ORDNANCE AND GUNNERY

A TEXT-BOOK

PREPARED FOR THE CADETS OF THE

UNITED STATES MILITARY ACADEMY, WEST POINT

BY

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PREFACE.

The material of war has undergone greater changes in the past thirty years than in the previous hundreds of years since the introduction of gunpowder. The weapons of attack and defense have become more numerous, more complicated, and vastly more efficient. The appurtenances to their use are more elaborate. The science of gunnery constantly requires of the officer greater knowledge and higher attainments, that he may thoroughly understand the powerful and important instruments that are put under his control and be prepared to obtain from them, in time of need, their full effect.

I have attempted in this text to set before the Cadets of the Military Academy the subjects of Ordnance and Gunnery in such manner as to give to the Cadets a thorough appreciation of the fundamental principles that underlie the construction and effective use of the instruments of war, and such practical knowledge of the material of today as should be possessed by every army officer.

The purpose held in view in the preparation of the text has been to present, in order, the theories that apply in the use of explosives and in the construction of Ordnance material, the methods pursued in the construction of the material, descriptions of the material, and the principles of its use.

The applications of the theoretical deductions to the investigation of the action of gunpowders and other explosives and to the construction and use of Ordnance material, are extensively illustrated by problems fully worked out in the text; the idea being that these solutions, in addition to making evident to the student the practical use of the theories, will serve as guides in solutions of similar problems encountered in practice.

When the theoretical deductions are applicable to other than ordnance constructions other problems inserted in the text indicate their more extended field.

In the chapter on interior ballistics, which is taken principally from the writings of Colonel James M. Ingalls, United States Army, the deduction and application of Colonel Ingalls' latest interior ballistic formulas are fully set forth. The determinations from these formulas have been found in practice to be more closely in accord with the actual results obtained in firings, than determinations from any ballistic formulas hitherto in use.

In the chapter on explosives the theoretical determination of the results from explosions, including the quantity of heat, the volume of the gases, the temperature, the pressure, etc., is explained and illustrated by examples. This demonstration has not hitherto been available in English.

A simplification has been introduced, by the author of the text, into the gun construction formulas of Clavarino. The simplification materially shortens these extended formulas and reduces the labor required in their application.

The graphic system of representing the pressures and shrinkages in cannon, devised by Lieut. Commander Louis M. Nulton, United States Navy, is also explained in connection with the deduction and application of the formulas of gun construction. The graphic system is a material help toward a ready understanding of the subject.

In the subject of exterior ballistics sufficient problems are introduced and fully worked out to illustrate the processes followed in the solutions of the principal problems of gunnery. This course has been adopted with the purpose of removing to a large extent the difficulties usually encountered in the practical application of the formulas of exterior ballistics.

An appendix to the chapter on exterior ballistics contains the deduction of the author's formulas for double interpolation. The formulas are more accurate and more convenient in applicacation than the interpolation formulas previously in use. Explanation of the use of the ballistic tables to which the interpolation formulas apply, follows the deduction of the formulas.

The chapter on armor contains information as to the general arrangement and thickness of the armor on ships of war, the expected targets of the heavy artillery.

A chapter on submarine mines, torpedos, and submarine torpedo boats concludes the text.

Acknowledgment is due for much assistance obtained from the text-book on Ordnance and Gunnery, by Captain L. L. Bruff, Ordnance Department, that has been in use at the Military Academy for the past eleven years. The plan of that work has been largely followed, many of its illustrations appear in this volume, and assistance has been derived from its text throughout.

I desire to express my indebtedness to Captain Edward P. O'Hern, Ordnance Department, Principal Assistant in the Department of Ordnance and Gunnery, whose valuable suggestions and helpful criticism have been of marked benefit to the text. Lieutenants Ennis, Bryant, and Selfridge, Artillery Corps, Assistant Instructors of the Department, have also, by their suggestions, added to the value of the text.

I desire, too, to thank Sergeant Carl A. Schopper, of the West Point Ordnance Detachment. The illustrations in the text are the products of his skill as a draftsman, of his knowledge of the illustrative arts, and of his unremitting labor.

ORMOND M. LISSAK.

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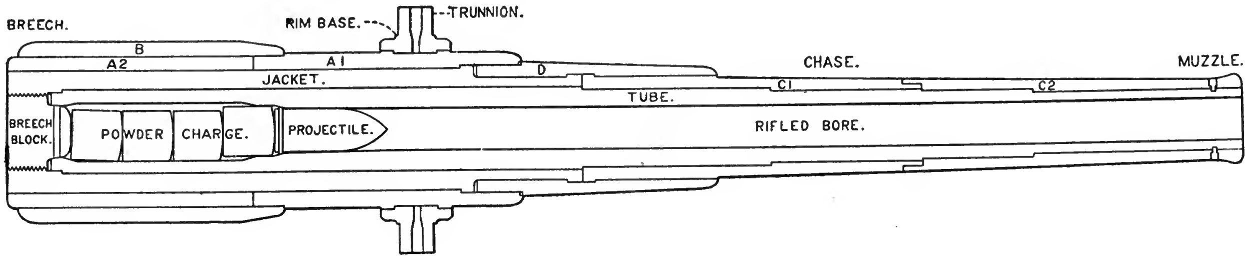


12-Inch Rifle, Model of 1900, 40 Calibers, 59.10 Tons.
(Diameters Exaggerated.)

ORDNANCE AND GUNNERY.

CHAPTER I.

GUNPOWDERS.

1. **Definitions.**—*Explosion,* in a general sense, may be defined as a sudden and violent increase in the volume of a substance. In a chemical sense, explosion is the extremely rapid conversion of a solid or a liquid to the gaseous state, or the instantaneous combination of two or more gases accompanied by increase of volume. Chemical explosion is always accompanied by great heat.

An explosion due to physical causes alone, as when a gas under compression is suddenly released and allowed to expand, causes cold.

The explosion of gunpowder may be divided into three parts: ignition, inflammation, and combustion.

Ignition is the setting on fire of a part of the grain or charge.

Gunpowder is ignited by heat, which may be produced by electricity, by contact with an ignited body, by friction, shock, or by chemical reagents.

An ordinary flame, owing to its slight density, will not ignite powder readily. The time necessary for ignition will vary with the condition of the powder. Thus damp powder ignites less easily than dry; a smooth grain less easily than a rough one; a dense grain less easily than a light one.

Powder charges in guns are ignited by primers, which are fired by electricity, by friction, or by percussion.

Inflammation is the spread of the ignition from point to point of the grain, or from grain to grain of the charge.

With small grain powders the spaces between the grains are small, and the time of inflammation is large as compared with the time of combustion of a grain; but with modern large grain powders the facilities for the spread of ignition and the time of burning of the grain are so great that the whole charge is supposed to be inflamed at the same instant, and the time of inflammation is not considered.

Combustion is the burning of the inflamed grain from the surface of ignition inward or outward or both, according to the form of the grain.

Experiment shows that powder burns in the air according to the following laws:

1. In parallel layers, with uniform velocity, the velocity being independent of the cross section burning.

2. The velocity of combustion varies inversely with the density of the powder.

When a charge of powder is ignited in a gun inflammation of the whole charge is rapidly completed. The gases evolved from the burning grains accumulate behind the projectile until the pressure they exert is sufficient to overcome the resistance of the projectile to motion. The accumulated gases, augmented by those formed by the continued burning of the charge, expand into the space left behind the projectile as it moves through the bore, exerting a continual pressure on the projectile and increasing its velocity until it leaves the muzzle.

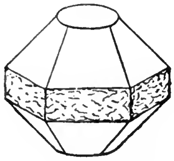
History.—The Chinese are said to have employed an explosive mixture, very similar to gunpowder, in rockets and other pyrotechny as early as the seventh century.

The earliest record of the use in actual war of the mixture of charcoal, niter, and sulphur called gunpowder, dates back to the fourteenth century. Its use in war became general at the beginning of the sixteenth century. Until the end of the sixteenth century it was used in the form of fine powder or dust. To overcome the difficulty experienced in loading small arms from the muzzle with powder in this form, the powder was at the end of the sixteenth century given a granular form. With the same end in view attempts at breech loading were made; but without success, as no effective gas check, which would prevent the escape of the powder gases to the rear, was devised.

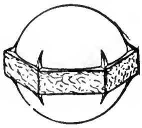
No marked improvement was made in gunpowder until 1860, when General Rodman, of the Ordnance Department, U. S. Army, discovered the principle of progressive combustion of powder, and that the rate of combustion, and consequently the pressure exerted in the gun, could be controlled by compressing the fine grained powder previously used into larger grains of greater density. The rate or velocity of combustion was found to diminish as the density of the powder increased. The increase in size of grain diminished the surface inflamed, and the increased density diminished the rate of combustion, so that, in the new form, the powder evolved less gas in the first instants of combustion, and the evolution of gas continued as the projectile moved through the bore. By these means higher muzzle velocities were attained with lower maximum pressures. To obtain a progressively increasing surface of combustion General Rodman proposed the perforated grain, and the prismatic form as the most convenient for building into charges. As a result of his investigations powder was thereafter made in grains of size suitable to the gun for which intended, small grained powder for guns of small caliber, and large grained powder for the larger guns. The powders of regular granulation, such as the cubical, hexagonal, and sphero-hexagonal, came into use, and finally for larger guns the prismatic powder in the form of perforated hexagonal prisms.

A further control of the velocity of combustion of powder was obtained in 1880 by the substitution of an underburnt charcoal for the black charcoal previously used. The resulting powder, called *brown* or *cocoa* powder from its appearance, burned more slowly than the black powder, and wholly replaced that powder in the larger guns.

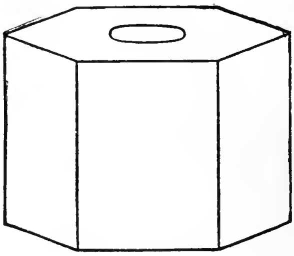
A still further advance in the improvement of powder was brought about in 1886 by the introduction of smokeless powders. These powders are chemical compounds, and not mechanical mixtures like the charcoal powders; they burn more slowly than the



Hexagonal.



Sphero-hexagonal.



Prismatic.

charcoal powders, and produce practically no smoke. Smokeless powders have now almost wholly replaced black and brown powders for charges in guns. Black powder is used in fuses, primers, and igniters, in saluting charges, and as hexagonal powder in the smaller charges for seacoast mortars.

2. **Charcoal Powders.**—Composition.—Black gunpowder is a mechanical mixture of niter, charcoal, and sulphur, in the proportions of 75 parts niter, 15 charcoal, and 10 sulphur.

The niter furnishes the oxygen to burn the charcoal and sulphur. The charcoal furnishes the carbon, and the sulphur gives density to the grain and lowers its point of ignition.

The distinguishing characteristic of charcoal is its color, being brown when prepared at a temperature up to 280°, from this to 340° red, and beyond 340° black.

Brown powder contains a larger percentage of niter than black powder, and a smaller percentage of sulphur. A small percentage of some carbohydrate, such as sugar, is also added. Its color is due to the underburnt charcoal.

Manufacture.—The ingredients, purified and finely pulverized, are intimately mixed in a wheel mill under heavy iron rollers. The mixture is next subjected to high pressure in a hydraulic press. The cake from the press is broken up into grains by rollers, and the grains are rumbled in wooden barrels to glaze and give uniform density to their surfaces. The powder is then dried in a current of warm dry air, and the dust removed. The powder is thoroughly blended to overcome as far as possible irregularities in manufacture.

For powders of regular granulation the mixture from the wheel mill was broken up and pressed between die plates constructed to give the desired shape to the grains. Prismatic powder was made by reducing the mill cake to powder and pressing it into the required form.

Smokeless Powders.—There are two classes of smokeless powders used in our service: nitroglycerine powder in small arms, and nitrocellulose powder in cannon. They are both made from guncotton, to which is added for the small-arm powder about 30 per cent by weight of nitroglycerine.

Comparison of Nitroglycerine and Nitrocellulose Powders.—The temperature of explosion of nitroglycerine powder is higher than that of nitrocellulose powder. As the erosion of the metal of the bore of the gun is found to increase with the temperature of the gases, greater erosion follows the use of nitroglycerine powder. The endurance, or life, of a modern gun is dependent on the condition of the bore, and on account of the great cost of cannon erosion becomes a more serious defect in cannon than in small arms. On this account, therefore, nitrocellulose powder is more suitable than nitroglycerine powder for cannon.

To produce a given velocity a larger charge of nitrocellulose than of nitroglycerine powder is required. This necessitates for nitrocellulose powder a larger chamber in the gun, and the increase in size of the chamber involves increased weight of metal in the gun. This is more objectionable in a small arm than in cannon, for the increased weight of the gun and of the charge adds to the burden of the soldier. For this reason nitroglycerine powder is more suitable than nitrocellulose powder in the small arm.

In the manufacture of nitroglycerine powders for cannon, a satisfactory degree of stability under all the conditions to which cannon powders are exposed was not obtained. In time the powder deteriorated, and exudation of free nitroglycerine occurred. Detonations and the bursting of guns followed. In the small-arm cartridge the powder is hermetically sealed, and as now manufactured appears to possess a satisfactory degree of stability.

For these reasons nitroglycerine powder has been selected for use in small arms in our service, and nitrocellulose powder for use in cannon.

A disadvantage attending the use of nitrocellulose powder arises from the fact that in the explosion there is not a sufficient amount of oxygen liberated to combine with the carbon and form CO_2. The reaction on explosion is approximately represented by the following equation.

$$2(C_6H_7O_2)O_3(NO_2)_3 = 9CO + 3CO_2 + 7H_2O + 3N_2$$

A large quantity of CO, an inflammable gas, is often left in the bore. On opening the breech more oxygen is admitted with the air, and should a spark be present the CO burns violently, uniting with the oxygen and forming CO_2. This burning of the gas is called a *flareback*. An instance of it has occurred with disastrous results in a turret gun aboard one of our men-of-war, the *Missouri*.

3. **Guncotton.**—Guncotton forms the base of most smokeless powders. When dry cotton, $C_6H_{10}O_5$, is immersed in a mixture of nitric and sulphuric acids part of the hydrogen of the cotton is replaced by NO_2 from the nitric acid. The sulphuric acid takes up the water formed during the reaction and prevents the dilution of the nitric acid. The nitrated cotton,

or nitrocellulose, may be of several orders of nitration, depending on the strength and proportions of the acids, and the temperature and duration of immersion; as mononitrocellulose, dinitrocellulose, trinitrocellulose, according as one or more atoms of hydrogen are replaced. All nitrocellulose is explosive, and the order of explosion produced is higher as the nitration is higher. Dinitrocellulose and trinitrocellulose are used in the manufacture of smokeless powders. The lower orders of nitrocellulose, containing less than 12.75 per cent of nitrogen, are soluble in a mixture of alcohol and ether. Trinitrocellulose contains a higher percentage of nitrogen, and is insoluble in alcohol and ether but soluble in acetone.

MANUFACTURE OF GUNCOTTON FOR SMOKELESS POWDERS.—The process followed is practically the same for all varieties, the nitration being stopped at the point desired in each case.

The cotton used is the waste or clippings from cotton mills. It is first finely divided and then freed from grease, dirt, and other impurities by boiling with caustic soda. After cleansing it is passed through a centrifugal wringer and then further dried in a dry-house.

The dry cotton is immersed in a mixture of about three parts sulphuric acid and two parts nitric acid for about fifteen minutes; after which the cotton is run through a wringer to remove as much acid as possible. It is then thoroughly washed or *drowned.*

After this washing the guncotton is reduced to a pulp and further washed to remove any trace of acid which may have been freed in pulping, carbonate of soda being added to neutralize the acid.

The water is then partially removed from the pulp by hydraulic pressure, and the dehydration is completed by forcing alcohol under high pressure through the compressed cake.

4. Nitroglycerine Small-arm Powder.—*Laflin and Rand, W. A.*—In the manufacture of this powder highly nitrated guncotton called *insoluble nitrocellulose* is used. It is insoluble in ether and alcohol but soluble in acetone.

The powder is composed of

Insoluble nitrocellulose	67.25 per cent
Nitroglycerine	30.00 per cent
Metallic salts	2.75 per cent

Forty pounds of acetone serve as solvent for 100 pounds of the above mixture.

The nitroglycerine and acetone are first mixed. The acetone makes the nitroglycerine less sensitive to pressure or shock, and therefore less dangerous to handle in the subsequent operations. The dried guncotton is spread in a large copper pan, the finely ground metallic salts are sifted over it, and the mixed nitroglycerine and acetone are sprinkled over both. The whole is mixed by hand by means of a wooden rake for a period of about ten minutes, the object of the mixing being to thoroughly moisten the guncotton for the purpose of eliminating the danger from the presence of dry guncotton in the next operation. The mixed mass is put into a mixing machine, where it is mechanically mixed for a period of three hours. It comes from the mixing machine in the form of a colloid or jelly like paste. It is then stuffed and compressed into brass cylinders, from which it is forced by hydraulic pressure through dies fitted with mandrels. It comes from the die in the form of a long hollow string or tube, and is received on a belt which carries it over steam pipes into baskets. The drying which it receives while on the belt strengthens the tube, and after remaining half an hour in the baskets it becomes sufficiently tough to be cut into grains. This is done in a machine provided with revolving knives. The resulting grains are bead-shaped single perforated cylinders and have a length of about one twentieth of an inch. The powder is dried for two or three weeks at a temperature not to exceed 110° F. It is then thoroughly mixed twice in the blending barrels and graphited at the same time. It is carefully screened to remove large grains, dust, and foreign matter, and is packed in muslin bags in metallic barrels holding 100 pounds.

Cordite.—This is an English nitroglycerine powder, composed of 58 per cent of nitroglycerine, 37 per cent of guncotton, and 5 per cent of vaseline. The vaseline serves to render the powder water proof and improves its keeping qualities. For small arms the powder is made in the form of slender cylindrical rods, the length of the chamber of cartridge. For cannon it is in thicker and longer rods, in tubular form, or in the form of perforated cylinders. For heavy guns a powder called *Cordite M. D.* has lately been introduced. The composition (30 parts nitroglycerine, 65 parts guncotton, 5 parts vaseline) is very similar to that of our small-arm powder. The reduction in the percentage of nitroglycerine was made for the purpose of lowering the temperature of explosion and reducing the erosion in the bore.

Wetteren Powder.—A nitroglycerine powder manufactured at the Royal Belgian Factory at Wetteren. The solvent used is amyl acetate.

5. **Manufacture of Nitrocellulose Powder.**—The guncotton used contains 12.65 per cent of nitrogen, and is soluble in the ether-alcohol mixture. It is prepared as previously described, the dehydration with alcohol being so conducted that when completed the proper proportion of alcohol for solution remains in the cake. The guncotton cake is broken up and ground until it is free from lumps, and is then placed in a mixing machine with the proper amount of ether, two parts of ether to one of alcohol. During the mixing the temperature is kept at 5° C. to prevent loss of the solvent.

The powder comes from the mixing machine as a colloid, and the remaining processes are similar to those described for nitroglycerine powder.

After graining, the solvent is recovered by forcing heated air over the powder. The ether and alcohol vapors are collected and afterwards condensed for further use. The powder is dried for a period varying from six weeks to three months, depending on the size of the grain. The drying is never complete, a small percentage of the solvent always remaining, but care is taken that the remaining percentage shall be uniform.

In the manufacture of all powders uniformity in the product can only be obtained by the strictest uniformity in the quantities and quality of the substances used, and in the conduct of the various processes.

Cannon powders are, as a rule, not graphited.

Other Smokeless Powders.—The length of time required for the drying of nitrocellulose powders has led to the development of other powders that require little or no time to dry.

Two such powders have been tested. One, *stabilite,* is composed of nitrocellulose with or without nitroglycerine and a solvent that is itself an explosive and not volatile. The other is similar to the present nitrocellulose powders except that dinitrocellulose is used in its manufacture instead of trinitrocellulose.

To make up for the insufficiency of oxygen in nitrocellulose, already referred to, a number of smokeless powders are made by a combination of nitrocellulose with nitroglycerine or with the nitrates of barium, potassium, and sodium. The nitroglycerine or the metallic nitrates furnish the oxygen which is deficient in the nitrocellulose.

E. C. Powder.—This powder contains both soluble and insoluble nitrocellulose and the nitrates of barium, potassium, and sodium. It is yellow in color and of fine granulation. It is an easily ignited quick burning powder and is used in our service in blank small-arm cartridges.

Schultze Powder, the type of smokeless sporting powders, is of constitution similar to that of E. C. powder.

Troisdorf Powder, used in the German service, and *B. N. Powder,* in the French service, are other powders similarly constituted. All these powders differ principally in the proportion of the ingredients, and also in the organic substance used as a cementing agent.

Maxim Powder is composed of nitrocellulose, both soluble and insoluble, nitroglycerine, and a small percentage of sodium carbonate.

Form and Size of Grain.—For most cannon in our service the powder is formed into a cylindrical grain with seven longitudinal perforations, one central and the other six equally distributed midway between the center of the grain and its circumference. A uniform thickness of web is thus obtained. The powder is of a brown color and has somewhat the appearance of horn. The length and diameter of the grain vary in powders for different guns, the size of grain increasing with the caliber of the gun. For the 3-inch rifle the grain has a length of about $\frac{3}{8}$ of an inch and a diameter of $\frac{2}{10}$ of an inch. For the 12-inch rifle the length is $1\frac{1}{2}$ inches and the diameter $\frac{7}{8}$ of an inch. For some of the smaller guns the grains are in the form of thin flat squares.

In other services cannon powders are made into grains of various shapes. Cubes, solid and tubular rods of circular cross section, flat strips, and rolled sheets are among the forms that have been used.

6. Proof of Powders.—All powders used by the Army are furnished by private manufacturers. The materials and processes employed in the manufacture are prescribed by the Ordnance Department in rigid specifications, and the manufacture in all its stages is under the inspection of the Department. The proof of the powder consists of tests made to determine its ballistic qualities, its uniformity, and its stability under various conditions. Its ballistic qualities and uniformity are determined from proof firings made in the gun for which the powder is intended. The required velocity must be obtained without exceeding the maximum pressure specified. The mean variation in velocity in a number of rounds must not exceed, in the small arm 12 feet per second, in cannon 1 per cent of the required velocity.

The stability of the powder under various conditions is determined by heat tests, and by a number of special tests. For small-arms powder the heat test consists in subjecting the powder, pulverized, to a temperature of 150° to 154° F. for 40 minutes. It must not in that time emit acid vapors, as indicated by the slightest discoloration of a piece of iodide of potassium starch paper partially moistened with dilute glycerine. The other tests consist in exposing the powder both loose and loaded in cartridges, to heat, cold, and moisture, for periods varying from six hours to one week. When fired the variations in velocities and pressures must not exceed specified limits.

Nitrocellulose cannon powders are subjected to a heat of 135° C. (275° F.) for five hours. Acid fumes, as indicated by the reddening of blue litmus paper, must not appear under exposure of an hour and a quarter, nor red nitrous fumes under two hours. Explosion must not occur under five hours. Other tests are made for the determination of the loss of weight when subjected to heat, of the moisture and volatile matter in the powder, of the quantities of nitrogen in the powder, and of ash in the cellulose.

For the proper regulation of the evolution of gas in the gun it is important that the grains of smokeless powder retain their general shape while burning. If they break into pieces under the pressure to which they are subjected, the inflamed surface is increased, gas is more quickly evolved, and the pressure in the gun is raised. The powder is therefore subjected to a physical test to determine that the grain has sufficient strength and toughness. The grains are cut so that the length equals the diameter, and are then subjected to slow pressure in a press. The grain must shorten 35 per cent of its length before cracking.

Powder grains incompletely burned, that have been recovered after firing, show that the burning proceeds accurately in parallel layers. The outer diameter of the grain is reduced and the diameter of the perforations increased in exactly equal amounts.

7. Advantages of Smokeless Powder.—The advantages obtained by the use of smokeless powder are due almost wholly to the fact that the powder is practically completely converted into gas. The experiments of Noble and Abel show that the gases evolved by charcoal powders amount to only 43 per cent of the weight of the powder, and part of the energy of this quantity of gas is expended in expelling the residue from the bore. A smaller quantity of smokeless powder will therefore produce an equal weight of gas, and with smaller charges we may give to the projectile equal or higher velocities. The smokelessness of the powder and the absence of fouling in the bore are also due to the complete conversion of the powder into gas.

Ignition and Inflammation of Smokeless Powder.—Though the temperature at which smokeless powder ignites, about 180° C., is much lower than that required for the ignition of black powder, 300° C., the complete inflammation of a charge composed only of smokeless powder takes place more slowly than the inflammation of a charge of black powder. This is due to the slower burning of the smokeless powder and the consequent delay in the evolution of a sufficient quantity of the heated gas to completely envelop the grains composing the charge. In the long chamber of a gun the gases first evolved at the rear of the charge may, in expanding, acquire a considerable velocity. The pressure due to their energy is added to the static pressure due to their temperature and volume, thus increasing the total pressure in the gun. The movement of the gases back and forth causes what are called *wave pressures*, and if the complete ignition of the charge is delayed until the projectile has moved some distance down the bore, there may result at some point in the gun a higher pressure than the metal of the gun at that point can resist.

For this reason and in order to insure the practically instantaneous ignition of the whole charge, small charges of black powder are added to every smokeless powder charge. The priming charges of black powder insure against hang-fires and misfires, and by producing uniformity of inflammation assist toward uniformity in the ballistic results.

In addition, in order to prevent as far as possible the production of wave pressures, the charge of powder, whatever its weight, is given when practicable a length equal to the length of the chamber.

8. Powder Charges.—The powder for a charge in the gun is inserted in one or more bags, depending upon the weight of the charge. The bags are made of special raw silk and are sewed with silk thread. The ends of each bag are double, and between the two pieces at each end is placed a priming charge of black powder, quilted in in squares of about two inches and uniformly spread over the surface.

The charge is inserted through an unsewed seam at one end, and the seam is then sewed. The bag, purposely made large, is then drawn tight around the charge by lacing drawn with a needle between two pleats on the exterior. Two priming protector caps are then drawn over the ends of the bag and fastened by draw strings. In the bottom of each cap is a disk of felt which serves to keep moisture from the priming charge and prevents the loss of the priming through wearing of the bottom of the bag. For convenience in handling the charge a cloth strap is attached to each protector cap. By means of the straps the protector caps may be pulled off without undoing the draw strings when the charge is to be inserted in the gun.

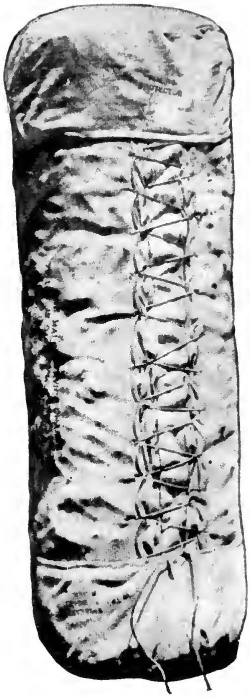
The illustrations show a bag filled ready for lacing, and a bag filled and laced and provided with the priming protector caps.

The weight of each portion of the charge should not be greater than can be readily carried by one man. Thus the charge of 360 pounds for the 12-inch rifle is put up in four bags each holding 90 pounds.

As previously stated, the charge whatever its weight is made up if practicable of a length nearly equal to that of the chamber, with a minimum limit of nine tenths of that length.

Raw silk does not readily hold fire. With powder bags made of cotton cloth it occasionally happens that a fragment of the bag remains burning in the bore, and to this fact is ascribed the flarebacks that have occurred. Powder bags treated with chemicals to render them non-inflammable have also been tried. Ammonium phosphate is found to be the best agent for this purpose.

Bag filled ready
for lacing



Bag laced and provided
with priming pro-
tector caps.

SECTION OF POWDER CHARGE FOR HEAVY GUNS.

A nitrocellulose cloth which will burn up completely and leave no residue has been used as a material for powder bags, but as the charge of powder enclosed in this material is much more subject to accidental ignition by a chance spark, the nitrocellulose cloth is not generally adopted.

The powder charge in fixed ammunition is placed loose in the cartridge case.

In fixed ammunition for cannon one or two wads of felt placed on top of the powder fill the space in the case behind the projectile. The priming charges of black powder are contained in the primer, which is inserted in the head of the cartridge case, and between two disks of quilted crinoline at the forward end of the charge.

Blank Charges.—If the same smokeless powder that is prescribed for use with the projectile in any piece is used in a blank charge, the grains are not subjected to the pressure under which they were designed to burn, and consequently they burn very slowly and many of them are ejected from the bore only partially consumed. The report made by the explosion under these circumstances is unsatisfactory for saluting purposes.

To produce a sharper report a more rapid evolution of gas is necessary, which requires, if smokeless powder is employed, the use of a smaller grain, or one that is porous through imperfect colloiding. It has been found that a satisfactory report can be obtained from a blank charge of smokeless powder only by the use of a grain so small or of such a nature that the rate of evolution of the gas becomes excessive. This has resulted, in several instances, in the bursting of the gun.

For this reason black powder only has been used in saluting charges. A nitrocellulose powder, called the *Thorn smokeless saluting powder*, has recently given satisfactory results in blank charges. The powder is in flat cross-shaped grains, about $\frac{3}{8}$ of an inch in length and breadth. It is of low density and has the appearance of blotting-paper.

COMBUSTION OF POWDER UNDER CONSTANT PRESSURE.

9. Quantity Burned when any Thickness has Burned.—Under constant pressure, as in the air, a grain of powder burns in parallel layers and with uniform velocity, in directions perpendicular to all the ignited surfaces.

Under the variable pressure in the gun powder burns with a variable velocity, but, as has been previously stated, modern smokeless powders burn accurately in parallel layers in the gun. A determination of the volume burned when any thickness of layer is burned will therefore be useful when we come to consider the burning of the powder in the gun.

Powders of irregular granulation may be considered as composed of practically equivalent grains of regular form.

Let l_0 be one half the least dimension of the grain,
- l the thickness of layer burned at the time t,
- S_0 the initial surface of combustion,
- S the surface of combustion at the time t, when a thickness l has been burned,
- S' the surface of combustion when $l=l_0$,
- V_0 the initial volume of the grain,
- V the volume burned at the time t,
- $F=V/V_0$ the fraction of grain burned at the time t.

The least dimension of the grain, $2l_0$, is called the *web* of the grain. As the burning proceeds equally in directions perpendicular to all the surfaces, the grain will, in most instances, be about to disappear when the thickness of layer burned is nearly equal to l_0. The surface S', corresponding to this thickness, is therefore called the *vanishing surface.*

A general expression may be written for the burning surface of a grain when a thickness l has been burned. Since a surface is a quantity of the second degree the expression will be of the form,

$$S=S_0+al+bl^2 \tag{1}$$

in which a and b are numerical coefficients whose values depend on the form and dimensions of the grain.

For grains that burn with a decreasing surface the sign of a in this equation will later be found to be negative, and for those that burn with an increasing surface the sign of b becomes negative.

The volume burned when any thickness l has been burned is

$$V=\int_0^l S dl$$

And substituting for S its value from equation (1),

$$V=S_0 l+\frac{a}{2}l^2+\frac{b}{3}l^3 \tag{2}$$

Dividing both members by V_0 and introducing l_0 by multiplication and division we have, for the fraction of the grain burned,

$$F=\frac{S_0 l+\frac{a}{2}l^2+\frac{b}{3}l^3}{V_0}=\frac{S_0 l_0}{V_0}\frac{l}{l_0}\left\{1+\frac{a l_0}{2S_0}\frac{l}{l_0}+\frac{b l_0^2}{3S_0}\frac{l^2}{l_0^2}\right\}$$

and making

$$\alpha=S_0 l_0/V_0 \qquad \lambda=a l_0/2S_0 \qquad \mu=b l_0^2/3S_0 \tag{3}$$

we obtain

$$F=\alpha\frac{l}{l_0}\left\{1+\lambda\frac{l}{l_0}+\mu\frac{l^2}{l_0^2}\right\} \tag{4}$$

This equation gives the value for the fraction of the grain burned when a length l has been burned; and as each grain in a charge of powder burns in the same manner, the equation also expresses the value for the fraction of the whole charge burned.

The quantities α, λ, and μ are called the *constants of form* of the powder grain. Their values depend wholly on the form and relative dimensions of the grain.

When $l=l_0$ the whole grain is burned, F becomes unity, and we have the relation

$$1=\alpha(1+\lambda+\mu) \tag{5}$$

which may always serve to test the correctness of the values of these constants as determined for any grain.

10. **Determination of the Values of the Constants of Form for Different Grains.**—In the values of α, λ, and μ, equations (3), the quantities S_0, l_0, and V_0 are known for any form of grain. We must know in addition the values of a and b.

When $l=l_0$ the volume burned is the original volume V_0 and equation (2) becomes

$$V_0=S_0l_0+\frac{a}{2}l_0^2+\frac{b}{3}l_0^3$$

The burning surface at this time, designated by S', is, from equation (1),

$$S'=S_0+al_0+bl_0^2$$

The values of a and b, if desired, may be derived from these two equations.

Combining the two equations with equations (3) we obtain the following values for α, λ, and μ.

$$\left.\begin{aligned}\alpha&=S_0l_0/V_0\\ \lambda&=3/\alpha-S'/S_0-2\\ \mu&=S'/S_0-2/\alpha+1\end{aligned}\right\} \tag{6}$$

The Vanishing Surface.—The quantity S', which represents the vanishing surface, or surface of combustion when $l=l_0$, requires explanation. A spherical grain burning equally along all the radii becomes a point as l becomes equal to l_0. S' for a sphere is therefore 0, and similarly for a cube. A cylindrical grain, of length greater than its diameter, becomes a line when $l=l_0$. S' is therefore 0 for this cylinder. A flat square grain remains flat throughout the burning, its thickness being reduced until as l becomes equal to l_0 there are two burning surfaces with no powder between them. S', in this case, is the sum of these two surfaces.

PARALLELOPIPEDON.—Let $2l_0$ be the least dimension, and m and n the other dimensions of the grain of powder, m being the longer.

$$S_0=4l_0m+4l_0n+2mn$$
$$S'=2(m-2l_0)(n-2l_0)$$
$$V_0=2l_0mn$$

Make x and y the ratios of the least dimension to the other dimensions of the grain

$$x=2l_0/m \qquad y=2l_0/n$$

With these values we get from (3) for α

$$\alpha=\frac{S_0l_0}{V_0}=2l_0/n+2l_0/m+1=1+x+y$$

Eliminating the common factors in the values of S' and S_0 we have,

$$\frac{S'}{S_0}=\frac{mn-2l_0n-2l_0m+4l_0^2}{2l_0m+2l_0n+mn}$$

and dividing each term by mn,

$$\frac{S'}{S_0}=\frac{1-2l_0/m-2l_0/n+4l_0^2/mn}{2l_0/n+2l_0/m+1}=\frac{1-x-y+xy}{1+x+y}$$

Substituting in equations (6),

$$\lambda=-\frac{x+y+xy}{1+x+y}$$

$$\mu=\frac{xy}{1+x+y}$$

For the parallelopipedon grain, the general expression for the fraction of the grain burned when a thickness l has been burned therefore becomes, by equation (4),

$$F=(1+x+y)\frac{l}{l_0}\left\{1-\frac{x+y+xy}{1+x+y}\frac{l}{l_0}+\frac{xy}{1+x+y}\frac{l^2}{l_0^2}\right\} \tag{7}$$

And by giving various values to x and y this equation may be applied to any form of the parallelopiped.

11. *Cube.*—For instance, for the cube $m=n=2l_0$, and x and y are unity. Therefore

$$\alpha=3 \qquad \lambda=-1 \qquad \mu=1/3$$

and

$$F=3\frac{l}{l_0}\left\{1-\frac{l}{l_0}+\frac{1}{3}\frac{l^2}{l_0^2}\right\}=1-\left(1-\frac{l}{l_0}\right)^3 \tag{8}$$

Strip.—For strips or ribbons of square cross section $n=2l_0$ and $y=1$,

$$\alpha=2+x \qquad \lambda=-\frac{1+2x}{2+x} \qquad \mu=\frac{x}{2+x}$$

If the strip is very long in comparison with the edge of cross section, x is practically zero and

$$\alpha=2 \qquad \lambda=-1/2 \qquad \mu=0$$

Square Flat Grains.—For square flat grains $x=y$ and

$$\alpha=1+2x \qquad \lambda=-\frac{x(2+x)}{1+2x} \qquad \mu=\frac{x^2}{1+2x}$$

If the grains are very thin, x is small compared with unity and

$$\alpha=1 \qquad \lambda=0 \qquad \mu=0$$

As the surface and volume of a burning sphere of powder vary with the diameter in precisely the same manner that the surface and volume of a cube vary with the edge of the cube, the values α, λ, and μ, see equations (6), will be the same for the sphere as for the cube. And similarly the values of these constants for a cylinder of length greater than its diameter will be the same as for the strips of square cross section, and the values for a flat cylinder will be the same as for the flat square grain.

SPHERE.—For the sphere,

$$\alpha=3 \qquad \lambda=-1 \qquad \mu=1/3$$

the same as for the cube.

12. SOLID CYLINDER.—For the solid cylinder of length greater than the diameter, $d=2l_0$ and $x=2l_0/m$,

$$\alpha=2+x \qquad \lambda=-\frac{1+2x}{2+x} \qquad \mu=\frac{x}{2+x}$$

If the diameter is very small compared with the length, as in the slender cylinders or threads of cordite, $2l_0$ is small with respect to m, x is small compared with unity, and approximately

$$\alpha=2 \qquad \lambda=-1/2 \qquad \mu=0$$

Therefore for cordite

$$F=2\frac{l}{l_0}\left\{1-\frac{1}{2}\frac{l}{l_0}\right\}=1-\left(1-\frac{l}{l_0}\right)^2 \tag{9}$$

FLAT CYLINDER.—$2l_0$=thickness, d=diameter, $x=2l_0/d$,

$$\alpha=1+2x \qquad \lambda=-\frac{x(2+x)}{1+2x} \qquad \mu=\frac{x^2}{1+2x}$$

the same as for the flat square grain.

SINGLE PERFORATED CYLINDER.—Let R be the outer radius of the grain, r the radius of the perforation, and m the length of the grain. Make $x=2l_0/m$. By proper substitution we find, for the tubular grain in general,

$$\alpha=1+x \qquad \lambda=-\frac{x}{1+x} \qquad \mu=0$$

If the grain is very long compared with its thickness of wall, x is small compared with unity. We then have

$$\alpha=1 \qquad \lambda=0 \qquad \mu=0$$

and

$$F=l/l_0 \tag{10}$$

This indicates for long tubes with thin walls a constant emission of gas during the burning of the grain, since F now varies directly with l.

13. MULTIPERFORATED CYLINDER.—A section of the service multiperforated grain before burning is shown in Fig. 1. The

FIG. 1.

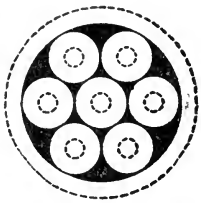


FIG. 2.

perforations are equal in diameter and symmetrically distributed. The web, $2l_0$, is the thickness between any two adjacent circumferences. When this thickness has burned the section is as shown in Fig. 2.

There remain now six interior and six exterior three-cornered pieces, called *slivers,* which burn with a decreasing surface until completely consumed.

The method previously followed cannot be used to find the value of F for the multiperforated grain because the law of burning for this grain changes abruptly when the grain is but partially consumed.

To find the value of F for this grain we proceed as follows.

Let R be the radius of the grain, r the radius of each perforation, m the length of the grain.

For the initial volume we have

$$V_0=\pi m(R^2-7r^2)$$

When a thickness l is burned, R, r, and m become respectively $R-l$, $r+l$, and $m-2l$, and the volume remaining is obtained from the above equation by making these substitutions. The difference between the two volumes will be the volume burned, and dividing this resulting volume by V_0 we have the value of F. This may be reduced to

$$F=\frac{2l_0\{R^2-7r^2+m(R+7r)\}}{m(R^2-7r^2)}\frac{l}{l_0}\left\{1+\frac{l_0\{3m-2(R+7r)\}}{R^2-7r^2+m(R+7r)}\frac{l}{l_0}\right.$$

$$\left.-\frac{6l_0^2}{R^2-7r^2+m(R+7r)}\frac{l^2}{l_0^2}\right\} \qquad (11)$$

For the service multiperforated grain we therefore have

$$\left.\begin{aligned}\alpha&=\frac{2l_0\{R^2-7r^2+m(R+7r)\}}{m(R^2-7r^2)}\\ \lambda&=\frac{l_0\{3m-2(R+7r)\}}{R^2-7r^2+m(R+7r)}\\ \mu&=-\frac{6l_0^2}{R^2-7r^2+m(R+7r)}\end{aligned}\right\} \qquad (12)$$

Equation (11) applies only while the web of the grain is burning and does not apply to the slivers.

The thickness of web bears the following relation to R and r

in our service grains, as may be readily seen by drawing a diameter through any three perforations, Fig. 1.

$$2l_0=\frac{D-3d}{4}=\frac{R-3r}{2} \tag{13}$$

We will take a specific grain for use later to illustrate the burning of the multiperforated cylinder. The grains of a lot of powder for the 8-inch rifle had the following dimensions, in inches.

$$R=0.256 \qquad r=0.0255 \qquad m=1.029$$

Therefore, from (13), $l_0=0.044875$.

Substituting in (11), we obtain for this grain

$$F=0.72667\frac{l}{l_0}\left\{1+0.19590\frac{l}{l_0}-0.02378\frac{l^2}{l_0^2}\right\} \tag{14}$$

When $l=l_0$, that is, when the grain is reduced to slivers,

$$F=0.85174$$

from which we see that the slivers form about 15 per cent of this particular grain.

14. Emission of Gas by Grains of Different Forms.—As the velocity of combustion under constant pressure is uniform, the time of burning will be proportional to the thickness of layer burned.

We may conveniently show the manner of burning of the different grains by dividing the half web into five layers of equal thickness, that is, by giving to the ratio l/l_0, in the value of the fraction burned, the values 1/5, 2/5, etc., in succession, and then tabulating the resulting values of F. The successive values of F obtained will be the fractional parts burned in 1/5, 2/5, etc., of the total time of burning; and the differences of the successive values of F will be the fractions burned in the successive intervals of time.

The following table is formed from equations (8), (9), and (14). For the multiperforated grain the fractions l/l_0 are fractions of the web only.

l/l_0	Cube.		Slender Cylinder.		Multiperforated Cylinder.	
	F.	Difference.	*F.*	Difference.	*F.*	Difference.
0.0	0.000		0.00		0.00	
		0.49		0.36		0.15
0.2	0.49		0.36		0.15	
		0.29		0.28		0.16
0.4	0.78		0.64		0.31	
		0.16		0.20		0.17
0.6	0.94		0.84		0.48	
		0.05		0.12		0.18
0.8	0.99		0.96		0.66	
		0.01		0.04		0.19
1.0	1.00	1.00	1.00	1.00	Web 0.85	0.85
						0.15
					Whole grain 1.00	1.00

Regarding the columns of differences in the table we see that nearly half of the cubical grain is burned in the first layer, and that the volume burned in the successive layers decreases continuously. The slender cylinder emits at first a less volume of gas than the cube and later a greater volume, that is, its burning is more progressive. We have seen, equation (10), that the long tubular grain burns with a constant surface. The quantity of gas given off in the burning of each layer is therefore the same, and the grain is more progressive than the slender cylinder. The multiperforated cylinder burns with a continually increasing surface until the web is consumed, and the volume of gas given off increases for each layer burned.

Whether the burning surface of the multiperforated grain increases or decreases depends on the relation between the length of the grain and the radii of the grain and of the perforations. Referring to equation (11) it will be seen that when

$$3m=2(R+7r) \tag{15}$$

the second term within the brackets disappears. m is the length of the grain. Giving to the multiperforated grain considered in equation (14) the length indicated in the last equation, we get $m=0.29$, and the value of F becomes

$$F=0.94892\frac{l}{l_0}\left\{1-0.08134\frac{l^2}{l_0^2}\right\}$$

A table formed from this equation will show that this grain burns with a continuously decreasing surface; the fractional volumes burned in the successive intervals being 0.189, 0.186, 0.178, 0.167, and 0.152. The sum of these, 0.872, is the fraction of the grain burned when the web ceases to burn.

It is apparent that since the manner of burning of a multiperforated grain depends upon the relation expressed in equation (15), a grain may start to burn with an increasing surface, and change, as the length is diminished, to burn with a decreasing surface.

The multiperforated grains used in our service are of lengths considerably greater than that indicated in equation (15). The length of the grain is about $2\frac{1}{2}$ times the outer diameter. The diameter of the perforations is about 1/10 the exterior diameter of the grain. The grains burn with a continuously increasing surface until the web is burned, and then with a decreasing surface.

The Weight of Charge Burned.—Assuming instant ignition of the whole charge, equation (4) expresses the value of the fraction of the charge burned when any thickness, l, has burned.

Let $\bar{\omega}$ be the weight of the charge,

y the weight burned at any instant.

The fraction of the charge burned is therefore $y/\bar{\omega}$, which we may write for F in equation (4), and obtain

$$y=\bar{\omega}\alpha\frac{l}{l_0}\left\{1+\lambda\frac{l}{l_0}+\mu\frac{l^2}{l_0^2}\right\} \qquad (16)$$

15. Consideration as to Best Form of Grain.—It would appear that the most desirable form of powder grain would be one that gives off gas slowly at first, starting the projectile before a high pressure is reached, and then with an increased burning surface and a more rapid evolution of gas maintaining the pressure behind the projectile as it moves down the bore.

It is this consideration that has led to the adoption in our service of the multiperforated grain, which in the preceding discussion is shown to be the only practicable form of grain that burns with an increasing surface emitting successively increasing volumes of gas. The facilities for complete inflammation of the charge are not as great in this grain as in some others, as the grains assume all positions in the cartridge bag, and do not present unobstructed channels to the flame from the igniter. We have seen, page 13, that when there is delay in the complete inflammation of the charge, excessive pressures, called wave pressures, may arise, due to the velocity acquired by the gases first formed.

The single perforated cylinder, or tubular grain, offers advantages in this respect. This grain when its length is great compared to the thickness of web, as when cut in lengths to fit the chamber, burns with a practically constant surface, as we have seen, equation (10). The charge is readily prepared by binding the grains in bundles, and when so prepared offers perfect facilities for the prompt spread of ignition through the uniformly distributed longitudinal air spaces within and between the grains.

While larger charges of powder in this form may be required, to produce a desired velocity, the advantages of greater uniformity in velocities and pressures, and decreased likelihood of excessive pressures, will probably be obtained by its use.

In the process of drying the tubular grain in manufacture the grain will warp excessively if too long with reference to its diameters. On this account and in order that the grain may serve for convenient building into charges its length is limited. The requirement of prompt ignition throughout the length of the grain also limits its length. Good results have been obtained with grains whose length was 85 times the outer diameter.

VARIOUS DETERMINATIONS.

16. To Determine the Number of Grains in a Pound.—Let

n be the number of grains in a pound of powder,
V_0 the volume of each grain in cubic inches,
δ the density of the powder.

The volume occupied by the solid powder in one pound is evidently nV_0; the volume of one pound of water is 27.68 cu. in.; and the volumes being inversely proportional to the densities, we obtain

$$n=\frac{27.68}{\delta V_0} \tag{17}$$

and when the number of grains in a pound is known, we have for the density

$$\delta=\frac{27.68}{nV_0} \tag{18}$$

To Determine the Dimensions of Irregular Grains.—Irregular grains may be considered as spheres, and the mean radius may be determined as follows. Retaining the above significations of n and V_0, let r be the mean radius of the grains in inches.

Then $V_0=4\pi r^3/3$. Substituting this in the above equation and solving for r we obtain

$$r=\frac{1.8766}{(\delta n)^{\frac{1}{3}}}$$

Comparison of Surfaces.—Let S_1 be the total initial surface of the grains in a pound of powder. As S_0 is the initial surface of each grain,

$$S_1=nS_0$$

Substituting the value of n from (17) and the value of S_0 from the first of equations (3) we obtain

$$S_1 = \frac{27.68\alpha}{\delta l_0} \qquad (19)$$

From which it appears that for two charges of equal weight, made up of grains of the same density and thickness of web, the initial surfaces of the two charges are to each other as the values of α for the two forms of grain. For charges of equal weights composed of grains of the same shape and density the initial surfaces will be inversely proportional to the least dimensions of the grains.

17. Density of Gunpowder.—The density, or specific gravity, of gunpowder is the ratio of the weight of a given volume of solid powder to the weight of an equal volume of water. The density of charcoal gunpowders is determined by means of an instrument called the mercury densimeter, in which is obtained the weight of a volume of mercury equal to the volume of the powder. From the known specific gravity of the mercury that of the powder is readily determined. Mercury is used in the instrument instead of water because mercury possesses the property of closely enveloping the grains of powder without being absorbed into their pores, and it does not dissolve the constituents of the powder.

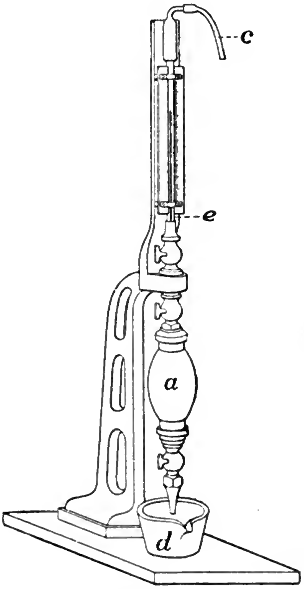


FIG. 3.

The densimeter is shown in the accompanying figure. The glass globe *a* is connected with an air pump by the rubber tube *c*. The lower outlet of the globe is immersed in mercury in the dish *d*.

As the globe is exhausted of air by means of the air pump, the mercury is drawn upward until it fills the globe and stands at a certain height in the glass tube *e*. The globe is then detached, full of mercury, and weighed. It is then emptied, and a given weight of powder placed in it. The globe is then returned to its original position, the air again exhausted, and mercury allowed to enter until it stands at the same height as before. The globe, now filled with mercury and powder, is again detached and weighed. With the difference of the two weights we may arrive at the weight of the mercury whose volume is equal to that of the powder, in the following manner.

Let w be the weight of the powder,
P the weight of the vessel filled with mercury,
P' the weight of the vessel filled with mercury and powder,
D the density of the mercury, about 13.56,
δ the density of the powder.

Then $P'-w=$ the weight of the mercury and vessel when the latter is partially filled with powder,

$P-(P'-w)=$ the weight of the volume of mercury displaced by the powder.

Since the weights of equal volumes are proportional to the densities, we have

$$w : P-P'+w :: \delta : D$$

or

$$\delta=\frac{wD}{P-P'+w}$$

The density of charcoal powders varies between 1.68 and 1.85.

Smokeless Powder.—The nitrocellulose smokeless powders are affected by mercury; therefore if the densimeter is used in the determination of the densities of these powders, water must be used in the instrument in place of mercury. The density of large grained powders may be determined by weighing a grain of the powder in air and in water. The difference of the weights in air and water is the weight of a volume of water equal to the volume of the grain. The density is then the weight in air divided by the difference of the weights.

The density of smokeless powders varies from 1.55 to 1.58.

CHAPTER II.

MEASUREMENT OF VELOCITIES AND PRESSURES.

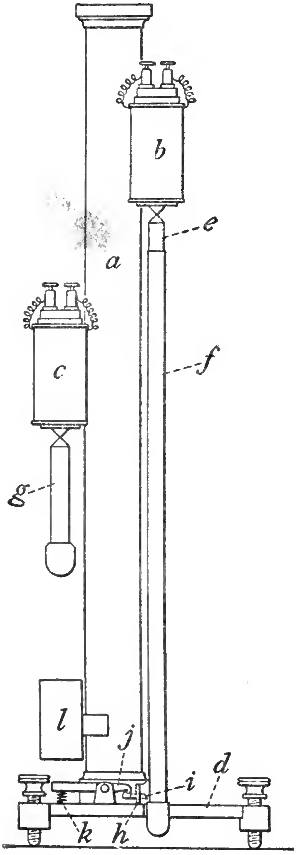


FIG. 4.

18. Measurement of Velocity.—In measuring the velocity of a projectile the time of passage of the projectile between two points, a known distance apart, is recorded by means of a suitable instrument. The calculated velocity is the mean velocity between the two points, and is considered as the velocity midway between the points. In order that this may be done without material error, the two points must be selected at such a distance apart in the path of the projectile that the motion of the projectile between the points may be considered as uniformly varying, and the path a right line.

Le Boulengé Chronograph.—The instrument generally employed for measuring the time interval in the determination of velocity was invented by Captain Le Boulengé of the Belgian Artillery, and is called the Le Boulengé Chronograph. It has been modified and improved by Captain Bréger of the French Artillery. The brass column, *a* Fig. 4, supporting two electromagnets, *b* and *c*, is mounted on the triangular bedplate *d* which is provided with levels and leveling screws. The magnet *b* supports the long rod *e*, called the *chronometer*, which is enveloped when in use by a zinc or copper tube *f*, called the *recorder*. A nut above the recorder, shown in Fig. 10, holds the recorder fixed in place on the chronometer rod. The magnet *c* which supports the short rod *g*, called the *registrar*, is mounted on a frame which permits it to be moved vertically along the standard. Fastened to the base of the standard is the flat steel spring *h* which carries at its outer end the square knife *i*. The knife is held retracted or cocked by the trigger *j* which is acted upon by the spring *k*. The chronometer *e* hangs so that one element of the enveloping tube or recorder is close to the knife. When the knife is released by pressure on the trigger it flies out under the action of the spring *h* and indents the recorder. The registrar *g* hangs immediately over the trigger. When the electric circuit through the registrar magnet is broken the registrar falls on the trigger and releases the knife. The tube *l* supports the registrar after it has fallen through it. Adjustable guides are provided to limit the swing of the two rods when first suspended. The stand or table on which the instrument is mounted is provided with a pocket which receives the chronometer when it falls, at the breaking of the circuit that actuates its magnet. A quantity of beans in the bottom of the pocket arrests the fall of the chronometer without shock.

In the use of the chronograph in measuring the velocity of a shot the following accessory apparatus is required: targets, rheostats, disjunctor, and measuring rule.

Targets.—Two wire targets, each made of a continuous wire, Fig. 5, are erected in the path of the projectile. The targets form parts of electric circuits which include the electromagnets of the chronograph. Each magnet has its own target and its own circuit independent of the other. The circuit from the nearer or first target includes the chronometer magnet; the circuit from the second target includes the registrar magnet. On the passage of the projectile through the first target the circuit is broken, the chronometer magnet demagnetized, and the long rod, or chronometer, falls. When the projectile breaks the circuit through the second target the short rod, or registrar, falls and, striking the trigger, releases the knife, which flies out and marks the recorder at the point which has been brought opposite the knife by the fall of the chronometer.

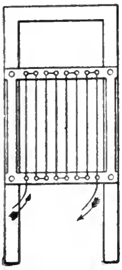


FIG. 5.

In some instruments the chronometer circuit is led through a contact piece not shown, carried by the spring *h*, and so arranged that the chronometer circuit cannot be closed until the knife is cocked. This arrangement prevents the loss of a record through failure to cock the knife when suspending the rods before the piece is fired.

The first target must always be erected at such a distance from the gun that it will not be affected by the blast. For small arms it is placed three feet from the muzzle and consists of fine copper wire wound backward and forward over pins very close together. For cannon it is placed from 50 to 150 feet from the muzzle, depending upon the size of the gun. For the measurement of ordinary velocities the targets are usually placed 100 feet apart for small arms and 150 feet for cannon.

The second target for small arms consists of a steel plate to stop the bullets, having mounted on its rear face, and insulated from it by the block *w*, Fig. 6, a contact spring *s*, contact pin *p*, and their binding screws. When the bullet strikes the plate the shock causes the end of the spring to leave the pin, and thus breaks the circuit, which is immediately reestablished by the reaction of the spring. By means of this device constant repairing of the target is avoided.

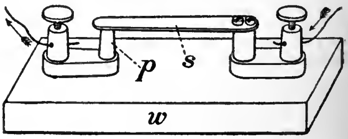


FIG. 6.

19. The Rheostat.—Both circuits are led independently through rheostats, by means of which the resistance in the circuits may be regulated, and the strength of the currents through the two magnets equalized. One form of rheostat is shown in Fig. 7. The current passes through the contact spring *a* and through a German silver wire wound in grooves on the wooden drum *b*. By turning the thumb nut *c* the contact spring is shifted, and more or less of the wire is included in the circuit.

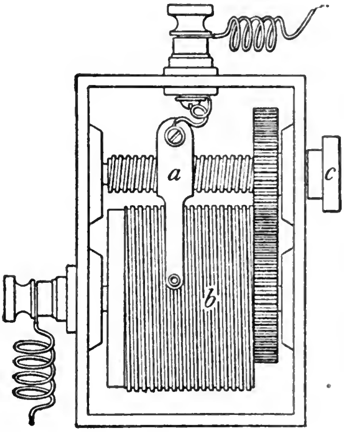


FIG. 7.

Another form of rheostat, through which both circuits pass independently, is shown in Fig. 8. Each current passes through a strip of graphite, *a*, and the resistance in the circuit may be increased or diminished by sliding the

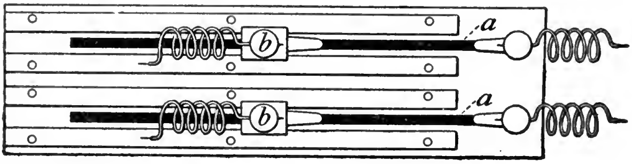


FIG. 8.

contact piece *b* so as to include a greater or less length of the graphite strip in the circuit.

The Disjunctor.—Both circuits also pass independently through an instrument called the disjunctor, by means of which they may be broken simultaneously. The disjunctor is shown in elevation and part section in Fig. 9. The two halves of the instrument are exactly similar. The two contact springs *c*, weighted at their free ends, bear against insulated contact pins *e*, supported in the same metal frame *d*. The frame is pressed upward against the spring catch *h* by two other contact springs, *f*. The electric circuit passes from one binding post through the parts *f*, *e*, *c*, and *a* to the other binding post.

On the release of the spring catch *h* the frame *d* flies upward under the action of the springs *f* until stopped by the pin *g*.

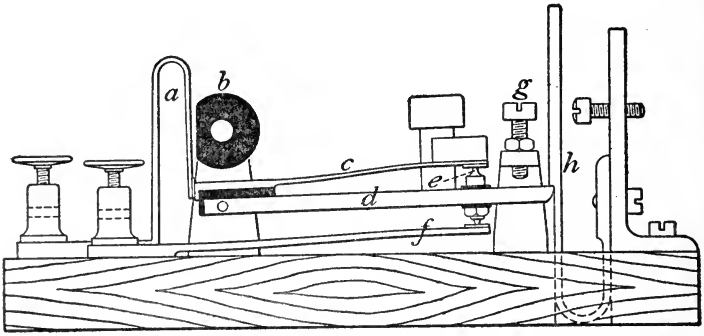


FIG. 9.

At the sudden stoppage of the movement the weighted ends of the contact springs simultaneously leave the contact pins, thus breaking both circuits momentarily. Mounted on a shaft are two hard rubber cams, *b*, which bear against other springs, *a*, in the two circuits. On turning the cam shaft the connection between the parts *a* and *c* is broken, breaking both electric circuits, but not necessarily simultaneously. The circuits are habitually broken in this manner except when taking disjunction or records in firing.

20. Disjunction.—By means of the disjunctor both circuits are broken at the same instant. The mark made by the knife under these circumstances is called the disjunction mark, and its height above a zero mark made by the knife when the chronometer is suspended from its magnet is evidently the height through which a free falling body moves in the time used by the instrument in making a record. This time includes any difference in the times required for demagnetization of the two magnets; the time occupied by the registrar in falling, and the time required for the knife to act.

From the height as measured we obtain the corresponding time from the law of falling bodies,

$$t=(2h/g)^{\frac{1}{2}}$$

Now when the circuits are broken by the projectile the chronometer begins to fall before the registrar. The mark made by the knife will therefore be found above the disjunction mark. If we measure the height of this second mark above the zero, the corresponding time is the whole time that the chronometer was falling before the mark was made, and to obtain the time between the breaking of the circuits we must subtract from this time the time used by the instrument in making a record, or the time corresponding to the disjunction. Let h_1 and h_2 represent the heights of the disjunction and record marks respectively, t_1 and t_2 the corresponding times. Let t be the time between the breaking of screens, then

$$t=t_2-t_1=(2h_2/g)^{\frac{1}{2}}-(2h_1/g)^{\frac{1}{2}}$$

It will be seen by the equation that *the difference of the times, and not the difference of the heights, must be taken.*

FIXED DISJUNCTION.—For the velocity at the middle point between targets we have, representing by s the distance between the targets,

$$v=s/t$$

Substituting for t its value, we have

$$v=\frac{s}{(2h_2/g)^{\frac{1}{2}}-(2h_1/g)^{\frac{1}{2}}}$$

From this equation we see that if the value of s, and of $(2h_1/g)^{\frac{1}{2}}$, the disjunction, be fixed, the values of v can be calculated for all values of h_2 within the limits of practice, and tabulated. This has been done for the values $s=100$ feet and $(2h_1/g)^{\frac{1}{2}}=0.15$ sec-

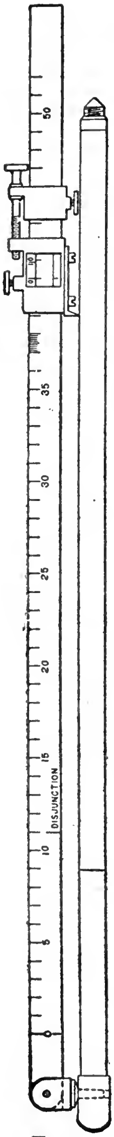


FIG. 10.

onds. This value of $(2h_1/g)^{\frac{1}{2}}$ is called *the fixed disjunction.* If such a table is not at hand, the fixed value of the disjunction avoids the labor of calculating $(2h_1/g)^{\frac{1}{2}}$ for each shot.

In this case

$$t = t_2 - 0.15 \text{ sec.} = (2h_2/g)^{\frac{1}{2}} - 0.15.$$

In ordinary practice it is better to take the disjunction at each shot, and to keep the disjunction mark near the disjunction circle, but not necessarily on it. The times corresponding to the heights of the disjunction and record marks are both read from the table, and with the difference of these times the velocity is taken from another table.

Measuring Rule.—For measuring the height of the mark on the recorder above the zero mark there is provided with the instrument a rule graduated in millimeters, and with a sliding index and vernier, the least reading being $\frac{1}{10}$ of a millimeter. The swivelled pin at the end of the rule, Fig. 10, is inserted in the hole through the bob of the chronometer, and the knife edge of the index is placed at the lower edge of the mark whose height is to be measured. The height is then read from the scale. Tables are constructed from which can be directly read the time corresponding to any height in millimeters within the limits of the scale. The maximum time that can be measured with this chronograph is limited by the length of the chronometer rod, and is about 0.15 of a second.

21. Adjustments and Use.—The instrument must be properly mounted on a stand at such a distance from the gun that it will not be affected by the shock of discharge. The electrical connections with the batteries and targets, through the rheostats r and disjunctor d, are made as shown in Fig. 11.

To adjust the instrument, first level it by the leveling screws, cock the knife, and suspend the chronometer rod, enveloped by the recorder, from its magnet. See that the recorder hangs close to the knife and that no part of the base of the rod touches any part of the instrument. The guides must be close to. but not touching, the bob of the chronometer. Depress the

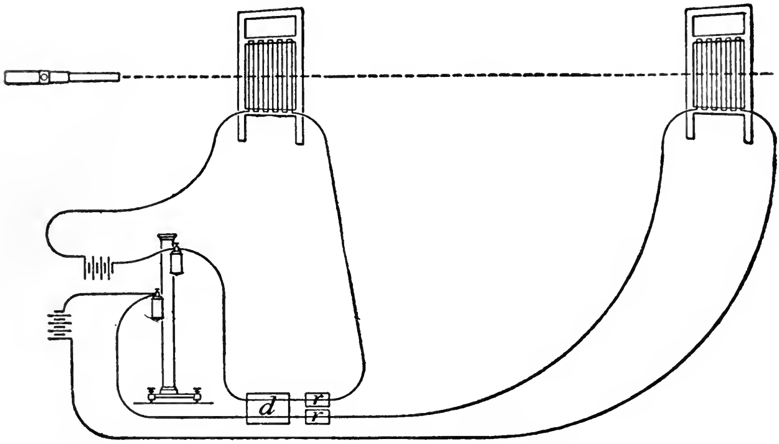


FIG. 11.

trigger. The knife will mark the recorder near the bottom. This mark is the zero from which all heights are measured, and the knife edge on the measuring rule index must be so adjusted that the zero of the vernier shall coincide with the zero of the scale when the knife edge is in the mark. The adjustment of the knife is made as follows. Place the sliding index so that the zero of the vernier is at the zero of the scale on the rule. Clamp the index and apply the rule to the chronometer. Loosen the screws that hold the knife and adjust the knife edge to the zero mark on the recorder. Tighten the knife screws. After this adjustment, slide the index to the mark *Disjunction* on the rule, and letting the knife edge bear against the recorder turn the recorder around the chronometer rod. The knife edge will scribe a circle on the recorder, and the mark made at disjunction should fall on or near this circle.

To regulate the strength of the magnets each of the rods is provided with a tubular weight, one tenth that of the rod. Place the proper weight on each rod and suspend the rods from their magnets. Increase the resistance in each circuit by slowly moving the contact piece of the rheostat until the rod falls. Remove the weights from the rods and again suspend the rods. Take the disjunction. If the bottom of the mark made by the knife does not lie on or near the circle previously scribed on the recorder, raise or lower the registrar magnet until coincidence is nearly obtained.

Test the disjunctor by shifting the two circuits. The height of disjunction should remain the same.

Test the circuits by suspending the rods and causing the circuits to be broken successively at the two targets. Note that the proper rod falls as each circuit is broken.

Always suspend the chronometer rod with the same side of the bob to the front, and always, before suspending it, press the recorder hard against the bob. After each record turn the recorder slightly on the rod to present a new element to the knife.

Circuits should always be broken at the disjunctor when the rods are not actually suspended, and the rods should be allowed to remain suspended as short a time as possible.

Measurement of Very Small Intervals of Time.—For the measurement of very small time intervals the registrar magnet is raised to near the top of the standard and placed in the circuit with the first target. The chronometer magnet is put in the circuit with the second target. Under this arrangement the disjunction mark will be made near the top of the recorder and the record mark under the disjunction. The interval of time measured is obtained by subtracting the time corresponding to the height of the record mark, from the time of disjunction. The object of this arrangement is to obtain the record when the chronometer has acquired a considerable velocity of fall, so that the scale of time will be extended, and small errors of reading will not produce large errors in time.

22. Schultz Chronoscope.—The Le Boulengé chronograph measures a single time interval only. When it is desired to measure the intervals between several successive events an instrument that provides a more extensive time scale is required.

The Schultz Chronoscope is an instrument of this class. An electrically sustained tuning fork, *c*, Fig. 12, whose rate of vibration is known, carries on one tine a quill point *b* which bears against the blackened surface of the revolving cylinder *a* and marks on it a sinusoidal curve which is the scale of time. By

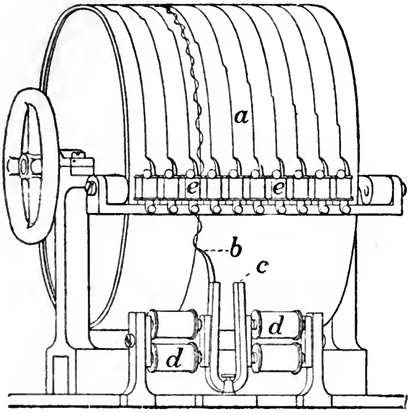


FIG. 12.

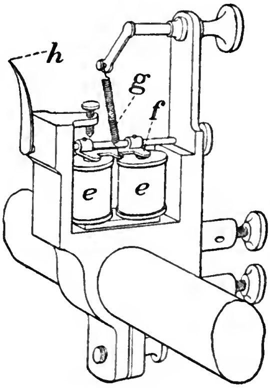


FIG. 13.

giving motion of translation to the cylinder past the fork the time scale may be extended helically over the whole length of the cylinder. The records of events, such as the passage of the shot through screens, are made by the breaking of successive circuits which pass through the Marcel Deprez registers shown at *e*, Fig. 12, and in Fig. 13. When the circuit is broken the magnet *e*, Fig. 13, is demagnetized, and the spring *g* rotates the armature *f* and the quill *h* attached to it. This marks a bend or offset in the trace of the quill on the revolving cylinder, and the point of the bend referred to the time scale marks the instant of the breaking of the circuit.

It will be noted that the tuning fork has a constant lead with respect to any register. The point of the time scale that corresponds to any point on a register record is found at the length of this lead from the point on the time scale opposite the given point on the register record.

The Sebert Velocimeter.—This instrument is used to record the movement of the gun in recoil. A blackened steel ribbon, *S*,

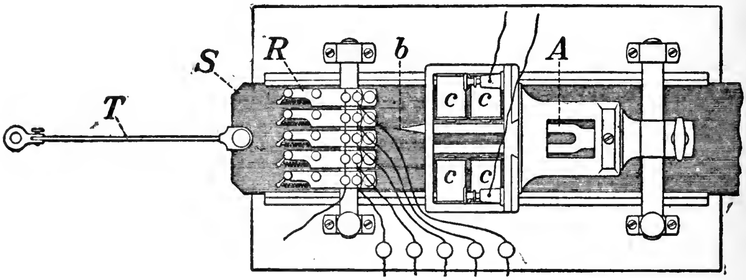


Fig. 14.

Fig. 14, is attached by the wire *T* to a bolt projecting from the trunnion of the gun. As the gun recoils it pulls the ribbon past the registers *R* and the tuning fork *A*, whose rate of vibration is known. The quill on the tuning fork marks the time scale on the blackened ribbon as shown by the curve *t*, Fig. 15. The time occupied by the gun in traversing any length is obtained by laying off this length on the time scale and counting the vibrations and parts of a vibration included. The right line through the centre of the time scale is made by pulling the ribbon past the fork when the fork is not vibrating. The line assists in the count of the number of double vibrations in any length.

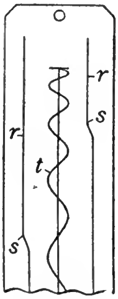


Fig. 15.

The time scale is therefore a complete record of the movement of the gun; and by measuring from it the length travelled by the gun during any vibration of the fork the velocity of the gun at the middle instant of the vibration may be determined.

When the gun moves in free recoil, that is, when it is so mounted that it recoils horizontally and with very little friction, the velocities of the projectile may be determined from the velocities of the gun; and the pressures necessary to produce these velocities in the prc ... ıy then be determined.

The registe hav ıo function in the measurement of the recoil proper, l ma ɔe used to record any event happening while the recoi. ·cor s being made. The instant of the departure of the p ect: from the bore is usually thus recorded, and independent ment of the velocity of the projectile between points i. the bore may also be made.

Two register records are shown by the lines *r*, Fig. 15, the event recorded by each register having occurred when the offset at *s* was made. The time that elapsed between the beginning of movement and the occurrence of the event recorded is obtained by laying off on the time scale the length from the origin of the register record to the offset.

Methods of Measuring Interior Velocities.—Two methods that have been used in determining the instant of the projec-

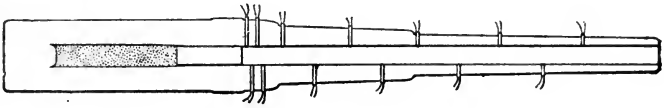


FIG. 16.

tile's passage past selected points in the bore are shown in Figs. 16 and 17.

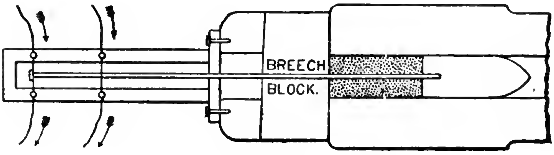


FIG. 17.

Some circuit breaking device is used at the points selected, and the electric wires are led to any suitable velocity instrument.

23. Measurement of Pressures.—Pressures in cannon are directly measured by means of the pressure gauge shown in Fig. 18.

In the steel housing *h* are assembled the steel piston *p* and the copper cylinder *c*, which is centered by the steel spring or rubber washer *w*. The housing is closed by the screw plug *s*. A small copper obturating cup *o* prevents the entrance of gas past the piston, and a copper washer performs the same office at the joint between the housing and the closing plug. A series of grooves *a*, called *air packing*, is sometimes cut near the bottom of the piston, and assists in obturation in the case of a defect in the copper cup. Any gas that may pass the cup has its tension materially reduced by expansion into the successive grooves.

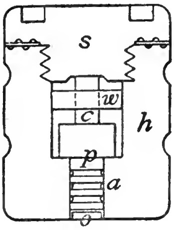


FIG. 18.

In another form of gauge the housing is threaded on the exterior and the gauge is screwed into a socket provided in the head of the breech block.

The gauge is placed in the gun behind the powder charge, or is inserted in its socket in the breech block. When the gun is fired the pressure of the powder gases is exerted against the end of the piston and the copper cylinder is compressed. The compression is manifestly due to the maximum pressure exerted in the gun. The length of the cylinder is measured both before and after firing, and the compression due to the pressure is determined. With the compression thus obtained the pressure per square inch that produced it is read at once from a *tarage* table previously constructed.

The Tarage Table.—The copper cylinders are cut in half-inch lengths from rods very uniformly rolled and carefully annealed. The compression of the cylinders under different loads is determined in a static pressure machine. It is assumed that the compression obtained in firing is due to a load on the piston of the pressure gauge equal to the load that produced the same compression in the static machine. The pressure per square inch in the gun may therefore be obtained by dividing the static load that corresponds to the observed compression by the area of the piston in the pressure gauge. Knowing the area of the piston used, the table of compressions and corresponding pressures per square inch is readily constructed from the results obtained in the machine.

The area of piston in cannon gauges is $^1/_{10}$ of a square inch, and in the small-arm pressure barrel, $^1/_{30}$ of a square inch.

Initial Compression.—When the pressure in the gun is high the compression of the copper is considerable, and the piston acquires an appreciable velocity during the compression. The energy of the piston due to this velocity adds to the compression that would result from the pressure alone, and consequently the measured compression is greater than the compression that corresponds to the true pressure. The energy of the piston may be reduced in two ways: by reducing its weight, and by limiting its travel and accompanying velocity. The piston is made as light as possible consistent with the duty it has to perform. To limit its travel the copper cylinders are initially compressed before using, by a load corresponding to a pressure somewhat less than that expected in the gun. Further compression of the copper will not occur until the load applied in the gun is close to that used in the initial compression.

The general practice is to use a copper initially compressed by a load corresponding to a pressure about 3000 lbs. less than that expected in the gun. Thus if a pressure of 35,000 lbs. is expected, a copper initially compressed by a load corresponding to 32,000 lbs. per square inch is used.

Small-arm Pressure Barrel.—In the measurement of pressures in small arms a specially constructed barrel whose bore is the same as that of the rifle barrel is used. The piston of the pressure gauge passes through a hole bored through the barrel over the chamber, and a steel housing erected over this part of the barrel serves as an anvil for the copper cylinder.

A hole is bored through the metallic cartridge case to permit the powder gases to act directly on the end of the piston.

24. The Micrometer Caliper.—The micrometer caliper, Fig. 19, is used for measuring the lengths of the copper cylinders before and after firing. This instrument is used generally for the measurement of short exterior lengths.

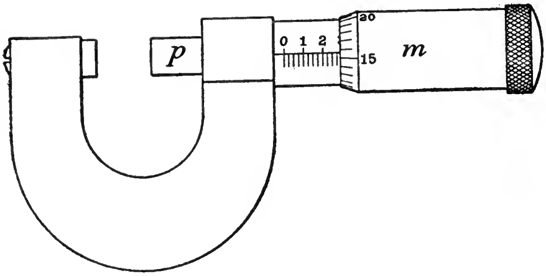


Fig. 19.

The movable measuring point *p* has a screw thread of forty turns to the inch cut on its shaft. One turn of the attached micrometer head *m* therefore moves the point one fortieth or 25 thousandths of an inch. By means of the scale on the spindle and the 25 divisions on the micrometer head *m* the distance that separates the measuring points can be read to the one-thousandth of an inch, and by further subdividing the divisions on the head by the eye, readings to the ten-thousandth of an inch may be made. The figure represents the points as separated by 0.2907 inches.

The Dynamic Method of Measuring Pressures.—This consists in determining the velocities of the gun in recoil, as by the Sebert velocimeter, or of the shot at different points of the bore. The differences of the velocities divided by the corresponding differences of the times give the accelerations, and the corresponding pressures are obtained by multiplying the accelerations by the mass. A pressure obtained in this manner is evidently only the pressure required to produce the observed acceleration in a body whose mass is that of the gun or of the projectile. That part of the pressure expended in overcoming the friction of the projectile in the bore and in giving rotation to the projectile is neglected. The measured pressure is consequently less than the true pressure exerted in the gun.

Comparison of the Two Methods.—When the same pressure in the bore is measured by the dynamic method and by the pressure gauge the result obtained dynamically is usually the greater, and this notwithstanding the fact, as just explained, that the dynamically measured pressure is less than the true pressure. This causes doubt as to the correctness of the pressures recorded by the gauge.

In the gun the compression of the copper is effected in a very small fraction of the time required in the static machine that produced the tarage, and as the maximum pressure in the gun is instantly relieved, it is held that the metal of the copper cylinder has not time to flow under this pressure, and consequently that the compression is less than it would be under the same load in the static machine. The pressure as obtained from the compression in the gauge is therefore less than the true pressure in the gun.

On the other hand Sarrau, an eminent French investigator, concludes from many experiments that with gunpowder, when the pressure gauge is placed in rear of the projectile, the compressions will agree with the tarage. The maximum pressure in the gun is reached in a very short time, but the time is appreciable. Therefore the application of the pressure resembles in some degree that of the force producing the tarage. When high explosives are used, or when with gunpowder the pressure gauge is placed anywhere in front of the base of the projectile so that the gas strikes it suddenly upon the passage of the projectile, the rate of application of the force is so great that as a general rule the true pressure is measured by the tarage corresponding to half the actual compression of the cylinder.

Though these differences of opinion as to the correctness of the pressure gauge exist, the gauge itself is in general use. It affords the most convenient method of getting a measure of pressure, and serves to compare the measured pressure with what is known from experience to be a safe pressure in the gun.

CHAPTER III.

INTERIOR BALLISTICS.

25. Scope.—*Ballistics* is the science that treats of the motion of projectiles.

Interior ballistics is concerned with the motion of the projectile while in the bore of the gun, and includes a study of the conditions existing in the bore from the moment of ignition of the powder charge to the moment that the projectile leaves the muzzle. The circumstances attending the combustion of the powder, the pressures exerted by the gases at different points of the bore, and the velocities impressed upon the projectile are the subjects of investigation; and the practical results of the study lie in the application of the deduced mathematical formulas which connect the travel of the projectile with the velocities and pressures. By means of the formulas we may determine the stresses to which a gun is subjected from the pressure of the powder gases, and the dimensions of chamber and of bore, and the weight of powder, to produce in a given projectile a desired velocity. The action of different powders may be compared and the most suitable powder selected for a particular gun. The interior pressures at all points along the bore being made known, the thickness required in the walls of the gun to safely withstand these pressures are determined from the principles of gun construction, to be studied later.

Early Investigations.—In 1743 Benjamin Robins described, before the Royal Society of England, experiments that he had made to determine the velocities of musket balls when fired with given charges of powder. To measure the velocities he invented the ballistic pendulum, which consisted simply of a large block of wood suspended so as to move freely. The bullet was fired into the block of wood, and the velocity impressed upon the pendulum was measured. By equating the expressions for the quantities of motion in the bullet before striking the pendulum, and in the pendulum after receiving the bullet, the velocity of the bullet was obtained. The gun pendulum, which consisted of a gun mounted to swing as a pendulum, was also invented by Robins. Among other deductions made from his experiments Robins announced the following. The temperature of explosion is at least equal to that of red-hot iron; the maximum pressure exerted by the powder gases is equal to about 1000 atmospheres; the weight of the permanent gases is about three tenths that of the powder, and their volume at atmospheric temperature and pressure about 240 times that occupied by the charge.

Dr. Charles Hutton, Professor in the Royal Military Academy, Woolwich, continued Robins's experiments, 1773 to 1791, improving and enlarging the ballistic pendulum so that it could receive the impact of one-pound balls. He verified Robins's deductions as to the nature of the gases, but put the temperature of explosion at double that previously deduced, and the maximum pressure at 2000 atmospheres. Hutton produced a formula for the velocity of a spherical projectile at any point of the bore, upon the assumption that the combustion of the charge is instantaneous and that the expansion of the gas follows Mariotte's law,—no account being taken of the loss of heat due to work performed—a principle which, at that time, was unknown.

In 1760 the Chevalier D'Arcy made the first attempt to determine dynamically the law of pressure in the bore by successively shortening the length of the barrel and measuring the velocity of the bullet for each length. The pressures were determined from the calculated accelerations.

In 1792 Count Rumford, born in the United States, endeavored to make direct measurement of the pressure exerted by fired gunpowder by measuring the maximum weights lifted by different charges fired in a small but very strong wrought iron mortar, or eprouvette. He determined a relation existing between the pressure of the powder gases and their density. The maximum pressure that would be exerted by the gases from a charge that completely filled the chamber was, as calculated by Rumford, about 100 tons to the square inch. Noble and Abel, in their later experiments, arrived at 43 tons per square inch as the maximum pressure under these conditions. Their value is now accepted as being very near the truth. The great difference in the two determinations is probably due to the fact that Rumford deduced his value for the maximum pressure from experiments with small charges that did not fill the chamber, so that the energy of the gases was greatly increased by the high velocity they attained before acting on the projectile.

Later Investigations.—In the years 1857 to 1860 General Rodman of the Ordnance Department, United States Army, conducted the experiments that resulted in the change of form of powder grains and their variation in size according to the caliber of the gun. He devised the pressure gauge for directly measuring the maximum pressures of the powder gases. His gauge differed from the pressure gauge now in use, only in the method employed to record the pressure. The piston of the gauge carried at its inner end a V-shaped knife, and the amount of the pressure was read from the length of the cut made by the knife in a disk of copper. General Rodman was also the author of the principle of interior cooling of cast iron cannon, by the application of which principle the metal surrounding the bore of a gun was put under a permanent compressive strain which greatly increased the resistance of the gun to the interior pressures.

In 1874 Noble and Abel announced the results of their experiments on the explosion of gunpowder in closed vessels. As the ballistic formulas now in use are based largely on the results of Noble and Abel's experiments, these will later be more fully described.

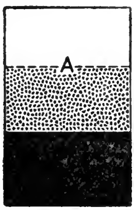
Let C' be the volume in cubic inches occupied by the solid powder of the charge; δ the density of the powder. $\delta C'$ will then be the volume of an equal weight of water, and

$$\varpi = \delta C'/27.68 \qquad (24)$$

which, substituted in equation (22), gives

$$\Delta = \delta C'/C \qquad (25)$$

The accompanying figure will serve to illustrate the difference between density, gravimetric density, and density of loading. The figure represents a section of the whole chamber of a gun charged with powder to the line A. The *density of loading* is in this case the weight of powder below the line A divided by the weight of water that will fill the *whole* chamber. The *gravimetric density* is the weight of the powder divided by the weight of water that will fill all that part of the chamber below the line A. Now considering the powder charge as compressed into a solid mass at the bottom of the chamber, represented by the black portion, the *density* of the powder will be its weight divided by the weight of water that will fill this black portion. As the weight of water that will fill each volume is equal to the volume in cubic inches divided by 27.68, we have:



$$\text{Density of Loading,} \quad \Delta = \frac{27.68\varpi}{\text{vol. of chamber}}$$

$$\text{Gravimetric Density,} \quad \gamma = \frac{27.68\varpi}{\text{vol. of charge}}$$

$$\text{Density,} \quad \delta = \frac{27.68\varpi}{\text{vol. of solid powder}}$$

Using metric units the factor 27.68 will be omitted.

28. Reduced Length of Powder Chamber.—For convenience in the mathematical deductions the volume of the powder chamber is reduced to an equal volume whose cross section is the same as the cross section of the bore. The length of this volume is called the reduced length of the powder chamber.

Let u_0 be the reduced length of the chamber,
ω the area of cross section of the bore,
C the volume of the chamber,
d the diameter of the bore.

Then

$$C = u_0\omega = u_0\pi d^2/4$$

and

$$u_0 = 4C/\pi d^2 \tag{26}$$

Reduced Length of Initial Air Space.—The air space in the loaded chamber, which includes all the space in the chamber not occupied by solid powder, is also reduced to a volume whose cross section is that of the bore. The length of this volume is called *the reduced length of the initial air space.*

Let z_0 be the reduced length of the initial air space, in inches.

Then, since C is the volume of the chamber and C' the volume of the solid powder,

$$z_0 = \frac{C - C'}{\omega}$$

Substituting for C and C' their values from equations (22) and (24)

$$z_0\omega = 27.68\varpi\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)$$

Make

$$a = \frac{\delta - \Delta}{\Delta\delta} \tag{27}$$

Then

$$z_0\omega = 27.68a\varpi \tag{28}$$

and since $\omega = \pi d^2/4$

$$z_0 = 35.2441 a\bar{\omega}/d^2 = [1.54709] a\bar{\omega}/d^2 \qquad (29)$$

the number in square brackets being the logarithm of 35.2441.

Problems.—1. The volume of the chamber of the 3-inch field rifle is 66.5 cu. in. The weight of the charge is 26 oz. Density of the powder 1.56. What is the density of loading, and what is the reduced length of the initial air space?

Ans. $\Delta = 0.6764$,
$z_0 = 5.33$ inches.

2. If the gravimetric density of the powder in the last example is unity, how many pounds will the chamber hold?

2.4 lbs.

3. The reduced length of the initial air space in the 8-inch rifle loaded with 80 lbs. of powder, density 1.56, is 43.72 inches. What is the capacity of the chamber?

$C = 3617$ cu. in.

4. The 5-inch siege gun has a chamber capacity of 402.5 cu. in. What is the density of loading with a charge of 5.37 lbs.?

$\Delta = 0.3693$.

5. The 4-inch rifle when loaded with 12 lbs. of sphero-hexagonal powder has a density of loading of 0.915. What is the chamber capacity?

$C = 363$ cu. in.

6. The 12-inch rifle has a chamber capacity of 17487 cu. in. The density of loading is 0.5936. What is the weight of the charge, and what is the volume of the solid powder in the charge? $\delta = 1.56$.

$\bar{\omega} = 375$ lbs.
Solid volume $= 6654$ cu. in.

7. What is the reduced length of the initial air space in the last example?

$z_0 = 95.79$ inches.

8. The chamber capacity of the 6-inch rifle is 2114 cu. in. What is the reduced length of the chamber?

$u_0 = 74.77$ inches.

PROPERTIES OF PERFECT GASES.

29. Mariotte's Law.—*At constant temperature* the tension, or pressure, of a gas is inversely as the volume it occupies.

As the density of a gas is inversely as its volume, this law may also be expressed: At constant temperature the pressure of a gas is proportional to its density.

Let v be the volume of a given mass of gas,

p its pressure in pounds per unit of area.

Then if the volume occupied by the gas be changed to v_0, the temperature of the gas being kept constant, the pressure will change according to the law

$$vp = \text{constant}$$

Let p_0 represent the normal atmospheric pressure, barometer 30 inches;

$p_0 = 14.6967$ pounds per square inch,

or 103.33 kilograms per square decimeter;

v_0 the volume of unit weight of a gas at 0° C. under normal atmospheric pressure.

Then by Mariotte's law, at 0° C.,

$$vp = v_0 p_0 \tag{30}$$

Specific Volume.—The specific volume of a gas is the volume of unit weight of the gas at zero temperature and under normal atmospheric pressure. v_0 in the above equation is the specific volume of the gas.

In English units the specific volume of a gas is the number of cubic feet occupied by a pound of the gas under the above conditions.

Specific Weight.—The specific weight of a gas is the weight of a unit volume of the gas at zero temperature and under normal atmospheric pressure. It is the reciprocal of the specific volume.

Gay-Lussac's Law.—The coefficient of expansion of a gas is the same for all gases; and is sensibly constant for all temperatures and pressures.

Let v_0 be the specific volume of a gas, v_t its volume at any temperature t, and α the coefficient of expansion. Then the *variation of volume under constant pressure* by Gay-Lussac's law will be expressed by the equation

$$v_t - v_0 = \alpha t v_0$$

or

$$v_t = v_0(1+\alpha t)$$

The value of α is approximately 1/273 of the specific volume for each degree *centigrade*. The above equation may therefore be written

$$v_t = v_0\left(1+\frac{t}{273}\right) \qquad (31)$$

30. Characteristic Equation of the Gaseous State.—The last equation, which expresses Gay-Lussac's law, may be combined with Mariotte's law, introducing the pressure p.

Let x be the volume that v_t would become at 0° C., under the pressure p_t. Then by Gay-Lussac's law

$$v_t = x(1+\alpha t)$$

but by Mariotte's law

$$p_t x = p_0 v_0$$

whence, eliminating x,

$$p_t v_t = p_0 v_0 (1+\alpha t) = \frac{p_0 v_0}{273}(273+t)$$

Since $p_0v_0/273$ is constant for any gas, put

$$R = p_0v_0/273 \tag{32}$$

whence, dropping the subscripts as no longer necessary,

$$pv = R(273 + t)$$

The temperature $(273 + t)$ is called the absolute temperature of the gas. It is the temperature reckoned from a zero placed 273 degrees below the zero of the centigrade scale. Calling the absolute temperature T there results finally

$$pv = RT \tag{33}$$

which is called *the characteristic equation of the gaseous state*, and is simply another expression of Mariotte's law in which the temperature of the gas is introduced.

Equation (33) expresses the relation that always exists between the pressure, volume, and absolute temperature of a *unit weight* of gas. To apply it to any gas, substitute for v_0 in the value of R, equation (32), the specific volume of the particular gas.

For any number w units of weight *occupying the same volume* the relation evidently becomes

$$pv = wRT \tag{34}$$

A gas supposed to obey exactly the law expressed in equation (33) is called a perfect gas, or is said to be theoretically in the perfectly gaseous state. This perfect condition represents an ideal state toward which gases approach more nearly as their state of rarefaction increases.

For a temperature T' equation (34) becomes

$$p'v' = wRT'$$

Dividing equation (34) by this equation we obtain

$$\frac{pv}{p'v'}=\frac{T}{T'} \tag{35}$$

from which we readily see that if the pressure of any mass of gas is constant the volume of the gas will vary with the absolute temperature, and if the volume is constant the pressure will vary with the absolute temperature.

Problems.—Equations (30) to (34) are used in solving the following problems.

Specific volumes:	Air................	$v_0=$ 12.391 cu. ft.
	Hydrogen.........	$v_0=$178.891 cu. ft.
	Coal gas	$v_0=$ 24.6 cu. ft.
	Water gas	$v_0=$ 18.09 cu. ft.

1. A volume of 3 cubic feet of air, confined at 59° F. (15° C.) and 30″ barometer, is heated to a temperature of 300° C. What pressure does it exert?

Vol. of 1 lb. air at 15°, equation (31), $v_t=v_0 288/273$.

$$3/v_t=w$$

Equation (34), $p=wRT/v=29.24$ lbs. per sq. in.

2. Two pounds of air confined in a volume of 1 cubic foot exerts a gauge pressure of 679.76 lbs. per square inch. What is its temperature by the centigrade and Fahrenheit scales?

The total pressure p is the gauge pressure plus the atmospheric pressure,

$$p=679.76+14.70=694.46$$

Equation (34), $T=pv/wR=520.54$

$$t=247°.54 \text{ C.}=477°.57 \text{ F.}$$

3. A spherical balloon 20 feet in diameter is to be inflated with hydrogen at 60° F., barometer 30.2 inches, so that gas may not be lost on account of expansion when the balloon has risen until

the barometer is at 19.6 inches and the temperature 40° F. How many cubic feet of gas must be put in the balloon?

The gas pressure in the balloon is in equilibrium with the atmospheric pressure. The weight of gas occupying the balloon must be such that at 40° F. the pressure will be in equilibrium with a barometric pressure of 19.6 inches.

$p = p_0 \times 19.6/30$ $\qquad v =$ volume of balloon

Equation (34), $\quad w = pv/RT = 15.05$ lbs.

Volume of w at 60° F. and 30″.2 barometer:

$$p = p_0 \times 30.2/30$$
$$v = wRT/p = 2827.4 \text{ cubic feet}$$

4. What is the lifting power at 70° F. (21°.11 C.) and 30 in. barometer of 1000 cubic feet of each of the gases whose specific volumes are given?

	Vol. 1 lb. at 70°. Equation (31).	Pounds in 1000 cu. ft.	Lifting power 1000 cu. ft. lbs.
Air.	13.35	74.91	
Hydrogen	192.73	5.19	69.72
Coal gas	26.5	37.73	37.18
Water gas.	19.49	51.31	23.60

5. The balloon in which Wellman intends to seek the North Pole has a capacity of 224,244 cubic feet, and weighs with its car and machinery 6600 lbs. What will be its lifting capacity when filled with hydrogen at 10° C. and 30 inches barometer?

Ans. 9647 lbs.

31. Thermal Unit.—The heat required to raise the temperature of unit weight of water at the freezing point one degree of the thermometer is called a thermal unit.

Mechanical Equivalent of Heat.—The mechanical equivalent of heat is the work equivalent of a thermal unit, that is it is the work that can be performed by the amount of heat required to raise the temperature of unit weight of water one degree. It will be designated by E. The unit weight of water being one pound, the value of E for the Fahrenheit scale is 778 foot-pounds; and for the centigrade scale, 1400.4 foot-pounds.

In metric units the value of E is 425 kilogrammeters.

Specific Heat.—The quantity of heat, expressed in thermal units, which must be imparted to *unit weight* of a given substance in order to raise its temperature one degree of the thermometer above the standard temperature is called the specific heat of the substance.

The specific heat of a gas may be determined in two ways: under constant pressure, and under constant volume.

Suppose heat to be applied to a unit weight of gas retained in a constant volume whose walls are impermeable to heat. The whole effect of the heat will be to raise the temperature of the gas. If, however, the gas is enclosed in an elastic envelope, supposed to maintain a constant pressure on the gas, the gas will expand on the application of heat, and part of the heat applied will perform the work necessary to expand the envelope. Therefore to raise the temperature of the gas one degree, a greater amount of heat must be applied when the gas is under constant pressure than when under constant volume; and the difference of these quantities, that is, the difference between the specific heat under constant pressure, c_p, and the specific heat under constant volume, c_v, will be the heat that performs the work of expansion. The mechanical equivalent of a heat unit being represented by E, we may write

$$\text{Work of expansion} = (c_p - c_v)E$$

Actually, part of the work that we have included in the work of expansion is internal work, used in overcoming the attractions between the molecules; but the quantity of work so absorbed is small and is omitted in the discussions.

The work of expansion is equal to the constant resistance multiplied by the path. We will assume the constant resistance to be the atmospheric pressure, p_0. The path is measured by the increase of volume of the gas. To determine the path we have from Gay-Lussac's law, for the centigrade scale equation (31),

$$v_t - v_0 = tv_0/273$$

and therefore for an increase of temperature of one degree there is an increase of volume equal to $v_0/273$. The work of expansion for one degree is therefore $p_0v_0/273$. Referring to equation (32),

$$p_0v_0/273 = R$$

The quantity R is therefore the external work of expansion performed under atmospheric pressure by unit weight of gas when its temperature is raised one degree centigrade. But this work of expansion has been found above to be equal to $(c_p - c_v)E$. Therefore we may write

$$(c_p - c_v)E = R = p_0v_0/273 \qquad (36)$$

From the definition of specific heat we deduce that the quantity of heat necessary to raise the temperature of unit weight of gas any number of degrees, as t, will be

$$Q = ct \qquad (37)$$

c representing either c_p or c_v.

Ratio of Specific Heats.—In the study of interior ballistics the particular values of c_p and c_v for the different gases which are formed by the explosion of gunpowder are of little importance. It suffices to know their ratio, which is constant for perfect gases and approximately so for all gases at the high temperature of combustion of gunpowder.

The ratio of the specific heats, c_p/c_v, is subsequently designated by n.

32. Relations between Heat and Work in the Expansion of Gases.—The relation which exists between the heat in a unit weight of gas and the work performed in the expansion of the gas may now be determined from equation (33),

$$pv=RT$$

which cortains the three variables p, v and T. If we suppose an element of heat, dq, to be applied to the gas, the effect will be generally an increase in the temperature, accompanied by an increase in the pressure, or in the volume, or in both the pressure and the volume.

Considering p constant, and differentiating, we get

$$dT=pdv/R$$

and the quantity of heat communicated to the gas will be, equation (37),

$$dq=c_pdT=c_ppdv/R$$

Considering v constant we obtain similarly

$$dq=c_vvdp/R$$

If p and v both vary, we obtain from the sum of the partial differentials, still representing by dq the element of heat applied to the gas,

$$dq=\frac{1}{R}(c_ppdv+c_vvdp) \tag{38}$$

The differential of equation (33) is

$$RdT=pdv+vdp \tag{38'}$$

Eliminating vdp between the last two equations we have

$$dq=c_vdT+\frac{c_p-c_v}{R}pdv \tag{39}$$

The quantity pdv represents the elementary work of the elastic force of the gas, while its volume increases by dv. The integral of pdv is therefore the total external work between the limits considered.

Representing by W the total external work we have

$$W=\int pdv \tag{40}$$

Represent by T_1 and T the initial and final temperatures.

Integrating equation (39) between the limits T and T_1 we obtain, since c_v, c_p, and R are constant for the same gas,

$$q=c_v(T-T_1)+\frac{c_p-c_v}{R}W \tag{41}$$

Isothermal Expansion.—If we suppose the initial temperature T_1 to remain constant, that is, that just sufficient heat is imparted to the gas while it expands to maintain its initial temperature, the quantity $T-T_1$ in equation (41) becomes 0, and solving with respect to W we obtain

$$W=\frac{R}{c_p-c_v}q$$

We see that in this case, since R, c_p, and c_v are constant for the same gas, the external work done is proportional to the quantity of heat absorbed by the gas.

Making q equal to one thermal unit, W becomes E, and we obtain, as before in equation (36),

$$E(c_p-c_v)=R$$

33. Adiabatic Expansion.—If a gas expands and performs work in such a manner that it neither receives heat from any extraneous body nor gives out heat to an extraneous body, the transformation is said to be *adiabatic*. In this case part of the heat in the gas is converted into work, the temperature and pressure of the gas both diminish, and the work performed will be less than for an isothermal expansion.

Since no heat is gained or lost, q becomes 0 in equation (41) and we have

$$W=R\frac{c_v}{c_p-c_v}(T_1-T)$$

Make $$c_p/c_v=n$$

Then $$W=\frac{R}{n-1}(T_1-T) \tag{42}$$

This equation gives the value of the external work done by a unit weight of gas whose temperature is reduced from T_1 to T in an adiabatic expansion. It will be seen that the external work done is proportional to the fall of temperature.

Law Connecting the Volume and Pressure.—In the adiabatic expansion, as no heat is received from an external source, dq in equation (38) becomes 0, and we have

$$0=c_p p dv+c_v v dp$$

Dividing through by $c_v pv$ we find, since $c_p/c_v=n$

$$n\frac{dv}{v}+\frac{dp}{p}=0,$$

and integrating, $$n \log_e v+\log_e p=\log_e c$$

whence $$v^n p=\text{constant}=v_1{}^n p_1$$

or $$p=p_1\left(\frac{v_1}{v}\right)^n \tag{43}$$

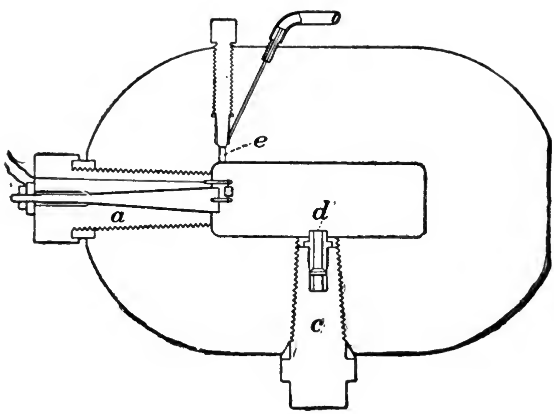
This equation expresses the relation between the volumes and pressures of a gas in an adiabatic expansion.

NOBLE AND ABEL'S EXPERIMENTS.

34. In 1874 and again in 1880 Captain Noble of the English Army and Sir Frederick Abel published the results of their experiments on the explosion of gunpowder in closed vessels. The purpose of their experiments was to determine definitely the nature of the products of combustion, the volume and temperature of the gases, and the pressures with different densities of loading.

Apparatus.—The steel vessel in which the powder was exploded was of great strength and capable of resisting very high pressures.

The charge of powder was introduced through the opening *a* which was then closed with a taper screw-plug. A pressure gauge



d was inserted in the plug *c* and an outlet was provided at *e* through which the gas could be drawn off if desired. The charge was fired by electricity.

The vessels were of two sizes. In the larger one a charge of 2.2 pounds of powder was fired, and the gases wholly retained. Black powder was used in the experiments.

The gravimetric density of the powder used was unity, so that when the chamber was completely filled the density of loading was also unity.

Results of the Experiments.—*Character of the Products.*—The products of combustion were found to consist of about 43 per cent by weight of permanent gases, and about 57 per cent of non-gaseous products. The non-gaseous products ultimately assume the solid form, but are liquid at the moment of the explosion. This was determined by tilting the vessel at an angle of 45 degrees, one minute after the explosion. Forty five seconds later it was returned to its original position. On opening the vessel the solid residue was found inclined to the walls at the angle of 45 degrees.

The permanent gases are principally CO_2, N, and CO, and the solids K_2CO_3, K_2S, K_2SO_4, and S. With the exception of the K_2S and the free sulphur, the products agree in character with those expressed in the formula generally adopted as approximately representing the reaction of black powder on explosion.

$$4KNO_3+4C+S=K_2CO_3+K_2SO_4+2N_2+2CO_2+CO$$

The formula, however, gives $35\frac{1}{2}$ per cent by weight of permanent gases and $64\frac{1}{2}$ per cent of solids.

It was found, as was to be expected, that in a closed vessel variations in the size, form, or density of the grains had practically no effect on the composition of the products of combustion, or on the pressures.

Volume of Gases.—Noble and Abel found that the gases, when brought to a temperature of 0° C. and under atmospheric pressure, occupied a volume of about 280 times the volume of the unexploded powder.

Specific Volume of Gunpowder Gases.—To simplify somewhat the discussions concerning the gases of fired gunpowder we will use as the specific volume the volume, at 0° C. and under atmospheric pressure, of the gases produced by the combustion of unit weight of powder. That is, we will consider this weight of gas as unit weight.

35. Relation between Pressure and Density of Loading.—The relation between the pressure, volume, and absolute temperature of the gases from ϖ units of weight of powder at the moment of explosion is given by equation (34).

$$pv = \varpi RT$$

Make $$f = RT \qquad (44)$$

and we obtain from (34), for the pressure exerted by the gases from ϖ pounds of powder, the gases occupying the volume v at the temperature of explosion,

$$p = f\varpi/v \qquad (45)$$

FORCE OF THE POWDER.—If we make both ϖ and v unity in this equation, p becomes equal to f. f is therefore the pressure per unit of surface exerted by the gases from unit weight of powder, the *gases* occupying unit volume at the temperature of explosion. f is called the *force of the powder.*

Let α be the volume of the residue from unit weight of powder, C the volume of the chamber.

Then the volume occupied by the gas from ϖ units of powder will be

$$v = C - \alpha\varpi$$

We may introduce the density of loading, using metric units by substituting for C in this equation its value ϖ/Δ from equation (23), and obtain

$$v = \frac{\varpi}{\Delta}(1 - \alpha\Delta)$$

Substituting this value of v in (45) we obtain

$$p = f\frac{\Delta}{1 - \alpha\Delta} \qquad (46)$$

This equation expresses the relation between the pressure of the gases from ϖ units of weight of powder and the density of loading.

When $$\frac{\Delta}{1-\alpha\Delta}=1, \text{ that is, when } \Delta=\frac{1}{1+\alpha} \qquad (46')$$

$$f=p$$

Comparing the value of Δ in equation (46′) with the general value, $\Delta=\bar{\omega}/C$, we see that in (46′) the weight of powder is unity, and the volume of the chamber $1+\alpha$. The volume occupied by the gas is therefore also unity. The pressure therefore becomes in this case the force of the powder as defined above.

By substituting in equation (46) two observed values of p corresponding to different values of Δ, the values of α and f were determined. As the means of many observations Noble and Abel finally adopted the values:

$\alpha=0.57$;
$f=18.49$ tons per square inch
$=291200$ kilograms per square decimeter

The pressure for any density of loading is given by the equation

$$p=18.49\frac{\Delta}{1-0.57\Delta} \text{ tons per square inch}$$

When $\Delta=1$ the equation gives $p=43$ tons per square inch.

The value of α, 0.57, means that the volume occupied at the temperature of explosion by the liquid residue from one kilogram of powder is 57/100 of one cubic decimeter. With gravimetric density unity one kilogram of powder occupies one cubic decimeter. Referring now to equation (21), we see that the solid powder, of ordinary density and of gravimetric density unity, occupies 57/100 of the volume of the charge in granular form. The volume of the residue at the temperature of explosion is therefore practically equal to the volume of the solid powder in the charge.

36. Temperature of Explosion.—The temperature of explosion may now be determined from equation (44), which with (32) gives

$$f=RT=\frac{p_0v_0}{273}T \qquad (47)$$

v_0 is the volume occupied by the gas from unit weight of powder. Since the volume of this quantity of gas is 280 times the volume of the powder, and one kilogram of powder occupies one cubic decimeter, $v_0=280$ cubic decimeters. p_0, the atmospheric pressure, is 103.33 kilograms per square decimeter. Substituting these with the value of f, 291200 kilograms per square decimeter, we find $T=2748°$ C. As this is the absolute temperature, subtracting 273 we find the temperature of explosion to be 2475° C.

Captain Noble later considered the absolute temperature as 2505° C.

The approximate correctness of these temperatures was verified by the introduction of pieces of fine platinum wire into the explosion chamber. The platinum, which melts at about 2000° C., was partially fused.

Mean Specific Heat of Products.—The quantity of heat given off by one kilogram of powder was found to be 705 calories, that is, the heat necessary to raise 705 kilograms of water one degree centigrade. From the relation $Q=ct$, equation (37), t being the actual temperature of explosion, not the absolute, a value was found for the mean specific heat of the products:

$$c=\frac{705}{2505-273}=0.316$$

Relations between Volume and Pressure in the Gun.—Noble and Abel found, contrary to their expectations, that the pressures in closed vessels did not differ greatly from the pressures in guns when the powder in the gun was wholly consumed or nearly so. They concluded from this that the expansion of the gases in the gun did not take place without the addition of heat; but that the gases received during the expansion the heat stored in the finely divided liquid residue.

Let c_1 be the specific heat of the residue, assumed to be constant. The elementary quantity of heat given up by each unit weight of residue will then be c_1dT. If there are w_1 units of weight of residue, w_1c_1dT units of heat will be yielded to the gases; and if there are w_2 units of weight of gas, each unit will receive, in heat units,

$$dq = -\frac{w_1}{w_2}c_1dT = -\beta c_1dT$$

β being the ratio w_1/w_2, and the negative sign being used because T decreases while q increases.

Substituting this value of dq in equation (39) it becomes

$$-(c_v+\beta c_1)dT = \frac{c_p - c_v}{R}pdv$$

Eliminate RdT by means of (38′); divide through by pv, and integrate, considering c_p, c_v, c_1 and β constant. We will obtain

$$p = p_1\left(\frac{v_1}{v}\right)^{\frac{\beta c_1 + c_p}{\beta c_1 + c_v}} \qquad (48)$$

When there is no residue β is 0, and the equation becomes identical with equation (43), which was deduced for an adiabatic expansion. In both these equations v_1 and v are the volumes actually occupied by the gases, exclusive of the residue.

Assume the gravimetric density and density of loading to be unity, that is, the chamber is filled with powder, and that the powder is all burned before the projectile moves. Then v_1 in equation (48) will be the volume occupied by the gases in the chamber of the gun, and p_1 the corresponding pressure. If we call v' the volume of the chamber, $\alpha v'$ will be the volume of the residue, and $v' - \alpha v' = v_1$ the volume of the gases; and if we call v'' the volume behind the projectile at any instant, the volume v occupied by the gases becomes $v'' - \alpha v' = v$. Equation (48) therefore becomes

$$p = p_1\left(\frac{v'(1-\alpha)}{v'' - \alpha v'}\right)^{\frac{\beta c_1 + c_p}{\beta c_1 + c_v}} \qquad (48')$$

These values for the constants were determined in the experiments.

$$p_1 = 43 \text{ tons per square inch}$$
$$\alpha = 0.57 \qquad v' = 27.68\ \bar{\omega}$$
$$\beta = 1.2957 \qquad c_p = 0.2324$$
$$c_1 = 0.45 \qquad c_v = 0.1762$$

From these values we find the ratio of the specific heats, $c_p/c_v = n = 1.32$. The value of the exponent in (48′) is 1.074.

37. Theoretical Work of Gunpowder.—The general expression for the work done by a gas expanding from a volume v_1 to a volume v is

$$W = \int_{v_1}^{v} p dv$$

Substituting for p its value from (43) and integrating,

$$W = \frac{p_1 v_1}{n-1}\left\{1 - \left(\frac{v_1}{v}\right)^{n-1}\right\}$$

Assuming that the powder is all burned before the projectile moves, and that the gravimetric density and density of loading are unity, the values v_1 and v in this equation may be replaced as indicated in equation (48′), and we obtain

$$W = \frac{p_1 v'(1-\alpha)}{n-1}\left\{1 - \left(\frac{v'(1-\alpha)}{v'' - \alpha v'}\right)^{n-1}\right\}$$

This is the expression for work under the adiabatic expansion for which $n = 1.32$. If we substitute for n the value 1.074, which is the value of the exponent in equation (48′), the equation will then apply to Noble and Abel's hypothesis.

Work at Infinite Expansion.—When the length of the bore is infinite, v'', which is the volume behind the projectile, is infinite, and we have

$$W = \frac{p_1 v'(1-\alpha)}{n-1}$$

To obtain the work of the gases from one pound of powder make $v' = 27.68$ cubic inches, the volume occupied by one pound, the gravimetric density being unity. Make $n = 1.32$, and substitute for the other constants the values given on page 73. Divide by 12 to reduce from inch-tons to foot-tons.

We find for the work of one pound of powder expanding adiabatically to infinity

$$W = 133.3 \text{ foot-tons per pound.}$$

Substituting for n the value of the exponent in equation (48′), 1.074, we obtain, under Noble and Abel's hypothesis that the gases received heat from the residue,

$$W = 576.35 \text{ foot-tons per pound.}$$

FORMULAS FOR VELOCITIES AND PRESSURES IN THE GUN.

38. Elements Considered. Assumptions.—Formulas connecting the velocity of the projectile with its travel in the bore may be deduced from the relations we have established involving the work of the powder; but these formulas, while they include the force of the powder, do not include consideration of the individual characteristics of different powders, such as form and size of grain, density, and velocity of combustion in the air; nor consideration of the effect on the combustion of the variable pressure in the gun.

M. Emile Sarrau, engineer-in-chief of the French powder factories, was the first to include these elements in ballistic formulas. He considers the progressive combustion of the charge under the influence of the varying pressure in the gun, regarding the powder as a variable in the formulas. The individual characteristics of the powder employed enter the formulas, which thereby become applicable to the determination, in advance, of the proper weight of charge, the kind of powder, the best form and size of grain to produce desired results in a given gun.

Sarrau assumes that the time required for complete inflammation of the charge is negligible compared with the time of combustion. He also assumes an adiabatic expansion of the gases.

This latter assumption, while incorrect according to the experiments of Noble and Abel, is now generally made by writers on interior ballistics; and whatever error is introduced through the assumption is later corrected in the determination, by experiment, of the constants in the formulas.

Principle of the Covolume.—Another assumption of important bearing in the deduction of the ballistic formulas will now be explained.

The characteristic equation for perfect gases, equation (33), combined with equation (47) gives for the pressure from unit weight of gas confined in the volume v,

$$p=f/v$$

But it has been found by experiment that for the gases of explosion the law expressed by this equation does not hold, and that to obtain the true value of the pressure we must diminish the volume v, which is the volume of the explosion chamber. The true equation must therefore be of the form

$$p=\frac{f}{v-\alpha} \tag{49}$$

We may call the volume $v-\alpha$ the effective volume of the gas.

Theoretical deductions indicate that the subtractive volume α is the actual volume of the incompressible molecules in a unit weight of powder gas; that is, it is the limiting volume beyond which a unit weight of gas cannot be compressed.

The volume α is called the *covolume*. Sarrau determined by experiment with different gases that the mean value of the covolume is one one-thousandth of the specific volume of the gas. Other writers take, for convenience, the reciprocal of the density of the powder as the covolume, this value not differing greatly from the other. We have seen, equation (20), that when the gravimetric density is unity the volume of the solid powder in unit volume of the charge is the reciprocal of the density of the powder. The assumption of the reciprocal of the density as the covolume is equivalent therefore to considering the covolume as the volume originally occupied by unit weight of solid powder.

Under this assumption the volume $v-\alpha$, equation (49), which is the effective volume of unit weight of the powder gases, becomes the volume of the powder chamber minus the volume of the solid powder in unit weight of the charge.

The effective volume of the gases from the whole charge will therefore be the volume of the powder chamber minus the volume of the solid powder in the whole charge.

But this is the initial air space in the chamber. Therefore *the effective volume occupied by the powder gases in the chamber is the initial air space.*

If the powder leaves a non-volatile residue, the volume of this residue at the temperature of explosion must be added to the covolume of the gases formed. α in equation (49) will then represent the covolume of the gases from unit weight of powder plus the volume of the residue from unit weight of powder.

39. Differential Equation of the Motion of a Projectile in a Gun.—Let

y be the weight of powder burned at the time t,
T_1 the absolute temperature of combustion,
T the absolute temperature of the gas at the time t.

The work of a unit weight of gas in an adiabatic expansion between the temperatures T_1 and T is given by equation (42). For a weight of gas y we have

$$W=\frac{yR}{n-1}(T_1-T)$$

From equation (44), since T_1 now represents the temperature of explosion, the value for the force of the powder is $f=RT_1$; and from equation (34), $pv=yRT$. With these substitutions the above equation becomes

$$(n-1)W=fy-pv \tag{50}$$

In this equation v is the volume occupied by the gases at the temperature T and at the time t.

Let u be the distance traveled by the projectile at the time t,

ω the cross section of the bore,

z_0 the reduced length of the initial air space.

Under the assumption of the volume originally occupied by unit weight of solid powder as the covolume of the gases, the initial air space in the chamber becomes the volume occupied by the powder gases in the chamber.

We therefore have, for the volume occupied by the gases at the time t,

$$v=\omega(z_0+u)$$

Substituting this value in equation (50) we have

$$(n-1)W=fy-\omega p(z_0+u) \tag{51}$$

an equation expressing the relation at each instant between the weight of powder burned, the pressure, the travel of the projectile, and the external work performed.

In introducing the velocity of the projectile we will assume that the whole work of the gas is expended in giving motion of translation to the projectile. Making w the weight of the projectile, *and representing now by v the velocity of the projectile,*

$$W=\frac{w}{2g}v^2=\frac{w}{2g}\left(\frac{du}{dt}\right)^2$$

p in (51) is the pressure per unit of area; ωp the total pressure on the base of the projectile. The acceleration of the projectile is d^2u/dt^2. The total pressure on the base of the projectile is equal to the product of the mass by the acceleration. Therefore

$$\omega p = \frac{w}{g}\frac{d^2u}{dt^2} \qquad 52)$$

Substituting these values of W and ωp in (51) we have

$$(z_0 + u)\frac{d^2u}{dt^2} + \frac{n-1}{2}\left(\frac{du}{dt}\right)^2 = fg\frac{y}{w} \qquad (53)$$

which is Sarrau's differential equation of the motion of a projectile in the bore of a gun.

In deducing this equation there were neglected the following energies.

The heat communicated by the gases to the walls of the gun,

The work expended on the charge, on the gun, and in giving rotation to the projectile,

The work expended in overcoming passive resistances, such as the forcing of the band, the friction along the bore, and the resistance of the air.

Dissociation of Gases.—The error committed by the omission of these energies may not be as great as would at first appear, for we have also omitted from consideration the heat supplied by the phenomenon called dissociation. According to Berthelot the composition of the complex gases from fired gunpowder is not permanent, and at the high temperature during the first instants of explosion these gases decompose into more simple combinations, perhaps into their elements. The increase in volume due to the displacement of the projectile causes a reduction in the temperature, which permits the dissociated gases to combine again with a consequent development of heat. The theory of dissociation forms the basis for the assumption of some writers on ballistics, notably Colonel Mata of the Spanish artillery, that by reason of this phenomenon the expansion of the gases in the gun takes place as though the gases received heat from the exterior, and not adiabatically.

It will be seen, however, from the form of equation (53) that the errors of assumption may be allowed for by giving to f a suitable value, and this without changing the form of the differential equation of motion. The force of the powder as it appears in equation (53) can therefore be considered only as a coefficient whose value must be determined by experiment.

Sarrau deduced from the differential equation of motion formulas for the velocity and pressure as functions of the travel of the projectile.

40. Ingalls' Formulas.—We will now follow Colonel Ingalls, United States Army, in the deduction of his formulas. These formulas are considered as giving more accurate results than Sarrau's formulas, for the velocity and pressures produced by modern powders in the bore of the gun: and the use of Sarrau's formulas is generally limited to the determination of muzzle velocities and maximum pressures.

Let v be the velocity of the projectile in the bore at the time t. Then

$$\frac{du}{dt}=v$$

and

$$\frac{d^2u}{dt^2}=\frac{dv}{dt}=\frac{vdv}{du}=\frac{d(v^2)}{2du} \tag{54}$$

Substituting these values in equation (53) it becomes

$$(z_0+u)\frac{d(v^2)}{du}+(n-1)v^2=\frac{2fg}{w}y \tag{55}$$

The true value of n, the ratio of the specific heats, c_p/c_v, is uncertain. For perfect gases its value is 1.41. Regarding the powder gases at the high temperature of explosion as perfect gases, earlier writers assumed this value for n. Recent investigations have shown that the value of 1.41 is too great. Some recent writers adopt the value unity for n. As we have seen, equation (35), the work of expansion is directly proportional to the difference of the specific heats; and if their ratio is unity and the difference between them zero, there can be no external work performed. The assumption of the value unity is made for convenience, and the error due to the assumption is compensated for, with the other errors, in the experimental determination of the values of the constants.

Ingalls assumes the value $n=4/3$, which is practically the value deduced from the experiments of Noble and Abel, see page 73.

Making $n=4/3$ in equation (55) we obtain

$$3(z_0+u)\frac{d(v^2)}{du}+v^2=\frac{6fg}{w}y \tag{56}$$

Make

$$x=u/z_0 \tag{57}$$

Under the assumption made that the covolume of the gases is equal to the volume occupied by the solid powder in the charge, the initial air space is the volume occupied by the gases in the powder chamber. Considering z_0, which is the reduced length of the initial air space, as the measure of this volume, x in equation (57), $x=u/z_0$, becomes *the number of expansions of the volume occupied by the powder gases in the chamber, when the projectile has traveled the distance* u.

It is important to bear in mind that x represents a number of expansions, and u the distance traveled by the projectile.

Making $x=u/z_0$, equation (55) becomes

$$3(1+x)\frac{d(v^2)}{dx}+v^2=\frac{6fg}{w}y \tag{58}$$

y, the weight of powder burned, is a function of the time and also of the travel u, and of x. The integration of this equation even when the simplest admissible form of y as a function of x is assumed has not yet been possible.

Considering y constant the equation may be integrated. Rearranging it,

$$\frac{d(v^2)}{v^2-\frac{6fgy}{w}}+\frac{dx}{3(1+x)}=0$$

And integrating,

$$\left\{v^2-\frac{6fgy}{w}\right\}(1+x)^{\frac{1}{3}}=C$$

When $x=0$, $v=0$, and $C=-6fgy/w$. Therefore

$$v^2=\frac{6fgy}{w}\left\{1-\frac{1}{(1+x)^{\frac{1}{3}}}\right\} \tag{59}$$

Making y constant in equation (58) is equivalent to assuming instantaneous combustion for that part of the charge that has burned at the time t. We know this to be in error since the combustion of the charge is progressive. If, however, we determine the values of the constants in the equations by substituting measured values of v, we obtain an equation that is true for the measured values, and may be true for other values of v at other points in the bore. Only by experiment can we determine whether results obtained under this supposition are correct; and experiment, as stated by Colonel Ingalls, is the final test of nearly all physical formulas.

41. Velocities in the Bore.—To make equation (59) applicable to points in the bore we must determine a relation between the quantity of powder burned at any instant and the corresponding travel of the projectile, that is, we must determine the value of y as a function of u or x. Then substituting for y in the equation this value, which for any powder will contain x as the only variable, we will have the desired equation expressing the relation between the velocity of the projectile and its travel in the bore.

Combustion under Variable Pressure.—We have previously deduced, page 26, an expression for the quantity of the powder burned, under constant pressure, as a function of the thickness of layer burned. This relation is given by equation (16) on that page.

$$y=\bar{\omega}\alpha\frac{l}{l_0}\left\{1+\lambda\frac{l}{l_0}+\mu\frac{l^2}{{l_0}^2}\right\} \tag{60}$$

in which y is the weight of the powder burned when a thickness of layer l has been burned, $\bar{\omega}$ is the weight of the charge, l_0 is half the least dimension of the powder grain, and α, λ, and μ are constants of form of the grain.

Representing by τ the time of combustion in air of the whole grain, or charge, the uniform velocity of combustion will be l_0/τ.

In the gun the powder burns under variable pressure, and the velocity of combustion is expressed by dl/dt. Assuming that the velocity of combustion varies as some power of the pressure, and representing by p_0 the pressure of the atmosphere under which the velocity of combustion is l_0/τ, we obtain the equation

$$\frac{dl}{dt}=\frac{l_0}{\tau}\left(\frac{p}{p_0}\right)^{\phi} \tag{61}$$

in which p represents the pressure on the base of the projectile at any instant.

The exponent ϕ is given different values by different writers. Sarrau assumes $\phi=1/2$. Recent experiments indicate a mean value of 0.8. The value unity is assumed by other writers. Ingalls assumes the value 1/2 with Sarrau.

The pressure per unit of area on the base of the projectile is, from equation (52),

$$p=\frac{w}{\omega g}\frac{d^2u}{dt^2} \tag{62}$$

Substituting this value of p in equation (61) and using equation

(54) and the relations

$$u = xz_0, \quad \therefore \frac{dx}{dt} = \frac{1}{z_0}\frac{du}{dt} = \frac{v}{z_0}$$

and

$$\frac{dl}{dt} = \frac{dl}{dx}\frac{dx}{dt} = \frac{dl}{dx}\frac{v}{z_0}$$

equation (61) may be brought to the form

$$\frac{dl}{dx} = \frac{l_0}{\tau}\left(\frac{wz_0}{2g\omega p_0}\right)^{\frac{1}{2}}\left(\frac{d(v^2)}{dx}\right)^{\frac{1}{2}}\frac{1}{v}$$

Integrating and dividing by l_0,

$$\frac{l}{l_0} = \frac{1}{\tau}\left(\frac{wz_0}{2g\omega p_0}\right)^{\frac{1}{2}}\int\left(\frac{d(v^2)}{dx}\right)^{\frac{1}{2}}\frac{1}{v}dx$$

Make

$$K = \frac{1}{\tau}\left(\frac{wz_0}{6g\omega p_0}\right)^{\frac{1}{2}} \tag{63}$$

$$X_0 = \sqrt{3}\int\left(\frac{d(v^2)}{dx}\right)^{\frac{1}{2}}\frac{1}{v}dx \tag{64}$$

Then

$$l/l_0 = KX_0 \tag{65}$$

Substituting this value in (60) we have

$$y = \bar{\omega}\alpha KX_0\{1 + \lambda KX_0 + \mu(KX_0)^2\} \tag{66}$$

42. Discussion of Values.—The value of K in this equation is composed wholly of constants. α, λ, and μ are the constants of form of the powder grain. By the differentiation of equation (59) and substitution in (64), *see foot-note, page 84*, we find for the value of X_0

$$X_0 = \int\frac{dx}{\sqrt{(1+x)\{(1+x)^{\frac{1}{2}} - 1\}}} \tag{67}$$

X_0 is therefore a function of x only, and x from its value, $x = u/z_0$, is itself a function of the travel of the projectile. Equation (66) therefore expresses, for powder of any particular granulation, the relation between the weight burned at any instant and the corresponding travel of the projectile.

This equation may be put into another form.

At the instant that the powder is all burned in the gun, $y = \bar{\omega}$ and $l = l_0$. We will distinguish the particular values of the various quantities *at the instant that the burning of the powder is completed* by putting a dash over the symbol.

When $y = \bar{\omega}$ and $l = l_0$, equations (65) and (66) then become

$$K\bar{X}_0 = 1 \tag{68}$$

$$1 = \alpha(1 + \lambda + \mu)$$

This last relation has been previously established in equation (5).

Substituting the value of K from (68) in (66), we obtain

$$\frac{y}{\bar{\omega}} = \frac{\alpha}{\bar{X}_0} X_0 \left\{ 1 + \frac{\lambda}{\bar{X}_0} X_0 + \frac{\mu}{\bar{X}_0^2} X_0^2 \right\} \tag{69}$$

We have now, in $\bar{X}_0$, introduced into the value of y the travel of the projectile at the specific instant that the burning of the charge is complete.

$$v^2 = \frac{6fgy}{w} \left\{ 1 - \frac{1}{(1+x)^{\frac{1}{3}}} \right\} \tag{59}$$

$$6fgy/w = A \qquad v^2 = A\left(1 - \frac{1}{(1+x)^{\frac{1}{3}}}\right)$$

$$d(v^2) = \frac{A}{3(1+x)^{\frac{4}{3}}}\,dx, \qquad v = \sqrt{A}\sqrt{\frac{(1+x)^{\frac{1}{3}} - 1}{(1+x)^{\frac{1}{3}}}}$$

$$\left(\frac{d(v^2)}{dx}\right)^{\frac{1}{2}} \frac{1}{v} = \frac{1}{\sqrt{3}\sqrt{(1+x)\{(1+x)^{\frac{1}{3}} - 1\}}}$$

From equation (64), $X_0 = \int \frac{dx}{\sqrt{(1+x)\{(1+x)^{\frac{1}{3}} - 1\}}}$

Make

$$X_1 = X_0 \left\{ 1 - \frac{1}{(1+x)^{\frac{1}{3}}} \right\} \tag{70}$$

and

$$X_1/X_0 = X_2 \tag{71}$$

whence

$$X_2 = \left\{ 1 - \frac{1}{(1+x)^{\frac{1}{3}}} \right\} \tag{72}$$

From equation (59) we obtain for the velocity at the instant that the burning of the charge is complete,

$$\bar{v}^2 = 6gf\frac{\bar{\omega}}{w}\bar{X}_2 \tag{73}$$

43. Velocity of the Projectile while the Powder is Burning.—Substituting in equation (59) the value of $6gf$ from (73) and the value of y from (69), using equation (71), and making

$$M = \frac{\alpha\bar{v}^2}{\bar{X}_1} \qquad N = \frac{\lambda}{\bar{X}_0} \qquad N' = \frac{\mu}{\bar{X}_0^2} \tag{74}$$

equation (59) reduces to the form

$$v^2 = MX_1\{1 + NX_0 + N'X_0^2\} \tag{75}$$

This equation expresses the value of the velocity of the projectile at any instant while the powder is burning, in terms of the variable travel of the projectile, and of its velocity and travel at the instant of the complete burning of the charge.

Velocity after the Powder is Burned.—Distinguish with the subscript a the values of v and p *after the charge is completely burned.* y is then equal to $\bar{\omega}$, and equation (59) when combined with (73) and (72) becomes

$$v_a^2 = \bar{v}^2 X_2/\bar{X}_2 \tag{76}$$

and making $$V_1^2 = \overline{v}^2/\overline{X}_2 \qquad (77)$$

we have $$v_a^2 = V_1^2 X_2 \qquad (78)$$

which is the formula for the velocity after the powder is all burned.

This equation is identical with equation (59), if in the latter we make $y = \bar{\omega}$. $V_1^2 = 6fg\bar{\omega}/w$, see (73) and (77), and X_2 is an abbreviation for the quantity in brackets, see (72).

As explained under equation (59), equation (78) is therefore the equation of the velocity under the supposition that the powder is all burned before the projectile moves.

The Velocity V_1.—From equation (78) we see that V_1 is what v_a becomes when X_2 is equal to unity; and, equation (72), X_2 is unity when x is infinite. *V_1 is therefore the velocity corresponding to an infinite travel of the projectile.*

44. Relation between the Velocities Before and After the Burning of the Charge.—Make

$$k = y/\bar{\omega} = \text{fraction of charge burned.}$$

Replacing M, N, and N' in equation (75) by their values, and combining with equations (69), (70), and (76) we may establish the relation

$$v = v_a\sqrt{k} \qquad (79)$$

That is, the velocity of the projectile before the charge is consumed is equal to what the velocity would have been at the same point if all the charge had been burned before the projectile moved, multiplied by the square root of the fraction of charge burned.

Relation between the Weight of Powder Burned and the Velocity and Travel of the Projectile.—Replacing v_a in equation (79) by its value from (78) we obtain

$$k = v^2/V_1^2 X_2 \quad \text{or} \quad y = \bar{\omega}v^2/V_1^2 X_2 \qquad (80)$$

equations that will be found convenient for determining the fraction of charge or weight of powder burned when the velocity and travel of the projectile are known.

By reason of the form assumed by the value of k for certain grains very simple relations may be established, for these grains, between the fraction of charge burned and the travel of the projectile.

Cubical, Spherical, and Spheroidal Grains.—For cubical grains $\alpha=3$, $\lambda=-1$, and $\mu=1/3$ (see page 20). These values apply also to spherical and spheroidal grains. Substituting them in equation (69) we obtain

$$k=1-\left(1-\frac{X_0}{\overline{X}_0}\right)^3 \qquad (81)$$

and

$$X_0=\overline{X}_0\{1-(1-k)^{\frac{1}{3}}\}$$

From the first equation we may obtain the fraction of charge burned for any travel of the projectile, and the converse from the second.

Slender Cylindrical and Prismatic Grains.—For long slender cylinders

$$k=1-\left(1-\frac{X_0}{\overline{X}_0}\right)^2 \qquad (82)$$

$$X_0=\overline{X}_0\{1-(1-k)^{\frac{1}{2}}\}$$

which also apply to grains in the form of long slender prisms of square cross-section.

For other forms of grain the solution of a complete cubic equation is necessary to determine X_0 when k is known.

45. Pressures.—The general expression for the pressure per unit of area on the base of the projectile is given in equation (62). Transforming this equation by means of (54) and (57) we obtain

$$p=\frac{w}{2g\omega z_0}\frac{d(v^2)}{dx} \qquad (83)$$

By substituting in succession the values of $d(v^2)/dx$ obtained from the equations for velocity before and after the complete burning of the charge we will obtain the values of p that apply before and after the charge is burned.

Pressure While the Powder is Burning.—Finding the value of $d(v^2)/dx$ from equation (75), *see foot-note,* and making

$$\left.\begin{aligned} X_3 &= dX_1/dx \\ X_4 &= X_0 + \frac{X_1 dX_0}{X_3 dx} \\ X_5 &= X_0^2 + \frac{2X_1 X_0 dX_0}{X_3 dx} \end{aligned}\right\} \quad (84)$$

$$M' = \frac{Mw}{2g\omega z_0} \quad (85)$$

we obtain for the pressure per unit of area on the base of the projectile while the powder is burning

$$p = M'X_3\{1 + NX_4 + N'X_5\} \quad (86)$$

It will be observed that X_3, X_4, and X_5 are all functions of x only. The logarithms of their values for various values of x will be found in Table I at the end of the volume.

Pressure After the Powder is Burned.—Finding the value of $d(v^2)/dx$ from equation (78), V_1^2 being constant, we obtain with the aid of (72)

$$\frac{d(v_a^2)}{dx} = \frac{V_1^2 dX_2}{dx} = \frac{V_1^2}{3(1+x)^{\frac{4}{3}}}$$

$$v^2 = MX_1\{1 + NX_0 + N'X_0^2\} \quad (75)$$

$$\frac{d(v^2)}{dx} = M\frac{dX_1}{dx} + MN\left(X_0\frac{dX_1}{dx} + X_1\frac{dX_0}{dx}\right) + MN'\left(X_0^2\frac{dX_1}{dx} + 2X_1X_0\frac{dX_0}{dx}\right)$$

Make $$\frac{dX_1}{dx} = X_3$$

$$\frac{d(v^2)}{dx} = MX_3\left\{1 + N\left(X_0 + \frac{X_1 dX_0}{X_3 dx}\right) + N'\left(X_0^2 + \frac{2X_1X_0dX_0}{X_3 dx}\right)\right\}$$

Substituting in (83) and making

$$P' = \frac{wV_1^2}{6g\omega z_0} \tag{87}$$

we obtain for the pressure per unit of area on the base of the projectile after the powder is all burned

$$p_a = \frac{P'}{(1+x)^{\frac{4}{3}}} \tag{88}$$

46. Maximum Pressure.—The maximum pressure in a gun occurs when the projectile has moved but a short distance from its seat, or when u and x are small. The position of maximum pressure is not fixed, but varies with the resistance encountered. As a rule it will be found that the less the resistance to be overcome by the expanding gases the sooner will they exert the maximum pressure and the less the maximum pressure will be. By the differentiation of equation (86) we may obtain the value for the maximum, but it is too complicated to be of practical use. Examination of the table of the X functions shows that X_3 is a maximum when $x=0.65$, nearly, while X_4 and X_5 increase indefinitely. The functions X_3, X_4, and X_5 are found to vary in such a manner that when λ, and therefore N, see (74), is negative, that is, when the powder burns with a decreasing surface, p will be a maximum when x is less than 0.65; and when λ and N are positive or when the powder burns with an increasing surface, p will be a maximum when x is greater than 0.65.

A function at or near its maximum changes its value slowly. Therefore a moderate variation of the position of maximum pressure will have no practical effect on the computed value of the pressure. It has been found by experiment that if we take $x=0.45$ for the position of maximum pressure when λ is negative, and $x=0.8$ when λ is positive, no material error results.

Therefore to obtain the maximum pressure make $x=0.45$, in equation (86) when the powder burns with a decreasing surface, *and make $x=0.8$ when the powder burns with an increasing surface.*

The Pressure P'.—Combining equations (87), (77), and (73) we obtain

$$P'=\frac{\bar{\omega}}{z_0\omega}f \qquad (89)$$

Comparing this with equation (45) we see that since $z_0\omega$ is the initial air space in the chamber, *P' is the pressure of the gases from $\bar{\omega}$ pounds of powder occupying the volume behind the projectile before the projectile has moved from its seat.* This volume is the initial air space. Equation (88) is therefore the equation of the pressure curve under the supposition that the powder is all burned before the projectile moves.

47. Values of the Constants in the Equations for Velocity, Pressure, and Fraction of Charge Burned.—We have now these equations which express the circumstances of motion of the projectile, and the fraction of charge burned at any instant. The original numbers of the equations are given on the left.

While the powder burns,

(75) $$v^2=MX_1\{1+NX_0+N'X_0^2\} \qquad (90)$$

(86) $$p=M'X_3\{1+NX_4+N'X_5\} \qquad (91)$$

After the powder is burned

(78) $$v_a^2=V_1^2X_2 \qquad (92)$$

(88) $$p_a=\frac{P'}{(1+x)^{\frac{4}{3}}} \qquad (93)$$

The fraction of charge burned, substituting N and N' for their values,

(69) $$\frac{y}{\bar{\omega}}=\frac{\alpha}{X_0}X_0\{1+NX_0+N'X_0^2\} \qquad (94)$$

The quantities M, N, N', M', V_1, P' and $\overline{X}_0$ in these five equations are constant for any experiment, and their values must be determined before the equations can be used. It will be seen in the equations that express the values of these constants, equations (74), (77), (85), and (87), that the quantities entering the values are of two kinds: the known elements of fire—by which is meant the constants of the powder, of the gun, and of the projectile—and quantities such as $\bar{v}$, $\overline{X}_0$, $\overline{X}_1$, etc., that involve the velocity and travel of the projectile at the instant that the powder is all burned.

When M and N are known all the constants are known.

The value of M given in equation (74) may be reduced by means of (77) and (71) to

$$M = \alpha V_1^2/\overline{X}_0 \tag{95}$$

We have, equation (74),

$$N = \lambda/\overline{X}_0 \tag{96}$$

M and N being known, $\overline{X}_0$ and V_1^2 are determined from these equations, and N', M', and P' become known from (74), (85), and (87).

Therefore when M and N are known the five equations, (90) to (94), are fully determined, and all the circumstances attending the movement of the projectile become known from them. For any assumed travel of the projectile u, the number of expansions, $x = u/z_0$, is obtained, and with this value of x the functions X_0 to X_5 are obtained from Table I. These substituted with the constants in the equations give the values of v, p, and y. Proceeding in this manner for a number of points along the bore complete curves may be constructed showing the values of v, p, and y for any point in the bore of the gun.

The value of x corresponding to $\overline{X}_0$ is obtained from the table. The value of $\bar{u}$ follows from the equation $\bar{u} = \bar{x}z_0$. This value $\bar{u}$ is the distance that the projectile has travelled at the moment that the charge is completely burned. For values of u less than this, equations (90), (91), and (94) apply; for greater values of u equations (92) and (93) apply.

48. *Determination of the Constants by Experiment.*—Regarding equation (90) and noting from equations (74) that N' is a function of N, it will be seen that if we measure two velocities at known points in the bore of the gun we can determine M and N from equation (90). x being known for each of the points the X functions are obtained from the table. With the two measured values of v we then form two equations in which M and N are the only unknown quantities. Determining M and N the other constants become known.

In using this method care must be exercised that the measured velocities are taken at points passed by the projectile before the powder has completely burned. If the powder is not wholly burned when the projectile leaves the gun one of the measured velocities may be taken at the muzzle.

Since M' is also a function of M, equation (85), we may make use of the two equations (90) and (91), or (92) and (91), and with a single measured velocity and a measured pressure determine M and N from these equations. But it has been shown in the chapter on powders that there is room to believe that the pressures as ordinarily measured with the crusher gauge are not reliable. Therefore results obtained in this way are not likely to be as satisfactory as those obtained from measured velocities, which can be determined with a high degree of accuracy.

It is found in fact that while the velocities obtained from the formulas agree very closely with those actually measured in practice, there is not as satisfactory an agreement between the pressures. The pressures are obtained in the formulas by the dynamic method and are usually higher than the measured pressures. This is in accord with what has already been said in our previous consideration of the subject of pressures, and adds to the evidence against the accuracy of the crusher gauge.

When τ and f are known all the constants are known.

From equations (63) and (68) we obtain

$$\tau=\left(\frac{wz_0}{6g\omega p_0}\right)^{\frac{1}{2}}\overline{X}_0 \tag{97}$$

From equations (73) and (77)

$$f=V_1^2w/6g\bar{\omega} \tag{98}$$

from which can be determined $\overline{X}_0$ and V_1^2. M and N follow from equations (95) and (96).

τ, the time of burning of the whole grain in air, is constant for the same powder.

The value of f, equation (98), is dependent on the value of V_1, a quantity determined by experiment in the gun. f for any powder is therefore constant, within the limits explained below, in the same gun only. It is practically constant for guns that do not differ greatly in caliber. Consequently when τ and f have once been determined for a powder and a gun, we may at once form the equations of motion and pressure for different conditions of loading, involving differences in the form and size of grain of the powder, in the weight of the charge, in the weight of the projectile, and in the size of the chamber and length of the gun.

49. The Force Coefficient f.—The quantity f at its first introduction, equation (45), was shown to be the pressure exerted by the gases from unit weight of powder, the gases occupying unit volume at the temperature of explosion. It was called the force of the powder. But in the ballistic formulas it has been affected by whatever errors there are in the assumptions made in deducing the formulas. It can consequently be regarded only as a coefficient, and it may conveniently be called *the force coefficient.*

Its value, when determined by experiment, may be considered constant in the same gun for charges of the same powder not differing in weight by more than about 15 per cent from the charge used in determining its value. The effective value of the force coefficient is measured in the formulas by projectile energy, and there has been omitted in deducing the formulas all consideration of the force necessary to start the projectile. As the charge decreases the portion of the developed force necessary to start the projectile bears a larger relation to the total force exerted; and if the charge is sufficiently small the projectile will not start at all. The effective force for a small charge must therefore be proportionally less than for a large charge, and the value of f determined from one charge must be modified for use with another that differs greatly in weight. The formula used by Ingalls for this modification will be found in equation (137), problem 3 of the applications which follow.

Values of the X Functions.—We may simplify the value of X_0 by means of circular functions. In equation (67) make

$$\sec\theta=(1+x)^{\frac{1}{6}}$$

we may then deduce, *see foot-note,*

$$X_0=6\int\sec^3\theta\, d\theta$$

The value of this integral, designated as (θ), is given in Table V of the book of ballistic tables for every minute of arc up to 87 degrees. We therefore have, simply

$$X_0=6(\theta)$$

Differentiating the equation $\sec\theta=(1+x)^{\frac{1}{6}}$

$$d\sec\theta=\sec\theta\tan\theta\, d\theta=\tfrac{1}{6}(1+x)^{-\frac{5}{6}}dx=dx/6\sec^5\theta$$

From the second and fourth members,

$$dx=6\sec^6\theta\tan\theta\, d\theta$$

$$\tan\theta=(\sec^2\theta-1)^{\frac{1}{2}}=[(1+x)^{\frac{1}{3}}-1]^{\frac{1}{2}}$$

Equation (67) becomes

$$X_0=\int\frac{6\sec^6\theta\tan\theta\, d\theta}{\sec^3\theta\tan\theta}=6\int\sec^3\theta\, d\theta$$

From the equations giving the values of the various X functions, (70), (71), and (84), first making

$$X=\frac{1}{1+\frac{1}{3}X_0\cos^4\theta\operatorname{cosec}\theta}$$

we may now deduce the following values:

$$X_1=X_0\sin^2\theta$$
$$X_2=\sin^2\theta$$
$$X_3=\sin\theta\cos^4\theta/X$$
$$X_4=X_0(1+X)$$
$$X_5=X_0^2(1+2X)$$

The logarithms of the values of the X functions for various values of x are found in Table I at the end of the volume.

The argument in the table is x. The value of x is obtained from the equation $x=u/z_0$, in which u is the travel of the projectile and z_0 the reduced length of the initial air space. Knowing z_0 and assuming the travel we obtain x and from the table find the corresponding values of the functions.

Interpolation, Using Second Differences.—It will often be necessary in determining values of the functions for values of x not given in the table to employ second differences in order to get the desired accuracy in the interpolated values of the functions.

In a table containing values of a function, the first differences are the differences between the successive values of the function. The second differences are the differences between the successive values of the first differences. Thus if the successive values of an increasing function are a, a', and a'', the first differences are $a'-a=\Delta_1$, and $a''-a'=\Delta_1'$. The second difference is then $\Delta_1'-\Delta_1=\Delta_2$.

The interpolation may be effected by the following formula. The sign of the last term in this formula is made + so that, in this particular table, only the numerical values of the second differences need be considered.

$$X = X_a \pm \frac{x - x_a}{h} \Delta_1 + \frac{x - x_a}{h}\left(1 - \frac{x - x_a}{h}\right)\Delta_2 \tag{99}$$

in which x is the given value of the argument, lying between the tabular values x_a and x_b,

$h = x_b - x_a$,

Δ_1 and Δ_2 are the first and second differences of the function under consideration,

X_a the tabular value of the function corresponding to x_a,

X the interpolated value of the function corresponding to x.

It will be observed that the difference between successive values of x varies in different parts of the table. In applying the formula we must use the same value of h in getting the two first differences from which the second difference is obtained.

The lower sign of the second term of the second member must be used when the function decreases as x increases. This sign will only be required for the values of the function X_3 when the value of x is greater than 0.65.

EXAMPLES.—1. What is the value of $\log X_0$ corresponding to $x = 1.17$?

$$x_a = 1.15 \quad x_b = 1.20 \quad h = .05 \quad x - x_a = .02$$

		1st diff.	2d diff.
$X_a = \log X_0(x = 1.15)$	0.52960	$792 = \Delta_1$	$36 = \Delta_2$
$\log X_0(x = 1.20)$	0.53752	756	
$\log X_0(x = 1.25)$	0.54508		

$$X = (0.52960) + \tfrac{2}{5}\,792 + \tfrac{2}{5} \times \tfrac{3}{5} \times 36 = (0.52960) + 316.8 + 8.6$$

The parentheses around 0.52960 indicate that this number is to be treated as a whole number in applying the corrections. Therefore

$$\begin{array}{r} 0.52960 \\ 316.8 \\ 8.6 \\ \hline X = \log X_0(x = 1.17) = 0.53285 \end{array}$$

2. What is the value of log X_1 when $x=0.563$?

Ans. Log $X_1=9.53337$.

3. Log X_3 for $x=0.275$. Log $X_3=9.82216$.
4. Log X_3 for $x=2.18$. Log $X_3=9.76089$.
5. Log X_5 for $x=0.772$. Log $X_5=1.15879$.

50. The Characteristics of a Powder.—The quantities f, τ, α, λ, and μ were called by Sarrau the characteristics of the powder, because they determine its physical qualities. Of these factors, f, the force coefficient of the powder, depends principally upon the composition of the powder. In the same gun it is practically constant for all powders having the same temperature of combustion. It increases with the caliber of the gun, and for this reason its value determined for one caliber cannot be depended upon for another. The factor τ, the time of combustion of the grain in air, depends upon the least dimension of the grain and upon the density; also, in smokeless powders, upon the quantity of solvent remaining in the powder. The factors α, λ, and μ depend exclusively upon the form of the grain, and for the carefully prepared powders now employed their values can be determined with precision. They are constant as long as the burning grain retains its original form.

