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ABSTRACT

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Within the context of the general linear regression model and the squared error loss measure for gauging estimator performance, preliminary test, Stein-like and least squares estimators are specified, evaluated and compared. It is noted that the Stein-James positive part and the modified Stein-James estimators, not commonly seen in the econometric literature, are uniformly superior over the range of the parameter space to the conventional and preliminary test estimators respectively.

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ON POST DATA MODEL EVALUATION

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In spite of the rapid advances over the last three decades of economic theory, econometric procedures, and data relating to economic processes and institutions, the search for quantitative economic knowledge still remains to some extent an essay in persuasion. In the process of non-experimental model building there are typically many admissible economic and statistical models which do not contradict our perceived knowledge of human behavior. Thus, in model specification there is usually uncertainty, for example, relative to the algebraic form, classification, number and timing of variables to be included in the behavioral and technical relations, and the corresponding stochastic assumptions. When econometric models are correctly specified, statistical theory provides procedures for obtaining point and interval estimates and evaluating the performance of various linear and usually unbiased (at least asymptotically) estimators. But, the applied worker must inevitably work with false models, where the true specification of the sampling model is unknown. Furthermore, the statistical model employed is usually determined by some preliminary test hypothesis using the data at hand. This search procedure, involving two stage or repeated significance test procedures applied to the same set of data and yielding an estimate after a preliminary test(s) of significance, is often used in applied work in economics with little or no information on the sampling properties of the resulting estimator and with little or no consideration to the possible distortion of subsequent inferences.

Within this context, the purpose of this article is to generalize and extend

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*Arnold Zellner read an early draft of this paper and made a large number of comments which were helpful in revising this article.

the results of Wallace and Ashar [12] relative to preliminary test or two stage estimating procedures and call attention to another class of estimators. In particular, we review the possible statistical consequences of using preliminary test or sequential estimators in the search process and suggest old and new estimators, that are superior, under a squared error loss measure for gauging estimator performance, to the conventional estimators usually employed. We also note that conventional estimating procedures currently used in applied work may not be appropriate for the problem at hand and, perhaps more importantly for the researcher, we show that better alternative estimators exist.

When making a choice between estimators the traditional solution is to restrict consideration to the class of unbiased estimators and hope that among the estimators in the restricted class one has uniformly smallest risk. Fortunately for many problems a best linear unbiased estimate exists. In this paper, in discussing the estimators that are alternatives to the conventional least squares estimator, we will leave the class of linear unbiased estimators. However, the notion of unbiasedness which has been accepted by or perhaps forced on applied workers, although intuitively plausible, is an arbitrary restriction or property and has no direct connection with the loss due to incorrect decisions. The economist who is interested in parameter estimates or predictions appropriate for choice purposes, may not care if he is right on the average, and thus the unbiasedness property may be unsatisfactory from a decision point of view. In any event our purpose, which is expository in nature, is to focus on point estimation and bring the statistical consequences of making use of these alternative estimators to the attention of the applied researcher.

In section II, we develop the regression statistical model, conventional estimators and tests, define a criterion for gauging estimator performance and

contrast the performance of restricted and unrestricted least squares estimators. In section III, we discuss the preliminary test or sequential estimator which results when one uses a preliminary test of hypothesis, using the data at hand to reach a decision concerning the choice of an estimate. In section IV, we discuss the class of Stein-James [3,5] estimators and the recent modification of these estimators proposed by Sclove, et al. [6]. The estimators are contrasted in section V and suggestions are made for choosing between the alternatives.

