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Manufacturing Processes

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Abstract

We formulate the problem of converting a labor-intensive batch
production process to one that incorporates flexible automation as a
finite-state Markov decision process. Interest rates and the level of
automated technology influence both operating and acquisition costs and are
treated as random variables. The model specifies the optimal level of capacity
to convert to flexible automation. The optimization criterion is the
minimization of the sum of expected, discounted costs incurred over a finite
planning horizon. The optimal acquisition strategy depends upon the time
period, the current interest rate, the current level of technology, and a
measure of the remaining capacity that is not automated. We investigate the
structure of optimal acquisition strategies using both mathematical analysis
and simulation. Our objective is to illustrate the qualitative characteristics
of optimal strategies for acquiring flexible automation. As a step toward the
implementation of the model, we examine the qualitative consequences associated
with specifying classes of inventory and acquisition cost functions.

OPTIMAL ACQUISITION OF AUTOMATED
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by

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An automated flexible manufacturing process is a set of computer controlled work stations connected by automated material handling, which is used to produce multiple variations of parts at low to medium volumes. Such a process may incorporate general purpose robots that interact with other general purpose equipment such as drill presses, lathes, and milling machines. A key feature of automated flexible manufacturing processes is the ability to achieve both the flexibility of a job shop and high throughput rates, while simultaneously reducing direct labor cost.

Although numerous benefits are associated with automated flexible manufacturing processes, such as reduced labor cost, faster throughput times, and faster responses to demand volume changes and to product design changes, the decision whether to invest in such a process is difficult. In practice, there is major uncertainty about implementation costs, date of on-line availability, and performance characteristics once on line. Many of the benefits typically associated with flexibility, such as improved quality control, reduced work-in-process inventories, and reduced lead times, are not yet fully substantiated and may be difficult to measure.

In this paper, we develop a dynamic, stochastic optimization model of the automated flexible manufacturing investment decision process. The model, a Markov decision process (MDP), explicitly considers uncertainties associated with the evolution of technology and the level of future interest rates. It also considers inventory reductions that can be obtained from using automated flexible manufacturing processes. While the model addresses issues related to

the acquisition of capital equipment, the intent is not to determine when a particular machine should be scrapped and replaced with a newer model. Rather our objective is to better understand the implications of uncertainty and other factors on long-run strategic plans for the acquisition of new technology in a batch manufacturing setting.

Our primary goal is to build a model that is rich enough to capture some of the salient features found in a complex decision environment and yet is amenable to analysis. We use our model to analyze the qualitative properties of optimal policies for the acquisition of flexible automation. These policies specify both the timing and level of conversion to flexible automation. We provide both analytical and simulation results to illustrate these properties and to partially validate the model specification. We specify particular functional forms for several of the relevant cost functions. The functional forms we employ are fairly general and are quite robust with respect to parameter estimation. As part of the validation process, however, we test the sensitivity of our results to the specification of these cost functions.

The remainder of the paper is organized as follows. Prior research relevant to our model is discussed in the next section. The model is developed in Section 2. In Section 3 we present several general analytical properties of optimal acquisition policies. In Section 4, we specify particular functional forms for various elements of the model. In Section 5 we describe a simulation experiment we use to investigate the structure of acquisition policies for the general model, and we discuss several results within the context of the simulation. Concluding remarks are in Section 6.

1. Prior Research

There are many papers in the economics, industrial engineering, and operations research literature that are related to the problem of acquiring automation. These papers can be segmented into two groups based upon the level of generality of the issues being studied. The first group deals with the general problem of the adoption of new technology. The second group focuses on the particular problem of converting to new types of production processes.

Papers in the first group give insight into the problem of selecting new technology as it becomes available. Dreyfus (1960), for example, considers a general equipment replacement problem that incorporates technological change. In the deterministic dynamic program that he formulates, Dreyfus considers replacing a machine of known age with a new machine. Technological improvements are modelled as reductions in machine maintenance cost and increases in revenues. Dreyfus' primary model is a (finite horizon) regenerative process that permits either the replacement of the entire production process with new equipment or doing nothing for at least one more period.

Balcer and Lippman (1982) develop a general dynamic, stochastic model of the timing and extent of technological adoption. Their model is quite comprehensive, encompassing much of the previous work on technological adoption found in the economics literature. They assume, as we do, that technology improves randomly over time. (Unlike us, however, they permit the time until the next improvement in technology to depend on factors other than the current level of technology.) In any period, the firm chooses to upgrade the technology currently in place to the level presently available or to do nothing. Technological improvements are measured by reductions in production cost. The focus of the paper is on the role of expectations of future

technological developments on current adoption policies. Balcer and Lippman show that under certain conditions a firm may choose to adopt a technology that had been available in past periods but had not been chosen.

Manne's (1961) work on capacity expansion investigates both deterministic and probabilistic models for determining the optimal timing and level of additional fixed capacity. His major results show that as the discount rate decreases and the economies of scale increase, more excess capacity is purchased for future demand (which was assumed to be linearly increasing). Further, he shows that as the variance of demand increases, the optimal capacity to purchase at one time increases -- similar to probabilistic economic order quantity results. Manne does not investigate the situation where conversion of capacity to more automated processes influences the relevant cost functions, but rather assumes that variable production costs remain the same for any level of capacity expansion.

The second group of papers deals with the specific problem of incorporating new technology into the production process. While papers in this group include details relevant to the acquisition of flexible automation, none of them permits the random evolution of technological development or uncertain interest rates. Gaimon (1985a,b), for example, formulates deterministic control models of the acquisition of automation. In both models, the control variables are the increase in automation and the increase or decrease in manual labor per unit time. The terms in each of the objective functions are a penalty for deviating from a planned production schedule, the cost of automation acquisition, the costs of increasing and decreasing the level of labor, and a salvage value term. In Gaimon (1985a), production cost is also included; in (1985b) the cost of maintaining both labor and automated equipment is in the objective function. Each paper establishes the optimality of not

simultaneously increasing and decreasing the rate of manual output.

Hutchinson (1976) models the choice between a transfer line that yields full capacity immediately and a computer-aided manufacturing (CAM) system that can be expanded incrementally over time. The choice between CAM and a transfer line is made on the basis of each system's net present value computed over a given planning horizon.

The model we develop in the next section is a hybrid of the models in the two groups of papers discussed above. It concerns the adoption of new technology, as in group one, and it permits the incremental acquisition of this technology, as in group two.

2. The Model

The choice of manufacturing technology is intimately related to decisions regarding the mix, scope, and volume of products that can or will be produced. (See Monahan and Smunt (1985) for an illustration of a framework for this model.) For now, however, we simplify the complex problem of jointly determining the production technology and the market basket of products to be manufactured. Our focus is only on the choice of technology under uncertainty. Therefore, we assume that mix, scope, and volume of products to be produced by an ongoing batch manufacturing firm is fixed over a finite horizon of N periods. (Typically, periods may represent years, and reasonable values of N may range from five to ten.) The firm is confronted with the problem of updating the technology of the process used to produce these products. New technology becomes available (randomly) over time. The cost of converting some or all of the present production process to the new technology depends not only on the technology available but on interest rates as well. Interest rates also vary randomly over the N period planning horizon.

