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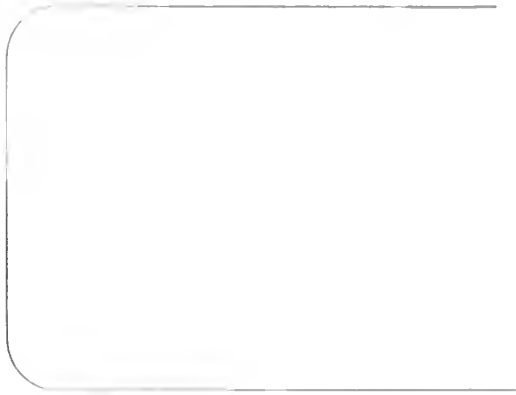
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AN OPTIMAL GROWTH PATH FOR THE MONEY
SUPPLY SUBJECT TO TARGET CONSTRAINTS

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An Optimal Growth Path for the Money
Supply Subject to Target Constraints

Dale J. Poirier*

Seemingly ingrained in the literature of monetary economics is the endless debate between the followers of Milton Friedman, who favor a steady and moderate rate of growth in the money supply, and those who favor using the money supply as an instrument together with fiscal policy to achieve desired objectives in areas such as national income and employment. Somewhere between these two groups are many economists who are sympathetic to both sides, acknowledging both the potential usefulness of using the money supply as an instrument for policy matters, and the potential dangers (in terms of induced cyclicalities) of a widely fluctuating rate of growth in the money supply. This paper suggests a growth path for the money supply which is optimal under a set of preferences likely to be representative of many economists in the middle-of-the-road group.

Suppose a policy maker desires the money supply (however defined) at times $t_0 < t_1 < \dots < t_k$ to equal e^{y_i} ($i = 0, 1, \dots, k$), respectively, where $k \geq 2$. These target levels can be part of the instrument solution to a rigorous mathematical optimization problem, or merely reflect the policy maker's own subjective preferences. Their exact origin is not of concern here.¹ In

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¹It is being implicitly assumed that the policy maker (e.g. the Fed) can actually achieve these targets. In the words of Friedman [2]: "No serious student of money--whatever his political views--denies that the Fed can, if it wishes, control the quantity of money. It cannot, of course, achieve a precise rate of growth from day to day or week to week. But it can come very close from month to month and quarter to quarter."

any case suppose the policy maker decides to follow a "pseudo-Friedman" doctrine and requires, subject to achieving these target levels, that changes in the rate of growth of the money supply be "minimized." This lexicographic preference ordering in which a steady rate of growth is a secondary goal to achieving the target levels seems to capture the sentiments of many middle-of-the-road economists.

The policy maker's dilemma of choosing an optimal path $e^{f(t)}$ which achieves the targets, yet minimizes changes in the growth rate, can be formulated mathematically as follows.

$$\min_{f(t)} \int_{t_0}^{t_k} |f'(t)|^2 dt \tag{1}$$

subject to $e^{f(t_i)} = e^{y_i} \quad (i = 0, 1, \dots, k)$

Loss function (1) weighs changes in the rate of growth (i.e., $f'(t)$) over the time period $[t_0, t_k]$ in the conventional quadratic fashion, and hence seems to be a reasonable criterion function for the pseudo-Friedman doctrine described previously.

One obvious candidate for the solution to this problem is to begin at e^{y_0} and let the money supply grow at a constant rate so as to hit the target e^{y_1} , then let it grow at the constant rate needed to achieve e^{y_2} , etc.. Such a path would imply that $f(t)$ take on the form of a continuous piecewise linear function. While integral (1) is then zero, the growth rate is not defined at t_1, t_2, \dots, t_{k-1} since in general the growth rate has jump discontinuities at these points. These discontinuities raise two problems. First, it is questionable whether in practice it is possible to bring about such instantaneous changes in the growth rate. Second, it is disturbing to envision the possible ramifications of such sudden, abrupt changes in the growth rate. Hence, it

seems desirable to restrict attention to the class of functions $f(t)$ which are "well-behaved," i.e., continuous functions with continuous first and second derivatives over $[t_0, t_k]$, hereafter denoted $C^2[t_0, t_k]$. This insures a "smooth" rate of growth.

Interestingly, this piecewise linear solution, as well as the solution when attention is restricted to those functions in $C^2[t_0, t_k]$ are both members of the family of spline functions.¹ Specifically, the solution in the class $C^2[t_0, t_k]$ of smooth functions is a natural cubic spline, i.e., (i) a piecewise function $S(t)$ in $C^2[t_0, t_k]$, (ii) whose pieces are cubic polynomials defined over the intervals $[t_{j-1}, t_j]$ ($j = 1, 2, \dots, k$) such that (iii) $e^{S(t_i)} = e^{y_i}$ ($i = 0, 1, \dots, k$), and (iv) $S''(t_0) = S''(t_k) = 0$.² The piecewise linear solution (for obvious reasons) is known as a linear spline. The linear spline and the cubic spline are similar in that their first and third derivatives, respectively, are step functions.

