Faculty Working Papers

THE OPTIMUM COLLECTIVE CONSUMING COMMUNITY

Leo Hillman

#166

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

February 28, 1974

THE OPTIMUM COLLECTIVE CONSUMING COMMUNITY

Leo Hillman

#166

THE OPTIMUM COLLECTIVE CONSUMING COMMUNITY[1]

by

Leo Hillman

University of Illinois
Urbana, Illinois

February, 1974

[1] This paper is based on Chapters Three, Four, and Five of my doctoral dissertation written at the University of Pennsylvania under Albert Ando and Wilfred Ethier. I should also like to thank Noel Edelson and Robert Inman for helpful comments and advice, as well as Takashi Takayama for a careful reading of the paper.

THE OPTIMUM COLLECTIVE CONSUMING COMMUNITY

Leo Hillman*

ABSTRACT

This paper focuses on a collective-consuming community such as would provide a locational option to consumers in a surrogate locational market in collective goods. A model is presented, wherein the provision of collective consumption to community members takes place via a collective consumption technology which generalizes the pure public good to allow for replication, congestion and rivalry. Each of these concepts is formally defined for the community with reference to its available collective consumption technology. The standard small country model of international trade describes production and intercommunal trade, with an intermediate and non-tradeable good linking the production and consumption technologies. The community is given the discretion to choose a consumption vector and its membership size. Comparative statics results indicate qualitative differences in optimum community configurations. The paper, by providing a reference optimum outcome for a collective consuming community, provides a frame of reference for considering market failures in a locational market for collective goods.

* University of Illinois at Urbana-Champaign

1. INTRODUCTION

Samuelson's pseudo-market algorithm for the attainment of Pareto efficiency in an economy with pure public goods demonstrated the public good market failure to be a consequence of non-revelation of consumer preferences.[1] In a classic paper Charles Tiebout[2] proposed that preference revelation would be evoked in the act of locational choice if communities provided diverse collective consumption offerings to their members. Then, with community membership acting as an exclusionary device, a market in collective goods via locational choice would function as a surrogate for Samuelson's centralized competitive pseudo-market with its omniscient Walrasian auctioneer.

The fundamental institutional characteristic of a locational market in collective goods is the existence of collective consuming communities providing locational options to consumers. This paper focuses on such a community. We describe the structure of a model of a collective consuming community and then proceed to consider the nature of the choice process entailed in establishing an optimum community. This optimum community provides a frame of reference against which to view prospective market failures. However, the analysis of market failures and corrective policies is beyond the scope of this paper.[3]

In considering the nature of a collective consuming community, we shall make three important departures from the way such a community was orginally envisaged by Tiebout. First, whereas Tiebout assumed that each community had a given collective good expenditure pattern[4], we shall be interested in the community's optimum level of collective consumption as a decision variable. Collective consumption will be viewed as provided via a collective consumption technology, which generalizes the polar pure public good to incorporate the implications of congestion, replication and rivalry.[5]

Secondly, Tiebout assumed all incomes to be derived from dividends; but we shall view the community as confronted with production and trading opportunities no different from those standardly attributed to a small trading nation.[6] The distribution of aggregrate income is presented as determined via a property rights assignment rule implicit in the community's tax schedule, which causes community members' incomes to be functionally dependent on the size of community membership.

Thirdly, optimum community membership size is determined explicitly in the process of the solution to an economically motivated optimization problem. Tiebout proposed on the other hand that, in order to avoid replication, a fixed factor was required to constrain community expansion;[7] while with constant returns in the provision of collective services, a sociological variable would explain the institutional existence of communities.[8] Our model does not begin with the allocation of a fixed amount of land to the community. We assume the shaddow price of land to be zero, the community always being able to physically accommodate a marginal member who, once settled, proceeds to purchase housing services as a component of his private good consumption. Rather than geographical boundaries or sociological motivations, it is efficiency in sharing with constrains community expansion.

The paper proceeds along the following lines. In part 2 the structure of the model of the collective consuming community is described. Part 3 formulates the community's optimum choice problem, with three discretionary variables, private good consumption, collective consumption and community membership size being chosen to maximize community social welfare. Comparative statics results are presented in part 4, and some concluding remarks are contained in part 5.

2. THE MODEL

2.1. Community Welfare

Our first need is to provide an interpretation of community welfare when community size is itself a decision variable.[9] Let us suppose that:

A.1. Community membership can be dichotomized into those consumers who are locationally committed to the community (denoted LCC's) and consumers who are not so committed (denoted non-LCC's).

A framework for motivating the dichotomy postulated in A.1 is provided by Albert Hirschman's "exit, voice, and loyalty."[10] LCC's are defined as "loyal" to the community, while non-LCC's are defined to display no commitment to community membership, being prepared to "exit" if superior alternative locational options present themselves. Along with "loyalty" comes effective "voice," and we assume:

A.2: only LCC preferences count in the determination of the community's social preference ordering over private and collective consumption.

We further assume:

A.3: all final consumption can be dichotomized into a composite commodity, private good consumption, and a flow of collective services provided by an aggregative collective consumption technology (to be defined).

A.4: LCC social preferences are representable by the continuous, strictly concave, twice differentiable strictly increasing function

$$(2.1) \qquad W = \tilde{W}(x, g)$$
$$W_i > 0, \{W_{ij}\} < 0, \; i,j = x, g$$

where g denotes the flow of collective services consumed by all community members and x denotes per capita LCC private good consumption. Until part 4 we assume neither good to be inferior.

Observe that community size does not enter directly into welfare determination, utility being defined in the traditional manner as an ordering exclusively over final goods consumed. Note further that $\tilde{W}$ is strictly concave. Any non-convexities arising out of the existence of collective consumption manifest themselves in the constraint binding on $\tilde{W}$ in $\{x, g\}$ space, not in the function $\tilde{W}(x, g)$ itself. The nature of the constraint on LCC welfare is determined jointly by the community's collective consumption

technology, its production and trading possibilities, and any community property rights assignment rule which brings the welfare of non-LCC's to bear on $\hat{W}(x, g)$. Together, these influences establish LCC income and the relative price of collective in terms of private consumption. This price-income information in turn specifies the LCC feasible choice set in $\{x, g\}$ space.

2.2 The Collective Consumption Technology[11]

A. 5: Assume there exists a continuum of consumers, with the units of measurement of community size defined so that N=1 is the number of LCC's and N=1 is the lower bound to N.

A. 6: It is possible to view the collective facilities provided by the community for its members as an aggregrative structure, denoted in dimension by Λ.

Definition 1: A collective consumption technology is a relationship

(2.2) $$g = g^*(N, \Lambda)$$

(2.2a) $$g \geq 0,\ \Lambda > 0,\ N \geq 1,\ g^*_N \leq 0,\ g^*_\Lambda > 0$$

which describes the nature of the collective consumption possibilities open to a collective consuming community.

I. e., for a given facility capacity and community membership, the collective consumption technology indicates the magnitude of the collective service flow to community members. An increase in facility capacity with community size unchanged increases the flow of collective services; while increasing community size and keeping facility capacity fixed fails to reduce the flow of collective services, only in the special pure public good case where $g^*_N = o$.

Let '^' denote the percentage change in a variable. Then totally differentiating (2.2) logarithmically yields

(2.3) $\hat{g} = \gamma\hat{N} + \xi\hat{\Lambda}$

where

(2.4) $\gamma \equiv \hat{g}\hat{N}^{-1} \Big|_{d\Lambda=0} = Ng^{-1}g^{*}_{N} \lesseqgtr 0$

(2.5) $\xi \equiv \hat{\Lambda}\hat{g}^{-1} \Big|_{dN=0} = \Lambda . g^{-1} . g^{*}_{\Lambda} > 0$

γ and ξ are two characteristic elasticities of the collective consumption technology. We can interpret γ as a congestion elasticity; it indicates the percentage decline in the collective service flow to community members when community size increases by one percent and no compensating addition is made to the community's collective facility capacity. ξ is on the other hand a replication elasticity indicating the percentage increase in collective services when, at a given community size, the collective facility capacity is enlarged by one percent.

A third characteristic elasticity of the collective consumption technology - and the one that we shall have most c use to be interested in - is established by rearranging terms in (2.3) to yield

(2.3a) $\hat{\Lambda} = \xi^{-1}\hat{g} - \xi^{-1}\gamma\hat{N}$

(2.3a) implicitly defines what we shall term the community's rivalry elasticity

(2.6) $\eta \equiv \hat{\Lambda}\hat{N}^{-1} \Big|_{dg=0} = -\xi^{-1}\gamma \geq 0$

The rivalry elasticity η accordingly denotes the percentage increase in community size necessary to sustain a given collective service flow to community members, when community membership size increases by one percent.

The terms congestion and rivalry have generally been used interchangeably in the collective consumption literature. γ and η however offer a formal distinction within the context of a community's application of its collective consumption technology. γ is a measure of crowding for a given collective facility capacity and thus suggests congestability. η on the other hand views crowding from the perspective of the maintainance of a given collective service flow, and this we take to be suggestive of the degree of rivalry in collective consumption. It is evident from (2.6) that the congestion and rivalry elasticities of the community's collective consumption technology are equal in absolute value, and so constitute symmetrical measures of crowding, only if $\gamma = \eta = 0$ or $\xi = 1$.

