THE OPTIMAL ACCUMULATION OF HUMAN CAPITAL OVER
THE LIFE CYCLE

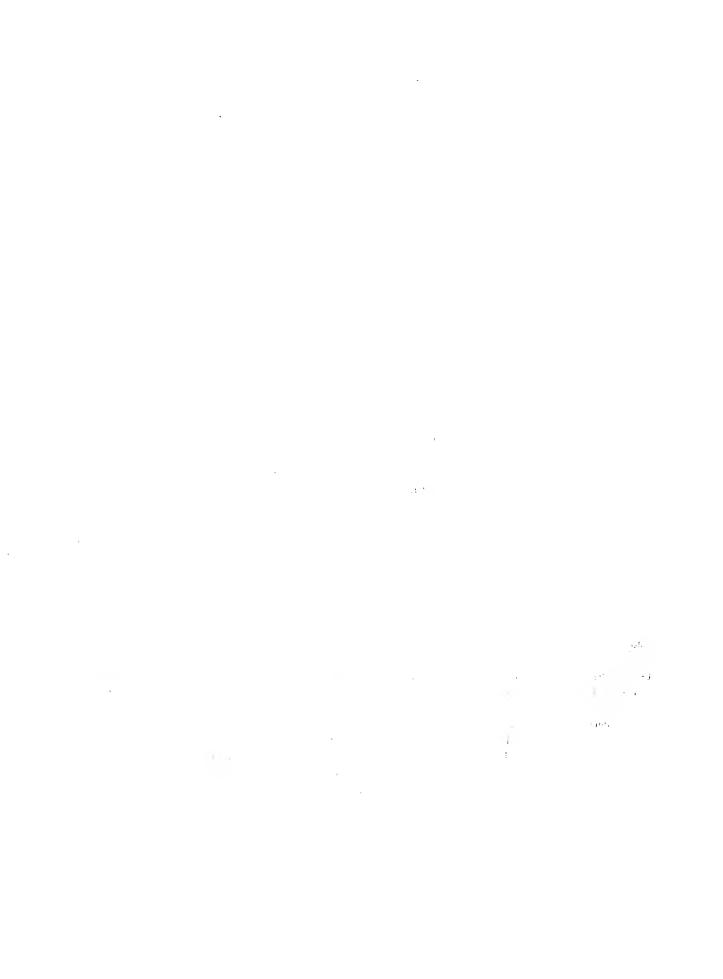John Graham, Assistant Professor, Department of
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Summary:

This paper summarizes the important contributions of the new life cycle human
capital literature and demonstrates that many of these results can be derived
more simply than in their original presentations. Within three period discrete-
time framework it is demonstrated how the optimal pattern of human capital in-
vestment over the life cycle depends upon the choice of the objective function,
the life cycle of leisure, and the extent of nonmarket benefits of human
capital. The paper offers sufficient conditions for the optimality of a profile
of monotonically declining investment activity over the life cycle.

The Optimal Accumulation of Human Capital Over the Life Cycle

I. Introduction

Although formal modeling of the process of human capital accumulation has been going on for some 15 years now, the development of such models within an explicit life cycle context has a far shorter history. Even as late as 1976, Blaug's comprehensive survey of the field offers no discussion of human capital accumulation over the life cycle.[1] Only since then have several articles appeared that attempt to solve for the optimal life time pattern of human capital investment.[2] This paper summarizes the important results of this new literature and demonstrates that most of these results can be derived more simply than in their original presentations and without some special assumptions previously imposed. Finally, this paper offers sufficient conditions for the optimality of a profile of declining investment time over the life cycle.

Early models of human capital accumulation were either static, or if dynamic, ignored the constraints imposed by the life cycle.[3] The development of models within a life cycle context represent a major advance for several reasons. First, the life cycle lends added realism to models of optimal accumulation. It forces the model builder to incorporate several essential characteristics of human capital investment that set it apart from other forms of consumer saving as well as from other types of physical investment. Unlike many other forms of consumer saving, acquisition of human capital is irreversible. Bonds acquired today can be sold tomorrow, but human capital acquired today cannot be so easily liquidated. By incorporating the process of aging, life cycle

models make explicity the necessarily sequential and one-directional nature of human capital accumulation. Unlike investment in most physical capital, investment in human capital is subject to constraints on the magnitude of accumulation. With perfect financal markets, firms can acquire as much capital as they want to during any period, but consumers cannot always acquire as much human capital as they desire.[4] The reason is that human capital is not purchased in the market at a constant price, but "produced" by the consumer with his finite resource of time. Since there exists no market in which more of this time input can be acquired immediately, the only alternative is to postpone some investment until the future.

One benefit of studying human capital within a life cycle context is that an examination of the pattern of accumulation that emerges can provide a check on the internal consistency of the model. For example, one should become wary of putting too much faith in the policy prescriptions that follow from a model in which monotonically increasing investment over the life cycle emerges as the optimal pattern of investment.