A distinguishing feature of the model is the inclusion of inventory related consequences of production process choices in an uncertain environment. Technological improvement is measured by reductions in the purchase cost of flexible automation. Technological uncertainty, therefore, creates an incentive to delay the acquisition of new technology. Inventory costs, while related to production costs and the level of interest rates, depend upon the production technology. In a batch production process, an important attribute of flexible automation is the ability to reduce inventory-related costs by decreasing both throughput and setup times. High interest rates and high inventory costs, therefore, create an incentive to acquire inventory cost reducing technology as quickly as possible. We use the model developed here to examine the consequences of these conflicting pressures.

The objective of the firm (and output of the model) is to determine an expected cost minimizing strategy the prescribes the optimal level of new technology to employ in each period as a function of the level of technology available, the mix of new and old technologies currently employed, and the current level of interest rates. We model the random improvement in technology and the random evolution of interest rates as a (time-homogenous) Markov chain. Suppose that interest rates are aggregated into L disjoint intervals, labelled 1,...,L. If the interest rate falls into the ith interval, we say that the interest rate level is i. We model technological change by supposing that there is a family of K technological cost functions, indexed by k, k=1,...,K. Each function specifies the cost of acquiring various quantities of flexible automation. For concreteness, we assume that "1" is the "lowest" level of technology, representing the highest per unit acquisition cost; K labels the "highest" level of technology.

Let $S = \{(i,k) : i=1,...,L \text{ and } k=1,...,K\}$. Let M denote the cardinality

of S. Let $P = [p_{ss'}]$ denote the $M \times M$ one-step transition matrix of the Markov chain $(X_t, t=1,2,\ldots,N)$ defined on S, where $p_{ss'} = Pr\{X_{t+1}=s' \mid X_t=s\}$ for $s,s' \varepsilon S$. X_t represents the interest-technology pair that prevails during period t. The time-homogeneity assumption is made solely for expository convenience.

Our assumption that the market basket of products to be produced remains fixed over the N period planning horizon implies that the capacity required over the N periods is known. We model the mix of old and new technology by specifying a_n as the proportion of that capacity that is being produced by flexible automation (new technology) at the beginning of period n. The <u>state</u> of the decision process is $(s,a) \varepsilon Sx[0,1]$, where a is the proportion of capacity that is automated at the beginning of the period and s is the interest-technology pair. Given $a_n=a$ and $X_n=s$, the problem in period n is to determine to what extent the proportion of capacity that is automated should be increased, if at all. The objective is to minimize the sum of discounted expected costs incurred over the entire planning horizon.

Let $p \varepsilon [0,1]$ denote the updated proportion of capacity that is automated (and used) in period n after the decision in period n has been made. If $a_n=a$, then $p \geq a$, and p-a is the new proportion of capacity that is automated and used in period n.

Total single-period cost when the state of the decision process is $(s,a)=(i,k,a)$ and "action" $p \geq a$ is chosen is denoted as $C(p,s,a)$ and is the sum of three components:

$$C(\hat{p},s,a) = \begin{matrix} c(p) & \text{production cost} \\ + I(p,i) & \text{inventory carrying cost} \\ + A(p,s,a) & \text{flexible automation acquisition cost} \end{matrix} \qquad (1)$$

for a ε [0,1], a $\leq$ p $\leq$ 1, and s ε S. In subsequent sections, structure is added to the problem by placing some restrictions on the forms of the three components of the single-period cost function.

Let $f_n(s,a)$ denote the discounted expected total cost incurred from the beginning of period n through the end of the planning horizon given that an optimal flexible automation acquisition policy is followed from period n onwards. Then $f_n(s,a)$ satisfies the following dynamic programming recursion:

$$f_n(s,a) = \min\{J_n(p,s,a)\} \qquad\qquad (2)$$
$$1 \geq p \geq a$$

where

$$J_n(p,s,a) = C(p,s,a) + E\lambda(s)f_{n+1}(a,s') \qquad\qquad (3)$$

and s' ε S denotes the random interest-technology pair next period, E denotes expectation, and $\lambda(s)$ is the one-period discount factor. Define

$$f_{N+1}(s,a) = g(s,a) \qquad\qquad (4)$$

as the salvage value of an automated process of "size" a. Additional structure is placed on $g(\cdot,\cdot)$ in subsequent sections.

An <u>optimal flexible automation acquisition strategy (or policy)</u> is specified as a sequence of functions $p_n(s,a)$, n=1,...,N, where $p_n(s,a)$ denotes the optimal proportion of flexible automation to have (and use) in period n given that the interest-technology pair is s and the proportion of flexibility is a at the end of period n-1, i.e., using (2), $f_n(s,a) = J_n(p_n(s,a),s,a)$, for n=1,...,N. Therefore, the optimal proportion of flexibility to <u>purchase</u> in period n is $p_n(s,a)$ - a.

3. Analytical Properties of Optimal Acquisition Strategies

In this section we place some mild restrictions on the single-period cost function given in (1) and the salvage value function given in (4) in order to establish properties of the optimal acquisition strategy and the optimal acquisition cost function.

Assumptions

1. $g(s,a)$ is convex and decreasing in a.

2. $C(p,s,a)$ is jointly convex in (p,a) and is   (5)
   continuously differentiable in p.

3. $A(p,s,a)$ is linearly separable in p and a
   and is decreasing in a.

In Section 5, we give explicit examples of production, inventory, and acquisition cost functions that satisfy the three, rather general, assumptions. The proofs of the results in this section are given in the Appendix.

The first result characterizes the optimal cost functions, given in (2) and (4). For any interest-technology pair s, the optimal cost incurred from period n through period N+1 decreases as the level of automation increases and is convex in the level of automation.

Proposition 1. For s ε S and n=1,...,N+1.

   i. $f_n(s,a)$ is decreasing in a ε [0,1], and

   ii. $f_n(s,a)$ is convex in a ε [0,1].

We demonstrate in the proof of Proposition 1 that $J_n(p,s,a)$ is convex in p. Let $p_n^*(s,a)$ denote a global (unconstrained) value of p that minimizes (3); i.e., $p_n^*(s,a)$ is a solution to the equation $J_n^{[1]}(p,s,a) = 0$.

The next result is useful in characterizing the optimal acquisition

policy.

Lemma 1. The unconstrained optimal level of automation in period n is independent of the current level of automation; i.e., $p_n^*(s,a) = p_n^*(s,0)$ for any a, $0 \leq a \leq 1$.

With the structure of the optimal cost function developed in Proposition 1 and the structure of the unconstrained solution given in Lemma 1, we can now describe the structure of the optimal acquisition policy. Generally, period n's optimal level of automation is independent of the current level of automation (unless the level of automation in place at the beginning of period n exceeds the desired level).

Proposition 2. For all a, s, and n, $p_n(s,a) = \max\{p_n^*(s,a),a\}$.

It follows from Lemma 1 and Proposition 2 that $p_n(s,a)$ is nondecreasing in a.

The next result asserts that the optimal level of automation declines as the planning horizon shortens.

Proposition 3. For all a, s, and n, $p_n(s,a) \geq p_{n+1}(s,a)$.

The combination of Lemma 1, Proposition 2, and Proposition 3 yields the following attribute of the optimal acquisition policy.

Proposition 4. For $a \in [0,1]$, $p_n^{[2]}(s,a) \leq p_{n+1}^{[2]}(s,a)$.