The freedom we have in specifying a collective consumption technology is analogous to that entailed in the specification of a production technology. As our general frame of reference we propose

<u>Definition 2:</u> A reference collective consumption technology is described by (2.2) with $g \geq 0$, $\Lambda > 0$, $N \geq 1$, $\overset{*}{\Lambda} > 0$, and additional attributes

(i) $g^*_N \Big|_{N=1} \lesseqgtr 0$; $g^*_N \Big|_{N>1} < 0$

(ii) $g^*_{NN} < 0$; $g^*_{\Lambda\Lambda} < 0$

(iii) $g^*_{N\Lambda} \gtreqless 0$, but $g^*_{N\Lambda} \leq 0$ at each level of g for some sufficiently large combination of N and Λ.

We assume

A. 7: the community's collective consumption technology is as described in definition 2.

The reference technology is depicted in $\{g, N\}$ space in figure 1. For any facility capacity, there exists a relationship between g and N, with successively higher contours representative of larger collective facility capacities. The congestion elasticity γ is the point elasticity on an iso - Λ contour. Attribute (i) of definition 2 permits a facility to be marginally noncongestible only at N = 1.

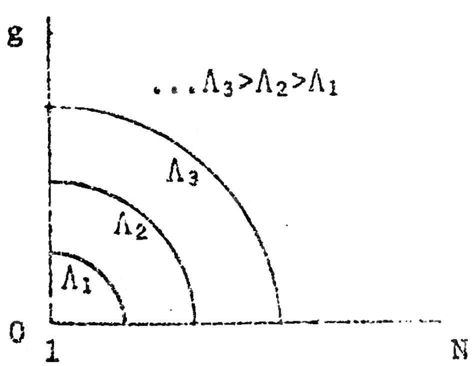

Figure 1

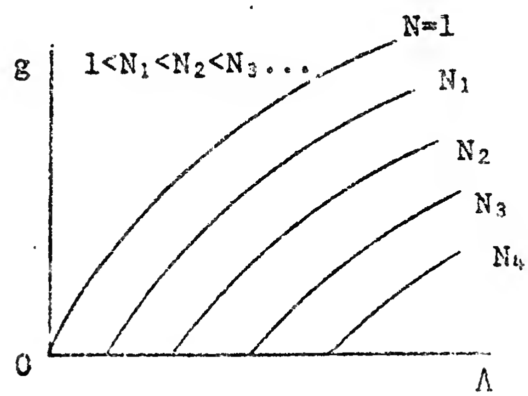

Figure 2

We can view figure 1 as depicting the collective consumption technology in its inverse form[12],

$$\Lambda = \Lambda^* (N, g). \tag{2.7}$$

Analogously, figure 2 depicts the collective consumption technology in its inverse form,[13]

$$N = N^* (g, \Lambda) \tag{2.8}$$

Each iso - N contour describes the transformation frontier between g and Λ at a given community size. The highest contour represents the minimal community membership size N = 1, with lower frontiers corresponding to

successively larger community sizes. ξ is the elasticity at a point on an iso - N contour.[14]

Now consider the third characteristic of the collective consumption technology, rivalry. We have observed that $\eta \equiv -\xi^{-1}\gamma$ and so rivalry is conceptually a combination of the congestion and replication characteristics of the collective consumption technology. Graphically, we depict rivalry as the point elasticity on an iso-g contour in $\{\Lambda, N\}$ space, as in figure 3. The properties of such iso-g contours are of fundamental importance, since they determine whether the collective consumption technology in itself endogenously constrains community expansion; for each contour indicates the circumstances under which a given collective service flow can be maintained when both community membership size and the congestible collective facility are enlarged. If both the latter can be increased without any constraining influence being exerted by the collective consumption technology, then we would need to resort to employing some fixed factor as the determinant of community size.

The iso-g contours in figure 3 exhibit $\Lambda^*_N > 0$ and $\Lambda^*_{NN} > 0$. From (2.2) and (2.7) we obtain

(2.9) $$\left.\frac{d\Lambda}{dN}\right|_{dg=0} = \Lambda^*_N = -g^*_N \cdot g^{*-1}_\Lambda > 0$$

Λ^*_N like its elasticity analogue η, is positive, since if g is to remain unchanged when membership size is enlarged, some compensatory marginal addition must be made to the community's collective facility.

From (2.9)

(2.10) $$\left.\frac{d^2\Lambda}{dN^2}\right|_{dg=0} = \Lambda^*_{NN} = g^*_N \cdot g^*_{\Lambda N} - g^*_{NN} \cdot g^{*-1}_\Lambda$$

It might be intuitively evident - or in any event we shall make it explicitly clear below when considering the relationship between the collective

consumption technology and the cost of collective consumption - that $\Lambda^*_{NN} > 0$ is a necessary condition for the collective consumption technology to mitigate against unbounded community expansion. I.e., just as $g^*_{NN} < 0$ indicates increasing congestion of a given facility capacity, we require $\Lambda^*_{NN} > 0$ to indicate increasing rivalry as the congestible facility is replicated and community membership size is enlarged.

In (2.10) the critical term is $g^*_{\Lambda N}$. If $g^*_{\Lambda N} > 0$, then $\Lambda^*_{NN} \gtreqless 0$; but if $g^*_{\Lambda N} \leq 0$, then $\Lambda^*_{NN} > 0$. Attribute (iii) of definition 2 guarantees that the reference technology exhibits $g^*_{\Lambda N} \leq 0$, and so $\Lambda^*_{NN} > 0$, for sufficiently large combinations of N and Λ on an iso-g contour. The justification for proposing

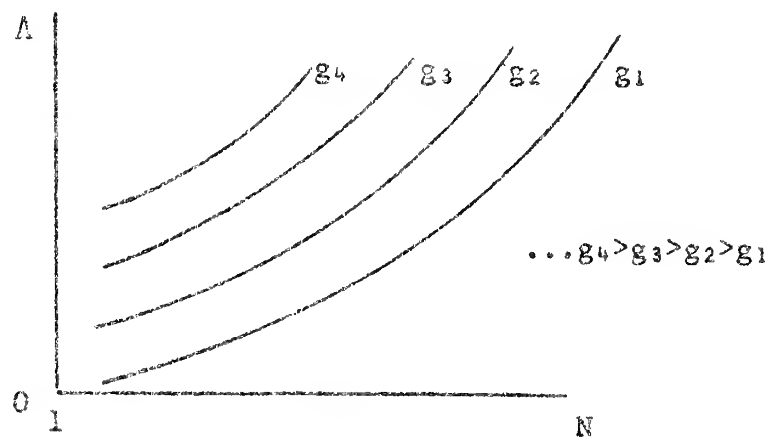

Figure 3

this relationship rests on the two perspectives on the cross-term $g^*_{\Lambda N}$. On the one hand, viewed as $dg^*_N/\partial\Lambda$, we would acknowledge the congestion reducing effects of increasing facility size and so suggest $g^*_{\Lambda N} > 0$. But from the perspective of dg^*_Λ / dN, a different set of circumstances suggest themselves. As N and Λ become large on a given iso-g contour, the negativity of $g^*_{\Lambda N}$ is suggested by the replication constraints imposed by the technology of providing collective services. I.e., at some point, efficiency in the provision of collective services dictates that LCC's turn away further consumers to initiate their own collective facility structure,

rather than have the community accommodate new members by enlarging its collective facilities. Before we formally consider the relationship between the costs of collective consumption and the collective consumption technology, we need to introduce production and prices.[15]

2.3 Production

The analogy between the small country engaged in international trade and a community appears quite complete from the viewpoint of production and markets. Accordingly, we shall adopt the wellknown small country frame of reference for the community.[16] We suppose that competitive community firms produce two outputs, the private good X and a durable intermediate good G. The latter is combined with cooperating labor to yield the collective facility

$$\Lambda = \bar{\Lambda}(N_\Lambda, \tilde{G}) \tag{2.11}$$

where N_Λ denotes labor employment in Λ and $\tilde{G}$ the stream of services from the quantity of G employed in Λ.

Λ is non-tradeable.[17] But the private good X is tradeable for the services of G in a world rentals market, in which G can be leased in exchange for a flow of π units of X per unit of time.[18] At any point in time the community is endowed with a historically given stock of G and LCC's engage in trade to satisfy their excess demands for G and X.

Production utilises two factor inputs. Labor inputs are for convenience assumed to be equal to community membership and are employed in conjunction with a generic non-labor input. Factors are paid the values of their marginal products. The community's total wage bill is given by wN in terms of the private good, and we denote aggregrate community non-wage income, derived from the services of the intermediate good G and the generic non-labor input, by R.

