



UNIVERSITY OF
ILLINOIS LIBRARY
AT URBANA-CHAMPAIGN
BOOKSTACKS

Digitized by the Internet Archive
in 2011 with funding from
University of Illinois Urbana-Champaign

<http://www.archive.org/details/overdraftcheckin677brya>

Faculty Working Papers

OVERDRAFT CHECKING AND BANK PROFITABILITY

William R. Bryan, Professor, Department of
Finance

Case M. Sprenkle, Professor, Department of
Economics

Morgan J. Lynge, Jr., Assistant Professor,
Department of Finance

#677

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

Random Coefficient Model

$\hat{\lambda}_1$	$\hat{m}_1 = \text{var}(\hat{\lambda}_{1j})$
.0044 (5.97)	.000048 (2.72)
.0100 (9.67)	.000094 (2.50)
-.0045 (-4.31)	.000041 (.93)

ent. The coefficients of dete
 mated values are significantly
 Hod Jan. 1969 to Dec. 1971.

y Bills are used to measure th

FACULTY WORKING PAPER

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

May 12, 1980

OVERDRAFT CHECKING AND BANK PROFITABILITY

William R. Bryan, Professor, Department of
Finance

Case M. Sprenkle, Professor, Department of
Economics

Morgan J. Lynge, Jr., Assistant Professor,
Department of Finance

#677

Summary

This paper investigates the impact of the introduction of an overdraft facility on bank's profitability. The deposit behavior of a bank customer is modeled and the customer's "maximizing behavior" is derived. Given this customer response the bank's change in revenues due to the overdraft facility are expressed. Using example parameter values it is shown that the introduction of an overdraft facility will reduce bank profits. This result can be overcome if a large increase in deposits is generated by the new facility -- an outcome deemed unlikely in a competitive environment.

OVERDRAFT CHECKING AND BANK PROFITABILITY

In recent years a growing number of commercial banks have offered an "overdraft" adjunct to their checking accounts. This new service has been introduced to attract new depositors, retain existing accounts, or both. The overdraft facility may affect customer behavior in a number of ways. For example, customers with an overdraft capability may reduce their holdings of bank deposits, both checking and time deposits, since they can conveniently borrow to meet expenditure needs. Or, customers may change the mix of their deposit holdings, presumably from non-interest earning demand deposits to interest earning time and savings deposits. These changes in customer behavior will affect the profitability of commercial banks. The focus of this study is the impact that an overdraft facility has on bank profits.¹

The introduction of bank overdraft facilities is not the only financial innovation affecting customer decisions about deposit balances to hold. Bank credit cards have also affected these deposit balance decisions. There are a number of studies that have explored the effect of bank credit cards on the transactions demand for money.² These studies conclude that bank credit cards cause people to hold smaller transactions balances. We reach the same conclusion for the impact of overdraft facilities on transactions balances. However, the focus here is on the impact of this behavior on bank profits.

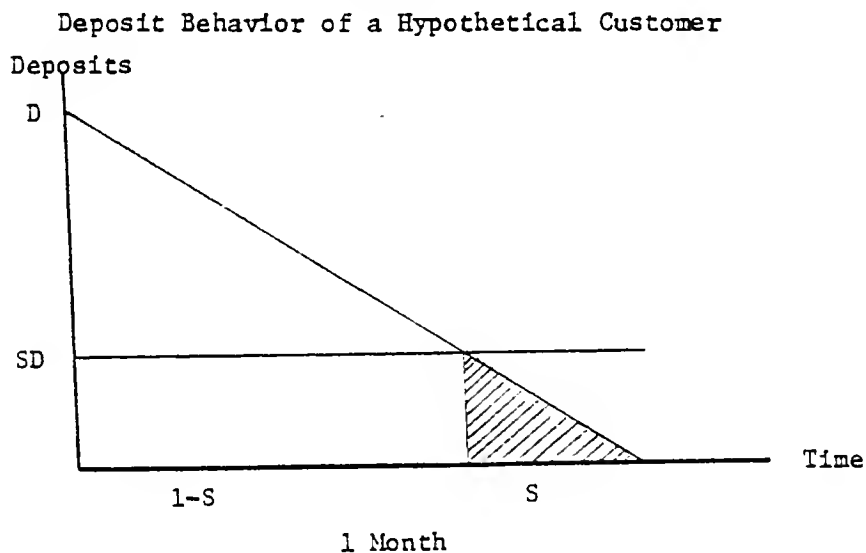
In summary, we find that if bank customers behave rationally, bank profits are reduced by the introduction of an overdraft facility, if no new customers are attracted. However, if a significant deposit increase occurs as a result of the facility, bank profits may increase. In the

