interior $\angle$s is double the sum of the exterior $\angle$s ?

84. The line joining the middle points of the non-parallel sides of a trapezoid is 13 feet 5 inches, and one of the bases is $2\frac{1}{2}$ times as long as the other ; find the length of the bases.

85. Find the length in metres of the line which bisects one side of a $\triangle$ and is parallel to a side whose length is 9 feet 10.11 inches.

86. If you should join the extremities of two parallel lines whose lengths are 7^{Km} and 4.375 miles respectively, what kind of a figure would be formed ? Why ?

87. How many sides has the polygon the sum of whose $\angle$s is $4\frac{1}{4}$ times those of a hexagon ?

88. Find in inches the bases of a trapezoid in which the line joining the middle points of the non-parallel sides = 40^{cm} and one base is 8^{cm} longer than the other.

89. How many sides has the polygon the sum of whose interior $\angle$s exceeds the sum of its exterior $\angle$s by 38 $\underline{|R}$?

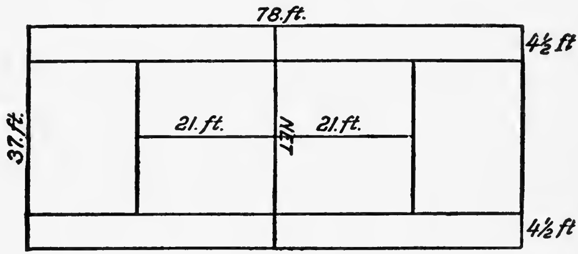
90. One base of a trapezoid is 5.1^{m}, the line joining the middle points of the non-parallel sides is $2\frac{1}{5}$ times the other base ; find the other base.

91. How many sides has the polygon each of whose interior $\angle$s exceeds its exterior $\angle$ by $\frac{22}{15}$ $\underline{|R}$?

92. How many sides has the polygon each of whose interior $\angle$s is 6 times its exterior $\angle$?

93. Find the difference in perimeter, in inches, between a square whose side is 1 foot 6 inches and a rectangle whose adjacent sides are 30^{cm} and 60.5^{cm} respectively.

94. Find the number of feet of lime-line of a tennis-court, as represented below. Reduce your answer to metres. (Log.)



95. Through the vertices of a △ A B C, lines are drawn parallel to the opposite sides of the △, thus forming a second △. Find the perimeter of the second △ in kilometres, if the sides of the first △ are 5 miles, 8 miles, and 11 miles.

96. How many sides has the polygon each of whose ∠s = 162°?

97. The perimeter of a rectangle is 8.04^m, and the sides are in the ratio of 1 to $1\frac{3}{5}$, find the lengths of the sides in inches.

98. How many sides has the polygon the sum of whose interior ∠s exceeds the sum of its exterior ∠s by 1080°?

99. A man owns a rectangular garden 55^m by 34^m; he makes a path 3.3^m wide around it; what is the perimeter of the part that remains?

100. Find the number of yards of lime-line for a foot-ball field, which is 330 feet by 160 feet, including all the five-yard lines. How long would it take a runner to cover the total distance, if he can make 110 metres in 12 seconds? (Log.)

BOOK II.

1. If the radii of two intersecting ⓢ are 3^m and 7^m respectively, what is the greatest possible distance, in feet and inches, between their centres? The least?

2. Four chords are 2^{Km} 5^{Dm} 8^{dm}, 0.15 miles, 0.25^{Km}, and 330 yards, respectively. If one is a diameter, which is it? Which of the others is nearest to the centre? Which farthest from it?

3. If a central ∠ of 28° intercepts an arc of 3.2^m, find, in feet and inches, the arc intercepted by an equal ∠ in an equal ⊙.

4. What can you say of the central ∠s of a ⊙ which intercept, and the chords which subtend, two arcs which are respectively 28 yards and 25^m?

5. In a given ⊙, the chord A B is 5 yards 2 feet, the chord C G is 4.9^m. Compare the arcs A B and C D, and the distances of the chords from the centre.

6. What can you say of two chords whose distances from the centre are 13^{cm} and 5 inches respectively?

7. One of the arcs intercepted by two chords, one of which is a diameter, intersecting at right angles, is 41° 18′ 4″; find the other arcs.

8. A secant parallel to a tangent subtends an arc of 117° 41′; find the arcs intercepted by the secant and the tangent.

9. One of the arcs intercepted by a diameter and a parallel secant is 37° 30′; find the length, in miles, of the

arc subtended by this secant, if a degree of the circumference is 24^{Km}. (Log.)

10. The line joining the centres of two ⓢ, tangent to each other externally, is $14^m\ 7^{dm}\ 3^{cm}$, and the radius of the less is $3^m\ 8^{dm}\ 5^{cm}$, find the radius of the greater.

11. If a central ∠ of 25° 15′ intercepts an arc of 15 feet 10 inches, find the length of the semi-circumference of the ⊙. (Log.)

12. How many degrees in an ∠ inscribed in $\frac{1}{8}$ of a circumference?

13. Find the length of the arc intercepted by an inscribed ∠ of 20° $22\frac{1}{2}'$ in a ⊙ whose circumference is $\frac{1}{6}$ of a mile. (Log.)

14. How many degrees in an inscribed ∠ which intercepts $\frac{8}{15}$ of a quadrant?

15. An ∠ formed by a tangent and a chord is $\frac{5}{27}$ ∟R; how many degrees in the intercepted arc?

16. Find the length of the arc intercepted by a central ∠ of 12° 15′ in a ⊙ whose circumference $= 1^{Km}$. (Log.)

17. If a central ∠ of 85° 40′ intercepts an arc of 32.5^m, how many degrees and minutes in the central ∠ which intercepts an arc of 65^{cm}? (Log.)

18. What part of a ∟R is an ∠ between a tangent and a chord intercepting an arc of $\frac{11}{24}$ of a semi-circumference?

19. The ∠ between two chords intersecting within the circumference is 35°, its intercepted arc is 25° 18′; find the arc intercepted by its vertical ∠.

20. Find the ∠ between a secant and a tangent when their intercepted arcs are respectively $\frac{1}{3}$ and $\frac{1}{5}$ of the circumference.

21. The ∠ between two secants, intersecting without the circumference, is 58° 41′, one of the intercepted arcs is 230°; find the other.

22. Find the ∠ between two tangents when the intercepted arcs are in the ratio 7 : 2.

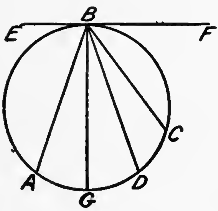


FIG. 2.

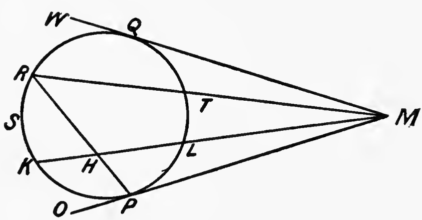


FIG. 3.

23. If, in Fig. 2, the ∠ A B C = 67°, and the arc D C is 25°, how many degrees in the ∠ A B D?

24. In the same figure, B G is a diameter, B C is 8° more than G C; find the ∠ E B C.

25. In the same figure, the arc D B is three and one-half times the arc D C, and the ∠ D B G = $13\frac{1}{2}$°; find the ∠ D B C.

26. In the same figure, if G D and B C are in the ratio 3 : 7 and the ∠ D B C = 15°, how many degrees in the ∠ G B C?

27. In Fig. 3, Q P is 24° less than a semi-circumference, how many degrees in the ∠ Q M P?

28. The ∠ R M K is 27°, the arc R K is 100°; how long is the arc T L, if a quadrant on this figure = 15^m?

29. The ∠ R H K is 70°, the arc R Q L is three times as long as the arc K P; find the number of degrees in K P.

30. The arc R P T is 10° less than two-thirds of a circumference, the ∠ Q M T is 17°; how many degrees in Q T?

31. How many degrees in the central ∠ which intercepts an arc of 17^{cm}, when a quadrant is 4^{dm} 2^{cm} 5^{mm}?

32. The ∠ between two tangents from the same point is 32° 30′; find the ratio of their intercepted arcs.

33. If a central ∠ of 65° intercepts an arc of 10 feet 5.984 inches, how many metres will there be in an arc of the same ⊙ intercepted by a central ∠ of 211° 15′? (Log.)

34. The ∠ between two tangents from the same point, to a ⊙ whose radius is 55^{cm}, is 120°; how many inches in the chord joining the points of tangency?

35. The centres of two ⓢ which are tangent to each other internally are 5 feet 8 inches apart, the radius of one is 1.1^{m}; find the radius of the other.

36. The chord joining the points of tangency of two intersecting tangents forms with one of them an ∠ of 17° 7′; find the ∠ between the tangents.

37. The radii of two concentric ⓢ are 8 feet 2.425 inches and 2.25^{m}, respectively; find the radius of a ⊙ tangent to both. (Two solutions.) Get one answer in metric units, the other in English units.

38. The ∠ between two chords, one of which is a diameter, is $\frac{7}{18}$ ∟R; find the arc subtended by the less chord.

39. Find the circumference, in metres, of a ⊙ in which

a central ∠ of 11° 15′ intercepts an arc of 3.5 inches. (Log.)

40. The ∠ between a tangent and a secant is 8° 11′, the smaller of the intercepted arcs is 56° 50′ 40″; find the larger.

41. In a certain ⊙ a central ∠ of 78° 45′ intercepts an arc of 168 miles; how long will it take a train moving 24 miles per hour to cover the circuit?

42. Two sides of an inscribed △ subtend $\frac{2}{15}$ and $\frac{1}{18}$ of the circumference, respectively; find the ∠s of the △.

43. One ∠ of an inscribed △ is 35°, one of its sides subtends an arc of 113°; find the other ∠s of the △.

44. The bases of a trapezoid subtend arcs of 100° and 140°, respectively; find its ∠s and the ∠ made by the non-parallel sides produced.

45. How long would it take a train running 40 miles an hour to go round a ⊙ in which a central ∠ of 15° intercepts an arc of 7.2^{Km}? (Log.)

46. The numbers of degrees in the arcs subtended by the sides of a pentagon, in order, are consecutive; find the ∠s of the pentagon.

47. The arcs subtended by three consecutive sides of a quadrilateral are 87°, 95°, 115°; find the ∠s of the quadrilateral; the ∠s made by the intersection of the diagonals; and the ∠s made by the opposite sides of the quadrilateral, when produced.

48. Find the ∠ made by the radii and the line joining the points of contact of two tangents drawn through a

point 6 inches from the circumference of a ⊙ of 6-inch radius.

49. Find the ∠s of an isosceles △, if the arc subtended by one of the equal sides is 33° more than 1.6 times the arc subtended by the base.

50. An ∠ formed by a diagonal and a base of an inscribed trapezoid is 20° 30′; find the ∠s made by the intersection of the diagonals.

51. Over how many degrees of arc of a ⊙ whose circumference is 435^{Km} will a train, moving 60 miles per hour, go in 15 minutes 5 seconds? (Log.)

52. Three consecutive ∠s of an inscribed quadrilateral are 140° 30′, 80° 30′, and 29° 30′; find the numbers of degrees in the arcs subtended by the four sides.

53. If it takes light 8 minutes to come from the sun to the earth, which distance is the same as 57.3° of the earth's orbit, how long would it take it to go the length of the entire orbit, supposing the orbit a ⊙? (Log.)

54. Three consecutive ∠s of a circumscribed quadrilateral are 85°, 122°, 111°; find the number of degrees in each ∠ of the inscribed quadrilateral made by joining the points of contact of the sides of the circumscribed quadrilateral.

55. Find the circumference of a ⊙ in which a train going 60 miles an hour goes over an arc of 1° 35′ in 17 seconds. (Log.)

56. Two arcs subtended by two adjacent sides of an inscribed quadrilateral are 127° and 68° 30′, and the ∠ between the diagonals, which intercepts the arc of 68° 30′, is 77° 30′; find the ∠s of the quadrilateral.

57. If a star makes a complete circuit of the heavens in 23 hours 56 minutes, through what arc will it go between 9.12 P.M. and 12.13 A.M.? (Log.)

58. If the earth in revolving about the sun moves 65,500 miles per hour in its orbit, find the entire length of this orbit, remembering that it takes 365 days 6 hours 9 minutes 9 seconds to make a complete revolution. (Log.)

59. If Jupiter is 476,000,000 miles from the sun, and the length of its orbit is three and one-seventh times the diameter of its orbit, and its period of revolution is 11 years, 315 days, what is its hourly motion in its orbit? (Log.)

60. If the earth's radius, 3,963 miles, is equal to the length of an arc of 57′ of the moon's orbit about the earth, what is the distance to the moon, considering the orbit a ⊙ and the circumference three and one-seventh times the diameter? (Log.)

BOOK III.

1. In Fig. 4, B C = 52^m, A C = 28^m, A′ B′ is ∥ to A B, C B′ = 13^m; find C A′ and A′ A.

2. If, in the same figure, C A′ = 10 feet, A′ A = 12 feet 4 inches, and B′ C = 16 feet 3 inches, what is the length of C B?

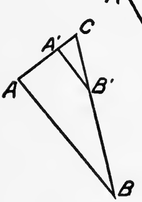


FIG. 4.

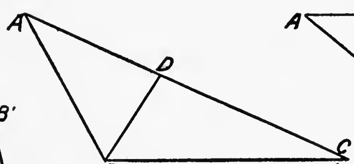


FIG. 5.

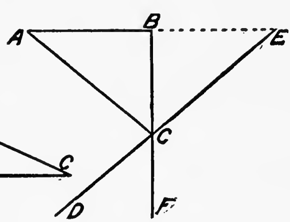


FIG. 6.

3. In Fig. 5, A B = 18.7^{m}, B C = 29.4^{m}, A C = 40.4^{m}, and B D is the bisector of the ∠ A B C; find A D and D C. (Log.)

4. If, in Fig. 5, A D = 3 feet 5 inches, A B = 4 feet 2 inches, and B C = 7 feet, find the length of A C.

5. In Fig. 6, C D is the bisector of the ∠ A C F, B E = 3.3^{dm}, A C = 6^{dm}, B C = 4.1^{dm}; find A B in yards. (Log.)

6. If, in Fig. 6, A C = 65 yards, A B = 48 yards, B C = 35 yards; find B E in metres. (Log.)

7. If, in Fig. 6, A E = 18 feet 6 inches, B C = 14 feet, and B E = 14 feet 2 inches; find in metres the lengths of A C and A B. (Log.)

8. The sides of a △ are $a = 15^{m}$, $b = 12^{m}$, $c = 10^{m}$; find the segments into which each side is divided by the bisector of the opposite ∠.

9. Find the segments into which each side is divided by the bisector of an exterior ∠ in the preceding problem.

10. The homologous sides of two similar △ are 5 feet 3 inches and 4 feet 5 inches, respectively. If the altitude to the given side of the first is 3 feet 9 inches, find the homologous altitude in the second.

11. The sides of a △ are 4^{m} 6^{dm}, 6^{m} 1^{dm}, and 8^{m}; the homologous sides of a similar △ are a, 305^{cm}, c; find a and c.

12. In the △ A B C and A′ B′ C′, A = 59° = A′, $b = 3$ feet 6 inches, $c = 13$ feet, $b' = 5.6^{m}$, $c' = 20.8^{m}$. Show what relation, if any, these △ bear to each other.

13. The perimeters of two similar polygons are 88^m and 396^m, respectively. One side of the first is 15 yards 4 feet 2.4 inches; find the homologous side of the second. (Log.)

14. The sides of two $\triangle$s are, respectively, 4^{Km}, 9^{Km}, 11^{Km}, and 1.2 miles, 2.7 miles, 3.3 miles. Show by your work any relation which may exist between these $\triangle$s.

15. One of the altitudes of a $\triangle = 1.5^m$; find the homologous altitude of a similar $\triangle$, if the perimeters of the two $\triangle$s are respectively 15 feet and 24 feet.

16. A series of straight lines passing through the point O intercept segments, on one of two parallel lines, of 15 feet, 18 feet, 24 feet, and 32 feet, the segment of the other parallel, corresponding to 24 feet, is 16 feet; find the other segments.

17. Two homologous sides of two similar polygons are 35^m and 50^m, respectively. The perimeter of the second is 8^{Hm}. What is the perimeter of the first?

18. The legs of a right $\triangle$ are 3^m and 4^m; find, in inches, the difference between the hypotenuse and the greater leg. Find also the segments of the hypotenuse made by the perpendicular from the vertex of the right $\angle$; and this perpendicular itself.

19. In a $\odot$ whose diameter is 16^m, find the length of the chord which is 4^m from the centre.

20. The sides of a $\triangle$ are 30^{cm}, 40^{cm}, and 45^{cm}; find the projection of the shortest side upon the longest.

21. Is the $\triangle$ of 20 acute, right, or obtuse? Which would it be if the sides were 30^{cm}, 40^{cm}, 55^{cm}? Find the

projection of the shortest side upon the medium side in the latter △.

22. A tangent to a ⊙ whose radius is 1 foot 6 inches, from a given point without the circumference, is 2 feet; find the distance from the point to the centre.

23. In the △ A B C, $a = 14^m$, $b = 17^m$, $c = 22^m$; is the ∠ C acute, right, or obtuse?

24. To find the altitude of a △ in terms of its sides.

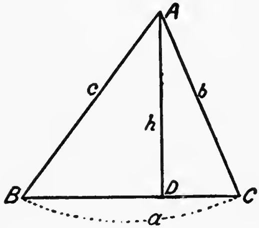


FIG. 6.

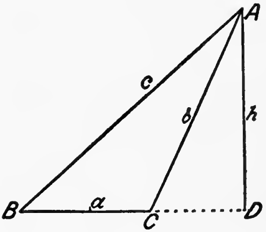


FIG. 7.

(1) $h^2 = c^2 - \overline{BD}^2$. (The square of either leg of a right △ is equal to the square of the hypotenuse minus the square of the other leg.)

$b^2 = a^2 + c^2 - 2a \times BD$ { The square of the side opposite the acute ∠ of a △ is equal to the sum of the squares of the other two sides minus twice one of them by the projection of the other upon it.

Solving for B D, $BD = \dfrac{a^2 + c^2 - b^2}{2a}$

Substituting in (1),

$$h^2=c^2-\left(\frac{a^2+c^2-b^2}{2a}\right)^2=\left(c+\frac{a^2+c^2-b^2}{2a}\right)\left(c-\frac{a^2+c^2-b^2}{2a}\right)$$

$$=\left(\frac{2ac+a^2+c^2-b^2}{2a}\right)\left(\frac{2ac-a^2-c^2+b^2}{2a}\right)$$

$$=\left[\frac{(a+c)^2-b^2}{2a}\right]\left[\frac{b^2-(a-c)^2}{2a}\right]$$

$$=\frac{(a+c+b)\ (a+c-b)}{2a}\times\frac{(b+a-c)\ (b-a+c)}{2a}.$$

Let $2s=a+b+c$,

Subtracting $2c=2c$,

$2s-2c=2(s-c)=a+b-c$.

Similarly, $2(s-a)=b+c-a$,

and $2(s-b)=a+c-b$.

Substituting we have

$$h^2=\frac{2s\times 2(s-b)}{2a}\times\frac{2(s-c)\times 2(s-a)}{2a}=\frac{4s(s-a)\ (s-b)\ (s-c)}{a^2}.$$

Extracting the square root, $h=\frac{2}{a}\sqrt{s(s-a)\ (s-b)\ (s-c)}$,

Similarly, $h'=\frac{2}{b}\sqrt{s(s-a)\ (s-b)\ (s-c)}$,

and $h''=\frac{2}{c}\sqrt{s(s-a)\ (s-b)\ (s-c)}$,

h' and h'' representing the altitude of the △ upon b and c, respectively.

25. To find the radius of the circumscribed ⊙ in terms of the sides of the △.

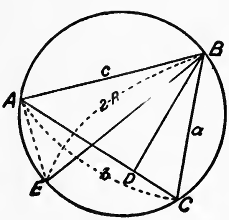


FIG. 8.

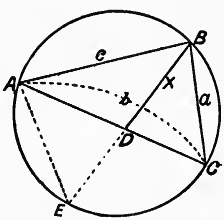


FIG. 9.

$ac = 2R \times BD$. (Fig. 8.)

(The product of two sides of a △ is equal to the diameter of the circumscribed ⊙ multiplied by the altitude to the third side.)

But by 24, $BD = \frac{2}{b}\sqrt{s(s-a)(s-b)(s-c)}$.

Hence $$ac = \frac{4R}{b}\sqrt{s(s-a)(s-b)(s-c)},$$

and $$R = \frac{a\,b\,c}{4\sqrt{s(s-a)(s-b)(s-c)}}.$$

26. To find the bisectors of the ∠s of a △ in terms of the sides.

$$(1)\ a\,c = x^2 + AD \times DC. \quad \text{(Fig. 9.)}$$

(The product of two sides of a △ is equal to the square of the bisector of the included ∠, plus the product of the segments of the third side made by the bisector.)

Transposing in (1), (2) $x^2 = a\,c - AD \times DC$.

