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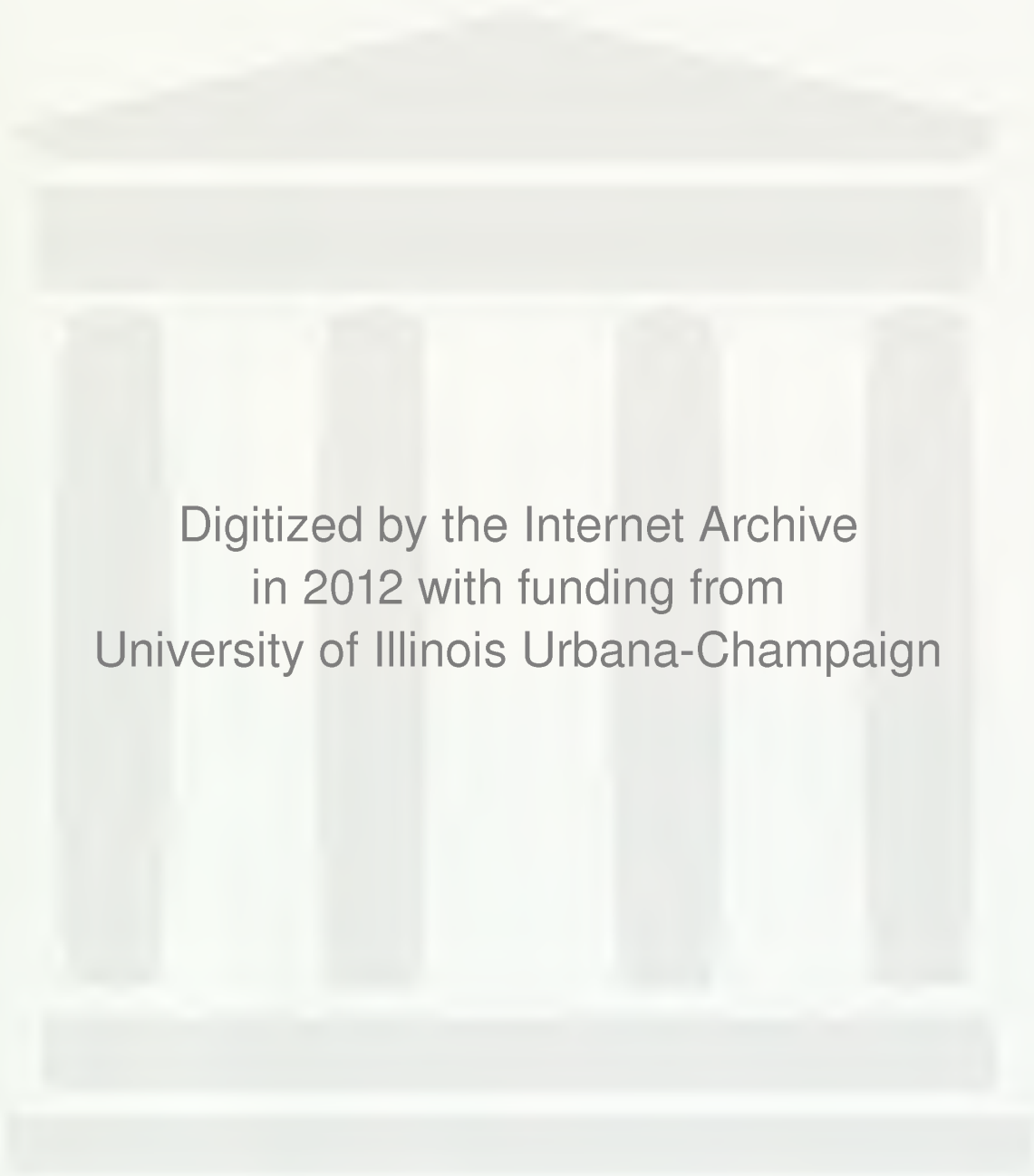
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October 3, 1975

# OPERATING LEVERAGE AS A DETERMINANT OF BUSINESS RISK

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# I. INTRODUCTION*

The theory of operating leverage has been receiving increased attention in the literature of business finance. Lev [6] and Percival [7] have related the concept to a security's risk in the context of the capital asset pricing model. Reilly and Bent [8], on the other hand, have related operating leverage to the more traditional measure of a firm's business risk, the coefficient of variation of operating income. In view of this recent interest in the subject and its importance to the financial manager, the purpose of this paper is to identify the important properties of operating leverage, develop and test its functional relationship to business risk, and elucidate what appears to be a popular misconception about the subject.

Section II reviews the most recent literature about operating leverage and the standard treatment afforded it by some current textbooks. Using both mathematical and graphical analyses, Section III develops the important properties of operating leverage. In Section IV the functional relationship between business risk, operating leverage, and sales volatility is derived. The next section presents the results of an empirical test of the functional relationship expressed in the preceding section. A cross-sectional test is also made to determine whether a change in business risk is more a function of a change in operating leverage or a change in sales volatility. The final section offers a brief review of the important findings, discusses some implications of these findings, and makes suggestions for future research.

*The authors wish to thank John M. Wachowicz for his helpful comments and suggestions.

## II. REVIEW OF THE LITERATURE

The treatments of operating leverage by Weston and Brigham and Van Horne appear typical of those textbooks that include discussions of the topic. After setting forth the elementary principles of break-even analysis and defining operating leverage as "the extent to which fixed costs are used in operations," Weston and Brigham [9, pp.60-64] derive a measure of how much operating leverage a firm is employing--the degree of operating leverage (DOL), which indicates the percentage change in operating income given a percentage change in quantity sold from a particular output level. Later in their book, they relate operating leverage to financial leverage and illustrate how the two combine to produce the same or different fluctuations in earnings per share.

Van Horne [10, pp.696-704] also approaches the subject of operating leverage by first introducing the break-even analysis topic. His definition of operating leverage, however, is slightly different, i.e. "the employment of an asset with a fixed cost in the hope that sufficient revenue will be generated to cover all fixed and variable costs." After developing the DOL variable, Van Horne proceeds to unite operating and financial leverage in order to demonstrate how the combination increases the risk of potential earnings per share.

