OUTLINES

Number Science

SECOND EDITION.

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From Prof. Boone
to
Florian Cajori

OUTLINES

—OF—

NUMBER SCIENCE

—BY—

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EXPLANATORY.

This little book is prepared to meet the author's convenience in giving Arithmetical instruction in the Normal School, and at the solicitation of a large number of pupil-teachers, who have received in the class-room and have afterwards used in their own schools, much of the matter herein contained. The work is but an elaboration of a series of articles written by the same hand in 1873, and published in an educational paper in this State. An examination of the order of procedure will show that the book is not made for *children*, but for the mature mind already conversant with most of the facts of number as presented in the text books on Arithmetic. "The aim of the Normal School is not so much to teach the facts of the common school branches as to make a thorough study of the relations of those facts to one another. The study of these relations opens up new lines of thought that make the common school branches intensely interesting studies to most students."

The province of the school as thus stated, being kept in view, the attempt has been made so to present the topics of number science that the lines which unify parts that are kindred, may be readily seen.

The discussion begins with a general definition of Mathematics, together with a brief consideration of the realms of *Space* and *Time*. These are defined as conditions:—Space as the condition of extension, and hence preliminary to Geometry in its various phases; and Time as the condition of succession, and hence preliminary to the idea of Number.

Starting with Time as the conditioning factor, it is sought to show the genesis of number in general. From this idea the mind readily passes to that of number in particular. The term *Integral unit* or *Unit one* (from Davies), is fixed upon to designate the primary idea of number in particular. It must be borne in mind that the elements which enter into a science are *mental* and not *material* objects. A pencil, a horse or a box, is not an element of number science—is not a *unit;* it is the *idea one* which the mind forms upon viewing the object as an entirety that constitutes the unit, or fundamental element in the science of numbers.

The student is next asked to re-think the nine general classes of numbers, to find the basis of each classification, and to give, by definition, the mark of each class.

The classification of numbers is discussed thus early not because of its logical relation to that which precedes or succeeds it, but because of the basal character which Number classification sustains in all computation.

Number Representation is next discussed. Under the Arabic Notation it is observed that the characters are used to represent numerical values thought in three systems of numbers. Each of these systems, together with its notation, is discussed.

Number Reduction is next considered. The kinds of reduction are determined, and applied to numbers thought in the decimal, the fractional and the compound systems. Attention is called to the fact that reduction descending is effected by *multiplication*, and that reduction ascending is effected by *division*, whether the number to be reduced be an abstract integer, a fraction or a denominate number.

Under Number Processes, the phases of synthesis and analysis are treated. The terms used, the mental acts involved, and the principles which guide the mind in computing are discussed.

In formulating definitions and principles it is sought in the main, to "bring before the mind the act or process by which the concept to be defined is supposed to be constructed." It has been a special aim to make every definition sufficiently inclusive to embrace all that the term covers wherever found in the work: e. g. The definitions of multiplication and division as given in most text books on Arithmetic. are but partial since they do not include multiplication and division by a *fraction*. The definitions in this book are believed to be ample.

Both common and decimal fractions are treated together, the principles of the one being the principles of the other.

The suggestions given for the treatment of Compound Numbers, will, it is hoped, enable the teacher to proceed more systematically and satisfactorily than by the methods usually presented.

Most of the definitions and discussions under the applications of Percentage are omitted, not because they are unimportant, but because they are so well given in text books on Arithmetic that their repetition here is deemed unnecessary.

Methods of solving representative problems have been freely inserted under these applications.

Involution and Evolution are treated Arithmetically instead of Geometrically and in a manner at once simple and exhaustive.

"Results in teaching depend upon the clearness with which distinctions are made;" and the pupil can be brought to make clear distinctions only by being held to rigidly logical modes of thinking. As an aid to this end, forms of solution are given for nearly all classes of arithmetical exercises. These forms or others equally logical should be *strictly adhered to in order to secure to the pupil the maximum culture which the subject can give.*

A number of errors, typographical and otherwise, were observed after the work was in print. Some of these have been corrected with the pen. Others still remain, but they are of such a character as not to mislead the attentive reader.

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OUTLINES

—OF—

NUMBER SCIENCE.

SECTION I.

NUMBER GENESIS.

1. Mathematics may be defined in a general way as the department of knowledge which exhibits the properties and relations of extension and number.

2. A consideration of extension leads into the realm of space. Geometry and kindred branches are evolved from the properties and relations of extension.

3. A consideration of number leads into the realm of time for the primal elements of the science.

The Basis of Number.

4. In General. *a.* Every conscious state of the mind is known to begin, to endure for a period and to end, giving way to another mental phenomenon.

b. Mental states are distinct from the mind and from one another. They are known to be distinct because experienced in different periods of time or because of a diversity of the states compared.

c. To give genesis to the idea of number it is necessary for a state to succeed a state in consciousness. The idea of number thus originates in the cognition of succession. Succession is possible only in time; hence time is conditional for the numerical idea.

d. **Arithmetic** is a branch of the mathematics of number; it is both a science and an art.

e. As a science, Arithmetic exhibits the facts of number, presents the relations sustained among the facts, formulates these relations into principles which bring the facts together until they appear in consciousness systematically arranged as an organic whole.

As an art, Arithmetic is the application of the science in computation.

5. In Particular. If the attention be directed to an object, as a tree, a house, an apple, etc., among the attributes observed is that of *oneness*. The idea *one* may be abstracted from the conception of the object which occasions it, and as thus abstracted, it may be thought apart from its object.

The Unit. The term *integral unit*, or *unit one* (from Davies) is fixed upon to designate the idea *one* as abstracted from the conception of an object thought as a whole, as not composed of identical parts, nor itself one of the identical parts which compose another whole.

[In mathematics equality is identity. The reader is referred to Everett's Science of Thought, pp. 98-105, inclusive.]

Remark. The primary, or integral *one*, arises in the mind as above stated; there are, however, two classes of *secondary ones* with which we shall have much to do in our investigations of number relations and processes.

1. *The Fractional Unit.* If an object be separated, into equal parts, and the attention be directed to a part thus found, the idea *one* arises.

Definition. The idea *one*, which is applicable to a part that results from the separation of an object into equal parts is called a *fractional unit.*

2. *The Multiple Unit.* If the mind think several objects as forming a group, the idea *one* arises.

Definition. The idea *one* which is applicable to a group of wholes may be called a *Multiple Unit.*

Remarks. 1. The integral unit, or *unit one*, is the primary idea in Arithmetic. All other units are definitely related to this primary unit through the objects from which the units are originally abstracted.

2. An object giving rise to the idea *one* may be called a *unit-object.* Most writers on Arithmetic call such an object a *unit.*

3. The object which gives rise to a fractional unit is one of the equal parts into which the unit-object of the integral unit is thought as separated. In view of this relation of part to whole, a fractional unit is often said to be derived from the integral unit by the analysis of the latter into equal parts.

The *unit*, which is the idea *one*, is a unity, and is incapable of division; its object, only, can be divided, a new idea *one* arising when a *part* instead of a *whole* engages the attention. [See *Unity* in Fleming's Vocabulary of Philosophy.]

A Number. A number may be defined as a unit or group of like units thought together.

Remarks. 1. The units composing a number may be integral, fractional or multiple.

2. Aristotle did not include *unity* in the idea of number. He considered unity as the element of number.

Locke included *unity* in the idea of number. This view is adopted by modern writers on Arithmetic.

SECTION II.

NUMBER CLASSIFICATION.

a. On the basis of integral or fractional units used in their formation, numbers are classified as *integers* and *fractions.*

6. An Integer. A number composed of one or more integral units is called an *integer.*

7. A Fraction. A number composed of one or more fractional units is called a *fraction.*

Remark. An integer and a fraction thought as combined are together called a *mixed* number.

b. On the basis of application to objects numbers are classified as *Abstract* and *Concrete.*

8. Abstract. A number thought as independent of, or separate from an object, is called an *abstract* number.

9. Concrete. A number thought as applied to an object is called a *concrete* number. Examples: 4 men, 12 horses, $\frac{3}{4}$ of a dollar, etc.

Remark. 4, 12 and $\frac{3}{4}$, in the examples are concrete numbers because their objects are named.

c. On the basis of divisibility, abstract integers are classified as *Composite* and *Prime.*

10. Composite. A number that can be divided into equal integers each greater than *one*, is a composite number.

11. Prime. A number that cannot be divided into equal integers each greater than *one*, is a prime number.

d. On the basis of distinct or assumed unit-object, concrete numbers are classified as *Simple* and *Denominate.*

12. Simple. A number whose unit-object is a distinct whole, is a *simple* number. As 5 books, 3 pens, 7 horses; the unit-objects being the distinct wholes, book, pen and horse, respectively.

Remark. An abstract number is often called a simple number.

13. Denominate. A number whose unit-object is an assumed time, extent or degree of intensity, is a *denominate* number. As 3 days, 5 feet, 7 pounds; the unit-objects being the day, the foot and the pound, respectively, each of which is an assumed unit-object.

14. Compound. Two or more denominate numbers having the same primary unit, are together called a *compound* number.

Remarks. 1. A denominate number is often defined as a number whose unit (object) is named, but such definition is applicable to all concrete numbers and is, therefore, too inclusive.

2. A compound number is often defined as a number consisting of two or more denominations. Under this definition 5 ft. 3 lb. 3 hr. is a compound number. The error in the definition is apparent.

3. The classification of fractions is deferred until the subject is treated in detail.

SECTION III.

NUMBER REPRESENTATION.

Notation.

15. Definition. Notation is a systematic method of representing numbers by symbols.

Kinds of Notation.

Remark Many kinds of notation have been in use in different times and places. But two of these, however, are usually presented in Arithmetic, viz., that used by the ancient Romans and that introduced into Europe by the Arabs.

The Roman Notation.

16. Characters. The alphabet of the Roman notation consists of the seven letters, viz.: I. V. X. L. C. D. M.

17. Signification. I represents *one*, V *five*, X *ten*, L *fifty*, C *one hundred*, D *five hundred*, M *one thousand.*

18. Relation.

V	represents	five	times	I.
X	"	two	"	V.
L	"	five	"	X.
C	"	two	"	L.
D	"	five	"	C.
M	"	two	"	D.

19. Limit. The Roman notation is limited to the representation of integers, and is chiefly used in numbering chapters, headings and divisions in books and papers.

20. Principles.

Remark. As now used the Roman notation is effected in accordance with the following principles:

I. If a letter be written with its equal or with a combination of its equals, the letters combined represent a value equal to the sum represented by the letters.

II. If a letter be placed at the left of a letter representing a greater value, the letters combined represent the difference between the values represented by the letters taken separately.

III. If a letter or combination of letters be placed at the right of a letter representing a greater value, the letters combined represent the sum of the two values.

IV. If a dash be placed over a letter or combination of letters the value represented is multiplied by one thousand.

21. Exercises. *Read each of the following:* IX XIV, XXVII, LIII, XCV, CXXVIII, XII, XCVI, MDCCL, MCI, MDCCCL, LXXXIII, $\overline{\text{D}}$IV.

Write in the Roman notation: Twenty-nine; thirty-three; fourteen; one hundred six; fifty-six; one thousand two hundred sixty-four; seventy-eight; one thousand seven hundred seventy-six; ninety-seven; forty-nine; five hundred thirteen; eleven hundred seventy. [Give additional exercises.]

The Arabic Notation.

22. Characters. The alphabet of the Arabic notation consists of the following ten characters called figures, viz.: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

23. Signification. The first nine of these figures represent one, two, three, &c., units to nine. They are called significant figures, or digits because they signify or point out numbers. The tenth figure is called zero or nought. It expresses no numerical value.

Remark. In Algebra the zero is classified among the symbols of quantity and is defined as the representative of an infinitely small quantity.

24. Systems. The Arabic figures are used in three distinct systems of numbers, viz.: (1.) The *decimal.* (2.) The *fractional.* (3.) The *compound.*

1.—The Decimal System.

25. A Scale. The series of units constituting the basis of a system of numbers is called a *scale.*

26. The Decimal Scale. The decimal system of numbers has for its basis a series of units each of which is ten times as great as the unit next below it in the series. This series of units is called the *decimal scale.*

27. Orders of Units. 1. The integral unit, or *unit one*, is called a unit of the *first order.*

2. Ten units of the first, or units' order are together called a unit of the *second order.*

3. Ten units of the second, or tens' order are together called a unit of the *third order*.

4. Ten units of the third, or hundreds' order are together called a unit of the *fourth order*.

5. Ten units of the fourth, or thousands' order are together called a unit of the *fifth order*.

6. Ten units of the fifth, or ten-thousands' order are together called a unit of the *sixth order*.

Remark. Higher orders of units are formed by thinking together ten units, respectively, of the sixth, seventh, eighth, ninth, tenth, &c.. orders indefinitely.

28. Periods. 1. The first three orders of units in the decimal scale, viz.: *units*', *tens*' and *hundreds*', constitute a period called *units' period.*

2. The three orders of units next higher than units' period, constitute thousands' period. *Units* of thousands, *tens* of thousands and *hundreds* of thousands are thought as composing this period.

3. The three orders of units next higher than thousands' period constitute millions' period. *Units* of millions, *tens* of millions and *hundreds* of millions are thought as composing this period.

4. Other periods of three orders each are formed of higher orders of units in the decimal scale. The periods above those named are those of billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, etc., to infinity.

5. Each period embraces three orders, viz.: *units*, *tens* and *hundreds* of that period.

29. Lower Orders.

Remarks. a. The decimal scale may include units lower than the unit one.

b. The division of a unit into tenths, hundredths, thousandths, ten-thousandths, etc., is called a decimal division of the unit.

1. Units which result from dividing the *unit one* into ten equal parts are called *tenths*, or units of tenths' order.

2. Units which result from dividing one-tenth into ten equal parts are called *hundredths*, or units of hundredths' order.

3. Units which result from dividing *one-hundredth* into ten equal parts are called *thousandths*, or units of thousandths' order.

4. Orders of decimal units called respectively *ten-thousandths, hundred-thousandths, millionths, ten-millionths*, etc., are formed by dividing into ten equal parts the unit next higher in the scale.

30. Principle of the Decimal Scale. Ten units of any order make a unit of the next higher order.

Notation of Numbers Thought in the Decimal Scale.

31. Characters. The Arabic characters already given, together with a point or period, called the decimal point, are used in representing numbers thought in the decimal scale.

32. Signification. The digits and the zero have the signification already stated, while the decimal point is used to mark the place in a written number from which to start in determining its value.

33. Units' Place. The first place at the left of the decimal point is units' place.

Remark. Any number of first order units from 1 to 9, may be represented by writing the proper figure in units' place.

34. Tens' Place. The place next at the left of units' place, is tens' place.

Remark. Any number of second order units from 1 to 9, may be represented by writing the proper figure in tens' place.

35. Hundreds' Place. The place next at the left of tens' place is hundreds' place.

Remark. Any number of third order units from 1 to 9, may be represented by writing the proper figure in hundreds' place.

36. Units of thousands, tens of thousands, and hundreds of thousands, respectively, may be represented in the next three places at the left of those already named. In the next three places may be represented, respectively, units of millions, tens of millions and hundreds of millions, etc.

37. Tenths, hundredths and thousandths, etc., may be represented by figures written at the right of units' place in places corresponding to tens', hundreds', thousands', etc., at the left.

38. Scale. The term scale is used to name the series of successive units which form the basis of a system of numbers. In notation, however, the series of successive places in which the different orders of units may be represented is called a scale. This scale may be called the representative scale to distinguish it from the thought scale defined above.

Remark. The *simple* value of a figure is determined by its form alone, while the *local* value of a figure is determined by both the form of the figure and the place it occupies in the representative scale.

39. Limit. The decimal system of writing numbers is limited to the representation of integers and such fractions as result from a decimal division of the the unit.

Numeration and Reading Numbers.

40. Numeration. Naming the successive places in a written number is called numeration.

Remark. In numerating a number it is both convenient and customary to begin with units' order.

41. Reading. Naming the numerical value represented by a written number is called reading the number.

Remarks. 1. In reading a number it is both convenient and customary to begin at the highest order in which numerical value is represented.

2. In reading a number the word *and* should never be used except between the integral and the fractional parts of a mixed number. Thus 325 is read, three hundred twenty-five; $4\frac{2}{3}$ is read, four *and* two thirds; 420.005 is read, four hundred twenty *and* five thousandths.

[Exercise in writing and reading numbers in the decimal scale.]

Read 46; 326; 460; 2346; 1785; 57689; 32567; 4567890; 45678834; 456784352; 46789716; 456.3; 3 4; 51.6; 317.04; 456 07; 5678.17; 45.041; 32.117; 2.3456; 4.055, 56.78946; .346; .3; .5678; .56789047; .03456789; .45678; .5678789;

Write each of the following in the decimal scale:— Two hundred thirty-four; seven thousand sixty-five; Five hundred forty-one; seven thousand seventy-six; two thousand ninety-eight; Four hundred thousand sixteen; forty-five thousand ten; sixty-six thousand ninety-four; seventeen thousand five; nineteen thousand nineteen; five hundred thousand six; six million four hundred thousand seventy-eight; three hundred million seven thousand six hundred nine; three tenths; nine tenths; fifteen hundredths; seven hundredths; fourteen thousandths; six thousandths; two ten-thousandths; three hundred eleven thousandths; one hundred two ten-thousandths; twenty-four hundred thousandths; five hundred and seven hundred thousandths; sixteen millionths; four hundred sixty-one millionths; fifty-seven ten-millionths; fifteen tenths; twenty-one tenths; two hundred thirteen tenths; five hundred tenths; seventy tenths; one thousand seventy-five tenths; twenty tenths; one hundred fifteen hundredths; two hundred six hundredths; four hundred forty-one hundredths; fifty hundredths; six hundred hundredths; four hundred fifty hundredths; four hundred seventeen tenths; three thousand hundredths; four thousand two hundred eighty-four thousandths; two thousand seven thousandths; one hundred tenths; forty-five thousand thousandths; forty tenths, ten thousand ten-thousandths; two hundred and six tenths; forty and seventeen hundredths; eleven and five thousandths: seventy-five and six tenths; one thousand and sixty-five ten-thousandths.

2.—The Fractional System.

42. The primary idea of a fraction is composed of the following elements:

a. The idea of a whole divided.
b. The idea of equality of the parts.
c. A number of the parts.

1. Two numbers are thus necessary to the conception of a fraction. *a.* The number of equal parts into which the unit is thought as separated. *b.* The number of those parts that are thought as constituting the fraction.

2. These two numbers are called the terms of the fraction. The number of equal parts into which the unit, or whole, is thought as separated is called the denominator of the fraction. The number of equal parts thought as constituting the fraction is called the numerator of the fraction.

3. *Denomination.* The fractional denomination of a fraction is the same as the ordinal of the denominator. This is true of all fractions except those having the number *two* for a denominater. The denomination of such fractions is *half* instead of *second*, the ordinal of the denominator.

The Notation of a Fraction.

43. Fractions which result from a decimal division of the unit may be written in the decimal (representative) scale. [Art. 37.] Other fractions have a notation peculiarly their own.

44. Since two numbers are necessary to the thought fraction, two written numbers are necessary to notate a fraction. These written numbers have the same names, respectively, as the terms of the *thought* fraction which they represent.

45. *a.* The written denominator of a fraction is the figure or figures representing the number of fractional units into which a unit is thought as separated.

b. The written numerator of a fraction is the figure or figures representing the number of fractional units composing the fraction.

46. The written denominator is placed below the written numerator and separated from it by a short line.

Remark. A fraction is thought as sustaining a definite relation to the integral unit; we, therefore, think of a written fraction as attached to units' place in the (representative) decimal scale. If the thought fraction require it, the written fraction may be attached to any other place in the representative scale.

[Exercise in writing and reading fractions.]

3.—The Compound System.

47. A compound number is thought in two or more different orders of units that have the same primary, or standard unit.

48. In compound numbers the number of orders (denominations) in any "measure" is limited to the number of different unit-objects agreed upon for measuring the attribute under consideration.

49. The compound system of numbers is based, not so much on the fact that the several scales are varying, as that each "measure" has its own scale. Some of these scales are varying, and some of them are uniform. Each of the common measures has a varying scale, while the "metric" measures, including the measure of U. S. money, has a uniform and decimal scale. In the old books will be found a duo-decimal "measure" which is, of course, uniform.

50. Each denominate number which forms part of a compound number, is thought in the decimal system, in the fractional system, or in both.

The Notation of a Compound Number.

51. The denominate numbers which compose a compound number, are written in a descending series, from left to right. Thus—4 bu. 3 pk. 5 qt. 1 pt.; 5 da. 16 hr. 47 min.; $\frac{2}{3}$ lb. $3\frac{3}{4}$ oz. $\frac{3}{8}$ pwt.

52. The names of the orders or denominations may be abbreviated, but the parts of a written compound number are not to be separated by any mark of punctuation.

In reading a compound number the word *and* should not be used between any two adjacent denominate numbers in the series composing the compound number.

Remark. The first compound number given under Art. 51 should be read—4 bushels 3 pecks 5 quarts 1 pint. The third should be read—$\frac{2}{3}$ of a pound $3\frac{1}{4}$ ounces $\frac{3}{8}$ of a pennyweight.

[Exercise in writing and reading compound numbers.]

SECTION IV.

NUMBER REDUCTION.

53. Reduction consists in the change by which a given numerical value is thought in another order or denomination.

54. Reduction Descending. Reduction descending consists in reducing a numerical value of any order or denomination to a lower order or denomination. It is effected by thinking the value of each unit of the given order or denomination in the number of units of the lower that are together equal to a unit of the higher.

55. Reduction Ascending. Reduction ascending consists in reducing a numerical value of any order or denomination to a higher order or denomination. It is effected by thinking as a unit of the higher the number of units of the lower order or denomination that are together equal to a unit of the higher.

Exercises.

Reduce to lower orders each of the following:

4 thousands; 14 hundreds; 7 tens; 3 units; 5 hundredths; 4 tenths; 2 thousandths, &c.

Reduce 1, 2, 3, 4, 5, each to 3ds, 4ths, 5ths, 6ths, etc.

Reduce $\frac{1}{2}$ to 4ths; $\frac{2}{3}$ to 6ths; to 9ths; to 12ths; to 15ths.

Reduce $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ each to 8ths; to 12ths; to 16ths.

Reduce $\frac{2}{4}$ to halves; $\frac{4}{6}$ to 3ds; $\frac{5}{10}$ to halves.

Reduce $\frac{15}{20}$ to 4ths; $\frac{10}{20}$ to halves; to 10ths; to 5ths.

Reduce $\frac{6}{3}$ to units; $\frac{4}{2}$ to units; $\frac{16}{4}$ to units.

Reduce 5 days to hr.; to min.; to sec.

Reduce 1 gal. to qt.; to pt.; to gills.

Reduce 3 bu. to pk.; to qt.; to pt.

Reduce 128 pt. to qt.; to pk.; to bu.

Reduce 96 gills to pt.; to qt.; to gal.

Reduce 500 units to tens; to hundreds.

Reduce .2500 to thousandths; to hundredths; to tenths.

Reduce .325 to hundredths; to tenths.

Reduce 220 units to tens; to hundreds; to tenths; to hundredths.

SECTION V.

NUMBER PROCESSES.

Computation.

56. Definition. Computation consists in obtaining a number or numbers from other numbers in the light of definite relations existing between or among them.

57. The Mental Acts Involved. (1.) The first act of the mind concerned in computation is an act of comparison. The numbers involved are compared in respect of one or more of the following points, viz.:

a. Concrete denomination.
b. Abstract denomination or order.
c. Equality.
d. Measurement.

(2.) The mental act which effects a computation is an act of synthesis or analysis as the conditions of a given case may require.

Remarks. 1. The mind thus performs but three operations upon numbers, viz.:

a. COMPARISON.
b. SYNTHESIS.
c. ANALYSIS.

The first of these is preliminary in its nature while the other two are the processes by means of which all numerical computation is effected.

2. *Synthesis*, as here used, means *putting together*, while *analysis* means *separating*, or *taking apart.*

58. The primary judgment in computation is a proposition of identity, *i. e.*, something *equals* something. This judgment is called an *equation.*

The values between which the relation of equality exists are called the *members* of the equation.

The Synthesis, or Combination of Numbers.

I—ADDITION.

59. Sum. The sum of two or more numbers is a number equal in value to them.

60. Addition. Finding the sum of numbers is called *addition.*

61. Addends. The numbers to be added are called *addends.*

62. The Mental Acts Involved. (1.) The mind compares two addends in respect of concrete denomination and order.

(2.) The mind begins with one of the addends and from its number of units counts until the units of the other addend are used. The number reached is the sum of the two addends.

Remarks. 1. An addition table is readily formed, the mastery of which enables the mind to give from memory the sum of any two addends within the limits of the table.

2. With the sum formed by the synthesis of two numbers, as above indicated, the mind may combine another number, and so may continue the act of combining one number with another so long as there are numbers to be added in a given case.

63. Principles. I. Only like numbers can be added together.

Remarks. 1. Like numbers are abstract numbers having like units or concrete numbers having like unit-objects. And since a collection of objects can be named from a common attribute only, so only like numbers can be named together or added.

2. If unlike numbers are to be added together a common name or denomination must be found for them. They are then thought together under that name.

II. The sum is of the same order or denomination as the addends.

III. The sum equals its addends.

IV. The sum exceeds any of its addends.

V. If the same numerical value be added to each of two equal values, the resulting sums are equal.

64. The Sign. A perpendicular cross (+) placed between two numbers indicates that they are to be added together. The sign is read *plus.*

General Remarks.

1. In adding numbers of any order in the decimal scale, a sum exceeding 9 is often found. Since such a sum cannot be represented in the place in the written scale which corresponds to the order of units composing the sum, a part or all of the sum must be reduced to units of one or more higher orders in the scale before it can be notated. Reduction ascending is involved.

2. The definitions, processes and principles stated above apply equally well whether the addends be thought in the decimal, the fractional or the compound system of numbers.

Exercises in Addition.

Remarks. For the exercises numbered in the left margin read across the page to the right.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(9)	34	67	47	27	17	16	65	$21\frac{1}{2}$
(10)	76	98	58	39	19	27	47	$18\frac{2}{3}$
(11)	26	46	74	66	28	94	86	$31\frac{3}{4}$
(12)	38	78	23	72	30	89	75	$48\frac{1}{4}$
(13)	46	96	17	87	48	67	49	$59\frac{1}{2}$
(14)	71	84	29	93	85	86	87	$74\frac{1}{2}$

	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
(23)	3.5	26½	4.6	5.1	3½	21½	4.7	21.3
(24)	4.6	47¼	3.7	6.7	5¼	46¾	8.5	46.7
(25)	7.4	38½	4.1	8.5	16	23¼	9.7	54.5
(26)	3.9	59¾	5.8	9.4	14	48½	6.5	68.7
(27)	6.2	78½	9.9	6.3	15½	56½	5.4	59.9
(28)	9.3	67¼	9.8	5.9	18¾	87¼	3.2	97.5

	(29)	(30)	(31)	(32)	(33)	(34)	(35)
(36)	412	567	2678	4678	567.83	56789	54347
(37)	316	898	4513	8764	387.65	98765	45678
(38)	578	596	4678	9876	123.45	13759	94352
(39)	963	347	3542	6789	543.21	24680	24536
(40)	447	598	2345	8765	234.56	46802	78579
(41)	567	678	5436	5678	654.32	20468	45678
(42)	324	345	7654	7654	345.67	57898	57890
(43)	568	234	4567	4567	765.43	85765	47875
(44)	718	432	8765	1234	456.78	45678	32567
(45)	678	561	5678	4321	876.54	57934	78579

II—MULTIPLICATION.

65. Product. The product of two numbers is a number that sustains the same relation to one of them that the other does to 1.

Remark. The term multiple is sometimes used for product.

66. Multiplication. Finding the product of two numbers is called multiplication.

67. Its Genesis. (1.) In addition it is not essential that the addends be compared in any respect except in that of denomination and order. They may, however, be compared in respect of equality. If the addends are equal their addition may be called *constant* addition.

(2.) If the sum of a constant addition be remembered so that it may be given when the number of equal addends and the value of each are known, the act is called *multiplication.*

68. The Mental Act. The act of synthesis involved is that of a constant addition previously performed, while the act called multiplication is but the recalling of the sum from memory.

69. Multiplicand. The multiplicand is usually defined as the number to be multiplied.

70. Multiplier. The multiplier is usually defined as the number by which the multiplicand is multiplied.

Remarks. 1. If the multiplier be an integer the multiplicand is one of the addends of a constant addition while the multiplier is the number of those addends.

2. If the multiplier be a fraction the process of multiplication consists in obtaining such part of the multiplicand as the multiplier is of 1.

71. Factors. The multiplicand and multiplier are called factors of the product.

A factor of a number is one of its makers by multiplication.

72. Principles.

I. The multiplicand may be abstract or concrete.

II. The product is of the same name and order as the multiplicand.

III. The multiplier is an abstract number.

IV. If both factors are abstract, they may be used interchangeably without affecting the value of the product.

V. If either or both factors be used in parts the sum of the partial products thus found equals the product of the factors as wholes.

VI. The product sustains the same relation to the multiplicand that the multiplier does to 1.

VII. If the multiplicr is 1 the product equals the multiplicand.

VIII. Multiplying either factor by a number multiplies the product by that number.

IX. If two equal values be respectively multiplied by the same number the resulting products are equal.

X. A number is multiplied by multiplying one of its factors.

Remark. The following principles are seen to be true in the light of definitions yet to be given.

a. A number expressed in the decimal system of notation is multiplied by any power of 10 by removing the decimal point as many places to the right as there are units represented by the index of the given power of 10.

b. Dividing any factor of a product by a number divides the product itself by that number.

c. Dividing both factors of a product by the same number divides the product by the square of that number.

d. If one of two factors be multiplied and the other be divided by the same number, their product is not changed.

73. The Sign of multiplication is an oblique cross (×). It is read *times* or *multiplied by*, and is placed between two factors whose product is required.

Remarks. 1. If both factors are abstract it is immaterial which reading be given to the sign; if, however, one of the factors be concrete, it is thereby made the multiplicand, [Prin III,] and if it precede the sign, the sign is read *multiplied by*, while if the multiplier precede the sign, the sign is read *times*.

2. If a numerical value be expressed within a parenthesis or other sign of aggregation and another number be written without the sign but not separated from it by any sign, the sign of multiplication is understood between them. $3(4+2)=3\times(4+2)=18$.

3. In effecting a multiplication a product exceeding 9 is often found. Since such a product cannot be represented in the place in the written scale which corresponds to the order of units composing the product, it becomes necessary to reduce to units of a higher order or orders so much of the product as is thus reducible without involving fractions. The reduction employed is reduction ascending.

[Exercises in multiplying numbers thought in the different systems.]

III—COMPOSITION.

74. The continued product of several factors is found by multiplying the product of two of them by a third, the product of the three by a fourth, and so on until all the factors are used.

75. A Composite Number. The product of two integral factors, each greater than 1, or the continued product of several such factors is a composite number.

76. Composition. The process of forming a composite number is called composition.

77. A Prime Number. A number that cannot be formed by composition is called a prime number.

78. Numbers are relatively prime if they have no common factor.

79. A *Common Multiple* of numbers is a product of which each of them is a factor.

80. The *Least Common Multiple* of numbers is the least product of which each of them is a factor.

81. A *Common Measure* of numbers is a factor that can be used in the formation of each of them.

82. The *Greatest Common Measure* of numbers is the greatest factor that can be used in the formation of each of them.

83. Principles.

I. Both prime and composite factors may be used in composition.

II. A multiple is the product of all its prime factors.

III. A common multiple of numbers has in its composition the product of all the prime factors of each of the given numbers.

IV. The least common multiple of numbers is the product of all the prime factors of each of the numbers, each factor being used in the composition the greatest number of times it occurs in the composition of any one of the numbers.

V. A factor of a number is a factor of any multiple of that number.

VI. A common factor of numbers is a factor of their sum.

VII. The greatest common factor of numbers is the product of all their common prime factors.

[Exercise in forming composite numbers, common multiples, etc.]

IV—INVOLUTION.

84. Power. A power of a number is the number itself or the product of the number by itself one or more times.

85. Involution. The process of forming a power is called *Involution.*

86. Root. One of the equal factors used to form a power is called a *root.*

87. Second Power. The product of two equal factors is called the second power, or square of either of them.

88. Second Root. Either of the two equal factors that compose a second power is called the second, or square root of the given power.

89. Higher powers are formed by the composition of a greater number of equal factors. Every such power is named by the ordinal of the number of equal factors used in the composition of the power.

90. A root is called the *third*, *fourth*, etc., if it be one of *three*, *four*, etc., equal factors that compose a power.

91. Any number is called both the first power and the first root of itself. A first power can enter into a synthesis with its equal and thus become a component of a higher power but, is itself not formed by involution.

92. The index of a power is a small symbol of number written at the right and above a given number and indicates that the number is one of as many equal factors as there are integral units expressed by the index. Thus $2^3=8$, $6^2=36$, etc.