next section customer reaction to an overdraft facility is examined, and "rational behavior" is specified. In the following sections we investigate the impact on bank profit that results from the introduction of the overdraft facility under conditions of (1) no new business and (2) an increase in customers.

Customer Reaction to an Overdraft Facility

To lay the basis for the analysis, we first depict an assumed (admittedly simplistic) behavior of a hypothetical customer. Suppose our customer receives and deposits his paycheck on the first day of each month, and writes checks continually during the month until his balance has reached zero.³ This behavior is depicted in Figure 1.

FIGURE 1



The average balance for this customer will be

(1) $\bar{D} = \frac{D}{2},$

where

\bar{D} = average deposit, and

D = deposit at the beginning of the month.

To the extent that a customer elects to hold a "safety margin" of demand deposits to avoid a negative balance or to reduce service charges, there is a dollar-for-dollar increase in average deposit holdings $(1 - R_D)$ percent of which can fund bank loans.

Suppose the bank introduces an overdraft system and that our hypothetical customer is a rational maximizer. To limit the problem, consider only those aspects of behavior contained within the bank-customer relationship. In a larger context the entire asset portfolio should be considered. We assume here that the customer has already optimized his portfolio in a non-overdraft environment, holding other financial and real assets besides demand deposit balances. Now the environment is changed by the introduction of an overdraft system. With this change we examine the customer's maximizing behavior with respect to alterations in the demand deposit portion or "transaction" balances of the portfolio. Assume that the customer considers the three-pronged decision of (1) how much of his transactions balances he should retain in demand deposits (shown as line segment D - SD in Figure 1), (2) how much he should place in savings deposits (SD in Figure 1),⁴ and (3) to what extent he should use his overdraft facility (depicted by the shaded triangle in Figure 1). In developing this decision, the following additional symbols will be employed:

R_c = net return to the customer.

s = portion of beginning deposit placed in savings account;
alternatively, the portion of the month during which
the customer has been in debt to the bank.

r_s = rate paid by bank on savings account.

r_b = rate charged by bank on overdraft borrowings.

The gross return to the customer on the portion of his deposit that he allocates to savings is sDr_s . To obtain the customer's net return, we subtract from this the costs associated with borrowing against his overdraft line. The ingredients of the cost of borrowing consist of (1) the average amount borrowed, $\frac{sD}{2}$, (2) the length of time during which the customer was in debt, s , and (3) the rate charged on overdraft borrowing, r_b . The net return to the customer is given by

$$(2) \quad R_c = sDr_s - \frac{sD}{2}(sr_b)$$

where the first term is the gross return on the customer's savings deposit and the second term is the interest paid on the average borrowing under the overdraft facility. If the customer sets s , the proportion of his deposit to be held in the savings account, to maximize his return, it can be shown (see Appendix) that s must be equal to:

$$(3) \quad s = \frac{r_s}{r_b}.$$

As might be expected, the optimal portion of funds placed in savings accounts is positively affected by the rate paid on savings accounts, and negatively affected by the rate charged on overdraft borrowing. In developing the impact of an overdraft facility on bank profits we will

assume that customers behave in a "maximizing" fashion, i.e., they set s according to equation (3). This will be referred to as "maximizing" behavior."

