ON THE

# MEANS OF COMPARING

THE

# RESPECTIVE ADVANTAGES OF DIFFERENT

# LINES OF RAILWAY;

AND ON THE USE OF

# LOCOMOTIVE ENGINES.

TRANSLATED FROM THE FRENCH

OF M. NAVIER,

INGENIEUR EN CHEF DES PONTS ET CHAUSSÉES, PARIS.

BY JOHN MACNEILL, CIVIL ENGINEER,
M.R.I.A., F.R.A.S., &c.

LONDON:
ROAKE AND VARTY, 31, STRAND.
1836.

LONDON:
ROAKE AND VARTY, PRINTERS, 31, STRAND.

TO THE

RIGHT HON. SIR HENRY PARNELL, Bart.,

TREASURER OF THE NAVY, PAYMASTER OF THE FORCES,

&c. &c. &c.

THIS ATTEMPT TO INTRODUCE INTO THE ENGLISH LANGUAGE

A TREATISE OF HIGH REPUTATION ON THE CONTINENT,

IS MOST RESPECTFULLY DEDICATED

BY HIS OBLIGED

AND VERY GRATEFUL SERVANT,

*JOHN MACNEILL.*

*January* 1836.

THE little work of M. Navier, which I now venture to introduce to the public, was translated during the intervals of professional duties, and written out more as an exercise, for the purpose of impressing the subject on my memory, than with any view to publication. Though always anxious to ascertain and preserve the meaning of the author, I now feel conscious that it may not be so well expressed as it deserves, or as it might have been had I been able to have devoted more time to the undertaking. Those persons who may be desirous of accurate information on a subject of such increasing importance, (and it is for such I principally intend it,) will, I am sure, forget the inexpertness of the translator, provided he be faithful, in the degree they become acquainted with the ability of the author.

Two reasons may be given for supposing that the publication of a work of this kind might be patronized at the present moment.

1st. From the large capital already expended in the construction of Rail-ways, and the daily extension of the system through every part of the kingdom, it was thought that any work which elucidated its principles, or removed the difficulties which attend a theoretical investigation of its various properties, might be of considerable interest and utility.

2nd. From the small amount of our real knowledge of the numerous circumstances which affect the power of locomotive engines, particularly on slopes, it was felt that any addition to the existing stock of information might be acceptable. That this was necessary was very apparent during the parliamentary examination of several Railway bills in the last and preceding sessions, and that it is equally so at the present moment may be inferred from the more recent fact, that two eminent mathematicians, who have studied the subject, have arrived at very different conclusions, and have published opinions quite at variance with each other.

The only formulæ that I am aware of, which relate to this branch of practical science, are those of the Rev. William Adamson, published in his *Sketches of our Information on Rail-ways*, in 1826, and those of Mr. Tredgold, in his *Treatise on Railways*, in 1825; the former are mentioned by Mr. Wood, in his *Practical Treatise on Railways;* but he does not make use of them, and it is to be presumed that he would have done so, had there not been some practical objection to them. Those of Mr. Tredgold were constructed before locomotive engines had arrived at the perfection to which they have now attained, and are therefore by no means applicable to the present state of steam-power on Rail-ways.

It is true that in Mr. Wood's work there are two Tables, Nos. X and XI, pages 418 and 419, which purport to give the gross weight that an engine of given power can draw up slopes of different rates of inclination, with different velocities; if these tables be founded on actual experiment and practice, there can be no doubt but that the velocity which he has supposed to be a function of the weight of the train and rate of inclination of the slope, may be deduced from them; this I have endeavoured to do, and to apply the formulæ thus obtained to a practical example of two proposed lines of Rail-way.* The result would indicate that an engine carrying a constant load of forty tons, can traverse one of these lines in less time than the other, but this superiority consists in time only: we cannot predict that this line is in other respects preferable or more economical than the other. Various other circumstances besides velocity should enter into the calculation,

* See Appendix 1.

before we can decide with certainty which is the best line; these tables therefore are inadequate to the purpose, and we have not at present any work in the language that I am aware of which will enable us to arrive at a satisfactory comparison.

During a fruitless search upon this subject, I had the gratification of receiving from my esteemed friend, M. Mallet of Paris, Ingenieur en Chef des Ponts et Chaussées under the French Government, the work of M. Navier. On its perusal, it soon appeared evident that the principles and laws of the motion of locomotive engines were investigated and explained with that acuteness and ability which might be expected from a mathematician of such high reputation, — that he had opened a road by which future enquirers might enter upon this difficult and important question and push the investigation to the degree now demanded by the matured experience and highly improved apparatus of the present time: and that he had pointed out not only a proper mode of calculation, but also various circumstances, which, modifying the process and affecting the result, should be carefully attended to in conducting all future experiments on the motion of locomotive engines on inclined planes.

M. Navier's formulæ will enable us to approximate very nearly to a correct comparison between two lines of Rail-way, and if data derived from recent actual practice be substituted for the assumed quantities in these formulæ, there is every reason to believe that they will enable us to estimate with great accuracy the absolute cost of conveying a given weight of merchandise or number of passengers over any line of Rail-way, a point apparently not hitherto attended to, but which must be one of the principal facts ascertained before any correct conclusion can be come to, as to the advantage which a line of Rail-way is likely to afford to the public, or before it can be compared with another line of Rail-way in the same direction.

The great advantage which Rail-ways possess over every other mode of transport, consists principally in the rapidity with which passengers may be conveyed from one town to another; and in some cases merchandise, for the latter can be carried cheaper by canals or even on roads, when a velocity not exceeding two or three miles an hour is considered sufficient. A Rail-way intended for the quick transit of passengers, should in my opinion be differently constructed from one intended for the carriage of merchandise, and both should be varied to suit the particular circumstances of the country they are intended to pass through, and the extent and description of the traffic expected to pass over them. There is also a limit which should be observed in forming the slopes on a line of Rail-way, but no general rule will suffice; for as the expense of conveying a passenger or a ton of goods along the line will depend on the original outlay in the construction of the works, the cuttings and embankments should be so managed, and the slopes so regulated, that the first cost of the works and the engine-power should be in proportion to the number of tons and passengers expected to traverse the line, and the importance attached to a rapid transit over it.

In discussing this subject, Mr. Tredgold states, in his Treatise already referred to, page 157, "It will readily appear from these equations that it is much less expensive to nearly follow the undulations of the surface, than to make either deep cutting or embankment beyond those limits, which are easily determined by half an hour's labour, in applying the equations to the case under consideration. We have inserted numbers and shown how to reduce it to the case of a road of average expense; and if a few examples be added, it will assist in removing those extravagant notions respecting cutting and embanking, which sink the capital of the country in unprofitable speculation."

It is to be hoped that this important part of the subject will not be suffered to remain much longer without a thorough examination, founded on actual observation of the working of locomotive engines on every possible degree of slope, ascending and descending, and under every circumstance that is likely to take place in practice. But as practical men have seldom the time, or the mathematical acquirements requisite for such an intricate investigation, a series of facts should be registered at all the different Rail-way establishments in the empire. These should embrace, 1st, the cost of the engines, their weight, a daily journal of their repairs, and the number of miles travelled; the weight carried, the water and fuel consumed; the pressure of the steam on each particular part of the line, &c. &c. If such facts were collected, and then given to some of our able mathematicians, there can be no doubt of their being able to furnish formulæ that would be of the utmost importance to the country; or it might be better to employ such persons to design and carry on, under their own superintendence, sets of experiments in any way they might think most adapted to obtain the information desired. Such persons might be easily found amongst our mathematical professors, and the expense of the experiments should be defrayed by the Rail-way Companies in some certain proportion.

Some of the tables and formulæ in M. Navier's work, and all the weights and measures, I have given both in French and English; and I have transferred the notes which are at the bottom of the pages in the original to the end of this work, an arrangement which appears to me to facilitate the clear understanding of the author, by allowing his chain of reasoning to be pursued unbroken.

The note which I have added on the different modes of estimating forces, and on the meaning of the expression *vis-viva* (*la force vive*) is extracted from the valuable work of Mr. Whewell *on Dynamics*, and from the *Elemens de Mécanique, par M. Boucharlat*, two authors distinguished for their works on mathematical and mechanical subjects. The note may, perhaps, by some be considered irrelevant or unnecessary; but I have thought it might be acceptable to others, for the terms *momentum* and *vis-viva* have been sometimes confounded with each other.

---

P.S.—I have rendered the word *pente* by *slope* in preference to *inclination*, *inclined plane*, or *gradient*, considering the two former, though generally used, as improper expressions, and the latter, to say the least of it, as having so very little to recommend it, that I hope it will have an extremely short existence in our nomenclature.

Judicious and appropriate terms are of the greatest importance in speaking and writing on scientific subjects, particularly where technical expressions must of necessity be introduced.*

A gentleman of high literary acquirements, to whom I applied, and who has taken the trouble to consider the subject, has suggested the term *clivity* as one that is of more legitimate etymology than *gradient*, and more appropriate than either *slope*, *inclined plane*, or *inclination*. I regret that I was not in possession of this term before I commenced the translation, the words *acclivity*, *declivity*, which may be so regularly derived from it, would have enabled me to have given the sense of the original with greater perspicuity.

* "It is highly desirable to keep scientific knowledge precise, and always to use the same terms in the same sense."—*Discourse on Natural Theology, by Henry Lord Brougham.*

# ERRATA.

Page 22, line 21, *for* $\frac{0.005\ P\eta}{}$ *read* $\frac{0.005\ P\eta}{i}$.

26 — 2, *for* 611.1 *read* 620.1.

27 — 5, *for* 637 *read* 641.

34 — 3, *for* $\pi\tau$ *read* $\pi r$.

37 — 9, *for* velosity *read* velocity.

38 — 17, *for* 00.21148 *read* 0.021148.

42 last line, *for* 81163 *read* 81272.

45 last line but one, *for* 138 *read* 238.

55 and 56 in Note*, *for* $57_0$ Farenheit *read* 25° Farenheit.

56, lines 5 and 12, and p. 57, last line but one, omit the initial C.

ON THE

# COMPARISON OF THE RESPECTIVE ADVANTAGES

OF

# DIFFERENT LINES OF RAIL-WAY,

# AND ON THE USE OF LOCOMOTIVE ENGINES.

---

1. *General ideas relative to the establishment of Rail-ways.*

2. *Principal elements in the comparison of different lines of Rail-ways.*

3. *Determination of the power required to draw a given train over a given Rail-way.*

4. *Determination of the weight of the train which can be drawn on a given Rail-way by a Locomotive Engine of a given power.*

5. *Examination of the uniform motion of the train on the different ascending or descending slopes which may form part of a Rail-way.*

6. *Examination of the motion of the train in passing from one slope to another.*

7. *Summary. Comparative estimate of the cost of transit on different lines of Rail-way.*

The following observations are partly taken from the course prepared for the students of the Board of Bridges and Highways *(Ponts et Chaussées,)* in France. It has been thought that they would perhaps be interesting to such engineers as may now be employed on Rail-way projects; and that they might throw some light upon those difficult and complicated questions, the examination of which has arisen from the introduction of these works.

## 1. *General ideas relative to the establishment of Rail-ways.*

Rail-ways are generally considered under two principal heads. 1st, As affording to commerce a more economical mode of transport; 2nd, as giving the means of carrying goods, and more especially passengers, with considerable speed; the mean rate of which may be stated, from what has taken place in England, at about eight leagues* an hour.

This great rapidity of transit being the characteristic property of Rail-ways, without which they would lose their principal advantage, and not produce the results that might be expected from them, it has been considered necessary to

* Or about 20 miles, a French post league being 2,000 toises, or $28\frac{1}{4}$ to a degree.

employ almost exclusively locomotive engines as the motive power. This system, moreover, presents other important advantages in its simplicity; and in being able, after the Rail-way is completed, to increase gradually the number of engines as the demands of commerce require it, and the number and the power of the apparatus may always be proportioned to the work which it really has to do, without the danger of incurring useless preliminary expenses, and with the advantage of profiting by improvements as they occur in the progress of the arts. One of the conditions therefore which must not be departed from in laying out great lines of Rail-way, is that these lines may be traversed along their whole extent by locomotive engines, and, as much as possible, in order to avoid interruptions and delays, that the same engine draw throughout the same train.

The preceding condition shows that very gentle slopes only can be admitted on Rail-ways, and such that the differences which exist between the powers required to draw the train on different parts of the line, shall not affect the working of the engines, or occasion any loss of power.

The mean tractive power* required on an hori-

* In England at present this is taken at 9lb. per ton, that is $\frac{2240}{9}$, 249 or the 250 part of the weight.—*Tr.*

zontal part of the Rail-way may be estimated at about the 200th part of the weight of the train, although some experiments, made under favourable circumstances, have given results rather less.

The weight of the train being taken as 1, this tractive power is represented by 0.005, and each millimetre in a metre (= 1 in 1000) of ascending slope, increases this number by .001, so that in a slope ascending 5 millimetres in a metre, (=1 in 200) for example, the tractive force would be represented by 0.01; that is to say, that it would be double of what it is on an horizontal part. But on a descending slope, on the contrary, each millimetre in a metre of slope diminishes the tractive power required by 0.001; so that the power required on a slope descending 5 millimetres in a metre (=1 in 200) becomes nothing.

It is evident from this, that the slopes on Rail-ways must be very gentle, and it may be said in general that the more the construction of a road or Rail-way is improved so as to diminish the tractive power required, the more is it necessary, in order to profit by the advantage thus obtained, to reduce the slopes.

But there exists a special reason for not forming, if it be possible, slopes above 1 in 200; since upon slopes of greater inclination the action of gravity becoming greater than the resistance arising from friction, the motion of the train becomes accelerated.

On account of the danger which this acceleration causes, it is necessary to prevent it by particular contrivances; and even to cause the train to descend at a moderate velocity. It is necessary, therefore, to destroy that portion of the action of gravity which produces acceleration, and which exceeds the tractive power. If the descending slope, for example, be 7 millimetres per metre, (1 in 143,) so that the gravity tends to cause the train to descend with a force represented by 0.007, $(=\frac{1}{143},)$ we employ only a part of this force, represented by 0.005, $(=\frac{1}{200},)$ which balances the tractive power, and we are obliged to destroy, by the use of breaks, or other means, the part represented by 0.002, $(=\frac{1}{500},)$ which produces the acceleration. It results from this, that a part of the power which is the effect of the descent is lost. In general, descents on a Rail-way will only produce a saving of power proportionate to the height from which the train has descended, when the descending slopes do not exceed in inclination 5 millimetres per metre, (=1 in 200,) the draught being supposed, as already stated, to be equal to the five thousandth $(\frac{1}{200})$ part of the weight of the train.

These considerations point out in a general way the view in which the establishment of Rail-ways has been considered. The possibility of establishing a mode of conveyance exceedingly rapid, the use of locomotive machines for the tractive power, the reducing of the slopes to the least inclination that is possible, and as much as can be to inclinations less than 0.005, (1 in 200,) have appeared to be considered the most essential conditions. It is superfluous to remark, besides, that the condition of diminishing, as much as possible, the time of transit between two given points, requires that we should endeavour to reduce also the length of the Rail-way which is proposed to be constructed between these two points. It would be committing a great error to suppose we may lengthen the line because the velocity of transport over it is great. The same principle which rendered the establishment of a Rail-way desirable, in order to obtain a mode of transport quicker than any other, requires that the shortest lines be sought after, and even to prefer them when sometimes they appear to be disadvantageous in other respects.