APPLICATION OF THE FORMULAS.

For convenience of reference the notation employed in the deduction of the formulas is here repeated, and the units customarily employed in our service are assigned to the different quantities. For most of these quantities specific units have not heretofore been designated.

a defined by equation (101) below.
C volume of powder chamber, cubic inches.
d caliber in inches.
D_1 outer diameter of powder grain, inches.

d_1 diameter of perforation of powder grain, inches.
f force coefficient of the powder, pounds per square inch.
F fraction of grain burned.
g acceleration due to gravity, 32.16 foot-seconds.
$k = y/\bar{\omega}$, fraction of charge burned.
l thickness of layer burned at any instant, inches.
l_0 one half least dimension of grain, inches.
L constant logarithms in the ballistic equations.
m length of powder grain, inches.
M ballistic velocity constant, foot-seconds.
M' ballistic pressure constant, pounds per square inch.
N, N' ballistic constants.
n number of powder grains in one pound.
P' ballistic pressure constant, pounds per square inch.
p pressure while powder burns, pounds per square inch.
p_a pressure after powder is burned, pounds per square inch.
p_m maximum pressure, pounds per square inch.
p_0 standard atmospheric pressure, 14.6967 lbs. per square inch.
S_1 initial surface of a pound of powder, square inches.
u travel of projectile, inches.
U total travel of projectile, inches.
v velocity of projectile while powder burns, foot-seconds.
v_a velocity of projectile after powder is burned, foot-secs.
V muzzle velocity of projectile, foot-seconds.
V_1 ballistic constant, velocity at infinity, foot-seconds.
v_c velocity of combustion of powder, foot-seconds.
v_0 specific volume of a gas, cubic feet.
V_0 initial volume of a powder grain, cubic inches.
w weight of projectile, pounds.
x number of expansions of volume of initial air space.
X_0, X_1, X_2, X_3, X_4, X_5, functions of x.
y weight of powder burned at any instant, pounds.
z_0 reduced length of initial air space, inches.

$\left.\begin{array}{l}\alpha\\ \lambda\\ \mu\end{array}\right\}$ constants of form of powder grain.

δ density of powder.

Δ density of loading.

ϖ weight of powder charge, pounds.

τ time of burning of whole grain in air, seconds.

ω cross section of bore, square inches.

Quantities topped with a bar, as $\bar{v}$, $\bar{x}$, $\bar{u}$, $\bar{X}_2$, etc., designate the particular values of the quantities at the instant of complete burning of the powder charge.

With the units assigned above the following working equations are, with the aid of equation (28), derived from the equations whose numbers appear on the left. The numbers in brackets are the logarithms of the numerical constants after reduction to the proper units.

(22) $$\Delta=[1.44217]\varpi/C \qquad (100)$$

(27) $$a=\frac{\delta-\Delta}{\Delta\delta} \qquad (101)$$

(29) $$z_0=[1.54708]a\varpi/d^2 \qquad (102)$$

(57) $$x=u/z_0 \qquad (103)$$

(73) $$\bar{v}^2=[4.44383]\,f\bar{X}_2\varpi/w \qquad (104)$$

(85) $$M'=[\bar{3}.82867]Mw/a\varpi \qquad (105)$$

(87) $$P'=[\bar{3}.35155]V_1^2w/a\varpi \qquad (106)$$

(89) $$P'=[1.79538]\,f/a \qquad (107)$$

(97) $$\tau=[\bar{2}.56006]\sqrt{aw\varpi\bar{X}_0}/d^2 \qquad (108)$$

(98) $$f=[\bar{5}.55617]V_1^2w/\varpi \qquad (109)$$

In addition to the above working equations the following formulas are needed or are useful in the solution of most problems.

(74) $$M = \alpha \bar{v}^2/\bar{X}_1 \quad N = \lambda/\bar{X}_0 \quad N' = \mu/\bar{X}_0^2 \tag{110}$$

(95) $$M = \alpha V_1^2/\bar{X}_0 \tag{111}$$

(75) $$v^2 = MX_1\{1 + NX_0 + N'X_0^2\} \tag{112}$$

(86) $$p = M'X_3\{1 + NX_4 + N'X_5\} \tag{113}$$

(78) $$v_a^2 = V_1^2 X_2 \tag{114}$$

(88) $$p_a = \frac{P'}{(1+x)^{\frac{4}{3}}} \tag{115}$$

(80) $$k = y/\bar{\omega} = v^2/V_1^2 X_2 \tag{116}$$

$$v_c = \frac{l_0}{\tau}\left(\frac{p}{p_0}\right)^{\frac{1}{3}} \tag{123}$$

$$l = l_0 X_0/\bar{X}_0 \tag{124}$$

$$f = f_0\left(\frac{\bar{\omega}}{\bar{\omega}_0}\right)^{\frac{1}{3}} \tag{137}$$

$$f = f_0\left(\frac{\bar{\omega}}{\bar{\omega}_0}\right)^{\frac{1}{3}}\left(\frac{w}{w_0}\right)^{\frac{1}{6}} \tag{138}$$

51. Transformation of the Formulas into the Forms (104) to (109).—In the deduction of the formulas the quantities have been expressed in general terms, no units having been assigned.

In assigning now to the velocity v the foot-second unit and to the weights the pound unit, we fix the units in the formulas as the foot, the pound, and the second. All dimensional quantities in the formulas must then be considered as expressed in feet, square feet, or cubic feet; pressures in pounds per square foot, and time in seconds. As appears on page 98, we intend now to preserve the foot-second as the unit of velocity, but to express the dimensional quantities, such as d, ω, z_0, u, etc., in terms of the inch as the unit, and the pressures in pounds per square inch. We must therefore introduce into the formulas such factors as will make them applicable to the new units.

This is accomplished as follows.

Equation (104). In the value of $\bar{v}^2$, equation (73), g is already in feet, $\bar{\omega}$ and w in pounds; $\overline{X}_2$ is dependent only on x, which is a ratio independent of the unit. f, which we now express in pounds per square inch, must, before being substituted for f pounds per square foot in (73), be converted into pounds per square foot by multiplying by 144. We therefore get for the numerical factor whose logarithm appears in (104) the quantity $6\,g\,144$.

Equation (105). The quantity $z_0\omega$ in (28) is expressed in cubic inches, and before substituting its value for $z_0\omega$ cubic feet in the formulas we must divide the value by 1728. This substitution is made in equation (85). M' is a pressure in pounds per square foot, as may be seen by substituting for M its value from (74). Equation (85) then becomes $M' = (w\bar{v}^2/2g) \times \alpha/\overline{X}_1\omega z_0$, work divided by a volume, or pressure, see equation (40). To reduce M' to pounds per square inch in order to convert into pounds per square inch the pressures determined from equation (86) we must divide it by 144. With these two operations we obtain, for the numerical factor in (105),

$$1728/(144 \times 2g\,27.68) = 6/g\,27.68$$

Equation (106). Substitute for $z_0\omega$ in (87) its value from (28) divided by 1728, and divide the value of P' by 144 to reduce P' to pounds per square inch. The numerical factor is $2/g\,27.68$.

Equation (107). Substitute for $z_0\omega$ in (89); multiply f now in pounds per square inch by 144, and divide by 144 to reduce P' to pounds per square inch. The numerical factor is 1728/27.68.

Equation (108). From (97), multiplying and dividing by $\omega^{\frac{1}{2}}$.

$$\tau = \frac{\left(w\,\frac{27.68a\bar{\omega}}{1728}\right)^{\frac{1}{2}}}{(6gp_0 144)^{\frac{1}{2}}\,\frac{\pi d^2/4}{144}}\bar{X}_0$$

The numerical factor becomes

$$4(27.68)^{\frac{1}{2}}/\pi(72gp_0)^{\frac{1}{2}} = (27.68/4.5\pi^2 gp_0)^{\frac{1}{2}}$$

Equation (109). Reduce (98) to pounds per square inch by dividing by 144. The numerical factor is $1/6g\,144$.

DETERMINATION OF THE BALLISTIC FORMULAS FROM MEASURED INTERIOR VELOCITIES.

52. As a test of the formulas that have been determined, and at the same time to illustrate their extensive use, we will follow Colonel Ingalls in his application of these formulas to the experiments made by Sir Andrew Noble in 1894 with a six-inch gun. The normal length of the gun was 40 calibers, but it could be lengthened as desired to 50, 75, or 100 calibers.

The length of a gun when expressed in calibers ordinarily means the length measured from the front face of the closed breech block to the muzzle of the gun. The travel of the projectile is the distance passed over by the base of the projectile, measured from its position in the gun when loaded. The length of the gun in calibers is therefore equal to the travel of the projectile plus the length of the powder chamber.

By means of a chronoscope not differing in principle from the Schultz chronoscope that has been described, the velocity of the shot could be measured at sixteen points in the bore. Noble gives the mean instrumental error of the chronoscope as three one-millionths of a second.

Problem 1.—A 100-pound projectile was fired from this 6-inch gun with a charge of 27½ lbs. of cordite. Diameter of grain 0″.4, density 1.56. Velocities measured at points corresponding to the different positions of the muzzle were as follows.

$u=199.2$ inches	$v=2794$ f. s.
259.2 "	2940 "
409.2 "	3166 "
559.2 "	3284 "

The volume of the chamber was 1384 cu. in.
Determine all the circumstances of motion.

Constants of the gun.	*Constants of the powder.*
$C=1384$	$\varpi=27.5$
$d=6$	$\delta=1.56$
$U=559.2$	$\alpha=2$
$w=100$	$\lambda=-\frac{1}{2}$ (see page 21)
	$\mu=0$
	$l_0=0.2$

From equation (100), $\Delta=0.55$
" " (101), $\log a=0.07084$
" " (102), $\log z_0=1.50096$
$z_0=31.693$

Method of Procedure.—With z_0 we may determine from equation (103) the value of x corresponding to any travel of the projectile, and with x we may obtain from Table I the corresponding values of the X functions.

We have now all the necessary data for the solution of the problem, and from this data we must determine the values of the constants in the five formulas (112) to (116). The procedure is as follows.

A. 1. Select two of the measured velocities and the corresponding values of the travel u, and assume that the velocities were reached before the powder was all burned.

2. Substitute successively in (112) the selected values of v

with the values of the X functions obtained with the corresponding travels.

We have then two equations in which only the constants are unknown. As N' is a function of N, there are but two constants, M and N, to be determined from the two equations.

3. Determine M and N from the two equations.

4. With the value of N find from the second of equations (110) the value of $\overline{X}_0$, and with this determine from the table the value of $\bar{x}$, and from (103) the value of $\bar{u}$.

5. The powder was all burned at this travel $\bar{u}$, and if the values of u corresponding to the selected velocities are less than $\bar{u}$, we were right in assuming these two velocities as having been reached before the powder was all burned.

Our determinations of M and N are therefore correct, and, as explained on page 91, all the other constants may be determined from these two.

53. *B.* If, however, one or both of the selected velocities were reached at a travel greater than $\bar{u}$, our assumption that they were both reached before the powder was burned was wrong and our values of M, N, and $\bar{u}$ obtained under that assumption are wrong.

We must therefore determine new values of M and N as follows.

Substitute the first of the selected velocities with the corresponding values of the X functions in (112) as before. Substitute the second selected velocity in (114) with the value of X_2 corresponding to the travel.

Determine V_1.

Replace N, N', and M in (112) by their values from (110) and (111). Then in (112) $\overline{X}_0$ is the only unknown quantity, and its value can be determined.

With $\overline{X}_0$ and V_1 the values of M and N are readily found.

C. The constants cannot be determined if both the selected velocities were reached after the powder was wholly burned.

Equation (114) should give the same value of V_1 for both the selected velocities.

Now to revert to the problem, which will be solved after the first method, designated *A*, and the steps of the solution will be numbered as in the explanation above.

We have to determine the ballistic constants for use in the velocity and pressure formulas.

Since $\mu=0$ we see from equation (110) that

$$N'=0$$

and that since λ is negative N is also negative.

Velocity formula (112) therefore becomes for this powder

$$v^2=MX_1(1-NX_0) \qquad (117)$$

from which with two measured values of v and the corresponding values of u, and hence of X_1 and X_0, we may determine M and N. We must use for this purpose two values of v while the powder is burning.

1. We will take the two measured values 2794 and 3166 and determine afterwards whether we are right in the selection.

2. The X functions for $u=199.2$ corresponding to $v=2794$ are found as follows.

Equation (103), $x=6.2853$, for $u=199.2$.

From the table of X functions, using first differences only,

$$\log X_0=0.82110$$

In the same way the other functions for this value of x, and the functions for the values of x corresponding to the other given values of u, are obtained from the table.

u	x	v	$\log X_0$	$\log X_1$	$\log X_2$
199.2	6.2853	2794	0.82110	0.50606	$\bar{1}.68496$
259.2	8.1784	2940	0.86213	0.58011	$\bar{1}.71799$
409.2	12.9112	3166	0.93117	0.69774	$\bar{1}.76657$
559.2	17.6446	3284	0.97710	0.77150	$\bar{1}.79440$

In equation (117), using two values v and v' and the values of X_0 and X_1 corresponding to each, and solving for N and M, we obtain

$$N=\frac{v'^2X_1-v^2X_1'}{v'^2X_0X_1-v^2X_0'X_1'}$$

$$M=\frac{v^2}{X_1-NX_0X_1}$$

3. Making $v=2794$ and $v'=3166$, we obtain with the corresponding values of X_0 and X_1

$$\log M=6.59155$$
$$\log N=\bar{2}.75465$$

With these, as has been shown on page 91, all the other ballistic constants are determined.

4. We will first determine from the second of equations (110)

$$\log \overline{X}_0=0.94432$$

and from the table find the corresponding value of x by interpolation, using first differences only,

$$\bar{x}=14.11$$

From equation (103) $\bar{u}=447.19$, that is, the burning of the powder was completed at the instant that the shot had travelled 447.19 inches.

5. The values of u for the points selected for the determination of the constants in the equations being less than $\bar{u}$ we find ourselves justified in the selection of these points.

From equation (105) $\log M'=4.91005$

(111) $\log V_1^2=7.23484$

(106) $\log P'=5.07622$

We now have all the constants that enter the equations (112) to (116) for velocity and pressure and fraction of charge burned. These equations become for this round

$$v^2=[6.59155]X_1(1-[\bar{2}.75465]X_0) \tag{118}$$

$$p=[4.91005]X_3(1-[\bar{2}.75465]X_4) \tag{119}$$

$$v_a^2=[7.23484]X_2 \tag{120}$$

$$p_a=\frac{[5.07622]}{(1+x)^{\frac{4}{3}}} \tag{121}$$

$$y=[\bar{6}.20449]v^2/X_2 \tag{122}$$

With these five equations we can determine the velocity, pressure, and weight of powder burned as the projectile passes any point in the bore, by substituting the values of the X functions determined from Table I for the value of x corresponding to the travel of the projectile at the point.

In this way we find from equation (118) for $u=259.2$, for which $x=8.1784$,—(the symbol L indicates a constant logarithm in the equation),

$\log X_0$	0.86213
L	$\bar{2}.75465$
0.41379	$\bar{1}.61678$
0.58621	$\bar{1}.76805$
$\log X_1$	0.58011
L	6.59155
$\log v^2$	6.93971
$\log v$	3.46985
$v=$	2950 foot-seconds

This differs from the measured velocity by 10 feet.

To find the velocity at the muzzle, for comparison with the measured velocity, we must make use of equation (114), since the powder was all burned before the projectile reached the muzzle.

$\log V_1^2$	7.23484
$\log X_2$	$\bar{1}.79440$
$\log V^2$	7.02924
$\log V$	3.51462
$V=$	3270.5 foot-seconds

This differs but 13.5 feet from the measured velocity of 3284 feet. The difference, $\frac{4}{10}$ of one per cent of the measured velocity, is negligible.

In the same way the velocity at any point may be determined and the curve v in Fig. 20 plotted.

54. Pressures.—The pressure at any point may be similarly obtained from equations (119) and (121). The pressures so obtained are plotted in the curve p, Fig. 20.

Maximum Pressure.—As the cylindrical grain burns with a decreasing surface the maximum pressure is obtained as explained on page 89 by making $x=0.45$ in equation (119),

$$\text{for } x=0.45 \qquad \log X_3=\bar{1}.85640 \qquad \log X_4=0.48444$$

With these values we get from equation (119)

$$p_m=48{,}276 \text{ lbs.}$$

Weight of Powder Burned.—From equation (122) we obtain the curve y, Fig. 20, which shows the weight of powder burned at each point of the travel. From this curve it is seen that at the point of maximum pressure, for which $u=14.26$ inches, about 12 of the 27.5 pounds of the charge were consumed. The charge was half consumed when the travel was 18 inches, and three-quarters consumed at a travel of about 68 inches.

The following table obtained from the three equations, (118), (119), and (122), is represented by the curves v, p, and y in Fig. 20

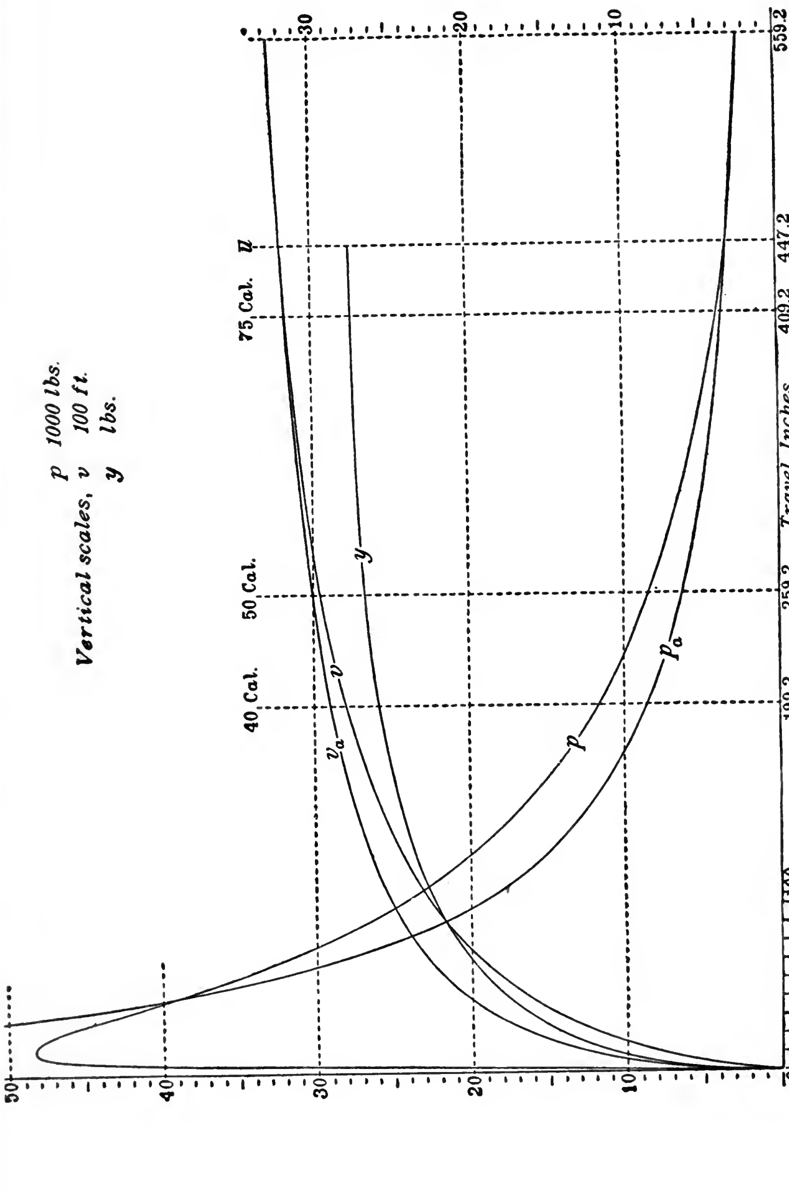


FIG. 20.—Charge, 27.5 pounds Cordite.

x	Travel u inches.	Veolcity v ft.-secs.	Pressure p pounds.	Powder burned y pounds.
0.2	6.34	564.99	43929	8.669
0.4	12.67	876.56	48183	11.597
0.6	19.02	1109.1	47558	13.584
0.8	25.36	1295.2	45569	15.097
1.0	31.69	1449.8	42895	16.315
1.5	47.54	1747.9	36632	18.589
2.0	63.38	1967.2	31386	20.209
2.5	79.24	2138.0	27158	21.442
3.0	95.08	2276.1	23738	22.419
4.0	126.77	2488.0	18600	23.873
5.0	158.46	2644.2	14975	24.898
6.2853	199.2	2794.0	11642	25.822
8.1784	259.2	2950.0	8329	26.677
12.9112	409.2	3166.0	3840	27.475
14.1100	447.2	3198.0	3191	27.500
17.6446	559.2	3271.0	2411	

In the figure the curve y stops at the travel $\bar{u}$ because equation (122) can only apply as long as the powder is burning. The powder, wholly burned at $\bar{u}$, is of course wholly burned at every point beyond $\bar{u}$.

The curves v_a and p_a in Fig. 20 are similarly obtained from equations (120) and (121). They represent the velocity and pressure under the supposition that the powder was wholly burned before the projectile moved, and from them are obtained the velocities and pressures in the gun after the powder is all burned, that is, after the travel $\bar{u}$.

The size of the page does not permit the representation of the first part of the curve p_a. This curve intersects the vertical axis at a point obtained by making $x=0$ in equation (121), for which value $p_a=119{,}180$ lbs. per sq. in. $=P'$, see (115). As explained on page 90, P' is the pressure per unit of surface exerted by ϖ pounds of powder confined in a volume equal to the initial air space.

The Force Coefficient f and Constant τ.—From equation

(109) $f=2247.4$ lbs. per sq. in.

(108) $\tau=0.50486$ seconds

f was originally considered as the force of the powder or, in the units assigned, the pressure exerted by a pound of a gas occupying a cubic foot at the temperature of explosion, see equation (45). But it has been affected by whatever errors there are in the assumptions made in the deduction of the formulas. It can consequently be regarded only as a coefficient, called the *force coefficient.*

τ is the total time of burning of the grain in air. The velocity of burning in air is, therefore, for this grain,

$$l_0/\tau = 0.39615 \text{ inches per second.}$$

55. Velocity of Combustion.—The velocity of combustion of the powder at any instant may be obtained from equation (61).

$$v_c = \frac{l_0}{\tau}\left(\frac{p}{p_0}\right)^{\frac{1}{3}} \tag{123}$$

by substituting the value of p corresponding to any point in the travel of the projectile.

Thus at the moment of maximum pressure, $p_m = 48{,}276$, and

$$v_c = 22.7 \text{ inches per second.}$$

At this rate of burning the charge would be consumed in about nine one-thousandths of a second.

Thickness of Layer Burned.—Combining equations (65) and (68) we obtain

$$l = l_0 X_0 / \overline{X}_0 \tag{124}$$

Substituting for any point the value of X_0 we obtain l.

Thus for $u = 199.2$, $\log X_0 = 0.82110$, and for the thickness of layer burned at this travel

$$l = 0.1506 \text{ inches.}$$

Variation in Size of Grain.—The thickness of layer burned at any travel of the projectile is evidently the half thickness of web of some whole grain of the same shape that would be completely burned at that point. We may therefore write in equation (124) l_0' for l and $\overline{X}_0'$ for X_0 and form the equation

$$2l_0'=2l_0\overline{X}_0'/\overline{X}_0 \qquad (125)$$

The web of a grain designed to be completely burned at any travel of the projectile under the same conditions of loading as in problem 1 will therefore have a thickness equal to twice the thickness of layer burned at the travel as obtained in that problem.

$$\text{For } u=199.2, \quad 2l_0'=0.3012 \text{ inches,}$$

which is twice the value we found for l at this length of travel.

Variation in Initial Surface of Charge for Same Shape of Grain.—From equations (19) and (125) we obtain

$$S_1'=S_1\overline{X}_0/\overline{X}_0' \qquad (126)$$

For the grain whose web we have just determined the initial surface of the charge would have the following relation to the same weight of charge of the powder used in problem 1.

$$S_1'=1.322\ S_1$$

56. Variations in Gun, Powder, or Projectile.—Having once determined the constants τ and f for any powder in a gun of any caliber, we may assume any variation in the gun except in caliber, or any variation in the powder or in the projectile, and determine the effect of the variation on the circumstances of motion. τ, the time of complete burning of the grain in air, is proportional to the web thickness. Its value for the same powder in grains of any other shape or size is equal to the determined value multiplied by the ratio of the web thicknesses of the new grain and of the grain used in the determination. For any assumed size of the chamber and fixed weight of charge or density of loading we may proceed exactly as in problem 1. For changes in the weight of the charge or of the projectile the procedure is the same as in that problem. For changes in the shape of the powder grain the method to be pursued will be best understood from an example.

Problem 2.—Suppose that the powder used in problem 1 instead of being made up into cylindrical grains was made into ribbons 0″.4 thick, 2″ wide, and 8″ long, of the same density as the cylindrical grains.

Determine the circumstances of motion with the same weight of charge, $27\frac{1}{2}$ pounds, as in that problem.

The thickness of web, 0″.4, is the same as for the cordite cylinder.

The values of the constants of form for the parallelopiped grain are, see page 19,

$$\alpha=1+x+y$$

$$\lambda=-\frac{x+y+xy}{1+x+y}$$

$$\mu=\frac{xy}{1+x+y}$$

in which $x=2l_0/m$ and $y=2l_0/n$.

Making $x=0.4/8=0.05$ and $y=0.4/2=0.2$ we find for the ribbon grain assumed in this problem

$$\alpha=1.25, \quad \lambda=-0.208, \quad \mu=0.008.$$

As the initial surfaces of two charges of equal weight composed of the same powder in grains of different shapes are to each other as the values of α for the two forms of grain, see equation (19), the initial surface of this charge will be $1.25/2=5/8$ of the initial surface of the charge in problem 1, and as the maximum pressure is dependent upon the initial surface we may expect a lower maximum pressure from this charge than from the first.

The values of f and τ determined in problem 1, being constant for the same powder and gun, are applicable to this round, and it will be seen from equations (100) to (109) that Δ, a, z_0, $\bar{v}^2$, P', $\bar{X}_0$, and V_1^2 have the same values as in that problem.

Therefore from equations (110), (111), and (105) we obtain at once the values of the constants in the formulas for velocity and pressure.

$$\log M = 6.38743$$
$$\log N = \bar{2}.37374$$
$$\log N' = \bar{4}.01445$$
$$\log M' = 4.70593$$

and with these values we may write the formulas for velocity and pressure while the powder is burning.

$$v^2 = [6.38743]X_1\{1 - [\bar{2}.37374]X_0 + [\bar{4}.01445]X_0^2\},$$
$$p = [4.70593]X_3\{1 - [\bar{2}.37374]X_4 + [\bar{4}.01445]X_5\}.$$

The formula for the weight of powder burned is the same as in problem 1, equation (122), but since the value of v for any value of x is now different the weights burned at the different travels will also be different.

The formulas for velocity and pressure after the charge is all burned are the same as in problem 1, equations (120) and (121), and the velocities and pressures beyond the point of complete consumption are the same. The point of complete consumption is the same as in that problem, since $\bar{X}_0$ has the same value.

The velocities and pressures and weight of powder burned under the conditions of this problem are shown in the subjoined table and in Fig. 21.

x	Travel u inches.	Velocity v f. s.	Pressure p pounds.	Powder burned y pounds.
0.2	6.34	458.86	29584	5.718
0.4	12.67	720.16	33587	7.828
0.6	19.02	919.33	34089	9.333
0.8	25.36	1081.6	33381	10.528
1.0	31.69	1218.6	32220	11.527
1.5	47.54	1489.7	28926	13.503
2.0	63.38	1696.1	25922	15.024
2.5	79.24	1862.3	23390	16.269
3.0	95.08	2005.6	21278	17.326
4.0	126.77	2223.2	18001	19.062
5.0	158.46	2397.2	15600	20.465
6.2853	199.2	2576.0	13324	21.947
8.1784	259.2	2780.3	10977	23.697
12.9112	409.2	3131.0	7559	26.871
14.1100	447.2	3198.0	7091	27.500
17.6446	559.2	3271.0	2411	

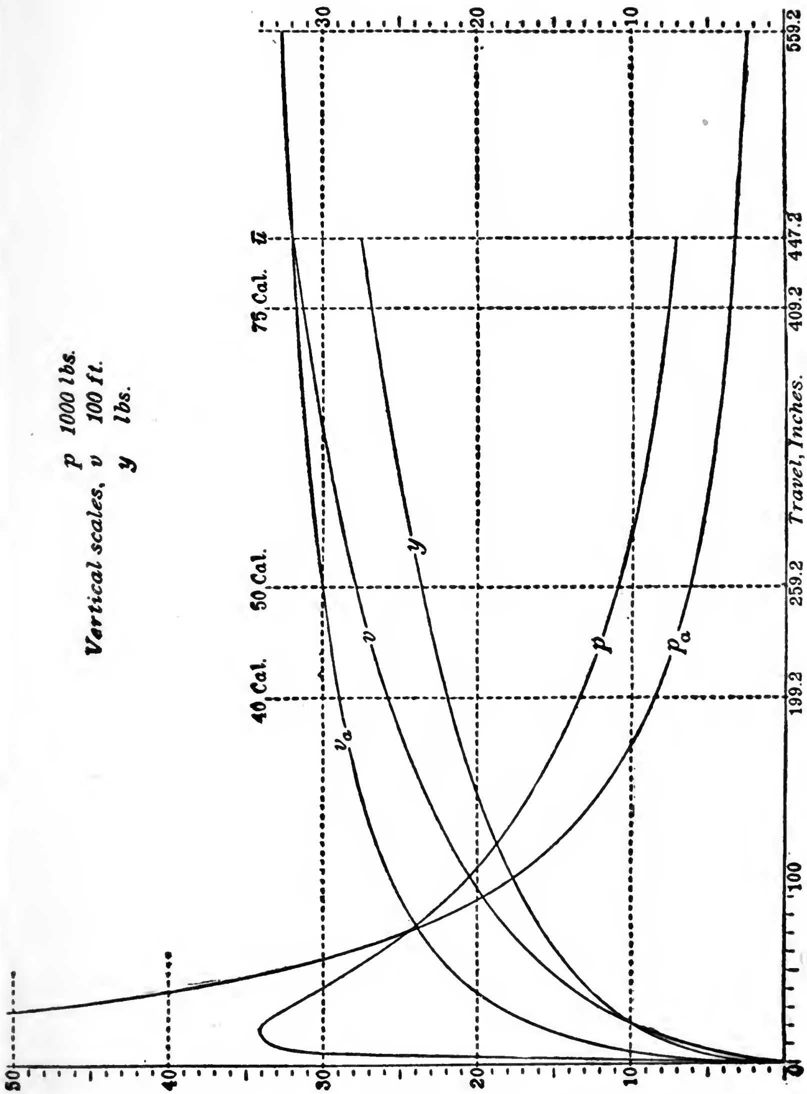


FIG. 21.—Charge, 27.5 pounds, Ribbon Grains.

Comparing this charge, by means of the tables or of the curves, with the charge in problem 1 we see that while the muzzle velocity is the same the maximum pressure is reduced from about 48,000 to about 34,000 lbs. The pressures along the chase are increased. The total area under the pressure curves, which represent the work expended upon the projectile, must be equal.

It is apparent from the powder curves that the powder burned more progressively in the second charge than in the first. This was to have been expected, for if we compare the rate of burning of the two grains in air by means of equations (9) and (7), dividing the half thickness of web into five equal parts, we find for the fraction burned in each layer:

Cordite grains	0.36	0.28	0.20	0.12	0.04
Ribbon grains	0.24	0.22	0.20	0.18	0.16

57. Velocities and Pressures after the Powder is Burned.— We have seen, pages 86 and 90, that equations (114) and (115) are the equations for the velocity and pressure under the supposition that the powder is all burned before the projectile moves.

The curves v_a and p_a in Figs. 20 and 21 are calculated from equations (120) and (121) for both shapes of grain. They are alike in the two figures since the weight of charge is the same. The curve v_a, from equation (120), shows what the velocities would be if the $27\frac{1}{2}$ pounds of powder were all burned before the projectile moved, and the curve p_a shows the pressures under the same condition.

We find in practice that the velocities measured beyond the point where the powder is all burned agree with the velocities obtained from the v_a formula. We are therefore warranted in using this formula for determining velocities after the powder is burned. And if the correct velocities are given by the v_a formula, the pressures obtained from the p_a formula must also be correct.

Therefore velocities and pressures after the powder is all burned are taken from the v_a and p_a curves or formulas.

From the manner of deduction of equations (112) and (114) these two equations will give the same value $\bar{v}$ for the value $\bar{u}$. The curves v_a and v therefore coincide at that value of the travel. It will be observed, however, in Fig. 21, that the curves p_a and p for the ribbon grain do not coincide at the travel $\bar{u}$.

It may be shown analytically that these curves coincide only for grains of such form that the vanishing surface is zero; such as the cube, sphere, or solid cylinder, see page 18. The vanishing surface of the ribbon grains of this problem is a finite surface that suddenly becomes zero at the travel $\bar{u}$. Coincidence of the two curves at this point could therefore not be expected.

The curves p_a and p in Fig. 20, for the cordite grain, coincide at $\bar{u}$, since the vanishing surface of the cordite grain is zero.

58. The Action of Different Powders.—In Fig. 22 the curves of velocity, pressure, and weight of powder burned, from problems 1 and 2, are shown together. This figure serves well to illustrate the action of different powders in the gun.

The curves with the subscript 1 are taken from problem 1, in which the charge was 27.5 lbs. of cordite. The curves with subscript 2 are from problem 2, in which the charge was of the same weight as in problem 1 and of powder of the same composition, but made up into ribbon-shaped grains with the same thickness of web as the cordite.

Regarding the curves y_1 and y_2 we see that the burning of the charge of powder was completed in each case at the same point of travel, $\bar{u} = 447.2$ inches. The quantity burned at any travel less than $\bar{u}$ was less for the ribbon grain than for the cordite.

The rate of emission of gas as a function of the travel of the projectile is shown by the tangents to the curves y_1 and y_2. For equal travels of the projectile the ribbons gave off gas less rapidly at first and until the projectile had traveled about 63 inches, at which point the curves y_1 and y_2 are farthest apart. From this point on the ribbon grains emitted gas more rapidly than the cordite.

We consequently find in the pressure curves lower pressures

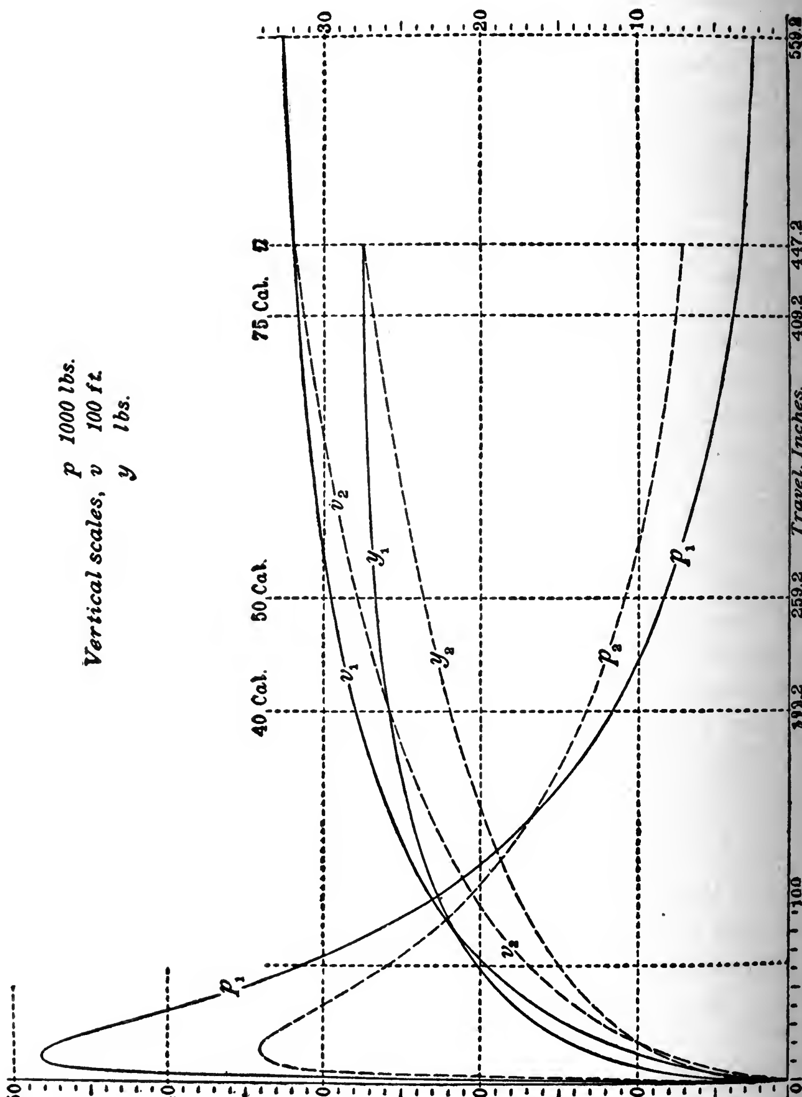


FIG. 22.—Comparison of Results with Cordite and Ribbon Grains.

from the ribbon grains over this part of the bore. The maximum pressure is lower and occurs later than the maximum pressure from the cordite. After the travel of 63 inches the pressure is better maintained by the more rapid evolution of gas from the ribbon grains and we find that the pressure curve p_2 falls off more slowly than the curve p_1, so that the two curves rapidly approach each other, and later cross at a travel of about 130 inches.

At the instant before the travel $\bar{u}$ is reached the area of the burning surface of the ribbon grains has a considerable value. It may readily be determined, from the given dimensions and density of the ribbon grains, that there are 76 of these grains in the charge of 27½ lbs. The initial surface of the charge is 3040 square inches.

The vanishing surface of each grain, determined by mensuration or by making $l = l_0$ in equation (1), is 24.32 square inches, and for the 76 grains, 1848 square inches. This is more than 6/10 of the original surface.

At the travel $\bar{u}$ this large burning area suddenly becomes zero. There is a sudden cessation of the emission of gas and a sharp drop in the pressure. As the burning surface of the cordite grain approaches zero gradually the pressure curve p_1 of this grain is continuous.

Since at the travel $\bar{u}$ the projectile has the same velocity from the two charges, the work done upon it is the same in each case, and the areas under the pressure curves to this point must be equal.

Corresponding with the sudden change in pressure at the travel $\bar{u}$ we find in the curve v_2 a sudden variation in the rate of change of the velocity of the projectile as a function of the travel, represented by the tangent to the curve.

The above considerations apply to the 100 caliber length of the gun.

Now if we consider the gun as 40, 50, or 75 calibers in length neither charge would have been wholly consumed in the bore; and we see from the curves that in each case the muzzle velocity would be less from the slower burning powder. It is therefore apparent that to produce in the gun of any of these lengths a given muzzle velocity, v_1, taken from the cordite curve, a larger charge of the slower powder would be required.

The maximum pressure from the larger charge of slow powder would remain less than that from the quicker powder, since the area under the two pressure curves must be equal and the pressure curve of the slow powder would be the higher at the muzzle.

As the gun is longer the difference in the weight of the two charges of the quick and slow powder that produce the same muzzle velocity is less, until at some length the difference becomes zero. The advantage of lower maximum pressure always remains with the slower powder.

59. **Quick and Slow Powders.**—It is apparent from Fig. 22 that if the maximum pressure and the muzzle velocities obtained from the cordite in the 40 and 50 caliber guns are satisfactory, the muzzle velocities produced by the same charge of powder in the form of ribbons would be too low. This powder would be too slow for guns of those lengths, while for the guns of 75 or more calibers it would be satisfactory.

The powder for a gun of any caliber and length has the greatest efficiency when in grains of such shape and dimensions that the charge of least weight produces the desired muzzle velocity within the allowed maximum pressure. The powder that produces these results may be considered the standard powder for the gun.

The maximum pressure is dependent on the initial surface of the powder charge. A powder with greater initial surface than the standard powder, that is a powder of smaller granulation, will produce a greater maximum pressure and therefore will be a quick powder for the gun, and a powder of larger granulation will be a slow powder.

In powder grains that are similar in shape but of different dimensions, the thickness of web will vary as the square root of the surface. We may therefore judge as to whether the powder is quick or slow for any gun by comparing its web thickness with that of the standard powder of the same shape.

It is also found that usually a powder that is satisfactory in a gun of a given caliber is slow for a gun of less caliber and quick for a gun of larger caliber. Therefore, as has been shown in the chapter on gunpowders, a special powder is provided for each caliber of gun and for markedly different lengths of the same caliber.

Effects of the Powder on the Design of a Gun.—In the design of a gun, the caliber, weight of projectile, and muzzle velocity being fixed, consideration must be given to the powder in order that the size of chamber, length of gun, and thickness of walls throughout the length may be determined. We have seen that to produce a given velocity in any gun we require a larger charge of a powder that is slow for the gun than of a quicker powder. The larger charge will require a larger chamber space, and will thus increase the diameter of the gun over the chamber. The maximum pressure being less than with the quicker powder the walls of the chamber may be thinner. The slow powder will give higher pressures along the chase, therefore the walls of the gun must here be thicker. The weight of the gun is increased throughout its length.

If we do not wish to increase the diameter of the chamber we must, for the slow powder, lengthen the gun in order to get the desired velocity.

On the other hand, with a powder that is too quick for the gun very high and dangerous pressures are encountered, requiring excessive thickness of walls over the powder chamber. The difficulties of obturation are increased. Excessive erosion accompanies the high pressures and materially shortens the life of the gun. The gun may be shorter and thinner walled along the chase.

It is evident from the above considerations that each gun must be designed with a particular powder in view, and that a gun so designed and constructed will not be as efficient with any other powder.

DETERMINATION OF THE BALLISTIC FORMULAS FROM A MEASURED MUZZLE VELOCITY AND MAXIMUM PRESSURE.

60. In the previous problems we determined the constants in the ballistic formulas by means of measured interior velocities. This method will usually not be available, as interior velocities can be measured only by special apparatus not usually at hand. The usual data observed in firing are the muzzle velocity and the maximum pressure.

The method of determining the constants with this data is illustrated in the following problem, and at the same time the method of applying the formulas to the multiperforated grain.

Problem 3.—Five rounds were fired from the Brown 6 inch wire wound gun at the Ordnance Proving Grounds, Sandy Hook March 14, 1905. The projectiles weighed practically 100 lbs. each. The charge was 70 lbs. of nitrocellulose powder in multi-perforated grains, with two igniters, each containing 8 ounces of black powder, at the ends of the charge. The multiperforated grains weighed 89 to the pound. They were of the form described on page 22. Their dimensions, corrected for shrinkage, were

$$D_1 = 0''.512 \qquad d_1 = 0''.051 \qquad m = 1''.029$$

The mean muzzle velocity of the five rounds was 3330.4 f. s.

The measured maximum pressure was 42,497 lbs. per sq. in.

The capacity of the powder chamber was 3120 cubic inches.

The total travel of the shot was 252.5 inches.

Determine the circumstances of motion.

Before we can proceed with the solution of the problem we must determine the constants of the powder. We will make no distinction between the two different kinds of powder, but consider the weight of charge as 71 pounds of multiperforated powder.

Dimensions of grains, $D_1 = 0''.512$, $d_1 = 0''.051$, $m = 1''.029$.

Weight of grain, 89 to 1 pound.

We will first determine the constants of form of the powder grain.

From equation (13)

$$2l_0 = 0.08975$$

and from equations (12) we find $\alpha = 0.72667$, $\lambda = 0.19590$, $\mu = 0.02378$. Equation (11), in which F is the fraction of grain burned when the web is burned, therefore becomes for this grain

$$F = 0.72667\frac{l}{l_0}\left\{1 + 0.19590\frac{l}{l_0} - 0.02378\frac{l^2}{l_0^2}\right\} \qquad (127)$$

Making $l = l_0$,

$$F = 0.85174 \qquad (128)$$

the fraction of grain burned when the burning of the web is completed. The slivers therefore form 0.14826 of this particular grain.

FICTITIOUS MULTIPERFORATED GRAIN.—The body of the grain burns with an increasing surface, while the slivers burn with a decreasing surface. To avoid the difficulties that would follow from the introduction of the two laws of burning into the ballistic formulas, we will substitute for the real grain a fictitious grain with such a thickness of web that when the web is burned the same weight of powder is burned as when the whole of the real grain is burned; that is, the body of the fictitious grain is equivalent to the whole of the real grain.

For the body of the fictitious grain F in the formula of the fraction burned must be unity when $l = l_0$. Making $F = 1$ in equation (127) and solving the cubic equation by Horner's Method, as explained in the algebra, we obtain for l/l_0

$$l/l_0 = 1.1524$$

The value of l/l_0 that will make $F = 1$ in equation (127) can be obtained more simply and with sufficient accuracy by trial as follows.

We have determined that when $l = l_0$ and $l/l_0 = 1$, $F = 0.85174$. This value is less than unity by 0.148. For a first trial we will increase the value of l/l_0 by 0.148 and obtain from (127),

with $l/l_0 = 1.148$ $\qquad F = 0.99568$

an increase in the value of F of 0.144. Therefore if we further increase l/l_0 by 0.005 we will get a value of F near unity;

$$\text{with } l/l_0=1.153 \qquad F=1.0006$$

Interpolating, by the rule of proportional parts, between these two sets of values we find that for $F=1$

$$l/l_0=1.1524$$

Substituting this value in (127) it becomes

$$1=0.837416(1+0.22573-0.031581)$$

Comparing this with equation (5), $1=\alpha(1+\lambda+\mu)$, which is derived from the formula for the fraction burned by making $l=l_0$, and which expresses the relations existing between the constants of form of the powder grain, we see that for the fictitious grain

$$\alpha=0.837416 \qquad \lambda=0.22573 \qquad \mu=-0.031581$$

The new value of l_0 must be the former value multiplied by the above ratio, $l/l_0=1.1524$, since we have multiplied all the quantities in equation (127) by this ratio to make $F=1$. Therefore $l_0=0.044875\times1.1524=0.051714$.

The volume of the real grain is

$$V_0=\tfrac{1}{4}\pi(D_1^2-7d_1^2)m=0.197144$$

Whence from equation (18) with $n=89$, $\delta=1.5776$.

61. Solution.—We have now all the data necessary for the solution of the problem. For convenience it is repeated here.

Constants of the Gun.	*Constants of the Powder.*
$C=3120$	$\varpi=\ \ 71$
$d=6$	$\delta=\ \ 1.5776$
$U=252.5$	$\alpha=\ \ 0.837416$
$w=100$	$\lambda=\ \ 0.22573$
Measured Data.	$\mu=-0.031581$
$V=3330.4$	$l_0=\ \ 0.051714$
$p_m=42497$	

From equation (100) $\Delta = 0.6299$

(101) $\log a = \bar{1}.97940$

(102) $\log z_0 = 1.82144$

$z_0 = 66.289$

On account of the thinness of web of the powder grain, and the high pressure, we may be certain that the charge was wholly consumed in the bore. Assuming that the maximum pressure was the maximum pressure on the base of the projectile we then have a pressure while the powder was burning and a velocity after the charge was all burned. As explained on page 92, equations (92) and (91), or (114) and (113), are applicable in this case.

Method of Procedure.—The procedure is as follows.

1. Substitute in (114) the measured muzzle velocity and the value of X_2 taken from the table with the value of x corresponding to the travel of the projectile at the muzzle.

2. Determine V_1.

3. Substitute in (113) the measured value of the maximum pressure and the values of the X functions corresponding to $x=0.8$ or $x=0.45$, according as the grain burns with an increasing or decreasing surface.

4. Assume a value for the travel at the moment of complete combustion and determine for this travel the values of $\bar{x}$ and $\overline{X}_0$.

5. With this value of $\overline{X}_0$ and the value of V_1, previously determined, find values for N, N', and M' from (110), (111), and (105).

6. Substitute these values in the second member of (113).

7. If the second member has then the same value as the first member, which is the measured maximum pressure, our assumption of the travel $\bar{u}$ is correct. If not we must make new assumptions for $\bar{u}$ and determine new values for M, N, and N' until we find values that will satisfy equation (112).

The successive steps of the solution which follows are numbered as in the preceding paragraph.

1. For the muzzle $U=252.5$ and, equation (103),

$$x=3.8091$$

From the table, for this value of x

$$\log X_2 = \bar{1}.61019$$

Therefore equation (114) becomes for the muzzle

$$v_a^2 = (3330.4)^2 = V_1^2\,[\bar{1}.61019] \qquad (131)$$

from which

2. $$\log V_1^2 = 7.43481$$

3. It was shown on page 90 that with a grain burning with an increasing surface the maximum pressure may be taken as occurring when

$$x = 0.8$$

which for this round corresponds to a travel $u = 53.03$ inches, see equation (103).

For this value of x we find from the table

$$\log X_3 = 9.86027 \qquad \log X_4 = 0.60479 \qquad \log X_5 = 1.17352$$

Equation (113) therefore becomes, since μ and N' are negative,

$$p_m = 42497 = [\bar{1}.86027]M'\{1 + [0.60479]N - [1.17352]N'\} \qquad (129)$$

From equation (105) we determine for this problem

$$M' = [\bar{3}.99801]\,M$$

and substituting this value of M' in equation (129) it becomes

$$p_m = 42497 = [\bar{3}.85828]M\{1 + [0.60479]\,N - [1.17352]N'\} \qquad (130)$$

4. The proper values of M, N, and N' must satisfy equation (130). But we see that equations (110) and (111) express fixed relations between these constants and V_1 at the moment of complete burning of the charge.

Therefore we will assume the travel at the moment of complete consumption, and with the corresponding value of $\bar{x}$, and therefore of $\bar{X}_0$, determine N and N' from equations (110) and M from (111).

Then substituting this set of values in equation (130) we will determine whether the values satisfy that equation. If not we will make other assumptions for $\bar{x}$ and proceed in the same way until we find satisfactory values of the constants.

The value of x at the muzzle is 3.8091. The value $\bar{x}$ must be less than this since we are assuming that the charge was all consumed in the gun. Let us assume $\bar{x}=2$.

5. Taking from the table the corresponding value of $\log X_0$ we find from equations (110) and (111) values of M, N, and N'.

6. These substituted in equation (130) make the second member equal to 45,746.

7. This is greater by 3249 pounds than the measured maximum pressure, 42,497 pounds; and we therefore conclude that we have assumed a too rapid combustion of the powder. The true value of $\bar{x}$ is therefore greater than 2.

Assume next	$\bar{x}=2.3$
From the table	$\log \overline{X}_0=0.65467$
From equation (111)	$\log M=6.70307$
From equation (110)	$\log N=\bar{2}.69892$
	$\log N'=\bar{3}.19009$

With these values in equation (130) we get

$$p_m=42{,}909 \text{ pounds}$$

As this differs from the given pressure, 42,497 pounds, by less than one per cent, we may without material error use these values of the constants as the true values.

The assumed value $\bar{x}=2.3$, by means of which the constants were determined, gives, from equation (103)

$$\bar{u}=152.5 \text{ inches}$$

We have from equations (105) and (106)

$$\log M'=4.70108$$

$$\log P'=4.95570$$

We may now from equations (112) to (116) form the five equations applicable to this round.

$$v^2=[6.70307]X_1\{1+[\bar{2}.69892]X_0-[\bar{3}.19009]X_0{}^2\} \qquad (132)$$

$$p=[4.70108]X_3\{1+[\bar{2}.69892]X_4-[\bar{3}.19009]X_5\} \qquad (133)$$

$$v_a{}^2=[7.43481]X_2 \qquad (134)$$

$$p_a = \frac{[4.95570]}{(1+x)^{\frac{4}{3}}} \qquad (135)$$

$$y = [6.41645] v^2 / X_2 \qquad (136)$$

With these equations we may determine the velocity, pressure, and weight of charge burned at any point in the bore. For any travel less than $152\frac{1}{2}$ inches equations (132) and (133) apply for the velocity and pressure, and equation (136) for the weight of powder burned. For any travel greater than $152\frac{1}{2}$ inches, equations (134) and (135) apply.

The table and curves which will follow are derived from these equations.

A convenient method of performing the work in constructing the table or curves is here shown. It is always best to assume values of x that are given in the table, rather than values of u, which would require interpolation in the table to find the values of the X functions.

The symbol L in the following work is used to designate the various constant logarithms in equations (132) to (136).

We will take for example the value $x=0.8$, corresponding to the travel at which we found the maximum pressure.

From the table:

$\log X_0 = 0.46075$	$\log X_1 = 9.71100$	$\log X_2 = 9.25025$
$\log X_3 = 9.86027$	$\log X_4 = 0.60479$	$\log X_5 = 1.17352$

Equation (103)	$\log x$	$\bar{1}.90309$	
	$\log z_0$	1.82144	
	$\log u$	1.72453	$u = 53.031$ inches

Equation (132)		$\log X_0$	0.46075		
		$\log N$	$\bar{2}.69892$		
$\log X_1$	$\bar{1}.71100$		1.15967	$\log X_0^2$	0.92150
$\log M$	6.70307	$+1$	1.14443	$\log N'$	$\bar{3}.19009$
	6.41407		0.01293		2.11159
	0.05365		1.13150		
$\log v^2$	6.46772				
$\log v$	3.23386				
$v =$	1713.4 foot seconds				

Equation (133)		$\log X_4$	0.60479		
		L	$\bar{2}.69892$		
$\log X_3$	$\bar{1}.86027$		$\bar{1}.30371$	$\log X_5$	1.17352
$\log M'$	4.70108	$+1$	1.20124	$\log N'$	$\bar{3}.19009$
	4.56135		0.02310		$\bar{2}.36361$
	0.07120		1.17814		
$\log p_m$	4.63255				
$p_m =$	42909 lbs. per sq. in.				

Equation (136)	$\log v^2$	6.46772	
	L	$\bar{6}.41645$	
	colog X_2	0.74945	
	$\log y$	$\bar{1}.63392$	$y = 43.045$ lbs.

And if we desire the values of v_a and p_a,

Equation (134)	$\log V_1^2$	7.43481	Equation (135)	log 1.8	0.25527
	$\log X_2$	$\bar{1}.25025$		$\times 4/3$	0.34036
	$\log v_a^2$	6.68586		$\log P'$	4.95570
	$\log v_a$	3.34253		$\log p_a$	4.61534
	$v_a = 2200.5$ f. s.			$p_a = 41{,}242$ lbs. per sq. in.	

These values of v_a and p_a are what the velocity and pressure would have been had the powder all burned before the projectile moved.

The calculations for velocity and pressure at any point of the bore beyond the point of complete combustion of the charge are extremely simple, being limited to the solving of the two equations (134) and (135), which require from the table the function X_2 only.

Proceeding as above for different values of x we obtain the data collected in the table on page 130, from which the curves in Fig. 23 are constructed.

62. Pressure Curves for Real and Fictitious Grains.—We have used in the deduction of the equations from which the table is produced a fictitious multiperforated grain the body of which, without the slivers, equals the whole of the real grain. The body of the real grain was, as shown by equation (128), 85.174 per cent of the whole grain, the slivers forming 14.826 per cent of the whole. The table and curve p show discontinuity of

x	Travel u inches.	Powder burned y lbs.	Velocity v f. s.	Pressure p lbs.	Velocity v_a f. s.	Pressure p_a lbs.
0.0	0.00	00.00	000.0	00000	000.0	90300
0.1	6.63	15.02	424.3	26018	922.5	79525
0.2	13.26	21.43	695.9	33698	1266.8	70814
0.4	26.52	30.48	1113.4	40316	1699.2	57657
0.6	39.77	37.35	1440.8	42500	1986.5	48254
0.8	53.03	43.05	1713.4	42909	2200.5	41242
1.0	66.29	47.98	1947.9	42500	2369.5	35836
1.3	86.18	54.43	2248.9	41223	2568.6	29742
1.6	106.06	60.04	2505.3	39659	2724.5	25258
1.9	125.95	65.04	2729.1	38052	2851.4	21836
2.3	152.47	71.00	2989.2	36002	2989.2	18380
3.1	205.50				3195.4	13761
3.8091	252.50				3330.4	11124

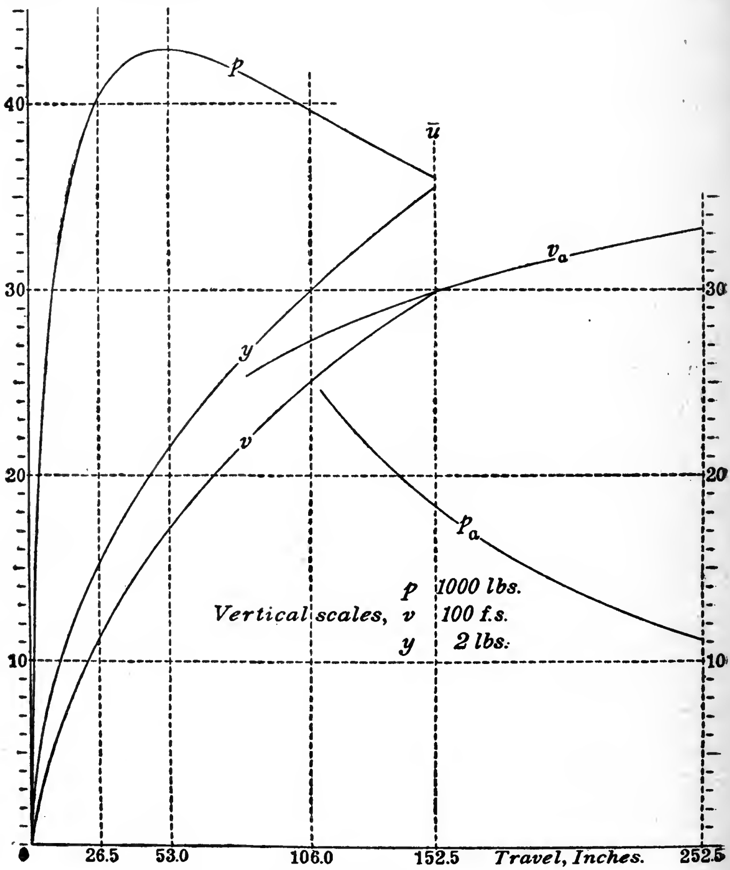


FIG. 23.—Charge, 71 pounds, Multiperforated Grains.

the pressure at the travel 152.5 inches when the burning of the whole charge is completed.

Actually there is no discontinuity in the true pressure curve. The web of the real grain was burned when 85.2 per cent of the body of the fictitious grain, or of the whole charge, was burned. This portion of the charge, 60.5 lbs., was burned at a travel of about 109 inches, as may be seen from the table. The charge burned with an increasing surface up to this point of travel and then with a decreasing surface which gradually approached the vanishing surface zero.

The pressure would therefore, at a travel of 109 inches, begin to fall off more rapidly, making a point of inflection in the true pressure curve. From this point, as the slivers burn, the pressure curve should gradually approach the curve p_a and join it at some point beyond the theoretical $\bar{u}=152.5$ inches, since the slivers, burning with a constantly decreasing surface, will require a longer time for complete consumption than the same weight in the body of the fictitious grain.

The Constant τ for this Powder.—From equation (108),

$$\tau=0.37477 \text{ seconds}$$

This is the time of burning of the whole grain in air.

The velocity of burning of this grain in air, l_0/τ, $=0.138$ inches per second.

The velocity of combustion in the gun is given by equation (123), and the thickness of layer burned at any travel by equation (124).

The Force Coefficient f.—From equation (109),

$$f=1379.5 \text{ lbs. per sq. in.}$$

It has been previously stated that f is constant for any powder in a given gun for charges not differing greatly in weight. The effective value of f, as measured in the formulas by projectile energy, must decrease as the charge decreases, for we have omitted in the formulas all consideration of the force necessary to start the projectile. It is apparent that if the charge were sufficiently reduced the projectile would not move, and f in the formula would be zero.

Therefore for any charge differing materially in weight from the charge used in the determination of f the value of f must be modified.

Ingalls adoots provisionally, this relation.

$$f = f_0 \left(\frac{\bar{\omega}}{\bar{\omega}_0} \right)^{\frac{1}{3}} \tag{137}$$

in which $\bar{\omega}_0$ is the weight of charge used in the determination of f_0; f is the modified value of f_0 for the charge $\bar{\omega}$; $\bar{\omega}$ is any charge differing in weight from the charge $\bar{\omega}_0$ by 15 per cent or more.

The value of f will be modified also by a marked change in the weight of the projectile. Ingalls uses for f in this case the value

$$f = f_0 \left(\frac{w}{w_0} \right)^{\frac{1}{8}}$$

and if both $\bar{\omega}$ and w change sufficiently,

$$f = f_0 \left(\frac{\bar{\omega}}{\bar{\omega}_0} \right)^{\frac{1}{3}} \left(\frac{w}{w_0} \right)^{\frac{1}{8}} \tag{138}$$

With the modified value of f from equation (137) we may now determine the velocities produced by reduced charges.

63. Problem 4.—What muzzle velocities should be expected from the 6 inch gun of problem 3, with charges (including igniters) of 59 and $33\frac{1}{4}$ lbs. of the powder used in that problem?

As these charges differ in weight by more than 15 per cent of the charge of 71 lbs. used in problem 3, we will obtain the value of f from equation (137), using for $\bar{\omega}_0$ and f_0 the values of problem 3.

We have as before

$$C = 3120 \qquad \delta = 1.5776 \qquad U = 252.5$$

The work may be conveniently performed as follows.

		Charge, 59 lbs.	*Charge, $33\frac{1}{4}$ lbs.*
Equation (137)	$\log \bar{\omega}$	1.77085	1.52179
	$\log \bar{\omega}_0$	1.85126	1.85126
	$\div 3$	$\bar{1}.91959$	$\bar{1}.67053$
		$\bar{1}.97320$	$\bar{1}.89018$
	$\log f_0$	3.13972	3.13972
	$\log f$	3.11292	3.02990
Equation (109)	$\log \bar{\omega}/w$	1.77085	1.52179
	L	4.44383	4.44383
	$\log V_1^2$	7.32760	6.99552
Equation (100)	Δ	$=0.5234$	0.2950
Equation (101)	$\log a$	$=0.10605$	0.44028
Equation (102)	$\log z_0$	$=1.86768$	1.95285
	z_0	$=73.736$	89.712
Equation (103) for the muzzle,	x	$=3.4244$	2.8146
From the table	$\log X_2$	$\bar{1}.59202$	$\bar{1}.55630$
Equation (114)	$\log V_1^2$	7.32760	6.99552
	$\log v_a^2$	6.91962	6.55182
	$\log v_a$	3.45981	3.27591
		$V=2883$ f. s.	$V=1888$ f. s.

The muzzle velocities actually obtained with charges of the above weights were 2879 and 1913 f. s. respectively. The calculated velocities show differences of 4 and 25 f. s. respectively. The latter difference, though practically not very great, shows that the modified value of f determined from the value deduced from one charge gives only approximate results when the second charge is, as in this case, less than 47 per cent of the first.

64. Problem 5.—What muzzle velocities should be expected from the 6 inch gun of problem 3, with charges (including igniters) of 68 and 75 lbs. of the powder used in that problem?

As these charges differ but little in weight from the charge of 71 lbs. used in problem 3, the value of f there determined will serve in this problem.

$f=1379.5 \quad C=3120, \quad \delta=1.5776 \quad U=252.5$

	Charge, 68 lbs.	Charge, 75 lbs.
Equation (100)	$\Delta=0.6033$	0.6654
Equation (101)	$\log a=0.01016$	1.93901
Equation (102)	$\log z_0=1.83344$	1.80486
	$z_0=68.146$	63.806
Equation (103)	$x=3.7052$	3.9573
Equation (109)	$\log V_1^2=7.41606$	7.45861
From the table	$\log X_2=\bar{1}.60555$	$\bar{1}.61648$
Equation (114)	$V=3242$ f. s.	$V=3448$ f. s.

The measured muzzle velocities with these charges were, respectively, 3236 and 3455 f. s. The differences between the calculated and measured velocities are immaterial.

We may make for this powder and gun any desired assumption as to the form of the powder grain, weight of charge, weight of projectile, size of powder chamber or length of gun, and with the values of f and τ from problem 3, determine the full circumstances of motion under the assumption.

Sufficient illustration has now been given of the remarkable accuracy, the simplicity and extensiveness of application of the ballistic formulas deduced by Colonel Ingalls. By their use we may obtain a more intimate knowledge of the conditions existing in the bore of a gun than has heretofore been attainable; and the knowledge so obtained will be applied in the manufacture of powder and of guns, and will result in the production of more efficient weapons.

United States Army Cannon.—A table containing data concerning the principal cannon now in service follows. The bursting charges for projectiles as given in the table are of rifle powder for the 1.457 inch and 3.2 inch guns, the 3.6 inch mortar, the 6 inch howitzer, and the two subcaliber tubes. For all other projectiles the bursting charges are of high explosive.

PRINCIPAL UNITED STATES ARMY CANNON.

Gun.	Weight.	Capacity of powder chamber.	Travel of projectile.	Weight of charge.	Weight of projectile.	Muzzle velocity.	Maximum pressure.	Density of loading.	Bursting charge.
Mountain, Field and Siege.	pounds.	cubic ins.	inches.	pounds.	pounds.	ft. secs.	pounds.		pounds.
1.457 inch pompon	410	4.86	40.68	0.075	1	1800	27000	0.4272	0.039
2.95 inch mountain gun	236	34.9	26.4	0.5	12.5	920	18000	0.3966	1.14
2.38 inch field rifle, 1905	440	34	59.05	0.72	7.5	1700	33000	0.5861	0.81
3 inch field rifle, 1905	788	66.5	74.54	1.625	15	1700	33000	0.6764	0.82
3.2 inch field rifle, 1897	830	50	75.05	1.12	13.5	1685	35000	0.6228	0.2
3.8 inch field rifle, 1905	1535	142.6	93.73	3	30	1700	33000	0.5823	2.1
3.8 inch field howitzer, 1906	391	66.34	30.01	1.2	30	900	15000	0.4966	2.1
3.6 inch field mortar, 1890	245	33.2	16.065	0.375	20	690	17000	0.3127	0.6
4.7 inch siege rifle, 1904	2688	251	114.9	5.94	60	1700	33000	0.6547	3.09
4.7 inch siege howitzer, 1906	....	....	....	....	60	900	15000	....	3.09
5 inch siege rifle, 1898	3639	402.5	119.8	5.37	45	1830	35000	0.3693	1.75
5 inch siege howitzer, 1900	1170	100	55.655	2.1	55	1000	23000	0.5660	2
6 inch siege howitzer, 1905	1660	281	62.325	4	120	900	15000	0.4	3.86
7 inch siege howitzer, 1898	3650	316.7	81.385	4.6	105	1100	28000	0.3933	7.4
7 inch siege mortar, 1892	1715	182.8	44.82	4	125	800	20000	0.6057	11.9
Seacoast.									
1.457 inch subcaliber tube	120	7.7	68.37	0.156	1.057	2000	25000	0.5617	0.1
2.95 inch subcaliber tube	236	34.9	26.4	0.44	18	750	18000	0.3470	0.81
2.24 inch rifle (6 pdr.), 1900	850	50	101.759	1.35	6	2400	34000	0.7183	0.25
3 inch rifle (15 pdr.), 1903	2692	296	139.28	6.06	15	3000	34000	0.6594	0.35
4.72 inch rifle, 50 cals	6160	496	205.1	10.5	45	2600	34000	0.5860	4.5
5 inch rifle, 1900	11120	1211	214.605	26	58	3000	36000	0.5478	2.3
6 inch rifle, 1905	21148	2122	256.285	42	106	2900	36000	0.5630	4.6
8 inch rifle, 1888	32218	3617	205.25	80	316	2200	38000	0.6122	19
10 inch rifle, 1900	76830	10040	329.62	224	604	2500	38000	0.6064	33
10 inch mortar, 1890	16734	1554	82.76	34	604	1150	33000	0.6056	33
12 inch rifle, 1900	132380	17487	395	367	1046	2500	38000	0.5697	58.3
12 inch mortar, C. I., 1886	31920	2021	91.64	33	824	1050	27500	0.4519	32.1
12 inch mortar, steel, 1890	29120	2674	98.92	54	1046	1150	33000	0.5590	58.3
14 inch rifle, 1906	111000	13363	401.4	280	1660	2150	36000	0.58	58.5
16 inch rifle, 1895	284500	29624	451.86	612	2400	2150	38000	0.6167	139.3

CHAPTER IV.

EXPLOSIVES.

65. Explosive and Explosion.—An *explosive* is a substance that is capable of sudden change from a solid or liquid state to a gaseous state, or a mixture of gases whose chemical combination, suddenly effected, results in a great increase of volume. A chemical explosion is always attended by the emission of great heat.

An explosion due to physical causes alone, as when a gas under compression is suddenly released and allowed to expand, causes cold.

Effects of Explosion.—The effects of an explosion are dependent on the quantity of gas evolved, on the quantity of heat, and on the rapidity of the reaction.

QUANTITY OF GAS. PRESSURE.—The volume of gas at the temperature of explosion determines the pressure exerted against the walls of the vessel containing the explosive.

Force.—The pressure per unit of surface exerted by the gases from unit weight of the explosive, the gases occupying unit volume at the temperature of explosion, is called the *force* of the explosive. The unit volume occupied by the gases is exclusive of the co-volume of the gases and the volume of any residue.

QUANTITY OF HEAT. WORK.—The quantity of heat determines the quantity of work that may be effected by the explosion. The bursting of the walls of the containing vessel and the projection of the fragments, or the projection of the shot from a gun, are effects produced by the conversion of the heat of explosion into work.

Potential.—The total work that can be performed by the gas from unit weight of the explosive under indefinite adiabatic expansion measures the *potential* of the explosive.

The theoretical potential of an explosive is never reached in practice. The potentials, however, afford the means of comparing the maximum theoretical quantities of work to be obtained from different explosives. The maximum practical effect obtained from explosives in firearms is from $\frac{1}{5}$ to $\frac{1}{6}$ of the potential.

Rapidity of Reaction.—An explosion starts with the explosion of a single molecule, or particle, of the explosive. The heat generated and the shock developed by the explosion of the first molecule are communicated to the surrounding molecules and by the explosion of these molecules are transmitted further into the mass.

The rapidity with which the explosive reaction is transmitted through the mass varies greatly in different explosives.

The explosion of gunpowders does not differ in principle from the burning of a piece of wood or other combustible. As we have seen in the chapter on gunpowders the combustion proceeds from layer to layer and the rate of combustion, in air and in the gun, and the quantity of powder burned at any time, may be determined by means of the formulas of interior ballistics.

The explosion of nitroglycerine, of guncotton, and of other explosives of like nature is effected with very much greater rapidity than the explosion of gunpowder. The theory of Berthelot is that in these explosives the spread of the explosive reaction is not confined to the exposed surfaces, but that the explosion of the initial molecule gives rise to an explosive wave which is transmitted with great velocity in all directions through the mass and causes the almost instantaneous conversion of the whole body into gas. The velocity of propagation of the explosive wave through a mass of guncotton has been determined experimentally by Sebert to be from 16,500 to 20,000 feet per second.

The progressive emission of gas from gunpowder produces a propelling effect by causing a gradual increase of pressure on the base of the projectile, while the sudden conversion into gas of nitroglycerine or guncotton produces the effect of a blow of great intensity.

66. Orders of Explosion.—The differences in the rapidity of reaction give rise to the division of explosives into two groups, high explosives and low or progressive explosives. Explosions are designated as detonations or explosions of the first order, and progressive explosions or explosions of the second order.

The high explosives are those of great rapidity of reaction. Their complete explosions are of the first order, and produce by reason of their quickness a crushing or shattering effect on any material exposed to them.

The principal high explosives in general use are nitroglycerine, the dynamites, guncotton, picric acid and its salts, the Sprengel mixtures, and the fulminate of mercury.

The cadets of the Military Academy have studied in their course in chemistry (Descriptive General Chemistry (Tillman), pages 369 to 385) the constitution, method of production, and characteristics of the principal high explosives. It is therefore unnecessary to further describe these explosives here.

The progressive explosives are those that consume an appreciable time in the explosion. They produce explosions of the second order. The explosion is slow, comparatively, and progressive, and produces a propelling or pushing effect.

The various gunpowders are progressive explosives. Gunpowders have been fully described in Chapter I.

Nitrocellulose.—The classification by Vielle of the nitrocelluloses of various degrees of nitration is shown in the following table. The higher the degree of nitration of the cellulose the greater is the power of its explosion.

VIELLE'S CLASSIFICATION OF NITROCELLULOSES.

Formula.	Designation.	c.c. of NO_2.	Per Cent of N.	Remarks.
$C_{24}H_{36}O_{20}(NO_2)_4$	Tetra-n.c.	108	6.76	
$C_{24}H_{35}O_{20}(NO_2)_5$	Penta-	128	8.02	
$C_{24}H_{34}O_{20}(NO_2)_6$	Hexa-	146	9.15	Only slightly attacked by acetic ether and ether-alcohol. } Friable cottons.
$C_{24}H_{33}O_{20}(NO_2)_7$	Hepta-	162	10.18	Becomes gelatinous in acetic ether and ether-alcohol. } Friable cottons.
$C_{24}H_{32}O_{20}(NO_2)_8$	Octo-	178	11.11	Soluble in ether-alcohol. Inferior colloid.
$C_{24}H_{31}O_{20}(NO_2)_9$	Ennea-	190	11.96	Highly soluble in ether-alcohol. Superior colloid.
$C_{24}H_{30}O_{20}(NO_2)_{10}$	Deca-	203	12.75	Highly soluble in ether-alcohol. Superior colloid.
$C_{24}H_{29}O_{20}(NO_2)_{11}$	Endeca-	214	13.47	Insoluble in ether-alcohol. Soluble in acetone. Guncotton.

It will be observed that the general formula for nitrocellulose is $C_{24}H_{40-n}O_{20}(NO_2)_n$.

The last four nitrocelluloses of the table are used in the manufacture of gunpowders.

67. Conditions that Influence Explosions.—The character of the explosion produced by any explosive is influenced by the physical condition of the explosive, by the external conditions, and by the nature of the exciting cause.

Physical Condition.—The influence of the physical condition of the explosive is seen in the sputtering of damp black powder when ignited, and in the insensitiveness to explosion of nitroglycerine when frozen.

External Conditions.—External conditions influence the explosion chiefly by the amount of confinement they impose. Confinement is necessary to obtain the full practical effect of all explosives. The more rapid the reaction the less the degree of confinement required. Thus blocks of iron may be broken by the explosion of nitroglycerine upon their surfaces in the open air. In this case the air imposes sufficient confinement, as the explosion is so quick that its effect on the iron is produced before the air has time to move.

Gunpowder, on the other hand, requires strong confinement if its complete explosion is desired. Thus, in firing a large charge of gunpowder under water, unless the case is strong enough to retain the gases until the reaction is complete the case will be broken by the pressure of the gases first given off, and a portion of the charge will be thrown out unburned. Large powder grains are frequently thrown out of the gun not wholly burned.

The confinement required by the slower explosives may be diminished by igniting the charge at many points, so that less time is required for the complete explosion.

Exciting Cause.—Heat is the immediate cause of all explosions, whether communicated to the explosive directly by a flame or heated wire, or indirectly through friction, or percussion, or chemical action. Each explosive has a specific temperature of explosion, to which one or more of the molecules must be raised before the explosion can begin. The heating of the initial molecule to the exploding point is not of itself sufficient to cause explosion of the entire mass, but this temperature must be transmitted from molecule to molecule throughout the mass.

The method of producing the explosion of the initial molecule has with many explosives an important influence on the character of the explosion. Nitroglycerine when ignited in small quantities burns quietly, but when struck it explodes violently. Similarly, guncotton when ignited by a flame burns progressively and the combustion may be extinguished by water, but when detonated by an explosive cap the explosion is of the first order. Most of the high explosives produce either detonations or explosions of lower order, depending upon the manner in which the explosion is initiated, and it is stated by Roux and Sarrau that even black gunpowder may be detonated by the use of nitroglycerine as an exploding charge.

Flame is sufficient to cause the complete explosion of the progressive explosives, though it may be necessary with some explosives that the flame be continuously applied. For some of the high explosives a percussive shock suffices to induce an explosion of the first order, while other high explosives are practically insensitive to shock and require for their explosion an initial explosion of some detonating substance.

68. Uses of Different Explosives.—It is apparent from what has been said concerning the differences in rapidity of reaction of the various explosives and the influences of external conditions that each class of explosives has its particular field of usefulness.

Thus the progressive explosives are more suitable for use in guns where a propelling rather than a shattering effect is desired from the explosion. A high explosive acts so quickly that if used in a gun its explosion would be completed practically before the projectile moved, with the result that the whole of its enormous force would be exerted upon the walls of the gun to produce rupture.

For the movement of masses of earth the slow explosive is better than the more rapid one, for here also a propelling rather than a shattering effect is desired.

In submarine mines the best results are obtained from dynamite No. 1, a dynamite consisting of 75 parts by weight of nitroglycerine absorbed into the pores of 25 parts of the siliceous earth called kieselguhr. The effect of the inert substance is to retard the explosion of the nitroglycerine, and the retarded explosion is of greater effect in a yielding substance like water than the more rapid explosion of pure nitroglycerine.

In hard rocks and metals the quickest explosive will give the best results, as in these hard substances the greatest intensity of blow is required to produce the shattering effect desired. Dynamite is ordinarily used for blasting purposes on account of its convenient form, its comparative safety in handling, and its ease of ignition.

Bursting Charges in Projectiles.—The explosives used as bursting charges in armor piercing projectiles must have a great shattering effect in order to break the projectile into fragments and to project the fragments with force; and at the same time the explosive must be practically insensitive to shock, so that it will not be exploded by the shock of discharge in the gun or the shock of impact on the ship's armor. The explosion of the bursting charge of an armor piercing projectile is effected by a detonating fuse so arranged as to cause the projectile to burst after it has perforated the armor.

The explosives used by the various foreign nations as bursting charges in projectiles are all composed principally of picric acid or its derivatives. The French melinite, the English lyddite, the Japanese shimose powder are examples.

Some of the picrates, as the picrates of lead, calcium, mercury, and others, are more sensitive to friction and percussion than picric acid itself. In order to prevent the formation by chemical action of any of these sensitive compounds when the bursting charge is composed of picric acid or of any of its derivatives, the walls of the projectile and all metal parts that come in contact with the bursting charge are covered with a protecting coat of rubber paint.

The walls of the cavity of the shell, the base plug, and the body of the fuse are so painted; also the screw threads of the base plug and fuse. Red or white lead or other metal lubricant must not be used on the screw threads.

69. Requirements for High Explosives for Projectiles.—The following requirements are considered essential for a high explosive to be used in filling shell. They have been found necessary as a result of a long series of tests.

SAFETY AND INSENSITIVENESS.—The explosive should be reasonably safe in manufacture and free from very injurious effects upon the operatives.

It must show a relatively safe degree of insensitiveness in an impact testing apparatus.

It must withstand the maximum shock of discharge under repeated firings in the shells for which it is intended.

It must withstand the shock of impact when fired in unfused shell, as follows:

(*a*) *Field Shell.*—With maximum velocity, against 3 feet of oak timber backed by sand. With the remaining velocity for full charge at 1000 yards range, against a seasoned brick wall.

(*b*) *Siege Shell.*—Against seasoned concrete thicker than the shell will perforate with remaining velocity for full charge at 500 yards range.

(*c*) *Armor Piercing Shell.*—Against a hard faced plate of thickness equal to the caliber of the projectile.

DETONATION AND STRENGTH.—It must be uniformly and completely detonated with the service detonating fuse.

It should possess the greatest strength compatible with a satisfactory fragmentation of the projectile. The average fragment of a projectile should be effective against the vulnerable material of a ship, such as the mechanisms of guns, gun mounts, engines, boilers, electric installations, and the like. With very quick and powerful explosives, as explosive gelatin and picric acid, the shattering effect is excessive and the fragments of the projectile are too small.

STABILITY.—It must not decompose when hermetically sealed and subjected to a temperature of 120° F. for one week.

It should preferably be non-hygroscopic, and its facility for detonation must not be affected by moisture that can be absorbed under ordinary atmospheric exposure necessary in handling.

It must not deteriorate or undergo chemical change in storage.

GENERAL CONDITIONS.—Loading must not be attended with unusual danger and should not require exceptional skill or tedious methods.

The explosive should be obtainable quickly in large quantities and at reasonable cost.

REMARKS.—The explosives used as shell fillers are more stable under severe heat treatment than the service smokeless powders. The explosives should therefore be correspondingly safer to store in large quantities.

Explosive D, used in our service, invented by Major Beverly W. Dunn, Ordnance Department, is safer to handle than black powder.

70. Exploders.—Fulminate of mercury is one of the most violent explosives. By reason of its sensitiveness to explosion by heat or percussion, and the intensity of the shock obtained by its explosion in small quantities, the fulminate of mercury is the most suitable substance for use in initiating detonations or explosions in other explosives.

It forms the principal or the only ingredient of the detonating composition in explosive caps, primers, and fuses. Other ingredients may be potassium chlorate or nitrate, or bisulphide of antimony, the proportions differing in order to produce the best results from the particular explosive with which the exploder is to be used.

DETONATORS.—A commercial detonating cap or fuse is shown in the accompanying figure. The fulminate of mercury, or detonating composition, *B,* is enclosed in a copper case closed with a plug of sulphur through which pass the bared ends of the electric wires. A platinum bridge connects the ends of the wires, and the heating of the bridge by the electric current fires the detonator.

In order to secure the best results it is necessary that the detonator be in intimate contact with the explosive. It is therefore usually placed in the midst of the mass, and the explosive is packed closely around it.

PRIMERS FOR GUNPOWDERS.—For the ignition of charges of gunpowder a large body of flame is of more advantage than an intense shock. Consequently in small-arm primers mercury fulminate has been replaced by a less violent composition of chlorate of potash and bisulphide of antimony, which produces a larger body of flame and is at the same time less sensitive to percussion and therefore safer for use in a small-arm cartridge. In primers for cannon the large body of flame is produced by the use of black powder for the priming charge in the primer, the ignition of the black powder being effected by the explosion of a small percussion cap or by the electric ignition of a small quantity of loose guncotton.

Explosion by Influence.—The detonation of a mass of explosive may under certain circumstances induce the explosion of another mass of the same or of a different explosive not in contact with the first. The induced explosion is called an explosion by influence or a *sympathetic explosion.*

The ability of one explosive to induce the sympathetic explosion of another not in contact with it appears to depend on the character of the shock communicated by the first explosive. Abel found that while the detonation of guncotton would cause the sympathetic detonation of nitroglycerine in close proximity to it, the detonation of nitroglycerine would not cause the detonation of guncotton, although nitroglycerine is more powerful than guncotton.

In explanation of this difference in action Abel advanced the theory of *synchronous vibrations.* It is a well established fact that certain vibrations will induce the decomposition of chemical compounds whose atoms are in a state of unstable equilibrium; and according to Abel sympathetic explosion is produced when the first explosive sets up in the connecting medium vibrations that are synchronous with those that would result from the explosion of the second explosive.

This theory is questioned by later investigators, and it is now generally held that sympathetic explosion is due to the transmission of a shock of sufficient intensity.

THEORETICAL DETERMINATIONS OF THE RESULTS FROM EXPLOSIONS.

71. In the theoretical determinations of the results from explosions metric units and the centigrade thermometric scale are usually employed.

Definitions. CALORIE.—*A small calorie* is the quantity of heat required to raise the temperature of 1 gram of water (1 cubic centimeter) from 0 degrees to 1 degree centigrade.

A large calorie is the quantity of heat required to raise the temperature of 1 kilogram of water (1 liter, 1 cubic decimeter) from 0 degrees to 1 degree. A large calorie is equal to 1000 small calories.

EXOTHERMIC AND ENDOTHERMIC REACTIONS.—An exothermic reaction gives off heat, an endothermic reaction absorbs heat.

MOLUGRAM.—The term molugram is used to designate a weight of as many grams as there are units in the molecular weight of the substance. Thus, the molugram of hydrogen, H_2, is 2 grams. Water or water vapor, H_2O, has a molecular weight of 18. The molugram of water is therefore 18 grams. The molugram of nitroglycerine, $C_3H_5(NO_2)_3O_3$, is 227 grams.

The molugram of a mixture has a weight in grams equal to the sum of the molecular weights of as many molecules of each constituent as appear in the formula for the mixture. Thus, the molugram of $10KNO_3+3S+C$ is 1119 grams.

Specific Heats of Gases.—The specific heat of a gas at constant pressure is the number of calories required to heat 1 gram of the gas from 0° to 1° while the gas is permitted to expand under the constant pressure.

The specific heat of a gas at constant volume is the number of calories required to heat 1 gram of the gas from 0° to 1°, the volume of the gas remaining unchanged.

When large calories are used the unit weight of gas is 1 kilogram.

MOLECULAR HEAT.—The molecular specific heat of a gas, or more simply the molecular heat, is the number of calories required to heat a molugram of the gas from 0° to 1°.

The molecular heat is obtained by multiplying the specific heat of the gas by its molecular weight. The molecular heat may be under constant pressure or under constant volume, depending upon whether the specific heat used as a multiplier is the specific heat at constant pressure or at constant volume.

Thus, carbon dioxide, CO_2; molecular weight, 44.

At constant pressure, specific heat, 0.2169; molecular heat, $0.2169 \times 44 = 9.5436$.

At constant volume, specific heat, 0.172; molecular heat, $0.172 \times 44 = 7.568$.

72. Specific Volumes of Gases.—The specific volume of a gas is the volume in cubic decimeters (liters) of 1 gram of the gas at 0° temperature and under atmospheric pressure (barometer, 760 millimeters; pressure, 103.33 kilograms per square decimeter).

MOLECULAR VOLUME.—*The molecular volume* is the volume, at 0° and 760 mm. pressure, of a molugram of the gas. It is obtained by multiplying the specific volume by the molecular weight.

Thus, CO_2, specific volume, 0.5073, molecular volume, $44 \times 0.5073 = 22.32$ cubic decimeters or liters.

The molecular volumes of all gases are the same, 22.32 liters, as will be shown.

LAW OF AVOGADRO.—*All gases under the same conditions of pressure and temperature have the same number of molecules in equal volumes.*

It follows from this law that the single molecules of all gases, whether simple or compound, occupy equal volumes under the same conditions of pressure and temperature.

The volume of the hydrogen atom is taken as the unit volume. The molecule of hydrogen and the molecules of the other simple gases as well are composed of two atoms. A molecule of any gas therefore occupies 2 unit volumes.

In the following reaction the number of volumes appears under each of the symbols

$$\underset{\text{1 vol}}{N} + \underset{\text{3 vols}}{H_3} = \underset{\text{2 vols.}}{NH_3}$$

That is, 1 volume of N combining with 3 volumes of H forms 2 volumes of ammonia, NH_3. The volumes may be expressed in any unit, as liters or cubic feet.

The atomic weight of nitrogen is 14 and of hydrogen 1. There are therefore in the molecule of NH_3 17 parts by weight occupying the same volume as 2 parts of hydrogen alone. The specific volume of NH_3, the volume of unit weight, is therefore 1/17 of the molecular volume of hydrogen, and the molecular volume of NH_3, which is the specific volume multiplied by the molecular weight, 17 in this case, is the molecular volume of hydrogen.

As the same is true for any other gaseous compound, it follows that the product of the specific volume of a gas by its molecular weight is a constant and is equal to the molecular volume of hydrogen.

The molecular volume of all gases is 22.32 liters.

By means of the molecular volume we may determine the volume of any weight of gas, or the weight of any volume, since we know that a mologram of any gas occupies 22.32 liters.

The specific volume, the number of liters occupied by 1 gram, is equal to 22.32 divided by the molecular weight.

The specific weight, the number of grams occupying one liter, is the reciprocal of the specific volume, or the molecular weight divided by 22.32.

Classification of Gases.—Compound gases such as CO_2, NH_3, C_2H_4, whose molecules contain more than two atoms, are called *gases with condensation,* as in their formation more than two atoms are condensed into the volume of two simple atoms. Compound gases such as CO, HCl, whose molecules contain two atoms, are called *gases without condensation.* Oxygen, hydrogen, and nitrogen are simple or perfect gases.

In the following determinations of the effects of explosion we will follow the methods described by Leon Gody in his work entitled *Matières Explosives.*

73. Quantity of Heat.—The heat given off in explosions can be measured experimentally by means of special calorimeters. Roux and Sarrau made use of a very strong cylindrical bomb, similar to the apparatus of Noble and Abel, illustrated on page 67. The bomb, charged with a few grams of explosive, was immersed in a known volume of water. After the explosion of the charge, effected electrically, the increased temperature of the body of water was noted and the quantity of heat necessary to produce the rise in temperature calculated.

The theoretical determination of the quantity of heat resulting from an explosion involves the application of certain principles of thermochemistry established by Berthelot.

Principle of the Initial and Final State.—The heat liberated (or absorbed) in any modification of a system of simple or compound bodies, effected under constant pressure or at constant volume and without any external mechanical effect, depends solely on the initial and final states of the system. It is completely independent of the series of intermediate transformations.

From this principle it follows that the heat liberated in any transformation accomplished through successive reactions is the algebraic sum of the heats liberated in the different reactions.

We may consider the formation of an explosive as an intermediate reaction in the formation of the products of explosion from simple elements. If we then subtract from the total heat of formation of the products of explosion the heat of formation of the explosive, the difference will be the heat liberated in the reaction of explosion.

Principle of Maximum Heat.—All chemical changes effected without the intervention of external energy tend toward the formation of the body or the system of bodies that liberates the most heat.

The quantity of heat liberated or absorbed in a reaction is independent of the time occupied in the reaction.

74. Heats of Formation.—The heats of formation *at constant pressure* of the principal explosives and of the gases resulting from explosion are given in Table II at the end of the volume. The heats are given in large calories for the molugram of each substance. Thus hydrochloric acid gives off in its formation 22 large calories; that is, 1 gram of hydrogen and 35.5 grams of chlorine in combining give off sufficient heat to raise the temperature of 22 kilograms of water from 0° to 1°. The heat of formation of 36.5 grams of HCl is therefore 22 large calories.

The heats of formation of endothermic bodies are preceded by the minus sign in the table.

The atomic and molecular weights in Tables II, III, and IV are those that were in use at the time these tables were formed. Atomic weights according to the latest determinations are given in Table V. In the examples which follow, involving the use of Tables II, III, and IV, the atomic and molecular weights as given in those tables are used.

Quantity of Heat at Constant Pressure.—In order to determine the quantity of heat given off in any chemical change the chemical reaction must be known. The composition of explosives is generally known and the products of explosion can be predicted, under the principle of maximum heat, when the body undergoes complete combustion; that is, when it contains sufficient oxygen to form stable compounds of the maximum oxidation.

The sum of the heats of formation of the products of explosion that appear in the formula for the reaction, minus the heat of formation of the explosive, is the quantity of heat liberated by the explosion.

Example 1.—As an example we will find the heat given off in the explosion of nitroglycerine under constant pressure, as in the open air.

The equation of the reaction is as follows:

$$\underset{454}{2C_3H_5(NO_2)_3O_3} = \underset{264}{6CO_2} + \underset{90}{5H_2O} + \underset{84}{3N_2} + \underset{16}{\tfrac{1}{2}O_2}$$

With the heats of formation from Table II for the molugram of each substance we obtain, for the numbers of molecules in the reaction,

$2C_3H_5(NO_2)_3O_3$,	$2\times 98 \quad = 196$,
$6CO_2$,	$6\times 94.3 = 565.8$,
$5H_2O$,	$5\times 58.2 = 291$.

The nitrogen and oxygen being simple elements add no heat.

We therefore have for the heat given off by the explosion under constant pressure of 2×227 grams of nitroglycerine

$$(565.8+291)-196=660.8 \text{ l. cal.,*}$$

* In other works the abbreviation used to designate a large calorie is *cal. k. d.* (kilogram-degree), and for a small calorie, *cal. g. d.* (gram-degree). The abbreviations *l. cal.* and *s. cal.* are used here, as they more plainly indicate the words abbreviated.

and for the heat given off by 227 grams of the explosive, a molu-gram,

$$Q_{mp}=660.8/2=330.4 \text{ l. cal.}$$

For the heat given off by a kilogram of the explosive,

$$Q_{kp}=\frac{330.4\times1000}{227}=1455.5 \text{ l. cal.}$$

75. **When Solid Products are Formed.**—If the explosion produces solid products the heats of formation of these bodies are added to the heats of formation of the gases in the determination of Q_{mp} and Q_{kp}.

Example 2.—A mixture of nitrobenzol with sufficient potassium chlorate to make the combustion of the nitrobenzol complete is exploded.

The reaction is

$$\underset{1266.8}{2C_6H_5NO_2+\tfrac{25}{3}KClO_3}=\underset{528}{12CO_2}+\underset{90}{5H_2O}+\underset{28}{N_2}+\underset{620.8}{\tfrac{25}{3}KCl}$$

A molugram of a mixture is the sum of the molecular weights in grams of as many molecules of each of the constituents as appear in the reaction. The molugram of this explosive mixture is therefore $2\times123+\frac{25}{3}\times122.5=1266.8$ grams.

Heats of formation:

$12CO_2$,	$12\times\ 94.3=$	1131.6
$5H_2O$,	$5\times\ 58.2=$	291
$\frac{25}{3}KCl$,	$\frac{25}{3}\times105\ =$	875
		2297.6
$2C_6H_5NO_2$,	$2\times\ 4.2=$	8.4
$\frac{25}{3}KClO_3$,	$\frac{25}{3}\times\ 94.6=$	788.3
		796.7

$$Q_{mp}=2297.6-796.7=1500.9 \text{ l. cal.}$$

$$Q_{kp}=\frac{1500.9\times1000}{1266.8}=1184.8 \text{ l. cal.}$$

Incomplete Combustion.—When an explosive does not contain sufficient oxygen for complete combustion the products formed vary with the temperature, the pressure, and the density of loading. Therefore no fixed formula can be written for the reaction. The products of combustion of these explosives are determined by analysis, and the heat given off may then be determined as above.

The explosion of guncotton under atmospheric pressure gives the following reaction.

$$C_{24}H_{29}O_{20}(NO_2)_{11}=15CO+9CO_2+9H_2O+5.5H_2+5.5N_2$$

Under high pressure the reaction is as follows.

$$C_{24}H_{29}O_{20}(NO_2)_{11}=12CO+12CO_2+6H_2O+8.5H_2+5.5N_2$$

76. Quantity of Heat at Constant Volume.—If the decomposition takes place at constant volume, for instance in a closed vessel, the heat developed is greater than in the open air under constant pressure. The gases developed in the open air perform the work of driving back the air, and this work absorbs some of the heat.

Let Q_{mp} be the heat given off by the molugram of the substance in the reaction at *constant pressure* at the surround-temperature t,

Q_{mv} the heat given off by the molugram of the substance in the reaction at *constant volume* at the surrounding temperature t,

W the work of expansion at constant pressure,

E the mechanical equivalent of heat, 425 kilogrammeters.

Then W/E is the heat expended in performing the work of driving back the air, and

$$Q_{mv}=Q_{mp}+W/E \tag{1}$$

But the work W due to the pressure of the gas against the constant pressure p is, as shown by equation (40), page 65,

$$W=\int_{v_b}^{v_1} pdv=p\int_{v_b}^{v_1} dv$$

v_b and v_1 representing the volumes of the gas before and after expansion.

Performing the indicated integration,

$$W = p(v_1 - v_b) \tag{2}$$

Taking the molecular volume at 0° and 760 mm., 22.32 liters, as the unit volume,

Let n_b represent the number of unit volumes before expansion,
n_1 the number of unit volumes after expansion to normal atmospheric conditions.

n_1 *will also represent the number of gaseous molecules,* since after expansion to the normal atmospheric conditions of temperature and pressure each unit volume is occupied by a molugram.

Then from Gay-Lussac's law, page 58, we have at the temperature t

$$v_1 = 22.32 n_1 (1 + \alpha t)$$
$$v_b = 22.32 n_b (1 + \alpha t)$$

Substituting these values in equation (2) we have

$$W = p\, 22.32 (n_1 - n_b)(1 + \alpha t)$$

Whence
$$\frac{W}{E} = \frac{p}{E} 22.32 (n_1 - n_b)(1 + \alpha t) \tag{3}$$

The value 425 for E, the mechanical equivalent of heat, is expressed in kilogrammeters. We must therefore express p, the normal atmospheric pressure in kilograms per square meter, 103.3×100, and the volume 22.32 liters (cubic decimeters) in cubic meters, 22.32/1000.

Equation (3) then becomes

$$\frac{W}{E} = \frac{10330 \times 22.32}{425 \times 1000} (n_1 - n_b)(1 + \alpha t)$$

or
$$W/E = 0.5424 (n_1 - n_b)(1 + \alpha t)$$

$$\alpha = 1/273 \quad \text{and} \quad 1/273 \times 0.5424 = 0.002, \text{ nearly}$$

Therefore
$$W/E = 0.5424 (n_1 - n_b) + 0.002 (n_1 - n_b) t \tag{4}$$

In the case of explosives the volume v_b is generally negligible with respect to v_1. v_b represents the volume of the explosive for those explosives that are completely converted into gas. n_b is therefore negligible with respect to n_1, and equation (4) becomes

$$W/E = 0.5424 n_1 + 0.002 n_1 t$$

Substituting this value in equation (1)

$$Q_{mv} = Q_{mp} + 0.5424 n_1 + 0.002 n_1 t$$

We will make $t = 15°$, since the heats of formation in Table II have been determined for that temperature, and Q_{mp} and Q_{mv} in the above equation will be determined from the table. We have, then, finally,

$$Q_{mv} = Q_{mp} + 0.5724 n_1 \qquad (5)$$

for the quantity of heat given off at constant volume by the molugram of the explosive.

77. Example 3.—Take, for example, nitroglycerine,

$$\underset{454}{2C_3H_5(NO_2)_3O_3} = \underset{264}{6CO_2} + \underset{90}{5H_2O} + \underset{84}{3N_2} + \underset{16}{\tfrac{1}{2}O_2}$$

We have found at constant pressure, example 1,

$$Q_{mp} = 330.4 \text{ l. cal.}$$

From the reaction we see that 2 molugrams of the explosive give off $6+5+3+0.5=14.5$ molecular volumes of gas. 1 molugram, therefore, gives

$$n_1 = 7.25 \text{ volumes}$$

Substituting in equation (5) we obtain

$$Q_{mv} = 330.4 + 0.5724 \times 7.25 = 334.5 \text{ l. cal.}$$

For 1 kilogram of the explosive, example 1,

$$Q_{kv} = \frac{334.5}{227} \times 1000 = 1473.6 \text{ l. cal.}$$

We found at constant pressure

$$Q_{kp} = 1455.5 \text{ l. cal.}$$

Potential.—The potential has been defined as the total work that can be performed by the gas from unit weight of the explosive under indefinite adiabatic expansion. The kilogram is taken as the unit weight in the determination of the potential, and the meter as the unit of length. The work unit is therefore the kilogrammeter. The total work from one kilogram of the explosive is equal to the maximum quantity of heat given off by one kilogram multiplied by the mechanical equivalent of heat.

The mechanical equivalent of heat is 425 kilogrammeters. Therefore representing the potential, the total work from a kilogram of the explosive, by W_k we have

$$W_k = Q_{kv} \times 425 \text{ kilogrammeters} \qquad (6)$$

78. Volume of Gases.—The volume of gases produced by explosion may be measured experimentally, the gases being drawn off from the calorimetric bomb for this purpose.

The volume of the gases may also be determined theoretically from the reaction.

As previously explained, the molecular volume (the volume of the molugram) of any gas, simple or compound, is 22.32 liters. Therefore in any reaction the molecular volume, at standard temperature and pressure, of the evolved gases is very simply obtained by multiplying the number of gaseous molecules in the formula for the reaction by 22.32.

Example 4.—A formula for the explosion of black gunpowder is

$$\underset{1010}{10KNO_3} + \underset{96}{3S} + \underset{96}{8C} = \underset{522}{3K_2SO_4} + \underset{276}{2K_2CO_3} + \underset{264}{6CO_2} + \underset{140}{5N_2}$$

The first two products of the reaction are solid. The gaseous products are 6 molecules of CO_2 and 5 of N. Therefore the molecular volume of the gases from 1202 grams of the explosive is, at 0° and 760 mm.,

$$V_m = 11 \times 22.32 = 245.52 \text{ liters}$$

and from 1 kilogram of explosive

$$V_k = \frac{245.52 \times 1000}{1202} = 204.26 \text{ liters}$$

The volumes at any other pressure or temperature may be obtained by means of equations (31) and (34), Chapter III.

79. Temperature of Explosion.—The method of Mallard and Le Chatelier for calculating the temperature of explosion at constant volume in a closed vessel is as follows.

The quantity of heat liberated by the explosion of the molugram of the explosive would, if the specific heat of the products were constant, be equal to the molecular specific heat multiplied by the *rise in temperature.* We would then have

$$Q_{mv} = C_{mv} \times t_1 \tag{7}$$

from which t_1, the rise in temperature, could be obtained. Assuming 15°, an ordinary temperature, as the temperature of the explosive when fired, the temperature of explosion would then be

$$t = t_1 + 15 \tag{8}$$

But it is known that the specific heat increases with the temperature. Assuming that the specific heat varies with the temperature in the manner represented by the linear expression,

$$C_{mv} = a + bt_1 \tag{9}$$

the values of a and b, and the consequent values of C_{mv}, have been deduced for certain gases as follows. The values are given in *small calories.*

	a	b	
For CO_2, SO_2,	6.26	0.0037	$C_{mv} = 6.26 + 0.0037\, t_1$
For H_2O,	5.61	0.0033	$C_{mv} = 5.61 + 0.0033\, t_1$
For gases without condensation,	4.80	0.0006	$C_{mv} = 4.80 + 0.0006\, t_1$

The values of a are the molecular heats of the gases in small calories at the temperature 15°, and the values of b are the increments of the molecular heats for each degree of rise in temperature.

Suppose that the products of an explosion are as follows:

$$\alpha\, CO_2 + \beta\, H_2O + \delta\, P$$

P representing a molecule of a perfect gas. The coefficients a and b for the products of explosion will then be

$$a = 6.26\,\alpha + 5.61\,\beta + 4.8\,\delta \tag{10}$$

$$b = 0.0037\,\alpha + 0.0033\,\beta + 0.0006\,\delta \tag{11}$$

Combining equations (7) and (9) and multiplying Q_{mv} by 1000, since it has been determined in large calories, and a and b are now in small calories, we obtain

$$1000Q_{mv} = at_1 + bt_1^2$$

Solving this equation for t_1 and substituting the resulting value in equation (8), we obtain, for the temperature of explosion

$$t = \frac{-a + \sqrt{a^2 + 4000bQ_{mv}}}{2b} + 15 \tag{12}$$

80. Example 5.—Nitroglycerine. $Q_{mv} = 334.5$ l. cal. (see example 3).

$$\underset{454}{2C_3H_5(NO_2)_3O_3} = \underset{264}{6CO_2} + \underset{90}{5H_2O} + \underset{84}{3N_2} + \underset{16}{0.5O_2}$$

Since the products, as given in the formula, are from two molecules of the explosive,

$$2a = 6.26 \times 6 + 5.61 \times 5 + 4.8 \times 3 + 4.8 \times 0.5 = 82.41$$
$$2b = 0.0037 \times 6 + 0.0033 \times 5 + 0.0006 \times (3 + 0.5) = 0.0408$$
$$a = 41.205 \qquad\qquad b = 0.0204$$

and from equation (12)

$$t = \frac{-41.205 + \sqrt{41.205^2 + 4000 \times 0.0204 \times 334.5}}{2 \times 0.0204} + 15 = 3178°$$

81. Temperature when Solid Products are Formed.—If the explosion gives rise to solid products the heat absorbed in raising the temperature of these products must be considered. In equation (7) C_{mv} must be the mean specific heat of the products of the explosion of a molugram of the explosive.

Suppose that in addition to the gaseous products assumed above, page 155, we have x molugrams of a solid product having a specific heat h referred to its molecular weight. Then a, equation (10), becomes

$$a = 6.26\alpha + 5.61\beta + 4.8\delta + hx \tag{13}$$

The specific heat of a solid product is assumed not to vary with the temperature, therefore the value of b as given by equation (11) is not affected.

The specific heats of substances will be found in Table III at the end of the volume.

Example 6.—Determine the temperature of explosion of the mixture of nitrobenzol and potassium chlorate of example 2.

The reaction is

$$\underset{1266.8}{2C_6H_5NO_2 + \tfrac{25}{3}KClO_3} = \underset{528}{12CO_2} + \underset{90}{5H_2O} + \underset{28}{N_2} + \underset{620.8}{\tfrac{25}{3}KCl}$$

From example 2, equation (5), page 152,

$$Q_{mp} = 1500.9 \text{ l. cal.}$$
$$Q_{mv} = Q_{mp} + 0.5724 n_1$$
$$n_1 = 12 + 5 + 1 = 18$$
$$Q_{mv} = 1511.2$$

From Table III, molecular specific heat of KCl, 12.89

eq. (13), $a = 6.26 \times 12 + 5.61 \times 5 + 4.8 + 12.89 \times 25/3 = 215.39$

eq. (11), $b = 0.0037 \times 12 + 0.0033 \times 5 + 0.0006 = 0.0615$

eq. (12), $$t = \frac{-215.39 + \sqrt{215.39^2 + 4000 \times 0.0615 \times 1511.2}}{2 \times 0.0615} + 15$$

$t = 3521°$.

82. Pressure in a Closed Chamber.—The pressure of the gases produced by explosion is a function of the volume occupied by the gases. In a closed chamber the volume available for the gases depends upon whether the products of explosion are wholly gaseous or whether they contain *non-gaseous* matter as well.

PRODUCTS WHOLLY GASEOUS.—We have deduced in equation (47), Chapter III, the following value for the force of an explosive.

$$f=p_0v_0T/273 \qquad (14)$$

in which, in the metric units that have been chiefly used in the previous calculations, the kilogram and the decimeter,

f is the pressure per square decimeter of the gases from 1 kilogram of explosive, the gases occupying at the temperature of explosion a volume of 1 cubic decimeter.

p_0 the normal atmospheric pressure, 103.3 kilograms per square decimeter,

v_0 the specific volume of the gas, now taken as the volume in cubic decimeters occupied by 1 kilogram of the gas at 0° and 760 mm.,

T the absolute temperature.

The volume V_k, as determined on page 154, is the volume in cubic decimeters, or liters, of the gaseous products from 1 kilogram of the explosive. Therefore

$$v_0=V_k \qquad (15)$$

The absolute temperature $T=273+t$, in which t, the temperature of explosion, is taken as the rise in temperature due to the explosion plus 15°, which is the assumed temperature of the explosive when fired.

Substituting the values of p_0, v_0, and T in equation (14) we obtain for the force of the powder

$$f=\frac{103.3V_k(273+t)}{273} \text{ kilograms per sq. dec.} \qquad (16)$$

RELATION BETWEEN PRESSURE, FORCE OF EXPLOSIVE, AND DENSITY OF LOADING.—We have, equation (49), Chapter III, for the pressure from unit weight of gas confined in the volume v,

$$p=\frac{f}{v-\alpha} \qquad (17)$$

in which α is the *covolume* of the gas.

By the process followed in Chapter III in deducing equation (46) from equation (45) this equation may be put in the form

$$P=\frac{f\Delta}{1-\alpha\Delta} \tag{18}*$$

in which P is the pressure per unit of surface of the gases from $\bar{\omega}$ units of weight of explosive,
Δ is the density of loading.

According to Sarrau the covolume is 1/1000 of the specific volume of the gases. Therefore when the products are wholly gaseous we have from equation (15)

$$\alpha=V_k/1000 \tag{19}$$

83. Non-gaseous Products.—When solid or liquid products result from the explosion, these products occupy part of the volume in the chamber and diminish the volume occupied by the gases.

Let y be the weight of gas from unit weight of explosive,
w_0 the volume at 0° and 760 mm., occupied by the gas from unit weight of explosive,
α' the volume, at temperature and pressure of explosion, of the non-gaseous residue from unit weight of explosive.

In this case if we consider as the specific volume of the gas the volume w_0 occupied by the gas from unit weight of the explosive instead of the volume v_0 occupied by unit weight of the gas, f, equation (14), becomes for the new specific volume

$$f=p_0w_0T/273 \tag{20}$$

And if we consider that α, the subtractive term in equation (14), includes the volume of the residue from unit weight of explosive as well as the covolume of the gases for the new specific volume,

$$\alpha=\alpha'+w_0/1000 \tag{21}$$

* This equation is identical with equation (46), Chapter III, deduced by Noble and Abel. They considered α as the volume of the solid residue from unit weight of powder, but later investigations show, as explained in Chapter III, that the covolume of the gases must appear in the equation. When solid products result the value of α must be modified to include the volume occupied by the solid products.

By definition we have

$$w_0 = v_0 y \tag{22}$$

With these new values of f and α equation (17) gives the pressure due to the gases from unit weight of the explosive, and equation (18) may be deduced from it as before.

Therefore when non-gaseous products result from the explosion the pressure is obtained from equation (18) by substituting for f and α the values given in equations (20) and (21).

The volume of the solid matter is easy to calculate, as from the formula of the decomposition we may obtain the weight of the residue from 1 kilogram of the explosive, and it is only necessary to divide this weight by the density.

The densities of substances are given in Table IV at the end of the volume.

84. Example 7.—What is the pressure in a closed chamber of a charge of the explosive of example 6, the density of loading being 0.9?

The reaction is

$$\underset{1266.8}{2C_6H_5NO_2 + \tfrac{25}{3}KClO_3} = \underset{528}{12CO_2} + \underset{90}{5H_2O} + \underset{28}{N_2} + \underset{620.8}{\tfrac{25}{3}KCl}$$

From example 6, $Q_{mp} = 1500.9$

$Q_{mv} = 1511.2$

$t = 3521$

$T = 273 + t = 3794$

Following example 4,

$$V_k = 18 \times 22.32 \times 1000/1266.8 = \underline{317.15 = v_0}, \text{ equation (15)}$$

	KCl.	Gas.
1266.8 kilos explosive produce, kilos	620.8	646
1 kilo explosive produces, kilos	0.49	$\underline{0.51 = y}$
Divide by density KCl, 1.94, Table IV	$\underline{0.2526 = \alpha'}$	

Eq. (22), $w_0 = 317.15 \times 0.51 = 161.75$

Eq. (21), $\alpha = 0.2526 + 0.1617 = 0.4143$

Eq. (20), $f=103.3\times161.75\times3794/273=232210$ kilos per sq. dec.

Eq. (18), $P=\frac{232210\times0.9}{1-(0.4143\times0.9)}=333240$ kilos per sq. dec.

For $\Delta=1$, $P=396460$ kilos per sq. dec.

Specific Heats and Densities of Non-gaseous Products.—It is assumed in the above discussion that the specific heats and densities of the non-gaseous products remain constant. This assumption is generally inaccurate, and the calculated values of force and pressure for explosives that yield non-gaseous products are therefore uncertain. For these explosives the most satisfactory determinations are made by experiment. Two or more charges of the explosive are exploded in a closed chamber and the values of P measured. Substituting these with the corresponding known values of Δ in equation (18) the values of f and α are determined.

85 Complete Calculation of the Effects of Explosion.—The formula of the reaction for the complete combustion of Sprengel's explosive acid, a mixture of picric acid and nitric acid, is as follows.

$$\underset{1145}{5C_6H_2(NO_2)_3OH}+\underset{819}{13HNO_3}=\underset{1320}{30CO_2}+\underset{252}{14H_2O}+\underset{392}{14N_2}$$

The molecular weight is $1145+819=1964$.

In the work that follows, the number of the page on which the process is explained, or the number of the equation from which the value is derived, appears on the left.

146, $Q_{mp}=(30\times94.3+14\times58.2)-(5\times49.1+13\times41.6)$
$=2857.5$ l. cal.

150, $Q_{kp}=2857.5\times\frac{1000}{1964}=1454.9$ l. cal.

(5) $Q_{mv}=2857.5+0.5724(30+14+14)=2890.7$ l. cal.

153, $Q_{kv}=2890.7\times\frac{1000}{1964}=1471.8$ l. cal.

(6) $W_k=1471.8\times425=625515$ kgm.

154, $V_m = (30+14+14)22.32 = 1294.56$ liters

154, $V_k = 1294.56 \times \frac{1000}{1964} = 659.14$ liters

(10) $a = 6.26 \times 30 + 5.61 \times 14 + 4.8 \times 14 = 333.54$

(11) $b = 0.0037 \times 30 + 0.0033 \times 14 + 0.0006 \times 14 = 0.1656$

(12) $t = \frac{-333.54 + \sqrt{333.54^2 + 4000 \times 0.1656 \times 2890.7}}{2 \times 0.1656} + 15$

$= 3306°$

(16) $f = \frac{103.3 \times 659.14(273+3306)}{273} = 892650$ kgm. per sq. dec.

(19) $\alpha = \frac{659.14}{1000} = 0.65914$

(18) $P = \frac{892650\Delta}{1-0.65914\Delta}$ kilograms per sq. dec.

For $\Delta = 0.8$, $P = 1510700$ kilograms per sq. dec.

CHAPTER V.

METALS USED IN ORDNANCE CONSTRUCTION.

86. Stress and Strain.—A proper understanding of these terms will be helpful in what follows.

When a force is applied to a body the effect produced depends upon whether or not the body is free to move. A force applied to a free body produces motion of the body. A force applied to a fixed body produces change of form of the body.

Stress is the name given to any force or part of a force that produces change of form of the body. The component forces or pressures induced in the interior of the body are also called stresses.

Strain is the effect of the force as measured by the change in form of the body to which the stress is applied.

Stresses are of different kinds, depending on the manner of application of the force; as tensile stress, compressive stress, torsional stress. A torsional stress is a compound stress and may be resolved into a tensile stress on some elements of the material and a compressive stress on others.

Each kind of stress produces a corresponding strain, or effect on the material, the tensile stress producing elongation, the compressive stress compression. As all stresses may be resolved into tensile and compressive stresses, all strains may be resolved into elongation and compression.

Physical Qualities of Metals.—The following qualities of metals are those with which we are most concerned in ordnance construction.

Fusibility.—The property of being readily converted into the liquid form by heat.

Malleability.—The property of being permanently extended in all directions without rupture when hammered or rolled.

Ductility.—The property of being permanently extended without rupture by a tensile stress, as in wire-drawing.

Hardness.—The property of resisting change of form under a compressive stress. A hard metal offers great resistance to such a stress, while a soft metal yields readily and changes its form without rupture. The terms hardness and softness are of course comparative only.

Toughness.—The property of resisting fracture under change of form produced by any stress.

Elasticity.—The property of resisting permanent deformation under change of form. This is one of the most important properties of gun metals, which under the high stresses due to the explosion are subjected to extensive deformation. Through this property the deformations disappear on the cessation of the stress and the metal resumes its original dimensions.

Strength of Metals.—The strength of metals is ordinarily determined by physical tests in a testing machine. As the tensile strength of metals is less than the compressive strength, usually a tensile test only is applied. A test specimen is cut from the metal to be tested and is prepared in suitable form to be inserted in the machine. The area of the cross section of the test specimen is usually a square inch or some aliquot part of a square inch.

In the machine the test piece is subjected to a tensile stress, the amount of which is recorded by a sliding weight on a scaled beam. The test piece stretches under the applied stress. With elastic metals it will be found that up to the application of a certain stress the test piece will resume its original length if the stress is removed, but on the application of a stress greater than this the test piece will remain permanently elongated. When permanent distortion occurs the metal is said to have a permanent set.

Elastic Limit.—The stress per square inch applied at the moment that the permanent set occurs is called the elastic limit of the metal. Within this limit the metal has practically perfect elasticity and does not suffer permanent deformation.

As the stress increases beyond the elastic limit the metal stretches permanently and more rapidly, the cross section at the weakest point reduces, and finally the test piece ruptures.

The elastic strength of metals will be found more fully treated in the discussion of the elastic strength of guns in Chapter VI.

87. Tensile Strength.—The stress per square inch that produces rupture of the metal is called the tensile strength.

Elongation at Rupture and Reduction of Area.—In ordnance structures the stresses are not expected to exceed the elastic limit of the metal, but should they by any chance exceed this limit the tensile strength of the metal and its capacity to permanently elongate before rupture become of importance. The permanent elongation will serve as a warning that the elastic strength has been exceeded. The reduction of area of cross section is intimately connected with the elongation. In the tests of metals for ordnance purposes these particulars are therefore always noted and limits are prescribed. For the measurement of the elongation at rupture the parts of the ruptured test piece are brought together and the distance is measured between two punch marks that were made on the test piece before insertion in the testing machine.

The tensile test therefore includes the determination of the elastic limit, the tensile strength, the elongation at rupture, and the reduction of area of cross section. The last two are recorded in percentages of the original dimensions.

The following table shows the physical requirements demanded by the Ordnance Department in the principal metals used in ord-

	Elastic Limit.	Tensile Strength.	Elongation at Rupture.	Contraction of Area.
	lbs. per sq. in.	lbs. per sq. in.	per cent.	per cent.
Copper	......	32,000	22.0	....
Bronze, No. 1	......	28,000	....	....
Bronze, No. 4	......	60,000	20.0	....
Tobin bronze	......	60,000	25.0	....
Cast iron, No. 1	......	22,000	....	....
Cast iron, No. 2	......	* 28,000	....	....
Wrought iron	22,000	50,000	25.0	35.0
Cast steel, No. 1	25,000	60,000	16.0	24.0
Cast steel, No. 3	45,000	85,000	12.0	18.0
Forged steel, No. 1	27,000	60,000	28.0	40.0
Forged steel (caps)	......	† 60,000	30.0	45.0
Forged steel (tubes)	46,000	86,000	17.0	30.0
Forged steel (jackets)	48,000	90,000	16.0	27.0
Forged steel (hoops)	53,000	93,000	14.0	20.0
Forged steel, D	100,000	120,000	14.0	30.0
Nickel steel	65,000	95,000	18.0	30.0
Steel wire (guns)	100,000	160,000	....	....

* Cast iron No. 2 must not show a tensile strength of more than 39,000 pounds per square inch.

† The tensile strength of steel used in caps for armor piercing projectiles must not exceed 60,000 pounds.

nance construction, the requirements varying for each kind of metal according to the use to which it is destined.

Testing Machine.—The standard government testing machine is at Watertown Arsenal, Mass. It has a testing capacity of 800,000 lbs.

A smaller testing machine, with a capacity of 50,000 lbs., is shown in Fig. 26. The specimen of the metal to be tested is turned to the shape shown by the piece marked 1. The ends of the test specimen are grasped by clamps fixed in the upper fixed head, *f*, of the machine and in the lower movable head *m*. Four heavy vertical screws pass through the corners of the movable head, and by their means the movable head is moved toward or from the fixed head, exerting on the specimen held between the clamps a force of compression or of extension as desired. The amount of this force is measured by a sliding weight, *w*, on a scaled beam in the same manner as a weight is determined on an ordinary scale. The total force divided by the area of cross section of the test specimen gives the force exerted per square inch.

A graphic representation of the relation between the force exerted and the change in length of the test specimen is made on the indicator card, *c*. An indicator card, showing the results of tensile tests on specimens of several metals, is shown in Fig. 25. Within the elastic limit of the metal the elongation of the test piece is proportional to the tensile stress. Up to this point, therefore, the line made by the indicator will be a straight line. At the elastic limit, where the bends occur in Fig. 25, permanent set occurs, and the test piece thereafter elongates more rapidly than the stress increases.

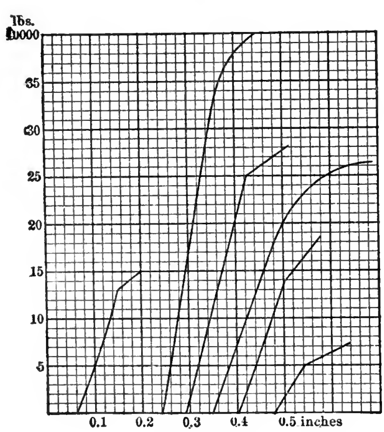


Fig. 25.

To prevent injury to the indicating apparatus by the shock that occurs when the test piece breaks, the indicator is usually disconnected after the elastic limit has been registered.

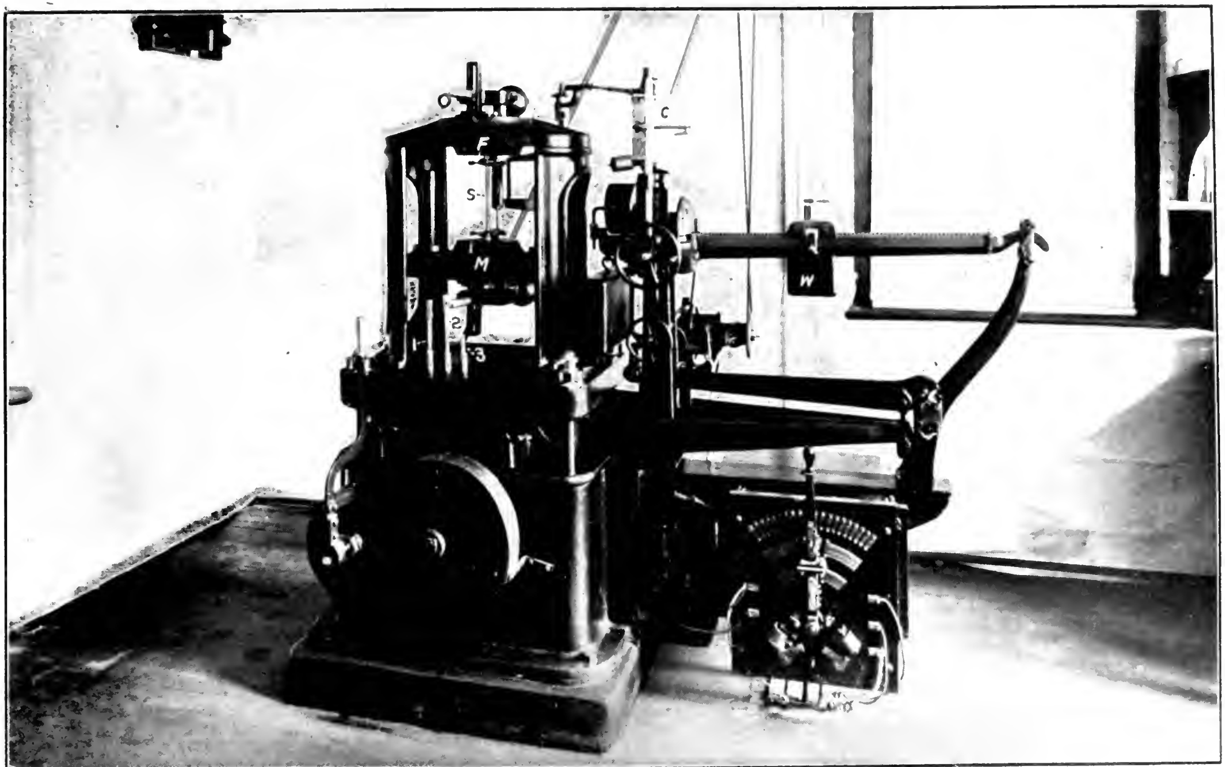


Fig. 26.—Riehlé Testing Machine. Capacity, 50,000 lbs.

Broken test-pieces are shown at 2 and 3 in Fig. 26. Comparing these with test piece 1, the effects of the test, the elongation at rupture, and the contraction of area are apparent.

88. Copper, Brass, Bronze.—Pure copper is used for the bands of projectiles. In alloys, as brass and bronze, it enters into the construction of parts of guns and gun carriages not usually subjected to great stress. In this form, too, it is extensively employed in the manufacture of cartridge cases, fuses, primers, gun sights, and instruments. Brass is an alloy of copper with zinc. Bronze is an alloy of copper with tin and usually a small quantity of zinc. The addition of zinc or tin produces a harder and stronger metal better suited than the soft copper for the uses to which these alloys are applied. By the addition of aluminum or manganese in the alloy the strong hard bronzes known as aluminum bronze and manganese bronze are produced.

Iron and Steel.—When iron ore is melted in the furnace the product obtained, called pig iron, is an alloy of iron with carbon, the carbon content being about 5 per cent. This alloy may be readily fused and cast, and is then called *cast iron*. By various processes in the furnace the amount of carbon in the iron may be reduced. When the quantity of contained carbon is between two per cent and two tenths of one per cent the product is *steel*. When there is less than two tenths of one per cent of carbon we have *wrought iron*.

As the amount of carbon is reduced the qualities of the metal change in a marked degree. Cast iron is easily fusible, is hard and not malleable or ductile, cannot be welded, and has a crystalline structure. Wrought iron, on the other hand, is practically infusible, is soft, and possesses great malleability and ductility. It is easily welded and has a fibrous structure.

The properties of steel lie between those of wrought iron and cast iron, and the steel partakes of the characteristics of one or the other according to the percentage of carbon contained. Thus low steel, that is, steel low in carbon, is comparatively soft and may be readily welded or drawn into wire, while high steels are harder and more brittle and weld with difficulty.

Chief Constituents.—When examined under the microscope iron and steel are found to be conglomerate in structure, consisting of microscopic particles chiefly of the following substances in widely varying proportions.

1. Pure or nearly pure metallic iron, called *ferrite;* soft, weak, and very ductile.

2. A definite iron carbide, Fe_3C, called *cementite,* which is extremely hard and brittle, but probably very strong under a tensile stress.

The character of the iron or steel depends upon the proportions of these two chief constituents. The steels which are especially soft and ductile, as rivet and boiler plate steels, consist chiefly of the soft ductile ferrite, the proportion of cementite in these steels not exceeding perhaps 1 per cent. The harder steels, like rail steels, which are called upon to resist abrasion, contain a much larger percentage of cementite, about 7 per cent, and about 93 per cent of ferrite. As the proportion of cementite increases and that of ferrite decreases the hardness increases and the ductility diminishes. The tensile strength increases to a maximum when the cementite amounts to about 15 per cent of the whole, and then decreases.

The percentage of carbon in the metal is $\frac{1}{15}$ the percentage of cementite the molecular weight of Fe_3C being 180, of which 12 parts are carbon.

Graphite. Cast Iron.—In *gray cast iron* there is present, in addition to the ferrite and cementite, a quantity of nearly pure carbon in the form of graphite. The graphite is in thin flexible sheets which form a more or less continuous skeleton running through the mass of gray cast iron. The graphite makes the metal weak and brittle.

White cast iron contains but little graphite, but has a much higher percentage of cementite than either gray cast iron or steel. The large percentage of cementite, over 60 per cent, brings the carbon content to about $4\frac{1}{2}$ per cent, making the iron extremely hard and brittle.

Slag. Wrought Iron.—Wrought iron contains, in addition to the matrix of ferrite and cementite common to all irons, a small quantity of slag, a silicate of iron formed in the process of puddling. The presence of this slag forms the chief difference between wrought iron and the low carbon steels.

89. Hardening and Tempering Steel.—The distinguishing characteristic of steel when compared with cast or wrought iron is the property it possesses of hardening when cooled quickly after being heated to a red heat, and of subsequently losing some of its added hardness when subjected to a lower heat.

There is more or less confusion in the use of the terms applied to the two processes. By some the first process, quick cooling from a high heat, is called *tempering,* and the second process, reheating to a lower heat, is called *annealing.* By others the first process is called *hardening* or *quenching,* and the second process, which mitigates or lets down the hardness, is called *tempering.* The more recent tendency is toward the use of the latter terms, and following what is perhaps the better practice, we will call the first process hardening and the second process tempering.

Effect of Heat.—In order to get a comprehensive idea of the processes of hardening and tempering it will be necessary to go somewhat further into the constitution of steel and to learn how its constitution is affected by heat. As before stated, the chief constituents of steel are ferrite (iron) and cementite (Fe_3C). These exist in different proportions, and the behavior of the metal under heat treatment is dependent to a certain extent on the proportions of these substances. The amount of carbon in the steel depends on the proportion of cementite. The results attending the application of heat to steel are chiefly due to the effect of the heat on the condition of the carbon.

Austenite.—When steel is heated to a temperature of from 700 to 1000 degrees centigrade, depending on the quantity of carbon contained, the ferrite and cementite of which it is composed are converted into a substance called *austenite,* which, according to Howe, Professor of Metallurgy in Columbia University and an eminent writer on steel, is a solid solution of carbon in iron. He defines a solid solution as a solid that bears the same relation to the definite solid chemical compounds that a liquid solution, salt water for instance, bears to the definite liquid chemical compounds, as water.

Austenite is a distinct substance with properties of its own. When it contains 0.75 per cent or more of carbon it is extremely hard and brittle. Its hardness and brittleness are approximately proportional to the percentage of carbon contained.

The temperature at which austenite forms depends upon the proportions of ferrite and cementite in the metal. When these proportions are such that there is 9/10 of 1 per cent of carbon in the metal, that is when the metal consists of $0.9 \times 15 = 13.5$ per cent of cementite and 86.5 per cent of ferrite, the transformation of these constituents into austenite takes place at a lower temperature than when they are present in any other proportions.

Pearlite. Eutectoid.—The mixture of ferrite and cementite containing 0.9 per cent of carbon is given a specific name, *pearlite*, and is characterized as a *eutectoid*, which means a solid mixture in the particular proportions that give to the mixture the lowest transformation point under the action of heat. The corresponding term applied to a liquid solution is *eutectic*. Thus the eutectic solution of salt in water contains 23.6 per cent of salt. When this percentage of salt is present the solution forms at the lowest temperature, and conversely the salt remains longest in solution as the temperature is lowered.

Steel containing less than 0.9 of one per cent of carbon is considered to be composed of pearlite and an excess of ferrite, while the steels higher in carbon contain pearlite and an excess of cementite.

Now referring to Fig. 27 we will see at what temperature the various mixtures are transformed into austenite. The proportions of carbon and iron in the metal are shown on the horizontal axis. The curves are worded to show the transformations that occur as the metal cools from the molten state.

When there is 0.9 per cent of carbon in the metal we have pearlite, which is converted into austenite at a temperature of about 680° C., as shown in the figure by the intersection of the line A_1 at the point S. In the steels lower in carbon, which are composed of pearlite and an excess of ferrite, the pearlite is transformed at the same temperature as before, but the excess of ferrite requires a higher temperature, as shown by the line SA_3, so that the transformation is not complete for any particular composition until that temperature is reached which is indicated by the intersection of the ordinate representing the composition with the line SA_3.

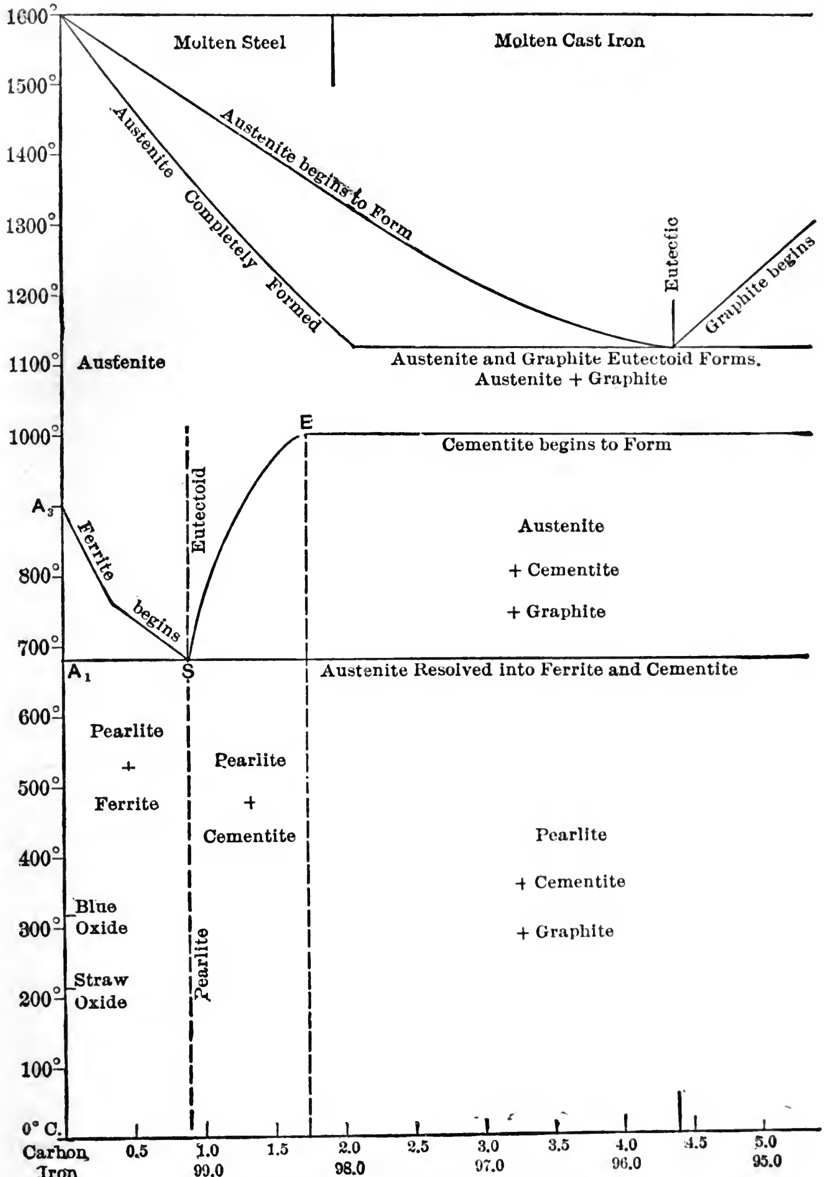


FIG. 27.—Effect of Heat on Iron and Steel.

And similarly for the higher carbon steels containing an excess of cementite; and for the cast irons, which, containing more than 2 per cent of carbon, are composed of pearlite, cementite, and graphite.

90. **Hardening.**—It will now be easy to understand the process of hardening steel by means of high heat followed by quick cooling. The high heat causes the formation of austenite in the metal. If the metal is allowed to cool slowly the austenite is retransformed into ferrite and cementite. This transformation requires an appreciable time, and if the metal is suddenly cooled from its high temperature the retransformation is prevented, and the hard austenite is preserved in the cold metal.

The change in the character of steel being due principally to the change in the condition of the carbon between its states in pearlite and cementite and in austenite, the effect of the heat treatment is greater as the proportion of carbon in the metal is greater. Thus the low-carbon steels containing from 0.06 to 0.10 per cent of carbon are in general but little affected by heat treatment and are practically incapable of being hardened, while the high-carbon steels and some cast irons are greatly affected and may be given extreme hardness.

The hardness and brittleness induced increase with the rapidity of cooling without limit, but they are apparently nearly independent of the temperature from which the sudden cooling begins, provided that this temperature is above the line of complete transformation, the line A_3SE, Fig. 27. If the metal is suddenly cooled from temperatures between the beginning and end of the transformation, that is at temperatures between the lines A_1 and A_3SE, the hardening increases as the quenching temperature rises. The range of temperature between the lines A_1 and A_3SE is called the *critical range*. In this range the hardness increases with the quenching temperature, but above the critical range the hardness is independent of the temperature.

The hardening of steel greatly increases its tensile strength and elastic limit, but it makes the steel brittle, thus reducing its toughness, as shown in test pieces by reduced elongation at rupture and diminished contraction of area of cross section.

The tensile strength of low-carbon steels increases with the rapidity of cooling without limit. In high-carbon steels the tensile strength at first increases with the rapidity of cooling, but reaches a maximum and then declines; that is, there is a certain rapidity of cooling that will give to any one of these steels its maximum tensile strength. This may be due to the fact that rapid cooling induces internal strains that may become so great as to be destructive.

The following table, taken from *Iron, Steel, and other Alloys*, by Henry Marion Howe, LL.D., well shows the effects of differences in the rapidity of cooling of steel containing 0.21 per cent of carbon.

Cooled in	Tensile Strength.	Elastic Limit.	Elongation.	Contraction of Area.
	lbs. per sq. in.	lbs. per sq. in.	per cent in 2 in.	per cent.
Iced brine	237,555	237,170	2.0	1.30
Cold water	216,215	....	1.5	1.67
Oil	174,180	....	2.9	1.403
Air	86,797	54,342	27.76	57.829
In furnace	80,103	44,221	28.15	54.749

91. Tempering.—Hardened steel is tempered by slight reheating, say to 200° or 300° C. This process lessens the hardness and brittleness of the steel, and thus increases its toughness. The austenite of the hardened steel is in a stable condition only when above the transformation temperature. As the temperature of the steel diminishes the austenite tends to change into ferrite and cementite. In the hardening process this tendency is resisted by the frictional resistance due to the sudden cooling, and the austenite is retained in an abnormal condition in the cold metal. The reheating of the metal in tempering appears to lessen the molecular rigidity of the austenite, and to afford opportunity for part of the austenite to follow the course that it would have taken in slow cooling through the transformation range and thus to be converted into pearlite. The higher the reheating the more does the change occur.

The rate of cooling after tempering has no effect on the steel, since the highest temperature of reheating has determined how far the change from austenite to pearlite may proceed, and no further change can occur at a lower temperature. It is therefore immaterial whether the cooling after tempering be slow or rapid.

Tempering has the effect of reducing somewhat the tensile strength and elastic limit of hardened steel, while it increases its toughness, as shown in test specimens by increased elongation at rupture and increased contraction of area of cross-section.

It will be seen that by proper regulation of the temperatures in the processes of hardening and tempering an extensive control of the properties of the metal is obtained, permitting the production of metal of the quality best suited to any particular purpose.

The tempering temperatures may be judged within limits by the color given to the steel, as it is heated, by the various oxides that form successively on the surface. The following table shows the temperatures at which the colors appear, and the tempering points for steels for various purposes.

220° C.,	straw; razors, surgical instruments.
245	yellow; penknives, taps, dies.
255	brown; cold chisels, hatchets.
265	brown with purple spots; axes.
275	purple; table knives, shears.
295	violet; swords, watch springs.
320	blue; saws.
525	incipient red.
700	dark red.
950	bright red.
1100	luminous yellow.
1300	incipient white.
1500	white.

Gun steel is tempered at temperatures between 600° and 675° C.

Annealing.—If the steel after being hardened is reheated to the critical temperature and then cooled slowly the austenite is completely converted into pearlite and ferrite or cementite, and the steel reverts to its original condition, losing all its added hardness and brittleness. This process is called *annealing*.

92. **Other Substances.**—In addition to the carbon in the metal, there are other substances, some of which are always present and others that may be added, that affect the qualities of steel.

Sulphur, phosphorus, manganese and silicon are usually present to a greater or less extent in all steels. If present in too large a percentage sulphur produces what is called *hot shortness* in the metal, that is brittleness when hot, while phosphorus makes the metal *cold short,* or brittle when cold. Manganese and silicon when present in proper percentages improve the qualities of the metal.

Chromium and *tungsten* give hardness to the steel without brittleness.

Nickel also greatly increases the toughness of the steel. Nickel steel for guns contains about 3½ per cent of nickel.

Uses.—Cast iron, wrought iron, cast steel and forged steel are all used in ordnance constructions. Cast iron on account of its cheapness and ease of manufacture in irregular shapes is used when practicable wherever great strength is not required, as in projectiles for the smaller guns and in parts of carriages not subject to wear or to high stresses.

Wrought iron is not now extensively used in ordnance constructions. The older seacoast carriages were almost wholly made of this metal.

Wherever great strength is required steel is employed. Cast steel is used in those parts that do not require the greater strength of forged steel, or that on account of their irregular shapes cannot be readily produced as forgings, such as the chassis and top carriages of seacoast gun carriages. Cast steel has also been used for projectiles and for guns, but without great success.

In structures or parts of structures requiring great strength, or subject to wear, forged steel only is used. Guns and armor and armor-piercing projectiles are now made of forged steel only, and the operative parts of gun carriages and of other structures are principally of this metal.

Gun Steel.—Of two steels, one high in carbon and the other low in carbon, the steel with the higher percentage of carbon will, with similar treatment, have the higher elastic limit. Since the elastic limit of the metal is the limit of the strength considered in the construction of guns, it would appear that the metal with the highest elastic limit would be the most desirable. But high steel is more difficult to manufacture than low steel, and in large pieces there is much greater liability to flaws, strains, and incipient cracks. After passing the elastic limit the hard steel has little remaining strength and breaks without warning, while the low steel, due to its greater toughness, yields considerably without fracture. For these reasons a low steel containing about one half of one per cent of carbon is used in the manufacture of guns.

MANUFACTURE OF STEEL FORGINGS FOR GUNS.

93. **Open Hearth Process.**—All gun steel at the present day is made by the open hearth process, which derives its name from the fact that the receptacle in which the steel is melted is open at the top and exposed to the flame of the fuel, which plays over the surface and performs a principal part in the formation of the steel. The product is called Siemens or Siemens-Martin steel, according to the ingredients contained.

The open hearth furnace, invented by Dr. Siemens, consists of the following essential parts:

1. The gas producer;
2. The regenerators;
3. The furnace proper.

The Gas-producer.—The fuel used in the Siemens furnace is gaseous, and is obtained from ordinary fuel by subjecting the fuel to a preliminary process in the gas producer. This apparatus, Fig. 28, consists of a rectangular chamber of fire-brick, one side, *B*, being inclined at an angle of from 45 to 60 degrees. *A* is the grate. The fuel, which may be of any kind, is fed into the producer through the hopper *C*. As the fuel slowly burns, the CO_2 rises through the mass above it and absorbs an additional portion of C, becoming converted into 2CO. This gas passes out of the opening *D* into a flue. In order to cause it to flow toward the furnace it is led through a long pipe, *E*, where it is partially cooled, and then descends the pipe *F* leading to the furnace. The gas in *F* being cooler than that in *E* and *D*, a constant flow of gas from producer to furnace is maintained.

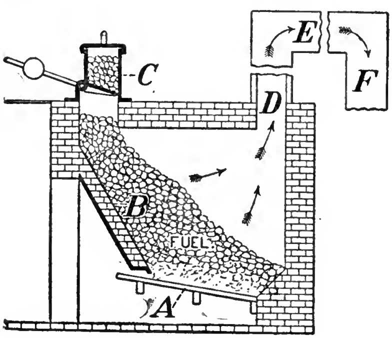


Fig. 28.

THE REGENERATORS.—The gas entering the furnace is, as has been stated, CO. To burn it to CO_2, air must be mixed with it. This mixture is made in the furnace proper, the CO and air being kept separate till they reach the point where they are to burn. The CO is cooled to some extent, as shown, before being admitted to the furnace.

To heat both air and CO before they are mixed and burned, and to accomplish this economically, and raise the gases to a high temperature, the waste heat of the furnace is employed. The heating of the gases is accomplished by means of the regenerators, Fig. 29. They consist of four large chambers, usually placed below

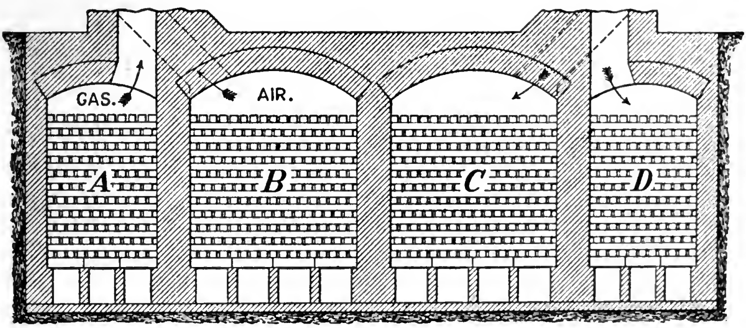


FIG. 29.

the furnace, filled with fire-brick. The fire-brick is piled so that there are intervals between the bricks to allow the passage of gas and air. When the furnace is started, CO is admitted through *A* and air through *B*, both *A* and *B* being cold. The gases pass between the fire-bricks in *A* and *B* and through flues at the top, and flow into the furnace proper, where they are lighted. The products of combustion are caused to pass through *C* and *D*, which are similar chambers. In doing so these products heat the fire-bricks in *C* and *D*. After some time—about one hour generally—by the action of valves controlled by the workmen, the CO and air are caused to enter the furnace through *D* and *C* respectively, and the products of combustion to pass out through *A* and *B*. In this case the CO and air, entering the heated chambers *D* and *C*, are raised to a high temperature before ignition, and the temperature of the furnace is thereby greatly increased. It is also evident that A and B will be more highly heated than C and D were, and therefore when the next change is made, the gas and air passing through A and B will be more highly heated than when they passed through D and C, and so on.

The action of the furnace is therefore *cumulative*, and its only limit in temperature is the refractoriness of the material. By regulating the proportions of gas and air, which is readily done, the temperature may be kept constant.

94. The Furnace.—The furnace proper, Fig. 30, is a chamber situated above the regenerating chambers. The dish-shaped cast

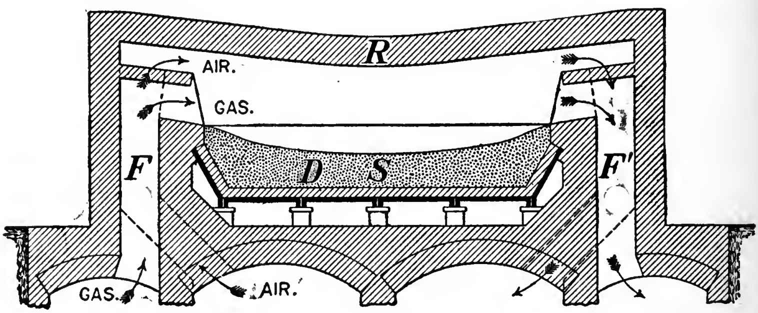


Fig. 30.

iron vessel D, lined with refractory sand S, constitutes the hearth of the furnace. The iron vessel is supported in such a manner that the air may circulate freely around it and keep it from melting. The iron that is to be converted into steel is piled on the hearth of the furnace.

The gaseous fuel and air enter by the flues F, and the products of combustion escape by the flues F', or the reverse, according to the position of the regulating valves. The gases are ignited as they enter the furnace. The sloping roof R, lined with fire-brick, deflects the flames over the metal on the hearth.

At opposite ends of the furnace are a charging door for admission of the metal, and a tap hole, closed with a plug of fire-clay, for drawing off the finished steel.

Operation.—The process consists in the decarbonizing of cast iron to the point at which the metal contains only that percentage of carbon that is desired in the steel, and in the partial removal from the iron of those impurities, such as silicon, manganese, and phosphorus, that are injurious to the steel if present in too large quantities.