Bierman and Hass [1, pp.93-98] discuss the theory of operating leverage in terms of its importance to the firm's business risk complexion. Specifically, by assuming both a constant cost structure and probability distribution of quantity sold, they are able to derive formulas for the mean, the standard deviation and the coefficient of variation of operating income. They then demonstrate how an increase in fixed costs increases

the coefficient of variation of operating income, thereby increasing business risk. Finally, they show that the fixed cost/variable cost mix eventually chosen will depend upon the financial manager's risk-return preference function.

The purpose of Reilly and Bent's working paper [8] is to define, measure, and analyze the concept of operating leverage. Arguing that the DOL specification measures only the effect of the fixed and variable cost mix, they define a firm's amount of operating leverage as the proportion of fixed operating costs to total operating costs. Because firms do not show a complete breakdown of fixed and variable costs, they derive proxies which they feel will be highly correlated with this proportion and use these proxies to analyze operating leverage at the levels of the aggregate economy, the industry, and the firm.

With respect to the aggregate economy, they interpret their measures as showing a consistent increase in operating leverage from 1946-72. At the industry level, they construe the ranges of their measures to mean that industries differ substantially in the use of operating leverage. They also conclude that depreciation coverage is the best measure of industry operating leverage because it has a significant negative correlation with their earnings volatility measures and their measures of the effect of operating leverage. Furthermore, their multivariate analysis indicates that operating leverage is more important than sales volatility in explaining industry business risk.

Finally, at the firm level, Reilly and Bent again find the earnings volatility measures to be most strongly associated with depreciation coverage, as are the measures of effect. Their multiple regression

results are mixed, however, depending upon which measure of earnings volatility is used as the dependent variable. When business risk is measured as the coefficient of variation of absolute earnings, the operating leverage variables are more important than the sales volatility variable. On the other hand, when the coefficient of variation of percent earnings changes is the dependent variable, sales volatility is more important, but the relationship is not statistically significant.

Two other articles on the subject deserve attention, since they discuss operating leverage in the context of the capital asset pricing model. The first, by Percival [7], is a purely theoretical development in which he demonstrates that an increase in operating leverage will increase the covariance of a security's return with that of the market by a factor which is proportional to the increase in the contribution margin but independent of the new break-even quantity. His analysis implies that an increase in fixed costs, the contribution margin held constant, does not increase the covariance despite the fact that the break-even point increases. Thus, a firm's position relative to its break-even point is a portion of diversifiable risk. Percival also disputes the use of the DOL variable as a measure of operating leverage because with an increase in fixed costs and decrease in variable costs, DOL may increase, decrease, or remain the same. Depending on the new break-even point, the result may be inconsistent with that under the capital asset pricing model. Finally, he suggests that the weakness of the DOL measure lies in the fact that it is not derived from a specific valuation function.

In the second article relating operating leverage to the capital asset pricing model, Lev [6] demonstrates that both the overall risk and the systematic risk of a common stock will be positively associated with

the firm's operating leverage, or negatively associated with the firm's level of variable costs. To test this hypothesis, he samples three industries (electric utility, steel, and oil production) and obtains each firm's estimate of average variable costs by running a simple time series regression of total costs against quantity in terms of physical output or dollar sales. He also calculates two risk measures for each firm: (1) overall risk is computed as the standard deviation of monthly returns, and (2) a systematic risk estimate (beta) is obtained using Sharpe's market model. Both risk measures are cross-sectionally regressed for each industry on the unit variable cost estimate. As expected, Lev finds that average variable cost is negatively associated with both risk measures and that it generally explains a larger portion of the cross-sectional variability of overall risk than it does for systematic risk. While the associations are statistically significant, the $R^2$s are modest, suggesting that operating leverage is not the only factor contributing to cross-sectional risk differentials.

## III. THEORY OF OPERATING LEVERAGE

Because understanding break-even analysis is crucial to understanding operating leverage, the very basics of this analytical tool are developed initially in this section.[1] After this, the important properties of operating leverage are set forth, and a popular misconception about the subject is clarified.

Under the assumption of linear cost and revenue functions, the operating profit equation can be stated as

$$\pi = Q(P - V) - F \,, \tag{1}$$

where P equals price per unit, V equals variable cost per unit, F equals

fixed expenses, and Q equals quantity produced and sold. At the break-even quantity, $Q_{BE}$, operating profits are equal to zero. Thus, setting Equation (1) equal to zero and solving for $Q_{BE}$ gives

$$Q_{BE} = F/(P - V) . \tag{2}$$

The above equation states that the firm's break-even quantity is determined by the absolute level of fixed costs and the contribution margin, P - V, which is the excess of price over variable cost per unit. Alternatively, the contribution margin is the rate of change of operating income per unit change in output:

$$d\pi/dQ = P - V . \tag{3}$$

As sales exceed the break-even point, a larger contribution margin will mean greater absolute increases in operating profits than a smaller contribution margin. A low contribution margin requires large increases in quantity sold to achieve noticeable increases in profits.

While the contribution margin determines what the absolute change in operating profits will be, operating leverage determines what the percentage change in operating profits will be as sales change. In this respect, the standard definition of operating leverage is the extent to which fixed expenses are important to the production of the firm's output. With the presence of any fixed charges, a percentage change in sales will be magnified into a greater percentage change in operating income because, as sales change in either direction, operating expenses will change less than proportionally. Fixed costs, then, are the lever that magnifies profit changes with respect to output changes, and the fulcrum of this lever is positioned at the break-even point.

While the "degree of operating leverage" has been the term commonly used to describe the measure of the effect of operating leverage, "operating elasticity" will be used here because it is an elasticity analogous to the familiar price elasticity of demand.[2] Thus, from Equation (1), operating elasticity at $Q$ units of output is

$$\varepsilon = \frac{Q}{\pi} \cdot \frac{d\pi}{dQ} = \frac{Q(P - V)}{Q(P - V) - F} \quad . \tag{4}$$

The interpretation of $\varepsilon$ is that it is a pure number which is unique to each output level $Q$, and it measures the percentage change in operating profits that will result from a percentage change in quantity sold from $Q$. As an example, if current sales of 100 units have an associated operating elasticity equal to 6, a 10 percent increase in sales to 110 units will result in a 60 percent increase in operating income. On the other hand, operating income will decline 60 percent if quantity sold drops to 90 units.

