UNIVERSITY OF ILLINOIS BULLETIN
Vol. III APRIL 15, 1906 No. 14
[Entered at Urbana, Illinois, as second-class matter]

TESTS OF
REINFORCED CONCRETE BEAMS
SERIES OF 1905

BY

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BULLETIN NO. 4 OF THE UNIVERSITY OF ILLINOIS.
ENGINEERING EXPERIMENT STATION

URBANA, ILLINOIS

PUBLISHED BY THE UNIVERSITY

THIS BULLETIN is the fourth of a series of bulletins published by the Engineering Experiment Station of the University of Illinois. It is the purpose of the Station to carry on investigations along various lines of engineering, and to study problems of importance to professional engineers and to the manufacturing, railway, mining, constructional and industrial interests of the State.

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The following have been issued:

Bulletin No. 1. Tests of Reinforced Concrete Beams, by A. N. Talbot. 1904.

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905.

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905.

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906.

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906.

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by A. N. Talbot. 1906.

The volume and number at the top of the title page of the cover refer to the general publications of the University of Illinois; at the bottom is given the number of the Engineering Experiment Station bulletins or circulars.

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No 4 APRIL 1906

TESTS OF REINFORCED CONCRETE BEAMS SERIES OF 1905.

BY ARTHUR N. TALBOT, PROFESSOR OF MUNICIPAL AND SANITARY ENGINEERING, IN CHARGE OF THEORETICAL AND APPLIED MECHANICS.

In Bulletin No. 1 of the University of Illinois Engineering Experiment Station, Tests of Reinforced Concrete Beams, the results of the investigations made in the Laboratory of Applied Mechanics of the University of Illinois in 1904 were recorded and discussed. Further tests were made in 1905. It is the purpose of this bulletin to describe and discuss the results of the series of 1905.

As was stated in Bulletin No. 1, the principles governing the strength and action of reinforced concrete construction have not been fully established, and the opinions and theories presented by engineers are somewhat conflicting. However, many of these points are being cleared up, and experimental work on reinforced concrete is gradually establishing the principles. The results given in Bulletin No. 1 have a bearing upon the action of reinforced concrete in simple flexure and upon the calculation of the strength of beams. The following topics and conclusions, among others, are there considered: the general action of the beam during the progress of the loading and the division of the phenomena included in the tests into four stages representing different conditions in the steel and concrete; the determination of deformations in the steel and concrete by careful instrumental work; the determination of the position of the neutral axis by experimental methods and the checking up of the tension developed in the steel by the use of the observed deformations and the position of the neutral axis thus obtained; an experimental determination of the amount of steel which will develop the full compressive strength of the concrete; for beams not having a sufficient amount of steel to develop the full compressive strength and not failing by secondary or web stresses, the conclusion that the maximum strength of the beam occurs at slightly above the yield point of the steel instead of the ultimate strength of the steel, and hence that the yield point of the steel used has an important bearing on the design of the beam; the conclusion that the form of the load-deformation curve goes to show that during the second stage the concrete fails in tension in such a way as to make this tension negligible in the calculation of the bending moment, instead of that the stretch is kept up in such a way as to continue to furnish strength to the beam, as has been contended. In general the results were such as to give confidence in the value of experimental methods in establishing principles for use in the design of reinforced concrete construction if only the tests are made in a systematic and scientific manner.

The Series of 1905 was undertaken with a view of further developing the fundamental principles governing reinforced concrete construction as well as of uncovering the field for future experimentation. It was considered best to restrict the tests of the year to matters bearing on the principles involved in beam action rather than to take up features involving details of construction. Among the topics taken up for consideration were the following: effect of amount of reinforcement; use of steel with high elastic limit but having a very smooth surface; failure by tension in steel, compression in concrete, bond, and diagonal tension; abnormal concretes, including lean mixtures, poorly mixed concrete, and so-called plane of set; effect of method of loading, of repetition of load, of rest, of retention of load, and of position of the reinforcing bars; position of neutral axis; plain concrete in compression; shearing strength of concrete; encased steel in tension; thermal conductivity of concrete. The tests were made in the Laboratory of Applied Mechanics of the University of Illinois. In common with a number of engineering schools, the work was done in cooperation with the Joint Committee on Concrete and Reinforced Concrete which was composed of representatives of several of the leading engineering societies of the country. The writer was chairman of the Committee on Tests for the Joint Committee and this series was the part of the work assigned to the University of Illinois. The Joint Committee furnished the cement, sand, stone, and steel. The work of testing the beams was done principally as thesis work. The data have now been worked over and the calculations checked, new drawings made, and results more fully studied and discussed. The following list gives the names of the members of the class of 1905 in civil engineering who presented theses in the line of reinforced concrete. They are entitled to credit for the care, skill, and interest and for the thorough and competent way in which they conducted their work.

D. A. Abrams,	Distribution of stresses.
Ralph Agnew and C. E. Sims,	Plain concrete in compression and encased steel in tension.
F. I. Blair,	Effect of repetition of load.
M. B. Case,	Effect of retention of load.
V. R. Fleming,	Comparison of methods of loading.
J. C. Gilmour,	Varying percentages of reinforcement.
S. C. Hadden,	Effect of removal of load.
E. T. Renner,	Effect of release of load.
W. H. Roney,	Effect of position of reinforcing bars.
J. E. Shoemaker and C. S. O'Connell,	Shearing strength of concrete.
W. H. Warner,	Abnormal concretes.

The investigation of plain concrete in compression and encased steel in tension and of shearing strength of concrete, as well as the investigation by L. A. Waterbury on thermal conductivity of concrete, is reserved for separate bulletins.

It will be noted that the problems are made as simple as possible with a view of getting data bearing on the establishment of principles, and for this reason the number of variables was made as small as possible. The experience gained in the tests of the Series of 1904 was taken advantage of. Supervision of this work was given by R. V. Engstrom, Instructor in Theoretical and Applied Mechanics, whose aid in this and in planning the tests and in interpreting the results has added much to the value of the work. Acknowledgment is also made to D. A. Abrams, Assistant in Laboratory of Applied Mechanics, for aid in the preparation of this bulletin.

As in the discussion of the results of the tests reference will be made to the formulas and methods of making computations, it seems well first to make a summary of the principles involved and the nomenclature used in the analytical treatment of flexure, as well as of the methods of failure of beams, and this is given under the head of Resistance of Beams to Flexure. The following division of the subject matter of the bulletin will be made: I. Resistance of Beams to Flexure. II. Materials, Test Pieces and Testing. III. Experimental Data and Discussion. Diagrams showing load-deformation curves and deflections of representative beams and positions of the neutral axis follow the text. The outline on the following page gives the order of presentation:—

I. Resistance of Beams to Flexure

1. *Preliminary.*—Flexure of reinforced concrete beams seems more complicated than is the case with steel and timber beams. In steel and timber beams it is usual to consider only horizontal tensile and compressive stresses with perhaps a check on horizontal or vertical shearing stresses. Other stresses exist which are ordinarily negligible. In reinforced concrete beams these secondary stresses may form the element of weakness. Unless it is known from the general dimensions and make-up of the beam that the secondary stresses do not control the strength of the beam, it will be necessary to calculate the stresses developed in the various ways. In general it may be said that a reinforced concrete beam may fail by one or more of the following methods: 1. Tension of steel; 2. Compression of concrete; 3. Shearing of concrete; 4. Bond or slip of bars; 5. Diagonal tension of concrete; 6. Miscellaneous methods, like the splitting of bars away from the concrete, the effect of the bearings, etc. What one of these methods of failure will govern the strength of a beam is dependent upon percentage of reinforcement, kind of steel, quality of concrete, relation of depth of beam to length of span, disposition of reinforcement, and other conditions. Generally, for a given beam, we may narrow down the number of probable methods of failure without much calculation.

Before discussing these methods of failure further, it will be well to go over the analytical treatment of the resistance of beams to flexure. This treatment will be made as brief as possible, a summary only of part of the work being given and no effort being made to cover the field thoroughly. The analysis governing the tension in the steel and the compression in the concrete follows the lines of the article of the writer in the Journal of the Western Society of Engineers, Vol. 9, August, 1904, page 394. The usual assumptions that the loads are applied at right angles to the length of the beam, that the supports will permit free longitudinal movement, that a plane section before bending remains a plane section after bending, and that the metal and surrounding concrete stretch together, are made. It is further assumed that the tensile strength of the concrete is negligible in the part of the beam where the bending moment is greatest, at least in the calculation of the resisting moment of the beam at the time of maximum load. The analysis is restricted to rectangular beams with reinforcement on the tension side only, and refers generally to simple beams free from end restraints.

2. *Notation.*—The following notation will be used:

b = breadth of rectangular beam.

d = distance from the compression face to the center of the metal reinforcement.

A = area of cross section of metal reinforcement.

$p = \frac{A}{bd}$ = ratio of area of metal reinforcement to area of concrete above center of reinforcement.

o = circumference or periphery of one reinforcing bar.

m = number of reinforcing bars.

E_s = modulus of elasticity of steel.

E_c = initial modulus of elasticity of concrete in compression, a term which will be defined.

$n = \frac{E_s}{E_c}$ = ratio of two moduli.

f = tensile stress per unit of area in metal reinforcement.

c = compressive stress per unit of area in most remote fiber of concrete.

c' = compressive stress per unit of area which causes failure by crushing.

ε_s = deformation per unit of length in the metal reinforcement.

ε_c = deformation per unit of length in most remote fiber of the concrete.

ε'_c = deformation per unit of length when crushing failure occurs; i. e., ultimate or crushing deformation.

$q = \frac{\varepsilon_c}{\varepsilon'_c}$ = ratio of deformation existing in most remote fiber to ultimate or crushing deformation.

k = ratio of distance between compression face and neutral axis to distance d.

z = distance from compression face to center of gravity of compressive stresses.

d' = distance from the center of the reinforcement to center of gravity of compressive stresses.

ΣX = summation of horizontal compressive stresses.

M = resisting moment at the given section.

s = horizontal tensile stress per unit of area in the concrete.

t = diagonal tensile stress per unit of area in the concrete.

u = bond stress per unit of area on the surface of the reinforcing bars.

v = vertical shearing stress and horizontal shearing stress per unit of area in the concrete.

3. *Relation between Stress and Deformation for Concrete in Compression.*—Concrete does not possess the property of proportionality of stress and deformation for wide ranges of stress as does steel; in other words, the deformation produced by a load is not proportional to the compressive stress. The relation between stress and deformation is not entirely uniform; there are even considerable differences in deformations for the same mixtures. Various curves have been proposed to represent the stress-deformation relation but the parabola is the most satisfactory general representation. Frequently the parabola expresses the relation almost exactly, and in nearly every case the parabolic relation will fit the stress-deformation diagram very closely throughout the part which is ordinarily developed in beams, the lack of agreement near the crushing point not being of importance. The analytical work with the parabola is not complicated, and this curve offers easy comparison with the straight-line relation and easy translation to this relation. Even if the straight-line relation be accepted as sufficient for use with ordinary working stresses, the parabolic or other variable relation must be used in discussing experimental data when any considerable deformation is developed in the concrete. The parabola will be adopted as the basis of the analytical work used in this bulletin.

Fig. 1 shows such a stress-deformation curve. For purposes of illustration, the crushing strength of the concrete is represented as 2000 lb. per sq. in. and the ultimate unit deformation as .002. The relation between proportionate stress or ratio of stress developed to ultimate compressive strength of the concrete $\left(\frac{c}{c'}\right)$ and proportionate deformation or ratio of deformation developed at the given stress to ultimate or crushing deformation $\left(\frac{\varepsilon_c}{\varepsilon'_c}=q\right)$, which forms the basis of this analysis, is also shown by the figure.

Modulus of elasticity is a term which has been used very loosely in connection with reinforced concrete. In the general theory of flexure it is defined to be the ratio of the unit stress to the unit deformation within the elastic limit of the material. As applied in this way to materials having the property of proportionality of stress and deformation the modulus of elasticity is a constant. For materials with a variable stress-deformation relation like concrete it may not be considered proper to call the variable ratio the modulus of elasticity and such a use in connection with formulas for flexure of concrete may lead to misunderstandings. However, it is important that a definite expression for this ratio be found. The writer obtains this relation from the initial modulus of elasticity and uses the term "initial modulus

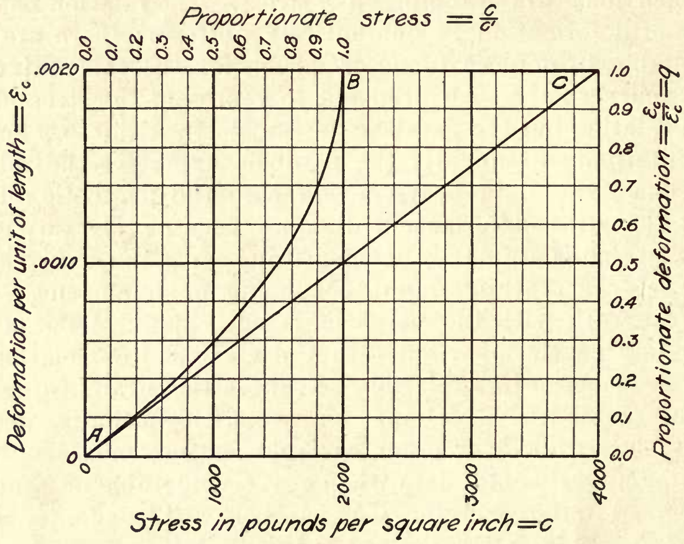

FIG. 1. STRESS-DEFORMATION CURVE FOR CONCRETE IN COMPRESSION.

of elasticity" to express the relation which would exist between stress and deformation if the concrete compressed uniformly at the rate it compresses for the lower stresses. The tangent of the angle which the line AC in Fig. 1 makes with the vertical gives this initial modulus of elasticity E_c. The line is tangent to the parabola at A, and its equation is $x = E_c\, y$. By means of this initial modulus of elasticity the parabolic stress-deformation relation may, from the properties of the parabola, be expressed as

$$c = E_c\,\varepsilon_c - \tfrac{1}{2}\,\frac{1}{\varepsilon'_c}\,E_c\,\varepsilon_c^2 = (1 - \tfrac{1}{2}\,q)\,E_c\,\varepsilon_c \,\ldots\ldots\ldots \quad (1)$$

in which q is the ratio of the deformation developed to the ultimate or crushing deformation of the concrete. From this the following equation is also true:

$$\frac{c}{c'} = (1 - \tfrac{1}{2}q)\,2q \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)$$

These relations are fundamental. The values of E_c, c, and ε_c must be obtained experimentally. The line for E_c should be taken as the line which will give a relation which will best fit throughout the range used in the test of beams and ε'_c should be taken as the abscissa of the vertex of the parabola which fits best and not necessarily as the actual crushing deformation of the concrete. It is the general relation which is important and not the values at the point of failure. Many stress-deformation diagrams have been gone over in this way, and this representation has been found quite satisfactory. It may be noted from Fig. 1 that while 2000 lb. per sq. in. will give a deformation of .002, it will take 1500 lb. per sq. in. to produce one-half of that deformation. For small stresses the stress-deformation curve does not differ much from the line of initial modulus of elasticity.

4. *Distribution of Stresses in Beams.*—Let Fig. 2 show the section of the beam. *kd* is the distance of the neutral axis below

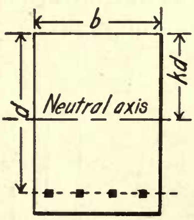

FIG. 2. SECTION OF BEAM.

the top of the beam, k being a ratio. In Fig. 3, the left diagram represents the deformations above and below the neutral axis. Consider that the upper fiber is stressed to the point of failure; the upper deformation will then be the ultimate or crushing deformation. Since the deformations are proportional to the distances from the neutral axis, the curve of compressive stresses shown on the right will be a parabola with its vertex at O. The horizontal distances to the "line for initial modulus of elasticity" represent the stresses which would exist for the same deformation with a constant modulus of elasticity equal to E_c. The stress in the steel is represented by a length proportional to the ratio of the modulus of elasticity of the steel to the initial modulus of elasticity of the concrete $\left(n = \frac{E_s}{E_c}\right)$. In like manner Fig. 4 gives the stress and deformation distribution for a deformation of the upper fiber equal to three-fourths of the ultimate deformation of the concrete and

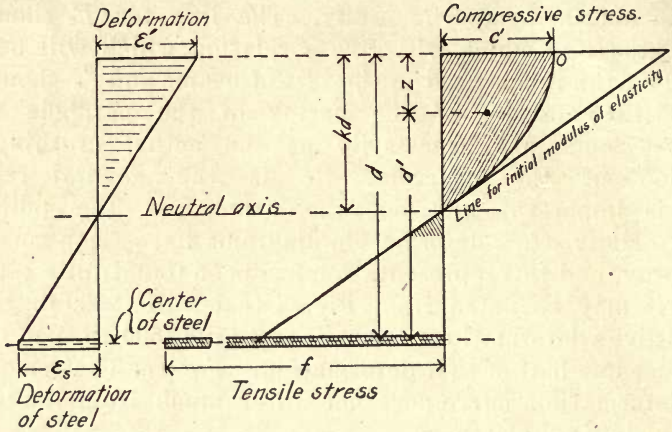

FIG. 3. STRESS AND DEFORMATION DISTRIBUTION AT ULTIMATE DEFORMATION OF CONCRETE.

a stress of fifteen-sixteenths of the crushing stress. Fig. 5 shows a similar distribution for one-half ultimate deformation and three-

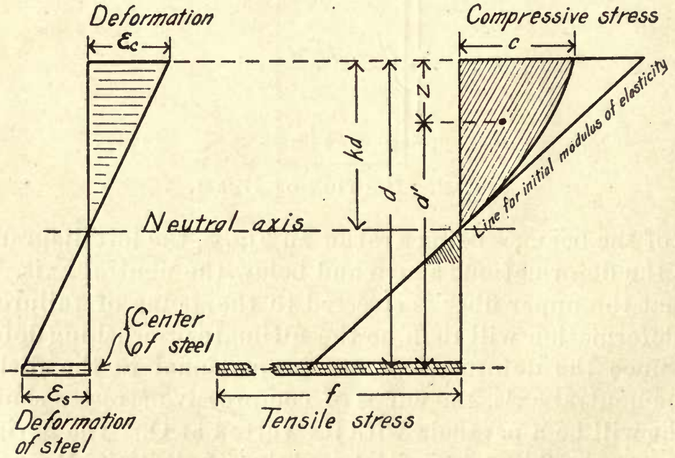

FIG. 4. STRESS AND DEFORMATION DISTRIBUTION AT THREE-FOURTHS ULTIMATE DEFORMATION OF CONCRETE.

fourths the crushing stress. It will be noted that the parabolic arc appears somewhat different from that in Fig. 3, and that it differs much less from the "line for initial modulus of elasticity."

5. *Relations in the Stress Diagram.*—In deriving formulas for resisting moment, position of neutral axis, and compressive stress at upper fiber, three relations in the stress diagram are needed: (1) the relation of the stress c and the deformation ε_c at the upper fiber; (2) the total compressive stress, here called ΣX; and (3) the position of the center of gravity of the compressive stresses given by the distance z. These relations vary for different

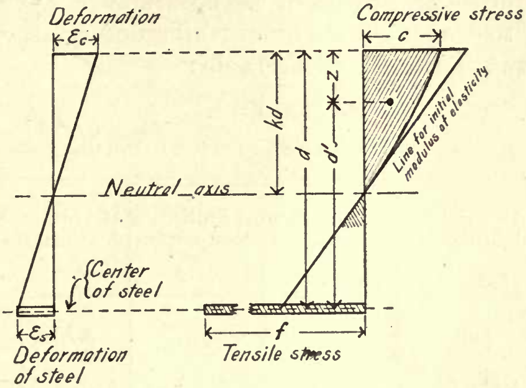

FIG. 5. STRESS AND DEFORMATION DISTRIBUTION AT ONE-HALF ULTIMATE DEFORMATION OF CONCRETE.

values of the deformation in upper fiber. Basing the variation on the parabolic stress-deformation law previously stated, and using

$q = \frac{\varepsilon_c}{\varepsilon'_c}$ as the ratio of the deformation developed in the upper

fiber to the ultimate deformation of the concrete, the following relations are readily deduced, though their derivation will not be given here.

$$\frac{c}{E_c \varepsilon_c} = 1 - \tfrac{1}{2} q \quad \text{.......} \quad (3)$$

$$\frac{\Sigma X}{\tfrac{1}{2} E_c \varepsilon_c kbd} = \frac{\text{Parabolic area}}{\text{Triangular area}} = 1 - \tfrac{1}{3} q \quad \text{.......} \quad (4)$$

$$\frac{z}{kd} = \frac{4-q}{12-4q} = \tfrac{1}{3}\left(1 + \tfrac{1}{4}\,\frac{q}{3-q}\right) \quad \text{.......} \quad (5)$$

Equation (3) gives the ratio of the compressive stress in the upper fiber to the stress which would exist for the same upper fiber deformation with a straight-line stress-deformation relation. Equation (4) gives the ratio of the summation of compressive stresses to the stress which would exist for the same upper fiber deformation with a straight-line stress-deformation relation. Equation (5) gives the ratio of the distance between the compression surface and the center of gravity of compressive stresses to the distance between that surface and the neutral axis.

Values for several ratios of the deformation developed in the upper fiber to the ultimate or crushing deformation of the concrete are given in the following table:

TABLE 1.

PROPERTIES OF THE STRESS DIAGRAM.

Property	At ultimate deformation $q=1$	At $\frac{3}{4}$ ultimate deformation $q=\frac{3}{4}$	At $\frac{1}{2}$ ultimate deformation $q=\frac{1}{2}$	At $\frac{1}{4}$ ultimate deformation $q=\frac{1}{4}$	By straight-line relation $q=0$
c	$\frac{1}{2}E_c\varepsilon_c$	$\frac{5}{8}E_c\varepsilon_c$	$\frac{3}{4}E_c\varepsilon_c$	$\frac{7}{8}E_c\varepsilon_c$	$E_c\varepsilon_c$
ΣX	$\frac{1}{3}E_c\varepsilon_c kbd$	$\frac{3}{8}E_c\varepsilon_c kbd$	$\frac{5}{12}E_c\varepsilon_c kbd$	$\frac{11}{24}E_c\varepsilon_c kbd$	$\frac{1}{2}E_c\varepsilon_c kbd$
z	$\frac{3}{8}kd$	$\frac{13}{36}kd$	$\frac{7}{20}kd$	$\frac{15}{44}kd$	$\frac{1}{3}kd$
$\dfrac{c}{\frac{2pf}{k}}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{9}{10}$	$\frac{21}{22}$	1

Fig. 6 shows graphically the relations given by equations (3), (4) and (5). It will be seen that the center of gravity of the compressive stresses ranges from $\frac{3}{8}$ distance down to neutral axis (the value for a deformation equal to the ultimate deformation) to $\frac{1}{3}$ distance down to neutral axis at the lower limit, $\left(\text{ratio}, \frac{z}{kd}\right)$. The position for $q=\frac{3}{4}$ is $\frac{13}{36}$, equal to .36. This is not far from the value $\frac{4}{11}$ which was used in the discussion of the experimental work in Bulletin No. 1, and which was obtained by another method of analysis. The position for $q = \frac{1}{4}$ is .341 and for $q = 0$ it becomes $\frac{1}{3}$ as in the straight-line relation. The other ratios are less nearly constant. The ratio for compressive stress at most remote fiber to that for direct proportionality with same deformation $\left(\frac{c}{E_c \varepsilon_c}\right)$ ranges from $\frac{1}{2}$ when ultimate deformation of concrete is developed to 1 for no deformation. The range for summation of compressive stress is from $\frac{2}{3}$ to 1. It should be remembered

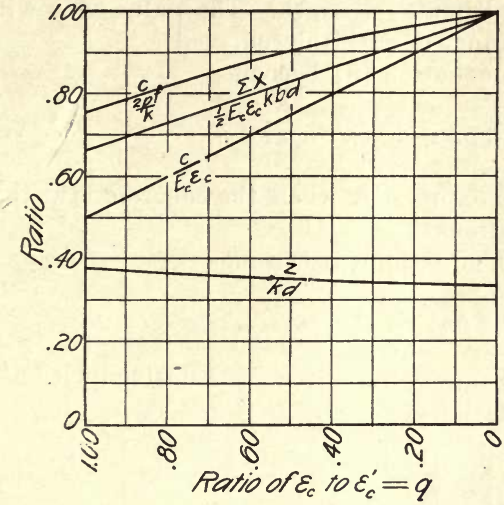

FIG. 6. VARIATION OF FUNCTIONS WITH q.

that these formulas are not applicable when tensile stresses of concrete need consideration.