In words, while the optimal desired level of automation increases as the

current level of automation increases (Proposition 2), the _rate_ of increase

declines as the planning horizon lengthens.

The final result in this section indicates how the marginal value of

additional automation changes over time. In particular, we show that the

marginal value of another unit of automation, while positive (Proposition 1

(i)), declines as the planning horizon shortens. When there are fewer periods

in which to utilize a given level of automation, the benefits that can be

generated by the automation decline.

Proposition 5. For a ε [0,1] $f_n^{[2]}(s,a) \leq f_{n+1}^{[2]}(s,a)$, for n=1,2,...,N.

The attributes of the optimal acquisition policy and the optimal cost function

stated in the Propositions above are illustrated in a discrete-state example in

Section 5.

Up to this point, we have established several properties of the optimal

acquisition strategy under fairly mild restrictions on the single-period cost

and salvage value functions. Several important issues relating to the

implementation of the model remain, however. What functional forms satisfy the

assumptions in (5)? What _might_ the components of the single-period cost

function look like? How sensitive are the qualitative results in this section

to the assumptions in (5)? We now investigate these and other related issues.

4. Particular Functional Forms

The application of the model developed here requires no a priori

restrictions on the form of the cost functions. Their specification is an

important issue, however. In this section we investigate the ramifications

associated with the specification of the cost functions. To do this, we

propose a class of inventory and acquisition cost functions that have the desirable properties of generality, require few parameters, and, under appropriate restrictions on the parameters, satisfy the assumptions in (5). In the absence of any restrictions on the parameters, analytical results analogous to those in Section 3 are difficult to obtain. To test the sensibility and credibility of the general functional forms, we specify particular functions and parameter values, compute optimal acquisition strategies, and simulate the application of those strategies in an environment that evolves according to the Markov chain (X_t). Finally, we test the sensitivity of our results to departures from these specified functional forms.

Inventory Carrying Cost

Inventory carrying cost is specified in the usual form as a variable cost function of production cost. First, we multiply the "value added" production cost, $c(p)$, by an "opportunity cost" multiplier $\gamma(i)$, where i is the interest rate level and $\gamma(i) \geq 0$ and $\gamma(i) \leq \gamma(i+1)$. This factor captures the capital cost of investing in inventory. Included in $\gamma(i)$ are the interest rate and other holding cost factors accounting for costs such as warehousing, obsolesence, shrinkage, and damage.

Inventory reductions may stem from flexibility because flexible automation usually provides lower setup costs and quicker response times. We model this effect by including another multiplier of $c(p)$, denoted $\alpha(p)$. We assume that $\alpha(p)$ multiplier is a decreasing and convex function of p. We further assume that the 0 percent flexible automation is the "base" case for inventory calculations; therefore $\alpha(0) = 1.0$. When the decision to purchase flexible automation is considered, current company cost data typically indicates the cost of inventory using the existing non-flexible production

process. Estimates of the reduction in lead times and the resulting decreases in inventory levels can be determined from simulation or analytic models. (See Monahan and Smunt (1987) for examples of using simulation models in this context.) Finally, we assume that inventory carrying costs cannot be completely eliminated by flexible automation but may be reduced to very low levels, i.e., $\alpha(1) > 0$.

In summary, the inventory carrying cost function is modeled as follows:

$$I(p,i) \equiv \alpha(p) \cdot \gamma(i) \cdot c(p), \qquad (6)$$

for $p \ \varepsilon \ [0,1]$ and $i=1,\ldots,L$.

Cost of Acquiring Flexible Automation

Let a denote the proportion of capacity that is automated. We assume that the cost of increasing this proportion to p, where $a \leq p \leq 1$, is:

$$P(p,s,a) \equiv T(p-a) \cdot \beta(s) \cdot c(0), \qquad (7)$$

where s represents the interest-technology pair, and $c(0)$ represents the "base-line" cost per period of producing with no automation. The multiplier $T(p-a)$ represents the diseconomies (economies) associated with purchasing flexible automation incrementally over time rather than all at once. To capture this phenomenon, we assume that $T(\cdot)$ is increasing and concave (convex), $T(0) = 0$, and $T(1) = 1$.

The multiplier $\beta(s)$ relates the actual cost of converting 100 percent of the capacity to flexible automation. Assume, for example, that for a given interest-technology pair s, it would cost a firm $c_F(s)$ to convert to 100 percent automation from no automation. Then $c_F(s) = \beta(s) \cdot c(0)$. The $\beta(s)$ term expresses the flexible automation acquisition cost as a multiple of the

cost to produce one period's demand with no flexible automation. We assign the acquisition cost in this way for generality. Managers often evaluate equipment acquisition costs in terms of a payback analysis, where the "years to payback" the fixed cost from variable cost saving are determined. Multipliers in the range of 1 to 5 are reasonable values for major capital investments. The $\beta(s)$ multiplier is a surrogate for the payback period and is a convenient way of parameterizing the acquisition cost. High β-values imply that flexible automation is costly relative to associated variable cost savings. Conversely, low β-values imply that flexible automation is relatively inexpensive.

5. Sensitivity Analysis: A Simulation Experiment

In this section we test the reasonableness of the general functional forms specified in the previous section. To do this we further select particular functions and parameter values that conform to the guidelines discussed in Section 4. We then generate a discrete-state analog of the MDP given in (2) and (4). For a given set of parameter values, we compute an optimal acquisition policy and implement that policy in our Markovian environment. The optimal policy $\{p_1(s,a),\ldots, p_{10}(s,a)\}$ is computed using a standard successive approximation algorithm, described for example in Hillier and Lieberman (1986, page 726). We use this simulation methodology to examine qualitative characteristics of optimal acquisition plans.

The characteristics we examine are related either to the effect of uncertainties in the external environment or to the direct costs effects peculiar to flexible manufacturing technology. Our goals are these:

· To better understand the influence of uncertainty on the acquisition process.

· To determine the significance of factors particularly relevant to

the acquisition of flexible automation, such as inventory cost reductions, on the acquisition decision.

5.1 Cost Functions

In this section, we assume that

$$c(p)=\rho_1 \exp(-\rho_2 p), \quad \alpha(p) = \exp(-\theta p), \text{ and } T(p-a) = (p-a)^\mu \tag{8}$$

where $\rho_1 > 0$, $\rho_2 \geq 0$, $\theta \geq 0$ and $\mu \geq 0$. For scaling, let $\beta(s) = \eta(s) \beta(1,1)$ for all i and k. We assume for each i and k that $\eta(i,\cdot)$ is nonincreasing and $\eta(\cdot,k)$ is nondecreasing. When $\mu \geq 1$, these structures satisfy the assumptions in (5). When $\mu < 1$, $T(p-a) > p-a$, and there are diseconomies of scale. In summary, the specific cost functions used in this section are:

$$c(p) \quad\quad = \rho_1 \exp(-\rho_2 p) \tag{9}$$

$$I(p,i) \quad = \rho_1 \exp[-(\theta + \rho_2)p]\gamma(i) \tag{10}$$

$$A(p,s,a) = \rho_1 (p-a)^\mu \eta(s)\beta(1,1), \tag{11}$$

and

$$g(s,a) = -\pi \cdot a \cdot \beta(s) \cdot c(0), \tag{12}$$

where $0 \leq \pi \leq 1$ represents the proportion of the acquisition cost that can be obtained from an automated process of size a when the interest-technology pair is s.

