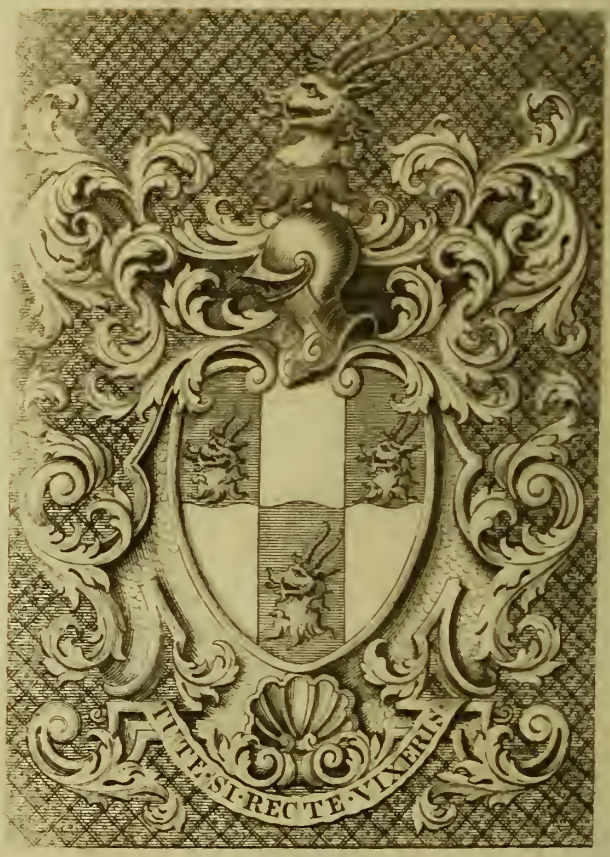

TUTE·SI·RECTE·VIXERIS

PHILOSOPHICAL TRANSACTIONS.

Giving Some

ACCOUNT

OF THE

Present Undertakings, Studies, and Labours

OF THE

INGENIOUS,

In Many

Considerable Parts of the WORLD.

Vol. XXIX. For the Years 1714, 1715, 1716.

LONDON:

Printed for W. INNYS, at the *Princes-Arms* in St. *Paul's Church-Yard.* MDCCXVII.

TO

Sir *Isaac Newton*, Kt.

PRESIDENT,

And to the

Council and Fellows

OF THE

Royal Society

OF

LONDON

Instituted for the

Advancement of *Natural Knowledge*;

THIS

Twenty Ninth VOLUME

OF

Philosophical Transactions

IS

HUMBLY DEDICATED,

BY

Edmond Halley, R. S. Secr.

Numb. 338. Beginning the 29th Volume.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *January*, *February* and *March*, 1714.

The CONTENTS.

THE

THE PREFACE.

THE Philosophical Transactions, (*begun to be Published by that indefatigably diligent Promoter of Learned Correspondence, Mr.* Oldenburgh, *the first who attempted any thing of this Nature) having been sufficiently known to the Curious, and always acceptable to the Learned, when due care has been taken in the choice of the Collections so recommended to the inquisitive and intelligent Reader; a long Preface must appear wholly unnecessary.*

However, it may not be amiss to inform the Learned and Ingenious, of some Particulars of the present Publisher's Design: Which is;

First, *To present the Publick with such short Tracts, as might otherwise be lost to Posterity, if printed in single Sheets or Pamphlets.*

Secondly, *To give the Extracts, for the most part, of the Material, omitting the Preambles and Conclusions, and the useless parts of such Letters as Correspondents shall please to honour him with, relating to the Improvement of Natural Philosophy, Mathematicks, or Mechanicks.*

Thirdly, *To give an exact account of such Experiments, as having been made before the Illustrious* ROYAL SOCIETY, *they shall please to order or permit the Publication of.*

Fourthly,

Fourthly, *To give, as Opportunity offers, the Abſtracts or Accounts of ſuch Books, relating to the Ends of the* Royal Society's *Inſtitution, as the Authors ſhall pleaſe to communicate themſelves; which ſhall be diſtinguiſhed from others: or of ſuch as are with difficulty to be procured in* England, *or extraordinary in their kind; upon which there ſhall ſometimes be added ſome Remarks.*

To this Undertaking the Publiſher invites and deſires all real Lovers of Knowledge to contribute their Aſſiſtance, by communicating their Obſervations, Diſcoveries, or Inventions; which ſhall not only be faithfully and duly Publiſhed, with due Acknowledgment; but ſome Returns made them, by Informations of what ſhall be thought acceptable to the Gentlemen, who ſhall think fit to entertain a Philoſophical Correſpondence: Who are deſired, in any Tract or Diſcourſe they ſhall ſo communicate, to omit all Perſonal Reflections; for if ſuch ſhould happen to be inſerted, the Publiſher ſhall take the liberty of leaving them out; it being his Opinion, that Diſputes on Philoſophical Subjects, may be managed with the utmoſt Candor, Reſpect and Friendſhip by the Diſputants, whoſe only Aim ought to be the Search of Truth.

The Gentlemen willing to encourage this Undertaking, are deſired to direct their Letters, To be left at the Royal Society's *Houſe, in* Crane-Court, *in* Fleetſtreet, London, *for the Secretary of the* ROYAL SOCIETY.

LOGO-

LOGOMETRIA

Auctore

ROGERO COTES,

Trin. Coll. Cantab. Soc.

Aſtr. & Ph. Exp. Profeſſore PLUMIANO, & R. S. S.

Eruditiſſimo Viro

EDMUNDO HALLEIO,

Societatis Regalis Secretario S. P.

MItto tibi, hortatu Illuſtriſſimi Præſidis NEWTONI, *quæ aliquot abhinc annis conſcripſeram de Rationibus dimetiendis. Tu vero, quum & Ipſe dudum in eodem Argumento præclare verſatus fueris, pro ſolito tuo candore, tentamen hoc qualecunque benigne accipies. Vale.*

AGITUR in hoc Tractatu de *Menſuris Rationum.* Hæ Menſuræ ſunt quantitates cujuſcunque generis, quarum magnitudines magnitudinibus rationum ſunt analogæ. In dato itaque Syſtemate, rationis ejuſdem eadem eſt menſura, duplicatæ dupla, triplicatæ tripla, ſubduplicatæ ſubdupla, ſeſquiplicatæ ſeſquialtera: denique quocunque modo per compoſitionem vel reſolutionem auctæ vel diminutæ rationis, ſimiliter

aucta eſt vel diminuta menſura. Æqualitatis ratio nullam habet magnitudinem, quia nullam addita vel detracta mutationem inducit; rationes quæ dicuntur majoris & minoris inæqualitatis contrarias habent magnitudinum ſuarum affectiones, quoniam in compoſitione & reſolutione contraria ſemper efficiunt: itaque ſi menſura rationis quam habet terminus major ad minorem poſitiva cenſeatur, menſura rationis quam habet terminus minor ad majorem erit negativa, menſura vero rationis inter æquales terminos nullius erit magnitudinis. Porro diverſa menſurarum oriuntur *Syſtemata*, prout modis diverſis exponitur analogia illa determinata & immutabilis quæ eſt inter magnitudines rationum. Inde vero patet, exhiberi poſſe numero infinita Syſtemata, minuendo vel augendo Syſtematis cujuſvis dati menſuras omnes in eadem data quacunque proportione, aut etiam pro menſuris adhibendo quantitates diverſi generis. In tanta autem varietate confuſionem aliquam oboriri neceſſe eſt, ni probe conſtiterit ad quodnam Syſtema referendæ ſint menſuræ ſingulæ de quibus contingat ſermonem inſtitui. Huic malo remedium optime parari poteſt ſi menſura datæ alicujus rationis, quæ commodiſſima videbitur, pro *Modulo* habeatur ad quem conſtanter in omni Syſtemate menſuræ reliquarum rationum exigantur. Id enim ſi fiat, ſtatim ex dato illo Modulo determinabitur Syſtema totum: nam ex menſuris conſtabit quæ Modulo erunt homogeneæ, quæque eo majores habebunt magnitudines vel minores quo major ille fuerit vel minor, ut ita menſurandarum rationum invariata magnitudinum ſervetur analogia inter ipſas menſuras. Patebit igitur in ſequentibus rationem quandam dari, dupli inter & tripli rationes intermediam, ad rationem vero tripli aliquanto propius accedentem, quæ propoſito noſtro non immerito aptiſſima judicetur, ſiquidem ipſa rei natura hujus uſum ſuadere ac non incertis indiciis efflagitare quodammodo videatur. Hanc ego, ex officio ejus deſumpto nomine, *Modularem Rationem* appellabo; quo autem pacto ipſa ſit accuratius definienda, oſtendetur inferius, nunc enim de Logarithmis pauca ſunt addenda.

Logarithmi ſunt rationum menſuræ Numerales: ſolent autem in Canone ſic diſponi, ut ſingulis numeris naturali ordine creſcentibus, & in ſerie continua poſitis adſcribatur Logarithmus, non quidem ipſius numeri uti vulgo dicitur, ſed rationis quam habet numerus ad Unitatem. Exinde vero rationis per quoſcunque terminos deſignatæ facilis eſt inventio Logarithmi. Nam cum ratio antecedentis ad conſequentem ſit exceſſus rationis antecedentis ad Unitatem

ſupra.

ſupra rationem conſequentis ad Unitatem: Logarithmus ejus ſimiliter erit exceſſus Logarithmi rationis quam habet antecedens ad Unitatem ſupra Logarithmum rationis quam conſequens habet ad Unitatem; hoc eſt, ut vulgari ſermone utamur, exceſſus Logarithmi antecedentis ſupra Logarithmum conſequentis; neutiquam enim diſplicet loquendi modus jam à multis annis receptus, ſi recte intelligatur. Exinde porro peregregium enaſcitur compendium ad operationes Arithmeticas. Datis enim duobus quibuſcunque numeris in ſe multiplicandis, ſi quæratur numerus ex multiplicatione productus; quoniam rationes numerorum datorum ad Unitatem, conficiunt ſimul additæ rationem producti ad Unitatem, & rationum componendarum menſuræ ſimul additæ conficiunt rationis compoſitæ menſuram: Logarithmus producti æquabitur Logarithmis numerorum datorum ſimul ſumptis. Ad eundem modum ſi quæratur numerus ex diviſione ortus; quoniam ratio diviſoris ad Unitatem è ratione dividendi ad Unitatem detracta relinquit rationem quoti ad Unitatem: habebitur quoti Logarithmus ſubducendo Logarithmum diviſoris è Logarithmo dividendi. Et eodem argumento, ſi quæratur dati cujuſvis numeri quælibet poteſtas; quoniam ratio dati numeri ad Unitatem per Indicem poteſtatis multiplicata rationem efficit quam habet numeri poteſtas ad Unitatem, & menſura prioris rationis multiplicata per eundem Indicem efficit pariter menſuram rationis poſterioris: Logarithmus poteſtatis æquabitur Logarithmo numeri dati per Indicem poteſtatis multiplicato. Et ſimiliter Logarithmus cujuſlibet radicis numeri dati æquabitur Logarithmo numeri dati per Indicem radicis diviſo. Igitur ope Canonis peragetur inventio poteſtatum & radicum per multiplicationem & diviſionem, multiplicatio autem & diviſio per additionem & ſubductionem. Ceterum de hiſce vulgo notis Logarithmorum uſibus non eſt mei inſtituti fuſius diſſerere: miſſis ergo ambagibus, ad alia nunc me confero & rem ipſam protinus aggredior.

Propositio I.

Invenire Mensuram Rationis cujuscunque propositæ.

PRoponatur Ratio inter AC & AB, cujus Mensuram oportet invenire. Terminorum differentia BC divisa concipiatur in particulas innumeras quam minimas PQ, atque ratio inter AC & AB in totidem rationes quam minimas inter AQ & AP: & si detur magnitudo rationis inter AQ & AP, dividendo dabitur ratio quam habet PQ ad AP; atque adeo data illa magnitudo rationis inter AQ & AP, per datam quantitatem $\frac{PQ}{AP}$ exponi potest. Manente AP, augeri vel minui intelligatur particula PQ in proportione quavis; & in eadem proportione augebitur vel minuetur magnitudo rationis inter AQ & AP: capiatur particula dupla vel tripla, subdupla vel subtripla, & evadet ratio duplicata vel triplicata, subduplicata vel subtriplicata; etiamnum igitur exponetur per quantitatem $\frac{PQ}{AP}$. Sed &, assumpta determinata quavis quantitate M, exponi potest per $M \times \frac{PQ}{AP}$: erit ergo quantitas $M \times \frac{PQ}{AP}$ mensura rationis inter AQ & AP. Hæc vero mensura diversam habebit magnitudinem, & ad Systema diversum accommodabitur, pro diversa magnitudine quantitatis assumptæ M, quæ adeo vocetur Systematis *Modulus*. Jam quemadmodum summa rationum omnium inter AQ & AP æqualis est propositæ rationi, quam utique habet AC ad AB: ita summa mensurarum omnium $M \times \frac{PQ}{AP}$ (per Methodos satis notas invenienda) æqualis erit ejusdem propositæ rationis mensuræ quæsitæ. *Q. E. I.*

Corol. 1. Terminis AP, AQ ita ad æqualitatem accedentibus, ut quam minima sit eorundem differentia PQ: erit $M \times \frac{PQ}{AP}$ vel $M \times \frac{PQ}{AQ}$ æqualis mensuræ rationis inter AQ & AP ad Modulum M.

Corol.

Corol. 2. Unde Modulus ille M eſt ad menſuram rationis inter terminos AQ & AP, ut terminorum alteruter AP vel AQ ad terminorum differentiam PQ.

Corol. 3. Data ratione inter AC & AB, datur ſumma omnium $\frac{PQ}{AP}$, & ſumma omnium $M \times \frac{PQ}{AP}$ eſt ut M. Itaque menſura datæ cujuſcunque rationis eſt ut Modulus Syſtematis ex quo deſumitur.

Corol. 4. Modulus ergo, in omni menſurarum Syſtemate, ſemper æqualis fit menſuræ rationis cujuſdam determinatæ atque immutabilis: Quam proinde *Rationem Modularem* vocabo.

Scholium 1.

Problematis ſolutio per Exemplum illuſtrabitur. Sit z quantitas determinata quævis & permanens, ſit vero x quantitas indeterminata fluxuque perpetuo variabilis, ejuſque fluxio ſit $\dot{x}$; & quæratur menſura rationis inter $z+x$ & $z-x$. Statuatur hæc ratio æqualis rationi inter y & 1, exponatur autem numerus y per AP, fluxio ejus $\dot{y}$ per PQ, 1 per AB: & ex Corollario primo colligetur fluxionem quæſitæ menſuræ rationis inter y & 1 eſſe $M \times \frac{\dot{y}}{y}$. Reponatur jam pro y valor ejus $\frac{z+x}{z-x}$, itemque pro $\dot{y}$ valoris fluxio $\frac{2z\dot{x}}{\overline{z-x}|^2}$: & fluxio menſuræ evadet $2M \times \frac{z\dot{x}}{zz-xx}$ vel $2M \times \frac{\dot{x}}{z-\frac{xx}{z}}$ ſive $2M$ in $\frac{\dot{x}}{z} + \frac{\dot{x}x^2}{z^3} + \frac{\dot{x}x^4}{z^5} +$ &c. Atque adeo menſura illa fiet $2M$ in $\frac{x}{z} + \frac{x^3}{3z^3} + \frac{x^5}{5z^5} +$ &c. Unde patet Corollarium ſequens.

Corol. 5. Si duarum quantitatum ſumma ſit z & differentia ſit x; & ſumatur $2M\frac{x}{z} = A$, $A\frac{xx}{zz} = B$, $B\frac{xx}{zz} = C$, $C\frac{xx}{zz} = D$, &c: Menſura rationis quam habet quantitas major ad quantitatem minorem, erit $A + \frac{1}{3}B + \frac{1}{5}C + \frac{1}{7}D +$ &c.

Scholium 2.

Non abſimili computo menſura rationis inter $1+v$ & 1 erit M in $v - \frac{1}{2}v^2 + \frac{1}{3}v^3 - \frac{1}{4}v^4 + \frac{1}{5}v^5 -$ &c. Unde ſi menſura illa vocetur m, erit $\frac{m}{M} = v - \frac{1}{2}vv + \frac{1}{3}v^3 - \frac{1}{4}v^4 + \frac{1}{5}v^5$, &c: ac proinde

inde $\frac{mm}{MM} = vv - v^3 + \frac{11}{12}v^4 - \frac{5}{6}v^5$, &c; ſimiliterque $\frac{m^3}{M^3} = v^3 - \frac{3}{2}v^4 + \frac{7}{4}v^5$, &c; quinetiam $\frac{m^4}{M^4} = v^4 - 2v^5$, &c; ac denique $\frac{m^5}{M^5} = v^5$, &c. Ut igitur viciſſim, ex data menſura m, inveniatur ratio quam metitur; addendo æqualia æqualibus habebitur $\frac{m}{M} + \frac{mm}{2MM} = v * - \frac{1}{6}v^3 + \frac{5}{24}v^4 - \frac{13}{60}v^5$, &c; atque iterum $\frac{m}{M} + \frac{mm}{2MM} + \frac{m^3}{6M^3} = v * * - \frac{1}{24}v^4 + \frac{3}{40}v^5$, &c; rurſumque $\frac{m}{M} + \frac{mm}{2MM} + \frac{m^3}{6M^3} + \frac{m^4}{24M^4} = v * * * - \frac{1}{120}v^5$, &c; atque tandem $\frac{m}{M} + \frac{mm}{2MM} + \frac{m^3}{6M^3} + \frac{m^4}{24M^4} + \frac{m^5}{120M^5} = v * * * *$, &c; id eſt, $\frac{m}{M} + \frac{mm}{2MM} + \frac{m^3}{6M^3} + \frac{m^4}{24M^4} + \frac{m^5}{120M^5} +$ &c. $= v$. Itaque ratio quæſita inter $1 + v$ & 1, eſt ea quam habet $1 + \frac{m}{M} + \frac{mm}{2MM} + \frac{m^3}{6M^3} + \frac{m^4}{24M^4} + \frac{m^5}{120M^5} +$ &c. ad 1. Ponatur $m = M$, ſive $\frac{m}{M} = 1$; & exinde Ratio Modularis erit ea quam habet $1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} +$ &c. ad 1.

Eodem modo, ſi detur ratio inter 1 & $1 - v$, menſura hujus rationis erit M in $v + \frac{1}{2}v^2 + \frac{1}{3}v^3 + \frac{1}{4}v^4 + \frac{1}{5}v^5$, &c. Et viciſſim ſi detur rationis menſura m, ratio erit ea quam habet 1 ad $1 - \frac{m}{M} + \frac{mm}{2MM} - \frac{m^3}{6M^3} + \frac{m^4}{24M^4} - \frac{m^5}{120M^5} +$ &c. Ponatur $m = M$, ſive $\frac{m}{M} = 1$; & exinde Ratio Modularis erit ea quam habet 1 ad $1 - \frac{1}{1} + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} +$ &c. Ex hiſce vero patet Corollarium ſequens.

Corol. 6. Expoſito termino R, ſi ſumatur $\frac{1}{1}R = A$, $\frac{1}{2}A = B$, $\frac{1}{3}B = C$, $\frac{1}{4}C = D$, $\frac{1}{5}D = E$, &c. in infinitum; & capiatur $S = R + A + B + C + D + E +$ &c: Ratio Modularis erit ea quæ eſt inter terminum minorem expoſitum R & majorem inventum S. Vel expoſito termino S, ſi ſumatur $\frac{1}{1}S = A$, $\frac{1}{2}A = B$, $\frac{1}{3}B = C$, $\frac{1}{4}C = D$, $\frac{1}{5}D = E$, &c. in infinitum; & capiatur $R = S - A + B - C + D - E +$ &c: Ratio Modularis erit ea quæ eſt inter terminum majorem expoſitum S & minorem inventum R. Porro eadem ratio eſt inter 2,718281828459 &c. et 1, vel inter 1 & 0,367879441171 &c.

Scho-

Scholium 3.

Si forte termini minores desiderentur, qui eandem proxime Rationem Modularem ita exhibeant, ut nulli ipsis non majores propius; instituenda erit operatio ad modum sequentem. Dividatur terminus major 2,71828 &c. per minorem 1, vel etiam major 1 per minorem 0,367879 &c. & rursus minor per numerum qui reliquus est, & hic rursus per ultimum residuum, atque ita porro pergatur: &

Rationes Vera Majores.		*Rationes Vera Minores.*	
1	0×2	0	1
2	1	2	0
3	1×2	2	1×1
8	3	6	2
11	4×1	8	3×1
76	28	11	4
87	32×1	19	7×4
106	39	87	32
193	71×6	106	39×1
1264	465	1158	426
1457	536×1	1264	465×1
21768	8008	1457	536
23225	8544×1	2721	1001×8
25946	9545	23225	8544
49171	18089×10	25946	9545×1
&c.	&c.	&c.	&c.

prodibunt quotientes 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1, 16, 1, 1, &c. His inventis, perficiendæ sunt binæ rationum columnæ, quarum altera terminos continet rationem habentes vera majorem, altera terminos quorum ratio est vera minor; ineundo computationem à rationibus 1 ad 0, 0 ad 1, quæ remotissimæ sunt à vera; inde autem exorsam deducendo ad rationes reliquas, quæ

quæ continuè ad veram propius accedunt. Multiplicentur itaque termini 1 & 0 per quotientem primum 2, & ſcribantur facti 2 & 0 infra terminos 0 & 1; & addendo prodibit ratio 2 + 0 ad 0 + 1, ſive 2 ad 1. Hujus termini multiplicentur per quotientem ſecundum 1, factique 2 & 1 addantur terminis 1 & 0; & habebitur ratio 2 + 1 ad 1 + 0, ſive 3 ad 1. Hujus termini multiplicentur per quotientem tertium 2, factique 6 & 2 addantur terminis præcedentibus 2 & 1; & habebitur ratio 8 ad 3. Hujus termini multiplicentur per quotientem quartum 1, factique 8 & 3 addantur terminis præcedentibus 3 & 1; & habebitur ratio 11 ad 4. Hujus termini multiplicentur per quotientem quintum 1, factique 11 & 4 addantur præcedentibus 8 & 3; & habebitur ratio 19 ad 7. Hujus termini rurſus multiplicentur per quotientem ſextum 4, factique 76 & 28 addantur præcedentibus 11 & 4, ad inveniendam rationem 87 ad 32; & ſic porro pergendum quouſque libuerit, tranſitu alternis facto in alteram columnam. Hiſce peractis, habebuntur rationes vera majores 3 ad 1, 11 ad 4, 87 ad 32, 193 ad 71, 1457 ad 536, 23225 ad 8544, 49171 ad 18089, &c. Vera autem minores erunt 2 ad 1, 8 ad 3, 19 ad 7, 106 ad 39, 1264 ad 465, 2721 ad 1001, 25946 ad 9545, &c. Atque hæ quidem ſunt præcipuæ & primariæ rationes, quibus ad rationem propoſitam continue appropinquatur.

Quod ſi exquiratur integra ſeries rationum omnium vera majorum quæ ita dari poſſint, ut nulla minoribus terminis deſignata ratio vera major ad veram propius accedat; & ſimiliter ſeries integra rationum omnium vera minorum quæ ita dari poſſint, ut nulla minoribus terminis deſignata ratio vera minor ad veram propius accedat: inter primarias illas modo inventas inſerendæ ſunt aliæ ſecundariæ rationes. Hæ vero locum habent ubi quotiens unitatem ſuperat. Inveniuntur autem mutata multiplicatione, quæ ſupra per quotientem facta eſt, in continuam additionem terminorum tot vicibus quot ſunt unitates in quotiente. Sic quia quotiens primus erat 2, termini 1 & 0 bis addendi ſunt terminis 0 & 1; & ſummæ dabunt rationes 1 ad 1, 2 ad 1. Hi ultimi termini 2 & 1, quia quotiens ſecundus erat 1, ſemel addendi ſunt terminis 1 & 0; & ſummæ dabunt rationem 3 ad 1. Hi termini 3 & 1, quia quotiens tertius erat 2, bis addendi ſunt terminis 2 & 1; & ſummæ dabunt rationes 5 ad 2, 8 ad 3. Hi ultimi termini 8 & 3, quia quotiens quartus erat 1, ſemel addendi ſunt terminis 3 & 1; & ſummæ dabunt rationem 11 ad 4. Hi termini 11 & 4, quia quotiens

tiens quintus erat 1, ſemel addendi ſunt terminis 8 & 3; & ſummæ dabunt rationem 19 ad 7. Hi denique termini 19 & 7, quia quo-

Rationes Vera Majores.		*Rationes Vera Minores.*	
1	0×2	0	1
2	1	1	0
3	1×2	1	1
8	3	1	0
11	4×1	2	1×1
19	7	3	1
30	11	5	2
19	7	3	1
49	18	8	3×1
19	7	11	4
68	25	19	7×4
19	7	87	32
87	32×1	106	39×1
&c.	&c.	&c.	&c.

tiens ſextus erat 4, quater addendi ſunt terminis 11 & 4; & ſummæ dabunt rationes 30 ad 11, 49 ad 18, 68 ad 25, 87 ad 32. Et ſic porro procedere licebit quouſque commodum videbitur. Iſta tandem operatione peracta, ſeries integra rationum omnium vera majorum, erit 1 ad 0, 3 ad 1, 11 ad 4, 30 ad 11, 49 ad 18, 68 ad 25, 87 ad 32, &c. ſimiliterque ſeries integra rationum omnium vera minorum, erit 0 ad 1, 1 ad 1, 2 ad 1, 5 ad 2, 8 ad 3, 19 ad 7, &c.

Harum approximationum utilitas ad alia multa ſeſe diffundit: quapropter earum inventionem aliquanto prolixius expoſitam dedi, per Methodum quæ mihi ſimpliciſſima & facillima videtur. Idem argumentum paulo aliter pertractarunt Viri celeberrimi *Walliſius* & *Hugenius*.

Propositio II.

Logarithmorum Canonem Briggianum *construere.*

NUmerorum Compositorum Logarithmi derivantur ex Logarithmis Primorum componentium, per additionem solam; horum autem investigatio pluribus modis institui potest: Exemplum unicum appono.

Per Corollarium quintum Propositionis superioris, scribendo 1 pro M, inveniantur Logarithmi rationum inter 126 & 125, 225 & 224, 2401 & 2400, 4375 & 4374; qui vocentur respective p, q, r, s: & Logarithmus denarii seu rationis decupli erit $239p + 90q - 63r + 103s$, sive 2,302585092994 &c. Itaque cum Logarithmus *Briggianus* denarii sit 1; fiat (per Corol. 3. Prop. 1.) ut denarii Logarithmus modo inventus 2,302585092994 &c, ad Modulum suum 1, ita denarii Logarithmus *Briggianus* 1, ad Modulum *Briggianum*, qui adeo erit 0,434294481903 &c. Ponatur ergo deinceps iste valor pro M, & erunt $M \times \overline{202p + 76q - 53r + 87s}$, $M \times \overline{167p + 63q - 44r + 72s}$, $M \times \overline{114p + 43q - 30r + 49s}$ Logarithmi *Briggiani* numerorum 7, 5, 3. Logarithmus numeri 2 habetur, subducendo Logarithmum numeri 5 à Logarithmo numeri 10. Atque ita dantur & Modulus *Briggianus* & Logarithmi Primorum omnium qui sunt minores denario.

Logarithmi numerorum sequentium Primorum 11, 13, 17, 19, 23, &c. ita computari possunt. Quæratur tum factus à numeris Primo proposito utrinque proxime adjacentibus, tum Primi ipsius quadratum, quod semper unitate factum illud superabit. Logarithmo rationis quadrati ad factum (per Corol. 5. Prop. 1. inveniendo) addatur ipsius facti Logarithmus, qui semper componetur ex datis Logarithmis Primorum qui proposito Primo sunt minores: & semisumma erit Logarithmus Primi quæsitus.

Corol. Canonis *Briggiani* Modulus est 0,434294481903 &c: Hujus vero Reciprocus est 2,302585092994 &c.

Scholium.

Ad hunc itaque modum perfici posset Logarithmorum Tabula amplissima, qualis edita est à *Briggio* vel *Vlacco.* Inventioni autem Numerorum & Logarithmorum sibi invicem congruentium, qui intermedii

termedii ſunt & ultra Tabulæ limites excurrunt, abunde ſufficiet terminus primus Seriei quæ in Corollario quinto Propoſitionis præcedentis exhibetur.

Si dato Numero intermedio quæratur ejus Logarithmus; pone a & e pro Numero intermedio propoſito atque huic proximo tabulari, ita ut a deſignet majorem, e minorem; ſit eorum ſumma z, differentia x; pone λ pro Logarithmo rationis quam habet a ad e, hoc eſt, pro exceſſu Logarithmi Numeri a ſupra Logarithmum Numeri e: & erit $\lambda = 2\,M\frac{x}{z}$ quamproxime.

Si quæratur Numerus qui congruit Logarithmo intermedio; quoniam eſt $\lambda = \frac{2Mx}{z} = \frac{2Mx}{2a-x}$ vel $\frac{2Mx}{2e+x}$; erit $x = \frac{\lambda}{M+\frac{1}{2}\lambda}a$ vel $\frac{\lambda}{M-\frac{1}{2}\lambda}e$ quamproxime.

Propositio III.

Syſtematis cujuſvis Logometrici conſtructionem exponere per Canonem Logarithmorum.

Caſ. 1. SI detur, è Syſtemate propoſito, menſura rationis alicujus determinatæ: rationis cujuſvis oblatæ menſura, erit ad menſuram illam datam determinatæ rationis, ut oblatæ rationis Logarithmus, ad Logarithmum rationis ejuſdem determinatæ.

Caſ. 2. Si non detur, è Syſtemate propoſito, menſura rationis alicujus determinatæ: inveniendus erit Modulus propoſiti Syſtematis, per Corollarium ſecundum Propoſitionis primæ. Et menſura cujuſvis oblatæ rationis, erit ad Modulum inventum, ut oblatæ rationis Logarithmus, ad Canonis Modulum.

Caſus hujus ultimi habentur Exempla in ſequentibus.

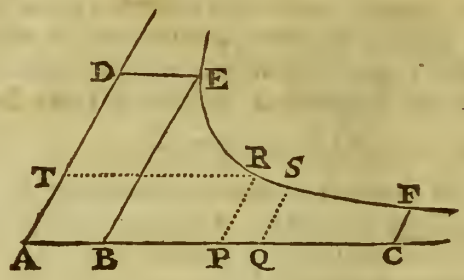
Propositio IV.

Spatium quodvis Hyperbolicum quadrare per Canonem Logarithmorum.

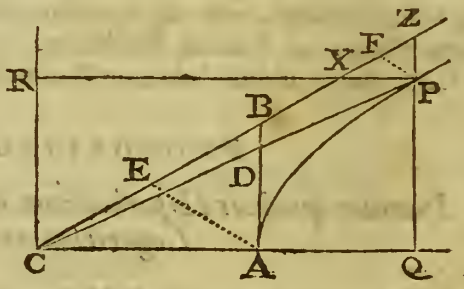
SIT Hyperbola quævis *ERSF* centro *A*, Aſymptotis *ABC*, *AD* deſcripta; & quæratur area *BEFC* quam claudunt rectæ *BE*, *CF* ad Aſymptoton *AD* parallelæ. Compleatur parallelogrammum *ABED*, & ad hunc Modulum inveniatur (per Propo-

 ſitionem

ſitionem tertiam) menſura rationis inter *AC* & *AB* vel inter *BE* & *CF*: Dico menſuram inventam æqualem fore magnitudini areæ quæſitæ *BEFC*. Nam diviſa concipiatur hujus areæ baſis *BC* in particulas innumeras quam minimas *PQ*, ea lege, ut ubique detur ratio illa quæ eſt inter *AQ* & *AP*, & ducantur Aſymptoto *AD* parallelæ *PR*, *QS*. Quoniam itaque eſt *AQ* ut *AP*; erit diviſim *PQ* ut *AP*, hoc eſt, ut *PR* reciproce. Unde data eſt area *PRSQ*, quæ proinde poteſt haberi pro menſura rationis datæ quæ eſt inter *AQ* & *AP*. Hujus autem menſuræ Modulus erit parallelogrammum *ABED*, per Corol. 2. Prop. 1. Nam ſi compleatur æquale parallelogrammum *APRT*; ſtatim intelligetur, ita illud ſe habere ad aream *PRSQ*, ut ſe habet *AP* ad *PQ*. Similes ergo ſummas arearum atque rationum utrinque colligendo; area tota *BEFC* erit menſura rationis totius quæ eſt inter *AC* & *AB*, vel inter *BE* & *CF*, ad eundem Modulum *ABED*.



Aliter. Sit rurſus Hyperbola quævis *AP*, centro *C* atque Aſymptoto *CB* deſcripta; & quæratur area Sectoris cujuſlibet *CAP*, ſemidiametris *CA*, *CP* curvæque *AP* interjecti. Producta ſemidiametro utravis *CAQ* ultra verticem *A*, ducatur illius conjugata *CR*; & ad ipſas *CQ*, *CR* ordinatim applicentur à puncto *P* rectæ *PQ*, *PR*, quæ Aſymptoto *CB* occurrant in *Z* & *X*; deinde agatur *AB* quæ Hyperbolam tangat in *A*, Aſymptoton ſecet in *B* rectamque *CP* in *D*: & Triangulo *ABC* exiſtente Modulo, area quæſita ſectoris *CAP* erit menſura rationis inter $\overline{QZ} + \overline{QP}$ & *AB*, ſive rationis inter *AB* & $\overline{QZ} - \overline{QP}$, ſive

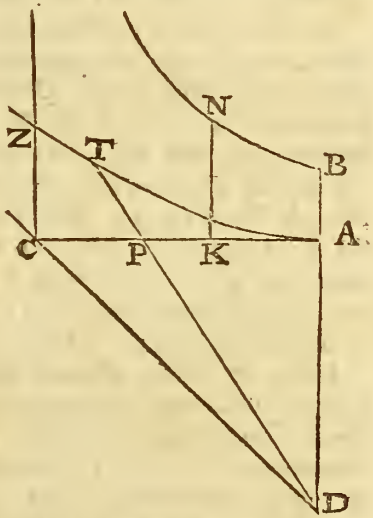


ſive ſubduplicatæ rationis inter $\underset{\sim}{OZ} + \underset{\sim}{OP}$ & $\underset{\sim}{OZ} - \underset{\sim}{OP}$, ſive ſubduplicatæ rationis inter $AB + AD$ & $AB - AD$; vel erit menſura rationis inter $RP + RX$ & CA, vel rationis inter CA & $RP - RX$, vel ſubduplicatæ rationis inter $RP + RX$ & $RP - RX$. Nam ſi ducantur rectæ AE, PF quæ ſecent Aſymptoton CB in E & F, alterique Aſymptoto parallelæ ſint: æquales erunt hæ omnes rationes rationi quam habet AE ad PF, vel CF ad CE; erit & ſector CAP areæ $EAPF$ æqualis; ſimiliterque triangulum ABC duplicato triangulo AEC, ſive parallelogrammo Aſymptotis & Hyperbolæ inſcripto æquabitur. Quare patet propoſitum ex ſupra demonſtratis.

Data vero per modum priorem area $BEFC$, vel per modum poſteriorem area CAP; dabitur alia quævis area Hyperbolica ad arcum EF, vel ad arcum AP terminata: quippe quæ ſemper eſt areæ modo inventæ & areæ alicujus rectilineæ vel ſumma vel differentia. $\underset{\sim}{Q}$. E. I.

Scholium.

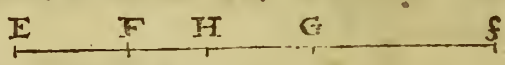
Hinc facilem habent ſolutionem Problemata omnia, quæcunque pendent ab Hyperbolæ quadratura. Exemplum ſatis luculentum præbebit deſcenſus gravium in Mediis, quorum reſiſtentia eſt in duplicata ratione velocitatis corporis moti. Sit V velocitas maxima quam corpus in hujuſmodi Medio, infinite deſcendendo, poteſt acquirere; T dimidium temporis quo corpus idem in eodem Medio, vi ſola ponderis ſui relativi, abſque reſiſtentia cadendo velocitatem illam acquiret; S ſpatium hocce caſu deſcriptum; R pondus relativum corporis in Medio reſiſtente: & quæratur ſpatium s quod corpus deſcendens, tempore quovis t, deſcribet in Medio reſiſtente; & reſiſtentia r quam patitur in fine illius temporis; & velocitas v ex iſto deſcenſu acquiſita.



Centro D, vertice A deſcribatur Hyperbola æquilatera AT, cujus una Aſymptotorum eſt DC & ad verticem tangens AC ſemiaxi AD æqualis. Capiatur area DAT ad dimidium trianguli DAC ut t ad T, ſecetque DT tangentem AC in P: &

& erit v ad V ut AP ad AC. Sit AK ipsis AC, AP tertia proportionalis: & erit r ad R ut AK ad AC. Ad tangentem AC erigantur normales CZ, KN, AB; centroque C & Asymptotis CA, CZ describatur Hyperbola quævis BN: & erit s ad S ut area $ABNK$ ad rectangulum CKN. Patent hæc omnia per Propositiones octavam & nonam Libri secundi Philosophiæ *Newtonianæ*.

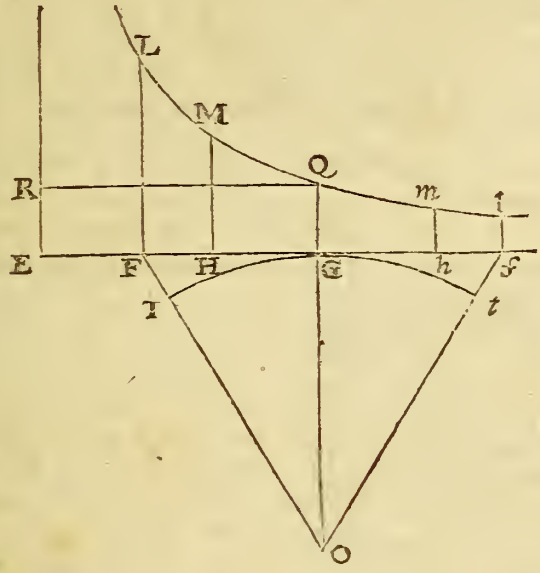
Est itaque t ad T ut area Hyperbolica DAT ad dimidium trianguli DAC, hoc est, ut dimidiata mensura rationis inter $AC+AP$ & $AC-AP$ ad illius mensuræ dimidiatum Modulum. Ergo si recta quævis EF producatur ad f, ita ut t sit mensura rationis inter Ef & EF ad Modulum T, & bisecetur Ff in G: erit GF ad GE ut AP ad AC, hoc est, ut v ad V. Sumantur GE, GF, GH continue proportionales: & erit GH ad GE ut AK ad AC, hoc est, ut r ad R. Erit insuper EG ad EH ut CA ad CK; unde cum sit s ad S ut area $ABNK$ ad rectangulum CKN, hoc est, ut mensura rationis inter CA & CK vel inter EG & EH ad mensuræ Modulum: erit s mensura rationis inter EG & EH ad Modulum S, atque inde dabitur.



Ex hisce porro facillime se prodit, per unicam quamvis Hyperbolam, constructio non inconcinna; quam & adscribere visum est ob dignitatem Problematis. In recta quavis GE sumatur utcunque punctum F inter E & G, & ab altera parte capiatur Gf ipsi GF æqualis, & sint GE, GF, GH continue proportionales. Deinde per puncta E, F, H, G, f ducantur sibi invicem parallelæ rectæ ER, FL, HM, GQ, fl, quas secet Hyperbola quævis $LMQl$ centro E, Asymptotis ER, EG descripta, & compleatur parallelogrammum $EGQR$. Jam si sit t ad T ut area Hyperbolica $LFfl$ ad parallelogrammum EQ: erit s ad S ut area $MHGQ$ ad EQ; v ad V ut GF ad GE; r ad R ut GH ad GE.

Libet & casum alterum adjicere ubi corpus ascendit; ne forte analogia illa, quæ inter utrumque servari debet, in allata constructione quodammodo perire videatur. Ergo eadem atque prius denotantibus V, R, T, S, ponantur v & r pro velocitate & resistentia sub ascensus initio, s pro spatio quod corpus ascendendo describere possit antequam tota velocitas amittatur, t pro tempore hujus ascensus. Ad EG erigatur perpendicularis GO ipsi EG æqualis, & sumendo puncta F, f, ad easdem distantias hinc inde à puncto G, jun-

jungantur *OF*, *Of*, quibus occurrat in *T* & *t* circuli arcus *TGt* centro *O* descriptus, & sint *Gh*, *Gf*, *GE* continue proportionales, & ducatur ipsi *ER* parallela *hm* Hyperbolæ occurrens in *m*. De-



inde si *t* sit mensura anguli *FOf* ad Modulum T, hoc est, si *t* sit ad T ut arcus *TGt* ad radium *OG*: erit *s* mensura rationis inter *Eh* & *EG* ad Modulum S, vel erit *s* ad S ut area Hyperbolica *mhGQ* ad *EQ*; & *v* erit ad V ut *Gf* ad *GE*; atque *r* ad R ut *Gh* ad *GE*.

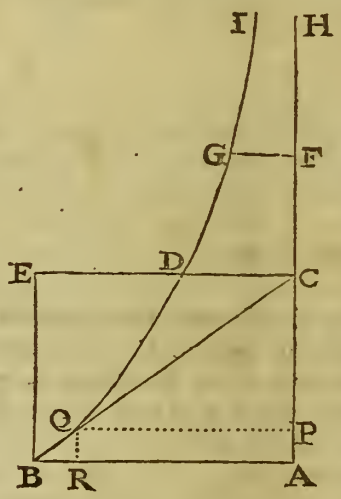
Propositio V.

Logisticam describere per Canonem Logarithmorum.

SI ad Logisticæ *BQDG* Asymptoton *APCF* ordinatim applicentur binæ quævis rectæ *AB*, *FG* intercludentes Asymptoti portionem quamvis *AF*: erit illa portio mensura rationis quam ad invicem habent ordinatæ; hæc utique est natura Curvæ notissima. Integrum ergo & perfectum Systema Logometricum per hanc Lineam exhibetur: id quod etiam de Hyperbola dici potest per Propositionem præcedentem, de Spirali Æquiangula per subsequentem; nam

nam omitto complures alias Figuras, quæ & ipsæ dudum sunt in Geometriam receptæ. Itaque si detur Asymptoti positio & simul duo puncta per quæ Curva transire debet, dabuntur puncta reliqua per casum priorem Propositionis tertiæ. Quod si data positione Asymptoti, detur insuper Systematis Modulus atque unicum punctum per quod ducenda erit Curva; invenientur puncta reliqua per Casum posteriorem Propositionis ejusdem. Iste vero Modulus quo pacto definiendus sit, & qualem habeat magnitudinem, jam oportet exponere.

Ducatur recta *BC* quæ Curvam tangat in *B* & Asymptoton secet in *C*. Dico primo, magnitudinem subtangentis *AC* eandem permanere ubicunque sumatur punctum *B*. Intelligatur enim Ordinata *PQ* viciniſſima Ordinatæ *ARB*, recta vero *QR* parallela Asymptoto *AC*, ac detur Ordinatarum intervallum illud quam minimum *AP*. Ob datam igitur lineolam *AP*, dabitur ratio quam habet *AB* ad *PQ*, & divisim ratio quam habet *AB* ad *RB*, atque adeo (propter similia triangula *BAC*, *BRQ*) ratio quam habet *AC* ad *RQ* sive *AP*, atque inde magnitudo ipsius *AC*.



Dico secundo, determinatam hanc & immutabilem subtangentem *AC*, esse Modulum ad quem exigendæ sunt mensuræ illæ interceptæ *AF*. Patet hoc per Corollarium secundum Propositionis primæ: nam dum termini *AB* & *PQ* ad æqualitatem proxime accedunt, erit *AC* ad *AP*, quæ metitur rationem inter *AB* & *PQ*, ut terminus *AB* ad terminorum differentiam *BR*. Unde data subtangente, facilis est descriptio Curvæ & solutio Problematum omnium quæ exhinc pendent.

Si Curva jam descripta habeatur, subtangentis magnitudo sic determinabitur. Producatur Ordinata quævis *CD* ad *E*, ita ut *CE* ad *CD* rationem habeat Modularem, per Corollarium sextum Propositionis primæ definitam; & recta *EB* quæ à puncto *E* parallela ducitur Asymptoto, quæque Curvæ occurrit in puncto *B*, æqualis erit subtangenti quæsitæ.

Corol.

Corol. 1. Area *ABIH*, quæ inter Curvam *BDI* & Asymptoton ejus *ACH* infinite versus *HI* extenditur, & ad alteram partem ab Ordinata *AB* terminatur, æqualis est parallelogrammo *ABEC* ab Ordinata eadem *AB* & subtangente *AC* comprehenso. Componuntur enim area & parallelogrammum ex elementis quæ sunt ut $AP \times AB$ & $AC \times RB$, quæque adeo æquantur propter analogiam inter *AP* & *RB*, *AC* & *AB*.

Corol. 2. Atque hinc, ob datam subtangentis magnitudinem, area illa indefinita erit ut Ordinata ad quam terminatur.

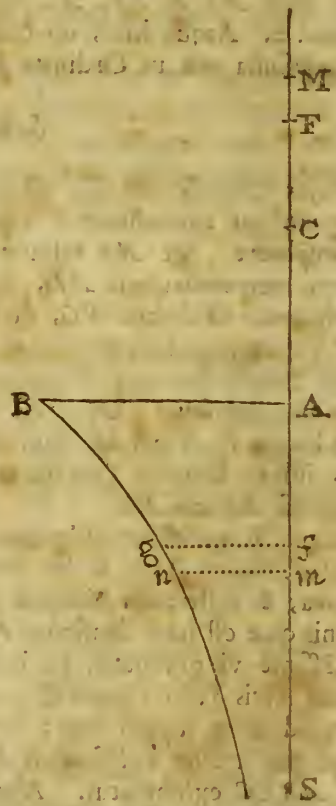
Scholium.

Hujus Propositionis usus per Exemplum declarabitur. Proponatur ad quamlibet altitudinem à superficie telluris, invenire densitatem Atmosphæræ. Sit *AB* telluris superficies, & abinde sursum producatur perpendicularis *AH*, atque ad hujus puncta singula ductæ concipiantur Ordinatæ *FG*, quæ sint ut Aeris densitates in locis *F*; & Ordinatarum termini omnes *G* in Linea Logistica *BDGI* siti erunt. Patet hoc per Corollarium secundum hujus Propositionis. Nam area indefinita *FGIH* est ut quantitas seu pondus Atmosphæræ supra locum *F*, & pondus illud est vis quæ comprimit Aerem in hoc loco, isthæc vero vis (uti docet Experientia multiplex) est ut Aeris compressi densitas *FG*.

Itaque si quotlibet altitudines sumantur in Arithmetica progressione: densitates Aeris in his altitudinibus erunt in progressione Geometrica; & differentia binarum quarumvis altitudinum, erit mensura rationis quæ est inter densitates Aeris in istis altitudinibus.

Cessante vi gravitatis, ita jam per vim aliquam extraneam intelligatur Aeris facta compressio, ut eandem habeat ubique densitatem quam ad terræ superficiem; & quantitas ejus, quæ modo erat exposita per aream indefinitam *HABI*, nunc per æquale rectangulum *ABEC* exhibebitur. Atmosphæræ hujus homogeneæ altitudo *AC*, est ad altitudinem Hydrargyri in tubo *Torricellii*, ut gravitas Hydrargyri ad gravitatem Aeris; atque inde datur. Huic autem datæ altitudini æquatur (per Corol. 1.) subtangens Curvæ *BDGI*, atque adeo Modulus Systematis mensurarum omnium *AF*. Est ergo Logarithmus rationis inter densitates Aeris in binis quibusvis altitudinibus, ad Modulum Canonis, ut altitudinum earundem differentia, ad Atmosphæræ prædictæ homogeneæ altitudinem illam datam *AC*.

Hæc ita se habent ex Hypothesi, quod vis gravitatis eadem sit ad omnes altitudines. Ceterum ex Philosophia *Newtoniana* constat eam diminui, in recessu à centro telluris, in duplicata ratione distantiæ: conclusio itaque paulo aliter se habebit. Sit *S* centrum telluris, & *AB* superficies ejusdem; sumatur ipsis *SF*, *SA* tertia proportionalis *Sf*, erigatur ordinata *fg* quæ sit ut Aeris densitas in *F*: & Curva *Bgn* quam punctum *g* perpetuo tangit, erit eadem atque prius Logistica, sed inverso situ. Augeatur enim altitudo *AF* particula quam minima *FM*, capiatur *Sm* ad *SA* ut *SA* ad *SM*, ducatur Ordinata *mn* quæ sit ut Aeris densitas in *M*; & erit *Sm* ad *Sf* ut *SF* ad *SM*, & divisim *fm* ad *FM* ut *Sf* ad *SM*, sive ut *Sf* ad *SF*, hoc est, ut *SAq* ad *SFq*. Unde *fm* est ut *SFq* inverse & *FM* directe, id est, ut gravitatio & moles Aeris inter *F* & *M* conjunctim; adeoque *fm*×*fg* sive area *fgnm* est ut gravitatio, moles & densitas ejusdem Aeris conjunctim, hoc est, ut pressio illius in Aerem inferiorem: & summa similium omnium arearum infra *fg* est ut summa pressionum omnium supra *F*, id est, ut Aeris in *F* densitas *fg*: & summarum differentia *fgnm* ut densitatum differentia *fg*—*mn*. Detur lineola *fm*; & erit *fg* ut area *fgnm*, adeoque ut *fg*—*mn*, atque inde (componendo) ut *mn*. Ergo data lineola *fm* erit mensura datæ illius rationis quæ est inter *fg* & *mn*: atque hinc patet Curvam *Bgn* esse Logisticam. Sed & eandem esse cum supra descripta Logistica, facile abinde colligitur, quod ordinatæ basi *AB* vicinissimæ & ad æqualia intervalla quam minima dispositæ, respective sint æquales in utraque Curva; ac proinde eadem curvatura, eadem inclinatio tangentis ad punctum *B*, eademque subtangentis magnitudo.



Ergo

Ergo si distantiæ *SF* à centro telluris, capiantur in Musica progressione; harum reciprocæ, nempe distantiæ *Sf*, erunt in progressione Arithmetica; & Aeris densitates *fg* erunt in progressione Geometrica.

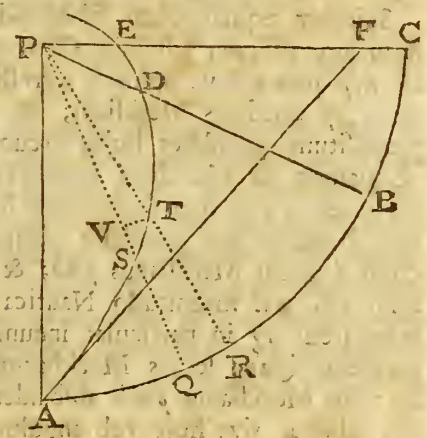
Ad inveniendam itaque densitatem in loco quovis *F*, minuenda est altitudo *AF* in ratione distantiæ *SF* ad telluris semidiametrum *SA*: & Logarithmus rationis inter densitates Aeris in *A* & *F*, erit ad Modulum Canonis, ut altitudo illa diminuta *Af*, ad Atmosphæræ homogeneæ altitudinem *AC*.

Quæ supra demonstrata sunt, accurate obtinebunt, si modo Atmosphæra ex Aere pariter Elastico tota constet: rationes igitur allatas paululum conturbabunt admisti vapores atque exhalationes, quibus etiam accedet Caloris Frigorisque diversa temperies ad altitudines diversas.