Various alternative expressions for operating elasticity can be derived from the above formulation, each having its own special advantages when discussing the effect of operating leverage for a given cost structure or the change in operating elasticity that takes place when the cost structure is altered. Thus, by dividing both the numerator and denominator of (4) by the contribution margin, $P - V$, operating elasticity is expressed in terms of the break-even point associated with a given cost structure:

$$\begin{aligned} \varepsilon &= Q/[Q - F/(P - V)] \\ &= Q/(Q - Q_{BE}) \quad . \end{aligned} \tag{5}$$

The equation of an equilateral hyperbola, with asymptotes parallel to the

coordinate axes, has the form

$$c = (x - h)(y - k) . \tag{6}$$

(h, k) is the center of the hyperbola, about which it is symmetric, and $x = h$ and $y = k$ are the asymptotes [3, pp.84-92]. Therefore, rewriting (5) as

$$Q_{BE} = (Q - Q_{BE})(\varepsilon - 1) \tag{7}$$

means that the graph of $\varepsilon$ against Q is an equilateral hyperbola with asymptotes $x = Q_{BE}$ and $y = 1$. Assuming the firm expects to operate above its break-even point, this graph will generally plot as

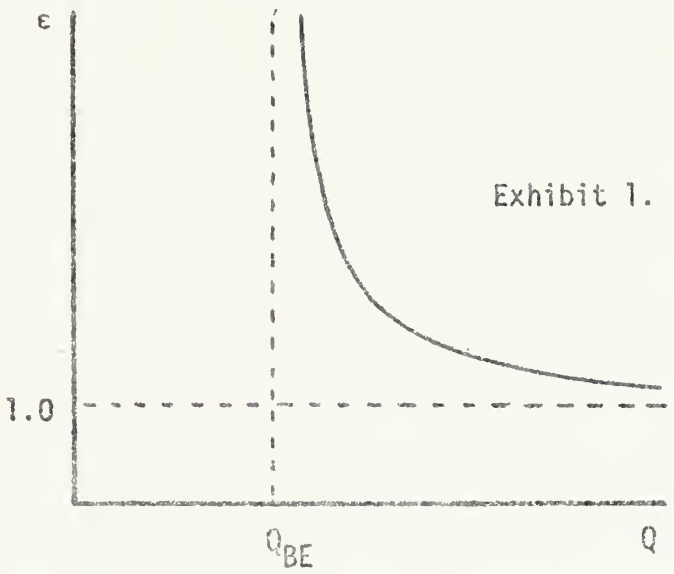


Exhibit 1.

Both Equation (5) and the graph in Exhibit 1 are helpful to understanding how operating elasticity will change at each output level if the contribution margin or fixed expenses change. Remembering that $Q_{BE} = F/(P - V)$, one can see that any change which raises the break-even quantity will increase operating elasticity at those levels of output above the new $Q_{BE}$, while a

change which lowers the break-even quantity will decrease operating elasticity at those levels of output above the old $Q_{BE}$. In terms of the graph in Exhibit 1, the asymptote, $x = Q_{BE}$, moves to the right and the curve moves outward from the new center. In the second instance, the asymptote, $x = Q_{BE}$, moves to the left and the curve moves inward toward the new center. It is also important to note that a substitution of fixed for variable expenses (or vice-versa) may leave the break-even point the same, resulting in no change in operating elasticity at each level of output; the hyperbola in Exhibit 1 would maintain its position. The table below summarizes the different combinations of change and their effect on $Q_{BE}$ and $\varepsilon$.

Exhibit 2.

| | | $\Delta(P - V)$ | | |
|---|---|---|---|---|
| | | − | 0 | + |
| $\Delta F$ | − | $\Delta Q_{BE} = ?$<br>$\Delta\varepsilon = ?$ | $\Delta Q_{BE} = -$<br>$\Delta\varepsilon = -$ | $\Delta Q_{BE} = -$<br>$\Delta\varepsilon = -$ |
| | 0 | $\Delta Q_{BE} = +$<br>$\Delta\varepsilon = +$ | $\Delta Q_{BE} = 0$<br>$\Delta\varepsilon = 0$ | $\Delta Q_{BE} = -$<br>$\Delta\varepsilon = -$ |
| | + | $\Delta Q_{BE} = +$<br>$\Delta\varepsilon = +$ | $\Delta Q_{BE} = +$<br>$\Delta\varepsilon = +$ | $\Delta Q_{BE} = ?$<br>$\Delta\varepsilon = ?$ |

The denominator of Equation (4) equals operating income while the numerator can be stated as operating income plus fixed expenses; therefore, another expression for operating elasticity is[3]

$$\varepsilon = (\pi + F)/\pi \ . \qquad (8)$$

This equation can also be rearranged to show that it is an equilateral hyperbola with $\pi = 0$ and $\varepsilon = 1$, the x and y asymptotes, respectively:

$$F = (\pi - 0)(\varepsilon - 1) \ . \qquad (9)$$

The graph of $\varepsilon$ against operating profits then generally resembles:

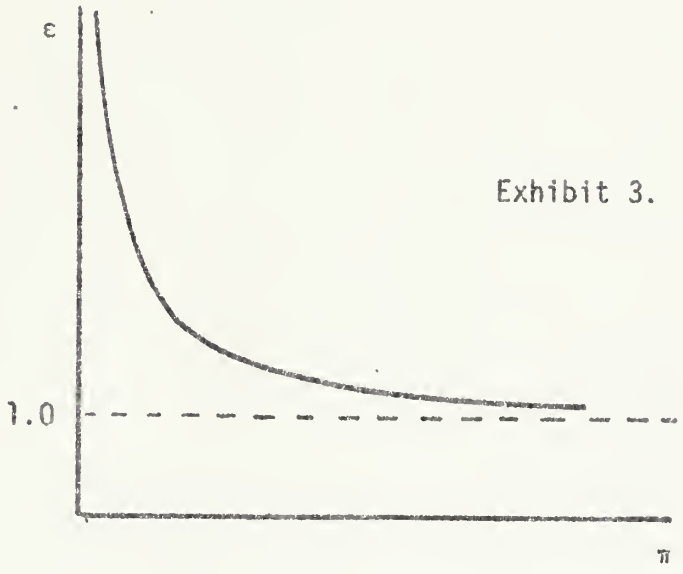


Exhibit 3.

This formulation emphasizes the fact that, for a given amount of fixed expenses, there is a unique operating elasticity associated with each level of operating income. Moreover, the contribution margin and quantity sold are not integral to the equation -- the hyperbola will occupy the same position as long as fixed expenses remain constant. It is certainly true, though, that the contribution margin and quantity sold will determine the firm's position on the hyperbola.

