Faculty Working Papers

OPTIMUM STRATEGIES FOR MANAGEMENT

INFORMATION PROCESSING AND CONTROL

Hirohide Hinomoto

University of Illinois

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana - Champaign

May 21, 1971

OPTIMUM STRATEGIES FOR MANAGEMENT INFORMATION PROCESSING AND CONTROL

Hirohide Hinomoto

University of Illinois

No. 12

INTRODUCTION

This paper is concerned with management's control over a group of productive activities whose performances tend to deteriorate over time. The objective of the manager is to maintain their performances at high levels through the execution of proper control actions at the right times.

In this paper performance is assumed to be measured by the net value produced per period. However, it could just as well be measured by the total value produced with a constant periodic inputs of labor and material, by the total cost of labor and material, computed at the standard prices, required for a constant periodic demand for the product, or by the labor or material quantity variance per unit product from the standard rate.

It is assumed that contingent changes in the performance of each activity are mostly changes for the worse owing to cumulative degrading conditions of operation. Such a situation is commonly found in mechanical processes where the rate of defective production increases as the machine gradually gets out of the original alignment, or as the machine's parts and tools accumulate their wear with time. Although the activity may occasionally improve its performance a little without external aids, the performance improvement is normally achieved by the execution of a control action. The control action is expected to produce a positive result, but it may sometimes result in no improvement in performance or even in further deterioration.

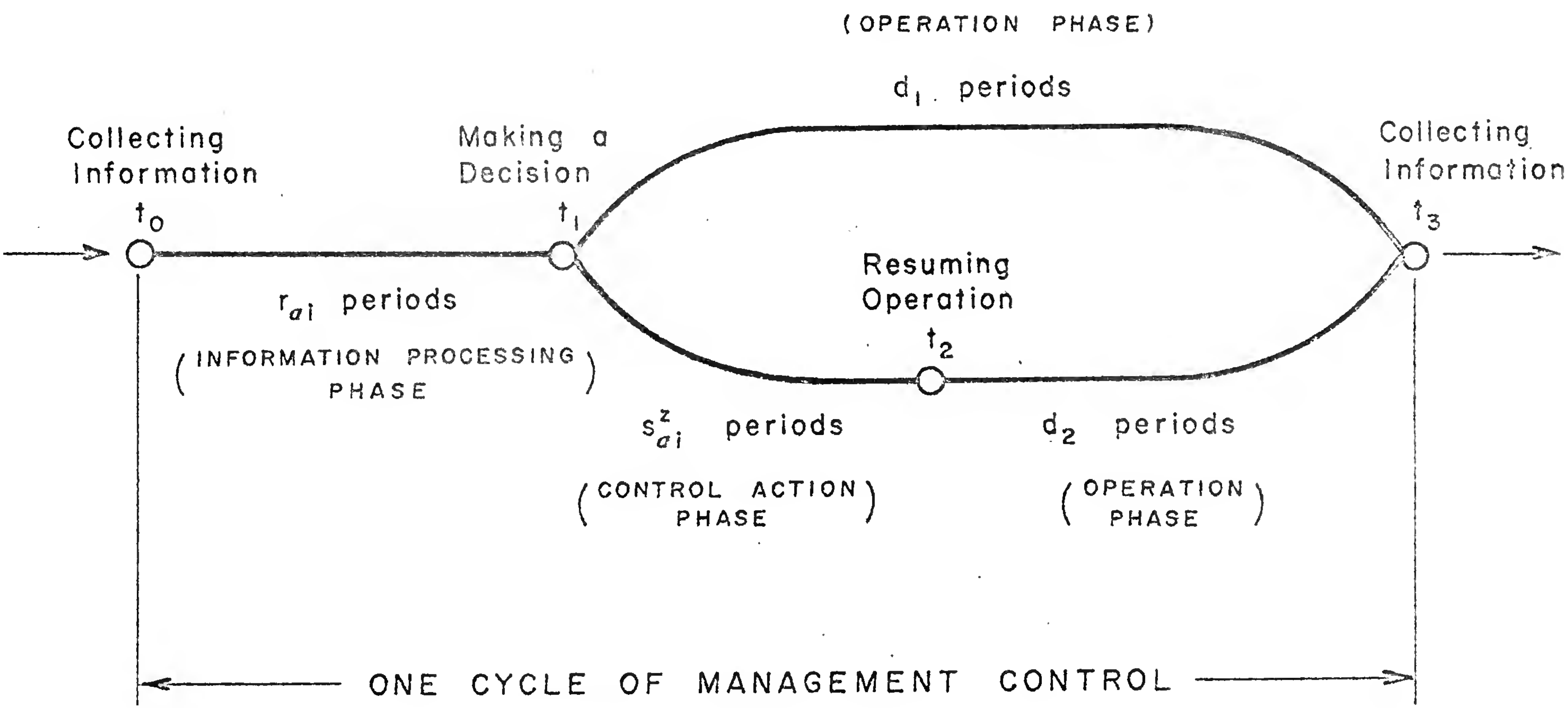
To describe the general tendency, all contingent changes in performance, including occasional improvements, will be aggregately referred to as "performance deterioration" and all performance changes after the execution

of a control action, whether successful or unsuccessful, by "performance improvement." The performance level of an activity in any period depends only on the level in the preceding period, unless a control action is initiated, in which case it will depend on the success of this action as well. Relying on this assumption, performance deterioration and improvement can be represented by discrete state Markov chains. The use of a Markov approach was originally proposed by Charnes and Stedry in the model describing a budget-aspiration-performance relationship [2, p. 212-229].