Propositio VI.

Logarithmorum Canonem ad Spiralem Æquiangulam accomodare.

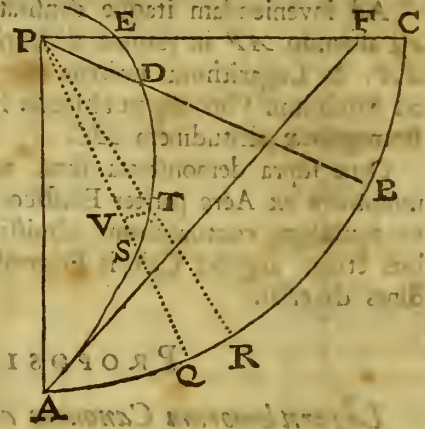
ÆQuiangula Spiralis appellatur Linea illa curva *ADE*, quæ polo *P* descripta, in eodem dato angulo secat exeuntes à polo radios *PA*, *PD*, *PE*, &c.



Si centro *P* & intervallo quovis *PA* describatur circulus *ABC*, qui radiis *PA*, *PD*, *PE* occurrat in *A*, *B*, *C*: Dico interceptum arcum *BC* mensuram fore rationis quam habet *PD* ad *PE*, & interceptum arcum *AB* mensuram rationis quam habet *PA* ad *PD*. Dividatur enim arcus *AB* in particulas quam minimas & æquales *QR*, & jungantur *PQ*, *PR* secantes Spiralem ad *S* & *T* in angulis datis *PST*, *PTS*: & ob datam particulam *QR*, dabitur angulus *QPR*, atque adeo species Figuræ *SPT*, & ratio laterum *PS*, *PT*. Data ergo particula *QR* mensura erit rationis datæ quam

 habet

habet *PS* ad *PT*; & ſumma particularum, nempe arcus *AB*, menſura erit ſummæ ſimilis rationum, hoc eſt, rationis quam habet *PA* ad *PD*. Et eodem argumento, erit arcus *BC* menſura rationis quam habet *PD* ad *PE*.

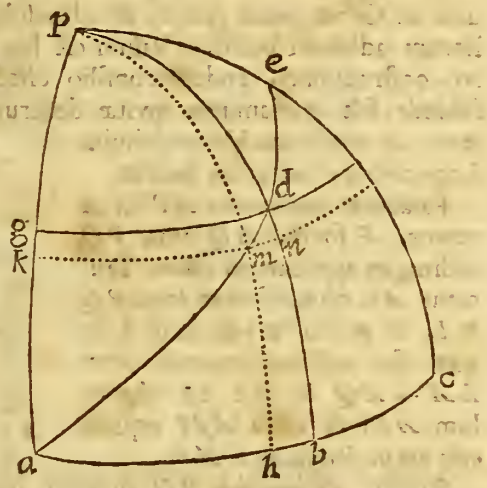


Ducatur *AF* Spiralem tangens ad Circuli & Spiralis interſectionem *A*, huic vero in *F* occurrat recta *PC* quæ ad radium *PA* normalis erigitur: & ſubtangens *PF* erit menſurarum Modulus, per Corol. 2. Prop. 1. Nam ſi in recta *PS* ſumatur *PV* ipſi *PT* æqualis, & jungantur puncta *V*, *T*; ſimilia erunt triangula *PAF*, *VST*. Unde *PF* eſt ad *VT* ut *PA* ad *VS*, ſed & *VT* eſt ad *QR* ut *PT* ad *PA*: ergo ex æquo perturbate, *PF* eſt ad *QR* quæ metitur rationem inter *PS* ad *PT*, ut terminus *PT* ad terminorum differentiam *VS*.

Scholium.

Spiralem æquiangulam, ad Meridianæ Nauticæ diviſionem demonſtrandam, feliciter adhibuit Geometra clariſſimus *Edmundus Halleius*. Sit *acp* pars octava Sphæræ terreſtris, *p* Polus, *ac* quadrans Æquatoris, *ap* quadrans Meridiani; & quæratur magnitudo rectæ, quæ propoſitum quemlibet hujus arcum deſignet in Planiſphærio. Per Æquatoris & Meridiani interſectionem *a*, ducta intelligatur linea Helicoeides *ade* quæ ſecet omnes Meridianos ad angulum ſemirectum, huic occurrat in *d* parallelus Æquatori circulus *gd*, per idem punctum *d* agatur Meridianus *pdb*; & longitudo interceptæ arcus Æquatoris *ab*, erit magnitudo Nautica quæſita arcus *ag*. Reſolvatur enim arcus *ag* in particulas innumeras quam minimas *gk*, ducatur parallelus *kmn*, ſecans Meridianum *pdb* in *n*, Lineam *dde* in *m*; & actus Meridianus *pmh* abſcindet Æquatoris particulam *bh*, quæ erit ad *mn*, ſive huic (ob angulum ſemirectum *mdn*) æqualem *dn* vel *gk*, ut peripheria Æquatoris ad peripheriam paralleli *kmn*. Eſt ergo particula *bh* magnitudo Nautica particulæ *gk*, & ſumma particularum omnium *bh*, nempe longitudo arcus *ab*, magnitudo Nautica

tica ſummæ particularum omnium *gk*, id eſt, arcus *ag*. Manente jam Æquatore *abc* vel *ABC*, concipiatur Sphærica ſuperficies in plano ejus Stereographice depingi; & Polo *p* occupante centrum *P*, projicientur Meridiani *pga*, *pdb*, *pec* in totidem rectas *PA*, *PDB*, *PEC* à centro *P* exeuntes, ita ut diſtantia abinde puncti cujuſvis *D* vel *A*, tangens ſit arcus dimidiati *pd* vel *pa* quem diſtantia illa repræſentat. Linea vero Helicoeides *ade* convertet ſe in Spiralem æquiangulam *ADE*, polo *P* deſcriptam, & ſecantem radios ſuos omnes ad angulum ſemirectum. Hoc ſiquidem poſtulat nota Lex hujuſce Projectionis, ut anguli omnes eandem in Plano ac in Sphærica ſuperficie magnitudinem ſervent. Arcus itaque propoſiti *ag* magnitudo Nautica *ab* vel *AB*, eſt ad ſubtangentem *PF* vel huic jam æqualem Sphæræ radium *PC*, ut Logarithmus rationis inter *PA* & *PD*, hoc eſt, inter tangentes dimidiatorum arcuum *pa* & *pd*, vel *pa* & *pg*, ad Modulum Canonis.

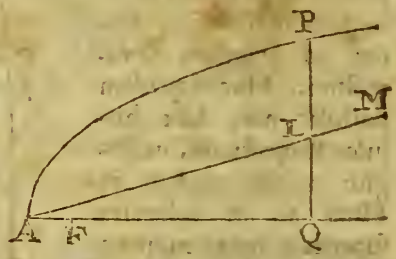


Hinc quoniam longitudo Radii eſt ad longitudinem arcus minuti unius primi, ut 3437,746770784939 &c ad 1, & reciprocus Moduli Canonis eſt 2,302585092994 &c, atque hi numeri in ſe multiplicati efficiunt 7915,704467897819 &c: ſi magnitudo illa Nautica *AB* in minutis primis exhibenda ſit, uti mos exigit; ſubducta tangente artificiali dimidiati arcus *pg* à tangente artificiali dimidiati arcus *pa*, multiplicetur reſiduum per numerum 7915,704467897819 &c, et factus dabit partes Meridionales deſideratas. Perinde vero ſe habebit concluſio, ſive in Æquatore, ſive extra hunc alibi ad utramvis partem locetur punctum *a*.

SCHO-

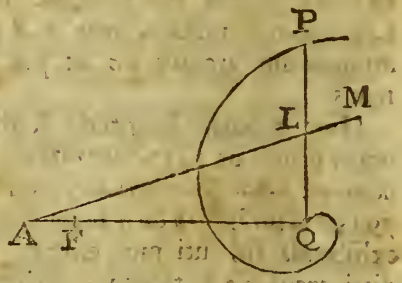
Scholium Generale.

IN eum potiſſimum finem præcedentia conſcripſi, ut allatis aliquot Exemplis oſtenderem, qua commodiſſima ratione Logarithmorum uſus in Geometriam recipi, & ad reſolutionem Problematum difficiliorum adhiberi poſſit. Viſum eſt hoc loco nonnullas adjicere porro conſtructiones, eodem conſilio effectas, quæ mihi iſta tractanti ſubinde ſeſe obviam non invitæ dederunt: ut ita, ex uberiore ſpecimine, de præſtantia Methodi hujus Logometricæ judicium feratur.



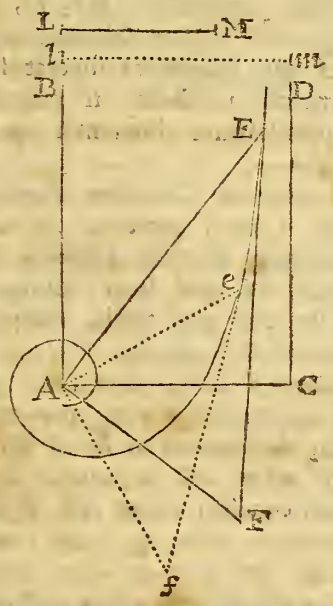
Parabolæ *Apollonianæ* AP ſit A vertex, F focus, AQ axis, PQ ordinatim applicata ad axem. Ducatur AL quæ bifariam ſecet PQ in L, & productæ adjiciatur LM quæ ſit menſura rationis inter $LA + AQ$ & QL ad Modulum AF: & recta AM æqualis erit arcui Parabolico AP.

Spiralis *Archimedea* PQ ſimilem habet extenſionem in rectam. Sit Q polus ejus, QP radius à polo ductus ad Curvæ quodlibet punctum P, & ad eum radium normalis QA. Ducatur LA parallela tangenti Spiralem in P, quæ radium PQ bifariam ſecet in L; & ponendo AF ad QL ut QL ad QA, ipſi AL adjiciatur LM quæ ſit menſura rationis inter $LA + AQ$ & QL ad Modulum AF: & recta AM æquabitur Spiralis arcui PQ.

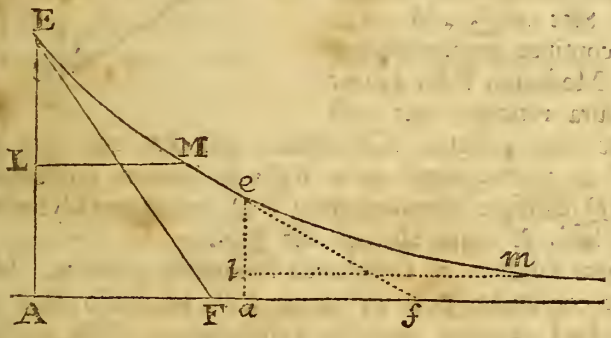


Spiralis Reciprocæ AeE ſit A polus, AB radius primus & infinitus, CD aſymptotos radio primo parallela ad diſtantiam AC; & invenienda proponatur hujuſce Curvæ longitudo. Inter Spiralem illam vulgarem *Archimedis* atque hanc, quam Reciprocam appello, iſthæc intercedit differentia, quod cum illius radii ſint ut anguli quos faciunt cum radio ſuo primo, hujus radii è contrario ſunt

ſunt reciproce ut iidem anguli: eandem utique proportionem habet radius *AE* ad radium *Ae* quam habet angulus *eAB* ad angulum *EAB*. Unde facile colligitur, ſi ad puncta *E* & *e* ducantur tangentes *EF*, *ef*, & ad radios *AE*, *Ae* erigantur normales *AF*, *Af*, fore normales iſtas ſibi invicem & Aſymptoti intervallo *AC* æquales. Invenietur autem longitudo cujuſvis arcus *Ee*, ponendo *LM* menſuram rationis inter *AE* & *EF* — *AF* ad Modulum *AF*, & ſimiliter *lm* menſuram rationis inter *Ae* & *ef* — *Af* ad æqualem Modulum *Af*. Nam ſi tangentium differentiæ *EF* — *ef* adjiciatur menſurarum differentia *lm* — *LM*, aggregatum æquabitur arcui *Ee*.



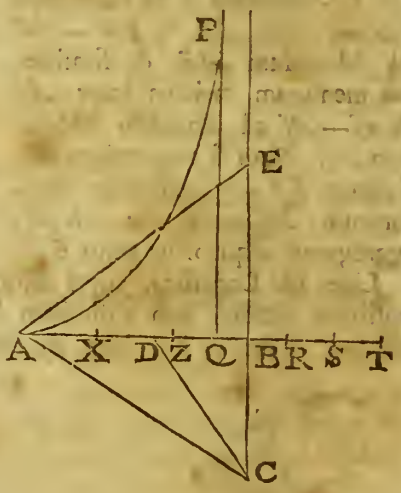
Linea illa Logiſtica, cujus aliquas expoſuimus affectiones in Propoſitione quinta, non abſimilem habet longitudinis ſuæ determinationem; quam & hoc loco apponam in eorum gratiam qui hujuſ-



modi contemplationibus delectantur. Oblata ſit igitur Logiſtica *EMem*, cujus Aſymptotos *AFaf*: & quæratur longitudo cujuſvis arcus *Ee*. Demiſſis in Aſymptoton perpendiculis *ELA*, *ela*, &

& ductis tangentibus EF, ef, capiatur AL æqualis excessui quo tangens EF superat subtangentem AF, & similiter al æqualis excessui quo tangens ef superat subtangentem af: & actis LM, lm Asymptoto parallelis, si tangentium differentiæ $EF-ef$ adjiciatur parallelarum differentia $lm-LM$, aggregatum æquabitur arcui Ee.

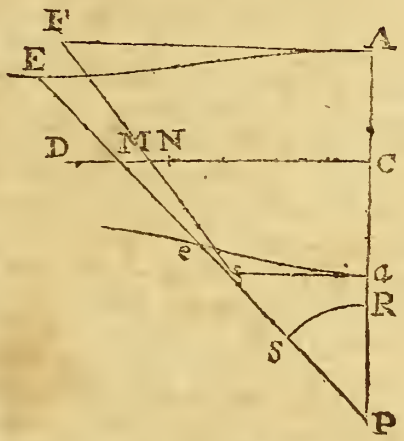
Accedo ad Cissoidem *Diocleam.* Sit A vertex ejus, AB diameter Circuli genitoris, BC Asymptotos, PQ perpendicularis in diametrum demissa, Cissoidi in P & diametro in Q occurrens. Agatur AC quæ secet Asymptoton in C ac faciat angulum BAC qui sit recti pars tertia, sumptaque inter BQ & BA media proportionali BD jungatur CD; denique per medium perpendiculum PQ ducatur AE recta, quæ occurrat Asymptoto in E: & Cissoidis arcus AP æquabitur duplicato excessui rectæ AE supra diametrum AB, & simul triplicatæ mensuræ rationis inter $BA+AC$ & $BD+DC$ ad Modulum BC.



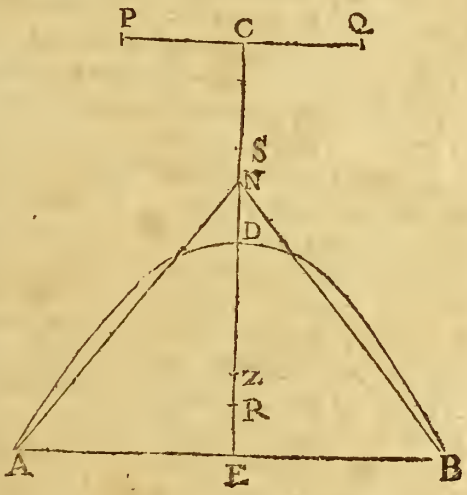
Si Cissoidis area APQ convertatur circum axem AQ; generabitur solidum cujus dimensio pendet à Logometria, & sic construitur. Sint AQ, AB, AR, AS, AT continue proportionales; deinde ad Modulum TS capiatur QX mensura rationis inter AB & BQ, & retro ponatur XZ æqualis ipsi SR una cum dimidio ipsius RB ac triente simul ipsius BQ: & solidum Cissoidale axem habens AQ basisque semidiametrum PQ, æquabitur Cylindro cujus eadem est basis & cujus altitudo est QZ.

Adjungam solidum ex Conchoide *Nicomedis* genitum. Sint AE, ae Curvæ conjugatæ, polo P, regula CD, intervallo CA vel Ca, axe $PaCA$ ad regulam normali, verticibusque A & a descriptæ. Per polum P ducatur ad libitum recta $PeDE$, regulæ occurrens in D, Lineæ vero in E & e: & ex natura Conchoidis, erunt segmenta DE, De intervallo CA vel Ca æqualia. Eodem intervallo centroque P describatur circuli arcus RS secans axem PC in R & rectam

rectam PD in S: & semisumma solidorum Conchoidalium quæ generantur ex conversione Figurarum $AEDC$, $aeDC$ circum axem AaP, erit ad sectorem Sphæræ genitum ex circuli sectore PRS circum axem eundem converso, ut $3PC \times PD + PRq$ ad PRq. Eorundem vero semidifferentia Cylindro æquatur, cujus basis est circulus diametro Aa descriptus, & cujus altitudo est mensura duplicata rationis inter PD & PC ad Modulum PC.



Area vero Figuræ totius $AEea$ æquatur rectangulo cujus basis est Aa, & cujus altitudo CM est mensura rationis inter $PD + DC$ & PC ad Modulum PC. Quod si desideretur quadratura partium $AEDC$, $aeDC$; ductis ad axem normalibus AF, af, in regula CD sumenda est CN quæ sit anguli CPD mensura ad eundem Modulum PC: & acta per punctum M recta FMf quæ parallela sit rectæ jungenti puncta P, N, quæque occurrat normalibus in F & f; erit area $AEDC$ æqualis Trapezio $AFMC$, & area $aeDC$ æqualis Trapezio $afMC$.

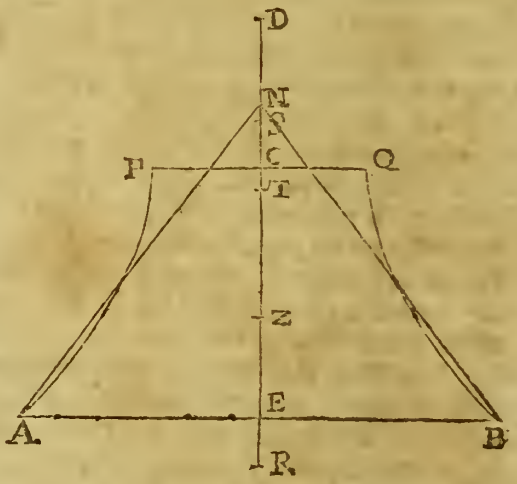


Hyperbolæ quadraturam in superioribus expositam dedi, eo modo, qui mihi visus est ad propositum quam maxime accommodatus. Libet aliam constructionem hoc loco apponere, & simul adjicere gravitatis centrum. Oblata sit portio interior ADB, intercluſa curvæ ADB & rectæ cuivis AB ad diametrum PQ parallelæ. A Figuræ centro C producatur diameter CDE, quæ basin AB bifariam secet in E; deinde si in diametro

producta ſumantur *CR* ad *CD*, & *CD* ad *CS*, ut baſis *AB* ad diametrum *PQ*, & ad Modulum *CS* fiat *CN* menſura rationis quam habet *CD* ad *ER*: triangulum rectilineum *ANB* æquabitur areæ curvilineæ *ADB*.

Hujus autem areæ centrum gravitatis *Z* invenietur, capiendo *CZ* ad *CR* ut 2 *CR* ad 3 *EN*.

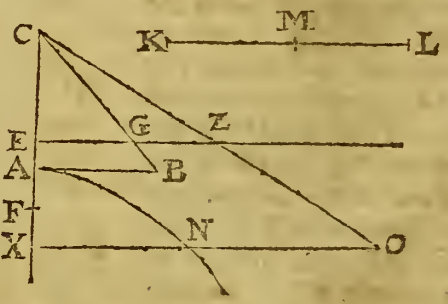
Sit nunc oblata portio exterior *APQB*, intercluſa curvis oppoſitis *AP*, *BQ*, diametro *PQ*, & rectæ curvis *AB* ad diametrum illam parallelæ. Eſto *CD* conjugatæ ſemidiametri longitudo extra portionem oblatam *APQB* poſita, quæ producta in contrariam partem centri *C* bifariam ſecet baſim *AB* in *E*. Deinde in diametro producta ſi ſumantur *CR* ad *CD*, & *CD* ad *CS*, & *CS* ad *CT*, ut baſis *AB* ad diametrum *PQ*, ponantur vero *CR* & *CT* ad eandem centri partem cum baſi *AB*; & ad Modulum *CS*, in contrariam centri partem, ſumatur *CN*



menſura rationis quam habet *CD* ad *ER*: triangulum rectilineum *ANB* æquabitur areæ curvilineæ *APQB*.

Hujus autem areæ centrum gravitatis *Z* invenietur, capiendo *CZ* ad *CR* ut 2 *TR* ad 3 *EN*.

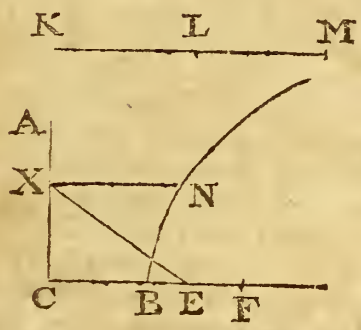
Pergo ad ſuperficies ab Hyperbola circum axes ſuos convoluta genitas. Sit *AN* Hyperbola deſcripta vertice *A*, centro *C*, Aſymptoto *CB*, foco *F*, ſemiaxe principali *AC*, ſemiaxe conjugato *AB* normali ad *AC*; & ad axis *AC* punctum quodvis *X* ſit *XN* ordinatim applicata, quæ Hyperbolæ occurrat ad *N*.



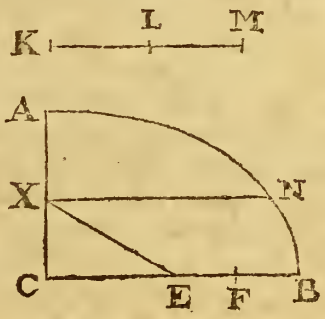
In

In axe *CA* capiatur *CE* ad *CA* ut *CA* ad *CF*; & ad eundem axem erecta perpendiculari *EZ*, quæ Asymptoto occurrat in *G*, angulo *CEZ* inscribatur æqualis ipsi *CX* recta *CZ*, quæ porro producta secet ordinatim applicatam *XN* ad *O*. Tum sumatur *KL* quæ sit æqualis excessui quo *XO* superat *AB*, atque *LM* quæ sit mensura rationis inter *GZ* + *ZE* & *CG* + *GE* ad Modulum *CE*: & superficies genita ex arcus *AN* conversione circum axem *AX*, erit ad Circulum semidiametro *AB* descriptum, ut excessus *KM* quo *KL* superat *LM*, ad semidiametrum illam *AB*.

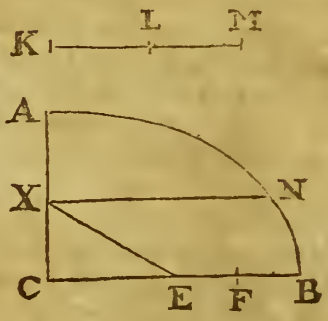
Sit rursus *BN* Hyperbola descripta vertice *B*, centro *C*, foco *E*, semiaxe principali *CB*, semiaxe conjugato *CA* normali ad *CB*; & ad axis *AC* punctum quodvis *X* sit *XN* ordinatim applicata, quæ Hyperbolæ occurrat ad *N*. In axe *CB* capiatur *CE* ad *CA* ut *CA* ad *CF*, & jungatur *EX*. Tum sumatur *KL* quæ sit ad *XC* ut *XE* ad *CE*, & *LM* quæ rationis inter *EX* + *XC* & *CE* mensura sit ad Modulum *CE*: & superficies genita ex arcus *BN* conversione circum axem *CX*, erit ad Circulum semidiametro *CB* descriptum, ut linearum *KL* & *LM* aggregatum *KM*, ad semidiametrum illam *CB*.



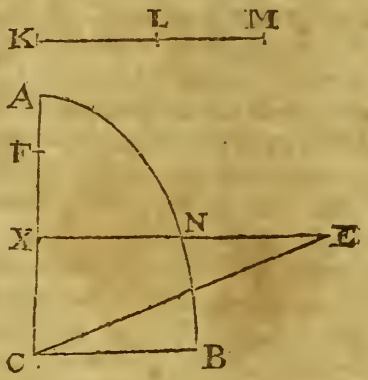
His addere licebit ab Ellipsi genitas superficies. Sit *ANB* Ellipsis descripta centro *C*, verticibus *A* & *B*, foco *F*, semiaxe principali *CB*, semiaxe conjugato *CA*; & ad axis *CA* punctum quodvis *X* sit *XN* ordinatim applicata, quæ Ellipsi occurrat ad *N*. In axe *CB* capiatur *CE* ad *CA* ut *CA* ad *CF*, & jungatur *EX*. Tum sumatur *KL* quæ sit ad *XC* ut *XE* ad *CE*, & *LM* quæ rationis inter *EX* + *XC* & *CE* mensura sit ad Modulum *CE*: & superficies genita ex arcus *BN*



conversione circum axem CX, erit ad Circulum semidiametro CB descriptum, ut linearum KL & LM aggregatum KM, ad semidiametrum illam CB. Ut hæc ultima constructio locum habeat, oportet semiaxem CA circa quem conversio facta est, minorem esse altero semiaxe CB; aliter enim Moduli CE quantitas $\frac{CAq}{\sqrt{CBq - CAq}}$ evadet impossibilis, & constructio illa Logometrica (quod in hujusmodi casibus fieri solet) convertet se in Trigonometricam, qualis illa est quæ jam sequitur.



Sit ANB Ellipsis descripta centro C, verticibus A & B, foco F, semiaxe principali CA, semiaxe conjugato CB; & ad axis CA punctum quodvis X sit XN ordinatim applicata, quæ Ellipsi occurrat ad N. Angulo CXN inscribatur recta CE, quæ sit ad CA ut CA ad CF. Tum sumatur KL quæ sit ad XC ut XE ad CE, & LM quæ anguli XEC mensura sit ad Modulum CE, hoc est, quæ sit æqualis arcui cujus sinus est XC ad radium CE: & superficies genita ex arcus BN conversione circum axem CX, erit ad Circulum semidiametro CB descriptum, ut linearum KL & LM aggregatum KM, ad semidiametrum illam CB. Posset hujus etiam superficiei dimensio per Logometriam designari, sed modo inexplicabili. Nam si quadrantis circuli quilibet arcus, radio CE descriptus, sinum habeat CX sinumque complementi ad quadrantem XE: sumendo radium CE pro Modulo, arcus erit rationis inter $EX + XC\sqrt{-1}$ & CE mensura ducta in $\sqrt{-1}$. Verum isthæc aliis, quibus operæ pretium videbitur, diligentius excutienda relinquo. Ceterum ex præcedentibus intelligi potest, quanta sit cognatio inter angulorum atque rationum



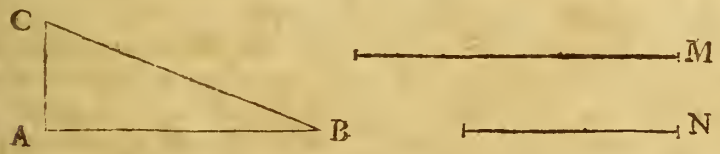
mensu-

menſuras, quamque levi mutatione in ſe invicem facillime convertantur pro variis ejuſdem Problematis caſibus. De Cubicarum æquationum radicibus dudum ab Analyſtis obſervatum eſt; vel eas exprimi poſſe per *Cardani* regulas, atque adeo per duarum mediarum proportionalium inventionem; vel per diviſionem arcus circularis in tres æquales partes, ſi forte fuerint inexplicabiles per memoratas regulas. * Hoc animadvertit *Carteſius*, ſed & ante *Carteſium* idem obſervavit *Franciſcus Vieta* ſub finem Supplementi Geometriæ. Exhinc autem aperte colligitur, qualis ſit ordo Naturæ tranſeuntis ad Anguli triſectionem à triſectione Rationis.

Mirabilem illam Harmoniam ulterius declarare lubet, Exempło deſumpto ab eadem Figura circum axes ſuos convoluta. Sit igitur $APBQ$ Ellipſis, axis ejus major AB, minor PQ, centrum C, focus F. Hæc circum axem utrumvis convoluta Solidum generet, cujus particulæ conſtantes ex materia homogenea, vires attractivas habeant in duplicata diſtantiarum ratione decreſcentes: & quæratur vis qua Solidum illud attrahit corpuſculum quodvis, in ejus ſuper-

---

* Sublato etenim termino ſecundo, tres habentur Æquationum caſus. Hi vero reſolvuntur ope trianguli rectanguli ABC, rectum habentis angulum ad A, in quo inſuper triangulo ſemper data ſunt duo latera.



Caſ. 1. Nam ſi ſit $x^3 + 3aax = \pm 2aab$: ponantur $AB = a$, $AC = b$; & ſumantur M & N binæ mediæ proportionales inter $BC + AC$ & $BC - AC$: & erit $M - N$ radix unica poſſibilis affirmativa, ſi habeatur $+2aab$; vel $N - M$ radix unica poſſibilis negativa, ſi habeatur $-2aab$.

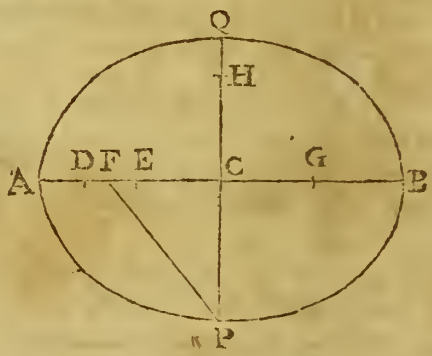
Caſ. 2. Si ſit $x^3 - 3aax = \pm 2aab$, exiſtente a minore quam b: ponantur $AB = a$, $BC = b$; & ſumantur M & N binæ mediæ proportionales inter $BC + AC$ & $BC - AC$: & erit $M + N$ radix unica poſſibilis affirmativa, ſi habeatur $+2aab$; vel $-M - N$ radix unica poſſibilis negativa, ſi habeatur $-2aab$.

Caſ. 3. Denique ſi ſit $x^3 - 3aax = \pm 2aab$, exiſtente a majore quam b: ponantur $AB = b$, $BC = a$; & ſumatur M ſinus trientis angulorum ſummæ $A + B$, atque N ſinus trientis angulorum differentiæ $A - B$, exiſtente radio $2BC$: & erunt $-M$, $-N$, & $M + N$ tres radices poſſibiles, ſi habeatur $+2aab$; vel M, N, & $-M - N$ tres radices poſſibiles, ſi habeatur $-2aab$.

Atque ita Problemata omnia Solida ſolutionem facilem recipiunt, vel per Canonem Logarithmicum, vel per Canonem Trigonometricum.

ficie

ficie locatum ad axis illius terminum. Jungantur puncta P, F, ac sumatur CD quæ sit mensura rationis inter $PF + FC$ & CP ad Modulum CA, pariterque sumatur CE quæ sit anguli CPF mensura ad Modulum CP; sitque FD excessus mensuræ CD supra CF, atque FE excessus ipsius CF supra mensuram CE: & Solidi convolutione circum axem majorem AB geniti vis in corpusculum ad A locatum, erit ad Sphæræ homogeneæ & eodem axe descriptæ vim in idem corpusculum, ut $3FD \times CPq$ ad $CFcub$; Solidi autem conversione circum axem minorem PQ geniti vis in corpusculum ad P locatum, erit ad Sphæræ homogeneæ & eodem axe descriptæ vim in idem corpusculum, ut $3FE \times CAq$ ad $CFcub$. Unde cum vis Sphæræ prioris in corpusculum ad A, sit ad vim Sphæræ posterioris in corpusculum ad P, ut CA ad CP: erit vis Solidi prioris in corpusculum ad A, ad vim Solidi posterioris in corpusculum ad P, ut $FD \times CP$ ad $FE \times CA$.

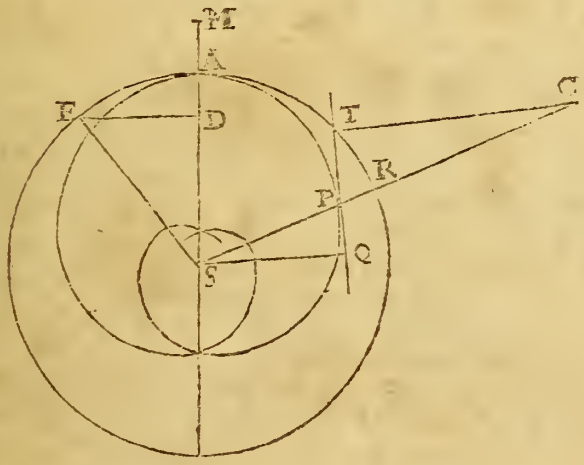


Hinc quoniam Solidum posterius medium est proportionale inter Solidum prius & Sphæram priorem: vis Solidi posterioris in corpusculum ad A, erit media proportionalis quamproxime inter vires Solidi prioris & Sphæræ prioris in idem corpusculum ad A, si modo axes Ellipseos sint prope æquales. Itaque in hoc casu, ponendo CG mediam proportionalem inter CF & $3FD$, & capiendo CH ad $3FE$ ut CA ad CF; posterioris Solidi vires ad A & P, vel ad B & Q, erunt ad invicem quamproxime ut CG ad CH. Id quod non inutile præbet compendium ad inventionem Figuræ Telluris, qualem eam subtiliter instituit celeberrimus *Newtonus*, summus ille Philosophiæ sanioris Instaurator.

Consideratio virium centripetarum aliud porro mihi suggerit Exemplum, in quo satis ampla se prodit mutationum varietas. Proponatur Trajectoriarum species enumerare, in quibus corpora moveri possunt, quæ à viribus centripetis in ratione distantiarum triplicata decrescentibus agitantur, quæque de loco dato, cum data velocitate, secundum datam rectam egrediuntur.

Cas. 1.

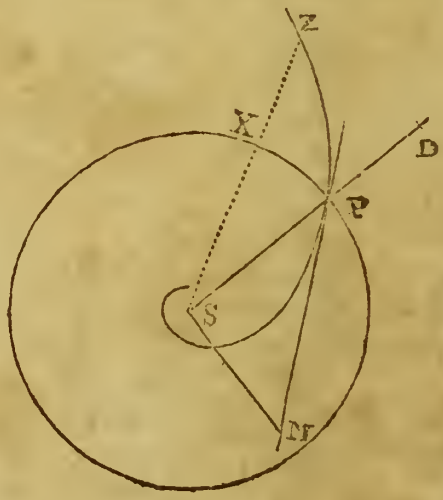
Caſ. 1. Sit S centrum virium, exeatque corpus de loco P ſecundum rectam PQ vel QP, ea cum velocitate quam acquirere poſſet ab iiſdem viribus, libere cadendo verſus centrum S de loco C, & caſu ſuo deſcribendo altitudinem CP. In datam rectam QPT demittantur perpendicula SQ, CT, centroque S & intervallo $\sqrt{SQq + QTq}$ deſcribatur circulus RTA, rectæ SPC occurrens in R: deinde ad Modulum $\sqrt{SCq - SRq}$ ſit arcus RA menſura rationis inter $SR \pm \sqrt{SRq - SPq}$ & SP, jaceant autem arcus ille RA & punctum Q ad diverſas partes rectæ SR; & punctum A erit Apſis ſumma Trajectoriæ. Exhinc vero Trajectoria dabitur, ſumendo SM æqualem ipſi $\sqrt{SCq - SRq}$, deinde in recta SA



capiendo longitudinem quamvis SD quæ ſit minor quam SA, ad eandem erigendo perpendiculum DE ſecans circulum in E, & jungendo SE. Nam ſi ad utraſque partes puncti A ponatur arcus circularis AR, cujus longitudo ſit menſura rationis inter $SE + ED$ & SD ad Modulum SM, & in ſemidiametris SR capiantur diſtantiæ SP æquales ipſi SD: erunt puncta P ad Trajectoriam deſcribendam. Tempus autem quo radius SP, à centro ad corpus motum ductus, percurret aream quamvis SAP, erit ut recta DE: nam area percurſa æquatur ipſi DE in Modulum dimidiatum $\frac{1}{2}SM$ ductæ. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem diſtantiam SP, cum iiſdem viribus revolvi

revolvi posset, ut $\sqrt{SCq - SPq}$ ad SC. Ex ipsa constructione patet, hanc Spiralem primam infinitis gyris circa centrum virium contorqueri, quin & seipsam infinitis vicibus decussare, & siti erunt Nodi omnes ad Apsidis lineam AS.

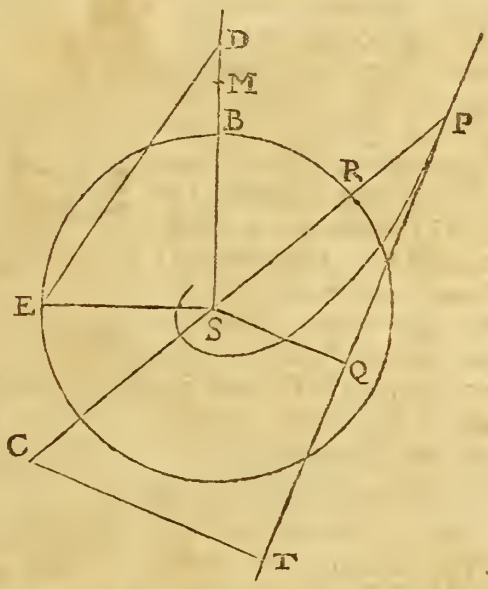
Cas. 2. Recedat punctum C ad infinitam distantiam à centro S; & corporis de loco P secundum rectam PM vel MP exeuntis ea sit velocitas, quam acquirere posset cadendo libere ad eundem locum P ab infinita distantia. Ad rectam SP ducatur normalis SM, quæ secet PM in M; deinde centro S & intervallo SP describatur circulus, & in ejus circumferentia capiatur arcus PX, cujus longitudo sit mensura rationis inter distantiam quamvis SD & distantiam datam SP ad Modulum SM, jaceant autem arcus ille PX & punctum M ad diversas partes rectæ SP si SD fuerit major quam SP, aliter ad easdem, inque semidiametro SX ponatur SZ æqualis ipsi SD; & punctum Z erit ad Trajectoriam describendam. Tempus autem quo radius SZ, à centro ad corpus motum ductus, percurret aream quamvis SPZ, erit ut differentia quadratorum ex SZ & SP: Nam area



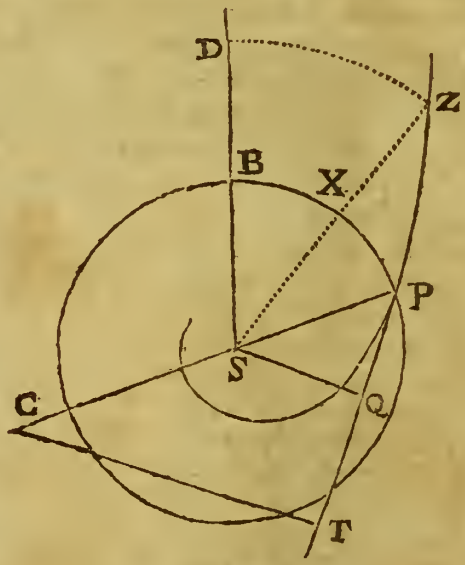
percursa, est ad illam differentiam, in data ratione Moduli dimidiati $\frac{1}{2}SM$ ad SP. Velocitas vero corporis in loco quovis P, æqualis erit velocitati qua in Circulo, ad eandem distantiam SP, cum iisdem viribus revolvi posset. Ex constructione patet hanc secundam Spiralem esse Æquiangulam illam Propositionis sextæ; ea vero migrabit in Circulum ubi angulus SPM sit rectus.

Cas. 3. Ut velocitas sit adhuc major, abeat jam punctum C ad distantiam plusquam infinitam à centro S, vel (quod perinde est) accedat à parte contraria eidem centro, ad finitam distantiam; & corporis de loco P secundum rectam PQ vel QP exeuntis, ea sit velocitas, quam acquirere posset ascendendo libere de loco C ad infinitam distantiam, & deinde ab infinita distantia ex altera centri parte

parte descendendo ad locum P, viribus centripetis inter ascendendum in æquales vires centrifugas conversis. In datam rectam PQT demittantur perpendicula SQ, CT; & erit TQ vel major, vel æqualis, vel minor quam SQ. Si TQ fuerit major quam SQ; centro S & intervallo $\sqrt{TQq - SQq}$ describatur circulus RBE rectæ SP occurrens in R, deinde ad Modulum $\sqrt{SCq - SKq}$ sit arcus RB mensura rationis inter $SR \pm \sqrt{SRq + SPq}$ & SP, jaceant autem arcus ille RB & punctum Q ad partes diversas rectæ SP. Ex hinc Trajectoria dabitur, sumendo SM æqualem ipsi $\sqrt{SCq - SRq}$, in recta SB capiendo longitudinem quamvis SD, ad eandem erigendo perpendiculum SE circulum secans in E, & jungendo DE. Nam si retro ponatur à puncto B circularis arcus BR, cujus longitudo mensura sit rationis inter $SE + ED$ & SD ad Modulum SM, & in semidiametro SR capiatur distantia SP æqualis ipsi SD: erit punctum P ad Trajectoriam describendam. Tempus autem quo radius SP, à centro ad corpus motum ductus, percurret aream quamvis hujus Trajectoriæ, erit ut incrementum vel decrementum rectæ DE per tempus illud factum: nam area percursa æquatur huic incremento vel decremento in Modulum dimidiatum $\frac{1}{2}SM$ ducto. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem distantiam SP, cum iisdem viribus revolvi posset, ut $\sqrt{SCq + SPq}$ ad SC. Ex constructione patet, hanc Spiralem tertiam infinitis gyris centrum cingere infra punctum datum P; at supra idem punctum vel non undique cinget, si arcus RB minor fuerit quam circumferentia tota $RBER$; vel toties cinget, quoties arcus ille circumferentiam excedit.



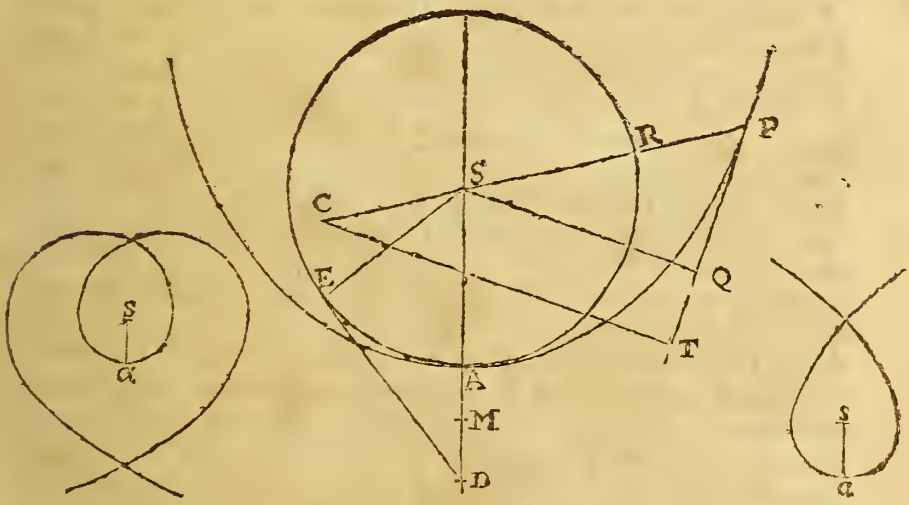
Caſ. 4. Reliquis manentibus, ſint jam TQ & SQ æquales. Centro S & intervallo SP deſcribatur circulus PXB, & ſit arcus PB æqualis ipſi SC, jaceant autem arcus PB & punctum Q ad partes diverſas rectæ SP. Exhinc Trajectoria dabitur, ſumendo in recta SB longitudinem quamvis SD, centroque S & intervallo SD deſcribendo circuli arcum DZ æqualem ipſi SC. Nam ſi ordine circulari contrario ponantur arcus PB à puncto P & arcus DZ à puncto D: erit punctum Z ad Trajectoriam deſcribendam. Tempus autem quo radius SZ, a centro ad corpus motum ductus, percurret aream quamvis SPZ, erit ut differentia radiorum SZ & SP: nam area percurſa æquatur huic differentiæ ductæ in ſemiſſem diſtantiæ SC. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem diſtantiam SP, cum iiſdem viribus revolvi poſſet, ut $\sqrt{SCq + SPq}$ ad SC. Ex conſtructione patet, hanc Spiralem quartam eſſe Reciprocam illam, cujus longitudinem ſupra dimenſam dedimus.



Caſ. 5. Reliquis adhuc manentibus, ſit jam TQ minor quam SQ. Centro S & intervallo $\sqrt{SQq - TQq}$ deſcribatur circulus RAE rectæ SP occurrens in R; deinde ſit arcus RA, ad ejuſdem circuli arcum cujus ſecans eſt SP, ut $\sqrt{SCq + SRq}$ ad SR; ponatur autem arcus ille RA ad eaſdem partes rectæ SP cum puncto Q: & A erit Apſis ima Trajectoriæ. Exhinc vero Trajectoria dabitur, ſumendo SM æqualem ipſi $\sqrt{SCq + SRq}$, in recta SA capiendo longitudinem quamvis SD quæ ſit major quam SA, ducendo DE quæ circulum tangat in E, & jungendo SE. Nam ſi ad utraſque partes puncti A ponatur arcus circularis AR, cujus longitudo menſura ſit anguli DSE ad Modulum SM, & in ſemidiametris SR capiantur diſtantiæ SP æquales ipſi SD: erunt puncta

P ad

P ad Trajectoriam describendam. Tempus autem quo radius *SP*, à centro ad corpus motum ductus, percurret aream quamvis *SAP*, erit ut recta *DE*: nam area percursa æquatur ipsi *DE* in Modulum dimidiatum $\frac{1}{2}SM$ ductæ. Velocitas vero corporis in loco quovis *P*, erit ad velocitatem qua in Circulo, ad eandem distantiam *SP*, cum iisdem viribus revolvi posset, ut $\sqrt{SCq + SPq}$ ad *SC*.



Ex constructione patet, hanc quintam Spiralem vel nullum habere Nodum, vel unicum, vel plures, pro varia proportione rectæ *SM* ad diametrum circuli *EAR*: toties enim Trajectoria sese decussabit, quoties illa recta diametrum excedit, & Nodi omnes siti erunt ad Apsidis lineam *AS*.

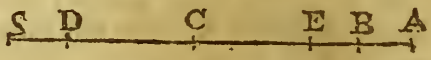
Sunt itaque Trajectoriarum quinque Species. Harum primam atque ultimam descripsit olim *Newtonus*, per Hyperbolæ & Ellipseos quadraturam.

Geometris integrum erit, ex adductis hactenus Exemplis de Methodo nostra judicare; quam quidem, si proba fuerit, ulterius excolere pergent & excolendo latius promovebunt. Patet utique campus amplissimus in quo vires suas experiri poterunt, præsertim si Logometriæ Trigonometriam insuper adjungant, quibus miram quandam affinitatem in se invicem euntibus intercedere notabam. Hisce quidem Principiis haud facile crediderim generaliora dari posse; cum

 tota

tota Mathesis vix quicquam in universo suo ambitu complectatur, præter angulorum & rationum Theoriam. Neque sane commodiora sperabit, qui animadverterit Effectionis facilitatem per amplissimas illas, omnibusque suis numeris absolutas, tum Logarithmorum tum Sinuum & Tangentium Tabulas, quas antecessorum nostrorum laudatissimæ solertiæ debemus acceptas. Ut vero tanti beneficii uberior nobis exsurgat fructus, id nunc exponendum restat, quibus artibus ad istiusmodi conclusiones rectissima perveniatur. In hunc finem Theoremata quædam, tum Logometrica tum Trigonometrica adjecissem, quæ parata ad usum asservo; ni consultius visum esset, quum absque nimiis ambagibus ea tradi non possent, intacta potius præterire atque aliis denuo investiganda relinquere. Ceterum isthoc apparatu non semper est opus; nam in Methodo Fluxionum sæpe evenit ut ipsæ Fluentes, omissis hujusmodi subsidiis, ad Logometriam satis commode revocentur: id quod uno atque altero Exemplo ostendam.

Egimus in præcedentibus de rectilineo Gravium descensu, per Medii resistentiam continuam retardato, ex Hypothesi quod illa resistentia esset in duplicata ratione velocitatis. Ex eadem Hypothesi resistentiam corporis penduli, in Cycloide oscillantis, jam sit propositum invenire. Cycloidis itaque in rectam explicatæ sit AC dimidium, C punctum infimum, B punctum à quo cadere incipit corpus pendulum, BC, CD arcus descensu ejus & subsequente ascensu descripti. Hisce positis, exquirenda est ratio quam habet resistentia corporis in loco quovis E, ad pondus ejus relativum in Medio resistente. Exponatur pondus illud per AC; & vis ab eodem oriunda, qua pendulum acceleratur ad E, exponetur per CE: quæ si dicatur x, & momentum ejus $+\dot{x}$; momentum arcus jam descripti BE erit $-\dot{x}$. Exponatur vis resistentiæ per z; & vis qua pendulum vere acceleratur, erit ut excessus vis prioris supra resistentiam, hoc est, ut $x-z$.



Itaque cum resistentia sit ut quadratum velocitatis, resistentiæ momentum $\dot{z}$ erit ut velocitas & velocitatis momentum, hoc est, ut $-\dot{x}$ & $x-z$, sive ut $z\dot{x}-x\dot{x}$. Nam si tempus in particulas æquales dividatur, erit velocitas ut arcus descripti momentum $-\dot{x}$, & velocitatis momentum ut vis acceleratrix $x-z$ quæ momentum illud generat. Quoniam ergo $\dot{z}$ est ut $z\dot{x}-x\dot{x}$, si capiatur

piatur quantitas invariabilis a, quæ ſit idoneæ magnitudinis: erit $a\dot{z} = z\dot{x} - x\dot{x}$.

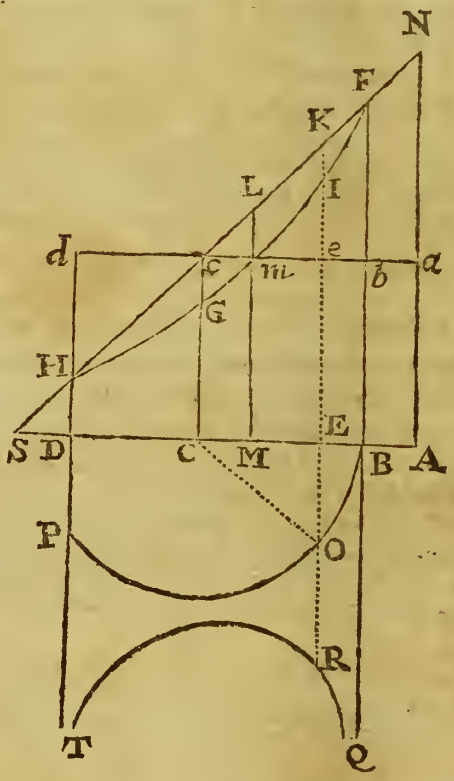
Ad hanc æquationem conſtruendam, aſſumatur quantitas v quæ ſit variabilis, & fingatur æquatio $z = p + qx + rv$, in qua notæ p, q, r deſignent alias novas quantitates invariabiles; & erit $\dot{z} = q\dot{x} + r\dot{v}$. Hiſce porro valoribus ipſarum z & $\dot{z}$ ſubſtitutis in æquatione prima $a\dot{z} = z\dot{x} - x\dot{x}$, habebitur $\overline{aq - p}, \dot{x} + ar\dot{v} = \overline{q - 1}, x\dot{x} + rv\dot{x}$. Ut hæc æquatio ſimplicior evadat, ponatur $q - 1 = 0$, & $aq - p = 0$; ſive $q = 1$, & $p = a$: & fiet $a\frac{\dot{v}}{v} = \dot{x}$, ac præterea $z = a + x + rv$. Jacentibus punctis D & S ad eandem partem puncti C, intelligatur CS æqualis ipſi a: & erit $z = SE + rv$, atque $CS\frac{\dot{v}}{v} = \dot{x}$. Sit c valor quantitatis v, dum incidit punctum E in punctum C: & quantitas x, ſive CE, æquabitur menſuræ rationis quam habet v ad c pro Modulo CS, per Propoſitionem primam: quam æqualitatem ſic deſignare ſoleo, $CE = CS\left|\frac{v}{c}\right.$. Tota ergo Problematis difficultas jam revocatur ad binas illas æquationes $CE = CS\left|\frac{v}{c}\right.$, atque $z = SE + rv$: hæ vero deduci non poterunt in uſum, priuſquam determinatæ fuerint quantitates r & CS. Ad hoc efficiendum, duæ reſtant conditiones nondum adimpletæ; oportet enim reſiſtentiam eſſe nullam, atque adeo quantitatem z ſive $SE + rv$ evaneſcere, ubi punctum E in puncta B & D inciderit.