The setting out of the line on the ground, when the country has been surveyed and laid down by plans and sections, does not require any new principles.

Suppose that it is intended to unite by a Rail-way the point A and the point B, which is more elevated than A; the most advantageous direction would evidently be in the right line A B; having one uniform slope. It is this line which ought to be obtained, or the nearest practicable one to it, both horizontally and vertically. If a uniform slope is impracticable, or if it requires too great a deviation from the direct line, it is necessary at least to endeavour to rise progressively from A to B, and never to ascend where we must descend again, and *vice-versâ*.

If such a line cannot be obtained, and there exists between the points A and B one or more lines of ridges and valleys which must be crossed, it is always necessary to endeavour to rise or to fall as little as possible, consequently to endeavour to cut the ridges in those points where the height is a minimum, and the valleys in the points where the height is a maximum, without lengthening the line too much. And it is very apparent that we shall generally be led to the ridge lines by following the secondary valleys which cut them, and which are always lines of less slopes. But if these lines of less slopes are still found to be too steep for a Rail-way, it will then become necessary to pass through below the ridge by means of a tunnel instead of ascend ing to the summit.

It often occurs that between the two given points A and B, several lines are to be found which appear to agree with the principles here laid down, and also that some one of these lines presents advantages of some other kind, such as that of passing near a considerable town, or through a district where important manufactories are carried on.

The choice to be made of these different lines, and which should always be founded upon considerations of the general interests of the country, may be difficult. We shall endeavour to explain some of the principal points which should influence a decision of this kind.

2. *Principal elements in the comparison of different lines of Rail-ways.*

The interest of the country is, in this respect; 1st, The establishment of a very rapid mode of transport,—a consideration which should give a preference to the shortest lines, the velocity being supposed to be the same on all; 2nd, The increase of wealth. The construction of a Railway, like that of a canal, or new road, is favourable to the advancement of wealth, in the first place, because the actual expense of transport in this direction is diminished; and in the second place, because this diminution in the cost of transport increases the value of the neighbouring properties, facilitates the establishment of new works, and increases production. The first of these two effects, that is to say, the diminution obtained on the actual cost of transport, is the cause of the second; so that this diminution is the principal circumstance, and that which should be especially considered.

We should even say that the rate of reduction which is obtained upon the actual cost of transport, by the establishment of a new communication, is almost the only circumstance which should be thought of, if it were not necessary to consider also the quantity of goods which is carried, or may be carried hereafter, in this direction; for it is evident that it may be less advantageous to the country to produce a great economy in the cost of transport upon a line where there is little to carry, and more advantageous to produce a less economy upon a line where a large quantity of merchandise is carried. It is therefore generally necessary to take into consideration, in the comparison of different lines, the quantity of traffic which may be established on each, and even the increase in the value of properties, and the developement of production, to which the establishment of these lines may give rise respectively, according to the nature of the countries which they traverse.

We shall not here undertake to go minutely into the influence of these last elements of the question, which rather belongs to statistics and to political economy, and with respect to which we cannot offer at present any precise opinions; we shall therefore confine ourselves to the consideration of the reduction which the establishment of a Rail-way can effect upon the actual cost of transport, a most important consideration, to which, as already remarked, it is always necessary to attend. This will form in every case the principal element of the comparison which is the subject of inquiry, and often lead to determinations purely geometrical or mechanical, and consequently exempt from arbitrary deductions.

The cost of transport upon a Rail-way, as upon a road or canal, depends on two principal points, which it is necessary to distinguish and consider separately. The first of these is the expense of constructing the Rail-way, and the second is the expense of conveying the goods on the Rail-way when it is constructed.

The expense of the construction of the Rail-way is independent of the quantity of merchandise or of passengers that will pass over it. The expense of transport, properly speaking, upon the Rail-way supposed to be constructed, depends, on the contrary, upon the quantity of merchandise or of passengers; that is to say, upon the tonnage ; all other things being equal, the expense will evidently be proportional to the tonnage.

As to the secondary expenses, such as the annual cost of repairs and management, it may be said that they are partly in proportion to the expense of the construction, and partly to the amount of tonnage.

We may therefore admit, without falling into any serious error, that the annual cost of transport on a Rail-way, is in all cases formed of two parts, the one proportional to the expense of the construction of the way, and the other proportional to the amount of tonnage.

We should also observe, that the cost of transport of one ton of merchandise cannot be specified, unless the number of tons which shall be carried annually from one extremity of the line to the other be known.

Suppose, for example, that we know, in one case, that, the road being constructed, the part of the expense which is proportional to the tonnage will amount to 0.30 fr. per ton, per league ; or [=1*d*,] per ton, per mile ; and in the other case, that the part of the expense which is proportional to the cost of construction, and which is independent of the tonnage, represents a capital of 1,200,000 fr. [=£48,000,] or an annual expense of 60,000 fr. per league, [£800 per mile.]

This annual expense, if the Rail-way has a traffic of 100,000 tons per annum, will amount to 0.60 fr. for each, and if the traffic be 200,000 tons, it will amount for each to 0.30 fr.; so that in the first case, the total cost of the transport of one ton over one league, is 0.90 fr., or threepence per ton, per mile; and in the second case, it is only 0.60 fr. per league, or twopence per ton, per mile.

The knowledge of the expense of construction of a Rail-way, and even, to a certain extent, that of the expense of repairs and management, are subjects which do not differ from those in which engineers are generally employed, and which do not require any particular consideration.

The knowledge of the expense of conveyance, properly speaking, requires an investigation similar to that which is made in the arts, for ascertaining the price of works executed by machines. It depends upon mechanical principles, to which we shall particularly apply ourselves.

### 3. *Determination of the power required to draw a given train over a given line of Rail-way.*

Let us observe, that upon an horizontal line the power required to draw a given weight, is considered as being equal to almost the two hundredth part of this weight, a result which we shall here suppose, (conformable to what is generally admitted,) to be independent of the absolute velocity of transit, although there is reason to believe that the tractive power increases with the velocity. We conclude from this, that in order to transport, with any velocity whatever, constant or variable, a weight P, to the distance represented by $a$, on an horizontal line, it is necessary to employ a power represented by $\frac{P}{200}\,a$; that is to say, the power necessary to raise the weight P to the height $\frac{a}{200}$. Thus, for example, it is the same thing to transport the weight P, to the distance of a league of 4,000 metres, [= 4374 yards,] upon an horizontal Railway, as to raise the same weight to a vertical height of 20 metres, [=21.87] yards.

If the transit had been over an ascending slope, forming with the horizon a very small angle $i$; in such a way that $i$ represents the rise on a unit of the length; the tractive power becomes P (0.005 + $i$,) and the power required to transport the weight P, to the distance $a$, becomes

$$P\,(0.005 + i)\,a\,;$$

and as the term $i\,a$ represents the vertical height to which the weight has been raised when it has passed over the distance $a$, we see that the power expended is here equal to what had been employed to transport the weight over a horizontal line added to that which is necessary to raise the weight to the vertical height to which it has really ascended.

If the transit had been over a descending slope, of which the inclination is in a similar manner represented by $i$; it is clear that the tractive power is reduced to $P\,(0.005 - i)$; and the power expended to travel over the distance $a$, to

$$P\,(0.005 - i)\,a.$$

From whence it results that the power expended is in this case obtained by subtracting from the power $0.005\,P\,a$, which is required to draw the train over the horizontal line, that which is represented by the descent of the weight from the vertical height $i\,a$, from which it has actually descended.

But it must be observed, that in the statement which has just been made, negative values of the quantity $P\,(0.005 - i)\,a$ must not be admitted, which would take place if the fraction $i$ was greater than 0.005; because the trains are not allowed to accelerate their motion, which they would have a tendency to do, and to acquire a velocity greater than would naturally result from their descent upon a slope more rapid than 0.005. Whenever the descending slopes are greater than $\frac{1}{200}$, a negative value of P (0.005—$i$) $a$ will be the result, in such cases zero must be substituted for this value.

These observations lead to a rule, exceedingly simple, for estimating the power which is required to move a train over a line of Railway. If we suppose that the same locomotive engine draws the same train over the whole line, and that there are not descending slopes more rapid than $\frac{1}{200}$, the power required to effect the transport of the weight P, from the point M to the point N, representing by A the length of the line MN, and by H the height of the point N above the point M, will evidently be represented by

$$P\left(\frac{A}{200}+H\right) \text{ in the direction M N, and}$$

$$P\left(\frac{A}{200}-H\right) \text{ in the direction N M,}$$

whatever be the distribution of the ascending and descending slopes, which the line may present. Consequently if the line MN were horizontal, or, more generally, if the two extremes M and N were at the same level, the transport of the weight P, from one extremity to the other, exactly equals the elevation of this weight to a height equal to the 200th part of the distance MN.

When the points M and N are not on the same level, it is sufficient to add or subtract from this height their difference of level, according as the transit is towards one extremity or the other.

The preceding rule is rigorously correct, when we suppose that the train starts from a state of rest, at one extremity of the line, and arrives at the other when the velocity is reduced to zero, and also considering the power as sufficient to effect the transit without regarding what is uselessly consumed by the friction, and other deteriorating causes, inherent in the working of the machinery of a locomotive engine.

This rule is, in fact, only a version of the general principle of the preservation of the *vis viva*,* from which it is known that a heavy body, made to pass over any given line whatever, always moves from one extremity to the other of this line, acquiring or losing a velocity due to the height from which it has ascended or descended, and in such a way that the body constantly returns to the same velocity, when it moves through points situated in the same horizontal

* See note A at end.

plane. We conclude from this that the length of the line remaining the same, the amouut of the quantity of power consumed to effect the transit depends entirely upon the length of the line, and the difference of the level of its extreme points.

But if, as it is convenient to do, we wish to value not only the power required to effect the transit, but also the total quantity of power really produced by the locomotive engine, we should observe that when the train, after being elevated to a certain height in passing up an ascending slope, descends an equal height in passing afterwards down a descending one, the descent restores the power which had been employed to raise the weight of the train; but not that which had been consumed by the friction produced whilst the machine was exerting the power necessary to effect the elevation. Whence it results, that whenever there is a *useless ascent*, that is to say, whenever an elevation is ascended, which must afterwards be descended, or that an elevation is ascended, which was before descended, it is necessary to keep in view the fraction of the power which is required to raise the train to the height in question, representing the effect of friction, and other resistances of the locomotive engine. By paying attention to this consideration, we perceive that the form of the section between the two extremes of the line is not unimportant, since there is a loss of power whenever there is a useless ascent.

Let us suppose that the point N is higher than the point M, and that in the transit in the direction M N, there will be a useless expenditure of power every time there is a descent. If, on the contrary, the point N is lower than the point M, there will be a useless expenditure of power whenever there is a rise. Whence we may conclude, that we should always endeavour to avoid adopting an alternation of ascending or descending slopes, but manage so that the line should ascend or descend progressively from one extremity of the road to the other. As to the manner of estimating the effect of the *useless rises*, it will always be easy to detect them by an inspection of the section. It will be sufficient for this purpose, to mark the points of *maximum* and *minimum* height. Let $h$ represent the sum of these *useless rises*. The total power expended by the locomotive engine being represented by unity, we may represent by $\mu$ the portion of this power which is not used to effect the transit on the elevation of the train, and which is consumed uselessly by the friction and other resistances to which the machinery is subject. It is evident that the part of the total power employed to effect the elevation of the weight P to the height

$h$, which is not restored by the descents, is expressed by $\mu$ P $h$. Thus, in adding this quantity to the expression which has been given before, the formula

$$P\,(0.005\ A \pm H + \mu\, h)$$

will represent for different lines of Rail-way, a number proportional to the quantity of power really produced to effect the transport of the weight P. We should take the sign + or —, according as the point of arrival is higher or lower than the point of departure.

The preceding result may be expressed by saying, that the quantity of power expended to produce the transit over any line whatever is equal to that which shall be necessary to elevate the weight of the train to a height expressed by

$$0.005\ A \pm H + \mu\, h\ ; \quad \ldots\ldots \quad (1)$$

that is to say, to a height equal to the 200th part of the length of the line, increased or diminished by the difference of the level of the extremes according to the direction of the transit, to which is to be added the sum of the *useless rises*, multiplied by the fraction expressing that part of the total quantity of power furnished by the steam engine consumed uselessly by friction, and which is not employed to effect the transit.

This result is also limited by the two hypotheses which we have before stated, that is to say, 1st, that there is no part of the descending slope more rapid than 0.005; 2ndly, that the same locomotive engine draws the train over the whole line. It is therefore necessary to consider the particular cases which do not coincide with these conditions.

1st. If there is in some part of the line a descending slope $i$, more rapid than 0.005, and of which the vertical height is $\eta$, we may remark, agreeably to what has been said before, that we only make use of the fraction 0.005 of the action of gravity in descending the slope, and that the part $i-0.005$ of this action is entirely lost, since we cannot allow the train to acquire the velocity which this would tend to give it.

The descent of such a slope cannot therefore be considered as producing the power $P\eta$ corresponding to this descent, but only to a power equal to $\frac{0.005 \, . \, P\eta}{i}$; and the quantity of power $\left(\frac{i-0.005}{i}\right) P\eta$ is lost. We therefore conclude that after having expressed as above by formula (1), the height to which the weight of the train is raised by the power which produces the transit, it is necessary to add to this height the quantity

$$\frac{-0.005}{i}\eta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

whenever there is a descending slope $i$ more rapid than $\frac{1}{200}$, the difference of level of the two extremes of this slope being $\eta$.

If there is upon the line an ascending slope so steep as to require the use of an auxiliary engine, it is necessary, in addition to the quantity of power already determined, agreeably to what has been said above, to add the quantity of power necessary to raise the auxiliary engine from the lower to the upper extremity of the slope. Let us designate by $a$ the length of the slope, and by $\eta$ its vertical height. Let us further suppose, that the weight of the auxiliary engine is equal to the fraction $k$, of the total weight of the train represented above by P. It is evident, that the quantity of power necessary to transport the auxiliary engine will be represented by $k$ P $(0.005.\ a+\eta)$. Whence we conclude that we arrive at this quantity of power, by adding to the height expressed by the formula (1) the quantity

$$k\,(0.005.\,a+\eta). \quad . \quad . \quad . \quad . \quad (3)$$

Further, there will not require any deduction to be made, for the descent of the machine when it returns along the slope in a contrary direction, the quantity of power which this descent would produce being entirely lost.

We can always, by means of these principles, easily estimate the amount of power necessary to effect the transit of a train from one extremity to the other of a Rail-way, an amount the value of which is thus expressed in extremely simple terms. However evident these principles may be, it will not, perhaps, be useless to give an example of the calculations to which they refer.

