INCLUDING THE METRIC SYSTEM

THOMSON'S RUDIMENTS OF ARITHMETIC

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Day and Thomson's Series.

RUDIMENTS

OF

ARITHMETIC;

CONTAINING

NUMEROUS EXERCISES

FOR THE

SLATE AND BLACKBOARD

FOR BEGINNERS.

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PREFACE.

EDUCATION, in its comprehensive sense, is the business of life. The exercises of the school-room lay the foundation; the superstructure is the work of after years. If these exercises are rightly conducted, the pupil obtains the rudiments of science, and what is more important, he learns *how* to *study*, how to *think* and *reason*, and is thus enabled to appropriate the means of knowledge to his future advancement. Any system of instruction, therefore, which does not embrace these objects, which treats a child as a mere *passive recipient*, is *palpably defective.* It is destitute of some of the most essential means of mental development, and is calculated to produce *pigmies*, instead of *giant intellects.*

The question is often asked, "What is the best method of proceeding with pupils commencing the study of Arithmetic, or entering upon a new rule?"

The old method.—Some teachers allow every pupil to cipher "on his own hook;" to go as fast, or as slow as he pleases, without reciting a single example or rule, or stopping to inquire the "why and the wherefore" of a single operation. This mode of teaching is a relic of by-gone days, and is *prima facie* evidence, that those who practice it, are *behind the spirit* of the times.

Another method.—Others who admit the necessity of teaching arithmetic in classes, send their pupils to their seats, and tell them to "study the rule." After idling away an hour or more, up goes one little hand after another with the despairing question:—"Please to show me how to do this sum, sir?" The teacher replies, "Study the rule;—that will tell you." At length, to silence their increasing importunity, he takes the slate, solves the question, and, without a word of explanation, returns it to its owner. He thus goes through the class. When the hour of recitation comes, the class is not prepared with the lesson. They are sent to their seats to make another trial, which results in no better success. And what is the consequence? They are discouraged and disgusted with the study.

A more excellent way.—Other teachers pursue a more excellent way, especially for young pupils. It is this:—The teacher reads over *with* the class the preliminary explanations, and after satisfying himself that they understand the meaning of the terms, he calls upon one to read and analyze the first example, and set it down upon the blackboard, while the rest write it upon their slates. The one at the board then performs the operation audibly, and those with their slates follow step by step.

Another is now called to the board and requested to set down the second example, while the rest write the same upon their slates, and solve it in a similar manner. He then directs them to take the third example, and lets them try their own skill, giving each such aid as he may require. In this way they soon get hold of the *principle*, and if now sent to their seats, will master the lesson with positive delight.

As to *assistance*, no specific directions can be given which will meet every case. The best rule is, to afford the learner just that *kind* and *amount*, which will secure the *greatest degree* of *exertion* on his part. Less than this discourages; more, enervates.

In conclusion, we would add, that this elementary work was undertaken at the particular request of several eminent practical teachers, and is designed to fill a niche in primary schools. It presents, in a cheap form, a series of progressive exercises in the simple and compound rules, which are adapted to the capacities of beginners, and are calculated to form habits of study, awaken the attention, and strengthen the intellect.

J. B. THOMSON.

NEW YORK, *January*, 1853.

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ARITHMETIC.

SECTION I.

ART. **1.** ARITHMETIC *is the science of numbers.*

Any single thing, as a peach, a rose, a book, is called a *unit*, or *one;* if another single thing is put with it, the collection is called *two;* if another still, it is called *three;* if another, *four;* if another, *five*, &c.

The terms, *one*, *two*, *three*, *four*, *&c.*, are the *names of numbers.* Hence,

2. NUMBER signifies a *unit*, or a *collection of units.*

Numbers are expressed by *words*, by *letters*, and by *figures.*

3. NOTATION is *the art of expressing numbers by letters or figures.* There are two methods of notation in use, the *Roman* and the *Arabic.*

I. ROMAN NOTATION.

4. The *Roman* Notation is the method of expressing numbers by *letters;* and is so called because it was invented by the ancient *Romans.* It employs seven capital letters, viz: I, V, X, L, C, D, M.

When standing alone, the letter I, denotes *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* M, *one thousand.*

QUEST.—1. What is Arithmetic? What is a single thing called? If another is put with it, what is the collection called? If another, what? What are the terms one, two, three, &c.? 2. What then is number? How are numbers expressed? 3. What is Notation? How many methods of notation are in use? 4. What is the Roman notation? Why so called? How many letters does it employ? What does the letter I, denote? V? X? L? C? D? M?

5. To express the intervening numbers from to *one a thousand*, or any number larger than a thousand, we resort to repetitions and various combinations of these letters, as may be seen from the following

TABLE.

I	denotes	one.
II	"	two.
III	"	three.
IV	"	four.
V	"	five.
VI	"	six.
VII	"	seven.
VIII	"	eight.
IX	"	nine.
X	"	ten.
XI	"	eleven.
XII	"	twelve.
XIII	"	thirteen.
XIV	"	fourteen.
XV	"	fifteen.
XVI	"	sixteen.
XVII	"	seventeen.
XVIII	"	eighteen.
XIX	"	nineteen.
XX	"	twenty.
XXI	"	twenty-one.
XXII	"	twenty-two.
XXIII	"	twenty-three.
XXIV	"	twenty-four.
XXV	"	twenty-five.
XXVI	"	twenty-six.
XXVII	"	twenty-seven.
XXVIII	"	twenty-eight.
XXIX	"	twenty-nine.
XXX	"	thirty.
XXXI	denotes	thirty-one.
XL	"	forty.
XLI	"	forty-one.
L	"	fifty.
LI	"	fifty-one.
LX	"	sixty.
LXI	"	sixty-one.
LXX	"	seventy.
LXXX	"	eighty.
XC	"	ninety.
XCI	"	ninety-one.
C	"	one hundred.
CI	"	one hund. and one.
CIV	"	one hund. and four.
CX	"	one hund. and ten.
CC	"	two hundred.
CCV	"	two hund. and five.
CCC	"	three hundred.
CCCC	"	four hundred.
D	"	five hundred.
DC	"	six hundred.
DCC	"	seven hundred.
DCCC	"	eight hundred.
DCCCC	"	nine hundred.
M	"	one thousand.
MC	"	one thousand and one hundred.
MM	"	two thousand.
MDCCCL	"	one thousand eight hundred and fifty.

QUEST.—5. What do the letters IV, denote? VI? VIII? IX? XI? XIV? XVI? XVIII? XIX? XXIV? XL? LXXX? XC? CIV? Express seven by letters on the slate or black-board. How express eleven? Thirteen? Twenty-five? Nineteen? Forty-four? Eighty-seven? Ninety-nine?

OBS. 1. Every time a letter is repeated, its *value* is repeated. Thus, the letter I, standing alone, denotes *one;* II, *two ones* or *two,* &c. So X denotes *ten;* XX, *twenty,* &c.

2. When two letters of different value are joined together, if the less is placed before the greater, the value of the greater is *diminished* as many *units* as the less denotes; if placed after the greater, the value of the greater is *increased* as many units as the less denotes. Thus, V denotes five; but IV denotes only four; and VI, six. So X denotes ten; IX, nine; XI, eleven.

Note.—The questions on the observations may be omitted, by beginners, till review, if deemed advisable by the teacher.

II. ARABIC NOTATION.

6. The *Arabic Notation* is the method of expressing numbers by *figures;* and is so called because it is supposed to have been invented by the *Arabs.* It employs the following *ten characters* or *figures,* viz:

1	2	3	4	5	6	7	8	9	0
one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	naught.

OBS. 1. The first nine are called *significant* figures, because each one always expresses a value, or denotes some number. They are also called *digits,* from the Latin word *digitus,* signifying a finger; because the ancients used to count on their fingers.

2. The last one is called *naught,* because when standing *alone,* it expresses *nothing,* or the *absence* of number. It is also called *cipher* or *zero.*

7. All numbers larger than 9, are expressed by different combinations of these *ten figures.* For example, to express *ten,* we use the 1 and 0, thus 10; to express *eleven,* we use two 1s, thus 11; to express *twelve,* we use the 1 and 2, thus 12, &c.

QUEST.—*Obs.* What is the effect of repeating a letter? If a letter of less value is placed before another of greater value, what is the effect? If placed after, what? 6. What is the Arabic notation? Why so called? How many figures does it employ? What are their names? *Obs.* What are the first nine called? Why? What else are they sometimes called? What is the last one called? Why? 7. How are numbers larger than nine expressed? Express ten by figures. Eleven. Twelve. Fifteen

The method of expressing numbers by figures from *one* to a *thousand*, may be seen from the following

TABLE.

1, one.	36, thirty-six.	71, seventy-one.
2, two.	37, thirty-seven.	72, seventy-two.
3, three.	38, thirty-eight.	73, seventy-three.
4, four.	39, thirty-nine.	74, seventy-four.
5, five.	40, forty.	75, seventy-five.
6, six.	41, forty-one.	76, seventy-six.
7, seven.	42, forty-two.	77, seventy-seven.
8, eight.	43, forty-three.	78, seventy-eight.
9, nine.	44, forty-four.	79, seventy-nine.
10, ten.	45, forty-five.	80, eighty.
11, eleven.	46, forty-six.	81, eighty-one.
12, twelve.	47, forty-seven.	82, eighty-two.
13, thirteen.	48, forty-eight.	83, eighty-three.
14, fourteen.	49, forty-nine.	84, eighty-four.
15, fifteen.	50, fifty.	85, eighty-five.
16, sixteen.	51, fifty-one.	86, eighty-six.
17, seventeen.	52, fifty-two.	87, eighty-seven.
18, eighteen.	53, fifty-three.	88, eighty-eight.
19, nineteen.	54, fifty-four.	89, eighty-nine.
20, twenty.	55, fifty-five.	90, ninety.
21, twenty-one.	56, fifty-six.	91, ninety-one.
22, twenty-two.	57, fifty-seven.	92, ninety-two.
23, twenty-three.	58, fifty-eight.	93, ninety-three.
24, twenty-four.	59, fifty-nine.	94, ninety-four.
25, twenty-five.	60, sixty.	95, ninety-five.
26, twenty-six.	61, sixty-one.	96, ninety-six.
27, twenty-seven.	62, sixty-two.	97, ninety-seven.
28, twenty-eight.	63, sixty-three.	98, ninety-eight.
29, twenty-nine.	64, sixty-four.	99, ninety-nine.
30, thirty.	65, sixty-five.	100, one hundred.
31, thirty-one.	66, sixty-six.	200, two hundred.
32, thirty-two.	67, sixty-seven.	300, three hundred.
33, thirty-three.	68, sixty-eight.	400, four hundred.
34, thirty-four.	69, sixty-nine.	900, nine hundred.
35. thirty-five.	70, seventy.	1000, one thousand.

QUEST.—How express fifteen? Twenty-five? Forty-seven? Thirty-six? Seventy-three? One hundred and one? One hundred and ten? One hundred and twenty? Two hundred and fifteen?

8. It will be perceived from the foregoing table, that the same figures, standing in different places, have different values.

When they stand *alone* or in the *right hand* place, they express *units* or *ones*, which are called units of the *first order*.

When they stand in the *second* place, they express *tens*, which are called units of the *second order*.

When they stand in the *third* place, they express *hundreds*, which are called units of the *third order*.

When they stand in the *fourth* place, they express *thousands*, which are called units of the *fourth order*, &c.

For example, the figures 2, 3, 4, and 5, when arranged thus, 2345, denote 2 thousands, 3 hundreds, 4 tens, and 5 units; when arranged thus, 5432, they denote 5 thousands, 4 hundreds, 3 tens, and 2 units.

9. Ten units make one ten, ten tens make one hundred, and ten hundreds make one thousand, &c.; that is, *ten* of any *lower* order, are equal to *one* in the next higher order. Hence, universally,

10. *Numbers increase from right to left in a tenfold ratio; that is, each removal of a figure one place towards the left, increases its value ten times.*

11. The different values which the same figures have, are called *simple* and *local* values.

The *simple* value of a figure is the value which it expresses when it stands alone, or in the right hand place.

Quest.—8. Do the same figures always have the same value? When standing alone or in the right hand place, what do they express? What do they express when standing in the second place? In the third place? In the fourth? 9. How many units make one ten? How many tens make a hundred? How many hundreds make a thousand? Generally, how many of any ower order are required to make one of the next higher order? 10. What is the general law by which numbers increase? What is the effect upon the value of a figure to remove it one place towards the left? 11. What are the different values of the same figure called? What is the simple value of a figure? What the local value?

The simple value of a figure, therefore, is the number which its name denotes.

The *local* value of a figure is the *increased* value which it expresses by having other figures placed on its right. Hence, the local value of a figure depends on its locality, or the place which it occupies in relation to other numbers with which it is connected. (Art. 8.)

OBS. This system of notation is also called the *decimal system*, because numbers increase in a *tenfold* ratio. The term *decimal* is derived from the Latin word *decem*, which signifies ten.

NUMERATION.

12. The art of reading numbers when expressed by figures, is called *Numeration.*

NUMERATION TABLE.

Hundreds of Trillions.	Tens of Trillions.	*Trillions.*	Hundreds of Billions.	Tens of Billions.	*Billions.*	Hundreds of Millions.	Tens of Millions.	*Millions.*	Hundreds of Thousands.	Tens of Thousands.	*Thousands.*	Hundreds.	Tens.	*Units.*
1	2	3	8	6	1	5	1	8	9	2	4	2	6	3
Period V. Trillions.			Period IV. Billions.			Period III. Millions.			Period II. Thousands.			Period I. Units.		

13. The different orders of numbers are divided into *periods* of three figures each, *beginning* at the *right hand.*

QUEST.—Upon what does the local value of a figure depend? *Obs.* What is this system of notation sometimes called? Why? 12. What is Numeration? Repeat the numeration table, beginning at the right hand. What is the first place on the right called? The second place? The third? Fourth? Fifth? Sixth? Seventh? Eighth? Ninth? Tenth, &c.? 13. How are the orders of numbers divided?

The first, or right hand period is occupied by units, tens, hundreds, and is called *units'* period; the second is occupied by thousands, tens of thousands, hundreds of thousands, and is called *thousands'* period, &c.

The figures in the table are read thus: One hundred and twenty-three *trillions*, eight hundred and sixty-one *billions*, five hundred and eighteen *millions*, nine hundred and twenty-four *thousand*, two hundred and sixty-three.

14. To read numbers which are expressed by figures.

Point them off into periods of three figures each; then, beginning at the left hand, read the figures of each period as though it stood alone, and to the last figure of each, add the name of the period.

OBS. 1. The learner must be careful, in *pointing off* figures, always to begin at the *right* hand; and in *reading* them, to begin at the *left* hand.

2. Since the figures in the first or right hand period always denote units, the name of the period is not pronounced. Hence, in reading figures, when no period is mentioned, it is always understood to be the right hand, or units' period.

EXERCISES IN NUMERATION.

Note.—At first the pupil should be required to apply to each figure the name of the place which it occupies. Thus, beginning at the right hand, he should say, "Units, tens, hundreds," &c., and point at the same time to the figure standing in the place which he mentions. It will be a profitable exercise for young scholars to write the examples upon their slates or paper, then point them off into periods, and read them.

QUEST.—What is the first period called? By what is it occupied? What is the second period called? By what occupied? What is the third period called? By what occupied? What is the fourth called? By what occupied? What is the fifth called? By what occupied? 14. How do you read numbers expressed by figures? *Obs.* Where begin to point them off? Where to read them? Do you pronounce the name of the right hand period? When no period is named, what is understood?

Read the following numbers:

Ex. 1.	97	16.	12 642	31.	7 620
2.	110	17.	20 871	32.	8 040
3.	256	18.	17 046	33.	9 638
4.	307	19.	43 201	34.	11 000
5.	510	20.	80 600	35.	12 100
6.	465	21.	4 203	36.	14 020
7.	1 248	22.	65 026	37.	10 001
8.	2 381	23.	78 007	38.	5 020
9.	4 026	24.	90 210	39.	18 022
10.	6 420	25.	5 025	40.	30 401
11.	8 600	26.	69 008	41.	2 506
12.	7 040	27.	100 000	42.	402 321
13.	8 000	28.	125 236	43.	65 007
14.	9 007	29.	6 005	44.	750 026
15.	10 000	30.	462 400	45.	804 420

46.	2 325 672	50.	7 289 405 287
47.	4 502 360	51.	185 205 370 000
48.	62 840 285	52.	6 423 691 450 896
49.	425 026 951	53.	75 894 128 247 625

EXERCISES IN NOTATION.

15. To express numbers by figures.

Begin at the left hand of the highest period, and write the figures of each period as though it stood alone.

If any intervening order, or period is omitted in the given number, write ciphers in its place.

Write the following numbers in figures upon the slate or black-board.

1. Sixteen, seventeen, eighteen, nineteen, twenty.
2. Twenty-three, twenty-five, thirty, thirty-three.
3. Forty-nine, fifty-one, sixty, seventy-four.
4. Eighty-six, ninety-three, ninety-seven, a hundred.

QUEST.—15. How are numbers expressed by figures? If any intervening order is omitted in the example, how is its place supplied?

5. One hundred and ten.

6. Two hundred and thirty-five.

7. Three hundred and sixty.

8. Two hundred and seven.

9. Four hundred and eighty-one.

10. Six hundred and ninety-seven.

11. One thousand, two hundred and sixty-three.

12. Four thousand, seven hundred and ninety-nine.

13. Sixty-five thousand and three hundred.

14. One hundred and twelve thousand, six hundred and seventy-three.

15. Three hundred and forty thousand, four hundred and eighty-five.

16. Two millions, five hundred and sixty thousand.

17. Eight millions, two hundred and five thousand, three hundred and forty-five.

18. Ten millions, five hundred thousand, six hundred and ninety-five.

19. Seventeen millions, six hundred and forty-five thousand, two hundred and six.

20. Forty-one millions, six hundred and twenty thousand, one hundred and twenty-six.

21. Twenty-two millions, six hundred thousand, one hundred and forty-seven.

22. Three hundred and sixty millions, nine hundred and fifty thousand, two hundred and seventy.

23. Five billions, six hundred and twenty-one millions, seven hundred and forty-seven thousand, nine hundred and fifty-four.

24. Thirty-seven trillions, four hundred and sixty-three billions, two hundred and ninety-four thousand, five hundred and seventy-two.

SECTION II.

ADDITION.

ART. **16.** Ex. 1. Henry paid 4 shillings for a pair of gloves, 7 shillings for a cap, and 2 shillings for a knife: how many shillings did he pay for all?

Solution.—4 shillings and 7 shillings are 11 shillings, and 2 shillings are 13 shillings. He therefore paid 13 shillings for all.

OBS. The preceding operation consists in finding a *single number* which is equal to the *several given* numbers *united* together, and is called *Addition.* Hence,

17. ADDITION *is the process of uniting two or more numbers in one sum.*

The *answer, or* number *obtained* by addition, is called the *sum* or *amount.*

OBS. When the numbers to be added are all of the *same kind, or denomination,* the operation is called *Simple Addition.*

18. *Sign of Addition* (+). The *sign* of addition is a *perpendicular cross* (+), called *plus,* and shows that the numbers between which it is placed, are to be added together. Thus, the expression 6 + 8, signifies that 6 is to be added to 8. It is read, "6 plus 8," or "6 added to 8."

Note.—The term *plus,* is a Latin word, originally signifying "more." In Arithmetic, it means "added to."

QUEST.—17. What is addition? What is the answer called? *Obs.* When the numbers to be added are all of the same denomination, what is the operation called? 18. What is the sign of addition? What does it show? *Note.* What is the meaning of the word plus?

19. *Sign of Equality* (=). The *sign* of equality is *two horizontal lines* (=), and shows that the numbers between which it is placed, are *equal* to each other. Thus, the expression 4+3=7, denotes that 4 added to 3 are equal to 7. It is read, "4 plus 3 equal 7," or "the sum of 4 plus 3 is equal to 7." 18+5=7+16.

ADDITION TABLE.

2 and			3 and			4 and			5 and		
1	are	3	1	are	4	1	are	5	1	are	6
2	"	4	2	"	5	2	"	6	2	"	7
3	"	5	3	"	6	3	"	7	3	"	8
4	"	6	4	"	7	4	"	8	4	"	9
5	"	7	5	"	8	5	"	9	5	"	10
6	"	8	6	"	9	6	"	10	6	"	11
7	"	9	7	"	10	7	"	11	7	"	12
8	"	10	8	"	11	8	"	12	8	"	13
9	"	11	9	"	12	9	"	13	9	"	14
10	"	12	10	"	13	10	"	14	10	"	15
6 and			**7 and**			**8 and**			**9 and**		
1	are	7	1	are	8	1	are	9	1	are	10
2	"	8	2	"	9	2	"	10	2	"	11
3	"	9	3	"	10	3	"	11	3	"	12
4	"	10	4	"	11	4	"	12	4	"	13
5	"	11	5	"	12	5	"	13	5	"	14
6	"	12	6	"	13	6	"	14	6	"	15
7	"	13	7	"	14	7	"	15	7	"	16
8	"	14	8	"	15	8	"	16	8	"	17
9	"	15	9	"	16	9	"	17	9	"	18
10	"	16	10	"	17	10	"	18	10	"	19

Note.—It is an interesting and profitable exercise for young pupils to recite tables in concert. But it will not do to depend upon this method alone. It is indispensable for every scholar who desires to be *accurate* either in *arithmetic* or *business*, to have the common

QUEST.—19. What is the sign of equality? What does it show?

arithmetical tables *distinctly* and *indelibly* fixed in his mind. Hence after a table has been repeated by the class in concert, or individually, the teacher should ask many promiscuous questions, to prevent its being recited *mechanically*, from a knowledge of the regular increase of numbers.

EXAMPLES.

20. *When the sum of a column does not exceed* 9.

Ex. 1. George gave 37 cents for his Arithmetic, and 42 cents for his Reader: how many cents did he give for both?

Directions.—Write the numbers under each other, so that *units* may stand under *units, tens* under *tens,* and draw a line beneath them. Then, beginning at the *right hand* or *units,* add each column *separately* in the following manner:—

Operation.

tens.	units.	
3	7	price of Arith.
4	2	" of Read.
7	9	*Ans.*

2 units and 7 units are 9 units. Write the 9 in units' place under the column added. 4 tens and 3 tens are 7 tens. Write the 7 in tens' place. The amount is 79 cents.

Write the following examples upon the slate or blackboard, and find the sum of each in a similar manner:

(2.)	(3.)	(4.)	(5.)
26	231	623	5734
42	358	145	4253

(6.)	(7.)	(8.)	(9.)
425	3021	5120	3521
132	1604	2403	1043
321	2142	1375	4215

10. What is the sum of 4321 and 2475?
11. What is the sum of 32562 and 56214?
12. What is the sum of 521063 and 465725?

21. *When the sum of a column exceeds* 9.

13. A merchant sold a quantity of flour for 458 dollars, a quantity of tea for 887 dollars, and sugar for 689 dollars: how much did he receive for all?

Operation.

458	price of flour.
887	" of tea.
689	" of sugar.
2034	dollars. *Ans.*

Having written the numbers as before, we proceed thus: 9 units and 7 units are 16 units, and 8 are 24 units, or we may simply say 9 and 7 are 16, and 8 are 24. Now 24 is equal to 2 tens and 4 units. We therefore set the 4 units or right hand figure in units' place, *because they are units;* and reserving the 2 tens or left hand figure in the mind, add it to the column of tens *because it is tens.* Thus, 2 (which was reserved) and 8 are 10, and 8 are 18, and 5 are 23. Set the 3 or right hand figure under the column added, and reserving the 2 or left hand figure in the mind, add it to the column of hundreds, *because it is hundreds.* Thus, 2 (which was reserved) and 6 are 8, and 8 are 16, and 4 are 20. Set the 0 or right hand figure under the column added; and since there is no other column to be added, write the 2 in thousands' place, *because it is thousands.*

N. B. The pupil must remember, in all cases, to set down the *whole sum* of the *last* or *left hand column.*

22. The process of *reserving the tens or left hand figure,* when the sum of a column exceeds 9, and *adding* it mentally to the next column, is called *carrying tens.*

Find the sum of each of the following examples in a similar manner:

(14.)	(15.)	(16.)	(17.)
856	364	6502	8245
764	488	497	4678
1620 *Ans.*	602	8301	362

23. From the preceding illustrations and principles we derive the following

GENERAL RULE FOR ADDITION.

I. *Write the numbers to be added under each other, so that units may stand under units, tens under tens, &c.*

II. *Beginning at the right hand, add each column separately, and if the sum of a column does not exceed* 9, *write it under the column added. But if the sum of a column exceeds* 9, *write the units' figure under the column and carry the tens to the next column.*

III. *Proceed in this manner through all the orders, and finally set down the whole sum of the last or left hand column.*

24. PROOF.—*Beginning at the top, add each column downward, and if the second result is the same as the first, the work is supposed to be right.*

EXAMPLES FOR PRACTICE.

(1.)	(2.)	(3.)	(4.)
Pounds.	Feet.	Dollars.	Yards.
25	113	342	4608
46	84	720	635
84	216	898	43

(5.)	(6.)	(7.)	(8.)
684	336	6387	8261
948	859	593	387
569	698	3045	13
203	872	15	7

9. What is the sum of 46 inches and 38 inches?

QUEST.—23. How do you write numbers for addition? When the sum of a column does not exceed 9, how proceed? When it exceeds 9, how proceed? 22. What is meant by carrying the tens? What do you do with the sum of the last column? 24. How is addition proved?

10. What is the sum of 51 feet and 63 feet?

11. What is the sum of 75 dollars and 93 dollars?

12. Add together 45 rods, 63 rods, and 84 rods.

13. Add together 125 pounds, 231 pounds, 426 pounds.

14. Add together 267 yards, 488 yards, and 625 yards.

15. Henry traveled 256 miles by steamboat and 320 miles by Railroad: how many miles did he travel?

16. George met two droves of sheep; one contained 461, and the other 375: how many sheep were there in both droves?

17. If I pay 230 dollars for a horse, and 385 dollars for a chaise, how much shall I pay for both?

18. A farmer paid 85 dollars for a yoke of oxen, 27 dollars for a cow, and 69 dollars for a horse: how much did he pay for all?

19. Find the sum of 425, 346, and 681.

20. Find the sum of 135, 342, and 778.

21. Find the sum of 460, 845, and 576.

22. Find the sum of 2345, 4088, and 260.

23. Find the sum of 8990, 5632, and 5863.

24. Find the sum of 2842, 6361, and 523.

25. Find the sum of 602, 173, 586, and 408.

26. Find the sum of 424, 375, 626, and 75.

27. Find the sum of 24367, 61545, and 20372.

28. Find the sum of 43200, 72134, and 56324.

29. A young man paid 5 dollars for a hat; 6 dollars for a pair of boots, 27 dollars for a suit of clothes, and 19 dollars for a cloak: how much did he pay for all?

30. A man paid 14 dollars for wood, 16 dollars for a stove, and 28 dollars for coal: how many dollars did he pay for all?

31. A farmer bought a plough for 13 dollars, a cart for 46 dollars, and a wagon for 61 dollars: what was the price of all?

32. What is the sum of 261+31+256+17?
33. What is the sum of 163+478+82+19?
34. What is the sum of 428+632+76+394?
35. What is the sum of 320+856+100+503?
36. What is the sum of 641+108+138+710?
37. What is the sum of 700+66+970+21?
38. What is the sum of 304+971+608+496?
39. What is the sum of 848+683+420+668?
40. What is the sum of 868+45+17+25+27+38?
41. What is the sum of 641+85+580+42+7+63?
42. What is the sum of 29+281+7+43+785+46?

43. A farmer sold 25 bushels of apples to one man, 17 bushels to another, 45 bushels to another, and 63 bushels to another: how many bushels did he sell?

44. A merchant bought one piece of cloth containing 25 yards, another 28 yards, another 34 yards, and another 46 yards: how many yards did he buy?

45. A man bought 3 farms; one contained 120 acres, another 246 acres, and the other 365 acres: how many acres did they all contain?

46. A traveler met four droves of cattle; the first contained 260, the second 175, the third 342, and the fourth 420: how many cattle did the four droves contain?

47. A carpenter built one house for 2365 dollars, another for 1648 dollars, another for 3281 dollars, and another for 5260 dollars: how much did he receive for all?

48. Find the sum of six hundred and fifty-four, eighty-nine, four hundred and sixty-three, and seventy-six.

49. Find the sum of two thousand and forty-seven, three hundred and forty-five, thirty-six, and one hundred.

50. In January there are 31 days, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, and December 31: how many days are there in a year?

24.*a.* *Accuracy* and *rapidity* in adding can be acquired only by *practice.* The following exercises are designed to secure this important object.

Obs. 1. In solving the following examples, it is recommended to the pupil simply to pronounce the result, as he adds each successive figure. Thus, in Ex. 1, instead of saying 2 and 2 are 4, and 2 are 6, &c., proceed in the following manner: "two, four, six, eight, ten, twelve, fourteen, sixteen, eighteen, twenty." Set down *naught* and carry *two.* "Two, (to carry) three, six, nine," &c.

2. When *two* or *three* figures taken together make 10, as 8 and 2, 7 and 3, or 2, 3, and 5, it accelerates the process to add their sum at once. Thus, in Ex. 4, the pupil should say: "ten (1+9), sixteen (6), twenty-six (5+5), thirty-six (2+8)," &c.

(1.)	(2.)	(3.)	(4.)
32	654	987	463
32	654	987	647
32	654	987	455
32	654	987	258
32	654	987	572
32	654	987	595
32	654	987	615
32	654	987	346
32	654	987	729
12	114	117	181

(5.)	(6.)	(7.)	(8.)
614	2140	8675	9244
452	8963	2433	1432
528	1232	6182	7234
539	7855	2921	2523
420	2123	2209	8440
385	3333	4863	4346
355	7674	6558	6704
134	4521	5434	1852
976	6589	5276	9258
468	2637	8789	8106

(9.)	(10.)	(11.)	(12.)
4360	9201	42671	62125
7046	7283	68439	31684
5724	4627	32074	22435
5385	6436	47616	16725
8275	9874	30045	94381
9342	8400	26765	25036
6768	6645	10850	85474
5020	4365	25232	10325
9384	8640	43679	42312

(13.)	(14.)	(15.)	(16.)
2720	5764	27856	47639
4382	5346	32534	23421
2640	3042	20631	34323
3047	5268	34327	71036
2163	3161	53102	62342
6741	2560	92763	57654
1360	7304	51834	32103
7056	2723	23452	53728
3554	8459	62327	61342
4275	6715	50632	23201

(17.)	(18.)	(19.)	(20.)
4521	6845	75360	89537
3432	3151	27838	23264
4327	2327	42627	41728
6238	4235	34872	74263
5494	2835	63538	21031
3217	5473	54321	53426
2382	9864	63054	91342
4723	3103	29872	23465
3604	7382	63541	38754
2352	5461	53279	94642

(21.)	(22.)	(23.)	(24.)
8564	56,430	84,703	341,725
4736	31,932	19,384	227,265
3405	29,754	21,705	311,265
5037	46,536	43,641	200,378
6571	86,075	27,469	421,850
7439	30,235	52,267	370,432
4525	41,623	61,383	174,370
3137	45,810	75,604	831,031
2743	56,239	43,876	580,456

(25.)	(26.)	(27.)	(28.)
7243	31,625	68,901	460,732
2034	51,482	50,345	804,045
3710	49,061	75,005	346,325
5634	80,604	29,450	450,673
1730	24,540	80,063	859,721
5613	67,239	91,700	236,548
3005	24,307	43,621	632,462
7206	58,392	47,834	753,324
4354	70,300	83,276	970,300
7821	56,749	25,327	267,436

(29.)	(30.)	(31.)	(32.)
6458	75,340	64,268	346,768
2435	6,731	405	21,380
4678	748	1,708	4,075
4962	68,451	43,671	126,849
5143	396	72,049	257
8437	7,503	492	1,305
7643	46,075	1,760	24,350
6850	1,290	25,357	439,871
7063	25,738	1,434	40,306
8324	46,803	84,162	601,734

(33.)	(34.)	(35.)	(36.)
423,674	632,153	317,232	412,783
307,316	420,432	203,671	631,432
730,248	323,680	334,263	572,316
506,213	507,325	210,600	231,254
110,897	383,734	356,237	673,323
206,341	634,156	264,871	217,067
324,563	450,071	531,634	306,421
185,174	803,463	342,106	764,315
364,230	160,705	768,342	207,254
150,176	300,430	407,821	843,552
843,204	461,007	311,289	321,634
370,679	297,313	564,735	502,543
445,168	813,792	470,334	617,405
370,432	200,406	436,216	506,032
5,338,315 *Ans.*	6,388,667 *Ans.*	621,353	762,573

(37.)	(38.)	(39.)	(40.)
674,326	783,457	863,725	958,439
453,403	675,306	755,387	843,670
561,734	858,642	964,845	784,561
789,867	246,468	836,450	976,435
645,275	587,649	645,265	833,406
576,182	523,731	783,842	797,624
934,922	445,372	532,653	845,358
423,641	832,148	647,412	978,262
561,232	465,363	481,735	784,643
143,671	642,742	824,364	865,343
238,406	830,423	537,572	976,736
453,762	256,372	463,489	853,974
984,651	662,456	827,343	467,852
845,359	572,834	642,536	948,685
967,423	864,213	725,342	896,872

SECTION III.

SUBTRACTION.

ART. **25.** Ex. 1. Charles having 15 cents, gave 6 cents for an orange: how many cents did he have left?

Solution.—6 cents taken from 15 cents leave 9 cents. Therefore he had 9 cents left.

OBS. The preceding operation consists in taking a *less* number from a *greater*, and is called *Subtraction.* Hence,

26. SUBTRACTION *is the process of finding the difference between two numbers.*

The *answer*, or number *obtained* by subtraction, is called the *difference* or *remainder.*

OBS. 1. The number to be subtracted is often called the *subtrahend*, and the number from which it is subtracted, the *minuend* These terms, however, are calculated to embarrass, rather than assist the learner, and are properly falling into disuse.

2. When the given numbers are all of the *same kind, or denomination*, the operation is called *Simple Subtraction.*

27. *Sign of Subtraction* (—). The *sign* of subtraction is a *horizontal line* (—), called *minus*, and shows that the number *after* it is to be subtracted from the one *before* it. Thus the expression $7-3$, signifies that 3 is to be subtracted from 7; and is read, " 7 minus 3," or " 7 less 3." Read the following: $18-7=20-9$. $23-10=16-3$. $35-8=31-4$.

Note.—The term *minus* is a Latin word signifying *less.*

QUEST.—26. What is subtraction? What is the answer called? Obs. What is the number to be subtracted sometimes called? That from which it is subtracted? When the given numbers are of the same denomination, what is the operation called? 27. What is the sign of subtraction? What does it show? *Note.* What is the meaning of the term minus?

SUBTRACTION TABLE.

2 from			3 from			4 from			5 from		
2	leaves	0	3	leaves	0	4	leaves	0	5	leaves	0
3	"	1	4	"	1	5	"	1	6	"	1
4	"	2	5	"	2	6	"	2	7	"	2
5	"	3	6	"	3	7	"	3	8	"	3
6	"	4	7	"	4	8	"	4	9	"	4
7	"	5	8	"	5	9	"	5	10	"	5
8	"	6	9	"	6	10	"	6	11	"	6
9	"	7	10	"	7	11	"	7	12	"	7
10	"	8	11	"	8	12	"	8	13	"	8
11	"	9	12	"	9	13	"	9	14	"	9
12	"	10	13	"	10	14	"	10	15	"	10
6 from			7 from			8 from			9 from		
6	leaves	0	7	leaves	0	8	leaves	0	9	leaves	0
7	"	1	8	"	1	9	"	1	10	"	1
8	"	2	9	"	2	10	"	2	11	"	2
9	"	3	10	"	3	11	"	3	12	"	3
10	"	4	11	"	4	12	"	4	13	"	4
11	"	5	12	"	5	13	"	5	14	"	5
12	"	6	13	"	6	14	"	6	15	"	6
13	"	7	14	"	7	15	"	7	16	"	7
14	"	8	15	"	8	16	"	8	17	"	8
15	"	9	16	"	9	17	"	9	18	"	9
16	"	10	17	"	10	18	"	10	19	"	10

OBS. This Table is the reverse of Addition Table. Hence, if the pupil has thoroughly learned that, it will cost him but little time or trouble to learn this. (See observations under Addition Table.)