Bank Returns from Overdraft Checking - No New Business

Now that we have established how customers will react to the introduction of an overdraft checking facility, let us turn to the impact on bank profits. Before the introduction of overdrafts the bank's return is simply⁶

$$(4) \quad R_B = \bar{D}(1 - R_D)r_L$$

where

R_B = returns to the bank,

R_D = reserve requirements on demand deposits, and

r_L = gross return on loans.

This expression presents the loan revenue earned by the bank on the funds available for lending from checking accounts.

The introduction of the overdraft facility brings about several changes that will affect revenues and costs. Savings balances are increased during the entire period. Demand balances are reduced from the outset, reaching zero $(1 - s)$ of the way through the period, after which the customer writes checks against his overdraft line (see Figure 1). Revenues are altered as lendable funds (after reserve requirements are met) from demand and time deposits have changed. Also overdraft loans provide a return (r_b) that we assume is greater than the return on other loans (r_L). Increased costs to the bank stem from two sources: One, there are increased interest payments to depositors resulting from the

shift from demand to time deposits. Two, there are costs to the bank as the size of the customer's overdraft borrowing moves beyond the size of loans which can be supported by the overdraft-related investment to his savings account--given our assumption about depositor behavior. As the customer's overdraft moves beyond $sD(1 - R_T)$, where R_T represents reserve requirements on time deposits, that borrowing must be funded by a reduction in other loans, or by bank borrowing. Note that we continue to assume that the customer does not remove his funds from the bank.

The streams of returns and costs associated with the overdraft facility are given by

$$(5) \quad R_B = [(1 - s)^2 \bar{D}(1 - R_D)r_L] + [2s\bar{D}(1 - R_T)r_L] + [s^2 \bar{D}(r_b - r_L)] - [2s\bar{D}r_s].$$

The first term represents the return to the bank from lending out of the (now reduced) customer's average demand deposit balance. The second term is loan revenue earned on the customer's savings deposit. The third term is the return from the customer's overdraft borrowing minus the foregone return by not making other loans.⁷ The fourth term is the interest cost on the customer's savings account.

Given that the bank will initiate an overdraft program, it has an incentive to identify the rate to be charged on overdraft loans which will achieve the best attainable profit position. Without the overdraft program, returns to the bank on existing customers are given by (4). Returns with the program are shown by (5). The change in returns is the difference between (4) and (5). This is given by

$$(6) \quad \Delta R_B = 2s\bar{D}(R_D - R_T)r_L - 2s\bar{D}r_s + s^2 \bar{D}(r_b - R_D r_L).$$

In words, the bank benefits from the program to the extent that there are lower reserve requirements on time deposits and from proceeds from overdraft borrowing. The chief decrease in returns results from the added interest payments on savings deposits.

Considering only that value of s wherein consumers maximize (i.e., $s = \frac{r_s}{r_b}$), the change in returns is

$$(7) \quad \Delta R_B = \frac{r_s}{r_b} [2(R_D - R_T)r_L - r_s - \frac{r_s}{r_b} R_D r_L] \bar{D}$$

It is easily seen that ΔR_B will be negative for any reasonable values of the variables since $2(R_D - R_T)r_L$ will be less than r_s . Thus, the bank cannot hope to make a profit from its old customers by the overdraft system.⁸ It should be noted that this result will hold no matter the level of r_b chosen by the bank. The higher is r_b , the lower will be the reduced profits. However, the bank cannot set its rate on overdraft loans above a competitive level. The rate the bank charges will have a bearing on its ability to attract new customers or to retain its present customers.

A numerical example may be helpful in clarifying the bank's situation. Assume the values shown in Table 1 for the interest rates and reserve requirements. Without an overdraft program, the return to the bank with an average deposit level (\bar{D}) of \$1 million is \$87,000 (from equation 4). With no increase in deposits after the introduction of an overdraft service, assuming maximizing behavior by customers, bank return is now \$76,553 (from equation 5). The reduction in return is \$10,447 or 1.0447% of the average deposit level (from equation 7).

