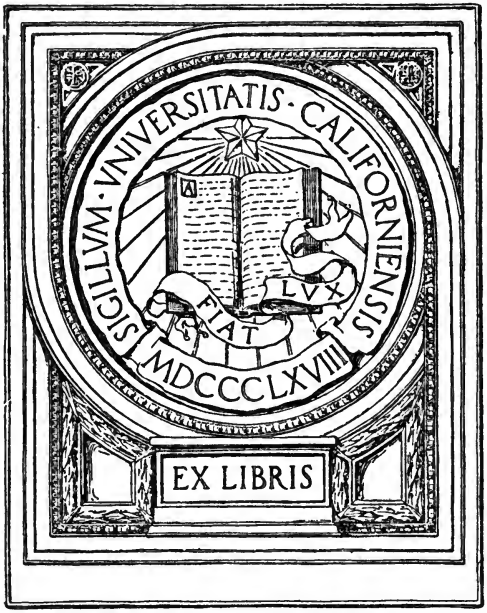

Florian Cajori

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ECLECTIC EDUCATIONAL SERIES.

ORAL LESSONS

IN

NUMBER

A MANUAL FOR TEACHERS

BY

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VAN ANTWERP, BRAGG & CO.
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PREFACE.

The great importance of the oral instruction in number given during the first three years of school, demands that it be wisely and carefully arranged. It should be not only the best possible for pupils at this age, but it should be the best possible preparation for the instruction which is to follow.

No one who has ever attempted to map out such a series of lessons in number, will claim that this work can be satisfactorily done by the overburdened teachers in our primary schools. The attempt to do this has been made on a wide scale, and the results have not been satisfactory.

It is not difficult for primary teachers to turn the crank of such a number-teaching machine as the so-called Grube method may be made, and especially when abstract numbers constitute the daily grist. It is also easy to drill pupils for weeks in counting to 100 by ones, by twos, etc., but it seems unnecessary to add that this monotonous drill is not the oral instruction in number which should be given the first years of school.

It is believed that primary teachers will welcome the guidance of a skillfully arranged series of oral lessons in number—lessons beginning at the first step and extending through the entire oral course. The careful study of such illustrative lessons will enable them, as a class, to give much more systematic and effective oral instruction in number than is possible in the absence of such needed assistance.

The oral lessons in number presented in this manual are the result of an earnest effort to supply this need. *They are illustrative lessons for the guidance of teachers.* The exercises are given in detail, to indicate clearly the nature and scope of the instruction which should be given each year.

These illustrative lessons are based on the fundamental principle in teaching, that all primary ideas and processes must be made clear and familiar before any successful advance can be made. They also recognize the important fact that children acquire primary ideas and processes *very slowly.* A failure to observe this principle and this fact is one of the most common errors in teaching the elements of arithmetic.

This manual also contains numerous blackboard and slate exercises to accompany the oral lessons; a concise statement of the principles involved in the first lessons in number; and suggestive methods of teaching elementary processes, definitions, and rules. It is designed to be a manual of elementary instruction in number.

CINCINNATI, O., *Nov.* 20, 1884.

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PRINCIPLES AND EXPLANATIONS.

THE instruction in number which should be given during the first two years of a school course, is primarily objective, and can best be given orally. When this objective oral instruction is completed, pupils are prepared to use an elementary arithmetic with advantage, and hence the oral instruction of the third year should introduce and accompany the lessons in the book used by the pupils. Nothing is gained by continuing *exclusive* oral instruction in number beyond the second school year.

The putting of an elementary arithmetic into the hands of pupils the third year, not only increases their interest, and otherwise promotes their progress in number, but it greatly relieves the teacher of unnecessary labor—not an unimportant consideration.

The use of a book the third year, and subsequently, is also a physical advantage to pupils. At this time in the school course, the lessons in language and other written exercises, many of which involve the copying of exercises from the blackboard, are a severe tax on the eyes and nerves of young children. It is feared that children are sometimes injured by the large amount of copying from the blackboard, and other slate work, required of them during the first three or four years of school.

First Year.

The aim of the lessons in number mapped out for the first year, is to teach objectively the numbers *from one to ten inclusive*—the digital or primary numbers.

The successive exercises include numbering, combining, separating, and taking away groups of objects, no group exceeding ten, and the making of the digital figures.

The exercises in numbering are intended to develop the power to recognize at sight, *without counting*, the number of objects in any group not exceeding ten—a power essential to the easy mastery of the other exercises. It is claimed by primary teachers of wide experience that the majority of children, when they first enter school, can not give *at sight* the number of objects in a group exceeding three. A few weeks of drill will, however, enable them to number instantly any group not exceeding ten. Dr. Thomas Hill, ex-President of Harvard College, says that fairly bright children will readily learn to number at sight as many as twelve to fifteen objects—a statement based on actual experiment. This may be done by an unconscious separation of the larger groups into two smaller groups, and the combining of these; but, howsoever done, the act is practically instantaneous.

The perceptive power necessary to number a group of objects from one to ten at sight, is not only fundamental in teaching objectively the combining and separating of numbers, but it is also of great value in practical life. Nearly every person has daily use for the power to recognize at sight the number of objects in small groups.

The purely objective exercises, stated above, should be followed by exercises in combining, subtracting, and

separating groups of objects not in sight, *but easily imagined.* This step should be taken only when pupils have acquired the skill to combine and separate groups of objects in sight. The true order is first the numbering, combining, and separating of groups of objects *in sight;* and, second, the combining and separating of groups of objects *not in sight.*

This step may be followed by the adding, subtracting, and separating of the corresponding abstract numbers, but so strong is the tendency of teachers to use abstract numbers to the neglect of needed objective exercises, that it is believed to be best *to exclude abstract numbers entirely from the first year's course.* There is no danger that the use of abstract numbers will be omitted or neglected in the succeeding years.

This tendency of teachers to use abstract numbers in primary lessons in arithmetic is largely due to the fact that it is easier to drill pupils on words and other symbols than it is to teach them real knowledge—a fact sadly illustrated in the memoriter, word, and figure drills which have so long characterized school instruction.

It may be a question whether children should be taught the figures and the making of them in connection with these first year lessons. This doubtless depends on the age of the children taught. If children are admitted to school as early as five years of age, the teaching of figures may be wisely deferred until the second year, or, at least, until the latter part of the first year. When pupils enter school at the age of six years and upwards, the figures may be taught at the close of the different series of lessons, but even then it may be well to defer such instruction *for a few weeks.*

The skill acquired in making figures the first year will promote the progress of the pupils the second year, and there seems to be little danger that the teaching of the figures in this manner will lead pupils into the

error of confounding figures with the numbers which they represent—an error common among pupils who, from the first, use figures as actual numbers.

The first step in teaching a number is to develop an idea of the number itself, and this can only be done by objects. A number is neither a word nor a figure, and hence it can not be taught by teaching its name or the figure or figures that express it. A child may learn the *names* of the numbers not only from one to ten, but from one to one hundred, and not have, as a result, a clear idea of a single number named. It is a great error to infer from the fact that the idea of number may be intuitive, that an idea of the different primary numbers need not be developed and taught. The idea of space is intuitive, but it does not follow that such particular space concepts as the line, the triangle, the square, the circle, the ellipse, the cube, the sphere, etc., are innate. Experience shows that a clear idea of these primary space or form concepts is slowly acquired by children, and that they need to be carefully taught. For a like reason, the primary numbers should be carefully taught, and this can best be done *by numbering, combining, and separating groups of objects.*

Special attention is called, in this connection, to the importance of avoiding in these primary processes the too common practice of *counting by ones.* The numbering, combining, and separating of groups of objects by counting, leads to the pernicious habit of adding and subtracting numbers by counting—a habit that must be overcome before a pupil can learn to add or subtract numbers as wholes. When a child can number a group of three objects at sight, he should be taught a group of four objects, as three and one, or one more than three, and not simply as four ones. It is not only unnecessary to number four objects by counting one, two, three, four, but this counting is likely to give the child

the erroneous idea that the *first* object is one, the *second* two, the *third* three, the *fourth* four. The child must see the entire group as four objects, and when he has learned that four objects are three objects and one object, or two objects and two objects, he has a clear idea of the number four.

The same is true of combining and separating groups of objects. The child has not learned to add 3 balls and 4 balls, for example, until he sees 7 balls the instant 3 balls and 4 balls are presented to the mind. The easy and quick perception of the sum of any two groups of objects, present or imagined, each not exceeding ten, is the first step in the art of adding and subtracting numbers.

It is admitted that first-year pupils can be taken over more ground than is covered by these primary lessons, but it is believed that nothing, in the end, will be gained by adding instruction that can be better given the second and third years. A good maxim in primary teaching is, "*Make haste slowly.*"

Second Year.

The several series of exercises that constitute the second year's course of oral instruction in number, involve and use the ideas and skill acquired by pupils the first year, and hence there is less occasion for the use of visible objects. In teaching the numbers from eleven to twenty inclusive, present and imagined objects are successively used; and the processes of adding, subtracting, and separating numbers are each introduced by objects in sight or easily imagined. The order observed is as follows:

1. The combining, taking away, and separating of groups of objects in sight or easily imagined.

2. The adding, subtracting, and analyzing of concrete numbers.

3. The adding, subtracting, and analyzing of abstract numbers.

The twofold aim of all the exercises in the first and second years is to impart a clear idea of the numbers from one to twenty, and to give the pupil the power to add and subtract the primary or digital numbers without counting. The entire series of exercises recognizes the fact that the power to perceive the sum or difference of any two digital numbers is the basis of the art of rapid and accurate computation. If this power be acquired the first two years of school, the time devoted to the teaching of number will be wisely and profitably employed.

It is at this point that the system of instruction in number, presented in this manual, differs most widely from the so-called Grube method, which, from the first, unites the four processes of addition, subtraction, multiplication, and division, as is shown in the following scheme for teaching the number seven to first-year pupils:

$6+1,\ 1+6,\ 7-1,\ 7-6;\ 5+2,\ 2+5,\ 7-2,\ 7-5;$ $4+3,\ 3+4,\ 7-3,\ 7-4;$ **II** $7\times1,\ 7\div1,\ \frac{1}{7}$ of $7;\ 2\times 3+1,\ 7\div2,\ \frac{1}{2}$ of $7;\ 3\times2+1,\ 7\div3,\ \frac{1}{3}$ of $7;\ 4\times1+3,$ $7\div4,\ \frac{1}{4}$ of $7;\ 5\times1+2,\ 7\div5,\ \frac{1}{5}$ of $7;\ 6\times1+1,\ 7\div 6,\ \frac{1}{6}$ of 7.

It is seen that the exercises which precede the vertical lines (**II**) involve addition and subtraction, and that those which follow these lines involve multiplication and division. Is there any such immediate and necessary connection between the concepts and processes of addition and subtraction and those of multiplication and division as requires the teaching of these four processes together? The concepts and processes of addi-

tion and subtraction relate to numbers as composed of *parts*, and, being inverse processes, should be taught together. The concepts and processes of multiplication and division relate to numbers as composed of *factors*, and, being inverse processes, should likewise be taught together. But there is nothing in the relation of these two sets of inverse processes *to each other* that necessitates or justifies the teaching of them *from the first* as correlates. On the contrary, there are strong reasons against the mixing up of these two sets of relations in the child's first lessons in number.

When the concepts and processes of addition and subtraction are familiar to pupils, those of multiplication and division are easily acquired. A knowledge of the former assists in acquiring the latter. Addition, for example, assists the pupil in determining the product of two digits, and the more familiar the pupil is with the process of addition, the more easily will he learn multiplication. On the contrary, multiplication can render a child little, if any, assistance in learning the sum of two digits. In the order of acquisition, the processes of multiplication and division follow those of addition and subtraction, and there is nothing gained by alternating these two sets of inverse processes in the *first* lessons in number.

It is admitted that these four processes can be taught simultaneously to children five years of age, and even without using objects. Primary teachers have accomplished even more difficult things, as the history of primary instruction sadly attests. Young children have been taught to spell orally many hundreds of words, most of which expressed no idea whatever to the speller. Many a child has committed the multiplication table before he could add 7 and 7, and hosts of children have learned to repeat pages of the text in their books without clearly comprehending a sentence

repeated. The question is not whether young children can do these things. The more vital question is, "Is this training the best possible for young children?"

A child might possibly be taught to walk by being put through daily a drill which would call into play, in succession, all the muscles in his legs, and give to each every possible variety of movement. Such a method might even claim to be "scientific," but nature's method of teaching a child to walk is to induce it to take one step, then two, and so on, *in walking*, and the process can not be hurried, strength and skill in walking being acquired *slowly*.

The primary and fundamental processes in number are ADDITION and SUBTRACTION, and the natural way to teach a child to add and subtract numbers is to give him exercises *involving these processes*. Exercises in multiplying and dividing numbers can render no assistance in these first lessons; and, if they could, such assistance is not needed, since the processes of addition and subtraction are easily taught without it.

It may be true that a child's knowledge of a given number is not perfect until he has viewed it in all possible relations to other numbers. A child's "grasp" of the size of numbers, exceeding say fifteen, may not be perfect until he has compared them with the primary or digital numbers, both with reference to their difference and to their quotient, but it does not follow that both of these comparisons should be made in the first lessons in number. It may be wisely taken for granted that the third and subsequent years of arithmetical instruction will do something to widen the pupil's grasp of numbers. It is seriously questioned whether a little child's grasp of the number 7 would be much broadened by the series of exercises which follow the vertical lines in the scheme above presented.

Third Year.

The preceding exercises in adding groups of objects and numbers and in separating groups of objects and numbers into parts, have prepared the way for the easy mastery of the primary processes of multiplication and division.

It seems important here to note that the adding of equal numbers is not multiplication, and that the separating of a number into equal parts is not arithmetical division. The fact that the *processes* of addition and multiplication are not the same, may be clearly shown by adding say twenty-four 25's and by multiplying 25 by 24. While the results reached are numerially the same, the two processes are obviously not the same. The distinction between dividing a number into equal parts and arithmetical division may be shown by separating, say, 15 blocks into 5 equal parts, and by dividing 15 cents by 3 cents. In the first process, there is neither a numerical divisor nor a numerical quotient; in the second process, 5 cents is the divisor, or measure, and 3 is the quotient. The first process divides 15 blocks into *parts*, which is division only in the primary meaning of the term; the second process divides 15 cents by a *factor* (5 cents) and finds the other factor (3), and this is arithmetical division.

It is thus seen that the processes of adding and separating numbers, previously taught, deal with numbers as composed of *parts*, but the processes of multiplication and division now to be taught deal with numbers as composed of *factors*. In multiplication two numbers are given, and it is required to find their product; in division two numbers are given, and it is required to find how many times one of these numbers is contained in the other;—that is, a product and one of its factors are given, and it is required to find the other

factor. These two processes are fundamental in arithmetical computations, and their thorough mastery is of prime importance.

The final end to be attained in the teaching of multiplication, is to impart to the pupil the ability to perceive the product of any two digital numbers, *without adding*, as soon as they are presented to the mind. The perception of these products must be as direct and as instantaneous as the recognition of the most familiar words in reading. When this ability is acquired, the pupil is prepared to multiply numbers expressed by two or more figures.

How should the multiplication of the primary or digital numbers be taught?

The old method, which required pupils to commit to memory a table of all the products of the digital numbers, two by two, is not satisfactory. Many children have great difficulty in memorizing and retaining so many products; and, when memory fails them, their only recourse is a reference to the printed multiplication table—a great hindrance and a great inconvenience. It is believed that full three fourths of the pupils who mechanically memorize the multiplication table, are never able to multiply large numbers without more or less dependence on the printed table.

A better method is to teach pupils how to determine the product of any two digits *by adding* before requiring them to commit the product to memory. When pupils see that the product of 4 and 5, for example, is the same numerically as the sum of four 5's or five 4's, they not only have a clear idea of the value of this product, but they have a key to it, on which they can depend if memory should ever fail to recall it. Besides, this method of learning the products assists in fixing them in memory, and the labor involved in memorizing the multiplication table is greatly reduced.

It should, however, be kept in mind that the finding of the numbers corresponding to these products by addition is an *introductory step*, and should be discontinued as soon as its object is attained. The next and essential step is to teach the products of the digital numbers *as products*, and as such to fix them in the memory. Memory drills should be continued until any two of the digital numbers are as clearly and as directly associated with their product as with their sum.

Division should be taught as the inverse of multiplication. The fact that 3 times 4 or 4 times 3 is 12, involves the facts that 4 is in 12 three times, and 3 in 12 four times; and when these facts are taught in connection with each other, the pupil sees the latter in the former, and is thus relieved of the necessity of committing the division results to memory.

There is nothing gained by the attempt to teach division as a method of subtraction. It is not a method of subtraction. It is true that the quotient shows how many times the divisor may be subtracted from the dividend, but this quotient is not found *by subtraction.* This may be clearly shown by dividing 125 by 25, and by subtracting 25 from 125 as many times as possible. The dissimilarity of the two processes is too obvious to require a formal comparison of them, and it is also as obvious that the results are not the same. The final result of the several subtractions of 25 from 125 is 0, and it is only *by counting* (or inspection) that the number of subtractions is determined. The quotient 5, obtained by division, is the number of times 25 is contained in 125, and, *as a consequence*, it shows that 25 can be taken from 125 five times. This consequence is an interesting fact, but it does not make division a method of subtraction. The fact that there are but two fundamental principles in

number, synthesis, and analysis does not show that there are but two fundamental *processes*. In practice there are two synthetic processes,—addition and multiplication; and two analytic processes,—subtraction and division. Addition and subtraction synthesize and analyze numbers *as parts*, and are inverse processes. Multiplication and division synthesize and analyze numbers *as factors*, and are likewise inverse processes.

FIRST-YEAR COURSE.

Aim.

To teach objectively the numbers from one to ten inclusive.

Steps.

1. The numbering of the objects in any group not exceeding ten, *without counting.*
2. The combining of any two groups whose sum does not exceed ten, *without counting.*
3. The taking from any group not exceeding ten each of the two smaller groups combined to form it.
4. The separating of any group not exceeding ten into the two smaller groups that compose it, and then taking successively each of the two smaller groups, thus found, from the original group.
5. The combining, separating, and subtracting of groups of objects not in sight but easily imagined,—groups not exceeding ten.
6. The comparing of two groups of objects, in sight or imagined, to see how much one group is greater or less than the other.
7. The applying of the processes learned to the solution of easy problems involving a simple exercise of the imagination and the judgment.
8. The teaching of the figures that express the digital numbers.

NOTE.—In teaching the successive numbers, it is not important that these steps be taken uniformly in the order above given. The second and third steps may properly be combined in one step as inverse processes. The first process in the fourth step may first be given separately, and then the two processes may be combined in one, as given.

Appliances.

In order to teach objectively the following lessons, the teacher should be supplied with a variety of objects of convenient sizes and of different shapes and colors, such as counters, small rods, blocks, shells, pebbles, buttons, beans, etc.

The objects used by the teacher in class instruction should be large enough to be easily seen by all the pupils, as blocks, shells, large buttons, pebbles, and rods of about the size of pencils or common pen-holders. The pupils can use at their seats such small objects as match-sticks, tooth-picks, small buttons, flat beans, etc.

The numeral frame can also be used by the teacher, but the balls are usually too small for use before large classes. A better aid would be two wires, with ten balls on each, the balls being from one to two inches in diameter, and the wires being stretched parallel and near each other, and in such a position that the teacher can readily move the balls with a pointer or with the hand. One wire with ten balls can be used the first year, but there are objections to the exclusive use of objects of one kind, and especially to the presenting of objects uniformly in lines or rows. The exclusive use of ten balls on a single wire would give many young pupils the idea that numbered objects stand *in rows*, and combine and separate *in straight lines*. The groups of objects numbered, combined, and separated by first-year pupils should be presented in all sorts of shapes—with the irregularity of groups in nature.

Much interest may be imparted to these objective drills by using, in their season, flowers and fruits of different kinds. Beautiful roses, apple-blossoms, pansies, dandelions, etc., are easily gathered or secured; also, apples, plums, peaches, cherries, apricots, etc. A little pains will secure the needed variety of objects for class instruction.

The objects used in class instruction (balls and other round objects excepted), should rest on a surface inclined enough to show all the objects well, and, at the same time, so nearly level that the objects used will not slide or roll. For this purpose, a large slate or a light board of convenient size may be placed on a table, or the table itself may be inclined a little by slipping blocks under two of the legs.

LESSON I.

The Numbers 1 and 2.

NOTE.—Nearly all children know the numbers *one* and *two* when they enter school, and hence but little time need be spent in teaching these numbers.

1. How many hands do I hold up? (Raise right hand.) How many hands do I now hold up? (Raise both hands.)

Hold up your right hand. Hold up your left hand. Hold up both hands. How many hands do you now hold up? "Two hands."

How many books do I hold up? "One book." How many now? "Two books." Hold up one thumb. Hold up two thumbs.

How many eyes have you? How many ears? How many chins? How many cheeks? How many tongues? How many feet?

2. Mary may bring me one book, and Kate may bring me one. How many books have I now?

How many books are one book and one book?

George may put one block on the table, and Charles may put one block with George's. How many blocks are on the table?

Frank may take away one of the blocks. How many blocks are left on the table?

How many blocks are one block and one block? Two blocks less one block?

How many boys are one boy and one boy? Two boys less one boy?

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3. Here are two books, and I now separate them into one book and one book. (Suit action to word.)

Two blocks are one block and —— block.

Two blocks less one block are —— block.

NOTE.—The teaching of the figures in this and succeeding lessons may be postponed several weeks, and then be taught in connection with a review of these lessons. See page 9.

4. This is the word that stands for one: *One.*

This is the figure that stands for one: *1.*

This is the word that stands for two: *Two.*

This is the figure that stands for two: *2.*

Make the figure 1 on your slate.

Make the figure 1 on your slate twice.

Make the figure 2 on your slate.

Make the figure 2 on your slate twice.

NOTE.—Devote an entire lesson to the teaching of these figures, and give the pupils practice in making them both on the board and on the slate. If the slates are not properly ruled, see that this is done, and have all figures made neatly.

LESSON II.

The Number 3.

1. How many hands do I hold up? "Two hands." How many fingers? "Two fingers." How many fingers do I now hold up? "Three fingers."

Hold up two fingers on your left hand. Hold up three fingers on your right hand.

NOTE.—The fingers are convenient objects for use in teaching the digital numbers, but they should not be used exclusively. See Lesson IV, note.

Mary may put two blocks on the table, and Kate may put one block with Mary's. How many blocks are now on the table?

Jane may take away one block. How many blocks are now on the table?

How many fingers do I hold up? "Three fingers." How many now? "Two fingers." How many books? "Two books." How many books now? "Three books."

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2. How many balls are two balls and one ball? One ball and two balls are how many balls?

How many acorns are two acorns and one acorn?

How many books are one book and one book? Two books and one book? One book and two books?

John may put two buttons and one button on the table. How many buttons are on the table?

Charles may take away two of the buttons. How many buttons are left on the table?

How many buttons are three buttons less two buttons?

How many blocks are one block and two blocks? Three blocks less one block?

How many pencils are three pencils less two pencils? Three pencils less one pencil?

How many chairs are three chairs less two chairs? Three chairs less one chair? Two chairs less one chair?

---

3. Three books are two books and —— book, or one book and —— books.

NOTE.—Separate the three books into two groups, and let the pupils see that three books are two books and one book, or one book and two books.

John may put three blocks on the table and divide them into two groups. How many blocks in each group?

Three blocks are two blocks and —— block, or one block and —— blocks.

Three blocks less two blocks are —— block; three blocks less one block are —— blocks.

Three books are one book, one book, and —— book.

Kate may put three blocks on the table and divide them into three parts. How many blocks in each part? "One block."

---

4. How many pigs are two pigs and one pig? Three pigs less one pig?

How many cents are one cent and two cents? Three cents less one cent? Two cents less one cent?

John may hold up three fingers, and James two fingers. How many more fingers does John hold up than James?

Jane may put three buttons and two pebbles on the table. How many more buttons than pebbles on the table?

Susan bought three sticks of candy and ate one stick. How many sticks of candy had she left?

Charles picked three pears and gave two of them to his mother. How many pears had he left?

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5. This is the word that stands for the number three: *Three.*

This is the figure that stands for three: *3.*

Make the figure 3 on your slate three times.

Make the figures 1, 2, 3 on your slate.

LESSON III.

The Number 4.

NOTE.—The teacher will need to multiply the exercises in this and the following lessons. No step should be left until the pupils can take it easily. It will often be necessary to devote several class exercises to one of the subdivisions of a lesson. The teacher should also vary the language used, making it less formal and more conversational and lively.

1. How many fingers do I hold up on my right hand? How many on my left hand? How many fingers on both hands?

Hold up three fingers. Hold up four fingers.

How many books do I hold up? "Three books." How many now? "Four books."

How many pencils in my right hand? "Four pencils." How many in my left hand? "Two pencils."

Mary may put three blocks on the table, and Kate may put one block with Mary's. How many blocks are on the table?

John may put two shells on the table, and Charles may put two shells with John's. How many shells are on the table?

How many pencils in my right hand? "Four pencils." How many pencils in my left hand? "Three pencils."

How many balls do I move on the upper wire? "Three balls." How many on the lower wire? "Four balls."

---

2. How many blocks are three blocks and one block? One block and three blocks? How many blocks are two blocks and two blocks?

Mary may put three shells on the table, and Jane may put one shell with Mary's. How many shells are on the table?

How many shells are three shells and one shell?

Charles may hold up two fingers on his right hand and two fingers on his left hand. How many fingers does Charles hold up?

How many fingers are two fingers and two fingers?

---

3. Jane may put two blocks and two blocks on the table. How many blocks on the table?

Kate may take away two of the blocks. How many blocks are left on the table?

How many blocks are four blocks less two blocks?

Susan may take three pencils in her left hand and one in her right hand. How many pencils has Susan?

Susan may give one pencil to Mary. How many pencils has Susan left?

How many pencils are four pencils less one pencil? Four pencils less three pencils?

How many birds are three birds and one bird? Four birds less one bird? One bird and three birds? Four birds less three birds? Two birds and two birds? Four birds less two birds?

Willie may hold up four fingers, and George two fingers. How many more fingers does George hold up than Willie?

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4. Let us divide four shells into two groups. How many shells in this group? "Three shells." How many shells in this group? "One shell."

Four shells are three shells and —— shell, or one shell and —— shells.

Let us now divide four shells into two equal groups. How many shells in each group? "Two shells."

Four shells are two shells and —— shells.

Kate may put four buttons on the table and divide them into two groups in two ways.

NOTE.—The two divisions will give these results:

Four buttons are { 3 buttons and 1 button, or 1 button and 3 buttons.
2 buttons and 2 buttons.

John may divide four pebbles into two groups in two ways.

Four birds are three birds and —— bird, or one bird and —— birds.

Four birds less one bird are —— birds; four birds less three birds are —— bird.

Four birds are two birds and —— birds; four birds less two birds are —— birds.

5. John has four marbles in his right hand and three marbles in his left hand. How many marbles in his right hand more than in his left?

Kate found four eggs in one nest and two eggs in another. How many more eggs did she find in the first nest than in the second?

Charles picked four peaches and gave two of them to his sister. How many peaches had Charles left?

Mary picked three roses from a bush and one rose from another bush. How many roses did she pick?

How many cents are three cents and one cent? Two cents and two cents?

How many cents are four cents less three cents? Four cents less one cent? Four cents less two cents?

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6. This is the word that stands for four: *Four.*
This is the figure that stands for four: *4.*
Make the figure 4 on your slate four times.
Make the figures 1, 2, 3, 4 on your slate.

NOTE.—See page 62, note.

LESSON IV.

The Number 5.

1. How many fingers do I hold up on my right hand? How many on my left hand? How many on both hands?

NOTE.—In giving these and similar finger exercises, put down the hands, and, as the question is asked, raise them (one or both, as the case may be) with the requisite number of fingers open, thus presenting the group of fingers to the eye *as a whole.* The number of fingers presented should be given *instantly* by the pupils.

Hold up four fingers. How many fingers more will make five fingers? Hold up five fingers.

George may put four blocks on the table. How many blocks must be put with the four blocks to make five blocks?

How many balls do I move on this wire? "Five balls." How many balls do I move on this wire? "Four balls."

Jane may put five shells on the table; John four blocks; and George three buttons.

How many more shells than blocks on the table? How many more shells than buttons?

NOTE.—Write a number of familiar words on the board—words composed of three, four, and five letters—and pointing to different words, ask: How many letters in this word? How many in this? etc. The reading chart may also be used, and the drill should be continued until the pupils can give the number of letters in each word *without counting.*

---

2. How many balls are four balls and one ball? One ball and four balls?

How many blocks are three blocks and two blocks? Two blocks and three blocks?

Kate may put three shells on the table, and Jane may put two shells with Kate's. How many shells are on the table?

How many shells are three shells and two shells?

George may put four pebbles on the table, and Charles may put enough more to make five pebbles.

How many pebbles are four pebbles and one pebble? One pebble and four pebbles?

How many cents are three cents and two cents? Two cents and three cents?

3. Mary may hold up four fingers on her right hand and one finger on her left hand. How many fingers does Mary hold up?

Mary may put down the one finger on her left hand. How many fingers does she now hold up?

How many fingers are five fingers less one finger? Five fingers less four fingers?

John may put three blocks and two blocks on the table. How many blocks are on the table?

Clarence may take away two of the blocks. How many blocks are left on the table?

How many blocks are five blocks less two blocks? Five blocks less three blocks?

How many chairs are four chairs and one chair? Five chairs less one chair? One chair and four chairs? Five chairs less four chairs?

How many desks are three desks and two desks? Two desks and three desks? Five desks less two desks? Five desks less three desks?

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4. Five acorns are four acorns and —— acorn, or one acorn and —— acorns.

Five acorns are three acorns and —— acorns, or two acorns and —— acorns.

NOTE.—Separate five objects of different kinds into two groups, in each of the two ways indicated above. The two divisions will give these results:

Five objects are: { 4 objects and 1 object, or 1 object and 4 objects. / 3 objects and 2 objects, or 2 objects and 3 objects.

John may divide five buttons into two groups in two ways.

NOTE.—After the first division, John should say, "Five buttons are four buttons and one button, or one button and four buttons." He should then combine the two groups and divide again, and

say, "Five buttons are three buttons and two buttons, or two buttons and three buttons."

Lucy may divide five shells into two groups in two ways.

Five desks are four desks and —— desk, or one desk and —— desks.

Five desks less one desk are —— desks; five desks less four desks are —— desk.

Five girls are three girls and —— girls, or two girls and —— girls.

Five girls less two girls are —— girls; five girls less three girls are —— girls.

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5. Kate may hold up five fingers, and Jane may hold up three fingers. How many fingers does Kate hold up more than Jane?

Harry may put five shells and four pebbles on the table. How many more shells than pebbles on the table?

How many pencils in my right hand? "Five pencils." How many pencils in my left hand? "Three pencils." How many more pencils in my right hand than in my left?

NOTE.—Continue the comparison of unequal groups by using books, pebbles, shells, etc.

How many lemons are two lemons and three lemons? Two lemons and two lemons? One lemon and four lemons? Three lemons and two lemons? Two lemons and one lemon?

How many oranges are five oranges less one orange? Four oranges less two oranges? Five oranges less two oranges? Four oranges less three oranges? Five oranges less three oranges?

6. This is the word that stands for five: *Five.*
This is the figure that stands for five: *5.*
Make the figure 5 on your slate five times.
Make the figures 1, 2, 3, 4, 5 on your slate.

LESSON V.

The Number 6.

1. How many fingers do I hold up? "Five fingers." How many now? "Six fingers." Five fingers and one finger are six fingers.

Hold up five fingers. How many more fingers must you hold up to make six fingers? Hold up six fingers.

Albert may put five blocks on the table, and Edward may put one block with Albert's. How many blocks are now on the table?

Clara may put four buttons on the table, and Agnes may put two buttons with Clara's. How many buttons are on the table?

How many balls do I move on this wire? "Six balls." How many on this wire? "Five balls."

How many pencils in my right hand? "Four pencils." How many in my left hand? "Six pencils."

How many books do I hold up? "Six books."

How many letters in the word "Susan"? In the word "Charles"?

NOTE.—If there is any hesitancy in numbering the different groups of objects, continue the drill, using various objects, and presenting promiscuously groups of three, four, five and six. Groups of the figures 1, 2, 3, 4, and 5, respectively, may be written on the board. Pointing to the different groups, ask: How many ones? How many threes? etc.

2. How many balls are five balls and one ball? One ball and five balls?

How many balls are four balls and two balls? Two balls and four balls?

How many balls are three balls and three balls?

Willie may put four blocks on the table, and Alice may put two blocks with Willie's. How many blocks are on the table?

How many blocks are four blocks and two blocks? Two blocks and four blocks?

George may hold up three fingers on his right hand, and three fingers on his left hand. How many fingers does George hold up?

Hold up two fingers and four fingers. How many fingers do you hold up?