EXAMPLES.

28. *When each figure in the lower number is smaller than the figure above it.*

1. A farmer raised 257 bushels of apples, and 123 bushels of pears: how many more apples did he raise than pears?

Directions.—Write the *less* number under the *greater*, so that *units* may stand under *units*, *tens* under *tens*, &c., and draw a line beneath them. Beginning with the *units* or *right hand* figure, subtract each figure in the *l*ower number from the figure above it, in the following manner: 3 units from 7 units leave 4 units. Write the 4 in units' place under the figure subtracted. 2 tens from 5 tens leave 3 tens; set 3 in tens' place. 1 hundred from 2 hundred leaves 1 hundred; write the 1 hundred in hundreds' place.

Operation.

	hund.	tens.	units.	
	2	5	7	apples.
	1	2	3	pears.
Rem.	1	3	4	bush.

Solve the following examples in a similar manner:

	(2.)	(3.)	(4.)	(5.)
From	45	68	276	698
Take	21	56	123	453

	(6.)	(7.)	(8.)	(9.)
From	54 dolls.	76 pounds.	257 yds.	325 shil.
Take	23 dolls.	64 pounds.	142 yds.	103 shil.

10. Samuel having 436 marbles, lost 214 of them: how many had he left?

29. *When a figure in the lower number is larger than the figure above it.*

11. A man bought 63 bushels of wheat, and afterwards sold 37: how many bushels had he left?

It is obvious that we cannot take 7 units from 3 units, for 7 is larger than 3; we therefore add 10 to the 3 units, and it will make 13 units; then 7 from 13 leaves 6; write the 6 in units' place under the figure subtracted. To compensate for the 10

First Method.

	63
	37
Rem.	26 bu.

units we added to the upper figure, we add 1 ten to the 3 tens or next figure in the lower number, and it makes 4 tens; and 4 tens from 6 tens leave 2 tens: write the 2 in tens' place. *Ans.* 26 bushels.

We may also illustrate the process of borrowing in he following manner:

63 is composed of 6 tens and 3 units. Taking 1 ten from 6 tens, and adding it to the 3 units, we have 63=50+13. Separating the lower number into tens and units, we have 37=30+7. Now, substracting as before, 7 from 13 leaves 6. Then as we took 1 ten from the 6 tens, we have but 5 tens left; and 3 tens from 5 tens leave 2 tens. The remainder is 26, the same as before.

Second Method.

$$63=50+13$$
$$37=30+\ 7$$
$$\textit{Rem.}=20+\ 6, \text{ or } 26.$$

30. The process of taking one from a higher order in the upper number, and adding it to the figure from which the subtraction is to be made, is called *borrowing ten,* and is the reverse of *carrying ten.* (Art. 22.)

OBS. When we *borrow ten* we must always remember to *pay* it. This may be done, as we have just seen, either by *adding* 1 to the *next figure* in the *lower* number, or by considering the *next figure* in the *upper* number 1 *less* than it is.

12. From 240 subtract 134, and prove the operation.

Since 4 cannot be taken from 0, we borrow 10; then 4 from 10 leaves 6. 1 added to 3 (to compensate for the 10 we borrowed) makes 4, and 4 from 4 leaves 0. 1 from 2 leaves 1.

Operation.

```
 240
 134
 ---
 106  Ans.
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PROOF.—We add the *remainder* to the *smaller* number, and since the *sum* is equal to the *larger* number, the work is right.

Proof.

134	less No.
106	remainder.
240	greater No.

Solve the following examples, and prove the operation

	(13.)	(14.)	(15.)	(16.)
From	375	5273	6474	8650
Take	238	2657	3204	5447

17. From 8461875, take 3096208.

31. From the preceding illustrations and principles we derive the following

GENERAL RULE FOR SUBTRACTION.

I. *Write the less number under the greater, so that units may stand under units, tens under tens, &c.*

II. *Beginning at the right hand, subtract each figure in the lower number from the figure above it, and set the remainder under the figure subtracted.*

III. *When a figure in the lower number is larger than that above it, add 10 to the upper figure; then subtract as before, and add 1 to the next figure in the lower number.*

32. PROOF.—*Add the remainder to the smaller number; and if the sum is equal to the larger number, the work is right.*

OBS. This method of proof depends upon the obvious principle, that if the *difference* between two numbers be added to the *less*, the *sum* must be equal to the *greater*.

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)
From	325	431	562	600
Take	108	249	320	231

	(5.)	(6.)	(7.)	(8.)
From	2230	3042	6500	8435
Take	1201	2034	3211	5001

QUEST.—31. How do you write numbers for subtraction? Where do you begin to subtract? When a figure in the lower number is larger than the one above it, how do you proceed? 32. How is subtraction proved?

(9.)	(10.)	(11.)
From 45100	826340	1000000
Take 10000	513683	999999

12. From 132 dollars subtract 109 dollars.
13. From 142 bushels subtract 85 bushels.
14. From 375 pounds subtract 100 pounds.
15. From 698 yards subtract 85 yards.
16. From 485 rods subtract 175 rods.
17. Take 230 gallons from 460 gallons.
18. Take 168 hogsheads from 671 hogsheads.
19. Take 192 bushels from 268 bushels.
20. From 275 dollars take 148 dollars.
21. From 468 pounds take 219 pounds.
22. From 3246 rods take 2164 rods.
23. From 45216 take 32200.
24. From 871410 take 560642.
25. From 926500 take 462126.
26. From 6284678 take 1040640.
27. 468—423.
28. 675—367.
29. 800—560.
30. 701—643.
31. 963—421.
32. 3263—1242.
33. 4165—2340.
34. 5600—3000.
35. 7246—4161.
36. 8670—7364.
37. 17265—13167.
38. 21480—20372.
39. 30671—26140.
40. 45723—31203.
41. 81647—57025.
42. 265328—140300.
43. 170643—106340.
44. 465746—241680.
45. 694270—590395.
46. 920486—500000.

47. A man having 235 sheep, lost 163 of them: how many had he left?

48. A farmer having 500 bushels of wheat, sold 278 bushels: how much wheat had he left?

49. A man paid 625 dollars for a carriage and 430

dollars for a span of horses: how much more did he pay for his carriage than for his horses?

50. A man gave 1263 dollars for a lot, and 2385 dollars for building a house: how much more did his house cost than his lot?

51. If a person has 3290 dollars in real estate, and owes 1631 dollars, how much is he worth?

52. A man gave his son 8263 dollars, and his daughter 5240 dollars: how much more did he give his son than his daughter?

53. A man bought a farm for 9467 dollars, and sold it for 11230 dollars: how much did he make by his bargain?

54. If a man's income is 10000 dollars a year, and his expenses 6253 dollars, how much will he lay up?

55. The captain of a ship having a cargo of goods worth 29230 dollars, threw overboard in a storm 13216 dollars' worth: what was the value of the goods left?

56. A merchant bought a quantity of goods for 12645 dollars, and afterwards sold them for 13960 dollars: how much did he gain by his bargain?

57. A man paid 23645 dollars for a ship and afterwards sold it for 18260 dollars: how much did he lose by his bargain?

58. The salary of the President of the United States is 25000 dollars a year; now if his expenses are 19265 dollars, how much will he lay up?

59. A general before commencing a battle, had 35260 soldiers in his army; after the battle he had only 21316: how many soldiers did he lose?

60. The distance of the sun from the earth is 95000000 miles; the distance of the moon from the earth is 240000 miles: how much farther from the earth is the sun than the moon?

EXAMPLES INVOLVING ADDITION AND SUBTRACTION.

61. Henry bought 63 oranges of one grocer, and 26 of another; he afterwards sold 72: how many oranges did he have left?

62. Charles had 47 marbles, and his father gave him 36 more; he afterwards lost 50: how many marbles did he then have?

63. A farmer having 158 sheep, lost 30 of them by sickness and sold 52: how many sheep did he have left?

64. Sarah's father gave her 60 cents, and her mother gave her 54 cents; if she spends 62 cents for a pair of gloves, how many cents will she have left?

65. A merchant purchased a piece of silk containing 78 yards; he then sold 18 yards to one lady, and 17 to another: how many yards had he left?

66. If a man has property in his possession worth 215 dollars, and owes 39 dollars to one person, and 54 dollars to another, how much money will he have left, when he pays his debts?

67. If a man's income is 185 dollars per month, and he pays 35 dollars for house rent, and 63 dollars for provisions per month, how many dollars will he have left for other expenses?

68. George having 74 pears, gave away 43 of them; if he should buy 35 more, how many would he then have?

69. If you add 115 to 78, and from the sum take 134, what will the remainder be?

70. If you subtract 93 from 147, and add 110 to the remainder, what will the sum be?

71. A merchant purchased 125 pounds of butter of one dairy-man, and 187 pounds of another; he afterwards sold 163 pounds: how many pounds did he have left?

72. A miller bought 200 bushels of wheat of one farmer, and 153 bushels of another; he afterwards sold 189 bushels: how many bushels did he have left?

73. A man traveled 538 miles in 3 days; the first day he traveled 149 miles, the second day, 126 miles: how far did he travel the third day?

74. A grocer bought a cask of oil containing 256 gallons; after selling 93 gallons, he perceived the cask was leaky, and on measuring what was left, found he had 38 gallons: how many gallons had leaked out?

75. A manufacturer bought 248 pounds of wool of one customer, and 361 pounds of another; he then worked up 430 pounds: how many pounds had he left?

76. A man paid 375 dollars for a span of horses, and 450 dollars for a carriage; he afterwards sold his horses and carriage for 1000 dollars; how much did he make by his bargain?

77. A grocer bought 285 pounds of lard of one farmer, and 327 pounds of another; he afterwards sold 110 pounds to one customer, and 163 pounds to another: how much lard did he have left?

78. A flour dealer having 500 barrels of flour on hand, sold 263 barrels to one customer and 65 barrels to another: how many barrels had he left?

79. Harriet wished to read a book through which contained 726 pages, in three weeks; the first week she read 165 pages, and the second week she read 264 pages: how many pages were left for her to read the third week?

80. A man bought a house for 1200 dollars, and having laid out 210 dollars for repairs, sold it for 1300 dollars: how much did he lose by the bargain?

81. A young man having 2000 dollars, spent 765 the first year and 843 the second year: how much had he left?

SECTION IV.

MULTIPLICATION.

ART. **33.** Ex. 1. What will three lemons cost, at 2 cents apiece?

Analysis.—Since 1 lemon costs 2 cents, 3 lemons will cost 3 times 2 cents; and 3 times 2 cents are 6 cents Therefore, 3 lemons, at 2 cents apiece, will cost 6 cents.

OBS. The preceding operation is a short method of finding how much 2 cents *will amount* to, when *repeated* or taken 3 times, and is called *Multiplication.* Thus, 2 cents + 2 cents + 2 cents are 6 cents. Hence,

34. MUTIPLICATION *is the process of finding the amount of a number repeated or added to itself, a given number of times.*

The number to be *repeated* or *multiplied*, is called the *multiplicand.*

The number by which we *multiply*, is called the *multiplier*, and shows how many times the multiplicand is to be repeated or taken.

The *answer*, or number *produced* by multiplication, is called the *product.*

Thus, when we say 5 times 7 are 35, 7 is the multiplicand, 5 the multiplier, and 35 the product.

OBS. When the multiplicand denotes things of *one kind, or denomination only*, the operation is called *Simple Multiplication.*

QUEST.—34. What is multiplication? What is the number to be repeated or multiplied called? What the number by which we multiply? What does the multiplier show? What is the answer called? When we say 5 times 7 are 35, which is the multiplicand? Which the multiplier? Which the product? *Obs.* When the multiplicand denotes things of one denomination only, what is the operation called?

35. The multiplier and multiplicand taken together, are often called *factors*, because they *make* or *produce* the product.

Note.—The term *factor*, is a Latin word which signifies an *agent*, a *doer*, or *producer*.

MULTIPLICATION TABLE.

2 times			3 times			4 times			5 times			6 times			7 times		
1	are	2	1	are	3	1	are	4	1	are	5	1	are	6	1	are	7
2	"	4	2	"	6	2	"	8	2	"	10	2	"	12	2	"	14
3	"	6	3	"	9	3	"	12	3	"	15	3	"	18	3	"	21
4	"	8	4	"	12	4	"	16	4	"	20	4	"	24	4	"	28
5	"	10	5	"	15	5	"	20	5	"	25	5	"	30	5	"	35
6	"	12	6	"	18	6	"	24	6	"	30	6	"	36	6	"	42
7	"	14	7	"	21	7	"	28	7	"	35	7	"	42	7	"	49
8	"	16	8	"	24	8	"	32	8	"	40	8	"	48	8	"	56
9	"	18	9	"	27	9	"	36	9	"	45	9	"	54	9	"	63
10	"	20	10	"	30	10	"	40	10	"	50	10	"	60	10	"	70
11	"	22	11	"	33	11	"	44	11	"	55	11	"	66	11	"	77
12	"	24	12	"	36	12	"	48	12	"	60	12	"	72	12	"	84

8 times			9 times			10 times			11 times			12 times		
1	are	8	1	are	9	1	are	10	1	are	11	1	are	12
2	"	16	2	"	18	2	"	20	2	"	22	2	"	24
3	"	24	3	"	27	3	"	30	3	"	33	3	"	36
4	"	32	4	"	36	4	"	40	4	"	44	4	"	48
5	"	40	5	"	45	5	"	50	5	"	55	5	"	60
6	"	48	6	"	54	6	"	60	6	"	66	6	"	72
7	"	56	7	"	63	7	"	70	7	"	77	7	"	84
8	"	64	8	"	72	8	"	80	8	"	88	8	"	96
9	"	72	9	"	81	9	"	90	9	"	99	9	"	108
10	"	80	10	"	90	10	"	100	10	"	110	10	"	120
11	"	88	11	"	99	11	"	110	11	"	121	11	"	132
12	"	96	12	"	108	12	"	120	12	"	132	12	"	144

Note.—1. It will be perceived that the several results of multiplying by 10, are formed by adding a *naught* or *cipher* to the figure that is to be multiplied. Thus, 10 times 2 are 20; 10 times 3 are 30, &c.

2. The results of multiplying by 5, terminate in 5 and 0 alternately. Thus, 5 times 1 are 5; 5 times 2 are 10; 5 times 3 are 15, 5 times 4 are 20, &c.

QUEST.—35. What are the multiplicand and multiplier together called? Why? *Note.* What does the term factor signify?

3. The first nine results of multiplying by 11, are formed by repeating the figure to be multiplied. Thus, 11 times 2 are 22; 11 times 3 are 33, &c.

4. In the successive results of multiplying by 9, the right hand figure regularly decreases by 1, and the left hand figure regularly increases by 1. Thus, 9 times 2 are 18; 9 times 3 are 27; 9 times 4 are 36, &c.

36. Multiplying by 1, is taking the multiplicand *once:* thus, 4 multiplied by 1=4.

Multiplying by 2, is taking the multiplicand *twice:* thus, 2 times 4, or 4+4=8.

Multiplying by 3, is taking the multiplicand *three times:* thus, 3 times 4, or 4+4+4=12, &c. Hence,

Multiplying by any whole number, is taking the multiplicand as many times, as there are units in the multiplier.

Note.—The application of this principle to *fractional* multipliers will be illustrated under fractions.

OBS. From the definition of multiplication, it is manifest that the *product* is of the *same kind* or *denomination* as the multiplicand; for, *repeating* a number or quantity does not alter its nature. Thus, if we repeat *dollars*, they are still *dollars;* if we repeat *yards*, they are still *yards;* if we repeat an *abstract number*, that is, a number which does not express any *sensible* object, the *product* will be an *abstract number*, &c.

37. *Sign of Multiplication* (×). The *sign* of multiplication is an *oblique cross* (×), and shows that the numbers between which it is placed, are to be multiplied together. Thus the expression 5×3, signifies that 5 and 3 are to be multiplied together, and is read, "5 multiplied by 3," or simply "5 into 3."

QUEST.—36. What is it to multiply by 1? By 2? By 3? What is it to multiply by any whole number? *Obs.* Of what kind or denomination is the product? Why? 37. What is the sign of multiplication? What does it show? How is the expression 9×6. read? How 6×7=42? 38. Does it make any difference in the product, which factor is taken for the multiplier? Illustrate this principle upon the blackboard.

38. *The product of any two numbers will be the same, whichever factor is taken for the multiplier.* Thus,

If a garden contains 3 rows of trees as represented by the number of horizontal rows of stars in the margin, and each row has 5 trees as represented by the number of stars in a row, it is evident, that the whole number of trees in the garden is equal either to the number of stars in a *horizontal* row, taken *three times*, or to the number of stars in a *perpendicular* row taken *five times*; that is, equal to 5 × 3, or 3 × 5.

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* * * * *
* * * * *
* * * * *
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EXAMPLES.

39. *When the multiplier contains but* ONE *figure.*

Ex. 1. What will 3 horses cost, at 123 dollars apiece?

Analysis. — Since 1 horse costs 123 dollars, 3 horses will cost 3 times 123 dollars.

Directions.—Write the *multiplier* under the *multiplicand*; then, beginning at the *right hand*, multiply each figure of the multiplicand by the multiplier.

Operation.

123	multiplicand.
3	multiplier.
Dolls. 369	*product.*

Thus, 3 times 3 units are 9 units, or we may simply say 3 times 3 are 9; set the 9 in units' place under the figure multiplied. 3 times 2 are 6; set the 6 in tens' place. 3 times 1 are 3; set the 3 in hundreds' place.

Note.—The pupil should be required to *analyze* every example, and to give the *reasoning* in full; otherwise the operation is liable to become mere *guess-work*, and a habit is formed, which is alike *destructive* to *mental discipline* and all *substantial improvement.*

Solve the following examples in a similar manner:

	(2.)	(3.)	(4.)	(5.)
Multiplicand	34	312	2021	1110
Multiplier	2	3	4	5

	(6.)	(7.)	(8.)	(9.)
Multiplicand,	4022	6102	7110	8101
Multiplier,	3	4	5	7

10. What will 6 cows cost at 23 dollars apiece.

Suggestion.—In this example the product of the different figures of the multiplicand into the multiplier, exceeds 9; we must therefore write the *units' figure* under the figure multiplied, and carry the *tens* to the next product on the left, as in addition. Thus, beginning at the right hand as before, 6 times 3 units are 18 units, or we may simply say 6 times 3 are 18. Now it requires two figures to express 18; we therefore set the 8 under the figure multiplied, and reserving the 1, carry it to the product of the next figure, as in addition. (Art. 23.) Next, 6 times 2 are 12, and 1 (to carry) makes 13. Since there are no more figures to be multiplied, we set down the 13 in full. The product is 138 dollars. Hence,

Operation.
23 dolls.
6
Ans. 138 dollars.

40. When the multiplier contains but *one* figure.

Write the multiplier under the multiplicand, units under units, and draw a line beneath them.

Begin with the units, and multiply each figure of the multiplicand by the multiplier, setting down the result and carrying as in addition. (Art. 23.)

Multiply the following numbers together.

11. 78×4.
12. 96×5.
13. 83×3.
14. 120×7.
15. 138×6.
16. 163×5.
17. 281×8.
18. 524×6.
19. 360×7.
20. 475×4.
21. 792×5.
22. 820×8.
23. 804×7.
24. 968×9.

25. What will 175 barrels of flour cost, at 6 dollars per barrel?

26. A man bought 460 pair of boots, at 5 dollars a pair: how much did he pay for the whole?

27. What cost 196 acres of land, at 7 dollars per acre?

28. What cost 340 ploughs, at 8 dollars apiece?

29. What cost 691 hats, at 7 dollars apiece?

30. What cost 865 heifers, at 9 dollars per head?

31. What cost 968 cheeses, at 8 dollars apiece?

32. What cost 1260 sheep, at 7 dollars per head?

33. What cost 9 farms, at 2365 dollars apiece?

41. *When the multiplier contains more than* ONE *figure.*

34. A man sold 23 sleighs, at 54 dollars apiece: how much did he receive for them all?

Suggestion.—Reasoning as before, if 1 sleigh costs 54 dollars, 23 sleighs will cost 23 times as much.

Operation.

	54	Multiplicand.
	23	Multiplier.
	162	cost of 3 s.
	108	" " 20 s.
Dolls.	1242	" " 23 s.

Directions.—As it is not convenient to multiply by 23 at once, we first multiply by the 3 units, then by the 2 tens, and add the two results together. Thus, 3 times 4 are 12, set the 2 under the figure 3, by which we are multiplying, and carry the 1 as above. 3 times 5 are 15, and 1 (to carry) makes 16. Next, we multiply by the 2 tens thus: 20 times 4 units are 80 units or 8 tens; or we may simply say 2 times 4 are 8. Set the 8 under the figure 2 by which we are multiplying, that is, in tens' place, because it is tens. 2 times 5 are 10. Finally, adding these two products together as they stand, units to units, tens to tens, &c., we have 1242 dollars, which is the whole product required.

Note.—When the multiplier contains more than one figure, the several products of the multiplicand into the separate figures of the multiplier, are called *partial* products.

35. Multiply 45 by 36, and prove the operation.

Beginning at the right hand, we proceed thus: 6 times 5 are 30; set the 0 under the figure by which we are multiplying; 6 times 4 are 24 and 3 (to carry) are 27, &c.

Operation.

```
  45 Multiplicand.
  36 Multiplier.
 ---
 270
135
----
1620 Prod.
```

PROOF.—We multiply the *multiplier* by the *multiplicand*, and since the result thus obtained is the *same* as the *product* above, the work is right.

Proof.

```
  36
  45
 ---
 180
144
----
1620 Prod.
```

36. What is the product of 234 multiplied by 165?

Suggestion.—Proceed in the same manner as when the multiplier contains but *two* figures, remembering to place the *right hand figure of each partial product directly under the figure by which you multiply.*

Operation.

```
  234
  165
 ----
 1170
1404
234
-----
38610 Ans
```

37. What is the product of 326 multiplied by 205?

Suggestion.—Since multiplying by a cipher produces nothing, in the operation we omit the 0 in the multiplier. Thus, having multiplied by the 5 units, we next multiply by the 2 hundreds, and place the first figure of this partial product under the figure by which we are multiplying.

Operation.

```
  326
  205
 ----
 1630
652
-----
66830 Ans.
```

42. From the preceding illustrations and principles we derive the following

GENERAL RULE FOR MULTIPLICATION.

I. *Write the multiplier under the multiplicand, units under units, tens under tens, &c.*

II. *When the multiplier contains but* ONE *figure, begin with the units, and multiply each figure of the multiplicand by the multiplier, setting down the result and carrying as in addition.* (Art. 23.)

III. *If the multiplier contains* MORE *than one figure, multiply each figure of the multiplicand by each figure of the multiplier separately, and write the first figure of each partial product under the figure by which you are multiplying.*

Finally, add the several partial products together, and the sum will be the whole product, or answer required.

43. PROOF.—*Multiply the multiplier by the multiplicand, and if the second result is the same as the first, the work is right.*

OBS. 1. It is immaterial as to the result which of the factors is taken for the multiplier. (Art. 38.) But it is more convenient and therefore customary to place the *larger* number for the multiplicand and the *smaller* for the multiplier. Thus, it is easier to multiply 254672381 by 7, than it is to multiply 7 by 254672381, but the product will be the same.

2. Multiplication may also be proved by *division*, and by *casting out the nines;* but neither of these methods can be explained here without anticipating principles belonging to division, with which the learner is supposed as yet to be unacquainted.

QUEST.—42. How do you write numbers for multiplication? When the multiplier contains but one figure, how do you proceed? When the multiplier contains more than one figure, how proceed? 41. *Note.* What is meant by partial products? What is to be done with the partial products? 43. How is multiplication proved?

EXAMPLES FOR PRACTICE.

1. Multiply 63 by 4.
2. Multiply 78 by 5.
3. Multiply 81 by 7.
4. Multiply 97 by 6.
5. Multiply 120 by 7.
6. Multiply 231 by 5.
7. Multiply 446 by 8.
8. Multiply 307 by 9.
9. Multiply 560 by 7.
10. Multiply 46 by 10.
11. Multiply 52 by 11.
12. Multiply 68 by 12.
13. Multiply 84 by 13.
14. Multiply 78 by 15.
15. Multiply 95 by 23.
16. Multiply 129 by 35.
17. Multiply 293 by 42.
18. Multiply 461 by 55.

19. If 1 barrel of flour costs 9 dollars, how much will 38 barrels cost?

20. If 1 apple-tree bears 14 bushels of apples, how many bushels will 24 trees bear?

21. In 1 foot there are 12 inches: how many inches are there in 28 feet?

22. In 1 pound there are 20 shillings: how many shillings are there in 31 pounds?

23. What will 17 cows cost, at 23 dollars apiece?

24. What will 25 tons of hay cost, at 19 dollars per ton?

25. What will 37 sleighs cost, at 43 dollars apiece?

26. What will a drove of 150 sheep come to, at 13 shillings per head?

27. What cost 105 acres of land, at 15 dollars per acre?

28. How much will 135 yards of cloth come to, at 18 shillings per yard?

29. In 1 pound there are 16 ounces: how many ounces are there in 246 pounds?

30. A drover sold 283 oxen, at 38 dollars per head: how much did he receive for them?

31. If you walk 22 miles per day, how far will you walk in 305 days?

32. In one day there are 24 hours: how many hours are there in 365 days?

33. Multiply 2345 by 175. 34. Multiply 6207 by 235.
35. Mult. 10645 by 1262. 36. Mult. 25271 by 2579.
37. Mult. 162537 by 21268. 38. Mult. 425231 by 30765.

27.* What will 15 caps cost at 18 shillings a piece?

Analysis.—The multiplier 15 is a composite number, the factors of which are 5 and 3. That is, $15=5\times3$. We first multiply the multiplicand by the factor 5, and this product by 3. The last product is the answer. Hence,

Operation.

```
 18
  5
 --
 90
  3
 --
270 shill.
```

44. To multiply by a composite number.

Multiply the multiplicand by one of the factors of the multiplier, and this product by another; and so on till you have multiplied by all the factors. The last product will be the answer.

OBS. 1.—A *composite* number is one which is produced by *multiplying two or more factors together.* Thus, $14=7\times2$; $55=11\times5$.

2. Some numbers may be resolved into *more* than *two* factors; and also into *different sets* of factors. Thus, $8=2\times2\times2=4\times2$.

28. What are the factors of 4, 6, 9, 10, 21, 35, 77?
29. Name the different sets of factors of 12, 16, 18, 20, 27, 32, 63.
30. Name the different sets of factors of 24, 30, 36, 48, 60, 72, 100.
31. Mult. 248 by 35 using the factors of 35. *Ans.* 8680.
32. Multiply 173 by 28. *Ans.* 4844.
33. Multiply 504 by 63. *Ans.* 31752.
34. Multiply 721 by 45. *Ans.* 32445.
35. Multiply 1048 by 56. *Ans.* 58688.
36. Multiply 2347 by 72. *Ans.* 188984.
37. Multiply 4630 by 96. *Ans.* 444480.
38. Multiply 25205 by 77. *Ans.* 1940785.
39. Multiply 36042 by 108. *Ans.* 3892536.

QUEST.—44. How do you multiply by a composite number? *Obs.* What is a composite number? 45. How multiply by 10, 100, 1000, &c.

45. To multiply by 10, 100, 1000, &c.

Annex as many ciphers to the multiplicand as there are ciphers in the multiplier, and the number thus formed will be the product required.

40. What will 10 dresses cost, at 18 dollars apiece?
Ans. 180 dolls.

41. 26×100.	46. 469×10000.
42. 37×100.	47. 523×100000.
43. 51×1000.	48. 681×1000000.
44. 226×1000.	49. 85612×10000.
45. 341×1000.	50. 960305×100000.

51. What will 20 wagons cost, at 67 dollars apiece?

Operation.
```
       67
       20
Ans. 1340 dollars.
```

Suggestion. — Since multiplying by *ciphers* produces *ciphers*, we omit multiplying by the 0, and placing the significant figure 2 under the right hand figure of the multiplicand, multiply by it in the usual way, and annex a cipher to the product. The answer is 1340 dollars. Hence,

46. When there are ciphers on the right hand of the multiplier.

Multiply the multiplicand by the significant figures of the multiplier, and to this product annex as many ciphers, as are found on the right hand of the multiplier.

(52.)	(53.)	(54.)	(55.)
85	97	123	234
200	3000	40000	50000

(56.)	(57.)	(58.)	(59.)
261	329	462	571
130	2400	35000	460000

QUEST.—46. When there are ciphers on the right of the multiplicand, how do you proceed?

60. In one hour there are 60 minutes: how many minutes are there in 125 hours?

61. What will 300 barrels of flour cost at 8 dollars per barrel?

62. What cost 400 yds. of cloth, at 17 shills. per yd.?

63. If the expenses of 1 man are 135 dollars per month, how much will be the expenses of 200 men?

64. If 1500 men can build a fort in 235 days, how long will it take one man to build it?

47. When there are ciphers on the right of the multiplicand.

Multiply the significant figures of the multiplicand by the multiplier, and to the product annex as many ciphers, as are found on the right of the multiplicand.

65. What will 43 building lots cost, at 3500 dollars a lot?

Having placed the multiplier under the significant figures of the multiplicand, multiply by it as usual, and to the product thus produced, annex *two ciphers,* because there are two ciphers on the right of the multiplicand.

Operation.

```
     3500
     43
    ----
    105
   140
   ------
Ans. 150500 dolls.
```

(66.)	(67.)	(68.)	(69.)
1300	2400	21000	25000
15	17	24	32

70. What is the product of 132000 multiplied by 25?

71. What is the product of 430000 multiplied by 34?

72. What is the product of 1520000 multiplied by 43?

73. What is the product of 2010000 multiplied by 52?

74. What is the product of 3004000 multiplied by 61?

QUEST.—47. When there are ciphers on the right of the multiplicand, how do you proceed?

48. When the multiplier and multiplicand *both* have ciphers on the right.

Multiply the significant figures of the multiplicand by the significant figures of the multiplier, and to this product annex as many ciphers, as are found on the right of both factors.

75. Multiply 16000 by 3200.

Having placed the significant figures of the multiplier under those of the multiplicand, we multiply by them as usual, and to the product thus obtained, annex *five ciphers*, because there are five ciphers on the right of both factors.

Operation.