Sint ergo b & d valores ipſius v, dum incidit punctum E in puncta B & D reſpective: & in his caſibus habebuntur $SB + rb = 0$, $SD + rd = 0$. Unde $r = -\frac{SB}{b}$, $r = -\frac{SD}{d}$, atque $z = SE + rv = SE - \frac{v}{b}SB = SE - \frac{v}{d}SD$. Porro erit $\frac{SB}{SD} = \frac{b}{d}$; atque adeo $CS\left|\frac{SB}{SD}\right. = (CS\left|\frac{b}{d}\right. = CS\left|\frac{b}{c}\right. - CS\left|\frac{d}{c}\right. = CB + CD =) BD$: unde dabitur punctum S.

Componetur itaque Problema hunc in modum. Producatur BD verſus D ad S, eo uſque, donec BD fuerit menſura rationis inter SB & SD ad Modulum CS. Deinde ad arbitrium poſita quantitate c, ita capiantur quantitates b & v; ut eodem Modulo CS, fiat CB menſura rationis quam habet b ad c, fiat quoque CE menſura rationis quam habet v ad c: & erit vis reſiſtentiæ in loco E, ad pondus relativum corporis penduli, ut $SE - \frac{v}{b}SB$, ad CA.

Hujus

Hujus Problematis ſolutio utilitatem habet in Phyſica non contemnendam: quapropter conſtructionem ejuſdem Linearem, ex eadem Analyſi deductam, ſubjungere viſum eſt. Invento uti ſupra puncto *S*; ad rectam *SA* erigantur perpendicula *DH*, *Cc*, *EK*, *BF*, *AN*, rectæ *SN* utcunque per *S* ductæ occurrentia in *H*, *c*, *K*, *F*, *N*. Per punctum *c* ducatur recta *da* parallela rectæ *DA*, quæ iiſdem perpendiculis occurrat in *d*, *c*, *e*, *b*, *a*; & ad Aſymptoton *SA* ducatur Logiſtica *HGIF*, quæ tranſeat per puncta *H* & *F*, ſecetque perpendicula *Cc*, *EK* in *G* & *I*, ac parallelam *da* in *m*: namque his poſitis, erit pondus relativum corporis penduli, ad vim illam qua pendulum acceleratur ad punctum *E* in Medio non reſiſtente, ut *aN* ad *eK*; erit autem ad vim reſiſtentiæ in loco *E*, ut *aN* ad *KI*; atque adeo ad vim qua pendulum acceleratur ad punctum *E* in Medio reſiſtente, ut *aN* ad *eI*. Porro, ſi per punctum *m* ducatur ad rectam *SMA* perpendicularis *LmM*, quæ ſecet *SN* in *L*: erit *M* locus ubi reſiſtentia fit maxima: atque adeo reſiſtentia illa maxima, erit ad pondus relativum penduli, ut *Lm* ad *Na*, hoc eſt, ut *CM* ad *CA*.

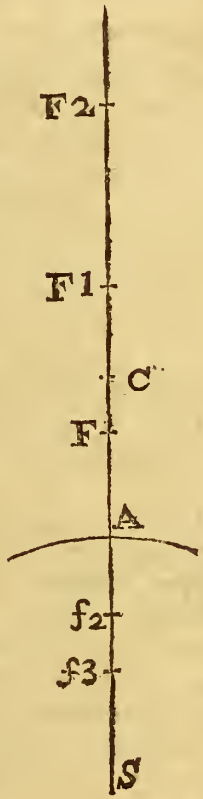


Ceterum ſi ita ducatur recta *SN*, ut abſcindat rectam *DH* quæ ſit dupla ipſius *SD*, centroque *C* & intervallo *CB* deſcribatur Circulus *BOP*, qui occurrat perpendiculo *KE* in *O*: erit penduli in Medio reſiſtente oſcillantis velocitas in loco *E*, ad velocitatem penduli ejuſdem ad eundem locum *E* delati per idem pondus relativum in Medio non reſiſtente, ut media proportionalis inter *CS* & *KI*, ad *EO*.

Adhæc

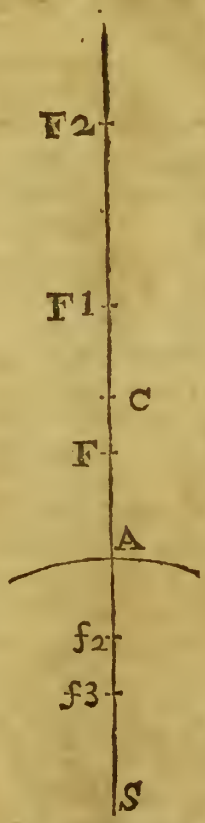
Adhæc si jungatur CO, & in perpendiculo KE sumatur ER, quæ sit ad CB ut CB ad mediam proportionalem inter Cc & KI; continuoque ductu rectæ ER in basim BE generetur area $BQRE$: erit tempus quo Cycloidis arcus BE describitur in Medio resistente, ad tempus quo idem arcus describeretur in Medio non resistente, ut area illa $BQRE$, ad Circuli sectorem BOC. Pergo nunc ad alia.

Densitatem Aeris invenimus ad quamvis altitudinem, ubi vis Gravitatis vel erat uniformis, vel decrescebat in recessu à centro telluris in duplicata ratione distantiæ: libet eandem exquirere denuo, ubi gravitatio vel augetur vel diminuitur in ratione datæ cujusvis dignitatis distantiæ. Sit S centrum telluris, A punctum in ejus superficie vel alibi utcunque situm, $SAF2$ recta à centro ad summitatem Atmosphæræ producta: & quærenda sit ratio densitatis in loco A, ad densitatem in loco quovis F, ex Hypothesi quod vis gravitatis in F sit ut distantiæ SF dignitas quæcunque SF^n, cujus index est n. Pro SF scribatur x, ac designent d & v densitates Aeris ad A & F; & cum densitas sit ubique ut pressura totius Aeris incumbentis, erit densitatis momentum ut momentum pressuræ, hoc est, $\dot{v}$ ut $v\dot{x}x^n$, atque adeo $\frac{\dot{v}}{v}$ ut $\dot{x}x^n$. Sit AC altitudo Atmosphæræ, cujus uniformis densitas eadem esset ac densitas loci A; vel sit AC ad altitudinem Hydrargyri barometrici in loco A, ut densitas Hydrargyri ad densitatem Aeris in eodem loco A: & si punctum F accedere intelligatur ad punctum A; erit altitudo Hydrargyri barometrici in loco A, ad altitudinem Hydrargyri barometrici in loco F, ut AC ad FC. Aeris ergo in loco A densitas d, est ad Aeris in loco F densitatem v, ut AC ad FC: unde consequitur ut sit $d-v$ sive $\dot{v}$, ad d sive v, ut AF sive $\dot{x}$, ad AC. Erit itaque, in hoc casu, $AC\frac{\dot{v}}{v}=\dot{x}=\frac{\dot{x}x^n}{SA^n}$. Quoniam ergo, ubicunque sumeretur punctum F, erat $\frac{\dot{v}}{v}$ ut $\dot{x}x^n$: erit porro $AC\frac{\dot{v}}{v}=\frac{\dot{x}x^n}{SA^n}$, ubicunque sumatur punctum F.



Jam

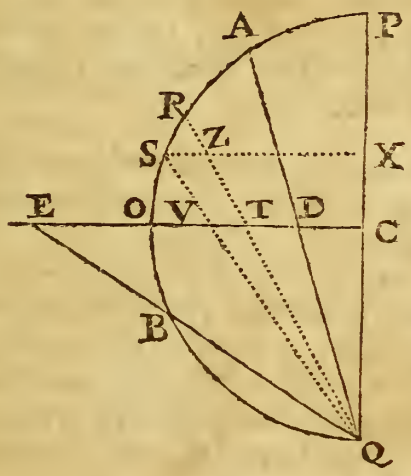
Jam ſi gravitatio ſit reciproce ut diſtantia à centro; ſive ut $\frac{1}{x}$ vel x^{-1}; erit $n = -1$, atque inde $AC\frac{\dot{v}}{v} = SA\frac{\dot{x}}{x}$; unde ſi Fluentes ſtatuantur æquales, menſura rationis inter denſitates d & v ad Modulum AC, æquabitur menſuræ rationis inter diſtantias SF & SA ad Modulum SA.



Si gravitationis ſit alia quævis Lex: quoniam eſt $AC\frac{\dot{v}}{v} = \frac{\dot{x}x^n}{SA^n}$; ſi Fluentes ſtatuantur æquales, erit $\frac{1}{n+1}$ in $\frac{SF^{n+1}}{SA^n} - SA$ menſura rationis inter denſitates d & v ad Modulum AC. Itaque ſi ſumantur in progreſſione Geometrica termini creſcentes SA, SF, $SF1$, $SF2$, &c: decreſcentes SF, SA, $Sf2$, $Sf3$, &c: menſura rationis inter denſitates Aeris in A & F ad Modulum AC, erit $\frac{1}{2}Af3$, ſi gravitatio ſit reciproce in triplicata ratione diſtantiæ; erit $Af2$, ſi gravitatio ſit reciproce in duplicata ratione diſtantiæ; erit AF, ſi gravitatio uniformis ſtatuatur; erit $\frac{1}{2}AF1$, ſi gravitatio ſit ut diſtantia; erit $\frac{1}{3}AF2$, ſi gravitatio ſit in duplicata ratione diſtantiæ. Et ſic proceditur in infinitum.

Denique ut plenius conſtet, Syntheticas etiam demonſtrationes ex elementis præmiſſis levi negotio concinnari poſſe; ſufficiet unicum inſuper addidiſſe Exemplum, tædet utique plura jam proferre. Repetatur itaque diviſio illa Nautica Meridianæ quam ſupra attigimus, & videamus etiam abſque ope Curvæ cujuſpiam Logometricæ, annon ſimplicior aliquanto ſit futura demonſtratio ad modum ſequentem. Sit $PXCQ$ Telluris axis, CO ſemidiameter Æquatoris, $PAOBQ$ Meridianus; & invenienda ſit in planiſphærio Nautico magnitudo cujuſvis arcus AB. Ad arcus illius terminos A & B ducantur ab alterutro Polorum P vel Q rectæ QA, QB, ſemidiametro CO occurrentes in D & E: Dico magnitudinem Nauticam arcus AB æqualem eſſe menſuræ rationis inter EC & DC ad Modulum OC. Nam diviſus intelligatur arcus AB in particulas

culas quam minimas *RS*, & jungantur *QR*, *QS* quæ ſecent *CO* in *T* & *V*; & demiſſo in axem perpendiculo *SX* quod rectæ *QR* occurrat in *Z*, erit lineola *SZ* æqualis particulæ *RS*. Itaque magnitudo Nautica naſcentis arcus *RS*, erit ad Sphæræ ſemidiametrum *OC*, ut arcus ille *RS* ſive lineola *SZ* ad *SX*, hoc eſt,



ut *VT* ad *VC*. Unde (per Corol. 2. Prop. 1.) magnitudo illa Nautica æquatur menſuræ rationis inter *VC* & *TC* ad Modulum *OC*: & ſimiles utrobique ſummas colligendo, magnitudo Nautica totius arcus *AB* æquabitur menſuræ totius rationis inter *EC* & *DC* ad eundem Modulum *OC*.

III. *An Extract from the* Acta Eruditorum *for the Month of* March, 1713. *Pag.* 111.

De Contagiosâ Epidemiâ, quæ in Patavino agro & totâ fere Venetâ ditione in Boves irrepsit, Dissertatio. Auctore *Bernardino Ramazzini*, Practicæ Medicinæ Professore Publico. *Patavii*, 1712. in 8vo.

A Dissertation concerning the dreadful Contagious Distemper, seizing the Black Cattle in the Venetian *Territories, and especially about* Padua.

IT is now (at the Publishing the Discourse) a Year and half, since a dreadful, unexpected and violent Contagion has seiz'd the *Black Cattle*, which, like an increasing Fire, could neither be extinguish'd nor stopt by any Human means.

This first began to be observ'd a little in *Agro Vincentino*, and soon discover'd it self more openly in the Country, spreading it self every way even to the very Suburbs of *Padua*, with a cruel Destruction of the *Cows* and *Oxen*. It has also been taken notice of in *Germany*, in many Places; nor has it been yet wholly conquer'd, Publick News informing us, that it still remains in the Territories of *Milan*.

Of this so threatning a Distemper, the famous Dr, *Ramazzini*, according to his yearly Custom, on *November* 9. 1711. made a particular Dissertation; in which he inquir'd into the Causes of the Distemper, and shew'd what Remedies might be us'd, to put a stop to its violent Course.

It is sufficiently evident, that this Distemper, in the Cow-kind, was a true Fever, from the coldness, rigor and standing up of the Hair of the Cattle at first, which was soon succeeded by a violent sharp burning, with a quick Pulse. That this Fever was malignant, mortal and pestilential, its concomitant Symptoms plainly shew'd; such as great uneasiness with difficulty

ficulty of breathing, great pantings with a ſort of ſnorting, and at the beginning a kind of *Stupor* and Drowzineſs, a continual Flux of a ſtrong ſmelling Matter from the Noſe and Mouth, a very fetid Dung, ſometimes with Blood, all Rumination ceaſing, Puſtules breaking out over the whole Body on the 5th or 6th day, like the *Small-Pox*; they all generally dy'd about the 5th or 7th day, very few of them eſcaping.

The Author deduces this Diſtemper from a contagious Original. He tells us, it is certain, that out of a great Drove, ſuch as the Merchants bring yearly into *Italy* out of *Dalmatia* and the bordering Countries, one Beaſt happen'd to ſtraggle from the reſt, and be left behind; which a Cowherd finding, brought to a Farm belonging to the Illuſtrious and Reverend Count *Borromeo*, Canon of *Padua*: This Beaſt infected all the Cows and Oxen of the Place where he was taken in, with the ſame Diſtemper he labour'd under; the Beaſt it ſelf dying in a few days, as did all the reſt, except one only, who had a Rowel put into his Neck.

'Tis no ſtrange thing therefore, if from the *Effluvia*, like an Atmoſphere, proceeding from the ſick Cattle, from thoſe dead, and from the Cowhouſes and Paſtures where they were fed, and by that means infected, and chiefly from the Cloaths of the Cowherds themſelves, this Infection falling upon a proper Subject, ſhould diffuſe it ſelf ſo largely. When therefore this ſubtile venomous Exhalation happens to meet with any of the Cow kind, joining it ſelf with the ſerous Juices and Animal Spirits, whilſt it is carry'd all over the Body, diſorders the natural conſiſtence of the Blood, and corrupts the Ferments of the *Viſcera*; whence it naturally follows, that the natural Functions of the *Viſcera* are vitiated, and the requiſite Secretions ſtop'd. For

Dr. *Ramazzini* not only ſuppoſes, but aſſerts, that this Poiſon is of that kind which rather fixes and coagulates, than diſſolves the Blood: For beſides the forementioned Symptoms accompanying the Diſeaſe, the Eye it ſelf is a Witneſs; ſince the dead Carcaſes being open'd while they are yet hot, little or no

Blood

Blood nevertheleſs runs out; thoſe Animals having naturally a thick Blood, eſpecially when the Fever has continued ſo many Days. Whether therefore this Plague came firſt from the foreign Beaſt, or any other way, it is the ſame thing, when at laſt it fell upon ſome Animal in which there was the morbid Seminary or Ground prepared for it.

In the dead Bodies of all the Cattle it was particularly obſerved, that in the *Omaſus*, or Paunch, there was found a hard compact Body, firmly adhering to the Coats of the Ventricle, of a large Bulk, and an intolerable Smell: In other Parts, as in the Brain, Lungs, *&c.* were ſeveral *Hydatides*, and large Bladders fill'd only with Wind, which being open'd gave a deadly Stink; there were alſo Ulcers at the Root of the Tongue, and Bladders fill'd with a *Serum* on the ſides of it. This hard and compact Body, like Chalk, in the *Omaſus*, the Author takes to be the firſt product of the contagious *Miaſma*. He adds a Prognoſtick, believing that from ſo many Attempts and Experiments, and the Method obſerv'd in the Cure of this Venom, at laſt a true and Specifick Remedy will be found out to extirpate the poiſonous Malignity wholly: He alſo expects ſome mitigation of it, from the approaching Winter and North Winds. He does not think this Contagion can affect Human Bodies, ſince even other *Species* of ruminating Animals, ſymbolizing with the Cow-kind, are yet untouch'd by it; nor was the Infection catch'd from the Air, provided due care was taken in the burying the dead Bodies.

As for the Cure of it: From the *Chirurgical* Part he commends Bleeding, burning on both ſides the Neck with a broad red-hot Iron, making Holes in the Ears with a round Iron, and putting the Root of *Hellebore* in the Hole, a Rowel or Seton under the Chin, in the Dew-laps; he alſo orders the Tongue and Palate to be often waſh'd and rub'd with Vinegar and Salt.

As for the *Pharmaceutical* Part; he recommends *Alexipharmicks*, and ſpecifick Cordials; and from the *Vegetable Kingdom*, three Ounces of *Jeſuits Bark*, infus'd in 10 or 12 Pints of Cordial Water or ſmall Wine, to be given in 4 or 5 Doſes; which

is

is to be done in the beginning of the Fever, when the Beast begins to be sick. From the *Animal*, two Drams of *Sperma Cæti* dissolv'd in warm Wine. From the *Mineral*, *Antimonium Diaphoreticum*. Against *Worms* breeding, an Infusion of *Quicksilver*, or *Petroleum* and Milk is to be given. And lastly, as to the *Food*, Drinks made with Barly or Wheat Flower, or Bread, like a *Ptisane*, fresh sweet Hay made in *May* and macerated in fair Water. In the mean time the Cattle must be kept in a warm Place, and cloath'd, to keep them as much as possible from the cold Air, daily making Fumigations in the Cowhouses with *Juniper Berries*, *Galbanum*, and the like. As to Prevention, he enjoyns Care in cleaning the Stalls, and scraping the Crust off from the Walls; Care also is to be taken of their Food, that it be good, the Hay and Straw not spoil'd by Rain in the making, and judges their Food ought to be but sparing; Friction, rubbing and currying, not only with the Hand, but with a Currycomb and Brush; with Setons under their Chin, made with a hot Iron run through the Part, and kept open with a Rope put through it.

IV. *A* Recipe: *Or the Ingredients of a Medicine for the spreading mortal Distemper amongst Cows: Lately sent over from* Holland, *where a like Distemper raged amongst the Black Cattle.*

R*ecipe Veronicæ, Pulmonariæ, Hyssopi, Scordii, ana M.* iv. *Rad. Aristolochiæ rotundæ, Gentianæ, Angelicæ, Petasitidis, Tormentillæ, Carlinæ, ana unc.* 12. *Bac. Lauri & Juniperi, ana unc.* 12. *Misc. fiat Pulvis.*

Bleed the Cow, and give her every Morning, for 3 or 4 Mornings, an Ounce of this Powder with a Horn in warm Beer.

If the Cow's Illness continues, after the omission of 2 or 3 Days, repeat the Medicine for 3 or 4 Days again.

F I N I S.

ERRATA. Pag. 43. Lin. 2. for *Cc*, read 4*CS*.

LONDON, Printed for WILLIAM INNYS, at the *Prince's-Arms* in St. *Paul's* Church-yard. 1714.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *April*, *May* and *June*, 1714.

The CONTENTS.

ERRATA, in the *Philos. Trans.* Numb. 337.

PAG. 80. lin. 3. lege *Societatis à secretis.* P. 88. l. 6. l. *rotundo.* P. 158. l. 25. l. *suspicor, agnescam.* P. 159. l. 9. l. *præstari.* P. 285. l. 25. l. *Legions* P. 288. l. 1. instead of *N N* there should be a Ligature ℕ, with the last Line raised a little higher than the first.

I. *The*

I. *A new Method for making* Logarithms, *and* vice versâ, *for finding the Number corresponding to a* Logarithm *given, by help of the following Table. Communicated by Mr.* John Long, S. Theol. Bacc. *and Fellow of* Corpus Christi *Coll.* Oxon.

Log.	*Nat. Num.*
0,9	7.943282347
0,8	6.309573445
0,7	5.011872336
0,6	3.981071706
0,5	3.162277660
0,4	2.511886432
0,3	1.995262315
0,2	1.584893193
0,1	1.258925412
0,09	1.230268771
0,08	1.202264435
0,07	1.174897555
0,06	1.148153621
0,05	1.122018454
0,04	1.096478196
0,03	1.071519305
0,02	1.047128548
0,01	1.023292992
0,009	1.020939484
0,008	1.018591388
0,007	1.016248694
0,006	1.013911386
0,005	1.011579454
0,004	1.009252886
0,003	1.006931669
0,002	1.004615794
0,001	1.002305238

Log.	*Nat. Num.*
0,0009	1.002074475
0,0008	1.001843766
0,0007	1.001613109
0,0006	1.001382506
0,0005	1.001151956
0,0004	1.000921459
0,0003	1.000691015
0,0002	1.000460623
0,0001	1.000230285
0,00009	1.000207254
0,00008	1.000184224
0,00007	1.000161194
0,00006	1.000138165
0,00005	1.000115136
0,00004	1.000092106
0,00003	1.000069080
0,00002	1.000046053
0,00001	1.000023026
0,000009	1.000020724
0,000008	1.000018421
0,000007	1.000016118
0,000006	1.000013816
0,000005	1.000011513
0,000004	1.000009210
0,000003	1.000006908
0,000002	1.000004605
0,000001	1.000002302

Log.	Nat. Num.
0,0000009	1.000002072
0,0000008	1.000001842
0,0000007	1.000001611
0,0000006	1.000001381
0,0000005	1.000001151
0,0000004	1.000000921
0,0000003	1.000000690
0,0000002	1.000000460
0,0000001	1 000000230

Log.	Nat. Num.
0,00000009	1 000000207
0,00000008	1.000000184
0,00000007	1.000000161
0,00000006	1.000000138
0,00000005	1.000000115
0,00000004	1.000000092
0,00000003	1.000000069
0,00000002	1.000000046
0.00000001	1.000000023

This Table is what I ſometimes make uſe of for finding the Logarithm of any Number propos'd, and *vice versâ*. For inſtance: Suppoſe I had occaſion to find the Logarithm of 2000. I look in the firſt Claſs of my Table (the whole Table conſiſts of 8 Claſſes) for the next leſs to 2, which is 1.995262315, and againſt it is 3, which conſequently is the firſt Figure of the Logarithm ſought. Again; dividing the Number propos'd 2, by 1.995262315 the Number found in the Table, the Quotient is 1.002374467; which being look'd for in the ſecond Claſs of the Table, and finding neither its equal, nor a leſſer, I add 0 to the part of the Logarithm before found, and look for the ſaid Quotient 1.002374467 in the third Claſs, where the next leſs is 1.002305238, and againſt it is 1, to be added to the part of the Logarithm already found; and dividing the Quotient 1.002374467, by 1.002305238, laſt found in the Table, the Quotient is 1.000069070; which being ſought in the fourth Claſs gives 0, but being ſought in the fifth Claſs gives 2, to be added to the part of the Logarithm already found; and dividing the laſt Quotient by the Number laſt found in the Table, *viz.* 1.000046053, the Quotient is 1.000023015, which being ſought in the ſixth Claſs, gives 9 to the part of the Logarithm already found: And dividing the laſt Quotient by the new Diviſor, *viz.* 1.000002072, the Quotient is 1 000000219, which being greater than 1.000000115, ſhews that the Logarithm already found, *viz.* 3.3010299 is leſs than the Truth by

more

more than half an Unit; wherefore adding 1, you have *Briggs*'s Logarithm of 2000, *viz.* 3.3010300.

If any Logarithm be given, suppose 3.3010300, throw away the Characteristic, then over against these Figures 3 ... 0 .. 1 ... 0 .. 3 .. 0 .. 0, you have in their respective Classes 1.995262315 ... 0 .... 1. 002305238 ..... 0 ..... 1 . 000069080 .... 0 ... 0 which multiplied continually into one another, the Product is 2.000000019966, which by reason the Characteristic is 3, becomes 2000.000019966, *&c.* that is, 2000, the Natural Number desired. I shall not mention the Method by which this Table is fram'd, because you will easily see that from the use of it.

It is obvious to the intelligent Reader, that these Classes of Numbers are no other than so many Scales of mean Proportionals: In the first Class, between 1 and 10; so that the last Number thereof, *viz.* 1,258925412 is the tenth Root of 10, and the rest in order ascending are the Powers thereof. So in the second Class, the last Number 1,023292992 is the hundredth Root of 10, and the rest in the same manner are Powers thereof. So 1,002305238 in the third Class, is the tenth Root of the last of the second, and the rest its Powers, *&c.* Or, which is all one, each Number in the preceding Class, is the tenth Power of the corresponding Number in the next following Class: Whence 'tis plain, that to construct these Tables requires no more than one Extraction of the fifth or sursolid Root for each Class, the rest of the Work being done by the common Rules of Arithmetick; and for extracting the fifth Root, you will find more than one very compendious Rule in *Num.* 210 of these *Transactions*, if any one shall desire to examin the *computus* of these Tables.

The Process is exactly the Reverse of Mr. *Briggs*'s Doctrine, in *Cap.* XII. of his *Arithmetica Logarithmica* of *Vlacq*'s Edition; and had *Briggs* been appriz'd hereof, it would have greatly eased the Labour of deducing the Logarithms of the first prime Numbers, which appear to have cost him so much Pains.

II. *An*

III. *An Extract of a Letter from* Mr. Anthony van Leeuwenhoeck, *F.R.S. Dated* October *the* 12*th.* 1713. *Concerning the Fibres of the Muſcles,* &c.

GENTLEMEN,

IN compliance with your deſires, I here ſend you a Copy of the Obſervations I communicated to the great Penſionary *Monſieur Heinſius*, concerning the Membranes with which the *Fibrillæ* of the Muſcles appear to be encompaſſed, both in the Fleſh of a Whale, Cod-fiſh, Salmon, and Smelt; and alſo in that from an Oxe to a Mouſe; in all which the appearance was the ſame.

The Fleſh of the Whale, was a ſmall piece cut off near the Tail of the Fiſh, which I deſired a Sea-Captain to bring me, that I might obſerve how the Fleſh in that part was joined to the Tendons in ſo large an Animal. Viewing this thro' the Microſcope, I judged the Fibres thereof to be four times as large as thoſe I had formerly obſerved in another piece of Whale's Fleſh, taken from another part of the Fiſh; which made me conſider, whether the Fibres of that part might not be, by Nature, furniſhed with larger Fibres for its greater ſtrength.

Cutting the ſaid Fleſh-parts length-wiſe, and a croſs the Fibres, I diſcovered more plainly than before, that each Particle or Fleſh-fibre, was enwrapt in a fine thin Membrane.

To have a better Idea of theſe Fleſh-fibres of a Whale, I cut a thin ſlice of it a-croſs, which I laid on a wetted piece of Glaſs, that the Fleſh which was very dry and ſhrunk, might, by the moiſture be ſwelled, and thereby diſtended to the natural ſize it had when on the Body of the Fiſh itſelf. In this ſtate, placed before the Microſcope, it appeared as I cauſed it to be drawn in the Figure. *A.B.C.D.* in which the Parts were ſo cloſe together, that their encompaſſing Membranes, repreſented by the black Lines, were but juſt diſcernable, ſome whereof appeared larger than others: theſe, if attentively viewed, ſeem'd

 plainly

plainly to be divided into multitudes of others, cut alſo tranſverſe, the bigneſs of which was no larger than a common Sand to the naked Eye. Theſe were ſo cloſe crowded together, that their Figure was very irregular, as well as their ſizes different; for tho' each ſeem'd encompaſſed with ſix others, yet ſome of them were twice as large as the other.

Having formerly mentioned the ſlenderneſs of theſe *Fibrillæ* in the Fleſh of a Whale, and judging theſe, as I ſaid before, to be four times as large, I took a thin ſlice of the formerly mentioned Whale-fleſh (which I had ſtill kept by me) and after having made it throughly wet, I viewed it with the ſame Microſcope as I had done this of the Tail. This appear'd as is repreſented (*Fig.* 2.) *E. F. G. H.* Letting the moiſture dry away from theſe ſlices, ſo ſtuck on to the Glaſs, the Particles became much ſmaller, and the Membranes with which each was encompaſſed, became very viſible; that is, thoſe which were not ſhrunk away; which was a very entertaining Object to the curious; and as often as I made new Cuts, a new Object preſented it ſelf.

A ſmall Particle of this Fleſh I cauſed to be drawn, as in (*Fig.* 3.) *I K L M.* Theſe Particles ſeem'd to touch and be joined to others; but now being dried, they ſhrunk in from the Membranes round about them; for the Membranes could not ſhrink, becauſe they were all joined to one another.

Along theſe Fleſh-fibres there runs ſome ſuch thick Membranes, that they equal the thickneſs of a Hair or more, which are ſcarce diſtant the breadth of a Sand from each other; from theſe larger Membranes other parts are ſpread, dividing each Fibre into numerous *Fibrillæ*; ſo that it may be ſaid, each fleſh-Fibre, no bigger than a Hair, is a little Muſcle encompaſſed in its peculiar Coat or Membrane, as I have ſaid before. Whereas the Deſigner had not the ſame apprehenſion of the ſize of theſe Fibres, as I and ſome other Perſons had, I made him draw a little piece as large as it appeared to my apprehenſion, as in (*Fig.* 4.) *N. O. P.* whence appears the difference of one Man's ſight from another.

I

I have alſo often ſeen ſome few of theſe Fibres, tho' joyned to others, yet but one Fourth of their bigneſs to which they were joined.

When I again moiſtend thoſe repreſented in the Third and Fourth Figures, (dryd up and ſhrunk) they would be again ſo ſwelled and diſtended, as to fill up the ſpaces between the Membranes, and re-aſſume the Shape they had before they were dried. Among ſeveral little pieces of Fleſh placed before another Microſcope, and moiſten'd as before, there was one, whoſe Particles were not ſeparated upon drying, which I ſuppoſed to be, from the ſplitting and tearing aſunder of a large Membrane that run thro' the middle of it, as may be ſeen in (*Fig.* 5.) *Q. R. S. T. V. W* where between *S T.* and *V.* the dried Particles remain unſeparated; theſe being cut a little thicker appeared alſo of a darker hew, and if they had been ſliced yet thicker would have appeared of a dark red. By *S. W.* is repreſented the thick Membrane dividing this piece, which was about the bigneſs of a Hair; this at *T.* ſent out a Branch, and near *W.* is ſplit into two, I apprehend that a great number of Blood-veſſels are ſpread over this Membrane, which by their ſmallneſs are not viſible; for it is by theſe the Nouriſhment is convey'd. Between *R. S.* and *Q. W.* the exceeding fine Membranes torn from the great are viſible.

Is it not amazing that in ſuch vaſt Animals as a Whale, ſuch exceeding ſmall *Fibrillæ* ſhould be found? nay, ſuch they are in ſmall Animals; and that the whole fifth Figure is not ſo large as a courſe grain of Sand.

This Whale was ſo large, that the upper part of its Body yielded 60 Quarteels of Blubber or Fat, which allowing 30 *Rotterdam* Stopes (making each about 3 Engliſh Quarts) to one Quarteel, it will nearly amount to 24000 Pound weight; beſides, there is a very great deal of Fat about the Entrails.

Then I cauſed a very little piece, conſiſting only of five *Fibrillæ*, to be drawn lengthwiſe, as they were ſeen thro' the Microſcope, as (in *Fig* 6.) *A. B. C. C. E. F.* in which Figure at *A.* and a little at that place, it is divided into two *Fibrillæ*. Between *C.*

 and

and *F.* are to be ſeen the little Membranes which incompaſs the *Fibrillæ*, which are here torn aſunder.

I have frequently, with pleaſure, obſerved theſe Fleſh-fibres lengthways, to be as it were corrugated or wrinkled, which I imagined to be the Repreſentation of their Reſt or unbent Poſture; and yet more, when the Part to which they belong is bowed together, or brought nearer; but when the Muſcle is extended, and its Antagoniſt acts, there is not the leaſt wrinkle obſervable in theſe *Fibrillæ*.

However, all the little Inequalities in theſe *Fibrillæ* muſt not be taken for thoſe Corrugations, ſince many of them are only the Particles torn off from the Membranes which encompaſs the *Fibrillæ*.

Figure (7) *G. H. I. K.* repreſents four little Fibres of a piece of Whales-Fleſh I had procured two Years ſince; this I cauſed to be drawn to ſhew the difference. By the two Figures 6 and 7, is viſible the Diameters of the Fibres are as big again in one, as in the other, therefore the Fibres muſt be four times as big in *Fig.* 6. as in *Fig.* 7. Now each Fleſh-fibre being compoſed of a great many ſmaller *Fibrillæ*, we may imagine each of theſe in-lying Fibres do likewiſe conſiſt of others of the like Nature.

I have a-freſh viewed ſeveral ſmall Fibres of Ox-Fleſh, and obſerved, that each of the *Fibrillæ* in them was encompaſſed with a thin Membrane. But I cannot ſhew theſe Membranes ſo clearly to other Perſons in Cows-Fleſh, as in Whales-Fleſh, becauſe the Parts of the former are of a much more compact and cloſe Texture than that of the Whale, from whence they do not ſhrink ſo much in drying.

I am of Opinion, that what I have ſaid of the Membranes (encompaſſing the Fibres and *Fibrillæ*) of the Fleſh in a Whale, will likewiſe hold true in other kinds of Fleſh; yea even down to that of a Rat or a Mouſe: Concerning which I ſhall profecute my Obſervations. I conclude, *&c.*

To

To these Observations of Mr. *Leewenhoeck* I shall join,

III. *An Extract from the* Journal Literaire, *Publish'd at the* Hague, *for the Months of* January *and* February, 1714. Pag. 238. *Being*,

An Account of several Observations concerning the Frame *and* Texture *of the* Muscles; *By Mr.* Muys *of* Franequer:

THE Celebrated Mr. *Muys*, who always acts for the Honour of the Academy of *Franequer*, and Advantage of Students in Physick and Anatomy, has made several Discoveries, as to the Mechanism and Texture of the Muscles of Animals; of which these are some.

He has observed, that the fleshy Fibres of the Muscles are composed of other smaller Fibres, which he calls *Fibrillæ*: that these *Fibrillæ* are of the size of a slender Hair, and that 500 or 600 of them, may be counted in one Fleshy-fibre, whose Diameter is no more than a 24th part of an Inch.

That each of these *Fibrillæ* also is made up of more than 300 little transparent *Tubuli*, but so slender, that if a Blood Globule (which, according to Mr. *Leuwenhoek*, is but the 1,000,000*th* part of a grain of Sand) were divided into 24 parts, one of these could hardly pass thro' these small Pipes.

He has shewn, that tho' the fleshy-Fibres of the Musces, are joined to the Tendons and tendinous Membrane of a Muscle; yet these tendinous Fibres are not a continuation of the fleshy ones, as most Anatomists suppose: which he proves thus; If by means of a wooden Knife, or only by pulling it, you separate the fleshy Fibres from the Tendon, the end of the Tendon to which they were joined, will remain smooth and even, and not rugged.

Having

Having made ſeveral Injections of warm Water into the crural Artery of a Lamb of a Year old, all the fleſhy-Fibres loſt all their redneſs, and became entirely white. The Fibres being whitened by this Injection, he injected a coloured Liquor by the ſame Artery; and then not only the ſmall Arteries appeared filled with this tinged Liquor, but he found alſo that the Liquor paſt thro' each Fibre, either in a Serpentine manner, or undulating, or frameing ſeveral Angles, or joined by a great number of *Anaſtomoſes.*

He obſerved alſo, that many ſmall Branches of the Arteries which before could not be ſeen, appeared viſibly, ſpread all round the little *Fibrillæ*, and tinged with the ſame Colour.

Having remarked, that the Parts of the fleſhy Fibres, which were near the Extremities of the Arteries, appeared tinged with the Liquor, he examined them with a Microſcope, and found the little *Fibrillæ* filled and tinged with the ſame Liquor; and yet there was not leaſt appearance of the Liquor in the *Interſtices* between the *Fibrillæ.*

Having made Injections by the crural Artery, of another coloured Liquor, in the Muſcles, whiten'd, as before, with Water, he ſaw not only the Fibres in ſome of the Muſcles, and the moſt part of them in the others filled with this matter; but having examined them with a good Microſcope, he found the *Fibrillæ*, and even the leaſt *Tubuli* which compoſe them, filled and tinged with the ſame Matter; and nevertheleſs the ſmall Ramifications of the Nerves appeared perfectly white.

It reſults from all theſe Obſervations,

1ſt. That the little Tubes, which make a *Fibrilla*, are really hollow, and that the Extremities of the Capillary Arteries open into them, and empty there a part of their Liquor, which is re-conveyed by the Veins to the Heart.

2d. That the Blood Globules muſt be divided into an almoſt infinite degree of ſmalneſs, before they can enter and paſs theſe *Tubuli.* That the Blood-Globules may be ſo divided, and when ſo divided paſs thro' the ſmall *Tubuli*, is evident from

from the redneſs of the Fibres and *Fibrillæ* of Animals, which have a red Fleſh; which will be no ſurprize to them who have read Mr. *Leeuwenhoeks* Letter 42, where he ſays, that theſe Globules do divide themſelves after this manner, to paſs thro' the laſt Extremities of the Capillary Arteries of the Brain; nor to those who know, that the Globules are extreame ſoft and eaſily ſeparable, as *Mounſieur Muys* has evinced by Arguments grounded on very curious Obſervations.

Monſieur Muys has added to his Obſervations very exact Figures, which contribute very much to the forming a clear and diſtinct Idea of the Structure of theſe Fibres of the Muſcles, and of the manner of the Arteries paſſing through them; but I dare not ſo far depend on my Skill in deſigning to venture to Copy them.

This knowing Perſon has alſo made ſeveral Diſcoveries of the Courſe and Ramifications of the Nerves in the Muſcles: But I wait for an Opportunity of informing my ſelf better of ſeveral Particularities, before I communicate them to you.

In my laſt I wrote to you concerning the Salts which Mr. *Muys* had diſcover'd in Human Blood; but I had forgot to inform you, that he had found out a way to ſeparate them from the Blood, without any Chymical *Analyſis*, and without making them undergo any change, and to form them into Criſtais, viſible without a Microſcope; as he has ſhewn to his Students in Phyſick.

IV. *An*

IV. *An Extract of ſeveral Letters from* Cotton Mather, *D. D. to* John Woodward, *M. D. and* Richard Waller, *Eſq*; S. R. Secr.

THE firſt Letter directed to Dr. *Woodward*, is dated at *Boſton* in *New England*, *Nov.* 17. 1712. In this the Writer gives an Account of a large Work in Manuſcript, in two Volumes in Folio, but does not name the Author. This, according to the account of it, is a large Commentary upon ſome Paſſages in the Bible, interſpers'd with large Philoſophical Remarks, taken out of Natural Hiſtorians, and the Obſervations of himſelf and others, more particularly as to Matters obſerv'd in *America*, whence he entitles the Work, *Biblia Americana*. This Work Dr. *Mather* recommends to the Patronage of ſome generous *Mecænas*, to promote the Publication of. As a Specimen of it, he tranſcribes a Paſſage out of it, being a Note on that Paſſage in *Gen Chap. 6. v.* 4. relating to *Giants*; and confirms the Opinion of there having been, in the *Antediluvian* World, Men of very large and prodigious Statures, by the Bones and Teeth of ſome large Animals, found lately in *Albany* in *New England*, which, for ſome Reaſons, he judges to be Human; particularly a *Tooth* brought from the Place where it was found to *New York*, 1705. being a very large Grinder, weighing 4 pounds and three quarters, with a Bone, ſuppos'd to be a Thigh-bone, 17 Foot long. He alſo mentions another Tooth, broad and flat like a fore-Tooth, four Fingers broad: the Bones crumble to pieces in the Air after they are dug up; they were found near a place call'd *Cluverack*, about 30 Miles on this ſide *Albany*. He then gives the Deſcription of one, which he reſembles to the Eye-Tooth of a Man; he ſays it has four Prongs, or Roots, flat, and ſomething worn on the top;

top it was ſix Inches high, lacking one eighth, as it ſtood upright on its Root, and almoſt thirteen Inches in circumference; it weigh'd two pounds four ounces *Troy* weight: There was another near a pound heavier, found under the Bank of *Hudſon*'s River, about fifty Leagues from the Sea, a great way below the Surface of the Earth, where the Ground is of a different Colour and Subſtance from the other Ground, for ſeventy five Foot long, which they ſuppoſe to be from the rotting of the Body, to which theſe Bones and Teeth did, as he ſuppoſes, once belong. *It were to be wiſh'd the Writer had given an exact Figure of theſe Teeth and Bones.*

The ſecond Letter to the ſame Perſon, is dated *Nov.* 18. 1712. from *Boſton*, as all the following are. In this he treats of the Plants of *America*; and, in the firſt place, offers a Conjecture of his concerning the *Shittim* Wood, mentioned in the Sacred Writings to be made uſe of for the Ark, *&c.* It is ſaid to be not as moſt other Woods, ſubject to rot: He judges that it was the black *Acacia*; that the *Gopher* Wood was the *Juniperus arbor tetragonophyllos*, frequent in the *Eaſt-Indies*, *&c.* He obſerves that the *Indians* often perform very great Cures with their Plants; of which there is a great variety, differing from the *European*, which he promiſes a Catalogue and Deſcription of; and, for the preſent, inſtances in ſome. As, a Plant efficacious in curing *Inflammations*, whence they call it *Antieriſypelas*; it grows plentifully in the Woods: A Chymical Oil extracted from it, taken inwardly, does Wonders in abſorbing ſcorbutick Salts. Another Plant, which goes by the Name of *Partridge-berries*, excellent in curing the *Dropſy*; a Decoction of the Leaves being drank ſeveral days as a Tea, diſcharging a vaſt quantity of Urine, as long as the Diſeaſe laſts; after which it may be drank without provoking Urine obſervably: Gouty Perſons drink it with benefit.

The Root call'd the *Bleeding Root*, curing the *Jaundies* in five or ſix days.

Another for *Gangrenes*, of which he does not give the Name.

Another Specifick for the Bite of the *Rattle-Snake*, and another for *Quinſies*, or ſore Throats. A Plant, call'd by the *Indians*, *Tauttuttipang*; infallible for the *Lues Venerea*, the Root being uſed in a Decoction, and drank half a Pint; a Cataplaſm of the ſame Root, bruiſed, apply'd to the Ulcers, cures them alſo.

A Thiſtle call'd the *Boar-Thiſtle*; very ſhort and prickly, with a large and long Root. To this they add a Root, call'd the *Cancer Root*, and a ſort of *Devils-Bit*: A Decoction of which three Roots is a Cure for the *King's-Evil*, tho' very far gone; a ſmall quantity being drank every day, and the bruiſed Roots apply'd to the Scrophulous Tumors.

But of theſe *American* Plants he promiſes a larger Crop.

The third Letter relates chiefly to the Birds of that Country; where, he ſays, they have many of the ſame *Species* with ours in *England*. He mentions very large wild *Turkies*, ſome weighing 50 or 60 pound, but the Fleſh is very tough and hard. He takes notice of a very large *Eagle* with a great Head, ſoaring very high, as all of that *Genus* do. As to the Itinerants; he takes notice of vaſt Flights of *Pigeons*, coming and departing at certain Seaſons: And as to this, he has a particular Fancy of their repairing to ſome undiſcovered Satellite, accompanying the Earth at a near diſtance.

The next Letter relates to *Antipathies*, and the Force of the *Imagination*. As to the firſt, he ſays, a Gentlewoman of his Neighbourhood ſwoons upon the ſeeing any one cut their Nails with a Knife; which if done with a Pair of Sciſſors has no Effect upon her. The Wife of a Perſon, vomiting

miting upon ſeeing her Husband take a Vomit; the Patient that took it being not mov'd, but forc'd to take a freſh Emetic.

Some Hiſtories are here related of the *Maculæ Maternæ*. One particularly, of a Woman longing for Peas, but refuſing to gratifie her deſire, for fear of a ſort of Bug, with which at that time moſt of their Peas were infeſted; this Woman's Child, when born, had an Excreſcence on the Forehead, reſembling one of thoſe Peas, with a black Speck, as the buggy Peas had, which after ſome time, dry'd away, and ſhell'd out they fancy'd, as the Bugs are obſerv'd to leave the Husk of the Pea.

This Letter concludes with an account of a Stone, generated under the Tongue, near the Root.

The fifth Letter gives an account of ſome *monſtrous Births*, but nothing very obſervable.

The ſixth Letter relates the Stories of ſome Perſons that had Informations of Medicines for the Diſtempers they lay under, in their Dreams; theſe Accounts relate little to Natural Philoſophy.

The next, and laſt to Dr. *Woodward*, relates the Cures of ſeveral Wounds in Perſons, which were judg'd mortal. In this little of Philoſophical Information.

The next Letter, being the firſt to Mr. *Waller*, is dated at *Boſton*, *Nov.* 24. 1712. In this the Writer obſerves, in the firſt place, That the *Indians* have no Diviſion of Time, except by *Sleeps*, *Moons* and *Winters*. Altho' the *Indians* have not divided or diſtinguiſhed the Stars into Conſtellations, yet it is obſervable that they call the Stars of *Urſa Major*, *Paukunawaw*, that is, *the Bear*; and this long before they had any Communication with *Europeans*. He ſays,

there is a Tradition among them, that in *November*, 1668. a Star appear'd below the Body of the Moon, within the Horns of it. In the next place he mentions the *Evening Glade*; first taken notice of by Dr. *Childrey*, to be constantly observ'd there in *February*, and a little before and after that Month; adding, that the Cause of that Appearance must be sought for above the Atmosphere. Then he gives a new Method of his own for finding the *Julian Period*, adding a Table for that purpose; which concludes this Letter.

The next relates the Appearances of several uncommon *Rainbows* and *Mock Suns*. On the 2d of *January*, in a clear Sky, but very cold; the Sun was from Ten o' Clock, for near Three Hours after, attended with four *Parhelia*, in the midst whereof were two Rainbows.

About six Weeks after this, in a Day much colder than used to be at that time of the Year, the Air a little hazy, a little after One o' Clock, for about half an hour, four Mock-Suns were seen.

He observes, that these Appearances with them are of great varieties, each usually differing in some respect from the other.

The next Letter dated, *Nov.* 26th. to the same Person, has the Relation of a strange Discovery of the Murder of a Person in *England*, to his Brother *Joseph Beacon*, at that time at *Boston*, in a Dream; the Person wounded appearing with the Wound on his Head; with the Attestations of several Persons, as to the Truth of it. The Information by the Dream was on the 2d of *May*, 1687, about Five o' Clock in the Morning; on the very same Day his Brother dy'd at *London*, of the Wounds he had receiv'd in *April* before: Of which Misfortune his Brother *Joseph Beacon* neither had, nor could have any notice, till the next Communication by Shipping, towards the latter end of *June* following; when he had a Letter of his Brother's Death, and the Cause of it,

it, agreeable to his Dream. but this not directly relating to Natural Philosophy, I omit the Particulars, tho' the Relation seems to be well attested.

The following Letter sent likewise to Mr. *Waller*, treats chiefly of the *Rattle-Snake*, hinting at the occasion of its Name, from the Rattles in its Tail, in which he says are sometimes twenty of those loose Rings, tho' he does not come in with the Opinion, that one is added every Year. Next follow these Observations. That the more Northerly they travel, these Snakes are less numerous, as well as less venomous; nor as it is said, are any seen to the North of *Merimack* River, which is about 40 miles North of *Boston*. Here he relates a Story, as he says, constantly affirmed by the *Indians*, that these Snakes frequently lie coiled at the Bottom of a great Tree, with their Eyes fixed on some Squirril above in the Tree; which tho' seeming by his cries and leaping about, to be in a Fright, yet at last runs down the Tree, and into the Jaws of the Devourer. Then he relates, that the Winter aboad of these Snakes is in the Clefts of inaccessible Rocks, from whence in the Spring, they come forth a Sunning themselves, at first very feeble, which is their chief time of destroying them. At this time the *Cystis* or Gall bladder in these Snakes is full of an acid azure coloured Juice, which they squeeze out into a Glass, but it is so Spirituous, that if the Glass be not immediately stopt, it will soon evaporate; this Liquor therefore they mix with a convenient quantity of powder'd Chalk or *Indian* Meal, and use it as a proper Medicine against the venemous Bite of this Snake; some have named it *Trochisci Connecticotiani*, from the Connecticot *Colony*. 'Tis observable when the Summer Heats come on, the Snakes have no longer this azure Liquor in their Gall-Bladders, in which there is only found a black thick Sediment, of no known use, at which time they think the forementioned spirituous Juice is carried to, and lodged in their Gums, and so conveyed or thrown by the hollow of the Teeth into the Wound, when they Bite, having received another Digestion,

ſtion, and higher Exaltation, by paſſing thro' ſeveral Strainers and Glands before it arrives to the Gums; as an inſtance of the virulence of this Liquor, he tells us, that a Traveller killing one of theſe Snakes, ſuffered the inraged dying Viper, to bite the end of his Switch, with the laſhes of which he had diſabled him; and a Fly by chance diſturbing one of his Temples as he rod on afterwards, he rub'd his Temple with the other end of the Switch, which as he relates it, immediately cauſed his whole Head to ſwell to a great exceſs, the Poiſon as he ſuppoſes permeating the whole length of the Switch. He adds another relation as to the penetrating quality of this Poiſon, a Perſon provoking a *Rattle-Snake* to bite the Edge of a broad Axe he had in his Hand; the colour, of the Steeled part bitten, was immediately changed, and at the firſt ſtroke he made with it in uſing his Ax, the ſo diſcoloured part broke out, leaving a gap in his Ax. But to return to the *Trochies* made of the Gall, he ſays it is a Cordial ſudorifick, and ſo good an Anodine, that he knows ſome who take 3 or 4 Grains of it to compoſe them to reſt after Travel. 'Tis good in all Fevers, eſpecially the Malignant. 'Tis an infallible Remedy for Obſtructions incident to Women upon catching cold in Childbed. Being taken in a convenient Quantity, 12 Hours before the Fit, it certainly cures a Quartan Ague. The Doſe is 14 Grains more or leſs according to the Circumſtances of the Patient in any Vehicle. The next Letter treats of the Effects of *Thunder* and *Lightning* very frequent with them, which from its frequent deſtroying Animals, without any viſible Hurt on the external Parts, he compares to the *Jewiſh* Puniſhment of pouring melted Lead down the Throats of the Condemn'd which they call'd *combuſtio animæ*. Tho' he likewiſe obſerves ſome have had their Hair ſinged with Marks on their Skin like thoſe made by ſmall Shot; ſome have had their Bones made limber like a Griſtle. The Captain of their Caſtle was found Dead in his Bed after a ſtorm of *Lightning* without any apparent Hurt. Here he relates a Paſſage of which an Account has ſometime

ſince

ſince been given in the *Philoſoph. Tranſact.* but is here confirm'd That *July* 24th, 1681. A Ship whereof one Mr. *Lad* was Maſter about 100 Leagues from *New England* in Lat. 38. met with a violent ſtorm of *Thunder*, which did much Damage to the Ship; at which time, a bituminous Matter fell on the Ship burning with that Violence, as not to be extinguiſh'd with Water till it was all burnt out, ſmelling ſtrongly like fired Gunpowder; and when they came to obſerve the Stars at Night, they found the Polarity or Direction of their Sea-Compaſſes to be changed; the North-Point being turned to the South, and ſo continued to do for the reſt of the Voyage for a 1000 Leagues. He adds farther, that one of theſe Compaſſes continues to do ſo ſtill, and was upon his Table before him at the time of his writing this preſent Letter. He makes a *Quere* whether this may be accounted for by Mr. *Boyl*'s Experiment of heating a Loadſtone red-hot, and by altering the Poſition in which it was cool'd, he could change its Polarity. Which ſome may ſay, might happen to this Needle, ſuppoſing it was made red-hot, and turned upon its Center in the Storm.