How many desks are five desks and one desk? One desk and five desks?

How many desks are four desks and two desks? Two desks and four desks? Three desks and three desks?

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3. Charles may put four shells and two shells on the table. How many shells are on the table?

Clara may take away two of the shells. How many shells are left on the table?

How many shells are six shells less two shells? Six shells less four shells?

Hold up six fingers, three on each hand; put down three fingers. How many fingers are left?

How many fingers are six fingers less three fingers?

How many oranges are six oranges less one orange? Six oranges less five oranges? Six oranges less two oranges? Six oranges less four oranges? Six oranges less three oranges?

How many tops are five tops and one top? Six tops less one top? One top and five tops? Six tops less five tops?

How many tops are four tops and two tops? Six tops less two tops? Two tops and four tops? Six tops less four tops?

How many tops are three tops and three tops? Six tops less three tops?

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4. Here are six shells; let us see in how many ways we can separate them into two groups:

Six shells are five shells and —— shell, or one shell and —— shells.

Six shells are four shells and —— shells, or two shells and —— shells.

Six shells are three shells and —— shells.

NOTE.—Take six shells and divide them successively into two groups as above indicated, and have the pupils say: "Six shells are five shells and one shell, or one shell and five shells," etc.

Mary may take six spools and separate them into two groups; Kate, six shells and separate them into two other groups; and Agnes, six buttons and separate them into two other groups.

John may take six blocks and divide them into two groups in three ways.

NOTE.—The three divisions will give these results:

Six blocks are: { 5 blocks and 1 block, or 1 block and 5 blocks.
4 blocks and 2 blocks, or 2 blocks and 4 blocks.
3 blocks and 3 blocks.

Kate may divide six shells into two equal groups. How many shells in each group? How many three shells in six shells?

Willie may divide six pencils into three equal groups. How many pencils in each group? How many two pencils in six pencils?

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5. Six apples are five apples and —— apple, or one apple and —— apples.

Six apples less one apple are —— apples; six apples less five apples are —— apple.

Six lemons are four lemons and —— lemons, or two lemons and —— lemons.

Six lemons less two lemons are —— lemons; six lemons less four lemons are —— lemons.

Six lemons are three lemons and —— lemons; six lemons less three lemons are —— lemons.

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6. How many men are:

Two men and two men?	Four men less two men?
Three men and two men?	Five men less two men?
Four men and two men?	Six men less two men?
One man and three men?	Four men less three men?
Two men and three men?	Five men less three men?
Three men and three men?	Six men less three men?
One man and four men?	Five men less four men?
Two men and four men?	Six men less four men?
One man and five men?	Six men less five men?

NOTE.—Let these exercises be first recited across the page; as, "Two men and two men are four men; four men less two men are two men." Then have the exercises in addition and subtraction recited separately. Also give the exercises in a miscellaneous manner.

7. Maud may hold up six fingers, and Mary may hold up four fingers. How many fingers does Maud hold up more than Mary?

There are six boys and four girls in a class. How many more boys than girls in the class?

John found six eggs in a nest, and Henry found five eggs in another nest. How many more eggs did John find than Henry?

I have six pencils in my right hand and three pencils in my left hand. How many more pencils in my right hand than in my left?

Clara found four eggs in one nest and two eggs in another nest. How many eggs did she find?

A boy bought six peaches and gave three of them to his sister. How many peaches had he left?

A man paid four dollars for a hat and two dollars for a cap. How many dollars did he pay for both?

Harry earned six cents and then gave two cents for a pencil. How many cents had he left?

Willie is six years old, and Fanny is four years old. How many years older than Fanny is Willie?

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8. This is the word that stands for six: *Six.*
This is the figure that stands for six: *6.*
Make the figure 6 on your slate six times.
Make the figures 1, 2, 3, 4, 5, 6 on your slate.

LESSON VI.

The Number 7.

1. How many fingers do I hold up on my right hand? "Four fingers." How many on my left hand? "Three fingers." How many on both hands?

Hold up six fingers. How many fingers more will make seven fingers? Hold up seven fingers.

Jane may put six shells on the table; Kate may put one shell with Jane's. How many shells are now on the table?

How many shells do I put on the table? "Four shells." How many now? "Seven shells."

NOTE.—The teacher should put down four shells, six shells, seven shells, five shells, etc., and continue until the pupils can give the number of shells *instantly*. Various other objects may be used. The pupils may also be required to give the number of panes of glass in the window, books on the desk, desks in a row, etc.

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2. How many tops are six tops and one top? Five tops and two tops?

How many tops are four tops and three tops?

Susan may put four shells on the table, and Mary may put three shells near Susan's. How many shells are on the table? (Sliding the groups together.)

How many shells are four shells and three shells?

James may hold up five counters and Willie may hold up two counters. How many counters do both hold up?

How many counters are five counters and two counters?

How many chairs are six chairs and one chair? Five chairs and two chairs? Four chairs and three chairs?

How many pencils are three pencils and four pencils? Two pencils and five pencils? One pencil and six pencils?

How many buttons do I put on the table? "Five buttons." How many more? "Two buttons." How many buttons are now on the table?

Harry may take up two buttons. How many buttons are left on the table?

How many buttons are seven buttons less two buttons?

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3. Jane may put four spools and three spools on the table. How many spools are on the table?

Mary may take away three of the spools. How many spools are left on the table?

How many spools are seven spools less three spools?

Herbert may put seven marbles in my hand, and Merrill may take four of them. How many marbles are left in my hand?

How many marbles are seven marbles less four marbles?

How many balls are seven balls less five balls? Seven balls less two balls?

How many shells are six shells less four shells? Five shells less three shells? Seven shells less three shells?

How many boys are six boys and one boy? Seven boys less one boy? One boy and six boys? Seven boys less six boys?

How many boys are five boys and two boys? Seven boys less two boys? Two boys and five boys? Seven boys less five boys?

How many girls are four girls and three girls? Seven girls less three girls? Three girls and four girls? Seven girls less four girls?

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4. Seven rings are six rings and —— ring, or one ring and —— rings.

Seven rings are five rings and —— rings, or two rings and —— rings.

Seven rings are four rings and —— rings, or three rings and —— rings.

NOTE.—Take the seven objects and divide them successively into the two groups above indicated, and have the pupils give the number of objects in each of the two groups.

John may take seven blocks and divide them into two groups; George, seven buttons and divide them into two other groups; and Willie, seven shells and divide them into two other groups. Each boy may tell us the result of his division.

Albert may take seven shells and divide them into two groups in as many ways as he can.

NOTE.—These divisions will give the following results:

Seven shells are: { 6 shells and 1 shell, or 1 shell and 6 shells. / 5 shells and 2 shells, or 2 shells and 5 shells. / 4 shells and 3 shells, or 3 shells and 4 shells.

Susan may take seven buttons and divide them into two groups in three ways.

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5. Seven chairs are six chairs and —— chair, or one chair and —— chairs.

Seven chairs less one chair are —— chairs; seven chairs less six chairs are —— chair.

Seven chairs are five chairs and —— chairs, or two chairs and —— chairs.

Seven chairs less two chairs are —— chairs; seven chairs less five chairs are —— chairs.

Seven chairs are four chairs and —— chairs, or three chairs and —— chairs.

Seven chairs less three chairs are —— chairs; seven chairs less four chairs are —— chairs.

Seven cents are five cents and —— cents; seven cents less five cents are —— cents.

6. How many lines are:

Two lines and two lines?	Four lines less two lines?
Three lines and two lines?	Five lines less two lines?
Four lines and two lines?	Six lines less two lines?
Five lines and two lines?	Seven lines less two lines?
One line and three lines?	Four lines less three lines?
Two lines and three lines?	Five lines less three lines?
Three lines and three lines?	Six lines less three lines?
Four lines and three lines?	Seven lines less three lines?

How many dots are:

One dot and four dots?	Five dots less four dots?
Two dots and four dots?	Six dots less four dots?
Three dots and four dots?	Seven dots less four dots?
One dot and five dots?	Six dots less five dots?
Two dots and five dots?	Seven dots less five dots?

NOTE.—For directions, see Lesson V, page 35, note.

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7. Willie may hold up seven fingers, and Harry five fingers. How many more fingers does Willie hold up than Harry?

James may put seven pebbles on the table, and George four pebbles. How many pebbles in James's group more than in George's? George may put enough pebbles in his group to make it equal to James's.

I put seven books in one pile and five books in another pile. How many books must I add to the second pile to make it equal in number to the first pile?

I give Kate six cents and Mary four cents. How many more cents have I given to Kate than to Mary?

How many cents in seven cents more than in five cents? In seven cents more than in four cents?

8. Samuel earned seven cents, and paid five cents for a book. How many cents had he left?

Sarah picked four roses from one bush and three roses from another bush. How many roses did she pick? If she should give three roses to her sister, how many would she have left?

A farmer has five horses in one field and two horses in another field. How many horses in both fields?

Jane is seven years old, and her sister is three years old. How many years older than her sister is Jane?

---

9. This is the word that stands for seven: *Seven.*
This is the figure that stands for seven: 7.
Make the figure 7 on your slate seven times.
Make the figures 1, 2, 3, 4, 5, 6, 7 on your slate.

LESSON VII.

The Number 8.

1. How many fingers do I hold up? "Seven fingers. How many fingers do I now hold up? "One finger." Seven fingers and one finger are eight fingers.

Hold up seven fingers. How many fingers more will make eight fingers? Hold up eight fingers.

Frank may put seven blocks on the table, and Willie may put one block with Frank's. How many blocks are now on the table?

How many books do I hold up? "Eight books." How many now? (Taking away one book.)

How many fingers do I hold up? "Six fingers." How many now? "Eight fingers." How many now? "Seven fingers."

How many pencils do I hold up? "Six pencils." How many now? "Seven pencils." How many now? "Eight pencils."

NOTE.—If there is any hesitancy in numbering the different groups of objects, continue the drill, using various objects, and presenting promiscuously groups of four, five, six, seven, and and eight. Write groups of the figures 1, 2, 3, 4, etc., on the board, and, pointing to the different groups, ask: How many twos in this group? How many fours in this? etc.

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2. How many balls are seven balls and one ball? Six balls and two balls? Two balls and six balls?

NOTE.—Present the groups of balls separately, and then slide them together as the question is asked.

George may put six shells on the table, and Charles may put two shells with George's. How many shells are on the table?

How many shells are six shells and two shells? Two shells and six shells?

Here are five balls, and here are three balls. How many balls do I slide together?

Jane may hand me five shells, and Lucy may hand me three shells. How many shells have I?

How many shells are five shells and three shells? Three shells and five shells?

How many fingers do I hold up on my right hand? "Four fingers." How many on my left hand? "Four fingers." How many on both hands?

Clara may hold up four fingers, and Helen may hold up four fingers. How many fingers do both hold up?

How many fingers are four fingers and four fingers?

How many birds are seven birds and one bird? Six birds and two birds? Five birds and three birds? Four birds and four birds?

How many boys are one boy and seven boys? Two boys and six boys? Three boys and five boys? Four boys and four boys?

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3. How many blocks do I put on the table? "Six blocks." How many blocks do I now put near them? "Two blocks." How many blocks are on the table?

I now take away the two blocks. How many blocks are left on the table?

Eight blocks less two blocks are how many blocks?

I again put the two blocks with the six blocks. How many blocks are on the table? I now take away six blocks. How many blocks are left on the table?

Eight blocks less six blocks are how many blocks?

Susan may put eight shells on the table, and Alice may take away three of the shells. How many shells are left on the table?

How many shells are eight shells less three shells? Eight shells less five shells?

Harry may take eight shells, and hand four of them to Thomas. How many shells has Harry left?

How many shells are eight shells less four shells?

How many plums are seven plums and one plum? Eight plums less one plum?

How many pears are six pears and two pears? Eight pears less two pears?

How many peaches are five peaches and three peaches? Eight peaches less three peaches?

How many oranges are four oranges and four oranges? Eight oranges less four oranges?

How many cents are five cents and three cents? Eight cents less three cents?

How many cents are four cents and four cents? Eight cents less four cents?

4. Let us take eight blocks and see in how many ways we can divide them into two groups:

(1) Eight blocks are seven blocks and —— block, or one block and —— blocks.

(2) Eight blocks are six blocks and —— blocks, or two blocks and —— blocks.

(3) Eight blocks are five blocks and —— blocks, or three blocks and —— blocks.

(4) Eight blocks are four blocks and —— blocks.

Here are eight pretty acorn cups. Mary may divide them into two groups in four ways, and after each division she may tell me the result:

NOTE.—The four divisions will give these results:

Eight cups are:
- 7 cups and 1 cup, or 1 cup and 7 cups.
- 6 cups and 2 cups, or 2 cups and 6 cups.
- 5 cups and 3 cups, or 3 cups and 5 cups.
- 4 cups and 4 cups.

I now put on the table eight blocks, eight shells, eight pebbles, and eight buttons. Harry may divide the blocks into two groups; Charles, the shells into two other groups; George, the pebbles into two other groups; and Frank, the buttons into two other groups. Each boy may now tell me the result of his division.

Agnes may take eight shells and divide them into two groups in as many ways as she can, telling me the result after each division.

Kate may take eight pencils and divide them into two *equal* groups. How many pencils in each group?

How many four pencils make eight pencils?

Jane may take eight shells and divide them into four equal groups. How many shells in each group?

How many two shells make eight shells?

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5. Eight birds are seven birds and —— bird, or one bird and —— birds.

Eight birds less one bird are —— birds; eight birds less seven birds are —— bird.

Eight men are six men and —— men, or two men and —— men.

Eight men less two men are —— men; eight men less six men are —— men.

Eight boys are five boys and —— boys, or three boys and —— boys.

Eight boys less three boys are —— boys; eight boys less five boys are —— boys.

Eight horses are four horses and —— horses.

Eight horses less four horses are —— horses.

---

6. Kate may hold up six fingers, and Lucy eight fingers. How many more fingers does Lucy hold up than Kate?

I have seven pencils in my right hand and four pencils in my left hand. How many more pencils in my right hand than in my left?

Kate picked eight roses and five tulips. How many more roses than tulips did she pick?

Clara has eight cents, and Ruth three cents. How many cents has Clara more than Ruth?

How many cents must I give to Ruth that she may have as many as Clara?

How many cents must be added to six cents to make eight cents?

There are eight sheep in one field, and five sheep in another field. How many more sheep in the first field than in the second?

Mary found six eggs in one nest and eight eggs in another nest. How many eggs in the first nest less than in the second? How many eggs in the second nest more than in the first?

7. There are four desks in one row, and four desks in another row. How many desks in both rows?

Clarence bought eight peaches and gave four of them to his sister. How many peaches had he left?

There are four boys and three girls in a class. How many pupils in the class?

John gave three cents for a pencil and five cents for paper. How many cents did he pay for pencil and paper?

There are two men plowing in one field, and five men in another field. How many men in both fields?

There were eight birds in a tree, and five of them flew away. How many birds were left in the tree?

Mary wrote eight words on her slate and then rubbed out three words. How many words were left?

Susan picked eight tulips and gave two of them to Kate. How many tulips had Susan left?

---

8. This is the word that stands for eight: *Eight.*
This is the figure that stands for eight: *8.*
Make the figure 8 on your slate eight times.
Make the figures 1, 2, 3, 4, 5, 6, 7, 8 on your slate.

REVIEW EXERCISES.

How many cents are:

Two cents and two cents?	Four cents less two cents?
Three cents and two cents?	Five cents less two cents?
Four cents and two cents?	Six cents less two cents?
Five cents and two cents?	Seven cents less two cents?
Six cents and two cents?	Eight cents less two cents?
Two cents and three cents?	Five cents less three cents?
Three cents and three cents?	Six cents less three cents?
Four cents and three cents?	Seven cents less three cents?
Five cents and three cents?	Eight cents less three cents?

How many figs are:

One fig and four figs?	Five figs less four figs?
Two figs and four figs?	Six figs less four figs?
Three figs and four figs?	Seven figs less four figs?
Four figs and four figs?	Eight figs less four figs?
One fig and five figs?	Six figs less five figs?
Two figs and five figs?	Seven figs less five figs?
Three figs and five figs?	Eight figs less five figs?
Two figs and six figs?	Eight figs less six figs?

LESSON VIII.

The Number 9.

1. How many balls do I move on this wire? "Eight balls." How many do I now move towards the eight balls? "One ball."

Eight balls and one ball (sliding them together) are nine balls. How many balls do I now move? "Nine balls."

How many books have I put on the table? "Nine books." I take away one book: how many books are left?

How many books must I add to eight books to make nine books?

Edward may put eight blocks on the table, and Charles may put one block with Edward's. How many blocks are on the table?

Jane may put nine shells on the table; Susan, nine rings; and Alice, nine counters.

I will write several groups of the figure 5 on the board. How many 5's in this group? "Seven 5's." How many 5's in this? "Nine 5's." How many in this? "Six 5's." How many in this? "Eight 5's."

NOTE.—Write groups of other figures on the board, and continue the drill until they are numbered without hesitation.

2. How many balls are eight balls and one ball? Seven balls and two balls? Six balls are three balls? Five balls and four balls?

NOTE.—First present the groups separately, and then slide them together as the question is asked.

Jane may put seven shells on the table, and Susan may put two shells near Jane's. How many shells on the table?

Seven shells and two shells are how many shells?

NOTE.—Slide the groups together as the question is asked.

Kate may put six rings on the table, and Mary may put three rings near Kate's. How many rings are on the table?

Six rings and three rings are how many rings?

Samuel may put five pebbles in my right hand and four pebbles in my left hand. How many pebbles in both of my hands?

How many pebbles are five pebbles and four pebbles?

How many chairs are eight chairs and one chair? Seven chairs and two chairs? Six chairs and three chairs? Five chairs and four chairs?

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3. Willie may put three pebbles on the table; Susan, three shells; and Charles, three cents. How many objects on the table?

Willie may take away his three pebbles. How many objects are left?

Susan may next take away her three shells. What is now left on the table?

How many balls have I moved on the wire? "Nine balls." How many balls do I take away? "Two balls." How many balls are left?

How many balls are nine balls less two balls?

Harry may put nine shells on the table, and Charles may take away three shells. How many shells are left?

Nine shells less three shells are how many shells?

I give George nine buttons, and he may give four buttons to James. How many buttons has George left?

Nine buttons less four buttons are how many buttons?

How many pens are eight pens and one pen? Nine pens less one pen? One pen and eight pens? Nine pens less eight pens?

How many apples are seven apples and two apples? Nine apples less two apples? Two apples and seven apples? Nine apples less seven apples?

How many lemons are six lemons and three lemons? Nine lemons less three lemons? Three lemons and six lemons? Nine lemons less six lemons?

How many figs are five figs and four figs? Nine figs less four figs? Four figs and five figs? Nine figs less five figs?

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4. I will take nine pencils, and see in how many ways I can divide them into two groups:

(1) Nine pencils are eight pencils and —— pencil, or one pencil and —— pencils.

(2) Nine pencils are seven pencils and —— pencils, or two pencils and —— pencils.

(3) Nine pencils are six pencils and —— pencils, or three pencils and —— pencils.

(4) Nine pencils are five pencils and —— pencils, or four pencils and —— pencils.

I now put on the table nine blocks, nine shells, nine pebbles, and nine buttons. Albert may divide the blocks into two groups; Edward, the shells into two

other groups; William, the pebbles into two other groups; and Calvin, the buttons into two other groups. Each boy may tell me the result.

Here are nine pretty acorn cups. Clara may divide them into two groups in four ways, and after each division she may tell me the result.

NOTE.—The four divisions will give these results:

Nine cups are:
- 8 cups and 1 cup, or 1 cup and 8 cups.
- 7 cups and 2 cups, or 2 cups and 7 cups.
- 6 cups and 3 cups, or 3 cups and 6 cups.
- 5 cups and 4 cups, or 4 cups and 5 cups.

Harry may take nine shells and divide them into two groups in as many ways as he can.

Charles may take nine blocks and divide them into three *equal* groups. How many blocks in each group?

How many three blocks in nine blocks?

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5. Nine plums are eight plums and —— plum, or one plum and —— plums.

Nine plums less one plum are —— plums; nine plums less eight plums are —— plum.

Nine pears are seven pears and —— pears, or two pears and —— pears.

Nine pears less two pears are —— pears; nine pears less seven pears are —— pears.

Nine peaches are six peaches and —— peaches, or three peaches and —— peaches.

Nine peaches less three peaches are —— peaches; nine peaches less six peaches are —— peaches.

Nine figs are five figs and —— figs, or four figs and —— figs.

Nine figs less four figs are —— figs; nine figs less five figs are —— figs.

6. I have nine pencils in my right hand and five pencils in my left hand. How many pencils in my right hand more than in my left?

Jane may put nine shells on the table, and Kate six shells. How many shells in Jane's group more than in Kate's? How many shells must I take from Kate's to make the two groups equal? How many shells must I add to Jane's to make the two groups equal?

I give John eight pencils and Harry five pencils. How many pencils has John more than Harry? How many pencils must I give to Harry to make his pencils equal John's in number?

Jane has nine cents, and Susan seven cents. How many cents has Jane more than Susan?

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7. Charles gave eight cents for raisins and five cents for an orange. How many more cents did he pay for the raisins than for the orange?

Orvil caught eight fishes and sold three of them. How many fishes had he left?

There are five trees in one row, and four trees in another row. How many trees in both rows?

Clarence has six marbles in his right hand and two marbles in his left hand. How many marbles in both of his hands?

George is eight years old, and Hiram is six years old. How many years older is George than Hiram?

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8. This is the word that stands for nine: *Nine.*

This is the figure that stands for nine: *9.*

Make the figure 9 on your slate nine times.

Make the figures 1, 2, 3, 4, 5, 6, 7, 8, 9 on your slate.

REVIEW EXERCISES.

How many cents are:

Four cents and two cents?	Six cents less two cents?
Five cents and two cents?	Seven cents less two cents?
Six cents and two cents?	Eight cents less two cents?
Seven cents and two cents?	Nine cents less two cents?
Three cents and three cents?	Six cents less three cents?
Four cents and three cents?	Seven cents less three cents?
Five cents and three cents?	Eight cents less three cents?
Six cents and three cents?	Nine cents less three cents?
Two pens and four pens?	Six pens less four pens?
Three pens and four pens?	Seven pens less four pens?
Four pens and four pens?	Eight pens less four pens?
Five pens and four pens?	Nine pens less four pens?
One pen and five pens?	Six pens less five pens?
Two pens and five pens?	Seven pens less five pens?
Three pens and five pens?	Eight pens less five pens?
Four pens and five pens?	Nine pens less five pens?
One pear and six pears?	Seven pears less six pears?
Two pears and six pears?	Eight pears less six pears?
Three pears and six pears?	Nine pears less six pears?
Two pears and seven pears?	Nine pears less seven pears?
Four pears and five pears?	Nine pears less five pears?
Six pears and three pears?	Nine pears less three pears?

LESSON IX.

The Number 10.

1. How many balls do I move on the wire? "Nine balls." How many balls do I now move? "One ball." Nine balls and one ball are ten balls.

How many balls on this wire? "Ten balls." How many balls on this wire? "Ten balls."

How many fingers do I hold up? "Eight fingers." How many thumbs? "Two thumbs." How many fingers and thumbs? "Ten fingers and thumbs."

Hold up ten fingers and thumbs.

Here is a pair of gloves. How many fingers and thumbs on one glove? How many on the two gloves?

How many nails on your right hand? "Five nails." How many on your left hand? How many on both hands?

Mary may put nine shells on the table, and Jane may put one shell with Mary's. How many shells on the table?

How many pencils must I put with nine pencils to make ten pencils?

NOTE.—Take different objects and drill the pupils until they can number a group of ten or less *instantly.*

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2. How many balls are nine balls and one ball? Eight balls and two balls? Seven balls and three balls? Six balls and four balls? Five balls and five balls?

NOTE.—Present groups of balls as indicated in the questions.

How many pencils are six pencils and two pencils? Six pencils and four pencils?

Charles may put eight blocks on the table. How many blocks must I put with them to make ten blocks?

How many blocks are eight blocks and two blocks?

Jonas may hand me seven pencils, and Henry may hand me three pencils. How many pencils have I now?

How many pencils are seven pencils and three pencils?

Here are six beautiful roses. How many roses must I put with them to make ten roses?

How many roses are six roses and four roses?

How many toes on your right foot? How many toes on your left foot? How many toes on both of your feet?

How many toes are five toes and five toes?

How many oranges are eight oranges and two oranges? Six oranges and four oranges? Seven oranges and three oranges? Five oranges and five oranges?

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3. How many balls on this wire? "Ten balls." How many balls do I move away? "Two balls." How many balls are left?

NOTE.—Continue this drill, taking from ten balls, three balls, four balls, etc.

Willie may put ten shells on the table, and John may take away three shells. How many shells are left on the table?

How many shells are ten shells less three shells?

Clara may put six shells on the table, and Martha may put four shells with Clara's. How many shells are on the table?

Martha may take away her four shells. How many shells are left on the table?

How many shells are ten shells less four shells?

Jane may hand Harry five pencils, and Kate may hand him five. How many pencils has Harry?

Harry may give Jane five of his pencils. How many pencils has Harry left?

How many apples are ten apples less one apple? Ten apples less two apples? Ten apples less three apples? Ten apples less four apples? Ten apples less five apples?

How many roses are nine roses and one rose? Ten roses less one rose? One rose and nine roses? Ten roses less nine roses?

How many roses are eight roses and two roses? Ten roses less two roses? Two roses and eight roses? Ten roses less eight roses?

How many chickens are seven chickens and three chickens? Ten chickens less three chickens? Three chickens and seven chickens? Ten chickens less seven chickens?

How many lambs are six lambs and four lambs? Ten lambs less four lambs? Four lambs and six lambs? Ten lambs less six lambs?

How many men are five men and five men? Ten men less five men?

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4. Here are ten red cherries, and let us see in how many ways we can divide them into two groups:

(1) Ten cherries are nine cherries and —— cherry, or one cherry and —— cherries.

(2) Ten cherries are eight cherries and —— cherries, or two cherries and —— cherries.

(3) Ten cherries are seven cherries and —— cherries, or three cherries and —— cherries.

(4) Ten cherries are six cherries and —— cherries, or four cherries and —— cherries.

(5) Ten cherries are five cherries and —— cherries.

Here are ten blocks, ten shells, ten pebbles, ten buttons, and ten cherries. Susan may divide the blocks into two groups; Kate, the shells into two other

groups; Lucy, the pebbles in two other groups; Helen, the buttons into two other groups; and Alice, the cherries into two other groups. Each girl may now tell us the result of her division.

James may take ten shells and divide them into two groups in five ways, telling us after each division the result.

NOTE.—The five divisions will give these results:

Ten shells are:
- 9 shells and 1 shell, or 1 shell and 9 shells.
- 8 shells and 2 shells, or 2 shells and 8 shells.
- 7 shells and 3 shells, or 3 shells and 7 shells.
- 6 shells and 4 shells, or 4 shells and 6 shells.
- 5 shells and 5 shells.

Clarence may take ten cents and divide them into two groups in as many ways as he can.

Frank may take ten blocks and divide them into two *equal* groups. How many blocks in each group?

How many five blocks make ten blocks?

Lucy may take ten shells and divide them into five equal groups. How many shells in each group?

How many two shells make ten shells?

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5. How many peaches are:

Ten peaches are nine peaches and —— peach, or one peach and —— peaches.

Ten peaches less one peach are —— peaches; ten peaches less nine peaches are —— peach.

Ten peaches are eight peaches and —— peaches, or two peaches and —— peaches.

Ten peaches less two peaches are —— peaches; ten peaches less eight peaches are —— peaches.

Ten pears are seven pears and —— pears, or three pears and —— pears.

Ten pears less three pears are —— pears; ten pears less seven pears are —— pears.

Ten pears are six pears and —— pears, or four pears and —— pears.

Ten pears less four pears are —— pears; ten pears less six pears are —— pears.

Ten oranges are five oranges and —— oranges; ten oranges less five oranges are —— oranges.

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6. There are nine birds on a tree, and seven birds on the ground. How many birds on the tree more than on the ground?

I have ten cents in my right hand and six cents in my left hand. How many cents in my right hand more than in my left?

How many pencils must be taken from ten pencils to leave seven pencils? To leave five pencils?

Harry gave ten cents for a book and eight cents for a slate. How many cents did the book cost more than the slate?

There are ten sheep in one field, and seven sheep in another field. How many sheep in the first field more than in the second?

There are ten girls and six boys in a class. How many more girls than boys in the class?

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7. There are five desks in one row, and five desks in another row. How many desks in both rows?

Jane found six eggs in one nest and four eggs in another nest. How many eggs did she find?

A boy gave ten cents for a slate and eight cents for a book. How many more cents did he pay for the slate than for the book?

A man paid three dollars for a hat and seven dollars for a pair of boots. How many dollars did he pay for both?

How many more dollars did the boots cost than the hat?

Henry earned ten cents, and paid six cents for two pencils. How many cents had he left?

There were nine cars in one train, and seven cars in another train. How many more cars in the first train than in the second?

---

8. This is the word that stands for ten: *Ten.*
These are the figures that stand for ten: *10.*
Make 10 on your slate ten times.
Make 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 on your slate.

NOTE.—Teach the name and value of the figure 0, and show the position of the 1 and 0 in 10.

REVIEW EXERCISES.

How many hands are:

Five hands and two hands?
Seven hands less two hands?
Six hands and two hands?
Eight hands less two hands?
Seven hands and two hands?
Nine hands less two hands?
Eight hands and two hands?
Ten hands less two hands?
Four hands and three hands?
Seven hands less three hands?

Five hands and three hands?
Eight hands less three hands?
Six hands and three hands?
Nine hands less three hands?
Seven hands and three hands?
Ten hands less three hands?

How many cents are:

Three cents and four cents?
Seven cents less four cents?
Four cents and four cents?
Eight cents less four cents?
Five cents and four cents?
Nine cents less four cents?
Six cents and four cents?
Ten cents less four cents?
Two cents and five cents?
Seven cents less five cents?
Three cents and five cents?
Eight cents less five cents?
Four cents and five cents?
Nine cents less five cents?
Five cents and five cents?
Ten cents less five cents?

How many birds are:

Two birds and six birds?
Eight birds less six birds?
Three birds and six birds?
Nine birds less six birds?
Four birds and six birds?
Ten birds less six birds?
One bird and seven birds?
Eight birds less seven birds?
Two birds and seven birds?
Nine birds less seven birds?
Three birds and seven birds?
Ten birds less seven birds?

LESSON X.

Separate each number from 1 to 10 inclusive into any two smaller numbers that compose it, and then subtract each of the two smaller numbers thus found from the original number, as follows:

2 cents are 1 cent and 1 cent ∴ 2 cents less 1 cent are 1 cent.

3 cents are 2 cents and 1 cent, or 1 cent and 2 cents ∴ 3 cents less 2 cents are 1 cent; 3 cents less 1 cent are 2 cents.

4 cents are 3 cents and 1 cent, or 1 cent and 3 cents ∴ 4 cents less 3 cents are 1 cent; 4 cents less 1 cent are 3 cents.

4 cents are 2 cents and 2 cents ∴ 4 cents less 2 cents are 2 cents.

5 cents are 4 cents and 1 cent, or 1 cent and four cents ∴ 5 cents less 4 cents are 1 cent; 5 cents less 1 cent are 4 cents.

5 cents are 3 cents and 2 cents, or 2 cents and 3 cents ∴ 5 cents less 3 cents are 2 cents; 5 cents less 2 cents are 3 cents.

6 cents are 5 cents and 1 cent, or 1 cent and 5 cents ∴ 6 cents less 5 cents are 1 cent; 6 cents less 1 cent are 5 cents.

6 cents are 4 cents and 2 cents, or 2 cents and 4 cents ∴ 6 cents less 4 cents are 2 cents; 6 cents less 2 cents are 4 cents.

6 cents are 3 cents and 3 cents ∴ 6 cents less 3 cents are 3 cents.

7 cents are 6 cents and 1 cent, or 1 cent and 6 cents ∴ 7 cents less 6 cents are 1 cent; 7 cents less 1 cent are 6 cents.

7 cents are 5 cents and 2 cents, or 2 cents and 5 cents ∴ 7 cents less 5 cents are 2 cents; 7 cents less 2 cents are 5 cents.

7 cents are 4 cents and 3 cents, or 3 cents and 4 cents ∴ 7 cents less 4 cents are 3 cents; 7 cents less 3 cents are 4 cents.

8 cents are 7 cents and 1 cent, or 1 cent and 7 cents ∴ 8 cents less 7 cents are 1 cent; 8 cents less 1 cent are 7 cents.

8 cents are 6 cents and 2 cents, or 2 cents and 6 cents ∴ 8 cents less 6 cents are 2 cents; 8 cents less 2 cents are 6 cents.

8 cents are 5 cents and 3 cents, or 3 cents and 5 cents ∴ 8 cents less 5 cents are 3 cents; 8 cents less 3 cents are 5 cents.