```
      16000
       3200
    -------
       32
      48
    -------
Ans. 51200000
```

Solve the following examples:

76. 2100×200.
77. 3400×130.
78. 12000×210.
79. 25000×2600.
80. 38000×19000.
81. 500000×42000.
82. 2800000×26000.
83. 140 yards×16000.
84. 1000 miles×140.
85. 20 dollars×35000.
86. 120 dollars×4200.
87. 75000 dolls.×365.
88. 867 pounds×424.
89. 6830 feet×562.
90. 6726 rods×627.
91. 7207 galls.×807.
92. 25268 pence×4005.
93. 36074 tons×4060.
94. 376245×3164.
95. 703268×5346.
96. 600400×7034.
97. 864325×6728.
98. 432467×30005.
99. 4567832×27324.
100. 680539×80406.
101. 7563057×62043.

102. Multiply seventy-three thousand and seven by twenty thousand and seven hundred.

103. Multiply six hundred thousand, two hundred and three by seventy thousand and seventeen.

QUEST.—48. When there are ciphers on the right of both the multiplier and multiplicand, how proceed?

SECTION V.

DIVISION.

ART. **49.** Ex. 1. How many lead pencils, at 2 cents apiece, can I buy for 10 cents?

Solution.—Since 2 cents will buy 1 pencil, 10 cents will buy as many pencils, as 2 cents are contained times in 10 cents; and 2 cents are contained in 10 cents, 5 times. I can therefore buy 5 pencils.

2. A father bought 12 pears, which he divided equally among his 3 children: how many pears did each receive?

Solution.—Reasoning in a similar manner as above, it is plain that each child will receive 1 pear, as often as 3 is contained in 12; that is, each must receive as many pears, as 3 is contained times in 12. Now 3 is contained in 12, 4 times. Each child therefore received 4 pears.

OBS. The object of the *first* example is to find how many times one given number is contained in another. The object of the *second* is to divide a given number into several *equal parts*, and to ascertain the *value* of these parts. The operation by which they are solved is precisely the *same*, and is called *Division.* Hence,

50. DIVISION *is the process of finding how many times one given number is contained in another.*

The number to be *divided*, is called the *dividend.*

The number by which we *divide*, is called the *divisor.*

The *answer*, or number *obtained* by division, is called the *quotient*, and shows how *many times* the divisor is contained in the dividend.

QUEST.—50. What is division? What is the number to be divided, called? The number by which we divide? What is the answer called? What does the quotient show?

Note.—The term *quotient* is derived from the Latin word *quoties*, which signifies *how often*, or *how many times.*

51. The number which is sometimes *left* after division, is called the *remainder.* Thus, when we say 4 is contained in 21, 5 times and 1 over, 4 is the divisor, 21 the dividend, 5 the quotient, and 1 the remainder.

OBS. 1. The remainder is always *less* than the divisor; for if it were equal to, or greater than the divisor, the divisor could be contained *once more* in the dividend.

2. The remainder is also of the same denomination as the dividend; for it is a part of it.

52. *Sign of Division* (÷). The *sign* of Division is a *horizontal line* between two dots (÷), and shows that the number *before* it, is to be divided by the number *after* it. Thus, the expression 24÷6, signifies that 24 is to be divided by 6.

Division is also expressed by placing the divisor *under* the dividend with a short line between them. Thus the expression $\frac{35}{7}$, shows that 35 is to be divided by 7, and is equivalent to 35÷7.

53. It will be perceived that division is *similar* in principle to subtraction, and may be performed by it. For instance, to find how many times 3 is contained in 12, subtract 3 (the divisor) continually from 12 (the dividend) until the latter is exhausted; then counting these repeated subtractions, we shall have the true quotient. Thus, 3 from 12 leaves 9; 3 from 9 leaves 6; 3 from 6 leaves 3; 3 from 3 leaves 0. Now, by counting, we find that 3 has

---

QUEST.—51. What is the number called which is sometimes left after division? When we say 4 is in 21, 5 times and 1 over, what is the 4 called? The 21? The 5? The 1? *Obs.* Is the remainder greater or less than the divisor? Why? Of what denomination is it? Why? 52. What is the sign of division? What does it show? In what other way is division expressed?

been taken from 12, 4 times; consequently 3 is contained in 12, 4 times. Hence,

Division is sometimes defined to be a short way of performing repeated subtractions of the same number.

OBS. 1. It will also be observed that division is the *reverse* of multiplication. Multiplication is the *repeated addition* of the same number; division is the *repeated subtraction* of the same number. The *product* of the one answers to the *dividend* of the other: but the latter is always *given*, while the former is *required.*

2. When the dividend denotes things of *one kind, or denomination only,* the operation is called *Simple Division.*

DIVISION TABLE.

1 is in		2 is in		3 is in		4 is in		5 is in	
1,	once.	2,	once.	3,	once.	4,	once.	5,	once.
2,	2	4,	2	6,	2	8,	2	10,	2
3,	3	6,	3	9,	3	12,	3	15,	3
4,	4	8,	4	12,	4	16,	4	20,	4
5,	5	10,	5	15,	5	20,	5	25,	5
6,	6	12,	6	18,	6	24,	6	30,	6
7,	7	14,	7	21,	7	28,	7	35,	7
8,	8	16,	8	24,	8	32,	8	40,	8
9,	9	18,	9	27,	9	36,	9	45,	9
10,	10	20,	10	30,	10	40,	10	50,	10
6 is in		7 is in		8 is in		9 is in		10 is in	
6,	once.	7,	once.	8,	once.	9,	once.	10,	once.
12,	2	14,	2	16,	2	18,	2	20,	2
18,	3	21,	3	24,	3	27,	3	30,	3
24,	4	28,	4	32,	4	36,	4	40,	4
30,	5	35,	5	40,	5	45,	5	50,	5
36,	6	42,	6	48,	6	54,	6	60,	6
42,	7	49,	7	56,	7	63,	7	70,	7
48,	8	56,	8	64,	8	72,	8	80,	8
54,	9	63,	9	72,	9	81,	9	90,	9
60,	10	70,	10	80,	10	90,	10	100,	10

QUEST.—*Obs.* When the dividend denotes things of one denomination only what is the operation called?

SHORT DIVISION.

ART. **54.** Ex. 1. How many yards of cloth, at 2 dollars per yard, can I buy for 246 dollars?

Analysis.—Since 2 dollars will buy 1 yard, 246 dollars will buy as many yards, as 2 dollars are contained times in 246 dollars.

Operation.

Divisor.	Dividend.
2)	246
Quot.	123 yds.

Directions.—Write the *divisor* on the left of the *dividend* with a curve line between them; then, beginning at the left hand, proceed thus: 2 is contained in 2, *once.* As the 2 in the dividend denotes hundreds, the 1 must be a hundred; we therefore write it in hundreds' place under the figure divided. 2 is contained in 4, 2 times; and since the 4 denotes tens, the 2 must also be tens, and must be written in tens' place. 2 is in 6, 3 times. The 6 is units; hence the 3 must be units, and we write it in units' place. The answer is 123 yards.

Solve the following examples in a similar manner:

2. Divide 42 by 2.
3. Divide 69 by 3.
4. Divide 488 by 4.
5. Divide 555 by 5.
6. Divide 684 by 2.
7. Divide 4488 by 4.
8. Divide 3963 by 3.
9. Divide 6666 by 6.

55. When the divisor is not contained in the *first* figure of the dividend, we must find how many times it is contained in the *first two* figures.

10. At 2 dollars a bushel, how much wheat can be bought for 124 dollars?

Operation.

2)124
Ans. 62 bu.

Since the divisor 2, is not contained in the first figure of the dividend, we find how many times it is contained in the *first two* figures. Thus 2 is in 12, 6 times; set the 6 under the 2. Next, 2 is in 4, 2 times. The answer is 62 bushels.

11. Divide 142 by 2.
12. Divide 129 by 3.
13. Divide 1648 by 4.
14. Divide 2877 by 7.

56. After dividing any figure of the dividend, if there is a *remainder*, prefix it mentally to the next figure of the dividend, and then divide this number as before.

Note.—To *prefix* means to place *before*, or at the *left hand.*

15. A man bought 42 peaches, which he divided equally among his 3 children: how many did he give to each?

When we divide 4 by 3, there is 1 remainder. This we prefix mentally to the next figure of the dividend. We then say, 3 is in 12, 4 times.

Operation.

$$3\underline{)42}$$
$$14 \text{ } Ans.$$

16. Divide 56 by 4.
17. Divide 125 by 5.
18. Divide 456 by 6.
19. Divide 3648 by 8.

57. Having obtained the *first quotient figure*, if the divisor is *not contained* in any figure of the dividend, place a *cipher* in the quotient, and *prefix* this figure to the next one of the dividend, as if it were a remainder.

20. If hats are 2 dollars apiece, how many can be bought for 216 dollars?

As the divisor is not contained in 1, the second figure of the dividend, we put a 0 in the quotient, and prefix the 1 to the 6 as directed above. Now 2 is in 16, 8 times.

Operation.

$$2\underline{)216}$$
$$Ans. \text{ } 108 \text{ hats.}$$

21. Divide 2545 by 5.
22. Divide 3604 by 4.
23. Divide 6402 by 6.
24. Divide 4024 by 8.

25. A man divided 17 loaves of bread equally between 2 poor persons: how many did he give to each?

Suggestion.—Reasoning as before, he gave each as many loaves as 2 is contained times in 17.

Thus, 2 is contained in 17, 8 times and 1 over; that is, after giving them 8 loaves apiece, there is one loaf left which is not divided. Now 2 is not contained in 1; hence the division must be represented by writing the 2 under the 1, thus $\frac{1}{2}$, (Art. 52,) which must be annexed to the 8. The true quotient, is $8\frac{1}{2}$. He therefore gave *eight* and *a half* loaves to each. Hence,

Operation.

$$2)\underline{17}$$

Quot. 8–1 remainder.

Ans. $8\frac{1}{2}$ loaves.

58. *When there is a remainder after dividing the last figure of the dividend, it should always be written over the divisor and annexed to the quotient.*

Note.—To *annex* means to place *after*, or at the *right hand.*

59. *When the process of dividing is carried on in the mind, and the quotient only is set down, the operation is called* SHORT DIVISION.

60. From the preceding illustrations and principles, we derive the following

RULE FOR SHORT DIVISION.

I. *Write the divisor on the left of the dividend, with a curve line between them.*

Beginning at the left hand, divide each figure of the dividend by the divisor, and place each quotient figure under the figure divided.

II. *When there is a remainder after dividing any figure, prefix it to the next figure of the dividend and divide this number as before. If the divisor is not contained in*

QUEST.—59. What is Short Division? 60. How do you write numbers for short division? Where begin to divide? Where place each quotient figure? When there is a remainder after dividing a figure of the dividend, what must be done with it? If the divisor is not contained in a figure of the dividend, how proceed? When there is a remainder, after dividing the last figure of the dividend, what must be done with it?

any figure of the dividend, place a cipher in the quotient, and prefix this figure to the next one of the dividend, as if it were a remainder. (Arts. 56, 57.)

III. *When there is a remainder after dividing the last figure, write it over the divisor and annex it to the quotient.*

61. Proof.—*Multiply the divisor by the quotient, to the product add the remainder, and if the sum is equal to the dividend, the work is right.*

Obs. Division may also be proved by subtracting the remainder, if any, from the dividend, then dividing the result by the quotient.

EXAMPLES FOR PRACTICE.

1. Divide 426 by 3.
2. Divide 506 by 5.
3. Divide 304 by 4.
4. Divide 450 by 6.
5. Divide 720 by 7.
6. Divide 510 by 9.
7. Divide 604 by 5.
8. Divide 760 by 8.
9. Divide 813 by 7.
10. Divide 3640 by 5.
11. Divide 6210 by 4.
12. Divide 7031 by 7.
13. Divide 2403 by 6.
14. Divide 8131 by 9.
15. Divide 7384 by 8.
16. Divide 8560 by 7.
17. Divide 7000 by 8.
18. Divide 9100 by 9.

19. How many pair of shoes, at 2 dollars a pair, can you buy for 126 dollars?

20. How many hats, at 4 dollars apiece, can be bought for 168 dollars?

21. A man bought 144 marbles which he divided equally among his 6 children: how many did each receive?

22. A man distributed 360 cents to a company of poor children, giving 8 cents to each: how many children were there in the company?

23. How many yards of silk, at 6 shillings per yard, can I buy for 450 shillings?

Quest.—61. How is division proved? *Obs.* What other way of proving division is mentioned?

24. A man having 600 dollars wished to lay it out in flour, at 7 dollars a barrel: how many whole barrels could he buy, and how many dollars would he have left?

25. If you read 9 pages each day, how long will it take you to read a book through which has 828 pages?

26. If I pay 8 dollars a yard for broadcloth, how many yards can I buy for 1265 dollars?

27. If a stage coach goes at the rate of 8 miles per hour, how long will it be in going 1560 miles?

28. If a ship sails 9 miles an hour, how long will it be in sailing to Liverpool, a distance of 3000 miles?

LONG DIVISION.

ART. **62.** Ex. 1. A man having 156 dollars laid it out in sheep at 2 dollars apiece: how many did he buy?

Analysis.—Reasoning as before, since 2 dollars will buy 1 sheep, 156 dollars will buy as many sheep as 2 dollars are contained times in 156 dollars.

Operation.

```
Divis. Divid. Quot.
  2)   156   (73
       14·
       ---
        16
        16
        --
```

Directions.—Having written the divisor on the left of the dividend as in short division, proceed in the following manner:

First. Find how many times the divisor (2) is contained in (15) the first two figures of the dividend, and place the quotient figure (7) on the right of the dividend with a curve line between them. *Second.* Multiply the divisor by the quotient figure, (2 times 7 are 14,) and write the product (14) under the figures divided. *Third.* Subtract the product from the figures divided. (The remainder is 1.) *Fourth.* Bringing down the next figure of the dividend, and placing it on the right of the remainder we have 16. Now 2 is contained in 16, 8 times; place the 8 on the right hand of the last quotient figure, and multiplying the divisor by it, (8 times 2 are 16,) set the product under the figures divided, and subtract as before. Therefore 156 dollars will buy 78 sheep, at 2 dollars apiece.

63. *When the result of each step in the operation is set down, the process of dividing is called* LONG DIVISION.

It is the *same* in *principle* as Short Division. The only difference between them is, that in *Long Division* the *result* of each step in the operation is written down, while in *Short Division* we carry on the whole process in the mind, simply writing down the *quotient.*

Note.—To prevent mistakes, it is advisable to put a dot under each figure of the dividend, when it is *brought down.*

Solve the following examples by Long Division:

2. Divide 195 by 3. *Ans.* 65.
3. Divide 256 by 2.
4. Divide 1456 by 4.
5. Divide 5477 by 3.
6. Divide 2665 by 5.
7. Divide 4392 by 6.
8. Divide 6517 by 7.

OBS. When the divisor is not contained in the first *two* figures of the dividend, find how many times it is contained in the first *three*, or the *fewest* figures which will contain it, and proceed as before.

9. How many times is 13 contained in 10519?

Operation.

```
13)10519(809 2/13 Ans.
   104
   ---
     119
     117
     ---
       2 rem.
```

Thus, 13 is contained in 105, 8 times; set the 8 in the quotient then multiplying and subtracting, the remainder is 1. Bringing down the next figure we have 11 to be divided by 13. But 13 is not contained in 11; therefore we put a *cipher* in the quotient, and bring down the next figure. (Art. 57.) Then 13 is contained in 119,

QUEST.—63. What is long division? What is the difference between long and short division?

9 times. Set the 9 in the quotient, multiply and subtract, and the remainder is 2. Write the 2 over the divisor, and *annex* it to the quotient. (Art. 58.)

64. After the first quotient figure is obtained, for *each figure of the dividend which is brought down,* either a *significant figure* or a *cipher* must be put in the *quotient.*

Solve the following examples in a similar manner:

10. Divide 15425 by 11. *Ans.* $1402\frac{3}{11}$.

11. Divide 31237 by 15. *Ans.* $2082\frac{7}{15}$.

65. From the preceding illustrations and principles, we derive the following

RULE FOR LONG DIVISION.

I. *Beginning on the left of the dividend, find how many times the divisor is contained in the fewest figures that will contain it, and place the quotient figure on the right of the dividend with a curve line between them.*

II. *Multiply the divisor by this figure and subtract the product from the figures divided; to the right of the remainder bring down the next figure of the dividend, and divide this number as before. Proceed in this manner till all the figures of the dividend are divided.*

III. *When there is a remainder after dividing the last figure, write it over the divisor, and annex it to the quotient, as in short division.*

OBS. 1. *Long Division* is *proved* in the same manner as *Short Division.*

2. When the divisor contains but *one* figure, the operation by *Short Division* is the most expeditious, and should therefore be practiced; but when the divisor contains *two* or *more* figures, it will generally be the most convenient to divide by *Long Division.*

QUEST.—65. How do you divide in long division? Where place the quotient? After obtaining the first quotient figure, how proceed? When there is a remainder after dividing the last figure of the dividend what must be done with it? *Obs.* How is long division proved? When should short division be used? When long division?

EXAMPLES FOR PRACTICE.

1. Divide 369 by 8.
2. Divide 435 by 9.
3. Divide 564 by 7.
4. Divide 403 by 10.
5. Divide 641 by 11.
6. Divide 576 by 12.
7. Divide 274 by 13.
8. Divide 449 by 14.
9. Divide 617 by 15.
10. Divide 675 by 25.
11. Divide 742 by 31.
12. Divide 798 by 37.
13. Divide 334 by 42.
14. Divide 960 by 48.
15. Divide 1142 by 53.
16. Divide 2187 by 67.
17. Divide 3400 by 75.
18. Divide 4826 by 84.

19. How many caps, at 7 shillings apiece, can I buy for 168 shillings?

20. How many pair of boots, at 5 dollars a pair, can be bought for 175 dollars?

21. A man laid out 252 dollars in beef, at 9 dollars a barrel: how many barrels did he buy?

22. In 12 pence there is 1 shilling: how many shillings are there in 198 pence?

23. In 20 shillings there is 1 pound: how many pounds are there in 215 shillings?

24. In 16 ounces there is 1 pound: how many pounds are there in 268 ounces?

25. How many trunks, at 15 shillings apiece, can be bought for 255 shillings?

26. If 27 pounds of flour will last a family a week, how long will 810 pounds last them?

27. How many yards of broadcloth, at 23 shillings per yard, can be bought for 756 shillings?

28. If it takes 18 yards of silk to make a dress, how many dresses can be made from 1350 yards?

29. How many sheep, at 19 shillings per head, can be bought for 1539 shillings?

30. A farmer having 1840 dollars, laid it out in land, at 25 dollars per acre: how many acres did he buy?

31. A man wishes to invest 2562 dollars in Railroad stock: how many shares can he buy, at 42 dollars per share?

32. In 1 year there are 52 weeks: how many years are there in 1640 weeks?

33. In one hogshead there are 63 gallons: how many hogsheads are there in 3065 gallons?

34. If a man can earn 75 dollars in a month, how many months will it take him to earn 3280 dollars?

35. If a young man's expenses are 83 dollars a month, how long will 4265 dollars support him?

36. A man bought a drove of 95 horses for 4750 dollars: how much did he give apiece?

37. If a man should spend 16 dollars a month, how long will it take him to spend 172 dollars?

38. A garrison having 2790 pounds of meat, wished to have it last them 31 days: how many pounds could they eat per day?

39. How many times is 54 contained in 3241, and how many over?

40. How many times is 68 contained in 7230, and how many over?

41. How many times is 39 contained in 1042, and how many over?

42. How many times is 47 contained in 2002, and how many over?

43. What is the quotient of 1704 divided by 56?

44. What is the quotient of 2040 divided by 60?

45. What is the quotient of 2600 divided by 49?

46. What is the quotient of 2847 divided by 81?

47. Divide 1926 by 75.

48. Divide 2230 by 85.

49. Divide 6243 by 96.

50. Divide 8461 by 99.

51. Divide 9423 by 105.

52. Divide 13263 by 112.

53. Divide 26850 by 123.

54. Divide 48451 by 224.

46.* How many balls at 15 pence apiece, can be bought for 146 pence?

Analysis.—This divisor being composite, whose factors are 3 and 5, we first divide by 3, and this quotient by 5.

Operation.

3)149
5)49, 2 rem.
9, 4 rem.
Ans. 9 and 14 r.

To find the *true remainder*, multiply each remainder by all the divisors preceding the division from which it arose, and to the sum of the products add the first remainder; the result will be the true remainder. In this example the only preceding divisor is 3; now the last remainder $4\times3=12$, and $12+2=14$ the true remainder. Hence,

66. To divide by a *composite* number.

Divide the dividend by one of the factors, then divide the quotient thus obtained by another factor, and so on till all the factors are employed. The last quotient will be the answer.

47. Divide 231 by 21, using its factors. *Ans.* 11.
48. Divide 195 by 16, using its factors. *Ans.* 12 and 3 r.
49. Divide 256 by 24, using its factors. *Ans.* 10 and 16 r.
50. Divide 365 by 48, using its factors. *Ans.* 7 and 29 r.
51. Divide 410 by 45, using its factors. *Ans.* 9 and 5 r.
52. Divide 217 by 63, using its factors. *Ans.* 3 and 28 r.
53. Divide 561 by 56, using its factors. *Ans.* 10 and 1 r.
54. Divide 893 by 72, using its factors. *Ans.* 12 and 29 r.
55. Divide 1275 by 96, using its factors. *Ans.* 13 and 2 r.

67. To divide by 10, 100, 1000, &c.

Cut off as many figures from the right hand of the dividend as there are ciphers in the divisor. The remaining figures of the dividend will be the quotient, and those cut off the remainder.

56. Divide 1325 by 10. 57. Divide 4626 by 100.
58. Divide 5633 by 1000. 59. Divide 8465 by 1000.
60. Divide 26244 by 1000. 61. Divide 136056 by 10000.

62. Divide 2443667 by 100000.

63. Divide 23454631 by 1000000.

68. When there are ciphers on the right hand of the divisor.

Cut off the ciphers from the divisor; also cut off as many figures from the right of the dividend. Then divide the remaining figures of the dividend by the remaining figures of the divisor, and the result will be the quotient.

Finally, annex the figures cut off from the dividend to the remainder, and the number thus formed will be the true remainder.

64. At 200 dollars apiece, how many carriages can be bought for 4765 dollars?

Operation.

2|00)47|65

Ans. 23—165 rem.

Having cut off the two ciphers on the right of the divisor, and two figures on the right of the dividend, we divide the 47 by 2 in the usual way.

65. Divide 2658 by 20.

Ans. 132 and 18 rem., or $132\frac{18}{20}$.

66. 3642 by 30.	67. 6493 by 200.
68. 76235 by 1400.	69. 82634 by 1600.
70. 93600 by 2000.	71. 14245 by 3000.
72. 23148 by 1200.	73. 42061 by 1500.
74. 50382 by 1800.	75. 88317 by 2100.
76. 894000 by 2500.	77. 9203010 by 3100.
78. 7450000 by 420000.	79. 9000000 by 300000.
80. 348676÷235.	81. 467342÷341.
82. 762005÷401.	83. 506725÷603.
84. 607507÷1623.	85. 736241÷2764.
86. 4367238÷2367.	87. 6203451÷3827.
88. 8230732÷3478.	89. 8235762÷42316.
90. 93670858÷67213.	91. 98765421÷84327.

QUEST.—68. When there are ciphers on the right of the divisor, how proceed? What is to be done with figures cut off from the dividend?

ARITHMETICAL TERMS.

71. Numbers are divided into *abstract* and *concrete*.

1. *Abstract* numbers are numbers used without application to any object; as *two*, *three*, *four*, *five*, &c.

2. *Concrete* numbers are numbers applied to some particular object; as *two* peaches, *three* pounds, &c.

3. Numbers are also divided into *prime* and *composite*.

4. A *prime* number is one which *cannot* be produced by multiplying any two or more numbers together; or which *cannot* be exactly divided by any *whole* number, except a *unit* and *itself*. Thus, 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, &c., are prime numbers.

OBS. 1. The *least* divisor of every number is a prime number.

2. One number is said to be *prime to another*, when a *unit* is the only number by which both can be divided without a remainder.

3. The number of prime numbers is unlimited. All below *fifty* are given above. The pupil can easily point out others.

5. A *composite* number is one which is produced by multiplying two or more *factors* together. Thus, $12 = 4 \times 3$.

6. An *even* number is one which can be divided by 2 without a remainder; as, 4, 6, 8, 10.

7. An *odd* number is one which cannot be divided by 2 without a remainder; as, 1, 3, 5, 7, 9, 15.

8. One number is a *measure* of another, when the *former* will divide the *latter*, without a remainder. Thus 2 is a measure of 4; 3 is a measure of 6.

9. A *common measure* is a number, which will divide *two* or *more* numbers, without a remainder. Thus, 2 is a common measure of 4, 6, and 8.

10. The *aliquot parts* of a number are the parts by which it can be divided without a remainder. Thus, 3 and 7 are aliquot parts of 21.

QUEST.—71. What are abstract numbers? Concrete? Prime? Composite? An even number? An odd number? A common measure?

GREATEST COMMON DIVISOR.

72. A *Common Divisor* is a number which will *divide two* or *more* numbers without a remainder. Thus, 2 is a common divisor of 4, 6, 8, 12, 16.

73. The *Greatest Common Divisor* of two or more numbers, is the *greatest* number which will divide each of them without a remainder. Thus, 6 is the greatest common divisor of 12, 18, and 24.

1. What is the greatest common divisor of 30 and 42?

Suggestion.—Dividing 42 by 30, the remainder is 12; then dividing 30 (the preceding divisor) by 12 (the last remainder) the remainder is 6; finally, dividing 12 (the preceding divisor) by 6 (the last remainder) nothing remains; consequently 6, the last divisor, is the greatest common divisor. Hence,

Operation.

```
30)42(1
   30
   --
   12)30(2
      24
      --
       6)12(2
         12
         --
```

74. To find the *greatest common* divisor of *two* numbers.

Divide the greater number by the less; then divide the preceding divisor by the last remainder, and so on, till nothing remains. The last divisor will be the greatest common divisor.

2. What is the greatest com. divisor of 56 and 140?
3. What is the greatest com. divisor of 116 and 203?
4. Find the greatest common divisor of 148 and 185.
5. Find the greatest common divisor of 237 and 395.
6. What is the greatest com. divisor of 122 and 427?

75. To find the greatest common divisor of *more* than *two* numbers?

First find the greatest common divisor of any two of

QUEST.—What are aliquot parts of a number? 72. A common divisor? 73. The greatest common divisor of two or more numbers? 74. How find the greatest common divisor of two numbers? 75. Of more than two?

the given numbers; then, that of the common divisor thus obtained and of another number, and so on through all the given numbers. The last common divisor found, will be the one required.

7. Find the greatest com. divisor of 45, 63 and 108.
8. Find the greatest com. divisor of 32, 48 and 200.
9. Find the greatest com. divisor of 256, 372 and 522.

LEAST COMMON MULTIPLE.

76. A *multiple* is a number which can be *divided* by another number *without a remainder.* Thus, 4 is a multiple of 2; 10 is a multiple of 5.

77. A *common multiple* is a number which can be *divided* by *two* or *more numbers* without a remainder. Thus, 12 is a common multiple of 2, 3, and 4.

78. The *continued product* of two or more numbers is always a common multiple of those numbers.

79. The *least common multiple* of two or more numbers, is the *least number which can be divided by each of them without a remainder.* Thus, 12 is the least common multiple of 4 and 6.

10. Find the least common multiple of 6, 10, and 12.

Operation.

```
2)6''10''12
  ---------
3)3'' 5'' 6
  ---------
  1'' 5'' 2
2 × 3 × 5 × 2 = 60
```

Suggestion.—Write the given numbers in a line, and, dividing by 2 the smallest number that will divide any two of them without a remainder, set the quotients 3, 5, and 6 in a line below. Next divide this line by 3 and set the quotients and undivided number 5 in a line as before. Finally, multiply all the divisors into the quo-

QUEST.—76. What is a multiple? 77. A common multiple? 79. What is the least common multiple of two or more numbers?

tients and undivided numbers in the last line, and the product is the answer required. Hence,

80. To find the least common multiple of two or more numbers.

I. *Write the given numbers in a horizontal line, and divide by the smallest number which will divide any two or more of them without a remainder, setting the quotients and undivided numbers in a line below.*

II. *Divide this line and set down the results as before; thus continue the operation till there are no two numbers which can be exactly divided by any number greater than* 1.

III. *Finally, multiply all the divisors and numbers in the last line together, and the product will be the least common multiple.*

11. Find the least common multiple of 16 and 20.
12. Find the least common multiple of 14, 21, and 28.
13. Find the least common multiple of 18, 5, 12, 10.
14. Find the least common multiple of 16, 15, 8, 36.
15. Find the least common multiple of 12, 18, 6, 8.
16. Find the least common multiple of 20, 32, 50, 35.
17. Find the least common multiple of 45, 18, 56, 64.
18. Find the least common multiple of 15, 17, 25, 51.
19. Find the least common multiple of 10, 33, 18, 90.
20. Find the least common multiple of 11, 22, 13, 39.
21. Find the least common multiple of 25, 36, 45, 96.
22. Find the least common multiple of 108, 256, 320.
23. Find the least common multiple of 8, 12, 16, 36, 44.
24. Find the least common multiple of 288, 360, 1728.
25. Find the least common multiple of 136, 458, 890.
26. Find the least common multiple of 256, 576, 2000.
27. Find the least common multiple of 820, 575, 3500.

QUEST.—80. How is the least common multiple of two or more numbers found?

SECTION VI.

FRACTIONS.

81. When a number or thing is divided into equal parts, the parts are called fractions. If divided into *two equal* parts, one of these parts is called *one half;* if divided into *three equal* parts, one of these parts is called *one third;* if divided into *four equal* parts, one of the parts is called *one fourth*, or *one quarter;* if into *ten*, *tenths;* if into a *hundred*, *hundredths*, &c. Hence,

82. A FRACTION *denotes a part or parts of a number or thing.* An *Integer* is a *whole* number.

Note.—The term *fraction*, is derived from the Latin *fractio*, which signifies the *act of breaking*, a *broken part* or *piece.* Hence, fractions are sometimes called *broken numbers.*

83. Fractions are divided into two classes, *Common* and *Decimal.* (For the illustration of Decimal Fractions see Practical Arithmetic.)

84. *Common* fractions are those which arise from dividing an integer into *any number* of equal parts.

They are expressed by two numbers, one placed over the other, with a line between them. For example, one half is written thus, $\frac{1}{2}$; one third, $\frac{1}{3}$; nine tenths, $\frac{9}{10}$.

The number below the line is called the *denominator*, and shows into *how many parts* the number is divided.

The number above the line is called the *numerator*, and shows *how many parts* are expressed by the fraction. Thus, in the fraction $\frac{2}{3}$, the denominator 3, shows that the number is divided into *three* equal parts; the numerator 2, shows that *two* of those parts are expressed by the fraction.

QUEST.—81. What is meant by one half? By one third? 82. What is a fraction? An integer?

The numerator and denominator together are called the *terms* of the fraction.

OBS. The number below the line is called the *denominator*, because it gives the name or denomination to the fraction; as halves, thirds, fifths, &c.

The number above the line is called the *numerator*, because it numbers the parts, or shows how many parts are expressed by the fraction.

85. Common Fractions are divided into *proper*, *improper*, *simple*, *compound*, *complex*, and *mixed* numbers.

A *proper* fraction is a fraction whose numerator is *less* than its denominator; as, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$.

An *improper* fraction is one whose numerator is *equal to*, or *greater* than its denominator; as, $\frac{3}{3}$, $\frac{7}{2}$.

A *simple* fraction is a fraction which has but *one* numerator and *one* denominator, and may be *proper* or *improper*; as, $\frac{3}{5}$, $\frac{9}{4}$.

A *compound* fraction is a fraction of a fraction; as, $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{8}$.

A *complex* fraction is one which has a fraction in its numerator or denominator, or in both; as, $\frac{2\frac{1}{2}}{5}$, $\frac{4}{5\frac{1}{3}}$, $\frac{2\frac{1}{3}}{8\frac{3}{4}}$, $\frac{\frac{3}{4}}{\frac{5}{6}}$.

A *mixed* number is a whole number and a fraction written together; as, $4\frac{2}{3}$, $25\frac{1}{12}$.

86. The *value* of a fraction is the *quotient* of the numerator divided by the denominator. Thus the value of $\frac{6}{3}$ is *two*; of $\frac{4}{4}$ is *one*; of $\frac{1}{3}$ is *one third*; &c. Hence,

If the numerator and denominator are *equal*, the value is a *unit* or *one*. Thus, $\frac{5}{5}=1$, $\frac{7}{7}=1$, &c.

If the numerator is *greater* than the denominator, the value is greater than *one*. Thus, $\frac{4}{2}=2$, $\frac{5}{3}=1\frac{2}{3}$.

If the numerator is *less* than the denominator, the value is less than *one*. Thus, $\frac{1}{3}=1$ *third* of 1, $\frac{4}{5}=4$ *fifths* of 1.

QUEST.—85. How are common fractions divided? What is a proper fraction? Improper? Simple? Complex? A mixed number?

REDUCTION OF FRACTIONS.

87. *Reduction of Fractions* is the process of *changing their terms* into others, without altering the *value* of the fractions.

CASE I.—*Reducing fractions to their lowest terms.*

88. A fraction is said to be reduced to its *lowest terms*, when its numerator and denominator are expressed in the *smallest* numbers possible.

Ex. 1. Reduce $\frac{4}{8}$ to its lowest terms.

Suggestion.—Dividing both terms of the fraction by 2, it becomes $\frac{2}{4}$. Then, dividing both by 2 again, we obtain $\frac{1}{2}$, whose terms are the lowest to which the given fraction can be reduced.

First Operation.

$2)\frac{4}{8}=\frac{2}{4}$: and $2)\frac{2}{4}=\frac{1}{2}$. *Ans.*

Or, divide both terms by their greatest common divisor, which is 4, and the given fraction will be reduced to its lowest terms by a single division. Hence,

Second Operation.

$4)\frac{4}{8}=\frac{1}{2}$. *Ans.*

89. To reduce a fraction to its *lowest* terms.

Divide the numerator and denominator by any number which will divide them both without a remainder; then divide this result as before, and so on until no number greater than 1 *will exactly divide them; the last two quotients will be the lowest terms to which the given fraction can be reduced.*

Or, divide both the numerator and denominator by their greatest common divisor; and the quotients will be the lowest terms of the given fraction.

OBS. The value of a fraction is not altered by reducing it to its

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QUEST.—87. What is reduction of fractions? 89. How is a fraction reduced to its lowest terms?

lowest terms; for the numerator and denominator are divided by the same number.

2. Reduce $\frac{5}{10}$ to its lowest terms. *Ans.* $\frac{1}{2}$.

3. Reduce $\frac{10}{15}$.
4. Reduce $\frac{8}{20}$.
5. Reduce $\frac{14}{35}$.
6. Reduce $\frac{17}{51}$.
7. Reduce $\frac{21}{56}$.
8. Reduce $\frac{35}{77}$.
9. Reduce $\frac{121}{847}$.
10. Reduce $\frac{84}{228}$.
11. Reduce $\frac{253}{385}$.
12. Reduce $\frac{1950}{2418}$.

CASE II.—*Reducing improper fractions to whole or mixed numbers.*

13. Reduce $\frac{16}{3}$ to a whole or mixed number.

Suggestion.—The value of a fraction is the quotient of the numerator divided by the denominator. We therefore divide the numerator by the denominator. Hence,

Operation.

$$\begin{array}{r} 3)\underline{16} \\ 5\frac{1}{3} \textit{ Ans.} \end{array}$$

90. To reduce an *improper* fraction to a *whole* or *mixed* number.

Divide the numerator by the denominator, and the quotient will be the whole or mixed number required.

OBS. The remainder, placed over the divisor, forms a part of the quotient, and must, therefore, be annexed to the integral figures.

14. Reduce $\frac{17}{5}$ to a whole or mixed number. *Ans.* $3\frac{2}{5}$.

15. Reduce $\frac{21}{7}$.
16. Reduce $\frac{34}{5}$.
17. Reduce $\frac{31}{8}$.
18. Reduce $\frac{43}{6}$.
19. Reduce $\frac{56}{9}$.
20. Reduce $\frac{25}{25}$.
21. Reduce $\frac{108}{36}$.
22. Reduce $\frac{256}{64}$.
23. Reduce $\frac{769}{242}$.
24. Reduce $\frac{1127}{221}$.

CASE III. — *Reducing mixed numbers to improper fractions.*

25. Reduce the mixed number $13\frac{2}{3}$ to an improper fraction.

QUEST.—90. How reduce an improper frac. to a whole or mixed number?

Suggestion.—Since in 1 unit there are 3 thirds, in 13, there are 13 times as many. We therefore reduce the 13 to thirds, by multiplying it by 3, because 3 thirds make a whole one; and adding the 2 thirds, we have 41 thirds. Hence,

Operation.

$$\begin{array}{r} 13\frac{2}{3} \\ 3 \\ \hline 41 \\ \hline 3 \end{array} \text{ Ans.}$$

91. To reduce a *mixed* number to an *improper* fraction.

Multiply the whole number by the denominator of the fraction, and to the product add the given numerator. The sum placed over the given denominator, will form the improper fraction required.

OBS. 1. A whole number may be expressed in the form of a fraction without altering its value, by *making* 1 *the denominator.*

2. A whole number may also be reduced to a fraction of any denominator, by *multiplying* the given number by the proposed denominator; the product will be the numerator of the fraction required. Thus, 25 may be expressed by $\frac{25}{1}$, $\frac{100}{4}$, or $\frac{400}{16}$, &c.

26. Reduce $7\frac{3}{4}$ to an improper fraction. *Ans.* $\frac{31}{4}$.

27. Reduce $8\frac{2}{5}$.

28. Reduce $14\frac{1}{7}$.

29. Reduce $25\frac{5}{6}$.

30. Reduce $30\frac{4}{9}$.

31. Reduce $43\frac{7}{11}$.

32. Reduce $61\frac{4}{13}$.

33. Reduce $108\frac{2}{5}$.

34. Reduce $210\frac{11}{16}$.

35. Reduce 63 to 4ths.

36. Reduce 225 to 11ths.

CASE IV. *Reducing compound fractions to simple ones.*

37. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ to a simple fraction.

Suggestion.—Multiplying the two numerators together, and the two denominators, we have $\frac{6}{12}$; and $\frac{6}{12}$ reduced to its lowest terms is equal to $\frac{1}{2}$, which is the answer required. Hence,

Operation.

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$.

and $\frac{6}{12} = \frac{1}{2}$ *Ans.*

QUEST.—91. How reduce a mixed number to an improper fraction? *Obs.* How express a whole number in the form of a fraction? How reduce a whole number to a fraction of a given denominator?

92. To reduce *compound* fractions to *simple* ones.

Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

OBS. If the example contains *whole* or *mixed* numbers, they must be reduced to *improper fractions*, before multiplying.

38. Reduce $\frac{3}{4}$ of $\frac{1}{7}$ of $2\frac{1}{4}$ of 4 to an improper fraction.

Solution.—The expression $2\frac{1}{4}$ of $4=\frac{9}{4}$ of $\frac{4}{1}$; and $\frac{3}{4}\times\frac{1}{7}\times\frac{9}{4}\times\frac{4}{1}=\frac{108}{112}$, or $\frac{27}{28}$. *Ans.*

39. Reduce $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{7}{8}$. 40. Reduce $\frac{1}{4}$ of $\frac{5}{7}$ of $\frac{10}{12}$ of 5.
41. Reduce $\frac{2}{7}$ of $\frac{19}{20}$ of $\frac{6}{11}$. 42. Reduce $\frac{5}{6}$ of $\frac{3}{10}$ of $\frac{8}{9}$ of $3\frac{1}{2}$.
43. Reduce $\frac{17}{18}$ of $\frac{28}{34}$ of $\frac{12}{56}$. 44. Reduce $\frac{7}{9}$ of $4\frac{1}{2}$ of 45.
45. Reduce $5\frac{2}{3}$ of $\frac{2}{51}$ of $\frac{3}{19}$ of 17.
46. Reduce $\frac{16}{45}$ of $\frac{9}{80}$ of $2\frac{1}{2}$ of $7\frac{3}{5}$.

Contraction by Cancellation.

47. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{7}$ to a simple fraction.

Suggestion.—Since the product of the numerators is to be divided by the product of the denominators, we cancel the factors 2 and 3, which are common to both. This divides the terms of the new fraction by the same number, and therefore does not alter its value. Then multiplying the remaining factors together, we have $\frac{5}{28}$, the answer required. Hence,

Operation.

$$\frac{1}{\not{2}} \text{ of } \frac{\not{2}}{\not{3}} \text{ of } \frac{\not{3}}{4} \text{ of } \frac{5}{7}=\frac{5}{28}$$

93. To reduce *compound* fractions to *simple* ones by CANCELLATION.

Cancel all the factors common to the numerators and denominators; then multiply the remaining factors together as before.

48. Reduce $\frac{2}{3}$ of $\frac{4}{7}$ of $\frac{6}{8}$ to a simple fraction. *Ans.* $\frac{2}{7}$.
49. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{7}$. 50. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{7}$ of $\frac{1}{2}$.
51. Reduce $\frac{2}{5}$ of $\frac{3}{4}$ of $\frac{5}{2}$ of $1\frac{1}{3}$. 52. Reduce $\frac{5}{7}$ of $\frac{2}{10}$ of $\frac{7}{11}$ of 3.

53. Reduce $\frac{7}{10}$ of $\frac{15}{14}$ of $\frac{2}{3}$ of 5. 54. Reduce $\frac{12}{15}$ of $\frac{20}{35}$ of $\frac{18}{36}$.
55. Reduce $\frac{13}{27}$ of $\frac{12}{39}$ of $\frac{33}{57}$. 56. Reduce $\frac{17}{36}$ of $\frac{84}{51}$ of $\frac{120}{132}$.
57. Reduce $\frac{19}{23}$ of 46 of $29\frac{1}{2}$.
58. Reduce $\frac{32}{45}$ of $\frac{90}{64}$ of $\frac{75}{100}$ of 48.
59. Reduce $\frac{63}{90}$ of $\frac{55}{35}$ of $\frac{72}{42}$ of 84.
60. Reduce $2\frac{1}{2}$ of $\frac{2}{7}$ of $\frac{7}{10}$ of 110.

Note.—For method of reducing *Complex Fractions* to *Simple* ones, see Art. 143.

CASE V.—*Reducing fractions to a common denominator.*

94. Two or more fractions have a *common denominator*, when they have the *same* denominator.

61. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{5}$ to a common denominator.

Suggestion.—After each numerator, we write all the denominators of the given fractions, except its own, with the sign × between them, and under each line place all the denominators in like manner. Then performing the multiplications indicated, the products are equal to the given fractions, and are the answer required. Hence,

Operation.

$$\frac{1}{2}=\frac{1\times3\times5}{2\times3\times5}=\frac{15}{30}.$$
$$\frac{2}{3}=\frac{2\times2\times5}{2\times3\times5}=\frac{20}{30}.$$
$$\frac{3}{5}=\frac{3\times2\times3}{2\times3\times5}=\frac{18}{30}.$$

95. To reduce fractions to a *common* denominator.

Multiply each numerator into all the denominators except its own, for a new numerator, and all the denominators together for a common denominator.

OBS.—*Compound* fractions must be reduced to *simple* ones, and *mixed* numbers to *improper* fractions, before attempting to reduce them to a common denominator.

62. Reduce $\frac{1}{2}$ of $\frac{2}{3}$, $2\frac{1}{4}$, and 2 to a com. denominator.

Suggestion.—$\frac{1}{2}$ of $\frac{2}{3}=\frac{2}{6}$, $2\frac{1}{4}=\frac{9}{4}$, and $2=\frac{2}{1}$. Reducing $\frac{2}{6}$, $\frac{9}{4}$, and $\frac{2}{1}$ to a common denominator, they become $\frac{8}{24}$, $\frac{54}{24}$, and $\frac{48}{24}$. *Ans.*

QUEST. 95.—How reduce fractions to a common denominator?

Reduce the following to a common denominator.

63. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{5}$.
64. Reduce $\frac{3}{7}$, $\frac{2}{5}$, and $\frac{12}{23}$.
65. Reduce $\frac{5}{6}$, $\frac{4}{7}$, and $\frac{9}{11}$.
66. Reduce $\frac{3}{14}$, $\frac{21}{44}$, and $\frac{37}{68}$.
67. Reduce $\frac{13}{18}$, $\frac{51}{73}$, and $\frac{100}{130}$.
68. Reduce $\frac{41}{58}$, $\frac{65}{90}$, and $\frac{83}{250}$.
69. Reduce $\frac{5}{16}$, $\frac{3}{4}$, and $\frac{1}{2}$ of 17.
70. Reduce $\frac{6}{9}$, $4\frac{1}{2}$, and $\frac{58}{65}$.

CASE VI.—*Reducing fractions to their least common denominator.*

71. Reduce $\frac{3}{4}$, $\frac{2}{6}$, and $\frac{5}{8}$ to the least com. denominator.

Operation.