TABLE 1

Example Values for Reserve Requirements and Interest Rates

r_s	= interest paid on savings accounts	= 5%
r_L	= interest earned on loans	= 10%
r_b	= interest earned on overdraft loans	= 18%
R_D	= reserve requirement on demand deposits	= 13%
R_T	= reserve requirement on savings deposits	= 5%

Even though the bank experiences a decline in earnings from existing depositors by initiating an overdraft facility, this loss can be more than offset if enough new deposits are attracted. We now turn to that area of consideration.

New Business and Overdraft Checking

What volume of new business is required to maintain bank profitability in the face of an overdraft checking facility? The answer depends upon the configuration of relevant interest rates and reserve requirements. Let \bar{D}_0 represent the "old" level of average deposits, and \bar{D}_N the increase in average deposits from offering the overdraft facility. The new deposits will increase the returns to the bank by equation (4), or $\bar{D}_N(1 - R_D)r_L$. However, if we assume that these new deposits also follow maximizing behavior, s percent of them will be converted to savings deposits, resulting in a reduced profit to the bank. Equation (7) can be expressed in terms of the change in returns per dollar of average

demand deposits, giving

$$(8) \quad \frac{\Delta R_B}{\bar{D}} = \Delta R'_B = \frac{r_s}{r_b} [2(R_D - R_T) r_L - r_s - \frac{r_s}{r_b} R_D r_L].$$

Thus the total change in bank returns from the conversion to an overdraft facility and the inflow of \bar{D}_N deposits is

$$(9) \quad \Delta R_B^N = \bar{D}_O \Delta R'_B + \bar{D}_N (1 - R_D) r_L + \bar{D}_N \Delta R'_B$$

where

$$\Delta R_B^N = \text{change in bank returns from conversion to overdraft facility plus inflow of } \bar{D}_N \text{ deposits.}$$

In equation (9) the first term represents the change in profit (ΔR_B is negative) due to the conversion to overdraft usage by the old customers (\bar{D}_O). The second term is the change in profit due to the inflow of new deposits (\bar{D}_N); the third term represents the profit change (ΔR_B is negative) due to the conversion of these new deposits to overdraft usage.

The bank should maximize its profit by varying the overdraft borrowing rate, but this would require knowledge of the elasticity of both D_O and D_N with respect to r_b . It is difficult to imagine a bank having such knowledge. Rather than pursue this, it seems more appropriate to consider the conditions under which the bank can make any profit from this system. From equation (9) positive changes in profit exist when

$$(10) \quad \frac{\bar{D}_N}{\bar{D}_O} > \frac{-\Delta R_B}{\Delta R_B + (1 - R_D) r_L}.$$

In other words, when the percentage increase in deposits (\bar{D}_N/\bar{D}_O) is greater than the right hand side of equation (10), positive profit changes will result.

Let us return to the example begun in the previous section with interest rate and revenue requirement values in Table 1. Using these values, the right hand side of equation (10) is .136; that is, if average deposit levels increase by 13.6% as the result of the introduction of an overdraft system, bank profits would remain unchanged. This estimate does not include the recovery of marketing costs or other direct costs associated with the system.

To continue the numerical example, let us examine the balance sheet changes that would result from the introduction of the overdraft facility. For a bank with average deposits of \$1 million, and assuming no new accounts, the following would occur: (1) average demand deposits would decline by \$479,000; (2) savings accounts would rise by \$556,000; and (3) overdraft borrowing would average \$77,000 for the month.

To offset the revenue loss associated with these developments it would be necessary to attract new deposits to increase the average level of deposits by \$136,000, or 13.6% of the initial average level. Assuming that new customers engage in maximizing behavior, some of these new demand deposits would be shifted to savings deposits. After all the shifting to achieve maximizing behavior is completed average demand deposits will be \$71,000 higher and savings deposits would be \$76,000 higher. Associated with these new deposits would be a rise in average overdraft borrowing of \$11,000.