93. Table of Squares from 1^2 to 25^2.

$1^2 = 1$	$13^2 = 169$
$2^2 = 4$	$14^2 = 196$
$3^2 = 9$	$15^2 = 225$
$4^2 = 16$	$16^2 = 256$
$5^2 = 25$	$17^2 = 289$
$6^2 = 36$	$18^2 = 324$
$7^2 = 49$	$19^2 = 361$
$8^2 = 64$	$20^2 = 400$
$9^2 = 81$	$21^2 = 441$
$10^2 = 100$	$22^2 = 484$
$11^2 = 121$	$23^2 = 529$
$12^2 = 144$	$24^2 = 576$

$25^2 = 625$

94. Table of Cubes from 1^3 to 9^3.

$1^3 = 1$	$5^3 = 125$
$2^3 = 8$	$6^3 = 216$
$3^3 = 27$	$7^3 = 343$
$4^3 = 64$	$8^3 = 512$

$9^3 = 729$.

95. Rules for Squaring Numbers.

I. *To Square a Number ending in* $\frac{1}{4}$.

1. Square the integer.
2. Add $\frac{1}{2}$ the integer.
3. Add $\frac{1}{16}$.

II. *To Square a Number ending in* $\frac{1}{2}$.

a 1. Square the integer.
2. Add the integer.
3. Add $\frac{1}{4}$.

b. Multiply the integer by the integer next greater and add $\frac{1}{4}$.

III. *To square a number ending in* $\frac{3}{4}$.

1. Square the integer.
2. Add $\frac{3}{2}$ of the integer.
3. Add $\frac{9}{16}$.

IV. *To square a number ending in* 5.

a { 1. Square the tens.
2. Add the tens.
3. Annex 25. }

b. Multiply the simple value of the tens by the number next greater and annex 25.

V. *To square a number between* 25 *and* 50.

1. Take 25 from the number.
2. Take the difference from 25.
3. Square the remainder.
4. Add the first difference as hundreds.

VI. *To square a number between* 50 *and* 75.

1. Take 50 from the number.
2. Add the difference to 25.
3. Call the result hundreds.
4. Add the square of the difference.

VII. *To square a number between* 75 *and* 100.

1. Take the number from 100.
2. Take the difference from the number.
3. Call the result hundreds.
4. Add the square of the first difference.

VIII. *To square a number ending in* 25.

1. Square the hundreds.
2. Add $\frac{1}{2}$ the hundreds.
3. Call the result ten-thousands.
4. Add 625.

IX. *To square a number ending in* 75.

1. Square the hundreds.
2. Add $\frac{3}{2}$ of the hundreds.
3. Call the results ten-thousands.
4. Add 5625.

X. *To multiply a number of two orders by* 11.

Think the sum of the terms of the multiplicand between them:

As—$34 \times 11 = 374$. $54 \times 11 = 594$.

[Exercises.]

96. Remarks on Synthesis.

1. Numerical synthesis classifies itself under four heads, viz, addition, multiplication, composition and involution.

There is, however, but a single method of synthesis, and that is *addition*. In multiplication, including composition and involution, a *sum* is remembered, this sum having been previously found by the synthesis called *addition*.

2. In addition two numbers are combined by one impulse of the mind without regard to the equality of the numbers.

3. In multiplication a given number of *equal* numbers may be thought as combined at once. The number of equal numbers is not limited to *two*, but may be any number whose sum, found by constant addition, can be given immediately from memory.

This remark holds only with an integral multiplier.

4. In composition a definite number of factors are used in continued multiplication.

5. In involution a definite number of equal factors are used in continued multiplication.

The Analysis, or Separation of Numbers.

Remark. Since a number may be obtained by adding together any two parts which form it, it follows that the number may again be resolved into its parts.

I—SUBTRACTION.

97. Difference. The difference between two numbers is a number which added to one of them will make a sum equal to the other.

Remark. In Arithmetic the term *difference* may be defined as—the numerical excess of one number over another.

98. Subtraction. Finding the difference between two numbers is called subtraction.

Remark. Subtraction may be defined as—taking a part of a number from the number.

99. The Minuend. The *sum* involved in subtraction is called the minuend.

Remark. The minuend is often defined as—the number to be diminished by the withdrawal of a part of it.

100. The Subtrahend. The known part of the minuend is called the subtrahend.

Remarks. 1. The subtrahend is usually defined as the number to be subtracted.

2. The term *remainder* is often used to designate the part of the minuend that is left after the withdrawal of the subtrahend. The number so designated is, however, the *difference* between the minuend and subtrahend.

101. *The Mental Acts.* 1. If the sum of two numbers and one of them be known, the other is found by the process called subtraction. The mental act consists in presenting the required part from memory, or it may consist in counting from the given part to the given sum.

2. If the difference between two separate numbers be required, the mind compares the two numbers in respect of concrete denomination and order. If the numbers are found to be similar, the mind proceeds to *withdraw* or *think away* from the greater, a part which equals the less. The number remaining is the excess of the greater over the less, and is, therefore, their difference.

102. The Sign. The sign of subtraction is a single dash, (—) placed after the minuend and before the subtrahend in an indicated subtraction.

103. Principles.

I. Only like numbers are used in subtraction.

II. The sum of the subtrahend and difference equals the minuend.

III. If the minuend and subtrahend be equally increased the difference between the sums thus obtained equals the difference between the minuend and subtrahend.

IV. If the same value be taken from two equal values the remainders are equal.

V. If either or both minuend and subtrahend be used in parts, the partial remainders combined equal the entire remainder.

Remarks. 1. Since the minuend is a sum, it is greater (in Arithmetic), than the subtrahend. It sometimes occurs, however, that there is a less number of units of some order in the minuend than of the same order in the subtrahend. In such case the minuend must be prepared before the subtraction can be effected. This preparation consists in reducing a unit of the order next higher in the minuend to units of the required order and combining them with the units of that order. If there be no units of the order next higher in the minuend, the work of reduction must begin at the first order up the scale in which numerical value is thought. A unit of that order is reduced to units of the next lower; one of the units resulting from this reduction is then reduced to units of the order next lower, and so on until the number of units of each order in the minuend equals or exceeds that of each order in the subtrahend. The subtraction is then readily effected. The reduction involved is *reduction descending.*

2. The reduction mentioned in remark 1, may be avoided by effecting the subtraction in the light of Principle III, adding 10 units of the deficient order in the minuend to the units of that order, and then compensating this addition by adding 1 unit of the next higher order to the subtrahend.

[Exercises in Subtraction.]

II—DIVISION.

104. Quotient. The quotient of one number by another is a number that sustains the same relation to the first number that 1 does to the second.

Remark. The quotient of one number by another is the factor which, used with the second number, will produce the first.

105. Division. Finding a quotient is called division.

Remark. In the light of remark under *Quotient,* division may be defined as finding one of two factors of a given product when the other factor is known.

106. Dividend. The number to be divided is called the dividend.

Remark. The dividend is the given product of which the divisor is the known factor.

107. Divisor. The term divisor is usually defined as the number by which the dividend is divided.

Remark. 1. The divisor is the factor given with the dividend to determine the quotient.

2. A particular problem may be—Given the dividend and quotient to find the divisor. This problem is solved by dividing the dividend by the quotient. The quotient of a preceding division thus becomes the divisor in the given problem.

108. Remainder. The term remainder is applied to a part of a given dividend that may remain undivided in any case.

The Genesis of Division.

109. (1.) If a given subtrahend be taken from a given minuend, and again be taken from the remainder, and again be taken from the second remainder, and so on until the given minuend is exhausted or gives a remainder less than the constant subtrahend, the several subtractions viewed together are called a constant subtraction.

(2.) If, when the minuend and constant subtrahend are known the mind gives from memory the number of subtractions necessary to exhaust the given minuend; or, if the minuend and the number of subtractions that can be made are known, and the mind gives from memory the constant subtrahend, the act is called *division.*

(3.) If, when a product and one of two factors that produce it are known, the mind gives from memory the other factor the act is called *division.*

110. The Mental Act.

(1.) The mental act which effects a division is the presentation (from memory) of the quotient when the dividend and divisor are known.

(2.) The act may be a memorized constant subtraction; or, it may be the recalling from memory of one of two factors when the other and their product are known.

Remark. In effecting the division of a decimal or a compound number, if the divisor be numerically greater than the number of units of the order or denomination to be divided, the latter number must be reduced to units of a lower order or denomination before the division can be effected without involving a fractional quotient. *Reduction descending* is involved.

III. Principles.

I. The product of the divisor and quotient equals the dividend.

II. If the dividend be divided in parts the sum of the several partial quotients obtained is the entire quotient.

III. If the dividend be divided by the factors of the divisor used in continued division, the final quotient is the quotient of the dividend by the entire divisor.

Remark. In applying this principle a remainder may occur upon dividing by one or more of the factors of the divisor. These partial remainders do not constitute the ultimate or true remainder.

The following is a method for determining the *true remainder.*

Example.—Divide 1377 by 294, using the prime factors of the divisor, and determine the true remainder.

Solution.—The prime factors of 294 are 2, 3, 7, 7.

		Partial Remainders	True Remainders.
2	1377		
3	688		1
7	229	1	2
7	32	5	30
	4	4	168
	True remainder		201

Explanation.

a. 1. The entire dividend divided by 2 gives a quotient of 688 and a remainder of 1.

2. 688, which is approximately $\frac{1}{2}$ the entire dividend, divided by 3 gives a quotient of 229 and a remainder of 1.

3. 229, which is approximately $\frac{1}{6}$ of the entire dividend, divided by 7 gives a quotient of 32 and a remainder of 5.

4. 32, which is approximately $\frac{1}{42}$ of the entire dividend, divided by 7 gives a quotient of 4 and a remainder of 4.

5. 4, the final quotient is approximately $\frac{1}{294}$ of the entire dividend, or the part required.

b. 1. If 1 remain upon dividing $\frac{1}{2}$ the dividend, 2 times 1, or 2, would remain upon dividing the entire dividend.

2. If 5 remain upon dividing $\frac{1}{6}$ of the dividend, 6 times 5, or 30 would remain upon dividing the entire dividend.

3. If 4 remain upon dividing $\frac{1}{42}$ of the dividend, 42 times 4, or 168, would remain upon dividing the entire dividend.

4. We thus find that upon dividing the entire dividend we should have remaining $1+2+30+168$, or 201, as the ultimate, or true remainder.

The accuracy of this result may be tested by dividing the entire dividend by the divisor as a whole.

IV. The quotient sustains the same relation to the dividend that 1 does to the divisor.

V. If the divisor is 1 the quotient equals the dividend.

VI. If a division require the number of times that one number is contained in another, the divisor and dividend are like numbers and the quotient is *abstract*.

VII. If a division require one of the equal parts of a number, the dividend and quotient are like numbers and the divisor is *abstract*.

VIII. If two equal numerical values be divided by the same number, the resulting quotients are equal.

112. General Principles of Division.

Remark. The following six principles are called *general* principles:

I. Multiplying the dividend by any number multiplies the quotient by that number.

II. Multiplying the divisor by any number divides the quotient by that number.

III. Multiplying dividend and divisor by the same number does not change the quotient.

IV. Dividing the dividend by any number divides the quotient by that number.

V. Dividing the divisor by any number multiplies the quotient by that number.

VI. Dividing dividend and divisor by the same number does not change the quotient.

[Exercises.]

III—DISPOSITION.

113. Definition. The analysis of a composite number into its factors is called *disposition*, or factoring.

Remarks. 1. Disposition is a phase of division viewed as the process of finding the factors that compose a multiple.

2. Disposition is the reverse of composition. In composition the factors are given to find their product, while in disposition the product is given to find its factors.

3. In disposition the factors are found by dividing the given multiple by any exact divisor of it. The quotient thus found

is divided by any exact divisor of itself. The second quotient is divided by any exact divisor of itself, etc., until the required factors are found. The several divisors used and the final quotient are the factors of the given multiple.

If the *prime* factors are required, the several divisors used and the final quotient must be prime numbers.

114. Principles.

I. If a number is divisible by two or more numbers in continued division, it is divisible by their product.

Remarks. 1. A number is said to be divisible by another if the quotient is an integer.

This is a limited meaning of the word *divisible.* In the light of the definitions and principles of division, any number is divisible by any other number.

2. In disposition a number is not considered as a factor of itself, nor is 1 considered as a factor of a number.

II. A common divisor of two numbers is a divisor of their difference.

III. If a number be divided by one of its prime factors or by the product of two or more of them, the quotient is the remaining prime factor or the product of the remaining prime factors of the number.

IV. If the product of two factors be divided by either of them the quotient is the other.

V. If the product of more than two factors be divided by one of them, the quotient is the product of the other factors of the number.

115. Divisibility of Numbers.

Remarks. 1. There is no general method devised whereby the factors of a multiple may be readily found ; nor is there any means whereby a number is known to be *composite.* Certain numbers, however, possess characteristic marks denoting that they are composite. A few of these will be discussed.

2. A number whose units' figure is 0, 2, 4, 5 or 8, is called an *even* number. All other numbers are called *odd* numbers.

I. An even number is divisible by 2.

Elucidation. The units' figure of every integer is one of the ten Arabic characters. If the number represented by any of these figures be multiplied by 2 the units' figure of the product is 0, 2, 4, 6 or 8.

II. If the sum represented by the digits of a number be divisible by 3, the number is a multiple of 3.

Elucidation.

$$4512=\begin{cases} 4000=4\times1000=4(999+1)=4\times999+4. \\ \ \ 500=5\times\ 100=5(\ 99+1)=5\times\ 99+5. \\ \ \ \ \ 10=1\times\ \ 10=1(\ \ 9+1)=1\times\ \ 9+1. \\ \ \ \ \ \ \ 2= \qquad\qquad\qquad\qquad\qquad\qquad 2. \end{cases}$$

Upon separating any number, as 4512, into parts as indicated above, it is observed that the last member of each of the continued equations is separated into two addends. The first of each of these parts is seen to be a multiple of 3. [Prin. V. page 33.] The other parts, together with the second member of the last equation, are represented by the several digits of the given number. If, therefore, the sum represented by these digits be divisible by 3, the given number is divisible by 3. [Prin. VI. page 33.]

III. A number is divisible by 4 if its two right hand figures are zeros or represent a multiple of 4. Why?

IV. A number is divisible by 5 if its units' figure is 0 or 5. Why?

V. A number is divisible by 6 if it be even and a multiple of 3. Why?

VI. A number is divisible by 7 if once its *units*+ 3 times its *tens* +2 times its *hundreds*+6 times its *thousands*+4 times its *ten-thousands*+5 times its *hundred-thousands*+the numbers represented by the succeeding figures multiplied, respectively, by the series of multipliers named above, be a multiple of 7.

VII. A number is divisible by 8 if its units', tens', and hundreds' figures are zeros or represent a multiple of 8. Why?

VIII. A number is divisible by 9 if the sum of the numbers represented by its digits be a multiple of 9.

Remark. This may be elucidated in a manner similar to that given for divisibility by 3.

IX. A number is divisible by 10 if its units' figure is 0. Why?

X. A number is divisible by 11, if the difference between the sum of the numbers represented by the digits in the odd places and the sum of the numbers represented by the digits in the even places is nothing or a multiple of 11.

XI. A number is disvisible by 12 if it be a multiple of 3 and 4. Why?

116. General Remarks.

1. A number is prime if it fail of division upon being tested by every prime number up to a divisor that gives a quotient less than the divisor. Why?

2. A table of prime numbers in a given series of natural numbers—as from 1 to 100, may be formed by checking off as *composite* every second number from 2, every third number from 3, every fifth number from 5, every seventh number from 7, every eleventh number from 11, etc., to the required limit. The numbers remaining are prime. The series of numbers with the composite numbers thus expunged, is called the seive of Eratosthenes.

3. In factoring a number the pupil should always test it by the conditions herein given, and not *guess* at its factors until the tests, as far as known, have been applied.

4. Pupils should learn the prime factors of every composite number from 4 to 100.

IV—EVOLUTION.

117. Definition. The analysis of a power into the equal factors which compose it is called *evolution*.

(1.) Each of the equal factors found by evolution is called a *root* of the power from which it is evolved.

(2.) A root is called the second, third, fourth, etc., according as it is one of two, three, four, etc., equal factors that compose a power.

Any number is called the first root of itself.

(3.) The index of a root is a fractional unit written at the right and a little above a written power and indicates by its denomination the root required.

(4.) The radical or root sign ($\sqrt{\ }$) is often used to indicate a root. If used alone before a number it indicates the second, or square root. If a root other than the second is required, the radical sign has placed above it the denominator of the fractional unit which denotes the required root. $8^{1/3}=\sqrt[3]{8}=2$.

(5) *Exponent.* The index of a power or of a root is often called an *exponent*.

(6.) If a number be affected by a fractional exponent other than a fractional unit, the ordinal of the numerator of the exponent is the power to which the number is to be involved, while the ordinal of the denominator is the root to be evolved from that power: or, the ordinal of the denominator of the exponent is the root to be evolved from the given number, while the ordinal of the numerator is the power to which that root is to be involved. Thus, $8^{2/3}$ indicates that the third root of the second power of 8 is required; or it indicates that the second power of the third root of 8 is required. In either case the result is 4.

[Exercises.]

118. Remarks on Analysis.

1. The analysis of numbers classifies itself under four phases, viz.: SUBTRACTION, DIVISION, DISPOSITION and EVOLUTION.

2. In subtraction a number is thought as separated into two parts without regard to the relative value of the parts.

3. In division a number is thought as separated into a definite number of equal parts.

4. In disposition the factors of a given composite number are found by continued division.

5. In evolution one of a definite number of equal factors which compose a power is found.

6. Any root is readily found by reversing the steps taken in forming the corresponding power.

SECTION VI.

APPLICATIONS.

Remarks. 1. In the preceding discussion all the processes and general principles involved in Arithmetic are presented.

The topics which remain are but applications of what has already been given. In some instances new terms will be used, but no new process and no new principles, except subordinate principles, will be found.

2. Great care should be given to both the "written form" and the "thought form" for the solution of examples under the various topics which follow. In many cases the thought form is so short and simple that the written form almost or fully expresses it. In such cases but one form is given—usually the written form.

3. The teacher should insist that the work be placed neatly upon the blackboard. It will be found good discipline to require all members of a class to apply the same form in the solution of similar problems. If pupils are weak, but a single form should be given for the same class of exercises. If strong, pupils may master several methods for doing the same work.

Measures and Multiples.

GREATEST COMMON DIVISOR.

119. Definition. The greatest common divisor of given numbers is the greatest number that is contained an integral number of times in each of them.

120. Principles.

I. The product of all the common prime factors of given numbers is their greatest common divisor.

II. A divisor of a number divides any multiple of it.

III. A common divisor of two numbers divides their difference.

IV. A common divisor of given numbers divides their sum.

121. Methods of finding g. c. d.

(1.) In the light of principle I, we find the prime factors of the given numbers and take the product of those factors that are common to all the numbers. This product is the greatest common divisor of the numbers.

(2.) The common factors of given numbers may be found by dividing them by a number that is seen to be a common factor of them. Divide the resulting quotients in the same manner, and so continue until quotients are obtained that are relatively prime.

The divisors used are the common factors sought and their product is the required greatest common divisor. [Prin. I.]

Example. Find the g. c. d. of 24, 30 and 42.

Written form.

2)24 ...30......42.
3)12....15 ...21.
4...... 5 ... 7.

$2 \times 3 = 6 =$ g. c. d.

(3.) The "division" method is effected by dividing the greater of two numbers by the less (comparing but two of the numbers at a time); and then dividing the divisor by the remainder, and so continuing to divide the last divisor by the last remainder until there is no remainder. The last divisor is the g. c. d. of the two numbers compared. This divisor is then compared with another of the given numbers (if there be more than two,) by division as above indicated, and so on until the numbers in any given case are all used. The last divisor thus found is the g. c. d. of the several given numbers.

Example. Find the g. c. d. of 260 and 716.

Written form.

260	2	716
196		520
64	3	196
64	16	192
		4

Thought form.

Since 260 is the greatest divisor of itself, if it divide 716, then is 260 the g. c. d. of itself and 716. Upon trial we find that 260 does not divide 716, but that the greatest multiple of 260 in 716 is 520. By prin. II we know that the g. c. d. will divide 520, and by prin. III, we know that it will divide 196, the difference between 716 and 520. 196 is the greatest divisor of itself. Now if it will divide 260, it will also divide 2 times 260+196, or 716, and hence will be the required g. c. d.

Upon trial we find that 196 does not divide 260, but by prin. III we know that the g. c. d. will divide the difference between 260 and 196, which is 64. 64 is the g. d. of itself. Now if it will divide 196 it will divide the sum of itself and 196, or 260, [Prin. IV.] and also 2 times 260+196, or 716, and hence will be the required g. c. d. 64 does not divide 196, but the greatest multiple of 64 in 196 is 192. By prin. II we know that the g. c. d will divide 192, and by prin. III we know that it will divide the difference between 196 and 192 which is 4. 4 is the greatest divisor of itself. Now if 4 divide 64 it will divide 3 times 64, or 192, and also the sum of 4 and 192 which is 196, and also 196+64, or 260; and also 2 times 260+196, or 716, and hence is the required g. c. d. 4 divides 64, hence 4 is the g. c. d. of 260 and 716.

Remark. The work may often be shortened by taking the multiple of the divisor that is nearest the value of the dividend, and finding the difference between that multiple and the dividend for a new divisor.

In the above example the multiple of 260 that is nearest the value of 716 is 780 and the difference between 780 and 716 is 64. We thus see that the g. c. d. of the given numbers may be found by two divisions instead of three as in the solution given.

[Exercises]

Find the g. c. d. of the following:

1.	35, 21, 9, 14.	9.	576 and 168.
2.	24, 16, 12, 20.	10.	121 " 495.
3.	63, 27, 36.	11.	21 " 77.
4.	20, 42, 54.	12.	260 " 416.
5.	15, 40, 55, 60, 75.	13.	125 " 500
6.	64, 72, 100, 96.	14.	294 " 472.
7.	75, 135, 400.	15.	108 " 146.
8.	72, 85, 132.	16.	1245 " 600.

Least Common Multiple.

122. Definition. The least common multiple of given numbers is the least product of which each of them is a factor.

123. Principle. The least common multiple of numbers is the product of all the prime factors of each of the numbers, each factor being used the greatest number of times that it occurs in any one of the numbers.

Example. Find the l. c. m. of 12, 15 and 25.

Written form.	*Thought form.*
$12=2\times2\times3.$ $15=3\times5.$ $25=5\times5.$ $2\times2\times3\times5\times5=300=$ l. c. m.	Since the required l. c. m. contains 12, it contains the prime factors of 12 which are 2, 2 and 3.

These we take as factors of the l. c. m. Since the l. c. m. contains 15, it contains the prime factors of 15 which are 3 and 5. We have 3 as a factor of the l. c. m. so we take 5 as one of its factors, and thus have the prime factors of 15 as factors of the l. c. m. Since the l. c. m. contains 25, it contains the prime factors of 25 which are 5 and 5. We have one 5 as a factor of the l. c. m., so we take another 5 as one of its factors and thus have the factors of 25 as factors of the l. c. m. We now have as factors of the required l. c. m. all the prime factors of 12, 15 and 25 and no other factor. The product of these factors=300, the required l. c. m.

Remark. If numbers are not readily factored, their l. c. m. may be found by either of the following methods The first comparison instituted in each method is between two of the given numbers. Next between the l. c. m. of the two numbers already compared and the third number. Next between the l. c. m. of the first three numbers and the fourth, and so on until all the numbers in any given case are used.

(1.) If one of the two numbers be divided by their g. c. d. the quotient contains those factors of the number divided that are not found in the other number. Now if the undivided number be multiplied by this quotient, the product contains all the prime factors of the two numbers and no other factor, and hence is their l. c. m.

A rule for this method may be formulated thus: *Divide one of two numbers by their g. c. d. and multiply the other number by the quotient. The product is the l. c. m. of the two given numbers.*

(2.) Since the l. c. m. of two numbers contains all the factors of one of the numbers and such factors of the other as are not found in the first, if the two numbers be multiplied together, the common multiple obtained is greater than their l. c. m. by a factor equal to their g. c. d

Hence the product of two numbers, divided by their g. c. d. equals their l. c. m.

[Exercises.]

Find the l. c. m.

1. 8, 14, 18.
2. 21. 16, 36.
3. 18, 36, 44.
4. 12, 28, 54.
5. 64, 84. 100.
6. 32, 75, 108.
7. 24, 32, 42, 50.
8. 15, 28, 40, 65.
9. 6, 8, 10, 12, 14.
10. 9, 14, 15, 16, 20.
11. 84, 76 90, 120.
12. 121, 200 324.

SECTION VII.

FRACTIONS.

124. A Fractional Unit. The idea *one* which is applicable to one of the equal parts into which a whole may be thought as separated, is called a fractional unit.

125. A Fraction. (1.) A fraction is a fractional unit or a number of like fractional units thought together.

(2.) A fraction is one or more of the equal parts of a unit.

Remarks. 1. The primary idea of a fraction is composed of the following elements, viz.:

a. The idea of a whole divided.

b. The idea of equality of the parts.

c. A number of the parts.

From this analysis of the primary idea it is apparent that the value of a fraction cannot exceed that of the unit which is applicable to the whole whence the fraction is derived.

The second definition is based upon the primary idea as thus analyzed.

2. Each of many like units may be thought as separated into the same number of equal parts and any number of these parts may be viewed together; hence the first definition.

3. The expression $\frac{10}{4}$ is not to be interpreted as representing ten fourths of 1, for *one* object can be thought into but *four* fourths. $\frac{10}{4}$ is to be viewed as expressing a synthesis of ten fractional units each of which is one-fourth of 1.

126. The Unit of a Fraction.

Definition. The unit, or whole which is thought as divided into equal parts is called the unit of the fraction.

127. The Notation of a Fraction. [See page 22.]

Classes of Fractions.

128. Based on the decimal or non-decimal division of the unit one, fractions are classified as *decimal* and *common.*

129. Decimal. A fraction whose fractional unit is a decimal part of the unit one, is called a decimal fraction.

Remarks. 1. A decimal fraction is usually expressed by the decimal notation, though it may be written in the fractional form.

2. A fraction expressed partly in the decimal and partly in the fractional notation is called a *complex decimal.* Examples, $.3\frac{1}{2}$; $.034\frac{3}{8}$.

130. Common. A fraction whose fractional unit is other than a decimal part of the unit one, is called a common fraction.

131. This classification of fractions is often based upon the method of notation used in expressing the fractions; those expressed in the decimal notation being called decimal fractions, and those expressed in the fractional form being called common fractions.

132. With 1 as a basis, or standard of comparison, fractions are classified as *proper* and *improper.*

133. Proper. A fraction whose value is less than 1 is called a *proper* fraction.

134. Improper. A fraction whose value equals or exceeds 1 is called an *improper* fraction.

Remarks. 1. The primary idea of a fraction is *a number of the equal parts of a unit.*

The classification of fractions as proper and improper is thus seen to be inconsistent with the primary idea of a fraction.

2 The definition of a fraction as *a number of like fractional units* is based upon a secondary idea, viz: A number of like parts of any number of like units. Under this definition any number of like fractional units is a fraction.

3. The two definitions of a fraction that we have given [Art. 125,] are fairly representative of the definitions found in the text books on Arithmetic. Under neither of these definitions is there ground for classifying fractions as proper and improper.

135. On the basis of form, fractions are classified as *simple*, *compound* and *complex*.

136. Simple. A simple fraction is defined as a fraction each of whose terms is a single integer.

137. Compound. A compound fraction is defined as a fraction of a fraction.

Remark. The simplification of a so-called compound fraction is effected in the light of principles which govern the multiplication of one fraction by another. $\frac{3}{4}$ of $\frac{2}{3}$ does not express a class of fractions, but simply indicates that $\frac{2}{3}$ is to be multiplied by $\frac{3}{4}$. [See Remark 2, under Art. 70.]

138. A complex fraction is defined as a fraction having a fraction in one or both of its terms.

Remarks. 1. A complex fraction is read as an expression of division: e. g. $\frac{2\frac{1}{2}}{3\frac{3}{4}}$ is read $2\frac{1}{2} \div 3\frac{3}{4}$ and $\frac{\frac{1}{3}}{\frac{5}{8}}$ is read $\frac{1}{3} \div \frac{5}{8}$.

In the light of the definition of denominator, the first of the above so-called complex fractions is derived from the division of a unit into $3\frac{3}{4}$ equal parts. The mind sees at once the impossibility of such a division. An examination of the second so-called complex fraction given, renders still more manifest the absurdity of calling these numerical values *fractions*.

2. Since a *simple* fraction is expressed by a form in distinction from *compound* and *complex*, it disappears upon the disappearance of these as *classes* of fractions.

3. If it is found convenient to use the terms proper, improper, simple, compound and complex to distinguish certain phases of fractional notation or indicated processes, it may be well to retain those terms; but they are certainly not necessary to name any of the essential thought elements of a fraction.

139. General Principles.

Remark. A fraction may be considered as a case of division. The numerator being the dividend, the denominator the divisor and the fraction itself the quotient. The general principles of division become, therefore, the general principles of fractions by a change of terminology.

I. If the numerator be multiplied the fraction is multiplied by the same number.

II. If the denominator be multiplied the fraction is divided by the same number.

III. If both terms of a fraction be multiplied by the same number greater than 1, the fraction is reduced to smaller fractional units but is not changed in value.

IV. If the numerator be divided the fraction is divided by the same number.

V. If the denominator be divided the fraction is multiplied by the same number.

VI. If both terms of a fraction be divided by the same number greater than 1, the fraction is reduced to larger fractional units, but is not changed in value.

VII. Fractions having a common denominator are to each other as their numerators.

Remark. This principle may be thus stated: Like parts of numbers are to each other as the numbers themselves. e. g. the relation of $\frac{2}{3}$ to $\frac{4}{3}$ is the same as that of 2 to 4; *i. e.* $\frac{1}{3}$ of 2 bears the same relation to $\frac{1}{3}$ of 4 that the whole of 2 bears to the whole of 4.

VIII. The numerator is as many times the value of the fraction as there are units in the denominator.

Remark. This principle is seen in the relation of a fraction to division. The numerator is dividend and hence is the product of the denominator (divisor) and the fraction (quotient.) A product is as many times either of the two factors which compose it as there are units in the other.

140. Reduction of Fractions.

Remarks. 1. For definitions and kinds of reduction, see page 24.

2. In the light of general principles, I, II, IV, V, pupils will readily multiply or divide a fraction by an integer.

The signs ∵ and ∴ are convenient to use in some of the written forms. The former is read "since" or "because," and the latter is read "hence" or "therefore."

141. Reduction Descending.

CASE I.

An integer or mixed number to a fraction.

Example. Reduce 3 to 8ths.

Written form	*Thought form.*
$3=\frac{3}{1}$. $\frac{3\times8}{1\times8}=\frac{24}{8}$. $\therefore 3=\frac{24}{8}$.	$3=\frac{3}{1}$; and by multiplying both terms of $\frac{3}{1}$ by 8 we have $\frac{24}{8}$. Hence $3=\frac{24}{8}$.

Remark. In reducing a mixed number to a fraction, the integer is reduced to the denomination of the fractional part by the above method and the fractional part is then added.

142. Other Forms.

Example. Reduce 3 to 8ths.

a $\begin{cases} \because 1=\frac{8}{8}, \\ \quad 3=3 \text{ times } \frac{8}{8}=\frac{24}{8}. \\ \therefore 3=\frac{24}{8}. \end{cases}$

b $\begin{cases} \because 1=8 \text{ times } \frac{1}{8}, \\ \quad 3=8 \quad \text{"} \quad \frac{3}{8}=\frac{24}{8}. \\ \therefore 3=\frac{24}{8}. \end{cases}$

c $\begin{cases} 1=\frac{8}{8}. \\ 3=\frac{24}{8}. \end{cases}$

Remarks on c. 1. Make $1=\frac{8}{8}$, and then multiply both members of the equation by 3. We thus find that $3=\frac{24}{8}$.

2. In any similar example make 1 the first member of the first equation; and for the second member take the equivalent of 1 in fractional units of the required denomination. Next multiply both members of the equation by the integer to be reduced.

Exercises. Perform each of the following reductions: 8 to 15ths; 19 to 4ths; 8 to 7ths; 6 to 3rds; 5 to halves. [Give additional exercises.]

Reduce each of the following to a fraction; $10\frac{1}{3}$; $14\frac{2}{7}$; $5\frac{1}{6}$; $3\frac{1}{2}$; $4\frac{1}{9}$; $7\frac{1}{4}$; $3\frac{2}{3}$; 2.5; 3.04; 5.2; 6.7.

143. *CASE II.*

A fraction to smaller fractional units.

Example. Reduce $\frac{2}{3}$ to 12ths.

Written form.

$$\frac{2\times4}{3\times4}=\frac{8}{12}$$

Thought form..

In the light of principle III, $\frac{2}{3}$ may be reduced to 12ths by multiplying both its terms by such a number as will produce 12 for the denominator of the resulting fraction.

Both terms of $\frac{2}{3}$ multiplied by $4=\frac{8}{12}$. $\therefore \frac{2}{3}=\frac{8}{12}$.

Example 2. Reduce $\frac{7}{8}$ to a decimal fraction.

Written form.

$$\frac{7\times125}{8\times125}=\frac{875}{1000}=.875.$$

Thought form.

In the light of Prin. III, $\frac{7}{8}$ is reduced to a decimal by multiplying both its terms by such a number as will produce a decimal denominator for the resulting fraction. Both terms of $\frac{7}{8}$ multiplied by $125=\frac{875}{1000}$. $\therefore \frac{7}{8}=.875$.