Management control is an iterative process involving three tasks: the collection of information on the conditions of activities, the evaluation of this information, and the execution of proper control actions when they are needed. Figure 1 shows the phases comprising the cycle of management control. The cycle starts with the "information processing" phase. It is followed by either the "control" and "operation" phases or the "operation" phase alone depending on whether a control action is executed. (These alternatives are indicated by the two paths between t_1 and t_3 in Figure 1.)

The tasks related to the collection and evaluation of information may be carried out at regular intervals irrespective of the conditions of activities. However, processing information on a regular basis may be undesirable depending on the time interval involved. For example, if the deterioration of the activity's performance is slow, the execution of the management control process at relatively short intervals is unnecessarily costly. But, if the deterioration is relatively fast, processing information

Figure 1. Two Alternative Time-Paths of Management Control



at relatively long intervals will result in ineffective control. There may be no single interval that can always resolve the above dilemma, in which case information processing should be executed at irregular intervals determined on the basis of observed conditions of the activities.

One simple strategy for control that is commonly practiced in industry is to set a threshold level for evaluating the performance of an activity and to execute a control action only if the performance is at or below this level. Such an approach assumes that future performances will remain below this level unless a control action is initiated. The main disadvantage with this approach is that it does not incorporate an evaluation of the benefits of a control action against the costs of implementation at each specific level. In the model below, optimal control strategies are developed for each of the possible performance levels at a decision point, regarding whether a control action should be executed and how long a period is allowed to elapse before the subsequent collection of information.

Solutions to the optimal stationary strategies are obtained by the linear programming method for sequential decisions proposed by Manne [7]. The objective of the model is to maximize the sum of the expected values produced per period by individual activities, net of the total cost of control covering the collection and evaluation of information regarding the conditions of the activities, and the selection and execution of control actions. In addition to the basic constraints, the model has a constraint on the average total time available for processing information or executing control actions.

Thus, the model determines both optimal strategies for control and an optimal allocation of the manager and his staff's time, or the processing time of computers to individual activities. On the basis of this time allocation, individual activities may be charged

with various costs of control not included in the model, such as the salaries of the manager and his staff, the rents or depreciation costs of computer facilities, and the materials and supplies used in information processing or execution of control actions.

The model determines optimum strategies for controlling each activity independent of the other implying the manager can take care of any number of activities simultaneously. This assumes that his capacity for processing information or executing control actions is considerably larger than the possible total requirement by all the activities. With or without such a time constraint, however, the formulation is valid only in terms of the average time requirement and simply ignores possible conflicts between activities simultaneously demanding the manager's service. In reality, such conflicts may arise, and a manager may have to determine priority rules for controlling the activities. For such cases, a state will then represent each possible combination of the performance levels of the activities, instead of each performance level of a single activity as in the present formulation. As a consequence, the state space in the new case will be substantially larger than the relatively small one in the present formulation. However, minor modifications in the present formulation should be able to incorporate these additions.

FORMULATION

A manager supervises n independent activities; the performance of each activity is described by one of m levels representing values produced in a period. The value produced by activity $\underline{a}$ at level i is $v_{\underline{a}i}$ given as the i^{th} element of vector $V_{\underline{a}}$, where $\underline{a}$ is an element of set A = {1, ..., n}, and i is an element of set J = {1, ..., m}.

(1) $$V_{\underline{a}} = (v_{\underline{a}1}, \ldots, v_{\underline{a}m}) \qquad \underline{a}\varepsilon A,$$

where

$$v_{\underline{a}1} > \ldots > v_{\underline{a}m} .$$

Performance deterioration from level i to lower levels in one period is given by a stationary transition probability matrix $P_{\underline{a}}$ for activity $\underline{a}$:

(2) $$P_{\underline{a}} = \begin{bmatrix} p_{\underline{a}i} \end{bmatrix} = \begin{bmatrix} p_{\underline{a}i,j} \end{bmatrix} \qquad \underline{a}\varepsilon A;\ i,j\varepsilon J.$$

In (2), $p_{\underline{a}i}$ is the i^{th} row vector and $p_{\underline{a}i,j}$ the $i\text{-}j^{th}$ element of the matrix such that

$$\Sigma_{j\varepsilon J}\, p_{\underline{a}i,j} = 1, \quad p_{\underline{a}i,j} \geq 0 .$$

In each row of $P_{\underline{a}}$, the stronger the skewness of distribution toward the right, the slower the tendency of deterioration. Positive entries would

normally be located on and above the main diagonal line of $P_{\underline{a}}$ indicating no change or no deterioration. However, positive entries may also exist below the diagonal line if there is slight natural performance improvement. The assumption of a stationary deterioration matrix implies that control actions produce no structural change in the matrix, (i.e., no technological change in machines and no learning by human operators).