2. The Statistical Model and Conventional Estimators

Assume the linear hypothesis model

(2.1) $$\underline{y} = X\underline{\beta} + \underline{e},$$

where $\underline{y}$ is a $(T \times 1)$ vector of observations, X is a $(T \times K)$ matrix of non-stochastic variables of rank K, $\underline{\beta}$ is a $(K \times 1)$ vector of unknown parameters and $\underline{e}$ is a $(T \times 1)$ vector of unobservable normal random variables with mean $\underline{0}$ and covariance $\sigma^2 I$. The unrestricted least squares estimator is

(2.2a) $$\underline{b} = (X'X)^{-1} = S^{-1}X'\underline{y},$$

where $\underline{b}$ is distributed normally with mean $\underline{\beta}$ and covariance $\sigma^2 S^{-1}$, and an unbiased estimate of σ^2 is given by $\hat{\sigma}^2 = \frac{(\underline{y}-X\underline{b})'(\underline{y}-X\underline{b})}{T-K}$. As is well known for the model (2.1), $\underline{b}$ is the maximum likelihood estimator, is unbiased and under a quadratic loss measure is minimax.

In addition to the sample information (2.1), suppose we either want to test in the conventional way or utilize additional information which consists of J general linear hypotheses about the unknown parameters in $\underline{\beta}$ which we specify as

(2.3) $R\underline{\beta} - \underline{r} = \underline{0}$ or $R\underline{\beta} - \underline{r} = \underline{\delta}$,

where $\underline{r}$ is a $(J \times 1)$ vector of known elements, R is a $(J \times K)$ known matrix with rank J, and $\underline{0}$ is a $(J \times 1)$ null vector and $\underline{\delta}$ is a $(J \times 1)$ vector representing specification errors in the perceived information, which are zero if that information is correct.

The restricted least squares or general linear hypothesis estimator, which makes use of both the sample and exact prior information or linear hypotheses, (2.1) and (2.3), is

(2.4a) $\hat{\underline{\beta}} = \underline{b} - S^{-1}R'(RS^{-1}R')^{-1}(R\underline{b}-\underline{r})$,

where $\hat{\underline{\beta}}$ is normally distributed with mean $[\underline{\beta}-S^{-1}R'(RS^{-1}R')^{-1}\underline{\delta}]$ and variance $\sigma^2[S^{-1}-S^{-1}R'(RS^{-1}R')^{-1}RS^{-1}]$.

If the restrictions (hypotheses) <u>are correct</u>, $\underline{\delta} = \underline{0}$, the restricted least squares estimators are unbiased and have smaller variances than do the unrestricted least squares estimators. If the prior restrictions are incorrect, $\underline{\delta} \neq \underline{0}$, the estimator $\hat{\underline{\beta}}$ has the following mean square error, involving bias and variance, associated with it:

(2.5)
$$E(\hat{\underline{\beta}}-\underline{\beta})(\hat{\underline{\beta}}-\underline{\beta})' = \sigma^2 S^{-1} - \sigma^2 S^{-1}R'(RS^{-1}R')^{-1}RS^{-1} + S^{-1}R'(RS^{-1}R')^{-1}\underline{\delta}\underline{\delta}'(RS^{-1}R')^{-1}RS^{-1}.$$

Given this general formulation for the statistical model and estimators, let us for expository purposes assume that the X's in (2.1) are orthogonal and thus $X'X = I_K$.[1/] Also assume for the linear hypotheses (2.3), $R = [I_J \;\; 0]$.[1/] Under this specification, the least squares estimator is

(2.2b) $\underline{b} = (X'X)^{-1}X'\underline{y} = X'\underline{y}$,

where $\underline{b}$ is normally distributed with mean $\underline{\beta}$ and covariance $\sigma^2 I$. The restricted estimator (2.4a) becomes

$$\hat{\beta} = \begin{bmatrix} r \\ b_{K-J} \end{bmatrix},$$

where $\hat{\beta}$ is normally distributed with mean $(\beta-\delta)$, covariance $\sigma^2[I_K - \begin{pmatrix} I_J & 0 \\ 0 & 0 \end{pmatrix}] = \sigma^2\begin{pmatrix} 0 & 0 \\ 0 & I_{K-J} \end{pmatrix}$ and mean square error $\sigma^2\begin{pmatrix} 0 & 0 \\ 0 & I_{K-J} \end{pmatrix} + \begin{bmatrix} \delta\delta' & 00 \\ 0 & 00 \end{bmatrix}$.