```
2)4 " 6 " 8
2)2 " 3 " 4
  1 " 3 " 2
2 × 2 × 3 × 2 = 24
```

Suggestion. — We first find the least common multiple of all the given denominators, which is 24; and this is the least common denominator required. The next step is to reduce the given fractions to *twenty-fourths* without altering their value. The denominator of the first fraction, is contained in 24, 6 times; and multiplying both terms of the fraction $\frac{3}{4}$ by 6, it becomes $\frac{18}{24}$. The denominator 6 is contained in 24, 4 times; and multiplying the fraction $\frac{2}{6}$ by 4, it becomes $\frac{8}{24}$. The denominator 8 is contained in 24, 3 times; and multiplying the fraction $\frac{5}{8}$ by 3, it becomes $\frac{15}{24}$. Now $\frac{18}{24}$, $\frac{8}{24}$, and $\frac{15}{24}$ are equal to the given fractions $\frac{3}{4}$, $\frac{2}{6}$, and $\frac{5}{8}$. Hence,

96. To reduce fractions to their *least* common denominator.

I. *Find the least common multiple of all the denominators of the given fractions, and it will be the least common denominator.*

II. *Divide the least common denominator by the denominator of each of the given fractions, and multiply the numerator by the quotient; the products will be the numerators required.*

QUEST. 96.—How reduce fractions to the least common denominator.

OBS.—If the example contains *compound* fractions, *whole* or *mixed* numbers, they must first be reduced to simple fractions, then all must be reduced to their lowest terms; otherwise the least common multiple of their denominators, may not be the least common denominator.

72. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to the least common denominator.

Suggestion.—First find the least common multiple of the denominators, which is $2 \times 3 \times 2 = 12$. Now

$12 \div 3 = 4$, and $4 \times 2 = 8$, the 1st numerator.
$12 \div 4 = 3$, and $3 \times 3 = 9$, the 2d "
$12 \div 6 = 2$, and $2 \times 5 = 10$, the 3d "

Operation.

2)3″4″6
3)3″2″3
1″2″1

Ans. $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$.

Reduce the following to the least common denominator.

73. Reduce $\frac{1}{4}$, $\frac{5}{6}$, and $\frac{3}{8}$.

74. Reduce $\frac{2}{5}$, $\frac{3}{4}$, and $\frac{7}{10}$.

75. Reduce $\frac{2}{6}$, $\frac{6}{8}$, and $\frac{4}{12}$.

76. Reduce $\frac{3}{9}$, $\frac{3}{6}$, $\frac{7}{8}$, and $\frac{5}{12}$.

77. Reduce $\frac{7}{24}$, $\frac{9}{15}$, and $\frac{17}{51}$.

78. Reduce $\frac{21}{35}$, $\frac{7}{12}$, $\frac{4}{7}$, and $\frac{5}{18}$.

79. Reduce $\frac{18}{45}$, $\frac{11}{15}$, and $\frac{7}{60}$.

80. Reduce $\frac{13}{27}$, $\frac{4}{9}$, $\frac{1}{3}$, and 10.

81. Reduce $\frac{4}{7}$ of 14, and $3\frac{1}{2}$.

82. Reduce $\frac{33}{40}$, $4\frac{2}{5}$, and 27.

83. Reduce $\frac{12}{35}$ of $\frac{7}{18}$, and $16\frac{2}{3}$.

84. Reduce $\frac{7}{8}$ of $2\frac{2}{5}$, and $4\frac{6}{10}$.

85. Reduce $\frac{3}{5}$, $\frac{7}{12}$, $\frac{11}{15}$, and $\frac{9}{20}$ to the common denominator 120.

86. Reduce $\frac{5}{8}$, $\frac{1}{12}$, $\frac{14}{16}$, and $\frac{3}{24}$ to the common denominator 144.

87. Reduce $\frac{15}{24}$, $\frac{21}{40}$, and $\frac{35}{36}$ to the common denominator 1080.

88. Reduce $\frac{5}{9}$, $\frac{10}{12}$, and $\frac{7}{18}$ to thirty sixths.

89. Reduce $\frac{8}{17}$, $\frac{21}{34}$, and $\frac{30}{51}$ to one hundred and seconds.

90. Reduce $\frac{19}{20}$, $\frac{16}{25}$, and $\frac{1}{2}$ of $\frac{2}{5}$ to hundredths.

91. Reduce $\frac{3}{4}$ of $\frac{1}{3}$, $\frac{20}{24}$, and $\frac{31}{48}$ to ninety-sixths.

92. Reduce $\frac{1}{2}$, $\frac{5}{8}$, and $\frac{7}{12}$ to twenty-fourths.

QUEST.—*Obs.* When the example contains compound fractions, whole or mixed numbers, how proceed?

ADDITION OF FRACTIONS.

97. If two or more fractions have a *common* denominator, the parts of a unit expressed by their numerators, are of the *same value* or *denomination*, and, therefore, may be added like whole numbers.

Ex. 1. A man gave $\frac{2}{8}$ of a dollar to one child, $\frac{3}{8}$ to another, and $\frac{5}{8}$ to another; how much did he give to all?

Suggestion.—Add the numerators, and place the sum over the common denominator. Thus 2 eighths and 3 eighths are 5 eighths, and 5 are 10 eighths. But $\frac{10}{8}=1\frac{2}{8}$, or $1\frac{1}{4}$ dollar.

Operation.

$\frac{2}{8}+\frac{3}{8}+\frac{5}{8}=\frac{10}{8}$, or $1\frac{1}{4}$d.

2. Add $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$.
3. Add $\frac{7}{9}$, $\frac{5}{9}$, and $\frac{8}{9}$.
4. Add $\frac{7}{10}$, $\frac{8}{10}$, and $\frac{3}{10}$.
5. Add $\frac{3}{15}$, $\frac{11}{15}$, and $\frac{13}{15}$.
6. A man bought $\frac{1}{2}$ a barrel of flour at one time, $\frac{2}{3}$ of a barrel at another, and $\frac{3}{4}$ of a barrel at another; how much did he buy in all?

Suggestion. — Reduce the fractions to a common denominator, and add the numerators. The result $\frac{46}{24}=1\frac{22}{24}$, or $1\frac{11}{12}$.

Operation.

$1\times3\times4=12$, 1st numerator
$2\times2\times4=16$, 2d "
$3\times2\times3=18$, 3d "
$2\times3\times4=24$, com. denom.
Ans. $\frac{46}{24}$, or $1\frac{11}{12}$ bar.

98. From these illustrations, we deduce the following

RULE FOR ADDITION OF FRACTIONS.

Reduce the fractions to a common denominator; add their numerators, and place the sum over the common denominator.

QUEST.—98. What is the rule for addition of fractions? *Obs.* What must be done with compound fractions, whole and mixed numbers?

OBS. 1. *Compound* fractions must be reduced to simple ones, *whole* and *mixed* numbers to improper fractions, and all of them to a common denominator, then add them as above.

2. In adding *mixed* numbers, it is generally more convenient to add the whole numbers and fractional parts separately, and then unite their sums.

3. The operation may frequently be shortened by reducing the given fractions to their *least common denominator*, and adding their numerators.

7. What is the sum of $\frac{1}{2}$ of $\frac{2}{3}$, $2\frac{1}{4}$, and 7?

Suggestion.—$\frac{1}{2}$ of $\frac{2}{3}=\frac{2}{6}$, $2\frac{1}{4}=\frac{9}{4}$, and $7=\frac{7}{1}$. Now reducing $\frac{2}{6}$, $\frac{9}{4}$, and $\frac{7}{1}$ to a common denominator as in the margin, and adding their numerators, the result is $\frac{230}{24}$, which is equal to $9\frac{14}{24}$, or $9\frac{7}{12}$.

Operation.

$$\frac{2}{6}=\frac{8}{24}$$
$$\frac{9}{4}=\frac{54}{24}$$
$$\frac{7}{1}=\frac{168}{24}.$$

Ans. $9\frac{14}{24}$, or $9\frac{7}{12}$.

8. Add $\frac{2}{3}$ and $\frac{3}{5}$.

9. Add $\frac{2}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$.

10. Add $\frac{4}{7}$, $\frac{8}{9}$, and $\frac{5}{6}$.

11. Add $\frac{3}{8}$, $\frac{7}{10}$, and $\frac{3}{4}$.

12. Add $\frac{11}{12}$, $\frac{13}{80}$, and $\frac{5}{8}$.

13. Add $\frac{4}{10}$, $\frac{5}{9}$, and $\frac{12}{27}$.

14. Add $\frac{5}{11}$, $\frac{6}{8}$, and $\frac{17}{12}$.

15. Add $\frac{20}{35}$, $\frac{17}{40}$, and $\frac{33}{75}$.

16. Add $\frac{4}{5}$ of $\frac{3}{4}$, $\frac{8}{9}$, and $7\frac{1}{2}$.

17. Add $4\frac{2}{3}$, $5\frac{1}{2}$, and $17\frac{1}{4}$.

18. Add $175\frac{5}{9}$, $20\frac{6}{7}$, and 43.

19. Add $\frac{5}{8}$ of 45, $\frac{10}{12}$ of $\frac{3}{4}$, and 51.

20. A grocer bought 3 chests of tea, one containing $125\frac{3}{4}$ pounds, another $95\frac{7}{8}$ pounds, and the other $113\frac{1}{6}$ pounds: how much did he buy?

21. A man gave $25\frac{3}{4}$ dolls. for a cow, $75\frac{5}{8}$ dolls. for a horse, $110\frac{1}{2}$ dolls. for a buggy, and $86\frac{9}{16}$ dolls. for a harness: how much did he pay for all?

22. If you spend $151\frac{3}{5}$ dolls. for clothing, $270\frac{5}{10}$ dolls. for board, and $83\frac{4}{5}$ dolls. for traveling, what is the amount of your expenses?

23. The smaller of two numbers is $251\frac{8}{10}$, and the difference between them is $135\frac{2}{15}$: what is the greater number?

SUBTRACTION OF FRACTIONS.

99. When two fractions have *a common denominator, the less numerator may be subtracted from the greater, as in whole numbers, and the result placed over the common denominator, will be the difference between them.*

Ex. 1. If I buy $\frac{27}{45}$ of an acre of land, and afterwards sell $\frac{19}{45}$ of an acre, how much shall I have left?

Suggestion.—Taking 19 forty-fifths from 27 forty-fifths, the remainder is $\frac{8}{45}$ of an acre.

Operation.
$\frac{27}{45}-\frac{19}{45}=\frac{8}{45}$

2. From $\frac{11}{13}$ take $\frac{7}{13}$.
3. From $\frac{15}{21}$ take $\frac{8}{21}$.
4. From $\frac{35}{63}$ take $\frac{24}{63}$.
5. From $\frac{71}{100}$ take $\frac{47}{100}$.
6. From $\frac{3}{5}$ of a yard of cloth, take $\frac{1}{4}$ of a yard.

Suggestion. — Since these fractions have not a common denominator, it is plain that one numerator cannot be taken from the other. We, therefore, reduce them to a common denominator, then subtract as above.

Operation.
$3\times4=12$ } numerators.
$1\times5=\ 5$ }
$5\times4=20$ com. denom.
$\frac{12}{20}-\frac{5}{20}=\frac{7}{20}$ yd. *Ans.*

100. From these illustrations, we deduce the following

RULE FOR SUBTRACTION OF FRACTIONS.

Reduce the fractions to a common denominator; subtract the less numerator from the greater, and place the remainder over the common denominator.

OBS. 1. *Compound* fractions must be reduced to simple ones,

QUEST.—100. What is the rule for subtraction? *Obs.* What must be done with compound fractions, whole and mixed numbers? How else are mixed numbers subtracted?

whole and *mixed* numbers to improper fractions, and all of them to a common denominator, as in addition.

2. In subtracting *mixed* numbers, it is sometimes more convenient to take the fractional part of the less from the fractional part of the greater, then the integral part of the less from that of the greater. (See Ex. 17.)

3. In subtracting a *proper* fraction from a whole number, we may borrow a unit and take the fraction from this, then diminish the whole number by 1.

7. From $\frac{11}{12}$ take $\frac{3}{5}$.
8. From $\frac{8}{9}$ take $\frac{2}{3}$.
9. From $\frac{13}{16}$ take $\frac{2}{5}$.
10. From $\frac{21}{30}$ take $\frac{11}{25}$.
11. From $\frac{31}{35}$ take $\frac{24}{30}$.
12. From $\frac{17}{43}$ take $\frac{11}{35}$.
13. From $\frac{42}{60}$ take $\frac{23}{44}$.
14. From $\frac{41}{70}$ take $\frac{28}{90}$.
15. From $\frac{85}{121}$ take $\frac{35}{99}$.
16. From $\frac{64}{100}$ take $\frac{17}{55}$.
17. From $7\frac{1}{3}$ take $2\frac{1}{4}$.

Suggestion. — Reducing the mixed numbers to improper fractions, then to a common denominator 12, they become $\frac{88}{12}$ and $\frac{27}{12}$.

First Operation.

$7\frac{1}{3}=\frac{22}{3}$, and $\frac{22}{3}=\frac{88}{12}$
$2\frac{1}{4}=\frac{9}{4}$, and $\frac{9}{4}=\frac{27}{12}$
$\frac{88}{12}-\frac{27}{12}=\frac{61}{12}$ or $5\frac{1}{12}$. *Ans.*

Or, reducing the *fractional parts* to a common denominator 12, then subtract the numerator of the less from that of the greater.

Second Operation

$7\frac{1}{3}=7\frac{4}{12}$
$2\frac{1}{4}=2\frac{3}{12}$
Ans. $5\frac{1}{12}$

18. From $9\frac{1}{4}$ take $3\frac{1}{2}$.
19. From $11\frac{2}{5}$ take $6\frac{2}{3}$.
20. From $12\frac{1}{6}$ take $8\frac{2}{3}$.
21. From $18\frac{2}{7}$ take $10\frac{3}{8}$.
22. From 4 take $\frac{2}{3}$. *Ans.* $3\frac{1}{3}$.
23. From 9 take $2\frac{1}{2}$.
24. From $15\frac{3}{4}$ take 7.
25. From $\frac{3}{5}$ of $\frac{1}{4}$ take $\frac{1}{6}$ of $\frac{1}{2}$. *Ans.* $\frac{4}{60}=\frac{1}{15}$.
26. From $\frac{2}{3}$ of $\frac{4}{5}$ of a yard, take $\frac{1}{5}$ of $\frac{3}{8}$ of a yard.
27. From $\frac{5}{7}$ of 40 pounds, take $\frac{1}{8}$ of 50 pounds.
28. From $\frac{1}{2}$ of $15\frac{1}{4}$ miles, take $\frac{5}{6}$ of 7 miles.

MULTIPLICATION OF FRACTIONS.

101. *Multiplying by a fraction is taking a certain* PORTION *of the multiplicand as many times, as there are like portions of a unit in the multiplier.* That is,

Multiplying by $\frac{1}{2}$, is taking 1 *half* of the multiplicand *once.* Thus, $6\times\frac{1}{2}=3$.

Multiplying by $\frac{1}{3}$, is taking 1 *third* of the multiplicand *once.* Thus, $6\times\frac{1}{3}=2$.

Multiplying by $\frac{2}{3}$, is taking 1 *third* of the multiplicand *twice.* Thus, $6\times\frac{2}{3}=4$.

Ex. 1. If 1 quart of condensed milk is worth $\frac{3}{8}$ of a dollar, how much are 4 quarts worth?

Analysis.—If 1 quart is worth $\frac{3}{8}$ dollar, 4 quarts are worth 4 times $\frac{3}{8}$ dollar, and 4 times $\frac{3}{8}$ are $\frac{12}{8}$, or $1\frac{1}{2}$ dollar. That is, we multiply the numerator by the whole number.

First Operation.

$$\frac{3\times4}{8}=\frac{12}{8},\text{ or }1\tfrac{1}{2}.$$

Ans. $1\frac{1}{2}$ dolls.

Or, we may divide the denominator by the whole number, when it can be done without a remainder, for dividing the denominator multiplies the value of the fraction. Hence,

Second Operation.

$$\frac{3}{8\div4}=\frac{3}{2},\text{ or }1\tfrac{1}{2}.$$

102. To multiply a FRACTION by a *whole* number.

Multiply the numerator of the fraction by the whole number, and write the product over the denominator.

Or, divide the denominator by the whole number, when this can be done without a remainder.

OBS. A fraction is multiplied into a number *equal* to its *denominator* by *canceling* the denominator. Thus, $\frac{4}{7}\times7=4$.

2. Multiply $\frac{5}{9}$ by 7.
3. Multiply $\frac{11}{12}$ by 9.
4. Multiply $\frac{8}{15}$ by 8.
5. Multiply $\frac{12}{17}$ by 10.

QUEST.—101. What is meant by multiplying by a fraction?

6. Multiply $\frac{9}{22}$ by 12.
7. Multiply $\frac{15}{23}$ by 21.
8. Multiply $\frac{24}{35}$ by 16.
9. Multiply $\frac{36}{47}$ by 23.
10. Multiply $\frac{75}{100}$ by 31.
11. Multiply $\frac{162}{500}$ by 83.

12. What cost 5 yards of cloth, at $2\frac{1}{2}$ dollars a yard?

Analysis.—Five yards will cost 5 times as much a 1 yard. Now, 5 times $\frac{1}{2}$ are $\frac{5}{2}=2\frac{1}{2}$; 5 times 2 are 10, and $2\frac{1}{2}$ are $12\frac{1}{2}$. Hence,

Operation.

$2\frac{1}{2}$
5
Ans. $12\frac{1}{2}$ dolls.

103. To multiply a *mixed* number by a *whole* one.

Multiply the fractional part and the whole number separately, and unite the products.

13. Multiply $12\frac{2}{3}$ by 5.
14. Multiply $21\frac{1}{4}$ by 9.
15. Multiply $35\frac{3}{4}$ by 7.
16. Multiply $29\frac{3}{5}$ by 10.
17. Multiply $45\frac{7}{8}$ by 12.
18. Multiply $53\frac{6}{7}$ by 21.

19. What cost $\frac{2}{3}$ of a pound of coffee, at 25 cents a pound?

Analysis.—Since 1 pound costs 25 cents, $\frac{2}{3}$ of a pound will cost $\frac{2}{3}$ of 25 cents. Now, 1 third of 25 is $8\frac{1}{3}$, and 2 thirds is 2 times $8\frac{1}{3}$, or $16\frac{2}{3}$ cents. We divide the whole number by 3, and multiply the quotient by 2.

First Operation.

3)25 cents.
$8\frac{1}{3}$
2
Ans. $16\frac{2}{3}$ cents.

Or, we may first multiply the whole number by 2, then divide this product by 3; for 1 third of 2 times 25 is obviously the same as 2 times 1 third of it. Hence,

Second Operation.

25 cents.
2
3)50
Ans. $16\frac{2}{3}$ cents.

QUEST.—102. How multiply a fraction by a whole number. *Obs.* How is a fraction multiplied by a number equal to its denominator? 103. How multiply a mixed number by a whole one?

104. To multiply a *whole* number by a *fraction.*

Divide the whole number by the denominator, and multiply the quotient by the numerator.

Or, multiply the whole number by the numerator, and divide the product by the denominator.

OBS. When the *whole* number cannot be divided by the denominator without a remainder, the latter method in generally preferred.

20. Multiply 21 by $\frac{3}{5}$.
21. Multiply 19 by $\frac{4}{7}$.
22. Multiply 31 by $\frac{4}{5}$.
23. Multiply 43 by $\frac{5}{6}$.
24. Multiply 39 by $\frac{3}{8}$.
25. Multiply 47 by $\frac{5}{7}$.
26. Multiply 75 by $\frac{9}{11}$.
27. Multiply 91 by $\frac{8}{15}$.

28. What cost $2\frac{1}{2}$ gallons of cider, at 31 cents a gall.?

Analysis. — If 1 gallon cost 31 cents, $2\frac{1}{2}$ gallons will cost $2\frac{1}{2}$ times 31 cents. Now, 2 times 31 are 62; and $\frac{1}{2}$ of 31 is $15\frac{1}{2}$, which added to 62, makes $77\frac{1}{2}$ cents. Hence,

2)31 cents.
$2\frac{1}{2}$
62
$15\frac{1}{2}$
Ans. $77\frac{1}{2}$ cents.

105. To multiply a *whole* by a *mixed* number.

Multiply first by the whole number, then by the fraction, and add the products together.

29. Multiply 22 by $3\frac{2}{3}$.
30. Multiply 46 by $4\frac{3}{5}$.
31. Multiply 58 by $9\frac{5}{9}$.
32. Multiply 65 by $11\frac{4}{9}$.

33. At $\frac{2}{5}$ of a dollar a pound, what will $\frac{3}{4}$ of a pound of tobacco cost?

Operation.

$\frac{2}{5}\times\frac{3}{4}=\frac{6}{20}$, or $\frac{3}{10}$. *Ans.*

Analysis. — Since 1 pound costs $\frac{2}{5}$ of a dollar, $\frac{3}{4}$ of a pound will cost $\frac{3}{4}$ of $\frac{2}{5}$ doll., and $\frac{3}{4}$ of $\frac{2}{5}=\frac{6}{20}$, or $\frac{3}{10}$ dollar. We multiply the numerators together and the denominators together, and the result $\frac{6}{20}$, reduced to the lowest terms is $\frac{3}{10}$ of a dollar. Hence,

QUEST.—104. How multiply a whole number by a fraction? 105. How multiply a whole by a mixed number?

106. To multiply a *fraction* by a *fraction*.

Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

OBS.—When the numerators and denominators contain *common* factors, they should be canceled.

34. What is the product of $\frac{4}{5}$ multiplied by $\frac{10}{12}$?

Suggestion.—Canceling the factors 4 and 5 which are common, the result is $\frac{2}{3}$.

Operation.

$\frac{\not{4}}{\not{5}}\times\frac{\not{1}\not{0}}{\not{1}\not{2}},\ \frac{2}{3}=\frac{2}{3}.$ *Ans.*

35. Multiply $\frac{2}{3}$ by $\frac{3}{5}$.
36. Multiply $\frac{4}{9}$ by $\frac{3}{8}$.
37. Multiply $\frac{7}{8}$ by $\frac{5}{9}$.
38. Multiply $\frac{9}{10}$ by $\frac{5}{12}$.
39. Multiply $\frac{8}{12}$ by $\frac{15}{24}$.
40. Multiply $\frac{7}{31}$ by $\frac{19}{28}$.
41. Multiply $\frac{55}{63}$ by $\frac{27}{44}$.
42. Multiply $\frac{72}{80}$ by $\frac{35}{48}$.

43. What is the product of $3\frac{2}{10}$ by $2\frac{1}{2}$?

Analysis.—The mixed number $3\frac{2}{10}=\frac{32}{10}$, and $3\frac{1}{2}=\frac{7}{2}$. Multiply the numerators together, and the denominators.

Operation.

$\frac{32}{10}\times\frac{5}{2}=\frac{160}{20}$, or 8.

Or, cancel the common factors, 8, then multiply. Hence,

$\frac{\not{1}\not{6},}{\not{2},}\ \frac{\not{3}\not{2}}{\not{1}\not{0}}\times\frac{\not{5}}{\not{2}}=8.$

107. To multiply a *mixed* number by a *mixed* number.

Reduce the mixed numbers to improper fractions, and proceed as in multiplying a fraction by a fraction.

43. Multiply $7\frac{3}{4}$ by $5\frac{1}{3}$.
44. Multiply $6\frac{4}{10}$ by $7\frac{1}{2}$.
45. Multiply $21\frac{1}{7}$ by $16\frac{1}{4}$.
46. Multiply $85\frac{5}{12}$ by $24\frac{3}{5}$.
47. Multiply $75\frac{4}{5}$ by $42\frac{3}{4}$.
48. Multiply $91\frac{5}{8}$ by $63\frac{6}{11}$.

QUEST.—106. How is a fraction multiplied by a fraction? 107. How multiply one mixed number by another?

DIVISION OF FRACTIONS.

Ex. 1. If 2 pounds of butter costs $\frac{4}{5}$ of a dollar, what will 1 pound cost?

Analysis.—Since 2 pounds costs $\frac{4}{5}$ doll., 1 pound will cost $\frac{1}{2}$ of $\frac{4}{5}$ doll., which is $\frac{2}{5}$ doll.; we divide the numerator of the fraction $\frac{4}{5}$ by the whole number 2, and set the quotient over the denominator.

Operation.

$$\frac{4 \div 2}{5} = \frac{2}{5} \text{ doll.}$$

2. If 3 quarts of chestnuts cost $\frac{2}{3}$ of a dollar, what will 1 quart cost?

Analysis.—In this case we cannot divide the numerator 2 by 3, without a remainder. We therefore multiply the denominator by it, which divides the fraction. Hence,

Operation.

$$\frac{2}{3 \times 3} = \frac{2}{9} \text{ doll.}$$

108. To divide a *fraction* by a *whole* number.

Divide the numerator by the whole number, when it can be done without a remainder. If not,

Multiply the denominator by the whole number.

3. What is the quotient of $\frac{6}{7}$ divided by 3? *Ans.* $\frac{2}{7}$.

4. Divide $\frac{10}{11}$ by 2.
5. Divide $\frac{20}{27}$ by 5.
6. Divide $\frac{11}{12}$ by 10.
7. Divide $\frac{19}{20}$ by 11.
8. Divide $\frac{75}{100}$ by 15.
9. Divide $\frac{120}{256}$ by 60.

10. At $\frac{1}{4}$ of a penny a yard, how many yards of tape can you buy for $\frac{3}{4}$ of a penny?

Analysis.—If $\frac{1}{4}$ of a penny will buy 1 yard, $\frac{3}{4}$ of a penny will buy as many yards as 1 fourth is contained times in 3 fourths, which is 3. That is, as the fractions have a common denominator, we divide one numerator by the other.

Operation.

$$\tfrac{3}{4} \div \tfrac{1}{4} = 3 \text{ yards.}$$

QUEST.—108. How is a fraction divided by a whole number?

11. At $\frac{2}{3}$ of a dollar a yard, how much silk can be bought for $\frac{4}{5}$ of a dollar?

Analysis.—Since these fractions have different denominators, we invert the divisor, and proceed as in multiplication of fractions. Hence,

Operation.
$\frac{4}{5}\times\frac{3}{2}=\frac{12}{10}$, or $1\frac{1}{5}$ yd.

109. To divide a *fraction* by a *fraction.*

I. *If the given fractions have a common denominator, divide the numerator of the dividend by the numerator of the divisor.*

II. *When the fractions have not a common denominator, invert the divisor, and proceed as in multiplication of fractions.*

OBS. 1. *Compound fractions* must be reduced to simple ones, and *mixed numbers* to improper fractions.

2. After inverting the divisor, the factors common to the numerators and denominators should be *canceled.*

12. What is the quotient of $\frac{10}{12}$ divided by $\frac{2}{3}$?

Suggestion.—Invert the divisor, and cancel the common factors.

$$\frac{\overset{5}{\cancel{10}}}{\underset{4}{\cancel{12}}}\times\frac{\cancel{3}}{\cancel{2}}=\frac{5}{4}, \text{ or } 1\frac{1}{4}.$$

13. Divide $\frac{18}{20}$ by $\frac{3}{5}$.
14. Divide $\frac{25}{30}$ by $\frac{10}{20}$.
15. Divide $\frac{33}{45}$ by $\frac{27}{35}$.
16. Divide $\frac{72}{100}$ by $\frac{24}{40}$.
17. Divide $\frac{2}{3}$ of $\frac{5}{7}$ of $5\frac{1}{4}$ by $\frac{5}{6}$ of $\frac{18}{4}$. *Ans.* $\frac{2}{3}$.
18. Divide $\frac{3}{4}$ of $\frac{10}{18}$ by $\frac{4}{5}$ of $\frac{3}{8}$.
19. Divide $\frac{4}{5}$ of $8\frac{1}{3}$ by $\frac{2}{3}$ of $\frac{6}{10}$.
20. Divide $\frac{5}{7}$ of 28 by $\frac{3}{5}$ of $\frac{5}{6}$ of $5\frac{1}{3}$.
21. Divide $\frac{4}{5}$ of $\frac{3}{12}$ of $7\frac{1}{2}$ by $\frac{10}{12}$ of 40.

22. At $5\frac{1}{2}$ dollars a yard, how much cloth can be bought for $26\frac{2}{5}$ dolls. *Ans.* $4\frac{4}{5}$ yds.

23. Divide $22\frac{1}{2}$ by $6\frac{3}{4}$.
24. Divide $38\frac{1}{2}$ by $\frac{3}{11}$.
25. Divide $15\frac{3}{4}$ by $\frac{3}{4}$ of 12.
26. Divide $\frac{4}{5}$ of 20 by $2\frac{5}{8}$.

QUEST.—109. How is one fraction divided by another when they have a common denominator? How, when they have not a common denominator?

27. At $\frac{3}{4}$ of a dollar a pound, how many pounds of tea will 25 dollars buy?

Analysis.—If $\frac{3}{4}$ of a dollar will buy 1 pound, 25 dollars will buy as many pounds as $\frac{3}{4}$ is contained times in 25. We multiply the whole number by the denominator, and divide the product by the numerator.

Operation.

$$\begin{array}{r} 25 \\ 4 \\ \hline 3)100 \\ \hline \end{array}$$

Ans. $33\frac{1}{3}$ p.

Or, we may reduce the whole number to the form of a fraction, and then divide it by the fraction. Thus, $25=\frac{25}{1}$, and $\frac{25}{1}\div\frac{3}{4}=\frac{25}{1}\times\frac{4}{3}=\frac{100}{3}$, or $33\frac{1}{3}$ pounds.

110. To divide a *whole* number by a *fraction.*

Multiply the whole number by the denominator, and divide the product by the numerator.

Or, reduce the whole number to the form of a fraction, and proceed according to the rule for dividing a fraction by a fraction.

OBS. When the divisor is a mixed number, it must be reduced to an improper fraction, then proceed as above.

28. Divide 115 by $\frac{5}{7}$.
29. Divide 147 by $\frac{7}{8}$.
30. Divide 180 by $\frac{11}{12}$.
31. Divide 218 by $\frac{9}{20}$.
32. Divide 228 by $15\frac{3}{4}$.
33. Divide 360 by $20\frac{5}{7}$.

34. Reduce the complex fraction $\dfrac{\frac{3}{4}}{\frac{2}{3}}$ to a simple one.

Analysis.—The denominator of a fraction is a divisor; hence the operation is the same as dividing $\frac{3}{4}$ by $\frac{2}{3}$. But to divide $\frac{3}{4}$ by $\frac{2}{3}$, we invert the divisor, then multiply the numerators together and the denominators. Hence,

$$\dfrac{\frac{3}{4}}{\frac{2}{3}}=\frac{3}{4}\div\frac{2}{3}.$$

$$\frac{3}{4}\times\frac{3}{2}=\frac{9}{8}, \text{ or } 1\tfrac{1}{8}.$$

111. To reduce a complex fraction to a simple one.

Consider the denominator as a divisor, and proceed as in division of fractions.

QUEST.—110. How is a whole number divided by a fraction?

OBS. When *complex* fractions are reduced to *simple* ones, they are *added, subtracted, multiplied*, and *divided*, according to the preceding rules.

35. Reduce $\frac{5\frac{1}{4}}{2\frac{1}{3}}$. 36. Reduce $\frac{6}{3\frac{3}{4}}$. 37. Reduce $\frac{7\frac{1}{2}}{9}$.

38. Reduce $\frac{\frac{4}{5}}{\frac{5}{9}}$. 39. Reduce $\frac{8\frac{2}{5}}{9\frac{3}{5}}$. 40. Reduce $\frac{25}{\frac{7}{10}}$.

EXERCISES IN FRACTIONS.

1. If a man can earn $\frac{9}{10}$ of a dollar per day, how much can he earn in $\frac{3}{4}$ of a day?

2. If water runs $3\frac{2}{3}$ miles per hour, how far will it run in $12\frac{1}{2}$ hours?

3. What cost $48\frac{4}{5}$ barrels of beef, at $16\frac{3}{8}$ dolls. a barrel?

4. How far can a man travel in $85\frac{1}{2}$ days, if he travels $33\frac{1}{2}$ miles per day?

5. What cost $58\frac{1}{4}$ cheeses, at $4\frac{3}{4}$ dollars apiece?

6. What cost $85\frac{3}{7}$ pounds of sugar, at $7\frac{1}{4}$ cents a pound?

7. What cost $67\frac{3}{4}$ bu. of barley, at $65\frac{1}{2}$ cents a bu.?

8. A grocer sold $\frac{3}{4}$ of 56 gallons of molasses, at $\frac{1}{5}$ of 3 dollars a gallon: how much did it come to?

9. If I pay $\frac{1}{3}$ of $\frac{6}{7}$ of 28 dollars for a barrel of flour, how much must I pay for $\frac{4}{5}$ of $3\frac{3}{4}$ barrels?

10. A grocer sold some figs for $\frac{19}{20}$ of a dollar, which was $\frac{2}{11}$ of a dollar a pound: how many did he sell?

11. At $6\frac{1}{4}$ cents apiece, how many oranges can be bought for $62\frac{1}{2}$ cents?

12. How much butter, at $18\frac{3}{4}$ cents, a pound, can be bought for $95\frac{1}{2}$ cents?

13. A man traveled $125\frac{4}{5}$ miles in $7\frac{3}{4}$ days: how far did he travel per day?

14. A lad having $87\frac{1}{2}$ cents, spent it in candy, at $37\frac{1}{2}$ cents a pound: how much did he buy?

15. A man spent $\frac{4}{5}$ of 720 dollars for tobacco, which was $\frac{2}{3}$ of $\frac{3}{4}$ of a dollar a pound: how much did he get?

SECTION VII.

COMPOUND OR DENOMINATE NUMBERS.

ART. **87.** SIMPLE *Numbers* are those which express *units* of the *same kind* or *denomination;* as, one, two, three; 4 pears, 5 feet, &c.

COMPOUND *Numbers* are those which express *units* of *different kinds* or *denominations;* as the divisions of money, weight, and measure. Thus, 6 shillings and 7 pence; 3 feet and 7 inches, &c., are compound numbers.

Note.—Compound Numbers are sometimes called *Denominate Numbers.*

FEDERAL MONEY.

88. *Federal Money* is the currency of the United States. Its denominations are, *Eagles*, *dollars*, *dimes*, *cents*, and *mills.*

10 mills (*m.*)	make	1 cent,	marked	*ct.*
10 cents	"	1 dime,	"	*d.*
10 dimes	"	1 dollar,	"	*doll.* or $.
10 dollars	"	1 eagle,	"	*E.*

89. The national coins of the United States are of three kinds, viz: gold, silver, and copper.

1. The gold coins are the *eagle*, *half eagle*, and *quarter eagle*, the *double eagle*,* and *gold dollar*.*

2. The silver coins are the *dollar*, *half dollar*, *quarter dollar*, the *dime*, *half dime*, and *three-cent-piece.*

QUEST.—87. What are simple numbers? What are compound numbers? 88. What is Federal Money? Recite the Table. 89. Of how many kinds are the coins of the United States? What are the gold coins? What are the silver coins?

* Added by Act of Congress, Feb. 20th, 1849.

3. The copper coins are the *cent* and *half cent*. Mills are not coined.

Obs. Federal money was established by Congress, August 8th, 1786. Previous to this, English or Sterling money was the principal currency of the country.

STERLING MONEY.

90. *English* or *Sterling Money* is the national currency of Great Britain.

4 farthings (*qr.* or *far.*)	make	1 penny, marked	*d.*
12 pence	"	1 shilling, "	*s.*
20 shillings	"	1 pound or sovereign,	£.
21 shillings	"	1 guinea.	

OBS. The Pound Sterling is represented by a gold coin, called a *Sovereign*. Its *legal* value, according to *Act of Congress*, 1842, is \$4.84; its *intrinsic* value, according to assays at the U. S. mint. is \$4.861. The *legal* value of an English shilling is $24\frac{1}{5}$ cents.

TROY WEIGHT.

91. *Troy Weight* is used in weighing gold, silver, jewels, liquors, &c., and is generally adopted in philosophical experiments.

24 grains (*gr.*)	make	1 pennyweight, marked	*pwt.*
20 pennyweights	"	1 ounce, "	*oz.*
12 ounces	"	1 pound, "	*lb.*

Note.—Most children have very erroneous or indistinct ideas of the *weights* and *measures* in common use. It is, therefore, strongly recommended for teachers to illustrate them *practically*, by referring to some visible object of equal magnitude, or by exhibiting the ounce, the pound; the *linear* inch, foot, yard, and rod; also a *square* and *cubic* inch, foot, &c.

QUEST.—What are the copper coins? *Obs.* When and by whom was Federal Money established? 90. What is Sterling Money? Repeat the Table. *Obs.* By what is the Pound Sterling represented? What is its legal value in dollars and cents? What is the value of an English shilling? 91. In what is Troy Weight used? Recite the Table.

AVOIRDUPOIS WEIGHT.

92. *Avoirdupois Weight* is used in weighing groceries and all coarse articles; as sugar, tea, coffee, butter, cheese, flour, hay, &c., and all metals except gold and silver.

16 drams (*dr.*)	make	1 ounce, marked	*oz.*
16 ounces	"	1 pound, "	*lb.*
25 pounds	"	1 quarter, "	*qr.*
4 quarters, or 100 lbs.	"	1 hundred weight,	*cwt.*
20 hund., or 2000 lbs.	"	1 ton, marked	*T.*

OBS. 1. *Gross* weight is the weight of goods with the boxes, casks, or bags which contain them, and allows 112 lbs. for a hundred weight

Net weight is the weight of the goods only.

2. Formerly it was the custom to allow 112 pounds for a hundred weight, and 28 pounds for a quarter; but this practice has become nearly or quite obsolete. The laws of most of the states, as well as general usage, call 100 lbs. a hundred weight, and 25 lbs. a quarter. In estimating duties, and weighing a few coarse articles, as iron, dye-woods, and coal at the mines, 112 lbs. are still allowed for a hundred weight. Coal, however, is sold in cities, at 100 lbs. for a hundred weight.

APOTHECARIES' WEIGHT.

93. *Apothecaries' Weight* is used by apothecaries and physicians in *mixing* medicines.

20 grains (*gr.*)	make	1 scruple, marked	*sc.* or ℈.
3 scruples	"	1 dram, "	*dr.* or ʒ.
8 drams	"	1 ounce, "	*oz.* or ℥.
12 ounces	"	1 pound, "	℔.

OBS. 1. The pound and ounce in this weight are the same as the *Troy* pound and ounce; the *subdivisions* of the *ounce* are different.

2. Drugs and medicines are bought and sold by *avoirdupois* weight.

QUEST.—92. In what is Avoirdupois Weight used? Recite the Table. *Obs.* What is gross weight? What is net weight? How many pounds were formerly allowed for a quarter? How many for a hundred weight? 93. In what is Apothecaries Weight used? Repeat the Table. *Obs.* To what are the Apothecaries' pound and ounce equal? How are drugs and medicines bought and sold?

LONG MEASURE.

94. *Long Measure* is used in measuring length or distances only, without regard to breadth or depth.

12 inches (*in.*)	make	1 foot,	marked	*ft.*
3 feet	"	1 yard,	"	*yd.*
5½ yards, or 16½ feet	"	1 rod, perch, or pole,		*r.* or *p.*