Remarks. 1. A decimal fraction may be transferred from the fractional to the decimal notation by omitting the written denominator and supplying the decimal point.

2. A decimal fraction may be transferred from the decimal to the fractional notation by writing the proper denominator under the written decimal and removing the decimal point from the written numerator.

3. The change of notation thus effected is not properly a reduction, for the size and number of fractional units in the fraction remain unchanged.

4. If a decimal fraction is to be reduced to smaller decimal units, the application of the above form is best shown by expressing the given decimal in the fractional notation.

144. Other Forms.

Example.—Reduce $\frac{2}{3}$ to 12ths.

$$a.\left\{\begin{array}{l} \because\ 1=\frac{12}{12}, \\ \quad \frac{1}{3}=\frac{1}{3} \text{ of } \frac{12}{12}=\frac{4}{12}. \\ \quad \frac{2}{3}=2 \text{ times } \frac{4}{12}=\frac{8}{12}. \\ \therefore\ \frac{2}{3}=\frac{8}{12}. \end{array}\right.$$

Written form

$$b.\left\{\begin{array}{ll} 1=\frac{12}{12}. & (1.) \\ \frac{1}{3}=\frac{4}{12}. & (2.) \\ \frac{2}{3}=\frac{8}{12}. & (3.) \end{array}\right.$$

Thought form.

For the first equation we take $1=\frac{12}{12}$. Equation (2) is found by dividing both members of equation (1) by 3. Equation (3) is obtained by multiplying both members of equation (2) by 2. We thus find that $\frac{2}{3}=\frac{8}{12}$.

Example 2. Reduce .2 to thousandths.

$$c.\left\{\begin{array}{l} \because 1=1.000, \\ \qquad .1=.1 \text{ of } 1.000=.100\,; \\ \text{and } .2=2 \text{ times } .100=.200. \\ \quad \therefore\ .2=.200. \end{array}\right.$$

[Exercises.]

Perform the following reductions, writing all the decimal fractions in the decimal notation:

$\frac{1}{5}$ to tenths; $\frac{7}{12}$ to 24ths; $\frac{8}{9}$ to 27ths; $\frac{3}{7}$ to 21sts; .5 to 100ths; .5 to 1000ths; $\frac{2}{5}$ to 15ths; $3\frac{1}{2}$ to 4ths; 3.2 to 100ths; $\frac{4}{5}$ to 20ths; $\frac{5}{4}$ to 100ths; $\frac{5}{6}$ to 18ths; 3.2 to 10000ths; $\frac{4}{9}$ to 54ths.

[Give additional exercises.]

145. *CASE III.*

Fractions to a common and to the least common denominator.

Remark. This case is one of reduction descending in which the reduction is effected as in Case II.

Definitions. *a.* Fractions have a common denominator if composed of like fractional units.

b. Fractions have their least common denominator if composed of the greatest possible like fractional units.

Remarks. 1. Before reducing fractions to equivalent fractions having the l. c. d. it is necessary that the terms of each fraction be relatively prime.

2. In case II, the denominator of the fraction resulting from the reduction is a multiple of the denominator of the given fraction. From the nature of the method employed in effecting the reduction, this must always be the case; hence a *common* denominator of fractions must be a common multiple of the denominators of the given fractions.

Principle. The least common denominator of given fractions is the l. c. m. of their denominators.

Example. Reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ to equivalent fractions having their least common denominator.

Thought form.

(1.) Prin. The l. c. d. of given fractions is the l. c. m. of their denominators.

(2.) The l. c. m. of 3, 4 and 6 is 12, hence each of these fractions must be expressed in 12ths.

(3.) In the light of general principle III, each of these fractions may be reduced to 12ths by multiplying both its terms by such a number as will produce 12 for the denominator of the resulting fraction.

(4.) Both terms of $\frac{2}{3}$ multiplied by $4=\frac{8}{12}$.
" " $\frac{3}{4}$ " " $3=\frac{9}{12}$.
" " $\frac{5}{6}$ " " $2=\frac{10}{12}$.

(5.) ∴ $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ reduced to equivalent fractions having their l. c. d. equal, respectively, $\frac{8}{12}$, $\frac{9}{12}$ and $\frac{10}{12}$.

Remark. Examples in this case may be solved by stating (1) and (2) as in the above thought form, and then reducing to the required denomination by any of the forms given under Case II.

[Exercises.]

Reduce to a common and l. c. d. the following:

$\frac{2}{5}$, $\frac{5}{6}$, $\frac{4}{5}$; $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$; $\frac{3}{4}$ $\frac{5}{7}$, $\frac{8}{9}$; $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$; $\frac{1}{3}$, $\frac{2}{9}$, $\frac{3}{7}$; $\frac{3}{4}$, $\frac{4}{7}$, .4, $\frac{1}{15}$; $\frac{4}{5}$, .5, $\frac{1}{4}$; .06, $\frac{1}{7}$, $\frac{1}{3}$, .05; $\frac{3}{8}$, $\frac{4}{7}$, $\frac{5}{6}$, $\frac{8}{9}$; $2\frac{1}{2}$, $\frac{3}{5}$, $\frac{3}{5}$, $4\frac{1}{3}$; $3\frac{1}{5}$, 2.5, $5\frac{1}{3}$.

146. Reduction Ascending.

CASE I.

A fraction to an integer or a mixed number; or to the decimal scale.

Example 1. $\frac{17}{5}$= what integer or mixed number?

Written form. | *Thought form.*

$$\frac{17 \div 5}{5 \div 5} = \frac{3\frac{2}{5}}{1} = 3\frac{2}{5}$$

Since a fraction is reduced to larger units by dividing both its terms by the same number, we may reduce $\frac{17}{5}$ to integral ones by dividing both its terms by 5. The result, $3\frac{2}{5} \div 1, = 3\frac{2}{5}$.

Remark. Dividing both terms of a fraction by its denominator reduces the fraction to the denomination of the unit 1.

Example 2. Reduce $\frac{32}{400}$ to a decimal fraction.

Written form. | *Thought form.*

$$\frac{32 \div 4}{400 \div 4} = \frac{8}{100} = .08.$$

$\frac{32}{400}$ is reduced to the denomination hundredths by dividing both its terms by 4; the result, $\frac{8}{100}$=.08, a decimal fraction.

Remark. The reduction of a common fraction to a decimal fraction may be effected by either reduction. The guiding thought is—to multiply or divide both its terms by such a number as will give a decimal denominator to the resulting fraction.

147. Other Forms.

Example 1. $\frac{17}{5}$= what integer or mixed number?

$$a. \begin{cases} \frac{17}{5}=17\div5. \\ 17\div5=3\frac{2}{5}. \\ \therefore\ \frac{17}{5}=3\frac{2}{5}. \end{cases}$$

$$b. \begin{cases} \because \frac{5}{5}=1, \\ \frac{17}{5}=\text{as many 1's as 5 is contained times in 17, or } 3\frac{2}{5}. \\ \therefore\ \frac{17}{5}=3\frac{2}{5}. \end{cases}$$

Example 2. Reduce $\frac{3}{4}$ to the decimal scale.

$$Form. \begin{cases} \frac{3}{4}=3\div4. \\ 3\div4=.75. \\ \therefore\ \frac{3}{4}=.75. \end{cases}$$

Remark. The denominator of a decimal fraction is a power of 10, the prime factors of which are 2 and 5. It follows, therefore, that if any fraction (in its lowest terms) have in its denominator a factor other than those of 10, such fraction cannot be reduced to a decimal. A common fraction is reducible to a decimal if the denominator of the given fraction is divisible by 2 or 5, or by 2 *and* 5 *and by no other prime number.* If a fraction, in its lowest terms, have in its denominator a prime factor other than those of a decimal denominator, a repetend, or circulate will result upon attempting to reduce the fraction to the decimal scale.

Exercises. Reduce to integers or mixed numbers the following:

$\frac{16}{5}$; $\frac{14}{6}$; $\frac{21}{3}$; $\frac{37}{5}$; $\frac{20}{4}$; $\frac{5}{2}$; $\frac{7}{3}$; $\frac{39}{13}$; $\frac{40}{7}$; $\frac{25}{7}$; $\frac{75}{4}$; $\frac{100}{3}$; $\frac{253}{7}$.

Reduce each of the following to the decimal scale or to a complex decimal.

$\frac{1}{2}$; $\frac{3}{4}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{1}{16}$; $\frac{3}{16}$; $\frac{5}{16}$; $\frac{7}{16}$; $\frac{9}{16}$; $\frac{11}{16}$; $\frac{13}{16}$; $\frac{15}{16}$; $\frac{3}{20}$; $\frac{7}{20}$; $\frac{9}{20}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{1}{7}$; $\frac{2}{7}$; $\frac{3}{7}$; $\frac{1}{12}$; $\frac{5}{12}$; $\frac{1}{13}$; $\frac{3}{13}$; $\frac{1}{15}$; $\frac{2}{15}$; $\frac{6}{16}$.

148. *CASE II.*

A fraction to larger units, or lower terms.

Example. Reduce $\frac{6}{9}$ to thirds.

Written form. *Thought form.*

$\frac{6 \div 3}{9 \div 3} = \frac{2}{3}$ | In the light of Prin. VI, $\frac{6}{9}$ may be reduced to 3rds by dividing both its terms by such a number as will give 3 for the denominator of the resulting fraction. Both terms of $\frac{6}{9}$ divided by 3, $= \frac{2}{3}$. $\therefore \frac{6}{9} = \frac{2}{3}$.

Remark. A fraction is reduced to its lowest terms by dividing both its terms by their g. c. d.

Exercises. Reduce each of the following to lower and lowest terms.

$\frac{16}{32}$; $\frac{144}{332}$; $\frac{105}{315}$; $\frac{1006}{4004}$; $\frac{325}{600}$; $\frac{117}{1746}$; $\frac{7}{21}$; $\frac{8}{56}$; $\frac{25}{40}$; $\frac{48}{60}$; $\frac{1246}{1684}$; $\frac{144}{1728}$.

149. General Remarks on Reduction of Fractions.

I. General principle III provides for the reduction of a fraction to smaller units; while general principle VI provides for the reduction of a fraction to larger units.

2. From the foregoing presentation of reduction of fractions we find that the several cases are readily classified under one or the other of the two kinds of reduction; and that each case involving reduction descending may be effected in the light of general principle III, while each case involving reduction ascending may be effected in the light of general principle VI.

150. Addition and Subtraction of Fractions.

Principle—Only like units can be added.

Principle—Only like units can be used in subtraction.

Remark. In adding or subtracting fractions it is to be observed that the numerators are the *numbers* with which we deal while the denominators only give name to those numbers.

Example. Add $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$.

Solution. Since only like units can be added, these fractions must be reduced to a common denominator before adding.

Written form.

$$\frac{2}{3}=\frac{8}{12},\ \frac{3}{4}=\frac{9}{12} \text{ and } \frac{5}{6}=\frac{10}{12}.$$

$$\frac{8}{12}+\frac{9}{12}+\frac{10}{12}=\frac{27}{12}=2\tfrac{1}{4}.$$

$$\therefore \tfrac{2}{3}+\tfrac{3}{4}+\tfrac{5}{6}=2\tfrac{1}{4}.$$

Remarks. 1. A form for subtracting one fraction from another is similar to that for addition.

2. If mixed numbers are to be added or subtracted, the integers and the fractions may be operated upon separately and the results combined, or, the mixed numbers may be reduced to fractions and the result found by the form given for examples in addition, and by a similar form for examples in subtraction.

3. It is not essential that fractions expressed in the decimal notation be reduced to the same fractional denomination before adding or subtracting. It is to be observed that only like orders of units can be added or subtracted.

Exercises.

(1.) Add $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$; $\frac{3}{5}, \frac{1}{8}, \frac{5}{12}$; $\frac{7}{8}, \frac{17}{20}, \frac{5}{16}, \frac{1}{4}$; $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{3}{7}$; $\frac{4}{11}, \frac{5}{9}$; $.2, 4, \frac{1}{2}, \frac{5}{6}$; $\frac{5}{16}, \frac{3}{100}, \frac{4}{25}$; $\frac{4}{7}, \frac{3}{10}, 2, 5, 5\frac{1}{6}$.

(2.) Perform the operation indicated in each of the following:

$\frac{3}{7}-\frac{1}{3}$; $\frac{7}{8}-\frac{2}{9}$; $\frac{3}{4}-\frac{1}{8}$; $\frac{7}{10}-\frac{2}{9}$; $\frac{3}{15}-\frac{3}{20}$; $3.4-1\frac{1}{5}$; $2\frac{1}{2}-\frac{6}{7}$; $2\frac{1}{16}-\frac{7}{22}$; $3.15-\frac{2}{25}$; $6\frac{1}{4}-\frac{1}{16}$; $25.04-21.002$; $364.006-162.1$; $32.5-1.0001$.

Multiplication of Fractions.

151. *CASE I.*

To multiply by an integer.

Example 1. Multiply $\frac{5}{16}$ by 3.

Thought form. Since a fraction is multiplied by multiplying its numerator, $\frac{5}{16}\times3=\frac{15}{16}$.

Example 2. Multiply $\frac{5}{16}$ by 2.

Thought form. Since a fraction is multiplied by dividing its denominator, $\frac{5}{16}\times2=\frac{5}{8}$.

Remark. If the product should not be in its lowest terms or if it be reducible to an integer or mixed number it should be reduced.

[Exercises.]

Multiplicands: $\frac{5}{16}$; $\frac{3}{5}$; $\frac{4}{7}$; $\frac{3}{4}$; $\frac{2}{15}$; $\frac{3}{20}$; $\frac{4}{15}$; $\frac{5}{17}$; $3\frac{1}{5}$; $\frac{9}{35}$; $4\frac{1}{2}$; $5\frac{1}{6}$; $15\frac{1}{3}$; $21\frac{3}{7}$.

Multipliers: 4, 5, 3, 2, 6, 7, 8, 10, 12, 15.

152. *CASE II.*

To multiply by a fraction.

Example 1. Multiply 6 by $\frac{2}{3}$

First form
- $6\times1=6$.
- $6\times\frac{1}{3}=\frac{1}{3}$ of $6=2$.
- $6\times\frac{2}{3}=2$ times $2=4$.
- $\therefore 6\times\frac{2}{3}=4$.

Sec'nd f'rm
- The multiplier $=\frac{1}{3}$ of 2.
- $6\times2=12$.
- $6\times\frac{1}{3}$ of $2=\frac{1}{3}$ of $12=4$.
- $\therefore 6\times\frac{2}{3}=4$.

Third form

Principle. The product sustains the same relation to the multiplicand that the multiplier does to 1.

In this example the multiplier is $\frac{2}{3}$ of 1; hence the product is $\frac{2}{3}$ of 6.

$\frac{1}{3}$ of $6=2$.

$\frac{2}{3}$ of $6=2$ times $2=4$.

$\therefore 6\times\frac{2}{3}=4$.

Fourth form

[Give principle and statement as in third form.]

$\frac{2}{3}$ of $6=\frac{1}{3}$ of 2 times 6, or 12.

$\frac{1}{3}$ of $12=4$.

$\therefore 6\times\frac{2}{3}=4$.

Fifth form

Written form.

(1.) $6=6$.

(2.) $\frac{2}{3}\times3=2$.

(3.) $6\times\frac{2}{3}\times3=6\times2$.

(4.) $6\times\frac{2}{3}=2\times2=4$.

$\therefore 6\times\frac{2}{3}=4$.

Thought form.

$6=6$.

Also the multiplier multiplied by its denominator equals its numerator.

Multiplying together the corresponding members of these equations gives equation (3).

Dividing both members of equation (3) by 3, gives equation (4).

The first member of equation (4) consists of the two factors whose product is required, while the second member is the result sought.

Example 2. Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

Remark. The forms of solution for this example do not differ from those given for example 1. Those forms are, however, applied to the solution of example 2.

First form

$\frac{3}{4}\times 1=\frac{3}{4}$.

$\frac{3}{4}\times\frac{1}{7}=\frac{1}{7}$ of $\frac{3}{4}=\frac{3}{28}$.

$\frac{3}{4}\times\frac{5}{7}=5$ times $\frac{3}{28}=\frac{15}{28}$.

$\therefore \frac{3}{4}\times\frac{5}{7}=\frac{15}{28}$.

Sec'nd f'rm

The multiplier is $\frac{1}{7}$ of 5.

$\frac{3}{4}\times 5=\frac{15}{4}$.

$\frac{3}{4}\times\frac{1}{7}$ of $5=\frac{1}{7}$ of $\frac{15}{4}=\frac{15}{28}$.

$\therefore \frac{3}{4}\times\frac{5}{7}=\frac{15}{28}$.

Third form

Principle. The product sustains the same relation to the multiplicand that the multiplier does to 1.

In this example the multiplier is $\frac{5}{7}$ of 1; hence the product is $\frac{5}{7}$ of $\frac{3}{4}$.

$\frac{1}{7}$ of $\frac{3}{4}=\frac{3}{28}$.

$\frac{5}{7}$ of $\frac{3}{4}=5$ times $\frac{3}{28}=\frac{15}{28}$.

$\therefore \frac{3}{4}\times\frac{5}{7}=\frac{15}{28}$.

Fo'rth form

[Give principle and statement as in third form.]

$\frac{5}{7}$ of $\frac{3}{4}=\frac{1}{7}$ of 5 times $\frac{3}{4}$, or $\frac{15}{4}$.

$\frac{1}{7}$ of $\frac{15}{4}=\frac{15}{28}$.

$\therefore \frac{3}{4}\times\frac{5}{7}=\frac{15}{28}$.

Fifth form

Written form.

(1.) $\frac{3}{4}\times4=3.$

(2.) $\frac{5}{7}\times7=5.$

(3.) $\frac{3}{4}\times\frac{5}{7}\times28=15.$

(4.) $\frac{3}{4}\times\frac{5}{7}=\frac{15}{28}.$

$\therefore$ etc.

Thought form.

The multiplicand multiplied by its denominator equals its numerator.

The multiplier multiplied by its denominator equals its numerator. Multiplying together the corresponding members of these equations gives equation (3); and dividing both members of equation (3) by 28 gives equation (4).

The first member of equation (4) consists of the two factors whose product is required, while the second member is the product sought.

153. General Remarks on Multiplication of Fractions.

1 Mixed numbers may be reduced to fractions before multiplying; or the multiplicand may be multiplied by the integral and the fractional parts of the multiplier (used separately) and the partial products combined.

2. If one or both factors be decimal fractions expressed in the decimal notation, any of the given forms may be used. The second form is, perhaps, the best to use in the multiplication of decimal fractions.

Example. Multiply 2.5 by .025.

Form. The multiplier equals .001 of 25.

$2.5\times25=62.5.$

$2.5\times.001$ of $25=.001$ of $62.5=.0625.$

$\therefore 2.5\times.025=.0625.$

3. The product of one fraction by another may be obtained by multiplying together the numerators of the factors for the numerator of the product, and the denominators of the factors for the denominator of the product. [See rule in any good text book]

Exercises.

Multiplicands. $\frac{3}{4}$; 8; $\frac{9}{16}$; $3\frac{1}{7}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{9}{10}$; $\frac{3}{5}$; $\frac{6}{7}$; 21; 16; 12; .05; .3; .025; 3.2; 21.004; $8\frac{1}{3}$; 4.07; $\frac{3}{8}$; .09; .5; .008; $3\frac{1}{2}$; $4\frac{1}{5}$; 6.7; 8.09; 6.3; 32.04; 2.004; 325.046; 4.0064; 31.75.

Multipliers. $\frac{6}{7}$; $\frac{3}{5}$; $\frac{3}{4}$; $\frac{4}{7}$; $\frac{5}{9}$; $\frac{3}{7}$; $\frac{2}{3}$; $\frac{7}{8}$; $\frac{2}{5}$; $\frac{5}{6}$ .5; .3; .7; .04; .005; .045; .008; $\frac{1}{3}$; 1.5; $\frac{3}{13}$; 2.3; 4.05; 6.25; 15.5; 7.5; .47; 31.056; .0045.

Division of Fractions.

154. *CASE I.*

To divide by an integer.

Example 1. Divide $\frac{6}{7}$ by 3.

Thought form. Since a fraction is divided by dividing its numerator, $\frac{6}{7} \div 3 = \frac{2}{7}$.

Example 2. Divide $\frac{6}{7}$ by 5.

Thought form. Since a fraction is divided by multiplying its denominator, $\frac{6}{7} \div 5 = \frac{6}{35}$.

Exercises.

Dividends. $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{6}{7}$; $\frac{7}{8}$; $\frac{3}{8}$; $\frac{8}{9}$; $\frac{10}{11}$; $\frac{12}{13}$; $\frac{15}{29}$; $\frac{13}{15}$; $\frac{21}{25}$; $\frac{18}{25}$; .2; .25; 3.25; 2.004; 36.78; 47.3; 17.08; 6.25; .0625; $3\frac{1}{2}$; $5\frac{2}{5}$; $24\frac{5}{6}$; $14\frac{1}{4}$; $16\frac{3}{4}$; $26\frac{7}{8}$; $32\frac{5}{8}$.

Divisors. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.

155. *CASE II.*

To divide by a fraction.

Example 1. Divide 5 by $\frac{3}{7}$.

First form.
$5\div1=5.$
$5\div\frac{1}{7}=7$ times $5=35.$
$5\div\frac{3}{7}=\frac{1}{3}$ of $35=11\frac{2}{3}.$
$\therefore 5\div\frac{3}{7}=11\frac{2}{3}.$

Sec'nd form
The divisor equals $\frac{1}{7}$ of 3.
$5\div3=\frac{5}{3}.$
$5\div\frac{1}{7}$ of $3=7$ times $\frac{5}{3}=\frac{35}{3}=11\frac{2}{3}.$
$\therefore 5\div\frac{3}{7}=11\frac{2}{3}$

Third form.
$1\div\frac{1}{7}=7.$
$1\div\frac{3}{7}=\frac{1}{3}$ of $7=\frac{7}{3}.$
$5\div\frac{3}{7}=5$ times $\frac{7}{3}=\frac{35}{3}=11\frac{2}{3}.$
$\therefore 5\div\frac{3}{7}=11\frac{2}{3}.$

Example 2. Divide $\frac{5}{7}$ by $\frac{3}{4}$.

Remark. For the first three forms see those given for the preceding example.

Fourth form.
$\frac{5}{7}=\frac{20}{28}.$
$\frac{3}{4}=\frac{21}{28}.$
$\frac{20}{28}\div\frac{21}{28}=20\div21=\frac{20}{21}.$

Fifth form.
One fraction may be divided by another by dividing the numerator of the dividend by the numerator of the divisor and the denominator of the dividend by the denominator of the divisor.

If the terms of the dividend be not respectively divisible by the corresponding terms of the divisor, they may be made so by multiplying both terms of the dividend by the product of the terms of the divisor.

Division of fractions is thus shown to be the reverse of multiplication of fractions.

156. General Remarks on Division of Fractions.

1. If the divisor is a mixed number, it is, perhaps, better to reduce it to a fraction before dividing.

2. If the divisor is written wholly in the decimal notation, the above forms are as applicable as though it were written in the fractional notation. The second form is preferred.

Example. Divide .0625 by 2.5.

Form. The divisor is .1 of 25.

.0625÷25=.0025.

.0625÷.1 of 25=10 times .0025=.025.

∴ .0625÷2.5=.025.

Exercises.

Dividends.—$\frac{3}{4}$; $\frac{8}{9}$; $\frac{2}{3}$; $\frac{4}{7}$; $\frac{3}{5}$; $\frac{5}{9}$; $\frac{5}{6}$; .5; .25; .025; 8.4; 3.2; 8.08; $10\frac{1}{4}$; $3\frac{1}{6}$; $5\frac{1}{2}$; $4\frac{3}{5}$; $6\frac{7}{8}$; 34.567; 325.568; 354.56; 1.2342; 56.67895.

Divisors.—$\frac{1}{5}$; $\frac{2}{3}$; $\frac{5}{7}$; $\frac{8}{15}$; $\frac{7}{9}$; $\frac{3}{5}$; $\frac{2}{7}$; $\frac{7}{12}$; $\frac{15}{16}$; $\frac{11}{12}$; $\frac{12}{17}$; $\frac{4}{9}$; .7; .5; .3; .07; .002; .626; 1.256; 3.4467; $2\frac{1}{2}$; $3\frac{1}{3}$; $4\frac{3}{4}$; $5\frac{5}{6}$; $7\frac{8}{9}$; $9\frac{7}{8}$.

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157. Exercises Involving Fractions.

1. 2 bu.=what part of 3 bu.?
2. $4=what part of $10?
3. 15 apples = what part of 50 apples?
4. $\frac{1}{2}$ acre = what part of 4 acres?
5. $\frac{2}{3}$ of a gallon = what part of 5 gallons?
6. $\frac{3}{5}$ of a bu.= what part of 10 bu?
7. $\frac{2}{3}$ of an orange = what part of $\frac{3}{4}$ of an orange?
8. $\frac{2}{3}$= what part of $\frac{3}{4}$?
9. $\frac{5}{6}$= what part of $\frac{7}{8}$?

10. $\frac{3}{7}$= what part of $\frac{5}{8}$?

11. $\frac{5}{11}$= what part of $\frac{5}{9}$?

12. $\frac{3}{8}$=what part of $\frac{5}{6}$?

13. How much water will fill 4 tubs if each tub holds $5\frac{1}{2}$ gallons?

14. What cost 9 apples at $1\frac{1}{3}$¢ each?

15. If 8lb. of sugar sell for $1, what is the price per pound?

16. At $6 per cord, required the cost of $\frac{3}{8}$ ot a cord of wood.

17. Required the cost of $\frac{5}{7}$ lb. of peaches at 35¢ per pound.

18. If coal is $3 per ton, required the cost of $4\frac{5}{6}$ tons.

19. If a train run 15 miles per hr., how far will it run in $3\frac{1}{3}$ hours?

20. At $$\frac{5}{6}$ per bushel, required the cost of $\frac{3}{8}$ of a bushel of corn.

21. What cost $\frac{2}{3}$ of a gallon of syrup, at $$1\frac{1}{5}$ per gallon?

22. If a man cut $\frac{3}{4}$ of a cord of wood in a day, how much can he cut in $\frac{2}{3}$ of a day?

23. A boy divided $\frac{7}{16}$ of a bushel of apples among 4 playmates; what part of a bushel did each receive?

24. A man had $\frac{2}{3}$ of a barrel of pork and sold $\frac{3}{4}$ of it; what part of a barrel remained?

25. A man lost $\frac{4}{5}$ of his money and found $\frac{1}{2}$ as much as he had after his loss; what part of his original sum had he then?

26. If 1 yard of ribbon cost $$\frac{3}{20}$, how many yards can be bought for $$\frac{3}{4}$?

27. How many yards of cloth will \$10 buy at \$$2\frac{1}{2}$ per yard?

28. If 2 men can do a piece of work in $4\frac{3}{4}$ days, how long will it take 8 men to do it?

29. What cost $3\frac{1}{3}$ boxes of oranges, if $2\frac{1}{2}$ boxes cost \$9?

30. What cost 30 bushels of corn, if $3\frac{1}{6}$ bushels cost \$1.20?

31. If a bushel of wheat cost \$$\frac{4}{5}$, what cost $\frac{5}{8}$ of a bushel?

42. If a bushel of wheat cost \$$\frac{4}{5}$, how many bushels can be bought for \$5.20?

33. A girl divided 10 apples among her companions, giving to each $\frac{2}{3}$ of an apple; how many companions had she?

34. $4=\frac{2}{3}$ of what number?

35. $\frac{2}{7}$ of $14=\frac{4}{9}$ of what number?

36. $\frac{8}{9}$ of $36=\frac{4}{5}$ of what number?

47. $\frac{3}{4}$ of \$$40=\frac{2}{5}$ of the cost of a horse; required its cost?

37. After spending $\frac{4}{7}$ of his money, John had \$42 remaining; how much had he at first?

39. If $\frac{5}{8}$ of an acre of land be worth \$15, what are 12 acres worth?

40. A boy sold lemons at the rate of 6 for 8 cents; how much did he receive for 3 lemons? For 8 lemons? For 12 lemons?

41. If $\frac{2}{3}$ of a yard of silk cost \$$3\frac{1}{3}$; what cost $4\frac{1}{2}$ yards?

42. If $\frac{2}{3}$ of a yard of cloth cost \$$\frac{4}{5}$, what cost $\frac{5}{8}$ of a yard?

43. A man gained \$15 by selling a watch for $1\frac{2}{7}$ times its cost; required its cost.

44. Mary, after losing $\frac{2}{3}$ of her flowers, had but 3 remaining; how many had she at first?

45. If to $\frac{1}{2}$ the cost of John's coat \$10 be added, the sum will be \$21; required its cost.

46. If to $\frac{5}{8}$ of William's age 8 years be added, the sum will be $1\frac{1}{4}$ times his age; how old is he?

47. Two men hire a wagon for \$9; A uses it 7 days, and B uses it 2 days; what should each pay?

48. John and James bought 22 apples for 11 cents; John paid 7 cents while James paid 4 cents; how many apples should each receive?

49. Anna has 5 pinks more than Ruth, and together they have 19; how many has each?

50. A boy said that 4 is 3 less than $\frac{1}{2}$ his number of marbles; how many has he?

51. 10 years are 6 years more than $\frac{2}{5}$ of John's age; how old is he?

52. If A and B do $\frac{3}{20}$ of a piece work in a day, how long will it take them to do the entire work?

53. George can plow a field in 8 days, and Henry can plow it in 12 days; how long would it take them to do the work, working together?

54. If $\frac{7}{9}$ of a barrel of flour cost $\$4\frac{2}{3}$, what cost $\frac{4}{5}$ of a barrel?

55. If A can do $\frac{2}{9}$ of a piece of work in a day, how much can he do in 2 days?

56. What cost 6 bushels of clover seed, if 2 bu. cost $\$12\frac{3}{4}$?

57. If .3 of a pound of coffee cost 9¢, what cost $\frac{4}{5}$ of a pound?

58. A has $13 which is $\frac{2}{3}$ of twice as much as B has; how much has B?

59. A horse is sold for $60 which is $\frac{3}{4}$ of $\frac{5}{6}$ of its value; required its value.

60. Henry and George bought 30 nuts; Henry paid 18 cents and George paid 12 cents; how should the nuts be divided?

61. A man failing in business can pay 40 cents on the dollar; what part of his debts can he pay?

62. $\frac{4}{5}=\frac{2}{3}$ of what number?

63. A has $3 more than B; and both together they have $$7\frac{1}{4}$; how many dollars has each?

64. $\frac{4}{5}=\frac{2}{3}$ of twice what number?

65. If $\frac{4}{9}$ of a box of berries cost $$\frac{7}{8}$, what cost $\frac{2}{3}$ of a box?

66. What is the number if its $\frac{1}{4}$ increased by 10 equals 21?

67. Required the number if its $\frac{1}{4}$ added to its $\frac{1}{2}$ equals $\frac{3}{4}$.

68. $\frac{2}{7}$ of a number $+5=26$; what is the number?

69. A farmer sold $\frac{4}{5}$ of his grain, and had 120 bushels remaining; how much had he at first?

70. Required the cost of 15 horses at the rate of $3\frac{1}{4}$ horses for $169.

71. How much will $3\frac{3}{4}$ acres of land cost at $64 for $1\frac{1}{4}$ acres?

72. The difference between $\frac{2}{3}$ of a number and $\frac{3}{4}$ of it is 6; what is the number?

73. If $2\frac{1}{2}$ times a number exceed 2 times the number by $3\frac{2}{3}$, what is the number?

74. If a man walk $\frac{3}{4}$ of a mile in $10\frac{2}{5}$ minutes, how long will it take him to walk 5 miles?

75. Required the cost of 4 dozen eggs at 25 cents for 10 eggs.

76. Required the cost of 45 apples at 10 cents per dozen.

77. $\$6=\frac{2}{3}$ of $\frac{1}{6}$ of a sum of money; required the sum.

78. How many yards of goods will make 3 dresses if 15 yards make $\frac{3}{5}$ of a dress?

79. $\frac{3}{4}$ of the length of a pole is in the water and $15\frac{1}{2}$ feet are out; what is the length of the pole?

80. If 19 boxes of berries are worth 57 cents, required the value of $\frac{2}{3}$ of a box.

81. If $\frac{5}{7}$ of a yd. of cloth cost $\$1\frac{5}{8}$, what cost $2\frac{2}{5}$ yd.?

82. $\frac{2}{3}$ of John's money equals $\frac{4}{5}$ of Harry's, and together they have \$55; how much has each?

83. If on 1 orange I lose $\frac{3}{10}$ of a cent, how many oranges must I sell to lose 6 cents?

84. William has twice as many cents as Herbert, and together they have 24; how many has each?

85. Two boys have 49 marbles; one has 7 more than the other; how many has each?

86. Henry received for his horse $\frac{7}{5}$ of its cost; what part of the cost was the gain?

87. A coat which cost \$12 was sold for \$16; the gain was how many hundredths of the cost?

88. Goods bought at \$12 were sold for \$10; how many hundredths of the cost was the loss?

89. A boy bought some apples for 72 cents and sold them for 84 cents; the gain was what part of the cost? How many hundredths of the cost?

90. A horse was bought for $60 and sold for $48; what part of the cost was the loss? How many hundredths of the cost was the loss?