5.2 Simulation Experiment Design

To make the computations practical in this simulation, we discretize the state space and assume that p and a can take only the values 0.0, 0.2, 0.4, 0.6, 0.8, or 1.0. In words, flexible automation can be acquired only in "bundles" whose size is at least 20% of the single-period capacity of the production facility.

The experiments use the discrete-state model and the functional forms specified in Section 5.1. For a given set of parameter values, the optimal policy is computed and then implemented in an environment that evolves according to the one-step transition matrix for the Markov chain. The implementation of each policy is repeated 50 times for each set of parameter values.

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Insert Tables 1-4 here

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Base parametric values used in the experiments are given in Tables 1 and 4. (A base parameter value is the value of the parameter in any experiment in which that parameter is not being varied.) The parameters correspond to those presented in Section 5.1. Our investigation of the structure of optimal acquisition policies includes the impact of various forms of the exogenous environment specified by the interest-technology transition matrix. (In this experiment we treat interest rates and technology levels seperately in order to isolate the influence of each factor.) We examine three environments, labelled "Bad," "Uncertain," and "Good," for each of these factors. The one-step transition matrices defining each of these environments are given in Tables 2 and 3 for interest rate and technology levels, respectively. With respect to interest rate levels, the label "Bad" ("Good") is used to imply that the

interest rate level next period is likely to be high (low) independent of the level this period.

The η matrix of multipliers appearing in the acquisition cost function given in (11) is given in Table 4. Note that the values in this matrix have the desired properties that in each column, the η_{ik}'s are increasing in i and in each row η_{ik}'s are decreasing in k. As the environment gets "better," costs of acquiring flexible manufacturing technology decrease.

We begin the analysis of the discrete-state version of the model with an example and a description of the corresponding optimal policy. The optimal acquisition policy in the example was computed using the parameter values given in Table 1, with the exception that $\mu = 1.0$. The interest rate and technology level transition matrices used in the example are the I_U and T_B matrices in Tables 3 and 4. The optimal solution is given in Table 5, in the more convenient form of $(p_n(s,a) - a)100$; that is, each entry in the table indicates the optimal percentage of flexible automation to purchase in the state (s,a) in period n. Zero levels of acquisition are recorded as blanks.

The optimal policy given in Table 5 is interpreted as follows: Suppose that at the beginning of period 1, there is currently no flexible automation and that the interest-techonology pair is (2,1), representing the "worst" possible state since the interest rate level is as high as it will ever be and technology is as low as it will ever be. From Table 5 we see that it is optimal not to purchase any flexible automation; in period 1, the entry in the row labelled (2,1) and the column labelled 0% is blank. Suppose now that the interest-technology state moves to (2,3) at the beginning of period 2. (This will occur with probability 0.10 (=0.50 x 0.20).) In period 2, the entry in the row labelled (2,3) and the column labelled 0% (the level of automation at the beginning of the period) is 40%; it is optimal to convert 40% of the

capacity to flexible automation. Similar interpretation can be applied to the remaining entries in Table 5.

The solution given in Table 5 illustrates two of the analytical results given in Section 3. Note that in any row of the table, the sum of the row element with its column heading is constant across the row. This illustrates Proposition 2: In any period, the optimal new level of flexibility is independent of the current level fo flexibility for any interest-technology pair. Note also that for any interest-technology pair and any level of current flexibility, the optimal amount to purchase does not increase as the period numbers increase. This illustrates Proposition 3.

This simple example demonstrates that while the optimal acquisition policy is easy to specify and to use, the consequences of the policy are difficult to determine. In the next subsection, we discuss the results of the simulation experiments used to investigate the structure of the optimal acquisition strategies when there are diseconomies of scale associated with the purchase of flexible automation.

5.3 Simulation Experiment Results

In order to illustrate qualitative characteristics of optimal acquisition plans, we present graphs of the levels of flexible automation as a function of some parameter. Each figure depicts the average level of automation at the end of periods 1, 3, 5, 7, and 9, given that acquisitions are made on the basis of the optimal acquisition policy. The average level of automation is taken over the 50 repetitions of implementing the optimal acquisition policy in the randomly evolving external environment. In all the experiments we assume that the initial state is (1,1,0); i.e. there is no flexible automation at the beginning of the planning horizon and the initial

interest-technology state is at the "low interest rate/low technology level.

Technological Uncertainty

The average acquisition of flexible automation purchased over the planning horizon using the optimal acquisition policy is graphed in Figure 1 as a function of $\beta = \beta(1,1)$ for the three technological environments given in Table 3. For any fixed β, the vertical distances between the plots for the "Bad" environment indicate the average level of acquisition over the relevant two period span. Notice that the average acquisition curves for Period 9 shift upwards as the technological improvements become more certain, that is as the technological environments move from "Bad" to "Good." Further, our detailed results indicated that less incremental purchases are made in the "Good" environment, suggesting that more flexible automation is purchased in total, and it is acquired in larger increments in the earlier periods. Hence, a "wait-and-see" effect occurs. Firms that anticipate a likely improvement of technology over time should be reluctant to purchase in early periods. This confirms our observation that firms currently seem to be unwilling to invest heavily in automated technology even though they are planning large future investments in flexible automation.

We found a similar effect for the Interest Rate Uncertainty factor. As the interest rate environment improves, there is a tendency to purchase more automation (in total).

Insert Figure 1 here

Diseconomies of Scale

The diseconomies of scale factor μ was varied from 0.8 to 1.0. The resulting acquisition strategies for this simulation are illustrated in Figure

2. As μ increases to 1.0, the penalty for incremental acquisition decreases and larger incremental purchases are made in later periods. The overall level of capacity that is eventually automated is fairly constant as a function of μ. This result implies that firms may want to consider large fixed investments in flexible automation if significant costs are incurred by a piecemeal purchase strategy. Integration of subsystems acquired over several periods is sometimes critical for the success of this new technology.

Insert Figure 2 here

Direct Production Cost

Figure 3 illustrates two important aspects of acquiring flexible automation as the amount of direct production cost reduction varies with implementation. The graphs show that it is optimal to make incremental purchases as ρ_2 increases from 0.2 to 1.0, or as greater direct production cost reductions occur. First, the total amount of flexible automation purchased increases with ρ_2, as expected. Second, and more important, is that more automation is purchased earlier. The ability to reduce direct production cost implies that it is advantageous to introduce flexible automation on a piecemeal basis in early periods. This counters the "wait-and-see" effect.

Insert Figure 3 here

Inventory Cost

In Figure 4 we show the average acquisition strategy as the inventory level reduction factor θ varies from 0.1 from 0.9 and γ is 0.4 and 0.6 (relativiely high inventory carrying costs). Figure 4 shows that more

purchases of flexible automation are made earlier and in larger amounts when the inventory carrying cost can be significantly reduced by the implemention of flexible automation. Firms should be aware that the reduction of inventory levels made possible by increased throughout times stemming from such flexibility can be a major advantage.

Insert Figure 4 here

5.4 Sensitivity of Simulation Results to Specific Functional Forms

We conclude with a brief discussion regarding the (lack of) sensitivity of the findings in Section 5.3 to the specification of the functional forms. We also discuss tests of parameter misspecification.