6. *Neutral Axis.*—The foregoing relations and the analytical condition that the total horizontal compressive stresses and the total horizontal tensile stress are equal will, if tensile stresses in the concrete be neglected, readily enable the position of the neutral axis to be determined for rectangular beams. From the proportionality of deformation (Fig. 3, 4, and 5),

$$\frac{\varepsilon_s}{1-k} = \frac{\varepsilon_c}{k} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Equating horizontal stresses,

$$Af = \tfrac{1}{2}(1-\tfrac{1}{3}q)E_c\varepsilon_c kbd \quad\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Dividing (7) by (6) and substituting $f = E_s\varepsilon_s$,

$$AE_s(1-k) = \tfrac{1}{2}(1-\tfrac{1}{3}q)E_c k^2 bd$$

Calling $\frac{E_s}{E_c} = n$ and $\frac{A}{bd} = p$,

$$pn(1-k) = \tfrac{1}{2}(1-\tfrac{1}{3}q)k^2$$

Solving,

$$k=\sqrt{\frac{2\,pn}{1-\frac{1}{3}q}+\frac{p^2\,n^2}{(1-\frac{1}{3}q)^2}}-\frac{pn}{1-\frac{1}{3}\,q}\dots\dots\dots\dots\dots\dots(8)$$

This gives the position of the neutral axis after tensile stresses in the concrete have become negligible and before the concrete reaches its ultimate strength. The value of k will vary somewhat for the range of q considered.

For $q=1$ equation (8) becomes

$$k=\sqrt{3\,pn+\frac{9}{4}p^2n^2}-\frac{3}{2}pn\dots\dots\dots\dots\dots\dots(9)$$

which is the expression when the concrete is at the limit of its compressive strength.

For $q=0$, equation (8) becomes

$$k=\sqrt{2\,pn+p^2n^2}-pn\dots\dots\dots\dots\dots\dots(10)$$

which is the same as the value of k derived with a straight-line stress-deformation relation.

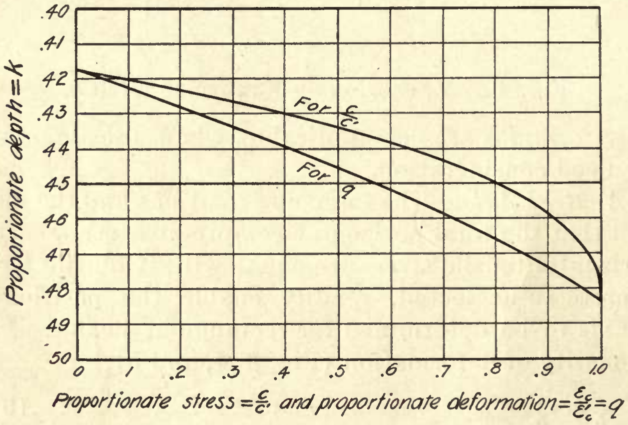

FIG. 7. VARIATION IN POSITION OF NEUTRAL AXIS FOR DIFFERENT VALUES OF q.

Fig. 7 shows the variation in k for $n=15$ and a 1 % reinforcement ($p=.01$), given both in terms of q and in terms of $\frac{c}{c'}$.

In this diagram the position of the neutral axis changes from .418 when $q=0$ to .484 when the full or crushing deformation is developed. It shows a slow change for increasing values of the compressive stress until two-thirds of the full compressive strength of the concrete is developed. Beyond this the neutral axis lowers rapidly. Ordinarily a 1 % reinforcement will not develop the full compressive strength of concrete, but the diagram serves to illustrate the change in the position of the neutral axis both in this and with other amounts of reinforcement. It is seen that the position remains nearly constant during what will be termed the third stage of beam action.

Of course for low values of q, the tensile strength of the concrete would modify the position somewhat.

For the calculations in this bulletin and for the reinforcements used, k for $q = \frac{1}{4}$ gives results which are representative for the range considered and will be used in the discussion. For $q = \frac{1}{4}$, equation (8) becomes

$$k = \sqrt{\tfrac{24}{11} pn + \tfrac{144}{121} p^2n^2} - \tfrac{12}{11} pn \ldots\ldots\ldots\ldots (11)$$

This equation gives the position of the neutral axis for deformations which correspond closely with those developed under working stresses.

Fig. 8, page 16, gives the position of the neutral axis based upon equation (11) ($q = \frac{1}{4}$) for $n = 10$, 12, 15, and 20. Calling the modulus of elasticity of steel, 30 000 000 lb. per sq. in., these ratios correspond to initial moduli of elasticity of concrete of 3 000 000, 2 500 000, 2 000 000 and 1 500 000 lb. per sq. in., respectively.

Attention is called to the change in position of the curves from that given in Bulletin No. 1, which was based upon $q = \frac{3}{4}$ and is applicable when the compressive stress is well developed, while that here used is more generally applicable for ordinary working stresses.

7. *Resisting Moment.*—When the tensile stresses in the concrete are neglected and the center of gravity of the compressive stresses is known, the value of the resisting moment of the beam (which it is readily seen is the moment of the couple formed by the tensile stress in the steel and the resultant of the compressive stresses in the concrete) is easily expressed as the product of the tensile stress in the steel and the distance from the center of the

steel to the center of gravity of the compressive stresses. Hence the formula for the resisting moment for a rectangular beam is

$$M = Af(d - z) \qquad (12)$$

It was shown that z varies slightly for different compressive stresses. The value of z when the concrete at the remote fiber is stressed three-fourths of its ultimate deformation ($q = \frac{3}{4}$) is

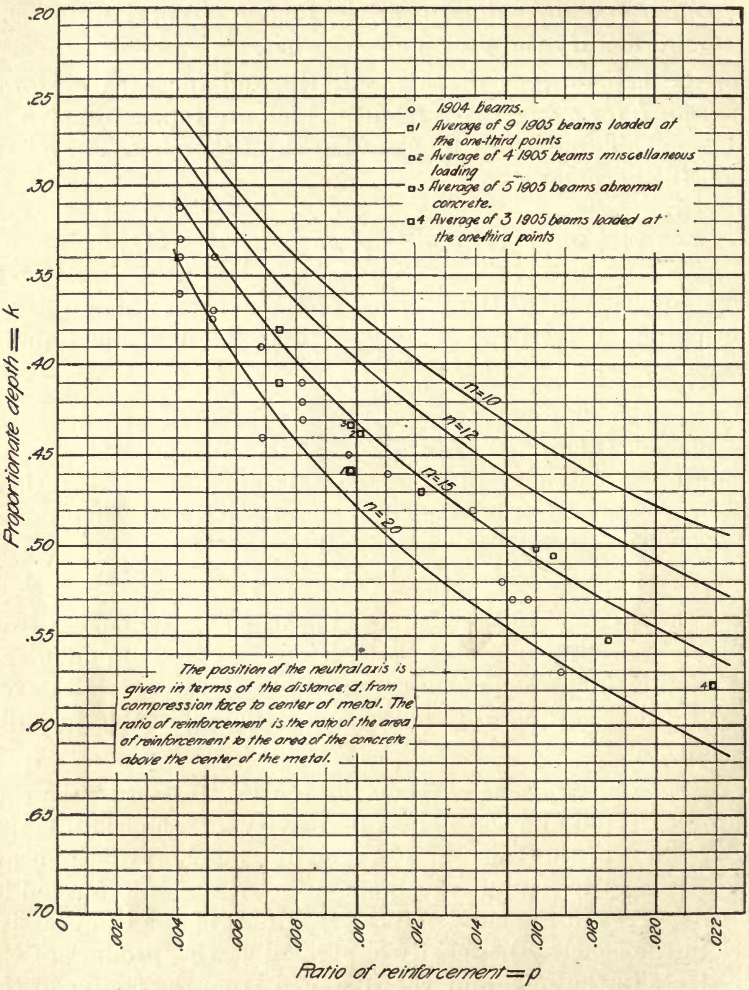

FIG. 8. POSITION OF NEUTRAL AXIS.

approximately $.36kd$; for $q = \frac{1}{2}$, $.35kd$, and for $q = \frac{1}{4}$, $.34kd$. For $q = 0$, $z = \frac{1}{3}kd$. This is the position when the straight-line stress-deformation is used; i. e., when the modulus of elasticity is constant and equal to the initial modulus of elasticity.

When the E_c of the concrete is known and the amount of reinforcement is fixed, equation (12) will take the form

$$M = A\,f\,d' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

where d' is the moment arm of the couple and may be expressed as a proportionate part of d. Thus for $q = \frac{1}{4}$, with $E_c = 2\,000\,000$ lb. per sq. in. ($n = 15$) and 1 % reinforcement ($p = .01$), $d' = .853d$. For 1.5 % reinforcement ($p = .015$), $d' = .831d$. The values of the resisting moment for these reinforcements become $.853A\,f\,d$ and $.831A\,f\,d$, respectively.

This method offers the most convenient way of calculating the resisting moment so far as it is controlled by the tension of the steel within its elastic limit. The position of the neutral axis may well be taken from a diagram like Fig. 8, and the value of d' is then easily obtained.

Generally it will be best to use the resisting moment in terms of the tension in the steel, but if it is desired to express it in terms of the compression in the concrete the following equation may be used.

$$M = \left\{\frac{1 - \frac{1}{3}q}{1 - \frac{1}{2}q}\right\} \frac{1}{2}\,c\,k\,b\,d\,(d - z) \ldots\ldots\ldots (14)$$

At least an approximate value of q will be known which may be used in equation (14). The fractional coefficient is the reciprocal of the function $\dfrac{c}{\frac{2pf}{k}}$ given in Fig. 6.

8. *Compressive Stress at Upper Fiber.*—The formulas for the position of the neutral axis and moment of resistance are based upon the assumption that the compressive stress in the upper fiber is within the crushing strength. To determine the value of the upper compressive stress substitute equation (3) in equation (7). This reduces to

$$c = \frac{2Af}{kbd} \cdot \frac{1 - \frac{1}{2}q}{1 - \frac{1}{3}q} = \frac{2pf}{k} \cdot \frac{1 - \frac{1}{2}q}{1 - \frac{1}{3}q} \quad \ldots\ldots (15)$$

For a deformation of upper fiber equal to three-fourths of deformation at crushing $\left\{ \text{or } c = \frac{15}{16} c' \right\}$, this becomes $c = \frac{5}{6} \frac{2pf}{k}$. For an upper deformation equal to one-half of ultimate deformation this becomes $c = \frac{9}{10} \frac{2pf}{k}$. For the crushing point of the concrete it becomes $c = \frac{3}{4} \frac{2pf}{k}$. As the upper deformation decreases, the value of c approaches $\frac{2pf}{k}$, which is the amount of the stress for a constant modulus of elasticity equal to the initial modulus of elasticity. By multiplying $\frac{2pf}{k}$, the stress found on the basis of a constant modulus of elasticity and a known position of the neutral axis, by this ratio $\frac{1 - \frac{1}{2}q}{1 - \frac{1}{3}q}$, the value of the compressive stress is found. The variation in this ratio may be seen in the upper line in Fig. 6 and also in the last line of Table 1. It will be seen that for high compressive stresses the stress developed is much less than that given by the straight-line relation using the value of the initial modulus of elasticity, being only three-fourths as much if the full compressive stress is developed. For low compressive stresses the discrepancy is much less.

It should be noted that when the compressive deformation developed is well up to the ultimate, the compressive stress calculated from equation (15) is much less than that found by using the formula $\frac{2pf}{k}$ (or any formula based on a straight-line stress-deformation relation), but when the load develops a deformation which is a small proportion of the ultimate, as may be the case for working loads, the coefficient found in equation (15) will not differ much from unity and the straight-line formula will be but little in error.

9. *Bond or Resistance to Slipping of Reinforcing Bars.*—In order to have beam action there must be a proper web connection between the tension and the compression portions of the beam. When there is no metallic web reinforcement the concrete of the beam acts as this web. Of course the amount of stress in the reinforcing bars and also in the compression area of the concrete

varies at different cross sections along the length of the beam. The increment between consecutive sections or increase in the tensile stresses of the reinforcing bars is transferred to or connected with the increments of the compressive stresses of the concrete by means of this web. In transmitting the increment of tension from the reinforcing rods to the surrounding concrete, there is developed a tendency of the rods to slip in the concrete, and the amount of resistance to slip thus developed is called bond and will be measured in terms of the area of the surface in contact with the concrete. It will be seen that the total bond developed on the surface of the bars in one inch of length is equal to the total change in total tensile stress in the bar for the same inch of length. Bond may be compared to the action of the rivets joining flange to web in a riveted steel girder, except that in the reinforced concrete beam the contact is continuous.

For horizontal reinforcement the formula for bond may be derived as follows: For any vertical section of the beam equation (13) ($M=A\,fd'$) gives the resisting moment. Differentiating this, $\frac{dM}{dx}=A\frac{df}{dx}d'$. By the principles of mechanics of beams $\frac{dM}{dx}=V$, where V is the total vertical shear at the given section (reaction at support minus loads between the support and the section considered). Substituting and transposing,

$$\frac{A\,df}{dx}=\frac{V}{d'} \qquad \text{(16)}$$

Now the derivative $\frac{A\,df}{dx}$ expresses the rate of change of the total tensile stress in the reinforcing bars at the section under consideration; it is given in terms of a unit of length of beam (lb. per inch of length) and measures what is transmitted to the concrete by the bond. Using m as the number of bars, o as the efficient circumference or periphery of one bar, the total surface of bar for one inch of length of beam is $m\,o$ and the bond stress developed is $m\,o\,u$, where u represents the bond developed per unit of area of surface of bar. Equating this to the value of the derivative in equation (16) and solving,

$$u=\frac{V}{mod'} \qquad \text{(17)}$$

Equation (17) is not applicable in just this form when the bars are bent up or inclined from the horizontal, since in this case d' is a variable and this fact will modify the differentiation.

10. *Vertical and Horizontal Shearing Stresses.*—It is shown in the mechanics of beams that there exist throughout a beam vertical and horizontal shearing stresses which vary in intensity, and that at any point in a beam the vertical shearing unit-stress is equal to the horizontal shearing unit-stress there developed. As noted under bond the total tension in the reinforcing bars varies along the length of the beam, as does also the total compressive stress. The horizontal shearing stress may be considered to transmit the increments or increase of the total tensile stresses in the reinforcing bars (which is transmitted to the surrounding concrete by the bond stresses) to the corresponding increments of compression in the compression area of the concrete, the concrete thus forming the stiffening web of the beam. The amount of this horizontal tensile stress so transmitted from the reinforcing bars per unit of length of beam is by equation (17) $m\,o\,u = \frac{V}{d'}$. Consider this distributed over a horizontal section just above the plane of the bars for a unit of length of beam, and call the horizontal unit shearing stress v. The shearing resistance per unit of length of beam thus developed is then $b\,v$, and equating this to $m\,o\,u$,

$$v = \frac{V}{bd'} \qquad \text{(18)}$$

This equation gives the horizontal shearing unit-stress, and therefore also the vertical shearing unit-stress, at a point just above the level of the reinforcing bars. As no tension is here considered as acting in the concrete, there will be no change in the intensity of the horizontal and vertical shearing stresses between this level and the neutral axis. For the part of the beam where tensile stresses extend well down to the reinforcement some modification of this treatment may be made. Above the neutral axis the intensity of the shearing stresses will decrease by the law of change of horizontal shearing stresses for homogeneous rectangular beams modified to suit the parabolic stress-deformation relation. The distribution of the intensity of the horizontal shearing stress over a vertical section is represented in Fig. 9.

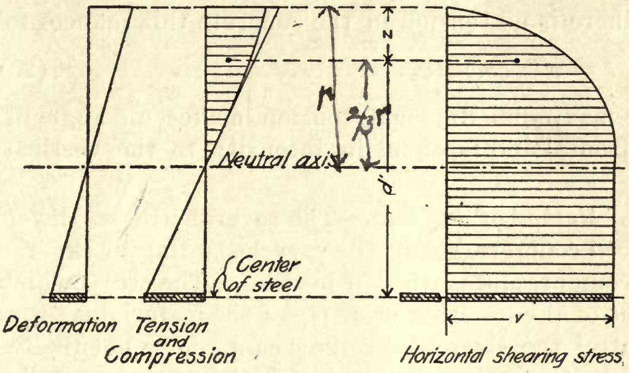

FIG. 9. DISTRIBUTION OF HORIZONTAL AND VERTICAL SHEAR.

As d' generally will not vary far from $.85d$, the shearing stress by equation (18) will be say 18% more than if considered to be uniformly distributed over a vertical section extending down to the center of the reinforcing rods. Even if tension is considered to exist in the concrete for a short distance below the neutral axis, the shearing stress will not be greatly modified thereby. If the bars are inclined or bent up from the horizontal, equation (18) must be changed to allow for a variable d'.

11. *Diagonal Tension in Concrete.*—In the flexure of a beam stresses are set up in the web which are sometimes called web stresses and sometimes secondary stresses. Besides the horizontal and vertical shearing stresses already discussed, tensile or compressive and shearing stresses exist in every diagonal direction. In determining the bending moment only the horizontal components of these are taken. When there is no metallic web reinforcement all the diagonal stresses are taken by the concrete. By the analysis of combined shear and tension the value of the maximum diagonal tensile unit-stress (see Merriman's Mechanics of Materials, p. 265, 1905 edition) is found to be

$$t = \tfrac{1}{2}s + \sqrt{\tfrac{1}{4}s^2 + v^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

where t is the diagonal tensile unit-stress, s is the horizontal tensile unit-stress existing in the concrete, and v is the horizontal or vertical shearing unit-stress. The direction of this maximum diagonal tension makes an angle with the horizontal equal to one-half the angle whose cotangent is $\frac{1}{2}\frac{s}{v}$.

If there is no tension in the concrete this reduces to

$$t = v \text{ ....} (20)$$

and the maximum diagonal tension makes an angle of 45° with the horizontal and is equal in intensity to the vertical shearing stress.

12. *Method of Failure.*—The several stresses developed in a reinforced concrete beam will vary according to the dimensions, reinforcement, and method of loading. The stress which reaches the limit of the resisting property of the material is the one which will control the strength of the beam. It is not likely that two or more of these stresses will reach their point of failure at the same time. It is not even generally feasible so to proportion a beam that its strength shall be the same in tension, compression, bond and diagonal tension. For other reasons the amount of reinforcement or depth of beam may be made the same in spans of different length or carrying different loads, and such a variation will change the relative value of tension, compression, bond, etc. While it may be well to calculate the various stresses, in many cases the relative dimensions and amount of reinforcement are such that the method of failure may be told without much calculation. In such cases only the formulas which determine the stress for the most probable methods of failure need be used.

13. *Primary and Ultimate Failure.*—In judging of the results of tests a distinction must be made between primary failure and ultimate failure. Some change or failure may take place in the beam during the test which will greatly modify the conditions, and we may not properly judge of the conditions existing at this time by what happens later. This early or critical failure may be named the primary failure and its cause should be called the cause of the failure of the beam. Thus, in a beam having a moderate or small amount of reinforcement, after the load is reached which stresses the steel beyond its yield point, the steel stretches rapidly, the neutral axis rises, and the compressive stresses are thereby materially increased until ultimate failure by compression may result. The real cause of failure, however, is the passing of the yield point of the steel, and the maximum

load is generally but little more than that carried at the yield point of the reinforcement. Again, slipping of bars may come after diagonal tension failure has occurred. Confusion has arisen from ultimate failures being reported instead of primary failures. It is not always possible to know positively the cause of failure, but generally a careful study of the test will give a trustworthy conclusion. Too frequently only an exterior appearance is reported which does not represent the true cause of failure and the report is likely to be misleading.

14. *Failure by Tension in Steel.*—Beams having shallow depth as compared with their length and having a moderate amount of reinforcement may when tested with usual way of loading be expected not to fail before the steel has been stretched to its yield point, and the maximum load carried will generally be but little higher than that carried when the yield point is reached. Fig. 10 (a) illustrates the typical form of failure by

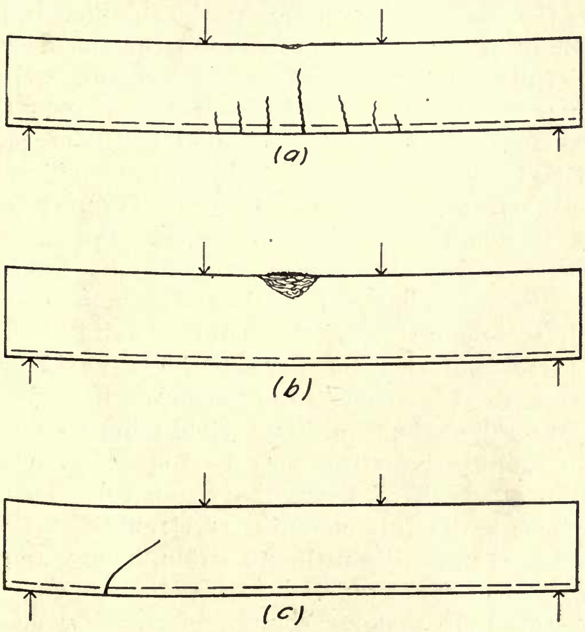

FIG. 10. TYPICAL FORMS OF FAILURE.
(*a*) Tension in Steel. (*b*) Compression of Concrete. (*c*) Diagonal Tension Failure.

tension in steel. When deformations have been measured, the plotted stress-deformation curve for tension will show a sudden and marked change at the yield point and there will be a correspondingly sudden change in the compression curve. As stated in Bulletin No. 1, for the 1–3–6 concrete used, failure by tension in steel occurred in beams having a reinforcement of not more than 1.5 % for steel of 33 000 lb. per sq. in. elastic limit and not more than 1.0% for steel of 55 000 lb. per sq. in. elastic limit. The calculated stress in the steel may be found by equations (12) or (13). Whether other forms of failure, as by diagonal tension, will cause failure before the yield point is reached is a matter for further consideration. It should be noted that tension cracks as shown in Fig. 10 (a) will appear considerably before the steel reaches its yield point. With other forms of failure, these cracks may appear but they do not grow to the extent they do in tension failures.

15. *Failure by Compression of Concrete.*—Beams having a large amount of reinforcement may fail by the crushing of the concrete at the top of the beam before the steel has been stressed to its elastic limit. The amount of reinforcement necessary so to develop the full compressive strength of the concrete will of course depend upon the quality of the concrete and upon the elastic limit of the steel. The 1 and 1.5 % noted in the preceding paragraph may be taken as tentative limits for the concrete here used. Fig. 10 (b) illustrates this form of failure. If stress-deformation diagrams are made, the line showing the shortenings of the upper fiber of the concrete will curve away rapidly from the usual straight-line position, but the steel deformation line will not be modified materially until near the point of failure. This condition of the stress-deformation curves is the best evidence that the crushing strength of the concrete has been reached without developing the strength of the steel to its yield point. The calculated value of the compressive stress may be found by equation (15). Whether the strength of beams having a reinforcement large enough to develop the full compressive strength of the concrete should be based upon the ultimate strength may require some discussion. The effect of repetition of load and even of retention of load upon concrete under considerable stress seems to lead to the choice of a lower stress than the ultimate strength of concrete for a limit to correspond to the yield point of steel.