We further assume that LCC's choice of N leaves the community at an interior point on its non-labor Rybczynski line; so the community remains

diversified in production and the exogenous terms of trade π establish factor rewards and relative factor intensities in production. The community's variable discretionary labor Rybczynski line is

$$a_{NX}(\pi)\ X + \left[a_{NG}(\pi) + r\left(\frac{w(\pi)}{\pi}\right)\right] G = N \qquad (2.12)$$

where a_{Ni} denotes a labor-input coefficient and r is the ratio of labor to the quantity of G employed in A. The community's production and trading possibilities, in conjunction with its collective consumption technology, determine the relative supply price of collective services. Once LCC income is established, the characteristics of the LCC feasible choice set binding on $\tilde{W}(x, g)$ are specified, and we can then proceed to consider the nature of the solution to the community's optimization problem.

2.4 The LCC Feasible Choice Set

In establishing the LCC feasible choice set, we assume:

A. 8: collective consumption is financed so that all community members pay an equal tax-price for collective services.

Let ρ denote the per capita[19] collective consumption tax price and let y denote LCC income, where both ρ and y are expressed in terms of the private good as numeraire. The LCC feasible choice set can now be denoted as $S(\rho, y)$.

A. 9: Let the community operate A with a balanced budget, such that

$$\theta g = p_A \cdot A \qquad (2.13)$$

where θ denotes the relative supply price of a unit of collective services and p_A is the relative price of the collective facility, given as a consequence of cost minimization by

$$p_A(\pi) = w(\pi)\ .\ a_{NA}\left(\frac{w(\pi)}{\pi}\right) + \pi . a_{GA}\left(\frac{w(\pi)}{\pi}\right) \qquad (2.14)$$

In (2.14) a_{NA} and a_{GA} are respectively the unit isoquant cost-minimizing labor and intermediate good input coefficients for the collective facility.

When we substitute (2.7) for the collective consumption technology into (2.13) and solve for θ , we obtain

(2.15) $\theta^*(g,N|\pi) = p_\Lambda(\pi) \cdot g^{-1} \cdot \Lambda^*(N,g)$

I.e., the supply price of collective services depends upon the collective service flow and membership size that LCC's might choose. Noting that $\rho \equiv \theta/N$, we obtain by differentiation of (2.15) the two dimensions of marginal cost in the community's provision of collective services to its members,

(2.16) $\theta^*_g = \theta g^{-1} (\xi^{-1}-1) \gtreqless 0$ as $\xi \lesseqgtr 1$

(2.17) $\theta^*_N = \theta N^{-1} \eta = \rho\eta \geq 0$

Observe that (2.17) provides us with another interpretation of the rivalry elasticity; η is equal to the elasticity of the supply price of collective services with respect to community membership size for a given level of collective consumption.

Our interest is in establishing the slope of the LCC budget constraint. From (2.15),

(2.18) $\rho^*(g,N/\pi) = \theta^*(g,N/\pi)N^{-1}$

So marginal variations in community collective consumption and membership size respectively affect ρ via

(2.19) $\rho^*_g = \rho g^{-1} (\xi^{-1}-1) \gtreqless 0$ as $\xi \lesseqgtr 1$

(2.20) $\rho^*_N = \rho N^{-1} (\eta-1) \lesseqgtr 0$ as $\eta \lesseqgtr 1$

Now consider the second aspect of the LCC feasible choice set, LCC income y. Suppose that the distributive content of the community's taxation system - as distinct from the per capita obligation to pay for collective services - can be expressed in the form

(2.21) $y(N) = w + \alpha(N)\ R(N)\ N^{-1}$

where $\{\alpha | 0 \leq \alpha \leq N\}$ is a variable income distribution parameter and $R_N \geq 0$. (2.21) is a general property rights assignment rule defining the dependence of LCC income on community size. The characteristics of the function $\alpha(N)$ are determined by the specific nature of the community's tax system. For example, $\alpha(N) = N$ is an extreme rule which grants all property rights to the community's non-wage income to LCC's, while $\alpha(N) = 1$ secures an egalitaran income distribution. Observe that $\alpha(N)/N$ is LCC's post-tax share of non-wage income and so $y \gtreqless \bar{y}$ as $\alpha \gtreqless 1$, where $\bar{y}$ denotes LCC per capita[20] income. Generally, from (2.21),

$$y_N = \alpha R N^{-2} \{E_{\alpha N} + E_{RN} - 1\} \tag{2.22}$$

where E_{ij} denotes an elasticity.

Substituting (2.18) and (2.21) into the LCC budget constraint, we obtain as the equation of the LCC feasible choice frontier

$$x + \rho^* (g, N | \pi) \; g = y(N) = w + \alpha R N^{-1} \tag{2.23}$$

(2.19), (2.20) and (2.22) describe variations in the feasible choice set that occur when marginal changes are made in the community's level of collective consumption and its membership size.

3. OPTIMAL CHOICE

3.1 The General Solution in $\{x, g\}$ Space.

LCC's choose x, g and N to maximize $\tilde{W}(x, g)$ subject to (2.23). The lagrangian is

$$L = \tilde{W}(x, g) + \lambda [x + \rho^* (g, N) \; g - y(N)] \tag{3.1}$$

where λ is a lagrangian multiplier, and the necessary first order conditions for a constrained local extremum are

(3.2) $$L_x = \tilde{W}_x + \lambda = 0$$

(3.3) $$L_g = \tilde{W}_g + \lambda(\rho^*_g g + \rho^*) = 0$$

(3.4) $$L_\lambda = x + \rho^* g - y = 0$$

(3.5) $$L_N = g\,\rho^*_N - y_N = 0 \quad \text{assuming } \lambda \neq 0$$

Eliminating λ from (3.2) and (3.3) and substituting (2.19) for ρ^*_g secures

(3.6) $$\rho\xi^{-1} = \tilde{W}_g \; \tilde{W}_x^{-1} \equiv MRS_{gx}$$

Since $\rho \equiv \theta/N$, (3.6) can be expressed as

(3.6a) $$\theta\xi^{-1} = N \cdot MRS_{gx} = \sum_N MRS_{gx}$$

$\theta\xi^{-1}$ is the marginal cost of collective consumption in terms of private consumption as determined via the collective consumption technology. (3.6a) is therefore the generalization of Samuelson's well-known necessary condition for a Pareto efficient combination of collective and private consumption to be realized. Observe however that the particular marginal rate of substitution being summed is that of LCC's whose preferences are imposed on the community at large.

Consider now the condition (3.5). The term $-g.\ \rho^*_N$ indicates the change in consumer surplus as measured by the compensating variation, when community size is marginally enlarged at a given level of collective consumption. Note that if $\rho^*_N < 0$, so that a marginal new community member depresses ρ, then consumer surplus increases; and vice-versa if $\rho^*_N > 0$. y_N indicates the change in LCC income when there is a marginal increase in community size. Accordingly (3.5) requires that in equilibrium the change in the compensating variation be equal to minus the LCC income

change when community size is marginally enlarged.

Finally (3.4) is the standard requirement that in commodity space optimal choices of x and g lie along the frontier of the feasible choice set, which in this case is itself specified by the choice of g and N.

The first order conditions are solved in a two step procedure. First solve (3.2) - (3.4) for the demand functions

(3.7) $$g_D(N,g) = g_D^*[\rho^*(N,g),\ y(N)]$$

(3.8) $$x_D(N,g) = x_D^*[\rho^*(N,g), y(N)]$$

and λ. Values of N and g specify a feasible choice set in commodity space via ρ and y, and (3.7) and (3.8) yield preferred consumption choices from a given feasible choice set.

The first order condition remaining, (3.5), establishes the community's efficient sharing locus in {N, g} space. So (3.5) and either one of the demand functions (3.7) and (3.8) constitute two equations in N and g. A solution for private good consumption x is then attainable via the budget constraint. Accordingly, an optimum solution, if it exists, simultaneously specifies an optimum choice set in commodity space and a choice of collective and private consumption and from the set.

3.2 Replication and Convexity of the Feasible Choice Set

As a prerequisite to obtaining a unique solution to the above problem, we require that from any one given feasible choice set $S[\rho,y]$, there exist a uniquely preferred consumption vector specified by the first order conditions; i.e., we require $S[\rho,y]$ to be a convex set.

Holding community membership size constant, we obtain for the slope of the feasible choice frontier

(3.9) $$\left.\frac{dx}{dg}\right|_{dN=0} = -\rho\xi^{-1} < 0$$

(3.10) $$\left.\frac{d^2x}{dg^2}\right|_{dN=0} = \rho\xi^{-2}\ \xi_\Lambda\ \Lambda_g^* - \xi^{-1}\ \rho_g^* \gtreqless 0$$

Thus ξ gives rise to potential nonconvexities in $S[\rho,y]$ and suitable restrictions on ξ will establish $S[\rho,y]$ as convex. The types of restrictions that are needed eliminate increasing proportionate returns to facility replication. For example, $\xi_\Lambda \leqq 0$ and $\xi \leqq 0$ together make (3.10) negative and ensure that $S[\rho,y]$ is a strictly convex set. A more stringent condition is to assume $\xi \equiv 1$, thus eliminating decreasing as well as increasing returns. In the latter case (2.19), (3.9) and (3.10) indicate the feasible choice frontier to be linear.

3.3 Existence and Uniqueness in an Expository Community.

We shall now provide a more exact exposition of the nature of the solution to the optimum collective consuming problem. We do this by making some specific expository assumptions about the community's collective consumption technology and its property rights assignment rule.