The course of action, z, available for activity $\underline{a}$ is an element of set $Z_{\underline{a}} = \{0, 1, \ldots, M_{\underline{a}}\}$. No control action is executed if $z = 0$, whereas a control action is executed if z is an element of subset $Z^{*}_{\underline{a}} = \{1, \ldots, M_{\underline{a}}\}$ of $Z_{\underline{a}}$. When a control action is executed, the performance improvement of activity $\underline{a}$ is given by the stationary transition matrix $Q^{z}_{\underline{a}}$:

$$(4) \qquad Q^{z}_{\underline{a}} = \begin{bmatrix} q^{z}_{\underline{ai}} \end{bmatrix} = \begin{bmatrix} q^{z}_{\underline{ai},j} \end{bmatrix} \qquad \underline{a}\varepsilon A;\ z\varepsilon Z_{\underline{a}};\ i,j\varepsilon J$$

where $q^{z}_{\underline{ai}}$ is the i^{th} row vector of the matrix and $q^{z}_{\underline{ai},j}$ is the $i\text{-}j^{th}$ element representing the conditional probability that the performance of activity $\underline{a}$ improves to level j after the execution of a control action, given that it was at level i before the execution. Since each row is a probability vector, it must satisfy

$$\Sigma_{j\varepsilon J}\, q^{z}_{\underline{ai},j} = 1, \qquad q^{z}_{\underline{ai},j} \geq 0$$

Although most control actions will produce performance improvements and will be indicated by positive entries below the main diagonal line of matrix $Q^{z}_{\underline{a}}$, positive values may exist on or above its diagonal line to

represent the probabilities of no improvement or inadvertent performance deterioration. The stationarity assumption on $Q_{\underline{a}}^{z}$ means that the effects of control actions stay constant, implying the skill of the manager has passed the learning stage.

The existing level of performance determines the amount of time required for processing its information. If the performance at time t_o is at level i, the process lasts $r_{\underline{ai}}$ periods. This is shown by the interval between t_0 and t_1 in Figure 1, during which the activity continues its operation usually with a declining performance. At the completion of information processing at t_1, the next course of action starting at t_1 is decided on the basis of the performance level at t_0. If no control action is executed following the information processing, the operation of the activity is continued with a farther decline in performance over the interval between t_1 and t_3 (illustrated by the upper time-path in Figure 1). But if a control action is executed starting at t_1, the operation is suspended until the completion of the control action at t_2. The time required for completing a control action is a function of the activity, its level, and the control action, and is represented by $s_{\underline{ai}}^{z}$. At time t_2, the activity resumes its operation presumably with an improved performance. Thereafter its performance gradually deteriorates over the interval between t_2 and t_3, as is illustrated by the lower time-path of Figure 1.

Let us now introduce a state variable, $x_{\underline{ai}}^{zd}$, for activity $\underline{a}$. This variable represents the probability of the manager making the following decision for activity $\underline{a}$ with performance level i at a decision point:

he executes action z lasting $s^z_{\underline{a}i}$ periods and then lets the activity operate for d periods before the subsequent collection of information on its performance. Specifically, $x^{zd}_{\underline{a}i}$ is given a positive value if the combination of i, z, and d is acceptable; otherwise, it is given zero. The objective now is to determine an optimum value for this variable satisfying

$$\sum_{i\varepsilon J}\sum_{z\varepsilon Z}\sum_{d\varepsilon D} x^{zd}_{\underline{a}i} = 1 \qquad \underline{a}\varepsilon A\,,$$

subject to

$$x^{zd}_{\underline{a}i} \geq 0\,.$$

The variable $x^{zd}_{\underline{a}i}$ is sometimes more appropriately called the probability decision variable. In Figure 1, if no control action is executed, $s^z_{\underline{a}i}$ equals 0 and d_1 represents d; and if a control action is executed, $s^z_{\underline{a}i}$ takes a positive value and d_2 represents d.

If the information collected at t_0 shows the performance during the period immediately preceding t_0, the expected net value associated with decision variable $x^{zd}_{\underline{a}i}$ can be given by one of the following two $E(w^{zd}_{\underline{a}i})$'s depending on whether a control action is to be executed:

$$(5) \qquad E(w^{zd}_{\underline{a}i}) = e_i(P_{\underline{a}} + \ldots + P_{\underline{a}}^{r_{\underline{a}i}+d})\, V_{\underline{a}} - g_{\underline{a}i} \quad \text{for } z=0,\ \underline{a}\varepsilon A,\ i\varepsilon J,\ d\varepsilon D;\ \text{or}$$

$$(6) \qquad E(w^{zd}_{\underline{a}i}) = e_i\{(P_{\underline{a}} + \ldots + P_{\underline{a}}^{r_{\underline{a}i}}) + P_{\underline{a}}^{r_{\underline{a}i}} Q^z_{\underline{a}}(I + P_{\underline{a}} + \ldots + P_{\underline{a}}^{d-1})\}\, V_{\underline{a}}$$

$$-g_{\underline{a}i} - h^z_{\underline{a}i} \qquad \text{for } z\varepsilon Z^*_{\underline{a}},\ \underline{a}\varepsilon A,\ i\varepsilon J,\ d\varepsilon D,$$

where

I = an identity matrix,

e_i = a unit row vector with 1 in the i^{th} element and 0 in the rest,

$g_{\underline{a}i}$ = the cost of information processing when the observed performance of activity $\underline{a}$ is at level i,

$h^z_{\underline{a}i}$ = the cost of control action z applied to activity $\underline{a}$ at level i,

$D = \{1, \ldots, d_{max}\}$, the set of alternative values considered for d, the number of periods of operation between the end of action z and the subsequent time for collecting information. d_{max} is an arbitrary maximum value considered for d.