Let there be a line of Rail-way defined by the following section, of which we can construct the figure.*

| References to the points of the section. | Distances in metres between these points. | Heights in metres of these points above the point M. |
|---|---|---|
| M | — | — |
| *a* | 4,000 | 8 |
| *b* | 5,000 | 50 |
| *c* * | 6,000 | 57 |
| *d** | 10,000 | 51 |
| *e* | 11,000 | 58 |
| *f* * | 8,000 | 62 |
| *g* * | 32,000 | 23 |
| N | 25,000 | 32 |
| | 101,000 | |

In the above table the points marked with an

* See note B.

asterisk, are those in which the heights are *maxima* or *minima*. In applying the principles already stated, we observe that the length of the line being 101,000 metres [=110.457 yards,] the transit along this line, supposing it to be horizontal, is in the first place equal to the elevation of the weight, to a height equal to $\frac{101,000}{200}$, or 505 metres [=552 3 yards].

Further, if the transit is made in the direction MN, the weight must be elevated to the height of 32 metres, [= 35 yards,] which is the quantity, the point N is more elevated than the point M. The power, therefore, expended to produce the transit will be equal to that which is necessary to raise the weight to a height of 537 metres, [=587.3 yds,] if there were no *(contre pentes)* descending slopes on the line.

But in consequence of these descending slopes, there is a useless ascent before the first *maximum* *c*, of 57 metres, [=62.3 yards,] less 51 metres, [=55.8 yards,] or 6 metres, [=6.5 yds.] and before the second *maximum f*, of 62 metres – 23 metres, or 39 metres [=42.7 yds.] Total of the useless ascents 45 metres [=49.2 yds.]

If we admit then that two thirds of the power produced by the locomotive engine is required to overcome its friction, and other resisting causes,* we must add 30 metres to the number before found, which will give 567 metres [=611.1 yards.]

We must also observe, that in the interval between the point *a* and the point *b*, which is a length of 5,000 metres, there is a rise of 42 metres which gives a slope of $\frac{42}{5,000}$ or 0.0084, =1 in 119, sufficiently steep to make it desirable, and, perhaps, necessary to place upon it an auxiliary engine.

Attending to this circumstance, we must specify in what proportion the weight of this engine increases the weight of the train, and this will depend upon the absolute power of the engine used, and the velocity with which it is proposed to ascend the slope.

If we admit, for example, that the weight of the auxiliary engine, is the fourth part of the weight of the train, we should increase this weight in the proportion of 5 to 4, and we get the amount of power required to move this engine, by adding to the number before found, 1st, the fourth part of $\frac{5,000}{200}$ or 6.25, metres [=6.8 yards,] for the transit over that part of the line supposed to be horizontal; 2dly, the

* Note C at the End.

fourth part of 42 metres, or 13 metres, [=14.2 yds.] for the elevation of the engine from the bottom to the top of the slope; these two sums added together make 19.25 metres [=21.0 yds,] which increases to 586.25 metres [=637 yds,] the height to which the weight of the train should be raised by expending a quantity of power equal to that which would effect the transit in the direction MN of the proposed line.

If now we examine the transit in the opposite direction NM, we have in the first place, as above, the height of 505 metres, [=552.3 yards,] which represents the transit along the line, supposing it horizontal, from which it is necessary to subtract the 32 metres, the amount of the descent from the point N to the point M, and this gives 505−32=473 metres [=517.3 yards.]

In the next place we observe that the amount of the *useless ascents* is expressed by the same number as before, (which is always the case,) so that we must take account of it in the calculation in the same way by adding 30 metres [=32.8 yds,] to the preceding number, which will give 503 metres [=550.1 yards.]

Finally, by observing that the train descends from the point *b* to the point *a*, a height of 42 metres, on a slope of 0.0082, which is greater than 0.005, we perceive that there is a part of the action of gravity which is not made use of during this descent, and which is represented by the fraction $\frac{32}{82}$; from which it results that we have taken the useful descent too much in the preceding number by a height equal to 42 metres, multiplied by the fraction $\frac{32}{82}$, that is to say, to 16.39 metres [=17.9 yds.] Adding then this last number, we shall find the total to be 519.39 metres, [=568 yds,] the height to which the train would be raised by the power which would effect the transit in the direction N M.

There is nothing to be reckoned for the descent of the auxiliary engine on the slope *b a*, so long as the mechanical value only of the quantity of power consumed is taken into account, but it is evident that the useless expenditure which is incurred by having this machine upon the line, during the whole time it is not employed to aid the ascent of the train, should be reckoned.

In summing up the considerations which have been just set forth, we perceive that the valuation of the power which it is necessary to provide to effect the transport of the weight P, from one extremity to another of a Rail-way, is reduced to determine, by means of the formula (1), and, if there be occasion, by employing the formulæ,

(2) and (3,) a certain vertical height Z, to which the weight P may be raised by employing the said quantity of power.

This power is then expressed by the product P Z; and if we represent by J the ratio of the height Z, to the length A of the line, or if we suppose

$$J = \frac{Z}{A}, \quad \ldots\ldots\ldots\ldots \quad (4)$$

the product P J will express very nearly the *mean tractive power*, which the locomotive engine should exert to accomplish the draught of the train.

4. *Determination of the weight of the train which can be drawn upon a line of Rail-way, by a locomotive engine of a given power.*

A locomotive engine being given, the power which it can exert is limited principally by two circumstances. 1st, By the quantity of steam which can be generated in a given time; 2ndly, by the tractive power which the machine can exert without slipping upon the rails.

It is evident that, in all possible cases, there will be a loss, if all the vaporating power of the fire-place and boiler were not employed; that is to say, if all the steam is not produced which could possibly be obtained. Thus the first condition, in the proper management of an engine, is to produce constantly the same quantity of heat. It results from this, as will be shown further on, that the weight of the train being given, there is for each slope a certain velocity which should be adopted, and *vice versâ.* Further, the weight of the train cannot exceed the limit corresponding to the adhesion of the engine on the rails.

The action of the engine, the motion of the train being supposed uniform, is also subjected to the condition, that the pressure of the steam should be in equilibrium with the tractive power, and which latter we should consider as applied to the circumference of the wheels of the locomotive engine. This condition determines the pressure which the steam must have in order to draw a given weight. If we previously fix a limit, which this pressure shall not exceed, the condition which this involves fixes a limit to the action of the locomotive engine.

The influences of these different conditions, and the results to which they lead, cannot well be explained without expressing them by means of formulæ.

Let us suppose, for a moment, that the expense of the heat of the fire-place is constant when the same weight of steam is produced, in each instant of time,—a supposition which is probably so near the truth, that we may admit it without inconvenience in calculations of this kind. Moreover, with the view of obtaining formulæ as simple as it is possible, we shall remark that within the limits of the pressure, under which we suppose the steam produced, we may, without committing any very great error, take

$$0.5\,n + 0.09 \quad . \; . \; . \; . \; . \quad (5)$$

for the expression of the weight in kilogrammes of the cube metres of steam produced under a pressure of $n$ atmospheres.*

This being supposed, we may represent by P the total weight of the train, comprising the engine, the tender, the waggons and their load, expressed in kilogrammes.

$\Omega$. The area of the two pistons of the locomotive engine.

$c$. The length of the strokes of the pistons.

$r$. The radius of the wheels.

F. The pressure exerted by the steam formed in the boiler, on a square metre, expressed in kilogrammes.

$\Upsilon$. The volume of the steam generated in a second.

$\Pi$. The weight of the steam.

U. The velocity of the train, or the distance it runs in a second.

* Note D.

The linear dimensions are supposed to be expressed in metres, as well as the velocity U. The weights and pressures are expressed in kilogrammes. We may observe, in the first place, that the pressure of one atmosphere on a square metre being 10,330 kilogrammes, the number of atmospheres, represented by $n$, in formula, (5,) becomes $n = \frac{F}{10330}$ *

We may, therefore, consider the weight of a cube metre of steam generated in the boiler under the pressure F, as represented by the following formula.

$$0.09 + 0.5 \frac{F}{10330}.$$

or more simply,

$$0.09 + 0.0000484\ F. \qquad . \quad . \quad . \quad . \qquad (6)$$

And from this will result between the numbers $\gamma$, $\Pi$, the relation

$$\gamma = \frac{\Pi}{0.09 + 0.0000484\ F} \qquad . \quad .. \qquad (7)$$

The question which we propose to resolve from these data is the following,

The weight, power, &c., of the locomotive engine are supposed to be given, and consequently

* See note E, where these formulæ are given in English measures.

we know, besides its dimensions, the weight $\Pi$, of the steam which it can generate constantly in a unit of time. The mean velocity U, with which it is desired to travel over the Rail-way is determined. And, finally, the line of Rail-way is supposed to have been submitted to the investigation which has been explained in the preceding article; by means of which we know the ratio J given by formula (4), and the *mean* tractive power, represented at the end of the same article by PJ, a power required to be produced to effect the transit of the weight P. It is required to determine the amount of the weight P, which can be drawn along the line by the engine.

This determination rests upon the two following circumstances: 1st. That the volume of steam expended in a unit of time, is equal to the volume of space passed through by the pistons. 2ndly. That the pressure of the steam in the boiler is such, that the power transferred to the circumference of the wheels is equal to the mean tractive power PJ.

The first condition is expressed by the equation.

$$\gamma = \frac{c}{\pi r}\,\Omega U. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

in which $\pi$ represents the ratio of the circumference to the diameter, and which by substitution in equation (7,) is changed into

$$\frac{\Pi}{0.09+0.0000484\text{F}}=\frac{c}{\pi r}\Omega\text{U}. \quad . \quad (9)$$

To express the second condition, it is necessary to fix the relation which exists between the pressure F, produced by the steam in the boiler, and the power transmitted to the circumference of the wheels; which effects the transit.

It follows from the principles explained in the *Treatise on Steam Engines*, by Tredgold, that the power transmitted by the piston rods, in high pressure engines without condensation, may be calculated by representing by $0.6n-1$, the number of atmospheres corresponding to this power, $n$ representing, as before, the number of atmospheres corresponding to the pressure which takes place in the boiler. In the engines here spoken of, it is necessary to observe that the steam is thrown into the chimney with considerable velocity, with the view of urging the combustion, and to some other circumstances, which lead us, after an attentive examination of known facts, to consider the power which is transmitted to the circumference of the wheels as answering to a pressure exerted on the pistons expressed by a number of atmospheres equal to $0.5n-1$.

The amount of this pressure expressed in kilogrammes, on a square metre, will therefore be *

$$0.5F - 10330; \quad . \quad . \quad . \quad , \quad . \quad . \quad (10)$$

and consequently the equilibrium which should exist between the pressure F, under which the steam is formed in the boiler, and the tractive power, will be expressed by the equation

$$\frac{c}{\pi r}\left(0.5F - 10330\right)\Omega = PJ. \quad . \quad . \quad (11)$$

The two equations (9) and (11) express the conditions of the movement of the locomotive engine when the whole of the power which this engine can produce is employed.

The value of F drawn from equation (9), is

$$F = \frac{1}{0.0000484}\left(\frac{\pi r}{c}\frac{\Pi}{\Omega U} - 0.09,\right) \quad . \quad . \quad (12)$$

which represents a pressure in the boiler, that cannot be exceeded when the train is moved with the velocity U.

The value of F, taken from equation (11), is

$$F = {}^{1}\left(\frac{\pi r}{c}\frac{JP}{\Omega} + 10330,\right) \quad . \quad . \quad . \quad . \quad . \quad (13)$$

which represents the pressure that should be in the boiler, to enable the engine to draw the weight P.

By equalizing those two values of F, we get the equation,

* Note F.

$$P = -\frac{1}{J}\left\{\frac{0.5}{0.0000484}\frac{\Pi}{U} - \left(\frac{0.5 \times 0.09}{0.0000484} + 10330\right)\frac{c\Omega}{\pi r}\right\}$$

or more simply,

$$P = \frac{1}{J}\left(10330\frac{\Pi}{U} - 11260\frac{c\Omega}{\pi r}\right); \quad . \quad . \quad . \quad (14)$$

and reciprocally,

$$U = \frac{10330\Pi}{PJ + 11260\frac{c\Omega}{\pi r}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Equation (14) expresses the greatest weight P, which can be drawn by the locomotive engine with the velocity U; and equation (15) expresses the greatest continued and permanent velocity which can be given to a train, the weight of which is P.

If we establish between P and U, the relation expressed by the equations (14) or (15), the action of the engine will be so regulated as to produce all the effect of which it is capable.

By multiplying equation (14) by U, or the equation (15) by P, we get

$$PU = \frac{1}{J}\left(10330\Pi - 11260\frac{c\Omega U}{\pi r}\right), \quad . \quad . \quad (16)$$

or

$$PU = \frac{10330\Pi}{J + 11260\frac{c\Omega}{\pi r P}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

for the expression of the useful effect which can be obtained in the unit of time, (in reference to the gross weight of the train) expressed in functions of the velocity of the transit, or of functions of the weight P of the train moved. It is seen, agreeably to known results, that this useful effect is increased, by increasing the radius of the wheels of the locomotive engine; or more generally, by diminishing the ratio $\frac{c\Omega}{r}$. Further, it is evident that the useful effect is greater, in proportion as the velocity of the transit is less, or as the weight carried is greater. But it must not be forgotten, that the amount of weight which can be carried is limited both by the condition, that the wheels of the locomotive engine do not slip upon the rails; and also by the condition of not subjecting the pressure F, under which the steam is formed, and of which the value is given by formula (13), to exceed a given limit.

It is not difficult to perceive, after this remark, that the load, which it is proposed to draw by a given engine, and the velocity with which the transit is proposed to be effected, cannot be varied to any great extent.

When it is desirable to change, in any considerable degree, these two conditions, it is necessary to change the proportions of the apparatus in such a way as to preserve very nearly the ratio $\frac{c\Omega}{rP}$; so that the useful effect obtained, in reference to the weight of the train, also preserves constantly the same value, or nearly so.

Equation (14) will give immediately, in each particular case, the solution of the question which is proposed in this article.

To give an example, suppose a locomotive engine is used similar to the machines at present used upon the Manchester and Liverpool Rail-way, and of a medium power. The weight of such an engine may be reckoned at 8 tons, that of its tender at 4 tons, and we shall have the

Diameter of the cylinders $0^{m}$ 28 ; the surface of the two pistons $0^{m.q.}$ $12315 = \Omega$.

Stroke of the pistons $0^{m}. 41 = c$.

Radius of the wheels, $0^{m}. 76 = r$.

From which we deduce $\frac{c\Omega}{\pi r} = 00.21148$.

Weight of the steam formed in a second,* 0 kil. $4 = \Pi$.

These values being substituted in the formulæ (13,) (14,) (15,) will give respectively

$$F = 94.57\ \text{JP} + 20660, \quad . \quad . \quad . \quad (18)$$

$$P = \frac{1}{J}\left(\frac{4132}{U} - 238\right) \quad . \quad . \quad . \quad (19)$$

$$U = \frac{4132}{\text{JP} + 238}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

* Note G.

Formula (19) is the expression of the total weight which can be drawn by this engine.

Formula (20) is the expression of the permanent velocity * which the train can acquire on a slope which requires a tractive force, JP.

Formula (18) shows the pressure under which the steam should be formed.