The significance of this one-to-one correspondence between $\varepsilon$ and $\pi$, for a given level of fixed expenses, is best understood by example. For purposes of illustration, assume that the firm substitutes fixed for

variable costs. In this instance, Equation (8) indicates that each level of positive profits will now have a higher elasticity associated with it, and in terms of Exhibit 3, the hyperbola will move outward from its center at (0, 1). Thus, if the firm is able to sell that quantity which gives it the same operating income as before the substitution, a percentage change in output from this quantity will result in a greater percentage change in operating income than will occur under the old cost structure. In other words, equal operating incomes under the two cost structures have different degrees of risk in terms of operating leverage, the cost structure having the larger fixed expenses also having the larger $\varepsilon$.

The preceding example may be generalized: Any change in fixed costs, no matter what the change in either price per unit or variable cost per unit, will alter the operating elasticity at each level of operating income. If fixed expenses increase, operating elasticity will increase at each level of $\pi$, with the reverse holding true for a decrease. The implication is that, for two firms having the same absolute dollar amount of operating income, the one having the larger fixed expenses is the more risky with respect to potential fluctuations in operating income. Furthermore, this conclusion is independent of the proportion of fixed to variable costs.

The final expression for operating elasticity can be derived by dividing both the numerator and denominator of Equation (4) by $Q(P - V)$, which results in

$$\varepsilon = 1/(1 - Q_{BE}/Q) \ . \qquad (10)$$

Again, this equation can be rearranged to take the form of an equilateral

hyperbola with asymptotes $x = 1$ and $y = 1$:[4]

$$1 = (Q/Q_{BE} - 1)(\varepsilon - 1) . \qquad (11)$$

The graph of this equation is exactly

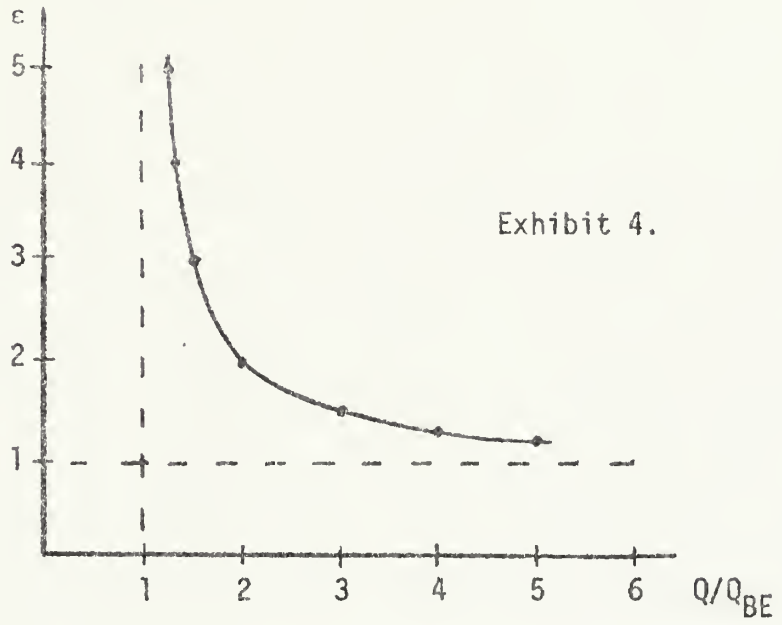


Exhibit 4.

The above formulation brings out three important points. First, the effect of using some fixed costs to produce output is greatest near the break-even point. As can be seen by inspecting Equation (10), if $Q = Q_{BE}$, the denominator equals zero and operating elasticity is undefined. As Q approaches $Q_{BE}$ from above, operating elasticity goes to infinity. Intuitively, any change from a quantity very near the break-even level will cause an extremely large percentage change in operating income because the base operating income is very close to zero. The second point brought out by Equation (10) is that for a given cost structure, operating income becomes less sensitive to volume changes as the firm's output increases from its break-even point. This is easily seen by noting that, as Q increases

from $Q_{BE}$, $Q_{BE}/Q$ approaches zero in the limit and $\varepsilon$ approaches one.

The final and most important point can be discerned by examining Equation (11). In this equation, c, referring to the general form of the equilateral hyperbola given by Equation (7), always equals one and, therefore, is not affected by a change in the firm's cost structure. For each $\varepsilon$ there is one, and only one, value of $Q/Q_{BE}$, which is a measure of a firm's level of output relative to its break-even point. This unique association means that the hyperbola in Exhibit 4 is the same for all firms that produce with some fixed costs. At a particular point in time, every firm plots somewhere on this hyperbola.[5] Thus, the riskiness of manufacturing a product using some fixed expenses depends upon the expected sales position relative to the break-even position. If, after the cost structure is altered, expected output is the same relative distance from the new break-even point as before the change, the risk due to operating leverage is not increased because the operating elasticity remains constant. The risk accompanying operating leverage is affected only if the relative distance between expected quantity and the break-even point rises or falls.

Supportive empirical results are supplied by Reilly and Bent [8]. Searching for a measure of the amount of operating leverage, they compute a depreciation coverage ratio as a proxy for the fixed/total costs ratio, and this measure exhibits significant inverse correlation with their measures of the effect of operating leverage. It can be demonstrated that depreciation coverage is in fact an approximation of $Q/Q_{BE}$. In terms of the standard symbols used in break-even analysis, their depreciation coverage ratio equals

$$Q(P - V) \div F .$$

Dividing both the numerator and denominator by (P - V) gives

$$\frac{Q(P - V)}{(P - V)} \div \frac{F}{(P - V)} = \frac{Q}{Q_{BE}} .$$

The $Q/Q_{BE}$ proportion is the determinant of the sensitivity of operating income to changes in output. While this point may appear obvious to most, it is usually glossed over in the literature, perhaps due to the conceptualization of operating leverage as the ratio of either fixed to total operating costs [4,8] or fixed to variable operating costs [6]. Since this definition is usually accompanied by a statement to the effect that the greater either ratio, the greater the use of operating leverage, the impression is left that when comparing two companies with the same sales volatility, the one having the greater proportion of fixed to total costs will have the more uncertain income stream. This conclusion is not necessarily correct in all cases. Consider the following example:

| Firm A | Firm B |
|---|---|
| P = \$10 | P = \$10 |
| V = \$5 | V = \$4 |
| F = \$10,000 | F = \$12,000 |
| $Q_{BE}$ = 2000 units | $Q_{BE}$ = 2000 units |

Since operating elasticity can be expressed as $\varepsilon = Q/(Q - Q_{BE})$, and since both firms have the same $Q_{BE}$, they also have the same $\varepsilon$ at all levels of output. However, at all levels of output it is also true that Firm B's ratio of fixed to total costs is greater than Firm A's. But even though B's proportion is higher, if both have the same expected sales position,

then risk in terms of operating elasticity is the same. Furthermore, the results of the next section can be used to show that if both firms have the same expected value and standard deviation of sales, they will have the same degree of business risk as measured by the coefficient of variation of operating income.