The objective of the model is to maximize the average net value produced per period given by the following W:

$$(7) \qquad W = \Sigma_{\underline{a}\varepsilon A} \left(\frac{\Sigma_{i\varepsilon J} \Sigma_{z\varepsilon Z_{\underline{a}}} \Sigma_{d\varepsilon D} \; E(w^{zd}_{\underline{a}i}) x^{zd}_{\underline{a}i}}{\Sigma_{i\varepsilon J} \Sigma_{z\varepsilon Z_{\underline{a}}} \Sigma_{d\varepsilon D} \; (r_{\underline{a}i} + s^z_{\underline{a}i} + d) x^{zd}_{\underline{a}i}} \right)$$

where $s^z_{\underline{a}i} = 0$ for $z = 0$ and

$x^{zd}_{\underline{a}i} \geq 0$ for all $\underline{a}$, i, z, and d.

In stationary state, a statistical equilibrium exists for each performance level of each activity. This is given by

$$(8) \qquad \Sigma_{i\varepsilon J} \Sigma_{z\varepsilon Z_{\underline{a}}} \Sigma_{d\varepsilon D} \left(f^{zd}_{\underline{a}i} \right)_j x^{zd}_{\underline{a}i} - \Sigma_{z\varepsilon Z_{\underline{a}}} \Sigma_{d\varepsilon D} x^{zd}_{\underline{a}j} = 0 \qquad \underline{a}\varepsilon A, \; j\varepsilon J,$$

where $\left(f^{zd}_{\underline{a}i}\right)_j$ is the j^{th} element of the following transition vector $F^{zd}_{\underline{a}i}$ and represents the conditional probability that, given initial level i, activity $\underline{a}$ moves into level j after $r_{\underline{a}i} + s^{z}_{\underline{a}i} + d$ periods during which the course of action z is executed:

$$(9) \qquad F^{zd}_{\underline{a}i} = \left(\left(f^{zd}_{\underline{a}i}\right)_j \right) = \begin{cases} e_i P_{\underline{a}}^{r_{\underline{a}i} + d} & \text{for } z = 0 \\ \\ e_i P_{\underline{a}}^{r_{\underline{a}i}} Q^{z}_{\underline{a}} P_{\underline{a}}^{d-1} & \text{for } z \varepsilon Z^{*}_{\underline{a}} \qquad i,j \varepsilon J,\ \underline{a} \varepsilon A,\ d \varepsilon D \end{cases}$$

Finally, since $x^{zd}_{\underline{a}i}$'s are probability variables, the following unitary constraint exists for each activity:

$$(10) \qquad \Sigma_{i \varepsilon J} \Sigma_{z \varepsilon Z_{\underline{a}}} \Sigma_{d \varepsilon D} \; x^{zd}_{\underline{a}i} = 1 \qquad \underline{a} \varepsilon A$$

The above formulation (7)-(10) completes the model under unlimited capacities for processing information and executing control actions. In reality various types of restrictions usually exist on those capacities. Some of the more important ones are on the total available time, the budget, and the maximum rate of digesting information. Here the first restriction is taken into consideration, assuming that the average amounts of time per period available to the manager and his staff for <u>processing information</u> and <u>executing control actions</u> are limited to H_r and H_s, respectively, expressed in fractions of the period. The budget restriction will not be considered, and the rate of digesting information is assumed to be fixed in each state of an activity.

Since the average numbers of hours required for processing information and executing control actions for all activities under selected decisions can not exceed H_r and H_s, the following constraints must be satisfied:

$$(11) \qquad \Sigma_{a\varepsilon A}\left(\frac{\Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_a}\Sigma_{d\varepsilon D}\ r_{ai}\,x_{ai}^{zd}}{\Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_a}\Sigma_{d\varepsilon D}\ (r_{ai} + s_{ai}^{z} + d)x_{ai}^{zd}}\right) \leq H_r, \text{ and}$$

$$(12) \qquad \Sigma_{a\varepsilon A}\left(\frac{\Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_a}\Sigma_{d\varepsilon D}\ s_{ai}^{z}\,x_{ai}^{zd}}{\Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_a}\Sigma_{d\varepsilon D}\ (r_{ai} + s_{ai}^{z} + d)x_{ai}^{zd}}\right) \leq H_s.$$