Pig cast iron heated to a red heat in a separate furnace is piled on the hearth of the Siemens furnace, and a quantity of steel or wrought iron scrap is usually added to the charge to reduce the percentage of carbon in the mass.

By the action of the furnace the charge is soon melted. Under the influence of the heat the carbon oxidizes to carbonic oxide gas, which escapes; the silicon oxidizes to silica and the manganese to manganous oxide. The silica and manganous oxide unite with the slag which floats in a thin layer on the molten metal.

The percentage of carbon in the steel at any stage of the process is determined by taking samples from the metal, cooling them, and observing their fracture on breaking; and by dissolving portions of the specimens in nitric acid and comparing the color with the colors of standard solutions of steel containing different percentages of carbon. In this way the composition of the steel can be exactly regulated, for the metal can be kept in a melted state without damage for a considerable time, and the character of the flame can be made oxidizing or reducing at will, according to the relative amounts of air and CO admitted.

The decarbonizing process is often continued until the percentage of carbon remaining in the steel is less than the percentage desired. The desired percentage is then obtained by the addition of pig iron containing a known percentage of carbon, such as spiegeleisen or ferromanganese, or by the addition of ore.

The lining of the hearth, *S* Fig. 30, is of sand when the iron to be reduced does not contain a harmful percentage of phosphorus. The process is then called the *acid process*, from the silicious or acid nature of the slag. When the iron contains a larger percentage of phosphorus a basic lining, as magnesia or calcined dolomite, is required for the removal of the phosphorus. The slag formed in the basic process is strongly retentive of phosphorus and removes the excess of this substance from the metal.

The reduction of a charge of metal in the Siemens furnace ordinarily takes about eight hours.

When the steel has attained its desired composition the furnace is tapped and the metal cast into ingots.

95. Other Processes.—*The crucible process* is used to some extent by Krupp in the production of gun steel. The ingredients of the steel are melted together in crucibles, and the resulting steel is poured from the crucibles into a common reservoir from which the ingots are cast.

The Bessemer Process, though important and producing large quantities of steel, is not used in making gun steel.

Casting.—The molten metal is drawn into an iron ladle which depends from a crane in front of the furnace. The ladle, Fig. 31, is lined with refractory sand. It is provided with trunnions, T', so that it may be tipped for pouring the metal into the mold, or it may have a tap hole, T, in the bottom, closed with a plug of fire-clay. The plug is lifted and replaced by means of a rod R also encased in refractory sand. There is an advantage in drawing the metal from the bottom of the ladle in that the scoria and impurities that float on the surface may be kept out of the mold. The metal if very hot is poured slowly into the mold in a thin stream, thus allowing opportunity for escape of the gases that it contains. If at a lower temperature it may be poured more quickly. It is frequently allowed to cool to the desired temperature in the ladle.

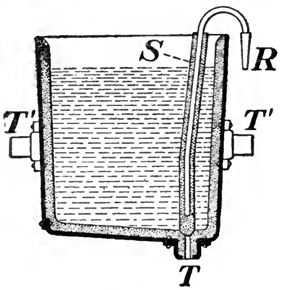


FIG. 31.

Molds.—In the casting of ordinary ingots, the iron or steel molds into which the metal is poured from the ladle are slightly conical in shape, Fig. 32, to facilitate their removal from the ingot. They are of various cross sections, depending on the shape of the ingot desired. The interior surface is covered with a wash of clay or plumbago.

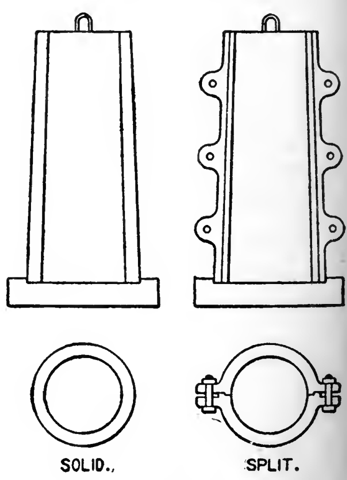


FIG. 32.

Sinking Head.—In all castings, whether of iron, steel, or other metal, an excess of metal, called the sinking head, is left at the top of the mold. The pressure due to the weight of this metal gives greater density to the casting. The sinking head also serves to collect the scoria and impurities which rise to the top, and it provides metal to fill any cracks or cavities that may form in the cooling of the ingot.

Defects in Ingots. *Blow Holes.*—The gases in the melted metal, unable to escape from the mold, form holes in the ingot, called blow holes. These cannot be detected, nor can they be removed by forging. Forging changes their form only without giving continuity to the metal. Blow holes are more prevalent in Bessemer than in open hearth steel and are more apt to occur at low temperatures of casting, when the metal hardens before the gas can escape.

Pipes.—The metal in contact with the molds cools first and solidifies. As the cooling and consequent contraction extends toward the center, the liquid metal is drawn away from the center, leaving cavities called pipes along the axis of the ingot. Pipes most frequently occur when the metal is cast too hot. Thus on the one hand too low a temperature causes blow holes and too high a temperature pipes.

Segregation.—As the various constituents of the steel (iron, silicon, manganese, etc.) solidify at different temperatures, it frequently happens that they separate from each other as the ingot cools, forming what is called segregation. This gives a different structure to the metal and greatly weakens it. Segregation is more likely to be found toward the center of the ingot and in the upper portions.

96. Whitworth's Process of Fluid Compression.—The purpose of this process, invented by Sir Joseph Whitworth of England, is to remove as far as possible the blow holes, pipes, and other defects from the ingot and to give the metal greater solidity and uniformity of structure than can be obtained in the ordinary method of casting. The object is accomplished, to a large extent, by the application of enormous pressure on the metal while in the fluid state in molds so constructed as to allow free escape of the gases.

The flask, *f* Fig. 33, made of cast steel, is of great strength to withstand the great pressure. It is built up of cylindrical sections which are bolted together to the desired length. The interior of the flask is lined with vertical wrought iron bars, *b*, whose long edges are cut away or beveled to form channels, *a*, by means of which the gas may escape: the interior and exterior channels thus formed being connected by grooves, *c*, cut in the sides of the bars at short intervals. The cylinder formed by the interior surfaces of the bars is lined with refractory sand. A cast iron plate, *d*, through which are continued the longitudinal gas channels closes the mold at the bottom. The mold rests on a car in the bottom of a pit.

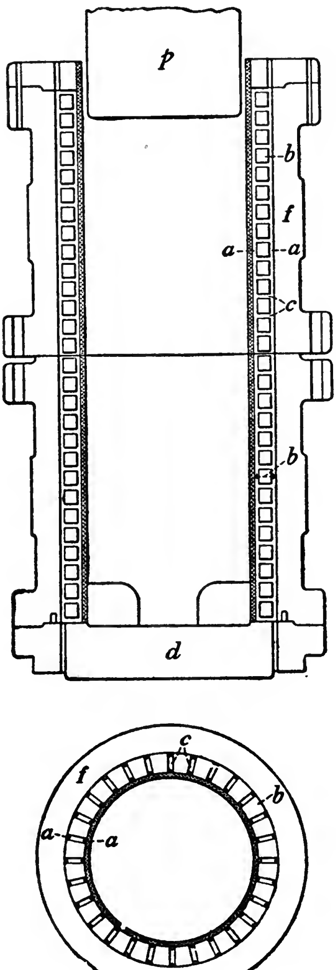


FIG. 33.

When the mold is filled with metal the car is run into a hydraulic press with an adjustable head. The head, *p*, of the press, of diameter only slightly less than the interior of the mold, is brought down against the molten metal and locked in that position. The metal wells up around the head of the press and, quickly cooling, forms a solid mass which with the head completely closes the top of the mold.

The press is constructed with its piston at the bottom so that the pressure may be applied on the bottom of the car that carries the mold.

By the pressure on the bottom of the car, gradually applied until a pressure of six tons to the square inch is reached, the car and mold are slowly forced upward. The molten metal is compressed by the applied pressure, and the gas, forced through the sand lining and the channels between the lining bars, issues from the top and bottom of the mold in a violent flow of flame. The pressure is continued until the column of metal has shortened one eighth of its length. A uniform pressure of about 1500 pounds to the square inch is then left on the ingot while it cools, to follow up the metal as it contracts and prevent the formation of cracks.

97. **Processes After Casting.**—The specifications for gun forgings require that the forgings be made from that part of the ingot that remains after 30 per cent by weight has been cut from the top of the ingot and 6 per cent from the bottom. These parts are cut off, as they are likely to contain most of the defects in the ingot.

For hollow forgings the center of the part selected is then bored out in a heavy lathe, or punched out if the ingot is short.

Heating.—The ingot is then heated preparatory to forging. The heating is accomplished in a furnace erected near the forging hammer or press, and is conducted with great care. The cooling of the ingot in the mold has left in the metal strains due to the successive contraction of the interior layers. Assisted by unequal expansion in heating the strains may cause cracks to develop in the ingot. Great care is therefore exercised that the heating shall proceed slowly and uniformly, thus avoiding the overheating of the exterior layers of metal before the heat has thoroughly penetrated to the interior.

Forging.—The heated ingot is forged either by blows delivered by a steam hammer or by pressure delivered by a hydraulic forging press. Under the slow pressure of the forging press the metal of the forging has more time to flow, the effect of the treatment is more evenly distributed, and the metal is more uniformly strained. This process is therefore preferred in the manufacture of gun forgings.

Fig. 34 is a reproduction from a photograph of a 5000-ton hydraulic forging press at the works of the Bethlehem Steel Co. The print shows a bored ingot for the tube of a 12-inch gun being forged on a mandrel. The outer diameter of the ingot is reduced by the forging and the length of the ingot increased. The diameter of the bore remains practically unchanged. The outer end of the ingot is supported from an overhead crane.

The ingot is turned on the anvil of the press, and advanced when desired, by means of the chain seen through the press. The method of turning is better shown in the plate following.

The movements of the head of the press are controlled by means of levers situated at a short distance to the right of the press. The operator at the lever sees recorded on the dial the pressure exerted at any instant.

Fig. 35 shows a 10-ton steam hammer forging a solid ingot for a 3-inch gun. The ingot is supported from an overhead crane and is nearly balanced in the sling chain by the bar of iron, called a porter bar, clamped to the ingot and extending to the rear. By bearing down on the porter bar the ingot is lifted off the anvil and may then be moved by the crane back and forth under the hammer. The ingot is turned under the hammer from the crane by means of the gearing shown in the upper part of the picture.

The movements of the hammer are controlled by the man at the left through the levers shown at his hand.

98. Hollow Forgings.—In forging bored ingots a solid steel shaft called a mandrel is passed through the bore of the heated ingot, and the method pursued in forging depends upon whether the length of the ingot is to be increased without change of interior diameter, as in forging a gun tube, or whether the diameters of the ingot are to be enlarged, as in forging hoops. In the first case the ingot, on a mandrel of proper diameter, is placed directly on the anvil of the press, as shown in Fig. 34. The effect of forging is then to increase the length of the ingot and decrease the outer diameter while maintaining the interior diameter unchanged. The mandrel is withdrawn from the forging by means of a hydraulic press.

In forging hoops, the mandrel rests on two supports on either side of the head of the press, Fig. 36, and is itself the anvil on which the forging is done. By turning the mandrel new surfaces of the hoop are presented to the press. The walls of the hoop are reduced in thickness by the forging, the diameters of the hoop being increased, while the length is not materially changed.

The specifications for gun forgings require that the part of a solid ingot used for a tube forging shall have before forging an area of cross section at least four times as great as the maximum

FIG. 34.—5,000-ton Hydraulic Forging Press.

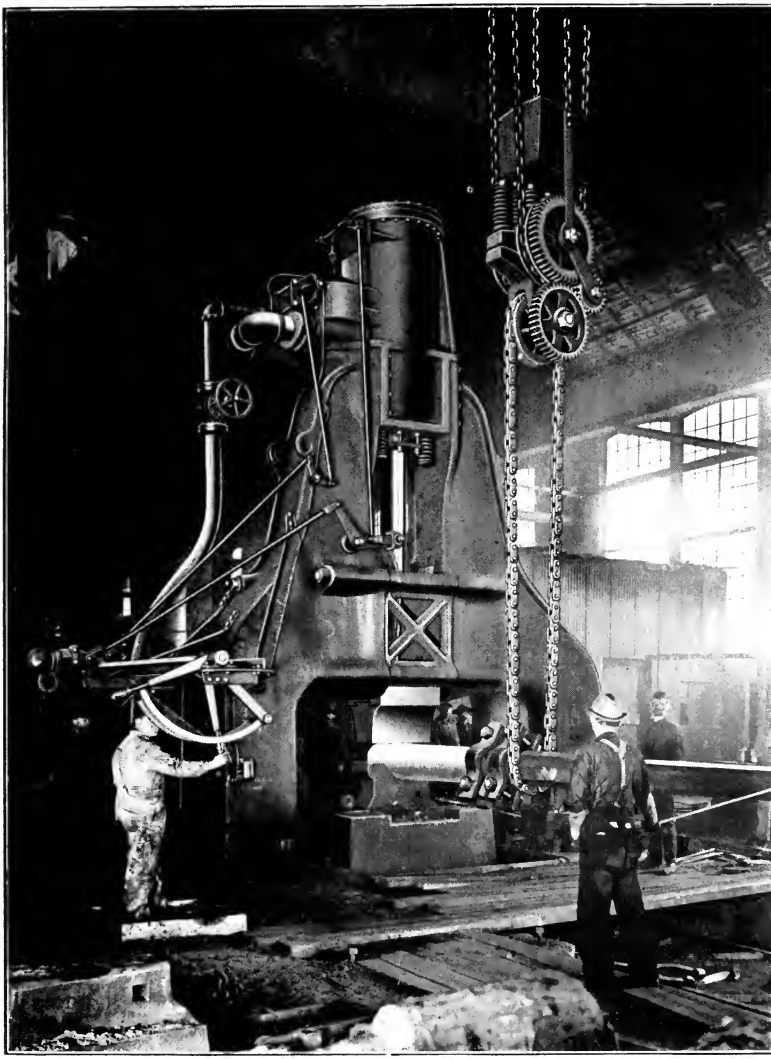


Fig. 35.—10-ton Steam Hammer.

area of cross section of the rough forging when finished, and for a jacket forging 3½ times as great. For forgings for guns 12 inches or more in caliber these figures are reduced to 3½ and 3 respectively. Forgings for lining tubes must be reduced 6 times in area.

If bored ingots are used the wall of the ingot must be reduced at least one half in thickness.

Annealing.—The working of the ingot in forging and the irregular cooling leaves the metal in a state of strain. The strains are removed by the process of annealing. For this purpose the

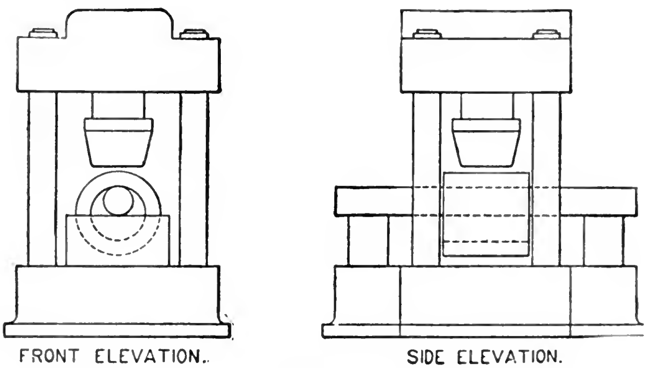


FIG. 36.

forging is usually laid in a brick-walled pit or furnace, and slowly and uniformly heated by wood fires, the burning logs being distributed along the pit as required to heat the forging uniformly. When the proper heat, usually a bright red, has been attained, the fires are allowed to die out, or are drawn, and the ingot remains in the furnace until both are cold. Three or four days may be required for the slow cooling of a large forging.

99. Hardening in Oil or Water.—Annealing removes the internal strains that exist in the forging, but, as before explained, it greatly reduces the tensile strength and elastic limit of the metal. To restore the strength to the metal and to produce in it the qualities required in gun forgings, the forging is next subjected to the process of hardening. Before hardening it is machined in a lathe nearly to finished dimensions. Specimens for tests are cut from the ends, and from their behavior in the testing machine the requirements of the subsequent treatment are determined.

The forging is then slowly and uniformly heated throughout. Large forgings, such as tubes and jackets, are heated in vertical furnaces, great care being exercised that the heating shall be uniform throughout the length of the piece in order that undue warping may not occur in the subsequent cooling. When the forging is at a uniform red heat the side of the furnace is opened and the forging is lifted out by a crane and immersed in a deep tank of oil or of water alongside the furnace. The oil tank is surrounded by another tank through which cold water is constantly running. The heat of the forging passes to the oil and thence to the water, and is thus gradually conducted away.

The Bethlehem Steel Co. of Bethlehem, Penn., and the Midvale Steel Co. of Philadelphia, the two principal manufacturers of gun forgings in this country, use different oils for oil tempering. The Bethlehem Co. uses petroleum oil once refined. The Midvale Co. uses cottonseed oil with flashing point not less than 360 degrees centigrade.

The temperature of the forging when immersed is very high compared with that of the oil. The cooling is therefore sudden at first, but as oil is a poor conductor of heat the heat of the forging is carried away slowly, leaving the metal with greater toughness than it would have if hardened in water and cooled more quickly.

Oil is customarily used in the hardening of gun forgings. Occasionally the qualities of the metal are such that better results are obtained by the quicker cooling in water.

Tempering.—The process of hardening greatly increases the elastic strength of the metal but reduces its toughness. At the same time it produces internal strains due to contraction in cooling. The strains are removed, the hardness reduced, and the toughness restored by the process of tempering, conducted in the same manner as the previous annealing, but at a lower heat, so that the gain in elastic strength is reduced but slightly and not entirely lost. The tempering temperature for gun forgings lies between 600 and 675 degrees centigrade, 1100 to 1250 degrees Fahrenheit.

Specimens are again taken from the ends of the forging and broken in the testing machine. If the specimens do not fulfil the requirements of the specifications the forging is again hardened and tempered, the temperature and conduct of the processes being so regulated as to improve those qualities in which the metal has proved defective in the tests.

Strength of Parts of the Gun.—The requirements in steel forgings for guns over 8 inches in caliber are shown in the table on page 165. It will be observed that the strength of the metal increases as we proceed outward from the center of the gun. Thus the elastic limit of the tube is 46,000 lbs., of the jacket 48,000, and of the hoops 53,000. It would be better if the strongest metal were in the tube, which has to endure the greatest strain. But the production of the high qualities required is much more difficult in large forgings than in smaller ones, and for this reason the requirements for the tubes and jackets must be lower than for the hoops. An additional reason for the difference in requirements is found in the fact that the metal of the tube has the advantage of greater elongation before rupture, as may be seen in the table on page 165. The greater elongation is difficult to produce with the higher elastic limit.

The tubes and jackets of guns under 8 inches in caliber have an elastic limit of 50,000 lbs.

Forged steel that has an elastic limit of over 110,000 lbs. is now produced.

CHAPTER VI.

GUNS.

ELASTIC STRENGTH OF GUNS.

100. The Elasticity of Metals.—In the chapter on metals the elastic limit of a metal has been defined as the minimum stress per unit of area of cross section that will produce in the metal a permanent set. For each kind of stress, whether of extension or compression, the metal has a distinct elastic limit. The elastic limit of extension, or the tensile elastic limit, is usually less than the elastic limit of compression. In gun steels the difference is not great and the two are considered equal. The tensile elastic limit is ordinarily used, as it is the limit usually measured.

Hooke's Law.—A tensile stress applied to a bar of metal causes elongation of the bar, and it is found by experiment that under stresses less than the elastic limit of the metal the elongation is proportional to the stress. In other words, within the elastic limit of the metal the ratio of the stress to the strain is constant. This law is known as Hooke's law, and is often expressed *ut tensio sic vis*.

Modulus of Elasticity.—If we measure the elongation of a bar caused by a tensile stress, and divide the measured elongation by the original length of the bar, we will obtain the elongation per unit of length, expressed as a numerical fraction.

Now if we divide any stress per unit of area within the elastic limit of the metal by the elongation per unit of length the result will be the constant ratio of stress to strain within the elastic limit. This ratio is called the modulus of elasticity.

Let E be the modulus of elasticity of the metal,

θ the elastic limit of the metal,

γ the elongation per unit of length at the elastic limit.

By definition we have

$$E=\theta/\gamma \tag{1}$$

If we assume that the elasticity of the metal continues indefinitely we see, by making γ equal to unity in the above equation, that the modulus of elasticity is the stress per unit of area that would extend a bar to twice its length.

When the elastic limit is expressed in pounds per square inch the modulus of elasticity of steel may, without sensible error, be taken as 30,000,000.

The modulus of elasticity is really a stress per unit of area, but it had best be considered as the abstract ratio between stress and strain.

Since by Hooke's law the ratio of the stress to the strain is constant within the elastic limit, we may write for θ and γ in equation (1) any other stress within the elastic limit and its corresponding strain.

Let S be a stress per unit of area within the elastic limit,
l the strain per unit of length due to the stress.

Then $$E=S/l \quad \text{and} \quad l=S/E \tag{2}$$

That is, the strain per unit of length due to any stress per unit of area within the elastic limit is equal to the stress divided by the modulus of elasticity.

101. Strains Perpendicular to the Direction of the Stress.—In the previous paragraphs we have considered only the strain produced in the direction of the stress. But we have seen in the chapter on metals that a tensile stress produces a reduction in area of cross section, and it is found by experiment that, for steel, the strain at right angles to the direction of a stress within the elastic limit of the metal is equal to one third of the strain in the direction of the stress. If the cube in Fig. 37 is subjected to the tensile stress represented by p, the edges, aa, bb, etc., parallel to the direction of the stress will be elongated, and the edges, ab, ac, etc., perpendicular to this direction will be shortened by an amount equal to one third the elongation of the parallel edges.

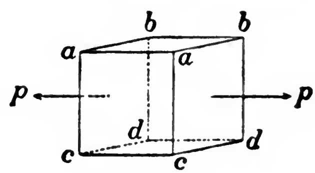


Fig. 37.

Equations of Relation between Stress and Strain.—If we consider that the cube is subjected at once to tensile stresses applied in the three directions perpendicular to its faces, the strain in each direction due to the stress in that direction will be diminished by the contrary strains due to the perpendicular stresses.

Let X, Y, and Z be three independent extraneous tensile forces perpendicular to the faces of the cube;

l_x, l_y, and l_z the strains in the directions of X, Y, and Z respectively.

The strain in the direction X due to the force X is from equation (2) X/E. It is diminished by $\frac{1}{3}\frac{Y}{E}$ and by $\frac{1}{3}\frac{Z}{E}$. Therefore, for the total strains in the three directions, we have

$$\left.\begin{aligned} l_x &= \frac{1}{E}\left(X - \frac{Y}{3} - \frac{Z}{3}\right) \\ l_y &= \frac{1}{E}\left(Y - \frac{X}{3} - \frac{Z}{3}\right) \\ l_z &= \frac{1}{E}\left(Z - \frac{X}{3} - \frac{Y}{3}\right) \end{aligned}\right\} \quad (3)$$

Problems. 1. A steel test specimen has an elastic limit of 59,000 lbs. What will be its elongation per unit of length at the elastic limit? 0.00197

2. The original diameter of the specimen being 0.505 inches, what is its diameter when the piece is stretched to its elastic limit? 0.5047 inches.

3. A vertical steel rod 20 feet long and $\frac{1}{2}$ inch square sustains at its lower end a load of 6000 lbs. The elastic limit of the steel is 72,000 lbs. What will be the elongation caused by the load? 0.192 inches.

4. Taking the modulus of elasticity of copper as 16,000,000, what will be the elongation of a copper bar 1 inch square and 10 feet long supporting a load of 5000 lbs.? 0.0375 inches.

102. Principal Stresses and Strains.—Since every stress applied to a solid produces stresses in directions perpendicular to the direction of the applied stress, at any point in a solid under stress there are always three planes at right angles to each other upon each of which the stress is normal. Thus in the cube we have just considered, the stresses at any point in the cube are normal to three planes parallel to the faces of the cube. The normal stresses are called the principal stresses at the point; and it may be shown by the ellipsoid of stress that one of the principal stresses is the greatest stress at the point. The corresponding strains are called the principal strains.

Stresses and Strains in a Closed Cylinder.—The following discussion of the elastic strength of cylinders is based on the theory of Clavarino, published in 1879, and modified through the results of experiments by Major Rogers Birnie, Ordnance Department, U. S. Army.

Consider a hollow metal cylinder, closed at both ends, to be subjected to the uniform pressure of a gas confined in the cylinder. The pressure acting perpendicularly to the cylindrical walls will tend to compress the walls radially. If we consider a longitudinal section of the cylinder by any plane through the axis, the pressure acting in both directions perpendicular to this plane will tend to disrupt or pull apart the cylinder at the section, and will therefore produce a tensile stress in a tangential direction on the metal throughout the section. The pressure acting against the ends of the cylinder will tend to pull it apart longitudinally.

The metal of any elementary cube in the cylinder is therefore subjected to three principal stresses: a radial stress of compression, a tangential stress of extension, and a longitudinal stress of extension.

If the cylinder be subjected to a uniform exterior pressure stresses will be similarly developed in the three directions.

In the following discussion we will always understand by the term stress, the stress per unit of area, and by the term strain, the strain per unit of length, unless these terms are qualified by the word total or other qualifying word.

Assume a closed cylinder affected by uniform interior and exterior pressures. At any point of the cylinder

Let t be the tangential stress,

p the radial stress,

q the longitudinal stress.

Substituting these letters in equations (3) for X, Y, and Z, respectively, and changing the sign of Y, since the interior and exterior pressures act toward each other radially, so that the stress, p, acts in a direction opposite to that assumed for Y in deducing equations (3), we obtain the following equations.

$$\left.\begin{aligned} l_t &= \frac{1}{E}\left(t+\frac{p}{3}-\frac{q}{3}\right) \\ l_p &= -\frac{1}{E}\left(p+\frac{t}{3}+\frac{q}{3}\right) \\ l_q &= \frac{1}{E}\left(q-\frac{t}{3}+\frac{p}{3}\right) \end{aligned}\right\} \qquad (4)$$

which express the values of the strains in the directions of the three stresses. These values may be positive or negative, depending upon the resultant of the stresses. A positive value of a strain represents elongation and a negative value contraction, as a positive value of a stress represents a tensile stress and a negative value a compressive stress.

103. Relations between the Stresses t, p, and q. Lamé's Laws.—The stresses and strains in equations (4) form six unknown quantities which cannot be determined from the three equations.

Lamé, a distinguished investigator in the subject of elasticity of solid bodies, has established relations between the stresses, by means of which the equations may be solved and the values of the stresses and strains determined. He assumes that the longitudinal stress q and the longitudinal strain l_q are constant throughout the cross section. The last of equations (4) may then be written

$$t-p=3(q-l_qE)=\text{constant} \qquad (5)$$

which equation is true whether q has a value or is zero. As t and p apply to any point in the walls of the cylinder, we have Lamé's first law.

In a cylinder under uniform pressure the difference between the tangential tension and the radial pressure is the same at all points in the section of the cylinder.

Now let us consider a right section of the cylinder, *of unit length*, Fig. 38.

Let P_0 be the pressure per unit of area acting on the interior of the cylinder,

P_1 the pressure per unit of area on the exterior,

R_0 the interior radius of the cylinder,

R_1 the exterior radius,

r the radius of any point in the cylinder.

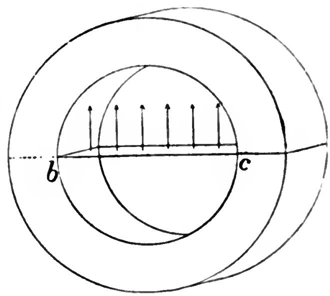


FIG. 38.

The total interior pressure acting normally on either side of the diametral plane bc is $2P_0R_0$. The total pressure acting on the outer circumference on either side of the plane and normal to it is $2P_1R_1$. The difference of these pressures is the resultant pressure acting on the metal in the sectional plane bc. The total tangential stress on the metal at the section will therefore be

$$2(P_0R_0 - P_1R_1)$$

But since t represents this stress per unit of area, the total stress is equal to $2\int_{R_0}^{R_1} t\,dr$. Therefore

$$\int_{R_0}^{R_1} t\,dr = P_0R_0 - P_1R_1$$

Assuming that t is a function of r, it must be such a function that $t\,dr$ when integrated between the limits R_1 and R_0 will be equal to $P_0R_0 - P_1R_1$. $t\,dr$ must then be equal to $-d(pr)$ because the integral of this expression taken between the given limits is $P_0R_0 - P_1R_1$. The substitution of the pressures P_0 and P_1 for the radial stress p, in integrating the expression $-d(pr)$, may be made because, as will be found later, p varies proportionately with P_0 and P_1.

We therefore have

$$tdr = -d(pr) = -pdr - rdp$$

From which by combination with equation (5) and integration, *see foot note,* we obtain

$$t+p=C/r^2 \tag{6}$$

in which C is a constant.

Equation (6) expresses Lamé's second law:

In a cylinder under uniform pressure the sum of the tangential tension and the radial pressure varies inversely as the square of the radius.

Both laws are based on the assumption that the longitudinal stress is constant or zero.

104. Stresses in the Cylinder.—By means of Lamé's laws we may now determine the values for the stresses at all points in the cylinder. We may write for t, p, and r in equations (5) and (6) the coordinate values referring to any point in the cylinder and thus form the equations

$$t-p=T_0-P_0=T_1-P_1$$

$$(t+p)r^2=(T_0+P_0)R_0^2=(T_1+P_1)R_1^2$$

Eliminating T_0 and T_1 from these equations we may obtain

$$t = \frac{P_0R_0^2-P_1R_1^2}{R_1^2-R_0^2} + \frac{R_0^2R_1^2(P_0-P_1)}{R_1^2-R_0^2}\frac{1}{r^2} \tag{7}$$

$$p = -\frac{P_0R_0^2-P_1R_1^2}{R_1^2-R_0^2} + \frac{R_0^2R_1^2(P_0-P_1)}{R_1^2-R_0^2}\frac{1}{r^2} \tag{8}$$

$$t\,dr=-pdr-rdp$$

$$-(t+p)dr=rdp$$

From equation (5) $\quad t+p=2p+k$

Therefore $\quad -\frac{dr}{r}=\frac{dp}{2p+k}$

Integrating

$$\log_e\frac{1}{r}=\frac{1}{2}\log_e(2p+k)+\log_e A$$

Replacing $2p+k$ by its value $t+p$ we obtain

$$1/r^2=A^2(t+p)$$

or $\quad t+p=1/A^2r^2$

From these equations we may obtain the values of the tangential and radial stresses at any point in the section of the cylinder by substituting for r its value for the point.

Longitudinal Stress.—The longitudinal stress has been assumed as constant over the cross section of the cylinder. Under this assumption when applied to a gun the total longitudinal stress due to the pressure on the face of the breech block is distributed uniformly over the cross section of the gun, producing a stress per unit of area that is small compared with the tangential and radial stresses. In the present discussion of the stresses acting on the cylinder the longitudinal stress will therefore be neglected, and q in equations (4) will be considered as zero. Later the value of the longitudinal stress will be deduced.

105. Resultant Stresses in the Cylinder.—Making $q=0$ in equations (4) and substituting for t and p their values from (7) and (8) we obtain

$$El_t=S_t=\frac{2}{3}\frac{P_0R_0^2-P_1R_1^2}{R_1^2-R_0^2}+\frac{4}{3}\frac{R_0^2R_1^2(P_0-P_1)}{R_1^2-R_0^2}\frac{1}{r^2} \qquad (9)$$

$$El_p=S_p=\frac{2}{3}\frac{P_0R_0^2-P_1R_1^2}{R_1^2-R_0^2}-\frac{4}{3}\frac{R_0^2R_1^2(P_0-P_1)}{R_1^2-R_0^2}\frac{1}{r^2} \qquad (10)$$

$$El_q=S_q=-\frac{2}{3}\frac{P_0R_0^2-P_1R_1^2}{R_1^2-R_0^2} \qquad (11)$$

In the above equations the first members are the respective strains multiplied by the modulus of elasticity. Referring to equation (2) we see that each product is equal to the stress which acting alone would produce the strain. The equations then give the values of the simple stresses that would produce the same strains as are caused by the stresses p and t acting together. Their values at any point in the cylinder are obtained from the above equations by giving to r the value for the point.

Basic Principle of Gun Construction.—The following principle is the foundation of the modern theory of gun construction.

No fiber of any cylinder in the gun must be strained beyond the elastic limit of the metal of the cylinder.

This principle is strictly adhered to in the construction of guns built up wholly of steel forgings. In the construction of wire-wound guns the tube is, in some constructions, purposely compressed beyond its elastic limit by the pressure exerted upon it by the wire.

The principle fixes a limit to the stresses to which any cylinder that forms part of a gun may be subjected. If we represent by

θ the tensile elastic limit of the metal,
ρ the compressive elastic limit of the metal,

the stresses represented by the first members of equations (9) to (11) may never exceed either θ or ρ, depending on whether the stress is one of extension or of compression; and the interior and exterior pressures, represented by P_0 and P_1 in those equations, must never have such values as to cause the stresses to exceed these limits.

106. Simplification of the Formulas of Gun Construction.—The formulas of gun construction are deduced from equations (9), (10), and (11). Heretofore, in the deduction, these equations have been used in the form in which they appear above, and the formulas resulting from them have been similarly extended and equally formidable in appearance, and much labor has been expended in writing them out.

We will introduce here, for the first time in any text, a simplification of equations (9), (10), and (11), which will result in a marked simplification of all the formulas of gun construction, making the formulas easier to handle, and greatly reducing the labor required in their use.

We will express in equations (9), (10), and (11) R_1^2 in terms of R_0^2, and in the future deductions we will always express R_2^2, R_3^2, R_n^2 in terms of R_0^2.

$$\text{Make } \left.\begin{array}{lll} R_1^2=aR_0^2 & \text{or} & a=R_1^2/R_0^2 \\ R_2^2=bR_0^2 & & b=R_2^2/R_0^2 \\ R_3^2=cR_0^2 & & c=R_3^2/R_0^2 \\ R_n^2=nR_0^2 & & n=R_n^2/R_0^2 \end{array}\right\} \quad (12)$$

For convenience in future discussion we will call a, b, c, n the *radius ratios*.

Now if we divide numerator and denominator of each term of equations (9), (10), and (11) by R_0^2 and substitute for R_1^2/R_0^2 its value a from equations (12) we obtain

$$El_t = S_t = \frac{2}{3}\frac{(P_0 - aP_1)}{(a-1)} + \frac{4}{3}\frac{a(P_0 - P_1)}{(a-1)}\frac{R_0^2}{r^2} \tag{13}$$

$$El_p = S_p = \frac{2}{3}\frac{(P_0 - aP_1)}{(a-1)} - \frac{4}{3}\frac{a(P_0 - P_1)}{(a-1)}\frac{R_0^2}{r^2} \tag{14}$$

$$El_q = S_q = -\frac{2}{3}\frac{(P_0 - aP_1)}{(a-1)} \tag{15}$$

Rules for Transformation.—We will notice here, with reference to the transformation, two facts on which we will base rules for future transformations. In what follows we will understand by the words *term factor* a factor that affects a whole term, in contradistinction to a factor that affects a part of a term only.

Comparing the first term of the second member of equation (13) with the corresponding term of equation (9) we can write the first rule.

Rule 1. The non-appearance of R_0^2 in any term involving the radius ratios indicates that the term from which it was formed had in the numerator the same number of *term factors involving the squares of the limiting radii* as in the denominator.

In the first term of the second member of equation (9) the numerator contains a single *term factor* involving the squares of the radii. The denominator similarly contains but one such term factor.

Comparing the last terms of equations (13) and (9) we see,

Rule 2. When R_0^2 appears in the numerator of a term involving the radius ratios, it indicates that the original term contained in the numerator one more *term factor involving the squares of the limiting radii* than in the denominator.

Though the last term of equation (13) contains in numerator and denominator the same number of term factors that involve the radius ratios, the presence of R_0^2 in the numerator indicates that the term from which it was formed had one more such term factor. That factor was R_0^2, and since $R_0^2/R_0^2 = 1$ the factor has disappeared from equation (13).

107. Stresses in a Simple Cylinder.—In a cylinder forming a part of a gun we have three cases to consider. There may be a pressure on the interior of the cylinder and none on the exterior, the atmospheric pressure being considered zero. There may be a pressure on the exterior of the cylinder and none on the interior. Or both exterior and interior pressures may be acting at once, the interior pressure being usually the greater. We will consider the simple cylinder under these circumstances.

Differentiating equation (13) we obtain

$$\frac{dS_t}{dr} = -\frac{8}{3}\,\frac{a(P_0-P_1)}{a-1}\,\frac{R_0^2}{r^3} \qquad (16)$$

and differentiating again,

$$\frac{d^2S_t}{dr^2} = \frac{8a(P_0-P_1)}{a-1}\,\frac{R_0^2}{r^4} \qquad (17)$$

Similarly from equation (14) we obtain

$$\frac{dS_p}{dr} = \frac{8}{3}\,\frac{a(P_0-P_1)}{a-1}\,\frac{R_0^2}{r^3} \qquad (18)$$

$$\frac{d^2S_p}{dr^2} = -\frac{8a(P_0-P_1)}{a-1}\,\frac{R_0^2}{r^4} \qquad (19)$$

First Case. Interior Pressure Only.—Making $P_1=0$ in equation (13) and remembering that r may vary between the limits R_0 and R_1 we see that the smaller the value of r the greater will be the value of the resultant tangential stress. This is more readily seen in equation (16) in which the first differential coefficient of the stress as a function of the radius is negative when $P_1=0$, showing that S_t decreases as r increases. R_0 being the least value of r the tangential stress is greatest at the interior of the cylinder. Since, when $P_1=0$, S_t in equation (13) is positive for all values of r, the stress is one of extension throughout the cross section of the cylinder. When $P_1=0$ in equation (17) the second member is positive, showing that the curve of stress is concave upwards, the axis of r being taken as horizontal. The curve of tangential stress due to an interior pressure only may then be represented in general by the curve t_1 in Fig. 39, the ordinates being the values of the stress, the abscissas the values of the radius.

The numbers at the extremities of the curve are the actual stresses due to an interior pressure $P_0 = 36{,}000$ pounds per square inch in a cylinder one caliber thick. They are calculated from equation (13) by making $P_1 = 0$ and $R_1 = 3R_0$. When $R_1 = 3R_0$ we have $a = R_1^2/R_0^2 = 9$. The equation becomes with these substitutions

$$S_t = \frac{P_0}{12}\left(1 + 18\frac{R_0^2}{r^2}\right) \tag{20}$$

Making $P_0 = 36{,}000$ and $r = R_0$ we obtain $S_t = 57{,}000$; and for $r = 3R_0$, $S_t = 9000$.

Similarly from equations (14), (18), and (19) we determine for the radial stress produced by an interior pressure the general curve

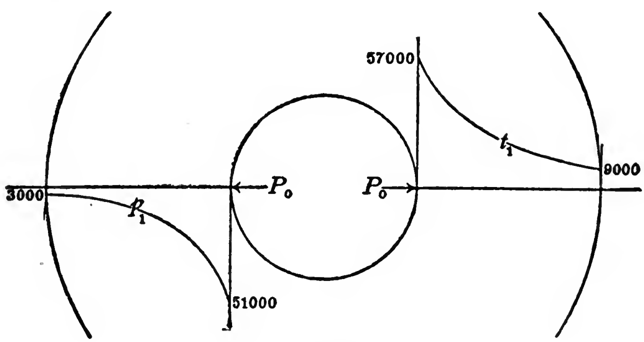


FIG. 39.

p_1, Fig. 39, which shows radial compression throughout the cross section with the greatest stress at the interior. Equations (14) and (15) become for the cylinder one caliber thick

$$S_p = \frac{P_0}{12}\left(1 - 18\frac{R_0^2}{r^2}\right) \tag{21}$$

$$S_q = -\frac{P_0}{12} \tag{22}$$

and comparing these with equation (20) we see that for equal values of r the radial stress from an interior pressure is always less than the tangential stress. The longitudinal stress is less than either.

The radial stresses produced by a pressure $P_0 = 36,000$ are noted on the curve p_1.

We may observe from equations (20), (21), and (22) that the thickness of the cylinder being expressed in calibers, or, what is the same thing, in terms of the interior radius, the stresses developed by an interior pressure are entirely independent of the caliber, and are the same for all cylinders the same number of calibers thick.

108. Second Case. Exterior Pressure Only.—Making $P_0 = 0$ in equations (13) to (19) we may determine the curves of stress for an exterior pressure acting alone. In this case the value of S_t, equation (13), is always negative. The stress is therefore compressive throughout the cylinder. dS_t/dr, equation (16), is positive. S_t therefore increases algebraically with r. d^2S_t/dr^2, equation (17), is negative. The curve is therefore concave downwards. The general curve t_2, in Fig. 40, therefore results.

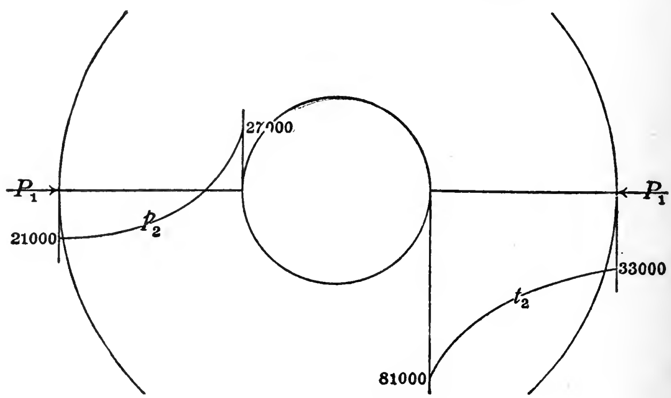


FIG. 40.

In the same way the general curve p_2 is obtained from equations (14), (18), and (19).

The numbers on the curves are the values for the stresses caused by an exterior pressure $P_1 = 36,000$ lbs. on a cylinder one caliber thick, for which $R_1 = 3R_0$ and $a = R_1^2/R_0^2 = 9$.

We see as before that the greatest stresses are at the interior of the cylinder, and that the tangential stress is greater than the radial. The tangential stress is one of compression throughout.

The radial stress is one of compression on the exterior and of extension on the interior.

109. Third Case. Interior and Exterior Pressures Acting.—The curves of stress due to interior and exterior pressures acting at once may be found from the equations, or by combination of the curves of stress due to the pressures acting separately. Thus in Fig. 41, in which the curves from Figs. 39 and 40 are repeated, the lines p_3 and t_3 represent the stresses due to the equal interior and exterior pressures, $P_0 = P_1 = 36{,}000$ lbs.

The position of the resultant curves of stress from interior and exterior pressures acting together will, of course, depend on the relative values of the two pressures. In Fig. 41 the pressures are

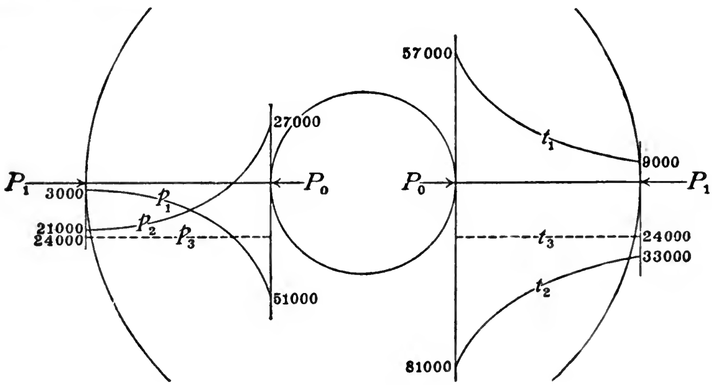


FIG. 41.

equal. In Fig. 42 are shown the curves resulting when the interior pressure is twice the exterior pressure; $P_0 = 36{,}000$, $P_1 = 18{,}000$.

We may see at once from these figures that the tangential resistance of a cylinder to an interior pressure may be greatly increased by the application of an exterior pressure. Assuming that the maximum ordinates of the curves t_1 and t_2, in Fig. 41, are the elastic limits θ and ρ respectively, the interior pressure acting alone would produce the limit of tangential extension. But with the exterior pressure acting the interior pressure has first to overcome the existing compression, and as ρ is usually greater than θ the interior pressure required to produce the stress $\rho + \theta$ would be more than twice as great as the pressure required to produce the stress θ alone. That is to say, that by the application of an exterior pressure we may more than double the tangential resistance of a cylinder to an interior pressure.

Similarly it is seen that the tangential resistance of a cylinder to an exterior pressure is increased by the application of an interior pressure.

110. Limiting Interior Pressures.—In determining the maximum safe pressure that can be applied to the interior of a cylinder there are two cases to be considered; for, as we have just seen, a greater interior pressure may be applied when there is an exterior pressure acting than when the interior pressure acts alone.

Interior and Exterior Pressures Acting.—In Figs. 41 and 42 we see that when both interior and exterior pressures are acting

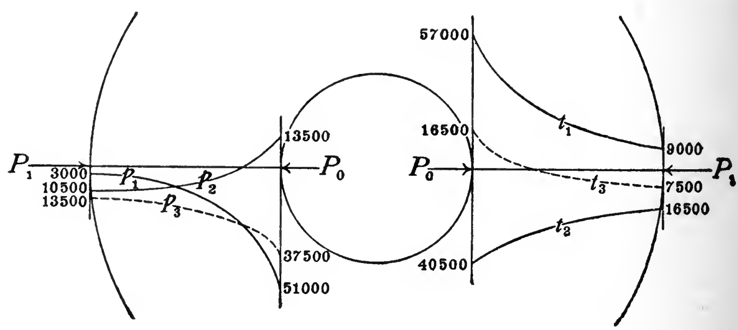


Fig. 42.

on a given cylinder the maximum values of the resultant tangential and radial stresses depend upon the relative values of the pressures. In Fig. 41 the maximum values of the two resultant stresses are equal. In Fig. 42 the resultant radial stress of compression has a greater maximum value than the resultant tangential stress of extension. Therefore when both pressures are acting, in order to determine the maximum permissible interior pressure we must find the values of the interior pressures that will produce the limiting stresses both of extension and of compression, and then adopt the smaller value as the greatest permissible pressure. The maximum stress in either case occurs when $r=R_0$. Therefore make this substitution in equations (13) and (14). Write θ for S_t and $-\rho$ for S_p and solve the equations for P_0. The negative sign is given to ρ, since ρ is an absolute value only, while S_p now represents a stress of compression, which is negative.

$$P_{0\theta}=\frac{3(a-1)\theta+6aP_1}{4a+2} \tag{23}$$

$$P_{0\rho}=\frac{3(a-1)\rho+2aP_1}{4a-2} \tag{24}$$

$P_{0\theta}$ is the interior pressure that acting with the exterior pressure P_1 will produce the limiting tangential stress of extension θ; and $P_{0\rho}$ is the interior pressure that acting with the exterior pressure P_1 will produce the limiting radial stress of compression ρ. The lesser of these two values should, according to our premises, always be used, but it will be seen later that in practice it is usual to neglect consideration of $P_{0\rho}$ and to make use of $P_{0\theta}$ even when it is the greater. Assuming that $\theta=\rho$ we will find by equating the second members of the above equations that $P_{0\theta}$ will be less than, equal to, or greater than $P_{0\rho}$ as follows.

$$P_{0\theta} \lesseqgtr P_{0\rho} \quad \text{as} \quad aP_1 \lesseqgtr \tfrac{3}{4}\theta \tag{25}$$

III. Interior Pressure Only.—We have seen in Fig. 39 that the greatest stress from an interior pressure acting alone is a tangential stress of extension at the interior of the cylinder. This must never exceed θ, the elastic limit for extension. Therefore to find the greatest permissible value of an interior pressure acting alone make $S_t=\theta$ in equation (13), $P_1=0$, $r=R_0$, and solve for P_0.

$$P_{0\theta}=\frac{3(a-1)}{4a+2}\theta \tag{26}$$

If the cylinder is one caliber thick $R_1=3R_0$, $a=9$, and

$$P_{0\theta}=0.63\theta$$

If the cylinder has infinite thickness $R_1=\infty$ and

$$P_{0\theta}=0.75\theta \tag{27}$$

From which we conclude that the greatest possible safe value for an interior pressure acting alone in a simple cylinder is 0.75θ;

and also that comparatively little benefit is derived by increasing the thickness of the cylinder to more than one caliber.

Now if we assume an exterior force applied to the cylinder and assume the effect of this force to be the stress ρ of compression, the tangential stress that must be produced by the interior pressure to reach the limit of safety becomes $\rho+\theta$, and this being substituted for θ in equation (26) it becomes

$$P_{0\theta}=\frac{3(a-1)}{4a+2}(\theta+\rho) \tag{28}$$

From equations (26) and (28) the advantage derived by the interior cooling of cast guns formed of a single cylinder becomes apparent. When the gun is cooled from the interior the layer of metal immediately surrounding the bore cools first and contracts. The cooling and contraction of the subsequent layers then produce a stress of compression on the layers of metal immediately surrounding the bore similar to the stress that would be produced by the application of an exterior pressure. The limiting interior pressure in this case would be obtained by substituting for ρ in equation (28) the value of the stress resulting from the initial compression.

112. Graphic Representation of Limiting Interior Pressures. —The system of graphics devised by Lieutenant Commander Louis M. Nulton, U. S. Navy, for the representation of the relation between the pressures and the shrinkages in cannon helps materially towards a ready understanding of the subject.

We will begin the study of the graphic system with the representation of the limiting interior pressures whose values are given by equations (23) and (24).

We will consider, as is customary in gun construction, that $\theta=\rho$.

Equations (23) and (24) may be put in the following forms, in which A, B, C, and D are constants for any given cylinder.

$$P_{0\theta}=A+BP_1 \tag{23a}$$

$$P_{0\rho}=C+DP_1 \tag{24a}$$

These are the equations of right lines that do not pass through the origin of coordinates. The lines may be constructed, as shown in

Fig. 43, from the axes of P_0 and P_1, the line marked $P_1 P_{0\theta}$ from equation (23*a*) and the line $P_1 P_{0\rho}$ from (24*a*).

The abscissa of any point of the line $P_1 P_{0\theta}$ is the value of P_0, which, acting together with the pressure P_1, whose value is represented by the ordinate of the point, will produce the limiting interior tangential stress of extension θ. Similarly the abscissa and ordinate of any point of the line $P_1 P_{0\rho}$ represent the pressures P_0 and P_1 that acting together on the cylinder will produce the limiting interior radial stress of compression ρ.

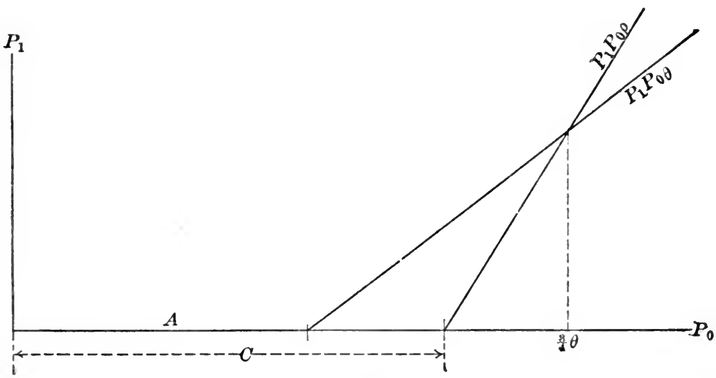


FIG. 43.

For any given value of either interior or exterior pressure, P_0 or P_1, we may at once determine from the figure the value of the corresponding exterior or interior pressure, P_1 or P_0, that will produce the limiting strain of compression or of extension.

For $P_0 = \frac{3}{4}\theta$ the pressure P_1, whose value is then $\frac{3}{4}\theta/a$, see equation (25), will produce in the interior of the cylinder the maximum permissible stresses both of extension and compression.

The figure also shows that the resistance of the cylinder to an interior pressure is increased by the application of an exterior pressure, since P_0 has its least value for $P_1 = 0$.

113. Limiting Exterior Pressure.—This is deduced only for the case of an exterior pressure acting alone, as we will have no occasion to use the limiting values of the exterior pressure when both interior and exterior pressures are acting.

From Fig. 40 we see that the greatest stress from an exterior pressure is a tangential stress of compression at the interior of the cylinder. This must not exceed ρ, the elastic limit for compression. Therefore make $S_t = -\rho$ in equation (13), $P_0 = 0$, $r = R_0$, and solve for P_1.

$$P_{1_\rho} = \frac{a-1}{2a}\rho \tag{29}$$

P_{1_ρ} being the exterior pressure that acting alone will produce the limiting tangential stress of compression ρ.

For the cylinder one caliber thick $R_1 = 3R_0$ in equation (29), $a = R_1^2/R_0^2 = 9$, and

$$P_{1_\rho} = 0.44\rho$$

For the cylinder of infinite thickness $R_1 = \infty$, and

$$P_{1_\rho} = 0.50\rho$$

again showing how little is gained by increasing the thickness of the cylinder beyond one caliber.

114. **Thickness of Cylinder.**—The thickness H needed in a simple cylinder to withstand an interior pressure $P_{C\theta}$ is obtained by replacing a in equation (26) by its value R_1^2/R_0^2, solving the equation for R_1 and then subtracting R_0 from each member.

$$R_1 - R_0 = H = R_0\left(\sqrt{\frac{3\theta + 2P_{0\theta}}{3\theta - 4P_{0\theta}}} - 1\right) \tag{30}$$

Similarly the necessary thickness to withstand an exterior pressure P_{1_ρ} is obtained from equation (29).

$$R_1 - R_0 = H = R_0\left(\sqrt{\frac{\rho}{\rho - 2P_{1_\rho}}} - 1\right)$$

Longitudinal Strength of a Simple Closed Cylinder.—The total pressure acting on each of the end walls is $\pi R_0^2 P_0$. This is assumed to be uniformly distributed over the cross section of the cylinder, $\pi(R_1^2 - R_0^2)$. The longitudinal stress per unit of area is therefore

$$q = \frac{P_0}{a-1}$$

Substituting this value of q in the third equation (4), and for t and p their values from (7) and (8), we obtain for the longitudinal stress in the cylinder

$$El_q = \frac{P_0 - 2aP_1}{3(a-1)}$$

Giving El_q its maximum value, θ or ρ, and solving for P_0, using θ we obtain

$$P_{0\theta} = 3(a-1)\theta + 2aP_1$$

for the interior pressure that will produce the maximum permissible longitudinal stress.

If $P_1 = 0$

$$P_{0\theta} = 3(a-1)\theta$$

a value considerably greater than that expressed in equation (26).

Problems.—1. What is the maximum permissible interior pressure on a steel gun hoop the interior diameter of which is 20 inches and the exterior diameter 28 inches, the elastic limit of the metal being 60,000 pounds per square inch?

Ans. 17,561 lbs. per sq. in.

2. The steel tubes of a water tube boiler are 2 inches in interior diameter and 2.4 inches in exterior diameter. The elastic limit of the metal is 30,000 lbs. per sq. in. What is the limiting interior water pressure? *Ans.* 5103.2 lbs. per sq. in.

3. Using a factor of safety of 1½, what is the limiting interior pressure in an air compressor tank with interior and exterior diameters of 15 and 17 inches respectively? The elastic limit of the metal is 30,000 lbs. per sq. in. *Ans.* 2391 lbs. per sq. in.

4. An iron tube 3 inches in interior diameter is subjected to exterior pressure, 1326.5 lbs. per sq. in. The elastic limit of the metal is 20,000 lbs. per sq. in. What must be the exterior diameter of the tube in order that it may safely withstand the pressure? *Ans.* 3.25 inches.

5. The 6-inch wire-wound gun has the following dimensions at the powder chamber: $R_0 = 4.5$ inches, $R_1 = 12$ inches. If the gun were constructed of a single forging with an elastic limit of 60,000 lbs. per sq. in. what would be the maximum permissible powder pressure? *Ans.* 36,132 lbs. per sq. in.

6. A boiler 6 feet in interior diameter is required to withstand

a steam pressure of 350 lbs. per sq. in. The elastic limit of the metal is 20,000 lbs. per sq. in. What is the maximum thickness required in the shell? *Ans.* 0.64 inches.

7. The cylinder of a hydraulic jack has an interior diameter of 10 inches and a maximum working pressure of 10,000 lbs. per sq. in. The elastic limit of the metal is 40,000 lbs. per sq. in. What thickness of wall is required in order that the factor of safety may be $1\frac{1}{2}$? *Ans.* 2.9 inches.

115. Compound Cylinder, Built-up Guns.—It has been shown that the resistance of a cylinder to an interior pressure may be greatly increased by the application of pressure on the exterior of the cylinder. This is accomplished in practice by shrinking a second cylinder over the first. The shrinkage causes a uniform pressure over the exterior of the inner cylinder and an equal uniform pressure on the interior of the outer cylinder.

The exterior pressure strengthens the inner cylinder against an interior pressure, and at the same time weakens the outer cylinder.

That the full strength of the compound cylinder may be utilized it is important that the shrinkage, and therefore the pressure at the surfaces in contact, be so regulated that under the action of an interior pressure the interior of the weakened outer cylinder will not be stretched to its elastic limit before the inner cylinder has reached that limit. Otherwise we cannot employ the full strength of the inner cylinder. And if the inner cylinder is strained to the elastic limit before the outer cylinder, we cannot employ the full strength of the outer cylinder.

We have seen in Fig. 39 that the tangential stress produced in a single cylinder by an interior pressure diminishes in value as the thickness of the cylinder increases. It is therefore apparent that the stress transmitted to the outer cylinder may, by giving proper thickness to the inner cylinder, be so reduced that when added to the initial stress existing in the outer cylinder this cylinder will not be strained beyond its elastic limit. And by adjusting the thicknesses of the two cylinders and the pressure produced by the shrinkage, the system may be so constructed that the cylinders composing it will both be strained to the elastic limit at the same time.

There is evidently then a relation between the thicknesses of the cylinders and the shrinkage that must be applied in order that the inner and outer cylinders shall be stretched to their elastic limits by the same interior pressure. This relation must be established if we desire to utilize the full elastic strength of the cylinders. And if a third and a fourth cylinder are added the proper relation between the thickness and the shrinkage must be established for these as well.

A modern gun is built up of a number of cylinders assembled by shrinkage, the number of the cylinders, from two to four, depending upon the size and power of the gun. The shrinkage of each cylinder is so adjusted that under the action of the powder pressure, if the pressure becomes sufficiently great, all the cylinders will be strained to the elastic limit at once.

When the powder pressure is acting in a compound cylinder the system is said to be *in action.* When the powder pressure is not acting the system is *at rest.* In action each elementary cylinder except the outer one is subjected to both interior and exterior pressures. At rest the inner cylinder is subjected to exterior pressure only, the outer cylinder to interior pressure only, and the intermediate cylinders to both pressures.

116. System Composed of Two Cylinders.—Assume a system so assembled that under the action of an interior pressure both cylinders will be strained to their elastic limits.

Let R_0, R_1, R_2, Fig. 44, be the radii of the successive surfaces from the interior outwards,

P_0, P_1, P_2, the normal pressures on the successive surfaces when the system is in action,

p_0, p_1, p_2, variations in P_0, P_1, P_2, as the system passes from a state of action to a state of rest,

θ_0, θ_1, the tensile elastic limits of the inner and outer cylinders respectively,

ρ_0, ρ_1, the compressive elastic limits.

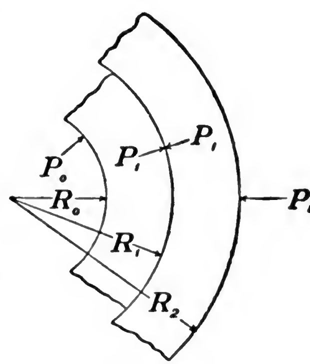


FIG. 44.

E, the modulus of elasticity, assumed the same for both cylinders,

P_{1s}, the normal pressure at the surface of contact *wher the system is at rest.*

Application of Formulas to Outer Cylinders.—It will be well, before proceeding further, to show how the formulas deduced for a single cylinder are made applicable to outer cylinders in compound systems.

Thus equation (23)

$$P_{0\theta}=\frac{3(a-1)\theta_0+6aP_1}{4a+2} \qquad (31)$$

gives the value of the limiting pressure in a single cylinder when the pressure P_1 acts on the exterior.

Let us make this apply to the second cylinder of a compound system.

Substituting for a its value R_1^2/R_0^2 and clearing of fractions in numerator and denominator,

$$P_{0\theta}=\frac{3(R_1^2-R_0^2)\theta_0+6R_1^2P_1}{4R_1^2+2R_0^2} \qquad (32)$$

Now to apply this equation to the second cylinder change all zero subscripts to 1, and subscripts 1 to 2. Making these changes, dividing numerator and denominator by R_0^2, we obtain, since $R_1^2/R_0^2=a$ and $R_2^2/R_0^2=b$,

$$P_{1\theta}=\frac{3(b-a)\theta_1+6bP_2}{4b+2a} \qquad (33)$$

Comparing this equation with (31), from which it has been deduced, we see that the transformation may be immediately made by substituting b for a, and by writing a after the numerical quantities that are affected when we substitute R_1^2/R_0^2 for a and clear of fractions.

We have made this transformation under transformation rule 1, page 197. In equation (31) the numerator forms but one *term factor* and the denominator another. As R_0^2 does not appear in

(31) we know that the equation from which it is derived, equation (32), is of the same form.

The following equation, which refers to pressures in the inner cylinder of a compound system,

$$p_1=\frac{(b-a)}{a(b-1)}p_0 \quad \text{becomes} \quad p_2=\frac{a(c-b)}{b(c-a)}p_1 \tag{34}$$

for the second cylinder, since the absence of R_0^2 in the first equation indicates that its original equation had two term factors involving the squares of the radii in the numerator as well as in the denominator. Therefore consider $1=R_0^2/R_0^2$ as present as a term factor in the numerator of the first equation, change to R_1^2/R_0^2, and write a for this quantity in the second equation.

The equation

$$P_{0\theta}=3(a-1)\theta_0+2aP_1$$

becomes, if made applicable to the second cylinder,

$$P_{1\theta}=\frac{3(b-a)\theta_1+2bP_2}{a} \tag{35}$$

since the absence of R_0^2 indicates that the original equation had one term factor in the denominator as well as in the numerator.

Equation (13) is, for the first cylinder,

$$El_t=S_t=\frac{2(P_0-aP_1)}{3(a-1)}+\frac{4a(P_0-P_1)}{3(a-1)}\frac{R_0^2}{r^2}$$

and becomes for the second cylinder

$$El_t=S_t=\frac{2(aP_1-bP_2)}{3(b-a)}+\frac{4ab(P_1-P_2)}{3(b-a)}\frac{R_0^2}{r^2} \tag{36}$$

Under transformation rule 2 the presence of R_0^2 in the numerator of the last term indicates that the original term had two term factors involving the squares of the limiting radii in the numerator and one in the denominator. Therefore supply the missing factor $1=R_0^2/R_0^2$, change to R_1^2/R_0^2, write a in (36) for this quantity and change the a in (13) to b. R_0^2 is itself not affected in the transformation, as in reality it disappears during the transformation and reappears later by reinsertion.

Whenever in doubt as to a transformation replace the radius ratios by their values, clear the resulting fractions, make the transformation, and rewrite the ratios.

117. System in Action.—When the system is in action the outer cylinder is strained to its elastic limit by an interior pressure. The limiting pressure is given by equation (26), changing the subscripts to conform to the nomenclature above.

$$P_{1\theta}=\frac{3(b-a)}{4b+2a}\theta_1 \tag{37}$$

The pressure $P_{1\theta}$ will extend the inner layer of the outer cylinder to its elastic limit. It is therefore the greatest safe pressure that can be applied to the interior of this cylinder.

The pressure $P_{1\theta}$ just found also acts upon the exterior of the inner cylinder, and the pressure P_0 upon the interior. For the limiting values of the interior pressure we have, under these circumstances, from equations (23) and (24),

$$P_{0\theta}=\frac{3(a-1)\theta_0+6aP_{1\theta}}{4a+2} \tag{38}$$

$$P_{0\rho}=\frac{3(a-1)\rho_0+2aP_{1\theta}}{4a-2} \tag{39}$$

The smaller of these values as determined by the test, equation (25), must be used as the limiting interior pressure. Acting with the pressure $P_{1\theta}$ it brings the inner layer of the inner cylinder to its elastic limit of tension or compression according as $P_{0\theta}$ or $P_{0\rho}$ is the less. At the same time the pressure $P_{1\theta}$ stretches the inner layer of the outer cylinder to its elastic limit.

Equation (37), containing in the second member known quantities only, is solved first, and the value of $P_{1\theta}$ obtained is substituted in equation (38) or (39) as determined by the test. The maximum permissible value of P_0 results.

118. System at Rest.—We have seen in Figs. 40 and 41 that an exterior pressure acting alone on a cylinder may produce a greater stress than when an interior pressure is also acting.

It may be, therefore, that the pressure $P_{1\theta}$ deduced as a safe pressure for the system in action may produce a higher pressure than the inner cylinder can safely withstand when the system is at rest, that is, when the interior pressure P_0 is zero. This must be determined before we can assume, as safe values for the pressures, the values obtained from the consideration of the system in action.

As the system passes from a state of action to a state of rest variations occur in the pressures acting, and consequent variations in the stresses at the various surfaces. p_0 and p_1 represent the variations in the pressures P_0 and P_1 respectively. Since the interior pressure changes from P_0 to 0 we have

$$p_0 = -P_0 \tag{40}$$

because $P_0 - P_0 = 0$; that is, the algebraic sum of the pressure in action and the variation in the pressure is the pressure at rest.

The variations in the tangential stresses due to the variations in the pressures may be determined from equation (13). For the exterior of the inner cylinder, the pressures $-P_0$ and p_1 acting, write $-P_0$ for P_0, p_1 for P_1 and make $r = R_1$.

It will be noticed that when $r = R_1$ in equations (13) and (14) the last factor becomes R_0^2/R_1^2 or $1/a$, which cancels the a in the numerator of the last term.

$$S_t = \frac{-6P_0 - (2a+4)p_1}{3(a-1)} \tag{41}$$

For the outer cylinder equation (13) takes the form of equation (36). For the interior of the outer cylinder, the pressure p_1 acting alone, write p_1 for P_1, make $P_2 = 0$, and $r = R_1$.

$$S_t = \frac{(2a+4b)p_1}{3(b-a)} \tag{42}$$

As the surfaces of contact of the two cylinders form virtually one surface the two values for the variation in the stress at this surface must be equal. Equating the second members of equations (41) and (42) and solving for p_1, we obtain

$$p_1 = -\frac{(b-a)P_0}{a(b-1)} \tag{43}$$

which expresses the relation between the variations in pressure at the interior and exterior of the inner cylinder.

We have designated the pressure at the surface of contact of the two cylinders, system at rest, by P_{1s}. The variation in pressure from the state of action to the state of rest must therefore be

$$p_1 = -(P_{1\theta} - P_{1s}) = P_{1s} - P_{1\theta} \tag{44}$$

because $P_{1\theta} - (P_{1\theta} - P_{1s}) = P_{1s}$. Solving (44) for P_{1s}

$$P_{1s} = P_{1\theta} + p_1$$

and substituting the value of p_1 from equation (43) we obtain

$$P_{1s} = P_{1\theta} - \frac{(b-a)P_0}{a(b-1)} \tag{45}$$

for the value of the pressure on the exterior of the inner cylinder, system at rest.

119. This value of P_{1s} must not exceed the maximum permissible value of an exterior pressure acting alone on the inner cylinder, as given by equation (29).

$$P_{1\rho} = \frac{a-1}{2a}\rho_0 \tag{46}$$

If it does the inner cylinder at rest will be crushed by the pressure applied to strengthen it in action.

The condition that P_{1s} shall not exceed $P_{1\rho}$ may be expressed

$$P_{1s} = P_{1\theta} - \frac{(b-a)P_0}{a(b-1)} \overline{\overline{<}} \frac{a-1}{2a}\rho_0 = P_{1\rho} \tag{47}$$

If the values of $P_{1\theta}$ from equation (37) and of P_0 from (38) or (39) do not fulfill the above conditions these values for the pressures cannot be used for the system in action.

To find the safe values for the pressures in this case we must reduce the value of the first member of (47), P_{1_θ}, until it is equal to the second member, P_{1_ρ}. $P_{1\theta}$ becomes then P_1 and we have

$$P_1 = P_{1_\rho} + \frac{(b-a)P_0}{a(b-1)} \tag{48}$$

This is the relation that must exist between P_1 and P_0 in order that these pressures may be safe for the system at rest.

Equations (38) and (39) express the relations between the safe pressures for the system in action.

If therefore we substitute the lesser value, P_1 from (48), for $P_{1\theta}$ in equations (38) and (39) and solve for P_0 we will obtain the values of P_0 that will be safe both in action and at rest.

$$P_{0\theta} = \frac{3(a-1)\theta_0 + 6aP_{1_\rho}}{(4a+2) - 6\dfrac{b-a}{b-1}} \tag{49}$$

$$P_{0_\rho} = \frac{3(a-1)\rho_0 + 2aP_{1_\rho}}{(4a-2) - 2\dfrac{b-a}{b-1}} \tag{50}$$

The lesser of these two values will be the limiting safe interior pressure that can be applied to the system.

Assuming θ and ρ equal, we will find by equating the second members of equations (49) and (50) that $P_{0\theta}$ will be less than, equal to, or greater than P_{0_ρ} according as

$$a(b-1)P_{1_\rho} \begin{smallmatrix}<\\=\\>\end{smallmatrix} \frac{3}{4}(a-1)\theta_0 \tag{51}$$

120. Graphic Representation. System at Rest and in Action.—Equation (43) expresses the value of the variation p_1 in the exterior pressure for a variation P_0 in the interior pressure. Dropping the negative sign for convenience this equation may be written, for a given cylinder,

$$p_1 = KP_0$$

and may be represented by the line p_1P_0 in Fig. 45. The variation in exterior pressure increases directly with the interior pressure at a rate represented by the inclination of the line p_1P_0.

The lines $P_1P_{0\theta}$ and $P_1P_{0\rho}$ represent, as in Fig. 43, the coordinate limiting pressures for the inner cylinder. $P_{1\theta}$ is the limiting pressure at the surface of contact in action obtained from equation (37). Considering only the tangential stresses, the abscissa of the point c, $P_0=42,955$, is the limiting value of the interior pressure in action. As the system passes from action to rest the exterior pressure falls at the rate represented by the inclination of the line

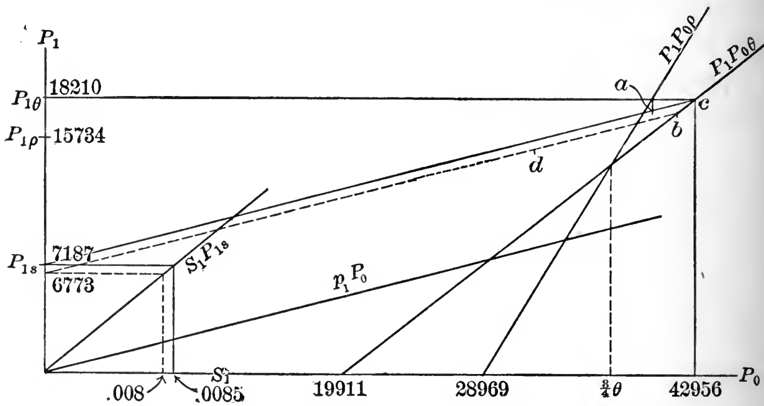


FIG. 45.

p_1P_0. Therefore drawing through c a line parallel to p_1P_0, the point where it cuts the axis P_1 will be the value of P_{1s}, the pressure at rest, P_0 being zero at this point. If the value of P_{1s} is less than $P_{1\rho}$, the limiting value of the pressure at rest calculated from equation (46), the value $P_{1\theta}$ is a safe value. If P_{1s} is greater than $P_{1\rho}$ we cannot use $P_{1\theta}$ in action. In this case we would find the permissible value of P_1 in action by drawing a line from $P_{1\rho}$ parallel to p_1P_0. Its intersection with $P_1P_{0\theta}$ would give the values of the coordinate limiting exterior and interior pressures in action.

121. Maximum Value of the Safe Interior Pressure in a Compound Cylinder.—The stresses and strains produced by any pressure applied to a compound cylinder are exactly the same as would be produced by the same pressure applied to a single cylinder of the same dimensions.

The resultant stresses in the compound cylinder are the algebraic sums of the stresses already existing in the cylinder and those induced by the application of the pressure, and similarly for the strains.

As the resultant stresses may never exceed the elastic limits of extension and compression, the maximum permissible pressure in any cylinder is given by equation (28).

Changing a into b to make of the compound cylinder a single cylinder whose outer radius is R_2, we have

$$P_{0\theta}=\frac{3(b-1)}{4b+2}(\theta+\rho)$$

Making $R_2=\infty$, and therefore $b=R_2^2/R_0^2=\infty$, we obtain

$$P_{0\theta}=\tfrac{3}{4}(\theta_0+\rho_0)$$

which is the greatest possible value of the safe interior pressure in a compound cylinder.

The same result is obtained by substituting $\theta_0+\rho_0$ for θ in equation (27).

122. Shrinkage.—*The absolute shrinkage* is the difference between the exterior diameter of the inner cylinder and the interior diameter of the outer cylinder before heated for assembling, $2ab$, Fig. 46.

The relative shrinkage is the absolute shrinkage divided by the diameter, or the shrinkage per unit of length, ab/R_1. The shrinkages are so small that it is unnecessary to distinguish between the lengths of the radii as affected by the shrinkage.

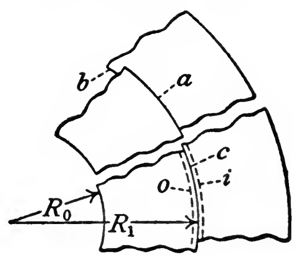


FIG. 46.

The shrinkage diminishes the exterior radius of the inner cylinder, when cold, and increases the interior radius of the outer cylinder, so that the radius R_1 of the surfaces in contact is of a length intermediate between the lengths of the original radii.

The relative shrinkage is, Fig. 46,

$$\phi=ab/R_1=\frac{ci+co}{R_1} \tag{52}$$

The relative compression ci/R_1 is the strain per unit of length produced by the pressure P_{1s} acting on the exterior of the inner cylinder. As the circumference is proportional to the radius the diminution of the circumference per unit of length will be the same as the unit shortening of the radius, and the value of the tangential strain produced by the pressure P_{1s} may be obtained from equation (13), by making $P_0=0$ and $r=R_1$.

$$ci/R_1=l_t=\frac{(2a+4)P_{1s}}{3E(a-1)}$$

The negative sign is omitted, as it simply indicates compression.

The tangential strain co/R_1 at the interior of the outer cylinder is similarly obtained from equation (13), which for the second cylinder takes the form of equation (36). Making $P_1=P_{1s}$, $P_2=0$, and $r=R_1$,

$$co/R_1=l_t=\frac{(2a+4b)P_{1s}}{3E(b-a)}$$

Therefore from equation (52) we have for the relative shrinkage

$$\phi=\frac{2a(b-1)P_{1s}}{E(a-1)(b-a)} \tag{53}$$

The absolute shrinkage is

$$S_1=2R_1\phi=\frac{4R_1a(b-1)P_{1s}}{E(a-1)(b-a)} \tag{54}$$

The exterior diameter of the inner cylinder before shrinkage should be

$$2R_1'=2R_1+S_1 \tag{55}$$

R_1 representing here the interior radius of the outer cylinder before assembling.

The relative tangential compression of the bore due to the shrinkage pressure P_{1s} is found from equation (13) by making $P_0=0$, $P_1=P_{1s}$, and $r=R_0$.

$$l_t=\frac{2aP_{1s}}{E(a-1)}$$

Substituting the value of P_{1s} from equation (54) and reducing we have

$$l_t = -\frac{(b-a)S_1}{(b-1)2R_1} \qquad (56)$$

from which we may obtain at once the tangential compression when the absolute shrinkage is known.

Since, equation (13), $El_t = S_t$ the tangential stress on the bore in pounds per square inch is found by multiplying the relative compression by the modulus of elasticity; 30,000,000 for gun steel.

123. Graphic Shrinkage.—Equation (54) becomes for a given compound cylinder

$$S_1 = MP_{1s}$$

It is represented in Fig. 45 by the line S_1P_{1s}, the axis of S_1 coinciding with the axis of P_0. Different scales are used on these two axes. The coordinates of any point of the line S_1P_{1s} represent, for the given compound cylinder, absolute shrinkage and the pressure produced by it at the surface of contact. Therefore to find the shrinkage necessary to produce the required pressure at rest, P_{1s}, draw the horizontal line from P_{1s} and the vertical line from its intersection with S_1P_{1s}. The intercept on the axis of S_1 is the value of the absolute shrinkage that will produce the pressure P_{1s}. $S_1 = 0.0085$ in the case illustrated.

124. Radial Compression of the Tube.—The value of the pressure on the exterior of the inner cylinder at rest is given by equation (45),

$$P_{1s} = P_{1\theta} - \frac{(b-a)P_0}{a(b-1)}$$

It will be seen from this equation that the larger the value of P_0 used the less will be the value of P_{1s}; and from equation (54) we see that the less the value of P_{1s} the less will be the shrinkage. Therefore if when $P_{0\theta}$ is greater than $P_{0\rho}$ we use $P_{0\theta}$ in equation (45), the resulting shrinkage will be less than if $P_{0\rho}$ were used, and as may be shown by equation (14) the resulting radial stress at the inner surface of the inner cylinder, system in action, will be increased. Now in deducing the value for the shrinkage we have used the pressures calculated to strain the metal to its elastic limit. Therefore with reduced shrinkage the pressure P_{0_ρ} will produce a stress of radial compression at the inner surface of the tube greater than the elastic limit of the metal.

But it is found that the metal of the inner cylinder supported as it is by the outer cylinder has greater strength to resist radial compression than is indicated by the tests of the detached specimens of the metal used in determining the elastic limits; and as the reduced shrinkage resulting from the use of $P_{0\theta}$ in equation (45) reduces all the stresses on the system in a state of rest, and those on the outer cylinder in a state of action, it is the practice to use $P_{0\theta}$ instead of P_{0_ρ} in calculating the shrinkage.

Guns as constructed yield by tangential extension, and the radial over-compression if it exists does not determine rupture. Consequently the tangential elastic resistance of the gun, even though frequently greater than the radial elastic resistance, is taken as the elastic strength of the gun.

125. Prescribed Shrinkage.—Equation (54) expresses the relation between the shrinkage and the pressure that it produces. When for any reason the compound cylinder is not assembled in such a manner as to offer the maximum elastic resistance, as, for instance, when a certain shrinkage less than the maximum permissible shrinkage is prescribed, the pressure due to the prescribed shrinkage may be found by solving equation (54) for P_{1s}. The elastic resistance of the compound cylinder assembled with the prescribed shrinkage will then be found from equations (49) and (50) by substituting for $P_{1\rho}$, which represents the pressure at rest, the value of P_{1s} from equation (54), which is the actual pressure applied.

The prescribed value of S_1 will give in equation (56) the resulting relative tangential compression of the bore.

Graphic Representation.—In Fig. 45 let the point 0.008 be the value of the prescribed shrinkage. By following the broken lines from this point we find on the axis P_1 the resulting pressure at the surface of contact, system at rest; and at b on the line $P_1P_{0\theta}$ the point whose coordinates are the limiting interior and exterior pressures, system in action.

126. Application of the Formulas.—Assuming the caliber of the bore and the thicknesses of the cylinders, to determine the shrinkage and the permissible pressures in the compound cylinder assembled to offer the maximum resistance.

The formulas usually required for a system composed of two cylinders are here assembled for convenience.

$$a = R_1^2/R_0^2, \quad b = R_2^2/R_0^2 \tag{12}$$

$$P_{1\theta} = \frac{3(b-a)}{4b+2a}\theta_1 \tag{37}$$

$$P_{0\theta} = \frac{3(a-1)\theta_0 + 6aP_{1\theta}}{4a+2} \tag{38}$$

$$P_{0\rho} = \frac{3(a-1)\rho_0 + 2aP_{1\theta}}{4a-2} \tag{39}$$

$$p_1 = \frac{(b-a)}{a(b-1)}p_0 \tag{43}$$

$$P_{1s} = P_{1\theta} - \frac{(b-a)P_{0\theta}}{a(b-1)} \leqq \frac{a-1}{2a}\rho_0 = P_{1\rho} \tag{47}$$

$$S_1 = \frac{4R_1 a(b-1)P_{1s}}{E(a-1)(b-a)} \tag{54}$$

$$l_t = -\frac{(b-a)S_1}{(b-1)2R_1} \tag{56}$$

$$P_{0\theta} = \frac{3(a-1)\theta_0 + 6aP_{1\rho}}{(4a+2) - 6\dfrac{b-a}{b-1}} \tag{49}$$

$$P_{0\rho} = \frac{3(a-1)\rho_0 + 2aP_{1\rho}}{(4a-2) - 2\dfrac{b-a}{b-1}} \tag{50}$$

$$El_t = S_t = \frac{2(P_0 - aP_1)}{3(a-1)} + \frac{4a(P_0 - P_1)}{3(a-1)}\frac{R_0^2}{r^2} \tag{13}$$

$$El_p = S_p = \frac{2(P_0 - aP_1)}{3(a-1)} - \frac{4a(P_0 - P_1)}{3(a-1)}\frac{R_0^2}{r^2} \tag{14}$$

In equation (43) above, $-P_0$ has been replaced by its value p_0 from equation (40) in order to make the equation general. $-P_0$ is a particular value of p_0.

In the first member of (47) $P_{0\theta}$ is written for P_0 to make the equation conform to the practice of using $P_{0\theta}$ in determining the shrinkage.

PROCESS.—Use the values of θ and ρ determined in the testing machine.

Find $P_{1\theta}$ from equation (37).

Find $P_{0\theta}$ and P_{0_ρ} from (38) and (39).

Make the test indicated in (47) and if either of the conditions are met use the value of the first member of (47) for P_{1s} in (54) and find S_1.

The values already found for $P_{1\theta}$ and $P_{0\theta}$ are then the limiting safe pressures.

If the first member of (47) is greater than the second,

Find $P_{0\theta}$ and P_{0_ρ} from (49) and (50).

Use P_{1_ρ} from (47) for P_{1s} in (54) to find S_1.

The stresses and strains produced by any pressures are found by means of equations (13) and (14); the tangential stresses and strains from equation (13), the radial from equation (14).

127. Problem 1.—The dimensions of the 4.7 inch siege rifle, at the section marked IV in Fig. 47, are:

$R_0=2.35$ inches, $R_1=3.86$, $R_2=6$. The prescribed elastic limit for both tube and jacket is 50,000 lbs. per sq. in. What will be the shrinkage when the cylinders are assembled to offer the maximum resistance, and what will be the maximum permissible interior pressure?

We have $$a=R_1^2/R_0^2=2.698$$

$$b=R_2^2/R_0^2=6.5187$$

$$b-a=3.8207$$

Equation (37) $$P_{1\theta}=\frac{3\times3.8207}{26.0748+5.396}\,50000=18210$$

(38) $$P_{0\theta}=\frac{3\times1.698\times50000+6\times2.698\times18210}{12.792}=42956$$

(39) $$P_{0_\rho}=\frac{5.094\times50000+5.396\times18210}{8.792}=40146$$

(47) $$P_{1_s}=18210-\frac{3.8207\times42956}{2.698\times5.5187}=7187<\frac{1.698}{5.396}50000$$

(54) $$S_1=\frac{4\times3.86\times2.698\times5.5187\times7187}{30,000,000\times1.698\times3.8207}=0.008489$$

The outer diameter of the tube must therefore be 0.0085 inches greater than the inner diameter of the jacket before assembling.

If P_{0_ρ} were used in place of $P_{0\theta}$ in the determination of P_{1_s}, equation (47), we would obtain $P_{1_s}=7909$, and from (54) $S_1=0.00934$.

128. Graphic Solution.—In Fig. 45 is shown the graphic solution of Problem 1. For this problem the equations take form as follows.

(38) $$P_{0\theta}=19910+1.325P_1$$

(39) $$P_{0_\rho}=28968+0.614P_1$$

(43) $$p_1=0.2566P_0$$

(47) $$S_1=0.0000118P_{1_s}$$

These equations are represented by the lines of the figure drawn to scale. Determine from equation (37) the limiting interior pressure on the jacket, $P_{1\theta}$. From this point on the axis of P_1 draw the horizontal line. It cuts $P_1P_{0\theta}$ at the point c, for which $P_0=42956$. Passing from action to rest the pressure P_1 varies at the rate indicated by the inclination of the line p_1P_0. Therefore draw from c a line parallel to this line. It cuts the axis of P_1 at P_{1_s}, which is the pressure at rest. P_{1_s} is less than P_{1_ρ}, equation (47), also represented in the figure. Therefore $P_{1\theta}$ in action is a safe pressure. Drawing the horizontal line from P_{1_s} and the vertical line from its point of intersection with $S_1P_{1_s}$ we find that the absolute shrinkage that will produce the pressure P_{1_s} is $S_1=0.0085$.

129. Problem 2.—What are the stresses on the inner and outer surfaces of the tube of the gun in the last problem, both at rest and in action, assuming the gun to be assembled with the shrinkage determined in that problem, and using the pressure $P_{0_\rho}=40146$, equation (39), as the interior pressure in action?

The pressure at rest, $P_{1_s}=7187$, determined in Problem 1, acts alone.

Tangential stresses, (13), $S_t(R_0)=-22839$ $S_t(R_1)=-13257$

Radial stresses, (14), $S_p(R_0)=+7613$ $S_p(R_1)=-1970$

In Problem 1 in determining by equation (47) the pressure at rest we used $P_{0\theta}=42956$ lbs. as the pressure in action. The pressure at the outer surface of the tube in action as given by equation (37), $P_{1\theta}=18210$, will therefore be produced only by the interior pressure $P_{0\theta}$. An interior pressure $P_{0_\rho}=40146$ lbs., less than $P_{0\theta}$, will produce a pressure on the exterior of the tube less than 18210 lbs. Equation (43) gives the value of the variation in the exterior pressure due to any variation p_0 in the interior pressure. Making $p_0=42956-40146=2810$ in equation (43) we find $p_1=721$. The pressure P_1 in action, due to the interior pressure P_{0_ρ}, is therefore $18210-721=17489$ lbs.

Making $P_0=40146$ and $P_1=17489$ we find

Tangential stresses, (13), $S_t(R_0)=+45236$ $S_t(R_1)=+15027$

Radial stresses, (14), $S_p(R_0)=-50764$ $S_p(R_1)=-20555$

Had the shrinkage in Problem 1 been determined by the use of $P_{0_\rho}=40146$ in equation (47), that pressure in action would have compressed the inner layer of the tube radially to its elastic limit, 50000 lbs. But with the reduced shrinkage due to the use of $P_{0\theta}$ in equation (47) the pressure of 40146 lbs. exerts a radial stress on the inner layer of the tube of 50764 lbs., which is in excess of the elastic limit.

130. Graphically.—The pressure P_1 in action, used in determining the stresses from equations (13) and (14), may be obtained from Fig. 45. The shrinkage being 0.0085, P_{1_s} is the pressure at rest. From P_{1_s} follow the line of variation in pressure to the point a, whose abscissa is $P_{0_\rho}=40146$. The ordinate of this point is the pressure P_1 in action when $P_0=40146$. Therefore $P_1=17489$.

131. Problem 3.—The shrinkage actually prescribed at the section of the 4.7 inch rifle used in Problem 1 is 0.008 of an inch. What is the elastic resistance of the gun, tangential and radial, at the section, and what is the relative compression of the bore and the stress of tangential compression at the surface of the bore?

(54) $$P_{1_s}=\frac{0.008\times30{,}000{,}000\times1.698\times3.8207}{4\times3.86\times2.698\times5.5187}=6773$$

(49) $$P_{0_\theta}=\frac{3\times1.698\times50000+6\times2.698\times6773}{12.792-6\dfrac{3.8207}{5.5187}}=42178$$

(50) $$P_{0_\rho}=\frac{3\times1.698\times50000+2\times2.698\times6773}{8.792-2\dfrac{3.8207}{5.5187}}=39319$$

(56) $$l_t=-\frac{3.8207\times0.008}{5.5187\times7.72}=-0.000717$$

(13) $$S_t=El_t=21510$$

132. GRAPHICALLY.—From the point 0.008, Fig. 45, on the axis of S_1 follow the broken lines and obtain successively the values found above for P_{1_s}, P_{0_ρ}, and P_{0_θ}.

133. Curves of Elastic Resistance.—In the same way the elastic resistances are found at various sections of the gun, and the curves of elastic resistance shown in Fig. 47 are constructed. By comparing the ordinates of these curves with the corresponding ordinates of the curve of powder pressures it will be seen that the gun has a factor of safety of about $1\frac{1}{4}$ over the part of its length that is subjected to the maximum pressure.

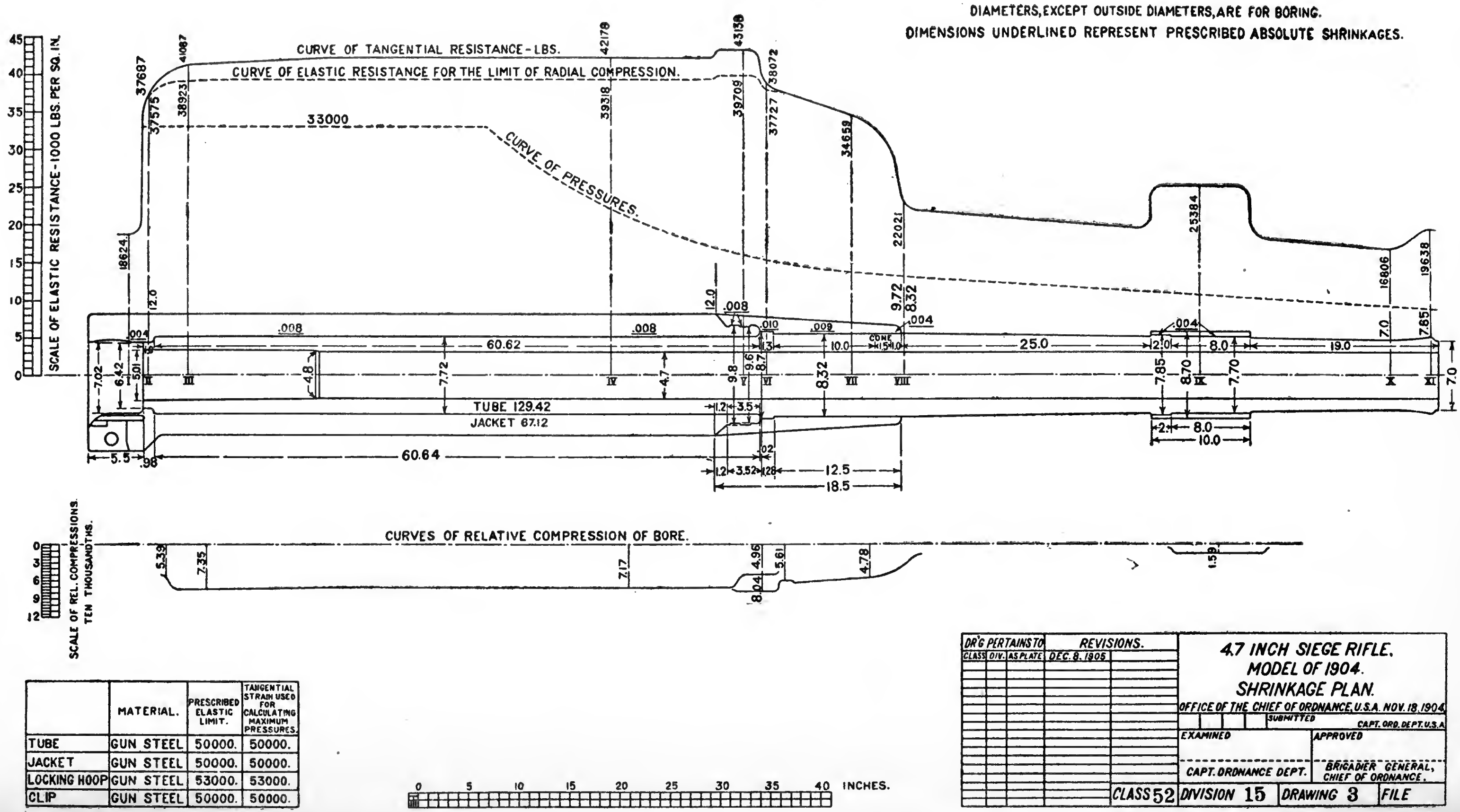
Problem 4.—What will be the tangential stresses in the system assembled as in Problem 3 under a powder pressure of 32,000 lbs. per sq. in.?

$$R_0=2.35 \qquad R_1=3.86 \qquad R_2=6 \quad \text{(See Problem 1)}$$

The pressure at rest, $P_{1_s}=6773$, determined in Problem 3, produces stresses as follows, equations (13) and (36).

Tube, (13), $S_t(R_0)=-21523$ $\qquad S_t(R_1)=-12493$

Jacket, (36), $S_t(R_0)=+18596$ $\qquad S_t(R_1)=+9566$



	MATERIAL.	PRESCRIBED ELASTIC LIMIT.	TANGENTIAL STRAIN USED FOR CALCULATING MAXIMUM PRESSURES.
TUBE	GUN STEEL	50000.	50000.
JACKET	GUN STEEL	50000.	50000.
LOCKING HOOP	GUN STEEL	53000.	53000.
CLIP	GUN STEEL	50000.	50000.

FIG. 47.—Shrinkages and Elastic Resistance of the 4.7-inch Siege Rifle, Model 1904.

The stresses within the elastic limit produced by an interior or exterior pressure on a compound cylinder are exactly the same as would be produced by the same pressure on a simple cylinder of the same dimensions. If therefore we consider the gun as a simple cylinder and calculate the stresses due to an interior pressure of 32,000 lbs., these stresses will be the variations in the stresses in the compound cylinder as it passes from rest to action, and the algebraic sums of the stresses at rest and the variations will be the stresses in action.

Considering the gun as a simple cylinder acted on only by the interior pressure, 32,000 lbs., we obtain from equation (13) for the stresses at the surfaces for which $r=R_0=2.35$, $r=3.86$, and $r=R_1=6$:

Inner surface of cylinder, $S_t=+54265$

At $r=3.86$, $S_t=+22546$

Outer surface of cylinder, $S_t=+11597$

Taking the algebraic sums of these stresses and those above determined for the system at rest, we find for the stresses in action:

Tube, $S_t(R_0)=+32742$, $S_t(R_1)=+10053$

Jacket, $S_t(R_0)=+41142$, $S_t(R_1)=+21163$

134. GRAPHICALLY.—As in the graphic solution of Problem 2, the pressure P_1 corresponding to the interior pressure $P_0=32,000$ is found from Fig. 45 by following the line of variation of pressure for $P_{1s}=6773$ to the point d whose abscissa is $P_0=32,000$. The ordinate of this point is P_1, and this being substituted with P_0 in equations (13) and (14), the values of the stresses are derived.

135. Curves of Stress in Section.—The curves of tangential stress in a section of a gun composed of two cylinders assembled to offer the maximum resistance are shown in Fig. 48. The curves s show the stresses in the cylinders produced by the shrinkage, the system being at rest. The curves r show the stresses in the cylinders for the system in action. The curve p shows the stresses that would result from the pressure P_0 in a single cylinder.

In each cylinder the ordinates of the curve r are the algebraic sums of the ordinates of the curves p and s.

The gain and loss of strength in the compound cylinder as compared with the single cylinder are shown in Fig. 49. The curve t is the curve of tangential stress due to the maximum permissible interior pressure in the single cylinder. The gain in strength in each cylinder of the compound cylinder is shown by the cross-shaded area marked

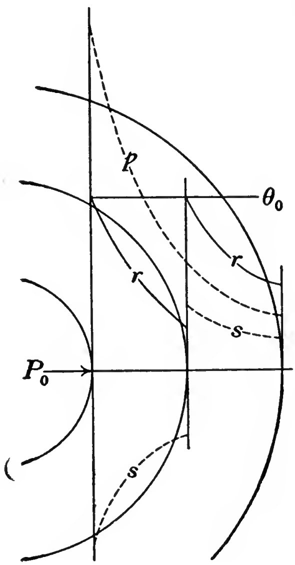


FIG. 48.

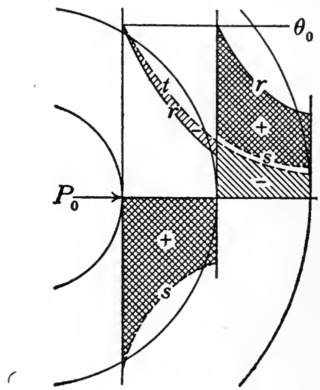


FIG. 49.

with the plus sign, and the loss in strength by the single-shaded area marked with the minus sign. The total tangential stress in the single cylinder is the area between the curve t and the horizontal axis. The inner cylinder of the compound cylinder gains over an equal portion of the single cylinder the shaded area below the axis, representing the compressive stress due to the shrinkage; and loses the area between the curves t and r, since the single cylinder would be under the stress t while the compound cylinder is subjected only to the lower stress r. The outer cylinder at rest being under the stress of extension represented by the area under the curve s, that area is lost to it in action, as compared with the single cylinder, while it gains the area lying between the curves r and t.

136. Problems.—5. A section of the 2.38 inch experimental field rifle, model of 1905, has the following dimensions: $R_0=1.19$

inches, $R_1=1.95$, $R_2=3$. What is the elastic resistance of this section assembled to offer the maximum resistance, and what is the absolute shrinkage? The elastic limit of the metal, nickel steel, is 65,000 lbs. per sq. in.

$P_{1\theta}=23243$ lbs. $P_{0\theta}=55184$ lbs.

$P_{0\rho}=51875$ lbs.

$P_{1s}=9158$ lbs. $S_1=0.00554$ in.

6. The prescribed shrinkage for the above section is 0.005 of an inch. What is the elastic resistance of the section with this shrinkage and what is the stress of tangential compression on the bore?

$P_{1s}=8271$ lbs. $P_{0\theta}=53527$ lbs.

$l_t=0.000879$ in. $S_t=26360$ lbs.

137. Systems Composed of Three and Four Cylinders.—The construction and elastic strength of the larger guns built up of three or four cylinders are determined by considerations similar to those explained in the foregoing discussion. Precaution is taken, by modifying the shrinkages if necessary, that the inner cylinders at rest shall not be injured by the shrinkage pressures of the outer cylinders. The elastic strength of the system, that is, the maximum permissible interior pressure, is the pressure that will bring any one of the elementary cylinders to its elastic limit of extension or compression. In a proper construction the tube is subjected to the greatest pressures both at rest and in action, and consequently if the elastic strength of the gun is exceeded by the powder pressure the tube will yield first.

In Fig. 50 are shown the curves of stress in a section through the powder chamber of the 8 inch gun, model of 1888.

The curves s_1 show the stresses due to the assembling of the jacket on the tube, the curves s_2 the stresses due to the shrinkage of the outer hoop. The curves s_r show the resultant stresses due to both shrinkages.

The numbers on all curves are the actual values of the stresses in tons per square inch due to an interior pressure $P_{0\theta}=23.2$ tons.

The curve p shows the stresses that would be produced by this pressure in a single cylinder of the same dimensions as the compound cylinder.

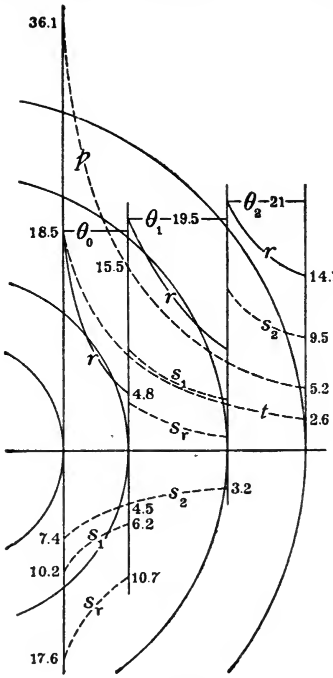


FIG. 50.

The curves r, the stresses in action, are the resultants of the curves s_r and p in each cylinder.

The curve t shows the stresses resulting in a single cylinder from the maximum interior pressure, 12.4 tons, permissible in a single cylinder of these dimensions.

The area between the curves p and t represents the gain in strength due to the compound construction.

Minimum Number of Cylinders for Maximum Resistance.—It will be noticed in Fig. 50 that although in action all the cylinders are stretched to their elastic limits the compression of the tube at rest is less than the elastic limit of compression ρ, assumed equal to θ. In this construction therefore there was not obtained the maximum resistance that the metal was capable of offering. The same conditions exist in the two cylinder gun, as may be seen in Problem 2. The stress of tangential compression at the surface of the bore at rest is found in that problem to be 22,839 lbs., while the elastic limit of the metal is 50,000 lbs.

It may be shown by the equations that in a two or three cylinder gun whose parts have essentially the same elastic limits the conditions that the parts shall be strained to the elastic limit in action and that the tube shall be compressed to its elastic limit at rest are incompatible. That both these conditions may be fulfilled the compound cylinder must be composed of at least four parts.

138. Graphic Construction. Three Cylinders.—The equations deduced for the compound cylinder of two parts are used for the cylinder of three parts, the subscripts and radius ratios in these equations being changed as required. Due to the application of the third cylinder the relation between the variations in pressure in the bore and at the first contact surface, equation (43), takes the form

$$p_1=\frac{(c-a)}{a(c-1)}P_0 \qquad (57)$$

and between the first and second contact surfaces, see equation (34),

$$p_2=\frac{a(c-b)}{b(c-a)}p_1 \qquad (58)$$

The shrinkage at the second surface of contact, equation (54), becomes

$$S_2=\frac{4R_2b(c-1)}{E(b-1)(c-b)}P_{2s} \qquad (59)$$

In addition we need for the graphic representation the pressure at the first contact surface due to the shrinkage pressure at the second surface. This is given by the equation

$$p_{12}=\frac{b(a-1)}{a(b-1)}P_{2s} \qquad (60)$$

in which p_{12} *represents that part of the pressure at the first contact surface that is due to* P_{2s} *only.*

Equation (60) also gives the value of the variation in the pressure at the first contact surface due to a variation in P_{2s}. The equation is deduced by equating the stresses at R_0 due to the pressures p_{12} and P_{2s}.

With the above equations we may now proceed to the graphic representation of the pressures and shrinkages shown in Fig. 51. We will call the three cylinders in order from the center outwards the tube, the jacket, and the hoop.

The first quadrant of the figure, similar to Fig. 45, refers to the tube and the shrinkage at the first contact surface. The second quadrant shows the pressures on the surface of the jacket. The shrinkage at the second contact surface is put in the third quadrant for convenience. The numbers of the equations

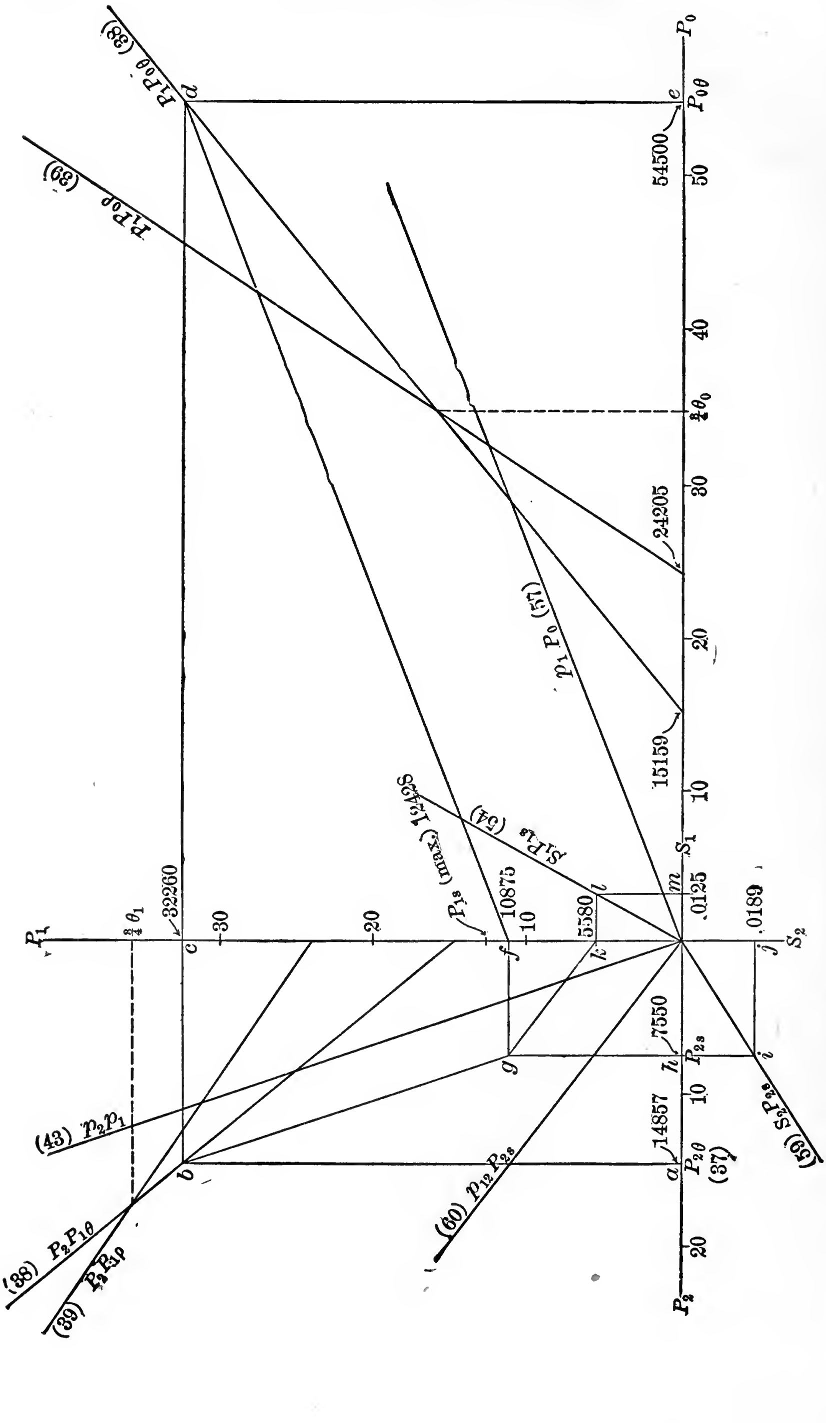


FIG. 51.—Pressures and Shrinkages in 6-inch Rifle, Model 1905, 3 Cylinders.

from which the lines are derived are shown on the lines. It will be understood that the subscripts and radius ratios in any equation must be such as make the equation refer to the particular cylinder to which it is applied.

$P_{2\theta}$ is first determined from equation (37). It will stretch the inner surface of the hoop to its elastic limit in action. It is therefore the greatest pressure that may be permitted on the exterior of the jacket. Draw *ab*, to $P_2P_{1\theta}$, and *bc*. *c* is the pressure P_1 that, acting on the interior of the jacket, will produce the limiting pressure $P_{2\theta}$ on the exterior. Draw *cd*, to $P_1P_{0\theta}$, and *de*. *e* is the value of P_0 in action that will produce the value *c* of P_1 and therefore the limiting pressure $P_{2\theta}$ on the interior of the hoop.

When the system passes from action to rest the pressure on the outer surface of the tube falls along the line *df* drawn parallel to p_1P_0. *f* is the total pressure on the exterior of the tube at rest. It is composed of the pressure P_{1s} due to the first shrinkage and the pressure p_{12} due to the second shrinkage.

The pressure on the outer surface of the jacket falls along the line *bg* parallel to p_2p_1, which line shows the relation existing between the variations in pressure at the two surfaces of the jacket. As the change in interior pressure on the jacket stops at *f* the change in the exterior pressure stops at *g*, and projecting *g* to *h* on the axis of P_2 we find the pressure P_{2s} on the exterior of the jacket at rest. This is the shrinkage pressure, and drawing *hi* and *ij* we find the shrinkage *j* that will produce the pressure P_{2s}.

The total pressure *f* on the exterior of the tube is composed of the pressure due to the first shrinkage and the pressure due to the second shrinkage. The variation in interior pressure on the jacket due to variation in the exterior pressure is given by equation (60), which is represented in the figure by the line $p_{12}P_{2s}$. If therefore we draw *gk* parallel to this line the point *k* will be the interior pressure on the jacket when the exterior pressure is 0, that is before the second shrinkage. The pressure *k* is therefore the pressure due to the first shrinkage only, and the shrinkage that will produce it is obtained by drawing the lines *kl* and *lm*.

For the system to be safe the total pressure *f* on the exterior of the tube must be less than the maximum permissible pressure as given by the last half of equation (47). We will now designate the maximum permissible pressure on the exterior of the tube by $P_{1s(max)}$, since P_{1_ρ} designates now an interior pressure on the jacket.

The values of the pressures and shrinkages marked on the figure apply to the chamber section of the 6-inch rifle, model 1905, the section being assembled to offer the maximum resistance. For the section,

$R_0=4$ inches	$\theta_0=46000$ lbs. per sq. in.
$R_1=5.9$	$\theta_1=48000$
$R_2=8.35$	$\theta_2=47700$ assumed, 53000 actual
$R_3=12$	$\theta=\rho$

The equations become with this data,

(37)	$P_{2\theta}=14857$	(38)	$P_{0\theta}=15159+1.2197P_1$
(38)	$P_{1\theta}=14425+1.2003P_2$	(39)	$P_{0_\rho}=24205+0.64920P_1$
(39)	$P_{1_\rho}=24023+0.6664P_2$	(57)	$p_1=0.39209P_0$
(43)	$p_2=0.33963p_1$	(54)	$P_{1_s}=446400S_1$
(60)	$p_{12}=0.70129P_{2_s}$	(47)	$P_{1s(max)}=12428$
(59)	$P_{2_s}=401610S_2$		

139. Wire Wound Guns.—As shown in Fig. 50 the various cylinders of a built up gun are strained to the elastic limit at the interior surfaces only. It is apparent that if the same thickness of wall is composed of a greater number of cylinders, each cylinder being brought to its elastic limit at the interior surface, more of the total strength of the metal will be utilized. It follows that with a greater number of cylinders the gun may be given the same elastic strength with less thickness of wall.

The most convenient method of increasing the number of cylinders is by winding wire under tension around the tube of the gun. The tension of the successive layers of wire may be so regulated that each layer will be strained to its elastic limit when the system is in action. Usually, however, the wire is wound with uniform tension. In the form of wire the metal in the gun is much more likely to be free of defects, and can be given a much higher elastic limit than when in the form of forged hoops. An elastic limit of over 100,000 pounds is obtained in steel gun wire.

But the elastic strength of the gun is determined by the elastic strength of the tube that forms the bore of the gun; and if the tube is worked only within its elastic limit the wire wound gun cannot be stronger than the built up gun. In the Brown wire wound gun shown in Fig. 5 on page 238, the wire is wound with a tension of 112,000 lbs. per sq. in., compressing the inner surface of the tube beyond its elastic limit without apparent injury. This gun is composed of a lining tube about which are wrapped overlapping sheets of steel 1/7 of an inch thick and of the shape shown in Fig. 6 on page 238. The steel sheets form, about the lining tube, an outer tube which is afterwards wrapped with wire from breech to muzzle. The wire wrapped overlapping sheets give longitudinal stiffness to the gun. Over the wire is shrunk a steel jacket with just sufficient tension to prevent its rotation upon the tube. The jacket is not depended upon to add to the tangential strength of the gun. It takes, however, a part of the longitudinal stress.

The Ordnance Department 6 inch wire wound gun is shown in Fig. 4, page 238. The wire, 1/10 of an inch square, is wound with a uniform tension of 47,400 lbs. per sq. in., much less than in the Brown gun. The wire winding extends over the breech and half way along the chase of the gun.

After 31 rounds had been fired from each of these guns with velocities of about 3280 feet and pressures of about 45,000 pounds, it was reported that the most notable result observed in the test of the guns was the considerable wear of the rifled bore near the seat of the projectile and near the muzzle of the gun. The wear of the bore was much greater than in built up guns of the same caliber fired with velocities of 2600 and 3000 feet.

This indicates that the life of the wire wound gun will be very short if fired with the higher velocities and pressures. In other words we are unable at present to take economical advantage of the greater strength of these weapons. The wire wound gun has, however, a greater reserve of strength when fired under ordinary pressures than has the gun of the same dimensions built up wholly of steel forgings.

No wire wound guns have yet been put in service in the United States. They have been extensively used for some years by the British Government.

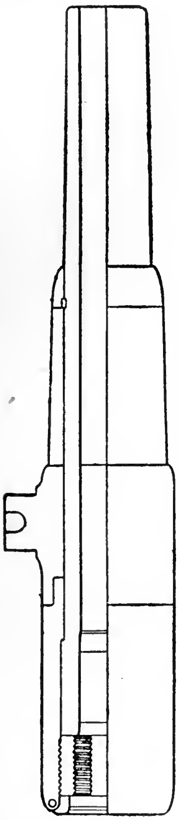
CONSTRUCTION OF GUNS.

140. **General Characteristics.**—The smaller guns in our service, such as the mountain gun, the field and siege howitzers and mortars, are made from single forgings. All other guns are built up. The smaller built up guns of caliber up to 5 inches consist of a central tube (see opposite page), a jacket surrounding the breech end of the tube, and a locking ring which locks the tube and jacket together. Guns of caliber greater than 5 inches have one or more layers of hoops surrounding the tube and jacket. The bore of the tube forms the powder chamber, the seat of the projectile, and the rifled bore. The jacket embraces the tube from the breech end forward nearly half the length of the tube and extends to the rear of the tube a sufficient distance to allow the seat of the breech block to be formed in the bore of the jacket. Through the bearing of the breech block in the jacket the longitudinal stress due to the pressure of the powder gases is transmitted to the jacket and the metal of the tube is thus relieved from this stress.

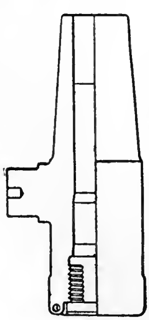
All guns of 6 inch caliber and above are hooped to the muzzle. The 6 and 8 inch guns have a single layer of hoops over the jacket. Guns of caliber larger than 8 inches have two layers of hoops over the jacket.

The construction of the several classes of guns and mortars of the latest models may be seen in the illustrations, pages 237 and 238.

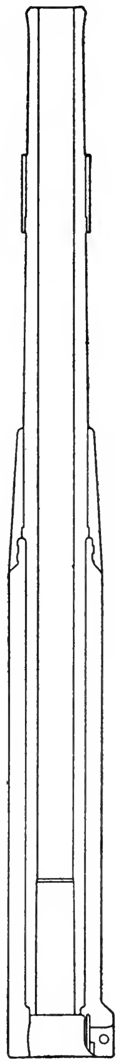
The forward end of the jacket of the field and siege rifles is threaded with a broad screw thread. The rear end of locking hoop is provided with a similar female thread, and the locking hoop is both screwed and shrunk on the jacket. The hoop is also shrunk to the tube, and by means of a bearing against a shoulder on the tube just forward of the jacket it holds the tube and jacket firmly together.



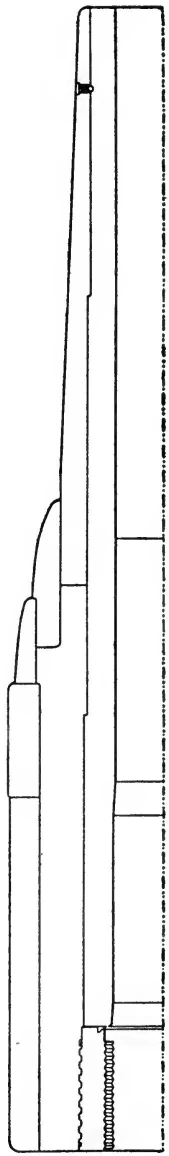
7-inch Howitzer, 1890.



Field and Siege Mortars.



Field and Siege Rifles.



12-inch Mortar.

A noteworthy difference will be observed in the construction of the two 12 inch rifles, Figs. 1 and 2, page 238. While the gun

12-INCH RIFLE, MODEL OF 1888, 34 CALS., 52 TONS.

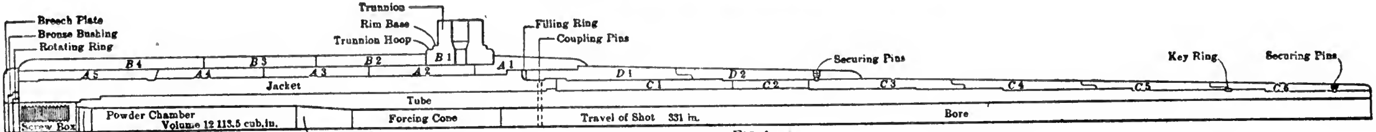


FIG. 1

12-INCH RIFLE, MODEL OF 1900, 40 CALS., 59.10 TONS.

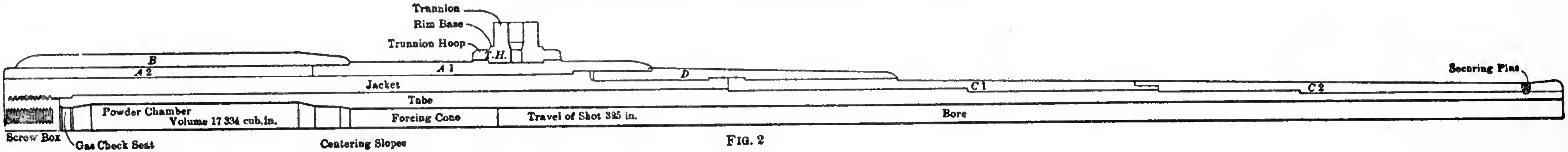


FIG. 2

6-INCH R.F. GUN, MODEL OF 1903, 50 CAL., 19 990 LB.

FIG. 3

INVOLUTE SHEET

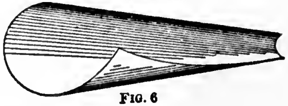


FIG. 6

6-INCH CROZIER, WIRE-WRAPPED GUN, 50 CAL., 20 600 LB.

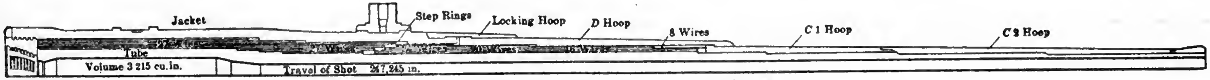


FIG. 4

SECTION ON A-A

FIG. 7

6-INCH BROWN, WIRE-TUBE GUN, 50 CAL., 19 300 LB.

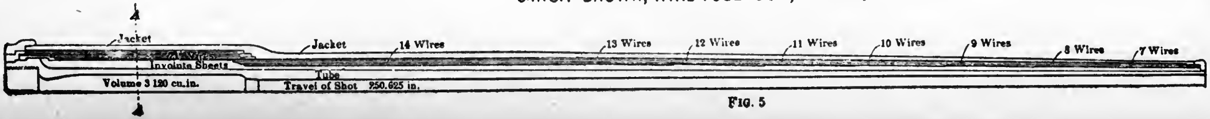


FIG. 5

of the older model, 34 calibers long, is composed of a tube and jacket and 17 hoops, the gun of later model, 40 calibers long, is composed of tube and jacket and but 7 hoops. The reduction in the number of the hoops by increasing their lengths has been made possible by the great advances that have been made in recent years in the production of large masses of steel of the requisite high quality. The improvement has been largely due to the demand of the Ordnance Department, and to the stringent and increased requirements in successive specifications for gun forgings.

By the increase in the size of the hoops there has been gained, in addition to ease and economy of manufacture, largely increased longitudinal strength and stiffness in the gun, which permits the construction of a longer gun without the tendency to droop at the muzzle.

The *D* hoop shown in Fig. 2, page 238, locks together the jacket and the C_1 hoop; and these, bearing against shoulders on the tube, in rear and in front, hold the tube firmly in place. The space behind the *D* hoop, left to accommodate the increase of length of the hoop when heated for shrinking, is filled with a steel filling ring as noted in the 1888 model. The joint between the C_1 and C_2 hoops is coned, as shown exaggerated in Fig. 52. Four securing pins passing through the C_2 hoop near the muzzle assist in preventing forward movement of the hoops under the vibration set up in the gun by the shock of discharge.

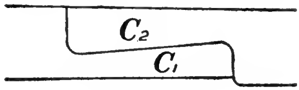


FIG. 52.

As the metal at the muzzle receives support from one side only the gun is thickened there to make the section of equal strength with those near it. The thickening of the metal produces what is called the swell of the muzzle.

141. Operations in Manufacture.—The steel forgings from which the parts of the guns are made are manufactured by private concerns and are delivered rough bored and turned to within about 3/10 of an inch of finished dimensions.

As the parts of the gun are of a general cylindrical form the principal operations in preparing them for assembling are the operations of boring and turning.

In making long bores of comparatively small diameter, as in the tubes of guns, special tools are necessary in order to insure straightness of the bore.

The tube is carefully mounted in the lathe and so centered that any bending or warping that may exist in the long forging will be wholly removed in the operations of boring and turning. The bore is started true with a small lathe tool and continued for a length of about three calibers. The tool shown in Fig. 53 is then

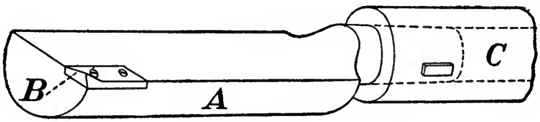


FIG. 53.

used to continue the bore. This tool, called a reamer, has a semi-cylindrical cast iron body, or bit, *A*, carrying the steel cutting tool *B*. It is supported in the boring bar *C*, which is pushed forward by the feed screw of the lathe. The semi-cylindrical bit exactly fits in the bore already started. As the tube rotates, the pressure against the cutting edge *B* forces the bit against the bottom of the bore. This together with the length of the bit prevents deviation of the cutting edge as the tool advances down the bore, and makes the bore a true cylinder.

In order to make the surface of the bore smooth and uniform the light finishing cuts are made with a packed bit or wood reamer, shown in Fig. 54.

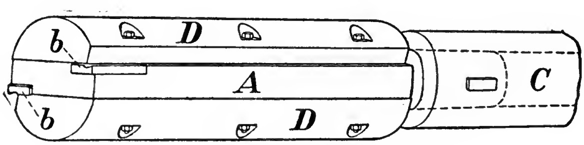


FIG. 54.

The cast iron bit *A* carries two cutters, *b*, at opposite extremities of a diameter. Two pieces *D* of hard wood packing are bolted to the bit and serve to guide the cutters accurately. The tool fits tightly in the bore. The light cut taken and the pressure of the oiled wood packing leaves the surfaces of the bore very smooth and uniform and highly polished.

142. Gun Lathe.—The general features of the lathe, by means of which the larger forgings are bored and turned, are shown in

Fig. 55. The principal parts are: the bed, *B*, made very strong and much larger than for the ordinary lathe; the head stock and cone pulley *C*; the face plate *F*, to which the work *T* is clamped; the slide rest *S*, carrying a cutting tool; the back rests *R*, forming intermediate supports for the tube *T*; the boring bed *O*, supported on the bed proper, *B*, and carrying the boring bar *P* with its tool *Q*; the feed screw *V*, which lies inside the boring bar *P*; and the gears *W*, by which the feed screw is driven.

Motion is communicated to all the parts by the belt *X*, acting on the cone pulley. This causes the face plate and tube to rotate and also communicates motion to the long shaft, not shown in the figure, upon the end of which is the lower gear wheel *W''*. The motion is transmitted through *W'* to *W*, and thence to

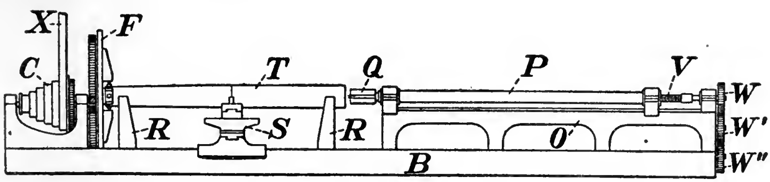


FIG. 55.

the feed screw *V*. By changing the gears any ratio between the velocity of rotation of the tube and that of translation of the tool *Q* can be obtained. It is necessary that there be only one source of motion, since if the feed screw or slide rest were driven independently of the cone pulley which drives the work, a change in the speed of one would not cause a corresponding change in the speed of the others, and damage to the tools, the work, or the machine might result.

The slide rest *S* is driven by a second feed screw not shown.

The back rests *R* can be adjusted to any diameter of forging.

The lathe is supplied with an oil pump, by means of which a stream of oil is forced into the bore while the work is in progress. The chips or cuttings come out at the opposite end of the tube from that at which the tool enters.

Boring and Turning Mill.—The smaller hoops are usually machined on a vertical boring and turning mill, shown in Fig. 56. The work is bolted to the slotted table *t*. The cutting tools are carried in the tool holders *o* at the lower ends of the boring bars *a*. In the illustration one of the boring bars is shown in a vertical position and the other inclined. The table rotates, carrying the work with it. By means of the feed mechanism the cutting tools are fed either vertically or horizontally or at an angle as desired.

On account of the greater difficulty of boring than of turning to prescribed dimensions, the bored shrinkage surface is always finished first. Allowance may then be made in turning the male surface for any slight error in the diameter of the bored surface. The desired shrinkage is thus obtained.

143. Assembling.—The interior diameter of the jacket, when bored to finished dimensions, is less than the exterior diameter of the tube by the amount of the shrinkage prescribed. In order to assemble the jacket on the tube it is therefore necessary to expand the jacket sufficiently to permit its being slipped over the tube into its place. The expansion is accomplished by heat. The jacket is placed in a vertical furnace heated by oil or other fuel to a temperature varying from 600 to 750 degrees Fahrenheit, depending upon the thickness of the forging and the amount of expansion required. Great care is exercised that the heating shall be uniform throughout the length of the forging. The requisite expansion, which in general is about 0.004 of an inch per inch of diameter, is determined by a gauge set to the exact diameter to which the bore should expand. The gauge, held at the end of a long rod, is tried in the bore of the forging in the furnace. When it enters the bore properly the requisite expansion has been attained. Care is taken to avoid overheating which might injuriously affect the qualities of the metal.

When the desired expansion has been attained the jacket is hoisted vertically from the furnace. It will be seen by reference to the figures on page 238 that the shoulders on the tubes of the 12-inch guns are so arranged that the jacket must be slipped over the breech end of the tube, while the arrangement of the shoulders on the wire wrapped tubes of the 6-inch guns require that the tube be inserted into the breech end of the jacket.

The method of assembling is called *breech insertion* or *muzzle insertion* according as the breech or muzzle end of the jacket first encircles the tube. For breech insertion, as in wire wrapped

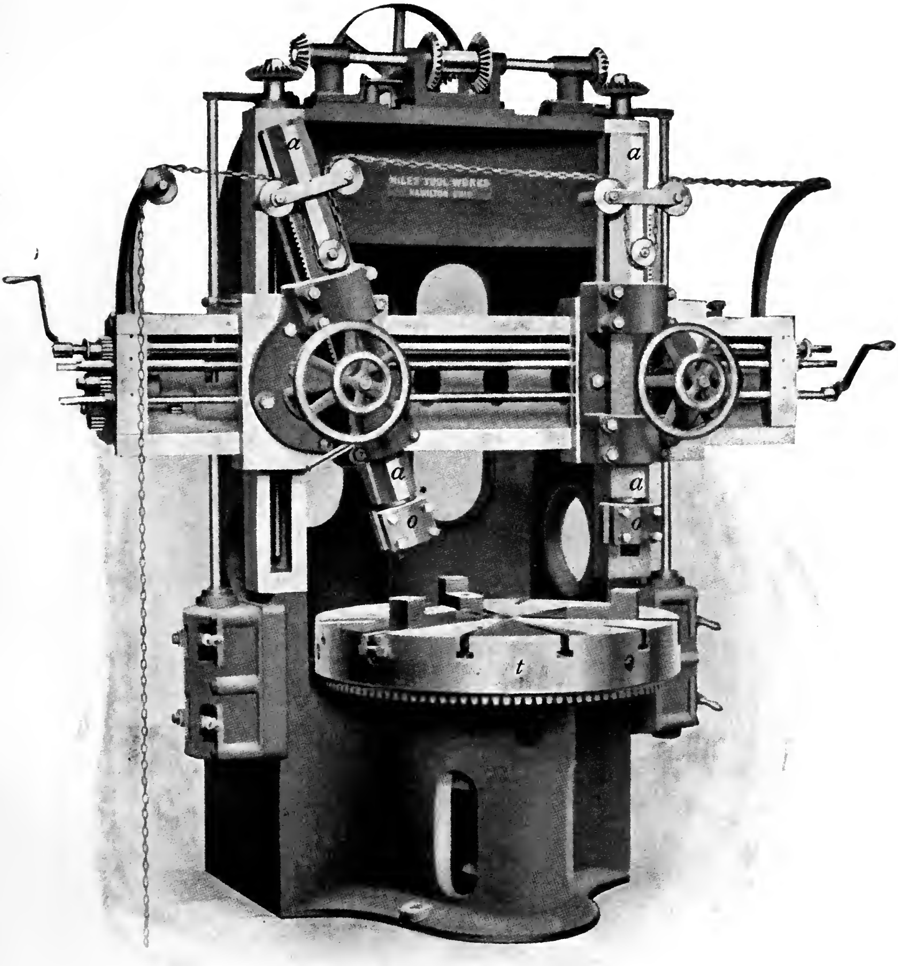


FIG. 56.—Vertical Boring and Turning Mill, 37-inch.

guns, the jacket after being lifted from the furnace is placed upright on a strong iron shelf supported at the mouth of a deep pit, Fig. 57. The tube is then carefully lowered into its seat in the jacket. For muzzle insertion, as in the 12-inch guns, the tube is supported upright in the pit, the breech end up, and the jacket is lowered over the tube.

Cooling of the heated jacket is accomplished by means of sprays of water directed against the forging from an encircling pipe as shown at *D* in Fig. 58. The cooling is begun at the section of the jacket which it is desired should take hold of the tube first,

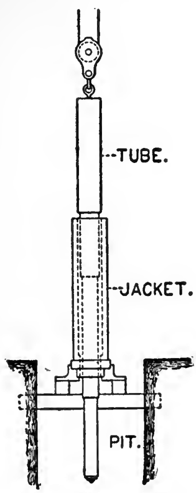


Fig. 57.

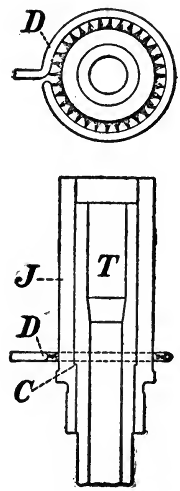


Fig. 58.

as at the shoulder *C*, Fig. 58. As the cooling of the remainder of the jacket progresses the metal is drawn toward the section first cooled, and thus a tight joint at the shoulder is insured. After the jacket has gripped at the shoulder the cooling pipe is moved very gradually upward toward the breech, care being exercised that the jacket shall grip at successive sections in order that longitudinal stresses due to unequal contraction may not be developed in the metal.

The shrinking on of hoops is conducted in practically the same manner as the shrinking of the jacket. When the hoops are small and can be handled quickly they are often assembled to the gun in a horizontal position. Cooling of the hoop is begun at the end toward the jacket, or toward the hoop already in place, in order that contraction shall take place in that direction and make a tight joint between the parts.

When the assembling of all the parts is completed the tube is finish smooth-bored and the exterior of the gun turned to prescribed dimensions.

144. **Rifling the Bore.**—The rifling of the bore is effected in the rifling machine, which is essentially similar to the boring and turning lathe previously described. The gun does not rotate in the rifling machine, but the cutting tool is given the combined movement of translation and rotation necessary to cut the spiral grooves in the bore. The rifling bar takes the place of the boring bar, *P* Fig. 55. The rifling bar, *m* Fig. 59, carrying at its forward

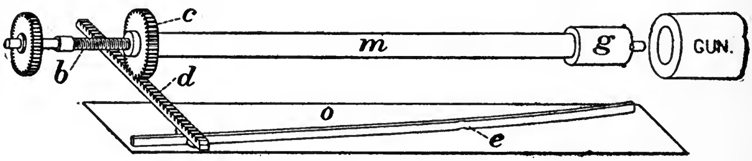


FIG. 59.

end the rifling tool *g* provided with cutters for the grooves, is moved forward and backward by means of the feed screw *b*. The desired motion of rotation is given to the rifling bar by means of the pinion *c* and the rack *d*, which engages on a guide bar *e* bolted to a table made fast to the side of the rifling bar bed. The bar *e* is flexible and is given the shape of the developed curve of the rifling. As the rack travels forward with the rifling bar it is forced to the left by the guide bar, imparting the proper amount of rotation to the rifling bar and cutting tools.

Cutting tools are carried at both ends of a diameter of the rifling tool. At the end of a cut the cutting tools are automatically withdrawn toward the center of the bar and the bar retracted for a new cut.

When a number of guns of the same design are to be manufactured, a spiral groove is cut in the rifling bar itself. A stud fixed in the forward support of the rifling bar works in the groove and gives to the bar the proper movement of rotation. The guide bar with rack and pinion is not then used.

MEASUREMENTS.

145. Necessity of Accurate Measurements.—In order that the gun may be assembled with the required shrinkages the surfaces of the various cylinders composing the gun must be accurately turned and bored to the prescribed dimensions. The dimensions of all parts of the gun must be in accord with the design. The tolerances, or allowed variations from prescribed dimensions, are in general two thousandths of an inch for the diameters of shrinkage surfaces, and one hundredth of an inch in lengths.

Accurate measurements of the various dimensions of every part of a gun are therefore essential.

The exact length of any dimension of a forging is usually obtained by means of one of two instruments, called measuring points and calipers. The points of the instrument used are adjusted until the distance between them is the exact length of the dimension to be determined. The length between the points of the instrument is then measured in a vernier caliper.

Vernier Caliper.—The vernier caliper is shown in Fig. 60. The steel blade *a* graduated in inches and decimal divisions is provided with a fixed jaw *b* and movable jaw *c*. By means of the clamp *d* and small motion screw *e* the movable jaw may be brought accurately to any distance from the fixed jaw. The distance between the jaws is read from the scale and vernier. The least reading of the vernier is one thousandth of an inch. The ends of the jaws *b* and *c* are usually one eighth of an inch wide so that the measurement between their outer edges is a quarter of an inch greater than the reading of the scale.

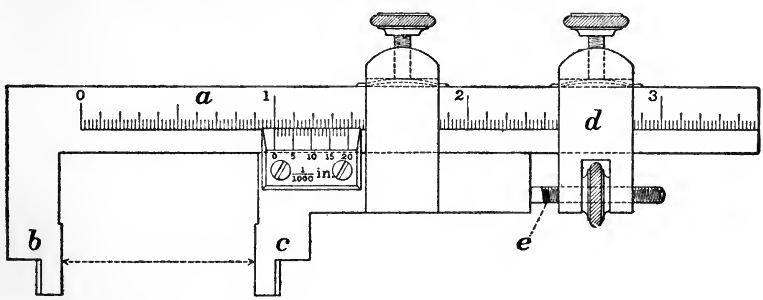


Fig. 60.

Measuring Points.—The measuring point consists ordinarily of a rod of wood into the ends of which are set metal points, Fig. 61. One of these points at least is capable of a small movement out and in. The rod is of wood in order that the heat of the hand may not affect its length. One of the metal points may

FIG. 61.

be provided with a micrometer head from which the movement of the point out and in from a fixed length may be read at once.

Measuring points are used in determining interior diameters and the distance between surfaces that face each other. In measuring an interior diameter at any point in a bore, as at *a*, Fig. 62, one end of the measuring point is placed at *a*. As the diameter is the longest line in the cross section, the end *b* must be moved out until the rod cannot be revolved about the end *a* in the plane of the cross section.

To determine, when touch is made at *b*, that the rod is truly in the cross sectional plane the rod must be revolved in a direction at right angles to this plane, for as seen in Fig. 63 the diameter is

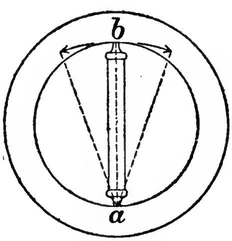


FIG. 62.

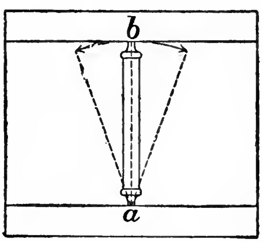


FIG. 63.

the shortest line in the longitudinal plane, and the rod when set to the proper length must be capable of revolution in that plane, touching only at the point *b*. In other words the measuring point has the length of the diameter when the measuring point is incapable of revolution in the cross sectional plane and at the same time capable of revolution in the longitudinal plane.

Similarly when applying the rod to the vernier caliper to read the length of the rod, the movable jaw of the caliper must be brought to such a distance from the fixed jaw that the rod when revolved about one end in two planes at right angles to each other will touch at one point only in each plane of movement. The length of the interior diameter may then be correctly read from the scale of the caliper.

In making measurements the sense of touch is depended upon to determine when contact exists. When the distance that separates a measuring point from a surface is so minute that light cannot be seen between the point and the surface, the lack of contact can be unerringly detected by the touch.

146. The Star Gauge.—In the case of long tubes all parts of which are not readily accessible some means must be adopted of making the measurements at a distance from the operator. The instrument used for this purpose is called a star gauge.

Its general features are shown in Fig. 64. The long hollow

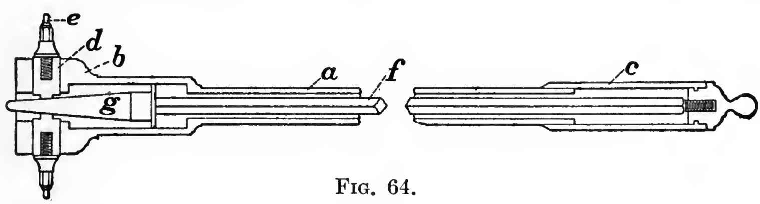


Fig. 64.

rod or staff *a* carries at its forward end the head *b*. Embracing the rear end of the staff is the handle *c* to which is attached the square steel rod *f*. The handle has a sliding motion or screw motion on the end of the staff, and any movement of the handle is communicated through the rod *f* to the cone *g* in which the square rod terminates at its forward end.

The head *b* has three or more sockets, *d*, which are pressed inward upon the cone *g* by spiral springs not shown in the figure. Into these sockets are screwed the star gauge points *e*. Three points are generally used, 120° apart. The points are of different lengths for the different calibers to be measured.

Any movement of the cone forward or backward causes a corresponding movement of the measuring points out or in. The cone has a known taper, and the change in its diameter under the measuring points due to any movement of the handle is marked on a scale at the handle end of the staff. The handle carries a vernier by means of which the scale may be read to a thousandth of an inch. The reading of the scale is the change in length of the diameter that is measured by the points when the handle is at the zero mark.

The staff *a* and rod *f* are made in sections, usually 50 inches long, so that the gauge may be given a length convenient for the measurement of any length of bore.

The star gauge is set for any measurement by means of a standard ring of the proper diameter. The standard rings are of steel, hardened and very carefully ground to the given diameter. If it is desired to measure a 10-inch bore for instance, measuring points of the proper length are inserted in the sockets *d* of the star gauge. The 10-inch ring is held surrounding the points, and the handle *c* of the star gauge is pushed in until the points touch the inner surface of the ring. The handle is then adjusted until the reading of the scale is zero. The instrument is now ready for use.

The gun or forging whose bore is to be measured is supported so that its axis is horizontal. The star gauge is also carefully supported in the axis of the bore prolonged, and in the bore when necessary. The distance of the measuring points from the face of the bore is read from a scale of inches marked on the staff. At each selected position of the gauge the handle is pushed forward until the measuring points touch the surface of the bore. The difference between the diameter of the bore at this point and the standard diameter for which the gauge is set is then read from the scale at the handle in thousandths of an inch.

147. Calipers.—For the measurement of outside diameters calipers are used. The ordinary calipers for measurement of short exterior lengths are shown in Fig. 65. For the measurement of the large exterior diameters of gun forgings, calipers as shown in Fig. 66 are employed. One of the points *a* or *b* is movable and may be provided with a micrometer head. As in the case of interior measurements the caliper must be revolved in two planes about the end that is held at the point from which the diameter is to be measured, and the distance between the points of the caliper must be adjusted until touch is made at one point only in each plane.

The distance between the points of the caliper, as determined by the length between the outer edges of the jaws of the vernier caliper, is then the true length of the exterior diameter.

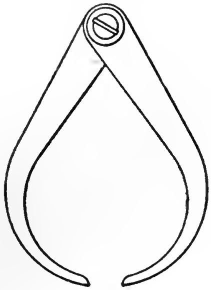


Fig. 65.

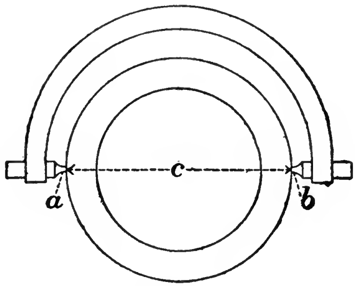


Fig. 66.

The frames of the large exterior calipers required for gun measurements must be made heavy in order that the calipers shall have sufficient stiffness and not be subject to change of form. In

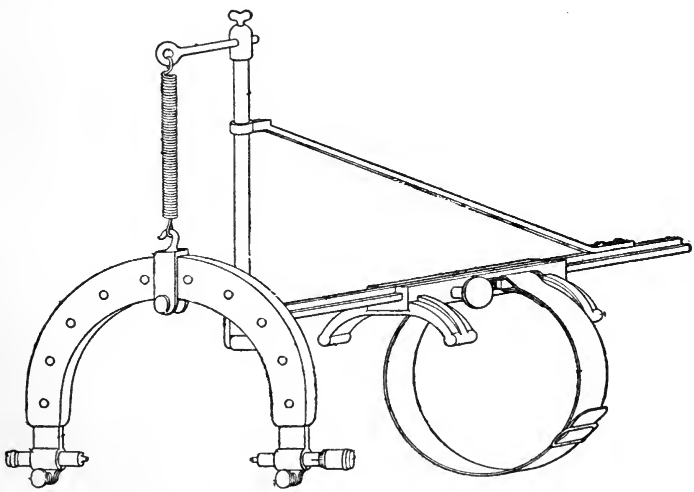


Fig. 67.

use these calipers are therefore supported from above by a spring connection with a frame that is secured to the piece being measured, Fig. 67.

Standard Comparator.—In order to insure accuracy in all measurements, all measuring scales are compared with a common standard. For this purpose the standard comparator is provided.

A heavy metal bar very accurately graduated in inches and decimal divisions rests in a very stiffly constructed cast iron bed. Sliding heads on the bed, one of which carries a reading microscope, may be set accurately at any determined distance apart.

RIFLING.

148. Purpose.—The purpose of the rifling in a gun is to give to the projectile the motion of rotation around its longer axis necessary to keep the projectile point on in flight. The rifling consists of a number of spiral grooves cut in the surface of the bore. The soft metal of a band on the projectile is forced into the grooves by the pressure of the powder gases, whereby a rotary motion is communicated to the projectile.

Twist.—The twist of the rifling at any point in the bore is the inclination of the tangent to the groove, at that point, to the axis

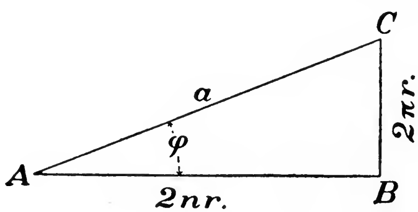


FIG. 68.

of the bore. Twist is usually expressed in terms of the caliber, as one turn in so many calibers. If the inclination of the groove is constant the rifling is of uniform twist. If the inclination of the groove increases from breech to muzzle the rifling has an increasing twist.

Let a, Fig. 68, be the development of one turn of a groove with uniform twist, n the twist in calibers, or the number of calibers in which the groove makes a complete turn, and r the radius of the bore. Then $AB=2nr$, $BC=2\pi r$, and we have

$$\tan \phi = 2\pi r/2nr = \pi/n \tag{61}$$

for the value of the tangent of the angle of the rifling. For the groove with increasing twist ϕ is variable, but at any point its tangent is π/n.

Let v denote the velocity of the projectile at any point of the bore, in feet per second,

ϕ the angle made by the tangent to one of the grooves with an element of the bore,

ω the angular velocity of the projectile,

r the radius of the bore, *in feet.*

The velocity of the projectile along the groove is the resultant of two components, v and $v \tan \phi$, at right angles to each other.

The actual velocity of rotation of a point on the surface of the projectile is $\omega r = \omega d/2$, and this is equal to the component $v \tan \phi$. Therefore

$$\omega d/2 = v \tan \phi \quad \text{and} \quad \omega = 2v \tan \phi/d \tag{62}$$

Increasing Twist.—When the twist is uniform the inclination of the grooves to the axis of the bore is the same throughout the length of the bore, and therefore it is greater at the breech than the inclination of the grooves of an increasing twist that is equal to the uniform twist at the muzzle. The pressure required to cause the projectile to take the grooves is therefore greater in the case of the uniform twist, and the greater resistance offered to the starting of the projectile serves to increase the maximum pressure in the gun. The total energy absorbed by the projectile in taking the rifling is greater with an increasing twist than with the uniform twist on account of the increased frictional resistance due to the continual change in the inclination of the grooves. The total energy absorbed is, however, small compared with that required to give the projectile its velocity of translation.

149. Equation of the Developed Curve of the Rifling.—If the twist increases from zero at the breech uniformly to the muzzle, the equation of the developed curve of the rifling will be of the form

$$y = ax + bx^2$$

which being differentiated twice gives

$$d^2y/dx^2 = 2b$$

That is, the rate of change in the tangent to the groove is constant.

A twist of this form would offer less resistance than the uniform twist to the initial rotation of the projectile. But to still further diminish this resistance, a twist that is at first less rapid than the uniformly increasing twist and later more rapid has been generally adopted for rifled guns. The equation of the semicubic parabola

$$x^{\frac{3}{2}}=2py \tag{63}$$

is generally adopted for the developed curve of the rifling. The twist is assumed at breech and muzzle and the curve between these points is obtained from the above equation.

The tangent to the curve at any point makes with the axis of x an angle whose tangent is dy/dx. The value of the tangent of the angle at any point is π/n, see equation (61), n representing the twist in calibers, the number of calibers in which the groove makes a complete turn.