Casual observation suggests that the most common pattern of human capital accumulation is a declining profile of investment time over the life cycle. Therefore, it should be interesting to investigate the conditions under which such a pattern is indeed optimal. Additionally, one would hope that a model might provide insight into other patterns of accumulation. How can one explain the not uncommon pattern of a return to full-time schooling after a period of full-time work? Is

such behavior simply irrational, or is it possible to provide a choice-
theoretic framework in which this behavior emerges as both rational
and optimal?

It is important at this stage to interject a word of caution.
While it is tempting to equate human capital accumulation with school-
ing, it is wrong to do so. The models developed in this paper and
in the related literature take a broader view. Human capital accu-
mulation is defined as all non-costless current activity that enhances
the value of future time. Thus, it includes not only schooling, but
also on-the-job training and investment in health or information.
Viewed in this wider context, it is not even clear that the "normal"
life cycle pattern has investment time declining with age.

II. Human Capital in the Life Cycle Literature

The first dynamic theories of optimal human capital accumulation
due to Ben-Porath and Becker represented significant advances over the
existing static models, but even they ignored any explicit life cycle
consideration. In other words, neither assumed the consumer to be making
choices that would maximize lifetime utility subject to the constraints
of age, lifetime income and intertemporal prices. Instead, the under-
lying hypothesis maintained by both Becker and Ben-Porath was that indi-
viduals would select a lifetime plan for human capital investment to
maximize total lifetime income, and then at some later stage would maxi-
mize lifetime utility subject to this value of lifetime income. It is
exactly this recursive two-step maximization that the recent life cycle
human capital literature demonstrates to be invalid. According to Ryder,
Stafford and Stephen:

The criterion of maximal present value for investment decisions is usually justified by what Hirshleifer terms the Separation Theorem. That is, 'given perfect and complete markets, the productive decision is to be governed solely by the objective market criterion represented by attained wealth--without regard to the individual's subjective preferences that enter into their consumption decisions.' Unlike physical capital, however, human capital is embodied in the human being and, hence, in the unit that makes decisions about the life-cycle allocation of time. One implication of this is that whenever time has an alternative use that produces utility the separation theorem holds only if restrictive assumptions are made.[5]

Unfortunately the gain in realism provided by **utility maximization** in a life cycle context has not been costless. When **consumption and** investment decisions are interrelated, the consumer's maximization prob-lem is very difficult to solve in general and even particular functional forms rarely provide closed form solutions. To obtain any **meaningful** results, the literature has been forced to impose potentially severe simplifying assumptions. For example, to obtain interpretable life cycle paths for leisure, investment and work time, Ryder, Stafford, and Stephen assume that the lifetime utility function can be written in logarithmic form. Binder and Weiss do not choose particular **functional** forms, but they do assume that their utility function exhibits separability of time and goods at each instant of time. Furthermore, their production function for human capital is assumed homogeneous **of degree** one in the input of time; in other words, doubling the **time devoted to** investment doubles the output of human capital produced. **Finally, in** Heckman's papers the existing stock of human capital enters both the utility and production functions in a very special way which Heckman dubs the "neutrality hypothesis." According to this **specification,**

additions to the stock of human capital equally augments the efficiency
of all uses of time—work, leisure, and investment.

In this study we first extend the work of Ryder, Stafford and
Stephen to examine how the optimal life cycle profile of human capital
accumulation depends upon the objective function of the consumer. In
Section III we derive the optimal investment profile for a wealth maxi-
mizer, and in Section IV the optimal profile for a utility maximizer.
Unlike earlier work our utility function is general in form and does
not assume separability of commodities within a given period. Our pro-
duction function exhibits diminishing returns to the time input. In
Section V we extend Heckman's results by allowing for non-neutral in-
fluences of human capital on the efficiency of alternative uses of time
by introducing a leisure technology parameter. We examine what influence
this parameter has on the profile of investment time. Throughout the
paper the results are expressed in terms of sufficient conditions for
the optimality of a "normal" declining investment time profile over the
life cycle.

III. Optimal Accumulation Within a Wealth-Maximization Model

Suppose an individual expects with certainty to live for three
identical and consecutive discrete time periods. During each per-
iod he must allocate his predetermined nonleisure time between two
activities—work time which earns a monetary reward and human capi-
tal investment time which enhances the value of future work time.
Let K_i denote the fraction of time during period i devoted to in-
vestment. The individual is endowed with an initial earnings potential

H_0 which can be augmented through human capital production according to the transition equation

$$H_i - H_{i-1} = F(K_i) \qquad (1)$$

For simplicity, assume the production function F satisfies the Inada conditions.[6] Assume that work time yields a return to the existing stock of human capital at the constant rental rate w. Therefore first period earnings equal $wH_0(1-K_1)$, that is, the return on the initial stock of human capital, wH_0, times the length of noninvestment time, $(1-K_1)$. Second period earnings equal $wH_1(1-K_2)$, where the new stock of human capital is obtained from equation (1) as determined by the first period's investment activity. Finally, third period earnings equal wH_2. K_3 equals zero since returns to any third period investment would not accrue until after the terminal date. The choice problem for the individual is to select K_1 and K_2 to maximize wealth, or discounted lifetime income:

$$(1+r)^2 wH_0(1-K_1) + (1+r)wH_1(1-K_2) + wH_2$$

where H_1 and H_2 are defined in equation (1), and r is the interest rate in the perfect financial capital market. Assuming interior solutions, wealth is maximized when

$$\frac{dF}{dK_2} = (1+r)H_1 \qquad (2)$$

$$(1+r)(1-K_2)\frac{dF}{dK_1} + \frac{dF}{dK_1} = (1+r)^2 H_0 \quad . \qquad (3)$$