Given these results, if we follow tradition and use a quadratic or square error loss measure for evaluating estimator performance [11], we have the following expected loss or risk for the least squares and restricted least squares estimator, respectively:

$$(2.6) \qquad \rho(b,\beta) = E(b-\beta)'(b-\beta) = \sigma^2 \mathrm{tr} I_K = \sigma^2 K$$

and

$$(2.7) \qquad \rho(\hat{\beta},\beta) = E(\hat{\beta}-\beta)'(\hat{\beta}-\beta) = \sigma^2 \mathrm{tr}\begin{pmatrix} 0 & 0 \\ 0 & I_{K-J} \end{pmatrix} + \mathrm{tr}\delta\delta'$$

$$= \sigma^2(K-J) + \delta'\delta = \sigma^2(K-J) + (\beta_J - r)'(\beta_J - r).$$

Under this expected squared error loss criterion an estimator $\hat{\theta}$ is said

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[1]/Instead of assuming conditions on the X'X and R matrices, one can reparameterize the model to hypotheses by multiplying β by a non-singular matrix W such that $\theta = W\beta$ and generate the equivalent formulation

$$y = Z\theta + e$$

and

$$[I_J \quad 0]\theta = r_o,$$

where $Z'Z = I_K$ and $W = QP$, where P is a non-singular matrix such that $P(X'X)^{-1}P' = I_K$ and Q is an orthogonal matrix such that $Q(P^{-1})'R'(RS^{-1}R')^{-1}RP^{-1}Q' = \begin{pmatrix} I_J & 0 \\ 0 & 0 \end{pmatrix}$. An estimator $\tilde{\theta}$ for θ yields an estimator $W^{-1}\tilde{\theta} = \tilde{\beta}$ for β. Transforming from β to θ also transforms the measure of goodness. The statistical implications of this transformation when making estimator comparisons are discussed in Bock, et. al. [2].

to be better than another $\tilde{\theta}$ if the risk of one estimator is equal to or less than the other, i.e., $E(\hat{\theta}-\theta)'(\hat{\theta}-\theta)-E(\tilde{\theta}-\theta)'(\tilde{\theta}-\theta) \leq 0$. In general risk functions for alternative estimators cross somewhere over the range of the parameter space and there is no risk function, for all θ, which is uniformly superior to (below) all others. In the case of the restricted and unrestricted least squares estimator

(i) if the hypotheses <u>are correct</u>, the risk of $\hat{\beta}$ is less than the risk of b, i.e.,

$$\rho(\hat{\beta},\beta) < \rho(b,\beta) \quad \text{or} \quad \sigma^2(K-J) < \sigma^2K;$$

(ii) if the hypotheses <u>are incorrect</u> whether the risk of $\hat{\beta}$ is equal to, greater than or less than the risk of b depends on δ or (β_J-r), the size of the specification error in the hypotheses. If, for example, we are considering the null hypothesis case where r is equal to a null vector then the size of the risk depends on the magnitude of β_J.

Making use of the risk functions (2.6) and (2.7), the risk of the restricted estimator is less than or equal to the risk of the conventional least squares estimator if

$$(2.8) \qquad \rho(b,\beta) - \rho(\hat{\beta},\beta) \geq 0$$

or

$$\sigma^2K - \sigma^2(K-J)+\delta'\delta \geq 0.$$

In order for (2.8) to be true, the condition

$$(2.9) \qquad \frac{\delta'\delta}{\sigma^2} = \frac{(\beta_J-r)'(\beta_J-r)}{\sigma^2} \leq J.$$

must be satisfied. We therefore have a somewhat typical situation where the risk functions of the two estimators cross.

The characteristics of typical risk functions for the restricted and unrestricted least squares estimators are given in Figure 1.