8 cents are 4 cents and 4 cents ∴ 8 cents less 4 cents are 4 cents.

9 cents are 8 cents and 1 cent, or 1 cent and 8 cents ∴ 9 cents less 8 cents are 1 cent; 9 cents less 1 cent are 8 cents.

9 cents are 7 cents and 2 cents, or 2 cents and 7 cents ∴ 9 cents less 7 cents are 2 cents; 9 cents less 2 cents are 7 cents.

9 cents are 6 cents and 3 cents, or 3 cents and 6 cents ∴ 9 cents less 6 cents are 3 cents; 9 cents less 3 cents are 6 cents.

9 cents are 5 cents and 4 cents, or 4 cents and 5 cents ∴ 9 cents less 5 cents are 4 cents; 9 cents less 4 cents are 5 cents.

10 cents are 9 cents and 1 cent, or 1 cent and 9 cents ∴ 10 cents less 9 cents are 1 cent; 10 cents less 1 cent are 9 cents.

10 cents are 8 cents and 2 cents, or 2 cents and 8 cents ∴ 10 cents less 8 cents are 2 cents; 10 cents less 2 cents are 8 cents.

10 cents are 7 cents and 3 cents, or 3 cents and 7 cents ∴ 10 cents less 7 cents are 3 cents; 10 cents less 3 cents are 7 cents.

10 cents are 6 cents and 4 cents, or 4 cents and 6 cents ∴ 10 cents less 6 cents are 4 cents; 10 cents less 4 cents are 6 cents.

10 cents are 5 cents and 5 cents ∴ 10 cents less 5 cents are 5 cents.

NOTE.—In some schools it may be deemed best to give the pupils slate exercises the latter part of the first year. Board and slate exercises may be easily provided, and by mentally adding the words balls, blocks, pears, etc., after the figures, the numbers in these exercises may be made concrete. The following board and slate exercises will serve as illustrations:

1. *Add*	1	7	2	6	3	5	4		*a*	*b*	*c*	*d*	*e*
	7	1	6	2	5	3	4		2	2	2	2	2
									2	2	3	1	1
From	8	8	8	8	8	8	8		2	2	2	3	2
Take	1	7	2	6	3	5	4	*Add*	2	1	1	2	3
2. *Add*	1	8	2	7	3	6	4	5	*a*	*b*	*c*	*d*	*e*
	8	1	7	2	6	3	5	4	3	2	2	3	2
									2	2	3	1	2
From	9	9	9	9	9	9	9	9	2	2	2	3	2
Take	1	8	2	7	3	6	4	5	*Add*	2	3	1	2	1

N. B.—When pupils are clearly able to advance beyond the foregoing lessons the first year, one or more of the lessons in the second-year course may be taught, but no attempt should be made to go beyond Lesson V. The first-year course should be *thoroughly mastered.*

SECOND-YEAR COURSE.

Aims.

1. *To teach the numbers from eleven to twenty inclusive, and their representation by figures.*

2. *To teach the adding, subtracting, and analyzing of numbers, the amounts, minuends, and numbers analyzed not exceeding twenty.*

Steps.

The steps taken to attain the second aim are as follows:

1. The adding of any two digital numbers without counting, and the subtracting of each from their sum.

2. The separating of each number, not exceeding eighteen, into any two digital numbers that compose it, and the subtracting of each number thus found from the original number.

3. The separating of any number, not exceeding twenty, that is composed of equal numbers greater than 1, into all the equal numbers that compose it.

4. The applying of the processes learned to the solution of practical problems, involving a simple exercise of imagination and judgment.

5. Blackboard and slate exercises in addition and subtraction, amounts and minuends not exceeding twenty.

LESSON I.

The Numbers 11 to 15.

NOTE.—The object of this lesson is to develop the idea of each number from eleven to fifteen inclusive, to teach its name, and its representation by figures. The teacher should be supplied with a numeral frame (or, in its absence, with twenty balls on two wires, ten on each), ten pencils or rods tied together in a bunch, and ten loose pencils or rods, and also a blackboard or large slate. The objects referred to in each question should be presented, and the action should otherwise be suited to the word.

1. How many balls on this wire? "Ten balls." How many balls do I now move below the ten balls? "One ball." Ten balls and one ball are *eleven* balls.

If I take one ball from the eleven balls, how many balls will be left?

How many pencils are ten pencils and one pencil? Eleven pencils less one pencil?

How many blocks are ten blocks and one block? Eleven blocks less one block?

I will make ten lines on the board. How many lines more will make eleven lines?

NOTE.—Make ten lines on the board, and then add one line at the right or below.

If I rub out one line, how many lines will be left?

Here are the figures that stand for eleven: *11.*

Make eleven lines on your slate, and write below them the figures that stand for eleven.

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2. Here are ten balls, and I now move two balls beneath them. Ten balls and two balls are *twelve* balls.

If I take two balls from twelve balls, how many balls will be left?

How many pencils are ten pencils and two pencils? Twelve pencils less two pencils?

How many blocks are ten blocks and two blocks? Ten lines and two lines?

How many are twelve blocks less two blocks? Twelve lines less two lines?

Here are the figures that stand for twelve: *12.*

Make twelve lines on your slate, and write the figures that stand for twelve below them.

|||||||||| ||
12

NOTE.—Show that the figure 1 stands for one ten (ten balls, ten pencils, etc.), and the figure 2 for the two added.

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3. I now move three balls beneath the ten balls. Ten balls and three balls are *thirteen* balls.

How many balls are ten balls and one ball? Ten balls and two balls? Ten balls and three balls?

How many pencils are ten pencils and three pencils? Thirteen pencils less three pencils?

How many balls are ten balls and three balls? Ten blocks and three blocks? Ten lines and three lines?

How many are thirteen balls less three balls? Thirteen blocks less three blocks? Thirteen lines less three lines.

Here are the figures that stand for thirteen: *13.*

Make thirteen lines on your slate, and write below them the figures that stand for thirteen.

|||||||||| |||
13

NOTE.—Show that the figure 1 stands for ten, and the figure 3 for three ones.

Make the figures 11, 12, and 13 on your slate.

4. I now move four balls below the ten balls. Ten balls and four balls are *fourteen* balls.

If four balls are taken from fourteen balls, how many balls will be left?

How many pencils are ten pencils and four pencils? Fourteen pencils less four pencils?

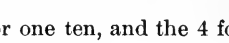
How many are ten blocks and four blocks? Ten shells and four shells? Ten lines and four lines?

How many are fourteen blocks less four blocks? Fourteen shells less four shells? Fourteen lines less four lines?

Here are the figures that stand for fourteen: *14.*

NOTE.—Show that the 1 stands for one ten, and the 4 for four ones. Illustrate by one dime and four cents.

Make fourteen lines on your slate, and write 14 below them.


14

Write 11, 12, 13, 14 on your slate.

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5. I now move five balls beneath the ten balls. Ten balls and five balls are *fifteen* balls.

Five balls taken from fifteen balls leave how many balls?

How many pencils are ten pencils and five pencils? Fifteen pencils less five pencils?

How many are ten shells and five shells? Ten lines and five lines? Ten cents and five cents?

How many are fifteen shells less five shells? Fifteen lines less five lines? Fifteen cents less five cents?

Here are the figures that stand for fifteen: *15.*

NOTE.—Show that the figure 1 stands for one ten, and the figure 5 for five ones. This may be illustrated by taking fifteen cents,—one dime and five cents. The dime or ten-cent piece

equals in value ten cents, and represents one ten or ten ones, and the five cents represent five ones. The dime and five cents represent fifteen ones.

Make fifteen lines on your slate, and write 15 below them.

15

Make 11, 12, 13, 14, 15 on your slate.

NOTE.—If the pupils are specially bright, and are interested, the teacher can profitably continue these exercises, teaching the numbers sixteen, seventeen, etc., to twenty; but, inasmuch as no number exceeding fifteen will be used for several weeks, it is believed to be wise to defer the teaching of these higher numbers until the sum of the two digital numbers used in any exercise is fifteen or more. See Lesson VII, page 88.

LESSON II.

1. John may take 2 books in his right hand, and 1 book in his left hand. How many books has John?

How many books are 1 book and 1 book? 2 books and 1 book? 3 books and 1 book?

How many are 1 and 1? 2 and 1? 3 and 1?

Kate had 3 plums, and she gave 1 plum to her brother. How many plums had Kate left?

How many plums are 2 plums less 1 plum? 3 plums less 1 plum? 4 plums less 1 plum?

How many are 2 less 1? 3 less 1? 4 less 1? 5 less 1?

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2. Charles has 4 marbles in his right hand and 1 in his left. How many marbles has Charles in both hands?

How many marbles are 4 marbles and 1 marble? 5 marbles and 1 marble? 6 marbles and 1 marble?

How many are 4 and 1? 5 and 1? 6 and 1?

John has 6 marbles: if he should give 1 marble to Willie, how many marbles would he have left?

How many marbles are 5 marbles less 1 marble? 6 marbles less 1 marble? 7 marbles less 1 marble?

How many are 5 less 1? 6 less 1? 7 less 1?

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2. Kate gave 7 cents for paper and 1 cent for a pencil. How many cents did she give for paper and pencil?

How many cents are 7 cents and 1 cent? 8 cents and 1 cent? 9 cents and 1 cent?

How many are 7 and 1? 8 and 1? 9 and 1?

Mary had 10 cents, and she gave 1 cent for a pencil. How many cents had she left?

How many cents are 8 cents less 1 cent? 9 cents less 1 cent? 10 cents less 1 cent?

How many are 8 less 1? 9 less 1? 10 less 1?

How many are 4 cents and 1 cent? 5 cents less 1 cent? 7 cents and 1 cent? 8 cents less 1 cent? 9 cents and 1 cent? 10 cents less 1 cent?

How many are 5 and 1? 6 less 1? 8 and 1? 9 less 1? 4 and 1? 5 less 1? 9 and 1? 10 less 1? 7 and 1? 8 less 1? 6 and 1? 7 less 1?

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4. How many are:

1 and 1?	2 less 1?
2 and 1?	3 less 1?
3 and 1?	4 less 1?
4 and 1?	5 less 1?
5 and 1?	6 less 1?
6 and 1?	7 less 1?
7 and 1?	8 less 1?
8 and 1?	9 less 1?
9 and 1?	10 less 1?

Read and complete:

$1+1=2$	$2-1=1$
$2+1=$	$3-1=$
$3+1=$	$4-1=$
$4+1=$	$5-1=$
$5+1=$	$6-1=$
$6+1=$	$7-1=$
$7+1=$	$8-1=$
$8+1=$	$9-1=$
$9+1=$	$10-1=$

NOTES.—1. The left-hand tables may first be recited together, thus: 1 and 1 are 2; 2 less 1 is 1, etc. They should then be recited separately, and the drill should be continued until they are each recited without hesitation.

2. Copy the two right-hand tables on the blackboard, and after teaching the signs +, — and =, drill until the pupils can read and complete in order or promiscuously with rapidity. The sign + may be read *plus* or *and*, as may be preferred, and the sign —, *less*. The two tables should finally be copied by the pupils on their slates and completed.

LESSON III.

1. I have 2 books in my right hand, and 2 books in my left. How many books in both of my hands?

How many books are 1 book and 2 books? 2 books and 2 books? 3 books and 2 books?

How many are 1 and 2? 2 and 2? 3 and 2?

John picked 3 pears, and gave 2 of them to his mother. How many pears had John left?

How many pears are 3 pears less 2 pears? 4 pears less 2 pears? 5 pears less 2 pears?

How many is 2 less 2? 3 less 2? 4 less 2? 5 less 2?

James found 4 eggs in one nest and 2 eggs in another nest. How many eggs did James find?

How many eggs are 4 eggs and 2 eggs? 5 eggs and 2 eggs? 6 eggs and 2 eggs?

How many are 4 and 2? 5 and 2? 6 and 2?

Jane picked 7 plums, and gave 2 of them to Susan. How many plums had Jane left?

How many plums are 6 plums less 2 plums? 7 plums less 2 plums? 8 plums less 2 plums?

How many are 6 less 2? 7 less 2? 8 less 2?

2. There are 7 birds on a branch of a tree, and 2 birds on another branch. How many birds on the two branches?

How many birds are 7 birds and 2 birds? 8 birds and 2 birds? 9 birds and 2 birds?

How many are 7 and 2? 8 and 2? 9 and 2?

There are 10 birds on a tree. If 2 birds fly away, how many birds will be left?

How many birds are 10 birds less 2 birds? 9 birds less 2 birds? 11 birds less 2 birds?

How many are 9 less 2? 10 less 2? 11 less 2?

How many are 5 cents and 2 cents? 7 cents less 2 cents? 7 cents and 2 cents? 9 cents less 2 cents?

How many are 6 cents and 2 cents? 8 cents less 2 cents? 4 cents and 2 cents? 6 cents less 2 cents?

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3. How many are:

1 and 2?	3 less 2?
2 and 2?	4 less 2?
3 and 2?	5 less 2?
4 and 2?	6 less 2?
5 and 2?	7 less 2?
6 and 2?	8 less 2?
7 and 2?	9 less 2?
8 and 2?	10 less 2?
9 and 2?	11 less 2?

Read and complete:

$1+2=$	$3-2=$
$2+2=$	$4-2=$
$3+2=$	$5-2=$
$4+2=$	$6-2=$
$5+2=$	$7-2=$
$6+2=$	$8-2=$
$7+2=$	$9-2=$
$8+2=$	$10-2=$
$9+2=$	$11-2=$

NOTE.—For directions, see page 69, notes 1 and 2.

How many are 3 and 2? 5 and 2? 7 and 2? 8 and 2? 4 and 2? 6 and 2? 9 and 2?

How many are 9 and 2? 11 less 2? 7 and 2? 9 less 2? 5 and 2? 7 less 2? 8 and 2? 10 less 2?

How many are 8 and 2? 10 less 2? 7 less 2? 6 less 2? 9 less 2? 5 less 2? 11 less 2?

4. Separate each of the numbers from 3 to 11 inclusive into 2 and another number, and then subtract 2 and its complement from the original number, thus:

3 is 2 and —, or — and 2 ∴ 3 less 2 is —; 3 less — is 2.
4 is 2 and — ∴ 4 less 2 is —.
5 is 2 and —, or — and 2 ∴ 5 less 2 is —; 5 less — is 2.
6 is 2 and —, or — and 2 ∴ 6 less 2 is —; 6 less — is 2.
7 is 2 and —, or — and 2 ∴ 7 less 2 is —; 7 less — is 2.
8 is 2 and —, or — and 2 ∴ 8 less 2 is —; 8 less — is 2.
9 is 2 and —, or — and 2 ∴ 9 less 2 is —; 9 less — is 2.
10 is 2 and —, or — and 2 ∴ 10 less 2 is —; 10 less — is 2.
11 is 2 and —, or — and 2 ∴ 11 less 2 is —; 11 less — is 2.

NOTES.—1. The character ∴ used in the above exercises is read *whence* or *hence*. It may be omitted in reciting these exercises.

2. These two series of inverse exercises may also be written on the blackboard and combined in one drill (the pupils filling the blanks), thus:

3 = 2 + , or + 2 ∴ 3 − 2 = ; 3 − = 2.
4 = 2 + ∴ 4 − 2 = .
5 = 2 + , or + 2 ∴ 5 − 2 = ; 5 − = 2.
6 = 2 + , or + 2 ∴ 6 − 2 = ; 6 − = 2.
7 = 2 + , or + 2 ∴ 7 − 2 = ; 7 − = 2.
8 = 2 + , or + 2 ∴ 8 − 2 = ; 8 − = 2.
9 = 2 + , or + 2 ∴ 9 − 2 = ; 9 − = 2.
10 = 2 + , or + 2 ∴ 10 − 2 = ; 10 − = 2.
11 = 2 + , or + 2 ∴ 11 − 2 = ; 11 − = 2.

---

5. How many books are 2 books + 2 books + 2 books? How many books are three 2 books? "Six books."

How many are two 2's? Three 2's? Four 2's? Five 2's?

How many 2 books make 4 books? 6 books? 8 books? 10 books? 12 books?

NOTE.—Take 4 books, 6 books, etc., and separate them into groups of 2 books.

How many 2's in 4? 2's in 6? 2's in 8? 2's in 10? 2's in 12?

NOTE.—Give the above and subsequent similar exercises without introducing the word "times" or the idea of *factor*. The equal numbers should be added or combined as *parts*, and the numbers should be separated into *equal parts*. These exercises not only afford practice in addition and analysis of numbers, but they will prepare the way for multiplication and division. It should be kept in mind that the adding of the equal parts of a number is not multiplication, and the separating of a number into its equal parts is not numerical division. See page 15.

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6. Jane picked 5 red roses from one bush, and 2 white roses from another bush. How many roses did she pick?

Willie earned 10 cents by selling papers, and then gave 2 cents for a pencil. How many cents had he left?

NOTE.—Do not teach any formal analysis of these and subsequent similar problems in this *second-year course*, and do not require pupils to use "because," "since," "therefore," etc. All that should be given is the answer and the process which gives it, as follows:

(1) *Seven roses:* 5 *roses and* 2 *roses are* 7 *roses.*

(2) *Eight cents:* 10 *cents less* 2 *cents are* 8 *cents.*

A train of cars contains 7 passenger cars and 2 baggage cars. How many cars in the train?

A house has 5 rooms on the first floor and 2 rooms on the second floor. How many rooms in the house?

Mary is 6 years old, and her brother is 2 years older than she. How old is her brother?

Carrie wrote 9 words on her slate, and then rubbed out 2 of them. How many words were left on her slate?

A man brought 10 melons to market, but 2 of them were stolen by a wicked boy. How many melons had he left?

A farmer having 11 hogs, sold 2 of them. How many hogs had he left?

Charles has 9 marbles and John 7 marbles. How many marbles has Charles more than John?

How many times 2 bananas make 8 bananas?

BOARD AND SLATE EXERCISES.

	a	*b*	*c*	*d*	*e*	*f*	*g*
	2	2	2	2	2	2	2
Add	3	5	4	6	7	9	8

	a	*b*	*c*	*d*	*e*	*f*	*g*
From	5	6	7	8	9	11	10
Take	2	2	2	2	2	2	2

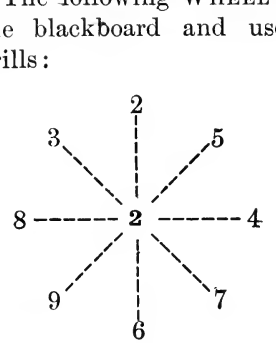
a	*b*	*c*	*d*	*e*	*f*
2	2	1	2	2	2
2	2	2	1	2	2
2	2	1	2	1	1
2	2	2	1	1	2
2	2	1	2	2	2
Add 2	1	2	2	2	1

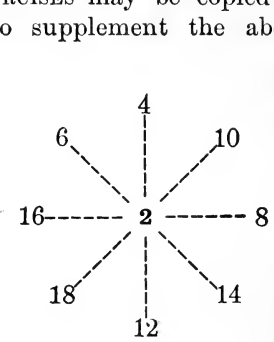
NOTES.—1. Write these exercises on the blackboard, and drill the pupils until the results are given instantly. The two left-hand series of exercises will be sufficient for one drill, and the right-hand or "column" series will afford another drill.

2. In adding the numbers in the column series, give only results, thus (*a*): 4, 6, 8, 10, 12. The pupils should not be permitted to say (*a*) 2 and 2 are 4; 4 and 2 are 6; 6 and 2 are 8, etc.

3. Have the pupils copy the above exercises on their slates, add or subtract the numbers, as the case may be, and write the sum or difference below.

The following WHEEL EXERCISES may be copied on the blackboard and used to supplement the above drills:





NOTES.—1. Point successively to the outer numbers in the left-hand diagram, and have the pupils add to the numbers thus designated the number 2 in the center.

2. In the use of the right-hand diagram, the number 2 in the center is subtracted from the outer numbers successively designated.

LESSON IV.

1. Mary picked 2 pears from one limb and 3 pears from another limb. How many pears did she pick?

How many pears are 2 pears and 3 pears? 1 pear and 3 pears? 3 pears and 3 pears?

How many are 1 and 3? 2 and 3? 3 and 3?

Mary picked 5 pears, and gave 3 of them to her father. How many pears had she left?

How many pears are 5 pears less 3 pears? 4 pears less 3 pears? 6 pears less 3 pears?

How many are 4 less 3? 5 less 3? 6 less 3?

---

2. Charles found 4 eggs in one nest and 3 eggs in another nest. How many eggs did he find?

How many eggs are 4 eggs and 3 eggs? 5 eggs and 3 eggs? 6 eggs and 3 eggs?

How many are 4 and 3? 5 and 3? 6 and 3?

Willie bought 7 peaches, and gave 3 of them to his playmate. How many peaches had he left?

How many peaches are 7 peaches less 3 peaches? 8 peaches less 3 peaches? 9 peaches less 3 peaches?

How many are 7 less 3? 8 less 3? 9 less 3?

---

3. 7 birds light on a bush, and 3 birds light on another bush. How many birds light on both bushes?

How many birds are 7 birds and 3 birds? 8 birds and 3 birds? 9 birds and 3 birds?

How many are 7 and 3? 8 and 3? 9 and 3?

George earned 10 cents, and gave 3 cents for a lead pencil. How many cents had he left?

How many cents are 10 cents less 3 cents? 11 cents less 3 cents? 12 cents less 3 cents?

How many are 10 less 3? 11 less 3? 12 less 3?

How many pens are 5 pens and 3 pens? 7 pens and 3 pens? 8 pens and 3 pens? 9 pens and 3 pens?

How many pens are 6 pens less 3 pens? 8 pens less 3 pens? 5 pens less 3 pens? 7 pens less 3 pens? 9 pens less 3 pens? 10 pens less 3 pens?

---

4. How many are:

1 and 3?	4 less 3?
2 and 3?	5 less 3?
3 and 3?	6 less 3?
4 and 3?	7 less 3?
5 and 3?	8 less 3?
6 and 3?	9 less 3?
7 and 3?	10 less 3?
8 and 3?	11 less 3?
9 and 3?	12 less 3?

Read and complete:

$1+3=$	$4-3=$
$2+3=$	$5-3=$
$3+3=$	$6-3=$
$4+3=$	$7-3=$
$5+3=$	$8-3=$
$6+3=$	$9-3=$
$7+3=$	$10-3=$
$8+3=$	$11-8=$
$9+3=$	$12-3=$

How many are 4 and 3? 6 and 3? 8 and 3? 5 and 3? 7 and 3? 9 and 3? 2 and 3? 3 and 3?

How many are 6 and 3? 9 less 3? 8 and 3? 11 less 3? 7 and 3? 10 less 3? 9 and 3? 12 less 3?

How many are 5 and 3? 8 less 3? 10 less 3? 12 less 3? 9 less 3? 7 less 3? 4 less 3? 6 less 3?

---

5. Separate each of the numbers, from 4 to 12 inclusive, into 3 and another number, and then subtract 3 and its complement from the original number, thus:

4 is 3 and —, or — and 3 ∴ 4 less 3 is —; 4 less — is 3.
5 is 3 and —, or — and 3 ∴ 5 less 3 is —; 5 less — is 3.
6 is 3 and — ∴ 6 less 3 is —.
7 is 3 and —, or — and 3 ∴ 7 less 3 is —; 7 less — is 3.
8 is 3 and —, or — and 3 ∴ 8 less 3 is —; 8 less — is 3.
9 is 3 and —, or — and 3 ∴ 9 less 3 is —; 9 less — is 3.
10 is 3 and —, or — and 3 ∴ 10 less 3 is —; 10 less — is 3.
11 is 3 and —, or — and 3 ∴ 11 less 3 is —; 11 less — is 3.
12 is 3 and —, or — and 3 ∴ 12 less 3 is —; 12 less — is 3.

NOTE.—See Lesson III, page 71, notes, for directions respecting the use of the above exercises as blackboard drills.

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6. How many blocks are 3 blocks and 3 blocks? 3 blocks, 3 blocks, and 3 blocks?

How many blocks are three 3 blocks? "Nine blocks."

How many are two 3's? Three 3's? Four 3's?

How many 3 blocks make 6 blocks? 9 blocks? 12 blocks?

NOTE.—Take 6 blocks, 9 blocks, and 12 blocks, and divide them into groups of 3 blocks.

How many 3's in 6? 3's in 9? 3's in 12?

How many 3 eggs in 7 eggs, and how many eggs over?

How many 3 eggs in 10 eggs, and how many over?

How many 3 eggs in 8 eggs, and how many over?

How many 3 eggs in 11 eggs, and how many over?

NOTE.—See Lesson III, page 72, note.

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7. There are 6 birds on the ground and 3 birds on the tree. How many birds in both places?

ANS.: *Nine birds: 6 birds and 3 birds are 9 birds.*

There are 8 boys and 3 girls in a class. How many pupils in the class?

A house has 8 windows in the first story and 3 windows in the second story. How many windows in the house?

A farmer brought 7 sacks of wheat and 3 sacks of corn to a mill. How many sacks of grain did he bring? How many more sacks of wheat than of corn?

Susan wrote 12 words on her slate, but misspelled 3 words. How many words did she write correctly?

Horace earned 10 cents, and then gave 3 cents for a top. How many cents had he left?

Clarence is 11 years old, and Henry 8 years old. How much older is Clarence than Henry?

Willie had 12 marbles, and gave 3 of them to James. How many marbles had he left?

Willie then gave 3 marbles to Harry. How many marbles had he left?

He next gave 3 marbles to Clarence. How many marbles had he then left?

A man paid 13 dollars for an overcoat, and 3 dollars for a hat. How many dollars did he pay for the coat more than for the hat?

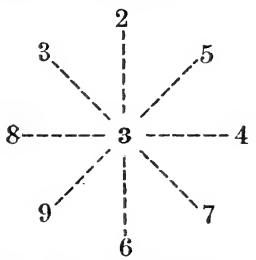
SLATE AND BOARD EXERCISES.

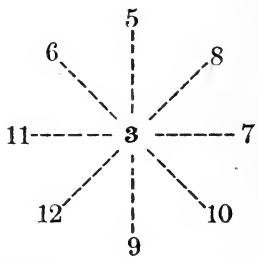
	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
	3	3	3	3	3	3	3	3	3
Add	2	4	1	3	5	7	6	8	9

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
From	5	7	4	6	8	10	9	11	12
Take	3	3	3	3	3	3	3	3	3

	a	*b*	*c*	*d*	*e*
	1	2	1	3	2
	3	2	2	2	3
	3	3	3	3	2
	3	3	3	2	3
	2	3	2	3	2
Add	1	1	1	2	1

NOTE.—See Lesson III, page 73, notes.





NOTE.—See Lesson III, page 74, notes.

LESSON V.

1. There are two boys sitting on one seat, and 4 boys on another seat. How many boys on both seats?

How many boys are 1 boy and 4 boys? 2 boys and 4 boys? 3 boys and 4 boys?

How many are 1 and 4? 2 and 4? 3 and 4?

There are 6 boys sitting on a seat. If 4 of the boys go away, how many boys will be left?

How many boys are 5 boys less 4 boys? 6 boys less 4 boys? 7 boys less 4 boys?

How many are 5 less 4? 6 less 4? 7 less 4?

2. How many shells are 4 shells and 4 shells? 8 shells less 4 shells? 5 shells and 4 shells? 9 shells less 4 shells? 6 shells and 4 shells? 10 shells less 4 shells?

NOTE.—If the pupils hesitate in giving these results, use the shells, combining the groups and taking away a group of four, as indicated.

How many quails are 5 quails and 4 quails? 4 quails and 4 quails? 6 quails and 4 quails?

How many are 4 and 4? 5 and 4? 6 and 4?

A hunter shot 5 quails in one field and 4 quails in another field. How many quails did he shoot in both fields?

A hunter had 8 quails, and he gave 4 of them to a neighbor. How many quails had he left?

How many quails are 8 quails less 4 quails? 9 quails less 4 quails? 10 quails less 4 quails?

How many are 8 less 4? 9 less 4? 10 less 4?

---

3. How many shells are 7 shells and 4 shells? 11 shells less 4 shells? 8 shells and 4 shells? 12 shells less 4 shells? 9 shells and 4 shells? 13 shells less 4 shells?

NOTE.—Do not permit the pupils to count the shells in the groups formed by combining 8 shells and 4 shells, or 9 shells and 4 shells. If they can not number the group *at sight*, show them how "to pick out" *ten* shells, and then combine the 10 shells with 3 shells or 4 shells, as the case may be. A little practice will enable children thus to number a group of 11 to 18 objects at sight.

A farmer has 7 cows in one field and 4 cows in another field. How many cows in both fields?

How many cows are 7 cows and 4 cows? 8 cows and 4 cows? 9 cows and 4 cows?

How many are 7 and 4? 8 and 4? 9 and 4?

A farmer has 11 cows in two fields. If there are 4 cows in one of the fields, how many cows are in the other field?

How many cows are 11 cows less 4 cows? 12 cows less 4 cows? 13 cows less 4 cows?

How many are 11 less 4? 12 less 4? 13 less 4?

How many cents are 5 cents and 4 cents? 9 cents less 4 cents? 7 cents and 4 cents? 11 cents less 4 cents? 9 cents and 4 cents? 13 cents less 4 cents?

How many cents are 4 cents and 4 cents? 8 cents less 4 cents? 6 cents and 4 cents? 10 cents less 4 cents? 8 cents and 4 cents? 12 cents less 4 cents?

---

4. How many are:

1 and 4?	5 less 4?
2 and 4?	6 less 4?
3 and 4?	7 less 4?
4 and 4?	8 less 4?
5 and 4?	9 less 4?
6 and 4?	10 less 4?
7 and 4?	11 less 4?
8 and 4?	12 less 4?
9 and 4?	13 less 4?

Read and complete:

$1+4=$	$5-4=$
$2+4=$	$6-4=$
$3+4=$	$7-4=$
$4+4=$	$8-4=$
$5+4=$	$9-4=$
$6+4=$	$10-4=$
$7+4=$	$11-4=$
$8+4=$	$12-4=$
$9+4=$	$13-4=$

How many are 7 and 4? 11 less 4? 6 and 4? 10 less 4? 8 and 4? 12 less 4? 9 and 4? 13 less 4?

How many are 3 and 4? 5 and 4? 7 and 4? 9 and 4? 2 and 4? 4 and 4? 6 and 4? 8 and 4?

How many are 6 and 4? 10 less 4? 8 less 4? 6 less 4? 12 less 4? 7 less 4? 9 less 4? 11 less 4? 13 less 4?

5. Separate each of the numbers, from 4 to 13 inclusive, into 4 and another number, and then subtract 4 and its complement from the original number, thus:

5 is 4 and —, or — and 4 ∴ 5 less 4 is —; 5 less — is 4.
6 is 4 and —, or — and 4 ∴ 6 less 4 is —; 6 less — is 4.
7 is 4 and —, or — and 4 ∴ 7 less 4 is —; 7 less — is 4.
8 is 4 and — ∴ 8 less 4 is —.
9 is 4 and —, or — and 4 ∴ 9 less 4 is —; 9 less — is 4.
10 is 4 and —, or — and 4 ∴ 10 less 4 is —; 10 less — is 4.
11 is 4 and —, or — and 4 ∴ 11 less 4 is —; 11 less — is 4.
12 is 4 and —, or — and 4 ∴ 12 less 4 is —; 12 less — is 4.
13 is 4 and —, or — and 4 ∴ 13 less 4 is —; 13 less — is 4.

NOTE.—See Lesson III, page 71, notes, for directions for blackboard drills.

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6. How many fingers are 4 fingers and 4 fingers? 4 fingers and 4 fingers and 4 fingers?

How many are two 4 fingers? Three 4 fingers?

How many are two 4's? Three 4's?

How many 4 shells in 8 shells? How many 4 shells in 12 shells?

NOTE.—Take 8 shells and 12 shells, and divide them into groups of 4 shells.

How many 4's in 8? 4's in 12?

How many 4 pins in 6 pins, and how many over?

How many 4 pins in 9 pins, and how many over?

How many 4 pins in 10 pins, and how many over?

How many 4 pins in 14 pins, and how many over?

---

7. Harry is 6 years old, and Frank is 4 years older than Harry. How old is Frank? ANS.: 10 *years old:* 6 *years and* 4 *years are* 10 *years.*

Mary is 9 years old, and Susan is 4 years younger than Mary. How old is Susan?

There are 8 peach trees and 4 pear trees in a garden. How many trees in the garden? How many more peach trees than pear trees?

Samuel caught 7 fishes and Henry caught 4 fishes. How many fishes did both boys catch? How many fishes did Samuel catch more than Henry?

A wagon contains 9 boys and 4 girls. How many children in the wagon? If 4 of the boys leave, how many children will be left in the wagon?

Lucy picked 3 bunches of pansies, with 4 pansies in each bunch. How many pansies did she pick? How many 4 pansies?

Frank has 8 cents, and he wishes to buy a slate worth 12 cents. How many more cents must he have to buy the slate?

Howard picked 13 peaches and ate 4 of them. How many peaches had he left?