With (11) and (12) in addition to (7)-(10), the formulation of the problem under time restrictions has been completed. However, since the fractional expressions in (7), (11), and (12) are not amenable to linear programming, original variables x_{ai}^{zd} are now transformed to new variables X_{ai}^{zd}, using the transformations suggested by Derman [3] and Klein [6]:

$$(13) \qquad X_{ai}^{zd} = \frac{x_{ai}^{zd}}{L_a} \qquad\qquad a\varepsilon A,\ i\varepsilon J,\ z\varepsilon Z_a,\ d\varepsilon D$$

where

$$(14) \qquad L_a = \Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_a}\Sigma_{d\varepsilon D}\ (r_{ai} + s_{ai}^{z} + d)\ x_{ai}^{zd} \qquad\qquad a\varepsilon A$$

By substituting X_{ai}^{zd} in place of x_{ai}^{zd}, the objective function (7) is changed to

$$(15) \qquad W = \Sigma_{a\varepsilon A}\Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_a}\Sigma_{d\varepsilon D}\ E(w_{ai}^{zd})X_{ai}^{zd}$$

subject to

$$X_{\underline{ai}}^{zd} \geq 0 \quad \text{for all } \underline{a}, i, z, \text{ and } d$$

By replacing $x_{\underline{ai}}^{zd}$ by $X_{\underline{ai}}^{zd}$, the constraint (6) is rewritten to

$$(16) \quad \Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_{\underline{a}}}\Sigma_{d\varepsilon D} \left(f_{\underline{ai}}^{zd} \right)_j X_{\underline{ai}}^{zd} - \Sigma_{z\varepsilon Z_{\underline{a}}}\Sigma_{d\varepsilon D} X_{\underline{aj}}^{zd} = 0 \qquad \underline{a}\varepsilon A,\ j\varepsilon J .$$

Multiplying both sides of (13) by $(r_{\underline{ai}} + s_{\underline{ai}}^{z} + d)$ and summing them for all i, z, and d for each $\underline{a}$, the following relation is obtained:

$$\Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_{\underline{a}}}\Sigma_{d\varepsilon D} (r_{\underline{ai}} + s_{\underline{ai}}^{z} + d)\ X_{\underline{ai}}^{zd} = \Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_{\underline{a}}}\Sigma_{d\varepsilon D} \frac{(r_{\underline{ai}} + s_{\underline{ai}}^{z} + d)x_{\underline{ai}}^{zd}}{L_{\underline{a}}} \qquad \underline{a}\varepsilon A.$$

Since the right hand side of the above equation equals 1 (owing to (14)), this equation is rewritten as the following:

$$(17) \quad \Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_{\underline{a}}}\Sigma_{d\varepsilon D} (r_{\underline{ai}} + s_{\underline{ai}}^{z} + d)X_{\underline{ai}}^{zd} = 1 \qquad \underline{a}\varepsilon A.$$

Constraints (11) and (12) can be rewritten by substituting $X_{\underline{ai}}^{zd}$ in place of $x_{\underline{ai}}^{zd}$ in a manner similar to that used in obtaining (15) from (7):

$$(18) \quad \Sigma_{\underline{a}\varepsilon A}\Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_{\underline{a}}}\Sigma_{d\varepsilon D} \quad r_{\underline{ai}}\ X_{\underline{ai}}^{zd} \leq H_r \text{ , and}$$

$$(19) \quad \Sigma_{\underline{a}\varepsilon A}\Sigma_{i\varepsilon J}\Sigma_{z\varepsilon Z_{\underline{a}}}\Sigma_{d\varepsilon D} \quad s_{\underline{ai}}^{z}\ X_{\underline{ai}}^{zd} \leq H_s$$

Thus the original non-linear formulation (7)-(12) has been transformed to the linear programming model (15)-(19) proposed by Manne [7]. The number of constraints given by (16) for each activity $\underline{a}$ is identical to the number of levels of the activity given by the possible combinations of i, z, and d. Since any one of these constraints is dependent on the rest, it can be taken out of the linear program. With (16) and (17), it is possible to obtain a solution maximizing the objective function in (5) which can handle cases without time restrictions on the information processing and the execution of control actions. Following Wagner's proof [11], it is always possible to find a solution with pure strategies if all levels of each activity are occupied by positive values. However, the imposition of the additional, non-redundant constraints (18) and (19) representing time restrictions usually deprive the model of the existence of a solution with all pure strategies.

Optimum values of $X_{\underline{ai}}^{zd}$ obtained by solving the above linear program are to be converted to the values of original variables $x_{\underline{ai}}^{zd}$. For this, first summing up both sides of (13) and substituting ([illegible]) into the righthand side of the summed-up equation, the following relation is found:

$$(20) \qquad \Sigma_{i\varepsilon J}\,\Sigma_{z\varepsilon Z_{\underline{a}}}\,\Sigma_{d\varepsilon D}\; X_{\underline{ai}}^{zd} = \frac{1}{L_{\underline{a}}} \qquad \underline{a}\varepsilon A .$$

Substituting optimum values of $X_{\underline{ai}}^{zd}$ on the lefthand side of (20) will give the value of $L_{\underline{a}}$. With this $L_{\underline{a}}$ and (13), individual $x_{\underline{ai}}^{zd}$'s are

converted to corresponding original variables x^{zd}_{ai}.