Needless to say, other examples can be devised in order to illustrate the inadequacy of using these ratios to distinguish the amount of operating leverage from the effect of operating leverage, as measured by operating elasticity. As demonstrated by the above example, a larger amount of operating leverage does not always indicate a greater effect. If, indeed, the distinction between amount and effect is necessary, either $Q/Q_{BE}$ or total fixed costs is a better definition of amount. This is because there is a definite correlation between $Q/Q_{BE}$ and $\varepsilon$ as well as between fixed costs and $\varepsilon$. The higher $Q/Q_{BE}$, the lower $\varepsilon$, and the greater fixed costs are, the greater $\varepsilon$ will be at all levels of operating income.

## IV. OPERATING LEVERAGE, SALES VOLATILITY, AND BUSINESS RISK

Business risk, the inherent uncertainty in the physical operations of the firm, has its impact on the variability of the operating income stream, and is a function of factors of both marketing and production. Business risk in marketing arises from general economic conditions, competitive market structure, product demand characteristics, pricing intricacies, and other factors, the sum of which combine to influence sales volatility. Business risk in production, on the other hand, arises primarily from changing labor conditions, raw materials prices, and administrative expenses, as well as from technological developments. In other words, the risk associated with this area results from those factors which affect fixed

and variable costs, the break-even position, and ultimately the operating elasticity at the expected level of sales. Business risk, then, is a function of both sales volatility and operating leverage.

The coefficient of variation of operating income, $CV(\pi)$, has wide acceptance as a measure of business risk. Adopting it as their measure, Reilly and Bent [8] seek to establish whether sales volatility or operating leverage is the greater contributor to business risk. They do not, however, specify the exact form of the relationship among the three variables. Instead, they use multiple regression, thereby assuming linearity, to test whether the explanatory power lies more with their operating leverage variables or their sales volatility variable. Bierman and Hass [1, pp.93-98], as mentioned previously, use the $CV(\pi)$ as the measure of business risk and illustrate the influence of operating leverage when the cost structure changes. They do not show, however, that the importance of operating leverage may also vary when the cost structure is held constant. In view of this and the conclusion of the preceding section that sales position relative to the break-even point determines the significance of operating leverage to financial planning, the analysis below will demonstrate that for a given cost structure, the coefficient of variation of operating income is the product of operating elasticity at the expected sales volume and the coefficient of variation of sales.

Assume sales volume, Q, is a normally distributed random variable with an expected value of $E(Q)$ and a standard deviation of $\sigma(Q)$. It is well known that a linear combination of a normally distributed random variable is itself normally distributed. Therefore, assuming a given cost structure

(P, V, and F are constants), the expected value of operating income, $E(\pi)$, is

$$E(\pi) = (P - V)E(Q) - F,$$

with a variance equal to

$$\sigma^2(\pi) = (P - V)^2\sigma^2(Q) .$$

Given the above, the coefficient of variation of operating income, $CV(\pi)$, a measure of business risk, can be expressed as

$$\begin{aligned} CV(\pi) &= \sigma(\pi)/E(\pi) \\ &= [(P - V)\sigma(Q)]/[(P - V)E(Q) - F] . \end{aligned} \tag{12}$$

The analysis up to this point is similar to that of Bierman and Hass. They end their derivation here, however, and illustrate the change in business risk that takes place if the firm changes its cost structure. For example, an inspection of Equation (12) shows that an increase in fixed costs will increase the $CV(\pi)$, as will a decrease in the contribution margin. The effect of a tradeoff between fixed costs and the contribution margin, though, is not so obvious. Here the influence on business risk depends upon whether the break-even quantity increases, decreases, or remains the same.

That the last is the case may be seen by refining Equation (12) further. Dividing both the numerator and denominator by $(P - V)E(Q)$ gives[6]

$$CV(\pi) = [\sigma(Q)/E(Q)]/[1 - Q_{BE}/E(Q)] . \tag{13}$$

Provided that $E(Q)$ and $\sigma(Q)$ remain constant, any combination of change in P, V, and F that increases $Q_{BE}$ will lower the denominator of the above equation and raise the $CV(\pi)$. This is consistent with the emphasis given in the previous section to $Q/Q_{BE}$ as the critical factor in determining the risk accompanying operating leverage. With an increase in $Q_{BE}$, all else held

constant, $E(Q)/Q_{BE}$ falls and the operating elasticity at $E(Q)$ rises. Intuitively, sales volatility is now magnified to a greater extent. The numerator of Equation (13) equals the coefficient of variation of sales, $CV(Q)$. The denominator can be simplified to[7]

$$1 - Q_{BE}/E(Q) = 1/\varepsilon_{E(Q)} .$$

When both are substituted back into (13), the coefficient of variation of operating income is the product of operating elasticity at the firm's expected sales level and its coefficient of variation of sales:

$$CV(\pi) = \varepsilon_{E(Q)} \times CV(Q) . \qquad (14)$$

If the coefficient of variation of operating income is the relevant measure of business risk, the above equation expresses the specific function relating a firm's business risk to its sales volatility and operating leverage. The contribution of operating leverage to business risk depends on the expected sales position relative to the break-even level of sales, for this determines the associated operating elasticity which magnifies the risk per expected sales volume into a greater risk per expected operating income.

One implication of Equation (14) is that firms may arrive at the same business risk classification by way of alternative routes. As an example, consider these two companies:

| Firm A | | Firm B | |
|---|---|---|---|
| $E(\pi)$ | = \$100,000 | $E(\pi)$ | = \$100,000 |
| $^{c}E(Q)$ | = 1.50 | $^{c}E(Q)$ | = 5.00 |
| $CV(Q)$ | = .50 | $CV(Q)$ | = .15 |
| $CV(\pi)$ | = 1.50 x .50 | $CV(\pi)$ | = 5.00 x .15 |
| | = .75 | | = .75 |

With their CV($\pi$)s equal, both firms are in the same business risk category. Nevertheless, their individual situations are very different. Firm A is subject to higher sales volatility with an expected quantity far from its break-even point. Firm B, in contrast, produces close to its break-even point but faces lower sales volatility.