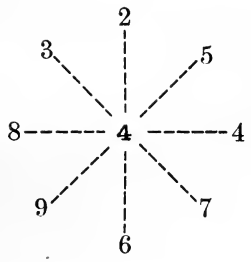
Charles paid 10 cents for a slate and 4 cents for a bunch of pencils. How much did he pay for slate and pencils? How many cents did the slate cost more than the pencils?

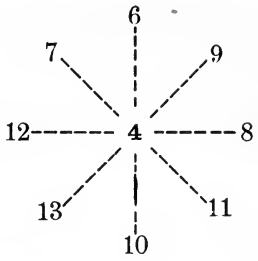
BOARD AND SLATE EXERCISES.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	4	4	4	4	4	4	4	4
Add	2	4	3	5	7	6	9	8

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
From	6	8	7	9	11	10	13	12
Take	4	4	4	4	4	4	4	4

	a	*b*	*c*	*d*	*e*
	4	2	4	4	4
	4	4	4	4	2
	1	4	3	2	1
	2	4	2	3	4
Add	1	1	1	2	3





NOTE.—See Lesson III, page 74, notes.

LESSON VI.

1. There are 2 men in one wagon, and 5 men in another wagon. How many men in both wagons?

How many men are 2 men and 5 men? 1 man and 5 men? 3 men and 5 men?

How many are 1 and 5? 2 and 5? 3 and 5?

There are 7 men in a wagon. If 5 men get out, how many men will be left in the wagon?

How many men are 6 men less 5 men? 7 men less 5 men? 8 men less 5 men?

How many are 6 less 5? 7 less 5? 8 less 5?

---

2. How many blocks are 4 blocks and 4 blocks? 8 blocks less 4 blocks? 5 blocks and 4 blocks? 9 blocks less 4 blocks? 6 blocks and 4 blocks? 10 blocks less 4 blocks?

There are 4 trees in a row, and 5 trees in another row. How many trees in both rows?

How many trees are 4 trees and 5 trees? 5 trees and 5 trees? 6 trees and 5 trees?

How many are 4 and 5? 5 and 5? 6 and 5?

A man brought 9 shade trees to town, and sold 5 of them. How many trees had he left?

How many trees are 9 trees less 5 trees? 10 trees less 5 trees? 11 trees less 5 trees?

How many are 9 less 5? 10 less 5? 11 less 5?

---

3. How many blocks are 7 blocks and 5 blocks? 12 blocks less 5 blocks? 8 blocks and 5 blocks? 13 blocks less 5 blocks? 9 blocks and 5 blocks? 14 blocks less 5 blocks?

A farmer has 7 sheep in one field, and 5 sheep in another field. How many sheep in both fields?

How many sheep are 7 sheep and 5 sheep? 8 sheep and 5 sheep? 9 sheep and 5 sheep?

How many are 7 and 5? 8 and 5? 9 and 5?

A farmer has 12 sheep: if he sells 5 of them, how many sheep will he then have?

How many sheep are 12 sheep less 5 sheep? 13 sheep less 5 sheep? 14 sheep less 5 sheep?

How many are 12 less 5? 13 less 5? 14 less 5?

---

4. How many peaches are 2 peaches and 5 peaches? 4 peaches and 5 peaches? 6 peaches and 5 peaches? 8 peaches and 5 peaches? 9 peaches less 5 peaches?

How many are:

1 cent and 5 cents?	6 cents less 5 cents?
2 cents and 5 cents?	7 cents less 5 cents?
3 cents and 5 cents?	8 cents less 5 cents?
4 cents and 5 cents?	9 cents less 5 cents?
5 cents and 5 cents?	10 cents less 5 cents?
6 cents and 5 cents?	11 cents less 5 cents?
7 cents and 5 cents?	12 cents less 5 cents?
8 cents and 5 cents?	13 cents less 5 cents?
9 cents and 5 cents?	14 cents less 5 cents?

5. How many are:

1 and 5?	6 less 5?
2 and 5?	7 less 5?
3 and 5?	8 less 5?
4 and 5?	9 less 5?
5 and 5?	10 less 5?
6 and 5?	11 less 5?
7 and 5?	12 less 5?
8 and 5?	13 less 5?
9 and 5?	14 less 5?

Read and complete:

$1+5=$	$6-5=$
$2+5=$	$7-5=$
$3+5=$	$8-5=$
$4+5=$	$9-5=$
$5+5=$	$10-5=$
$6+5=$	$11-5=$
$7+5=$	$12-5=$
$8+5=$	$13-5=$
$9+5=$	$14-5=$

How many are 4 and 5? 9 less 5? 6 and 5? 11 less 5? 8 and 5? 13 less 5? 3 and 5? 8 less 5? 5 and 5? 10 less 5? 7 and 5? 12 less 5?

How many are 3 and 5? 5 and 5? 7 and 5? 9 and 5? 2 and 5? 4 and 5? 6 and 5? 8 and 5?

How many are 8 less 5? 10 less 5? 12 less 5? 14 less 5? 7 less 5? 9 less 5? 11 less 5? 13 less 5?

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6. Separate each of the numbers, from 6 to 14 inclusive, into 5 and another number, and then subtract 5 and its complement from the original number, thus:

6 is 5 and —, or — and 5 ∴ 6 less 5 is —; 6 less — is 5.
7 is 5 and —, or — and 5 ∴ 7 less 5 is —; 7 less — is 5.
8 is 5 and —, or — and 5 ∴ 8 less 5 is —; 8 less — is 5.
9 is 5 and —, or — and 5 ∴ 9 less 5 is —; 9 less — is 5.
10 is 5 and — ∴ 10 less 5 is —.
11 is 5 and —, or — and 5 ∴ 11 less 5 is —; 11 less — is 5.
12 is 5 and —, or — and 5 ∴ 12 less 5 is —; 12 less — is 5.
13 is 5 and —, or — and 5 ∴ 13 less 5 is —; 13 less — is 5.
14 is 5 and —, or — and 5 ∴ 14 less 5 is —; 14 less — is 5.

NOTE.—See Lesson III, page 71, notes.

7. How many balls are 5 balls and 5 balls? 5 balls and 5 balls and 5 balls?

How many balls are two 5 balls? Three 5 balls?
How many are two 4's? Three 4's?
How many 5 balls in 10 balls?
How many 5 balls in 15 balls?
How many 5's in 10? 5's in 15?
How many 5 pens in 7 pens, and how many over?
How many 5 pens in 11 pens, and how many over?
How many 5 pens in 13 pens, and how many over?
How many 5 pens in 12 pens, and how many over?
How many 5 pens in 14 pens, and how many over?
How many 5's in 15? 5's in 10?
How many 3's in 9? 3's in 12? 3's in 15?
How many 4's in 8? 4's in 12?
How many 2's in 10? 2's in 12? 2's in 8?

---

8. A hen had 12 pretty chickens, and a hawk carried off 5 of them. How many chickens were left?

ANS.: 7 *chickens:* 12 *chickens less* 5 *chickens are* 7 *chickens.*

There are 8 windows in the first story of a house, and 5 windows in the second story. How many windows in the house? How many more windows in the first story than in the second?

Albert is 9 years old, and Frank is 5 years older than Albert. How old is Frank?

A man gave 13 dollars for a watch, and 5 dollars for a chain. How many dollars did the watch cost more than the chain?

A young pear tree bore 12 pears, and 5 of them were shaken off by the wind. How many pears were left on the tree?

Henry paid 7 cents for an inkstand, and 5 cents for a pen. How many cents did he pay for both?

There are 10 bananas in one bunch, and 5 bananas in another bunch. How many bananas in both bunches? How many more bananas in the first bunch than in the second?

Charles gave 15 cents for a knife, and 5 cents for a comb. How many cents did the knife cost more than the comb?

Kate found 11 eggs in one nest, and 5 eggs in another nest. How many eggs in the first nest more than in the second?

There are three rows of peach trees in an orchard, and 5 peach trees in each row. How many peach trees in the orchard?

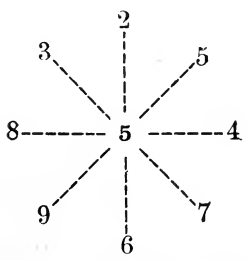
BOARD AND SLATE EXERCISES.

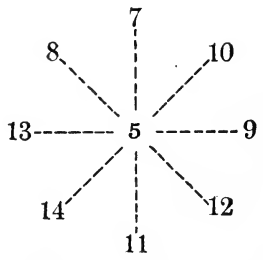
	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	5	5	5	5	5	5	5	5
Add	3	5	2	4	6	8	7	9

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
From	8	10	7	9	11	13	12	14
Take	5	5	5	5	5	5	5	5

	a	*b*	*c*	*d*	*e*
	5	2	5	3	5
	5	2	4	2	3
	2	5	3	3	2
	2	5	2	2	3
Add	1	1	1	5	2

NOTE.—See Lesson III, page 73, notes.





NOTE.—See Lesson IV, page 74, notes.

LESSON VII.

The Numbers 16 to 20.

NOTE.—The object of this lesson is to develop the idea of each number from 16 to 20 inclusive, to teach its name and its representation by figures. See Lesson 1, page 67, note.

1. A few weeks ago, children, I taught you the numbers from 10 to 15. I now wish to teach you the numbers from 16 to 20. We shall need the numeral frame, a package of 10 pencils and 10 loose pencils, and 20 blocks.

How many balls on this wire? "10 balls." If I put 2 balls with the 10 balls (suiting action to word), how many balls will there be?

10 balls and 3 balls are how many balls? 10 balls and 4 balls? 10 balls and 5 balls?

Now, if I put 6 balls with 10 balls, how many balls will there be? "16 balls." Right. 10 balls and 6 balls are 16 balls.

Here are 10 pencils. How many pencils must I put with them to make 16 pencils? "6 pencils." Right. 10 pencils and 6 pencils are how many pencils? "16 pencils."

I now make 10 lines on the board. How many more lines must I add to make 16 lines? "6 lines." 10 lines and 6 lines are how many lines?

How many are 10 and 6?

Here are the figures that stand for sixteen: *16.*

What does the 1 stand for? "One ten." What does the 6 stand for? "Six ones."

Make 16 lines on your slate, and write 16 below them.

Make 11, 12, 13, 14, 15, 16 on your slate.

2. Here are 10 balls again. If I add 7 balls (suiting action to word), how many balls will there be? (If no pupil can tell, the teacher should give the answer.) 10 balls and 7 balls are 17 balls.

How many pencils are 10 pencils and 7 pencils? "17 pencils."

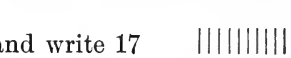
How many lines must I add to 10 lines to make 17 lines? "7 lines." Right. I will make the 7 lines.

How many are 10 lines and 7 lines? 10 boys and 7 boys? 10 cents and 7 cents? 10 and 7?

Here are the figures that stand for seventeen: *17.*

What does the 1 stand for? What does the 7 stand for?

Make 17 lines on your slate, and write 17 below them.



Make 11, 12, 13, 14, 15, 16, 17 on your slate.

17

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3. Here are 10 balls, and how many balls do I move beneath them? "8 balls." 10 balls and 8 balls are 18 balls.

How many balls must I put with 10 balls to make 18 balls? "8 balls."

How many pencils are 10 pencils and 8 pencils? "18 pencils."

I put 10 blocks on the table, and Kate may put with them enough blocks to make 18. How many blocks has she put on the table? "8 blocks."

10 blocks and 8 blocks are how many blocks? "18 blocks."

10 and 8 are how many?

Mary may put 10 blocks on the table, and Kate 8 blocks. How many blocks on the table?

Kate may take away her 8 blocks. How many blocks are left on the table? "10 blocks."

8 pears taken from 18 pears leave how many pears?

Here are the figures that stand for eighteen: *18.*

What does the 1 stand for? What does the 8 stand for?

Make 18 lines on your slate, and write 18 below them.

Make 13, 14, 15, 16, 17, 18 on your slate.

18

---

4. Here are 10 balls, and how many balls do I move beneath them? "9 balls." 10 balls and 9 balls are 19 balls.

How many pencils must I put with 10 pencils to make 19 pencils? "9 pencils." 10 pencils and 9 pencils (suiting action to word), are how many pencils?

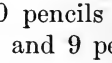
Jane may put 10 blocks in a row on the table, and Samuel may put 9 blocks in another row. How many blocks are 10 blocks and 9 blocks?

How many boys are 10 boys and 9 boys? How many are 10 and 9?

These are the figures that stand for nineteen: *19.*

What does the 1 stand for? What does the 9 stand for?

Make 19 lines on your slate, and write 19 below them.



Make 11, 12, 13, 14, 15, 16, 17, 18, 19 on your slate.

19

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5. We have at last reached 20! How many balls on this wire? "10 balls." How many balls on this wire? "10 balls." How many 10 balls on both wires? "Two 10 balls." Two tens are called *twenty.* How many balls on the two wires?

How many pencils in this bunch? "10 pencils." Let us tie 10 more pencils in a second bunch.

How many pencils in my right hand? "10 pencils." How many pencils in my left hand? "10 pencils." How many 10 pencils in both hands? "Two 10 pencils."

What are two 10's called? "Twenty." Then how many pencils in both my hands?

How many lines are 10 lines and 10 lines?

How many blocks are 10 blocks and 10 blocks?

Here are 2 dimes. How many cents in this dime? "10 cents." How many cents in this? "10 cents." How many 10 cents in two dimes? "Two 10 cents." How many cents in two 10 cents? "20 cents."

How many cents in 2 dimes? "20 cents."

These are the figures that stand for twenty: *20.*

What does the 2 stand for? "Two 10's. What does the 0 stand for? "No one." Right, and so 20 stands for two 10's or twenty.

Make 20 lines on your slate, in two groups of 10 lines each.

Make 10 and 20 on your slate, each five times.

20

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6. How many are 10 and 2? 10 and 4? 10 and 7? 10 and 1? 10 and 3? 10 and 5? 10 and 8? 10 and 9? 10 and 6? 10 and 10? Two 10's?

2 from 12 leaves how many? 3 from 13? 5 from 15? 7 from 17? 9 from 19? 4 from 14? 6 from 16? 8 from 18? 10 from 20?

LESSON VIII.

1. How many fingers are 1 finger and 6 fingers? 2 fingers and 6 fingers? 8 fingers less 6 fingers? 3 fingers and 6 fingers? 9 fingers less 6 fingers?

There are 2 baggage cars and 6 passenger cars in a train. How many cars in the train?

How many cars are 2 cars and 6 cars? 1 car and 6 cars? 3 cars and 6 cars?

How many are 1 and 6? 2 and 6? 3 and 6?

There are 8 kites flying in the air. If the boys haul down 6 kites, how many kites will be left flying?

How many kites are 7 kites less 6 kites? 8 kites less 6 kites? 9 kites less 6 kites?

How many are 7 less 6? 8 less 6? 9 less 6?

---

2. How many balls are 4 balls and 6 balls? 10 balls less 6 balls? 5 balls and 6 balls? 11 balls less 6 balls? 6 balls and 6 balls? 12 balls less 6 balls?

NOTE.—See Lesson V, page 79, note.

There are 4 trees in one row and 6 trees in another row. How many trees in both rows?

How many trees are 4 trees and 6 trees? 5 trees and 6 trees? 6 trees and 6 trees?

How many are 4 and 6? 5 and 6? 6 and 6?

Kate picked 10 pansies, and gave six of them to her mother. How many pansies had Kate left?

How many pansies are 10 pansies less 6 pansies? 12 pansies less 6 pansies? 11 pansies less 6 pansies?

How many are 4 and 6? 10 less 6? 11 less 6? 12 less 6? 8 less 6? 9 less 6? 7 less 6?

---

3. How many balls are 7 balls and 6 balls? 13 balls less 6 balls? 8 balls and 6 balls? 14 balls less 6 balls? 9 balls and 6 balls? 15 balls less 6 balls?

NOTE.—Use the numeral frame, and number the large groups by "picking out" *ten*.

Jane picked 7 roses from one bush and 6 roses from another bush. How many roses did she pick from both bushes?

How many roses are 7 roses and 6 roses? 8 roses and 6 roses? 9 roses and 6 roses?

How many are 7 and 6? 8 and 6? 9 and 6?

Jane picked 13 roses, and gave 6 of them to her sister. How many roses had Jane left?

How many roses are 13 roses less 6 roses? 14 roses less 6 roses? 15 roses less 6 roses?

How many are 13 less 6? 14 less 6? 15 less 6?

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4. How many oranges are 5 oranges and 6 oranges? 11 oranges less 6 oranges? 7 oranges and 6 oranges? 13 oranges less 6 oranges? 9 oranges and 6 oranges? 15 oranges less 6 oranges?

How many peaches are 4 peaches and 6 peaches? 10 peaches less 6 peaches? 6 peaches and 6 peaches? 12 peaches less 6 peaches? 8 peaches and 6 peaches? 14 peaches less 6 peaches?

How many are:

2 lemons and 6 lemons?
3 lemons and 6 lemons?
4 lemons and 6 lemons?
5 lemons and 6 lemons?
6 lemons and 6 lemons?
7 lemons and 6 lemons?
8 lemons and 6 lemons?
9 lemons and 6 lemons?

8 lemons less 6 lemons?
9 lemons less 6 lemons?
10 lemons less 6 lemons?
11 lemons less 6 lemons?
12 lemons less 6 lemons?
13 lemons less 6 lemons?
14 lemons less 6 lemons?
5 lemons less 6 lemons?

How many pears are 7 pears and 6 pears? 9 pears and 6 pears? 8 pears and 6 pears?

How many pears are 13 pears less 6 pears? 15 pears less 6 pears? 14 pears less 6 pears?

5. How many are:

1 and 6?	7 less 6?
2 and 6?	8 less 6?
3 and 6?	9 less 6?
4 and 6?	10 less 6?
5 and 6?	11 less 6?
6 and 6?	12 less 6?
7 and 6?	13 less 6?
8 and 6?	14 less 6?
9 and 6?	15 less 6?

Read and complete:

$1+6=$	$7-6=$
$2+6=$	$8-6=$
$3+6=$	$9-6=$
$4+6=$	$10-6=$
$5+6=$	$11-6=$
$6+6=$	$12-6=$
$7+6=$	$13-6=$
$8+6=$	$14-6=$
$9+6=$	$15-6=$

How many are 2 and 6? 8 less 6? 4 and 6? 10 less 6? 6 and 6? 12 less 6? 8 and 6? 14 less 6?

How many are 3 and 6? 9 less 6? 5 and 6? 11 less 6? 7 and 6? 13 less 6? 9 and 6? 15 less 6?

How many are 9 less 6? 11 less 6? 13 less 6? 15 less 6? 14 less 6? 12 less 6? 10 less 6? 8 less 6?

---

6. Separate each number from 7 to 15 into 6 and another number, and then subtract 6 and its complement from the original number, thus:

7 is 6 and —, or — and 6 ∴ 7 less 6 is —; 7 less — is 6.
8 is 6 and —, or — and 6 ∴ 8 less 6 is —; 8 less — is 6.
9 is 6 and —, or — and 6 ∴ 9 less 6 is —; 9 less — is 6.
10 is 6 and —, or — and 6 ∴ 10 less 6 is —; 10 less — is 6.
11 is 6 and —, or — and 6 ∴ 11 less 6 is —; 11 less — is 6.
12 is 6 and —, ∴ 12 less 6 is —.
13 is 6 and —, or — and 6 ∴ 13 less 6 is —; 13 less — is 6.
14 is 6 and —, or — and 6 ∴ 14 less 6 is —; 14 less — is 6.
15 is 6 and —, or — and 6 ∴ 15 less 6 is —; 15 less — is 6.

NOTE.—See Lesson III, page 71, notes 1 and 2.

7. How many are 6 balls and 6 balls?
How many balls are two 6 balls?
How many are two 6's?
How many 6 shells in 12 shells?
How many 6's in 12?
How many 6 tops in 8 tops, and how many over?
How many 6 tops in 10 tops, and how many over?
How many 6 tops in 13 tops, and how many over?
How many 6 tops in 15 tops, and how many over?
How many 6 tops in 16 tops, and how many over?
How many 2's in 4? 2's in 8? 2's in 12?
How many 3's in 6? 3's in 9? 3's in 12?
How many 4's in 8? 4's in 12? 4's in 16?

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8. A school yard contains 5 maple trees and 6 elm trees. How many trees in the yard?

A huckster bought 8 turkeys of one farmer, and 6 turkeys of another farmer. How many turkeys did he buy?

How many more turkeys did he buy of the first farmer than of the second?

John found 9 eggs in one nest, and 6 eggs in another nest. How many eggs did he find in both nests?

How many eggs in the first nest more than in the second?

There are 13 persons in a street car: if 6 of them leave the car, how many will be left in it?

Charles is 11 years old, and his sister is 6 years old. How much older is Charles than his sister?

A man paid 8 dollars for a pair of boots, and 6 dollars for a hat. How much did he pay for both boots and hat?

How much did he pay for the boots more than for the hat?

There are 9 cherry trees in a row, and 6 plum trees in another row. How many trees in the two rows? How many more cherry trees than plum trees?

How many radishes in 2 bunches, if there be 6 radishes in each bunch?

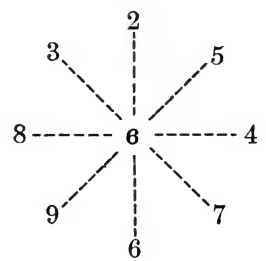
BOARD AND SLATE EXERCISES.

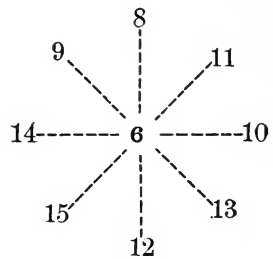
	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	6	6	6	6	6	6	6	6
Add	3	4	2	5	7	6	8	9

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
From	9	10	8	11	13	12	14	15
Take	6	6	6	6	6	6	6	6

	a	*b*	*c*	*d*	*e*
	6	4	5	6	3
	6	6	4	2	6
	3	2	3	4	2
	2	2	2	3	3
Add	1	2	1	3	4

NOTE.—See Lesson III, page 73, notes.





NOTE.—See Lesson III, page 74, notes 1 and 2.

LESSON IX.

1. How many blocks are 1 block and 7 blocks? 8 blocks less 7 blocks? 2 blocks and 7 blocks? 9 blocks less 7 blocks? 3 blocks and 7 blocks? 10 blocks less 7 blocks?

How many boys are 1 boy and 7 boys? 2 boys and 7 boys? 3 boys and 7 boys?

How many are 1 and 7? 2 and 7? 3 and 7?

A boy caught 10 fishes, and sold 7 of them. How many fishes had he left?

How many fishes are 10 fishes less 3 fishes? 10 fishes less 2 fishes? 10 fishes less 1 fish?

How many are 8 less 7? 9 less 7? 10 less 7?

---

2. How many balls are 4 balls and 7 balls? 11 balls less 7 balls? 5 balls and 7 balls? 12 balls less 7 balls? 6 balls and 7 balls? 13 balls less 7 balls?

There are 4 roses on one bush, and 7 roses on another bush. How many roses on both bushes?

How many roses are 4 roses and 7 roses? 5 roses and 7 roses? 6 roses and 7 roses?

How many are 4 and 7? 5 and 7? 6 and 7?

Clara picked 12 roses, and gave 7 of them to her mother? How many roses had Clara left?

How many roses are 12 roses less 7 roses? 11 roses less 7 roses? 13 roses less 7 roses?

How many are 11 less 7? 12 less 7? 13 less 7?

---

3. How many balls are 7 balls and 7 balls? 14 balls less 7 balls? 8 balls and 7 balls? 15 balls less 7 balls? 9 balls and 7 balls? 16 balls less 7 balls?

NOTE.—See Lesson V, page 79, note.

There are 7 trees in one row, and 7 trees in another row. How many trees in both rows?

How many trees are 7 trees and 7 trees? 8 trees and 7 trees? 9 trees and 7 trees?

How many are 7 and 7? 8 and 7? 9 and 7?

A man brought 14 melons to market, and sold 7 of them. How many melons had he left?

How many melons are 14 melons less 7 melons? 15 melons less 7 melons? 16 melons less 7 melons?

How many are 14 less 7? 15 less 7? 16 less 7?

---

4. How many are:

3 oranges and 7 oranges?	10 oranges less 7 oranges?
5 oranges and 7 oranges?	12 oranges less 7 oranges?
7 oranges and 7 oranges?	14 oranges less 7 oranges?
9 oranges and 7 oranges?	16 oranges less 7 oranges?
2 lemons and 7 lemons?	9 lemons less 7 lemons?
4 lemons and 7 lemons?	11 lemons less 7 lemons?
6 lemons and 7 lemons?	13 lemons less 7 lemons?
8 lemons and 7 lemons?	15 lemons less 7 lemons?

---

5. How many are:

1 and 7?	8 less 7?
2 and 7?	9 less 7?
3 and 7?	10 less 7?
4 and 7?	11 less 7?
5 and 7?	12 less 7?
6 and 7?	13 less 7?
7 and 7?	14 less 7?
8 and 7?	15 less 7?
9 and 7?	16 less 7?

Read and complete:

$1+7=$	$8-7=$
$2+7=$	$9-7=$
$3+7=$	$10-7=$
$4+7=$	$11-7=$
$5+7=$	$12-7=$
$6+7=$	$13-7=$
$7+7=$	$14-7=$
$8+7=$	$15-7=$
$9+7=$	$16-7=$

How many are 3 and 7? 10 less 7? 5 and 7? 12 less 7? 7 and 7? 14 less 7? 9 and 7? 16 less 7?

How many are 4 and 7? 11 less 7? 6 and 7? 13 less 7? 8 and 7? 15 less 7? 2 and 7? 9 less 7?

How many are 10 less 7? 10 less 3? 12 less 7? 12 less 3? 16 less 7? 16 less 9? 13 less 7? 13 less 6? 15 less 7? 15 less 8?

How many are 3 and 7? 5 and 7? 7 and 7? 9 and 7? 2 and 7? 4 and 7? 6 and 7? 8 and 7?

How many are 10 less 7? 12 less 7? 14 less 7? 16 less 7? 9 less 7? 11 less 7? 13 less 7? 15 less 7?

---

6. Separate each number, from 8 to 16 inclusive, into 7 and another number, and subtract 7 and its complement, thus found, from each original number, thus:

8 is 7 and —, or — and 7 ∴ 8 less 7 is —; 8 less — is 7.
9 is 7 and —, or — and 7 ∴ 9 less 7 is —; 9 less — is 7.
10 is 7 and —, or — and 7 ∴ 10 less 7 is —; 10 less — is 7.
11 is 7 and —, or — and 7 ∴ 11 less 7 is —; 11 less — is 7.
12 is 7 and —, or — and 7 ∴ 12 less 7 is —; 12 less — is 7.
13 is 7 and —, or — and 7 ∴ 13 less 7 is —; 13 less — is 7.
14 is 7 and — ∴ 14 less 7 is —.
15 is 7 and —, or — and 7 ∴ 15 less 7 is —; 15 less — is 7.
16 is 7 and —, or — and 7 ∴ 16 less 7 is —; 16 less — is 7.

NOTE.—See Lesson III, page 71, notes.

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7. How many balls are 7 balls and 7 balls, or two 7 balls?

How many 7 balls in 14 balls?

How many 7 balls in 15 balls, and how many balls over?

How many 7 balls in 10 balls, and how many balls over?

How many 7 balls in 16 balls, and how many balls over?

How many 7 balls in 12 balls, and how many balls over?

How many 4's in 10, and how many ones over?

How many 4's in 14, and how many ones over?

How many 4's in 11, and how many ones over?

How many 5's in 17, and how many ones over?

How many 5's in 13, and how many ones over?

How many 7's in 15, and how many ones over?

How many 7's in 17, and how many ones over?

---

8. Harry is 6 years old, and James is 7 years older than Harry. How old is James?

There are 8 girls and 7 boys in a class. How many pupils in the class?

There are 9 keys in one bunch, and 7 keys in another bunch. How many keys in both bunches?

Harry has 13 marbles, and Mark has 7 marbles. How many more marbles has Harry than Mark?

There are 7 ducks in the water, and 7 ducks on the land. How many ducks in the flock?

A hunter brought home 14 quails, and gave 7 of them to a neighbor. How many quails had he left?

James caught 5 fishes, and Clarence caught 7 fishes. How many fishes did both catch?

A little boy earned 15 cents by doing errands, and then paid 7 cents for a present to his sister. How many cents had he left?

A drover bought 16 cows, and then sold 7 of them. How many cows had he left?

A boy picked 13 pears, and gave 7 of them to his father. How many pears had he left?

How many cherries in 2 bunches, of 7 cherries each?

Mary picked 14 roses, and tied them in bunches of 7 roses each. How many bunches did they make?

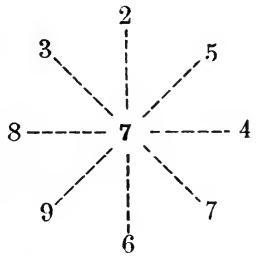
BOARD AND SLATE EXERCISES.

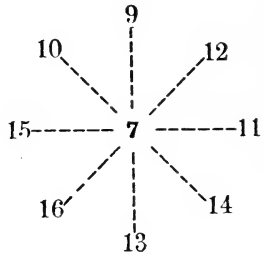
	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	7	7	7	7	7	7	7	7
Add	2	4	3	5	7	8	6	9

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
From	9	11	10	12	14	15	13	16
Take	7	7	7	7	7	7	7	7

a	*b*	*c*	*d*	*e*	*f*
2	2	3	1	2	3
2	7	2	1	4	3
7	1	7	7	3	7
2	2	3	5	3	2
2	4	2	4	3	2
Add 3	3	1	2	3	1

NOTE.—See Lesson III, page 73, notes.





NOTE.—See Lesson III, page 74, notes.

LESSON X.

1. How many shells are 1 shell and 8 shells? 9 shells less 8 shells? 2 shells and 8 shells? 10 shells less 8 shells? 3 shells and 8 shells? 11 shells less 8 shells?

Willie found 2 eggs in one nest and 8 eggs in another nest. How many eggs did he find?

How many eggs are 1 egg and 8 eggs? 2 eggs and 8 eggs? 3 eggs and 8 eggs?

How many are 1 and 8? 2 and 8? 3 and 8?

A man bought 10 oranges, and gave 8 of them to his children. How many oranges had the man left?

How many oranges are 9 oranges less 8 oranges? 10 oranges less 8 oranges? 11 oranges less 8 oranges?

How many are 9 less 8? 10 less 8? 11 less 8?

---

2. How many balls are 4 balls and 8 balls? 12 balls less 8 balls? 5 balls and 8 balls? 13 balls less 8 balls? 6 balls and 8 balls? 14 balls less 8 balls?

There are 4 boys and 8 girls in a class. How many pupils in the class?

How many apples are 4 apples and 8 apples? 5 apples and 8 apples? 6 apples and 8 apples?

How many are 4 and 8? 5 and 8? 6 and 8?

Mary picked 12 plums, and gave 8 of them to her mother. How many plums had Mary left?

How many plums are 4 plums and 8 plums? 5 plums and 8 plums? 6 plums and 8 plums?

How many are 4 and 8? 5 and 8? 6 and 8?

---

3. How many balls are 7 balls and 8 balls? 15 balls less 8 balls? 8 balls and 8 balls? 16 balls less 8 balls? 9 balls and 8 balls? 17 balls less 8 balls?

NOTE.—See Lesson V, page 79, note.

There are 7 girls in one class and 8 girls in another class. How many girls in both classes?

How many girls are 7 girls and 8 girls? 8 girls and 8 girls? 9 girls and 8 girls?

How many are 7 and 8? 8 and 8? 9 and 8?

How many balls are left when 8 balls are taken from 15 balls? 8 balls from 16 balls? 8 balls from 17 balls?

How many lemons are 15 lemons less 8 lemons? 16 lemons less 8 lemons? 17 lemons less 8 lemons?

How many are 15 less 8? 16 less 8? 17 less 8?

---

4. How many are:

3 roses and 8 roses?	11 roses less 8 roses?
5 roses and 8 roses?	13 roses less 8 roses?
7 roses and 8 roses?	15 roses less 8 roses?
9 roses and 8 roses?	17 roses less 8 roses?
2 pears and 8 pears?	10 pears less 8 pears?
4 pears and 8 pears?	12 pears less 8 pears?
6 pears and 8 pears?	14 pears less 8 pears?
8 pears and 8 pears?	16 pears less 8 pears?

How many are 9 cents less 8 cents? 11 cents less 8 cents? 13 cents less 8 cents? 15 cents less 8 cents? 17 cents less 8 cents?

How many are 10 cents less 8 cents? 12 cents less 8 cents? 14 cents less 8 cents? 16 cents less 8 cents?

---

5. How many are:

1 and 8?	9 less 8?
2 and 8?	10 less 8?
3 and 8?	11 less 8?
4 and 8?	12 less 8?
5 and 8?	13 less 8?
6 and 8?	14 less 8?
7 and 8?	15 less 8?
8 and 8?	16 less 8?
9 and 8?	17 less 8?