In general, the changes in functional form or misspecification of a parameter value changed the optimal acquisition levels but not the overall direction or shape of the strategies. As an example of a parameter misspecification, we changed θ to 0.3 from the base value of 0.9, representing a higher level of inventory reduction obtained from using flexible automation. The total acquisition of the new technology increased by only 10%. The incremental acquistion during the planning horizon was similar to the acquisition in the original experiment. In another test of parameter misspecification, μ was lowered from 0.95 to 0.7 to indicate greater diseconomies of scale. As expected, the early acquisition of flexible automation was similar to that of the original results, but the purchases in latter periods were delayed slightly. Although delayed, the latter purchases tended to be larger than they were originally since technology tended to be higher. As a final example, we changed the fixed cost acquisition function from a concave function to a cubic function. Since the incremental purchase

cost was relatively flat for moderate acquisition percentages, most purchases were made in the early periods; the size of each acquisition ranged from 60% to 80% of capacity by period 3. In no case, however, did the optimal policies indicate purchases above 80% due to the high incremental cost of the last 20%.

Our sensitivity results are similar to results reported in Manne (1961). He illustrated that the total cost function is relatively flat near the optimal value of capacity expansion size, so misspecification of the relevant variables may not lead to a disastrous choice for capacity increment. Changes in the functional form or misspecification of parameter values can obviously introduce errors into the model. Based upon the preliminary tests we conducted, however, it appears that the qualitative features of the optimal acquisition strategies are quite robust with respect to these errors.

6. Conclusions

The decision to invest in flexible automation requires the consideration of a number of complex, interrelated factors over which the firm may or may not have control. Technological development and the level of interest rates are beyond the control of the firm and greatly influence the production process investment decision.

We conclude (from our model) that technological and interest rate uncertainty influence the investment decision and that greater uncertainty regarding the cost of new technology delays the acquisition of flexible automation. Flexible automation is purchased, however, in periods of high interest rates when it is expected that future interest rates will remain high and inventory costs are a significant portion of total cost. Inventory cost reductions resulting from flexible automation counter the "wait-and-see" effect associated with uncertain interest rates and uncertain technological

advancement, but do so only when inventory costs are relatively high. Potential for significant reductions in production cost due to flexible automation also counters the "wait-and-see" effect. Finally, these general results appear robust in that major changes in the cost parameter functional forms change only the level of optimal acquisition plans but not their direction.

Future model development for implementation in manufacturing firms will require the addition of factors such as demand fluctuation, product-mix change, technological obsolescence, and response time to design changes. Our current research addresses these issues.

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APPENDIX

Proof of Proposition 1 (i) (by induction on n).

Since

$$f_{N+1}(s,a) = -\pi \cdot a \cdot \beta(s) \cdot c(0) \tag{A1}$$

is decreasing in a, the result is true for N+1. Assume $f_{n+1}(s,a)$ is decreasing

in a for some n, $1 \leq n \leq N$. For $0 \leq a_1 \leq a_2 \leq 1$,

$$J_n(p_n^*(s,a_2),s,a_2) \leq J_n(p_n^*(s,a_1),s,a_2)$$

$$\tag{A2}$$

$$\leq J_n(p_n^*(s,a_1),s,a_1),$$

where the first inequality follows from the optimality of p_n^* and the second

from the fact that C(p,s,a) is decreasing in a. Furthermore,

$$J_n(a,s,a) = C(a,s,a) + E \lambda(s) f_{n+1}(s',a)$$

is decreasing in a by the induction hypothesis.

Since

$$f_n(s,a) = \begin{matrix} J_n(p_n^*(s,a),s,a) & \text{if } p_n^*(s,a) \geq a \\ J_n(a,s,a) & \text{otherwise,} \end{matrix}$$

it follows from (A2) and the preceeding sentence that $f_n(s,a)$ is decreasing in

a.

Proof of Proposition 1 (ii)

To prove that $f_n(s,a)$ is convex in a it is sufficient to prove that

$J_n(p,s,a)$ is jointly convex in (p,a); see, e.g., Heyman and Sobel (1984,

Appendix B). Since $f_{N+1}(s,a)$, given in (A1), is linear in a, it is convex in

a. Suppose $f_{n+1}(s,a)$ is convex in a for some n, n = 1,2,...,N. Since C(p,s,a)

is jointly convex in (p,a), it follows that $J_n(p,s,a)$ is jointly convex in (p,a) and that $f_n(s,a)$ also has this property.

Proof of Lemma 1.

Since $\dfrac{\partial^2 J_n}{\partial p \partial a} = \dfrac{\partial^2 C}{\partial p \partial a} = 0$, it follows that the solution to $J_n^{[1]}(p,s,a) = 0$ is independent of a.

Proof of Proposition 2.

It is optimal to choose $p_n^*(s,a)$ if it is feasible to do so (i.e., if $p_n^*(s,a) \geq a$). Since $J_n(p,s,a)$ is convex in p, it is optimal to choose a if $p_n^*(s,a) < a$; convexity implies that $J_n(p,s,a)$ continues to increase as p gets larger than a.

Proof of Proposition 3-5.

Since $f_{N+1}^{[2]}(s,a) < 0$ and $f_{N+1}^{[2]}(s,a) = 0$ (since $f_{N+2}(s,p) \equiv 0$), Proposition 5 is true for N. Note that

$$J_n^{[1]}(p,s,a) = C^{[1]}(p,s,a) + E\ \lambda(s)f_n^{[1]}(s',p) \qquad (A3)$$

for $n=1,\ldots,N+1$. Therefore, from the relations above,

$$J_N^{[1]}(p,s,a) \leq J_{N+1}^{[1]}(p,s,a)$$

and $p_N^*(s,a) \geq p_{N+1}^*(s,a)$ follows from the convexity of $J_n(\cdot,s,a)$. Therefore, Proposition 3 holds for N. Assume that $f_{n+1}^{[2]}(s,a) \leq f_{n+2}^{[2]}(s,a)$ for some n, $n=1,\ldots,N-1$. This induction hypothesis, in conjunction with (A3), implies that $p_n^*(s,a) \geq p_{n+1}^*(s,a)$, which in turn implies that $p_n(s,a) \geq p_{n+1}(s,a)$. To

complete the proofs of Propositions 3 and 5, it remains to show that

$$f^{[2]}_n(s,a) \leq f^{[2]}_{n+1}(s,a). \qquad (A4)$$

We do this by examining three cases.

Case 1: $p_n(s,a) = p^*_n(s,a)$ and $p_{n+1}(s,a) = p^*_{n+1}(s,a)$.

In this case,

$$\begin{aligned}
f^{[2]}_n(s,a) &= [C^{[1]}(p_n(s,a),s,a) \\
&\quad + E \ \lambda(s) \ f^{[2]}_{n+1}(s',p_n(s,a))] \cdot p^{[2]}_n(s,a) + C^{[3]}(p_n(s,a),s,a) \\
&= C^{[3]}(p_n(s,a),s,a),
\end{aligned}$$

since the term in [ ] is zero. (The term in [ ] is $J^{[1]}_n(p,s,a)$ evaluated at p^*_n, the global minimizing value of p.) Similarly, it can be shown that

$$f^{[2]}_{n+1}(s,a) = C^{[3]}(p_{n+1}(s,a),s,a). \qquad (A6)$$

Since $C^{[3]}(p,s,a)$ is independent of p, (A4) follows from (A5) and (A6).