91. $\frac{2}{3}$ of $6=how many hundredths of $20.

92. For what must goods costing $50 be sold to gain ten hundredths of the cost?

93. A paid $80 for a horse and sold it so as to lose $\frac{3}{100}$ of its cost; for what did he sell it?

94. If a merchant sold goods at $2 per yard and thereby gained $\frac{25}{100}$ of the cost, required the cost.

95. A cow was bought for $25 and sold for $30; what part of the cost was the gain?

96. Henry can make $\frac{2}{3}$ of a pair of boots in a day, and James can make $\frac{4}{5}$ of a pair in a day; how long will it take both to make 2 pairs of boots?

97. In how many days can 3 men cut 15 cords of wood, if 1 man in 1 day cut $\frac{2}{3}$ of a cord?

98. A boy bought a certain number of apples at 2 cents each, and the same number at 4 cents each, and then sold out at the rate of 3 for 5 cents; did he gain or lose and how much?

99. If $\frac{3}{7}$ of an orange cost as much as $\frac{1}{6}$ of a pineapple, required the price of two oranges in pineapples.

100. A man gained $10 by selling his horse for $1\frac{2}{5}$ times its cost; what was the cost?

101. A can do a piece of work in 5 days and B can do it in 3 days; in what time can they together do it?

102. M. bought $\frac{5}{8}$ of $15\frac{1}{4}$ yd. of cloth for $\frac{3}{7}$ of $\$24\frac{3}{8}$; required the price of the cloth per yd.

103. If 95 bushels of apples cost \$110; what is the value of $3\frac{1}{8}$ bushels.?

104. A lumber dealer bought siding at \$18.75 per M and sold it at \$2.875 per C; how much did he gain per M?

Aliquot Parts.

158. Table of Aliquots.

$2\frac{1}{2} = \frac{1}{4}$ of 10.	$18\frac{3}{4} = \frac{3}{16}$ of 100.
$3\frac{1}{3} = \frac{1}{3}$ of 10.	$20 = \frac{1}{5}$ of 100.
$6\frac{1}{4} = \frac{1}{16}$ of 100.	$25 = \frac{1}{4}$ of 100.
$8\frac{1}{3} = \frac{1}{12}$ of 100.	$33\frac{1}{3} = \frac{1}{3}$ of 100.
$12\frac{1}{2} = \frac{1}{8}$ of 100.	$62\frac{1}{2} = \frac{5}{8}$ of 100.
$16\frac{2}{3} = \frac{1}{6}$ of 100.	$66\frac{2}{3} = \frac{2}{3}$ of 100.
$125 = \frac{1}{8}$ of 1000.	

159. Forms of Solution.

Example 1. At $18\frac{3}{4}$ cents per lb. required the cost of 32 lb. of butter.

Solution. At \$1 per lb. 32 lb. of butter cost \$32, but at $18\frac{3}{4}$ cents, or $\$\frac{3}{16}$, 32 lb. cost $\frac{3}{16}$ of \$32, or \$6. $\therefore$ etc.

Example 2. At $12\frac{1}{2}$ cents per lb. how many lb. of rice will \$24 buy?

Solution. At $12\frac{1}{2}$ cents, or $\$\frac{1}{8}$ per lb., \$1 will buy 8 lb., and \$24 will buy 24 times 8 lb., or 192 lb. $\therefore$ etc.

Example 3. At $\$87\frac{1}{2}$ per acre, how many acres of land can be bought for \$4900?

Solution. At \$100 per acre, \$4900 will buy 49 acres, but $\$87\frac{1}{2}$, or $\frac{7}{8}$ of \$100, \$4900 will buy 8 times $\frac{1}{7}$ of 49 acres, or 56 acres.

160. *Exercises.*

1. At $6\frac{1}{4}$¢ each what cost 16 oranges?
2. At $6.25 per barrel what cost 7 barrels of flour?
3. At $2\frac{1}{2}$¢ apiece what cost 20 pencils?
4. At $2.50 a box what cost 60 boxes of potatoes?
5. At $8\frac{1}{3}$¢ per yd. what cost 56 yd. of muslin?
6. At $$8.33\frac{1}{3}$ each what cost 30 calves?
7. At $12\frac{1}{2}$¢ a lb. what cost 64 lb. of sugar?
8. $12.50 per acre what cost 17 acres of corn?
9. At $16\frac{2}{3}$¢ per lb. what cost 40 lb. of butter?
10. At $16\frac{2}{3}$¢ each what cost 4 slates?
11. At $18\frac{3}{4}$¢ per lb. what cost 32 lb. of steak?
12. At $18.75 each what cost 80 ponies?
13. If 1 doz. eggs cost 20¢ what cost 15 doz.?
14. At 20¢ each what cost 7 books.?
15. If 1 cow cost $25, required the cost of 16 cows.
16. At 25¢ each what cost 28 collars?
17. At $3\frac{1}{3}$¢ apiece what cost 9 apples?
18. At $3.50 each what cost 15 hats?
19. At $$33\frac{1}{3}$ per acre what cost 30 acres of land?
20. At 50¢ each what cost 7 books?
21. At $62\frac{1}{2}$¢ each what cost 17 pitchers?
22. At $$66\frac{2}{3}$ per head what cost 12 horses?
23. At $1.25 per rod what cost 10 rods of fencing?
24. At $125 per head what cost 14 horses.?
25. At $20 per acre what cost 15 acres of land?
26. At $37\frac{1}{2}$¢ per yd. what cost 7 yd. silk cord?
27. At $75 per head required the cost of 12 horses.
28. At $6\frac{1}{4}$¢ per spool required the cost of 11 spools of thread.

SECTION VIII.

COMPOUND NUMBERS.

161. Compound numbers are classified on the basis of the kind of attribute measured, as—

1. MEASURE OF DURATION.
2. MEASURES OF EXTENSION.
3. MEASURES OF FORCE.

162. *Diagram exhibiting in classified form the "measures" usually treated under compound numbers.*

Remarks. 1. The decimal measures are embraced in the diagram , but are treated separately.

2. The measure of value in most civilized countries is derived from the force of gravity, or weight. The primary units of value were weight units.

Measures.

- 1. *Of Duration.*—Time measure.
- 2. *Of Extension*
  - *Length*
    - Long measure.
    - Surveyors' long measure.
    - Mariners' " "
    - Decimal " "
    - Circumference "
  - *Surface*
    - Square measure.
    - Surveyor's square "
    - Decimal " "
  - *Volume*
    - Cubic measure.
    - Decimal cubic measure.
    - Dry measure.
    - Liquid measure.
    - Decimal capacity "
- 3. *Of Force.*
  - *Weight*
    - Troy Weight.
    - Apothecaries' weight.
    - Avoirdupois "
    - Decimal "
  - *Value*
    - United States money.
    - English "
    - Etc.

163. Order of Study.

Remark. The following order should be followed in the study of each measure.

(1.) The primary, or standard unit. How determined.

(2.) Other units and their relative value.

(3.) Scale and table.

(4.) Reduction.

a. Descending.

1. Integers to integers of lower denomination.
2. Fractions " " " " " "
3. " " fractions " " " "

b. Ascending.

1. Integers to integers of higher denomination.
2. " " fractions " " " "
3. Fractions " " " " " "

(5.) Synthesis.

a. Addition.

b. Multiplication.

(6.) Analysis.

a. Subtraction.

b. Division.

164. Tables.

For the tables of compound numbers and many interesting and useful facts the pupil is referred to text books on Arithmetic and to Encyclopedias.

Applications of Compound Numbers.

Remark. The "order of study" will be applied, in part, to "time measure." Each of the other measures should be treated in a similiar manner.

165. Time Measure.

Time. That which renders succession possible is called *time.*

The Primary Unit. The average solar day is taken as the primary unit of time.

(1.) A *Solar* day is the interval of time between two successive transits of the vertical rays of the sun across a given meridian. This interval varies at different times.

(2.) The sidereal day is the interval between two successive transits of a fixed star across a given meridian. This interval is the same at one time that it is at another.

(3.) The solar and the sidereal days would be of equal length if the earth did not revolve around the sun. While the earth is rotating upon its axis it is also moving forward in its orbit; so that when it has made a complete rotation, it must make part of another before the sun's rays are vertical a second time upon any given meridian. The solar day is thus a little longer than the sidereal day. [About four minutes.]

Other Denominations. The other time units are either multiples or divisors of the day.

The multiples of the day are the week, the month, the year and the century.

The divisors of the day are the hour, the minute and the second.

Relations.

Multiples.

1. The week equals 7 days.
2. The month equals $4\frac{2}{7}$ weeks.

Remark. The average calendar month is a little more than $4\frac{2}{7}$ weeks, or 30 days, while the lunar month is a small fraction more than 4 weeks.

3. The year equals 12 calendar months.
4. The century equals 100 years.

Divisors.

1. The hour equals $\frac{1}{24}$ of the day.
2. The minute equals $\frac{1}{60}$ of the hour.
3. The second equals $\frac{1}{60}$ of the minute.

Scale. The units used in time measure may be written in a scale as follows:

	100	12	$4\frac{2}{7}$	7	24	60	60
cen.	yr.	mo.	wk.	da.	hr.	min.	sec.
1	1	1	1	1	1	1	1

Remarks. 1. The values expressed by the respective units of this scale increase from right to left in the written scale. In this respect the time scale is like the decimal scale.

2. Since the rate of increase varies, the time scale is called a varying scale. In this respect it is unlike the decimal scale, whose rate of increase is uniformly 10.

3. The time scale consists of but eight units. In this respect it is unlike the decimal scale, whose orders of units may be repeated in periods indefinitely.

4. In the time scale the units extend both above and below the primary unit. In this respect it is like the decimal scale.

Table. For convenience the relations of the time units may be tabulated thus:

60	sec.	=	1 min.
60	min.	=	1 hr.
24	hr.	=	1 da.
7	da.	=	1 wk.
$4\frac{2}{7}$	wk.	=	1 mo.
12	mo.	=	1 yr.
100	yr.	=	1 cen.

166. Reduction Descending.

Example 1. Reduce 2 wk. 3 da. 12 hr. to min.

First form. ∵ 1 wk. = 7 da.,
2 " = 2 times 7 da. = 14 da.
14 da.+3 da. = 17 da.
∵ 1 da. = 24 hr.,
17 " = 17 times 24 hr. = 408 hr.
408 hr.+12 hr. = 420 hr.
∵ 1 " = 60 min.,
420 " = 420 times 60 min. = 25200 min.
∴ 2 wk. 3 da. 12 " = 25200 min.

Second form. ∵ 1 wk. = 7 times 1 da.,
2 " = 7 " 2 " = 14 da.
14 da.+3 da. = 17 da.
∵ 1 " = 24 times 1 hr.,
17 " = 24 " 17 " = 408 hr.
408 hr.+12 hr. = 420 hr.
∵ 1 " = 60 times 1 min.,
420 " = 60 " 420 " = 25200 min.
∴ 2 wk. 3 da. 12 " = 25200 min.

Example 2. Reduce $\frac{5}{9}$ wk. to smaller denominate units.

First form. ∵ 1 wk. = 7 da.,
$\frac{5}{9}$ " = $\frac{5}{9}$ of 7 da. = $3\frac{8}{9}$ da.
∵ 1 da. = 24 hr.,
$\frac{8}{9}$ " = $\frac{8}{9}$ of 24 hr. = $21\frac{1}{3}$ hr.
∵ 1 hr. = 60 min.,
$\frac{1}{3}$ " = $\frac{1}{3}$ of 60 min = 20 min.
∴ $\frac{5}{9}$ wk. = 3 da. 21 hr. 20 min.

Second form. $\because$ 1 wk. = 7 times 1 da.,
$\frac{5}{9}$ " = 7 times $\frac{5}{9}$ da. = $3\frac{8}{9}$ da.
$\because$ 1 da. = 24 " 1 hr.,
$\frac{8}{9}$ " = 24 " $\frac{8}{9}$ " = $21\frac{1}{3}$ hr.
$\because$ 1 hr. = 60 " 1 min.,
$\frac{1}{3}$ " = 60 " $\frac{1}{3}$ " = 20 min.
$\therefore$ $\frac{5}{9}$ wk. = 3 da. 21 hr. 20 min.

Example 3. Reduce $\frac{3}{700}$ of a wk. to the fraction of an hour.

First form. $\because$ 1 wk. = 7 da.,
$\frac{3}{700}$ " = $\frac{3}{700}$ of 7 da. = .03 da.
$\because$ 1 da. = 24 hr.,
.03 " = .03 of 24 hr. = $\frac{18}{25}$ hr.
$\therefore$ $\frac{3}{700}$ wk. = $\frac{18}{25}$ hr.

Second form. $\because$ 1 wk. = 7 times 1 da.,
$\frac{3}{700}$ " = 7 " $\frac{3}{700}$ " = .03 da.
$\because$ 1 da. = 24 " 1 hr.,
.03 " = 24 " .03 " = $\frac{18}{25}$ hr.
$\therefore$ $\frac{3}{700}$ wk. = $\frac{18}{25}$ hr.

(1.) Two forms of solution have been given for a problem in each case under reduction descending. The first form in each case is that usually given for such problems and needs no comment other than the observation that the multiplier is not taken from the table but is the number (taken abstractly) to be reduced.

(2.) The second form rests upon the following:

Principle.—*The numerical relation that exists between given units exists between like multiples and also between like parts of those units.*

[See remark under Prin. VII, page 58.]

The first step consists in the statement of the relation existing between a unit of the denomination to be reduced and a unit of the denomination to which the reduction is to be made. The second step is made in the light of the above principle. e. g. Since 1 hr. =60 times 1 min., 420 hr. (a multiple of 1 hr.) equal 60 times 420 min. (a like multiple of 1 min.) It is observed that the multiplier is, in every instance, taken from the table. This form of solution is uniform and general in its application to the solution of all problems in the several cases of reduction descending in all the measures.

(3.) A careful study of the second form given for the solution of problems in reduction *ascending*, will show the form to be uniform and of general application in all the measures.

(4.) Any reduction, either descending or ascending, may be made by a direct use of the equation. The first statement under the second form in each of the given examples is taken as the first equation. The second equation is obtained from the first by multiplying both its members by such a number as will give the number to be reduced for the first member of the second equation. In transforming the first equation it is to be observed that its second member consists of two factors, and that the member is to be multiplied by multiplying its second facfor. [Prin. X, page 31.]

167. Reduction Ascending.

Example 1. Reduce 25200 min. to integers of higher denomination.

First form. $\because$ 60 min. = 1 hr.,
25200 " = as many hr. as 25200 min. are times 60 min. which = 420.
$\because$ 24 hr. = 1 da.,
420 hr. = as many da. as 420 hr. are times 24 hr. which = 17, with a remainder of 12 hr.
$\because$ 7 da. = 1 wk.,
17 da. = as many wk. as 17 da. are times 7 da. which = 2, with a remainder of 3 da.
$\therefore$ 25200 min. = 2 wk. 3 da. 12 hr.

Second form. $\because$ 1 min. = $\frac{1}{60}$ of 1 hr.,
25200 " = $\frac{1}{60}$ of 25200 hr. = 420 hr.
$\because$ 1 hr. = $\frac{1}{24}$ of 1 da.,
420 " = $\frac{1}{24}$ of 420 da. = 17 da. 12 hr.
$\because$ 1 da. = $\frac{1}{7}$ of 1 wk.,
17 " = $\frac{1}{7}$ of 17 wk. = 2 wk. 3 da.
$\therefore$ 25200 min. = 2 wk. 3 da. 12 hr.

Example 2. Reduce 3 da. 21 hr. 20 min. to the fraction of a wk.

First form. $\because$ 60 min. = 1 hr.,
20 " = as many hr. as 20 min. are times 60 min. which = $\frac{1}{3}$.
21 hr. + $\frac{1}{3}$ hr. = $21\frac{1}{3}$ hr.
$\because$ 24 hr. = 1 da.,
$24\frac{1}{3}$ hr. = as many da. as $21\frac{1}{3}$ hr. are times 24 hr. which = $\frac{8}{9}$.
3 da. + $\frac{8}{9}$ da. = $3\frac{8}{9}$ da.
$\because$ 7 da. = 1 wk.,
$3\frac{8}{9}$ da. = as many wk. as $3\frac{8}{9}$ da. are times 7 da. which = $\frac{5}{9}$.
$\therefore$ 3 da. 21 hr. 20 min. = $\frac{5}{9}$ wk.

Remark. The foregoing form is the same as the first form used in solving Ex. 1. The following form is more in accordance with the facts involved.

$\because$ 60 min. = 1 hr.,

20 " = such part of 1 hr. as 20 is part of 60 which is $\frac{1}{3}$.

21 hr. + $\frac{1}{3}$ hr. = $21\frac{1}{3}$ hr.

$\because$ 24 hr. = 1 da.,

$21\frac{1}{3}$ hr. = such part of 1 da. as $21\frac{1}{3}$ is part of 24 which is $\frac{8}{9}$.

3 da. + $\frac{8}{9}$ da. = $3\frac{8}{9}$ da.

$\because$ 7 da. = 1 wk.,

$3\frac{8}{9}$ da. = such part of 1 wk. as $3\frac{8}{9}$ is part of 7 which is $\frac{5}{9}$. $\therefore$ etc.

Second form. $\because$ 1 min. = $\frac{1}{60}$ of 1 hr.,

20 " = $\frac{1}{60}$ of 20 hr. = $\frac{1}{3}$ hr.

21 hr. + $\frac{1}{3}$ hr. = $21\frac{1}{3}$ hr. = $\frac{64}{3}$ hr.

$\because$ 1 hr. = $\frac{1}{24}$ of 1 da.,

$\frac{64}{3}$ hr. = $\frac{1}{24}$ of $\frac{64}{3}$ da. = $\frac{8}{9}$ da.

3 da. + $\frac{8}{9}$ da. = $3\frac{8}{9}$ da. = $\frac{35}{9}$ da.

$\because$ 1 da. = $\frac{1}{7}$ of 1 wk.,

$\frac{35}{9}$ da. = $\frac{1}{7}$ of $\frac{35}{9}$ wk. = $\frac{5}{9}$ wk.

$\therefore$ 3 da. 21 hr. 20 min. = $\frac{5}{9}$ wk.

Remark. The several denominate numbers constituting the given compound number, may be reduced to the lowest denomination (min.); and this number of minutes may then be reduced to the fraction of a week. Mixed numbers are thus avoided.

Example 3. Reduce $\frac{18}{25}$ hr. to the fraction of a wk.

First form. [Use either the first form given under example 2 or that given in the remark under example 2.]

Second form. $\because$ 1 hr. $= \frac{1}{24}$ of 1 da.,
$\frac{18}{25}$ hr. $= \frac{1}{24}$ of $\frac{18}{25}$ da. $= .03$ da.
$\because$ 1 da. $= \frac{1}{7}$ of 1 wk.,
.03 da. $= \frac{1}{7}$ of .03 wk. $= \frac{3}{700}$ wk.
$\therefore \frac{18}{25}$ hr. $= \frac{3}{700}$ wk.

168. General Remarks.

1. Such examples as the following are often given.

2 da. 5 hr. 15 min. equal what part of 2 wk. 4. da. 3 hr.?

In such cases each compouud number should be reduced to the lowest denomination given in either.

Solution.

2 da. 5 hr. 15 min. = 3195 min.

2 wk. 4 da. 3 hr. = 26100 min.

$\because$ 1 min. $= \frac{1}{26100}$ of 26100 min.,

3195 min. = 3195 times $\frac{1}{26100}$ of 26100 min. $= \frac{213}{1740}$ of 26100 min.

$\therefore$ etc.

2. The following two solutions are given for the purpose of exhibiting a method that shall obviate the use of a complex fraction in stating the relation of the less unit to the greater in each example.

Example 1. Reduce 33 yd. to rd.

The first step (using the second form given in examples 1, 2 and 3 in Reduction Ascending,) is to state the relation between 1 yd. and 1 rd., thus:

$\because$ 1 yd. $= \frac{1}{5\frac{1}{2}}$ of 1 rd., etc.

Since the relation as stated cannot be read as a fraction, it is well to express the relation by $\frac{2}{11}$ which is the equivalent of $\frac{1}{5\frac{1}{2}}$.

The solution should be as follows:

Form. $\because$ 1 yd. $= \frac{2}{11}$ of 1 rd.,
33 " $= \frac{2}{11}$ of 33 rd. $= 6$ rd.
$\therefore$ 33 " $= 6$ rd.

Example 2. Reduce 132 ft. to rd.

Form.
$$\begin{cases} \because \quad 1 \text{ ft.} = \frac{2}{33} \text{ of } 1 \text{ rd.}, \\ \quad 132 \text{ `` } = \frac{2}{33} \text{ of } 132 \text{ rd.} = 8 \text{ rd.} \\ \therefore 132 \text{ `` } = 8 \text{ rd.} \end{cases}$$

3. Addition, multiplication, subtraction and division of compound numbers may be effected in the same manner and in obedience to the same principles that govern the synthesis and the analysis of numbers expressed in the decimal scale. No new principle is introduced and but one new fact, viz.:—a varying scale is employed instead of a uniform scale.

4. The subject of *time* is placed first in the classification of the measures because the primary units of extension and weight are derived from a time unit.

The primary unit of the common measures of extension is the yard, which is a definite portion of the length of a pendulum that beats *seconds* under certain conditions.

The primary unit of weight is the *pound* which is the force of gravity that acts upon a certain volume of water under certain conditions. The primary unit of extension is derived directly from a time unit (the second), and the primary unit of weight is derived directly from a measure of extension (a volume) and through that from the same time unit, (the second.)

The primary unit of value is a certain weight of silver or gold.

[NOTE—Circumference measure and the decimal (metric) measures are exceptions to the above remark.]

169. Exercises.

1. Reduce 5 wk. 3 da. 4 hr. to min.
2. Reduce $\frac{2}{3}$ da. $\frac{2}{15}$ hr. $\frac{7}{10}$ min. to sec.
3. Reduce 2 bu. 3 pk. 5 qt. to pt.
4. Reduce $\frac{3}{8}$ bu. $\frac{2}{5}$ pk. to qt.
5. Reduce 5 yd. 2 ft. 7 in. to in.
6. Reduce 2 mi. to fathoms.
7. Reduce 60 acres to sq. ft.
8. Reduce 3 hhd. 24 gal. 3 qt. to gills.
9. Reduce 4 lb. 15 pwt. to grains.
10. Reduce $\frac{5}{8}$ oz. $\frac{5}{6}$ pwt. to grains.
11. Reduce 7℥ 2℈ to grains.

12. Reduce $\frac{3}{4}$℥ $\frac{4}{5}$ʒ $\frac{2}{3}$℈ to grains.

13. Reduce 5 cwt. 14 lb. 7 oz. to drams.

14. Reduce $\frac{5}{8}$ ton $\frac{2}{5}$ cwt. $\frac{5}{9}$ lb. $\frac{11}{12}$ oz. to oz.

15. Reduce 7 wk. 3 da. 1 hr. 15 min. to seconds.

16. Reduce 3 gal. 2 qt. 1 gi. to gills.

17. Reduce 3 yd. 2 ft. 5 in. to inches.

18. Reduce 5 bu. 2 pk. 7 qt. to pints.

19. Reduce $\frac{4}{5}$ lb. Troy, to integers of lower denominations.

20. Reduce .3 da. to integers of lower denominations.

21. Reduce $\frac{5}{8}$ yd. to integers of lower denominations.

22. Reduce .875 gal. to integers of lower denominations.

23. Reduce $\frac{1}{1635}$ da. to the denomination minutes.

24. Reduce .007 gal. to the denomination pints.

25. Reduce $\frac{1}{47}$ yd. to the denomination inches.

26. Reduce $\frac{3}{95}$ bu. to the denomination pints.

27. Reduce 54960 min. to integers of higher denominations.

28. Reduce 186 pt. to a compound number of higher denominations.

29. Reduce 57648 sec. to a compound number of higher denominations.

30. Reduce $35\frac{1}{2}$ qt. to a compound number composed of pk. and bu.

31. Reduce 211 inches to a compound number composed of in. ft. and yd.

32. Reduce 23400 grains to Troy integers of higher denominations.

33. Reduce 600 gr. to a compound number composed of pwt. and ounces.

34. Reduce 8211 oz. to a compound number composed of lb. and cwt.

35. Add together 3 bu. 3 pk. 6 qt. 1 pt.; 5 bu. 2 pk. 5 qt. 1 pt.; 6 bu. 1 pk. 7 qt. 1 pt.; 1 bu. 3 pk. 2 qt.

36. Add 34 cwt. 17 lb. 11 oz. 13 dr.; 19 cwt. 46 lb. 7 oz. 4 dr.; 71 cwt. 10 lb. 15 oz. 12 dr.

37. Add 5 mi. 6 fur. 21 rd. 4 yd. 2 ft. 9 in.; 9 mi. 7 fur. 17 rd. 5 yd. 1 ft. 11 in.; 14 mi. 21 fur. 14 rd. 2 yd. 2 ft. 4 in.

38. From 24 sq. yd. 7 sq. ft. 110 sq. in. take 16 sq. yd. 6 sq. ft. 136 sq. in.

39. From 63 bu. 2 pk. 5 qt. take 54 bu. 3 pk. 6 qt.

40. From a hhd. of molasses $\frac{3}{4}$ leaked out. How much remained?

41. What is the time from March 16, 1884 until July 4, 1884?

42. How long has it been since February 4, 1884?

43. A note was given Dec. 18, 1883 and was paid March 28, 1884; how long did it run?

44. Multiply 5 lb. 6 oz. 3 pwt. 11 gr. by 6.

45. Multiply 4 bu. 3 pk. 7 qt. 1 pt. by 5.

46. Multiply 4 rd. 3 yd. 2 ft. 7 in. by 4.

47. Divide 5 da. 13 hr. 44 min. 18 sec. by 6.

48. Divide 6 cwt. 44 lb. 12 oz. 10 dr. by 4.

49, Divide 3 bu. 2 pk. 5 qt. by 2 bu. 6 qt. 1 pt.

50. Divide 6 bl. 21 gal. 5 qt. by 5 bl. 12 gal. 7 qt.

51. Reduce $\frac{2}{3}$ of a ft. to the fraction of a rd.

52. Reduce .05 of a mi. to lower denominations.

53. $\frac{7}{8}$ of a dr. to the fraction of a lb.

SECTION IX.

TIME AND LONGITUDE.

Distance measured on a parallel of latitude is called longitude.

Remarks. 1. Since longitude is measured on the arc of a circle it is estimated in units of the circumference, viz.: degrees, minutes and seconds.

2. Unless otherwise designated, the meridian of Greenwich is taken as the prime from which longitude is reckoned.

170. Because of the daily rotation of the earth on its axis, any place on the surface of the earth except the poles, moves—

In 24 hours	through	360°	of	space.
" 1 hour	"	15°	"	
" 1 min.	"	15′	"	
" 1 sec.	"	15″	"	

Hence if 15 units of longitude lie between two places, the time registered at the places, respectively, differs by 1 time unit; hours corresponding to degrees, minutes to minutes, and seconds to seconds.

On the other hand, if the time of two places is found to differ by 1 time unit, the places are distant from each other (east and west) 15 corresponding longitude units.

171. All places on the earth's surface have the same absolute time, but not the same relative time.

When the vertical rays of the sun reach the meridian of a place it is noon at all places on that meridian; while it is afternoon, or later in the day, at all places east, and before noon, or earlier in the day, at all places west of the given meridian. Thus a place east of a given meridian has later relative time and a place west has earlier relative time than that on the given meridian.

172. Standard Time.

For the purpose of estimating time for the railway service, the country of the United States is divided into belts or strips marked by meridians of longitude 15° apart. The 75th meridian from Greenwich marks the middle of the eastern belt. The 90th meridian marks the middle of the central belt. The 105th meridian marks the middle of the mountain belt. The 120th meridian marks the middle of the Pacific belt.

The local time on each of these meridians is taken as the "Standard" time for railway purposes at all places within the given belt; and since these meridians are 15° apart, the time in each belt is registered one hour earlier than in the belt next at the east. e g. When it is noon on the 75th meridian, it is noon at all places within the eastern belt, 11 a. m. on the 90th meridian and at all places within the central belt, 10 a. m. on the 105th meridian and at all places within the mountain belt and 9 a. m. on the 120th meridian and at all places within the Pacific belt.

Remark. In each belt near its prime meridian workshops, manufactories, public schools and many other branches of business are now generally carried on by "Standard" instead of local time.

173. Forms of Solution.

Example 1. The time between two places is 6 hr. 12 min. 10 sec.; required their difference in longitude.

Form.

∵ 1 hr. 1 min. 1 sec. correspond to 15 times 1° 1′ 1″, 6 hr. 12 min. 10 sec. correspond to 15 times 6° 12′ 10″ = 93° 2′ 30″.

∴ their difference in longitude is 93° 2′ 30″.

Example 2. The difference in longitude between New York and Greenwich is 74° 3″; required their difference in time.

Form.

∵ 1° 1′ 1″ correspond to $\frac{1}{15}$ of 1 hr. 1 min. 1 sec.,
74° 3″ " " $\frac{1}{15}$ " 74 " 3 sec. = 4 hr. 56 min. .2 sec. ∴ etc.

Example 3. When it is noon at New York, what is Greenwich time?

Remark. Their time difference is given above

Form. Since their time difference is 4 hr. 56 min. .2 sec., and since Greenwich is east of New York, its time is later than that of New York by 4 hr. 56 min .2 sec., or 56 min. .2 sec. past 4 p. m.

Example 4. When it is noon at Greenwich what is New York time?

Remark. Their time difference is given above.

Form. Since their time difference is 4 hr. 56 min. .2 sec., and since New York is west, its time is earlier by 4 hr. 56 min. .2 sec., or 3 min. 59.8 sec. past 7 a. m.

Exercises.

1. Two places differ in time 6 hr. 7 min. 10 sec.; required their difference in longitude.

2. Two places are distant from each other 1 quadrant; what is their time difference? When it is 10 a. m. at the place the farther west what time is it at the other? When it is 1 o'clock p. m. at the place the farther east what time is it at the other?

3. A man travels westward 5°17′11″; is his watch too fast or too slow and how much?

4. A man travels until his watch is 19 min. 55 sec. too fast; has he traveled east or west, and how far?

5. A man travels until his watch is too slow by 11 hr. 5 min. 16 sec.; has he traveled east or west, and how far?

6. The longitude of St. Louis is 90°10′, while that of Cincinnati is 84°26′; required their difference in time. When it is noon at either place what is the time at the other?

7. The longitude of Bangor Me., is 68°45′, and that of San Francisco is 122°25′; required their time difference. When it is 7 a. m. at either place what time is it at the other?

8. A vessel sailed due north at the rate of 14 knots per hour, while the sun apparently moved through 1 sextant 5°18′; how long did she sail, and how many statute miles?

9. Washington is in longitude 77°2′48″ west and Cincinnati is in longitude 84°26′ west; when it is 6 a. m. at either place what time is it at the other?

10. If my watch keeps Terre Haute time and indicates 17 min. 10 sec. past 1 o'clock p. m. when it is noon by local time how far and in which direction am I from Terre Haute?

11. A man travels from Pittsburg in longitude 80° 2′ west, until his watch is 1 hour and 45 minutes too fast; how far and in which direction has he traveled?

12. The longitude of Galveston is 94°50′ west while Constantinople is in longitude 28°49′ east; when it is noon at either place what time is it at the other?

13. A ship's chronometer, set at Greenwich, indicates 5 hr. 45 min. 24 sec. p. m. when the sun is on the meridian; required the longitude of the vessel.

14. A degree of longitude in the latitude of Boston is 44½ geographic miles. How many more statute miles in 7° of longitude at the equator than in the latitude of Boston ?

15. At 3 o'clock and 35 min. a. m. in London, what is the time at Boston ?

16. At 5 o'clock p. m. in Rome, what time is it in Terre Haute ?

17. At noon in Pekin what time is it in San Francisco ?

18. How many degrees of east or west longitude may a place have ? Why ?

19. What is the difference between the local and "standard" time of Terre Haute ?

20. When it is 10 o'clock at Indianapolis, what is "standard" time at the same place ?

21. When it is 4 p. m., local time, at Omaha, what is 'standard" time at New York City ?

22. When it is 5 hr. 15 min. 10 sec. a. m., "standard" time at San Francisco, what is local time at Pittsburg ?

23. At 6 hr. 30 min., a. m. local time, Jacksonville, Fla., what is the "standard" time at San Antonio, Texas ?

24. When it is noon at Cincinnati, local time, is it before or afternoon and how much by "standard" time at Cleveland, Toledo, Detroit, Ft. Wayne, Mobile, Richmond ?

SECTION X.

AREAS AND VOLUMES.

174. Areas.

Example 1. A room is 12 ft. long and 8 ft. wide; how many sq. ft. are in the floor?

Solution.

A surface 1 ft. l. and 1 ft. w. = 1 sq. ft.
A surface 12 ft. l. and 1 ft. w. = 12 times 1 sq. ft. = 12 sq. ft.
A surface 12 ft. l. and 8 ft. w. = 8 times 12 sq. ft. = 96 sq. ft.

∴ etc.

Remark. Length, width and area are always expressed in concrete units, hence length cannot be multiplied by width; even if such multiplication were possible, the product would not be *area*, which is different in name from the multiplicand.