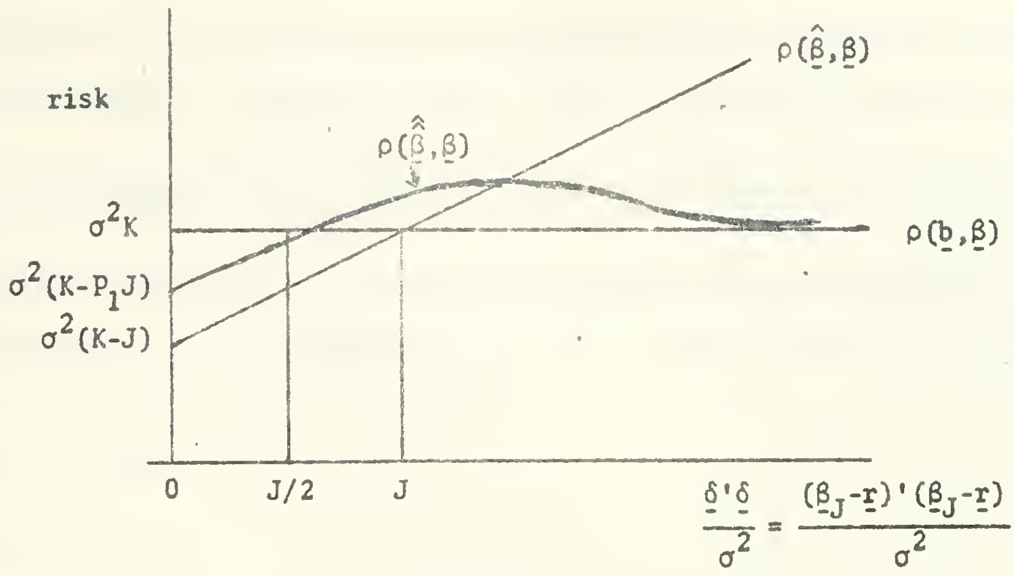

Figure 1.
Typical Risk Functions for the Least Squares, Restricted Least Squares and Preliminary Test Estimators.

As the errors of the hypotheses get larger and larger, i.e., the magnitude of $\underline{\delta}$ or $(\underline{\beta}_J-\underline{r})$ increases, the risk of the restricted least squares estimator $\hat{\underline{\beta}}$ is unbounded.

3. The Preliminary Test Estimator

As noted in the introduction, in applied work there is generally uncertainty concerning the statistical model that generated the data and the investigator usually proceeds by using the procedure of statistical testing followed by estimation. Thus, it is conventional when deciding questions concerning the inclusion or deletion of a variable, the algebraic form of the relation, etc., to use likelihood ratio procedures and test the hypothesis $H:\underline{\beta}_J = \underline{r}$ against not H, using the test statistic

(3.1) $$u = (\underline{b}_J-\underline{r})'(\underline{b}_J-\underline{r})/J\hat{\sigma}^2$$

and reject H if u is greater than some critical value c, where u is distributed as the non-central F distribution with J and T-K degrees of freedom and non-centrality parameter $\frac{(\underline{\beta}_J-\underline{r})'(\underline{\beta}_J-\underline{r})}{2\sigma^2}$. Thus, the hypothesis H, that $\underline{\beta}_J = \underline{r}$, is tested against not H, by using the test statistic u and rejecting H if $u \geq F^{\alpha}_{(J,T-K)} = c$. The value of c is determined, for a given level of the test, α, by

(3.2) $$\int_c^{\infty} dF(u) = \alpha,$$

where F is a central F distribution with J and T-K degrees of freedom. By accepting H, we take $\hat{\underline{\beta}}$ as our estimate of $\underline{\beta}$, and by rejecting H, we use $\underline{b}$, the unrestricted least squares estimate.