Let us suppose that we had found by a process similar to that which has been described in the preceding article, that the *mean tractive force* which should be employed on a given Rail-way is represented by the fraction 0.006 of the weight to be moved, which is the same thing as to admit that the power expended in moving the train, is the same as if the Rail-way presented throughout its whole extent an uniform ascending slope of 1 in 1000: we should then make

$$J = 0.006.$$

If we suppose also that the transit is to be made with a mean velocity of about 8 leagues per hour, we must put $U = 9$ metres [= 9.84 yards.]

These last values substituted in formula (19,) will give $P = 36850$ kilogrammes or 81272 lbs. for the amount of the total weight of the train, or about 37 tons. Subtracting 12 tons for the weight of the locomotive engine and its tender, there remains 25 tons for the weight of the

* Note H.

wagons and their loads. The load or useful weight is about the $\frac{2}{3}$ of this last weight, or 17 tons.

If we substitute in formula (18) for JP the value 221.1 kilogrammes, F will become = 41570 kilogrammes, for the pressure exerted by the steam on a square metre, which is a little more than 4 atmospheres, or a little more than 3 atmospheres, above the exterior atmosphere.

5. *Investigation of the uniform motion of the train, on the various ascending or descending slopes, which may constitute part of a Railway.*

The whole weight of a train supposed to be drawn by a locomotive engine of a given power, having been determined in the manner which has been just explained in the two preceding articles. There is no doubt that the results obtained might be realized in actual practice, if the line of rail-way presented a uniform slope requiring a constant tractive power JP, to draw the weight P, in which case the velocity of the motion would be also constant. But as a line of rail-way presents generally unequal slopes, it is necessary to examine in each particular case, if the existence of these slopes do not alter these results, and in what limits the slopes should be confined, in order that these results may be applicable.

It may be said in general, 1st, that the result of the preceding article is applicable, or, which is the same thing, that any loss upon the action of the locomotive engine will not take place in consequence of the existence of an ascending slope, *when the engine can draw the train upon this slope ;* that is to say, when the tractive power, which is required on the slope does not render it necessary to raise the pressure under which the steam is formed too high, or will not cause the wheels of the locomotive engine to slip on the rails.

2ndly. That also there will be no loss in consequence of a descending slope, *when the action of gravity on the train does not surpass the resistances, including the power necessary to propel the locomotive engine when unattached to the train.*

If, therefore, we represent in general any ascending slope whatever by $i$, so that the power necessary to draw the weight P upon this slope should be expressed by $(0.005 + i)$ P, we must make $J = 0.005 + i$ in formula (13), in order to verify if the value which results from it for the pressure F does not surpass the proper limit. Further, we must also examine if the power $(0.005 + i)$ P would not surpass the power which the locomotive engine can exert without slipping, and which may generally be estimated at the twentieth part of the weight of the engine.* If the slope *i* satisfies these two conditions, it will not impede the transit of the train, or cause any loss of power.

If now *i* represents any descending slope whatever, so that the power necessary to draw the weight P upon this slope will become $(0.005-i)$. P, we must first examine if this quantity, when put in the place of J P in formula (15), gives to the denominator of this formula a negative or zero value, or only leads to a value of U, surpassing the greatest velocity which it is proper to allow the train to take. In this case the descending slope requires the use of the break, and will occasion a loss of power; in all other cases it produces none.

To make this more clear, we will now apply these principles to the example which has been given at the end of the preceding article, and to which the formulæ (18), (19), and (20), relate. The following table is formed on the principle that the values of F are calculated by formula (18), and the values of U by formula (20), in giving to P the value P = 36850 kilogrammes, or 81163lbs. as before found.

* Note I.

| Indication of the slopes. | Ratio of the tractive force to the weight drawn = J. | Pressure in the boiler on a square metre = F. | Permanent velocity. = U. |
|---|---|---|---|
| | | Kilogrammes. | Metres. |
| Descending 0.006 | — 0.001 | 17,170 | 20.5 |
| 0.005 | 0.000 | 20,660 | 17.4 |
| 0.004 | + 0.001 | 24,150 | 15.0 |
| 0.003 | 0.002 | 27,630 | 13.3 |
| 0.002 | 0.003 | 31,120 | 11.8 |
| 0.001 | 0.004 | 34,600 | 10.7 |
| Zero 0.000 | 0.005 | 38,090 | 9.8 |
| Ascending 0.001 | 0.006 | 41,570 | 9.0 |
| 0.002 | 0.007 | 45,060 | 8.3 |
| 0.003 | 0.008 | 48,540 | 7.8 |
| 0.004 | 0.009 | 52,030 | 7.3 |
| 0.005 | 0.010 | 55,510 | 6.8 |
| 0.006 | 0.011 | 59,000 | 6.4 |

We see by this table, that on an ascending slope of 6 *millimètres per mètre*, or 1 in 166, the pressure of the steam in the boiler would not amount to 5 atmospheres beyond the exterior pressure, and that the train would maintain upon this slope a velocity of more than 6 metres per second, or nearly six leagues, or 14½ miles per hour. We also perceive that on a descending slope, having the same inclination, the velocity of the train would not exceed $20^{m}.5$ per second, or about 18 leagues or 45 miles per hour. Also the ascending slope of 6 *millimètres per mètre*, or of 1 in 166, requires for a weight of 36850 kilogrammes, a tractive force of 405 kilogrammes, which is a quantity very nearly equal to the 20th part of the weight of the locomotive engine, which is 8 tons. It appears then, from these results, that in the circumstances which have been supposed, that is to say, in admitting that the mean tractive force on the line of Rail-way corresponds to an uniform ascending slope of 1 in 1000, and a mean velocity of 8 leagues per hour, the motion of a train of about 37 tons gross, drawn by a locomotive engine of 8 tons weight, should not be impeded by ascending or descending slopes of 1 in 200, or even 1 in 166, and that the existence of these slopes would not occasion any loss of power. But it is also evident, that there is a reason that the slope of 1 in 200 has been considered as the limit beneath which the slopes should be endeavoured to be kept.

It is necessary also to remark, that the results to which we have just come are derived from data which have been used, and particularly to the mean velocity of transit which has been assumed.

If we suppose a less velocity, the same engine will be capable of drawing a greater weight, and the tractive power augmenting in consequence, shows that there will be a slipping of the wheels, on a slope less than 0.005 or 1 in 200.

When the same engine draws a greater

weight, with a less velocity, the expense of transport is diminished, which arises principally from this, that the useful weight carried forms in this case a greater portion of the total weight of the train. It is useful to form a correct idea of the amount of this diminution.

Let us suppose, for example, that a line of Rail-way does not present any slopes steeper than 0.003, we can allow to the train, drawn by the locomotive engine, such as that already spoken of, a weight corresponding to the limit fixed by the slipping of the wheels on such a slope; that is to say, a weight equal to $\frac{400^{\mathrm{k.}}}{0.008} = 50{,}000$ kilogrammes, or 50 tons.

If at the same time we are desirous that the traction of such a weight should not raise the pressure of the steam higher than in the preceding case, where the weight was only 37 tons, it will be necessary to increase the length of the stroke, or the diameter of the pistons, or to diminish the diameter of the wheels of the locomotive engine, so that the quantity $\frac{c\Omega}{\pi r \mathrm{P}}$ should preserve the same value. It will result from this that the number 138, which is found in the denominator of formula (20), will become $238 \frac{50}{37} = 322$.

This formula (20) is then changed into

$$U = \frac{4132}{JP + 322},$$

and if we make $P = 50000$ kilogrammes and $J = 0.006$, it will give $U = 6^{m.} 64$ for the mean velocity with which the locomotive engine will draw over the whole extent of the line, the new train of 50 tons. To compare the expense of the transit in these two cases, we shall consider therefore in the case of the train of 37 tons that the net weight is proportional to 27 — 12, or 25, and in the case of the 50 tons that it is 50−12, or 38.

Besides, in the first case the mean velocity was 9 metres, or 9.84 yards per second, and in the second case that it was 6.64 metres, or 7.26 yards. The expense of conveyance is therefore greater in the first case in the proportion of $\frac{38}{25} \cdot \frac{6.64}{9}$, or 1.122 to 1.

When Rail-ways of a certain extent are under consideration, it can scarcely be hoped to reduce the greatest slope to less than a 3 thousandth, or 1 in 333 ; and consequently, if we wish that the same engine should draw the same weight over the whole line, we cannot make an engine weighing 8 tons draw a train, the total weight of which exceeds 50 tons.

The moving of such a train with a velocity of $6^m$ 64, found above, or nearly 7 leagues an hour, is therefore the most economical arrangement that can be adopted. Now it is shown, by the preceding calculation, that in carrying the velocity to 9 metres per second, or a little more than 8 leagues per hour, which requires the weight of the train to be reduced to 37 tons, is the expense of carriage increased, upon the line of Rail-way which has been taken for the example, by about 12 per cent. But then this advantage is acquired, which in certain cases may be very important, that the train will overcome without difficulty, not only the slopes of 1 in 333, but even slopes exceeding 1 in 200.

All these circumstances lead to the establishment of this fundamental principle, that the essential character of Rail-ways is to afford the means of a very rapid conveyance.

In order to render it convenient and advantageous to affect the transit at a low velocity, it would be necessary that the line should not have any ascending slope in the direction of the transit.

Let us remark that the velocity of the transit is here considered as given and fixed at 9 metres per second, so that we should deviate from the conditions laid down, if we reduced the velocity to $6^m$ 64, with a view of increasing the load, and obtaining a saving of 12 per cent mentioned above. It is true that the velocity of 9 metres per second, or 20 miles an hour, is considered as necessary only for the conveyance of passengers, and that if the conveyance of merchandize is carried on separately, the velocity of the trains loaded with the latter might without inconvenience be less.

This circumstance leads to the conclusion that there will be a certain advantage in not having slopes greater than 1 in 333, which would permit the conveyance of merchandize with a less velocity, and by allowing a greater weight to be moved, would produce a certain saving in the cost of carriage, the amount of which can be ascertained from the preceding calculations.

We should consider, also, that if we are not rigorously tied down to the condition, that the same locomotive engine should draw the same weight over the whole line, we may in most cases, much diminish the injurious influence of slopes greater than 1 in 333, or even 1 in 200, by diminishing the weight of the trains on these slopes only, or in placing there auxiliary engines.

They could then be passed over with velocities much greater than what has been here stated, a circumstance which would compensate in part for the increased expense which would arise from the plan here suggested.

We may presume, from what has been stated, that there is often little to gain, by increasing to any great extent the expense of constructing a line of Rail-way, with a view of reducing the limit of the slopes from 1 in 200 to 1 in 333. It is impossible to establish on this point absolute and general rules, but the principles which have been here stated, afford the means of choosing in each particular case, that arrangement which is the most advantageous.

It is easy to perceive to what extent the difficulty and expense of carriage rapidly increases on long ascending slopes, which exceed 1 in 200. The following table*

| Rate of ascending slopes. | Ratio of the tractive power to the weight drawn = J. | Total weight of the train which the engine weighing 8 tons can draw without slipping = P. | Net weight carried $= \frac{2}{3}$ (P— 12,000.) |
|---|---|---|---|
| | | kilogrammes. | kilogrammes. |
| 0 005 | 0.01 | 40,000 | 18,667 |
| 0.0075 | 0.0125 | 32,000 | 13,333 |
| 0.01 | 0.015 | 26,667 | 9,778 |
| 0.0125 | 0.0175 | 22,222 | 6,815 |
| 0.015 | 0.02 | 20,000 | 5,333 |
| 0.0175 | 0.0225 | 17,778 | 3,852 |
| 0.02 | 0.025 | 16,000 | 2,667 |

* See Note K.

points out the gross weight of a train, that the engine, to which the preceding calculations have been applied, can draw without slipping upon different slopes, and of the net weight which can be carried. As the tractive power is in all cases supposed equal to the $\frac{1}{20}$ of the weight of the engine, that is to say to 400 kilogrammes, or 882 lbs. the pressure under which the steam ought to be generated, should be the same in all cases, and about 4.8 atmosphere, independent of the exterior atmosphere. The velocity of the train should then also be the same in all cases, and about $6\frac{1}{2}$ metres or 7 yards per second. Thus the useful effect obtained on each slope is proportional to the corresponding number found in the last column. Upon a slope of 0.02 or 1 in 50, the engine cannot draw more than a waggon.

We have seen in the preceding article that the train of 37 tons, drawn by the engine on an ascending slope of 1 in 1000, with the velocity of about 8 leagues per hour, carries a net load of about 17 tons. As the net load corresponding to a slope of 1 in 100, in the preceding table, is not much more than the half of this, we see that the passage of the train over this slope, requires an auxiliary engine of almost an equal force; this agrees with known facts.

If, besides, the pressure of the steam is not raised in the two engines to about 4½ atmospheres, above the exterior atmosphere, and if, for example, we do not exceed the ordinary limit of 3½ atmospheres, the tractive power being inferior to the resistance, the velocity with which the train commences to ascend the slope will progressively diminish, towards a state of rest. But by raising the pressure to 4½ atmospheres, the velocity would not be reduced to less than about 6½ metres, or 7 yards per second.

The principles which have been just explained, appear to indicate, that on great lines of Railway, where we are confined not to surpass slopes of 1 in 200, and where it is proposed to establish a rapid conveyance, the use of auxiliary engines will not generally be required. Besides, we can always, by means of these principles, perceive distinctly if the use of auxiliary engines be necessary, or if the descending slopes require the use of a break; and consequently if the case be one of those which have been pointed out as exceptions in Art. 3, in which we cannot employ alone the formula (1) for the calculation of the power consumed by the transit, and in which this calculation requires the use of the additional terms given in the formulæ (2) and (3).

## 6. *Examination of the velocity of the train during its passage from one slope to another.*

The principles laid down in the two preceding articles, are founded on a consideration of the *permanent velocity*, which can be produced and maintained by the locomotive engine on each slope, and the trains have been considered as passing over the entire lengths of the various slopes with this velocity. This supposition being adopted, the conditions expressed in Art 5, to show whether an ascending or descending slope, shall occasion or not a loss of power in the working of a locomotive engine as determined by Art. 4, may be admitted without fear of error.

But the supposition of which we have just spoken is not altogether conformable to natural effects, for the train cannot at once change the velocity due to one slope to that of another, whether on account of the *inertiæ*, or because that on every slope the permanent velocity given by formula (15), (and for the particular case which has been taken for an example by formula (20),) supposes the existence of a certain pressure of the steam, a pressure the value of which is given by formula (12). Now we cannot, all at once, increase the pressure under which the steam is generated, since this increase is inseparable from an elevation in temperature of the water in the boiler.

The train must pass gradually from its actual permanent velocity to the permanent velocity due to the new slope upon which it is about to enter, in the same time that the water and the steam contained in the boiler passes gradually from their actual temperature, to the temperature under which the steam should be generated, in order that its pressure be in equilibrium with the tractive power which is necessary to this new slope, and thus maintain uniform velocity.

We must now investigate if these necessary changes can take place, without occasioning on the one part a greater consumption of fuel, and on the other part without diminishing the velocity of the train.