Therefore, differentiating equation (63),

$$dy/dx=\tan\phi=3x^{\frac{1}{2}}/4p=\pi/n \tag{64}$$

Problem 1.—Determine the equation of the developed rifling curve, and the part of the curve to be used, for the 3 inch rifle, model 1905. The twist is 0 at the breech end, 1 turn in 25 calibers at a point 12.52 inches from the muzzle, and from this point uniform to the muzzle. The length of the rifled bore is 72.72 inches.

The twist at the breech is 0, or one turn in an infinite number of calibers. Therefore n in equation (64) is infinite, $\tan\phi$ is 0 and $x=0$; and from equation (63) y is also 0. The origin of the curve is therefore at the breech.

At 12.52 inches from the muzzle, $x=72.72-12.52=60.2$, and the twist $n=25$.

Substituting these values in equation (64) and solving for p,

$$p=3(60.2)^{\frac{1}{2}}25/4\pi=46.31$$

Substituting in (63) we have for the equation of the developed groove of the rifling from the breech to a point 12.52 inches from the muzzle

$$x^{\frac{3}{2}}=92.62y$$

and the part of the curve to be used lies between the origin and the ordinate for which the abscissa is $x=60.2$. From this point to the muzzle the curve is a straight line making with the axis of x an angle whose tangent is $\pi/25$.

The curve is shown numbered 1 in Fig. 69.

150. Problem 2.—Determine the equation of the developed rifling curve, and the part of the curve to be used, for the 4.7 inch Armstrong gun, 50 calibers long. The twist is 1 turn in 600 calibers at the breech, and 1 turn in 30 calibers at the muzzle. The length of the rifled bore is 203.12 inches.

At the breech $n=600$ and $\tan\phi=\pi/600$

At the muzzle $\tan\phi=\pi/30$

The curve represented by equation (64) passes through the origin of coordinates.

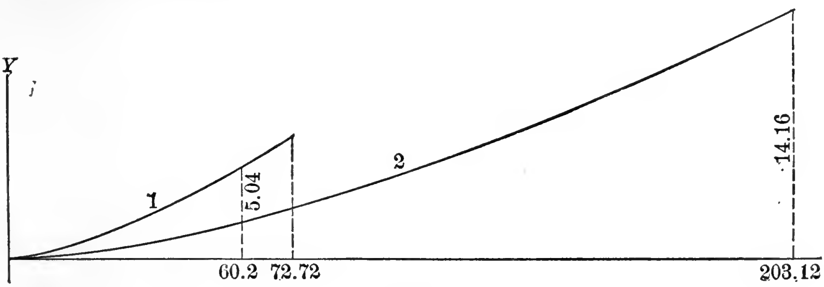


Fig. 69.

Let x_1 be the abscissa of the point of the curve at which the tangent is $\pi/600$. Then $x_2=x_1+203.12$ will be the abscissa of the point at which the tangent is $\pi/30$.

From equation (64)

$$\pi/600=3x_1^{\frac{1}{2}}/4p \qquad \pi/30=3(x_1+203.12)^{\frac{1}{2}}/4p$$

We have two equations involving x_1 and p. Solving we find

$$p=102.2 \qquad x_1=0.51 \qquad x_2=203.63$$

The equation of the developed curve of the rifling is, equation (63),

$$x^{\frac{3}{2}}=204.4y$$

And the abscissas of the extremities of the part of the curve to be used are the values determined for x_1 and x_2.

The curve is shown numbered 2 in Fig. 69.

Service Rifling.—An increasing twist is adopted for the guns in our service. In all guns of recent model the twist is one turn in 50 calibers at the breech, and increases to one turn in 25 calibers at a point about 2½ calibers from the muzzle. The purpose of the uniform twist for a short length at the muzzle is to give steadiness to the projectile as it issues from the bore.

A right handed twist is used in all guns in our service.

The number of grooves depends on the caliber of the gun. In the siege and seacoast guns the number is six times the caliber of the gun in inches. Thus the 5 inch gun has 30 grooves and the 10 inch gun 60. The 3 inch field rifle has 24 grooves.

The shape of the grooves is shown in Fig. 70. The widths of

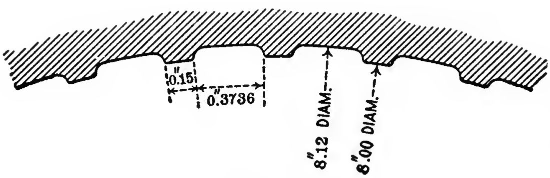


Fig. 70.

land and groove noted in the figure are the same for all guns of 5 inch caliber and greater. The depth of the groove varies from 0.03 of an inch in the 3 inch gun to 0.06 in the seacoast rifles, and 0.07 in the seacoast mortars.

A form of groove called the hook section groove, used in Navy rifles, is shown in Fig. 71. The view is from the breech end.

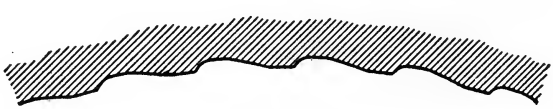


Fig. 71.

The driving edge of the groove makes a sharp angle with the surface of the bore, and the other edge has a gradual slope to that surface.

The depth of the groove in the larger naval guns is 0.05 of an inch.

In the service 30 caliber rifle the depth of the grooves is 0.004 of an inch. It is desirable in small arms to limit the depth of the grooves to the minimum, in order to lessen the thickness of barrel and to permit ready cleaning of the bore. There are four grooves each 0.1767 inches wide. The lands are one third as wide. The twist is uniform, one turn in 10 inches.

BREECH MECHANISM.

151. General Characteristics.—The breech mechanism comprises the breech block, the obturating device, the firing mechanism, and the mechanism for the insertion and withdrawal of the block.

The breech block closes the bore after the insertion of the charge and transmits the pressure of the powder gases as a longitudinal stress to the walls of the gun.

There are two general methods of closing the breech. In the first method the block is inserted from the rear. The block is provided with screw threads on its outer surface which engage in corresponding threads in the breech of the gun. In order to facilitate insertion and withdrawal of the block the threads on block and breech are interrupted.

The surface of the block is divided into an even number of sectors and the threads of the alternate sectors are cut away. Similarly the threads in the breech are cut away from those sectors opposite the threaded sectors on the block. The block may then be rapidly inserted nearly to its seat in the gun, and when turned through a comparatively small arc, say 1/8 or 1/12 of a circle, depending upon the number of sectors into which the block is divided, the threads on the block and in breech are fully engaged and the block locked.

In the second method a wedge-shaped block is seated in a slot cut in the breech of the gun at right angles to the bore, and slides in the slot to close or open the breech.

Variations of these two methods will be noted in the descriptions of the breech mechanism of some of the guns in service.

The breech block is usually supported in the jacket of the gun or in a base ring screwed into the jacket. The seat in the jacket being of greater diameter than could be provided in the tube, the bearing surface of the screw threads on the block is increased, and the length of the block may be diminished.

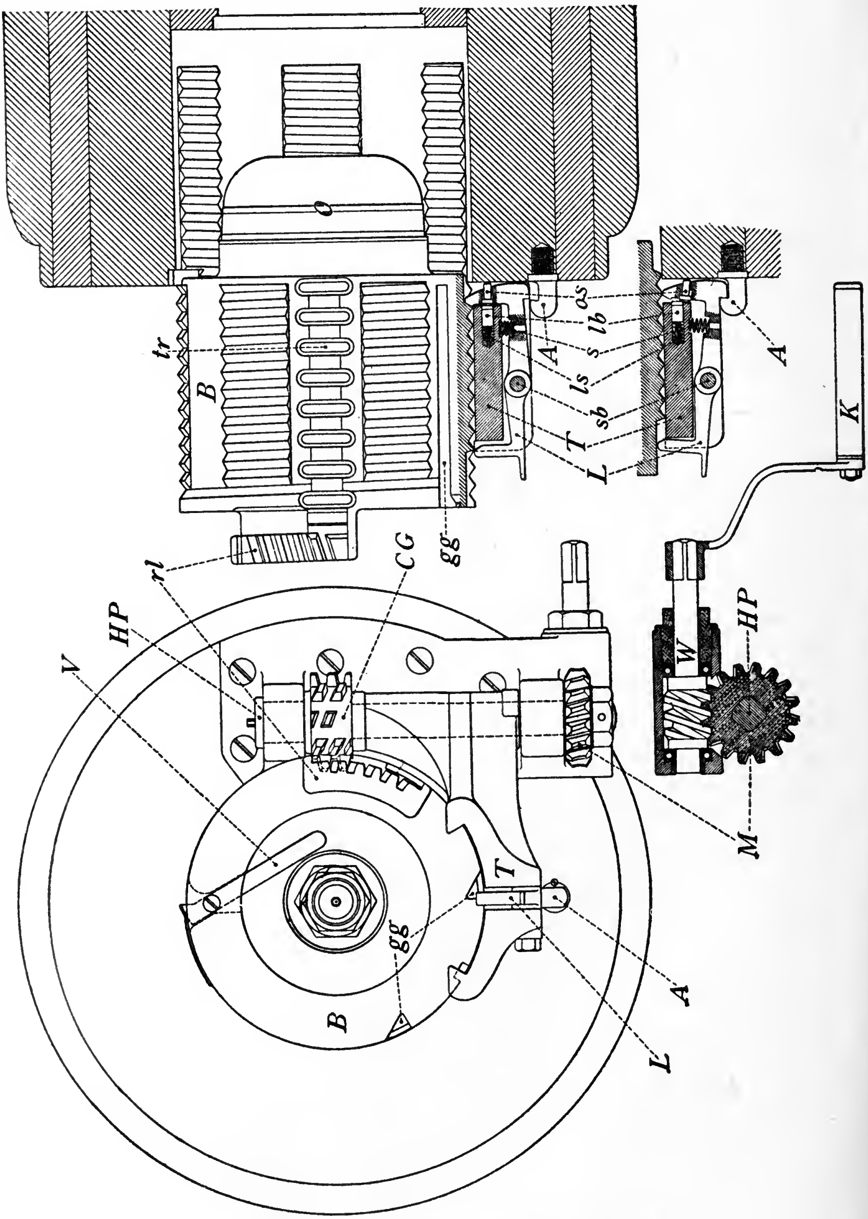


Fig. 72.—Breech Mechanism for Heavy Guns.

The Slotted Screw Breech Mechanism.—The slotted screw breech mechanism is better adapted than any other for use in heavy guns. It is also used in most of the field and siege guns of our service. The form used in the field and siege guns is described with the 3-inch field gun in Chapter VIII.

An example of the slotted screw breech mechanism as used in the heavier guns is shown in Figs. 72 to 74, which represent the breech mechanism of the 12-inch rifle. The breech block *B* has six threaded and six slotted sectors. When the breech is closed the threads on block engage with the threads in the breech. The breech is opened by turning the crank *K* mounted on the shaft *W*. The movement of the crank is transmitted through the worm gear to the hinge pin *HP*, and through the compound gear *CG* to the rotating lug *rl* formed on the rear of the block. The block is thus rotated one twelfth of a turn, and its threaded sectors then lie in the slotted sectors of the breech. Further movement of the crank causes the teeth of the compound gear *CG* to engage in the teeth of the translating rack *tr* cut in a slotted sector of the block. The block is thereby caused to slide to the rear on to the tray *T*, the guide rails of the tray engaging in the grooves *g g* in the block. When the block is sufficiently withdrawn the bottom of the block depresses the rear end of the tray latch *L* and lifts the forward end of the latch out of the catch *A*, where it has been held by the pressure of the spring *s*. The tray is now unlocked from the breech. The upper front toe of the latch *L* engages in a groove in the breech block, locking the block and tray together. The further action of the compound gear on the last teeth of the translating rack *tr* then causes the tray to swing to the right about the hinge pin, carrying the block clear of the breech. As the tray swings clear of the breech the locking bolt *lb* forces forward the operating stud *os* and enters a seat in the latch. The latch is thus locked in its raised position and secures the breech block against being pushed forward off the tray when open.

In closing the breech the operations are reversed in order. When the tray comes in contact with the face of the breech the operating stud *os* forces the locking bolt *lb* from its seat in the latch. The latch is depressed by the spring *s* and thus unlocks the block from the tray.

The two plugs shown in the obturator head of the breech mechanism, Fig. 74, are in the seats provided for the insertion of pressure gauges when it is desired to measure the pressure in the gun.

In recent mechanisms of this type there is added a locking device which locks the block in position when closed and insures against the opening of the block by the pressure of the powder gases. The locking bolt is withdrawn by hand before opening the block.

152. Bofors Breech Mechanism.—The mechanism shown in Figs. 75 to 78, known as the Bofors breech mechanism, is most suitable for guns of medium caliber. It is applied to the 6-inch gun in our service. The block, *b* Fig. 75, is ogival in shape and

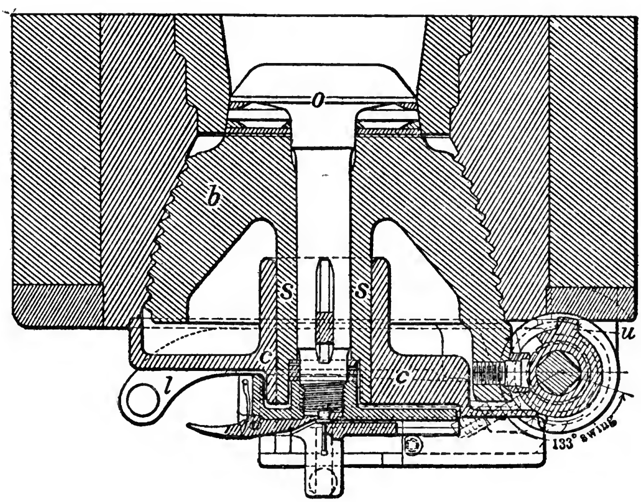


FIG. 75.

has six threaded and six slotted sectors. With the ogival shape a very small retraction to the rear is necessary before the block may be swung open. In the 6-inch gun this retraction is 1.2 inches, just sufficient to withdraw the obturator *o* from its seat in the bore. The block is supported when the breech is opened by the block carrier *c* provided with a central tube which embraces a spindle *s* formed in the block.

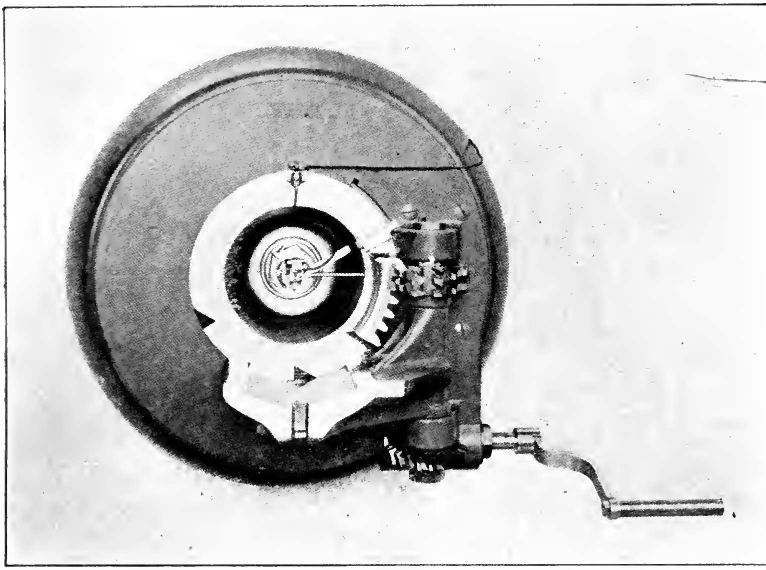


Fig. 73.—Closed.

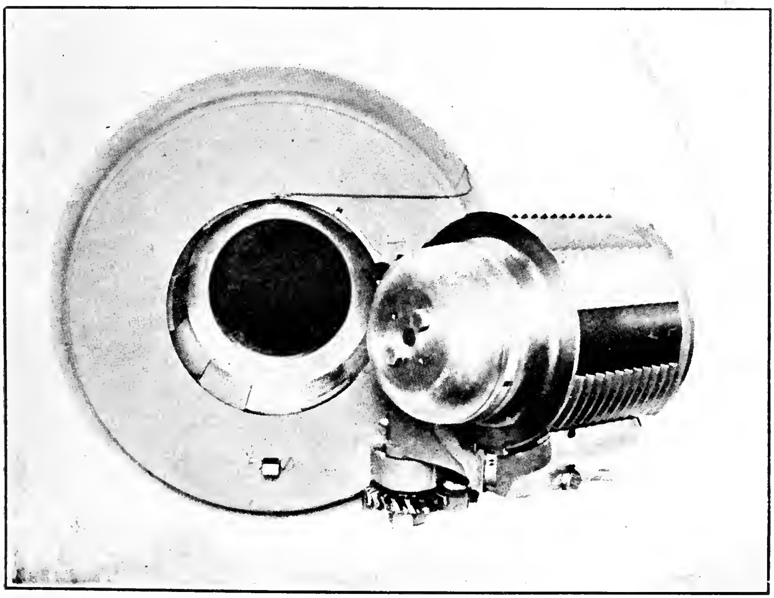


Fig. 74.—Open.

Breech Mechanism for Heavy Guns.

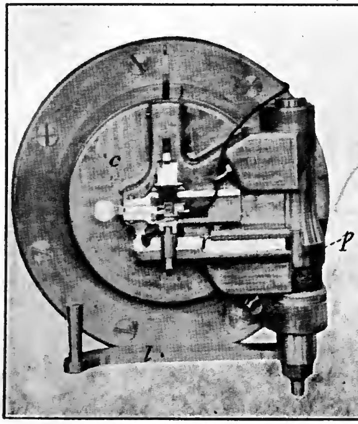


Fig. 76.—Closed.

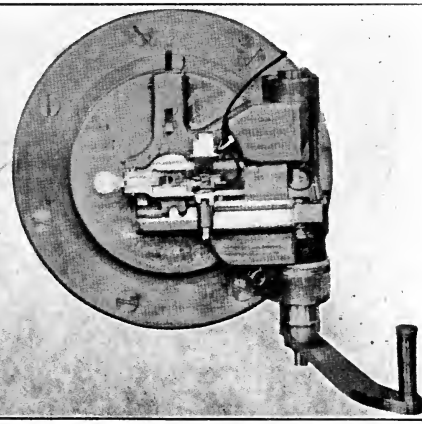


Fig. 77.—Block Unlocked, Ready to Swing Open.

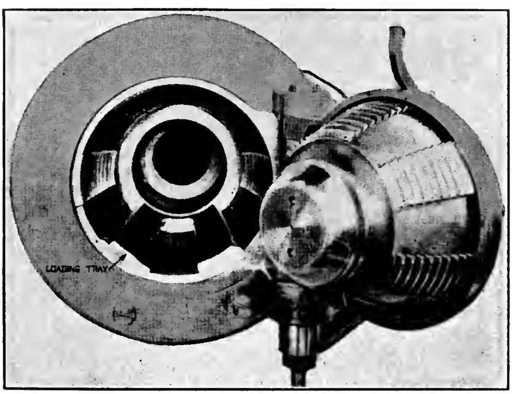


Fig. 78.—Open.

Bofors Rapid Fire Breech Mechanism.

This mechanism is not applicable to the larger guns because the greater weight of the breech blocks in these guns requires better support than can be conveniently given by this method.

The mechanism is actuated by means of the lever *l*, Fig. 76, which is attached to the lower end of the hinge pin. A spool *p* mounted on the hinge pin has teeth cut near its lower end which engage in the rack *r*. The rack slides in a horizontal groove cut in the block carrier *c*, and the teeth at its left mesh with corresponding teeth on the hub of the breech block which projects through the rear face of the carrier.

When rotation of the block is completed a lug, *u* Fig. 75, on the spool engages in a slot at the rear end of the block and translates the block slightly to the rear. Before this translation is complete the block carrier is unlocked from the gun, and swings to

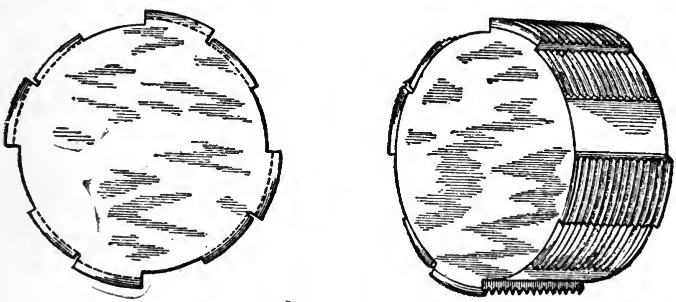


FIG. 79.

the rear with the block, fully uncovering the bore. The loading tray, shown in Fig. 78, the purpose of which is to protect the threads of the breech from injury as the shot is put into the bore, remains permanently in the breech. When the block is entered and rotated the tray is pushed aside by the threads on the block until it covers the slotted sector. On opening the block it is brought back into the position shown.

In the breech mechanism shown in Fig. 74 the loading tray is a separate piece placed in the breech by hand when loading, and removed before closing the block.

153. The Welin Breech Block.—The Welin breech block, largely used in naval ordnance, has the threaded sectors arranged in steps at different distances from the center of rotation, as shown in Figs. 79 and 80. By this means the threaded area may cover two thirds, three fourths, or even a larger portion of the surface of the block. A large increase in threaded area is thus secured over that obtained on a cylindrical block with alternate threaded sectors, and the block may therefore be made smaller. The amount of rotation required in locking and unlocking is also diminished, one twelfth of a turn sufficing for the block shown in Fig. 79, and one sixteenth for the block of Fig. 80.

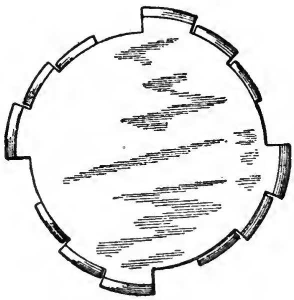


Fig. 80.

Obturation.—There must be provided at the breech of the gun some device that will prevent the powder gases from passing to the rear into the threads and other parts of the breech mechanism. If any passage is open to the gases they are forced through it with great velocity by the high pressure existing in the bore. Their velocity together with their high temperature gives to them great erosive power, and the threads and other parts of the breech mechanism subject to their action are eroded, channeled, and worn away to such an extent that the breech mechanism is soon ruined and the gun is rendered useless.

In guns that use fixed ammunition the obturation is performed by the cartridge case, which expands under the pressure in the bore to a tight fit against the walls of the gun. The breech mechanism of these guns contains, therefore, no obturator parts.

With the slotted screw breech block two systems of obturation are used. They are known by the names of their inventors, DeBange and Freyre.

154. **The DeBange Obturator.**—This system is in the most general use. It is seen at *o*, Figs. 72 and 75, in the breech mechanisms already described. The details are shown in Fig. 81. The obturator consists of the steel mushroom head *h* with the spindle *s*, the pad *p*, the split steel rings *r*, and the steel filling-in disk *d*. The pad *p* is made of asbestos, tallow, and paraffine or other substance, that together form a plastic mixture that melts only at a high heat. The ingredients are mixed and then pressed into

shape under a hydraulic press and protected by a cover made of canvas or of asbestos wire cloth. The split rings, *r* Fig. 81 and

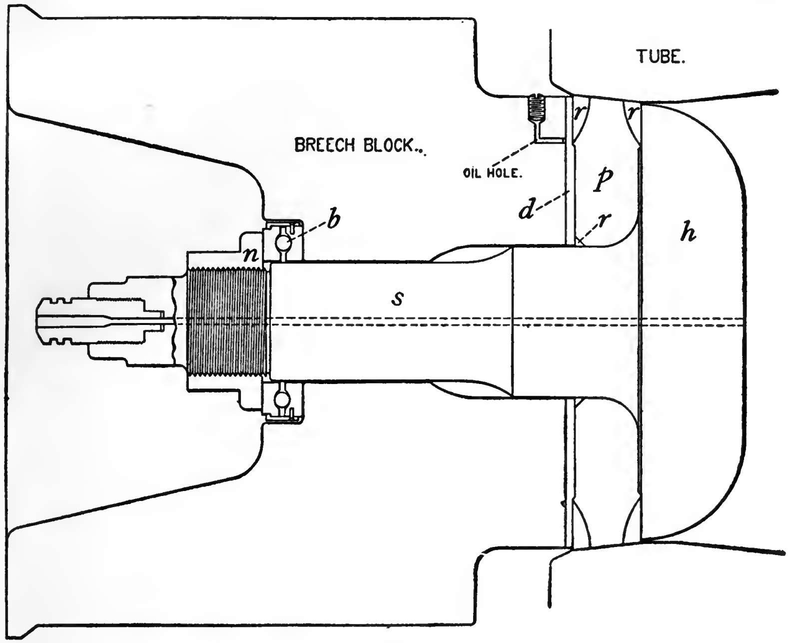


Fig. 81.

Fig. 82, are hardened, and their outer surfaces, which are coned toward the front, are very carefully ground, so that their diameters when the rings are free are 0.01 of an inch larger than the diameters of the conical seat in the bore. The edges of the rings therefore always bear against the walls of the bore.

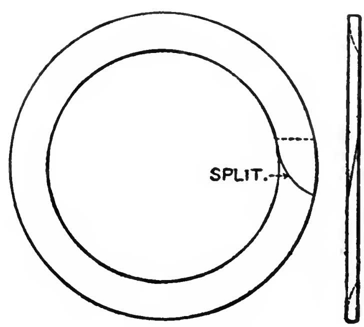


Fig. 82.

The pressure of the gases against the mushroom head compresses the elastic pad and further presses the split rings against the walls of the bore, thus effectually preventing the passage of gas to the rear.

The smaller split ring surrounding the spindle serves to prevent escape of the pad composition between the filling-in disk and the spindle.

The spindle *s* passes through a central hole in the breech block. The obturator parts are held in place by the split nut *n* clamped on the spindle. The nut bears against a shoulder in the block through the ball bearing *b*. It will be seen that the breech block may rotate independently of the obturator parts, so that in opening the breech the rotation of the block is not affected by any sticking of the obturator to its seat in the gun. On retraction of the block the obturator is readily withdrawn from its conical seat.

A vent is drilled the full length of the obturator spindle to afford a passage for the flames from the primer to the powder charge in the gun. The two grooves at the rear end of the spindle serve for the attachment of the firing mechanism.

The Freyre Obturator.—The Freyre obturator shown in Fig. 83 is used in the 3.6 inch field mortar. The head *g* is cone shaped.

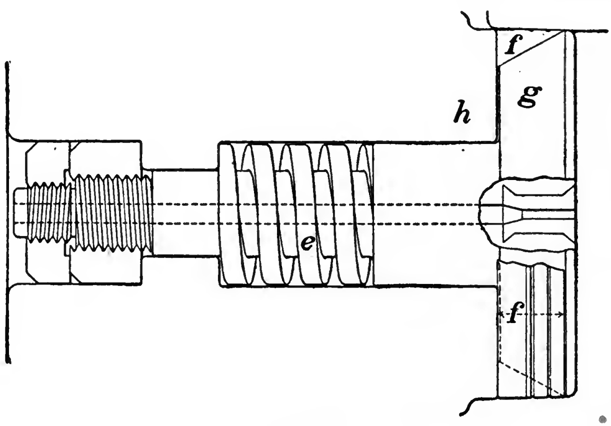


Fig. 83.

In rear of it resting against the head of the breech block *h* is the cone shaped steel ring *f*. The head *g* is constantly pressed forward by the spring *e*. Under the action of the powder pressure the head is forced to the rear and expands the ring *f* against the walls of the bore.

With this obturator the breech mechanism is comparatively short and light in weight, which is an important advantage in a field mortar. The obturator ring with its thin front edge is, however, readily subject to accidental injury, which would render the obturation imperfect.

155. Firing Mechanism.—A seat for the firing mechanism is formed on the rear end of the obturator spindle by two grooves, *g* Fig. 84, cut in the spindle. A hinged collar *k* embraces the end of the spindle. The housing *h* screws over the collar and is locked

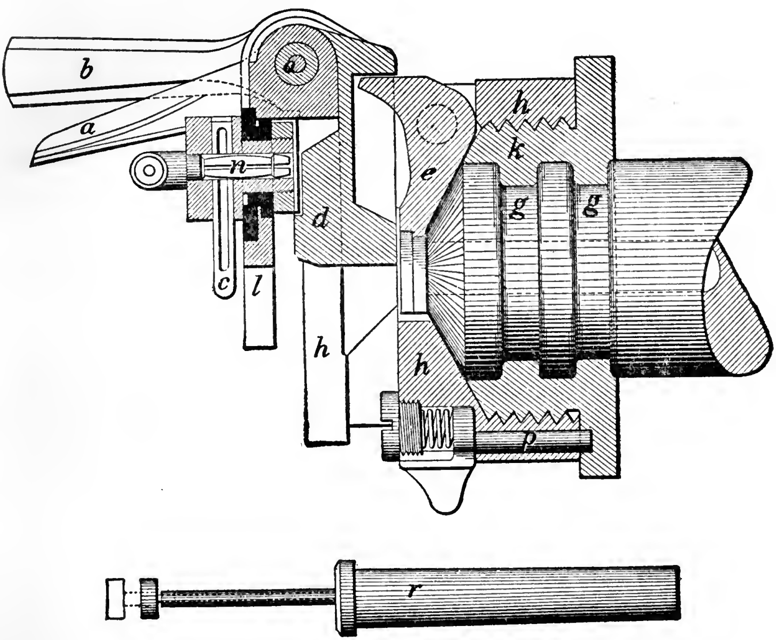


Fig. 84.

to it by the spring pin *p*. The ejector *e* pivoted in the housing has at its lower end a forked seat for the head of the primer. Projecting ribs on the front face of the housing form guides for the slide, *d* Fig. 84 and Fig. 85. The slide is moved up or down by means of the handle *b*, the catch lever *a* being first pressed to release a holding catch. Pivoted at *o* in the slide is the slotted firing leaf *l*, which carries the insulated brass contact clip *c* and is provided with an eye into which the hook of the lanyard engages.

The slide being at its uppermost position, the primer *r* is inserted in the vent in the obturator spindle, the head of the primer resting in its seat in the ejector. The slide is then pushed down. The firing leaf *l*, by means of the slot, embraces the insulated primer wire just in front of the button at its outer end. The two halves of the contact clip *c* spring apart and embrace the uninsulated button.

If the breech is closed, a pull on the lanyard rotates the firing leaf *l* about its axis *o*, drawing out the primer wire and firing the primer by friction; or the closing of the electric circuit, which enters the mechanism through the electric terminal *n*, will fire the primer electrically. The electric current passes through insulated parts to the platinum firing bridge inside the primer and thence through the body of the primer to the metal of the gun and to the ground.

Firing by either of these methods cannot be accomplished unless the slide *d* is all the way down and the breech is fully closed.

A safety lug on the right side of the housing engages in a groove in the firing leaf and prevents the latter being drawn to the rear before the slide is all the way down. The contact clip engages the primer button only in the last part of the downward movement of the slide.

The inner end of the safety bar, *s* Fig. 85, also engages the firing leaf. The outer end of the safety bar embraces a stud projecting from the safety bar slide, *i* Fig. 87, and the safety bar slide carries at its outer end a stud that engages in a groove cut in the gun. The groove is so shaped as to withdraw the safety bar only at the last part of the movement of the block in closing. At this moment also the parts of the electric circuit breaker, fixed one to the block and the other to the gun, Fig. 87, come into contact.

It will be seen therefore that the primer cannot be fired until the breech block is locked.

We have seen that the breech block rotates independently of the obturator spindle. In order then that the firing mechanism may always be in an upright position when the breech is closed, a guide bar, *m* Fig. 87, fixed at one end to the housing and at the other end to the block, causes the mechanism to rotate on the spindle with the block.

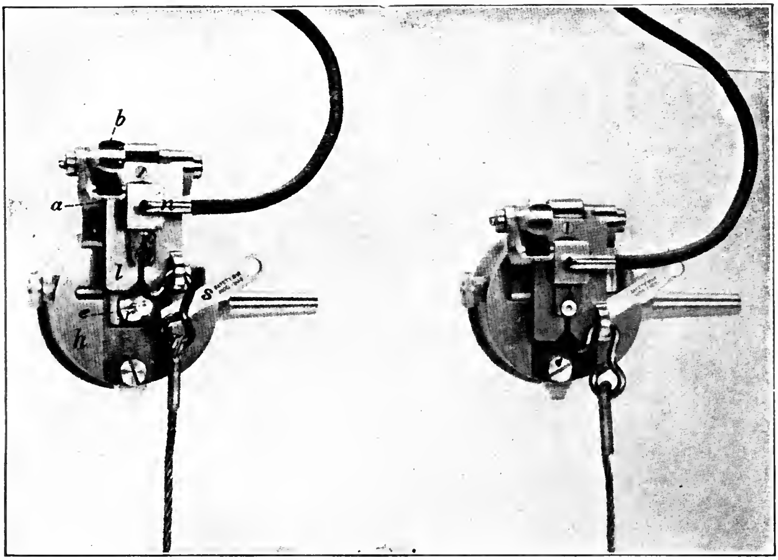


Fig. 85.—Slide Raised and Primer Inserted.

Fig. 86.—Slide Lowered Ready for Firing.

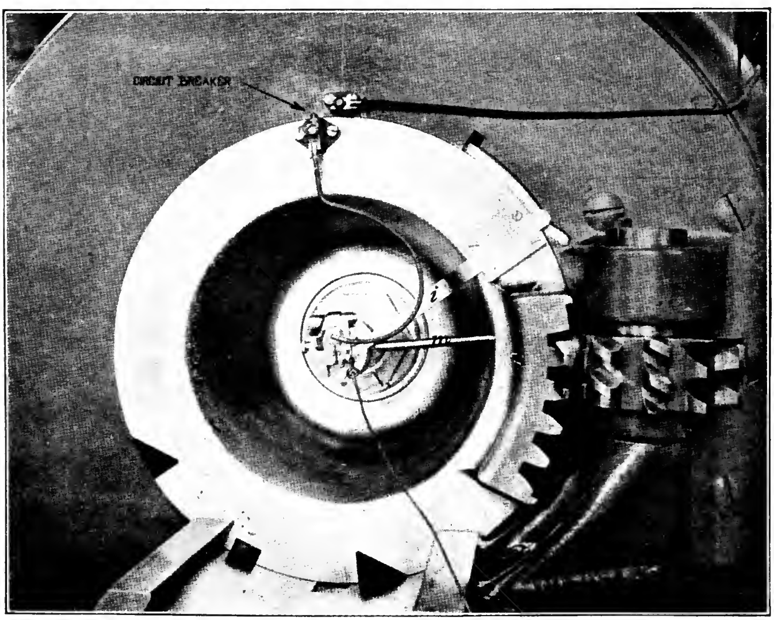


Fig. 87.—Breech Partially Unlocked. Safety Bar Forced in by Cam Slot, and Electric Circuit Broken.

Firing Mechanism for Heavy Guns.

The fired primer is ejected by lifting the slide. The lug on the slide, *d* Fig. 84, strikes the upper part of the ejector lever, giving to the lower end a sharp movement to the rear, which throws the primer clear of the piece.

156. Sliding Wedge Breech Mechanism.—The method of closing the breech by means of a sliding wedge-shaped block is used principally by Krupp, and to some extent by other makers. The jacket of the gun, *a* Fig. 88, extends to the rear of the tube, and the bore of the gun is continued through the extension. A slot cut transversely through the jacket just in rear of the tube forms a seat for the sliding breech block *k*. The front surface of the slot is a plane surface perpendicular to the axis of the bore, the rear surface is cylindrical and inclined to the axis of the bore. Two guides *b b′* similarly inclined guide the breech block in its movements. The breech block is of the same shape as the slot and slides in and out to close and open the breech. The greater part of the movement of the block is accomplished rapidly by means of the translating screw *c*, which is held in two bearings at the ends of the block and works in a half nut *d* on the gun. The screw is turned by means of the handle *e*, which is removed from the position in which it is shown and applied to the end of the screw *c*. The final movement in closing and the initial movement in opening are effected more slowly and more powerfully by the locking screw *g*. A nut *f* carried on the locking screw locks the block when closed.

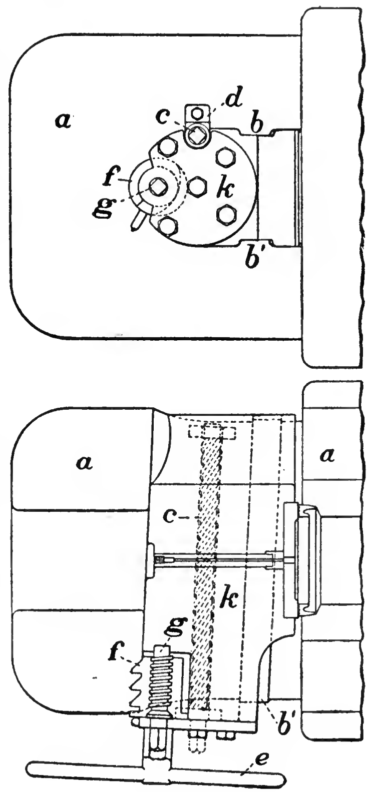


FIG. 88.

Obturation.—Obturation is effected with the sliding breech block by means of a steel obturator plate, *b* Fig. 89, carried in the block, and a steel cup-shaped ring, *a*, called the Broadwell ring, seated in the end of the bore. The pressure of the gases forces the ring back tightly against the plate and at the same time presses the thin lip *c* against the walls of the bore. The grooves shown in the rear surface of the ring serve as air packing and also to collect any dirt that may be on the surface of the plate. The hollow *e* in the plate also serves to collect fouling and to remove it from the bearing surface. The plate is forced tightly against the ring by the last movement of the locking screw in closing.

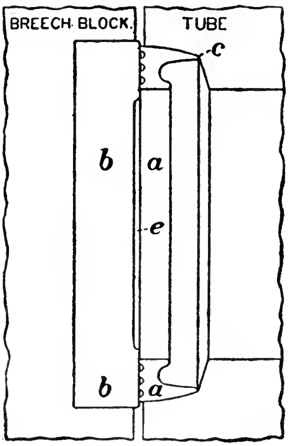


Fig. 89.

This mechanism is better adapted to small than to large guns. The light breech block of a small gun may be pushed to its seat by hand. Only a limited screw motion is then necessary to firmly seat and lock the block. Better obturation is also obtained when a cartridge case is used with this mechanism than when dependence is placed on the Broadwell ring.

In guns using fixed ammunition, if the breech block closes from the rear less care is required in inserting the round than if the breech is closed from one side. In the latter case if the round is not sufficiently inserted, the block in closing strikes the cartridge case and a temporary jamming of the mechanism occurs.

157. Older Forms of Breech Mechanism.—There are mounted in our fortifications many guns equipped with the breech mechanism shown in Fig. 90.

The block is revolved by means of one crank fixed to the gun, and withdrawn and swung aside by a second crank attached to the tray. The shaft of the revolving crank carries at its end the pinion *p*, Fig. 91, which works in the rack of the rotating ring *b*. The rotating ring revolves in bearings provided in the face plate, and communicates its motion of rotation to the block through the lug *a*, which engages in one of the slotted sectors. When the rotation of the block is completed the translating stud at the bottom of the block has entered one of the threads of the double threaded translating roller. The other thread of the roller works in a corresponding thread cut in the tray. Rotation of the translating

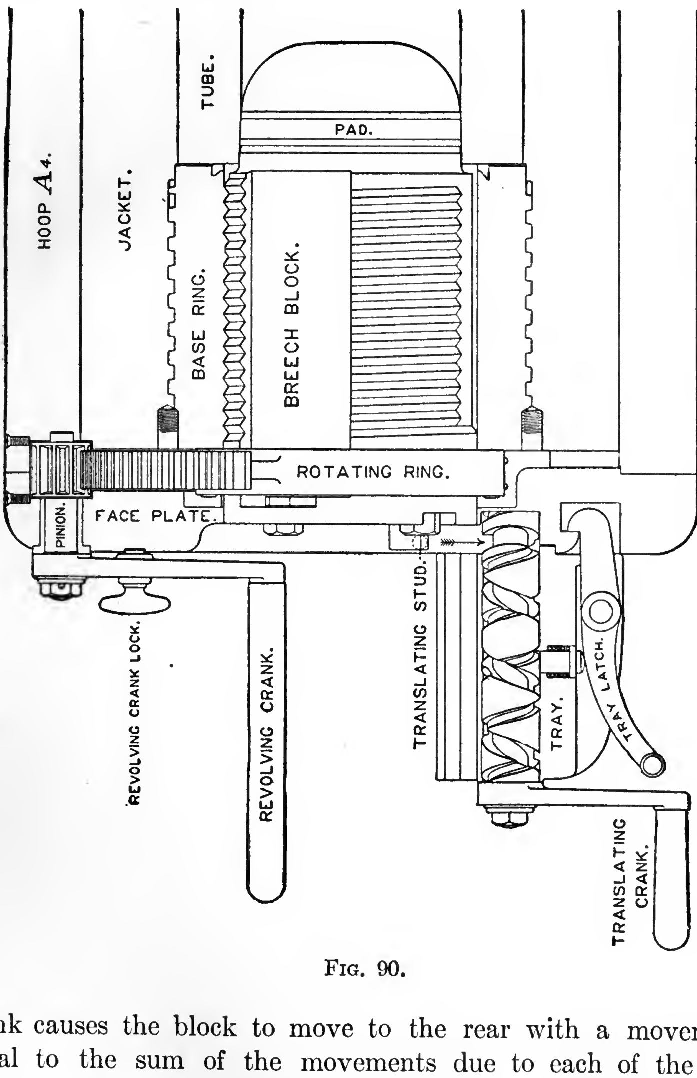


Fig. 90.

crank causes the block to move to the rear with a movement equal to the sum of the movements due to each of the two threads. When the front of the roller passes to the rear of the stud shown acting on the tray latch, the block is brought to a stop on the tray, and the shock of its arrest is sufficient to release the tray latch from its hold on the lip of the recess in the gun. The tray then swings aside, carrying the block clear of the breech.

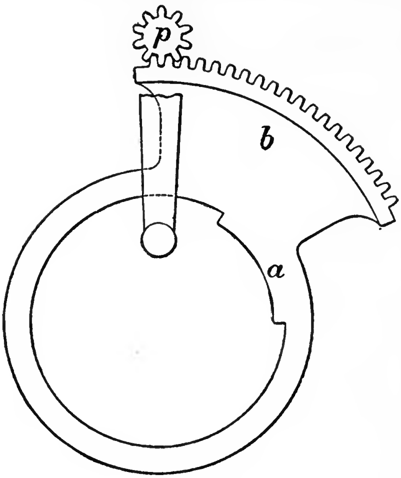


FIG. 91.

The tray is similar in general shape to the tray of the more modern mechanism shown in Fig. 72.

12-inch Mortar Breech Mechanism.—The 12-inch mortars are provided with the mechanism shown in Fig. 92. It differs from the mechanism just described only in the method of rotating the breech block. A steel plate *k* is fixed to the rear face of the breech block and extending upwards provides journals for the pinions *a*, *b*, and *c* of the rotating gear. The pinion *c* meshes in the rack *e* fixed to the gun, and when the crank *d* is turned the

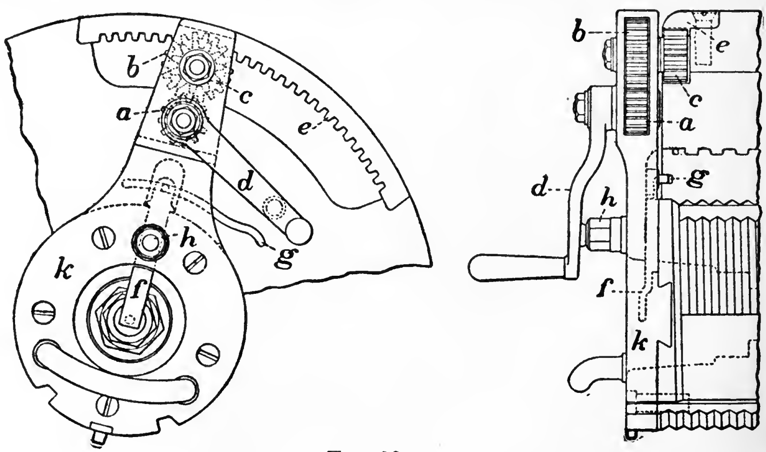


FIG. 92.

block is rotated to open or close. The block is withdrawn on a tray as described above. The translating stud that engages in the translating roller is seen at the bottom of the block.

The vent shield *f*, cut shorter than shown in the figure, is provided with a stud at its lower end that engages with the safety bar of the firing mechanism already described. The stud at its upper end works in the groove *g* cut in the gun, withdrawing the safety bar as the breech is fully closed.

Automatic and Semi-automatic Breech Mechanisms.—In guns provided with automatic breech mechanism the energy of recoil or the pressure of the powder gases is utilized to open the breech, withdraw the fired shell, insert a new cartridge and close the breech. After the firing of the first round the only operation necessary for firing the succeeding rounds is pulling the trigger. The automatic mechanism is at present applied only to guns of small caliber that use the small arm cartridge or fire a projectile weighing not more than a pound.

The semi-automatic mechanism is applied to guns of medium caliber, up to 6 inches, and efforts are being made to adapt it to the larger guns. The breech is opened by mechanism that is operated during the recoil or counter recoil of the piece, and if fixed ammunition is used the fired shell is ejected. At the same time power is stored in a spring to be later used in closing the breech.

In some mechanisms the insertion of the succeeding round by hand operates the breech closing mechanism. In others the pulling of a lever after the insertion of the round actuates this mechanism.

158. The 2.38-inch Field Gun Breech Mechanism.—The semi-automatic breech mechanism of the 2.38-inch light field gun is shown in Figs. 93 to 95.

The wedge shaped breech block *b* is seated in a vertical slot cut through the extension of the jacket. Projecting guide ribs, *t* Fig. 94, in the slot engage in grooves cut in the sides of the block. The block is lowered or raised to open or close the breech by means of the crank *c*. A stud at the end of the crank engages in the cam groove *g* on the right side of block, the groove being so shaped that the crank gives vertical movement to the block. On the outer end of the crank shaft is the operating lever, *l* Fig. 95, attached to which is the operating bar *r*, and the coiled operating spring.

The forward end of the operating bar embraces the pin protruding from the sliding piece *s*, which slides in an undercut groove

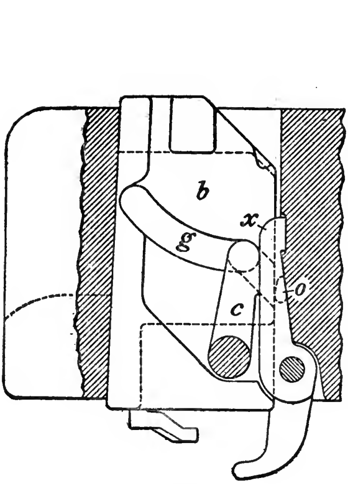


FIG. 93.

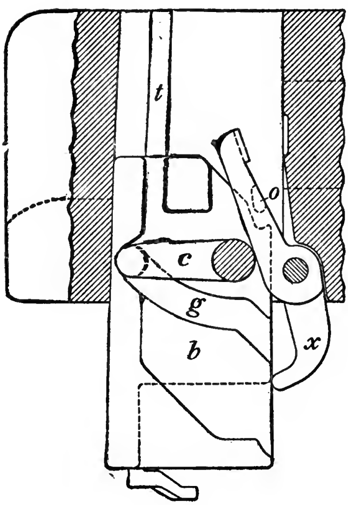


FIG. 94.

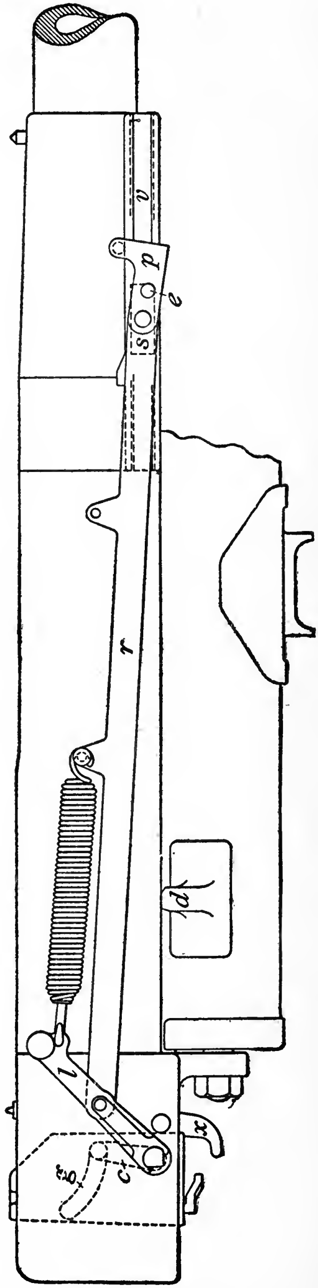


FIG. 95.

2.38-inch Field Gun, Semi-automatic Breech Mechanism.

v in the locking ring of the piece. The pawl *p*, pivoted on the same pin, has at its upper end a stud which rests on a shoulder above the groove. The end of a spring pin, *e*, in the pawl works in a slot cut in the sliding piece *s* and limits the motion of the pawl.

The mechanism above described is fixed to the piece and moves with the piece in recoil.

A stud, *d*, is fixed on the recoil cylinder of the carriage. When the piece recoils, carrying the mechanism with it, the pawl *p* is lifted by the stud and falls back into the position shown as soon as it has passed the stud. As the piece returns in counter recoil the pawl is engaged by the stud and held. The piece continues its forward movement. The slide *s* moves, relatively, to the rear in its slot, causing the bar *r* to rotate the operating lever *l* against the tension of the coiled spring.

The rotation of the lever lowers the breech block and opens the breech. The block in the last part of its movement operates the forked extractor *x* which ejects the empty cartridge case.

The stud on the upper end of the pawl *p* has now moved up the incline at the rear end of the shoulder on which it slides, lifting the pawl, disengaging it from the stud *d* on the carriage, and allowing the piece to finish its movement into battery. The pawl *p* being disengaged from the stud the breech block moves upward under the action of the operating spring until the curved locking studs *o* on each arm of the extractor, Fig. 94, engage in the corresponding recesses cut in the sides of the block. The curved shape of the locking studs and recesses, together with the directions in which the engaging parts are constrained to move, prevent further movement of the parts and the block is therefore locked open against the tension of the operating spring.

The rear part of the jacket extension is trough shaped to permit the ready insertion of the cartridge into the breech. As the cartridge is pushed into the breech with force its flanged head engages the extractor arms and forces the locking studs *o* out of the recesses. The action of the operating spring through the lever *l* and the crank *c* then lifts the block and closes the breech.

The firing mechanism is similar to that of the 3-inch field gun which is fully described in Chapter VIII.

159. THE 3-INCH SEACOAST GUN BREECH MECHANISM.—The operating parts of the U. S. Ordnance Co.'s semi-automatic breech mechanism, applied to the 3-inch seacoast gun, are shown in Figs. 96 and 97. Attached to the gun is the actuating rod *a*, its front

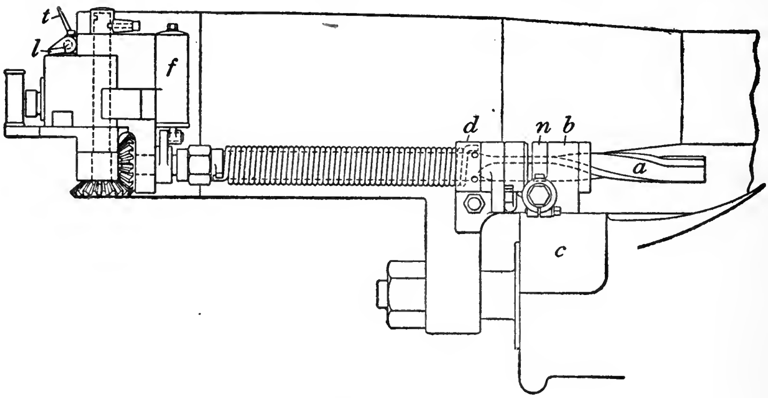


FIG. 96.

end provided with three twisted ribs which are practically screw threads with a very long pitch. The nut *n* similarly threaded is held in the bearing *b* which is fixed on the recoil cylinder *c* of the carriage.

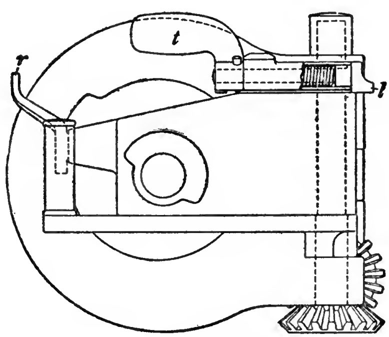


FIG. 97.

When the gun recoils the nut *n* is turned through 128 degrees by the actuating rod, but in counter recoil the nut is held by a pawl and the actuating rod turns clockwise, looking from the rear, in passing through the nut. The turning of the actuating rod operates the miter gears at its rear end and through them opens the breech and ejects the fired shell.

The operating spring, one end of which is held in the adjusting nut *d* which is carried in a bearing on the gun, is wound up by the movement of the actuating rod during counter recoil, and the energy stored in the spring is later utilized to close the breech. A small hydraulic buffer, *f*, modifies the action of the spring and relieves the mechanism of violent shock. The block is held open by the lug *l*, which under the action of a spring falls inside the carrier when the breech is open.

After the insertion of the cartridge, hand pressure on the tripping lever *t* lifts the lug *l* from inside the carrier. The operating spring, then free to act, closes the breech block.

The firing mechanism is similar to that described in Chapter VIII in the 3-inch field gun. The trigger is seen at *r*, Fig. 97.

Automatic breech mechanisms are described in Chapter XVI, in the descriptions of the guns in which they are used.

CHAPTER VII.

RECOIL AND RECOIL BRAKES.

160. Stresses on the Gun Carriage.—The stresses to which a gun carriage is subjected are due to the action of the powder gases on the piece. Gun carriages are constructed either to hold the piece without recoil or to limit the recoil to a certain convenient length. In the first case the maximum stress on the carriage is readily deduced from the maximum pressure in the gun. In the second case it becomes necessary to determine all the circumstances of recoil in order that the force acting at each instant may be known, and the parts of the carriage designed to withstand this force and to absorb the recoil in the desired length.

Velocity of Free Recoil.—Suppose the gun to be so mounted that it may recoil horizontally and without resistance. On explosion of the charge the parts of the system acted upon by the powder gases are the gun, the projectile, and the powder charge itself, the latter including at any instant both the unburned and the gaseous portions. While the projectile is in the bore, if we neglect the resistance of the air, none of the energy of the powder gases is expended outside the system. The center of gravity of the system is therefore fixed and the sum of the quantities of motion in the different parts is zero. The movement of the powder gases will be principally in the direction of the projectile. We may therefore write

$$Mv_f = mv + \mu v_c \qquad (1)$$

in which M, m, and μ are the masses of the gun, projectile, and charge of powder, respectively; and v_f, v, and v_c the velocities of the same parts. The mass of the charge is the same whether the charge is unburned or partially or wholly burned.

The velocity of the projectile at any point in the bore of the gun may be determined from the formulas of interior ballistics, equations (112) to (115), page 100. The velocity of the center of mass of the products of combustion is unknown. The velocity of the products varies from zero near the breech to v at the base of the projectile, and we may, without material error, consider the velocity of the center of mass of the products as equal to half the velocity of the projectile.

Writing $v/2$ for v_c in equation (1), replacing masses by weights, and solving for v_f we obtain

$$v_f = \frac{w + \frac{1}{2}\varpi}{W} v \tag{2}$$

W, w, and ϖ being the weights of the gun, projectile, and charge.

At the muzzle of the gun v becomes the initial velocity V, and for the velocity of free recoil at that instant

$$v_f' = \frac{w + \frac{1}{2}\varpi}{W} V \tag{3}$$

This value v_f' is not the maximum velocity of free recoil, though it is the maximum value reached while the velocities of the gun and of the projectile are connected. At the departure of the projectile the bore of the gun is still filled with gases under tension, which continue to exert pressure on the breech and increase the velocity of recoil. The value v_f' obtained by the above equation is about 7/10 of the maximum velocity of free recoil.

It has been determined by experiment with the Sebert velocimeter that the maximum velocity of free recoil may be obtained from equation (3) by substituting for the quantity $\frac{1}{2}\varpi V$ the quantity 4700ϖ. The equation then becomes

$$V_f = \frac{wV + 4700\varpi}{W} \tag{4}$$

V_f being the maximum velocity of free recoil.

The coefficient 4700 applies to smokeless powders. The coefficient for black powders was 3000.

161. Determination of the Circumstances of Free Recoil.—In the above equations the velocity of free recoil is expressed as a function of the velocity of the projectile, and we have in the ballistic formulas the velocity of the projectile expressed as a function of the travel of the projectile. We might therefore now determine the velocity of free recoil as a function of the travel of the projectile. But in the determination of all the circumstances of recoil it is necessary to know the relations between the velocity, time, and length of recoil; and in order to arrive at these relations by means of equation (2), we must obtain an expression for the velocity of the projectile as a function of the time.

With the velocity of the projectile expressed as a function of the time, equation (2) will then express the velocity of free recoil as a function of the time, and with the velocity of recoil so expressed we may obtain the length of recoil from the equation

$$x=\int v_f dt \qquad (5)$$

x representing the length of free recoil.

We thus obtain the complete relations between the velocity, time, and length of free recoil.

162. Velocity of the Projectile as a Function of the Time.—The velocity of the projectile as a function of the time is obtained in the following manner. Representing the travel of the projectile by u, we have

$$v=du/dt, \quad \text{from which} \quad t=\int \frac{1}{v}du \qquad (6)$$

That is, t is the area under the curve whose ordinates are values of $1/v$ and whose abscissas are values of u.

Therefore if we construct such a curve the area under the curve from the origin to any ordinate will be the time corresponding to the velocity whose reciprocal is represented by the ordinate.

Construct the curve v, Fig. 98, from the ballistic formulas, the abscissas representing travel, the ordinates velocity of the projectile.

Take the value of v as expressed by any ordinate and lay off its reciprocal on the same ordinate, to any convenient scale. The curve $1/v$ in the figure is obtained in this way. Its ordinates are values of $1/v$, its abscissas are values of u. The areas under the curve are therefore values of t, equation (6).

For very small values of v the ordinates $1/v$ will be very large and will not fall within the limits of an ordinary drawing. We cannot determine, then, from the drawing, the area under the first part of the curve. But we can obtain a sufficiently close approximation to this area in the following manner. We may assume,

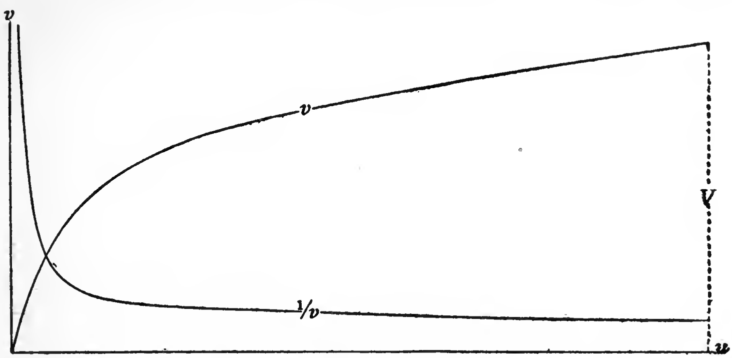


FIG. 98.

without material error in the determination of this small area, that the velocity of the projectile as a function of the time is expressed by the equation of a parabola

$$v=\sqrt{2pt} \tag{7}$$

Multiplying by dt and integrating, we have, since $\int v\,dt=u$,

$$u=\int\sqrt{2pt}\,dt=\tfrac{2}{3}\,(2p)^{\frac{1}{2}}t^{\frac{3}{2}} \tag{8}$$

At the instant at which the shot leaves the bore, v in equation (7) becomes the initial velocity V, and denoting the corresponding time by t' we obtain from that equation

$$V=\sqrt{2pt'} \quad \text{or} \quad \sqrt{2p}=V/\sqrt{t'}$$

Substituting this value of $(2p)^{\frac{1}{2}}$ in equation (8), t in that equation becoming t' and u the total travel of the projectile U, we obtain

$$t'=\frac{3}{2}\frac{U}{V}$$

t' is then the total area under the curve $1/v$, Fig. 98, and subtracting from t' the area that can be measured we obtain the area under that part of the curve near the origin that is not plotted.

Having now from the v curve the values of $v=f(u)$ and from the areas under the $1/v$ curve the values of $t=f(u)$ we may, by combination, determine the desired values of $v=f(t)$.

Using as abscissas the areas under the curve $1/v$, which are the values of t, and as ordinates the corresponding ordinates of the curve v, which are the velocities, we obtain the curve of the velocity of the projectile as a function of the time, Fig. 99.

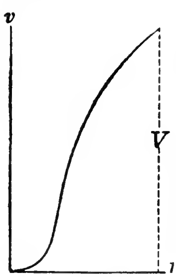


FIG. 99.

Since the velocity of free recoil as given by equation (2) is equal to the velocity of the projectile multiplied by a constant, the curve in Fig. 99 becomes at once the curve of velocity of free recoil, if we consider the scale of the ordinates as multiplied by the coefficient of v in equation (2).

163. Maximum Velocity of Free Recoil.—The curve shown in Fig. 99 gives the velocity of free recoil only while the projectile is in the bore, and as previously explained the velocity of recoil has not reached its maximum when the projectile leaves the piece. The value of the maximum velocity of recoil is given by equation (4). With this value as an ordinate, Fig. 100, draw a line parallel to the axis of t and continue the curve of velocity already drawn until it is tangent to this line. It is reasonable to infer that the rate of change in the curvature of the curve of recoil will continue uniform from the point corresponding to the muzzle of the gun to the point of maximum velocity, and the curve so continued will with sufficient exactness express the circumstances of motion. A slight error made in the selection of the point of tangency will be without practical effect on the determinations to be later made from this curve. The abscissa of the point of tangency is the time corresponding to the maximum velocity of free recoil.

As, by assumption, there is no resistance to recoil, the maximum velocity attained will never be reduced, and the curve will extend indefinitely parallel to the axis of t.

The tangent to the curve at any point is a value of dv_f/dt, and therefore represents the acceleration at the instant of time represented by the abscissa of the point. The tangent has a maximum value at the point of inflexion of the curve, the point where the curve ceases to be convex toward the axis of t, and becomes concave. This point is therefore the point of maximum acceleration.

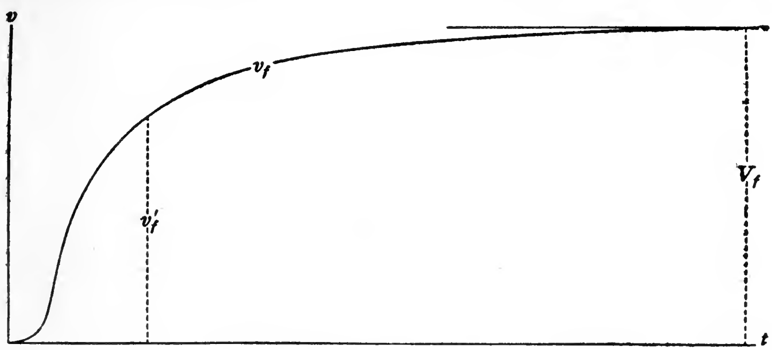


Fig. 100.

The maximum acceleration being due to the maximum powder pressure in the gun the abscissa of the point of inflexion is the time of the maximum pressure.

Since, equation (5), $x=\int v_f dt$, the area under the curve v_f, Fig. 100, from the origin to any ordinate is the length of free recoil corresponding to the velocity represented by the ordinate.

Retarded Recoil.—In the discussion thus far we have neglected all resistances and have considered the movement of the gun in recoil as unopposed. When the gun is mounted on a carriage the recoil brakes, of whatever character, begin to act as soon as recoil begins, and consequently the velocity of recoil is less at each instant than the velocity shown by the curves just determined.

The manner of obtaining the velocity of retarded recoil will be explained later.

Recoil Brakes.—To absorb the energy of recoil and to bring the gun to rest in a convenient length, all gun carriages which permit movement of the gun in recoil are provided with recoil brakes.

These are of two general classes, friction brakes and fluid brakes. Friction brakes were formerly used on seacoast carriages, but are now confined exclusively to wheeled carriages. Fluid brakes are either hydraulic or pneumatic. Pneumatic brakes, depending for their resistance on the compression of air, have been used in England to some extent on seacoast carriages. On account of the difficulty of preventing loss of pressure in the brakes through leakage of the air these brakes are not satisfactory.

164. Hydraulic Brakes.—A hydraulic recoil brake consists of a cylinder filled with liquid, and a piston. Relative movement is given to the cylinder and piston by the recoil, and provision is made for the passage of the liquid from one side of the head of the piston to the other by apertures cut in the piston or in the walls of the cylinder. The power of the brake lies in the pressure produced in the cylinder by the resistance offered by the liquid to motion through the apertures.

If the area of the apertures is constant it is evident that the resistance to flow will be greater as the velocity of the piston or the velocity of recoil is greater. Therefore the pressure in the cylinder, which measures the resistance offered, will vary with the different values of the velocity of recoil. If, however, the apertures are constructed in such a manner that the area of aperture increases when the velocity of the piston increases and diminishes when that velocity diminishes, the variation in the area of aperture may be so regulated that the pressure in the cylinder will be constant or will vary in such a manner as to keep the total resistance to recoil constant.

Both of these methods have been used in the construction of recoil brakes for gun carriages. The brakes with constant orifices and variable pressures were used on the old carriages for 15-inch smooth bore guns.

For a fixed length of recoil a constant resistance will have a lower *maximum* value than a variable resistance, and consequently will produce a less strain on the gun carriage. For this reason and for other advantages that will appear in the discussion which follows, the brake with variable orifices, and constant or variable pressure as circumstances may require, is at present used to the exclusion of all others on gun carriages.

Hydraulic Brake with Variable Orifice.—The mode of action of the hydraulic brake with variable orifices will be understood

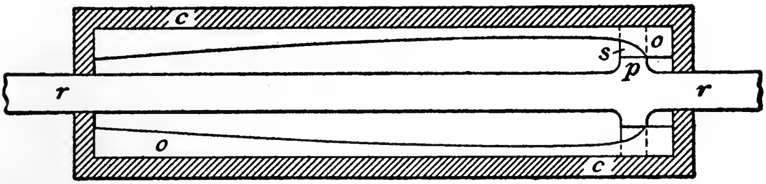


Fig. 101.

from Fig. 101, which represents a longitudinal section through a recoil cylinder of the form used in our seacoast carriages.

Fig. 102 represents a cross section through the cylinder.

To the walls of the cylinder *c* are fastened two bars *o* called throttling bars, of varying cross section as shown. The piston *p* is stationary, the piston rod *r* being fixed to a stationary part of the carriage. The cylinder *c* is attached to the gun and moves to the rear in recoil.

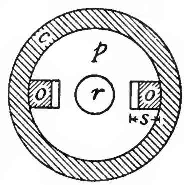


Fig. 102.

The direction of the movement of the cylinder is to the right in the figure. The figure shows the relative positions of cylinder and piston at the beginning of recoil.

Through the piston head are cut two slots or apertures, *s*, through which the liquid is forced from one side of the piston to the other as the *cylinder* moves in recoil. Each slot has the dimensions of the maximum section of the throttling bar, with just enough clearance to permit operation. The area of orifice open for the flow of liquid at any position of the piston is therefore equal to the area of the slots minus the area of cross section of the throttling bars at that point; and the profile of the throttling bars is so determined that the resistance to the flow of the liquid, or the pressure in the cylinder, is made constant or variable as desired.

165. Total Resistance to Recoil.—The total resistance to recoil is composed of the resistance opposed by the brake, the resistance due to friction, the resistance—either plus or minus—due to the inclination of the top of the chassis, and the resistance due to the counter recoil springs if there are such included in the recoil system. The function of the counter recoil springs is to return the gun to battery after recoil.

The resistance of the counter recoil springs varies with the degree of compression. Therefore to maintain a constant total resistance when springs are included in the system the resistance of the brake must also vary, the other resistances being constant.

Let W be the weight of the moving parts,
M the mass of the moving parts,
f the coefficient of friction,
α the angle of inclination of the chassis rails,
S the resistance of the springs at any time t,
P the total resistance of the hydraulic brake, or the total pressure in the cylinder, at the time t,
R the total resistance to motion,
v_r the velocity of *retarded recoil* at the time t,
V_r the maximum velocity of retarded recoil.

The resistance due to friction will be $fW\cos\alpha$; that due to the inclination of the chassis rails will be $W\sin\alpha$. The total resistance at the time t is therefore

$$R = W(\sin\alpha + f\cos\alpha) + S + P \qquad (9)$$

Dividing the total resistance by the mass, we have, for the retardation,

$$-dv/dt = R/M \qquad (10)$$

When the total resistance to recoil is constant, the retardation R/M is constant, and we may substitute it for g in the equation that expresses the law of constant forces,

$$v^2 = 2gh$$

Assuming the origin of movement as at the maximum velocity of recoil, V_r, and designating by l' the length of recoil from this point to the end, the above equation becomes

$$V_r^2 = 2l'R/M$$

or

$$l' = V_r^2 M/2R \qquad (11)$$

l' is the length in which the constant resistance R will overcome a velocity of recoil V_r.

For the velocity at any point whose distance from the origin is x, we have the relation

$$l' - x = v_r^2 M/2R \qquad (12)$$

since $l' - x$ is the length in which the constant resistance must overcome the velocity v_r.

Values of the Total and Partial Resistances and Velocities of Recoil.—In the construction of a gun carriage the length of recoil is usually fixed by the design of the carriage. We will therefore assume a length l as the total length of recoil. We must now determine the total constant resistance that will restrict the recoil to this length and then determine the portion of this resistance that is to be contributed by the brake. In so doing we will arrive at the values of the velocities of recoil at all points in the path.

166. Total Constant Resistance.—The curve v_f in Fig. 103, which as far as the point m is the curve v_f in Fig. 100 drawn to a

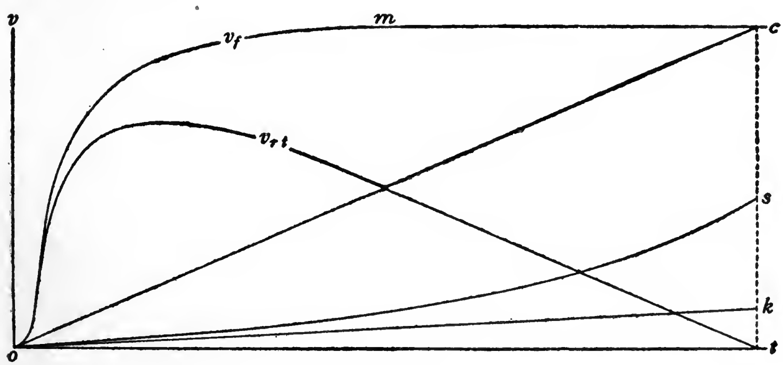


FIG. 103.

different scale, represents the velocity of free recoil as a function of the time. We have seen that the tangent to the curve at any point represents the acceleration at that point.

We may represent the negative velocities due to a constant resistance by the ordinates of some straight line oc, whose abscissas are the corresponding times. The tangent of the constant angle *toc* is therefore equal to $-dv/dt$, the retardation due to the force.

The line *oc* is for convenience drawn above the axis of t. As its ordinates represent the negative velocities due to the resistance the line properly belongs below the axis.

Now if we subtract from the velocities of free recoil, represented by the ordinates of the curve v_f, the velocities due to the retarding force, the ordinates of *oc*, the ordinates of the resulting curve v_{rt} will be the velocities of retarded recoil. The curve v_{rt} is therefore the curve of the velocity of retarded recoil as a function of the time. The abscissas of the curve being values of t, the area under the curve will be the total length of retarded recoil, see equation (5).

We have assumed a total length of recoil, l, and if the area measured under the curve of retarded recoil, as obtained above, does not give this length, we must change the angle *toc*, draw a new line *oc*, and construct a new curve. After a few trials the proper direction of *oc* will be determined and the area under the curve of retarded recoil, v_{rt} Fig. 103, will be the length l.

Then the retardation represented by the line *oc* is given, see equation (10), by the equation

$$-\tan toc = -dv/dt = R/M \tag{13}$$

from which, after measuring the angle *toc*, we may determine R, the total constant resistance that will limit the recoil to the length l.

The length of retarded recoil corresponding to any velocity of retarded recoil represented by an ordinate of the curve v_{rt} is the area under the curve from the origin to the given ordinate.

We may now construct the curve of retarded recoil as a function of the distance recoiled. To construct a point of the curve measure the area under the curve v_{rt} in Fig. 103 from the origin to any ordinate; use the value of this area as an abscissa, and use the selected ordinate of the curve v_{rt} as an ordinate. The curve v_{rx} in Fig. 104, constructed in this manner from the curve v_{rt} in Fig. 103, represents the velocity of retarded recoil as a function of the distance recoiled.

Minor Constant Resistance.—The total resistance R is composed, equation (9), of the constant part $W(\sin\alpha + f\cos\alpha) = k$

and the two variable parts S and P. The value of $W(\sin\alpha+f\cos\alpha)$ may be readily determined. The retardation due to this resistance is equal to k/M, and is represented in Fig. 103 by a line *ok* drawn so that the tangent of the angle *tok* is equal to k/M.

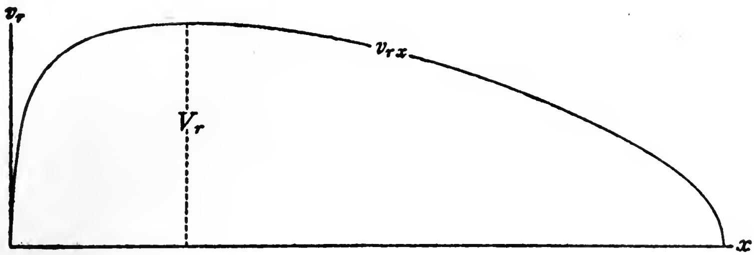


FIG. 104.

167. Resistance of the Spring.—The resistance S of a coiled spring varies directly with the compression of the spring.

Representing by G the force required to compress the spring, when free, over the first unit of length, the resistance of the spring at any length of compression x is

$$S=Gx$$

If the spring has an initial compression so that it exerts a resistance G', the resistance after further compression over a length x becomes

$$S=G'+Gx \qquad (14)$$

For the counter recoil springs of a gun carriage, G' represents the residual pressure in the spring when the gun is in battery, and x represents any length of recoil.

The resistance of the spring at any point may therefore be determined from equation (14).

To find the velocities taken out of the system by the spring, we proceed as follows.

Representing by v' the velocity in the mass M due to the spring alone, the retardation due to the spring is

$$-dv'/dt=(G'+Gx)/M$$

In order to integrate we must express dt in terms of dx. $dx=v'dt$. Therefore

$$dt=dx/v',$$

and $$-dv'/dt=-v'dv'/dx=(G'+Gx)/M$$

and integrating,

$$-v'^2/2=(G'x+Gx^2/2)/M$$

the constant of integration being O, since when x is O, v' is O.

The values of v' are obtained from this equation in terms of x. We may find from the curves v_{rx} and v_{rt} the value of t corresponding to any value of x. The values of v' obtained above may then be laid off in Fig. 103 as the true ordinates of the curve *os*. These ordinates are laid off in the figure from the line *ok* so that in the figure the ordinates of *os* are the sums of the true ordinates of *ok* and *os*. The ordinates of *os* are therefore the velocities taken out of the system by resistances other than the hydraulic brake.

As the ordinates of the line *oc* are the velocities taken out by the total constant resistance, the ordinates between the lines *os* and *oc* represent the velocities to be taken out of the system by the brake alone.

Resistance of the Hydraulic Brake,—Pressure in the Cylinder.—The pressure in the brake cylinder at any point of the recoil may now be determined from equation (9)

$$P=R-W(\sin\alpha+f\cos\alpha)-S \tag{15}$$

if we substitute for R its constant value from equation (13), for S its value at the given point from equation (14), and for the remaining term its constant value.

168. Relation Between the Pressure, Area of Orifice, and Velocity of Recoil.—In this discussion we will designate by the term *aperture* the cut through the piston, and by the term *orifice* that portion of the aperture open to the flow of the liquid; and we will consider for simplicity that there is but one aperture and one orifice.

Let A be the *effective* area of the piston in square feet, that is, the area of the piston minus the area of the piston rod and aperture. The square foot is taken as the unit of area, because in the velocities involved in the discussion the foot is the unit of length.

Let a be the area of the orifice at any time t,

V_r the maximum velocity of retarded recoil,

v_r the velocity of retarded recoil at any time t,

v_l the velocity of the liquid through the orifice at the time t,

γ the weight of a cubic foot of the liquid,

P the total pressure on the piston at the time t.

The cylinder being full of liquid the volume that passes through the orifice is the volume displaced by the piston. We therefore have at any instant

$$v_r A = v_l a$$

or, for the velocity of flow,

$$v_l = v_r A/a \tag{16}$$

From Torricelli's law for the flow of liquids through orifices we know that the pressure required to produce this velocity of flow is the pressure due to a column of liquid whose height h is given by the equation

$$v^2 = 2gh \tag{17}$$

Substituting for v the value of v_l from equation (16) and solving for h we obtain

$$h = v_r^2 A^2/2ga^2 \tag{18}$$

The weight of a cubic foot of the liquid being γ, the weight of the column whose area of cross section is unity will be γh, and the weight of the column whose area of section is equal to that of the piston will be $A\gamma h$. $A\gamma h$ is therefore the pressure on the piston, and substituting in this expression the value of h from equation (18) we have, for the total pressure on the piston, for any velocity v_r

$$P = \gamma A^3 v_r^2/2ga^2 \tag{19}$$

This equation is general and expresses the relation that exists between P, A, and a for any given velocity of recoil.

Solving for a^2 we obtain

$$a^2 = \gamma A^3 v_r^2/2gP \tag{20}$$

169. Area of Orifice.—With the relations established in equations (14), (15), and (20), which are here repeated, and the curve v_{rx} in Fig. 104, we are now prepared to determine the variable area of orifice in the piston.

(14) $$S = G' + Gx$$

(15) $$P = R - W(\sin \alpha + f \cos \alpha) - S$$

(20) $$a^2 = \gamma A^3 v_r^2 / 2gP$$

The dimensions of the recoil cylinder will be fixed within narrow limits by the design of the carriage, and by the requirement that the pressure per unit of area must not be so great as to render difficult the effective packing of the stuffing boxes through which the piston rod passes. We will therefore assume that the diameters of the cylinder and piston rod are given, and as the relation between the total area of piston and the effective area may be readily established we will assume that the effective area A of the piston is known.

Brake with Variable Pressure.—The value of P at any point in the cylinder, for which the length of recoil is x, is obtained from equation (15), the proper value of S for the point having been first determined from (14). The value of v_r is taken from the curve v_{rx} in Fig. 104 at the ordinate whose abscissa is x. The values of P and v_r thus determined are substituted in equation (20). The resulting value of a is the area of orifice at the given point.

170. Constant Pressure.—If P in equations (19) and (20) is constant we will have in a given cylinder, for any other values of v_r and a, as V_r and a_o, respectively the maximum velocity of recoil and the maximum area of orifice

$$a_o^2 = \gamma A^3 V_r^2 / 2gP \qquad (21)$$

and by combining equations (20) and (21) we obtain for any given cylinder

$$a/a_o = v_r / V_r \qquad (22)$$

from which we see that to maintain a constant pressure in the cylinder the area of the orifice must vary directly with the velocity of recoil.

Assuming the maximum velocity of recoil as the origin of movement and substituting in equation (22) the value of v_r/V_r obtained by combining equations (11) and (12), in which l' represents the total length of recoil *after the maximum velocity has been reached,* we obtain

$$a = a_o\sqrt{1 - \frac{x}{l'}} \qquad (23)$$

that is, with constant pressure in the cylinder the area of orifice varies as the ordinates of a parabola.

Equation (23) and all equations in which l' appears refer only to that part of the recoil from the maximum velocity to the end of recoil.

Brake with Constant Pressure.—When there are no springs or other variable resistance in the recoil system, S becomes 0 in the value of P, equation (15), and a constant resistance will be required in the brake.

To determine the area of orifice we have, for this case,

$$P = R - W(\sin\alpha + f\cos\alpha)$$

(21) $$a_o^2 = \gamma A^3 V_r^2 / 2gP$$

(22) $$a/a_o = v_r/V_r$$

Find the value of P from the first equation in the manner already explained on page 286.

The maximum ordinate of the curve v_{rx}, Fig. 104, is the value of V_r in equation (21). A is known. The maximum area of orifice a_o may be now determined from equation (21) and the area of orifice at all other points more simply by means of equation (22), using the values of v_r taken from the curve v_{rx}. The areas from the maximum velocity to the end may also be obtained from equation (23).

Horizontal Chassis.—If the chassis rails are horizontal and the top carriage is mounted on rollers, so that we may neglect the friction, the term $W(\sin\alpha + f\cos\alpha)$ in the value of P, equation (15), also becomes zero, and P reduces to R. Substituting for R in equation (11) the value of P from (21) and solving for a_o we obtain

$$a_o^2 = \gamma l' A^3 / W \qquad (24)$$

The maximum area of orifice is in this case independent of the velocity of recoil, and is dependent only on the length of recoil. Therefore for a given maximum area of orifice the length of recoil will be the same no matter what the initial velocity of the projectile, the charge of powder, or the angle of fire may be.

Under these conditions the brake requires no adjustment for varying conditions of fire, and in this respect it possesses further advantage over the brake with constant orifices and variable pressure.

The explanation of the independence, under the given conditions, of the length of recoil and the velocity will appear if we substitute P for R in equation (11). We obtain

$$l'=MV_r^2/2P \tag{25}$$

In equation (21) we see that for a given maximum area of orifice the pressure P must vary directly as V_r^2 varies. Therefore in (25), P varying with V_r^2, l' will remain constant.

171. Profile of the Throttling Bar.—Suppose there are n similar apertures cut in the piston. The area of each orifice at any point in the cylinder will then be a/n, a being determined for the particular point from equation (20). Let b, Fig. 105, be the width and d the depth of each aperture. The throttling bar has the same depth, and a variable width y.

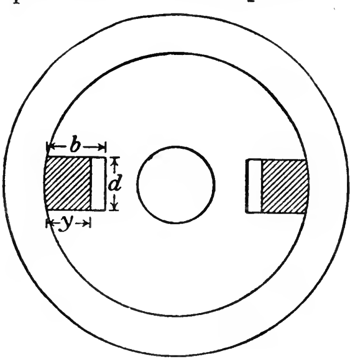


FIG. 105.

Then for the area of each orifice at the given point in the cylinder we have

$$a/n=d(b-y)$$

For the brake with constant pressure the profile of the throttling bar from the point of maximum velocity to the end will be a parabola. Its equation, obtained by substituting the value of a from the above equation in equation (23) and reducing, is

$$y=b-\frac{a_0}{nd}\sqrt{1-\frac{x}{l'}}$$

Neglected Resistances.—In the foregoing discussion we have neglected the resistance due to the friction of the liquid and the contraction of the liquid vein. It has been found by experiment that the error due to the neglect of these resistances may be corrected by assigning to v_l, the velocity of the flow through the orifices, equation (16), a value greater than the actual value as expressed in equation (17). The value to be substituted is determined by experiment for each class of carriage and takes the form $v_s = av_l + b$, a and b being constants. The result of the substitution is an increase in the area of orifice for any given pressure in the cylinder, see equation (20).

172. Recoil System of Seacoast Carriages.—The arrangement of the parts of the recoil system on our seacoast disappearing carriages, and on barbette carriages for guns 8 inches or more in caliber, is shown in Fig. 106.

The two cylinders c are integral parts of the top carriage, the top carriage, including the cylinders, forming a single steel casting in the sides of which above the cylinders are trunnion seats, for the gun trunnions in a barbette carriage, and for the gun lever trunnions in a disappearing carriage.

The piston rods of the recoil cylinders are fixed to the chassis in front and supported in the rear. They enter the cylinders through stuffing boxes. On discharge of the piece the top carriage and recoil cylinders move to the rear with the gun, forcing the liquid in the cylinders through the orifices in the stationary pistons.

The direction of the movement of the cylinders is to the *right* in Fig. 106.

To equalize the pressure in the two cylinders their interiors are connected at the front by the pipe a and at the rear by the two pipes d and f. Each half of the pipes d and f has unobstructed communication with the other half of the same pipe through a valve box v. A cross pipe b connects the pipe a with the valve box. A path is afforded through the pipes a, b, and d and f for the flow of liquid from one side of the piston to the other, which path, as well as the orifices in the pistons, must be considered in determining the area of orifice.

The area of orifice, and consequently the length of recoil, is calculated for standard conditions of loading. Any variation in

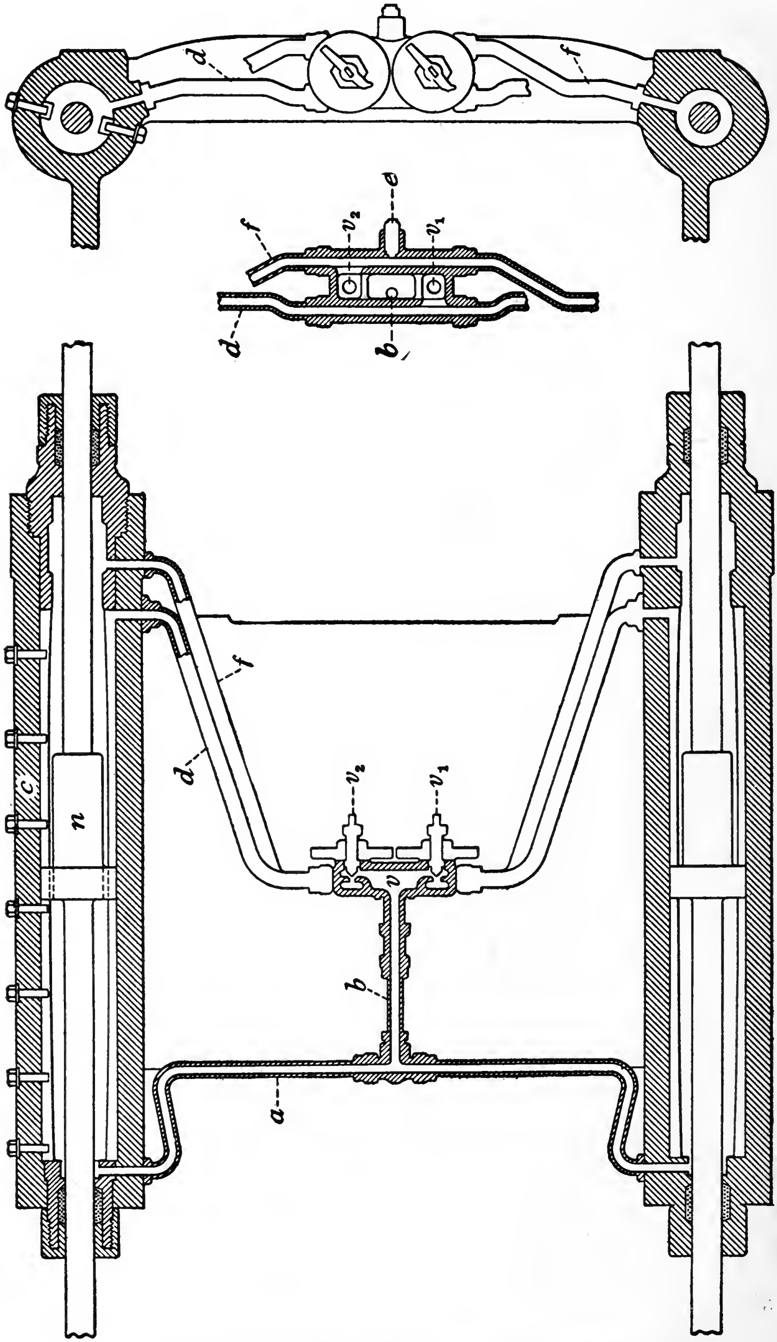


FIG. 106.—Recoil System, Seacoast Carriages.

these conditions will vary the length of recoil, and thus, in disappearing carriages, vary the height of the breech of the gun above the loading platform. Standard conditions of loading do not always exist, and it is therefore desirable to have means for varying the resistance in the cylinders in order that the prescribed length of recoil may be obtained under any conditions, as for instance when reduced charges are being used.

For the purpose of varying the area of orifice, and therefore the resistance in the cylinders, adjustable valves called throttling valves are provided at v_1 and v_2. The flow from the pipe b into the pipe d communicating with the body of the cylinder is regulated by the valve v_1, and the area open to the flow is affected to increase or diminish the pressure in the cylinder as desired. The pipe d and its valve v_1 are for the control of the recoil.

To control the counter recoil and to bring the gun and top carriage to rest without shock as they come into battery under the action of gravity, the counter recoil buffer is provided. The rear cylinder head is provided with a cylindrical recess into which the enlargement n of the piston rod, just in rear of the piston, enters as the carriage approaches its position of rest in battery. The lug n is slightly conical, so that the escape of the liquid from the recess is gradually obstructed. The pipe f with its valve v_2 assists in the regulation of this part of the counter recoil.

The valves v_1 and v_2 are moved to increase or diminish the area of orifice by means of the handles seen in the rear view, at the right of Fig. 106.

The cylinders are filled, through holes provided in the top, with a mineral oil called hydroline. The freezing point of the oil is below 0° F. Its specific gravity is about 0.85. The oil may be drawn off through a hole e in the valve box, ordinarily stopped with a screw plug.

The throttling bars are fastened to the cylinders by screw bolts through the cylinder walls, as shown in Fig. 106.

Modification of Recoil System.—In the recoil system just described it will be noticed that, at the beginning of recoil, as the enlargements n of the piston rods emerge from the recesses in the rear cylinder heads there is around the enlargements but little clearance by which the oil displaced by their bulk in the cylinders proper may enter the vacated recesses. Consequently if the cylinders are full of oil the liquid will be forced with great velocity through the clearances, and the pressure in the cylinders will be correspondingly high.

To prevent this high pressure, oil is withdrawn from the cylinders in sufficient quantity to leave an air space in the cylinders nearly equal to the space occupied by the enlargements of the piston rods, and on emerging from the recesses the enlargements occupy the air space without giving to the liquid an excessive velocity of flow.

The removal of oil from the cylinders is objectionable in that if the cylinders are not completely filled with oil the uncovered parts of the piston and of the cylinder walls are attacked by rust.

It will be noticed, too, that any movement of *either* of the throttling valves that control the recoil and counter recoil affects the area of orifice. Therefore the regulation of the counter recoil affects also the recoil.

For these reasons it has been found desirable to separate the two systems so as to have independent control of both recoil and counter recoil; and in a 6-inch disappearing carriage now being tested an additional recoil cylinder is fixed in the counterweight of the carriage. The control of the recoil is effected wholly by this large cylinder, and the counter recoil is controlled by smaller cylinders whose pistons are acted on by the top carriage in the last part of its movement into battery.

Other advantages of this arrangement will appear in the description of the carriage in the next chapter.

173. Wheeled Carriages, Recoil.—To arrive at the effect of the recoil on a wheeled carriage we must consider the effects of all the forces that act upon the carriage. These forces include the weight of the system composed of the carriage and gun, and the various forces developed by the transmission of the powder pressure to the points of support of the carriage.

In Fig. 107 is represented the trail of a wheeled carriage with the wheel and spade. For the purpose of discussion we will assume that the carriage is a rigid body, that the wheels are locked, and that the pressure developed in the gun, or the pressure developed in the recoil system when the gun recoils on the carriage, is transmitted to the carriage at the point *o*.

The points of application and the directions of the forces acting on the carriage and of the reactions at the points of support are represented in the figure.

ϕ is any angle of elevation,
P the transmitted pressure.

Let M be the mass of the system composed of the gun and carriage,
W its weight,
$F = F' + F''$, the total friction on the ground.

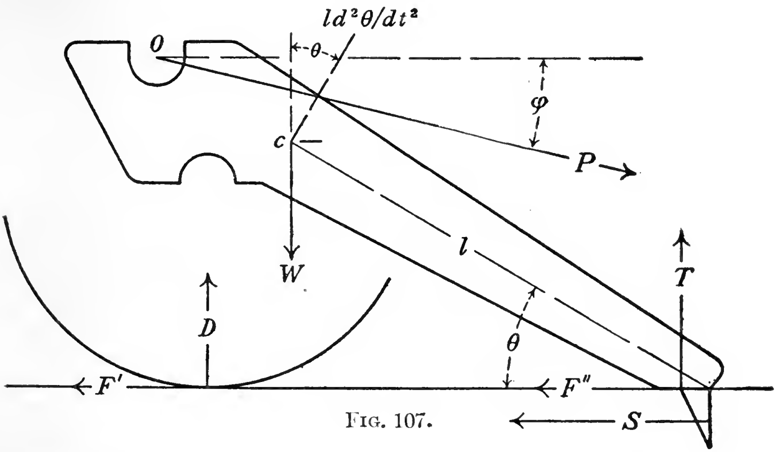


FIG. 107.

The center of gravity of the system is represented at *c*.

The forces acting on the carriage are symmetrically disposed with respect to the axial plane, and therefore their resultant acts in that plane.

A system of forces acting in a plane is completely known when its components in the direction of two rectangular axes in the plane and the moments about any axis perpendicular to the plane are determined.

We will assume the rectangular axes as horizontal and vertical, the vertical axis through the center of gravity and the horizontal axis on the surface of the ground.

The effect of the forces acting on the carriage will be, under the most general consideration, a movement of the carriage to the rear, and at the same time, since the resistance to motion is greatest at the point of support of the trail, there will occur a movement of rotation of the carriage about the point of support.

Applying to the carriage, in the manner shown in Fig. 107, all the forces that act upon it, we may consider the carriage as a free body and may then determine the values that the forces must have in order to produce in the free body the actual movement of the carriage in recoil.

The movement of a free rigid body acted on by forces may be considered as composed of a movement of translation of the center of gravity and a movement of rotation of the body about the center of gravity. The movements of translation and of rotation may be considered separately.

We have for the equations of motion of the center of gravity

$$\frac{P\cos\phi - F - S}{M} = \frac{d^2x}{dt^2} \tag{26}$$

$$\frac{D + T - W - P\sin\phi}{M} = \frac{d^2y}{dt^2} \tag{27}$$

The sum of the moments of the applied forces with reference to an axis through the center of gravity is the same whether the center of gravity is in motion or at rest, and is equal to the product of the acceleration of rotation into the moment of inertia of the body about the axis. Therefore, representing with small letters the lever arms of the forces with respect to an axis through the center of gravity, we have the equation

$$\frac{Pp + Ff + Dd + Ss - Tt}{Mk_1^2} = \frac{d^2\theta}{dt^2} \tag{28}$$

k_1 representing the principal radius of gyration of the body.

174. Condition of Movement.—Now to introduce into the three general equations of motion, (26), (27), and (28), the condition that the movement of the free body shall be the same as the movement of the carriage in recoil, we may write

$$y = l\sin\theta$$

since this condition holds in the actual movement of the carriage; that is, as long as the point of the trail is on the ground the center of gravity is at the distance $l \sin \theta$ from the ground.

Differentiating y twice we obtain

$$dy = l \cos \theta d\theta$$

$$d^2y = l \cos \theta d^2\theta - l \sin \theta d\theta^2$$

and dividing by dt^2

$$\frac{d^2y}{dt^2} = l \cos \theta \frac{d^2\theta}{dt^2} - l \sin \theta \frac{d\theta^2}{dt^2}$$

$d\theta/dt$ is the angular velocity of the carriage about the point of the trail. $ld\theta/dt$ is therefore the linear velocity of the center of gravity about the same point. Representing this linear velocity by v we obtain from the above equation after multiplying the last term by l/l

$$\frac{d^2y}{dt^2} = l \frac{d^2\theta}{dt^2} \cos \theta - \frac{v^2}{l} \sin \theta \tag{29}$$

This equation expresses that the vertical acceleration of the center of gravity rotating about the point of the trail is equal to the vertical component of the linear acceleration $ld^2\theta/dt^2$ about that point, see Fig. 107, minus the vertical component of the acceleration along the radius l.

Any change in the angle that the trail makes with the ground is accompanied by an equal change in the angle of revolution of the body about the center of gravity, see the two angles θ in Fig. 107. Therefore the quantities $d^2\theta/dt^2$ in equations (29) and (28) are the same.

Substituting the value of d^2y/dt^2 from equation (29) in equation (27) we introduce into the general equations the actual condition of motion. We then have, for the gun carriage, the three equations

$$\frac{P \cos \phi - F - S}{M} = \frac{d^2x}{dt^2} \tag{30}$$

$$\frac{D + T - W - P \sin \phi}{M} = l \cos \theta \frac{d^2\theta}{dt^2} - \frac{v^2}{l} \sin \theta \tag{31}$$

$$\frac{Pp + Ff + Dd + Ss - Tt}{Mk_1^2} = \frac{d^2\theta}{dt^2} \tag{32}$$

We may determine any three of the quantities in these equations if we establish, or assume, values for the other quantities; and in this way we may determine the effects that follow from variations in the values of any of the quantities that enter the equations.

The above equations are applicable only while $y=l\sin\theta$; that is, as long as the point of the trail remains on the ground.

As the linear velocity of the center of gravity is usually small the value of the term $v^2\sin\theta/l$ in equation (31) is very small and is generally neglected in computations. In the computations of the stresses before movement begins v is 0.

175. Application of the Equations.—The general equations (26), (27), and (28) are applicable in the solution of all problems that involve the determination of the stresses, and of the movement, produced by the application of a force or a system of forces to any body or structure.

The equations have been deduced under the most general considerations, and while the number of quantities that appear in them is greatly in excess of the number of equations, it will be found, in practical application under given conditions, that equations of relation between the various quantities may be readily established in sufficient number to reduce the number of unknown quantities in the equations to three, whose values may then be determined.

Thus to apply the general equations, under given conditions, to any given construction, such as the gun carriage represented in Fig. 107.

The intensity and direction of the applied force or forces are usually known or assumed. We will therefore assume that in equations (26), (27), and (28) P and ϕ are known.

For the gun carriage, the condition $y=l\sin\theta$ eliminates the quantity d^2y/dt^2 and brings the equations into the forms (30), (31), and (32). *A similar condition of restraint will ordinarily be found in all constructions that are free to move in given directions only.*

In the modified equations, P, ϕ, W, and M are known. All dimensional quantities such as l, p, t, etc., are determined from the known dimensions of the construction. k_1 may be determined. θ is known.

D and T being parallel forces their intensities have a relation to each other dependent on the distances of their points of application from the directions of the vertical components of the applied forces, which relation may be determined from the known dimensions of the construction.

Representing by f' the coefficient of friction we have $F = F' + F'' = f'D + f'T$. This equation and the established relation between D and T provide two equations by means of which two of the quantities, D and F for instance, may be expressed in terms of the third, T.

Neglecting the term $v^2 \sin \theta / l$, there are now left unknown in the original equations the quantities T, S, d^2x/dt^2, $d^2\theta/dt^2$.

If a value of any one of these quantities is established by the given conditions the values of the others may be determined from the equations. For instance, the problem may specify that the pressure on the spade shall not exceed a certain limit. Then S would be known. Or it may be specified that there shall be no horizontal movement. This would make $d^2x/dt^2 = 0$. Or that there shall be no rotation; $d^2\theta/dt^2 = 0$.

Integrating the expression for the value of d^2x/dt^2 we obtain $dx/dt = v$, the velocity in the direction of x as a function of the time, and integrating again we obtain x, the distance passed over, also as a function of the time. Similarly, if the term d^2y/dt^2 remains among the unknown quantities.

Integrating $d^2\theta/dt^2$ we obtain the velocity of rotation, and integrating a second time we obtain the angular displacement, both as functions of the time.

The problem is now completely solved.

If there is no movement of the body the problem is much simplified, as under that condition the terms involving the differentials and the velocity v become 0.

The equations are also applicable in determining the relations that must exist, in order that any given condition may be fulfilled, between the dimensions and weight of a construction and the forces applied to it. This will be shown in the following problem.

176. Problem.—Determine, for the 3-inch field carriage, the relations that must exist between the constant resistance in the recoil system and the weight and dimensions of the carriage in order that there may not be any movement of the carriage when the firing is at zero elevation.

In the three equations (30) to (32), ϕ, the angle of elevation, becomes 0; and since there is to be no movement of the carriage the terms involving the accelerations and the linear velocity become 0. Without movement there will be no friction and F will also be 0.

The three equations then reduce to

$$P-S=0$$
$$D+T-W=0$$
$$Pp+Dd+Ss-Tt=0$$

which express the relations that must exist between the resistance P to recoil, the weight, and the dimensions of the carriage under the condition of stability imposed.

As the center of gravity of the system moves to the rear when the gun recoils on the carriage, the most unfavorable position of the center of gravity must be used in the equations. This will be the rearmost position.

Design of a Field Carriage to Fulfil the above Conditions.—Using the equations established in the preceding problem, W, the weight of the system composed of the gun and gun carriage must be such that when the weight of the limber filled with ammunition is added, the weight behind each horse of the team shall not exceed 650 pounds. The length of the trail l will be limited by considerations of draft and of the turning angle of the limbered carriage. The height of the carriage, $f+p_{(\phi=0)}$, must be such that the gun may be readily served and not too easily overturned. The area of the spade must be such that the pressure against it will not exceed 80 pounds per square inch, as it is found that in average ground the spade will not satisfactorily prevent movement of the carriage when the pressure against the spade exceeds this limit. Therefore the area of the spade $=S/80$.

By carefully weighing these and other considerations, and assuming successive values for the various quantities in the established equations, satisfactory dimensions for the carriage as a whole are finally determined.

Similar equations are established for each of the individual parts of the carriage in exactly the same manner as explained for the carriage as a whole. The stresses to which each part is subjected and the necessary strength and best form of the part to perform its functions are thus determined.

The pressure P determined from the above equations is the greatest pressure that may be transmitted to the carriage under the condition of stability imposed. The 3-inch gun recoils on its carriage and the recoil is controlled by a hydraulic brake and counter recoil springs. If we neglect the friction of the moving parts, P becomes at once the maximum constant resistance that may be permitted in the recoil controlling system. It is a value of R in equations (9) and (15). We will then determine, as explained under hydraulic brakes, the length of the recoil when opposed by this resistance, and the length so determined will be the minimum length of recoil that may be permitted on the carriage.

177. 3-inch Field Carriage Recoil System.—A longitudinal section through the gun recoil system of the 3-inch field carriage is shown in Fig. 108, drawn to a distorted scale in order to show the parts more clearly.

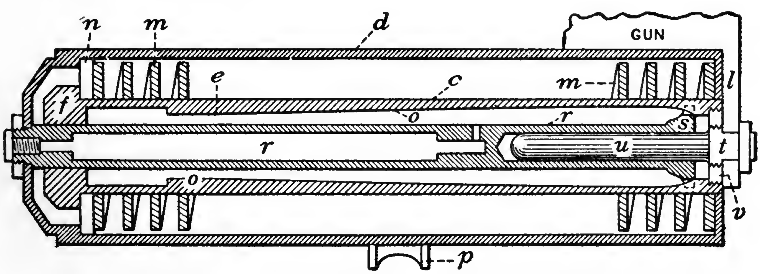


Fig. 108.

A cylindrical cradle d, of cross-section as shown in Fig. 109, is pintled by the pintle p in a part of the carriage called the rocker, not shown. The grooves a of the pintle are engaged by clips provided on the rocker. The rocker embraces the axle of the carriage and has a movement in elevation which is transmitted to the gun by the cradle.

The gun is provided with clips *k* which engage the upper flanges of the cradle; and when fired, the gun slides to the rear on the upper surface of the cradle. The lug *l*, Fig. 108, is an integral part of the gun. The counter recoil buffer *u* is attached to the lug by a bolt *t*, and the recoil cylinder *c* is attached to the same bolt by means of the screw *v*. Integral with the walls of the cylinder are three throttling bars *o*. The piston head *s* is provided with three corresponding apertures, Fig. 109.

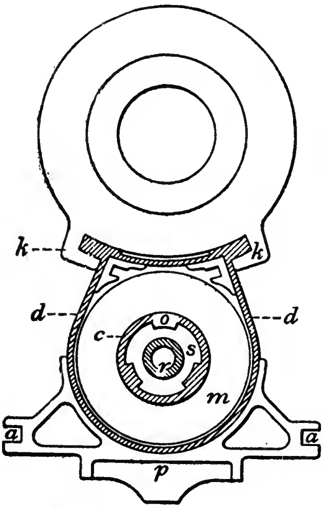


FIG. 109.

The hollow piston rod *r* is held to the front end of the cradle by a nut screwed on the forward end of the rod. The rod terminates at its rear end in the piston head *s*. The outer shoulder formed on the front head *f* of the recoil cylinder receives the thrust of the counter recoil springs *m* transmitted through the annular spring support *n*, which also serves to center the cylinder in recoil. The flat coiled springs *m* extend continuously from the front end to the rear end of the recoil cylinder.

The gun in recoiling draws with it, by means of the lug *l*, the recoil cylinder *c*, filled with oil, and the counter recoil buffer *u*. The piston, attached to the cradle, does not move. When the forward end *e* of the curve of the throttling bar reaches the piston head *s*, the apertures in the piston are completely closed against the flow of the liquid, and recoil ceases. The counter recoil buffer *u* has now been drawn all the way out of the piston rod.

Under the action of the springs *m*, which have been compressed by the recoil, the gun returns to battery. The first part of the counter recoil, during which the counter recoil buffer is out of the hollow piston rod, is unobstructed. When the buffer enters the piston rod the escape of oil from inside the rod is permitted only through the narrow clearance between the rod and the buffer. The resistance thus offered gradually diminishes the velocity of counter recoil and brings the gun to rest without shock as it comes into battery. The buffer is cylindrical for the greater part of its length, with a clearance in the piston rod of 0.025 of an inch on the diameter. The diameter of the buffer gradually enlarges over a length of three inches at the rear until the clearance is but 1/1000 of an inch on the diameter.

The pressure on the piston due to the recoil is transmitted through the cradle to the pintle *p* and thence to the carriage.

The length of recoil is 45 inches.

Recoil System of Other Carriages.—The recoil-controlling parts of the carriages for siege guns, and of the barbette carriages for seacoast guns six inches or less in caliber, embody the same principles as the system described above.

CHAPTER VIII.

ARTILLERY OF THE UNITED STATES LAND SERVICE.

178. Classification.—Service artillery may be broadly divided into two classes: mobile artillery and artillery of position.

Mobile artillery consists of the guns designed to accompany or to follow armies into the field, and comprises mountain, field, and siege artillery.

Artillery of position consists of the guns permanently mounted in fortifications. As the fortifications of the United States are all located on the seacoasts, the guns that form their armament are usually designated *seacoast guns.*

Mobile Artillery.—The mobile artillery of the United States as at present designed will consist of the following guns:

Gun.	Caliber.	Projectile.
Mountain gun	2.95 inch	18 lbs.
Light field gun	2.38 inch	7½ lbs.
Field gun	3.0 inch	15 lbs.
Field howitzer	3.8 inch	30 lbs.
Heavy field gun	3.8 inch	30 lbs.
Heavy field howitzer	4.7 inch	60 lbs.
Siege gun	4.7 inch	60 lbs.
Siege howitzer	6.0 inch	120 lbs.

The selection of these calibers is based on the following principles. The field gun, the principal artillery weapon of an army in the field, must have sufficient mobility to enable it to accompany the rapidly moving columns of the army. Long experience indicates that to attain the desired degree of mobility the weight behind each horse of the team should not exceed 650 pounds. A six horse team is used with the field gun. The total weight of the gun, carriage, limber, and equipment, with a suitable quantity of ammunition, is therefore limited to 3900 pounds. Limited by this requirement the power of the gun should be as great as it can be made. The shrapnel being the most important projectile of the field gun the caliber of the gun should be such as to give the shrapnel the greatest efficiency. Consideration of these requirements has led to the adoption of the 3-inch caliber for the field gun of our service.

A gun of greater power will, on those occasions when it can be brought into action, be more effective than the 3-inch gun. The heavy 3.8-inch field gun, firing a 30-pound projectile and possessing sufficient mobility to enable it to accompany the slower moving columns of the army, is therefore provided. The weight behind the six horse team is limited to 4800 pounds. With this weight the gun is capable of rapid movement for short distances.

The caliber of the siege gun is limited by the requirement that the weight of the gun shall not exceed the draft power of an eight horse team. The draft power of this team, for the siege gun, is taken as 8000 pounds.

Allowing for bad roads and rough usage and for the occasional necessity of covering considerable distances at high speed, the draft power of a horse for artillery purposes is taken as considerably less than the draft power of the horse used in ordinary commerce.

The guns above named are intended for the attack of targets that can be reached by direct fire, that is, by fire at angles of elevation not exceeding 20 degrees. For the attack of targets that are protected against direct fire and for use in positions so sheltered that direct fire cannot be utilized, curved fire, that is, fire at elevations exceeding 20 degrees, is necessary. There is therefore provided, corresponding to each caliber of gun, a howitzer of an equal degree of mobility. The howitzer is a short gun designed and mounted to fire at comparatively large angles of elevation.

In order to reduce to the minimum the number of calibers of the mobile artillery and thus simplify as far as possible the supply of ammunition in the field, the calibers of the guns and howitzers have been so selected that, while both guns and howitzers fulfil the requirements as to weight and power for each degree of mobility, the caliber of each howitzer is the same as that of the gun of the next lower degree of mobility. That is, the howitzer corresponding in mobility to one of the guns is of the same caliber as the next heavier gun and uses the same projectile.

As there may be occasions when profitable use can be made of a gun throwing a lighter projectile than that of the 3-inch field gun, the light field gun, 2.38-inch caliber, is provided. The weight of the projectile is 7½ pounds, this weight being considered the lowest limit for an efficient shrapnel. The 2.38-inch gun will probably be used for the movable defense of seacoast fortifications.

179. Advantages of Recent Carriages.—The chief difference between the latest and earlier designs of carriages for mobile artillery lies in the provision made in the later carriages for recoil of the gun on the carriage. By this means a part of the force produced by the discharge is absorbed in controlling the recoil of the gun on the carriage, leaving only a part available to produce motion of the carriage; and by the addition to the end of the trail of a spade which is sunk in the ground the carriage is enabled to withstand the transmitted force without motion to the rear. When the spade is once fixed firmly in the earth further firing of the gun does not produce recoil of the carriage. Rapidity of fire is thereby greatly increased, and the soldier is relieved from the fatiguing labor of running the carriage back into battery after each round.

Rapidity of fire is also increased by the use of fixed ammunition, and by the provision for a slight movement in azimuth of the gun on the carriage. The movement in azimuth permits a change in the pointing of the gun of three or four degrees to either side without disturbing the carriage after the spade is set in the ground.

In addition, the gun sights on all modern constructions are fixed to some non-recoiling part of the carriage so that they are not affected by the recoil. The operation of sighting may therefore go on continuously, independently of the loading and firing.

Our service field and siege carriages, with the exception of the 6-inch siege howitzer carriage, are so designed that the wheels will not be lifted from the ground under firings at zero elevation.

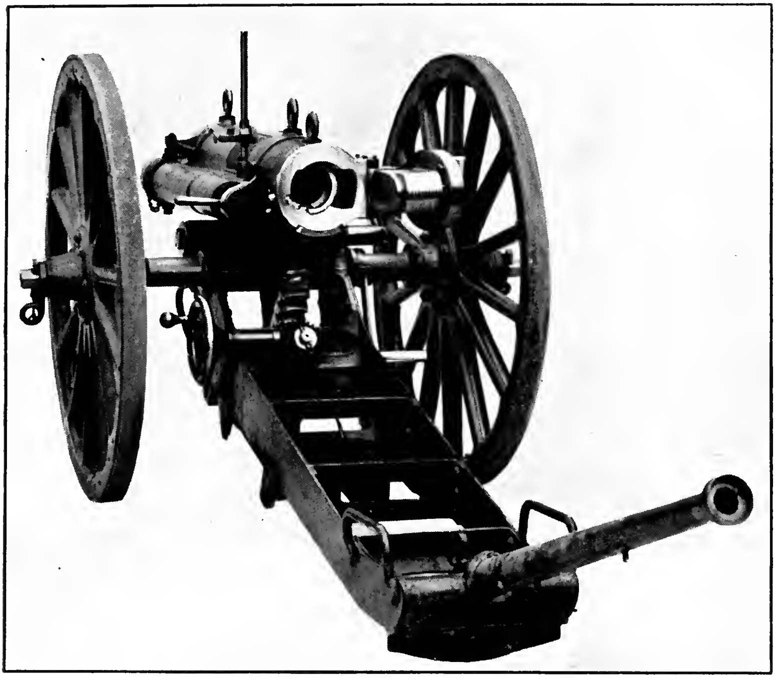


FIG. 110.—2.95-inch Mountain Gun.

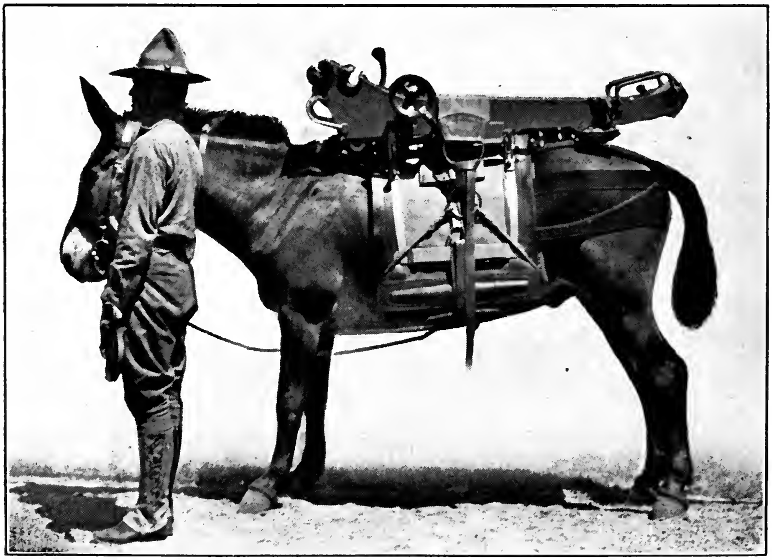


FIG. 111.—Transport of Trail.

The Mountain Gun.—For mountain service the system composed of gun and carriage must be capable of rapid dismantling into parts, no one of which will form too heavy a load for a pack mule. The weight of the load, including the saddle and equipment of the mule, should not exceed 350 pounds. The system must be capable of rapid reassembling for action.

The mountain gun used in our service, originally made by Vickers Sons and Maxim of England, has a caliber of 75 millimeters, or 2.95 inches, and fires projectiles weighing 12½ and 18 pounds. The caliber of this piece will probably soon be changed to 3 inches so that it may use the same projectile as the 3-inch field gun.

The gun is made from a single forging, and weighs complete with breech mechanism 236 pounds. Fixed ammunition is used in it. The breech mechanism, Fig. 110, is of the interrupted screw type. The block has two threaded sectors separated by flat surfaces. It is provided with percussion firing mechanism so arranged that the gun cannot be fired until the breech block is fully closed and locked. The trigger to which the firing lanyard is attached is seen to the left in the figure outside the breech. In case of a misfire the mechanism may be recocked without opening the breech.

180. The Carriage.—A low wheeled carriage is provided for the mountain gun. The wheels are 36 inches in diameter and have a track of 32 inches. The principal parts of the carriage are the cradle, the trail and elevating gear, the wheels and axle.

The Cradle.—The cradle is a bronze casting, with a central cylindrical bore and a smaller cylinder on each side. The central cylinder embraces the gun to within a few inches of the muzzle and forms a support in which the gun slides in recoil. The side cylinders are hydraulic buffers the piston rods of which are secured to lugs on the gun by interrupted screws so that the gun may be readily separated from the cradle. Grooves of varying width and depth cut in the interior walls of the buffer cylinders allow passage of oil from one side of the piston to the other in recoil. Constant pressure is maintained in the cylinder throughout the length of recoil, 14 inches. Spiral springs surrounding the piston rods return the gun to battery.

The cradle is secured to the trail by a bolt, seen above the axle in Fig. 110, which passes through two lugs formed on the under side of the cradle, the outer ends of the bolt fitting into two bearings or sockets provided at the forward upper end of the trail. The cradle moves in elevation about this bolt.

Light lifting bars are provided for use in dismantling and assembling the gun and carriage. They are passed through the two eye bolts on the top of the cradle, and through one on the gun.

Front and rear sights are attached to the cradle. The rear tangent sight is detachable.

THE TRAIL.—The trail consists of two outside plates or flasks of steel joined together by a shoe and three transoms. The shoe is provided with a spade on the under side to assist in checking recoil, and with a socket on the upper side, in which a handspike may be fitted, or the shafts attached when traveling on wheels. At the front end of the trail are the bearings for the cradle bolt and further to the rear are bearings for the axle. The bearings are open at the top, Fig. 112, the openings having a width less than the diameter of the bearing. The cradle bolt and axle tree are cylindrical, with flats cut on them so that they can only enter their bearings at a certain angle. When in position in the bearings they are turned through 90 degrees and thus secured. The crank secured to the axle at the right, Fig. 110, is for the purpose of turning the axle, in dismantling the carriage, to bring the flats of the axle in line with the openings of the bearings. When assembled the axle is locked in position by a spring latch bolt in the crank handle which engages in a slot provided in the trail.

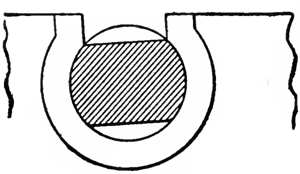


FIG. 112.

THE ELEVATING GEAR.—The elevating gear is permanently attached to the trail. Motion of the hand wheel, Fig. 110, is communicated to the gun through bevel gears, *b* Fig. 113, a worm, *w*, and a toothed quadrant, *q*, attached at its rear end to the cradle. An arm formed on the forward end of the quadrant embraces the cradle bolt and revolves around it. A cross bar, *c*, on each side near the upper end of the arm keeps the quadrant in a central position, and two spiral springs fastened to the front transom and acting on the arm maintain practically a uniform weight on the elevating gear while the gun is being elevated or depressed.

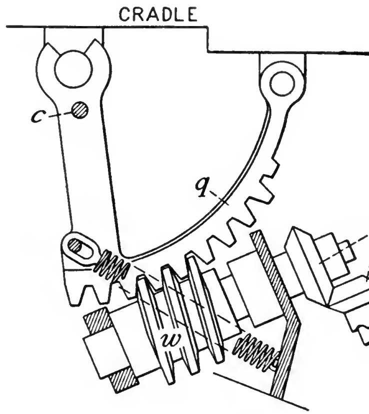


Fig. 113.

The gun may move in elevation from minus 10 degrees to plus 27 degrees.

181. Ammunition. — Fixed ammunition is used. The charge is about 8 ounces of smokeless powder. The 110-grain percussion primer is used in the cartridge case and a front igniter of about $\frac{1}{8}$ ounce of black rifle powder. Three kinds of projectiles are provided: canister, shrapnel, and high explosive shell. The canister and shrapnel weigh $12\frac{1}{2}$ lbs., the high explosive shell 18 lbs. The canister contains 244 cast iron balls each $\frac{5}{8}$ of an inch in diameter. The shrapnel contains 234 balls. The bursting charge for the shell is 2.07 lbs. of high explosive.

The muzzle velocity of the $12\frac{1}{2}$-lb. projectile is 850 feet. The maximum pressure in the bore is 18,000 lbs.

The gun has an effective range of about 4000 yards.

Transportation.—For purposes of transportation the gun and carriage, with tools, implements, and equipments, are divided into four loads, the principal items of which are the gun, the cradle, the trail, the wheels and axle. These loads, without the pack equipment, weigh approximately 250 lbs. each. The pack saddle and equipment weigh 90 lbs., so that the total weight carried by the mule is about 340 lbs.

The trail, which forms the most inconvenient load, is shown in Fig. 111, loaded on the pack animal.

The ammunition is carried in nine loads of 10 or 12 rounds each, according as the projectiles weigh 18 or $12\frac{1}{2}$ lbs. A box holding 5 or 6 rounds is slung on hooks on each side of the pack saddle by loops formed in wire straps about the box. The boxes open at the end so that the ammunition may be removed from them without disturbing the pack.

Field Artillery.—The field artillery as at present designed will consist of the 2.38-inch gun, the 3-inch gun, the 3.8-inch gun, and the 3.8-inch and 4.7-inch howitzers. It is also the intention to modify the carriage of the mountain gun so that the piece may be fired at high angles of elevation and be used as a light field howitzer. The caliber of the gun will then be changed to 3 inches so that the projectiles of the 3-inch field gun may be used in it. There is also at present in service a 3.6-inch field mortar.

Fixed ammunition is used in all field pieces except the mortar.

The following table contains data relating to the guns and carriages of the field artillery.

	Guns.			Howitzers.		Mortar.
Caliber, inches	2.38	3	3.8	3.8	4.7	3.6
Date of Model	1905	1905	1905	1906	1906	1890
Charge, lbs.	0.72	1.62	3	1.2	1.3	0.38
Projectile, lbs.	7.5	15	30	30	60	20
Bursting charge, lbs.	0.8	0.82	2.1	2.1	3.1	0.6
Cartridge complete, lbs.	9.5	18.75	38	35	65	..
Shrapnel balls, number	118	252	526	526	1063	..
Muzzle velocity, f. s.	1700	1700	1700	900	900	690
Maximum pressure, lbs.	33000	33000	33000	15000	15000	17000
Weight, limbered, lbs.	2400	3900	4800	3900	4800	..
AT MAXIMUM ELEVATION.						
Elevation, degrees	15	15	15	45	45	45
Time of flight, seconds	19.4	21.9	21	36.3	37.4	21.2
Remaining velocity, f. s.	664	737	769	707	752	515
Range, yards	5800	6100	6900	6300	6850	3360

Other data concerning the guns of the field artillery will be found in the table on page 135.

The velocities and pressures are fixed at the low figures given in the table in order that the guns and carriages may be kept within the limits as to weight.

With velocities of 400 feet the service shrapnel balls are effective against men, and with velocities of 880 feet, against animals. As the velocity of the balls is increased by from 250 to 300 feet at the bursting of the shrapnel, it will be seen from the table that shrapnel fire from the field pieces is effective at all ranges.

The designs of the field guns of different caliber, with their mounts, differ practically only in the size of the parts. A description of one will therefore answer for all.

182. The 3-inch Field Gun.—The 3-inch field gun is the principal weapon of the field artillery. The gun, of nickel steel, is built up in the manner described on page 236. A hoop called the clip is shrunk on near the muzzle. On the under side of this hoop, and of the locking hoop and jacket, are formed clips, *k* Fig. 117, which embrace the guide rails of the cradle of the carriage. The gun slides in recoil on the upper surface of the cradle. A downwardly extending lug, *l* Figs. 116 and 117, at the rear of the jacket serves for the attachment of the recoil cylinder, which moves with the gun in recoil.

THE BREECH MECHANISM.—The breech mechanism, model 1904, is shown in Fig. 114, in the locked position. The mechanism is of the slotted screw type.

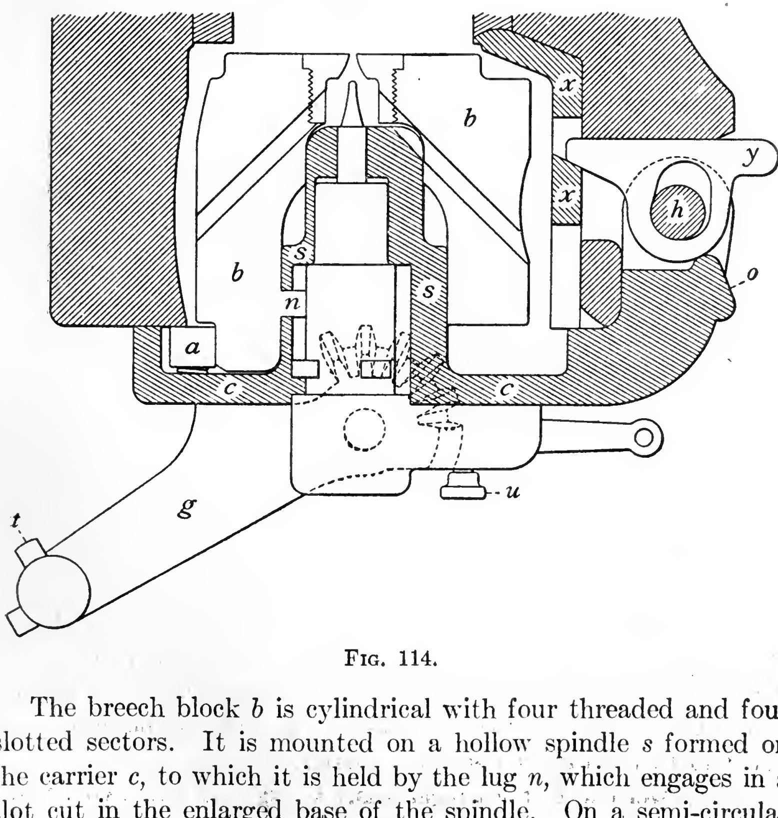


FIG. 114.

The breech block *b* is cylindrical with four threaded and four slotted sectors. It is mounted on a hollow spindle *s* formed on the carrier *c*, to which it is held by the lug *n*, which engages in a slot cut in the enlarged base of the spindle. On a semi-circular boss formed on the rear face of the block is cut a toothed rack, outlined at z, Fig. 117. The teeth of a bevel pinion formed on the inner end of the operating lever *g* mesh in the teeth of the rack. The lever is pivoted on a pin which passes through two lugs formed on the rear face of the carrier. On grasping the handle of the lever the pressure against a latch *t* in the handle unlocks the lever from the face of the breech. Swinging the lever to the rear rotates the block until it is stopped by a lug inside the carrier and locked in position by the spring stud *a*. Further movement of the lever causes both block and carrier to rotate together about the hinge pin *h*. When the movement is nearly complete the surface *o* of the carrier bears against the arm of the extractor lever *y*, which causes the extractor *x* to move sharply to the rear and eject the empty cartridge case.

183. The Firing Mechanism.—The firing mechanism, Fig. 115, is contained in the firing lock case *f*, which is inserted into the

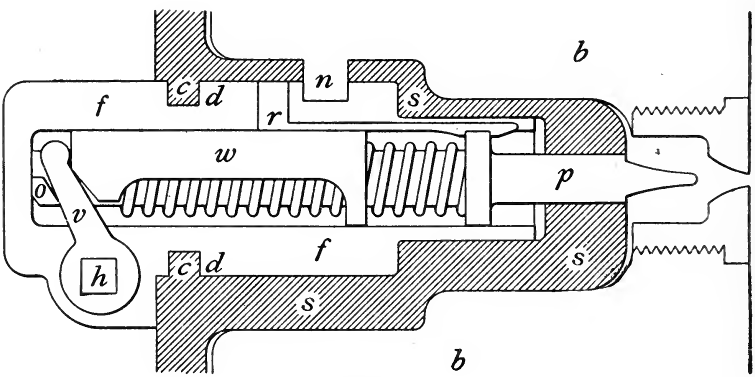


Fig. 115.

hollow spindle from the rear, the interrupted lugs *d* on the lock case engaging behind corresponding interrupted lugs *c* on the carrier. Assembled in the lock case are the firing pin *p*, the spiral firing spring, the firing pin sleeve *w*, and the trigger fork *v*, the latter fitting over the squared end of the trigger shaft *h*, which is journaled in an arm of the lock case *f*, Fig. 117, extending downward and to the right outside the carrier.

At the lower end of the trigger shaft *h*, Fig. 117, are two levers at right angles to each other, one marked trigger provided with an eye for the hook of the lanyard, the other acted upon by an upwardly extending lug on the end of the firing lever shaft.

A narrow section of the forward end of the lock case, Fig. 115, is cut out for the flat sear spring *r*. A notch in the sear engages the shoulder formed on the firing pin. The sleeve *w* at its rear end bears upon the last coil of the firing pin spring. When the trigger shaft *h* is turned by a pull on the lanyard, or by means of the firing lever, the trigger fork *v* forces the sleeve *w* to the front, compressing the firing spring. The forward end of the sleeve pushes the sear spring aside from its engagement on shoulder of firing pin, and the compressed spring then drives the firing pin forcibly forward until arrested by the shoulder striking the inner surface of the spindle. When the pull on the lanyard has ceased, the firing spring, still compressed, exerts a pressure against the rear end of the sleeve *w*, thence on the fork *v*, and on the head *o* of the firing pin; and the construction of these parts is such that the spring can regain its extended length only when the parts are in the position shown in the figure. The firing pin is therefore immediately withdrawn, on the cessation of the lanyard pull, until caught again by the sear.

The system of cocking and firing the piece by one movement is called the continuous pull system. The firing spring is compressed only at the moment of firing, whereas in the mechanism that is cocked in opening the breech the firing spring is compressed whenever the breech is opened and may remain compressed for a long time.

Safety Devices.—Safety against discharge before the breech is fully closed is secured as follows. The axis of the spindle *s* on the carrier, Fig. 114, lies $\frac{3}{10}$ of an inch below and $\frac{3}{10}$ of an inch to the right of the axis of the gun. The breech block which revolves on this spindle is therefore eccentric with the bore. The firing mechanism is eccentric with the block, the axis of the firing mechanism being fixed in the axis of the bore. When the block is locked the hole in its front end through which the firing pin protrudes in firing is also in the axis of the bore, but as the block is rotated in opening, the hole rotates out of the axis of the bore and the flat surface at its rear end comes in front of the firing pin and prevents movement of the firing pin until the breech is locked.

The headed spring pin *u*, Fig. 117, enters a hole in the carrier and retains the firing mechanism in its position in the carrier. By withdrawing this pin and rotating the firing lock case *f* upward through 45 degrees the interrupted lugs *d*, Fig. 115, on the firing lock case disengage from behind the interrupted lugs *c* on the carrier, and the firing mechanism may be withdrawn from the gun. The breech block is then readily removed. The breech mechanism may thus, without the use of tools, be readily dismantled for repair, or the gun may be quickly disabled in the event of imminent capture.

Four holes are drilled rearwardly through the breech block, *b* Fig. 114, to permit the escape of gas without injury to the screw threads of the mechanism in case the primer in the cartridge is punctured by the blow of the firing pin.

The 3-inch Gun, Model 1905.—The 3-inch gun, model 1905, is 50 lbs. lighter than the 1902 and 1904 models, the outside diameters being slightly diminished. The twist of the rifling, which in the earlier models increases from 1 turn in 50 calibers at the breech to 1 in 25 at the muzzle, increases from zero at the breech to 1 in 25 at 9¾ inches from the muzzle, from which point it is uniform to the muzzle. The purpose of the change in twist is to diminish the resistance encountered by the projectile in the first part of its movement and thereby diminish the maximum pressure. The short length of uniform twist at the muzzle steadies the projectile as it issues from the bore.

184. The Carriage.—The principal parts of the carriage are the cradle, the rocker, the trail, the wheels and axle.

The Cradle.—The cradle, *c* Figs. 116 and 117, is a long steel cylinder, which contains the recoil controlling parts. These parts are fully described in the chapter on recoil, and illustrated in Figs. 108 and 109 of that chapter. The gun slides in recoil on the upper surface of the cradle, the clips of the gun, *k* Fig. 117, engaging the flanged edges. A pintle plate fastened to the bottom of the cradle is provided with the pintle *p*, Fig. 117, and the grooved arc *a*, which serve to connect the cradle to the rocker.

The Rocker.—The rocker *r* embraces the axle between the flasks of the trail by the bearings at its ends. The cradle pintle fits in a seat provided in the rocker above the axle, and the clips

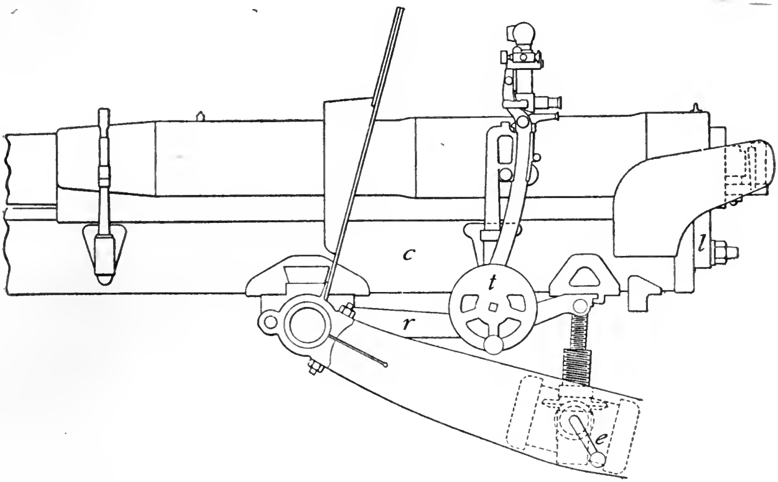


FIG. 116.

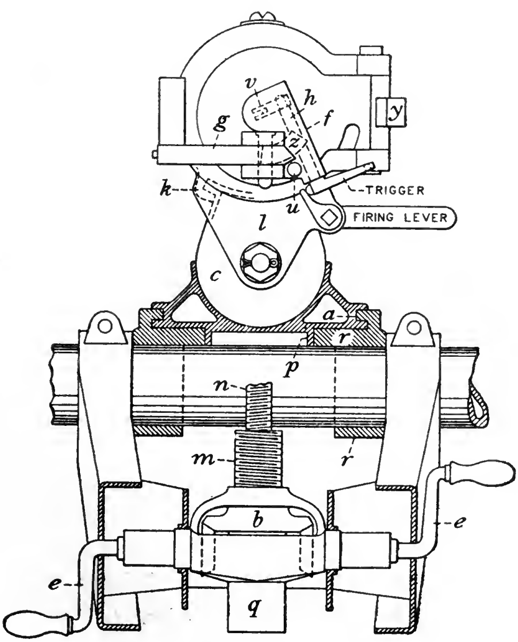


FIG. 117.

on the rocker engage in the grooved arc *a* of the cradle. This construction permits movement of the cradle and gun in azimuth on the rocker, while the rocker itself revolves about the axle and thus gives movement in elevation to the cradle and the gun. The movement in azimuth, 4 degrees either way, is produced by a screw on the shaft of the hand wheel *t*, Fig. 116. The shaft is fixed in bearings in the rocker arms and the screw works in a nut pivoted in a bracket fastened under the cradle.

The double elevating screw, actuated by either of the crank shafts *e* fixed in bearings in the trail, rotates the rocker and cradle about the axle. The bevel pinion on the end of each shaft *e* rotates the bevel pinion *b* in its bearings. The pinion *b* is splined to the outer screw *m* and causes the outer screw to turn in the fixed nut *q* which is supported below the pinion *b* by a transom. The outer screw *m* has a left handed thread on the exterior and a right handed thread in the interior. When turned it travels up or down in the nut *q*, and at the same time causes the inner screw *n* to move into or out of the outer screw, the inner screw being prevented from turning by its connection with the rocker arms, *r* Fig. 116. The movement of the inner screw for each turn of the pinion *b* is thus equal to the sum of the pitches of the outer and inner screws.

THE TRAIL.—The trail, Fig. 119, composed of two flanged steel flasks connected by transoms and top and bottom plates, terminates at its lower end in a fixed spade provided with a float or wings which prevent excessive burying of the spade in the ground. The lower edge of the spade is of hardened steel riveted on so that it may be readily replaced when worn out. The lunette, a stout eye bolt fixed in the end of the trail, engages over the pintle of the limber when the carriages are connected for traveling. Seats for two cannoneers who serve the piece in action are attached to the trail one on either side near the breech of the piece; and two other seats on the axle, facing toward the muzzle, are occupied in traveling by two cannoneers, one of whom manipulates the lever of the wheel brakes.

THE WHEELS AND AXLE.—The axle of forged steel is hollow. The axle arms are given a set so as to bring the lowest spoke of each wheel vertical.

The wheels are a modified form of the Archibald pattern, 56 inches in diameter with 3-inch tires. The hub, Fig. 118, consists of a steel hub box *h* and hub ring *r* assembled by bolts through the flanges, between which the spokes of the wheel are tightly clamped. The hub box is lined with a bronze liner forced in. A steel cap *c* is screwed on the outer end of the hub box. Riveted to the cap is a self closing oil valve, by means of which the wheels are oiled without removal from the axle. The hollow axle forms a reservoir for the oil.

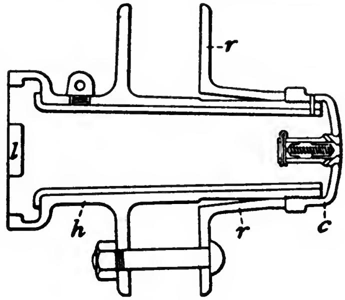


FIG. 118.

The wheels are secured to the axle by the wheel fastening, a bronze split ring, hinged for assembling around the axle. The ring revolves freely in a groove in the axle. Interrupted lugs on its exterior engage behind corresponding interrupted lugs, *l* Fig. 118, in the inner end of hub box, and hold the wheel on the axle. A hasp connects the hub and the wheel fastening so that they cannot revolve independently and disengage the lugs.

185. THE SHIELD.—The cannoneers serving the piece are protected by a shield of hardened steel $\frac{2}{10}$ of an inch thick. It is in three parts. One part, the apron, depends from the axle and is swung up forward under the cannoneers' seats when traveling. The main shield, rigidly attached to the frame of the carriage, extends upwards from the axle to $2\frac{1}{4}$ inches below the tops of the wheels. The top shield is hinged to the main shield. When raised its upper edge is 62 inches from the ground, a height sufficient to afford protection from long range and high angle fire to cannoneers on the trail seats. In traveling the top shield is folded over so that should the carriage turn over on the march the shield is partially protected from injury. Each shield before being attached to the carriage is tested at a range of 100 yards with a bullet from the service rifle. The plate must not be perforated, cracked, broken, or materially deformed in the test.

SIGHTS.—The piece is provided with three different means of sighting. Two fixed sights on the upper element of the gun, Fig. 116, determine a line of sight parallel to the axis, for use in giving general direction to the piece. For more accurate sighting a tangent rear sight and a front sight with crossed wires are provided. They are seated in brackets attached to the cradle. A telescopic panoramic sight is seated on the stem of the tangent sight. This sight is used for direct aiming and for indirect aiming, which consists in pointing the gun by means of a line of sight considerably divergent from the line of fire. By means of the panoramic sight any object in view from the gun may be used as an aiming point.

A range quadrant, seated on the cradle of the carriage, provides the means of determining the elevation in indirect fire.

The sights are fully described in the chapter on sights, Chapter XIII, and the range quadrant in Chapter XIV.

The Limber.—The limber, Fig. 120, is practically wholly of metal, the neck yoke and pole, and spokes and felloes of the wheels, being the only wooden parts. The body of the limber is a steel frame, composed of three rails riveted to lugs formed on the axle and braced by steel tie rods. The middle rail is in the form of a split cylinder, one half passing below the axle and the other above. The halves unite in front forming a socket for the pole, which is held firmly in place by a clamp. Similarly in the rear the middle rail forms a seat for the pintle hook. The pintle hook is swiveled in its seat, so that if at any time the gun carriage turns over the pintle will turn without overturning the limber as well.

The ammunition chest, of sheet steel, is fastened to the outer rails. The front of the chest and the door which forms the rear are strengthened by vertical corrugations. The door opens downward and is then supported by chains. The metallic ammunition is supported in the chest by three diaphragms each perforated with 39 holes. The middle and rear diaphragms are connected by flanged brass tubes cut away on top to reduce the weight. The tubes support the front ends of the cartridge cases and enable blank ammunition and empty cases to be carried.

Seats made of sheet steel are provided for three cannoneers on the limber chest, and a steel foot-plate rests on the rails in front of the chest.

The wheels of the limber and the wheels of all other carriages

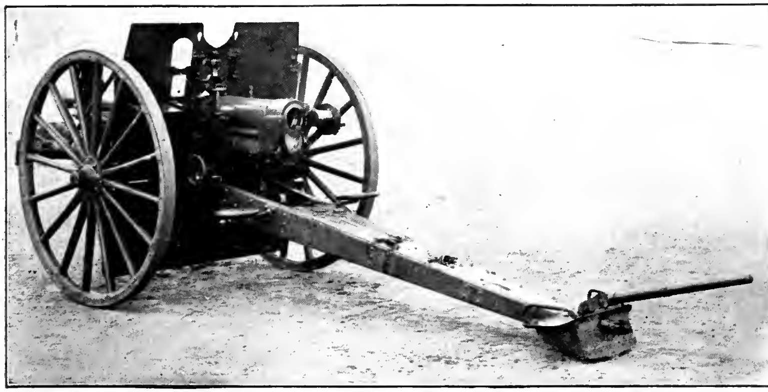


Fig. 119.—3-inch Field Gun, Model 1902.

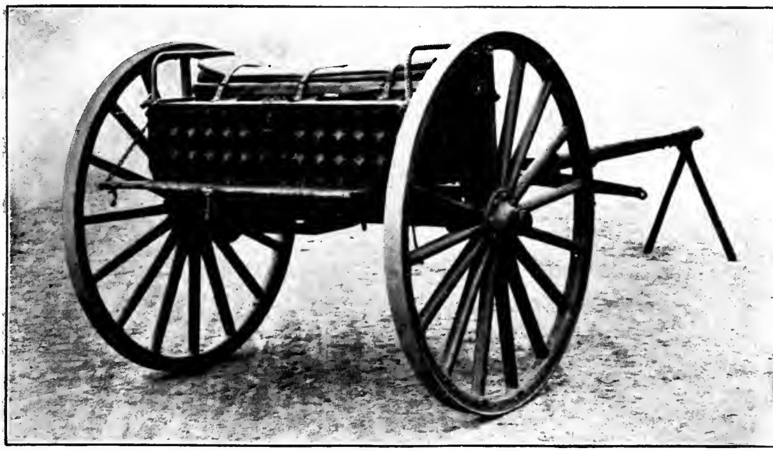


Fig. 120.—3-inch Field Limber.

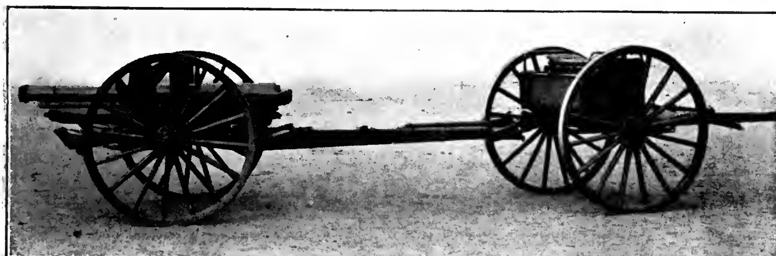


Fig. 121.—3-inch Field Gun, Limbered.

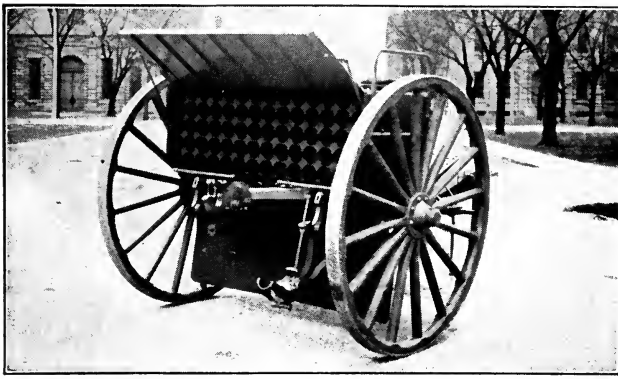


Fig. 122.—3-inch Field Caisson.

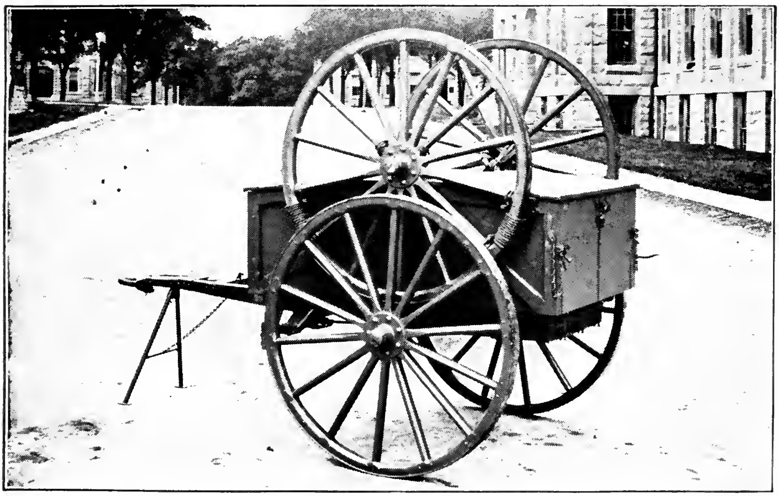


Fig. 123.—3-inch Field Battery Wagon.

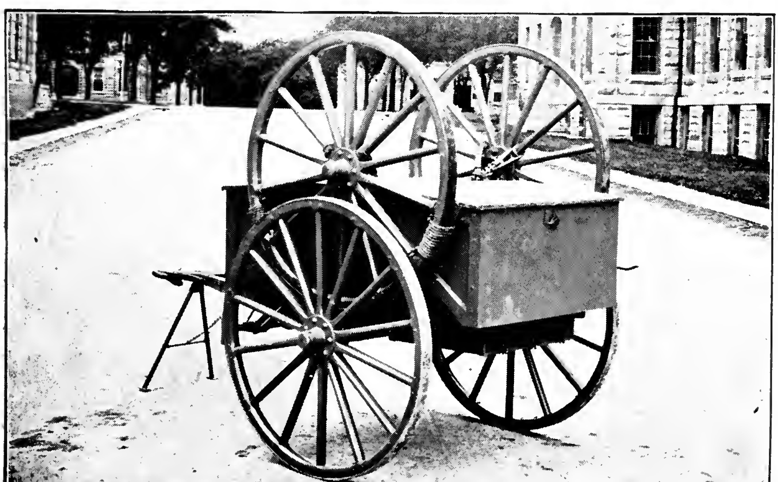


Fig. 124.—3-inch Field Store Wagon.

that form part of a field battery are interchangeable with the wheels of the gun carriage.

186. The Caisson and other Wagons.—The construction of the caisson, Fig. 122, does not differ materially from that of the limber. The ammunition chest is larger and carries 70 rounds of ammunition. The front of the chest is of armor plate $\frac{2}{10}$ of an inch thick; and the door at the rear, which opens upward to an angle of about 30 degrees above the horizontal, is of armor plate $\frac{15}{100}$ of an inch thick. A $\frac{2}{10}$-inch plate also depends from the axle as in the gun carriage. The cannoneers serving the caisson are thus afforded protection for a height of 63 inches from the ground.