The equations can be solved, in principle, for K_1^* and K_2^*, the optimal fractions of nonleisure time devoted to human capital accumulation. Together these equations represent the familiar requirement that investment time be chosen to equate the marginal cost of production (on the right hand side) with its discounted lifetime marginal benefits (on the left hand side).

To evaluate the life cycle pattern of investment time in this model it is sufficient to compare K_1^* to K_2^*. At this point it should be clear why we have selected a three period life cycle. With $K_3=0$, three periods represent the minimum life cycle needed to compare two unconstrained periods. In principle the analysis could be extended to any number of discrete periods.

We shall prove and then discuss the following theorem.

Theorem One: $K_1 > K_2$ is the only feasible investment path that satisfies (2) and (3).

We proceed with a proof by contradiction. Suppose $K_1=K_2=K$. Then $\frac{dF}{dK_1} = \frac{dF}{dK_2} = \frac{dF}{dK}$. Combine (2) and (3) to obtain:

$$(1+r)^2(1-K)H_1 + (1+r)H_1 \overset{?}{=} (1+r)^2 H_0 \qquad (4)$$

It is straightforward to show that (4) cannot hold with equality. To see this, differentiate the left hand side of (4) with respect to K obtaining

$$(1+r)^2(1-K) \frac{dF}{dK}$$

which is nonnegative for $0 \le K \le 1$. Therefore, the left hand side of (4) is an increasing function of K. Evaluating this expresion at

$K=0$, its smallest value, yields $(1+r)^2 H_0 + (1+r)H_0$. Since this already exceeds the value of the right hand side, (4) cannot hold with equality for any $0 \leq K \leq 1$. Therefore, the assumption $K_1 = K_2$ has led to a contradiction.

Now suppose $K_2 > K_1$. This implies that $\frac{dF}{dK_2} < \frac{dF}{dK_1}$. Now when (2) is substituted into (3), $\frac{dF}{dK_1}$ must be replaced by even more than $(1+r)H_1$. This only makes the left hand side of (4) larger, so the contradition stands.

Finally, suppose $K_1 > K_2$. This means $\frac{dF}{dK_1} < \frac{dF}{dK_2}$, so when substituting (2) into (3), $\frac{dF}{dK_1}$ will be replaced by less than $(1+r)H_1$. This is the only case for which equality in (4) can possibly hold. For some K, $K_1 > K_2$ does not imply a contradiction. Q.E.D.

The importance of Theorem One is that it demonstrates that aside from an ad hoc change in the form of production function for human capital over the life cycle, there is no way to explain a nondecreasing investment profile when individuals select investment time to maximize lifetime wealth. Intuitively, investments are made early rather than late in life for two reasons. First, the earlier the investment is undertaken, the greater are the remaining number of periods for the benefits to accrue. Investment late in the life cycle leaves insufficient time to reap the benefits. The other reason why earlier investment is more profitable is that is when the costs of production are the lowest. Investment not only increases the value of future work time, but it also increases the cost of future investment. These rising costs and declining benefits reinforce each other to make a declining life cycle of investment activity the only feasible profile.

IV. Optimal Accumulation Within a Utility-Maximization Model

Assume the identical set-up to the previous problem, except
that now the individual selects K_1 to maximize lifetime utility.
This means he has several additional choices to make. Let ℓ_1 be the
fraction of total time devoted to leisure in period i, where leisure is
defined as all nonwork, noninvestment time, and the only use of time
that yields utility directly. In addition to his allocation of time,
the individual must decide upon his pattern of purchases of goods over
the life cycle. Let X_i be a composite nondurable commodity purchased
in period i at a price of one. The individual's choice problem can
be summarized as selecting X_1, X_2, X_3, ℓ_1, ℓ_2, ℓ_3, K_1, and K_2 to
maximize lifetime utility

$$u(X_1,\ell_1) + v(X_2,\ell_2) + z(X_3,\ell_3),$$

subject to: $(1+r)^2(1-\ell_1-K_1)wH_0 + (1+r)(1-\ell_2-K_2)wH_1$

$$+ wH_2(1-\ell_3) - (1+r)^2X_1 - (1+r)X_2 - X_3 = 0 .$$

The only assumption imposed on preferences is that the lifetime utility
function exhibits intertemporal separability. The complete set of
first-order conditions appears in Appendix A.