From *Thunder* he proceeds to *Earth-Quakes*, which tho' he ſays they have not done with them the Miſchiefs frequent in *Sicily*, *Italy*, &c. yet they have had ſeveral very ſenſible and affrightning. In the Year 1663, they had 6 or 7 violent ſhakes in the ſpace of 3 Days: a Town lying on the River *Connecticut*, has had ſcores of them in a Year, for many Years together. The *Indians* affirm, that ſeveral Rivers have not only been ſtopt in their courſe and diverted, but ſome wholly ſwallowed up by *Earth-Quakes*. He farther adds, a paſſage out of *Joſſelin* who dwelt in the Neighbourhood, that in the Year 1670, at a place called *Kenebunch*, near the ſide of the River, a piece of clay Ground was thrown up over the tops of high Oaks, growing between it and the River, which it thereby ſtopt, and left a Hole in the Place from which it was thrown forty Yards ſquare, *&c.* Next as to Storms of Hail, he relates that they have had very extraordinary ones, inſomuch that they have lain 3 or 4 Foot thick on the Ground,

ſome

ſome as big as Hens Eggs, others five times as big. He mentions, as an Accident ſometimes happening to them in the Winter, that it has rain'd plentifully, and at Night frozen ſo extremely, that the weight of the Icicles has broken the Limbs of the Trees, and not unfrequently ſplit their Trunks. Tho' they have not thoſe *Hurricanes* to which the *Caribbe* Iſlands are ſubject; yet they have had *Whirlwinds*, or Guſts, drive along a particular narrow Tract, for divers Miles together, with a violence not to be oppoſed by any thing on Earth; that if their Towns had ſtood in the way, they muſt undoubtedly have been deſtroy'd. Of theſe, he ſays, a thick dark, ſmall Cloud has aroſe, with a Pillar of Light in it, of about 8 or 10 Foot Diameter, and paſt along the Ground in a Track not wider than a Street, horribly tearing up Trees by the Roots, blowing them up in the Air like Feathers, and throwing up Stones of a great weight to a conſiderable height in the Air, throwing down all in its paſſage; the Noiſe this Cloud made was ſo great all the while, that the Noiſe of the Miſchiefs done by it, was thereby quite drown'd.

The remainder of this Letter relates to ſome very ancient Remains, at a Place call'd *Ammuskeag*, a little above the hideous Falls of *Merimack* River. There is a huge Rock in the midſt of the Stream, on the Top of which are a great number of Pits, made exactly round, like Barrels or Hogſheads of different Capacities, ſome ſo large as to hold ſeveral Tuns; the Natives know nothing of the Time they were made; but the Neighbouring *Indians* have been wont to hide their Proviſions in them, in their Wars with the *Maqua*'s; affirming, God had cut them out for that uſe for them. They ſeem plainly to be artificial.

In the next place, he gives an account of a ſtrange Inſcription found on a Rock, in theſe Words. *At* Taunton, *by the ſide of a Tiding River, part in part out of the River, there is a large Rock, on the perpendicular ſide of which, next to the Stream, are 7 or 8 Lines, about 7 or 8 Foot long, and about a*

Foot,

Foot wide, each of them ingraven with unaccountable Characters, not like any known Character. He has not yet been able to procure the whole, which he hopes to be Maſter of before long, and has herewith ſent a Copy of two of them, promiſing the reſt; they are as is repreſented, *Fig.* 8.

The laſt Letter of this Collection, dated *Nov.* 29. 1712. gives a Calculation of the poſſible Increaſe of the Deſcendants of *Adam*; and from this Introduction proceeds to the Account of ſome long-liv'd Perſons there, as likewiſe of their Fruitfulneſs. He ſays, 'tis no rare thing with them to have an aged Gentlewoman ſee many more than 100 of her Offspring. He mentions one Woman that had 23 Children, of which 19 liv'd to Man's Eſtate. Another that had 27; another 26, of which 21 were Sons, one whereof was Sir *William Phipps*; another 39 Children. Here he gives ſeveral Inſtances of Perſons living, with them, to above 100 Years of Age. One *Clement Weaver* lived to 110, his Wife being upwards of 100. This Man, to the laſt Year, could carry a Buſhel of Wheat to the Mill, above 2 Miles. He relates the Caſe of an old Man, above 100, that loſt the memory of ſeveral of the latter Years of his Life, but very well retain'd the Remembrance of what paſt in his younger days. I do not find, by any of theſe Relations, that the Perſons obſerv'd any Regularity, or Method, in their manner of Diet, Exerciſe, or the like.

V. *An Account, or History, of the Procuring the* SMALL POX *by Incision, or Inoculation; as it has for some time been practised at* Constantinople.

Being the Extract of a Letter from Emanuel Timonius, Oxon. & Patav. M. D. S. R. S. *dated at* Constantinople, December, 1713.

Communicated to the Royal Society *by* John Woodward, M. D. Profes. Med. Gresh. *and* S. R. S.

THE Writer of this ingenious Discourse observes, in the first place, that the *Circassians, Georgians,* and other *Asiaticks,* have introduc'd this Practice of procuring the *Small-Pox* by a sort of Inoculation, for about the space of forty Years, among the *Turks* and others at *Constantinople.*

That altho' at first the more prudent were very cautious in the use of this Practice; yet the happy Success it has been found to have in thousands of Subjects for these eight Years past, has now put it out of all suspicion and doubt; since the Operation having been perform'd on Persons of all Ages, Sexes, and different Temperaments, and even in the worst Constitution of the Air, yet none have been found to die of the *Small-Pox*; when at the same time it was very mortal when it seized the Patient the common way, of which half the affected dy'd. This he attests upon his own Observation.

Next he observes, they that have this Inoculation practised upon them, are subject to very slight Symptoms, some being scarce sensible they are ill or sick; and what is

is valued by the Fair, it never leaves any Scars or Pits in the Face.

The Method of the Operation is thus. Choice being made of a proper Contagion, the Matter of the Pustules is to be communicated to the Person proposed to take the Infection; whence it has, metaphorically, the name of Insition or Inoculation. For this purpose they make choice of some Boy, or young Lad, of a sound healthy Temperament, that is seized with the common *Small-Pox* (of the distinct, not Flux sort) on the twelfth or thirteenth day from the beginning of his Sickness; they with a Needle prick the Tubercles (chiefly those on the Shins and Hams) and press out the Matter coming from them into some convenient Vessel of Glass, or the like, to receive it; it is convenient to wash and clean the Vessel first with warm Water: A convenient quantity of this Matter being thus collected, is to be stop'd close, and kept warm in the Bosom of the Person that carries it, and, as soon as may be, brought to the place of the expecting future Patient.

The Patient therefore being in a warm Chamber, the Operator is to make several little Wounds with a Needle, in one, two or more places of the Skin, till some drops of Blood follow, and immediately drop out some drops of the Matter in the Glass, and mix it well with the Blood issuing out; one drop of the Matter is sufficient for each place prick'd. These Punctures are made indifferently in any of the fleshy Parts, but succeed best in the Muscles of the Arm or *Radius*. The Needle is to be a three-edg'd Surgeon's Needle; it may likewise be perform'd with a Lancet: The custom is to run the Needle transverse, and rip up the Skin a little, that there may be a convenient dividing of the Part, and the mixing of the Matter with the Blood more easily perform'd; which is done, either with a blunt Stile, or an Ear-picker: The Wound is cover'd with half a Walnut-shell, or the like Concave Vessel, and bound over, that the Matter be not rub'd off by the Garments; which is all

removed in a few Hou s. The Patient is to take care of his Diet. In this place the Cuſtom is to abſtain wholly from Fleſh and Broath for 20 or 25 days.

This Operation is perform'd, either in the beginning of the Winter, or in the Spring.

Some, for caution, order the Matter to be brought from the Sick by a third Perſon, leſt any Infection ſhould be convey'd by the Cloaths of the Operator; but this is not material.

As to the Proceſs of this Matter, in reſpect of the *Idioſyncraſie*; the *Small-Pox* begins to appear ſooner in ſome than in others, in ſome with greater, in others with leſſer Symptoms; but with happy Succeſs in all. In this Place the Efflorescence commonly begins at the end of the ſeventh day. which ſeems to favour the Doctrin of *Criſes*.

It was obſerv'd, in a Year when the common *Small-Pox* was very mortal, that thoſe by Inciſion were alſo attended with greater Symptoms. Of 50 Perſons, who had the Inciſion made upon them almoſt in the ſame day, four were found in whom the Eruption was too ſudden, the Tubercles more, and Symptoms worſe. There was ſome ſuſpicion, that theſe four had caught the common *Small Pox* before the Inciſion was made. It is enough for our preſent purpoſe, that there was not one but recovered after the Inciſion: In thoſe four the *Small-Pox* came near the confluent ſort. At other times the inoculated are diſtinct, few and ſcatter'd; commonly 10 or 20 break out; here and there one has but 2 or 3, few have 100: There are ſome in whom no Puſtule riſes, but in the Places where the Inciſion was made, which ſwell up into purulent Tubercles; yet theſe have never had the *Small-Pox* afterwards in their whole Lives; tho' they have cohabited with Perſons having it.

It is to be noted, that a no ſmall quantity of Matter runs for ſeveral days, from the place of the Inciſion.

The Pocks ariſing from this Operation are dry'd up in a ſhort time, and fall off, partly in thin Skins, and partly con-

contrary to the common ſort, vaniſh by an inſenſible waſting.

The Matter is hardly a thick *Pus*, as in the common, but a thinner kind of *Sanies*; whence they rarely pit, except at the place of the Inciſion, where the Cicatrices left are not to be worn out by time, and whoſe Matter comes near the nature of *Pus*.

If an Apoſteme breaks out in any (which Infants are moſt ſubject to) yet there is nothing to be fear'd, for it is ſafely heal'd by Suppuration. If any other Symptom happens, 'tis eaſily cur'd by the common Remedies.

Obſerve, they ſcarce ever make uſe of the Matter of the Inciſious Pox, for a new Inciſion. If this Inoculation be made on Perſons who have before had the *Small-Pox*, they find no alteration, and the places prick'd preſently dry up; except in an ill Habit of Body, where poſſibly a ſlight Inflammation and Exulceration may happen for a few days.

To this time, he ſays, I have known but one Boy, on whom the Operation was perform'd, and yet he had not the *Small-Pox*, but without any miſchief; and ſome Months after catching the common ſort, he did very well. It is to be obſerv'd, that the places of the Inciſion did not ſwell. I ſuſpect this Child prevented the inſertion of the Matter, for he ſtrugled very much under the Operation, and there wanted help to hold him ſtill. The Matter to be inſerted will keep in the Glaſs very well for 12 Hours. He goes on.

I have never obſerv'd any miſchievous Accident from this Inciſion hitherto; and altho' ſuch Reports have been ſometimes ſpread among the Vulgar, yet having gone on purpoſe to the Houſes whence ſuch Rumors have ariſen, I have found the whole to be abſolutely falſe.

It is now eight Years ſince I have been an Eye-witneſs of theſe Operations; and to give a greater Proof of the Sedulity I have uſed in this Diſquiſition, I ſhall relate two Hiſtories.

There

There was, in a certain Family, a Boy of 3 Years old, afflicted with the *Falling-Sickness*, the *King's-Evil*, an *Hereditary Pox*, and a long *Marasmus*. The Parents were desirous to have the Incision made upon him; the *Small-Pox* were thrown off with ease; about the 40th day he dy'd of his *Marasme*. In another Family, a Girl of 3 Years old, troubled with the like Fits, strumous, attended with an *Hereditary Lues*, and labouring under a colliquative Loosness for three Months. The Operation was perform'd on this Child; she came off very well of the *Small-Pox*, which was all over the 15th day; on the 32d she dy'd of her Loosness, which had never left her the whole time

But it is true, I never maintain'd the Inoculation as a *Panacæa*, or Cure for all Diseases; nor do I think it proper to be attempted on Persons like to die. Some more quick-sighted, imagin'd these two Children were, as useless Shades, sent to *Charon* by any means that could be made use of. If I could have collected any more concerning this Matter, I should have imparted it candidly.

The rest of Dr. *Timone*'s Letter contains his Reasons for this Method of Practice; which being the *Ætiological* Part, is publish'd in his own Words, as follows.

ÆTIOLOGIA.

Contagium Variolarum per puris infusionem propagari haud equidem mirabitur qui Æsculapii templum vel à primo limine salutavit, & fermentationis doctrinam subodoratus est: Nec obscurior est insitionis modus, quàm panificium, aut ars cerevisiaria, in quibus ex admixto fermento massæ fermentandæ turgescunt; conciliato nimirum motu intestino minimarum particularum principiis active pollentium. Si quis quærit interim cur variolæ periculosæ alioquin & persæpe lethales, ex insitione sine ullo periculo excludantur. Dico: Variolæ communes vel concurrente pravâ aliquâ speciali aeris diathesi suscitantur, vel ab effluviis à varioloso cor-

corpore emanantibus per contagium propagantur. Primus casus in paucis individuis accidit, & concurrente quidem vel insigni cacochymia, vel saltem varioloſi seminii in talibus individuis latitantis acerrimâ exaltatione: Secundus casus communissimus est. In primo casu miasma malignum aereum, in secundo virulenta contagii corpuscula indolis (probabiliter) salino-sulphureæ sed specificam fracedinem seu ranciditatem nacta statim ac per respirationem hauriuntur spiritus ipsos, & labe quidem teterrima inficiunt; subsequenter autem massam sanguineam & lympham vitiari manifestum est: Spiritus statim infici rationi consentaneum est, tum quia in fontes spirituum, cor scilicet & cerebrum, statim ingressum habent virulentem aporriæ, tum ratione analogismi inter miasmata & effluvia ista ipsosque spiritus, cum utraque spirituoso-aereæ texturæ sint. Deducitur etiam cita & prava spirituum infectio à tot tantisque norvosi systematis symptomatibusque, quæ malas plerumque comitantur variolas, & præcipuè à convulsionibus epilepticis quæ infantibus accidunt ipso momento, quo varioloso inficiuntur contagio multo antequam febris illos corripiat. Massam autem sanguineam inquinari præter febrem purulenta tuberculorum exclusio testatur. Lymphæ verò vitiatæ fidem faciunt glandularum in faucibus tumor, screatus, & enormis multoties ptyalismus. Inter hæc circularis etiam sequitur noxa. Sed præcipuè sanguinis particulæ ab indebita spirituum irradiatione in plures ataxias & anomalias perducuntur. Duobus tamen potissimum modis in variolis communibus mortem contingere observavi.

Primus est quando paucis erumpentibus variolis, & tardè ad maturitatem procedentibus, mala alia oboriuntur symptomata; secundus quando nimia tuberculorum copia cadaverosam putredinem inducit. In primo casu malignæ vulgo dicuntur variolæ: causa autem est vel nimia fusio & dissolutio massæ sanguineæ, vel ejusdem coagulatio & grumescentia. Si enim impetus spirituum explosivus justo plus augeatur, particulæ massæ sanguineæ nimium ad invicem atteruntur, comminuuntur, & tenuissimas nanciscuntur acrotitas: sanguis in hoc statu sollertis naturæ mechanismum eludit, cumque nil fæculentioris in glandulis secretoriis cribrisque deponat, œconomiæ animalis functionibus requisitas filtrationes & transcolationes

ones celebrari haud patitur: improportionata etenim est figura particularum liquidi ad configurationem pororum in tubulis & colatoriis ratione subtilitatis nimiæ: filtratione enim defæcarentur particulæ sanguinis si naturalem servarent schematismum & molem: hinc dicitur pepsim fieri per incrassationem. Præterhoc celeritas ipsa transitûs sanguinis in causa est ut nihil deponatur in colatoriis. Torrens ubi nimio impetu & præcipiti cursu fertur aquas turbidas defæcari haud patitur; quia vis centripeta gravitatem admixti terrei sequens superatur à fortiorum pulsoria virtute æquorum globulorum rapide ruentium: virtus enim fortis, verbi gratia, ut unum non poterit lineam perpendicularem describere ubi virtus fortis ut duo ad lineam horizontalem protrudit: sic etiam haud pluit vento flante intensissimo; eadem geometrica proportione (probabiliter loquendo) sanguinis particulæ aucto ab effrænibus spiritibus motu, tubulos colatorios præterfluunt nullâ factâ fæcum depositione. Hæc probabilia fiunt à summa pulsûs celeritate, febre intensissima, sudore nullo, & urina cruda. E contra quandoque contingit ut ab acutis, & scindentibus deleterii fermenti particulis frangatur, corrodatur, vel saltem relaxetur elater spirituum: elanguescente igitur spirituum motu torpidiores etiam hebetioresque fiunt sanguinis lymphæque particulæ: igitur dum in labyrinthæis tubulorum anfractibus moram indebitam contrahunt alias turmatim invicem complicari, alias autem, congestione factâ, super alias incidere, & diverso ad invicem superficierum suarum contactu à naturali configuratione descifcere, & novas induere angulorum dimensiones necesse est. Sic igitur diversa ab illa, quam superius narravimus, figurarum ad tubulorum meatus improportione, pari tamen calamitatis eventu dædaleæ naturæ machinationes irritas fieri contingit. Hæc probabilia fiunt à pulsu tardo & raro, ac febris carentia quandoque in summa malignitate observatis, paucis & tardè erumpentibus voriolarum pustulis. Ulterius à trepidatoria, seu subsultoria ac tumultuosa furentium spirituum irradiatione inæqualis eodem tempore in diversis partibus massæ sanguineæ, & arteriarum etiam venarumque contingere potest impulsus. Sive igitur fibrillæ aliquæ (ut quidem volunt) reperiantur in sanguine, seu chili nondum bene assimilati sint portiones usibus peculiaribus

dica-

dicatæ; probabiliter istarum motum turbari contingit: has enim in circulatorio motu secundum longitudinem suam naturaliter moveri necesse est: ab inæquali autem pressione dicta rectilineam figuram perdere, & in spiras ac semicirculos crispari coguntur: has igitur sic contortas transversaliter postmodum in circulatione raptari, ad invicem implicatas convolvi, &, ramosis schematibus obortis, racematim adeo conglobari necesse est, ut in majusculos tandem grumos coalescant, sive demum fibrillæ illæ non dentur, certè cujuscumque figuræ sint massæ sanguineæ particulæ, illas à naturali descisscere situatione ex hac motûs inæqualitate contingit: Confusæ igitur particulæ istæ & ad invicem implicatæ statim vehiculi sui, seri scilicet globulis per expressionem à suo contubernio explosis, majorem, ratione molis auctæ gravitatem nanciscuntur, ideoque impulsivæ circulatoriæ facultatis vim superant: Has igitur hîc illic restitare ac stagnare necesse est, prout in hoc vel illo loco prima mutua cohæsio forte contigerit: Hinc livida stigmata, & simul (quod sæpe observavi in variolis cum petechiis erumpentibus) frequens sequitur mictus, quo limpidissimum serum in magna copia excluditur. En fusio, & coagulatio. Hinc mirum non est cur moriantur aliqui in variolis cum petechiis, convulsionibus syncope, vigiliis nimiis, emorrhegiis, delirio, vomitibus, enormibus, dysenteriis, &c. quamvis haud multa pustularum putrilagine perfundantur: In stygium enim veluti characterismum variolarum fermentum multoties evehitur, ita ut quamvis haud magnam crassi puris copiam progignere aptum sit, spiritibus tamen, liquidis & solidis suprarecensita mala modis vel explicatis vel aliis consimilibus communicare possit, sicque mortem inferre; & hoc ante undecimum plerumque. Veniamus nunc ad secundum modum. Diversa enim aliquando contingit pernicies & longè alterius generis tragœdia: quamvis enim absint illa symptomata, nimia tamen puris, materiæ scilicet cadaverisatæ, copia corpus obruitur. Pus autem generari probabile est quando sulphureis oleosisque massæ sanguineæ particulis in fracedine & fusione constitutis acido-salinarum particularum coaffusio contingit. Huic asserto facem accendunt innumera chymica experimenta quibus manifestè edocemur solutionibus pinguium sulphureorum per alkalia factis acido quoli-

 bet

bet coaffuſo ſtatim maſſam albicantis coloris emergere. Multoties igitur miaſma ſeu fermentum variolarum per reſpirationem hauſtum ratione indolis propriæ acerrimæ & fortaſſis ſepticæ tales in maſſam ſanguineam particularum acido-ſalinarum & oleoſo-ſulphurearum producere poteſt combinationes, ut non ſeminia ſolum variolarum, quæ omnibus individuis (mole tamen minima) à nativitate indita ſunt, agitentur, actuentur, & in purulentam abeant putrilaginem, ſed maſſa ipſa ſanguinea tota acorem contrahat, & motu quodam corruptorio putreſcat & cadaveriſetur. Sic igitur, incendio veluti cohorto, ulterius furere fermenteſcentes particulas contingit, quam varioloſis ſeminiis per deſpumationem eliminandis opus ſit: hic motus non eſt depuratorius heterogeneis ſecernendis inſerviens, ſed deſtructivus & corruptorius, fermento nempe maſſam totam ſuperante & invertente; fracidis ſcilicet rebellibuſque particulis victoria potitis, & omnes alias in ſua caſtra migrare cogentibus. Hoc manifeſtè obſervamus in variis potulentis, in quibus fermentatione aliquando excitatâ, motus ſuccedit corruptivus liquores totaliter vitians: hinc videmus aliquos quamvis ſuprarecenſitis ſymptomatibus immunes, immenſo tamen, ut ita dicam, putredinis oceano ſuffocatos: Et hoc periculum uſque ad vig ſimum ſecundum protrahitur. Ultimo loco conſiderandum ſolida etiam & nobiliores partes in hiſce caſibus peſſimè afficiri, & in ſpaſmos inordinatos fieri: variis horum diſtortionibus tubulorum meatus vitiari, at functionum munera depravari neceſſe eſt: Ecce igitur continentia, contenta, & impetum facientia, quorum triumviratu humani corporis reſpublica regitur, una eademque ruina ut plurimum involuta: mirabiturne quiſpiam malorum inde Iliadem in hominis perniciem pullulare? Obſervandum ulterius multis, qui peſte laboraverint, communibus variolis etiam poſt annum correptis bubones coſdem intumuiſſe, qui antea in peſte eruperant: nonne hoc etiam ſummam malignitatem teſtatur. Inſitionem modo ad rationis trutinam revocemus. At hercule longe aliter in hoc contagionis modo rem procedere quis eſt qui non fateatur? Primum enim Spiritus nullatenus infici manifeſtum eſt: deinde non lymphæ, non ſanguini labes illa teterrima inuritur, non ſolidis vitium aliquod communicatur. Hinc ſymptomata omnia levia

via, nulla pessima, nulli infantibus epileptici insultus. Contagionis enim hujusce fermentum non spiritale, non aereum & acutum est, sed humorale, iners, ac pigrum: venena autem quo subtiliora eo pejora: Ratione igitur improportionis nulla inter fermentum hoc & spiritus esse poterit lucta. Pus equidem variolarum in ipsa substantia sanguini immediatè infusum statim in largum veluti pelagus exceptum diluitur, involvitur, absorbetur, obtunditur: sic illud mitescit, sic in mansuetiorem indolem cicuratur. Contagiosæ istæ particulæ sanguinem ingressæ statim sibi congeneres varioloſi seminii particulas sanguini à nativitate inditas inveniunt; iis igitur confermentescunt, sed invicem combinatæ ac complexæ haud amplius sui juris sunt ut ulteriores excitent turbas, regiam vitæ petant, spirituum thesauros diripiant; nam mutuis compedibus constrictæ fixantur, præcipitantur, crassioresque & hebetiores fiunt, quam antea fuerint. Statim igitur volubilioribus aquearum particularum globulis tamquam aptis vehiculis superincumbentes, sanguinis motu à centro ad peripheriam tendente, secundo veluti amne, ad ambitum corporis protruduntur, eliminantur. Nonne manifestè videmus haud pus generari in insititiis variolis, sed saniosam, dilutiorem videlicet aqueamque magis materiam? Nonne ex hoc phænomeno palam est acido-salinas fermenti contagiosi particulas haud oleosas passim sanguinis particulas in cadaverosam purulentiam pervertere, sed blandioribus potius lævioribusque aqueis particulis easdem dilutas & saturatas foras asportari? Ex negatione fovearum & cicatricum nonne manifestum est acres, aculeatas, pungentes & corrosivas salini fermenti particulas à balsamicis statim sanguinis globulis obtundi, spiculis suis orbari, & hebetiori figura modificatas, vi veluti mochlica, extra propelli? Integra interim servatur massæ sanguineæ textura, inviolata consistentia. Nullam hîc vides fusionem, nullam grumescentiam, nullum corruptorium aut destructivum motum. In insitione enim tantum solummodo sanguis fermentescit, quantum impuro à puri consortio separando, ac per despumationem extrudendo satis est. In hoc fermentationis motu solum per undulationem quandam leviter aliquando afficiuntur spiritus, lympha, & solidæ partes, & siquæ ad ista

contagii particulæ perveniunt, certè (quod insitionis adumbrat metaphora) non nisi sylvestri acrimonia privatæ, ac veluti dulcificatæ pervenire possunt. Hæc tenuitatis meæ satis conscius haud præfricta fronte obtrudo: non me latet longè meliora emanatura ab illis, queis meliore luto finxit præcordia Titan: In historica tamen insitionis hujusce narratione aliquatenus me bene meritum spero.

Constantinopoli, Anno 1713. *Mense Decembre.*

Emanuel Timonius, Constantinopolitanus. In Universitatibus Oxoniensi & Patavina Philosophiæ & Medicinæ Doctor.

VI. *Theoremata quædam infinitam Materiæ Divisibilitatem spectantia, quæ ejusdem raritatem & tenuem compositionem demonstrant, quorum ope plurimæ in Physica tolluntur difficultates.*

A Johanne Keill, *M. D. Profes. Astron. Savil. Oxon. & S. R. S.*

JAmdudum sequentia *Theoremata* in lucem emisi, omissis quidem *Demonstrationibus*, eo quod arbitrabar eas, utpote non admodum involutas, à quovis in *Geometriâ*, vel etiam in *Arithmeticâ* mediocriter versato, facile elici potuisse; Sed quoniam video, D. *Christianum Wolfium* in *Academiâ Fredricianâ* Mathematum Professorem, reliquosque Actorum *Lipsiensium* Authores, hæc *Theoremata* non rectè intellexisse, cumque eorum in *Philosophiâ* explicandâ usus non sit exiguus; libet ea nunc denuo, adjectis *Demonstrationibus*, *Reipublicæ Philosophicæ* impertiri.

Suppono *Materiam omnem divisibilem esse in infinitum, eamq; posse formam quamcunque seu figuram induere, & ad quamcunque tenuitatem, seu crassitiem quamcunque exiguam reduci.*

Lemma

Lemma.

Datâ quâvis materiæ quantitate, ex eâ, vel ex quâvis ejus parte, formari poteſt ſphera concava, cujus ſemidiameter ſit datæ rectæ æqualis.

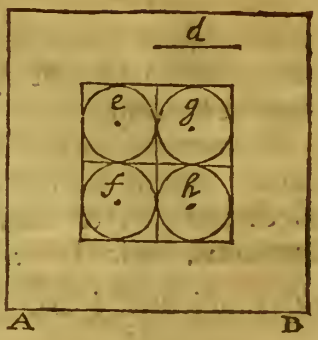
Sit materiæ particula a^3 & data recta ſit b. Ratio peripheriæ circuli ad Radium ſit p ad r. dicatur ſemidiameter concavitatis x, & craſſities, pelliculæ concavitatem ſphæræ ambientis, erit $b-x$ & Cylindrus ſphæræ circumſcriptus cujus radius eſt b erit $\frac{p \times b^3}{r}$, unde ſphæra cylindro inſcripta erit $\frac{2 \times p b^3}{3 \quad r}$, Eâdem ratione ſphæra cujus radius eſt x erit $\frac{2 \times p x^3}{3 \quad r}$ quarum differentia $\frac{2p}{3 \times r} \times b^3 - x^3$ ponenda eſt ſphæricæ lamellæ æqualis, ſeu materiæ particulæ datæ; hoc eſt erit $\frac{2p}{3r} \times b^3 - x^3 = a^3$ ſeu $b^3 - x^3 = \frac{3ra^3}{2p}$ unde $x^3 = b^3 - \frac{3ra^3}{2p}$ & $x = \sqrt[3]{b^3 - \frac{3ra^3}{2p}}$ adeoque craſſities lamellæ ſphæricæ ſeu $b-x$ erit $= b - \sqrt[3]{b^3 - \frac{3ra^3}{2p}}$.

Eâdem ratione fieri poſſunt ex data materiæ quantitate Cubi *concavi,* Cylindri *concavi, vel corpora etiam alterius cujuſvis figuræ concavæ, quorum latera ſunt datæ rectæ æqualia.*

Theorema Primum.

Datâ quavis materiæ quantitate quantumvis exiguâ, & dato ſpatio quovis finito utcunque amplo; quod v. gr. *ſit cubus, qui ſphæram Saturni circumſcriberet: Poſſibile eſt ut materia iſtius Arenulæ per totum illud ſpatium diffundatur, atque*

atque ipſum ita adimpleat, ut nullus ſit in eo porus cujus diameter datam ſuperet lineam.



Sit datum ſpatium Cubus cujus latus ſit recta *A B*, diametro ſcil. orbitæ Saturni æqualis, deturque materiæ particula cujus quantitas ſit b^3, & data recta (quâ pororum diametri non majores eſſe debent) ſit *d* Dividi concipiatur recta *A B* in partes æquales rectæ *d*, quarum numerus finitus erit, cum nec recta *A B* ponitur infinitè magna, nec recta *d* infinitè parva: ſit numerus ille *n*, hoc eſt ſit $nd = AB$, adeoque erit n^3d^3 æqualis cubo rectæ *AB*. Concipiatur item ſpatium datum dividi in cubos quorum ſingulorum latera ſunt æqualia rectæ *d*, eritque cuborum numerus n^3, & hi cubi per ſpatia *e f g h* in figura repreſententur. Dividi porro ſupponatur particula b^3 in partes quarum numerus ſit n^3, & in unoquoque ſpatio cubico ponatur una harum particularum, & hac ratione materia b^3 per omne illud ſpatium diffundetur. Poteſt præterea unaquæque ipſius b^3 particula in ſua quaſi cellâ locata in ſphæram concavam formari, cujus diameter ſit æqualis datæ rectæ *d*; unde fiet, ut ſphæra quælibet proximam quamque tangat, & data materiæ particula utcunque exigua b^3 ſpatium datum ita adimplebit, ut nullus fiet in eo porus cujus diameter datam rectam *d* ſuperat. *Q. E. D.*

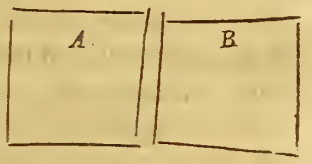
Cor. Hinc dari poteſt corpus, cujus materia, ſi in ſpatium abſolutè plenum redigatur, ſpatium illud fieri poteſt prioris magnitudinis pars quælibet data.

Theorema Secundum.

Poſſunt eſſe duo corpora mole æqualia, quorum materiæ quantitates ſint utcunque inæquales, & datam quamvis ad ſe invicem

invicem obtineant rationem, pororum tamen summæ, seu spatia vacua inter corpora, ad rationem æqualitatis ferè accedant. Vel in stilo Cartesiano: *Spatium omne, quod à materiâ subtili intra unius corporis poros occupatur, posset esse fere æquale spatio quod à simili materiâ intra alterum corpus tenetur. Licet materia propria unius corporis decies millies vel centies millies superat materiam propriam alterius Corporis, & Corpora sint mole æqualia.*

Ex. gr. Sit Digitus cubicus *Auri* & Digitus cubicus *Aeris* vulgaris non condensati. Certum est quantitatem materiæ in *Auro* vicies millies circiter superare materiam aëris, attamen fieri potest, ut spatia in *auro* vel absolutè vacua, vel materiâ subtili repleta, sint ferè æqualia spatiis in aëre, vel vacuis, vel materiâ tantum subtili repletis.



Sint *A* & *B* corpora duo, magnitudine æqualia: utrumque *v. gr.* sit cubus uniûs digiti Et corpus *A* decies millies sit gravius corpore *B*, unde & corpus *A* quantitate materiæ decies millies superabit corpus *B*. Ponamus jam materiæ quantitatem in *A* redigi in spatium absolutè plenum, quod sit digiti cubici pars centies millesima; (liquet enim ex corolli præcedentis Theorematis id fieri posse). Unde cum materia in *A* decies millies superat materiam in *B*, materia illa in *B*, si in spatium absolutè plenum compingatur, occupabit tantum digiti cubici partem $\frac{1}{1000000000}$ seu millies decies centies millesimam; Adeoque partes reliquæ 999999999 vel erunt absolutè vacuæ, vel materiâ aliqua subtili, qualis supponitur Cartesiana, tantum repletæ. Porro, cum materiæ quantitas in *A* impleat tantum digiti partem centies millesimam, erunt in corpore *A* partes 99999 centies millesimæ, vel vacuæ, vel materia subtili repletæ, hoc est reducendo fractionem ad denominatorem prioris fractionis, erunt in *A* partes vacuæ 999990000 millies decies centies millesimæ. Adeoque vacuitates in *A* erunt ad vacuitates in *B*, ut numerus 999990000 ad numerum 999999999, qui numeri sunt ad se invicem ferè in

ratione

ratione æqualitatis, nam eorum differentia, parvam admodum ad ipſos numeros obtinet rationem. Adeoque ſpatia vacua, vel materiâ ſubtili tantum repleta, quæ ſunt in duobus corporibus *A* & *B*, eandem cum ipſis numeris, ad ſe invicem rationem obtinentes, ſunt etiam ferè in ratione æqualitatis. *Q. E. D.*

Corpora autem omnia eſſe rariſſima, hoc eſt pro mole ſua parvam admodum continere materiæ quantitatem, ex diaphanorum proprietatibus certiſſimè conſtat, nam *Radii Lucis* intra vitrum, vel aquam non ſecus ac in aere per rectas lineas diffunduntur; quæcunque luci expoſita ſit corporis Diaphani facies; Adeoque à minimâ quâvis aſſignabili Diaphani parte, ad aliam quamvis ejuſdem partem, ſemper extenditur in his corporibus porus rectilineus, per quem tranſiverit lux, atque hoc fieri non poteſt niſi Materia Diaphani ad ejus molem, parvam admodum obtineat rationem, nec fortaſſe materiæ quantitas in vitro, ad ejus magnitudinem majorem habet rationem, quam magnitudo unius Arenulæ ad totam Terreni orbis molem: Hoc autem non eſſe impoſſibile, ſuperius oſtenſum eſt. Unde cum Aurum non ſit octuplo denſius Vitro; ejus quoque materia, ad propriam molem, exiguam admodum obtinebit rationem.

Hinc ratio reddi poteſt, cur effluvia magnetica eadem ferè facilitate denſum Aurum & tenuem aerem pervadunt.

Ex his etiam propoſitionibus, & ex maximâ lucis celeritate, ratio reddi poteſt, cur *Lucis* radii ex pluribus objectis prodeuntes & per tenue foramen tranſmiſſi, ſe mutuo non impediunt, ſed per eandem rectam in motu ſuo perſeverant: Quod per motum ſeu impulſum fluidi, plenum efficientis, vix explicari poteſt; *corpus enim omne à pluribus potentiis, ſecundum diverſas directiones, ſimul impulſum, unam tantum & determinatam directionem accipit ex omnibus compoſitam.*

London, Printed for W. Innys, at the *Princes'-Arms* in St. *Paul's Church-yard*. 1714.

Fig. I.

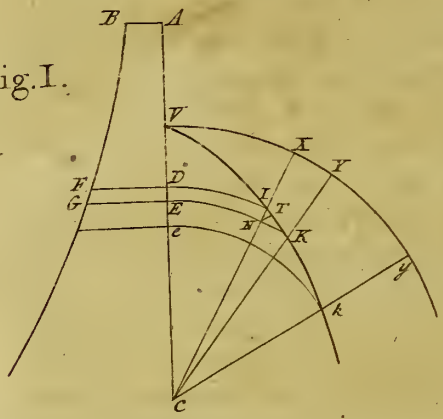


Fig. II.

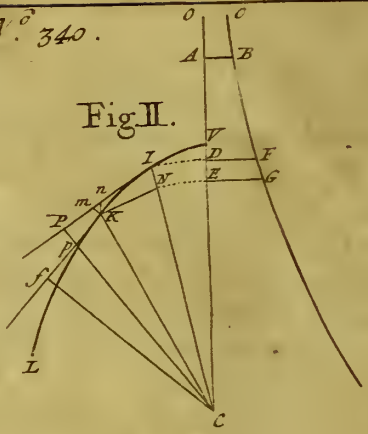


Fig. V.

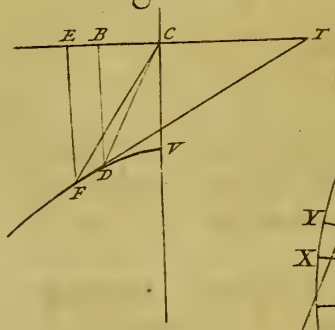


Fig. VI

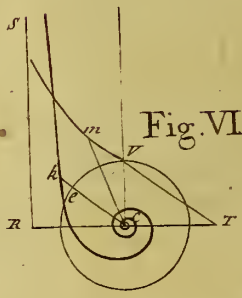


Fig. III

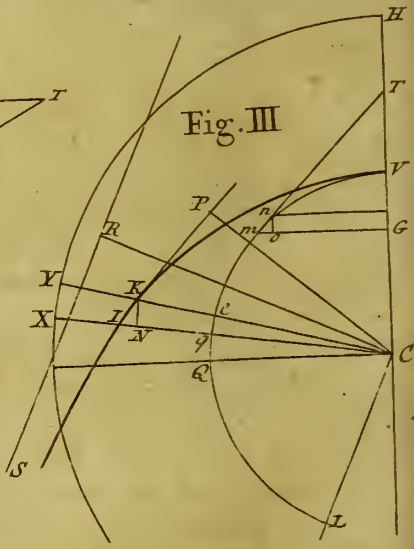


Fig. VII.

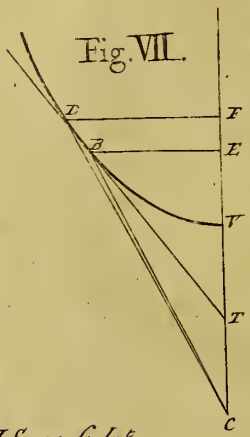


Fig. VIII.

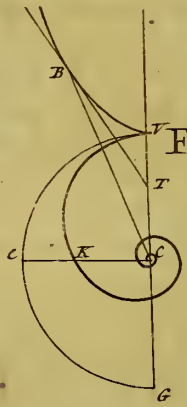
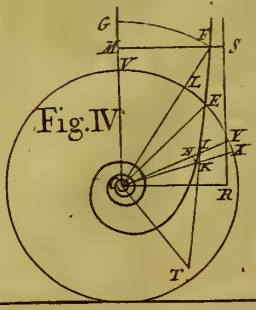


Fig. IV



I. Senex sculp.t

PHILOSOPHICAL TRANSACTIONS.

For the Months of *July*, *August* and *September* 1714.

The CONTENTS.

I. *Observationes occultationis Stellæ τ in origine cornu Borei* Tauri, *sub disco Lunæ, simulque Eclipseos Lunæ statim insecutæ, Anno* 1713 Novemb 21. *mane* St. vet. Romæ *habitæ, & à præstantissimo Astronomo* D. Francisco Blanchino. *R. S. S. communicatæ. Cum Emersionibus nonnullis Intimi Jovialium ex umbra Jovis, ibidem eodem anno observatis.*

H. post Merid. h ′ ″	
12 53 34	STELLA *Bayero* τ Tauri proxime appellit ad limbum Lunæ, observata per Telescopium duodecim palmorum.
12 54 34	Eadem jam occultata est ab ea parte Lunaris Limbi, quæ media ferme est inter maculas Aristarchi & Galilei. Parallelus diurnus à centro Lunæ descriptus apparet Australior quam stella τ partibus Micrometri $7\frac{1}{2}$, qualium Lunæ diameter subtendit 37. Stellæ igitur τ declinatio Borealior est declinatione apparente Lunaris centri minutis circuli maximi $5'\frac{3}{8}$ circiter.
14 0 14	Sirius attingit Meridianum: unde verificata sunt tempora.
14 32 57	Stella τ, quæ aliquot minuta excesserat è limbo Lunæ, in revolutione diurna præcedit limbum occidentalem Lunæ secundis horariis 0′. 33″, eademque præcedit centrum Lunæ secundis 103″ sive 1′. 43″.
14 42 50	Eadem præcedit limbum Lunæ secundis 48″, & centrum 1′. 58″.
14 50 37	Differentia Ascensionis rectæ stellæ & limbi est 1′. 03″, centri vero Lunæ & ejusdem stellæ 2′. 13″.
15 0 0	In limbo Lunæ Penumbra, quæ antea erat dilutior, sensim fit densior.
15 2 20	Penumbra fit evidentior, sed nondum apparet Umbra vera.
15 4 20	Initium incidentiæ Lunæ in Umbram veram, ea in parte limbi quæ proxima est maculæ *Schiccardi*.

Umbra

h	′	″	
15	5	21	Umbra vera jam obtegit partem unam, qualium Lunæ diameter in Micrometro obtinet 37.
15	7	20	Jam partes duæ obteguntur qualium Lunæ diameter est 37.
15	16	20	Jam obteguntur Lunaris diametrio $\frac{3}{5}$.
15	31	20	Latent Lunaris diametri $\frac{2}{5}$.
16	12	0	Jam latent in diametro partes $\frac{11}{17}$
16	17	20	Partes latentes 15, ut antea.
16	50	20	Jam partes latentes, $\frac{12}{17}$.
16	54	35	Incipit emergere prior limbus Tychonis.
16	56	9	Jam totus Tycho emergit.
17	13	30	Latent Lunaris diametri partes 5 è 37.
17	27	45	Umbra vera excedit è limbo Lunæ, in loco designato per diametrum ductam inter *Aristarchum* & *Platonem* situ inter-medio.

N. B. *Hæc observatio pluris æstimanda, quod occultatio stellæ τ acciderit tam vicina Opposito Solis, ut inde locus Solis inter Fixas rite examinari poterit.*

Emersiones Satellitis intimi Jovialium ex umbra Jovis *observatæ* Romæ, *anno* 1713.

Sept $\frac{11}{21}$ post meridiem 8^h. 38′. 20″ Intimus Jovis Satelles incipit emergere, è regione spatii inter utramque Jovis fasciam protensi. Observatio peragebatur Telescopio D. *Andreæ Chiarelli* longitudinis 40 palm. Romanorum. Deinde 8^h 44′. Tertius Satelles ita apparebat Quarto conjunctus, ut ambo viderentur unicus Satelles, Distabant à centro Jovis diametris Jovialibus circiter 5 $\frac{1}{4}$. Hora vero 9^h. 4′, Jam disjuncti videbantur. Quartus situ inverso apparuit paulo depressior Tertio, & paulo elongatior a Jove: quare erat Tertio Borealior.

Sept. $\frac{18}{21}$ 10^h. 36′. 23″, Primus seu intimus Satelles incipit emergere ab Umbra, Tubo 25 Palm. Domini *Campani.*

Novemb. $\frac{12}{23}$ 7^h 32′ 22″, Primus Satelles incipit emergere, conspectus per Tubum Domini *Chiarelli* palm. 40. Deinde eadem

dem nocte $7^h 46'$ Primus & Secundus proximi ſunt, & $7^h. 53'$ iidem ita ſunt vicini ut vix punctulo diſtinguantur.

Decemb. 9 *N.S.* vel Novemb. 28. *V.S.* $5^h. 45'. 45''$, Primus Satelles incipit emergere ab umbra Jovis.

Decemb. 21. *V.S.* $5^h. 50'. 22$, iterum viſus eſt primus Satelles incipiens emergere ex umbra.

Ex his obſervationibus accurato calculo ſubjectis, manifeſtum eſt æquationem ſecundam, quam a motu Luminis progreſſivo ortam ſupponimus, neceſſario locum habere. Nam poſt 57 ſatellitis intimi revolutiones, quibus Jupiter a Terra pluſquam Radio Orbis magni receſſit, novem fere minutis tardius conſpecta eſt Eclipſis ultima, quam debuit juxta tenorem Obſervationis primæ : quod quidem Hypotheſibus D. Caſſini *conſonum eſt.*

Ex iiſdem etiam confirmatur (quod nos quoque antea annotavimus, nempe) *quod motus Intimi Satellitis Jovis paulo celerior ſit quam in Tabulis elaboratiſſimis D. Caſſini, ante viginti annos cum publico communicatis, & calculi facilitate plurimum ſe commendantibus. Erroculus autem iſte vix excedere videtur duo temporis minuta in ſingulis Jovis revolutionibus, ſive duodecim annis ; quibus cælum anticipat* Caſſini *calculum. Hac vero adhibita correctione, ſatis accuratus habebitur conſenſus.*

H

II. Joannis Keill *M. D. & in Academia* Oxoniensi *Astronomiæ Professoris* Saviliani, *Observationes in ea quæ edidit Celeberrimus Geometra* Johannes Bernoulli *in Commentariis Physico Mathematicis* Parisiensibus *Anno* 1710. *de inverso Problemate Virium Centripetarum. Et ejusdem Problematis solutio nova.*

NObilissimum est problema Datâ lege Vis centripetæ invenire Curvam quam describit Mobile, de loco dato, secundum datam rectam,& cum data velocitate egrediens Concessis figurarum curvilinearum quadraturis, ejus solutionem perfectam olim dedit Dominus *Newtonus* in *Principiis Philosophiæ Mathematicis*. Hoc ipsum Problema denuo aggressus est vir clarissimus & Geometra celeberrimus Dominus *Johannes Bernoulli* in Academia *Basiliensi* Matheseos Professor *, qui non pauca eaque egregia ingenii sui specimina jam pridem edidit, quibus Geometriam reconditiorem non parum ditavit. Unde à tanti viri acumine novam pulchramque Problematis solvendi methodum expectabam. Gestiebam itaque solutionem Bernoullianam perlegere, & cum Newtoniana comparare; quibus tandem diligentius perlectis & examinatis, hæc quæ sequuntur annotavi.

Dominus Bernoulli eandem præmittit propositionem quam Newtonus problemati demonstrando prius adhibuit: estque ea in *Principiis* XL, non minus pulchra quam demonstratu facilis. Scil.

Si corpus cogente vi quacunque centripeta moveatur utcunque, & corpus aliud recta ascendat vel descendat, sintque eorum velocitates, in aliquo æqualium altitudinum casu,æquales; velocitates eorum in omnibus æqualibus altitudinibus erunt æquales.

* Vide Commentarios Physico-mathematicos Parisienses Anno 1710.

Hujus propositionis Demonstrationem Newtonianam ait Bernoullius esse nimis implicatam, & suam, quam simpliciorem vocat, ejus loco substituit. At pace tanti viri liceat mihi dicere, si quid discriminis sit inter demonstrationem Bernoullianam & Newtonianam, id in eo situm est, quod hæc multo facilior esse videtur minusque perplexa quam illa. *Fig.* I. Nam si centro *C* describantur circuli *DI*, *EK*, quorum intervallum *DE* est quam minimum, sintque corporum in *D* & *I* velocitates æquales, & ab *N* ad *IK* demittatur perpendiculum *NT*, fuse ostendit Newtonus vim accelerátricem secundum *DE*, esse ad vim acceleratricem secundum *IK* ut *IN* ad *IT*. Nimirum si vis secundum *DE* vel *IN* exponatur per rectas *DE* vel *IN*, vis illa secundum *IN* resolvitur in duas *TI*, *TN*, quarum illa solum quæ est ut *TI* motum secundum directionem *IK* accelerat: accelerationes autem seu velocitatum incrementa sunt ut vires & tempora quibus generantur conjunctim. At tempora ob æquales velocitates in *D* & *I*, sunt ut viæ descriptæ *DE*, *IK*; quare accelerationes in decursu corporum per lineas *DE* & *IK*, sunt ut *DE* ad *IT* & *DE* ad *IK* conjunctim; *i. e.* ut *DE* quad. quod est *IN* quad. ad rectang. $IT \times IK$. adeoque ob *IN* quad. $= IT \times IK$, incrementa velocitatum sunt æqualia: æquales igitur sunt velocitates in *E* & *K*, & eodem argumento semper reperientur æquales in æqualibus distantiis. Hæc est summa demonstrationis Newtoni quæ tam dilucide ab eo exponitur, ut inter propositiones elementares paucas faciliores invenies. At non sic procedit Dominus Bernoullius, sed illi sufficit dicere, Mechanicam ostendere vim secundum *DE* esse ad vim secundum *IK*, ut *IK* ad *DE*. Mechanicam etiam ostendere incrementa velocitatum esse in ratione virium & temporum conjunctim; & initio motus positis velocitatibus æqualibus tempora sunt ut viæ descriptæ *DE*, *IK*; & hinc, (argumento prorsus simili ei quo utitur Newtonus) concludit incrementum velocitatis, quod acquirit corpus dum describit *IK*, esse ad incrementum velocitatis dum describitur *DE*, ut $DE \times IK$ ad $IK \times DE$, & proinde velocitatum incrementa ubique in distantiis æqualibus esse æqualia.

At

At si Tyronibus facilem voluisset tradere demonstrationem, debuisset Propositionem Mechanicam citare, eamque ad præsentem casum accommodare. Et quidem pluribus verbis opus est, ut hoc fiat per theorema quod innuere videtur, in quo agitur de descensu Gravium in planis inclinatis : nullum enim est hic Planum datum quod recto corporum descensui obstat ; immo tantum abest ut corpus à plano cohibeatur, ut è contra à Plano seu Tangente per vim quandam continuo retrahitur. Proculdubio igitur manifesta magis foret ejus ratiocinii vis, si demissis Mechanicæ propositionibus, rem omnem ex propriis principiis demonstrasset, uti fecit Newtonus. Nam resolvendo triang. rectang. KNI in duo triangula æquiangula, est KI ad IN ut IN ad IT, adeoque loco rationis IN ad IT ponere potuisset rationem KI ad IN vel ad DE.