Attached to the caisson by a hinged bracket at the rear is an automatic fuse setter, by means of which the cannoneer at the caisson may quickly set the fuse of the projectile to the time of burning corresponding to any range ordered by the battery commander. The fuse setter is described in the chapter on primers and fuses, and is illustrated in Fig. 229.

Three caissons with their limbers accompany each gun into the field.

The wagons of a battery include also the forge limber, which, as its name indicates, carries a blacksmith's forge and set of tools; and the battery wagon, Fig. 123, which carries carpenter's and saddler's tools and supplies; materials for cleaning and preservation; spare parts of gun, of carriage, and of harness; tools and implements; miscellaneous supplies and two spare wheels.

A wagon called the store wagon, Fig. 124, is for use in carrying such stores, spare parts, and materials as cannot be carried in the battery wagon.

Experiments are now being conducted toward the development of an automobile battery wagon.

Field Howitzers and Mortars.—The 3.8-inch and 4.7-inch field howitzers have not yet been constructed. The principles of construction of the guns and carriages will be understood from the description of the 6-inch howitzer and carriage which follows later.

There is at present in service a 3.6-inch field mortar shown in Fig. 125. The piece is a short gun intended for vertical fire against troops protected by intrenchments or other shelter. The Freyre obturator described on page 262 is used in the breech mechanism to save weight. The gun weighs 245 lbs. and its mount 300 lbs. more, so that the gun with its mount may be readily moved in the field. The mount is a single steel casting. The gun is held at any desired elevation by means of a clamp which acts on a steel arc attached to the under side of the gun.

When in use the carriage rests on a wooden platform, and recoil is checked by a heavy rope attached to stakes driven into the ground in front.

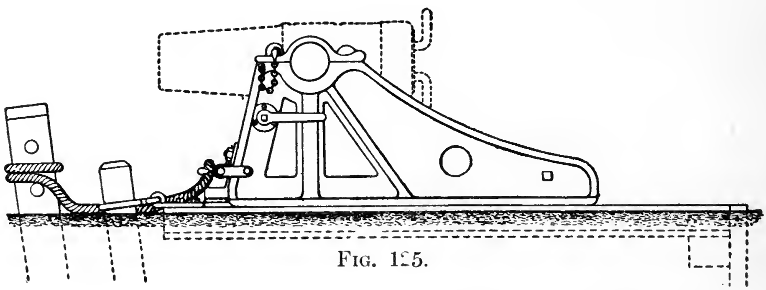


Fig. 125.

187. Siege Artillery.—The new siege artillery comprises the 4.7-inch gun and the 6-inch howitzer. The older siege pieces now in service are the 5-inch gun, the 7-inch howitzer, and the 7-inch mortar.

The following table contains data relating to the guns and carriages of the siege artillery.

	Guns.		Howitzers.		Mortar.
Caliber, inches	4.7	5	6	7	7
Date of model	1904	1898	1905	1898	1892
Charge, lbs.	5.94	5.37	4	4.6	4.0
Projectile, lbs.	60	45	120	105	125
Bursting charge, lbs.	3.1	1.75	3.86	7.4	11.9
Cartridge complete, lbs.	$73\frac{3}{4}$	..	..	..	..
Shrapnel balls, number	1063	..	2150	..	..
Muzzle velocity, f. s.	1700	1830	900	1100	800
Maximum pressure, lbs.	33000	35000	15000	28000	20000
Weight, limbered, lbs.	8000	8800	7900	..	..
AT MAXIMUM ELEVATION.					
Elevation, degrees	15	31	45	35	45
Time of flight, seconds	21.8	38.2	37.5	34.3	32.9
Remaining velocity, f. s.	971	638	764	749	641
Range, yards	7600	10000	7000	7700	5200

Other data concerning the guns of the siege artillery will be found in the table on page 135.

The 4.7-inch Siege Gun.—The gun is similar in construction and in breech mechanism to the 3-inch field gun. Fixed ammunition is used in it.

THE CARRIAGE.—The carriage is, in general, similar in construction to the 3-inch field carriage. The greater weight of the gun and the increased force of recoil render necessary certain changes in the parts. In the 3-inch carriage the recoil cylinder and counter recoil springs are assembled together in a single cylinder in the cradle. The cradle of the 4.7-inch carriage, Figs. 127, 128, and 129, consists of three steel cylinders bound together by broad steel bands, the middle band provided with trunnions. The middle cylinder contains the mechanism for the hydraulic control of recoil. Each of the outer cylinders contain three concentric columns of coiled springs for returning the gun to battery. The front end of each of the outer two spring columns is connected to the rear end of the next inner column by a steel tube, flanged outwardly at the front end and inwardly at the rear end. A headed rod passes through the center of the inner coil and is fixed to a yoke that is fastened to the lug at the breech of the gun, see Fig. 128. The head of the rod acts on the inner coil only, and the pressure is transmitted through the flanged tubes or stirrups to the outer coils. In this way the springs work in tandem and have a long stroke with short assembled length.

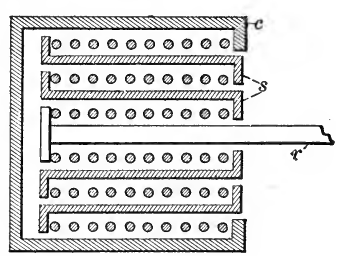


FIG. 126.

The arrangement of the springs will be understood by reference to Fig. 126, in which *r* represents the headed rod, *s* the tubular stirrups, and *c* the walls of the cradle cylinder.

The length of recoil is 66 inches.

The gun is supported, and slides in recoil, on rails *r* fixed on top of the spring cylinders. The distance apart of the rails broadens the bearing of the gun and gives it steadiness both in action and in transportation. An extension piece, bolted to the front end of the cradle and readily detachable, continues the rails to the front clip of the gun. When traveling this extension piece is detached and carried in fastenings under the trail.

THE PINTLE YOKE.—The cradle is trunnioned in a part called the pintle yoke, *y* Fig. 127, which is itself pintled in a seat, *p*, called the pintle bearing, mounted between the forward ends of the trail flasks, its rear end embracing the hollow axle *x*. A traversing bracket, *b*, is attached to the bottom of the pintle yoke and extending to the rear under the axle forms a support for the

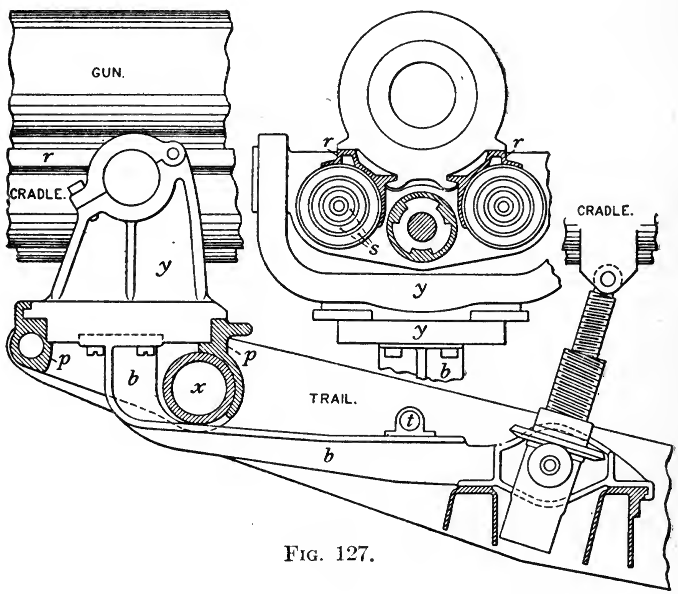


FIG. 127.

traversing shaft *t* and for the elevating mechanism. The rear end of the traversing bracket slides on supporting transoms between the flasks of the trail, motion being given to the bracket by means of a screw on the traversing shaft which works in a nut suitably attached to the trail. The gun may be moved in azimuth on the carriage 4 degrees either way. The elevating mechanism is carried on the traversing bracket and moves with the gun in azimuth. It is therefore not subjected to any cross strains. The gun may be moved in elevation from minus 5 to plus 15 degrees.

188. THE WHEELS AND THE TRAIL.—The wheels are 60 inches in diameter with 5-inch tires. Exhaustive tests recently con-

Fig. 128.—4.7-inch Siege Gun, Firing Position.

Fig. 129.—4.7-inch Siege Gun, Traveling Position.

cluded indicate that no practical advantage is gained by the use of wider tires on vehicles of this class and weight.

The trail is of the usual construction, two pressed steel flasks of channel section tied together by transoms and plates. The front ends of the flasks are riveted to cast steel axle bearings which extend to the front of the axle and support between them the pintle bearing *p*. The location of the pintle socket in front of the axle permits the use of a shorter trail and reduces the weight at end of trail to be lifted in limbering.

Bearings are provided at about the middle of the trail, in the opening seen in Fig. 128, for a detachable geared drum which is used in giving initial compression to the counter recoil springs in assembling, and in withdrawing the gun to its traveling position. When not in use the drum is kept in the tool-box in the trail.

The spade with its horizontal floats is hinged to the trail on top. For traveling it is turned up and rests on top of the trail, see Fig. 129; for firing it is turned down. In either position it is locked in place by a heavy key bolt.

A bored lunette plate is riveted to the bottom of the trail, for engagement on the pintle of the limber.

The Limber.—The limber, Fig. 130, is merely a wheeled turn-

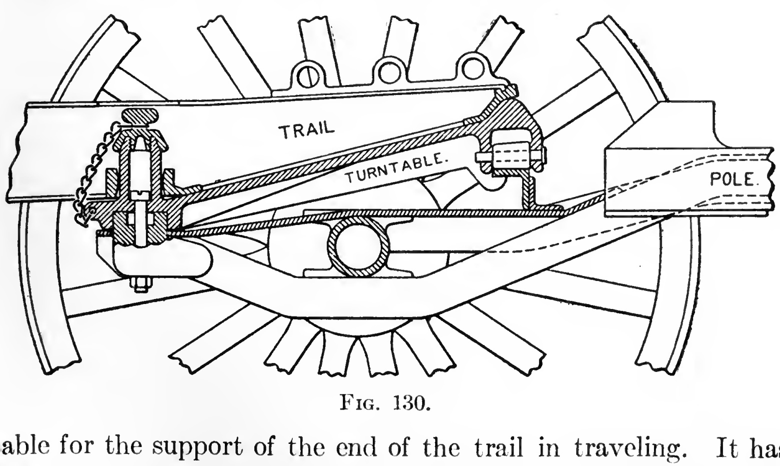


Fig. 130.

table for the support of the end of the trail in traveling. It has the usual arrangements for the attachment of the team. Its wheels are interchangeable with those of the carriage. The turntable, shaped to fit the end of the trail, is mounted on a frame fixed to the axle. It forms a seat for the trail. The seat is pivoted at the rear end and its front end rests on rollers which travel on a circular path on the limber. A pintle on the seat engages in the lunette in the bottom of the trail.

When traveling, in order to distribute the weight as evenly as possible between the front and rear wheels of the limbered carriage, the gun is disconnected from the piston rod and spring rods, and drawn back 40 inches to the rear, Fig. 129. In this position the recoil lug is secured between two stout braces attached to a heavy trail transom. The breech of the gun is thus supported and rigidly held in traveling, and the elevating and traversing mechanisms are relieved from all strains. The braces referred to are pivoted in the trail, and when not in use are turned down inside the trail.

189. Weights.—The weight of the gun carriage complete is 4440 lbs., and that of the gun and carriage, 7170 lbs. The weight at the end of the trail, gun in firing position, or the weight to be lifted in limbering, is 400 lbs.; with the gun in traveling position, this is increased to 1150 lbs., which is the part of the weight of the gun carriage sustained by the limber.

Siege Limber Caisson.—For the transportation of ammunition for siege batteries there is provided a vehicle called the siege limber caisson. As the name indicates, this vehicle is composed of two parts. Each part supports an ammunition chest arranged to carry 28 rounds of 4.7-inch ammunition or 18 rounds of 6-inch ammunition, thus making 56 rounds of 4.7-inch ammunition or 36 rounds of 6-inch ammunition per vehicle. For each siege battery of 4 guns 16 limber caissons are provided.

The 6-inch Siege Howitzer.—This is a short piece, 13 calibers long, mounted on a wheeled carriage so constructed that the piece can be fired at angles of elevation from minus 5 to plus 45 degrees. This wide range of elevation on a wheeled mount introduces into the carriage requirements not encountered in the construction of the carriages previously described, which provide for a maximum elevation of 15 degrees.

The piece is made from a single forging, Fig. 131. A lug, *l*, extends upward from its breech end for the attachment of the recoil piston rod and the yoke for the rods of the spring cylinders.

Flanged rails *r* formed above the piece support it on the cradle of the carriage, on which the piece slides in recoil.

The operating lever of the breech mechanism of the gun, Figs. 132 and 133, is above the axis of the gun instead of below it as in other guns. It is so placed for the purpose of increasing the clearance in recoil and for convenience in operating.

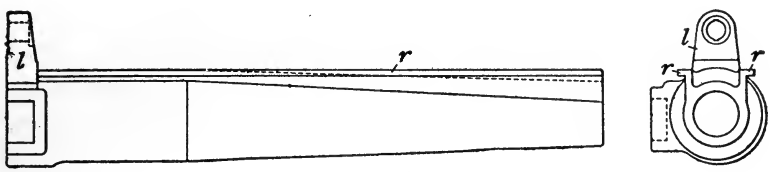


Fig. 131.

190. The Carriage.—The cradle, Figs. 132 and 133, is provided with recoil and spring cylinders. The arrangement of the springs in the spring cylinders is the same as shown in Fig. 126 for the 4.7-inch siege gun. The gun is placed below the cylinders in order that the center of gravity of the system may be as low as possible. The trunnions of the cradle rest in beds in the top carriage, which in turn rests on and is pintled in the part called the pintle bearing. Flanges on the top carriage engage under clips on the pintle bearing. The forward ends of the trail flasks are riveted to the pintle bearing, which forms a turntable on which the top carriage, and the parts supported by it, have a movement of three degrees in azimuth to either side. The traversing is accomplished by means of the hand-wheel *t* on the left side. The traversing shaft is supported in a bracket, *a*, fixed to the left flask, and its worm works in a nut, *o*, pivoted to the top carriage.

The Rocker.—The rear part of the rocker is a U-shaped piece that passes under the gun and is attached to the cradle by the hook *k*, pivoted in the cradle. Arms extend forward from the sides of the U and embrace the cradle trunnions between the cradle and the cheeks of the top carriage, so that the rocker may rotate about the cradle trunnions. The sights are seated on a bar supported on the left vertical arm of the rocker. The upper end of the elevating screw *n* is attached to the bottom of the rocker, while the lower end of the screw and the elevating gear are sup-

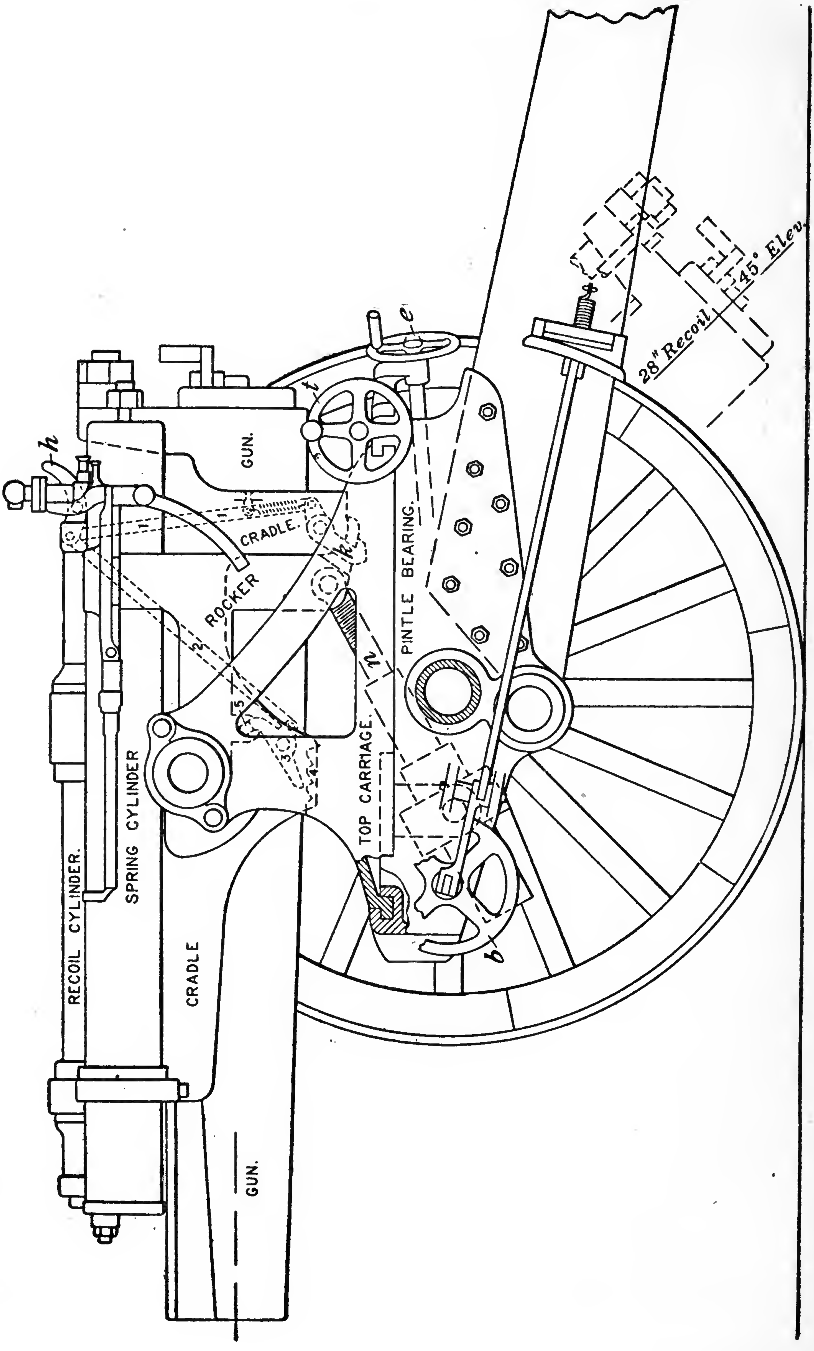


FIG. 132.—6-inch Siege Howitzer.

ported by trunnions in lugs on the under side of the top carriage. The rocker therefore moves in elevation in the top carriage and gives elevation to the gun-supporting cradle fastened to the rocker by the hook *k*. The elevating apparatus is operated by a hand-wheel *e* on either side.

The Trail.—The flasks of the trail extend separately to the rear a sufficient distance to permit free movement between them

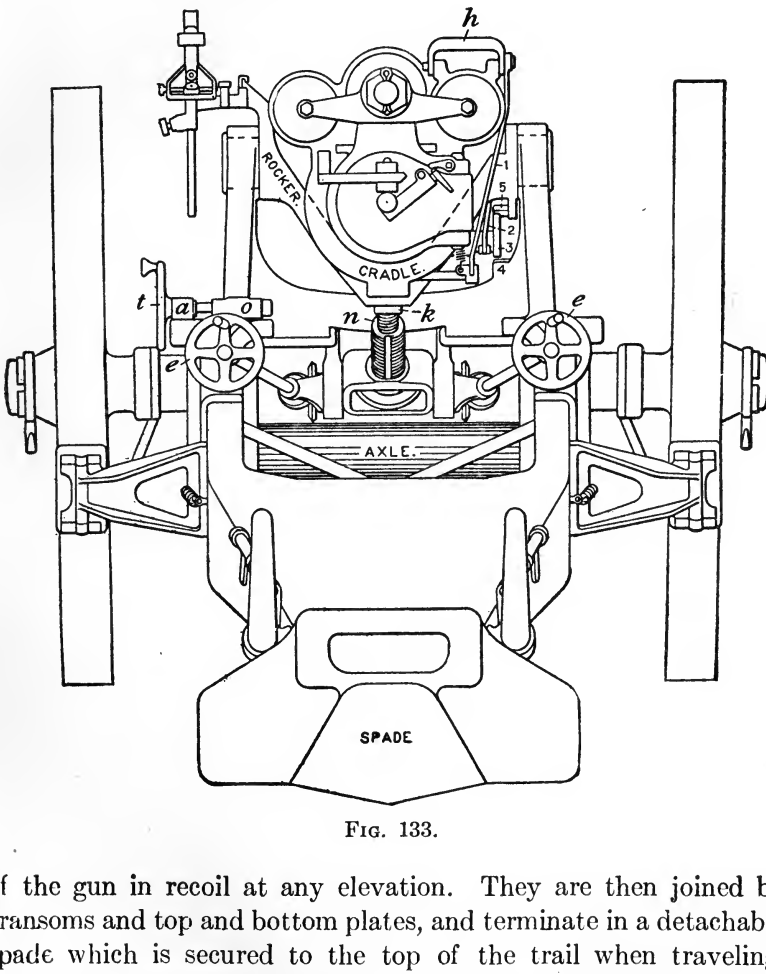


Fig. 133.

of the gun in recoil at any elevation. They are then joined by transoms and top and bottom plates, and terminate in a detachable spade which is secured to the top of the trail when traveling. Sockets are provided for two handspikes at the end of the trail. Two lifting bars are also fixed to the trail. In order to permit the desired movement of the cradle in elevation the axle is in three parts, the middle part lower than the two axle arms. The three parts are held by shrinkage in cylinders formed in the sides of the pintle bearing.

The wheel brakes, used both in firing and in traveling, are manipulated by hand-wheels *b* in front of the axle.

191. Recoil Controlling System.—The feature of this carriage which chiefly differentiates it from other carriages described is the provision for the automatic shortening of recoil as the elevation of the gun is increased. From minus 5 degrees to 0 elevation the gun has a recoil of 50 inches. As the elevation increases from 0 to 25 degrees the length of recoil diminishes continuously from 50 inches to 28 inches. For elevations between 25 and 45 degrees the length of recoil rmeains at 28 inches. The variation in length of recoil is necessitated by the approach of the breech to the transoms and to the ground as the piece is elevated.

The automatic regulation of recoil is produced in the following manner. Four apertures are cut in the piston of the recoil cylinder and two longitudinal throttling grooves in the walls of the cylinder. The total area of apertures and deepest section of the grooves is the proper maximum area of orifice for the 50-inch length of recoil, while the grooves alone furnish the proper continuous area of orifice for a recoil of 28 inches. A disk rotatably mounted on the piston rod against the front of the piston, and provided with apertures similar to those in the piston and similarly placed, is rotated on the piston rod during the recoil of the piece by two lugs projecting into helical guide slots cut in the walls of the recoil cylinder. The rotating disk gradually closes the apertures in the piston, and the twist of the guiding slots is such that the area of orifice is varied as required for limiting to 50 inches the recoil of the gun when fired at 0 elevation.

The recoil cylinder is rotatably mounted in the cradle. Teeth cut on its outer surface, Fig. 134, mesh in the teeth of a ring surrounding the right spring cylinder, and the teeth of the ring also mesh, at any elevation between 0 and 25 degrees, in a spiral gear cut on the cylindrical block *s*, which is seated in the hollow trunnion of the cradle and is fast to the right cheek of the top carriage. As the gun is elevated from 0 to 25 degrees the spiral teeth of the gear cause the ring to rotate clockwise and the cylinder counter clockwise. The rotating recoil cylinder carries with it the disk in front of the piston, causing the disk to close the piston apertures more and more until at 25 degrees elevation they are completely closed. The throttling grooves in the walls of the cylinder then provide the proper area of orifice for the 28-inch length of recoil permitted to the gun at elevations between 25 and 45 degrees.

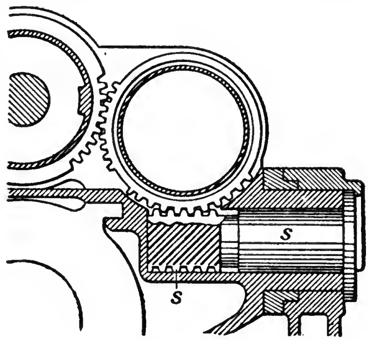


FIG. 134.

LOADING POSITION.—To load the piece after firing at high angles the hook *k*, which holds the cradle to the rocker, is disengaged by means of a handle, *h*, conveniently placed on top of the cradle, and the cradle and gun are swung by hand to a convenient position for loading. The center of gravity of the tipping parts is in the axis of the trunnions. A pawl, 3, attached to the cradle automatically engages teeth, 4, on the top carriage and retains the gun in the loading position until released by means of the same handle *h* that was used to disengage the cradle hook.

As the sights and elevating screw are attached to the rocker, their positions are not affected by the position of the piece in loading. The operations of laying the piece may therefore be performed at the same time as the loading.

STABILITY OF THE CARRIAGE.—The piece is set low in the carriage to diminish as far as possible the overturning moment; but the maximum velocity of free recoil of this light piece is so great that stability of the carriage at all angles of elevation could not be obtained without exceeding the limit of weight and making the recoil unduly long. The carriage will be stable for angles of elevation greater than about 10 degrees. The wheels are expected to rise from the ground in firings at angles of elevation less than 10 degrees.

THE LIMBER.—The limber is the same as the limber of the 4.7-inch siege carriage previously described. When limbered the rear end of the cradle is locked to the trail in order to relieve the elevating and traversing mechanisms from strain. The short length of the howitzer renders it inadvisable to move the gun to a more rearward traveling position.

WEIGHTS.—The weight of gun and carriage is about 6900 pounds, and the weight of the limber 1000 pounds. The total

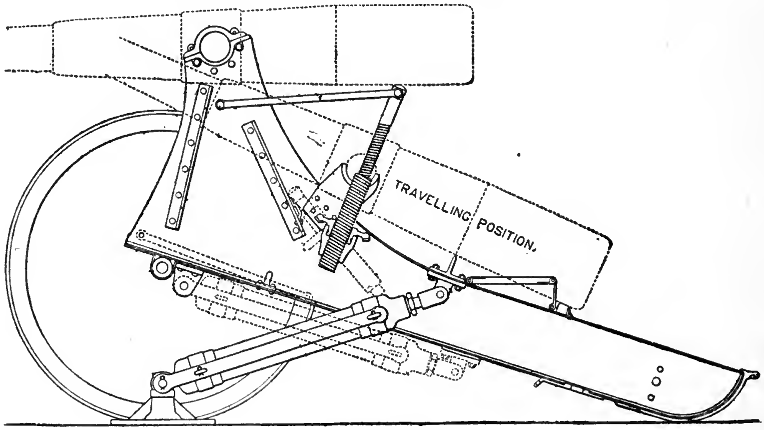


FIG. 135.

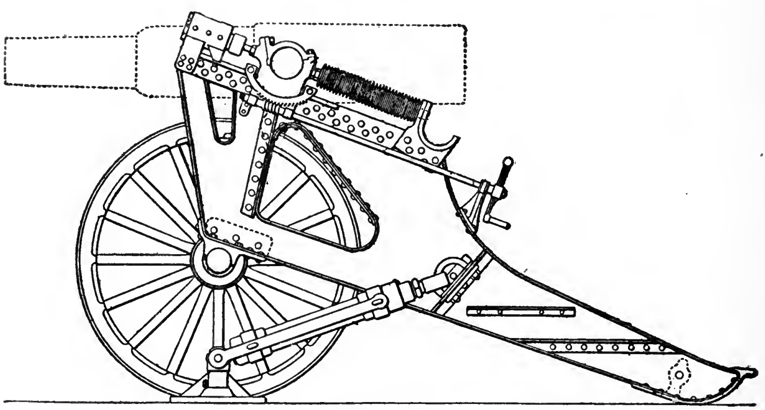


FIG. 136.

weight is slightly less than the limit of 8000 pounds, considered as a maximum load for a siege team.

192. Siege Artillery in Present Service.—The wheeled siege pieces in present service are the 5-inch gun, shown in Fig. 135, and the 7-inch howitzer, Fig. 136.

When emplaced in a siege battery the carriage for either piece rests on a wooden platform. Recoil is limited by means of a hydraulic buffer attached to the trail and pintled in front to a heavy pintle fixed to the platform. The howitzer also recoils on the carriage, the recoil of the piece being controlled by hydraulic buffers one on each side in front of the trunnions. Springs, strung on rods in rear of the trunnions, return the gun to the firing position. The springs are either coiled or Belleville springs, the latter being saucer shaped disks of steel strung face to face and back to back.

The pieces are mounted at a height of about six feet above the ground to enable the guns to be fired over a parapet of sufficient height to shelter the gunners.

For traveling, the guns are shifted to the rear into trunnion beds provided in the trail.

The 7-inch siege mortar and carriage are shown in Fig. 137.

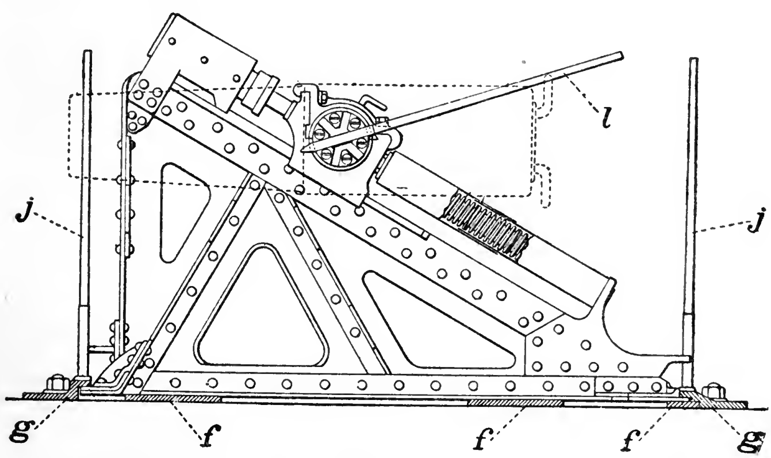


FIG. 137.

The carriage rests on three traverse circle segments *f* bolted to the platform. It is held to the paltform by the overhanging flanges of the segments *g*. Elevation is given to the gun by means of the handspike *l* which, for the purpose, is seated in a slot in the trunnion; and direction is given by means of the handspikes *j* which are engaged against lugs on the carriage. The means of controlling the recoil of the piece are similar to those employed with the 7-inch howitzer.

193. Seacoast Artillery.—Comprised in the seacoast artillery are guns ranging in caliber from 2.24 inches to 16 inches, their projectiles ranging in weight from 6 pounds to 2400. The 2.24-inch and 3-inch guns, called the 6-pounder and the 15-pounder, are used for the defense of the sea fronts of fortifications against landing parties and for the defense of the submarine mine fields. The guns of medium caliber, from 4 to 6 inches, are best used for the protection of places subject to naval raids, and for the defense of mine fields at distant ranges. Their fire is effective against unarmored or thinly armored ships.

The 8- and 10-inch guns are effective against armored cruisers and against the thinly armored parts of battleships.

The proper target for guns 12 inches or more in caliber is the heavy water line armor of the enemy's battleship.

The 12-inch gun is the largest gun at present mounted in our fortifications. One 16-inch gun has been manufactured and satisfactorily tested, but no guns of this caliber are mounted. The latest model of 12-inch gun was designed to give the 1000 pound projectile a muzzle velocity of 2550 feet, which would insure perforation, at a range of 8700 yards, of the 12-inch armor carried by the latest type of battleship. But it has been found that in the production of this high muzzle velocity in a heavy projectile the erosion due to the heat and great volume of the powder gases is so great as to materially shorten the life of the gun. It has been decided therefore as a measure of economy to reduce the muzzle velocities of the larger guns from 2550 feet to 2250, and to build for the defense of such wide waterways as cannot be properly defended by the 12-inch guns with the reduced velocity, 14-inch guns which will give to a 1660-pound projectile a muzzle velocity of 2150 feet, sufficient to insure perforation of 12-inch armor at a range of 8700 yards.

The wide channels that exist at the entrances to Long Island Sound, Chesapeake Bay, Puget Sound, and Manila Bay will require these 14-inch guns for their defense.

The table following contains data relating to seacoast guns.

Gun.	Date of Model.	Charge, lbs.	Projectile, lbs.	Bursting Charge, lbs.	Muzzle Velocity, f. s.	Maximum Pressure, lbs.	For Maximum Range.			
							Elevation, deg.	Range, yds.	Time of Flight, sec.	Remaining Velocity f. s.
2.24-inch	1900	1.35	6	0.25	2400	34000	18	7600	25.1	695
3-inch	1903	6.06	15	0.35	3000	34000	15	8500	24.1	776
4.72-inch	Armstrong	10.5	45	1.96	2600	34000	15	10000	26.4	718
5-inch	1900	26	58	2.75	3000	36000	15	10900	27.0	865
6-inch	1905	42	106	4.6	2900	36000	15	12400	29.4	926
8-inch	1888	80	316	19	2200	38000	12	11000	23.5	1080
10-inch	1900	224	604	33	2500	38000	12	12300	24.7	1148
12-inch	1900	367	1046	58.3	2500	38000	10	11600	21.5	1269
14-inch	1906	280	1660	58.5	2150	36000	10	11300	20.9	1302
16-inch	1895	612	2400	139.3	2150	38000	10	12800	22.4	1373
Mortar.										
10-inch	1890	34	604	33	1150	33000	45	11500	48.1	975
12-inch	1890	54	1046	58.3	1150	33000	45	13400	52.7	1055

The bursting charges given in the table are for shell. The bursting charge for a shot is about one third of the bursting charge for a shell of the same caliber.

Other data concerning the seacoast guns will be found in the table on page 135.

Seacoast Guns.—The seacoast guns and mortars are constructed as shown on pages 237 and 238. As the considerations that limit the weights of the guns of the mobile artillery do not apply to seacoast guns mounted on fixed platforms, and as with longer guns higher muzzle velocities may he obtained without increasing the maximum pressure, the seacoast guns are much longer, in calibers, than are the field and siege pieces. This may be noted in the table on page 135.

All seacoast guns up to 4.7 inches in caliber use fixed ammunition. In guns of greater caliber the projectile is inserted first and is followed by the powder charge made up in one or more bags. In general the breech mechanism of the guns using fixed ammunition is of the type described with the 3-inch field gun. Guns five and six inches in caliber are provided with the Bofors of similar mechanism. Larger guns have the cylindrical slotted screw mechanism described on page 256.

194. Seacoast Gun Mounts.—The mounts for the seacoast guns, commonly called carriages, are distinguished as barbette or disappearing carriages according as they hold the gun always exposed above the parapet or withdraw the gun behind the parapet at each round fired. The disappearing carriage has the advantage of excellent protection for the carriage and gun crew, and, for guns of the larger calibers, the added advantage of greatly increased rapidity of fire. The increased rapidity of fire is due to the lowering of the gun to a height convenient for loading, so that the heavy projectiles and charges of powder need not be lifted in loading. On high sites the disappearing carriage is not necessary to secure protection for the gunners, for behind the parapets the gunners can only be reached by high angle fire from the enemy's ship, and on account of the excessive strain on the decks that would accompany such fire guns aboard ship are not so mounted that they can be fired at high angles. Disappearing carriages, emplaced, are more costly than barbette carriages, but the advantage of the more rapid fire from the disappearing carriage has determined its use in this country for all seacoast guns above six inches in caliber, on high sites as well as on low sites.

Many of the 6-inch guns and all guns below six inches in caliber are mounted on barbette carriages provided with shields of armor plate for the protection of the gunners.

Seacoast guns being permanently emplaced the weights of the gun and the carriage, and simplicity of mechanism in both gun and carriage, are not matters of such importance as they are in the field and siege artillery. We consequently find adapted to the seacoast guns and carriages every mechanism that will assist in increasing the rapidity of fire. Fixed ammunition is used in guns up to 4.7 inches in caliber and its use will probably be extended to larger calibers. Experiments are being made with mechanisms for the automatic or semi-automatic opening and closing of the breech. The mechanisms for elevating the gun and for traversing the carriage are arranged to be operated from either side of the carriage, and in the carriages for the larger guns provision is made for the operation of these mechanisms both by hand and by electric power. Sights are provided on both sides of the gun, and the operations of aiming and loading may proceed together.

Finally the magazines and shell rooms in the walls of the fortifications are so arranged with regard to the gun emplacement, and so equipped, as to insure a rapid delivery of ammunition to every gun.

The seacoast gun mounts differ for guns of different caliber. A description of one mount of each distinct type will follow and will serve to show the principles that govern in similar constructions.

General Characteristics.—In general, the mount consists of a fixed base bolted to the concrete platform of the emplacement, and of a gun-supporting superstructure resting on the base and capable of revolution about some part of it. The superstructure supports, in addition to the gun, all the recoil controlling parts and the necessary mechanisms for elevating, traversing, and retracting the gun.

Fastened to the fixed base or to the platform around the base is an azimuth circle graduated to half degrees, and on the movable part of the carriage is fixed a pointer, with vernier reading to minutes, that indicates the azimuth angle made by the gun with a meridian plane through its center of motion.

The gun, supported by means of its trunnions on the superstructure of the carriage or contained in a cradle which is itself so supported, has movement in elevation about the axis of the trunnions. The elevating mechanisms, or the sights, are provided with graduated scales which usually indicate the range corresponding to each position of the gun.

Protecting guards are provided wherever necessary for the protection of the gunners against accident, or for the protection of the mechanisms of the carriage against the entrance of dust or water.

195. Pedestal Mounts.—Seacoast guns up to six inches in caliber are mounted in barbette on carriages similar in construction to the carriage shown in Figs. 138 and 139.

A conical pedestal of cast steel, *p* Fig. 138, is bolted to the concrete platform. A pivot yoke *y* free to revolve is seated in the pedestal. In the upwardly extending arms of the pivot yoke are seats for the trunnions of the cradle *c*. The gun is supported and slides in recoil in the cradle. The weight of all the revolving parts is supported by a roller bearing *r* on a central boss in the base of the pedestal. In the lower rear portion of the cradle are formed a central recoil cylinder and two spring cylinders, Fig. 139, similar to the corresponding cylinders described in the 4.7-inch siege carriage, but much shorter. As the seacoast gun mounts are firmly bolted to platforms and as they may be made as strong as desired without limit as to weight, these mounts will stand much higher stresses, without movement or rupture, than can be imposed on a wheeled carriage. We therefore find that shorter recoil is allowed to the seacoast guns than to the lighter field and siege guns. Thus the recoil of the 5-inch gun on the pedestal mount is but 13 inches, and of the 6-inch gun 15 inches, while the 4.7-inch siege gun recoils 66 inches on its carriage and the 3-inch field gun 45 inches.

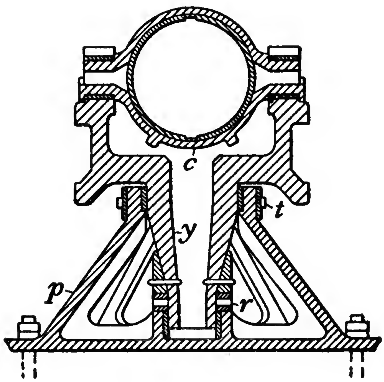


FIG. 138.

Bolted to the arms of the pivot yoke, on each side, are brackets to which are attached platforms for the gunners. The platforms move with the gun in azimuth and carry the gunners undisturbed in the operations of pointing and of manipulating the breech mechanism.

The carriage may be traversed from either side. The shafts of the traversing hand-wheels extend downward toward the pedestal and actuate a horizontal shaft held in bearings on the pivot yoke. A worm on this shaft acts on a circular worm-wheel surrounding the top of the pedestal, *t* Fig. 138.

Elevation is given by the upper hand-wheel, on the left side only. The elevating gear is supported by a bracket bolted to the platform bracket and works on an elevating rack attached to the cradle, the center of the rack being in the axis of the trunnions.

The traversing rack, or worm-wheel, surrounding the upper part of the pedestal is held to the pedestal by an adjustable friction band; and a worm-wheel in the elevating gear, contained in the gear casing fixed to the elevating bracket, Fig. 139, is held between two adjustable friction disks. These friction devices are so adjusted as to enable the gun to be traversed or elevated without

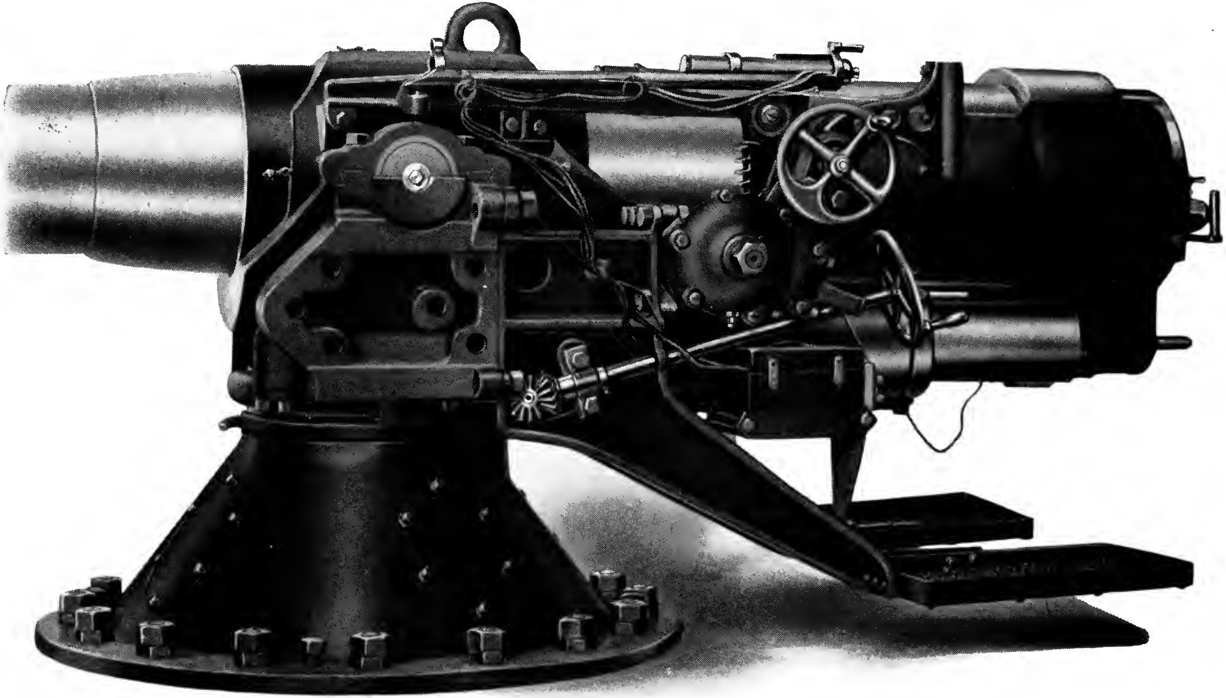


Fig. 139.—Pedestal Mount for 6-inch Gun, Shield Removed.

slipping of the mechanism, and yet to permit slipping in case undue strain is brought on the teeth of the worm-wheels.

A shoulder guard is attached to the cradle on each side of the gun to protect the gunners from injury during movement of the piece in recoil.

Open sights and a telescopic sight are seated in brackets on the cradle on each side of the gun. Dry batteries in two boxes held in brackets secured to the platform brackets supply electric power for firing the piece and for lighting the electric lamps of the sights.

The shield, of hardened armor plate, 4½ inches thick, is fastened by two spring supports to the sides of the pivot yoke. The bolt holes for the shield support are seen in Fig. 139. The shield is pierced with a port for the gun and with two sight holes, and is inclined at an angle of 40 degrees with the horizon, see Fig. 201.

196. The Balanced Pillar Mount.—A variation of the mount just described is found in the balanced pillar mount, also called the *masking parapet mount*. This mount is constructed for guns up to 5 inches in caliber. The purpose of this mount is to afford a means of withdrawing the gun, when not in use, behind the parapet and out of the view of the enemy. The gun is withdrawn behind the parapet only after the firing is completed, and not after each round. Guns mounted on the disappearing carriages later described are withdrawn from view after each round fired.

The construction of the balanced pillar mount will be understood from Fig. 140. The pintle yoke, with all the parts supported by it, rests on the top of a long steel cylinder which has movement up and down in an outer cylinder. The base of the pintle yoke is circular. It embraces a heavy pintle formed on the top of the cylinder and rests on conical rollers which move on a path provided on the cylinder top. Clips attached to the base of the pivot yoke engage under the flanges of the roller path and hold the top carriage to the cylinder.

Imbedded in the concrete of the platform is the outer cast iron cylinder in which the inner cylinder slides up and down. The weight of the inner cylinder and supported parts is balanced by lead and iron counterweights strung on a central rod which is connected to brackets on the inside of the inner cylinder by three chains. The pulleys over which the chains pass are supported on posts that pass through holes in the counterweight and rest in sockets formed in the bottom of the cylinder. For lifting and lowering the inner cylinder with the gun and top carriage, a ver-

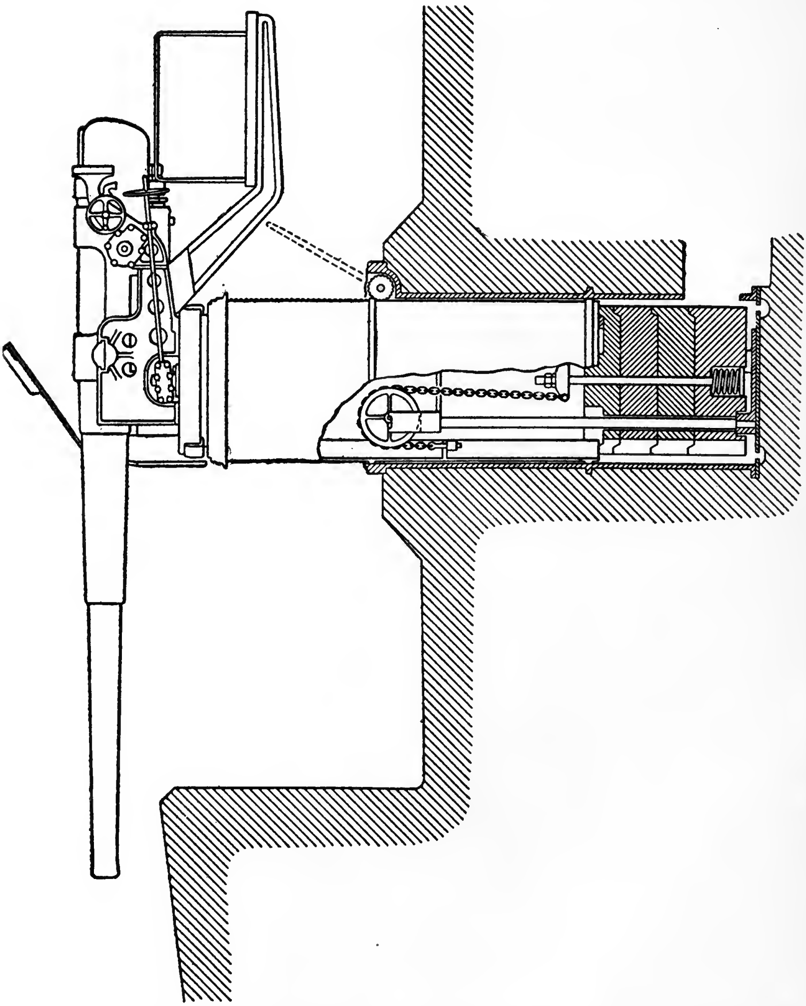


FIG. 140.—5-inch Seacoast Gun on Balanced Pillar Mount.

tical toothed rack is fixed to the exterior of the inner cylinder. A pinion is seated in bearings provided at the top of the outer cylinder and engages in the rack. The pinion is turned by means of two detachable levers mounted on the ends of the pinion shaft.

By means of a friction clamp the pinion is made to hold the elevated carriage against any sudden downward shock.

The construction permits a vertical movement of the gun and carriage of about $3\frac{1}{2}$ feet.

When firing, the muzzle of the gun projects over the parapet; and before lowering, the gun is turned parallel to the parapet.

In a similar mount provided for 3-inch guns the outer cylinder is a double cylinder. The counterweight is annular and occupies the space between the two cylinders composing the double outer cylinder. The lifting levers are applied directly to the shaft of one of the chain pulleys, over which pass the chains that connect the counterweight to brackets on the outside of the inner cylinder. The brackets move in slots provided in the interior of the double cylinder.

197. Barbette Carriages for the Larger Guns.—Guns from 8 to 12 inches in caliber are mounted in barbette on carriages similar in construction to that shown in Fig. 141. The carriages are made

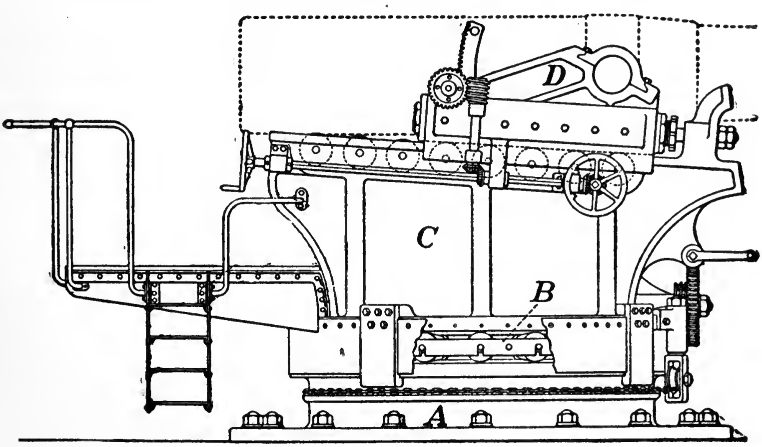


FIG. 141.

principally of cast steel, all the larger parts with the exception of the base ring being of that metal. The cast iron base ring, *A* Fig. 142, has formed on it a roller path, *b*, on which rest the live conical rollers *E* of forged steel. The rollers are flanged at their inner ends and kept at the right distance apart by outside and inside distance-rings *B*. The central upwardly extending cylinder *c* forms a pintle about which the upper carriage revolves. Embracing the pintle and resting on the rollers is an upper circular plate called the racer. Clips attached to the racer, see Fig. 141, and engaging under the flange of the lower roller path hold the parts together under the shock of firing. The two cheeks, *C* Fig. 141, of the chassis are cast in one piece with the racer for the

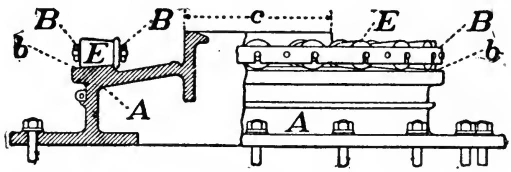


FIG. 142.

smaller carriages and separately for the larger carriages, and are connected together by transoms and strengthened by inner and outer ribs. A groove or recess is formed in the upper part of each cheek, see Fig. 143, for the series of rollers seen in Fig. 141, on which the top carriage moves in recoil. The axles of the rollers are fixed in the walls of the grooves at such a height that the tops of the rollers are just above the top of the chassis.

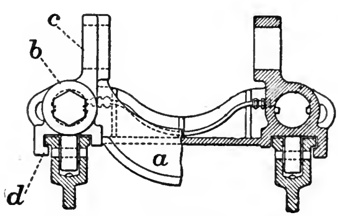


FIG. 143.

The top carriage, *D* Fig. 141 and *a* Fig. 143, rests on the rollers and is held to the chassis by means of the clips *d*, Fig. 143. The top carriage is cast in one piece. It consists of two side frames united by a transom *a* passing under the gun. The side frames contain the trunnion beds *c* for the gun trunnions and the two recoil cylinders *b*. The piston rods of the recoil cylinders are held in lugs formed on the front of the chassis.

Elevation from minus 7 to plus 18 degrees is given by means of the hand-wheel seen near the breech of the gun, Fig. 141, or by the hand-wheel just under the top carriage. The carriage is traversed by means of the crank handle in front of the chassis. Through a worm and worm-wheel the crank actuates a sprocket-wheel fixed in bearings on the chassis. A chain that encircles the base ring and that is fast to the base ring at one point passes over the sprocket-wheel. When the sprocket-wheel is turned it pulls on the chain and causes the chassis to revolve.

In later carriages the chain is replaced by a circular toothed rack attached to and surrounding the base ring, and the sprocket-wheel is replaced by a gear train whose end pinion meshes in the rack. There is less friction and less lost motion with this construction.

The shot is hoisted to the breech by means of a crane attached to the side of the carriage.

When the gun is fired, the gun and top carriage recoil to the rear on the rollers. The length of recoil is limited by the length of the recoil cylinder, and on this type of carriage is about five calibers. The recoil is absorbed partly in lifting the gun and top carriage up the inclined chassis rails and partly by friction, but principally by the resistance of the recoil cylinders, as explained in the chapter on recoil.

On cessation of the recoil the gun returns to battery under the action of gravity, the inclination of the chassis rails, four degrees, being greater than the angle of friction.

198. Disappearing Carriages.—The importance of the function of the heavy seacoast guns, the difficulty in the way of quick or extensive repairs to their mounts, the great cost of the guns and their carriages, are all considerations that point to the desirability of giving to these guns and carriages the greatest amount of protection practicable.

The guns are therefore emplaced in the fortifications behind very thick walls of concrete, which are themselves protected in front by thick layers of earth. Additional protection is obtained by mounting the guns on carriages which withdraw the guns from their exposed firing position above the parapet to a position behind the parapet and below its crest, where the gun and every part of the carriage except the sighting platforms and sight standards are protected from a shot that grazes the crest at an angle of seven degrees with the horizontal.

An additional and very important advantage gained by the use of these carriages is the increased rapidity of fire obtained from the guns mounted upon them. The guns in their lowered positions are at a convenient level for loading, and the time and labor that must be expended in lifting the heavy projectiles and powder charges to the breech of a gun of the same caliber mounted in barbette are practically eliminated.

12-inch Disappearing Carriage, Model 1901.—The annular base ring, *b* Fig. 144, surrounds a well left in the concrete of the emplacement. The racer *a* rests on live rollers on the base ring and is pintled on a cylinder formed by the inner wall of the base ring. The racer supports the superstructure as in the carriage just described. It is held to the base ring by clips *c*, which engage under a flange on the inside of the pintle. A working platform, or floor, of steel plates is fixed to brackets *x* fastened to the racer, and moves with the carriage in azimuth.

The forward ends of the chassis cheeks are continued upward, and on the inside of the cheeks and of the upward extensions are formed vertical guideways for the crosshead *k*, from which the counterweight *w* is suspended.

Gun Lifting System.—The top carriage, similar in construction to that of the barbette carriage, rests on flanged live rollers which roll on the rails of the chassis. The rollers are connected together by side bars in which the axles of the rollers are fixed.

The gun levers *l* are trunnioned in the trunnion beds of the top carriage. They support the gun between their upper ends, and between their lower ends, the crosshead *k* from which the counterweight is suspended.

The crosshead is provided with clips that engage the vertical guides formed on the inside of the chassis cheeks. Cut on the front faces of the clips of the crosshead are ratchet teeth in which pawls *p* engage to hold the counterweight up after the gun has recoiled. The pawls are pivoted on the chassis. Levers *v* pivoted on the ends of a shaft across the front of the chassis serve as means for releasing the pawls when it is desired to put the gun in battery.

The counterweight consists of 102 blocks of lead of varying size, weighing approximately 164,700 pounds. It is piled on the bottom plate *m*, which is suspended by four stout rods from the crosshead. The preponderance of the counterweight may be adjusted, within limits, by the addition or removal of small weights at the top.

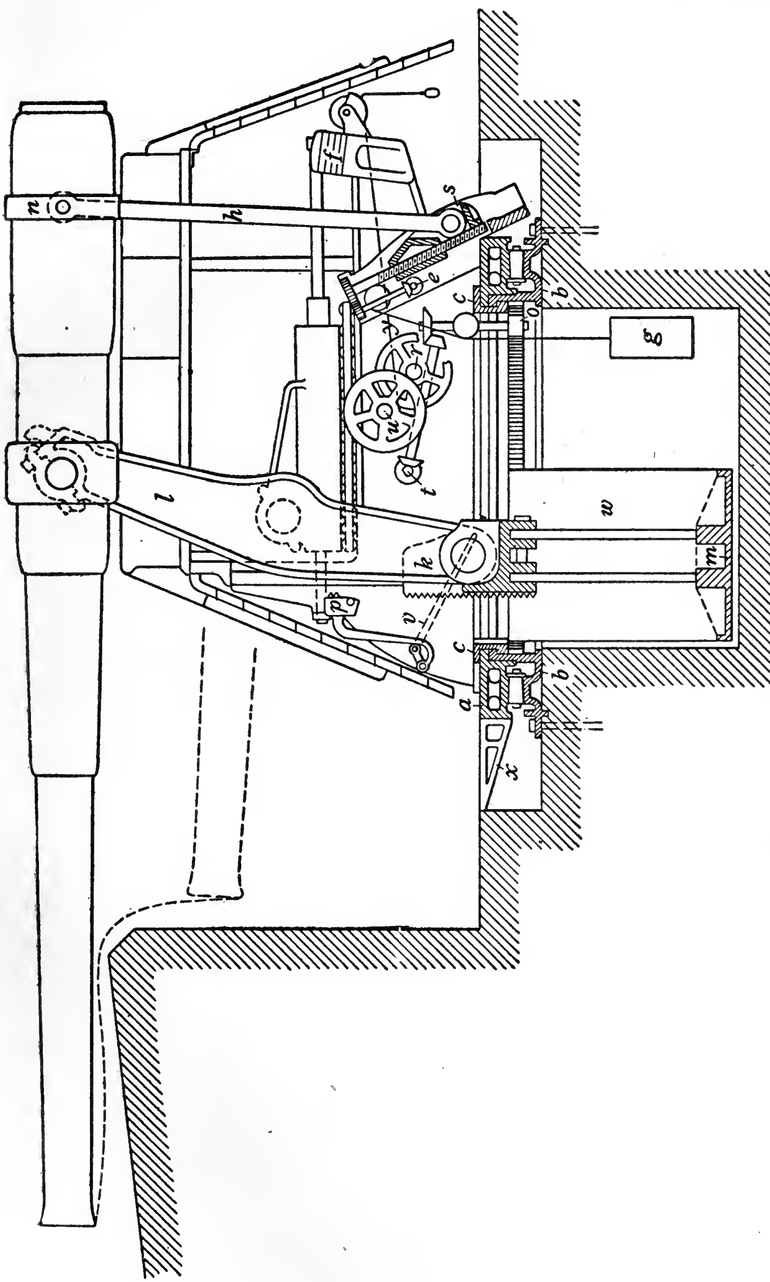


FIG. 144.—12-inch Gun on Disappearing Carriage, Firing Position.

199. Elevating System.—The gun elevating system consists of the band *n* dowelled to the gun and provided with trunnions that are engaged by the forked ends of the elevating arm *h*. The elevating arm has at its lower end a double ended pin which rotates in bearings in the elevating slide *s*. The elevating slide has a movement up and down on an inclined guideway machined on the rear face of the rear transom. Movement is given to the slide by means of a large axial screw on which the slide moves as a nut prevented from turning. The screw is turned by gearing on the shaft *e* actuated by hand-wheels outside the carriage. In order to counterbalance the weight of the elevating arm and band, and to equalize the efforts required in elevating and depressing the gun, a wire rope passes from the elevating slide over pulleys and supports a counterbalancing weight *g*. The gun moves in elevation from minus 5 degrees to plus 10 degrees.

Traversing System.—Crank-handles on the traversing shaft *t* actuate, through gearing, a vertical shaft carrying at its lower end a pinion *o* which works in a circular rack on the inside of the base ring. In a convenient position on the racer near the azimuth pointer is placed the lever of a traversing brake, not shown, which works against the base ring. By its means traversing is retarded as the carriage approaches any desired azimuth.

Retracting System.—Means are provided to bring the gun down from its firing position when for any reason it has been elevated into battery and not fired. Detachable crank-handles mounted on the ends of the shaft *r* turn two winding drums on the shaft *u* inside the chassis. A wire rope *y* leads from each drum around a pulley at the rear end of the chassis to the top of the gun lever, a loop in the end of the rope engaging over the hook of the lever.

Sighting System.—Elevated platforms are provided on each side of the carriage. The telescopic sight, see Fig. 145, is mounted above the left platform on a hollow standard that rises from the floor of the racer. A vertical rod passing through the standard is connected at the top to a pivoted arm carrying the sight, and at the bottom the rod is so geared to the elevating shaft that the same movement in elevation is given to the sight arm as is given to the gun. Within reach of the gunner at the sight are two

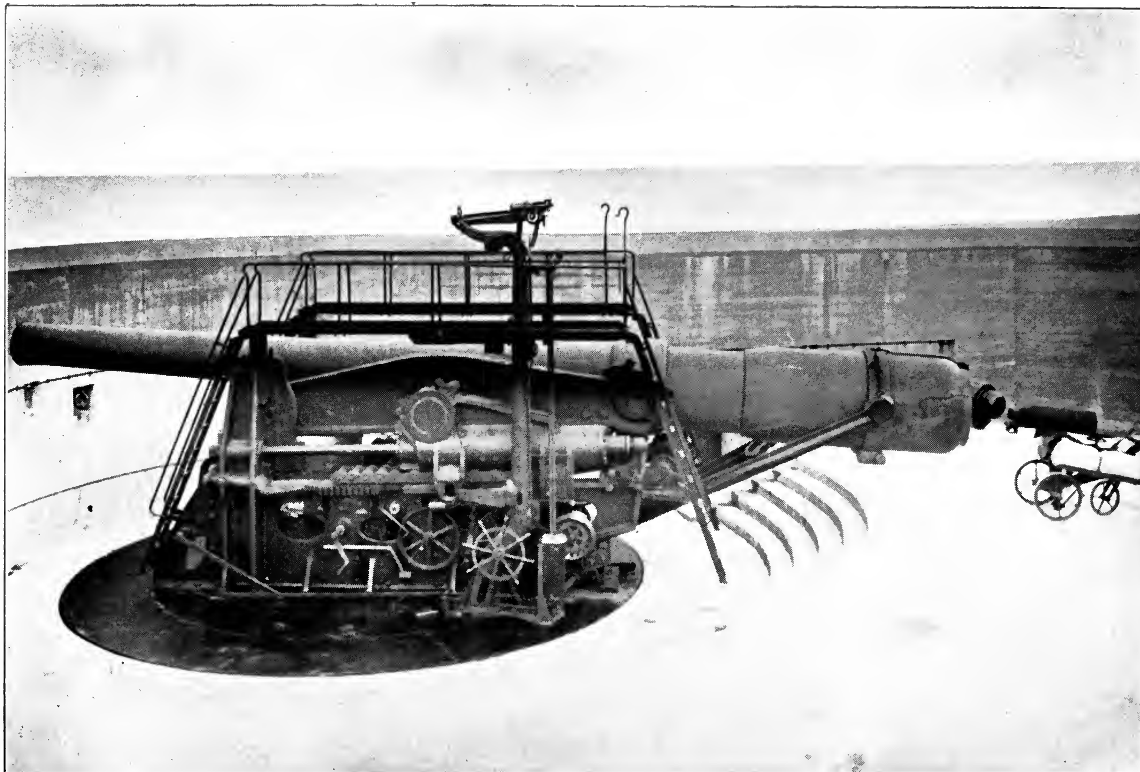


Fig. 145.—12-inch Gun on Disappearing Carriage, Loading Position.

crank-handles, at the upper ends of vertical shafts, by means of which the gunner has electric control of the elevating, traversing, and retracting mechanisms.

Trials are being made of the panoramic sight fitted to disappearing carriages. The vertical tube of the sight is made very long and the sight is attached to the side of the carriage in such a position that the eye piece is convenient to the gunner standing on the racer platform, while the head piece of the sight is above the parapet.

OPERATION.—The operation of the carriage for firing is as follows. The gun is loaded in its retracted position, Fig. 145, being held in that position by the pawls *p* engaged in the notches on the crosshead *k*. After the gun is loaded the tripping levers *v* are raised, releasing the pawls from the notches in the crosshead. The counterweight falls and the top carriage moves forward on its rollers, the last part of its motion being controlled by the counter-recoil buffers in the recoil cylinders, so that the top carriage comes to rest without shock on the chassis. By the movement of the gun levers the gun is lifted to its elevated position above the parapet.

When the piece is fired the movements are reversed in direction. The recoil forces the gun to the rear, the top carriage rolls back on the chassis rails and the counterweight rises vertically under the restraint of the guides engaged by the crosshead.

In the movement either way the upper end of the gun lever describes an arc of an ellipse. The path of the muzzle of the gun, indicated in Fig. 144, is affected by the constraint of the elevating arm. The ellipse is the most favorable figure to follow in the movement of a gun on a disappearing carriage. From the firing position the movement of the gun is at first almost horizontally backward, and the movement downward occurs principally in the latter part of the path. Therefore the carriage that moves the gun in an elliptical path can be brought nearer to the parapet and thus receive better protection than any other carriage.

The recoil is controlled principally by the recoil cylinders, and the shock at the cessation of motion is mitigated by two buffers *f* which receive the ends of the gun levers. The buffers are composed of steel plates alternating with sheets of balata.

Balata is a substance that resembles hardened rubber. It has not as great elasticity as rubber but does not deteriorate as rapidly under exposure to the weather.

200. Modification of the Recoil System.—In the chapter on recoil it was pointed out that there is a disadvantage in having the control of the counter recoil in the same hydraulic cylinders that control the recoil. The adjustment of the counter-recoil system affects the adjustment of the recoil system.

It will also be observed in the carriage just described that in the latter part of the movement in recoil the gun is moving almost vertically downward. Consequently the movement of the top carriage to the rear is very slight during this part of the recoil, and the slight movement affords little opportunity for the close control by the recoil cylinders of the final movement of the gun. But it is in the last part of the recoil that complete control of the movement of the gun is most desirable, in order that the gun may be brought to rest at any desired position for loading, and without shock to the carriage.

While the movement of the top carriage is least rapid at the latter end of recoil the counterweight has then its most rapid movement. Therefore a recoil cylinder fixed so as to move with the counterweight will afford the best control of the final movement of the gun.

The top carriage has its most rapid movement at the latter part of the movement of the gun into battery, while the counterweight has its least rapid movement at that time. The control of the counter recoil is therefore best effected through the top carriage.

By retaining therefore, to act on the top carriage, recoil cylinders adapted for the control of the counter recoil only, and by adding to the counterweight a cylinder adapted for control of the recoil, we will obtain the advantage of completely separating the two systems, thus making them capable of independent adjustment, and the advantage of obtaining from each system the greatest control of the movement to which it is applied.

201. 6-inch Experimental Disappearing Carriage, Model 1905.—The modification of the recoil system as above indicated has been applied to a 6-inch experimental carriage.

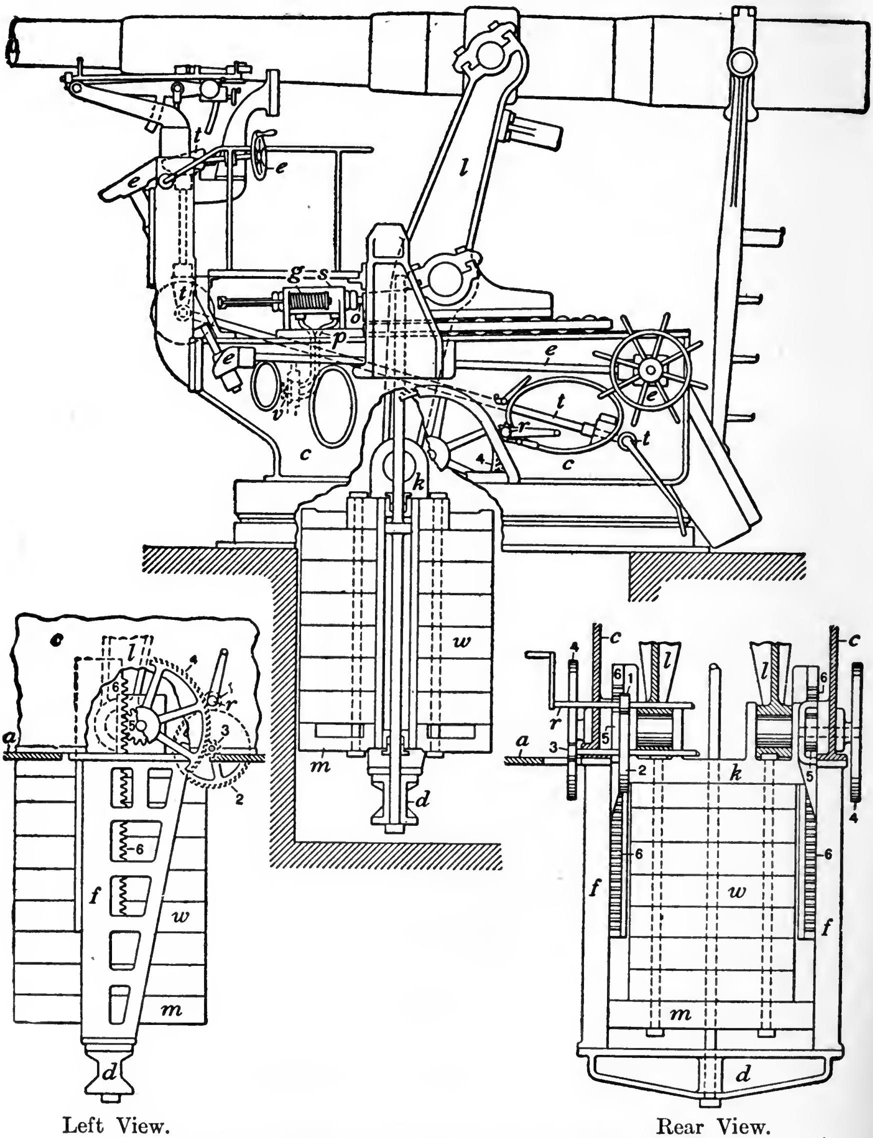
The recoil cylinder is held in the center of the counterweight, Fig. 146. The lower end of the piston rod is fixed in the lower member *d* of a frame whose sides *f* are bolted to the bottom of the racer *a*, as shown in the left and rear views. Grooves cut in the walls of the recoil cylinder permit the flow of the liquid from one side of the piston to the other. For the regulation of the extent of the recoil, and therefore of the height of the gun when in loading position, two diagonal channels pass through the center of the piston head from one face to the other, and the flow through them is controlled by a conical valve enclosed in the upper piston rod, which is hollow. The stem of the valve projects above the end of the piston rod.

The counter recoil is checked by the short cylinders *s* mounted on each chassis rail in front of the top carriage. The pistons of the counter-recoil cylinders are not provided with apertures for the flow of the liquid from one side of the piston to the other, but the flow of the liquid takes place through the pipes *p* which are led from both cylinders to a valve *v*, by which the area of orifice is controlled and through which the pressure in the two cylinders is equalized. The pressure in the counter-recoil cylinders does not exceed 500 pounds per square inch, while the pressure in the recoil cylinder is 1800 pounds.

As the top carriage comes into battery the front of the carriage strikes the rear end *o* of the piston rod and forces the piston through the cylinder against the liquid resistance and against the action of springs *g* mounted on each side of the cylinder. The springs act on central rods connected to the forward end of the piston, and as the top carriage moves from battery the springs move the piston to the rear in position to be acted on by the top carriage as it comes back into battery.

There are other points of difference between this carriage and the carriage last described.

The retraction of the gun from the firing position is accomplished without the use of wire ropes by the vertical racks 6, shown in the left and rear views, attached to bars that connect the cross-head *k* and the bottom section *m* of the counterweight. The end pinions 5 of two trains of gears, one on each side, mesh in the rack, the gear trains being actuated by the cranks on the shaft *r*. The



Left View. Rear View.

FIG. 146.—6-inch Experimental Disappearing Carriage, Model 1905.

retracting mechanism is partially shown in the smaller views. The parts are similarly numbered in all the figures. The mechanism is thrown out of gear when not in use.

The rollers of the top carriage are geared to the top carriage so that they are compelled to move with the top carriage and there can be no slipping of the top carriage on the rollers. In present service carriages this slipping sometimes occurs as the gun recoils, so that on counter recoil the rollers reach their position in battery before the top carriage, and prevent the top carriage from coming fully into battery.

The sight standard is moved to the front of the chassis in order to get better protection for the gunner, for the sight, and for the elevating and traversing mechanisms under control of the gunner. Through the upper hand-wheel *e* and the shafts and gears also marked *e* the gunner has control of the elevating mechanism; and through another hand-wheel at his right hand, covered by the wheel *e* in the figure, and the shafts and gears marked *t* he controls the traversing mechanism.

Firings from this 6-inch carriage have shown that the gunner on the sighting platform is so near the muzzle of the gun that he is injuriously affected by the blast. The sighting platforms will therefore be removed to the rear end of the carriage, in which position they will also afford means of access to the breech when the gun is up.

202. Seacoast Mortars.—The thick armored sides of ships of war protect the ships to a greater or less extent against the direct fire from high powered guns. The great weight of armor that would be required for complete deck protection is prohibitive. The decks of war ships are therefore thin and practically unarmored, the heaviest protective deck on any battleship being not more than two inches thick over the flat part. The decks therefore offer an attractive target.

As the elevation above sea level of the sites of the guns in most fortifications is not sufficient to permit direct fire against the decks, there are provided for use against this target the 12-inch seacoast mortars, short guns so mounted that they can be fired at high angles only. The heavy projectiles fired from these guns carry large bursting charges of high explosive. Descending almost vertically on the deck of a ship they easily overcome the slight resistance offered, and penetrating to the interior of the ship burst there with enormous destructive effect.

The mortar carriages permit firing only at angles of elevation between 45 and 70 degrees. With a fixed charge of powder a limited range only would be covered by fire between these angles. Charges of several different weights are therefore used in the mortars. With each charge a certain zone in range may be covered by the fire, and the charges are so fixed that the range zones overlap. Any point within the limits of range may thus be reached by the projectile. The least range with the smallest charge provided is about a mile and a half. Mortar batteries are therefore usually erected at not less than this distance from the channels or anchorages that are under their protection.

The 12-inch Mortar Carriage, Model 1896.—The construction of the 12-inch mortar carriage, model 1896, will be understood from Fig. 147. The mortar is supported by the upper ends of the

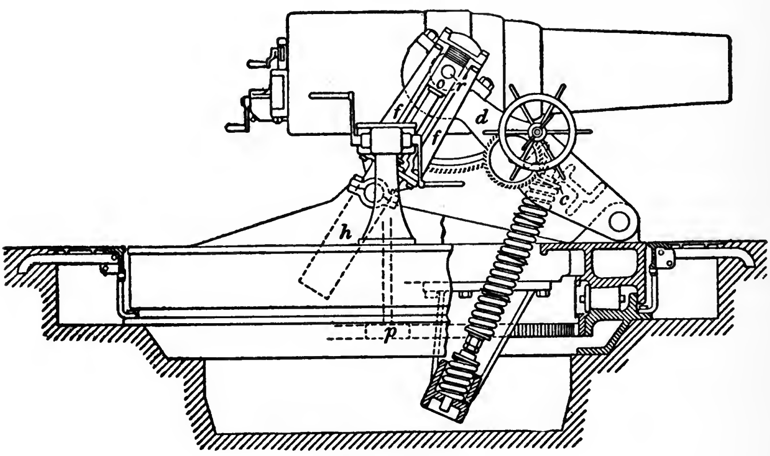


Fig. 147.

two arms of a saddle *d* which is hinged on a heavy bolt to the front of the racer. The arms of the saddle are connected by a thick web. Extending across under the web is a rocking cap-piece, *c*, against which five columns of coiled springs act, supporting the gun in its position in battery and returning it to battery after recoil.

The lower ends of the springs rest in an iron box trunnioned in two brackets bolted to the bottom of the racer. The box oscillates as required during the movement of the saddle in recoil and counter recoil. Holes in the bottom of the box and in the cap-piece and saddle web permit the ends of the rods on which the springs are strung to pass through during the movement.

The recoil cylinders *h* are trunnioned in bearings fixed to the top of the racer. Bolted to the top of each cylinder is a frame *f* which serves as a guide for the crosshead *o* at the upper end of the piston rod. The crosshead embraces the stout pin *r* which extends outward from the trunnion of the mortar and communicates the motion of the piece in recoil to the piston rod.

The provision for the flow of liquid in the recoil cylinder from one side of the piston to the other differs in this carriage from that described in other carriages. A small cylinder, *A* Fig. 148, is formed outside the recoil cylinder proper, *H*. Holes *a*, bored through the dividing wall, form passages through which the oil may pass from the front of the piston to the rear. The piston head in its movement closes the holes successively. Thus as the velocity of recoil decreases the area open to the flow of the liquid is reduced. The area of aperture is also regulated by screw throttling plugs *b* that are seated in the outer wall of the small cylinder. These plugs have stems of different diameters, and are used to partially or wholly close any of the passages in the proper regulation of the recoil. The recoil cylinders on each side of the carriage are connected by the equalizing pipe *p*.

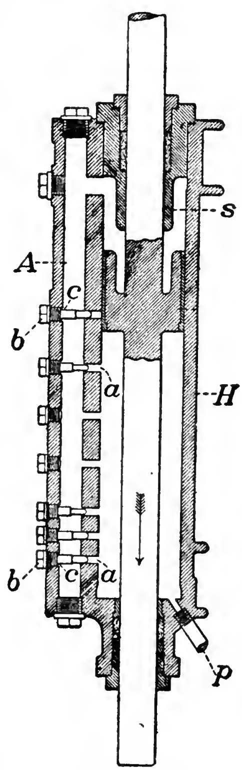


Fig. 148.

The counter recoil is checked and the gun brought into battery without shock by the counter-recoil buffer *s*, an annular projection formed on the cylinder head surrounding the piston rod. The buffer enters, with a small clearance, an annular cavity in the head of the piston, and the liquid in the cavity escapes slowly through the clearance. As an added precaution against shock when the gun returns to battery, buffer stops composed of alternate layers of balata and steel plates are held between the crosshead guides of the frame *f*, Fig. 147, under the cap.

The gun is elevated by the mechanism shown mounted on the saddle, Fig. 147, and traversed by means of the crank shaft and mechanism supported in a vertical stand on the racer. A pinion *p* on the end of a vertical shaft engages in a circular rack bolted to the inner surface of the base ring.

The movement of the saddle in recoil causes the gun to rotate on its trunnions. To prevent excessive rotation of the gun and excessive strain on the elevating mechanism, a friction collar is provided in the large gear wheel of the elevating mechanism. The collar slips in the gear wheel when the strain is excessive.

For determining elevation, a quadrant, similar to the gunner's quadrant described in the chapter on sights, is permanently attached to a seat prepared on the right rim base of the mortar.

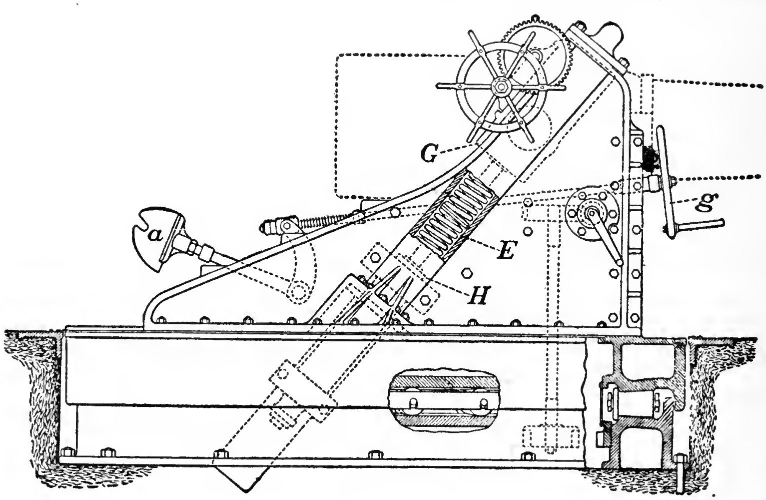


Fig. 149.

203. The 12-inch Mortar Carriage, Model 1891.—The 12-inch mortar carriage, model 1891, on which many 12-inch mortars are mounted in our fortifications, is shown in Figs. 149 and 150.

The spring cylinders *E* are formed in the vertical cheeks bolted to the racer. Inside the cheeks are inclined guideways for sliding crossheads *G*. The crossheads receive the trunnions of the gun. The pistons *h* of the recoil cylinders project downward from the crossheads and enter the recoil cylinders *H* attached to the lower parts of the spring cylinders. The recoil cylinders are of the type shown in Fig. 148. The crosshead *G* has at its upper end an arm, *r* Fig. 150, which projects outwardly into the spring cylinder and carries at its outer end the adjusting screw *k*, which rests on top of the column of springs. The springs are compressed when the gun recoils, and return the gun to battery on the cessation of recoil. By means of the adjusting screw *k* the height of the trunnion carriages *G* may be adjusted to bring the mortar to the proper height for loading.

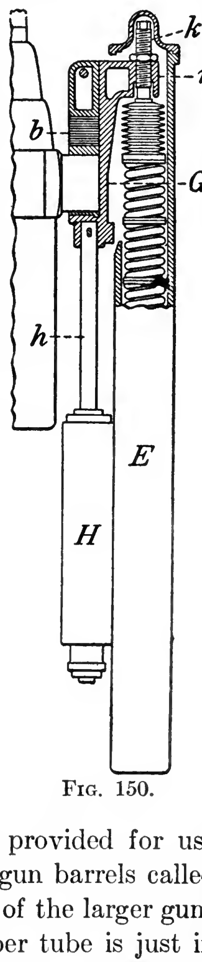


Fig. 150.

The hand-wheel *g*, Fig. 149, works the shot hoist *a*, by means of which the shot is lifted to the breech of the gun for loading.

204. Subcaliber Tubes.—For the purpose of enabling troops to become familiar with the operation of the guns and carriages by actual firing, yet without the expense attendant upon the use of the regular ammunition, there are provided for use inside the various service guns smaller guns or gun barrels called subcaliber tubes. These are seated in the bores of the larger guns in such position that the breech of the subcaliber tube is just in front of the breech block of the gun when closed. The subcaliber tube is loaded with fixed ammunition arranged to be fired by the firing mechanism of the larger gun. Three calibers of subcaliber tubes are provided: one of 0.30-inch caliber, using the small arm cartridge, for guns that use fixed ammunition; one of 1.475-inch caliber, using 1-pounder ammunition, for use in all guns 5 inches or more in caliber; and one of 75 mm. (2.95 inches) caliber, using 18-pounder ammunition, for use in the 12-inch mortar.

For those guns that use fixed ammunition *the 30-caliber subcaliber tube,* a 30-caliber rifle barrel, is fixed in a metal mounting that has the shape and dimensions of the complete cartridge used in the piece. Fig. 151 shows the subcaliber tube for the 3-inch rifle.

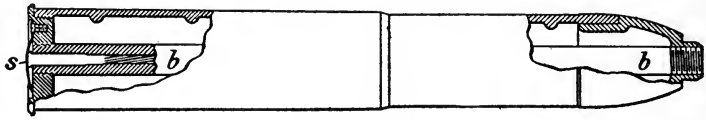


FIG. 151.

The 30-caliber small arm cartridge is inserted in the barrel *b* and is fired by the percussion firing mechanism of the piece. It is extracted, far enough to be grasped by the hand, by the extractor, two bowed springs *s* which are under compression when the small arm cartridge is forced to its seat by the breech block of the gun. A special primer is used in the small arm cartridge, strong enough to withstand without puncture the heavy blow of the firing pin of the gun.

The head of the subcaliber cartridge is permitted longitudinal movement in the body in order to allow for expansion of the 30-caliber barrel in firing.

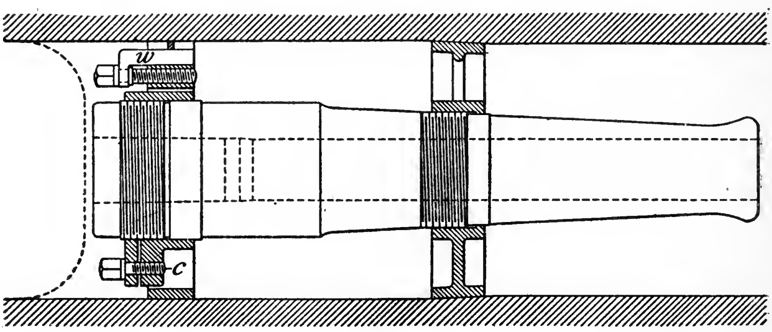


FIG. 152.

The 1-pounder tube is provided with different fittings to adapt it to the particular gun in which it is to be used. It is fitted in the gun in the manner shown in Fig. 152, which represents the 75 mm. subcaliber tube in the 12-inch mortar.

The 75 mm. tube is a gun similar to the mountain gun, without its breech mechanism. The cartridges for the mountain gun are used in it.

The wheel-shaped fittings, called adapters, are screwed on the gun. The front adapter fits against the centering slope in the bore for the band of the projectile. The outer rim of the rear adapter is cut through at the top and the rim is expanded against the sides of the bore by the wedge *w*, which is forced between the parts of the rim by means of the screw seated in one of them. The tube is prevented from turning in the adapters by the clamp screw *c*.

The firing mechanism of the guns in which the two larger subcaliber tubes are used is not of the percussion type. The cannon cartridges used in these two tubes are therefore provided with the 110-grain igniting primer, described in the chapter on primers, in place of the usual percussion primer. The igniting primer in the cartridge is ignited by the flame from the ordinary primer seated in the rear end of the breech mechanism of the gun.

Drill Cartridges, Projectiles, and Powder Charges.—For ordinary use at drill, without firing, dummy cartridges are provided for guns that use fixed ammunition, and dummy projectiles and powder charges for other guns. The dummies have the dimensions and weights of the parts they represent.

The drill cartridge for guns using fixed ammunition are hollow bronze castings, Fig. 153, of the shape of the service cartridge

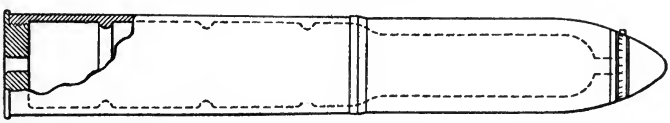


FIG. 153.

loaded with shrapnel. For the instruction of cannoneers in fuse setting there is fitted at the head of the cartridge a movable ring graduated in the same manner as the time scale on the combination time and percussion fuse.

Drill projectiles, for guns separately loaded, are of the construction shown in Fig. 154. A bronze band, *b*, is inset at the bourrelet to prevent wearing of the rifling in the gun by frequent insertion of the projectile. The rotating band *r*, made in two or more sections with spaces between, is pressed to the rear on a sloping seat by springs *s*. When the projectile is rammed with force into the gun the band is likely to stick in its seat and thus to resist efforts to withdraw the projectile. The method of attachment of the band is for the purpose of affording a means of readily overcoming this resistance. The extractor, a hook on the

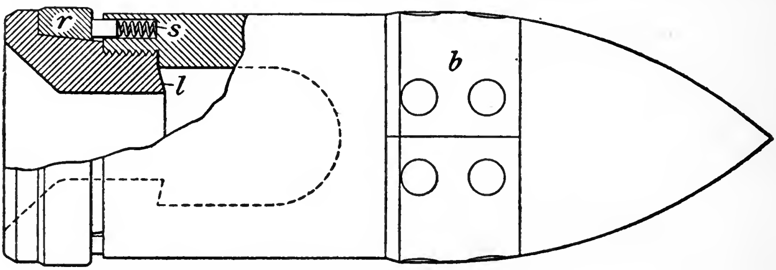


FIG. 154.

end of a pole, is engaged over the inner lip *l*. A pull on the pole will, if the band is stuck, first move the remainder of the projectile to the rear until it strikes and dislodges the band.

The dummy powder charge, Fig. 155, circular in section, is

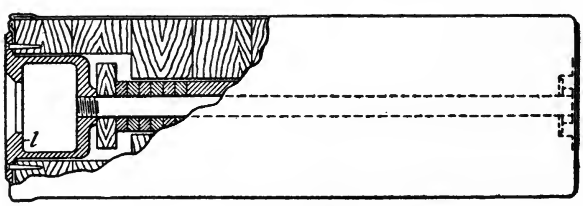


FIG. 155.

made up of a core of metal surrounded by disks of wood, the whole covered with canvas. The parts are assembled by means of a central bolt. An inner lip *l* formed in the hollow metal base piece is engaged by the hook of the extractor.

CHAPTER IX.

EXTERIOR BALLISTICS.

205. Definitions.—Exterior Ballistics treats of the motion of a projectile after it has left the piece.

In the discussions the dimensions of the gun are considered negligible in comparison with the trajectory.

The *Trajectory, bdf,* Fig. 156, is the curve described by the center of gravity of the projectile in its movement.

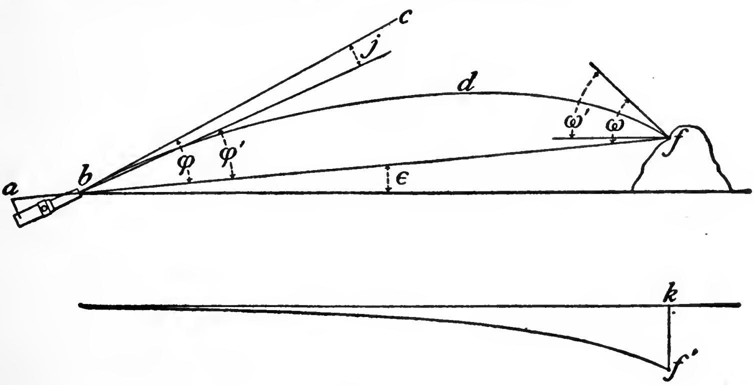


Fig. 156.

The *Range, bf,* is the distance from the muzzle of the gun to the target.

The *Line of Sight, abf,* is the straight line passing through the sights and the point aimed at.

The *Line of Departure, bc,* is the prolongation of the axis of the bore at the instant the projectile leaves the gun.

The *Plane of Fire,* or *Plane of Departure,* is the vertical plane through the line of departure.

The *Angle of Position*, ε, is the angle made by the line of sight with the horizontal.

The *Angle of Departure*, ϕ, is the angle made by the line of departure with the line of sight.

The *Quadrant Angle of Departure*, $\phi+\varepsilon$, is the angle made by the line of departure with the horizontal.

The *Angle of Elevation*, ϕ', is the angle between the line of sight and the axis of the piece when the gun is aimed.

The *Jump* is the angle j through which the axis of the piece moves while the projectile is passing through the bore. The movement of the axis is due to the elasticity of the parts of the carriage, to the play in the trunnion beds and between parts of the carriage, and in some cases to the action of the elevating device as the gun recoils. The jump must be determined by experiment for the individual piece in its particular mounting. It usually increases the angle of elevation so that the angle of departure is greater than that angle.

The *Point of Fall*, f, or *Point of Impact*, is the point at which the projectile strikes.

The *Angle of Fall*, ω, is the angle made by the tangent to the trajectory with the line of sight at the point of fall.

The *Striking Angle*, ω', is the angle made by the tangent to the trajectory with the horizontal at the point of fall.

Initial Velocity is the velocity of the projectile at the muzzle.

Remaining Velocity is the velocity of the projectile at any point of the trajectory.

Drift, kf', is the departure of the projectile from the plane of fire, due to the resistance of the air and the rotation of the projectile.

Direct Fire is with high velocities, and angles of elevation not exceeding 20 degrees.

Curved Fire is with low velocities, and angles of elevation not exceeding 30 degrees.

High Angle Fire is with angles of elevation exceeding 30 degrees.

206. The Motion of an Oblong Projectile.—The projectile as it issues from the muzzle of the gun has impressed upon it a motion of translation and a motion of rotation about its longer axis. The guns of our service are rifled with a right handed twist, and the rotation of the projectile is therefore from left to right when regarded from the rear. After leaving the piece the projectile is a free body acted upon by two extraneous forces, gravity and the resistance of the air.

When the projectile first issues from the piece, its longer axis is tangent to the trajectory. The resistance of the air acts along this tangent, and is at first directly opposed to the motion of translation of the projectile.

The longer axis of the projectile being a stable axis of rotation tends to remain parallel to itself during the passage of the projectile through the air, but the tangent to the trajectory changes its inclination, owing to the action of gravity. The resistance of the air acting always in the direction of the tangent, thus becomes inclined to the longer axis of the projectile, and in modern projectiles its resultant intersects the longer axis at a point in front of the center of gravity.

In Fig. 157, G being the center of gravity, and R the resultant

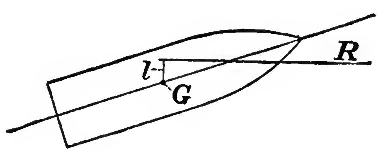


FIG. 157.

resistance of the air, this resultant acts with a lever arm l, and tends to rotate the projectile about a shorter axis through G perpendicular to the plane of fire.

The resultant effect of the resistance of the air on the rotating projectile is a precessional movement of the point of the projectile to the right of the plane of fire. After the initial displacement of the point to the right the direction of the resultant resistance changes slightly to the left with respect to the axis of the projectile, and produces a corresponding change in the direction of the precession, which diverts the point of the projectile slightly downward.

If the flight of the projectile were continued long enough the point would describe a curve around the tangent to the trajectory; but actually the flight of the projectile is never long enough to permit more than a small part of this motion to occur.

The precession of the point is greater as the initial energy of rotation is less. It is therefore necessary to give to the projectile sufficient energy of rotation to make the divergence of the point small. Otherwise the precessional effect may be sufficient to cause the projectile to tumble.

When the point of the projectile leaves the plane of fire the side of the projectile is presented obliquely to the action of the resistance of the air, and a pressure is produced by which the projectile is forced bodily to the right out of the plane of fire. It is to this movement that the greater part of the deviation of the projectile is due.

DRIFT.—The departure of the projectile from the plane of fire, due to the causes above considered, is called drift.

207. Form of Trajectory.—It may be shown analytically that the drift of the projectile increases more rapidly than the range. The trajectory is therefore a curve of double curvature, convex to the plane of fire.

The trajectory ordinarily considered is the projection of the actual curve upon the vertical plane of fire. This projection so nearly agrees with the actual trajectory that the results obtained are practically correct; and the advantage of considering it, instead of the actual curve, is that we need consider only that component of the resistance of the air which acts along the longer axis of the projectile and which is directly opposed to the motion of translation.

Determination of the Resistance of the Air.—The relation between the velocity of a projectile and the resistance opposed to its motion by the air has been the subject of numerous experiments.

In the usual method of determining this relation the velocity of the projectile is measured at two points in the trajectory. The points are selected at such a distance apart that the path of the projectile between them may be considered a right line, and the action of gravity may be neglected. The resistance of the air is then regarded as the only force acting to retard the projectile, and is considered as constant over the path between the two points.

The loss of energy in the projectile, due to the loss of velocity, is the measure of the effect of the resistance of the air, and is equal to the product of the resistance into the path. The resistance thus obtained is the mean resistance, and corresponds to the mean of the two measured velocities.

EARLY EXPERIMENTS.—The first experiments were those of Robins in 1742. For the measurement of velocities he used the ballistic pendulum. His conclusions were, that up to a velocity of 1100 foot seconds the resistance is proportional to the square of the velocity; beyond 1100 f. s. the resistance is nearly three times as great as if calculated by the law of the lower velocities.

Hutton in 1790, with the improved ballistic pendulum, made numerous experiments with large projectiles. His conclusions were that the resistance increases more rapidly than the square of the velocity for low velocities, and for higher velocities that it varies nearly as the square.

General Didion made a series of experiments at Metz in 1840 with spherical projectiles of varying weights. His conclusions were that the resistance varied as an expression of the general form $a(v^2+bv^3)$, a and b being constants. This formula held for low velocities only.

Experiments were again made at Metz in 1857. Electro-ballistic instruments were now used for the measurement of velocities. The conclusions from these experiments were that the resistance varies as the cube of the velocity. Experiments by Prof. Helie at Gavre in 1861 gave practically the same results.

The experiments above described were made principally with spherical projectiles. The difference in the nature of the resistance experienced by oblong and spherical projectiles, together with the difference in the velocities, then and later, may account for the wide difference in the results obtained from these and from later experiments.

LATER EXPERIMENTS.—The Rev. Francis Bashforth made exhaustive experiments in England, in 1865 and again in 1880, using comparatively modern projectiles and accurate ballistic instruments. His conclusions were, that for velocities between

900 and 1100 f. s. the resistance varied as the sixth power of the velocity; between 1100 and 1350 f. s., as the cube of the velocity; and above 1350 f. s., as the square of the velocity.

The most recent experiments are those made by Krupp in 1881 with modern guns, projectiles, and velocities. The results of these experiments were used by General Mayevski in the deduction of the formulas for the resistance of the air which are now generally used.

Conclusions from the Experiments.—The experiments have shown that the resistance of the air varies with the form of the projectile, with its area of cross section, with the velocity of the projectile, and with the density of the air. Considering the form of the projectile the resistance is affected principally by the shape of the head, and by the configuration at the junction of the head and body. The ogival head encounters less resistance than any other form of head. The resistance was found to increase directly with the area of cross section of the projectile, and directly with the density of the air.

208. Mayevski's Formulas for Resistance of the Air.—In expressing the relation between the resistance of the air and the velocity of the projectile, General Mayevski placed the retardation, as determined in Krupp's experiments, equal to an expression which involves, together with an unknown power of the velocity, quantities whose values are dependent on the weight, form, and cross section of the projectile, and on the density of the air.

Calling ρ the resistance of the air,

w the weight of the projectile in pounds,

g the acceleration of gravity,

the retardation is $\rho g/w$

Representing by R the retardation of the projectile, make

$$R=\rho g/w=v^n A/C \qquad (1)$$

in which A is a constant and n some power of the velocity, both to be determined from the experiments.

The Ballistic Coefficient, C.—The quantity C in the equation was given a value

$$C=\frac{\delta_1}{\delta}\frac{w}{cd^2}$$

in which δ_1 is the standard density of the air,

δ the density at the time of the experiment,

c the coefficient of form,

d the diameter of the projectile in inches,

w the weight of the projectile in pounds.

By the introduction of this coefficient into the value of the retardation, the effect of variations in weight, form, and cross section of the projectile, and in the density of the air, may be considered.

The coefficient of form c was taken as unity for the standard projectiles. For projectiles of a form that offers greater resistance the value of c will be greater than unity. Examination of equation (1) shows that as c increases, and C decreases, the retardation is increased; a result also obtained by increase in d or δ, that is in the cross section of the projectile or in the density of the air; while by an increase in w, C is increased and the retardation is diminished. The coefficient C is therefore the measure of the ballistic efficiency of the projectile.

The value of c for all projectiles in our service is usually taken as unity.

The density of the air is a function of the temperature and of the atmospheric pressure. The values of δ_1/δ for different atmospheric pressures and temperatures are found in Table VI of the ballistic tables.

Mayevski determined, from Krupp's experiments, values for n and A for different velocities as follows.

Velocities, f. s.	n	log A	Velocities, f. s.	n	log A
Above 2600	1.55	$\bar{3}.6090480$	1230 to 970	5	$\overline{14}.8018712$
2600 to 1800	1.7	$\bar{3}.0961978$	970 to 790	3	$\bar{8}.7734430$
1800 to 1370	2	$\bar{4}.1192596$	Below 790	2	$\bar{5}.6698914$
1370 to 1230	3	$\bar{8}.9809023$			

209. Trajectory in Air. Ballistic Formulas.—In the deduction of the ballistic formulas the trajectory is considered as a plane curve. The line of sight is taken as horizontal. The angle of elevation is taken as the angle of departure, and the striking angle becomes the angle of fall.

The trajectory so considered is called *The Horizontal Trajectory.*

Considering the motion of translation only, and that the resistance of the air is directly opposed to this motion, let, Fig. 158,

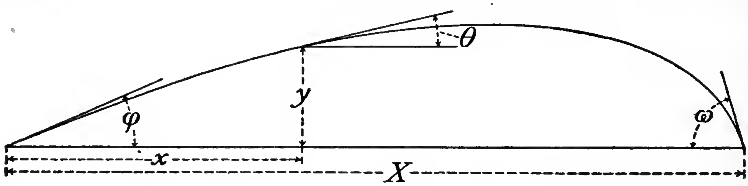


FIG. 158.

R be the retardation due to the resistance of the air, its value being given by equation (1);

V, the initial velocity;

v, the velocity at any point of the trajectory whose co-ordinates are x and y;

v_1, the component of v in the direction of x;

ϕ, the angle made with the horizontal by the tangent to the trajectory at the origin, or the angle of departure;

θ, the value of ϕ for any other point of the trajectory;

ω, the angle of fall;

x and y, the co-ordinates of any point of the trajectory, *in feet;*

X, the whole range, *in feet.*

EQUATIONS OF MOTION.—The only forces acting on the projectile after it leaves the piece are the resistance of the air and gravity.

The resistance of the air is directly opposed to the motion of the projectile, and continually retards it. Gravity retards the projectile in the ascending portion of the trajectory, while it accelerates it in the descending portion.

Considering the ascending portion of the trajectory, the velocity in the direction of x is

$$v \cos \theta = v_1 = dx/dt \qquad dx = v_1 dt \tag{2}$$

The velocity in the direction of y is

$$v \sin \theta = v_1 \tan \theta = dy/dt \qquad dy = v_1 \tan \theta \, dt \tag{3}$$

The retardation in the direction of y is therefore

$$-d(v_1 \tan \theta)/dt = g + R \sin \theta \tag{4}$$

Since gravity has no component in a horizontal direction, the retardation in the direction of x is

$$-dv_1/dt = R\cos\theta \qquad dt = -dv_1/R\cos\theta \tag{5}$$

Substituting this value of dt in (2), (3), and (4), and performing the differentiation indicated in (4), $d\tan\theta$ being $d\theta/\cos^2\theta$, we obtain

$$dx = -v_1 dv_1/R\cos\theta \tag{6}$$

$$dy = -v_1\tan\theta\, dv_1/R\cos\theta \tag{7}$$

$$d\theta = g\cos\theta\, dv_1/Rv_1 \tag{8}$$

The four equations (5) to (8) are the differential equations of motion of the projectile, and if they could be integrated directly they would give the values of t, x, y, and θ for any point of the trajectory. But as they are expressed in terms of R, v, and θ, three independent variables, the direct integration is impossible.

The value of R is given by Mayevski's formulas, $R = Av^n/C$, n representing the exponent of v for any particular velocity. Substituting this value of R in (6), the equation may, by means of the relation $v\cos\theta = v_1$, be put in the form

$$dx = -C\cos^{n-1}\theta dv_1/Av_1^{\,n-1} \tag{9}$$

The second member would be an exact integral were it not for the factor $\cos^{n-1}\theta$. In direct fire $\cos\theta$ differs but little from unity, and it might be taken as unity without appreciable error. $\cos^{n-1}\theta$ would then be unity and the expression would be integrable. A closer approximation, however, as shown by Siacci, results from making

$$\cos^{n-1}\theta = \cos^{n-2}\phi$$

Making this substitution equation (9) may be brought by reduction, *see foot note,* to the form

$$dx = -\frac{C}{A}\frac{d(v_1\sec\phi)}{(v_1\sec\phi)^{n-1}} \tag{10}$$

$$\cos^{n-2}\phi = 1/\sec^{n-2}\phi = \sec\phi/\sec^{n-1}\phi$$

ϕ is constant, therefore $\sec\phi dv_1 = d(v_1\sec\phi)$.

Make

$$v_1 \sec \phi = v \cos \theta / \cos \phi = u$$

$$V_1 \sec \phi = V \cos \phi / \cos \phi = V$$

Making these substitutions in equation (10) and integrating between the limits u and V we obtain

$$x = \frac{C}{(n-2)A}\left[\frac{1}{u^{n-2}} - \frac{1}{V^{n-2}}\right] \qquad (11)$$

And similarly equations (5) and (8) may be brought to the forms

$$t = \frac{C}{(n-1)A \cos \phi}\left[\frac{1}{u^{n-1}} - \frac{1}{V^{n-1}}\right] \qquad (12)$$

$$\tan \phi - \tan \theta = \frac{gC}{nA \cos^2 \phi}\left[\frac{1}{u^n} - \frac{1}{V^n}\right] \qquad (13)$$

210. To simplify equations (11) to (13), make

$$\left.\begin{aligned} S(u) &= \frac{1}{(n-2)Au^{n-2}} + Q \\ S(V) &= \frac{1}{(n-2)AV^{n-2}} + Q \\ T(u) &= \frac{1}{(n-1)Au^{n-1}} + Q' \\ I(u) &= \frac{2g}{nAu^n} + Q'' \end{aligned}\right\} \qquad (14)$$

The reason for the addition of the constants will appear.

Making these substitutions, equations (11) to (13) become

$$x = C\{S(u) - S(V)\} \qquad (15)$$

$$t = \frac{C}{\cos \phi}\{T(u) - T(V)\} \qquad (16)$$

$$\tan \theta = \tan \phi - \frac{C}{2 \cos^2 \phi}\{I(u) - I(V)\} \qquad (17)$$

Making in the last equation $\tan \theta = dy/dx$, and making

$$A(u) = -\frac{1}{A}\int \frac{I(u)du}{u^{n-1}} \qquad (14')$$

equation (17) may be brought to form, *see foot note,*

$$\frac{y}{x}=\tan\phi-\frac{C}{2\cos^2\phi}\left\{\frac{A(u)-A(V)}{S(u)-S(V)}-I(V)\right\} \tag{18}$$

Equations (15) to (18), with the equations

$$u=v\frac{\cos\theta}{\cos\phi} \tag{19}$$

and

$$C=f\frac{\delta_1}{\delta}\frac{w}{\beta cd^2} \tag{20}$$

are the fundamental equations of Exterior Ballistics, and constitute the method of Siacci, an eminent Italian ballistician. The essence of the method lies in the use of u, called by Siacci the *pseudo velocity*, for v, the actual velocity.

In all problems of direct fire, since the difference between ϕ and θ is not great, u may be used for v with sufficient accuracy. In problems in curved and high angle fire, and in direct fire when greater accuracy is desired, we pass from the value of u to the value of v by means of equation (19). It will be seen from this equation that, since $u\cos\phi=v\cos\theta$, u is the component of v parallel to the line of departure.

The Ballistic Coefficient.—The ballistic coefficient, like the *force coefficient* in the interior ballistic formulas, affords a convenient means of introducing into the exterior ballistic formulas any correction necessary to make the formulas applicable to conditions differing from the conditions for which the formulas were deduced.

From (17),

$$dy=\tan\phi\,dx-\frac{C}{2\cos^2\phi}\{I(u)dx-I(V)dx\} \tag{17a}$$

From (10), and $v_1\sec\phi=u$, $dx=Cdu/Au^{n-1}$

Substitute this value in the second term of the second member of (17*a*). Integrate the equation between the limits u and V with the help of (14′), and divide through by x.

$$\frac{y}{x}=\tan\phi-\frac{C}{2\cos^2\phi}\left\{\frac{C\{A(u)-A(V)\}}{x}-I(V)\right\}$$

Substitute for C/x its value from (15).

For general use with the formulas of exterior ballistics Mayevski's value for C, page 362, is changed by the introduction of two quantities, f and β, so that the value of the ballistic coefficient takes the form written in equation (20).

f is called the *altitude factor,* and brings into consideration the diminution in the density of the air as the altitude of the trajectory increases. The value of f is greater than unity and depends upon the mean altitude of the trajectory, which is taken as two-thirds of the maximum altitude.

β is an *integrating factor,* and corrects for the error due to certain assumptions made in deducing the primary equations, when these equations are applied to a trajectory whose curvature is considerable. β is approximately unity in all problems of direct fire. The product βc is called the *coefficient of reduction.*

When in the statements of ballistic problems the data required to determine δ_1/δ, β *or* c *is not given, the value unity is assumed for the factor.* f *is also assumed as unity unless a correction for altitude is desired.* When all these factors are unity the ballistic coefficient becomes

$$C=w/d^2$$

211. The Functions.—The functional expressions in equations (15) to (18) are called: $S(u)$ the space function, $T(u)$ the time function, $I(u)$ the inclination function, and $A(u)$ the altitude function. Their values are given by the equations (14) and (14′). The values of these functions for values of u from 3600 to 100 foot seconds have been calculated, and form Table I of the Ballistic Tables.

Since V is a particular value of u the values of the functions of V are included in the table as values of the functions of u. For example, to find the value of $S(V)$, V being given, enter the table with the value of V as a value of u and take out the corresponding value of $S(u)$.

The quantities Q, Q', and Q'', in the values of the functions, equations (14), are arbitrary constants; and the purpose of including them is to provide a means for avoiding abrupt changes in the tables at those points where in Mayevski's formulas the values of A and n change.

CALCULATION OF THE FUNCTIONS.—The method of employing the constants in forming the tables is best shown by an example. The value of the S function is, equation (14),

$$S(u)=\frac{1}{(n-2)Au^{n-2}}+Q$$

For va'ues of v greater than 2600 f. s., we have from Mayevski's formulas, $n=1.55$. Therefore for a velocity greater than 2600 f. s.

$$S(u)=-\frac{u^{0.45}}{0.45A_1}+Q=-\frac{1}{0.45A_1}(u^{0.45}+Q_1)$$

In order to avoid the use of large numbers Table I of the latest ballistic tables, published in 1900, is so constructed that the S, A, and T functions reduce to zero for $u=3600$. $I(u)$ reduces to zero for $u=\infty$. We have then for $S(u)$, when $u=3600$

$$S(u)=0=-\frac{1}{0.45A_1}(\overline{3600}^{0.45}+Q_1)$$

and therefore

$$Q_1=-(3600)^{0.45}$$

For any other value of u down to 2600

$$S(u)=\frac{1}{0.45A_1}(\overline{3600}^{0.45}-u^{0.45})=K-K'u^{0.45} \qquad (21)$$

For velocities between 2600 and 1800 f. s., $n=1.7$, and

$$S(u)=-\frac{1}{0.3A_2}(u^{0.3}+Q_2)$$

Q_2 must have such a value as to make the value of $S(u)$ for $u=2600$ the same as the value determined from equation (21) with this value of u. Therefore

$$-\frac{1}{0.3A_2}(\overline{2600}^{0.3}+Q_2)=K-K'\overline{2600}^{0.45}$$

from which the value of Q_2 can be determined.

The same process is followed at each change in the values of n and A.

When $n=2$ equation (11) becomes indeterminate and the values of the functions cannot be determined as above; but making $n=2$ in equation (10) and integrating we obtain

$$x=-\frac{C}{A}(\log_e u-\log_e V)$$

$S(u)$ becomes in this case

$$S(u)=-\frac{\log_e u}{A}+Q$$

INTERPOLATION IN TABLE I.—This is effected by the ordinary rules of proportional parts. The difference between successive values of u varies from unity in one part of the table to 2, 5, and 10 in other parts. This difference must be carefully noted in interpolating.

212. Formulas for the Whole Range.—Designate the whole range, Fig. 158, by X, the corresponding time of flight by T, the angle of fall (considered positive for convenience) by ω, and use the subscript $_\omega$ to designate the values of u and v at the point of fall.

At the point of fall $y=0$ and $\theta=-\omega$; and after combining equations (17) and (18) to eliminate $I(V)$ from (17), equations (15) to (19) become, respectively,

$$X=C\{S(u_\omega)-S(V)\} \tag{22}$$

$$T=\frac{C}{\cos\phi}\{T(u_\omega)-T(V)\} \tag{23}$$

$$\tan\omega=\frac{C}{2\cos^2\phi}\left\{I(u_\omega)-\frac{A(u_\omega)-A(V)}{S(u_\omega)-S(V)}\right\} \tag{24}$$

$$\sin 2\phi=C\left\{\frac{A(u_\omega)-A(V)}{S(u_\omega)-S(V)}-I(V)\right\} \tag{25}$$

$$u_\omega=v_\omega\cos\omega/\cos\phi \tag{26}$$

At the summit of the trajectory $\theta=0$. Using the subscript $_o$ to designate the summit, equations (17) and (19) become, after reduction,

$$I(u_o)=\sin 2\phi/C+I(V) \tag{27}$$

$$u_o=v_o/\cos\phi \tag{28}$$

Combining (27) and (25) we have

$$I(u_o)=\frac{A(u_\omega)-A(V)}{S(u_\omega)-S(V)} \tag{29}$$

Therefore (24) and (25) become

$$\tan\omega=\frac{C}{2\cos^2\phi}\{I(u_\omega)-I(u_o)\} \tag{30}$$

$$\sin 2\phi=C\{I(u_o)-I(V)\} \tag{31}$$

213. The Ballistic Elements.—The quantities C, u, V, ϕ, θ, ω, T, and X in the previous equations are called the ballistic elements. When referring to the end of the range they are written as capitals, or with the subscript $_\omega$. For any other point of the trajectory they are written as small letters, with suitable subscript if desired. The subscript $_o$ always refers to the summit of the trajectory. The equations, by reason of Siacci's assumption for the value of $\cos^{n-1}\theta$, express the relations existing between these elements in direct fire only.

When three or more of the elements are given the others may be determined.

The Rigidity of the Trajectory.—According to the principle of the rigidity of the trajectory, which is mathematically demonstrated, the relations existing between the trajectory and the chord representing the range are sensibly the same whether the chord be horizontal or inclined to the horizon, provided that the *quadrant angle of departure* and the angle of position are small or that the difference between them is small. That is to say that, considering $\phi+\varepsilon$ and ε as small, in Fig. 156, if the trajectory bdf and its chord bf were revolved about the point b until bf were horizontal, the relation of the trajectory to bf would not change. A trajectory calculated for a *horizontal* range equal to bf would then answer as the trajectory for the actual *inclined* range bf.

Therefore when the quadrant angle of departure, $\phi+\varepsilon$, is small we may consider bf, or any other chord of the trajectory, as a horizontal range; and we may apply to the trajectory subtended by the chord the formulas deduced for a horizontal range.

If however the quadrant angle of departure is large, the principle of the rigidity of the trajectory applies only when the angle of position is also large, that is when $\phi+\varepsilon$ does not differ much from ε. Therefore in any complete high angle trajectory for a horizontal range the principle of the rigidity of the trajectory applies only to a part of the trajectory near the origin. This part may be treated as a horizontal range whose angle of departure is the difference between the quadrant angle of departure of the horizontal trajectory and the angle of position.

When the difference between $\phi+\varepsilon$ and ε is small, ϕ must be small. It is therefore evident that, in direct fire, the principle of the rigidity of the trajectory applies whenever the angle of departure is small.

This principle enables us to use the elements calculated for a horizontal range when firing at objects situated above or below the level of the gun.

214. Use of the Formulas.—The method of using the formulas may best be shown by considering a problem.

Problem 1.—What is the time of flight of a 3-inch projectile weighing 15 lbs., for a range of 2000 yards; muzzle velocity, 1700 f. s.?

The given data are $C=15/9$, $V=1700$, and $X=6000$, *the range being always taken in feet.* T is required.

These formulas apply:

$$T=\frac{C}{\cos\phi}\{T(u_\omega)-T(V)\} \tag{23}$$

$$\sin 2\phi=C\left\{\frac{A(u_\omega)-A(V)}{S(u_\omega)-S(V)}-I(V)\right\} \tag{25}$$

$$X=C\{S(u_\omega)-S(V)\} \tag{22}$$

Take the T, S, A, and I functions of V from Table I.

Determine $S(u_\omega)$ from (22).

Find u_ω from Table I, and take from the Table $T(u_\omega)$ and $A(u_\omega)$.

Find ϕ from (25).

Find T, required, from (23). *Ans.* $T=4.48$ *seconds.*

215. Secondary Functions.—The most important problems in gunnery may be solved by means of equations (22) to (31) and ballistic Table I, but some of the solutions are indirect and tentative and therefore very laborious. The processes of solution have been greatly abbreviated and the labor greatly reduced by the introduction of secondary functions, whose values, for all the requirements of modern gunnery, have been calculated and collected in Table II of the ballistic tables.

The development of the science of exterior ballistics to its present accuracy and comparative simplicity is principally due to *Colonel James M. Ingalls,* U. S. Army, whose interior ballistics are set forth in Chapter III.

From equation (15) we have

$$S(u)=x/C+S(V)$$

and substituting the values of $S(u)$ and $S(V)$, see (14),

$$\frac{1}{(n-2)Au^{n-2}}=\frac{x}{C}+\frac{1}{(n-2)AV^{n-2}}$$

From this equation it is apparent that the value of the pseudo velocity u, *at any point, is a function of* x/C *and* V *only,* and is independent of the height of the point in the trajectory.

Make

$$z=x/C \qquad\qquad Z=X/C$$

It will be seen in equations (16), (17), and (18) that t, θ, and y are functions of u and therefore also functions of z and of V.

The secondary functions, whose values are here given, are all functions of Z and V, and are tabulated with Z and V as arguments.

$$\left.\begin{aligned}A&=\frac{A(u)-A(V)}{S(u)-S(V)}-I(V)\\B&=I(u)-\frac{A(u)-A(V)}{S(u)-S(V)}\\A'&=A+B=I(u)-I(V)\\T'&=T(u)-T(V)\\B'&=B/A\end{aligned}\right\}\qquad(32)$$

The subscripts are dropped in these expressions since they only serve to indicate particular values of u, while the table contains the values of A, B, etc., for all the values of u.

The table also contains, in the column u, the values of u for all values of Z and V.

Equations (23), (24), and (25) may now be put, by reduction, into the following exceedingly simple forms.

$$T = CT'/\cos\phi \qquad (33)$$

$$\sin 2\phi = AC \qquad (34)$$

$$\tan\omega = BC/2\cos^2\phi = B'\tan\phi \qquad (35)$$

Equations (17) and (18) may also be put in the forms

$$\tan\theta = \frac{\tan\phi}{A}(A - a') \qquad (36)$$

$$y = \frac{x\tan\phi}{A}(A - a) \qquad (37)$$

In these equations a and a' are the values of A and A' corresponding to $z = x/C$ for the particular point of the trajectory considered, while A and A' are the values corresponding to $Z = X/C$ for the whole range.

216. *At the summit* $\tan\theta$ reduces to zero; and we obtain from equation (36), writing a_0' for a' at the summit,

$$a_0' = A \qquad (38)$$

Equation (37) then becomes

$$y_0 = \frac{x_0\tan\phi}{a_0'}(a_0' - a_0) \qquad (38')$$

From the third equation (32) we have for the summit $b_0 = a_0' - a_0$. With this relation and the relation $z_0 = x_0/C$, and making

$$a_0'' = b_0 z_0 / a_0'$$

equation (38′) reduces to the form

$$y_0 = a_0'' C \tan\phi \qquad (39)$$

y_0 representing the *maximum ordinate.*

To obtain a_0'' for use in this equation we find in Table II, in the A' column, the value of A as determined for the whole range. With this value as A' and the given value of V we find a_0'' in the A'' column.

Write
$$Z=X/C \tag{40}$$
$$v=u\cos\phi/\cos\theta \tag{41}$$
$$C=f\frac{\delta_1}{\delta}\frac{w}{cd^2} \tag{42}$$

and

$$\text{Drift (yds.)}=\begin{cases}[\bar{3}.79239]\,C^2D'/\cos^3\phi\ \text{(seacoast guns)}\\ [\bar{3}.92428]\,C^2D'/\cos^3\phi\ \text{(field guns)}\end{cases} \tag{43}$$

which is Mayevski's formula for drift, abbreviated for tabulation by Colonel Ingalls. The values of D' are found in Table II.

We have in the equations (33) to (43) the principal formulas required for the solution of nearly all the problems of direct fire.

While the formulas apply strictly to *direct fire* only, where the values of ϕ and θ are such as to permit the use of Siacci's value of $\cos^{n-1}\theta$ without appreciable error, they give sufficiently accurate results for *curved fire,* and they are used for curved fire as well.

They are made applicable to *high angle fire* by giving to the coefficient c in the ballistic coefficient such values as will make the results obtained from the formulas agree with the results obtained in actual firings. For the *low velocities* used in mortars and howitzers the formulas are simplified, as will later be shown.

Ballistic Tables.—The Ballistic Tables, which are issued by the War Department, consist of three volumes, entitled: *Artillery Circular M,* Series of 1893 (printed in 1900), *Supplement to Artillery Circular M* (1903), and *Supplement No. 2 to Artillery Circular M* (1904). The supplements extend Tables II, IV, and V of *Artillery Circular M.*

In addition there has appeared a simplification of Table IV in the *Journal of the United States Artillery,* number for January and February, 1905.

Artillery Notes, No. 25, issued by the War Department, 1905, contains a corrected table to replace Table VI of *Artillery Circular M,* the latter table having been found to be based on incorrect data.

The ballistic formulas are found assembled on page VIII of the first book of tables, *Artillery Circular M*, so that the books of tables contain all that is needed for the solution of most of the problems of gunnery.

Under the heading *Formulas to be used with Table II*, on page VIII of *Artillery Circular M*, appears the formula

$$S(u)=Z+S(v)$$

which is another form of

$$X=C\{S(u)-S(V)\}$$

This formula, which is sometimes convenient to use, requires the use of Table I.

To understand the additional formulas under this heading on page VIII of the ballistic tables it is only necessary to know that ε represents the angle of position of a target, not on the same level with the gun, whose horizontal distance from the gun is x, and that ϕ_x is the angle of departure for the horizontal range x. a is the particular value of A that corresponds to the value of x.

These formulas express the relations that exist between ϕ, ε, and ϕ_x. They are used to determine the *quadrant angle of elevation* for a target situated so much above or below the level of the gun and at such a range *that the principle of the rigidity of the trajectory cannot be applied.*

EXTERIOR BALLISTIC FORMULAS.

The formulas required in the solutions of most ballistic problems are here assembled for convenience. There are included the formulas already deduced and others which are deduced later.

DIRECT FIRE.

$$V>825 \text{ f. s.} \quad \phi<20°$$

$C=f\dfrac{\delta_1}{\delta}\dfrac{w}{cd^2}$	(42)	$Z=X/C$	(40)
$\sin 2\phi=AC$	(34)	$T=CT'/\cos\phi$	(33)
$\tan\omega=B'\tan\phi$	(35)	$v=u\cos\phi/\cos\theta$	(41)
$y=x\tan\phi\,(A-a)/A$	(37)	$a_0'=A=\sin 2\phi/C$	(38)
$\tan\theta=\tan\phi\,(A-a')/A$	(36)	$y_0=a_0''\,C\tan\phi$	(39)

CORRECTION FOR ALTITUDE.

$$\log(\log f) = \log y_0 + \bar{5}.01765 \qquad (44)$$

DANGER SPACE AND DANGER RANGE.

$$(A-a)z = 2y\cos^2\phi/C^2 \qquad (51) \qquad f = f_0 + \frac{e_0}{e_0+e_1}(f_1 - f_0) \qquad (53)$$

$$\varDelta X = X - x \qquad (54) \qquad a_0'a_0'' = 2\,y_0/C^2 \qquad (55)$$

DRIFT.

$$\left.\begin{array}{ll}\text{Seacoast Guns.} & \text{Drift (yds.)} = [\bar{3}.79239]C^2D'/\cos^3\phi \\ \text{Field Guns.} & \text{Drift (yds.)} = [\bar{3}.92428]C^2D'/\cos^3\phi\end{array}\right\} \qquad (43)$$

WIND EFFECT—RANGE.

$$\varDelta V = W_p\cos\phi \qquad (45) \qquad V' = V \pm \varDelta V \qquad (46)$$

$$\sin\varDelta\phi = W_p\sin\phi/V' \qquad (47) \qquad \phi' = \phi \mp \varDelta\phi \qquad (48)$$

$$\varDelta X\,(\text{ft.}) = X' - (X \pm W_pT) \qquad (49)$$

WIND EFFECT—DEVIATION FOR 8, 10, 12-INCH PROJECTILES.

$$\text{Deviation (yds.)} = [7.00000]\sin\alpha W(\text{m.p.h.})\left(\frac{T\,(\text{sec.})}{33000 + X\,(\text{yds.})}\right)^2 \qquad (50)$$

CURVED FIRE.

Always correct for altitude.

For $V > 825$ f. s. and ϕ, 20° to 30°, use formulas for direct fire.

Use the following formulas when

$$V < 825\ \text{f. s.} \qquad \phi < 30^\circ$$

$$C = f\frac{\delta_1}{\delta}\frac{w}{cd^2} \qquad (42) \qquad Z = X/C \qquad (40)$$

$$\log(\log f) = \log y_0 + \bar{5}.01765 \qquad (44)$$

$$\sin 2\phi = [5.80618]\,AC/V^2 \qquad (56) \qquad \tan\omega = B'\tan\phi \qquad (35)$$

$$v_\omega = [\bar{3}.09691]u_\omega\cos\phi\;V/\cos\omega \qquad (57)$$

$$T = [2.90309]\,CT'/V\cos\phi \qquad (58)$$

HIGH ANGLE FIRE.

$\phi > 30°$

Always correct for altitude.

When the coefficient of reduction c is known use Table IV.

When the coefficient of reduction is not known use the formulas for direct fire and Table II, or Table I in those problems for which Table II is not sufficiently extended.

CURVATURE OF EARTH.

$$\text{Curvature (ft.)} = [\bar{7}.33289] X^2 \text{ (yds.)} \qquad (59)$$

217. Interpolation in Table II.—Exact formulas for interpolation in Table II are deduced and explained in the appendix to this chapter. These formulas greatly facilitate the solution of ballistic problems. A thorough understanding of the interpolation formulas, and facility in their use, should be acquired before proceeding further. These formulas, which are here written, will be used in place of the interpolation formulas given on page VIII of the ballistic tables, as the latter formulas are approximate only.

Double Interpolation Formulas—Ballistic Table II.

f = non-tabular value of any function corresponding to the non-tabular values V and Z.

f_0 = tabular value of function corresponding to tabular values V_0 and Z_0 *always next less than* V *and* Z.

h = difference between velocities given in caption of table.

Δv_0 and Δz_0 = tabular differences for f_0.

Δv_1 = tabular difference next following Δv_0 in same table.

$f\binom{-V}{+Z}$ indicates that function decreases as V increases, and increases as Z increases.

Use the following formulas for the functions A, A', B, T', log C', and D' throughout the table. They also apply for some values of the functions A'' and log B' when $V > 2500$.

$$f\binom{-V}{+Z} = f_0 + \frac{Z-Z_0}{100}\Delta z_0 - \frac{V-V_0}{h}\Delta v_0 - \frac{Z-Z_0}{100}\cdot\frac{V-V_0}{h}(\Delta v_1 - \Delta v_0)$$

$$V = V_0 + \frac{\left(f_0 + \frac{Z - Z_0}{100}\Delta z_0\right) - f}{\Delta v_0 + (\Delta v_1 - \Delta v_0)\frac{Z - Z_0}{100}} \times h$$

$$Z = Z_0 + \frac{f - \left(f_0 - \frac{V - V_0}{h}\Delta v_0\right)}{\Delta z_0 - (\Delta v_1 - \Delta v_0)\frac{V - V_0}{h}} \times 100$$

Use the following formulas for the functions A'' and $\log B'$ when $V < 2500$, and for some values beyond that point.

$$f\binom{+V}{+Z} = f_0 + \frac{Z - Z_0}{100}\Delta z_0 + \frac{V - V_0}{h}\Delta v_0 + \frac{Z - Z_0}{100} \cdot \frac{V - V_0}{h}(\Delta v_1 - \Delta v_0)$$

$$V = V_0 + \frac{f - \left(f_0 + \frac{Z - Z_0}{100}\Delta z_0\right)}{\Delta v_0 + (\Delta v_1 - \Delta v_0)\frac{Z - Z_0}{100}} \times h$$

$$Z = Z_0 + \frac{f - \left(f_0 + \frac{V - V_0}{h}\Delta v_0\right)}{\Delta z_0 + (\Delta v_1 - \Delta v_0)\frac{V - V_0}{h}} \times 100$$

Use the following formulas for the function u.

$$f\binom{+V}{-Z} = f_0 - \frac{Z - Z_0}{100}\Delta z_0 + \frac{V - V_0}{h}\Delta v_0 - \frac{Z - Z_0}{100} \cdot \frac{V - V_0}{h}(\Delta v_0 - \Delta v_1)$$

$$V = V_0 + \frac{f - \left(f_0 - \frac{Z - Z_0}{100}\Delta z_0\right)}{\Delta v_0 - (\Delta v_0 - \Delta v_1)\frac{Z - Z_0}{100}} \times h$$

$$Z = Z_0 + \frac{\left(f_0 + \frac{V - V_0}{h}\Delta v_0\right) - f}{\Delta z_0 + (\Delta v_0 - \Delta v_1)\frac{V - V_0}{h}} \times 100$$

Inspect the tables to determine how the function varies with V and Z, and select the proper group of formulas.

Exercise great care in the use of the plus and minus signs.

As the numbers in the difference columns of the table are written as whole numbers we must, when using the interpolation formulas, treat the tabular values of the functions as whole numbers, and afterwards put the decimal point where it belongs.

Regarding the interpolation formulas we will note that the proportional parts of the differences Δz_0 and Δv_0 are always applied to the tabular value of the function, f_0, with a sign indicated by the manner of variation of the function with Z and V respectively; positive if the function is increasing, negative for a decreasing function. The sign of the last term of the f formulas is positive if the signs of the preceding terms are similar, and negative if they are dissimilar.

In the formulas for V and Z the fractional coefficients of h and 100 are equal respectively to $\frac{V-V_0}{h}$ and $\frac{Z-Z_0}{100}$. These coefficients will always indicate by their values whether we are working with the proper tabular values. *Numerator and denominator of the fraction should always be positive, and the value of the fraction less than unity.*

218. The Solution of Problems.—With the ballistic formulas and the tables, the solutions of the problems of gunnery become very simple. We will remember that all the functions in Table II are functions of V and of $Z=X/C$, the arguments of the table. Therefore, given any two of the three quantities, V, Z, and a value of a function, the third may be determined from the table, and also the corresponding value of any other function in the table. For instance, suppose V and A' are given and the corresponding values of A'', log B' and T' are required. With V and A' we may obtain Z from the table, and with V and Z we obtain A'', log B' and T'.

Inspecting the formulas, pages 376 and 377, we select those that contain the given quantities, and such other formulas as, with Table II, will enable us to pass to the formula containing the required quantity.

It must be remembered that in the formulas the large letters represent values of the quantities for the whole range, or complete horizontal trajectory; while the small letters represent values of the same quantities for particular points of the trajectory. In the tables all these values are gathered in columns headed with the large letters, which are thus used in a general sense.

In what follows, either in general discussions or when demonstrating the use of the tables, the large letters will be used.

To show the advantages derived from the use of Table II with the abbreviated formulas, let us consider the problem whose solution by means of Table I has been indicated on page 372.

219. Problem 1.—What is the time of flight of a 3-inch projectile weighing 15 lbs., for a range of 2000 yards; muzzle velocity, 1700 feet?

$C=15/9$, $V=1700$, and $X=6000$ are given. T is required.

These formulas apply:

$$T=CT' \sec \phi \tag{33}$$

$$\sin 2\phi=AC \tag{34}$$

$$Z=X/C \tag{40}$$

Determine Z from (40).
With Z and V take A and T' from Table II.
Determine ϕ from (34).
Determine T from (33). *Ans.* $T=4.48$ seconds.
Compare this with the process indicated on page 372.

To show the most convenient method of performing the work, the solution of a problem is here given in full.

220. Problem 2.—A 575 lb. projectile is fired from a 10-inch gun at a target 8000 yds. distant; muzzle velocity, 2540 f. s. Assuming the atmospheric conditions as normal, determine the angle of elevation required and the other ballistic elements.

No data being given for the determination of δ_1/δ, and the correction for altitude not being required, the value $C=w/d^2$ is taken for the ballistic coefficient.

	$\log w$	2.75967
	$2 \log d$	2.00000
	$\log C$	0.75967
$Z=X/C$	$\log X$	4.38021
	$\log Z$	3.62054

$Z=4173.9$

To find the *angle of departure*, use $\sin 2\phi = AC$.
From Table II, with $V=2540$ and $Z=4174$,

$$A=(0.03054)+.74\times107-.4\times243-.3\times10=0.03033$$

The inclusion of the number in parentheses is to indicate that in applying the corrections this number is treated as a whole number.

$$\begin{array}{rl} \log A & \bar{2}.48187 \\ \log C & 0.75967 \\ \hline \log \sin 2\phi & \bar{1}.24154 \end{array} \qquad \begin{array}{r} 2\phi = 10^\circ\ 2'.6 \\ \phi = 5^\circ\ 1' \end{array}$$

ϕ, after being accurately determined, is used to the nearest minute only.

To find the *time of flight*, use $T=CT'\sec\phi$.
From Table II, with V and Z,

$$T'=(2.145)+.74\times68-.4\times89-.3\times3=2.1588$$

$$\begin{array}{rl} \log T' & 0.33421 \\ \log C & 0.75967 \\ \hline & 1.09388 \\ \log \cos\phi & \bar{1}.99833 \\ \hline \log T & 1.09555 \end{array} \qquad T=12.46 \text{ seconds}$$

To find the *angle of fall*, use $\tan\omega = B'\tan\phi$.
From Table II, with V and Z, $(\Delta v_1 - \Delta v_0)$ being negative,

$$\log B'=(0.1513)+.74\times38-.4\times12+.3=0.15366$$

$$\begin{array}{rl} \log B' & 0.15366 \\ \log \tan\phi & \bar{2}.94340 \\ \hline \log \tan\omega & \bar{1}.09706 \end{array} \qquad \omega = 7^\circ\ 8'$$

To find the *striking velocity*, use $v=u\cos\phi\sec\theta$.
θ in this case becomes ω. From Table II, with V and Z,

$$u=1481-.74\times20+.4\times66=1492.6$$

$$\begin{array}{rl} \log u & 3.17394 \\ \log \cos\phi & \bar{1}.99833 \\ \hline & 3.17227 \\ \log \cos\omega & \bar{1}.99663 \\ \hline \log v & 3.17564 \end{array} \qquad v=1498 \text{ f. s.}$$

It is evident from these values of u and v that no material error is made by considering, for this shot, that $u=v$.

To find the *maximum ordinate*, use $y_0=a_0'' C \tan \phi$.

As already explained, see equation (39), we find the value of a_0'' in this equation by means of the value A obtained from the equation $\sin 2\phi=AC$. At the summit, see equation (38),

$$a_0'=A=\sin 2\phi/C$$

This value of A is therefore the value of A' for the summit. Using this value of A in the A' column of Table II, with the given value of V, we obtain from the A'' column the value of a_0''.

The value of A obtained above is 0.03033

From Table II, with $V=2540$ and $A'=0.0303$,

$$\frac{\overset{2200}{Z-Z_0}}{100}=\frac{303-(300-.4\times24)}{18-.4}=.71$$

$$a_0''=1200+.71\times59=1241.9$$

$\log a_0''$	3.09409	
$\log C$	0.75967	
$\log \tan \phi$	$\bar{2}.94340$	
$\log y_0$	2.79716	$y_0=626.8$ feet

221. Problem 3.—Compute the drift for the shot in Problem 2. Use Mayevski's formula, D (yds.) $=[\bar{3}.79239]\ C^2D'/\cos^3 \phi$.

$V=2540 \qquad Z=4174 \qquad \phi=5° 1' \qquad \log C=0.75967$

From Table II $\quad D'=81+.74\times5-.4\times6=82.3$

$\log D'$	1.91540	
$2 \log C$	1.51934	
const. log	$\bar{3}.79239$	
	1.22713	
$3 \log \cos \phi$	$\bar{1}.99499$	
$\log D$	1.23214	$D=17$ yards

222. Correction for Altitude.—The altitude factor f in the ballistic coefficient, see equation (42), takes into account the diminution in the density of the air as the projectile rises, and it corrects with sufficient exactness for the error that arises from the use of the standard density with which Table II is computed. When accuracy is desired the altitude factor is calculated and applied to the ballistic coefficient in all firings at angles greater than about 5 degrees.

Under the assumption of the mean height of the trajectory as two thirds of the maximum ordinate, the value of the altitude factor is given by the equation

$$\log(\log f) = \log y_0 + \bar{5}.01765 \qquad [44]$$

The summit ordinate is, equation (39),

$$y_0 = a_0'' C \tan \phi$$

As C enters the value of y_0 we must assume, for an approximation in the determination of the altitude factor by means of equations (39) and (44), the value of C obtained by considering the altitude factor as unity. Call this value C_1. With C_1 compute ϕ as explained in Problem 2, determine y_0 from equation (39) and f from (44). Call these values ϕ_1, y_{01}, and f_1. Then applying the value f_1, thus determined, to the assumed value C_1, a new value of C, C_c, is obtained. This value C_c will be close to the true value and may usually, with sufficient accuracy for practical purposes, be used as C. If greater accuracy is desired a second determination (of ϕ_c, y_{0c}, and f_c) is made. The resulting value, f_c, is applied to the value C_1 first assumed, and the process is repeated until there is no material change between the corrected values of C_1 resulting from the last two operations. The final corrected value is then used as C.

223. Problem 4.—Correct the ballistic coefficient for altitude, and determine the angle of elevation required in order that a 1048 lb. projectile fired from the 12 inch rifle with a muzzle velocity of 2350 f. s. may strike a target distant 12,000 yds.; the atmospheric conditions at the time of firing being barometer 29″.5, thermometer 67° F.,

$$d = 12 \qquad w = 1048 \qquad X = 36{,}000 \qquad V = 2350$$

The process may be indicated as follows:

$$C = f\frac{\delta_1}{\delta}\frac{w}{cd^2} \qquad Z = X/C \qquad \text{Table II, } A,\ a_0'' \qquad \sin 2\phi = AC$$

$$y_0 = a_0'' C \tan \phi \qquad \log(\log f) = \log y_0 + \bar{5}.01765$$

Table VI $\qquad \delta_1/\delta = 1.037 - 0.5\ (1.037 - 1.003) = 1.02$

$C = f\frac{\delta_1}{\delta}\frac{w}{cd^2}$	$\log \delta_1/\delta$	0.00860	
Consider $c = 1$, $f = 1$	$\log w$	3.02036	
		3.02896	
	$\log d^2$	2.15836	
	$\log C_1$	0.87060	(1st approximation)
$Z = X/C$	$\log X$	4.55630	
	$\log Z$	3.68570	$Z = 4849.5$

Table II, $A = (0.04589) + .495 \times 146 - .5 \times 396 - .248 \times 13 = 0.044601$

While using the table we will take out for future use the value of a_0'' corresponding to $a_0' = A = 0.044601$.

With $a_0' = 0.044601$, we obtain from the A' column

$$\frac{Z - \overset{2600}{Z_0}}{100} = \frac{446 - (447 - .5 \times 38)}{24 - .5 \times 2} = .783$$

Note tnat in this operation we have taken a tabular value 0.0447 for A larger than the given value 0.0446 because the tabular value when corrected for the variation in V becomes less than the given value.

$$a_0'' = 1444 + .783 \times 61 = 1491.8$$

$\sin 2\phi = AC$	$\log A$	$\bar{2}.64934$	
	$\log C_1$	0.87060	
	$\log \sin 2\phi_1$	$\bar{1}.51994$	$2\phi_1 = 19° 20'.1$
			$\phi_1 = 9° 40'$
$y_0 = a_0'' C \tan \phi$	$\log \tan \phi_1$	$\bar{1}.23130$	
	$\log C_1$	0.87060	
	$\log a_0''$	3.17371	
	$\log y_{01}$	3.27561	

$$\begin{array}{rl}
\log y_{01} & 3.27561 \\
\log\ (\log f) = \log\ y_0 + \bar{5}.01765 & \bar{5}.01765 \\
\hline
\log\ (\log f_1) & \bar{2}.29326 \\
\hline
\log f_1 & 0.01965 \\
\log C_1 & 0.87060 \\
\hline
\log C_c & 0.89025 \quad \text{(1st correction)}
\end{array}$$

With the corrected value of C we repeat the process followed after the determination of C_1, the first approximation.

$$\begin{array}{lrl}
Z = X/C & \log X & 4.55630 \\
 & \log C_c & 0.89025 \\
\hline
 & \log Z & 3.66605 \qquad Z = 4635
\end{array}$$

Table II, $A = (0.04306) + .35 \times 140 - .5 \times 372 - .175 \times 12 = 0.041669$

Take out for future use the value of a_0'' corresponding to $a_0' = A = 0.04167$

$$\frac{Z - \overset{2500}{Z_0}}{100} = \frac{416.7 - (424 - .5 \times 36)}{23 - .5 \times 2} = .486$$

$$a_0'' = 1383 + .486 \times 61 = 1412.6$$

$$\begin{array}{lrl}
\sin 2\phi = AC & \log A & \bar{2}.61981 \\
 & \log C_c & 0.89025 \\
\hline
 & \log \sin 2\phi_c & \bar{1}.51006 \qquad 2\phi_c = 18^\circ\ 53'.0 \\
 & & \qquad\qquad \phi_c = 9^\circ\ 26'.5 \\
y_0 = a_0''\, C \tan \phi & \log \tan \phi_c & \bar{1}.22088 \\
 & \log C_c & 0.89025 \\
 & \log a_0'' & 3.15002 \\
\hline
 & \log y_{0c} & 3.26115 \\
\log\ (\log f) = \log y_0 + \bar{5}.01765 & & \bar{5}.01765 \\
\hline
 & \log\ (\log f_c) & \bar{2}.27880 \\
\hline
 & \log f_c & 0.01900 \\
 & \log C_1 & 0.87060 \\
\hline
 & \log C_{cc} & 0.88960 \quad \text{(2d correction)}
\end{array}$$

As this value of $\log C_{cc}$ does not differ greatly from the value $\log C_c = 0.89025$, obtained by the first correction, further correction is unnecessary and we will use $\log C_{cc}$ as $\log C$ in determining the angle of departure.

$$Z = X/C \qquad \text{Table II, } A \qquad \sin 2\phi = AC$$

$$\begin{array}{lll} & \log X & 4.55630 \\ & \log C & 0.88960 \\ \hline & \log Z & 3.66670 \qquad\qquad Z = 4641.9 \end{array}$$

$$A = (0.04306) + .419 \times 140 - .5 \times 372 - .21 \times 12 = 0.041761$$

$$\begin{array}{lll} \sin 2\phi = AC & \log A & \bar{2}.62077 \\ & \log C & 0.88960 \\ \hline & \log \sin 2\phi & \bar{1}.51037 \qquad 2\phi = 18° \, 53'.8 \\ & & \qquad\qquad\quad \phi = 9° \, 26'.9 \end{array}$$

This value of ϕ is practically the same as the value ϕ_c previously obtained. It is obvious therefore that we have carried the correction for altitude sufficiently far.

224. Angle of Departure Constant.—When the angle of departure ϕ is fixed, instead of the range X as in the last problem, the correction for altitude is made and the range found as here indicated.

$$C = f \frac{\delta_1}{\delta} \frac{w}{cd^2} \quad A = \sin 2\phi / C \quad \text{Table II, } a_0'' \quad y_0 = a_0'' C \tan \phi$$

$$\log (\log f) = \log y_0 + \bar{5}.01765 \qquad X = ZC$$

Determine C_1 from $C = w\delta_1 / \delta d^2$, as in Problem 4 (1st approximation).

Find $a_0' = A$ from $\sin 2\phi = AC$

Find a_0'' corresponding to a_0' from Table II

Find y_{01} from $y_0 = a_0'' C \tan \phi$

Find f_1 from $\log (\log f) = \log y_0 + \bar{5}.01765$

Find C_c from $C_c = f_1 C_1$ (1st correction)

and proceed in the same way to find C_{cc} or C_{3c} as required.

Find the range from $X = ZC$ with the final corrected value of C.

225. The Effect of Wind.—In considering the wind we assume that the air moves horizontally, and that the effect on the velocity of the projectile is due to the component of the wind in the plane of fire only. We also assume as practically correct that the time of flight of the projectile is not influenced by the wind.

Let W be the velocity of the wind in foot seconds,

W_p the component of W in the plane of fire,

α the angle, reckoned from the target, between the direction of the wind and the plane of fire.

Then

$$W_p = W \cos \alpha.$$

Call W_p positive for a wind opposed to the projectile, and negative for a wind with it.

THE EFFECT ON RANGE. *Ingall's Method.*—We will assume that the effect of the wind component, W_p, is simply to increase or diminish the resistance encountered by the projectile; and that therefore this resistance, instead of being due to the velocity v, is due to the velocity $(v \pm W_p)$. Represent by ΔX the correction to be applied to the range in a calm to produce the true range, this correction being the variation in range, with its sign changed, caused by the wind. We may put equations (23) and (22), when ϕ is small and $\cos \phi$ nearly unity, in the following forms, using the upper signs when the direction of W_p is toward the gun and the lower signs when it is toward the target.

$$T(v \pm W_p) = T/C + T(V \pm W_p)$$

$$\Delta X = C\{S(v \pm W_p) - S(V \pm W_p)\} - (X \pm TW_p)$$

in which $T(v \pm W_p)$ and $S(v \pm W_p)$ are the T and S functions in Table I.

Compute the range X and the time of flight T without considering the wind. Then from the first of the foregoing formulas find $v \pm W_p$, and from the second the desired value of ΔX.

226. *Another Method.*—Let ob, Fig. 159, represent the initial direction of the projectile and its velocity V. Let bc represent the velocity W_p of the wind component in the plane of fire, reversed in direction While the projectile moves from o to b the air particle b moves to the left a distance equal to bc. The direction of movement of the projectile relative to this particle of air is therefore oc, which is also the relative velocity, V', of the projectile. ϕ' is the relative inclination, and $\Delta\phi$ the relative change in inclination. Draw *cd* perpendicular to *ob*, and call *bd* ΔV. Then, using the upper signs only,

$$\Delta V = W_p \cos \phi \qquad (45)$$

$$V' = V \pm \Delta V \text{ (nearly)} \qquad (46)$$

$$V' \sin \Delta\phi = W_p \sin \phi \qquad (47)$$

$$\phi' = \phi \mp \Delta\phi \qquad (48)$$

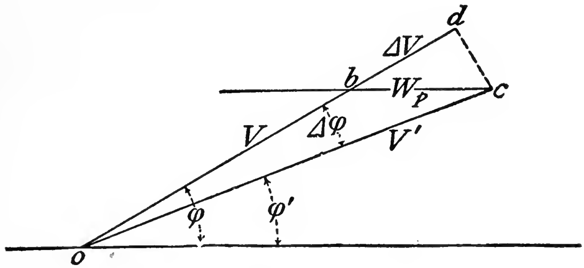


FIG. 159.