The proof that the natural cubic spline $S(t)$ is the solution to (1) in the class $C^2[t_0, t_k]$ is well-known in approximation theory literature.³ The ingredients of the proof are of no particular interest to the discussion here, and hence will be omitted. Rather, for the purpose of illustration, a numerical example is presented.

As stated earlier the exact origin of the targets e^{y_i} ($i = 1, 2, \dots, k$) is not of importance here. Their selection is likely to reflect in a very compli-

¹Applications of spline functions in economics, both from a theoretical and an empirical standpoint, have received the intensive attention of the author. The interested reader is encouraged to consult Poirier [3] and [4] for more details.

²The adjective "natural" is used in spline theory to a cubic spline which satisfies not only (i)-(iii), but which also satisfies (iv).

³See for example Ahlberg, Nilson, and Walsh [1, pp. 3, 75-7].

cated way the policy maker's philosophy concerning seasonal variations in the demand for money and the desirability of integrated fiscal-monetary stabilization measures. Here the following situation is considered. At time t_0 (July, 1972) the policy maker's horizon is one year. He has target values for the money supply (defined to be seasonally unadjusted M_2) at the end of the next four quarters, i.e., at $t_1 = 3$ (October, 1972), $t_2 = 6$ (January, 1973), $t_3 = 9$ (April, 1973), and $t_4 = 12$ (July, 1973), where time (t) is measured in months. Using hindsight the target values were selected as follows. The first and last targets, $e^{y_0} = 503.6$ and $e^{y_4} = 547.1$, measured in billions of dollars, are the actual unadjusted M_2 levels achieved in the respective periods. The intermediate targets, $e^{y_1} = 513.4$, $e^{y_2} = 532.0$, and $e^{y_3} = 540.1$, reflect seasonal demand variations from a constant rate of growth path over the period. These targets were determined by (i) equating the adjusted and unadjusted series in July, 1972 and July, 1973; (ii) computing the resulting ratios of unadjusted to adjusted values in the intermediary quarters; and (iii) and then multiplying these ratios by the constant rate of growth amount (implied by e^{y_0} and e^{y_4}) in the respective quarters. The sensitivity of the resulting optimal growth path to the selection of these targets is discussed subsequently.

Given these targets the optimal path $S(t)$ which solves (1) is shown in Figure 1. Also shown is the constant rate of growth path $F(t)$, the targets (denoted by "o"), and the actual observed unadjusted M_2 values (denoted by "x").

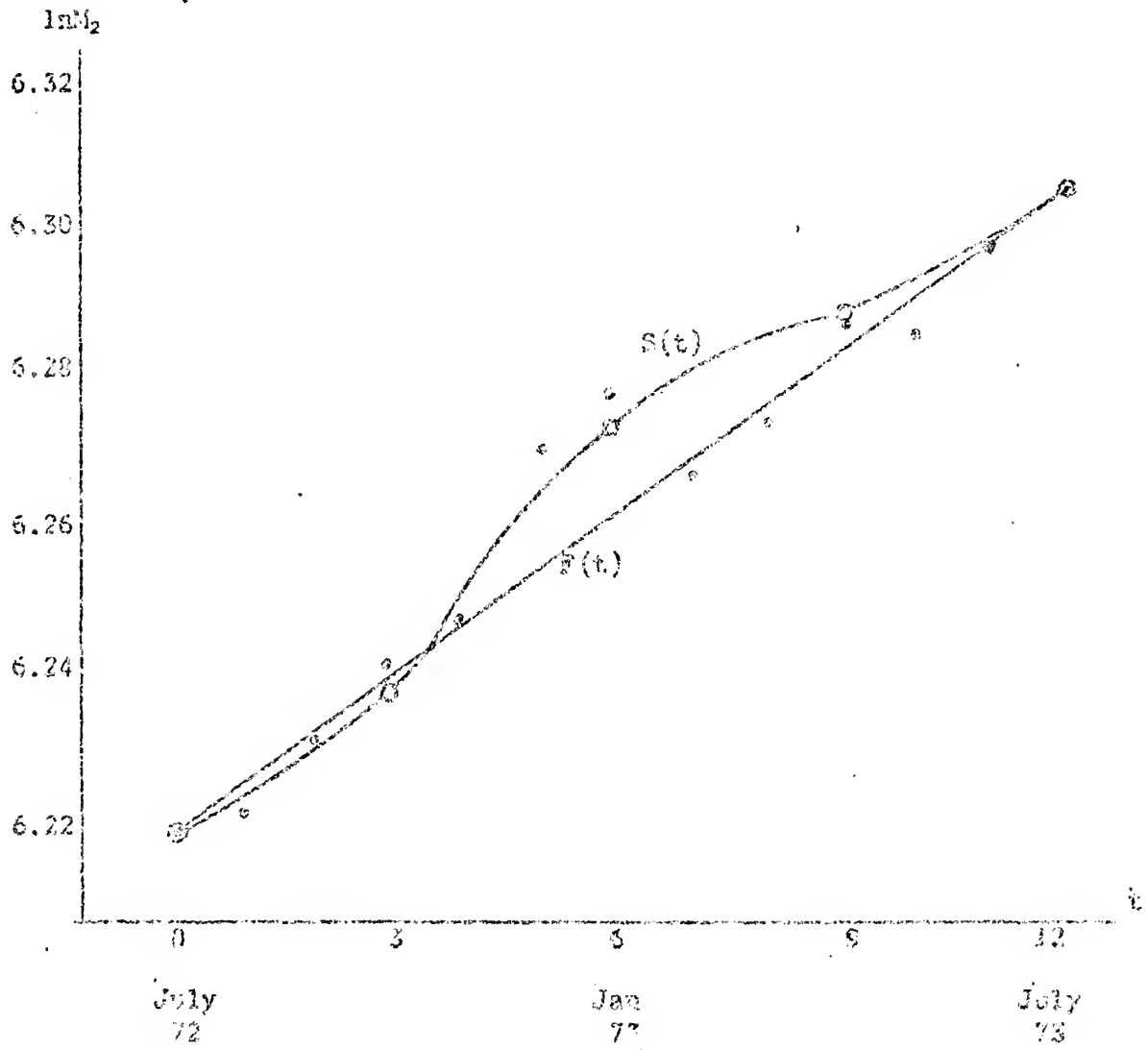
Although the natural cubic spline is a piecewise cubic function of time, it is much easier to parameterize in terms of the targets. Poirier [3, pp. 27-36] shows that at any point t in time, it is possible to write $S(t)$ as a linear function of the log targets, i.e.,