16. *Failure of Bond Between Steel and Concrete.*—Primary failure by the breaking of the bond between steel and concrete is

unusual for beams having the proportions of ordinary test beams. The bond stress developed in such beams at their ultimate load as calculated by equation (17) $\left(u = \frac{V}{mod'}\right)$, ranges from 70 to 193 lb. per sq. in., and the bond tests on plain mild steel rods give values from 200 to 500 lb. per sq. in. and on some forms of deformed bars from 300 to 1000 lb. per sq. in. It is true that conditions under which the bond tests are made differ from those in the beam and also that bond stresses may not be distributed in the beam exactly as assumed and considerable allowance should be made for these. Besides, the effect of time and of repetition of stress upon bond resistance is not known. For bars bent up out of the horizontal a much higher stress is brought into action near the end of the bar than with the bars laid horizontally throughout the length of the beam. The value of the bond resistance will depend upon the smoothness of the surface of the bar, the uniformity of its diameter, the adhesive strength of the concrete, and the shrinkage grip developed in setting. In most of the failures reported to be caused by slipping of the bars, it seems certain that this slipping occurred subsequent to diagonal tension failures or other changes which were the primary causes of failure. For mild steel reinforcement placed horizontally in beams of ordinary dimensions, the diagonal tensile strength of the beam will be a much weaker element than the bond stress between steel and concrete. An interesting illustration of the reverse of this condition is found in the tests of beams reinforced with tool steel described herein.

17. *Failure by Shearing of Concrete.*—The horizontal and vertical shearing unit-stresses obtained by the use of equation (18) $\left\{v = \frac{V}{bd'}\right\}$ are low, the highest value developed for the beams herein tested being 151 lb. per sq. in. Even if we consider a point in a beam at which the concrete is carrying stress in tension up to its ultimate strength, the value of the diagonal shearing stress will scarcely reach twice the vertical shearing stress. The shearing strength of concrete is much higher than this,—probably from 50 to 75% of the compressive strength. Tests made at the University of Illinois and elsewhere show as great strength as this.

The low values frequently quoted, which range from 15 to 35% of the compressive strength, are from tests made in such a way that bending action controls and the failures are more nearly tension failures. It can hardly be said, then, that reinforced concrete beams fail by shear. What have been called shearing failures are really diagonal tension failures.

18. *Failure by Diagonal Tension in Concrete.*—When the diagonal tensile stresses developed become as great as the tensile strength of the concrete, the beam will fail by diagonal tension, provided there is no metallic web reinforcement. Fig. 10 (c) gives the typical form which this failure takes. As the value of the maximum diagonal tensile stress developed in a beam is by equation (19) dependent upon the horizontal tensile stress developed at the same point it is difficult to compute its actual amount. The best method seems to be to compute the horizontal and vertical shearing unit-stress and make all comparisons on the basis of this value. Beams which fail by diagonal tension and which are without metallic web reinforcement give a value of 100 to 150 lb. per sq. in. for the vertical shearing unit-stress when calculated by equation (18) (and lower values for poorer concrete), the limit depending upon the strength of the concrete. When these values are combined in equation (19) with the probable horizontal tensile stress developed in the concrete below the neutral axis, the resulting diagonal tensile stress is evidently the full tensile strength of the concrete. Diagonal tension failures are frequently characterized by sudden breaks, without much warning, as is the case in the failure of plain concrete beams. A variation from this gives a slower failure, part of the shear being carried through the reinforcing bars, and the ultimate failure involving the splitting and stripping of the bars from the beam above as described under the next heading.

It is evident, since the vertical or external shear is independent of the resisting moment, that the relation between the depth and length of a beam will determine whether the beam will fail by diagonal tension or by tension of steel or compression of concrete. In relatively short and deep beams the diagonal tensile strength will fix the strength of the beam, while in long shallow beams this element may be disregarded.

Since the diagonal tension may be resolved into horizontal and vertical or other components, the concrete may be relieved of a part of the diagonal tensile stress by one or both of two means: (1) by bending the reinforcing rods or strips sheared from them into a diagonal position, and (2) by making use of stirrups to take the vertical component of the diagonal tension. The necessity and efficacy of these metallic web reinforcements can not be discussed here but will be taken up in a later bulletin.

19. *Failure by Splitting of Bars away from Upper Portion of Beam.*—Failures sometimes occur, either after a diagonal crack has appeared or at the same time that such a crack is observed, in which the reinforcing bars and the concrete below the level of the bars are split away from the remainder of the beam, the crack running horizontally for some distance. This stripping is caused by vertical tension in the concrete transmitted to it by the stiffness of the reinforcing bars after the concrete fails to carry its assignment of diagonal tension. In Fig. 11 consider that a diagonal crack CD has been formed. Take a vertical section through AD. On account of the diagonal crack normal beam action does not exist and part of the vertical shear from the main portion of

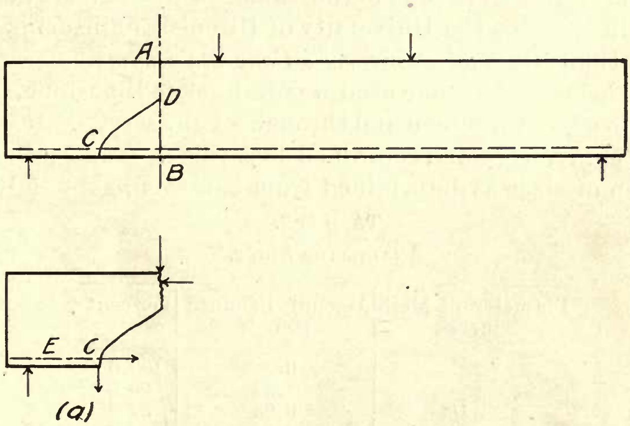

FIG. 11. FAILURE WHEN BARS SPLIT AWAY FROM UPPER PORTION OF BEAM.

the beam is transmitted by the projecting portion of the beam DCB acting as a cantilever and the flexural stiffness of the bars to the point C and there applied to the left portion of the beam as a downward force. (*a*) shows the part at the left of AD acting as a free body. The part of the vertical shear applied at C tends to split the bars from the beam, starting at C and running toward

E. This action is resisted by the tensile strength of the concrete in a vertical direction, and when this is exceeded the bars will split from the concrete above. This may happen without any horizontal movement or slip of the bars. Splitting of the bars from the beam presupposes a failure in diagonal tension, for as long as true beam action exists vertical tension is not developed. After the diagonal crack is formed this part of the beam takes on the nature of a truss. This form of failure is then a secondary failure, though under some conditions the load carried before splitting occurs may be considerably more than that at which the diagonal crack appeared. This explanation shows why the concrete at the bottom of the bars continues to adhere to the bars. There is no evidence of shearing failure in these cases.

Attention should also be called to the danger from spacing bars too closely or with not sufficient concrete below the bars.

II. Materials, Test Pieces and Testing

The materials, method of making test pieces, and manner of testing the beams were much the same as in the tests described in Bulletin No. 1 of the University of Illinois Engineering Experiment Station, *Tests of Reinforced Concrete Beams.*

20. *Stone.*—The stone used was Kankakee limestone, ordered screened over a ¼-in. screen and through a 1-in. screen. It weighed 87 lb. per cu. ft. loose and contained 45% voids. Table 2 shows the proportion of sizes as determined from two samples of 15 lb. each.

TABLE 2.

Analysis of Stone.

Diameter of Mesh inches	Amount Retained pounds	Per cent passing
2	0	100.0
1	.53	96.5
1/2	9.03	37.2
1/4	4.06	9.2
No. 10 sieve	1.12	1.7
Dust	.25	....

In the determination of voids in both stone and sand, the material was poured slowly into the water so that the voids became filled with water, and no air was entangled.

21. *Sand.*—The sand was furnished by the Garden City Sand Co., of Chicago, and was of the quality known locally as torpedo

sand. It was clean and sharp, and was screened through a sieve of ¼-in. mesh before using, except for Beams No. 1 to 11 in which it was used unscreened. The sand weighed 103 lb. per cu. ft. loose and contained 28 to 30% voids. Table 3 gives the result of the me-

TABLE 3.

ANALYSIS OF SAND.

Sieve No.	Diameter of Mesh inches	Per cent passing
4	.208	100.0
10	.073	79.8
20	.034	54.5
50	.011	17.1
74	.0078	6.0
100	.0045	2.0

chanical analysis of the sand. The gradation of particles is not as desirable as that found in the sand used in the Series of 1904.

22. *Cement*—The cement was furnished by the Joint Committee and was formed of a mixture of five standard brands of American Portland cement mixed in equal proportions at one of the mills in Pennsylvania. Table 4 gives the tensile strength of

TABLE 4.

TENSILE STRENGTH OF CEMENT.

Ref. No.	Ultimate Strength, lb. per sq. in.			
	Age 7 Days		Age 75 Days	
	Neat	1-3 Mortar	Neat	1-3 Mortar
1	685	397	760	500
2	710	345	720	610
3	630	325	580	560
4	880	330	640	540
5	700	340	760	490
6	735	385	....	500
Aver.	723±56	354±20	692±54	533±31

neat cement and 1-3 mortar. The briquettes were stored in damp air for one day and under water for the remainder of the time.

23. *Steel.*—In accordance with the recommendation of the Committee on Tests of the Joint Committee on Concrete and Reinforced Concrete, that the operations of the year be restricted to beams reinforced with plain bars, no deformed bars were used. The bars used were round rods, ½ and ¾ inches in diameter. Both

mild steel and tool steel were used. Although it was thought that care had been exercised in getting the mild steel of the same grade, it was found that the ½-in. rods could be divided into two classes, one in which the yield point was about 33 000 lb. per sq. in.

TABLE 5.

TENSION TESTS OF MILD STEEL.

Specimens Taken From	Nominal Diameter	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.	Elongation in 8 inches per cent	Specimens Taken From	Nominal Diameter	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.	Elongation in 8 inches per cent
G. S.*	3/4	28400	46700	37.0	Beam No. 20	1/2	45300	62800	28.5
G. S.	3/4	29000	46400	35.0	Beam No. 20	1/2	45500	62900	27.
G. S.	3/4	28800	46000	36.0	Beam No. 20	1/2	44900	62900	30.
G. S.	3/4	28300	45900	38.0	Beam No. 20	1/2	41200	61300	29.
Average		28600	46250	36.5	Beam No. 21	1/2	42400	61700	27.5
					Beam No. 21	1/2	42600	61700	29.
G. S.	1/2	33100	51600	33.	Beam No. 21	1/2	43800	62400	27.
G. S.	1/2	34100	52900	30.	Beam No. 21	1/2	42900	62400	27.
G. S.	1/2	34000	52900	31.	Beam No. 23	1/2	43100	61200	29.
Average		33700	52400	31.2	Beam No. 23	1/2	44200	62400	29.5
					Beam No. 23	1/2	44600	62200	27.5
					Beam No. 23	1/2	42900	62300	28.
Beam No. 36	1/2	33400	53200	31.5	Average		43600	62350	28.3
Beam No. 36	1/2	32400	50200	33.					
Beam No. 36	1/2	31100	50200	32.	Beam No. 11	1/2	42200	59200	27.5
Beam No. 37	1/2	31700	50300	34.	Beam No. 11	1/2	35300	50000	30.5
Beam No. 37	1/2	32700	52200	31.5	Beam No. 11	1/2	39700	58000	28.
Beam No. 37	1/2	34700	53000	34.	Beam No. 11	1/2	41400	58200	29.
Beam No. 37	1/2	32900	52000	32.	Beam No. 18	1/2	34500	49900	34.5
Beam No. 37	1/2	33500	50800	32.5	Beam No. 18	1/2	35900	50000	34.
Beam No. 47	1/2	34100	52000	32.	Beam No. 18	1/2	42200	61600	27.
Beam No. 47	1/2	32600	51200	30.5	Beam No. 18	1/2	41700	61500	26.5
Beam No. 47	1/2	34100	51000	32.5	Beam No. 19	1/2	35900	50000	33.
Beam No. 47	1/2	33400	51100	33.	Beam No. 19	1/2	34500	49600	33.5
Beam No. 48	1/2	36700	55100	30.	Beam No. 19	1/2	45300	62900	27.
Beam No. 48	1/2	33900	51800	28.	Beam No. 19	1/2	45000	62900	27.5
Beam No. 48	1/2	33800	51500	33.5	Beam No. 27	1/2	33900	49300	34.5
Beam No. 48	1/2	32700	51000	33.5	Beam No. 27	1/2	41500	59700	30.
Beam No. 54	1/2	34200	54400	31.	Beam No. 27	1/2	39800	58200	....
Beam No. 54	1/2	33300	50900	30.5	Beam No. 30	1/2	34600	49700	33.
Beam No. 54	1/2	33500	50800	33.5	Beam No. 30	1/2	41500	58700	27.5
Beam No. 54	1/2	33700	50900	32.	Beam No. 30	1/2	42500	59200	....
Beam No. 58	1/2	32300	51600	28.	Beam No. 30	1/2	42000	59300	27.5
Beam No. 58	1/2	33800	54700	28.	Beam No. 31	1/2	35400	49200	32.
Beam No. 58	1/2	32400	51100	33.	Beam No. 31	1/2	34600	50200	32.5
Beam No. 58	1/2	33800	54500	28.	Beam No. 31	1/2	41900	58700	30.
Beam No. 59	1/2	32400	51400	31.5	Beam No. 31	1/2	42500	59200	29.5
Beam No. 59	1/2	32800	51900	30.					
Beam No. 59	1/2	32700	52100	28.					
Beam No. 59	1/2	31600	51600	31.					
Average		33200	51900	31.4					

* G. S. refers to general stock.

and another averaging 43 600 lb. per sq. in. Unfortunately some of the beams contained a mixture of these two classes. Table 5 gives the results of tests made on rods taken from near the ends of some of the tested beams, as well as on bars taken from the general stock. Some of the beams were destroyed before the variation in the steel was discovered, and the strength of this steel is not fully known. The ¾-in. mild steel was quite soft, the yield point averaging 28 600 lb. per sq. in. The tests of the tool steel are given in Table 6. Its elastic limit averaged 52 900 lb. per sq. in.

TABLE 6.

TENSION TESTS OF TOOL STEEL.

Specimen Taken From	Nominal Diameter inches	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.	Elongation in 8 inches per cent
Gen'l Stock.	3/4	52500	83900	25.4
Gen'l Stock.	3/4	53200	85400	25.5
Gen'l Stock.	3/4	53000	85400	23.2
Gen'l Stock	3/4	53000	84000	25.4
Average	....	52900	84700	24.9

24. *Concrete.*—Table 7 gives the results of compression tests of 6-in. concrete cubes. The concrete used in making cubes was taken from the mix used in making a test beam. The number of the test beam corresponding to the cubes is given in the first column. The cubes were left exposed in a room, and were not moistened in any way. It seems evident that they became very dry and that their strength suffered from this cause even more than did the concrete beams. It is felt that the compressive strength of the concrete in the test beams was considerably greater than that developed in the cubes.

Table 8 gives the flexural strength of three plain concrete beams. The effect of the weight of the beam and of the loading apparatus is included in determining the modulus of rupture. The results are about the same as those for the concrete beams tested in 1904.

25. *Test Beams.*—The size of the test beam used was that adopted by the committee on Plan and Scope of the Joint Committee, i. e., 8 inches wide, 11 inches deep and 13 feet long, with test span of 12 feet. The center of the steel reinforcement was placed 10 inches below the top surface, except that in special cases

TABLE 7.

COMPRESSION TESTS OF 6-INCH CONCRETE CUBES.

Concrete as in	Kind of Concrete	Age days	Crushing Load pounds	Compressive Strength lb. per sq. in.
			48000	1330
Beam No. 35	–3–6	66	47980	1330
	1		57400	1590
			38400	1065
Beam No. 44	1–3–6	67	39800	1105
			29400	816
			47200	1310
Beam No. 65	1–3–6	59	41600	1156
			48850	1355
Average				1230
Beam No. 59	1–2–4	58	45400	1290
			37800	1050
			64500	1790
Beam No. 60	1–2–4	58	73200	2070
			64200	1780
			54560	1510
Beam No. 66	1–2–4	57	55000	1530
			41240	1140
Average				1520
			28200	783
Beam No. 48	1–3–6	69	28300	786
			28000	777

TABLE 8.

TESTS OF PLAIN CONCRETE BEAMS.

Dimensions, 8 in. x 11 in. x 13 ft. Span, 12 ft.

Beam No.	Kind of Concrete	Age at Test days	Maximum Applied Load pounds	Modulus of Rupture lb. per sq. in.
64	1–3–6	58	1200	357
65	1–3–6	59	1260	366
66	1–2–4	59	1320	375

it was bent up at the ends. These beams were much more easily made and handled than the larger size of test beams (12 in. $\times 13\frac{1}{2}$ in. $\times$ 15 ft. 4 in.) used in 1904. The amount of reinforcement given is in terms of the area of the concrete above the center of the metal, no deduction being made for the area taken by the metal.

26. *Making of Beams.*—Beams were made directly on the concrete floor of the testing laboratory, a strip of building paper being first spread on the floor. The forms used for the sides and ends were in general similar to those used in 1904 which are shown in Fig. 5, of Bulletin No. 1. They were of dressed pine with braces and bolts as before. They proved entirely satisfactory, except for some warping of the timber which was overcome by soaking the boards in water between applications. The concrete was mixed by hand with shovels, a man experienced in mixing concrete assisting in the work. All materials were measured by loose volume. Cement and sand were thoroughly mixed on a large sheet-steel mixing board before stone was added. The stone, cement, and sand were then mixed, water was added, and the mixture was turned until it had a uniform consistency. A moderately wet concrete was used. Table 9 shows the probable

TABLE 9.

AMOUNT OF WATER USED IN MIXING CONCRETE.

Beam No.	Weight of Dry Material, pounds				Weight of Water pounds	Per cent of Water
	Stone	Sand	Cement	Total		
24	657	369	105	1131	85.5	7.6
45	667	394	96.5	1157.5	101.5	8.8

range of the amount of water used. For Beam No. 24 the sand was already moist and in No. 45 it was dry. With few exceptions batches of concrete were mixed just large enough for one beam, and for three 6-inch cubes. Concrete was deposited in layers of about 3 inches, each layer being thoroughly tamped. The side faces of the beams were spaded as the work progressed. Forms were allowed to remain in place about 48 hours. The proportions for most of the beams were 1 cement, 3 sand, 6 stone by loose volume. A few beams were made of concrete with the proportions 1-2-4. The beams were numbered in the order of making. The average weight of the beams was about 1200 lb., making the concrete weigh about 150 lb. per cu. ft.

27. *Storage*—The temperature of the room during the curing of the beams ranged from 60 to 70 degrees, F. and no protection.

from drying out was used, except that the beams were sprinkled with water for a few days after being made. It seems probable that the beams dried out too much to secure the best quality of concrete. The general age of the beams at testing was 60 days; the exact age is given in the table. The storage space of the room being limited it was necessary to pile the beams in tiers five beams high. The beams were moved from their place of construction, generally after more than two weeks had elapsed, and were piled in tiers with thin strips between. An unfortunate accident happened to the first fifty beams. Not being securely braced, the last tier fell over sideways and struck the next tier and thus caused all to tumble like a pile of bricks. The age at time of fall ranged from two to six weeks. Fifteen beams were broken by the fall, the blow producing tension in the upper side. The older beams being in the rear had the severest fall. Several of the broken beams were tested, interesting results being obtained. These results are of value in showing the abuse which such beams may stand. Beam No. 18 which was broken in the fall at a point 2½ ft. from the middle carried a load of 18000 lbs., the highest load carried. In this test the loads were applied at two points 7½ ft. apart, and it should be borne in mind that between these two points no shear existed and hence no diagonal tension was developed. The loads held by the broken beam and their action during the test are to the credit of reinforced concrete construction. Broken beams which were not tested or which showed untrustworthiness in the tests are omitted from the tables.

28. *Details of Tests.*—The method of testing followed in general the plan used in 1904 tests as described in Bulletin No. 1, page 9, under Details of Tests. The tests were made on the 200,000-lb. Olsen testing machine. The span length was 12 feet. The supports at the end of the span rested on the table of the machine, their bases being cylindrical surfaces of 12-inch radius and their tops being curves of small radius, thus allowing a rocking action with changes in the length of the lower surface of the beam. For a beam loaded at two points, the load was transferred from the machine by a 10-inch I-beam and two turned rollers. Iron bearing plates ⅜ x 3 x 8 inches were placed above and below the beam for the bearing of the rollers and pedestals. A layer of plaster of paris was placed between these bearing plates and the beam to overcome unevenness of surface and was allowed to set un-

der such load as came from the weight of beam and the apparatus used in loading. It is thought that this method of loading brings very little longitudinal stress before an adjustment results.

The loads were applied at a slow speed of the machine, the increase of deflection averaging about .03 in. per minute. Repeated loads were applied at, say, .3 in. per minute and the release was made at, perhaps, .6 in. per minute. The time of the ordinary test was a half hour to one hour. Load was increased by increments of 1000 lb., except when the loads were being released or reapplied when readings were taken at intervals of 2000 or 3000 lb.

Deflections at the middle of the span were observed. The deflections were obtained by means of a fine thread stretched at constant tension between points over the supports and at the middle of the depth of the beam, and passing in front of a paper scale attached to the side of the beam at the middle of the span. The scale was pasted on the face of a mirror and readings were obtained by lining up the thread and its reflection. These readings were accurate to .01 in. The ordinary deflectometer could not be used satisfactorily on account of the deflection of the table of the machine under the load and the yielding of the plaster of paris over the lower bearing plates. On part of the tests a cathetometer was used.

To obtain the longitudinal elongation and shortening at the top and bottom of the beam (fiber deformations), the extensometer device shown in Fig. 12 was used. This device was the outgrowth of the experience with the apparatus used in the tests of 1904 and which was described in Bulletin No. 1. The two yokes were placed symmetrically with respect to the center of the span and generally 42 inches apart. The upper pair of contact points of a yoke was applied to the side of the beam at points one-half inch below the top of the beam, and the lower pair at the level of the center of the steel reinforcement, i. e., ten inches below the top of the beam. The rollers and dials, similar to those of the Johnson extensometer, were attached to the yoke in such a way that the middles of the rollers were in direct vertical line with the contact points, and the axes of the rollers were at right angles to the plane of the side of the beam. The rollers were 20¼ inches apart vertically, the upper one being 5¼ inches above the upper contact points. The rollers were .5 in. in circumference, the dials

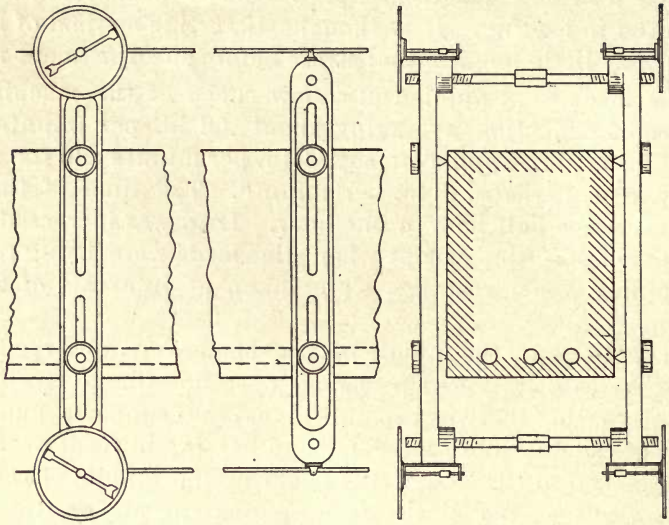

FIG. 12. EXTENSOMETER DEVICE.

four inches in diameter, and the graduations such that readings were obtained to .0001 in. The second yoke was provided with a fixed pin in a position corresponding to the four rollers. Connection was made between this pin and the corresponding roller by a horizontal rod which consisted of a ¼-in. brass pipe and a steel strip about 9 inches long. A V-shaped notch at one end of the brass pipe engaged the pin of the fixed end. The steel strip at the other end had a rounding surface which rested and rolled on the roller of the measuring device. The calibration of the appliance by means of standard screw micrometer showed a possible error of 1% in the extensometer measurements. The extensometer device was generally removed before ultimate failure of the beam was reached. This apparatus gave satisfactory results except that in the time tests the changes in the lengths of the brass rods due to variations in temperature affected the results.