In the first place we may remark that there is not generally any loss of power in the system which we are considering, when there is no escape of steam from the safety-valves. Whenever the steam generated does not leave the engine until after having acted on the pistons, the heat which it has been necessary to transmit to it has been employed to overcome the resistances which are opposed to the motion of the train, or to give to the mass of this train the *vis viva* equivalent to the effect of these resistances.

It is, besides, very evident that we continue to neglect here, as a secondary object, the consideration of the loss of heat which takes place at the exterior surfaces of the heated parts of the the apparatus, or rather of the slight differences which these losses may occasion according to the elevation of temperature in the water of the boiler. After this remark the question proposed is simply to examine if the passage from one slope to another can be made without a loss of steam.

This passage may be considered under two heads. 1st, when a slope is arrived at where the resistance to motion will be greater; 2ndly, where the resistance will be less.

Let us admit, for the sake of example, that the train travelling over a level, is just about to ascend a slope of 0.005. In continuing to take for an example the train, of which the conditions of its motion have been determined in Article 4, it is seen by the table at page 49, that this train will travel upon the level line with the velocity of $9^{m}.8$, [= 10.72 yards] the steam being generated under a pressure of about 3.8 atmospheres, and consequently at the temperature of about 144°.* When its motion is regulated by the slope of 0.005, it will move upon it with the velocity of $6^{m}.8$, [= 7.44 yards] the steam being generated under a pressure of about 5.6 atmospheres, which answers to a temperature of about 158°.†

* 291° of Farenheit.

† 317 of Farenheit.

The temperature should therefore be elevated 14°* in the boiler; and we may suppose that this elevation of temperature may be effected in about 6 minutes, if all the heat transmitted by the furnace was employed, since we know that it requires an hour or more to elevate the temperature of the water in the boiler to 150°.† But as it is necessary to furnish at once the heat required to form the steam employed to keep up the motion, and to obtain the elevation of the temperature required, we ought to suppose that even in urging the fire it will require more than 6 minutes to produce it.

Be this as it may, there is nothing to prevent us, as long as we please, before entering upon the slope, 1st, to increase the fire; 2ndly, to load the safety-valves, so as to obtain a pressure of 5.6 atmospheres; 3rdly, to diminish gradually the size of the orifice of communication by which the steam passes from the boiler to the cylinders. The first disposition tends to raise the temperature; the second establishes the limit, suitable to the pressure which the steam should acquire in consequence of increased temperature; the third has for its object to regulate the quantity of steam sent to the cylinders, so that although the pressure is raised in the boiler, the action on the pistons remains nevertheless always the same, in such a manner that the velocity is not accele-

* 57° Farenheit. † 302 Farenheit.

rated. In operating thus, the steam expended only carries with it the same quantity of heat, and the extra heat produced is entirely employed to raise the temperature. When the temperature shall be thus raised 14° * C. in the boiler, it will not be necessary to urge the fire further. The apparatus may be thus made ready and kept indefinitely in this state; that is to say, the temperature being 158°† in the boiler, and nevertheless by a proper diminution of the orifice of communication, the action of the pistons is not greater than when this temperature was 144°‡ C. The train will thus arrive at the commencement of the slope; it will begin to ascend with its velocity of $9^{m}.8$, [= 10.7 yards] which will diminish gradually, towards a state of rest, if the orifice of communication was not then progressively opened, in such a way, that at the instant the velocity is reduced to $6^{m}.8$, [= 7.44 yards] the cylinders would receive the whole of the steam which the fire could generate, under the pressure of 5.6 atmospheres.

Now it is evident, that unless the fire has been urged more than was necessary to promote the elevation of the temperature to the required degree, there will not be any loss of steam through the safety-valves, since the velocity of

* 57° of Farenheit. † 317° Farenheit.

‡ 292° Farenheit.

the train diminishing progressively from $9^m.8$ to $6^m.8$, is constantly more than is necessary to enable the pistons to use all the steam which the machine can generate under the pressure of 5 6 atmospheres, regulated by the weight on the safety-valves. If such a loss should be feared, it is necessary that the velocity of the train be diminished below the speed of $6^m.8$, which accords to the slope to be passed over: this can only take place through the negligence of the conductor, who has not opened the orifice sufficiently soon after the entry of the train upon the slope.

Suppose, now, that the train has arrived at the summit of the slope of 0.005, and just about to enter a level line. As the safety-valves remain loaded for the pressure of 5.6 atmospheres, the fire should be lowered for some minutes. As soon as the train is on the level line, the actual pressure of the steam exceeding the resistance, the velocity is immediately increased. From whence we see in the first place, that the steam cannot escape by the safety-valves, even when the fire is not slackened, unless the orifice of communication is not so regulated as to allow all the steam which can be generated to go to the pistons. But in thus lowering the fire we, in order to form steam, avail ourselves more certainly of the heat which was given to the water and the boiler to raise them to 14° C. of more elevated temperature.

The velocity of the train will cease to be accelerated, 1st, when the pressure of the steam shall not be greater than is necessary to balance the resistances; 2nd, when the motion of the pistons takes all the steam that can be formed by the furnace restored to its ordinary state. These two circumstances taking place when the temperature is 144° in the boiler, and the velocity of the train $9^m.8$, this state will establish itself spontaneously with the single precaution of giving a sufficient passage for the steam from the boiler to the cylinders. The safety-valves can then be unloaded if it be desired, and regulated to the pressure of 3.8 atmospheres.

The remarks which have just been made are applicable to all analogous cases. We ought to observe, that by the passage from one slope to another, it does not at all follow that there is a necessity for a loss of steam, and that such a loss takes place only through the fault of the fireman or the conductor, which is equally liable to occur in the ordinary working of the engine. From this it may be concluded, agreeably to what has been said above, that this passage does not occasion any loss in the action of the engine, and in fact, we see distinctly the compensations which take place, the heat which had been employed to elevate the temperature in the boiler being given out again when the temperature is lowered, (except a slight difference due to the effect of the loss by the exterior surfaces;) and the diminished velocity which takes place after the train has overcome the superior extremity of the slope being compensated by the greater velocity with which it commenced to ascend the slope.

The exact determination of the velocity of the train in the circumstances here considered, presents a very complex question, of which a part of the elements is arbitrary, or cannot be exactly appreciated. In even seeking to simplify this question by convenient hypothesis, it still remains very complicated. The approximate solutions which can be obtained from it, require, besides, some developements which would extend this paper to too great a length.

We shall confine ourselves to giving the formulæ by which may be determined the varied velocity of a train, in the particular case of the pressure of the steam in the boiler being kept constantly the same: formulæ which it may be useful to have at hand.

The condition which regulates this velocity is, that the *vis viva* varies in each element of time by a quantity equal to double the quantity of action exerted by the power resulting from the production of the steam, diminished by the quantity of action destroyed by the resistances.

We may represent, in preserving the denominations of the preceding articles, by $p$, the weight of the wheels belonging to the waggons, and to the locomotive engine which form the train, and by $u$, the velocity of the train at the end of the time $t$. The vis-viva of the part $P-p$, of the weight of the train will be expressed by $\frac{P-p}{g}u^2$; as to the vis-viva of the wheels, it must be observed that every point in a wheel may be supposed to turn round that point which rests on the rail, the centre of the wheel turns round this point with the velocity $u$. Then let $x$ represent the distance of the element $dp$, of the weight of the wheel at the point of its circumference which rests upon the rail, and $r$, the radius of the wheel; the actual velocity of the element $dp$ is $\frac{ux}{r}$; and consequently the actual vis-viva of the wheel is $\frac{u^2\int dp.x^2}{gr^2}$, in taking the integral $\int dp.x^2$, for the whole wheel. Now we know that in calling $\delta$ the length of the simple pendulum, the oscillations of which are of the same duration as that of a wheel suspended from a point in its circumference, we have $\int dp.x^2=pr\delta$, $p$ being the weight of the wheel, then the preceding expression of the vis-viva of a wheel may be written $\frac{\delta}{r}\ \frac{p}{g}u^2$. From whence we conclude, in supposing that the ratio $\frac{\delta}{r}$ is the same value for all the wheels of the train, the total vis-viva of this train is expressed by

$$\frac{1}{g}\left(\mathrm{P}-p+\frac{\delta}{r}p\right)u^2,$$

$p$ representing, as stated before, the weight of all the wheels.

On the other side, the power exerted by the steam, diminished by the resistances is expressed by

$$\frac{c\Omega}{\pi r}(0.5\mathrm{F}-10330)-(0.005\pm i)\ \mathrm{P},$$

$i$ being the slope on which the train is moving, and the sign + or the sign — being taken according as this slope is ascending or descending.

We therefore have for the equation of velocity

$$\frac{\left(\mathrm{P}-p+\frac{\delta}{r}p\right)}{g}\frac{du}{dt}=\frac{c\Omega}{\pi r}(0.5\mathrm{F}-10330)-(0.005\pm i)\mathrm{P}, \qquad \ldots\ldots\ (21)$$

of which the integral (F being supposed constant as well as $i$), is

$$\frac{\mathrm{P}-p+\frac{\delta}{r}p}{g}(u-u_o)=\left[\frac{c\Omega}{\pi r}(0.5\mathrm{F}-10330)-(0.005\pm)i\mathrm{P}\right]t, \qquad \ldots\ldots\ (22)$$

in representing by $u_o$ the initial velocity, or that which exists at the moment we reckon $t=o$.

According to what takes place in practice, the weight of the wheels is very nearly the half of the weight of the waggons when empty, and consequently very nearly the $\frac{1}{6}$ of the total weight of the train. On the other hand the ratio $\frac{\delta}{r}$ differs very little from $\frac{3}{2}\cdot$ Thus we have very nearly

$$P - p + \frac{\delta}{r} p = \left[\frac{5}{6} + \frac{3}{2} \cdot \frac{1}{6}\right] P = \frac{13}{12} P.$$

We can therefore, without much error, apply the approximate formula.

$$\frac{13}{12} \frac{P}{g} (u - u_0) = \left[\frac{c\Omega}{\pi r}(0.5F - 10330) - (0.005 \pm i)P\right] t,$$

or if we wish

$$u - u_0 = \frac{12}{13}\left[\frac{c\Omega}{\pi r} \frac{0.5F - 10330}{P} - (0.005 \pm i)\right] gt. \quad (23)$$

This formula may serve to determine the velocity of the train, at all times that the apparatus is supposed to be so regulated that the pressure F of the steam in the boiler is kept constant, which requires the condition that the heat transmitted in every unit of time increases with the velocity, in order that the boiler may always furnish a supply equal to the variable consumption of the steam.

We could, for example, employ the formula in question in order to ascertain if a train, by means of a certain acquired velocity, could surmount a steep slope of a given length. In fact, the equation (23) shows that a train commencing to move from a state of rest should have acquired the velocity $u$, after having run over a space expressed by

$$\frac{u^2}{\frac{12}{13}\left[\frac{c\Omega}{\pi r} \frac{0.5F - 10330}{P} - (0.005 \pm i)\right] . 2g}; \quad . \quad . \quad (24)$$

and reciprocally that the velocity $u$ will be reduced to zero, when the denominator of the formula (24) becomes negative, after the train shall have run over a space expressed by this formula.

If we still take for example the train as determined in Art. 4, we have $\frac{c\Omega}{\pi r} = 0.021148$, and P = 36850. The expression (24) becomes

$$\frac{u^2}{\frac{12}{13}\left[0.005928\ (0.5n-1)-\ (0.005 \pm i)\right].2g}\ ; \quad . \ (25)$$

$n$ being the number of atmospheres which expresses the constant pressure of the steam which is generated. Suppose, for example, that this pressure being extended to 5 atmospheres, and that the combustion is increased sufficiently to allow the consumption of steam which should take place at a velocity of 15 metres [= 16.4 yards] per second, which supposes a quantity of heat transmitted in a unit of time, more than double of the mean quantity which has been taken in the preceding calculations. Admit also that the train commences with this velocity of 15 metres, [= 16.4 yards] to ascend a slope of a centimetre per metre, [= 1 in 100]. Putting in the formula (25) $n = 5$, $u = 15$ metres, $i = 0.01$, we shall have 2034 metres [= 2224 yards] for the distance which the train can run up the slope before it stops and descends again in a contrary direction.

If we suppose the slope 2 centimetres per metre, [= 1 in 50], or if we make i=0.02, the same formula will give 771 metres [= 843 yards] for the distance in question.

If besides we wish to know what distance the train, starting from a state of rest, ought to run upon a horizontal line to acquire a velocity of 15 metres [= 16.4 yards] per second, we must make $i=o$, in the formula (25), and we shall have the distance 3192 metres [= 3491 yards].

### *7th. Summary. Comparative Estimate of the Cost of Transit on different lines of Rail-way.*

We have shown in Article 2, that the degree of advantage which a line of Rail-way can present, depends in a great measure on the reduction that can be effected in the actual price of the cost of transport.

We have further remarked that the price of transport upon a line of Rail-way, results from two principal elements; that is to say, 1st, the amount of the cost of construction, to which is to be added a part of the cost of management and repairs. 2nd, The cost of transport, properly so called, to which is also to be added a part of these same secondary expenses.

The annual sum produced by the interest and the first installment of the expense of construction, increased by the expense of repairs and management which relate to it, being divided by the number of tons of merchandise supposed to pass annually by the road, will give the proportionate expense of each ton.

As to the cost of transport, properly so called, we distinguish two parts; 1st, the expense of the locomotive engine, comprehending the purchase and the repairs of this engine and its tender, the fuel, and the water consumed, the workmen which attend it; 2nd, the expense of the wagons comprehending their purchase and repairs, and the workmen or attendants employed in the management and care of the train.

To these expenses should be added that of warehouse and offices, as well as that of the workmen and other agents employed for warehousing, loading, unloading, and the regulating the transport of the goods and passengers.

The 3rd article, and those which follow it, have for their object the determination of the most important part of the expense of which we have just spoken, that of the locomotive engine.

We have given in the 3rd article a general rule for finding the amount of the quantity of action necessary to effect the transport of a given weight, upon a given line of Rail-way; in article 4, the manner of deducing from the result obtained the total weight of the train, which a locomotive engine can draw upon this line with a given volocity, and consequently the weight of payable merchandise which should be carried by this engine. We have afterwards, in articles 5 and 6, exhibited the use of this rule by a particular examination of the motion of the train upon unequal slopes which might appertain to the line of Rail-way, and shown in what limit of slopes it may be applied without error, or what the slopes should be which would require the employment of auxiliary engines.

The result in question may be summed up in the following manner. In preserving the denominations employed in the preceding articles, (see page 31,) we shall designate by

A, the length of the line of Rail-way, valued in metres.

U, the mean velocity with which this line should be traversed, expressed in metres per second.

P, the total weight of the train, determined conformably to what has been said in article 4, which can be drawn by the locomotive engine, with the mean velocity U, expressed in tons.

Q, the weight of this locomotive engine and its tender, also expressed in tons.

Δ, the expense of working this engine in each unit of time, (which we suppose to be one second.)

Observing also that if the weight of the payable merchandise, is about $\frac{2}{3}$ (P—Q), we shall evidently have

$$\frac{A\,\Delta}{\frac{2}{3}\,(P-Q)\,U} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

for a close approximate expression of the expense of the locomotive engine for every ton, transported from one extremity of the line to the other.