$$S(t) = wy \tag{2}$$

where $y = [y_0, y_1, \dots, y_k]'$ is the vector of targets measured in natural loga-

FIGURE 1

Comparison of Growth Paths



rithms, and $w = [w_0, w_1, \dots, w_k]$ is a row vector whose elements depend on t and the "knots" t_0, t_1, \dots, t_k . The linearity of $S(t)$ in terms of y makes it quite easy to analyze the sensitivity of the optimal path to changes in the targets. Specifically, letting Δ be the finite change operator,

$$S(t) = w_0 \Delta y_0 + w_1 \Delta y_1 + \dots + w_k \Delta y_k \quad (3)$$

Since the weights w_0, w_1, \dots, w_k depend only on t and the knots, they do not have to be recomputed for each new set of targets.

Table 1 contains the monthly weights for the numerical problem considered earlier. These weights are valid for any case involving a one year horizon and quarterly targets. As to be expected, at quarter j , $w_j = 1$, and all other weights are zero. Note also that at other times the closest target receives the largest weight.

In summary, this study has introduced a growth path for the money supply which is optimal under a set of preferences likely to be representative of many economist torn between a target oriented monetary policy and a constant rate of growth path. From a spline theory viewpoint it has been shown once again (cf. Poirier [3]) that economic theory can justify the use of spline functions. In fact under the preferences implied by (1), a spline function is the optimal path.

TABLE 1

Monthly Weights

t	w_0	w_1	w_2	w_3	w_4
0	1.000	.0000	.0000	.0000	.0000
1	.5875	.5132	-.1270	.03175	-.005291
2	.2541	.8915	-.1587	.3968	-.006614
3	.0000	1.000	.0000	.0000	.0000
4	-.07804	.7646	.3862	-.08730	.01455
5	-.05291	.3545	.8042	-.1270	.02116
6	.0000	.0000	1.0000	.0000	.0000
7	.02116	-.1270	.8042	.3545	-.05291
8	.01455	-.08730	.3862	.7646	-.07804
9	.0000	.0000	.0000	1.0000	.0000
10	-.006614	.03968	-.1587	.8915	.2541
11	-.005291	.03175	-.1270	.5132	.5875
12	.0000	.0000	.0000	.0000	1.000

References

- [1] Ahlberg, J.H., E.N. Nilson, and J.L. Walsh, The Theory of Splines and Their Applications, New York: Acadmeic Press, 1967.
- [2] Friedman, Milton, "The Case for a Monetary Rule," Newsweek (February 7, 1972), 67.
- [3] Poirier, Dale J., "Applications of Spline Functions in Economics," Madison: unpublished University of Wisconsin Ph.D. thesis, 1973.
- [4] _____, "Piecewise Regression Using Cubic Splines," Journal of the American Statistical Association, LXVIII (September, 1975).



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