The beginning (before overdraft), intermediate (with overdraft but without new deposits) and final (with new deposits) balance sheets and profit figures are shown in Figure 2. As a result of the overdraft facility and the new deposits the bank has grown to \$1,224,000 in total

FIGURE 2

Three Balance Sheet Positions and Related Profits
(Using assumed values from Table 1)

Before

Reserves	130,000	Average DD	1,000,000
Overdraft loans	-	SD	-
Other loans	870,000		
TOTALS	1,000,000		1,000,000

Profit = \$87,000

With overdraft - no new deposits

Reserves	96,000	Average DD	521,000
Average over-draft loans	77,000		
Other loans	904,000	SD	556,000
TOTALS	1,077,000		1,077,000

Profit = \$76,553

With overdraft - and new deposits

Reserves	109,000	Average DD	592,000
Average over-draft loans	88,000		
Other loans	1,027,000	SD	632,000
TOTALS	1,224,000		1,224,000

Profit = \$86,940

assets. The increased cost of the savings deposits is offset by increased revenue from higher loans and the new overdraft loans. Part of the increase in loans was funded by the redirection in required reserves.

In the analysis developed above, overdraft checking is profitable to the customer chiefly because borrowing costs are based on daily-average "negative" balances.⁹ In contrast, the funds placed in savings accounts

earn interest throughout the month. As a result average borrowings for the month are much smaller than the funds placed in savings. Thus, even though the interest rate charged for overdraft borrowing may be substantially higher than the rate on savings, the depositor's returns on savings exceed his cost of borrowing.

Program Modifications

If competitive conditions preclude the possibility of a substantial (e.g., 13.6%) increase in deposits modifications to the previously presented overdraft program may be utilized to reduce the reduction in profit from present customer's accounts. Two possible modifications come readily to mind: (1) charging interest on overdrafts not for the period actually borrowed, but rather for the whole payment period, and (2) allowing overdraft borrowing only in the form of lump sums. Both of these modifications effectively increase the cost of borrowing to the customer and hence will decrease the optimal s , or the portion of the month that borrowing takes place. While these modifications will reduce the bank's profit reduction from old customers, they will also diminish the attractiveness of the overall program to new customers. Whether the less attractive modified program will be able to attract sufficient new customers to keep profits unchanged is doubtful.

The implications of the first modification are easily seen. If interest on the overdraft is now charged for the whole period, the return to the customer is now (instead of equation 2) simply

$$(2a) \quad R_c = sD(r_s - r_b)$$

A customer now maximizes (2a) by simply setting s equal to zero, given that negative values of s are impossible. That is, the customer will not use his overdraft facility at all for transactions needs, saving its use solely to meet uncertainty. The bank's loss from old customers will be reduced to zero, but the overdraft facility will also be much less attractive to new customers.

The second possibility for offsetting losses from old customers is to lend on overdraft only in lumps. When an account goes negative, the customer is loaned, say, \$100, as the account goes just below minus \$100 an additional \$100 is loaned, and so on.

If x is the size of the lump in dollars, while borrowing the customer will on the average be borrowing $\frac{x}{2}$ more dollars than without the lump. Since the customer will be borrowing for a time equal to s percent of the period, his average extra borrowing caused by the lump will be $\frac{sx}{2}$ and the extra cost will therefore be $\frac{r_b sx}{2}$.

With the lump, the return to the customer is

$$(11) \quad R_c = sDr_s - \frac{r_b s^2 D}{2} - \frac{r_b sx}{2}$$

where the third term represents the cost of the increased borrowing caused by the lump. The optimal s for the customer is now (see Appendix for derivation):

$$(12) \quad s = \frac{r_s}{r_b} - \frac{x}{4D}$$

The customer will put less into his savings account and borrow later in the month on overdraft than without the lumps.