Si de loco quovis A in recta AC cadat corpus, deque loco ejus E erigatur semper perpendicularis EG vi centripetæ proportionalis, sitque BFG linea curva quam punctum G perpetuo tangit ; demonstrat Newtonus velocitatem corporis in loco quovis E esse ut Areæ curvilineæ $ABGE$ latus quadratum. Adeoque si velocitas dicatur v, erit v^2 ut Area $ABGE$: & si P sit altitudo maxima, ad quam corpus in Trajectoria revolvens, deque quovis ejus puncto eâ quam ibi habet velocitate sursum projectum ascendere possit : sitque quantitas A distantia corporis à centro, in alio quovis orbitæ puncto ; & vis centripeta sit semper ut ipsius A dignitas quælibet, scil. ut A^{n-1}, Velocitas corporis in omni altitudine A erit ut $\sqrt{nP^n - nA^n}$.

Similiter Dominus Bernoullius ostendit, si distantia à centro dicatur x, velocitas v & vis centripeta φ, esse $v = \sqrt{ab - \int \varphi x}$ ubi ex Quadraturis constat esse Aream $ABGE = ab - \int \varphi x$. Perinde itaque est sive exprimatur quadratum velocitatis per Aream $ABGE$, sive per quantitatem huic æqualem $ab - \int \varphi x$. Et si vis centripeta φ sit ut nA^{n-1} seu $n\ x^{n-1}$, fit $ab = P^n$ &

---

[2] Vide prop. 39. & 40. Principiorum.

 $\int \varphi x$

$\int \varphi \dot{x} = A^n$, adeoque $ab - \int \varphi \dot{x}$ eſt ut quantitas $P^n - A^n$.

Deſcribat corpus Curvam VK, vi centripeta tendente ad C, deturque circulus VXY, centro C intervallo quovis CV deſcriptus. Q ſit quantitas conſtans, atque $\frac{Q}{A} = z$. Sitque KI elementum Curvæ; IN vel DE elementum altitudinis, XY elementum arcus: demonſtrat Newtonus Elementum arcus ſeu XY exprimi poſſe per hanc formulam $\frac{Q \times IN \times CX}{AA\sqrt{ABGE - z^2}}$ Similiter ex præmiſſis Dominus Bernoullius, poſito Arcu $UX = z$, & altitudine ſeu diſtantia $= x$, elementum arcus ad hanc reducit formulam ſcil. $\dot{z} = \frac{a^2 c \dot{x}}{\sqrt{abx^4 - x^4 \int \varphi \dot{x} - a^2 c^2 x^2}}$ Et primo quidem aſpectu videbatur formula Newtoniana quodammodo ſimplicior Bernoullianâ, eo quod paucioribus conſtat terminis; at re diligentius exploratâ, vidi Bernoullianam formulam omnino cum Newtoniana coincidere; nec niſi in notatione quantitatum ab ea differre. Nam ſi pro $ab - \int \varphi \dot{x}$ ponatur $ABGE$, pro ac ponatur Q, & x pro A, a pro CX, & $\dot{x}$ pro IN, fit

$$\frac{a^2 c \dot{x}}{\sqrt{abx^4 - x^4 \int \varphi \dot{x} - a^2 c^2 x^2}} = \frac{Q \times CX \times IN}{\sqrt{A^4 \times ABGE - \frac{Q^2 A^4}{A^2}}} =$$

$\frac{Q \times CX \times IN}{AA\sqrt{ABGE - Q^2}}$ ſeu ponendo z^2 loco $\frac{Q^2}{A^2}$, (quod facit Newtonus commodioris notationis gratia,) Formula Bernoulliana evadit $\frac{Q \times CX \times IN}{A^2\sqrt{ABGE - z^2}}$: unde conſtat formulam illam non magis à Newtoniana diſcrepare, quam verba Latinis literis expreſſa differunt ab iiſdem verbis ſcriptis in Græcis characteribus.

Poſt traditam generalem formulam; deſcendit Dominus Bernoullius ad caſum particularem, ubi vis centripeta eſt reci-

procè

procè ut quadratum diſtantiæ; & per varias reductiones & operationes ſatis moleſtas, conſtructionem oſtendit Curvarum quæ urgente eâ vi centripeta deſcribi poſſunt, eaſque ad æquationes reducendo probat eſſe Sectiones Conicas. Deinde queritur Dominum Newtonum ſupponere ſine demonſtratione Curvas à tali vi deſcriptas eſſe Sectiones Conicas.

Impoſſibile eſt ut credat nullam Newtono notam fuiſſe hujus rei demonſtrationem; Noverit enim eum primum & ſolum fuiſſe qui hanc omnem de vi centripeta doctrinam geometricè tractavit, quique eam ad tantam perfectionem perduxit, ut poſt plures quam viginti annos, parum admodum à præſtantiſſimis Geometris ei additum ſit. Noverit etiam Bernoullius Newtonum, præter generalem problematis inverſi ſolutionem, oſtendiſſe modum quo formari poſſunt Curvæ, quæ vi centripeta decreſcente in triplicata diſtantiæ ratione deſcribuntur, adeoque alterum illum caſum ignorare non potuiſſe. Nec profecto intelligo qua ratione Bernoullius Newtono objiciat, eum hujus caſus demonſtrationem prætermiſiſſe; cum ipſe non pauca ſæpius propoſuit Theoremata, quorum demonſtrationes nuſquam dedit; & quidni liceat Newtono ad alia feſtinanti hoc idem facere. Interim in nova *Principiorum* Editione, facilior multo & magis clara, licet tribus verbis, extat hujus rei demonſtratio, quam eſt Bernoulliana.

Tandem Bernoullius ut neceſſitatem ſuæ demonſtrationis inverſi Problematis in hoc particulari caſu oſtendat, hæc addit. Conſiderandum eſt, inquit, quod vis quæ facit ut corpus in Spirali Logarithmica moveatur, debet eſſe reciprocè ut cubus diſtantiæ à centro; at non inde ſequitur talibus viribus ſemper deſcribi debere tales Curvas, cum ſimiles etiam vires facere poſſunt ut corpus in Spirali Hyperbolica moveatur.

Miror ſane quod Vir Cl. ſuſpicetur Newtonum talem unquam duxiſſe conſequentiam. Nam præter Spiralem logarithmicam, oſtendit Newtonus qua ratione aliæ Curvæ, numero infinitæ & diverſæ, formari poſſunt, quæ omnes deſcribantur eadem vi centripeta qua Spiralis Logarithmica; interque eas reponi debet hæc ipſa Spiralis Hyperbolica, ut in ſequentibus oſtendemus.

Exinde

Exinde autem concludit Newtonus Sectiones tantum Conicas necessario describi debere per vim centripetam quadrato distantiæ reciprocè proportionalem: Nempe quod Curvatura orbitæ cujuscunque ex datis velocitate, vi centripeta & positione Tangentis datur; datis autem umbelico, puncto contactus & positione tangentis, semper describi possit Sectio Conica quæ curvaturam illam datam habeat. Hoc à me ostensum est in Actis Philosophicis Londinensibus Anno 1708. In hac igitur Sectione, urgente illa vi corpus movebitur, & in nulla alia; cum corpus de eodem loco, secundum eandem directionem, eadem cum velocitate, & urgente eadem vi centripeta exiens, non possit diversas semitas describere.

Liceat jam mihi Dominum Bernoullium imitari, & inversum de vi centripeta problema longe diversa methodo resolvere, & ad casum particularem applicare; ubi scil. vis est reciprocè ut cubus distantiæ, simulque ostendere demonstrationem Cor. 3. prop. 41. *Principiorum* Newtoni.

Quod ut fiat, quædam ex iis quæ in Actis Philosophicis No. 317. exposui hic præmittenda sunt. *Fig.* II.

Sit *VIL* Curva quævis, quam corpus urgente vi centripeta ad centrum *C* tendente describit: hanc Curvam in duobus punctis infinitè vicinis *I* & *K* tangant rectæ *IP*, *Kp*, ad quas è centro demittantur perpendiculares *CP*, *Cf*; centro item *C* describantur *KE*, *ID*, & ducatur *CI*.

Erit vis centripeta ut Quantitas $\frac{Pp}{PC^3 \times IN}$ Quod Theorema licet in prædicto loco demonstravimus, ecce aliam ejus demonstrationem. Ex *K* ducantur *Km* ad *CP* & *Kn* ad *CI* parallelæ. Et ob æquiangula triangula *ICP*, *IKN*, *nKm*, Item que ob *IKm* & *IpP* æquiangula. Erit,

$$Ip \text{ vel } IP : IK :: pP : Km$$
$$PC : IP :: Km : mn$$
$$IN : IK :: mn : nK$$

unde ex æquo fiet $PC \times IN : IK^2 :: pP : nK$, & erit $nK = \frac{pP \times IK^2}{PC \times IN}$. Præterea tempus quo describitur arcus *IK* est ut

Area

Area ſeu triangulum ICK, vel ejus duplum $PC \times IK$; adeoque ſi tempus detur erit $PC \times IK$ quantitas conſtans. Dato autem tempore, vis centripeta eſt ut lineola Kn quæ ſub urgente vi illa deſcribitur, adeoque vis centripeta eſt ut lineola illa Kn ducta in quantitatem conſtantem $\frac{1}{PC^2 \times IK^2}$, hoc eſt, erit vis centripeta ut $\frac{1}{PC^2 \times IK^2} \times \frac{Pp \times IK^2}{PC \times IN}$ ſeu ut quantitas $\frac{Pp}{PC^3 \times IN}$. Quod erat demonſtrandum.

Velocitas corporis in quovis loco eſt ut via in minimo quovis tempore percurſa directè & ut tempus illud inversè; adeoque & ut $IK \times \frac{1}{PC \times IK}$, hoc eſt, velocitas erit reciprocè ut Perpendicularis è centro in Tangentem.

Si diſtantia corporis à centro dicatur x, & Perpendicularis in tangentem p, erit $IN = \dot{x}$ & $Pp = \dot{p}$ & vis centripeta exponi poteſt per quantitatem $\frac{f^4 \dot{p}}{p^3 \dot{x}}$, aſſumendo quantitatem quamlibet pro f^4.

Adeoque ſi cum Domino Bernoullio vim centripetam nominemus φ, erit $\frac{f^4 \dot{p}}{p^3 \dot{x}} = \varphi$ & $\frac{f^4 \dot{p}}{p^3} = \dot{x}\varphi$; & capiendo harum quantitatum fluentes erit $\frac{f^4}{2p^2}$ = Fluenti quantitatis $\dot{x}\varphi$.

At cum velocitas corporis ſit reciprocè ut perpendicularis p, ejus quadratum exponi poteſt per $\frac{f^4}{2p^2}$. Si itaque velocitas dicatur v, erit $v^2 = \frac{f^4}{2p^2}$ = Fluenti quantitatis $\dot{x}\varphi$. Quod ſi A ſit locus de quo cadere debet corpus ut acquirat in D vel I veloci-

velocitatem v, deque loco corporis D erigatur perpendicularis $DF = \varphi$ erit rectangulum $DE \times DF = x\varphi$. Sit jam BFG linea curva cujus ordinatæ exponant vires centripetas, seu quantitates φ. Fluens quantitatis $x\varphi$ erit Area curvilinea $ABFD = v^2 = \frac{f^4}{2p^2}$, adeoque erit v ut Areæ $ABFD$ latus quadratum. Quod si velocitas ea sit quæ ab infinita distantia cadendo acquiritur, erit v^2 seu fluens ipsius $x\varphi$ æquale areæ $ODFO$ indefinitè protensæ.

Hinc semper dabitur quantitas p in terminis finitis, quando Area illa curvilinea terminis finitis exponi potest. Sit, verbi gratia, vis centripeta reciprocè ut distantiæ dignitas m, hoc est, sit $x\varphi = \frac{gx}{x^m}$. Si velocitas corporis sit ea quæ acquiritur cadendo ab infinita distantia, erit $v^2 = \frac{g}{m-1 \times x^{m-1}} = \frac{f^4}{2p^2}$ & in hisce omnibus casibus Area indefinitè protensa est quantitas finita. Potest autem corpus in trajectoria revolvi velocitate cujus quadratum vel majus fieri potest, vel minus quantitate $\frac{g}{m-1 \times x^{m-1}}$, vel huic æquale. Adeoque erit $v^2 = \frac{f^4}{2p^2} = \frac{g}{m-1 x^{m-1}} \pm e^2$

Hinc urgentibus his viribus, tria Curvarum genera describi possunt; prout e^2 est quantitas positiva vel negativa vel nulla.

V.G. Si Velocitas major sit ea quæ acquiritur ab infinita distantia cadendo, fit $\frac{f^4}{2p^2} = \frac{g}{m-1 x^{m-1}} + e^2$: si velocitas sit minor erit $\frac{f^4}{2p^2} = \frac{g}{m-1 x^{m-1}} - e^2$: si æqualis, erit $\frac{f^4}{2p^2} = \frac{g}{m-1 x^{m-1}}$

Sit

Sit $\frac{1}{2} f^4 = a^2 e^2$ & $\frac{1}{m-1} \times g = b^2 e^2$. Et ſi velocitas corporis ſit ea quæ ab infinito cadendo acquiritur, erit $p^2 = \frac{a^2 x^{m-1}}{b^2}$ ſeu $p = \frac{a x^{\frac{m-1}{2}}}{b}$.

At ſi velocitas major ſit aut minor hac velocitate, fiet uti oſtenſum eſt $\frac{f^4}{2p^2} = \frac{g}{m-1 \, x^{m-1}} \pm e^2 = \frac{\frac{1}{m-1} g \pm e^2 x^{m-1}}{x^{m-1}}$. Unde pro $\frac{1}{2} f^4$ & $\frac{g}{m-1}$ ponendo earum valores $a^2 e^2$ & $b^2 e^2$, erit $\frac{a^2 e^2}{p^2} = \frac{b^2 e^2 \pm e^2 x^{m-1}}{x^{m-1}}$ ſeu $\frac{a^2}{p^2} = \frac{b^2 \pm x^{m-1}}{x^{m-1}}$, & fiet $p^2 = \frac{a^2 x^{m-1}}{b^2 \pm x^{m-1}}$.

Adeoque ſi Vis centripeta ſit reciprocè ut cubus diſtantiæ, hoc eſt, ſi ſit $m = 3$ & $m - 1 = 2$. Erit $p^2 = \frac{a^2 x^2}{b^2}$, vel $p^2 = \frac{a^2 x^2}{b^2 + x^2}$, vel denique $p^2 = \frac{a^2 x^2}{b^2 - x^2}$.

In primo caſu conſtat Curvam eſſe Spiralem Logarithmicam: nam fit $p = \frac{a x}{b}$, & $b : a :: x : p$. adeoque ob conſtantem rationem b ad a, erit angulus $C I P$ ubique conſtans.

Ponamus jam eſſe $p^2 = \frac{a^2 x^2}{b^2 + x^2}$ & ex hac ſuppoſitione tres oriuntur diverſæ Curvarum ſpecies, prout a major eſt quam b, aut ei æqualis, aut minor. *Fig.* III.

Et primo ſit a major quàm b. Centro C & ad diſtantiam quamvis datam deſcribatur circulus HTX, cui rectæ CK, CI productæ occurrant in T & X. Et eſt $IN^2 : KN^2 :: IP^2 : PC^2$

 & ita

& ita $CI^2 - PC^2 : PC^2 :: x^2 - p^2 : p^2 :: x^2 - \frac{a^2 x^2}{b^2 + x^2}$ $: \frac{a^2 x^2}{b^2 + x^2} :: 1 - \frac{a^2}{b^2 + x^2} : \frac{a^2}{b^2 + x^2} :: b^2 + x^2 - a^2 : a^2$. Quare erit $\sqrt{x^2 + b^2 - a^2} : a :: IN : KN : \dot{x} : \frac{a\dot{x}}{\sqrt{x^2 + b^2 - a^2}}$ $= KN$. Et quoniam est a major quam b, erit $b^2 - a^2$ quantitas negativa. Sit illa $-c^2$, unde fit $KN = \frac{a\dot{x}}{\sqrt{x^2 - c^2}}$ Dicatur radius circuli HY h, & est $CK : KN :: CY : YX$ hoc est $x : \frac{a\dot{x}}{\sqrt{x^2 - c^2}} :: h : \frac{ha\dot{x}}{x\sqrt{x^2 - c^2}} = YX = \dot{y}$, si arcus HY vocetur y.

Sit $x = \frac{c^2}{z}$ unde $\dot{x} = -\frac{c^2 \dot{z}}{z^2}$ & $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$. Item erit $x^2 - c^2$ $= \frac{c^4}{z^2} - c^2 = \frac{c^4 - c^2 z^2}{z^2} = \frac{c^2}{z^2} \times c^2 - z^2$: unde $\sqrt{x^2 - c^2} = \frac{c}{z} \times \sqrt{c^2 - z^2}$: quibus valoribus substitutis, erit $\frac{ha\dot{x}}{x\sqrt{x^2 - c^2}} = \frac{-ha\dot{z}}{c\sqrt{c^2 - z^2}}$ Sit $a : c :: n : 1$. hoc est, sit $a = nc$, & fiet XY seu $\dot{y} = -\frac{nh\dot{z}}{\sqrt{c^2 - z^2}}$. Est verò $\frac{nh\dot{z}}{\sqrt{c^2 - z^2}}$ ad $\frac{c\dot{z}}{\sqrt{c^2 - z^2}}$ ut nh ad c; hoc est in ratione data: adeoque eorum fluentes, si simul incipiunt, erunt in eadem ratione, hoc est erit HY seu y ad fluentem quantitatis $\frac{c\dot{z}}{\sqrt{c^2 - z^2}}$ ut nh ad c.

Quod si centro C radio $CV = c$ describatur circulus VL, & CG sit $= z$, & $no = \dot{z}$, fiet arcus $mn = \frac{c\dot{z}}{\sqrt{c^2 - z^2}} =$ fluxioni arcus Qm, quando fluxio est quantitas positiva: sed quando est negativa

tiva, ejus fluens eſt arcus Vm prioris complementum. Arcus enim ejuſque complementum eandem habent quantitatem fluxionem denotantem, diverſis tantum ſignis affectam; quia creſcente uno decreſcit alter.

Hinc eſt HT ad Vm ut nh ad c: ſed eſt CV ad CH ut Ve: HT, hoc eſt $c : h :: Ve : \frac{h \times Ve}{c} = HT$, quare erit $\frac{h \times Ve}{c} : Vm :: nh : c$, unde $Ve : Vm :: n : 1$.

Præterea ex natura circuli erit $CG : CV :: CV : CT$, quando mT circulum tangit: hoc eſt erit $z : c :: c : \frac{c^2}{z} = CT = x$. Hinc ſi capiatur angulus VCe ad angulum VCm ut n ad 1, & producatur Ce ad K ut ſit CK = ſecanti CT, erit K punctum in Curvâ quæſitâ.

Hic obiter notandum eſt, ſi n ſit numerus, hoc eſt, ſi ſit a ad c vel a ad $\sqrt{a^2 - b^2}$ ut numerus ad numerum, Curva VI fiet Algebraica: nam in hoc caſu relatio mG ad ſinum anguli VCe æquatione definitur, & inde habebitur relatio ſinus anguli VCe ad CT vel CK per æquationem determinatam, & inde demum dabitur æquatio quæ exprimet relationem inter ordinatam & interceptam à puncto C incipientem. Harum Curvarum ordines & gradus in Scala æquationum Algebraica diverſi erunt pro magnitudine numeri n. In his omnibus Curvis ſic deſcriptis Aſympoti poſitio hac ratione determinatur: Fiat angulus VCL ad rectum angulum ut n ad 1. In eo angulo diſtantia corporis à centro evadit infinita. Jam quad. perpendicularis in Tangentem $PC = \frac{a^2 x^2}{b^2 + x^2}$, ubi x eſt infinita, fit $PC^2 = \frac{a^2 x^2}{x^2}$, ſeu $PC = a$. Ducatur itaque CR ad CL perpendicularis & æqualis rectæ a, & ſi per R ducatur RS rectæ CL parallela, hæc Curvam tanget ad infinitam diſtantiam, ſeu erit Curvæ Aſymptotos.

Si corpus in quavis harum Curvarum deſcendendo, ad Apſidem imam pervenerit; Hinc rurſus aſcendet in infinitum,

& aliam Curvam priori ſimilem, ſeu potius ejuſdem Curvæ ſimilem portionem, aſcendendo deſcribet.

Curvæ hæ poſſunt pluribus revolutionibus circa centrum torqueri, priuſquam ad Aſymptoton convergere incipiant, & motus angularis rectæ CK erit æqualis totidem rectis quot numerus n conſtat Unitatibus. *v. g.* ſi n ſit 100, perficientur viginti quinque integræ revolutiones priuſquam diſtantia à centro evadat infinita.

Aucto numero n, eadem manente a, minuitur c: eſt enim $\frac{a}{n} - c$ & $\frac{a^2}{n^2} = c^2 = a^2 - b^2$, unde fiet $n^2 - 1 \times a^2 = n^2 b^2$. Et proinde fiet $a^2 : b^2 :: n^2 : n^2 - 1$; adeoque ſi b^2 ad æqualitatem accedat ipſius a^2, perveniet quoque $n^2 - 1$ ad rationem æqualitatis cum n^2, & proinde augebitur n & in eadem ratione minuetur c. Ponatur itaque eſſe b^2 fere æquale ipſi a^2; adeo ut cum differentia ſit infinite parva, fiat n numerus infinite magnus, & radius circuli c fiet infinite parvus, ſeu circulus in ſuum centrum contrahetur. At ſic evaneſcente c, non pariter evaneſcit CT, ſi angulus VCM ſit propemodum rectus: nam in omni circulo, etiam minimo, ſecans anguli recti eſt quantitas infinita. Curva itaque hæc, ob n numerum infinitum, infinitis numero revolutionibus centrum ambibit, priuſquam ad Aſymptoton convergere incipiet.

Evaneſcente autem c fit $b = a$ & $p = \frac{ax}{\sqrt{x^2 + a^2}}$. Et quoniam in omni caſu eſt $\dot{y} = \frac{ha\dot{x}}{x\sqrt{x^2 + c^2}}$, evaneſcente c fiet $\dot{y} = \frac{ha\dot{x}}{x^2}$, unde capiendo Fluentes fiet $y = \frac{ha}{x}$ ſeu $xy = ha =$ datæ quantitati. *Fig.* IV.

Hæc Curva eſt Spiralis Hyperbolica, quæ plures habet notabiles proprietates. Si ducatur radius quilibet CIT Curvæ occurrens in I, & peripheriæ circuli in T, & ex C ad CI excitetur perpen-

perpendicularis CT, atque IT tangat Curvam in I, & rectæ CT occurrat in T: erit CT constans recta, æqualis scil. arcui VE; qua proprietate Logarithmicam æmulatur, cum CT Curvæ Subtangens dici possit. Sit enim Radius circuli $CE = h$, arcus $VE = a$, dicatur CI x & VT sit y. Quia est $ha = x \times y$ erit $\frac{ha}{x} = y$ & $\frac{ha\dot{x}}{x^2} = \dot{y}$. Porro est $CT : CI :: TX : NK$ hoc est $h : x :: \frac{ha\dot{x}}{x^2} : NK$: quæ proinde est $\frac{a\dot{x}}{x}$. Et quoniam est $IN : NK :: CI : CT$. hoc est $\dot{x} : \frac{a\dot{x}}{x} :: x : CT$, erit $CT = a$.

Si centro C, intervallo quovis CG, describatur circuli arcus GF, hic arcus inter rectam CV & curvam interceptus erit semper æqualis constanti rectæ CT vel a. Nam quoniam est $VL \times CF = CV \times VE$, erit $VL : VE :: CV : CF :: VL : GF$ unde æquantur VE & GF. Si ad CG ex C excitetur normalis $CR = VE$ vel FG vel a, & per R agatur RS rectæ CV parallela, erit RS Curvæ Asymptotos. Nam est recta MS æqualis arcui GF, & proinde FS distantia Curvæ ab RS est semper æqualis excessui quo arcus superat suum sinum: at cum distantia crescat in infinitum, excessus ille minuetur in infinitum, & fiet tandem data quavis recta minor, & proinde RS erit Curvæ Asymptotos.

Sit jam b major quam a; & similiter, ut in priore casu, invenietur $KN = \frac{a\dot{x}}{\sqrt{x^2 + b^2 - a^2}}$: at quoniam b superat a, erit $c^2 = b^2 - a^2$ quantitas positiva, & KN fiet $= \frac{a\dot{x}}{\sqrt{x^2 + c^2}}$ & ponendo radium circuli $HT = h$, invenietur $XT = \frac{ha\dot{x}}{x\sqrt{x^2 + c^2}}$. Ponatur $x = \frac{c^2}{z}$, & erit $\dot{x} = -\frac{c^2\dot{z}}{z^2}$ & $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$. Erit quoque $x^2 = \frac{c^4}{z^2}$ & $x^2 + c^2 = \frac{c^4}{z^2} + c^2 = \frac{c^4 + c^2 z^2}{z^2} = \frac{c^2}{z^2} \times c^2 + z^2$: unde

$\sqrt{x^2 + c^2}$

$\sqrt{\dot{x}^2 + c^2} = \frac{c}{z} \times \sqrt{c^2 + z^2}$. His itaque valoribus ſubſtitutis ſit $\frac{h a \dot{x}}{x\sqrt{x^2 + c^2}} = -\frac{h a \dot{z}}{c\sqrt{c^2 + z^2}} = -\dot{y}$. Nam tale ſumi poteſt initium arcus HY, ut ſimul cum Fluente quantitatis $\frac{-h a \dot{z}}{c\sqrt{c^2 + z^2}}$ creſcat & decreſcat. Fiat $nc = a$ & erit $\frac{n h \dot{z}}{\sqrt{c^2 + z^2}} = \dot{y}$, & $\frac{\frac{1}{2} n h^2 \dot{z}}{\sqrt{c^2 + z^2}} = \frac{1}{2} h \dot{y} =$ ſectori CXY.

Eſt autem $\frac{\frac{1}{2} n h^2 \dot{z}}{\sqrt{c^2 + z^2}} : \frac{\frac{1}{2} c^2 \dot{z}}{\sqrt{c^2 + z^2}} :: nh^2 : c^2$, hoc eſt in data ratione. Adeoque erit ſector CXY ad $\frac{\frac{1}{2} c^2 \dot{z}}{\sqrt{c^2 + z^2}}$ ſemper in data ratione. Harum itaque quantitatum fluentes erunt in eadem ratione, cum ſimul incipere ponantur. Fluens autem ſectoris CXY eſt ſector CVY, & fluens quantitatis $\frac{\frac{1}{2} c^2 \dot{z}}{\sqrt{c^2 + z^2}}$ eſt ſector Hyperbolæ, quod ſic oſtenditur. *Fig.* V.

Centro C ſemiaxe tranſverſo $CV = c$ deſcribatur Hyperbola æquilatera, & ex duobus punctis vicinis D & F ordinentur ad axem conjugatum rectæ DB, EF; ducantur item CD, CF. Et incrementum ſeu fluxio trianguli BCD æquale erit $BE \times BD$ — ſectore DCF: unde ſector DCF (qui eſt Fluxio ſectoris CVD) æqualis erit $BE \times BD$ — incremento trianguli BCD. Et ſi BC dicatur z, ob Hyperbolam, eſt $BD^2 = BC^2 + CV^2 = z^2 + c^2$: unde $BD = \sqrt{c^2 + z^2}$, & $BE \times BD = z \times \sqrt{c^2 + z^2}$. Triangulum autem BCD eſt $\frac{1}{2} z \times \sqrt{c^2 + z^2}$, cujus fluxio eſt $\frac{1}{2} \dot{z} \times \sqrt{c^2 + z^2} + \frac{\frac{1}{2} \dot{z} \times z^2}{\sqrt{c^2 + z^2}}$. Subtrahatur hæc quantitas ab $\dot{z} \times \sqrt{c^2 + z^2}$, & reſtabit ſector Hyperbolæ minimus CDF

=

$= \frac{1}{2}\dot{z} \times \sqrt{c^2 + z^2} - \frac{\frac{1}{2}\dot{z} \times z^2}{\sqrt{c^2 + z^2}} = \frac{\frac{1}{2}\dot{z} \times c^2 + z^2 - \frac{1}{2}\dot{z} \times z^2}{\sqrt{c^2 + z^2}} =$

$\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2 + z^2}}$. Adeoque fluens ſectoris CDF eſt æqualis fluenti quantitatis $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2 + z^2}}$. Proinde erit ſector CVD fluens quantitatis $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{c^2 + z^2}}$. Præterea DT recta tangat Hyperbolam & occurrat Axi conjugato in T. Eſt ex natura Hyperbolæ $BC : CV :: CV : CT$, hoc eſt $z : c :: c : \frac{c^2}{z} = CT = x$. Atque hinc oritur conſtructio quæ ſequitur. *Fig.* VI.

Centro C ſemiaxe tranſverſo CV, deſcribatur Hyperbola æquilatera Vm, item circulus Ve. Capiatur ſector circularis CVe ad ſectorem Hyperbolicam CVm ut n ad 1; tangat Hyperbolam in m recta Tm, occurrens Axi conjugato in T: producatur Ce ad k ut ſit $Ck = CT$, & punctum k erit in Curva quæſitâ. Nempe talis eſt ea Curva, ut ſi Ck dicatur x, Perpendicularis à C in tangentem ejus demiſſa erit ſemper æqualis $\frac{ax}{\sqrt{b^2 + x}}$. Quando x eſt infinita evaneſcit b^2, & perpendicularis fit $= a$, & tunc coincidit CR cum CV. Si itaque capiatur in axe conjugato $CR = a$, & ducatur RS ipſi CV parallela, erit hæc Curvæ Aſymptotos.

Si eo uſque augeatur a ut fiat quantitas $b^2 - a^2$ infinite parva, tunc evaneſcet c^2, & quantitas $\frac{ha\dot{x}}{x\sqrt{x^2 + c^2}}$ fit $\frac{ha\dot{x}}{x^2}$ $= \dot{y}$. Unde ſi capiantur harum quantitatum fluentes, habebimus $\frac{ha}{x} = y$, & $ha = xy$, hoc eſt rectangulum ſub arcu circulari & diſtantia Curvæ à centro erit ſemper data quantitas; atque hac ratione migrabit curva in ſpiralem Hyperbolicam. Eſt itaque ſpiralis Hyperbolica Curva media ſeu quaſi limes, inter eas Curvas quæ conſtruuntur per ſectores circulares & eas quæ conſtruuntur per ſectores Hyperbolicos. Itaque ſpiralis illa Hyperbolica conci-

pi-

pi poteſt formari vel per ſectorem Circuli aut Ellipſis, vel per ſectorem Hyperbolæ, cujus Axis tranſverſus minuitur in infinitum, & in eadem ratione augetur numerus n.

Ad eum jam devenimus caſum ubi velocitas corporis minor eſt eâ quæ acquiritur cadendo ab infinita diſtantia, & ubi $p^2 = \frac{a^2 x^2}{b^2 - x^2}$. Et hic ſimili ratiocinio ac in priori caſu, invenietur $KN = \frac{a\dot{x}}{\sqrt{b^2 - a^2 - x^2}}$, ubi neceſſe eſt ut ſit b^2 majus quam a^2. Hinc ſi $b^2 - a^2$ dicatur c^2, fit $KN = \frac{a\dot{x}}{\sqrt{c^2 - x^2}}$; & proinde XY ſeu $\dot{y} = \frac{ha\dot{x}}{x\sqrt{c^2 - x^2}}$.

Sit jam $x = \frac{c^2}{z}$, & fiet $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$ ſeu $\frac{ha\dot{x}}{x} = -\frac{ha\dot{z}}{z}$ & $c^2 - x^2$ erit $= \frac{c^2}{z^2} \times z^2 - c^2$, quibus valoribus ſubſtitutis fit $\frac{-ha\dot{z}}{c\sqrt{z^2 - c^2}} = \frac{ha\dot{x}}{x\sqrt{x^2 - c^2}} = -\dot{y}$. Nam tale ponendum eſt initium arcus VX, ut ſimul cum fluente quantitatis $\frac{ha\dot{z}}{c\sqrt{z^2 - c^2}}$ incipiat: unde erit $\frac{\frac{1}{2}h^2 a\dot{z}}{c\sqrt{z^2 - c^2}} = \frac{1}{2}h\dot{y} =$ ſectori $CXY = \frac{\frac{1}{2}nh^2\dot{z}}{\sqrt{z^2 - c^2}}$, ponendo $nc = a$. Eſt vero $\frac{\frac{1}{2}nh^2\dot{z}}{\sqrt{z^2 - c^2}}$ ad $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2 - c^2}}$ ut nh^2 ad c^2, hoc eſt in ratione conſtanti. Quare harum quantitatum Fluentes ſunt in eadem ratione, hoc eſt Fluens quantitatis $\frac{1}{2}h\dot{y}$ ſeu $\frac{\frac{1}{2}nh^2\dot{z}}{\sqrt{c^2 - z^2}}$ erit ad fluentem quantitatis $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2 - c^2}}$ ut nh^2 ad c^2. Eſt autem fluens quantitatis

$\frac{1}{2}$

$\frac{1}{2} b y =$ sectori CVX, & Fluens quantitatis $\frac{\frac{1}{2} c^2 \dot{z}}{\sqrt{z^2 - c^2}}$ est sector Hyperbolæ, quod sic ostenditur. *Fig.* VII.

Centro C semiaxe transverso $CV = c$ describatur Hyperbola æquilatera, & ex duobus punctis infinite vicinis B & D ad axem ordinentur duæ rectæ BE, DF; ducantur item CB, CD. Et erit Fluxio seu incrementum trianguli $CBE =$ triangulo $CBD + BE \times EF$; unde triangulum CBD, seu sector minimus CBD, erit $=$ incremento trianguli $CBE - BE \times EF$. Dicatur CE z, & erit $BE = \sqrt{z^2 - c^2}$, & $BE \times EF = \dot{z} \sqrt{z^2 - c^2}$. Est quoque triangulum $CBE = \frac{1}{2} z \sqrt{z^2 - c^2}$, cujus Fluxio est $\frac{1}{2} \dot{z} \times \sqrt{z^2 - c^2} + \frac{\frac{1}{2} \dot{z} \times z^2}{\sqrt{z^2 - c^2}}$; à quo si subtrahatur quantitas $\dot{z} \times \sqrt{z^2 - c^2}$, fit sector minimus $CBD = \frac{\frac{1}{2} \dot{z} \times z^2}{\sqrt{z^2 - c^2}} - \frac{1}{2} \dot{z} \times \sqrt{z^2 - c^2} = \frac{\frac{1}{2} \dot{z} \times z^2 - \frac{1}{2} \dot{z} \times \overline{z^2 - c^2}}{\sqrt{z^2 - c^2}} = \frac{\frac{1}{2} c^2 \dot{z}}{\sqrt{z^2 - c^2}}$: unde constat sectorem CBE esse fluentem quantitatis $\frac{\frac{1}{2} c^2 \dot{z}}{\sqrt{z^2 - c^2}}$. Præterea si BT tangens Hyperbolam Axi transverso occurrat in T, ex natura Hyperbolæ fit $CE : CV :: CV : CT$, hoc est $z : c :: c : \frac{c^2}{z} = CT = x$. Fig. VIII.

Hinc deducimus sequentem constructionem. Centro C, semiaxe transverso $CV = c$, describatur Hyperbola æquilatera VB, & circulus Ceg ex centro C. Ad hyperbolam ducatur recta CB, & hyperbolæ Tangens BT axi transverso occurrat in T. Capiatur circuli sector CVe, qui sit ad sectorem Hyperbolicum CVB ut n ad 1. In Ce capiatur $CK = CT$, & erit K punctum in Curva quæsita, cujus perpendiculum è centro C ad Tangentem in K demissum, si CK dicatur x, est æquale $\frac{a x}{\sqrt{b^2 - x^2}}$.

Et in hac Curva, urgente vi centripeta quæ ſit reciproce ut cubus diſtantiæ, movebitur corpus, ſi ſecundum directionem Tangentis cum juſta velocitate exeat. Qualis autem debet eſſe velocitas quæ faciat ut corpus harum Curvarum quamvis deſcribat, ſic invenietur.

Cum velocitas qua corpus in trajectoria quacunque movetur ſit reciproce ut quantitas p, aſſumendo conſtantem quamvis a, ea ſemper exponi poteſt per $\frac{a}{p}$. Et ſi ad Axem CV ordinentur rectæ quæ ſint reciproce ut cubi diſtantiarum à centro, ſeu ut vires centripetæ, & hac ratione formetur Figura curvilinea, ejus Area indefinite extenſa ſemper exponi poteſt per $\frac{b^2}{x^2}$, ut ex Quadraturis conſtat. At Area illa eſt ut quadratum velocitatis quæ acquiritur ab infinita diſtantia cadendo, adeoque velocitas hoc caſu acquiſita erit ut $\frac{b}{x}$. Hinc ſi velocitas illa dicatur y, & velocitas qua corpus in Trajectoria movetur dicatur v, taleſque aſſumantur quantitates a & b, ut in una aliqua à centro diſtantia ſit $y : v :: \frac{b}{x} : \frac{a}{p}$, erit ubique in omnibus diſtantiis $y : v :: \frac{b}{x} : \frac{a}{p} :: p : \frac{ax}{b}$. Unde ſi $y = v$, erit $p = \frac{ax}{b}$, & Curva hac velocitate deſcripta erit Spiralis Nautica; vel Circulus exiſtente $p = x$ & $a = b$.

Si y ſit major quam v, tunc p major erit quam $\frac{ax}{b}$; eritque illa, ut ex præcedentibus conſtat, $= \frac{ax}{\sqrt{b^2 - x^2}}$. Curva autem conſtruetur per ſectorem Hyperbolicum, ut in ultimo caſu oſtenſum fuit, ubi diſtantia corporis à centro per concurſum Tangentis Hyperbolæ cum Axe tranſverſo determinatur. Si

y ſit minor quam v, at in tantilla ratione ut maneat b major quam a, Curva formabitur per eundem ſectorem hyperbolicum. At diſtantia corporis à centro deſumitur ex concurſu Tangentis cum Axe conjugato.

Si ſit $y : v :: p : x$, erit in eo caſu $a = b$, & Curva evadit Spiralis Hyperbolica, ubi eſt $p = \frac{ax}{\sqrt{a^2 + x^2}}$. Hinc ſi de loco quovis projiciatur corpus ſecundum datam rectam, cum ea velocitate quæ ſit ad velocitatem ab infinito cadendo acquiſitam, ut diſtantia corporis à centro ad perpendicularem è centro ad lineam directionis demiſſam, movebitur illud corpus in Spirali Hyperbolica. Si denique ſit v tantò major quam y, ut ſit etiam a major quam b, Curva conſtruetur per Sectores Circulares. Atque hac ratione datâ velocitate ſemper determinari poſſit relatio quantitatum a & b, ac proinde Curva deſcribetur in qua corpus cum illa velocitate movebitur: & viciſſim data Curva, ſeu datis quantitatibus a & b, invenietur velocitas qua Curva illa deſcribitur.

Omnium Curvarum Areæ (ſi circulum excipias) quæ urgente hac vi centripetâ deſcribi poſſunt, ſunt perfecte quadrabiles. Nam primo, in ſpirali Logarithmica, quia eſt $p = \frac{ax}{b}$, erit $KN = \frac{a\dot{x}}{\sqrt{b^2 - a^2}} = \frac{a\dot{x}}{c}$, ponendo $b^2 - a^2 = c^2$: *vid. Fig.* II. adeoque erit triangulum $CKI = \frac{\frac{1}{2}ax\dot{x}}{c}$, cujus Fluens eſt $\frac{ax^2}{4c}$ = Areæ Curvæ.

Si p ſit $\frac{ax}{\sqrt{b^2 + x^2}}$, & a major quam b, oſtenſum eſt eſſe $KN = \frac{a\dot{x}}{\sqrt{x^2 - c^2}}$, unde $KN \times \frac{1}{2} CI = \frac{\frac{1}{2}a\dot{x}\dot{x}}{\sqrt{x^2 - c^2}}$, cujus Fluens eſt $\frac{1}{2} a \times \sqrt{x^2 - c^2}$ = Areæ Curvæ. At ſi a minor ſit quam b, fit $KN =$

$\frac{a\dot{x}}{\sqrt{x^2+c^2}}$, & $KN \times \frac{1}{2} CI = \frac{\frac{1}{2} a x \dot{x}}{\sqrt{x^2+c^2}}$, cujus Fluens eſt $\frac{1}{2} a \sqrt{x^2+c^2} - Q =$ Areæ Curvæ. Ponatur $x = 0$, & fiet $\frac{1}{2} ac - Q = 0$, unde $Q = \frac{1}{2} ac$, & Area Curvæ fit $= \frac{1}{2} a \sqrt{x^2+c^2} - \frac{1}{2} ac$.

In Spirali Hyperbolica evaneſcit quantitas c, & Area Curvæ fit $\frac{1}{2} ax$.

Si p ſit $= \frac{a\dot{x}}{\sqrt{b^2-x^2}}$, oſtenſum eſt eſſe $KN = \frac{a\dot{x}}{\sqrt{c^2-x^2}}$, unde $\frac{1}{2} CI \times KN = \frac{\frac{1}{2} a \times x \dot{x}}{\sqrt{c^2-x^2}}$, cujus fluens eſt $Q - \frac{1}{2} a \sqrt{c^2-x^2}$ = Areæ. Fiat $x = 0$, & erit $Q - \frac{1}{2} ac = 0$, ſeu $Q = \frac{1}{2} ac$; unde erit Area Curvæ ſemper æqualis $\frac{1}{2} ac - \frac{1}{2} a \sqrt{c^2-x^2}$. Fiat $c^2 - x^2 = 0$ ſeu $c = x$, & Area curvæ fit $\frac{1}{2} ac$. Unde ſi initium Areæ non capiatur ab initio ipſius x, ſeu ubi x eſt $= 0$, ſed ubi $x = c$ eſt maxima, hoc eſt ſi Area ab V incipiat, (*vid. Fig.* VII.) erit Area ſemper æqualis $\frac{1}{2} a \sqrt{c^2-x^2}$.

De Areis quas deſcribunt corpora radiis ad centrum ductis, urgente vi centripeta quæ ſit reciproce ut diſtantiarum cubi, ſequentia adnotavit Collega meus peritiſſimus Geometriæ Profeſſor *Halleius*. Nempe ſi corpora diverſos circulos vel diverſas Spirales Hyperbolicas hac lege deſcribunt; erunt areæ ſectorum, tam in Circulis quam in Spiralibus illis omnibus; æqualibus temporibus deſcriptæ, ſemper æquales: Nam velocitates corporum in circulis motorum ſecundum hanc legem, debent eſſe radiis ſeu diſtantiis reciproce proportionales, adeoque arcus ſimul percurſi erunt quoque in eâdem radiorum reciprocâ ratione, unde ſtatim patebit ſectores ſimul deſcriptos eſſe æquales.

In reliquis omnibus Curvis cum ſit velocitas ad velocitatem corporis in eadem diſtantia in circulo moti ut $\frac{a}{b} \times x$ ad p, (*vide* Fig. III.) ſeu ut $\frac{a}{x} \times IK$ ad KN; interea dum corpus in Trajecto-

ria

ria percurrit lineolam IK, corpus aliud in circulo in eadem distantia motum percurret arcum $\frac{b}{a} \times KN$; & Area sectoris Circuli & Trajectoriæ simul descriptæ erunt $\frac{b}{a} \times KN \times \frac{1}{2} CN$ & $KN \times \frac{1}{2} CN$, quæ duæ Areæ sunt in ratione data, scil. ut b ad a. Adeoque ubi est $a = b$, uti fit in Spirali Hyperbolica, Area sic descripta erit semper æqualis Areæ sectoris circularis in æquali tempore descriptæ.

November 24. 1713.

III. *Rules for correcting the usual Methods of computing Amounts and present Values, by Compound as well as Simple Interest; and of stating Interest Accounts. Offer'd to Consideration, by* Thomas Watkins, *Gent. F. R. S.*

I. Of Compound Interest.

THE Supposition whereon the Method of computing by Compound Interest is founded; *viz.* That all Interest Money, Rents, *&c.* are or may be constantly receiv'd, and put out again at Interest, the Moment they become due, without any Charge, or Trouble, being impracticable; therefore all Computations by this Method (except of Fee-Simples or other Perpetuities) must needs be erroneous. Thus for Instance, the Amount of a Sum of Money, or Annuity, for want of Deductions out of the Profits, for the unavoidable Trouble, Charge, and Delay in the Management, will be too great: and for the same reason, the present value of a Sum of Money payable in any time to come, will be too little; also the present value of an Annuity (being only the Amount of the difference between the Annuity, and Interest of the said present value) will be too much. But in long terms of Years, as that difference becomes less so does the Error, as the term is

great-

greater; and in Fee-Simples it vaniſhes; the contrary to which happens in Amounts of Sums of Money, and Annuities.

All which is propos'd to be rectify'd, only by a juſt reduction of the Rate, and Annuity; (which is done by deducting ſo much *per Cent.* thereout, as the whole Trouble, and Charge of Management is ſuppos'd to amount to, and reducing the Remainder, by a Diſcount equivalent to the ſuppos'd loſs of time) and then by working with the Rate ſo reduc'd, for Sums of Money, and with the Rate and Annuity reduc'd in the like proportion for Annuitys, according to the common Method of Compound Intereſt; as follows. Put r for the rate of Intereſt of 1 *l.* c for the Charge and Trouble of the Management of 1 *l.* Then is $r - cr =$ the Rate after deducting the ſaid Charge, $=$ (putting d for $1 - c$) dr. And for the Diſcount put t for the time loſt, that is for ſuch part of the Period of time in which the Payments are made (whether Yearly, ½ Yearly, Quarterly, or otherwiſe) as is ſuppos'd to be ſpent in receiving and putting them out again at Intereſt. Then, dtr, being $=$ the Intereſt of 1 *l.* for that time; ſay, as $1 + dtr : 1 :: dr : \frac{dr}{1 \times dtr} =$ (putting e for $1 + dtr$) $\frac{dr}{e}$, which is equal to the reduc'd Rate, near enough for practice, for which put $\mathfrak{r}$. But if the utmoſt accuracy be requir'd, the Diſcount itſelf muſt be made with regard to the like loſs of time, which is done by a Series of Diſcounts rais'd thus; $e(= 1 + tdr) : tdr :: dr : \frac{td^2 r^2}{e} :: \frac{t d^2 r^2}{e} : \frac{t^2 d^3 r^3}{e^2}$ &c.

Whence $dr - \frac{td^2 r^2}{e} + \frac{t^2 d^3 r^3}{e^2} - \frac{t^3 d^4 r^4}{e^3}$ &c. $=$ (putting q for $\frac{tdr}{e}$) $\overline{1 - q + q^2 - q^3 \text{ \&c.}} \times dr. = \frac{dr}{e}(= \overline{1 - q} \times dr) + \overline{q^2 - q^3 \text{ \&c.}} \times dr$, is $= \mathfrak{r}$, $=$ the true Rate reduced. Put $s = 1 + \mathfrak{r}$, $n =$ the time, $p =$ the preſent Sum or Value, $m =$ the Amount.

Then

Then will $\overline{1 + \frac{dr}{e} + q^2 - q^3 \&c. \times dr}|^n \times p = \overline{1 + \mathfrak{r}}|^n \times p = p s^n$, be exactly $= m$: But $\overline{1 + \frac{dr}{e}}|^n \times p = m$ is sufficient for practice.

And for the Amounts and present values of Annuitys.

Put A = Annuity *per annum*.
a = Annuity $\frac{1}{2}$ yearly, quarterly, *&c.*
R = Yearly rate of Interest of 1 *l.*
r = Rate $\frac{1}{2}$ yearly, quarterly, *&c.*
$\mathfrak{r}$ = Reduc'd rate yearly, $\frac{1}{2}$ yearly, quarterly, *&c.*
n = Number of Years, $\frac{1}{2}$ years, quarters, *&c.*

Then will $A \frac{\mathfrak{r}}{R} = a \frac{\mathfrak{r}}{r}$ be = reduc'd Annuity taken yearly $\frac{1}{2}$ yearly, quarterly, or otherwise; and by Compound Interest twill be $\frac{\overline{1 + \mathfrak{r}}|^n - 1}{\mathfrak{r}} \times a \frac{\mathfrak{r}}{r} = \frac{\overline{1 + \mathfrak{r}}|^n - 1}{r} a = \frac{\overline{1 + \mathfrak{r}}|^n - 1}{R} A = \frac{s^n - 1}{R} A = m$, and $\frac{s^n - 1}{R s^n} A \left(= \frac{m}{s^n}\right) = p$. Whence the Theorems for solving all the other Cases are easily deduc'd. And if the Rate be requir'd, when 'tis for a Sum of Money, the Solution is obvious: when for the Amount or Value of an Annuity, since $\overline{1 + r}|^n = \frac{mr + a}{a} = \frac{a}{a - pr}$ are the Equations whence Theorems for the Rate are usually deriv'd, which by this Correction become $\overline{1 + \mathfrak{r}}|^n = \frac{m r + a}{a} = \frac{a}{a - pr}$. That the same $\mathfrak{r}$ may be had on both sides the Equation, put $u\mathfrak{r}$ for r, and 'twill be $\overline{1 + \mathfrak{r}}|^n = \frac{m u \mathfrak{r} + a}{a} = \frac{a}{a - pu\mathfrak{r}}$: then, by the Rate assum'd as near the truth as may be, find the value of $u \left(= \frac{1}{d} + t r = \frac{1}{d - td\mathfrak{r}}\right)$ and in any Theorem for the

Rate,

Rate, putting mu for m, and pu for p, the result will be the Rate reduced nearly: and by repeated Operations correcting r and thereby n, the true r, and thence R the whole Rate, may be found.

The only difficulty that remains, is the right assuming the Quantities c and t, the impossibility of doing which with perfect Exactness, I suppose to be the reason why neither this, nor any Method of Correction to the like purpose, has yet been taken notice of by the Writers on this Subject; and what may therefore be very likely to be objected to this. But the same Objection I take to be of equal force against the Estimates of any other Uncertainties whatsoever, as Estates for Lives, Insurances *&c.*

First then, for the Quantity c, which is put for the Trouble and Charge of Management, *viz.* of collecting and placing out the Money on good Security, together with all Contingencies attending the same, as travelling Charges, Expences, Attorney's Bills, *&c.* of all which, the principal Article is the Charge of Collection or Receiver's Fees, which is commonly a fix'd Rate, customarily allow'd in the Place, or upon the Estate it self out of which the Purchase is made, if it be a Rent; and for Interest Money, or any other Annuity, the like Estimate is to be made, whether the Proprietor acts for himself, or by another. Then for the Charge of placing out the Mony at Interest when receiv'd; though this be for the most part defray'd by the Borrower, yet because it highly concerns the Lender to see it be securely done, there are usual Allowances made to Agents and Scriveners, to encourage their Care and Fidelity therein; besides the Time, Expence and Trouble of the Proprietor himself, in finishing Contracts, inspecting Securities *&c.* and whatever is sav'd in this Article, we must suppose to be fully made up by an Equivalent Degree of Risque in the Security.