But $$\frac{CD}{DA} = \frac{a}{c}.$$

(The bisector of an ∠ of a △ divides the opposite side into segments proportional to the adjacent sides.)

By composition $$\frac{DC + AD}{DC} = \frac{a+c}{a},$$

and $$\frac{DC + AD}{AD} = \frac{a+c}{c};\ \text{or}$$

$$\frac{b}{DC} = \frac{a+c}{a},\ \text{and}\ \frac{b}{AD} = \frac{a+c}{c}.$$

Whence $$DC = \frac{a\,b}{a+c},\ \text{and}\ AD = \frac{b\,c}{a+c}.$$

Substituting in (2) we have

$$x^2 = a\,c - \frac{a\,b^2\,c}{(c+a)^2} = a\,c\left[1 - \frac{b^2}{(c+a)^2}\right] = a\,c\left[\frac{(c+a)^2 - b^2}{(c+a)^2}\right].$$

$$= \frac{a\,c(c+a+b)\,(c+a-b)}{(a+c)^2}.$$

(Substituting as in 24.)

$$= \frac{a\,c \times 2s \times 2\,(s-b)}{(a+c)^2}.$$

Extracting the square root,

$$x = \frac{2}{a+c}\sqrt{a\,c\,s\,(s-b)}\,.$$

Similarly, $$x' = \frac{2}{b+c}\sqrt{b\,c\,s\,(s-a)}\,,$$

and $$x'' = \frac{2}{a+b}\sqrt{a\,b\,s\,(s-c)}\,.$$

NOTE.—In a right △ (hypotenuse c and legs a, b) the formula $a = \sqrt{c^2 - b^2}$ and $b = \sqrt{c^2 - a^2}$, should be written $a = \sqrt{(c+b)\,(c-b)}$, and $b = \sqrt{(c+a)\,(c-a)}$, when logarithms are to be employed.

27. The chord A B, which is 4.2^m long, divides the chord C D into segments which are 1.4^m and 2.1^m, respectively. Find the segments of A B made by C D.

28. The sides of a △ are 25 yards, 30 yards, 35 yards. Find the length of the median * to the side of 30 yards, and its projection upon the same.

29. Find the diameter of the ⊙ circumscribed about the △ two of whose sides are 3 feet 4 inches and 4 feet 6 inches, and the perpendicular to the third side from the opposite vertex is 2 feet 3 inches.

30. Find the length of the bisector of the opposite ∠ to the least side in the △ whose sides are 24^{cm}, 20^{cm}, 11^{cm}; the

* A median is a line from a vertex of a △ to the middle point of the opposite side.

three altitudes of the △; and the radius of the circumscribed ⊙. (Log.)

31. Two secants from the same point without a ⊙ are 25^{cm} and 35^{cm}. If the external segment of the less is 7^{cm}, find the external segment of the greater.

32. A secant from a given point without a ⊙ and its external segment are 2 feet 4 inches and 7 inches, respectively; find the length of the tangent to the ⊙ from the same point.

33. The greatest distance of a chord of 11 feet from its arc is 6 inches; find the diameter of the ⊙.

34. Two sides of a △, inscribed in a ⊙ whose radius is 15 inches, are 9 inches and 25 inches; find the perpendicular to the third side from the opposite vertex.

35. Find the greater segments of a line of 36^{cm} when it is divided internally and externally in extreme and mean ratio.

36. Find a mean proportional to two lines which are 5^{dm} and 2^{m} long, respectively.

37. Find a fourth proportional to the lines a, b, c, when $a=65^{cm}$, $b=42^{cm}$, $c=26^{cm}$.

38. Find a third proportional to m and n, when $m=17^{Km}$ and $n=51^{Km}$.

39. The chords A B and C D intersect at E; A E$=15^{dm}$, B E$=46^{dm}$, C D$=115^{dm}$; find C E and D E.

40. Find the distance from a given point to the circumference of a ⊙ whose radius is 9 inches, if the tangent to the ⊙ from the given point$=$1 foot.

41. If, in the preceding problem, another tangent were

drawn from the same point, what would be the length of the line joining the points of contact of these two tangents?

42. The segments of a transversal made by lines passing through a common point are 1 foot 3 inches, 1 foot 9 inches, and 2 feet 11 inches, respectively. If the least segment of a parallel to this transversal, intercepted by the same lines, is 30^{cm}, find the other segments.

43. If a gate-post 5 feet high casts a shadow 17 feet long, how high is a house which, at the same time, casts a shadow 221 feet long?

44. A baseball diamond is a square with 90 feet to a side; find the distance across from first base to third.

45. The projections of the legs of a right △ upon the hypotenuse are 8^{cm} and 9^{dm}; find the shorter leg.

46. In a ⊙ whose radius is 41 feet are two parallel chords, one 80 feet, the other 18 feet. Find how far apart these two chords are. (Two solutions.)

47. If a chord of 75^{cm} subtends an arc of $m°$ in a ⊙ whose radius is 415^{cm}, how long a chord will subtend an arc of $m°$ in a ⊙ whose radius is 33.20^{m}? (Log.)

48. The sides of a △ are $1{,}789^{m}$, $4{,}231^{m}$, and $3{,}438^{m}$; find the three altitudes and the diameter of the circumscribed ⊙. (Log.)

49. The altitude of an equilateral △ is 45 feet, what is the length of a side in feet and inches?

50. Find the radius of the ⊙ in which a chord of 40.5^{m} is 14.4^{m} from the centre. Find also the distances from one end of this chord to the ends of the diameter perpendicular to it.

51. The greater segments of a line divided internally in extreme and mean ratio is 1 foot 6 inches; find the length of the line.

52. The projections of the legs of a right △ upon the hypotenuse are 27^{cm} and 48^{cm}; find the lengths of the legs.

53. Find the width of a street, where a ladder 95.8 feet long will reach from a certain point in the street to a window 67.3 feet high on one side, and to one 82.5 feet high on the other side. (Log.)

54. Find the diameter of a ⊙ in which the chord of half the arc subtended by a chord of 30^{cm} is 17^{cm}.

55. Find the altitude of an equilateral △ whose side $= 2.2^{m}$.

56. What is the diameter of a ⊙ when the point from which a tangent of 6 feet is drawn is 8 inches from the circumference?

57. The sides of a △ are 185^{m}, 227^{m}, and 242^{m}; find the three altitudes, the bisectors of the three ∠s, and the radius of the circumscribed ⊙. (Log.)

58. The sides of a trapezoid are 437.3 feet, 91 feet, 291.7 feet, and 91 feet; find the altitude of the trapezoid and the diagonals.

59. The sides of a parallelogram are $24\frac{1}{2}$ miles and $31\frac{1}{2}$ miles, and one of the diagonals is 28 miles; find the number of kilometres in the other diagonal.

60. If a chord of 2 feet is 5 inches from the centre of a ⊙, what is the distance of a chord whose length is 10 inches?

61. One side of a △ is 136^{cm}, the altitude of the △ to the second side is 102^{cm}, the diameter of the circumscribed ⊙ is 184^{cm}; find the third side of the △.

62. The common chord of two intersecting ⊙s whose radii are 2 feet 1 inch and 1 foot 9 inches is 1 foot 2 inches; find the distance between their centres.

63. Is the △ whose sides are 38^m, 36^m, 12^m, acute, right, or obtuse?

64. In the △ whose sides are 11^m, 13^m, 14^m, find the segments into which the side 14 is divided by the perpendicular from the opposite vertex.

65. Find the legs of a right △ when their projections upon the hypotenuse are 11.16 feet and 19.84 feet.

66. The sides of a △ are 23 feet, 27 feet, 38 feet; find the length of the median to the longest side and its projection upon the longest side.

67. What is the longest and shortest chord that can be drawn through a point 15^{cm} from the centre of a ⊙ whose radius is 39^{cm}?

68. How long is the shadow of a house 23^m high, when a stake 4 feet high casts a shadow 2 feet 6 inches long? (Log.)

69. Find the length of the common tangent of two ⊙s which cuts the line joining their centres, when this line is 2 feet and the radii of the ⊙s are 5 inches and 3 inches.

70. The greater leg of a right △ is 1 inch, and the difference between the hypotenuse and the less leg is $\frac{1}{2}$ inch; find the hypotenuse, the less leg, the perpendicular from the vertex of the right ∠ to the hypotenuse, and the segments of the hypotenuse made by this perpendicular.

71. Find the product of the segments of any chord passing through a point 8^m from the centre of a ⊙ whose diameter is 20^m.

72. Through a point 21^{cm} from the circumference of a ⊙ is drawn a secant 84^{cm} long. The chord part of this secant is 51^{cm}. Find the radius of the ⊙.

73. The diagonals A C and B D of an inscribed quadrilateral intersect at E, A C is 59^{m}, B E 35^{m}, and D E 18^{m}; find A E and C E.

74. What is the length of a tangent drawn from a point 4 inches from the circumference of a ⊙ whose radius is 3 feet 9 inches?

75. Find the diameter of a ⊙ in which two chords, 30 feet and 40 feet long, parallel and on opposite sides of the diameter, are 35 feet apart.

76. The smaller segment of a line divided externally in extreme and mean ratio is 12^{cm}; find the length of the greater segment.

77. Two sides of a △ are 16^{Km} and 9^{Km}, and the median to the first side is 11^{Km}; find the length of the third side in miles.

78. In the preceding problem, find the lengths of the projections of the median and the second and third side upon the first side.

79. Find the lengths of the projections of each side upon the other two sides in a △ whose sides are 6^{m}, 8^{m}, and 12^{m}.

80. How far apart are two parallel chords 48 feet and 14 feet long in a ⊙ whose diameter is 50 feet, if they are on the same side of the centre?

BOOK IV.

1. Find the area of a rectangle whose base and altitude are 37 feet and 14 feet.

2. What is the area of a parallelogram whose base and altitude are 13^{m} and 18^{m}?

3. How many hektares in a rectangular field 53^{Dm} by 29^{Dm}?

4. Find the width of a rectangular field containing an acre, if the length is 176 yards.

5. How many acres in a parallelogram whose base and altitude are 17^{Hm} and 13^{Hm}? (Log.)

6. How many rods in the side of a square field containing a hektare? (Log.)

7. How many metres in the side of a square field containing an acre? (Log.)

8. A rectangle which is 7 times as long as it is wide contains 32 square rods; find its width and length.

9. Find the area of the surface of a flower-bed 4.55^{m} long and 2.75^{m} wide.

10. The perimeter of a rectangle is 24^{m}, and the length is 9.2^{m}; find the breadth and the area of the rectangle.

11. What is the ratio of the areas of two rectangular fields, one of which is 231^{m} long and 87^{m} wide, and the other 58^{m} wide and 110^{m} long?

12. Two rectangles have the same altitude, and the area of the first is 62 acres and the area of the second 38 acres.

If the base of the first is 570 rods, what is the base of the second?

13. What part of a mile is the perimeter of a square hektare ($1^{Km} = \frac{5}{8}^{mile}$)?

14. The perimeter of a rectangle is 6 feet, and the length is 3 times the breadth; find the length, the breadth, and the area of the rectangle.

15. If the perimeter of a rectangle is 26^m and its length is 2.5^m more than its breadth, find its length, breadth, and area.

16. A parallelogram whose area is one acre has a base of 60 rods; and one whose area is 1^{Ha} has the same altitude; find the base of the latter. (Log.)

17. Find the side of a square equivalent in area to a rectangle whose base is 4 feet 6 inches and whose altitude is 6 inches.

18. What is the altitude of a rectangle whose base is 23^m, equivalent to a square whose area is 5.06^a?

19. Find the area of a $\triangle$ whose base and altitude are respectively 3 feet 2 inches and 5 yards 1 inch.

20. Find the base and altitude of a rectangle whose perimeter is 54^m and whose area is 182^{qm}.

21. Find the side of a square whose area is 18 square yards 7 square feet.

22. Find the area, in acres, of a rectangle whose perimeter is 156^{Dm} and whose dimensions are to each other as 6 : 7. (Log.)

23. A $\triangle$ whose base is 35^{cm} contains $.525^{ca}$; how many square inches in a $\triangle$ whose homologous base is 14^{cm}? (Log.)

24. Find the area of an equilateral △ whose side is 8 feet.

25. Find the difference in area between a △ whose base and altitude are each 1 yard, and a △ whose sides are each 1^m. (Log.)

26. The bases of a trapezoid are 7.32^m and 8.45^m, and the altitude is 4.4^m; find the area in ares.

27. The altitude of an equilateral △ is 6 feet 3 inches; find the area.

28. To find the area of a △ in terms of its sides.

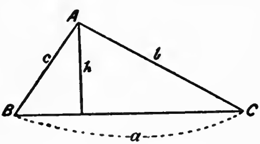


FIG. 10.

Let K = the area of the △.

(1) $K = \frac{1}{2} ah.$

(The area of a △ is equal to one-half the product of its base and altitude.)

By 24, Book III., $h = \frac{2}{a}\sqrt{s(s-a)(s-b)(s-c)}$.

Substituting in (1), $K = \frac{a}{2} \times \frac{2}{a}\sqrt{s(s-a)(s-b)(s-c)}$; or,

$$K = \sqrt{s(s-a)(s-b)(s-c)}.$$

29. To find the area of a △ in terms of its sides and the radius of the circumscribed ⊙.

By 25, Book III., $R = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$.

Substituting K for its value as found in the preceding article,

$$R^* = \frac{abc}{4K}.$$

Solving,
$$K = \frac{abc}{4R}.$$

30. To find the area of a △ in terms of its sides and the radius of the inscribed ⊙.

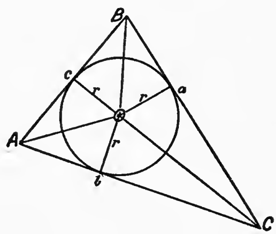


FIG. 11.

By drawing lines from the centre of the ⊙ to the vertices we form three ⊿ whose common vertex is O, whose bases are a, b, c, the sides of the given △, and whose altitudes are each r, the radius of the inscribed ⊙.

Now, Area AOC $= \frac{1}{2}br$,

" AOB $= \frac{1}{2}cr$,

" BOC $= \frac{1}{2}ar$.

Adding, " ABC $= \frac{1}{2}(a + b + c)r$.

Substituting K for area ABC, and s for $\frac{1}{2}(a + b + c)$,

$$K = rs.$$

From this equation, $r = \frac{K}{s}$; *i.e.*, the radius of the inscribed ⊙ equals the area of the △ divided by one-half the perimeter.

* Hereafter this form of the formula for R should be held in mind.

31. To find the area of a △ in terms of its sides and the radius of an escribed ⊙.*

(1) Area AOB $= \frac{1}{2}\, cr'$,
(2) " AOC $= \frac{1}{2}\, br'$,
(3) " BOC $= \frac{1}{2}\, ar'$.

Subtracting (3) from the sum of (1) and (2),

Area ABC $= \frac{1}{2}(b + c - a)r'$; or,

$$K = (s - a)r'.$$

Similarly, $K = (s - b)r''$,

$$K = (s - c)r''',$$

r'' and r''' representing the radii of the escribed ⊙ tangent to b and c, respectively.

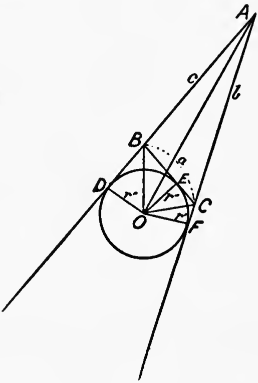


Fig. 12.

These three formulæ give $r' = \frac{K}{s-a}$, $r'' = \frac{K}{s-b}$, $r''' = \frac{K}{s-c}$, from which the radii of the escribed ⊙ can be found when the sides of a △ are given.

32. The sides of a △ are: $a = 21^m$, $b = 17^m$, $c = 10^m$; find the area of the △ and the radii of the circumscribed, inscribed, and escribed ⊙.

33. Find the difference in area between a rectangle 4 times as long as wide, with a perimeter of 100 yards and a square whose perimeter is 80 yards.

34. A man has a rectangular piece of ground 55^m by 110^m. After a path 4.5^m wide is made around it, is the part left more or less than an acre? How much?

35. The bases of a trapezoid are 13.2^{cm} and 15.6^{cm}, and the altitude is 1 yard 2 inches; find the area in centares.

* An escribed ⊙ is a ⊙ tangent to one side of a △ and the prolongations of the other two sides.

36. The side of a square is 2 feet; find the sides of an equivalent rectangle whose base is 4 times its altitude.

37. The area of a △ is 112^{qm}, its base is 14^{m}; find the area of a similar △ whose homologous base is 8^{m}.

38. Find the dimensions of a rectangle whose perimeter is 8 feet 4 inches, and whose area is 4 square feet 13 square inches.

39. Through the middle of a rectangular garden, 156^{m} by 140^{m}, run two paths at right angles to each other and parallel to the sides, the longer one 0.8^{m} wide, the shorter 1.2^{m} wide; find the area not taken up by the paths.

40. The sides of a △ are: $a = 588$ feet, $b = 708$ feet, $c = 294$ feet; find the area of the △ and the radii of the circumscribed, inscribed, and escribed ⓢ. (Log.)

41. The area of a rhombus is 360^{ca}, one diagonal is 7.2^{Dm}; find the other.

42. The area of a polygon is $5\frac{4}{9}$ times the area of a similar polygon. If the longest side of the larger polygon is 40^{m}, what is the longest side of the smaller polygon?

43. Find the area of a square whose diagonal is 30 feet.

44. Find the number of square feet in an equilateral △ whose side is one metre. (Log.)

45. Find the side in kilometres, of an equilateral △ whose area is 47 acres. (Log.)

46. Find the side of a square equivalent to the difference of two squares whose sides are 115^{m} and 69^{m}.

47. Find the area, in square feet, of an isosceles right △ if the hypotenuse is 25^{m}. (Log.)

48. The sides of a △ are 10 feet, 17 feet, and 21 feet. Find the areas of the two parts into which the △ is divided by the bisector of the ∠ formed by the first two sides.

49. The side of a rhombus is 39^{m}, and its area is 540^{ca}; find its diagonals.

50. The area of a trapezoid is 13 acres, and the sum of its bases is 813 yards; find its altitude.

51. Find the area, in acres, of a right △ whose hypotenuse is 36^{Hm} and one leg 28.8^{Hm}. (Log.)

52. Find the ratio of the areas of two ▵ which have a common ∠, when the sides including this ∠ in the first are 131^{m} and 147^{m}, and in the second are 211 feet and 287 feet. (Log.)

53. Two homologous sides of two similar polygons are 21^{Hm} and 35^{Hm}; the area of the greater polygon is 525^{Ha}; what is the area of the smaller polygon?

54. Find the area of a quadrilateral whose sides are 8^{m}, 10^{m}, 12^{m}, 6^{m}, and one of whose diagonals is 14^{m}.

55. On a map whose scale is 1 inch to a mile, how many hektares would be represented by a square centimetre? (Log.)

56. The homologous altitudes of two similar ▵ are 9^{m} and 21^{m}, and the area of the smaller is 405 square feet; find the area of the larger.

57. The area of a trapezoid is 84^{Ha}, its altitude 3.5^{Hm}, and one base 20^{Hm}; find the other base.

58. The areas of two ⚠ are 144 square yards and 108 square yards. Two sides of the second are 12 yards and 21 yards, and one side of the first is 9 yards. Find a second side of the first, which, with the side 9 yards includes an ∠ equal to the ∠ of the second included by the sides 12 yards and 21 yards.

59. Find the area of a square whose diagonal is 8^m.

60. Find the difference in perimeter between a rectangle whose base is 16 feet and an equivalent square whose side is 12 feet.

61. Find the diagonals of a rhombus whose side is 6 feet 1 inch and whose area is 9 square feet 24 square inches.

62. Find the area of a trapezoid whose parallel sides are 28^m and 33^m, and whose non-parallel sides are 12^m and 13^m.

63. Find the dimensions of a rectangle whose area is 1,452 square feet and one of whose sides is $\frac{4}{5}$ its diagonal.

64. The sides of a △ are 26^m, 28^m, 30^m; find its area, the three altitudes, and the radii of the inscribed, escribed, and circumscribed ⓢ.

65. How many tiles, 6 inches by $4\frac{1}{2}$ inches, will it take to cover a swimming pool 40 feet by 27 feet?

66. Find the sides of an isosceles right △ whose area is 98^a.

67. Find the area (in centares) and one side of a rhombus, if the sum of the diagonals is 34 feet and their ratio is 5 : 12.

68. The bases of a trapezoid are 197.3^{m} and 142.7^{m}, and its area 37.57^{a}; find its altitude. (Log.)

69. Find the area, in square feet, of a right △, when the sides are in the ratio 3 : 4 : 5, and the altitude to the hypotenuse is 1.2^{Dm}.

70. In the quadrilateral A B C D, A B = 10^{m}, B C = 17^{m}, C D = 13^{m}, D A = 20^{m}, and A C = 21^{m}; find the area in hektares, and the perpendiculars from B and D to A C.