The returns to the bank in terms of s will be as before except for two added terms. Because the customer will have increased demand deposits (equal to the amount of the expanded overdraft borrowing caused by the lump) the bank must fund the reserves required against those deposits. The cost to the bank of those borrowings is $\frac{sx}{2}R_D r_L$. Also, the bank will have additional income, $\frac{sx}{2}r_b$, from the expanded overdraft loan to the customer (related to the lump). The sum of these two terms is $\frac{sx}{2}(r_b - R_D r_L)$. Taking this term into account converts the change in returns from that given in equation (6) to

$$(13) \quad \Delta R_B = 2s\bar{D}(R_D - R_T)r_L - 2s\bar{D}r_s + (s^2\bar{D} + \frac{sx}{2})(r_b - R_D r_L).$$

At the optimal s of $\frac{r_s}{r_b} - \frac{x}{4\bar{D}}$ with the lump equation (7) now becomes

$$(14) \quad \Delta R_B = \frac{r_s}{r_b}[2(R_D - R_T)r_L - r_s - \frac{r_s}{r_b}R_D r_L]\bar{D} \\ + \frac{x}{2}[r_s - (R_D - R_T)r_L - \frac{x}{2\bar{D}}(r_b - R_D r_L)]$$

Maximizing ΔR_B with respect to the lump yields an optimal ratio for

$\frac{x}{\bar{D}}$ of

$$(15) \quad \frac{x}{\bar{D}} = \frac{4[r_s - (R_D - R_T)r_L]}{r_b - R_D r_L}.$$

At this value for x (or $\frac{x}{\bar{D}}$) or any smaller x , ΔR_B from equation (14) is greater than ΔR_B from equation (7). That is, the lump decreases the loss to the bank from old customers. In fact at the optimal $\frac{x}{\bar{D}}$ it is possible that by setting the size of the lump optimally the bank can prevent any loss at all from its old customer's behavior. However, the optimal size of the lump is going to be unrealistically large. For our

hypothetical values of the parameters optimal $\frac{x}{D}$ is $\frac{.160}{.167}$ or about one hundred percent. This is clearly substantially above any practical limit, and rules out the possibility of preventing all losses from the old customers.¹⁰ Substantially smaller and more realistic values of x will not prevent at least some losses from old customers. Thus it can be seen that modifications to the overdraft program designed to reduce losses from old customers will also diminish the attractiveness of the program and reduce the chances of attracting sufficient new customers to increase overall profits.

Automatic Transfer Accounts

The framework developed here can be applied equally well to an analysis of the recently approved automatic transfer accounts (AT's) for commercial accounts. Under an AT arrangement the bank customer can have funds automatically transferred from a savings account to a checking account to maintain an agreed-upon balance in the checking account. The funds transferred from the savings account are equivalent to overdraft borrowing and the cost of "borrowing" (r_b) is simply the foregone interest on the savings account (i.e., $r_b = r_s$). With no restrictions the optimal s for the customer is 1 (from equation 3), or maintenance of a zero balance checking account and all funds in the savings account. The impact on bank profits is severe, reducing R_B from 7.66% per dollar of deposits (for $s = .278$) to 4.0% (for $s = 1$) for the reserve requirements and interest rates assumed in Table 1. To reduce this profit shrinkage banks have resorted to minimum balance requirements, minimum (lump) transfers, and transfer charges.¹¹ These devices all reduce s below 1, or generate other revenues to offset the profit shrinkage. However,

evaluating equation (15) using $r_s = r_b$ shows that the minimum lump sum transfer to avoid reducing profits is 4.5 times the average deposit level, clearly unworkable. Conclusions about the profit impact of an overdraft facility also apply to the profit impact of an AT system for a bank.

Conclusions

The overall effect on a bank from adding an overdraft program is quite likely to be lower bank profits unless very substantial new customer business is generated by the program. Bank costs generated by old customer behavior switching out of demand deposits into time deposits outweigh revenues from overdraft loans. That is, for every dollar so switched by old customers the bank will gain only $(R_D - R_T)r_L$ and lose r_s . This net loss cannot be offset in the case of an overdraft program through the returns from increased overdraft borrowing. The attraction of new customers by the program must therefore be substantial enough to offset these losses from the bank's old customers. Modifying the program to eliminate the profit shrinkage from old customers holds little promise. Some combination of loss reduction with some gain in customers seems to hold the best hope for maintaining profit levels, but this result would be difficult to achieve in competitive market conditions.