Reproduced below are the two first order conditions from Appendix
A for choosing K_1 and K_2:

$$(1-\ell_3^*) \frac{dF}{dK_2} = (1+r)H_1 \tag{5}$$

$$(1+r)(1-\ell_2^*-K_2) \frac{dF}{dK_1} + (1-\ell_3^*) \frac{dF}{dK_1} = (1+r)^2H_0 \tag{6}$$

Like (2) and (3), these equations say that investment time should be chosen to equate foregone marginal costs with marginal benefits. But unlike in the earlier pair of equations, benefits now depend upon the planned life cycle consumption of leisure time. This means that the optimal profile of human capital investment cannot be derived solely from the pair of equations above, but requires the two additional first order conditions for leisure in the second and third periods. However, we shall see that equations (5) and (6) by themselves are sufficient to obtain a sense of the lifetime profile of investment as long as we assume that the future time path of leisure is always being chosen optimally (and denoted ℓ_2^* and ℓ_3^*).

The major result of this section is that Theorem One which guaranteed a declining life cycle of investment time will no longer hold unconditionally. When individuals maximize lifetime utility and not wealth, for certain parameter values any investment profile is feasible. At best we can offer the following sufficient condition for the optimality of the "normal" decreasing pattern of investment time over the life cycle.

Theorem Two: $K_1 > K_2$ is the only feasible investment path that satisfies (5) and (6) if $\ell_2^* \leq \ell_3^*$.

We again offer a proof by contradiction. Suppose $K_1 = K_2 = K_2$. Then $\frac{dF}{dK_1} = \frac{dF}{dK_2} = \frac{dF}{dK}$. Substituting from (5) into (6) to eliminate $\frac{dF}{dK}$, we obtain:

$$(1+r)^2 \frac{(1-\ell_2^*-K)}{(1-\ell_3^*)} H_1 + (1+r)H_1 \overset{?}{=} (1+r)^2 H_0 \qquad (7)$$

Differentiation of the left hand side of (7) shows that the expression is a nondecreasing function of K:

$$(1+r)^2 \frac{(1-\ell_2^*-K)}{(1-\ell_3^*)} \frac{dF}{dK} \geq 0.$$

Now evaluate (7) at K=0, where the left hand side will be the smallest:

$$(1+r)^2 \frac{(1-\ell_2^*)}{(1-\ell_3^*)} H_0 + (1+r) H_0 \overset{?}{=} (1+r)^2 H_0 \qquad (8)$$

Equality in (8) cannot hold if $\ell_2^* \leq \ell_3^*$, but it can possibly hold for some $\ell_3^* < \ell_2^*$. Therefore, as long as $\ell_2^* \leq \ell_3^*$, the assumption that $K_1 = K_2$ leads to a contradiction. Just as in the proof to theorem one, another contradiction is reached (given $\ell_2^* \leq \ell_3^*$) if it is assumed that $K_1 < K_2$ since $\frac{dF}{dK_1}$ in equation (6) will be replaced by even more than $\frac{(1+r) H_1}{(1-\ell_3^*)}$ from equation (5), making the left hand side of (7) even larger. Only $K_1 < K_2$ produces no contradiction. Q.E.D.

Theorem Two can be explained intuitively. The first order conditions (5) and (6) for selecting the investment profile say that an important determinant of human capital investment is planned consumption of leisure. Since returns to human capital investment accrue only when the individual is working, the individual's preferences over leisure will influence the extent of future benefits from current investment. It is easy to see how an "abnormal" investment profile might occur. If consumption of leisure is falling and work time is increasing over the life cycle, the discounted benefits from investing during period two might be greater than the discounted benefits from investing during

period one if the returns in period three (large relative to those in period two because of the declining life cycle of leisure time) are sufficiently heavily discounted.

But, of course, the life cycle of leisure itself is not independent of the individual's production of human capital. Therefore, it is necessary to re-examine the first-order conditions to see if a declining life cycle of leisure (which could produce an increasing investment profile) is likely or even feasible. From Appendix A, the first order conditions for ℓ_2 and ℓ_3 are:

$$\frac{\partial v(\ell_2, X_2)}{\partial \ell_2} - (1+r)wH_1 \lambda = 0 \tag{9}$$

$$\frac{\partial z(\ell_3, X_3)}{\partial \ell_3} - wH_2 \lambda = 0 \tag{10}$$

where λ is the lagrangian multiplier associated with the budget constraint. Without imposing further assumptions on the form of the utility function, very little can be said about the life cycle of leisure. However, if we are willing to assume separability of leisure and goods within any period, and further to assume that the utility function for leisure differs across periods only by the individual's subjective rate of discount which also equals the market interest rate (i.e., $\frac{\partial v}{\partial \ell_2} = (1+r) \frac{\partial z}{\partial \ell_2}$), then we can divide (9) by (10), obtaining the condition:

$$\frac{\frac{\partial z}{\partial \ell_2}}{\frac{\partial z}{\partial \ell_3}} = \frac{H_1}{H_2} \tag{11}$$

Now as long as the second order conditions for utility maximization hold ($\frac{\partial^2 z}{\partial \ell_1^2} < 0$), then according to equation (11), ℓ_3 is less than ℓ_2 whenever H_2 exceeds H_1. Whenever K_2 is nonzero, H_2 will exceed H_1, so leisure consumption will fall from period two to period three. This likely occurance is exactly the circumstances under which the investment profile can be increasing over some portion of the life cycle.