III. Experimental Data and Discussion

29. *Outline.*—Fifty-four reinforced concrete beams were tested. Table 10 gives general data of the beams. Further information is given in the tables referred to in the column headed Classification. The methods of calculation of deformations and of position of neutral axis will be first described, followed with an explanation of the load-deformation curves, deflection curves, etc. A description is then given of Tables 11 to 16 including an explanation of the classification and contents of these tables. The following topics are then taken up: failure by tension in steel; failure by compression of concrete; failure by diagonal tension; failure of bond; effect of artificial cracks and spaces; effect of method of loading; effect of repetition of load; progressively applied and released loads; effect of rest after release of load; effect of retention of load; effect of position of reinforcing bars; effect of lean and abnormal concrete; effect of exposing reinforcing bars; position of neutral axis and value of modulus of elasticity.

30. *Calculation of Deformation and of Position of Neutral Axis.*—The calculation of the position of the neutral axis and of the deformations at the extreme fibers was based upon the assumption that a plane section before bending remains a plane section after bending. This work was done graphically from the observed readings of the extensometers and the position of the rollers with respect to the beam. The deformation per unit of length was calculated by dividing the total deformation by the gauged length or distance between corresponding contact points, and this average unit-deformation is used in the diagrams and tables. In general, also, the values of the deformations used refer to the zero or initial position of the beam under its own weight and that of the I-beam at the time the load was first applied. In other words, any set which the beam may have taken has not been considered in this calculation, nor has the effect of the breaking of the concrete in tension.

31. *Diagrams.*—Load-deformation curves, deflection curves and position of the neutral axis are shown in Fig. 20 to 58 at the end of the text. The data are presented in the same way as in the diagrams given in Bulletin No. 1. The curve marked "Upper Fiber" represents the shortening per unit of length at the com-

TABLE 10.

DATA ON BEAMS.

Beam No.	Kind of Concrete	Amount of Reinforcement	Per cent of Reinforcement	Age at Test days	Classification
5	1-3-6	4 ½-in. Round	.98	69	Table 11
11	"	" "	.98	65	" 11
13	"	" "	.98	62	" 11
14	"	" "	.98	63	" 12
15	"	" "	.98	63	" 12
16	"	" "	.98	63	" 11
17	"	" "	.98	65	" 11
18	"	" "	.98	72	" 12
19	"	" "	.98	63	" 11
20	"	" "	.98	64	" 11
21	"	" "	.98	62	" 12
22	"	" "	.98	63	" 11
23	"	" "	.98	62	" 11
24	"	4 ¾-in. Round	2.21	63	" 14
26	"	4 ½-in. Round	.98	64	" 11
27	"	3 ½-in. Round	74	68	" 15
28	"	4 ¾-in. Round	2.21	60	" 14
29	"	" "	2.21	62	" 14
30	"	4 ½-in. Round	.98	62	" 12
31	"	" "	.98	62	" 11
33	"	3 ¾-in. Round	1.66	60	" 15
34	"	" "	1.66	60	" 15
35	"	3 ½-in. Round 2 ¾-in. Round	1.84	60	" 15
36	"	3 ½-in. Round	.74	62	" 15
37	"	5 ½-in. Round	1.24	59	" 15
38	"	2 ¾-in. Round 2 ½-in. Round	1.60	57	" 15
39	Abnormal	4 ½-in. Round	.98	61	" 13
40	"	" "	.98	61	" 13
41	"	" "	.98	61	" 13
42	"	" "	.98	62	" 13
43	"	" "	.98	61	" 13
44	"	" "	.98	61	" 13
45	1-3-6	3 ½-in. Round 2 ¾-in. Round	1.84	61	" 15
46	"	5 ¾-in. Round	2.76	61	" 15
47	"	4 ½-in. Round	.98	60	" 12
48	"	" "	.98	64	" 16
49	"	2 ¾-in. Round	1.10	60	" 17
50	"	4 ½-in. Round	.98	387	" 11
51	"	3 ¾-in. Round	1.66	63	" 17
52	"	" "	1.66	60	" 17
53	"	2 ¾-in. Round	1.10	61	" 17
54	"	4 ½-in. Round	.98	63	" 16
55	"	3 ¾-in. Round	1.66	59	" 17
56	"	2 ¾-in. Round	1.10	59	" 17
57	"	" "	1.10	59	" 17
58	1-2-4	4 ½-in. Round	.98	59	" 16
59	"	" "	.98	59	" 16
60	1-3-6	2 ¾-in. Round	1.10	59	" 17
61	"	3 ¾-in. Round	1.66	59	" 17
62	"	2 ¾-in. Round	1.10	59	" 17
63	"	" "	1.10	58	" 16
67	"	" "	1.10	57	" 17

pression face of the beam. The curve marked "Steel" indicates the elongation per unit of length in the plane of the reinforcement and considers that the steel elongates the same as the concrete at the same depth. The curve of deflection has a separate scale of abscissas. The applied load is here used, and no account is taken of the weight of the beam, which has already stressed the fibers at the time the extensometers are read at zero load, nor of the I-beam, etc., used in transmitting the load. In the figures following, (Fig. 56 to 58), positions of the neutral axis are given as ordinates and correspond to the load given on the scale of abscissas. The position of the neutral axis is given in per cent of the distance from the compression face of the beam to the center of the metal reinforcement. The detailed record of the observed readings of extensometers and deflection measurements is so voluminous and is covered so well by these diagrams that it does not seem necessary to reproduce it here.

32. *Explanation of Tables 11 to 16.*—The beams are classified in Tables 11 to 16 according to the nature of the reinforcement, the loading and the kind of concrete. The neutral axis is given as a proportionate part of the distance from the upper fiber to the center of the reinforcement for the first part of the third stage as hereafter discussed. The columns headed Maximum Applied Load and Load Considered do not include the weight of the beam and the loading apparatus. The beams weighed about 1200 lb. each and the loading apparatus for loads applied at two points about 300 lb. The bending moment due to this, about 25 000 in.-lb. if the overhang is considered, is not included in the moments used in calculating the stress in the steel, since one use of these stresses is the comparison of observed and computed values. The inclusion of these weights will for 1% beams add about 3600 lb. per sq. in. to the tension in the steel calculated from the resisting moment. The values given in column headed From Resisting Moment are calculated by equation (13) from the resisting moment due to the applied load, using the position of the neutral axis given in the table. The values given under From Deformations are obtained by multiplying the observed unit deformations by the modulus of elasticity of steel, 30 000 000 lb. per sq. in. The two columns of calculated stresses in steel are not exactly comparable, since there is included in the deformations an amount due to the part of the weight of the beam which was originally taken by

TABLE 11.

1 PER CENT MILD STEEL REINFORCEMENT.

LOADING AT ONE-THIRD POINTS.

Beam No.	Method of Applying Load	Neutral Axis k	Load at First Visible Crack pounds	Max. Applied Load pounds	Load Considered pounds	Calculated Stress in Steel lb. per sq. in. From Resisting Moment	Calculated Stress in Steel lb. per sq. in. From Deformations	Manner of Failure
5	Continuous	.450	6000	11000	11000	40000	39600	Tension in steel
11	Continuous	.490	9000	11000	11000	40600	46000	Tension in steel
13	Progressively applied and realeased	.490	5000	11800	11800	43600	41400	Tension in steel
16	8000 lb. retained 28 hours	.420	4500	11600	11600	41600	48000	Tension in steel
17	9000 lb. repeated 10 times	.500	7000	11100	11100	41200	36900	Tension in steel followed by diagonal tension
19	6000 lb. repeated 8 times, 10000 lb. 6 times	.450	6000	10800	10000	36400	....	Tension in steel
20	Progressively applied and released	.445	7000	10200	10000	36200	36900	Diagonal tension
22	9000 lb. retained 20 hours	.400	5000	12700	12500	44500	49500	Tension in steel
23	13 hours' rest after 6000 lb., 9000 lb. repeated twice	.450	9000	12800	12000	43700	44000	Tension in steel
26	5000 lb. retained 25 hours	.470	4800	11450	11000	40000	44100	Tension in steel
31*	15 hours' rest after 8000 lb.	.430	.....	11400	11000	39600	41700	Tension in steel
50†	Continuous	.440	6000	10250	10250	37100	40800	Tension in steel

* Beam No. 31 was cracked before test.

† Beam No. 50 was loaded up to 2000 lb. at age of 60 days and observations made on the effect of rest; at the age of 387 days load was applied continuously to failure.

tensile stresses and deformations in the concrete. The amount of stress thus added for a beam with 1% reinforcement would perhaps be in the neighborhood of 2000 lb. per sq. in. The manner of failure assigned in the tables is based on considerations of the appearance of fracture, the behavior of the stress-deformation diagram, and other evidence.

33. *1% Mild Steel Reinforcement. Loading at One-third Points.*—Beams having .98% reinforcement are here for convenience called 1% beams. Table 11 gives the results of the twelve beams which were thus reinforced with mild steel. This amount of reinforcement was taken as representing an ordinary amount and one for which under usual methods of loading, failure would come through steel being stressed beyond the yield point,—a form of failure which is here called a steel-tension failure. The beams included under this heading had the bars in a horizontal position throughout the length of the span. Those in which the bars were bent up or inclined toward the ends of the beam are given under a separate heading. These 1% beams were tested in different ways and with different objects in view,—continuous application, progressively applied and released loads, repetition of the same load, retention of load for a period of time, and time effect after release of load. These phases of the subject are treated under separate headings further along. In general these beams failed by tension in steel as might be expected from the relation of depth, span, and amount of reinforcement. The variation in the stress at which the steel began to stretch considerably is explained by the variation found in the yield point of the bars.

34. *1% Mild Steel Reinforcement. Miscellaneous Loading.*—Table 12 includes the beams with 1% reinforcement which were tested with loads applied at other than the one-third points. The load was applied continuously to failure. The differences in manner of failure and in deformations developed will be discussed under Effect of Method of Loading.

35. *1% Mild Steel Reinforcement with Abnormal Concrete.*—Table 13 gives the results of six beams made of concrete mixed or proportioned, or built by other than the methods employed generally in making the test beams. These are discussed under Abnormal Concrete and Diagonal Tension.

36. *2.2% Mild Steel Reinforcement.*—Table 14 gives the results of beams having 2.2% reinforcement loaded at one-third

TABLE 12.

1 Per Cent Mild Steel Reinforcement.

Miscellaneous Loading.

Beam No.	Loading	Neutral Axis k	Load at First Visible Crack pounds	Max. Applied Load pounds	Load Considered pounds	Calculated Stress in Steel lb. per sq. in. From Resisting Moment	Calculated Stress in Steel lb. per sq. in. From Deformations	Manner of Failure
14*	At two points 7½ ft. apart	.602	......	15400	15000	32700	27500	Diagonal tension
15	At two points 7½ ft. apart	.464	14000	17600	17600	35400	32700	Diagonal tension
18*	At two points 7½ ft. apart	.458	......	18800	18000	36800	39000	Diagonal tension
21	At middle	.420	5000	9800	9500	51200	47000	Tension in steel
30	At middle	.430	5000	8000	7000	37900	31800	Tension in steel
47	At eight p'nts	.440	8000	13000	13000	47100	37200	Tension in steel

* Beams No. 14 and 18 were cracked before test.

TABLE 13.

1 Per Cent Mild Steel Reinforcement.

Abnormal Concrete.

Loads Applied Continuously at One-Third Points.

Beam No.	Kind of Concrete	Neutral Axis k	Load at First Visible Crack pounds	Max. Applied Load pounds	Load Considered pounds	Calculated Stress in Steel lb. per sq. in. From Resisting Moment	Calculated Stress in Steel lb. per sq. in. From Deformations	Manner of Failure
39	Poorly mixed	.423	7000	7000	6000	21500	20700	Diagonal tension
40	Poorly mixed	.490	8800	8800	8000	29500	30900	Diagonal tension
41	Lean concrete at bottom	.390	8800	8800	8800	31200	27900	Diagonal tension
42	Lean concrete at bottom	.520	6000	6000	6000	22400	19200	Diagonal tension
43	Plane of set 1 in. from bottom	.390	9100	9300	9000	31900	37800	Tension in steel
44	Plane of set 5½ in. from bottom	.404	8000	9400	9000	32000	41400	Diagonal tension

points. The amounts of the loads at time of release are given in the text of the discussion farther on and are also shown in the load-deformation diagrams of these beams.

TABLE 14.

2.21 PER CENT MILD STEEL REINFORCEMENT.

LOADING AT ONE-THIRD POINTS.

Beam No.	Method of Applying Load	Neutral Axis k	Load at First Visible Crack pounds	Max. Applied Load pounds	Load Considered pounds	Calculated Stress in Steel lb. per sq. in. From Resisting Moment	From Deformations	Manner of Failure
24	Progressively applied and released	.516	8000	15600	15600	25800	25500	Diagonal tension
28	Progressively applied and released	.610	7000	14300	14000	24200	29400	Diagonal tension
29	12000 lb. repeated 15 times	.605	10000	15900	15000	25900	30000	Diagonal crack followed by compression

TABLE 15.

MISCELLANEOUS MILD STEEL REINFORCEMENT.

LOAD APPLIED CONTINUOUSLY AT ONE-THIRD POINTS.

Beam No.	Per cent reinforcement	Neutral Axis k	Load at First Visible Crack pounds	Max. Applied Load pounds	Load Considered pounds	Calculated stress in steel lb. per sq. in. From Resisting Moment	From Deformations	Manner of Failure
27*	.74	.380	5000	8000	8000	37400	44100	Tension in steel
36†	.74	.410	6000	7400	7000	33100	36000	Tension in steel
37	1.23	.470	8000	13000	12000	35000	36000	Tension in steel
38	1.60	.501	7000	13950	13000	29500	35400	Diagonal tension
33	1.66	.505	10000	14400	14000	30600	30000	Diagonal tension
34	1.66	.374	12800	12800	12000	24900	37200	Diagonal tension
45‡	1.84	.606	......	12400	12400	25600	27200	Diagonal tension
35	1.84	.552	10000	15000	14000	28300	30000	Tension in steel
46	2.76	.680	10000	15200	15000	21400	26400	Compression in concrete

* Beam No. 27 was tested for effect of rest after a load of 6000 lb. had been applied.

† A load of 6000 lb. was retained 22 hours.

‡ Beam No. 45 was cracked before test.

37. *Miscellaneous Reinforcement with Mild Steel.*—Table 15 includes beams made with a variety of percentages of reinforcement. These are discussed under Failure by Tension in Steel, Failure by Compression of Concrete, Failure by Diagonal Tension, Effect of Repetition of Load, and Progressively Applied and Released Loading.

38. *Beams with Rods Bent out of Horizontal.*—Table 16 gives five beams in which the reinforcing rods were bent into a parabolic or trapezoidal form, as shown in Fig. 15. It is evident from

TABLE 16.

BEAMS WITH RODS BENT OUT OF HORIZONTAL.
1 PER CENT REINFORCEMENT.
LOADS APPLIED CONTINUOUSLY.

Beam No.	Loading	Neutral Axis k	Load at First Visible Crack pounds	Max. Applied Load pounds	Load Considered pounds	Calculated Stress in Steel lb. per sq. in. From Resisting Moment	Calculated Stress in Steel lb. per sq. in. From Deformations	Manner of Failure
48	At one-third points	.440	6000	9300	9000	32500	37800	Tension in steel
54	At middle	.445	6000	6600	6000	32600	30600	Tension in steel
58	At one-third points	.374	4000	8900	8000	28200	......	Tension in steel
59	At one-third points	.360	4000	7900	7900	27600	36000	Tension in steel
63*	At one-third points	.410	4000	7400	7000	25000	30000	Tension in steel

* Beam No. 63 had 1.10% reinforcement.

the results of the tests that the bending of these rods should commence at points nearer the ends of the beam and that some of the rods should remain horizontal. This is discussed under Effect of Position of Reinforcing Bars.

39. *Tool Steel Reinforcement.*—Table 17 gives eleven beams which were reinforced with round rods of tool steel $\frac{3}{4}$-inch in diameter having an elastic limit of 52 000 lb. per sq. in. These were used to find the effect of high elastic limit metal. Owing to the extremely smooth surface and uniform cross-section of this steel, these beams did not develop as much strength as those reinforced with mild steel, the tensile stress developed being well below the elastic limit of the material. The beams failed by slipping of the

rods, followed by vertical and longitudinal cracks. The form of failure is so marked and the conditions are so unusual, however, that the results may prove of more value than if the beams had broken in the manner which might otherwise be expected. The discussion of these tests will be taken up under Failure of Bond.

40. *Failure by Tension in Steel.*—As shown in Table 11, all the 1% beams which were loaded at the one-third points except Beam No. 20 failed by tension in steel, i. e., by a primary failure (soon followed by the ultimate failure of the beam) which came when the steel was stretched beyond its yield point and without any other sign of failure appearing until the greatly increased stretch of the steel beyond the yield point brought entirely new conditions into action. Unfortunately, the steel used was quite variable in elastic limit, tests made afterward showing that ½-in. bars in the same beam gave yield points at about 33 000 and 45 000 lb. per sq. in., and hence the breaking values of the beams are not as uniform as would otherwise be the case. Evidently, two lots of bars must have become mixed in shipping. The yield point of the ¾-in. bars was 28 000 lb. per sq. in. However, it is evident from the results that beams made of 1-3-6 concrete of the quality here used, reinforced with 1% of mild steel may be expected to fail by tension, unless, of course, the relation of depth to length of span is such that failure by diagonal tension occurs. For beams failing by tension in steel, the resisting moment of the beam may well be calculated by multiplying the total stress permitted in the steel by the distance from the center of the steel to the center of gravity of the compression area of the concrete. Of the beams in Table 15 having more than 1% reinforcement, Beam No. 37 (1.24% reinforcement) and No. 35 (1.84%) failed by tension in steel, and No. 46 (2.76%) failed by compression of the concrete. No. 38 (1.60%), No. 33 (1.66%), No. 34 (1.66%), and No. 45 (1.84%) failed by diagonal tension in concrete before the elastic limit of the steel was reached and before the full compressive strength of the concrete had a chance to develop. The same is true of the beams in Table 14. The beams in Table 16 failed by tension in steel. It seems evident, therefore, as noted in the next paragraph, that the conclusion given in Bulletin No. 1, that beams made of 1-3-6 concrete of good quality reinforced with 1.5% of steel of say 33 000 lb. per sq. in. elastic limit will fail by steel-tension, is

correct provided the dimensions of the beam are such that the failure is not by diagonal tension.

41. *Failure by Compression of Concrete.*—None of the beams failed primarily through the development of the ultimate compressive strength of the concrete except No. 46, which had 2.76% reinforcement. Some of them, of course, crushed at the top with the rapid rise in the neutral axis after the steel had passed the yield point, but this must not be considered a compression failure. The beams having a large enough reinforcement of mild steel to counterbalance the compressive strength of the concrete gave diagonal tension failures before the full compressive strength of the concrete was reached. However, the load-deformation diagrams and computations of the compressive stresses throw some light on the effect of the larger reinforcement. The load-deformation curve for the upper fiber shows no change of direction up to a unit shortening of .0010 and .0012 in beams where the deformation reached this amount. In Beams No. 28 and 29 (Table 14) in which the unit shortening curves went to the highest values, the concrete did not show sign of failure when the unit-deformation reached .0020 and .0026. In Beam No. 26, the deformation at upper

TABLE 17

TOOL STEEL REINFORCEMENT.

LOADS APPLIED AT ONE-THIRD POINTS.

Beam No.	Per cent Reinforcement	Neutral Axis k	Load at First Visible Cracks	Max. Applied Load pounds	Load Considered pounds	Calculated Stress in Steel lb. per sq. in. From Resisting Moment	Calculated Stress in Steel lb. per sq. in. From Deformations	Manner of Failure
49	1.10	.430	9000	11300	11000	35400	37800	Bond
53	1.10	.510	6000	8000	8000	26600	29700	Bond
56*	1.10	....	....	860	.....	....	.....	Bond
57	1.10	.472	2000	7350	7000	22900	28500	Bond
60†	1.10	.380	6000	13200	13000	23500	27000	Bond
62	1.10	.480	2000	8100	8000	26300	22500	Bond
67	1.10	.381	6000	8600	8000	25200	27500	Bond
51*	1.66	.500	5000	11800	11000	24100	32400	Bond
52	1.66	.520	6000	15000	15000	33100	40800	Bond
55	1.66	.590	8000	9950	9950	22600	21000	Bond
61	1.66	.481	5000	6600	6600	14300	16200	Bond

* Beam was cracked before test. In Beam No. 56 reinforcing bars were wrapped in oiled tissue paper.

† Beam No. 60 was loaded at two points 7½ ft. apart.

fiber at maximum load was .0026. A study of these curves in connection with the amount of reinforcement used shows that these tests confirm the conclusions reached in Bulletin No. 1, that with 1-3-6 concrete of the quality here used, a reinforcement of 1.5% of steel of an elastic limit of say 33 000 lb. per sq. in., will not develop the full compressive strength of the concrete. This conclusion of course is independent of whether the diagonal tensile strength of the beam is large enough to resist that method of failure. A calculation of the compressive stress developed with 1.5% reinforcement of 33 000 lb. per sq. in. metal, based upon equation (15) using 2 000 000 lb. per sq. in. for the modulus of elasticity and $q = .6$, gives 1600 lb. per sq. in. A comparison of the deformations with the ultimate deformations of the concrete cylinders tested indicates that this value is about 80% of the ultimate compressive strength of this concrete.

42. *Failure by Diagonal Tension in Concrete.*—As noted in the statement made on page 25, it is believed that the shearing strength of concrete is great enough to resist any shearing stresses which come on beams of ordinary dimensions. What are frequently called diagonal shearing failures are really diagonal tension failures. Since the actual amount of the diagonal tensile stress can not be calculated without knowing the horizontal tensile stress developed in the concrete at the same place, it may be best to make comparisons through the medium of the horizontal and vertical shearing stresses, as given by equation (18). The amount of the diagonal tensile unit-stress may under some circumstances be two or more times as much as the vertical shearing unit-stress, as may be seen by a study of equation (19). Perhaps in the beams under consideration the maximum diagonal tensile stress may be considered to be in the neighborhood of two and a half times the vertical shearing stress. Bearing this in mind, we may use the value of the vertical shearing unit-stress as calculated by the formula $v = \frac{V}{bd'}$ (where d' is the distance from the center of the steel to the center of gravity of the compressive stresses in the concrete) for making comparisons of the diagonal tensile stresses developed. Table 18 gives the calculated values of the vertical shearing unit-stresses developed in beams which gave diagonal tension failures, as calculated by the above formula. The highest value is 151 lb. per sq. in. for Beam 18 which failed

under an applied load of 18 800 lb. Evidently the diagonal tensile stress developed in this beam was between 300 and 400 lb. per sq. in. The lowest values for normal concrete were Beam No. 34 which failed with $v = 104$ lb. per sq. in. and No. 20 with 86 lb per sq. in. The average value for failures of this type was $v = 123$ lb. per sq. in. Beam No. 20 gave especially low results. Of the beams not failing by diagonal tension, two developed a vertical shearing stress of 123 lb. per sq. in., and four reached 100 lb. per sq. in. None of these beams, then, developed a stress higher than the average given above.

It should be borne in mind that these results are with 1-3-6 concrete, that the bars were laid horizontally throughout the length of the beam, and that there was no vertical or diagonal steel reinforcement used.

TABLE 18

VALUES OF VERTICAL SHEARING STRESS AND BOND DEVELOPED IN BEAMS FAILING BY DIAGONAL TENSION.

Beam No.	Vertical Shearing Stress lb. per sq. in. $v = \frac{V}{bd'}$	Bond lb. per sq. in. of surface of bars $u = \frac{V}{mod'}$	Remarks *
14	133	170	Loaded at two points $7\frac{1}{2}$ ft. apart
15	143	182	Loaded at two points $7\frac{1}{2}$ ft. apart
18	151	193	Loaded at two points $7\frac{1}{2}$ ft. apart
20	86	109	
24	130	110	2.21% mild steel reinforcement
28	124	106	2.21% mild steel reinforcement
29	137	117	2.21% mild steel reinforcement
33	120	135	1.66% mild steel reinforcement
34	104	116	1.66% mild steel reinforcement
38	117	143	1.60% mild steel reinforcement
45	109	112	1.84% mild steel reinforcement
Av.	123	136	
39	62	80	Abnormal concrete
40	78	100	Abnormal concrete
41	75	95	Abnormal concrete
42	57	73	Abnormal concrete
44	80	101	Abnormal concrete
Av.	70	90	

* Unless otherwise stated, all beams in this table were of 1% mild steel reinforcement, loaded at one-third points.