Referring to Fig. 160, let *b* represent the position of the gun, and *bd* the range X in calm air. In the head wind the range is reduced to *bc*. *cd* is therefore the variation in range due to the wind. While the projectile travels from *b* to *c* the air particle travels from *b* to *a*, the distance W_pT. *ac*, or X', is therefore the distance that separates the projectile and the air particle at the

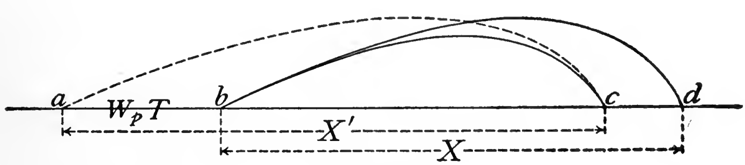


FIG. 160.

end of the time T; that is, it is the relative range of the projectile with respect to the air particle. The relative initial velocity of the projectile is as shown in Fig. 159, its velocity in a calm, V, increased by the component ΔV of the air's velocity in the direction of motion. $V' = V + \Delta V$ is therefore the initial velocity necessary to produce the relative range, and similarly $\phi' = \phi - \Delta\phi$ is the necessary angle of departure.

It is apparent from Fig. 160 that

$$cd = bd - bc = bd - (ac - ab)$$

or

$$cd = X - (X' - W_pT)$$

and calling cd with its sign changed ΔX, we have

$$\Delta X = X' - (X + W_pT)$$

Compute the relative range X' with the values V' and ϕ', using the formulas with Table II. While the projectile is traversing this relative range the air particle moves over a distance W_pT. The actual range traversed by the projectile is therefore $X' \mp W_pT$, and the variation in range due to the wind is

$$X - (X' \mp W_pT)$$

Changing the sign and rearranging, we get

$$\Delta X = X' - (X \pm W_pT) \qquad (49)$$

in which X and T are computed from V and ϕ without considering the wind.

The upper signs in the above equations apply when the wind blows toward the gun, the lower signs when it blows toward the target.

Application of Methods.—The first method of obtaining the variation in range due to wind is useful only when the angle of departure is small. The second method may be used in all problems of direct fire.

227. Problem 5.—What will be the effect of a one o'clock wind, blowing 30 miles an hour, on the range of the shot in Problem 1?

Velocity in miles per hour$\times 44/30$ = velocity in foot seconds.

$$W = 30 \times 44/30 = 44 \text{ f. s.} \qquad \alpha = 30°$$

$$W_p = W \cos \alpha = 38.1 \text{ f. s.}$$

From Problem 1: $\log C = 0.22185$, $X = 6000$, $V = 1700$, $T = 4.48$, $\phi = 2° \ 42'$

Therefore $W_pT = 170.7$, and $X + W_pT = 6170.7$

First Method. $V + W_p = 1738.1$

From Table I, $S(1738.1) = 6220.2 - .81 \times 43.8 = 6184.7$

$T(1738.1) = 2.508 - .81 \times .025 = 2.4878$

$\log T$	0.65128
$\log C$	0.22185
$\log T/C$	0.42943

T/C	2.6880
$T(1738.1)$	2.4878
$T(v+W_p)$	5.1758

From Table I,

$$v + W_p = 1112 + \frac{5.189 - 5.176}{.018} \times 2 = 1113.4$$

and $$S(1113.4) = 9860.0 - \frac{14}{20} \times 20.6 = 9845.6$$

$S(1113.4)$	9845.6		
$S(1738.1)$	6184.7		
log	3660.9		3.56359
		$\log C$	0.22185
log	6101.5		3.78544
$X + W_pT$	6170.7		
$\Delta X =$	-69.2 feet		

228. *Second Method.*—Find

Equation (45)	$\Delta V = 38.06$
(46)	$V' = 1738.1$
(47)	$\Delta\phi = 3'.6$
(48)	$\phi' = 2°\ 38'.4$
From $\sin 2\phi' = AC$	$A = 0.05521$
From Table II	$Z = 3671.5$
From $Z = X'/C$	$X' = 6119.1$
	$X + W_pT = 6170.7$
Equation (49)	$\Delta X = -51.6$ feet

Note the difference in the results of the two methods. **Neither** method is wholly satisfactory.

229. The Effect of Wind on Deviation.—The component of the wind perpendicular to the plane of fire, $W \sin \alpha$, is alone considered as producing deviation. The deviation due to the wind can only be determined by experiment for each kind of projectile.

The following formula for the deviation of 8, 10, and 12 inch projectiles is given, in another form, in the Coast Artillery Drill Regulations.

$$\text{Deviation (yards)} = [7.00000] \sin \alpha W(\text{m.p.h.}) \left(\frac{T(\text{sec})}{33000 + X(\text{yds.})} \right)^2 \quad (50)$$

in which W is the velocity of the wind in miles per hour,
α its angle with the plane of fire,
T is the time of flight in seconds,
X the range in yards.

Problem 6.—Compute the deviation of the shot in Problem 2 for a two o'clock wind blowing 20 miles an hour.

$W = 20$ m.p.h. $\quad \alpha = 60°$ $\quad W \sin \alpha = 17.32$ $\quad T = 12.46$

$$\text{Deviation} = [7.00000] 17.32 \left(\frac{12.46}{33000 + 8000} \right)^2 = 16 \text{ yards, left.}$$

230. The Danger Space.—The danger space is the horizontal distance over which an object of a given height will be struck. It is the horizontal length of those portions of the trajectory for which the ordinates are equal to and less than the given height. Usually the danger space at the further end of the range is alone considered.

The elements of the trajectory are assumed to be known.

Let abc, Fig. 161, be the known trajectory for the range X, and

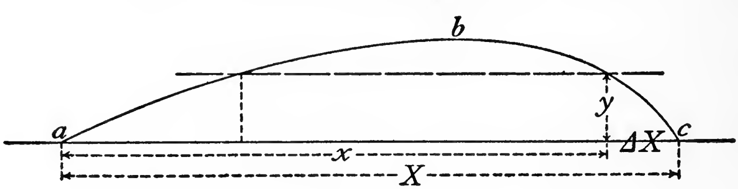


Fig. 161.

let y represent the height of the object for which the danger space is to be determined. The danger space for this height is evidently so much of the range as lies beyond the ordinate y. It is equal to the whole range minus the abscissa x corresponding to the ordinate y. Calling the danger space ΔX we obtain $\Delta X = X - x$.

The problem of determining the danger space therefore consists in finding the value of x corresponding to the given value of y and subtracting from the given range.

Substituting Cz for x in equation (37) and combining with equation (34) we obtain

$$(A-a)z = 2y \cos^2\phi / C^2 \tag{51}$$

in which A is the value of the function for the whole range X, and a the particular value of the same function for the abscissa x corresponding to the ordinate y. The elements of the whole range being known, and y given, the second member of the above equation is known, and A in the first member. There remain two quantities, a and z, to be determined from the equation. This is done by applying the method of double position.

231. METHOD OF DOUBLE POSITION.—Enter Table II with the known value of V. Inspect the table and find a value of Z which when substituted with its corresponding value of a from the A column in the first member of equation (51) will give to that member a value close to the known value of the second member. The difference between the first and second members is the error. Repeat this operation until two *successive values* of Z are found, Z_0 and Z_1, that give values for the first member, *one value greater and one less than the value of the second member.*

Let Z_0 and Z_1, Fig. 162, represent these values of Z; F_0 and F_1 the resulting values of the first member of equation (51); and S the known value of the second member. e_0 and e_1 will represent the errors obtained with F_0 and F_1. It is evident from the figure that the true value of Z lies between Z_0 and Z_1 and that its distance from the smaller trial value Z_0 is given by the proportion

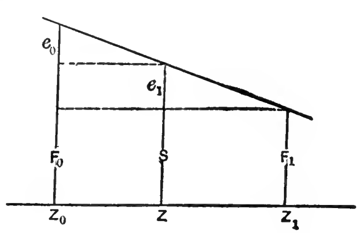


FIG. 162.

$$\frac{Z-Z_0}{Z_1-Z_0} = \frac{e_0}{e_0+e_1}$$

Solving for Z

$$Z = Z_0 + \frac{e_0}{e_0+e_1}(Z_1 - Z_0) \tag{52}$$

In the application of this method to equation (51) we are assuming that $(A-a)z$ varies proportionately with z between the values Z_0 and Z_1. This is not a true assumption, but the results are sufficiently approximate for practical use.

To make this demonstration general we may consider that z and $(A-a)$ in equation (51) represent any two functions, f and f', whose product is known. We then have

$$ff'=k$$

We may write either f or f' for Z in equation (52) and obtain the general formula

$$f=f_0+\frac{e_0}{e_0+e_1}(f_1-f_0) \qquad (53)$$

We may now, employing the method of double position, determine from equation (52) the value of z in (51), and from the equation $z=x/C$ we obtain the value of x corresponding to the given ordinate y. We then have for the danger space

$$\Delta X=X-x \qquad (54)$$

232. Problem 7.—What is the danger space, for an infantryman, in the 1000 yard trajectory of the service 0.30 caliber rifle; muzzle velocity, 2700 f. s.; bullet, 150 grains?

This assumes that the rifle is fired from the ground.

The height of a man is assumed at 5′ 8″ $=5.67$ feet $=y$.

The value of the coefficient of form c, in the ballistic coefficient, as determined by experiment for the 150 grain bullet is $c=0.5694$, *see foot-note.*

$$w=150/7000 \qquad d=0.3 \qquad V=2700 \qquad X=3000$$

The coefficient of form is determined for the small arms bullet by means of actual measurements of the velocity of the bullet at the ends of a long range, as, for instance, 500 yards. With the measured values of V and v, the latter corrected for the effect of wind if there is any, and the measured range, the value of C is determined from the equation $x=C\{S(v)-S(V)\}$ by means of ballistic Table I. The coefficient of form c is then obtained from the equation

$$C=\frac{\delta_1}{\delta}\frac{w}{cd^2}$$

For the projectiles of large guns the coefficient c is determined by means of measured values of ϕ, V, and X, see Problem 12.

The steps in the operation are indicated as follows:

$C = w/cd^2$ $\quad Z = X/C$ $\quad$ Table II, A $\quad \sin 2\phi = AC$

$(A-a)z = 2y\cos^2\phi/C^2$ $\quad x = zC$ $\quad \Delta X = X - x$

$C = w/cd^2$	log 7000	3.84510	
	log c	$\bar{1}.75542$	
	log d^2	$\bar{2}.95424$	
		2.55476	
	log 150	2.17609	
$Z = X/C$	log C	$\bar{1}.62133$	
	log X	3.47712	
	log Z	3.85579	$Z = 7174.5$

Table II, $\quad A = (0.06201) + 0.745 \times 158 = 0.063187$

$\sin 2\phi = AC$	log A	$\bar{2}.80063$	
	log C	$\bar{1}.62133$	
	log sin 2 ϕ	$\bar{2}.42196$	$2\phi = 1° 30'.8$
			$\phi = 45'.4$
$(A-a)z = 2y\cos^2\phi/C^2$	log $2y$	1.05461	
	log $\cos^2\phi$	$\bar{1}.99992$	
		1 05453	
	log C^2	$\bar{1}.24266$	
		1.81187	$(A-a)z = 64.844$

Applying the method of double position to find the values of z and a that will satisfy this equation, we find by inspection of Table II for $V = 2700$ that the value of $Z = 6500$ with the corresponding value of A, 0.05307, will when substituted in the last equation give a close approximation to 64.844.

With $Z = 6500$ we obtain

$$(0.063187 - 0.05307)6500 = 65.761$$

$$e_0 = 65.761 - 64.844 = 0.917$$

With $Z = 6600$

$$(0.063187 - 0.05449)6600 = 57.4$$

$$e_1 = 64.844 - 57.4 = 7.444$$

The results obtained with these values of Z are greater and less than 64.844.

Then from $Z=Z_0+\frac{e_0}{e_0+e_1}(Z_1-Z_0)$

$$z=6500+\frac{0.917}{0.917+7.444}\times 100=6511$$

$x=zC$

$$\begin{array}{ll} \log z & 3.81365 \\ \log C & \bar{1}.62133 \\ \hline \log x & 3.43498 \end{array} \qquad x=2722.6$$

$\Delta X=X-x$ $\qquad \Delta X=3000-2722.6=277.4$ ft. $=92.5$ yds.

For $V=2700$ we will also find that the value $Z=1122.7$ with the corresponding value of a will nearly satisfy the equation $(A-a)z=64.844$. This value of z gives $x=469.5$ feet, which is at once the danger space at the inner end of the trajectory, see Fig. 161.

233. The Danger Range.—When the danger space is continuous and coincides with the range it is called *the danger range.* Thus the danger range for an infantryman is the range at every point of which an infantryman would be struck. The maximum ordinate of the trajectory is therefore 5 feet 8 inches.

To determine the danger range we compute the horizontal trajectory whose maximum ordinate y_0 is given.

Combining equations (34) and (39) and making cos ϕ unity, since ϕ for all danger ranges is very small, we obtain

$$a_0'a_0''=2y_0/C^2 \qquad (55)$$

From this we determine a_0' by trial by the method of double position, using the A' and A'' columns of Table II. Since at the summit $a_0'=A$, see (38), with this value of a_0' we go to the A column of Table II for the given value of V and find the corresponding value of Z, from which the required X is obtained.

234. Problem 8.—What is the danger range, for a cavalryman, of the service rifle fired from the ground? The height of a cavalryman is assumed as 8 feet.

$$V=2700 \qquad \log C=\bar{1}.62133 \qquad y_0=8$$

The successive steps are indicated as follows:

$a_0'a_0''=2y_0/C^2$ Table II, Z $X=ZC$

$a_0'a_0''=2y_0/C^2$

$\log 2y_0$	1.20412
$\log C^2$	$\bar{1}.24266$
$\log a_0'a_0''$	1.96146

$a_0'a_0''=91.508$

By inspection of Table II for $V=2700$ we find that the product of a_0' and a_0'' for $Z=3400$ will give a close approximation.

For $Z=3400$ $a_0'a_0''=0.0467\times1938=90.504$

$$e_0=91.508-90.504=1.004$$

For $Z=3500$ $a_0'a_0''=0.0488\times2002=97.697$

$$e_1=97.697-91.508=6.189$$

The first product obtained is less than 91.508 and the second product greater. In $f=f_0+\frac{e_0}{e_0+e_1}(f_1-f_0)$ write a_0' for f; 0.0467, the smaller trial value of a_0', for f_0; and 0.0488 for f_1.

$$a_0'=(0.0467)+\frac{1.004}{1.004+6.189}\times21=0.04699$$

or it may sometimes be more convenient to find the value of Z and then the value of a_0'. Thus

$$Z=3400+\frac{1.004}{1.004+6.189}\times100=3414$$

and $$a_0'=(0.0467)+.14\times21=0.04699$$

Using this value of a_0' in the A column, we obtain

$$Z=6000+\frac{4699-4634}{129}\times100=6050.4$$

$X=ZC$

$\log Z$	3.78178
$\log C$	$\bar{1}.62133$
$\log X$	3.40311

$X=2529.9$ ft. $=843.3$ yds.

The trajectory for this range is, at its highest point, 8 feet from the ground. A cavalryman at any point of the range would therefore be struck.

235. Curved Fire.—Problems involving angles of departure less than 30 degrees, and initial velocities less than 825 f. s., are solved by means of the first part of Table II, pages 14 to 16, Ballistic Tables. The formulas to be used are collected on page VIII of the tables under the heading "Formulas to be used with the first part of Table II." They will also be found under the heading *Curved Fire* on page 377, *ante*.

For velocities less than 825 f. s. the resistance of the air is assumed to vary as the square of the velocity, or, as it is called, according to the *Quadratic Law of Resistance*. Under this law the formulas for direct fire are capable of modification into the forms that we are now considering.

It may be shown that under the quadratic law of resistance the function A, for the same value of $Z=X/C$, that is, for the same range and projectile, will vary for different values of V in the ratio V_1^2/V^2. If therefore we obtain the values of A with the value V_1 and all the necessary values of Z, we can pass by means of the above ratio to the value of A for any other velocity. The value $V_1=800$ was used in calculating the part of Table II that refers to velocities less than 825 f. s.

The value of $\sin 2\phi$, see equation (34), calculated for $V_1=800$ becomes for any other velocity

$$\sin 2\phi = AC\left(\frac{800}{V}\right)^2 = [5.80618]\frac{AC}{V^2} \qquad (56)$$

the form in which it appears among the formulas we are considering.

Under the quadratic law the other functions vary according to different ratios of V_1 and V, as shown by the formulas in which they appear. Under this law the function B' becomes independent of the muzzle velocity, and therefore V does not appear in the formula for $\tan \omega$.

Correction for Altitude.—In curved fire the correction of the ballistic coefficient for altitude is made by the same process as in direct fire, but using the value of $\sin 2\phi$ given by equation (56) instead of that given by equation (34).

236. Problem 9.—A shot is fired from the 4.7 inch siege howitzer at a target 4000 yards distant; $w=60$ lbs., $V=820$ f. s., barometer 29″.6, thermometer 63°. Correct the ballistic coefficient once for altitude and find the angle of departure and the time of flight.

The process of correcting for altitude may be indicated as follows:

$$C=f\frac{\delta_1}{\delta}\frac{w}{cd^2} \qquad Z=X/C \qquad \text{Table II, } A,\ a_0'' \qquad \sin 2\phi=[5.80618]AC/V^2$$

$$y_0=a_0''C\tan\phi \qquad \log(\log f)=\log y_0+\bar{5}.01765$$

Table VI, $\delta_1/\delta=1.029-0.6(1.029-0.994)=1.008$

$C=f\frac{\delta_1}{\delta}\frac{w}{cd^2}$

$f=1 \qquad c=1$

	$\log \delta_1/\delta$	0.00346	
	$\log w$	1.77815	
		1.78161	
	$\log d^2$	1.34420	
	$\log C_1$	0.43741	(1st approximation)
$Z=X/C$	$\log X$	4.07918	
	$\log Z$	3.64177	$Z=4383$

Table II, $A_1=(0.24821)+.83\times 662=0.25370$

$A=a_0' \qquad$ With $a_0'=0.25370$ find a_0''.

$$\frac{Z-\overset{2200}{Z_0}}{100}=\frac{2537-2456}{123}=.66$$

$$a_0''=1138+.66\times 53=1173$$

$\sin 2\phi=[5.80618]AC/V^2$

$\log A$	1.40432
$\log C_1$	0.43741
const.	5.80618
	5.64791

$\log V^2$	5.82762	
$\log \sin 2\phi$	$\bar{1}.82029$	$2\phi_1 = 41° \; 23'.2$
		$\phi_1 = 20° \; 41'.6$
$\log a_0''$	3.06930	
$\log C_1$	0.43741	
$\log \tan \phi_1$	$\bar{1}.57719$	
$\log y_{01}$	3.08390	
$\log (\log f) = \log y_0 + \bar{5}.01765$	$\bar{5}.01765$	
$\log (\log f_1)$	$\bar{2}.10155$	
$\log f_1$	0.01263	
$\log C_1$	0.43741	
$\log C_c$	0.45004	

We will use this as log C in determining the angle of departure and time of flight.

$Z = X/C$ Table II, A, T'

$\sin 2\phi = [5.80618] AC/V^2$ $T = [2.90309] CT'/V \cos \phi$

$Z = X/C$ $\log X$	4.07918	
$\log C$	0.45004	
$\log Z$	3.62914	$Z = 4257.4$

Table II, $A = (0.24163) + .574 \times 658 = 0.24541$

$\sin 2\phi = [5.80618] AC/V^2$

$\log A$	$\bar{1}.38989$	
$\log C$	0.45004	
const.	5.80618	
	5.64611	
$\log V^2$	5.82762	
$\log \sin 2\phi$	$\bar{1}.81849$	$2\phi = 41° \; 10'.7$
		$\phi = 20° \; 35'.4$

$T = [2.90309] CT'/V \cos \phi$

Table II, $T' = (5.801) + .574 \times 152 = 5.8882$

log T'	0.76998
log C	0.45004
const.	2.90309
	4.12311
log $V \cos \phi$	2.88514
log T	1.23797

$T = 17.3$ seconds

237. **High Angle Fire.**—Problems in high angle fire are solved by means of Table IV. This table was computed under the quadratic law of resistance and is practically a range table, for velocities less than 825 feet, for a projectile whose ballistic coefficient is unity. To make it applicable to other projectiles the tabular numbers involve the value of the ballistic coefficient with the values of the different elements. Therefore with C known, and applied as indicated in the headings of the columns, we may, with any other known element of the trajectory in addition to the elevation, obtain from the different columns the values of the remaining elements.

Thus C, ϕ, and V being known, find $V/\sqrt{C}$ and take out of Table IV, for the particular value of ϕ, the values of X/C, $T/\sqrt{C}$, etc., corresponding to $V/\sqrt{C}$ as obtained. X, T, etc. may then be obtained. If ϕ is not a tabular value, solve the problem for the tabular values of ϕ on either side of the given value and interpolate between the results.

To correct for altitude use the formulas log (log f) given at the head of each table. The value of the maximum ordinate is also there given in the terms of the range.

The Coefficient of Reduction.—While the quadratic law of resistance applies to velocities less than 825 f. s., Table IV may be used for the higher velocities now obtained from our mortars by the introduction of the coefficient of reduction c into the ballistic coefficient. Compensation may thus be made for the errors arising from the use of the table for higher velocities. The coefficient of reduction is actually a quantity required to make the results obtained from the formulas and Table IV agree with the results obtained in experiment.

The values of c for the 1046 lb. mortar projectile have been calculated from actual firings for different ranges and angles of elevation. The determinations were made from firings with the 12 inch cast iron steel hooped mortar. The values of c which are given in the following table therefore apply only to projectiles fired with the velocities used in this mortar. In the steel mortar, model 1890, higher velocities are attained.

The method employed in the calculation of the coefficient of reduction is shown in Problem 12.

VALUES OF THE COEFFICIENT OF REDUCTION, c, FOR THE 1046 LB. PROJECTILE IN THE 12 INCH MORTAR; DETERMINED FROM ACTUAL FIRINGS.

Elevation, Degrees.	Range in Yards.					
	3000	4000	5000	6000	7000	8000
45	1.59	2.11	1.93	1.76	1.53	1.25
46	1.77	2.20	1.94	1.76	1.55	1.28
47	1.93	2.28	1.94	1.77	1.57	1.32
48	2.07	2.34	1.95	1.78	1.59	1.36
49	2.19	2.38	1.95	1.79	1.61	1.40
50	2.29	2.41	1.96	1.80	1.63	1.44
51	2.39	2.42	1.97	1.81	1.66	1.48
52	2.48	2.42	1.98	1.82	1.68	1.52
53	2.56	2.42	1.99	1.83	1.71	1.56
54	2.62	2.42	1.99	1.84	1.74	1.61
55	2.66	2.42	2.00	1.85	1.77	1.65
56	2.65	2.41	2.01	1.86	1.79	1.70
57	2.64	2.40	2.02	1.87	1.82	1.75
58	2.62	2.38	2.04	1.88	1.85	1.80
59	2.59	2.37	2.05	1.89	1.88	1.85
60	2.56	2.35	2.07	1.90	1.91	1.91
61	2.53	2.34	2.09	1.92	1.95	1.97
62	2.49	2.32	2.11	1.94	1.99	2.04
63	2.45	2.30	2.13	1.97	2.04	2.11
64	2.41	2.28	2.15	2.01	2.09	2.18
65	2.37	2.26	2.17	2.07	2.15	2.25

238. Problems in High Angle Fire.—When C, ϕ, and V or X are given, to determine the remaining elements.

I. *Given C, V, and X, to determine ϕ and the other elements.*

METHOD. 1. With the given data find $C_1 = w/d^2$, $V/\sqrt{C_1}$, and X/C_1.

2. With the value of $V/\sqrt{C_1}$ enter Table IV and find by inspection in consecutive tables two values of X/C, one value greater and one value less than the trial value already determined.

3. Assume the lesser of the elevations for the two tables as a first trial value of ϕ, determine f from the formula at the top of the table for this value of ϕ and compute C_c from $C_c = fw/cd^2$.

4. Then, using the value of C_c as C, redetermine $V/\sqrt{C}$ and X/C.

5. With these values reenter Table IV and redetermine as before a second trial value of ϕ.

6. With this value of ϕ and the given value of X compute V.

7. If the computed value be greater than the given value, recompute with the next lesser value of ϕ; if less, recompute with the next greater value. The given value of V will usually lie between the two values thus computed, if not continue the process until this result is attained.

8. Then interpolate for ϕ, assuming it to vary directly with V.

9. To find the other elements, T, ω, and v_ω, use the tables for the values of ϕ on each side of the value just determined. Find the values of these elements from each table, and interpolate between the values so determined for the values corresponding to the determined value of ϕ.

Problem 10.—A projectile weighing 1046 lbs. is to be fired from a 12 inch mortar, model 1888, to reach a target at a range of 7180 yards. Assuming the muzzle velocity to be 950 f. s., determine the angle of elevation required.

$$w = 1046 \qquad d = 12 \qquad V = 950 \qquad X = 21540$$

1. $C_1 = w/d^2$ $\qquad \log C_1 = 0.86117$

$$V/\sqrt{C_1} = 352.48 \qquad X/C_1 = 2965.4$$

2. From Table IV,

for $\phi = 59°$ and $V/\sqrt{C} = 352.48$ $\qquad X/C = 2971.7$

$\phi = 60°$ $\qquad V/\sqrt{C} = 352.48$ $\qquad X/C = 2914$

3. Assume $\phi = 59°$ $\qquad$ Page 402, $c = 1.88 - \dfrac{180}{1000} \times .03 = 1.8746$

$\log(\log f) = \log X - 5.32914$	$\log X$	4.33325
	const.	5.32914
	$\log(\log f)$	$\bar{1}.00411$
	$\log f$	0.10095
$C = fw/d^2c$	$\log w/d^2$	0.86117
		0.96212
	$\log c$	0.27291
	$\log C_c$	0.68921
4.	$\log V$	2.97772
	$\log \sqrt{C_c}$	0.34461
	$\log V/\sqrt{C}$	2.63311 $V/\sqrt{C} = 429.65$
	$\log X$	4.33325
	$\log C_c$	0.68921
	$\log X/C$	3.64404 $X/C = 4406$

5. From Table IV,

for $\phi = 55°$ and	$V/\sqrt{C} = 429.65$	$X/C = 4436.1$
$\phi = 56°$	$V/\sqrt{C} = 429.65$	$X/C = 4375.1$
Computed	$V/\sqrt{C} = 429.65$	$X/C = 4406.0$

6. Assume $\phi = 55°$ $c = 1.77 - .18 \times .12 = 1.7484$

$\log(\log f) = \log X - 5.40257$	$\log X$	4.33325
	const.	5.40257
	$\log(\log f)$	$\bar{2}.93068$
	$\log f$	0.08525
$C = fw/d^2C$	$\log w/d^2$	0.86117
		0.94642
	$\log c$	0.24264
	$\log C_c$	0.70378
	$\log X$	4.33325
	$\log X/C$	3.62947 $X/C = 4260.6$

Table IV, $V/\sqrt{C}=410+\frac{158.6}{170}\times 10=419.33$

$\log V/\sqrt{C_c}$	2.62256	
$\log \sqrt{C_c}$	0.35189	
$\log V$	2.97445	$V=942.87$

7. Assuming $\phi=56°$ $c=1.79-.18\times .09=1.7738$

$\log(\log f)=\log X-5.38029$

$\log X$	4.33325	
const.	5.38029	
$\log(\log f)$	$\bar{2}.95296$	
$\log f$	0.08974	
$\log w/d^2$	0.86117	
	0.95091	
$\log c$	0.24890	
$\log C_c$	0.70201	
$\log X$	4.33325	
$\log X/C$	3.63124	$X/C=4278$

$C=fw/d^2c$

Table IV, $V/\sqrt{C}=420+\frac{65}{168}\times 10=423.87$

$\log V/\sqrt{C_c}$	2.62723	
$\log \sqrt{C_c}$	0.35101	
$\log V$	2.97824	$V=951.13$

8. For $V=942.87$, $\phi=55°$, and for $V=951.13$, $\phi=56°$. Therefore for $V=950$

$$\phi=55°+\frac{713}{826}\times 60'=55°\ 51'.8$$

9. To obtain the values of T, ω, and v_ω, corresponding to $\phi=55°\ 51'.8$, enter Table IV for $\phi=55°$ and $\phi=56°$, using as arguments the values of $V/\sqrt{C}$ obtained above in steps 6 and 7.

For $\phi=55°$:

$V/\sqrt{C}=419.33$

$T/\sqrt{C}=19.81+0.93\times 0.44=20.219$

$\omega=58°\ 59'+0.93\times 10'=59°\ 8'.3$

$v_\omega/\sqrt{C}=355+0.93\times 6=360.58$

For $\phi=56°$:

$V/\sqrt{C}=423.87$

$T/\sqrt{C}=20.656$

$\omega=60°\ 7'.9$

$v_\omega/\sqrt{C}=364.73$

From these values we derive, using the values of $\sqrt{C}$ as determined in steps 6 and 7,

$$T=45.462 \qquad T=46.351$$
$$\omega=59^\circ\ 8'.3 \qquad \omega=60^\circ\ 7'.9$$
$$v_\omega=810.76 \qquad v_\omega=818.43$$

Interpolating between these values, that correspond to $\phi=55^\circ$ and $\phi=56^\circ$, we find for $\phi=55^\circ\ 51'.8$

$$T=45.46+\frac{51.8}{60}(46.35-45.46)=46.2 \text{ seconds}$$

$$\omega=59^\circ\ 8'.3+\frac{51.8}{60}\times 59'.6=59^\circ\ 59'.8$$

$$v_\omega=810.8+\frac{51.8}{60}\times 7.61=817.4 \text{ foot seconds}$$

239. II. *Given C, V, and ϕ, to determine X and the other elements.*

METHOD. To determine the value of the coefficient c from the table on page 402 we must know both ϕ and X. In this problem X is unknown.

1. We will therefore first determine from Table IV an approximate value of X, designated X_1, using for this purpose $C_1=w/d^2$ and the tabular value of ϕ next greater than the given value.

2. Take from the table for c the value of c corresponding to the value X_1 and to the value of ϕ used in step 1. Call this value c_1.

3. Determine a second approximate value for the ballistic coefficient $C_2=w/c_1d^2$. Correct for altitude by means of Table IV, using ϕ as in step 1; and with the corrected coefficient, C_3, determine a corrected range, X_2. This corrected range will be sufficiently close to the true range to enable us to obtain approximately the correct values of c from the table. This has been the object of the foregoing steps.

4. With the corrected range, X_2, and the tabular values of ϕ on each side of the given value take new values of c from the table. Call these values c_2 and determine with them two new values for C, designated $C_4=w/c_2d^2$.

5. By means of Table IV, for the values of ϕ on each side of the given value, correct both values of C_4 for altitude. Call the resulting values C_5.

6. Using the values C_5 as C find the corresponding values of $V/\sqrt{C}$ and then, from Table IV, the corresponding values of X and the other elements.

7. Interpolate between the values thus found for the values corresponding to the given value of ϕ.

Problem 11.—Assume $d=12$ inches, $w=1046$ lbs.
$\phi=55° 40'$ $V=950$ f. s.

Determine X, T, ω, and v_ω.

1. $C_1=w/d^2=[0.86117]$

$$\begin{array}{rl} \log V & 2.97772 \\ \log \sqrt{C_1} & 0.43059 \\ \hline \log V/\sqrt{C_1} & 2.54713 \quad V/\sqrt{C_1}=352.48 \end{array}$$

With this value we find from Table IV, for $\phi=56°$,

$$X/C_1=3084+.25\times 156=3123$$

$$\begin{array}{rl} \log X/C_1 & 3.49457 \\ \log C_1 & 0.86117 \\ \hline \log X_1 & 4.35574 \quad X_1=22685 \text{ ft.} \\ & \qquad\qquad =7561.7 \text{ yds.} \end{array}$$

2. From the table of values of c, with $X=7562$ yds. and $\phi=56°$,

$$c_1=1.79-.562\times 0.9=1.739$$

3. $C_2=w/c_1d^2=C_1/c_1=[0.62087]$

For use in Table IV,

$$\begin{array}{rl} \log V & 2.97772 \\ \log \sqrt{C_2} & 0.31044 \\ \hline \log V/\sqrt{C_2} & 2.66728 \quad V/\sqrt{C_2}=464.81 \end{array}$$

From Table IV, for $\phi=56°$,

$$X/C_2=4890+.48\times 173=4973$$

$$\begin{array}{rl} \log X/C_2 & 3.69662 \\ \log C_2 & 0.62087 \\ \hline \log X & 4.31749 \end{array}$$

$\log(\log f) = \log X - 5.38029$		5.38029
	$\log(\log f)$	$\bar{2}.93720$
	$\log f$	0.08654
	$\log C_2$	0.62087
	$\log C_3$	0.70741
Determine $V/\sqrt{C_3}$	$\log V$	2.97772
	$\log\sqrt{C_3}$	0.35371
	$\log V/\sqrt{C_3}$	2.62401 $V/\sqrt{C_3}=420.74$

From Table IV, for $\phi=56°$,

$$X/C_3 = 4213 + .07\times 168 = 4224.8$$

$\log X/C_3$	3.62581
$\log C_3$	0.70741
$\log X_2$	4.33322 $X_2 = 21539$ ft. $= 7179.7$ yds.

4. Since, in mortar fire, X will vary but little for a variation of one degree in ϕ, we may without material error use this value X_2 in the determination of c for 55° as well as for 56°.

Therefore, from the table of values of c, with $X=7180$ yds. and

$\phi=55°$,	$\phi=56°$,
$c_2=1.77-.18\times .12=1.748$	$c_2=1.79-.18\times .09=1.774$
$C_4=w/c_2d^2=C_1/c_2=[0.61863]$	$C_4=[0.61222]$

5. For use in Table IV,

$\log V$	2.97772	$\log V$	2.97772
$\log\sqrt{C_4}$	0.30932	$\log\sqrt{C_4}$	0.30611
$\log V/\sqrt{C_4}$	2.66840	$\log V/\sqrt{C_4}$	2.67161
$V/\sqrt{C_4}=466.02$		$V/\sqrt{C_4}=469.47$	

From Table IV,

$X/C_4 = 4959 + .6 \times 176 = 5064.6$		$X/C_4 = 5060.4$	
$\log X/C_4$	3.70455	$\log X/C_4$	3.70418
$\log C_4$	0.61863	$\log C_4$	0.61222
$\log X$	4.32318	$\log X$	4.31640
const.	5.40257	const.	5.38029
$\log (\log f)$	$\bar{2}.92061$	$\log (\log f)$	$\bar{2}.93611$
$\log f$	0.08329	$\log f$	0.08632
$\log C_4$	0.61863	$\log C_4$	0.61222
$\log C_5$	0.70192	$\log C_5$	0.69854

6. For use in Table IV,

$\log V$	2.97772	$\log V$	2.97772
$\log \sqrt{C}$	0.35096	$\log \sqrt{C}$	0.34927
$\log V/\sqrt{C}$	2.62676	$\log V/\sqrt{C}$	2.62845
$V/\sqrt{C} = 423.41$		$V/\sqrt{C} = 425.06$	

From Table IV,

$X/C = 4272 + .34 \times 170 = 4329.8$	$X/C = 4298.7$
$T/\sqrt{C} = 20.25 + .34 \times .43 = 20.396$	$T/\sqrt{C} = 20.704$
$\omega = 59° \; 9' + .34 \times 10' = 59° \; 12'.4$	$\omega = 60° \; 9'.1$
$v_\omega/\sqrt{C} = 361 + .34 \times 7 = 363.38$	$v_\omega/\sqrt{C} = 365.57$

From the above values we derive

$X = 21797$	$X = 21472$
$T = 45.763$	$T = 46.272$
$\omega = 59° \; 12'.4$	$\omega = 60° \; 9'.1$
$v_\omega = 815.3$	$v_\omega = 817.03$

7. Interpolating between these values, that correspond to $\phi = 55°$ and $\phi = 56°$, we find for $\phi = 55° \; 40'$

$$X = 21580 \text{ ft.} = 7193.3 \text{ yards}$$
$$T = 46.1 \text{ seconds}$$
$$\omega = 59° \; 46'.9$$
$$v_\omega = 816.5 \text{ foot seconds}$$

It will be seen that the approximate range, $X_2=7179.7$ yards, used in determining the value of c, is very close to the true range, 7193.3 yards.

240. Calculation of the Coefficient of Reduction.—A recent addition to Table IV, printed in the *Journal of the United States Artillery*, Jan.–Feb., 1905, provides a simple method of computing the coefficient of reduction for any projectile, when ϕ, V, and X are determined from actual firings.

A column containing values of V^2/X, obtained by combining the two columns $V/\sqrt{C}$ and X/C, is added to the table. With ϕ and V^2/X as arguments, we may obtain C from the value in the column $V/\sqrt{C}$. The value of C thus obtained is the complete value, $C=f\frac{\delta_1}{\delta}\frac{w}{cd^2}$. Determine f from the formula at the head of the table, and δ_1/δ from Table VI. c is then readily determined.

When the additional column giving the values of V^2/X is not at hand, the value of $V/\sqrt{C}$ corresponding to any value of V^2/X may be readily determined from Table IV by trial. Square the values in the $V/\sqrt{C}$ column and divide by the corresponding values in the X/C column until two values of V^2/X are found, one value greater and one less than the given value. By interpolation the value of $V/\sqrt{C}$ corresponding to the given value of V^2/X may then be found.

241. Problem 12.—The range of the 1046 lb. projectile from the 12 inch steel mortar, model 1890 MI, is limited to 11,215 yards. The muzzle velocity of the projectile is 1150 feet, the velocity being limited by the requirement that the maximum pressure shall not exceed 33,000 lbs. In order to extend the range of the mortar a projectile weighing 824 lbs. is provided, for which, without exceeding the allowed pressure, the muzzle velocity is increased to 1325 feet and the range to 12,713 yards.

Compute the value of the coefficient of reduction, c, for that projectile with the following data obtained in experiment.

$$d=12 \quad w=824 \quad V=1325 \quad \phi=45° \quad X=38{,}139 \text{ feet}$$

Barometer, 30″.5 Thermometer, 65° F.

The process of solution is indicated as follows:

V^2/X Table IV, C from $V/\sqrt{C}$, $\log(\log f)=\log X-$const. log.

$$c=f\frac{\delta_1}{\delta}\frac{w}{Cd^2}$$

From the given data, $V^2/X=46.03$

From Table IV we find with this value

$$V/\sqrt{C}=639.25$$

$\log V$	3.12222
$\log V/\sqrt{C}$	2.80567
$\log \sqrt{C}$	0.31655
$\log C_c$	0.63310

$\log(\log f)=\log X-5.55099$

$\log X$	4.58137
const.	5.55099
$\log(\log f)$	$\bar{1}.03038$
$\log f$	0.10725
$\log \delta_1/\delta$	$\bar{1}.99211$
$\log w$	2.91593
	3.01529
$\log Cd^2$	2.79146
$\log c$	0.22383

$c=1.6743$

242. Perforation of Armor.—The following empirical formulas are used by the Ordnance Department, U. S. Army, for calculating perforation of the earlier Krupp armor.

Uncapped projectiles,

$$t^{1.5}=[\bar{7}.16459]\frac{wv^2}{d}$$

Capped projectiles,

$$t^{0.7}=[\bar{4}.84060]\frac{w^{0.5}v}{d^{0.75}}$$

in which t=thickness perforated, in inches;
w=weight of projectile, in pounds;
v=striking velocity, in foot seconds;
d=diameter of projectile, in inches.

The following formula has been proposed by the Ordnance Board for capped projectiles against thin plates:

$$\left(\frac{t}{\sin \alpha}\right)^{0 \cdot 7} = [\bar{4}.92665]\frac{w^{0 \cdot 5}v}{d^{0 \cdot 75}}$$

in which α is the angle of impact, that is to say, the angle between the axis of the projectile and the face of the plate. This formula is applicable to tempered nickel steel plates from 3 to $4\frac{1}{2}$ inches thick, and for angles of impact varying from normal to 50 degrees.

The following formulas are used by the Bureau of Ordnance, U. S. Navy, for calculating the perforation of face hardened armor without backing. They apply to Harvey armor only. No formula satisfactory to the Bureau has yet been developed for the perforation of the most modern Krupp armor.

Uncapped projectiles,

$$v = [3.34512]\frac{d^{\frac{1}{2}}t^{\frac{3}{4}}}{w^{\frac{1}{2}}}$$

Capped projectiles,

$$v = [3.25312]\frac{d^{\frac{1}{2}}t^{\frac{4}{5}}}{w^{\frac{1}{2}}}$$

in which the letters represent the same quantities as in the formulas above.

The formula for capped projectiles is tentative only.

Range Tables.—The elements of the trajectories for different ranges are calculated for each gun in the service and embodied with other information in a range table. The standard muzzle velocity and standard weight of projectile are used in the construction of the table for each gun. The range is the argument in the table, the successive entries in the range column differing from each other by 200 yards. The perforation of armor, and the logarithm of the ballistic coefficient corrected for altitude at standard temperature and pressure, are entered at intervals of 1000 yards.

The construction of range tables will be understood from the following data taken from the first line of the range table for the 10-inch rifle.

Muzzle Velocity, 2250 f. s. *Projectile, capped, 604 lbs.*

Range, X	1000 yards
Angle of departure, ϕ	0° 34′.1
Change in elevation for 10 yds. in range	0′.4
Time of flight, T	1.37 seconds
Angle of fall, ω	0° 36′
Slope of fall	1 on 95
Maximum ordinate, y_0	8 feet
Striking velocity, v	2116 f. s.
Perforation of Krupp armor, impact normal	13.3 inches
" " " " 30° with normal	11.2 inches
Ballistic coefficient, log C	0.78112

Curvature of the Earth.—The angle of elevation is affected by the curvature of the earth about 15 seconds of arc for each 1000 yards of range.

The amount of curvature, in feet, is approximately two thirds the square of the range in miles, or

$$\text{Curvature (ft.)} = [\bar{7}.33289]X^2(\text{yds.}) \tag{59}$$

ACCURACY AND PROBABILITY OF FIRE.

243. Accuracy.—The accuracy of a gun at any range and under any given conditions of loading and firing is determined as follows.

A number of shots are fired under the given conditions, care being exercised to make the circumstances of all the rounds as nearly alike as possible. The point of fall of each shot is plotted on a chart or marked on the target when practicable. The target may be either horizontal or vertical. We will assume a vertical target.

The coordinates x and y of each shot-mark, or impact, are measured with respect to two rectangular axes X and Y drawn through an assumed origin conveniently placed. The sum of the abscissas divided by the number of shots, which is the mean abscissa, and the sum of the ordinates divided by the same number, the mean ordinate, are the coordinates of the mean point of fall, called the *center of impact.*

A representation of a target of 8 shots from the 10-inch rifle is shown in Fig. 163. The range was 3000 yards. The center of impact is at the center of the crossed circle.

The distance, in the direction of the axis of Y, of any impact from the center of impact is the vertical deviation for the shot. The deviation is plus if the shot-mark lies above the center of impact, and minus if below. The distance of the shot-mark from the center of impact in the direction of the axis of X is the lateral deviation of the shot, plus if to the right, minus if to the left.

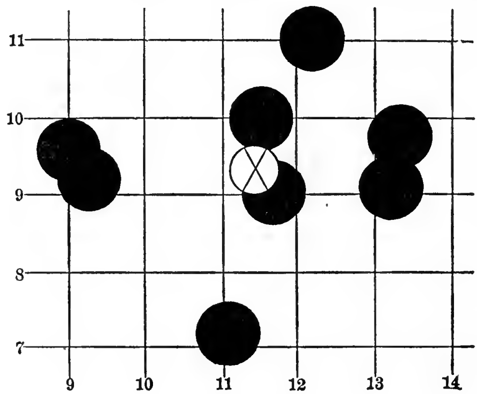


FIG. 163.

The numerical sum of the horizontal deviations divided by the number of shots is the mean horizontal deviation. The mean vertical deviation is similarly obtained from the numerical sum of the vertical deviations.

The actual distance of each shot from the center of impact is the *absolute deviation* for the shot, and the mean of the absolute deviations is the *mean absolute deviation* for the group.

The mean absolute deviation is usually computed from the mean horizontal and vertical deviations by taking the square root of the sum of their squares. The value computed in this more convenient way differs slightly from the mean of the absolute deviations.

By comparing the mean absolute deviations of different groups of shots we may arrive at the comparative accuracy of different guns or of the same gun under different conditions of loading or firing.

The measure of the ability of a gunner is the absolute distance of the center of impact of the group of shots from the point of the target aimed at.

244. Example.—In a test of the 10-inch rifle for accuracy 8 shots were fired at a vertical target distant 3000 yards. The coordinates of the shots measured from a point on the target, see Fig. 163, are given below. Find the center of impact and the mean absolute deviation.

No. of Shot.	Coordinates, Feet.		Deviations.	
	Horizontal.	Vertical.	Horizontal.	Vertical.
1	12.20	11.00	0.80	1.65
2	11.50	9.90	0.10	0.55
3	13.30	9.75	1.90	0.40
4	11.70	9.10	0.30	0.25
5	13.20	9.15	1.80	0.20
6	9.00	9.55	2.40	0.20
7	11.05	7.15	0.35	2.20
8	9.25	9.20	2.15	0.15
8	91.20	74.80	9.80	5.60
	11.40	9.35	1.23	0.70

The coordinates of the center of impact are: horizontal, 11.40 feet; vertical, 9.35 feet.

The mean deviations from the center of impact are: horizontal, 1.23 feet; vertical, 0.70 feet.

$$\text{The mean absolute deviation} = \sqrt{\overline{1.23}^2 + \overline{0.70}^2} = 1.42 \text{ feet.}$$

245. Probability of Fire.*—Suppose that a large number of shots have been fired at a target, under conditions as nearly alike as possible, and that the center of impact of the group of shot-marks on the target has been determined.

If we count the number of impacts that lie within any given distance from the center of impact and divide this number by the

* The greater part of the discussion of the subject of Probability of Fire follows the method set forth by Professor Philip R. Alger, U. S. Navy, in an article appearing in the *Proceedings of the U. S. Naval Institute*, Whole No. 108, 1903, and in the *Journal of the United States Artillery*, March–April, 1904.

whole number of shots, the resulting fraction will express the probability that any shot will fall within the given distance.

Probability is thus always expressed as a fraction of unity. If an event may happen in a ways and may fail in b ways, the probability of its happening is $a/(a+b)$, and of its failing to happen, $b/(a+b)$. The sum of these two fractions, unity, represents the certainty that the event will either happen or fail. Unity therefore indicates certainty.

By examination of many groups of shots we learn that as we approach the center of impact the impacts become more numerous, also that both the vertical and horizontal deviations are as likely to be on one side of the center of impact as on the other.

We also learn that the vertical and horizontal deviations are entirely independent of each other, and that any vertical deviation is just as likely to occur with one horizontal deviation as with another. This makes it necessary in considering probabilities that we consider the horizontal and vertical deviations separately.

Let O, Fig. 164, represent the center of impact of any group of

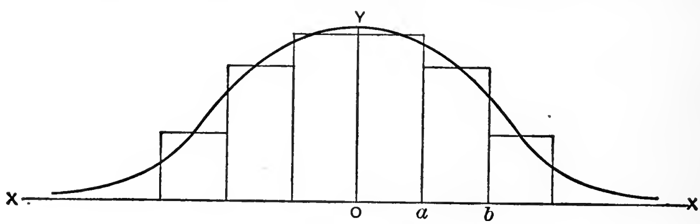


FIG. 164.

shots used as a criterion. Considering only lateral deviations, lay off on the axis of X successive distances representing lateral deviations.

Count the number of impacts on the target that lie within the distance Oa to the right of the center of impact. Erect at a an ordinate of such length that the area of the rectangle between the ordinate and the axis of Y represents the number of impacts found within the distance.

Proceed in the same manner for the distance ab and for the other distances represented by the other divisions of the axis of X.

The area of any rectangle divided by the area of all the rectangles will then be the probability that any shot will lie within the limits of deviation between the limiting ordinates. As the total area of all the rectangles is a constant, the probabilities with respect to deviations within any limits represented by different portions of the axis of X are proportional to the rectangles erected on those portions.

246. Probability Curve.—If we consider that a very large number of shots have been fired and make the rectangles very small, so that the base of each becomes dx, we obtain the area in the figure bounded by the curve and the axis of X.

The curve is called the *probability curve* and the area under any part of it divided by the whole area is the probability that any shot will deviate from the center of impact within the limits between the limiting ordinates.

If we consider the whole area under the curve as unity, the area under any part of the curve will represent at once the probability of a deviation within the limits between the limiting ordinates.

As the ordinates may be considered as areas infinitely small in width any ordinate will represent the probability of a specific deviation represented by the abscissa; that is, it will represent the probability that a shot will fall *at a specific distance* on either side of the center of impact. The area of the ordinate being infinitely small the chance that a shot will have any specific deviation is infinitesimal and not worthy of consideration. If we were dealing with events that could happen only in a finite number of ways, each ordinate would be an area that would have a finite relation to the sum of all the ordinates or areas, and would then represent the probability of the happening of a particular event.

Characteristics.—The curve is symmetrical with respect to the axis of Y, since the probability is the same for equal deviations on either side. The ordinate has its greatest value at the center of impact, since the center of impact is the mean position of all the shots and the probability of the deviations increases continuously as the deviations are less. The curve is theoretically an asymptote to the axis of X, since all deviations between $+\infty$ and $-\infty$ are possible. Practically it may be considered as meeting the axis of X at a short distance from the center, since with events happening under the same conditions large variations from the mean are not to be expected.

While the curve as deduced applies to the deviations, or errors, of shot, the laws that are expressed by it are general in character and apply to accidental errors of any kind.

247. Equation of the Probability Curve.—The equation of the curve must be such as to express the characteristics just enumerated. Deduced by means of the theory of accidental errors, taking as its basis the axiom that the arithmetical mean of observed values of any quantity, the values occurring under similar circumstances, is the most probable value of the quantity, the equation takes the form

$$y=\frac{1}{\pi\gamma}e^{-x^2/\pi\gamma^2} \qquad (60)$$

in which γ is the mean error, in our case the mean deviation, and $e=2.71828$ the base of the Napierian system of logarithms. The factor $1/\pi\gamma$ is introduced to make the whole area under the curve unity, $\left(\int_{-\infty}^{+\infty} e^{-x^2/\pi\gamma^2}dx=\pi\gamma\right)$, thus obviating the necessity of dividing a partial area by the whole area whenever a probability is to be computed.

As stated above, the area under any part of the curve divided by the whole area under the curve is the probability that the deviation of any shot will lie between the limits of deviation represented by the part of the axis of X between the limiting ordinates. The area under the curve is $\int ydx$, and since we have introduced into y in equation (60) the factor required to make the whole area unity, the integral taken between limits will represent at once the probability for any limit of deviation.

Thus the probability that any shot will have a deviation less than the numerical value Oa, Fig. 164, is

$$P=2\int ydx=\frac{2}{\pi\gamma}\int_0^a e^{-x^2/\pi\gamma^2}dx \qquad (61)$$

the factor 2 appearing since the ordinate at the end of the distance Oa occurs at equal distances on either side of the center.

The values of P in this equation for various values of a and γ are arranged in the following table with a/γ as an argument. Knowing the mean lateral or vertical deviation γ, to find the probability of a shot striking within the distance a to the right or left of the center of impact, it is only necessary to take from the table the value of P that corresponds to the argument a/γ.

PROBABILITY OF A DEVIATION LESS THAN a IN TERMS OF THE RATIO a/γ.

$\frac{a}{r}$	P	$\frac{a}{r}$	P	$\frac{a}{r}$	P	$\frac{a}{r}$	P
0.1	0.064	1.1	0.620	2.1	0.906	3.1	0.987
0.2	0.127	1.2	0.632	2.2	0.921	3.2	0.990
0.3	0.189	1.3	0.700	2.3	0.934	3.3	0.992
0.4	0.250	1.4	0.735	2.4	0.945	3.4	0.994
0.5	0.310	1.5	0.768	2.5	0.954	3.5	0.995
0.6	0.368	1.6	0.798	2.6	0.962	3.6	0.996
0.7	0.424	1.7	0.825	2.7	0.969	3.7	0.997
0.8	0.477	1.8	0.849	2.8	0.974	3.8	0.998
0.9	0.527	1.9	0.870	2.9	0.979	3.9	0.998
1.0	0.575	2.0	0.889	3.0	0.983	4.0	0.999

248. Illustration of the Use of the Table.—On December 17, 1880, at Krupp's proving ground at Meppen, 50 shots were fired from a 12 cm. siege gun at 5° elevation, giving a mean range of 2894.3 meters. The points of fall were marked on the ground and their distances from assumed axes measured. The center of impact was thus determined. The lateral deviations were measured from the center of impact. The mean lateral deviation was 1.07 meters.

We will find from the table the probability that any shot should have a deviation of less than one meter from the center of impact.

The deviation is $a=1$. The mean lateral deviation is $\gamma=1.07$. Therefore $a/\gamma=1/1.07=0.935$, and from the table, $P=0.544$, the probability that any shot will fall within 1 meter of the center of impact.

For 50 shots the probability is that $P\times 50$ shots will be found within this limit of deviation, $P\times 50=0.544\times 50=27$. This number of shots actually fell within the limit of deviation of 1 meter in the experiment.

Making $a=2$ meters, $a/\gamma=2/1.07=1.87$, $P=0.864$, and $50\times 0.864=43$. The probability is that 43 shots out of the 50 will be found within 2 meters, laterally, of the center of impact. Forty-three shots were actually so found.

249. Probable Zones and Rectangles.—Since P is the probability that the deviation of any shot will not be greater than a, $100P$ represents the number of shots in 100 that will probably fall on both sides of the mean impact within the limit of the deviation a. It is therefore the percentage of hits that will probably be found in the zone defined by the limits at the distance a in both directions from the center of impact. From the table we find that for $P=0.25$, or $100P=25$ per cent, $a/\gamma=0.4$, or $a=0.4\gamma$. The half width of the zone that probably contains 25 per cent of hits is therefore 0.4γ and the full width of the zone is $2a=0.8\gamma$.

This zone is called the *25 per cent zone.*

Similarly for the zone that probably contains 50 per cent of hits, the *50 per cent zone,* $a=0.846\gamma$ and $2a=1.69\gamma$.

Knowing the mean deviation, vertical or horizontal, we may at once from these relations find the width of either zone.

The 50 per cent zone is also called the *probable zone* and its half width is the *probable error,* or deviation, since it is the error that is just as likely to be exceeded as not to be exceeded.

The 25 per cent rectangle is the rectangle formed by the intersection of the 50 per cent zones for lateral and vertical deviations. The probability of each of these zones being 1/2 the probability of the rectangle will be $1/2\times1/2$.

Similarly *the 50 per cent rectangle* is that formed by the intersection of the zones for each of which $P=\sqrt{1/2}$. It is also called *the probable rectangle.*

COMPARISON OF THE ACCURACY OF GUNS.—The rectangles of probability may be used in comparing the accuracy of different guns. The probable rectangle is generally used when this method is employed.

For small arms and high powered guns using direct fire the probable rectangle is taken in the vertical plane, since the targets for these guns usually offer a vertical front.

For guns using curved or high angle fire the probable rectangle is taken in the horizontal plane.

Probability of Hitting any Area.—The probability of hitting any area whose width is $2b$ and whose height is $2h$, and which is symmetrical with respect to the center of impact, as the area *abcd,* Fig. 165, assuming O as the center of impact, is equal to the product of the two values of P taken from the table with b/γ_x and h/γ_y as arguments, the subscripts indicating lateral and vertical deviations.

If the center of impact lies in the given area, or on its edge, the probability of hitting the area is readily obtained by dividing the area into parts by lines passing through the center of impact and taking the sum of the probabilities of hitting the parts.

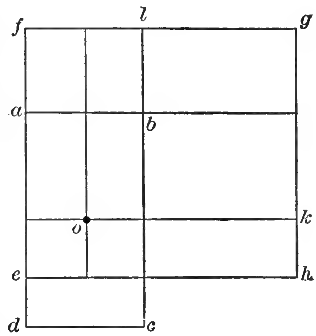


FIG. 165.

Thus the probability of hitting the area *efgh*, Fig. 165, is the sum of the probabilities of hitting the four rectangles into which it is divided by lines through the center of impact. The probability for any one of these rectangles is 1/4 the probability for the area, symmetrical to the center of impact, that is formed by four rectangles equal to the one considered.

If the center of impact lies wholly without the area, the probability of hitting the area is obtained by extending the area to include the center of impact and then taking the difference of the probabilities for the whole area and for the part added to the original area.

Thus the probability for the rectangle *bg* is equal to the probability for the rectangle *og* minus the sum of the probabilities for the rectangles *ol* and *bk*.

APPENDIX TO CHAPTER IX.

THE USE OF TABLE II—INGALLS' BALLISTIC TABLES.

250. Description of Table II.—The several functions in this table are functions of two independent variables, V and Z. Each function varies with V and Z according to the law expressed by the equation which gives the value of the function, and the several functions vary differently. Thus the functions A and A' and others decrease as V increases and increase as Z increases throughout the table. The functions A'' and log B' increase with V and increase with Z up to a value of $V=2500$, beyond which point they will be found to increase with V for certain values of Z and to decrease with V for other values of Z. The function u increases with V and decreases with Z throughout the table.

The values of any function given in the table are the computed values obtained by assuming successive values for V and Z in the equation of the function. The constant difference 100 is taken between the successive values of Z. As most of the functions vary more rapidly when V is small, the computed values are taken close together for the lower values of V and at greater intervals for the larger values of V. Thus for values of V below 1000 the computations were made for values of V differing from each other by 25. Between $V=1000$ and $V=2000$, the difference between the tabular values of V is 50, and above $V=2000$ the difference is 100. The purpose of this course was to obtain in the tables correct values of the functions so close to each other as to permit the assumption, without material error, that the function varies uniformly between the tabulated values. This assumption enables us to interpolate between the given values with comparative ease.

251. Deduction of Formulas for Double Interpolation.—To obtain a formula for interpolation we will proceed as follows. A function of two independent variables may be graphically represented by the length of a line drawn perpendicular to the plane which contains the axes of the variables. The variables in the tables are V and Z. Let us take from the table a value of any one of the functions, as A, and call this value f_0, the corresponding values of V and Z being called V_0 and Z_0. Let the axis of V be horizontal and the axis of Z vertical. From the point V_0Z_0 on the plane, Fig. 166, draw a line perpendicular to the plane, and lay off on it the length f_0 equal to the value of the function taken from the table. Lay off the distance Z_0Z_2 parallel to the axis of Z and equal to 100. From Z_2 draw a line perpendicular to the plane and lay off on it the value of the function given in the same table for the next greater value of Z. Lay off V_0V_2 parallel to the axis of V and equal to the difference between the two velocities given in the caption of the table, and call this distance h. On a perpendicular to the plane at V_2 lay off the value of the function taken from the next succeeding table for the first value of Z, and from a point at a distance of 100 below V_2 lay off the next succeeding value of the function from this table. Complete the figure shown by the heavy lines. The solid represented by this figure is made up of all the values of the function lying between the four tabular values.

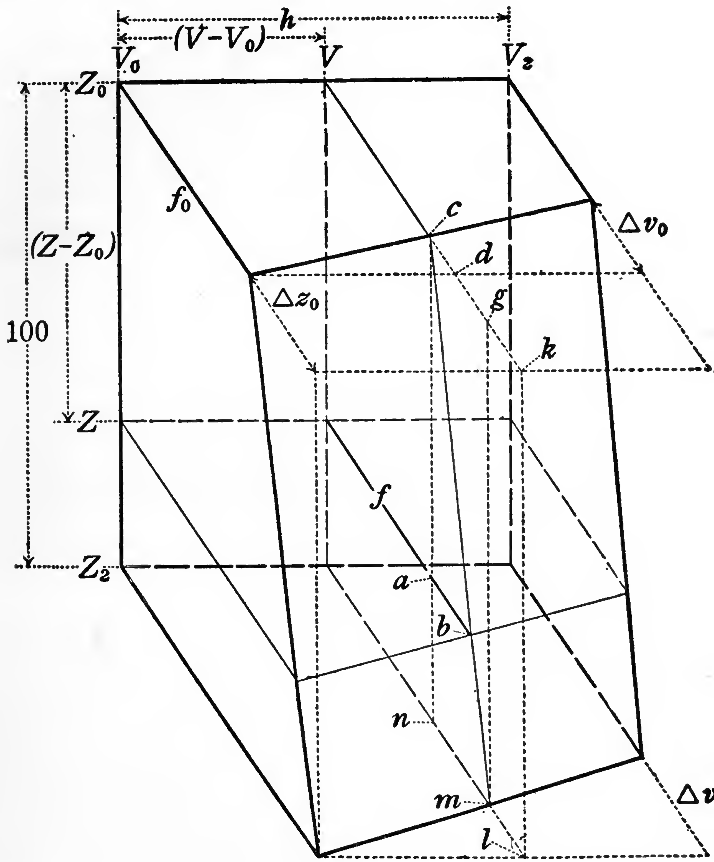


FIG. 166.

Let us cut the solid by a plane through V in the figure at a distance $V-V_0$ from V_0, and by another plane through Z in the figure at a distance $Z-Z_0$ from Z_0. The intersection of these two planes, f, will be the value of the function corresponding to the values V and Z. In the column marked Δz in the table, opposite the value of each function, appears the difference between this value and the value next below. This difference for f_0, called Δz_0, is represented in the figure; and similarly the corresponding difference in the Δv column, which is the difference between the values of the function for the same value of Z and successive tabular values of V, is shown as Δv_0 in the figure; and the next succeeding difference in the same column is shown as Δv_1 at the bottom of the figure. Draw vertical lines from c, m, and l.

From the figure:

$$f_0 - dc + ab = f$$

$$h : \Delta v_0 :: V - V_0 : dc \qquad dc = \frac{V - V_0}{h} \Delta v_0$$

From the triangle cnm we have:

$$100 : nm :: Z - Z_0 : ab$$

$$ab = \frac{Z - Z_0}{100} nm$$

$$nm = nl - ml \qquad nl = ck = dk + dc = \Delta z_0 + \frac{V - V_0}{h} \Delta v_0$$

$$\Delta v_1 : ml :: h : V - V_0 \qquad ml = \frac{V - V_0}{h} \Delta v_1$$

$$nm = \Delta z_0 + \frac{V - V_0}{h} \Delta v_0 - \frac{V - V_0}{h} \Delta v_1 = \Delta z_0 - (\Delta v_1 - \Delta v_0) \frac{V - V_0}{h}$$

$$ab = \frac{Z - Z_0}{100} \left\{ \Delta z_0 - (\Delta v_1 - \Delta v_0) \frac{V - V_0}{h} \right\}$$

$$f = f_0 - \frac{V - V_0}{h} \Delta v_0 + \frac{Z - Z_0}{100} \left\{ \Delta z_0 - (\Delta v_1 - \Delta v_0) \frac{V - V_0}{h} \right\}.$$

The above expression having been deduced under the conditions that the function decreases with V and increases with Z, we will indicate this by writing $f^{(-V)}_{(+Z)}$ for f. Transposing the terms of this formula, for convenience, it may be written:

$$f^{(-V)}_{(+Z)} = f_0 + \frac{Z - Z_0}{100} \Delta z_0 - \frac{V - V_0}{h} \Delta v_0 - \frac{Z - Z_0}{100} \cdot \frac{V - V_0}{h} (\Delta v_1 - \Delta v_0),$$

and by changing the signs according to the manner of the variation of the function with V and Z, we may write the formulas for those functions that vary in a different manner.

The formula gives the value of the function corresponding to the values of V and Z between the tabular values. If we solve it for V we obtain an expression for the value of V when non-tabular values of the function and of Z are given; and similarly, solving it for Z, the resulting formula will give the value of Z corresponding to non-tabular values of the function and of V.

The formulas will be of the form given below.

252. Double Interpolation Formulas—Ballistic Table II.

f = non-tabular value of any function corresponding to the non-tabular values V and Z.

f_0 = tabular value of function corresponding to tabular values V_0 and Z_0, *always the nearest values less than V and Z.*

h = difference between velocities given in caption of table.

Δv_0 and Δz_0 = tabular differences for f_0.

Δv_1 = tabular difference next following Δv_0 in same table.

$f^{(-V)}_{(+Z)}$ indicates that function decreases as V increases and increases as Z increases.

Use the following formulas for the functions A, A', B, T', log C', and D' throughout the table. They also apply for some values of the functions A'' and log B' when $V > 2500$.

$$f^{(-V)}_{(+Z)} = f_0 + \frac{Z-Z_0}{100}\Delta z_0 - \frac{V-V_0}{h}\Delta v_0 - \frac{Z-Z_0}{100}\cdot\frac{V-V_0}{h}(\Delta v_1 - \Delta v_0)$$

$$V = V_0 + \frac{\left(f_0 + \frac{Z-Z_0}{100}\Delta z_0\right) - f}{\Delta v_0 + (\Delta v_1 - \Delta v_0)\frac{Z-Z_0}{100}} \times h$$

$$Z = Z_0 + \frac{f - \left(f_0 - \frac{V-V_0}{h}\Delta v_0\right)}{\Delta z_0 - (\Delta v_1 - \Delta v_0)\frac{V-V_0}{h}} \times 100$$

Use the following formulas for the functions A'' and log B' when $V < 2500$, and for some values beyond that point.

$$f_{(+Z)}^{(+V)} = f_0 + \frac{Z-Z_0}{100}\Delta z_0 + \frac{V-V_0}{h}\Delta v_0 + \frac{Z-Z_0}{100}\cdot\frac{V-V_0}{h}(\Delta v_1 - \Delta v_0)$$

$$V = V_0 + \frac{f - \left(f_0 + \frac{Z-Z_0}{100}\Delta z_0\right)}{\Delta v_0 + (\Delta v_1 - \Delta v_0)\frac{Z-Z_0}{100}} \times h$$

$$Z = Z_0 + \frac{f - \left(f_0 + \frac{V-V_0}{h}\Delta v_0\right)}{\Delta z_0 + (\Delta v_1 - \Delta v_0)\frac{V-V_0}{h}} \times 100$$

Use the following formulas for the function u.

$$f_{(-Z)}^{(+V)} = f_0 - \frac{Z-Z_0}{100}\Delta z_0 + \frac{V-V_0}{h}\Delta v_0 - \frac{Z-Z_0}{100}\cdot\frac{V-V_0}{h}(\Delta v_0 - \Delta v_1)$$

$$V = V_0 + \frac{f - \left(f_0 - \frac{Z-Z_0}{100}\Delta z_0\right)}{\Delta v_0 - (\Delta v_0 - \Delta v_1)\frac{Z-Z_0}{100}} \times h$$

$$Z = Z_0 + \frac{\left(f_0 + \frac{V-V_0}{h}\Delta v_0\right) - f}{\Delta z_0 + (\Delta v_0 - \Delta v_1)\frac{V-V_0}{h}} \times 100$$

Inspect the tables to determine how the function varies with V and Z, and select the proper group of formulas.

Exercise great care in the use of the plus and minus signs.

Double Interpolation in Simple Tables.—Regarding Fig. 166, from which the above formulas have been deduced, we will see that the interpolated value f of the function may be obtained from the four tabular values represented by the four heavy corner lines of the figure. Interpolating by the rule of proportional parts between the value f_0 of the function and the value immediately below it in the same table for V, which value is represented at Z_2 in the figure, we obtain the value of the function at V_0Z in the figure. Proceeding in the same manner in the table for the next value of V we obtain the value of the function at V_2Z in the figure.

Interpolating between the values at V_0Z and V_2Z we obtain the desired value f.

This method is the most convenient method of double interpolation in simple tables, such as Table VI of the Ballistic Tables. The numbers in that table are simple and the data is all found together on one page.

USE OF THE FORMULAS.

253. Given Non-Tabular Values of V and Z, to Find f.—Select the f formula applicable to the particular function. Take from the table the value of the function corresponding to the tabular values of V and Z next less than the given values. The tabular values of V and Z are V_0 and Z_0 in the formula. Express the fractions $\frac{V-V_0}{h}$ and $\frac{Z-Z_0}{100}$ decimally. If we take from the table at the same time with the function the corresponding numbers in the Δz and Δv columns, also the number next following in the Δv column, called respectively Δz_0, Δv_0, and Δv_1 in the formula, we have all the data necessary for the solution of the problem. The numbers in the different columns of the table are obtained by considering the values of the functions as whole numbers. *The corrections therefore must be applied to the function as if it were a whole number.*

In the examples which follow we will indicate *by enclosing the decimal values of functions in parentheses* that they are to be considered as whole numbers in applying the corrections.

EXAMPLE.

1. *Given* $V=1015$ $\quad Z=3742$ $\quad$ *What is the value of* A'?

$$\frac{V-V_0}{h}=\frac{15}{50}=.3 \qquad \frac{Z-Z_0}{100}=.42 \qquad f_0=0.2946$$

$$\Delta z_0=96 \qquad \Delta v_0=223 \qquad \Delta v_1=230$$

$$\begin{aligned} f&=(0.2946)+.42\times 96-.3\times 223-.42\times .3\times 7\\ &=(0.2946)+40.32-66.9-.88\\ &=(0.2946)-27.5\\ &=0.29185\end{aligned}$$

2. *Given* $V=887$ $Z=7275$ *What is the value of log* B'?

$$\frac{V-V_0}{h}=\frac{12}{25}=.48 \qquad \frac{Z-Z_0}{100}=.75$$

$$f=(0.09779)+.75\times133+.48\times59-.75\times.48\times1=0.099067$$

To help in fixing the formulas for f in the mind, we will note that the correction for Δz is applied with a positive sign if the function increases with Z, and with a negative sign if the function decreases with Z. The correction for Δv is similarly applied according as the function varies with V. The sign of the last term is positive if the signs of the two preceding terms are similar, and negative if they are dissimilar. The difference between the two values of Δv in the last term is usually positive and no attention need be paid to the sign of this difference except when dealing with the functions log B' and log C'.

The formulas used in the above examples, which we will call the f formulas, and which give the values of the functions for non-tabular values of V and Z, indicate the simplest and quickest method of arriving at the correct value of an interpolated function. *This method should therefore always be followed in solving problems of this nature.*

3. Given $V=1630$	$Z=3781$	Find D'	*Ans.* 155.9
4. Given $V=972.4$	$Z=9569$	Find A	*Ans.* 0.464181
5. Given $V=2790$	$Z=1255$	Find log C'	*Ans.* 4.65946
6. Given $V=2790$	$Z=8473$	Find log C'	*Ans.* 4.97732

Note the difference in the signs of the last term of the formula in the two preceding examples; also the sign of the same term in the following example.

7. Given $V=1217$ $Z=8778$ Find log B' *Ans.* 0.138514

Note that in the following example A'' decreases with V.

8. Given $V=3040$ $Z=4926$ Find A'' *Ans.* 2952.4

254. Given Non-Tabular Values of the Function and of V, to Find Z.—Select the Z formula applicable to the particular function. Inspect the table on the page that contains the given value of V to find the proper values to substitute in the formula for f_0, Z_0, and the tabular differences. To arrive at accurate results this requires some little care, and is best done in the following manner. By rapid inspection of the table find the two values of the function between which the given value lies. Apply to the tabular value corresponding to the *larger* value of Z the correction $\frac{V-V_0}{h}\Delta v_0$. By comparing the corrected tabular value with the given value we determine on which side of the corrected tabular value the given value lies, and thereby determine which value of Z to use for f_0 and the differences in the formula. An example will illustrate this.

9. *Given* $A=0.06121$ $\quad V=2192 \quad$ *Find* Z.

$$\frac{V-V_0}{h}=.92$$

Looking in the table for $V=2100$ we find that the given value of A lies between the values corresponding to $Z=5100$ and $Z=5200$. Applying to the value of the function corresponding to the *larger* value of Z the correction $\frac{V-V_0}{h}\Delta v_0=.92\times 571=525$ we have $(0.06263)-525=0.05738$ as the value of the function for $V=2192$ and $Z=5200$. This value is less than the given value by about 380, and as the function increases with Z the given value lies below it in the table.

The tabular Δz for the value of the function, 0.06263, that we have taken from the table, is about 190; that is the function is here increasing by about 190 for each tabular value of Z. The tabular function when corrected gave us a value too small by 380. Consequently if we take the second value of Z greater than 5200, the one we have used, we shall probably have the value we seek.

We will therefore take the function for $Z=5400$ and apply the correction to get its value for $V=2192$. The corrected value is $(0.06639)-.92\times 602=0.060852$. As this is less than the given value of A and close to it, we know that the given value lies between $Z=5400$ and $Z=5500$, and we will use $Z=5400$ and the corresponding tabular values in the formula.

It will be observed in each of the formulas for Z and V that, in the numerator of the last term, there is a term in parentheses containing f_0 plus or minus a correction. This term in parentheses is the tabular value of the function corrected for the difference between the given value of V or Z and the next less tabular value. *It is essential, in order to arrive at correct results, that the value of this term be found first;* for, as shown above, it is only by this means that we can determine the true tabular values of Z or V between which the required value lies. It will be shown later that without these values correct results cannot be obtained.

In this example we have found the value of the term in parentheses to be $(0.06639) - .92 \times 602 = 0.060852$. Using this in the formula with the given value of the function and the tabular quantities corresponding to f_0, the process becomes exceedingly simple, and the required value is easily and quickly and accurately obtained.

$$Z_0 = 5400 \quad f_0 = 0.06639 \quad \Delta z_0 = 193 \quad \Delta v_0 = 602 \quad \Delta v_1 = 618$$

$$Z = 5400 + \frac{6121 - 6085.2}{193 - 16 \times .92} 100$$

$$Z = 5400 + \frac{358}{1782} 100 = 5420.1$$

If we had not pursued the above course, but had used for Z_0 the smaller value of Z obtained at our first inspection of the table, the result would have been as follows.

$$Z = 5100 + \frac{6121 - 5568.4}{184 - 16 \times .92} 100 = 5432.6$$

The difference in the results is due to the fact that in using the value $Z = 5100$ we assume that the function varies uniformly between this value and the obtained value, a difference of 332, while our process of interpolation is based on the assumption that the variation is uniform for a difference in Z of 100 only.

The effect of the difference in the values of Z obtained by the two methods may be seen in the problem from which the above data were taken. The value of the ballistic coefficient, C, was 4.7859 and the range X was required. $X = ZC$.

With $Z = 5420.1$ $\qquad X = 25940$ ft.
With $Z = 5432.6$ $\qquad X = 26000$ ft.

It may sometimes be more convenient, *after having found the proper value of Z for use in the formula,* to obtain from the table the corrected values of the function for that value of Z and for the next greater value of Z. The given value of the function will lie between these two corrected tabular values, and the true value of Z may be found by the method of proportional parts.

$$\begin{array}{llll} \text{For } V=2192 & Z=5400 & A=(0.06639)-.92\times 602 = 0.060852 \\ & Z=5500 & A=(0.06832)-.92\times 618 = \underline{0.062634} \\ & & \qquad\qquad\qquad\qquad\qquad\qquad 1782 \end{array}$$

$$\begin{array}{ll} A, \text{ given,} & .06121 \\ & \underline{.060852} \end{array}$$

$$Z=5400+\frac{358}{1782}100=5420.1$$

The results given by the two methods are the same. Indeed the methods are the same, for through the agency of Δz_0 and Δv_1 in the formula we make use of the tabular values of the function corresponding to the second value of Z. It will be seen in the examples above that the fractions to be reduced are exactly alike.

In problems in the text books on exterior ballistics the value of Z is nearly always determined to the nearest tenth. This indicates that it is important to obtain the correct value. The correct value can be obtained, from the tables, *only* by interpolating between the nearest tabular values on each side. The importance of the preliminary application of the correction $\frac{V-V_0}{h}\Delta v_0$ to the tabular values of the function, for the purpose of determining the proper value of Z to use, is therefore apparent.

In using the formulas for Z and V the fractional coefficients of 100 and of h in the last terms will always inform us whether we are in the proper place in the tables. **Both numerator and denominator of the fraction must be positive, and the value of the fraction must be less than unity.** A negative value of the fraction or a value greater than unity indicates that we have not used the nearest values of f_0 and V_0 or Z_0 and the differences. The result is therefore approximate only, and the degree of approximation varies with the number of units in the value of the fraction.

The formulas for V and Z may be easily fixed in the memory if we observe that the numerator of the last term is the difference between the given value of the function and the nearest corrected tabular value, the correction being applied to the tabular value with a sign indicated by the manner of variation of the function with Z or V. The first term of the denominator is Δv_0 in the V formulas, and Δz_0 in the Z formulas. The sign of the second term of the denominator is the same as the sign inside the parentheses of the numerator. The value of the second term of the denominator is positive for all the functions except log B' and log C'. For some value of log B', and for most values of log C', Δv_1 is less than Δv_0, so that $(\Delta v_1 - \Delta v_0)$ becomes negative and causes a change of sign for the second term of the denominator in the V and Z formulas, and for the last term in the f formulas.

10. *Given* $u=991$ $\quad V=1630 \quad$ *Find* Z.

$$\frac{V-V_0}{h}=\frac{30}{50}=.6$$

This value of u apparently lies between the values of $Z=4600$ and $Z=4700$, but applying the correction $\frac{V-V_0}{h}\Delta v_0=.6\times 15=9$ to 987, the tabular value of the function for $Z=4700$, adding it since u increases with V, we find that the value of u for $V=1630$ and $Z=4700$ is 996. This being greater than our given value, and the function decreasing with Z, the given value corresponds to a value of Z greater than 4700. Similar inspection shows that the proper value of Z is less than 4800. We therefore use the values for $Z=4700$ in the formula.

$Z_0=4700 \quad f_0=987 \qquad \Delta z_0=6 \quad \Delta v_0=15 \quad \Delta v_1=15$

$$Z=4700+\frac{996-991}{6+0}100=4783.3$$

11. *Given* $A''=2158$ $\quad V=979 \quad$ *Find* Z.

$$\frac{V-V_0}{h}=.16$$

The change in the function here is very slight for a change in V, and we see at once that this value of A'' lies between $Z=4000$ and $Z=4100$.

$$Z=4000+\frac{2158-2138.5}{57+0}100=4034.2$$

12. Given $B=0.0341$	$V=2763$	Find Z	*Ans.* 4053.4
13. Given $D'=790$	$V=1784.6$	Find Z	*Ans.* 7278.1
14. Given $\log B'=0.07140$	$V=1146$	Find Z	*Ans.* 3894.9
15. Given $A'=0.2252$	$V=970$	Find Z	*Ans.* 2813.1

255. Given Non-Tabular Values of the Function and of Z, to Find V.—This problem is slightly more troublesome than the one just explained, because as V is not given we cannot turn directly to the page on which the nearest tabular value of the function will be found.

Select the V formula applicable to the particular function. With the next tabular value of Z less than the given value look through the table until two consecutive tables are found which, for this value of Z, give values of the function less and greater than the given value. Apply the correction $\frac{Z-Z_0}{100}\Delta z_0$ to the tabular value corresponding to the larger value of V and determine, from the corrected tabular value, the side on which the given value lies, and the proper table to use.

16. *Given* $B=0.32386$ $\quad Z=5887 \quad$ *Find* V.

$$\frac{Z-Z_0}{100}=.87$$

Inspecting the tables with the value $Z=5800$ we find that tabular values of the function greater and less than the given value are found in the consecutive tables for $V=900$ and $V=925$, these values being respectively 0.3388 and 0.3230. Apparently then the value of V for the given function lies between 900 and 925, and the values for f_0, V_0, etc., in the formula, should be taken from the table for $V=900$. But applying the correction $\frac{Z-Z_0}{100}\Delta z_0=.87\times 77=67$ to the tabular value of the function for $Z=5800$ and $V=925$, adding it since B increases with Z, we obtain for the function at $V=925$ and $Z=5887$, the value 0.3297, which is greater than the given value. Since B decreases with V the given value must therefore lie to the right of the value for $V=925$, and as the difference between the two is considerably less than the Δv in the table, 144, we know without further inspection that the value for V lies between 925 and 950, and in the formula we will use the quantities taken from the table for $V=925$.

$$V_0=925 \qquad Z_0=5800 \qquad f_0=0.3230$$

$$\Delta z_0=77 \qquad \Delta v_0=144 \qquad \Delta v_1=147$$

$$V=925+\frac{3297-3238.6}{144+3\times .87}25=925+\frac{584}{1466}25=935$$

In a manner similar to that explained in the first problem under the previous heading this same value of V can be obtained, *after having found the value of the function for* $Z=5887$ *and* $V=925$, by finding the value of the function corresponding to $Z=5887$ and the next tabular value of V, 950, and determining the true value of V by the method of proportional parts.

For $Z=5887$	$V=925$	$B=(0.3230)+.87\times 77=0.3297$
	$V=950$	$B=(0.3086)+.87\times 74=0.31504$
		1466

	3297
B, given,	32386

$$V=925+\frac{584}{1466}25=935$$

17. *Given* $T'=9.130$ $\qquad Z=9378$ $\qquad$ *Find* V.

$$\frac{Z-Z_0}{100}=.78$$

Inspecting the table with $Z=9300$, we find that the given value of T' lies between the tabular values for $V=1600$ and $V=1650$. Adding to 9.046, the value of T' for the larger value of V, the correction $.78\times 128$, we find that T' for $Z=9378$ is 9.146. We know then that the value of V sought is greater than 1650; and since $9.146-9.130$ is less than the Δv in the table, 152, we know that V lies between 1650 and 1700. We therefore use in the formula the values from the table for $V=1650$.

$$V=1650+\frac{9146-9130}{152+.78\times 1}50=1655.2$$

18. *Given* $\log B'=0.1652$ $\quad Z=4625 \quad$ *Find* V.

$$\frac{Z-Z_0}{100}=.25$$

From the value of tan ω, equation (35), we have $B'=\frac{\tan \omega}{\tan \phi}$. The same range may be attained by different shots fired with different velocities at different angles of elevation. The angles of fall will also be different. But the changes in the angle of elevation and angle of fall may be such that the ratio of the tangents of the angles will remain constant. We may therefore get similar values for B', and for its logarithm, with one value of X and widely different values of V. When, therefore, log B' is given and a value of Z, since Z contains X as a factor, we may find in the tables more than one value of V corresponding to these given values. Should this case be encountered in the solution of a ballistic problem, the proper value of V to use would be determined after consideration of the other data of the problem.

With the data given above we find the two following solutions, in the tables for $V=1900$ and $V=2900$ respectively; using in the first the formula for V when log B' corresponds to a value of $V<2500$, and in the second the formula for V when log B' corresponds to a value of $V>2500$.

$$\log B'=0.1652 \quad Z=4625 \quad \frac{Z-Z_0}{100}=.25$$

$$V=1900+\frac{1652-1627.5}{28+2\times .25}50=1943$$

$$V=2900+\frac{1655.5-1652}{11+0}100=2931.8$$

As we have before noted, the functions A'' and log B', for some values of Z, increase with V when $V<2500$ and decrease with V beyond that point. Therefore we may expect to find, for these values of Z, equal values of either function on both sides of $V=2500$.

19. Given $u=931.3$	$Z=8122.7$	Find V	*Ans.* 2187.5
20. Given $B=0.16801$	$Z=6345$	Find V	*Ans.* 1832.0
21. Given $T'=3.7943$	$Z=4852$	Find V	*Ans.* 1747.0
22. Given $\log B'=0.23376$	$Z=7318$	Find V	*Ans.* 2226.0

256. Given One Function and V or Z, to Find the Corresponding Value of Another Function.—Inspecting the formulas for V and Z we see that the fractional coefficients of h and 100, in the last terms, are respectively equal to $\frac{V-V_0}{h}$ and $\frac{Z-Z_0}{100}$. We therefore take out this coefficient from the Z formula if V is given with the function, and from the V formula if Z is given, using the formula applicable to the given function. Substitute the value thus obtained for $\frac{Z-Z_0}{100}$ or for $\frac{V-V_0}{h}$ in the f formula applicable to the required function, using for f_0 and the differences in this formula the tabular values for the required function corresponding to the same values of V and Z as were used in the previous operation.

23. *Given $A''=3150$ $V=1929.5$ Find u.*

$$\frac{V-V_0}{h}=.59$$

From the Z formula for A'' when $V<2500$

$$\frac{Z-\overset{5200}{Z_0}}{100}=\frac{3150-(3116+5.9)}{65+.59}=.43$$

It will always be well when taking from the table the quantities required in computing the coefficient $(Z-Z_0)/100$ *from the Z formula* to write above Z_0 the tabular value used, as it is written in the above equation. This will serve as a memorandum as to what value of Z_0 to use when computing the value of the required function.

The memorandum is not necessary when computing $(V-V_0)/h$, as the value of V_0 is indicated on the page at which the table is open.

Substituting the value of this coefficient, obtained above, in the f formula for the function u, and using for f_0 and the differences in this formula the tabular quantities for the function u for the same values of V and Z used in computing the coefficient,

$$u=1041-.43\times8+.59\times14-0=1045.8$$

24. *Given* $D'=125$ $V=3018$ *Find* A''.

$$\frac{V-V_0}{h}=.18$$

$$\text{for } D' \quad \frac{\overset{5500}{Z-Z_0}}{100}=\frac{125-120.4}{7-.18}=.67$$

Since V is greater than 2500 we must inspect the table to see how A'' varies for the value of Z used. We find that A'' is here diminishing with V and increasing with Z. The first of the f formulas is therefore appropriate.

$$A''=3364+.67\times73-.18\times6-0=3411.8$$

25. *Given* $A'=0.0401$ $Z=540$ *Find* T'.

$$\frac{Z-Z_0}{100}=.4$$

For $Z=500$ this value of A' lies between the values given for $V=900$ and $V=925$. Applying the correction for Z to the value corresponding to $V=925$, we find that 925 is the proper value of V to use in the formula.

$$\frac{V-V_0}{h}=\frac{418-401}{19+4\times.4}=.825$$

$$T'=(0.548)+.4\times111-.825\times14-.4\times.825\times3=0.5799$$

26. *Given* $\log B'=0.0809$ $Z=2565$ *Find* $\log C'$.

$$\frac{Z-Z_0}{100}=.65$$

$$\text{for } \log B' \quad \frac{V-V_0}{h}=\frac{809-786.65}{44+2\times.65}=.493$$

$$\log C'=(5.3076)+.65\times34-.493\times274-.65\times.493\times2=5.29624$$

27. Given $A'=0.2485$ $V=2180.4$ Find B *Ans.* 0.15578

28. Given $T'=7.698$ $Z=5728$ Find D' *Ans.* 1013.3

29. Given $\log B'=0.1832$ $V=1832$ Find u *Ans.* 954.2

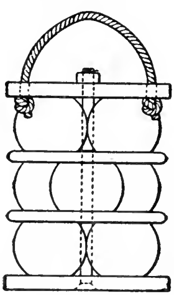
30. Given $A=0.01669$ $Z=1224.5$ Find $\log C'$ *Ans.* 5.1347

CHAPTER X.

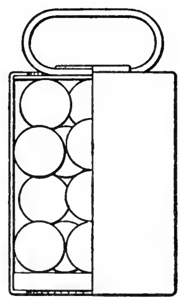
PROJECTILES.

257. Classification.—Projectiles are classed as shot, shell, and case shot. The shell is a hollow shot designed to be filled with a bursting charge that by means of a fuse may be exploded at a selected time. The case shot consists of a number of shot held together by an enclosing envelope which may be ruptured by the shock of discharge or by a bursting charge in flight. The envelopes of canister and grape shot are ruptured by shock in the gun. The envelope of shrapnel is ruptured by a bursting charge.

Old Forms of Projectiles.—In the old smooth bore cannon round cast iron shot and shell of diameter nearly equal to the caliber of the gun were used. The grape, canister, and shrapnel for these

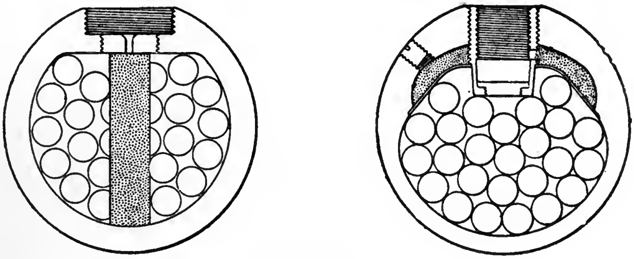


GRAPE.



CANISTER.

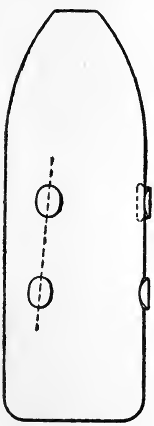
guns are shown in the illustrations. The shrapnel was invented about 1803 by Colonel Shrapnel of the British Army. In its first form it contained a number of lead balls with loose powder in the interstices. The walls of the shell were made thick to resist deformation by the movement of the contained balls. In its later forms the spaces between the balls were filled with melted sulphur, and a chamber for the bursting charge was provided as shown. By this arrangement the walls were no longer subject to the impact from the loose balls, and therefore could be made thinner,



SHRAPNEL.

thus providing room for a greater number of bullets. The confining of the bursting charge in a chamber made its explosive effect greater and permitted a reduction in its weight.

Chain shot and bar shot, made up of two projectiles connected by a chain or bar, were occasionally used in early times; and in-



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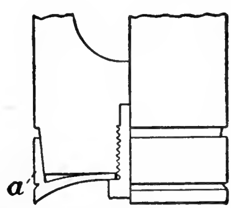
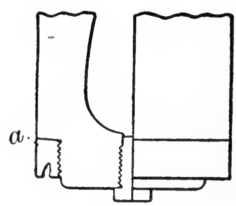


EUREKA.



BUTLER.

cendiary shell, called *carcasses,* which were ordinary shell filled with combustible material, the flames from which issued through holes drilled through the walls of the shell.

Smooth bore guns were succeeded by muzzle loading rifled guns. The introduction of rifling brought about the use of elongated projectiles of increased weight. The capacity of the gun in weight of metal thrown was largely increased and much greater accuracy of fire was obtained.

For the projectiles for the muzzle loading rifled cannon some device was necessary to cause the projectile to take the rifling. The several devices that were employed are shown in the illustrations on the preceding page.

The studs on the projectile shown in the first figure were fitted into the grooves of the rifling as the projectile was inserted at the muzzle. In the other projectiles shown the parts *a* are of brass, and in firing were expanded outward into the rifling by the pressure of the powder gases. Other means that were employed are shown in Figs. 167, 168, and 169.

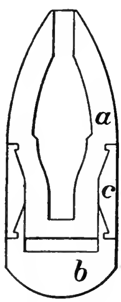


FIG. 167.

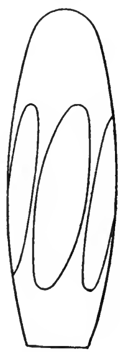


FIG. 168.

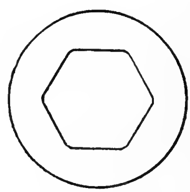


FIG. 169.

Fig. 167 shows the Hotchkiss projectile. The parts *a* and *b* are of iron and are held apart by the ring of lead *c*. The gas pressure acting on the part *b* forced the lead outward into the rifling.

Fig. 168 shows the Whitworth projectile. The bore of the Whitworth gun was a twisted prism of hexagonal cross section as shown in Fig. 169. The projectile was fashioned to fit the bore, its sides being provided with surfaces of a similar prism.

258. Modern Projectiles. BANDING.—With the introduction of breech loading in arms of all kinds the problem of giving rotation to the projectile was much simplified As the chamber of the gun is larger than the bore, a projectile provided with a soft metal band, *b* Fig. 170, of diameter larger than the diameter of the bore, may be inserted through the chamber. On the explosion of the charge the pressure causes the sloping ends *d* of the lands of the rifling to force their way through the rotating band, causing the band to conform in shape to the section of the rifling, and assuring the proper rotation in the projectile. As the band completely fills the cross section of the bore it serves also as a check to prevent the escape of gas past the projectile, and in addition it

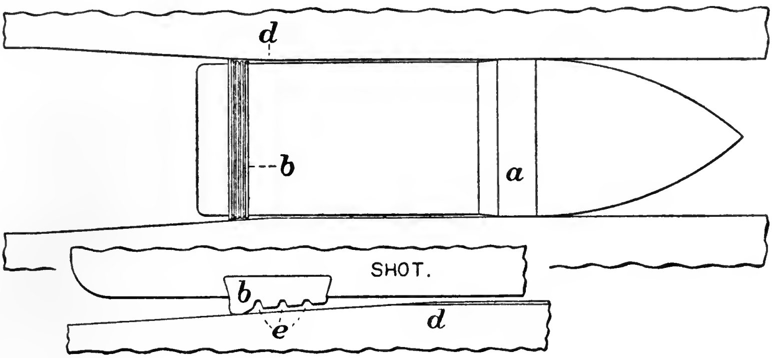


FIG. 170.

serves to center the projectile in the bore, and to determine a fixed position of the projectile when rammed into the gun.

The banding of projectiles is practically the same for all calibers. An undercut groove, *b* Fig. 171, is cut around the projectile near the base. A straight band of copper, of cross section as shown at *a*, is hammered into the groove and completely fills it, as shown at *e*. The ends of the band are beveled lengthwise and make a scarf joint where they meet. The bands for projectiles of small caliber are solid rings of metal forced into the grooves of the projectile under hydraulic pressure. The bottom of the groove *b* is scored with vertical cuts into which the copper enters when the band is hammered on. These

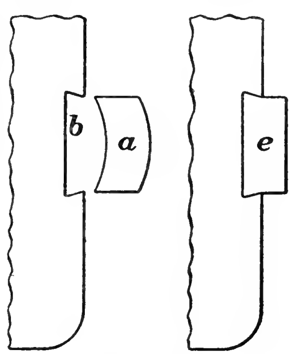


FIG. 171.

prevent the rotation of the band independently of the projectile. The width of the band depends upon the caliber of the projectile and is greater for the larger calibers. The outer surface of the band is smooth in projectiles for siege and smaller caliber guns. In the wider bands of the larger projectiles a number of grooves are cut, as shown in section at *e*, Fig. 170, to diminish the resistance to forcing and to provide space for the metal forced aside by the lands of the rifling.

In the latest 6-inch wire wound guns, in which velocities of over 3400 feet have been produced, difficulty has been experienced on account of the tendency of the jointed rotating bands to strip from the projectile during flight, due to the effect of the centrifugal force. A band made by winding a thin copper ribbon on edge and filling the groove has been tried with these projectiles but without success.

It is probable that the method of banding with solid rings seated by hydraulic pressure will ultimately be used with these and with larger projectiles.

259. Form of Projectile.—With the exception of the canister all modern projectiles are of the same general shape, a cylindrical body with ogival head. The ogival head is found by experiment to be the most advantageous, as it offers little resistance to the air and at the same time provides enough metal at the point of the projectile to give to the point the requisite strength to perform the work of penetration.

The ogive is struck from a center on a line perpendicular to the axis of the projectile, Fig. 172, and with a radius usually ex-

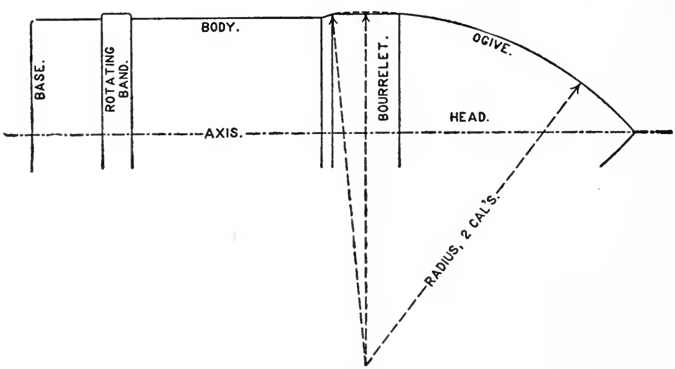


Fig. 172.

pressed in calibers. The radius of the head varies in different projectiles from $1\frac{1}{2}$ to 3 calibers.

The lower part of the ogive is turned off to make a cylindrical bearing surface for the front part of the projectile. This surface, called *the bourrelet*, has a diameter 1/100 of an inch less than the diameter of the gun.

Below the bourrelet the diameter of the projectile is diminished, for ease of manufacture and to prevent bearing in the gun, to about 7/100 of an inch less than the caliber. The band is placed from $1\frac{1}{2}$ to $2\frac{1}{2}$ inches from the base, depending on the caliber, the greatest diameter of the band exceeding the caliber by from 1/10 to 3/10 of an inch.

The length of projectile varies between $2\frac{1}{2}$ and 5 calibers. The length of most of the seacoast projectiles is $3\frac{1}{2}$ calibers.

Canister.—Canister projectiles are for use at very short range, when the guns of a battery are being charged by the enemy. The projectile consists of a number of small balls contained in a metallic envelope so constructed that it will break into pieces at the shock of discharge. In our service, canister are provided for the mountain guns only. The canister for the 75 m|m Vickers Maxim gun is shown in Fig. 173.

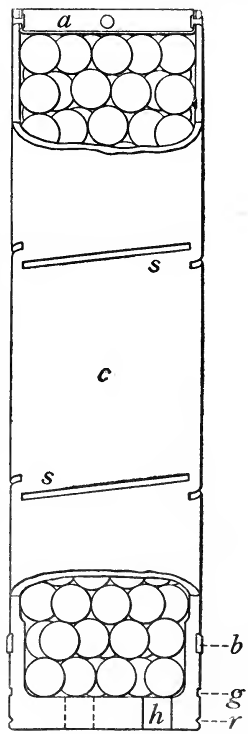


FIG. 173.

The case, *c*, made of malleable iron, is solid at the bottom and open at the top. It is weakened by two series of cuts, *s*, each series consisting of three oblique cuts, each of which extends over an arc of 120 degrees. The case contains 244 iron balls $\frac{5}{8}$ of an inch in diameter and weighing 30 to the pound. The balls are confined in the case by the tin cup, *a*, riveted in. Three holes, *h*, drilled through the bottom of the case admit the powder gases to assist in rupturing the case. The metallic cartridge case is attached to the projectile by being crimped at several points into the groove *r*. The copper band, *b*, forms a stop for the head of the cartridge case, and serves as a gas check in the gun. The groove *g*, in other projectiles, is filled with grease for the purpose of preventing the entrance of moisture into the cartridge case.

It is the present intention of the Ordnance Department not to manufacture any more canister. Their place will be taken by shrapnel, which are so constructed that they may be burst within 25 feet of the muzzle of the gun.

260. Shrapnel.—The modern shrapnel is a projectile designed to carry a number of bullets to a distance from the gun and there to discharge them with increased energy over an extended area. It is particularly efficacious against troops in masses and is not used against material. The shrapnel is the principal field artillery projectile. It is also provided for mountain and siege artillery and for use in the small caliber guns in seacoast fortifications in repelling land attacks.

In the earlier models the case of the shrapnel was so constructed as to break into a number of fragments on explosion of the bursting charge, with the idea of thus practically increasing the number of bullets carried. With the same end in view the spaces between the balls were filled with the parts of cast metal diaphragms that separated the layers of balls and broke up into additional fragments at the bursting of the projectile. The bursting charge was placed sometimes in the head and sometimes in the base of the projectile. It was found with these shrapnel that a very large percentage of the numerous fragments had not sufficient energy to inflict serious injury. The shrapnel is therefore at present constructed of a stout case which, except for the blowing out of the head, remains intact at the explosion of the bursting charge, and from which the balls are expelled in a forward direction and with increased velocity by the bursting charge in the base. By these means, while the number of fragments is less, a greater number possess the required energy and the effective range of these is increased.

Fig. 174 represents the shrapnel for the 3-inch field gun. The case, *c*, is a steel tube drawn in one piece with a solid base. A steel diaphragm, *d*, rests on a shoulder near the base, forming a chamber for the bursting charge in the base of the projectile, and a support for a central steel tube which extends through the head, *h*. A small quantity of guncotton in the bottom of the tube is ignited by the flame from the fuse, and in turn ignites the bursting charge. The balls, of lead hardened with antimony, are 252 in number. Each ball is 49/100 of an inch in diameter and weighs approximately 167 grains, or 42 to the pound. After the balls are inserted a matrix of mono-nitro-naphthalene is poured into the case, filling the interstices between the balls in the lower half of the case. When cool this substance is a waxy solid. It gives off a dense black smoke in burning. The purpose of its introduction is to render the burst of the shrapnel visible from the gun so that the gun commander may determine whether his projectiles are attaining the desired range. Resin is used as the matrix in the forward half of the case.

The matrix forms a solid mass with the balls and prevents their deformation by the pressure that they would exert upon each other, on the shock of discharge in the gun, if they were loose in the case. Resin gives better support to the balls than naphthalene and therefore no more of the naphthalene is used than is necessary to produce the desired amount of smoke.

On being expelled from the case the matrix burns and breaks up, leaving the balls free.

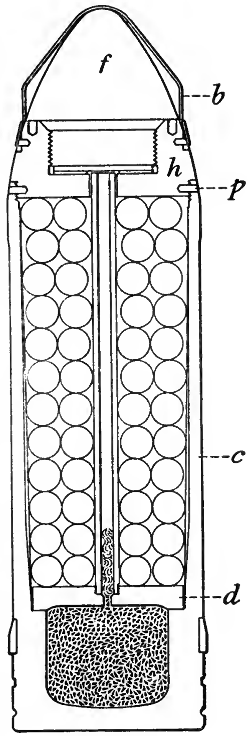


FIG. 174.

To prevent rotation of the contained mass in the case the interior of the case is fluted lengthwise, so that its cross section is as shown in Fig. 175; and to reduce the friction to a minimum, particularly in the chamber for the bursting charge, the interior of the case is coated with a smooth asphalt lacquer.

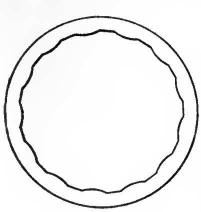


FIG. 175.

The head, *h*, of steel is given a cellular form to make it as light as possible. The weight of the projectile complete is fixed at 15 lbs., and weight is saved as far as possible in all parts of the case in order that the greatest number of balls may be carried. The head is screwed into the body and fixed by two brass pins, *p*. The combination time and percussion fuse, *f*, is screwed into the head. It is protected against injury or tampering by the spun brass cap, *b*, soldered on to the head of the projectile.

The projectile is fixed in the cartridge case as explained for the canister.

Shrapnel forms 80 per cent of the ammunition supply of the field gun.

261. The Bursting of Shrapnel.—When the shrapnel bursts the balls are expelled forward with increased velocity, and as they have at the same time the movement of rotation of the projectile they are dispersed more or less to the right and left. Their paths form a cone, called the cone of dispersion, about the prolongation of the trajectory. The section of this cone at the ground is an irregular oval with its longer axis in the plane of fire. The dimensions of the area will vary, as is evident from Fig. 176, with the

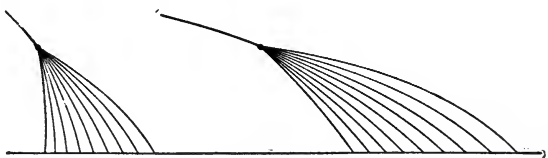


Fig. 176.

angle of fall, the height of burst, and the relation between the velocities of translation and rotation at the moment of burst.

It is assumed that when a shrapnel ball has an energy of 58 foot pounds it has sufficient force to disable a man, and with 287 foot pounds of energy it will disable a horse. These energies correspond in the service shrapnel bullet to velocities of about 400 and 880 foot seconds. An increased velocity of from 250 to 300 feet is imparted to the balls by the bursting charge. Knowing the velocity of the projectile and the weight of the balls the space within which the balls will be effective may be determined for any range.

Point of Burst.—The best point of burst for a shrapnel is assumed to be that point from which the burst of the shrapnel will produce practically one hit per square yard of vertical surface at the target. It is determined from the cone of dispersion by finding the right section that contains as many square yards as there are bullets in the shrapnel. The distance in front of the target at which the burst occurs is called the *interval of burst*. On account of the variation at different ranges in the velocities of translation and of rotation the interval of burst which will produce one hit per square yard of vertical surface at the target varies with the range, decreasing as the range increases.

Practically it is found best to consider the height of burst rather than the interval of burst, since the battery commander can more readily estimate the height than the interval. Suitable cross hairs in the field of the battery commander's telescope facilitate this estimation.

In our service a height of 3/1000 of the range, called 3 *mils*, is adopted as the most favorable mean height of burst. The point of burst at this height gives, over a large part of the range, very approximately the correct interval of burst. For short ranges this height of burst is excessive, and for long ranges it is insufficient.

The following table shows for the 3-inch shrapnel the results obtained at different ranges from bursts at the correct interval of burst, and also at a height of burst of 3 mils. The front of target that should be covered depends upon the number of balls in the shrapnel. For the 3-inch shrapnel with 270 bullets, a former model, the front to be covered with one hit per square yard is 18.5 yards.

Range.	One Hit per Square Yard.		Height of Burst, 3 Mils.	
	Interval.	Front Covered.	Interval.	Front Covered.
Yards.	Yards.	Yards.	Yards.	Yards.
1000	81.4	18.5	118.2	27.0
2000	73.0	18.5	83.4	21.2
2500	68.98	18.5	73.5	19.55
3000	65.84	18.5	66.6	18.76
3500	63.28	18.5	60.9	18.84
4000	61.07	18.5	56.4	17.12
4500	58.97	18.5	51.3	16.13

It will be observed that between 2000 and 4500 yards the height of burst of 3 mils gives approximately the desired density of fire at the target. At ranges less than 2000 yards the front covered is largely increased and the density of fire therefore diminished.

The figures refer to a single shrapnel bursting at the mean point of burst. In a group of shrapnel the bursts above and below the mean point would largely make up the discrepancies in distribution and density.

FUSE.—The fuse used in the shrapnel is the combination time and percussion fuse of which a full description will be found in the chapter on fuses. The fuse is arranged in such a manner that if the projectile is not burst in flight it will be burst soon after impact, a short time being allowed by the delay element in the fuse, during which the projectile may rise on a graze and its burst be accomplished in the air.

The fuse is also constructed to permit of using the shrapnel as canister. When the fuse is set at zero of the time scale, the projectile will burst within 25 feet of the muzzle of the gun.

262. **Shot and Shell.**—Solid shot are no longer used in modern cannon except for target practice, at least in our service. Certain hollow projectiles with thick walls designed principally for the perforation of armor are denominated shot to distinguish them from shell, which name is given to thinner walled projectiles that have not as great a penetrative power but carry larger bursting charges, and have consequently greater destructive effect after penetration.

Shell were formerly made of cast iron, being cast in one piece and subsequently bored for the fuse, Fig. 177.

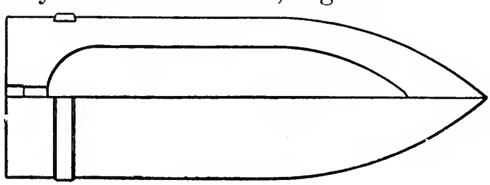


FIG. 177.

With the adoption of high explosives for bursting charges, greater strength in the walls of shell became desirable in order to insure against accidental explosion of the projectile while in the gun. With the exception of some of the projectiles for guns of minor caliber in which black powder is used for the bursting charge, all projectiles are now made of forged steel.

Fig. 178 represents a steel shell for the 5-inch siege rifle. The steel projectiles for mountain, field and siege artillery are similarly constructed.

The base of the shell is closed by a steel base plug, *p*, which is screwed in after the explosive charge has been packed in the projectile. The plug is bored and tapped for the base fuse, *f*, which when inserted is flush with the rear surface of the projectile. The wrench holes in base plug and in head of fuse are filled with lead in order to make a continuous bearing surface for the copper cup, *c*. The cup is applied to the base of the shell to prevent the powder gases in the gun from penetrating to the interior of the projectile by way of the joints of the screw threads. The edge of the cup

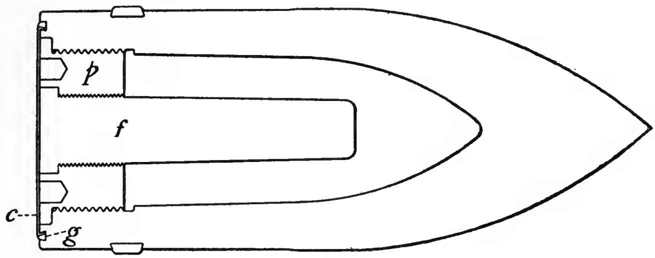


Fig. 178.

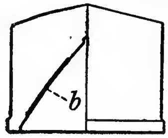
fits into the circular undercut groove, *g*, and the joint there is sealed and the cup held in place by lead wire hammered in.

Armor Piercing Projectiles.—Armor piercing projectiles are of the same general construction as the steel shell just described. Their distinguishing feature is a soft metal cap embracing the point of the projectile for the purpose of increasing the power of the projectile in the perforation of hard armor.

The head and point of an armor piercing projectile are extremely hard, the hardness being attained in the process of manufacture by any one of several secret tempering processes. The metal of the projectile before being subjected to the secret process has a tensile strength of about 85,000 pounds per square inch, which is undoubtedly increased by the tempering. The cap, on the other hand, has a tensile strength not exceeding 60,000 pounds, with a large percentage of elongation, and reduction of area, as may be seen in the table on page 165. The metal of the cap is therefore very soft compared with the metal in the head of the projectile.

A 10-inch armor piercing shot is shown in Fig. 179 and a 10-inch shell in Fig. 180.

The shot has thicker walls and head, and a less capacity for



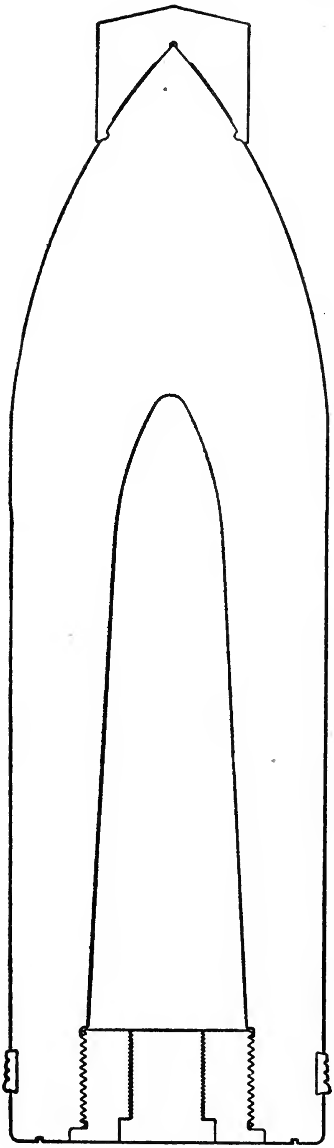


Fig. 179.
10-in. Armor Piercing Shot.

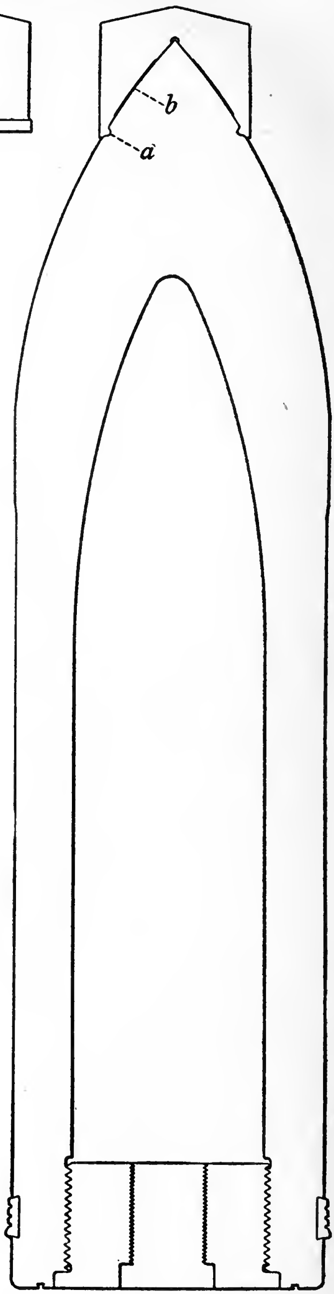


Fig. 180.
10-in. Armor Piercing Shell.

the bursting charge. The outer diameters of the two projectiles are the same, and the weight of each when ready for firing is the same, 604 pounds. To maintain uniformity of weight the shot is made about $4\frac{1}{2}$ inches shorter than the shell.

The cap is fixed to the head of the projectile by means of the circular groove, *a*, cut around the head of the projectile. The cap before affixing is of the shape shown half in section and half in elevation in the figure between the projectiles. A shallow recess, *b*, is filled with graphite to lubricate the projectile as it passes through the cap and armor. To fasten the cap, the projectile with the cap on its point is put in a lathe, and the excess metal at the base of the cap is hammered into the groove of the projectile by means of pneumatic hammers.

In naval projectiles the caps are sometimes fastened on by passing two wires through holes drilled in the cap and notches cut in the projectile.

263. Action of the Cap.—The soft steel cap increases the power of penetration to the projectile in hard faced armor, at

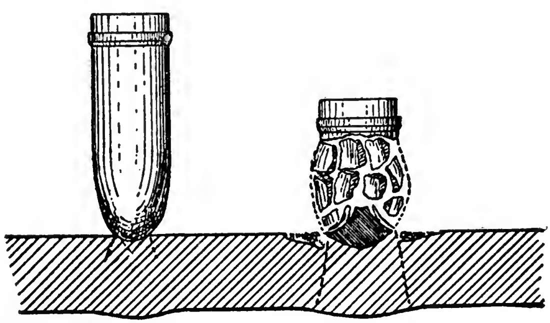


Fig. 181.

normal impact and up to an angle of 30 degrees from the normal, about 15 per cent with respect to the velocity of the projectile and more than 20 per cent with respect to the thickness of plate.

Among the several theories advanced as to the action of the cap, the following appears the most satisfactory.

When an uncapped projectile strikes the extremely hard face of a modern armor plate, the whole energy of the projectile is applied at the point, and the high resistance of the face of the plate puts upon the very small area at the point of the projectile a stress greater than the metal can resist, however highly tempered it may be. The point is therefore broken or crushed and the head of the projectile flattened, Fig. 181. The flattening of the head brings loss of penetrative power, and the energy of the projectile is expended largely in shattering the projectile itself. The head of the projectile adheres to the plate and is practically welded to it.

The effect on a plate of thickness equal to the caliber of the projectile may be the partial or complete punching out of a cylindrical piece, Fig. 182. But even if the plate is completely perforated, the projectile does not get through as a whole; and behind the plate are found only fragments of the projectile and of the metal forced from the plate.

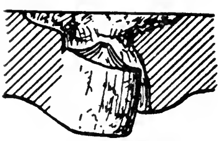


FIG. 182.

When a projectile provided with a cap strikes a hard faced plate, the pressure due to the resistance of the plate is not confined simply to the point of the projectile, but is distributed uniformly over a comparatively large cross section. In

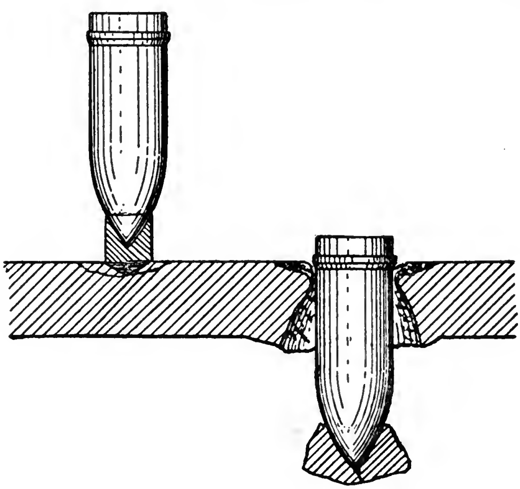


FIG. 183.

addition the point of the projectile is firmly supported on all sides by the metal of the cap. As a consequence the point is not deformed, and passing easily through the cap it finds the hard face of the plate dished and severely strained and more or less crumbled by the impact of the cap. The unexpended energy of the projectile forces the point through the weakened face and through the softer metal of the back.

The face of the plate is crumbled, and a conical hole made through the softer metal, through which the projectile passes practically intact and in condition for effective bursting, Fig. 183.

The form of the cap has not apparently a great effect on the results. Many different shapes are used by different manufacturers, some of which are shown in Fig. 184.

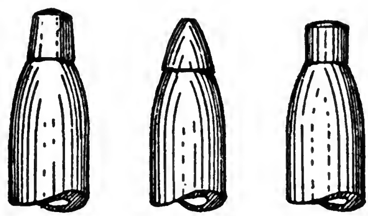


FIG. 184.

The cap increases *the biting angle* of the projectile, the limiting angle of impact at which the projectile will perforate the plate.

The following results have been obtained in comparative tests of capped and uncapped projectiles against tempered nickel steel plates. The angle of impact is measured from the normal to the plate.

Gun.	Thickness of Plate.	Angle of Impact.	Striking Velocity.	Projectile.	Effect.
	Inches.	Degrees			
8-inch rifle......	3.5	60	1074	Capped	Perforated plate
	......	60	1073	Uncapped	Indented plate ½ inch
	......	65	1066	Capped	Perforated plate
	......	65	1077	Uncapped	Indented plate 1⅛ inches
12-inch mortar...	4.5	40	711	Capped	Nearly perforated. Indentation 6 inches deep. Fragment nearly punched out
	......	40	711	Uncapped	Glanced from plate. Indentation 1¾ inches deep

It is stated that the addition of the cap to the projectile and the consequent moving of the center of gravity of the projectile toward the point favorably influences the trajectory, increasing both the accuracy and range.

All projectiles for seacoast guns above 3 inches in caliber will probably be provided with caps.

264. Deck Piercing and Torpedo Shell.—These projectiles are provided for the 12-inch mortars. The torpedo shell is longer and of greater interior capacity than the deck piercing shell, and carries a larger bursting charge of high explosive. The bursting charge for the deck piercing shell is 64 pounds and for the torpedo shell 134 pounds.

Latest Form of Base of Shell.—A form of base with which good results have been obtained is shown in Fig. 185. The metal of the shell is cut away, beginning at a short distance behind the band, leaving only a narrow ring to support the band. In the perforation of armor the band and the supporting ring are sheared off, thus relieving the projectile of the resistance due to the greater diameter of the band.

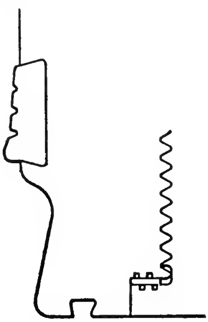


Fig. 185.

Shell Tracers.—Experiments are now being conducted toward the development of a projectile that will indicate its line of flight by the emission of flame, or by the emission of some substance that will be visible from the gun; the purpose of the projectile being to enable the gun commander to follow the flight of a projectile from his gun and thus determine whether the gun is properly directed.

The tracer for use at night consists of a short metal cylinder filled with a slow burning substance that emits a bright flame during the flight of the projectile through the air. It may be screwed into a seat prepared in the base of any projectile. Ignition of the compound occurs in the gun.

For day tracing a special shell is prepared. The cavity of the shell is partly filled with a mixture of lampblack and water, the mixture having the consistency of thick paint. A small orifice is made through the base of the projectile on one side. The powder gases enter this orifice under the pressure in the gun, and filling the cavity in the shell force from the orifice during flight a spray of black liquid. In recent experiments the flight of a 6-inch day tracing shell was followed for over 7200 yards.

Hand Grenades.—The hand grenade is a metal bomb filled with high explosive and provided with one or more percussion caps or fuses, which cause its explosion on striking after being thrown. Hand grenades were effectively used by both sides in the Russo-Japanese war.

265. Volumes of Ogival Projectiles.—Assume a solid cylinder, Fig. 186, of the length and diameter of a given solid shot.

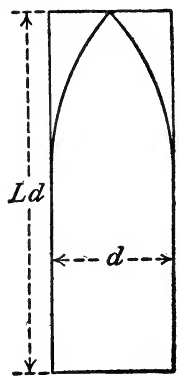


FIG. 186.

Let d represent the diameter of the shot, usually taken as equal to the caliber of the gun,

L, the length of the shot in calibers.

The volume of the cylinder is $(\pi d^2/4)Ld$.

Let B represent, *in calibers*, the length of a cylinder whose diameter is d and whose volume, $(\pi d^2/4)Bd$, is equal to that part of the cylinder in Fig. 186 that is outside the shot.

Subtracting this volume from the volume of the whole cylinder and representing by V_s the volume of the solid shot, we have

$$V_s = \frac{\pi d^2}{4}(L-B)d = \frac{\pi d^3}{4}(L-B)$$

$(L-B)d$, or $L-B$ calibers, is the length of a solid cylinder whose diameter is the diameter of the shot and whose volume is equal to the volume of the shot. $L-B$ is called the *reduced length* of the projectile in calibers, as it is the length of a cylinder of equal diameter and volume.

B is a function of the radius of the ogive expressed in calibers. Its value, obtained by means of the calculus, is given by the equation

$$B = 2n^2(2n-1)\sin^{-1}\frac{\sqrt{4n-1}}{2n} - \frac{6n^2-2n-1}{3}\sqrt{4n-1}$$

in which n is the radius of the ogive in calibers. When $n=2$, the usual radius of head in seacoast projectiles, $B=0.58919$.

For cored shot the reduced length is less than for solid shot by the length of the cylinder whose volume is that of the interior cavity. Representing by B' the length of this cylinder in calibers, the solid volume of the cored shot, or volume of the metal, is given by the equation

$$V_c=\frac{\pi d^3}{4}\{L-(B+B')\}$$

Weights of Projectiles.—Representing the reduced length by l, and dividing the expression for the volume of one projectile by a similar expression for another, we have

$$V_s/V_s'=d^3l/d'^3l'$$

Since the weights are proportional to the volumes:

The weights of ogival projectiles are proportional to the products of the cubes of their diameters by their reduced lengths.

The weights of ogival projectiles of the same caliber are proportionate to their reduced lengths.

As the standard projectiles for most of our guns are similar, their dimensions when expressed in terms of the caliber are the same. The reduced length is therefore the same for all these projectiles, and the weights of the projectiles are proportional to the cubes of the calibers.

266. Thickness of Walls.—The maximum stress sustained in the gun by the walls of a cored projectile, at any section of the projectile, is due to the pressure to which the walls are subjected in transmitting to that part of the projectile in front of the section the maximum acceleration attained in the gun. The maximum acceleration is due to the maximum pressure in the gun; and this pressure being known the acceleration is determined by dividing the pressure by the mass of the projectile.

$$\alpha=P/M=Pg/w$$

α being the acceleration, P the total maximum pressure on the base of the projectile, and w the weight of the projectile. Substituting the values of the known quantities α may be determined.

α being known, if we substitute for w the weight of that part of the projectile in front of the given section and solve the equation for P, the value obtained, which we will call P_1, will be the pressure sustained by the walls of the section. The area of the section is $\pi(R^2-r^2)$. The pressure per unit of area is therefore P_1 divided by $\pi(R^2-r^2)$.

This pressure must not exceed the elastic limit of the metal for compression, divided by a suitable factor of safety; nor must it cause excessive flexure in the walls. If it does the walls must be made thicker.

Thickening the walls will increase the weight in front of the section and therefore a new value of w must be obtained for a second determination.

In shrapnel it is desirable to make the walls as thin as possible in order to increase the number of bullets that may be carried. The longitudinal pressure of the contained bullets is borne by the thicker base of the projectile, and the walls sustain only the pressure due to the centrifugal force and that proceeding from the weight of the head and fuse. Their thickness will therefore be determined by the requirement that they must resist rupture by the pressure exerted by the gases from the bursting charge when the head of the projectile is blown off. The pressure required to blow off the head is equal to the resistance offered to shearing by the screw threads and shear pins of the head.

A much greater thickness of wall than is needed in the gun is required to enable a projectile to withstand the shock of impact on the face of an armor plate. The retardation in this case is much greater than the acceleration in the gun and consequently the stresses on the walls are correspondingly greater. As there is no means of determining the retardation at impact, the proper thickness of walls of armor piercing projectiles cannot be calculated, but must be determined by experiment.

We may, however, by assuming that the plate offers a constant resistance to the penetration of the projectile, determine the thickness of wall necessary in the projectile to enable it to pass through the plate and have any required velocity on emerging.

Thus, to determine the thickness of wall of an armor piercing shell that is required, with a striking velocity v, to perforate an armor plate of given thickness and to have on emerging a remaining velocity v_1.

Let S be the constant resistance offered by the plate
l the thickness of the plate, in feet,
α the constant retardation of the projectile during penetration.

The work performed by the resistance over the path l is equal to the energy abstracted from the projectile while traversing this path. Therefore

$$Sl=\frac{M}{2}(v^2-v_1^2) \qquad S=\frac{M}{2l}(v^2-v_1^2)$$

The retardation due to the resistance is equal to the resistance divided by the mass. Therefore

$$\alpha=\frac{S}{M}=\frac{v^2-v_1^2}{2l}$$

The pressure sustained by any section of the projectile during penetration is equal to the mass of that portion of the projectile behind the section multiplied by the retardation. Denoting by w' the weight of that part of the projectile behind any given section, we have for the pressure sustained per unit of area at the section

$$p=\frac{w'}{g}\frac{\alpha}{\pi(R^2-r^2)}=\frac{w'(v^2-v_1^2)}{2lg\pi(R^2-r^2)}$$

R and r must be given such values, that is, the thickness of the walls must be such that p will not exceed the elastic limit of the metal for compression, or that the flexure of the walls, considering the shell as a hollow column, will not be sufficient to cause rupture.

267. Sectional Density of Projectiles.—It has been found by experiment, as explained in exterior ballistics, that the *retardation* in the velocity of a fired projectile, due to the resistance of the air, is expressed by an equation that, for any fixed atmospheric conditions and standard form of projectile, may be put in the form

$$R=A\frac{d^2}{w}f(v)$$

R representing the *retardation*, A a constant, d the diameter of the projectile, w its weight, and $f(v)$ some function of its velocity.

For a given velocity it is apparent that the retardation will increase directly with the square of the diameter of the projectile and inversely with its weight; or, more concisely, the retardation will increase directly with the fraction d^2/w.

The reciprocal of this fraction, or w/d^2, will therefore be the measure of the capacity of the projectile to resist retardation, that is, to overcome the resistance of the air.

The fraction w/d^2 is called the *sectional density* of the projectile. $w/\frac{1}{4}\pi d^2$ is the weight of the projectile per unit area of cross section, and w/d^2 is taken as the measure of this weight, $\pi/4$ being constant.

The sectional density is of importance in considering the motion of the projectile both in the air and in the gun.

Effect on the Trajectory.—The greater the sectional density of the projectile, the less the value of its reciprocal, the factor d^2/w in the above equation, and consequently the less is the value of the retardation of the projectile.

Of two projectiles fired with the same initial velocity and elevation, the projectile with the greater sectional density will therefore lose its velocity more slowly and will attain a greater range. For any given range it will be subjected for a less time to the action of gravity and other deviating causes, and will therefore have a flatter trajectory and greater accuracy.

The advantages of increased sectional density are therefore increased range, greater accuracy, and a flatter trajectory.

The sectional density may be increased by increasing the weight of the projectile or by decreasing its diameter. The weight of a projectile for any gun may be increased by increasing its length. This has been done with modern projectiles for large guns until the length is from $3\frac{1}{2}$ to 4 calibers. In small arms the weight is increased by the use of lead in the bullet. Increase in sectional density by decrease in diameter is found in the modern small arms of reduced caliber, the weight and diameter of the projectile having been reduced in such proportions as to increase its sectional density.

Effect on the Gun.—An increase in the weight of the projectile requires an increased pressure in the bore of the gun if the initial velocity is to be maintained. The maximum pressure for any gun being fixed, it has been possible to increase the weight and sectional density of projectiles only by the use of improved powders, which while they exert no greater maximum pressures exert higher pressures along the bore of the gun. The mean pressure on the projectile is therefore greatly increased, and to withstand the increased pressure the chase of the gun is made stronger.

MANUFACTURE OF PROJECTILES.

268. Cast Projectiles.—A wooden pattern of the shape of the projectile is first made, the dimensions of the pattern being slightly greater than the dimensions desired in the projectile, in order to allow for contraction of the metal in cooling. The pattern is in one or more parts, depending upon its size. The pattern shown in Fig. 187 is in two parts separated at the line *b*. The parts are slightly coned from this line to facilitate withdrawal from the mold. For hollow projectiles a core box is also made similar in its interior dimensions to the cavity in the shell. The core, *e* Fig. 187, made of core sand mixed with adhesives, is formed in the core box around a hollow metal spindle wound with tow. The heat of the casting burns the tow, and the gases from the core pass out through the hollow spindle.

Fig. 188 shows a mold prepared for casting a shell. The outer box, called the flask, is in two sections parting at the line *xy*. In the lower part the sand is molded around the pattern, which is also divided into two parts on the same line. In the upper part of the flask the remainder of the mold is made and the core attached in its proper position by means of the frame *a* bolted to the flask. The gate *b* and the riser *c* are also formed in the mold, the riser being considerably greater in diameter than shown in the figure. The patterns are withdrawn and the parts of the mold brought together and bolted.

The molten metal enters through the gate *b*, generally in a tangential direction, so that the metal in the mold has a circular motion which assists in the escape of the gases and brings the impurities to the center and top. The mold is filled with the metal to the top of the riser, where the impurities collect. The pressure of the liquid metal in the riser assists in making the casting sound, and affords a means of adding molten metal as the casting shrinks in cooling.

Solid shot are cast head down in order that the dense metal may be in the head of the shot. Shells are cast base down, that the base of the shell may be sound and free from cavities that would allow the powder gases to pass into the interior and ignite the bursting charge.

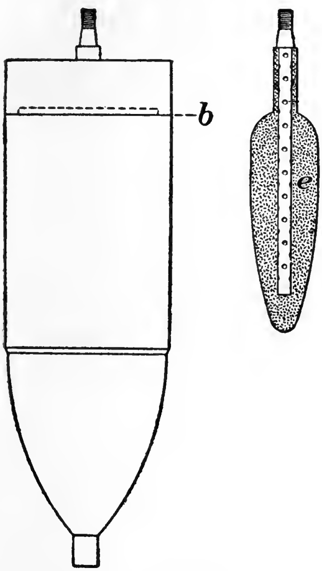


FIG. 187.

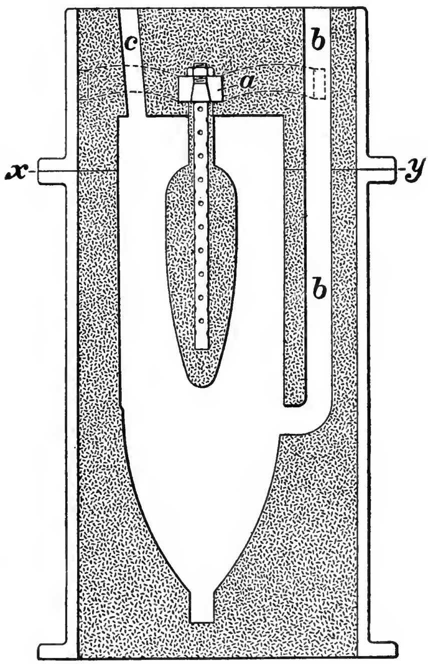


FIG. 188.

Chilled Projectiles.—For use against wrought iron armor the heads of cast projectiles were hardened in casting by the process of chilling. A comparatively thin iron mold the shape of the head and in contact with it was fixed in the sand around the head of the projectile. This served to rapidly conduct the heat away from the head of the projectile, causing it to cool rapidly and giving it great hardness. These projectiles are no longer used.

Forged Projectiles.—The steel for a forged projectile is cut from a cast ingot, and is then bored, forged, and turned to finished dimensions. Armor piercing projectiles are in addition treated with some secret process of tempering to give them the hardness and toughness necessary for the perforation of armor.

269. Requirements in Manufacture.—The qualities of the metal of the projectile are prescribed as follows: For cast iron, tensile strength 27,000 lbs. per square inch; for steel, in what are called common shell, that is, those of the smaller calibers, tensile strength 85,000 lbs. For armor piercing projectiles the tensile strength or elastic limit is not specified, further than by the requirement that the projectiles in a lot shall not vary in tensile strength by more than 20,000 lbs. The strength of these shells is determined by actual firing against armor. The cap must be of steel whose tensile strength does not exceed 60,000 lbs., with an elongation at rupture of 30 per cent, and a reduction in area of 45 per cent.

The base plugs of all projectiles are made of forged steel.

Inspection of Projectiles.—The dimensions of the projectiles are tested by means of calipers, and profile and ring gauges. The slight variations, called *tolerances*, allowed from the standard dimensions are specified for each dimension, and the gauges for any projectile are constructed for the maximum and minimum of the particular dimension. Thus for the diameter of the band there are two ring gauges, one a maximum, the other a minimum, and similarly for other diameters. Maximum and minimum plug gauges are applied to the threads of the fuse hole. A ring gauge is shown in Fig. 189. A profile gauge or templet is shown at *a* in Fig. 190.

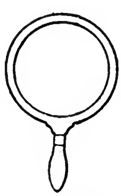


Fig. 189.

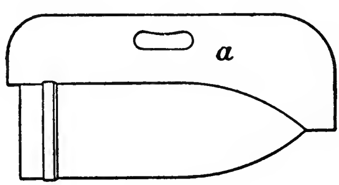


Fig. 190.

Eccentricity in the cavity of the projectile is determined by rolling the projectile along two rails, *a* Fig. 191, placed on a flat surface. Irregular movement of the projectile denotes eccentricity, which may be measured by means of the calipers, *d*, shown in the figure.

For the detection of holes or cracks through the walls of hollow projectiles all such projectiles are subjected to an interior hydraulic pressure. A pressure of 500 lbs. per sq. in. is applied for one minute to steel projectiles, and a pressure of 300 lbs. for two minutes to those of cast iron.

To determine whether the treatment received by the armor piercing shot in the tempering process has left in the shot initial strains that might cause rupture in store or in firing, these shot are cooled to a temperature of 40 degrees F. and then suddenly heated

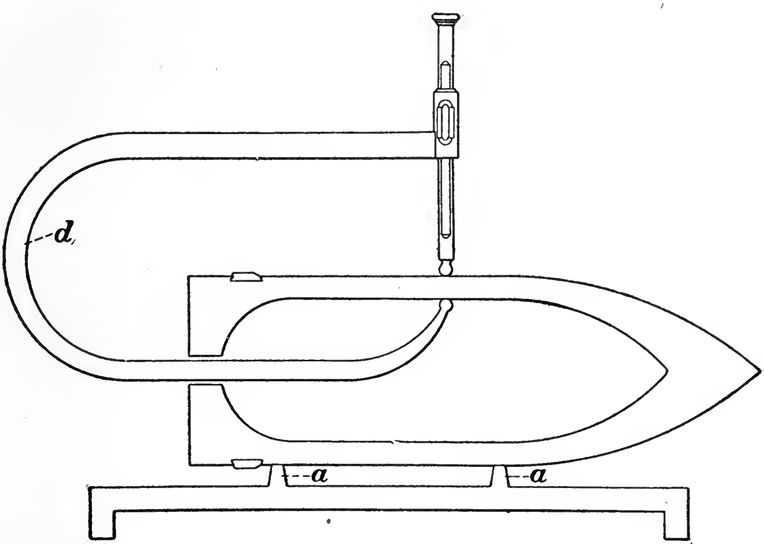


Fig. 191.

by being plunged into boiling water. When thoroughly heated by the water, the projectile is suddenly cooled by being half inserted, with its axis horizontal, in a bath of water at 40 degrees F. After a brief interval it is turned 180 degrees for a like immersion of the other half. Three days must elapse after the tempering of the projectile before this test is applied. The necessity of the test is indicated by the not infrequent bursting of the projectiles in the shops after tempering. This test is not applied to armor piercing shell. The thinner walls of these projectiles are more uniformly affected by the tempering process.

The interior walls of hollow projectiles are coated with a lacquer of turpentine and asphalt for the purpose of making them smooth and of reducing the friction between the walls and the bursting charge.

Ballistic Tests.—Each class of projectile is subjected to a allistic test under conditions assimilating the conditions of service. For the purpose of the test two or more projectiles are selected from each lot presented. The projectiles tested are filled with sand in place of a bursting charge, and after the test must be in condition for effective bursting.

Armor piercing shot are fired against hard faced Krupp armor plate, from 1 to 1⅓ calibers thick, secured to timber backing. The striking velocities of the shot from 8, 10, and 12 inch rifles against plates one caliber thick are near to 1750 feet, which corresponds to ranges of about 3000, 4000, and 5000 yards, respectively, from the three guns. The shot is required to perforate the plate unbroken and then be in condition for effective bursting.

Armor piercing shells must meet similar conditions, the thickness of the plate being one half the caliber of the shell, and the striking velocities, 1420 f. s. for 5-inch shell, 1220 f. s. for 6-inch shell, and 920 f. s. for 8-, 10-, and 12-inch shell.

12-inch deck piercing shell must perforate a 4½-inch nickel steel protective deck plate at an angle of impact of 60 degrees.

12-inch torpedo shell are fired into a sand butt from a gun in which the chamber pressure must be 37,000 lbs.

Common steel shell for seacoast guns of small caliber are tested with service velocities against tempered steel plates from 3 to 5 inches thick, depending on the caliber and service velocity of the projectile.

The shell for field and mountain guns are fired into sand, with a pressure in the gun 12 per cent greater than the service pressure and with at least the service velocity.

Tests are also made to determine whether the fragmentation of the projectile on bursting is satisfactory.

The Painting of Projectiles.—Projectiles are so painted as to indicate the metal of which they are formed and the character of the bursting charge. The greater part of the body is black. A broad colored band around the projectile over the center of gravity indicates by the color whether the projectile is of iron, cast or chilled, or of steel, cast or forged.

The color of the base indicates whether the projectile is charged with powder or with high explosive. In assembled ammunition the base color is painted in a band just above the band of the projectile.

CHAPTER XI.

ARMOR.

270. History.—The use of armor for the protection of ships of war began in France in 1855 and soon became general. The first armor was of wrought iron. This metal opposed a sufficient resistance to the round cast iron projectiles of that time and to the elongated cast iron shot of a later date. As the power of guns increased and chilled projectiles came into use wrought iron armor became ineffective. It was replaced about 1880 by compound armor, which consisted of a wrought iron back and a hard steel face. Compound armor was made either by running molten steel on the previously prepared wrought iron back or by welding a plate of steel to another of wrought iron by running molten steel between them, both plates being previously brought to a welding heat. The hard steel face opposed a great resistance to penetration of the shot and caused the shot to expend its energy in shattering itself. At the same time it distributed the stress over an increased section of the iron back, and the toughness of the wrought iron served to hold the plate together. The chief defect of the compound plate was due to the difficulty of obtaining intimate union between the two metals, and lay in the tendency of the steel face to flake off over considerable areas. The basic principle of this armor, the hard face and the tough back, is still maintained in the construction of the most modern armor.

NOTE.—This chapter is largely derived from the chapter on armor by Lieutenant Commander Cleland Davis, U. S. Navy, in Fullam and Hart's Text Book of Ordnance and Gunnery, 1905.

At the same time that the compound plate was used by Great Britain and other powers the all steel plate was being used by France, the effectiveness of the two plates being about equal.

In 1889 the homogeneous nickel-steel plate, markedly superior to the steel plate in toughness and resisting power, was introduced. The Harvey treatment of the nickel-steel plate, developed in the United States in 1890, still further increased the resisting power of armor, and in 1895 the Krupp process followed with further improvement.

Harvey and Krupp Armor.—The principle employed in the manufacture of armor by these two processes is the same. In both, the face of the plate is made extremely hard by supercarbonization and subsequent chilling. The superiority of the Krupp plate appears to be due to the composition of the steel. The Harvey plate is made of a manganese nickel steel, while in the Krupp plate chromium is also present, and in greater quantity than the manganese. The composition of the two plates, in percentages, is given as follows:

	C.	Mn.	Si.	P.	S.	Ni.	Cr.
Harvey....	0.30	0.80	0.10	0.04	0.02	3.25	0.00
Krupp	0.35	0.30	0.10	0.04	0.02	3.50	1.90

The nickel, and to a certain extent the manganese, give great strength and toughness to the metal, while the chromium makes the metal more susceptible to the treatment that gives the desired qualities to the finished plate. First, it permits the attainment of a very tough fibrous condition throughout the body of the plate that makes it less liable to crack; second, it gives the metal an affinity for carbon which enables supercarbonization to a greater depth; third, it increases the susceptibility of the metal to tempering, which gives a greater depth of chill. These are the qualities that mark the superiority of Krupp armor.

Even when carbonization of the plates is effected in the same manner, carbon will be absorbed to a greater depth in the Krupp than in the Harvey armor, giving a greater depth of hardened face and an increased resistance to penetration of about 20 per cent.

271. Manufacture of Armor.—The steel, of proper composition, is made in the open hearth furnace and cast into an ingot of the shape shown in Fig. 192. The head of the ingot affords a means for the attachment of the chains of the cranes employed in handling it. A long heavy beam is used to counterbalance the weight of the plate when slung in the chains.

When stripped from the mold and cleaned, the ingot is heated in a furnace and then forged, as shown in Fig. 193, under an immense hydraulic press capable of exerting a total pressure of about 15,000 tons. The forging reduces the thickness of the plate

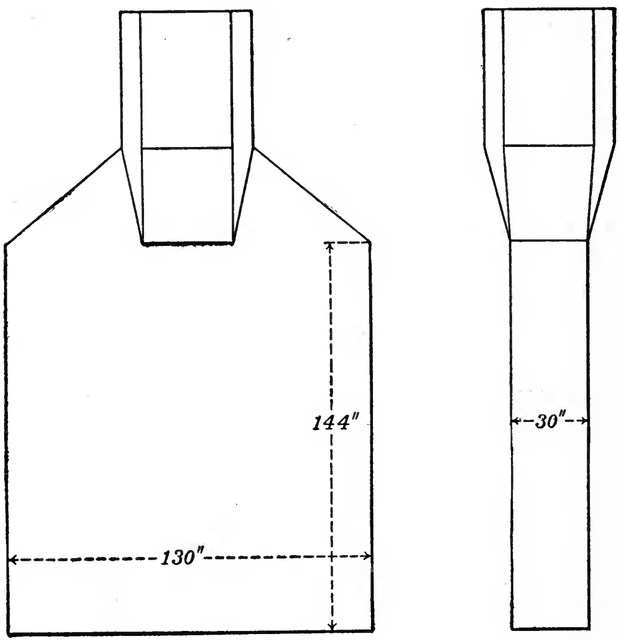


FIG. 192.

and increases its length and breadth. The plate is then rough machined approximately to finished dimensions.

CARBONIZING.—The carbonization of the face of the plate is effected by one of two methods: the cementation process, or the gas carbonizing process. The cementation process consists in covering the surface of the plate with carbonaceous material, usually a mixture of wood and animal charcoal, heating the plate to a temperature of about 1950 degrees, and maintaining it at this temperature for a sufficient time to accomplish the required

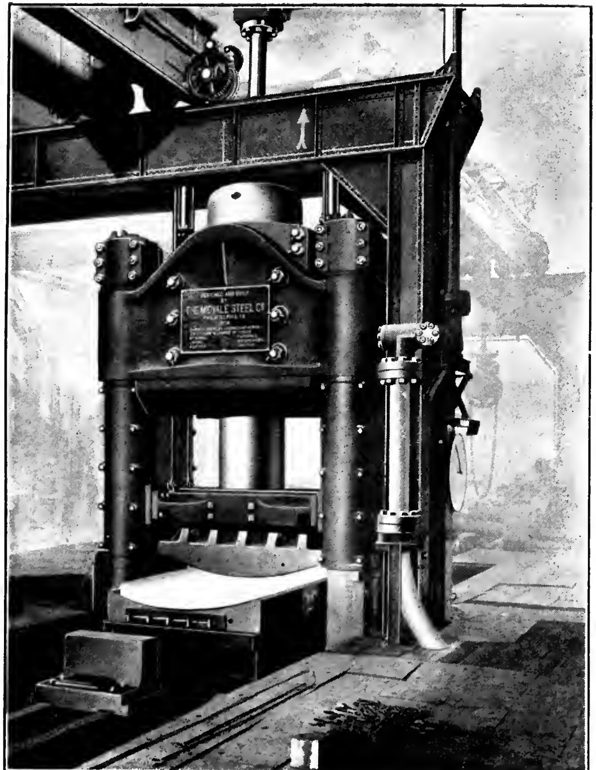


FIG. 194.—9,000-ton Press, Bending Armor.

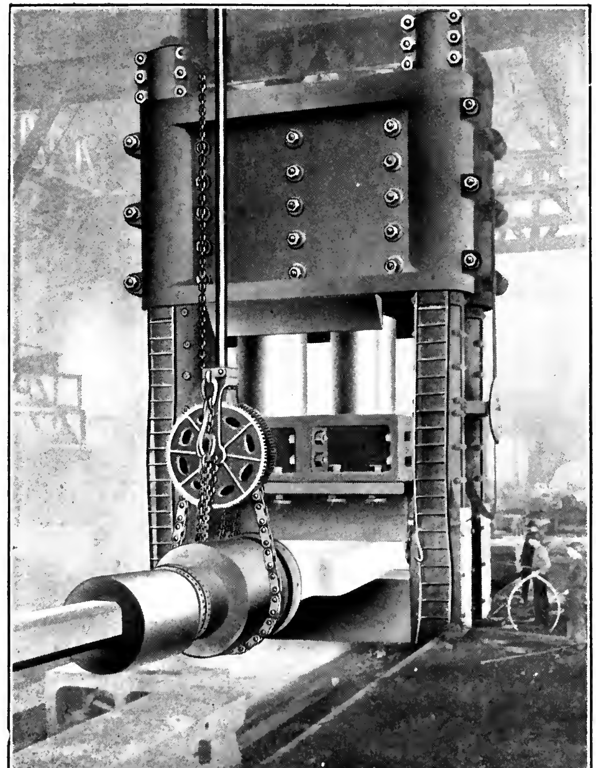


FIG. 193.—15,000-ton Hydraulic Forging Press.

degree of carbonization. A covering of sand protects the face of the plate and the carbonizing material from the flames of the furnace, and excludes the air. From four to ten days, depending on the thickness of the plate, are required to bring the plate to the desired temperature, and a further period of from four to ten days to effect the carbonization of the face. Under the action of the heat the carbon is absorbed into the face of the plate, and penetrates into the interior, the quantity of the absorbed carbon diminishing from the surface inward.

The gas carbonizing process consists in passing coal gas along the face of the plate heated in a furnace to about 2000 degrees. The heat decomposes the gas, which deposits carbon on the face of the plate, and the carbon is absorbed as in the cementation process.

Reforging and Bending.—After being cleaned of the scale that is formed on it in the process of carbonization the plate is reforged to its final thickness. It is then annealed and bent to the desired shape in a hydraulic press. The operation of bending an armor plate in a 9000 ton press is shown in Fig. 194.

Hardening.—For tempering, the plate is uniformly heated to a high temperature and quickly cooled or chilled by cold water sprayed upon it under a pressure of about 23 pounds to the square inch.