V. Optimal Accumulation Within A Utility-Maximization Model With Consumption Benefits From Human Capital

Robert Michael formalized the notion that investment in human capital might not only enhance future market earnings, but also provide direct consumption benefits.[7] We shall investigate how the intensity of such consumption benefits might influence the optimal profile of lifetime human capital investment. Our particular specification of consumption benefits of human capital is due to James Heckman.[8]

The only modification from the previous problem is in the form of the utility function. Let lifetime utility be written as

$$u(X_1, \ell_1 H_0^a) + v(X_2, \ell_2 H_1^a) + z(X_3, \ell_3 H_2^a) \tag{12}$$

where intertemporal separability is again assumed to prevail, but not separability of time and goods within any period. Michael suggests that if human capital enhances efficiency in market production, it may also increase the efficiency of nonmarket production. In (12), the parameter $\underline{a}$ captures the relative change in efficiency of work time compared to leisure time. For $a=0$, (12) reduces to the utility function of the previous section in which human capital provided no

consumption benefits. For a=1, human capital accumulation increases

the efficiency of both work and leisure equally. To see this, notice

that the marginal cost of leisure is constant in all three periods

when a=1. In other words, even though the market wage increases as

a result of investment, the opportunity cost of leisure time remains

unchanged because leisure increases in efficiency at exactly the

same rate as the market wage. Heckman has dubbed this case "Michael-

neutrality". For 0<a<1, efficiency still increases with human

capital accumulation, but the increase is not as fast as the increase

in market efficiency, so the opportunity cost of leisure increases

with investment. For a>1, the increase in the efficiency of leisure

time is greater than that in work time, so the opportunity cost of

leisure actually falls with investment in human capital. We want to

investigate how this leisure-technology parameter influences the

optimal profile of human capital accumulation.

The formal choice-problem for the individual is to select a

life cycle profile for consumption of goods and leisure and invest-

ment in human capital to maximize the lifetime utility function in

(12) subject to the lifetime budget constraint and production func-

tion for human capital. The formal solution to this problem appears

in Appendix B. As before, our interest focuses upon the two first-

order conditions for investment time:

$$(1 - \ell_3^* + a\ell_3^*) \frac{dF}{dK_2} = (1+r)H_1 \tag{13}$$

$$(1+r)(1 - \ell_2^* - K_2 + a\ell_2^*) \frac{dF}{dK_1} + (1 - \ell_3^* + a\ell_3^*) \frac{dF}{dK_1} = (1+r)^2 H_0 \tag{14}$$

where ℓ^* again means that leisure is being chosen optimally.

These equations differ from the marginal conditions in the previous section by the addition of one (or more) extra term of benefits on the left hand side. For example, $a\ell_3^* \frac{dF}{dK_2}$ in (13) represents the consumption benefits received in period three (in terms of increased efficiency of leisure time) from investing in human capital during period two. Ceteris paribus, the presence of nonmarket benefits increases the lifetime production of human capital.

Two interesting cases should be noted. When a=0, (13) and (14) reduce to the marginal conditions of the previous section, as should be expected. Perhaps less expected, when a=1, (13) and (14) reduce to the marginal conditions of the wealth-maximization model. In this case planned lifetime consumption of leisure disappears from the calculation of marginal benefits of investment, because even in leisure the individual captures the returns from human capital investment.

Again we want to examine the life cycle profile of investment time that emerges from this model. The following theorem summarizes the influence of the leisure technology parameter on the investment profile:

Theorem Three: $K_1 > K_2$ is the only feasible investment profile that satisfied (13) and (14) when

    (a) a=1

or (b) $0 \leq a < 1$ and $\ell_3^* \geq \ell_2^*$

or (c) a>1 and $\ell_3^* \leq \ell_2^*$

Proof: We proceed with a proof by contradiction. Suppose $K_1 = K_2 = K$, so $\frac{dF}{dK_1} = \frac{dF}{dK_2} = \frac{dF}{dK}$. Combining (13) and (14) to eliminate $\frac{dF}{dK}$:

$$(1+r)^2 \frac{(1 - \ell_2 - K_2 + a\ell_2)}{(1 - \ell_3 + a\ell_3)} H_1 + (1+r)H_1 \overset{?}{=} (1+r)^2 H_0 \qquad (15)$$

Differentiation of the left hand side of (15) with respect to K shows it is a nondecreasing function of K since

$$\frac{(1+r)^2 (1 - \ell_2 - K + a\ell_2)}{(1 - \ell_3 + a\ell_3)} \frac{\partial H_1}{\partial K} \geq 0 \; .$$

Now evaluate the left hand side of (15) at K=0, where the expression attains its smallest value:

$$(1+r)^2 H_0 \frac{(1 + (a-1)\ell_2)}{(1 + (a-1)\ell_3)} + (1+r)H_0 \qquad (16)$$

It is evident that the value of (16) varies both with the value of $\underline{a}$ and relation between ℓ_2 and ℓ_3.