The so-called abnormal concrete gave lower results. Beams No. 39 and 40, made of concrete which was unevenly and insufficiently mixed, broke at $v = 66$ and 78 lb. per sq. in. Beams No. 41 and 42, made with the lean mixture (1-6-12) at the bottom, broke at $v = 75$ and 57 lb. per sq. in., respectively. Beam No. 43, made with a "plane of set" above the bars, did not fail by diagonal tension, but Beam No. 44 broke at 80 lb. per sq. in.

It seems apparent from these tests that the richness and the tensile strength of the concrete enter into the diagonal tensile strength of a beam in a way not usually recognized and that for beams having a short length in comparison with the depth it may be the controlling element of strength, unless, of course, some metallic form of web reinforcement is used.

43. *Failure of Bond.*—Failure of the bond between the reinforcing rods and the concrete is difficult to detect. The fact that a rod has been found after failure of the beam to have slipped is not evidence that slipping occurred before failure began and hence was the primary cause of failure. In some instances reported as failure by slipping, the slipping evidently occurred as a consequence of the new conditions brought into play by whatever was the primary cause of failure, and slipping may not be considered the primary failure.

The smooth and almost polished surface and uniform cross-section of the tool steel used in eleven beams gave an opportunity to study failure of bond or slip of bars. These bars were ¾-in. round tool steel of about 52 000 lb. per sq. in. elastic limit, bought of the Crescent Steel Co. The surface of these bars was dense and smooth, the finishing work leaving the surface almost like a glaze. The cross-section of the rods was very nearly uniform; for example, measurement of the diameter of a rod taken at intervals of ¼ inch were as follows: .7590, .7590, .7590, 7588, .7589, .7589 in. Measurements of mild steel rods taken at two points ¼ inch apart will vary as much as .0015 in.

All these beams failed by bond of steel and concrete or slipping of the bars. Fig. 13 shows their appearance after failure. In this figure, the numbers for Beams No. 52 and 53 should be transposed. Table 19 gives the bond developed in lb. per sq. in. at the time of failure, as calculated by equation (17) $u = \frac{V}{mod'}$, and also the vertical shearing stress developed with the same load. The weight of the beam and loading apparatus is included in these calculations. Beam No. 49 failed suddenly. The failure shows a nearly vertical crack with a horizontal crack extending along the plane of the reinforcement toward the support. Fig. 14 is from a photograph. It seems likely that slipping occurred from the end of the rods to the vertical crack and also that the horizontal crack developed at the time of slipping and in connection with the vertical tension coming on the rod. The bond stress developed, 161 lb. per sq. in. of surface of bar, is the largest except one devel-

TABLE 19

VALUES OF VERTICAL SHEARING STRESS AND BOND DEVELOPED IN BEAMS REINFORCED WITH TOOL STEEL.

Beam No.	Vertical Shearing Stress lb. per sq. in. $v = \frac{V}{bd'}$	Bond lb. per sq. in. of surface of bar $u = \frac{V}{mod'}$	Remarks *
49	95	161	1.10% reinforcement
53	72	123	1.10% reinforcement
57	66	112	1.10% reinforcement
60	107	181	1.10% reinforcement
62	73	124	1.10% reinforcement
61	73	124	1.10% reinforcement
51	101	114	1.66% reinforcement
52	126	143	1.66% reinforcement
55	107	120	1.66% reinforcement
Av.	91	133	
61†	61	69	1.66% reinforcement
56	....	....	1.10% reinforcement

* See Table 17 for additional notes.

† Beam No. 61 had artificial cracks outside of the load points.

oped in this series. The vertical crack was closer to the support than was the case with the other beams. The record of Beam No. 60 is not definite enough to give the exact conditions of failure. It developed the highest bond stress of the tool steel series, 181 lb. per sq. in. Probably the slipping and consequent failure were sudden. The fact that the loads were closer to the supports than in the other beams may have a bearing on the high value developed. It seems probable also that the additional anchorage of 3 inches of rod which in all the beams projected beyond the point of support would act to raise the calculated bond stress for beams in which slipping occurred from the ends.

Eight beams may be described as slipping and failing gradually. At a load of 75% to 95% of the maximum, a crack, vertical or nearly vertical in position, appeared between the load point and the support and not very far from the former, and gradually increased in height until the maximum load was reached. The load then fell off, and this crack grew until suddenly failure occurred at a load from 1000 to 4000 lb., less than the maximum. In Beam No. 52 the critical crack appeared at 13000 lb., 87% of the maximum load. The direction and position of the critical crack are indications that slipping of the rods was the primary cause of failure. The cracks shown in Fig. 13 and in Fig. 14 are as they

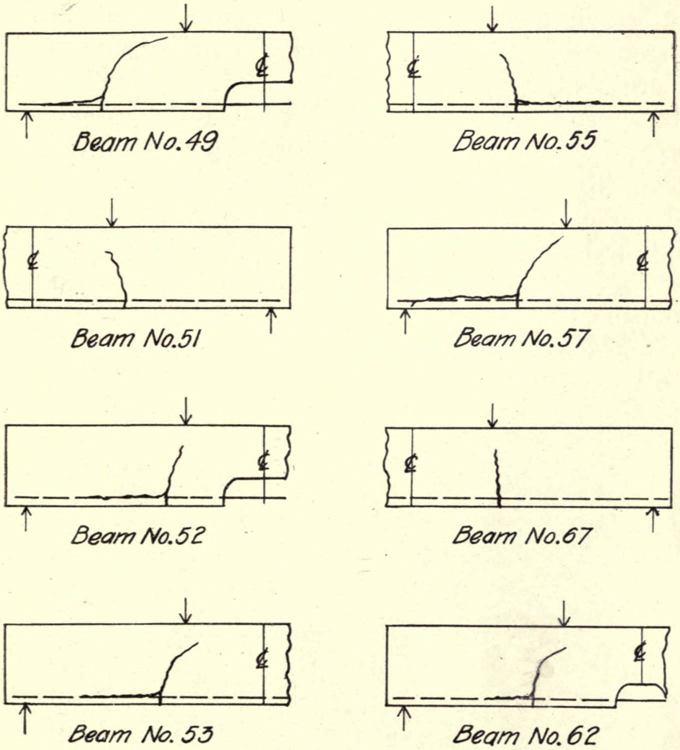

FIG. 13. SKETCH SHOWING FAILURE OF BEAMS REINFORCED WITH TOOL STEEL BARS.

appeared near the time of final failure. At first appearance only the vertical portion showed. It seems likely that this slipping occurred from the crack to a point under the load, there being no shear and hence no bond stress on the portion of the beam between the two loads. The calculated bond stress at maximum loads for these beams ranged from 114 to 143 lb. per sq. in. Bond tests made with this tool steel by Mr. Kirk, the rods being imbedded 6 inches in the concrete, gave values of 153, 147, 154, and 141 lb. per sq. in. of surface, averaging 149 lb. per sq. in. It may be noted that at the first appearance of the critical crack in these eight beams, the bond stress developed ranged from 90 to 125 lb. per sq. in. The position of the critical crack and the manner of failure in this group of beams are materially different from the conditions accompanying diagonal tension failures. It must not be overlooked, however, that the presence of this initial crack does weaken the resistance of the beam to diagonal tension, and thus increases the web stresses above the crack and also the vertical tension transmitted from the rod just beyond the crack, which together cause the final failure to be of the form shown.

Beam No. 61, (Fig. 14) is interesting as showing the effect of artificial cracks in a part of the beam where vertical shear exists. In making the beam, strips of tin covered with paper were placed in the beam forming vertical cracks running from the reinforcing rods four inches in height. In this beam there were thirteen of these cracks 8 inches apart, six of them being outside of the load points. Failure occurred in a crack 16 inches outside of a load point in a manner quite similar to that described in the preceding paragraph. The crack showed at the bottom of the beam at a load of 5000 lb. and extended to within 3 inches of top of beam at the maximum applied load of 6600 lb. The load fell off, the crack extended, and final failure occurred at a lower load. The value of the bond stress developed was 69 lb. per sq. in. Mortar below the rods was not broken except at these artificial cracks. This is lower than found in the other beams, but it must be remembered that the presence of the vertical artificial crack gave a different distribution of web stresses from the beginning.

The failures here discussed indicate that there are two types of failure of bond. 1. Slip from the direction of the middle of the span, with a slowly developing crack slightly inclined from the vertical which extends upward as the load is increased to the maximum load, growing still more as the test is continued at a dropping load, and finally breaking by splitting below and cracking diagonally above. 2. Slip from the end of the beam and a sudden

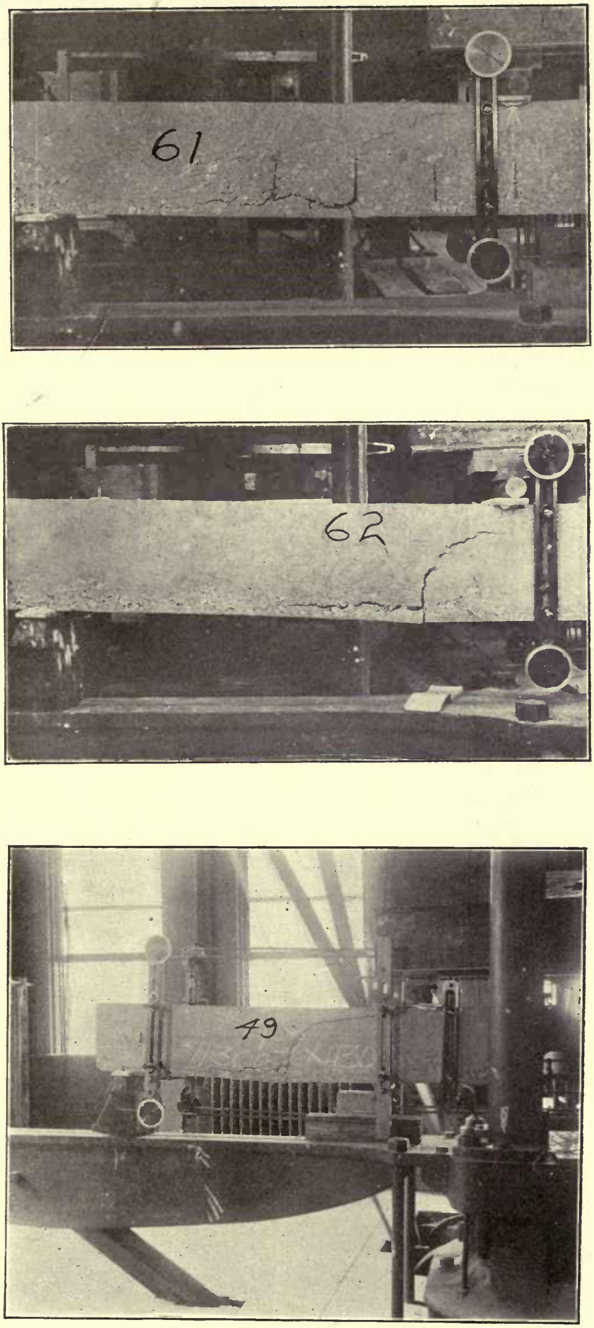

FIG. 14. VIEWS SHOWING FAILURE OF BEAMS REINFORCED WITH TOOL STEEL BARS.

failure at maximum load by the formation of a crack slightly inclined from the vertical and near to the support together with accompanying splitting and diagonal cracking to the top of the beam. The characteristic of the first is slow failure along a crack which is nearly vertical and which gradually grows with increasing load, and of the latter a sudden failure through a crack in a nearly vertical position not visible until time of failure is reached. It is likely that both are variations of a single form of failure, the former appearing when the vertical tensile strength of the concrete is exceeded. In failures by diagonal tension, the cracks formed are inclined more from the vertical than are these cracks. Again, attention should be called to the fact that the rods were laid horizontally and that there was no vertical or diagonal steel reinforcement used. It may be added that in none of the beams made with mild steel bars placed horizontally was there any evidence of slip of bar, and Beam No. 18 developed 193 lb. per sq. in. bond stress. (Table 18.)

An effort was made to discover whether bond is affected by grip of the concrete as distinguished from adhesion. In Beam No. 56 the rods were wrapped in thin oiled paper. Unfortunately, in the accident before described this beam was broken in two, and it seems likely that any bond which existed was greatly disturbed. Bond tests made with the same steel wrapped in paper in the same way gave 60 and 56 lb. per sq. in. of surface, so that it may be expected that if the beam had not been injured the load carried would have been higher.

44. *Effect of Artificial Vertical Cracks and Spaces.*—In Beams No. 51 and 52 artificial vertical cracks were formed as in No. 61 except that there were only five cracks and these were placed between the load points where the shear is zero. Outside the load points the beam then acted as a normal beam. Another point of difference was that these cracks extended to the bottom of the beam. A comparison of the deformations with those with normal beams shows little difference, a result which was to be expected if the tensile strength of the concrete in a horizontal direction is not to be relied on.

Another variation consisted in cutting out a space 24 inches long and 5 inches high across the full width of the beam,—thus leaving the rods exposed, as was done in Beams No. 49 and 52, Fig. 16, Art. 52. Beam No. 62 was similar, the portion cut away being

16 inches long and 3 inches high. Although cracks appeared at the corners of these spaces, failure occurred outside of the load points, and the cutting out of these spaces apparently was not a source of weakness. A discussion of the deformations observed on the exposed bars will be given under Effect of Exposing Reinforcing Bars.

45. *Effect of Method of Loading.*—Considerable interest has been manifested by engineers in the effect of method of loading test beams. The opinion has been advanced that a uniformly distributed load will allow a higher moment of resistance to be developed, and cases of tests with loads formed of sacks of sand or bars of iron have been cited in support of this. On the other hand the moment of resistance developed in beams loaded at the middle has been found to be higher than was to be expected if the distribution of stress is as assumed in the ordinary theory of flexure. A few beams were tested with the view of finding out something concerning the effect of changing the point of application of loads.

These beams had 1% reinforcement (Tables 11 and 12). No. 5 and No. 11 were loaded at the one-third points. No. 21 and No. 30 were loaded at the middle of the span (concentrated load). No. 47 was loaded at eight points 1½ feet apart, the load being divided equally among these eight points. In No. 14, 15, and 18, the load was applied equally at two points 7½ feet apart. The behavior of the beams which were loaded at the one-third points was of course similar to what has already been described. In the beams loaded at the middle, cracks appeared under the load on the tension side early in the test and extended vertically higher until within 3 and 4 inches of the top of the beam at the maximum load. As was to be expected these beams gave steel-tension failures. No. 47, loaded at eight points, failed at the middle by steel-tension, the crack at the middle of the span finally extending vertically to within 2 inches of the top of the beam. Fig. 32 gives its load-deformation diagram. The beams which were loaded at two points 7½ feet apart failed by diagonal tension in the concrete at the highest loads carried by any of the standard size of beams tested and hence developed the highest vertical and horizontal shearing unit-stress. In general, then, it may be said that all of these beams failed in the manner which would have been predicted.

The moment of resistance developed in the beams which were loaded at the middle, as calculated from the maximum loads by the usual formulas, was higher than that developed by the other methods of loading, the excess in No. 21 being particularly marked. The fact that in the tests made afterward on steel taken from the beams, one rod of No. 30 was found to have a yield point of 34 600 lb. per sq. in., while those from No. 21 were above 42 000 lb. per sq. in. may be sufficient explanation for No. 30 not developing so high a moment of resistance as No. 21. An attempt was made to find the distribution of the stresses through the length of the beams by placing an extensometer on an 8-inch gauged length at the middle of the span in addition to the regular extensometer which was used with a gauged length of 42 inches, but the extensometer device which was rigged up proved not to be serviceable for the purpose, although it did give information of value in other ways. However, it is evident from the deformations measured, as well as from the resisting moments developed, that the distribution of the stresses in a center-loaded reinforced concrete beam is not as assumed in the ordinary theory of flexure. The stress in the steel at the middle of the span evidently is less than the amount given by calculations, and at points somewhat away from the middle the stress in the steel is greater than the calculated amount. The fact that the load is applied at the top of the beam affects the distribution. The results of tests made with center loading by various experimenters agree in showing that the moment of resistance developed in tests by center loading is considerably in excess of that obtained from calculations by the theory of flexure and that such results may not be relied on for ordinary loading. Tests of beams loaded at or near the one-third points agree in general very closely with the calculations based on the theory of flexure. The one tested with load applied at eight points compares well with the other loading in a general way, although the moment of resistance developed was somewhat lower than for the beams loaded at one-third points. This was due in part at least to the fact that all four of the reinforcing bars in Beam No. 47 were of steel with yield point of only 33 000 lb. per sq. in. It seems proper to say, then, that the results of tests made with center loading are not comparable with other loading and that, as center loading is not ordinarily assumed in designing, this method of loading test beams should not be used. It seems proper to add also, that a loading at the one-third points or thereabout is an allowable method of testing and gives results fairly comparable with uniform loading so far as the development of tensile and compressive stresses is concerned. It does not give as high shearing stresses and hence as high diagonal tensile stresses as uniform loading, but this is counteracted by the fact that with uniform loading these stresses remain high only a short distance from the supports and that in this distance the resistance to such stresses is greater than beyond, and also that at a point one-eight the span length from the supports these stresses decrease to less than those of the beam loaded at the one-third points. The convenience of the latter method of loading makes it, all things considered, the best general form of loading test beams.

46. *Effect of Repetition of Load.*—Most tests of reinforced concrete beams have been made by applying the load increasingly until rupture takes place. It is known that when the load is taken off, or released, it will not return to its original position. Part of the effect is due to the fact that a portion of the weight of the beam which had produced tension in the concrete in the lower part of the beam must after the failure of the tension of the concrete have the effect of giving additional tension to the steel. A part may be due to the overcoming of initial or shrinkage stresses in the concrete and steel. It would seem that a considerable part must be due to the failure of the concrete on the tension side properly to interlock or mesh so as to occupy its original position, thus leaving tension in the steel during the release of the load, and on the compression side to the inability of the concrete to spring back to its original place. How much of this effect may properly be called "set" can not be discussed here.

It is not so generally known that when a load is reapplied the second application produces a different effect, both on the compression side and on the tension side of the beam. It seemed important to learn the effect of a number of applications or repetitions of the same load on a beam, and repetitions of different loads were made on Beams No. 17, 19, and 29. In the diagrams, to secure clearness only a part of the repetitions have been platted. The number of the application is indicated by a figure.

Beam No. 17 (1% reinforcement) was loaded with 9000 lb. eleven times. This load and the weight of beam and loading apparatus gave a stress of, say, 37 000 lb. per sq. in. in the steel, 1400 lb. per sq. in. compression in the extreme fiber of the concrete, and

77 lb. per sq. in. horizontal and vertical shear. The load-deformation diagram (Fig. 35) shows little increase or change in the steel deformation after the second application, going and returning on the same line, but the compressive deformation of the concrete increased with successive applications and releases of the load. It should be noted that the compressive stress reached with this load was fairly high for the quality of concrete used. Upon the eleventh application, the load was run up to a maximum of 11 100 lb., failing immediately after at a load of 10 500 lb. Although a diagonal crack outside of the one-third point was the outward cause of failure it seems likely from the high stress in the steel (over 40 000 lb. per sq. in.), the shape of the load-deformation curve, and the position of the cracks, that these cracks were not the primary cause of failure, but that failure should be attributed to the stretch of the steel beyond the yield point. The cracks appeared at 6000 lb. at the first application of the load, and the one under consideration became somewhat more prominent with the repetition of the loading.

Beam No. 19 (1% reinforcement) (Fig. 33) was loaded with 6000 lb. eight times and then with 10 000 lb. six times. The sixth time the load was run past 10 000 to a maximum load of 10 800, falling off to 10 000 and failing by tension in the steel in the middle third. The stresses at the load of 6000 lb. together with the weight of beam and loading apparatus may be estimated to be 25 000 lb. per sq. in. tension in the steel and 1000 lb. per sq. in. compression in the concrete; and at the applied load of 10 000 lb., 38 000 lb. per sq. in. tension in the steel and 1500 lb. per sq. in. in the concrete. These are high stresses at which to test repetition of loading. The stress-deformation curves indicate that the concrete was compressed well up toward its crushing point at the time of maximum load. Two diagonal cracks appeared just outside the middle third at the first application of the 6000 lb. load, and upon the third application, another became visible. The crack in the middle which finally became the seat of failure appeared at the first application of 8000 lb.

Beam No. 29 (2.2% reinforcement) (Fig. 34) was loaded with 12 000 lb. fifteen times, and upon the sixteenth application the load was increased to the maximum of 15 900 lb , failing soon after at 15 900 lb. The stresses at the load of 12 000 lb. together with the weight of beam and loading apparatus were, say, 23 000 lb. per sq. in. tension in the steel, 1500 lb. per sq. in. compression in the concrete, and 110 lb. per sq. in. vertical and horizontal shear. Repetition of the loading gave greatly increased shortenings in the upper fiber of the concrete. In considering this, the high percentage of reinforcement and the large values of the compressive stresses developed in the concrete by virtue of this reinforcement should be borne in mind. The final failure of the beam was by compression at, say, 1700 lb. per sq. in. and at a time when failure by diagonal tension seemed imminent.

The tests of these beams throw considerable light on the effect of repeated application of loads under high stresses and point to important conclusions. The deflections under the last repetition of load were 12%, 15% and 30% in excess of those obtained with the first application. The source of this increase is mainly on the compression side, for the deformation observed in the remote fiber of the concrete increased from 30% to 50% beyond that of the first load, while the deformation at the level of the steel increased only 7% to 9% and in one beam the steel deformation ran backward and forward along the same lines after the second application. The increments of these changes in general decreased somewhat with the repetitions. It would be interesting to know, with such high stresses in the concrete, what would be the final effect of continued application of the load. It seems plain that the deformation at the level of the steel does not change much, and would not change except as it is modified by the changed distribution of the compressive stresses. It would also be interesting to know the effect of repetition of loads at lower stresses in the concrete. Since in the phenomena of failure of such materials as concrete the amount of the deformation is correlative in effect with amount of stress, it would likewise be of interest to know the effect of repetition of load upon both denser and more porous mixtures. The results here given indicate that the compressive stress to be taken as the basis of ultimate load in designing beams should be somewhat less than the ultimate strength of the concrete. It may be added that there was no general marked growth in the cracks with the repetition of load, and a comparison of the determinations at the maximum load with those of other beams of the same make-up does not show any special difference in results due to the repetition of the application of the load.

The deflection retained on the first release of load seems to be nearly the same proportion of the deflection due to that load as the deflection retained after the last application bears to that under the last load. This retained deflection is from 20% to 35% of the deflection under load. The larger part of this is due to failure of the concrete to return to the original condition, the retained deformation of the upper fiber being 30% to 50% of the deformation under load.

47. *Progressively Applied and Released Loads.*—The usual method of testing beams is to apply to load by increments until failure occurs. To determine the effect of removing the load as the testing progresses, in the tests of four beams the load was released after each application, the load being increased each time, generally in increments of 1000 lb. Beam No. 13 (Fig. 36) had 1% reinforcement and failed by tension in steel. Beam No. 20 (1%) failed by diagonal tension, the diagonal crack appearing at a load of 9000 lb., starting from a point at the bottom one foot from a support. The beam failed suddenly at a load of 10 200 lb., giving the lowest shearing stress (86 lb. per sq. in.) of any of the beams made of normal concrete which failed by diagonal tension. No explanation is offered for this unusually low value. Beams No. 24 (Fig. 38) and No. 28 (Fig 39) had 2.2% reinforcement and failed by diagonal tension of concrete with an average vertical shearing stress of 116 lb. per sq. in. in the concrete and at a high compressive stress. The phenomena accompanying the tests of these beams did not differ from those of beams of similar make-up tested in other ways; appearance of cracks, position of neutral axis, manner of failure, amount of deflection, etc., were not noticeably different. The effect on the load-deformation curves and deflection curves is interesting. These curves conform to the general outline of the curves given on page 48 of Bulletin No. 1. Upon release of load the beam does not return to its original shape, but retains a part of the deformations and deflection. The amount of this retained deformation increases with the amount of the load, though the amount of increase for the deformation at the level of the steel is less than that at the upper fiber of the concrete. In general it may be said that the retained deflection after a given load ranged from 20% to 35% of the total deflection for that load. The retained deformation for the upper fiber ranged from 25% to 40% of the deformation under load, this percentage increasing somewhat with the increase of the load. The retained

deformation of the steel ranged from 8% to 30%, this percentage being smaller for the larger loads.