Read and complete:

$1+8=$	$9-8=$
$2+8=$	$10-8=$
$3+8=$	$11-8=$
$4+8=$	$12-8=$
$5+8=$	$13-8=$
$6+8=$	$14-8=$
$7+8=$	$15-8=$
$8+8=$	$16-8=$
$9+8=$	$17-8=$

How many are 1 and 8? 3 and 8? 5 and 8? 7 and 8? 9 and 8?

How many are 2 and 8? 4 and 8? 6 and 8? 8 and 8? 10 and 8?

How many are 3 and 8? 11 less 8? 13 less 8? 15 less 8? 17 less 8? 10 less 8? 12 less 8? 14 less 8? 16 less 8?

---

7. Separate each of the numbers, from 9 to 17 inclusive, into 8 and another number, and then subtract 8 and its complement from the original number, thus:

9 is 8 and —, or — and 8 ∴ 9 less 8 is —; 9 less — is 8.
10 is 8 and —, or — and 8 ∴ 10 less 8 is —; 10 less — is 8.
11 is 8 and —, or — and 8 ∴ 11 less 8 is —; 11 less — is 8.
12 is 8 and —, or — and 8 ∴ 12 less 8 is —; 12 less — is 8.
13 is 8 and —, or — and 8 ∴ 13 less 8 is —; 13 less — is 8.
14 is 8 and —, or — and 8 ∴ 14 less 8 is —; 14 less — is 8.
15 is 8 and —, or — and 8 ∴ 15 less 8 is —; 15 less — is 8.
16 is 8 and — ∴ 16 less 8 is —.
17 is 8 and —, or — and 8 ∴ 17 less 8 is —; 17 less — is 8.

NOTE.—See Lesson III, page 71, notes.

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8. How many balls are 8 and 8 balls, or two 8 balls?
How many 8 balls in 16 balls?
How many are two 8's?
How many 8's in 16?
How many 8 balls in 10 balls, and how many over?
How many 8 balls in 12 balls, and how many over?
How many 8 balls in 17 balls, and how many over?
How many 8 balls in 18 balls, and how many over?
How many 4's in 8? 4's in 12? 4's in 16?

How many 5's in 10? 5's in 15? 5's in 20?
How many 5's in 17, and how many over?
How many 5's in 14, and how many over?

---

9. A boy caught 2 fishes, and sold 1 of them for 5 cents and the other for 8 cents. How many cents did he receive?

ANS.: 13 *cents:* 5 *cents and* 8 *cents are* 13 *cents.*

Harry bought a box of figs, and gave 8 figs to his brother, and had 8 figs left. How many figs did he buy?

A boy was carrying a basket with 15 eggs in it, and a large dog ran against the basket and broke 8 of the eggs. How many eggs were left unbroken?

Clarence is 14 years old and Edgar is 8 years old. How much older is Clarence than Edgar?

Susan wrote 18 words on her slate, and then rubbed out 8 words. How many words were left on her slate?

Frank caught 17 fine trout, and sold 8 of them to Edward. How many trout had Frank left?

A grocer bought a dozen brooms, and sold 8 of them. How many brooms had he left?

Edward's father gave him 10 cents, and his mother gave him 5 cents. How many cents had he?

If Edward should pay 8 cents for a slate, how many cents would he have left?

How many more cents in 12 cents than in 8 cents? In 15 cents than in 8 cents?

Charles has 18 cents, and William has 8 cents. How many more cents has Charles than William?

Helen has 17 roses, and Kate has 8 roses. How many many more roses has Helen than Kate?

John has 9 marbles, and James 8 marbles. How many marbles have both of the boys?

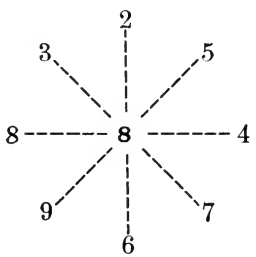
BOARD AND SLATE EXERCISES.

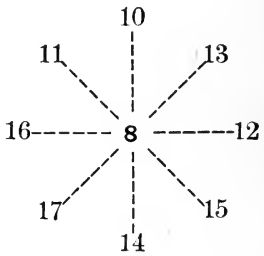
	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
	8	8	8	8	8	8	8	8	8
Add	2	1	3	5	4	6	8	7	9

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
From	10	9	11	13	12	14	16	15	17
Take	8	8	8	8	8	8	8	8	8

	a	*b*	*c*	*d*	*e*	*f*	*g*
	1	1	2	1	2	3	3
	2	3	2	2	3	3	2
	4	2	2	1	6	2	3
	8	8	8	8	4	4	5
Add	3	5	6	7	3	5	6

NOTE.—See Lesson III, page 73, notes.





NOTE.—See Lesson III, page 74, notes.

LESSON XI.

1. How many shells are 1 shell and 9 shells? 10 shells less 9 shells? 2 shells and 9 shells? 11 shells less 9 shells? 3 shells and 9 shells? 12 shells less 9 shells?

There are 9 sheep in one field, and 3 sheep in another field. How many sheep in both fields?

How many sheep are 3 sheep and 9 sheep? 2 sheep and 9 sheep? 1 sheep and 9 sheep?

How many are 1 and 9? 2 and 9? 3 and 9?

Kate wrote 12 words on her slate, and then rubbed out 9 of them. How many words were left on her slate?

How many words are 10 words less 9 words? 11 words less 9 words? 12 words less 9 words?

How many are 10 less 9? 11 less 9? 12 less 9?

---

2. How many balls are 4 balls and 9 balls? 13 balls less 9 balls? 5 balls and 9 balls? 14 balls less 9 balls? 6 balls and 9 balls? 15 balls less 9 balls?

NOTE.—See Lesson V, page 79, note.

There are 4 boys and 9 girls in a reading class. How many pupils in the class?

How many pupils are 4 pupils and 9 pupils? 5 pupils and 9 pupils? 6 pupils and 9 pupils?

How many are 4 and 9? 5 and 9? 6 and 9?

There are 13 pupils in a class, and 9 of them are girls. How many boys in the class?

How many pupils are 13 pupils less 9 pupils? 14 pupils less 9 pupils? 15 pupils less 9 pupils?

How many are 13 less 9? 14 less 9? 15 less 9?

---

3. How many balls are 7 balls and 9 balls? 16 balls less 9 balls? 8 balls and 9 balls? 17 balls less 9 balls? 9 balls and 9 balls? 18 balls less 9 balls?

John has 7 marbles in his left hand and 9 marbles in his right hand. How many marbles has John in both hands?

How many marbles are 7 marbles and 9 marbles? 8 marbles and 9 marbles? 9 marbles and 9 marbles?

How many are 7 and 9? 8 and 9? 9 and 9?

Charles picked 16 cherries, and gave 9 of them to his sister. How many cherries had he left?

How many cherries are 16 cherries less 9 cherries? 17 cherries less 9 cherries? 18 cherries less 9 cherries?

How many are 16 less 9? 17 less 9? 18 less 9?

How many are 3 men and 9 men? 5 men and 9 men? 7 men and 9 men? 9 men and 9 men?

How many are 2 sheep and 9 sheep? 4 sheep and 9 sheep? 6 sheep and 9 sheep? 8 sheep and 9 sheep?

---

4. How many are:

2 figs and 9 figs?	11 figs less 9 figs?
4 figs and 9 figs?	13 figs less 9 figs?
6 figs and 9 figs?	15 figs less 9 figs?
8 figs and 9 figs?	17 figs less 9 figs?
1 cent and 9 cents?	10 cents less 9 cents?
3 cents and 9 cents?	12 cents less 9 cents?
5 cents and 9 cents?	14 cents less 9 cents?
7 cents and 9 cents?	16 cents less 9 cents?
9 cents and 9 cents?	18 cents less 9 cents?

---

5. How many are:

1 and 9?	10 less 9?
2 and 9?	11 less 9?
3 and 9?	12 less 9?
4 and 9?	13 less 9?
5 and 9?	14 less 9?
6 and 9?	15 less 9?
7 and 9?	16 less 9?
8 and 9?	17 less 9?
9 and 9?	18 less 9?

Read and complete:

$1+9=$	$10-9=$
$2+9=$	$11-9=$
$3+9=$	$12-9=$
$4+9=$	$13-9=$
$5+9=$	$14-9=$
$6+9=$	$15-9=$
$7+9=$	$16-9=$
$8+9=$	$17-9=$
$9+9=$	$18-9=$

How many are 2 and 9? 4 and 9? 6 and 9? 8 and 9? 3 and 9? 5 and 9? 7 and 9? 9 and 9?

How many are 11 less 9? 13 less 9? 15 less 9? 17 less 9? 12 less 9? 14 less 9? 16 less 9? 18 less 9?

6. Separate each of the numbers, from 10 to 18 inclusive, into 9 and another number, and then subtract 9 and its complement, thus found, from the original numbers, thus:

10 is 9 and —, or — and 9 ∴ 10 less 9 is —; 10 less — is 9.
11 is 9 and —, or — and 9 ∴ 11 less 9 is —; 11 less — is 9.
12 is 9 and —, or — and 9 ∴ 12 less 9 is —; 12 less — is 9.
13 is 9 and —, or — and 9 ∴ 13 less 9 is —; 13 less — is 9.
14 is 9 and —, or — and 9 ∴ 14 less 9 is —; 14 less — is 9.
15 is 9 and —, or — and 9 ∴ 15 less 9 is —; 15 less — is 9.
16 is 9 and —, or — and 9 ∴ 16 less 9 is —; 16 less — is 9.
17 is 9 and —, or — and 9 ∴ 17 less 9 is —; 17 less — is 9.
18 is 9 and — ∴ 18 less 9 is —.

NOTE.—See Lesson III, page 71, notes.

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7. How many balls are 9 balls and 9 balls, or two 9 balls?

How many are two 9's?
How many 9 balls in 18 balls?
How many 9's in 18?
How many 9 balls in 12 balls, and how many over?
How many 9 balls in 15 balls, and how many over?
How many 9 balls in 20 balls, and how many over?
How many 9 balls in 19 balls, and how many over?
How many 6's in 12? 6's in 18?
How many 6's in 16, and how many over?
How many 7's in 16, and how many over?
How many 5's in 16, and how many over?
How many 4's in 15, and how many over?

---

8. There are 5 cows in one field and 9 cows in another field. How many cows in both fields? How many more cows in the second field than in the first?

Jane read 7 verses, and Kate read 9 verses more than Jane. How many verses did Kate read?

A farmer has 13 acres of wheat and 9 acres of corn. How many more acres of wheat than of corn has he?

There are 15 yards of ribbon on a spool: if a clerk sell 9 yards of the ribbon, how many yards will be left on the spool?

A huckster bought 8 chickens of one farmer and 7 chickens of another. How many chickens did he buy? If he sell 9 chickens, how many chickens will he then have?

A man started to walk 14 miles to reach a railroad station, and when he had walked 9 miles he stopped at a hotel for dinner. How many miles was he from the station?

A hunter saw 15 quails in a flock, and 9 of them flew away. How many quails were left?

A lady gave 16 dollars for a shawl and 9 dollars for a dress. How many dollars did the shawl cost more than the dress?

A boy earned 5 cents on Thursday, 4 cents on Friday, and 9 cents on Saturday. How many cents did he earn in the three days?

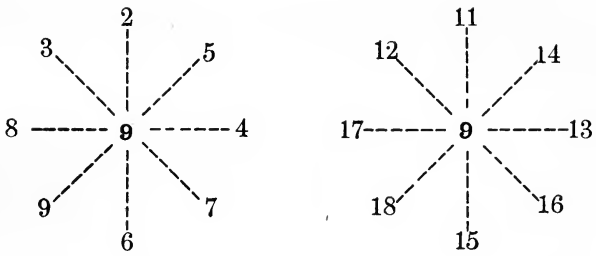
SLATE AND BOARD EXERCISES.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
	9	9	9	9	9	9	9	9	9
Add	1	3	5	2	4	6	8	7	9

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
From	10	12	14	11	13	15	17	16	18
Take	9	9	9	9	9	9	9	9	9

	a	*b*	*c*	*d*	*e*	*f*
	2	3	2	1	1	2
	2	2	2	2	2	1
	3	1	9	9	9	9
	9	9	3	3	3	3
Add	3	4	2	3	4	5

NOTE.—See Lesson III, page 73, notes.



NOTE.—See Lesson III, page 74, notes.

LESSON XII.

Primary Combinations in Addition.

1. There are only forty-five primary combinations in addition, and these are produced by adding each digital number (10 not included) to *itself* and to each of the *higher* digital numbers, as follows:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
1+1	2+2	3+3	4+4	5+5	6+6	7+7	8+8	9+9
2+1	3+2	4+3	5+4	6+5	7+6	8+7	9+8	
3+1	4+2	5+3	6+4	7+5	8+6	9+7		
4+1	5+2	6+3	7+4	8+5	9+6			
5+1	6+2	7+3	8+4	9+5				
6+1	7+2	8+3	9+4					
7+1	8+2	9+3						
8+1	9+2							
9+1								

NOTE.—The above combinations should be written on the board, and the pupils drilled in adding each from left to right, and from right to left (the first in each column excepted), thus: "b" = 3 + 2 = 5; 2 + 3 = 5. The two may be combined, thus: 3 + 2 or 2 + 3 = 5. This is a condensed addition table.

2. These primary combinations in addition, and the inverse processes in subtraction, may be written on the board, as follows:

$1+1=2$	$\therefore$	$2-1=1$	
$2+1$ or $1+2=$	$\therefore$	$3-1=$	; $3-2=$
$3+1$ or $1+3=$	$\therefore$	$4-1=$	; $4-3=$
$4+1$ or $1+4=$	$\therefore$	$5-1=$	; $5-4=$
$5+1$ or $1+5=$	$\therefore$	$6-1=$	; $6-5=$
$6+1$ or $1+6=$	$\therefore$	$7-1=$	; $7-6=$
$7+1$ or $1+7=$	$\therefore$	$8-1=$	; $8-7=$
$8+1$ or $1+8=$	$\therefore$	$9-1=$	; $9-8=$
$9+1$ or $1+9=$	$\therefore$	$10-1=$	; $10-9=$
$2+2=4$	$\therefore$	$4-2=2$	
$3+2$ or $2+3=$	$\therefore$	$5-2=$	; $5-3=$
$4+2$ or $2+4=$	$\therefore$	$6-2=$	; $6-4=$
$5+2$ or $2+5=$	$\therefore$	$7-2=$	; $7-5=$
$6+2$ or $2+6=$	$\therefore$	$8-2=$	; $8-6=$
$7+2$ or $2+7=$	$\therefore$	$9-2=$	; $9-7=$
$8+2$ or $2+8=$	$\therefore$	$10-2=$	; $10-8=$
$9+2$ or $2+9=$	$\therefore$	$11-2=$	; $11-9=$
$3+3=$	$\therefore$	$6-3=$	
$4+3$ or $3+4=$	$\therefore$	$7-3=$	; $7-4=$
$5+3$ or $3+5=$	$\therefore$	$8-3=$	; $8-5=$
$6+3$ or $3+6=$	$\therefore$	$9-3=$	; $9-6=$
$7+3$ or $3+7=$	$\therefore$	$10-3=$	; $10-7=$
$8+3$ or $3+8=$	$\therefore$	$11-3=$	; $11-8=$
$9+3$ or $3+9=$	$\therefore$	$12-3=$	; $12-9=$
$4+4=$	$\therefore$	$8-4=$	
$5+4$ or $4+5=$	$\therefore$	$9-4=$	; $9-5=$
$6+4$ or $4+6=$	$\therefore$	$10-4=$	; $10-6=$
$7+4$ or $4+7=$	$\therefore$	$11-4=$	; $11-7=$
$8+4$ or $4+8=$	$\therefore$	$12-4=$	; $12-8=$
$9+4$ or $4+9=$	$\therefore$	$13-4=$	; $13-9=$

$5+5=$ $\therefore$ $10-5=$

$6+5$ or $5+6=$ $\therefore$ $11-5=$; $11-6=$

$7+5$ or $5+7=$ $\therefore$ $12-5=$; $12-7=$

$8+5$ or $5+8=$ $\therefore$ $13-5=$; $13-8=$

$9+5$ or $5+9=$ $\therefore$ $14-5=$; $14-9=$

$6+6=$ $\therefore$ $12-6=$

$7+6$ or $6+7=$ $\therefore$ $13-6=$; $13-7=$

$8+6$ or $6+8=$ $\therefore$ $14-6=$; $14-8=$

$9+6$ or $6+9=$ $\therefore$ $15-6=$; $15-9=$

$7+7=$ $\therefore$ $14-7=$

$8+7$ or $7+8=$ $\therefore$ $15-7=$; $15-8=$

$9+7$ or $7+9=$ $\therefore$ $16-7=$; $16-9=$

$8+8=$ $\therefore$ $16-8=$

$9+8$ or $8+9=$ $\therefore$ $17-8=$; $17-9=$

$9+9=$ $\therefore$ $18-9=$

LESSON XIII.

Separate each number, from 11 to 19 inclusive, into two digital numbers (including 10), and take from it each digital number found, thus:

11 is 10 and —, or 1 and — $\therefore$ 11 less 1 is —; 11 less 10 is —.
11 is 9 and —, or 2 and — $\therefore$ 11 less 2 is —; 11 less 9 is —.
11 is 8 and —, or 3 and — $\therefore$ 11 less 3 is —; 11 less 8 is —.
11 is 7 and —, or 4 and — $\therefore$ 11 less 4 is —; 11 less 7 is —.
11 is 6 and —, or 5 and — $\therefore$ 11 less 5 is —; 11 less 6 is —.

---

12 is 10 and —, or 2 and — $\therefore$ 12 less 2 is —; 12 less 10 is —.
12 is 9 and —, or 3 and — $\therefore$ 12 less 3 is —; 12 less 9 is —.
12 is 8 and —, or 4 and — $\therefore$ 12 less 4 is —; 12 less 8 is —.
12 is 7 and —, or 5 and — $\therefore$ 12 less 5 is —; 12 less 7 is —.
12 is 6 and —, $\therefore$ 12 less 6 is —.

13 is 10 and —, or 3 and — ∴ 13 less 3 is —; 13 less 10 is —.
13 is 9 and —, or 4 and — ∴ 13 less 4 is —; 13 less 9 is —.
13 is 8 and —, or 5 and — ∴ 13 less 5 is —; 13 less 8 is —.
13 is 7 and —, or 6 and — ∴ 13 less 6 is —; 13 less 7 is —.

---

14 is 10 and —, or 4 and — ∴ 14 less 4 is —; 14 less 10 is —.
14 is 9 and —, or 5 and — ∴ 14 less 5 is —; 14 less 9 is —.
14 is 8 and —, or 6 and — ∴ 14 less 6 is —; 14 less 8 is —.
14 is 7 and —, ∴ 14 less 7 is —.

---

15 is 10 and —, or 5 and — ∴ 15 less 5 is —; 15 less 10 is —.
15 is 9 and —, or 6 and — ∴ 15 less 6 is —; 15 less 9 is —.
15 is 8 and — or 7 and — ∴ 15 less 7 is —; 15 less 8 is —.

---

16 is 10 and —, or 6 and — ∴ 16 less 6 is —; 16 less 10 is —.
16 is 9 and —, or 7 and — ∴ 16 less 7 is —; 16 less 9 is —.
16 is 8 and —, ∴ 16 less 8 is —.

---

17 is 10 and —, or 7 and — ∴ 17 less 7 is —; 17 less 10 is —.
17 is 9 and —, or 8 and — ∴ 17 less 8 is —; 17 less 9 is —.

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18 is 10 and —, or 8 and — ∴ 18 less 8 is —; 18 less 10 is —.
18 is 9 and —, ∴ 18 less 9 is —.

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19 is 10 and —, or 9 and — ∴ 19 less 9 is —; 19 less 10 is —.

NOTE.—The above exercises may be written on the board, using the signs +, —, and =, respectively, in place of "and," "less," and "is." They may also be copied by the pupils on their slates, and the blank spaces filled with the proper numbers.

THIRD-YEAR COURSE.

NOTE.—The first fourteen lessons in the ELEMENTARY ARITHMETIC (now used by the pupils) are chiefly devoted to addition and subtraction; and, since the needed preparatory oral training is given in the second-year course, but little oral instruction will be required the first half of the third school year.

The exercises in the first lesson (page 116) are introductory to the third year lessons in addition and subtraction, as well as to those in multiplication and division, and hence the exercises in this first lesson should precede Lesson III in the ELEMENTARY ARITHMETIC. The "Supplemental Blackboard Exercises" in addition and subtraction given in this manual (on page 155), should accompany the blackboard exercises in the ELEMENTARY ARITHMETIC.

With the above exceptions, the following lessons are intended to be introductory to and to accompany the exercises in multiplication and division in the ELEMENTARY ARITHMETIC, and hence may be given the last half of the third school year. It is suggested that the exercises in this manual precede the corresponding exercises in the book used by the pupils.

In some schools, it may be found practicable and desirable to give a part of this oral course in multiplication and division in the latter part of the *second* school year, before an elementary book is put into the hands of the pupils. When this is done, no attempt should be made to advance beyond Lesson V.

ORAL LESSONS IN MULTIPLICATION AND DIVISION.

Aims.

To teach (1) *the product of any two digital numbers, and* (2) *the division of this product by each of its two factors.*

Steps.

1. The finding of the number corresponding to the product of any two digital numbers, by adding one of them to itself continuously as many times as there are units in the other given number, *less one;* or (2) by adding one of the numbers to its product by a number *one less* than the other given number.

2. The fixing of the product of any two digital numbers in the memory, so that it may be recalled instantly, *without adding*, when its two factors are given.

3. The teaching of the division of any product by each of its two digital factors as the *inverse* of the process of their multiplication.

LESSON I.

The Numbers 20 to 100.

NOTE.—The object of this lesson is to develop the numbers from 20 to 100 inclusive, teach their names, and their representation by figures.

The teacher should be supplied with a numeral frame, ten bunches of rods or cards of ten each, and ten loose rods or cards; also a blackboard.

In asking the questions, the teacher should present the appropriate objects, and otherwise "suit the action to the word."

---

1. Here is the numeral frame. Let us see how many balls on it.

How many wires in the numeral frame? "10 wires."

How many balls on the first wire? On the second wire? On each wire? "10 balls."

How many 10 balls on the two upper wires? "Two 10 balls." How many balls on these two wires?

Here are the figures that stand for twenty: *20.*

How many 10 balls on the three upper wires? "Three 10 balls." How many balls? "30 balls." Three tens are how many?

Here are the figures that stand for thirty: *30.*

How many 10 balls on the four upper wires? "Four 10 balls." Four 10 balls are how many balls?

How many balls on any 4 wires? Four tens are how many?

Here are the figures that stand for forty: *40.*

How many 10 balls on the 5 upper wires? Five 10 balls are how many balls?

How many balls on any 5 wires? Five tens are how many?

Here are the figures that stand for fifty: *50.*

NOTE.—In like manner, teach the numbers 60, 70, 80, 90, 100.

There are 10 rods (or cards) in each of these bunches. How many rods in 2 bunches? 3 bunches? 5 bunches? 6 bunches? 4 bunches? 7 bunches? 9 bunches? 8 bunches? 10 bunches?

NOTE.—Write on the blackboard the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90, 100; and, pointing to them promiscuously, have the pupils read them. Next call attention to the fact that the figures 1, 2, 3, etc., *denote* the number of tens, 40 being 4 tens, 50 being 5 tens, etc.

Write neatly on your slates the numbers 10, 20, 30, etc., to 100.

---

2. How many balls on the two upper wires? I will now slide these 20 balls to the right end of the wires, and all the other balls to the left end of the wires.

If I now slide 1 ball on the third wire to the right under the 20 balls, how many balls will there be at the right? "21 balls."

If I slide 2 balls on the third wire under the 20 balls, how many balls will there be?

NOTE.—Slide back the 1 ball and slide 2 balls under the 20 balls. Slide successively 3 balls, 4 balls, etc., under the 20 balls, and have pupils give the number of balls. Then ask the following questions, sliding in each case the designated number of balls under the 20 balls.

How many balls are 20 balls and 1 ball? 20 balls and 2 balls? 20 balls and 3 balls? 20 balls and 4 balls? 20 balls and 5 balls? 20 balls and 6 balls? 20 balls and 7 balls? 20 balls and 8 balls? 20 balls and 9 balls? 20 balls and 10 balls?

NOTE.—Take the pencils (or cards), and holding 2 bunches in your left hand, add successively 2 pencils, 5 pencils, 3 pencils, etc., and have the pupils give in each case the number of pencils.

---

3. How many are:

20 and 1?	20 and 6?
20 and 2?	20 and 7?
20 and 3?	20 and 8?
20 and 4?	20 and 9?
20 and 5?	20 and 10?

I will now write on the board a line of ten 2's, thus:

2 2 2 2 2 2 2 2 2 2

What figure must I make at the right of the first 2 to express *twenty?* "0." At the right of the second 2 to express *twenty-one?* At the right of the third 2 to express *twenty-two?* At the right of the fourth 2 to ex-express *twenty-three?*

NOTE.—In like manner, write the figures 4, 5, etc., at the right of the remaining 2's in the line, and teach what number the two figures in each case express.

What number do I now write on the board? (Write 23 on the board.) How many does the figure 2 denote? "2 tens or twenty." How many does the figure 3 denote? "3 ones."

NOTE.—In like manner, teach what each figure in 24, 25, etc., expresses. Write on the blackboard promiscuously the numbers from 20 to 29 inclusive, and, pointing to them, have the pupils read the numbers designated.

Write neatly on your slates, in a column, all the numbers from 20 to 29 inclusive.

NOTE.—In like manner, teach the numbers between 30 and 40, and between 40 and 50. When this is done, the pupils will be able to write the numbers between 50 and 60, 60 and 70, 70 and 80, 80 and 90, and 90 and 100.

LESSON II.

1. How many times do I say *la*, children: *La*, *la*. How many times now: *La*, *la*, *la*. Say *la* twice. Say *la* three times. Say *la* four times.

Clap your hands twice. Clap them 3 times. 4 times. 5 times.

How many times do I write the figure 2 on the board? (Write it 3 times in one group, 4 times in another, etc.)

How many times 1 finger do I hold up? (Hold up 3 fingers, 4 fingers, etc.)

How many pencils are 3 times 1 pencil? 4 times 1 pencil? 6 times 1 pencil?

How many are three 1's? 3 times 1? Five 1's? 5 times 1? Seven 1's? 7 times 1?

2. How many are:

2+2, or 2 twos?
2+2+2, or 3 twos?
2+2+2+2, or 4 twos?
2+2+2+2+2, or 5 twos?
2+2+2+2+2+2, or 6 twos?
2+2+2+2+2+2+2, or 7 twos?
2+2+2+2+2+2+2+2, or 8 twos?
2+2+2+2+2+2+2+2+2, or 9 twos?
2+2+2+2+2+2+2+2+2+2, or 10 twos?

3. How many are:

Two	2 balls?	2 times 2 balls?
Three	2 balls?	3 times 2 balls?
Four	2 balls?	4 times 2 balls?
Five	2 balls?	5 times 2 balls?
Six	2 balls?	6 times 2 balls?
Seven	2 balls?	7 times 2 balls?
Eight	2 balls?	8 times 2 balls?
Nine	2 balls?	9 times 2 balls?
Ten	2 balls?	10 times 2 balls?

NOTE.—Take the numeral frame and slide all the balls to the left-hand side, and slide to the right-hand side 2 balls on the first wire, 2 balls on the second wire, 2 balls on the third wire, etc., to correspond with the question. If in any case the pupil can not give the product, let him add 2 balls to the product next preceding. Continue the drill, asking the questions promiscuously, and sliding the balls in a corresponding manner.

How many blocks are 2 times 2 blocks? 3 times 2 blocks? 5 times 2 blocks? 7 times 2 blocks? 9 times 2 blocks?

How many blocks are 4 times 2 blocks? 6 times 2 blocks? 8 times 2 blocks? 10 times 2 blocks?

4. How many are:	How many times:
3 times 2 horses?	2 horses in 6 horses?
5 times 2 horses?	2 horses in 10 horses?
2 times 2 bushels?	2 bushels in 4 bushels?
6 times 2 bushels?	2 bushels in 6 bushels?
4 times 2 yards?	2 yards in 8 yards?
8 times 2 yards?	2 yards in 16 yards?
7 times 2 yards?	2 yards in 14 yards?
9 times 2 dollars?	2 dollars in 18 dollars?
10 times 2 dollars?	2 dollars in 20 dollars?

5.	
1 time 2?	2 in 2?
2 times 2?	2 in 4?
3 times 2?	2 in 6?
4 times 2?	2 in 8?
5 times 2?	2 in 10?
6 times 2?	2 in 12?
7 times 2?	2 in 14?
8 times 2?	2 in 16?
9 times 2?	2 in 18?

NOTES.—1. Write the above tables on the board, and have them recited in three ways, as follows: *First*, The two tables together, thus (taking third line): 3 times 2 are 6, 2 in 6 three times. *Second*, The two tables separately, drilling on the table of products until they are recited *without the least hesitation. Third*, The tables promiscuously.

2. There are advantages in forming and learning the tables of products *in consecutive order*, but, in determining an unknown product, pupils should be required *to add* the multiplicand to the product next *preceding.*

How many are 4 times 2? 6 times 2? 3 times 2? 5 times 2? 7 times 2? 9 times 2? 8 times 2? 10 times 2?

How many times 2 in 8? 2 in 12? 2 in 6? 2 in 10? 2 in 14? 2 in 18? 2 in 16? 2 in 20?

6. There are 2 pints in a quart: how many pints in 5 quarts?

ANS.: 10 *pints:* 5 *times* 2 *pints are* 10 *pints.*

How many pints in 4 quarts? In 6 quarts? In 8 quarts? In 10 quarts? In 7 quarts?

How much will 3 oranges cost at 2 cents apiece? 7 oranges? 9 oranges?

What is the cost of 4 two-cent postage stamps? 6 two-cent stamps? 10 two-cent stamps?

There are 2 horses in a span: how many horses in 3 spans? 5 spans? 7 spans?

SLATE AND BOARD EXERCISES.

Copy and fill:		Copy and fill:	
$1 \times 2 =$	$2 \div 2 =$	$3 \times 2 =$	$6 \div 2 =$
$2 \times 2 =$	$4 \div 2 =$	$5 \times 2 =$	$10 \div 2 =$
$3 \times 2 =$	$6 \div 2 =$	$6 \times 2 =$	$12 \div 2 =$
$4 \times 2 =$	$8 \div 2 =$	$8 \times 2 =$	$16 \div 2 =$
$5 \times 2 =$	$10 \div 2 =$	$4 \times 2 =$	$8 \div 2 =$
$6 \times 2 =$	$12 \div 2 =$	$7 \times 2 =$	$14 \div 2 =$
$7 \times 2 =$	$14 \div 2 =$	$9 \times 2 =$	$18 \div 2 =$
$8 \times 2 =$	$16 \div 2 =$	$2 \times 2 =$	$4 \div 2 =$
$9 \times 2 =$	$18 \div 2 =$	$7 \times 2 =$	$14 \div 2 =$
$10 \times 2 =$	$20 \div 2 =$	$9 \times 2 =$	$18 \div 2 =$

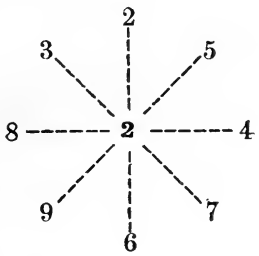
NOTE.—Read the sign $\times$ *times*, and the sign $\div$, *divided by.*

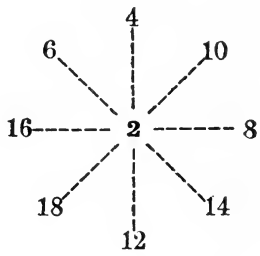
---

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	2	2	2	2	2	2	2	2
Multiply	2	4	6	3	7	9	5	8

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Divide	2)4	2)8	2)12	2)6	2)14	2)18	2)10	2)16

The following "Wheel Exercises" may be used to supplement the above board drills:





NOTE.—In the left-hand table the number 2, at the center, is to be multiplied by each of the outer numbers; in the right-hand table, each of the outer numbers is to be divided by the number 2 at the center.

LESSON III.

1. How many balls are:

Two	3 balls?	2	times 3 balls?
Three	3 balls?	3	times 3 balls?
Four	3 balls?	4	times 3 balls?
Five	3 balls?	5	times 3 balls?
Six	3 balls?	6	times 3 balls?
Seven	3 balls?	7	times 3 balls?
Eight	3 balls?	8	times 3 balls?
Nine	3 balls?	9	times 3 balls?
Ten	3 balls?	10	times 3 balls?

NOTE.—Slide all the balls to the left-hand side of the numeral frame, as in Lesson II, and then slide to right-hand side as many 3 balls as are designated in each question. Find each unknown product by adding 3 balls to the product next preceding.