If, however, the *numbers* representing the dimensions of a parallelogram be multiplied together, the product is the *number* representing the square units in the given surface.

Example 2. A lot contains 192 sq. rd. and is 16 rd. long; how wide is it?

Solution. A surface 1 rd. l. and 1 rd. wide = 1 sq. rd.
A surface 16 rd. l. and 1 rd. wide = 16 times 1 sq. rd. = 16 sq. rd.
A surface 16 rd. l. must be as many rd. wide to contain 192 sq. rd. as 192 sq. rd. are times 16 sq. rd. which = 12.
∴ The lot is 12 rd. wide.

Remark. If the *number* of square units in a parallelogram be divided by the *number* of corresponding linear units in either dimension, the quotient is the *number* of linear units in the other dimension.

Example 3. The base of a plane triangle is six inches and its altitude is 4 inches; what is its area?

(1.) The area of a plane triangle is one-half the area of a parallelogram having the same base and altitude as the triangle; hence find the area of such parallelogram as in Ex. 1, and divide it by 2.

(2.) If the *number* of linear units in the base or altitude of a plane triangle be multiplied by one-half the *number* of like linear units in the other dimension, the product is the *number* of corresponding square units in the triangle.

Example 4. The diameter of a circle is 14 inches; what is its area?

(1.) The circumference of a circle is nearly $3\frac{1}{7}$ times the diameter.

(2.) Any circle is practically equal to a rectangle whose length is the semi-circumference and whose width is the radius of the given circle.

Solution. $3\frac{1}{7}$ times 14 inches = 44 in. = the approximate circumference of the given circle.

Its equivalent rectangle is, therefore 22 in. by 7 in. 7 times 22 = 154. ∴ the approximate area of the given circle is 154 sq. in.

(3.) The area of any circle is .7854 of the area of its circumscribed square.

Each side of the square that circumscribes the given circle is 14 inches, the diameter of the circle. The area of the square is found by multiplying 14 by 14 and giving the denomination *square inches* to the product. .7854 of this product equals 154+ sq. in., the area of the given circle.

The rule is usually formulated thus:—*To find the area of a circle: Multiply the square of the diameter by* .7854.

175. Volumes.

Example 1. A common brick is 8 in. long, 4 in. wide and 2 in. thick; how many cu. in. does it contain?

Solution.

A solid 1 in. l. 1 in. w. 1 in. th. = 1 cu. in.

A solid 8 in. l. 1 in. w. 1 in. th. = 8 times 1 cu. in. = 8 cu. in.

A solid 8 in. l. 4 in. w. 1 in. th. = 4 times 8 cu. in. = 32 cu. in.

A solid 8 in. l. 4 in. w. 2 in. th. = 2 times 32 cu. in. = 64 cu. in.

∴ a brick contains 64 cu. in.

Remark. If the *numbers* representing the three dimensions of a rectangular solid be multiplied together, the product is the *number* representing the corresponding volume units in the given solid.

Example 2. A bin containing 315 cu. ft. is 9 ft. long and 5 ft. wide, how deep is it?

Solution.

A solid 1 ft. l. 1 ft. w. 1 ft. d. = 1 cu. ft.

A solid 9 ft. l. 1 ft. w. 1 ft. d. = 9 times 1 cu. ft. = 9 cu. ft.

A solid 9 ft. l. 5 ft. w. 1 ft. d. = 5 times 9 cu ft. = 45 cu. ft.

A solid 9 ft. l. 5 ft. w. must be as many ft. deep to contain 315 cu. ft. as 315 cu. ft. are times 45 cu. ft. which equal 7. ∴ the bin is 7 ft. deep.

Remark. If the *number* representing the volume of a rectangular solid be divided by the product of the *numbers* representing two of its dimensions, the quotient is the *number* representing its third dimension.

Exercises.

1. How many square feet in a surface 12 ft. by 5 ft.?

2. How many sq. yd. in a lot 15 yd. by 11 yd.?

3. A field is 40 rods long and 18 rods wide; how many acres in it?

4. A room is 15 ft. 6 in. long by 24 ft. 4 in, wide; how many square yards in the ceiling?

5. A floor contains 192 sq. yd, and is 12 yards wide; how long is it?

6. A field contains 18 acres and is 60 rods long; how wide is it?

7. A brick walk is $\frac{3}{4}$ yd. wide; how long must it be to contain 25 square yards?

8. A triangular wall is 6 ft. long and $5\frac{1}{2}$ ft. high at its widest end; what is the area of one side?

9. A block is 5 in. by 3 in. by 2 in.; what is its volume?

10. A piece of stone is 7 ft. long, $5\frac{1}{2}$ ft. wide by $3\frac{1}{2}$ ft. thick; how many cu. ft. in it?

11. A bin is 8 ft. by 6 ft. by $4\frac{1}{2}$ ft.; how many bushels of wheat will it hold?

12. A cistern is 9 ft. by 5 ft. by 5 ft.; how many gallons does it hold?

13. How many perch of masonry in a wall 350 ft by 18 ft. by 2 ft.?

14. A block contains 64 cu. ft., and is 4 ft. wide and 2 ft. thick; how long is it?

15. A box is 5 ft. long and 3 ft. wide; how deep must it be to contain 60 cu. ft.?

16. A tank is 10 ft. long and 4 ft. wide; how deep must it be to hold 3 tons of water?

17. A haystack is 36 ft. long and 9 ft. wide; what is its hight if it contain 11 tons of hay?

18. A horse is tied to a stake by a rope 100 ft. long; over how much ground can he walk?

SECTION XI.

THE DECIMAL SYSTEM OF MEASURES.

Remark. The history of the decimal system of measures and numerous facts and arguments showing its simplicity, are found in the publications of the American Metric Bureau of Boston. The purpose of this section is to exhibit the system and to present methods of computing by means of it.

176. Primary Units.

Of length,	—the	meter.	marked m.
" Capacity,—	"	liter (leeter.)	" l.
" weight, —	"	grăm,	" g.

Remarks. 1. The ar is the unit for measuring land. Other surfaces are measured by the sq. m. or by decimal parts or decimal multiples thereof. The ar is marked a.

2. The ster is the unit for measuring wood. Other volumes are measured by the cubic meter or by decimal parts or decimal multiples thereof. The ster is marked s.

3. In measuring great weights the quintal and the tonneau (Metric ton) are used. These are marked Q. and T., respectively.

177. Secondary Units.

Each of the secondary units used is either a decimal part or a decimal multiple of a primary unit, and is designated by using one of the following prefixes with the name of a primary unit.

Decimal parts.			*Decimal Multiples.*		
Dĕcĭ,	meaning	.1.	Dĕka,	meaning	10.
Cĕntĭ,	"	.01.	Hĕktō,	"	100.
Mĭllĭ,	"	.001.	Kĭlō,	"	1000.
			Myria,	"	10000.

Remarks. 1. In combining any one of the foregoing prefixes with the name of a primary unit, each word retains its own pronunciation and accent.

2. In abbreviating a word formed by combining a prefix with the name of a primary unit, it is customary to use the initial letter of each word, using a small letter to designate the primary unit or any of its decimal parts, and a capital letter to designate any of its decimal multiples.

Scales.

178. Of Length Measure.

	10	10	10	10	10	10	10
Mm.	Km.	Hm.	Dm.	m.	dm.	cm.	mm.
1	1	1	1	1	1	1	1

179. Of Surface Measure.

	100	100	100	100	100	100	100
Sq. Mm.	sq. Km.	sq. Hm.	sq. Dm.	sq. m.	sq. dm.	sq. cm.	sq. mm.
1	1	1	1	1	1	1	1

180. Of Land Measure.

	10	10	10	10	10	10	10
Ma.	Ka.	Ha.	Da.	a.	da.	ca.	ma.
1	1	1	1	1	1	1	1

181. Of Volume Measure.

	1000	1000	1000	1000	1000	1000	1000
Cu. Mm.	cu. Km.	cu. Hm.	cu. Dm.	cu. m.	cu. dm.	cu. cm.	cu. mm.
1	1	1	1	1	1	1	1

182. Of Wood Measure.

Remark. Only three units are used.

	10	10
Ds.	s.	ds.
1	1	1

183. Of Liquid and Dry Measure.

	10	10	10	10	10	10	10
Ml.	Kl.	Hl.	Dl.	l.	dl.	cl.	ml.
1	1	1	1	1	1	1	1

184. Of Weight.

	10	10	10	10	10	10	10	10	10
T.	Q.	Mg.	Kg.	Hg.	Dg.	g.	dg.	cg.	mg.
1	1	1	1	1	1	1	1	1	1

Relation of Decimal and Common Measures.

185. Of Length.

1 cm.	=	.3937 in.
1 dm.	=	3.937 in.
1 m.	=	39.37 in.
1 Dm.	=	32.8 ft.
H m	=	328 ft. 1 in:
1 Km.	=	3280 ft. = $\frac{3}{5}$ mi., nearly.
1 Mm.		6.2137 miles.

186. Of Surface.

1 sq. dm.	=	15½ sq. in.
1 sq. m.	=	1550 sq. in.
1 ar.	=	119.6 sq. yd. / 1 sq. Dm.
1 Ha.	=	2.47 acres.

187. Of Volume.

1 cu. cm	=	.06 cu. in.
1 cu. dm.	=	61 cu. in.
1 cu. m. or 1 ster,	=	35.317 cu. ft., or 1.308 cu. yd.

188. Of Capacity.

1 ml.	= 1 cu. cm.	=	.27 fl. dr.	= .061 cu. in.
1 cl.	= 10 cu. cm.	=	.338 fl. oz.	= .61 " "
1 dl.	= .1 cu. dm.	=	.845 gi.	= 6.1 " "
1 l.	= 1 cu. dm.	=	1.05 qt. liq.	= .9 qt. dry.
1 Dl.	= 10 cu. dm.	=	2.64 gal. liq.	= 9.08 qt. dry.
1 Hl.	= .1 cu. m.	=	26.41 gal. liq.	= 2.76 bu.
1 Kl.	= 1 cu. m.	=	264.17 gal. liq.	= 1.308 cu. yd.

189. Of Weight.

1 mg.	= the wt. of	1 cu. mm.	of water	=	.015 gr. Av.
1 cg.	= " " "	10 "	" " "	=	.154 " "
1 dg.	= " " "	.1 " cm.	" "	=	1.54 " "
1 g.	= " " "	1 "	" " "	=	15.43 " "
1 Dg.	= " " "	10 "	" " "	=	.35 oz. "
1 Hg.	= " " "	1 dl.	" " "	=	3.53 " "
1 Kg.	= " " "	1 l.	" "	=	2.2 lb. "
1 Mg.	= " " "	10 "	" "	=	22.04 " "
1 Q.	= " " "	1 Hl.	" "	=	220.4 " "
1 T.	= " " "	1 cu. m.	" "	=	2204.6 " "

Remark. The fractions in the above tables of relative value are not exact, but they are sufficiently approximate to meet most applications. Indeed where no great accuracy is required it is sufficient to use the following table of—

190. Approximate Values.

1 dm.	=	about 4 in.
1 m.	=	" $39\frac{1}{2}$ in.
5 m.	=	" 1 rod.
1 Km.	=	" $\frac{5}{8}$ mi.
1 sq. m.	=	" $10\frac{3}{4}$ sq. ft.
1 Ha.	=	" $2\frac{1}{2}$ acres.
1 cu. m. or ster.	=	" $1\frac{1}{3}$ cu. yd., or $\frac{1}{4}$ cord.
1 l.	=	" 1 qt.
1 Hl.	=	" $2\frac{1}{2}$ bu.
1 g.	=	" $15\frac{1}{2}$ gr.
1 Kg.	=	" $2\frac{1}{5}$ lb.
1 T.	=	" 2200 lb.

191. Table of Specific Gravities; Water Being 1.

Air,	.001	Ice,	.93
Alcohol, pure,	.79	Iron, cast,	7.1
" commercial,	.83	" wrought,	7.7
Brass,	7.6	Lead,	11.35
Brine,	1.04	Lime,	1.8
Coal, soft,	1.25	Marble,	2.7
" hard,	1.5	Mercury,	13.6
Copper,	9.	Milk,	1.03
Cork,	$\frac{1}{6}$.	Silver,	10.5
Gold,	19.2	Zinc,	7.1

192. Important Facts.

1. The ar is an area equivalent to 1 sq. Dm.

2. The ster (pronounced *stair*) is a volume equivalent to 1 cu. m.

3. The liter is a volume equivalent to 1 cu. dm.

4. The gram is the weight of a cu. cm. of distilled water at its greatest density, i. e., at 4° centigrade, or 39.2° Fahrenheit. The water is balanced in a vacuum.

5. 1 liter, or 1 cu. dm. of distilled water weighs 1 kilogram, while 1 cu. m. of water weighs 1 metric ton.

6. The legal letter weight in the United States is ½ oz. Av. The Postmaster General is authorized to substitute a 15 gram weight for the ½ oz. weight in all postoffices that exchange mails with foreign nations and in other postoffices at his discretion.

7. A freshly coined nickel 5 cent piece (not the new nickel), is 2 cm. in diameter and weighs 5 grams. The silver coinage of the United States is worth 4 cents per gram. Three 5 cent nickels or 60 cents in silver coin constitute one letter weight.

193. Applications of Decimal Measures.

Length.

Write	and	read	1	m. as	dm.	cm.	mm.	Dm.	Hm.	Km.
"	"	"	25.4 m.	"	"	"	"	"	"	
"	"	"	2.64m.	"	"	"	"	"	"	
"	"	"	3 dm.	"	"	"	"	"	"	
"	"	"	.5 "	"	"	"	"	"	"	
"	"	"	5 mm.	"	"	"	"	"	"	
"	"	"	15 Dm. "m	"	"	"	"	"	"	

34m.+42 dm.+134 cm. = how many m.? dm.? cm.? etc

Capacity.

Write	and	read	1	l. as	dl.	cl.	ml.	Dl.	Hl.	Kl.	Ml.
"	"	"	34.5 l.	"	"	"	"	"	"	"	
"	"	"	56.34 l.	"	"	"	"	"	"	"	
"	"	"	25 cl.	"	"	"	"	"	"	"	
"	"	"	1.2 Dl.	"	"	"	"	"	"	"	

Weight.

Write and read	1	g. as	dg.	cg.	mg.	Dg.	Hg.	Kg.	Q.	T.
" " "	36	"	"	"	"	"	"	"	"	"
" " "	3.4 cg		"	"	"	"	"	"	"	"
" " "	54.3 dg.		"	"	"	"	"	"	"	"
" " "	69 Kg.		"	"	"	"	"	"	"	"

[Exercises.]

Remarks. 1. Areas and volumes are found as in the common measures.

2. In finding the capacity of a given volume it is necessary to remember that a cu. dm. = 1 liter.

3. In finding the weight of a given volume or capacity of a substance, it is necessary to remember that a cu. cm. of distilled water weighs 1 g., that 1 cu. dm. of distilled water weighs 1 Kg and fills a liter cup, and that 1 cu. m. of distilled water weighs one metric ton. If the weight of a substance other than water be required, we find the weight of an equal volume of water and multiply it by the specific gravity of the substance.

Example 1. A tank is 4 m. long, 3 m. wide and 2.5 m. deep; how many Kg. of brine will fill it?

Solution. $4 \times 3 \times 2.5 = 30$ = the *number* of cu. m. in the tank.

Since 1 cu. m. of distilled water weighs 1 metric ton, 30 cu. m. of distilled water weigh 30 metric tons; and since 1 T. = 1000 Kg., 30 T. = 30000 Kg. = the weight of the water necessary to fill the tank.

Since brine is 1.04 times as heavy as water, the weight of this volume of brine = 1.04 times 30000 Kg., or 31200 Kg. ∴ etc.

2. Find the area of the walls of a room 6.2 m. long, 5.05 m. wide and 3.5 m. high.

3. How many rolls of paper 45 cm. wide and 8 m. long, allowing 11.2 sq. m. for openings will be required to paper those walls?

4. Find the cost of plastering the room at 50¢ per sq. m.

5. How many sq. m. in a board 8 m. by 25 m.? 5 m. by 25 cm.? 7 dm. by 156 mm.?

6. If wood is cut into 120 cm. lengths, and a pile is 43.7 m. long and 1.6 m. high, how many sters does it contain?

7. A bin is 11.4 m. long, 4.15 m. wide and 2.8 m. deep; how many hektoliters of barley does it hold?

8. If the specific gravity of grain is .81, what is the weight of the grain that fills the bin?

9. A vat is 186 cm. long, 7.7 m. wide and 48 dm. deep; how many tonneaus of water does it hold?

10. What is the weight in kilograms, of a cu. cm. of water? Of a Hl. of water? Of Mercury? Of milk? Of lime? Of a cu. m. of cork? Of brass? Of zinc?

11. What weight of water will fill a vat 92 cm. by 76 cm. by 4.2 dm.? What weight of milk will fill it?

12. If the above vat be filled with brine weighing 1.04 Kg. per liter, required the weight of the brine?

13. How many liters of air in a room 6.3 m. by 5.17 m. by 2.9 m.? What is the weight of the air in grams? In kilograms?

14. How many pills of .36 g. each, can be made from a mass weighing .72 Kg.?

15. What is the weight of 7.1 Hl. of pure alcohol?

16. An irregularly shaped mass of copper displaces .88 l. of water; what is its weight in kilograms?

17. A piece of iron 125 cm. long, 56 cm. wide, and 6.2 cm. thick weighs 256.4 kilograms; what is the specific gravity of the iron?

18. A piece of ore weighing 5.6 kilograms, weighs in water only 3.12 kilograms; what is its s. g.?

19. If a tap running 2.7 l. per min., fill a tub in 14 minutes, how long would it take a tap running 4.2 l. per min. to fill it?

20. A cistern will hold 18 tonneaus of rain water; what depth of rain must fall upon a flat roof 20 m. long by 15 m. wide, to fill the cistern?

21. Find the weight in Kg. of a block of ice 4.5 m. by 3.2 m. by 2 dm.

22. What is the weight of a bar of lead 2.3 dm. by 2 cm. by 1.2 cm.?

23. What is the weight of a piece of copper 4 dm. by 1.6 dm. by 3 cm.?

24. What is the value of a piece of silver 3.2 dm. by 7 cm. by 2 cm. at 3¢ per gram?

25. How many bullets, each weighing 2.7 dekagrams can be made from a cubical block of lead whose edge is .74 dm.?

26. What is the s. g. of a substance that weighs 20 Kg. in air and 18 Kg. in water?

27. How many jets, each burning 110 liters of gas per hour for $2\frac{3}{4}$ hours each night, would consume the contents of a gasometer containing 300000 cu. m. of gas?

28. What is the weight of a block of marble .72m. by 12 dm. by 1.2 dm.?

29. A box is 2.3 m. by 9 dm. by 25 cm.; how many liters does it hold?

30. If a wheel is 90 cm. in circumference, how many times does it revolve in going 5 kilometers?

31. If a cu. cm. of ore weigh 6.2 g., required its s. g.

32. How many meters of carpeting .7 m. wide, will cover a floor 6 m. by 7 m., the strips extending the longer way of the floor?

33. Required the area of a circle 3.2 m. in diameter.

34. If a cu. m. of earth weigh 1268 Kg., what is its s. g.?

35. A cistern is 4 m. deep by 3 m. wide, how long must it be to hold 60 metric tons of water?

36. What is the Troy weight of 3 cu. dm. of mercury?

SECTION XII.

PERCENTAGE.

194. Percentage is a method of computing by hundredths.

The Terms Employed.

195. The Base. The base is the number of which a number of hundredths is reckoned.

196. The Rate Per Cent. The rate per cent. is the fraction which indicates the number of hundredths involved.

197. The Percentage. The percentage is a number that bears the same relation to the base that the rate % does to 1. It is a number of times .01 of the base if the rate per % is .01 or more; it is a part of .01 of the base if the rate % is less than .01.

Remarks. 1. The term *amount* is applied to the sum of the base and the percentage.

The base is $\frac{100}{100}$ of itself and the percentage is a number of hundredths of the base, hence their sum, the amount, is a number of hundredths of the base, and comes within the definition of the *percentage*.

The term *difference* is applied to the part of the base that remains after the withdrawal of the *percentage* from the base. The difference is, therefore, a number of hundredths of the base and comes within the definition of the *percentage*.

198. Relations of Percentage. (1.) The percentage, being a number of hundredths of the base, is found by obtaining the number of hundredths of the base that the rate per cent. indicates; *i. e.* by multiplying the base by the rate per cent. The base and the rate per cent. are thus seen to be factors of the *percentage.*

Percentage is related to multiplication in the signification of its terms; the base being *multiplicand*, the rate per cent. being *multiplier* and the *percentage* being *product.*

(2.) *Principle.* If two or more factors are given, their product is found by multiplying the factors together.

Principle. If the product of two factors and one of them be given, the other is found by dividing the product by the given factor.

In the light of one or the other of the above principles of factoring is seen the process to be performed in finding any one of the three terms of percentage if the other two are known.

Percentage is related to factoring in the principles which determine the processes to be performed.

(3.) The rate per cent. is always given in the denomination of *hundredths.*

Percentage is related to fractions in that one of its terms (the rate per cent.) is a fraction.

General Cases of Percentage.

Remark. All special problems in percentage may be classified under three cases.

199.

CASE I.

Given the base and the rate per cent. to find the percentage.

Solution. Since the percentage is the product of the base and the rate per cent., the percentage is found by multiplying the base by the rate per cent.

200.

CASE II.

Given the base and the percentage to find the rate per cent.

Solution. Since the percentage is the product of the base and the rate per cent., the rate per cent. is found by dividing the percentage by the base, expressing the quotient in the denomination of hundredths.

201.

CASE III.

Given the percentage and the rate per cent. to find the base.

Solution. Since the percentage is the product of the base and the rate per cent., the base is found by dividing the percentage by the rate per cent.

Remark. It is observed that Case I is solved by multiplication in the light of the first of the two principles stated under the second relation of percentage, while Cases II and III are both solved by division in accordance with the second of the two principles referred to. On the basis of processes employed, two cases will be found to embrace all problems in percentage.

202. Forms of Solution. (1.) What is 7% of 35 ?

First form. We have given the base, 35, and the rate per cent., .07, to find the percentage. Since the percentage is the product of the base and the rate per cent., the percentage in this example, is found by multiplying 35 by .07. The product, 2.45, is the required percentage.

Second form. 1% of 35 = .01 of 35 = .35.
7% of 35 = 7 times .35 = 2.45.
$\therefore$ 7% of 35 = 2.45.

(2.) 4.5 = what % of 75 ?

First form. We have given the base, 75, and the percentage, 4.5, to find the rate per cent. Since the percentage is the product of the base and the rate per cent., the rate per cent., in this example, is found by dividing 4.5 by 75, expressing the quotient as hundredths. The quotient, .06, is the required rate per cent.

Second form. 1% of 75 = .01 of 75 = .75 ; hence 4.5 equal as many times 1% of 75 as .75 is contained times in 4.5 which = 6. 6 times 1% = 6%. Therefore 4.5 = 6% of 75.

Third form. $\because 1 = \frac{1}{75}$ of 75, 4.5 = 4.5 times $\frac{1}{75}$ of 75 = $\frac{4.5}{75}$ of 75. Multiplying both terms of $\frac{4.5}{75}$ by $1\frac{1}{3}$, we have .06. $\therefore$ 4.5 = 6% of 75.

(3.) What per cent. of a number is $\frac{7}{20}$ of it ?

Solution. $\frac{7}{20} = \frac{35}{100}$. Hence $\frac{7}{20}$ of a number = .35, or 35% of it.

(4.) 36 equal 9% of what number?

First form. We have given the percentage, 36, and the rate per cent., .09, to find the base. Since the percentage is the product of the base and the rate per cent., the base in this example, is found by dividing 36 by .09. The quotient, 400, is the required base.

(5.) 48 = 20% more than what number?

First form. Since 48 = 20% more than some number, 48 = 120% of that number. We now have given the percentage, 48, and the rate per cent., 1.20, to find the base. Since the percentage is the product of the base and the rate per cent., the base in this example, is found by dividing 48 by 1.20. The quotient, 40, is the required number.

Second form. Since 48 = 20% more than some number, 48 = 120% of that number; and 1% of the number must = $\frac{1}{120}$ of 48 = .4, and 100% of the number must = 100 times .4 = 40. ∴ 48 = 20% more than 40.

Third form. Since 20% of a number = $\frac{1}{5}$ of it, then 48 is $\frac{6}{5}$ of some number.

$\frac{1}{5}$ of that number must = $\frac{1}{6}$ of 48 = 8; and
$\frac{5}{5}$ of the number = 5 times 8 = 40.
∴ 48 = 20% more than 40.

(6.) $\frac{3}{5}$ = 10% less than what number?

First form. Since $\frac{3}{5}$ = 10% less than some number, $\frac{3}{5}$ = 90% of that number.

We now have given the percentage, $\frac{3}{5}$, and the rate %, .90, to find the base.

Since the percentage is the product of the base and the rate per cent., the base in this example is found by dividing $\frac{3}{5}$ by .90. The quotient, $\frac{2}{3}$, is the required base.

Second form. Since 10% less than some number = $\frac{3}{5}$
90% of the number must = $\frac{3}{5}$;
and 1% " " " " = $\frac{1}{90}$ of $\frac{3}{5}$ = $\frac{1}{150}$.
" 100% " " " " = 100 times $\frac{1}{150}$ = $\frac{2}{3}$.
∴ $\frac{3}{5}$ = 10% less than $\frac{2}{3}$.

Exercises.

	Base.	*Rate.*	*Percentage.*
1.	45.	5 %	?
2.	15.	9 "	?
3.	1.25	12 "	?
4.	45	$\frac{1}{2}$ "	?
5.	$\frac{3}{5}$	$\frac{2}{3}$ "	?
6.	60	?	15
7.	450	?	45
8.	1250	?	600
9.	2.5	?	5
10.	$\frac{3}{4}$	?	$\frac{1}{4}$
11.	$\frac{7}{8}$	?	.875
12.	15	?	2.25
13.	?	4 %	48
14.	?	15 "	750
15.	?	24 "	96
16.	?	$\frac{4}{5}$ "	$\frac{3}{4}$
17.	?	$\frac{5}{8}$ "	.375
18.	?	$66\frac{2}{3}$ "	540
19.	?	75 "	16
20.	?	110 "	64
21.	?	125 "	150
22.	?	134 %	2680
23.	?	$101\frac{1}{2}$ "	140
24.	?	118 "	236
25.	$\frac{3}{4}$	$\frac{5}{6}$ "	?
26.	235	11 "	?
27.	4.5	2.5 "	?
28.	.004	.04 "	?
29.	54	.3 "	?
30.	160	?	48
31.	515	?	3
32.	$\frac{2}{5}$	?	.005

Applications of Percentage.

203. There are two classes of applications of percentage. The first class includes all those problems in which the percentage is the product of the base and the rate per cent.

The second class includes those problems in which the percentage is the product of three factors, viz., the base, the rate per cent., and the number representing the time (in years) involved in the transaction under consideration.

204. The principal applications of the first class are—Profit and Loss, Commission and Brokerage, Stocks, Insurance, Taxes and Customs.

The principal applications of the second class are—Interest, Discount, Bonds, Exchange, Equation of Payments and Accounts.

Remarks. 1. In every problem of the first class the three terms of percentage are represented.

2. Many of the problems to be solved are compound, being composed of two or more problems, some of which may not involve percentage.

Applications of the First Class.

205. Profit and Loss. (1.) The number on which the gain or loss is estimated is the *base*. In most examples the cost price is the base.

(2.) The gain, loss or selling price is the *percentage*.

(3.) The rate of gain, rate of loss or rate of selling is *rate per cent.*

206. Forms of Solution.

Example 1. A man paid $110 for a horse and sold it at a profit of 20 per cent.; required the gain.

First form. We have given the cost, \$110, which is *base*, and the rate of gain, .20, which is *rate per cent.*, to find the gain, which is *percentage.*

Since the percentage is the product of the base and the rate per cent., the percentage in this example is found by multiplying \$110 by .20. The product, \$22, is the required percentage, which is the gain.

Second form. The gain is estimated on the cost.

1% of \$110 = .01 of \$110 = \$1.10, and
20% of \$110 = 20 times \$1.10 = \$22.
$\therefore$ The gain was \$22.

Third form. $\because$ 20% of a number $= \frac{1}{5}$ of it,
20% of \$110 $= \frac{1}{5}$ of 110 = \$22.
$\therefore$ the gain was \$22.

Example 2. If a merchant pay 15 cents per yard for muslin, for how much does he sell it to lose 25%?

First form. Since he sells at 25% below cost, he sells for 75% of the cost; and we have given the cost price, 15¢, which is base, and the rate of of selling, .75, which is rate %, to find the selling price, which is the *percentage.* Since the percentage is the product of the base and the rate per cent, the percentage in this example is found by multiplying 15¢ by .75; the product, $11\frac{1}{4}$¢ is the required percentage, which is the selling price.

Second form. Since he sells at a loss of 25%, he sells for 75%, or $\frac{3}{4}$ of the cost.

$\frac{1}{4}$ of 15¢ $= \frac{15}{4}$¢.
$\frac{3}{4}$ of 15¢ = 3 times $\frac{15}{4}$¢ $= 11\frac{1}{4}$¢
$\therefore$ etc.

Example 3. A hat costing \$8 was sold for \$9, what was the rate of gain?

First form. (1.) \$9, the selling price, *minus* \$8, the cost, = \$1, the gain.

(2.) We now have given the cost, \$8, which is *base*, and the gain, \$1, which is the *percentage*, to find the rate of gain, which is *rate per cent.*

Since the percentage is the product of the base and the rate per cent., the rate per cent. in this example is found by dividing \$1 by \$8, expressing the quotient as hundredths. The quotient, $.12\frac{1}{2}$, is the required rate per cent. which is the rate of gain. ∴ etc.

Second form. \$9, the selling price, *minus* \$8, the cost price, = \$1, the gain.
\$1 = $\frac{1}{8}$ of \$8.
$\frac{1}{8}$ of a number = $.12\frac{1}{2}$ of it.
∴ the rate of gain was $12\frac{1}{2}\%$.

Example 4. A grocer sold coffee at 8¢ above cost, and gained 20 per cent.; required the cost.

Form. We have given the gain, 8¢, which is the percentage, and the rate of gain, .20; which is rate per cent., to find the cost, which is base.

Since the percentage is the product of the base and the rate per cent., the base in this example is found by dividing 8¢ by .20; the quotient, 40¢, is the required base which is the cost. ∴ etc.

Example 5. A merchant marked goods at 20% above cost, and then sold at 20% less than the marked price. Did he gain or lose and how much %?

Form. (1.) Assume \$1 as the cost.

(2.) Since he marked the goods at 20% above cost, the marked price was 1.20 of the cost; and we have given the cost, $1, which is *base*, and the rate of marking, 1.20, which is *rate per cent.*, to find the marked price which is *percentage.* [Solving by Case I, the marked price is found to be $1.20.]

(3.) Since he sold at 20% less than the marked price, he sold for .80 of the marked price, and we have given the marked price, $1.20, which is *base*, and the rate of selling, .80, which is *rate per cent.*, to find the selling price which is *percentage.*

Since the percentage is the product of the base and the rate per cent., the percentage in this example is found by multiplying $1.20 by .80, the product, $.96 is the required percentage, which is the selling price.

(4.) The cost, $1, *minus* the selling price, $.96 = $.04 = the loss.

(5.) We now have given the cost, $1, which is base, and the loss, $.04, which is percentage, to find the rate of loss, which is *rate per cent.* This is found by dividing $.04 by $1, expressing the quotient as hundredths. The quotient, .04, is the required rate per cent., which is rate of loss.

Exercises.

1. 14 barrels of flour are bought at $3.87½ each and sold at $4.12½ each; required the gain. The rate of gain.

2. A horse is bought for $94, and sold at 4% gain; required the gain. The selling price.

3. A lot is bought for $2256; for what must it be sold to gain 20 per cent.?

4. There was a gain of $14 realized on a certain sale; the rate of gain was .15; what was the purchase price? The selling price?

5. A grocer bought 148 gallons of molasses at 26¢ per gal., and sold it for $64; did he gain or lose? How much and at what rate?

6, A farm was sold for $7400 which was 5% more than it was worth; required its value.

7. A man sold two horses for $120 each; on one he gained 15% and on the other he lost 15%; did he gain or lose on the entire operation, and how much?

8. I sold a piece of property for $1000 gaining 16%; I then invested the $1000 in another piece of property which I was forced to sell at a loss of 16%; did I gain or lose on the series of transactions and at what rate?

9. A quantity of wheat was sold for $1248 which was 12 per cent. more than its cost; required the cost.

10. If the cost aud rate of gain are known how find the gain? The selling price? Form and solve a problem.

11. If the gain and the rate of gain or loss be given what can be found and how? Form and solve problems.

12, If cost and selling price are known what can be found and how? Form and solve problems.

13. A man bought corn at 30¢ per bu. and sold it at a gain of $16\frac{2}{3}$%; what was the selling price?

14. If land cost $48 per acre, how much must be asked for it, that a 10% abatement may be made and a profit of 14% still be realized?

15. A merchant asked an advance of 35%, but afterward sold at 25% less than his asked price; did he gain or lose and how much on goods that cost $72.25?