40 rods	"	1 furlong,	marked	*fur.*
8 furlongs, or 320 rods	"	1 mile,	"	*m.*
3 miles	"	1 league,	"	*l.*
60 geographical miles, or 69½ statute miles	"	1 degree,	"	*deg.* or °.

360 *deg.* make a great circle, or the circum. of the earth.

OBS. 1. 4 inches make a hand; 9 inches, 1 span; 18 inches, 1 cubit; 6 feet, 1 fathom; 4 rods, 1 chain; 25 links, 1 rod.

2. Long measure is frequently called *linear* or *lineal* measure. Formerly the inch was divided into 3 *barleycorns;* but the barleycorn, as a measure, has become *obsolete.* The inch is commonly divided either into *eighths*, or *tenths;* sometimes it is divided into *twelfths*, which are called *lines.*

CLOTH MEASURE.

95. *Cloth Measure* is used in measuring cloth, lace, and all kinds of goods, which are bought or sold by the yard.

2¼ inches (*in.*)	make	1 nail,	marked	*na.*
4 nails, or 9 in.	"	1 quarter of a yard,	"	*qr.*
4 quarters	"	1 yard,	"	*yd.*
3 quarters	"	1 Flemish ell,	"	*Fl. e.*
5 quarters	"	1 English ell,	"	*E. e.*
6 quarters	"	1 French ell,	"	*F. e.*

QUEST.—94. In what is Long Measure used? Repeat the Table. Draw a line an inch long upon your slate or black-board. Draw one two inches long. Draw another a foot long. Draw one a yard long. How long is your teacher's desk? How long is the school-room? How wide? *Obs.* What is Long Measure frequently called? How is the inch commonly divided at the present day? 95. In what is Cloth Measure used? Repeat the Table.

OBS. *Cloth* measure is a species of *long* measure. The yard is the same in both. Cloths, laces, &c., are bought and sold by the *linear* yard, without regard to their width.

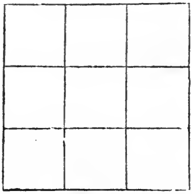
SQUARE MEASURE.

96. *Square Measure* is used in measuring surfaces, or things whose *length* and *breadth* are considered without regard to *height* or *depth ;* as land, flooring, plastering, &c.

144 square in. (*sq. in.*)	make	1 square foot,	marked	*sq. ft.*
9 square feet	"	1 square yard,	"	*sq. yd.*
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet	"	1 sq. rod, perch, or pole,	"	*sq. r.*
40 square rods	"	1 rood,	"	*R.*
4 roods, or 160 sq. rds.	"	1 acre,	"	*A.*
640 acres	"	1 square mile,	"	*M.*

OBS. 1. A *square* is a figure, which has *four equal* sides, and all its angles *right angles*, as seen in the adjoining diagram. Hence,

9 *sq. ft.*=1 *sq. yd.*



2. A *Square Inch* is a square, whose sides are each a *linear* inch in length.

A *Square Foot* is a square, whose sides are each a *linear* foot in length.

A *Square Yard* is a square, whose sides are each a *linear* yard or three *linear* feet in length, and contains 9 *square* feet, as represented in the adjacent figure.

3. In measuring land, surveyors use a *chain* which is 4 rods long, and is divided into 100 links. Hence, 25 links make 1 rod, and $7\frac{92}{100}$ inches make 1 link.

This chain is commonly called ***Gunter's Chain***, from the name of its inventor.

4 Square Measure is sometimes called *Land Measure*, because it is used in measuring land.

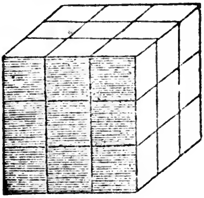
QUEST.—*Obs.* Of what is Cloth Measure a species? 96. In what is Square Measure used? Repeat the Table. *Obs.* What is a square? What is a square inch? What is a square foot? A square yard? Can you draw a square inch? Can you draw a square foot? A square yard?

CUBIC MEASURE.

97. *Cubic Measure* is used in measuring solid bodies, or things which have *length, breadth,* and *thickness;* such as timber, stone, boxes of goods, the capacity of rooms, &c.

1728 cubic inches (*cu. in.*)	make	1 cubic foot,	marked	*cu. ft.*
27 cubic feet	"	1 cubic yard,	"	*cu. yd.*
40 feet of round, or 50 ft. of hewn timber,	"	1 ton,	"	*T.*
42 cubic feet	"	1 ton of shipping,	"	*T.*
16 cubic feet	"	1 cord foot, or a foot of wood,	"	*c. ft.*
8 cord feet, or 128 cubic feet	"	1 cord,	"	*C.*

OBS. 1. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains 1 cord. For 8 into 4 into 4=128. 27 cu. ft.=1 cu. yd.



2. A *Cube* is a solid body bounded by *six equal squares.* It is often called a *hexaedron.* Hence,

A *Cubic Inch* is a cube, each of whose sides is a *square* inch, as represented by the adjoining figure.

A *Cubic Foot* is a cube, each of whose sides is a *square* foot.

3. The *Cubic Ton* is chiefly used for estimating the cartage and transportation of timber. By a *ton of round* timber is meant, such a quantity of timber in its rough or natural state, as when hewn, will make 40 cubic feet, and is supposed to be equal in weight to 50 feet of hewn timber.

4. The *cubic ton* or *load,* is by no means an *accurate* or *uniform* standard of estimating weight; for, different kinds of timber, are of very different degrees of density. But it is perhaps sufficiently accurate for the purposes to which it is applied.

QUEST.—97. In what is Cubic Measure used? Recite the Table. How long, wide, and high, must a pile of wood be to make a cord? What is a cube? What is a cubic inch? What is a cubic foot? Can you draw a cubic inch on your slate?

WINE MEASURE.

98. *Wine Measure* is used in measuring wine, alcohol, molasses, oil, and all other liquids except beer, ale, and milk.

4 gills (*gi.*)	make	1 pint,	marked	*pt.*
2 pints	"	1 quart,	"	*qt.*
4 quarts	"	1 gallon,	"	*gal*
$31\frac{1}{2}$ gallons	"	1 barrel,	"	*bar.* or *bbl.*
42 gallons	"	1 tierce,	"	*tier.*
63 gallons, or 2 bbls.	"	1 hogshead,	"	*hhd.*
2 hogsheads	"	1 pipe or butt,	"	*pi.*
2 pipes	"	1 tun,	"	*tun.*

Obs. The wine gallon contains 231 cubic inches.

BEER MEASURE.

99. *Beer Measure* is used in measuring beer, ale, and milk.

2 pints (*pt.*)	make	1 quart,	marked	*qt.*
4 quarts	"	1 gallon,	"	*gal.*
36 gallons	"	1 barrel,	"	*bar.* or *bbl.*
54 gals. or $1\frac{1}{2}$ bbls.	"	1 hogshead,	"	*hhd.*

Obs. The beer gallon contains 282 cubic inches. In many places milk is measured by wine measure.

DRY MEASURE.

100. *Dry Measure* is used in measuring grain, fruit, salt, &c.

2 pints (*pts.*)	make	1 quart,	marked	*qt.*
8 quarts	"	1 peck,	"	*pk.*
4 pecks, or 32 qts.	"	1 bushel,	"	*bu.*
8 bushels	"	1 quarter,	"	*qr.*
32 bushels	"	1 chaldron,	"	*ch.*

Note.—In England, 36 bushels of coal make a chaldron.

Quest.—98. In what is Wine Measure used? Recite the Table. *Obs.* How many cubic inches in a wine gallon? 99. In what is Beer Measure used? Repeat the Table. *Obs.* How many cubic inches in a beer gallon?

TIME.

101. *Time* is naturally divided into *days* and *years;* the former are caused by the revolution of the Earth on its axis, the latter by its revolution round the Sun.

60 seconds (*sec.*)	make	1 minute,	marked	*min.*
60 minutes	"	1 hour,	"	*hr.*
24 hours	"	1 day,	"	*d.*
7 days	"	1 week,	"	*wk.*
4 weeks	"	1 lunar month,	"	*mo.*
12 calendar months, or 365 days, 6 hrs., (nearly,)	"	1 civil year,	"	*yr.*
13 lunar mo., or 52 weeks,	"	1 year,	"	*yr.*
100 years	"	1 century,	"	*cen.*

OBS. 1. Time is measured by clocks, watches, chronometers, dials, hour-glasses, &c.

2. A *civil* year is a *legal* or *common* year; a period of time established by government for civil or common purposes.

3. A *solar* year is the time in which the earth revolves round the sun, and contains 365 days, 5 hours, 48 min., and 48 sec.

4. A *leap* year, sometimes called *bissextile*, contains 366 days, and occurs once in *four* years.

It is caused by the excess of 6 hours, which the civil year contains above 365 days, and is so called because it *leaps* or *runs* over one day more than a common year. The odd day is added to February, because it is the shortest month. Every leap year, therefore, February has 29 days.

102. The names of the days are derived from the names of certain Saxon deities, or objects of worship. Thus,

Sunday is named from the *sun*, because this day was dedicated to its worship.

Monday is named from the *moon*, to which it was dedicated.

QUEST.—100. In what is Dry Measure used? Recite the table. 101. How is Time naturally divided? How are the former caused? How the latter? Repeat the Table. *Obs.* How is Time measured? What is a civil year? A solar year? A leap year? How is Leap Year caused? To which month is the odd day added? From what are the names of the days derived?

Tuesday is derived from *Tuisco*, the Saxon god of war.
Wednesday is derived from *Woden*, a deity of northern Europe.
Thursday is from *Thor*, the Danish god of thunder, storms, &c.
Friday is from *Friga*, the Saxon goddess of beauty.
Saturday is from the planet *Saturn*, to which it was dedicated.

103. The following are the names of the 12 calendar months, with the number of days in each:

January,	(Jan.)	the *first*	month,	has	31	days.
February,	(Feb.)	" *second*	"	"	28	"
March,	(Mar.)	" *third*	"	"	31	"
April,	(Apr.)	" *fourth*	"	"	30	"
May,	(May)	" *fifth*	"	"	31	"
June,	(June)	" *sixth*	"	"	30	"
July,	(July)	" *seventh*	"	"	31	"
August,	(Aug.)	" *eighth*	"	"	31	"
September,	(Sept.)	" *ninth*	"	"	30	"
October,	(Oct.)	" *tenth*	"	"	31	"
November,	(Nov.)	" *eleventh*	"	"	30	"
December,	(Dec.)	" *twelfth*	"	"	31	"

OBS. 1. The number of days in each month may be easily remembered from the following lines:

"Thirty days hath September,
April, June, and November;
February twenty-eight alone,
All the rest have thirty-one;
Except in Leap Year, then is the time,
When February has twenty-nine."

2. The names of the calendar months were borrowed from the Romans, and most of them had a fanciful origin. Thus,

January was named after *Janus*, a Roman deity, who was supposed to preside over the year, and the commencement of all undertakings.

February was derived from *februo*, a Latin word which signifies to purify by sacrifice, and was so called because this month was devoted to the purification of the people.

QUEST.—103. What is the origin of the names of the months?

March was named after *Mars*, the Roman god of war; and was originally the first month of the Roman year.

April, from the Latin *aperio, to open*, was so called from the opening of buds, blossoms, &c., at this season.

May was named after the goddess *Maia*, the mother of *Mercury*, to whom the ancients used to offer sacrifices on the *first day* of this month.

June was named after the goddess *Juno*, the wife of Jupiter.

July was so called in honor of *Julius Cæsar*, who was born in this month.

August was so called in honor of *Augustus Cæsar*, a Roman Emperor, who entered upon his first consulate in this month.

September, from the Latin numeral *septem, seven*, was so called, because it was originally the *seventh* month of the Roman year. It is the *ninth* month in our year.

October, from the Latin *octo, eight*, was so called because it was the *eighth* month of the Roman year.

November, from the Latin *novem, nine*, was so called because it was the *ninth* month of the Roman year.

December, from the Latin *decem, ten*, was so called because it was the *tenth* month of the Roman year.

104. The year is also divided into *four seasons* of three months each, viz: *Spring*, *Summer*, *Autumn* or *Fall*, and *Winter*.

Spring comprises March, April, and May; *Summer*, June, July, and August; *Autumn* or *Fall*, September, October, and November; *Winter*, December, Jan. and Feb.

CIRCULAR MEASURE.

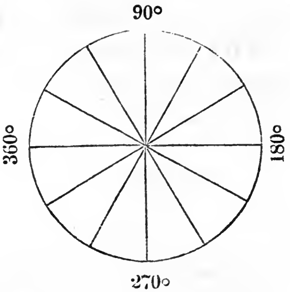
105. *Circular Measure* is applied to the divisions of the circle, and is used in reckoning latitude and longitude, and the motion of the heavenly bodies.

60 seconds (″)	make 1 minute,	marked	′
60 minutes	" 1 degree,	"	°
30 degrees	" 1 sign,	"	*s.*
12 signs, or 360°	" 1 circle,	"	*c.*

Quest.—104. Name the seasons. 105. To what is Circular Measure applied?

OBS. 1. Circular Measure is often called *Angular Measure*, and is chiefly used by astronomers, navigators, and surveyors.

2. The circumference of every circle is divided, or supposed to be divided, into 360 equal parts, called *degrees*, as in the subjoined figure.



3. Since a degree is $\frac{1}{360}$ part of the circumference of a circle, it is obvious that its *length* must depend on the *size* of the circle

MISCELLANEOUS TABLE.

106. The following denominations not included in the preceding Tables, are frequently used.

12 units	make	1 dozen, (*doz.*)
12 dozen, or 144	"	1 gross.
12 gross, or 1728	"	1 great gross.
20 units	"	1 score.
56 pounds	"	1 firkin of butter.
100 pounds	"	1 quintal of fish.
30 gallons	"	1 bar. of fish in Mass.
200 lbs. of shad or salmon	"	1 bar. in N. Y. and Ct.
196 pounds	"	1 bar. of flour.
200 pounds	"	1 bar. of pork.
14 pounds of iron, or lead	"	1 stone.
21½ stone	"	1 pig.
8 pigs	"	1 fother.

OBS. Formerly 112 pounds were allowed for a quintal.

QUEST.—*Obs.* What is Circular Measure sometimes called? By whom is it chiefly used? Into what is the circumference of every circle divided? On what does the length of a degree depend? 106. How many units make a dozen? How many dozen a gross? A great gross? How many units make a score? Pounds a firkin?

PAPER AND BOOKS.

107. The terms, *folio, quarto, octavo,* &c., applied to books, denote the *number* of leaves into which a sheet of paper is folded.

24 sheets of paper	make	1 quire.
20 quires	"	1 ream.
2 reams	"	1 bundle.
5 bundles	"	1 bale.

A sheet folded in two leaves, is called a *folio.*
A sheet folded in four leaves, is called a *quarto,* or 4*to.*
A sheet folded in eight leaves, is called an *octavo,* or 8*vo.*
A sheet folded in twelve leaves, is called a *duodecimo.*
A sheet folded in sixteen leaves, is called a 16*mo.*
A sheet folded in eighteen leaves, is called an 18*mo.*
A sheet folded in thirty-two leaves, is called a 32*mo.*
A sheet folded in thirty-six leaves, is called a 36*mo.*
A sheet folded in forty-eight leaves, is called a 48*mo.*

108. Previous to the adoption of Federal money in 1786, accounts in the United States were kept in pounds shillings, pence, and farthings.

In New England currency, Virginia, Kentucky, Tennessee. Indiana, Illinois, Missouri, and Mississippi,	6 shil. make $1.
In New York currency, North Carolina, Ohio, and Michigan,	8 shil. make $1.
In Pennsylvania currency, New Jersey, Delaware, and Maryland,	7s. 6d. make $1.
In Georgia currency and South Carolina,	4s. 8d. make $1.
In Canada currency, and Nova Scotia,	5 shil. make $1.

QUEST.—107. When a sheet of paper is folded in two leaves, what is it called? When in four leaves, what? When in eight? In twelve? In sixteen? In eighteen? In thirty-six? 108. Previous to the adoption of Federal Money, in what were accounts kept in the U. S.? How many shillings make a dollar in N. E. currency? In N. Y. currency? In Penn. currency? In Georgia currency? In Canada currency?

OBS. At the time Federal money was adopted, the *colonial currency* or *bills of credit* issued by the colonies, had more or less *depreciated* in value: that is, a colonial pound was worth less than a pound Sterling; a colonial shilling, than a shilling Sterling, &c. This depreciation being greater in some colonies than in others gave rise to the *different values* of the *State currencies*.

ALIQUOT PARTS OF $1 IN FEDERAL MONEY.

50 cents = $\$\frac{1}{2}$	$12\frac{1}{2}$ cents = $\$\frac{1}{8}$
$33\frac{1}{3}$ cents = $\$\frac{1}{3}$	10 cents = $\$\frac{1}{10}$
25 cents = $\$\frac{1}{4}$	$8\frac{1}{3}$ cents = $\$\frac{1}{12}$
20 cents = $\$\frac{1}{5}$	$6\frac{1}{4}$ cents = $\$\frac{1}{16}$
$16\frac{2}{3}$ cents = $\$\frac{1}{6}$	5 cents = $\$\frac{1}{20}$

PARTS OF $1 IN NEW YORK CURRENCY.

4 shillings = $\$\frac{1}{2}$	1 shil. 4 pence = $\$\frac{1}{6}$
2 shil. 8 pence = $\$\frac{1}{3}$	1 shilling = $\$\frac{1}{8}$
2 shillings = $\$\frac{1}{4}$	6 pence = $\$\frac{1}{16}$

OBS. 1. In New York currency, it will be seen, (Art. 108,) that

A six-pence,	written	6d. =	$6\frac{1}{4}$ cents.
A shilling,	"	1s. =	$12\frac{1}{2}$ "
One (shil.) and 6 pence,	"	1/6. =	$18\frac{3}{4}$ "
Two shillings,	"	2s. =	25 "

PARTS OF $1 IN NEW ENGLAND CURRENCY.

3 shillings = $\$\frac{1}{2}$	1 shilling = $\$\frac{1}{6}$
2 shillings = $\$\frac{1}{3}$	9 pence = $\$\frac{1}{8}$
1 shil. and 6d. = $\$\frac{1}{4}$	6 pence = $\$\frac{1}{12}$

OBS. 2. In New England currency, it will be seen, that

A four-pence-half-penny,	written	$4\frac{1}{2}$d. =	$6\frac{1}{4}$ cents.
A six-pence,	"	6d. =	$8\frac{1}{3}$ "
A nine-pence,	"	9d. =	$12\frac{1}{2}$ "
A shilling,	"	1s. =	$16\frac{2}{3}$ "
One (shil.) and six-pence,	"	1/6. =	25 "
Two shillings,	"	2s. =	$33\frac{1}{3}$ "

QUEST.—What are the aliquot parts of $1 in Federal Money? In New York currency? In New England currency? What are the aliquot parts of a pound Sterling? Of a shilling?

ALIQUOT PARTS OF STERLING MONEY.

Parts of £1.		*Parts of 1s.*	
10 shil.	= £$\frac{1}{2}$	6 pence	= $\frac{1}{2}$ shil.
6s. 8d.	= £$\frac{1}{3}$	4 pence	= $\frac{1}{3}$ shil.
5 shil.	= £$\frac{1}{4}$	3 pence	= $\frac{1}{4}$ shil.
4 shil.	= £$\frac{1}{5}$	2 pence	= $\frac{1}{6}$ shil.
3s. 4d.	= £$\frac{1}{6}$	$1\frac{1}{2}$ pence	= $\frac{1}{8}$ shil.
2s. 6d.	= £$\frac{1}{8}$	1 penny	= $\frac{1}{12}$ shil.
2 shil.	= £$\frac{1}{10}$	1 far.	= $\frac{1}{4}$ penny.
1s. 8d.	= £$\frac{1}{12}$	2 far.	= $\frac{1}{2}$ penny.
1 shil.	= £$\frac{1}{20}$	3 far.	= $\frac{3}{4}$ penny.

ALIQUOT PARTS OF A TON.

10 hund. lbs. = $\frac{1}{2}$ ton.	2 hund. 2 qrs. = $\frac{1}{8}$ ton.
5 hund. lbs. = $\frac{1}{4}$ ton.	2 hund. lbs. = $\frac{1}{10}$ ton.
4 hund. lbs. = $\frac{1}{5}$ ton.	1 hund. lbs. = $\frac{1}{20}$ ton.

ALIQUOT PARTS OF A POUND AVOIRDUPOIS.

8 ounces = $\frac{1}{2}$ pound.	2 ounces = $\frac{1}{8}$ pound.
4 ounces = $\frac{1}{4}$ pound.	1 ounce = $\frac{1}{16}$ pound.

ALIQUOT PARTS OF TIME.

Parts of 1 year.	*Parts of 1 month.*
6 months = $\frac{1}{2}$ year.	15 days = $\frac{1}{2}$ month.
4 months = $\frac{1}{3}$ year.	10 days = $\frac{1}{3}$ month.
3 months = $\frac{1}{4}$ year.	6 days = $\frac{1}{5}$ month.
2 months = $\frac{1}{6}$ year.	5 days = $\frac{1}{6}$ month.
$1\frac{1}{2}$ month = $\frac{1}{8}$ year.	3 days = $\frac{1}{10}$ month.
$1\frac{1}{3}$ month = $\frac{1}{9}$ year.	2 days = $\frac{1}{15}$ month.
1 month = $\frac{1}{12}$ year.	1 day = $\frac{1}{30}$ month.

QUEST.—How many shillings in half a pound Ster.? In a fourth? A fifth? A tenth? A twentieth? How many pence in half a shilling? In a third? A fourth? A sixth? A twelfth? How many hundreds in half a ton? In a fourth? A fifth? A tenth? How many ounces in half a pound? In a fourth? An eighth? A sixteenth? How many months in half a year? In a third? A fourth? A sixth? A twelfth?

SECTION VIII.

FEDERAL MONEY.

110. Accounts in the United States are kept in *dollars*, *cents*, and *mills*. Eagles are expressed in dollars, and dimes in cents. Thus, instead of 4 eagles, we say, 40 dollars; instead of 5 dimes, we say, 50 cents, &c.

111. Dollars are separated from cents by placing a *point* or *separatrix* (.) between them. Hence,

112. To read Federal Money.

Call all the figures on the left of the point, dollars; the first two figures on the right of the point, are cents; the third figure denotes mills; the other places on the right are parts of a mill. Thus, $5.2523 is read, 5 dollars, 25 cents, 2 mills, and 3 tenths of a mill.

OBS. 1. Since *two* places are assigned to *cents*, when no cents are mentioned in the given number, *two ciphers* must be placed before the mills. Thus, 5 dollars and 7 mills are written $5.007.

2. If the given number of cents is *less* than ten, a *cipher* must always be written before them. Thus, 8 cents are written .08, &c.

1. Read the following expressions: $83.635; $75.50. $126.607; $268.05; $382.005; $2160.

2. Write the following sums: Sixty dollars and fifty cents. Seventy-five dollars, eight cents, and three mills. Forty-eight dollars and seven mills. Nine cents. Six cents and four mills.

QUEST.—88. What is Federal Money? What are its denominations? Recite the Table. 110. How are accounts kept in the United States? How are Eagles expressed? Dimes? 111. How are dollars distinguished from cents and mills? 112. How do you read Federal Money? *Obs.* How many places are assigned to cents? When the number of cents is less than ten, what must be done? When no cents are mentioned, what do you do?

REDUCTION OF FEDERAL MONEY.

CASE I.

Ex. 1. How many cents are there in 65 dollars?

Suggestion.—Since in 1 dollar there are 100 cents, in 65 dollars there are 65 times as many. Now, to multiply by 10, 100, &c., we annex as many ciphers to the multiplicand, as there are ciphers in the multiplier. (Art. 45.) Hence,

Operation.
```
65
 100
6500 cents.
```

113. *To reduce dollars to cents, annex* TWO *ciphers.*
To reduce dollars to mills, annex THREE *ciphers.*
To reduce cents to mills, annex ONE *cipher.*

OBS. To reduce dollars and cents to cents, *erase the sign of dollars and the separatrix.* Thus, $25.36 reduced to cents, become 2536 cents.

2. Reduce $4 to cents. *Ans.* 400 cents.
3. Reduce $15 to cents.
4. Reduce $27 to cents.
5. Reduce $85 to cents.
6. Reduce $93 to cents.
7. Reduce $96 to mills.
8. Reduce $12.23 to cents.
9. Reduce $86.86 to cents.
10. Reduce $9.437 to mills.

CASE II.

1. In 2345 cents, how many dollars?

Suggestion.—Since 100 cents make 1 dollar, 2345 cents, will make as many dollars as 100 is contained times in 2345. Now to divide by 10, 100, &c., we cut off as many figures from the right of the dividend as there are ciphers in the divisor. (Art. 67.) Hence,

Operation.
```
1|00)23|45
Ans. $23.45
```

QUEST.—113. How are dollars reduced to cents? Dollars to mills? Cents to mills? *Obs.* Dollars and cents to cents?

114. To reduce cents to dollars.

Point off TWO *figures on the right; the figures remaining on the left express dollars; the two pointed off, cents.*

115. To reduce mills to dollars.

Point off THREE *figures on the right; the remaining figures express dollars; the first two on the right of the point, cents; the third one, mills.*

116. To reduce mills to cents.

Point off ONE *figure on the right, and the remaining figures express cents; the one pointed off, mills.*

2. Reduce 236 cts. to dolls.
3. Reduce 2163 cts. to dolls.
4. Reduce 865 mills to dolls.
5. Reduce 906 mills to cts.
6. Reduce 2652 cts. to dolls.
7. Reduce 3068 cts. to dolls.

ADDITION OF FEDERAL MONEY.

Ex. 1. What is the sum of $8.125, $12.67, $3.098, $11?

Suggestion.—Write the dollars under dollars, cents under cents, mills under mills, and proceed as in Simple Addition. From the right of the amount point off *three* figures for cents and mills.

Operation.

	$ 8.125
	12.67
	3.098
	11.00
Ans.	$34.893

117. Hence, we derive the following general

RULE FOR ADDING FEDERAL MONEY.

Write dollars under dollars, cents under cents, mills under mills, and add each column, as in simple numbers.

From the right of the amount, point off as many figures for cents and mills, as there are places of cents and mills in either of the given numbers.

QUEST.—114. How are cents reduced to dollars? 115. Mills to dollars? 117. How do you add Federal Money? How point off the amount?

OBS. If either of the given numbers have no cents expressed, supply their place by ciphers.

(2.)	(3.)	(4.)	(5.)
$375.037	$4869.45	$760.275	$4607.375
60.20	344.00	897.008	896.084
843.462	6048.07	965.054	95.873

(6.)	(7.)	(8.)	(9.)
$782.206	$609.352	$2903.76	$4668.253
84.60	830.206	453.06	430.064
379.007	408.07	25.89	307.60
498.015	631.107	6842.07	7452.349

10. What is the sum of $63.072, $843.625, and $71.60?

11. Add $873.035, $386.23, $608.938, $169.176.

12. Add 463 dolls. 7 cts.; 248 dolls. 15 cts.; 169 dolls. 9 cts. 7 mills.

13. Add 89 dolls. 8 cts.; 97 dolls. 10 cts. 3 mills; 40 dolls. 6 cts.; 75 dolls.

14. Add 365 dolls. 20 cts. 2 mills; 68 dolls. 6 cts. 5 mills; 7 cts. 3 mills; 286 dolls.; 80 dolls. 6 mills; 30 dolls. 15 cts.

15. A man bought a cow for $16.375, a calf for $4.875, and a ton of hay for $13.50: how much did he pay for the whole?

16. A lady paid $23 for a cloak, $7.625 for a hat, $25.75 for a muff, and $18 for a tippet: how much did she pay for all?

17. A farmer sold a cow for $16.80, a calf for $4.08, a horse for $78, and a yoke of oxen for $63.18: how much did he receive for all?

QUEST.—*Obs.* When any of the given numbers have no cents expressed how is their place supplied?

SUBTRACTION OF FEDERAL MONEY.

Ex. 1. What is the difference between $845.634, and $86.087?

Suggestion.—Write the less number under the greater, dollars under dollars, &c., then subtract, and point off the answer as in addition of Federal Money.

Operation.

	$845.634
	86.087
Ans.	$759.547.

118. Hence, we derive the following general

RULE FOR SUBTRACTING FEDERAL MONEY.

Write the less number under the greater, with dollars under dollars, cents under cents, and mills under mills; then subtract, and point off the answer as in addition of Federal Money.

	(2.)	(3.)	(4.)	(5.)
From	$856.453	$960.78	$605.607	$6243.760
Take	$387.602	$463.05	78.36	327.053

	(6.)	(7.)	(8.)	(9.)
From	965.005	840.000	483.853	4265.76
Take	87.85	378.457	48.75	2803.98

10. From $86256.63 take $4275.875?
11. From $100250, take $32578.867?
12. From 1 dollar, subtract 11 cents.
13. From 3 dolls. 6 cts. 7 mills, take 75 cents.
14. From 110 dolls. 8 mills, take 60 dolls. and 8 cents.
15. From 607 dolls. 7 cents, take 250 dolls. and 3 cts.
16. A lad bought a cap for $2.875, and paid a five-dollar-bill; how much change ought he to receive back?
17. Henry has $7.68, and William has $9.625: how much more has the latter than the former?
18. From $865275.60, take $340076.875.

QUEST.—106. How do you subtract Federal Money? How point off the answer?

MULTIPLICATION OF FEDERAL MONEY.

Ex. 1. What will 3 caps cost, at $1.625 apiece?

Suggestion.—Since 1 cap costs $1.625, 3 caps will cost 3 times as much. We therefore multiply the price of 1 cap by 3, the number of caps, and point off three places for cents and mills. Hence,

Operation.

	$1.625
	3
Ans.	$4.875

119. When the multiplier is a whole number, we have the following

RULE FOR MULTIPLYING FEDERAL MONEY.

Multiply as in simple numbers, and from the right of the product, point off as many figures for cents and mills, as there are places of cents and mills in the multiplicand.

OBS. 1. In Multiplication of Federal Money, as well as in simple numbers, the *multiplier* must always be considered an *abstract number.*

2. In business operations, when the mills are 5 or over, it is customary to call them a cent; when under 5, they are disregarded.

	(2.)	(3.)	(4.)	(5.)
Multiply	$633.75	$805.625	$879.075	$9071.26
By	8	9	24	37

	(6.)	(7.)	(8.)	(9.)
Multiply	$4063.36	$5327.007	$6286.69	$8265.68
By	63	86	123	264

10. What cost 8 melons, at 17 cents apiece?
11. What cost 12 lambs, at 87 cents apiece?
12. What cost 8 hats, at $3.875 apiece?
13. At $8.75 a yard, what will 9 yards of silk come to?
14. At $1.125 apiece, what will 11 turkeys cost?

QUEST.—119. How do you multiply Federal Money? How point off the product? *Obs.* What must the multiplier always be considered? When the mills are 5, or over, what is it customary to call them? When less than 5, what may be done with them?

15. At $2.63 apiece, what will 15 chairs come to?

16. What costs 25 Arithmetics, at 37½ cents apiece?

17. What cost 38 Readers, at 62½ cents apiece?

18. What cost 46 over-coats, at $25.68 apiece?

19. What cost 69 oxen, at $48.50 a head?

20. At $23 per acre, what cost 65 acres of land?

21. At $75.68 apiece, what will 56 horses come to?

22. At 7½ cents a mile, what will it cost to ride 100 miles?

23. A farmer sold 84 bushels of apples, at 87½ cents per bushel: what did they come to?

24. If I pay $5.37½ per week for board, how much will it cost to board 52 weeks?

DIVISION OF FEDERAL MONEY.

Ex. 1. If you paid $18.876 for 3 barrels of flour, how much was that a barrel?

Operation.

3) $18.876

Ans. $6.292.

Suggestion.—Since 3 barrels cost $18.876, 1 barrel will cost 1 third as much. We therefore divide as in simple division, and point off *three* places for cents and mills, because there are three in the dividend. Hence,

120. When the divisor is a whole number, we have the following

RULE FOR DIVIDING FEDERAL MONEY.

Divide as in simple numbers, and from the right of the quotient, point off as many figures for cents and mills, as there are places of cents and mills in the dividend.

OBS. When the dividend contains no cents and mills, if there is a remainder *annex three ciphers* to it, then divide as before, and point off *three figures* in the quotient.

QUEST.—120. How do you divide Federal Money? How point off the quotient? *Obs.* When the dividend contains no cents and mills, how proceed?

Note.—For a more complete development of *multiplication* and *division* of Federal Money, the learner is referred to the author's Practical and Higher Arithmetics.

When the *multiplier* or *divisor* contain decimals, or cents and mills, to understand the operation fully, requires a thorough knowledge of Decimal Fractions, a subject which the limits of this work will not allow us to introduce.

(2.)	(3.)	(4.)
6) $856.272.	8) $9567.648.	9) $7254.108.

5. Divide $868.36 by 27.
6. Divide $3674.65 by 38.
7. Divide $486745 by 49.
8. Divide $634.075 by 56.
9. Divide $6634.25 by 60.
10. Divide $5340.73 by 78.
11. Divide $7643.85 by 83.
12. Divide $4389.75 by 89.
13. Divide $836847 by 94.
14. Divide $94321.62 by 97.

15. A man paid $2563.84 for 63 sofas: what was that apiece?

16. A miller sold 86 barrels of flour for $526.50: how much was that per barrel?

17. If a man pays $475.56 for 65 barrels of pork, what is that per barrel?

18. A man paid $1875.68 for 93 stoves: how much was that apiece?

19. If $2682.56 are equally divided among 100 men, how much will each receive?

20. A cabinet-maker sold 116 tables for $968.75: how much did he get apiece?

21. A farmer sold 168 sheep for $465: how much did he receive apiece for them?

22. A miller bought 216 bushels of wheat for $375.50: how much did he pay per bushel?

23. If $2368.875 were equally divided among 348 persons, how much would each person receive?

SECTION IX.

REDUCTION.

Art. **121.** Reduction is the process of changing *Compound Numbers* from one denomination into another, without altering their *value*.

REDUCING HIGHER DENOMINATIONS TO LOWER.

122. Ex. 1. Reduce £2, to farthings.

Suggestion.—First reduce the given pounds (2) to shillings, by multiplying them by 20, because 20s. make £1. Next reduce the shillings (40) to pence, by multiplying them by 12, because 12d. make 1s. Reduce the pence (480) to farthings, by multiplying them by 4, because 4 far. make 1d.

Operation.

£2
20s. in £1.
40 shillings.
12d. in 1s.
480 pence.
4 far. in 1d.
Ans. 1920 farthings.

2. Reduce £1, 2s. 4d. and 3 far. to farthings.

Suggestion.—In this example there are shillings, pence, and farthings. Hence, when the pounds are reduced to shillings, the given shillings (2) must be added mentally to the product. When the shillings are reduced to pence, the given pence (4) must be added; and when the pence are reduced to farthings, the given farthings (3) must be added.

Operation.

£ *s.* *d.* *far.*
1 2 4 3
20s. in £1.
22 shillings.
12d. in 1s.
268 pence.
4 far. in 1d
Ans. 1075 farthings

Quest.—121. What is reduction? 122. Ex. 1. How reduce pounds to shillings? Why multiply by 20? How are shillings reduced to pence? Why? How pence to farthings? Why?

OBS. In these examples it is required to reduce *higher* denominations to *lower*, as pounds to shillings, shillings to pence, &c.

123. The process of reducing *higher* denominations to *lower*, is usually called *Reduction Descending.*

It consists in *successive multiplications*, and may with propriety be called *Reduction by Multiplication.*

124. From the preceding illustrations we derive the following

RULE FOR REDUCTION DESCENDING.

Multiply the highest denomination given by the number required of the next lower denomination to make ONE *of this higher, and to the product, add the given number of this lower denomination.*

Proceed in this manner with each successive denomination, till you come to the one required.

EXAMPLES.

3. Reduce 4 miles, 2 fur., 8 rods and 4 feet to feet.

Operation.