Finally, the expected amounts of time required per period for processing information and executing control actions for activity a are given by the following $E(\bar{r}_a)$ and $E(\bar{s}_a)$, expressed in a fraction of the period:

(21) $E(\bar{r}_a) = \frac{1}{L_a} \Sigma_{i\varepsilon J} \Sigma_{z\varepsilon Z_a} \Sigma_{d\varepsilon D}\, r^z_{ai} x^{zd}_{ai}$ $\quad a\varepsilon A$, and

(22) $E(\bar{s}_a) = \frac{1}{L_a} \Sigma_{i\varepsilon J} \Sigma_{z\varepsilon Z_a} \Sigma_{d\varepsilon D}\, s^z_{ai} x^{zd}_{ai}$ $\quad a\varepsilon A$.

Using $E(\bar{r}_a)$ and $E(\bar{s}_a)$ obtained in (21) and (22), the total amounts of time required for processing information and executing control actions are allocated to individual activities in the following proportions α_a and β_a, respectively:

(23) $\alpha_a = E(\bar{r}_a)/\Sigma_{a\varepsilon A}\, E(\bar{r}_a)$ $\quad a\varepsilon A$, and

(24) $\beta_a = E(\bar{s}_a)/\Sigma_{a\varepsilon A}\, E(\bar{s}_a)$ $\quad a\varepsilon A$.

The above α_a or β_a may be used to establish the standard rates for distributing among individual activities the portions of the manager and his staff's salaries assignable to the tasks of processing information or executing control actions. Furthermore, these rates may be used to charge those activities with the indirect costs of additional labor, equipment and facilities, or materials and supplies necessary for the manager and his staff in carrying out the information processing and control tasks.

[illegible]

In this example a manager supervises 3 production lines -- lines 1, 2, and 3. The performance of each line is described by one of 4 levels representing net value produced by the line per day. The value produced by line $\underline{a}$ at level i is given by the ith entry of the following vector $V_{\underline{a}}$:

$$V_1 = (5.0,\ 4.0,\ 3.0,\ 2.0)$$
$$V_2 = (5.0,\ 3.5,\ 2.0,\ 1.0) \qquad \text{in \$1000's/day}$$
$$V_3 = (5.0,\ 3.0,\ 1.5,\ 0.)$$

The performance of line $\underline{a}$ deteriorates day by day with transition matrix $P_{\underline{a}}$:

$$P_1 = \begin{bmatrix} .60 & .25 & .10 & .05 \\ .00 & .70 & .20 & .10 \\ .00 & .00 & .80 & .20 \\ .00 & .00 & .00 & 1.00 \end{bmatrix},\quad P_2 = \begin{bmatrix} .73 & .15 & .08 & .04 \\ .00 & .80 & .13 & .07 \\ .00 & .00 & .85 & .15 \\ .00 & .00 & .00 & 1.00 \end{bmatrix},\quad P_3 = \begin{bmatrix} .87 & .09 & .03 & .01 \\ .00 & .83 & .12 & .05 \\ .00 & .00 & .78 & .22 \\ .00 & .00 & .00 & 1.00 \end{bmatrix}$$

There are 3 corrective actions -- action 1, action 2, and action 3 -- available to the manager for improving the deteriorated performance of each production line. The effectiveness of action z (z = 1, 2, 3) is represented by a single improvement matrix, $Q_{\underline{a}}^{z}$ ($\underline{a}$ = 1, 2, 3), common to all production lines:

$$Q_{\underline{a}}^{1} = \begin{bmatrix} .90 & .07 & .02 & .01 \\ .85 & .09 & .04 & .02 \\ .60 & .22 & .14 & .04 \\ .30 & .36 & .24 & .10 \end{bmatrix},\quad Q_{\underline{a}}^{2} = \begin{bmatrix} .88 & .09 & .02 & .01 \\ .82 & .10 & .05 & .03 \\ .66 & .22 & .09 & .03 \\ .43 & .32 & .17 & .08 \end{bmatrix},\quad Q_{\underline{a}}^{3} = \begin{bmatrix} .85 & .09 & .04 & .02 \\ .80 & .11 & .06 & .03 \\ .70 & .20 & .07 & .03 \\ .46 & .33 & .16 & .05 \end{bmatrix}$$

$$\underline{a} = 1, 2, 3$$

The length of time $s_{\underline{a}i}$ days and the cost $h_{\underline{a}i}$ dollars required for executing corrective action for line $\underline{a}$ are assumed to be independent of the performance level i and given by the a^{th} entries of the following vectors $S_{\underline{a}}$ and $T_{\underline{a}}$ ($\underline{a}$ = 1, 2, 3):

$$S_1 = (1, \ 2, \ 3) \qquad T_1 = (1.0, \ 1.2, \ 1.4)$$
$$S_2 = (2, \ 3, \ 4) \qquad T_2 = (1.3, \ 1.5, \ 1.7)$$
$$S_3 = (2, \ 2, \ 3) \qquad T_3 = (1.6, \ 1.7, \ 1.8)$$

in days $\qquad$ in \$1000's.