In Krupp plates as first made the tempering produced cracks over the whole hard surface of the plate, some of them a quarter of an inch wide and extending some distance into the plate. The cracks were characteristic of the plate and were not considered abnormal, the resistance of the plate even with the cracks being greater than that of plates made by other processes. With improvement in the process of manufacture smoother plates were produced, and in many of the latest plates the surface appears continuous to the naked eye. When etched with acid, however, the face is found to be covered with a network of fine lines and presents an appearance similar to that of crackled glass.

272. Armor Bolts.—The armor plates are fastened to the sides of ships by means of nickel-steel bolts. These are of such strength that they are not broken by the impact of projectiles that badly crack the plate. The bolts pass through the sides of the ship and are screwed into the soft back of the armor plate. To insure a good fit of the plate, and at the same time to lengthen the armor bolt so that its deformation per unit of length under the stresses of impact may not be excessive, wood backing is used between the armor plate and the ship's side. The wood backing is being reduced in thickness and the tendency is to discard it altogether. Figs. 195 and 196 show types of bolts for armor with and without wood backing.

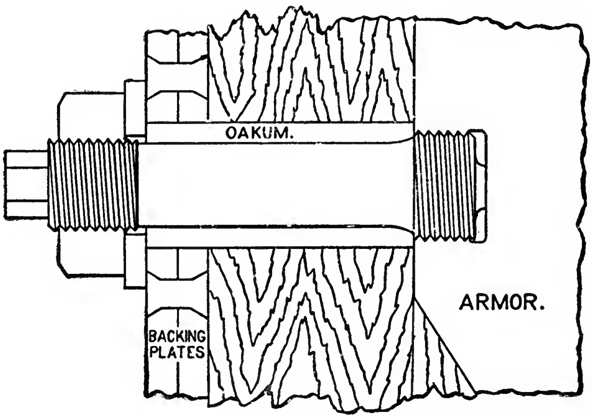


FIG. 195.

The threads on the bolts are all plus threads, so that the bolt is of uniform strength. A calking of marline or oakum surrounds the bolt to prevent leakage through the bolt hole. A steel washer is under the head of the bolt. A rubber washer has also been used under the steel washer to diminish the suddenness of any strain on the bolt head.

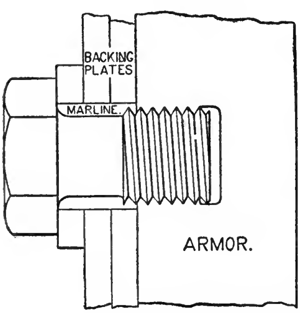


FIG. 196.

Armor bolts vary in diameter from 1.5 inches for plates 5 inches thick or less to 2.4 inches for plates 9 inches thick and upward.

In number they are provided one for every five square feet of surface as far as the framing of the ship will permit.

Ballistic Test of Armor.—The U. S. Navy specifications require as a test, before acceptance of Krupp and Harvey armor, three impacts of capped shells against a specimen plate, with velocities as given in the following table.

Caliber of Gun, Inches.	Capped Projectile, Pounds.	Plate Thickness, Inches.	Striking Velocity, f. s.
6	105	5	1416
6	105	6	1608
6	105	7	1791
7	165	6	1416
7	165	7	1578
7	165	8	1732
8	260	7	1412
8	260	8	1552
8	260	9	1685
10	510	9	1458
10	510	10	1569
10	510	11	1676
12	870	11	1412
12	870	12	1501

The first impact in the center of the plate must not develop a through crack to an edge of the plate, and no part of the projectile shall get entirely through the plate and backing. On the second and third impacts no part of the projectile shall get entirely through the plate and backing. The impacts shall not be nearer than $3\frac{1}{2}$ calibers to each other or to an edge of the plate.

Comparing the requirements for plates attacked by the 8, 10, and 12 inch guns with the requirements of the ballistic tests of armor piercing projectiles for the land service, page 464, it will be seen that the armor plates one caliber thick are tested with velocities about 200 feet less than those at which the projectiles from land guns are required to perforate similar plates.

Characteristic Perforations.—Characteristic perforations in hardened and unhardened armor are shown in Figs. 197 and 198, the front face of the plate being uppermost in each figure. The face of the hardened armor, Fig. 197, breaks and crumbles under impact, while the metal of the unhardened plate, Fig. 198, being softer and more tenacious, flows under the pressure of the projectile in the direction of least resistance and forms a combing in front of the plate. When the projectile reaches the back of the hardened armor the metal of the back, being prevented from flowing by the hard face, breaks out in one or more pieces, leaving

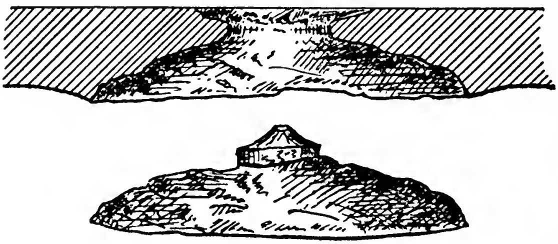


FIG. 197.

a broad based conical hole through the back and producing but slight bulging of the rear surface of the plate.

As the metal of the unhardened plate is of the same constitution throughout, the perforation does not exhibit the marked

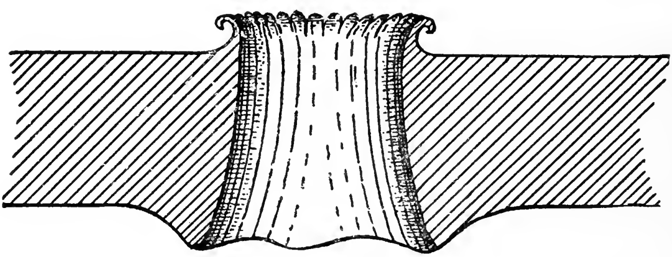


FIG. 198.

differences shown in the hardened plate. The metal of the back part of the plate flows to the rear, producing a greater bulging of the rear surface.

273. **Armor Protection of Ships.**—The armor carried by ships of war is of various thicknesses, depending upon the size and purpose of the ship and on the position of the armor on or in the ship. The thickest armor is used to protect the water line and the vital parts of battleships. The present practice in the United States is to protect the whole length of the water line with a belt of armor 8 feet wide extending $4\frac{1}{2}$ feet above the water line and $3\frac{1}{2}$ feet below it.

This belt, see Fig. 199, has its maximum thickness over that part of the ship that contains the machinery and the magazines. The thickness diminishes from the mid-ship section and is least at the bow and stern.

The gun turrets are protected in front by the thickest armor. Armor of less thickness covers the casemates, barbettes, and sides of the turrets, the thickness depending upon the importance of the part protected and upon its exposure to hostile fire.

An armored deck of a thickness to prevent penetration by the fragments of exploded shell extends the whole length of the ship. This deck, the berth deck, Figs. 199 and 200, is flat over the machinery and boiler spaces and slopes downward at the sides and at the bow and stern to the bottom of the belt armor. On the heaviest ships the armored deck has a thickness of two inches over the flat part and four inches on the slopes, the thickness being reduced over the flat part in order to reduce the weight. The gun deck, next above the armored deck, is sometimes an armored splinter deck one inch thick.

Across the main body of the ship, bow and stern, extends heavy athwartship armor, which, with the armored barbettes and turrets, provides protection to the body of the ship from fire from the front or rear. Thus with the side armor the main body of the ship becomes an armored box, within which the crew, the machinery, the magazines, and the guns are protected.

With the improvements that have taken place in armor within the last fifteen years there has been a gradual reduction in the thickness of armor carried by ships of the various classes.

The battleship Oregon, built in 1893, has a water line belt 18 inches in thickness, while the battleship Connecticut, commissioned in September, 1906, has but 11 inches of armor at her water line.

The arrangement of the armor on the battleship Connecticut is shown in Figs. 199 and 200.

Definitions.—The following definitions will assist toward a ready understanding of the figures.

Turret.—A revolving armored structure in which one or two guns are mounted. The guns revolve with the turret and are completely enclosed with the exception of the chase of the gun,

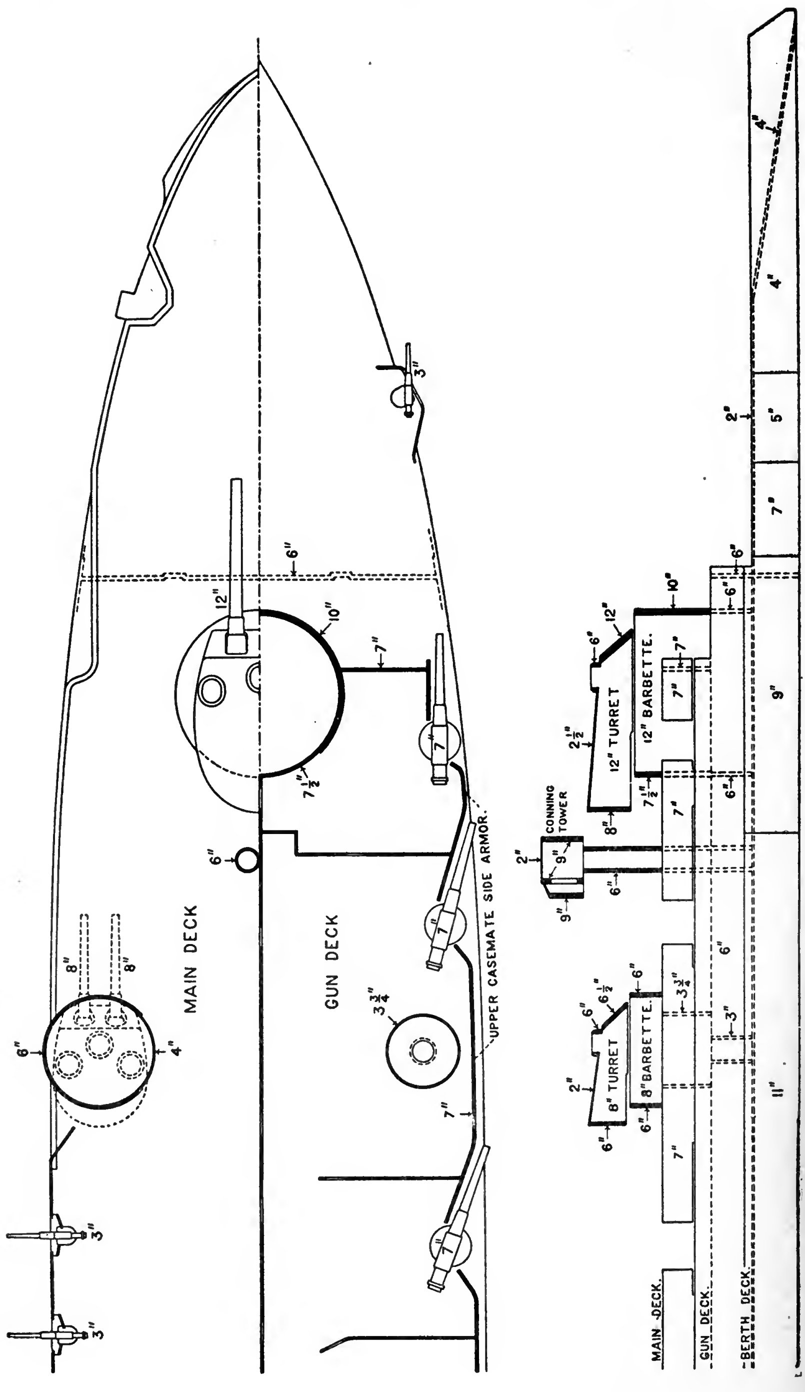


FIG. 199.—Distribution of Armor, United States Battleship Connecticut.

which projects through a port hole in the front plate of the turret.

Barbette.—A fixed circular structure, armored, which protects the mechanism for the ammunition supply of the gun mounted above it and the mechanism of the turret containing the gun.

Casemate.—An isolated gun position for a broadside gun with fixed armor protection. The casemate completely encloses the gun with the exception of the chase, which projects through a port hole.

Central Citadel.—Armor enclosing a series of broadside guns. There may or may not be splinter bulkheads between the guns. With the bulkheads completely enclosing the guns the citadel becomes a series of casemates.

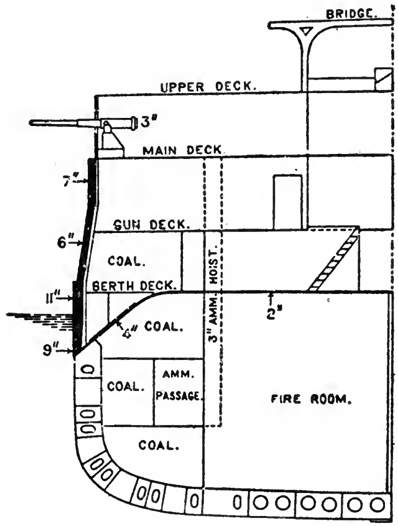


Fig. 200.

274. Chilled Cast Iron Armor.—This armor on account of its thickness and great weight is used only on land. It is manufactured by Gruson of Germany. It is cast in large blocks whose outer faces are made very hard by chilling. The blocks are then built into turrets, usually of rounded shape.

On account of the great weight and hardness of the metal and the rounded shape of the turrets, this armor affords better protection than any other armor.

Gun Shields.—Guns of 6 inches caliber and less mounted in barbette in seacoast fortifications are provided with shields permanently attached to their carriages. The shields are made of Krupp plate $4\frac{1}{2}$ inches thick. The requirements of the ballistic test for these shields are as follows.

The shield, firmly supported by a backing of oak timbers, is subjected to three shots from a 5-inch gun. The striking velocity of the shot is 1500 feet and the impact normal. On the first impact, near the center of the shield, no portion of the projectile shall get through the shield, nor shall any through crack develop to an edge of the shield. The other two impacts are so located that no point of impact shall be less than three calibers of the projectile from another point of impact or from an edge of the shield. At the second and third impacts no projectile or fragment of projectile shall go entirely through the shield.

The supports that hold the shield to the carriage are very heavy ribbon-shaped springs, which reduce the stress on the carriage from the impact on the shield. The springs are of great strength in order to withstand the shock of impact. They are made of steel with a tensile strength of 110,000 lbs., elastic limit 75,000 lbs., e ongation at rupture 15 per cent, contraction of area 25 per cent.

The fastening bolts must have a tensile strength of 80,000 lbs., and an elongation at rupture of 27 per cent.

The shields are curved around the front of the carriage and are inclined upward and to the rear at an angle of 40 degrees. The chase of the gun protrudes through a hole in the shield and other holes are provided for sighting purposes.

Fig. 201 shows the arrangement of the shield on a 6-inch barbette carriage.

Shields will probably be provided for all barbette carriages.

It is still a matter of discussion as to whether advantage is derived by the use of gun shields, for while they serve to keep out the smaller projectiles they also serve to determine the bursting of larger projectiles whose destructive power may be sufficient to disable the gun and wholly destroy the gun detachment. Without the shields these projectiles would in many instances pass by, doing little or no harm.

Field Gun Shields.—Shields of hardened steel plate two-tenths of an inch thick are attached to the gun carriage and caisson for the 3-inch field gun. These shields are tested by firings, at a range of 100 yards, with the 30 caliber rifle, using steel jacketed bullets with 2300 feet muzzle velocity. The plate must not be perforated, cracked, broken, or materially deformed.

The front of the caisson chest is made of the same material as the shields and has the same thickness. The door of the chest, which opens upward to an angle of 30 degrees, is made of hardened steel plate $\frac{15}{100}$ of an inch thick.

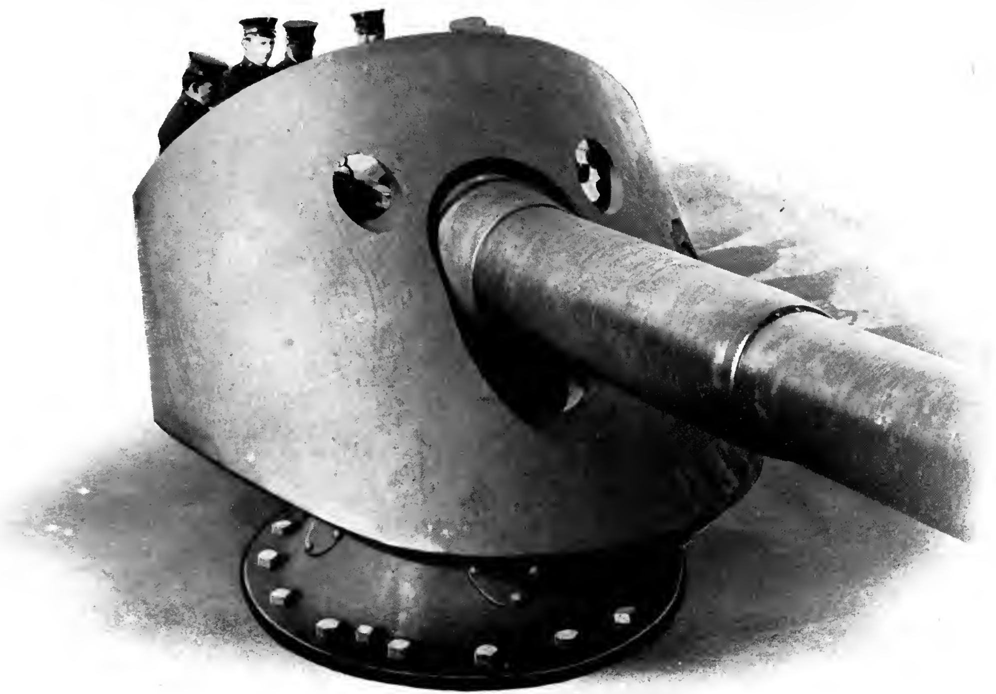


Fig. 201.—6-inch Gun on Pedestal Mount with Shield.

CHAPTER XII.

PRIMERS AND FUSES FOR CANNON.

275. Classification.—Primers are the means employed to ignite the powder charges in guns.

They may be divided, according to the method by which ignition is produced, into three classes:

Friction primers,
Electric primers,
Percussion primers.

Combination primers are those so constructed that they may be fired by any two of the above methods. Primers that close the vent against the escape of the powder gases are called *obturating* primers.

All primers should be simple in construction, safe in handling, certain in action and not liable to deterioration in store. Electric primers in addition should be uniform as to the electric current required for firing.

Common Friction Primer.—The primer known as the *common friction primer,* formerly used in all cannon, is shown in Fig. 202.

The body *b* and the branch *d* are copper tubes. The tube *b* is filled with rifle powder, and is closed at its lower end by a wax stopper *a*. The tube *d* is filled with the friction composition, whose ingredients are chlorate of potash, sulphide of antimony, ground glass, and sulphur mixed with a solution of gum arabic. Imbedded in the friction composition is the serrated end of the copper wire *c*, the other end of the wire being formed into a loop for attachment of the hook of the lanyard. The outer end of the tube *d* is closed over the flattened end of the wire, which is bent over into a hook, as shown, and serves to hold the wire securely in place except when a stout pull is given to the lanyard. The pull on the lanyard straightens out the hook and draws the serrated wire through the friction composition, igniting it. The fire is communicated to the rifle powder in the tube *b*, and thence through the vent to the powder charge in the gun.

For use in axial vents, in order to prevent the primer being blown to the rear among the men of the gun detachment, a coiled copper wire *e* is added to the primer, one end of the wire being

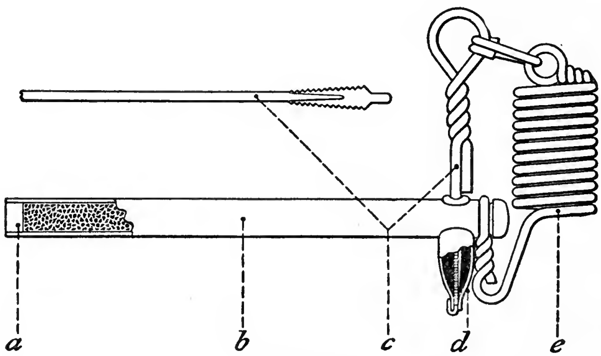


FIG. 202.

made fast to the top of the primer body, the other end to the loop for lanyard hook. The coil is extended by the pull of the lanyard, and the primer when blown to the rear remains attached to the lanyard.

Service Primers.—The primer above described is blown out of the gun by the explosion of the powder charge, leaving the vent open for the escape of gas. This disadvantage is overcome in modern practice by the use of obturating primers. The breech mechanisms of all guns now made are adapted to obturating primers, and the primer just described is no longer used in service cannon.

The firing mechanism described in the chapter on guns, page 263, is fitted to most of the cannon in our service that do not use fixed ammunition. The firing mechanism is adapted to receive the primer and hold it firmly, and is provided with means for firing the primer either by the pull of a lanyard or by electricity.

276. The Service Combination Primer.—The principal primer used in our service is a combination primer which is arranged to be fired either by friction or by electricity. The primer is shown complete in Fig. 203. The igniting elements are shown on a larger

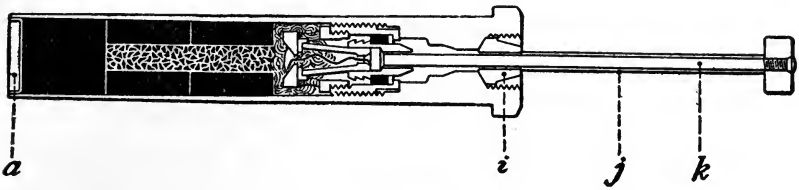


FIG. 203.

scale in Fig. 204. The igniting elements are assembled in the brass case *f*, which is screwed to its seat in the primer.

FRICTION ELEMENTS.—For firing by friction there is pressed into the case *f* an annular pellet of friction composition, shown in black in Fig. 204, which rests on a vulcanite washer, *g*. The washer supports the composition and prevents it from crumbling when the pull which fires the primer is applied. The inner end of the firing wire, *k*, is loosely surrounded by the serrated cylinder *h*, which is imbedded up to the serrations in the friction composition. The headed inner end of the firing wire fits in a seat inside the serrated cylinder, and the parts are held securely in place by the forked metal support *e* and the closing nut *b*.

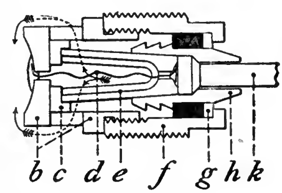


FIG. 204.

When the firing wire is pulled the serrated cylinder is drawn through the composition and ignites it. The conical end of the cylinder *h* is drawn to its seat in the rear part of the primer and prevents escape of gas to the rear. The flame from the friction composition passes through vents in the closing nut, *b*, and ignites the priming charge of compressed and loose black powder in the body of the primer.

The mouth of the primer is stopped by the brass cup, *a*, shellacked in place. This cup is blown out by the explosion of the primer charge, and the flames from the primer pass through the vent in the breech block and ignite the powder charge in the gun. The pellet of powder near the mouth of the primer is also blown through the vent and insures the ignition of the charge in the gun.

Electric Elements.—For electric firing the wire *k* is covered with an insulating paper cylinder *j* and enters the primer body through a vulcanite plug *i*. The wire is in electric contact with the serrated cylinder *h*, Fig. 204, but this is insulated from the primer body by the vulcanite washer *g* and the pellet of friction composition, a non-conductor of electricity.

The electrical elements of the primer are assembled in the metal case *f*. The head of the forked metal support *e* is in contact with the headed end of the wire *k*, but not fastened to it. The forked end of the support is held in the vulcanite cup *c*. The brass contact nut *b*, screwed into the end of the case *f*, presses the assembled parts into intimate electrical contact. A platinum wire *d* is soldered to the head of the support *e* and to the contact nut *b*. An igniting charge of guncotton surrounds the wire.

When the primer is inserted in the gun the uninsulated button at the end of the wire *j* is grasped by the parts of an electric contact piece through which the electric firing current passes. The current passes through the wire *j*, the platinum bridge, and the body of the primer to the walls of the gun and thence to the ground.

The passage of the electric current heats the platinum wire, igniting the guncotton and the priming charge of powder.

It will be observed that the friction elements of the combination primer are independent of the electrical elements, and that when one of these primers fails to fire by electricity it may still be fired by friction.

If, however, the primer fails in an attempt to fire it by friction, it will not generally be possible to fire it electrically since the cylinder *h*, which has been pulled into the head of the primer, is out of contact with the part *e* and the platinum wire bridge. The current will then pass directly from *h* through the primer body and gun to the ground.

The primer should in this case be at once removed from the vent and not be again used.

The outer button and wire *k* may be turned without danger of breaking the platinum wire bridge *d*.

When an electric or friction primer fails to fire it should be removed from the vent and the wire bent down and around the primer to prevent attempts to use it again.

The metal parts of the primer are tinned to prevent corrosion.

Other Friction and Electric Primers.—Primers arranged for firing by friction alone are shown in Figs. 205 and 206. The primer

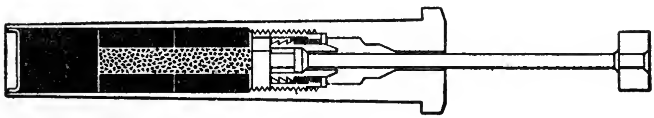


FIG. 205.

shown in Fig. 206, of simple and cheap construction, is for drill purposes only.

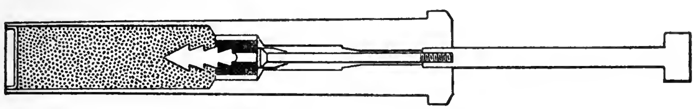


FIG. 206.

The friction primer shown in Fig. 207 and the electric primer

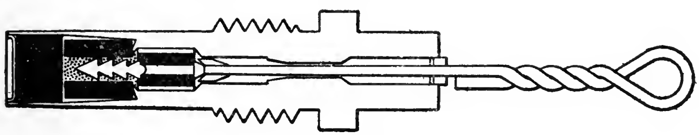


FIG. 207.

shown in Fig. 208 are for use in the 3.6-inch and 7-inch mortars,

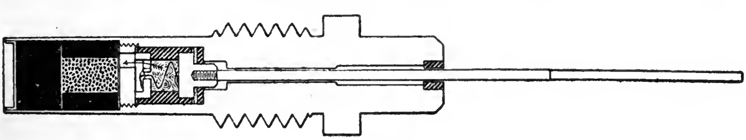


FIG. 208.

these guns not being provided with firing mechanisms. The primers are screwed into the vents in the breech blocks.

277. **Percussion Primers.**—The friction and electric primers described are used in guns in which the projectiles and powder charges are loaded separately, the primer being separately inserted in the breech block. Percussion primers, and the electric primer described with them, are, on the other hand, inserted in cartridge cases, in which are usually assembled both the projectile and the powder charge.

The essential parts of a simple percussion primer such as the cap in a small arm cartridge, are the primer cup, the anvil, and the percussion composition.

Formerly the percussion composition of all service primers contained a large percentage of fulminate of mercury. On account of the danger involved in handling mixtures containing the fulminate of mercury, its use as a primer ingredient in service primers manufactured at the Frankford Arsenal has been abandoned, and a mixture known as the H-48 composition is now employed.

This mixture contains the same ingredients as the friction composition, but in different proportions, as follows:

Chlorate of potash, 49.6.	Ground glass, 16.6.
Sulphide of antimony, 25.1.	Sulphur, 8.7.

To insure the practically instantaneous ignition of smokeless powder charges, the addition of a small charge of quick-burning black powder is required. This may be inserted in the base of the smokeless powder charge, or may be contained in the primer. It is desirable, on account of the smoke produced by black powder and the fouling of the bore, that the quantity of black powder used be limited to the smallest amount that will produce prompt and complete ignition of the smokeless powder. The minimum amounts required for different charges have been determined and, for fixed ammunition, are contained in the percussion and igniting primers. These primers are inserted in the head of the cartridge case, in the position occupied by the primer in the small arm cartridge.

Two sizes of percussion primers, the 110-grain and the 20-grain, have been adopted for all guns from the 1-pounder to the 6-inch Armstrong inclusive.

110-GRAIN PERCUSSION PRIMER.—The body *f* is of brass, 2.93 inches long, Fig. 209. A pocket is formed in the head of the case for the reception of the metal cup *e* containing the percussion composition *d*. Projecting up from the bottom of the pocket is the anvil *c* against which the percussion composition is fired. Two vents are drilled through the bottom of the pocket. The priming charge consists of 110 grains of black powder inserted under high pressure into the primer body around a central wire. The withdrawal of the wire after the compression of the powder leaves a longitudinal hole the full length of the primer. Six sets of radial holes are drilled through the walls of the primer and through the compressed powder. The compression of the powder increases the time of burning of the priming charge and causes the primer to burn with a torch-like rather than an explosive effect, making the

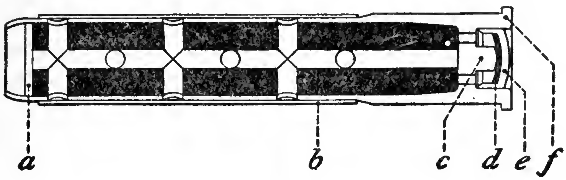


Fig. 209.

ignition of the smokeless powder charge more complete. The holes through the priming charge increase the surface of combustion and the mass of flame, and direct the flames to different parts of the charge of powder, thus facilitating its complete ignition. The paper wad, *a*, shellacked in the mouth of the primer and the tin-foil covering, *b*, serve to keep out moisture and to protect the primer from the impact of the powder grains when transported assembled in cartridge cases.

This primer is used in cartridge cases for guns from the 6-pounder to the 6-inch Armstrong gun, inclusive.

20-grain Percussion Primer.—The 20-grain percussion primer, shown in Fig. 210, length 1.1 inches, is used in cartridge cases for 1-pounder subcaliber tubes, 1-pounder machine guns, and 1.65-inch Hotchkiss guns.

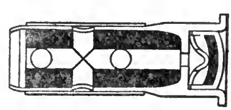


Fig. 210.

20-grain Saluting Primer.—This primer, Fig. 211, costing less to manufacture than the 110-grain primer, is to be used in place of the latter with blank charges only. The primer contains a charge of 20 grains of loose rifle powder. As black powder only is used in blank charges, a smaller igniting charge answers.

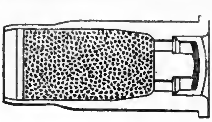


Fig. 211

110-grain Electric Primer.—This primer, Fig. 212, is similar in form to the 110-grain percussion primer just described, and has the same priming charge similarly arranged. Ignition is produced electrically through the brass cup *g*, to which one end of the platinum wire *e* is soldered. A small quantity of guncotton surrounds the wire. Electric contact is made with the cup *g* by the insulated firing pin of the gun. The cup is insulated from the body of the primer by the cylinder *f* and bushing *d*, both of vulcanite. The brass contact bushing *c*, to which the other end of the platinum wire is soldered, completes the electrical connection.

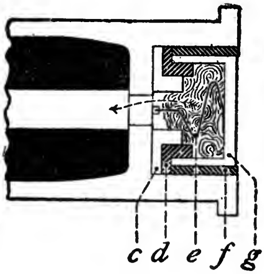


FIG. 212.

278. Combination Electric and Percussion Primer.—In Fig. 213 is shown a combination electric and percussion primer used in rapid-fire guns in the U. S. Navy. Its construction can be readily understood from the figure. The insulation is shown by the heavy black lines. When fired by percussion the percussion cap is not directly struck by the firing pin, but by the point of a plunger forced inward by the blow.

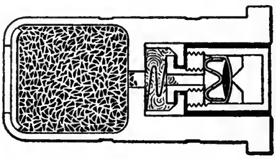


FIG. 213.

Igniting Primers.—The igniting primers are for use in cartridge cases for subcaliber tubes for seacoast cannon not provided with percussion firing mechanism. They contain no means of ignition within themselves, but require for their ignition an auxiliary friction or electric primer which is inserted in the vent of the piece in the same manner as for service firing. The flame passes from the service primer through the vent in the breech block to the igniting primer in the head of the cartridge case. The flame from the service primer would not be sufficient to ignite properly the smokeless powder charge in the cartridge case, and therefore the igniting primer is added.

The 110-grain and the 20-grain igniting primers, Figs. 214 and 215, differ from the corresponding percussion primers in the substitution of the obturating cup *a* and obturating valve *b*, both of brass, for the percussion cup and anvil. The obturating cup *a* is provided with a central vent to allow passage for the flame from the auxiliary primer. The obturating valve *b* is cup-shaped, and has three sections of metal cut away from its top and sides to allow passage of the flame. The valve *b* has a sliding fit in the cup *a*, and when the pressure is greater in front of the valve than behind it, the valve is forced to the rear and the solid top of the valve closes the vent in the outer cup.

The valve is shown in section in Fig. 214, in the position it assumes after firing; and in elevation in Fig. 215, in its position before firing.

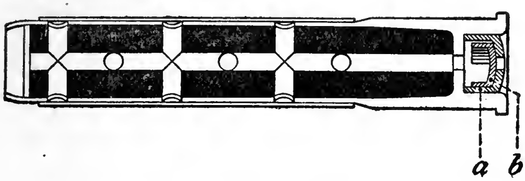


FIG. 214.

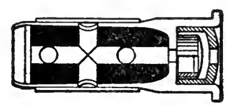


FIG. 215.

Insertion of Primers in Cartridge Cases.—The percussion primers and igniting primers and the electrical primers of the same form are so manufactured as to have a driving fit in their seats in the cartridge cases to which they are adapted, the diameter of the primer being from one-and-a-half to two thousandths of an inch greater than the diameter of the seat. Special presses for the insertion of the primers are provided. The primer must not be hammered into the cartridge case. The primer seats in all cartridge cases using these primers are rough bored to a diameter about 20 per cent less than the finished size, and then mandrelled to finished dimensions with a steel taper plug, to toughen the metal of the cartridge case around the primer seat. The toughening is necessary to prevent expansion of the primer seats under pressure of the powder gases, and consequent loose fitting of the primers in subsequent firings.

FUSES.

279. Classification.—Fuses are the means employed to ignite the bursting charges of projectiles at any point in the flight of the projectile, or on impact.

They are of three general classes:

Time fuses,
Percussion fuses,
Combination time and percussion fuses.

All fuses should be simple in construction, safe in handling, certain in action, and not liable to deterioration in store. In addition the rate of burning of the time train of the fuse must be uniform.

The time fuse alone, that is, without percussion element, is no longer used in modern ordnance.

Percussion Fuses.—A percussion fuse is one that is prepared for action by the shock of discharge, and that is caused to act by the shock of impact.

When ready to act, as after the shock of discharge, the fuse is said to be *armed.*

Percussion fuses are inserted at the point or in the base of the projectile. In the projectiles for 1- and 2-pounder guns the fuse is inserted at the point. The percussion fuses for field, siege, and seacoast projectiles are base insertion fuses.

The percussion fuse consists essentially of the case or body, of brass, which contains and protects the inner parts and affords a means of fixing the fuse in the projectile; the plunger, carrying the firing pin and provided with devices to render the fuse safe in handling; the percussion composition, which is fired by the action of the plunger on impact; and the priming charge of black gunpowder.

The percussion composition of all service fuses manufactured at Frankford Arsenal is the same. The ingredients are chlorate of potash, sulphide of antimony, sulphur, ground glass, and shellac. The thoroughly pulverized ingredients are mixed dry, and alcohol is added to dissolve the shellac. The percussion pellets are formed by pressing the mixture while in a plastic state into the percussion-primer recess. Upon the evaporation of the alcohol the shellac causes the pellet to adhere to the metal of the recess.

A fulminate of mercury percussion composition was formerly used in fuse primers, but on account of the danger incident to handling this compound it has been abandoned as a primer ingredient.

It is still used abroad, and the percussion composition of both the Ehrhardt and Krupp combination time and percussion fuses contains fulminate of mercury.

Point Percussion Fuse.—Point percussion fuses are adapted to the projectiles for 1-pounder and 2-pounder guns only.

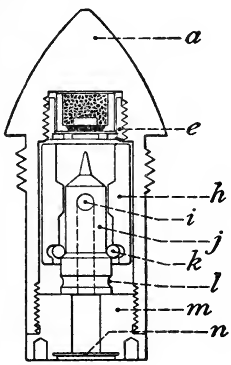


FIG. 216.

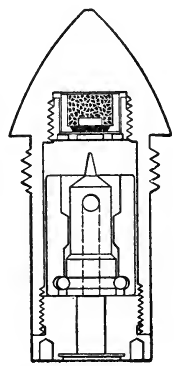


FIG. 217.

The body, *a* Fig. 216, is of brass. The percussion composition and the priming charge of black powder are assembled in a vented case, *e*, which is screwed into a recess formed in the head of the fuse. A thin brass disk, the primer shield, protects the percussion composition from the firing pin in the body of the fuse. It prevents any dislodgment of the composition during transportation or by shock of discharge and also restrains the firing pin during the flight of the projectile.

Contained in the body of the fuse is the plunger, which consists of the firing pin *j*, the cylindrical sleeve *h*, and the split-ring spring *k*, all of brass. The firing pin has an enlarged rear part joined to the forward part by a conical slope and provided near the bottom with a groove, *l*, of diameter slightly larger than the diameter of the forward part of the pin. A radial hole, *i*, through the pin near its forward end, and an axial hole from this point to the rear end of the pin, provide a passage for the flame from the priming charge. The rear part of the bore through the sleeve *h* is of diameter just sufficient to admit the spilt ring which rests against the forward shoulder of the counterbored recess in the sleeve and holds the firing pin so that its point is wholly within the sleeve. The front part of the sleeve is counterbored to permit ready entrance of the flame from the priming charge into the passage through the firing pin. The plunger thus assembled is placed in the fuse body, which is closed by the brass closing screw *m* provided with a central vent which is in turn closed by the brass disk *n*. To prevent pressure of the closing screw on the plunger, which might cause expansion of the split ring and the arming of the fuse, the plunger is allowed a longitudinal play in the fuse body of from one to two hundredths of an inch. With the parts of the fuse in this position the point of the firing pin is prevented from coming into contact with the percussion composition, and therefore the fuse cannot be fired.

If sufficient force is applied rearwardly to the sleeve *h*, the split ring *k* will be forced over the enlarged portion of the firing pin until it rests in the groove *l* near the bottom; and the sleeve, moving to the rear, will expose the point of the firing pin. The fuse is then armed, as shown in Fig. 217.

To insure arming of the fuse when fired the resistance of the split ring to expansion is made less than the force necessary to give the sleeve the maximum acceleration of the projectile. Therefore when the piece is fired and while the projectile is attaining its maximum acceleration, the pressure of the sleeve will force the ring over the enlarged part of the firing pin into the groove at the rear.

The diameter of this groove being greater than the diameter of the front part of the firing pin, the ring is now expanded into the counterbored recess in the sleeve and locks the sleeve and firing pin together, with the point of the firing pin projecting beyond the sleeve.

As the plunger of the fuse does not encounter the atmospheric resistance which retards the projectile in its flight, it is probable that during the flight of the projectile the plunger moves slowly forward until the point of the firing pin rests against the brass primer shield.

At impact of the projectile the combined weight of the plunger parts acts to force the point of the firing pin through the primer shield and into the percussion composition, igniting the composition.

The flame from the priming charge passes through the forward vents, through the passages in the plunger, and through the vent in the closing screw, blowing out the closing disk and igniting the bursting charge in the shell.

280. Base Percussion Fuse, for minor caliber shell. This fuse, as well as the point percussion fuse, is adapted to the projectiles for 1-pounder and 2-pounder guns. The fuse for the projectiles of the 6-pounder gun and of the 2.38-inch field gun is similar in construction.

The fuse, Fig. 218, is similar in construction and action to the point percussion fuse. As the primed end of the fuse is toward the interior of the shell the flame from the priming charge passes directly to the bursting charge in the shell without passing through the body of the fuse. The flame passages through the plunger parts are therefore omitted. The primer cup *b*, containing the percussion composition and priming charge, is closed at its outer end by the brass disk *a*, which is secured in place by crimping over it a thin wall left on the brass closing cap screw *c*.

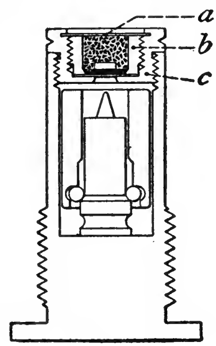


Fig. 218.

The act of arming a ring-resistance percussion fuse shortens the plunger and increases materially its longitudinal play in the fuse body. This fact permits a ready and simple means of inspecting for premature arming without dismantling the fuse. If the fuse be held close to the ear and shaken, the marked difference between the play of the plunger in an armed fuse and in an unarmed one can be readily discerned.

Centrifugal Fuses.—The centrifugal fuse of service pattern is the result of a long series of experiments made for the purpose of developing a fuse that would fulfill the requirements of absolute safety in handling and transportation, and certainty of action.

In the case of ring-resistance fuses, or any fuse the action of which depends on the longitudinal stresses developed by the pressure in the gun, the conditions of safety in handling and certainty of action are opposing ones.

It was impossible to meet successfully both sets of conditions in all cases, the stress developed in the direction of the axis by accidental dropping of a fuse being in many cases higher than that developed in the gun.

A fuse which is armed by the centrifugal force developed by the rotation of the projectile, and which is safe until the maximum velocity of rotation is nearly attained, has been developed at the Frankford Arsenal and is now used in the projectiles for low velocity guns; the mountain gun, and all howitzers and mortars. In these guns the maximum acceleration of the projectile in the bore is so low that the ring-resistance fuse must be very sensitive in order to insure arming, with the result that it becomes too sensitive for safety in handling and transportation. For the projectiles of other guns the fuses are similar, but are provided with ring-resistance plungers instead of centrifugal plungers.

The centrifugal fuse, before arming, is shown in Fig. 219. Fig. 220 is a view of the plunger after arming.

The fuse body, or stock, and the primer parts of the centrifugal fuse do not differ materially from the corresponding parts of the ring-resistance fuses. To better protect the priming charge the closing cap screw *b* is lengthened and the vented primer-closing screw *a* is added.

The body of the centrifugal plunger is in two parts, nearly semi-cylindrical in shape, which when the fuse is at rest are held together by the pressure of a spiral spring *g* contained in the cylindrical bushing *e* which is secured to one of the plunger halves. The spring exerts its pressure on the other half of the plunger through the bolt *f*. Pivoted in a recess in one half of the plunger is the firing pin *d*, which when the fuse is at rest is held with its point below the front surface of the plunger by the lever action of the link *c* which is pivoted in the other half. Under the action of the centrifugal force developed by the rapid rotation of the projectile the two halves of the plunger separate. The separating movement causes the rotation of the firing pin *d*, the point of which is now held in advance of the front surface of the plunger, Fig. 220, ready, on impact of the projectile, to pierce the brass primer shield and ignite the percussion composition. When the fuse is armed the end of the link *c* rests on the pivot of the firing pin, thus affording support to the firing pin when it strikes the percussion primer. The separation of the plunger parts is limited by the nut *i* coming to a bearing on a shoulder in the bushing *e*, so

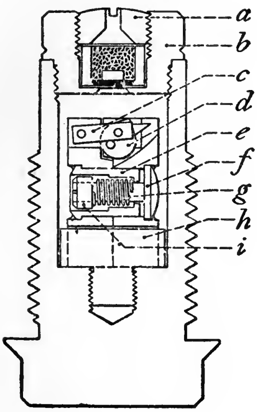


Fig. 219.

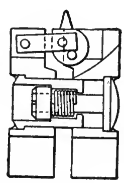


Fig. 220.

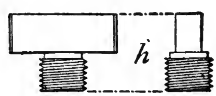


Fig. 221.

as not to permit the diameter of the expanded plunger to equal the interior diameter of fuse stock, see Fig. 222.

A rotating piece, *h* Figs. 219 and 221, screwed into head of fuse stock, engages in a corresponding slot cut through the bottom of both plunger-halves and insures rotation of the plunger with the shell.

The strength of the spring *g* is so adjusted that the fuse will not arm until its rapidity of revolution is a certain percentage of that expected in the shell in which it is to be used, and that it will certainly arm when the rapidity of revolution approximates that expected in the shell. Should the parts of the plunger be accidentally separated and the fuse armed by a sudden jolt or jar in transportation or handling, the reaction of the spring will immediately bring the plunger to the unarmed condition.

The fuse just described is called the *F fuse*.

The fuse shown in Fig. 222, the *S fuse,* is for use with 3.6- and 7-inch mortar shell, powder-charged. The additional priming in end of fuse gives a greater body of flame than is emitted from the *F* fuse.

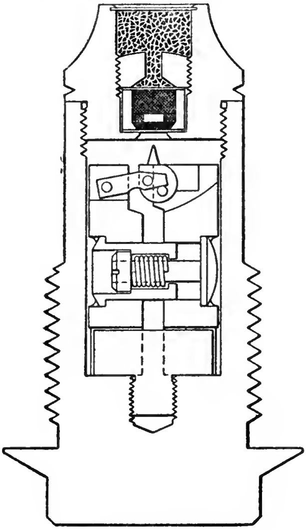


FIG. 222.

A similar fuse of larger size is used in powder-charged shell of 8-inch caliber and over.

A fuse, called the *12 M fuse,* is provided for use in the 12-inch mortar deck-piercing and torpedo shell. This fuse is similar in construction to the other centrifugal fuses, but on account of the low velocity of rotation of mortar projectiles and their low striking velocity a much heavier plunger is needed to provide the force necessary for arming the fuse, and for puncturing the primer-shield on impact.

281. Combination Time and Percussion Fuses.—All combination fuses used in the service are point insertion and combine the elements of time and percussion arranged to act independently in one fuse body.

Combination fuses contain two plungers and two primers. One plunger, the time plunger, is armed by the shock of discharge and fires its primer immediately, igniting the time train of the fuse. The other plunger, the percussion plunger, is also armed by the shock of discharge but fires its primer on impact of the projectile.

Service Combination Fuse.—The upper part of the fuse, Fig. 223, contains the time elements, the lower part the percussion elements. The time elements consist of the concussion or time plunger *b,* the firing pin *c,* and the time train. The firing pin is fixed in the body of the fuse, and the plunger carries the percussion composition and a small igniting charge of black powder. The plunger is held out of contact with the firing pin by the split resistance-ring *a.* On the shock of discharge the inertia of the plunger acting through the conical surface in contact with the split ring expands the ring so that the plunger can pass through it and carry the percussion composition to the firing pin.

The time train of the fuse is composed of two rings of powder, *f* and *h*, contained in grooves cut in the two time-train rings *m* and *n*. The grooves are not cut completely around the rings, but a solid portion is left between the ends of the groove in each ring.

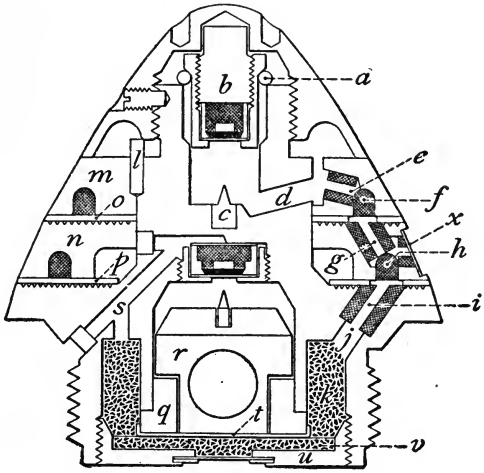


Fig. 223.

Mealed powder is compressed into the grooves under a pressure of 70,000 pounds per square inch, forming a train 7 inches long, the combined length of the two grooves.

The flame from the percussion composition passes through the vent *d*, igniting the compressed tubular powder pellet *e*, which in turn ignites one end of the upper time train *f*. When the fuse is set at zero the flame passes immediately from the upper time train through the powder pellet *g* to one end of the lower time train *h*; thence through the pellet *i* and vent *j* to the powder *k* in the annular magazine at the base of the fuse.

Under each of the time rings is a felt washer, *o* and *p*, that closes the joint under the ring against the passage of flame, except through the hole in the washer directly over the vent in the part below. The upper washer *o* is glued to the upper corrugated surface of the lower time ring *n* and moves with that ring. The lower washer *p* is glued to the fuse body and is stationary. The upper time ring *m* is fixed in position by two pins *l* halved into the fuse body and the ring. The lower time ring is movable, and any of the graduations on its exterior, see Fig. 224, which correspond to

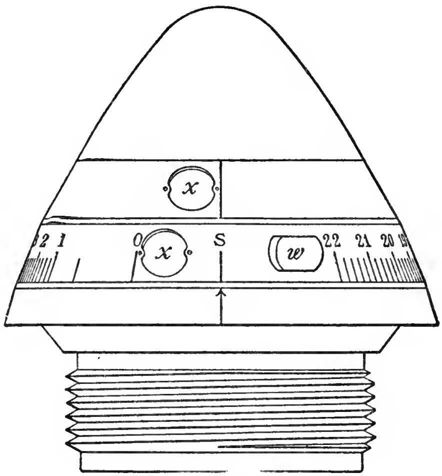


FIG. 224.

seconds and fifths of seconds of burning, may be brought to the datum line marked on body of fuse below the ring. The ring is moved, in setting, by means of a wrench applied to the projecting stud *w*.

To set the fuse for any time of burning, say 20 seconds, move the lower time ring *n* until the mark 20 is over the datum line. On ignition of the primer the flame ignites the upper time train *f*, which burns clockwise, looking from base to point of fuse, until the hole through the washer over the zero mark of the lower ring *n* is encountered. The flame then passes through the vent *g* to the lower time train *n*, which burns anti-clockwise until the mark 20 is reached. This mark being over the vent *i* in the body of fuse, the flame now passes to the magazine *k*. The setting of the fuse consists in fixing the position of the passage from the upper to the lower time train, so as to include a greater or less length of each train between the vent *e* and the vent *i*.

In each time ring a vent opens from the initial end of the powder train to the exterior. The vent contains a pellet of powder and is covered by a thin brass cup. The vent in the lower time ring is seen at x in Fig. 223. The caps, x, of both vents are shown in Fig. 224. The blowing out of the cap affords a passage to the open air for the flame from the burning time train, thus preventing the bursting of the fuse by the pressure of the contained gases.

When the fuse is set at safety, indicated by the letter S stamped on the lower time ring, the position shown in Fig. 224, the solid metal between the ends of the upper time train is over the vent g to the lower train, and the solid metal between the ends of the lower train is over the vent i leading to the magazine. In case of accidental firing by the time plunger, the upper train will be completely consumed without communicating fire to the lower train and to the magazine. The fuse is habitually carried at this setting, which serves also when it is desired to explode the shell by impact only.

For percussion firing the fuse is now provided with a ring-resistance plunger similar to that shown in Fig. 218. Better results are obtained with the ring-resistance plunger than with the centrifugal plunger, which was formerly used in these fuses and is shown at r in Fig. 223. A vent s leads from the percussion primer to the annular magazine k. A thin brass cap t separates the lower plunger-recess from the powder in the four radial chambers v cut in the bottom closing screw. The central vent in the closing screw is closed by a piece of shellacked linen, held in place by a brass washer.

These fuses are issued fixed in the loaded projectiles. For protection in transportation the fuse is covered by a spun brass cap, soldered on to the head of the projectile. The soldering strip is torn off and the cover removed before using the projectile.

A 21-second fuse of this pattern is now in service, and a 31-second fuse is being developed.

282. COMBINATION FUSE, OLD PATTERN.—As the former model of combination fuse may perhaps still be encountered in service, it is illustrated here. The time train, b Fig. 225, is made by filling a lead tube with mealed powder and then drawing the filled tube through dies until its diameter has been reduced to the desired dimension. The powder train is thereby given practically uniform density, so that it burns more uniformly than the time trains of previous fuses. The results, however, were not so good as the results obtained with fuses of the present service model.

The time train, *b*, incased in the lead tube, is wound spirally around the lead cone *c*. To set the fuse for any time of burning the time train and lead cone are punctured, by means of a tool provided for the purpose, at the point on the scale marked on the cover of fuse corresponding to the time of burning desired. The puncture passes completely through the time train and the lead cone behind it, forming a channel from the annular space in which

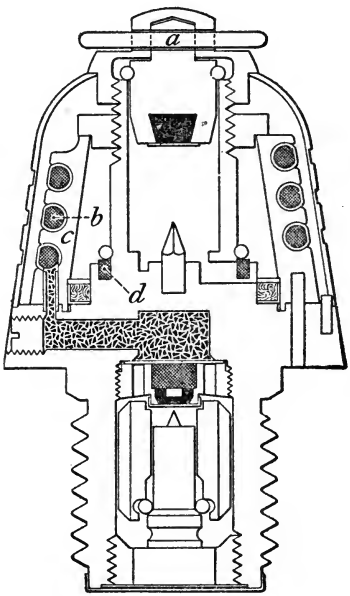


FIG. 225.

the letter *b* appears to the powder in the time train. When the projectile is fired the flame from the percussion composition ignites the compressed powder ring *d*, and the flame from this ring ignites the time train at the point at which it has been punctured. The safety pin *a* retains the time plunger in its unarmed position, and must be withdrawn before placing the projectile in the gun.

Two fuses of this pattern were made, one with a 15-second time train and the other with a 28-second time train.

Ehrhardt Combination Fuse.—This fuse is similar in construction to the Frankford Arsenal fuse, latest pattern, described above and differs only in details.

The arming of the time plunger of the Ehrhardt fuse, Fig. 226, is resisted by the U-shaped spring *a*, the upper ends of which are sprung out into a counterbored recess in the closing cap, and by

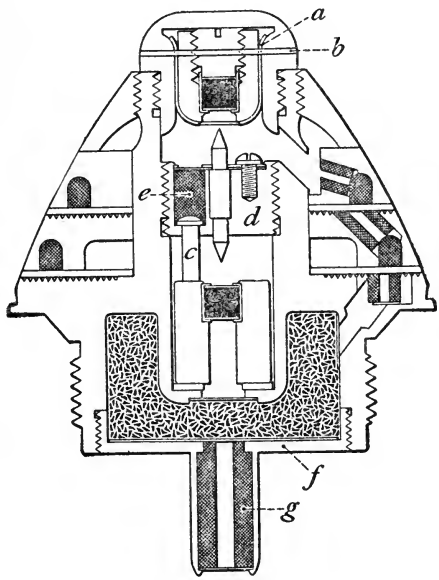


Fig. 226.

the slender brass pin *b*, which passes through the plunger and both sides of the closing cap. At discharge of the piece the inertia of the plunger shears the pin *b* and straightens the U-shaped spring *a*, permitting the plunger to strike the firing pin.

In the percussion mechanism the composition is carried in the plunger and the firing pin is fixed in the diaphragm *d* in body of fuse. The plunger is held away from the firing pin, before firing, by the brass restraining pin *c*. The pin is let into a hole in the diaphragm *d*, the head of the pin abutting against a shoulder near the bottom of the hole. The restraining pellet of powder *e* is pressed in to fill the recess above the pin. A perforated brass disk and a piece of linen close the hole at its upper end and prevent the powder pellet from being jarred out of place. The burning of this pellet on ignition from the time plunger leaves the restraining pin and percussion plunger free to move forward at impact.

A compressed charge of black powder, *g*, is inserted into the extension of the closing screw *f* to reinforce the magazine charge and effectually to carry the flame to the base charge in the shrapnel.

The Krupp combination fuse does not differ essentially from the Ehrhardt fuse. The shear pin through time plunger is omitted, the U-shaped spring being made strong enough to offer sufficient resistance against accidental arming. The percussion plunger, carrying the percussion composition, is held away from the firing pin, before firing, by a sleeve and an inverted U-shaped resistance spring. A spiral spring between plunger and firing pin prevents the creeping forward of the plunger during the flight of the projectile.

Detonating Fuses.—These fuses are for use in shell containing high explosives.

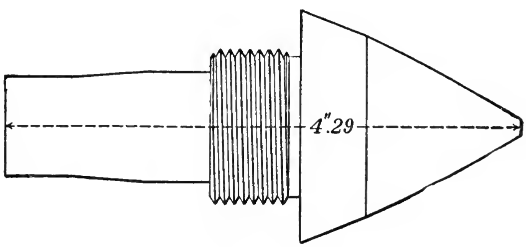


FIG. 227.

Fig. 227 shows the form of detonating fuse for point insertion in field shell. Fig. 228 shows the form of fuse for base insertion in siege and seacoast projectiles.

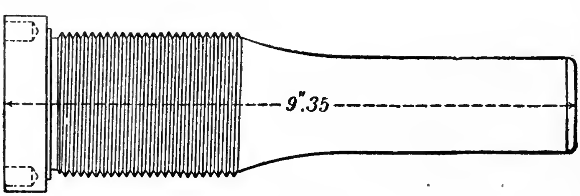


FIG. 228.

In order to prevent the unscrewing of the fuse during flight of the projectile, all point insertion fuses are provided with right-handed screw threads and base insertion fuses with left-handed threads.

283. The Fuse Setter.—The fuse setter is a device for the rapid and accurate setting of the time fuse in the field gun projectile. It is attached to a hinged bracket on the caisson for the field gun, see Fig. 122, in a position convenient for the cannoneer who serves the caisson.

The base of the fuse setter, Fig. 229, is fixed to the bracket on the caisson. Mounted on the base are two movable rings called the corrector ring and range ring. The range ring carries the range scale graduated in yards, and the corrector ring carries an index or pointer that moves between the corrector scales that are fastened to the fixed cover. The base and the two rings are bored out conically to fit over the combination time and percussion fuse used in the 3-inch projectile. The corrector ring is notched to receive the rotating stud, *w* Fig. 224, which projects from the time train ring of the fuse. A spring plunger projects inwardly from the range ring of the fuse setter.

A guide fixed to the base serves to direct the point of the projectile into the socket of the fuse setter and to support the cartridge during the operation of fuse setting.

To set the fuse for the time of burning corresponding to any range, as 1000 yards, the range ring is turned by means of the range-worm handle until the 1000 mark on the range scale is opposite the datum line marked on the corrector scale, see Fig. 229. The weather-proof cover of the time fuse in the projectile is stripped off and the point of the projectile is then placed in the fuse setter, the rotating stud on the fuse engaging in the notch in the corrector ring. The cartridge is then turned slowly in a clockwise direction until the spring plunger, which has been pushed in by the insertion of the fuse in the fuse setter, is forced out into a notch prepared for it in the body of the fuse. The plunger prevents further rotation of the cartridge, the time fuse of which has now been set to the proper time of burning for 1000 yards.

The rate of burning of different fuses of the same lot will be uniform, but it may vary slightly from the rate of burning used in the graduation of the scale of the fuse setter. This must be determined by actual firings, and if after a few shots it is found that the projectiles burst short of or beyond the range for which the time fuse is set, or if the height of burst is not exactly as desired,

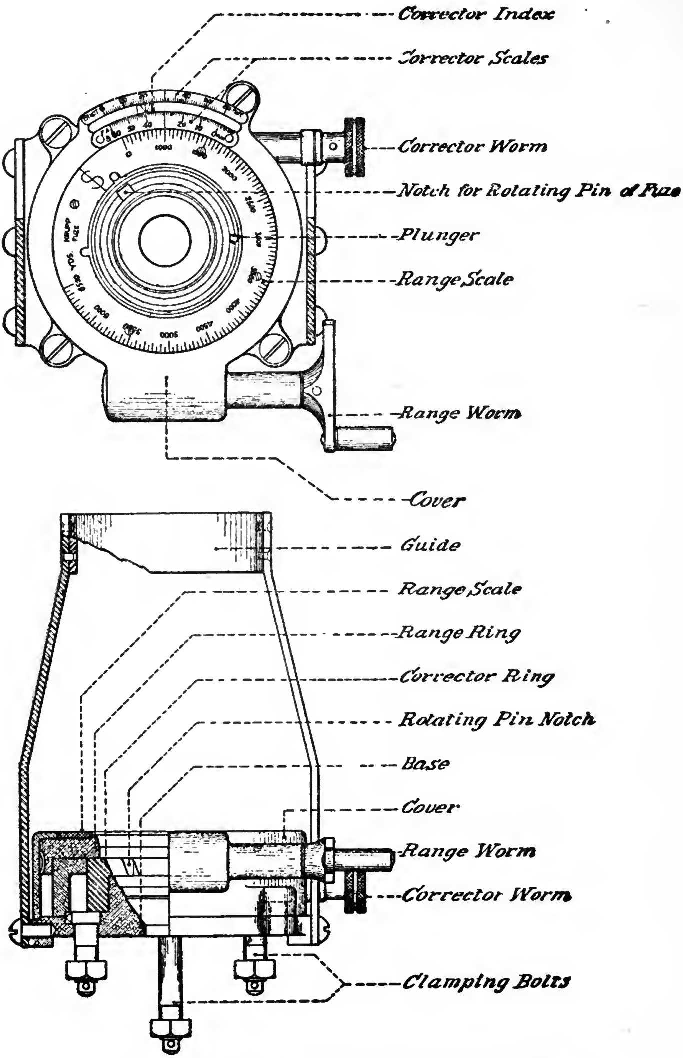


FIG. 229.—Fuse Setter for 3-inch Projectiles.

a correction is made in the setting of the fuse by means of the corrector ring in the fuse setter.

The height of burst may be increased or diminished by turning the corrector ring, by means of the corrector-worm thumb nut, to increase or diminish the corrector scale reading.

A point on the corrector scale corresponds to a difference of one mil in the height of burst.

The fuse setters now issued are provided with two corrector scales, one for use with Frankford Arsenal and Krupp fuses, and the other for use with Ehrhardt fuses.

284. Arming Resistance of Fuse Plungers. RING RESISTANCE FUSES.—The arming resistance of the ring resistance fuse, Fig. 216, is the resistance offered by the split ring k to movement over the enlarged base of the firing pin.

As the projectile is accelerated in the bore of the gun the split ring imparts the acceleration to the sleeve h of the plunger. If the resistance that the split ring offers to rearward motion over the slope of the firing pin is less than the pressure that the ring must impart to the sleeve to give to the sleeve the maximum acceleration of the projectile, the rearward movement of the ring will occur and the fuse will arm.

Problem 1. Determine the maximum permissible arming resistance for the ring-resistance fuse in the projectile for the 3-inch gun, for which we have the following data.

Maximum pressure,	$P=33{,}000$ lbs. per sq. in.
Weight of projectile,	$w=15$ lbs.
Weight of plunger sleeve,	$w_s=464$ grains $=464/7000$ lbs.
Diameter of projectile,	$d=3$ inches.

Neglecting friction and the rotation of the projectile we will assume that the pressure is wholly employed in giving motion of translation to the projectile.

The maximum acceleration of the projectile is

$$\alpha=\frac{g}{w}P\frac{\pi d^2}{4}=\frac{32.16\times 33000\times\pi\times 9}{15\times 4}=500120$$

If the split ring of the fuse plunger imparts this acceleration to the sleeve, the pressure on the ring will be

$$F=\alpha\frac{w_s}{g}=\frac{500120\times 464}{32.16\times 7000}=1030.8 \text{ pounds}$$

Therefore the plunger with sleeve weighing 464 grains will arm in the gun if the arming resistance of the fuse is anything less than 1030.8 pounds.

285. *Problem 2.* The actual arming resistance of the fuse for the 3-inch projectile is 220 pounds. What pressure per square inch is required in the gun in order to arm the fuse?

Equating the values of α in the equations established in the preceding problem, and writing p for P to indicate any pressure per square inch, we obtain

$$p\frac{\pi d^2}{4}\frac{1}{w}=\frac{F}{w_s}$$

The total pressure on the projectile at any instant divided by the weight of the projectile is equal to the pressure on the sleeve at the instant divided by the weight of the sleeve.

Making $F=220$, and substituting for the other quantities the values as given in the preceding problem, we find

$$p=\frac{4\times15\times220\times7000}{\pi\times9\times464}=7043 \text{ lbs. per sq. in.}$$

The fuse will arm under any pressure in excess of this.

Problem 3. What is the minimum effective powder pressure that will arm the ring-resistance fuse described below, when fired from the 6-inch gun?

Weight of projectile, $w=106$ lbs.
Weight of plunger sleeve, $w_s=700$ grains $=0.1$ lbs.
Ring resistance to arming, $=220$ lbs.

Ans. $p=8248$ lbs. per sq. in.

286. CENTRIFUGAL FUSE.—The arming resistance of the centrifugal fuse, Fig. 219, is the pressure exerted by the spring g, which holds the plunger halves together. The centrifugal force due to the rotation of the projectile tends to separate the plunger halves. In order that the fuse may be armed when the projectile strikes, the arming resistance must be less than the centrifugal force developed by the rotation in the projectile at impact. For simplicity we will consider that the projectile's velocity of rotation at impact is the same as at the muzzle of the gun.

Problem 4. Determine the maximum permissible arming resistance for the centrifugal fuse in the 12-inch mortar projectile, for which we have the following data.

Weight of plunger complete, 660 grains.

Weight of plunger half, $w_s = 330$ grains $= 330/7000$ lbs.

Radius of center of gravity of plunger half, $r = 0.4$ ins. $= 0.4/12$ ft.

Twist at muzzle, $n = 25$.

Muzzle velocity of projectile, $V = 950$ f. s.

Diameter of projectile, $d = 12$ inches $= 1$ ft.

Combining equations (62) and (61), page 250, we find for the velocity of rotation of the projectile at the muzzle

$$\omega = 2V\pi/dn = 2\ \pi\ 950/25 = 238.76$$

The centrifugal force acting on each plunger half is

$$F = m_s v^2/\rho = w_s \omega^2 r^2/g\rho$$

in which v is the linear velocity of the center of gravity of the plunger half, due to the rotation,

r the radius of the center of gravity,

ρ the radius of its path.

At the beginning of movement $\rho = r$, and we have

$$F = w_s \omega^2 r/g = \frac{330 \times \overline{238.76}^2 \times 0.4}{7000 \times 32.16 \times 12} = 2.79 \text{ lbs.}$$

for the force tending to move each plunger half.

If the resistance of the spring is less than 2.79 lbs. the fuse will start to arm.

As the plunger halves separate, the resistance of the spring increases in the manner shown by equation (14), page 285.

$$S = G' + Gx$$

It will be seen, from the value of F above, that F increases directly with r. In order that the fuse, after starting to arm, may arm completely, the values of G' and G must be such, that is, the spring must be of such construction, that S will not increase more rapidly than F.

287. *Problem 5.* Assume that the spring in the plunger of the fuse for the 12-inch mortar projectile is under a tension of $1\frac{1}{2}$ lbs. What muzzle velocity is required in the projectile to arm the fuse?

We have

$$\omega = 2V\pi/dn$$

$$F = w_s \omega^2 r/g$$

from which

$$\omega = (Fg/w_s r)^{\frac{1}{2}} = 2V\pi/dn$$

Solving for V

$$V = \frac{dn}{2\pi}\left(\frac{Fg}{w_s r}\right)^{\frac{1}{2}}$$

The force required for arming is in this case 1.5 pounds. Substituting 1.5 for F, and for the other quantities the values as given in the preceding problem, we have

$$V = \frac{25}{2\pi}\left(\frac{1.5 \times 32.16 \times 7000 \times 12}{330 \times 0.4}\right)^{\frac{1}{2}} = 697.14 \text{ f. s.}$$

The fuse will arm for any muzzle velocity of the projectile exceeding 697.14 foot seconds.

Problem 6. What is the minimum muzzle velocity that will arm the centrifugal fuse described below, when fired from a 6-inch howitzer?

Weight of plunger half, $w_s = 400$ grains $= 4/70$ lbs.

Radius of center of gravity of plunger half, $r = 0.5$ in. $= 0.5/12$ ft.

Spring resistance to arming, $F = 2$ lbs.

Twist of rifling at muzzle, $n = 25$.

Diameter of projectile, $d = 6$ in. $= 0.5$ ft.

Ans. $V = 327$ foot seconds.

CHAPTER XIII.

SIGHTS.

288. Purpose.—It has been shown in exterior ballistics that in order that the projectile from any gun may hit the target the gun must be fired at a certain angle of elevation, depending upon the range and upon the relative level of the gun and target, and must be given such direction to the right or left of the target as to neutralize the deviation of the shot from the plane of fire due to the drift and wind.

The elevation in the plane of fire and the allowance for deviation from the vertical plane containing gun and target are determined beforehand either by calculation or estimate. Direction is given to the axis of the gun by whatever means may be provided. The axis of the gun when given the determined elevation and deviation has a fixed relation to the line from the gun to the target.

The sights of the gun provide the means of determining when the axis of the gun has the predetermined direction with respect to the line from gun to target.

Principle and Methods.—The principle of sighting is simple. It consists in determining, by means of the sights, a line to which the axis of the gun has the fixed relation already determined as being required between the axis and the line to the target; and then, by looking through the sights, making the line of the sights and the line to the target coincide.

The line of sight on a gun may be fixed in one of two ways: first, by means of two plain or open sights, the rear one of which has a peep or notch capable of adjustment in vertical and horizontal directions; second, by means of a telescope, whose axis or line of collimation may be given any direction desired.

In Fig. 230 O represents the peep of the rear sight in its zero position, the line from O to the front sight A being parallel to the axis of the piece. Or the line OA may represent the line of collimation of a telescope, the telescope being pivoted at A. If now we calculate that to reach the target at F, under the conditions prevailing, a certain angle of elevation is required and a certain deviation to the left, we lift the peep of the rear sight to the point C so that OAC is the required angle of elevation, and then move the peep horizontally from C to E to obtain the required deviation. The line of sight is now the line EA, and if the gun is maneuvered so that this line is made to pass through the target, the axis has

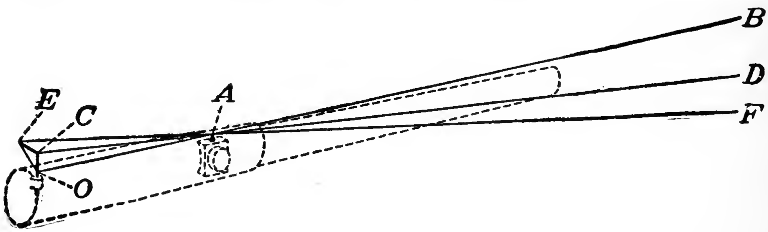


FIG. 230.

then the elevation and deviation required under the existing conditions.

The gun is aimed at the target F, but its axis, parallel to the line CB, is practically pointed at B, which is above F by the vertical distance BD and to the left of F by the horizontal distance DF.

TARGET NOT IN VIEW.—In the foregoing the target has been assumed to be in view. If the target is not in view the required position of the axis of the gun with respect to a horizontal line in the vertical plane through gun and target is determined. The vertical angle between this line and the axis is the angle of elevation. This angle is laid off by the sights as before and the gun is elevated until the line of sight AC is horizontal as determined by means of a spirit level mounted on the rear sight. Other means must be employed for determining the direction in this case.

289. Graduation of Rear Sights.—The graduations of the rear sight for elevation may be, and often are, in degrees and minutes of arc, the center of the arc being at the center of motion of the rear sight. But as the powder charges of guns are made up to give certain fixed muzzle velocities to the projectiles, the angle of elevation required to attain any range with the given muzzle velocity under standard atmospheric conditions may be determined in advance, and the rear sight be graduated for range instead of angular elevation.

The range graduation is the more convenient, for the range may usually be readily determined, and the graduation on the rear sight indicates at once the proper elevation.

The horizontal deflection scale, by means of which allowance is made for deviation to the right or left, is graduated, in sights for field artillery, to thousandths of the range. These graduations are called mils, from the French *millièmes*. It is apparent from Fig. 230 that if EC is n thousandths of AC, the horizontal distance DF will be n thousandths of AD and practically of the range AF. In sights for seacoast artillery the least division of the deflection scale is three minutes of arc, which corresponds to a deflection of 0.00087 of the range, approximately 1/1000.

Correction for Drift.—The deviation of the projectile due to drift, which is caused by the rotation of the projectile and the resistance of the air, may be determined for any range by the formulas of exterior ballistics, and thus the curve of drift may be constructed for any gun. If then the rear sight is so constructed that as the peep is lifted in elevation to any range it is automatically moved horizontally just enough to compensate for the drift at that range, the sight makes automatic correction for the drift, and need be further adjusted only for the wind or other atmospheric deviating influences.

In all service guns the drift of the projectile is to the right. The drift increases with the range. The rear sight with automatic drift correction therefore moves to the left as it is raised in elevation. In our service, automatic drift correction will be found only in sights for small arms.

It is well to bear in mind that the projectile follows the movement of the *rear sight*, going higher as the sight is raised, and to the right or left as the sight is moved to the right or left.

290. Correction for Inclination of Site.—The angle of elevation of a gun is the angle, *in a vertical plane*, that the axis of the gun makes with the horizontal. In Fig. 231 let r be the point to which the rear sight must be raised, in the vertical plane of the axis, to give to the gun a desired angle of elevation equal to ofr, f representing the front sight. h is a horizontal line in the vertical plane of the axis. Now suppose the gun to be revolved to the left about its axis. The axis of the gun remains in the vertical plane, but the points r, o, and f revolve to the left out of the plane; and as r is farther from the center than o and f, its movement is greater than the equal movements of o and f. We may therefore consider that, relatively to o and f, r takes some position r'. Pro-

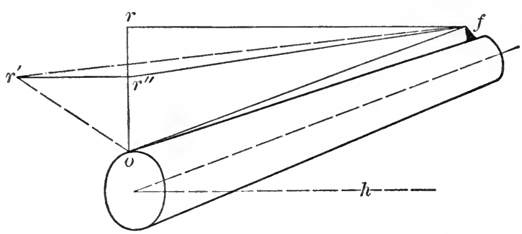


FIG. 231.

jecting r' on the vertical plane, at r'', we see that the angle of sight ofr' produces an angle of elevation ofr'', which is less than the desired angle ofr. It is apparent too that the line of sight through r' will cause the gun to be pointed to the left of the plane of ofr.

If, however, the sight is pivoted at o so that it has movement in a plane perpendicular to the axis of the gun, we are enabled, when the gun has been revolved, to make the sight arm or' vertical; and since the points o and f have revolved together, or', now coincident with or, will subtend the desired vertical angle ofr.

It is therefore essential that the rear sights for guns that are likely to be fired on uneven sites shall be so constructed that the sight arm may revolve about the zero point of the elevation scale in a plane perpendicular to the axis of the gun. We will find that the rear sights for all guns mounted on wheeled carriages are constructed in this manner.

Guns of position are mounted on carriages that rest on level platforms, and their sights are so adjusted as to always move in a vertical plane.

Location of Sights.—Sights for all guns are now placed on some non-recoiling part of the gun carriage, and the elevating and traversing mechanisms are under the control of the cannoneer at the sights, so that the operation of sighting may go on continuously during the loading and firing of the piece.

LINE SIGHTS.—Most guns are provided with line sights fixed to the gun. They serve only to give general direction to the piece, and consist of a front stud with conical point, and a notched bar on the top of the breech. The line extending from a point over the center of the notch at the level of the top of the bar to the point of the front sight is parallel to the axis of the piece.

The most recent service sights and other appliances used in gun pointing will now be described. The sights mounted on the various guns of older model will readily be understood after a study of these.

291. Sights for Mobile Artillery.—The appliances provided for sighting the 3-inch field piece, and other pieces on wheeled carriages, include line sights, the adjustable or tangent sight, the panoramic sight and the range quadrant.

The line sights are fixed to the gun as already described.

The Adjustable or Tangent Sight.—The adjustable sight consists of a fixed front sight and an adjustable rear sight.

The front sight, supported in a bracket on the cradle, is a short tube, Fig. 232, whose axis is marked by the intersection of two cross wires set in the tube at angles of 45 degrees with the horizon. A bead on top of the tube serves for approximate determination of direction.

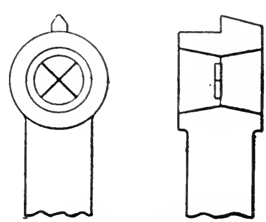


FIG. 232.

The rear sight, Fig. 233, is shown viewed from the left in the left-hand figure, and from the rear in the figure on the right. The rear sight bracket is seated in a socket attached to the cradle of the carriage, on the left side. At the upper end of the bracket two seats are formed for the attachment of the socket for the sight. The seats are faced in a plane perpendicular to the axis of the piece and circular guides are cut on them, with the zero index of the elevation scale as a center.

The shank socket which holds the rear sight is mounted on the bracket and has circular motion on the guides under the action of

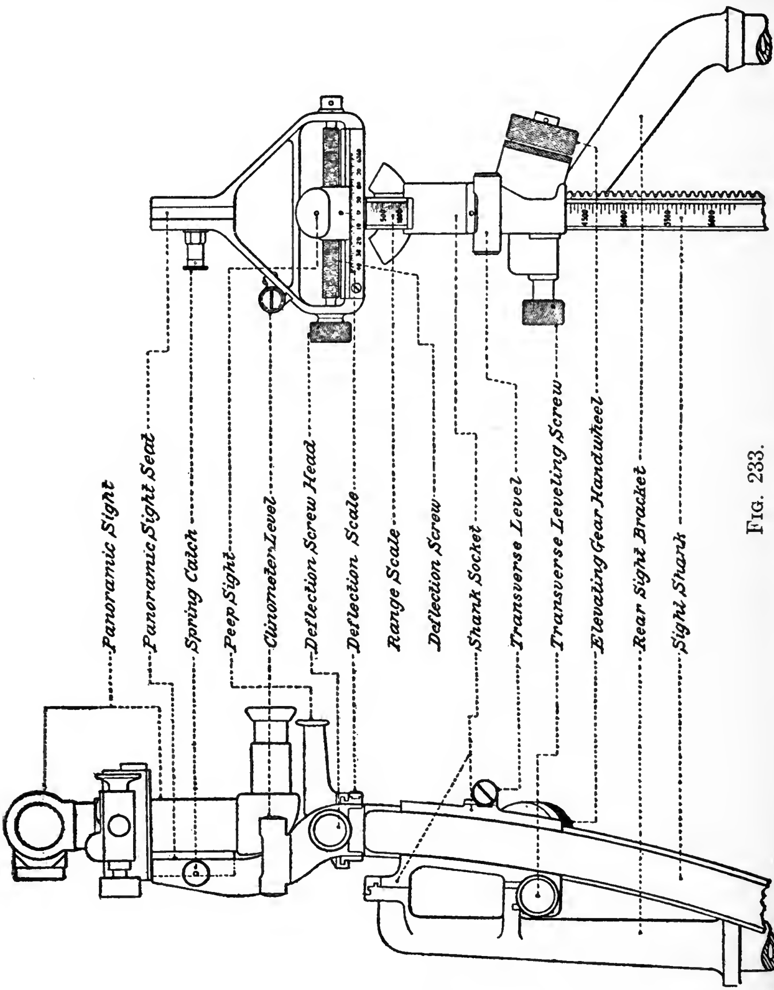


FIG. 233.

the transverse leveling screw. This arrangement permits the correction for inclination of site by revolution of the rear site in a plane perpendicular to the axis of the gun until the sight is vertical, as indicated by the transverse level fixed to the socket.

The sight shank is an arm curved to the arc of a circle whose center is the front sight. The shank slides up and down in guides in the socket, its movement being effected by the thumb nut, called the elevating gear hand wheel, through a scroll gear wheel which acts on the teeth of the rack cut on the right face of the shank. The scroll gear is held in mesh by a spring. By pulling out the thumb nut the gear is disengaged from the rack, and a large change in elevation may then be rapidly made by sliding the shank through the socket by hand.

The range scale is marked on the rear face of the shank, and is read at the index at the upper end of the socket. The smallest division of the scale corresponds to 50 yards of range. but this may be readily subdivided by the eye.

On the upper end of the shank is a frame in which is mounted the peep of the rear sight. The peep is moved to the right or left by means of the deflection screw. The peep hole is 1/10 of an inch in diameter. The divisions of the deflection scale correspond to one mil, 1/1000 of the range. The scale is marked from left to right as follows:

40 30 20 10 0 90 80 70 6360

The deflection readings are uniform with those of the panoramic sight and battery commander's telescope. They will be explained later in the description of the panoramic sight.

The sight is continued upward above the seat for the peep to form a seat for the panoramic sight.

The axis of the clinometer level is parallel to the line of sight, and thus permits the use of the sight as a quadrant in giving elevation to the piece when the target is not in view.

In the sight for the 6-inch howitzer, see Fig. 132, the front sight is mounted on the same bar as the rear sight, and the bar revolves in elevation about a point between the two sights. The rear sight has a sliding movement in deflection on the end of the bar.

The adjustable sight is often called a tangent sight from its similarity to the sights with straight shanks formerly much used with cannon. The peep of the tangent sight moves on the tangent of an arc instead of on the arc itself. The rear sight for the 30-caliber rifle is a tangent sight.

For field howitzers the seats for the front and rear sights are alike, so that the positions of the sights may be reversed for indirect sighting, which consists in directing the line of sight at any object other than the target.

292. **The Panoramic Sight.**—The fire from modern field guns is so accurate and destructive that it has been found necessary in recent battles to establish field batteries always in positions out of view of the enemy, in order to protect the batteries from the fire of the enemy's guns.

Indirect sighting becomes then of necessity the usual method of sighting guns in battle.

The panoramic sight affords the means of aiming the gun by directing the line of sight on any object in view from the gun. At the same time it offers the advantages of a telescopic sight in direct or indirect aiming.

The panoramic sight is a telescope so fitted with reflectors and prisms that a magnified image of an object anywhere in view may be brought to the eye without change in the direction of sight.

The panoramic sight for the field and siege guns is shown in Fig. 234. The rays of light from the object viewed enter the sight through the plain glass window in the head piece and are bent downward by the prism of total reflection *A*, rectified vertically by the prism *B*, focussed by the object lens *C*, and rectified laterally by the gabled prism *D*, so that there is presented to the eyepiece *E* a rectified image of the object, which image is magnified by the two lenses of the eyepiece.

The magnifying power of the instrument is 4 and the field of view is 10 degrees.

The Rotating Prism.—The interior tube containing the prism *B* and the objective *C* is mounted so that it may rotate in the body of the telescope.

The prism *B* is rectangular in cross section. Its upper and lower faces are oblique to its axis, and its length is such that a ray that enters the prism axially emerges axially. Every ray entering parallel to the axis therefore emerges at an equal distance on the other side of the axis. A vertical ray entering the prism at *a*, Fig. 235, is reflected by the back of the prism and emerges at *c*. Now

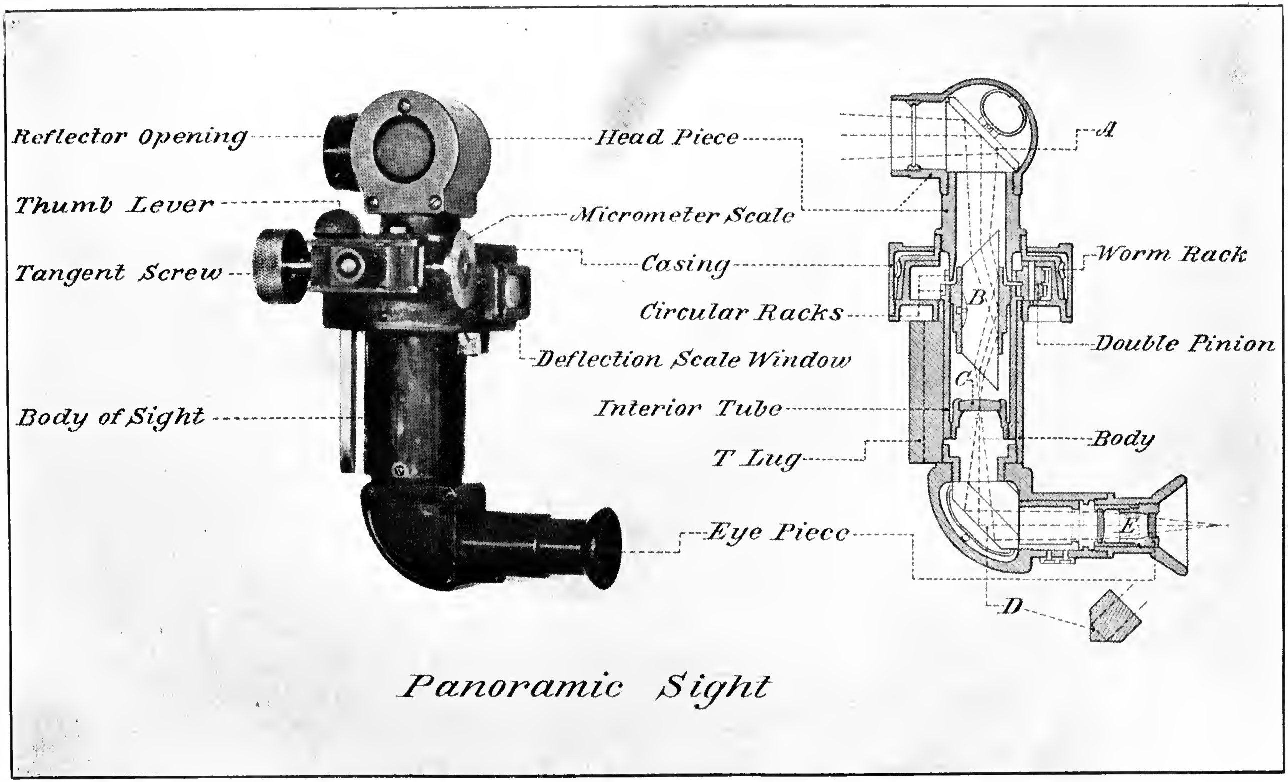


FIG. 234.—Panoramic Sight for Mobile Artillery.

if the prism is revolved through any angle, *say 45 degrees*, as represented in the figure by the position shown in broken lines, the ray *a* will emerge at *e*, the back of the prism now being at the angle of 45 degrees with its original position; and the angle through which the ray has moved, measured from the axis of the prism, which is the axis of rotation, is *90 degrees*. The angular movement of the ray is therefore double the angular movement of the prism. Consequently the image of an object seen through the prism rotates through twice the angle of rotation of the prism.

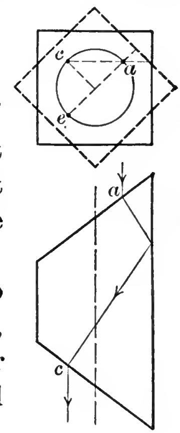


FIG. 235.

The head piece containing the prism *A* is also mounted to rotate on the body of the telescope, and in order to counteract the doubled angular movement of the image by the prism *B*, the head piece is made to rotate twice as fast as the prism. The image of any object then rotates through the same angle as the head piece, and the relative positions of objects in the field of view are not changed.

The different movements of *A* and *B* are accomplished by means of one tangent screw through gearing contained in the cylindrical casing seen at the junction of the rotating parts.

THE GRADUATED SCALE.—The angular movement of the head piece is indicated by a graduated scale on its perimeter, visible through a window in the rear of the casing. When the index on the casing is at the zero of the scale, the line of sight of the panoramic sight is in the vertical plane parallel to the axis of the piece. If at the same time the tangent sight on which the panoramic sight is mounted is at the zero of the elevation scale, the line of sight of the panoramic sight is parallel to the axis of the piece.

In the scale on the head piece the circle is divided in 64 equal parts, numbered clockwise. One complete turn of the tangent screw moves the head piece through one of these angles. A micrometer scale mounted on the shaft of the tangent screw has 100 equal divisions. A movement of the tangent screw through one of the divisions of the micrometer scale therefore moves the head piece through 1/6400 of a circle, which angle corresponds very closely to 1/1000 of the range. The reading of the scales is in 6400ths of the circle. The hundreds are read from the scale on the head piece, and tens and units from the scale on the tangent screw. Thus if the index has passed the mark 27 on the head scale, and the index of the micrometer scale stands at 18, the reading is 2718.

Referring now to the readings on the deflection scale of the tangent sight, page 511, we see that the first reading to the left of the zero, which is 10, indicates a position of the tangent sight parallel to the position of the panoramic sight when the index of the scale on the head of the panoramic sight is between 0 and 1 of the scale, and the index of the micrometer scale is at 10. Similarly the reading 90, to the right of the zero, indicates the position of the panoramic sight between 63 and 64 of the head scale with the micrometer scale at 90. The reading of the panoramic sight is then 6390.

Use as a Range Finder.—As horizontal angles may be measured with the panoramic sight the sight may be used as a range finder. Using the line between the sights of the flank guns of a battery as a base the triangle formed by the two sights and the target may be determined.

On Seacoast Carriages.—Trials are now being made of the panoramic sight applied to disappearing carriages. The sight is attached to the left cheek of the chassis with the eye end of the sight at a height convenient for the gunner standing on the racer platform. The vertical tube of the sight is of length sufficient to bring the head of the sight above the crest of the parapet.

293. The Range Quadrant.—In rapid firing, the duties of setting the sight for range and deflection, and laying the piece by manipulating the elevating and traversing mechanisms would, if attended to by a single cannoneer, frequently delay the firing much beyond the time required to load. Since in the carriages for mobile artillery the elevating and traversing mechanisms are entirely independent of each other, the pointing of the piece may be much simplified and the time required be considerably lessened by assigning to one cannoneer the pointing of the piece for direction and to a second the elevation of the piece for range. Such a division of duties is provided for by the elevating crank at the right side of the trail and by the range quadrant attached to the right of the cradle. By this arrangement, the gunner on the left of the piece, using the open or panoramic sight, lays for direction

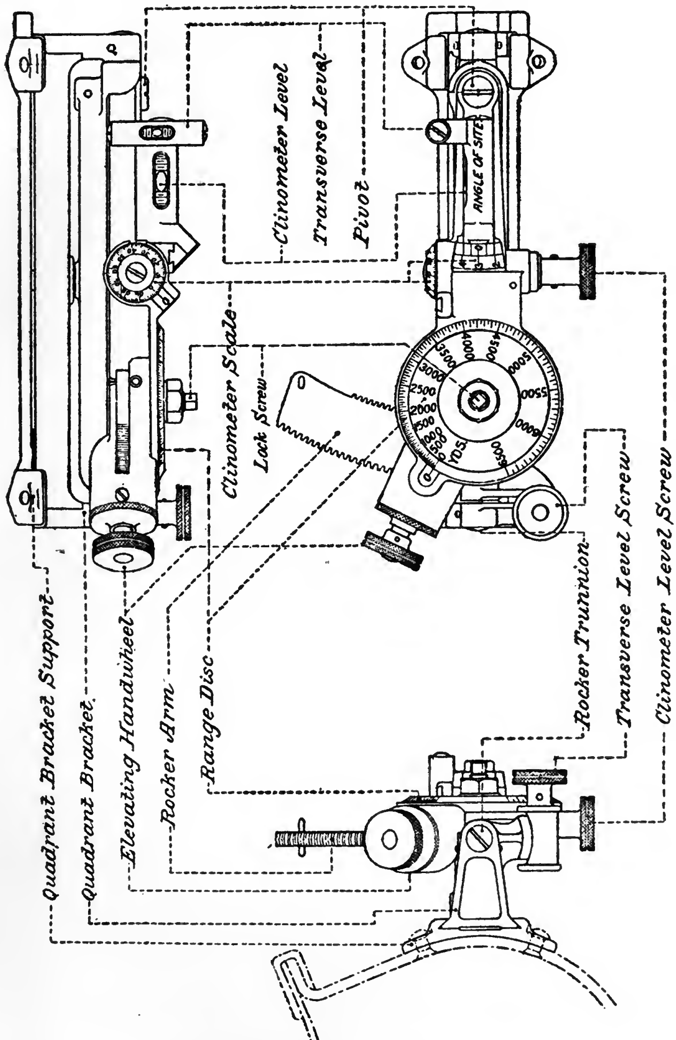


FIG. 236.—Range Quadrant for Mobile Artillery.

only, while the cannoneer on the right of the piece gives quadrant elevations.

The range quadrant, Fig. 236, is supported in a bracket on the right side of the cradle of the carriage with its axis parallel to the vertical plane containing the axis of the piece; and provision is made for rotation of the quadrant about its axis in order that the curved rocker arm of the quadrant may be made vertical when the wheels of the carriage are on different levels. The vertical position of the quadrant arm is indicated by the transverse level.

The quadrant consists of a fixed arm of which the rocker arm is a part; and a movable arm, in front of the fixed arm in the figure, carrying a range disk, a clinometer level, and the mechanism for elevating the movable arm. The fixed arm has at the rear an upwardly extending arc, called the rocker arm, with toothed racks on front and rear edges. The movable arm, pivoted at the front to the fixed arm, is given motion about its pivot by a gear actuated by the elevating hand wheel and meshing in the rearmost rack. A pinion on the shaft of the range disk meshes in the forward rack, and the movement of the arm in elevation is indicated by the scale on the range disk in terms of the corresponding range.

THE CLINOMETER.—The clinometer level is pivoted on the axis of the movable arm, and may be moved relatively to the arm by the clinometer level screw, the upper end of which carries a micrometer scale. A short circular scale is marked on the left edge of the piece carrying the level. The level scale is in 64ths of a circle, and the micrometer scale in 6400ths, similar to the scales of the panoramic sight.

The purpose of the clinometer is to make correction for difference in level of the gun and target. The angle subtended at the target by the difference in level is called the angle of site, as may be seen by the words on the clinometer level in the figure. In exterior ballistics we have called this angle the angle of position, which is a better term, first in better expressing what is meant, and second in not leading to confusion through similarity to the word sight, and to the term angle of sight, in frequent use.

The readings on the clinometer scale are 2, 3, and 4, read 200, 300, and 400, to which are added the readings of the micrometer scale. 300 corresponds to the horizontal position of the axis of the gun. The angle of position, expressed in 6400ths of the circle, is obtained by subtracting the reading of the scales from 300. If the reading is greater than 300 the result is negative and the target is above the gun.

294. Use of the Quadrant.—The quadrant is used as follows. The gun is pointed at the target by means of the line sights, the quadrant being set at the zero of the range scale. The quadrant is leveled transversely, and the clinometer level is leveled by means of its screw. The angle indicated on the clinometer scale is the angle of position of the target. Further movement of the gun in elevation is, by means of the clinometer, measured from this position of the gun as zero. The movable arm of the quadrant is elevated until the range of the target is recorded on the range scale. The piece is then elevated until the clinometer level is again level. The piece has now the proper angle of elevation for the range increased or diminished by the angle of position, according as the target is higher or lower than the gun.

It will be noted that in the use of the clinometer in correcting the angle of elevation by adding or subtracting the angle of position we are applying the principle of the rigidity of the trajectory.

The Battery Commander's Telescope and Ruler.—The battery commander's telescope and the battery commander's ruler, used as aids in determining the elements of sighting for pieces employed in indirect fire, should perhaps be classed as range and position finders rather than as appliances for sighting. They will be described in the chapter on range and position finding, which follows this chapter.

Telescopic Sights.—The advantages gained by the use of a telescope in laying a piece consist in an increased power of vision and a large decrease in personal error. The telescope renders distinct an object that may be barely visible to the naked eye and enables the gunner to lay the gun on such an object with accuracy and facility.

Telescopic sights are now used on all guns of position. They are fixed to the non-recoiling cradle of the barbette mount, and to the chassis of the disappearing mount. Hand wheels, or electric controllers, for the manipulation of the mechanisms for laying the piece are in positions convenient to the gunner at the sight, and in addition an electric firing pistol is placed at his hand so that all the operations of aiming and firing the piece are under his control.

295. Telescopic Sight, Model 1904.—The latest pattern of telescopic sight, model 1904, for guns mounted on disappearing carriages, is shown in Fig. 237; see also Fig. 145. Sights of the same model are provided also for barbette carriages. They differ from the sight described only in the method of attachment to the carriage.

The sight arm *a* is pivoted at its forward end on the sight standard of the carriage and is supported, by a pin through the hole near its rear end, on a vertical rod so connected with the elevating mechanism of the gun that it gives to the sight arm the same movement in elevation that is given to the gun, see Figs. 145 and 146. A curved guide *g*, moving in a groove in the standard, keeps the sight arm in the vertical plane. A cradle *c* carrying the telescope *t* is pivoted to the forward end of the sight arm in such a manner that the cradle has both vertical and horizontal movement about its pivot. Vertical movement is given by the hand wheel *e* which actuates a gear mounted on the sight arm and meshing in the rack on the shank *s*. The cradle is given horizontal movement on the head of the shank by the deflection screw *d*. On the rear face of the shank is an elevation scale graduated to degrees and minutes of arc, the least reading being 6 minutes. A deflection scale on the rear end of the cradle under the telescope extends over 4 degrees of arc. The degree marks are numbered from 1 on the right to 5 on the left, the 3-degree mark corresponding to no deflection. The least reading of the deflection scale is 3 minutes, which corresponds approximately to a deflection of one mil.

When the sight is set at the zero of the elevation and deflection scales the axis of the telescope is parallel to the axis of the piece.

A range drum *m* connected with the elevating gear of the sight indicates the range corresponding to any position of the sight. The range drum contains a coiled ribbon spring arranged to equalize the efforts in elevating and depressing the sight.

A peep sight *p* is mounted above the eye end of the telescope, and an open front sight *f*, with crossed wires, is mounted above the forward end of the cradle.

Electric lamps *l* illuminate, in night sighting, the elevation and deflection scales and the cross hairs in the telescope.

THE TELESCOPE.—The construction of the telescope will be understood from Fig. 238. The achromatic object glass *o*, com-

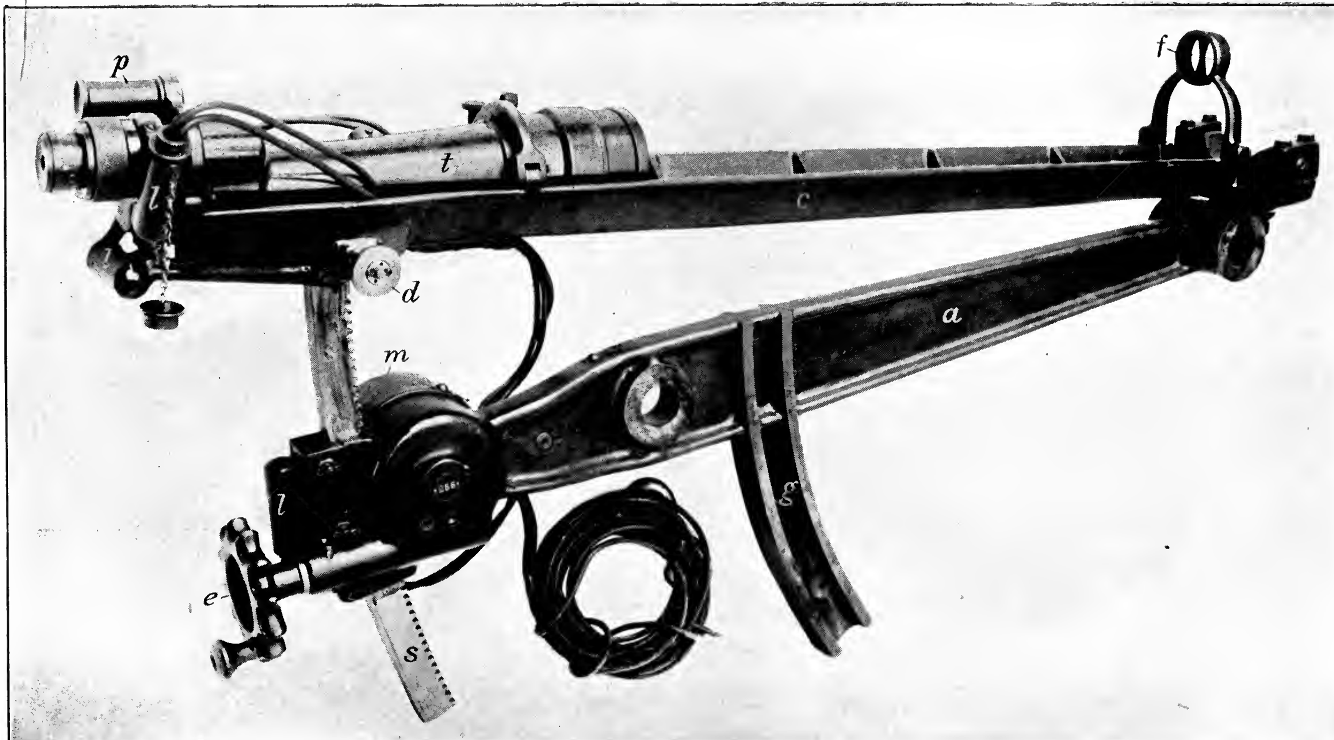


Fig. 237.—Telescopic Sight, Model 1904.

posed of three lenses, has a clear aperture 3 inches in diameter and a focal length of 17.25 inches. The length of the telescope is diminished and an erect image presented to the eyepiece by means of the two Porro prisms *p*. In the figure the prisms appear to be so placed that each intercepts a ray of light entering or issuing from the other, but in reality the prisms are offset from each other so that the light has unobstructed passage to and from them. One prism is horizontal and the other stands vertically. The lower prism by its inclined surfaces bends the ray twice through angles of 90 degrees, reflecting it back to the upper prism, which again bends it twice and reflects it into the field of the eyepiece. The image, rectified horizontally and vertically by the prisms, is

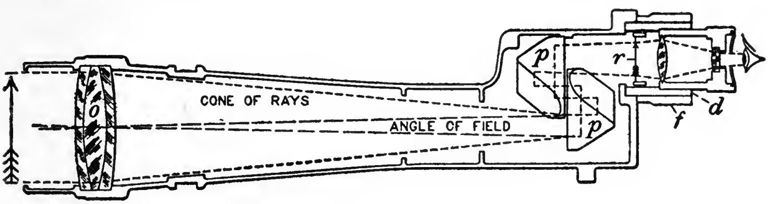


FIG. 238.

focussed in a plane marked by horizontal and vertical cross wires *r* carried in a ring, and is magnified by the two lenses of the eyepiece. The ring carrying the cross wires is mounted in a tube *d* called the draw tube which may be given movement in and out by rotation of the focussing ring *f*. The eyepiece has a screw motion out and in.

Two different eyepieces are provided with the telescope, their magnifying powers being 12 and 20 diameters respectively. The field of view of the telescope with the 12-power eyepiece is 3.6 degrees, and with the 20-power eyepiece 2.6 degrees.

In the use of the instrument the eyepiece is first adjusted until the cross wires are distinctly defined. The cross wires are then brought into the focal plane of the objective by turning the focussing ring until the object viewed is also distinctly defined and does not appear to move when the eye is shifted from side to side. An objective once focussed is correct for all observers, but the eyepiece requires focussing for each individual.

Small electric lamps of about 2 candle power, *l* Fig. 237, illuminate, in night sighting, the cross wires at *r* and the elevation and deflection scales in the vicinity of the indexes. The lamp that illuminates the cross wires is attached outside the draw tube and its light is reflected by two mirrors through two slits cut through the tube at right angles to each other. The light from each mirror is thrown upon the full length of a cross wire, and the wires appear as bright lines in a dark field.

296. Telescopic Sight, Model 1898.—The telescopic sight, model 1898, illustrated in Fig. 240, is provided for the 8-, 10-, and 12-inch barbette carriages and for disappearing carriages of the earlier models. A seat for the sight is attached to the chassis. When mounted in this seat the sight is used to give to the gun direction in azimuth only.

A seat is also provided on the trunnion of the gun, and in this seat the sight may be used in giving both elevation and direction. The bracket *b*, Fig. 240, is screwed to the trunnion. The telescope is mounted in a frame whose trunnions *t* rest in notches in the bracket. The frame and telescope are leveled transversely by the screw *l* which bears against a lug projecting from the trunnion shaft of the frame.

Erecting Prisms.—To rectify the image of the object there is mounted in the telescope between the objective and the eyepiece a Hastings-Brashear compound erecting prism, Fig. 239. The

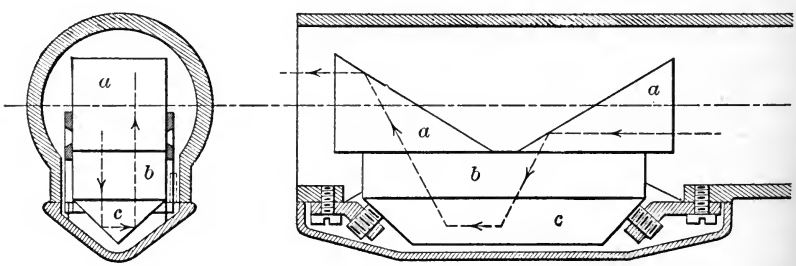


Fig. 239.

compound prism is composed of two prisms, *a*, whose angles are 30, 60, and 90 degrees, laid with their 30-degree angles toward each other on a parallel-sided glass plate *b*. On the other side of the plate is fixed a gabled prism *c* with a 90-degree angle. The upper prisms rectify the image vertically, and the lower prism

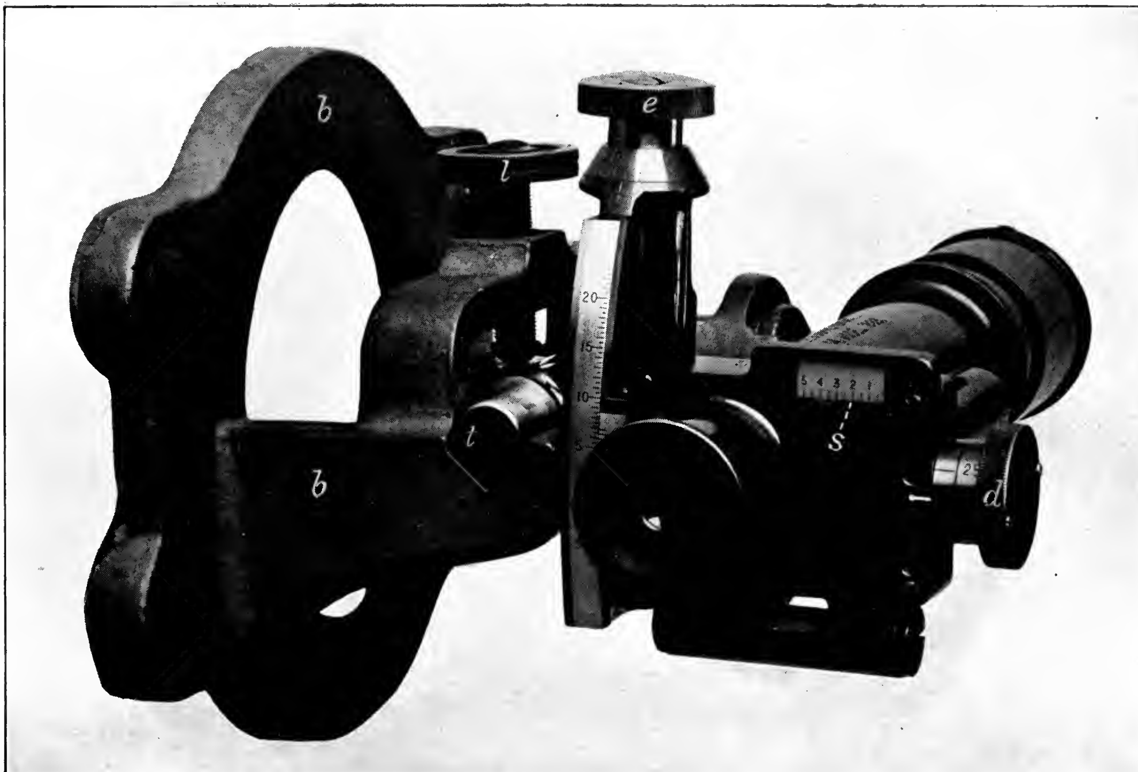


Fig. 240.—Telescopic Sight, Model 1898.

horizontally, as may be seen by following the course of the ray of light shown in the figure.

The telescope is pivoted at its forward end to the frame and is given movement in elevation by the screw *e*, Fig. 240. The elevation scale is read to one minute by a micrometer scale under the screw head.

Deflection is given by moving the vertical cross wire in the telescope to the right or left by means of the deflection screw *d*, and then moving the gun until the intersection of the vertical and horizontal cross wires covers the point aimed at.

There are two deflection scales, one inside the telescope and one outside. The inside scale, of horn, is in the focal plane of the telescope and is seen at the same time with the object viewed. The scale is graduated in divisions of 3 minutes, and the degrees are numbered from 1 on the right to 5 on the left as in the model 1904 telescopic sight. The cross wires in the telescope appear in front of the scale. The vertical cross wire is attached to a sliding diaphragm which is actuated by the deflection screw *d* and moves the vertical wire to any desired degree of deflection to the right or left.

In sighting, the intersection of the cross wires is brought in line with the object sighted.

The outside deflection scale, *s* Fig. 240, corresponds in movement with the scale inside the telescope. Both scales are read to minutes by the graduations on the micrometer head *d*.

In a telescopic sight the cross wires inside the telescope form virtually the front sight, and the aperture of the eyepiece forms the rear sight. With the telescope just describedd eflection is given by moving the vertical cross wire to the right or left, and this movement is equivalent to moving the front sight to the right or left. We have seen on page 507 that with the front sight fixed the projectile follows the movement of the rear sight. When the rear sight is fixed a movement of the front sight is equivalent to a movement of the rear sight in the opposite direction. Therefore with the telescopic sight, model 1898, the projectile will be moved to the right by movement of the vertical cross wire to the left, and to the left by movement of the vertical wire to the right.

297. The Power and Field of View of Telescopes.—The power of a telescope, the ratio of the apparent angle subtended by any object to the actual angle which the object subtends, may be obtained by dividing the aperture of the object lens by the aperture of the eye lens. The telescope of the model 1904 sight has an objective with an aperture of 3 inches. The eye lens of one of the eyepieces provided has an aperture of $\frac{1}{4}$ of an inch. The power of the telescope with this eyepiece is therefore 12. In the telescope of the model 1898 sight the aperture of the objective is $1\frac{1}{4}$ inches and of the eye lens $\frac{1}{6}$ of an inch. The telescope has therefore approximately a power of 8.

The eye receives the maximum amount of light through a telescope when the diameter of the pencil of light emerging from the eyepiece is equal to the diameter of the pupil of the eye. In the normal eye the diameter of the pupil varies approximately from $\frac{1}{6}$ of an inch to $\frac{1}{4}$ of an inch, according as there is much light or little.

The field of view of a telescope is equal to the field of the eyepiece divided by the power of the telescope. The telescope of the model 1898 sight has a power of 8 and its eyepiece has a field of 48 degrees. The field of view of the telescope is therefore 6 degrees.

The field of view of the same telescope with different eyepieces varies practically in inverse ratio to the power of the telescope.

298. Aiming Mortars.—Mortars, both field and seacoast, are as a rule located out of view of their targets and usually behind high shelter. Seacoast mortars are permanently emplaced. Their carriages are provided with graduated azimuth circles by means of which the piece may be laid at any given angle with the meridian plane. The angle made with the meridian plane by the line to the target is determined by means of range and position finders. The piece is then laid at that angle by means of the graduations on the azimuth circle, and correction is made for drift and deviation due to the wind.

For giving direction to field and siege mortars the vertical plane through gun and target is established by stakes, or by trestles with plumb lines, set up either in front of or behind the mortar in such a position that both gun and target are in view. The axis of the mortar is brought into this plane or into any determined position with respect to the plane, and the first round is fired. Correction for error in direction is afterwards made by means of marks on the platform.

The Gunner's Quadrant.—Elevation is given to mortars by means of the gunner's quadrant shown in Fig. 241. The movable

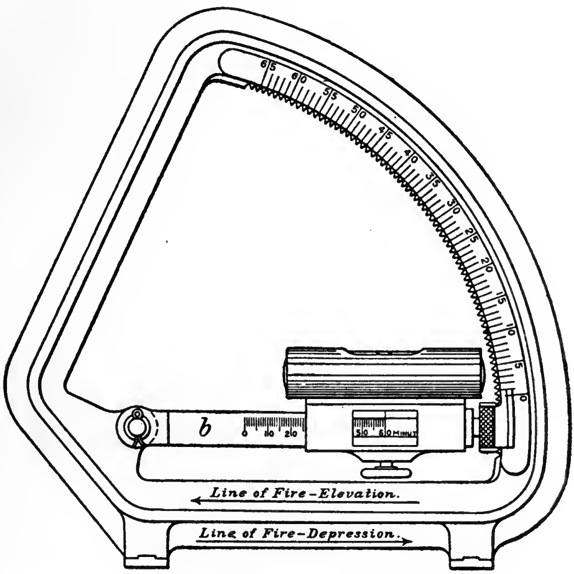


FIG. 241.

arm *b* carries a spirit level and may be set at any desired angle with the base of the instrument up to 65 degrees. The notched scale fixes positions for the arm *b* at whole degrees. Minutes are obtained by sliding the level along the scale on the curved arm *b*. The principle of the sliding level on the curved arm will be readily understood by reference to Fig. 242.

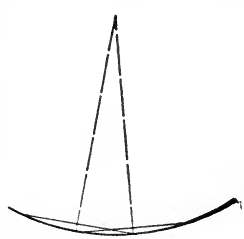


FIG. 242.

The quadrant may be used to measure angles of elevation or of depression from 0 to 65 degrees.

The quadrant, set to any desired angle of elevation, is placed on the gun on a seat prepared for it parallel to the axis of the piece. The instrument is so placed that the proper arrow on its base points in the direction of the line of fire. The piece is then elevated until the bubble of the level is in the middle of the tube.

By placing the instrument on a vertical seat, as for instance the face of the breech or muzzle of a gun, angles greater than 25 degrees from the vertical may be measured. The angle is obtained by subtracting the reading of the quadrant from 90 degrees.

To facilitate the elevating of the mortar the quadrant is now, on mortars mounted on the model 1896 carriage, permanently fixed to a seat provided on the right rimbase of the mortar. The level is fixed on the movable arm of the quadrant, and minutes of elevation are obtained through movement of the arm by means of a tangent screw at its end.

CHAPTER XIV.

RANGE AND POSITION FINDING.

299. Definitions.—A range finder is an instrument for determining the range from the observer to any distant object.

A position finder is an instrument for determining the position of an object with respect to any plane or line, as the meridian plane for guns of position or the front of a battery for mobile artillery.

An instrument adapted to perform both functions becomes a range and position finder.

Range Finders.—With all practical range finders the determination of the range comes from the solution of a triangle. The target is the apex of the triangle. The base of the triangle is laid off either vertically or horizontally from the instrument, and the angles at the extremities of the base are determined, one or both of them, by means of the instrument.

In determining any fixed range the effect of an error in the measurement of an angle at the base of the triangle will diminish as the length of the base increases. This is apparent from Fig. 243. A given range *ot* is less affected by an angular error *tbe* made at the end of the base *ob* than by an equal error *tac* made at the end of the shorter base *oa*.

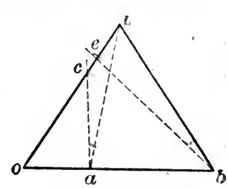


Fig. 243.

It is therefore always desirable to use as long a base as can be conveniently obtained. For this reason horizontal base lines are preferred, since the vertical base of any range finder is limited in length to the height of the instrument above the water.

Consequently in seacoast fortifications, if the surroundings afford convenient sites for the angle measuring instruments, the range finding system consists of two transits or azimuth instruments established at the ends of a long base. Observations are made on the target from both ends of the base. The position of the target is plotted on a chart, and its range and position determined for any gun. If the target is moving, simultaneous observations are made from both ends of the base at periodic intervals. The readings of the instruments are transmitted by telephone or telegraph to a plotting room in the fortification, where the successive positions of the target are marked on the chart. From the plotted course prediction may be made as to the position the target will occupy at some determined instant in advance, and the range and azimuth of the target at the selected instant may be determined for any gun or battery in the fortification.

300. Depression Range Finders.—The principle employed in the depression range finder will be understood from Fig. 244. The instrument, at a known height above the sea level, measures the vertical angle to any object. From the fixed height each angle corresponds to a certain length of base, which is the horizontal range to the object.

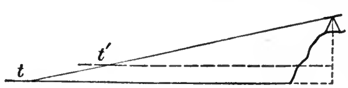


FIG. 244.

The range in yards is indicated on a scale which is moved past an index by the same mechanism that gives angular movement to the line of sight.

A difference in the sea level due to the action of tides will affect the height of the instrument above the sea level and consequently the range corresponding to any angle, *t* and *t'*, Fig. 244. Means are therefore provided for adjustment of the instrument for variations in its height above sea level.

The instrument is made a position finder by being mounted so as to revolve on a fixed base which is graduated in degrees and hundredths, the zero graduation being placed in the meridian plane.

Swasey Depression Range and Position Finder.—The depression range and position finder now used in our service is shown in Fig. 245. The observing telescope, similar in construction to

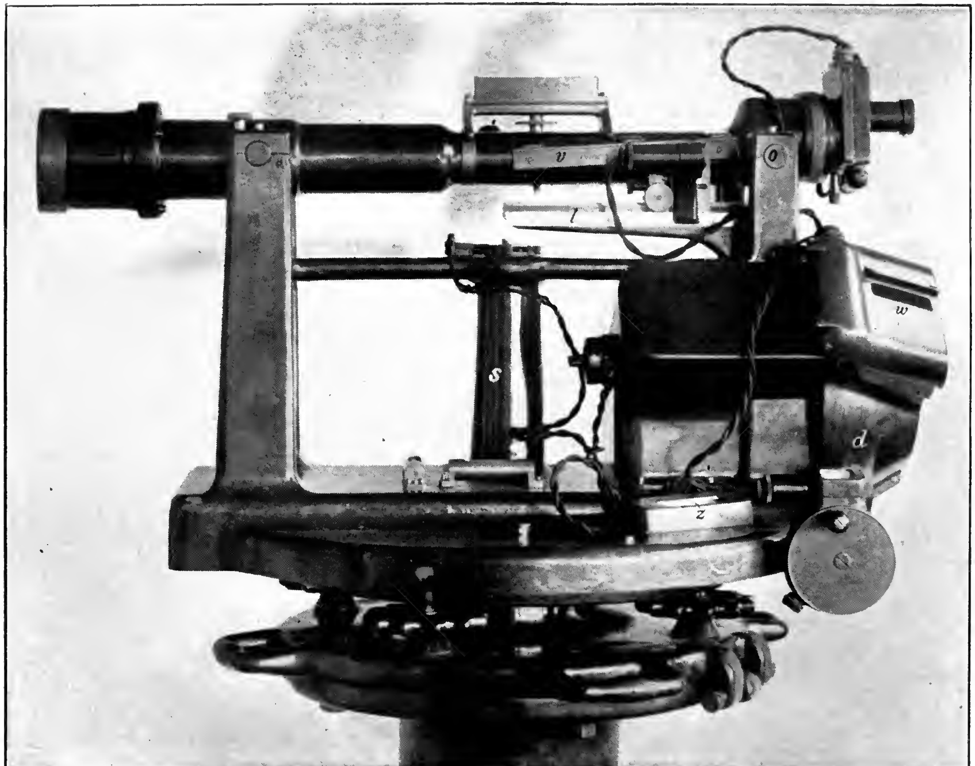


Fig. 245.—Swasey Depression Range and Position Finder

the telescope of the model 1904 sight, is mounted in a frame which revolves about a central spindle *s* projecting upward from the pedestal. The telescope is pivoted near its front end, and is supported near its rear end by the attached bar *v* which rests on a stud projecting from the carriage *a*. The carriage *a* is mounted on the forward arm of a bent lever *l* which is pivoted at *o*. The lower vertical arm of the lever is connected by gearing with the operating shaft, not seen in the figure. Turning the operating shaft moves the lower end of the lever *l*, and thus gives vertical movement to the telescope about its forward pivot. The range drum enclosed in the casing *d*, and visible through the window *w* in the casing, is given motion by the same shaft, and the scale on the drum indicates the range corresponding to any position of the telescope. The azimuth is read from a graduated scale seen through the window *z*.

The carriage *a* may be moved along the upper arm of the lever to adjust the position of the telescope for any height above sea level. The height scale along which the carriage moves reads from 40 to 400 feet. Corrections may be made, by moving the carriage, for the change in height of the instrument due to the change in sea level caused by the tides.

301. The Plotting Room.—The range and azimuth of any selected target, as determined by either range finder system, is communicated to the plotting room. In this room are assembled all the instruments necessary for the complete determination of the elements of sighting for the directing gun in the battery whose fire is directed from the room. The corrections to be applied to the observed range to compensate for the effect of the wind, of the thermometric and barometric conditions, of differences in tide level, and of the motion of the target, are quickly determined from the instruments for a predicted position of the target at some instant in advance. The deviation due to the wind and drift and motion of the target are also determined. The corrected range, azimuth, and deviation are sent to the gun, and the gun is then pointed according to the instructions received. The command to fire is given at such a moment as to cause the shot to arrive at the predicted position of the target at the same instant as the target.

The instruments used are as follows.

The wind component indicator gives the components of the wind for range and deflection for use on the range and deflection boards. The azimuth of the wind's direction, taken from the wind dial, and the velocity of the wind, taken from an anemometer, are laid off on the instrument. The azimuth of the target is also laid off, and the instrument then indicates by a pointer the range and deflection components of the wind with respect to the line from the gun to the target.

The atmosphere board indicates the correction to be applied at the range board for thermometric and barometric changes.

The range board, with the data supplied by the foregoing instruments and other data indicated below, gives the corrections in yards to be applied to the range for wind, atmosphere, tides, and variations from the standard muzzle velocity, and indicates the sum of these corrections.

The plotting board converts the range and position of the target as determined from the reports of the range and position finders, to the range and position for the particular battery or gun, with the correction for range determined by the range board.

The deflection board indicates, for the corrected range and azimuth from the plotting board, the sum of the deflections to be applied to the sight, or to the azimuth of the piece, to correct for the deviating effect of wind, drift, and the motion of the target.

By means of these instruments, which have been devised by artillery officers of our army, the correct setting of a gun may be determined, the gun aimed, and the shot sped on its way, in an interval of 15 seconds. The instruments are simple in construction and manipulation, and their use is entrusted to the enlisted soldier.

302. Field Range and Position Finding.—For range and position finding in the field there are provided the Weldon range finder, the battery commander's telescope, the battery commander's ruler, and the field plotting board. The uses of these instruments will be understood from their descriptions.

The Weldon Range Finder.—The Weldon range finder, Fig. 246, consists of three triangular prisms mounted in a metal frame. The silvered base of each prism rests against the metal. The angle at the apex of each prism is as follows.

The upper or first prism, 90 degrees
The second prism, 88° 51′ 15″
The third prism, 74° 53′ 15″

Now if we construct, as in Fig. 247, the first two of the above angles at the end of a base whose length is unity, and the third

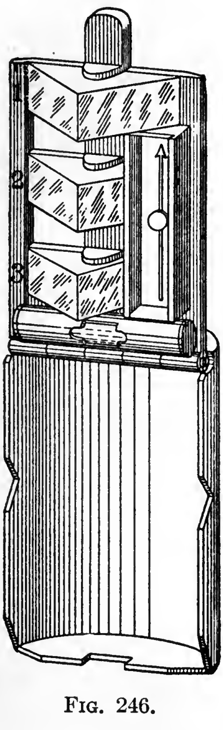


FIG. 246.

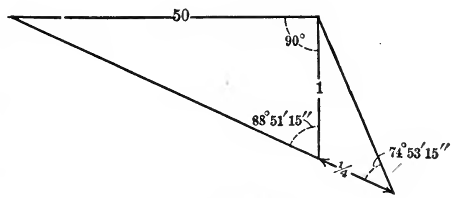


FIG. 247.

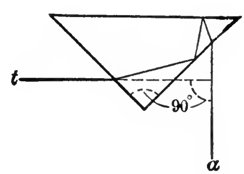


FIG. 248.

angle as shown in the figure, the sides of the resulting triangles will be of the lengths marked on them in the figure, the sides being proprtional to the sines of the opposite angles.

Each prism diverts a ray of light through an angle equal to the angle at its apex, as may be seen from Fig. 248. A ray entering the first prism from *t* or *a* issues from the prism in a direction perpendicular to its original direction. And similarly a ray will issue from the second prism at an angle of 88° 51′ 15″ with its original direction.

Standing at *a*, Fig. 249, and looking into the *first prism*, we see the image of the object *t* in the direction *ad*, perpendicular to *at*, and at the same time looking over the prism we see the object *d* in line with the image of *t*. Now moving back on the line *da* there will be some point *b* on this line where the target *t*, seen in the *second prism*, will again align with the object *d* seen over the prism. The angle *tba* is then 88° 51′ 15″ and *at*, the range to the target, is 50 times the base *ab*, see Fig. 247.

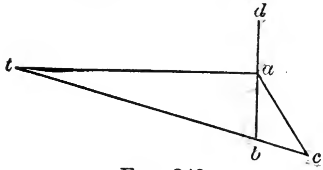


Fig. 249.

The *second prism* may be used at both ends of the base. The triangle *abt* will then be an isosceles triangle, the angle at *a* being equal to the angle at *b*, and the length of the sides *at* and *bt* will be 25 times the length of the base.

The *third prism* is provided for use when the base *ab* is inconveniently long or when through the interposition of a gulch or other obstacle the length of the base can not be directly measured.

The points *a* and *b*, Fig. 249, having been determined, the observer moves on the line *tb* to some point *c* from which, looking in the *third prism*, he sees the image of the point *a* covering the object at *t* seen over the prism. The angle at *c* is then 74° 53″ 15′, and as shown in Fig. 247 the base *cb* is one quarter of the base *ab* or 1/200 of the range *at*.

It is apparent from Fig. 248 that the instrument may be used with the apex of the prism toward the eye or toward the target, since both *t* and *a* may represent either target or eye. The position of the image in either case with respect to the apex of the prism is indicated in the figure.

The true *refracted* image may always be distinguished from images *reflected* from the face of a prism by revolving the instrument about a vertical axis. Reflected images revolve with the instrument, but as the lateral refraction is a fixed one *the refracted image remains stationary* when the instrument is revolved.

When the instrument is held with the compass needle pointing north, the bottoms of the two notches in the middle of the cover mark the east and west line; and these two notches together with the two at the end of the cover mark diagonal lines running northeast and northwest.

303. The Battery Commander's Telescope.—The battery commander's telescope, Fig. 250, is mounted on a tripod in the same

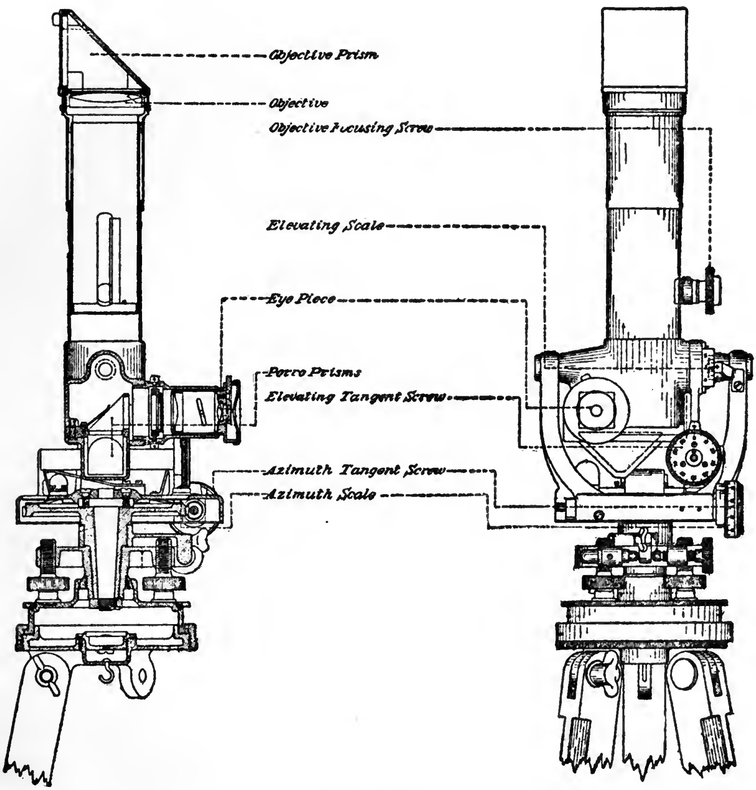


FIG. 250.

manner as the telescope of a transit instrument. It has movement about horizontal and vertical axes. The amounts of the movements about the axes are indicated by scales graduated to 6400ths of the circle, or mils, corresponding for horizontal movement to the deflection scale of the panoramic sight, and for vertical movement to the clinometer scale of the range quadrant.

The telescope forms an erect magnified image of the object. The ray of light enters the window in front of the objective prism, is reflected downward by this prism, which is one of total reflection, passes through the objective, is rectified by the two Porro prisms, and forms the image in the plane of the cross hairs in front of the eyepiece.

The objective has a clear aperture of $1\frac{3}{4}$ inches, and a focal length of 11 inches. The power of the telescope is 10, and the field of view is 4 degrees.

The battery commander's telescope is used for measuring both horizontal and vertical angles; horizontally, the azimuths between the target, gun, and aiming point, the azimuth of the front of a hostile position, the correction in azimuth required to bring the shots from a battery on to the target; and vertically, the angle of position of the target, the correction in elevation required to bring the projectile to the target or the burst of the shrapnel to the proper height above the target.

304. The Battery Commander's Ruler.—The battery commander's ruler, Figs. 251 and 252, constructed after the manner of the slide rule, provides on the front, Fig. 251, a scale for quickly measuring azimuths and a slide rule for determining the height of the trajectory in mils at any point of the range, and on the back, Fig. 252, a table of parallaxes, computed for a base of 20 yards, for several ranges and for different angles of obliquity of base to range.

The instrument is of brass about 6 inches long, 1 inch wide, and $\frac{1}{8}$ of an inch thick.

A cord about 2 feet long passes through a hole in the ruler. One end of the cord is fastened to a button on the observer's coat so that when the ruler is held out until the cord is taut the ruler is 20 inches from the observer's eye.

The scales on either edge of the front of the ruler are graduated to read azimuths in mils. To measure any angle in azimuth, as for instance from the target to the aiming point, the ruler is held horizontally at the length of the cord with the zero at the end marked *T* in line with the target. The azimuth to the aiming point is indicated on the scale at the point where the line from the eye to the aiming point cuts the edge of the ruler. It will be seen that azimuths to the right of the target read from 0 to 300, and azimuths to the left read from 6100 to 6400, corresponding to the deflection scales of the sights. The ruler is always held with that

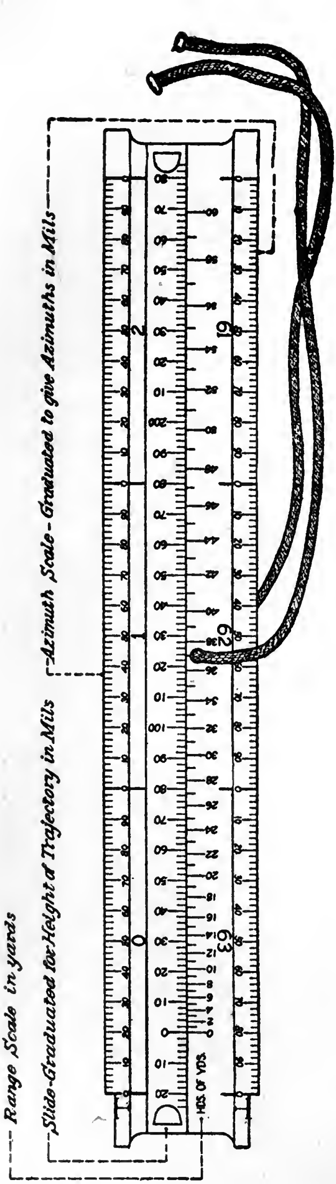


FIG. 251.

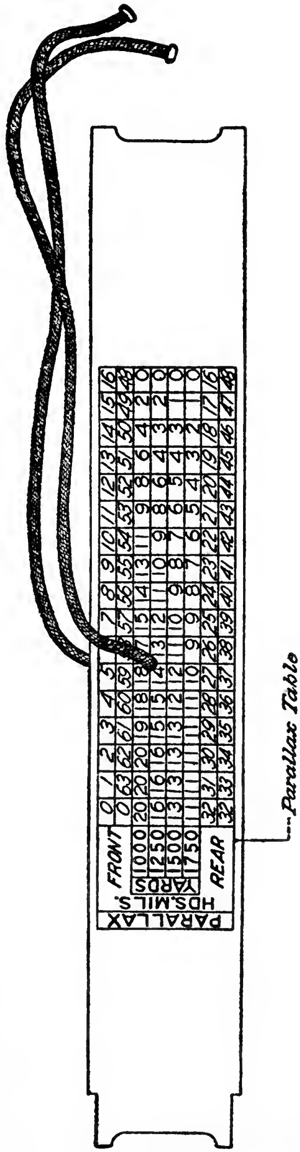


FIG. 252.
Battery Commander's Ruler.

edge up that will give a reading in the desired direction from the mark *T* on the scale. All desired azimuths are similarly measured. The ruler will be used for these measurements when the more accurate battery commander's telescope is not at hand.

THE SLIDE.—The slide and the adjacent range scale on the ruler provide the means for determining the height of the trajectory in mils at any given point of the range. This information may be frequently required for use in ascertaining whether an intervening obstacle such as a hill, or woods, or a tower, will interfere with the fire at a given target, or in determining the extent behind the obstacle that is masked from the fire of the gun. The slide is graduated in mils from -24 through 0 to $+284$. The adjacent range scale on the ruler is in hundreds of yards.

To use the instrument, first determine the angle of position of the target, in mils, by the battery commander's telescope or otherwise. Move the slide so as to place the slide graduation that indicates the angle of position of the target over the range of the obstacle as indicated on the range scale. The height of the trajectory at the obstacle, in mils, is then indicated on the slide opposite the range of the target on the range scale. If the height indicated is greater than the angle of position of the obstacle, obtained in the same manner as the angle of position of the target, the projectile will clear the obstacle.

The principle involved in the use of the slide will be understood from Fig. 253, in which the 6000-yard trajectory of the 3-

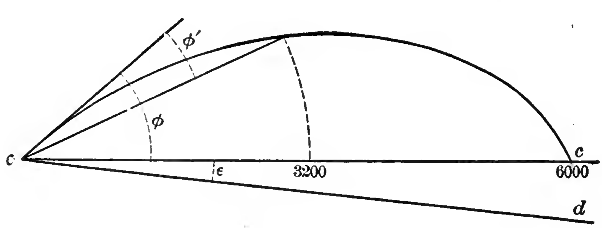


FIG. 253.

inch rifle is represented. The angular heights of the successive points of the trajectory, measured from the origin, evidently diminish from the angle of departure ϕ at the origin to zero at the end of the range. Under the principle of the rigidity of the trajectory we may assume with sufficient exactness that within the limits of direct fire any portion of the trajectory from the origin is the true trajectory for the range represented by its chord. We may therefore assume the portion of the trajectory subtended by the shorter chord in the figure as the true trajectory for the range 3200 yards, and from the figure we see that the angular height of the 6000-yard trajectory at 3200 yards is the angle of departure ϕ for 6000 yards minus the angle of departure ϕ' for 3200 yards.

On the range scale under the slide, Fig. 251, the zero of the two scales being together, each range is indicated opposite its corresponding angle of departure as indicated in mils on the slide. Thus the angle of departure for a range of 3200 yards is nearly 100 mils, 100/6400 of 360 degrees, or 5° 37′.

A movement of the slide in either direction will cause the reading above any range to be increased or diminished; that is, the movement adds an angle to the angle of departure for the range, or subtracts an angle. If the zero of the slide is moved to the 3200-yard mark on the range scale, the angle of departure for 3200 yards is subtracted from the reading over every range on the scale. Therefore the angle of departure for, say, the 6000-yard range is diminished by the angle of departure for 3200 yards, and as shown in Fig. 253 this difference, indicated on the slide over the 6000-yard mark on the range scale, is the height of the 6000-yard trajectory at 3200 yards.

Now if we assume that the line *od*, Fig. 253, is horizontal and that the target at *c* is elevated above *d* by the angle of position ε, say 20 mils, it is evident that 20 mils must be added to every reading on the slide. We therefore move the zero of the slide back until the 20 on the slide instead of the zero is now over the range 3200. The reading over every range is increased by 20.

We have now put the angle of position of the target over the range of the obstacle, and over the range of the target we read the height of the trajectory at the obstacle.

305. The Parallax Table.—On the back of the ruler, Fig. 252, is inscribed what is called the parallax table. By parallax is meant here the angle, in mils, subtended by the front of a platoon, 20 yards, from any point outside the battery. Thus in Fig. 254, *a* being the aiming point and *t* the target, the parallax of the aiming point is the angle at a subtended by the two guns, and the parallax of the target is the angle at t subtended by the guns.

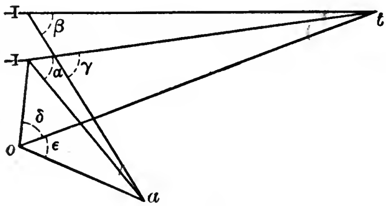


FIG. 254.

The parallax of a point that lies in a direction normal to the front of the battery is, since 1 mil is 1/1000 of the range, equal to 20 divided by the number of thousands of yards in the range. Thus for 4000 yards the parallax is 5 mils. If the point lies in a direction oblique to the front of the battery, the parallax is equal to the normal parallax multiplied by the cosine of the angle which the direction of the point makes with the normal to the battery front.

The parallax has been calculated for different ranges and different directions of the point and tabulated on the back of the ruler. The upper two lines of the table, Fig. 252, give the angles of obliquity in hundreds of mils in the two quadrants in front of the battery, the lower two lines give similar angles for the two quadrants in rear. The parallax of any point at any one of the four ranges marked at the left is found in the line of the range and in the column that indicates, to the nearest hundred mils, the obliquity of the point's direction. The parallax in any fixed direction is an inverse function of the range, therefore for any range not given in the table it may be readily determined by means of the parallax for some range in the table. Thus the parallax for 3000 yards is half that for 1500 yards or $\frac{1}{3}$ that for 1000 yards.

By means of the parallax the proper setting of the sight in indirect firing may be determined for one gun from the sighting of the adjacent gun. Thus in Fig. 254 if the gun on the right has found the target, at the angle α from the aiming point, the angle β for the second gun is readily obtained. Representing by p_a and p_t the parallax angles at a and t respectively, we see from the figure that, since

$$\gamma = \alpha + p_a = \beta + p_t$$
$$\beta = \alpha + p_a - p_t$$

306. Plotting Board for Mobile Artillery.—The plotting board, Fig. 255, 16 inches wide by 39 inches long, is covered with rubber cloth. Across the middle of the board is a grooved guideway *g*, its edges graduated in yards. The protractor *o* slides in the guideway. The protractor is graduated in 64ths of a circle and by a vernier may be read to mils. The outer graduated rim of the protractor turns about the fixed central part. Fixed to the outer rim of the protractor is the arm *f*, and pivoted to the center of the protractor is the arm *m*, both graduated in yards. On each arm

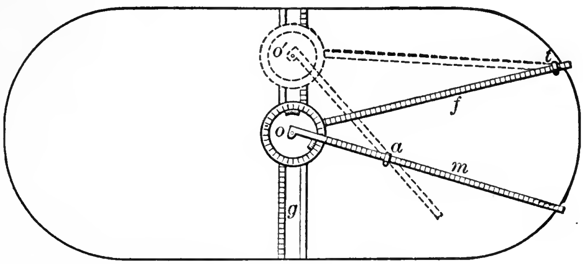


FIG. 255.

is a sliding index, *a* and *t*, provided with a pin which may be stuck into the board to hold the index in a fixed position.

The plotting board is used at the observing station to determine, for the directing gun, the position of the target with respect to the point selected as an aiming point. Thus in Fig. 254, *o* is the observing point from which the aiming point *a*, the target *t*, and the directing gun are visible. The ranges from the observer to the three points are determined, and the angles made by the lines to the points with the line from the observer to the gun. This line to the gun is the datum line, and is represented on the plotting board by the center line of the grooved guideway. The scale on the edge of the guideway is graduated to yards.

With the protractor in the center of the board, *o* Fig. 255, the arm *m* is placed at an angle with the guideway equal to the angle $\delta+\epsilon$, Fig. 254, and the sliding index on the arm is placed at the range *oa* on the scale. Similarly the arm *f* is revolved to make the angle δ with the guideway, and its index is placed at the range *ot* on the scale. The pins of the two indexes are stuck into the board.

The protractor is now moved along the guideway to the point on the guideway scale, o' Fig. 255, that marks the distance from the observer to the gun. The two arms slide through the indexes and assume the positions of the lines from the gun to the aiming point and to the target, Fig. 254. The angle α between the arms is read from the protractor, and the ranges from the gun to the aiming point and target are read from the scales on the arms.

307. Other Range Finders.—Other range finders have been constructed on the principle of the Weldon range finder, using prisms with different angles or producing the deflection of the ray by means of mirrors.

The Berdan Range Finder.—The Berdan range finder consists of two telescopes permanently mounted 6 or 12 feet apart on the bed of a wagon, and provided with graduated circular bases by means of which the angles between each of the telescopes and the base are measured. The short base renders excessive the effect of a slight error in the measurement of an angle, and for this reason principally the instrument has not been found satisfactory in service.

The Barr and Stroud Range Finder.—The Barr and Stroud range finder, used on the ships of our own and foreign navies, and now being tested for our field service, is constructed, optically, in the manner shown in Fig. 256. The tube containing the optical

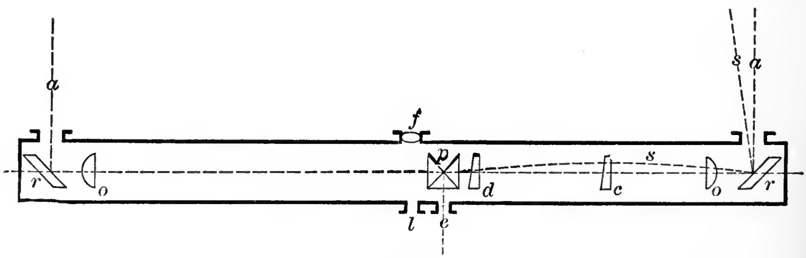


FIG. 256.

parts is so mounted on the deck of the ship, that the target may be kept in view during heavy rolling or pitching of the ship.

Two reflectors r, marking the ends of a base line $4\frac{1}{2}$ feet long, divert the rays from the target through the objectives o and thence through the prisms p to the observer's right eye at e. The field of view of the right eyepiece is divided horizontally by a dark line, Figs. 258 and 259. The image from the objective on the right is formed above this line and that from the left below it.

A deflecting prism, *d* Fig. 256, has a sliding movement in the right telescope. When in position at *d* the prism has no deflecting effect on the ray from the objective, and in this position of the prism the parallel rays *a* from an object at a great distance, as from the sun or moon, will form a continuous image in the field of the right eyepiece. Now if a nearer object, on the same line from the left reflector, be viewed, the direction of the ray to the light reflector will be changed from *a* to *s* and the image from the right telescope will not be continuous with that from the left, Fig. 259.

Fig. 257.

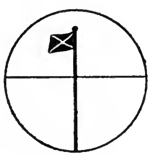


Fig. 258.

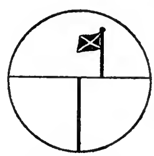


Fig. 259.

Fig. 260.

Continuity in the image is obtained by sliding the deflecting prism *d* to the position *c*. The amount of the movement of the deflecting prism is dependent on the range to the object; and the ranges corresponding to the various positions of the prism are marked on a scale that is carried by the prism. A movement of the deflecting prism over a length of six inches corresponds to a change in range from infinity to 250 yards.

The observer looks with his left eye through the eyepiece *l*, Fig. 256, and through the finder objective *f* opposite. The left eyepiece and the object lens *f* form a low powered telescope with a large field of view. The object viewed, Fig. 257, is seen through this telescope, and in the field of view above the object appear a pointer and a portion of the scale that is attached to the deflecting prism *d*. The range to the object is read from the scale at the pointer.

For use at night in obtaining the range to any target that bears a light an optical appliance called an *astigmatizer* is provided in the instrument. The astigmatizer lengthens the images of a point of light into vertical streaks, Fig. 260, and the streaks are brought into coincidence. The astigmatizer is moved aside when not in use.

The Le Boulengé Telemeter.—The Le Boulengé telemeter is an instrument by means of which the velocity of sound is used for measuring distance. The instrument is a glass tube filled with liquid. In the tube is a loose glass piece or traveler whose specific gravity is but slightly greater than that of the liquid, so that when the tube is held vertical the traveler falls through the liquid slowly and with approximately uniform motion. The time between the flash of a gun and the arrival of the report is measured by turning the tube from a horizontal to a vertical position when the flash is seen, and back to the horizontal when the report is heard. The range corresponding to the distance that the traveler has fallen in the interval is read from a scale on the tube.

As the velocity of sound, 1100 feet per second in calm air, varies with the velocity and direction of the wind, this method of measuring ranges is not satisfactory.

CHAPTER XV.

SMALL ARMS AND THEIR AMMUNITION.

308. Service Small Arms.—The present service small arms are the .38 caliber revolver, model 1903, and the .30 caliber rifle, model 1903. Automatic pistols have been issued to the service for trial within recent years, but the results of the trials have not been sufficiently favorable to bring about the adoption of any of these arms for the military service. Automatic and semi-automatic rifles have also been submitted to the Ordnance Department for test. The tests are now in progress.

The .38 Caliber Revolver.—The service revolver is made by the Colt's Patent Fire Arms Manufacturing Co. of Hartford, Conn., and is known as the Colt's double action revolver, caliber .38.

A double action revolver is one that may be fired in either of two ways: by separately cocking the hammer and pulling the trigger; or by performing both operations with a single pull on the trigger. When the separate movements are employed the piece is said to be used in single action; and in double action, when cocked and fired by the pull on the trigger alone. The service revolver may be used either in single action or in double action. Much greater rapidity of fire can be attained using the revolver in double action, but on account of the increased effort required in firing in this manner, and the consequent unsteadiness of the hand holding the revolver, the fire is not likely to be as accurate as when the revolver is fired in single action.

The mechanism of the revolver is shown in Fig. 261. The operation of the mechanism is briefly as follows. In single action the piece is cocked by pressure of the thumb on the head of the hammer, 18. The lower end of the hammer moves the upper end of the trigger forward and upward until the upper edge of the trigger engages under the lip at the lower end of the hammer and holds the hammer in the cocked position. A pull on the trigger will then release the hammer, which, under the action of the mainspring 32, falls and explodes the cartridge. The pressure on the trigger being released, the rebound-lever spring 37

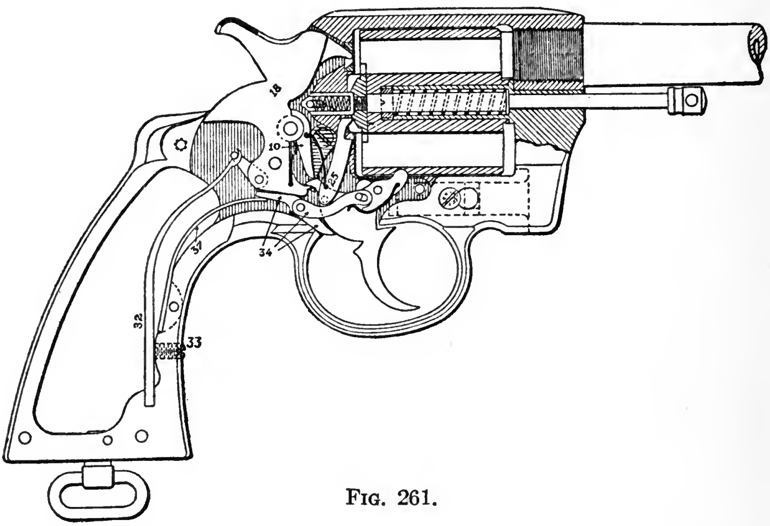


Fig. 261.

acting on the rebound-lever 34 moves the hammer back slightly to its safety position and at the same time moves the trigger forward.