Suppose a=1, as in part (a) of the theorem. (15) reduces to

$$(1+r)^2 H_0 + (1+r)H_0 \overset{?}{=} (1+r)^2 H_0$$

which cannot hold with equality. And since the left hand side of (15) only increases as K increases, the assumption $K_1 = K_2$ always produces a contradiction. Following the previous two proofs, the assumption $K_2 > K_2$ only serves to make matters worse, so $K_1 > K_2$ is the only possible assumption that does not produce a contradiction.

Now as in part (b) of the theorem, suppose $0 \leq a < 1$. For K=0 and ℓ_3 sufficiently less than ℓ_2, the first term of (16) can be made arbitrarily small, so (15) can possibly hold with equality. However, for $\ell_3 \geq \ell_2$

equality would be again impossible, and only worse for $K > 0$ or $K_2 > K_1$. Thus $\ell_3 \geq \ell_2$ is a sufficient condition for $K_1 > K_2$ given $0 \leq a < 1$.

As in part (c) of the theorem, suppose $a>1$. For $K=0$ and ℓ_2 sufficiently less than ℓ_3, the first term of (16) can be made arbitrarily small, so (15) can hold with equality. However, for $\ell_3 \leq \ell_2$, equality is again impossible, and only worse for $K > 0$ or $K_2 > K_1$. Thus $\ell_3 \leq \ell_2$ is a sufficient condition for $K_1 > K_2$ given $a>1$. Q.E.D.

The importance of Theorem Three is that it demonstrates that the "normal" declining investment time profile is not the only possible life cycle pattern to emerge when individuals behave as utility maximizers and human capital provides consumption benefits. When these consumption benefits are small ($0<a<1$), so that human capital augments the efficiency of work more than leisure time, then a sufficient condition for the "normal" declining investment time profile to be optimal is that leisure time is nondecreasing. The intuition for the possibility of an "abnormal" increasing profile of investment time is similar to that for Theorem Two. Since most returns to investment in human capital (but not all returns, if $a>0$) are enjoyed only when the individual is working, if work time is most intensive late in life and if the future is heavily discounted, then later rather than earlier investment may be more profitable. A similar (but reversed) argument follows when $a>1$. Now, human capital augments the efficiency of leisure time more than work time. Since most returns to human capital now accrue during leisure time, if leisure time is increasing over the life cycle, the incentives to invest in human capital might increase with age if the future is sufficiently discounted.

The most interesting result is that if human capital is "neutral," so that investment in human capital equally augments the efficiency of work and leisure, then the "normal" declining investment time profile is the only feasible pattern. And since (13), (14) and (15) are all monotonic in a, we can say that the closer human capital is to being neutral, the more likely a normal investment profile emerges regardless of the life cycle of leisure. Again, the intuition should be clear. For a=1, the first-order conditions are formally the same as in the wealth maximization model where the "normal" investment profile was the only feasible pattern. Since human capital is neutral, the allocation of time between work and leisure activities over the life cycle does not affect the returns from investing in human capital so it cannot affect the timing of such investment over the life cycle.

Theorem Three assumes that leisure time is always being chosen optimally to satisfy the system of first order conditions in Appendix B. What does this life cycle of leisure look like? Just as in Section IV, very little can be said without imposing additional structure to the utility function. If we once again assume separability of time and goods within each period and if we assume that the utility function differs across time only by the individual's subjective rate of discount, then we can write

$$\frac{\dfrac{dz^{*}}{d\ell_2 H_1 a}}{\dfrac{dz^{*}}{d\ell_3 H_2 a}} = \frac{H_1}{H_2} \tag{17}$$

where z^{*} is the utility operator on leisure time.

Notice that this condition is not quite identical to equation (11) in Section IV because of the presence of efficiency effects on leisure. If we assume that z^* is homogeneous of degree ϵ in $H_1{}^a$, then

$$\frac{dz^*}{d\ell_2 H_1 a} = H_1^{a\epsilon} \frac{dz^*}{d\ell_2}$$

so (17) can be rewritten as

$$\frac{\dfrac{dz^*}{d\ell_2}}{\dfrac{dz^*}{d\ell_3}} = \frac{H_1^{1-a\epsilon}}{H_2^{1-a\epsilon}} \tag{18}$$

Equation (18) suggests that the life cycle of leisure depends in part upon the leisure technology parameter $\underline{a}$. Suppose $\underline{a}$ is sufficiently small so that $1 - a\epsilon > 0$. Since $H_2 \geq H_1$, then $\ell_3 \leq \ell_2$ or in other words, leisure time declines over the life cycle. But according to part (b) of Theorem Three, these are the exact conditions under which the optimality of the "abnormal" increasing profile of investment time can not be ruled out. Now suppose $\underline{a}$ exceeds one by a sufficient amount so that $1 - a\epsilon < 0$. For $H_2 \geq H_1$, $H_2^{1-a\epsilon} \leq H_1^{1-a\epsilon}$ and $\ell_3 \geq \ell_2$; or in other words, leisure time increases over the life cycle. According to part (c) of Theorem Three, these conditions also are consistent with the possible optimality of the "abnormal" increasing investment profile.