An important feature shown in the load-deformation curves and one which has a bearing on the calculation of stresses in beams is the effect of the release of load upon the general form of the curve. Although upon the reapplication of a load the deformation is greater than that at the first application of this amount, yet upon the application of larger load, the deformations return to the general form of the curve. In fact, the outline of the curves conform very closely both in shape and amount to those formed without release of load, all differences being explainable by variations in properties of the concrete. This is further evidence that in the calculation of stresses, in the determination of the position of the neutral axis, and in the determination of the modulus of elasticity, gross deformations and not net or elastic deformations give results most nearly representative of conditions in the beam and should be used in the design of beams. The effect found in repetition of loading adds weight to this conclusion.

No difference in method of failure or in appearance of cracks from that of similar beams was found by the use of this method of applying loads. The position of neutral axis, calculated from gross deformations, agrees with other methods of applying loads. The set on the tension side is probably mostly due to the failure of the particles of concrete to interlock, and tension is thus left in the steel. Some part of this failure to return to original position is due to the added stress in the steel due to its taking a further part of the weight of the beam after the concrete has failed in tension, and a part may be due to the removal of initial stresses existing in the concrete. On the compression side most of the set is due to the plastic nature of concrete, though part seems to be of the nature of a final stress in the material. Whatever it is, the application of a greater stress carries the material to the same point it would have gone to with a continuously applied load. This phenomenon is worthy of further study.

48. *Effect of Rest after Release of Load.*—An effort was made to determine to what extent the deflections and deformations remaining in the beam after the load had been released were permanent, or in other words to find whether the beam returned toward the original position within a given interval of time.

Beam No. 23 (1% reinforcement) was loaded with 6000 lb.; the load was then released and the deflections and deformations measured at intervals extending over a number of hours. The calculated stress in the steel at this load including weight of beams was 25 000 lb. per sq. in. and that in the concrete, say, 1000 lb. per sq. in. Beam No. 27 (.74% reinforcement, 6000 lb. load, calculated stress in steel 33 000 lb. per sq. in. and in concrete 1150 lb. per sq. in.), No. 31 (a broken beam, load of 8000 lb.) and No. 50 (1% reinforcement, 2000 lb. load, calculated stress in steel 11 000 lb. per sq. in., and in concrete 450 lb. per sq. in.) were tested in the same way. The results of these tests are in some respects not satisfactory. It was not appreciated in advance that changes in temperature would have so great an effect upon the brass rods used in the extensometer device. The cathetometer observations of deflections gave constant readings except for one or two sudden changes which it seems must have been due to a change at the instrument rather than in the beam. Greater reliance may be placed on the results by the thread method of determining deflections of the beam. From observations made with the thread the indications are that the beams made no appreciable recovery of the set formed in the beam even after periods of 15 to 40 hours of rest. This is not what might be expected, and it should not be accepted as a conclusion without confirmation by other tests.

49. *Effect of Retention of Load.*—To determine the effect of retaining a load for a time longer than than that of the ordinary test, four beams were kept in the machine for periods ranging from 20 to 38 hours and the deflections and deformations observed. It was found as was to be expected that the load indicated on the scale beam of the machine usually dropped down somewhat, decreasing during the first three hours as much as 1200 lb. for an original load of 8000 lb. Part of this was due to a decrease in the deflection of the beam, although no motion of the gears of the machine could be detected. Each time that observations were made the load remaining on the beam was first noted, and then the extensometers and the deflection were read. A careful search was then made for cracks which might have developed, and then the original load was applied, after which the extensometers and deflection were read again. The load was retained as nearly

as possible in this manner for various lengths of time, after which the load was increased until the beam failed.

Perhaps the following condensed log of the tests will best state the conditions found.

Beam No. 16 (1% reinforcement) (Fig. 49). The first crack was discovered at a load of 4500 lb. one foot north of the middle of the span length. It was visible only on one side of the beam and extended vertically to within 7 inches of the top. At 7000 lb. there were three fine cracks a few inches apart just outside both load points and inclined slightly toward them. These were all on the west side of the beam and extended to within 6½ inches of the top. A load of 8000 lb. was then retained for 38 hours with no perceptible change in the cracks. With increased load, none of them increased much until the load reached 11 000, the maximum load being 11 600 lb.

Beam No. 22 (1% reinforcement) (Fig. 40). The first crack was discovered under the north load on the west side at 5 000 lb. At 8000 lb. two cracks appeared near the middle on the west side and one just outside the south load. None of these cracks extended to within 6 inches of the top. The cracks had not risen materially at 9000 lb. After 9000 had been retained three hours, the cracks near the middle had become visible on the east side and a new one 6 inches south of the middle on both sides was found, but no change was found in the remainder of the 20 hours' retention of load. At the maximum load of 12 700 lb. the crack 6 inches south of the middle had risen to within 4 inches of the top and had opened nearly 1/32 inch at the bottom. The cracks in the middle were approximately vertical. Those outside the load points inclined slightly towards them. The load was applied until the deflection was more than two inches.

Beam No. 26 (1% reinforcement) (Fig. 41). A load of 5000 lb. was retained 25 hours. No cracks were observed in the application of this load and none was observed until the load of 5000 lb. had remained on the beam 25 hours, when three cracks were found along the middle third extending to within 7 inches of the top. The load was then increased to 10 000 when several more cracks appeared under both load points. The maximum was reached at 11 450 when a crack 6 inches south of the middle began to open up, the deflection being then .72 inch. The final breaking load was 10 000 lb. which occurred with a deflection of 2.14 in.

Beam No. 36 (.74% reinforcement) (Fig. 42). The first crack appeared at a load of 6000 lb. just outside the north load and extended within 6 inches of top on both sides. This load of 6000 lb. was retained for 18 hours, and there was no apparent change in cracks during this time. After the maximum load of 7400 lb. had been passed a crack was discovered 14 inches north of the center, which extended almost vertically to within 5 inches of the top, and failure occurred at this crack.

It will be seen that while the stress in the steel at the retained loads was say 29 000, 32 000, 18 000, and 28 000 lb. per sq. in., respectively, (not including weight of beam), there was little effect shown in the stretched concrete by changes in appearance or growth of cracks. In Beam No. 26 the first crack became visible after the load of 5000 lb. had been retained 25 hours, but the stretch in the steel at this load is about the average stretch at which cracks became visible in these beams. In Beam No. 22 a crack which had been noted on one side of the beam became visible on the other side and a new crack was found during the retention of the load, but this was at a load which indicates a stress of 32 000 lb. per sq.in. in the steel. In general, little effect in the appearance and growth of cracks was noted.

Fig. 43 shows the change in load indicated on the scale beam for a constant deflection during the first fourteen minutes for Beams No. 26 and 36. As the deflections are approximately proportional to the loads, it may be judged from the diagram how rapidly the beam would deflect if a constant load were applied.

The change in deformations and deflections during the retention of load are not reproduced, as the changes in temperature of the measuring apparatus rendered the observations irregular and somewhat untrustworthy. The increase in the deflection during the retention of the load was for Beam No. 16 18%, for Beam No. 22 12%, for Beam No. 26 35%, and for Beam No. 36 28% of the amount of the deflection when the load was first applied. The observations show that the deformation in the steel and that in the upper fiber of the concrete are both increased, the increase being greater for the compression side. The average of the increase in the steel deformation for Beams No. 16, 26 and 36 is 15%, and that for the upper fiber of the concrete is 41% of the amount of the deformations when the load is first applied. It appears that at the beginning of the test the deformation in the steel increases more rapidly than that in the compression side of the beam and the neutral axis rises somewhat. Later the deformation in the steel decreases and that on the compression side increases considerably, and the neutral axis reaches a position lower than its first position. While the values observed may not be quantitatively correct, it is felt that the general results are worthy of consideration. It may be added that no effect of the retention of the load was apparent in either the form of failure or the amount of the maximum load. It is worthy of note that except in Beam No. 36, in which the retained load was well up toward the maximum, the load-deformation curves and deflection curves rise upward upon the application of larger loads after a load has been retained and finally take the general shape of such curves for progressively applied loads, much as was found to be the case with the curves for released loads.

50. *Effect of Position of Reinforcing Bars.*—A few test beams were made to learn something of the effect of bending the reinforcing bars into parabolic and trapezoidal form. Fig. 15

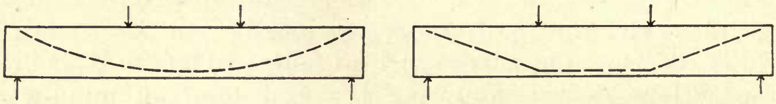

FIG. 15. SKETCH SHOWING FORM IN WHICH BARS WERE BENT.

shows the two positions used. These beams were not designed with an amount of reinforcement or a relation of depth to span which would develop the full diagonal tensile strength of the concrete, and hence these tests have no bearing upon the efficacy of bending up bars to aid in taking the diagonal component of the stresses. Moreover, all of the bars were bent up. The bars, particularly in the parabolic form, were bent up from points too near the middle of the span to get high loads. The tests are chiefly of value in the peculiarity of the place and form of failure. Although failure took place at vertical cracks with an outward appearance of steel-tension failures, these cracks appeared generally outside the load points. In Beam No. 48 a vertical crack appeared at a point about half way between one load point and the support at a load of 6000 lb. and extended upward to the steel. At 7000 lb. this crack had risen further and a small crack branched out from it and ran along the line of steel for about 10 inches. At the same time a second vertical crack appeared at a point about half way between the other support and load point. At the maximum load, 9 300 lb., these vertical cracks had reached nearly to the top of the beam. The load then rapidly decreased. The cracks along the steel were fine cracks and did not open up. The calculated stress in the steel within the middle third was 33 000 lb. per sq. in., which is about the elastic limit of the steel used in these five beams. It would seem that the stress at the vertical crack must have been less than this. Beam No. 63 failed in the same way except that the maximum load was 7400 lb. and the stress must have been less than in No. 48. Beam No. 58 having bars bent in parabolic form failed in a similar manner with a maximum load of 8900 lb. Beam No. 59, also with bars bent in parabolic form, failed at cracks near the load points at a maximum of 8900 lb. Beam No. 54, also with bars bent up in parabolic form, was, unlike the foregoing, tested with center loading and failed at a vertical crack at a load of 6600 lb. The information concerning these beams is not explicit enough to tell whether any of them failed by other cause than failure of the steel in tension. It would be interesting to know whether slipping of the bars occurred in any of these beams.

51. *Effect of Lean and Abnormal Concretes.*—These tests are of interest in showing the direction of the effect of poor workmanship and lean mortar. Beams No. 39 and 40 were made of concrete which was mixed about one-third as much as that for the ordinary beams and was not so well rammed. On account of the poor mixing, patches of unmixed material could be detected and the sand grains and stone were not well coated. They failed suddenly by diagonal tension at maximum applied loads of 7000 lb. and 8800 lb. (vertical shearing stress of 62 and 78 lb. per sq. in. including weight of beam, etc.). As normal beams with the same reinforcement broke by failure of steel at loads from 9500 to 11 000 lb., and as diagonal tension corresponding to a vertical shear of 125 lb. per sq. in. was developed in the normal beams before failure by diagonal tension occurred, it will be seen that the effect of poor mixing on the resistance to diagonal tension is quite marked.

Beams No. 41 and 42 were made to see the effect of using lean concrete in the lower half of the beam, the part whose chief function is to transmit stresses from the tension of the steel to the compression area of the concrete; in other words, to act as a web.

In these two beams the lower $5\frac{1}{2}$ inches of the beam was made of concrete with 1 part cement 6 parts sand and 12 parts stone by loose volume and the upper $5\frac{1}{2}$ inches was made of the usual 1-3-6 mixture. The top layer was placed over the leaner concrete in the usual manner. Beam No. 41 broke by diagonal tension of the concrete (sudden) at a maximum load of 8800 lb. Beam No. 42 broke in the same way at a maximum load of 6000 lb. The calculated vertical shearing stresses (including weight of beam, etc.) are 75 and 57 lb. per sq. in., respectively, as compared with, say, 125 lb. per sq. in. in normal beams. Considering that the diagonal tensile resistance developed is proportional to these vertical shearing stresses, it is evident that the richness and the strength of the concrete have much to do with its ability to resist diagonal or web stresses. The importance of quality of concrete for the purpose of resisting diagonal stresses is not usually recognized, and this element should be considered even when metallic web reinforcement is used.

Beams No. 43 and 44 (1-3-6 concrete) were made with "planes of set"; that is, the bottom of the beam was made and allowed to set before finishing the construction of the beam. In Beam No. 43 a 1-inch layer of concrete was placed in the bottom of the form, the rods were imbedded half their thickness, the layer left untamped, and this layer left to set for 24 hours The remainder of the depth of the beam was then built as usual. It will be seen that this is a severe condition. In Beam No. 44 the bottom $5\frac{1}{2}$ inches of the beam was built as usual, except that the top surface was roughened with the point of a trowel and was left to set for 24 hours, when it was completed in the usual way. Beam No. 43 failed at a vertical crack at 1 foot from the center at a maximum applied load of 9300 lb. in a manner and with a calculated stress which indicates a steel-tension failure. The load is no lower than that in some normal beams made with the same low steel bars. How much more stress the "plane of set" would have stood is not known. The bond developed is calculated as 101 lb. per sq. in. of steel surface. Beam No. 44 failed by diagonal tension of the concrete at a maximum applied load of 9400 lb. A vertical crack extending to the level of the steel appeared 3 ft. from one support at 8000 lb., and the diagonal crack causing failure originated at this point. The calculated vertical shearing stress at a load of 9400 lb. is 80 lb. per sq. in. Nothing is known which would connect the diagonal tensile stress corresponding to this low value of vertical shear with the manner of making the beam, and no explanation is offered of the cause of the failure of this beam by diagonal tension of the concrete at so low a stress.

52. *Effect of Exposing Reinforcing Bars*—In the construction of three beams, no concrete was put in a space at the bottom on either side of the middle, thus making an arch-like opening and leaving the reinforcing rods exposed for a length of 16 to 24 inches. Fig. 16 shows the form of the opening. Deformations

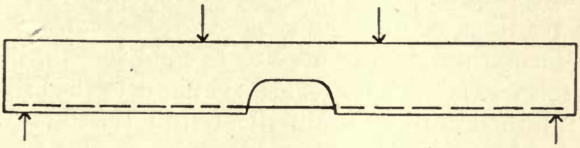

FIG. 16. SKETCH SHOWING EXPOSED RODS.

were taken both on the exposed portion of the reinforcing rods and on the usual gauged length of 42 inches. The gauged length used for the extensometer on the exposed rods was 8 inches. Table 20 gives the stresses calculated from the observed deforma-

TABLE 20.

STRESSES IN STEEL IN BEAMS HAVING TOOL STEEL REINFORCEMENT WITH BARS EXPOSED.

Beam No.	Load Considered	Stress in Steel lb. per sq. in.		
		From Resisting Moment	From Elongations in	
			Exposed Steel	Exposed and Encased Steel
49	8000	25900	23000	27000
52	8000	26100	25200	29400
62	8000	26100	25800	23100
Average		26000	24700	26600

tions for the usual gauged length and for 8 inches on the exposed rods, as well as the stresses calculated from the bending moments by the formulas heretofore given. The three stresses compare very favorably. If allowance were made for the portion of the weight of the beam which after the concrete has broken in tension has the effect of adding to the deformation of the part of the steel bar which is embedded in the concrete, the stress in the last column would be smaller, and the agreement would be closer.

It should be noted that this effect of the breaking of the concrete in tension does not affect the exposed steel observation, and hence the stress in the steel should compare with that calculated from the moment of the applied load only. This agreement is what is to be expected from the usual assumption that the deformation in the concrete at the level of the steel is the same as that in the steel.

It is of interest to compare the effect upon the exposed steel and upon the encased steel when the load is released. In Beam No. 49, after a load of 9000 lb. had been applied, the load was released and readings taken. As usual the observations indicated retained deformations in the 42-inch gauged length, but the instrument on the exposed rods returned almost to the original reading. The retained deformation indicated by the extensometer on the exposed rod amounted to .000023, corresponding to a stress of, say, 690 lb. per sq. in.

53. *Position of Neutral Axis and Value of Modulus of Elasticity.*—The successive positions of the neutral axis within the middle third of the span length, as determined experimentally by the method already explained, are shown for the various beams on the diagrams given in Fig. 56, 57 and 58 at the end of the bulletin. In general, the change in the position of the neutral axis as the applied load is increased follows the law outlined on page 29 of Bulletin No. 1. The neutral axis rises during the second stage and remains nearly stationary during the third stage. (For the use of the terms second stage and third stage see page 21 and Fig. 15 of Bulletin No. 1.) For beams in which by reason of a large amount of reinforcement high compressive stresses in the concrete are developed, the neutral axis falls considerably during the stage of rapid deformation of the upper fiber. An example of this is shown in Beam No. 46. (Fig. 56.) Beams with low or medium amounts of reinforcement, as for example the 1% beams, do not of course develop large deformations in the upper fiber. That a slight lowering of this position of the neutral axis during the third stage is not always markedly noticeable, as would be indicated by theory, is probably due to the decrease in the total amount of tension in the concrete in a given section as the load is increased, the moiety of tension remaining in the concrete affecting the position of the neutral axis much more than it does the resisting moment of the beam.

The position of the neutral axis given in Tables 11 to 17 is for the first part of the third stage of flexure. Generally the value there recorded represents an average of the positions above and below a load which produces a deformation in the upper fiber of .0004 or .0005 per unit of length. These deformations correspond to something like .2 to .25 of the ultimate deformation of the concrete. It will be recalled that the value q=.25 was selected for the formula for position of neutral axis for the calculations of this bulletin. (Page 15.)

In Fig. 8, page 16, the proportionate depth of the neutral axis for the beams made with normal 1-3-6 concrete and reinforced with mild steel are platted. The values for the 1904 beams, which are also platted, do not differ much from these.

The agreement of the points platted is as close as may be expected when variations in quality of concrete, lack of knowledge of exact position of reinforcing bar, effect of a contact point resting against one edge of a large stone, and other variations are taken into account. It will be seen that the positions range somewhat below the line drawn for position of neutral axis with an initial modulus of elasticity of 2 000 000 lb. per sq. in. (n=15) and q=.25, as calulated from equation (11) given on page 15. As the concrete grows older, the modulus of elasticity will increase somewhat and the neutral axis will rise accordingly, although the change may not be great after an age of 60 days. However, it would seem proper with 1-3-6 limestone concrete of the kind here used to use the line for n=15 shown in Fig. 8 for getting the position of neutral axis and for use in calculating stresses, and to call the initial modulus of elasticity 2 000 000 lb. per sq. in. Of course for high reinforcements, a larger value than q=.25 will be developed, but this change will not seriously affect the position of the neutral axis. If it is desired to use the straight-line deformation relation, a lower value than 2 000 000 for the constant modulus of elasticity should be selected.

The straight line given on Fig. 16 of Bulletin No. 1 gives fair results up to a reinforcement of 1.5%, but it seems better to use the analytical determination, especially as the proportionate depth may be easily found by means of a diagram. It may be added that the value of the initial modulus of elasticity of 2 000-000 lb. per sq. in., n=15, corresponds closely with results found in compression tests on concrete of the character used in these tests.

54. *Summary.*—It is difficult to summarize the results of the investigation, since it covers considerable ground and the results are intimately connected with a variety of conditions and many-sided phenomena. A careful perusal of the data and discussion will give a clearer insight and perhaps a fairer grasp of the results than can be gained from a summarized statement. The following statements are not intended to stand forth as conclusions, but rather as partial interpretations of some of the general phenomena of the tests:

1. The general phenomena of the tests, like stages of flexure, the failure of tension in the concrete, moment of resistance of the beam, effect of elastic limit of the steel, etc., agree well with those described in Bulletin No. 1.

2. The tests confirm the conclusion given in Bulletin No. 1 that for normal 1-3-6 concrete of the quality used a reinforcement of 1.5% of steel of 33 000 lb. per sq. in. elastic limit will not develop the full compressive strength of the concrete. In beams having a smaller amount of reinforcement, failure will be due to the stretching of the steel beyond its yield point. This assumes that the beam is so proportioned or so reinforced that bond or diagonal tension will not be the cause of failure.

3. Beams which failed by diagonal tension developed an average vertical shear of 123 lb. per sq. in., as calculated by equation (18). The diagonal tensile stress corresponding to the shearing stress may be considered to be the tensile strength of the concrete. The results have a bearing upon the importance of web reinforcement.

4. Beams reinforced with tool steel failed by slipping of the bars with 133 lb. per sq. in. as the average bond stress developed. These beams showed a characteristic type of failure. The special tests to determine the bond resistance of these tool steel rods averaged 149 lb. per sq. in. In the beams reinforced with mild steel placed horizontally, there was no evidence of slip, although in one beam a bond stress of 193 lb. per sq. in. was developed.

5. Center loading may be expected to give results which are higher than those found by the ordinary beam formula. Moments of resistance derived from results of center loading tests may not properly be used as a basis of calculation for other forms of load-

ing. The results with loading at the one-third points compare favorably with multiple-point loading, and are comparable with uniform and other distributed loading.

6. Repeated applications of a load which sets up high compressive stresses in the concrete give increasing deformations. The deflections after ten to fifteen applications were found to be 12% to 30% in excess of the deflection at the first application.

7. Beams which were loaded to give a stress of 15 000 lb. per sq. in. in the steel and 800 lb. per sq. in. in the concrete, or more, failed to return to their original position upon the removal of the load, the amount of the retained deflection being 20% to 35% of the deflection. No appreciable recovery of the set was apparent after periods of 15 to 40 hours.

8. Beams loaded so as to develop stresses of 18 000 to 32 000 lb. per sq. in. in the steel and compressive stresses of 800 to 1400 lb. per sq. in. in the concrete gave little perceptible change in appearance or growth of cracks after the load had been retained 20 to 38 hours, and upon the application of greater loads the load-deformation curves and deflection curves rose upward and took the general shape for such curves for progressively applied loads. During the retention of load, the deflection increased 12% to 35%, the principal cause of this increase evidently being the increased compression of the concrete.

9. The general form of the stress-deformation curves for continually increasing loads, progressively applied and released loads, and repeated loads is the same, until the compressive deformations of the concrete become large.

10. Poorly-mixed concrete and lean concrete when used in the lower half of beams gave failures by diagonal tension at shearing stresses not much more than one-half the stresses developed in beams made with normal concrete which failed in the same manner. Tensile strength of mortar is therefore of importance, at least when metallic web reinforcement is not provided. No noticeable effect from the so-called "plane of set" was found.

11. The tests of beams having artificial cracks and exposed rods give results which tend to confirm the analytical basis used for determining stress in steel and for explaining slipping of bars and splitting of bars away from upper portion of beam.

12. The position of the neutral axis found for the various beams indicates that for a limestone concrete of the proportions

used the value of the initial modulus of elasticity is 2 000 000 lb. per sq. in. or less. It may not be much higher than this for older concrete or richer concrete. If a straight-line stress-deformation relation, (constant modulus of elasticity), is assumed, the value may well be less than that of the initial modulus of elasticity here used.

It may not be out of place to add a few comments on the analytical treatment of the resistance of beams to flexure and on the discussion of methods of failure.

13. The parabolic stress-deformation relation for concrete in compression offers a satisfactory solution for use with beams in which any considerable deformation is developed in the concrete. The algebraic work based upon the proportion of the ultimate deformation developed, (q), is not particularly complicated, and the final formulas are not more difficult of use than are those based upon the straight-line stress-deformation relation (constant modulus of elasticity) and are easily transformed into the straight-line relation for use or comparison.