71. Find the area of a △ if the perimeter is 82 feet and the radius of the inscribed ⊙ 1.3 feet.

72. Find the ratio of the areas of two equilateral △s if the side of one is 10^{m} and the altitude of the other is 10^{m}.

73. Find the area, the altitudes, and the radii of the inscribed, escribed, and circumscribed ⊙ of the isosceles △ whose leg is 5 feet 5 inches and whose base is 10 feet 6 inches.

74. The bases of a trapezoid are 13^{m} and 61^{m}; the non-parallel sides are 25^{m} each; find the area of the trapezoid.

75. How many yards of carpet $\frac{3}{4}$ of a yard wide will it take to carpet a room 15 feet by 18 feet?

76. Find the area of a rhombus whose perimeter is 6^{m} and one of whose diagonals is 1.2^{m}.

77. The altitude of a given △ is $.32^{Km}$; find the homologous altitude, in miles, of a similar △ 49 times as large.

78. Find the area of a pentagon whose perimeter is 5.18^{m}, circumscribed about a ⊙ whose diameter is 1.1^{m}.

79. Find the area in square metres of a right △ in which a perpendicular from the vertex of the right ∠ to the hypotenuse divides the hypotenuse into segments of $39\frac{12}{17}$ feet and $11\frac{5}{17}$ feet. (Log.)

80. Upon the diagonal of a rectangle 6^m by 8^m a △ whose area is three times the area of the rectangle is constructed; find the altitude of the △.

81. Find the side of an equilateral △ equivalent to the sum of two equilateral △s whose sides are respectively 5^m and 12^m.

82. Find the area of a trapezoid whose bases are 26 feet and 40 feet, and whose other sides are 13 feet and 15 feet.

83. The three sides of a △ are 417.31 feet, 589.72 feet, and 389.6 feet; find its area in ares. (Log.)

84. Find the radii of the inscribed, escribed, and circumscribed ⊙s. (Log.)

85. Find the three altitudes. (Log.)

86. Find the median to the longest side.

87. Find the bisectors of the three ∠s. (Log.)

88. The base of a △ is 25^m, its altitude 12^m; find the area of the △ cut off by a line parallel to the base and two-thirds of the way from the vertex to the base.

89. Two homologous sides of two similar △s are 12 feet and 35 feet, respectively; find the homologous side of a similar △ equivalent to their sum.

90. The bases of a given △ and ▱ are equal, and the altitude of the △ is 2^m and the altitude of the ▱ 5^m; find the ratio of their areas.

91. How many yards of wall paper are required to paper a room 25 feet long, 22 feet wide, and 12 feet high, allowing for a chimney which projects into the room 1 foot, one door 5 feet by 7 feet, another 10 feet by 10 feet, a mantel 4 feet by 6 feet, and a window 6 feet by 11 feet?

92. The homologous altitudes of two similar $\triangle$s are 5^{m} and 15^{m}, respectively; what fraction of the second is the first?

93. Find the legs of a right $\triangle$ whose hypotenuse is 25^{Hm} and whose area is 150^{Ha}.

94. In a $\triangle$ whose base is 22 feet, find the length of the line parallel to the base and dividing the $\triangle$ into two equal parts. (Log.)

95. Find the area of the $\triangle$ whose sides are to each other as 5 : 12 : 13, and whose altitude to the greater side is $23\frac{1}{3}$ inches.

96. The area of the polygon P is 735.8^{qm}, and of the similar polygon Q is 98.47^{qm}; find the side of Q homologous to a side of P equal to 81.41^{m}. (Log.)

97. If two sides of a $\triangle$ whose area is 9 acres are 165 rods and 201 rods, what is the length of the portions of these sides cut off by a line parallel to the base and cutting off a $\triangle$ of 4 acres?

98. Find the area of a right $\triangle$ whose hypotenuse is 70^{m} and one of whose $\angle$s is 60°. (Log.)

99. The side of a square is 12^{m}; find the side of a square having the ratio 8 to 3 to this square.

100. In a trapezoid whose altitude is 10 feet and whose bases are 21 feet and 29 feet, what is the length of a line parallel to the bases and $2\frac{1}{2}$ feet from the smaller base.

BOOK V.

NOTE I.—The answers to a large number of the problems of this Book may be left in an expressed form, if desired. For example : What is the area of a hexagon inscribed in a ⊙ whose radius is 15 feet ? Ans. $\frac{6 \times \overline{15}^2}{4}\sqrt{3}$.

NOTE II.—Quite a number of problems in this Book which seem difficult, on a mere reading, are rendered quite easy by drawing figures representing the given conditions and requirements.

NOTE III.—In many of these problems it is well to represent the number in terms of which the answer is to be gotten by a letter, and then replace the letter by its value in the final form of the result, as in finding the area, etc., of circumscribed and inscribed polygons in terms of the radius.

1. How many degrees in each ∠ of a regular octagon ? Of a regular dodecagon ? Of a regular polygon of 27 sides ?

2. How many degrees in the ∠ at the centre of a regular polygon of 15 sides ? Of 16 sides ?

3. Find the side of a square inscribed in a ⊙ whose radius is 91 feet.

4. Find the radius of a ⊙ circumscribed about a regular hexagon whose perimeter is 5.1^{m}.

5. How many degrees in each exterior ∠ of a regular polygon of 18 sides ? Of 25 sides ? Of 35 sides ?

6. How many sides has the regular polygon whose ∠ at the centre is $17° \; 8' \; 7\frac{5}{7}''$?

7. How many sides has the regular polygon whose interior and exterior ∠s are in the ratio of 18 to 4?

8. Find the side of an equilateral △ inscribed in a ⊙ whose diameter is 35.8^{cm}.

9. Find the perimeter of a regular decagon inscribed in a ⊙ whose diameter is 7 feet.

10. Find the radius of the ⊙ circumscribed about a regular hexagon whose apothem is $12\sqrt{3}^{Hm}$; also the area of the hexagon.

11. Find the area of an equilateral △ inscribed in a ⊙ whose radius is 15 feet. (Log.)

12. Find the radius of a ⊙ circumscribed about a square whose area is 1 square yard 7 square feet.

13. Find the apothem of a regular hexagon whose area is $54\sqrt{3}^{qm}$.

14. Find the radius of a ⊙ circumscribed about an equilateral △ whose area is $27\sqrt{3}$ square feet.

15. Find the area of a regular hexagon whose perimeter is 78^{Km}.

16. The apothem of an inscribed square is $10\sqrt{2}$ feet; find the area of an equilateral △ circumscribed about the same ⊙.

17. Find the area of a regular polygon whose apothem is 3.75^{Hm}, and whose perimeter is 15^{Hm}. Express the result in acres. (Log.)

18. Find the side of a regular decagon inscribed in a ⊙ whose radius is 35 feet.

19. Find the ratio of the areas of two equilateral △, one inscribed in, the other circumscribed about, a ⊙ whose radius is 5 inches.

20. Find a mean proportional between the areas of problem 19. Find the area also of a regular hexagon inscribed in the same ⊙ (5-inch radius). Compare the two results. (Log.)

21. Find the radius of a ⊙ circumscribed about a regular hexagon whose apothem is $\frac{15}{2}\sqrt{3}$ feet.

22. Find the area, in acres, of a regular hexagon circumscribed about a ⊙ whose radius is 7^{Hm}. (Log.)

23. The area of an equilateral △ circumscribed about a given ⊙ is 87^{Ha}; find the area of a square inscribed in the same ⊙. (Log.)

NOTE.—It is customary to use the value $3\frac{1}{7}$ for π in problems involving English units, and 3.1416 where metric units are employed.

24. Find the circumference and area of a ⊙ whose radius is 11 feet.

25. Find the diameter and area of a ⊙ whose circumference is $53\frac{3}{7}$ feet.

26. Find the circumference of a ⊙ whose area is $502{,}656^{\text{ca}}$.

27. Two circumferences are in the ratio 3 : 5, and the radius of the larger is 35^{m}; what is the radius of the smaller?

28. Find the radius of a ⊙ equivalent to two ⊙ whose radii are respectively 5.6^{Dm} and 4.2^{Dm}.

29. What is the length of an arc of 75° of a ⊙ whose radius is 21 feet?

30. The areas of two ⊙s are in the ratio of $1 : 5\frac{4}{9}$. If the radius of the larger is 4 feet 1 inch, what is the radius of the smaller?

31. Find the difference in area between a square and an equilateral △ each inscribed in a ⊙ whose radius is 15^m. (Log.)

32. Find the area of a segment of a ⊙ of 31-foot radius cut off by the side of a regular inscribed hexagon. (Log.)

33. Find the difference in length between the circumference of a ⊙ whose area is 15836.8056^a and the perimeter of the inscribed hexagon.

34. Find the circumference of a ⊙ circumscribed about a square field containing 700 acres. (Log.)

35. Find the area of a ⊙ whose circumference is 29.53104^{Dm}.

36. What is the area of a segment whose arc is 120°, in a ⊙ whose radius is 4.3^{Hm}?

37. Find the number of degrees in an arc equal in length to the radius of its ⊙.

38. What is the ratio of the areas of two ⊙ whose radii are 50 feet and 65 feet?

39. Find the apothem, the side, and the area of a regular octagon inscribed in a ⊙ whose radius is 1^m. (Log.)

40. How many metres in the diameter of a ⊙ whose area is one acre?

41. What is the area of a sector whose arc is 175° in a ⊙ whose radius is 24 feet?

42. Find the radius of a ⊙ in which the arc subtended by the side of a regular inscribed dodecagon is 3.1416$^{\text{Dm}}$.

43. How many acres in a ⊙, if a quadrant is one mile in length?

44. What is the ratio of the areas of two ⓢ whose circumferences are 35$^{\text{m}}$ and 40$^{\text{m}}$, respectively?

45. Find the side, the apothem, and the area of a regular dodecagon inscribed in a ⊙ whose diameter is 3$^{\text{Km}}$. (Log.)

46. How far apart are the circumferences of two concentric ⊙ which contain 5 acres and 10 acres, respectively? (Log.)

47. Find the circumferences of the ⓢ circumscribed about and inscribed in a square whose side is 14$^{\text{m}}$. (Log.)

48. Find the ∠ at the centre subtended by an arc of 13 inches in a ⊙ whose radius is $14\frac{7}{22}$ inches.

49. What is the area between three ⓢ, each tangent to the other two, if each has a radius of 440 yards?

50. Find the side of a square equivalent to a ⊙ whose radius is 19 feet.

51. Find the length of a side and the area of a regular octagon circumscribed about a ⊙ whose radius is a mile. (Log.)

52. How far apart are two parallel chords in a ⊙ whose radius is 33 feet, if these chords are the sides of regular inscribed polygons, one a hexagon, the other a dodecagon? (Log.)

53. How many rotations to the mile does a wheel whose diameter is 5 feet 6 inches make?

54. Find the side of a regular pentagon equivalent to the sum of three regular pentagons whose sides are 8^m, 9^m, and 12^m.

55. How much more fence would it take to enclose 500 acres in the shape of a square than it would if it were in circular shape?

56. Find the perimeter of a sector whose area is 77 square inches and whose arc is 45°.

57. Find the area of that part of a ⊙ whose radius is 7^{Km} included between two parallel chords, one of which is the side of a regular inscribed △ and the other the side of an inscribed square. (Log.)

58. If a bicycle wheel makes 680 rotations to the mile, what is its diameter?

59. Find the side and area of a regular pentagon inscribed in a ⊙ whose radius is 8^m.

60. Find the area of a ⊙ in which is inscribed a rectangle 6 feet by 8 feet.

61. Find the area of the regular hexagon formed by joining the alternate vertices of a regular hexagon whose side is 20 feet.

62. Find the ratio of the areas of the two hexagons in problem 61.

63. What is the radius of a ⊙ whose area is doubled by increasing its radius 7 feet?

64. Find the side and the area of a regular dodecagon circumscribed about a ⊙, whose circumference is 31.416^{Hm}. (Log.)

65. Find the radius of a ⊙ equivalent to three ⊙, whose diameters are 54 feet, 56 feet, and 72 feet.

66. What is the difference in area between an equilateral △ and a regular decagon each of which has a perimeter of 3 miles? (Log.)

67. The area of a segment cut off by the side of a regular inscribed hexagon is 413^{Ha}; what is the perimeter of this segment? (Log.)

68. Find the side of a square equivalent to a ⊙, in which a chord of 30 feet has an arc whose height is 5 feet.

69. Find the radius of a ⊙ three times as large as a ⊙ whose radius is 3 feet.

70. What is the area of a regular octagon whose perimeter is 28^{Dm}? (Log.)

71. Find the area of the sector whose arc is 175 feet in a ⊙ whose radius is 133 feet.

72. What must be the width of a walk which contains 1^{Ha} made around a circular plot of ground containing 5^{Ha}?

73. Find the area of the sector whose arc is the side of a regular inscribed dodecagon in a ⊙ in which a chord of 70 feet is 12 inches from the centre.

74. An acre of ground lies between three ⓢ, each tangent to the other two; find the radius of one of these ⓢ.

75. Find the radius of a ⊙ 36 times as large as a ⊙ whose radius is 14^m.

76. If a meridian circle of the earth is 25,000 miles, what is the length of the diameter in kilometers?

77. If the circumference of a ⊙ is 34.5576^{Dm}, what is the diameter of a concentric ⊙ which divides it into two equivalent parts ?

78. If the side of a regular inscribed hexagon cuts off a segment whose area is 25^{a}, what is the apothem of this hexagon ? (Log.)

79. A wheel whose radius is 3 feet 6 inches makes 20 rotations per second ; how many miles will a point on the circumference go in a day ? (Log.)

80. The difference between the area of a ⊙ and its inscribed square is 3 acres, find the area of the square ?

81. If an 8-inch pipe will fill a certain cistern in 2 hours 40 minutes, how long will it take a 2-inch pipe ?

82. Find the radius of a ⊙ in which an arc of 18° has the same length as an arc of 45° has in a ⊙ whose radius is 56 feet.

83. If the radius of the earth is 3,963 miles, how many metres is it from the pole to the equator, measured on a meridian ? (Log.)

84. Upon each side of a 7-foot square as a diameter, semicircumferences are described within the square, forming four leaves, or lobes ; find the area of one of these leaves.

85. Find the number of acres between two concentric circumferences which are 2 miles and 1 mile long, respectively. (Log.)

86. Find the height of an arc subtended by the side of an inscribed dodecagon in a ⊙ whose area is 154 square feet.

87. Find the area of a ⊙ inscribed in a quadrant of a circle whose radius is 61^m.

88. Find the area of each part of the quadrant of problem 87, outside the inscribed ⊙.

89. If the circumference of a ⊙, whose diameter is 18^m, is divided into six equal parts, and arcs are described within the ⊙, with these points of division as centres, what is the area of the six leaf-shaped figures thus formed?

90. If a bridge in the form of a circular arch 18 feet high spans a stream 150 feet wide, what is the length of the whole circumference of which this arch is an arc ?

91. The area inclosed by two tangents and two radii is 140^{Ha}. If one of the tangents = 7^{Hm}, find the distance from the centre to the meeting of the tangents ; also the area of the ⊙, in acres.

92. Find the sum of the areas of the crescents formed by describing semicircumferences on the legs and hypotenuse of a right △ (all on one side), if the legs are 5 feet and 12 feet respectively. How does this compare with the area of the △ ?

93. If the sides of a △ are 40^m, 50^m, and 60^m, what is the length of the circumference of the circumscribed ⊙ ?

94. Find the sum of the areas of two segments, cut off by two chords, 15 feet and 20 feet respectively, drawn from the same point to the extremities of the diameter of their ⊙.

95. If the radius of the earth is 3,963 miles, how high must a light-house light be to be seen 30 miles off at sea ?

96. The areas of two concentric ⊙s are to each other as 5 to 8. Find the radii of the two ⊙s, if the area of that part of the ring which is contained between two radii making the angle 45° is 300 square feet.

97. If two tangents, including an ∠ of 60° and drawn from the same point without a ⊙, with two radii drawn to their points of contact, inclose an area of $162\sqrt{3}^{ca}$, find the length of these tangents and the area of the sector formed by these two radii and their arc.

98. Find the area of the segments of the ⊙ in the preceding problem made by a chord perpendicular to its radius at its middle point.

99. If a track, having two parallel sides and two semicircular ends, each equal to one of the parallel sides, measures exactly a mile at the curb, what distance does a horse cover running ten feet from the curb? How many acres within the circuit he makes?

100. Three ⊙s, each tangent to the other two, inclose with their convex arcs 1^{Ha} of ground. How far is it from the centres of these ⊙s to the middle point of this piece of ground?

NUMERICAL PROBLEMS, EXERCISES, PROPOSITIONS, AND OTHER QUESTIONS

SELECTED FROM THE

ENTRANCE EXAMINATION PAPERS OF A NUMBER OF THE LEADING COLLEGES AND SCIENTIFIC SCHOOLS.

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1. From any point in the base of an isosceles triangle perpendiculars are drawn to the sides; prove their sum to be equal to the perpendicular drawn from either basal vertex to the opposite side.—*Boston University.*

2. The angle at the vertex A of an isosceles triangle A B C is equal to twice the sum of the equal angles B and C. If C D is drawn perpendicular to B C, meeting A B produced at D, prove that the triangle A C D is equilateral.—*Wesleyan University.*

3. If from one of the vertices (A) of a triangle (A B C) a distance (A D) equal to the shorter one of the two sides (A B and A C) meeting in A be cut off on the longer one (A B), prove that $\angle DCB = \frac{1}{2} [\angle ACB - \angle ABC]$.—*U. of Cal.*

4. Show that the angle included between the internal bisector of one base angle of a triangle and the external bisector of the other base angle is equal to half the vertical angle of the triangle.—*Harvard.*

5. If A B C be an equilateral triangle, and if B D, C D bisect the angles B, C, the lines D E, D F parallel to A B, A C, divide B C into three equal parts.—*Cornell.*

6. What is a polygon? Prove that the sum of the interior angles of an n-gon is $n - 2$ straight angles.—*Dartmouth.*

7. A D and B C are the parallel sides of a trapezoid A B C D, whose diagonals intersect at E. If F is the middle point of B C, prove that E F produced bisects A D.—*Mass. Inst. Tech.*

8. If perpendiculars be drawn from the angles at the base of an isosceles triangle to the opposite sides, the line from the vertex to the intersection of the perpendiculars bisects the angle at the vertex and the angle between the perpendiculars. Prove.—*Boston University.*

9. Prove that a parallelogram is formed by joining the midpoints of the (adjacent) sides of any quadrilateral. Hint, draw the diagonals of the quadrilateral.—*Bowdoin.*

10. In any triangle A B C, if A D is drawn perpendicular to B C, and A E bisecting the angle B A C, the angle D A E is equal to one-half the difference of the angles B and C.—*Cornell.*

11. Show that in any right-angled triangle the distance from the vertex of the right angle to the middle point of the hypotenuse is equal to one-half the hypotenuse.—*School of Mines.*

12. If D is the middle point of the side B C of the triangle A B C, and B E and C F are the perpendiculars from B and C to A D, prove that B E = C F.—*Wesleyan University.*

13. If in a right-angled triangle one of the acute angles is one-third of a right angle, the opposite side is one-half the hypotenuse.—*U. of Cal.*

14. Prove that the diagonals and the line which joins

the middle points of the parallel sides of a trapezoid meet in a point.—*Harvard.*

15. How many degrees in one angle of an equiangular docedagon ?—*Dartmouth.*

16. If the opposite sides of a pentagon be produced to intersect, prove that the sum of the angles at the vertices of the triangles thus formed is equal to two right angles.—*Cornell.*

17. The interior angle of a regular polygon exceeds the exterior angle by 120°. How many sides has the polygon? —*Mass. Inst. Tech.*

18. If one diagonal of a quadrilateral bisects both angles whose vertices it connects, then the two diagonals of the quadrilateral are mutually perpendicular. Prove.—*Boston University.*

19. In a given polygon, the sum of the interior angles is equal to four times the sum of the exterior. How many sides has the given polygon ?—*Wesleyan University.*

20. What is the greatest number of re-entrant angles a polygon may have compared to the number of its sides ? What is the value of the re-entrant angles of a pentagon in terms of the interior angles not adjacent ?—*Cornell.*

21. Show what the sum of the opposite angles of a quadrilateral inscribed in a circle is equal to.—*Columbia.*

22. When and why may an arc be used as the measure of an angle ? The vertex of an angle of 60° is outside a circle and its sides are secants ; what is the relation between the intercepted arcs ?—*Dartmouth.*

23. Show that two angles at the centres of unequal circles are to each other as their intercepted arcs divided by the radii.—*U. of Cal.*

24. Prove that in any quadrilateral circumscribed about a circle the sum of two opposite sides is equal to the sum of the other two opposite sides.—*Harvard.*

25. Construct a common tangent to two circles.—*Boston University.*

26. Three consecutive sides of a quadrilateral inscribed in a circle subtend arcs of 82°, 99°, and 67° respectively. Find each angle of the quadrilateral in degrees, and the angle between its diagonals.—*Yale.*

27. If A C and B C are tangents to a circle whose centre is O, from a point C without the circle, prove that the centre of the circle which passes through O, A, and B, bisects O C.—*Mass. Inst. Tech.*

28. Fix the position of a given circle that touches two intersecting lines.—*Vanderbilt University.*

29. Through a given point in the circumference of a circle chords are drawn. Find the locus of their middle points.—*Cornell.*

30. Give contractions for the *inscribed, escribed,* and *circumscribed* circles of any triangle.—*Sheffield S. S.*

31. Construct a circle that shall pass through two given points and shall cut from a given circle an arc of given length.—*Vassar.*

32. Prove that the circumference of a circle may be passed through the vertices of a quadrilateral provided two of its opposite angles are supplementary.—*Boston University.*

33. A and B are two fixed points on the circumference of a circle, and P Q is any diameter. What is the locus of the intersection of P A and Q B ?—*Harvard.*

34. The length of the straight line joining the middle

points of the non-parallel sides of a circumscribed trapezoid is equal to one-fourth the perimeter of the trapezoid.—*Mass. Inst. Tech.*

35. The points of tangency of a quadrilateral, circumscribed about a circle, divide the circumference into arcs, which are to each other as 4, 6, 10, and 16. Find the angles of the quadrilateral.—*Harvard.*

36. Given three indefinite straight lines in the same plane, no two of which are parallel, show that four circles can be described to touch the three lines.