## V. EMPIRICAL TESTS AND RESULTS

In the previous section, it is proved that the coefficient of variation of operating income is the product of operating elasticity at the firm's expected sales level and the coefficient of variation of sales:

$$CV(\pi) = \epsilon_{E(Q)} \times CV(Q) .$$

In this section, the procedures and results of two empirical tests using this equation are summarized. The first test is concerned with establishing empirical support for the equation at the firm level. For each firm in an industry identified by a four-digit SIC number, these statistics are calculated for the time periods 1963-67 and 1968-72, using data drawn from the COMPUSTAT Annual Industrial Tape: (1) the coefficient of variation of operating income, (2) the coefficient of variation of sales, and (3) the five-year average of the absolute values of the operating elasticities, where the operating elasticity at each year's sales is computed using the formula:

$$\epsilon_{t-1} = \frac{\pi_t - \pi_{t-1}}{\pi_{t-1}} \div \frac{Sales_t - Sales_{t-1}}{Sales_{t-1}}$$

The third statistic differs from its counterpart in the first equation in two ways. First, it is not operating elasticity at the average sales level over the period, the computation of which requires estimates of the

revenue and cost functions. Secondly, the absolute value transformation is made because, for firms in a multiproduct situation or for those producing under conditions of nonlinear revenue and cost functions, negative operating elasticities at positive profit levels are a reality. If the average of the pure elasticities is computed, the negatives and positives cancel each other, and the average is dampened. An incorrect picture is presented because in a nonlinear framework, it can be shown that the closer the firm is to a break-even point, the greater the absolute value of the operating elasticity. Intuitively, the magnification of sales volatility will be greater in such a situation. For these reasons, the average of the absolute values of the elasticities is used as a proxy for the elasticity at the average sales volume.

Given the first equation, it is true that

$$CV(\pi)/CV(Q) = \varepsilon_{E(Q)} .$$

If, for each firm in an SIC grouping, both $CV(\pi)/CV(Q)$ and $\varepsilon_{E(Q)}$ could be measured exactly, the cross-section simple and rank correlations between them should equal one. An estimate of $CV(\pi)/CV(Q)$ is easily calculated. As discussed above, while an estimate of $\varepsilon_{E(Q)}$ is not readily available, an intuitively appealing substitute can be computed. Therefore, as a test of the equation, Pearson product moment ($r$) and Spearman rank ($r_s$) correlations are computed, for each time period, between the estimate of $CV(\pi)/CV(Q)$ and the five-year average of the absolute values of the operating elasticities. The a priori expectation is that the larger the average of the absolute values of the elasticities, the greater the ratio of the estimate of $CV(\pi)$ to the estimate of $CV(Q)$ will be.

The results for ten industries are summarized in Exhibits 5 and 6. The sample sizes differ between the periods due to missing data and because

Exhibit 5. Spearman Rank and Pearson Product Moment Correlations for Period 1963-67[a,b,c]

| SIC# | Industry Name | Spearman Rank Correlation | Pearson Product Moment Correlation |
|---|---|---|---|
| 1311 | Crude Petroleum and Natural Gas (16) | .535** | .703 |
| 2300 | Textile Apparel Manufacturers (37) | .526 | .659 |
| 2899 | Chemicals and Chemical Preparations (17) | .500** | .821 |
| 3311 | Minor Steel (21) | .549 | .676 |
| 3550 | Specialty Machinery (16) | .309* | .249* |
| 3679 | Electronic Components (22) | .689 | .913 |
| 3714 | Motor Vehicle Parts and Accessories (35) | .599 | .678 |
| 4210 | Trucking, Local and Long Distance (17) | .664 | .639 |
| 5311 | Department Stores (22) | .691 | .836 |
| 5411 | Grocery Stores (31) | .597 | .741 |

[a]The sample size is in parentheses following the industry name.

[b]All correlations are significant at the .01 level unless otherwise indicated.

[c]A "*" means correlation is insignificant at the .05 level, while "**" means correlation is significant at the .05 level.

Exhibit 6. Spearman Rank and Pearson Product Moment Correlations for Period 1968-72[a,b,c]

| SIC# | Industry Name | Spearman Rank Correlation | Pearson Product Moment Correlation |
|---|---|---|---|
| 1311 | Crude Petroleum and Natural Gas (30) | .680 | .296* |
| 2300 | Textile Apparel Manufacturers (46) | .652 | .435 |
| 2899 | Chemicals and Chemical Preparations (25) | .496 | .103 |
| 3311 | Minor Steel (22) | .522 | .405 |
| 3550 | Specialty Machinery (20) | .475** | .867 |
| 3679 | Electronic Components (21) | .735 | .793 |
| 3714 | Motor Vehicle Parts and Accessories (40) | .618 | .593 |
| 4210 | Trucking, Local and Long Distance (20) | .761 | .845 |
| 5311 | Department Stores (26) | .809 | .563 |
| 5411 | Grocery Stores (30) | .607 | .344** |

[a]The sample size is in parentheses following the industry name.

[b]All correlations are significant at the .01 level unless otherwise indicated.

[c]A "*" means correlation is insignificant at the .05 level, while "**" means correlation is significant at the .05 level.

only those firms with positive operating profits in all years are included. Equal sample sizes are not felt to be necessary because there is no intention of making comparisons between the two periods. Moreover, the maximum possible sample sizes are desired to reduce the probability of Type I and II errors when testing the significance of the correlations.

An examination of these exhibits shows that all the correlations are positive and that an overwhelming majority are greater than .50 . Except in three industries, all correlations are significant at the 5% level or better, when the null hypothesis is that r or $r_s$ is equal to zero and the alternate hypothesis is that each is greater than zero.