To summarize, our investigation of the life cycle of leisure permits many feasible paths for leisure time over the life cycle. It is not possible to show that the only optimal paths for leisure coincide with the sufficient conditions stated in Theorem Three for the optimality of the "normal" declining investment profile. The leisure profile can reasonably follow paths that might be consistent with nondecreasing investment time.

VI   Summary and Extensions

This paper offers sufficient conditions (in terms of three theorems)
under which the optimal amount of time devoted to human capital produc-
tion declines over the life cycle. The results have been stated in
this form since there is some casual empirical evidence that a declining
profile is the "normal" life cycle—particularly when human capital
production take the form of schooling. To summarize these conditions,
a declining investment profile is the only feasible path if

1.   Individuals maximize lifetime income, or

2.   Individuals maximize lifetime utility and human capital is
      "neutral" in its effects on the efficiency of work and leisure
      time (a=1), or

3.   Individuals maximize lifetime utility, human capital is
      "biased" towards market efficiency ($0 \leq a < 1$), and leisure time
      increases over the life cycle, or

4.   Individuals maximize lifetime utility, human capital is
      "biased" towards leisure efficiency ($a > 1$), and leisure time
      decreases over the life cycle.

If none of these conditions is satisfied, then other investment profiles
may be possible, including increasing investment time over the life cycle.

How likely is it that one of these four conditions will be satisfied?
The recent human capital literature dismisses the first possibility of
wealth maximization as an unrealistic paradigm. If individuals behave
as utility maximizers, then the question reduces to an examination of
the neutrality or nonneutrality of human capital (the value of a).
Unfortunately there have been very few attempts at estimating the value

of the leisure technology parameter $\underline{a}$.[9] If it could be demonstrated, for example, that human capital was approximately neutral, then there would be evidence that the second condition is satisfied. This is obviously a question that cannot be resolved without a good deal more evidence.

There is another way in which the current work is unsatisfactory. Our treatment of nonmarket benefits of human capital in Section V is really assymmetric. If an increase in the stock of human capital makes both work **and** leisure time more efficient, then why doesn't it also raise the efficiency of the other use of time—investment time? The production function could and should be amended to include the possibility of neutral or nonneutral increases in the efficiency of investment time. One could rewrite equation (1) as

$$H_i - H_{i-1} = F(K_i H_{i-1}^b) \tag{19}$$

where $\underline{b}$ is a positive learning technology parameter. If b=1, equation (19) suggests that part of the current stock of human capital is itself an input in the production of additional human capital. This neutral specification was introduced by Ben-Porath. If $0 < b < 1$, acquisition of human capital still augments the production of future human capital but not as much as it increases market earnings—human capital is biased toward work. For $b > 1$, human capital is biased toward investment since the efficiency of investment time rises faster than market wages.

Just as we investigated the effect of the leisure technology parameter $\underline{a}$ on the life cycle profile of investment, so too could we, in

principle, investigate the effect of the learning technology parameter b. Unfortunately, this does not turn out to be a very easy task. It is impossible to offer any closed form conditions that summarize the influence of b on the investment profile. It is easy to see why this is so: raising b has two opposing effects. On the one hand, raising b increases the incentives to invest now because it raises the marginal benefit of current human capital production since returns will accrue not only as higher future earnings, but also as lower costs of future production. On the other hand, raising b increases the incentives to postpone some investment since future marginal costs will be lower.

While it may not be possible to explicitly derive any theorems on the effect of b on the investment profile, Paula Stephan has investigated the influence of b through computer simulation analysis. She shows how life cycle investment and earnings profiles will be affected when human capital has a market bias ($0 \le b < 1$) and when it has a production bias ($b > 1$). Unfortunately, however, she assumes that the individual selects his investment plan to maximize lifetime income, not lifetime utility. It would be interesting to investigate how Stephen's conclusions are affected by a change in the objective function. This paper suggests her results may well be very sensitive to the wealth maximization criterion. As yet, the question remains unresolved.

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Footnotes

[1] See Blaug [3].

[2] Five articles are noteworthy. They are Blinder and Weiss [4], Ryder, Stafford and Stephen [10], Ghez and Becker [5], and two by Heckman [7,8].

[3] For example, Becker [1] and Ben-Porath [2].

[4] Of course, even firms can be constrained in the presence of internal or external adjustment costs. Blinder and Weiss point out the formal similarity between a firm's investment behavior faced with adjustment costs and human capital accumulation.

[5] Ryder, Stafford, and Stephen [10], page.