If two of the three lines are parallel, show that the four circles reduce to two.—*Cornell.*

37. From a fixed point O of a given circumference are drawn two chords, O P, O Q, so as to make equal angles with a fixed chord, O R, between them. Prove that P Q will have the same direction whatever the magnitude of the angles.—*Harvard.*

38. Draw a straight line tangent to a given circle and parallel to a given straight line.—*Yale.*

39. Given two parallel lines and a secant line, also two circles each tangent to both parallels and to the secant; prove that the distance between the centres equals the segment of the secant line intercepted between the two parallels.—*Boston University.*

40. The vertices of a quadrilateral inscribed in a circle divide the circumference into arcs which are to each other as 1, 2, 3, and 4. Find the angles between the opposite sides of the quadrilateral.—*Harvard.*

41. Show how to construct an isosceles triangle with a given base and a given vertical angle.—*School of Mines.*

42. Two circumferences intersect at A and B. Through B any secant is drawn so as to cut the circumferences in C

and D respectively. Show that the angle C A D is the same for all secants drawn through B. What value has this angle when the circumferences intersect each other orthogonally ?—*Harvard.*

43. The perimeter of the circumscribed equilateral triangle is double that of the similar inscribed triangle.—*Sheffield S. S.*

44. The radius of a circle is 13 inches. Through a point 5 inches from the centre a chord is drawn. What is the product of the two segments of the chord ? What is the length of the shortest chord that can be drawn through that point ?—*Wesleyan University.*

45. A B is the hypotenuse of a right triangle A B C. If perpendiculars be drawn to A B at A and B, meeting A C produced at D, and B C produced at E, prove the triangles A C E and B C D similar.—*Yale.*

46. Prove that the diagonal of a square is incommensurable with its side. When are two quantities said to be incommensurable ?—*Bowdoin.*

47. A B C D is an inscribed quadrilateral. The sides A B and D C are produced to meet at E. Prove triangles A C E and B D E similar.—*Mass. Inst. Tech.*

48. A chord 18 inches long is bisected by another chord 22 inches long. Find the segments of the latter.—*N. J. State College.*

49. In any given triangle, if from two of the vertices perpendiculars be drawn to the opposite sides, the triangle cut off by the line joining the feet of the perpendiculars is similar to the given triangle.—*U. of Cal.*

50. The diagonals of a certain trapezoid, which are 8 and 12 feet long respectively, divide each other into segments

which in the case of the shorter diagonal are 3 feet and 5 feet long. What are the segments of the other diagonal ?—*Harvard.*

51. The sides of a triangle are 5, 6, and 8. Find the segments of the last side made by a perpendicular from the opposite angle.—*Rutgers S. S.*

52. In a plane triangle what is the square on the side opposite to the obtuse angle equal to ? Demonstrate.—*School of Mines.*

53. The sides of a triangle are 9, 8, 13. Is the greatest angle acute, obtuse, or right ?—*Vassar.*

54. Given $AB = xy$, write five resulting proportions. Need not prove.—*Boston University.*

55. The radii of two circles are 8 inches and 3 inches, and the distance between their centres is 15 inches. Find the length of their common tangents.—*Wesleyan University.*

56. The bases of two similar triangles are respectively 12.34 and 18.14 metres. The altitude of the first is 6.12 metres ; find the altitude of the second. (Use logarithms.)—*Yale.*

57. If A B and C D are equal chords of a circle and intersect at E, prove that $AE = ED$ and $BE = EC$.—*Mass. Inst. Tech.*

58. One segment of a chord drawn through a point 7 units from the centre of a circle is 4 units. If the diameter of the circle is 15 units, what is the other segment ?—*Brown.*

59. Two parallel chords of a circle are d and k in length, and their distance apart is f; what is the radius ?—*Vanderbilt University.*

60. In a certain circle a chord is 10 inches long, while another chord twice as far from the centre as the first is 5 inches long; find the radius of the circle and the distances of the chords from the centre.—*Harvard.*

61. When is a line said to be divided *harmonically?* From the point P without a circle a secant through the centre is drawn cutting the circle in A and B. Tangents are drawn from P and the points of contact connected by a line cutting A B in Q. Show that P and Q divide A B harmonically.—*Sheffield S. S.*

62. Two sides of a triangle are 17 and 10; the perpendicular from their intersection to the third side is 8; what is the length of the third side?—*Mass. Inst. Tech.*

63. Prove that the sum of the squares of the sides of a parallelogram is equal to the sum of the squares of its diagonals.—*School of Mines.*

64. In a triangle whose sides are 48, 36, and 50, where do the bisectors of the angles intersect the sides? What are the lengths of the bisectors?—*Rutgers S. S.*

65. The distance from the centre of a circle to a chord 10 inches long is 12 inches. Find the distance from the centre to a chord 24 inches long.—*Wesleyan University.*

66. The diameter of a circle is 20 inches, the least distance from a certain point upon the circumference to a diameter is 8 inches; find the distances from this point to the ends of the above diameter.—*Boston University.*

67. Let A B C be a right triangle. The two sides about the right angle C are respectively 455 and 1,092 feet. The hypotenuse A B is divided into two segments A E and B E by the perpendicular upon it from C. Compute the lengths of A E, B E, and C E.—*Yale.*

68. C is any point on the straight portion, A B, of the boundary of a semicircle. C D, drawn at right angles to A B, meets the circumference at D. D O is drawn to the centre, O, of the circle, and the perpendicular dropped from C upon O D meets O D at E. Show that D C is a mean proportional to A O and D E.—*Harvard.*

69. The length of one side of a right triangle is 12, and the length of the perpendicular from its extremity to the hypotenuse is $4\frac{8}{13}$. Find the lengths of hypotenuse and other side.—*Mass. Inst. Tech.*

70. The three sides of a triangle are 6, 8, 10 units long; compute the lengths of the three medial lines.—*Cornell.*

71. The area of a rectangle is 64, the difference of two adjacent sides is 12; construct the rectangle.—*Bowdoin.*

72. Prove that if any point on one of the diagonals of a parallelogram be joined to the vertices, of the triangles thus formed, those having the same base are equivalent.—*U. of Cal.*

73. In a triangle A B C, let O be the point in which the medians (lines drawn from the vertices to the middle points of the opposite sides) intersect. Prove that the triangles O A B, O A C, O B C are equivalent.—*Amherst.*

74. If two equivalent triangles have a common base, and lie on opposite sides of it, the base, or the base produced, will bisect the line joining the vertices.—*Dartmouth.*

75. If the perimeter of a rectangle is 72 feet, and the length is equal to twice the width, find the area.—*Johns Hopkins University.*

76. The area of a certain isosceles triangle is 50 square feet, and each of its equal sides is 10 feet long; find the angles of the triangle.—*Cornell.*

77. Two mutually equiangular triangles are similar. The base of a triangle is 32 feet, its altitude 20 feet. What is the area of the triangle cut off by drawing a line parallel to the base and at a distance of 15 feet from the base ?—*Wesleyan University.*

78. The perimeter of a trapezoid is 56 inches. If each of the non-parallel sides is 13 inches long, and the area is 180 square inches, what are the respective lengths of the parallel sides ?—*Mass. Inst. Tech.*

79. The area of a certain polygon is 5 square feet. Find the area of a similar polygon whose perimeter is in the ratio of M to N to that of the given polygon.—*Sheffield S. S.*

80. A vertex of a parallelogram and the middle points of the two sides adjacent to it form the vertices of a triangle whose area is equal to one-eighth the area of the parallelogram.—*Boston University.*

81. (*a.*) If two triangles are on equal bases and between the same parallels, a line parallel to their bases cuts off equal areas.

(*b.*) Lines joining the non-adjacent extremities of two parallel chords are equal.

(*c.*) State and prove the converse of the preceding proposition.—*Yale.*

82. Given $\frac{2}{x} = \frac{x}{3}$. Construct x.—*Cornell.*

83. Find the area of a triangle in terms of its sides.—*Vanderbilt University.*

84. Prove that, if in the triangle A B C the line drawn from the vertex C to the middle point of the opposite side is equal to half the latter, the area of the triangle is nu-

merically equal to half the product of A C by B C.—*Harvard.*

85. Given three rectangles, find a square whose area is equal to the sum of the areas of the larger two minus the area of the smallest one.—*U. of Cal.*

86. Prove that the square described upon the altitude of an equilateral triangle has an area three times as great as that of a square described upon half of one side of the triangle.—*Cornell.*

87. A D and B C are the parallel sides of the trapezoid A B C D, whose diagonals intersect at O. Prove

$$\text{area } AOD : \text{area } BOC = \overline{AO}^2 : \overline{OC}^2.$$

—*Mass. Inst. Tech.*

88. Construct a square whose area is 3 times that of a given square.—*Sheffield S. S.*

89. Draw a hexagon having one re-entrant angle, and construct a triangle equivalent to this polygon.—*Cornell.*

90. The parallel sides of a trapezoid are 12 and 18, the non-parallel sides are each 5 ; find its area and the altitude of the triangle formed by producing the non-parallel sides until they meet.—*Dartmouth.*

91. Through a point in one side of a triangle draw a line parallel to the base which shall bisect the area of the triangle.—*Cornell.*

92. The area of a polygon is 160 square feet, one side is 6 feet long ; find the homologous side of a similar polygon whose area is 800 square feet.—*Boston University.*

93. The base of a triangle is 16 feet, and the two other sides are respectively 12 and 10 feet. Find the altitude of the triangle, and also the area.—*Yale.*

94. In a certain triangle A B C, $\overline{AC}^2 - \overline{BC}^2 = \frac{1}{2}\overline{AB}^2$; show that a perpendicular dropped from C upon A B will divide the latter into segments which are to each other as 3 to 1.—*Harvard.*

95. Construct a parallelogram equivalent to a given triangle and having one of the diagonals equal to a given line.—*U. of Cal.*

96. Construct a polygon similar to a given polygon and having two and a half times its area.—*Cornell.*

97. How many degrees in each angle of a regular decagon?—*Yale.*

98. If the diagonals A C and B G of the regular octagon A B C D E F G H intersect at O, how many degrees are there in the angle A O B?—*Mass. Inst. Tech.*

99. Show that the sum of the alternate angles of an inscribed hexagon (not necessarily regular) is equal to four right angles.—*School of Mines.*

100. An equilateral triangle is inscribed in a circle. Find its side, apothem, and area in terms of the radius R. —*Dartmouth.*

101. Find the ratio of the area of a regular hexagon inscribed in a circle to that of a regular hexagon circumscribed about the same circle.—*Johns Hopkins University.*

102. What regular polygon has each angle equal to five thirds of a right angle?—*U. of Cal.*

103. A certain equilateral triangle has sides $8\sqrt{3}$ inches long; what is the radius of the circumference circumscribed about this triangle?—*Harvard.*

104. Compute the area of a regular hexagon whose side is 5 feet. Construct a triangle of equivalent area.—*Sheffield S. S.*

105. The area of the regular inscribed hexagon of a circle is three-fourths of that of the regular circumscribed hexagon.—*Cornell.*

106. Find the number of degrees in an angle of a regular pentagon and give proof of the process.—*Bowdoin.*

107. If the interior angles of any quadrilateral be bisected and each bisector produced to meet two others, the quadrilateral formed may be inscribed in a circle. Prove.—*Boston University.*

108. The diagonals of a regular pentagon divide each other in mean and extreme ratio.—*U. of Cal.*

109. Show that an equiangular polygon inscribed in a circle is regular if the number of its sides is odd.—*Cornell.*

110. The radius of a certain circle is 9 inches; find the area of that one of all the regular polygons inscribed in it which has the shortest perimeter. How long a perimeter can a regular polygon inscribed in this circle have?—*Harvard.*

111. A regular hexagon, A B C D E F, is inscribed in a circle whose radius is 2; find the length of the diagonal A C.—*Mass. Inst. Tech.*

112. To compute the area of a circle whose radius is unity.—*Dartmouth.*

113. Find the area of a circle inscribed in a square containing 400 square feet.—*N. J. State College.*

114. Find the side of a square equivalent to a circle whose radius is 56 feet. (Use logarithms.)—*Yale.*

115. The area of a certain regular hexagon is 294 $\sqrt{3}$ square inches; find the area and the circumference of the circumscribed circle.—*Harvard.*

116. The circumference of a circle is 78.54 inches ; find (1) its diameter, and (2) its area.—*Rutgers S. S.*

117. If the areas of two regular pentagons be as 16 to 25, and the perimeter of the first pentagon be 50 inches, what is the perimeter of the second ?—*Cornell.*

118. If the radius of a circle is 5, find the area of the sector whose central angle is 50°.—*Wesleyan University.*

119. The angle of a sector is 30° ; the radius is 12. Find the area of the sector.—*Amherst.*

120. Prove that the area of the regular inscribed dodecagon is equal to three times the square of the radius.—*U. of Cal.*

121. If the diameter of a circle is 3 inches, what is the length of an arc of 80° ?—*Mass. Inst. Tech.*

122. In a circle whose radius is 8, what is the length of the arc of a sector of 45° ? What is the area of this sector ?—*Rutgers S. S.*

123. If the radius of a circle is 5 inches, compute its circumference and its area ; also the perimeter, the area, and the apothem of an inscribed square.—*Yale.*

124. The perimeter of a regular hexagon is 480 feet, and that of a regular octagon is the same. Which is the greater in area, and by how much ?—*Cornell.*

125. The area of a certain circle is 154 square inches ; what angle at the centre is subtended by an arc of the circumference $5\frac{1}{2}$ inches long ?—*Harvard.*

126. Find the length of the arc of 75° in the circle whose radius is 5 feet.—*N. J. State College.*

127. A M and B N are perpendiculars from points A and B to the line M N. Find a point P on the line M N such

that the sum of the distances A P, B P, is the least possible.—*Wellesley.*

128. Two circles are tangent internally, the ratio of their radii being 2 : 3. Compare their areas, and also the area left in the larger circle with each.—*Sheffield S. S.*

129. A kite-shaped racing-track is formed by a circular arc and two tangents at its extremities. The tangents meet at an angle of 60°. The riders are to go round the track, one on a line close to the inner edge, the other on a line everywhere $5\frac{1}{4}$ ft. outside the first line. Show that the second rider is handicapped by about 22 feet.—*Harvard.*

130. The diameters of two water-pipes are 6 and 8 inches respectively. What is the diameter of a pipe having a capacity equal to their sum ?—*Rutgers S. S.*

131. (*a.*) There are two gardens : one is a square and the other a circle ; and they each contain a hectare. How much farther is it around one than the other ?

(*b.*) If the area of each is 2 hectares, what will be the difference of their perimeters ?—*Yale.*

132. Inscribe a square in a scalene triangle.—*Cornell.*

133. A horse is tethered to a hook on the inner side of a fence which bounds a circular grass-plot. His tether is so long that he can just reach the centre of the plot. The area of so much of the plot as he can graze over is $\frac{98}{3}$ $(4\pi - 3\sqrt{3})$ sq. rd. ; find the length of the tether and the circumference of the plot.—*Harvard.*

134. If the apothem of a regular hexagon is 2, find the area of its circumscribed circle.—*Wesleyan University.*

135. Of all polygons formed of given sides the maximum may be inscribed in a circle.—*Sheffield S. S.*

136. If the radius of a circle is 6, what is the area of a segment whose arc is 60° ? (Take $\pi = 3.1416$.)—*Mass. Inst. Tech.*

137. A stone bridge 20 ft. wide has a circular arch of 140 ft. span at the water level. The crown of the arch is $140\ (1 - \frac{1}{2}\sqrt{3})$ ft. above the surface of the water. How many square feet of surface must be gone over in cleaning so much of the under side of the arch as is above water ?—*Harvard.*

138. Of all isoperimetric figures the circle has the greatest area.—*Cornell.*

139. Compute by logarithms the value of

$$\sqrt{\frac{(2.3456)^3 \times (.301456)^2}{(4.02356)^4}}$$ —*Yale.*

SELECTED EXAMINATION PAPERS IN PLANE GEOMETRY SET FOR ADMISSION TO A NUMBER OF THE LEADING COLLEGES AND SCIENTIFIC SCHOOLS IN THE UNITED STATES.

Harvard, June, 1892.

[In solving problems use for π the approximate value $3\frac{1}{7}$.]

1. Prove that if two sides of a triangle are unequal, the angle opposite the greater side is greater than the angle opposite the less side.

In a certain right triangle one of the legs is half as long as the hypotenuse; what are the angles of the triangle?

2. Show how to find on a given indefinitely extended straight line in a plane, a point O which shall be equidistant from two given points A, B in the plane. If A and B lie on a straight line which cuts the given line at an angle of 45° at a point 7 inches distant from A and 17 inches from B, show that O A will be 13 inches.

3. Prove that an angle formed by a tangent and a chord drawn through its point of contact is the supplement of any angle inscribed in the segment cut off by the chord. What is the locus of the centre of a circumference of given radius which cuts at right angles a given circumference?

4. Show that the areas of similar triangles are to each other as the squares of the homologous sides.

5. Prove that the square described upon the altitude of an equilateral triangle has an area three times as great as that of a square described upon half of one side of the triangle.

6. Find the area included between a circumference of radius 7 and the square inscribed within it.

Harvard, June, 1893.

[In solving problems use for π the approximate value $3\frac{1}{7}$.]

1. Prove that two oblique lines drawn from a given point to a given line are equal if they meet the latter at equal distances from the foot of the perpendicular dropped from the point upon it.

How many lines can be drawn through a given point in a plane so as to form in each case an isosceles triangle with two given lines in the plane?

2. Prove that in the same circle, or in equal circles, equal chords are equally distant from the centre, and that of two unequal chords the less is at the greater distance from the centre.

Two chords of a certain circle bisect each other. One of them is 10 inches long; how far is it from the centre of the circle?

A variable chord passes, when produced, through a fixed point without a given circle. What is the locus of the middle point of the chord?

3. A common tangent of two circumferences which touch each other externally at A, touches the two circumferences at B and C respectively; show that B A is perpendicular to A C.

4. Assuming that the areas of two triangles which have an angle of the one equal to an angle of the other are to each other as the products of the sides including the equal angles, prove that the bisector of an angle of a triangle divides the opposite side into parts which are proportional to the sides adjacent to them.

5. Prove that the circumferences of two circles have the same ratio as their radii.

6. A quarter-mile running track consists of two parallel straight portions joined together at the ends by semicircumferences. The extreme length of the plot enclosed by the track is 180 yards. Find the cost of sodding this plot at a quarter of a dollar per square yard.

Harvard, June, 1894.

[In solving problems use for π the approximate value $3\frac{1}{7}$.]

1. Prove that any quadrilateral the opposite sides of which are equal, is a parallelogram.

A certain parallelogram inscribed in a circumference has two sides 20 feet in length and two sides 15 feet in length; what are the lengths of the diagonals?

2. Prove that if one acute angle of a triangle is double another, the triangle can be divided into two isosceles triangles by a straight line drawn through the vertex of the third angle.

Upon a given base is constructed a triangle one of the base angles of which is double the other. The bisector of the larger base angle meets the opposite side at the point P. Find the locus of P.

3. Show how to find a mean proportional between two given straight lines, but do not prove that your construction is correct.

Prove that if from a point, O, in the base, B C, of a triangle, A B C, straight lines be drawn parallel to the sides, A B, A C, respectively, so as to meet A C in M and A B in N, the area of the triangle A M N is a mean proportional between the areas of the triangles B N O and C M O.

4. Assuming that the areas of two parallelograms which have an angle and a side common and two other sides unequal, but commensurable, are to each other as the unequal sides, prove that the same proportion holds good when these sides have no common measure.

5. Every cross-section of the train-house of a railway station has the form of a pointed arch made of two circular arcs the centres of which are on the ground. The radius of each arc is equal to the width of the building (210 feet); find the distance across the building measured over the roof, and show that the area of the cross-section is $3,675\ (4\pi - 3\sqrt{3})$ square feet.