2. How many are:

3 times 3 peaches?
5 times 3 peaches?
7 times 3 pears?
9 times 3 pears?
2 times 3 melons?
4 times 3 melons?
6 times 3 dollars?
8 times 3 dollars?
10 times 3 dollars?

How many times:

3 peaches in 9 peaches?
3 peaches in 15 peaches?
3 pears in 21 pears?
3 pears in 27 pears?
3 melons in 6 melons?
3 melons in 12 melons?
3 dollars in 18 dollars?
3 dollars in 24 dollars?
3 dollars in 30 dollars?

3.

1 time 3?	3 in 3?
2 times 3?	3 in 6?
3 times 3?	3 in 9?
4 times 3?	3 in 12?
5 times 3?	3 in 15?
6 times 3?	3 in 18?
7 times 3?	3 in 21?
8 times 3?	3 in 24?
9 times 3?	3 in 27?
10 times 3?	3 in 30?

NOTE.—See Lesson II, page 121, notes.

How many are 5 times 3? 7 times 3? 9 times 3? 6 times 3? 8 times 3? 10 times 3?

How many are 4 times 3? 3 in 12? 6 times 3? 3 in 18? 8 times 3? 3 in 24? 10 times 3? 3 in 30? 7 times 3? 3 in 21? 9 times 3? 3 in 27?

---

4. There are 3 feet in a yard: how many feet in 4 yards? In 6 yards? 8 yards?

A stem of clover has 3 leaves: how many leaves on 5 stems? 4 stems? 9 stems?

John walks 3 miles a day in going to school: how many miles does he walk in 5 days? In 10 days?

How much will 5 lemons cost at 3 cents apiece? 8 lemons at 3 cents apiece?

How many lemons, at 3 cents each, can be bought for 15 cents? For 24 cents?

How much will 6 pencils cost at 3 cents each? 10 pencils?

How many pencils, at 3 cents each, can be bought for 18 cents? For 30 cents?

There are 3 bushels of wheat in a sack: how many bushels in 10 sacks? In 7 sacks?

Kate is 3 years old, and her father is 9 times as old as she: how old is her father?

A mason earns 3 dollars a day: how much will he earn in 6 days? In 10 days?

How many times 3 dollars in 15 dollars? In 30 dollars? In 18 dollars?

5.

$6 = 2\times$	, or $3\times$	$\therefore$	$6\div 3 =$	; $6\div 2 =$
$9 = 3\times$		$\therefore$	$9\div 3 =$	
$12 = 4\times$	, or $3\times$	$\therefore$	$12\div 3 =$	; $12\div 4 =$
$15 = 5\times$	, or $3\times$	$\therefore$	$15\div 3 =$	; $15\div 5 =$
$18 = 6\times$	, or $3\times$	$\therefore$	$18\div 3 =$	; $18\div 6 =$
$21 = 7\times$	, or $3\times$	$\therefore$	$21\div 3 =$	; $21\div 7 =$
$24 = 8\times$	, or $3\times$	$\therefore$	$24\div 3 =$	; $24\div 8 =$
$27 = 9\times$	, or $3\times$	$\therefore$	$27\div 3 =$	; $27\div 9 =$
$30 = 10\times$	, or $3\times$	$\therefore$	$30\div 3 =$	; $30\div 10 =$

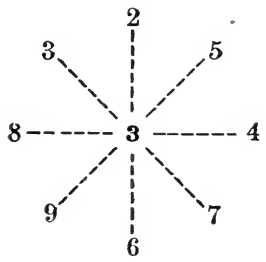
NOTE.—Write the above inverse tables on the blackboard, and have the pupils recite them, thus: (3) 12 is 3 times 4 or 4 times 3 $\therefore$ 4 in 12 three times; 3 in 12 four times. Or (if preferred): 12 equals 3 times 4 or 4 times 3 $\therefore$ 12 divided by 4 equals 3; 12 divided by 3 equals 4. Finally, have the pupils copy the tables neatly on their slates, filling the blanks.

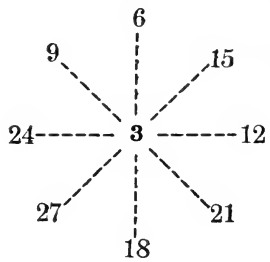
SLATE AND BOARD EXERCISES.

$1 \times 3 =$	$3 \div 3 =$	$2 \times 3 =$	$6 \div 3 =$
$2 \times 3 =$	$6 \div 3 =$	$4 \times 3 =$	$12 \div 3 =$
$3 \times 3 =$	$9 \div 3 =$	$6 \times 3 =$	$18 \div 3 =$
$4 \times 3 =$	$12 \div 3 =$	$8 \times 3 =$	$24 \div 3 =$
$5 \times 3 =$	$15 \div 3 =$	$3 \times 3 =$	$9 \div 3 =$
$6 \times 3 =$	$18 \div 3 =$	$5 \times 3 =$	$15 \div 3 =$
$7 \times 3 =$	$21 \div 3 =$	$7 \times 3 =$	$21 \div 3 =$
$8 \times 3 =$	$24 \div 3 =$	$9 \times 3 =$	$27 \div 3 =$
$9 \times 3 =$	$27 \div 3 =$	$1 \times 3 =$	$3 \div 3 =$
$10 \times 3 =$	$30 \div 3 =$	$10 \times 3 =$	$30 \div 3 =$

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	3	3	3	3	3	3	3	3
Multiply	3	2	4	6	8	5	7	9

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Divide	3)9	3)6	3)12	3)18	3)24	3)15	3)21	3)27





LESSON IV.

1. How many balls are:

Two 4 balls?	2 times 4 balls?
Three 4 balls?	3 times 4 balls?
Four 4 balls?	4 times 4 balls?

Five 4 balls?	5 times 4 balls?
Six 4 balls?	6 times 4 balls?
Seven 4 balls?	7 times 4 balls?
Eight 4 balls?	8 times 4 balls?
Nine 4 balls?	9 times 4 balls?
Ten 4 balls?	10 times 4 balls?

NOTE.—See Lesson II, page 120, note.

---

2. How many are:

	How many times:
2 times 4 balls?	4 balls in 8 balls?
4 times 4 balls?	4 balls in 16 balls?
6 times 4 balls?	4 balls in 24 balls?
3 times 4 balls?	4 balls in 12 balls?
5 times 4 balls?	4 balls in 20 balls?
7 times 4 balls?	4 balls in 28 balls?
9 times 4 balls?	4 balls in 36 balls?
8 times 4 balls?	4 balls in 32 balls?
10 times 4 balls?	4 balls in 40 balls?

3.

1 time 4?	4 in 4?
2 times 4?	4 in 8?
3 times 4?	4 in 12?
4 times 4?	4 in 16?
5 times 4?	4 in 20?
6 times 4?	4 in 24?
7 times 4?	4 in 28?
8 times 4?	4 in 32?
9 times 4?	4 in 36?
10 times 4?	4 in 40?

NOTE.—See Lesson II, page 121, notes.

How many are 3 times 4? 5 times 4? 7 times 4? 4 times 4? 6 times 4? 8 times 4? 7 times 4? 9 times 4? 10 times 4?

How many are 2 times 4? 4 in 8? 4 times 4? 4 in 16? 6 times 4? 4 in 24? 3 times 4? 4 in 12? 5 times 4? 4 in 20? 7 times 4? 4 in 28? 9 times 4? 4 in 36? 8 times 4? 4 in 32?

---

4. There are 4 quarts in a gallon: how many quarts in 2 gallons? In 4 gallons?

There are 4 pecks in a bushel: how many pecks in 3 bushels? In 5 bushels?

How many pecks in 6 bushels? In 8 bushels?

A horse has 4 hoofs: how many hoofs have 10 horses? 7 horses? 9 horses?

How many shoes does it take to shoe a horse? To shoe 4 horses?

How much will 6 lemons cost at 4 cents apiece? 10 lemons at 4 cents apiece?

At 4 cents apiece, how many lemons can be bought for 12 cents? For 24 cents?

Kate bought 8 spools of thread at 4 cents a spool: how much did they cost?

How many spools of thread, at 4 cents each, can be bought for 20 cents? For 32 cents?

What will 6 bunches of beets cost at 4 cents a bunch?

How many bunches of beets, at 4 cents each, can be bought for 24 cents? For 28 cents?

How many beets in 5 bunches, if there be 4 beets in each bunch?

What will 6 pencils cost at 4 cents apiece?

At 4 cents apiece, how many pencils can be bought for 24 cents?

At 4 cents apiece, how many oranges can be bought for 28 cents?

What will 10 oranges cost at 4 cents apiece?

5. $4 = 1\times$, or $4\times$ $\therefore$ $4\div 4 =$; $4\div 1 =$
$8 = 2\times$, or $4\times$ $\therefore$ $8\div 4 =$; $8\div 2 =$
$12 = 3\times$, or $4\times$ $\therefore$ $12\div 4 =$; $12\div 3 =$
$16 = 4\times$ $\therefore$ $16\div 4 =$
$20 = 5\times$, or $4\times$ $\therefore$ $20\div 4 =$; $20\div 5 =$
$24 = 6\times$, or $4\times$ $\therefore$ $24\div 4 =$; $24\div 6 =$
$28 = 7\times$, or $4\times$ $\therefore$ $28\div 4 =$; $28\div 7 =$
$32 = 8\times$, or $4\times$ $\therefore$ $32\div 4 =$; $32\div 8 =$
$36 = 9\times$, or $4\times$ $\therefore$ $36\div 4 =$; $36\div 9 =$
$40 = 10\times$, or $4\times$ $\therefore$ $40\div 4 =$; $40\div 10 =$

NOTE.—See Lesson III, page 125, note.

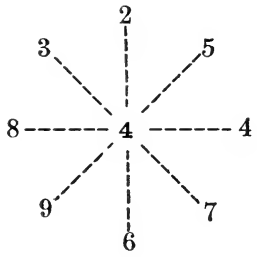
SLATE AND BOARD EXERCISES.

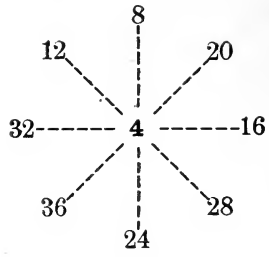
$1\times 4 =$; $4\div 4 =$	$3\times 4 =$; $12\div 4 =$
$2\times 4 =$; $8\div 4 =$	$5\times 4 =$; $20\div 4 =$
$3\times 4 =$; $12\div 4 =$	$7\times 4 =$; $28\div 4 =$
$4\times 4 =$; $16\div 4 =$	$4\times 4 =$; $16\div 4 =$
$5\times 4 =$; $20\div 4 =$	$2\times 4 =$; $8\div 4 =$
$6\times 4 =$; $24\div 4 =$	$6\times 4 =$; $24\div 4 =$
$7\times 4 =$; $28\div 4 =$	$8\times 4 =$; $32\div 4 =$
$8\times 4 =$; $32\div 4 =$	$7\times 4 =$; $28\div 4 =$
$9\times 4 =$; $36\div 4 =$	$9\times 4 =$; $36\div 4 =$
$10\times 4 =$; $40\div 4 =$	$1\times 4 =$; $4\div 4 =$

---

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Multiply	4	4	4	4	4	4	4	4
	2	6	4	5	3	9	7	8

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Divide	4)8	4)24	4)16	4)20	4)12	4)36	4)28	4)32





LESSON V.

1. How many balls are:

Two	5 balls?	2 times 5 balls?
Three	5 balls?	3 times 5 balls?
Four	5 balls?	4 times 5 balls?
Five	5 balls?	5 times 5 balls?
Six	5 balls?	6 times 5 balls?
Seven	5 balls?	7 times 5 balls?
Eight	5 balls?	8 times 5 balls?
Nine	5 balls?	9 times 5 balls?
Ten	5 balls?	10 times 5 balls?

NOTE.—See Lesson II, page 120, note.

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2. How many are:	How many times:
3 times 5 chairs?	5 chairs in 15 chairs?
5 times 5 chairs?	5 chairs in 25 chairs?
2 times 5 books?	5 books in 10 books?
4 times 5 books?	5 books in 20 books?
6 times 5 pens?	5 pens in 30 pens?
8 times 5 pens?	5 pens in 40 pens?
7 times 5 cents?	5 cents in 35 cents?
9 times 5 cents?	5 cents in 45 cents?
10 times 5 cents?	5 cents in 50 cents?

3.

1 time 5?	5 in 5?
2 times 5?	5 in 10?
3 times 5?	5 in 15?
4 times 5?	5 in 20?
5 times 5?	5 in 25?
6 times 5?	5 in 30?
7 times 5?	5 in 35?
8 times 5?	5 in 40?
9 times 5?	5 in 45?
10 times 5?	5 in 50?

NOTE.—See Lesson II, page 121, notes.

How many are 5 times 5? 4 times 5? 6 times 5? 8 times 5? 10 times 5? 7 times 5? 9 times 5? 3 times 5?

How many are 3 times 5? 5 in 15? 5 times 5? 5 in 25? 7 times 5? 5 in 35? 9 times 5? 5 in 45? 4 times 5? 5 in 20? 6 times 5? 5 in 30? 8 times 5? 5 in 40? 10 times 5? 5 in 50?

---

4. There are 5 nails on a man's hand: how many nails on 2 hands? On 4 hands?

There are 5 desks in a row: how many desks in 4 rows? In 6 rows?

There are 5 cents in a "nickel": how many cents in 5 nickels? In 10 nickels?

If there be 5 radishes in a bunch, how many radishes in 7 bunches? In 9 bunches?

How much will 6 oranges cost, at 5 cents apiece? 8 oranges at 5 cents apiece?

How many oranges, at 5 cents each, can be bought for 30 cents? For 40 cents?

How much will 4 yards of ribbon cost at 5 cents a yard? 8 yards at 5 cents a yard?

How many yards of ribbon, at 5 cents a yard, can be bought for 40 cents? For 50 cents?

A boy put 35 beets into bunches of 5 beets each: how many bunches did they make?

5.

5 = 1 ×	, or 5 ×	∴	5 ÷ 5 =	;	5 ÷ 1 =
10 = 2 ×	, or 5 ×	∴	10 ÷ 5 =	;	10 ÷ 2 =
15 = 3 ×	, or 5 ×	∴	15 ÷ 5 =	;	15 ÷ 3 =
20 = 4 ×	, or 5 ×	∴	20 ÷ 5 =	;	20 ÷ 4 =
25 = 5 ×	,	∴	25 ÷ 5 =		
30 = 6 ×	, or 5 ×	∴	30 ÷ 5 =	;	30 ÷ 6 =
35 = 7 ×	, or 5 ×	∴	35 ÷ 5 =	;	35 ÷ 7 =
40 = 8 ×	, or 5 ×	∴	40 ÷ 5 =	;	40 ÷ 8 =
45 = 9 ×	, or 5 ×	∴	45 ÷ 5 =	;	45 ÷ 9 =
50 = 10 ×	, or 5 ×	∴	50 ÷ 5 =	;	50 ÷ 10 =

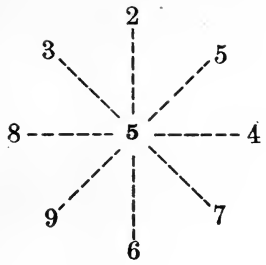
NOTE.—See Lesson III, page 125, note.

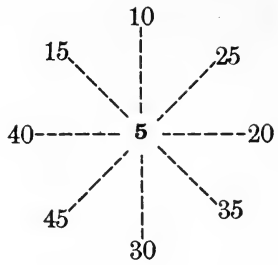
BOARD AND SLATE EXERCISES.

1 × 5 =	5 ÷ 5 =	3 × 5 =	15 ÷ 5 =
2 × 5 =	10 ÷ 5 =	1 × 5 =	5 ÷ 5 =
3 × 5 =	15 ÷ 5 =	4 × 5 =	20 ÷ 5 =
4 × 5 =	20 ÷ 5 =	6 × 5 =	30 ÷ 5 =
5 × 5 =	25 ÷ 5 =	5 × 5 =	25 ÷ 5 =
6 × 5 =	30 ÷ 5 =	7 × 5 =	35 ÷ 5 =
7 × 5 =	35 ÷ 5 =	2 × 5 =	10 ÷ 5 =
8 × 5 =	40 ÷ 5 =	8 × 5 =	40 ÷ 5 =
9 × 5 =	45 ÷ 5 =	10 × 5 =	50 ÷ 5 =
10 × 5 =	50 ÷ 5 =	9 × 5 =	45 ÷ 5 =

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	5	5	5	5	5	5	5	5
Multiply	4	3	2	8	6	9	5	7

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Divide	5)20	5)15	5)10	5)40	5)30	5)45	5)25	5)35





LESSON VI.

1. How many balls are:

Two	6 balls?	2 times	6 balls?
Three	6 balls?	3 times	6 balls?
Four	6 balls?	4 times	6 balls?
Five	6 balls?	5 times	6 balls?
Six	6 balls?	6 times	6 balls?
Seven	6 balls?	7 times	6 balls?
Eight	6 balls?	8 times	6 balls?
Nine	6 balls?	9 times	6 balls?
Ten	6 balls?	10 times	6 balls?

NOTE.—See Lesson II, page 120, note.

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2. How many are:	How many times:
3 times 6 boys?	6 boys in 18 boys?
5 times 6 boys?	6 boys in 30 boys?
2 times 6 slates?	6 slates in 12 slates?
4 times 6 slates?	6 slates in 24 slates?
6 times 6 knives?	6 knives in 36 knives?
8 times 6 knives?	6 knives in 48 knives?
7 times 6 words?	6 words in 42 words?
9 times 6 words?	6 words in 54 words?
10 times 6 cents?	6 cents in 60 cents?

3.	1 time 6?	6 in 6?
	2 times 6?	6 in 12?
	3 times 6?	6 in 18?
	4 times 6?	6 in 24?
	5 times 6?	6 in 30?
	6 times 6?	6 in 36?
	7 times 6?	6 in 42?
	8 times 6?	6 in 48?
	9 times 6?	6 in 54?
	10 times 6?	6 in 60?

How many are 3 times 6? 5 times 6? 7 times 6? 9 times 6? 4 times 6? 6 times 6? 8 times 6? 10 times 6?

How many times 6 in 12? 6 in 24? 6 in 36? 6 in 48? 6 in 60? 6 in 18? 6 in 42? 6 in 30? 6 in 54?

---

4. A beetle has 6 legs: how many legs have 3 beetles? 5 beetles?

An orchard contains 6 rows of peach trees, and there are 6 trees in each row: how many peach trees in the orchard?

A school-room has 7 rows of desks, with 6 desks in each row: how many desks in the room?

Jane wrote 10 words, and each word contained 6 letters: how many letters did she write?

John made 8 columns of figures on his slate, and each column had 6 figures: how many figures did he make?

How much will 6 slates cost at 6 cents apiece? 8 slates at 6 cents apiece?

How many slates, at 6 cents each, can be bought for 30 cents? For 60 cents?

What will 7 heads of cabbages cost at 6 cents a head?

How many melons, at 6 cents each, can be bought for 42 cents? For 54 cents?

5.

$6 = 1 \times$, or $6 \times$ $\therefore$ $6 \div 6 =$; $6 \div 1 =$
$12 = 2 \times$, or $6 \times$ $\therefore$ $12 \div 6 =$; $12 \div 2 =$
$18 = 3 \times$, or $6 \times$ $\therefore$ $18 \div 6 =$; $18 \div 3 =$
$24 = 4 \times$, or $6 \times$ $\therefore$ $24 \div 6 =$; $24 \div 4 =$
$30 = 5 \times$, or $6 \times$ $\therefore$ $30 \div 6 =$; $30 \div 5 =$
$36 = 6 \times$ $\therefore$ $36 \div 6 =$
$42 = 7 \times$, or $6 \times$ $\therefore$ $42 \div 6 =$; $42 \div 7 =$
$48 = 8 \times$, or $6 \times$ $\therefore$ $48 \div 6 =$; $48 \div 8 =$
$54 = 9 \times$, or $6 \times$ $\therefore$ $54 \div 6 =$; $54 \div 9 =$
$60 = 10 \times$, or $6 \times$ $\therefore$ $60 \div 6 =$; $60 \div 10 =$

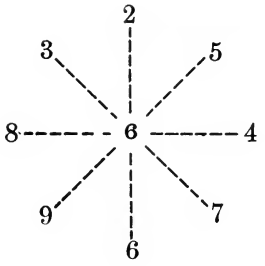
NOTE.—See Lesson III, page 125, note.

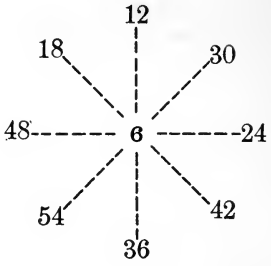
SLATE AND BOARD EXERCISES.

$1 \times 6 =$	$6 \div 6 =$
$2 \times 6 =$	$12 \div 6 =$
$3 \times 6 =$	$18 \div 6 =$
$4 \times 6 =$	$24 \div 6 =$
$5 \times 6 =$	$30 \div 6 =$
$6 \times 6 =$	$36 \div 6 =$
$7 \times 6 =$	$42 \div 6 =$
$8 \times 6 =$	$48 \div 6 =$
$9 \times 6 =$	$54 \div 6 =$
$10 \times 6 =$	$60 \div 6 =$

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	6	6	6	6	6	6	6	6
Multiply	3	2	5	4	6	9	7	8
	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Divide	6)18	6)12	6)30	6)24	6)36	6)54	6)42	6)48

	3	6	1	4	2	5	9	10
	6	6	6	6	6	6	6	6
Multiply	3	6	1	4	2	5	9	10
	18	36	6	24	12	30	54	60
Divide	6)18	6)36	6)6	6)24	6)12	6)30	6)54	6)60





LESSON VII.

1. How many balls are:

Two	7 balls?	2 times	7 balls?
Three	7 balls?	3 times	7 balls?
Four	7 balls?	4 times	7 balls?
Five	7 balls?	5 times	7 balls?
Six	7 balls?	6 times	7 balls?
Seven	7 balls?	7 times	7 balls?
Eight	7 balls?	8 times	7 balls?
Nine	7 balls?	9 times	7 balls?
Ten	7 balls?	10 times	7 balls?

NOTE.—See Lesson II, page 120, note.

2.

How many are:	How many times:
3 times 7 marbles?	7 marbles in 21 marbles?
5 times 7 marbles?	7 marbles in 35 marbles?
7 times 7 pencils?	7 pencils in 49 pencils?
4 times 7 pencils?	7 pencils in 28 pencils?
6 times 7 soldiers?	7 soldiers in 42 soldiers?
8 times 7 soldiers?	7 soldiers in 56 soldiers?
2 times 7 figures?	7 figures in 14 figures?
9 times 7 dollars?	7 dollars in 63 dollars?
10 times 7 dollars?	7 dollars in 70 dollars?

3.

1 time 7?	7 in 7?
2 times 7?	7 in 14?
3 times 7?	7 in 21?
4 times 7?	7 in 28?
5 times 7?	7 in 35?
6 times 7?	7 in 42?
7 times 7?	7 in 49?
8 times 7?	7 in 56?
9 times 7?	7 in 63?
10 times 7?	7 in 70?

How many are 3 times 7? 5 times 7? Twice 7? 4 times 7? 8 times 7? 6 times 7? 9 times 7? 7 times 7? 10 times 7?

How many times 7 in 21? 7 in 35? 7 in 14? 7 in 28? 7 in 56? 7 in 42? 7 in 63? 7 in 49? 7 in 70?

---

4. There are 7 days in a week: how many days in 4 weeks? In 8 weeks? 6 weeks?

If there are 7 boards in a length of fence, how many boards will make 5 lengths? 9 lengths?

There are 7 hills of tomatoes in a row: how many hills in 7 rows? In 10 rows?

If a slate cost 7 cents, how many cents will 6 slates cost? 9 slates?

How many slates, at 7 cents each, can be bought for 35 cents? 49 cents? 63 cents?

How much will 6 melons cost at 7 cents apiece? 10 melons at 7 cents apiece?

How many yards of muslin, at 7 cents a yard, can be bought for 42 cents? For 70 cents?

How many pounds of sugar, at 7 cents a pound, can be bought for 63 cents? For 56 cents?

5.

7 = 1 ×	, or 7 ×	∴	7 ÷ 7 =	;	7 ÷ 1 =
14 = 2 ×	, or 7 ×	∴	14 ÷ 7 =	;	14 ÷ 2 =
21 = 3 ×	, or 7 ×	∴	21 ÷ 7 =	;	21 ÷ 3 =
28 = 4 ×	, or 7 ×	∴	28 ÷ 7 =	;	28 ÷ 4 =
35 = 5 ×	, or 7 ×	∴	35 ÷ 7 =	;	35 ÷ 5 =
42 = 6 ×	, or 7 ×	∴	42 ÷ 7 =	;	42 ÷ 6 =
49 = 7 ×		∴	49 ÷ 7 =		
56 = 8 ×	, or 7 ×	∴	56 ÷ 7 =	;	56 ÷ 8 =
63 = 9 ×	, or 7 ×	∴	63 ÷ 7 =	;	63 ÷ 9 =
70 = 10 ×	, or 7 ×	∴	70 ÷ 7 =	;	70 ÷ 10 =

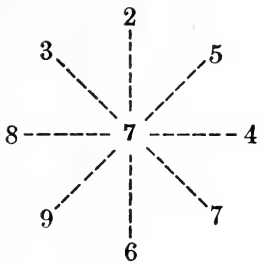
SLATE AND BOARD EXERCISES.

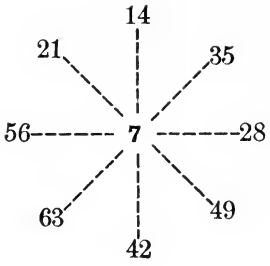
1 × 7 =	7 ÷ 7 =	2 × 7 =	14 ÷ 7 =
2 × 7 =	14 ÷ 7 =	4 × 7 =	28 ÷ 7 =
3 × 7 =	21 ÷ 7 =	1 × 7 =	7 ÷ 7 =
4 × 7 =	28 ÷ 7 =	3 × 7 =	21 ÷ 7 =
5 × 7 =	35 ÷ 7 =	5 × 7 =	35 ÷ 7 =
6 × 7 =	42 ÷ 7 =	7 × 7 =	49 ÷ 7 =
7 × 7 =	49 ÷ 7 =	6 × 7 =	42 ÷ 7 =
8 × 7 =	56 ÷ 7 =	8 × 7 =	56 ÷ 7 =
9 × 7 =	63 ÷ 7 =	10 × 7 =	70 ÷ 7 =
10 × 7 =	70 ÷ 7 =	9 × 7 =	63 ÷ 7 =

---

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	7	7	7	7	7	7	7	7
Multiply	3	4	2	5	7	9	6	8

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Divide	7)21	7)28	7)14	7)35	7)49	7)63	7)42	7)56





LESSON VIII.

1. How many balls are:

Two 8 balls?	2 times 8 balls?
Three 8 balls?	3 times 8 balls?
Four 8 balls?	4 times 8 balls?
Five 8 balls?	5 times 8 balls?
Six 8 balls?	6 times 8 balls?
Seven 8 balls?	7 times 8 balls?
Eight 8 balls?	8 times 8 balls?
Nine 8 balls?	9 times 8 balls?
Ten 8 balls?	10 times 8 balls?

NOTE.—See Lesson II, page 120, note.

2. How many are:	How many times:
3 times 8 quarts?	8 quarts in 24 quarts?
5 times 8 quarts?	8 quarts in 40 quarts?
2 times 8 yards?	8 yards in 16 yards?
4 times 8 yards?	8 yards in 32 yards?
6 times 8 pounds?	8 pounds in 48 pounds?
8 times 8 pounds?	8 pounds in 64 pounds?
10 times 8 pounds?	8 pounds in 80 pounds?
7 times 8 dimes?	8 dimes in 56 dimes?
9 times 8 dimes?	8 dimes in 72 dimes?

3.

1 time 8?	8 in 8?
2 times 8?	8 in 16?
3 times 8?	8 in 24?
4 times 8?	8 in 32?
5 times 8?	8 in 40?
6 times 8?	8 in 48?
7 times 8?	8 in 56?
8 times 8?	8 in 64?
9 times 8?	8 in 72?
10 times 8?	8 in 80?

How many are 3 times 8? 7 times 8? 5 times 8? 9 times 8? 4 times 8? 8 times 8? 2 times 8? 6 times 8? 10 times 8?

How many times 8 in 24? 8 in 56? 8 in 40? 8 in 72? 8 in 32? 8 in 64? 8 in 16? 8 in 48? 8 in 80?

---

4. There are 8 quarts in a peck: how many quarts in 5 pecks? In 7 pecks? In 10 pecks?

If there be 8 panes of glass in 1 window, how many panes of glass in 4 windows? 8 windows?

A school-room has 6 rows of desks, with 8 desks in each row: how many desks in the school-room?

Helen has written 9 columns of figures on her slate, with 8 figures in each column: how many figures has she written?

A railroad car has 8 wheels: how many wheels in a train of 6 cars?

How many pecks in 40 quarts? In 80 quarts?

If a man work 8 hours a day, how many hours will he work in 6 days?

How many melons, at 8 cents each, can be bought for 56 cents? For 64 cents?

How many pounds of sugar, at 8 cents a pound, can be bought for 40 cents? For 72 cents?

5.

$8 =$	$1 \times$	, or $8 \times$	$\therefore$	$8 \div 8 =$	;	$8 \div 1 =$
$16 =$	$2 \times$	, or $8 \times$	$\therefore$	$16 \div 8 =$	;	$16 \div 2 =$
$24 =$	$3 \times$	, or $8 \times$	$\therefore$	$24 \div 8 =$	;	$24 \div 3 =$
$32 =$	$4 \times$	, or $8 \times$	$\therefore$	$32 \div 8 =$	;	$32 \div 4 =$
$40 =$	$5 \times$	, or $8 \times$	$\therefore$	$40 \div 8 =$	;	$40 \div 5 =$
$48 =$	$6 \times$	, or $8 \times$	$\therefore$	$48 \div 8 =$	;	$48 \div 6 =$
$56 =$	$7 \times$	, or $8 \times$	$\therefore$	$56 \div 8 =$	;	$56 \div 7 =$
$64 =$	$8 \times$		$\therefore$	$64 \div 8 =$		
$72 =$	$9 \times$	, or $8 \times$	$\therefore$	$72 \div 8 =$	;	$72 \div 9 =$
$80 =$	$10 \times$	, or $8 \times$	$\therefore$	$80 \div 8 =$	;	$80 \div 10 =$

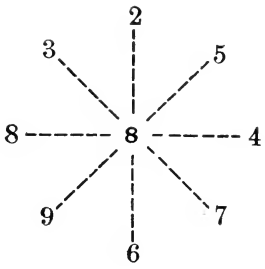
SLATE AND BOARD EXERCISES.

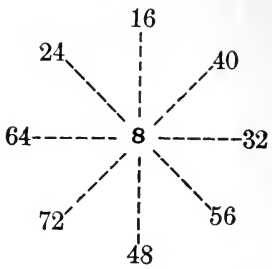
$1 \times 8 =$	$8 \div 8 =$	$3 \times 8 =$	$24 \div 8 =$
$2 \times 8 =$	$16 \div 8 =$	$1 \times 8 =$	$8 \div 8 =$
$3 \times 8 =$	$24 \div 8 =$	$4 \times 8 =$	$32 \div 8 =$
$4 \times 8 =$	$32 \div 8 =$	$2 \times 8 =$	$16 \div 8 =$
$5 \times 8 =$	$40 \div 8 =$	$5 \times 8 =$	$40 \div 8 =$
$6 \times 8 =$	$48 \div 8 =$	$8 \times 8 =$	$64 \div 8 =$
$7 \times 8 =$	$56 \div 8 =$	$6 \times 8 =$	$48 \div 8 =$
$8 \times 8 =$	$64 \div 8 =$	$9 \times 8 =$	$72 \div 8 =$
$9 \times 8 =$	$72 \div 8 =$	$7 \times 8 =$	$56 \div 8 =$
$10 \times 8 =$	$80 \div 8 =$	$10 \times 8 =$	$80 \div 8 =$

---

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	8	8	8	8	8	8	8	8
Multiply	3	2	4	5	7	9	6	8

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Divide	8)24	8)16	8)32	8)40	8)56	8)72	8)48	8)64





LESSON IX.

1. How many balls are:

Two 9 balls?	2 times 9 balls?
Three 9 balls?	3 times 9 balls?
Four 9 balls?	4 times 9 balls?
Five 9 balls?	5 times 9 balls?
Six 9 balls?	6 times 9 balls?
Seven 9 balls?	7 times 9 balls?
Eight 9 balls?	8 times 9 balls?
Nine 9 balls?	9 times 9 balls?
Ten 9 balls?	10 times 9 balls?

NOTE.—See Lesson II, page 120, note.