```
 m. fur. r. ft.
 4   2   8  4
 8
 --
 34 furlongs.
 40
 ----
2)1368 rods.
   16½
 ----
 8212
1368
  684
 -----
Ans. 22576 feet.
```

Suggestion.—Having reduced the miles and furlongs to rods, we have 1368 rods. We then multiply by $16\frac{1}{2}$, because $16\frac{1}{2}$ feet make 1 rod. (Art. 94.) Now $16\frac{1}{2}$ is a mixed number; we therefore first multiply by the whole number (16), then by the fraction ($\frac{1}{2}$), and add the products together. (Art. 84.)

QUEST.—123. What is reducing compound numbers to lower denominations usually called? Which of the fundamental rules is employed in reduction descending? 124. What is the rule for Reduction Descending?

4. In £5, 16s. 7d., how many farthings? *Ans.* 5596 far.
5. In £18 how many pence?
6. In £23, 9s., how many shillings?
7. In 17s. 2d. 3 far., how many farthings?
8. Reduce 5 lbs. 6 oz. Troy weight, to grains. *Ans.* 31680 grs.
9. Reduce 13 lbs. Troy, to ounces.
10. Reduce 4 lbs. 3 oz. Troy, to penny weights.
11. Reduce 15 oz. 6 pwts. 4 grs. Troy, to grains.
12. In 2 cwt. 3 qrs. 7 lbs. 4 oz. 3 drams, avoirdupois weight, how many drams? *Ans.* 72259 drams.
13. In 13 lbs. 4 oz. avoirdupois, how many ounces?
14. In 2 qrs. 17 lbs. avoirdupois, how many pounds?
15. In 6 lbs. 12 oz. avoirdupois, how many drams?
16. In 12 cwt. 1 qr. 6 lbs. avoir., how many ounces?
17. In 16 miles, how many rods?
18. In 28 rods and 2 feet, how many inches?
19. In 19 fur. 4 rods and 2 yds., how many feet?
20. In 25 leagues and 2 m., how many rods?
21. Reduce 14 yards cloth measure to quarters.
22. Reduce 21 yards 2 quarters to nails.
23. Reduce 17 yards 3 quarters 2 nails, to nails.
24. How many quarts in 23 gallons, wine measure?
25. How many gills in 30 gallons and 2 quarts?
26. How many gills in 63 gallons?
27. How many quarts in 41 hogsheads?
28. How many pecks in 45 bushels?
29. How many pints in 3 pecks and 2 quarts?
30. How many quarts in 52 bu. and 2 pecks?
31. How many hours in 15 weeks?
32. How many minutes in 25 days?
33. How many seconds in 265 hours?
34. How many minutes in 52 weeks?
35. How many seconds in 68 days?

REDUCING LOWER DENOMINATIONS TO HIGHER.*

125. Ex. 1. Reduce 1920 farthings to pounds.

Suggestion.—First reduce the given farthings (1920) to pence, the next higher denomination, by dividing them by 4, because 4 far. make 1d. Next reduce the pence (480) to shillings, by dividing them by 12, because 12d. make 1s. Finally, reduce the shillings (40) to pounds, by dividing them by 20, because 20s. make £1. The answer is £2. That is, 1920 far. are equal to £2.

Operation.