The length of time $r_{\underline{a}i}$ days and the cost $g_{\underline{a}i}$ dollars required for processing information regarding the performance of production line $\underline{a}$ at level i are given by the i^{th} entries of the following vectors $R_{\underline{a}}$ and $G_{\underline{a}}$ ($\underline{a}$ = 1, 2, 3):

$$R_1 = (1, \ 1, \ \text{[illegible]}, \ 2) \qquad G_1 = (1.0, \ 1.1, \ 1.2, \ 1.3)$$
$$R_2 = (1, \ 1, \ 2, \ 2) \qquad G_2 = (1.0, \ 1.2, \ 1.4, \ 1.6)$$
$$R_3 = (1, \ 2, \ 2, \ 3) \qquad G_3 = (1.2, \ 1.3, \ 1.4, \ 1.5)$$

in days $\qquad$ in \$1000's.

Nine days is the maximum allowable time to elapse between the end of a control action and the collection of new information on any production line. Finally, the average amounts of time per day allowable for processing information and executing corrective actions for all production lines are .6 day and .5 day, respectively. Summarily, those allowable amounts of time are

$d_{max} = 9$ days

$H_r = .6$ day

$H_s = .5$ day

Given the above conditions, this linear program has 12 constraints in (16) and 3 constraints in (17), without the time restrictions H_r and H_s. With these restrictions, it has two additional constraints in (18) and (19). In either case, it has 432 variables formed by 3 production lines, 4 performance levels for each production line, 4 control decisions selecting either no control action or one of 3 control actions, and 9 alternative periods of operation before collecting new information.

The program without the time constraints was computed by MPS (an IBM linear programming code) on an IBM 360/75, producing a solution with a single strategy at each level of each production line and the total expected net value of $7,076. Then, this linear program was computed with the two time constraints. Its solution produced the total expected net value of $6,705 and gave mixed strategies at level 4 of line 1, levels 1 and 3 of line 2, and level 1 of line 3, as is shown in Table 1. At each of these levels, the manager should randomly select one of its strategies in proportion to the values of their decision variables. For example, if the performance of line 2 at a decision point is at level 3, Table 1 indicates strategies (z=1, d=6) and (z=1, d=7) are available to him. Therefore, he should select one of them by a random method in the following proportions:

$$\text{Strategy } (z=1,\ d=6):\quad \frac{X_{23}^{16}}{X_{23}^{16} + X_{23}^{17}} = \frac{.015}{.028} = .536$$

Table 1. Optimum Strategies and Amounts of Time Required for Management Control at Various Performance Levels.

Production Line	Performance Level	Optimum Strategy		Decision Variable		Required No. of Days Per Strategy		Expected Amounts of Time Required Per Day	
		Control Action	Days of Operation	Transformed Variable*	Original Variable*	Information Processing	Control Action	Information Processing*	Control Action
a	i	z	d	x_{ai}^{zd}	$X_{ai}^{zd} = L_1 x_{ai}^{zd}$	r_{ai}	s_{ai}^{z}	$r_{ai} x_{ai}^{zd}$	$s_{ai}^{z} x_{ai}^{zd}$
			days			days	days	day	day
1	1	1	8	.002	.019	1	1	.002	.002
	2	1	8	.012	.110	1	1	.012	.01[illegible]
	3	1	7	.027	.263	1	1	.027	.027
	4	1 1	6 7	.013 .050	.129 .479	1 1	1 1	.013 .050	.0[illegible] .0[illegible]
	Sum		$\frac{1}{L_1}$ =	.104	1.000			.104	.104
2	1	0 1	6 7	.006 .005	.062 .051	1 1	0 2	.006 .005	.000 .0[illegible]
	2	1	7	.023	.225	1	2	.023	.0[illegible]
	3	1 1	6 7	.015 .013	.145 .133	2 2	2 2	.0[illegible] .027	.02[illegible] .027
	4	2	5	.039	.384	2	3	.077	.1[illegible]
	Sum		$\frac{1}{L_2}$ =	.101	1.000			.167	.2[illegible]
3	1	0 0 0	4 5 6	.060 .015 ε	.377 .094 ε	1 1 1	0 0 0	.060 .015 ε	.0[illegible] .0[illegible] .0[illegible]
	2	2	4	.040	.250	2	2	.080	.080
	3	2	3	.023	.143	2	2	.046	.046
	4	2	1	.022	.136	3	2	.065	.04[illegible]
	Sum		$\frac{1}{L_3}$ =	.160	1.000			.256	.16[illegible]

*: ε represents an insignificant positive value.

Strategy (r=1, d=7): $$\frac{x_{23}^{17}}{x_{23}^{16} + x_{23}^{17}} = \frac{.013}{.028} = .464$$

The last two columns of Table 1 list the expected amounts of time required by the strategies selected at various levels of each production line. The sums of these values are listed in Table 2, giving the expected amounts of time required per day for processing information and executing control actions for each production line. These values show that the proportions of the total time spent on production lines 1, 2, and 3 for processing information are 19.4%, 31.1%, and 49.5%, respectively. Those proportions for executing control actions are 20.8%, 45.4%, and 33.8%. The total time required for processing information is .537 day against the time restriction of H_r = 0.6 day, whereas the total time required for executing control actions is .5 day, consuming the entire amount of the available time, or H_s=.5 day. Thus, the time available for control actions is the primary constraint on this management control.