A 10:(i) First, in the light of section 3.2 above, we assume that $\xi \equiv 1$. Thus the scale of provision of g does not enter into the determination of the costs collective consumption.

(ii) Secondly, we assume the collective consumption to exhibit $\eta(1) = 0$, $\eta_N > 0$ and $\eta_{NN} > 0$; i.e., collective consumption is marginally non-rivalrous at N = 1, but beyond N = 1 rivalry as measured by η increases at an increasing rate.

(iii) Thirdly, suppose the community's tax system implies a property rights assignment rule for $\alpha(N)$, such that

(3.11) $$E_{\alpha N} \gtreqless 1 - E_{RN} \quad \text{as} \quad \alpha(N) \lesseqgtr 1$$

and assume that $y > \bar{y}$. LCC's accordingly suffer an income loss whenever community size is marginally expanded. We shall further assume that in (3.11) the elasticities $E_{\alpha N}$ and E_{RN} are constant.

For a collective-consuming community described by A10, we can establish the following propositions:

Proposition 1: A10(i) implies that the feasible choice set's position in $\{x,g\}$ space is independent of the choice of collective consumption and is completely specified by a choice of community size.

Proof: From (2.19), $\rho^*_g \big|_{\xi=1} = 0$. However, with the exception of $\rho^*_N(1)$, $\rho^*_N \neq 0$, so ρ varies with N. y depends upon N only via (2.21), so the feasible choice set is specified by $S[\rho(N), y(N)] = \tilde{S}(N)$.
Q.E.D.

Proposition 2: A10(i) and A10 (ii) imply that ρ depends only upon the community size chosen, not the level of collective consumption, and $\rho(N)$ attains a unique minimum as depicted in figure 4.

Proof: Since $\rho^*_g \big|_{\xi=1} = 0$, $\rho^*(N,g) = \rho(N)$. From (2.20), $\rho(N)$ attains an extremum where N=1. Since $\Pi_N > 0$, the extremum is a global minimum.
Q. E. D.

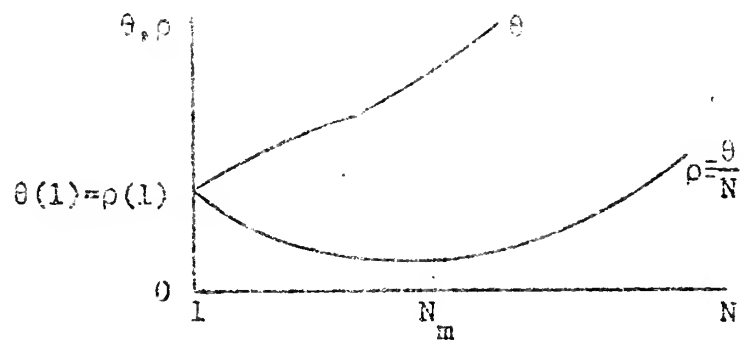

Figure 4

Since in (2.17) $\theta^*_N = \rho\eta$, the supply price of collective services is an increasing function of N for $\eta > 0$. But although θ is monotonically increasing as community size is enlarged, ρ is decreasing over that range of N where $\eta<1$. For over the latter range of N the gains from sharing the costs of collective consumption with a marginal new community member exceed the costs incurred, via the increase in θ, in enlarging the collective facility to maintain a given collective service flow to community members.

Following from proposition 2 we have

Proposition 3: Let N_m denote the community membership size at which ρ attains a minimum; i.e., $\min \rho = \rho(N_m)$. Then the community's optimal choice of membership size is less than N_m.

Proof: An optimal choice of N must satisfy the first order condition (3.5). Since by A10 (iii) $y_N < 0$, the optimal choice of community size must lie in the range of N where $\rho_N < 0$; i.e., optimal N must be less than N_m. Q.E.D.

Clearly if $y_N < 0$, optimal N can be no greater than N_m, since for $N > N_m$ we have $\rho_N > 0$ and the change in the compensating variation due to a marginal expansion in community size is negative. This would mean that community members are prepared to forego some income to prevent the tax-price change induced by the addition of a marginal consumer to the community from coming into effect. Symmetrically, they would be prepared to bribe some of their fellow members to leave the community.

Proposition 1 implies that the demand for collective services given by (3.7) can be expressed as

(3.7a) $$\tilde{g} = \tilde{g}[\rho(N), y(N)] = g_D(N)$$

I.e., for a given value of N and specification of a feasible choice set $\tilde{S}(N)$, some collective demand $\tilde{g}$ is chosen. The first order condition (3.5) can be expressed as

(3.5a) $$g = y_N \rho^{-1}_N = g_E(N)$$

where $g_E(N)$ is the minimal level of collective consumption required for sharing efficiency over the range of N. As described in the general case in section 3.1 above, the solution to the community's optimal choice problem is obtained by solving the two functions in the space of g and N, $g_D(N)$ and $g_E(N)$.

Consider first the nature of the function $g_E(N)$:

Proposition 4: The sharing efficiency function $g_E(N)$ is bounded at $N = N_m$ and is either monotonically increasing for $\{1 \leq N < N_m\}$ or has a global minimum. $g_E(1)$ is positive and finite.
See figure 5.

Proof: Substituting (2.20) and (2.22) into (3.5a) and setting $c \equiv E_{\alpha N} + E_{RN} - 1$, we obtain

(3.5b) $g_E(N) = \dfrac{c\ \alpha(N)\ R(N)}{\theta(\eta-1)}$

From (3.5b), $g_E(1) = -c\ \alpha(1)\ R(1)\ \theta^{-1} > 0$ and $\lim_{N \to N_m} g_E = \infty$.

Logarithmically differentiating (3.5a) we obtain in elasticity terms, using primes now to denote differentiation with respect to N,

(3.12) $$g_E'(N) = g_E\ N^{-1}\ (E_{\rho' N} - E_{y' N})$$

Therefore $g_E'(N) \gtreqless 0$ as $E_{\rho' N} \gtreqless E_{y' N}$.

Consider $E_{\rho' N}$. Evaluating this elasticity yields

(3.13) $$E_{\rho' N} \equiv -N\ \rho''\rho' = [(2-\eta) + \eta' N(1-\eta)^{-1}]\Big|_{\eta<1} > 0$$

Differentiation of (3.13) with respect to N then yields after collection of terms

(3.14) $$E_{\rho' N}' = (1-\eta)^{-1}\ [\eta\eta' + \eta'' N + \eta' N(1-\eta)^{-1}] > 0$$

Now consider $E_{y'N}$. Multiplying out this elasticity yields

(3.15) $$E_{y'N} \equiv -N\, y''\, y' = -c > 0$$

Since the elasticities $E_{\alpha N}$ and E_{RN} have been assumed constant and $c \equiv E_{\alpha N} + E_{RN} - 1$ we obtain upon differentiating (3.15), $E'_{y'N} = 0$.

Figure 5 depicts the two possible outcomes with respect to $E_{\rho'N}$ and $E_{y'N}$. In figure 5a $E_{\rho'N}(1) < E_{y'N}(1)$, so an extremum for $g_E(N)$ exists. Since the second order condition for a minimum for $g_E(N)$ is $E'_{\rho'N} > E'_{y'N}$, the unique extremum is a minimum. In figure 5b $E_{\rho'N}(1) > E_{y'N}(1)$, so $g_E(N)$ is monotonically increasing.