In the next place, for the loss of time; though 'tis also impossible for this to be exactly ascertain'd, nor perhaps so nearly as the former, since it depends very much on the Diligence of the Manager: Yet if the usual times of Payment of

of the particular Rent or Annuity to be purchaſed, with a moderate degree of diligence in the Manager, and the uſual indulgence practis'd by Men of Buſineſs in this Caſe towards one another be obſerv'd, a reaſonable Eſtimate may be alſo made of the loſs of time; In which 'tis to be noted, that Intereſt Money being uſually paid in ſmall Sums, when any Sum of Money to be made up of ſeveral ſuch Payments, is intended to be put out at Intereſt, the whole muſt lie dead till the laſt Payment be made; alſo that the Principal lies dead ſometimes as well as the Intereſt; and that on the other hand, to ſave time, Borrowers may be found out, and treated with during the time of Collection; but this Advantage is in a great Meaſure loſt by the difficulty of fixing the time or *Quantum* of a Loan, till the whole be paid in. Note alſo, that if the Charge of Collection, or Loſs of time, on the Rent, or Annuity, of any particular Eſtate or Place, be found to differ from that of the Intereſt of the Purchaſe Mony, and ſo much exactneſs be required, as that the Computation be made with regard to ſuch difference: It muſt be done, either by aſſuming a proper Medium for both, or more accurately thus: For the reduc'd Rate of the $\left\{\begin{matrix}\text{Annuity}\\ \text{Intereſt-Mony}\end{matrix}\right\}$ put $\left\{\begin{matrix}\mathfrak{z}\\ \mathfrak{r}\end{matrix}\right\}$ Then $r : \mathfrak{z} :: a : a\frac{\mathfrak{z}}{r}$, and $\mathfrak{r} : \overline{1+\mathfrak{r}}|^n - 1 :: a\frac{\mathfrak{z}}{r} : \frac{\overline{1+\mathfrak{r}}|^n - 1}{r} a \times \frac{\mathfrak{z}}{\mathfrak{r}} = m = \overline{1+\mathfrak{r}}|^a \times p$.

To give a Specimen of this Method in Numbers, firſt the Quantities c and t are to be aſſum'd, which are not here to be accomodated to any particular Place or Eſtate, but to be taken in general: And firſt for c the Charge of Management; the uſual Rates of Collectors Fees in theſe Kingdoms, as I am inform'd, are 6*d.* 12*d.* and 18*d. per* Pound, but the moſt uſual 12*d.* which is 5 *per Cent.* However to be within compaſs, I ſhall take 4 *per Cent.* for the Medium, including what further

 trou-

trouble and charge may attend the Receipt of the Mony, besides Receivers Fees; and 2 *per Cent.* for all the other Charges before mention'd, in placing it out at Interest, both which make 6 *per Cent.* so that c is $=0,06$, and d $(=1-c)=0,94$. Next for t the Loss of Time; since few Annuities are paid yearly, and a Discount being given for the Loss of Time, we are to lose no more than is discounted for; therefore I choose Half-yearly Payments for the Examples, being the most usual, which with little Alteration may serve for quarterly; and considering the before-mention'd Circumstances relating to the Time, I look upon two Months the least, and seven or eight Months the most, that can well be suppos'd to be spent, one Time with another, in receiving and putting out the Mony, upon a moderate Management; between which the Medium is about four Months and half, which being $\frac{3}{4}$ of $\frac{1}{2}$ a Year, gives $\frac{3}{4}=t$: and if $\frac{dr}{e}$ be $=\mathfrak{r}$, the yearly Rates of 4, 5, 6, 8 and 10 *per Cent.* will produce for Half-yearly Rates reduc'd of 1 *l.* 0.018539, 0.023093, 0.027616, 0.036569 and 0.0454 each $=\mathfrak{r}$. But if $\mathfrak{r}$ be $=\frac{dr}{e}+\overline{q^2-q^1}\,\&c.\times dr$, 'twill be, 0.01854219, 0.0230999155, 0.027627775, 0.036596289 and 0.04545235, each $=\mathfrak{r}$, (so that each Rate loses by this Estimate about $\frac{1}{12}$ Part.) Whence the following Amounts, and present values of 1 *l. per Annum* computed Half-yearly, are produc'd, and compared with those of the usual Method computed yearly, to agree with the common Tables.

Years

Years.	Amounts of 1 l. per An. at 5 per Cent. by Comp. Int.	Co. Int. cor.	Differences.	Amounts of 1 l. per An. at 6 per Cent. by Comp. Int.	Co. Int. cor.	Differences.
5	5,52563	5,13104	,39459	5,63709	5 22138	,41571
10	12,57789	11,57846	,99943	13,18079	12,07854	1,10225
20	33,06595	29,85996	3,20599	36,78559	32,91054	3,87505
30	66,43885	58,72546	7,71339	79,05819	68,83976	10,21843
40	120,79977	104,30079	16,49898	154,76197	130,80733	23,95464
60	353,58372	289,88164	63,70208	533,12818	422,01429	111,11389
80	971,22882	752,53436	218,69446	1746, 5999	1288, 2484	458,3515
100	2610,02516	1905,92671	704,09845	5638, 3680	3864, 9753	1773,3927
	The same at 8 *per Cent.*			The same at 10 *per Cent.*		
5	5,86660	5,40633	,46027	6,10510	5,59705	,50805
10	14,48656	13,15092	1,33564	15,93743	14,32680	1.61063
20	45,76196	40,13759	5,62437	57,27500	49,17930	8,09570
30	113,28321	95,51623	17,76698	164,49402	133,96428	30,52974
40	259,05652	209,15737	49,89915	442,59257	340,21900	102,37357
60	1253,2133	920,90040	332, 3129	3034,81648	2062,57167	972,24481
80	5886,9354	3918,0578	1968, 8776	20474,0027	12255,3348	8218, 6679
100	27484,5157	16539,0989	10945,4168	137796,127	72575,3926	65220,7344

Years.	Pref. Values of 1 l. per An. at 5 per Cent. by Comp. Int.	Co. Int. cor.	Differences	Pref. Values of 1 l. per An. at 6 per Cent. by Comp. Int.	Co. Int. cor.	Differences
5	4,32948	4,08343	,24605	4,21236	3,97582	,23654
10	7,72174	7,33314	,38860	7,36009	7,00322	,35687
15	10,37966	9,91935	,46031	9,71225	9,30843	,40382
20	12,46221	11,97753	,48468	11,46992	11,06373	,40619
30	15,37245	14,91902	,45343	13,76483	13,41805	,34678
40	17,15909	16,78200	,37709	15,04630	14,78309	,26321
50	18,25593	17,96190	,29403	15,76186	15,57456	,18730
70	19,34268	19,18247	,16021	16,38454	16,29953	,08501
100	19,84791	19,79231	,05560	16,61755	16,59510	,02245
F.S.	20,00000	20,00000	,00000	16,66666	16,66666	,00000

Note, That this Correction is alſo applicable to the Valuations of Eſtates for Lives, in which the firſt Step being to find an Equivalent in Years of Continuance, brings them to the Caſe of Eſtates for Years.

II. Of Simple-Intereſt.

The abſurdity of the Suppoſition, on which the uſual Method of computing preſent Values by Simple-Intereſt, is founded, *viz.* That the Rent or Annuity is conſtantly received, and put out again at Intereſt, as it becomes due; but that the Intereſt of the Purchaſe Mony lies dead during the whole Term, is ſo apparent, and the Errors ariſing from it ſo groſs, that the Writers who have laid down this Method, have at the ſame time caution'd againſt the Uſe of it for any more than 6 or 7 Years, the Error for that time being not conſiderable.

The ſame Suppoſition does alſo occaſion the miſcomputation of Amounts, or rather the miſapplication of them to their proper Caſes. Wherefore, ſince the Simple-Intereſt of Mony is of equal Value *pro rata*, and of the ſame regard with Rents or Annuitys, being each the Original Profits iſſuing alike from a principal Stock, Eſtate or Value, and equally improveable; This general Rule may ſerve for a juſt Correction of this Method, *viz.* That ſuppoſing in any Caſe an Intereſt ought to be, or not to be allowed to either of thoſe Profits, the ſame be done in the like Caſe to the other. Thus, in the Caſe of Debts, or Amounts of Sums of Mony, Rents, or Annuitys for the time paſt, its uſual in practiſe to allow no Intereſt to either: For though the Law, to curb the exorbitant Avarice of Uſurers, and for other Reaſons, does more expreſly diſallow Intereſt upon Intereſt for a Debt; our Courts both of Law and Equity, as I am inform'd, will be as far from allowing the Charge of Intereſt againſt a Tenant for Rent in Arrear; except on a *nomine pœnæ* (which is now become almoſt obſolete) ſo that in this Caſe (putting a and r for the Annuity and Rate yearly, Half-yearly, or otherwiſe) as $prn + p$ is the Amount of a Sum of Mony, ſo is an the Amount of an Annuity or Rent in Arrear, and not $\frac{n-1}{2} r + 1 \times an$ as Arithmeticians commonly make it. But in the Computation of preſent Values, or Amounts for the time to come,

come, the ſame being made on the Expectation of a conſtant regular Income of the Profits, without any extraordinary Interruption, an Intereſt ought to be allow'd to both, eſpecially in preſent Values, which are found by ſetting the Amounts of both againſt each other: ſo that in theſe Caſes, putting $x = n - 1$, if $\overline{\frac{1}{2}xr + 1} \times an$, be made the Amount of a Rent or Annuity; then $\overline{\frac{1}{2}xr + 1} \times prn + p$, will be the proper Amount of a Sum of Money, and not $prn + p$: and conſequently $\frac{\overline{\frac{1}{2}xr + 1} \times an}{\overline{\frac{1}{2}xr + 1} \times rn + 1} = p$ will be the preſent Value of a Rent or Annuity, the ſubſequent Intereſt being remitted on both ſides in lieu of the Loſs of Time and Charge of Management; which ſuch as are apt to depreciate long Futurities, may think the propereſt Method of approaching the true Value; but I rather look upon the former Method of Compound Intereſt corrected as more exact, as well as more general, the Intereſt remitted in this, being in ſhort Terms leſs, and in long Terms more than an Equivalent for the Trouble, Charge and Delay in the Management. But it is however the moſt exact of any of the Methods, that have yet been deduc'd from Simple-Intereſt. The reduc'd Rate may alſo in ſome Caſes be properly made uſe of for Amounts, but not for preſent Values, except for ſhort Terms; and then, ſince $r : a :: \overline{\frac{1}{2}x\mathfrak{r} + 1} \times \mathfrak{r}n : \overline{\frac{1}{2}x\mathfrak{r} + 1} \times an \frac{\mathfrak{r}}{r}$, 'twill be $\overline{\frac{1}{2}x\mathfrak{r} + 1} \times an \frac{\mathfrak{r}}{r} = m = \overline{\frac{1}{2}x\mathfrak{r} + 1} \times p\mathfrak{r}n + p$, and $\frac{\overline{\frac{1}{2}x\mathfrak{r} + 1} \times an}{\overline{\frac{1}{2}x\mathfrak{r} + 1} \times \mathfrak{r}n + 1} \times \frac{\mathfrak{r}}{r} = p$.

Exam-

Examples of this Method compar'd with those of the former, will stand as follows; in which all is computed Half-yearly, except the last Column of Compound Interest.

Amounts of 1 *l.* at 5 *per Cent.* computed 6 several Ways.

Years.	1	2	3	4	5	6
	Simple Int.	*Id. for Bonds*	*Sim. Int. cor.*	*Id. by the red. rate*	*Co. Int. cor.*	*Comp. Int.*
	$prn + p = m$	Id. till $prn = p$	$\frac{1}{2} \times r + 1 \times prn + p = m$	$\frac{1}{2} \times r + 1 \times pr^n + p = m$	$\overline{1+r}^n \times p = m$	$\overline{1+R}^N \times p = m$
5	1,25	1,25	1,27813	1,25501	1,25655	1,27628
10	1,5	1,5	1,61875	1,56338	1,57892	1,62889
20	2,	2,	2,48750	2,34021	2,49300	2,65330
30	2,5	2,	3,60625	3,33048	3,93625	4,32194
40	3,	2,	4,97500	4,53419	6,21504	7,04000
60	4,	2,	8,46250	7,58194	15,49408	18,67919
80	5,	2,	12,95000	11,48345	38,62672	49,56144
100	6,	2,	18,43750	16,23874	96,29634	131,50126
	Amounts of 1 l. *at* 6 per Cent. *by the same Theorems.*					
5	1,3	1,3	1,3405	1,31063	1,31328	1,33822
10	1,6	1,6	1,7710	1,69758	1,72471	1,79084
20	2,2	2,	2,9020	2,70048	2,97463	3,20713
30	2,8	2,	4,3930	4,00870	5,13039	5,74349
40	3,4	2,	6,2440	5,62223	8,84844	10,28572
60	4,6	2,	11,0260	9,76525	26,32086	32,98769
80	5,8	2,	17,2480	15,12954	78,29488	105,79599
100	7,	2,	24,9100	21,71510	232,89852	339,30208
	Amounts of 1 l. *at* 10 per Cent. *by the same Theorems.*					
5	1,5	1,5	1,6125	1,54749	1,55970	1,61051
10	2,	2,	2,4750	2,30157	2,43268	2,59374
20	3,	2,	4,9500	4,42951	5,91793	6,72750
30	4,	2,	8,4250	7,38381	14,39643	17,44940
40	5,	2,	12,9000	11,16449	35,02190	45,25925
60	7,	2,	24,8500	21,20493	207,25717	304,48165
80	9,	2,	40,8000	34,55084	1225,5335	2048,4003
100	11,	2,	60,7500	51,20222	7258,5398	13780,6127

Years

Years.	*Pref. Values of* 1 l. per An. *at* 5 per Cent. *by*			*Pref. Values of* 1 l. per An. *at* 6 per Cent. *by*		
	Sim. In. cor.	*Co. Int. cor.*	*Comp. Int.*	*Sim. In. cor.*	*Co. Int. cor.*	*Comp. Int.*
5	4,35208	4,08343	4,32948	4,23349	3,97582	4,21236
10	7,64478	7,33314	7,72174	7,25579	7,00322	7,36009
15	10,10821	9,91935	10,37966	9,39341	9,30843	9,71225
20	11,95980	11,97753	12,46221	10,92350	11,06373	11,46992
30	14,45407	14,91902	15,37245	12,87275	13 41805	13,76483
40	15,97990	16,78200	17,15909	13,99744	14,78309	15,04630
50	16,96682	17,96190	18,25593	14,69544	15,57456	15,76186
70	18,10986	19,18247	19,34268	15,47252	16,29953	16 38454
100	18,91525	19,79231	19,84791	15,99759	16,59510	16,61755
F.S	20,00000	20,00000	20,00000	16,66666	16,66666	16,66666

The Theorems to the preceding Columns of Amounts (of which the fourth aud fifth are infinitely variable in the Result, by assuming *c* and *t* in the reduc'd Rates at Pleasure) may serve to answer all the simple Cases of Amounts that occur in Business: To instance in some,

1. The first Column contains the Amounts of such Debts, or Sums of Mony as carry a simple Interest till the Principal be paid.

2. The second Column answers the common Case of Debts due by Bond, that by Law are allow'd not to exceed the Penalty, which is generally double the principal Debt.

3. The third Column answers the Case of a Security or joynt Obligor, that has duly and constantly paid the Interest, and at last the principal Sum of a Debt, from which he has a Counter-Bond from the principal Debtor to save him harmless, against whom he may make his Charge from this Column.

4. In case the Parties shall agree that the Debt shall lie for any time certain, or uncertain; and for the much greater ease, advantage and satisfaction of both of them, no Interest to be call'd for, till the Principal it self is paid; but to carry Interest as it becomes due, the Lender allowing for the time and charge he must have been at, in receiving and putting out his Interest, the fourth Column will fit this Case, or else the fifth

as

as a greater or leſs degree of Lenity is agreed upon in favour of the Borrower.

5. The fifth Column is alſo proper in the following Caſe, *viz* if it be demanded, what Eſtate in reverſion, after a certain number of Years, any Sum in Hand will Purchaſe; the firſt ſtep being to find the Amount of that Sum to the time the Reverſion commences, its had in this Column.

6. The laſt Column gives the Amount of a Sum of Money, according to the common Method of Compound Intereſt, but being computed with that extraordinary rigor as has been ſaid, (except ſome ſmall allowance for the loſs of time, by being done Yearly) 'tis hardly ſuitable to any Caſe.

Other Caſes might be enumerated, to which the foregoing Theorems might be equitably apply'd; beſides ſuch extraordinary ones, wherein it may appear to Arbitrators, or a Court of Equity, that either Partie deſerves Favour, either by way of Compenſation for Injuries ſuffer'd from the other, by means of any fraudulent or oppreſſive Practices, or otherwiſe, for which no other redreſs is provided.

III. Of Intereſt Accounts.

The Inequality of the uſual Method of ſtating Intereſt Accounts, as practis'd in our Courts of Equity, will beſt appear by an Example, for which I ſhall take the common general Caſe of an Intereſt Account to be ſtated on a Mortgage, *viz.* ſuppoſe 1000 *l.* to be let out at 6 *per Cent.* on a Mortgage of 120 *l. per annum,* payable Half Yearly, and the Mortgagee after five Years end, to have Poſſeſſion till the Arrear of Intereſt, accruing Intereſt and Principal be diſcharged: *Quære,* How long that will be? ſuppoſing alſo, for the ſake of brevity in the Account, the Payments to be equally, and punctually made as they become due. By the Chancery Method, the Rent is firſt apply'd to diſcharge the Arrear of Intereſt; and then the remainder of every Half-Year's Rent, after deducting the ſame Half-Years Intereſt, is apply'd towards the Diſcharge of

of the Principal, and thereby the Principal and Intereſt continually leſſens, till the whole be paid off. Now by this means the Mortgagee, after the Arrear is diſcharged, pays Comp. Intereſt, with the utmoſt rigour, for ſo much *per annum* of the Rent, as exceeds the Intereſt of the whole Principal Mony, and receives but Simple Intereſt for his Debt; which, however ſtrange it may ſeem, is eaſily prov'd, by applying the proper Theorems of Simple and Compound Intereſt to this Caſe, in which the Annuity, Principal Money, Rate and Arrear of Intereſt are given, and the time requir'd; the reſult being the ſame with that of the Chancery Method, except a very ſmall difference only when any part of the time is expreſs'd by a Fraction: *viz.* putting L for Logarithm, $\alpha = a - pr = 30$, $s = 1 + r = 1{,}03$, $t =$ time of contracting the Arrear $= 10$ Half-years, $n =$ any number of $\frac{1}{2}$ Years ſpent in diſcharging the whole or any part, $N =$ number of Years required; the Equation for the Arrear will be $prt + prn = an$; and for the Principal and accrueing Intereſt $prn + p = prn + \frac{s^n - 1}{r} \times \alpha$. Whence

$\frac{prt}{2\alpha} + \frac{La - L\alpha}{2Ls} = N = 16{,}7249$ Years $=$ the time demanded;

i.e. $\frac{prt}{2\alpha} = 5$ Years $=$ the time of diſcharging the Arrear, and

$\frac{La - L\alpha}{2Ls} = 11{,}7249$ years, $=$ the time in which the Principal and accrueing Intereſt is diſcharged; during which its evident, the Mortgagee pays full Compound Intereſt for 60 *l. per annum* of the Rent. For the Correction of which inequality, in the firſt Place, to the end that neither Branch may exceed, or be depriv'd of its due Profits; This general Rule is propos'd as neceſſary to be always obſerv'd, *viz.* That Amounts of the Produce on each ſide be ſtated ſeparately, and ſet againſt each other in the Account, in order to a Ballance. And in the common Caſes of Mortgages, Government, and

Stock Securities, *&c.* where the Debt is paid off by a Rent, Annuity, Pension, Dividend, or other Payments issueing in the same manner, and with the like trouble, charge, *&c.* as Interest Money does; I presume this Rule will also be easily admitted, *viz.* That the same equitable Advantage be impartially allow'd on both sides; for which the Method of Simple Interest, as corrected under the foregoing Head, seems truly adapted; whereby the Original Profits on each side are supposed to be deem'd, either as Interest, or else as Principal Mony; and since the Amounts both of an Annuity and Sum of Mony, for the time past, as there stated, on the first of these Suppositions (t being there $= 0$) are likewise vouch'd by our Laws, and the practice of our Courts, to be good when separately us'd; I think its very evident, that the Account ought to be stated by setting those Amounts against each other thus, $prt + prn + p = an$, (whence $\frac{1 + tr}{2a} p = N, = 21,6666$ Years) and that this Method is most proper for general Use, in the Cases mention'd: Unless it should be thought fit, in consideration of the various Ways found out for the ready improvement of Mony, to allow a further Advantage on both sides, by charging the Original Profits as Principal Mony, and giving a Simple Interest there to, which still falls short of the Advantage allow'd to Rents by the Chancery Method. And this is to be done two ways, *viz.* either by applying an Amount of Rent to pay off the Arrear first, and afterwards another Amount of Rent to discharge the Principal, and accrueing Interest; or else by letting the Profits with all Arrears and other Charges run on at Simple Interest on each side, till the end of the Term: *viz.* putting $x = n - 1$, $y = t - 1$, $a = a - pr$, $f = \overline{2 - r} \times a$, $g = f - 2ptr^2$, $\mu = \overline{\frac{1}{2}yr + 1} \times prt =$ Arrear of Interest; by the first of these 'twill be, for the Arrear, $\mu + ptr^2n = \overline{\frac{1}{2}xr + 1} \times an$, and for the Principal and accruing Interest, $\overline{\frac{1}{2}xr + 1} \times an = p$; Whence

$\sqrt{8\mu a}$

$$\frac{\sqrt{8\mu\alpha r+g^2}-\sqrt{8p\alpha r+f^2}-f+g}{4\alpha r}=N=18{,}7653$$

Years. By the other 'twill be, for the whole, $\mu+ptr^2n+p=\overline{\tfrac{1}{2}xr+1}\times\alpha n$, Whence $\frac{\sqrt{\overline{\mu+p}\times 8\alpha r+g^2}-g}{4\alpha r}=N=$ 18,1648 Years. The Lender will have to alledge for the first of these two Ways, that as the Rent is not hindred by any other parallel Charge from making the utmost produce it can, so for that reason ought his Principal Mony to have the Advantage of the Arrears being first discharged, which also agrees with the Chancery Method in this particular.

Lastly, another way of stating this Account, may be taken from that Notion of Simple-Interest, whereby the Annuity only is charged as Principal Mony: and then 'twill be, for the Arrear, $prt+prn=an$, and for the Principal and accruing Interest, $\overline{\tfrac{1}{2}xr+1}\times\alpha n=p$; Whence $\frac{\sqrt{8p\alpha r+f^2}-g}{4\alpha r}\left(=\frac{prt}{2\alpha}+\frac{\sqrt{8p\alpha r+f^2}-f}{4\alpha r}\right)=N=$ 17,3072 Years; which appears to be same with the Chancery Method, only that the Compound Interest in that, is turned into Simple in this; and as it still retains part of the same inequality, to the Advantage of the Borrower, it seems only fit to be observed in such Cases wherein the Borrower may be thought to merit favour, as when the Debt is paid out of the Profits of Trade, arising by extraordinary Risque or Industry. But since such a Rule of distinction is hardly possible to be reduc'd to general practice, the use of this Theorem seems restrained to such Cases only, wherein the Parties themselves, or a Court of Equity shall think it reasonable.

For a further illustration of these Rules, the following Specimen is added, which shews at sight, how the Results of the several Methods differ, as the Rent, Arrear, or Rate of Interest, is greater or less, and consequently of how much more

or

or leſs concern it is to the Parties, as well as to the due Adminiſtration of Juſtice, to have regard thereto.

The time requir'd in	Years (computed Half-yearly.)			
To diſcharge a Mort. of 1000 *l.* by a Rent of	120 *l. per annum.*		90 *l. per annum.*	
At the Rate of Int. *per annum* of	5 *per C.*	6 *per C.*	5 *per C.*	6 *per C.*
No Arrear of Intereſt.				
By the Chancery Method	10,9141	11,7249	16,4205	18,5835
By the ſame turning the Comp.Int. into Sim.	11,2579	12,3072	17,5335	20,7400
By Simp.Int.cor. theOrignal Profits charged as Princ. and the Arrear Intereſt, contin^d at Int. first diſcharg'd	14,2857	16 6666	25,0000	33,3333
5 Years Arrear of Intereſt.				
By the Chancery Method	14,4855	16,7249	22,6705	28,5835
By the ſame turning the Comp.Int. into Sim.	14,8293	17,3072	23,7835	30,7400
By Simp.Int.cor. theOriginalProfits charged as Princ. and the Arrear Intereſt, contin^d at Int. first diſcharg'd	15,3191	18,1648	24,7148	32,7664
	15,5897	18,7653	25 4899	34,8052
	17,8571	21,6666	31,2500	43,3333
10 Years Arrear of Intereſt.				
By the Chancery Method	18,0569	21,7249	28,9205	38,5835
By the ſame turning the Comp. Int. into Sim.	18,4007	22,3072	30,0335	40,7400
By Simp.Int.cor. theOriginal Profits charged as Prine. and the Arrear Intereſt, contin^d at Int. first diſcharg'd	20,3161	25,5998	33,6320	48,0874
	21,2895	27,5587	36,1932	53,8107
	21,4286	26,6666	37,5000	53,3333
15 Years Arrear of Int.				
By the Chancery Method	21,6283	26,7249	35,1705	48,5835
By the ſame turning the Comp. Int. into Sim.	21,9721	27,3072	36,2835	50,7400
By Simp.Int.cor. theOriginal Profits charged as Prine. and the Arrear Intereſt, contin^d at Int. first diſcharg'd	26,1353	34,2913	43,9764	65,8386
	28,0209	37,7141	48,5159	74,5664
	25,0000	31,6666	43,7500	63,3333

I muſt alſo obſerve for the ſake of ſuch as are unacquainted with ſpecious Arithmetic, that though for brevitys ſake, the foregoing Theorems and Examples are laid down, and wrought in Algebraic Terms: Yet the ſame Accounts may be ſtated after the Chancery manner it ſelf, according to the ſeveral Principles before deliver'd, and with the ſame Reſults, with this only caution, that (inſtead of a continual deduction of the Rent or Annuity out of the Principal and Intereſt of the Debt, which occaſions the Error before mention'd) the preceding General Rule of ſtating ſeparate Amounts be obſerv'd, which may be done by continually adding the profits together on each ſide, in the ſame manner, as if the Parties were

to

make a ſeparate Charge againſt each other, which is the rather to be noted, as being the only Courſe that can be taken, in caſe the Sums or Times of payment ſhould differ, but the reſpective Reſults will notwithſtanding be analogous to the above Examples.

All which is ſubmitted to the conſideration of more diſcerning Judgments, eſpecially the Applications of the Rules to particular Caſes, for exemplifying the Theorems. But if any of thoſe Rules or Theorems ſhould be objected againſt, meerly becauſe they tend to introduce ſome Alterations in the preſent practice; I ſhall for anſwer only add, with ſubmiſſion, to what is before ſaid, that in former Ages, when our Laws relating to theſe matters had their riſe, (the Profits of *England* ariſing chiefly from Husbandry and Tillage, and little from Trade,) the Caſh of the Kingdom was but low, the Rates of Intereſt very high to the Advantage of Uſurers, and thoſe ways for the ready Improvement of Money accomodated to all Peoples uſe, not known; (much like to which we are told was the State of the Jewiſh Affairs, when they were forbidden to take Uſury of any but Strangers.) But latter Ages have produc'd vaſt Alterations in all theſe Reſpects, which having happen'd by inſenſible degrees, may be one reaſon why neither our Legiſlature, nor Courts of Judicature have yet taken ſuch notice thereof, as time and leiſure, with the Tender of proper and practical Methods of Computation, may hereafter induce them to do.

F I N I S.

LONDON: Printed for W. Innys at the *Prince's Arms* in St. *Paul's Church-Yard*. MDCCXV.

Intrat. Depositum		Sors	*Exit.* Depositum		Sors	Deposit.	
$n+1$	0	Z	$n+1+p$	1	H	$n+1+$	$A=Z$
$n+1+p$	1	Y	$n+1+2p$	2	K	$n+1+p$	$C=Y$
$n+1+2p$	2	X	$n+1+3p$	3	L	$n+1+2p$	$D=\frac{1}{2}X+\frac{1}{2}\times\overline{Y+yp}$
$n+1+3p$	3	V	$n+1+4p$	4	M	$n+1+3p$	$E=\frac{1}{4}V+\frac{1}{4}\times\overline{X+xp}+\frac{1}{2}\times\overline{Y+2yp}$
$n+1+4p$	4	T				$n+1+4p$	$F=\frac{1}{8}T+\frac{1}{8}\times\overline{V+up}+\frac{1}{4}\times\overline{X+2xp}+\frac{1}{2}\times\overline{Y+3yp}$

N.° 1

N.° 2

$$Z=\tfrac{1}{2}\times\overline{H-p}+\tfrac{1}{4}\times\overline{H-p+hp}+\tfrac{1}{8}\times\overline{H-p+2hp}+\tfrac{1}{16}\times\overline{H-p+3hp}+\cdots\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp-hp}+\tfrac{1}{2^n}\times\overline{np+n+1}$$

$$Y=\tfrac{1}{2}\times\overline{K-p}+\tfrac{1}{4}\times\overline{H-p+2hp}+\tfrac{1}{8}\times\overline{H-p+3hp}+\tfrac{1}{16}\times\overline{H-p+4hp}+\cdots\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp}+\tfrac{1}{2^n}\times\overline{np+p+n+1}$$

$$X=\tfrac{1}{2}\times\overline{L-p}+\tfrac{1}{4}\times\overline{H-p+3hp}+\tfrac{1}{8}\times\overline{H-p+4hp}+\tfrac{1}{16}\times\overline{H-p+5hp}+\cdots\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp+hp}+\tfrac{1}{2^n}\times\overline{np+2p+n+1}$$

$$V=\tfrac{1}{2}\times\overline{M-p}+\tfrac{1}{4}\times\overline{H-p+4hp}+\tfrac{1}{8}\times\overline{H-p+5hp}+\tfrac{1}{16}\times\overline{H-p+6hp}+\cdots\cdots\tfrac{1}{2^n}\times\overline{H-p+nhp+hp}+\tfrac{1}{2^n}\times\overline{np+3p+n+1}$$

N.° 3

$$H=\tfrac{1}{2^{n-1}}\times\overline{C+ncp-cp}+\tfrac{1}{2^{n-2}}\times\overline{D+ndp-2dp}+\tfrac{1}{2^{n-3}}\times\overline{E+nep-3ep}+\tfrac{1}{2^{n-4}}\times\overline{F+nfp-4fp}+\cdots\cdots$$

$$K=\tfrac{1}{2^{n-2}}\times\overline{D+ndp-dp}+\tfrac{1}{2^{n-3}}\times\overline{E+nep-2ep}+\tfrac{1}{2^{n-4}}\times\overline{F+nfp-3fp}+\cdots\cdots$$

$$L=\tfrac{1}{2^{n-3}}\times\overline{E+nep-ep}+\tfrac{1}{2^{n-4}}\times\overline{F+nfp-2fp}+\cdots\cdots$$

$$M=\tfrac{1}{2^{n-4}}\times\overline{F+nfp-fp}+\cdots\cdots$$

N.° 4 N.° 6 N.° 8

$$Y-Z=\tfrac{1}{2}K-\tfrac{1}{2}H+\tfrac{1}{4}hp+\tfrac{1}{8}hp+\tfrac{1}{16}hp+\cdots\cdots\tfrac{1}{2^n}\times hp=\tfrac{1}{2}K-\tfrac{1}{2}H+zp-\tfrac{1}{2}hp=-\tfrac{1}{2^{n-0}}\times C-\tfrac{ncp}{2^n}+zp$$

$$X-Y=\tfrac{1}{2}L-\tfrac{1}{2}K+\tfrac{1}{4}hp+\tfrac{1}{8}hp+\tfrac{1}{16}hp+\cdots\cdots\tfrac{1}{2^n}\times hp=\tfrac{1}{2}L-\tfrac{1}{2}K+zp-\tfrac{1}{2}hp=-\tfrac{1}{2^{n-1}}\times D-\tfrac{ndp}{2^{n-1}}-\tfrac{cp}{2^n}+zp$$

$$V-X=\tfrac{1}{2}M-\tfrac{1}{2}L+\tfrac{1}{4}hp+\tfrac{1}{8}hp+\tfrac{1}{16}hp+\cdots\cdots\tfrac{1}{2^n}\times hp=\tfrac{1}{2}M-\tfrac{1}{2}L+zp-\tfrac{1}{4}hp=-\tfrac{1}{2^{n-2}}\times E-\tfrac{nep}{2^{n-2}}-\tfrac{dp}{2^{n-1}}-\tfrac{cp}{2^n}+zp$$

N.° 5 N.° 7

$$K-H=-\tfrac{1}{2^{n-1}}\times\overline{C+ncp-cp}+\tfrac{1}{2^{n-2}}\times dp+\tfrac{1}{2^{n-3}}\times cp+\tfrac{1}{2^{n-4}}\times fp+\cdots\cdots=-\tfrac{1}{2^{n-1}}\times C-\tfrac{ncp}{2^{n-1}}+hp$$

$$L-K=-\tfrac{1}{2^{n-2}}\times\overline{D+ndp-dp}+\tfrac{1}{2^{n-3}}\times cp+\tfrac{1}{2^{n-4}}\times fp+\cdots\cdots=-\tfrac{1}{2^{n-2}}\times D-\tfrac{ndp}{2^{n-2}}-\tfrac{cp}{2^{n-1}}+hp$$

$$M-L=-\tfrac{1}{2^{n-3}}\times\overline{E+nep-ep}+\tfrac{1}{2^{n-4}}\times fp+\cdots\cdots=-\tfrac{1}{2^{n-3}}\times E-\tfrac{nep}{2^{n-3}}-\tfrac{dp}{2^{n-2}}-\tfrac{cp}{2^{n-1}}+hp$$

N.° 9.

$$Z=A$$
$$Y=C$$
$$X=2D-C-cp$$
$$V=4E-2D-C-2dp-2cp$$

N.° 10 N.° 8

$$C-A=Y-Z=-\tfrac{1}{2^n}\times C-\tfrac{ncp}{2^n}+zp$$

$$2D-2C-cp=X-Y=-\tfrac{1}{2^{n-1}}\times D-\tfrac{ndp}{2^{n-1}}-\tfrac{cp}{2^n}+zp$$

$$4E-4D-2dp-cp=V-X=-\tfrac{1}{2^{n-2}}\times E-\tfrac{nep}{2^{n-2}}-\tfrac{dp}{2^{n-1}}-\tfrac{cp}{2^n}+zp$$

N.° 11

$$C=\frac{A\times 2^n+ap\times 2^n-ncp}{1+2^n}$$

$$D=\frac{C\times 2^n+cp\times\overline{2^{n-1}-\frac{1}{2}}+ap\times 2^{n-1}-ndp}{1+2^n}$$

$$E=\frac{D\times 2^n+dp\times\overline{2^{n-1}-\frac{1}{2}}+cp\times\overline{2^{n-2}-\frac{1}{4}}+ap\times 2^{n-2}-nep}{1+2^n}$$

N.° 12

$$=\frac{\overline{A+ap}\times 2^n-ncp}{1+2^n}$$

$$=\frac{\overline{C+cp}\times 2^n-ndp}{1+2^n}$$

$$=\frac{\overline{D+dp}\times 2^n-nep}{1+2^n}$$

I. Senex sculp.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *October*, *November* and *December*, 1714.

The CONTENTS.

I. *An Account of the* Rain *which fell every Year at* Upminster *in* Essex, *the last Eighteen Years, with Remarks upon that of the Year* 1714. *By* W. Derham, *F. R. S. Also a Comparison of what has been observed of that kind at* Paris, *by M.* De la Hire.

LAST Year having been so remarkably Dry, that Ponds hereabouts are for the most part dry, and the Springs generally either very low or quite failing, I had the Curiosity to make an Extract (out of my Registers of the Weather, *&c.*) of the *Quantity of Rain* which fell at *Upminster* the last 18 Years. The Particulars of which, every Year, may be seen in the following Table. In one Column of which, the Weight of the Rain in Pounds *Troy* and Centesimals of Pounds, may be seen; in the other, the Depth of it in Inches and Centesimals of Inches, or what Height it would have been, had it not been imbibed by the Earth, or lessened by Exhalations, but been suffered to have stagnated on the Ground.

Among the *Dry Years*, 1704 was complained of for one; which I remember the News-Papers reported to have been so considerable at *Venice*, that they were forced to fetch their Water in Barks five Leagues off, as far as the *Brenta*; so that publick Prayers were put up for Rain. Yet we may observe that several other Years were drier than that with us at *Upminster*. But among them all, none comparable to the last Year 1714. In which the whole Quantity of Rain was no more than 55 *l.* 95 Hundredths, or 11 Inches 19 Hundredths; whereas the least Quantity of any of the preceding 18 Years, exceeded 15 Inches in Depth.

What Effects this Drought hath had in the Bodies of Animals, I leave others to judge. It is well known how contagious and fatal a Distemper hath raged among, not only

only our own Black Cattle, but in many other Parts of *Europe*. And I obſerved the *Itch* was epidemical among the poorer ſort, at the beginning of the Year; that the *Meaſles* were very common, ſome Parts of the Year; and that *Pleuriſies* and *Malignant Feavers* infeſted a great many, eſpecially in the Summer Months. But how far theſe Diſtempers might be owing to the Dry Seaſon, I leave to the Judgment of our learned Phyſicians.

With greateſt Duty and Reſpect I am the Society's

Moſt humble Servant, &c.

A TABLE of Rain which fell at *Upminſter*, from the Year 1697, to the Year 1714.

Year	Weight.		Depth.	
	l.	Cent.	inch.	Cent.
1697	77	60	15	52
1698	122	32	24	46
1699	75	54	15	11
1700	95	13	19	03
1701	93	45	18	69
1702	101	89	20	38
1703	119	94	23	99
1704	79	02	15	81
1705	84	62	16	93
1706	121	43	24	29
1707	81	55	16	31
1708	96	09	19	22
1709	132	82	26	56
1710	91	84	18	37
1711	118	02	23	60
1712	118	78	23	76
1713	115	80	23	16
1714	55	95	11	19

To compare with these, we have collected out of the *Memoirs* of the *Royal Academy* of *Sciences*, the Quantity of *Rain* and Dissolved *Snow* which has fallen at the *Observatory* at *Paris* for 23 Years together; according to the accurate Observation of M. *De la Hire*. And that the Comparison might be made more justly, we have reduced the *French* Measure to our own. But it is to be observed that the Diversity of Stile makes the Years not exactly the same, though, as to this Matter, the difference may seem very inconsiderable. We have forborn to make any Remarks upon this Comparison, leaving it to the Consideration of the curious Reader.

	French		*English*	
Anno.	Inch.	Lin	Inch.	Cent.
1689	18	11½	20	23
1690	23	3¾	24	87
1691	14	5¼	15	40
1692	22	7½	24	14
1693	22	8	24	18
1694	19	9	21	07
1695	19	7¾	20	96
1696	19	5½	20	76
1697	20	3	21	60
1698	21	9	23	20
1699	18	8¼	19	93
1700	20	0½	21	38
1701	21	4¼	22	78
1702	16	4	17	42
1703	17	4¼	18	51
1704	19	10½	21	20
1705	13	10¾	14	82
1706	15	3⅝	16	32
1707	17	11	19	11
1708	18	3½	19	51
1709	21	9⅛	23	21
1710	15	8¾	17	10
1711	25	2	26	84

II. *Solutio*

II. *Solutio Generalis Problematis* XV. *propositi à* D. de Moivre, *in tractatu* de Mensura Sortis *inserto Actis Philosophicis Anglicanis* No 329. *pro numero quocunque Collusorum*: *per* D. Nicolaum Bernoulli, Basiliensem, Reg. Soc. Sodalem.

CUM Methodus synthetica, quâ usus est *D. de Moivre* ad inveniendam cujusque Collusoris sortem, in usum verti nequeat tunc quando plures quam tres sunt collusores, ob vix perspiciendam legem progressionis serierum quæ se offerunt; ostendam hic quo modo Analysis in ejusmodi Problematibus, ubi depositum continuo augetur, adhiberi queat: eumque in finem demonstrationem dabo analyticam trium Theorematum, quæ inveni, & quidem diu ante visum D. *Moyvræi* libellum de Mensura Sortis, occasione triplicis quæstionis mihi ab amico circa ludum hunc, quem Galli vocant *le Jeu de la Poule*, propositæ, pro inveniendis scil. probabilitate vincendi, lucro item vel damno cujusque Collusoris, & duratione certaminis.

THEOREMA I.

Si Collusores aliquot *A*, *B*, *C*, *D*, *E*, &c. quorum numerus est $n+1$ & dexteritates sunt æquales, deponant singuli 1, & istis conditionibus certent. 1°. Ut illorum duo *A* & *B* ludum incipiant. 2°. Ut victus locum suum tertio *C* cedat, ita ut ille tertius *C* jam cum victore contendat, quique ex hoc certamine victor evaserit cum quarto *D* ludat, & ita deinceps. 3°. Ut ille depositum totum obtineat, qui omnes collusores successivè vicerit. Dico probabilitates vincendi duorum quorumlibet collusorum sese immediate in ordine ludendi sequentium esse in ratione $1+2^n$ ad 2^n, adeoque expectationes lusorum *A* (*B*), *C*, *D*, *E*, &c esse in progressione Geometrica.

Demonstratio.

Ponantur expectationes vincendi ipsius A vel $B = a$, ipsius $C = c$, ipsius $D = d$, ipsius $E = e$, &c. Porro cum accidere possit, ut collusor aliquis prima vice in ludum intrans inveniat adversarium qui vel nondum, vel semel, vel bis, vel ter, &c. jam successive victor extitit, vocetur expectatio lusoris illius primo casu $= z$, secundo $= y$, tertio $= x$, quarto $= u$, quinto $= t$, &c. Item cum collusor aliquis vinci possit ab adversario qui antea jam vel nullum, vel unum, vel duos, vel tres, &c. collusores successive vicit, ita ut exiens è ludo relinquat adversarium qui vel semel, vel bis, vel ter, vel quater &c. victor extitit, vocetur expectatio seu probabilitas vincendi ejus qui exit è ludo primo casu $= h$, secundo $= k$, tertio $= l$, quarto $= m$, &c. Hisce omnibus positis habebuntur sequentes novem series æquationum signatæ N°. 1. N°. 2. N°. 3, &c. usque ad N°. 9. *Tab.* I. Ratio eas inveniendi breviter hæc est. Inter æquationes N. 1°. reperitur *ex. gr.* $f = \frac{1}{8}t + \frac{1}{8}u + \frac{1}{4}x + \frac{1}{2}y$. Nam collusor F certabit vel cum collusore A, vel B, vel C, vel D, vel E: ut primum vel secundum contingat, oportet ut vel A vel B quater successive victor existat, cujus eventus probabilitas est $\frac{2}{16}$ seu $\frac{1}{8}$: Ut tertium contingat oportet ut C ter victor existat, cujus eventus probabilitas est etiam $\frac{1}{8}$: Ut quartum contingat oportet ut D bis successive vincat, quod probabilitatem habet $\frac{1}{4}$; Ut quintum contingat, oportet ut E semel vincat, cujus eventus probabilitas est $\frac{1}{2}$;

ergo

ergo luſoris F probabilitas vincendi eſt $= \frac{1}{8}t + \frac{1}{8}u + \frac{1}{4}x + \frac{1}{2}y$. Sic inter æquationes No. 2. eſt, *ex. gr.* $x = \frac{1}{2}l + \overline{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \frac{1}{2^n}} \times h + \frac{1}{2^n} \times 1$. Colluſor enim qui certat cum adverſario qui jam bis ſucceſſive victor extitit, vincere poteſt vel omnes colluſores, vel aliquos, vel nullum. Prioris eventus probabilitas eſt $\frac{1}{2^n}$, ſecundi $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \frac{1}{2^n}$, & tertii $\frac{1}{2}$; ſi primus eventus contingat, probabilitas vincendi evadit certitudo integra ſeu 1; ſi ſecundus, exit è ludo relinquens colluſorem qui ſemel vicit; ſi tertius, exit è ludo relinquens colluſorem qui ter ſucceſſive vicit; adeoque ſors ejus totalis eſt $\frac{1}{2} \times l + \overline{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} \ldots \frac{1}{2^n}} \times h + \frac{1}{2^n} \times 1$. Simili ratiocinio inveniuntur æquationes N°. 3. Colluſor enim qui victus ab adverſario exit è ludo, relinquens *ex.gr.* colluſorem unius tantum ludi victorem, acquirit ſortem vel ipſius C, vel ipſius D, vel ipſius E, vel ipſius F, &c. prout adverſarius à quo victus eſt vincit vel omnes colluſores præter unum, vel omnes præter duos, vel omnes præter tres, &c. unde ſequitur quod $h = \frac{1}{2^{n-1}} \times c + \frac{1}{2^{n-2}} \times d + \frac{1}{2^{n-3}} \times e + \frac{1}{2^{n-4}} \times f +$ &c. Æquationes N°. 4. inveniuntur ſubtrahendo æquationes N°. 2. ab invicem: & æquationes N°. 5. ſubtrahendo æquationes N°. 3. ab invicem. Æquationes N°. 6. inveniuntur ſubſtituendo in æquationibus N°. 4. valores inventos per æquationes N°. 5. Æquationes N°. 7. inveniuntur quærendo valores ipſarum z, y, x, u, &c. per æquationes N°. 1. Et his valoribus ſubſtitutis in æquationibus N°. 4. habebuntur æquationes No. 8, quæ comparatæ cum æquationibus N°. 6:

dant æquationes N°. 9. ex quibus ſequitur quod $1 + 2^n . 2^n :: :: a : c :: c : d :: d : e$, &c. *Q. E. D.*

Corollarium.

Hinc faci'e inveniuntur probabilitates vincendi ſingulorum Colluſorum, quas habent tum ante ludum inceptum, tum in quolibet ſtatu in quem ludum proſequendo pervenire poſſunt. Si ſint, *ex gr.* tres colluſores A, B, C, erit $n = 2$, & $1 + 2^n : 2^n :: 5 . 4 :: a . c$: id eſt, probabilitates vincendi ipſorum A, B, C, antequam A vicerit B, vel B vicerit A, ſe habent ut numeri 5, 5, 4, adeoque ipſæ probabilitates ſunt $\frac{5}{14}, \frac{5}{14}, \frac{4}{14}$: omnes enim ſimul ſumptæ facere debent 1 ſeu certitudinem integram. Poſtquam A vicerit B, probabilitates vincendi ipſorum B, C, A, erunt h, y ſeu c, & (quia A æqualem habet expectationem ad victoriam, & ad ſortem ipſius B obtinendam) $\frac{1+h}{2}$ reſpectivè, *hoc eſt*, quia per æq. 1. N°. 3. $h = \frac{1}{2^{n-1}} \times c = \frac{1}{2} c$, & $c = \frac{4}{14} = \frac{2}{7}$ ut modo inventum, hæ probabilitates erunt $\frac{1}{7}, \frac{2}{7}, \frac{4}{7}$, ut D. *de Moivre* invenit Coroll. 1. Prop. 15. pag. 242.

Si ſint quatuor colluſores A, B, C, D, erit $n = 3$, & $1 + 2^n . 2^n :: 9, 8$, adeoque probabilitates colluſorum ab initio ludi erunt ut 9, 9, 8, $\frac{8 \times 8}{9}$, ſive ut 81, 81, 72, 64, *hoc eſt*, ipſæ a, a, c, d, erunt $\frac{81}{298}, \frac{81}{298}, \frac{72}{298}$ & $\frac{64}{298}$. Poſtquam A vicerit B, probabilitates ipſorum B, D, C, A, erunt h, d, c, $\frac{1+3h}{4}$, eſt autem per æq. 1. N°. 3. $h = \frac{1}{2^{n-1}} \times c + \frac{1}{2^{n-2}} \times d = \frac{1}{4} c + \frac{1}{2} d$,

$c =$

$e = \frac{72}{298} = \frac{36}{149}$, & $d = \frac{64}{298} = \frac{32}{149}$, ut modo inventum: ergo hæ probabilitates erunt $\frac{25}{149}$, $\frac{32}{149}$, $\frac{36}{149}$, $\frac{56}{149}$ respective. Postquam A vicerit B & C, probabilitates vincendi ipsorum C, B, D, A, erunt k, $\frac{c}{2}$, x, $\frac{1+h}{2}$, seu (quia per æq 2. N°. 3. $k = \frac{1}{2^{n-2}} \times d = \frac{1}{2} d$, & per æq. 3. N°. 7. $x = 2d - c$) $\frac{16}{149}$, $\frac{18}{149}$ $\frac{28}{149}$, $\frac{87}{149}$. Et nota quod calculi bonitas confirmetur ex eo, quod summæ harum probabilitatum, *hoc est*, $\frac{1}{7} + \frac{2}{7} + \frac{4}{7}$ in priori exemplo, & $\frac{25}{149} + \frac{32}{149} + \frac{36}{149} + \frac{56}{149}$, nec non $\frac{16}{149} + \frac{18}{149} + \frac{28}{149} + \frac{87}{149}$ in posteriori exemplo, singulæ sint $= 1$ seu certitudini integræ.

THEOREMA II.

Positis quæ prius & insuper hac conditione, ut victus semper mulctetur summa p, quæ deposito augendo inserviat; quod depositum sic gradatim auctum illi soli cedat, qui omnium successive colluſorum victor extiterit; denotatis etiam ut antea per literas minusculas a, c, d, e, &c. probabilitatibus vincendi ipsorum A (vel B), C, D, E, &c. respective: per easdem vero literas majusculas A, C, D, E, &c. ipsorum A (vel B), C, D, E, &c. expectationibus, *hoc est*, portionibus depositi quas singuli expectant: Dico, fore semper $C = \frac{\overline{A + ap} \times 2^n - ncp}{1 + 2^n}$

$$D = \frac{\overline{C + cp} \times 2^n - ndp}{1 + 2^n}, \quad E = \frac{\overline{D + dp} \times 2^n - nep}{1 + 2^n}, \text{ \&c.}$$

Demonstratio.

Denotetur ut prius per literas minusculas z, y, x, u, t, &c. probabilitas vincendi ludentis cum adversario, qui jam vel nullum, vel unum, vel duos, *&c* collusores successive vicit; per easdem vero literas majusculas Z, Y, X, U, T, &c. ejus expectatio, quam scil. habet diversis illis casibus, deposito existente $n + 1$, $n + 1 + p$, $n + 1 + 2p$, $n + 1 + 3p$, &c. respectivè. Sic etiam per literas minusculas h, k, l, m, &c. denotetur probabilitas vincendi lusoris victi ab adversario, qui antea vel nullum, vel unum, vel duos, &c. collusores successive vicerat; quemadmodum per literas majusculas H, K, L, M, &c. ejusdem expectatio diversis illis casibus, deposito existente $n + 1 + p$, $n + 1 + 2p$. $n + 1 + 3p$, &c. respective. His positis iisdem quibus antea ratiociniis invenientur sequentes duodecim æquationum series in *Tab.* II. signatæ N°. 1. N°. 2. N°. 3, &c. Inter æquationes N°. 1. *ex. gr.* est $E = \frac{U}{4} + \frac{X + xp}{4} + \frac{T + 2yp}{2}$. Lusor enim E ludet vel cum lusore A, vel lusore B, vel C, vel D. Si ludit cum A vel B, expectatio ejus erit $= U$, quia ludit cum adversario qui jam tres adversarios vicit, deposito existente $n + 1 + 3p$. Si ludit cum lusore C, expectatio ejus erit $= X + xp$, ludit enim cum adversario qui jam duos collusores vicit, adeoque si depositum esset $n + 1 + 2p$ ejus expectatio esset $= X$: verum quia ludente E depositum est $= n + 1 + 3p$, ob tres collusores victos & summa p mulctatos, addenda est expectationi X portio illa mulctæ unius p, quam lusor E sperare potest: est autem hæc portio (quia probabilitas ejus vincendi est x) $= xp$, ejus igitur expectatio totalis tunc erit $= X + xp$. Sic si ludit cum lusore D, expectatio ejus erit $= T + 2yp$: additur ad T (quæ esset ejus expectatio deposito existente $n + 1 + p$) portio $2yp$, quæ ipsi debetur de duabus mulctis

mulctis $2p$, quibus depositum $n+1+3p$ majus est quam $n+1+p$. Simili modo habentur æquationes N°. 2. 3 4. & 5. Substituendo autem primam æquationem N°. 2. *Tab.* I. in æquationibus N°. 4, habentur æquationes N°. 6. Et substituendo primam æquationem N°. 3. *Tab.* I. in æquationibus N°. 5. habentur æquationes. N°. 7. quibus dein in æquationibus No 6. substitutis habentur æquationes No. 8. Æquationes N°. 9. inveniuntur quærendo valores ipsarum Z, T, X, U, &c per æquationes N°. 1. *Tab.* I. & II. vel N° 2. *Tab* II. & N°. 7. *Tab.* I. Et his valoribus substitutis in æquationibus N°. 4. habentur æquationes N°. 10. Quæ comparatæ cum æquationibus N°. 8 (in quibus pro z substituatur a, per 1. æq. Tab. I.) dant æquationes N°. 11. Et hæ æquationes N°. 11. comparatæ cum æquationibus N°. 9. *Tab.* I. dant æquationes N°. 12, quæ constituunt Theorema, quod demonstrandum erat.