Harvard, June, 1895.

One question may be omitted.

[In solving problems use for π the approximate value $3\frac{1}{7}$.]

1. Prove that if two straight lines are so cut by a third that corresponding alternate-interior angles are equal, the two lines are parallel to each other.

2. Prove that an angle formed by two chords intersecting within a circumference is measured by one-half the sum of the arcs intercepted between its sides and between the sides of its vertical angle.

Two chords which intersect within a certain circumference divide the latter into parts the lengths of which, taken in order, are as 1, 1, 2, and 5 ; what angles do the chords make with each other?

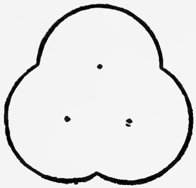
3. Through the point of contact of two circles which touch each other externally, any straight line is drawn terminated by the circumferences; show that the tangents at its extremities are parallel to each other.

What is the locus of the point of contact of tangents drawn from a fixed point to the different members of a system of concentric circumferences?

4. Prove that, if from a point without a circle a secant and a tangent be drawn, the tangent is a mean proportional between the whole secant and the part without the circle.

Show (without proving that your construction is correct) how you would draw a tangent to a circumference from a point without it.

5. Prove that the area of any regular polygon of an even number of sides ($2n$) inscribed in a circle is a mean proportional between the areas of the inscribed and the circumscribed polygons of half the number of sides. If n be indefinitely increased what limit or limits do these three areas approach?



6. The perimeter of a certain church window is made up of three equal semi-circumferences, the centres of which form the vertices of an equilateral triangle which has sides $3\frac{1}{2}$ feet long. Find the area of the window and the length of its perimeter.

Harvard, June, 1896.

One question may be omitted.

[In solving problems use for π the approximate value $3\frac{1}{7}$.]

1. Prove that if two oblique lines drawn from a point to a straight line meet this line at unequal distances from the foot of the perpendicular dropped upon it from the given point, the more remote is the longer.

2. Prove that the distances of the point of intersection of any two tangents to a circle from their points of contact are equal.

A straight line drawn through the centre of a certain circle and through an external point, P, cuts the circumference at points distant 8 and 18 inches respectively from P. What is the length of a tangent drawn from P to the circumference?

3. Given an arc of a circle, the chord subtended by the arc and the tangent to the arc at one extremity, show that the perpendiculars dropped from the middle point of the arc on the tangent and chord, respectively, are equal.

One extremity of the base of a triangle is given and the centre of the circumscribed circle. What is the locus of the middle point of the base?

4. Prove that in any triangle the square of the side opposite an acute angle is equal to the sum of the squares of the other two sides diminished by twice the product of one of those sides and the projection of the other upon that side.

Show very briefly how to construct a triangle having given the base, the projections of the other sides on the base, and the projection of the base on one of these sides.

5. Show that the areas of similar triangles are to one another as the areas of their inscribed circles.

The area of a certain triangle the altitude of which is $\sqrt{2}$, is bisected by a line drawn parallel to the base. What is the distance of this line from the vertex?

6. Two flower-beds have equal perimeters. One of the beds is circular and the other has the form of a regular hexagon. The circular bed is closely surrounded by a walk 7 feet wide bounded by a circumference concentric with the bed. The area of the walk is to that of the bed as 7 to 9. Find the diameter of the circular bed and the area of the hexagonal bed.

Yale, June, 1892.

TIME ALLOWED, ONE HOUR.

1. Construct accurately, by ruler and compass, a parallelogram A B C D having the angle A 45°, the side A B 6 units in length, and the altitude 3 of the same units.

Calculate the length of A C.

2. (*a*) State the converse of the following proposition:

If a triangle is isosceles and if a straight line is drawn through the vertex parallel to the base, it bisects an exterior angle of the triangle.

(*b*) Prove the converse as you have stated it.

Make the demonstration as full and clear as possible.

3. Prove two of the following propositions: The work may be limited to drawing a figure and giving a synopsis of the demonstration.

(*a*) *If the area of a regular polygon is equal to the product of the perimeter by one-half the apothegm, it follows that the area of a circle* $= \pi R^2$.

(*b*) *If two lines are drawn through the same point across a circle, the products of the two distances on each line from this point to the circumference are equal to each other.*

(*c*) *If the radius of a circle be divided in extreme and mean ratio, the greater segment is equal to one side of a regular inscribed decagon.*

Yale, June, 1893.

1. Prove that if the diagonals of a quadrilateral bisect each other the figure is a parallelogram.

2. Prove that in any right-angled triangle the square on the side opposite to the right angle is equal to the sum of the squares on the other two sides.

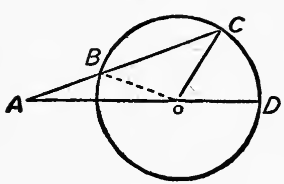
A purely geometrical proof is preferred.

State fully each principle employed in the proof.

3. Given a straight line A B, of indefinite length, and a point C without it. Find a point in A B equally distant from A and C.

Make the necessary construction accurately with ruler and compass.

In what case is the solution impossible?



4. Given an angle C O D at the centre of a circle and the line C A meeting D O produced in A so that A B is equal to the radius of the circle. Prove that the angle A is equal to one-third of the angle C O D.

Yale, June, 1894.

GEOMETRY (A).

TIME ONE HOUR.

1. If the diagonals of a quadrilateral bisect each other, the figure is a parallelogram.

2. To draw a tangent to a given circle, so that it shall be parallel to a given straight line.

3. If A B is a chord of a circle, and C E is any chord drawn through the middle point C of the arc A B cutting the chord A B at D, prove that the chord A C is a mean proportional between C D and C E.

4. The areas of two similar triangles are to each other as the squares of any two homologous sides.

5. The area of a circle is equal to one-half the product of its circumference and radius.

Yale, June, 1894.

GEOMETRY (B).

TIME FORTY-FIVE MINUTES.

1. What is the number of degrees in each angle of a regular decagon?

2. Find the area in square feet of an equilateral triangle whose side is 3 metres.

3. A B C is a right triangle. The sides A C and B C about the right angle C are respectively 50 and 120 feet. Divide the triangle into two parts equal in area by a line D F parallel to B C. Compute the length of the three sides of the triangle A D F.

4. The area of a circle is a hectare. What is its diameter?

5. Calculate in metres the length of a degree on the circumference of the earth, assuming the section of the earth to be a circle whose radius is 3,963 miles. [Those taking the preliminary examinations must use logarithms.]

[*For preliminary candidates only.*]

6. Find the value of the following expression by logarithms:

$$\sqrt[3]{\frac{(.06342)^2 \times 187.32}{.34216 \times 6.0372}}.$$

Yale, June, 1895.

GEOMETRY (A).

TIME ALLOWED, SIXTY MINUTES.

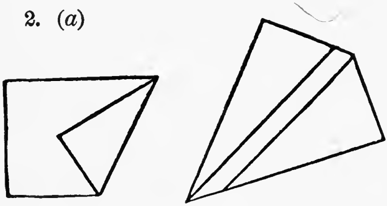
1. (*a*) Define the terms "locus" and "limit of a variable" and give an example of each.

(*b*) Prove that two triangles are similar if their homologous sides are proportional.

(*c*) Through a given point A within a circle draw two equal chords.

[*Both the construction (with ruler and compass), and also the proof, are required.*]

2. (*a*)



Prove that if each of two angles of a quadrilateral is a right angle, the bisectors of the other angles are either perpendicular, or parallel, to each other.

(*b*) Prove that if the radius of a circle is divided in extreme and mean ratio, the greater part is equal to the side of a regular inscribed decagon.

[*The construction is not required.*]

Yale, June, 1895.

GEOMETRY (B).

TIME ALLOWED, FORTY-FIVE MINUTES.

One question may be omitted. Logarithmic tables should be used in calculating the answers of two questions.

1. The base of a triangle is 14 inches and its altitude is 7 inches. Find the area of the trapezoid cut off by a line 6 inches from the vertex.

Express the result in square metres.

2. Find the number of feet in an arc of 40° 12′ if the radius of the circle is 0.7539 metres.

3. The length of a chord is 10 feet, and its greatest distance from the subtending arc is 2 feet 7½ inches. Find the radius of the circle.

4. Find the area, and also the weight in grams, of the largest square that can be cut from a circular sheet of tin 16 inches in diameter and weighing 8.2 ounces per square foot.

Yale, June, 1896.

GEOMETRY (A).

TIME, ONE HOUR.

1. The sum of the three angles of a triangle is equal to two right angles.

2. Construct a circle having its centre in a given line and passing through two given points.

3. The bisector of the angle of a triangle divides the opposite side into segments which are proportional to the two other sides.

4. If two angles of a quadrilateral are bisected by one of its diagonals, the quadrilateral is divided into two equal triangles and the two diagonals of the quadrilateral are perpendicular to each other.

5. The circumferences of two circles are to each other as their radii. (Use the method of limits.)

Yale, June, 1896.

GEOMETRY (B).

TIME ALLOWED, FORTY-FIVE MINUTES.

1. A tree casts a shadow 90 feet long, when a vertical rod 6 feet high casts a shadow 4 feet long. How high is the tree?

2. The distance from the centre of a circle to a chord 10 inches long is 12 inches. Find the distance from the centre to a chord 24 inches long.

3. The diameter of a circular grass plot is 28 feet. Find the diameter of a grass plot just twice as large. (Use logarithms.)

4. Find the area of a triangle whose sides are $a = 12.342$ metres $b = 31.456$ metres $c = 24.756$ metres, using the formula Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$. (Use logarithms.)

Princeton, June, 1894.

What text-book have you read?

1. Prove that the sum of the three angles of a triangle is equal to two right angles. Define triangle, right angle, right triangle, scalene triangle.

2. Prove that the opposite sides and angles of a parallelogram are equal. Define a parallelogram, a rectangle.

3. Prove that an angle inscribed in a circle is measured by one-half of the arc intercepted by its sides.

Consider all cases.

4. Show how to construct a triangle, having given two sides and the angle opposite one of them.

Is the construction always possible? If not, state when and why it fails.

5. Prove that if any chord is drawn through a fixed point within a circle, the product of its segments is constant in whatever direction the chord is drawn.

6. Prove the ratio between the areas of two triangles which have an angle of the one equal to an angle of the other.

Define area.

7. Define a regular polygon and prove that two regular polygons of the same number of sides are similar.

Define similar figures.

Princeton, June, 1895.

What text-book have you read?

1. Prove that every point in a perpendicular erected at the middle of a given straight line is equidistant from the extremities of the line, and every point not in the perpendicular is unequally distant from the extremities of the line.

2. Prove that the sum of the interior angles of a polygon is equal to two right angles taken as many times less two as the figure has sides.

Define a polygon, also a right angle.

3. Prove that the tangents to a circle drawn from an exterior point are equal, and make equal angles with the secant drawn from this point through the centre; also that either tangent is a mean proportional between the secant and its external segment.

Define circle, tangent, secant, chord, mean proportional.

4. Show how to circumscribe a circle about a given triangle, giving reasons for the process.

5. Prove what the area of a triangle is equal to; also the area of a trapezoid.

Define triangle, trapezoid, area.

6. Prove that the area of a circle is equal to one-half the product of the circumference by the radius.

Express the area of a circle in terms of π.

Define π and give its numerical value.

Princeton, June, 1896.

State what text-book you have read and how much of it.

1. Prove that the sum of the three angles of a triangle is equal to two right angles; and that the sum of all the interior angles of a polygon of n sides is equal to $(n-2)$ times two right angles.

2. Show that the portions of any straight line intercepted between the circumferences of two concentric circles are equal.

3. Define similar polygons and show that two triangles whose sides are respectively parallel or perpendicular are similar polygons according to the definition.

4. Prove that, if from a point without a circle a secant and a tangent are drawn, the tangent is a mean proportional between the whole secant and its external segment.

5. Prove what the area of a triangle is equal to;—also of a trapezoid;—also of a regular polygon. Define each of the figures named.

6. Explain how to construct a triangle equivalent to a given polygon.

7. Prove that of all isoperimetric polygons of the same number of sides, the maximum is equilateral.

Princeton, September, 1896.

State what text-book you have read and how much of it.

1. Name and define six quadrilateral figures.

Prove that in a parallelogram the opposite sides are equal, and the diagonals bisect each other.

2. Define and show how to construct the inscribed circle and the three escribed circles of a given triangle.

3. Prove that, if the base of a triangle is divided, either internally or externally, into segments proportional to the other two sides, the line joining the point of section and the opposite vertex of the triangle is the bisector of the angle (either internal or external) at that vertex.

4. Prove what the area of a parallelogram is equal to, and show how to construct a square equivalent to a given parallelogram.

5. Prove that if a circle is divided into any number of equal parts, the chords joining the successive points of division form a regular inscribed polygon, and the tangents drawn at the points of division form a regular circumscribed polygon.

6. Prove that the maximum of all isoperimetric polygons of the same number of sides is a regular polygon.

Columbia, June, 1896.

TIME ALLOWED, TWO AND ONE-HALF HOURS.

Omit one question from each of the groups, A, B, C.

State what text-book you have used in preparation.

A.

1. Prove that, in a circle, a diameter is greater than any other chord.

2. Prove that, in any triangle, a line drawn parallel to the base divides the other sides proportionally.

3. Prove that an angle formed by a tangent and a chord of a circle meeting at the point of contact of the tangent, is measured by one-half of the included arc.

B.

4. Prove that if four quantities are in proportion, they are in proportion by composition and by division.

5. Show how to construct a triangle equal to a given pentagon.

6. Show how to inscribe a regular decagon in a circle.

C.

7. Let A, B, C, D be four points lying in the order named upon a certain circumference. The arcs A B, B C, and C D, are of 76°, 53°, and 118° respectively. Find the angle between the chords A C and B D, and also the angle between A B and C D, produced.

8. Prove that the difference of the diagonals of any quadrilateral is less than the sum of either pair of opposite sides.

9. Find a point in the base of a triangle such that lines drawn from it parallel to the other side of the triangle shall be equal to each other.

School of Mines, June, 1896.

TIME ALLOWED, TWO AND ONE-HALF HOURS.

1. Prove that if a straight line, E F, has two of its points, E and F, each equally distant from two points, A and B, it is perpendicular to the line A B at its middle point.

2. In equal circles incommensurable angles at the centre are proportional to their intercepted arcs : demonstrate.

3. In the parallelogram A B C D straight lines join the

middle point E of side B C with the vertex A, and the middle point F of side A D with the vertex C. Show that A E and F C are parallel and that the diagonal B D is trisected.

4. Show that the areas of similar triangles are to each other as the squares of their homologous sides.

5. How do you divide a line in extreme and mean ratio?

6. What are the immediate propositions which lead up to the determination of the area of the circle of radius unity, and how is this area determined? No demonstrations are required.

University of Pennsylvania, June, 1893.

TWO HOURS.

1. If two straight lines intersect each other, the opposite (or vertical) angles are equal.

The straight lines which bisect a pair of adjacent angles formed by two intersecting straight lines are perpendicular to each other.

2. If each side of a polygon is extended, the sum of the exterior angles is four right angles.

3. In the same circle, or in equal circles, equal chords are equally distant from the centre, and of two unequal chords, the less is at the greater distance from the centre.

The least chord that can be drawn in a circle through a given point is the chord perpendicular to the diameter through the point.

4. Two triangles are similar when they are mutually equiangular.

5. Show how to find a mean proportional between two given lines.

6

6. The square described upon the hypotenuse of a right-angled triangle is equivalent to the sum of the squares described upon the other two sides. (*Give the pure geometric proof.*)

7. In a triangle any two sides are reciprocally proportional to the perpendiculars let fall upon them from the opposite vertices.

8. The area of the regular inscribed triangle is half the area of the regular inscribed hexagon.

University of Pennsylvania, June, 1895.

TIME : ONE HOUR AND A HALF.

Give all the work.

1. The interior and exterior bisectors of any angle of a triangle divide the opposite side into segments which are proportional to the adjacent sides.

2. If two of the medial lines of a triangle are equal, the triangle is an isosceles.

3. The area of a rhombus is 240 and its side is 17, find its diagonals.

4. Construct a square whose area shall be five times the area of a given square.

5. The parallelogram formed by lines joining the middle points of the adjacent sides of a quadrilateral is equivalent to one-half the quadrilateral.

6. If the interior bisector of the angle C and the exterior bisector of the angle B of a triangle A B C meet at D, prove that angle B D C = $\frac{1}{2}$ A.

7. In any triangle the product of two sides is equal to the diameter of the circumscribed circle multiplied by the perpendicular to the third side from its opposite vertex.

8. Define π. Give a method for computing an approximate value of π.

9. If the radius of a circle is r, what is the side of the inscribed decagon?

University of Pennsylvania, September, 1895.

TIME: ONE HOUR AND A HALF.

Give all the work.

1. The lines joining the middle points of the adjacent sides of any quadrilateral form a parallelogram whose perimeter is equal to the sum of the diagonals of the quadrilateral.

2. Prove that the bisectors of the angles of a rectangle form a square.

3. The three medial lines of a triangle intersect in one point which divides each medial line in the ratio 1 : 2.

4. If from a point a tangent and a secant to a circle are drawn, the tangent is a mean proportional between the whole secant and its external segment.

5. Similar triangles are to each other as the squares of two homologous sides.

6. Divide a given straight line in extreme and mean ratio.

7. Construct a triangle which shall be similar to, and three times as large as, a given triangle.

8. From a given point without a circle draw a secant whose external and internal segments shall be equal.

9. If the radius of a circle is 2, what is the area of a sector whose central angle is 152°?

University of Pennsylvania, June, 1896.

TIME : TWO HOURS.

1. Define : Altitude of a triangle, medial line, regular polygon, inscribed angle, segment and sector of a circle.

2. If two parallels are cut by a straight line, the alternate exterior angles are equal.

3. Either side of a triangle is greater than the difference of the other two.

4. The sum of the angles of any polygon is equal to twice as many right angles as the polygon has sides, less four right angles.

5. The areas of similar triangles are to each other as the squares of their homologous sides.

6. The lines joining the middle points of the sides of any quadrilateral is a parallelogram.

7. Construct a square equivalent to a given triangle.

8. The line joining the middle points of the two non-parallel sides of a trapezoid is $12\frac{3}{4}$ inches, the distance between the parallel sides is $8\frac{5}{6}$ inches, what is the side of a regular hexagon equivalent to the trapezoid?

9. Define π. Outline a method for computing π.

University of Pennsylvania, September, 1896.

TIME : TWO HOURS.

1. Define : An angle (right, acute, and obtuse), tangent to a circle, regular polygon, mention all different kinds of parallelograms.

2. If two straight lines are cut by a third, making the alternate-interior angles equal, the two sides are parallel.

3. In any triangle the greater angle lies opposite the greater side.

4. What is each angle in a regular pentagon, regular hexagon, regular dodecagon?

5. If in a right triangle a perpendicular be drawn from the vertex of the right angle to the hypotenuse, the perpendicular is a mean proportional between the segments of the hypotenuse.

6. The lines joining the middle points of the sides of a rhombus form a rectangle.

7. Construct a square equivalent to a given pentagon.

8. The base of a triangle is 7.345 inches and the altitude 4.756 inches, what is the side of a regular triangle which has the same area as the given triangle?

9. Find the area of a regular hexagon inscribed in a circle whose radius is 11.529 inches.

Cornell, 1894.

1. If two triangles have two sides of the one equal, respectively, to two sides of the other, but the included angle of the first greater than the included angle of the second, then the third side of the first is greater than the third side of the second. Prove this; and state the converse.

2. Prove that lines drawn through the vertices of a triangle to the middle points of the opposite sides meet in a point.

How do the areas of the three triangles formed by joining this point to the vertices of the original triangle compare? Why?

3. If equilateral triangles be constructed upon each side of any given triangle, prove that the lines drawn from their outer vertices to the opposite vertices of the given triangle are equal.

4. From any point P, outside of a circle whose centre is at O, two tangents are drawn touching the circle at A and B; at Q, a variable point in the smaller arc A B, a tangent is drawn cutting the other two tangents in H and K. Prove that the perimeter of the triangle P H K is constant, and also that the angle H O K is constant. Compare this angle with the angle P.

5. If similar parallelograms be described upon the three sides of a right triangle as homologous sides, prove that the parallelogram described upon the hypotenuse is equivalent to the sum of those described upon the other two sides.

6. Prove that the sum of the perpendiculars drawn to the sides of a regular polygon from any point P *within* the figure, is equal to the apothem of the polygon multiplied by the number of its sides.

State this proposition, so modified, that the point P may be *without* the polygon.

7. Of all isoperimetric triangles having the same base, that which is isosceles has the maximum area.

Cornell, 1895.

One question may be omitted.

1. The sum of the lines which join a point within a triangle to the three vertices is less than the perimeter, but greater than half the perimeter.

2. Two triangles are equal if the three sides of one are equal respectively to the three sides of the other.

3. Construct through a point, P, exterior to a circle, a secant P A B so that $\overline{AB}^2 = PA \times PB$.

4. The radius of a circle is 6 inches; through a point 10 inches from the centre tangents are drawn. Find the lengths

of the tangents, also of the chord joining the points of contact.