Q. E. D.

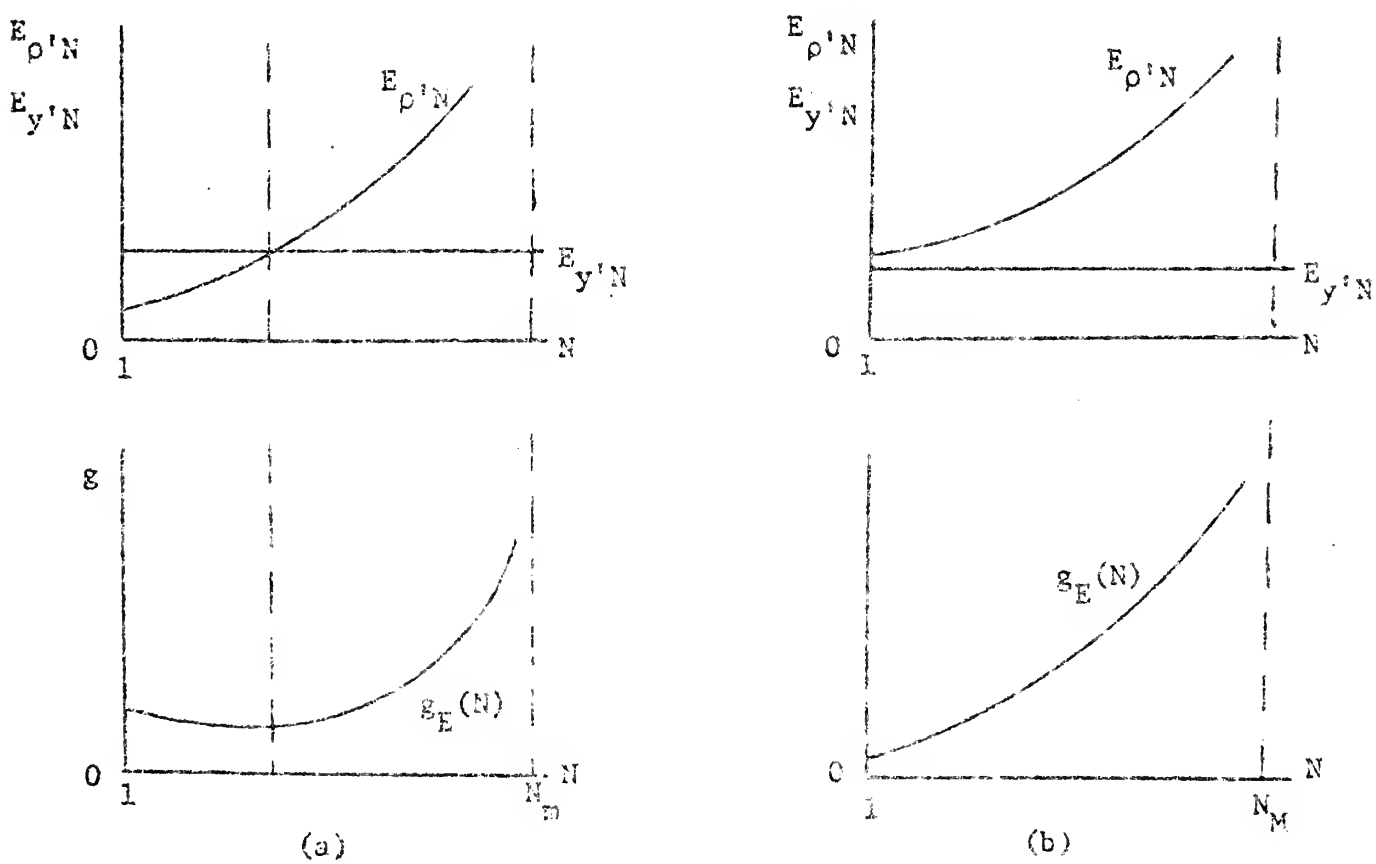

Figure 5

Now consider the demand function for collective services in the space of g and N.

Proposition 5: The function $g_D(N)$ is characterized by

(i) $g'_D(N) > 0$ if $g_D(N) > g_E(N)$

(ii) $g'_D(N) \gtreqless 0$ if $g_D(N) < g_E(N)$ and $\eta<1$

(iii) $g'_D(N) < 0$ if $\eta \geq 1$

Proof: Differentiation of (3.7a) yields

$$(3.16) \qquad g'_D(N) = \tilde{g}_\rho \, \rho_N + \tilde{g}_y \, y_N$$

Applying the Slutsky - Hicks decomposition to $\tilde{g}_\rho$, we obtain after rearrangement of terms

$$(3.16a) \qquad g'_D(N) = \rho_N \cdot g_\rho \Big|_{dy=0} + \tilde{g}_y (y_N - \tilde{g} \ \rho_N)$$

(3.16a) decomposes the response of demand for collective services by LCC's to a marginal increase in community size into a substitution effect and an income effect. If $\eta<1$ then we can substitute (3.5a) into (3.16a) to obtain

$$(3.16b) \qquad g'_D(N) \Big|_{\eta<1} = -\rho_N \left\{ -g_\rho \Big|_{dy=0} + \tilde{g}_y \big(g_D(N) - g_E(N)\big) \right\}$$

i) Consider (3.16b). Since $\eta<1$, we have $(-\rho_N) > 0$. The substitution effect $(-g_\rho)$ is positive, a marginal increase in N causing consumers to substitute collective for private goods at a constant level of income. Collective consumption is not an inferior good, so $\tilde{g}_y > 0$: accordingly if $g_D(N) > g_E(N)$, then $g'_D(N) > 0$.

ii) But if in (3.16b) $g_D(N) < g_E(N)$, then the substitution and income effects of a marginal increase in community size operate in different directions and $g'_D(N) \gtreqless 0$.

(iii) In (3.16a) if $\eta \geq 1$, then $\rho_N \geq 0$ and so the substitution effect is negative. The income effect is also negative, so $g'_D(N) < 0$

Q. E. D.

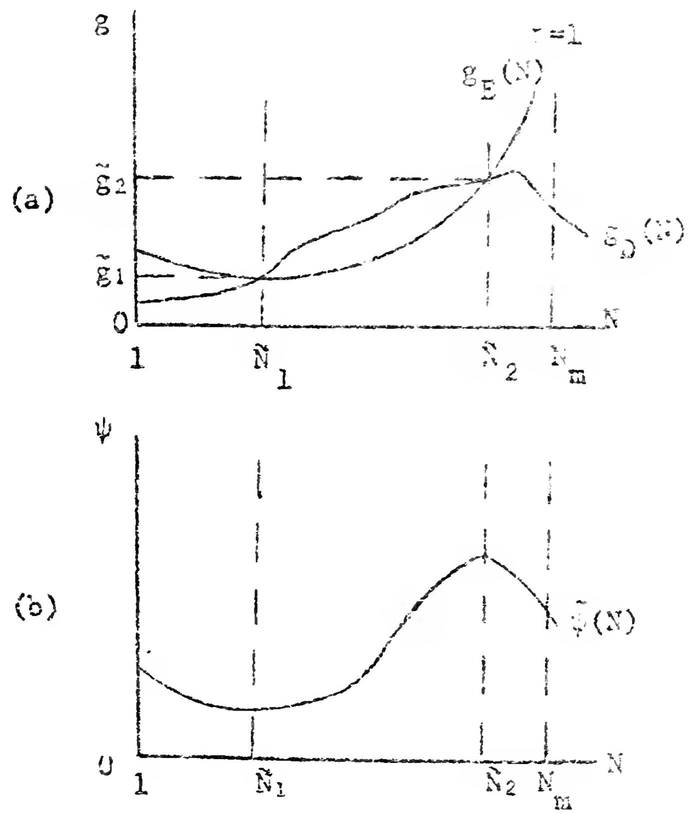

Figure 6

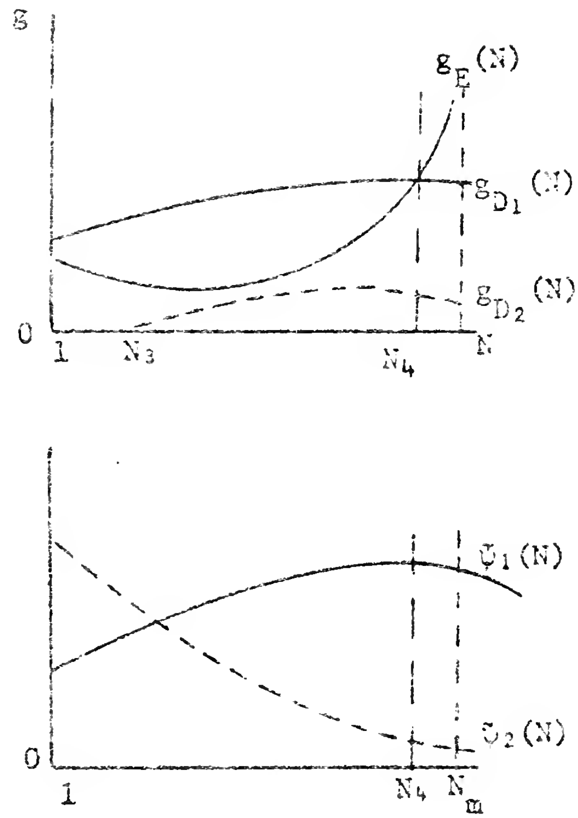

Figure 7

The three characteristic outcomes to the solution of $g_E(N)$ and $g_D(N)$ are depicted in figures 6a and 7a. In figure 6a there are two solutions at $[\tilde{N}_1, \tilde{g}_1]$ and $[\tilde{N}_2, \tilde{g}_2]$. In figure 7a the function $g_{D1}(N)$ yields a unique solution, while in the case of $g_{D2}(N)$ no solution exists.

With reference to figures 6b and 7b, we can state

Proposition 6: (i) If two solutions exist as in figure 6, then LCC welfare attains a local maximum at $[\tilde{N}_2, \tilde{g}_2]$ and a local minimum at $[\tilde{N}_1, \tilde{g}_1]$.

(ii) If the solution is unique as in the case of $g_{D1}(N)$ in figure 7, then the solution indicates a global maximum for LCC welfare.

(iii) If no solution exists as in the case of $g_{D2}(N)$ in figure 7, then LCC welfare is a monotonically declining function of community size. This is also so if a unique solution is obtained as a tangency of $g_D(N)$ and $g_E(N)$.

Proof: Consider the indirect LCC social welfare function

(3.17) $$\tilde{\psi}(N) = \psi[\rho(N), y(N)]$$

Differentiating (3.17) we obtain

(3.18) $$\tilde{\psi}_N = \psi_\rho \rho_N + \psi_y y_N$$

(3.18) implies that for $\eta \geq 1$, $\tilde{\psi}_N < 0$.