16. I sold $\frac{3}{4}$ of an article for what the entire article cost; what was my rate of gain?

17. A grocer sold a hogshead of molasses for $30 which was 15% more than it cost; required the cost per gallon.

18. A merchant marks a piece of goods $12, but takes off 6% for cash. If he still makes 10% profit, what was the cost of the goods?

19. A grocer gains 14% by using a false weight, required the weight of his pound weight.

20. A man bought a horse for $55 and sold him for $70; required his rate of gain.

21. If $\frac{2}{7}$ of an article be sold for what $\frac{2}{3}$ of it cost, what is the rate of gain?

22. If butter be bought at $24 per cwt., for what price per lb. must it be sold to gain 15%, and allow a discount of 8% for cash?

207. Commission and Brokerage.

Remark. The following are the most common corresponding terms involved.

(1.) The sum representing the amount of business transacted is *base.*

(2.) The commission or brokerage is the *percentage.* The *amount* involved, or the sum of the investment and the commission is the *percentage* in some instances.

(3.) The rate of commission or brokerage is *rate per cent.* If the sum of the investment and commission be the percentage, the rate per cent. is $\frac{100}{100}$ + the rate of commission.

208. Forms of Solution.

Example 1. An agent sold $1560 worth of goods and charged 4% for his services; required his commission.

Form. We have given the sum representing the amount of business done, $1560, which is *base*, and the rate of commission, .04, which is rate per cent., to find the commission, which is the percentage. Since the percentage is the product of the base and the rate per cent., the percentage in this example is found by multiplying $1560 by .04. The product, $62.40, is the percentage, which is the required commission.

[Give other forms of solution.]

Example 2. An agent was paid $24 for buying $1440 worth of wheat; required his rate of commission.

Form. We have given the commission, $24, which is *percentage*, and the sum representing the amount of business done, $1440, which is base, to find the rate of commission which is rate per cent.

Since the percentage is the product of the base and the rate per cent., the rate per cent. in this example is found by dividing $24 by $1440, making the quotient hundredths. The quotient, .01⅔ is the rate per cent. which is the required rate of commission. ∴ etc.

[Give other forms of solution.]

Example 3. A banker received $40 for making a collection at 4%; required the sum collected.

Form. We have given the commission, $40, which is *percentage*, and the rate of commission, .04, which is *rate per cent.*, to find the sum representing the amount of business done, which is *base.*

Since the percentage is the product of the base and the rate per cent., the base in this example is found by dividing $40 by .04. The quotient, $1000, is the base, which is the sum representing the amount of business done. ∴ etc.

[Give other forms of solution.]

Example 4. A commission merchant received \$4160 with which to purchase goods after deducting his commission of 4%; required the sum invested.

First form. Since the rate of commission was .04, the sum received was 1.04 of the sum invested; and we have given the amount received, \$4160, which is *percentage,* and the relation of this sum to the sum invested, 1.04, which is *rate per cent.*, to find the sum invested which is *base.*

Since the percentage is the product of the base and the rate per cent., the base in this example is found by dividing \$4160 by 1.04. The quotient, \$4000, is the base which is the required sum invested.

Second form. Every dollar's worth of goods purchased cost the sender of the money \$1 for the goods +4¢ for agent's commission, or \$1.04; hence as many dollars were spent for goods as \$1.04 are contained times in \$4160, or 4000. ∴ the sum spent for goods was \$4000.

Third form. Since the sum invested + 4% of itself = \$4160, 104% of the sum invested = \$4160,
and 1% of the sum invested = $\frac{1}{104}$ of \$4160 = \$40;
" 100% of " " " = 100 times \$40 = \$4000.
∴ the investment was \$4000.

[Exercises.]

1. At 3% commission what does an agent receive for selling \$15360 worth of goods?

2. A lawyer collected 65% of a debt of \$348; his rate of commission was 4%; what did he receive?

3. A merchant sent his agent \$1250 with instructions to buy goods; how much was expended for goods after the agent deducted his commission at 2½%?

4. How many barrels of flour at $4.50 per barrel can be bought for $684, if 3% commission be deducted before purchasing?

5. An agent received $240 as commission at 4%; what amount of business did he transact?

6. The rate of commission was 5%; the sum sent the owner as proceeds of the sale was $2275; what was the commission?

7. What amount of business is done if $36.50 is the commission at $2\frac{1}{2}$%?

8. I sent my agent $516 to invest in corn after deducting his commission at $3\frac{1}{2}$%; what sum was paid for corn?

9. An agent receives a remittance of $758 with which to buy goods, deducting his commission at $1\frac{3}{4}$%; what is his commission?

10. If the merchant for whom the business is done (in ex. 9) send check for the commission, what is its face?

11. A consignment of grain was sold for $15650, of which $15540 were net proceeds; required the rate of commission.

12. An agent received 2% commission for selling goods. His entire commission was $126; what sum did he remit his employer?

13. I sold goods on commission at 4% through a broker who charged me $2\frac{1}{2}$%; my commission after paying the broker was $216; required the net proceeds of the sales.

14. A real estate agent retains as commission $124, sending the employer $6575; what was the amount of the sale and the rate of commission?

15. An agent bought 25 horses on commission at $3\frac{1}{2}\%$. His commission was $63; required the cost of each horse.

16. If I pay an attorney $48.50 for making a collection at 5%; what was the claim?

17. How many barrels of flour at $5 each can an agent buy for $324, deducting his commission of 3%?

18. An agent received $3440 with which to buy pork after deducting his commission of $1\frac{1}{4}\%$; required the number of pounds of pork purchased at 3¢ per pound?

19. A shipment of hay sold for $14 per ton; the rate of commission was 3%; incidental charges $200; the net proceeds were $6290. Required the number of tons.

209. Stocks.

1. The par value of stocks is *base*. Sometimes the conditions are such that the cost or another number becomes *base*.

2. The premium, the discount, or the cost, is *percentage*.

3. The rate of premium, rate of discount, rate of purchase, or rate of sale is *rate per cent*.

210. Forms of Solution.

Example 1. A company pays a dividend of 5%; what does that man receive who owns 12 shares?

Form. 12 shares of $100 each are worth $1200. We now have given the par value of the stock, $1200 which is *base* and the rate of premium, .05, which is *rate per cent.*, to find the premium, which is *percentage.*

Since the percentage is the product of the base and the rate per cent., the percentage in this example is found by multiplying $1200 by .05. The product, $60, is the percentage, which is the required dividend.

[Give other forms.]

Example 2. How much bank stock at 10% discount, can be bought for $27945?

First form. At 10% discount, the stock is bought at 90% of the par value; and we have given the purchase price, $27945, which is percentage, and the rate of purchase, .90, which is *rate per cent.*, to find the par value, which is *base.*

Since the percentage is the product of the base and the rate per cent.,the base in this example is found by dividing $27945 by .90. The quotient, $31050, is the base, which is the required par value.

Second form. Since the stock is bought at 10% discount, every $1 worth of stock is bought for $.90. As many dollars' worth of stock can be bought for $27945 as $.90 is contained times in $27945,which equal 31050. ∴ $31050 worth of stock can be bought for $27945 at 10% discount.

[Give other forms.]

Example 3. Stock was bought at 120 and sold at 128; what was the rate of gain?

Form. At 120, $1 worth of stock was bought for $1.20, and at 128 it was sold for $1.28. The gain was $.08. We have given the cost of the stock, $1.20, which in this case is *base,* and the gain, $.08, which is *percentage.* Since the percentage is the product of the base and the rate per cent., the rate per cent. in this example is found by dividing $.08 by $1.20, expressing the quotient as hundredths. The quotient, .06⅔, is the rate percent., which is the required rate of gain.

Example 4. A broker bought stock at 3% discount and sold at 3% premium, and gained \$420; required the face of the stock bought.

Form. Since he bought at 3% of the face less than the face and sold for 3% of the face more than the face, he gained 6 % of the face, but \$420 equaled his gain; hence 6% of the face of the stock equaled \$420, then 1% of the face $=\frac{1}{6}$ of \$420 = \$70, and 100% of the face = 100 times \$70 = \$7000. ∴ etc.

[Give other forms.]

Exercises.

1. What cost 60 shares railroad stock at 4% premium?

2. Midland stock bought at 3% discount was sold at 4% premium; required the gain on 75 shares.

3. 9 shares of I. & St. L. stock at 96 are exchanged for mining stock at 104; how many \$100 shares of mining stock are purchased?

4. The net earnings of a street railway are \$1500; the capital invested is \$30000; required the rate of dividend that can be declared.

5. How many shares of canal stock at 94 can be bought for \$118800?

6. At 20% premium how many shares of stock will \$4800 buy?

7. If Vandalia stock is quoted at 102¼ how much stock can be bought for \$3246, brokerage being ¾%?

8. Pan Handle stock bought at 110 and sold at 116 yields what rate of gain?

9. Wm. Smith receives \$630 as a 7% dividend; how many \$50 shares does he own?

10. A man owned 25 shares of rolling mill stock of \$50 each; the company declared a dividend of 8%, payable in stock: how many additional shares were issued to him?

11. A bridge company whose stock was \$12500 required an assessment of \$625; what rate did they declare?

12. Mr. Wells owns 35 shares of \$100 each in a turnpike company; his dividend was \$201.25; required the rate of dividend.

211. Insurance.

1. The sum insured is *base.*

2. The premium or the sum insured $\pm$ the premium is a *percentage* of the sum insured.

3. The rate of premium or $\frac{100}{100}$ $\pm$ the rate of premium is the *rate per cent.*

Remark. If the premium is viewed as the percentage, the rate of premium is rate per cent.; but if the amount or difference be viewed as percentage, the rate per cent. is $\frac{100}{100}$ $\pm$ the rate of premium.

212. Forms of Solution.

Examples. 1. A house is insured for \$800 at 4%; required the premium.

2. The premium paid for insuring a barn for \$1200 is \$6; required the rate of insurance.

3. A man paid \$150 for insuring goods at 3 per cent.; required the value insured.

Remark. The above problems come within Cases I, II, and III, respectively, and are readily solved by forms similar to those given under the preceding applications of percentage.

4. A stock of goods worth $19600 is insured at 2%, so as to recover both the value of the goods and the premium in case of loss; required the sum insured.

First form. Since the premium is 2% of the sum insured, the value of the goods, $19600, must be 98% of the sum insured. We have given $19600, the value of the goods, which is *percentage*, and .98, the rate of difference, which is *rate per cent.*, to find the sum insured, which is *base*. Since the percentage is the product, &c.

Second form. Since 2% of the sum insured = the premium, 98% of the sum insured must = the value of the property = $19600; then 1% of the sum insured = $\frac{1}{98}$ of $19600 = $200, and 100% of the sum insured = 100 times $200 = $20000. ∴ etc.

5. A shipper took out a policy for $34200 to include the value of the goods shipped and also the premium at 6 per cent.; required the value of the goods.

Form. Since the premium was 6% of the face of the policy, the value of the goods must have been 94% of its face. We have given the sum insured, $34200, which is *base*, and the rate of difference, .94, which is *rate per cent.*, to find the value of the goods, which is *percentage*.

Since the percentage is the product of the base and the rate per cent., etc.

[Give other forms.]

Exercises.

1. Property is insured for $2250 at $3\frac{1}{2}$%; what is the premium?

2. A stock of goods is insured for $4800 at $\frac{2}{5}$%; what is the premium?

3. A man pays $24 for insuring a house at ½ per cent.; what is the face of the policy?

4. The rate of insuring a house was 1½% and the premium was $12; what was the face of the policy?

5. $56 is the premium paid on a policy of $11200; what is the rate?

6. A shipment of grain worth $15600 is insured at 2%, so as to include the premium as well as the value of the grain in case of loss; what is the face of the policy?

7. The rate of premium is 3½% and the value of property insured is $2488; what face of policy will include both in case of loss?

8. To include the premium at 1¾% and the value ot the goods, the face of a policy is $6300; what is the value of the goods?

9. A vessel's cargo is insured for $17600; the premium at 4% is included; what is the value of the cargo?

10. What will it cost to insure a house for $900 at 1⅓ per cent.? At ¾ per cent.?

11. A piece of property was insured for $5000; the premium paid was $30; what was the rate?

12. At 2¼% what sum must be insured on property worth $3648 to include both property and premium in case of loss?

13. A merchant insured a consignment of goods for $13728 so as to include both property and premium; required the premium.

14. If a man pays $38.80 for insuring ⅔ of the value of his house, what is its value if the rate of insurance is 2½%?

15. A building is insured so as to include $\frac{3}{5}$ of its value and the entire premium. The value of the building is $24500, and the rate of insurance $1\frac{1}{4}\%$; what is the premium?

213. Taxes.

1. The assessed value of the property taxed is *base.*

2. The rate of taxation is *rate per cent.*

3. The tax is the *percentage.*

Remark. Use forms of solution similar to those already given.

Exercises.

1. What sum must be assessed in order that $12500 may remain after paying a commission of 4% for collection?

2. The valuation of the property in a certain district is $2364748. A tax of 12560 is required. What tax must that man pay whose property is assessed at $8000?

3. If the rate of tax was $12 on $1000 and the tax levied was $14674, what was the valuation?

4. Required A's tax on property worth $2460 at $\frac{2}{5}\%$?

5. What sum must be assessed to raise a net tax of $7400, and pay a commission of 2% for collection?

6. In a school district a school is maintained by a tax on the property of the district, which is valued at $648750. A teacher is paid $45 per month for 6 months, and other expenses are $64.50; required the tax on property worth $4865.

7. Find A's tax from the following items:

His district paid in teachers' salaries, .	$1200.00
" " for fuel, ...	57.60
" " " incidentals, ...	38.00

The money received from the school fund was $258. The remaining expense was paid by a rate bill. The aggregate attendance was 9568 days, while A sent 4 pupils 46 days each.

8. The cost of a public work was $1260. The rate of taxation was 3 mills on the dollar, and the collector's commission was $3\frac{1}{2}\%$; what was the valuation?

214. Customs, or Duties.

1. The net quantity of goods is the *base* in computing specific duties.

2. The cost of the goods in the country whence they were exported is the *base* in computing Ad Valorem duties.

Remark. Duties are assessed upon the goods actually imported. All deductions are made previous to the assessment.

3. The rate of duty is *rate per cent.*

4. The duties assessed constitute the *percentage.*

Remark. Use forms of solution similar to those already given.

Exercises.

1. An importation was 56 casks of wine, each containing 36 gallons; The net duty at 30% Ad Valorem, amounted to $907.20; required the invoice per gallon.

2. A quantity of lace was invoiced at $816.54. The merchant paid the duties and a freight bill of $22.50, and found that the total cost was $980.50; required the rate of duty.

3. Required the duty at $2\frac{1}{2}$ cents per pound on 2700 pounds of cloves, tare being 5%.

4. If the duty on opium is 100%, required the import tax on 236 lb. of opium invoiced at $3.75 per lb.

5. Required the Ad Valorem duty at 30% on 125 boxes of tea, each containing 70 lb. and invoiced at 85¢ per lb., tare being 8 lb. per box.

Applications of the Second Class.

215. Interest.

1. The principal is *base*.

2. The rate of interest is *rate per cent*. The rate of the amount or of the difference, i. e. $\frac{100}{100} \pm$ the rate of interest may be *rate per cent.*

3. The interest is *percentage*. The amount is also a percentage of the principal.

4. The number representing the time in years is a factor used with the principal and the rate of interest to determine the interest.

Remarks. 1. The time unit in interest is 1 year of 12 months of 30 days each.

2. Interest has involved in it four terms, viz:—the principal, the rate of interest, the time in years, and the interest. The first three of these are factors of the fourth.

216. *CASE I.*

Given the principal, the rate of interest and the time to find the interest.

Example. Required the interest of $500 for 2 yr. 3 mo. 15 da. at 6%.

First form. 2 yr. 3 mo. 15 da. = $2\frac{7}{24}$ yr.

Since the interest is the product of the principal, the rate of interest and the number representing the time in years, the interest in this example is found by multiplying together \$500, .06 and $2\frac{7}{24}$. The product, \$68.75, is the required interest.

Remark. The following expressions for the above form indicate a method for finding interest for months or days:

For months. $\left\{ \frac{\text{Principal} \times \text{rate} \times \text{time in months}}{12} \right.$ = Interest

For days. $\left\{ \frac{\text{Principal} \times \text{rate} \times \text{time in days}}{12 \times 30} \right.$ = Interest.

Second form.

Int. of \$500 for 1 yr. at 6% =	\$30.00
" " 500 " 1 mo. " 6% = $\frac{1}{12}$ of \$30 =	2.50
" " 500 " 1 da. " 6% = $\frac{1}{30}$ of \$2.50 =	.08⅓
" " 500 " 2 yr. " 6% = 2 times \$30 =	\$60.00
" " 500 " 3 mo. " 6% = 3 " \$2.50 =	7.50
" " 500 " 15 da. " 6% = 15 " .08⅓ =	1.25
" " 500 " 2 yr. 3 mo. 15 da. at 6% =	\$68.75

Exercises.

1. The principal is \$150, the rate of interest 6%, the time 3 yr.; required the interest.

2. Find the interest of \$750 for 2 yr. 8 mo. at 5%.

3. Find the interest of \$6750 for 5 yr. 10 mo. 15 da. at 8%.

4. Find the interest of \$75 for 11 mo. at 9%.

5. Find the interest of \$956 for 7 mo, at 10%.

6. Find the interest of \$1278 for 5 mo. 3 da. at 6 per cent.

7. Find the interest of $16 for 90 da. at 12%.

8. Find the interest of $800 for 5 yr. 4 mo. 20 da. at 10 per cent.

9. Find the interest of $1750 for 6 yr. 2 mo. 28 da. at 8 per cent.

10. Find the interest of $9.50 for 20 da. at 10%.

11. Find the interest of $17850 for 40 da. at 11%.

12. Find the interest of $675 from July 3, 1881, to August 7, 1883, at 8 per cent.

13. Find the interest of $95 from May 10, 1880, to April 6, 1884, at 7 per cent.

14. A note was drawn for $850 on January 8, 1882; a payment of $200 was made September 18, 1882; what was due January 8, 1883, rate of interest being 5 per cent?

15. A note was drawn for $1000 on June 6, 1878; its rate of interest was 6%. A payment of $450 was made June 6, 1879; another of $200, September 21, 1879; another of $350, May 10, 1880; what was due December 25, 1881?

16. A note was drawn for $67 on October 17, 1882; the rate of interest being 9%. A payment of $16 was made December 1, 1882; another of $22, March 12, 1883; another of $18, June 11, 1883; another of $7, September 17, 1883; what was due October 17, 1883?

17. What is the compound interest of $156 for 4 yr. 6 mo. at 6 per cent.?

18. What is the compound interest of $1350 for 3 yr. 8 mo. at 6 per cent.?

19. What is the compound interest of $1100 for 1 yr. 6 mo. at 8%; interest compounded quarterly?

20. What is the compound interest of \$800 for 3 yr, at 5%; interest compounded semi-annually?

21. What is the annual interest of \$750 for 3 yr. 4 mo. at 6 per cent.?

22. Find the annual interest accruing on \$65 for 5 yr. 8 mo. 12 da. at 7 per cent.

217. *CASE II.*

Given, the principal, the time and the interest, to find the rate of interest.

Example. The principal is \$500, the time $2\frac{7}{24}$ yr. and the interest \$68.75; required the rate.

First form. Since the interest is the product of the principal, the rate and the number representing the time in years, the rate of interest in this example is found by dividing \$68.75 by the product of \$500 and $2\frac{7}{24}$ and making the quotient hundredths. The quotient, .06, is the required rate.

Second form. The interest of \$500 for $2\frac{7}{24}$ yr. at 1% $= \$11\frac{11}{24}$. Now since $\$11\frac{11}{24} =$ the interest of \$500 for $2\frac{7}{24}$ yr. at 1%, \$68.75 = the interest of \$500 for $2\frac{7}{24}$ yr. at as many times 1% as 68.75 is times $11\frac{11}{24}$ which = 6. 6 times 1% = 6%. ∴ the required rate of interest is .06.

Exercises.

1. The principal is \$380, the time 3 yr. 4 mo. and the interest \$76.76; required the rate.

2. A man received \$218.40 interest on a loan of \$780 for 4 yr. 8 mo.; required the rate.

3. The principal is \$4760, the time 2 yr. 8 mo. 20 da. the interest, \$864; required the rate.

4. The interest is \$456.84, the time 5 yr. 9 mo. 15 da.; what is the rate, the principal being \$986?

5. The principal was \$960, the time 7 yr. 5 mo. and the interest \$520.80; required the rate.

6. The principal is \$480, the time 6 yr. 3 mo. and the interest \$210; what is the rate?

7. The time is 8 yr. 9 mo. 12 da., the interest \$17.46, and the principal \$26.50; required the rate.

8. The principal is \$2015, the interest \$575.40 and the time 5 yr. 8 mo. 15 da.; required the rate.

218. *CASE III.*

Given the principal, the interest and the rate, to find the time.

Example. In what time will \$500 yield \$68.75 at 6%?

First form. Since the interest is the product of the principal, the rate and the number representing the time in years, the time in this example is found by dividing \$68.75 by the product of \$500 and .06. The quotient, $2\frac{7}{24}$, is the number representing the time in years. ∴ the time is $2\frac{7}{24}$ yr. or 2 yr. 3 mo. 15 da.

Second form. Since \$500 at interest for 1 yr. at 6% yield \$30, \$500 must be on interest for as many times 1 yr. at 6% to yield \$68.75 as \$68.75 are times \$30, which $=2\frac{7}{24}$. $2\frac{7}{24}$ time 1 yr. $=2\frac{7}{24}$ yr, $=$ 2 yr. 3 mo. 15 da. ∴ etc.

Exercises.

1. The principal is \$4080, the interest \$668.10, and the rate 5%; required the time.

2. The principal is \$176, the interest \$22; and the rate 7%; required the time.

3. The principal is \$1300, the interest \$274 and the rate 8%; required the time.

4. How long will it take any principal to double itself at 4%? at 5%? at 6%? at 10%?

219. *CASE IV.*

Given the interest or amount, the time, and the rate of interest, to find the principal.

Example. What principal will yield \$68.75 int. in 2 yr. 3 mo. 15 da. at 6%?

First form. Since the interest is the product of the principal, the rate of interest, and the number representing the time in years, the principal in this example is found by dividing \$68.75 by the product of .06 and $2\frac{7}{24}$. The quotient, \$500, is the required principal.

Second form. A principal of \$1 will yield $\$.13\frac{3}{4}$ interest in $2\frac{7}{24}$ yr. at 6%; and to yield \$68.75 interest in the same time at the same rate would require a principal as many times \$1 as \$68.75 are times $\$.13\frac{3}{4}$, which = 500. 500 times \$1 = \$500. ∴ etc.

Exercises.

1. Required the principal if the time is 3 yr. 8 mo., the rate .06, and the interest \$462.

2. Required the principal if the time is 6 yr. 3 mo., the rate .07, and the interest \$64.26.

3. What principal will in 7 yr. 4 mo. at 8% amount to \$749.70?

4. What sum will yield \$185 int. in 18 mo. at 5%?

5. What sum must be invested in 6% stocks to yield an income of \$1500?

6. What principal will amount to \$200 in 14 yr. $3\frac{3}{7}$ mo. at 7%?

7. What principal will amount to \$355.60 in 2 yr. 7 mo. at 8%?

Review Exercises in Interest.

	Principal.	*Rate.*	*Time.*	*Interest.*	*Amount.*
1.	$420	6%	3 yr. 6 mo.	?	?
2.	492 15	6 "	1 yr. 3 mo. 18 da.	?	?
3.	?	6 "	4 yr.	$24	?
4.	300	6 "	?	36	?
5.	?	6 "	1 yr. 6 mo.	72	?
6.	?	8 "	2 yr. 11 mo. 27 da.		$845
7.	12.80	7 "	3 yr. 4 mo. 3 da.	?	?
8.	?	5 "	2 yr. 4 mo. 12 da.	40	
9.	750	4 "	?	120	
10.	3542	? "	2 yr. 6 mo. 15 da.		
11.	?	9 "	9 yr. 9 mo.		900
12.	1475	10 "	5 yr. mo. 5 da.		?
13.	50	8 "	?	20	
14.	500	? "	2 yr. 6 mo.	50	
15.	?	6 "	3 yr. 2 mo.		5
16.	?	6 "	2 yr. 6 mo.		76
17.	1000	4 "	1 yr. 8 mo.		?
18.	?	5 "	30 da.		2072
19.	530	6 "	8 yr. 2 mo. 21 da.	?	?
20.	176.25	? "	1 yr. 11 mo. 5 da.	25.52	
21.	185.85	3½ "	3 yr. 5 mo. 15 da.		?
22.	?	12 "	90 da.		412
23.		6 "	2 yr. 6 mo.	?	690
24.		7 "	6 yr.	157.50	?
25.	?	8 "	1 yr. 6 mo. 24 da.	30.24	
26.	82.50	6 "	5 yr. 8 mo. 12 da.	?	
27.	450	? "	1 yr. 8 mo. 12 da.	61.20	
28.	600	9 "	?		798
29.	375	8 "	?	90	
30.	?	5 "	3 yr.	341.25	
31.	400	7 "	?	68.60	
32.	700	9 "	?	924.70	

220. Discount.

Discount is treated under three heads, viz: True Discount, Bank Discount and Commercial Discount.

a. True Discount. True Discount is an application of Case IV in interest.

1. In *true discount*, the *present worth* is the *principal.*

2. The debt is *amount*, and may be considered as *percentage* if the time element be reduced to 1 yr.

Remark. The rate of interest and the number representing the time in years being factors, if one of them be divided and the other multiplied by the same number, the value of the product is not affected. e. g. If the time should be 3 yr. and the rate .06, we may reduce the time element to 1 yr. by dividing it by 3 if at the same time we multiply the rate, .06, by three.

It is evident that the interest of a given principal for 3 yr. at 6 per cent. equals the interest of the same principal for 1 yr. at 18 per cent.

3. The discount is the percentage. (Interest).

4. The rate of interest is *rate per cent.* The relation (expressed in hundredths) of the debt to the present worth may be the rate per cent.

5. If the debt be considered as *percentage*, the rate per cent. becomes $\frac{100}{100}$ + the given rate multiplied by the number representing the time in years; the time element being divided by itself and thus reduced to 1 year.

6. The time named is the time element, except as stated in 2 and 5.

Example. Required the present worth and true discount of $568.75 due in 2 yr. 3 mo. 15 da. if money is worth 6%.

First form. The time element is reduced to 1 yr. by dividing it by $2\frac{7}{24}$. By multiplying the rate, .06, by $2\frac{7}{24}$, it becomes $.13\frac{3}{4}$.

If the debt, $568.75, be considered as *percentage*, the rate per cent. becomes $1.13\frac{3}{4}$.

We thus have given the percentage, $568.75, and the rate per cent., $1.13\frac{3}{4}$., to find the *base*, which is the required present worth.

Since the percentage is the product of the base and the rate per cent., the base in this example is found by dividing $568.75 by $1.13\frac{3}{4}$. The quotient, $500, is the required base, or present worth.

Second form. $1 at interest for 2 yr. 3 mo. 15 da. at 6%, amounts to $\$1.13\frac{3}{4}$. $1 is, therefore, the present worth of $\$1.13\frac{3}{4}$ due in 2 yr. 3 mo. 15 da. without interest, money being worth 6%. Hence the present worth of $568.75 is as many times $1 as $568.75 are times $\$1.13\frac{3}{4}$, which = 500. 500 times $1 = $500. ∴ etc.

Exercises.

1. Required the present worth and true discount of $436 due 3 years hence, if money is worth 12%.

2. A note of $4800 is due in 4 years. What is its cash value if money is worth 5% per annum?

3. A was offered a lot for $225 cash or $230 in 3 mo. Did he make or lose, and how much by accepting the latter offer?

4. Required the discount of a debt of $864 due in 8 mo. if paid now?

5. A grocer bought 62 barrels of molasses of $31\frac{1}{2}$ gal. each, at 26¢ per gal., on 90 days' time, and sold immediately for $615; how much did he gain if money was worth 8%?

6. What is the present worth and true discount of $27.50 due in 20 months, if money is worth 6%?

7. Hogs were purchased to the value of $1574, one-half payable in 3 mo. and the remainder in 6 mo., without interest. What is the cash value of the stock if money is worth 7%?

8. Which is worth the most $640 in 12 mo., $620 in 6 mo. or $600 in cash, if money is worth 8%?

b. Bank Discount. Bank Discount is the simple interest of a given sum for the time elapsing between the date of discounting and the date of legal maturity. This time is called the term of discount.

1. The face of the obligation is *principal* or *base.*

2. The rate of discount, or rate of proceeds is *rate per cent.*

3. The discount or proceeds is *percentage.*

4. The term of discount is the *time* element.

Remarks. I. If an interest bearing note be discounted at a bank, the *amount* of the note is found at the given rate of interest for the time elapsing between the date of the note and its legal maturity. The *amount* thus found is then discounted at the rate of discount for the term of discount.

2. If the face of the note, (principal), the rate of discount, (rate of interest), and the term of discount (the time) be given to find the discount, (interest), the problem is solved under Case I in interest.

3. If the face of a note, (principal), rate of discount, (rate of interest), and term of discount, (time element), be given to find the proceeds, the discount, which is interest, is found under Case I in interest. The discount is then subtracted from the face of the note; the remainder being the required proceeds; or the rate of proceeds may be used as rate per cent. and the problem solved directly by Case I in interest.

4. If the proceeds, rate of discount or rate of proceeds, and term of discount be given to find the face of the note, the problem is solved under Case IV in interest, considering the proceeds as interest, and the rate of proceeds as rate of interest, first reducing the time element to 1, and increasing the given rate of discount in the same ratio.

Example. Given the proceeds \$493, the term of discount, 63 days, and the rate of discount, .08, to find the face of the note.

Solution. The time element, $\frac{63}{360}$, divided by itself is reduced to 1.

The given rate of discount, .08, multiplied by $\frac{63}{360}$ $=.01\frac{2}{5}$.

The rate of proceeds is, therefore, $.98\frac{3}{5}$.

The following form exhibits the work.

$$\frac{\$493}{1\times.986}=\$500=\text{the face of the note.}$$

[NOTE.—For practical purposes the form of solution usually given in the text books for the above problem is preferable to the solution here given.]

Exercises.

1. Find the bank discount and proceeds of a note of \$80 payable in 60 days, discounted at 8%?

2. A note of \$56 dated Jan. 1, 1884, and payable May 1, 1884, is discounted in bank at 6%; required the proceeds.

3. A note of \$500, dated Dec. 15, 1883, and payable Feb. 18, 1884, is discounted at 9%; required the proceeds and the discount.

4. A note of \$650 with interest at 8% is dated Nov. 30, 1883, and is payable in 90 days. It is discounted Jan. 5, 1884, at 10%; required the proceeds.

5. A note of \$1400 with interest at 10% is dated Jan. 16, 1884, and is payable May 18, 1884. It is discounted April 7, 1884, at 12%; required the proceeds.

6. For what sum must a 60 days note be drawn that when discounted in bank at 6% the proceeds may be \$1000?

7. A owes B $1500; for what sum must a 90 days note be drawn that when discounted in bank at 6%, B may obtain his money?

8. A merchant bought goods for $1621.20 cash, obtaining the money from a bank on a 60 days 7% note; what was the face of the note?

9. Required the cash value of a note of $6780 discounted in bank for 4 mo. 15 da. at 6%?

10. Required the face of a note given in bank that when discounted for 5 mo. 21 da. at 7%, the proceeds shall be $57.97?

c. Commercial Discount. Commercial Discount is a deduction from the face of a bill or other obligation without regard to time; It is also called *per cent. off*, and is effected under Case I in *Percentage.*

Exercises.

1. A merchant bought $1200 worth of goods on 6 mo. time; but paid cash on obtaining a discount of 8% off. What did he pay?

2. 6% off was allowed on a bill of goods amounting to $1878.50; what sum did they cost?

3. A country merchant purchased a bill of goods amounting to $3675 on 4 mo.; but was offered 5% off for cash. Would he gain by borrowing the money from a bank at 8% per annum for the time and paying the cash?

4. What is the cash value of goods listed at $5650, 10% off for wholesale and 5% off for cash?

5. A merchant paid $1.14 per yd. for goods after a discount of 6% had been made from the marked price. What was the marked price?

6. What was the invoice price of goods for which I paid $39 after a discount of 40% had been made?

221. Exchange.

1. The face of a draft is *base.*

2. The exchange or the cost of a draft is *percentage.*

3. The rate of exchange or the rate of cost is rate per cent. The rate of cost is also called the *course of exchange.*

4. In time drafts the time named plus 3 days is the time element.

5. The interest accruing on a time draft is deducted from the face of the draft, for, the bank having the use of the money during the time should, in equity, pay the interest.

6. Make each problem an application of either the first or the second class of the applications of percentage, according as the element of time is or is not involved.

Exercises.

1. Required the cost of a draft on New York at $\frac{1}{4}$% premium.

2. What is the cost of a draft on Chicago at $\frac{3}{8}$% discount?

3. Required the cost of a draft for $560 payable 30 days after sight, exchange $\frac{1}{2}$% premium, and interest 6%.

4. What is the face of a 30 days draft which cost $352.62, exchange being $1\frac{1}{2}$% discount, and interest 6 per cent.?

5. A New York merchant sold $1284 worth of goods. Would he better draw on the purchaser, paying $1\frac{3}{4}$% for collection, or require a sight draft on New York for the bill, exchange being $2\frac{1}{4}$% premium?