Case 2: $p^*_n(s,a) = p_n(s,a)$ and $p_{n+1}(s,a) = a$.

In this case, $f^{[2]}_n(s,a)$ satisfies (A5) and

$$f^{[2]}_{n+1}(s,a) = [C^{[1]}(a,s,a) + E \ \lambda(s)f^{[2]}_{n+2}(s',a)] \qquad (A7)$$

$$+ C^{[3]}(a,s,a).$$

The term in [ ] is simply $J^{[1]}_{n+1}(a,s,a)$, which is positive since $p_{n+1}(s,a) \neq p^*_{n+1}(s,a)$ and $J_{n+1}(p,s,a)$ is convex in p. Therefore, $f^{[2]}_{n+1}(s,a) > C^{[3]}(a,s,a) = C^{[3]}(p_n(s,a),s,a) = f^{[2]}_n(s,a)$, and (A4) again

holds. The first equality follows since $C^{[3]}(p,s,a)$ is independent of p.

Case 3: $p_n(s,a) = a$.

If $p_n(s,a) = a$, then $p_{n+1}(s,a) = a$. In this case, since
$p_n^{[2]}(s,a) = p_{n+1}^{[2]}(s,a) = 1$,

$$
\begin{aligned}
f_n^{[2]}(s,a) &= C^{[1]}(a,s,a) + C^{[3]}(a,s,a) + E\ \lambda(s)\ f_{n+1}^{[2]}(s',a) \\
&\leq C^{[1]}(a,s,a) + C^{[3]}(a,s,a) + E\ \lambda(s)\ f_{n+2}^{[2]}(s',a) \\
&= f_{n+1}^{[2]}(s,a),
\end{aligned}
$$

where the inequality follows from the induction hypothesis. Again, (A4) holds.

Since Cases 1-3 exhaust all possible relationships between p_n and p_{n+1},

Proposition 5 is proven.

Proposition 4 follows directly from Proposition 3 and its proof is

omitted.

TABLE 1

BASE PARAMETER VALUES

$N = 10$	Ten period planning horizon
$L = 2$	Two levels of interest rates: $1 \equiv$ Low, $2 \equiv$ High.
$T = 4$	Four levels of technology: $1 \equiv$ lowest level (highest cost) $4 \equiv$ highest level (lowest cost)
$(\lambda(1),\lambda(2)) = (.9, .7)$	Risk-adjusted discount factors as a function of interest rates.
$(\gamma(1),\gamma(2)) = (.2, .4)$	Inventory cost multiplier as a function of interest rates.
$\rho_2 = 10000$	Single-period direct production cost with no automation.
$\rho_2 = 0.5$	Measure of direct production cost reduction resulting from automation.
$\theta = 0.3$	Inventory level reduction multiplier.
$\beta(1,1) = 2.6$	Base acquisition cost multiplier.
$\mu = 0.95$	Diseconomies of scale parameter.
$\pi = 0.2$	Salvage value parameter.

TABLE 2

INTEREST RATE ENVIRONMENTS

$$
I_B = \begin{array}{cc} 0.2 & 0.8 \\ 0.2 & 0.8 \end{array}
\qquad
I_U = \begin{array}{cc} 0.5 & 0.5 \\ 0.5 & 0.5 \end{array}
\qquad
I_G = \begin{array}{cc} 0.8 & 0.2 \\ 0.8 & 0.2 \end{array}
$$

"BAD"   "UNCERTAIN"   "GOOD"

TABLE 3

TECHNOLOGICAL ADVANCEMENT ENVIRONMENTS

$$
T_B = \begin{array}{cccc}
0.5 & 0.3 & 0.2 & 0.0 \\
0.0 & 0.5 & 0.4 & 0.1 \\
0.0 & 0.0 & 0.7 & 0.3 \\
0.0 & 0.0 & 0.0 & 1.0
\end{array}
\qquad
T_U = \begin{array}{cccc}
0.25 & 0.25 & 0.25 & 0.25 \\
0.0 & 0.33 & 0.33 & 0.33 \\
0.0 & 0.0 & 0.50 & 0.50 \\
0.0 & 0.0 & 0.0 & 1.00
\end{array}
$$

"BAD"   "UNCERTAIN"

$$
T_G = \begin{array}{cccc}
0.1 & 0.2 & 0.3 & 0.4 \\
0.0 & 0.1 & 0.3 & 0.6 \\
0.0 & 0.0 & 0.3 & 0.7 \\
0.0 & 0.0 & 0.0 & 1.0
\end{array}
$$

"GOOD"

TABLE 4

ACQUISITION COST MULTIPLIERS

$$
\eta = \begin{array}{cccc}
1.00 & 0.80 & 0.70 & 0.60 \\
1.10 & 1.00 & 0.80 & 0.65
\end{array}
$$