The analysis here is concerned with the impact on the profits of an individual bank from introducing an overdraft facility. While a single bank may be able to increase its profits, primarily by increasing its deposits, clearly the banking system as a whole will suffer a reduction in profit. Deposit shifts among banks will reduce the profit-generating ability of the deposit-losing banks. Hence the overdraft

facility, while providing an increased return to bank customers, cannot be profit-justified in its own right for the commercial banking system.

M/E/188

FOOTNOTES

¹One author states that "unsecured, openend revolving credit is one of the fastgrowing forms of consumer credit." He claims that this type of lending "... can become the most profitable." See William B. McNeill, "Primary Areas of Consumer Credit," in The Banker's Handbook, edited by William H. Baughn and Charles E. Walker, revised edition, Dow Jones-Irwin, 1978, pp. 750-767.

²See Hester, Donald D., "Monetary Policy in a 'Checkless Society'," Journal of Finance, 26 (May 1972), pp. 279-293; Lewis, Kenneth A., "A Note on the Interest Elasticity of the Transactions Demand for Cash," Journal of Finance 29 (September 1974), pp. 1149-52; Sastry, A. S. Rama, "The Effect of Credit on Transactions Demand for Cash," Journal of Finance 24 (September 1970), pp. 777-81; and White, Kenneth J., "The Effect of Bank Credit Cards on the Household Transactions Demand for Money," Journal of Money, Credit and Banking (February 1976), pp. 51-61.

³While the transactions demand literature has rejected this simplistic behavior pattern, the emphasis of this literature has been on business accounts. Despite this literature, the behavior depicted here remains a useful approximation of household behavior.

⁴The analysis in this study is confined to the bank-customer relationships. We do not consider the effects of the movement of funds outside the bank.

⁵At this point we ignore any (non pecuniary) return to the customer (net of service charges) on demand deposits. If this return were included, R_c would of course be larger, and the optimal s , equation (3), would be smaller.

⁶For simplification, the "cost" of demand deposit funds to the bank (net of service charge income) has been omitted. Had this cost been included the values for the change in bank returns, equations (6) and (7), would be larger. However, the thrust of the results would not be changed.

⁷To simplify the analysis, we ignore the option of funding from non-deposit sources. Put differently, we assume (contrary to fact) that gross returns on loans, r_L , is equal to the cost of, say borrowing in the federal funds market.

⁸Recall that the return on demand deposits to customers and the costs of demand deposits to banks have been omitted. Had these been included, the shift from demand to savings accounts would be less, and the bank's return would not fall as much, although it would still fall if reasonable values are assigned to the two missing elements.

⁹Strictly speaking the balances do not go negative because the depositor receives an automatic loan to restore a zero balance.

¹⁰Such large values for the lump also make nonsense of our implicit assumption that the customer will end the period with his last lump completely used up by his payment needs.

¹¹For an example of how these devices affect bank profits see Alfred Broaddus, "Automatic Transfers from Savings to Checking: Perspective and Prospects," Economic Review, Federal Reserve Bank of Richmond, November/December 1978, pp. 3-13.

APPENDIX

Optimal s for the overdraft customer

To derive the optimal s, equation (2), the return to the customer is differentiated with respect to s, which is the customer's decision variable, giving

$$(2a) \quad \frac{dRc}{ds} = Dr_s - sDr_b.$$

Setting (2a) to zero, thereby maximizing the customer's return gives

$$(3) \quad s = \frac{r_s}{r_b}.$$

Optimal s with lump transfers

The return to the customer using a lump transfer is shown in equation (11). Again, differentiating (10) with respect to s yields

$$(11a) \quad \frac{dRc}{ds} = Dr_s - r_b sD - r_b \frac{x}{2}.$$

Setting (10a) equal to zero and solving for x gives

$$(11b) \quad s = \frac{r_s}{r_b} - \frac{x}{2D}.$$

Expressing 10b in terms of \bar{D} , or average level of demand deposits gives

$$(12) \quad s = \frac{r_s}{r_b} - \frac{x}{4\bar{D}}.$$



UNIVERSITY OF ILLINOIS-URBANA



3 0112 060296271