```
 4)1920 far.
12)480d.
20)40s.
   £2 Ans.
```

2. In 1075 farthings, how many pounds?

Suggestion.—In dividing the given farthings by 4, there is a remainder of 3 far., which should be placed on the right. In dividing the pence (268) by 12, there is a remainder of 4d., which should also be placed on the right. In dividing the shillings (22) by 20, the quotient is £1 and 2s. over. The last quotient with the several remainders is the answer. That is, 1075 far. are equal to £1, 2s. 4d. 3 far.

Operation.

```
 4)1075 far.
12)268d. 3 far. over.
20)22s. 4d. over.
   £1, 2s. over.
Ans. £1, 2s. 4d. 3 far.
```

Obs. In the last two examples, it is required to reduce *lower* denominations to *higher*, as farthings to pence, pence to shillings, &c. The operation is exactly the *reverse* of that in Reduction Descending.

126. The process of reducing *lower* denominations to *higher* is usually called *Reduction Ascending*.

It consists in *successive divisions*, and may with propriety be called *Reduction by Division*.

Quest.—125. Ex. 1. How are farthings reduced to pence? Why divide by 4? How reduce pence to shillings? Why? How shillings to pounds? Why? 126. What is reducing compound numbers to higher denominations usually called? Which of the fundamental rules is employed in Reduction Ascending?

127. From the preceding illustrations we derive the following

RULE FOR REDUCTION ASCENDING.

Divide the given denomination by that number which it takes of this denomination to make ONE *of the next higher. Proceed in this manner with each successive denomination, till you come to the one required. The last quotient, with the several remainders, will be the answer sought.*

128. PROOF.—*Reverse the operation; that is, reduce back the answer to the original denominations, and if the result correspond with the numbers given, the work is right.*

OBS. Each remainder is of the same denomination as the dividend from which it arose. (Art. 51, Obs. 2.)

EXAMPLES.

3. In 429 feet, how many rods?

Suggestion.—We first reduce the feet to yards, then reduce the yards to rods by dividing them by $5\frac{1}{2}$. (Art. 86.)

Or, we may divide the given feet by $16\frac{1}{2}$, the number of feet in a rod, and the quotient will be the answer.

Operation.

3)429 feet.
$5\frac{1}{2}$)143 yds.
2 2
11)286
Ans. 26 rods

Proof.

26 rods.
$5\frac{1}{2}$
130
13
143 yds.
3
429 feet.

We first reduce the rods back to yards, (Art. 84,) then reduce the yards to feet. The result is 429 feet, which is the same as the given number of feet.

Or, we may multiply the 26 by $16\frac{1}{2}$, and the product will be 429.

4. Reduce 256 pence to pounds. *Ans.* £1, 1s. 4d.

5. Reduce 324 pence to shillings.

QUEST.—127. What is the rule for reduction ascending? 128. How is reduction proved? *Obs.* Of what denomination is each remainder?

6. Reduce 960 farthings to shillings.
7. Reduce 1250 farthings to pounds.
8. In 265 ounces Troy weight, how many pounds?
9. In 728 pwts., how many pounds Troy?
10. In 548 grains, how many ounces Troy?
11. In 638 oz. avoirdupois weight, how many pounds?
12. In 736 lbs. avoirdupois, how many quarters?
13. In 1675 oz. avoirdupois, how many hundred weight?
14. In 1000 drams avoirdupois, how many pounds?
15. In 4000 lbs. avoirdupois, how many tons?
16. How many yards in 865 inches?
17. How many rods in 1000 feet?
18. How many miles in 2560 rods?
19. How many miles in 3261 yards?
20. How many leagues in 2365 rods?

EXAMPLES IN REDUCTION ASCENDING AND DESCENDING.

129. In solving the following examples, the pupil must first consider whether the question requires *higher* denominations to be reduced to *lower*, or *lower* denominations to *higher*. Having settled this point, he will find no difficulty in applying the proper rule.

FEDERAL MONEY. (ART. 88.)

1. In 3 dollars and 16 cents, how many cents?
2. In 81 cents and 2 mills, how many mills?
3. In 245 cents, how many dollars?
4. In 321 mills, how many dimes?
5. In 95 eagles, how many cents?
6. In 160 dollars, how many cents?
7. In 317 dollars, how many dimes?
8. In 4561 mills, how many dollars?
9. In 8250 cents, how many eagles?
10. In 61 dolls., 12 cts., and 3 mills, how many mills?

STERLING MONEY. (ART. 90.)

11. Reduce £17, 16s. to shillings.
12. Reduce 19s. 6d. 2 far. to farthings.
13. Reduce 1200 pence to pounds.
14. Reduce 3626 farthings to shillings.
15. Reduce £19 to farthings.
16 Reduce 2880 farthings to shillings.
17 Reduce £21, 3s. 6d. to pence.
18 Reduce 3721 farthings to pounds.

TROY WEIGHT. (ART. 91.)

19. In 7 lbs., how many ounces?
20. In 9 lbs. 2 oz., how many pennyweights?
21. In 165 oz., how many pounds?
22. In 840 grains, how many ounces?
23. In 3 lbs. 5 oz. 2 pwts. 7 grs., how many grains?
24. In 6860 grains, how many pounds?

AVOIRDUPOIS WEIGHT. (ART. 92.)

25. In 200 oz., how many pounds?
26. In 261 lbs., how many ounces?
27. In 3 tons, 2 cwt., how many pounds?
28. In 1 cwt. 2 qrs., how many ounces?
29. In 1000 oz., how many pounds?
30. In 4256 lbs., how many tons?

APOTHECARIES' WEIGHT. (ART. 93.)

31. Reduce 45 pounds to ounces.
32. Reduce 71 oz. to scruples.
33. Reduce 93 lbs. 2 oz. to grains.
34. Reduce 165 oz. to pounds.
35. Reduce 962 drams to pounds.

LONG MEASURE. (ART. 94.)

36. In 636 inches, how many yards?
37. In 763 feet, how many rods?

38. In 4 miles, how many feet?
39. In 18 rods 2 feet, how many inches?
40. In 1760 yards, how many miles?
41. In 3 leagues, 2 miles, how many inches?

CLOTH MEASURE. (ART. 95.)

42. How many yards in 19 quarters?
43. How many quarters in 21 yards and 3 quarters?
44. How many nails in 35 yards and 2 quarters?
45. How many Flemish ells in 50 yards?
46. How many English ells in 50 yards?
47. How many French ells in 50 yards?

SQUARE MEASURE. (ART. 96.)

48. In 65 sq. yards and 7 feet, how many feet?
49. In 39 sq. rods and 15 yds., how many yards?
50. In 27 acres, how many square feet?
51. In 345 sq. rods, how many acres?
52. In 461 square yards, how many rods?
53. In 876 sq. inches, how many sq. feet?

CUBIC MEASURE. (ART. 97.)

54. In 48 cubic yards, how many feet?
55. In 54 cubic feet, how many inches?
56. In 26 cords, how many cubic feet?
57. In 4230 cubic inches, how many feet?
58. In 3264 cubic feet, how many cords?

WINE MEASURE. (ART. 98.)

59. Reduce 94 gallons 2 qts. to pints.
60. Reduce 68 gallons 3 qts. to gills.
61. Reduce 10 hhds. 15 gallons to quarts.
62. Reduce 764 gills to gallons.
63. Reduce 948 quarts to hogsheads.
64. Reduce 896 gills to gallons.

BEER MEASURE. (ART. 99.)

65. How many quarts in 11 hogsheads of beer?
66. How many pints in 110 gallons 2 qts. of beer?
67. How many hogsheads in 256 gallons of beer?
68. How many barrels in 320 pints of beer?
69. How many pints in 46 hhds. 10 gallons?
70. How many hhds. in 2592 quarts?

DRY MEASURE. (ART. 100.)

71. In 156 pecks, how many bushels?
72. In 238 quarts, how many bushels?
73. In 360 pints, how many pecks?
74. In 58 bushels, 3 pecks, how many pecks?
75. In 95 pecks, 2 quarts, how many quarts?
76. In 373 quarts, how many bushels?
77. In 100 bushels, 2 pecks, how many pints?

TIME. (ART. 101.)

78. How many minutes in 16 hours?
79. How many seconds in 1 day?
80. How many minutes in 365 days?
81. How many days in 96 hours?
82. How many days in 3656 minutes?
83. How many seconds in 1 week?
84. How many years in 460 weeks?

CIRCULAR MEASURE. (ART. 105.)

85. Reduce 23 degrees, 30 minutes to minutes.
86. Reduce 41 degrees to seconds.
87. Reduce 840 minutes to degrees.
88. Reduce 964 minutes to signs.
89. Reduce 2 signs to seconds.
90. Reduce 5 signs, 2 degrees to minutes.
91. Reduce 960 seconds to degrees.
92. Reduce 1800 minutes to signs.

93. In 45 guineas, how many farthings?
94. In 60 guineas, how many pounds?
95. In 62564 pence, how many guineas?
96. In £84, how many guineas?
97. How many grains Troy, in 46 lbs. 7 oz. 5 pwts.?
98. How many pounds Troy, in 825630 grains?
99. Reduce 62 lbs. 10 pwts. to grains.
100. In 16 tons, 11 cwt. 9 lbs., avoir., how many pounds?
101. Reduce 782568 ounces to tons.
102. In 18 rods, 2 yds. 3 ft. 10 in., how many inches?
103. How many feet in 3 leagues, 2 miles, 12 rods?
104. In 2738 inches, how many rods?
105. In 2 tons, 3 cwt. 2 qrs. 15 lbs., how many ounces?
106. Reduce 53 lbs. 11 pwts. 10 grs. Troy, to grains.
107. How many English ells in 45 yards?
108. How many yards in 45 English ells?
109. How many Flemish ells in 54 yards?
110. How many French ells in 60 yards?
111. In 13 m. 2 fur. 6 ft. 7 in., how many inches?
112. In 84256 feet, how many leagues?
113. In 135 bu. 3 pks. 2 qts. 1 pt. how many pints?
114. In 84650 pints, how many quarters?
115. How many gills in 48 hhds. 18 gal. wine measure?
116. How many pipes in 98200 quarts?
117. How many seconds in 15 solar years?
118. How many weeks in 8029200 seconds?
119. How many square feet in 82 acres, 36 rods, 8 yds.?
120. How many cords of wood in 68600 cubic inches?
121. How many inches in 10 cords and 6 cubic feet?
122. In 246 tons of round timber, how many inches?
123. In 65200 square yards, how many acres?
124. In 8 signs, 43 deg. 18 sec., how many seconds?
125. In 75260 minutes, how many signs?

COMPOUND ADDITION.

ART. **129.** *Compound Addition* is the process of uniting two or more *compound* numbers in one sum.

Ex. 1. What is the sum of £2, 3s. 4d. 1 far.; £1, 6s. 9d. 3 far.; £7, 9s. 7d. 2 far.

Operation.

	£	*s.*	*d.*	*far.*
	2 "	4 "	4 "	1
	1 "	6 "	9 "	3
	7 "	9 "	7 "	2
Ans.	11 "	0 "	9 "	2

Suggestion.—First write the numbers under each other, pounds under pounds, shillings under shillings, &c. Then, beginning with the lowest denomination, we find the sum is 6 farthings, which is equal to 1 penny and 2 far. over. Write the 2 far. under the column of farthings, and carry the 1d. to the column of pence. The sum of the pence is 21, which is equal to 1s. and 9d. Place the 9d. under the column of pence, and carry the 1s. to the column of shillings. The sum of the shillings is 20, which is equal to £1 and nothing over. Write a *cipher* under the column of shillings, and carry the £1 to the column of pounds. The sum of the pounds is 11, which is set down in full.

130. Hence, we derive the following general

RULE FOR COMPOUND ADDITION.

I. *Write the numbers so that the same denominations shall stand under each other.*

II. *Beginning at the right hand, add each column separately, and divide its sum by the number required to make* ONE *of the next higher denomination. Setting the remainder under the column added, carry the quotient to the next column, and thus proceed as in Simple Addition.* (Art. 23.)

PROOF.—*The proof is the same as in Simple Addition.*

QUEST.—129. What is Compound Addition? 130. How do you write compound numbers for addition? Where do you begin to add, and how proceed? How is Compound Addition proved?

(2.)

£	s.	d.	far.
1	3	6	2
3	0	8	3
9	18	9	1
14	3	0	2 *Ans.*

(3.)

lb.	oz.	pwt.	gr.
2	5	7	4
2	0	5	19
6	8	0	3
11	1	13	2 *Ans.*

(4.)

m.	r.	ft.	in.
7	15	20	8
6	4	8	7
9	6	4	4
22	27	0	7 *Ans*

(5.)

£	s.	d.	far.
10	17	0	1
19	6	5	2
7	8	2	0
3	2	6	3

(6.)

lb.	oz.	pwt.	gr.
17	10	13	5
8	9	2	8
10	4	11	3
21	11	16	6

(7.)

r.	yd.	ft.	in.
4	4	2	6
6	6	0	2
6	8	1	4
2	3	2	5

(8.)

cwt.	qr.	lb.	oz.
5	3	4	5
6	2	9	8
8	1	7	2
6	0	9	6

(9.)

wk.	d.	hr.	min.
13	4	19	30
1	5	13	16
7	3	5	10
12	0	14	25

(10.)

yd.	qr.	na.	in.
6	3	1	2
3	2	3	1
7	0	2	4
5	1	1	2

11. Add 4 tons, 5 cwt. 3 qrs. 2 lbs. 10 oz. 4 drs.; 6 tons, 4 cwt. 17 lbs. 15 oz. 9 drs.; 3 tons, 2 cwt. 1 qr. 15 lbs.

12. Add 4 hhds. 10 gals. 3 qts. 1 pt.; 15 hhds. 19 gals. 2 qts.; 8 hhds. 7 gals. 2 qts. 1 pt. wine measure.

13. Add 1 pipe, 1 hhd. 8 gals. 2 qts. 1 pt. 2 gills; 1 pipe, 6 gals. 1 qt.; 3 pipes, 1 hhd. 3 gals. 3 qts. 1 pint.

14. A man sold the following quantities of wheat: 5 bu. 3 pks. 2 qts.; 10 bu. 1 pk. 4 qts.; 21 bu. 2 pks. 5 qts.: how much did he sell in all?

15. A merchant bought 3 pieces of silk, one of which contained 21 yds. 2 qrs. 3 nails; another 19 yds. 3 qrs. 1 nail; and the other 26 yds. 1 qr. and 2 nails: how many yards did they all contain?

COMPOUND SUBTRACTION.

ART. **131.** *Compound Subtraction* is the process of finding the difference between two *compound* numbers.

Ex. 1. From £11, 8s. 5d. 3 far., subtract £5, 10s. 2d. 1 farthing.

Operation.

£	s.	d.	far.
11	8	5	3
5	10	2	1
5	18	3	2

Suggestion.—Write the less number under the greater, pounds under pounds, shillings under shillings, &c. Then, beginning with the lowest denomination, proceed thus: 1 far. from 3 far. leaves 2 far. Set the remainder 2 under the farthings. Next, 2d. from 5d. leave 3d. Write the 3 under the pence. Since 10 shillings cannot be taken from 8 shillings; we borrow as many shillings as it takes to make *one* of the next higher denomination, which is pounds; and £1, or 20s., added to the 8s. make 28 shillings. Now 10s. from 28s. leave 18s., which we write under the shillings. Finally, carrying 1 to the next number in the lower line, we have £6; and £6 from £11 leave £5, which we write under the pounds. The answer is £5, 18s. 3d. 2 far.

132. Hence, we derive the following general

RULE FOR COMPOUND SUBTRACTION.

I. *Write the less number under the greater, so that the same denominations may stand under each other.*

II. *Beginning at the right hand, subtract each lower number from the number above it, and set the remainder under the number subtracted.*

III. *When a number in the lower line is larger than that above it, add as many units to the upper number as it*

QUEST.—131. What is Compound Subtraction? 132. How do you write compound numbers for subtraction? Where begin to subtract, and how proceed? When a number in the lower line is larger than that above it, what is to be done?

takes to make ONE *of the next higher denomination; then subtract as before, and adding* 1 *to the next number in the lower line, proceed as in Simple Subtraction.*

PROOF.—*The proof is the same as in Sim. Subtraction.*

	(2.)	(3.)
From	£13, 7s. 8d. 3 far.	19 lbs. 3 oz. 7 pwts. 12 grs.
Take	£ 6, 5s. 11d. 1 far.	15 lbs. 8 oz. 3 pwts. 4 grs.

	(4.)	(5.)
From	12 T. 7 cwt. 1 qr. 3 lbs.	15 m. 3 fur. 10 r. 8 ft. 4 in.
Take	7 T. 9 cwt. 3 qrs. 4 lbs.	9 m. 6 fur. 3 r. 4 ft. 7 in.

6. From 24 yds. 2 qrs. 3 nails, take 16 yds. 3 qrs. 2 nails.

7. A lady having £18, 4s. 7d. in her purse, paid £8, 7s. 3d. for a dress: how much had she left?

8. If from a hogshead of molasses you draw out 19 gals 3 qts. 1 pt., how much will there be left in the hogshead

9. A person bought 8 tons, 3 cwt. 19 lbs. of coal, and having burned 3 tons, 6 cwt. 45 lbs. sold the rest: how much did he sell?

10. From 17 years, 7 mos. 16 days, take 15 years, and 4 months.

11. From 39 yrs. 3 mos. 7 days, 4 min., take 23 yrs. 5 mos. 3 days, 16 hrs.

12. From 43 A. 2 roods, 15 rods, take 39 acres and 11 rods.

13. From 38 leagues, 2 miles, 5 fur. 17 rods, take 29 leagues, 2 miles, 7 fur. 13 rods.

14. From 125 bushels, 3 pecks, 4 quarts, 2 pints, take 108 bushels, 2 pecks, 7 quarts.

15. From 85 guineas, 13 shillings, 4 pence, 2 far. take 39 guineas, 15 shillings, 8 pence.

QUEST.—How is Compound Subtraction proved?

COMPOUND MULTIPLICATION.

ART. **133.** *Compound Multiplication* is the process of finding the amount of a *compound* number repeated or added to itself, a given number of times.

Ex. 1. What will 3 barrels of flour cost, at £1, 7s. 5d. 2 far. per barrel?

Operation.

£	s.	d.	far.
1 "	7 "	5 "	2
			3
4	2	4	2

Suggestion.—Write the multiplier under the lowest denomination of the multiplicand, and proceed thus: 3 times 2 far. are 6 far. which are equal to 1d. and 2 far. over. Write the remainder 2 far. under the denomination multiplied, and carry the 1d. to the next product. 3 times 5d. are 15d., and 1 to carry makes 16d., equal to 1s. and 4d. over. Write the 4d. under the pence, and carry the 1s. to the next product. 3 times 7s. are 21s. and 1 to carry makes 22s., equal to £1, and 2s. Write the 2 under the shillings and carry the £1 to the next product. Finally, 3 times £1 are £3, and 1 to carry makes £4. Write the £4 under the pounds. The answer is £4, 2s. 4d. 2 far.

134. Hence, we derive the following general

RULE FOR COMPOUND MULTIPLICATION.

Beginning at the right hand, multiply each denomination of the multiplicand by the multiplier separately, and divide its product by the number required to make ONE *of the next higher denomination, setting down the remainder and carrying the quotient as in Compound Addition.*

2. Multiply £4, 6s. 2d. 3 far. by 15.

3. Multiply 19 lbs. 8 oz. 9 pwts. 7 grs. by 12.

4. If a man walks 3 miles, 3 fur. 18 rods in 1 hour, how far will he walk in 10 hours?

QUEST.—133. What is Compound Multiplication? 134. What is the rule for Compound Multiplication?

5. Multiply 7 leagues, 1 m. 31 rods, 12 ft. 3 in. by 9.

6. Multiply 18 tons, 3 cwt. 10 lbs. 7 oz. 3 drs. by 11.

7. A man has 7 pastures, each containing 6 acres, 25 rods, 51 square feet: how much do they all contain?

8. A man bought 9 loads of wood, each containing 1 cord and 21 cu. ft.: how much did they all contain?

9. Multiply 17 yds. 3 qrs. 2 nails by 35.

10. Multiply 53 days, 19 min. 7 sec. by 41.

11. Multiply 36 years, 3 weeks, 5 days, 12 hours, by 63.

12. Multiply 65 hhds. 23 gals. 3 qts. 1 pt. by 72.

COMPOUND DIVISION.

135. *Compound Division* is the process of dividing *compound* numbers.

Ex. 1. A father divided £10, 5s. 8d. 2 far. equally among his 3 sons: how much did each receive?

Operation.

	£	s.	d.	*far.*
3)	10 "	5 "	8 "	2
Ans.	3 "	8 "	6 "	$3\frac{1}{3}$

Suggestion.—Write the divisor on the left of the dividend, and proceed as in Short Division. Thus, 3 is contained in £10, 3 times and £1 over. We write the 3 under the pounds, *because it denotes pounds;* then reducing the remainder £1 to shillings and adding the given shillings 5, we have 25s. Again, 3 is in 25s. 8 times and 1s. over. We set the 8 under the shillings, *because it denotes shillings;* then reducing the remainder 1s. to pence and adding the given pence 8, we have 20d. Now 3 is in 20d. 6 times and 2d. over. We set the 6d. under the pence, *because it denotes pence.* Finally, reducing the rem. 2d. to farthings and adding the given far. 2, we have 10 far.; and 3 is in 10, 3 times and 1 far. over. Write the 3 under the far

QUEST.—135. What is Compound Division?

136. Hence, we derive the following general

RULE FOR COMPOUND DIVISION.

I. *Beginning at the left hand, divide each denomination of the dividend by the divisor, and write the quotient figures under the figures divided.*

II. *If there is a remainder, reduce it to the next lower denomination, and adding it to the figures of the corresponding denomination of the dividend, divide this number as before. Thus proceed through all the denominations, and the several quotients will be the answer required.*

OBS. 1. Each quotient figure is of the same denomination as that part of the dividend from which it arose.

2. When the divisor exceeds 12, and is a composite number, we may divide first by one factor and that quotient by the other.

2. Divide 14 lbs. 5 oz. 6 pwts. 9 grs. by 3.

3. Divide £5, 17s. 8d. 1 far. by 4.

4. Divide 25 lbs. 3 ounces, 8 pwts. 7 grs. by 5.

5. Divide 15 T. 15 cwt. 3 qrs. 10 lbs. by 6.

6. Divide 23 yards, 2 qrs. 1 nail, by 7.

7. Divide 35 leagues, 1 m. 3 fur. 17 rods by 8.

8 Divide 45 hhds. 18 gals. 39 qts. 1 pint by 9.

9 A farmer had 34 bu. 3 pks. 1 qt. of wheat in 9 bags: ow much was in each bag?

10. If you pay £25, 17s. 8½d. for 5 cows, how much will that be apiece?

11. Divide 38 tons, 5 cwt. 2 qrs. 15 lbs. by 17.

12. Divide 41 hhds. 13 gals. 2 qt. wine measure by 23.

13. Divide 54 acres, 2 roods, 25 rods, by 34.

14. Divide 29 cords, 19 cu. feet, 18 cu. inches by 41.

15. Divide 78 years, 17 weeks, 24 days, by 63.

QUEST.—136. What is the rule for Compound Division? *Obs.* Of what denomination is each quotient figure?

MISCELLANEOUS EXERCISES.

1. From the sum of 463279 + 734658, take 926380.
2. To the difference of 856273 and 46719, add 420376.
3. To 476208 add 5207568 − 4808345.
4. Multiply the sum of 863576 + 435076 by 287.
5. Multiply the difference of 870358 − 640879 by 365.
6. Divide the sum of 439409 + 87646 by 219.
7. Divide the difference of 607840 − 23084 by 367.
8. Divide the product of 865060 × 406 by 1428.
9. Divide the quotient of 55296 ÷ 144 by 89.
10. What is the sum of 4845 + 76 + 1009 + 463 + 407?
11. What is the sum of 836 × 46, and 784 × 76?
12. What is the sum of 1728 ÷ 72, and 2828 ÷ 96?
13. What is the sum of 85263 − 45017, and 68086?
14. What is the difference between 38076 + 16325, and 20268 + 45675?
15. What is the difference between 40719 + 6289, and 31670 − 18273.
16. What is the difference between 378 × 96, and 9419?
17. What is the difference between 7560 ÷ 504, and 7560 × 504?
18. From 145 × 87, take 12702 ÷ 87.
19. Multiply 83 × 19 by 75 × 23.
20. How many times can 34 be subtracted from 578?
21. How many times can 1512 be taken from 7569?
22. How many times can 63 × 24 be taken from 27640?
23. How many times is 68 + 31 contained in 45600?
24. Divide 832 + 1429 by 45 + 84.
25. Divide 467 + 2480 by 346 − 187.
26. Divide 68240 − 16226 by 10405 − 6200.
27. Divide 320 × 160 by 2125 − 960.
28. Divide 826340 − 36585 by 126 × 84.
29. From 62345 + 19008, take 2134 × 38.
30. From 2631 × 216, take 576 ÷ 36.

33. A young man having 50 dollars, bought a coat for 15 dollars, a pair of pants for 8 dollars, a vest for 5 dollars, and a hat for 3 dollars: how much money did he have left?

34. A farmer sold a cow for 18 dollars, a calf for 4 dollars, and a lot of sheep for 35 dollars: how much more did he receive for his sheep than for his cow and calf?

35. A man having 90 dollars in his pocket, paid 27 dollars for 9 cords of wood, 35 dollars for 7 tons of coal, and 11 dollars for carting both home: how much money had he left?

36. A young lady having received a birthday present of 100 dollars, spent 17 dollars for a silk dress, 26 dollars for a crape shawl, and 8 dollars for a bonnet: how many dollars did she have left?

37. A dairy-woman sold 23 pounds of butter to one customer, 34 pounds to another, and had 29 pounds left: how many pounds had she in all?

38. A lad bought a pair of boots for 16 shillings, a pair of skates for 10 shillings, a cap for 17 shillings, and had 20 shillings left: how many shillings had he at first?

39. A grocer having 500 pounds of lard, sold 3 kegs of it; the first keg contained 43 pounds, the second 45 pounds, and the third 56 pounds: how many pounds did he have left?

40. A man bought a horse for 95 dollars, a harness for 34 dollars, and a wagon for 68 dollars, and sold them all for 225 dollars: how much did he make by his bargain?

41. A person being 1000 miles from home, on his return, traveled 150 miles the first day, 240 miles the second day, and 310 miles the third day: how far from home was he then?

42. George bought a pony for 78 dollars and paid 3 dollars for shoeing him; he then sold him for 100 dollars: how much did he make by his bargain?

43. A man bought a carriage for 273 dollars, and paid 27 dollars for repairing it; he then sold it for 318 dollars: how much did he make by his bargain?

44. A man bought a lot for 275 dollars, and paid a carpenter 850 dollars for building a house upon it: he then sold the house and lot for 1200 dollars: how much did he make by the operation?

45. A farmer having 150 sheep, lost 17 and sold 65; he afterwards bought 38: how many sheep had he then?

46. A man bought 27 cows, at 31 dollars per head: how many dollars did they all cost him?

47. A miller sold 251 barrels of flour, at 8 dollars a barrel: how much did it come to?

48. A merchant sold 218 yards of cloth, at 8 dollars per yard: how much did it come to?

49. A merchant sold 18 yards of broadcloth, at 4 dollars a yard, and 21 yards of cassimere, at 2 dollars a yard: how much did he receive for both?

50. A farmer sold 12 calves, at 5 dollars apiece, and 35 sheep, at 3 dollars apiece: how much did he receive for both?

51. A grocer sold to one person 25 firkins of butter, at 7 dollars a firkin, and 13 to another, at 8 dollars a firkin: how much did both lots of butter come to?

52. A shoe dealer sold 100 pair of coarse boots to one customer, at 4 dollars a pair, and 156 pair of fine boots to another, at 5 dollars a pair: what did both lots of boots come to?

53. A miller bought 165 bushels of corn, at 5 shillings a bushel, and 286 bushels of wheat, at 9 shillings a bushel: how much did he pay for both?

54. A man bought 45 clocks, at 3 dollars apiece, and sold them, at 5 dollars apiece: how much did he make by his bargain?

55. A bookseller bought 87 books, at 7 shillings apiece, and afterwards sold them, at 6 shillings apiece: how much did he lose by the operation?

56. How many yards of calico, at 18 cents a yard, can be bought for 240 cents?

57. A little girl having 326 cents, laid it out in ribbon, at 25 cents a yard: how many yards did she buy?

58. If a man has 500 dollars, how many acres of land can he buy, at 15 dollars per acre?

59. How many cows, at 27 dollars apiece, can be bought for 540 dollars?

60. How many barrels of sugar, at 23 dollars per barrel, can a grocer buy for 575 dollars?

61. Henry sold his skates for 87 cents, and agreed to take his pay in oranges, at 3 cents apiece: how many oranges did he receive?

62. William sold 80 lemons, at 4 cents apiece, and took his pay in chestnuts, at 5 cents a quart: how many chestnuts did he get for his lemons?

63. A milkman sold 110 quarts of milk, at 6 cents a quart, and agreed to take his pay in maple sugar, at 11 cents a pound: how many pounds did he receive?

64. A farmer bought 25 yards of cloth, which was worth 6 dollars per yard, and paid for it in wood, at 2 dollars per cord: how many cords did it take?

65. A pedlar bought $4\frac{1}{2}$ pieces of silk, at 24 dollars apiece: how much did he pay for the whole?

66. A farmer sold $8\frac{1}{4}$ bushels of wheat, at 96 cents per bushel: how much did he receive for his wheat?

67. A man sold a lot of land containing $15\frac{3}{4}$ acres, at 16 dollars per acre: how much did he receive for it?

68. If a man can walk 45 miles in a day, how far can he walk in $20\frac{4}{5}$ days?

69. What cost 75 yds. of tape, at $\frac{3}{5}$ of a cent per yd.?

70. What will 100 pair of childrens' gloves come to, at $\frac{3}{10}$ of a dollar a pair?

71. What will 160 boys' caps cost, at $\frac{7}{8}$ of a dollar apiece?

72. What will 210 pair of shoes cost, at $\frac{5}{7}$ of a dollar a pair?

73. How many childrens' dresses can be made from a piece of lawn which contains 54 yards, if it takes $4\frac{1}{2}$ yards for a dress?

74. A farmer wishes to pack 100 dozen of eggs in boxes, and to have each box contain $6\frac{1}{4}$ dozen: how many boxes will he need?

75. A lad having 275 cents, wishes to know how many miles he can ride in the Railroad cars, at $2\frac{1}{2}$ cents per mile: how many miles can he ride?

76. How many apples, at $\frac{1}{2}$ a cent apiece, can Horatio buy for 75 cents?

77. If Joseph has to pay $\frac{3}{4}$ of a cent apiece for marbles, how many can he buy for 84 cents?

78. At $\frac{7}{8}$ of a dollar apiece, how many parasols can a shopkeeper buy for 168 dollars?

79. If I am charged $\frac{5}{7}$ of a dollar apiece for fans, how many can I buy for 265 dollars?

80. How many yards of silk, which is worth $\frac{9}{10}$ of a dollar a yard, can I buy for 227 dollars?

81. How many pair of slippers, at $\frac{7}{8}$ of a dollar a pair, can be bought for 448 dollars?

82. In £45, 13s. 6d., how many pence?

83. In £63, 7s. 8d. 2 far., how many farthings?

84. How many yards of satin can I buy for £75, 10s., if I have to pay 5 shillings per yard?

85. How many six-pences are there in £100?

86. A grocer sold 10 hogsheads of molasses, at 3 shillings per gallon: how many shillings did it come to?

87. A milkman sold 125 gallons of milk, at 4 cents per quart: how much did he receive for it?

88. A man made 30 barrels of cider which he wished to put into pint bottles: how many bottles would it require?

89. How much would 85 bushels of apples cost, at 12 cents a peck?

90. What will 97 pounds of snuff cost, at 8 cents per ounce?

91. What will 5 tons of maple sugar come to, at 11 cents a pound?

92. A farmer sold 34 tons of hay, at 65 cents per hundred: how much did he receive for it?

93. A blacksmith bought 53 tons of iron for 3 dollars per hundred: how much did he pay for it?

94. A young man returned from California with 50 pounds of gold dust, which he sold for 16 dollars per ounce Troy: how much did he receive for it?

95. A man bought 36 acres of land for 3 dollars per square rod: how much did his land cost him?

96. John Jacob Astor sold five building lots in the city of New York, containing 560 square rods, for 13 dollars per square foot: how much did he receive for them?

97. A laboring man engaged to work 5 years for 16 dollars per month: what was the amount of his wages?

98. What will 17 cords of wood cost, at 6 cents per cubic foot?

99. If it takes 35 men 18 months to build a fort, how many years would it take 1 man to build it?

100. If it takes 1 man 360 days to build a house, how many weeks would it take 15 men to build it, allowing 6 working days to a week?

ANSWERS TO EXAMPLES.

ADDITION.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
	Art. 20.	4.	5286 yards.	28.	171658.
1.	Given.	5.	2404.	29.	57 dollars.
2.	68.	6.	2765.	30.	58 dollars.
3.	589.	7.	10040.	31.	120 dollars.
4.	768.	8.	8668.	32.	565.
5.	9987.	9.	84 inches.	33.	742.
6.	878.	10.	114 feet.	34.	1530.
7.	6767.	11.	168 dollars.	35.	1779.
8.	8898.	12.	192 rods.	36.	1597.
9.	8779.	13.	782 pounds.	37.	1757.
10.	6796.	14.	1380 yards.	38.	2379.
11.	88776.	15.	576 miles.	39.	2619.
12.	986788.	16.	836 sheep.	40.	1020.
		17.	615 dollars.	41.	1418.
	Art. 22.	18.	181 dollars.	42.	1191.
13, 14.	Given.	19.	1452.	43.	150 bushels.
15.	1454.	20.	1255.	44.	133 yards.
16.	15300.	21.	1881.	45.	731 acres.
17.	13285.	22.	6693.	46.	1197 cattle.
		23.	20485.	47.	12554 dollars.
	Art. 24.	24.	9726.	48.	1282.
1.	155 pounds.	25.	1769.	49.	2528.
2.	413 feet.	26.	1500.	50.	365 days.
3.	1960 dollars.	27.	106284.		

Art. 24.a.	10. 65471.	20. 551452.	30. 279,075.
1. 300.	11. 327371.	21. 46157.	31. 295,306.
2. 6000.	12. 390497.	22. 424634.	32. 1,606,895.
3. 9000.	13. 37938.	23. 430032.	35. 6,140,704.
4. 4861.	14. 50342.	24. 3458772.	36. 7,569,904.
5. 4871.	15. 449458.	25. 48350.	37. 9,253,854.
6. 47067.	16. 466789.	26. 514299.	38. 9,247,176.
7. 53340.	17. 40290.	27. 595522.	39. 10,531,960.
8. 59139.	18. 50676.	28. 5781566.	40. 12,811,860
9. 61304.	19. 508302.	29. 61993.	

SUBTRACTION.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
	ART. **28.**	14.	275 pounds.	48.	222 bushels.
1.	Given.	15.	613 yards.	49.	195 dollars.
2.	24.	16.	310 rods.	50.	1122 dollars.
3.	12.	17.	230 gallons.	51.	1659 dollars.
4.	153.	18.	503 hhds.	52.	3023 dollars.
5.	245.	19.	76 bushels.	53.	1763 dollars.
6.	31 dollars.	20.	127 dollars.	54.	3747 dollars.
7.	12 pounds.	21.	249 pounds.	55.	16014 dollars.
8.	115 yards.	22.	1082 rods.	56.	1315 dollars.
9.	222 shillings.	23.	13016.	57.	5385 dollars.
10.	222 marbles.	24.	310768.	58.	5735 dollars.
		25.	464374.	59.	13944 soldiers
	ART. **30.**	26.	5244038.	60.	94760000 m.
11,	12. Given.	27.	45.	61.	17 oranges.
13.	137.	28.	308.	62.	33 marbles.
14.	2616.	29.	240.	63.	76 sheep.
15.	3270.	30.	58.	64.	52 cents.
16.	3203.	31.	542.	65.	43 yards.
17.	5365667.	32.	2021.	66.	122 dollars
		33.	1825.	67.	87 dollars.
	ART. **32.**	34.	2600.	68.	66 pears.
1.	217.*	35.	3085.	69.	59.
2.	182.	36.	1306.	70.	164.
3.	242.	37.	4098.	71.	149 pounds.
4.	369.	38.	1108.	72.	164 bushels.
5.	1029.	39.	4531.	73.	263 miles.
6.	1008.	40.	14520.	74.	125 gallons.
7.	3289.	41.	24622.	75.	179 pounds.
8.	3434.	42.	125028.	76.	175 dollars.
9.	35100.	43.	64303.	77.	339 pounds.
10.	312657.	44.	224066.	78.	172 barrels.
11.	1.	45.	103875.	79.	297 pages.
12.	23 dollars.	46.	420486.	80.	110 dollars.
13.	57 bushels.	47.	72 sheep.	81.	392 dollars.

* It is an excellent exercise for the pupil to *prove* all the examples. This is one of the best means to give him confidence in his own powers.

MULTIPLICATION.

Ex.	Ans.
ART. 39.	
1.	Given.
2.	68.
3.	936.
4.	8084.
5.	5550.
6.	12066.
7.	24408.
8.	35550.
9.	56707.
10.	Given.
ART. 40.	
11.	312.
12.	480.
13.	249.
14.	840.
15.	828.
16.	815.
17.	2248.
18.	3144.
19.	2520.
20.	1900.
21.	3960.
22.	6560.
23.	5628.
24.	8712.
25.	1050 dollars.
26.	2300 dollars.
27.	1372 dollars.
28.	2720 dollars.
29.	4837 dollars.
30.	7785 dollars.
31.	7744 dollars.
32.	8820 dollars.
33.	21285 dollars.
ART. 41.	
34—37.	Given.
ART. 43.	
1.	252.
2.	390.
3.	567.
4.	582.
5.	840.
6.	1155.
7.	3568.
8.	2763.
9.	3920.
10.	460.
11.	572.
12.	816.
13.	1092.
14.	1170.
15.	2185.
16.	4515.
17.	12306.
18.	25355.
19.	342 dollars.
20.	336 bushels.
21.	336 inches.
22.	620 pounds.
23.	391 dollars.
24.	475 dollars.
25.	1591 dollars.
26.	1950 shil.
27.	1575 dollars.
28.	2430 shil.
29.	3936 ounces.
30.	10754 dollars.
31.	6710 miles.
32.	8760 hours.
33.	9100 weeks.
34.	23760 min.
35.	28350 gallons.
36.	34675 dolls.
37.	33840 sq. in.
38.	26070 miles.
ART. 45.	
40.	Given.
41.	260.
42.	3700.
43.	51000.
44.	226000.
45.	341000.
46.	4690000.
47.	52300000.
48.	681000000.
49.	856120000.
50.	96030500000.
51.	Given.
ART. 46.	
52.	17000.
53.	291000.
54.	4920000.
55.	11700000.
56.	33930.
57.	789600.
58.	16170000.
59.	262660000.
60.	7500 minutes
61.	2400 dollars.
62.	6800 shillings.
63.	27000 dollars.
64.	352500 days.

MULTIPLICATION CONTINUED.—ARTS. 47, 48.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
65.	Given.	78.	2520000.	91.	5816049 galls.
66.	19500.	79.	65000000.	92.	101198340 d.
67.	40800.	80.	722000000.	93.	146460440 T.
68.	504000.	81.	21000000000.	94.	1190439180.
69.	800000.	82.	72800000000.	95.	3759670728.
70.	3300000.	83.	2240000 yds.	96.	4223213600.
71.	14620000.	84.	140000 miles.	97.	5815178600.
72	65360000.	85.	700000 dolls.	98.	12976172335.
73	104520000.	86.	504000 dolls.	99.	124811441568
74.	183244000.	87.	27375000 d.	100.	54719418834.
75.	Given.	88.	367608 lbs.	101.	469234745451
76.	420000.	89.	3838460 ft.	102.	197118900.
77.	442000.	90.	4217202 r.	103.	420152303451.

SHORT DIVISION.

ART. 54.

1. Given.
2. 21.
3. 23.
4. 122.
5. 111.
6. 342.
7. 1122.
8. 1321.
9. 1111.

ART. 55.

10. Given.
11. 71.
12. 43.
13. 412.
14. 411.

ART. 56.

15. Given.
16. 14.
17. 25.
18. 76.
19. 456.

ART. 57.

20. Given.
21. 509.
22. 901.
23. 1067.
24. 503.
25. Given.

ART. 61.

1. 142.
2. $101\frac{1}{5}$.
3. 76.
4. 75.
5. $102\frac{6}{7}$.
6. $56\frac{8}{9}$.
7. $120\frac{4}{5}$.
8. 95.
9. $116\frac{1}{7}$.
10. 728.
11. $1552\frac{2}{4}$.
12. $1004\frac{3}{7}$.
13. $400\frac{3}{6}$.
14. $903\frac{4}{9}$.
15. 923.
16. $1222\frac{6}{7}$.
17. 875.
18. $1011\frac{1}{9}$.
19. 63 pair.
20. 42 hats.
21. 24 marbles.
22. 45 children.
23. 75 yards.
24. 85 barrels, and 5 dolls. over.
25. 92 days.
26. $158\frac{1}{8}$ yards.
27. 195 hours.
28. $333\frac{3}{4}$ hours

LONG DIVISION.

Ex.	Ans.
	ART. 62.
1, 2.	Given.
3.	128.*
4.	364.
5.	$1825\frac{2}{3}$.
6.	533.
7.	732.
8.	931.
9—11.	Given.
	ART. 65
1.	$46\frac{1}{8}$.
2.	$48\frac{3}{9}$.
3.	$80\frac{4}{7}$.
4.	$40\frac{3}{10}$.
5.	$58\frac{3}{11}$.
6.	48.
7.	$21\frac{1}{13}$.
8.	$32\frac{1}{14}$.
9.	$41\frac{2}{15}$.
10.	27.
11.	$23\frac{29}{31}$.
12.	$21\frac{21}{37}$.
13.	$19\frac{36}{42}$.
14.	20.
15.	$21\frac{29}{53}$
16.	$32\frac{43}{67}$.
17.	$45\frac{25}{75}$.
18.	$57\frac{38}{84}$.
19.	24 caps.
20.	35 pair.
21.	28 barrels.
22.	$16\frac{6}{12}$ shillings.
23.	$10\frac{15}{20}$ pounds.
24.	$16\frac{12}{16}$ pounds.
25.	17 trunks.
26.	30 weeks.
27.	$32\frac{20}{23}$ yards.
28.	75 dresses.
29.	81 sheep.
30.	$73\frac{15}{25}$ acres.
31.	61 shares.
32.	$31\frac{28}{52}$ years.
33.	$48\frac{41}{53}$ hhds.
34.	$43\frac{55}{75}$ months.
35.	$51\frac{32}{83}$ months.
36.	50 dollars.
37.	$10\frac{12}{16}$ months.
38.	90 pounds.
39.	60, and 1 over.
40.	106, and 22 over.
41.	26, ana 28 over.
42.	42, and 28 over.
43.	$30\frac{24}{56}$.
44.	34.
45.	$53\frac{3}{49}$.
46.	$35\frac{12}{81}$.
47.	$25\frac{51}{75}$.
48.	$26\frac{20}{85}$.
49.	$65\frac{3}{48}$.
50.	$85\frac{46}{99}$.
51.	$89\frac{78}{105}$.
52.	$118\frac{47}{112}$.
53.	$218\frac{36}{123}$.
54.	$216\frac{67}{224}$.
	ART. 67.
55, 56.	Given.
57.	$46\frac{26}{100}$.
58.	$5\frac{633}{1000}$.
59.	$8\frac{465}{1000}$.
60.	$26\frac{244}{1000}$.
61.	$13\frac{6056}{10000}$.
62.	$24\frac{43667}{100000}$.
63.	$23\frac{454631}{1000000}$.
	ART. 68.
64, 65.	Given.
66.	$121\frac{12}{30}$.
67.	$32\frac{93}{200}$.
68.	$54\frac{635}{1400}$.
69.	$51\frac{1034}{1600}$.
70.	$46\frac{1600}{2000}$.
71.	$4\frac{2245}{3000}$.
72.	$19\frac{348}{1200}$.
73.	$28\frac{61}{1500}$.
74.	$27\frac{1782}{1800}$.
75.	$42\frac{117}{2100}$.
76.	$357\frac{1500}{2500}$.
77.	$2968\frac{2210}{3100}$
78.	$17\frac{310000}{420000}$
79.	30.
80.	$1483\frac{171}{235}$.
81.*	$1370\frac{172}{341}$.
82.	$1900\frac{105}{401}$.
83.	$840\frac{205}{603}$.
84.	$374\frac{505}{1623}$.
85.	$266\frac{1017}{2764}$.
86.	$184\frac{1710}{2367}$.
87.	$1620\frac{3711}{3827}$.
88.	$2366\frac{1784}{3478}$.
89.	$194\frac{26451}{42316}$
90.	$1393\frac{43149}{67213}$.
91.	$1171\frac{18504}{84327}$.

GREATEST COMMON DIVISOR. ART. 74.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
2.	28.	4.	37.	6.	61.	8.	8.
3.	29.	5.	79.	7.	9.	9.	2.

LEAST COMMON MULTIPLE.

ART. **80.**

10. Given.
11. 80.
12. 84.
13. 180.
14. 720.
15. 72.
16. 5600.
17. 20160.
18. 1275.
18. 990.
20. 858.
21. 7200.
22. 34560.
23. 1584.
24. 8640.
25. 13859080.
26. 288000.
27. 3300500.

REDUCTION OF FRACTIONS. ART. 89.

1. Given.
2. Given.
3. $\frac{2}{3}$.
4. $\frac{2}{5}$.