14. The resisting moment of a beam which does not have an excess of reinforcement may well be expressed in terms of the total tensile stress in the steel and the distance from the steel to the center of the compressive stresses as given by equation (13). Its calculated value will not differ greatly whether the parabolic or the straight-line stress-deformation relation be used, or for any ordinarily assumed position of the neutral axis. The moment arm will, however, decrease somewhat as the amount of reinforcement increases. Whether the position of the neutral axis is obtained from a formula or taken from a diagram, the calculation by equation (8) is not more difficult with the parabolic than with the straight-line relation.

15. For any considerable compression in the concrete, the formula based on the straight-line stress-deformation relation gives too high a value for the compressive stress. If the straight-line relation is to be used, even for calculations at working loads, the modulus of elasticity selected should be lower than the value of the initial modulus of elasticity for the same concrete, or a higher limiting compressive stress may be chosen than is actually developed in the beam.

16. Equation (17) for bond and equation (18) for shear are applicable to the method of reinforcement here discussed. A direct calculation of diagonal tensile stress may not be made, since the intensity is dependent upon an unknown horizontal tension; but comparison of diagonal tensile stresses may well be made upon the basis of the vertical shearing stresses developed.

17. Failure by splitting of the bars away from the upper portion of beam must be a secondary form of failure, following failure by diagonal tension or other form of primary failure.

18. In all tests involving a determination of the cause of failure, care should be taken to distinguish primary failure from secondary or ultimate failure. While such distinction can not always be made, the value of the conclusions must depend upon the accuracy of the information on this point.

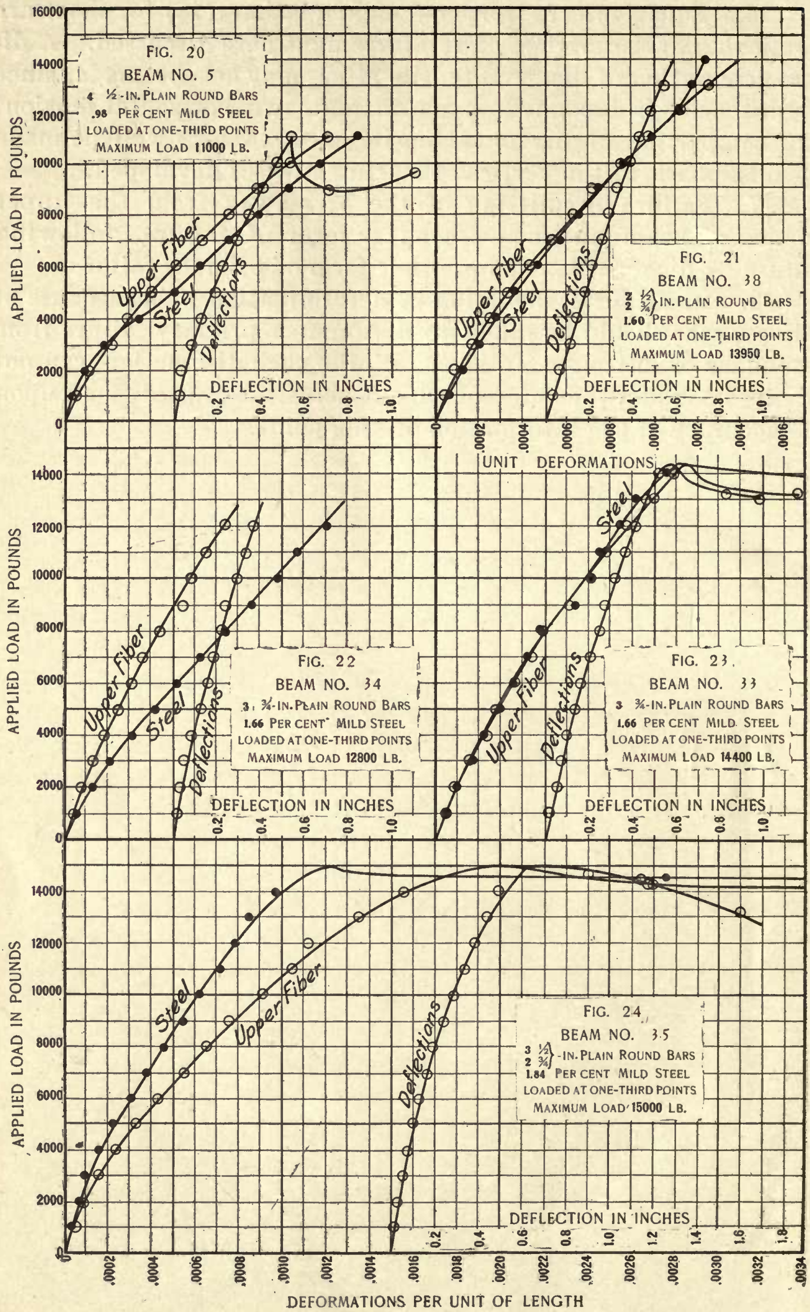
FIG. 20
BEAM NO. 5
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 11000 LB.
FIG. 21
BEAM NO. 38
2 ½ 2 ¾ IN. PLAIN ROUND BARS
1.60 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 13950 LB.
FIG. 22
BEAM NO. 34
3 ¾-IN. PLAIN ROUND BARS
1.66 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 12800 LB.
FIG. 23
BEAM NO. 33
3 ¾-IN. PLAIN ROUND BARS
1.66 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 14400 LB.
FIG. 24
BEAM NO. 35
3 ½ 2 ¾ -IN. PLAIN ROUND BARS
1.84 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 15000 LB.
Upper Fiber
Steel
Deflections
DEFLECTION IN INCHES
UNIT DEFORMATIONS
APPLIED LOAD IN POUNDS
DEFORMATIONS PER UNIT OF LENGTH

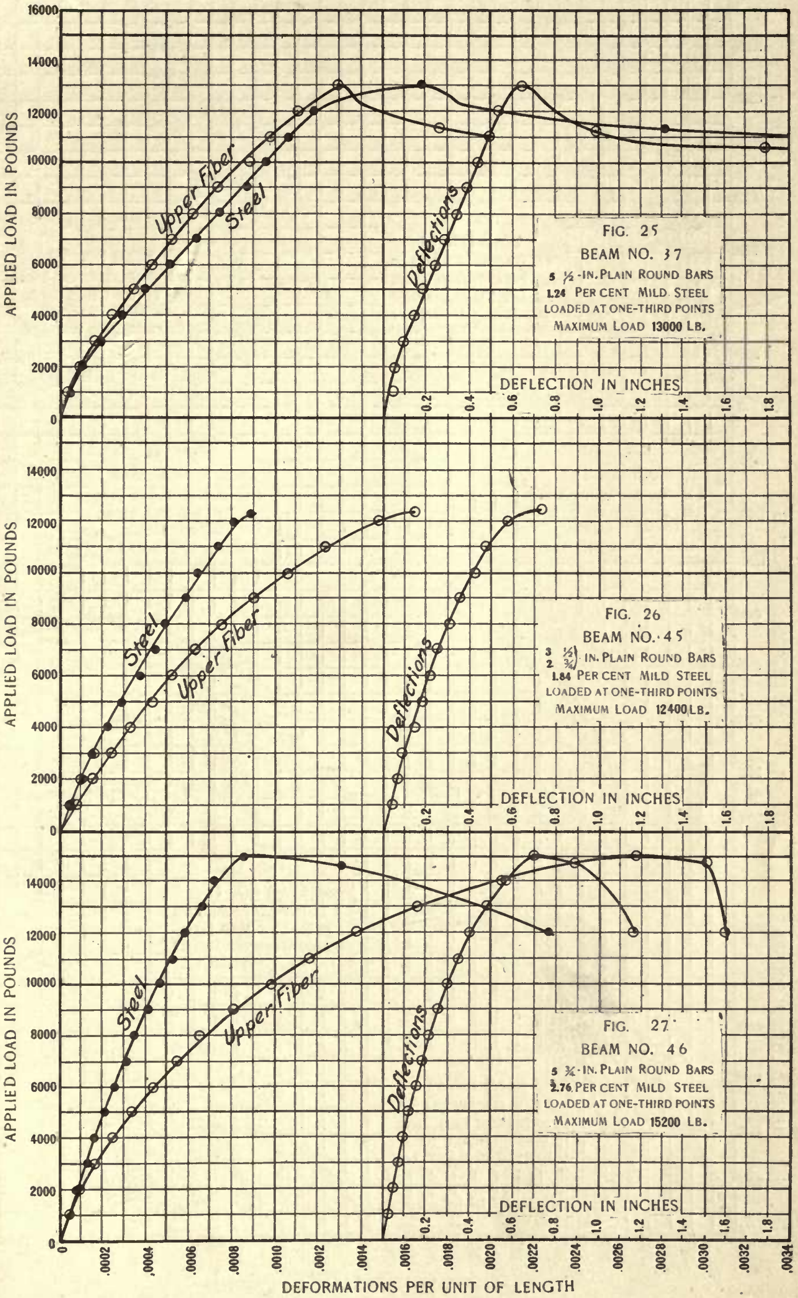
FIG. 25
BEAM NO. 37
5 ½-IN. PLAIN ROUND BARS
1.24 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 13000 LB.
FIG. 26
BEAM NO. 45
3 ½ 2 ¾ IN. PLAIN ROUND BARS
1.84 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 12400 LB.
FIG. 27
BEAM NO. 46
5 ¾-IN. PLAIN ROUND BARS
2.76 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 15200 LB.
Upper Fiber
Steel
Deflections
DEFLECTION IN INCHES
APPLIED LOAD IN POUNDS
DEFORMATIONS PER UNIT OF LENGTH

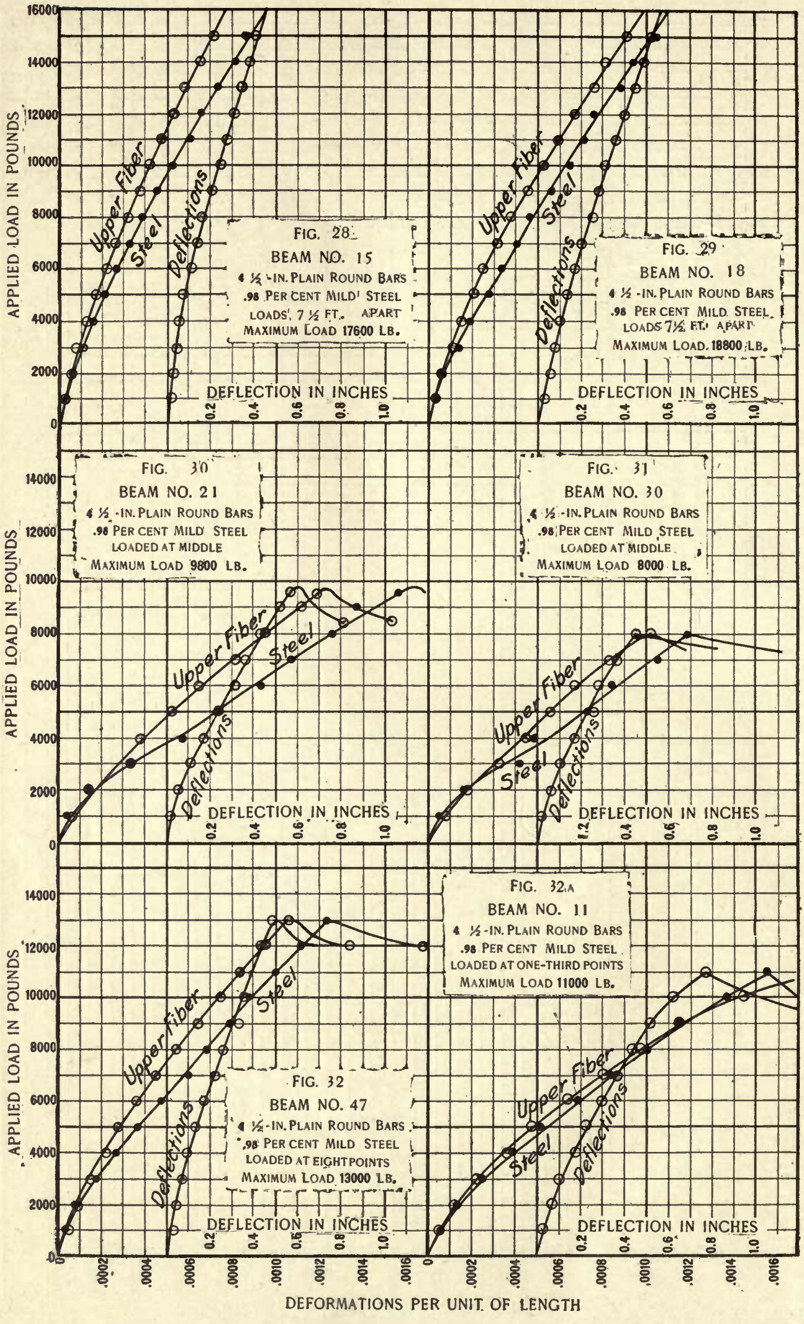
FIG. 28
BEAM NO. 15
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADS 7 ½ FT. APART
MAXIMUM LOAD 17600 LB.
FIG. 29
BEAM NO. 18
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADS 7½ FT. APART
MAXIMUM LOAD 18800 LB.
FIG. 30
BEAM NO. 21
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT MIDDLE
MAXIMUM LOAD 9800 LB.
FIG. 31
BEAM NO. 30
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT MIDDLE
MAXIMUM LOAD 8000 LB.
FIG. 32
BEAM NO. 47
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT EIGHT POINTS
MAXIMUM LOAD 13000 LB.
FIG. 32A
BEAM NO. 11
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 11000 LB.
Upper Fiber
Steel
Deflections
DEFLECTION IN INCHES
0.2
0.4
0.6
0.8
1.0
APPLIED LOAD IN POUNDS
16000
14000
12000
10000
8000
6000
4000
2000
0
0
.0002
.0004
.0006
.0008
.0010
.0012
.0014
.0016
DEFORMATIONS PER UNIT OF LENGTH

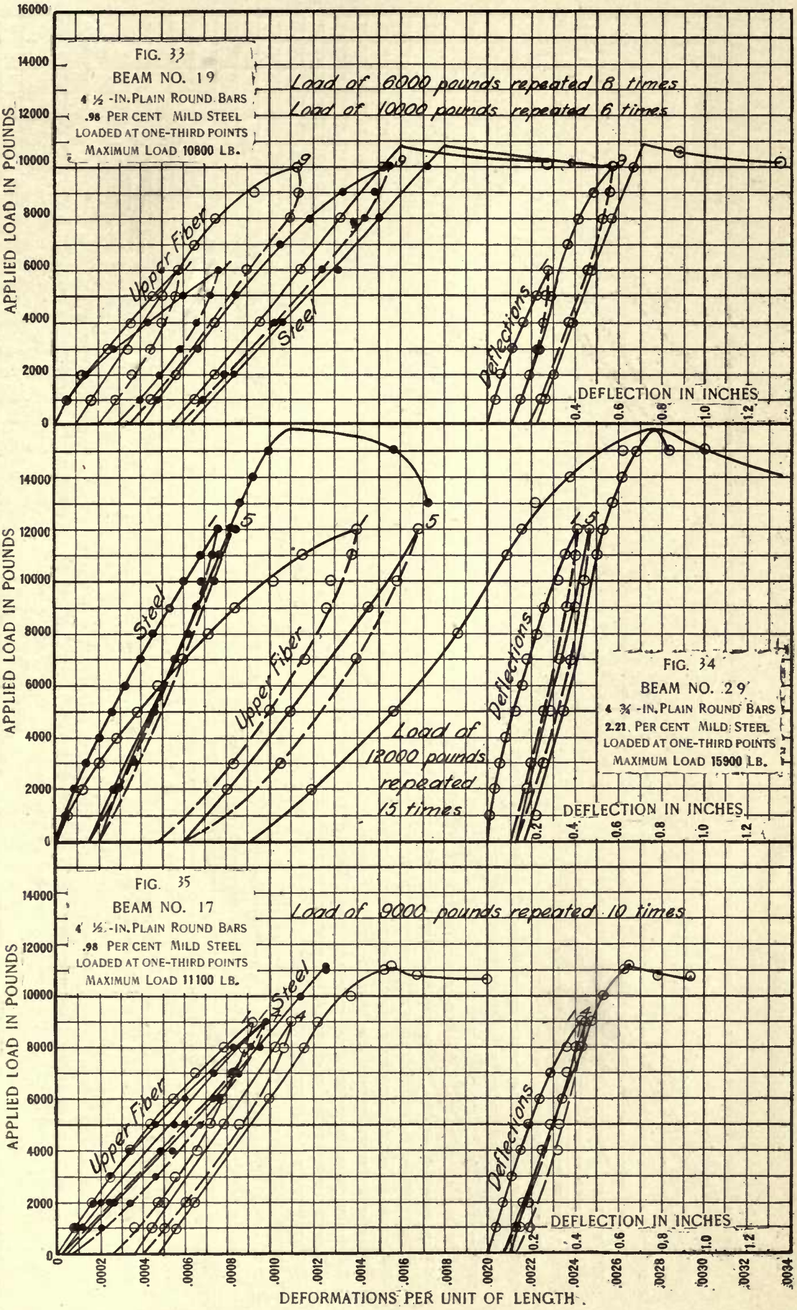
FIG. 33
BEAM NO. 19
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 10800 LB.
Load of 6000 pounds repeated 8 times
Load of 10800 pounds repeated 6 times
Upper Fiber
Steel
Deflections
DEFLECTION IN INCHES
FIG. 34
BEAM NO. 29
4 ¾-IN. PLAIN ROUND BARS
2.21 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 15900 LB.
Load of 12000 pounds repeated 15 times
FIG. 35
BEAM NO. 17
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 11100 LB.
Load of 9000 pounds repeated 10 times
APPLIED LOAD IN POUNDS
DEFORMATIONS PER UNIT OF LENGTH

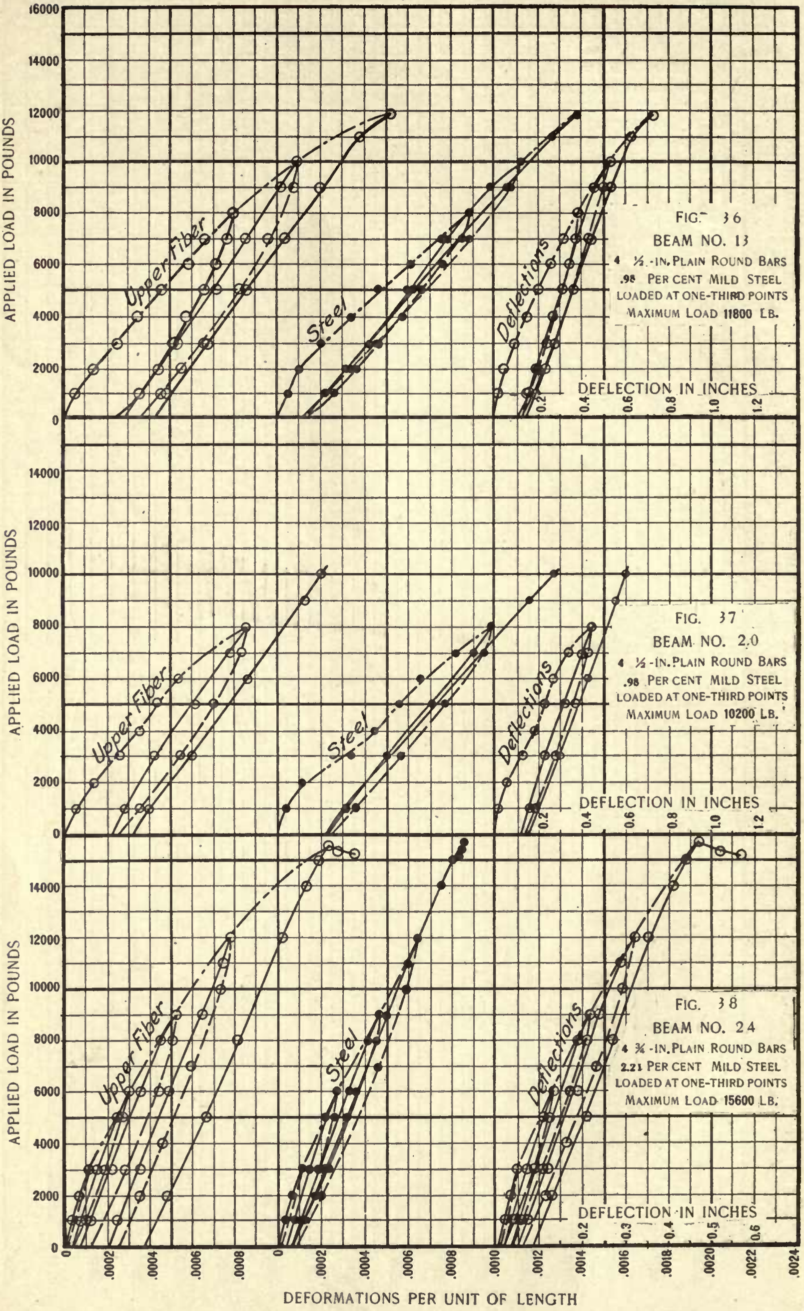
FIG. 36
BEAM NO. 13
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 11800 LB.
FIG. 37
BEAM NO. 20
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 10200 LB.
FIG. 38
BEAM NO. 24
4 ¾-IN. PLAIN ROUND BARS
2.21 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 15600 LB.
Upper Fiber
Steel
Deflections
DEFLECTION IN INCHES
APPLIED LOAD IN POUNDS
DEFORMATIONS PER UNIT OF LENGTH

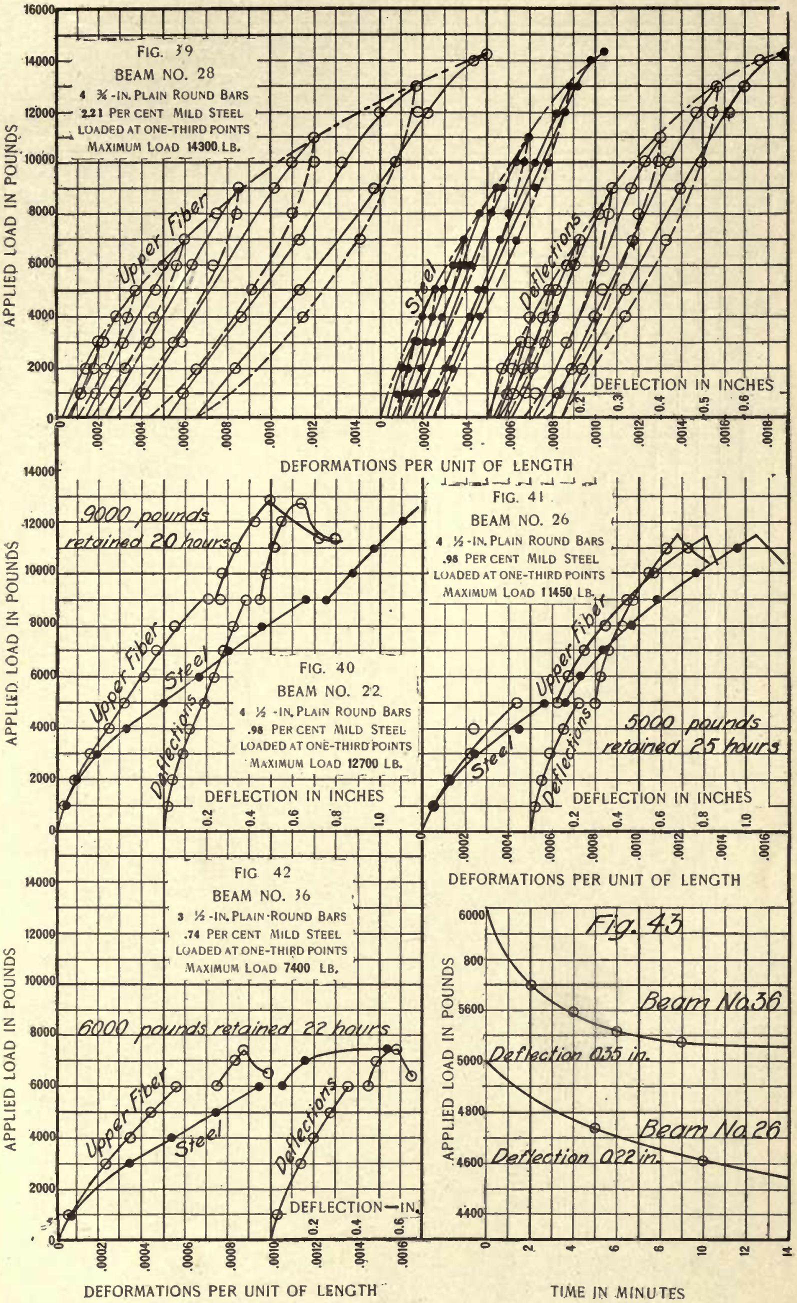
FIG. 39
BEAM NO. 28
4 ¾-IN. PLAIN ROUND BARS
2.21 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 14300 LB.
Upper Fiber
Steel
Deflections
DEFLECTION IN INCHES
APPLIED LOAD IN POUNDS
DEFORMATIONS PER UNIT OF LENGTH
FIG. 40
BEAM NO. 22
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 12700 LB.
9000 pounds retained 20 hours
FIG. 41
BEAM NO. 26
4 ½-IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 11450 LB.
5000 pounds retained 25 hours
FIG 42
BEAM NO. 36
3 ½-IN. PLAIN ROUND BARS
.74 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 7400 LB.
6000 pounds retained 22 hours
DEFLECTION—IN.
Fig. 43
Beam No. 36
Deflection .035 in.
Beam No. 26
Deflection .022 in.
TIME IN MINUTES