In this conventional two stage testing procedure, estimation is dependent on a preliminary test of significance, which implies the use of the preliminary test estimator,

(3.3) $$\hat{\hat{\underline{\beta}}} = I_{(0,c)}(u)\hat{\underline{\beta}} + I_{[c,\infty)}(u)\underline{b},$$

where $I_{(0,c)}(u)$ and $I_{[c,\infty)}(u)$ are indicator functions which are one if u falls in the interval subscripted and zero otherwise.

Since this is the actual estimator used by researchers in the process of post data model construction or in their attempt to learn from data, this is the estimator whose properties should be of interest to us, and we should at least want to know how it compares with the conventional least squares estimator which does not make use of preliminary tests of significance.

Sclove, et al. [6] and Bock, Yancey and Judge [2] have shown that the risk for the preliminary test estimator may be written as

$$(3.4) \qquad \rho(\hat{\hat{\beta}},\beta) = E(\hat{\hat{\beta}}-\beta)'(\hat{\hat{\beta}}-\beta) = \sigma^2K - J\sigma^2P_1 + 2(p_1-2p_2)\frac{\delta'\delta}{\sigma^2},$$

where P_1 and P_2 are probabilities that non-central F random variables are equal to or less than the critical value c. Since σ^2K is the risk of the conventional estimator this means that if the risk of the preliminary test estimator is to be smaller than the risk of the least squares estimator the sum of the last two expressions in (3.4) must be negative. As Sclove, et al. [6] and Bock, et al. [2] have shown, the condition for this to occur that is least favorable to the preliminary test estimator is that

$$(3.5) \qquad \frac{\delta'\delta}{\sigma^2} = \frac{(\beta_J-r)'(\beta_J-r)}{\sigma^2} \leq \frac{J}{2}.$$

Thus, once again we have two estimators which yield risk functions that cross. If the general linear hypotheses are correct, δ or $(\beta_J-r) = 0$, then the risk of the preliminary test estimator from (3.4) is $\sigma^2(K-Jp_1)$ where p_1 is a probability and is thus less than or equal to 1. In this situation, as was the case with the restricted estimator, the risk of the preliminary test estimator is less than the conventional estimator. When $\frac{\delta'\delta}{\sigma^2} = \frac{(\beta_J-r)'(\beta_J-r)}{\sigma^2} \geq J$, the preliminary test has a risk that is greater than that of the conventional estimator. As the specification error, δ, gets larger and larger, the risk of the preliminary test estimator reaches a maximum and for very large values of δ, the risk of the preliminary test estimator approaches that of the conventional least squares estimator. The characteristics of a typical risk function for the preliminary test estimator is depicted in Figure 1.

Since the probabilities p_1 and P_2 in (3.4) are a function of the critical value c, the level of the preliminary test risk function is a function of α,

the level of the test. As α approaches 1, the p_i's approach zero and the preliminary test risk function approaches that of the conventional least squares. As α approaches 0 and the p_i's approach 1, the risk function for the preliminary test estimator approaches that of the restricted least squares estimator. Therefore, the choice of α affects the relative gains and losses that one, using the preliminary test estimator, may incur.

In summary, let us note that in the neighborhood of the region where the hypotheses are correct, and in particular if $\frac{(\underline{\beta}_J-\underline{r})'(\underline{\beta}_J-\underline{r})}{\sigma^2} \leq \frac{J}{2}$, the risk for the preliminary test estimator is less than that for the conventional estimator. However, for a wide range of the parameter space, i.e., $\frac{(\underline{\beta}_J-\underline{r})'(\underline{\beta}_J-\underline{r})}{\sigma^2} \geq \frac{J}{2}$, the risk for the preliminary test estimator is greater than the risk for the least squares estimator. Since $\underline{\beta}_J$ or $\underline{\beta}_J-\underline{r}$ is unknown, if one chooses the preliminary test estimator, and this is the estimator most often used by the economic researcher, he chooses an estimator which does well if his hypotheses are correct at the expense of doing very badly over a considerable interval of the parameter space. This possible loss in a decision context or distortion concerning subsequent inferences raises serious questions concerning the use of conventional hypotheses testing procedures for post data model construction. In addition, these results point to the fact that when preliminary tests are employed, the researcher faces a decision problem concerning the optimum level of the test and/or an assumption concerning the degree of or distribution of hypothesis specification error.