6. What is the face of a draft on Indianapolis at 45 days, exchange being at a premium of 3%?

7. How much must I pay in Paris for a draft on Chicago for $4500 at $18\frac{3}{4}$¢ per franc?

8. What must be paid in New York for a draft on London of 560£ at 8% premium?

9. What is the face of a 60 days draft that when sold will yield $1000, exchange $\frac{1}{8}$% discount, and interest 6%?

10. Required the cost of a 30 days draft for $1920 at $\frac{3}{8}$% discount, interest 7%.

11. What is the face of a 60 days draft that can be bought for $3195.20, interest 8% and exchange $1\frac{1}{4}$% premium.

222. Equation of Payments.

Remark. A problem in equation of payments usually contains a number of problems under Case I in interest; the object being to find an equitable time for the payment of several sums of money due at different times.

Exercises.

Remark. For methods of solution see text books.

1. Required the average term of credit for the following debts: $400 due in 3 mo., $500 due in 5 mo. and $700 due in 8 mo.

2. A debt of $2400 is subject to the following conditions: $800 is due in 4 mo., $600 is due in 6 mo., and the remainder is due in 8 mo. What is the average term of credit?

3. A man owes $240 due in 20 days, and $560 due in 30 days. At the end of 16 days he pays $300 and at the end of 24 days he pays $350; when, in equity, should he pay the remainder?

4. A merchant bought goods as follows: April 1, $280 on 3 mo. time, $200 on 4 mo., $300 on 5 mo., and $560 on 6 mo. On what date will a single payment discharge the debts?

5. Wm Smith owes $30 due in 60 days, $100 due in 120 days, and $750 due in 180 days; what is the equated time of payment?

6. Mr. Wallace bought grain on a credit of 90 days to the following amounts:

25th of Jan. . . .	$3750
10th of Feb.	3000
6th of March	2400

On the first day of May he wishes to give his note for the amount. At what time will it mature?

7. A merchant bought goods as follows: Feb. 10, $364; March 12, $375; April 15, $554; May 18, $622. He obtains 6 months' credit on each purchase; at what time can the whole be equitably discharged?

8. What is the average date for paying three bills, due as follows; Jan. 31, $477; Feb. 28, $377; March 31, $777?

9. Required the average time of the following bills, allowing to each term of credit 3 days of grace. —April 3, $500 on 3 mo.; April 4, $200 on 2 mo.; April 4, $200 cash; and April 10, $500 on 3 months.

10. Find the equated time for the payment of the following notes: $350 dated July 12, 1883, for 60 days; $720 dated Sept. 10, 1883, for 90 days; and $1200 dated Nov. 5, for 120 days.

11. A owes $600 due in 6 months, but at the end of 3 months he desires to make a payment sufficiently large that the remainder may not be payable until 6 months after its first date of maturity; how large must be the payment?

12. On tho first day of Jan. B takes a house at a rental of $300 per annum, agreeing to pay the rent quarterly in advance; required the equated time for the payment of the whole.

SECTION XIII.

RATIO AND PROPORTION.

Ratio.

223. Ratio is the relation of one number to another considered as a measure.

Examples. The ratio of 6 to 3 = 2; i. e. if 3 be applied as a measure to 6, the number of applications that can be made is 2. 2 is, therefore, the relation that 6 sustains to 3 considered as a measure.

The ratio of 1 to 2 = $\frac{1}{2}$; of 2 to 3 = $\frac{2}{3}$, etc.

Remarks. 1. The number of times that a divisor is contained in a dividend is the ratio of the dividend to the divisor. A quotient if abstract, may be viewed as a ratio

A multiplier is the ratio of the product to the multiplicand.

2. The part that one number is of another is called the ratio of the first to the second. Ratio is thus related to fractions in that it is the relation of a part to a whole.

3. Since ratio is the relation of *measure*, the two numbers between which a ratio exists are like numbers.

224. The Terms Used. Three terms are concerned in thinking a ratio: viz.—a dividend, a divisor, and a quotient. These are called, respectively, *Antecedent*, *Consequent* and the *Ratio*.

Antecedent. The dividend, or first term of a ratio is called the *antecedent.*

Consequent. The divisor, or second term of a ratio is called the *consequent.*

The Ratio. The quotient of the antecedent by the consequent is called the *ratio*.

225. The Notation of a Ratio.

A colon is used to separate the written terms of a ratio, thus: 6:3. This expression is read the ratio of 6 to 3; it expresses the quotient of 6 by 3.

226. Classes.

Simple. A ratio each of whose terms is a single number either integral, fractional or mixed is called a simple ratio. As $6 : 2$; $\frac{1}{2} : \frac{2}{3}$; $2\frac{1}{2} : 3\frac{1}{3}$.

Compound. Two or more simple ratios, viewed together as to the product of their corresponding terms, constitute a compound ratio.

227. Principles.

Remark. Since the terms *antecedent*, *consequent* and *ratio*, signify dividend, divisor and quotient, respectively, the general principles of division become, by a change in terminology, the principles of ratio.

I. Multiplying the antecedent multiplies the ratio.

II. Multiplying the consequent divides the ratio.

III. Multiplying both antecedent and consequent by the same number does not change the ratio.

IV. Dividing the antecedent divides the ratio.

V. Dividing the consequent multiplies the ratio.

VI. Dividing both antecedent and consequent by the same number does not change the ratio.

Remark. Prin. III may be thus stated: The ratio between like multiples of two numbers equals the ratio between the two numbers. Prin. VI may be thus stated: The ratio between like parts of two numbers equals the ratio between the two numbers.

Proportion.

228. Two equal ratios constitute a proportion.

Remark. The ratios which form a proportion may both be simple, both compound, or one simple and the other compound.

A Simple Proportion. A proportion that consists of two simple ratios is called a simple proportion.

A Compound Proportion. A proportion containing a compound ratio is called a compound proportion.

229. Notation.

A proportion is notated by writing a double colon between the two equal ratios and interpreting it by the word *as*. The proportion, 2:4::5:10, is read 2 is to 4 as 5 is to 10. A proportion is often notated by writing the two equal ratios with the sign of equality between them. The proportion, $2:3 = 4:6$, is read—The ratio of 2 to 3 equals the ratio of 4 to 6.

Remark. The antecedent of the first ratio and the consequent of the second ratio of a proportion are called the *extreme* terms and the other terms the *mean* terms of the proportion.

230. Principle.

The product of the mean terms of any proportion equals the product of its extreme terms.

Demonstration. In a ratio the antecedent is the product of two factors, viz.: the consequent and the ratio. In a proportion the product of the mean terms is composed of three factors, viz.: The ratio and the consequent of the second ratio (which compose the antecedent of the second ratio,) and the consequent of the first ratio.

The product of the extreme terms is composed of three factors, viz.: the ratio and the consequent of the first ratio (which compose the antecedent of the first ratio) and the consequent of the second ratio.

It is thus observed that the factors composing the product of the means are identical with the factors composing the product of the extremes; hence the two products are equal.

231. Forms of Solution.

Remarks. 1. Any problem that is solvable by proportion is readily solved by analysis.

2. It is to be remembered that a ratio exists between like numbers only.

3. In solving a problem by proportion two distinct steps are taken.

a. The arrangement of the ratios, called the statement of the proportion.

b. The reduction of the proportion, or the finding of the unknown term.

Example 1. If 11 bu. of wheat cost $9, what cost 17 bu.?

Written form. *Thought form.*

9:x::11:17 | Since a ratio exists between like numbers only, a ratio exists, in this example, between 11 bu. and 17 bu., and between $9, the cost of 11 bu., and the cost of 17 bu., and these ratios must be equal. 9, which represents the cost of 11 bu., may be made either term of either ratio, and the required number, which we will represent by x, will be the other term of the same ratio. We choose to make 9 the antece dent of the first ratio; then is x the consequent of the first ratio. \$9 is the cost of 11 bu., while \$$x$ is the cost of 17 bu., a greater number than 11, hence the consequent, x, of the first ratio is greater than its antecedent, 9; and since the two ratios must be equal to form a proportion, the consequent of the second ratio must be greater than its antecedent. Hence we make 11 (bu.) the antecedent and 17 (bu.) the consequent of the second ratio.

We now have the two extremes and one mean of a proportion. Since the product of the extremes equals the product of the means, the required mean is found by dividing the product of the extremes by the given mean, (using the numbers abstractly in performing the operation.) The required mean is 13.91. ∴ 17 bu. cost \$13.91.

Example 2. If 75 men can build a wall 50 ft. long, 8 ft. high and 3 ft. thick, in ten days, how long will it take 100 men to build a wall 150 ft. long, 10 ft. high and 4 ft. thick?

Written form.

$$\begin{array}{r} x:10::75:100 \\ 150:50 \\ 10:8 \\ 4:3 \end{array}$$

Thought form.

In this problem a ratio exists between 75 men and 100 men; 50 ft. length and 150 ft. length; 8 ft. hight and 10 ft. hight; 3 ft. thickness and 4 ft. thickness; 10 days and the required number of days which we may represent by x.

10 (days) may be made either term of either ratio, and x (days) will be the other term of the same ratio. We will make 10 the consequent of the first ratio, then will x be the antecedent of the first ratio.

10 days are required for 75 men to do a work while x days are required for 100 men to do the work. 100

men require a less number of days than 75 men, hence the antecedent, x, of the first ratio is less than its consequent, 10; and since the two ratios must be equal to form a proportion, the antecedent of the second ratio is less than its consequent; we therefore, write 75 as the antecedent and 100 as the consequent of the second ratio.

Again,—10 days are required to build a wall 50 feet long, while x days are required to build a wall 150 feet long, a greater length than 50 feet, hence x is greater than ten, and as the two ratios must be equal to form a proportion, the antecedent of the second ratio must be greater than its consequent; we therefore write 150 as the antecedent and 50 as the consequent of the second ratio.

Again,—10 days are required to build a wall 8 ft. high, while x days are required to build a similar wall 10 ft. high, a greater hight than 8 ft., hence x is more than ten, and since the two ratios must be equal to form a proportion, the antecedent of the second ratio is greater than its consequent; we therefore write 10 as the antecedent and 8 as the consequent of the second ratio.

Again,—10 days are required to build a wall 3 feet thick, while x days are required to build a similar wall 4 ft. thick, a greater thickness than 3 ft., hence x is more than 10, and, since the two ratios must be equal to form a proportion, the antecedent of the second ratio is greater than its consequent; we therefore, write 4 as the antecedent and 3 as the consequent of the second ratio.

We now have given the factors of the means and the factors of one extreme of a compound proportion to find the other extreme.

Since the product of the extremes equals the product of the means, the required extreme is found by dividing the product of the factors of the means by the product of the factors of the given extreme. [The work of reducing the proportion may be shortened by considering the factors of the means as factors of a dividend and the factors of the given extreme as factors of a divisor, and canceling common factors.] The required extreme is found to be 37½. ∴ the required time is 37½ days.

Exercises.

1. If 70 horses cost $3500, what cost 160 horses?

2. If 5 lb. of coffee cost $1.35, what cost 9 lb.?

3. If 12 tons of hay cost $87, what cost 17 tons?

4. If a farm cost $4800, what cost $\frac{3}{5}$ of it?

5. If $\frac{7}{8}$ yd. of silk cost $1.65, what cost 8 yd.?

6. If 19 acres of land sell for $570, required the price of 40 acres.

7. How many men will do as much work in 84 days as 8 men do in 126 days?

8. If I borrow $3500 for 30 days, for what time may I return $900 to requite the favor?

9. If a 10 cent loaf weighs 1 lb. 2 oz. when flour is $7½ per barrel, what should it weigh when flour is $6 per barrel?

10. What cost 147 bu. of corn if 35 bu. cost $28½?

11. If 15 sheep cost $15.90, how many sheep can be bought for $155.82?

13. At 36 d. per $½, what is 1£ worth in U. S. money?

14. In what time can a man pump 54 barrels of water, if he pump 24 barrels in 1 hr. 14 min?

15. If 10 bales of cotton can be carried 115 miles for $8, how far can 15 bales be carried for the same money?

16. If $1200, in 2 yr. 3 mo. gain $162 interest at 6%, how much will $800 gain in 3 yr. 3 mo. at 8%?

17. A mill makes 1265 barrels of flour by running 10 hr. per day for 13 days; how many barrels would it make in 26 days, running 15 hours per day?

18. If it cost $28 to carpet a room 12 ft. by 7 ft., what will it cost to carpet a room 30 ft. by 18 ft.?

19. If 60 men dig a canal 80 rd. long, 12 ft. wide and 6 ft. deep in 18 days, working 16 hr. per day, how many men will be required to dig a ditch 30 rd. long, 9 ft. wide and 4 ft. deep in 24 days, working 12 hours per day?

10. A man gained $3155 on the sale of 91 horses; how much would he have gained on 245 horses sold at the same rate?

21. If a tank $17\frac{1}{2}$ ft. long, $11\frac{1}{2}$ ft. wide and 13 ft. deep, contain 546 barrels, how many barrels will a tank hold that is 16 ft. long, 15 ft. wide and 7 ft. deep?

22. If the interest $346 for 1 yr. 8 mo. is $32, what is the interest of $215.25 for 3 yr. 4 mo. 24 da. at the same rate?

23. A certain bin is 8 ft. by $4\frac{1}{5}$ ft. by $2\frac{1}{2}$ ft. and its capacity is 75 bu.; how deep must a bin be to contain 450 bu., if it be 28 ft. long by $3\frac{1}{2}$ ft. wide?

24. If 366 men in 5 days of 10 hr. each, can dig a trench 70 yards long, 3 yards wide, and 2 yards deep; what length of trench 5 yards wide and 3 yards deep, can 240 men dig in 9 days of 12 hr. each?

25. If it cost $64 to pave a walk 3 ft. wide and 32 ft. long, what will it cost to pave a walk 5 ft. wide and 64 ft. long?

26. If 7 men, working 10 hr. per da., make 6 wagons in 21 days, how many wagons can 12 men make in 16 days working 9 hours per day?

27. If $8 yield $2 interest in 3 mo. 12 da, at 9%, how much int. will $140 yield in 9 mo. 22 da. at 6%?

28. If it cost $320 to buy the provisions consumed in 8 mo. by a family of 7 persons, how much at the same rate will it cost to feed a family of twice the number of persons for ¼ as much time?

29. If a stone 2 ft. long 10 in. wide, and 8 in. thick, weigh 72 lb., required the weight of a similar stone 6 ft. long, 15 in. wide and 6 in. thick.

30. If 300 lb. of wool at 28¢ per lb. are exchanged for 36 yd. of cloth 1⅓ yd. wide, how many lb. of wool at 35¢ per lb., should be given for 20 yd. of cloth ¾ yd. wide?

Proportional Parts.

232. A number is divided into proportional parts if separated into parts whose ratio equals the ratio of given numbers.

Example. Divide 40 into two parts that are to each other as 3 to 5.

40 is to be divided into parts that are respectively, equi-multiples of 3 and 5; i. e., one of the required parts of 40 is as many times 3 as the other part is times 5.

40, the sum of the required parts is, therefore, the *same* number of times the sum of 3 and 5. 40 is 5 times the sum of 3 and 5. 5 times $3 = 15$ and 5 times $5 = 25$. Hence 15 and 25 are the required parts into which 40 is to be divided.

Principles. *a*. The sum of the proportionals is to the sum of the required numbers as the less proportional is to the less of the required numbers.

b. The sum of the proportionals is to the sum of the required numbers as the greater proportional is to the greater of the required numbers.

SECTION XIV.

INVOLUTION AND EVOLUTION.

233. Table for Determining the Number of Terms (Orders) in the Square Root of a Given Square Number.

$1^2=1$
$9^2=81$
$10^2=100$
$99^2=9801$
$100^2=10000$
$999^2=998001$
$1000^2=1000000$
$9999^2=99980001$
$.1^2=.01$
$.9^2=.81$
$.01^2=.0001$
$.99^2=.9801$
$.001^2=.000001$
$.999^2=.998001$
etc.

From the marginal table it is seen that a square consisting of one or two terms contains one term in its square root; a square of three or four terms contains two terms in its square root; a square of five or six terms contains three terms in its square root, etc.

The principle may be formulated thus:

If a square number expressed in the decimal scale be separated into periods of two terms each, beginning at the interval between units and tenths, there are as many terms in its square root as there are periods in the square.

234. Table Indicating the Lowest Place which a Significant Figure can Occupy in the written Product of the Given Factors.

Remark. The following abbreviations are used —u for units, t for tens, h for hundreds, th for thousands, tth for ten thousands, .t for tenths, .h for hundredths, .th for thousandths, .tth for ten thousandths, etc.

u² = u.	t × u = t.	u × .t = .t
t² = h.	h × u = h.	u × .h = .h
h² = tth.	th × u = th.	u × .th = .th
th² = m.	h × t = th.	.t × .h = .th
.t² = .h	th × t = tth.	.t × .th = .tth
.h² = .tth	etc.	etc.
etc.		

235. A Number Consisting of Tens and Units Involved to the Second Power.

Example. $\overline{34}^2 =$ what?

Remark. In the light of the definition of a second power, 34 is squared by multiplying it by itself. In order that the partial products be readily seen, they should not be combined by addition until they are all found.

The involution may be shown in the form that follows.

$$\begin{array}{l} 34 \\ 34 \\ \hline 16 = 4^2 = u^2. \\ \left.\begin{array}{l} 120 = 30 \times 4 \;\; = t. \times u \\ 120 = \;\; 4 \times 30 = t \times u \end{array}\right\} = 2t \times u. \\ 900 = \overline{30}^2 = t^2. \\ \hline 1156 = 34^2 = t^2 + 2t \times u + u^2. \end{array}$$

Remark. If any number consisting of tens and units be squared, it is readily seen that the same steps will be taken as in the above example, i. e., the square of a number consisting of tens and units is composed of the square of the tens plus twice the tens multiplied by the units plus the square of the units.

236. A Number Consisting of Hundreds, Tens and Units Involved to the Second Power.

Example. $\overline{234}^2 =$ what?

$$
\begin{array}{r}
234 \\
234 \\
\hline
16 = 4^2 = u^2. \\
120 = 30 \times 4 = t \times u. \\
800 = 200 \times 4 = h \times u. \\
120 = 4 \times 30 = t \times u. \\
900 = \overline{30}^2 = t^2. \\
6000 = 200 \times 30 = h \times t. \\
800 = 4 \times 200 = h \times u. \\
6000 = 30 \times 200 = h \times t. \\
40000 = \overline{200}^2 = h^2. \\
\hline
54756 = \overline{234}^2 = h^2 + 2\,h \times t + t^2 + 2[h+t] \times u + u^2.
\end{array}
$$

Remark. If any number consisting of h, t and u be squared the same steps will be taken as in the above example; i. e., the square of a number consisting of h, t and u is composed of the square of the h plus twice the h multiplied by the t plus the square of the t plus twice the sum of the h and t multiplied by the u plus the square of the u.

237. A Number Consisting of Tenths and Hundredths Involved to the Second Power.

Example. $\overline{.24}^2 =$ what?

$$
\begin{array}{l}
.24 \\
.24 \\
\hline
.0016 = \overline{.04}^2 = .h^2. \\
.008 \;\; = .2 \times .04 = .t \times .h. \\
.008 \;\; = .04 \times .2 = .t \times .h. \\
.04 \;\;\;\; = .2^2 = .t^2. \\
\hline
.0576 = \overline{.24}^2 = .t^2 + 2.t \times .h + .h^2.
\end{array}
$$

Remark. It is thus seen that the square of a number consisting of tenths and hundredths is composed of the square of the tenths plus twice the tenths multiplied by the hundredths plus the square of the hundredths.

238. A Number Consisting of Tenths, Hundredths and Thousandths Involved to the Second Power.

Example. $\overline{.246}^2 =$ what?

$$
\begin{array}{lll}
 & .246 & \\
 & .246 & \\
\hline
.000036 & = \overline{.006}^2 = .th^2. \\
.00024 & = .04 \times .006 = .h \times .th. \\
.0012 & = .2 \times .006 = .t \times .th. \\
.00024 & = .006 \times .04 = .h \times .th. \\
.0016 & = .04^2 = .h^2. \\
.008 & = .2 \times .04 = .t \times .h. \\
.0012 & = .006 \times .2 = .t \times .th. \\
.008 & = .04 \times .2 = .t \times .h. \\
.04 & = .2^2 = .t^2. \\
\hline
.060516 & = .246^2 = .t^2 + 2.t \times .h + .h^2 + 2[.t + .h] \times .th + .th^2.
\end{array}
$$

Remark. It is thus seen that the square of a number consisting of *tenths*, *hundredths* and *thousandths* is composed of the square of the tenths *plus* twice the tenths multiplied by the hundredths, *plus* the square of the hundredths, *plus* twice the sum of the tenths and the hundredths multiplied by the thousandths, *plus* the square of the thousandths.

239. A Number Consisting of Tens, Units and Tenths Involved to the Second Power.

Example. $\overline{32.6}^2 =$ what?

$$
\begin{array}{rl}
32.6 & \\
32.6 & \\
\hline
.36 & = .6^2 = .t^2. \\
1.2 & = 2 \times .6 = u \times .t. \\
18. & = 30 \times .6 = t \times .t. \\
1.2 & = .6 \times 2 = u \times .t. \\
4. & = 2^2 = u^2. \\
60. & = 30 \times 2 = t \times u. \\
18. & = .6 \times 30 = t \times .t. \\
60. & = 2 \times 30 = t \times u. \\
900. & = \overline{30}^2 = t^2. \\
\hline
1062.76 & = \overline{32.6}^2 = t^2 + 2t \times u + u^2 + 2[t + u] \times .t + .t^2.
\end{array}
$$

Remark. It is thus seen that the square of a number consisting of *tens*, *units* and *tenths* is composed of the square of the tens, *plus* twice the tens multiplied by the units, *plus* the square of the units, *plus* twice the sum of the tens and the units multiplied by the tenths, *plus* the square of the tenths.

240. General Formula Embodying the Involution of any Number in the Decimal Scale to the Second Power.

A general principle for exhibiting the elements which compose the second power of a number consisting of any number of terms (orders) in the decimal scale may be thus stated:—

The square of a number consisting of any number of terms is composed of the square of the first, or highest term *plus* twice the first term multiplied by the second, *plus* the square of the second, *plus* twice the sum of the first two terms multiplied by the third, *plus* the square of the third, *plus* twice the sum of the first three terms multiplied by the fourth, *plus* the square of the fourth, *plus* twice the sum of the first four terms multiplied by the fifth, *plus* the square of the fifth, *plus*, etc.

Remark. A fraction is involved to the second power by squaring both its terms.

241. Evolution of the Second Root.

Example 1. $\sqrt{1156}$ = what?

Written form.	*Thought form.*
1156(34 900 6t)256 240 16 16	Since there are four terms in the square there are two terms in its square root, viz.—*tens* and *units*. The square of a number consisting of t and u is composed of the $t^2+2t\times u+u^2$.

The sq. of t = h; hence the h of the power contain the sq. of the t of the root. The greatest square number of h in 11 h is 9 h, the sq. root of which is 3 t. 9 h taken from the power leave 256. This remainder contains a product of which twice the tens of the root, or 6 t, is one factor and the units of the root is the other factor. Since t×u = t, the 25 t of the remaining part of the power contain the required product. Since 25 t contain a product of which one factor is 6 t, the other factor is found by dividing 25 t by 6 t; the quotient, 4, is supposed to be the units of the root. The product of 6 t by 4 = 24 t, which taken from the remaining part of the power leave 16. This remainder must contain the square of the u of the root. The square of 4 = 16, which taken from the remaining part of the power leave nothing. 1156 is thus found to be a square number of which 34 is the square root.

Example 2. $\sqrt{54756}$ = what?

Written form.

```
     54756(234
     4
4h)  14756
     12
      2756
         9
46t)  1856
      184
        16
        16
```

Thought form.

Since there are five terms in the given square there are three terms in its square root, viz.—h, t and u.

The square of a number consisting of h, t and u, is composed of the square of the h, *plus* twice the h multiplied by the t, *plus* the square of the t, *plus* twice the sum of the h and the t multiplied by the u, *plus* the square of the u.

The sq. of h = tth; hence the tth of the power contain the sq. of the h of the root. The greatest square number of tth in 5tth is 4tth; the square root of which is 2h. 4tth taken from the power leave 14756. This remainder contains the product of two factors, one of which is twice the h of the root, or 4 h, and the other the t of the root. Since $h \times t = th$, the th of the remaining part of the power contain the required product . . . . Since 14 th contain a product of which one factor is 4 h, the other factor is found by dividing 14 th by 4 h, the quotient, 3 t, is supposed to be the tens of the root. The product of 4h by $3t = 12$ th, which, taken from the remaining part of the power leave 2756. This remainder contains the sq. of the t of the root. The square of $3t = 9h$, which taken from the remaining part of the power leave 1856. This remainder contains the product of two factors, one of which is twice the sum of the h and the t of the root, or 46 tens, and the other the units of the root.

Since $t \times u = t$, the tens of the remaining part of the power contain the required product.

Since 185 tens contain a product of which one factor is 46t, the other factor may be found by dividing 185 tens by 46 tens, the quotient, 4, is supposed to be the u of the root. The product of 46t by $4 = 184t$, which taken from the remaining part of the power leave 16. This remainder must contain the square of the u of the root. The sq. of $4 = 16$, which taken from the remaining part of the power leave nothing.

54756 is thus found to be a square number the square root of which is 234.

Remark. It is observed that the *tens* and *units* of the root as found are *supposed* to be the correct numbers for those orders, respectively. Sometimes the obtained quotient may be so great that its product by the divisor will, when subtracted from the remaining part of the power, leave a remainder too small to contain the partial products that are yet to be taken out. If a quotient is too great a less quotient must be used.

Exercises.

Evolve the second root of 4096; 9216; 7569; .0676; 54.76; 5476; 10201; 7744; 10509; 6889; 11025; 11236; 3844; .9409; 53.1441; 21025; 173056; 998001; 67305616.

242. Table for Determining the Number of Terms in the Cube Root of a Given Cube Number.

$1^3=1$	
$9^3=729$	
$10^3=1000$	
$99^3=970299$	
$100^3=1000000$	
$999^3=997002999$	

From the table it is seen that a cube number consisting of three terms or less, contains one term in its cube root; that a cube number consisting of four, five or six terms contains two terms in its cube root; that a cube number consisting of seven, eight, or nine terms contains three terms in its cube root; etc.

243. Principle. There are as many terms in the cube root of a cube number as there are periods of three terms each in the number, counting from the interval between units' and tenths' orders.

Remarks. 1. The highest period in a cube integer may contain but one or two terms.

2. The terms of the highest period of a cube decimal fraction may be wholly or partly represented by zeros.

3. The lowest period of a cube integer may be wholly or partly represented by zeros. Every period in a cube number may be partly represented by zeros.

244. Table Indicating the Lowest Place in which a Significant Figure can Occur in the Written Cube of Numbers in the Orders Named.

$u^3 = u.$	$.t^3 = .th.$
$t^3 = th.$	$.h^3 = .m.$
$h^3 = m.$	$.th^3 = .b.$
$th^3 = b.$	Etc., etc.

[For other indicated products see corresponding table, page 160.]

245. A Number Consisting of Tens and Units Involved to the Third Power.

Example. $34^3 =$ what ?

In the light of the definition of a 3d power, a number is cubed by multiplying its square by the number itself.

$$
\begin{array}{rlr}
(30+4)^2 = & 30^2 + 2 \text{ times } 30 \times 4 + 4^2 = & 1156 \\
& 30 + 4 & 34 \\
\hline
& 30^2 \times 4 + 2 \text{ times } 30 \times 4^2 + 4^3 & 4624 \\
30^3 + 2 \text{ times } 30^2 \times 4 & + 30 \times 4^2 & 3468 \\
\hline
30^3 + 3 \text{ times } 30^2 \times 4 + 3 \text{ times } 30 \times 4^2 + 4^3 = & & 39304
\end{array}
$$

Hence.—The cube of a number consisting of tens and units is composed of the cube of the tens, *plus* 3 times the square of the tens multiplied by the units *plus* 3 times the tens multiplied by the square of the units, *plus* the cube of the units.

246. A Number Consisting of Hundreds, Tens and Units, Involved to the Third Power.

Example. $\overline{234}^3$ = what?

$234 = 200 + 30 + 4.$

$(200 + 30 + 4)^2 = 200^2 + 2$ times $200 \times 30 + 30^2 + 2$ times $(200 + 30) \times 4 + 4^2$ [See Art. 236.]

Re-writing this formula after removing the parenthesis we have—

$200^2 + 2$ times $200 \times 30 + 30^2 + 2$ times $200 \times 4 + 2$ times $30 \times 4 + 4^2$

Multiply this formula by. . . . . $200 + 30 + 4$

and write the partial products below.

$$
\begin{array}{r}
4^3 \\
2 \text{ times } 30 \times 4^2 \\
2 \text{ times } 200 \times 4^2 \\
30^2 \times 4 \\
2 \text{ times } 200 \times 30 \times 4 \\
200^2 \times 4 \\
30 \times 4^2 \\
2 \text{ times } 30^2 \times 4 \\
2 \text{ times } 200 \times 30 \times 4 \\
30^3 \\
2 \text{ times } 200 \times 30^2 \\
200^2 \times 30 \\
200 \times 4^2 \\
2 \text{ times } 200 \times 30 \times 4 \\
2 \text{ times } 200^2 \times 4 \\
200 \times 30^2 \\
2 \text{ times } 200^2 \times 30 \\
200^3
\end{array}
$$

Upon comparing these partial products we find that we have $200^3 + 3$ times $200^2 \times 30 + 3$ times $200 \times 30^2 + 30^3 + 3$ times $200^2 \times 4 + 6$ times $200 \times 30 \times 4 + 3$ times $30^2 \times 4 + 3$ times $200 \times 4^2 + 3$ times $30 \times 4^2 + 4^3$.

Factoring the 5th, 6th and 7th terms of this formula reduces them to 3 times $(200^2 + 2$ times $200 \times 30 + 30^2) \times 4$. Observing that the parenthetical quantity equals $(200 + 30)^2$, the reduced expression becomes 3 times $(200 + 30)^2 \times 4$.

Factoring the 8th and 9th terms of the formula above, reduces them to 3 times $(200 + 30) \times 4^2$.

Re-writing the formula, substituting 3 times $(200 + 30)^2 \times 4$ for the 5th, 6th and 7th terms, and 3 times $(200 + 30) \times 4^2$ for the 8th and 9th terms, we have as the completed formula— $200^3 +$ 3 times $200^2 \times 30 +$ 3 times $200 \times 30^2 + 30^3 +$ 3 times $(200 + 30)^2 \times 4 +$ 3 times $(200 + 30) \times 4^2 + 4^3$.

Any number consisting of h, t and u may be cubed in the same manner as the above; hence, the cube of a number consisting of hundreds, tens and units is composed of the cube of the hundreds, *plus* 3 times the square of the hundreds multiplied by the tens, *plus* 3 times the hundreds multiplied by the square of the tens, *plus* the cube of the tens, *plus* 3 times the square of the sum of the hundreds and tens multiplied by the units, *plus* 3 times the sum of the hundreds and tens multiplied by the square of the units, *plus* the cube of the units.

This formula is abbreviated thus: $h^3 + 3h^2 \times t + 3\,h \times t^2 + t^3 + 3\,[h + t]^2 \times u + 3\,[h + t] \times u^2 + u^3$.

The cube of a number consisting of th, h, t and u is composed of $th^3 + 3\,th^2 \times h + 3\,th \times h^2 + h^3 + 3(th + h)^2 \times t + 3(th + h) \times t^2 + t^3 + 3(th + h + t)^2 \times u + 3(th + h + t) \times u^2 + u^3$.

Remarks. 1. A careful study of the above forms will discover the law by which a formula may be constructed for the cube of a number consisting of any nnmber of orders in the decimal scale.

2. The cube of a fraction is found by cubing both its terms.

Evolution of the Third Root.

Example. $\sqrt[3]{103823}$ = what?

Written form.	*Thought form.*
103823(47 64 48h)39823 336 6223 588 343 343	Since there are six terms in the power, there are two terms in its cube root; viz.—t and u. The cube of a number consisting of tens and units is composed of $t^3 + 3t^2 \times u + 3t \times u^2 + u^3$. The cube of tens is thousands, hence the th of the power contain

the cube of the t of the root. The greatest cube number of th in 103 th is 64 th, the cube root of which is 4t. The power diminished by 64th = 39823, which contains a product of which one factor is 3 times the square of the t of the root and the other is the u of the root. 3 times the square of 4 t = 48h. Since the product of h by u = h, the hundreds of the remaining part of the power contain the required product. 398h ÷ 48h = 7, which is supposed to be the u of the root. 7 times 48h = 336 h, which taken from the remaining part of the power leave 6223. This remainder contains the product of 3 times the tens of the root by the square of the u of the root. 12 t × 49 = 588t, which taken from the remaining part of the power leave 343. This remainder must contain the cube of the u of the root. The cube of 7 = 343, which taken from the remaining part of the power leaves nothing. We thus find that 103823 is a cube number and that 47 is its cube root.

Remarks. 1. In the light of the foregoing form the observant pupil will readily evolve the third root of any cube number.