5. $\frac{2}{5}$.
6. $\frac{1}{3}$.
7. $\frac{3}{8}$.
8. $\frac{5}{11}$.
9. $\frac{1}{7}$.
10. $\frac{7}{19}$.
11. $\frac{23}{35}$.
12. $\frac{25}{31}$.

ART. **90.**

14. Given.
15. 3.
16. $6\frac{4}{5}$.
17. $3\frac{7}{8}$.
18. $7\frac{1}{6}$.
19. $6\frac{2}{9}$.
20. 1.
21. 3.
22. 4.
23. $3\frac{43}{242}$.
24. $5\frac{22}{221}$.

ART. **91.**

26. Given.
27. $\frac{42}{5}$.
28. $\frac{99}{7}$.
29. $\frac{155}{6}$.
30. $\frac{274}{9}$.
31. $\frac{480}{11}$.
32. $\frac{797}{13}$.
33. $\frac{542}{5}$.
34. $\frac{3371}{16}$.
35. $\frac{252}{4}$.
36. $\frac{2475}{11}$.

ART. **92.**

38. Given.
39. $\frac{7}{20}$.
40. $\frac{125}{168}$.
41. $\frac{57}{385}$.
42. $\frac{7}{9}$.
43. $\frac{1}{6}$.
44. $157\frac{1}{2}$.
45. $\frac{34}{57}$.
46. $\frac{19}{25}$.

ART. **93.**

48. Given.
49. $\frac{2}{7}$.
50. $\frac{3}{14}$.
51. 1.
52. $\frac{3}{11}$.
53. $\frac{5}{2}$.
54. $\frac{8}{35}$.
55. $\frac{44}{513}$.
56. $\frac{70}{99}$.
57. 1121.
58. 36.
59. 154.
60. 55.

ART. **95.**

63. $\frac{40}{60}$, $\frac{45}{60}$, $\frac{12}{60}$.
64. $\frac{345}{805}$, $\frac{322}{805}$, $\frac{420}{805}$.
65. $\frac{385}{462}$, $\frac{264}{462}$, $\frac{378}{462}$.
66. $\frac{8976}{41888}$, $\frac{19992}{41888}$, $\frac{22792}{41888}$.
37. $\frac{123370}{170820}$, $\frac{119340}{170820}$, $\frac{131400}{170820}$.
68. $\frac{922500}{1305000}$, $\frac{942500}{1305000}$, $\frac{433260}{1305000}$.
69. $\frac{40}{128}$, $\frac{96}{128}$, $\frac{1088}{128}$.
70. $\frac{780}{1170}$, $\frac{5265}{1170}$, $\frac{1044}{1170}$.

ART. **96.**

71. Given.
72. Given.
73. $\frac{6}{24}$, $\frac{20}{24}$, $\frac{9}{24}$.
74. $\frac{10}{20}$, $\frac{15}{20}$, $\frac{14}{20}$.
75. $\frac{3}{24}$, $\frac{18}{24}$, $\frac{8}{24}$.
76. $\frac{24}{72}$, $\frac{36}{72}$, $\frac{63}{72}$, $\frac{30}{72}$.
77. $\frac{1785}{6120}$, $\frac{4212}{6120}$, $\frac{2040}{6120}$.
78. $\frac{756}{1260}$, $\frac{735}{1260}$, $\frac{720}{1260}$, $\frac{350}{1260}$.

REDUCTION OF FRACTIONS CONTINUED.—ART. 96.

Ex.	Ans.	Ex.	Ans.
79.	$\frac{72}{180}$, $\frac{132}{180}$, $\frac{21}{180}$.	86.	$\frac{90}{144}$, $\frac{12}{144}$, $\frac{126}{144}$, $\frac{18}{144}$.
80.	$\frac{13}{27}$, $\frac{12}{27}$, $\frac{9}{27}$, $\frac{270}{27}$.	87.	$\frac{675}{1080}$, $\frac{567}{1080}$, $\frac{1050}{1080}$.
81.	$\frac{16}{2}$, $\frac{7}{2}$.	88.	$\frac{20}{36}$, $\frac{30}{36}$, $\frac{14}{36}$.
82.	$\frac{33}{40}$, $\frac{176}{40}$, $\frac{108}{40}$.	89.	$\frac{48}{102}$, $\frac{63}{102}$, $\frac{60}{102}$.
83.	$\frac{2}{15}$, $\frac{250}{15}$.	90.	$\frac{95}{100}$, $\frac{64}{100}$, $\frac{20}{100}$.
84.	$\frac{21}{10}$, $\frac{46}{10}$.	91.	$\frac{24}{96}$, $\frac{80}{96}$, $\frac{62}{96}$.
85.	$\frac{72}{120}$, $\frac{70}{120}$, $\frac{88}{120}$, $\frac{54}{120}$.	92.	$\frac{12}{24}$, $\frac{15}{24}$, $\frac{14}{24}$.

ADDITION OF FRACTIONS. ART. 98.

2. $2\frac{1}{7}$.
3. $2\frac{2}{9}$.
4. $1\frac{4}{5}$.
5. $1\frac{4}{5}$.
8. $1\frac{4}{15}$.
9. $1\frac{29}{60}$.
10. $2\frac{37}{126}$.
11. $1\frac{33}{40}$.
12. $1\frac{169}{240}$.
13. $1\frac{2}{5}$.
14. $2\frac{41}{66}$.
15. $1\frac{611}{1400}$.
16. $8\frac{89}{90}$.
17. $27\frac{5}{12}$.
18. $239\frac{26}{63}$.
19. $79\frac{3}{4}$
20. $334\frac{19}{24}$.
21. $298\frac{7}{16}$.
22. $505\frac{9}{10}$.
23. $386\frac{14}{15}$.

SUBTRACTION OF FRACTIONS. ART. 100.

1. Given.
2. $\frac{4}{13}$.
3. $\frac{7}{21}$.
4. $\frac{11}{63}$.
5. $\frac{24}{100}$.
7. $\frac{19}{60}$.
8. $\frac{2}{9}$.
9. $\frac{33}{80}$.
10. $\frac{13}{50}$.
11. $\frac{3}{35}$.
12. $\frac{122}{1505}$.
13. $\frac{117}{660}$.
14. $\frac{173}{630}$.
15. $\frac{380}{1089}$.
16. $\frac{91}{275}$.
18. $5\frac{3}{4}$.
19. $4\frac{11}{15}$.
20. $3\frac{1}{2}$.
21. $7\frac{51}{56}$.
23. $6\frac{1}{2}$.
24. $8\frac{3}{4}$.
26. $\frac{11}{24}$.
27. $22\frac{9}{28}$.
28. $3\frac{1}{4}$.

MULTIPLICATION OF FRACTIONS. ART. 102.

1 Given.
2. $3\frac{8}{9}$.
3. $8\frac{1}{4}$.
4. $4\frac{4}{15}$.
5. $7\frac{1}{17}$.
6. $4\frac{10}{11}$.
7. $13\frac{16}{23}$.
8. $10\frac{34}{35}$.
9. $17\frac{29}{47}$.
10. $23\frac{1}{4}$.
11. $26\frac{223}{250}$.
12. Given.

ART. **103.**

13. $63\frac{1}{3}$.
14. $191\frac{1}{4}$.
15. $250\frac{1}{4}$.
16. 296.
17. $550\frac{1}{2}$.
18. 1131.

ART. **104.**

20. $12\frac{3}{5}$.
21. $10\frac{6}{7}$.
22. $24\frac{4}{5}$.
23. $35\frac{5}{6}$.
24. $14\frac{5}{8}$.
25. $33\frac{4}{7}$.
26. $61\frac{4}{11}$.
27. $48\frac{8}{15}$.

ART. **105.**

29. $80\frac{2}{3}$.
30. $211\frac{3}{5}$.
31. $554\frac{2}{9}$.
32. $743\frac{8}{9}$.

ART. **106.**

35. $\frac{2}{5}$.
36. $\frac{1}{6}$.
37. $\frac{35}{72}$.
38. $\frac{3}{8}$.
39. $\frac{5}{12}$.
40. $\frac{19}{124}$.
41. $\frac{15}{28}$.
42. $\frac{21}{32}$.
43. $41\frac{1}{3}$.
44. 48.
45. $343\frac{4}{7}$.
46. $2101\frac{1}{4}$.
47. $3240\frac{9}{20}$.
48. $5822\frac{31}{88}$.

DIVISION OF FRACTIONS.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
ART. **108.**	9. $\frac{5}{64}$.	17. Given.	26. $6\frac{2}{21}$.
1. Given.	ART. **109.**	18. $1\frac{7}{18}$.	ART. **110.**
2. Given.	10. Given.	19. $16\frac{2}{3}$.	27. Given.
3. Given.	11. Given.	20. $7\frac{1}{2}$.	28. 161.
4. $\frac{5}{11}$.	12. Given.	21. $\frac{9}{200}$.	29. 168.
5. $\frac{4}{27}$.	13. $1\frac{1}{2}$.	22. Given.	30. $196\frac{4}{11}$.
6. $\frac{11}{120}$.	14. $1\frac{2}{3}$.	23. $3\frac{1}{3}$.	31. $484\frac{4}{9}$.
7. $\frac{19}{220}$.	15. $\frac{77}{81}$.	24. $141\frac{1}{6}$.	32. $14\frac{10}{21}$.
8. $\frac{1}{20}$.	16. $1\frac{1}{5}$.	25. $1\frac{3}{4}$.	33. $17\frac{11}{29}$.

COMPLEX FRACTIONS.

ART. **111.**	2. $\frac{9}{4}$.	4. $\frac{5}{6}$.	6. $\frac{42}{48}$, or $\frac{7}{8}$.
1. Given.	3. $\frac{8}{5}$.	5. $\frac{36}{25}$.	7. $\frac{250}{7}$.

EXERCISES IN FRACTIONS.

1. $\$\frac{27}{40}$.	5. $\$276\frac{11}{16}$.	9. $24.	13. $16\frac{36}{155}$ m.
2. $45\frac{5}{6}$ m.	6. $\$6.19\frac{5}{14}$.	10. $5\frac{9}{40}$ lbs.	14. $2\frac{1}{3}$ lbs.
3. $\$799\frac{1}{10}$.	7. $\$44.37\frac{5}{8}$.	11. 10 or.	15. 1152 lbs.
4. $2864\frac{1}{4}$ m.	8. $\$25\frac{1}{5}$.	12. $5\frac{7}{75}$ lbs.	

ADDITION OF FEDERAL MONEY. ART. 117.

2. $1278.699.	6. $1743.828.	10. $978.297.	14. $829.496.
3. $11261.52.	7. $2478.735.	11. $2037.379.	15. $34.75.
4. $2622.337.	8. $10224.78.	12. $880.317.	16. $74.375.
5. $5599.332.	9. $12858.266.	13. $301.243.	17. $162.06.

SUBTRACTION OF FEDERAL MONEY. ART. 118.

2. $468.851.	6. $877.155.	10. $81980.755.	14. $49.928.
3. $497.73.	7. $461.543.	11. $67671.133.	15. $357.04.
4. $527.247.	8. $435.103.	12. $0.89.	16. $2.125.
5. $5916.707.	9. $1461.78.	13. $2.317.	17. $1.945.

MULTIPLICATION OF FEDERAL MONEY. ART. 119.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
1. Given.	7. $458122.602	13. $78.75.	19. $3346.50
2. $5070.	8. $773262.87.	14. $12.375.	20. $1495.
3. $7250.625.	9. $2182139.52	15. $39.45.	21. $4238.08
4. $21097.80.	10. $1.36.	16. $9.375.	22. $7.50.
5. $335636.62	11. $10.44.	17. $23.75.	23. $73.50.
6. $255991.68	12. $31.	18. $1181.28	24. $279.50.

DIVISION OF FEDERAL MONEY. ART. 120.

1. Given.	7. $9933.57.	13. $8902.627.	19. $26.82.
2. $142.712.	8. $11.322.	14. $972.38.	20. $8.35.
3. $1195.956.	9. $110.57.	15. $40.69.	21. $2.767.
4. $806.012.	10. $68.47.	16. $6.12.	22. $1.738.
5. $32.16.	11. $92.09.	17. $7.31.	23. $6.807.
6. $96.70.	12. $49.32.	18. $20.16.	

REDUCTION DESCENDING.

ART. **124.**	11. 7348 gr.	20. 24640 r.	28. 180 pk.
1–4. Given.	13. 212 oz.	21. 56 qrs.	29. 52 pts.
5. 4320d.	14. 67 lbs.	22. 344 na.	30. 1680 qts.
6. 469s.	15. 1728 dr.	23. 286 na.	31. 2520 hrs.
7. 827 far.	16. 19696 oz.	24. 92 qts.	32. 36000 m.
8. Given.	17. 5120 r.	25. 976 g.	33. 954000 s.
9. 156 oz.	18. 5568 in.	26. 2016 g.	34. 524160 m.
10. 1020 pwt.	19. 12612 ft.	27. 10334 q.	35. 5875200 s.

REDUCTION ASCENDING.

ART. **127.**	10. 1 oz. 2 pwts. 20 grs.	15. 2 tons.
1–4. Given.	11. 39 lbs. 14 oz.	16. 24 yds. 1 in.
5. 27 shillings.	12. 29 qrs. 11 lbs.	17. 60 r. 10 ft.
6. 20 shillings.	13. 1 cwt. 4 lbs. 11 oz.	18. 8 miles.
7. £1, 6s. 0d. 2 far.	14. 3 lbs. 14 oz. 8 drs.	19. 1 m. 6 fur. 32 r. 5 yds.
8. 22 lbs. 1 oz.		20. 2 lea. 1 m. 3 fur. 5 r.
9. 3 lbs. 0 oz. 8 pwts.		

REDUCTION ASCENDING AND DESCENDING.

Ex.	Ans.
	ART. **129.**
1.	316 cents.
2.	812 mills.
3.	2 dolls. 45 cts.
4.	3 dimes 2 cts. 1 mill.
5.	95000 cents.
6.	16000 cents.
7.	3170 dimes.
8.	4 dolls. 56 cts. 1 mill.
9.	8E.2dolls.50c.
10.	61123 mills.
11.	356 shillings.
12.	938 farthings.
13.	£5.
14.	75s. 6d. 2 far.
15.	18240 far.
16.	60 shillings.
17.	5082 pence.
18.	£3, 17s 6d. 1 far.
19.	84 ounces.
20.	2200 pwts.
21.	13 lbs. 9 oz.
22.	1 oz. 15 pwts.
23.	19735 grains.
24.	1 lb. 2 oz. 5 pwts. 20 grs.
25.	12 lbs. 8 oz.
26.	4176 ounces.
27.	6200 lbs.
28.	2400 ounces.
29.	62 lbs. 8 oz.
30.	2 tons, 2 cwt. 2 qrs. 6 lbs.
31.	540 ounces.
32.	1704 scruples.
33.	536640 grs.
34.	13 lbs. 9 oz.
35.	10 lbs. 0 oz. 2 drs.
36.	17 yds. 2 ft.
37.	46 rods 4 ft.
38.	21120 feet.
39.	3588 inches.
40.	1 mile.
41.	696960 in.
42.	4 yds. 3 qrs.
43.	87 qrs.
44.	568 nails.
45.	66 Fl. e. 2 qrs.
46.	40 E. e.
47.	33 F. e. 2 qrs.
48.	592 sq. ft.
49.	$1194\frac{3}{4}$ sq. yds.
50.	1176120 sq.ft.
51.	2 A. 25 sq. r.
52.	15 sq. r. $7\frac{1}{4}$ sq. yds.
53.	6 sq. ft. 12 sq. in.
54.	1296 cu. ft.
55.	93312 cu. in.
56.	3328 cu. ft.
57.	2 cu. ft. 774 cu. in.
58.	25 cords, 64 cu. ft.
59.	756 pts.
60.	2200 gills.
61.	2580 qts.
62.	23 gals. 3 qts. 1 pt.
63.	3 hhds. 48 gls
64.	28 gals.
65.	2376 qts.
66.	884 pints.
67.	4 hhds. 40 gls.
68.	1 bbl. 4 gals.
69.	19952 pts.
70.	12 hhds.
71.	39 bushels.
72.	7 bu. 1 pk. 6 qts.
73.	22 pks. 4 qts.
74.	235 pecks.
75.	762 quarts.
76.	11 bu. 2 pks. 5 qts.
77.	6432 pints.
78.	960 minutes.
79.	86400 sec.
80.	525600 min.
81.	4 days.
82.	2 days 12 hrs. 56 min.
83.	604800 sec.
84.	8 yrs. 11 mo.
85.	1410′.
86.	147600″.
87.	14°.
88.	0s. 16° 4 .
89.	216000″.
90.	9120′.
91.	0° 16′.
92.	1 sign.

REDUCTION ASCENDING AND DESCENDING.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
93.	45360 far.	104.	13 r. 13 f. 8 i.	116.	194 p. 1 h. 43 gals.
94.	£63.	105.	69840 oz.	117.	473353920 s.
95.	248 G. 5s. 8d.	106.	205554 grs.	118.	13 wks. 1 d. 22 hrs. 20 min.
96.	80 G.	107.	36 E. ells.	119.	3581793 s. ft.
97.	268440 grs.	108.	56 yds. 1 qr.	120.	39 ft. 1208 i.
98.	143 l. 4 o. 1 p. 6 g.	109.	72. Fl. ells.	121.	2222208 c. in.
99.	357360 grs.	110.	40 F. ells.	122.	17003520 in.
100.	33109 lbs.	111.	839599 in.	123.	13 A. 75 r. 11¼ yds.
101.	24 T. 9 cwt. 10 lbs. 8 oz.	112.	5 l. 306 r. 7 f.	124.	1018818 sec.
102.	3682 in.	113.	8693 pts.	125.	41 S. 24°, 20′.
103.	58278 ft.	114.	165 qrs. 2 bu. 2 pks. 5 qts.		
		115.	97344 gills.		

COMPOUND ADDITION.

5. £40, 14s. 2d. 2 f.	9. 35 w. 4 h. 21 m.	13. 6 pi. 18 gals. 3 qts. 2 gi.
6. 59 l. 2 p. 22 g.	10. 23 yds. 3 na.	14. 37 bu. 3 pks. 3 qts.
7. 22 r. 1 yd. 5 in.	11. 13 T. 12 c. 1 qr. 10 l. 9 o. 13 d.	15. 67 y. 3 q. 2 na.
8. 26 cwt. 3 qrs. 5 lbs. 5 oz.	12. 27 hhds. 38 g.	

COMPOUND SUBTRACTION.

2. £7, 1s. 9d. 2 far.	6. 7 yds. 3 qrs. 1 n.	12. 4 A. 2 roods, 4 rods.
3. 3 lbs. 7 oz. 4 pwts. 8 grs.	7. £9, 17s. 4d.	13. 8 lea. 2 mi. 6 fur. 4 r.
4. 4 T. 17 cwt. 1 qr. 24 lbs.	8. 44 gals. 1 pt.	14. 17 bu. 5 q. 2 p.
5. 5 m. 5 fur. 7 r. 3 ft. 9 in.	9. 4 T. 16 c. 74 lb.	15. 45 G. 18s. 8d 2 far.
	10. 2 y. 3 mo. 16 d.	
	11. 15 y. 10 mo. 3 d. 8 h. 4 m.	

COMPOUND MULTIPLICATION.

1. Given.	5. 66 l. 285 r. 11 f. 3 i.	9. 625 y. 2 q. 2 n.
2. £64, 13s. 5d. 1 farthing.	6. 199 T. 14 c. 14 l. 15 o. 1 d.	10. 2173 d. 13 h. 3 m. 47 s.
3. 236 l. 5 o. 11 p. 12 g.	7. 43 A. 16 r. 84¾ f.	11. 2272 y. 30 w 3 d. 12 h.
4. 34 mi. 2 f. 20 r.	8. 10 cords, 61 c. f.	12. 4707 h. 18 g.

COMPOUND DIVISION.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
1.	Given.	6.	3 y. 1 q. $1\frac{6}{7}$ na.	12.	1 hhd. 49 gals. $3\frac{13}{23}$ qts.
2.	4 l. 9 oz. 15 p. 11 g.	7.	4 l. 1 m. 2 f. $17\frac{1}{8}$ r.	13.	1 A. 2 roods, $17\frac{7}{34}$ r.
3.	£1, 9s. 5d. $0\frac{1}{4}$ f.	8.	5 h. 2 g. 4 q. $0\frac{7}{9}$ p.	14.	91 c. f. $0\frac{18}{41}$ i.
4.	5 l. 13 p. $15\frac{4}{5}$ g.	9.	3 b. 3 p. $3\frac{6}{9}$ q.	15.	1 yr. 12 wks. $4\frac{59}{63}$ d.
5.	2 T. 12 c. 2 q. $14\frac{1}{6}$ l.	10.	£5, 3s. 6d. 2 f.		
		11.	2 T. 5 c. $3\frac{14}{17}$ l.		

MISCELLANEOUS EXERCISES.

1.	271557.	23.	$460\frac{60}{99}$.	51.	$279.	76.	150 ap.
2.	1229930.	24.	$17\frac{68}{129}$.	52.	$1180.	77.	112 mar.
3.	875431.	25.	$18\frac{35}{159}$.	53.	3399 s.	78.	192 par.
4.	372713-124.	26.	$12\frac{1554}{4205}$.	54.	$90.	79.	371 fans.
		27.	$43\frac{1105}{1165}$.	55.	87 s.	80.	$252\frac{2}{9}$ yds.
5.	837598-35.	28.	$74\frac{6539}{10584}$.	56.	$13\frac{6}{18}$ yds.	81.	512 pair.
		29.	261.	57.	$13\frac{1}{25}$ yds.	82.	10962d.
6.	$2406\frac{141}{219}$.	30.	568280.	58.	$33\frac{5}{15}$ a.	83.	60850 f.
7.	$1593\frac{125}{367}$.	33.	$19.	59.	20 cows.	84.	302 yds.
8.	245948, 616 rem.	34.	$13.	60.	25 bar.	85.	4000.
		35.	$17.	61.	29 or.	86.	1890 s.
9.	$4\frac{28}{89}$.	36.	$49.	62.	64 quarts.	87.	$20.
10.	6800.	37.	86 lbs.	63.	60 lbs.	88.	7560 bot.
11.	98040.	38.	63 s.	64.	75 c.	89.	$40.80.
12.	$53\frac{44}{96}$.	39.	356 lbs.	65.	$108.	90.	$124.16.
13.	108332.	40.	$28.	66.	$7.92.	91.	$1100.
14.	11542.	41.	300 m.	67.	$252.	92.	$442.
15.	33611.	42.	$19.	68.	936 m.	93.	$3180.
16.	26869.	43.	$18.	69.	45 cents.	94.	$9600.
17.	3810225.	44.	$75.	70.	$30.	95.	$17280.
18.	12469.	45.	106 sh.	71.	$140.	96.	$1981980.
19.	2720325.	46.	$837.	72.	$150.	97.	$960.
20.	17.	47.	$2008.	73.	12 dress's.	98.	$130.56.
21.	5 and 9 r.	48.	$1744.	74.	16 boxes.	99.	52 y. 6 m.
22.	18 and 424 over.	49.	$114.	75.	110 miles.	100.	4 weeks.
		50.	$165.				

APPENDIX.

METRIC WEIGHTS AND MEASURES.

Art. **1.** The *Metric System* of Weights and Measures is founded upon the *decimal notation*, and is so called because its primary *unit* or *base* is the *Meter*.

2. The *meter* is the *unit of length*, and is equal to *one ten-millionth* part of the distance on the earth's surface from the equator to the pole, or 39.37 inches nearly.

3. From the meter are derived the *unit of surface* called the *are*, the *unit of capacity* called the *liter*, and the *unit of weight* called the *gram*.

4. The several ascending and descending denominations increase and decrease regularly by the scale of ten, according to the *law of simple numbers*. (Art. 9.)

5. The names of the *higher* denominations are formed by prefixing to the several *units* the *Greek* numerals, *dec'a*, *hec'to*, *kil'o*, and *myr'ia*, which denote 10, 100, 1000, 10000; as dec'ameter, hec'tometer, kil'ometer, myr'iameter.

The names of the *lower* denominations are formed by prefixing to the *units* the *Latin* numerals, *dec'i*, *cen'ti*, and

Quest.—1. Upon what is the Metric System founded? Why so called? 2. What is the meter? 3. From what are the units of surface, capacity, and weight derived? 4. How do the ascending and descending denominations increase? 5. How are the names of the higher denominations formed? The lower?

mil′li, which denote $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$; as dec′imeter, centimeter, and mil′limeter.

NOTE.—The numeral prefixes are the key to the system, and therefore should be thoroughly learned at the outset.

LINEAR MEASURE.

6. The *unit of length* is the *meter*. The denominations are the *mil′limeter*, *cen′timeter*, *dec′imeter*, *me′ter*, *dec′ameter*, *hec′tometer*, *kil′ometer*, and *myr′iameter*.

Denominations			Equivalents.*
10 mil′li-me-ters (mm.)	= 1 cen′ti-me-ter,	(cm.)	= .3937 in.
10 cen′ti-me-ters	= 1 dec′i-me-ter,	(dm.)	= 3.937 in.
10 dec′i-me-ters	= 1 me′ter	(m.)	= 39.37 in.
10 me′ters	= 1 dec′a-me-ter,	(Dm.)	= 393.7 in.
10 dec′a-me-ters	= 1 hec′to-me-ter,	(Hm.)	= 328 ft. 1 in.
10 hec′to-me-ters	= 1 kil′o-me-ter,	(Km.)	= 3280 ft. 10 in.
10 kil′o-me-ters	= 1 myr′i-a-me-ter,	(Mm.)	= 6.2137 miles.

Approximate values, expressed in round numbers, are often useful in comparing Metric Weights and Measures with those in common use. The following are proposed:—

Consider a *meter* 40 inches; a *decimeter* 4 inches; 5 meters 1 rod; a kilometer 200 rods, or $\frac{5}{8}$ of a mile, &c.

NOTES.—1. The term *meter*, is from the Greek *metron*, a *measure.* The *standard* meter is a scale made of platinum, and is preserved in the National Archives.

2. The denominations most used in linear measure, are the *centimeter*, *meter*, and *kilometer.* Long distances, as roads, canals, &c., are reckoned in kilometers; short distances, as cloths, ribbons, &c., are estimated by the meter and centimeter. The millimeter, decimeter, decameter, &c., like mills, dimes, and eagles, in Federal money, are seldom used.

In reciting the tables the last column may be omitted.

QUEST.—6. What are the denominations in linear measure? Repeat the table. *Note.* What are the denominations most used in this measure?

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* Act of Congress 1866.

SQUARE MEASURE.

7. The *square meter* is the *unit* for measuring ordinary surfaces, as floors, ceilings, &c. For smaller surfaces, the *square decimeter*, *centimeter*, &c., are employed.*

As the meter contains 10 decimeters, a *square meter* must contain 100 *square decimeters*, for $10 \times 10 = 100$. For the same reason a *square decimeter* must contain 100 *square centimeters*, &c.

Denominations.				Equivalents.†
100 sq. mil'li-me-ters	=	1 sq. cen'ti-me-ter,	=	0.155 sq. in.
100 sq. cen'ti-me-ters	=	1 sq. dec'i-me-ter,	=	15.5 sq. in.
100 sq. dec'i-me-ters	=	1 sq. me'ter,	=	1550 sq. in.

Approximate values.—Consider a sq. meter 10 sq. ft.; a sq. decimeter, 15 sq. in., &c.

NOTE.—Since square decimeters are *hundredths* of a square meter; square centimeters are *hundredths* of a square decimeter, &c.,—it follows, that, in writing them, *sq. decimeters, sq. centimeters*, &c., must each occupy *two decimal* places. Hence, if the number of sq. decimeters be less than 10, a *cipher* must be prefixed to the figure denoting them. Thus, 5 sq. meters and 7 sq. decimeters are written 5.07 sq. meters; 37 sq. meters, 3 sq. decimeters and 9 sq. centimeters are written 37.0309 sq. meters.

8. The *unit* for *Land Measure* is called the *āre*, which is equal to a *sq. decimeter*, and therefore contains 100 sq. meters. The denominations of Land Measure are the *cen'ti-āre*, the *āre*, and the *hect'āre.**

Denominations.				Equivalents.†
100 cen'ti-āres (ca.)	=	1 āre, (a.)	=	119.6 sq. yards.
100 āres	=	1 hect'āre, (Ha.)	=	2.471 acres.

QUEST.—7. What is the unit for measuring ordinary surfaces? 8. What in measuring land? *Note.* Why are there no deciare and decare in land measure?

* Lamotte.

† Act of Congress, 1866.

Approximate values.—Call a centiare 10 sq. ft.; an are 4 sq. rods, a hectare $2\frac{1}{2}$ acres.

NOTES.—1. The term *are*, is from the Latin *area*, a *surface*. Small pieces of land, as grass plats, court yards, &c., are commonly estimated by square meters.

2. It will be observed that there is no *deciare* ($\frac{1}{10}$ of an are) nor *decare* (10 ares). The reason is that the *decimal scale* is applied to the *length* of the sides of the squares, instead of their surfaces. Thus, the sides of the *centiare*, the *are*, and the *hectare*, are respectively 1 meter, 1 decameter, and 1 hectometer in length; and their surfaces are 1 sq. meter, 100 sq. meters, and 1000 sq. meters.

Had the decimal scale been applied to the *surface*, the sides of surfaces containing 10 square meters, 1000 square meters, &c., could not be expressed with exactness in decimals, and to obtain them it would be neccessary to extract the *square root*.

CUBIC MEASURE.

9. The *unit* for measuring ordinary solids, as embankments, excavations, &c., is the *cubic meter*. Smaller bodies are estimated in cubic *decimeters*, *centimeters*, or *millimeters*.

Since *each side* of a *cubic meter* is 10 *decimeters* in length, it follows that a cubic meter must contain 1000 *cubic decimeters;* for $10 \times 10 \times 10 = 1000$. Also, that a *cubic decimeter* contains 1000 cubic centimeters, &c.

Denominations.				Equivalents.*
1000 cu. mil'limeters	=	1 cu. cen'timeter,	=	0.061 cu. in.
1000 cu. cen'timeters	=	1 cu. dec'imeter,	=	61.026 cu. in.
1000 cu. dec'imeters	=	1 cu. me'ter,	=	35.316 cu. ft

NOTE.—As cubic decimeters are *thousandths* of a cubic meter, cubic centimeters *thousandths* of a cubic decimeter, &c., it follows that cubic decimeters, centimeters, &c., must each occupy *three decimal* places; consequently, if the number of cubic decimeters, &c., is

QUEST.—9. What is the unit for measuring ordinary sol'ds? *Note.* How many places do cu. decimeters occupy?

* Act of Congress, 1866.

less than 100, ciphers must be prefixed to the figure or figures denoting them. Thus, 73 cubic meters and 5 cubic decimeters are written 73.005 cubic meters.

10. The *unit* for measuring *wood* and *timber* is called the *stêre*, which is equal to a *cubic meter*. The stêre has only *one subdivision*, which is called the *dec'i-stere*, and *one multiple*, called the *dec'a-stere*.

Denominations.		Equivalents.*
10 dec'i-steres =	1 stere	= 35.316 cu. ft., or 1.308 cu. yds.
10 steres =	1 dec'a-stere	= 2.759 cords, or 353.6 cu. ft.

Approximate values.—Call a decistere 3¼ cubic feet; a stere ¼ cord; a decastere 2½ cords.

NOTES.—1. The term *stere*, is from the Greek *steros*, *solid*.

2. In France, *fire-wood* is commonly measured in a *cubical box* or *crib*, whose *length*, *breadth*, and *height* are each 1 meter.

3. In computing large quantities of wood, it is customary to reckon by *steres* or *decasteres*.

DRY AND LIQUID MEASURE.

11. The *unit* for measuring *liquids* and *dry articles* as oil, wine, grain, fruit, &c., is the *cubic decimeter*, which is called the *lï'ter*. The liter has the form of a cylinder, and is equal to 1.0567 wine quarts. The denominations are the *mil'lilïter*, *cen'tilïter*, *dec'ilïter*, *lï'ter*, *dec'alïter*, *hec'tolïter* and *kil'olïter*.

Denominations.		Dry Measure.*	Liquid Measure.*
10 mil'li-lï-ters =	1 cen'ti-lï-ter =	0.6102 cu. in. =	0.338 fld. oz
10 cen'ti-lï-ters =	1 dec'i-lï-ter =	6.1022 cu. in. =	0.845 gill
10 dec'i-lï-ters =	1 lï'ter =	0.908 quart =	1.0567 quart.
10 lï'ters =	1 dec'a-lï-ter =	9.08 " =	2.6417 galls.
10 dec'a-lï-ters =	1 hec'to-lï-ter =	2.8375 bush. =	26.417 "
10 hec'to-lï-ters =	1 kil'o-lï-ter =	28.372 " =	264.18 "

QUEST.— 10. What is the unit for measuring wood? Name the denominations? 11. What is the unit of dry and liquid measure? The denominations? *Note.* What denominations are used most?

* Act of Congress, 1866.

Approximate values.—Call a liter 1 quart, and a hectoliter 2½ bushels.

NOTES.—1. The term *liter* is from the Greek *litron, a measure of capacity.*

2. The denominations chiefly used in liquid measure, are the *liter, decaliter,* and *deciliter;* in dry measure the *liter, decaliter, hectoliter,* and *kiloliter.*

3. Since the liter is equal to a cubic decimeter, it follows that the kiloliter (1000 liters) contains a cubic meter: that the deciliter (the 10th of a liter) contains 100 cubic centimeters, &c.

4. A *milliliter* of water weighs 1 *gram;* a *liter* 1 *kilogram;* and a *kiloliter,* or *cubic meter,* 1 *tonneau,* or *ton.*

WEIGHTS.

12. The *unit* of weight is called the *gram,* which is equal to the weight of a *cubic centimeter* of distilled water in a vacuum, at the temperature of 39.83° Fahrenheit,* or 15.432 grains. The denominations are the *mil'li-gram, cen'ti-gram, dec'i-gram, gram, dec'a-gram, hec'to-gram, kil'-o-gram, myr'i-a-gram, quin'tal,* and *mil'lier* or *ton.*†

Denominations.				Equivalents.‡		
10 mil'ligrams	=	1 cen'tigram	=	0.1543		grains.
10 cen'tigrams	=	1 dec'igram	=	1.5432		"
10 dec'igrams	=	1 gram	=	15.432		"
10 grams	=	1 dec'agram	=	0.3527	oz.	avoir.
10 dec'agrams	=	1 hec'togram	=	3.5274	"	"
10 hec'tograms	=	1 kil'ogram	=	2.2046	lbs.	"
10 kil'ograms	=	1 myr'iagram	=	22.046	"	"
10 myr'iagrams	=	1 quin'tal	=	220.46	"	"
10 quin'tals	=	1 mil'lier or ton	=	2204.6	"	"

QUEST.—12. What is the unit of weight? Name the denominations.

* This temperature is equivalent to 4° of the Centigrade Thermometer, and is the point at which water attains its maximum density.

† Contraction of tonneau.

‡ Act of Congress, 1866.

Approximate values.—Call a *gram* 15 grains; a *kilogram* 2½ pounds; a *quintal* 220 pounds, and a *tonneau* 2200, or a long ton.

NOTES.—1. The term *gram* is from the Greek *gramma*, a standard.

2. The denominations of weight most in use, are the *gram* and *kilogram.** The gram with its subdivisions is used in mixing medicines, and other cases requiring minuteness and accuracy. The kilogram, somtimes contracted to *kilo*, is the ordinary weight of commerce. In weighing heavy articles the quintal and ton are used.

FRENCH CURRENCY.

13. The currency of France, like the weights and measures, is based upon the *decimal system.* The denominations are the *franc*, the *dec'ime*, and *cen'time.*

Denominations.				Equivalents.	
10 cen'times	=	1 dec'ime	=	0.0186	dollar.
10 dec'imes	=	1 franc	=	0.186	"

NOTE.—The *franc* is the *unit.* The *decime*, like our dime, is seldom used; its value being expressed in *centimes* or *hundredths* of a franc. Thus, 85 francs, 4 decimes, and 3 centimes, are written 85.43 francs. *Centimes*, being *hundredths* of a franc, require two decimal places.

14. The *coins* of France are of three kinds,—*gold*, *silver*, and *bronze.*

The *gold* coins are 40 franc, 20 franc, and 5 franc pieces.

The *silver* coins are the franc, 2 franc, and 5 franc pieces.

The *bronze* coins are 1 centime, 2 centimes, 5 centimes, and 10 centimes; which weigh, 1, 2, 5, and 10 grains, respectively.

NOTE.—The standard of the gold and silver coins is $\frac{9}{10}$ pure metal, and $\frac{1}{10}$ alloy.

1. Write 125 francs and 7 centimes.

Ans. 125.07 francs

2. Write 260 francs and 4 decimes.

3. Write 907 francs, 3 decimes, 8 centimes.

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* The standard kilogram adopted a as model for weights, is made of platinum, and preserved in the archives of the government.

METRIC NOTATION.

15. Ex. 1. Write 7 kilometers, 5 hectometers, 4 meters, 2 decimeters, and 8 centimeters, in meters.

Analysis.—Since the denominations of the Metric System increase and decrease by the decimal scale, it is plain they may be written one after another like *simple numbers*, placing a *decimal* point between the denomination regarded as the *unit*, and those below it, to show that the latter are *decimal parts.* There being no decameters, a cipher is put in its place. The above distance is therefore equivalent to 7504.28 meters. Hence,

Operation.

7504.28 meters.

16. To express *Metric* Weights and Measures.

Write the given denominations one after another, beginning with the highest, and place a decimal point between the one taken as the unit, and those below it.

NOTES.—1. If any intervening denominations are omitted in the given number, their places must be supplied by ciphers. (Art. 15.)

2. In *Metric* as well as in *Compound* Numbers, convenience requires that the *measure* employed as the *unit,* should be *proportionate* to the thing measured. Thus, long distances, as from New York to San Francisco, should not be stated in meters, for the reason that the number would be too large. Nor should the meter be employed to measure the thickness of paper, because its thickness is too small a part of that unit.

1. Express 5 kiloliters, 8 hectoliters, 7 liters, and 4 centiliters in hectoliters, in liters, and centiliters, respectively.

Ans. 58.0704 hectoliters; 5807.04 liters; 580704 centiliters.

2. Write 13 quintals, 4 myriagrams, 1 kilogram, 5 grams, and 25 centigrams, making the kilogram the unit.

QUEST.—16. How write metric weights and measures? *Note.* If any denomination is ommitted. what is to be done?

3. Write 18 sq. meters and 5 sq. decimeters. *Ans.* 18.05 sq. m.

4. Write 17 hectares, 6 ares, and 3 centiares, in ares. *Ans.* 1706.03 ares.

5. Express in cubic meters, 19 cubic meters and 17 cubic decimeters. *Ans.* 19.017 cu. m

6. Express in liters, 61 hectoliters, 7 liters, 3 centiliters and 5 milliliters.

REDUCTION OF METRIC WEIGHTS AND MEASURES.

Case I.

17. *To reduce Metric Weights and Measures from a higher denomination to a lower.*

Ex. 1. Reduce 46.3275 kilometers to meters.

Analysis.—Since a *unit* of a higher denomination equals ten in the next lower, it is plain, to reduce a higher denomination to the next lower, we must multiply by 10; to reduce it to the next lower still, we must multiply it again by 10, and so on.

Operation.

46.3275 Km. = 46327.5 m.

But to multiply by 10, we remove the decimal point one place to the right, &c. (Art. 192.) Hence, the

Rule.—*Remove the decimal point one place to the right for each lower denomination to which the given number is to be reduced, annexing ciphers if necessary.*

Note.—If the given number has no decimal figures, the decimal point is supposed to occupy the first place on its right.

Quest.—17. How reduce metric numbers from higher to lower denominations?

2. Reduce 867 kilograms to grams.

Ans. 867000 grams.

3. Reduce 264.42 hectoliters to centiliters.

4. In 2561 ares, how many square meters?

5. In 8652 cubic meters, how many cubic decimeters?

6. In 63240 cubic decimeters, how many cubic centimeters. *Ans.* 63240000 cu. Cm

7. Reduce 4256.25 kilograms to grams.

8. Reduce 845 francs to centimes.

CASE. II.

18. *To reduce Metric Weights and Measures from a lower denomination to a higher.*

9. Reduce 84526.3 meters to kilometers.

Operation.

84526.3 M. = 84.5263 Km.

Analysis.—Since it takes *ten* of each *lower* denomination to make a *unit* in the next *higher*, it follows that, to reduce a number from a lower to the next higher denomination, it must be divided by 10; to reduce it to the next higher still, it must be divided again by 10, and so on. (Art. 9.) But to divide by 10, we remove the decimal point one place to the *left*, &c. (Art. 195.) Hence the

RULE.—*Remove the decimal point one place to the left for each higher denomination to which the number is to be reduced, prefixing ciphers if necessary.*

10. Reduce 87 meters to kilometers.

Ans. 0.087 Km.

11. In 1482.35 grams, how many kilograms?

QUEST.—18. How reduce metric weights and measures from lower to higher denominations?

12. In 39267.5 liters, how many kiloliters?

13. Reduce 812067 centiares to hectares.

14. In 1000000 cubic centimeters, how many cubic meters? *Ans.* 1 cu. m.

15. In 605349 cubic meters, how many kiloliters?

Case III.

19. *To reduce Metric Weights and Measures to the common system.*

1. Reduce 3 hectometers, 6 decameters, and 5 decimeters to inches.

Analysis.—3 hectometers, 6 decameters, and 5 decimeters =360.5 meters. Now as there are 39.37 inches to every meter, there must be 39.37 times as many inches as meters; and 360.5 × 39.37 = 14192.885 in. or 1182.74 + ft. Hence the

Operation.

360.5	meters.
39.37	
14192.885 in.	

Rule.—*Express the given metric number decimally in the denomination of the principal unit of the table, and multiply it by the value of that unit; the product will be the answer.*

Notes.—1. The product will be in the same denomination as that in which the value of the principal unit of the table is expressed, and may be reduced to any other denomination required. (Arts. 161, 162.)

2. The principal unit of dry and liquid measure is the *liter*; that of weight is the *gram* or *kilogram*, &c.

2. Reduce 573 kilograms to pounds.

Ans. 1263.2358 pounds

Quest.—19. How reduce metric weights and measures to the common system?

3. In 1285 liters, how many wine gallons?

4. In 391 kiloliters, how many bushels?

5. Reduce 1865 meters and 25 centimeters of cloth to yards. *Ans.* 2039.61206 + yards.

6. In 35260 ares of land, how many acres?

7. Reduce 508.85 francs to Federal money.

CASE IV.

20. *To reduce common weights and measures to the Metric System.*

8. In 48 rods, 6 feet, 5 inches, how many meters?

Analysis.—48 rods, 6 feet, 5 inches = 9581 inches.

Now as 39.37 inches make 1 meter, 9581 inches will make as many meters as 39.37 is contained times in 9581, which is 243.357 in. Hence the

Operation.

39.37) 9581.00000.
243.357 + m.

RULE.—*Reduce the compound number to units and decimals of a unit of the same denomination as that in which the principal metric unit of the table is expressed, and divide it by the value of this unit; the quotient will be the answer.*

NOTE.—The quotient will be in the denomination of the principal unit of the table, whose value has been employed as a divisor.

9. In 3 cwt. 15 lbs. 12 oz., how many kilograms?

Solution.—3 cwt. 15 lbs. 12 oz. = 315.75 lbs., and $315.75 \div 2.0246 = 156.944 +$ kilograms.

10. Reduce 1917 miles to the metric system.

Ans. 1191.160 + Km.

QUEST.—20. How reduce common weights and measures to the metric system? *Note.* Of what denomination is the quotient?

11. In 13750 pounds, how many kilograms?
12. Reduce 2056 bushels, 3 pecks to kiloliters.
13. Reduce 9256 sq. rods to ares.
14. Reduce 14506 cu. feet to cu. meters.
15. Reduce $357.375 to francs.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF METRIC WEIGHTS AND MEASURES.

21. Since the denominations of Metric Weights and Measures *increase* and *decrease* by the scale of *ten*, those *above units* occupying the place of *tens*, *hundreds*, *thousands*, *&c.*, those below, *tenths*, *hundredths*, *&c.*, it is plain they may be *added*, *subtracted*, *multiplied*, and *divided*, by the corresponding rules of Decimal Fractions.

22. Ex. 1. What is the sum of 7358.356 meters, 86.142 decameters, 95 centimeters, and 450.6 meters.

Analysis.—Reducing the given numbers to the same denominations, *viz.*, to meters, 86.142 decameters become 861.42 meters, and 95 centimeters become 0.095 meters. *We now write the numbers units under units, tenths under tenths, &c., and proceed as in Addition of Decimals.* (Art. 187.)

Operation.

7358.356	meters.
861.42	"
0.095	"
450.6	"
8670.471	"

2. What is the sum of 2358.35 liters, 861.45 liters, 98.831 liters, and 643.5 liters? *Ans.* 3962.131 liters.

QUEST.—22. How add metric numbers.

3. Find the sum of 145.19 kilograms of sugar, 168.45 kilograms, 431 kilograms, 8.60 kilograms, 36.1 kilograms, and 465.81 kilograms.

4. Find the sum of 2346.43 meters of cloth, 45.3 meters, 156.21 meters, and 236.8 meters.

Ans. 2784.74 meters.

5. What is the sum of 67.2560789 kilometers, 346.852 decameters, 905.204 meters, and 5670 millimeters.

Ans. 71630.3699 meters.

23. Ex. 6. From 6725.724 meters, subtract 4.16631 kilometers.

Analysis.—Reducing the numbers to the same denomination, 4.16631 kilometers = 4166.31 meters.

We now write the less number under the greater, units under units, tenths under tenths, &c., and proceed as in Subtraction of Decimals. (*Art.* 189.)

Operation.

6725.724	m.
4166.31	
12559.414	m.

7. What is difference between 6843.94 liters and 394.203 liters? *Ans.* 6449.737 liters.

8. Find the difference between 931 kilograms and 391.275 kilograms.

9. Find the difference between 6125 ares and 61.54 ares. *Ans.* 606.346.

10. The difference between 563 myriameters and 265346 decimeters?

11. What is the difference between 568 steres and 101 decasteres?

QUEST.—23. How subtract metric numbers?

24. Ex. 11. How much silk is there in 117½ pieces, each of which contains 83.75 meters?

Analysis.—Since 1 piece contains 83.75 meters, 117½ pieces will contain 117½ times as much.

We multiply, and point off the product as in Multiplication of Decimals. (Art. 191.)

Operation.

83.75	meters.
117.5	
9840.625	"

12. What cost 4125.63 kiloliters of wheat, at $12.50 a kiloliter? *Ans.* 51570.375 Kl.

13. What cost 361 hectoliters of wine, at 5.4 francs per liter? *Ans.* 194940 f.

14. How many square feet in a room whose length is 6.2 meters, and its width 4.56 meters?

15. At $1.75 a square meter, what will it cost to carpet a hall, whose length is 16.5 meters, and whose breadth is 7.4 meters? *Ans.* 213.675.

16. If 1 are of land cost 86.95 francs, what will 350.28 ares cost?

17. If 1 stere of wood cost 6.25 francs, what will 79 steres cost? *Ans.* 493.75 f.

18. What will it cost to dig a cellar 12.2 meters long, 5.4 meters wide, and 2.8 meters deep, at 45 cents per cubic meter?

19. What cost 65 hectares of land, at $15½ per are?

20. How many ares in a field 365 hectometers long, and 243 decameters wide?

21. What will 7 hectoliters of brandy come to, at 7.03 francs per liter?

QUEST.—24. How multiply metric numbers.

25. Ex. 22. A man divided 980.5 kilograms of flour equally among 185 soldiers; how much did each receive?

Analysis.—If 185 soldiers receive 980.5 kilograms, 1 soldier must receive as many kilograms as 185 is contained times in 980.5.

Operation.

185) 980.5
5.3 Kg.

We divide, and point off the quotient as in Division of Decimals. (Art. 194.)

23. A merchant paid $1872.40 for 234.45 meters of broadcloth; what was that per meter? *Ans.* $7.986 +

24. Paid 216.15 francs for 35.5 liters of molasses; how much was that per liter?

25. A man traveled 5682.5 kilometers in $7\frac{1}{2}$ days, how far did he travel per day? *Ans.* 757.66 + Km.

26. How many cloaks can be made from 425.8 meters of cloth, allowing to each cloak 15.4 meters.

27. A farmer, having 58.65 ares of land, wishes to fence it into fields of 3.45 ares each; how many fields can he make? *Ans.* 17 fields.

28. How many boxes, each holding 12.05 kilograms, will it take to pack 759.15 kilograms of butter?

39. The area of a grass plat is 21.06 sq. meters, and its length 6.48 meters; what is its width?

30. If I pay $276 for 92 meters of silk, how much is that per meter? *Ans.* $3.

31. A man laid out 3175 francs in flour, at 25 francs per barrel; how many barrels did he buy.

QUEST.—25. How divide metric numbers?