4. The Stein-Like Estimators

Given the questionable virtue of the use of preliminary tests of significance and the corresponding preliminary test estimators as one goes through the process of data dredging, we wish to now review and evaluate a class of estimators initially suggested by Stein [8] and Stein and James [3]. What we would like, of course, is an estimator that has a smaller risk than the conventional one over all of the parameter space. However, this was not true for the preliminary test and restricted estimators and as Stein and James [3, 7] have shown, this is impossible in the one or two regressor case, i.e., K or J $\leq$ 2. Fortunately, for the usual several parameter (hypothesis) case, Stein and James [3, 8] have shown that in estimation under squared error loss that <u>the conventional estimator $\underline{b}$ is dominated</u> (uniformly inferior over the entire parameter space) <u>by the Stein-James estimator</u>

$$(4.1) \qquad \underline{\beta}^* = (1-c/u)(\underline{b}-\hat{\underline{\beta}}) + \hat{\underline{\beta}} = \underline{b} - c/u(\underline{b}-\hat{\underline{\beta}}),$$

under the conditions $J \geq 3$ and $0 < c < 2(J-2)/(T-K+2)$, where c, u, $\underline{b}$ and $\hat{\underline{\beta}}$ are defined in (3.2), (2.3), (2.6) and (?.1), respectively. They further show that the optimal (minimum risk) choice of the critical value c is $c = (J-2)/T-K+2$, and thus the Stein-James estimator (4.1) becomes

$$(4.2) \qquad \underline{\beta}^* = [1 - (J-2)/(T-K+2)u](\underline{b}-\hat{\underline{\beta}}) + \hat{\underline{\beta}}.$$

This means when the above conditions on J and c are fulfilled the risk of the Stein-James estimator is less than the risk of the least squares estimator over the range of the parameter space, i.e., $\rho(\underline{\beta}^*,\underline{\beta}) < \rho(\underline{b},\underline{\beta})$ and therefore the least squares estimators are inadmissible. Stein [7] has shown for $J \leq 2$ there is no other estimator that has a smaller risk than the least squares estimator over

the parameter space. The risk for $\underline{\beta}^*$ when $(\underline{\beta}_J-\underline{r})$ or $\underline{\delta} = \underline{0}$ (correct hypothesis), when using the optimum value of c, is $\sigma^2(K-J+2) + \frac{\sigma^2 2(J-2)}{(T-K+2)}$, and the risk goes to $\sigma^2 K$, the risk of the conventional estimator, as the hypothesis error gets larger and larger. These characteristics of the typical risk function for the Stein-James estimator are reflected in Figure 2.