The strongest rank correlation in each period is registered by the department store group, .69 in the first period and .81 in the second period. The specialty machinery group has the weakest rank correlations; $r_s$ equals .31 in 1963-67 and is not significantly different from zero, while $r_s$ equals .48 in 1968-72 and is significant at the 5% level. This same industry has the highest Pearson r in the second period, .87 which is significant at the 1% level. In the first period, electronic components has a Pearson r equal to .91 which is also significant at the 1% level. Overall, the association between the variables is significantly positive and somewhat strong. The results tend to support the theory that the closer a firm is to a break-even point, the larger its operating elasticity, and the greater the differential between the coefficient of variation of sales and the coefficient of variation of operating income.

The second test utilizes the first equation to investigate whether a change in business risk between the periods 1964-68 and 1969-73 is more the result of a change in operating leverage or a change in sales volatility.

A change in operating leverage, in the sense used here, means that the firm moves closer to or farther away from its break-even point and its operating elasticity rises or falls. In this test both $CV(\pi)$ and $CV(Q)$ are estimated for each firm in an SIC four-digit grouping for both five-year periods. The estimate of each firm's $\varepsilon_{E(Q)}$ for a period is forced by dividing the estimate of $CV(\pi)$ by the estimate of $CV(Q)$. Since it is true by definition that

$$\log[CV(\pi)] = \log[\varepsilon_{E(Q)}] + \log[CV(Q)] ,$$

computing the natural logarithms and taking the differences in each variable from one period to the next gives the compound percentage change in each. In other words, the compound percentage change in the coefficient of variation of operating income equals the compound percentage change in operating elasticity plus the compound percentage change in the coefficient of variation of sales.

After calculating these percentage changes for each firm in the industry, cross-section simple correlations are computed in order to estimate the coefficients of determination ($r^2$s). The latter variables are necessary in order to answer the question: Does the percentage change in $\varepsilon_{E(Q)}$ or the percentage change in $CV(Q)$ explain more of the cross-sectional variability in the percentage change in $CV(\pi)$?

The results are presented in Exhibits 7 and 8. Exhibit 7 contains the mean percentage changes in the three variables and their standard deviations for ten SIC industries. In all but one case, the standard deviation is much larger than the mean, suggesting that there is a wide variation of changes in the variables among the firms in any given industry.

Exhibit 7. Mean Compound Percentage Changes
Between 1964-68 and 1969-73[a,b]

| SIC# | Industry Name | Operating Elasticity | Coefficient of Variation of Sales | Coefficient of Variation of Operating Income |
|---|---|---|---|---|
| 1311 | Crude Petroleum and Natural Gas (16) | .073 (.708) | -.029 (.968) | .044 (1.236) |
| 2300 | Textile Apparel Manufacturers (32) | .216 (.864) | -.409 (.802) | -.194 (.650) |
| 2899 | Chemicals and Chemical Preparations (16) | .165 (.569) | -.286 (.901) | -.121 (.835) |
| 3311 | Minor Steel (18) | -.157 (.720) | .238 (.699) | .081 (.557) |
| 3550 | Specialty Machinery (14) | .338 (.776) | -.075 (.627) | .264 (.606) |
| 3679 | Electronic Components (15) | .533 (.569) | -.512 (.673) | .022 (.640) |
| 3714 | Motor Vehicle Parts and Accessories (35) | .066 (.674) | -.237 (.726) | -.171 (.766) |
| 4210 | Trucking, Local and Long Distance (14) | .219 (.726) | .019 (.430) | .238 (.621) |
| 5311 | Department Stores (22) | .258 (1.054) | -.347 (.828) | -.090 (.733) |
| 5411 | Grocery Stores (23) | .231 (1.033) | -.099 (.787) | .132 (.931) |

[a]The sample size is in parentheses following the industry name.

[b]The standard deviation is in parentheses following the mean percentage change.

Exhibit 8. Coefficients of Determination[a,b]

| SIC# | Industry Name | Operating Elasticity | Coefficient of Variation of Sales |
|---|---|---|---|
| 1311 | Crude Petroleum and Natural Gas (16) | .39** | .67** |
| 2300 | Textile Apparel Manufacturers (32) | .16* | .10 |
| 2899 | Chemicals and Chemical Preparations (16) | .05 | .62** |
| 3311 | Minor Steel (18) | .17 | .13 |
| 3550 | Specialty Machinery (14) | .38* | .04 |
| 3679 | Electronic Components (15) | .15 | .39* |
| 3714 | Motor Vehicle Parts and Accessories (35) | .25** | .35** |
| 4210 | Trucking, Local and Long Distance (14) | .65** | .01 |
| 5311 | Department Stores (22) | .39** | .01 |
| 5411 | Grocery Stores (23) | .47** | .08 |

[a]The sample size is in parentheses following the industry name.

[b]A "*" means coefficient of determination is significant at the .05 level, while "**" means it is significant at the .01 level.

Generally, a close examination of the exhibit indicates that the average change in sales volatility between the two periods is negative, while that in operating elasticity is positive. The average change in business risk is also upward. More specifically, in six of the ten industries, the mean percentage change in $CV(\pi)$ is positive. This increase is due to a mean percentage increase in $\varepsilon_{E(Q)}$ that more than offsets the usual mean percentage decline in $CV(Q)$. The $CV(Q)$ shows a mean percentage decrease in eight of the ten industries, while the percentage change in $\varepsilon_{E(Q)}$ is downward only for the minor steel category.

Exhibit 8 shows the calculated coefficients of determination between the percentage change in $CV(\pi)$ and each of the other variables. Generally, one or the other of the $r^2$s is significantly different from zero. Only in the minor steel group are both insignificant. In four of the ten industries, the percentage change in $CV(Q)$ explains more of the cross-sectional variability in the percentage change in $CV(\pi)$ than does the percentage change in $\varepsilon_{E(Q)}$. As an example, in the crude petroleum and natural gas group, 67% of the total variation in the change in business risk among firms in the industry is explained by a change in sales volatility; only 39% is explained by a change in operating elasticity. In the remaining six industries, the variation in the change in business risk is explained more by the change in operating elasticity. For the trucking industry, 65% of the total variation in the percentage change in $CV(\pi)$ is explained by the percentage change in $\varepsilon_{E(Q)}$.

Finally, it is interesting to note that the results are independent of the fixed cost/variable cost mix in an industry. In some of those industries where the expectation is that fixed costs are very important to production, a change in operating elasticity is not important in

explaining the change in business risk between periods, e.g., in the crude petroleum, chemicals, electronic components, and motor vehicle parts industries. In contrast, for department and grocery stores, industries in which fixed costs are not dominant, the change in operating elasticity contributes the most to explaining the change in business risk. These results are interpreted to be consistent with the argument that proximity to break-even point, rather than the fixed cost/variable cost proportion, is the primary determinant of the risk of operating leverage.