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2. How many are:	How many times:
3 times 9 miles?	9 miles in 27 miles?
7 times 9 miles?	9 miles in 63 miles?
4 times 9 inches?	9 inches in 36 inches?
6 times 9 inches?	9 inches in 54 inches?
2 times 9 pints?	9 pints in 18 pints?
5 times 9 pints?	9 pints in 54 pints?
8 times 9 rings?	9 rings in 72 rings?
10 times 9 rings?	9 rings in 90 rings?
9 times 9 rings?	9 rings in 81 rings?

3.

1 time 9?	9 in 9?
2 times 9?	9 in 18?
3 times 9?	9 in 27?
4 times 9?	9 in 36?
5 times 9?	9 in 45?
6 times 9?	9 in 54?
7 times 9?	9 in 63?
8 times 9?	9 in 72?
9 times 9?	9 in 81?
10 times 9?	9 in 90?

How many are 3 times 9? 7 times 9? 5 times 9? 9 times 9? 4 times 9? 2 times 9? 8 times 9? 6 times 9? 10 times 9?

How many times 9 in 27? 9 in 63? 9 in 45? 9 in 81? 9 in 36? 9 in 18? 9 in 72? 9 in 54? 9 in 90?

---

4. An orchard has 9 rows of trees, and 9 trees in each row: how many trees in the orchard?

Five of the rows of trees in the orchard are apple trees, and 4 of them are peach trees: how many apple trees in the orchard? How many peach trees?

If a man work 9 hours a day, how many hours will he work in 6 days? In 10 days?

How many dollars will 7 sheep cost at 9 dollars a head? 8 sheep? 5 sheep?

At 9 dollars a head, how many sheep can be bought for 45 dollars? For 63 dollars? For 36 dollars?

How many cents will 6 pounds of sugar cost at 9 cents a pound? 8 pounds?

At 9 cents a pound, how many pounds of sugar can be bought for 54 cents? For 72 cents?

How many bunches will 63 radishes make, if 9 radishes be put in each bunch?

5.

9 =	1 ×	, or 9 ×	∴	9 ÷ 9 =	;	9 ÷ 1 =
18 =	2 ×	, or 9 ×	∴	18 ÷ 9 =	;	18 ÷ 2 =
27 =	3 ×	, or 9 ×	∴	27 ÷ 9 =	;	27 ÷ 3 =
36 =	4 ×	, or 9 ×	∴	36 ÷ 9 =	;	36 ÷ 4 =
45 =	5 ×	, or 9 ×	∴	45 ÷ 9 =	;	45 ÷ 5 =
54 =	6 ×	, or 9 ×	∴	54 ÷ 9 =	;	54 ÷ 6 =
63 =	7 ×	, or 9 ×	∴	63 ÷ 9 =	;	63 ÷ 7 =
72 =	8 ×	, or 9 ×	∴	72 ÷ 9 =	;	72 ÷ 8 =
81 =	9 ×		∴	81 ÷ 9 =		
90 =	10 ×	, or 9 ×	∴	90 ÷ 9 =	;	90 ÷ 10 =

SLATE AND BOARD EXERCISES.

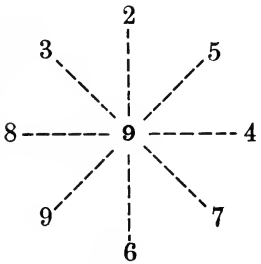
1 × 9 =	9 ÷ 9 =	2 × 9 =	18 ÷ 9 =
2 × 9 =	18 ÷ 9 =	4 × 9 =	36 ÷ 9 =
3 × 9 =	27 ÷ 9 =	6 × 9 =	54 ÷ 9 =
4 × 9 =	36 ÷ 9 =	1 × 9 =	9 ÷ 9 =
5 × 9 =	45 ÷ 9 =	3 × 9 =	27 ÷ 9 =
6 × 9 =	54 ÷ 9 =	5 × 9 =	45 ÷ 9 =
7 × 9 =	63 ÷ 9 =	8 × 9 =	72 ÷ 9 =
8 × 9 =	72 ÷ 9 =	10 × 9 =	90 ÷ 9 =
9 × 9 =	81 ÷ 9 =	7 × 9 =	63 ÷ 9 =
10 × 9 =	90 ÷ 9 =	9 × 9 =	81 ÷ 9 =

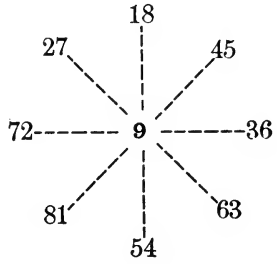
NOTE.—The successive products of 9 by the other digital numbers are 9, 18, 27, 36, 45, 54, 63, 72, 81, and 90. It is an interesting fact that the sum of the digits that express each of these products is 9. It is also seen that, in the successive products, the ten figure increases by one, and the unit figure decreases by one.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	9	9	9	9	9	9	9	9
Multiply	2	4	7	3	5	9	6	8

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Divide	9)18	9)36	9)63	9)27	9)45	9)81	9)54	9)72

NOTE.—Additional blackboard exercises in multiplying numbers, written in horizontal lines, may be easily provided by writing a horizontal line of figures on the board, and having the number expressed by each figure, beginning at the right, multiplied by a designated digital number, as 4, 7, 9, etc. The division of the successive products by each of its factors will afford excellent practice in division.





LESSON X.

1. How many balls are:

Two	10 balls?	2	times 10 balls?
Three	10 balls?	3	times 10 balls?
Four	10 balls?	4	times 10 balls?
Five	10 balls?	5	times 10 balls?
Six	10 balls?	6	times 10 balls?
Seven	10 balls?	7	times 10 balls?
Eight	10 balls?	8	times 10 balls?
Nine	10 balls?	9	times 10 balls?
Ten	10 balls?	10	times 10 balls?

NOTE.—See Lesson II, page 120, note.

2. How many are:	How many times:
3 times 10 dimes?	10 dimes in 30 dimes?
5 times 10 dimes?	10 dimes in 50 dimes?
7 times 10 dollars?	10 dollars in 70 dollars?
4 times 10 dollars?	10 dollars in 40 dollars?
2 times 10 fingers?	10 fingers in 20 fingers?
6 times 10 fingers?	10 fingers in 60 fingers?
8 times 10 cents?	10 cents in 80 cents?
9 times 10 cents?	10 cents in 90 cents?
10 times 10 cents?	10 cents in 100 cents?

3.	
1 time 10?	10 in 10?
2 times 10?	10 in 20?
3 times 10?	10 in 30?
4 times 10?	10 in 40?
5 times 10?	10 in 50?
6 times 10?	10 in 60?
7 times 10?	10 in 70?
8 times 10?	10 in 80?
9 times 10?	10 in 90?
10 times 10?	10 in 100?

How many are 3 times 10? 7 times 10? 4 times 10? 2 times 10? 6 times 10? 8 times 10? 5 times 10? 9 times 10? 10 times 10?

How many times 10 in 30? 10 in 70? 10 in 40? 10 in 20? 10 in 60? 10 in 80? 10 in 50? 10 in 90? 10 in 100?

---

4. There are 10 cents in a dime: how many cents in 5 dimes? 3 dimes? 10 dimes?

How many dimes in 50 cents? In 30 cents? In 60 cents? In 100 cents?

There are 10 dimes in a dollar: how many dimes in 4 dollars? In 6 dollars? In 8 dollars?

How many dollars in 40 dimes? In 60 dimes? In 80 dimes? In 100 dimes?

There are 10 postage stamps in a row, and 10 rows in a sheet: how many postage stamps in a sheet?

How many postage stamps in 5 rows or one half of a sheet? In 7 rows? In 9 rows?

What is the cost of 6 yards of calico, at 10 cents a yard? 7 yards? 10 yards?

How many pounds of cheese, at 10 cents a pound, can be bought for 60 cents? For 70 cents? 100 cents?

5.

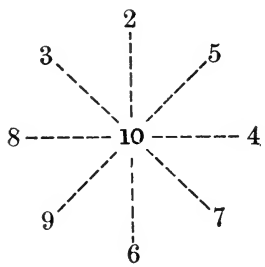
$10 = 1\times$	, or $10\times$	$\therefore$	$10\div 10 =$;	$10\div 1 =$
$20 = 2\times$	, or $10\times$	$\therefore$	$20\div 10 =$;	$20\div 2 =$
$30 = 3\times$	, or $10\times$	$\therefore$	$30\div 10 =$;	$30\div 3 =$
$40 = 4\times$	, or $10\times$	$\therefore$	$40\div 10 =$;	$40\div 4 =$
$50 = 5\times$	, or $10\times$	$\therefore$	$50\div 10 =$;	$50\div 5 =$
$60 = 6\times$	, or $10\times$	$\therefore$	$60\div 10 =$;	$60\div 6 =$
$70 = 7\times$	, or $10\times$	$\therefore$	$70\div 10 =$;	$70\div 7 =$
$80 = 8\times$	, or $10\times$	$\therefore$	$80\div 10 =$;	$80\div 8 =$
$90 = 9\times$	, or $10\times$	$\therefore$	$90\div 10 =$;	$90\div 9 =$
$100 = 10\times$		$\therefore$	$100\div 10 =$	

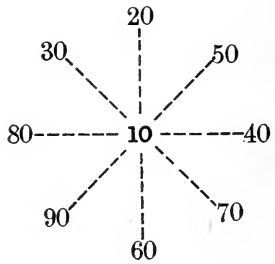
SLATE AND BOARD EXERCISES.

$1\times 10 =$	$10\div 10 =$	$3\times 10 =$	$30\div 10 =$
$2\times 10 =$	$20\div 10 =$	$6\times 10 =$	$60\div 10 =$
$3\times 10 =$	$30\div 10 =$	$2\times 10 =$	$20\div 10 =$
$4\times 10 =$	$40\div 10 =$	$5\times 10 =$	$50\div 10 =$
$5\times 10 =$	$50\div 10 =$	$1\times 10 =$	$10\div 10 =$
$6\times 10 =$	$60\div 10 =$	$4\times 10 =$	$40\div 10 =$
$7\times 10 =$	$70\div 10 =$	$8\times 10 =$	$80\div 10 =$
$8\times 10 =$	$80\div 10 =$	$10\times 10 =$	$100\div 10 =$
$9\times 10 =$	$90\div 10 =$	$7\times 10 =$	$70\div 10 =$
$10\times 10 =$	$100\div 10 =$	$9\times 10 =$	$90\div 10 =$

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	10	10	10	10	10	10	10	10
Multiply	2	4	7	3	5	9	6	8

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
Divide	10)20	10)40	10)70	10)30	10)50	10)90	10)60	10)80





LESSON XI.

1. There are only forty-five primary combinations in multiplication, and these are formed by multiplying each digital number (10 not included) by *itself* and by each of the *higher* digital numbers, as follows:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
1×1	2×2	3×3	4×4	5×5	6×6	7×7	8×8	9×9
2×1	3×2	4×3	5×4	6×5	7×6	8×7	9×8	
3×1	4×2	5×3	6×4	7×5	8×6	9×7		
4×1	5×2	6×3	7×4	8×5	9×6			
5×1	6×2	7×3	8×4	9×5				
6×1	7×2	8×3	9×4					
7×1	8×2	9×3						
8×1	9×2							
9×1								

NOTE.—These forty-five combinations may be written on the board, and the pupils drilled until they give the products instantly.

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2. These primary combinations in multiplication, and the corresponding inverse processes in division, may be recited as follows:

$1 \times 1 = 1 \quad \therefore \quad 1 \div 1 = 1$

2×1 or $1 \times 2 = \quad \therefore \quad 2 \div 1 = \; ; \quad 2 \div 2 =$
3×1 or $1 \times 3 = \quad \therefore \quad 3 \div 1 = \; ; \quad 3 \div 3 =$
4×1 or $1 \times 4 = \quad \therefore \quad 4 \div 1 = \; ; \quad 4 \div 4 =$
5×1 or $1 \times 5 = \quad \therefore \quad 5 \div 1 = \; ; \quad 5 \div 5 =$
6×1 or $1 \times 6 = \quad \therefore \quad 6 \div 1 = \; ; \quad 6 \div 6 =$
7×1 or $1 \times 7 = \quad \therefore \quad 7 \div 1 = \; ; \quad 7 \div 7 =$
8×1 or $1 \times 8 = \quad \therefore \quad 8 \div 1 = \; ; \quad 8 \div 8 =$
9×1 or $1 \times 9 = \quad \therefore \quad 9 \div 1 = \; ; \quad 9 \div 9 =$

$2 \times 2 = 4 \quad \therefore \quad 4 \div 2 = 2$

3×2 or $2 \times 3 = \quad \therefore \quad 6 \div 2 = \; ; \quad 6 \div 3 =$
4×2 or $2 \times 4 = \quad \therefore \quad 8 \div 2 = \; ; \quad 8 \div 4 =$
5×2 or $2 \times 5 = \quad \therefore \quad 10 \div 2 = \; ; \quad 10 \div 5 =$
6×2 or $2 \times 6 = \quad \therefore \quad 12 \div 2 = \; ; \quad 12 \div 6 =$
7×2 or $2 \times 7 = \quad \therefore \quad 14 \div 2 = \; ; \quad 14 \div 7 =$
8×2 or $2 \times 8 = \quad \therefore \quad 16 \div 2 = \; ; \quad 16 \div 8 =$
9×2 or $2 \times 9 = \quad \therefore \quad 18 \div 2 = \; ; \quad 18 \div 9 =$

$3 \times 3 = 9 \quad \therefore \quad 9 \div 3 = 3$

4×3 or $3 \times 4 = \quad \therefore \quad 12 \div 3 = \; ; \quad 12 \div 4 =$
5×3 or $3 \times 5 = \quad \therefore \quad 15 \div 3 = \; ; \quad 15 \div 5 =$
6×3 or $3 \times 6 = \quad \therefore \quad 18 \div 3 = \; ; \quad 18 \div 6 =$
7×3 or $3 \times 7 = \quad \therefore \quad 21 \div 3 = \; ; \quad 21 \div 7 =$
8×3 or $3 \times 8 = \quad \therefore \quad 24 \div 3 = \; ; \quad 24 \div 8 =$
9×3 or $3 \times 9 = \quad \therefore \quad 27 \div 3 = \; ; \quad 27 \div 9 =$

$4 \times 4 = 16 \quad \therefore \quad 16 \div 4 = 4$

5×4 or $4 \times 5 = \quad \therefore \quad 20 \div 4 = \quad ; \quad 20 \div 5 =$

6×4 or $4 \times 6 = \quad \therefore \quad 24 \div 4 = \quad ; \quad 24 \div 6 =$

7×4 or $4 \times 7 = \quad \therefore \quad 28 \div 4 = \quad ; \quad 28 \div 7 =$

8×4 or $4 \times 8 = \quad \therefore \quad 32 \div 4 = \quad ; \quad 32 \div 8 =$

9×4 or $4 \times 9 = \quad \therefore \quad 36 \div 4 = \quad ; \quad 36 \div 9 =$

$5 \times 5 = 25 \quad \therefore \quad 25 \div 5 = 5$

6×5 or $5 \times 6 = \quad \therefore \quad 30 \div 5 = \quad ; \quad 30 \div 6 =$

7×5 or $5 \times 7 = \quad \therefore \quad 35 \div 5 = \quad ; \quad 35 \div 7 =$

8×5 or $5 \times 8 = \quad \therefore \quad 40 \div 5 = \quad ; \quad 40 \div 8 =$

9×5 or $5 \times 9 = \quad \therefore \quad 45 \div 5 = \quad ; \quad 45 \div 9 =$

$6 \times 6 = 36 \quad \therefore \quad 36 \div 6 = 6$

7×6 or $6 \times 7 = \quad \therefore \quad 42 \div 6 = \quad ; \quad 42 \div 7 =$

8×6 or $6 \times 8 = \quad \therefore \quad 48 \div 6 = \quad ; \quad 48 \div 8 =$

9×6 or $6 \times 9 = \quad \therefore \quad 54 \div 6 = \quad ; \quad 54 \div 9 =$

$7 \times 7 = 49 \quad \therefore \quad 49 \div 7 = 7$

8×7 or $7 \times 8 = \quad \therefore \quad 56 \div 7 = \quad ; \quad 56 \div 8 =$

9×7 or $7 \times 9 = \quad \therefore \quad 63 \div 7 = \quad ; \quad 63 \div 9 =$

$8 \times 8 = 64 \quad \therefore \quad 64 \div 8 = 8$

9×8 or $8 \times 9 = \quad \therefore \quad 72 \div 8 = \quad ; \quad 72 \div 9 =$

$9 \times 9 = 81 \quad \therefore \quad 81 \div 9 = 9$

LESSON XII.

Parts of Numbers.

The previous lessons have made the pupils familiar with the process of separating a number into *equal parts*. The step now to be taken is to teach pupils to find directly *one* of the equal parts of a number, and this involves the developing of the idea of *one half*, *one third*, *one fourth*, and other fractional parts of a unit.

1. Take an apple and cut it into two *equal* pieces, and teach that one of these pieces is *one half* of the apple. Write the fraction $\frac{1}{2}$ on the board.

Cut the apple into four equal pieces by cutting each half into two equal pieces, and teach that each one of these pieces is *one fourth* of the apple; that two pieces are *two fourths;* three pieces, *three fourths*, etc. Write the fraction $\frac{1}{4}$ on the board.

How many halves in an apple? How many fourths? How many fourths in one half?

Cut the apple into eight equal pieces by cutting each fourth into two equal pieces, and teach that each one of these eight pieces is *one eighth* of the apple; that two pieces are *two eighths;* three pieces, *three eighths*, etc. Write the fraction $\frac{1}{8}$ on the board.

How many eighths in an apple? How many eighths in one half of it? In one fourth?

Draw a line and a circle on the board, and divide each into halves, fourths, and eighths, and have the pupils name the parts as each is divided.

Next take an apple and divide it into three equal pieces, and teach that each piece is *one third* of the apple. Write the fraction $\frac{1}{3}$ on the board.

In like manner teach *one sixth*, *one ninth*, *one fifth*, *one tenth*, etc.

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2. How much is $\frac{1}{2}$ of 2 blocks? $\frac{1}{2}$ of 4 blocks? $\frac{1}{2}$ of 8 blocks? $\frac{1}{2}$ of 10 blocks? $\frac{1}{2}$ of 12 blocks?

NOTE.—Take the number of blocks named in the question, and separate them into two equal groups.

How much is $\frac{1}{4}$ of 4 blocks? $\frac{1}{4}$ of 8 blocks? $\frac{1}{4}$ of 12 blocks? $\frac{1}{4}$ of 16 blocks? $\frac{1}{4}$ of 20 blocks?

How much is $\frac{1}{3}$ of 3 blocks? $\frac{1}{3}$ of 9 blocks? $\frac{1}{3}$ of 12 blocks? $\frac{1}{3}$ of 18 blocks? $\frac{1}{3}$ of 24 blocks?

How much is $\frac{1}{6}$ of 12 balls? $\frac{1}{6}$ of 18 balls? $\frac{1}{6}$ of 30 balls? $\frac{1}{6}$ of 24 balls? $\frac{1}{6}$ of 36 balls? $\frac{1}{6}$ of 48 balls?

How much is $\frac{1}{5}$ of 10? $\frac{1}{5}$ of 20? $\frac{1}{5}$ of 30? $\frac{1}{5}$ of 40? $\frac{1}{5}$ of 35? $\frac{1}{5}$ of 45? $\frac{1}{5}$ of 50?

NOTE.—The above exercises should be multiplied until the pupils are familiar with the process of taking $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc., of a number.

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3. The following exercises may be written on the blackboard, and used for oral drills in the analysis of numbers into equal parts, and then finding one of the equal parts.

	4 = two 2's	∴	$\frac{1}{2}$ of 4 = 2	
6 =	2's or 3's	∴	$\frac{1}{3}$ of 6 =	; $\frac{1}{2}$ of 6 =
8 =	2's or 4's	∴	$\frac{1}{4}$ of 8 =	; $\frac{1}{2}$ of 8 =
10 =	2's or 5's	∴	$\frac{1}{5}$ of 10 =	; $\frac{1}{2}$ of 10 =
12 =	2's or 6's	∴	$\frac{1}{6}$ of 12 =	; $\frac{1}{2}$ of 12 =
14 =	2's or 7's	∴	$\frac{1}{7}$ of 14 =	; $\frac{1}{2}$ of 14 =
16 =	2's or 8's	∴	$\frac{1}{8}$ of 16 =	; $\frac{1}{2}$ of 16 =
18 =	2's or 9's	∴	$\frac{1}{9}$ of 18 =	; $\frac{1}{2}$ of 18 =
	9 = three 3's	∴	$\frac{1}{3}$ of 9 = 3	
12 =	3's or 4's	∴	$\frac{1}{4}$ of 12 =	; $\frac{1}{3}$ of 12 =
15 =	3's or 5's	∴	$\frac{1}{5}$ of 15 =	; $\frac{1}{3}$ of 15 =
18 =	3's or 6's	∴	$\frac{1}{6}$ of 18 =	; $\frac{1}{3}$ of 18 =
21 =	3's or 7's	∴	$\frac{1}{7}$ of 21 =	; $\frac{1}{3}$ of 21 =
24 =	3's or 8's	∴	$\frac{1}{8}$ of 24 =	; $\frac{1}{3}$ of 24 =
27 =	3's or 9's	∴	$\frac{1}{9}$ of 27 =	; $\frac{1}{3}$ of 27 =
	16 = four 4's	∴	$\frac{1}{4}$ of 16 = 4	
20 =	4's or 5's	∴	$\frac{1}{5}$ of 20 =	; $\frac{1}{4}$ of 20 =
24 =	4's or 6's	∴	$\frac{1}{6}$ of 24 =	; $\frac{1}{4}$ of 24 =
28 =	4's or 7's	∴	$\frac{1}{7}$ of 28 =	; $\frac{1}{4}$ of 28 =
32 =	4's or 8's	∴	$\frac{1}{8}$ of 32 =	; $\frac{1}{4}$ of 32 =
36 =	4's or 9's	∴	$\frac{1}{9}$ of 36 =	; $\frac{1}{4}$ of 36 =

	25 = five 5's	∴	$\frac{1}{5}$ of 25 = 5	
30 =	5's or 6's	∴	$\frac{1}{6}$ of 30 = ;	$\frac{1}{5}$ of 30 =
35 =	5's or 7's	∴	$\frac{1}{7}$ of 35 = ;	$\frac{1}{5}$ of 35 =
40 =	5's or 8's	∴	$\frac{1}{8}$ of 40 = ;	$\frac{1}{5}$ of 40 =
45 =	5's or 9's	∴	$\frac{1}{9}$ of 45 = ;	$\frac{1}{5}$ of 45 =
	36 = six 6's	∴	$\frac{1}{6}$ of 36 = 6	
42 =	6's or 7's	∴	$\frac{1}{7}$ of 42 = ;	$\frac{1}{6}$ of 42 =
48 =	6's or 8's	∴	$\frac{1}{8}$ of 48 = ;	$\frac{1}{6}$ of 48 =
54 =	6's or 9's	∴	$\frac{1}{9}$ of 54 = ;	$\frac{1}{6}$ of 54 =
	49 = seven 7's	∴	$\frac{1}{7}$ of 49 = 7	
56 =	7's or 8's	∴	$\frac{1}{8}$ of 56 = ;	$\frac{1}{7}$ of 56 =
63 =	7's or 9's	∴	$\frac{1}{9}$ of 63 = ;	$\frac{1}{7}$ of 63 =
	64 = eight 8's	∴	$\frac{1}{8}$ of 64 = 8	
72 =	8's or 9's	∴	$\frac{1}{9}$ of 72 = ;	$\frac{1}{8}$ of 72 =
	81 = nine 9's	∴	$\frac{1}{9}$ of 81 = 9	

LESSON XIII.

Supplemental Drills in Rapid Combinations.

The foregoing exercises may be supplemented by oral and blackboard drills in the rapid combination of numbers by addition, subtraction, multiplication, and division. The rapidity secured by these drills has won for them the appellation of "Lightning Combinations."

1. The following are illustrations of oral drills:

1. Take 5, add 7, subtract 4, multiply by 7, add 4, divide by 6, subtract 3, multiply by 9, add 5, add 4, divide by 8. What is the result?

2. Take 9, multiply by 6, add 7, subtract 5, divide by 8, add 2, multiply by 6, add 9, divide by 9, multiply by 5, add 7, divide by 7, multiply by 8. Result?

3. Take 65, subtract 9, divide by 8, multiply by 6, add 9, subtract 6, divide by 9, add 4, multiply by 8, add 9, divide by 9, divide by 3. Result?

4. Take 15, add 12, add 9, subtract 4, divide by 8, add 4, multiply by 9, subtract 8, divide by 8, multiply by 7, add 9, add 7, divide by 8, divide by 3. Result?

NOTE.—At first the class may be permitted to give orally each result; but, as soon as the pupils are sufficiently familiar with the process, only the *final* result should be given. The teacher should dictate slowly at first, increasing in rapidity as skill is acquired by the class.

2. The drills may be varied by writing the combinations on the blackboard, and pointing successively to those to be made.

The following exercises are illustrations of these blackboard drills, the operations being performed from left to right in order:

1. $19+8,\ +9,\ -4,\ \div 8,\ +4,\ \times 9,\ -8,\ \div 8,\ \times 7,\ +9,\ +7,\ \div 8,\ \div 3 =$ what?

2. $8\times 9,\ \div 12,\ \times 7,\ +7,\ \div 7,\ +3,\ \times 9,\ -6,\ \div 7,\ \div 6,\ =$ what?

3. $56\div 7,\ \times 9,\ -5,\ -4,\ \div 7,\ \times 9,\ +3,\ \div 12,\ \times 6,\ +6,\ \div 8,\ \div 6 =$ what?

4. $42+6,\ +9,\ -3,\ \div 9,\ \times 8,\ +9,\ +7,\ \div 8,\ \times 10,\ +1,\ \div 9,\ \div 3 =$ what?

5. $6+5,\ \times 7,\ +4,\ \div 9,\ \times 7,\ -5,\ +6,\ \div 8,\ \times 3,\ +9,\ +7,\ \div 10,\ \times 12 =$ what?

6. $47+9,\ \div 7,\ \times 5,\ +8,\ -3,\ \div 9,\ \times 12,\ +3,\ \div 7,\ \times 8,\ \div 9,\ \times 4 =$ what?

7. $100-25,\ +6,\ \div 9,\ \times 6,\ +10,\ \div 8,\ \times 7,\ +7,\ \div 9,\ \times 7,\ -1,\ \div 8 =$ what?

NOTE.—The comma (,) is used in the above exercises to indicate that the operations expressed by the signs $+$, $-$, $\times$, and $\div$ are to be performed in the order *in which they occur from left to right.* If the comma were not thus used, the operations indicated by $\times$ and $\div$ would take precedence over the $+$ or $-$ immediately preceding. The operations in the first exercise, if written without the comma, would be performed in this order: $(5+7)-(4\times 7)+(4\div 6)-(3\times 9)+5+(4\div 8)$. The sign $+$ or $-$ would include all that lies *between it and the next sign of* $+$ *or* $-$.

Arithmeticians are not agreed respecting the order in which the operations indicated by the signs $\times$ and $\div$ are to be performed. $8 \times 6 \div 4 \times 3$ may equal 36 or 4, since the operations may be performed in their order from left to right, or 8×6 may be divided by 4×3.

SUPPLEMENTAL BLACKBOARD EXERCISES.

ADDITION.

The exercises in adding columns given in the ELEMENTARY ARITHMETIC (pages 18, 21, 23, 26, 28, etc.), present numerous combinations; and, if properly used, will afford sufficient slate practice for third-year pupils.

1. These exercises may be easily reproduced and multiplied on the blackboard by writing in a single column the additive number given, and then writing successively beneath the column each of the *lower* digits.

Suppose, for illustration, that the additive number is 4. Write a column of 4's on the blackboard (as at the right), and under the column (or at the left) write successively 1, 2, and 3.

The use of 1 as the initial number will give as results 5, 9, 13, 17, 21, etc.; the use of 2 as the initial number will give 6, 10, 14, 18, 22, etc.; and the use of 3 will give 7, 11, 15, 19, 23, etc. In practice, the teacher will find it convenient to erase the initial figure used, and write the new initial figure at the foot of the column. The columns should also be added downwards, beginning with the proper initial figure.

1	2	3	4
			4
			4
			4
			4
			4
			4
			4
			4
			4
			4
			4
			4
			4
			4
1	2	3	4

2. These exercises are all included in the following table:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

In adding any column in the above table, the initial numbers of the columns at its left are successively used; thus, the column of 6's is added by beginning successively with 1, 2, 3, 4, and 5.

3. Exercises in adding columns of numbers *promiscuously arranged* may be readily produced on the black-board by writing a column of the lower digital numbers and the number reached, arranged promiscuously, and then writing successively at the foot of this column, as an initial number, each of the lower digital numbers.

These promiscuous exercises are shown by the table on the opposite page.

In adding any column in this table, begin successively with the initial figures of the columns at its left. In adding the sixth (*f*) column, for illustration, begin successively with 1, 2, 3, 4, 5, and 6.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
	2	3	3	5	6	3	4	3
	1	2	4	4	5	7	8	7
	2	1	3	5	3	6	6	8
	2	2	2	2	4	1	7	9
	1	2	1	1	6	5	8	6
	1	3	2	2	3	7	2	9
	2	3	3	4	4	4	1	4
	2	1	4	3	5	4	2	9
	2	2	4	5	6	7	2	5
	1	3	3	5	3	3	8	9
	1	2	3	4	1	7	3	3
	2	3	2	4	6	2	4	9
	1	1	2	3	2	7	8	7
	2	2	1	5	3	6	5	9
	1	3	4	2	4	7	6	2
	1	2	2	2	6	4	7	9
	2	3	4	3	3	3	8	4
	1	2	3	1	2	2	6	9
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

The numbers in the above table may also be added horizontally by adding the different lines of numbers, first from left to right, and then from right to left, or inversely. By changing the initial numbers, the exercises may be readily multiplied.

The above table may also be used for drills in adding the digital numbers two and two. The first number in each column may be successively added to each number in the column, progressing upwards. Thus, taking the fourth (*d*) column for illustration, the number 5 may be successively added to 1, 3, 2, 2, 5, etc.

The first number at the top of each column may also be successively added to each number in the column, progressing downwards. The number 5 in the fourth (*d*) column may, for example, be successively added to 4, 5, 2, 1, etc.

NOTES.—1. *It is not necessary for the teacher to take the time to copy drill exercises on the blackboard.* Every third-year class contains pupils who can easily be trained to copy plainly and neatly on the blackboard all the drill exercises in this manual and also in the ELEMENTARY ARITHMETIC, and this copying will both please and benefit the pupils who do it, provided the work be properly distributed.

2. The exercises daily produced on the blackboard for class drill are fresher and otherwise better than those printed on cards or charts. It is also questioned whether the apparatus devised by different persons for the teaching of addition is not more ingenious than useful. Experience shows that the use of such devices soon becomes very mechanical, and that the interest of the pupils lessens as the sense of novelty fades out.

II. SUBTRACTION.

The first and second year exercises have given pupils the ability to subtract instantly any digital number from the sum of it and any other digital number. The third year exercises in subtraction involve the taking of the digital numbers from numbers not exceeding 100, and the only difficulty here is when the subtrahend number is greater than the number denoted by the unit figure of the minuend, as in subtracting 7 from 31, 42, 53, 74, 65, or 86.

This difficulty is easily mastered if pupils are taught first to subtract the subtrahend number from *the unit number of the minuend increased by ten*, and then take one from the ten number of the minuend. Thus, 31 less 7 is 11 less 7 plus 30 less 10, which is 20, plus 4, which is 24. 42 less 7 is 12 less 7 plus 40 less 10, which is 30, plus 5, which is 35.

Skill in these processes may be imparted by writing on the blackboard series of numbers with the same unit figure, and then subtracting from the numbers in each series, from right to left, the several digital numbers successively, as in the table at top of opposite page.

82	92	62	72	52	32	42	22	12
3	3	3	3	3	3	3	3	3
82	92	62	72	52	32	42	22	12
4	4	4	4	4	4	4	4	4

In like manner, 5, 6, 7, 8, and 9 may be used as subtrahends. The exercises may be continued by changing the unit figure of the minuend, and taking each of the *higher* digital numbers as a subtrahend number.

The following table represents all of these difficulties, one of each:

20	20	20	20	20	20	20	20	20
9	8	7	6	5	4	3	2	1
	41	41	41	41	41	41	41	41
	9	8	7	6	5	4	3	2
		52	52	52	52	52	52	52
		9	8	7	6	5	4	3
			73	73	73	73	73	73
			9	8	7	6	5	4
				84	84	84	84	84
				9	8	7	6	5
					35	35	35	35
					9	8	7	6
						56	56	56
						9	8	7
							67	67
							9	8
								88
								9

Each of these combinations can be readily expanded into a series of blackboard drills.