It does not appear possible to give a more simple rule, if, as it is convenient to do, we conceive that the locomotive engines shall be constantly managed so as to obtain from them all the mean action of which they are capable.

We should, in fact, perceive, that if we do not restrict ourselves to this condition, losses would follow in some cases, upon the purchase and repairs of the engine, and upon the labour of the workmen if not upon the value of the fuel consumed, by the effect of which the comparisons we have in view would cease to present the requisite accuracy.

This accuracy cannot be obtained, but by a calculation of the kind which is presented in this work, bringing into the calculation in every particular case, the proportion of the weight of the locomotive engine, to the total weight of the train which it draws, and always keeping in view this proportion, as an essential element of the result.

As to the second part, which forms the cost of transport, properly so called, that is to say, the expense of the wagons, it appears that we might regard it, as being for every ton of merchandise transported proportional to the length of the line.

This expense will be so much per ton per mile, and estimated accordingly.

Finally, as to the secondary expenses, of warehousing and despatching, it may be said that they differ little for two lines of which the lengths are not very unequal; but we cannot doubt, that in general they increase with the length of the lines, and it appears convenient, when Railways of a great extent are considered, to appreciate them, as well as the preceding expense, at so much per ton per mile.

In recapitulating what has been just said, we see that the total price of transport of a ton, from one extremity to another of a Rail-way, will consist of,

1st. The annual interest of the expenses of construction, and the annual expenses of management and repairs, divided by the number of tons transported annually.

2nd. The expense of the locomotive engine expressed by the formula (26).

3rd. The expense of the wagons, which is proportional to the length of the Rail-way.

4th. The expense of warehousing and despatching which we shall also consider as being proportional to the length of the Rail-way.

We then see that the valuation of this total price is thus reduced, in each particular case, to the determination of a very small number of elements, that is to say, the expense of construction and repairs, for which data is given by the formation of the project, the estimate of the annual tonnage, the determination of the weight of the train which should be drawn by a locomotive engine of a given power; and lastly, the length of the line of Rail-way.

We see further, and sometimes this remark will be very important, that if, on making a comparison between two lines, all the elements which we have just specified are found in favour of one of them, the preference which it merits, in respect to the economy which it would produce into the cost of transport is evident, without having occasion to value in money the relative influence of each of these elements, a valuation which always presents some uncertainty, seeing the little extent of information which is possessed on this subject, and the difficulty of knowing exactly the annual tonnage.

Thus, 1st, if a line is less expensive in construction; 2nd, if the ratio $\frac{A}{(P-g)\,U}$ is less; 3rd, and lastly, if the length of the line is less; we are certain that the expense of transport will be less on this line. The result of the comparison depends therefore entirely upon the determination of geometrical or mechanical quantities, in estimating which there is nothing arbitrary or uncertain. But if the three elements upon which the comparison rests, that is to say, the annual expense of the cost of construction, the repair, and the management, the quantity of action necessary to transport a given weight from one extremity of the line to the other, lastly, the length of this line do not all give a favourable result to one of the lines, which are compared, it becomes necessary, in order to decide the question, to value in money each of the parts of expense of transport, and consequently to specify the quantity of merchandize, and the number of passengers, which may be presumed would be carried annually along the lines.

---

Since the foregoing observations went to press, the following additional remarks by M. Navier have appeared in the *Annales des Ponts et Chaussées.*

According as we examine with more care the circumstances depending on the use of locomotive engines on Rail-ways, our ideas become more extended and correct. I shall therefore return to the subject, on account of a remark which M. de Prony has been pleased to communicate to me upon some parts of the observations published in the preceding work.

There has been given, page 17, a rule for the calculation of the quantity of power required for the transit of a train of the weight P, from one extremity to another of a Rail-way, acording to which this power is very simply expressed by the formula $P\left(\frac{A}{200} \pm H\right)$.

P being the weight of the train, A the length of the line, H the height of the extremity N, above the extremity M and the signs + or — are used according as the transit is made in the direction MN, or in the direction NM. This expression is independent of the figure of the section between the two extremities M and N of the line.

It has been further observed, at p. 19, that where it is wished to comprehend in the calculation the power employed to surmount friction, the expression of the whole quantity of power employed in the transport, must then depend on the figure of the section, and that the influence of the descending slopes should be considered. It is with this view that the term $\mu h$ has been introduced in formula (1) of page 21: the addition of this term was necessary, because in the locomotive engine a part of the power expended is constant, and will exist even when the effort to be overcome is reduced to zero, so that the power expended is actual loss, and the useful effect does not always remain in the same ratio.

Nevertheless, in examining this subject more attentively, it will be perceived that we have attributed too great an influence to the effects of the descending slopes, and even that within the limits which comprehend the effects of this kind submitted to calculation, the term $\mu h$ may be entirely suppressed in the formula which we have just mentioned. This is seen immediately, from the constitution of the approximate expression which is employed for estimating the work performed by the locomotive engines.

Let us take again the equation (15) page 36; according to which the permanent velocity U, which can be given to a locomotive engine drawing a train of the weight P upon different slopes, is represented by

$$U=\frac{10330\,\Pi}{PJ+11260\,\frac{c\Omega}{\pi r}}.$$

In this formula PJ expresses the effort which it is necessary to produce to draw the train; $\frac{c\Omega}{\pi r}$ is a constant, of which the value is given for every locomotive engine. The quantity $\Pi$, which is the weight of the steam produced in a second is also supposed constant.

We may then write more simply

$$U=\frac{m}{PJ+n},$$

$m$ and $n$ being constants.

This granted, let there be a line of Rail-way MN, and let us represent by A its length, and by H the height of the extremity N, above the extremity M; let us admit at first, that the slope is uniform from one extremity to the other. The velocity which the locomotive engine will acquire on it, will be expressed by

$$U = \frac{m}{P\left(0.005+\frac{H}{A}\right)+n.}$$

Let now T represent the duration of the transit, or the duration of the work of the locomotive engine. We shall have

$$T = \frac{A}{U}, \quad \text{or } T = \frac{P\,(0.005A+H)+nA}{m}.$$

Let us further admit that the length A is composed of several parts of which the respective lengths are $a$, $a'$, $a''$, &c; and that in passing the length $a$, a height $h$ is attained; that in passing the length $a'$, a height $h'$; in the length $a''$ a height $h''$; and so on. The duration of the transit for each of these intervals will be respectively,

$$t = \frac{P\,(0.005a+h')+na}{m},$$

$$t' = \frac{P\,(0.005a'+h)+na'}{m},$$

$$t'' = \frac{P\,(0.005a''+h'')+na''}{m},$$

&c. &c. &c.

and the sum of these durations, or the total time of the transit, will be $t+t'+t''+\&=$

$$\frac{P\,[0.005\,(a+a'+a''+\&)+h+h'+h''\&]+n\,(a+a'+a''+\&).}{m}$$

Now we have $a+a'+a''+\&=A$, and $h+h'+h''+\&=H$, then

$$t+t'+t''+\&=T.$$

It is superfluous to remark, that if a descent is made in running over some of the parts, $a$, $a'$, $a''$, &c. it will be necessary to give the sign — to the differences of the level, $h$, $h$,' $h'$, &c. of the extreme points of these parts. The preceding result would still subsist.

From this result, which is important for the establishment of Rail-ways, the duration of the work of the locomotive engine is assumed, and consequently the portion of the expense of the transport which arises from it, will always be the same, whatever be the figure of the section between the two extremities of the same line, provided that the length of this line is not altered. This conclusion supposes, moreover, that the same locomotive engine draws throughout the same train, and that there are no descending slopes sufficiently steep to require the use of the break.

It is seen, by what precedes, that in tracing a line of Rail-way, there is no inconvenience in rising higher, to re-descend afterwards, as long as that does not make it necessary to extend the limit of the slopes. Thus, for example, several lines uniting two given extreme points, upon which it is admitted that the same locomotive engine draws throughout the same train, will be perceptibly equal in respect to the expense of the transport, whatever be the heights to which they rise or from which they descend, if their lengths be equal ; and if upon any of these lines the steepest slopes do not surpass $0^{m}.005$. (= 1 in 200.) But a line, where the limit of the slopes should be less, would present an advantage conformable to what has been said page 9, in permitting, without losing any thing on the action of the engine, heavier convoys to be drawn with less velocities. It appears then, in tracing lines of Rail-ways, that especial care should be taken to diminish the length of the transit, and to lower the limit of the slopes.

The suppression of the term $\mu h$, in formula (1) of page 21, will moreover, simplify the examination and calculation necessary to compare different lines in making use of the ideas which have been suggested.

# NOTES.

## Note A.

The *vis-viva* of any system of points, is the product of the mass of each point into the square of its velocity.

The *moving force*, or *quantity of motion*, of a body, is generally understood to mean the product of the mass into the velocity, and is the same as the *momentum:* and the conservation of the quantity of force thus measured is proved by proving the conservation of the motion of the centre of gravity. But if the force of a body in motion be measured by the *whole* effect which it will produce before the velocity is destroyed, or by the whole effort which has been exercised in generating it, without regard to the time, it must be measured by the mass multiplied into the square of the velocity. Thus balls of the same size projected into a resisting substance, as a bed of clay, will go to the same depth, so long as their weights, multiplied into the squares of the velocities, are the same. Force thus measured is called *vis-viva,* in opposition to force measured by momentum, which is proportional to the pressure, or *dead pull,* producing it. And it will appear, that forces will always produce a certain quantity of *vis-viva,* by acting through a given space, whatever be the manner in which the bodies are constrained to move.—*On the Motion of Points constrained and resisted, and on the Motion of a Rigid Body. Second Part of a Treatise on Dynamics, by W. Whewell, M.A.* See Appendix 2.

## NOTE B.

| Reference to the points of Section. | Distances in yards between these points. | Height in feet of these points above the point M. |
|---|---|---|
| M | — | — |
| a | 4,374.5 | 26.24 |
| b | 5,468.0 | 164.01 |
| c * | 6,561.6 | 186.97 |
| d * | 10,936.0 | 167.29 |
| e | 12,029.6 | 190.25 |
| f * | 8,748.8 | 203.37 |
| g * | 34,995.2 | 75.44 |
| N | 27,340.0 | 104.97 |

## NOTE C.

IT will be seen further on, that we adopt the following rule for estimating the power of a locomotive engine : $n$ being the number of atmospheres corresponding to the pressure under which the steam is generated, and consequently the power resulting from the production of this steam being proportional to $n-1$, we suppose the effort which is transmitted to produce the draught is proportional to $0.5n-1$.

Hence the fraction $\frac{0.5n-1}{n-1}$ expresses the part of the power produced which is used, and the part of this power which is lost by the effect of resistances is expressed by $1-\frac{0.5n-1}{n-1}$, or by $\frac{0.5n}{n-1}$. The value of this expression is different, according to the magnitude of the number $n$. If we suppose $n=4$, which agrees with the pressure that most commonly takes place in the working of locomotive engines, this value is $\frac{2}{3}$.

In taking this number to estimate in an approximate manner the effect of the descending slopes, we shall estimate rather above than below its true value, the quantity of power which they destroy.

## Note D.

The degree of exactness of this formula may be judged of by the following table.

| Temperature in degrees of the centegrade thermometer. | Pressure in atmospheres. | Weight of a cube metre of steam, calculated | |
|---|---|---|---|
| | | by the known formulæ. | by the proposed formulæ. |
| | | kilogr. | kilogr. |
| 100 | 1 | 0.590 | 0.59 |
| 121 | 2 | 1.117 | 1.09 |
| 135 | 3 | 1.615 | 1.59 |
| 145 | 4 | 2.101 | 2.09 |
| 153 | 5 | 2.574 | 2.59 |
| 160 | 6 | 3 047 | 3.09 |

This table, reduced to English measures, is as follows:—

| Temperature in degrees of Fahrenheit's thermometer. | Pressure in atmospheres. | Weight of 61,023 cube inches of steam calculated | |
|---|---|---|---|
| | | by the known formulæ. | by the proposed formulæ. |
| | | lbs. | lbs. |
| 212 | 1 | 1.301 | 1.301 |
| 249 | 2 | 2.463 | 2.404 |
| 276 | 3 | 3.563 | 3.507 |
| 293 | 4 | 4.634 | 4.609 |
| 307 | 5 | 5.677 | 5.712 |
| 319 | 6 | 6.720 | 6.815 |

## Note E.

$n = \frac{F}{22783}$, in which F represents the pressure in lbs. on a square metre, or 1,550 square inches.

We may therefore consider the weight of a cube metre or 61,027 cube inches of steam generated in the boiler under the pressure F, as represented by the following formula:

$$0.1985 + 1.10275 \frac{F}{22783}$$

or more simply, $0.1985 + 0.0000484\,F$. . . . . (6)

and $\gamma = \frac{\Pi}{0.1985 + 0.0000484F}$. . . . . . . . . . . (7)

in which $\Pi$ equals the weight of the steam in lbs. And $\gamma =$ cube metres, or $\gamma \times 60{,}027 = \frac{\Pi \times 60.027}{0.1985 + 0.0000484F}$ equals the volume in cube inches of steam generated in a second.

---

## Note F.

The expression which we here adopt, (from Tredgold) for the value of the power transmitted, differs from the rule very generally admitted, of considering the power used as being a determinate fraction of the power represented by the production of the steam, and which will lead us to express the power transmitted by $k\,(n - 1)$, $k$ being a fractional coefficient. But the formula proposed by Tredgold seems better adapted to the nature of the question, since it is necessary to pay attention to this circumstance, that even whilst the tractive power is nothing, as it would in fact be on a descending slope of 5 millimètres per metre, [= 1 in 200,] the pressure of the steam in the boiler should nevertheless produce the power necessary to make an equilibrium with the exterior atmosphere and to propel the apparatus without a load.

Now the expression $k\ (n-1)$ being equalized to zero, gives $n=1$; a result which does not satisfy the condition of which we have just spoken; whilst the expression $0.5n-1$, equalized to zero, gives $n=2$, which on the contrary is a satisfactory result.

The formula $0.5n-1$, besides, does not differ, for the values of the number $n$, which correspond to circumstances the most common in the working of locomotive engines, from results known by experience, and from the mean values admitted by engineers. Mr. Stevenson, (*Observations on the Comparative Merits of Locomotive and Fixed Engines, &c.* page 29,) takes the pressure which produces the draught equal to the half of the pressure which is in the boiler independent of the exterior atmosphere, which makes $k=6.5$ in the expression $k\ (n-1)$. But this valuation seems too much, especially when the pressure of the steam is not very great. Mr. Wood admits that the useful effect produced by locomotive engines is the $\frac{3}{10}$ of the power produced by the formation of the steam, (*Practical Treatise on Rail-ways,* see pages 227 and 231 of the French translation,) which makes $k=0.3$, a value which on the contrary appears a little low, when we examine with attention all the experiments stated by the author, and especially when we mean to express by $n$ the true pressure under which the steam is produced, a pressure which is generally below that which corresponds with the load on the safety valves. In making $k=\frac{1}{3}$ the formulæ $k(\ n-1)$ and $0.5n-1$, will agree in the case where $n=4$, that is to say, for the pressure which most commonly takes place in the working of the engines.