5. Construct a polygon similar to two given similar polygons, and equivalent to their sum.

6. The bisector of an angle of a triangle divides the opposite side into segments proportional to the other two sides.

7. The perimeter of an inscribed equilateral triangle is equal to half the perimeter of the circumscribed equilateral triangle.

8. If one of the acute angles of a right triangle is double the other, the hypotenuse is double the shorter side.

Johns Hopkins University, October, 1896.

1. Prove that the bisectors of the two pairs of vertical angles formed by two intersecting lines are perpendicular to each other.

2. Show that through three points not lying in the same straight line one circle, and only one, can be made to pass.

3. The bases of a trapezoid are 16 feet and 10 feet respectively; each leg is 5 feet. Find the area of the trapezoid. Also find the area of a similar trapezoid, if each of its legs is 3 feet.

4. Define regular polygon. Prove that every equiangular polygon circumscribed about a circle is a regular polygon.

5. Prove that the opposite angles of a quadrilateral inscribed in a circle are supplements of each other.

6. Construct a square, having given its diagonal.

7. Prove that the area of a triangle is equal to half the product of its perimeter by the radius of the inscribed circle.

8. What is the area of the ring between two concentric circumferences whose lengths are 10 feet and 20 feet respectively?

Sheffield Scientific School, June, 1892.

[NOTE.—State at the head of your paper what text-book you have studied on the subject and to what extent.]

1. Prove the two propositions relating to the sum of the *interior* angles of a convex polygon, and the sum of the *exterior* angles formed by producing each side in one direction.

2. In a circle the greater chord subtends the greater arc, and conversely.

3. When is a line said to be divided *harmonically*? From the point P without a circle a secant through the centre is drawn cutting the circle in A and B. Tangents are drawn from P and the points of contact connected by a line cutting A B in Q. Show that P and Q divide A B harmonically.

4. Derive an expression for the area of a regular polygon.

5. When two sides of a triangle are given at what angle must they intersect if the area shall be maximum? Prove your answer.

Sheffield Scientific School, June, 1896.

[NOTE.—State at the head of your paper what text-book you have studied on the subject and to what extent.]

1. Two angles whose sides are parallel each to each are either equal or supplementary. When will they be equal, and when supplementary?

2. An angle formed by two chords intersecting within the circumference of a circle is measured by one-half the sum of the intercepted arcs.

3. A triangle having a base of 8 inches is cut by a line parallel to the base and 6 inches from it. If the base of the

smaller triangle thus formed is 5 inches, find the area of the larger triangle.

4. Construct a parallelogram equivalent to a given square, having given the sum of its base and altitude. Give proof.

5. What are regular polygons? A circle may be circumscribed about, and a circle may be inscribed in, any regular polygon.

Wesleyan University, September, 1896.

$1\frac{1}{2}$ HOURS.

1. The exterior angles of a polygon, made by producing each of its sides in succession, are together equal to four right angles.

The sum of the interior angles of a polygon is ten right angles. How many sides has the polygon?

2. An angle inscribed in a circle is measured by one-half of the arc intercepted between its sides.

3. Show how to bisect a given angle.

4. The radius of a circle is 6 feet. What are the radii of the circles concentric with it whose circumferences divide its area into three equivalent parts?

5. Show how to inscribe in a given circle a regular polygon similar to a given regular polygon.

6. If two polygons are composed of the same number of triangles, similar each to each, and similarly placed, the polygons are similar.

The University of Chicago, September, 1896.

TIME ALLOWED, ONE HOUR AND FIFTEEN MINUTES.

[When required, give all reasons in full, and work out proofs and problems in detail.]

1. Show that if on a diagonal of a parallelogram two points be taken equally distant from the extremities, and these points be joined to the opposite vertices of the parallelogram, the four-sided figure thus formed will be a parallelogram.

2. State and prove the converse of the following theorem : In the same circle, equal chords are equally distant from the centre.

3. Given a circle, a point, and two straight lines meeting in the point and terminating in the circumference of the circle. State what four lines or segments form a proportion and in what order they must be taken :

(1) When the point is outside the circle, and
 (*a*) both lines are secants,
 (*b*) one line is a secant, and the other a tangent,
 (*c*) both lines are tangents.

(2) When the point is within the circle, and the two lines are chords.

Prove in full (1) (*a*). Show that (1) (*c*) is a limiting case of (1) (*a*).

4. To a given circle draw a tangent that shall be perpendicular to a given line.

5. Show how to construct a triangle, having given the base, the angle at the opposite vertex, and the median from that vertex to the base. Discuss the cases depending upon the length of the given median.

Massachusetts Institute of Technology, June, 1896.

[Every reason must be stated in full.]

1. If straight lines are drawn to the extremities of a straight line from any point in the perpendicular erected at its middle point, they make equal angles with the line and with the perpendicular.

2. Two right triangles are equal when the hypotenuse and a side of one are equal, respectively, to the hypotenuse and a side of the other.

3. Prove the formula for the sum of the angles of any polygon. Define a regular polygon. How many degrees in each angle of a regular heptagon?

4. In the same circle or in equal circles chords equally distant from the centre are equal.

5. Two triangles are similar when their homologous sides are proportional.

6. A hexagon is formed by joining in succession the middle points of the sides of a given regular hexagon. Find the ratio of the areas of these two hexagons.

7. If AB and A_1B_1 are any two chords of the outer of two concentric circles, which intersect the circumference of the inner circle at P, Q, and P_1, Q_1, respectively, prove: $AP.\ PB = A_1P_1.\ P_1B_1$.

Brown University, June, 1896.

1. Have you been over all the required work?

2. The exterior angle of a triangle is equal to the sum of the opposite interior angles.

3. Find a point equidistant from two given points P and Q, and at a given distance C D from a given line A B.

4. If a secant and a tangent be drawn from a point without a circle, the tangent is a mean proportional between the secant and its external segment.

5. Similar triangles are to each other as the squares of their homologous sides.

6. The diagonals drawn from a vertex of a regular pentagon to the opposite vertices trisect that angle.

Vassar College, September, 1895.

1. Find the area of a right triangle if the perimeter is 60 feet, and its sides are as 3 : 4 : 5.

2. The sides of a triangle are 8, 9, 13; is the greatest angle acute, right, or obtuse?

3. The perpendicular erected at the middle point of the base of an isosceles triangle passes through the vertex and bisects the angle at the vertex.

4. If two circles touch internally, and the diameter of the smaller is equal to the radius of the larger, the circumference of the smaller bisects every chord of the larger which can be drawn through the point of contact.

5. If two similar triangles A B C, D E F, have their homologous sides parallel, the lines A D, B E, C F which join their homologous vertices meet in the same point.

Vassar College, June, 1896.

1. Define similar triangles.

State all the cases of similar triangles, and prove one.

2. Construct a right triangle, having given the hypotenuse and the sum of the legs.

3. Prove that the radius of a circle inscribed in an equilateral triangle is equal to one-third of the altitude of the triangle.

4. Construct the fourth proportional when three are given.

5. Find the area of an isosceles triangle if the base is equal to 36 feet and one leg is equal to 30 feet.

6. To divide a given line in extreme and mean ratio. What regular inscribed polygons may be constructed by means of this division? Prove your statement.

Amherst College, June, 1895.

1. To construct a square that shall have to a given square the ratio of 3 to 2.

2. The circumference of a circle is the limit of the perimeter of a regular circumscribed polygon, as the number of sides of the polygon is indefinitely increased.

3. If two polygons are composed of the same number of similar triangles, similarly placed, the polygons are similar.

4. The sum of the squares on two sides of a triangle is equal to twice the square on half the third side increased by twice the square on the median to that side.

5. Find the locus of all points, the perpendicular distances of which from two intersecting lines are to each other as 3 to 2.

Amherst College, June, 1896.

1. Two triangles having an angle of the one equal to an angle of the other, and the including sides proportional are similar.

2. Inscribe a circle in a given triangle.

3. (1) When are two lines said to be *incommensurable?* (2). Are $3\frac{2}{5}$ and $5\frac{1}{11}$ incommensurable? Give the reason for your answer. (3). Define a *limit.* Mention some propositions to which the method of limits is applied.

4. In an isosceles right triangle either leg is a mean proportional between the hypotenuse and the perpendicular upon it from the vertex of the right angle.

5. The area of an inscribed regular hexagon is equal to $\frac{3}{4}$ of that of the circumscribed regular hexagon.

Dartmouth College, 1894.

1. Name the different classes of triangles.

2. What are the conditions of similarity in triangles?

3. The diameter of a circle is 25 feet. What is the perpendicular distance to the circumference from a point in the diameter 5 feet from either end.

4. One angle of a parallelogram is $\frac{4}{5}$ of a right angle. What values have the remaining angles?

5. The segments of a given line are 4, 6, 7. Divide any other line in the same proportion.

6. In any triangle the product of any two sides is equal to the product of the segments of the third side formed by the bisector of the opposite angle, plus the square of the bisector. Demonstrate.

Wellesley College, June, 1895.

1. An angle formed by two tangents is how measured? Prove.

2. The diagonals of a rhombus bisect each other at right angles.

3. (*a*) If a line bisects an angle of a triangle and also bisects the opposite side the triangle is isosceles.

(*b*) State and demonstrate the general case for the ratio of the segments of the side opposite to a bisected angle.

4. With a given line as a chord, construct a circle so that this chord shall subtend a given inscribed angle.

5. (*a*) On a circle of 4 feet radius, how long is an arc included between two radii forming an angle of $20°$? Prove, deriving the formula employed.

(*b*) Find the area of the regular circumscribed hexagon of a circle whose radius is 1.

6. Two similar triangles are to each other as the squares of their homologous sides.

Bowdoin College, June, 1895.

1. The perpendiculars from the vertices of a triangle to the opposite sides meet in a common point.

2. Upon a given straight line describe an arc of a circle which shall contain a given angle.

3. In any triangle the square of a side opposite an acute angle equals the sum of the squares on the other two sides minus twice the product of one of these sides by the projection of the other upon it.

4. The length of a tangent to a circle, from a point eight units distant from the nearest point on the circumference, is twelve units. Find the diameter of the circle.

5. Two triangles having an angle of one equal to an angle of the other are to each other as the product of the sides including the equal angles.

6. Find the ratio of the radius of a circle to the side of the inscribed square.

7. The area of a sector of sixty degrees is two hundred nine, and forty-four hundredths square inches. Find the length of the radius.

Bowdoin College, June, 1896.

1. The bisectors of the three angles of a triangle meet in the centre of the inscribed circle.

2. The circumference of a circle described on one of the equal sides of an isosceles triangle as a diameter passes through the middle point of the base.

3. If two chords be drawn through a fixed point within a circle, the product of the segments of one chord equals the product of the segments of the other.

4. The radius of a circle is 10; inscribe within it a regular decagon and compute the length of its side.

5. In an acute-angled triangle the side $AB = 10$, side $AC = 7$, the projection of AC on AB is 3.4. Construct the triangle and compute the third side, BC.

6. The area of one circle is 100; find the circumference of another circle described on the radius of the first as a diameter.

University of California, August, 1896.

1. Prove that if two sides of one of two triangles be equal to two sides of the other, and the angles opposite one pair of equal sides be equal, the angles opposite the other pair of sides are either equal or supplementary.

2. To construct a triangle having given the base, one angle at the base and the altitude.

3. Prove that the straight lines drawn at right angles to the sides of a triangle at their middle points meet in a point.

4. Prove that, if an angle at the centre of a circle and an angle at the circumference be subtended by the same arc, the angle at the circumference is one-half of the angle at the centre.

5. If the middle points of adjacent sides of a convex quadrilateral be connected by straight lines what figure is formed? What is the relation between the areas of this figure and the quadrilateral? Prove your statements.

6. To divide a given straight line internally in extreme and mean ratio. What regular polygons may be inscribed in a circle by means of this construction? Show (without proof) how one of these polygons is constructed.

7. Present, in the clearest language and most perfect form you can command, some proposition of your own choosing.

Bryn Mawr College, September, 1896.

TWO AND ONE-HALF HOURS.

1. Show how to draw a perpendicular from a given point to a given line, the point not lying on the line. Show that only one such perpendicular can be drawn.

2. Prove that if two parallel lines are cut by a third straight line, the two interior angles on one side of the transversal are together equal to two right angles.

Prove that the lines bisecting the angles of a parallelogram form a rectangle.

3. Define a parallelogram ; prove that the opposite sides and angles are equal, and that the diagonals bisect one another.

Prove that *any* line through the intersection of the diagonals of a parallelogram bisects the figure.

4. Prove that in any circle angles at the centre have the same ratio as the arcs on which they stand.

Show how to divide the circumference of a circle into three parts that shall be in the ratio 1 : 2 : 3.

5. Prove that an angle formed by two chords intersecting within a circle is measured by one-half the sum of the intercepted arcs.

A B C D is a quadrilateral in a circle ; P, Q, R, S, are the points of bisection of the arcs A B, B C, C D, D A. Show that P R is perpendicular to Q S.

6. Prove that the sum of the squares of two sides of a triangle is equal to twice the square of half the base increased by twice the square of the distance from the vertex to the bisection of the base. Apply this to find a line whose extremities shall lie one on each of two given concentric circles, the line itself being bisected at a given point.

7. Prove that three lines drawn through the vertices of a triangle to bisect the opposite sides meet in a point, and de-

termine the position of this point on any one of the three bisectors.

Show how to construct a triangle when the lengths of the three medians are given.

8. Define the tangent to a circle at a point ; and prove that the tangent at a point is perpendicular to the diameter through the point.

Two circles whose centres are A, B, meet at a point P. Prove that if A P touch the circle whose centre is B, then B P will touch the circle whose centre is A.

9. State and prove the relation between the segments of intersecting chords of a circle. Apply this to find a mean proportional to two given lines.

Boston University, June, 1896.

TIME 1 H. 30 M.

[Candidates will quote authority for each step.]

1. The extremities of the base of an isosceles triangle are equally distant from the opposite sides. Prove.

2. Two unequal circles have a common centre. Prove that chords of the greater circle, which are tangent to the lesser circle, are equal.

3. The sides of a triangle are 4, 7, 10 ; find the sides of a similar triangle having nine times the area of the first. Prove the principle employed.

4. Homologous altitudes of similar triangles have the same ratio as any two homologous sides. Prove.

5. The sum of the perpendiculars from any point within an equilateral triangle to the three sides is equal to the altitude of the triangle. Prove.

Boston University, September, 1896.

TIME 1 H. 30 M.

[Candidates will quote authority for each step.]

1. Connect the mid points of the adjacent sides of a rhombus and prove character of the figure formed.

2. Chords meeting a diameter at the same point and making the same angle with it are equal. Prove.

3. The radius of a circle is 10 feet. Find the side of an equilateral triangle having the same area as the circle.

4. In any triangle the square of the side opposite an acute angle is equal to the sum of the squares of the other two sides diminished by twice the product of one of these sides and the projection of the other side upon it. Prove.

5. Two *equivalent* triangles have a common base and lie on opposite sides of it. Prove that the line joining their vertices is bisected by the base, produced, if necessary.

Vanderbilt University, May 24, 1894.

1. The circles described on two sides of a triangle as diameters intersect on the third side.

2. The diagonals of a trapezoid divide each other into segments which are proportional.

3. Similar triangles are as the squares of their homologous sides.

4. Two quadrilaterals are equivalent when the diagonals of one are respectively equal and parallel to the diagonals of the other.

5. The area of a ring bounded by two concentric circumferences is equal to the area of a circle having for its diameter

a chord of the outer circumference tangent to the inner circumference.

6. A swimmer whose eye is at the surface of the water can just see the top of a stake a mile distant; the stake proves to be eight inches out of the water; required the radius of the earth.

New Jersey State College for the Benefit of Agriculture and the Mechanic Arts, New Brunswick, N. J., June, 1891.

1. Define the various kinds of triangles and quadrilaterals.

2. If two straight lines cut each other, the vertical angles are equal.

3. An angle formed by a tangent and a chord from the point of contact is measured by one-half the intercepted arc.

4. If a variable tangent meets two parallel tangents it subtends a right angle at the centre.

5. The bisector of an angle of a triangle divides the opposite side into segments proportional to the adjacent sides.

6. A parallelogram is divided by its diagonals into four triangles of equal area.

7. The areas of two similar segments are to each other as the squares of their radii.

8. The diameter of a circle is 5 feet; find the side of the inscribed square.

9. Find a side of the circumscribed equilateral triangle, the radius of the circle being $\sqrt{3}$.

10. Find the radius of the circle in which the sector of 45° is .125 square inches.

LOGARITHMS.

1. The **logarithm** of a number is the **exponent of the power** to which an *assumed number* must be raised to produce the first number.

2. Since logarithms are **exponents,** the principles established in Theory of Exponents in Algebra, hold in logarithms, and are the very principles which make logarithms serviceable ; as follows :

I. *The logarithm of a product is equal to the sum of the logarithms of its factors.*

II. *The logarithm of a quotient is equal to the logarithm of the dividend minus the logarithm of the divisor.*

III. *The logarithm of any power of a number is equal to the logarithm of the number multiplied by the exponent of the power.*

IV. *The logarithm of a root of a number is equal to the logarithm of the number divided by the index of the root.*

3. The only kind of logarithms with which we have to do here are those in which the *assumed number,* called the **base,** is **10.**

Such logarithms are termed **Common Logarithms.**

$$10^4 = 10000 \qquad 10^{-1} = \frac{1}{10^1} = .1$$

$$10^3 = 1000 \qquad 10^{-2} = \frac{1}{10^2} = .01$$

$$10^2 = 100 \qquad 10^{-3} = \frac{1}{10^3} = .001$$

$$10^1 = 10 \qquad 10^{-4} = \frac{1}{10^4} = .0001$$

$$10^0 = 1$$

Thus, by definition, log 10000 = 4 ; log 1000 = 3, etc.

But all numbers which are not integral powers of 10, as the above are, must have a fractional, decimal, part to their logarithms.

Thus, the logarithm of any number between

$$10 \text{ and } 100$$

would lie between

$$1 \text{ and } 2,$$

that is, it would be

$$1 + \text{a decimal.}$$

Of any number between

$$1 \text{ and } 10,$$

the logarithm would be

$$0 + \text{a decimal;}$$

between .1 and 1,

$$-1 + \text{a decimal;}$$

between .01 and .1,

$$-2 + \text{a decimal;}$$

and so on.

This decimal part of a logarithm is called the **mantissa**; the integral part, the **characteristic.**

From the above it is seen that all mantissas are *positive.* And to show that a negative sign belongs to the characteristic only, it is placed above the characteristic, thus:

$$\log .03152 = \bar{2}.49859.$$

4. Moving the decimal point to right or left in any number multiplies or divides that number by ten or some integral power of ten. And as the logarithm of a product is equal to the logarithm of the multiplicand plus the logarithm of the multiplier, and the logarithm of a quotient is equal to the logarithm of the dividend minus the logarithm of the divisor, and the logarithm of the multiplier and divisor in such cases (moving the decimal point) is an integer, the only part of a logarithm affected by a change of the decimal point in a number is the integral part, the *characteristic.*

Then all numbers which differ only in the position of the decimal point have the same mantissa.

5. A careful study of Art. **3** will make plain the following rules in regard to the characteristic :

I. *If the number is greater than* 1, *the characteristic is one less than the number of places to the left of the decimal point.*

II. *If the number is less than* 1, *the characteristic is negative, and is one more than the number of zeros between the decimal point and the first significant figure of the decimal.*

Thus, the characteristic	of	log	378.37	is	2 ;
"	"	"	"	.0917	"	— 2 ;
"	"	"	"	5.391	"	0 ;
"	"	"	"	.8395	"	— 1.

6. The rules for determining the position of the decimal point in a number corresponding to any given logarithm are just the converse of the above.

I. *When the characteristic is positive, the number of places to the left of the decimal point is one more than the number of units in the characteristic.*

II. *When the characteristic is negative, the number is a decimal, and the number of zeros between the decimal point and the first significant figure is one less than the number of units in the characteristic.*

7. To avoid certain difficulties in the use of logarithms *every* logarithm which has a *negative* characteristic should have 10.00000 — 10 (equal to 0) added to it.

Thus, $\bar{2}.37931$ should be written 8.37931 — 10;

$\bar{4}.92012$ " " " 6.92012 — 10;

$\bar{1}.72082$ " " " 9.72082 — 10.

8. The **cologarithm** of a number, or the *arithmetical complement* of the logarithm of the number, is the logarithm of the reciprocal of that number.

Thus, colog 317 = log $\frac{1}{317}$; but by II., Art. **2**, log $\frac{1}{317}$ = log 1 — log 317 = 0 — log 317 = — log 317.

And so the *cologarithm* of any number is equal to the *negative logarithm* of that number.

9. Since to subtract a quantity is to add that quantity with its sign changed, rule II., Art. **2**, may be stated:

The logarithm of a quotient is equal to the logarithm of the dividend plus the negative logarithm, or cologarithm, of the divisor.