For $\eta<1$ (3.18) can be rewritten as

(3.19) $$\tilde{\psi}_N\Big|_{\eta<1} = [\psi_\rho \psi_y^{-1} + y_N \rho_N^{-1}] \psi_y \rho_N ,$$

Since $\tilde{g} = \psi_\rho \psi_y^{-1}$ and from (3.5a) $g_E(N) = y_N \rho_N^{-1}$, we obtain from (3.19)

(3.19a) $$\tilde{\psi}_N\Big|_{\eta<1} = [g_D(N) - g_E(N)] (-\rho_N)\psi_y$$

Therefore $\tilde{\psi}_N \gtreqless 0$ as $g_D(N) \gtreqless g_E(N)$. Q.E.D.

Observe that the sharing efficiency function $g_E(N)$ is specified by the community's collective consumption technology and its property rights assignment rule, while the collective demand function $g_D(N)$ reflects LCC preferences in consumption. A collective demand function such as $g_{N1}(N)$ in figure 7 is representative of preferences relatively biased towards collective consumption. At N = 1, although LCC's alone pay for collective consumption, $g_D(1) > g_E(1)$. As N is enlarged, the indirect social welfare function $\psi_1(N)$ attains a global maximum at a community size of N_4.

On the other hand, preferences underlying a collective demand function such as $g_{D2}(N)$ in figure 7 are biased in favor of private consumption. LCC's demand no collective consumption at all until collective costs can be shared with at least N_3 community members. An optimum community size in excess of N = 1 need then not necessarily exist. In the case depicted, the substitution effect towards collection consumption as community size is enlarged beyond N_3 is not sufficiently strong to lift $g_D(N)$ above $g_E(N)$. So

in figure 7b $\tilde{\Psi}_2(N)$ is monotonically decreasing and optimal choice of community size entails maintaining $N = 1$.

In figure 6 however, although $g_D(1) < g_E(1)$, the substitution effect does come to dominate the income effect sufficiently for collective demand to attain a minimal efficient sharing level. Then so long as $g_D(N) > g_E(N)$, LCC welfare increases as community size is enlarged and $\tilde{\Psi}(N)$ reaches a maximum at a community size of $\tilde{N}_2$. $\tilde{\Psi}(\tilde{N}_2)$ will then be a global maximum if preferences are sufficiently biased in favor of collective consumption to secure $\tilde{\Psi}(1) < \tilde{\Psi}(\tilde{N}_2)$[21].

The direct utility, commodity space analogue of $\tilde{\Psi}(N)$ is depicted in figure 8. A choice of community size specifies a feasible choice set. From the three feasible choice sets $\tilde{S}(1)$, $\tilde{S}(\tilde{N}_2)$ and $\tilde{S}(N_m)$ the indicated preferred consumption choices are respectively s_1, s_2 and s_m. $s_1 s_2 s_m$ is then the expansion path of consumption choice as N is enlarged over the range $\{N | 1 \leq N \leq N_m\}$. Direct utility $\tilde{W}(\tilde{x}, \tilde{g})$ along $s_1 s_2 s_m$ for a choice of $(\tilde{x}, \tilde{g})$ from $S(\tilde{N})$ corresponds to indirect utility $\tilde{\Psi}(\tilde{N})$.

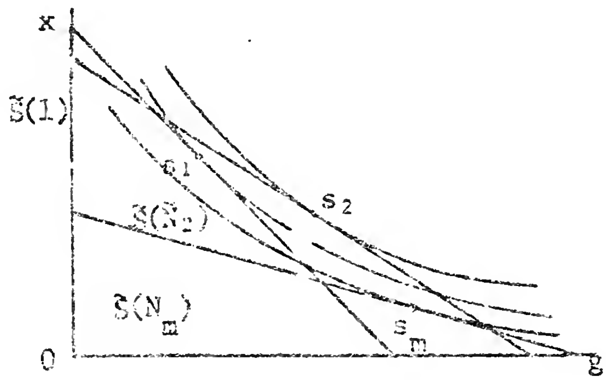

Figure 8

3.4 The Special Case of No Income Loss

There are two constraining influences on community size expansion, the collective consumption technology and the property rights assignment rule. Suppose however that LCC income were independent of community size; i.e.,

the property rights assignment rule as community size was marginally enlarged entailed $y_N = o$. Then the sole mitigating constraint to community expansion is rivalry in collective consumption. Non-wage income R(N), which could be viewed as a surrogate for a fixed factor, no longer plays a role.

If $y_N = o$, the first order condition (3.5) becomes $g.\rho^*_N = o$. Accordingly, a solution requires g_E to be such that $\rho^*_N = o$; i.e., $N = N_m$ and $\eta = 1$. The community chooses that value of N which minimizes ρ and $g_E(N) = \infty$ at $N = N_m$ as depicted in figure 9. Any level of collective consumption chosen so that $\eta = 1$ on the iso-g contour is sharing-efficient.

In the collective demand function, substitute $y_N = o$ into (3.16). Then $g'_D(N) = \tilde{g}_\rho \ \rho_N$. Since g is a normal good, it is necessarily not a Giffen good so $\tilde{g}_\rho < 0$. Therefore $g'_D(N) \lesseqgtr 0$ as $\rho_N \gtreqless 0$; i.e., $g'_D(N) \gtreqless 0$ as $N \lesseqgtr N_m$ or $\eta \lesseqgtr 1$. So collective demand attains a unique maximum at $N = N_m$. See figure 9,

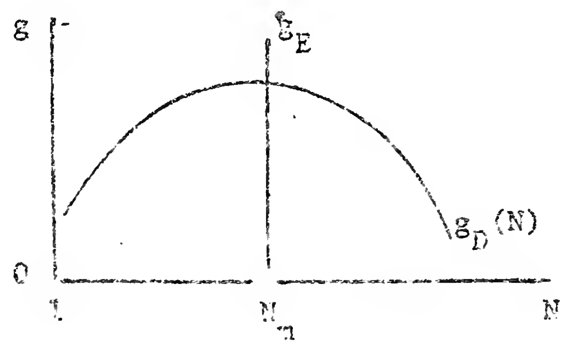

Figure 9

Setting $y_N = o$ in (3.18) we obtain as the first order condition for a maximum of $\tilde{\psi}(N)$, $\tilde{\psi}_N = \psi_\rho \ \rho_N$. Since $\psi_\rho < 0$, $\tilde{\psi}_N \gtreqless 0$ as $\rho_N \lesseqgtr 0$ and so $\tilde{\psi}(N)$ attains a global maximum at $N = N_m$ where $\eta = 1$.

In this degenerate case of $y_N = o$, optimum community size is therefore independent of preferences over private and collective consumption. The solution is always at $N = N_m$ without regard to the properties of the collective demand function. Moreover, a solution always exists if for some community

size less than N_m, a positive demand for collective services exists. The solution is unique and necessarily indicates a global maximum for LCC welfare.

Although optimal N is independent of preferences, the optimum level of collective consumption does of couse depend upon collective demand. We observe then that in the degenerate case of $y_N = o$, the essential simultaneity of the solution has a recursive form. Optimal N is determined first by $g_E(N)$ via the collective consumption technology; and then given optimal N, preferences determine optimal g.

4. COMPARATIVE STATICS

We shall now prove some propositions about the comparative statics properties of the solution obtained in section 3.3. First we assume that a global maximum for $\Psi(N)$ exists at some $N > 1$. In the neighborhood of this equilibrium, we shall consider changes in the supply price of collective consumption, in the community's non-wage income, in the degree of rivalry in collective consumption, and in preferences. All comparative statics results follow from effects on the functions $g_D(N)$ and $g_E(N)$ of changes in the appropriate exogenous varable.

<u>Lemma</u>: Let $(\tilde{N},\tilde{g})$ denote an optimal solution such that $\{1<\tilde{N}<N_m\}$, and let β denote an arbitrary exogenous variable.

Then

$$(4.1) \qquad \frac{d\tilde{N}}{d\beta} \gtreqless 0 \quad \text{as} \quad \left.\frac{dg_D}{d\beta}\right|_{\tilde{N}} \gtreqless \left.\frac{dg_E}{d\beta}\right|_{\tilde{N}}$$

$$(4.2) \qquad \frac{d\tilde{g}}{d\beta} = \left.\frac{d\tilde{g}}{d\beta}\right|_{\tilde{N}} + \frac{d\tilde{g}}{d\tilde{N}}\,\frac{d\tilde{N}}{d\beta}$$

I.e., the net shift of the functions $g_D(N)$ and $g_E(N)$ determines the change in optimum community size, while the optimum collective consumption choice is

altered via the shift in $g_D^{\prime}(N)$ and the consequent adjustment along $g_D(N)$ to the new equilibrium.

The lemma is evident from an inspection of figure 6 and requires no formal proof.

3.1 Supply Price of Collective Consumption

Consider now the effect on optimal choice of N and g when the supply price of collective services is marginally decreased. Such a decrease could occur because of a technical change in the collective consumption technology which permitted a greater flow of collective services for any $\{\Lambda, N\}$ combination. Or τ, the relative price of G, might fall. If the latter is the case, suppose that the community imports G in equilibrium.[22]

Proposition 7: (i) A decrease in the supply price of collective services increases optimum membership size if demand for collective consumption is price inelastic in the neighborhood of the equilibrium; and vice-versa if demand is elastic.