Corollarium.

Hinc quoque facile inveniuntur singulorum Collusorum sortes seu expectationes, ipsorumque adeo lucra vel damna. Sint *ex. gr.* tres collusores A, B, C: erit $C = \frac{\overline{A+ap} \times 2^n - ncp}{1+2^n}$ $= (\text{ob } n = 2)\ \frac{4A+4ap-2cp}{5} = (\text{ob } a = \frac{5}{14}\ \&\ c = \frac{2}{7}$ per coroll. Theor. 1.) $\frac{4A+\frac{6}{7}p}{5}$. Unde cum omnium trium expectationes simul sumptæ, *id est*, $A+A+C$ æquare debeant id quod ab initio depositum fuit, *id est* 3, erit $2A + \frac{4A+\frac{6}{7}p}{5} = \frac{14A+\frac{6}{7}p}{5} = 3$, & $14A = 15 - \frac{6}{7}p$, & $A = \frac{15}{14} - \frac{3}{49}p =$ expectationi lusoris A vel B: proinde C expectatio lusoris tertii $C = \frac{4A+\frac{6}{7}p}{5} = \frac{6}{7} + \frac{6}{49}p$. A qui-

bus expectationibus si subtrahatur 1, id quod ab initio singuli deposuerunt, remanebit ibi $\frac{1}{14} - \frac{3}{49} p$, hic $\frac{6}{49} p - \frac{1}{7}$; quemadmodum D. *de Moivre* invenit. *Exempl.* 2. Sint collusores 4, A, B, C, D, erit $C = \frac{\overline{A + ap} \times 2^n - ncp}{1 + 2^n} = (\text{ob } n = 3)$ $\frac{8A + \overline{8ap - 3cp}}{9} = (\text{ob } a = \frac{81}{298} \text{ \& } c = \frac{36}{149}$, per coroll. Theor. 1.) $\frac{8A + \frac{216}{149}p}{9}$; item $D = \frac{\overline{C + cp} \times 2^n - ndp}{1 + 2^n} =$ $\frac{8C + 8cp - 3dp}{9} = (\text{ob } d = \frac{32}{149} \text{ per id. corr.}) \frac{8C + \frac{192}{149}p}{9} =$ $\frac{64A + \frac{3456}{149}p}{81}$: unde habebitur æquatio $2A + C + D = 2A +$ $\frac{8A + \frac{216}{149}p}{9} + \frac{64A + \frac{3456}{149}p}{9} = \frac{298A + \frac{5400}{149}p}{81} = 4$, sive $149A$ $+ \frac{2700}{149}p = 162$, & $A = \frac{162}{149} - \frac{2700}{22201}p$. Hinc $C =$ $\frac{8A + \frac{216}{149}p}{9} = \frac{144}{149} + \frac{1176}{22201}p$, & $D = \frac{64A + \frac{3456}{149}p}{81} = \frac{128}{149}$ $+ \frac{4224}{22201}p$. Subtracta autem unitate 1, quam singuli ab initio ludi deposuerunt, remanebit $\frac{13}{149} - \frac{2700}{22201}p$ pro lusore A vel B, $\frac{1176}{22201}p - \frac{5}{149}$ pro C, & $\frac{4224}{22201}p - \frac{21}{149}$ pro D; quæ singula indigitabunt lucrum vel damnum, prout pars affirmata præpollet negatæ, vel contra. Simili ratione habebuntur etiam sortes quas acquirunt in quolibet statu in quem ludum prosequendo pervenire possunt.

THEO-

THEOREMA 3.

Positis quæ prius, si adsint spectatores Q, R, S, T, U, &c. quorum numerus sit n unitate minor quam numerus collusorum, quorumque prior Q affirmet certamen finitum iri post $n+p$ ludos peractos, R post $n+p-1$, S post $n+p-2$, T post $n+p-3$, U post $n+p-4$, &c præcise, non antea; sintque q, r, s, t, u, &c. sortes ipsorum Q, R, S, T, U, &c. Dico fore $q = \frac{1}{2}r + \frac{1}{4}s + \frac{1}{8}t + \frac{1}{16}u +$ &c.

Demonstratio.

Vocetur A collusor ille, qui post $n+p$ ludos vincere supponitur: hic intrare debet in ludum post p ludos peractos, & tum ludet contra adversarium, qui jam vel unum vel duos, vel tres, &c. collusores successive vicit. Jam cum, ut primus casus contingat, & ut collusor A omnes suos collusores præter unum, *id est*, $n-1$ collusores successive vincat, æque probabile sit quam ut adversarius ejus vincat $n-1$ collusores, *id est*, (quia jam unius collusoris victor fuit) ut certamen finiat post $n+p-1$ ludos peractos; hujusque eventus probabilitas sit $= r$: erit probabilitas ut collusor A unum adhuc collusorem vincat, *id est*, certamen finiat post $n+p$ ludos $= \frac{1}{2}r$. Sic, ut secundus; casus existat, & ut A omnes collusores præter duos vincat, æquè probabile est quam ut certamen finiatur post $n+p-2$ ludos, adeoque ut tunc A vincat adhuc duos collusores, *id est*, ut certamen finiat post $n+p$ ludos, probabilitas erit $= \frac{1}{4}s$. Eodem modo ut, tertio casu existente, A vincat omnes collusores, probabilitas est $= \frac{1}{8}t$; ut quarto $= \frac{1}{16}u$, &c. Quare ut

 indiffe-

indifferenter certamen finiatur post $n+p$ ludos, probabilitas est $\frac{1}{2}r+\frac{1}{4}s+\frac{1}{8}t+\frac{1}{16}u+\&c.=q$. Q. E. D.

Corollarium 1.

Facile hinc invenitur quænam sit probabilitas ut certamen finiatur intra datum quemvis ludorum numerum. Series enim fractionum incipientium à fractione $\frac{1}{2^{n-1}}$, quarum denominatores crescant in continua proportione dupla, numerator autem cujusque fractionis sit summa numeratorum tot fractionum immediate præcedentium quot sunt unitates in $n-1$, dabit omnes successive probabilitates, ut certamen finiatur peractis præcise n, $n+1$, $n+2$, $n+3$ &c. ludis: & per consequens si addantur tot termini hujus seriei quot sunt unitates in $p+1$, summa ipsorum exprimet probabilitatem ut certamen finiatur ad minimum ludis $n+p$ peractis. *Ex. gr.* Si sint collusores 4, adeoque $n=3$, habebitur hæc series $\frac{1}{4}, \frac{1}{8}, \frac{2}{16}, \frac{3}{32}, \frac{5}{64}, \frac{8}{128}, \frac{13}{256}, \frac{21}{512}$ &c. E qua si fiat alia $\frac{1}{4}, \frac{3}{8}, \frac{8}{16}, \frac{19}{32}, \frac{43}{64}, \frac{94}{128}, \frac{201}{256}$, *&c.* cujus termini sint summæ terminorum præcedentis seriei, denotabunt iidem termini qualis sit probabilitas ut certamen finiatur ad minimum 3, 4, 5, 6, *&c.* ludis.

Corollarium 2.

Potest terminus quicunque prioris seriei (excepto primo termino,) ut & summa omnium terminorum, *id est*, terminus quicunque posterioris seriei, per formulam generalem exprimi hoc modo. Si $n+1$ sit numerus collusorum, & p sit numerus terminorum, erit ultimus terminus prioris seriei

$\frac{1}{2^n}$

$$\frac{1}{2^n} - \frac{p-n+1}{1\times 2^{2n}} + \frac{p-2n\times p-2n+3}{1\times 2\times 2^{3n}} -$$
$$-\frac{p-3n\times p-3n+1\times p-3n+5}{1\times 2\times 3\times 2^{4n}} +$$
$$+\frac{p-4n\times p-4n+1\times p-4n+2\times p-4n+7}{1\times 2\times 3\times 4\times 2^{5n}}, -\&c.$$ Et

ſumma omnium terminorum ſive ultimus terminus poſterioris

$$\text{ſeriei} = \frac{p+1}{1\times 2^n} - \frac{p-n\times p-n+3}{1\times 2\times 2^{2n}} + \frac{p-2n\times p-2n+1\times p-2n+5}{1\times 2\times 3\times 2^{3n}}$$
$$-\frac{p-3n\times p-3n+1\times p-3n+2\times p-3n+7}{1\times 2\times 3\times 4\times 2^{4n}} + \&c.$$

Tabula I.

Intrat		Exit.		N°. 1
	Sors		Sors	
0	z	1	h	$a = z$
1	y	2	k	$c = y$
2	x	3	l	$d = \frac{1}{2}x + \frac{1}{2}y$
3	u	4	m	
4	t			$e = \frac{1}{4}u + \frac{1}{4}x + \frac{1}{2}y$
				$f = \frac{1}{8}t + \frac{1}{8}u + \frac{1}{4}x + \frac{1}{2}y$

N°. 2.

$$z = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \frac{1}{2^n}\times h + \frac{1}{2^n}\times 1$$

$$y = \frac{1}{2}k + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \frac{1}{2^n}\times h + \frac{1}{2^n}\times 1.$$

$$x = \frac{1}{2}l + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \frac{1}{2^n}\times h + \frac{1}{2^n}\times 1.$$

$$u = \frac{1}{2}m + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \frac{1}{2^n}\times h + \frac{1}{2^n}\times 1$$

No. 3.

N°. 3.

$$b = \frac{1}{2^{n-1}} \times c + \frac{1}{2^{n-2}} \times d + \frac{1}{2^{n-3}} \times e + \frac{1}{2^{n-4}} \times f + \cdots$$

$$k = \frac{1}{2^{n-2}} \times d + \frac{1}{2^{n-3}} \times e + \frac{1}{2^{n-4}} \times f + \cdots$$

$$l = \frac{1}{2^{n-3}} \times e + \frac{1}{2^{n-4}} \times f + \cdots$$

$$m = \frac{1}{2^{n-4}} \times f + \cdots$$

N°. 4.	N°. 6.	N°. 8.
$z - y = \frac{1}{2} h - \frac{1}{2} k$	$= \frac{1}{2^n} \times c$	$= a - c$
$y - x = \frac{1}{2} k - \frac{1}{2} l$	$= \frac{1}{2^{n-1}} \times d$	$= 2c - 2d$
$x - u = \frac{1}{2} l - \frac{1}{2} m$	$= \frac{1}{2^{n-2}} \times e$	$= 4d - 4e$

N°. 5.	N°. 7.	N°. 9.
$h - k = \frac{1}{2^{n-1}} \times c$	$z = a$	$c = a \times \frac{2^n}{1 + 2^n}$
$k - l = \frac{1}{2^{n-2}} \times d$	$y = c$	$d = c \times \frac{2^n}{1 + 2^n}$
$l - m = \frac{1}{2^{n-3}} \times e$	$x = 2d - y = 2d - c$	$e = d \times \frac{2^n}{1 + 2^n}$
	$u = 4e - x - 2y = 4e - 2d - e$	

Corollarium 3.

Potest quis priusquam ludus inchoetur in se suscipere, ut summam $n+1$ de qua collusores contendunt, & mulctas omnes pendat, si sibi initio in manus datum sit $n + 1 + 2^n - 1 \times p$.

Demonstrationem duorum præcedentium corollariorum curiosis indagandam relinquo.

III. *Solutio generalis altera præcedentis Problematis, ope Combinationum & Serierum infinitarum, per* D. Abr. de Moivre. Reg. Soc Sodalem.

Designationes.

SI *B* & *C* colluſores duo ſimul certent, ad deſignandum *B* victorem eſſe, *C* victum, ſcribatur *BC*; atque viciſſim ad deſignandum *C* victorem eſſe, *B* victum; ſcribatur *CB*: & ſic de cæteris.

Ponatur 1° *B* vincere *A*, certamenque concludi tribus ludis

$$\left.\begin{array}{c} BA \\ \hline BC \\ BD \end{array}\right\}\text{Sic patet } B \text{ victorem neceſſario evadere.}$$

Ponatur 2° *B* vincere *A*, certamenque concludi quatuor ludis

$$\left.\begin{array}{c} BA \\ \hline CB \\ CD \\ CA \end{array}\right\}\text{Sic patet } C \text{ victorem neceſſario evadere.}$$

Ponatur 3° *B* vincere *A*, certamenque concludi quinque ludis

$$\left.\begin{array}{cc} BA & BA \\ \hline CB^{*} & BC \\ DC & DB \\ DA & DA \\ DB & DC \end{array}\right\}\text{Sic patet } D \text{ victorem neceſſario evadere, idque duplici modo.}$$

Ponatur 4° *B* prima vice vincere *A*, certamenque concludi ſex ludis.

BA	BA	BA
CB	CB*	BC
DC*	CD*	DB
AD	AC	AC
AB	AB	AD
AC	AD	AB

Sic patet *A* victorem necessario evadere, idque triplici modo.

Ponatur 5° certamen concludi septem ludis, ponaturque semper *B* prima vice vincere ipsum *A*.

BA	BA	BA	BA	BA
CB	CB	CB*	BC	BC
DC	DC*	CD	DB	DB
AD*	DA	AC	AD*	DA
BA	BD	BA	CA	CD
BC	BC	BD	CB	CB
BD	BA	BC	CD	CA

Sic patet *B* vel *C* necessario victores evadere, *B* triplici modo, *C* duplici.

Ponatur 6° certamen concludi octo ludis,

BA	BA	BA	BA	BA	BA	BA	BA
CB	CB	CB	CB	CB*	BC	BC	BC
DC	DC	DC*	CD	CD	DB	DB	DB
AD	AD*	DA	AC	AC	AD	AD*	DA
BA*	AB	BD	BA*	AB	CA*	AC	CD
CB	CA	CB	DB	DA	BC	BA	BC
CD	CD	CA	DC	DB	BD	BD	BA
CA	CB	CD	DA	DC	BA	BC	BD

Sic patet *C* victorem evadere triplici, *D* duplici, *B* triplici modo, *&c.*

Nunc ordine scribantur literæ quibus victores designantur.

3, | 1 *B*
4, | 1 *C*
5, | 2 *D*
6, | 3 *A*
7, | 3 *B* + 2 *C*
8, | 3 *C* + 2 *D* + 3 *B*.
9, | 3 *D* + 2 *A* + 3 *C* + 3 *D* + 2 *A*
10, | 3 *A* + 2 *B* + 3 *D* 3 *A* + 2 *B* + 3 *A* + 2 *C* + 3 *D*
&c.

Per-

Perſpecta illarum formatione, patebit 1° literam *B* in ordine aliquo ſemper toties reperiri, quoties *A* in ordine ultimo & penultimo reperitur: 2° *C* in ordine aliquo toties reperiri quoties *B* in ordine ultimo & *D* in penultimo reperiuntur: 3° *D* in ordine aliquo toties reperiri quoties *C* in ultimo & *B* in penultimo: 4° *A* in ordine aliquo ſemper toties reperiri quoties *D* in ordine ultimo & *C* in penultimo reperiuntur.

Sed numerus variationum dato cuilibet ludorum numero competens, duplus eſt numeri variationum omnium dato ludorum numero unitate diminuto competentis: adeoque Probabilitas quam habet Colluſor *B* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *A* ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat idem *A*, ut vinceret dato ludorum numero minus duobus: & ſic de cæteris.

Probabilitas quam habet *C*, ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *B*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *D*, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet *D* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *C*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *B*, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet *A* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *D*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *C* ut vinceret dato ludorum numero minus duobus.

Ex jam obſervatis facile eſt componere Tabulam Probabilitatum, quas *B*, *C*, *D*, *A* habent ut victores evadant dato ludorum numero, atque etiam illorum ſortium ſeu expectationum.

Tabula Probabilitatum, &c.

		B	C	D	A
'	3	$\frac{1}{4} \times \overline{4+3p}$	—	—	—
''	4	—	$\frac{1}{8} \times \overline{4+4p}$	—	—
'''	5	—	—	$\frac{2}{16} \times \overline{4+5p}$	—
''''	6	—	—	—	$\frac{3}{32} \times \overline{4+6p}$
V	7	$\frac{3}{64} \times \overline{4+7p}$	$\frac{2}{64} \times \overline{4+7p}$	—	—
V'	8	$\frac{3}{128} \times \overline{4+8p}$	$\frac{3}{128} \times \overline{4+8p}$	$\frac{2}{128} \times \overline{4+8p}$	—
V''	9	—	$\frac{3}{256} \times \overline{4+9p}$	$\frac{6}{256} \times \overline{4+9p}$	$\frac{4}{256} \times \overline{4+9p}$
V'''	10	$\frac{4}{512} \times \overline{4+10p}$	$\frac{2}{512} \times \overline{4+10p}$	$\frac{6}{512} \times \overline{4+10p}$	$\frac{9}{512} \times \overline{4+10p}$
'X	11	$\frac{13}{1024} \times \overline{4+11p}$	$\frac{10}{1024} \times \overline{4+11p}$	$\frac{2}{1024} \times \overline{4+11p}$	$\frac{9}{1024} \times \overline{4+11p}$
X	12	$\frac{18}{2048} \times \overline{4+12p}$	$\frac{19}{2048} \times \overline{4+12p}$	$\frac{14}{2048} \times \overline{4+12p}$	$\frac{4}{2048} \times \overline{4+12p}$
		&c.	&c.	&c.	&c.

Jam vero Series istæ sunt convergentes, adeoque singulæ summari possunt per vulgarem Arithmeticam ; & obtinebuntur vel summæ accuratæ si possint, vel saltem approximatæ, si non liceat, terminos multos adhibere.

Inveni-

Invenire ſummas Probabilitatum ad infinitum uſque pergentium, quas Colluſores habent ut victores evadant.

Sint Probabilitates omnes ipſius B ad infinitum, nempe
$B' + B'' + B''' + B'''' + B^{v} + B^{vi}$ &c. $= y$

Probabilitates ipſius C
$C' + C'' + C''' + C'''' + C^{v} + C^{vi}$ &c. $= z$

Probabilitates ipſius D
$D' + D'' + D''' + D'''' D^{v} + D^{vi}$ &c. $= v$

Probabilitates ipſius A
$A' + A'' + A''' + A'''' A^{v} + A^{vi}$ &c. $= x$

Scribantur autem in Scala perpendiculariter deſcendente, ad hunc modum.

$B' = B'$
$B'' = B''$
$B' = \frac{1}{2}A'' + \frac{1}{4}A'$
$B'''' = \frac{1}{2}A''' + \frac{1}{4}A''$
$B^{v} = \frac{1}{2}A'''' + \frac{1}{4}A'''$ Proinde $y = \frac{1}{4} + \frac{3}{4}x$.
$B^{vi} = \frac{1}{2}A^{v} + \frac{1}{4}A''''$
Ergo $y = \frac{1}{4} + \frac{1}{2}x + \frac{1}{4}x$.

Demonſtratio.

Etenim prima columna perpendicularis $= y$, ex Hypotheſi
Eſt vero $A' + A'' + A''' + A'''' + A^{v}$ &c. $= x$, ex hypotheſi;
Ergo $\frac{1}{2}A' + \frac{1}{2}A'' + \frac{1}{2}A''' + \frac{1}{2}A'''' + \frac{1}{2}A^{v}$ &c. $= \frac{1}{2}x$.
Proinde $\frac{1}{2}A'' + \frac{1}{2}A''' + \frac{1}{2}A'''' + \frac{1}{2}A^{v}$ &c. $= \frac{1}{2}x - \frac{1}{2}A'$.
Et $B' + B'' + \frac{1}{2}A'' + \frac{1}{2}A''' + \frac{1}{2}A''''$ &c. $= \frac{1}{2}x - \frac{1}{2}A' + B' + B''$.

Sed

Sed $\frac{1}{2} A' = \bar{0}, B'' = 0$ & $B' = \frac{1}{4}$, ut patet ex Tabula:

Ergo ſecunda columna perpendicularis $= \frac{1}{4} + \frac{1}{2} x$.

Sed tertia columna perpendicularis $= \frac{1}{4} x$.

erit igitur $y = \frac{1}{4} + \frac{3}{4} x$.

Simili modo ſcribantur

$$\begin{aligned}
C' &= C' \\
C'' &= C'' \\
C''' &= \tfrac{1}{2} B'' + \tfrac{1}{4} D' \\
C'''' &= \tfrac{1}{2} B''' + \tfrac{1}{4} D'' \quad \text{hoc eſt } z = \tfrac{1}{2} y + \tfrac{1}{4} v. \\
C^{V} &= \tfrac{1}{2} B'''' + \tfrac{1}{4} D''' \\
C^{VI} &= \tfrac{1}{2} B^{V} + \tfrac{1}{4} D'''' \\
&\&c.
\end{aligned}$$

Ergo $z = \frac{1}{8} + \frac{1}{2} y - \frac{1}{8} + \frac{1}{4} v$.

Scribantur etiam

$$\begin{aligned}
D' &= D' \\
D'' &= D'' \\
D''' &= \tfrac{1}{2} C'' + \tfrac{1}{4} B' \\
D'''' &= \tfrac{1}{2} C''' + \tfrac{1}{4} B'' \quad \text{\& pari Argumento patebit} \\
D^{V} &= \tfrac{1}{2} C'''' + \tfrac{1}{4} B''' \quad v + \tfrac{1}{2} z + \tfrac{1}{4} y \\
D^{VI} &= \tfrac{1}{2} C^{V} + \tfrac{1}{4} B'''' \\
&\&c.
\end{aligned}$$

Scribantur denique

$$\begin{aligned}
A' &= A' \\
A'' &= A'' \\
A''' &= \tfrac{1}{2} D'' + \tfrac{1}{4} C' \\
A'''' &= \tfrac{1}{2} D''' + \tfrac{1}{4} C'' \\
A^{V} &= \tfrac{1}{2} D'''' + \tfrac{1}{4} C''' \quad \text{Unde concludetur } x = \tfrac{1}{2} v + \tfrac{1}{4} z \\
A^{VI} &= \tfrac{1}{2} D^{V} + \tfrac{1}{4} C'''' \\
&\&c.
\end{aligned}$$

Resolutis autem quatuor istis æquationibus, reperietur

$$B' + B'' + B''' + B'''' \&c. = y = \frac{56}{149}$$

$$C' + C'' + C''' + C'''' \&c. = z = \frac{36}{149}$$

$$D' + D'' + D''' + D'''' \&c. = v = \frac{32}{149}$$

$$A' + A'' + A''' + A'''' \&c. = x = \frac{25}{149}$$

Valoribus istis inventis, ponatur jam $\frac{56}{149} = b$, $\frac{36}{149} = c$, $\frac{32}{149} = d$, $\frac{25}{149} = a$.

Iterum sit.

$3B'p + 4B''p + 5B'''p + 6B''''p$ &c. $= py$.
$3C'p + 4C''p + 5C'''p + 6C''''p$ &c. $= pz$.
$3D'p + 4D''p + 5D'''p + 6D''''p$ &c. $= pv$.
$3A'p + 4A''p + 5A'''p + 6A''''p$ &c. $= px$.

$3B' = 3B'$
$4B'' = 4B''$
$5B''' = \frac{5}{2}A'' + \frac{5}{4}A'$
$6B'''' = \frac{6}{2}A''' + \frac{6}{4}A''$
$7B^{V} = \frac{7}{2}A'''' + \frac{7}{4}A'''$
$8B^{VI} = \frac{8}{2}A^{V} + \frac{8}{4}A''''$

Ergo $y = \frac{3}{4} + \frac{3}{4}x + a$

Etenim prima Columna perpendicularis $= y$, ex Hypothesi:
$3B' + 4B'' = \frac{3}{4}$: Nam est $B' = \frac{1}{4}$, & $B'' = 0$.
$3A' + 4A'' + 5A'''$ &c. $= x$ ex Hypothesi.
$A' + A'' + A'''$ &c. $= a$, ut repertum est.

Est igitur $4A' + 5A'' + 6A''' + 7A''''$ &c. $= x + a$
Et $\frac{4}{2}A + \frac{5}{2}A'' + \frac{6}{2}A''' + \frac{7}{2}A''''$ &c. $= \frac{1}{2}x + \frac{1}{2}a$.

Sed $A' = 0$
Ergo ſecunda Columna perpendicularis $= \frac{3}{4} + \frac{1}{2} x + \frac{1}{2} a$.

$$3 A' + 4 A'' + 5 A''' + 6 A'''' \text{ \&c.} = x$$
$$2 A' + 2 A'' + 2 A''' + 2 A'''' \text{ \&c.} = 2 a$$

Eſt igitur $5 A' + 6 A'' + 7 A''' + 8 A''''$ &c. $= x + 2 a$.
Et $\frac{5}{4} A' + \frac{6}{4} A'' + \frac{7}{4} A''' + \frac{8}{4} A''''$ &c. $= \frac{1}{4} x + \frac{1}{2} a$.
Eſt igitur tertia Columna perpendicularis $= \frac{1}{4} x + \frac{1}{2} a$.
Erit igitur $y = \frac{3}{4} + \frac{1}{2} x + \frac{1}{2} a + \frac{1}{4} x + \frac{1}{2} a$
ſive $y = \frac{3}{4} + \frac{3}{4} x + a$, quod erat probandum.

$$3 C' = 3 C'$$
$$4 C'' = 4 C''$$
$$5 C''' = \tfrac{5}{2} B'' + \tfrac{5}{4} D'$$
$$6 C'''' = \tfrac{6}{2} B''' + \tfrac{6}{4} D''$$
$$7 C^{v} = \tfrac{7}{2} B'''' + \tfrac{7}{4} D'''$$
$$8 C^{vi} = \tfrac{8}{2} B^{v} + \tfrac{8}{4} D''''$$
&c.

Ergo $z = \frac{1}{2} y + \frac{1}{2} b + \frac{1}{4} v + \frac{1}{2} d$.
Etenim prima Columna perpendicularis $= z$, ex Hypotheſi.

$$3 C' + 4 C'' = \tfrac{1}{2}.$$
$$3 B' + 4 B'' + 5 B''' + 6 B'''' \text{ \&c.} = y.$$
$$B' + B'' + B''' + B'''' \text{ \&c.} = b.$$

Eſt igitur $4 B' + 5 B'' + 6 B''' + 7 B''''$ &c. $= y + b$.
Sed $4 B' = 1$.
Ergo $5 B'' + 6 B''' + 7 B''''$ &c. $= y + b - 1$.
$\frac{5}{2} B'' + \frac{6}{2} B''' + \frac{7}{2} B''''$ &c. $= \frac{1}{2} y + \frac{1}{2} b - \frac{1}{2}$.
Ergo ſecunda Columna perpendicularis $= \frac{1}{2} + \frac{1}{2} y + \frac{1}{2} b - \frac{1}{2}$
$= \frac{1}{2} y + \frac{1}{2} b$.
Iterum, $3 D' + 4 D'' + 5 D''' + 6 D''''$ &c. $= v$
$2 D' + 2 D'' + 2 D''' + 2 D''''$ &c. $= 2d$
Eſt igitur $5 D' + 6 D'' + 7 D''' + 8 D''''$ &c. $= v + 2 d$.
Et $\frac{5}{4} D' + \frac{6}{4} D'' + \frac{7}{4} D''' + \frac{8}{4} D''''$ &c. $= \frac{1}{4} v + \frac{1}{2} d$.
Ergo tertia Columna perpendicularis $= \frac{1}{4} v + \frac{1}{2} d$
Eſt igitur $z = \frac{1}{2} y + \frac{1}{2} b + \frac{1}{4} v + \frac{1}{2} d$, quod erat probandum.

Eodem prorſus ordine ſcribantur.

$$\begin{aligned} 3\,D' &= 3\,D' \\ 4\,D'' &= 4\,D'' \\ 5\,D''' &= \tfrac{5}{2}\,C'' + \tfrac{5}{4}\,B' \\ 6\,D'''' &= \tfrac{6}{2}\,C''' + \tfrac{6}{4}\,B'' \\ 7\,D^{V} &= \tfrac{7}{2}\,C'''' + \tfrac{7}{4}\,B''' \\ 8\,D^{VI} &= \tfrac{8}{2}\,C^{V} + \tfrac{8}{4}\,B'''' \\ &\&c. \end{aligned}$$

$$\begin{aligned} 3\,A' &= 3\,A' \\ 4\,A'' &= 4\,A'' \\ 5\,A''' &= \tfrac{5}{2}\,D'' + \tfrac{5}{4}\,C' \\ 6\,A'''' &= \tfrac{6}{2}\,D''' + \tfrac{6}{4}\,C'' \\ 7\,A^{V} &= \tfrac{7}{2}\,D'''' + \tfrac{7}{4}\,C''' \\ 8\,A^{VI} &= \tfrac{8}{2}\,D^{V} + \tfrac{8}{4}\,C'''' \\ &\&c. \end{aligned}$$

Unde $v = \frac{1}{2}z + \frac{1}{2}c + \frac{1}{4}y + \frac{1}{2}b$. Et $x = \frac{1}{2}v + \frac{1}{2}d + \frac{1}{4}z + \frac{1}{2}c$.

Quæ quidem Concluſiones eodem modo demonſtrantur ac ſuperiores.

Solutis autem quatuor iſtis æquationibus, elicietur $y = \frac{45536}{149|^2}$, $z = \frac{38724}{149|^2}$, $v = \frac{37600}{149|^2}$ $x = \frac{33547}{149|^2} = \frac{33547}{22201}$

Ergo, ſi velint B, C, D, A vendere Spectatori cuidam R ſummas quas ſinguli obtinere ſperant, æquum erit ut emptor R pendat

ipſi B $4 \times \frac{56}{149} + \frac{45536}{22201}p$, ipſi C $4 \times \frac{36}{149} + \frac{38724}{22201}p$.

ipſi D $4 \times \frac{32}{149} + \frac{37600}{22201}p$, ipſi A $4 \times \frac{25}{149} + \frac{33547}{22201}p$.

Invenire Probabilitates quas habent B, C, D, A, ut mulctentur, dato ludorum numero.

Si Ludi duo tantum ſint, erunt hoc modo.

$\left.\begin{matrix} BA \\ CB \end{matrix}\quad \begin{matrix} BA \\ BC \end{matrix}\right\}$ Unde patet B vel C neceſſario mulctari.

Si Ludi tres fuerint, hoc modo ſe res habet.

$\left.\begin{matrix} BA \\ CB \\ DC \end{matrix}\quad \begin{matrix} BA \\ CB \\ CD \end{matrix}\quad \begin{matrix} BA \\ BC \\ DB \end{matrix}\quad \begin{matrix} BA \\ BC \\ BD \end{matrix}\right\}$ Hinc patet C, vel D vel B neceſſario mulctari.

Si vero quatuor Ludi fuerint:

$\frac{BA}{CB}$	$\frac{BA}{CB}$	$\frac{BA}{CB}$	$\frac{BA}{CB}$	$\frac{BA}{BC}$	$\frac{BA}{BC}$
DC	DC	CD	CD	DB	DB
AD	DA	AC	CA	AD	DA

Debet igitur A triplici modo, D duplici, C ſimplici, mulctari.

Et ſic de cæteris. Ex quibus manifeſta eſt Compoſitio Tabulæ ſubjunctæ Probabilitatum quas B, C, D, A habent ut mulctentur, dato ludorum numero.

	Num Lud.	B	C	D	A
'	2	$\frac{1}{2}$	$\frac{1}{2}$		
''	3	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	
'''	4		$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$
''''	5	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{3}{16}$
V	6	$\frac{6}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
VI	7	$\frac{6}{64}$	$\frac{8}{64}$	$\frac{8}{64}$	$\frac{4}{64}$
		&c.			

Sint autem y, z, v, x ſummæ omnium Probabilitatum quas B, C, D, A habent reſpective ut mulctentur.

Scribantur eodem ordine ac in præcedentibus.

$B' = B'$
$B'' = B''$
$B''' = \frac{1}{2}A'' + \frac{1}{4}A'$
$B'''' = \frac{1}{2}A''' + \frac{1}{4}A''$
$B^{V} = \frac{1}{2}A'''' + \frac{1}{4}A'''$
$B^{VI} = \frac{1}{2}A^{V} + \frac{1}{4}A''''$
&c.

$C' = C'$
$C'' = C''$
$C''' = \frac{1}{2}B'' + \frac{1}{4}D'$
$C'''' = \frac{1}{2}B''' + \frac{1}{4}D''$
$C^{V} = \frac{1}{2}B'''' + \frac{1}{4}D'''$
$C^{VI} = \frac{1}{2}B^{V} + \frac{1}{4}D''''$
&c.

Ergo $y = \frac{3}{4} + \frac{1}{2}x + \frac{1}{4}x$.

Ergo $z = \frac{1}{2} + \frac{1}{2}y + \frac{1}{4}v$.

Ergo $y = \frac{3}{4} + \frac{3}{4}x$.

Scri-

Scribantur deinde

$$D' = D' \qquad A' = A'$$
$$D'' = D'' \qquad A'' = A''$$
$$D''' = \tfrac{1}{2}C'' + \tfrac{1}{4}B' \qquad A''' = \tfrac{1}{2}D'' + \tfrac{1}{4}C'$$
$$D'''' = \tfrac{1}{2}C''' + \tfrac{1}{4}B'' \qquad A'''' = \tfrac{1}{2}D''' + \tfrac{1}{4}C''$$
$$D^{v} = \tfrac{1}{2}C'''' + \tfrac{1}{4}B''' \qquad A^{v} = \tfrac{1}{2}D'''' + \tfrac{1}{4}C'''$$
$$D^{vi} = \tfrac{1}{2}C^{v} + \tfrac{1}{4}B'''' \qquad A^{vi} = \tfrac{1}{2}D^{v} + \tfrac{1}{4}C''''$$
&c. &c

Ergo $v = \tfrac{1}{4} + \tfrac{1}{2}z + \tfrac{1}{4}y$. Ergo $x = \tfrac{1}{4}v + \tfrac{1}{4}z$.

Resolutis autem quatuor istis æquationibus, invenietur

$$y = \frac{243}{249} \qquad z = \frac{252}{149} \qquad v = \frac{224}{149} \qquad \& x = \frac{175}{149}$$

Ergo si velit Spectator aliquis S mulctas omnes sustinere, æquum erit ut ipsi S

B tradat $\frac{243}{149}p$ C $\frac{252}{149}p$ D $\frac{224}{149}p$. & A $\frac{175}{149}p$.

Sublatis itaque summis probabilitatum quas singuli Collusores habent ut mulctentur, è summis expectationum quas habent iidem si victores abeant, restabunt sortes eorum respective: nempe

B recipit ab R $\frac{4 \times 56}{149} + \frac{45536}{22201}p$

B tradit ipsi S $\frac{243}{149}p$

Ergo ipsi B superest $\frac{224}{149} + \frac{9329}{22201}p$

Sed B deposuerat 1 priusquam ludus inciperetur.

Ergo B lucratur $\frac{75}{149} + \frac{9329}{22201}p$.

C recipit ab R $\frac{4 \times 36}{149} + \frac{38724}{22201} p$

C tradit ipsi S $\frac{252}{149} p$

Ergo ipsi C superest $\frac{144}{149} + \frac{1176}{22201} p$.

Sed C deposuerat 1.

Ergo C lucratur $- \frac{5}{149} + \frac{1176}{22201} p$.

D recipit ab R $\frac{4 \times 32}{149} + \frac{37600}{22201} p$

D tradit ipsi S $\frac{224}{149} p$

Ergo ipsi D superest $\frac{128}{149} + \frac{4224}{22201} p$

Sed D deposuerat 1.

Ergo D lucratur $- \frac{21}{149} + \frac{4224}{22201} p$.

A recipit ab R $\frac{4 \times 25}{149} + \frac{33547}{22201} p$

A tradit ipsi S $\frac{175}{149} p$

Ergo ipsi A superest $\frac{100}{149} + \frac{7472}{22201} p$

Sed A deposuerat $1 + p$, nempe 1 priusquam ludus inchoaretur, & p postquam semel victus fuerat à B:

Ergo A lucratur $- \frac{49}{149} = \frac{14729}{22201} p$.

Lucrum

Lucrum ipsius $B = +\frac{75}{149} + \frac{9329}{22201}p$

ipsius $C = -\frac{5}{149} + \frac{1176}{22201}p$

ipsius $D = -\frac{21}{149} + \frac{4224}{22201}p$

ipsius $A = -\frac{49}{149} - \frac{14729}{22201}p$

Summa Lucrorum $= \quad 0 \quad 0$

Summa autem lucrorum ipsorum B & $A = \frac{26}{149} - \frac{5400}{22201}p$; sed posueramus B vicisse ipsum A semel, priusquam Collusores pacta inirent cum R & S. Priusquam vero ludus inchoaretur, A poterat æqua sorte expectare ut vinceret ipsum B, adeoque summa lucrorum $\frac{26}{149} - \frac{5400}{22201}$ in duas partes æquales est dividenda, ita ut utriusque lucrum censendum sit $\frac{13}{149} - \frac{2700}{22201}p$.

Ponatur $\frac{13}{149} - \frac{2700}{22201}p = 0$, & erit $p = \frac{1937}{2700}$.

Ergo si sit mulcta p ad summam quam singuli deponunt ut 1937 ad 2700, A & B nihil lucrantur, nihil perdunt. Verum hoc in Casu C lucratur $\frac{1}{225}$, quam D perdit.

Coroll. 1. Spectator R, priusquam ludus inchoetur, id suscipere in se poterit, ut summam 4 de qua Collusores contendunt, & mulctas omnes pendat, si sibi initio in manus darentur $4 + 7p$.

Coroll. 2. Si dexteritates Collusorum sint in ratione data, sortes Collusorum eadem ratiocinatione determinabuntur.

Coroll. 3. Si Series aliqua ita ſit conſtituta, ut continuò decreſcat, & terminus quivis ad præcedentes quoſlibet habeat rationes datas, ſive eaſdem ſive diverſas, ſeries iſta accurate ſummabitur. Inſuper ſi termini omnes hujus Seriei multiplicentur per terminos progreſſionis Arithmeticæ, ſinguli per ſingulos, Series nova reſultans accurate ſummabitur.

Coroll 4. Si ſint Series plures collaterales, ita relatæ ut terminus quilibet cujuſque Seriei ad præcedentes quoſlibet aliarum Serierum habeat rationes datas, ſive eaſdem ſive diverſas, ita ut Series iſtæ collaterales ſe decuſſent data qualibet lege conſtanti, Series iſtæ accurate ſummabuntur. Inſuper ſi termini omnes harum Serierum multiplicentur ordinatim per terminos Progreſſionis Arithmeticæ, ſinguli per ſingulos, Series novæ ex hac multiplicatione reſultantes etiamnum accurate ſummabuntur.

Clavis ad Problema generale.

Si ſint Colluſores quotcunque *v. g.* Sex, B, C, D, E, F, A & Probabilitates quas habent ut victores evadant, ſive ut mulctentur, dato Ludorum numero, denotentur reſpective B, C, D, E, F & A; & Probabilitates dato Ludorum numero his proximo & minori competentes, per $B_{\prime\prime}, C_{\prime\prime}, D_{\prime\prime}, E_{\prime\prime}, F_{\prime\prime}, A_{\prime\prime}$; & Probabilitates dato Ludorum numero his itidem noviſſimis proximo & minori competentes, per $B_{\prime\prime\prime}, C_{\prime\prime\prime}, D_{\prime\prime\prime}, E_{\prime\prime\prime}, F_{\prime\prime\prime}, A_{\prime\prime\prime}$, & ſic deinceps; erit ſemper,

$$
\begin{aligned}
B_{\prime} &= \tfrac{1}{2}A_{\prime\prime} + \tfrac{1}{4}A_{\prime\prime\prime} + \tfrac{1}{8}A_{\prime\prime\prime\prime} + \tfrac{1}{16}A_{V} \\
C_{\prime} &= \tfrac{1}{2}B_{\prime\prime} + \tfrac{1}{4}F_{\prime\prime\prime} + \tfrac{1}{8}E_{\prime\prime\prime\prime} + \tfrac{1}{16}D_{V} \\
D_{\prime} &= \tfrac{1}{2}C_{\prime\prime} + \tfrac{1}{4}B_{\prime\prime\prime} + \tfrac{1}{8}F_{\prime\prime\prime\prime} + \tfrac{1}{16}E_{V} \\
E_{\prime} &= \tfrac{1}{2}D_{\prime\prime} + \tfrac{1}{4}C_{\prime\prime\prime} + \tfrac{1}{8}B_{\prime\prime\prime\prime} + \tfrac{1}{16}F_{V} \\
F_{\prime} &= \tfrac{1}{2}E_{\prime\prime} + \tfrac{1}{4}D_{\prime\prime\prime} + \tfrac{1}{8}C_{\prime\prime\prime\prime} + \tfrac{1}{16}B_{V} \\
A_{\prime} &= \tfrac{1}{2}F_{\prime\prime} + \tfrac{1}{4}E_{\prime\prime\prime} + \tfrac{1}{8}D_{\prime\prime\prime\prime} + \tfrac{1}{16}C_{V}
\end{aligned}
$$

Et fiat ſemper retrogreſſus ordinatim ad tot literas minus duobus quot ſunt Colluſores, omittaturque ſemper litera A, prima æquatione excepta, ubi litera A terminos omnes præter primam occupat.

IV.

IV. *An Account of several extraordinary Meteors or Lights in the Sky. By Dr.* Edmund Halley, Savilian *Professor of Geometry at* Oxon, *and Secretary to the* Royal-Society.

THE *Theory* of the Air seemeth, at present, to be perfectly well understood, and the differing Densities thereof at all Altitudes, both by Reason and Experiment are sufficiently defined; for supposing the same Air to occupy Spaces reciprocally proportional to the Quantity of the superior or incumbent Air, I have elsewhere proved that at 40 Miles high the Air is rarer than at the Surface of the Earth about 3000 times; and that the utmost Height of the Atmosphere, which reflects Light in the *Crepusculum*, is not fully 45 Miles Notwithstanding which, 'tis still manifest that some sort of Vapours, and those in no small Quantity, arise nearly to that Height. An Instance of this may be given in the great Light the Society had an account of (*vid. Transact.* Sept. 1676) from Dr. *Wallis*, which was seen in very distant Counties almost over all the South Part of *England*. Of which though the Doctor could not get so particular a Relation, as was requisite to determine the Height thereof, yet from the distant Places it was seen in, it could not but be very many Miles high.

So likewise that Meteor which was seen in 1708. on the 31*st* of *July*, between Nine and Ten a Clock at Night, was evidently between 40 and 50 Miles perpendicularly high, and as near as I can gather, over *Sherenes*s and the *Buoy on the Nore*. For it was seen at *London* moving horizontally from *E. by N.* to *E. by S.* at least 50 Degrees high, and at *Redgrave* in *Suffolk*, on the *Yarmouth* Road, about 20 Miles from the East Coast of *England*, and at least 40 Miles to the Eastward of *London*, it appeared a little to the Westwards of the South, suppose *S. by W.* and was seen about 30 Degrees high, sliding obliquely downwards. I was shown in both Places the Situation thereof, which was as described, but could wish some Person skill'd in Astronomical Matters had seen it,

 that

that we might pronounce concerning its Height with more certainty: Yet, as it is, we may ſecurely conclude, that it was not many Miles more Weſterly than *Redgrave*, which, as I ſaid before, is above 40 Miles more Eaſterly than *London*. Suppoſe it therefore, where perpendicular, to have been 35 Miles Eaſt from *London*, and by the Altitude it appear'd at in *London*, *viz.* 50 Degrees, its Tangent will be 42 Miles, for the Height of the Meteor above the Surface of the Earth; which alſo is rather of the leaſt, becauſe the Altitude of the Place ſhewn me, is rather more than leſs than 50 Degrees: and the like may be concluded from the Altitude it appear'd in at *Redgrave*, near 70 Miles diſtant. Though at this great Diſtance, it appear'd to move with an incredible Velocity, darting, in a very few Seconds of Time, for about 12 Degrees of a great Circle from North to South, being very bright at its firſt Appearance; and it died away at the End of its Courſe, leaving for ſome time a pale Whiteneſs in the Place, with ſome Remains of it in the Track where it had gone; but no hiſſing Sound as it paſt, or Bounce of an Exploſion were heard.

It may deſerve the Honourable Society's Thoughts, how ſo great a Quantity of Vapour ſhould be raiſed to the very Top of the Atmoſphere, and there collected, ſo as upon its Accenſion or otherwiſe Illumination, to give a Light to a Circle of above 100 Miles Diameter, not much inferior to the Light of the Moon; ſo as one might ſee to take a Pin from the Ground in the otherwiſe dark Night. 'Tis hard to conceive what ſort of Exhalations ſhould riſe from the Earth, either by the Action of the Sun or ſubterranean Heat, ſo as to ſurmount the extream Cold and Rareneſs of the Air in thoſe upper Regions: But the Fact is indiſputable, and therefore requires a Solution.

Like to this, but much more conſiderable, was that famous Meteor which was ſeen to paſs over *Italy* on the 21*ſt* of *March O. S. Anno* 1676. about an Hour and Three Quarters after Sun ſet, which happen'd to be obſerved and well conſider'd by the famous Profeſſor of Mathematicks in *Bononia*

Geminian Montanari, as may be ſeen in his *Italian* Treatiſe about it, ſoon after publiſhed at *Bononia*. He obſerves that at *Bononia*, its greateſt Altitude in the *S. S. E.* was 38 Degrees, and at *Siena* 58 to the *N. N. W*: that its Courſe by the Concurrence of all the Obſervers was from *E. N. E* to *W. S. W.* that it came over the *Adriatick* Sea as from *Dalmatia*: that it croſt over all *Italy*, being nearly vertical to *Rimini* and *Savigniano* on the one Side, and to *Leghorn* on the other: that its perpendicular Altitude was at leaſt 38 Miles: that in all Places near this Courſe, it was heard to make a hiſſing Noiſe as it paſſed, *Ronzare, Far ſtrepito comme un fuoco artificiale, Fiſciare per aria comme un Raggio di polve*; that having paſt over *Leghorn* it went off to Sea towards *Corſica*, and laſtly that at *Leghorn* it was heard to give a very great Blow, *Tuono di maggior rumore di groſſa Cannonata*; immediately after which another ſort of Sound was heard like the rattling of a great Cart running over Stones, which continued about the time of a *Credo*.

He concludes from the apparent Velocity it went on with at *Bononia*, at above 50 Miles Diſtance, that it could not be leſs ſwift than 160 Miles in a Minute of Time, which is above Ten times as ſwift as the diurnal Rotation of the Earth under the Equinoctial, and not many times leſs than that wherewith the annual Motion of the Earth about the Sun is performed. To this he adds the Magnitude thereof, which appeared at *Bononia* bigger than the Moon in one Diameter, and above half as big again in the other; which with the given Diſtance of the Eye, makes its real leſſer Diameter above half a Mile, and the other in proportion. This ſuppoſed, it cannot be wondred that ſo great a Body moving with ſuch an incredible Velocity through the Air, though ſo much rarified as it is in its upper Regions, ſhould occaſion ſo great a hiſſing Noiſe, as to be heard at ſuch a Diſtance as it ſeems this was. But 'twill be much harder to conceive, how ſuch an *impetus* could be impreſſed on the Body thereof, which by many Degrees exceeds that of any Cannon Ball; and how

 this

this *impetus* ſhou'd be determined in a Direction ſo nearly parallel to the Horizon; and what ſort of Subſtance it muſt be, that could be ſo impelled and ignited at the ſame time: there being no *Vulcano* or other *Spiraculum* of ſubterraneous Fire in the *N. E.* parts of the World, that we ever yet heard of, from whence it might be projected.

I have much conſidered this Appearance, and think it one of the hardeſt things to account for, that I have yet met with in the *Phænomena* of *Meteors*, and am induced to think that it muſt be ſome Collection of Matter form'd in the *Æther*, as it were by ſome fortuitous Concourſe of Atoms, and that the Earth met with it as it paſt along in its Orb, then but newly formed, and before it had conceived any great *Impetus* of Deſcent towards the Sun. For the Direction of it was exactly oppoſite to that of the Earth, which made an Angle with the Meridian at that time (the Sun being in about 11 Degrees of *Aries*) of 67 Gr. that is, its Courſe was from *W. S. W.* to *E. N. E.* wherefore the *Meteor* ſeem'd to move the contrary Way: And beſides falling into the Power of the Earth's Gravity, and loſing its Motion from the Oppoſition of the *Medium*, it ſeems that it deſcended towards the Earth, and was extinguiſh'd in the *Tyrrhene Sea*, to the *W. S. W.* of *Leghorn*. The great Blow being heard upon its firſt Immerſion into the Water, and the rattling like the driving a Cart over Stones being what ſucceeded upon its quenching; ſomething like which is always obſerved upon quenching a very hot Iron in Water. Theſe Facts being paſt diſpute, I would be glad to have the Opinion of the Learned thereon, and what Objection can be reaſonably made againſt the aboveſaid *Hypotheſis*, which I humbly ſubmit to their Cenſure.

P. S. Since this was written, there has fallen into my Hands an Account of much ſuch another Appearance, ſeen in *Germany*, in the Year 1686, at *Leipſic*, by the late Mr. *Gottfreid Kirch*, who was for many years a very diligent Obſerver of the Heavens'

Heavens, and perfectly well instructed in Astronomical Matters. He in an Appendix to his *Ephemerides* for the Year 1688, gives us this remarkable Relation in the following Words.

Die 9 Jul. st. vet. *Hora* 1 ½ *matutina, Globus ardens cauda præditus in* 8 ½ *Gr. Aquarii & 4 Gr. Sept. apparuit, qui per semiquadrantem Horæ immotus perstitit, cujus diameter semidiametrum Lunæ circiter æquabat. Primo lux tanta erat, ut ejus ope sine candelis legere potuissemus: postea pedetentim in loco suo evanescebat. Phænomenon istud dicto tempore multis aliis in locis pariter visum est, præsertim* Schlaizii, *oppido undecim milliaribus Germanicis abhinc (i. e. a* Lipsiâ) *versus Meridiem distante, altitudine circiter 60 Gr. ab Horizonte meridiano.*

At the time of this Appearance the Sun was in 26 ½ Gr. of ♋, and by the given Place of the Meteor, 'tis plain, it was seen about ¾ of an Hour past the Meridian, or in *S. by W.* and by its Declination it could not be above 24 Degrees high at *Leipsic*, though the same, at *Schlaize* was about 60 Gr. high: The Angle therefore at the Meteor was about 36 Gr. Whence by an easy *calculus* it will be found, that the same was not less than 16 German Miles distant in a right Line from *Leipsick*, and above 6 ½ such Miles perpendicular above the Horizon, that is at least 30 English Miles high in the Air. And though the Observer says of it, *immotus perstitit per semiquadrantem horæ*; 'tis not to be understood that it kept its Place like a Fixt Star, all the time of its Appearance; but that it had no very remarkable progressive Motion. For himself has at the End of the said *Ephemerides* given a Figure of it, which he has marked *Fig. D.* whereby it appears that it darted downwards obliquely to the Right-Hand, and where it ended, left two Globules or Nodes, not visible but by an Optick Tube.