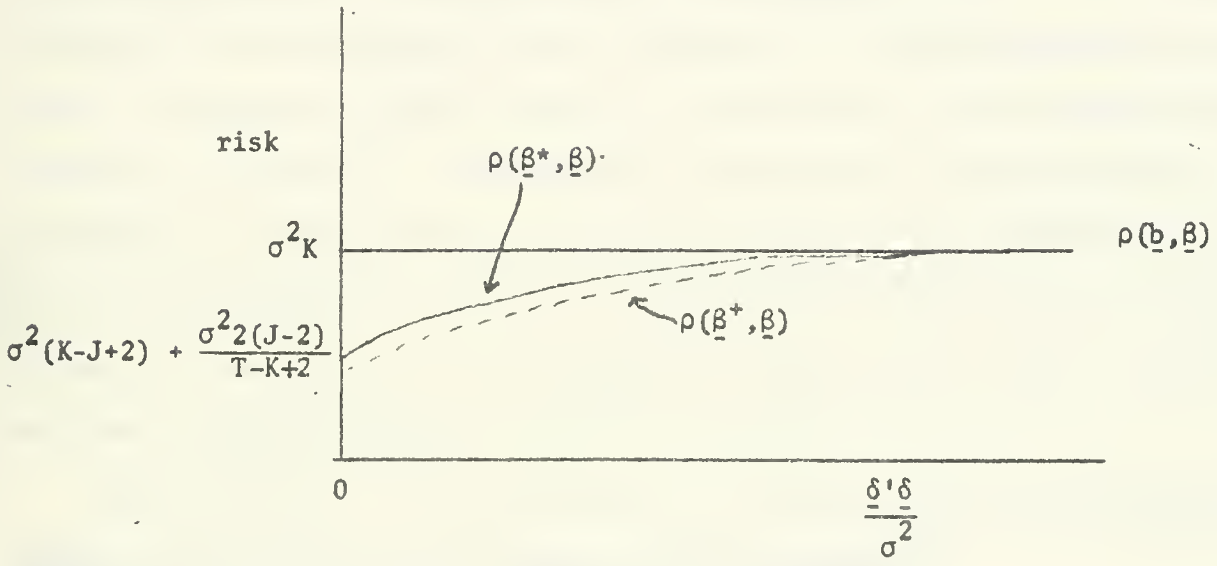

Figure 2.
Comparison of the Risk Functions
for the Stein-James and Least Squares Estimators

Baranchik [1] and Stein [9] generalized this result and have shown that the Stein-James estimator (4.1), (4.2) is dominated by the corresponding positive part estimator

(4.3) $\underline{\beta}^+ = (1-c^*/u)^+(\underline{b}-\hat{\underline{\beta}}) + \hat{\underline{\beta}} = I_{[c^*,\infty)}(u)(1-c^*/u)(\underline{b}-\hat{\underline{\beta}}) + \hat{\underline{\beta}}$,

when the restrictions $J \geq 3$ and $0 < c^* < 2(J-2)/(T-K+2)$ are fulfilled. This estimator has the form

$$\underline{\beta}^{+} = \hat{\underline{\beta}} \qquad \text{if } u < c^*$$

and

$$\underline{\beta}^{+} = (1-c^*/u)(\underline{b}-\hat{\underline{\beta}}) + \hat{\underline{\beta}} \qquad \text{if } u > c^* .$$

Thus, when u is large relative to c*, then $\underline{\beta}^{+}$ is approximately equal to $\underline{b}$. Unlike (4.1), no one value of c is uniformly optimal in the positive part estimator (4.3). Given this result, Sclove, et al. [6] conjectured and Bock proved under the conditions noted above that the positive part estimator (4.3) is uniformly superior to the preliminary test estimator (3.3). Thus, the positive part estimator provides a minimax substitute for the preliminary test estimator. This result, although important in establishing the risk ranking of the hierarchy of estimators is conditioned by the fact that the risk of the two estimators are approximately the same over the range of the parameter space and for the usual values of T and K ($T < 20$ and $K < 10$), and the risks of the preliminary test and positive part estimators are approximately equal to that of the least squares estimator.

Building on this work, Sclove, et al. [6] have shown that the following modified version of the Stein-James positive part estimator

(4.4) $$\underline{\beta}^{**} = I_{[c,\infty)}(u)(1-c^*/u)(\underline{b}-\hat{\underline{\beta}}) + \hat{\underline{\beta}}$$

is uniformly superior to the preliminary test estimator (2.10) over the range of the parameter space and over all values of c, the critical value of the test. The estimator has the form

$$\underline{\beta}^{**} = \hat{\underline{\beta}} \qquad \text{if } u \leq c$$

and

$$\underline{\beta}^{**} = (1-c^*/u)(\underline{b}-\hat{\underline{\beta}}) + \hat{\underline{\beta}} , \text{ if } u > c.$$

When $c \leq c^*$, the modified version is the same as the original positive part

estimator (4.3). As was the case in going from (4.1) to (4.3), the modified Stein estimator (4.4) can be improved upon by its positive part. The characteristics of the modified positive part estimator are reflected in Figure 3.