(ii) If collective demand is price-inelastic in the neighborhood of the equilibrium, a decrease in the supply price of collective consumption increases the optimal collective consumption level; and if demand is elastic, the change in optimal collective consumption is qualitatively indeterminate.

Proof: (i) Let $d\beta$ denote a marginal change such that $d\theta/d\beta<0$.
Differentiating $g_D(N)$ with respect to β and noting that $dy/d\beta=0$, we obtain

$$(4.3)\qquad \frac{dg_D}{d\beta} = \frac{d\tilde{g}}{d\rho}\frac{d\rho}{d\theta}\frac{d\theta}{d\beta}$$

We have

$$(4.4)\qquad \frac{dg_E}{d\beta} = \frac{-g_E}{\theta}\frac{d\theta}{d\beta}$$

So using the lemma and observing that in equilibrium $g_D = g_E$, we establish

(4.5) $$\frac{d\tilde{N}}{d\beta} \gtreqless 0 \quad \text{as} \quad -\frac{d\tilde{g}}{d\rho}\frac{\rho}{g} \lesseqgtr 1$$

(ii) From (4.2) and (4.5)

(4.6) $$\frac{d\tilde{g}}{d\beta} = \frac{dg_D}{d\beta}\bigg|_{\tilde{N}} + \frac{dg_D}{d\tilde{N}}\frac{d\tilde{N}}{d\beta}$$

$dg_D(\tilde{N})/d\beta > 0$ and from proposition 5 $d\tilde{g}/d\tilde{N} > 0$.

Accordingly $d\tilde{g}/d\beta > 0$ if $d\tilde{N}/d\beta > 0$; and $d\tilde{g}/d\beta \gtreqless 0$ if $d\tilde{N}/d\beta < 0$.

Q. E. D.

3.2 Non-wage income

We consider now the effect on the community's optimal choice of N and g of a marginal increase in its aggregrate non-wage income. We shall here relax the assumption that both x and g are both normal goods.

Proposition 8: (i) Let m_g denote LCC's marginal budget share of collective consumption. Then

$$\frac{d\tilde{N}}{dR} \gtreqless 0 \quad \text{as} \quad m_g \gtreqless c(\eta-1)^{-1}$$

(ii) If collective consumption is inferior in the neighborhood of the equilibrium, then $d\tilde{g}/dR < 0$; if collective consumption is not inferior, then $d\tilde{g}/dR \gtreqless 0$.

Proof:

(i) (4.7) $$\frac{d\tilde{g}}{dR}\bigg|_{\tilde{N}} = \frac{\partial\tilde{g}}{\partial y}\frac{\partial y}{\partial R} = \frac{\partial\tilde{g}}{\partial y}\alpha(\tilde{N})\tilde{N}$$

and

(4.8) $$\frac{dg_E}{dR}\bigg|_{\tilde{N}} = \frac{c\,\alpha(\tilde{N})}{\theta(\tilde{N})[\eta-1]}$$

Noting that $\rho \equiv \theta/N$ and $m_g \equiv \rho\,\partial\tilde{g}/\partial y$, proposition 8(i) follows from the lemma.

(ii) (4.9) $$\frac{d\tilde{g}}{dR} = \frac{d\tilde{g}}{dR}\bigg|_{\tilde{N}} + \frac{d\tilde{g}}{d\tilde{N}}\frac{d\tilde{N}}{dR}$$

In the neighborhood of the optimum, $d\tilde{g}/d\tilde{N} > 0$. If g is inferior, $d\tilde{g}(\tilde{N})/dR < 0$ and $d\tilde{N}/dR < 0$, so $d\tilde{g}/dR > 0$. If g is not inferior, $d\tilde{g}(\tilde{N})/dR < 0$ and $d\tilde{N}/dR < 0$, so $d\tilde{g}/dR \gtreqless 0$.

Q. E. D.

Proposition 8(i) indicates that the response of $\tilde{N}$ to a marginal increase in R depends upon the collective consumption technology via η; or whether or not collective consumption is inferior via m_g; and on the property rights assignment rule via c.

To provide some motivation for this result, consider the special case of no rivalry and an egalitaran property rights assignment. Then $\eta = 0$ and $c = -1$, so $d\tilde{N}/dR \gtreqless 0$ as $m_g \gtreqless 1$. I. e., the outcome depends only upon whether or not private consumption is inferior. Should x be inferior, then $m_g < 1$ and $\tilde{N}$ is enlarged; and if x is normal, $m_g < 1$ and $\tilde{N}$ declines. Since we generally expect private consumption to be normal, the presumption is that optimum community size declines if community non-wage income is marginally increased.

Maintaining $c = -1$ and allowing $\eta > 0$, we observe that for $d\tilde{N}/dR < 0$ to result, x need not be a normal good. The greater is rivalry in equilibrium, the higher is the benchmark value of m_g - i. e., the greater the inferiority of x at which $d\tilde{N}/dR = 0$.

On the other hand, the effect of the property rights rule is discerned by permitting c to vary and maintaining $\eta=0$. Then we observe that $d\tilde{N}/dR \gtreqless 0$ as $m_g \gtreqless -c$. The smaller c, the larger the income loss to LCC's as community size is marginally enlarged and the more inferior is x at the benchmark value of m_g where $d\tilde{N}/dR = 0$.

3.3 Rivalry in Collective Consumption

Consider a change in the collective consumption technology which results in a new rivalry elasticity in equilibrium. I. e., suppose in figure 10 that the

community's optimum position is at A and the iso-g contour shifts from g_1 to g_2.

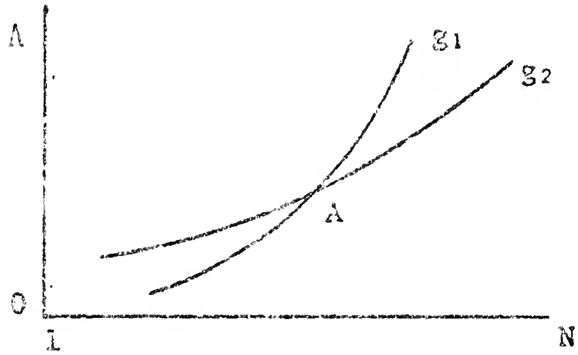

Figure 10

Proposition 9: A decrease in rivalry - as exemplified by the shift from g_1 to g_2 in Figure 10 - increases both the community's optimum membership size and its optimum level of collective consumption.

Proof: At A, θ and y are unaltered by the decrease in rivalry and we have $d\tilde{g}(\tilde{N})/d\eta = 0$. However the efficient sharing level of collective consumption increases:

(4.10) $$\frac{1}{g_E}\left.\frac{dg_E}{d\eta}\right|_{\tilde{N}} = \frac{1}{1-\eta} > 0$$

Therefore from the lemma $d\tilde{N}/d\eta < 0$.

The change in the optimum level of collective consumption is given by

(4.11) $$\frac{d\tilde{g}}{d\eta} = \left.\frac{d\tilde{g}}{d\eta}\right|_{\tilde{N}} + \frac{d\tilde{g}}{d\tilde{N}}\frac{d\tilde{N}}{d\eta} < 0$$

Since $d\tilde{g}(\tilde{N})/d\eta = 0$, $d\tilde{g}/d\tilde{N} > 0$ and $d\tilde{N}/d\eta < 0$, it follows that $d\tilde{g}/d\eta < 0$.

Q. E. D.

3.4 Prefences

Finally, we have the expected result that:

Proposition 10: A change in LCC preferences in favor of collective consumption increases the optimum community size and optimum level of collective

consumption; and vice-versa for a change in preferences in favor of private consumption.

Proof: $g_E(N)$ is independent of preferences. A change in preferences in favor of g shifts $g_D(N)$ vertically upwards in the neighborhood of the equilibrium. So, by the lemma, both $\tilde{N}$ and $\tilde{g}$ are increased. Conversely, a change in preferences in favor of private consumption shifts $g_D(N)$ downward decreasing $\tilde{N}$ and $\tilde{g}$.

Q. E. D.

5. CONCLUSION

The model presented in this paper constitutes a general frame of reference for describing a collective consuming community and analysing its optimal choice problem.[23] The comparative statics results can be viewed as characterizing the qualitative differences in the choice of discretionary variables among communities which differ only in the exogenous attribute considered.

The model is intended to be quite general in its applicability. Conceptually, the frame of reference provided is sufficiently broad to encompass any institutional collectivity, ranging from a Tiebout-type local community to an autonomous national state. Differences would manifest themselves in the nature of the community's aggregate collective consumption technology and its property rights assignment rule. The theory of fiscal federalism suggests a hierarchy of optimum collectivities based on the nature of the collective consumption technology motivating communal action. The property rights assignment rule is in turn implied by a collectivity's approach to vertical equity in its taxation structure. For example, if the collectivity is a nation-state, ethical judgements about income distribution tend to manifest themselves in a progressive tax system, which is then implicit in the specification of $\alpha(N)$ as community size is marginally enlarged. A property tax on the other hand implies a different specification for $\alpha(N)$. In each case the income of pre-existing community members is affected via the impact on the tax base when

a new consumer joins the community, and by the position the new community member takes in the structure of progressivity - or otherwise - of the tax schedule. It is these effects which we have subsumed into the property rights assignment rule.