TABLE 5

OPTIMAL % ACQUISITION BY STATE AND TIME PERIOD
% CAPACITY AUTOMATED AT BEGINNING OF PERIOD

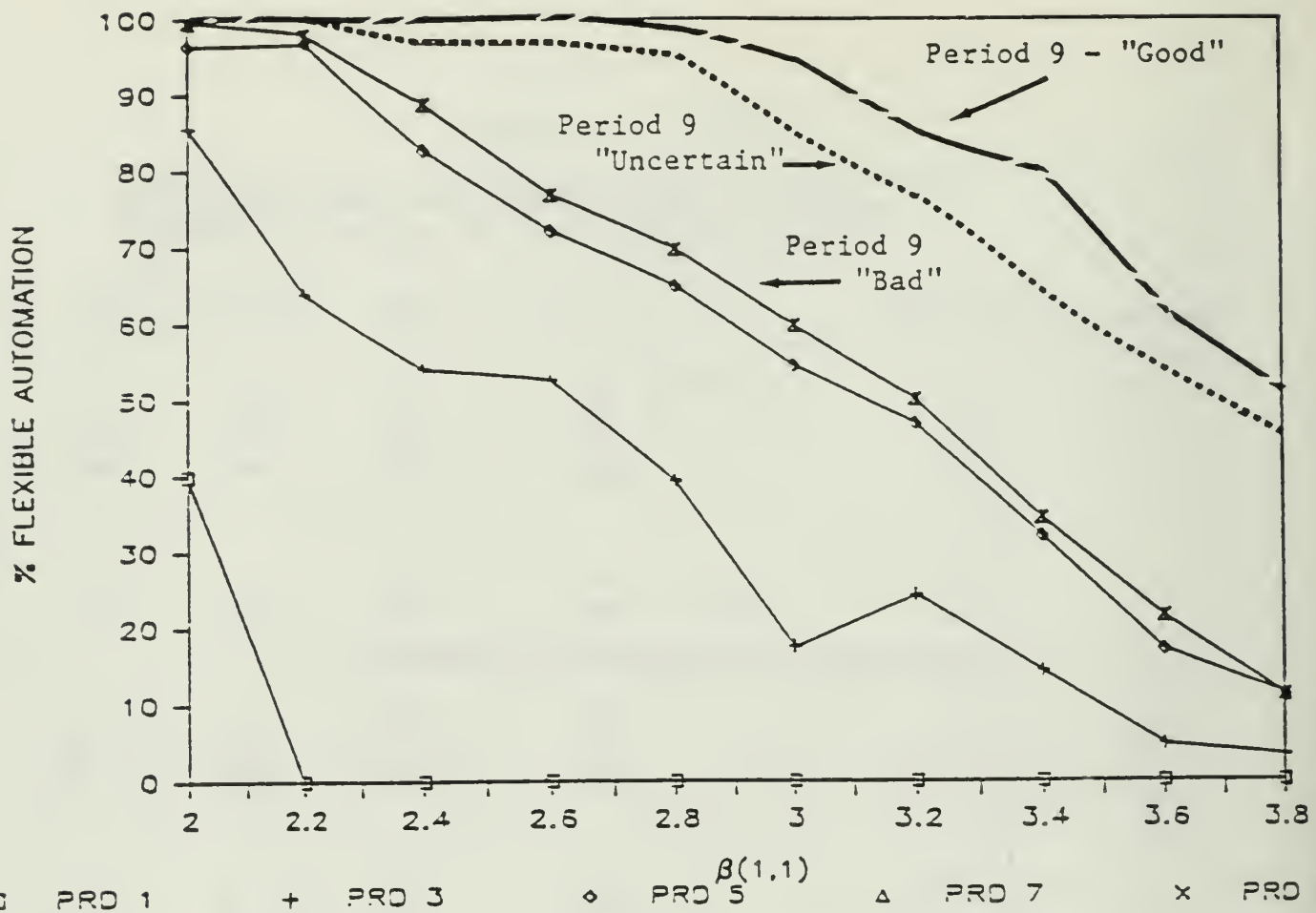| PERIOD	(INT, TECH)	0%	20%	40%	60%	80%
1-3	(1,1)					
	(1,2)	60	40	20		
	(1,3)	80	60	40	20	
	(1,4)	100	80	60	40	20
	(2,1)					
	(2,2)					
	(2,3)	40	20			
	(2,4)	100	80	60	40	20
4-5	(1,1)					
	(1,2)	40	20			
	(1,3)	80	60	40	20	
	(1,4)	100	80	60	40	20
	(2,1)					
	(2,2)					
	(2,3)	40	20			
	(2,4)	100	80	60	40	20
6	(1,1)					
	(1,2)	40	20			
	(1,3)	60	40	20		
	(1,4)	80	60	40	20	
	(2,1)					
	(2,2)					
	(2,3)	40	20			
	(2,4)	80	60	40	20	
7	(1,1)					
	(1,2)	20				
	(1,3)	40	20			
	(1,4)	60	40	20		
	(2,1)					
	(2,2)					
	(2,3)	20				
	(2,4)	60	40	20		
8	(1,1)					
	(1,2)					
	(1,3)	20				
	(1,4)	40	20			
	(2,1)					
	(2,2)					
	(2,3)					
	(2,4)	40	20			
9-10		All zeros				

Figure 1. Effect of Technological Uncertainty – plots of periods 1-9 for "Bad" environment and period 9 for "Uncertain" and "Good" environments.
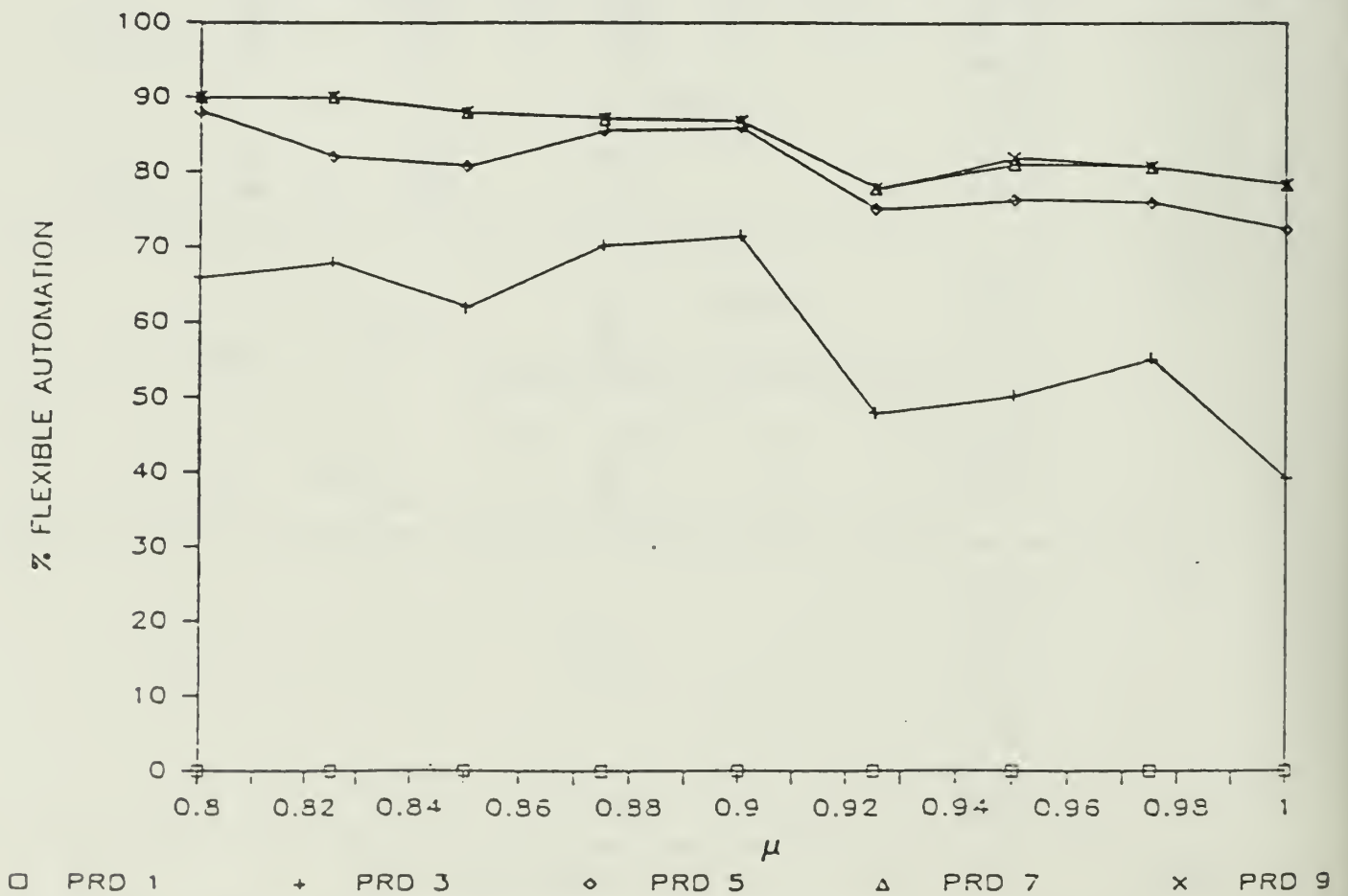