## NOTE G.

The quantity of steam produced in a unit of time, is the principal element which determines the power of an engine. Mr. Wood's work, in general so useful, still leaves, perhaps, more precise determinations on this point to be desired. From what is said at page 231 of the French translation, the author appears to consider the engine, to which table 11 in the preceding page refers, to be like the *Planet*, which, according to table 9, page 226, vaporates 1249 kilogrammes = 2755 lbs. of water per hour, or $0^k.347$ = .767 lbs. per second.

The dimensions of the *Planet* are besides conformable to those of the machine which we take for an example, and according to Mr. Wood, page 228. The Planet weighs from 5 to 6 tons. We have taken the weight of the steam carried off in a second by the motion of the pistons, as much as $0^k.4$ = .88 lbs., and this estimate does not appear to us to be too high, it being remembered that we have given the engine rather a greater weight. The estimation of which we speak is intermediate between the result which Mr. Wood seems to admit, and that which is taken by Mr. Pouillet for the base of the calculations which he has given in the 3*rd No. of the Portefeuille Industriel du Conservatoire des arts et mètiers*. M. Pouillet takes the weight of steam at 28.75 per min. or $0.479^k$ as the product of an engine; the weight of which is 6.5 tons; and with the water in the boiler and the coal on the bars is 8 tons.

It has appeared to us preferable to keep nearer to the result admitted by Mr. Wood, in order to avoid exaggeration on the power of the engines.

It seems to us besides, that we have not estimated too low, the action of the engine taken as an example, when we remark that the results to which the proposed formulæ lead, correspond to some useful effects which surpass considerably the results given by Mr. Wood in his table 11, page 230 of the translation.

## NOTE H.

It may be remembered, that in making $P=o$, in formulæ (20), U is found $=17^{m}.36,=$ 18.98 yards for the last limit of the permanent velocity which the locomotive engine can take, unless on a descending slope so steep that the action of gravity overcoming the resistance of the friction, the term JP may become negative.

As it is known that the locomotive engines have sometimes, run over extensive portions of the Rail-way from Liverpool to Manchester, or even the entire length, with velocities which were not much below $17^{m}.36=$ 18.98 yards per second, this circumstance might give rise to some doubt, as to the exactness of the expressions which we here employ. But it must be remarked, that the extraordinary velocities in question have been certainly obtained by increasing the combustion, much more than was done in the regular work of the engines, and consequently producing in a unit of time, a quantity of steam greater than we have supposed. Formulæ (15) shows, that the velocity is, all other things being equal, proportional to this quantity of steam.

The greatest velocities cited are those of a journey made by the *Planet* with some electors from Manchester to Liverpool, in 58 minutes, which corresponds to a velocity of about $13^{m}.8=15.09$ yards per second; and that which took place when Mr. Huskisson was carried wounded to Manchester, after the accident which caused his death. They then ran, it is said, a space of 15 miles in 25 minutes, which is equal to about 16 metres=17.5 yard per second.

This last observation is, perhaps, not so much to be depended on as the other.

## Note I.

According to the result given in the *Practical Treatise on Rail-ways*, by Mr. Wood, (page 169, and following, of the translation,) the limit of the effort which an engine can exert without slipping on the wheels, is fixed at $\frac{1}{20}$ of its weight in the case where the four wheels are acted on by the pistons, and at $\frac{1}{25}$ in the case where the pistons only act on the two wheels. It may also be perceived that the limits relate to a particular state of the rails, which seldom occurs, and but for a very short period in France. Further in the calculations which lead to the results of which we speak, Mr. Wood estimates the tractive power at 10 English pounds for each ton of the weight drawn, that is to say, at $\frac{1}{224}$ of this weight, whilst we estimate it here at $\frac{1}{200}$. Finally, Mr. Wood supposes the wheels to be 4 feet in diameter, whilst the wheels that we take are 5 feet; and it is known, that the resistance to slipping increases with the diameter of the wheels. It appears, therefore, that the estimate which we have adopted may be admitted without fear. We find also among the experiments mentioned by Mr. Wood, many cases where the draught has surpassed perceptibly this estimate. For example, we see (page 219 of the translation) that the *Fury* has drawn 46.25 tons over a space more than a league. As her weight was 4.25 tons, and the tender 3.2 tons, the total weight of the train being 53.7 tons, gives a tractive power equal to 0.2685 tons, in reckoning it at $\frac{1}{200}$ of the weight drawn, notwithstanding which the 20th part of the weight of the engine is only 0.2125 ton. We see also, (page 250,) that the *Planet* has taken from Liverpool to Manchester a train weighing in all

86 tons, of which we consequently reckon the tractive force on the level parts at 0.43 tons; nevertheless the engine weighing 6 tons, the 20th of its weight was only 0.3 ton.

---

## NOTE K.

The table given at page 49, converted into English measure, is as follows.

| Rate of ascending slopes. | Ratio of the tractive power to the weight drawn = J. | Total weight of the train which the engine weighing 8 tons can draw without slipping = P. | Net weight carried = $\frac{2}{3}$ (P − 12.000). |
|---|---|---|---|
| | | lbs. | lbs. |
| 1 in 200 | 0.01 | 88,219 | 41,170 |
| 1 in 133 | 0.0125 | 70,575 | 29,406 |
| 1 in 100 | 0.015 | 58,814 | 21,565 |
| 1 in 80 | 0.0175 | 49,010 | 15,030 |
| 1 in 66 | 0.02 | 44,110 | 11,362 |
| 1 in 57 | 0.0215 | 39,209 | 8,496 |
| 1 in 50 | 0.025 | 35,288 | 6,882 |

See table at the end calculated by the late Mr. G. Dodds, Engineer of the Monkland and Kirkintilloch Rail-ways.

# APPENDIX I.

The tables alluded to in the preface are those of Mr. Wood, pages 418, 419, of his work on Rail-ways. These tables are intended to show the gross load which locomotive engines, capable of taking a certain number of tons at a certain number of miles an hour, will drag at different velocities in miles an hour, on different ascending slopes.

The formula by which these tables are calculated is not given, but it appears to be derived in the manner hereafter stated; and as a good deal of discussion has arisen on the subject, I take the liberty to give it at length, with the following extract from Mr. Wood's work, which appears to bear on the subject, and a table, which I have calculated from the aforesaid formula, showing the time required to traverse two given lines of Rail-way, with an engine supposed capable of drawing 40 tons at 15 miles an hour on the horizontal.

Mr. Wood says, "in the construction of these tables (10 and 11), we have supposed the power of the engine to be constantly the same, or the production of steam in the boiler to be constant and regular, or equal quantities in equal times; but a little consideration of the mode by which the steam is generated, will show this is not the case.

" The supply of air for the support of combustion, is almost entirely produced by the exit of the steam into the chimney;

if this is capable of producing a certain effect at twelve miles per hour, when the engine is moving at twenty-four miles per hour, double the same number of cylinders, full of steam, pass into the chimney *in the same time,* and therefore the draught will be greater, and the generation of steam more rapid; and hence we should have, at greater velocities, an increase of effect with the same engine. But as in this case, the piston moves at a correspondingly increased velocity, thereby producing a diminution of effect; and there being also an increase of resistance from the air at greater velocities, perhaps in the absence of experiment, to prove the amount of all these different forces, we ought in practice to suppose the power of the engine constant, and capable of producing, at the higher velocities, an effect equal to that shown in Table XI.

" These tables are formed on the supposition that the load is equal in both directions, as they show the load which the engines are capable of drawing *up* planes of the inclination given; the load *down* the respective planes will be as much greater as the assistance which gravitation affords."

The formula by which these tables are supposed to be calculated may be thus derived. Let $x =$ the gross weight in lbs., and $r$, the denominator of the fraction which represents the ratio of the length of the plane to its height. Then

$\frac{x}{r} =$ the gravity down the plane.

$\frac{x}{2240} =$ gross weight in tons.

$\frac{x}{2240} \times 10 = \frac{x}{224} =$ the resistance in lbs. Then

$\frac{x}{224} + \frac{x}{r} =$ power required up the inclined plane,

$\frac{x}{224} - \frac{x}{r} =$ power required down the same plane.

This formula becomes $rx \pm 224\,x = 224\,pr$; by putting $p =$ power and $x = \frac{224\,pr}{r \pm 224} = \frac{224 \times 450\,r}{r \pm 224}$ in lbs.

or by dividing by 2240

$\frac{x}{2240} = \frac{45r}{r \pm 224}$ in tons, . . . . . . . . . . . . . (1)

Let C= the constant resistance arising from the friction of the working parts of the engine, and which is not influenced by the velocity of the engine. If the absolute power of the engine $=$ P + C. at twelve miles an hour. Then $\frac{(P+C)12}{v} =$ absolute power of the engine at the velocity $v$, and $\frac{(P+C)12}{v} - C =$ the effective power or disposable force of the engine at the velocity $v$, and $\frac{(P+C)12-Cv}{fv} =$ effective power in tons, on the horizontal at the velocity $v$; this value substituted for 45, in equation (1) gives $\frac{12P+12C-vC}{vf} \times \frac{r}{r \pm 224} =$ tons that can be drawn up the inclined plane $r$, with the velocity $v$, and generally $\frac{v^1 P + v^1 C - v C}{vf} \times \frac{r}{2 \pm 224} =$ tons, . . . (2) that can be drawn up the inclined plane represented by $r$, with the velocity $v$, by an engine that is capable of exerting an effective power of P, with the velocity $v^1$.

Supposing, as in Table 10, that the engine is capable of drawing forty-five tons on the horizontal at a velocity of twelve miles an hour, the power it then exerts is 450lbs. taking the resistance to be $\frac{1}{224}$ part of the weight. Then by making P=450; $v^1 = 12$; C= 300 $= 10$; this formula is reduced to

$$\frac{12 \times 450 + 12 \times 300 - v300}{10v} \times \frac{r}{r+224} =$$

$$\frac{540 + 360 - v30}{v} \times \frac{r}{r+224};$$ which will give the numbers in Table 10. Thus suppose $r$, the rate of inclination, to be 200, and the velocity $v = 12$, the above formula becomes

$\frac{540 + 360 - 360}{12} \times \frac{200}{424} = 45 \times \frac{200}{424} = 21.22$ as given in the tables in the column of velocity 12 and opposite the rate of inclination 200. From the formula $\frac{v^1 P + v^1 c - vc}{vf} \times \frac{r}{r-224} =$ tons $=$ T, we have the velocity $v = \frac{v^1 Pr + v^1 cr}{Trf + r\,c - 224 f\,T}$ . . . . . . (3)

and by putting $v^1 = 15$, $P = 40$, $c = 400$, this formula becomes

$$= \frac{15r}{112+r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the time required to traverse each plane will be expressed by $.75D\frac{(112+r)}{15r} = t''$; . . . . . . . . . . . (5)

in which D = the length of the plane in chains.

From these formulæ the time required to traverse each of two proposed lines of Rail-way between the points A and B, but by different routes, have been calculated. See Tables 1 and 2.

From those tables it appears that an engine with a given load will traverse the line (AD) in both directions in $14.1^h.\ 53^m.\ 53^s.$ and the line AB in $13^h.\ 57'.\ 37^s$; the difference in time will therefore be 14. 1. 53 — 13. 57. 37 = 4′ 16″ in favour of the line AB.

As far as time alone is concerned, this will probably be a very near approximation ; but no further : it does not enable us to say which line is actually the best, or over which a given weight can be carried at the least expense.

# TABLE No. 1.

*Showing the Time that an Engine capable of drawing 40 Tons on the horizontal at 15 miles an hour will traverse the Rail-way A D, with the same load.*

| Distances in Chains. | Differences in Level, A to D. Rise in Feet. | Fall in Feet. | Rate of Inclination. | Velocity in miles per hour. A to D. | D to A. | Time occupied on each plane in minutes. From A to D. | From D to A. |
|---|---|---|---|---|---|---|---|
| 169 | ... | ... | Level | 15·00 | 15·00 | 8·45 | 8·45 |
| 397 | 15 | ... | 1 in 1746 | 14·09 | 16·02 | 21·13 | 12·34 |
| 263 | 31 | ... | 1 - 560 | 12·5 | 18·75 | 15·78 | 10·52 |
| 305 | ... | ... | Level | 15·00 | 15·00 | 15·22 | 15·22 |
| 534 | 174 | ... | 1 in 202 | 9·64 | 33·66 | 41·54 | 11·89 |
| 488 | 64 | ... | 1 - 500 | 12·25 | 19·33 | 29·86 | 18·62 |
| 257 | 46 | ... | 1 - 264 | 10·53 | 26·05 | 18·30 | 7·39 |
| 326 | ... | ... | Level | 15·00 | 15·00 | 16·3 | 16·3 |
| 588 | ... | 155 | 1 in 250 | 27·17 | 10·35 | 16·23 | 42·60 |
| 528 | ... | ... | Level | 15·00 | 15·00 | 26·4 | 26·4 |
| 202 | ... | 40 | 1 in 330 | 22·70 | 11·19 | 6·67 | 13·53 |
| 501 | ... | ... | Level | 15·00 | 15·00 | 25·05 | 25·05 |
| 61 | ... | 10 | 1 in 400 | 20·83 | 11·71 | 2·19 | 3·90 |
| 290 | ... | ... | Level | 15·00 | 15·00 | 14·5 | 14·5 |
| 110 | ... | 22 | 1 in 330 | 22·70 | 11·19 | 3·63 | 7·37 |
| 393 | ... | ... | Level | 15·00 | 15·00 | 19·65 | 19·65 |
| 170 | ... | 34 | 1 in 330 | 22·70 | 11·19 | 5·63 | 11·39 |
| 152 | ... | ... | Level | 15·00 | 15·00 | 7·6 | 7·6 |
| 40 | ... | 4 | 1 in 660 | 18·06 | 12·82 | 1·66 | 2·84 |
| 392 | ... | ... | Level | 15·00 | 15·00 | 19·6 | 19·6 |
| 600 | ... | 120 | 1 in 330 | 22·70 | 11·19 | 19·82 | 40·21 |
| 113 | ... | 17 | 1 - 440 | 20·12 | 11·95 | 4·04 | 7·09 |
| 268 | ... | ... | Level | 15·00 | 15·00 | 13·4 | 13·4 |
| 136 | ... | 20 | 1 in 449 | 19·98 | 12·00 | 5·10 | 8·5 |
| 75 | ... | 8 | 1 - 610 | 18·37 | 12·67 | 3·02 | 4·38 |
| 230 | ... | ... | Level | 15·00 | 15·00 | 11·5 | 11·5 |
| 54 | 5 | ... | 1 in 715 | 12·96 | 17·78 | 3·12 | 2·27 |
| 436 | ... | ... | Level | 15·00 | 15·00 | 21·8 | 21·8 |
| 40 | ... | 8 | 1 in 330 | 22·70 | 11·19 | 1·32 | 2·68 |
| 87 | ... | ... | Level | 15·00 | 15·00 | 4·35 | 4·35 |
| 96 | ... | 19 | 1 in 330 | 22·70 | 11·19 | 3·17 | 6·43 |
| 186 | ... | 24 | 1 - 500 | 19·33 | 12·25 | 7·21 | 11·38 |
| | | | | | Sum. | 413·21 | 428·65 |
| | | | | | | h. m. sec. 6 53 14 | h. m. sec. 7 8 39 |

| | h. | m. | sec. |
|---|---|---|---|
| From A to D . | 6 | 53 | 14 |
| D to A . | 7 | 8 | 39 |
| | 14 | 1 | 53 |