The notion of locationally committed consumers has allowed us to approach the problem of the collectivity making decisions about its constituent members. We can interpret N = 1 as the number of consumers deciding at a given point in time how many additional consumers they would optimally prefer to have join them instantaneously as community members. If the number of consumers that does join them is different from the number preferred, then from the perspective of locationally committed consumers, there has been a market failure in the locational market for collective goods. The institutional nature of the community would then determine the types of policy instruments availabe to meet the market failure. An autonomous state would of course have much more discretion in its policy possibilites than, say, a local school district. An analysis of such market failures and correctionary policies is an interesting extension of this paper.[24] Here a frame of reference has been developed for the consideration of such problems -- in addition to the interest per se in the formulation of a model of an optimum collective consuming community in a locational market for collective goods.

References

1. James M. Buchanan, "An Economic Theory of Clubs," Economica, LIV, (February 1965): 1-14.

2. James M. Buchanan and Charles J. Goetz, "Efficiency Limits of Fiscal Mobility: An Assessment of the Tiebout Model," Journal of Public Economics, I (April 1972): 25-43.

3. Noel M. Edelson, "Congestion Tolls Under Monopoly," American Economic Review, LXI (December 1971): 873-82.

4. Bryan Ellickson. "A Generalization of the Pure Theory of Public Goods," American Economic Review, LXIII (June 1973): 417-43.

5. Leo Hillman, Collective Consumption, Optimum Resident Size and Locational Choice, Ph.D. dissertation, University of Pennsylvania, 1973.

6. Albert O. Hirschman, Exit, Voice and Loyalty, (Cambridge: Harvard University Press, 1970).

7. Robert P. Inman, "The Specification and Application of a Shared Good Technology," paper read before the Econometric Society, Winter Meeting, December 1973.

8. Morton I. Kamien, Nancy L. Schwartz and Donald J. Roberts, "Exclusion, Externalities and Public Goods," Journal of Public Economics, II (July 1973): 217-30.

9. Murray C. Kemp, The Pure Theory of International Trade and Investment, (Englewood Cliffs: Prentice-Hall, 1969).

10. James E. Meade, The Theory of International Economic Policy; Vol. II, Trade and Welfare, Mathematical Appendix (New York: Oxford University Press, 1955).

11. William H. Oakland, (a) "Congestion, Public Goods and Welfare," Journal of Public Economics, I (November 1972): 339-57.

12. William H. Oakland, (b) "Public Goods and Locational Choice," Unpublished, (December 1972).

13. Mark V. Pauly and Michael R. Redisch, "The Not-For-Profit Hospital as Physicians' Cooperative," American Economic Review, LXII (March 1973):87-9.

14. Paul A. Samuelson, "The Pure Theory of Public Expenditure," Review of Economics and Statistics, XXXXI (November 1954):387-89.

15. Paul A. Samuelson, "Pure Theory of Public Expenditure and Taxation," in Public Economics, ed. J. Margolis and M. Guitton (New York:Macmillan, 1969).

16. Julian L. Simon, "The Welfare Effect of an Additional Child Cannot be Stated Simply," Faculty Working Paper No. 80, College of Commerce and Business Administration, University of Illinois at Urbana-Champaign, January 1973.

17. Charles M. Tiebout, "A Pure Theory of Local Expenditures," Journal of Political Economy, LXIV, (October 1956):416-24.

Footnotes

1. See Samuelson (1954) and (1969).

2. See Tiebout (1956).

3. On locational market failures and corrective policies, see Hillman (1973), chapter 7. Note that the type of market failure referred to here is specific to the community. The question is whether the locational market in collective goods results in that configuration for the community that members would optimally choose if they had appropriate discretion. See also Buchanan and Goetz (1972) and Oakland (1972b), who are concerned whether the locational market results in a Pareto-optimal allocation of consumers between two collective-consuming communities.

4. Tiebout has "each (community) announce a different pattern of expenditives on public goods." P. 421.

5. There have been a number of suggested generalizations of Samuelson's polar pure public good. These have taken a number of different directions. See for example Oakland (1972a), Ellickson (1973) and Kamien, Schwartz and Roberts (1973). The approach presented below has most in common with Buchanan (1965) but is essentially different in that rivalry is not a manifestation of a consumer's subjective preference ordering, but constitutes an objective technological aspect of collective consumption. For an approach similar to that developed in this paper, see Inman (1973).

6. There is a natural symmetry between a small trading nation facing exogenous prices in a world market and a community of consumers. Moreover, the conception of a community adopted in this paper encompasses the nation-state as the highest level in a hierarchy of fiscal federalism. Insofar as the application is to a nation-state, the country has to be small. It is an easy modification to introduce discretionary power over prices, but that would add an unnecessary complication by involving us in optimum tariff arguments.

7. I.e., Tiebout suggested that: "The factor may be the limited land area of a suburban community, combined with a set of zoning laws against apartment buildings. It may be the local beach, whose capacity is limited. Anything of this nature will provide a restraint . . . some factor or resource is fixed. If this were not so, there would be no logical reason to limit community size.." P. 419.

8. In his model comparing the efficiency properties of the locational market with Samuelson's pseudo-market, Tiebout assumes "that the costs of additional services are constant. Further, assume that a doubling of the population means doubling the amount of services required." Then he contends that under these conditions, with the potential number of communities unbounded, "there is no reason why the number of communities will not be equal to the population . . . Unless some sociological variable is introduced, this may reduce the solution of the problem of allocating public goods to the trite one of making each person his own municipal government." P. 421.

9. For a classic perspective on this problem, see Meade (1955). Meade's concern was that if social welfare were to be defined as an ordering over a predetermined number of people, some resulting optimality criterion could deny existence to some part of the group. The possibility of expelling some part of the group does not arise for us because of the way in which our problem is formulated. For some additional implications of the sorts of issues raised by Meade, see Simon (1973).

10. See Hirschman (1970). Tiebout's approach was to posit "a city manager who follows the preferences of the older residents of the community.." P. 419.

11. See also Inman (1973).

12. The existence of this inverse is secured by $g^*_\Lambda > 0$.

13. This inverse exists for $N>1$ because $g^*_N < 0$.

14. Note that the iso - Λ and iso - N contours in figures 2 and 3 respectively could be viewed as extending into the fourth quadrant. However, we have assumed in (1a) that $W_g > 0$. A rational society would accordingly never choose an application of its collective consumption technology which resulted in $g < 0$. So there is no loss in generality in defining the technology for $g \geq 0$ only and thereby confining the functions Λ^* to N^* to the positive quadrant.

15. Samuelson's polar pure public good is of course a limiting case of a collective consumption technology which exhibits neither congestion nor rivalry; i.e. $\eta \equiv 0 \equiv \gamma$, or $g^*_N \equiv 0 \equiv \Lambda^*_N$. The iso-Λ contours in $\{g, N\}$ space are horizontal because, with no congestion, the collective service flow is invariant with respect to community size. Similarly, the iso-g contours in $\{\Lambda, N\}$ space are horizontal, no addition to the community's collective facility being necessary to accommodate a marginal community member at a given level of collective consumption.

16. Since the model is familar, we shall not elaborate it here. We assume all the standard properties of production functions - i.e., linear homogeneity of degree one, strict concavity and the Inada conditions - and we assume competitive pricing and instantaneous clearing of all markets. See Kemp (1969).

17. Clearly the collective facility cannot be traded, since it is defined as employing domestic labor to provide collective services to community members.

18. We are concerned in this paper with describing the community only at a point in time. However, with perfect intertemporal markets there should be a complete indifference between purchasing the durable good or purchasing its services.

19. Note that per capita here is defined under the rescaled units of measurement of N, where N = 1 is the number of LCC's.

20. See footnote 19.

21. Since $g_D^i(N) = 0$ for $\{N_2 > N > N_m\}$ there can in principle be more than two solutions. However, it appears justifiable to assume that the negative income effect will dominate the substitution effect over this range of N. We can expect consumers to display an increasing reluctance to trade private for collective consumption as y falls with community size expansion. In any event, the existence of more than two solutions does not affect any of the above results in any substantive manner. All it does is give us additional local extrema in $\psi(N)$.

22. We make this assumption for expository purposes. It means that both the substitution and income effects of an increase in π operate in the same direction. If G were exported, we would have to contend with the complication of a positive income effect. This can be readily handled. See Hillman (1973), chapters 4 and 5.

23. Since we have been concerned with an optimal sharing problem, our model is similar in spirit to models such as Pauly and Redisch's 'not-for profit hospital as physician's cooperative' (1973) and Edelson's highway sharing problem with congestion tolls under monopoly (1971).

24. I.e., the optimum can be compared with some other outcome and appropriate correctionary policy -- if required -- can be formulated. For this we also require a complete theory of individual consumer behavior in a market for collective goods via locational choice. See Hillman (1973), chapters 6 and 7.