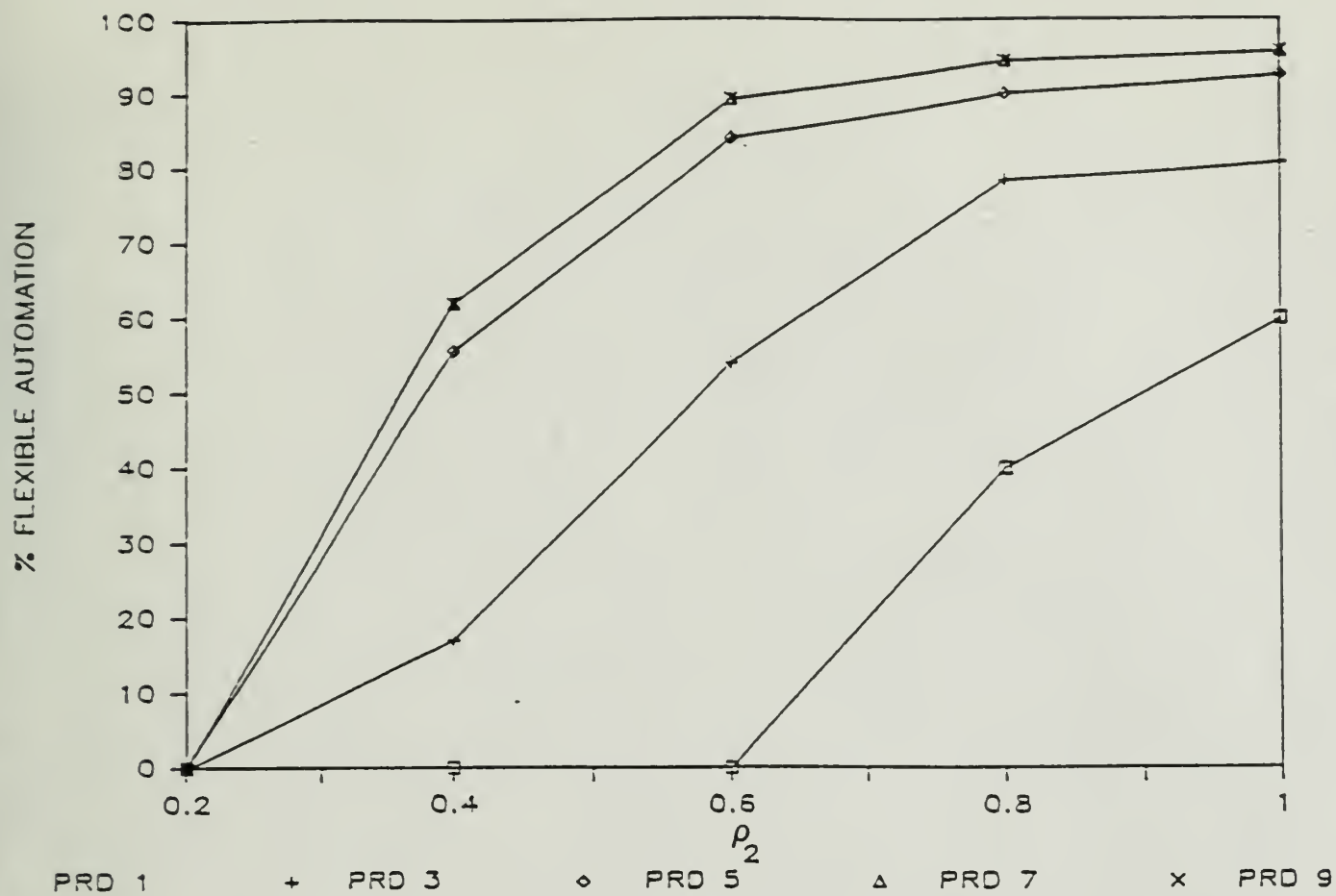Figure 2. Effect of Diseconomies of Scale Factor.

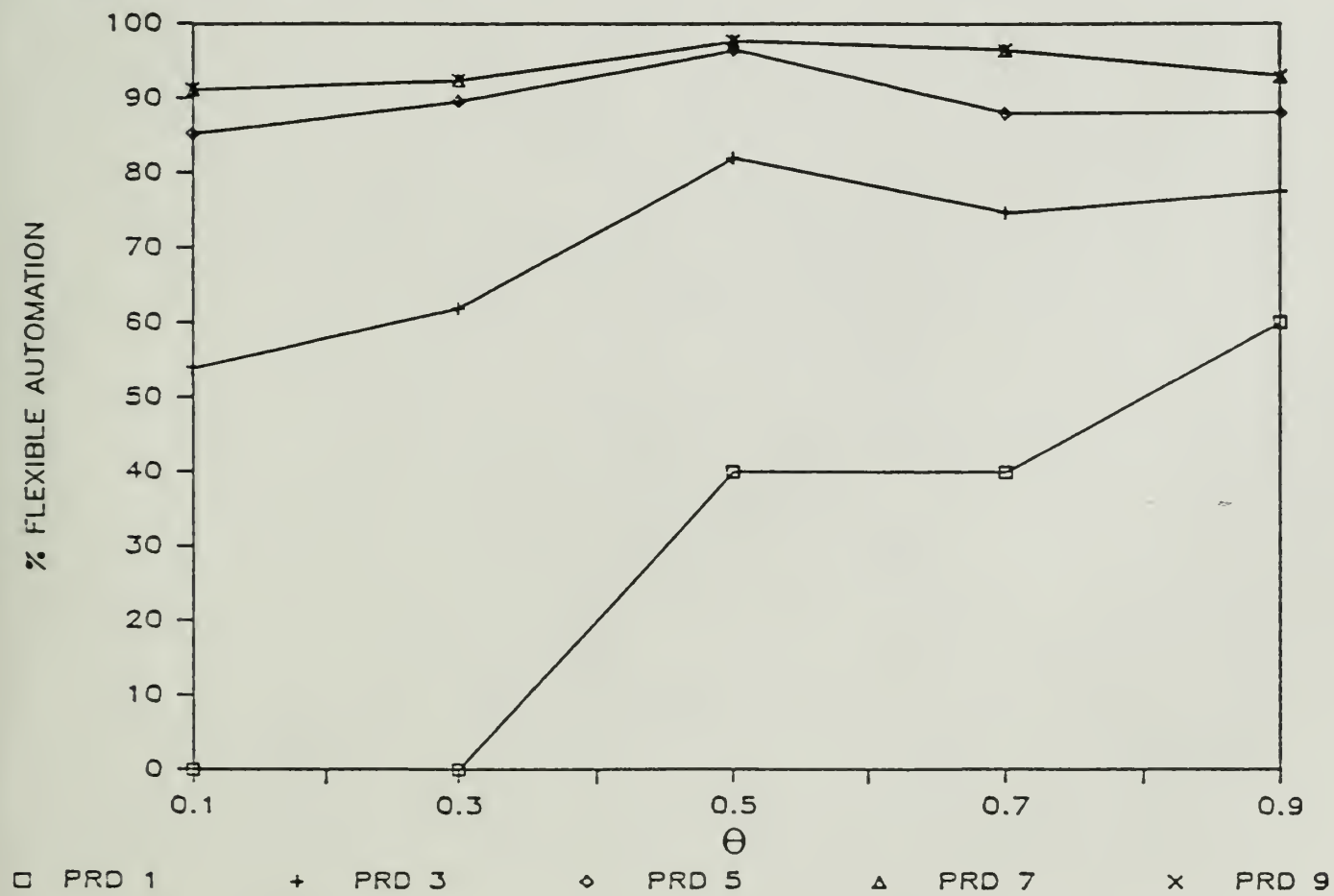Figure 3. Effect of Production Cost Factor.

Figure 4. Effect of Inventory Cost Multiplier – High Opportunity Cost.