This is the form of the rule that should be invariably applied in practice.

10. Negative logarithms should always have zero in the form 10.00000 — 10 added to them before they are employed otherwise in an example. *This altered form of the negative logarithm* may well be distinguished by the name *cologarithm*, and is so distinguished hereafter.

Thus,

$$\log\frac{1777}{8943} = \left\{\begin{matrix} \log 1777 + (-\log 8943) = \log 1777 + \text{colog } 8943 \\ \| \qquad\quad \| \qquad\qquad\quad \| \qquad\quad \| \\ 3.24969 + (-3.95148) = 3.24969 + 6.04851 - 10 \end{matrix}\right\} =$$

$$9.29820 - 10.$$

11. This method of using logarithms avoids all subtraction of logarithms, except in finding cologarithms; and these are very easily found by the following rule:

Begin with the characteristic of the logarithm and subtract each figure from 9, *except the last significant figure, and subtract that from* 10.

Thus, log 8409 = 3.92474; and colog 8409 =
$10 - 3.92474 - 10 = 6.07526 - 10.$

By subtracting from left to right in this way the cologarithm of any number of four figures or less can be read right from the table almost as easily as the logarithm itself, after some practice.

12. The following points should be carefully noted in using logarithms that have negative characteristics:

1. In getting the cologarithm, the 10 following the mantissa destroys the second 10 of the 10.00000 — 10 added.

Thus,

$$\text{colog } .7231 = \left\{ \begin{array}{rr} -(9.85920 - 10) = & -9.85920 + 10 \\ 10.00000 - 10 = & \underline{10.00000 - 10} \\ & 0.14080. \end{array} \right\}$$

2. In adding or multiplying, superfluous tens should be dropped.

Thus: adding,

$$\begin{array}{r} 9.87349 - 10 \\ \underline{8.96454 - 10} \\ 18.83803 - 20 \end{array} = 8.83803 - 10;$$

Multiplying,

$$\begin{array}{r} 9.76604 - 10 \\ \underline{3} \\ 29.29812 - 30 \end{array} = 9.29812 - 10.$$

3. In dividing, a sufficient number of tens should be added, before and after the mantissa, to make the number of tens after the mantissa equal to the number of units in the divisor.

Thus, $\frac{9.76155 - 10}{3} = \frac{29.76155 - 30}{3} = 9.92052 - 10;$

$$\frac{8.98304 - 10}{4} = \frac{38.98304 - 40}{4} = 9.74576 - 10.$$

HOW TO USE THE TABLE.

13. The first page of the table gives the characteristics and mantissas of numbers up to 100. The remainder of the table gives only mantissas. The characteristic is to be supplied by the rules of Art. **5**. The first three figures of the number are found in the left-hand column, marked **N** at the top and the bottom. The fourth figure of the number is found in the first line at the top and the bottom. The mantissa is then found in the same horizontal line with the first three figures, and in the same vertical column with the fourth figure. The first two figures of the mantissa are printed only in the first column. *In every case where an asterisk is found the first two figures of the mantissa are found in the first column of the next line below.*

14. *To find the logarithm of a number.*

1. To find the logarithm of a number of four figures, as 8713.

By Art. **5**, the characteristic = 3.
By the table as explained above the mantissa = .94017.
Hence log 8713 = 3.94017.

2. To find the logarithm of a number of five or more figures, as 35647.

The characteristic = 4.
The mantissa for 3564 = .55194.
" " " 3565 = .55206.

That is, an increase of one unit in the number, at this point in the table, makes an increase of .00012 in the mantissa. Then an increase of .7 of a unit (7 in the fifth place is .7 of 1 in the fourth place) in the number will make an increase of .7 of .00012 in the mantissa = .000084.

$$\text{Therefore log } 35647 = \begin{cases} 4.55194 \\ .00008 \\ \hline 4.55202. \end{cases}$$

NOTE 1.—The difference between any two consecutive mantissas, as .00012 above, is called the *tabular difference,* and is printed in the right-hand column of the table under **D**.

NOTE 2.—When all these tabular differences are multiplied by the nine significant digits expressed as tenths, they give a table of *proportional parts.* This table furnishes, ready-made, the amounts to be added to obtain logarithms of five-figure numbers. Only a portion (the most helpful, however) of such a table of proportional parts, is given with this table of logarithms, p. 115. It is sufficient to make their use and meaning plain.

NOTE 3.—In calculating proportional parts and in all calculations with tabular differences they are treated as whole numbers, as they bear the same relation to their mantissas that whole numbers do to whole numbers.

NOTE 4.—In calculating additions to be made to a logarithm all figures that follow the fifth are rejected. When the sixth figure is 5, or greater, the fifth figure is increased by 1. When the last significant figure of a logarithm is $\bar{5}$, it means that such an increase has been made for rejected figures following the fifth place.

3. To find the logarithm of 18.7432.

The characteristic = 1.

[As already explained in Art. **4**, the position of the decimal point does not affect the mantissa in the least.]

Mantissa for 1874 = 27274.
" " 1875 = 27300.

That is an increase of one in the number here makes an increase of 26 in the mantissa. Then an increase of .32 of one (32 following the fourth place is .32 of 1 in the fourth place) in the number will make an increase of .32 of 26 in the mantissa = 8.32.

$$\text{Hence log } 18.7432 = \begin{cases} 1.27274 \\ \quad\quad\;\; 8 \\ \overline{1.27282.} \end{cases}$$

NOTE 5.—The process employed in finding the logarithm of a number of more than four figures is called *interpolation.*

15. *How to find the number corresponding to the logarithm.*

1. To find the number corresponding to the logarithm 0.56514.

The mantissa increases constantly throughout the table. Follow the first column of mantissas till 56 is found, as the first two figures of the mantissa. Continuing 514 is easily found in the same horizontal line with 367 and in the column under 4.

Hence the number (placing the decimal point by Art. **6**) = 3.674.

2. To find the number corresponding to the logarithm 8.26470 — 10.

This mantissa cannot be found in the table.

The nearest mantissa less than 26470 = 26458.
" " " larger " 26470 = 26482.

The number corresponding to mantissa 26458 (disregarding the decimal point) is 1839. For a mantissa 24 greater (26482) the corresponding number is 1840, that is, an increase of 24 in the mantissa, at this point in the table, means an increase of 1 in the number. Then an increase of 12, which is the amount the given mantissa, 26470, exceeds the mantissa 26458, would mean an increase of $\frac{12}{24}$ of 1, = .5.

Hence the number = .018395.

3. To find the number corresponding to the logarithm 1.71895.

The next smaller mantissa = 71892.

Then the given mantissa is 3 larger; and as the tabular difference is 8, $\frac{3}{8}$ of 1 = .375 must be added to 5235 the number corresponding to mantissa 71892.

Hence the number = 52.3538.

Note 1.—Numbers corresponding to given logarithms should not be carried to more than five or six significant figures, in a five-place table.

Note 2.—Art. **4** makes it clear that the mantissa for 200 is the same as the mantissa for 2000; for 375, the same as for 3750, etc. So the mantissa for any number of three figures is found in the 0 column and in the same horizontal line with these three figures in the **N** column.

Note 3.—A negative quantity cannot be a power of a positive quantity, and hence a negative quantity, as such, has no logarithm. Hence when negative quantities occur in any example worked by logarithms, the negative sign is absolutely disregarded, except so far as it affects the sign of the result.

EXAMPLES.

16. Find by logarithms the values of the following:

1. Given $x = \frac{394.1 \times .9385}{.02003}$; find x.

$$\begin{aligned} \log 394.1 &= 2.59561 \\ \log .9385 &= 9.97243 - 10 \\ \text{colog } .02003 &= 1.69832 \\ \hline \log x &= 4.26636 \qquad = \log 18465.4 \\ \therefore x &= 18465.4 \end{aligned}$$

2. Given $x = \frac{(801.012)^2 \times (.0315)^{\frac{3}{2}}}{(1.3907)^{\frac{1}{5}}}$; find x.

$$\begin{aligned} \log (801.012)^2 &= 2.90364 \times 2 = 5.80728 \\ \log (.0315)^{\frac{3}{2}} &= (8.49831 - 10) \times \tfrac{3}{2} = 7.74747 - 10 \\ \text{colog } (1.3907)^{\frac{1}{5}} &= (9.85677 - 10) \times \tfrac{1}{5} = 9.97135 - 10 \\ \hline \log x &= 3.52610 \\ \therefore x &= 3358.2 \end{aligned}$$

3. $95.37 \times .0313$.

4. $(-93985) \times 1.0484$.

5. $.0008601 \times 1.28865$.

6. $\frac{5008.4}{9.394}$.

7. $\frac{.93284 \times 91.3009}{10.1029}$.

8. $\frac{-314}{9.8743}$.

9. $\dfrac{.03494 \times (-9432)}{.00411 \times 3753.6}$.

10. $\dfrac{-111.121}{-4.943}$.

11. $(3.1835)^{3}$.

12. $\dfrac{5}{(1197)^{\frac{2}{3}}}$.

13. $(.311)^{\frac{1}{6}}$.

14. $\sqrt[7]{.0000009431}$.

15. $\left(-\dfrac{8}{5}\right)^{\frac{9}{7}}$.

16. $\dfrac{34985 \times (.00039)^{\frac{2}{5}}}{(-91)^{\frac{1}{3}}}$.

17. $(51\tfrac{1}{13})^{\frac{3}{4}}$.

18. $\dfrac{(-419)^{\frac{2}{3}} \times (-90.071)}{(10016)^{3} \times (-.11101) \times 1399}$.

19. $\sqrt[3]{\dfrac{(1.0642)^{2} \times .1098}{(683.51)^{\frac{2}{5}}}}$.

20. $\sqrt{917 \sqrt[3]{110021}}$.

21. $\sqrt{\dfrac{.02053 \times .0010997 \times .32024}{.091352}}$.

22. $\dfrac{(15)^{5}}{(5)^{15}}$.

23. $\left(\dfrac{311 \times 497 \times 7.3}{(19843000)^{\frac{2}{7}}}\right)^{\frac{4}{5}}$.

24. $\left(\dfrac{18}{37}\right)^{\frac{2}{9}} \div \left(\dfrac{4301}{23001}\right)^{\frac{4}{7}}$.

25. $\left\{(3\tfrac{1}{11})^{\frac{4}{7}} \times \left(\dfrac{8\frac{1}{14}}{7\frac{23}{31}}\right)^{\frac{1}{2}}\right\}^{3}$.

26. $\sqrt[4]{\tfrac{8}{9}} \div \sqrt[9]{\tfrac{4}{7}}$.

27. $\left\{(1000)^{\frac{3}{7}} \div (80009)^{\frac{1}{5}}\right\}^{\frac{2}{3}}.$

28. $(911 \times 10003)^{\frac{3}{8}}.$

29. $\sqrt[3]{\dfrac{.40071 \times (.00352)^{\frac{1}{4}}}{(.09045321)^{\frac{1}{7}}}}.$

30. $(-.1)^{\frac{1}{5}} \times (1000)^{\frac{2}{3}} \times \sqrt[7]{.01}.$

31. $(3\frac{1}{7})^{2\frac{1}{7}}.$

32. $\left(\dfrac{4\frac{1}{13}}{13\frac{1}{4}}\right)^{3.1}.$

33. $(21\frac{1}{7})^{3.2} \div (80\frac{1}{3})^{1.3}.$

34. $\sqrt[2.1]{\dfrac{(444)^2 \times (.00041007)^{3.7}}{(9.8563)^{\frac{3}{8}}}}.$

35. $\sqrt[5]{\dfrac{(15.434)^2 \times (3897.3)^{\frac{1}{10}} \times .41984}{(.000372)^{2.3} \times (784.96)^3 \times 5013.4 \times (.003)^{\frac{1}{2}}}}.$

TABLES.

COMMON LOGARITHMS OF NUMBERS

GIVING CHARACTERISTICS AND MANTISSAS OF LOGARITHMS OF NUMBERS FROM 1 TO 100, AND MANTISSAS ONLY OF NUMBERS FROM 100 TO 10000.

LOGARITHMS OF NUMBERS.

N	Log.	N	Log.	N	Log.	N	Log.
1	0.00000	26	1.41497	51	1.70757	76	1.88081
2	0.30103	27	1.43136	52	1.71600	77	1.88649
3	0.47712	28	1.44716	53	1.72428	78	1.89209
4	0.60206	29	1.46240	54	1.73239	79	1.89763
5	0.69897	30	1.47712	55	1.74036	80	1.90309
6	0.77815	31	1.49136	56	1.74819	81	1.90849
7	0.84510	32	1.50515	57	1.75587	82	1.91381
8	0.90309	33	1.51851	58	1.76343	83	1.91908
9	0.95424	34	1.53148	59	1.77085	84	1.92428
10	1.00000	35	1.54407	60	1.77815	85	1.92942
11	1.04139	36	1.55630	61	1.78533	86	1.93450
12	1.07918	37	1.56820	62	1.79239	87	1.93952
13	1.11394	38	1.57978	63	1.79934	88	1.94448
14	1.14613	39	1.59106	64	1.80618	89	1.94939
15	1.17609	40	1.60206	65	1.81291	90	1.95424
16	1.20412	41	1.61278	66	1.81954	91	1.95904
17	1.23045	42	1.62325	67	1.82607	92	1.96379
18	1.25527	43	1.63347	68	1.83251	93	1.96848
19	1.27875	44	1.64345	69	1.83885	94	1.97313
20	1.30103	45	1.65321	70	1.84510	95	1.97772
21	1.32222	46	1.66276	71	1.85126	96	1.98227
22	1.34242	47	1.67210	72	1.85733	97	1.98677
23	1.36173	48	1.68124	73	1.86332	98	1.99123
24	1.38021	49	1.69020	74	1.86923	99	1.99564
25	1.39794	50	1.69897	75	1.87506	100	2.00000

N	0	1	2	3	4	5	6	7	8	9	D
100	00 000	043	087	130	173	217	260	303	346	389	43
101	432	475	518	561	604	647	689	732	775	817	43
102	860	903	945	988	*030	*072	*115	*157	*199	*242	42
103	01 284	326	368	410	452	494	536	578	620	662	42
104	703	745	787	828	870	912	953	995	*036	*078	42
105	02 119	160	202	243	284	325	366	407	449	490	41
106	531	572	612	653	694	735	776	816	857	898	41
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302	40
108	03 342	383	423	463	503	543	583	623	663	703	40
109	743	782	822	862	902	941	981	*021	*060	*100	40
110	04 139	179	218	258	297	336	376	415	454	493	39
111	532	571	610	650	689	727	766	805	844	883	39
112	922	961	999	*038	*077	*115	*154	*192	*231	*269	39
113	05 308	346	385	423	461	500	538	576	614	652	38
114	690	729	767	805	843	881	918	956	994	*032	38
115	06 070	108	145	183	221	258	296	333	371	408	38
116	446	483	521	558	595	633	670	707	744	781	37
117	819	856	893	930	967	*004	*041	*078	*115	*151	37
118	07 188	225	262	298	335	372	408	445	482	518	37
119	555	591	628	664	700	737	773	809	846	882	36
120	918	954	990	*027	*063	*099	*135	*171	*207	*243	36
121	08 279	314	350	386	422	458	493	529	565	600	36
122	636	672	707	743	778	814	849	884	920	955	35
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307	35
124	09 342	377	412	447	482	517	552	587	621	656	35
125	691	726	760	795	830	864	899	934	968	*003	35
126	10 037	072	106	140	175	209	243	278	312	346	34
127	380	415	449	483	517	551	585	619	653	687	34
128	721	755	789	823	857	890	924	958	992	*025	34
129	11 059	093	126	160	193	227	261	294	327	361	34
N	0	1	2	3	4	5	6	7	8	9	D

PP	44	43	42	41	40	39	38	37	36
1	4.4	4.3	4.2	4.1	4.0	3.9	3.8	3.7	3.6
2	8.8	8.6	8.4	8.2	8.0	7.8	7.6	7.4	7.2
3	13.2	12.9	12.6	12.3	12.0	11.7	11.4	11.1	10.8
4	17.6	17.2	16.8	16.4	16.0	15.6	15.2	14.8	14.4
5	22.0	21.5	21.0	20.5	20.0	19.5	19.0	18.5	18.0
6	26.4	25.8	25.2	24.6	24.0	23.4	22.8	22.2	21.6
7	30.8	30.1	29.4	28.7	28.0	27.3	26.6	25.9	25.2
8	35.2	34.4	33.6	32.8	32.0	31.2	30.4	29.6	28.8
9	39.6	38.7	37.8	36.9	36.0	35.1	34.2	33.3	32.4

N	0	1	2	3	4	5	6	7	8	9	D
130	11 394	428	461	494	528	561	594	628	661	694	33
131	727	760	793	826	860	893	926	959	992	*024	33
132	12 057	090	123	156	189	222	254	287	320	352	33
133	385	418	450	483	516	548	581	613	646	678	33
134	710	743	775	808	840	872	905	937	969	*001	32
135	13 033	066	098	130	162	194	226	258	290	322	32
136	354	386	418	450	481	513	545	577	609	640	32
137	672	704	735	767	799	830	862	893	925	956	32
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270	31
139	14 301	333	364	395	426	457	489	520	551	582	31
140	613	644	675	706	737	768	799	829	860	891	31
141	922	953	983	*014	*045	*076	*106	*137	*168	*198	31
142	15 229	259	290	320	351	381	412	442	473	503	31
143	534	564	594	625	655	685	715	746	776	806	30
144	836	866	897	927	957	987	*017	*047	*077	*107	30
145	16 137	167	197	227	256	286	316	346	376	406	30
146	435	465	495	524	554	584	613	643	673	702	30
147	732	761	791	820	850	879	909	938	967	997	29
148	17 026	056	085	114	143	173	202	231	260	289	29
149	319	348	377	406	435	464	493	522	551	580	29
150	609	638	667	696	725	754	782	811	840	869	29
151	898	926	955	984	*013	*041	*070	*099	*127	*156	29
152	18 184	213	241	270	298	327	355	384	412	441	29
153	469	498	526	554	583	611	639	667	696	724	28
154	752	780	808	837	865	893	921	949	977	*005	28
155	19 033	061	089	117	145	173	201	229	257	285	28
156	312	340	368	396	424	451	479	507	535	562	28
157	590	618	645	673	700	728	756	783	811	838	28
158	866	893	921	948	976	*003	*030	*058	*085	*112	27
159	20 140	167	194	222	249	276	303	330	358	385	27
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PP	35	34	33	32	31	30	29	28	27
1	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.7
2	7.0	6.8	6.6	6.4	6.2	6.0	5.8	5.6	5.4
3	10.5	10.2	9.9	9.6	9.3	9.0	8.7	8.4	8.1
4	14.0	13.6	13.2	12.8	12.4	12.0	11.6	11.2	10.8
5	17.5	17.0	16.5	16.0	15.5	15.0	14.5	14.0	13.5
6	21.0	20.4	19.8	19.2	18.6	18.0	17.4	16.8	16.2
7	24.5	23.8	23.1	22.4	21.7	21.0	20.3	19.6	18.9
8	28.0	27.2	26.4	25.6	24.8	24.0	23.2	22.4	21.6
9	31.5	30.6	29.7	28.8	27.9	27.0	26.1	25.2	24.3

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160	20 412	439	466	493	520	548	575	602	629	656	27
161	683	710	737	763	790	817	844	871	898	925	27
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192	27
163	21 219	245	272	299	325	352	378	405	431	458	27
164	484	511	537	564	590	617	643	669	696	722	26
165	748	775	801	827	854	880	906	932	958	985	26
166	22 011	037	063	089	115	141	167	194	220	246	26
167	272	298	324	350	376	401	427	453	479	505	26
168	531	557	583	608	634	660	686	712	737	763	26
169	789	814	840	866	891	917	943	968	994	*019	26
170	23 045	070	096	121	147	172	198	223	249	274	25
171	300	325	350	376	401	426	452	477	502	528	25
172	553	578	603	629	654	679	704	729	754	779	25
173	805	830	855	880	905	930	955	980	*005	*030	25
174	24 055	080	105	130	155	180	204	229	254	279	25
175	304	329	353	378	403	428	452	477	502	527	25
176	551	576	601	625	650	674	699	724	748	773	25
177	797	822	846	871	895	920	944	969	993	*018	25
178	25 042	066	091	115	139	164	188	212	237	261	24
179	285	310	334	358	382	406	431	455	479	503	24
180	527	551	575	600	624	648	672	696	720	744	24
181	768	792	816	840	864	888	912	935	959	983	24
182	26 007	031	055	079	102	126	150	174	198	221	24
183	245	269	293	316	340	364	387	411	435	458	24
184	482	505	529	553	576	600	623	647	670	694	24
185	717	741	764	788	811	834	858	881	905	928	23
186	951	975	998	*021	*045	*068	*091	*114	*138	*161	23
187	27 184	207	231	254	277	300	323	346	370	393	23
188	416	439	462	485	508	531	554	577	600	623	23
189	646	669	692	715	738	761	784	807	830	852	23
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1	2.7	2.6	2.5	2.4	2.3	2.2
2	5.4	5.2	5.0	4.8	4.6	4.4
3	8.1	7.8	7.5	7.2	6.9	6.6
4	10.8	10.4	10.0	9.6	9.2	8.8
5	13.5	13.0	12.5	12.0	11.5	11.0
6	16.2	15.6	15.0	14.4	13.8	13.2
7	18.9	18.2	17.5	16.8	16.1	15.4
8	21.6	20.8	20.0	19.2	18.4	17.6
9	24.3	23.4	22.5	21.6	20.7	19.8