## VI. CONCLUSIONS, IMPLICATIONS, AND SUGGESTIONS FOR FUTURE RESEARCH

One of the major findings of this research is that all firms have the same graph when operating elasticity is plotted against $Q/Q_{BE}$, a measure of the firm's output position relative to its break-even position. This confirms a point about operating leverage which has been recognized before, but not adequately emphasized -- that a firm's sales volume relative to its break-even level of sales determines its risk due to operating leverage. Thus, two firms, one producing with $1 and the other with $1,000,000 of fixed costs, can have the same operating elasticity and therefore the same risk due to operating leverage, if both have sales at the same relative distance from their respective break-even points. The operating elasticity variable, then, can be used as a measure to compare firms with respect to the risk that accompanies their use of operating leverage.

Another important finding is that a change in fixed costs, no matter what the corresponding change in the contribution margin, alters the operating elasticity at all levels of operating income. If fixed costs increase, the operating elasticity at each profit level increases, with the

reverse holding if fixed costs decrease. The implication is that when comparing two firms having equal operating incomes, the one with the greater fixed expenses has the greater operating elasticity and is potentially more risky with respect to fluctuations in operating earnings if demand changes from its current level.

Either $Q/Q_{BE}$ or total fixed expenses are also advocated as more correct definitions of a firm's amount of operating leverage. As shown, the use of a definition such as either the fixed to total costs ratio or the fixed to variable costs ratio can lead to an erroneous conclusion regarding a firm's operating earnings volatility. It is not always true that the higher either of these ratios, the higher operating elasticity will be. It is true, however, that the smaller $Q/Q_{BE}$, the larger $\varepsilon$ will be, and the larger fixed expenses are, the larger $\varepsilon$ will be at all levels of operating income.

The last important finding concerns the relationship among a firm's business risk, operating leverage, and sales volatility. Here, the equation for the coefficient of variation of operating income is modified to include the operating elasticity variable explicitly. Specifically, it is demonstrated that a firm's coefficient of variation of operating income equals the product of operating elasticity at the expected sales level and the coefficient of variation of sales. In order to establish empirical support for this relationship, two tests are made. The first shows the ratio of $CV(\pi)$ to $CV(Q)$ to be positively correlated with a proxy for $\varepsilon_{E(Q)}$. The second test uses the equation to determine whether a change in business risk between time periods is more a function of a change in sales volatility or a change in operating elasticity. The results show that in six of ten industries, a change in operating elasticity explains more of the cross-

sectional variability in the change in business risk, while in the remaining four, a change in sales volatility is more important.

The conclusion that $CV(\pi)$ equals $\varepsilon_{E(Q)}$ times $CV(Q)$ has important implications, one of which is that companies can achieve the same business risk category in a number of different ways. Identical coefficients of variation of operating income can be arrived at through high sales volatility far from the break-even point or low sales volatility close to the break-even point. Furthermore, when the equation is coupled with the conclusion that proximity to the break-even point is the primary determinant of the risk associated with operating leverage, it is readily apparent that a change in $CV(\pi)$ will occur, due to a change in cost structure, only if quantity sold relative to the break-even quantity rises or falls.

The interrelationships suggested by the equation are also important when planning financial structure. Certainly, sales volatility is a consideration in the debt-equity decision. However, it will be the more important consideration only when the sales level is far from the break-even point so that the operating elasticity is small. When sales are close to the break-even level, the operating elasticity may be the more important factor in the decision, particularly when sales are stable.

Because acceptance of a project may affect the firm's output position relative to its break-even point, useful applications may also be made in the capital budgeting area. First, the financial manager might conceivably use both a project's operating elasticity and coefficient of variation of sales to determine its appropriate risk class in order to choose a relevant discount rate. Secondly, given that the cost of equity capital is, conceptually, the risk-free rate plus premiums for business and financial risk, it seems worthwhile to ask if the premium for business risk is a

function of only the coefficient of variation of operating income, or is it a more complex function of both $\epsilon$ and CV(Q)? While different combinations of $\epsilon$ and CV(Q) may lead to the same CV($\pi$), those same combinations may lead to different business risk premiums. Investors may view low sales volatility at a level close to the break-even point as being more risky than high sales volatility far from the break-even point.

Conclusions similar to those proven here about operating leverage can be shown to hold true with respect to financial leverage as well. As an example, there is a unique correspondence between the coverage ratio and financial elasticity (degree of financial leverage), the equation for which is independent of the debt ratio. Finally, the coefficient of variation of common stock earnings can be shown to equal the product of operating elasticity, financial elasticity, and the coefficient of variation of sales. Given that the variability in common stock earnings is a measure of total risk, this equation can be used to explore the question of which factor contributes most to total risk.

FOOTNOTES

[1]See Horngren [5, pp.39-57] for an excellent discussion of the theory of break-even analysis.

[2]As pointed out by Dilbeck [2], "operating elasticity" is one of three elasticities positioned throughout the income statement, the other two being "financial elasticity" and "tax elasticity."

[3]The proof that the numerator equals operating income plus fixed expenses is:

$$Q(P - V) = [Q(P - V) - F] + F$$

$$= \pi + F .$$

[4]Note the shift from $Q_{BE}/Q$ to $Q/Q_{BE}$.

[5]Strictly speaking, this statement applies only to those firms which operate under the assumption of linear cost and revenue functions or whose operating profit equation is linear over the relevant range of output. Nevertheless, the essential conclusion, that proximity to break-even point determines whether operating leverage is important to risk, holds if nonlinear cost and revenue functions are appropriate.

[6]The proof of this is:

$$CV(\pi) = \frac{[(P - V)\sigma(Q)/(P - V)E(Q)]}{[(P - V)E(Q) - F]/[(P - V)E(Q)]}$$

$$= [\sigma(Q)/E(Q)]/[1 - F/(P - V)E(Q)]$$

$$= [\sigma(Q)/E(Q)]/[1 - Q_{BE}/E(Q)] .$$

[7]The proof of this is:

$$1 - Q_{BE}/E(Q) = [E(Q) - Q_{BE}]/E(Q)$$

$$= 1/[E(Q)/(E(Q) - Q_{BE})]$$

$$= 1/\epsilon_{E(Q)} .$$

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