When firing in double action the pull on the trigger causes the upper end of the trigger to bear against the end of the strut 10 which is pivoted on the pivot of the hammer and bears against the hammer above the pivot. The pull on the trigger thus lifts the hammer until, when the hammer is nearly at full cock, the strut escapes from the end of the trigger and the hammer falls. As the rear part of the trigger moves upward, whether in single or in double action, the upper end of the hand 25 engages in a notch on the rear face of the cylinder and causes the cylinder to revolve through one-sixth of a turn. At the last part of the movement of the trigger a projecting lug forward on its upper surface passes through a slot in the frame and engaging in a notch in the cylinder prevents further movement of the cylinder.

The mechanism includes safety devices which allow the piece to be cocked only when the cylinder is fully closed and latched in the proper position.

309. THE MAINSPRING TENSION SCREW.—The mainspring tension screw 33 is an important part of the mechanism whose functions are not usually understood. Its purpose is to vary the tension of the mainspring in order to adjust the force of the blow delivered by the hammer on the primer of the cartridge. When the revolver is used in double action the hammer is not retracted as far as in single action and consequently delivers a lighter blow on the primer. It is a difficult matter to manufacture a primer suitable for both methods of firing. If the cap of the primer is made thin enough to insure firing of the primer under the lighter blow in double action, the metal of the cap is likely to be pierced by the point of the hammer under the heavier blow in single action. The pierced primer allows the powder gases to escape to the rear, perhaps to the injury of the soldier. If on the other hand the primer cap be made sufficiently thick to insure its not being punctured by the heavier blow, the primer may not be sufficiently sensitive to be always fired by the lighter blow. The importance of a proper adjustment of the tension of the mainspring is therefore apparent. If it is found that failures to fire in double action are frequent the screw 33 should be screwed in slightly to increase the tension of the mainspring and produce a heavier blow of the hammer. But the tension must not be increased more than absolutely necessary, for otherwise puncture of the primer may occur when the revolver is fired in single action.

THE BARREL.—The barrel of the revolver has a length of 6 inches, and a diameter between the lands of the rifled bore of 0.357 of an inch. It is rifled with 6 grooves 0.003 of an inch deep and with a uniform twist of one turn in 16 inches. The rifling has a left handed twist in order that the drift of the bullet to the left may counteract to some extent the tendency that exists to pull to the right in firing.

Ammunition and Ballistics.—The ball and blank cartridges used in the revolver are shown in Fig. 262. The charge in the ball cartridge is about $3\frac{1}{2}$ grains of a nitroglycerine powder, and produces in the bullet a muzzle velocity of 750 feet. The bullet, of lead, weighs 148 grains. Its greatest diameter is 0.357 of an inch, which is the diameter between the lands of the rifled bore. The powder gases entering a conical cavity in the base of the bullet expand the base of the bullet into the grooves of the rifling. The grooves of the bullet are filled with Japan wax as a lubricant. The wax also serves, together with the crimping of the front end of the cartridge case against the bullet, to keep out moisture and render the cartridge waterproof.

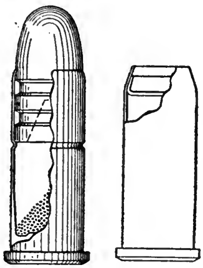


Fig. 262.

While the bullet has sufficient energy to inflict a disabling wound at a range of 200 yards, the revolver cannot be relied upon for accurate firing beyond 75 yards.

The blank cartridge contains 7 grains of E. C. powder held in the case by a paper wad crimped in place and shellacked.

310. The Colt Automatic Pistol.—In the Colt automatic pistol the recoil of a movable barrel and slide is utilized to eject the fired shell, cock the firing mechanism, and load a new cartridge into the barrel; so that after the first shot is fired the only operation necessary to fire the remaining cartridges in the magazine is a pull of the trigger for each cartridge.

The pistol is made in three calibers, .32, .38, and .45. The magazines of the two smaller pistols hold 8 cartridges; that of the .45 caliber pistol holds 7 cartridges. The .45 caliber pistol is represented in Figs. 263 to 265. The rear part of the frame or receiver *r* forms a hollow handle which encloses the magazine and the firing mechanism. The magazine, Fig. 264, is a light metal case containing a spring and follower. The cartridges are inserted one at a time by sliding in at the top. The sides of the magazine curve slightly over the upper cartridge, which may be removed only by being pushed out to the front. The magazine when filled is inserted into the handle of the pistol from below and is held in place by a spring catch.

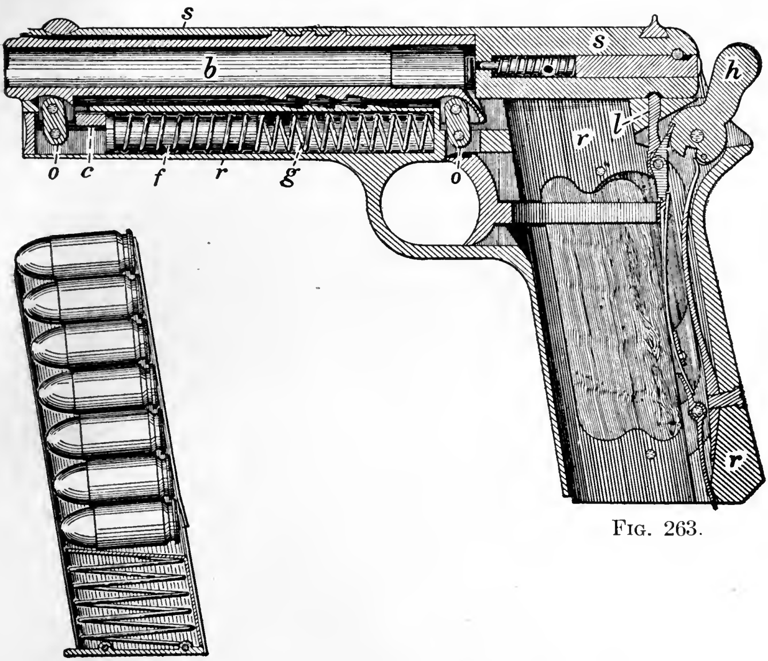


Fig. 263.

Fig. 264.

Fig. 265.

Colt Automatic Pistol, Caliber .45.

The forward extension of the receiver *r* contains the retractor spring *g* and has formed on its sides guides for the reciprocating slide *s*. The barrel *b* is attached to the receiver by two links *o*. The forward part of the slide *s* covers the barrel, and the rear part forms the breech bolt and carries the firing pin. Three lugs formed on the top of the barrel engage in notches in the slide and lock barrel and slide together. The slide lock *c*, a straight bar, holds the slide to the receiver. It passes through longitudinal slots in the sides of the receiver, and its ends are engaged in notches in the slide. The head of the retractor-spring follower *f* presses against a recessed seat in the middle of the slide lock *c*, and thus holds slide and barrel in firing position.

Operation.—The operation of the pistol when fired is as follows. The powder gases acting rearwardly against the bolt force the slide to the rear against the pressure of the retractor spring. The barrel, carried to the rear with the slide, revolves about the lower pivots of the two links *o*, its axis always remaining parallel to the top of the receiver. The downward movement of the barrel soon disengages it from the slide, but not until after the bullet has left the muzzle. The momentum acquired by the slide causes it to continue to the rear. Its rear end cocks the hammer *h*. An extractor carried by the slide withdraws the fired shell which, striking an ejector, is thrown out to the right through a slot in the slide. When the front of the bolt has passed to the rear of the top cartridge in the magazine this cartridge is forced upward into the path of the bolt by the magazine spring.

As the slide returns under the action of the retractor spring the bolt forces the top cartridge forward out of the magazine into the barrel in its lowered position, and then raises the barrel into its locked position for firing. A pull on the trigger now causes the cocked hammer to strike the firing pin and fire the cartridge.

When the last cartridge has been fired the slide remains to the rear, thus warning the soldier that the magazine is empty.

The safety lever, *l* Fig. 263, prevents movement of the trigger until the slide and barrel are in proper position for firing.

To load the first cartridge into the barrel, the rearward movement of the slide is produced by hand, the slide being grasped by the disengaged hand at the roughened surfaces on its sides, and pulled to the rear.

The necessity of using two hands to load the first cartridge into the barrel is one objection to the pistol as a military arm.

HOLSTER.—The pistol holster is a light steel frame covered with leather, and is arranged to be attached to the butt of the pistol in such a manner as to serve as a stock by means of which the pistol can be fired from the shoulder.

AMMUNITION.—The .45 caliber bullet, of lead with a cupro-nickel jacket, weighs 200 grains. The charge of powder is 5.1 grains. The muzzle velocity of the bullet is 900 feet.

Five shots may be fired from the pistol in a second.

311. Modern Military Rifles.—The modern military rifle differs from its predecessors chiefly in caliber and in the use of the magazine. The caliber of the rifle in our service has been reduced from 0.45 to 0.30 of an inch, with an accompanying reduction in the weight of the bullet from 500 grains to 220 grains, and recently to 150 grains. The maximum pressure in the bore has been increased, with the change in caliber, from 25,000 pounds per square inch to 44,000 pounds.

INCREASED VELOCITY.—The increased pressure, better sustained along the bore by modern powders, produces in the lighter bullet a velocity very much greater than that attained in the rifles of larger caliber. The muzzle velocity of the bullet from the .45 caliber rifle was 1300 feet per second, while the present service rifle gives to the 220-grain bullet a muzzle velocity of 2200 feet, and to the 150-grain bullet a muzzle velocity of 2800 feet. At the same time, since the weight of the gun has not materially changed, the ratio of weight of bullet to weight of gun has greatly diminished. On this ratio principally depends the maximum velocity of free recoil of the gun for any given velocity of the projectile, see equation (4), page 275. We may consider the velocity of recoil, or better its square, as a measure of the shock of recoil. In the modern rifle the ratio of weight of bullet to weight of gun is diminished to such an extent that, even with the increased velocity of the bullet, the velocity of recoil is diminished. In consequence of the lighter shock of recoil on the soldier's shoulder, he is enabled to longer continue his fire without fatigue.

Other Advantages. — The increased muzzle velocity increases the range and accuracy of the rifle and flattens the trajectory, thus increasing the danger space for any range. The increased velocity has been attained with a shorter barrel, thus diminishing the weight of the gun and facilitating the handling of the gun by the soldier.

The reduced weight of the bullet and of the charge of powder reduces the weight of the cartridge, thereby enabling the soldier to carry a greater number of cartridges on his person.

The Jacketed Bullet.—In order that the metal of the bullet shall not be stripped by the rifling as the bullet passes with high velocity through the bore, it is necessary to cover the soft lead of the bullet with a jacket of tougher material. The modern bullet is therefore composed of a lead core enclosed in a jacket made of cupro-nickel or of nickeled steel. The lead gives weight to the bullet and increases its sectional density, see page 458, while the tougher jacket enables the bullet to take the rifling without material deformation, and also gives to the bullet greater penetration in any resisting material.

The Magazine.—Ease and rapidity of fire are greatly increased by the use of the magazine. At the first introduction of magazine guns the cartridges in the magazine were considered as in reserve, to be used only in cases of emergency. The gun was habitually used as a single loader. In the latest weapons the filling of the magazine may be accomplished more readily than the insertion of a single cartridge into the barrel, since the cartridges are carried by the soldier in packets adapted to magazine loading only. Magazine fire is therefore used habitually, though the guns are adapted for single loading as well.

The mechanism of the magazine is usually arranged to lock the bolt of the gun open when the magazine is empty, so that in the excitement of battle the soldier may not continue to go through the motions of firing with an unloaded gun.

312. **Requirements.**—That the military arm may stand the rough usage incident to service in war it is essential that it be strongly constructed. Its mechanisms must be strong, simple, and easily dismantled for repair in the field without the use of

tools. The mechanisms must not be seriously affected by a moderate amount of rust or dust.

To lessen the chances of injury to the rifle as few of the parts as possible should project beyond its general outline. This latter consideration forms one of the objections to the attachment to military rifles of telescopic sights and other devices for increasing the accuracy of fire. The military rifle can rarely get the care necessary to keep the more delicate and more complicated sporting and target rifles in condition. Especially is this so in time of war when armies, those of the United States particularly, are largely composed of untrained volunteers most of whom have never previously carried a rifle. The arm that is put into their hands must be of such a character that it will be serviceable under almost all conditions, and as accurate as it may be made under this requirement.

Tests.—Before the adoption into our service of a rifle of new model the arm is subjected to tests as follows.

Endurance Test.—The arm is tested for endurance by firing from each of several rifles 5000 rounds, in forty lots of 100 rounds each and two lots of 500 rounds each.

At various stages of the endurance test the ballistic qualities of the arm are tested by firing for *velocity* and *accuracy*, and the working of the mechanism by tests for *rapidity of fire.*

Dust Test.—The rifle, with the breech block closed, is subjected to a blast of fine sand for two minutes, first with the magazine empty and again with the magazine filled with cartridges. After each exposure to the blast the surplus sand is removed by blowing, by wiping with the bare hands only, and by tapping the butt and muzzle on the ground. The rifle must then be capable of operation in single loading and in magazine fire.

Rust Test.—The rifle is thoroughly cleaned and all oil and grease removed by washing in soda water. The muzzle and chamber are tightly corked and the rifle is immersed in a saturated solution of sal ammoniac for ten minutes and then exposed to a damp atmosphere for 48 hours. The rifle must then be capable of operation as before.

Defective Cartridge Test.—Cartridges cut through at the head, others cut through at the extractor groove, and others slit throughout their length are fired in the rifle.

Fig. 266.—Erosion in Rifle Barrels.

Fig. 267.—Effects of Erosion on Bullets.

Excessive Charge Test.—Five rounds are fired with cartridges loaded to produce a maximum pressure in the chamber one third greater than the maximum pressure attained in service.

313. Life of the Rifle. Erosion.—Although the rifle remains serviceable, as far as the operation of its mechanism is concerned, after endurance tests of 5000 rounds or more, its accuracy diminishes markedly after a number of rounds considerably less than 5000, the number depending on the conditions of the firing. With its accuracy seriously impaired the rifle ceases to be suitable for service. The service life of the rifle must therefore be measured by the number of rounds that can be fired from it with accuracy, and not by the number fired in tests for endurance.

The accuracy of the rifle is principally affected through the erosion of the barrel by the powder gases. The gases, highly heated and moving with high velocity under great pressure, attack the walls of the bore, which are probably softened by the great heat, and cut irregular channels in the metal, destroying the surface of the bore and the rifling. The erosion is greatest at the seat of the bullet immediately in front of the cartridge case, and extends forward into the barrel for several inches. Beyond this the walls of the bore are practically unaffected.

The effect of erosion is well shown in the enlarged photographs, Fig. 266, of rifle barrels from which 3500, 4000, and 5000 rounds have been fired.

When the erosion has become marked, the bullet is forced against an irregular surface and the metal of the bullet jacket, probably also softened by the heat, is unequally stretched on different sides, producing a decided eccentricity of the point of the bullet and great irregularity of the base. The sides of the bullet are deeply scored by the powder gases escaping past the bullet and by the irregularities of the bore.

In Fig. 267 are shown enlarged photographs of a service 220-grain bullet, model 1903, recovered after being fired into sawdust from a new rifle barrel, and of bullets fired from barrels that had been previously fired 3500 and 4500 times.

The deformation of the bullet is the chief cause of its inaccuracy. At the same time its muzzle velocity is reduced by the escape of the gases past the bullet in the bore.

Velocity and Pressure.—The erosive effect of the gases appears to depend more on their velocity than on the maximum pressure. Thus in tests that were made with the service rifle with 220-grain bullets fired with muzzle velocities of 2300 and 2200 feet, the maximum pressures in the two cases not being very different, the first appreciable falling off in accuracy occurred after 2000 rounds with the 2300-foot velocity and after 4000 rounds with the velocity of 2200 feet; and the accuracy after 7000 rounds with the lower velocity was better than after 4000 rounds with the higher.

Ammunition loaded to produce a muzzle velocity of 2300 feet was originally used in the service rifle, but after the above mentioned tests the muzzle velocity was reduced to 2200 feet and the accuracy life of the rifle increased from 2000 to 4000 rounds.

The 150-grain bullet recently adopted for the new rifle was intended originally to have a muzzle velocity of 2800 feet, the maximum pressure being considerably less than with the 220-grain bullet. It is doubtful whether, on account of the rapid erosion, this high velocity can be fixed as the standard.

Erosion, the cause of the reduction in the muzzle velocity in the small arm, is also the cause of the recent reduction of the muzzle velocities in the 10- and 12-inch seacoast guns from 2500 to 2250 feet.

314. The U. S. Magazine Rifle, Model 1903.—The present service rifle fulfils all the requirements enumerated in a previous paragraph as essential for a military rifle. As the Cadets of the Military Academy are armed with the rifle and familiar with its operation through daily use, an extended description of the weapon is not necessary here. Consideration of some of its parts may be of advantage.

Two views of the mechanism of the rifle, with bolt in closed position, are shown in Fig. 268.

The Receiver.—The receiver is that part of the gun that contains the breech closing bolt. It is held to the stock by the two guard screws, front and rear. The barrel is screwed into the front of the receiver.

Trigger Pull.—It will be observed that the rounded upper edge of the trigger bears against the bottom of the rear part of the

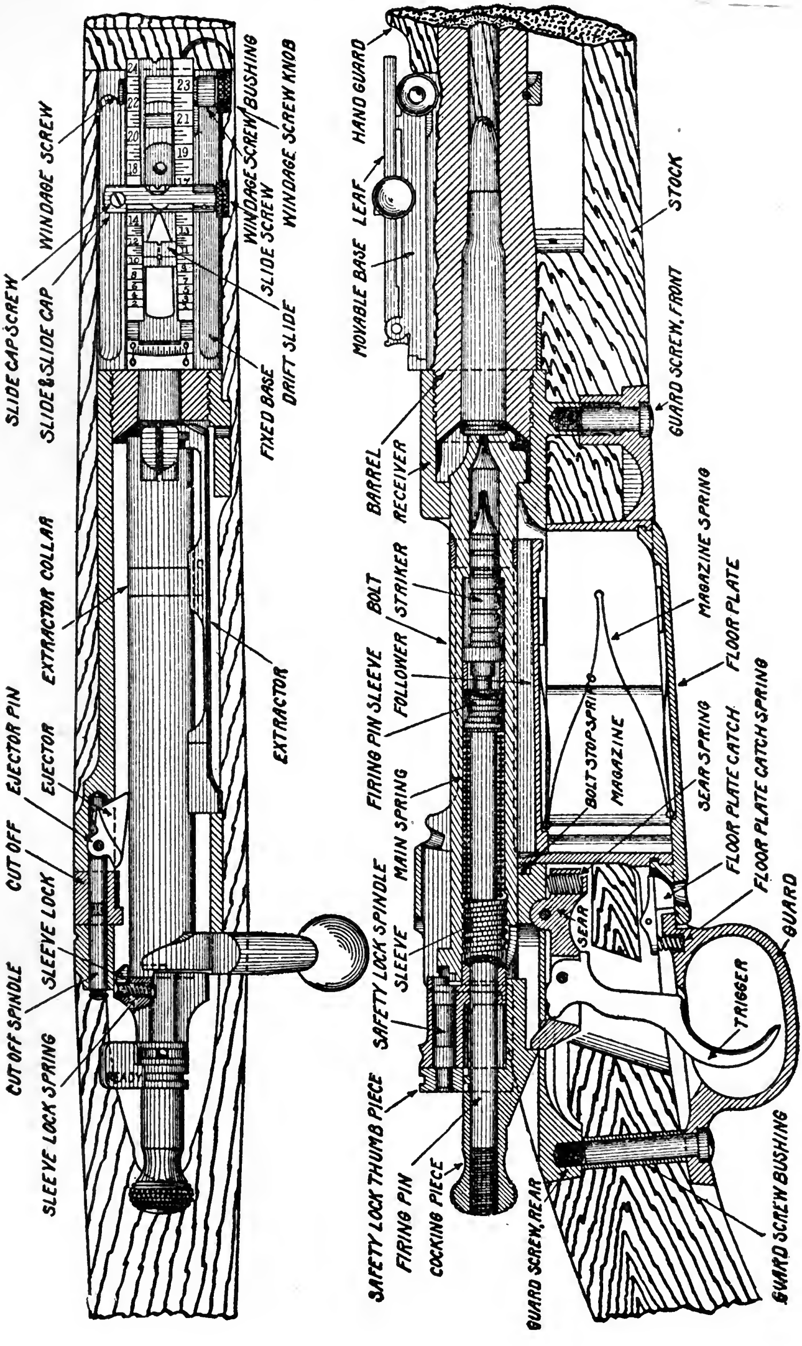


Fig. 268.—Mechanism of U. S. Magazine Rifle, Model 1903.

receiver, against which it is held by the pressure of the sear spring, the trigger being pivoted in the slotted sear. When the trigger is pulled it has comparatively free movement until the rear point, or heel, of the trigger bears against the receiver. The nose of the sear, its rear part which projects upward through a slot in the receiver, is by this movement partially withdrawn from the sear notch in the cocking piece. When the heel of the trigger bears against the receiver the trigger leverage is reduced and a short but more decided pull is required to further withdraw the sear from the sear notch. The purpose of the first movement of the trigger, against slight resistance, is to prevent accidental discharge of the piece as the soldier first feels the trigger, and to increase the accuracy of fire by enabling the soldier to partially withdraw the sear while aiming, and to complete its withdrawal at the proper moment by a slight movement of the finger.

Cams.—In the operation of the mechanism the most decided resistances are encountered in the compression of the mainspring and, at times, in the insertion of a cartridge into the barrel and in the extraction of the fired shell. In order that these operations may be accomplished with the least fatigue to the soldier they are all performed by means of cams.

The mainspring is partially compressed in the movement of unlocking the bolt by the action of a cammed surface of the bolt against the cocking cam on the firing pin, and the compression of the spring is completed on the closing of the bolt by the action of the two locking lugs at front end of bolt against the cammed locking shoulders in the receiver. The cammed movement of rotation also forces the cartridge to its seat in the chamber. In the rotation of the bolt in opening, the extracting cam at upper end of bolt handle works against a cammed surface in the receiver and moves the bolt slightly to the rear, starting the fired shell from the chamber.

The Barrel.—The rifling of the barrel consists of four grooves 0.004 of an inch deep. The grooves are three times as wide as the lands. The twist is uniform, one turn in 10 inches, and right handed. The length of the barrel, measured from end to end, is 24.206 inches, a length that permits the use of this arm by the cavalry, and makes their fire as efficient as that of the infantry.

Formerly the cavalry were provided with carbines, short guns with the same mechanism as the longer rifle and using the same ammunition.

The muzzle of the barrel is rounded to protect the rifling. Any irregularity of the muzzle end of the bore will seriously affect the accuracy of the arm by causing unequal pressure on the sides of the bullet as it is about to leave the bore.

315. The Sights, Model 1905.—The sight seats or bases for front and rear sights are bands that encircle the barrel, to which they are fixed by splines and pins. This method of attachment is preferable to the method formerly employed of screwing the sight seats directly to the barrel, as the sights are now more securely held and there is less likelihood of their adjustment being disturbed.

The windage screw, Fig. 268, which gives the movement in deflection to the rear sight, is acted on by a spring which prevents lost motion due to wear in the parts of the rotating mechanism.

Each division or point of the deflection scale of the rear sight corresponds to a lateral deviation of 4 inches in 100 yards.

The leaf of the sight is graduated for elevations from 100 to 2500 yards, the sight for the latter range being taken through the notch on upper end of leaf.

With the leaf down the sights are set at 400 yards, battle range, at all positions of the slide on the leaf.

In the movement of the slide up the leaf, the drift slide, Fig. 268, in which are cut the sighting notches and peep, follows a drift curve cut in the leaf and thus compensates for the lateral deviation of the trajectory from the line of sight as adjusted on the piece. Explanation of the drift and of the adjustment of the line of sight will be found in a later paragraph entitled *Deviation*.

The front sight is fitted in a stud that before being screwed to its seat is adjusted laterally to its proper position on the individual rifle. The proper adjustment is obtained by actual firing with each rifle. The firings are done by expert marksmen over a covered 200-yard range provided at the armory.

The sight radius of the piece, the horizontal distance between the point of the front sight and the rear edge of the notch or peep of the rear sight, is 22.3254 inches.

Rapidity of Fire.—With single loading, 23 aimed shots have been fired from the rifle in one minute, and with magazine fire, 25 shots in one minute. With the rifle held at the hip, 27 unaimed shots, loaded singly, have been fired in one minute, and with magazine fire, 35 shots.

The Bayonet.—The tang *B* of the bayonet, Fig. 269, is of one piece with the blade. In a recess in the tang is mounted the catch *H* which engages under the bayonet stud on the gun, locking the bayonet to the gun; and the catch *E* which secures the bayonet in its scabbard by engaging a hook provided in the scabbard. Either catch is released by pressure on the thumb piece *E*.

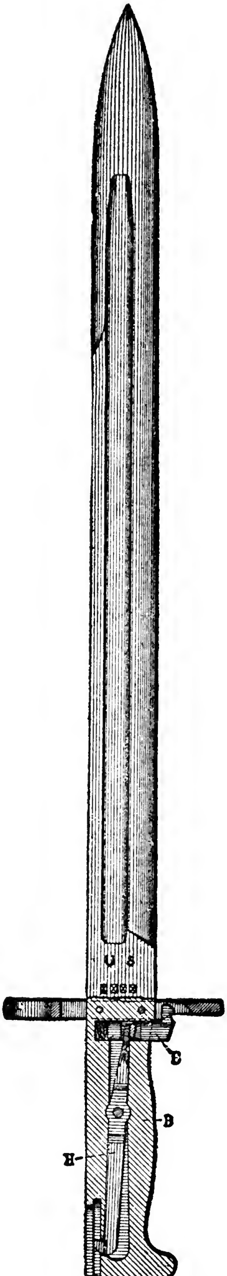


Fig. 269.

Appendages.—Among the appendages provided for the care of the piece is a *bullet jacket extractor*, Fig. 270, a cylindrical steel plug rifled on the exterior to fit the bore. This is pushed down the bore from the muzzle until it rests on the bullet jacket, which may then be forced out of the barrel.


Fig. 270.

A headless shell extractor consists of a steel plug, Fig. 271 of the general shape of the inside of the cartridge case, with a head like that of the cartridge. A steel ball rolls freely in a groove at the point, the groove being inclined outward toward the point. The extractor is roughened on the side opposite the groove. The extractor is pushed into the

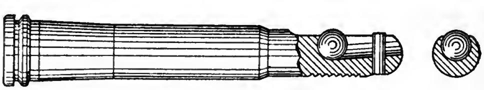


Fig. 271.

headless shell by the bolt of the gun, the gun being held with the muzzle up. The muzzle of the gun is then pointed down and the bolt withdrawn, extracting the extractor and the headless shell.

An aiming device is also provided for purposes of instruction in aiming. It consists of the circular steel clip *a*, Fig. 272, which embraces the gun in rear of the rear sight and supports the standard *b* to which the cage *c* may be fixed at any desired height. The cage contains a reflector so arranged that the instructor sees in the reflector the images of the gun sights and of the object aimed at. He may therefore correct the soldiers' aim.

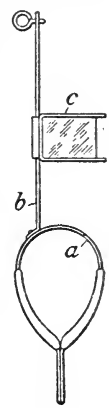


FIG. 272.

A cleaning thong and brush are contained in a metal case carried in the butt of the stock. The case is arranged to contain also a quantity of oil and a metal oil-dropper. A brass cleaning rod, a steel front sight cover, and a suitable screw driver are provided with each piece.

316. Deviation. Drift.—The rifle has a right-handed twist. The drift proper is therefore to the right. But at the moment that the bullet leaves the bore the muzzle of the gun is actually pointed to the left of its aimed position. The movement of the muzzle is probably due to vibrations of the barrel caused by the passage of the bullet through the bore. The barrel being held firmly at the bands the vibrations will take place about these points as nodes. The vibratory movement of the barrel is such that at the moment that the bullet leaves the bore the muzzle is pointed to the left of its aimed position.

The horizontal deviation of the bullet from the axial plane of sight is therefore the resultant of the drift due to the rifling and the deviation due to the vibration of the barrel. Following custom, we will call the resultant horizontal deviation the drift.

As determined by experimental firings the drift of the 220-grain bullet, fired from the service rifle, is to the left of the axial plane of sight up to a range of 850 yards, and beyond that range the drift is to the right.

In order to minimize the deviation at the most important ranges the drift slot in the leaf of the model 1905 sight is so cut as to make the trajectory cross the adjusted line of sight at a range of 1530 yards. Within that range the drift is to the left of the line of sight, its maximum value being 1.8 inches at the range of 1200 yards. After the trajectory crosses the line of sight the drift is to the right and increases rapidly from 1.1 inches at 1600 yards to 39.4 inches at 2500 yards.

Vertical Deviation.—The angles of elevation of the rifle as determined from actual firings at different ranges are all greater than the computed angles of elevation for the ranges. This indicates that at the moment that the bullet leaves the bore the position of the muzzle due to the vibratory movement of the barrel is below as well as to the left of its aimed position. The difference between the observed and computed elevations increases with the range, as it should since the effect of a constant difference of the angles will be less as the range increases.

The .22-caliber Gallery Practice Rifle.—The gallery practice rifle differs from the U. S. magazine rifle, model 1898, known as the *Krag-Jorgensen* rifle, only as to the barrel and the receiver. The barrel of the gallery practice rifle is a .22-caliber rifled barrel adapted to fire commercial .22-caliber, rim-fire, short or long cartridges. The barrel is issued assembled with a suitable extractor to a modified receiver. Any model 1898 rifle may be converted into a gallery practice rifle by dismounting the .30-caliber barrel and receiver and mounting in their stead the .22-caliber barrel and receiver.

With .22-caliber long cartridges, a range of 50 feet requires the sight to be set at 100 yards, and a range of 100 feet requires a sight setting of 225 yards.

AMMUNITION FOR THE .30-CAL. MAGAZINE RIFLE.

317. The Ball Cartridge.—The ball cartridge, Fig. 273, consists of the cartridge case, the primer, the charge of powder, and the bullet.

The Cartridge Case.—The cartridge case is made from a circular disk of brass cut from a flat ribbon 0.13 of an inch thick. The disk is first bent into the form of a cup and then drawn out in successive operations by being forced by punches through dies successively diminishing in diameter. In each draw press the length of the cartridge is increased and its diameters and thickness of wall diminished. Six draws are required to bring the cartridge to the desired size. After the cupping operation and after

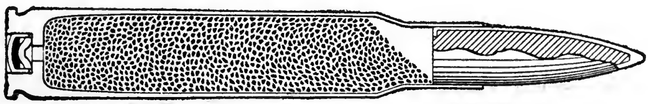


FIG. 273.

each of the first four draws the case is softened by annealing, which removes the brittleness of the metal caused by the drawing process. The cases are trimmed as required. The head of the cartridge case and the primer pocket are formed in a press. The mouth of the case is then annealed and the reduction of the neck and shoulder is accomplished in three operations in another press. The extractor groove is turned in the head, and the vent is punched through the bottom of the primer pocket.

BODY.—The body of the cartridge is of greater diameter than the rifled bore of the gun, in order to provide the necessary chamber space in the shortest practicable length. The enlarged body is a disadvantage in that it increases the bulk of the cartridge, and requires a larger chamber in the gun and greater thickness in the working parts of the gun. But in the present development of powders it has not yet been possible to produce from a cylindrical cartridge of reasonable length the desired ballistics for the rifle.

HEAD SPACE.—The space in the rifle between the head of the bolt and the surface against which the cartridge bears is called the head space. The head space in the rifle is of a length to allow proper clearance between the bolt and the head of the cartridge when the cartridge is fully inserted in the chamber. The head of the cartridge should always occupy the same position in the rifle, in order that the blow of the firing pin on the primer may be uniform, thus reducing the chances of misfires and punctured primers.

In order that the position of the primer in the gun shall vary the least the head space should be as short as possible, that is, the bearing surface of the cartridge should be close to the head of the cartridge, since in the manufacture of the cartridge the variations in a short dimension are likely to be less than in a longer one.

The cartridge with flanged head, Fig. 274, used in former service rifles, has an advantage over the present cartridge in this respect. The head space with the flanged cartridge measured from the seat for the front edge of the flange, was about $\frac{1}{10}$ of an inch

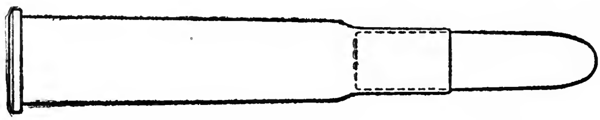


FIG. 274.

long, while the head space in the present rifle which is measured from the seat for the sloping shoulder of the cartridge, is nearly two inches long. In addition the bearing surface of the present cartridge is sloped, so that more extensive variations in the position of the head of the cartridge are likely to occur.

THE PRIMER.—The primer, Fig. 273, consists of the cup, the anvil, and the percussion composition. A pellet of moist percussion composition is put into the cup which is previously shellacked so that the composition will adhere. A shellacked disk of paper is pressed in tightly over the composition to keep out moisture. The anvil of hard brass is then forced into the cup. The primers are dried for several days in a dry house.

The cup of the primer is made of gilding metal, an alloy of copper much softer than the brass of the cartridge case. The metal of the cup must be sufficiently soft, and of the proper thickness, to permit a large part of the blow of the firing pin to be transmitted to the percussion composition, thus insuring explosion of the primer. At the same time the metal must be sufficiently hard to resist puncture by the firing pin. The firing pin strikes the primer with an energy of about 17 inch-pounds.

The priming composition is as follows:

Chlorate of potash,	632 parts
Sulphide of antimony,	320 parts
Ground glass,	212 parts
Sulphur,	110 parts

The finely pulverized ingredients are thoroughly mixed wet, and the composition is always handled wet, in which condition it is safe to handle. The composition is called the *H 48* composition.

This composition is safe, sufficiently sensitive, and emits a large body of flame. The large body of flame makes the composition superior for use with smokeless powders to the fulminate of mercury formerly used in all primers and still largely used in the primers in sporting cartridges and others.

The primer is seated slightly below the head of the cartridge in order to diminish the liability to accidental explosion of the cartridge in handling.

The Powder Charge.—The powder charge consists of about 51 grains of nitroglycerine powder. The weight of powder required to produce the muzzle velocity of 2800 feet varies in different lots of powder. The weight of charge therefore varies slightly in different cartridges.

318. Bullets.—The core of the bullet, Fig. 273, is an alloy of 16 parts of lead and one part of tin. The jacket, of cupro-nickel, is drawn from a disk in the same manner as the cartridge case. The lead slug is forced into the jacket, the point of the bullet shaped in a press, and the rear end of jacket turned squarely over the base of the bullet.

The 220-grain bullet is shown in Fig. 275, and the recently adopted 150-grain bullet in Fig. 276. The 220-grain bullet had a muzzle velocity of 2200 feet, the maximum pressure in the bore of the rifle being about 49,000 lbs. The 150-grain bullet is given a muzzle velocity of 2800 feet with a maximum pressure of 45,600 pounds. The great increase in the muzzle velocity makes the trajectory of the lighter bullet very much flatter than that of the 220-grain bullet, and thus correspondingly increases the accuracy of the rifle. It might be expected that the lighter bullet would suffer greater retardation in flight from the resistance of the air, but this bullet with its sharp point encounters less resistance than the heavier bullet with its rounded point. Greater accuracy at all ranges therefore results from the lighter bullet, with its higher velocity and sharp point.

Fig. 275.

Fig. 276.

The *bearing surface* of a bullet, that part of the bullet that comes in contact with the walls of the bore, should end abruptly, in order that as the bullet leaves the muzzle the bearing against the walls of the bore will cease at the same instant on all sides, and the bullet will not be deflected by the longer contact of any one point with the walls of the bore. The bearing surface of the service bullet terminates at the base. The base of the bullet should therefore be square with the axis, and the edge of the base should be as sharp as the metal of the jacket will permit.

In Fig. 277 is shown in full size a bullet recently tested. The bullet, of copper, weighed 175 grains. The bearing surface began about $\frac{3}{4}$ of an inch from the point and extended to about $\frac{1}{4}$ of an inch from the base, terminating on the rear slope of the bullet, the diameter of the base being less than the caliber. In tests for comparative accuracy at 500 yards the radius of the circle of shots was 4.2 inches for the 150-grain service bullet, 5.6 inches for the 220-grain service bullet, and 25.6 inches for the experimental copper bullet. On examination of the copper bullets, recovered after firing, the marks of the rifling were found extending farther to the rear on one side of the bullet than on the others. The difference in length of bearing on the different sides is sufficient to account for the inaccuracy.

Fig. 277.

319. The Blank Cartridge.--The bullet of the ball cartridge guides the cartridge from the magazine into the chamber of the rifle. In order that blank cartridges may be loaded from the magazine, a hollow paper bullet, Fig. 278, replaces the metal bullet

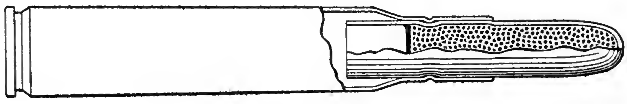


Fig. 278.

of the ball cartridge. The paper bullet is charged with 5 grains of E. C. powder held in place by a drop of shellac. The bullet is made by rolling a strip of paper into a tube of proper length, the end of the tube being afterwards closed into the rounded head by pressure in a machine. The strip of paper that forms the tube is gummed only on the outside edge so that the charge may readily burst the bullet at the muzzle of the gun. If the paper were gummed over its entire length the bullet would be so stiff that it might act as a rocket and do injury at some distance from the muzzle.

The propelling charge in the cartridge case is 10 grains of E. C. powder.

The blank cartridge is made $\frac{1}{10}$ of an inch shorter than the ball cartridge, to prevent the accidental assembling of a ball cartridge into a clip with blank cartridges. The machine in which this operation is performed is adapted for cartridges of one length only.

The Dummy Cartridge.—In order that the dummy cartridge, Fig. 279, may be readily distinguished from the ball cartridge both.

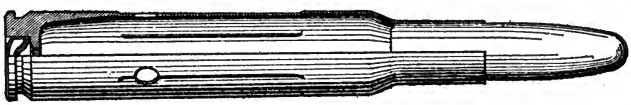


Fig. 279.

by sight and touch, the case of the dummy cartridge is tinned and corrugated, and three holes are bored through the bottoms of the corrugations. These are means intended to diminish the chances of the insertion of a ball cartridge in the rifle when drilling with dummy cartridges.

The Guard Cartridge.—The long range of the bullet of the ball cartridge and its great penetrative power render the ball cartridge unsuitable for the use of guards in times of peace, and for use in cities or other crowded places at times of riot and dis-

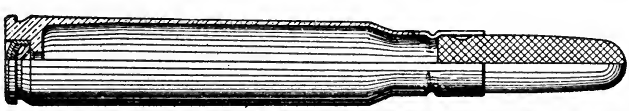


Fig. 280.

turbance. The guard cartridge, Fig. 280, is provided for these uses. The unjacketed lead bullet weighs 117 grains and is given a velocity of 1150 feet. The cartridge gives good results at 100 yards and has sufficient accuracy for use at 150 and 200 yards.

The lead bullet is deformed on striking and has little penetrative power, so that it is not likely to cause injury at a distance to innocent persons.

320. Proof of Ammunition.—Ammunition is proved by velocity and accuracy tests made with the arm in which the ammunition is to be used. Service rifle cartridges are also tested to determine whether they are waterproof.

VELOCITY TEST.—The velocity is measured at 53 feet from the muzzle, the first velocity screen being placed 3 feet from the muzzle and the two screens 100 feet apart. The mean velocity of 10 shots must not differ more than 15 feet from the standard.

ACCURACY TEST.—The accuracy test for rifle ammunition consists of several series of 10 shots each fired at a target 500 yards from the muzzle. The gun is fixed in a rest. The target is a heavy steel plate about 20 feet square, painted white and marked with horizontal and vertical black lines 2 feet apart.

The horizontal and vertical coordinates of each shot mark are measured from a convenient origin. The means of the horizontal and vertical coordinates are respectively the horizontal and vertical coordinates of the *center of impact.*

The distance of each shot from the center of impact is measured and the mean of these distances is the *mean radius* of the group of shots, or, as it is sometimes called, the radius of the circle of shots.

The mean of the vertical distances of the shots from the center of impact is the *mean vertical deviation,* and the mean of the horizontal distances from the center of impact is the *mean horizontal deviation.*

In the proof of ammunition the mean horizontal deviation is not measured, as the horizontal deviation depends upon the atmospheric conditions rather than upon the ammunition.

The results of recent comparative tests of the 220-grain and 150-grain bullets in the service rifle are shown in the following table.

Bullet.	Charge, Grains.	Pressure, Lbs.	Velocity, f. s.		Accuracy 500 Yards.		Penetration 500 Yards, Inches.
			Muzzle.	1000 Yards.	Rad.	M.V.D.	
220-grain, 1903...........	44	49000	2200	980	5.6	4.2	23.3
150-grain, 1906...........	51	45000	2730	1130	4.2	2.5	32.5

Equipment for Accuracy Test.—As it would often be most inconvenient to make on the target the measurements necessary for the determination of the mean radius and deviations of a group of shots, the ammunition proof range is provided with a *camera obscura* in a building in front and to one side of the target and near it. The lens of the camera forms an image of the target on a paper facsimile of the target constructed to the proper scale so that the lines of the image coincide with the lines of the target facsimile. An observer in the camera marks with a pencil the image of each shot mark made on the target, and the desired measurements are then conveniently made from the paper facsimile.

WATERPROOF TEST.—Cartridges from each lot manufactured are immersed in water at a depth of 8 inches for a period of 24 or 48 hours, and are then tested for velocity. There must be no falling off in velocity due to the entrance of moisture into the case.

CHAPTER XVI.

MACHINE GUNS.

321. Service Machine Guns.—The machine guns in our service are the Gatling machine gun and the Maxim automatic machine gun. The guns are of the same caliber as the infantry rifle and use the same ammunition.

In the Gatling machine gun the operations of loading, firing, and extracting the empty shell are effected through mechanisms actuated by a crank. The crank is turned by the gunner at a rate to produce any desired rapidity of fire. The greatest efficiency is obtained from the gun at a rate of fire of 600 rounds per minute. In an emergency this rate can be greatly increased.

In the Maxim automatic machine gun the operating mechanism is actuated by the recoil, so that after the first shot is fired the firing continues without effort on the part of the gunner as long as the trigger is pressed. The rate of fire from the gun depends upon the condition of the barrel and mechanism. In a new gun 250 cartridges in a single belt are fired at the rate of 650 shots a minute. After 8000 rounds this rate is reduced to about 325 shots a minute. In the continuous firing of 1000 rounds the rate of fire from a new gun is about 400 rounds a minute.

The Gatling gun has the advantages of a more rapid rate of continuous fire, and of a complete control of the rate of fire at all times. The fire of the automatic gun is however sufficiently rapid, the aiming is not interfered with by the operation of a crank, and the gun is lighter and more readily transported. It has therefore been adopted as the principal machine gun for our service.

Machine gun fire has recently become of such importance in battle that a machine gun platoon, armed with two automatic machine guns, is organized in each battalion of infantry and in each squadron of cavalry, so that six machine guns now accompany each regiment into the field.

The Gatling Machine Gun.—Fixed to a central shaft *S*, Fig. 281, are the ten .30-caliber rifled barrels *B* held in the barrel

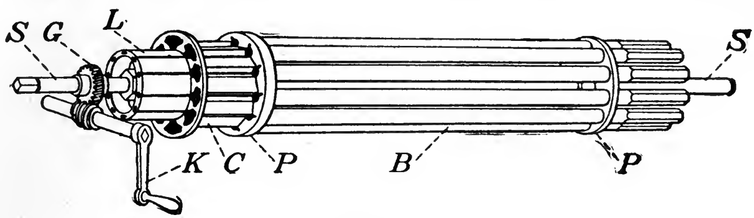


Fig. 281.

plates *P*; the carrier block *C*, provided with grooves which receive the cartridges successively and guide them into the barrels, the lock cylinder *L*, provided with guide slots in which the breech blocks for the barrels slide to close and open the breech; and the worm wheel *G*, by means of which the shaft and attached parts are rotated. The shaft is supported at each end in a frame, the sides of which also support the shaft of the rotating crank *K*.

The parts behind the rear barrel plate are completely inclosed in a cylindrical bronze casing which keeps out dust and protects the operating parts against injury. Within the casing is a hollow cylinder, called the cam cylinder, on the interior surface of which a continuous cam groove is cut.

The breech bolt, Fig. 282, one for each barrel, carries the firing

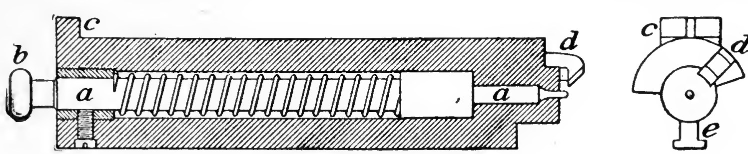


Fig. 282.

pin *a*, and its spring, and the extractor *d*. The guide rib *e* at the bottom of the bolt engages in a guide slot of the lock cylinder, *L* Fig. 281. The lug *c* on top of the bolt engages in the cam groove cut in the walls of the cam cylinder.

The cam groove, represented in Fig. 283 as though visible through the casing and cam cylinder, extends continuously around the interior of the cylinder. The top and bottom parts of the groove, *a* and *b*, follow lines cut from the cylinder by planes at right angles to its axis. These parts of the groove are joined by the inclined parts *cd*. The cam cylinder is fixed to the casing and does not revolve.

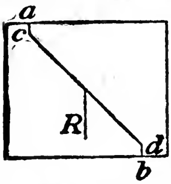


FIG. 283.

322. Operation.—As the lock cylinder, *L* Fig. 281, rotates with the barrels in a clockwise direction, the uppermost breech bolt is in its rearmost position, being held there by the lug *c* of the bolt moving in the circular part *a* of the groove. While the bolt is in this position a cartridge is placed by the feed mechanism in the top groove of the carrier block *C* in front of the bolt. As the bolt in its rotation moves downward on the right side it is

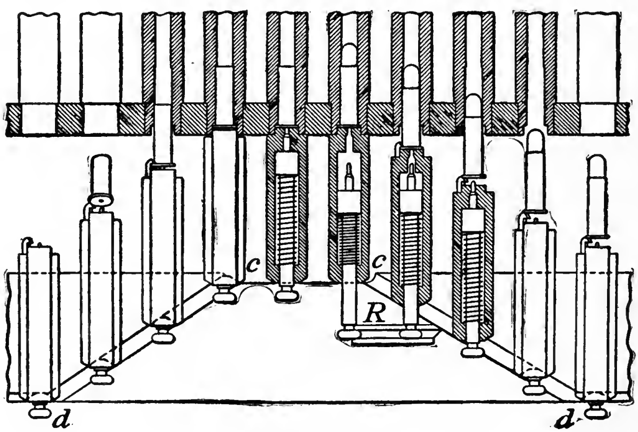


FIG. 284.

moved forward by the cam groove *cd* and pushes the cartridge into the barrel. During this movement the cocking head of the firing pin, *b* Fig. 282, is caught by a grooved rib, *R* Figs. 283 and 284, and the firing pin is prevented from moving forward with the bolt. The method of operation will be understood from Fig. 284, which shows a development of the cam groove and rotating parts. The lines *dd* and *cc* in Fig. 284 represent respectively the developments of the parts *a* and *b* of the groove as shown in Fig. 283.

When the barrel is in its lowest position the head of the firing pin leaves the rib *R*, and the firing pin, under the action of its spring, strikes and fires the cartridge. As the breech bolt moves upward on the left side it is drawn to the rear by the cam groove, extracting the fired shell from the barrel and ejecting it to the left through a slot in the casing.

THE FEED.—A hopper is formed in the top of the bronze casing immediately over the carrier block, *C* Fig. 281 and *e* Fig. 285. The device, called the Bruce feed, for feeding cartridges to the gun, is fixed in a socket at the mouth of the hopper. Pivoted on the standard, *ac* Fig. 285, is a swinging piece *b*, provided with two flanged grooves which engage the heads of the cartridges: by the flange of the 1898 cartridge, and by the groove of the 1903 cartridge. The grooves in *b* are quickly filled by stripping the cartridges from the paper boxes in which they are packed. The cartridges from one of the grooves in *b* pass immediately through the groove in *c* and are fed one at a time to the carrier block *e* by the wheel *d* which is caused to revolve by the carrier block. When one of the grooves in *b* is empty the weight of the cartridges in the other groove causes the piece *b* to swing to one side and bring the full groove over the groove in *c*.

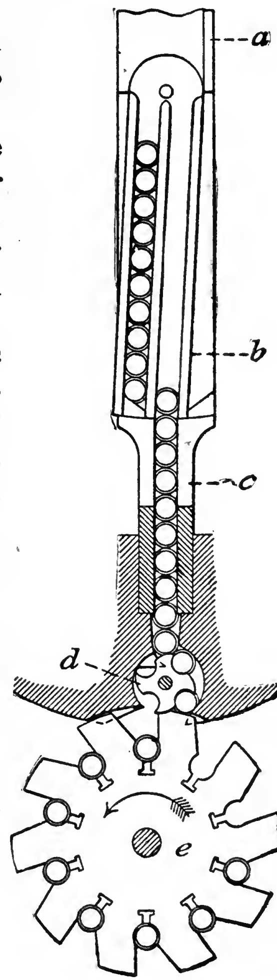


FIG. 285.

MOUNTS.—The Gatling gun is mounted, for field service, on a shielded wheeled carriage with limber. When mounted in the casemates of permanent or temporary fortifications for use in repelling landing parties and in protecting the land approaches, a fixed mount is provided.

Blank Cartridge for Gatling Gun.—When the blank cartridge for the infantry rifle is used in the Gatling gun the blunt end of

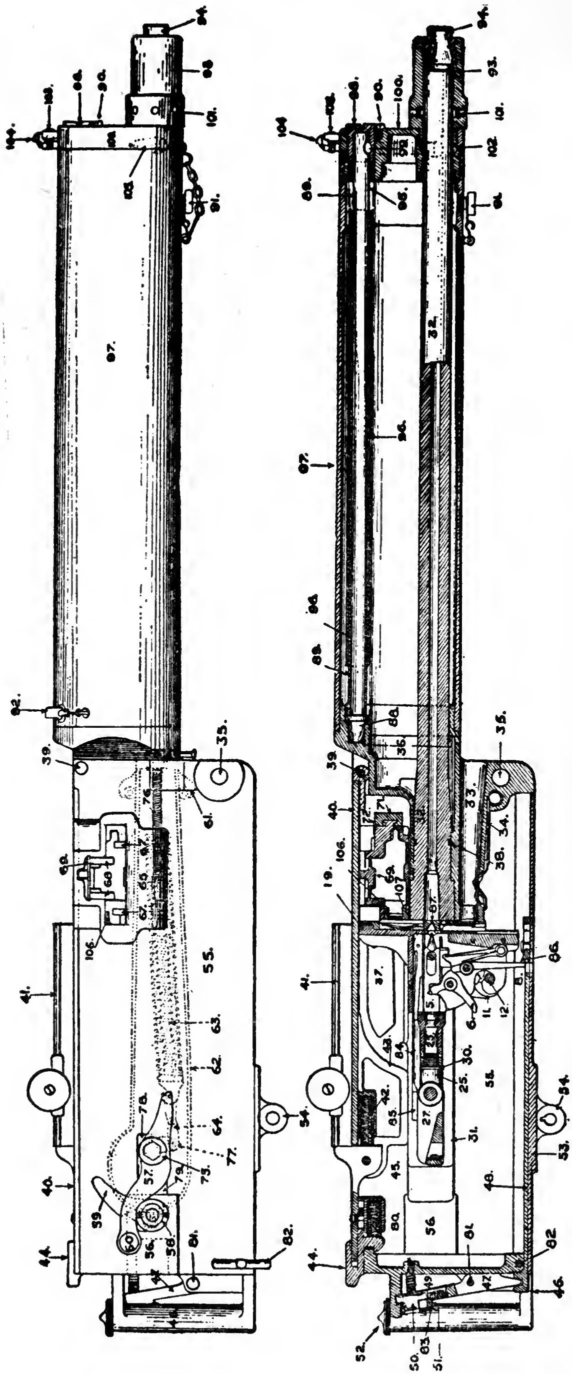


FIG. 286.—Maxim Automatic Machine Gun.

the paper bullet often catches on a shoulder at the rear end of the barrel, thus preventing insertion of the cartridge and causing the mechanism to jam.

A special blank cartridge is therefore made for the gun. The cartridge case is extended to the length of the complete ball cartridge and, after the insertion of the powder charge, the mouth of the case is closed into the rounded form of the point of the 220-grain bullet.

323. The Maxim Automatic Machine Gun.—The Maxim automatic machine gun has a single barrel, and the recoil of the barrel and attached mechanism is utilized to perform the operations necessary in continuous firing.

The barrel, 32 Fig. 286, is inclosed in a cylindrical water jacket 97, and slides in its bearings in stuffing boxes at each end of the water jacket. Fixed to the rear end of the water jacket is the breech casing 55, a rectangular steel box that incloses the operating mechanism and provides means, 35 and 54, for the attachment of the gun to its mount.

Method of Action.—The barrel and the breech mechanism recoil together until after the bullet has left the bore. When the barrel has reached the end of its recoil the breech mechanism continues to the rear, opens the breech, and extracts the fired shell; and, returning under the action of a spring, inserts a new cartridge in the barrel and fires the piece. These actions are repeated as long as the trigger is pressed.

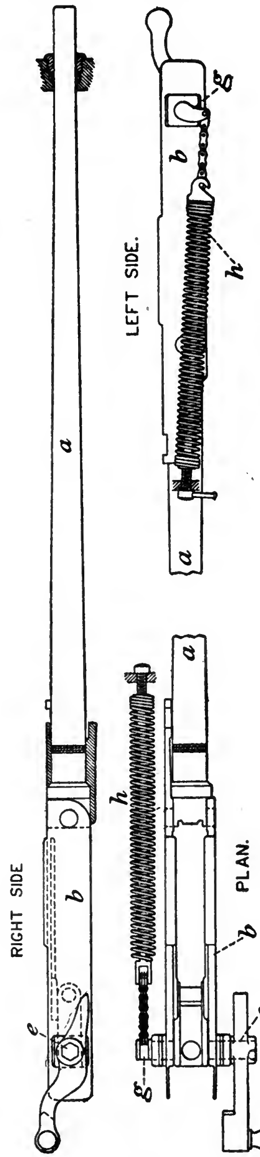


Fig. 287.

The cartridges are fed to the gun in a belt, see Fig. 291, which is automatically drawn through the feed mechanism above the breech in such manner as to present a new cartridge after each discharge.

Recoiling Parts.—The recoiling parts, Fig. 287, comprise the barrel *a*, the two recoil plates *b* fixed to the breech of the barrel, the operating crank shaft *e* fixed in bearings in the recoil plates, and the breech mechanism which slides between the recoil plates and is operated by means of the crank shaft *e*.

The recoil plates slide in grooves provided in the sides of the breech casing 55, Fig. 286. The left recoil plate extends to the front of the breech and operates the feed mechanism above the barrel. The crank shaft 75 projects on both sides through slots 79 in the casing. The movement of the recoiling parts to the

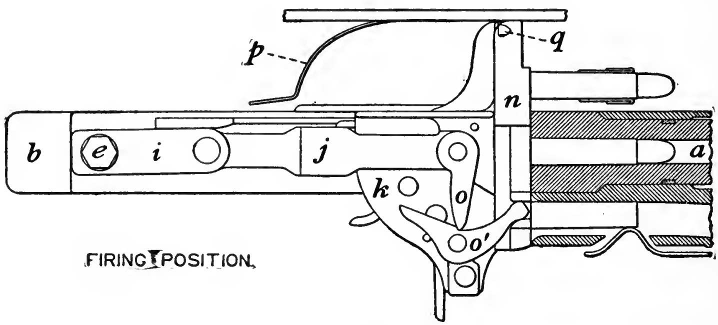


Fig. 288.

rear is stopped when the crank shaft strikes the rear edges of the slots. Fixed to the right end of the shaft is the cam lever 57. During the recoil, and after the shot has left the bore, the lower surface of the cam lever bears on the roller 58, and as the recoil continues the cam lever, riding on the roller, is rotated upward, thus producing a downward movement to the crank on the shaft between the recoil plates. The crank is seen in Fig. 287 and at *i* Figs. 288 and 289. Attached by links to the fusee, *g* Fig. 287, on the crank shaft outside the breech casing, is the operating spring *h* which at its forward end is attached to the breech casing. On recoil and rotation of the shaft the spring is extended, and at the end of the recoil the reaction of the spring returns the parts to the firing position.

324. The Breech Mechanism.—The breech mechanism is shown in Figs. 288 and 289. It consists of the lock *k* which contains the firing mechanism; the carrier *n*, a narrow piece which slides up and down the front of the lock and is provided in front with a flanged groove to engage the head of the cartridge; and the forked link *j* pivoted at its rear end to the crank *i* on the operating shaft *e*. The breech mechanism slides back and forth between the recoil plates *b* in grooves cut in the sides of the recoil plates.

The parts being in the firing position the flanged groove of the carrier *n* engages the head of a cartridge in the feed belt above the barrel and also the head of the cartridge in the barrel. When the piece is fired the barrel and breech mechanism start to the

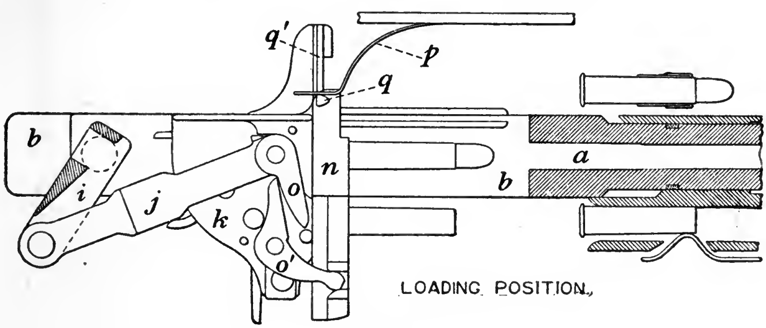


Fig. 289.

rear together. At the end of the movement of the barrel, the breech mechanism is drawn farther to the rear between the recoil plates by the rotation of the crank *i* as shown in Fig. 289.

In this movement the carrier *n* is guided by its bearings *q* which move on the upper surfaces of solid cams, 37 Fig. 286, fixed to the side plates of the breech casing. The movement of the carrier is at first straight to the rear withdrawing the cartridge from the belt and the fired shell from the chamber. The carrier is then depressed by a guide lug, 43 Fig. 86 and *p* Figs. 288 and 289, attached to the top plate of the breech casing. The loaded cartridge is thus brought opposite the barrel and the fired shell opposite the ejector tube 33. The reaction of the coiled spring now returns the parts to the firing position, the carrier *n*, Figs. 288 and 289, moving straight to the front in its depressed position. After the cartridge has been placed in the chamber, the carrier is slid upward by the action of the finger *o* against the lifting lever *o′*, the finger *o* being fixed to the link *j*. The carrier leaves the fired shell in the ejector tube where it is held by a spring to prevent its falling back into the mechanism. It is ejected from the tube by the next succeeding shell.

THE FIRING MECHANISM.—The firing mechanism, shown in Fig. 290, is contained between two plates *k*. The solid part of the forked link *j* acts in its downward movement against the projecting end of the tumbler *c*, withdrawing the firing pin until it is caught by the safety catch *e*. At the same time the sear *d* en-

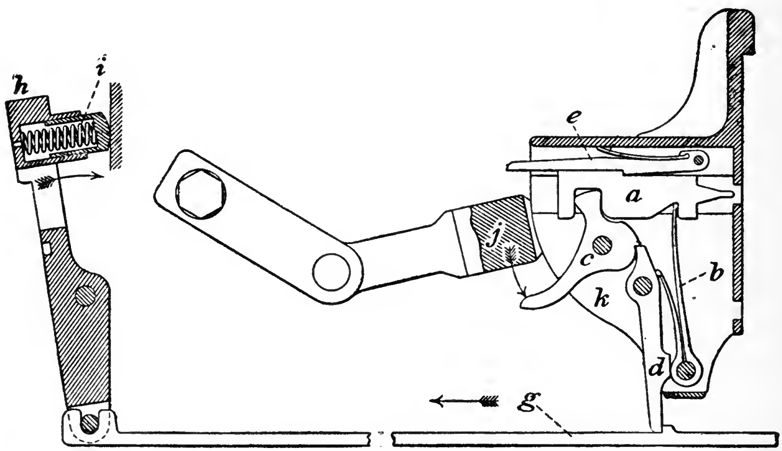


FIG. 290.

gages in the notch of the tumbler where it is held by one leaf of the spring *b*. The trigger *h* is placed at the rear outside the breech casing, between the two gun handles. A forward pressure against its upper end moves the trigger bar *g* to the rear. When the trigger is pressed the lug on the trigger bar that engages the sear *d* releases the sear from the notch in the tumbler as the breech mechanism moves forward in closing, and holds it released after the breech is closed. After the release of the sear the firing pin is held back by the safety catch *e*. The link *j* in the last part of its movement upward lifts the projecting end of the safety catch and releases the firing pin, which under the action of the spring *b* flies forward and fires the cartridge.

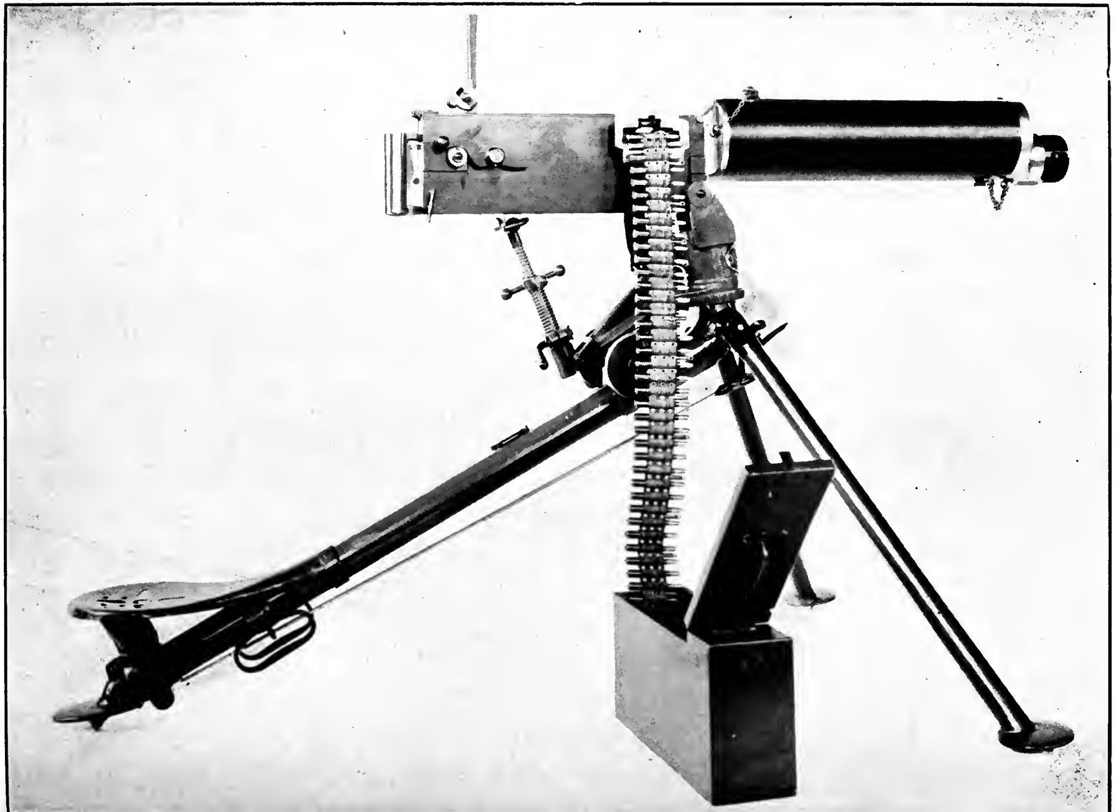


FIG. 291.—Maxim .30-Caliber Automatic Machine Gun.

The trigger is constantly pressed to the rear by the spring *i* and is provided with a safety catch to guard against accidental firing. The trigger cannot be pressed forward for firing until the safety catch is lifted.

325. The Water Jacket.—In continuous firing the barrel of an automatic rifle becomes very highly heated and if not cooled in some way may even attain a red heat. The walls of the bore are so softened by the heat that the lands of the rifling are soon worn away and the gun loses its accuracy. The accuracy is completely destroyed after about 1000 rounds fired with the water jacket empty. The necessity of cooling the barrel during firing is therefore apparent, and the gun should never be fired, except in emergency, without water in the jacket.

The water jacket of the Maxim gun holds 12 pints of water. The barrel of the gun is coated with copper on the exterior as a protection against rust. The stuffing boxes through which the barrel passes are packed with asbestos packing.

A steam tube, 89 Fig. 286, is fitted in the upper part of the water jacket to provide a means of escape for the steam that is formed in the water jacket during continuous firing. Near each end of the steam tube is a hole 89 for the admission of steam, and at the front end a hole 99 through both tube and water jacket permits escape of steam to the exterior. The steam tube is surrounded by the tubular valve 96 which slides on the steam tube and closes the forward or rear steam port according as the gun is depressed or elevated, thus preventing the entrance of water into the steam tube while permitting the entrance of steam.

The Cartridge Belt.—The cartridge belt, Fig. 291, is formed of two pieces of flax webbing connected by brass strips and eyelets between adjacent cartridges, every third strip projecting about an inch beyond the bullet edge of the belt to guide the belt properly through the feed mechanism of the gun. A flat brass handle 4 inches long is attached to each end of the belt.

Each belt holds 250 cartridges.

The cartridges are quickly and evenly inserted into the belt pockets by means of a small belt-filling machine, Fig. 292, which is attached to a bench and operated by hand.

Mounts.—For service with the infantry and cavalry the automatic gun is mounted on a tripod, Figs. 291 and 293. It is transported by means of pack animals. For transportation the legs of the tripod fold together and the rear leg telescopes. A complete outfit consists of five packs. The gun and tripod form one pack which weighs, with the equipment of the animal, 275 pounds. Each of the other four packs consists of 1500 rounds of ammunition, and accessories for the gun including water for refilling the water jacket. These packs weigh complete about 290 pounds each.

The gun with tripod, and water jacket filled with water, weighs 152 pounds. It may therefore be readily transported by hand over short distances in the field. The legs of the tripod fully extended to the front and rear form convenient shafts for carrying.

For use in fortifications the gun is mounted on a two-wheeled carriage provided with shields. The parts of the mount connecting with the gun are alike in the carriage and in the tripod mount, so that the guns may be fitted to either type of mount as desired.

Blank Firing Attachment.—The pressure produced in the discharge of a blank cartridge is not sufficient to operate the mechanism of the gun. There is therefore provided for use in drill with blank cartridges an attachment called the drill and blank firing attachment. The attachment, Fig. 293, is affixed to one of the rear gun handles and acts, through the continuous turning of a crank by hand, to operate the crank shaft of the recoil mechanism in the same manner as when operated by the explosion of a ball cartridge.

326. The Maxim One-pounder Automatic Gun.—This gun, called the *Pompom* from the noise of its explosions, is constructed on the same principles as the .30-caliber automatic gun above described.

On account of the greater size and weight of the parts and the increased total force of recoil, an additional coiled recoil spring, *s* Fig. 294, surrounds the barrel in the water jacket. The spring, as well as the barrel, is coated with copper. A small hydraulic cylinder *c* also assists in checking the recoil. The cylinder is held in the rear plate of the breech casing, the piston *p* of the cylinder being connected with a cross bar *x* held between the rear ends of the recoil plates.

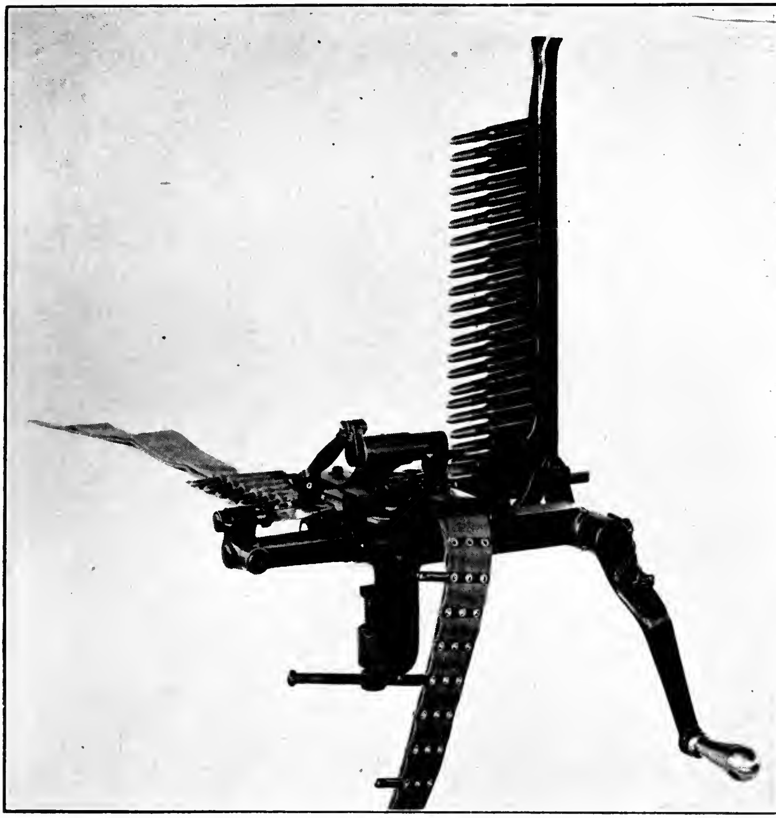


Fig. 292.—Belt Filling Machine.

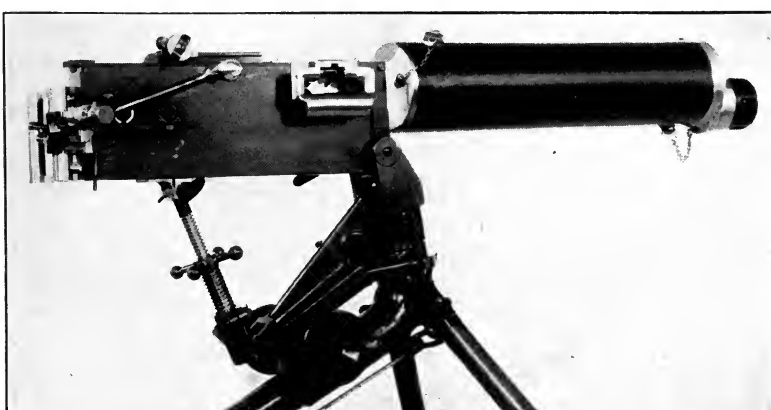


Fig. 293.—Attachment for Firing Blank Cartridges.

Maxim .30-Caliber Automatic Machine Gun.

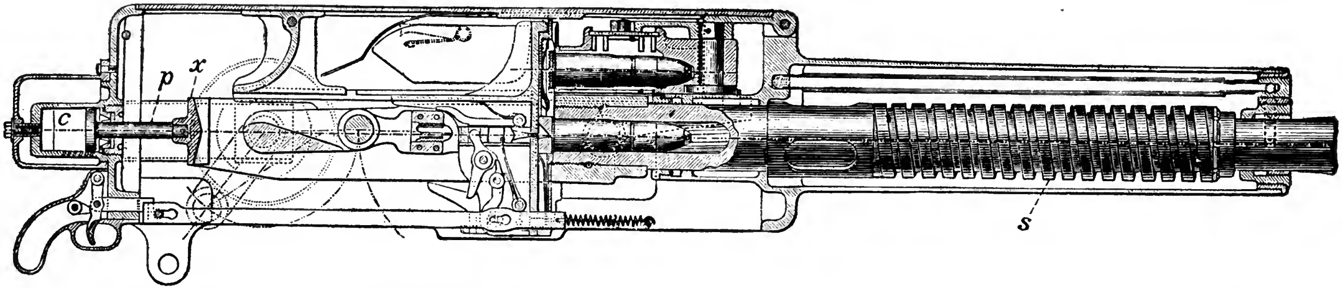


FIG. 294.—Maxim Automatic One-pounder. Pompom.

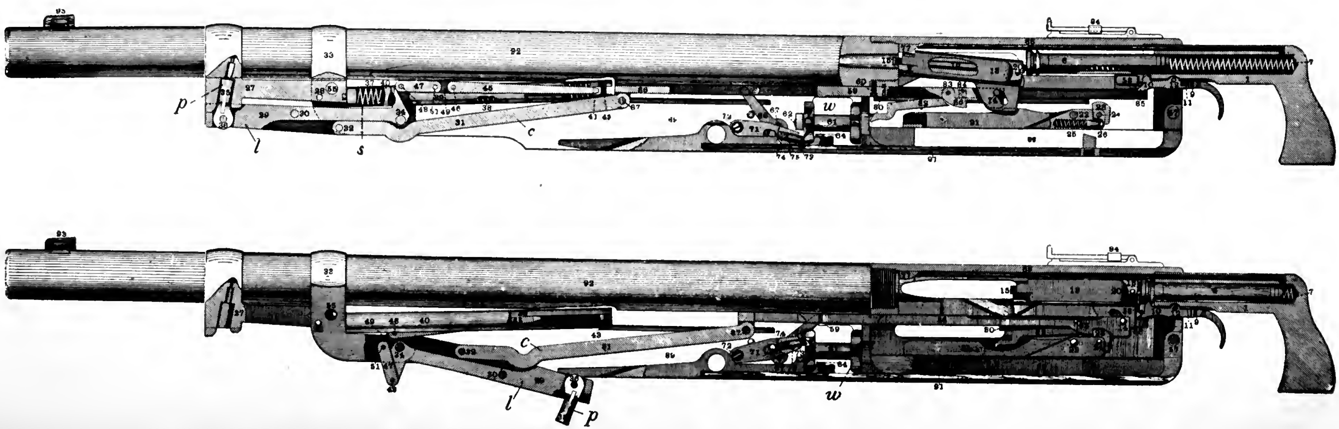


FIG. 295.—Colt .30-Caliber Automatic Machine Gun.

The caliber of the gun is 1.457 inches. It fires a shell weighing one pound, with a bursting charge of 4/10 of a pound.

The Colt Automatic Machine Gun.—The operation of the Colt automatic machine gun, Fig. 295, is effected through the direct action of the powder gases on the end of a swinging lever *l*. A vent is cut through the bottom of the stationary barrel a short distance in rear of the muzzle. When the bullet has passed the vent a portion of the powder gases enter the vent and impinge on a piston *p* attached to the lever *l*. The blow on the piston causes the lever to revolve downward and to the rear against the action of a coiled spring *s* which at the end of the movement returns the lever to its former position.

The movement of the lever is communicated by the connecting bar *c* to the mechanisms in the rear, and actuates these mechanisms to perform the successive operations necessary for the maintenance of continuous fire.

The cartridges are fed to the gun in a belt similar to that described for the Maxim gun. The feeding of the belt is accomplished by the feed wheel *w* under the rear end of the barrel.

CHAPTER XVII.

SUBMARINE MINES AND TORPEDOES. SUBMARINE TORPEDO BOATS.

327. Submarine Mines and Torpedoes.—*A submarine mine* is a charge of explosive confined in a strong case anchored in position under the surface of the water.

A torpedo is a submarine vehicle charged with explosive. The term torpedo formerly included fixed as well as moving mines, and still includes, to a certain extent, both these classes.

History.—The first recorded experiments with submarine mines were made by *David Bushnell* of Connecticut, in 1775. His mines contained charges of black powder, and explosion was effected by means of clockwork, which, after being set in motion, allowed sufficient time before the explosion for the operator to get clear.

Bushnell also constructed a submarine boat for the purpose of conveying his mines to hostile vessels. The boat, Fig. 296, was formed of two sides, each shaped like the upper shell of a tortoise. Entrance was gained through a hatch in the top. It carried but one operator, who moved the craft by means of screw propellers. The explosive was carried in a case with the firing mechanism, on the back of the boat, and was fastened by a rope to the stem of a wood screw which projected through the top of the boat. The operator was expected to bring the craft under the hostile ship, and fasten the wood screw in the ship's wooden bottom. This effected, the moving away of the submarine boat would release the mine and set the clockwork in motion, to explode the charge after a sufficient interval of time.

An attempt was actually made in 1776 with this boat against the English man-of-war *Eagle* in the harbor of New York. The operator claimed that he found the vessel, and that in attempting to fasten the screw in her bottom he struck iron. In looking for a better location he lost the vessel. He released the magazine in the harbor, and an hour afterward the explosion occurred.

Bushnell also attacked the English fleet, at Philadelphia in 1777, with drifting torpedoes. This attempt was also unsuccessful.

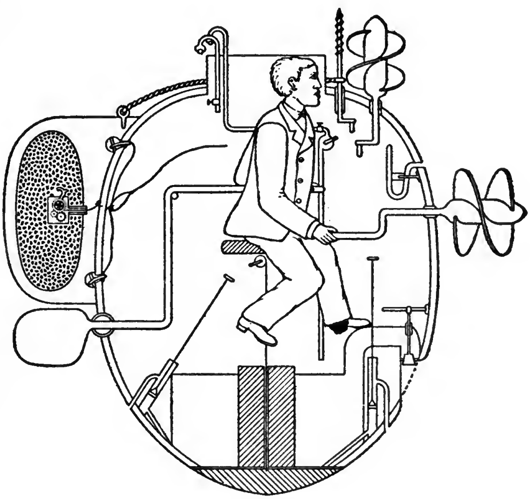


FIG. 296.

Robert Fulton experimented with torpedoes from 1797 to 1810. In 1801 he succeeded in sinking the first vessel, a small one, with a submarine mine. The mine contained 20 pounds of gunpowder. In 1804 he conducted, for the English, an unsuccessful attack with mines against the French fleet in the harbor of Boulogne. The mines exploded but did no harm to the French ships.

In 1842 *Samuel Colt* applied electricity to the firing of submarine mines, and in the following years was successful in numerous experiments in the explosion of mines at great distances from the operator.

Mines and torpedoes were first successfully used in war by the Confederates in our Civil War. With imperfect appliances they succeeded in sinking or seriously damaging more than thirty United States ships. Their success attracted the attention of the world to this method of naval attack and defense, with the result that there has followed great improvement in the appliances and methods employed, and the means for submarine warfare are now given earnest consideration by all maritime nations.

328. Confederate Mines.—The mines used by the Confederates were of various forms. The simplest and one of the most effective mines was made of a barrel, which was partially filled with black gunpowder. The charge was usually about 100 pounds. The barrel, Fig. 297, was provided with pointed ends to prevent its being overturned by the current. It was moored to float 5 or 6

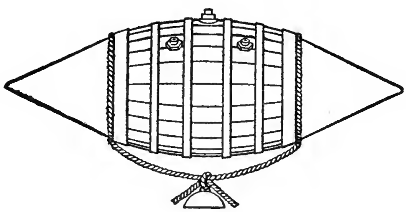


FIG. 297.

feet below the surface of the water, and a depending weight kept the top of the barrel uppermost. Screwed into sockets on top of the barrel were a number of percussion or chemical fuses. A vessel striking one of these would explode the mine.

The chemical fuse consisted of a small glass tube filled with sulphuric acid and surrounded by a mixture of chlorate of potash and white sugar, the whole enclosed in an outer lead tube. The lead tube was crushed by the blow of a striking vessel and the glass tube broken. The action of the sulphuric acid on the mixture of chlorate of potash and white sugar produced fire, which was communicated to the powder charge of the mine by a priming of black powder.

Another very effective buoyant mine, known as the Singer mine, is shown in Fig. 298. The case, made of tin, was of size sufficient to hold from 50 to 100 pounds of gunpowder, and to provide sufficient air space *a* for flotation. A percussion cap was held in a cup in the lug *e* in the midst of the powder charge, and the upper end of the rod *d* was close to the cap. A firing bolt *b* was held back against the pressure of a spiral spring by the pin *g*. A heavy iron cap *c*, connected by a wire to the pin, rested on the top of the mine. When the mine was struck the cap was knocked off. The cap in falling pulled out the pin *g*. The firing mechanism would then act and explode the mine.

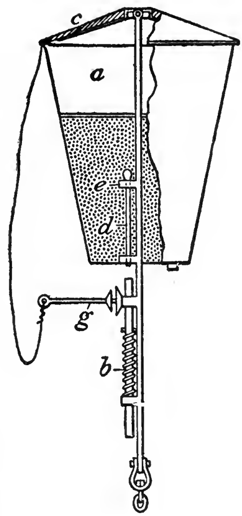


Fig. 298.

In shallow waters, frame and spar, or pile, torpedoes were used. The frame torpedo, Fig. 299, consisted of a number of inclined timbers framed together and supporting at their upper ends explosive shell provided with percussion caps.

Two forms of the spar torpedo are shown in Figs. 300 and 301. The spar torpedo was also used for offensive operations in boats. The spar, with torpedo at the end, was carried projecting from the bow of a launch.

The most noteworthy exploit with a spar torpedo was that of Lieut. W. B. Cushing, U. S. Navy, who in 1864 attacked in a launch the Confederate ironclad *Albemarle* which was tied to a dock in the river at Plymouth, N. C. The *Albemarle* was sunk by

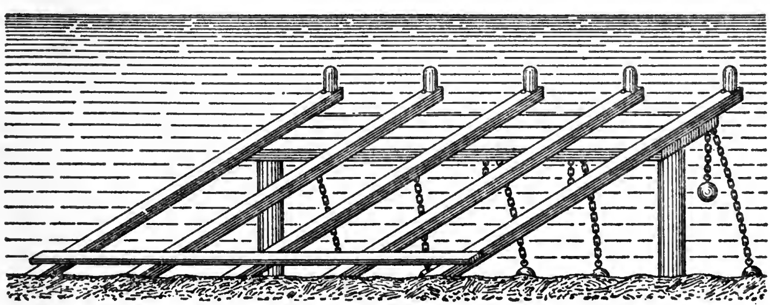


Fig. 299.

the explosion of the torpedo. So was the launch. Lieutenant Cushing and one member of his crew of thirteen escaped.

The Confederates also made use of submarine boats carrying torpedoes, and they sunk by these means the United States frigate *Housatonic* in Charleston Harbor in 1864. The submarine boat used on this occasion was worked by a crew of nine men who

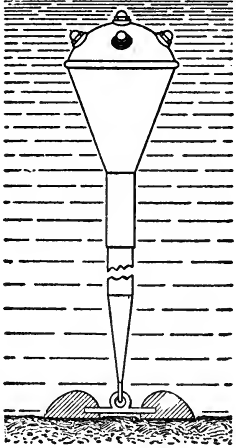


Fig. 300.

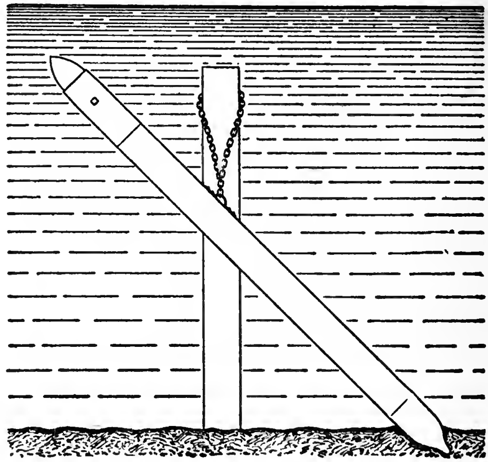


Fig. 301.

operated the propellers by hand. The boat and her crew were carried down with the *Housatonic*.

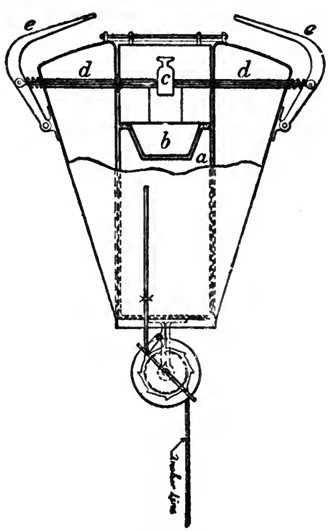


Fig. 302.

Spanish Mechanical Mine.—Fig. 302 represents a Bustamente contact mine. Seventeen of these mines were removed by our Navy from the harbor of Guantanamo, Cuba, after the capture of the harbor in 1898.

The mine is circular in cross section. It carried a charge of 100 pounds of wet guncotton in the cylinder *a* and a priming charge of dry guncotton in the chamber *b*. Against the chemical fuse *c*, a bottle containing sulphuric acid and surrounded by a mixture of chlorate of potash and sugar, rest the ends of six iron rods or plungers *d* whose outer ends are connected to the six pivoted contact arms *e*. A blow on any one of the arms *e* would cause a plunger to break the fuse. Ignition of the priming charge and explosion of the bursting charge would follow.

329. Electric Mines.—Mechanical mines such as those described above, when once planted, render the waterways dangerous to friend and foe alike. This great disadvantage is overcome in modern practice by the use of electrically controlled mines which may be made instantly operative or harmless at the will of an operator on shore.

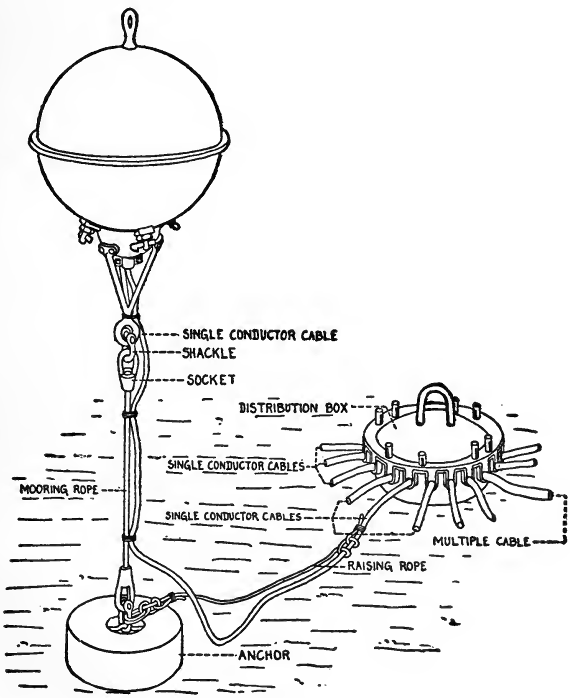


FIG. 303.

BUOYANT MINES.—A modern buoyant mine is shown in Fig. 303. The spherical case of steel contains the explosive and the circuit-closing and firing devices, with sufficient air space for flotation. A continuous insulated cable extends from the mining casemate in the fortification to the mine in position. The firing circuit is broken at the mine, and the electrical arrangements are such that the mine may be fired by the operation of the circuit-closer when the mine is struck by a vessel, or at any time at the will of the operator in the mining casemate. Or the striking of a mine may be automatically signaled to the operator, who may then fire it at once, or after a few moments delay, in order to allow a ship to get well over it, or not fire it at all.

Buoyant mines are moored at a submergence of about 5 feet at low water, so that they may be near enough to the surface to be struck by passing vessels and yet not near enough to be readily seen. They are not in general used in water less than 20 feet deep. They may be operated successfully in water 150 feet deep. In order to obtain the necessary buoyancy the mines used in waters of the greatest depths are cylindrical in shape with hemispherical ends.

Ground Mines.—Ground mines are used when the depth of the water does not exceed 35 feet. They rest on the bottom. A heavy mushroom-shaped case contains the charge of explosive and the electric firing device. The circuit-closing device is carried by a buoyant case similar in shape to the buoyant mine. The buoy is moored, with proper submergence, to the ground mine. When the buoy is struck by a passing vessel the circuit-closer within it acts in precisely the same manner as the circuit-closer in the buoyant mine, and, if desired, completes the firing circuit that fires the charge in the mine resting on the bottom.

330. The Explosive.—Dynamite and guncotton are the principal explosives used in submarine warfare.

Dynamite has been used in the mines of the United States service. It has the advantages of cheapness and ease of ignition. Its disadvantages are danger in handling, liability to explosion when a derelict mine is struck by a vessel, and changing sensibility to the action of the detonator when freezing and thawing. If the dynamite becomes wet, through a leak in the mine case, the nitroglycerine separates from the absorbent.

Guncotton has the advantage of being perfectly safe in storage and in handling, and of detonating when wet if a small amount of dry guncotton be present. The dry cotton must be in close contact with the wet. Too much water will make the detonation uncertain. The explosive force of guncotton is less than that of dynamite.

Excellent results have recently been obtained in submarine work with the explosive *tri-nitro-toluol.*

The Charge.—Charges varying from 100 to 1000 pounds of explosive have been used in mines. A charge of 100 pounds exploded in contact with a warship's bottom will disable and probably sink the ship.

In recent experiments with a submerged target built in exact representation of the bottom of a battleship, the explosion of a 12-inch mortar shell containing 63 pounds of high explosive, at a distance of 20 feet from the target and at a depth of 15 feet, produced serious injury to the target; 64 pounds at a distance of 15 feet nearly disrupted the target and caused bad leakage, producing dangerous injury; while 130 pounds at a distance of 15 feet disrupted the double bottom and caused the target to sink immediately. The results showed the utility of this method of attack on vessels, and the desirability of using as large an explosive charge as possible in the projectiles for the seacoast mortars.

General Henry L. Abbott, Corps of Engineers, U. S. Army, conducted a very extensive series of subaqueous experiments with different explosives. He deduced the following formulas for the energy and pressure delivered at a distance by a subaqueous explosion.

$$W=\frac{58C}{(D+0.01)^{2\cdot 1}}$$

$$P=\left(\frac{1,832,000C}{(D+0.01)^{2\cdot 1}}\right)^{\frac{2}{3}}$$

in which W represents the energy per square inch,

P the pressure in pounds per square inch,

C the weight of charge, in pounds,

D the distance in feet.

Applying the pressure formula to the explosions of the three mortar shell in the vicinity of the battleship target, we find that the pressures on the target were, in order, 3574, 5401, and 8662 pounds per square inch.

331. **Defensive Mine Systems.**—The submarine mine system is used as an auxiliary in the defense of a river or harbor in connection with the land fortifications, and its chief purpose is to so limit and obstruct the approach of the enemy's vessels that

they will be compelled to make frontal attack on the fortifications and be held exposed to the fire of the heaviest guns.

In order that the most effective fire may be employed the outer lines of mines are planted at a distance from the fortifications, not exceeding the most effective range of the guns.

The usual mine system for the defense of a harbor is illustrated in Fig. 304. Concealed and protected in a fortification is the

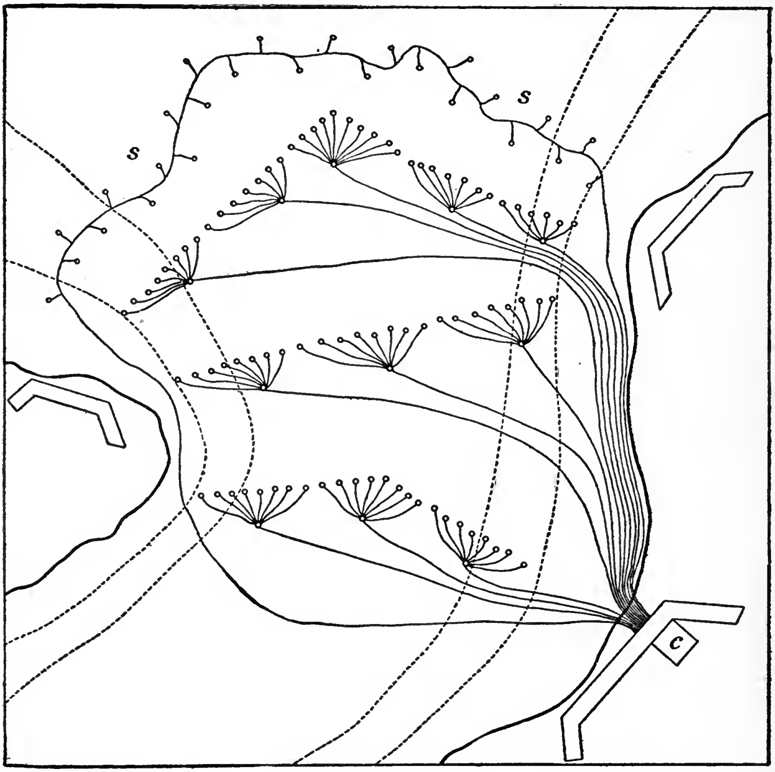


Fig. 304.

mining casemate *C* which contains the electric generators, batteries, and instruments needed in the service of the mines. From this point the mines are controlled.

The mines are planted, in the waterways to be defended, in groups, for convenience of service.

Multiple conductor cables, one for each group, lead from the mining casemate to junction or distribution boxes similar to that shown in Fig. 303. In the junction box the conductors of the multiple cable are separated and joined to the conductors of single conductor cables which lead to the individual mines of the group. Thus each mine has its own cable and may be operated independently of all the other mines.

The course of a hostile vessel approaching or moving through the mine fields is observed by means of the range and position finding system of the fortification, and the operator in the mining casemate is apprised of the proximity of the vessel to any mine.

In addition to the groups of mines, other mines, called skirmish mines, *s* Fig. 304, may be laid on single cables in irregular lines about the groups. The skirmish mines may be made active or safe at the will of the operator, but cannot, on account of their arrangement on a single cable, be fired singly by judgment.

The arrangement of all the mines is such that a vessel can follow no reasonable course into the harbor without encountering several mines. Gaps, left between the groups in the various lines, form a more or less tortuous channel which allows passage to friendly vessels. Guide boats are employed to conduct friendly vessels through the safe passages.

Subsidiary waterways not of service to the defense may be closed to the enemy by mechanical mines, which contain within themselves the electric batteries that provide the firing current.

In the fortifications, gun batteries, usually of 3-inch guns, cover the mine fields, and protect them against attempts of the enemy to clear the fields by countermining from boats.

Search lights are provided to illuminate the mine fields at night.

332. **Countermining.**—Countermining consists in exploding and cutting adrift the fixed mines of the enemy and destroying their cable connections by the explosion of other mines distributed among them. The purpose of countermining is to make a safe channel through the mines of the defense. Countermining is usually done at night from small boats.

The Removal of Mines.—The experience had in clearing the harbors of the United States of mines after the Spanish War indicates that the safest way to remove the mines is to explode them in place.

Mobile and Automobile Torpedoes.—The mobile torpedo conveys the explosive charge under the water and explodes the charge against the bottom of the enemy's ship. Mobile torpedoes are now used exclusively by navies, and all such torpedoes are self-propelling or automobile. The necessity of erecting on shore, at the water's edge, special plants for the service of the torpedoes, and the necessity of protecting such plants, are considerations that militate against the use of mobile torpedoes for harbor defense.

The Sims-Edison Torpedo.—A long series of experiments were made a number of years ago with the Sims-Edison torpedo, Fig. 305, to determine whether this torpedo was adapted for harbor defense.

The torpedo consists of a cylindrical hull with conical ends. It is 28 feet long, 21 inches in diameter, and is supported at a depth

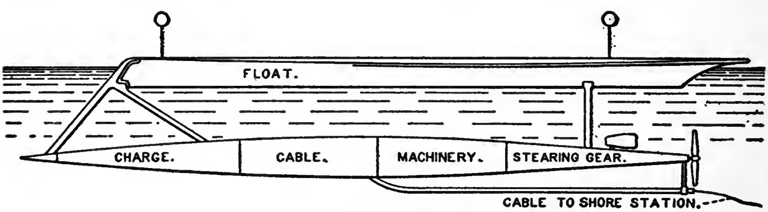


Fig. 305.

of 5 feet under the water by a float, to which it is connected by steel rods. Two balls carried above the float enable the operator on shore to observe the position of the torpedo and to direct its movement. The torpedo is propelled, steered, and exploded by electricity. The power is generated at a station on shore and is communicated to the torpedo through a cable which is carried coiled in a central chamber and is paid out as the torpedo moves.

A charge of 300 pounds of explosive is carried in the head of the torpedo.

The results obtained in the experiments were not sufficiently satisfactory to warrant the adoption of this torpedo for the harbor defense service.

333. **The Whitehead Torpedo.**—The Whitehead torpedo, Fig. 306, is now used by all the navies of the world. Its motive power is furnished by compressed air which is stored, at a pressure of about 1100 pounds per square inch, in a tank carried by the torpedo.

The torpedo is fired, by compressed air or by gunpowder, from launching tubes that are mounted on the ship's deck or built into the ship below the water line. A torpedo tube arranged for firing with compressed air is shown mounted on the deck of a torpedo boat, in Fig. 307.

The explosive charge, carried in the head of the torpedo, is fired by percussion when the torpedo strikes.

Submersion Mechanism.—In a chamber in rear of the air tank is the mechanism for regulating the depth of the torpedo. The head of a piston, acted on by springs, protrudes through a central hole in the rear wall of the chamber into another narrow chamber to which the water has access through the holes in the walls of the torpedo. The water pressure thus acts on one side of the piston

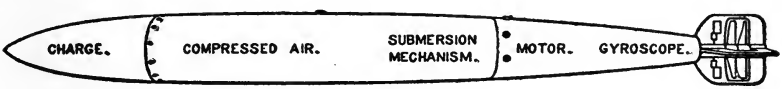


Fig. 306.

and the springs on the other. The springs may be regulated to exert a pressure on the piston equal to the pressure of the water at any desired depth. At that depth the piston will be stationary, while at any other depth it will be moved forward or backward. The piston is connected with horizontal diving rudders at the tail of the torpedo, one on each side. Any movement of the piston caused by the departure of the torpedo from the depth for which it is adjusted is communicated to these rudders, which act to return the torpedo to the desired depth.

The piston ceases to act when the torpedo is at the fixed depth, whatever may be the position of the longitudinal axis of the torpedo. As the axis will not be horizontal when the depth is reached the torpedo, if controlled by the piston alone, will overrun the depth and then return again to it, and will continue in this way rising and descending. To prevent this action a heavy pendulum, in the chamber with the piston, is also connected with the diving rudders. The pendulum remains vertical, and at any departure of the axis of the torpedo from the horizontal, the diving rudders are turned to correct the departure. The piston and pendulum together thus serve to keep the torpedo on an even keel at the desired submergence.

THE MOTIVE ENGINES.—The motive engines in the next compartment are supplied with compressed air through pipes that lead from the tank forward. The engines actuate two shafts, one within the other, that carry the propellers. The propellers turn in opposite directions. This arrangement of the propellers serves better than any other arrangement to prevent rolling of the torpedo.

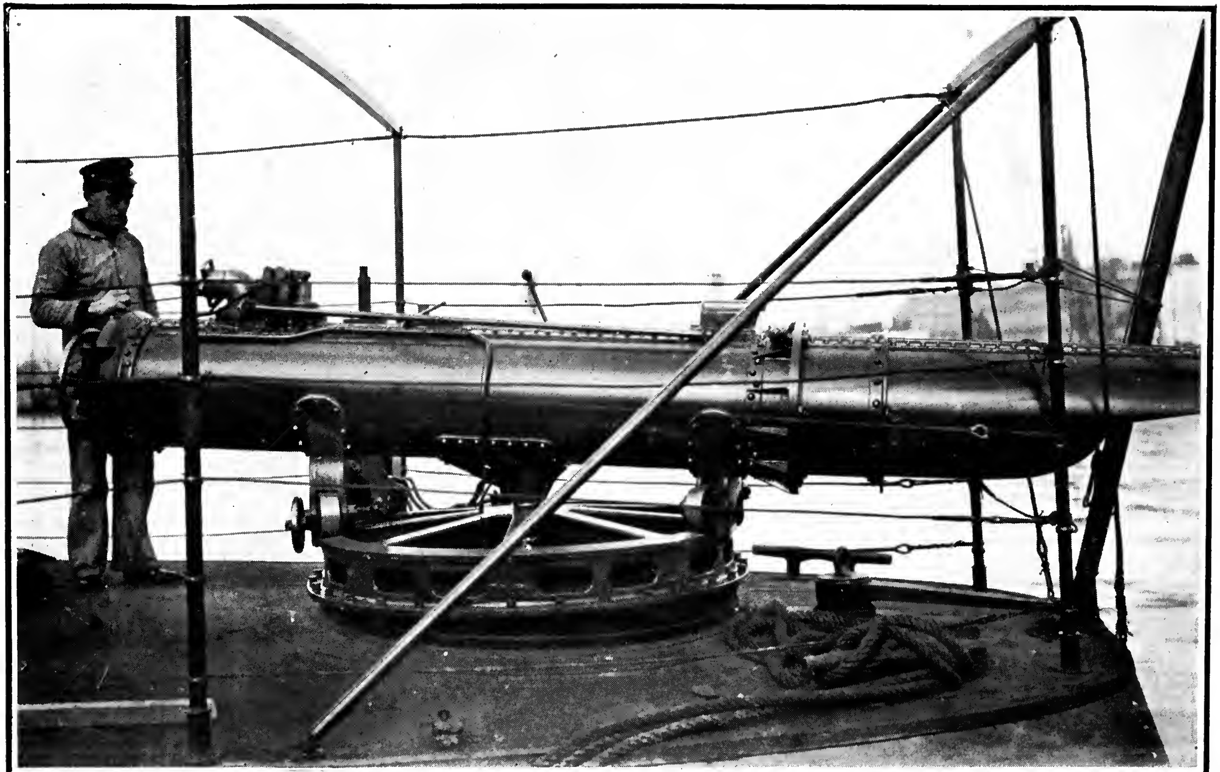
DIRECTING MECHANISM.—The compartment in rear of the engine contains the device for correcting any deviation of the torpedo from a straight course. A small gyroscope, with wheel about 3 inches in diameter, is mounted under the propeller shaft with its axis parallel to the axis of the torpedo. The gyroscope is set in motion by a spring-actuated mechanism at the launching of the torpedo. The axis of the gyroscope tends always to remain parallel to its original direction, and at any departure of the axis of the torpedo from its original direction the gyroscope actuates the valve of a small air steering-engine which moves the vertical rudders of the torpedo in such manner as to bring the torpedo back to its course.

SINKING MECHANISM.—In order to sink the torpedo at the end of its course, if it does not strike its target, and thus to prevent its falling into the hands of the enemy or doing injury to friends, a mechanism is provided which opens a sea-valve into the comparatively empty chamber that contains the gyroscope. The water fills the chamber and sinks the torpedo.

DATA.—The Whitehead torpedo has a diameter of 18 inches, and a length of about 16 feet. It has a mean velocity of 28 knots an hour over a range of 2200 yards. The charge of explosive weighs 60 pounds.

The Schwarzkopf torpedo differs from the Whitehead only in that the body of the torpedo is made of bronze instead of steel.

334. The Bliss-Leavitt Torpedo.—The Bliss-Leavitt torpedo, a recent American construction, and in use in the United States Navy, is of the same general construction as the Whitehead torpedo. Improvements in the mechanisms give to this torpedo greater range and greater accuracy.



Courtesy of the SCIENTIFIC AMERICAN.

FIG. 307.—Torpedo in Tube, Ready for Launching.

The air tank is charged to a pressure of 2225 pounds per square inch. The motor engine is of the Curtis turbine type and makes 10,000 revolutions a minute, operating the two propellers at the rate of 900 turns a minute. A large gain in power is obtained by a superheating process applied to the compressed air. An alcohol flame, automatically ignited when the torpedo is launched, greatly increases the expansive power of the compressed air as it enters the engine. The expansion is so great that trouble has been encountered from the freezing of the mechanism. Temperatures of 40° below zero have been registered in some runs.

The gyroscope controlling the vertical rudders is also of a turbine construction, and is rotated by compressed air at the rate of 18,000 revolutions a minute. It is much more effective in maintaining the torpedo in a fixed course than the spring-actuated gyroscope in the Whitehead torpedo. The accuracy of the torpedo is therefore greatly increased.

The Bliss-Leavitt torpedo is made in two sizes, 18 and 21 inches in diameter. The 21-inch torpedo is about 16½ feet long. It has an extreme range of 3500 yards and a mean speed over that range of 28 knots an hour. Over a range of 1200 yards its mean speed is 36 knots.

The explosive charge consists of 132 pounds of wet guncotton containing 25 per cent of water.

The firing mechanism in the point is the same as in the Howell torpedo described below.

The Howell Torpedo.—The Howell torpedo was invented by Admiral John A. Howell, United States Navy. The motive power of the Howell torpedo is a solid flywheel, *w* Fig. 308, which is

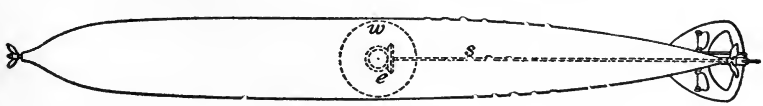


Fig. 308.

caused to revolve at a rate of 10,000 revolutions a minute, before the torpedo is launched, by a small turbine engine located in the launching tube. The rotation of the flywheel is communicated to two propellers, one on each side, through the bevel gears *e* and shafts *s*

A device applied to the propellers increases the pitch of the blades as their velocity of rotation diminishes, thus better maintaining the speed of the torpedo at the latter end of its course.

The gyroscopic power of the rotating flywheel gives to the torpedo great rigidity of direction in the horizontal plane.

The submergence is regulated by a hydrostatic piston and pendulum that act on the horizontal rudders at the tail, the mechanism being similar to that described in the Whitehead torpedo.

The small screw at the nose of the torpedo locks the firing mechanism in the safety position until the torpedo has traveled 30 or 40 yards through the water. The rotation of the screw during this travel arms the firing mechanism.

The Howell torpedo carried a charge of 174 pounds of guncotton. It was fired by gunpowder from the launching tube. Its extreme effective range, 1000 yards, was so limited that the torpedo never came into general use.

Towing Torpedoes.—Towing torpedoes are so arranged that they may be made to diverge to a considerable extent on either side of the wake of the towing vessel, so that this vessel may pass clear of the ship attacked and yet cause the torpedo to strike. Towing torpedoes were used by the Russians in their war with Turkey, 1877, but in no case with success.

335. Submarine Torpedo Boats.—While submarine torpedo boats are now used only by the navy, it has been recommended that they be used by the Coast Artillery as adjuncts to the submarine mine systems. They will perform a twofold function in the mine fields, first, in the inspection and repair of the mines and cables and other subaqueous material, to which access will be gained through a diving compartment or caisson provided in the boat; and second, in supplementing the fixed mines by defending with the torpedo those channels or passages that by reason of the great depth or the strength of the current cannot be closed by fixed mines.

Submarine boats are of two general classes, the *diving* boat and the *submersible* boat. The diving boat submerges by inclination of its longitudinal axis effected through horizontal rudders. It rises by the same means. The submersible boat sinks and rises

Fig. 309.—Holland Submarine Torpedo Boat.

bodily with even keel, the movements being effected by the vertical component of the water pressure against inclined hydroplanes projecting from both sides of the boat and symmetrically disposed with respect to the center of gravity.

Both classes of boats are provided with gasoline engines for propulsion on the surface, and with electric motors for use when submerged. When on the surface the motors may be used as dynamos to charge the storage batteries, the power being supplied by the gasoline engines.

To adjust the buoyancy, water is pumped into or out of the ballast tanks by pumps actuated by the engines or motors.

Air compressors and tanks are also provided. The compressed air is used for the discharge of the torpedoes, and to supplement the pumps in the discharge of water ballast.

The compressed air may also be used to renew the air supply in the vessel when submerged. The renewal of the air supply is, however, usually not necessary. Tests have shown that the crew does not suffer from bad air when the boat is hermetically sealed for long periods. In one test 7 men remained under water for 15 hours without change of air and without discomfort. In another test the boat, fully manned, remained totally submerged for 12 hours without change of air. In a recent test the boat, with 13 men aboard, remained submerged at a depth of about 40 feet for a period of 24 hours. During the last hours air was drawn from the compressed air supply. The test showed that the boat could remain under water for three days before exhausting the supply of air.

The Holland Submarine Boat.—The Holland submarine boat is the latest and most successful boat of the diving type of submarine.

The boat, Fig. 309, is spindle-shaped, circular in cross-section, with its greatest diameter about one third of its length from the bow. The single propeller is actuated by gasoline engines when the boat is on the surface, and by electric engines when the boat is awash or submerged.

Submergence is effected by means of horizontal diving rudders at the tail, arranged similarly to the diving rudders of the Whitehead torpedo.

The internal arrangements of the craft do not differ materially from those of the Lake submarine boat illustrated in Fig. 311, except that the Holland boat contains no diving caisson. The conning tower projects very slightly above the general outline of the boat.

At a recent government test of the Holland boat *Octopus* an average speed of 11 knots an hour was maintained by the boat in cruising condition on the surface, and 10 knots an hour when awash and submerged.

336. The Lake Submarine Boat.—The Lake submarine boat is of the *submersible* type. An exterior view of the *Protector*, the first torpedo boat of this type, is shown in Fig. 310, and an interior view of the boat submerged is shown in Fig. 311.

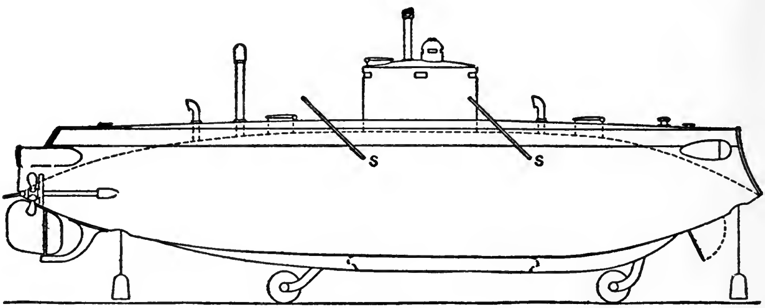
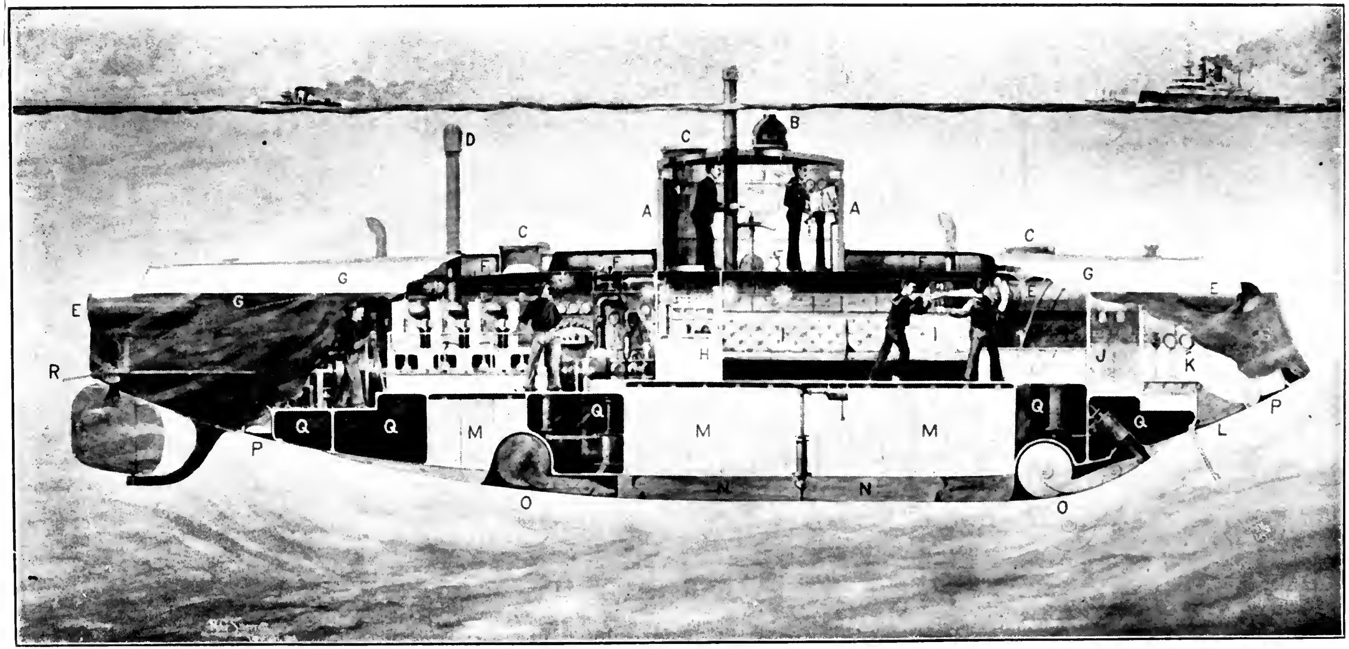


FIG. 310.

The hull is spindle-shaped, 67½ feet long with 14 feet beam. The draught, in cruising condition on the surface, is 12 feet. The displacement is 136 tons in cruising trim and 175 tons when submerged. A superstructure is erected on the hull, the top of the superstructure forming the deck of the boat. The space between the superstructure and the hull is occupied by the air, oil, and ballast tanks, and by the tanks for the gasoline used in the engines. The storage of the gasoline outside the hull greatly diminishes the chances of explosion from leaking gasoline, or of the asphyxiation of the crew from the same cause.

A conning tower rises from the hull. A sighting hood projects above the conning tower, and the omniscope, through which vision is obtained in all directions, rises 3 or 4 feet above the sighting hood.



A, A, Bronze Conning Tower.
B, Sighting Hood.
C, C, Hatches.
D, Exhaust from Engines.
E, E, Torpedo Tubes.
F, F, Gasoline Tanks.
G, G, Line of Spindle Hull.
H, Galley Compartment.
I, I, Crew Space.
J, Air Lock.
K, Diving Compartment.
L, Diving Door.
M, M, Storage Batteries.
N, N, Drop Keel.
O, O, Wheels.
P, P, Anchor Weights.
Q, Q, Ballast Tanks.
R, Horizontal Rudder.

Fig. 311.—Lake Submarine Torpedo Boat.

The boat is built to withstand an exterior pressure of 75 pounds to the square inch, which corresponds to a depth of about 150 feet.

The boat is provided with twin screws.

SUBMERSION.—Submergence is effected on an even keel; when under way, by inclining the four hydroplanes, *s* Fig. 310, downward and forward; and when the boat is stationary by dropping the anchors at each end, reducing the buoyancy to less than the combined weight of the anchors, and then pulling the boat downward by the anchor chains. All these operations are simply effected from the conning tower.

The horizontal rudder, *R* Fig. 311, is used only to counteract the pressure of the water on the front of the conning tower when the boat is running submerged.

The buoyancy of the boat is increased or diminished by pumping water out of or into the ballast tanks. A reserve of about 300 pounds of buoyancy is always maintained except when running on the bottom, and the boat is held submerged either by the anchors or, when moving, by the water pressure on the hydroplanes. It may be kept at any desired submergence, whether moving or at rest.

For running on the bottom, wheels are provided which are ordinarily carried in pockets in the keel and which are brought into position under the keel by hydraulic mechanism. The wheels are simple rollers and the propellers move the boat, the chief function of the wheels being to protect the bottom of the boat against injury from obstacles on the bottom.

When the buoyancy has been destroyed and when, through any accident to the pumps, it cannot be regained by discharging ballast, two sections of the keel, *N* Fig. 311, weighing together 5 tons, may be dropped from the boat by the turn of a wrench. Should this not be sufficient to cause the boat to rise, the two anchors, weighing half a ton each, may be let go. As a last resource the crew may escape through the diving chamber.

THE DIVING CHAMBER.—The diving chamber in the forward compartment is a feature of this boat that makes the boat especially valuable for submarine mine work. An air lock affords access to the chamber from the interior, and a downwardly-opening watertight door in the hull affords egress to the bottom. The diving chamber has telephonic communication with the conning tower.

Armament and Speed.—The boat carries three torpedoes, two in the tubes in the bow and one in the stern tube. The torpedoes are discharged from the tubes by compressed air. Extra torpedoes may be carried in the living room.

The first boat of this type made, in the official trials by the Russian Government, a speed of 9.3 knots an hour on the surface, under engines and motors combined, and 8.5 knots under engines alone. With conning tower awash and under engines alone the speed was 7.4 knots; and totally submerged, under electric motors alone, the speed was 5.4 knots. The cruising radius on the surface at full speed is about 350 knots. The submerged cruising radius, with motors, is about 20 knots at full speed and 30 knots at economical speed.

A Lake boat, with a displacement of 235 tons, is now (May, 1907) undergoing test by the United States Government, and boats with 500 tons displacement are projected.

TABLES.

TABLE I.

LOGARITHMS OF THE X FUNCTIONS.

Subtract 10 from each characteristic greater than 2.

x	$\log X_0$	$\log X_1$	$\log X_2$	$\log X_3$	$\log X_4$	$\log X_5$
0.001	9.03899	5.56162	6.52263	8.73764	9.16405	8.30001
0.010	9.53911	7.05911	7.52000	9.23296	9.66437	9.30059
0.05	9.88671	8.09440	8.20769	9.56059	0.01322	9.99778
0.10	0.03494	8.53009	8.49515	9.68493	0.16295	0.29663
0.15	0.12078	8.77897	8.65819	9.74798	0.25023	0.47060
0.20	0.18111	8.95170	8.77059	9.78653	0.31194	0.59347
0.25	0.22750	9.08291	8.88541	9.81206	0.35965	0.68834
0.30	0.26509	9.18802	8.92293	9.82962	0.39851	0.76552
0.35	0.29661	9.27522	8.97861	9.84191	0.43127	0.83052
0.40	0.32372	9.34942	9.02570	9.85051	0.45956	0.88660
0.45	0.34746	9.41375	9.06630	9.85640	0.48444	0.93587
0.50	0.36855	9.47036	9.10181	9.86028	0.50663	0.97980
0.55	0.38750	9.52077	9.13327	9.86260	0.52665	1.01937
0.60	0.40469	9.56610	9.16141	9.86371	0.54488	1.05539
0.65	0.42041	9.60719	9.18678	9.86386	0.56161	1.08840
0.70	0.43489	9.64471	9.20982	9.86325	0.57705	1.11887
0.75	0.44829	9.67918	9.23089	9.86201	0.59140	1.14715
0.80	0.46075	9.71100	9.25025	9.86027	0.60479	1.17352
0.85	0.47241	9.74052	9.26812	9.85811	0.61733	1.19821
0.90	0.48334	9.76802	9.28468	9.85562	0.62913	1.22143
0.95	0.49363	9.79373	9.30010	9.85284	0.64027	1.24332
1.00	0.50334	9.81784	9.31450	9.84984	0.65081	1.26404
1.05	0.51255	9.84053	9.32798	9.84664	0.66082	1.28369
1.10	0.52128	9.86193	9.34065	9.84329	0.67034	1.30239
1.15	0.52960	9.88217	9.35258	9.83981	0.67942	1.32020
1.20	0.53752	9.90136	9.36384	9.83623	0.68809	1.33721
1.25	0.54508	9.91958	9.37449	9.83256	0.69640	1.35348
1.30	0.55234	9.93693	9.38459	9.82882	0.70436	1.36908
1.35	0.55929	9.95346	9.39417	9.82503	0.71201	1.38406
1.40	0.56597	9.96926	9.40329	9.82119	0.71936	1.39846
1.45	0.57238	9.98436	9.41198	9.81732	0.72644	1.41230
1.50	0.57856	9.99884	9.42028	9.81343	0.73328	1.42569
1.55	0.58452	0.01272	9.42820	9.80953	0.73988	1.43858
1.60	0.59026	0.02605	9.43579	9.80561	0.74625	1.45104
1.65	0.59582	0.03887	9.44305	9.80169	0.75242	1.46310
1.70	0.60119	0.05122	9.45003	9.79777	0.75840	1.47478
1.75	0.60639	0.06311	9.45672	9.79386	0.76419	1.48608
1.80	0.61143	0.07459	9.46316	9.78996	0.76981	1.49705
1.85	0.61632	0.08567	9.46935	9.78607	0.77527	1.50770
1.90	0.62106	0.09638	9.47532	9.78219	0.78057	1.51803
1.95	0.62567	0.10675	9.48108	9.77833	0.78573	1.52808
2.0	0.63015	0.11678	9.48663	9.77449	0.79075	1.53788
2.1	0.63875	0.13591	9.49717	9.76687	0.80040	1.55668
2.2	0.64691	0.15395	9.50704	9.75939	0.80958	1.57456
2.3	0.65467	0.17097	9.51630	9.75193	0.81833	1.59158
2.4	0.66207	0.18708	9.52501	9.74461	0.82668	1.60783

LOGARITHMS OF THE X FUNCTIONS—*Continued.*

Subtract 10 from each characteristic greater than 2.

x	log X_0	log X_1	log X_2	log X_3	log X_4	log X_5
2.5	0.66914	0.20236	9.53322	9.73740	0.83467	1.62338
2.6	0.67589	0.21687	9.54098	9.73031	0.84232	1.63824
2.7	0.68237	0.23070	9.54833	9.72333	0.84966	1.65250
2.8	0.68859	0.24389	9.55531	9.71645	0.85673	1.66623
2.9	0.69457	0.25650	9.56194	9.70969	0.86353	1.67945
3.0	0.70032	0.26858	9.56826	9.70304	0.87009	1.69216
3.1	0.70587	0.28014	9.57427	9.69650	0.87642	1.70442
3.2	0.71122	0.29124	9.58001	9.69007	0.88252	1.71627
3.3	0.71639	0.30190	9.58551	9.68374	0.88842	1.72773
3.4	0.72140	0.31217	9.59077	9.67752	0.89416	1.73882
3.5	0.72624	0.32205	9.59582	9.67140	0.89970	1.74956
3.6	0.73093	0.33159	9.60066	9.66538	0.90508	1.75997
3.7	0.73548	0.34079	9.60532	9.65946	0.91027	1.77004
3.8	0.73990	0.34969	9.60979	9.65363	0.91537	1.77989
3.9	0.74419	0.35829	9.61410	9.64790	0.92037	1.78955
4.0	0.74836	0.36662	9.61825	9.64225	0.92510	1.79872
4.2	0.75637	0.38250	9.62613	9.63122	0.93432	1.81656
4.4	0.76398	0.39745	9.63348	9.62053	0.94308	1.83349
4.6	0.77121	0.41157	9.64036	9.61015	0.95143	1.84962
4.8	0.77810	0.42492	9.64682	9.60008	0.95939	1.86500
5.0	0.78469	0.43759	9.65290	9.59029	0.96700	1.87971
5.2	0.79099	0.44963	9.65864	9.58079	0.97430	1.89379
5.4	0.79703	0.46110	9.66407	9.57153	0.98130	1.90730
5.6	0.80284	0.47205	9.66921	9.56252	0.98803	1.92028
5.8	0.80842	0.48251	9.67409	9.55375	0.99450	1.93277
6.0	0.81379	0.49253	9.67874	9.54521	1.00074	1.94479
6.2	0.81897	0.50213	9.68316	9.53687	1.00676	1.95640
6.4	0.82397	0.51136	9.68738	9.52874	1.01257	1.96760
6.6	0.82881	0.52022	9.69142	9.52081	1.01819	1.97844
6.8	0.83349	0.52875	9.69528	9.51306	1.02363	1.98891
7.0	0.83801	0.53698	9.69897	9.50549	1.02890	1.99905
7.2	0.84241	0.54492	9.70252	9.49809	1.03402	2.00892
7.4	0.84667	0.55259	9.70592	9.49085	1.03898	2.01847
7.6	0.85081	0.56000	9.70919	9.48377	1.04379	2.02776
7.8	0.85483	0.56717	9.71234	9.47683	1.04848	2.03677
8.0	0.85873	0.57411	9.71538	9.47004	1.05304	2.04552
8.2	0.86254	0.58084	9.71830	9.46341	1.05748	2.05408
8.4	0.86625	0.58737	9.72112	9.45689	1.06180	2.06240
8.6	0.86986	0.59371	9.72385	9.45050	1.06601	2.07050
8.8	0.87338	0.59986	9.72648	9.44424	1.07012	2.07841
9.0	0.87682	0.60585	9.72903	9.43809	1.07413	2.08612
9.2	0.88017	0.61167	9.73150	9.43206	1.07804	2.09345
9.4	0.88345	0.61734	9.73390	9.42614	1.08187	2.10100
9.6	0.88665	0.62286	9.73621	9.42033	1.08560	2.10819
9.8	0.88978	0.62824	9.73846	9.41462	1.08926	2.11502
10.0	0.89284	0.63349	9.74065	9.40901	1.09283	2.12209
10.2	0.89584	0.63860	9.74276	9.40349	1.09633	2.12882
10.4	0.89877	0.64360	9.74482	9.39807	1.09976	2.13540
10.6	0.90165	0.64848	9.74683	9.39274	1.10312	2.14186
10.8	0.90447	0.65324	9.74877	9.38749	1.10640	2.14818

LOGARITHMS OF THE X FUNCTIONS—*Continued.*

Subtract 10 from each characteristic greater than 2.

x	log X_0	log X_1	log X_2	log X_3	log X_4	log X_5
11.0	0.90723	0.65790	9.75067	9.38233	1.10963	2.15437
11.2	0.90993	0.66245	9.75252	9.37725	1.11279	2.16045
11.4	0.91259	0.66691	9.75432	9.37225	1.11589	2.16642
11.6	0.91520	0.67127	9.75607	9.36732	1.11893	2.17227
11.8	0.91776	0.67554	9.75778	9.36247	1.12192	2.17801
12.0	0.92027	0.67972	9.75945	9.35770	1.12485	2.18364
12.2	0.92274	0.68381	9.76108	9.35301	1.12772	2.18916
12.4	0.92516	0.68783	9.76267	9.34836	1.13057	2.19462
12.6	0.92754	0.69176	9.76422	9.34379	1.13335	2.19996
12.8	0.92989	0.69562	9.76574	9.33928	1.13609	2.20522
13.0	0.93219	0.69941	9.76722	9.33484	1.13877	2.21039
13.2	0.93446	0.70313	9.76867	9.33045	1.14142	2.21547
13.4	0.93669	0.70678	9.77009	9.32613	1.14402	2.22047
13.6	0.93888	0.71036	9.77148	9.32186	1.14659	2.22539
13.8	0.94104	0.71388	9.77284	9.31766	1.14911	2.23023
14.0	0.94317	0.71734	9.77417	9.31350	1.15159	2.23400
14.2	0.94527	0.72074	9.77547	9.30940	1.15403	2.23970
14.4	0.94733	0.72408	9.77675	9.30535	1.15644	2.24433
14.6	0.94936	0.72736	9.77800	9.30136	1.15882	2.24888
14.8	0.95137	0.73059	9.77922	9.29741	1.16115	2.25337
15.0	0.95334	0.73377	9.78043	9.29351	1.16346	2.25780
15.2	0.95529	0.73689	9.78160	9.28966	1.16573	2.26216
15.4	0.95721	0.73997	9.78276	9.28585	1.16797	2.26647
15.6	0.95910	0.74301	9.78391	9.28208	1.17018	2.27073
15.8	0.96097	0.74599	9.78501	9.27837	1.17236	2.27495
16.0	0.96282	0.74892	9.78610	9.27470	1.17450	2.27912
16.2	0.96463	0.75181	9.78718	9.27107	1.17663	2.28309
16.4	0.96643	0.75466	9.78823	9.26748	1.17872	2.28711
16.6	0.96820	0.75747	9.78927	9.26393	1.18078	2.29108
16.8	0.96995	0.76024	9.79029	9.26042	1.18282	2.29500
17.0	0.97168	0.76297	9.79129	9.25695	1.18483	2.29886
17.2	0.97338	0.76566	9.79227	9.25352	1.18682	2.30268
17.4	0.97507	0.76831	9.79324	9.25012	1.18879	2.30645
17.6	0.97673	0.77093	9.79419	9.24676	1.19072	2.31017
17.8	0.97838	0.77351	9.79513	9.24344	1.19264	2.31385
18.0	0.98001	0.77606	9.79605	9.24015	1.19454	2.31750
18.2	0.98161	0.77856	9.79696	9.23689	1.19640	2.32108
18.4	0.98320	0.78104	9.79785	9.23367	1.19825	2.32463
18.6	0.98477	0.78349	9.79872	9.23048	1.20008	2.32814
18.8	0.98632	0.78591	9.79959	9.22732	1.20188	2.33161
19.0	0.98785	0.78829	9.80044	9.22419	1.20367	2.33504
19.2	0.98937	0.79065	9.80128	9.22109	1.20543	2.33843
19.4	0.99086	0.79296	9.80210	9.21803	1.20717	2.34177
19.6	0.99235	0.79527	9.80292	9.21499	1.20891	2.34510
19.8	0.99382	0.79754	9.80372	9.21198	1.21062	2.34838
20.0	0.99527	0.79978	9.80451	9.20900	1.21230	2.35162

Table II.

HEATS OF FORMATION, AT 15° C. AND NORMAL ATMOSPHERIC PRESSURE (760 MM). LARGE CALORIES.

Name.	Formula.	Molecular Weight.	Heat given off, the product being			
			Gaseous	Liquid.	Solid.	Dissolved.
Hydrochloric acid	HCL	36.5	22.			39.3
Hydrobromic acid	HBr	81.	9.5			29.5
Water	H_2O	18.	58.2	69.	70.4	
Hydrogen sulphide	H_2S	34.	4.8			
Nitric acid	HNO_3	63.	34.4	41.6	42.2	48.8
Hyposulphurous acid	$H_2S_2O_3$	114.				67.2
Sulphur dioxide	SO_2	64.	69.2			
Sulphur trioxide	SO_3	80.	91.8		103.6	141.
Sulphuric acid	H_2SO_4	98.		124.	124.8	
Hypochlorous acid anhydride	Cl_2O	86.	−15.2			−5.8
Perchloric acid	$HClO_4$	100.5		−30.8		
Carbon dioxide	CO_2	44.	94.3			
Carbon monoxide	CO	28.	25.8			
Nitrous oxide	N_2O	44.	−20.6	−16.2		
Nitrogen dioxide	NO	30.	−21.6			
Nitrous anhydride	N_2O_3	76.	−22.2			
Nitrogen peroxide	NO_2	46.	−2.6	1.8		
Nitric anhydride	N_2O_5	108.	−1.2	3.6	11.8	28.6
Potassium oxide	K_2O	94.			97.2	164.6
Sodium oxide	Na_2O	62.			100.2	145.2
Antimonous oxide	Sb_2O_3	287.2			167.4	
Antimonic oxide	Sb_2O_5	329.2			228.8	
Potassium chloride	KCl	74.6			105.	100.8
Sodium chloride	$NaCl$	58.5			97.3	96.2
Ammonium chloride	NH_4Cl	53.5			76.7	72.7
Calcium chloride	$CaCl_2$	110.			170.	187.
Potassium sulphide	K_2S	110.2			102.2	112.4
Sodium sulphide	Na_2S	78.			88.4	103.2
Antimony sulphide	Sb_2S_3	335.2			34.	
Ammonium sulphide	$(NH_4)_2S$	68.				56.8
Potassium nitrate	KNO_3	101.1			118.7	
Sodium nitrate	$Na\ No_3$	85.			110.6	
Ammonium nitrate	NH_4NO_3	80.			87.9	
Potassium sulphate	K_2SO_4	174.			342.2	
Sodium sulphate	Na_2SO_4	142.			326.4	
Potassium carbonate	K_2CO_3	138.			278.8	
Sodium carbonate	Na_2CO_3	106.			274.8	
Nitronaphthalene	$C_{10}H_7NO_2$	173.			−14.7	
Binitronaphthalene	$C_{10}H_6(NO_2)_2$	218.			−5.7	
Trinitronaphthalene	$C_{10}H_5(NO_2)_3$	263.			3.3	
Potassium chlorate	$KClO_3$	122.5			94.6	
Ammonia	NH_3	17.	12.2			
Nitrogen sulphide	NS	46.	−19.	−25.4	−31.9	
Cyanogen	CN	26.	−37.3			−33.9
Hydrocyanic acid	HCN	27.	−29.	−23.8		23.4
Potassium cyanide	KCN	65.			30.3	27.4
Acetylene	C_2H_2	26.	−61.4			

HEATS OF FORMATION—*Continued.*

Name.	Formula.	Molecular Weight.	Heat given off, the product being			
			Gaseous	Liquid.	Solid.	Dissolved.
Ethylene	C_2H_4	28.	−15.4			
Methane	CH_4	16.	18.5			
Benzene	C_6H_6	78.	−10.2	−3.2	−0.9	
Terebenthene	$C_{10}H_{16}$	136.	8.6	−17.		
Naphthalene	$C_{10}H_8$	128.			−23.7	
Anthracene	$C_{14}H_{10}$	178.			−42.4	
Methyl alcohol	CH_3OH	32.	53.6	62.		64.
Ethyl alcohol	C_2H_5OH	46.	60.7	70.5		73.
Propyl alcohol	C_3H_7OH	60.		67.		70.
Phenol	C_6H_5OH	94.		34.5	36.8	32.
Glycerine	$C_3H_5(OH)_3$	92.		165.5	169.4	164.
Mennite dulcite	$C_6H_{14}O_6$	172.			320.	315.
Glucoses and isomers	$C_6H_{12}O_6$	180.			306.	303.
Saccharose and isomers	$n(C_6H_{12}O_6)$	n(180.)			n(269.)	
Cellulose (cotton)	$C_6H_{10}O_5$	162.			227.	
Aldehyde	C_2H_4O	44.	50.5	56.5		60.1
Ethyl nitrate	$C_2H_5NO_3$	91.		49.3		50.3
Nitroglycerine	$C_3H_5(NO_2)_3O_3$	227.		98.		
Nitromannite	$C_6H_8(NO_3)_6$	452.			149.	
Mercury fulminate	$C_2N_2O_2Hg$	284.			−62.9	
Nitrocellulose (N_{11})	$C_{24}H_{29}N_{11}O_{42}$	1143.			624.	
Nitrobenzene	$C_6H_5NO_2$	123.		4.2	6.9	
Dinitrobenzene	$C_6H_4(NO_2)_2$	168.			12.7	
Picric acid	$C_6H_2(NO_2)_3OH$	229.			49.1	41.
Potassium picrate	$C_6H_2(NO_2)_3OK$	267.			117.5	107.5
Ammonium picrate	$C_6H_2(NO_2)_3ONH_4$	246.			80.1	71.4
Sodium picrate	$C_6H_2(NO_2)_3ONa$	251.			105.3	98.9
Diazonitrobenzol	$C_6H_5N_3O_3$	167.			−47.4	
Ether	$(C_2H_5)_2O$	74.	65.3	72.		78.
Methyl nitrate	CH_3NO_3	77.		39.9		
Dinitroglycol	$C_2H_4N_2O_6$	152.		66.9		
Propyl glycol	$C_3H_8O_2$	76.		127.		
Nitrocellulose (N_8)	$C_{24}H_{32}N_8O_{36}$	1008.			706.	
Amyl alcohol	$C_5H_{11}OH$	88.	82.3	93.		95.8
Amyl nitrate	$C_5H_{11}NO_3$	133.	7.	71.		
Glycol	$C_6H_6O_2$	62.			111.7	113.4
Sodium oxalate	$(CO_2Na)_2$	134.			313.8	

Table III.

SPECIFIC HEATS.

Name.	Formula.	Molecular Weight.	Specific heats referred to	
			One Gram.	Molecular Weight.
Sulphur (fused)	S_2	64.	0.203	12.8
Phosphorus	P_4	124.	0.190	11.8
Arsenic	As_2	150.	0.081	12.1
Antimony	Sb_2	244.	0.051	12.4
Carbon	C_2	24.	0.202	4.8
Mercury	Hg	200.	0.033	32.56
Lead	Pb_2	414.	0.031	13.2
Silver	Ag_2	216.	0.057	12.4
Magnesia	MgO	40.	0.244	9.76
Chromic oxide	Cr_2O_3	152.8	0.190	29.00
Aluminum oxide	Al_2O_3	103.	0.217	22.40
Ammonium chloride	NH_4Cl	53.	0.373	20.00
Potassium chloride	KCl	74.6	0.173	12.89
Sodium chloride	NaCl	58.5	0.214	12.5
Barium chloride	$BaCl_2$	207.	0.090	18.6
Calcium chloride	$CaCl_2$	111.	0.104	18.4
Silver chloride	AgCl	143.	0.091	13.1
Potassium sulphide	K_2S	110.	0.091	19.00
Sodium sulphide	Na_2S	78.	0.091	19.00
Iron sulphide	FeS	88.	0.136	11.94
Potassium ferro cyanide	$K_4Fe(CN)_6$	430.	0.280	118.00
Potassium nitrate	KNO_3	101.1	0.239	24.20
Sodium nitrate	$NaNO_3$	85.	0.278	23.70
Barium nitrate	$Ba(NO_3)_2$	261.	0.150	38.00
Strontium nitrate	$Sr(NO_3)_2$	211.	0.180	38.00
Lead nitrate	$Pb(NO_3)_2$	330.	0.110	36.4
Silver nitrate	$AgNO_3$	170.	0.143	24.4
Ammonium nitrate	NH_4NO_3	80.	0.455	36.4
Potassium sulphate	K_2SO_4	174.	0.190	33.2
Sodium sulphate	Na_2SO_4	142.	0.229	32.4
Calcium sulphate	$CaSO_4$	136.	0.180	25.4
Strontium sulphate	$SrSO_4$	183.5	0.140	24.8
Copper sulphate	$CuSO_4$	159.5	0.134	21.4
Potassium bichromate	$K_2Cr_2O_7$	294.	0.187	36.4
Potassium carbonate	K_2CO_3	138.	0.210	30.0
Sodium carbonate	Na_2CO_3	106.	0.270	29.0
Calcium carbonate	$CaCO_3$	100.	0.200	21.0
Barium carbonate	$BaCO_3$	197.	0.110	21.4
Lead carbonate	$PbCO_3$	26C.	0.141	39.4
Potassium chlorate	$KClO_3$	122.5	0.210	25.7
Potassium perchlorate	$KClO_4$	138.5	0.190	26.3
Water	H_2O	18.	1.000	18.0
Nitric acid	HNO_3	63.	0.445	28.0
Sulphuric acid	H_2SO_4	98.	0.340	33.4
Benzene	C_6H_6	78.	0.440	34.0
Alcohol	C_2H_5OH	46.	0.595	27.3
Glycerine	$C_3H_5(OH)_3$	92.	0.591	54.4
Antimony oxide	Sb_2O_3	287.2	0.090	25.85
Silica	SiO_2	60.3	0.195	11.76

TABLE IV.

DENSITIES AND MOLECULAR VOLUMES.

Name.	Formula.	Molecular Weights, M	Density. D	Molecular Volume in c.c. $\frac{M}{D}$
Sulphur	S_2	64.	2.04	31.36
Carbon	C_2	24.	2.50 diamond 2.27 graphite 1.67 amorph.	6.85 10.66 15.28
Potassium chloride	KCl	74.6	1.94	38.70
Sodium chloride	$NaCl$	58.5	2.10	27.20
Barium chloride	$BaCl_2$	207.	3.70	56.0
Strontium chloride	$SrCl_2$	158.5	2.80	59.0
Ammonium chloride	NH_4Cl	53.	1.53	35.0
Potassium nitrate	KNO_3	101.	2.06	49.0
Sodium nitrate	$NaNO_3$	85.	2.24	39.0
Barium nitrate	$Ba(NO_3)_2$	261.	3.25	82.0
Lead nitrate	$Pb(NO_3)_2$	330.	4.40	76.0
Silver nitrate	$AgNO_3$	170.	4.35	39.0
Ammonium nitrate	NH_4NO_3	80.	1.71	41.0
Strontium nitrate	$Sr(NO_3)_2$	211.	2.93	71.30
Potassium carbonate	K_2CO_3	138.	2.26	62.0
Sodium carbonate	Na_2CO_3	107.	2.47	43.0
Barium carbonate	Ba_2CO_3	197.	4.30	46.0
Strontium carbonate	$SrCO_3$	147.5	3.62	40.0
Calcium carbonate	$CaCO_3$	100.	2.71	36.0
Potassium sulphate	K_2SO_4	174.	2.66	66.0
Sodium sulphate	Na_2SO_4	142.	2.63	54.0
Barium sulphate	$BaSO_4$	233.	2.45	52.0
Strontium sulphate	$SrSO_4$	183.5	3.59	52.0
Calcium sulphate	$CaSO_4$	136.	2.93	46.0
Potassium chlorate	$KClO_3$	122.5	2.33	52.6
Potassium bichromate	$K_2Cr_2O_7$	294.	2.69	110.0
Antimony oxide	Sb_2O_3	292.	5.53	53.0
Antimony sulphide	Sb_2S_3	334.	4.42	75.0
Calcium oxide	CaO	56.	3.15	18.0
Ammonium sulphate	$(NH_4)_2SO_4$	132.	1.76	75.0
Copper nitrate	$Cu(NO_3)_2$	192.	2.03	94.5
Mercuric oxide	HgO	216.	11.14	19.38
Potassium sulphide	K_2S	110.	2.97	37.0
Sodium sulphide	Na_2S	78.	2.17	36.0
Silica	SiO_2	60.	2.65	23.0
Potassium cyanide	KCN	65.0	1.52	43.0

Table V.

ATOMIC WEIGHTS.

The atomic weights in this table are the International Atomic Weights (1906) modified to make the atomic weight of hydrogen unity.

Element.	Symbol	Atomic Weight.	Element.	Symbol	Atomic Weight.
Aluminum	Al	26.9	Neon	Ne	19.9
Antimony	Sb	119.3	Nickel	Ni	58.3
Argon	A	39.6	Niobium	Nb	93.3
Arsenic	As	74.4	Nitrogen	N	13.9
Barium	Ba	136.4	Osmium	Os	189.6
Beryllium	Be	9.	Oxygen	O	15.9
Bismuth	Bi	206.9	Palladium	Pd	105.7
Boron	B	10.9	Phosphorus	P	30.8
Bromine	Br	79.4	Platinum	Pt	193.3
Cadmium	Cd	111.6	Potassium	K	38.9
Cæsium	Cs	132.	Praseodymium	Pr	139.4
Calcium	Ca	39.8	Radium	Ra	223.3
Carbon	C	11.9	Rhodium	Ro	102.2
Cerium	Ce	139.	Rubidium	Rb	84.8
Chlorine	Cl	35.2	Ruthenium	Ru	100.9
Chromium	Cr	51.7	Samarium	Sm	148.9
Cobalt	Co	58.5	Scandium	Sc	43.8
Copper	Cu	63.1	Selenium	Se	78.6
Erbium	E	164.8	Silicon	Si	28.2
Fluorine	F	18.9	Silver	Ag	107.1
Gadolinium	Gd	155.	Sodium	Na	22.9
Gallium	Ga	69.5	Strontium	Sr	87.
Germanium	Ge	71.9	Sulphur	S	31.8
Gold	Au	195.7	Tantalum	Ta	181.6
Helium	He	4.	Tellurium	Te	126.6
Hydrogen	H	1.	Terbium	Tb	158.8
Indium	In	113.1	Thallium	Tl	202.6
Iodine	I	125.9	Thorium	Th	230.8
Iridium	Ir	191.5	Thulium	Tm	169.7
Iron	Fe	55.5	Tin	Sn	118.1
Krypton	Kr	81.2	Titanium	Ti	47.7
Lanthanum	La	137.9	Tungsten	W	182.6
Lead	Pb	205.4	Uranium	U	236.7
Lithium	L	7.	Vanadium	V	50.8
Magnesium	Mg	24.2	Xenon	Xe	127.
Manganese	Mn	54.6	Ytterbium	Yb	171.7
Mercury	Hg	198.5	Yttrium	Y	88.3
Molybdenum	Mo	95.3	Zinc	Zn	64.9
Neodymium	Nd	142.5	Zirconium	Zr	89.9

Table VI.

CONVERSION: METRIC AND ENGLISH UNITS, TEMPERATURES.

English to Metric.		Metric to English.	
To Convert	Multiply by	To Convert	Multiply by
Inches to centimeters	2.539978	Centimeters to inches	0.39370428
Inches to meters..........	0.02539978	Meters to inches.........	39.370428
Feet to meters..........	0.3047973	Meters to feet............	3.280869
Yards to meters.........	0.9143918	Meters to yards..........	1.093623
Miles to kilometers.......	1.609329	Kilometers to miles.......	0.6213769
Square inches to square centimeters...........	6.451484	Square centimeters to square inches	0.155003
Square feet to square meters..................	0.09290138	Square meters to square feet..................	10.76410
Square yards to square meters................	0.8361126	Square meters to square yards.................	1.196011
Cubic inches to cubic centimeters	16.38663	Cubic centimeters to cubic inches...............	0.06102537
Cubic feet to cubic meters	0.02831609	Cubic meters to cubic feet.	35.31561
Cubic yards to cubic meters................	0.7645345	Cubic meters to cubic yards................	1.307985
Quarts, liquid, to liters....	0.9463279	Liters to quarts (liq.).....	1.056716
Gallons (231 cu. in.) to dekaliters.............	0.3785311	Dekaliters to gallons.......	2 641791
Grains to grams.........	0.06479887	Grams to grains.........	15.43236376
Ounces (avoir.) to grams..	28.34951	Grams to ounces (avoir.)..	0.03527398
Pounds (av.) to kilograms.	0.4535922	Kilograms to pounds (av.)	2.20462339
Foot-pounds to kilogrammeters.................	0.1382537	Kilogrammeters to footpounds.................	7.233080
Pounds per sq. in. to kilograms per sq. cent.......	0.07030S2	Kilograms per sq. cent. to pounds per sq. in........	14.22309
Pounds per sq. in. to kilograms per sq. decimeter	7.03082	Kilograms per sq. decimeter to pounds per sq. in.................	0.1422309

TEMPERATURES.—T_f=temperature Fahrenheit: T_c=temperature centigrade.

Fahrenheit to centigrade, $T_c=\frac{5}{9}(T_f-32°)$.

Centigrade to Fahrenheit, $T_f=\frac{9}{5}T_c+32°$.