III. Multiplication.

Blackboard exercises in multiplication are easily made by writing a line of the digital numbers, and then requiring the pupils to multiply each number, from right to left, by a given digital number. There is, however, an advantage in writing the two numbers to be multiplied under each other, that they may be associated in vision as well as mentally. The several series of combinations are presented in the following table:

1.	5	8	3	9	4	2	6	7	1
	1	1	1	1	1	1	1	1	1
2.	5	8	3	9	4	2	6	7	1
	2	2	2	2	2	2	2	2	2
3.	5	8	3	9	4	2	6	7	1
	3	3	3	3	3	3	3	3	3
4.	5	8	3	9	4	2	6	7	1
	4	4	4	4	4	4	4	4	4
5.	5	8	3	9	4	2	6	7	1
	5	5	5	5	5	5	5	5	5
6.	5	8	3	9	4	2	6	7	1
	6	6	6	6	6	6	6	6	6
7.	5	8	3	9	4	2	6	7	1
	7	7	7	7	7	7	7	7	7
8.	5	8	3	9	4	2	6	7	1
	8	8	8	8	8	8	8	8	8
9.	5	8	3	9	4	2	6	7	1
	9	9	9	9	9	9	9	9	9

Note.—The above tables may be used for drills in division requiring the pupils to divide each product by each of its two factors.

IV. Division.

The separate blackboard drills in division should include the division of numbers by digital numbers which are not their factors, as is shown in the following table:

2)	3	9	5	11	7	9	13	15	17	19
3)	7	11	8	13	17	20	23	25	26	29
4)	9	10	11	13	17	22	29	35	38	39
5)	11	12	24	36	38	41	43	46	48	49
6)	13	19	25	31	45	38	49	28	17	32
6)	15	50	52	35	38	44	45	56	58	59
7)	15	19	23	27	30	37	40	45	50	57
7)	33	39	44	46	48	51	54	58	65	69
8)	18	20	26	30	33	37	41	45	50	54
8)	47	43	47	58	61	63	66	70	75	79
9)	19	21	26	30	32	37	40	42	47	49
9)	55	57	59	66	69	71	76	78	85	89
10)	32	43	56	64	78	82	96	77	88	99

The numbers in the above table should be divided from left to right, and the pupils should give only results; thus, taking third series for example, the results are: 2 and 1 over; 2 and 2 over; 2 and 3 over; 3 and 1 over; 4 and 1 over, etc.

Each series of numbers may be increased at the pleasure of the teacher.

MISCELLANEOUS LESSONS AND SUGGESTIONS.

UNITED STATES MONEY.

The more common money units, as the cent and the dollar, may be taught as early as the second year, and pupils may also be made familiar with all the smaller coins in common use, including the *cent*, *two-cent piece*, *five-cent piece* (nickel), *ten-cent piece* (dime), *twenty-five cent piece* (quarter-dollar), *fifty-cent piece* (half-dollar), and *dollar*.

Such instruction should be given incidentally, and the lessons should be repeated until the pupils are familiar with the different coins and their comparative value.

The first lesson may be devoted to the cent and two-cent piece; the second, to the half-dime or "nickel;" the third, to the dime; the fourth, to the quarter-dollar; the fifth, to the half-dollar; and the sixth, to the dollar, as indicated in the following outlines. The actual coins should be used.

---

1. How many cents equal in value a two-cent piece? How many cents in 2 two-cent pieces? 4 two-cent pieces? 5 two-cent pieces?

How many cents in 2 two-cent pieces and 1 cent? 3 two-cent pieces and 1 cent? 5 two-cent pieces and 1 cent? etc.

2. How many cents in this half-dime? (Present a silver half-dime or a "nickel.") How many cents in 3 half-dimes? In 5 half-dimes?

How many cents in 2 half-dimes and 3 cents? 5 half-dimes and 2 cents?

NOTE.—Multiply these questions; and, if practicable, let the pupils "count" the pieces of money. Make up little problems involving the making of change, etc.

3. How many cents in this dime? In 2 dimes? 4 dimes? 6 dimes? 8 dimes? 10 dimes?

How many cents in 2 dimes and 1 half-dime? 3 dimes, 2 half-dimes, and 4 cents? etc.

4. How many cents in this quarter-dollar? How many cents in 2 quarter-dollars? In 3 quarter-dollars? In 4 quarter-dollars?

How many cents in 2 quarter-dollars, 1 dime, and 2 half-dimes?

How many cents in 1 quarter-dollar, 2 dimes, and 4 half-dimes?

5. How many cents in this half-dollar? How many cents in 2 half-dollars?

How many cents in 1 half-dollar, 1 quarter-dollar, and 1 dime?

How many cents in 1 half-dollar, 2 quarter-dollars, and 5 dimes?

6. How many cents in this dollar? How many cents in 2 dollars? 3 dollars? 5 dollars?

How many half-dollars in a dollar? How many quarter-dollars in a dollar? How many dimes?

How much money in 1 dollar, 2 half-dollars, 2 quarter-dollars, and 1 dime?

Pupils may also be early taught to write sums of money, and to add, subtract, multiply, and divide sums of money. Such instruction is best given *orally*, and

it should be introduced at such times in the first three years of instruction in number as the teacher may deem best. There should be no elaborate explanation of the decimal notation as applied to the writing of dollars and cents, but the pupils should be shown *how* to write dollars and cents, the figures denoting dollars being preceded by \$, called the dollar sign, and separated from those denoting cents by a period or point.

\$ 1.25
\$ 5.18
\$ 8.40
\$10.65
\$25.50
\$30.25
\$47.80
\$52.05
\$71.08

Write on the board, as at the right, several sums of money, and teach the pupils to read the figures denoting dollars and those denoting cents separately and together. Dictate similar sums of money for the pupils to write, and continue the drill until they can both read and write small sums of money with accuracy and ease.

When the pupils reach and master the "Written Exercises" on page 38 of the ELEMENTARY ARITHMETIC, repeat the exercises there given, changing the numbers to sums of money. They may be dictated, or written on the blackboard, or both. The first fifteen exercises may be changed as follows:

	2.	**3.**	**4.**	**5.**	**6.**	**7.**
\$1.25	\$1.08	\$1.35	\$0.98	\$2.23	\$4.63	\$1.48
\$2.48	\$2.09	\$1.46	\$0.99	\$3.39	\$0.78	\$2.29
\$1.05	\$1.76	\$1.78	\$0.88	\$1.07	\$1.39	\$1.06
\$2 23	\$1.84	\$1.99	\$0.77	\$0.88	\$0.65	\$1.57
\$1.09	\$1.19	\$1.38	\$1.43	\$0.67	\$1.73	\$1.89
\$1.08	\$1.0′	\$1.47	\$2.08	\$0.78	\$0.89	\$1.05

8.	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**	**15.**
\$1.45	\$2.47	\$3.65	\$4.73	\$5.86	\$6.93	\$2.87	\$3.64
\$0.36	\$1.43	\$2.46	\$3.44	\$2.77	\$4.78	\$1.79	\$2.46

The pupils will readily change the remaining exercises on the page in a similar manner; also, exercises 27 to 32, inclusive, on page 31.

Exercises in multiplying and dividing sums of money may be readily provided by changing the multiplicands and dividends on pages 54, 56, and 58 to dollars and cents, by taking the two right-hand figures in each number for cents, and the left-hand figure for dollars, separating by a period, and prefixing the dollar sign, when necessary so to do. The pupils will make these changes readily, and thus relieve the teacher of the task of writing the exercises on the board.

COMMON MEASURES.

The more common measures may be taught *objectively* in connection with third-year lessons in number, but it seems best to make such instruction incidental, introducing it gradually to afford variety, and at such times as the pupils are best prepared to receive it and profit by it. If deferred until the close of the third year, the following lessons can be given consecutively.

MEASURES OF LIQUIDS.

The teacher should be supplied with a gill measure, a pint measure, a quart measure, and a gallon measure. A gill cup, a pint cup, a quart cup, and a gallon milk-can, with a bucket of water, will answer every purpose. These can be easily procured.

1. Teach the names of each of the measures, and take the gill cup and show the pupils how to find how many gills in a pint.

Count, children, and see how many times I fill this little gill cup with water, and pour it into the pint cup to fill it. How many times? "Four times."

Now let us see how many times a pint of water will fill a gill cup. Count as I fill the gill cup from the pint cup. How many times? "Four times."

How many gills make a pint?

2. Susan may now take the pint cup and find how many times she must fill it with water and pour it into the quart cup to fill it. How many times? "Twice."

Jane may find how many times a quart of water will fill the pint cup. How many? "Twice."

How many pints make a quart?

3. Charles may now take the quart cup and find how many times he can pour it full of water into the gallon measure. How many? "Four times." The gallon measure is full.

Now Harry may take the gallon of water and find how many times it will fill a quart cup. How many times? "Four times."

How many gills make a pint? How many pints make a quart? How many quarts make a gallon?

TABLE.

4 *gills are* 1 *pint.*
2 *pints are* 1 *quart.*
4 *quarts are* 1 *gallon.*

4. How many pint bottles will hold 3 quarts of vinegar?

How many times must you fill a pint cup to measure 2 quarts of milk?

A milkman has a three-gallon can full of milk: how many quarts of milk can he sell? When he sells 12 quarts, how many quarts will be left in the can?

A housekeeper filled 6 two-quart cans and 8 quart cans with peaches: how many quarts of peaches did she have? How many gallons?

DRY MEASURES.

The teacher should be supplied with a pint measure, a quart measure, a peck measure, and at least a peck of some kind of grain or small fruit. These can be easily procured for temporary use. It is very important that the pupils see and handle these different measures; and, when practicable, a bushel measure.

The first step is to teach the several measures, and their names, and the next step is to teach the number of pints in a quart, quarts in a peck, and pecks in a bushel.

---

1. John may take the pint measure and measure 2 pints of grain; 4 pints.

Charles may take the quart measure and measure 3 quarts of grain; 6 quarts.

Mary may take the pint measure and find how many pints of grain will fill a quart measure. How many? "Two pints."

Nellie may take a quart of grain and find how many times it will fill a pint measure. How many? "Two times."

Then how many pints make a quart?

2. Clarence may take the quart measure and find how many quarts of grain will fill a peck measure. How many? "Eight quarts."

Now John may find how many times a peck of grain will fill a quart measure. How many? "Eight times."

Then how many quarts make a peck?

If we had a bushel measure here, it would take 4 pecks of grain to fill it. How many pecks make a bushel?

3. How many pint baskets can I fill with one quart of cherries?

How many quart bags can I fill with a peck of nuts?

How many peck measures can I fill with a bushel of wheat?

A boy picked 2 pecks of chestnuts, and sold them at 10 cents a quart: how many quarts of nuts did he sell?

How many pint bottles will 5 quarts of seed fill?

TABLE.

2 *pints are* 1 *quart.*
8 *quarts are* 1 *peck.*
4 *pecks are* 1 *bushel.*

MEASURES OF LENGTH OR DISTANCE.

The teacher should be supplied with a foot-rule, a yard-stick, and a piece of tape at least a rod long, and accurately divided into feet. Each pupil should be supplied with a foot-rule, or with a narrow strip of strong paper, one foot in length, and plainly divided into inches.

1. The first step is to develop the idea of length or distance by comparing objects of different lengths, and training pupils in estimating the length of objects "by the eye."

Hold up two objects nearly of the same length, and have the pupils judge which is the longer. Test by putting the objects close together.

Draw two lines on the blackboard of different lengths and in different positions, and have pupils judge which is the longer. Test by measuring lines with the rule.

Draw horizontal lines from one to twelve inches long, and have pupils estimate their length in inches. Test by applying the rule.

Draw vertical and oblique lines on the board, and have their lengths estimated by the pupils and then measured with the rule.

Have pupils estimate the length and width of panes of glass, of slates of different sizes, books, etc., and then test accuracy by applying the rule.

2. In like manner, the length of lines, as the length and width of the table, blackboard, etc., may be estimated in feet and measured, and the length of the sides of the room, long strings, etc., may be estimated in yards and measured.

The rod measure may also be introduced, and the length and width of the school-yard, the width of the street, etc., may be measured by a line one rod in length.

3. This is a foot-rule: how long is it? "One foot." How many inches long is it? "Twelve inches."

Here is a line just 24 inches long: how many feet long is it?

Here is a string 1 yard long: if I cut it into pieces, each 1 foot long, how many pieces will it make?

How many feet make a yard?

Here is a board 9 feet long: what is its length in yards?

How many feet in a ribbon 4 yards long?

TABLE.

12 *inches are* 1 *foot.*
3 *feet are* 1 *yard.*
$5\frac{1}{2}$ *yards are* 1 *rod.*

MEASURES OF TIME.

1. The portion of time called a *day* begins at 12 o'clock at night, or at midnight, and ends at 12 o'clock the next night.

When did to-day begin? "At 12 o'clock last night." When will to-day end? "At 12 o'clock to-night."

What is 12 o'clock at night called? "It is called midnight." How long, then, is a day? "From midnight to the next midnight."

What is the middle of the day called? "It is called midday, or noon." The noon divides the day into how many equal parts? "Two equal parts."

2. A day is divided into twenty-four equal parts called *hours.* How many hours from midnight to midnight? From midnight to noon? From noon to midnight?

When does the clock strike? "At the end of each hour." How many times does the clock strike in a a day? "Twenty-four times." How many times does it strike from midnight to noon? From noon to midnight?

How many hours in a day?

3. An hour is divided into sixty equal parts called *minutes.* The pupils may all keep still for one minute. How many times this silence would make an hour? "Sixty times." How many minutes in the school recess? How many minutes in this class exercise?

How many minutes in an hour?

Put your finger on your pulse and count silently sixty "beats." It has taken a little less than a minute. Count to sixty as I beat seconds with my hand. (This may be done by the guidance of the second-hand of a watch.) How long has it taken? "One minute."

How many seconds in a minute?

TABLE.

60 *seconds are* 1 *minute.*
60 *minutes are* 1 *hour.*
24 *hours are* 1 *day.*

The teacher may also teach the number of days in a week; the names of the days of the week in their order; the number of days in a year; the number of

months in a year; the names of the months in their order; the division of the year into four seasons; the names of the months in each season, etc.

MEASURES OF WEIGHT.

In teaching weight, the teacher should be supplied with a common balance, the usual weights, and several parcels, each weighing one pound.

1. The first step is to give the pupils an idea of the weight called a *pound*. Let the pupils take a pound parcel in one hand, and a pound weight in the other, and "heft" them. Tell them that a pint of cold water weighs a pound (nearly).* Let the pupils "heft" two-pound parcels, three-pound parcels, etc.

2. Teach the division of the pound into sixteen equal parts called *ounces*. Show an ounce weight, a four-ounce weight, and an eight-ounce weight. Let the pupils weigh different articles.

How many ounce weights weigh as much as a pound weight? How many four-ounce weights are a pound? How many eight-ounce weights?

What part of a pound is an ounce? What part of a pound are four ounces? Eight ounces?

3. Teach the pupils that one hundred pounds is a hundred-weight, and that twenty hundred-weights are a ton.

How many pounds in a hundred-weight of hay? Three hundred-weights?

How many pounds in a ton? In two tons? Four tons? One half of a ton? One fourth of a ton?

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*A gallon of ice-cold water weighs a little more than $8\frac{1}{3}$ pounds, and hence a pint weighs $1\frac{1}{24}$ pounds.

TABLE.

16 *ounces are* 1 *pound.*
100 *pounds are* 1 *hundred-weight.*
20 *hundred-weights are* 1 *ton.*

METRIC MEASURES.

Since the metric measures are not in common use, it is not necessary to teach them as a part of the elementary course in number. Pupils should first be made familiar with the common measures.

When the metric measures are introduced, care should be taken to make the pupils practically familiar with the metric units, and this can best be accomplished *by the actual use of the metric measures in measuring.* To this end, the school should be supplied with a meter measure, a liter measure, and gram and kilogram weights. When pupils learn that a new nickel five cent piece weighs 5 grams, and two of them 10 grams, they will soon have a very clear idea of the gram.

The pupils should have frequent practice in the use of these measures. With the meter they should measure the length and width of the school-room floor, the teacher's desk, the blackboard, etc.; also the distance between objects in the school-room, in the school-yard, etc. With the liter they should measure water, grain, etc., and with the gram and kilogram they should weigh different articles.

These exercises in metric measurements should be introduced one or two years before the systematic teaching of the metric system as presented in the NEW COMPLETE ARITHMETIC. It is only by long practice that pupils can be made as familiar with the metric measures as they are with the common measures, and

all such instruction must be given orally. It is a mistake to cumber the pages of an elementary text-book for pupils with such introductory and preparatory drills.

In these preparatory lessons no attention should be given to the metric equivalents, and the only comparisons made between the metric measures and the common measures should be *by the eye.* By placing a meter and a yard-stick together, the pupils will see that the meter is a little longer than the yard; by pouring a liter of water or of grain into a dry quart measure, and also into a liquid quart measure, the pupils will see the liter is a little less than the dry quart, and a little more than the liquid quart; and by placing a kilogram weight in one pan of a balance, and a two-pound weight in the other pan, they will see that a kilogram is a little more than two pounds. They will thus learn and can easily remember that the meter is a little more than a yard; the liter, about a quart; and a kilogram, a little more than two pounds.

It will be time enough to teach the exact numerical equivalents, when pupils are so familiar with the metric measures that they can think of them without any reference to the common measures. When a pupil is told, for example, that a room is 8 meters long and 5 meters wide, he should be able to comprehend its dimensions without reducing them to yards or feet; and this result can only be attained by the continued use of the meter in measuring distances.

The reduction of the numbers composed of metric units to equivalent numbers composed of common units, and *vice versa*, is the *final* step in teaching the metric system. The early introduction of the metric equivalents and the reductions of metric numbers to like common denominate numbers, are mistakes which have resulted in much confusion.

SUBTRACTION: WRITTEN PROCESS.

There are two methods or processes of subtracting one number from another when a term of the subtrahend is greater than the corresponding term of the minuend. These processes are as follows:

1. The adding of 10 to the term of the minuend, and then subtracting 1 from the next higher term of the minuend, or considering it 1 less.

Thus, in subtracting 487 from 659, ten (10) is added to the 5 (making 15), and 1 is subtracted from the 6 (leaving 5); that is, the 6 is considered 5.

```
659
487
172
```

2. The adding of 10 to the term of the minuend, and 1 to the next higher term of the subtrahend.

Thus, in subtracting 487 from 659, 10 is added to the 5 (making 15), and 1 is added to the 4 (making 5).

```
659
487
172
```

There are two methods of explaining each of these processes, as follows:

The first process may be explained by showing that the adding of 10 to a term of the minuend, and subtracting 1 from the next higher term, increases and decreases the minuend *equally*, and hence *its numerical value is not changed.* The subtracting of 1 from the next higher term of the minuend offsets the 10 added to the lower term.

The first process may also be explained by showing that the 10 added to the term of the minuend is obtained by taking 1 from the next higher term and reducing it to 10 of the next lower order. Instead of changing the higher minuend *figure*, the term is considered 1 less; that is, 1 is mentally subtracted from it.

NOTE.—When the next higher term of the minuend is 0, 1 is taken from the next higher term whose value is one or more, and the necessary reductions to lower orders are made until 10

is obtained to increase the minuend term. Then each 0 of the minuend is considered 9, and the first significant figure of it reached is considered 1 less. This second explanation seems to be a favorite one with teachers, though few, if any, accountants make such reductions when subtracting numbers.

The second process may be explained by showing that the adding of 10 to a term of the minuend, and 1 to the next higher term of the subtrahend, increases both minuend and subtrahend *equally*, and hence their difference is not changed.

This may be illustrated by taking any two numbers, as 7 and 4, and adding say 8 to each. The difference between 7 and 4 is 3, and the difference between 15 and 12 is 3.

The second process may also be explained by supposing that the 10 added to the minuend is obtained by taking 1 from the next higher term of the minuend, and then showing that, instead of changing the higher minuend figure, or considering the term 1 less (that is, subtracting mentally 1 from it), the 1 may be added to the higher term of the subtrahend *before* subtracting, and thus the 1 and the subtrahend term be *both* taken from the higher minuend term *at the same time.*

This may be illustrated by subtracting 487 from 659. The 10 added to the 5 (tens) may be considered as obtained from the 6 hundreds; but, instead of subtracting the 1 from the 6 (considering 6 diminished by 1), and then taking 4 from 5, both the 1 and the 4 may be taken from the 6 by *one* subtraction by taking the *sum* of the 1 and the 4, which is 5, from the 6.

It is thus seen that in the first process *two* subtractions are made, while in the second process the 1 is added to the subtrahend term, and but *one* subtraction is made.

The term "borrow" should not be used in the description or explanation of either process. The 10

added to the minuend figure is not obtained by "borrowing."

There is a difference of opinion among arithmeticians respecting the comparative merits of these two processes. The writer uses and prefers the second process, possibly the result of early training. It seems to him more natural and more easily explained than the first process. The difference, however, is so slight that pupils who have learned either process should not be required to learn the other.

NOTE.—There is an obvious advantage in using the word "term," in descriptions of elementary processes, to denote the value of each of the successive figures which express a number. It avoids the alternative of an incorrect use of the word "figure," or an awkward use of the phrase "the number denoted by each figure," or its modified equivalent. A figure is not a number. *Figures* can not be added, subtracted, multiplied, or divided. The *numbers* denoted by figures are added, subtracted, etc.

The advantage of using the word *term*, instead of the expression, *the number denoted by a figure*, is shown by substituting the latter phrase, with needed modification, for the word term as used in the above descriptions of the process of subtraction. For example, the expression "when a term of the subtrahend is greater than the corresponding term of the minuend," becomes "when the number denoted by a figure of the subtrahend is greater than the number denoted by the corresponding figure of the minuend." There is no justification of the use of the word *figure* for *term* in such expressions.

The use of the word term to designate a part of a number is analagous to its subsequent use in arithmetic and algebra, as in the expressions, *the terms of a compound number*, *the terms of a fraction*, *the terms of a ratio or proportion*, *the terms of a series*, *the terms of a polynomial*, etc.

DIVISION.

SHORT AND LONG DIVISION.

It is the practice in some schools to teach long division before short division. It is claimed that the writing of the partial products and dividends, as is done in long division, makes the process of division easier than it is when these successive partial products and dividends are kept in mind, as in short division.

This may be true, and yet the training which the pupils have received has so prepared them for this mental process that it presents little, if any, difficulty. If this be not true, the pupils need this very training. Besides, short division is nearly, if not quite, as easily mastered before as after long division, while its previous mastery assists the pupil in learning long division—certainly the more difficult process when the divisor contains two or more figures. But whether short or long division be first taught, only digital numbers should be used for divisors until the process of short division is mastered.

Place of the Quotient.

The difference between short and long division is best shown by writing the quotient in each process *above the dividend*, as in the examples below:

SHORT DIVISION.

672, *Quotient.*
7)4704, *Dividend.*

LONG DIVISION.

672, *Quotient.*
7)4704, *Dividend.*
42
50
49
14
14

SHORT DIVISION.

392, *Quotient.*
12)4704, *Dividend.*

LONG DIVISION.

392, *Quotient.*
12)4704, *Dividend.*
36
110
108
24
24

It is thus clearly shown that the only difference between the two processes is that the partial products and partial dividends are formed mentally and kept in mind in short division, while they are written in long division. The pupil should be required to use short division when the divisor is 10 or less.

The preceding method of writing the quotient in long division is used by some teachers. If adopted in long division, there will be an advantage in using it also in short division, thus giving the quotient the same position in both processes.

If the quotient be written above the dividend in the division of integral numbers, it should also be written above in the division of decimals.

There is, however, good reason for the almost universal practice of writing the quotient below the dividend in short division, and at the right in long division. In the solution of problems, it often becomes necessary to divide a sum, difference, or product, and this can best be done by writing the quotient under or at the right. The rewriting of the dividend, to make room for the writing of the quotient above it, would break the continuity of the written solution, as well as lengthen the process. The reduction of denominate numbers to a lower denomination, and the solution of problems in partial payments, when the rate is more or less than six, afford good illustrations of this difficulty.

The Determining of Quotient Figures.

The principal difficulty in long division is in determining the successive figures of the quotient, especially when the divisor is a large number. This difficulty may be greatly lessened by the following method:

1. Begin with divisors that are expressed by one figure.

2. Next take examples in which each successive quotient figure is found *by dividing the left-hand term of each successive partial dividend by the left-hand term of the divisor.*

3. Then take examples in which the two left-hand terms are 1 and 0 respectively, *thus making* 10 *the trial divisor.*

The following solutions illustrate these three steps:

(1)	(2)	(3)
8)5088(636	224)51968(232	1036)452732(437
48	448	4144
28	716	3833
24	672	3108
48	448	7252
48	448	7252

In the second solution, the successive quotient figures are found by dividing 5, 7, and 4 respectively by 2, the left-hand term of the divisor, and in the third solution the successive quotient figures are found by dividing 45, 38, and 72 respectively by 10, the number expressed by the two left-hand figures of the divisor.

By solving several examples of each of these three kinds, pupils will not only become familiar with the long division process, but they will be prepared to use

as a *trial divisor* a number expressed by one, two, or three of the left-hand figures of the divisor.

NOTE.—The first fifteen problems in long division in the ELEMENTARY ARITHMETIC (page 105) are arranged in accordance with the above method.

ORAL SOLUTIONS.

In the lessons in number given the first three years, no attempt should be made to teach the logical analysis of problems, and generally nothing is gained by stating formally the *reasons* for processes and results.

In solving the numerous little problems which embody the applications of processes, the pupils should first give the answer, and then state the process or processes by which it is reached, as follows:

(1) 2 pears: 9 pears less 7 pears are 2 pears.

(2) 13 cents: 8 cents and 5 cents are 13 cents.

(3) 15 cents: 5 times 3 cents are 15 cents.*

(4) 4 rows: 4 trees are contained in 16 trees 4 times.

The oral solutions of problems in the fourth and fifth years should also be concise and simple. Young pupils are not helped by an attempt to give a minute and formal statement of every condition involved in a problem; and, at no stage of their advancement, is the reasoning faculty trained by the repetition of what has

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* It is absurd to require young children to give such a solution as this:

Teacher.—"What will five apples cost at three cents apiece?"

Pupil.—"Five apples will cost five times as much as one apple. Hence, if one apple costs three cents, five apples will cost five times three cents, which is fifteen cents. Therefore, if one apple costs three cents, five apples will cost fifteen cents."

been aptly called "logical verbiage." It is now admitted that the elaborate logical analyses of problems which pupils were formerly required to give in what is called "mental arithmetic," was a serious hindrance to the mastery of the processes and principles of arithmetic, and it is equally evident that it was an injury to the thinking power of children. Much of the glibbest logical analysis, once the pride of so many teachers, was the result of the worst form of rote teaching, the analyses being committed to memory by the pupils, and repeated without any wholesome exercise of the logical faculty.

This wide abuse of the so-called "mental arithmetic" has led many teachers to underestimate the value of analytic drills in teaching arithmetic; and, as a consequence, they have a small place, if any, in their instruction. The truth is, the clear analysis of problems has a very important place in arithmetical instruction, and hence the so-called "oral problems" in the arithmetics used should be taught with as much thoroughness as the written problems, especial care being always taken to adapt the form of analysis required to the capacity and advancement of the pupils. The oral problems in the higher book are not only more difficult than those in the lower book, but the analyses are properly more logical and formal. A careful study of the model solutions given in the ELEMENTARY ARITHMETIC, will assist teachers in avoiding the error referred to above.

WRITTEN PROCESSES.

All the written processes in the elementary course should be taught inductively. The oral exercises which precede the written problems, are often so complete an introduction to the corresponding written pro-

cesses that pupils can pass from one to the other without difficulty. All that is necessary, in most cases, is to put on the blackboard the written solutions of two or three of the oral problems in connection with the oral solutions. But this step should not be taken *until the inductive oral exercises have all been recited orally.* The pupils should first master the oral processes, and then be led to pass from or through these to the written processes.

When the inductive problems have been solved both orally and on the slate, the pupils are prepared to pass to the solution of the so-called "written problems." All the written problems, assigned for a lesson, should be solved by the pupils on slate or paper, *and the solutions should be brought to the recitation for the teacher's inspection and approval.* The solutions should be made in an *approved form*, and they should be arranged in a neat and systematic manner. A little instruction will enable pupils to make an economic use of space in slate and blackboard work, and, at the same time, present each solution in an intelligible form. When the solutions of problems are properly arranged and written, two to three minutes will suffice to inspect the slate work of a large class, and often this may be done in less than a minute. The accuracy and the neatness of the written solutions should receive attention.

RULES.

The old method of teaching arithmetical processes by requiring pupils, first, to commit to memory a formal rule, and then to solve the problems "according to the rule," and with constant reference to it, was long since discarded by the most successful teachers. Experience has shown that the rule is not only useless as a means

of teaching numerical processes, but that it is an actual hindrance. It has also shown that a knowledge of the process is essential to the proper teaching of the rule. Hence, "*processes before rules*," and "*rules through processes*," have been generally accepted as wise maxims for the teaching of elementary arithmetic.

Since the rule is to be taught after the process, the author's rule should be placed *after* the problems in all elementary arithmetics. The placing of the author's rule before the problems, or after the first four or five problems, is a violation of the true order of teaching processes and rules, and it leads to an improper use of the rule in the solution of the remaining problems. Whether all the problems should be solved before a rule is generalized, is a question for each teacher to decide, but the author's statement of the rule is best taught after the given problems have been solved and the process thoroughly mastered. In teaching any process, the successive steps will, of course, be taught, and the pupils will be required again and again to describe these steps in words, but all this will be done with direct reference to the mastery of the processes as such. Attention will also be given to the primary principles involved, but this will not be chiefly directed to their concise statement in language. In other words, the primary facts involved in principles and rules should be taught incidentally in connection with the solution of the problems. The teaching of principles and rules inductively, with the memorizing of their best possible statement, is the final step.

When the formal rule is taught, it should be derived from the process by the pupils under the guidance of the teacher. The true order of the successive steps is as follows:

1. A mastery of the process without reference to the author's rule.

2. The recognition and statement of the successive steps of the process in their order.

3. The combination of these several statements into a general statement.

4. A comparison of the general statement thus formed with the author's rule.

5. The memorizing of the approved rule.

The rule for addition, for illustration, may thus be taught:

What is the first step in the addition of numbers? "To write the numbers to be added."

How are the numbers to be written? "So that the figures which denote the same order of units shall be in the same column."

NOTE.—Take an example, and solve it on the blackboard by doing just what each answer describes. Lead the pupils to correct all such wrong expressions as "figures of the same order," the writing of the figures "under each other," etc. The correct expressions have, of course, been previously used in describing solutions of problems.

What is the next step? "Draw a line underneath."

Now put your three statements together in one sentence, and I will write it on the blackboard:

"*Write the numbers to be added so that figures which denote the same order of units shall be in the same column, and draw a line underneath.*"

What is the next step? "Add the numbers in each column."

With which column do you begin? "With the units' column."

What is done with the sum of each column? "Write the sum underneath the column added."

When can this be done? "When the sum is less than ten."

Now put these four statements in one sentence, and I will write it on the blackboard:

"*Beginning with units' column, add the numbers in each column, and write the sum, when less than ten, underneath.*"

What is done when the sum of the numbers in any column is ten or more?

"*When the sum of any column is ten or more, write the right-hand figure under the column added, and add the number denoted by the left-hand figure or figures with the next column.*"

I have written this statement on the blackboard.

How is the sum of the numbers in the left-hand column written?

"*Write the entire sum of the left-hand column, placing the right-hand figure under the column added.*"

These four sentences make the *rule* for addition.

When the rule has thus been developed and corrected, have the several steps repeated in connection with the solution of a problem on the board, one pupil giving the first sentence, another the second, and so on. Finally, perfect the rule by comparing it with the author's rule, and then require the pupils to repeat the entire rule with accuracy and without hesitation.

DEFINITIONS.

The definitions should, in like manner, be taught inductively, and they should first be stated by the pupils under the teacher's guidance.

The definition of addition, for illustration, may be taught by writing several examples on the blackboard (using small concrete numbers), and by questions leading the pupils to see that the sum contains as many units or ones as all the numbers added taken together. The general principle may thus be stated:

The sum of two or more numbers contains as many units as all the given numbers.

The fact that addition is a *process* may next be taught, and the pupils then led to the following definition:

Addition is the process of finding the sum of two or more numbers.

NOTE.—More advanced pupils may be shown that the facts in the above principle and definition may be united in one sentence, as follows:

Addition is the process of finding a number that contains as many units as two or more given numbers.

In elementary classes, it is believed to be better to define the term *sum* (as above), and define addition by the use of this known term.

SUMMARY.

The above suggestions combined give the following order for teaching each general process in elementary arithmetic:

1. The oral drill on the inductive oral exercises.
2. The induction of the written process from the oral solutions, under the teacher's guidance.
3. The solution of the oral problems on slate or paper by the written process.
4. The solution of the written problems on slate or paper.
5. The induction of the rule from the written process, and the memorizing of the approved rule.
6. The induction and memorizing of definitions and principles.

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them, which was often not the case when pupils were required to complete the so-called Mental Arithmetic before beginning the study of Written Arithmetic.

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