2. For the application of cube root in determining lines see text books on Arithmetic.

Exercises.

Evolve the third root of—32768; 68921; 148877; 250047; 287.496; 328.509; .389017; .438976; 1061208; 1092727; 1124864; 1157.625; 1.191016; 870983875; 12977875; 5735339; 300763; 912673.

Cube Root by Endings.

Example. 12167 is a cube number; what is its cube root?

Solution. *a.* Since 7 is the units of the cube, 3 must be the units of its cube root.

b. The greatest cube in 12, (the left period) is 8, the cube root of which is 2; hence 2 is the tens of the cube root. The required root is, therefore, 23.

SECTION XV.

TEST PROBLEMS.

1. If a merchant mark his goods 25% above cost, and sell them at 25% below the marked price, does he gain or lose and at what rate %?

2. A load of hay weighs 15 cwt. 21 lb.; if 2 cwt. 11 lb. be sold, what part of the load remains?

3. Multiply .025 by 2.5.

4. A owned $\frac{3}{7}$ of a store and sold to B $\frac{4}{9}$ of his share and to C $\frac{2}{5}$ of his share; what part of it did he still own?

5. Required the cost of $\frac{7}{8}$ of a yard of cloth, if $\frac{5}{6}$ of a yard cost \$$7\frac{2}{3}$.

6. The longitude of Washington is 76° 56′ west of London, what change would it be necessary to make in a time-piece in coming from London to Washington?

7. At what rate per cent. will $380 in 7 yr. 3 mo. yield $165.30 interest?

8. $\frac{3}{4}$ = what part of 9? What per cent. of 9?

9. If 9½ eggs weigh a pound, and a pound of eggs equal a pound of steak as food, at what price per dozen must eggs be bought in place of steak at 22 ¢ per lb.?

10. What is the weight of the air in a room 25 ft. by 20 ft. by 12 ft., water weighing 770 times as much as air, and a cubic foot of water weighing 1000 oz. Avoirdupois?

11. If lead is 11.445 times as heavy as water, what is the weight of a piece of lead 1 m. by 2 dm. by 5 cm.?

12. A lot of goods was marked 40% above cost; if sold at 30% less than the marked price, was there a gain or loss and at what rate per cent.?

13. Which would yield the better pay, a 7% bond at 115 or a 6% bond at 98, and at what rate better?

14. If a merchant sell flour at $9 per barrel, and wait 6 months for his pay, at what price could he afford to sell for cash if money is worth 2% a month?

15. For what sum must a 3 months' note be drawn so that when discounted by a bank at 7 per cent. I may get $400?

16. At what quotation must I buy a 6% stock to make as good an investment as from a 4% stock at 75?

17. A traveling salesman is allowed 12% commission on his sales; his employer's rate of profit is 20% on the goods sold; what is the first cost of goods which the salesman sells for $7.66?

18. A water tank is 3 ft. deep, 4 ft. long, and 4 ft. wide; it is supplied from a flat roof 20 ft. by 30 ft.; what depth of rain must fall to fill the tank?

19. Required the present worth of $1320 due in 3 yr. 4 mo. without interest, if money is worth 6%.

20. A man bought a horse for $72 and sold it for 25 per cent. more than it cost and 10% less than the asking price; what did he ask for the horse?

21. 2 ft. 9 in. = what part of a rod?

22. Cincinnati is 7° 49′ west of Baltimore; when it is noon at Baltimore, what time is it at Cincinnati?

23. A man bought stock at 25% below par, and and sold it at 25% above par; required his rate of gain.

24. How many yards of carpet $\frac{3}{4}$ of a yard wide will cover a floor 18 ft. by 15 ft.?

25. $\frac{7}{8} \div \frac{3}{5} =$ what?

26. Reduce to equivalent fractions with l. c. d. $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$.

27. How find the area of a circle if only the radius be given?

28. If a wheel turn 17° 30′ in 35 minutes, in what time will it make a revolution?

29. If rosin be melted with 20% of its weight of tallow, what % of the weight of the mixture is tallow?

30. Required the weight in kilograms of a bar of iron 3.6 m. long, 6 cm. wide, and 2 cm. thick, if iron is 7.8 times as heavy as water.

31. With gold at 103 what rate of interest do I make on a $1000 5-20 bond bought at 106?

32. A and B are partners for 1 yr., A putting in $2000 and B $800; how much more must B put in at the end of 6 months to receive one-half the profits?

33. How many grams does a Dl. of water weigh?

34. Demonstrate the *principle:*—The product of the means of a proportion equals the product of the extremes.

35. The surface of the earth contains about 144000000 sq. mi. of water, and about 53000000 sq. mi. of land. What % of the earth's surface is water?

36. $\frac{3}{8} \times \frac{5}{7} =$ what?

37. If a watch sell for $60 at a loss of 22%, for what should it sell to gain 30 per cent.?

38. A broker bought stock at 8% premium and sold it at 9% discount thereby losing $510; how many shares did he buy?

39. What is the net tax in a town whose taxable property is assessed at $430000, at 12 mills per dollar, 5 per cent. being paid for collection?

40. The difference of time between two places is 2 hr. 15 min. 10 sec.; required their difference in longitude.

41. Reduce .037 lb. Av. to drams.

42. Reduce 84.5 ars. to square meters.

43. An agent received $484.50 with which to buy sheep after deducting his commission at 2%; how much money did he spend for sheep?

44. A policy for $2675 cost $53.30; find the rate of insurance.

45. The difference of time between two places is 45 min. 30 sec., and the place having the earlier time is in longitude 85° 40′ west; required the longitude of the other place.

46. Reduce .096 of a bu. to the decimal of a pint.

47. A note of $125 dated May 3, 1883, and payable in sixty days, with interest at 5%, was discounted June 18, 1883, at 10%; required the proceeds.

48. How many bu. of wheat will fill a bin 8 ft. by 5 ft. by 4 ft.?

49. How many meters of carpet .7 m. wide will cover a floor 4 m. by 4.5 m.?

50. Add $\frac{2}{3}$ $\frac{3}{5}$ and $\frac{4}{9}$.

51. Multiply $\frac{3}{7}$ by 2.3.

52. Divide 2.3 by $\frac{3}{7}$.

53. $\frac{3}{7}$ = what part of 2.3? What % of 2.3?

54. Reduce 135 sq. rd. 54 sq. ft. to the decimal of an acre.

55. How much money must I remit to my agent to buy goods and pay himself $24 commission at $1\frac{1}{3}$%?

56. A sold a horse to B and gained $\frac{1}{5}$ of its cost; B sold it for $80 and lost $\frac{1}{6}$ of what it cost him; how much did A pay for the horse?

57. Divide 3.45 by 1.5.

58. A garden contains 800 sq. rd. and is $33\frac{1}{3}$ rd. long; how wide is it?

59. What per cent. of $\frac{4}{5}$ is $\frac{2}{3}$?

60. What is $\frac{1}{2}$ per cent. of .5?

61. 428 is 7% more than what number?

62. A man owes $300 due in 4 mo., $600 due in 5 mo., and $100 due in 6 mo.; if he pay $\frac{1}{2}$ of his indebtedness in 2 mo., when in equity should he pay the balance?

63. What is the difference between the true and the bank discount of $359.50 for 90 days without grace, money being worth 8%?

64. A board is 20 ft. long and 9 in. wide; what is it worth at $30 per M?

65. At what time between 6 and 7 o'clock are the hour and minute hands of a watch together?

66. A broker bought 60 shares of stock at $106\frac{1}{2}$, received a 5% dividend and then sold at 104; did he gain or lose and how much?

67. How many ft., board measure, each board being 18 inches wide, can be cut from a squared log 16 ft. long, 18 in. wide and 10 in. thick, allowing $\frac{1}{8}$ in. for each cut of the saw?

68. If 12 men can do a piece of work in $5\frac{1}{2}$ days, in how many days can 8 men and 5 boys do it, 1 man doing the work of $2\frac{1}{2}$ boys?

69. If a note of $5000, for 4 mo., at 6% int. per annum, be discounted in bank, on day of making, at at 8% per annum, what will be the proceeds?

70. If it cost $312 to fence a field 216 rd. by 24 rods, what cost the fence of a sq. field of equal area?

71. A merchant bought goods at 20 cents per yd. and sold them at 40% profit, after allowing his customers 12½% discount off; what was the marked price?

72. A vessel, at noon, sails due north; after a certain time an observation shows the sun to have sunk toward the west 2 signs 15 degrees; how long has the vessel been sailing?

73. I sell a bill on London for £1675, at the rate of 24.3 cents per shilling; how much do I receive?

74. If $6000 of 6% stock be sold at 90, and the proceeds invested in 10% stock at 105, what will be the change in the income?

75. I buy $1500 worth of goods at 4 mo., $850 at 3 mo., $1750 at 5 mo.; what is the equated time for the payment of the whole?

76. The square root of .1369 *plus* the square root of 1296 equals what?

77. A owes B $1800; B offers to allow 5% off for cash; A pays $1425, how much is still due?

78. Evolve the second root of 15625.

79. What is the surface of a cube which contains 8 times the volume of a cube whose edge is $\frac{1}{7}$ of a foot?

80. What is the capacity of a cylinder 20ft. long, whose radius is 2 ft.?

81. I bought goods in Europe, paid 20% duties, a commission of 2% upon duties and cost, and sold them at $10 per yard, clearing 34% on invoice price; required the entire cost per yard.

82. A travels 5½ hours at the rate of 6 miles per hr., B then follows from the same point at the rate of 9 mi. per hr.; how long will it take B to overtake A?

83. At 24.2 cents per shilling, what cost £1050?

84. What principal in 3 yr. 4 mo. 24 da. at 5% will amount to $761.44?

85. For what sum must a note dated April 5, 1883, for 90 da. be drawn, that when discounted at 7%, April 21, 1883, the proceeds may be $650?

86. A room is 26 ft. long, 16 ft. wide and 12 ft. high; what is the distance from one of the lower corners diagonally to the opposite upper corner?

87. St. Petersburg is 30° 19′ east longitude, and Indianapolis is 86° 5′ west longitude; when it is 3 a. m. at St. Petersburg, what is Indianapolis time?

88. Reduce 492 dekagrams to quintals.

89. How many bricks, each 8 in. long, 4 in. wide and $2\frac{1}{2}$ in thick, will be required for a wall 120 ft. long 8 ft. high and 1 ft. 4 in. thick, no allowance being made for mortar?

90. If 6 men can build a wall 20 ft. long, 6 ft. high and 4 ft. thick in 16 days, in what time can 24 men build a wall 200 ft. long, 8 ft. high and 4 ft. thick?

91. A man bought a square farm containing 140 acres and 100 sq. rd.; required the length of one side.

92. Evolve the 3rd root of 592704. Of 2985984.

93. A farmer exchanged 100 bu. of wheat at $1.25 per bu. for corn at $.37½ per bu.; how many bu. of corn did he receive?

94. The sum of two numbers is 785; their difference is 27; what are the numbers?

95. Divide $\frac{108}{11}$ by $\frac{4}{11}$.

96. Reduce $\frac{9}{25}$ to a decimal fraction.

97. How many loads are contained in a pile of wood 40.16 ft. long, 7.04 ft. high and 4 ft. wide, if each load contains $1\frac{1}{4}$ cords?

98. If I exchange $12000 of 8% stock at 115 for

5% stock at 69, do I gain or lose on annual income, and how much?

99. How large a sight draft can be bought for $259.52, exchange being $1\frac{3}{8}$% premium?

100. What are the cubical contents of a cylinder that will just enclose a sphere 9 in. in diameter?

101. Evolve the second root of $\frac{72}{280}$ to two decimal places.

102. Three farms contain respectively, 356, 898, and 1254 acres, which I desire to cut into building lots of the largest equal size possible; how many acres will each lot contain?

103. Divide $\frac{4}{5}$ by $\frac{2}{3}$.

104. Multiply .303 by .03.

105. The sun at 12 o clock is over the meridian at Washington; over what meridian will it be after traveling through 5 signs and 5 degrees? What time will it then be at Washington?

106. How many grams does a liter of rain water weigh?

107. At 7 cents per square foot, required the cost of a brick walk 6 feet wide around a lot 200 ft. by 300 ft.

108. What sum of money loaned at 6% for 10 mo. will yield as much interest as $750 loaned at 4% for 11 months?

109. Required the area of a circle whose radius is 10 feet.

110. A, B and C eat 8 loaves of bread of which A furnishes 3, and B 5. C pays A and B 8 pieces of money of equal value; how should they divide the money?

111. For what sum must I make a bank note for 60 days, which discounted at 10% will pay a $1000 debt now due?

112. A street 60 ft. wide is crossed at right angles by another 80 ft. wide, what is the distance between diagonal corners?

113. A wall is 83 meters long, 2 Dm. high, and 5 dm. thick; what is its value at $3.30 per cu. meter?

114. *a.* What is the value of the N. E. ¼ of the N. W. ¼ of section 16, at $12 per acre?

b. Make a plat of the congressional township and indicate the part described.

115. A miller takes for toll 4 quarts from every 5 bu. of grain; what per cent. does he get?

116. In what time will $375.40 yield $37 54 interest at 6% per annum?

117. A barn is 40 ft. wide; the comb is 15 ft. from the plate and the rafters are of equal length; what is the length of each rafter?

118. If money is worth 12%, what is the true discount of $235.10, due one year hence?

119. In a cube whose edge is ¾ in., how many cubes each ¼ of an inch wide?

120. A rectangular field 15 rods wide contains 3 acres; how long is it?

121. How many yards of carpet 27 inches wide will cover a floor 18 ft. long by 14 ft. wide?

122. If an article be sold for twice its cost, what is the rate per cent. of gain?

123. If an article be sold for one-half its cost, what is the rate per cent. of loss?

124. If an article is sold for one third its cost, what is the rate per cent. of loss?

125. Property worth $8760 is rented for $650 per annum; what rate of interest does the investment yield?

126. In what time will $2450 yield $725 int. at 8%?

127. A man receives $280 interest, annually, on a 7% loan; what is the face of his loan?

128. What principal will amount to $5750 in 3 yr. 5 mo. 17 da. at 6%?

129. For what sum must property worth $6000 be insured at 5% to cover $\frac{3}{4}$ of the property and the premium ?

130. What is the selling price of corn whose first cost is 40 cts. per bu., freight 8% and rate of gain $16\frac{2}{3}$%?

131. A merchant sold two bills of goods for $50 each; on one he gained 15%, and on the other he lost 15%; required his gain or loss.

132. What is the rate of interest on an investment in U. S. 4 per cents. at 95 ?

133. A merchant is offered goods for $2500 cash, or for $2650 on 60 days time; which is the better offer if money is worth 8% per annum?

134. For what must 5% stock be bought that it may yield 7% interest on the investment ?

135. U. S. $3\frac{1}{2}$'s bought at 98 pay what rate of interest on the investment ?

136. What sum of money will yield as much interest in 10 months at 6%, as $1500 will yield in 12 mo. at 4% ?

137. How much must be paid in U. S. currency for a draft of £210 10s. 6d., exchange being 102, brokerage $\frac{1}{8}$%, and gold at 104?

138. Reduce $\frac{3}{5}$, $\frac{4}{9}$, $\frac{7}{15}$ to equivalent fractions having the l. c. d.

139. Multiply $\frac{2}{3}$ by $\frac{6}{7}$; $\frac{3}{11}$ by .04; 2.06 by .0013.

140. Divide $\frac{5}{7}$ by $\frac{2}{3}$; .004 by $\frac{3}{5}$; 3.064 by .08.

141. Reduce 5 yd. 2 ft. 7 in. to inches.

142. Reduce $\frac{3}{4}$ mile to lower integers.

143. A box is $4\frac{3}{4}$ m. long, 8 dm. wide and 3.5 dm. deep; how many Kg. of distilled water will it hold ?

144. A pile of wood is 15.5 m. long, 12 dm. wide, and 1.8 meters high; how many sters does it contain?

145. $\frac{3}{5}$ = what % of $\frac{7}{8}$?

146. Reduce $\frac{3}{40}$ to the decimal scale.

147. Two men paid $150 for a horse; one paid $90 and the other paid $60; they sold it so as to gain $75; what was the share of each?

148. A man owes A $105, B $75. and C $120; he has only $125 ? how much should he pay to each?

149. Find g. c. d. of 348 and 1116.

150. What is the face of a note payable in 90 days, on which $3500 can be obtained at a bank, discounting at 6%?

151. An irregular mass of metal immersed in a vessel full of water caused 2.25 l. of water to overflow the sides of the vessel; what was the weight of the metal if its specific gravity was 7.2 ?

152. What is the price, at $11.65 per kilogram, of 1 liter of alcohol, its s. g. being .79?

153. What is the capacity in liters, of a cylindrical cup 16 cm. in diameter and 1.3 dm. deep?

154. A & B gain in business $4160, of which A is to have 8% more than B, how much will each receive?

155. A merchant insures a cargo of goods for $3456 at 3½%' the policy covering both property and premium; what was the value of the property?

156. If a stack of hay 8½ ft. high, weigh 7 cwt., what weighs a similar stack 15 ft. high ?

157. For what sum must a note be drawn at 3 mo. that the proceeds, when discounted, without grace, at a bank, at 8% shall be $1274?

158. The contents of a cubical block of stone are 4913 cu. cm.; required its superficial contents in sq. m.

159. I imported 12 casks of wine, each containing 48 gallons invoiced at $2.75 per gal.; paid $108 for freight, and an Ad Valorem duty of 36%; what is my rate of gain if I sell the whole for $3258?

160. A New York merchant gave $960 for a bill of £250 on London, required the rate of exchange.

161. What is the largest square stick that can be cut from a log 4 ft. in diameter?

162 A sea captain set his watch at London, and found after traveling that it was 4 hours faster than the local time of the place he had reached; how far had he traveled and in what direction?

163. What is the area of a field whose parallel sides are 90 rods and 124 rods long, respectively, and the perpendicular distance between them is 50 rods?

164. A has $550, and B has $330; what per cent. of the money of each is the money of the other?

165. A farmer's wagon loaded with wheat weighed 4912 lb., and his wagon alone weighed 920 lb., what was his load of wheat worth at 95 cts. per bu.?

166. If a bolt of paper is 8 yd. long and ½ yd. wide, how much will the paper cost at 30 cts. per bolt, to paper the walls and ceiling of a room 18 ft. long, 15 ft. wide and 10 ft. high?

167. A man paid $30.09 for the use $204 for 11 mo.? what was the rate of interest?

168. What is now due on a note given Sept. 1, 1881, the principal being $430, the rate 7%, per annum, endorsed Aug. 4, 1882, $34, July 7, 1883, $118?

169. A railroad right of way 4 rods wide passes diagonally through a quarter section of land; how many acres remain unoccupied by the road?

170. Reduce ⅜ da., 3 min. to the decimal of a day.

171. A man owned a farm of 40 A. 3 R. 22 Sq. rd., and sold it at $45.50 per acre; what sum did he realize?

172. The selling price of a house was 130% of the cost; the gain was $240; required the cost.

173. The interest was $45, the rate 8% per annum, the time 1 yr. 3 mo. 18 da.; required the principal.

174. Reduce 6 oz., Av. to the decimal of a cwt.

175. How many boards 16 ft. long will be required to fence a lot 125 ft. by 80 ft., the fence being 5 boards high? Required its cost at $27 per M, each board being 5 in. wide and 1 in. thick.

176. Divide sixteen ten-millionths by twenty-five ten-thousandths.

177. How many bricks can be laid in a pavement 25 ft. 4 in. long by 15 ft. 3 in. wide, each brick being 8 in. by 4 in.?

178. When it is 2 o'clock at Terre Haute, what is the time 47° 13′ 27″ west of Terre Haute?

179. The capacity of a cubical cistern is 74088 cu. in.; how many sq. ft. in the bottom of it?

180. What cost 2 lb. 4 oz. of cloves at $87\frac{1}{2}$ cents per lb.?

181. At $11 per rod, what will be the cost of fencing a lot in the form of a right angled triangle, whose sides forming the right angle are 264 rods and 23 rods, respectively?

182. A grocer bought a hhd. of sugar weighing 3 cwt. 2 qr. 20 lb. for $30, and sold it at $7\frac{1}{2}$ lb. for $1; what per cent. does he gain?

183. A 6 months' note for $5000, drawing interest at 6% per annum, was discounted—not in bank—3 mo. after date at 10% per annum; required the proceeds.

184. A physician bought 2 lb. 9 oz. of quinine at $38 per lb., Avoirdupois, and sold it in 10 grain doses at 20 cts per dose; did he gain or lose and how much?

185. Find the Amt. of $3350 for 3 yr. 9 mo. at 8%.

186. If you travel 360 miles in 12 days of 8 hr. each, how many miles can you travel, at the same rate, in 60 days of 6 hr. each?

187. What is the side of a cube which contains as many cu. ft. as a box 8 ft. 3 in. long, 3 ft. wide, and 2 ft. 7 in. deep?

188. What per cent. of 14 bu. is 5 bu, 3 pk. 5 qt.?

189. Find the g. c. d. of 368 and 612.

190. $\frac{5}{8}$ = what decimal fraction?

191. If 50 men build 50 rods of wall in 75 days how many men will build 80 rods of similar wall $\frac{3}{2}$ as thick and $\frac{4}{5}$ as high in 40 days?

192. A farm is 125 rods square and a rectangular field containing the same area is 130 rods long; how wide is it?

193. By what must $\frac{3}{8}$ be multiplied that the product may be 12?

194. A flash of lightning is seen 47 sec. before its peal of thunder is heard; required the distance of the thunder cloud.

195. What is the difference betweeen the simple interest of $1000 at 6%, for 5 yr., and the true discount of the same sum for the same time at the same rate? Which is the greater and how much?

196. What is the value in English money, of $500 at $4.84 per £?

197. How many square yards of canvas would be required to make a balloon of globular form, 20 yards in diameter?

198. If linen is bought at 2 s. 9 d. per yd., for what must it be sold to gain 25%? What is the selling price in U. S. money?

199. If the fore wheels of a wagon be 4 feet in diameter and the hind wheels 5 feet in diameter, how many more revolutions will the former make than the latter in going a mile?

200. On a base of 120 rods, a surveyor wishes to lay off a rectangular field that shall contain 60 A.; how far from his base line must he run out?

201. A sells to B, tea worth 45¢ for 48¢; what should B charge A for sugar worth 9¢ to balance the transaction?

SECTION XVI.

REVIEW QUESTIONS AND TOPICS.

Define mathematics.—What is conditional for extension?—What branches of knowledge involve a study of extension?—What is known of every conscious mental state?—How are mental states known to be distinct?—How does the idea of number arise?—What is conditional for the numerical idea?—Define Arithmetic as a science; as an art.—What attribute furnishes the basis of number in particular?—By what process does the mind obtain the idea *one?*—Define the *integral unit*, or *unit one*.—How many classes of secondary ones are mentioned?—Define a fractional unit; a multiple unit.—What is the primary idea in Arithmetic?—How are other units related to the *unit one?*—What is a unit object?—Define a number. —Of what units may a number be composed?—On what basis are numbers classified as integers and fractions?—Define an integer; a fraction.—On what basis are numbers classified as abstract and concrete?—Define an abstract number; a concrete number.—On what basis are abstract integers classified as prime and composite?—On what basis are numbers classified as simple and denominate?—Define a simple number; a denominate number; a compound number.—How is a denominate number sometimes defined?—What objection?—How is a compound number sometimes defined?—What objection?—Define notation.—What kinds are usually presented in Arithmetic? —What is the alphabet of the Roman notation?—What does each letter signify?—What are the limits of the Roman notation?—State the principles.—What is the alphabet of the Arabic notation?—What is the signification of each character?—How many and what distinct systems of numbers make use of the Arabic characters?—Define a scale; the decimal scale?—What is a unit of the first order?—How are units of different orders formed?—What are periods in the decimal system of numbers?—What orders are embraced by any period?—What is meant by a decimal division of 1?—How are lower orders of units formed?—State the principle of the decimal scale.—What is the office of the decimal point?—Describe units' place.—How many and what kind of units may be expressed therein?—Describe tens' place.—How many and what kind of units may be expressed therein?—How may higher decimal units be expressed?—How may lower decimal units be expressed?—What is the representative scale?—What is the simple value of a figure?—What is the local value of a figure?—What are the limits of the decimal system of numbers?—Define numeration; reading numbers.—In reading a number where should the word *and* be used?—What elements constitute the primary idea of a fraction?—How many and what numbers are necessary to the idea of a fraction?—What are these numbers together called?—Define the denominator; the numerator.—What is the denomination of a fraction?—What exception?—How may decimal fractions be notated?—How are other fractions notated?—What names are applied, respectively, to the terms of a written fraction?—How is a compound number thought?—In compound numbers the number of denominations is how limited?—What kind of a scale has each of the common measures?—Each denominate number forming a part of a compound number is how thought?—How is a compound number notated?—How are the parts of a compound number written as to abbreviation and punctuation?—How is a compound number read?—Define reduction; descending; ascending; and state how each is effected?—Define computation?—How are numbers compared previous to computing?—What mental act effects a computation?—How many and what operations does the mind perform upon numbers?—What is the primary judgment in computation?—What is an equation?—What are the members of an equation?—Define sum, addition, addends. —State the mental acts involved in addition.—State and illustrate the principles of addition.—Describe the sign of addition and state its use.—Show by an example that reduction may be necessary in addition.—Which reduction may be involved in addition?—Define product.—What other term means the same?—Define multiplication.—State the genesis of multiplication.—What act of synthesis is involved?—What is the act of multiplication?—Define multiplicand; multiplier.—If the multiplier is an integer what relation do multiplicand and multiplier sustain?—If the multiplier is a fraction in what consists the multiplication?—Define a factor.—State and illustrate each of the ten principles of multiplication.—Describe and state the use of the sign of multiplication.—Which reduction is often involved in multiplication?—Show by example how reduction may be necessary in multiplication.—What is meant by "continued product?"—Define a composite number; define composition; a prime number.—Under what conditions are numbers relatively prime?—Define a common multiple; the l. c. m.; a common measure; the greatest common measure.—State and illustrate each of the principles given under composition.—Define a power; involution; a root; second power; second root.—How are higher powers formed and named? —How are roots named?—What is the first power and first root of a number?—Describe and define the index of a power.—Repeat the table of squares given; the table of cubes.—State and apply rules for squaring numbers.—Under how many and what heads is the synthesis of numbers discussed?—How many and what methods of synthesis are there?—What relation do the other processes called synthetic sustain to the one method of synthesis?—State in substance the last four remarks given under synthesis?—On what ground is a number resolvable into parts?—Define difference, giving two definitions.—Define subtraction; minuend; subtrahend; remainder.—State the mental acts involved in subtraction.—How is the difference between two separate numbers found?—Describe the sign of subtraction and state its use.—State and illustrate each of the principles of subtraction.—Under what condition must the minuend be prepared before a subtraction is effected?—In what consists the preparation?—Which reduction is involved?—How may reduction be avoided in subtraction?—Define quotient; division; dividend; divisor.—Define each of these terms in the light of its relation to multiplication.—What is meant by a constant subtraction?—State how division grows out of subtraction?—How is division related to multiplication?—In what consists the mental act of division?—Which reduction is sometimes involved in division?—Illustrate.—State and illustrate each of the principles of division.—State and illustrate each of the six general principles of division.—Define disposition.—How is disposition related to composition?—How are the factors of a number found?—State and illustrate each of the principles of disposition.—Under what condition is a number known to be prime?—Define evolution; the index of a root.—Describe and state the use of the radical sign; exponent.—What is signified by a fractional exponent other than a fractional unit?—Under how many and what heads is the analysis of numbers discussed?—State the substance of the last four general remarks given under analysis.—Define g. c. d.—State and illustrate the principles under this head.—How many methods are given for finding g. c. d.?—Solve an example illustrative of each method.—Define l. c. m.—State the principle and solve an example, using the form given on page 53.—If given numbers are not readily factored how may their l. c. m. be found? —Why?—Define a fraction, giving two definitions. —According to the primary idea of a fraction, what is the maximum value of a fraction?—How is an expression like $\frac{7}{3}$ to be interpreted?—How is a fraction notated?—On what basis are fractions classified as decimal and common?—Define a decimal fraction.—How is a decimal fraction usually notated?—How *may* it be notated?—What is a complex decimal?—Define a common fraction.—On what basis are fractions classified as proper and improper?—Define a proper fraction; an improper fraction.—State wherein there is no ground for this classification.—On what basis are fractions classified as simple, compound and complex?—Define a simple fraction; a compound fraction; a complex fraction.—Show wherein this classification is not well founded.—State the likeness of a fraction to division.—State and illustrate each of the general principles of fractions.—State the principle under which reduction descending of fractions may be effected.—Under what principle may reduction ascending of fractions be effected?—How may a fraction be reduced to its lowest terms?—Under what condition is a common fraction *not* reducible to a decimal?—Under what principle is addition of fractions effected?—Under what principle is subtraction of fractions effected?—State the two cases under which multiplication of fractions is presented.—Solve an example under each case.—State the two cases under which division of fractions is presented.—Solve an example under each case, using —*a*., common fractions; *b*., decimal fractions.—Show that division of fractions is the reverse of multiplication of fractions.—Divide $\frac{9}{10}$ by $\frac{2}{7}$, effecting the division by constant subtraction.—Into what classes are compound numbers divided?—On what basis?—Write the diagram.—State the order to be pursued in the study of a "measure."—Recite the tables of compound numbers.—How many and what cases of reduction descending in compound numbers?—Solve an example under each case. How many and what cases of reduction ascending in compound numbers?—Solve an example under each case.—Define Longitude.—What meridian is generally taken as prime?—Make a table of corresponding time and longitude units.—What is meant by relative time?—Absolute time?—Show that all places have not the same relative time.—Explain what is meant by "Standard time,"—Solve examples in time and longitude.—What is area?—What is a volume, or solid?—Solve an example in which area is computed.—Solve an example in which volume is computed.—Show that *length* and *breadth* are not multiplied together.—If they could be would the product be *surface?*—Why?—Name the three primary units of the decimal system of measures.—Which of these is the fundamental unit of the *system?*—How does the meter compare in length with the yard?—Name the land unit.—What is its area?—By what units are most surfaces measured?—What is the wood unit?—What is its volume? By what units are most volumes measured?—What units are used in measuring great weights?—How does each of these compare with the gram?—Name each prefix designating a secondary unit.—Construct tables for the measures of length, surface, volume, capacity, and weight, in the decimal system.—What relations exist by means of which capacity and weight are readily found from extension?—What is percentage?—Name the essential terms of percentage.—Define each.—Define *amount* and *difference*, and state which of the essential terms includes them.—State the relations of percentage.—*a*. To multiplication.—*b*. To factoring.—*c*. To fractions.—State each of the general cases of percentage.—Give the solution of each.—How many and what elements are involved in the first class of applications of percentage?—Name the principal applications of the first class.—What elements are involved in the second class of applications of percentage?—Name the principal applications of the second class.—Define profit and loss; cost; selling price; gain; loss.—State the relation of profit and loss to percentage by naming the corresponding terms.—What is an agent?—A factor? —A broker?—A commission merchant?—Define commission; brokerage.—State the relation of commission and brokerage to percentage by naming the corresponding terms.—Define *stock* as used in percentage; a corporation; a firm; a company; a share; a certificate of stock; a dividend; par value; market value; premium; discount.—How is stock quoted?—State the relation of stock to percentage by naming the corresponding terms. —Define insurance; fire insurance; marine insurance; life insurance; policy; face of policy; policy-holder; underwriter; premium.—State the relation of insurance to percentage by naming the corresponding terms.—Define a tax; a property tax; a poll tax; an income tax; an excise tax; an assessor; his duties; an invoice; tare; leakage; breakage; gross weight; net weight; specific duty; advalorem duty.—Define interest; principal; amount; rate; legal rate.—What is the time unit in interest?—Define simple interest; a partial payment.—State the United States rule for computing interest in partial payments; the merchants' rule. —How many elements are involved in simple interest?—What relations are sustained among the elements?—Define compound interest; annual interest; discount; true discount; present worth.—State the relation of true discount to simple interest by naming the corresponding terms.—Define bank discount; term of discount; proceeds; days of grace; face of a note; legal maturity; a protest. —If an interest bearing note be discounted in bank, on what is the discount computed?—State the relation of bank discount to simple interest by naming the corresponding terms.—Define commercial discount.—State its relation to percentage.—Define exchange; a draft; a sight draft; a time draft; the face of a draft; the course of exchange. —Who is the drawer or maker of a draft?—The drawee?—The payee?—State the relation of exchange to simple interest by naming the corresponding terms.—What is meant by the equation or average of payments?—Show the relation of equation of payments to simple interest.—Define Ratio.—How many and what terms are used in thinking a ratio?—Define each.—How is a ratio notated?—Define a simple ratio; a compound ratio; an inverse ratio.—State each of the six principles of ratio.—Define proportion? a simple proportion; a compound proportion.—How is a proportion notated?—State and demonstrate the principle of proportion. —Under what condition is a number divided into proportional parts? State the principles.—If two sides of a right-angled triangle be known, how find the third side?—The areas of similar surfaces are to each other as what? How find the area of a trapezoid?—How find the two sides of a rectangle, if the area and the ratio of the sides are given?—How find the three dimensions of a rectangular solid, if the solid contents and the ratio of the sides are given?—How find the convex surface of a cylinder?—How find the volume of a cylinder?