[6] Specifically, it is assumed that $\frac{dF}{dK} > 0$, $\frac{d^2F}{dK^2} < 0$, $F(0) = 0$,

$\lim_{K \to 0} \frac{dF}{dK} (0) = \infty$, and $\lim_{K \to 1} \frac{dF}{dK} (1) = 0$.

[7] See Michael [9].

[8] See Heckman [7].

[9] Graham [6] cannot reject the hypothesis that $a=1$ based upon a panel study of the human capital investment behavior of some 900 families surveyed by the Michigan Survey Research Center.

[10] See Stephen [11].

Appendix A

The individual selects X_1, X_2, X_3, ℓ_1, ℓ_2, ℓ_3, K_1, and K_2 to maximize

$$u(X_1,\ell_1) + v(X_2,\ell_2) + z(X_3,\ell_3)$$

subject to the budget constraint

$$(1+r)^2(1-\ell_1-K_1)wH_0 + (1+r)(1-\ell_2-K_2)wH_1$$

$$+ (1-\ell_3)wH_2 - (1+r)^2X_1 - (1+r)X_2 - X_3 = 0$$

and the human capital production functions

$$H_1 = H_0 + F(K_1)$$

$$H_2 = H_1 + F(K_2)$$

The first-order necessary conditions (where λ is the multiplier associated with the budget constraint) are:

$$\partial u/\partial X_1 - \lambda(1+r)^2 = 0 \qquad\qquad (A.1)$$

$$\partial v/\partial X_2 - \lambda(1+r) = 0 \qquad\qquad (A.2)$$

$$\partial z/\partial X_3 - \lambda = 0 \qquad\qquad (A.3)$$

$$\partial u/\partial \ell_1 - \lambda(1+r)^2 wH_0 = 0 \qquad\qquad (A.4)$$

$$\partial v/\partial \ell_2 - \lambda(1+r)wH_1 = 0 \qquad\qquad (A.5)$$

$$\partial u/\partial \ell_3 - \lambda wH_2 = 0 \qquad\qquad (A.6)$$

$$(1+r)(1-\ell_2-K_2)\frac{dF}{dK_1} + (1-\ell_3)\frac{dF}{dK_1} = (1+r)^2 H_0 \qquad (A.7)$$

$$(1-\ell_3)\frac{dF}{dK} = (1+r)H_1 \tag{A.8}$$

$$(1+r)^2(1-\ell_1-K_1)wH_0 + (1+r)(1-\ell_2-K_2)wH_1$$
$$+ (1-\ell_3)wH_2 - (1+r)^2X_1 - (1+r)X_2 - X_3 = 0 \tag{A.9}$$

Appendix B

The individual selects X_1, X_2, X_3, ℓ_1, ℓ_2, ℓ_3, K_1, and K_2 to maximize

$$u(X_1, \ell_1 H_0{}^a) + v(X_2, \ell_2 H_1{}^a) + z(X_3, \ell_3 H_2{}^a), \qquad a \geq 0$$

subject to the budget constraint and production functions in Appendix A.

The first order conditions are:

$$\partial u / \partial X_1 - \lambda(1+r)^2 = 0 \tag{B.1}$$

$$\partial v / \partial X_2 - \lambda(1+r) = 0 \tag{B.2}$$

$$\partial z / \partial X_3 - \lambda = 0 \tag{B.3}$$

$$\frac{\partial u}{\partial \ell_1 H_0{}^a} H_0{}^a - \lambda(1+r)^2 w H_0 = 0 \tag{B.4}$$

$$\frac{\partial v}{\partial \ell_2 H_1{}^a} H_1{}^a - \lambda(1+r) w H_1 = 0 \tag{B.5}$$

$$\frac{\partial z}{\partial \ell_3 H_2{}^a} H_2{}^a - \lambda w H_2 = 0 \tag{B.6}$$

$$\frac{\partial v}{\partial \ell_2 H_1{}^a} a \ell_2 H_1^{a-1} \frac{dF}{dK_1} + \frac{\partial z}{\partial \ell_3 H_2{}^a} a \ell_3 H_2^{a-1} \frac{dF}{dK_1}$$

$$+ \lambda\left(-(1+r)^2 w H_0 + (1+r)(1-\ell_2-K_2)w\frac{dF}{dK_1}\right.$$

$$\left. + (1-\ell_3)w\frac{dF}{dK_1} \right) = 0 \tag{B.7}$$

$$\frac{\partial z}{\partial \ell_3 H_2{}^a} \, a\ell_3 H_2^{a-1} \frac{dF}{dK_2} + \lambda(-(1+r)wH_1$$

$$+ (1-\ell_3)w\frac{dF}{dK_2}) = 0 \qquad\qquad (B.8)$$

$$(1+r)^2(1-\ell_1-K_1)wH_0 + (1+r)(1-\ell_2-K_2)wH_1$$

$$+ (1-\ell_3)wH_2 - (1+r)^2 X_1 - (1+r)X_2 - X_3 = 0 \qquad (B.9)$$

To obtain equation (13) in the text, substitute (B.5) and (B.6) into (B.7). To obtain equation (14) in the text, substitute (B.6) into (B.8).

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