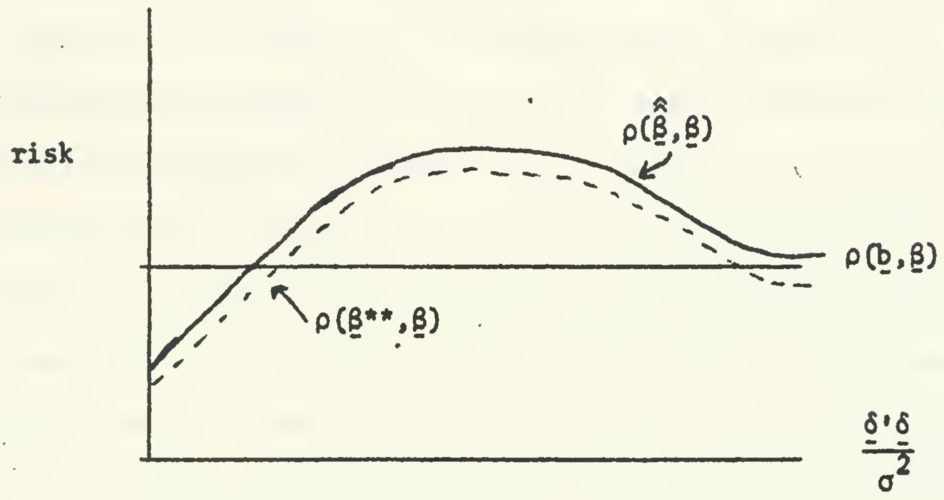

Figure 3
Comparison of Typical Risk Functions for the
Preliminary Test and Modified Positive Part Estimators

5. Concluding Remarks

These estimators and their corresponding sampling characteristics clearly point to the unsatisfactory nature of conventional search procedures involving preliminary tests of significance based on the data at hand. Alternatively, the Stein-like estimators which involve only minor addition computational burdens are superior to conventionally used estimators over a range of conditions often found in practice. In particular, the following conclusions seem to emerge:

(i) If the number of regressors (hypotheses) is greater than 2, the Stein-James positive part estimator, $\underline{\beta}^{+}$, should be used in preference to the conventional least squares estimator since the risk of this

estimator is uniformly smaller over the range of the parameter space

(ii) If one engages in the economist's favorite past-time of post data model construction (e.g., using a preliminary test of significance based on the data at hand to decide whether to include or exclude a particular variable from the relation) then the modified Stein-James estimator, $\underline{\beta}^{**}$, is uniformly superior over the range of the parameter space to the conventional preliminary test estimator, $\hat{\hat{\underline{\beta}}}$.

While the Stein-like estimators are superior to those conventionally employed, they, like the least squares estimator, fail to satisfy the conditions necessary for a generalized Bayes estimator and thus are not admissible [10]. Given this situation with the sampling theory estimators, one might be led to consider a Bayesian approach to this problem. Within this context, Zellner and Vandaele [13] have developed Bayesian interpretations and alternatives to the preliminary test and Stein-like estimators and the posterior mean estimators that result minimize average risk and are admissible and consistent. In contrast to the sampling theory estimators discussed in this paper which emphasize point estimation, the Bayes pre test estimator yields a posterior distribution which may be used for inference purposes. However, in the search for an acceptable model, the problem of how to formulate an appropriate prior for the Bayes pre test estimator remains just as the problem of the optimal level of the test with the sampling theory preliminary test estimators.

What we need to combat this state of affairs, as Leamer [4] has noted, is a new set of statistical procedures for drawing inferences based on false models and a new set of decision rules to use in model evaluation and construction. Until such time arrives, it would appear that if the investigator wants to stay with sampling theory estimators and is willing to leave the class of linear unbiased estimators and use mean square error or risk as a basis for gauging estimator performance, he would be well advised to consider a version of the Stein-James estimator.

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