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190	875	898	921	944	967	989	*012	*035	*058	*081	23
191	28 103	126	149	171	194	217	240	262	285	307	23
192	330	353	375	398	421	443	466	488	511	533	23
193	556	578	601	623	646	668	691	713	735	758	22
194	780	803	825	847	870	892	914	937	959	981	22
195	29 003	026	048	070	092	115	137	159	181	203	22
196	226	248	270	292	314	336	358	380	403	425	22
197	447	469	491	513	535	557	579	601	623	645	22
198	667	688	710	732	754	776	798	820	842	863	22
199	885	907	929	951	973	994	*016	*038	*060	*081	22
200	30 103	125	146	168	190	211	233	255	276	298	22
201	320	341	363	384	406	428	449	471	492	514	22
202	535	557	578	600	621	643	664	685	707	728	21
203	750	771	792	814	835	856	878	899	920	942	21
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154	21
205	31 175	197	218	239	260	281	302	323	345	366	21
206	387	408	429	450	471	492	513	534	555	576	21
207	597	618	639	660	681	702	723	744	765	785	21
208	806	827	848	869	890	911	931	952	973	994	21
209	32 015	035	056	077	098	118	139	160	181	201	21
210	222	243	263	284	305	325	346	366	387	408	21
211	428	449	469	490	510	531	552	572	593	613	20
212	634	654	675	695	715	736	756	777	797	818	20
213	838	858	879	899	919	940	960	980	*001	*021	20
214	33 041	062	082	102	122	143	163	183	203	224	20
215	244	264	284	304	325	345	365	385	405	425	20
216	445	465	486	506	526	546	566	586	606	626	20
217	646	666	686	706	726	746	766	786	806	826	20
218	846	866	885	905	925	945	965	985	*005	*025	20
219	34 044	064	084	104	124	143	163	183	203	223	20
220	242	262	282	301	321	341	361	380	400	420	20
221	439	459	479	498	518	537	557	577	596	616	20
222	635	655	674	694	713	733	753	772	792	811	19
223	830	850	869	889	908	928	947	967	986	*005	19
224	35 025	044	064	083	102	122	141	160	180	199	19
225	218	238	257	276	295	315	334	353	372	392	19
226	411	430	449	468	488	507	526	545	564	583	19
227	603	622	641	660	679	698	717	736	755	774	19
228	793	813	832	851	870	889	908	927	946	965	19
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154	19
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230	36 173	192	211	229	248	267	286	305	324	342	19
231	361	380	399	418	436	455	474	493	511	530	19
232	549	568	586	605	624	642	661	680	698	717	19
233	736	754	773	791	810	829	847	866	884	903	19
234	922	940	959	977	996	*014	*033	*051	*070	*088	18
235	37 107	125	144	162	181	199	218	236	254	273	18
236	291	310	328	346	365	383	401	420	438	457	18
237	475	493	511	530	548	566	585	603	621	639	18
238	658	676	694	712	731	749	767	785	803	822	18
239	840	858	876	894	912	931	949	967	985	*003	18
240	38 021	039	057	075	093	112	130	148	166	184	18
241	202	220	238	256	274	292	310	328	346	364	18
242	382	399	417	435	453	471	489	507	525	543	18
243	561	578	596	614	632	650	668	686	703	721	18
244	739	757	775	792	810	828	846	863	881	899	18
245	917	934	952	970	987	*005	*023	*041	*058	*076	18
246	39 094	111	129	146	164	182	199	217	235	252	18
247	270	287	305	322	340	358	375	393	410	428	18
248	445	463	480	498	515	533	550	568	585	602	18
249	620	637	655	672	690	707	724	742	759	777	17
250	794	811	829	846	863	881	898	915	933	950	17
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123	17
252	40 140	157	175	192	209	226	243	261	278	295	17
253	312	329	346	364	381	398	415	432	449	466	17
254	483	500	518	535	552	569	586	603	620	637	17
255	654	671	688	705	722	739	756	773	790	807	17
256	824	841	858	875	892	909	926	943	960	976	17
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	17
258	41 162	179	196	212	229	246	263	280	296	313	17
259	330	347	363	380	397	414	430	447	464	481	17
260	497	514	531	547	564	581	597	614	631	647	17
261	664	681	697	714	731	747	764	780	797	814	17
262	830	847	863	880	896	913	929	946	963	979	16
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	16
264	42 160	177	193	210	226	243	259	275	292	308	16
265	325	341	357	374	390	406	423	439	455	472	16
266	488	504	521	537	553	570	586	602	619	635	16
267	651	667	684	700	716	732	749	765	781	797	16
268	813	830	846	862	878	894	911	927	943	959	16
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120	16
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270	43 136	152	169	185	201	217	233	249	265	281	16
271	297	313	329	345	361	377	393	409	425	441	16
272	457	473	489	505	521	537	553	569	584	600	16
273	616	632	648	664	680	696	712	727	743	759	16
274	775	791	807	823	838	854	870	886	902	917	16
275	933	949	965	981	996	*012	*028	*044	*059	*075	16
276	44 091	107	122	138	154	170	185	201	217	232	16
277	248	264	279	295	311	326	342	358	373	389	16
278	404	420	436	451	467	483	498	514	529	545	16
279	560	576	592	607	623	638	654	669	685	700	16
280	716	731	747	762	778	793	809	824	840	855	15
281	871	886	902	917	932	948	963	979	994	*010	15
282	45 025	040	056	071	086	102	117	133	148	163	15
283	179	194	209	225	240	255	271	286	301	317	15
284	332	347	362	378	393	408	423	439	454	469	15
285	484	500	515	530	545	561	576	591	606	621	15
286	637	652	667	682	697	712	728	743	758	773	15
287	788	803	818	834	849	864	879	894	909	924	15
288	939	954	969	984	*000	*015	*030	*045	*060	*075	15
289	46 090	105	120	135	150	165	180	195	210	225	15
290	240	255	270	285	300	315	330	345	359	374	15
291	389	404	419	434	449	464	479	494	509	523	15
292	538	553	568	583	598	613	627	642	657	672	15
293	687	702	716	731	746	761	776	790	805	820	15
294	835	850	864	879	894	909	923	938	953	967	15
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114	15
296	47 129	144	159	173	188	202	217	232	246	261	15
297	276	290	305	319	334	349	363	378	392	407	15
298	422	436	451	465	480	494	509	524	538	553	15
299	567	582	596	611	625	640	654	669	683	698	15
300	712	727	741	756	770	784	799	813	828	842	14
301	857	871	885	900	914	929	943	958	972	986	14
302	48 001	015	029	044	058	073	087	101	116	130	14
303	144	159	173	187	202	216	230	244	259	273	14
304	287	302	316	330	344	359	373	387	401	416	14
305	430	444	458	473	487	501	515	530	544	558	14
306	572	586	601	615	629	643	657	671	686	700	14
307	714	728	742	756	770	785	799	813	827	841	14
308	855	869	883	897	911	926	940	954	968	982	14
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	14
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310	49 136	150	164	178	192	206	220	234	248	262	14
311	276	290	304	318	332	346	360	374	388	402	14
312	415	429	443	457	471	485	499	513	527	541	14
313	554	568	582	596	610	624	638	651	665	679	14
314	693	707	721	734	748	762	776	790	803	817	14
315	831	845	859	872	886	900	914	927	941	955	14
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	14
317	50 106	120	133	147	161	174	188	202	215	229	14
318	243	256	270	284	297	311	325	338	352	365	14
319	379	393	406	420	433	447	461	474	488	501	14
320	515	529	542	556	569	583	596	610	623	637	14
321	651	664	678	691	705	718	732	745	759	772	14
322	786	799	813	826	840	853	866	880	893	907	13
323	920	934	947	961	974	987	*001	*014	*028	*041	13
324	51 055	068	081	095	108	121	135	148	162	175	13
325	188	202	215	228	242	255	268	282	295	308	13
326	322	335	348	362	375	388	402	415	428	441	13
327	455	468	481	495	508	521	534	548	561	574	13
328	587	601	614	627	640	654	667	680	693	706	13
329	720	733	746	759	772	786	799	812	825	838	13
330	851	865	878	891	904	917	930	943	957	970	13
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	13
332	52 114	127	140	153	166	179	192	205	218	231	13
333	244	257	270	284	297	310	323	336	349	362	13
334	375	388	401	414	427	440	453	466	479	492	13
335	504	517	530	543	556	569	582	595	608	621	13
336	634	647	660	673	686	699	711	724	737	750	13
337	763	776	789	802	815	827	840	853	866	879	13
338	892	905	917	930	943	956	969	982	994	*007	13
339	53 020	033	046	058	071	084	097	110	122	135	13
340	148	161	173	186	199	212	224	237	250	263	13
341	275	288	301	314	326	339	352	364	377	390	13
342	403	415	428	441	453	466	479	491	504	517	13
343	529	542	555	567	580	593	605	618	631	643	13
344	656	668	681	694	706	719	732	744	757	769	13
345	782	794	807	820	832	845	857	870	882	895	13
346	908	920	933	945	958	970	983	995	*008	*020	13
347	54 033	045	058	070	083	095	108	120	133	145	13
348	158	170	183	195	208	220	233	245	258	270	12
349	283	295	307	320	332	345	357	370	382	394	12
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351	531	543	555	568	580	593	605	617	630	642	12
352	654	667	679	691	704	716	728	741	753	765	12
353	777	790	802	814	827	839	851	864	876	888	12
354	900	913	925	937	949	962	974	986	998	*011	12
355	55 023	035	047	060	072	084	096	108	121	133	12
356	145	157	169	182	194	206	218	230	242	255	12
357	267	279	291	303	315	328	340	352	364	376	12
358	388	400	413	425	437	449	461	473	485	497	12
359	509	522	534	546	558	570	582	594	606	618	12
360	630	642	654	666	678	691	703	715	727	739	12
361	751	763	775	787	799	811	823	835	847	859	12
362	871	883	895	907	919	931	943	955	967	979	12
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	12
364	56 110	122	134	146	158	170	182	194	205	217	12
365	229	241	253	265	277	289	301	312	324	336	12
366	348	360	372	384	396	407	419	431	443	455	12
367	467	478	490	502	514	526	538	549	561	573	12
368	585	597	608	620	632	644	656	667	679	691	12
369	703	714	726	738	750	761	773	785	797	808	12
370	820	832	844	855	867	879	891	902	914	926	12
371	937	949	961	972	984	996	*008	*019	*031	*043	12
372	57 054	066	078	089	101	113	124	136	148	159	12
373	171	183	194	206	217	229	241	252	264	276	12
374	287	299	310	322	334	345	357	368	380	392	12
375	403	415	426	438	449	461	473	484	496	507	12
376	519	530	542	553	565	576	588	600	611	623	12
377	634	646	657	669	680	692	703	715	726	738	11
378	749	761	772	784	795	807	818	830	841	852	11
379	864	875	887	898	910	921	933	944	955	967	11
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	11
381	58 092	104	115	127	138	149	161	172	184	195	11
382	206	218	229	240	252	263	274	286	297	309	11
383	320	331	343	354	365	377	388	399	410	422	11
384	433	444	456	467	478	490	501	512	524	535	11
385	546	557	569	580	591	602	614	625	636	647	11
386	659	670	681	692	704	715	726	737	749	760	11
387	771	782	794	805	816	827	838	850	861	872	11
388	883	894	906	917	928	939	950	961	973	984	11
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	11
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390	59 106	118	129	140	151	162	173	184	195	207	11
391	218	229	240	251	262	273	284	295	306	318	11
392	329	340	351	362	373	384	395	406	417	428	11
393	439	450	461	472	483	494	506	517	528	539	11
394	550	561	572	583	594	605	616	627	638	649	11
395	660	671	682	693	704	715	726	737	748	759	11
396	770	780	791	802	813	824	835	846	857	868	11
397	879	890	901	912	923	934	945	956	966	977	11
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086	11
399	60 097	108	119	130	141	152	163	173	184	195	11
400	206	217	228	239	249	260	271	282	293	304	11
401	314	325	336	347	358	369	379	390	401	412	11
402	423	433	444	455	466	477	487	498	509	520	11
403	531	541	552	563	574	584	595	606	617	627	11
404	638	649	660	670	681	692	703	713	724	735	11
405	746	756	767	778	788	799	810	821	831	842	11
406	853	863	874	885	895	906	917	927	938	949	11
407	959	970	981	991	*002	*013	*023	*034	*045	*055	11
408	61 066	077	087	098	109	119	130	140	151	162	11
409	172	183	194	204	215	225	236	247	257	268	11
410	278	289	300	310	321	331	342	352	363	374	11
411	384	395	405	416	426	437	448	458	469	479	11
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413	595	606	616	627	637	648	658	669	679	690	11
414	700	711	721	731	742	752	763	773	784	794	10
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416	909	920	930	941	951	962	972	982	993	*003	10
417	62 014	024	034	045	055	066	076	086	097	107	10
418	118	128	138	149	159	170	180	190	201	211	10
419	221	232	242	252	263	273	284	294	304	315	10
420	325	335	346	356	366	377	387	397	408	418	10
421	428	439	449	459	469	480	490	500	511	521	10
422	531	542	552	562	572	583	593	603	613	624	10
423	634	644	655	665	675	685	696	706	716	726	10
424	737	747	757	767	778	788	798	808	818	829	10
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596	525	532	539	546	554	561	568	576	583	590	7
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609	462	469	476	483	490	497	504	512	519	526	7
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611	604	611	618	625	633	640	647	654	661	668	7
612	675	682	689	696	704	711	718	725	732	739	7
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615	888	895	902	909	916	923	930	937	944	951	7
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636	346	353	359	366	373	380	387	393	400	407	7
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650	291	298	305̄	311	318	325̄	331	338	345̄	351	7
651	358	365̄	371	378	385̄	391	398	405̄	411	418	7
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653	491	498	505̄	511	518	525̄	531	538	544	551	7
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679	187	193	200	206	213	219	225	232	238	245	6
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693	073	080	086	092	098	105	111	117	123	130	6
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697	323	330	336	342	348	354	361	367	373	379	6
698	386	392	398	404	410	417	423	429	435	442	6
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825	645	651	656	661	666	672	677	682	687	693	5
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833	065	070	075	080	085	091	096	101	106	111	5
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835	169	174	479	184	189	195	200	205	210	215	5
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841	480	485	490	495	500	505	511	516	521	526	5
842	531	536	542	547	552	557	562	567	572	578	5
843	583	588	593	598	603	609	614	619	624	629	5
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845	686	691	696	701	706	711	716	722	727	732	5
846	737	742	747	752	758	763	768	773	778	783	5
847	788	793	799	804	809	814	819	824	829	834	5
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849	891	896	901	906	911	916	921	927	932	937	5
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878	349	354	359	364	369	374	379	384	389	394	5
879	399	404	409	414	419	424	429	433	438	443	5
880	448	453	458	463	468	473	478	483	488	493	5
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885	694	699	704	709	714	719	724	729	734	738	5
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887	792	797	802	807	812	817	822	827	832	836	5
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896	231	236	240	245	250	255	260	265	270	274	5
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956	046	050	055	059	064	068	073	078	082	087	5
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971	722	726	731	735	740	744	749	753	758	762	4
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975	900	905	909	914	918	923	927	932	936	941	4
976	945	949	954	958	963	967	972	976	981	985	4
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988	476	480	484	489	493	498	502	506	511	515	4
989	520	524	528	533	537	542	546	550	555	559	4
N	0	1	2	3	4	5	6	7	8	9	D

N	0	1	2	3	4	5	6	7	8	9	D
990	564	568	572	577	581	585	590	594	599	603	4
991	607	612	616	621	625̄	629	634	638	642	647	4
992	651	656	660	664	669	673	677	682	686	691	4
993	695̄	699	704	708	712	717	721	726	730	734	4
994	739	743	747	752	756	760	765̄	769	774	778	4
995	782	787	791	795	800	804	808	813	817	822	4
996	826	830	835̄	839	843	848	852	856	861	865	4
997	870	874	878	883	887	891	896	900	904	909	4
998	913	917	922	926	930	935̄	939	944	948	952	4
999	957	961	965	970	974	978	983	987	991	996	4
N	0	1	2	3	4	5	6	7	8	9	D

THE METRIC TABLES OF WEIGHTS AND MEASURES.

The **Metric System** is a decimal system of weights and measures.

The basis of the whole system is the **metre.**

The length of a metre is defined by a platino-iridium bar kept in the International Metric Bureau at Paris. The metre was meant to be one ten-millionth of the distance from the equator to the pole, but a slight error in the calculation has been discovered.

The Latin prefixes indicate the denominations smaller than the unit, and the Greek prefixes the denominations larger than the unit. Thus:

Deci	designates	tenth.
Centi	"	hundredth.
Milli	"	thousandth.
Deka	"	ten.
Hekto	"	hundred.
Kilo	"	thousand.
Myria	"	ten thousand.

The denominations in more frequent use are denoted by heavier type.

LENGTH.

TABLE.

10 **millimetres** (mm)	= 1 **centimetre** (cm).
10 centimetres	= 1 decimetre (dm).
10 decimetres	= 1 **metre** (m).
10 metres	= 1 dekametre (Dm).
10 dekametres	= 1 hektometre (Hm).
10 hektometres	= 1 **kilometre** (Km).
10 kilometres	= 1 myriametre (Mm).

SURFACE.

The units of surface are squares whose dimensions are the corresponding linear units; hence it takes 10 times 10, or 100, of one denomination to make one of the next higher. For measuring small surfaces the principal unit is the **square metre.**

TABLE.

100 square millimetres (sq mm)	= 1 **square centimetre** (sq cm).
100 square centimetres	= 1 square decimetre (sq dm).
100 square decimetres	= 1 **square metre** (sq m).
100 square metres	= 1 square dekametre (sq Dm).
100 square dekametres	= 1 square hektometre (sq Hm).
100 square hektometres	= 1 **square kilometre** (sq Km).

LAND.

TABLE.

100 centares (ca)	= 1 are (a).
100 ares	= 1 hektare (Ha).

A centare is a square metre, an are a square dekametre, and a hektare a square hektometre.

VOLUME.

The units of volume are cubes whose dimensions are the corresponding linear units; hence it takes 10 times 10 times 10, or 1000, of one denomination to make one of the next higher.

TABLE.

1000 cubic millimetres (cu mm)	= 1 **cubic centimetre** (cu cm).
1000 cubic centimetres	= 1 cubic decimetre (cu dm).
1000 cubic decimetres	= 1 **cubic metre** (cu m).

WOOD.

TABLE.

10 decisteres (dst)	= 1 stere (st).
10 steres	= 1 dekastere (Dst).

A stere is a cubic metre.

CAPACITY.

The unit of capacity is a **litre**, which equals a cubic decimetre.

TABLE.

10 millilitres (ml)	= 1 **centilitre** (cl).
10 centilitres	= 1 decilitre (dl).
10 decilitres	= 1 **litre** (l).
10 litres	= 1 dekalitre (Dl).
10 dekalitres	= 1 **hektolitre** (Hl).
10 hektolitres	= 1 kilolitre (Kl).

WEIGHT.

The unit of weight is a **gram,** which equals the weight of a cubic centimetre of water at its greatest density.

TABLE.

10 **milligrams** (mg)	= 1 **centigram** (cg).
10 centigrams	= 1 decigram (dg).
10 decigrams	= 1 **gram** (g).
10 grams	= 1 dekagram (Dg).
10 dekagrams	= 1 hektogram (Hg).
10 hektograms	= 1 **kilogram,** or **kilo** (K).
1000 kilograms	= 1 **ton** (T).

METRIC EQUIVALENTS.

1 **metre**	= **39.37 in.** = 1.0936 yd.	1 yard	= .9144 m.
1 **kilometre**	= **.62138 mile**	1 mile	= 1.6093 kilometres.
1 **hektare**	= **2.471 acres**	1 acre	= .4047 Ha.
1 litre	= .908 qt. dry	1 qt. dry	= 1.101 l.
	1.0567 qt. liq.	1 qt. liq.	= .9463 l.
1 gram	= 15.432 grains	1 grain	= .0648 gram.
1 **kilogram**	= **2.2046 lbs.**	1 pound	= .4536 K.
1 stere	= .2759 cord	1 cord	= 3.625 steres.

APPROXIMATE METRIC EQUIVALENTS.

1 **cm.**	= $\frac{2}{5}$ **in.**	1 Hl.	= $2\frac{5}{6}$ bush.
1 **Km.**	= $\frac{5}{8}$ **mile.**	1 K.	= $2\frac{1}{5}$ lbs.
		1 T.	= 2200 lbs.

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