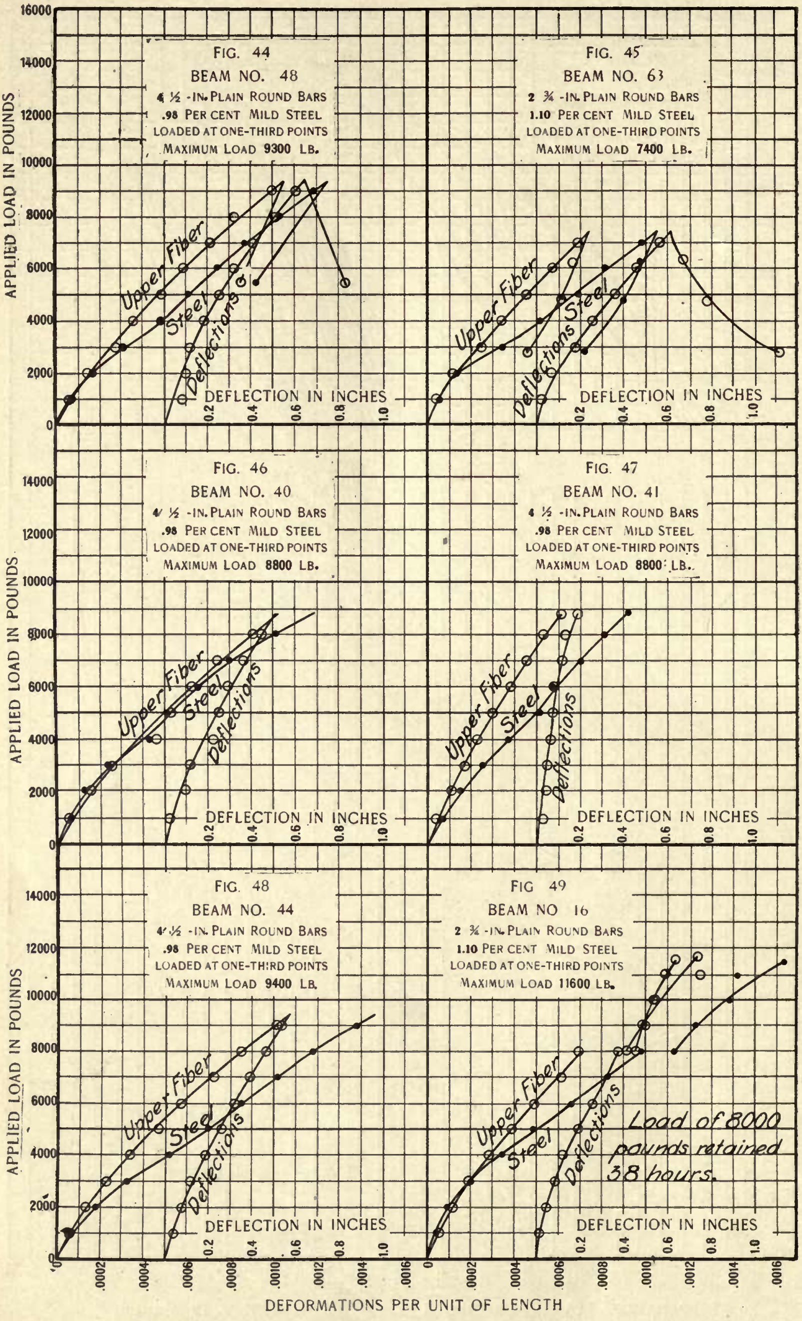
FIG. 44
BEAM NO. 48
4 ½ -IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 9300 LB.
FIG. 45
BEAM NO. 63
2 ¾ -IN. PLAIN ROUND BARS
1.10 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 7400 LB.
FIG. 46
BEAM NO. 40
4 ½ -IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 8800 LB.
FIG. 47
BEAM NO. 41
4 ½ -IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 8800 LB.
FIG. 48
BEAM NO. 44
4 ½ -IN. PLAIN ROUND BARS
.98 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 9400 LB.
FIG 49
BEAM NO 16
2 ¾ -IN. PLAIN ROUND BARS
1.10 PER CENT MILD STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 11600 LB.
Upper Fiber
Steel
Deflections
DEFLECTION IN INCHES
Load of 8000 pounds retained 38 hours.
APPLIED LOAD IN POUNDS
DEFORMATIONS PER UNIT OF LENGTH

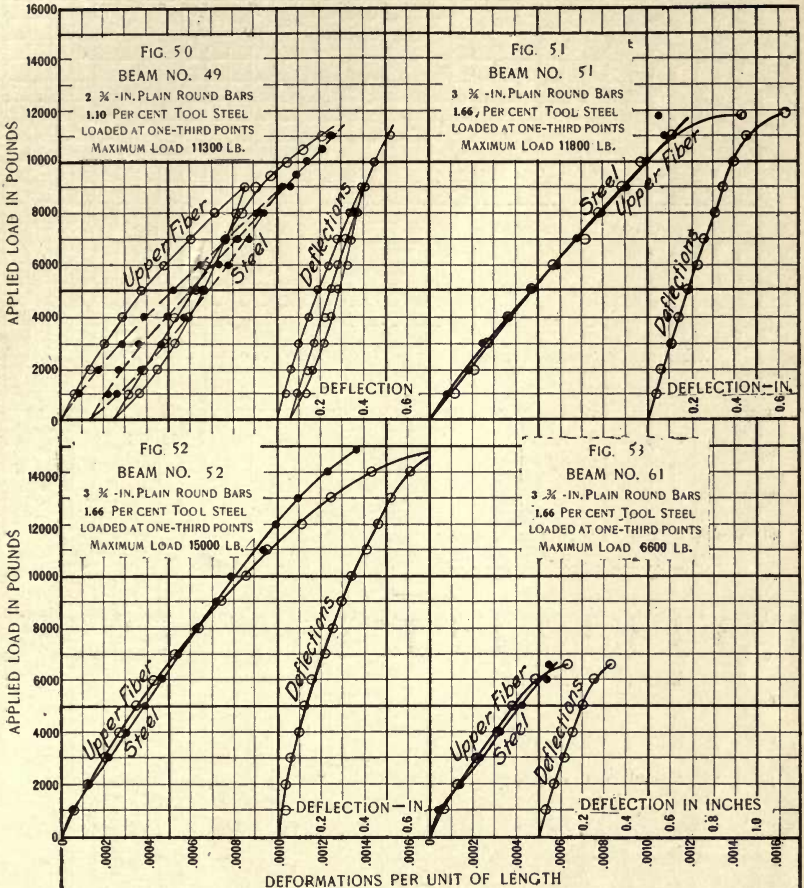
FIG. 50
BEAM NO. 49
2 ¾-IN. PLAIN ROUND BARS
1.10 PER CENT TOOL STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 11300 LB.
FIG. 51
BEAM NO. 51
3 ¾-IN. PLAIN ROUND BARS
1.66 PER CENT TOOL STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 11800 LB.
FIG. 52
BEAM NO. 52
3 ¾-IN. PLAIN ROUND BARS
1.66 PER CENT TOOL STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 15000 LB.
FIG. 53
BEAM NO. 61
3 ¾-IN. PLAIN ROUND BARS
1.66 PER CENT TOOL STEEL
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 6600 LB.
Upper Fiber
Steel
Deflections
DEFLECTION
DEFLECTION—IN.
DEFLECTION IN INCHES
APPLIED LOAD IN POUNDS
0
2000
4000
6000
8000
10000
12000
14000
16000
0.2
0.4
0.6
0.8
1.0
0
.0002
.0004
.0006
.0008
.0010
.0012
.0014
.0016
DEFORMATIONS PER UNIT OF LENGTH

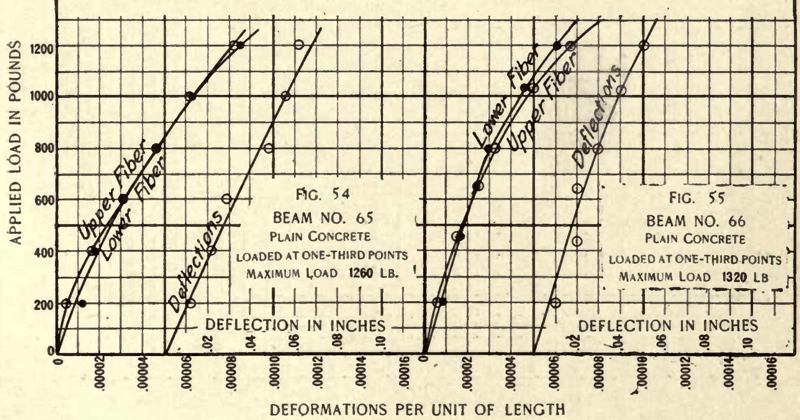
FIG. 54
BEAM NO. 65
PLAIN CONCRETE
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 1260 LB.
FIG. 55
BEAM NO. 66
PLAIN CONCRETE
LOADED AT ONE-THIRD POINTS
MAXIMUM LOAD 1320 LB
Upper Fiber
Lower Fiber
Deflections
DEFLECTION IN INCHES
APPLIED LOAD IN POUNDS
0
200
400
600
800
1000
1200
.02
.04
.06
.08
.10
0
.00002
.00004
.00006
.00008
.00010
.00012
.00014
.00016
DEFORMATIONS PER UNIT OF LENGTH

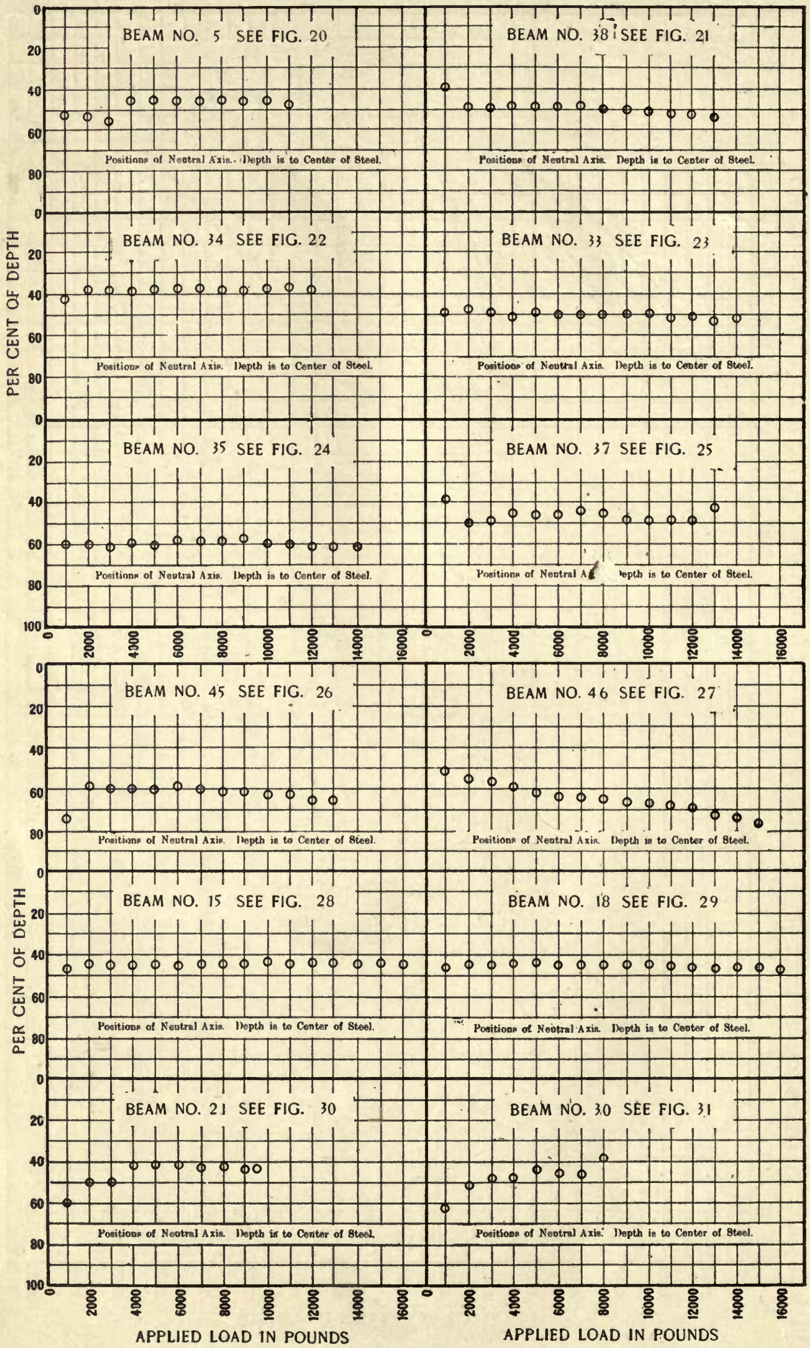
BEAM NO. 5 SEE FIG. 20
BEAM NO. 38 SEE FIG. 21
BEAM NO. 34 SEE FIG. 22
BEAM NO. 33 SEE FIG. 23
BEAM NO. 35 SEE FIG. 24
BEAM NO. 37 SEE FIG. 25
BEAM NO. 45 SEE FIG. 26
BEAM NO. 46 SEE FIG. 27
BEAM NO. 15 SEE FIG. 28
BEAM NO. 18 SEE FIG. 29
BEAM NO. 21 SEE FIG. 30
BEAM NO. 30 SEE FIG. 31
Positions of Neutral Axis. Depth is to Center of Steel.
PER CENT OF DEPTH
0
20
40
60
80
100
0
2000
4000
6000
8000
10000
12000
14000
16000
APPLIED LOAD IN POUNDS

FIG. 56

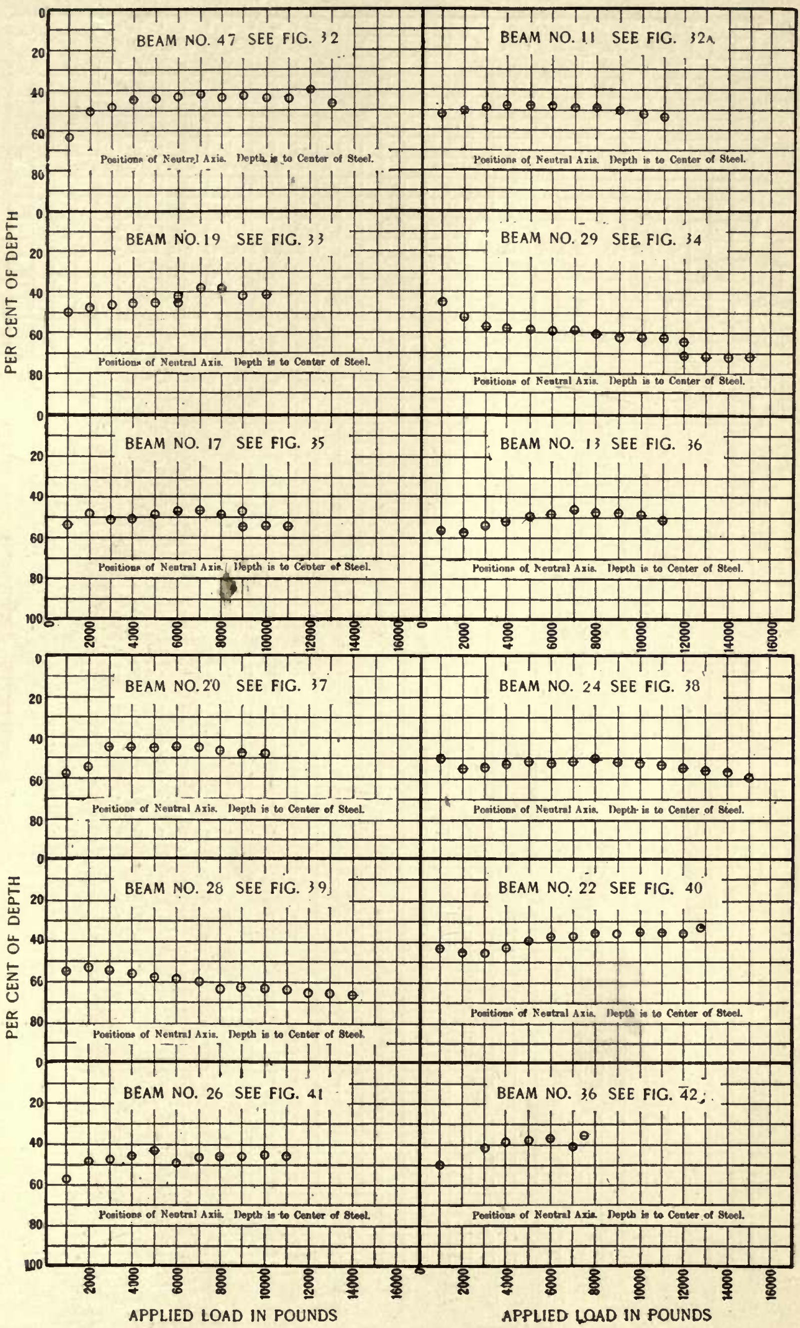
BEAM NO. 47 SEE FIG. 32
BEAM NO. 11 SEE FIG. 32A
BEAM NO. 19 SEE FIG. 33
BEAM NO. 29 SEE FIG. 34
BEAM NO. 17 SEE FIG. 35
BEAM NO. 13 SEE FIG. 36
BEAM NO. 20 SEE FIG. 37
BEAM NO. 24 SEE FIG. 38
BEAM NO. 28 SEE FIG. 39
BEAM NO. 22 SEE FIG. 40
BEAM NO. 26 SEE FIG. 41
BEAM NO. 36 SEE FIG. 42
Positions of Neutral Axis. Depth is to Center of Steel.
PER CENT OF DEPTH
APPLIED LOAD IN POUNDS

FIG. 57

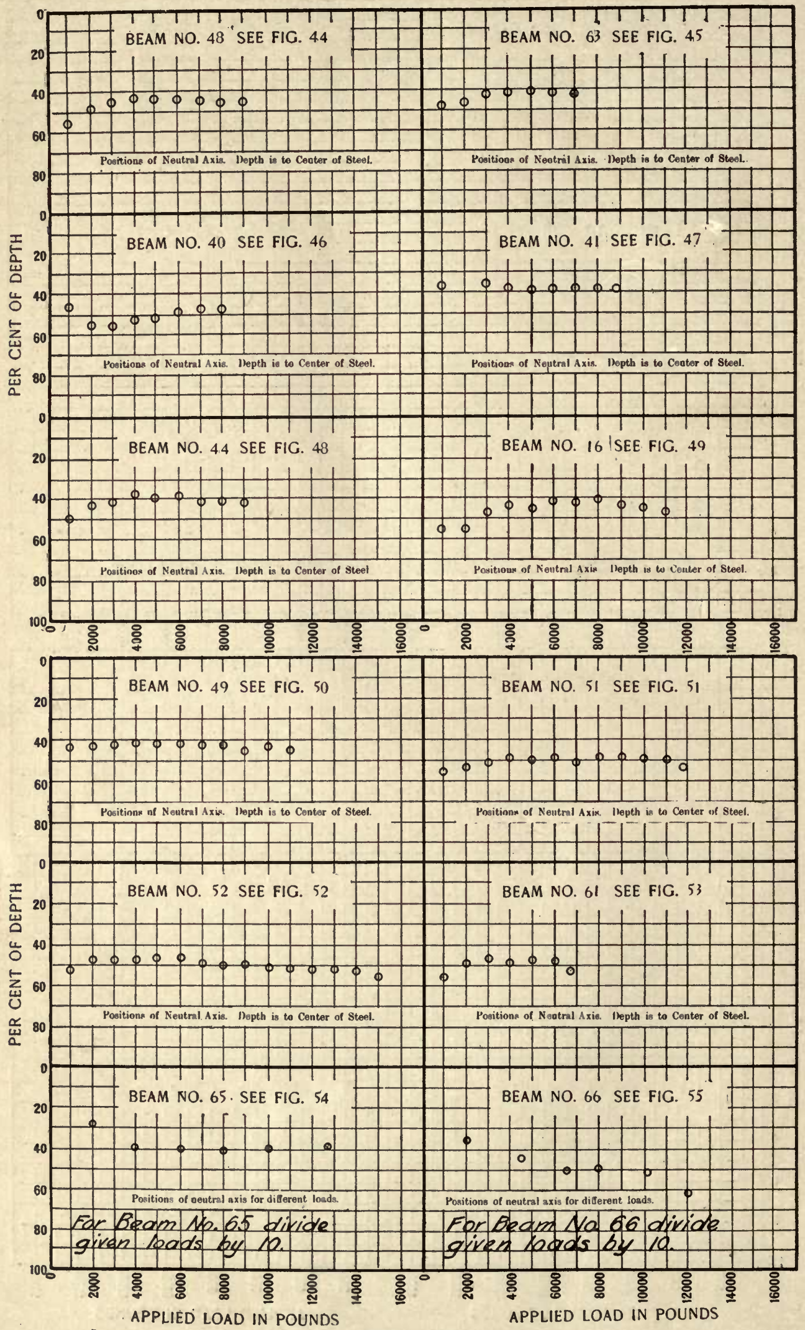
BEAM NO. 48 SEE FIG. 44
Positions of Neutral Axis. Depth is to Center of Steel.
BEAM NO. 63 SEE FIG. 45
Positions of Neutral Axis. Depth is to Center of Steel.
BEAM NO. 40 SEE FIG. 46
Positions of Neutral Axis. Depth is to Center of Steel.
BEAM NO. 41 SEE FIG. 47
Positions of Neutral Axis. Depth is to Center of Steel.
BEAM NO. 44 SEE FIG. 48
Positions of Neutral Axis. Depth is to Center of Steel
BEAM NO. 16 SEE FIG. 49
Positions of Neutral Axis Depth is to Center of Steel.
BEAM NO. 49 SEE FIG. 50
Positions of Neutral Axis. Depth is to Center of Steel.
BEAM NO. 51 SEE FIG. 51
Positions of Neutral Axis. Depth is to Center of Steel.
BEAM NO. 52 SEE FIG. 52
Positions of Neutral Axis. Depth is to Center of Steel.
BEAM NO. 61 SEE FIG. 53
Positions of Neutral Axis. Depth is to Center of Steel.
BEAM NO. 65 SEE FIG. 54
Positions of neutral axis for different loads.
For Beam No. 65 divide given loads by 10.
BEAM NO. 66 SEE FIG. 55
Positions of neutral axis for different loads.
For Beam No. 66 divide given loads by 10.
PER CENT OF DEPTH
0 20 40 60 80 100
0 2000 4000 6000 8000 10000 12000 14000 16000
APPLIED LOAD IN POUNDS

Fig. 58

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THE STATE UNIVERSITY

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