Table 2. Expected Amounts of Time Required for Management Control of Production Lines.

Management Control Tasks	Expected Amounts of Time Required Per Day				
	Production Line	Line 1	Line 2	Line 3	Total
Processing Information	Expected Fraction of Day	.104	.167	.266	.537
	Proportion of Total Time	.194	.311	.495	1.000
Executing Control Actions	Expected Fraction of Day	.104	.227	.169	.500
	Proportion of Total Time	.208	.454	.338	1.000

SUMMARY

This paper [illegible] a management control problem in which the manager controls activities whose performances deteriorate with time. The control is carried out in a sequence of two control tasks: the processing of information regarding the condition of each activity and the execution of a proper control action if it is needed. The processing of information regarding the current condition creates a time lag between the collection of this information and the selection of a control action. The evaluated current condition and the selected control action together determine the subsequent time for collecting information. Consequently, information on each activity will be collected most probably at irregular intervals. Processing information does not interrupt the activity's operation, but executing a control action does. Thus the control action incurs both the explicit cost of its execution and the implicit cost of values not produced by the activity during the execution.

The assumed natural performance deterioration of each activity and the improvement of the deteriorated performance are represented by discrete state Markov chains. The problem is formulated as a linear programming problem for sequential decisions along the line proposed by Manne [7] so as to maximize an objective function representing the expected values produced per period by the activities, net of various costs of the control tasks. The basic formulation is then given additional constraints regarding the amounts of time available for processing information and executing control actions.

Solutions to the linear program give optimum stationary strategies which determine for each activity the corrective action to execute and a time interval to elapse before collecting new information. Without the time constraints, the solutions determine for each activity the expected ideal amounts of time necessary for the control tasks; With those constraints, they determine optimum allocations of the available amounts of time to individual activities.

Although in the present formulation the time constraints H_r and H_s in (11)-(12) or (18)-(19) are imposed against $r_{\underline{ai}}$ and $s^z_{\underline{ai}}$ representing the entire lengths of the "information processing" and "control action" phases, they may be imposed against certain segments of these phases. For example, consider a case where the information processing phase includes $r^c_{\underline{ai}}$ hours of computation by a computer. If the average computer time available to the entire group is H_r hours per day, the constraint in this case is obtained by replacing $r_{\underline{ai}}$ in the numerator on the lefthand of (11) or that in (18) with $r^c_{\underline{ai}}$. In such a case, unlike the entire length of the phase $r_{\underline{ai}}$, the computing time $r^c_{\underline{ai}}$ or the time of any other element in this phase need not be an integral multiple of the unit period of the Markov process. This also applies to the control-action phase.

With the exclusion of the time constraints (18)-(19), the formulation (15)-(17) is composed of the independent linear programs for individual activities. When those constraints are added to the formulation, they unite the independent programs into an integral linear program having a decomposable form. Therefore, if the new linear program has a large number of

constraints, it may be reformulated using the decomposition principle so as to reduce its computing time.[1]

[1]Discussions on the decomposition principle are found in G. B. Danzig, Linear Programming & Extensions, p. 448-470, or G. Hadley, Linear Programming, p. 400-411.

REFERENCES

1. Bonini, C. P., R. K. Jaedicke, and H. M. Wagner, eds., Management Controls: New Directions in Basic Research, New York, McGraw-Hill, 1964.

2. Charnes, A., and A. Stedry, "Exploratory Models in the Theory of Budget Control," in W. W. Cooper, H. J. Leavitt, and M. W. Shelly II, eds., New Perspectives in Organization Research, New York, Wiley, 1964, pp. 212-249.

3. Derman, C., "On Sequential Decisions and Markov Chains," Management Science, Vol. 9, No. 1, October 1962, pp. 16-24.

4. Feller, W., An Introduction to Probability Theory and its Applications, Vol. I, New York, Wiley, 1957.

5. Kemeny, J. G., and J. L. Snell, Finite Markov Chains, New York, Van Nostrand, 1960.

6. Klein, M., "Inspection-Maintenance-Replacement Schedules under Markovian Deterioration," Management Science, Vol. 9, No. 1, October 1962, pp. 25-32.

7. Manne, A. S., "Linear Programming and Sequential Decisions," Management Science, Vol. 6, No. 1, April 1960, pp. 259-267.

8. Marschak, J., "Towards an Economic Theory of Organization and Information," Chap. XIV in Thrall, Davis, and Coombs, eds., Decision Processes, New York, Wiley, 1954.

9. ______________, "Elements for a Theory of Teams," Management Science, Vol. 1, No. 2, Jan. 1955, pp. 127-37.

10. Marschak, T., "Economic Theories of Organization," in J. G. March, ed., Handbook of Organizations, New York, Wiley, 1965, pp. 423-50.

11. Wagner, H., "On the Optimality of Pure Strategies," Management Science, Vol. 6, No. 1, April 1960, pp. 268-269.