# TABLE No. II.

*Showing the Time that an Engine capable of drawing* 40 *Tons on the horizontal at* 15 *miles an hour, will traverse the Rail-way A B, with the same load.*

| Distances in Chains. | Differences in Level, A to B. | | Rate of Inclination. | Velocity in miles per hour. | | Time occupied on each plane in minutes. | |
|---|---|---|---|---|---|---|---|
| | Rise in Feet. | Fail in Feet. | | A to B. | B to A. | From A to B. | From B to A. |
| 21 | . . . | 4·6 | 1 in 310 | 23·48 | 11·01 | 0·66 | 1·43 |
| 30 | 0·6 | . . . | 1 3960 | 15·17 | 12·13 | 1·48 | 1·85 |
| 382 | 43·0 | . . . | 1 590 | 18·51 | 12·60 | 15·47 | 22·73 |
| 196 | 121·6 | . . . | 1 107 | 33·66 | 7·32 | 4·36 | 20·00 |
| 165 | . . . | 22·3 | 1 490 | 19·44 | 12·20 | 6·36 | 10·14 |
| 260 | . . . | 36·3 | 1 473 | 19·65 | 12·12 | 9·92 | 16·06 |
| 195 | . . . | 0·3 | Level | 15·00 | 15·00 | 9·75 | 9·75 |
| 48 | 1·3 | . . | 1 in 2530 | 14·36 | 15·69 | 2·50 | 2·25 |
| 1096 | 147·6 | . . . | 1 490 | 12·20 | 19·44 | 67·37 | 42·28 |
| 707 | . . . | 49·0 | 1 950 | 17·00 | 13·42 | 31·19 | 39·51 |
| 727 | . . . | 68·0 | 1 700 | 17·85 | 12·93 | 30·54 | 42·10 |
| 42 | . . . | 1·0 | 1 2600 | 15·67 | 14·38 | 2·01 | 2·19 |
| 187 | . . . | 13·3 | 1 930 | 16·68 | 13·09 | 8·40 | 10·71 |
| 324 | . . . | 31·9 | 1 670 | 18·01 | 12·85 | 13·49 | 18·91 |
| 872 | . . . | 21·0 | 1 2740 | 15·63 | 14·41 | 41·20 | 45·38 |
| 82 | . . . | 4·9 | 1 1140 | 16·63 | 13·65 | 3·69 | 4·50 |
| 433 | . . . | 10·9 | 1 2662 | 15·65 | 14·39 | 20·75 | 22·56 |
| 149 | 6·9 | . . . | 1 1460 | 13·93 | 16·25 | 8·02 | 6·87 |
| 289 | . . . | 4·3 | 1 4700 | 15·36 | 14·65 | 14·11 | 14·79 |
| 611 | . . . | 42·0 | 1 960 | 16·98 | 13·43 | 26·98 | 34·12 |
| 677 | . . . | 3·6 | Level | 15·00 | 15·00 | 33·85 | 33·85 |
| 203 | 15·0 | . . . | 1 in 890 | 13·32 | 17·15 | 11·43 | 8·87 |
| 41 | . . . | . . . | Level | 15·00 | 15·00 | 2·05 | 2·05 |
| 159 | . . . | 11·0 | 1 in 960 | 16·98 | 13·43 | 7·02 | 8·88 |
| 75 | . . . | 0·6 | Level | 15·00 | 15·00 | 3·75 | 3·75 |
| 352 | 17·0 | . . . | 1 1360 | 13·85 | 16·34 | 19·06 | 16·15 |
| | | | | | Sum. | 395·41 | 441·80 |
| | | | | | | h. m. sec. 6 35 25 | h. m. sec. 7 21 48 |

| | h. | m. | sec. |
|---|---|---|---|
| From A to B | 6 | 35 | 25 |
| B to A | 7 | 21 | 48 |
| | 13 | 57 | 37 |

# APPENDIX II.

*On the different Methods of Estimating Forces, and on what is understood by* la force vive, (vis viva.)

It has already been shown that two forces, F and F′, applied to the same moveable body, were to each other as the velocities they communicated to that body. Let us now consider these forces when they are applied to different masses, and let us suppose at first that two equal forces directly opposed to each other, act on two masses, M and M′ equal and spherical; they will communicate to these masses two velocities $v$ and $v'$, which will be equal.

M and M′ in meeting will press mutually, and will be in equilibrium, because every thing is equal on each side. But if we had $M = nM'$, and that V′ was greater than V, we should consider M as composed of the masses $m'$, $m''$, $m''$, . . . $m^{(n)}$, each equal to $M^1$. It is certain that in virtue of the mutual connexion of the parts which compose solids, one of these masses cannot move without drawing the others along with it, so that if the body M should move, for example, through three metres, per second, each of the masses $m'$, $m''$, $m'''$, &c., should also move through three metres per second, which is the same thing as to say that if V is the velocity of M, the masses $m'$, $m''$, $m'''$, &c., shall each have the velocity V. Now if $m'$, moving with the velocity V, impinges against the mass M′, which, by hypothesis, is equal to it, it will destroy in V′ a velocity equal to V; and if in the same time $m''$, acting by the interposition of the other masses strike also against M′, they will again destroy a part V, of V′, and so of the rest. So that all these masses united will spontaneously destroy in V′, a velocity equal to $n$V; and in supposing that the velocity V′ is then exhausted, there will be an equilibrium: it is necessary then that, in this case, $V' = nV$. Eliminating $n$, between this equation and the equation $M = nM'$, we shall have the proportion, $M : M' :: V' : V$, which shows that when a mass M contains $n$ times the matter of another, these masses ought to be in the inverse ratio of their velocities, in order that they may be in equilibrium.

This proposition still holds good when M does not contain M′ an exact number of times, which would be easily demonstrated by infinitessimals, or the method of limits.

Consequently from what precedes that, since the velocities of two bodies are in the inverse ratio of the numbers of the material particles they contain, it follows that when these bodies are of the same volumes and of different densities, that their velocities are in the inverse ratio of their densities.

Suppose now that the force F, which communicates to the mass M the velocity V, acts upon a mass which is M times less, and which may consequently be represented by $\frac{M}{M} = 1$, this force will communicate to the mass 1 a velocity which is equal to M times that which F will communicate to M; this velocity will then be expressed by MV.

For the same reason, the force F′, which communicates to M′ the velocity V′, will communicate to the mass $\frac{M'}{M} = 1$, a velocity M′ V′.

The velocities MV and M′ V′, being those which are communicated by the forces F and F′, to the same mass 1, it follows from the principles of the velocities being proportional to the forces, that we have $F : F' :: MV : M'V'$.

The expressions MV and M′V′, are what we call the *quantity of motion* communicated by the forces F and F′ to the moving bodies M and M′.

The unit of force being arbitrary, we can represent it by the quantity of motion which it communicates to the moving body. Thus, in supposing that F′ is this unit of force, we shall replace F′ by M′V′, in the preceding proportion, and we have by it $F = MV$.

If we consider the force $\phi$, which acts instantaneously, it has been shown that this force is represented by the velocity which it will communicate to the moving body in the unit of time, if the movement should become all at once uniformly accelerated, we shall then have, in putting for V its value $\phi$ $F = MQ$.

This equation shows further, that $\phi$ is the force which should act upon a unit of the mass; for if we make $M = 1$, we have $F = \phi$. $\phi$ being the accelerating force, F is that which is called the moving force.

It has been shown, that in calling $g$ the force of gravity, P the weight of the body, and M its mass, we had $P = Mg$; if, between this equation and the preceding, we eliminate M, we shall find $F = P\frac{\phi}{g}$; so that when the accelerating force $\phi$ is that of gravity, $\phi = g$, and the preceding equation is reduced to $F = P$, in this case the accelerating force is therefore estimated by the weight of the body on which it acts.

There was formerly a celebrated dispute amongst geometricians on the measure of forces. This dispute, as many others, originated from a misunderstanding of the definition of the words.

A force being known to us only by its effects, may be measured in different manners, according to the use to which it is desired to appropriate it. For example, if it be proposed to determine the burden which a man can sustain for an instant, it is evident that the force of this man will be proportional to the weight which he will be capable of sustaining, and consequently can be represented by this weight; but if it be desired to measure the force of this man by the work he can execute in a given time, there will be another manner of estimating his force; and which will be entirely different from the other, because we feel that a weaker man with a greater disposition to sustain a longer fatigue might produce in his work a greater result, and in this point of view be considered as endowed with a greater force than the other.

In this second mode of considering the power of a man, we shall regard it as proportional to the weight which he lifts, and to the height to which he shall have raised it in a given time; it being understood that we do not suppose that the effort varies in proportion to the height, because, in fact, this height only represents the number of times that a certain work is repeated. Thus, in supposing that two men raising in one day's work the same weight, the one 600 metres high, and the other 200 metres, in this manner of estimating the force, we shall look upon one of these men as having three times as much power as the other.

It follows further from this, that if, in the same day's work, two men raising the first 20 kilogrammes to 200 metres, and the second 10 kilogrammes to 400 metres, we should regard them, according to the present hypothesis, as having equal powers, although really the intrinsic power of these men may be very different; but here we regard them only in proportion to the work done.

It is in this manner that Descartes estimated the power of a man, or of any other motive power, the dispute which caused a difference of opinion between him and other geometricians, only rested upon the definition of the word *power*. He maintained that a power should be measured by the mass into the square of the velocity. We proceed to show how, when we consider bodies in motion, that this definition of Descartes of the word power (force), leads to this consequence.

Suppose P the weight raised, and $h$ the height to which it ought to be raised in a given time; the power in the hypothesis of Descartes will then be measured by the product $P \times h$.

We can in this expression replace P, by its value $Mg$,* and we shall have $Ph = Mgh$; multiplying by 2, it will become $2Ph = M \times 2gh$; observing that the square of the velocity $v$ due to the height $h$, has for expression $2gh$, we can replace $2gh$ by $v^2$; this gives $2Ph = Mv^2$.

Having defined the word power (*force*) differently from Des-

* In a former part of this work the author shows that the coefficient $g$ represents the force of gravity.

cartes, we shall not say as he does, that $Mv^2$ is the measure of a power (*force*), because we have shown that a power (*force*) should be represented by the quantity of motion $Mv$ that it produces. Thus, to avoid all ambiguity, we shall employ a new denomination in giving, according to usage, the name of *force vive,* to the product $Mv^2$ of the mass by the square of the velocity.

*La force vive* (*vis-viva*) is of great utility when the effect of a machine is desired to be known. If it be required, for example, to apply a flow of water, to move a carriage on a given road, to compress a mass of air, to draw from a mine a certain quantity of coals, &c., in all these cases we can *compare* the effect of the moving power, to the product of a certain weight by a given length; consequently, to an expression of the form $Ph$, of which the double, as we have just demonstrated, becomes the product $Mv^2$.

Through the kindness of my friend Thomas Graham, Esq., of Nantes, I am enabled to lay before the public the following valuable table, calculated by the late W. George Dodds, Engineer of the Monkland and Kirkintilloch Rail-ways in Scotland.

| Inclination of Rail-way from a level to 1 in 25. | Weight of locomotive engine exclusive of the tender, &c. | Power of engine in pounds by adhesion in the worst state of weather. | Power of engine in pounds by adhesion in the best state of weather. | Gravity per ton at the differentinclinations of the Rail-way. | Powerper ton to overcome the friction of the carriages and load in the worst state of weather for the adhesion of the engine wheels. | Powerper ton to overcome the friction of the carriages and load in the best state of weather for the adhesion of the engine wheels. | Load of engine at a speed of 4 miles per hour worst weather. | Load of engine at a speed of 4 miles per hour best weather. | Load of engine at a speed of 8 miles per hour worst weather. | Load of engine at a speed of 8 miles per hour best weather. | Load of engine at a speed of 16 miles per hour worst weather. | Load of engine at a speed of 16 miles per hour best weather. | Load of engine at a speed of 20 miles per hour worst weather. | Load of engine at a speed of 20 miles per hour best weather. | Remarks<br>* Signifies a weight less than that of the engine itself. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | tons | pounds | pounds | | pounds | pounds | tons | tons | tons | tons | tons | tons | tons | tons | |
| Level | 7½ or 16,80.lbs. | 672 | 1810 | .00 | 8 | 10.8 | 84. | 167.75 | 42. | 83.7 | 21. | 41.9 | 16.8 | 33.5 | The gravity of a ton (or any other given weight) is found by multiplying the weight of the body by the height of the plane and dividing the sum by its length. |
| 1 in 800 | | | | 2.8 | | | 62.2 | 133. | 35.7 | 74.1 | 19.3 | 39.3 | 15.7 | 31.8 | |
| 1 in 400 | | | | 5.6 | | | 49.4 | 110. | 31.1 | 66.5 | 17.8 | 37. | 14.7 | 30.3 | |
| 1 in 200 | | | | 11.2 | | | 35. | 82.2 | 24.6 | 55.1 | 15.5 | 33.2 | 13.1 | 27.7 | |
| 1 in 100 | | | | 22.4 | | | 22.1 | 54.5 | 17.5 | 41.1 | 12.3 | 27.5 | 10.7 | 23.6 | |
| 1 in 96 | | | | 23.3 | | | 21.4 | 53. | 17. | 40.3 | 12.1 | 27.2 | 10.6 | 23.4 | |
| 1 in 50 | | | | 44.8 | | | 12.7 | 32.5 | 11. | 27.2 | 8.7 | 20.5 | 7.9 | 18.3 | |
| 1 in 25 | | | | 89.6 | | | 6.8 | 18. | 6.3 * | 16.2 | 5.5 * | 12.6 | 5.1 * | 12.5 | The friction of the carriages is taken at one two hundredth part of their weight; and is a constant quantity so long as the speed remains the same, but increases as the square of the velocity. |
| 1 in 96 | engine weighing 5 tons, or 11,200 lbs | 448 | 1206 | 23.3 | 8 | 10.8 | 56.9 | 111.8 | 28. | 55.8 | 14. | 27.9 | 11.2 | 22.3 | |
| 1 in 96 | engine weighing 6 tons, or 13440 | 537.6 | 1448 | 23.3 | 8 | 10.8 | 67.2 | 134.2 | 33.6 | 66.9 | 16.8 | 33.5 | 13.4 | 26.8 | |

This table is made from the two extremes, the best and worst state of the weather for the locomotive engines. The loads will of course vary through all the intermediate states of the Rails, &c. The weather being always uncertain, the lowest load is the only one that can be guaranteed.

(Signed) GEO. DODDS. *Jan.* 26, 1831.

LONDON:
ROAKE AND VARTY, PRINTERS, 31, STRAND.