The same Mr. *Gottfried Kirch* in the beginning of a *German* Treatise of his, concerning the great Comet which appeared in the Year 1680, intituled **Newe Himmels Zeitung,**

printed at *Nurenburg anno* 1681, (of which perhaps we ſhall have further occaſion to make mention) gives us a Relation of ſuch another luminous Meteor ſeen likewiſe at *Leipſick* on the 22*d* of *May* 1680. *ſt. vet.* about three in the Morning: which though himſelf ſaw not, was yet there obſerv'd by divers Perſons who made various Reports of it, but the more intelligent agreed that it was ſeen deſcending in the North, and left behind it a long white Streak where it had paſt. At the ſame time at *Haarburgh* the like Appearance was ſeen in *N.E.* or rather *N N.E*; as alſo at *Hamburg*, *Lubeck* and *Stralſund*, all which are about 40 *German* Miles from *Leipſick:* but in all theſe Places, by Perſons unacquainted with the manner of properly deſcribing things of this kind. So that all we can conclude from it is, that this Meteor was exceeding high above the Earth, as well as the former.

All the Circumſtances of theſe *Phænomena* agree with what was ſeen in *England* in 1708, but it commonly ſo happens that theſe contingent Appearances eſcape the Eyes of thoſe that are beſt qualified to give a good Account of them. 'Tis plain however that this ſort of luminous Vapour is not exceedingly ſeldom thus collected; and when the like ſhall again happen, the Curious are entreated to take more Notice of them than has been hitherto done, that we may be enabled thereby better to account for the ſurprizing Appearances of this ſort of *Meteor*.

V. *Some Remarks on the* Variations *of the* Magnetical Compaſs *publiſhed in the* Memoirs *of the* Royal Academy of Sciences, *with regard to the General Chart of thoſe* Variations *made by* E. Halley; *as alſo concerning the true Longitude of the* Magellan *Streights.*

IT muſt be acknowledged that the Gentlemen of the *Royal Academy of Sciences* in *France*, have, for ſome Years paſt, apply'd themſelves with much Candour, as well as Diligence, to examine the Chart I publiſh'd in the Year 1701, for ſhewing at one View the Variations of the Magnetical Compaſs, in all thoſe Seas with which the *Engliſh* Navigators are acquainted: And, to my no ſmall Satisfaction, I find that what I did ſo long ago, has been ſince abundantly verified by the concurrent Reports of the *French* Pilots, who of late have had frequent Opportunities of enquiring into the Truth thereof. So that I am in hopes I have laid a ſure Foundation for the future Diſcovery of an Invention, that will be of wonderful Uſe to Mankind when perfected; I mean that of the Law or Rule by which the ſaid Variations change, in Appearance regularly, all the World over. Of this I have adventured long ſince to give my Thoughts in N° 148 and N° 195 of theſe *Tranſactions*, and as yet I ſee no Cauſe to retract what I there offer for a Reaſon of this Change; but of this we might be more certain, had we a good Collection of Obſervations made in that Ocean which divides *Aſia* and *America*, and occupies about two fifths of the whole Circumference of the Globe. This, we hope, from the natural Curioſity of the *French* (who want no Means of performing it) may be effectually ſupply'd by ſuch of that Nation who may return from *Peru* by the *Eaſt-Indies*.

In the mean time I cannot omit to take Notice of two Particulars, ſeeming to call in Queſtion the Truth of my aforeſaid Map, which I have lately obſerved in the *Memoirs* of the *Royal Academy of Sciences.*

The one is in the *Memoirs* of the Year 1700, concerning the Variation obſerved at *Paraïba* in *Braſile,* about 25 Leagues to the Northwards of *Pernambouc,* by M. *Couplet le fils,* whoſe Words are theſe,

Le 20 *Mai,* 1698. *ayant auparavant tracé ſeigneuſement une ligne* Meridienne, *dont je m'etois ſervi pour les Obſervations Aſtronomiques, jobſervai la declinaicon de l' aiguille aimantée de* 5° 35′ Nordoueſt. And the ſame Obſerver tells us, that he found the Latitude of the Town of *Paraïba* 6° 38′ 18″. Now it ſo fell out, that my ſelf was in the River of *Paraïba,* in the Month of *March,* 1699. and there fitted and cleaned my Ship, ſo that I had full Opportunity to obſerve the Variation both on Board and on Shore, and found it conſtantly to be above 4 Gr *North-Eaſt*; ſo that I am willing to believe this to be an Error of the Preſs, putting N. W. for N. E; or rather of the Memory of M. *Couplet,* who, it ſeems, loſt all his Papers by Shipwreck in his Return. The like may be ſaid of the Latitude of *Paraïba,* which, though I did not obſerve my ſelf, yet at the Fort of *Cabo Dello,* at the Mouth of the River, and which is about 3 Leagues more Northerly than the Town, I found the Latitude not leſs than 6° 55′ South, and by Conſequence that of the Town more than 7 Degrees.

The other is in a Diſcourſe of M. *de Liſle,* in the *Memoirs* of 1710; where he compares the Variations obſerved in ſome late Voyages, with my Map of the Variations. Among other Things, 'tis there ſaid, that on the Eaſt-ſide of the Iſland St *Thomas,* under the Equinoctial Line, M. *Bigot de la Canté,* ſecond Lieutenant of the King's Ship *la Sphere,* had, in the beginning of the Year 1708, found the Variation 11½ Gr. whereas my Chart makes it but 5½ Gr. 'Tis true, that I never obſerved my ſelf in thoſe Parts; and 'tis

from

from the Accounts of others, and the Analogy of the whole, that in ſuch Caſes I was forc'd to ſupply what was wanting; and 'tis poſſible that there may be more Variation on that Coaſt than I have allowed. But conſulting my Chart (which was fitted to the Year 1700) I find I then make the Variation at the Iſle of St *Thomas* full $7\frac{1}{2}$ Gr. and not $5\frac{1}{2}$ Gr. the which, by the Year 1708, might well ariſe to near r. So that the Difference will become very tolerable; ereas an Error of 6 Degrees, ſuch as is here repreſented, wou'd render the Credit of my Chart juſtly ſuſpected, and the ſame by conſequence wholly uſeleſs, as not to be confided in.

But a further Thing I might complain of is, that in the ſame *Memoire* of M. *de Liſle*, the Geography of my Chart is called in Queſtion; and we are told that I have placed the Entrance of the *Magellan* Straights at leaſt 10 Degrees more Weſterly than I ought to have done: for that the Ship St. *Louis*, in the Year 1708, ſailing from the Mouth of *Rio Gallega*, in about the Latitude of 52 Gr. South, and not far from Cape *Virgin*, directly for Cape *Bonne Eſperance* (which Courſe perhaps was never run before) had found the Diſtance between the two Lands not more than 1350 Leagues, which, he concludes, is much leſs than my Chart of the Variation makes it. I know not from what Computation M. *de Liſle* has deduced this Conſequence, but I find by my Chart that I have made the Longitude of *Rio Gallega* 75 Gr. Weſt from London, and that of Cape *Bonne Eſperance* $16\frac{1}{2}$ Eaſt from it; that is in all $91\frac{1}{2}$ Gr. difference of Longitude. This, with the two Latitudes, gives the Diſtance, according to the Rhumb-line 1364 Leagues, but according to the Arch of a great Circle, no more than 1287 Leagues; ſo that inſtead of invalidating what I have there laid down, it does abſolutely confirm it, as far as the Authority of one ſingle Ship's Journals can do it.

I do not pretend that I have had Obſervations made with all the Preciſion requiſite, to lay down inconteſtably the *Magellan* Straights in their true Geographical Site; but yet it has not been without good Grounds that I have placed them as I have done. For when Sir *John Narborough*, in the Year 1670, wintered in Port St. *Julian*, on the Coaſt of *Patagonia*, Capt. *John Wood*, then his Lieutenant, and an approved Artiſt in Sea Affairs, did obſerve the beginnir of an Eclipſe of the Moon, *Sept.* 18vo *Stil. vet.* at juſt 8ane Night: And the ſame beginning was obſerv'd by M. *Hevelius* at *Dantzick* at 14^h 22′; whence Port St. *Julian* is more Weſterly than *Dantzick* 6^h 22′, or than *London* 5^h 6, that is 76½ Gr. Beſides, I have had in my Cuſtody a very curious Journal of one Capt. *Strong*, who went into the *South Seas* in queſt of a rich Plate-wreck, and who diſcover'd the two Iſlands he called *Falkland's Iſles*, lying about 120 Leagues to the Eaſtwards of the *Patagon* Coaſt, about the Lat. of 51½. This Capt. *Strong* had a quick Paſſage from the Iſland of *Trinadada* (in 20½ South) to the *Magellan* Straights; and in this Journal, which was very well kept, I found that Cape *Virgin* was, by his Account, 45 Degrees of Longitude more Weſterly than that Iſland, whoſe Longitude I know to be juſt 30 Degrees from *London*: that is in all 75 Gr.

From theſe concurrent Teſtimonies, wanting better, I adventured to fix the Longitude of this Coaſt as I have done; and I can by no means grant an Error of 10 Degrees to be poſſible in it, though perhaps it may need ſome ſmaller Correction. I will however readily grant, that thoſe that go thither from *Europe*, ſhall find the Land more Eaſterly than is here expreſs'd, by reaſon of a conſtant Current ſetting to the Weſtward near the Equator, where Ships are many times long detained by Calms, whilſt the Stream carries them along with it; which thing befalls all Ships bound to any Part of the Eaſt Coaſt of the *South America*.

Printed for William Innys, *at the* Prince's-Arms *in St.* Paul's Church-yard, MDCCXV.

Fig. I.

Cursus Cometæ Anni 1680 Observationes primæ, Coburgi Saxoniæ a D.ns Gottfried Kirch habitæ.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *January* and *February*, 171 4/5.

The CONTENTS.

I. *Observationes quædam accuratæ insignis Cometæ sub finem anni* 1680 *visi*, Coburgi Saxoniæ *a Domino* Gottfried Kirch *habitæ*; *decimo tertio die antequam a quoquam alio observatus sit.*

COMETA ille qui anno exeunte 1680 visus est, plurimis de causis præcipuus merito habendus est; tam ob Cursum ejus quadrimestrem, quo novem integra Signa percurrit; quam ob immensam Caudæ magnitudinem & claritatem: maxime vero ob insignem Orbitæ Curvitatem, cujus ope tandem patefacta est Cometarum Theoria. Nam dum Astronomi omni adhibita diligentia motum ejus observationibus definiendo desudarunt, conatus eorum secundavit D.*Newtoni* in Geometricis vis pene divina, qui primus mortalium CometasOrbes Parabolicis maxime affines describere probavit, ac datis tribus locis accurate observatis eosdem construere docuit, remque hujusce Cometæ exemplo illustravit: id quod sub finem *Lib.* III. *Principiorum Philosophiæ* videre est.

Accidit autem, nescio quo fato, ut Cometa hic (quem vespertinum tantopere prosecuti sunt Cœlispices) antequam Solem attigerat matutinus, nec *Parisiis* neque *Grenvici* ne semel quidem observatus sit: quique eum viderunt & observarunt, incongrua & inter se pugnantia, ac pro rei subtilitate parum idonea prodidere: neque ante *Novemb.* 17. mane a quopiam Observatorum visus est. Unde factum ut Orbitæ pars illa, qua ad Solem descendit Cometa, paulo incertius definiri potuit. Bona autem fortuna nuper incidimus in Librum meritissimi Astronomi Domini *Gottfried Kirch*, *Germani*, anno 1681 *Noribergæ* impressi, cui titulus Neue Himmels Zeitung, hoc est *Novus Nuncius cœlestis*; ubi auctor diligentissimus nobis exponit, quo casu ductus Cometam hunc nondum adnata Cauda obscurum, ac vix nudis oculis conspicuum detexe-

detexerit; dum scilicet Lunam & Martem ei vicinum observaturiens circumlustraret, die *Novemb.* 4. *S. V.* mane, idque *Coburgi Saxoniæ*, quod oppidum XI. Grad. *Londino* orientalius est, sub altitudine poli 50° 20′ circiter. Excitatus autem, ut ait, rumoribus Cometæ in *Germania* visi, vultu in Orientem verso pernoctavit, ut cœlo tum forte perquam sereno, si quid novi oriretur, situm ejus notaret. Lunâ vero jam ad stellam aliquam *Tychoni* incognitam applicatâ, (sed quæ in Catalogo *Flamsteedii Britannico* habetur, estque numero 44ta *Leonis*) voluit dictæ stellæ locum ex circumvicinis Fixis determinare; dumque Tubum trium graduum capacem hinc inde circumrotat, incidit in Luculam quandam nebulosam, speciem insolitam præ se ferentem, quamque vel novum Cometam esse, vel Stellam nebulosam ad instar ejus quæ in *Cingulo Andromedæ* est, statim conclusit.

Primo autem Cometam vidit, Hora $4\frac{1}{2}$ matutinâ, paulo altiorem duabus stellulis Telescopicis, quæ in *Figura* I. literis *a* & *c* signantur, cum quibus tamen Hora 6ta visus est in lineâ accurate rectâ; unde constabat eum moveri, idque motu directo. Inter horas vero 5. & 6. Tubo decempedali Phænomenon hoc contemplatus est, vidirque duas alias stellulas contiguas prioribus minores, literisque *e* & *d* notatas, & supra has tertiam *g*. Erat autem distantia Cometæ ab *e* paulo minor quam ab *a*, major vero distantia *d e*. 6h. 38′ distantia Cometæ ab *e* dupla erat intervalli inter ipsas *d e*, ac linea *d e* producta reliquit Cometam* infra se, sic tamen ut marginem ejus superiorem attingeret. 6h. 45′. Cometa jam sensibiliter remotior erat ab *e* quam ab *a*, distabatque ab *a* paulo plus quam dimidio distantiæ stellularum *a* & *g*.

Notandum vero est Horologium totis 14 minutis cœlum anticipasse, uti ex altitudinibus *Cordis Leonis* tum captis patuit.

Nobilis sane est hæc observatio, adeoque Stellularum Cometæ tunc adjacentium loca non una methodo perquisivimus, exercitatissimam manum suam & instrumenta perelegantia

* Tubo scilicet objecta invertente.

præ-

præbente Reverendo Viro D. *Jacobo Pound*, R. Soc. Sodali. Unde constabat stellulas illas tum temporis infrascriptos habuisse situs, nempe

	Long.	Lat.
a	♌ 29°. 54′. 20″	1. 29. 20 Bor.
d	29. 27. 20	1. 8. 00
e	29. 34. 30	1. 10. 45

Circulus autem maximus per *a* & *c* ductus, deprehensus est per *Ultimam Caudæ Ursæ majoris* transire, adeoque angulum cum circulo Longitudinis ad *a* esse 15° 36′ ½. Cumque distantia Cometæ ab *a* versus *c* paulo major fuerat dimidio distantiæ *a g*, (quam Tubo sedecim pedum & Micrometro deprehendimus 2 ⅔ min.) ponamus eam fuisse duodecim minutorum; & ex datis proveniet Cometæ locus ♌ 29° 51′ cum Lat. Boreâ 1° 17′ ¾. Hora scilicet Horologii 6ta, sed *Londini* 5h 2′ Temp. Appar.

Deinde *Novemb.* 6. mane, 4h 42′, Tubo bipedali deprehendit Cometam omnino in linea recta inter *Martem* & Stellulam *N*; quæ quidem in *Catalogo Britannico* 45ta Leonis est, & tunc erat in ♍ 2° 42′ cum Lat. Aust. 0° 16′ ½. *Mars* autem tum temporis habuit (ex collatis observationibus paulo antea & post factis) ♍ 3° 46′ ½ cum Lat. Bor. 1° 56′. Unde, ob datam ejus viam, Cometa occupavit ♍ 3°. 23′ cum Lat. Bor. 1° 6′. *Londini* Temp. App. 3h 58′ mane.

Novembris quoque 11mo 5h 15′ mane, Cometa æqualiter distabat à Stellis Leonis σ & τ *Bayero*, nondum vero attigit rectam easdem jungentem, sed parum abfuit ab ea. In *Catalogo Britannico* σ tunc habuit ♍ 14° 15′. cum Lat. Bor. 1° 41′ ferè, τ vero ♍ 17° 3′ ½ cum Lat. Aust. 0° 34′. Proinde Cometæ Latitudo paulo minor erat medio inter illas, nempe quam 0° 33′ ½ Bor. ac Longitudo quam ♍ 15°. 39′. Sed huic non utique fidendum, cum pendeat ab æstimata æqualitate distantiarum, quæ res lubrica est. Cauda autem jam cœpta non nisi dimidio gradu longa Tubo decempedali visa est.

Qui plura cupit adeat Librum ipsum *Germanicè* editum.

II.

II. *An Account of the Book entituled* Commercium Epiſtolicum Collinii & aliorum, De Analyſi promota; *publiſhed by order of the* Royal-Society, *in relation to the Diſpute between Mr.* Leibnitz *and Dr.* Keill, *about the Right of Invention of the Method of* Fluxions, *by ſome call'd* the Differential Method.

SEveral Accounts having been publiſhed abroad of this *Commercium*, all of them very imperfect: It has been thought fit to publiſh the Account which follows.

This *Commercium* is compoſed of ſeveral ancient Letters and Papers, put together in order of Time, and either copied or tranſlated into Latin from ſuch Originals as are deſcribed in the Title of every Letter and Paper; a numerous Committee of the Royal-Society being appointed to examin the Sincerity of the Originals, and compare therewith the Copies taken from them. It relates to a general Method of reſolving finite Equations into infinite ones, and applying theſe Equations, both finite and infinite, to the Solution of Problems by the Method of Fluxions and Moments. We will firſt give an Account of that Part of the Method which conſiſts in reſolving finite Equations into infinite ones, and ſquaring curvilinear Figures thereby. By Infinite Equations are meant ſuch as involve a Series of Terms converging or approaching the Truth nearer and nearer *in infinitum*, ſo as at length to differ from the Truth leſs than by any given Quantity, and if continued *in infinitum*, to leave no Difference.

Dr. *Wallis* in his *Opus Arithmeticum* publiſhed *A. C.* 1657. *Cap.* 33. *Prop.* 68. reduced the Fraction $\frac{A}{1-R}$ by perpetual Diviſion into the Series $A + AR + AR^2 + AR^3 + AR^4 + \&c.$

Viſcount *Brounker* ſquared the Hyperbola by this Series $\frac{1}{1\times2} + \frac{1}{3\times4} + \frac{1}{5\times6} + \frac{1}{7\times8} + \&c.$ that is by this, $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \&c.$ conjoyning every two Terms into one. And the Quadrature was publiſhed in the *Philoſophical Tranſactions* for *April* 1668.

Mr. *Mercator* ſoon after publiſhed a Demonſtration of this Quadrature by the Diviſion of Dr. *Wallis*; and ſoon after that Mr. *James Gregory* publiſhed a Geometrical Demonſtration thereof. And theſe Books were a few Months after ſent by Mr. *John Collins* to Dr. *Barrow* at *Cambridge*, and by Dr. *Barrow* communicated to Mr. *Newton* (now Sir *Iſaac Newton*) in *June* 1669. Whereupon Dr *Barrow* mutually ſent to Mr. *Collins* a Tract of Mr. *Newton*'s entituled *Analyſis per æquationes numero terminorum infinitas.* And this is the firſt Piece publiſhed in the *Commercium*, and contains a general Method of doing in all Figures, what my Lord *Brounker* and Mr. *Mercator* did in the Hyperbola alone. Mr. *Mercator* lived above ten Years longer without proceeding further than to the ſingle Quadrature of the Hyperbola. The Progreſs made by Mr. *Newton* ſhews that he wanted not Mr. *Mercator*'s Aſſiſtance. However, for avoiding Diſputes, he ſuppoſes that my Lord *Brounker* invented, and Mr. *Mercator* demonſtrated, the Series for the Hyperbola ſome Years before they publiſhed it; and, by conſequence, before he found his general Method.

The aforeſaid Treatiſe of *Analyſis* Mr. *Newton*, in his Letter to Mr. *Oldenburgh*, dated *Octob.* 24. 1676. mentions in the following Manner. *Eo ipſo tempore quo* Mercatoris *Logarithmotechnia prodiit, communicatum eſt per amicum* D. Barrow (*tunc* Matheſeos *Profeſſorem* Cantab.) *cum* D. Collinio *Compendium quoddam harum Serierum, in quo ſignificaveram Areas & Longitudines Curvarum omnium, & Solidorum ſuperficies & contenta ex*

datis

datis rectis ; & vice versa ex his datis rectas determinari posse : & methodum indicatam illustraveram diversis seriebus. Mr. *Collins* in the Years 1669, 1670, 1671 and 1672 gave notice of this Compendium to Mr *James Gregory* in *Scotland*, Mr. *Bertet* and Mr. *Vernon* then at *Paris*, Mr. *Alphonsus Borelli* in *Italy*, and Mr. *Strode*, Mr. *Townsend*, Mr. *Oldenburgh*, Mr. *Dary* and others in *England*, as appears by his Letters. And Mr. *Oldenburg* in a Letter, dated *Sept.* 14. 1669. and entred in the Letter-Book of the Royal-Society, gave notice of it to Mr. *Francis Slusius* at *Liege*, and cited several Sentences out of it. And particularly Mr. *Collins* in a Letter to Mr. *James Gregory* dated *Novemb.* 25. 1669. spake thus of the Method contained in it. Barrovius *Provinciam suam publicè prælegendi remisit cuidam nomine* Newtono Cantabrigiensi, *cujus tanquam viri acutissimo ingenio præditi in Præfatione Prælectionum Opticarum, meminit : quippe antequam ederetur* Mercatoris *Logarithmotechnia, eandem methodum adinvenerat, eamque ad omnes Curvas generaliter & ad Circulum diversimode applicârat.* And in a Letter to Mr. *David Gregory* dated *August* 11. 1676. he mentions it in this manner. *Paucos post menses quam editi sunt hi Libri* [viz. *Mercatoris* Logarithmotechnia & Exercitationes Geometricæ *Gregorii*] *missi sunt ad* Barrovium Cantabrigiæ. *Ille autem responsum dedit hanc infinitarum Serierum Doctrinam à* Newtono *biennium ante excogitatam fuisse quam ederetur* Mercatoris *Logarithmotechnia & generaliter omnibus figuris applicatam, simulque transmisit D.* Newtoni *opus manuscriptum.* The last of the said two Books came out towards the End of the Year 1668, and Dr. *Barrow* sent the said Compendium to Mr. *Collins* in *July* following, as appears by three of Dr. *Barrow*'s Letters. And in a Letter to Mr. *Strode*, dated *July* 26. 1672, Mr. *Collins* wrote thus of it. *Exemplar ejus* [Logarithmotechniæ] *misi* Barrovio Cantabrigiam, *qui quasdam* Newtoni *chartas extemplo remisit : E quibus & aliis quæ prius ab authore cum* Barrovio *communicata fuerant, patet illam methodum à dicto* Newtono *aliquot annis antea excogitatam & modo univer-*

sali applicatam fuisse : Ita ut ejus ope, in quavis Figura Curvilinea proposita, quæ una vel pluribus proprietatibus definitur, Quadratura vel Area dictæ figuræ, accurata si possibile sit, sin minus infinitè verò propinqua, Evolutio vel longitudo Lineæ Curvæ, Centrum gravitatis figuræ, Solida ejus rotatione genita & eorum superficies; sine ulla radicum extractione obtineri queant. Postquam intellexerat D. Gregorius *hanc methodum à D.* Mercatore *in Logarithmotechnia usurpatam & Hyperbolæ quadrandæ adhibitam, quamque adauxerat ipse* Gregorius, *jam universalem redditam esse, omnibusque figuris applicatam; acri studio eandem acquisivit multumque in ea enodanda desudavit. Uterque D.* Newtonus *&* Gregorius *in animo habet hanc methodum exornare : D.* Gregorius *autem D.* Newtonum *primum ejus inventorem anticipare haud integrum ducit.* And in another Letter written to Mr. *Oldenburgh* to be communicated to Mr. *Leibnitz*, and dated *June* 14 1676. Mr. *Collins* adds: *Hujus autem methodi ea est præstantia ut cum tam late pateat ad nullam hæreat difficultatem.* Gregorium *autem aliosque in ea fuisse opinione arbitror, ut quicquid uspiam antea de hac re innotuit, quasi dubia diluculi lux fuit si cum meridiana claritate conferatur.*

This Tract was first printed by Mr. *William Jones*, being found by him among the Papers and in the Hand-writing of Mr. *John Collins*, and collated with the Original which he afterwards borrowed of Mr. *Newton*. It contains the above-mention'd general Method of *Analysis*, teaching how to resolve finite Equations into infinite ones, and how by the method of Moments to apply Equations both finite and infinite to the Solution of all Problems. It begins where Dr. *Wallis* left off, and founds the method of Quadratures upon three Rules.

Dr. *Wallis* published his *Arithmetica infinitorum* in the Year 1655, and by the 59*th* Proposition of that Book, if the *Abscissa* of any curvilinear Figure be called x, and m and n be Numbers, and the Ordinates erected at right Angles be $x^{\frac{m}{n}}$, the Area of the Figure shall be $\frac{n}{m+n} x^{\frac{m+n}{n}}$. And this is assumed by

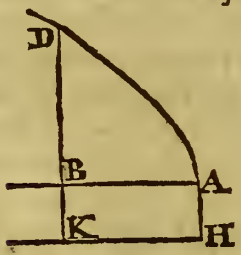
by Mr. *Newton* as the first Rule upon which he founds his Quadrature of Curves. Dr. *Wallis* demonstrated this Proposition by Steps in many particular Propositions, and then collected all the Propositions into One by a Table of the Cases. Mr. *Newton* reduced all the Cases into One, by a Dignity with an indefinite Index, and at the End of his Compendium demonstrated it at once by his method of Moments, he being the first who introduced indefinite Indices of Dignities into the Operations of *Analysis*.

By the 108*th* Proposition of the said *Arithmetica Infinitorum*, and by several other Propositions which follow it; if the Ordinate be composed of two or more Ordinates taken with their Signes + and —, the Area shall be compos'd of two or more Areas taken with their Signes + and — respectively. And this is assumed by Mr. *Newton* as the second Rule upon which he founds his Method of Quadratures.

And the third Rule is to reduce Fractions and Radicals, and the affected Roots of Equations into converging Series, when the Quadrature does not otherwise succeed; and by the first and second Rules to square the Figures, whose Ordinates are the single Terms of the Series. Mr *Newton*, in his Letter to Mr. *Oldenburgh* dated *June* 13. 1676. and communicated to Mr. *Leibnitz*, taught how to reduce any Dignity of any Binominal into a converging Series, and how by that Series to square the Curve, whose Ordinate is that Dignity. And being desired by Mr. *Leibnitz* to explain the Original of this Theoreme, he replied in his Letter dated *Octob.* 24. 1676, that a little before the Plague (which raged in *London* in the Year 1665) upon reading the *Arithmetica Infinitorum* of Dr. *Wallis*, and considering how to interpole the Series x, $x - \frac{1}{3}x^3$, $x - \frac{2}{3}x^3 + \frac{1}{5}x^5$, $x - \frac{3}{3}x^3 + \frac{3}{5}x^5 - \frac{1}{7}x^7$, *&c.* he found the Area of a Circle to be $x - \frac{\frac{3}{2}x^3}{3} - \frac{\frac{3}{8}x^5}{5} - \frac{\frac{3}{16}x^7}{7} - \frac{\frac{5}{128}x^9}{9} - \&c.$ And by pursuing the Method of Interpolati-

on he found the Theoreme abovemention'd, and by means of this Theoreme he found the Reduction of Fractions and Surds into converging Series, by Division and Extraction of Roots; and then proceeded to the Extraction of affected Roots. And these Reductions are his third Rule.

When Mr. *Newton* had in this Compendium explained these three Rules, and illustrated them with various Examples, he laid down the Idea of deducing the Area from the Ordinate, by considering the Area as a Quantity, growing or increasing by continual Flux, in proportion to the Length of the Ordinate, supposing the Abscissa to increase uniformly in proportion to Time And from the Moments of Time he gave the Name of Moments to the momentaneous Increases, or infinitely small Parts of the Abscissa and Area, generated in Moments of Time. The Moment of a Line he called a Point, in the Sense of *Cavallerius*, tho' it be not a geometrical Point, but a Line infinitely short, and the Moment of an Area or Superficies he called a Line, in the Sense of *Cavallerius*, tho' it be not a geometrical Line, but a Superficies infinitely narrow. And when he consider'd the Ordinate as the Moment of the Area, he understood by it the Rectangles under the geometrical Ordinate and a Moment of the Abscissa, tho' that Moment be not always expressed. *Sit ABD*, saith he, *Curva quævis, & AHKB rectangulum cujus latus AH* vel *KB est unitas. Et cogita rectam DBK uniformiter ab AH motam areas ABD & AK describere; & quod* [recta] *BK* (1) *sit momentum quo* [area] *AK* (*x*), *&* [recta] *BD* (*y*) *momentum quo* [area curvilinea] *ABD gradatim augetur; & quod ex momento BD perpetim dato possis, per præcedentes* [tres] *Regulas, aream ABD ipso descriptam investigare, sive cum area AK* (*x*) *momento* 1 *descripta conferre.* This is his Idea of the Work in squaring of Curves, and how he applies this to other Problems, he expresses in the next Words. *Jam qua ratione*, saith he, *superficies ABD ex momento suo per-*



petim

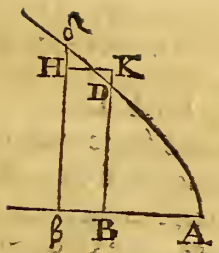
petim dato per præcedentes [tres] *Regulas elicitur, eâdem quælibet alia quantitas ex momento ſuo ſic dato elicietur. Exemplo res fiet clarior.* And after ſome Examples he adds his Method of Regreſſion from the Area, Arc, or ſolid Content, to the Abſciſſa; and ſhews how the ſame Method extends to Mechanical Curves, for determining their Ordinates, Tangents, Areas, Lengths, *&c.* And that by aſſuming any Equation expreſſing the Relation between the Area and Abſciſſa of a Curve, you may find the Ordinate by this Method. And this is the Foundation of the Method of Fluxions and Moments, which Mr. *Newton* in his Letter dated *Octob.* 24, 1676 comprehended in this Sentence. *Data æquatione quotcunque fluentes quantitates involvente, invenire Fluxiones; & vice verſa.*

In this Compendium Mr. *Newton* repreſents the uniform Fluxion of Time, or of any Exponent of Time by an Unit; the Moment of Time or of its Exponent by the Letter *o*; the Fluxions of other Quantities by any other Symbols; the Moments of thoſe Quantities by the Rectangles under thoſe Symbols and the Letter *o*; and the Area of a Curve by the Ordinate incloſed in a Square, the Area being put for a Fluent and the Ordinate for its Fluxion. When he is demonſtrating any Propoſition he uſes the Letter *o* for a finite Moment of Time, or of its Exponent, or of any Quantity flowing uniformly, and performs the whole Calculation by the Geometry of the Ancients in finite Figures or Schemes without any Approximation: and ſo ſoon as the Calculation is at an End, and the Equation is reduced, he ſuppoſes that the Moment *o* decreaſes *in infinitum* and vaniſhes. But when he is not demonſtrating but only inveſtigating a Propoſition, for making Diſpatch he ſuppoſes the Moment *o* to be infinitely little, and forbears to write it down, and uſes all manner of Approximations which he conceives will produce no Error in the Concluſion. An Example of the firſt kind you have in the End of this Compendium, in demonſtrating the firſt of the three Rules laid down in the Beginning of the Book.

Examples of the ſecond kind you have in the ſame Compendium, in finding the Length of Curve Lines *p.* 15. and in finding the Ordinates, Areas and Lengths of Mechanical Curves *p.* 18. 19. And he tells you, that by the ſame Method, Tangents may be drawn to mechanical Curves *p.* 19. And in his Letter of *Decemb.* 10. 1672. he adds, that Problems about the Curvature of Curves Geometrical or Mechanical are reſolv'd by the ſame Method. Whence its manifeſt, that he had then extended the Method to the ſecond and third Moments. For when the Areas of Curves are conſidered as Fluents (as is uſual in this *Analyſis*) the Ordinates expreſs the firſt Fluxions, the Tangents are given by the ſecond Fluxions, and the Curvatures by the third, And even in this *Analyſis p.* 16. where Mr. *Newton* ſaith, *Momentum eſt ſuperficies cum de ſolidis, & Linea cum de ſuperficiebus, & Punctum cum de lineis agitur*, it is all one as if he had ſaid, that when Solids are conſidered as Fluents, their Moments are Superficies, and the Moments of thoſe Moments (or ſecond Moments) are Lines, and the Moments of thoſe Moments (or third Moments) are Points, in the Senſe of *Cavallerius*. And in his *Principia Philoſophiæ*, where he frequently conſiders Lines as Fluents deſcribed by Points, whoſe Velocities increaſe or decreaſe, the Velocities are the firſt Fluxions, and their Increaſe the ſecond. And the Probleme, *Data æquatione fluentes quantitates involvente fluxiones invenire & vice verſa*, extends to all the Fluxions, as is manifeſt by the Examples of the Solution thereof, publiſhed by Dr. *Wallis Tom.* 2. *p* 391, 392, 396. And in *Lib.* II. *Princip. Prop.* XIV. he calls the ſecond Difference the Difference of Moments.

Now that you may know what kind of Calculation Mr. *Newton* uſed in, or before the Year 1669. when he wrote this Compendium of his *Analyſis*, I will here ſet down his Demonſtration of the firſt Rule abovementioned. *Sit*

Curva



Sit Curvæ alicujus A D δ *Basis* A B $= x$, *perpendiculariter applicata* B D $= y$, *& area* A B D $= z$, *ut prius. Item sit* B β $= o$, B K $= v$, *& Rectangulum* β β H K (ov) *æquale spatio* B β δ D. *Est ergo* A β $= x + o$, *&* A δ β $= z + ov$, *His præmissis, ex relatione inter* x *&* z *ad arbitrium assumpta, quæro* y *ut sequitur.*

Pro lubitu sumatur [æquatio] $\frac{2}{3} x^{\frac{3}{2}} = z$, *sive* $\frac{4}{9} x^3 = zz$. *Tum* $x + o$ (A β) *pro* x, *&* $z + ov$ (A δ β) *pro* z *substitutis, prodibit* $\frac{4}{9}$ *in* $x^3 + 3x^2o + 3xo^2 + o^3 =$ (*ex natura Curvæ*) $z^2 + 2zov + o^2v^2$. *Et sublatis* $\frac{4}{9} x^3$ *&* zz *æqualibus, reliquisque per* o *divisis, restat* $\frac{4}{9}$ *in* $3x^2 + 3xo + o^2 = 2zv + ov^2$. *Si jam supponamus* B β *in infinitum diminui & evanescere, sive* o *esse nihil, erunt* v *&* y *æquales, & termini per* o *multiplicati evanescent; ideoque restabit* $\frac{4}{9} \times 3xx = 2zv$, *sive* $\frac{2}{3} xx (= zy) = \frac{2}{3} x^{\frac{5}{2}} y$, *sive* $x^{\frac{5}{2}} \left(= \frac{x^2}{x^{\frac{1}{2}}}\right) = y$. *Quare è contra, si* $x^{\frac{1}{2}} = y$, *erit* $\frac{2}{3} x^{\frac{3}{2}} = z$.

Vel generaliter, Si $\frac{n}{m+n} \times a x^{\frac{m+n}{n}} = z$; *sive ponendo* $\frac{na}{m+n} = c$, *&* $m + n = p$, *Si* $c x^{\frac{p}{n}} = z$, *sive* $c^n x^p = z^n$: *Tum* $x + o$ *pro* x, *&* $z + ov$ *sive* (*quod perinde est*) $z + oy$ *pro* z *substitutis, prodit* c^n *in* $x^p + poxp^{-1}$ &c. $= z^n + noyz^{n-1}$ &c. *reliquis nempe* [Serierum] *terminis, qui tandem evanescerent, omissis. Jam sublatis* $c^n x^p$ *&* z^n *æqualibus, reliquisque per* o *divisis, restat* $c^n p x^{p-1} = nyz^{n-1} \left(= \frac{nyz^n}{z} = \frac{nyc^n x^p}{c x^{\frac{p}{n}}}\right)$ *sive dividendo per* $c^n x^p$, *erit* $px^{-1} = \frac{ny}{c x^{\frac{p}{n}}}$ *sive* $pcx^{\frac{p-n}{n}} = ny$; *vel restituendo* $\frac{na}{m+n}$ *pro* c *&* $m + n$ *pro* p, *hoc est* m *pro* $p - n$, *&* na

pro

pro pc, *fiet* $ax^{\frac{m}{n}} = y$. *Quare è contra, si* $ax^{\frac{m}{n}} = y$ *erit* $\frac{n}{m+n} ax^{\frac{m+n}{n}}$ $= z$ Q. E. D.

By the ſame way of working the ſecond Rule may be alſo demonſtrated. And if any Equation whatever be aſſumed expreſſing the Relation between the Abſciſſa and Area of a Curve, the Ordinate may be found in the ſame manner, as is mentioned in the next Words of the *Analyſis*. And if this Ordinate drawn into an Unit be put for the Area of a new Curve, the Ordinate of this new Curve may be found by the ſame Method: And ſo on perpetually. And theſe Ordinates repreſent the firſt, ſecond, third, fourth and following Fluxions of the firſt Area.

This was Mr. *Newton*'s Way of working in thoſe Days, when he wrote this Compendium of his *Analyſis*. And the ſame Way of working he uſed in his Book of Quadratures, and ſtill uſes to this Day.

Among the Examples with which he illuſtrates the Method of Series and Moments ſet down in this Compendium, are theſe. Let the Radius of a Circle be 1, and the Arc z, and the Sine x, the Equations for finding the Arc whoſe Sine is given, and the Sine whoſe Arc is given, will be

$$z = x + \tfrac{1}{6}x^3 + \tfrac{3}{40}x^5 + \tfrac{5}{112}x^7 + \tfrac{35}{1152}x^9 + \&c.$$
$$x = z - \tfrac{1}{6}z^3 + \tfrac{1}{120}z^5 - \tfrac{1}{5040}z^7 + \tfrac{1}{362880}z^9 - \&c.$$

Mr. *Collins* gave Mr. *Gregory* notice of this Method in Autumn 1669, and Mr. *Gregory*, by the Help of one of Mr. *Newton*'s Series, after a Year's Study, found the Method in *December* 1670; and two Months after, in a Letter dated *Feb.* 15. 1671. ſent ſeveral Theorems, found thereby, to Mr. *Collins*, with leave to communicate them freely. And Mr. *Collins* was very free in communicating what he had received both from Mr. *Newton* and from Mr. *Gregory*, as appears by his Letters printed in the *Commercium*. Amongſt the Series which Mr *Gregory* ſent in the ſaid Letter, were

theſe

these two. Let the Radius of a Circle be r, the Arc a, and the Tangent t, the Equations for finding the Arc whose Tangent is given, and the Tangent whose Arc is given, will be these.

$$a = t - \frac{t^3}{3r^2} + \frac{t^5}{5r^4} - \frac{t^7}{7r^6} + \frac{t^9}{9r^8} - \&c.$$

$$t = a + \frac{a^3}{3r^2} + \frac{2a^5}{15r^4} + \frac{17a^7}{315r^6} + \frac{62a^9}{2835r^8} + \&c.$$

In this Year (1671) Mr. *Leibnitz* published two Tracts at *London*, the One dedicated to the Royal-Society, the Other dedicated to the Academy of Sciences at *Paris*; and in the Dedication of the First he mentioned his Correspondence with Mr. *Oldenburgh*.

In *February* $167\frac{2}{3}$ meeting Dr. *Pell* at Mr. *Boyle's*, he pretended to the differential Method of *Mouton*. And notwithstanding that he was shewn by Dr. *Pell* that it was *Mouton's* Method, he persisted in maintaining it to be his own Invention, by reason that he had found it himself without knowing what *Mouton* had done before, and had much improved it.

When one of Mr. *Newton's* Series was sent to Mr. *Gregory*, he tried to deduce it from his own Series combined together, as he mentions in his Letter dated *December* 19. 1670. And by some such Method Mr *Leibnitz*, before he left *London*, seems to have found the Sum of a Series of Fractions decreasing *in Infinitum*, whose Numerator is a given Number and Denominators are triangular or pyramidal or triangulo-triangular Numbers, *&c.* See the Mystery! From the Series $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \&c.$ subduct all the Terms but the first (*viz* $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ *&c*) and there will remain $1 = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5}$ *&c*) $= \frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \frac{1}{4\times5} + \&c.$ And from this Series take all the Terms but the first, and there will remain $\frac{1}{2} = \frac{2}{1\times2\times3} + \frac{2}{2\times3\times4} + \frac{2}{3\times4\times5} + \frac{2}{4\times5\times6} + \&c.$ And from the first Series take all the Terms but the two first, and there will remain $\frac{3}{2} = \frac{2}{1\times3} + \frac{2}{2\times4} + \frac{2}{3\times5} + \frac{2}{4\times6} + \&c.$

In the End of *February* or beginning of *March* 167$\frac{2}{3}$. Mr. *Leibnitz* went from *London* to *Paris*, and continuing his Correſpondence with Mr. *Oldenburg* and Mr. *Collin*, wrote in *July* 1674. that he had a wonderful Theoreme, which gave the Area of a Circle or any Sector thereof exactly in a Series of rational Numbers; and in *October* following, that he had found the Circumference of a Circle in a Series of very ſimple Numbers, and that by the ſame Method (ſo he calls the ſaid Theoreme) any Arc whoſe Sine was given might be found in a like Series, though the Proportion to the whole Circumference be not known. His Theoreme therefore was for finding any Sector or Arc whoſe Sine was given. If the Proportion of the Arc to the whole Circumference was not known, the Theoreme or Method gave him only the Arc; if it was known it gave him alſo the whole Circumference: and therefore it was the firſt of Mr. *Newton*'s two Theoremes above-mention'd. But the Demonſtration of this Theoreme Mr *Leibnitz* wanted. For in his Letter of *May* 12. 1676. he deſired Mr. *Oldenburgh* to procure the Demonſtration from Mr *Collins*, meaning the Method by which Mr. *Newton* had invented it.

In a Letter compos'd by Mr. *Collins* and dated *April* 15. 1675. Mr. *Oldenburgh* ſent to Mr. *Leibnitz* Eight of Mr. *Newton*'s and Mr. *Gregory*'s Series, amongſt which were Mr. *Newton*'s two Series above mention'd for finding the Arc whoſe Sine is given, and the Sine whoſe Arc is given; and Mr. *Gregory*'s two Series above mentioned for finding the Arc whoſe Tangent is given, and the Tangent whoſe Arc is given. And Mr. *Leibnitz* in his Anſwer, dated *May* 20. 1675. acknowledged the Receipt of this Letter in theſe Words. *Literas tuas multa fruge Algebraica refertas accepi, pro quibus tibi & doctiſſimo* Collinio *gratias ago. Cum nunc præter ordinarias curas Mechanicis imprimis negotiis diſtrahar, non potui examinare Series quas miſiſtis ac cum meis comparare. Ubi fecero perſcribam tibi ſententiam meam: nam aliquot jam anni ſunt quod inveni meas via quadam ſic ſatis ſingulari.* But

But yet Mr. *Leibnitz* never took any further notice of his having received these Series, nor how his own differed from them, nor ever produced any other Series then those which he received from Mr. *Oldenburgh*, or numeral Series deduced from them in particular Cases. And what he did with Mr. *Gregory*'s Series for finding the Arc whose Tangent is given, he has told us in the *Acta Eruditorum mensis Aprilis* 1691. pag. 178. *Jam anno* 1675, saith he, *compositum habebam opusculum Quadraturæ Arithmeticæ ab amicis ab illo tempore lectum*, &c. By a Theoreme for transmuting of Figures, like those of Dr. *Barrow* and Mr. *Gregory*, he had now found a Demonstration of this Series, and this was the Subject of his *Opusculum*. But he still wanted a Demonstration of the rest: and meeting with a Pretence to ask for what he wanted, he wrote to Mr. *Oldenburg* the following Letter, dated at *Paris May* 12. 1676.

Cum Georgius Mohr *Danus nobis attulerit communicatam sibi à Doctissimo* Collinio *vestro expressionem rationis inter arcum & sinum per infinitas Series sequentes; posito sinu* x, *arcu* z, *radio* 1,

$$z = x + \tfrac{1}{6}x^3 + \tfrac{3}{40}x^5 + \tfrac{5}{112}x^7 + \tfrac{35}{1152}x^9 + \&c.$$

$$x = z - \tfrac{1}{6}z^3 + \tfrac{1}{120}z^5 - \tfrac{1}{5040}z^7 + \tfrac{1}{362880}z^9 - \&c.$$

Hæc, INQUAM, cum nobis attulerit ille, quæ mihi valde ingeniosa videntur, & posterior imprimis Series elegantiam quandam singularem habeat: ideo rem gratam mihi feceris, Vir clarissime, si demonstrationem transmiseris. Habebis vicissim mea ab his longe diversa circa hanc rem meditata, de quibus jam aliquot abhinc annis ad te perscripsisse credo, demonstratione tamen non addita, quam nunc polio. Oro ut Clarissimo Collinio *multam a me salutem dicas: is facile tibi materiam suppeditabit satisfaciendi desiderio meo.* Here, by the Word *INQUAM*, one would think that he had never seen these two Series before, and that his *diversa circa hanc rem meditata* were something else than one of the Series which he had received from

Mr. *Oldenburgh* the Year before, and a Demonſtration thereof which he was now poliſhing, to make the Preſent an acceptable Recompence for Mr. *Newton*'s Method.

Upon the Receipt of this Letter Mr. *Oldenburg* and Mr. *Collins* wrote preſſingly to Mr. *Newton*, deſiring that he himſelf would deſcribe his own Method, to be communicated to Mr. *Leibnitz*. Whereupon Mr. *Newton* wrote his Letter, dated *June* 13. 1676, deſcribing therein the Method of Series, as he had done before in the Compendium above-mentioned; but with this Difference: Here he deſcribed at large the Reduction of the Dignity of a Binomial into a Series, and only touched upon the Reduction by Diviſion and Extraction of affected Roots: There he deſcribed at large the Reduction of Fractions and Radicals into Series by Diviſion and Extraction of Roots, and only ſet down the two firſt Terms of the Series into which the Dignity of a Binomial might be reduced. And among the Examples in this Letter, there were Series for finding the Number whoſe Logarithm is given, and for finding the Verſed Sine whoſe Arc is given: This Letter was ſent to *Paris*, *June* 26. 1676. together with a MS. drawn up by Mr. *Collins*, containing Extracts of Mr. *James Gregory*'s Letters.

For Mr. *Gregory* died near the End of the Year 1675; and Mr. *Collins*, at the Requeſt of Mr *Leibnitz* and ſome other of the Academy of Sciences, drew up Extracts of his Letters, and the Collection is ſtill extant in the Hand Writing of Mr. *Collins* with this Title; *Extracts of Mr.* Gregory's *Letters, to be lent to Mr.* Leibnitz *to peruſe, who is deſired to return the ſame to you.* And that they were ſent is affirmed by Mr. *Collins* in his Letter to Mr. *David Gregory* the Brother of the Deceas'd, dated *Auguſt* 11. 1676. and appears further by the Anſwers of Mr. *Leibnitz* and Mr. *Tſchurnhauſe* concerning them.

The Anſwer of Mr. *Leibnitz* directed to Mr *Oldenburgh* and dated *Auguſt* 27. 1676, begins thus; *Literæ tuæ die* Julii 26. *datæ plura ac memorabiliora circa rem Analyticam continent quam*

multa volumina spissa de his rebus edita. Quare tibi pariter ac clarissimis viris Newtono *ac* Collinio *gratias ago; qui nos participes tot meditationum egregiarum esse voluistis.* And towards the End of the Letter, after he had done with the Contents of Mr. *Newton*'s Letter, he proceeds thus. *Ad alia tuarum Literarum venio quæ doctissimus* Collinius *communicare gravatus non est. Vellem adjecisset appropinquationis* Gregorianæ *linearis demonstrationem. Fuit enim his certe studiis promovendis aptissimus.* And the Answer of Mr. *Tschurnhause*, dated *Sept.* 1. 1676, after he had done with Mr. *Newton*'s Letter about Series, concludes thus. *Similia porro quæ in hac re præstitit eximius ille Geometra* Gregorius *memoranda certe sunt. Et quidem optimè famæ ipsius consulturi, qui ipsius relicta Manuscripta luci publicæ ut exponantur operam navabunt.* In the first Part of this Letter, where Mr. *Tschurnhause* speaks of Mr. *Newton*'s Series, he saith that he looked over them cursorily, to see if he could find the Series of Mr. *Leibnitz* for squaring the Circle or Hyperbola. If he had searched for it in the Extracts of *Gregory*'s Letters he might have found it in the Letter of *Febr.* 15. 1671. above-mentioned. For the MS. of those Extracts with that Letter therein is still extant in the Hand-Writing of Mr. *Collins*.

And tho' Mr *Leibnitz* had now received this Series twice from Mr. *Oldenburgh*, yet in his Letter of *August* 27. 1676. he sent it back to him by way of Recompence for Mr. *Newton*'s Method, pretending that he had communicated it to his Friends at *Paris* three Years before or above; that is, two Years before he received it in Mr. *Oldenburgh*'s Letter of *April* 15. 1675; at which Time he did not know it to be his own, as appears by his Answer of *May* 20. 1675 above-mentioned. He might receive this Series at *London*, and communicate it to his Friends at *Paris* above three Years before he sent it back to Mr. *Oldenburg*: but it doth not appear that he had the Demonstration thereof so early. When he found the Demonstration, then he compos'd it in his *Opusculum*, and communicated that also to his Friends; and he himself has

 told

told us that this was in the Year 1675. However, it lies upon him to prove that he had this Series before he received it from Mr. *Oldenburgh*. For in his Answer to Mr. *Oldenburgh* he did not know any of the Series then sent him to be his own; and concealed from the Gentlemen at *Paris* his having received it from Mr. *Oldenburgh* with several other Series, and his having seen a Copy of the Letter in which Mr. *Gregory* had sent it to Mr. *Collins* in the Beginning of the Year 1671.

In the same Letter of *August* 27. 1676, after Mr. *Leibnitz* had described his Quadrature of the Circle and Equilateral Hyperbola, he added: *Vicissim ex seriebus regressuum pro Hyperbola hanc inveni. Si sit numerus aliquis unitate minor* $1 - m$, *ejusque logarithmus Hyperbolicus* l. *Erit* $m = \frac{l}{1} - \frac{l^2}{1 \times 2} + \frac{l^3}{1 \times 2 \times 3} - \frac{l^4}{1 \times 2 \times 3 \times 4} +$ &c. *Si numerus sit major unitate, ut* $1 + n$, *tunc pro eo inveniendo mihi etiam prodiit Regula quæ in* Newtoni *Epistola expressa est: scilicet erit* $n = \frac{l}{1} + \frac{l^2}{1 \times 2} + \frac{l^3}{1 \times 2 \times 3} + \frac{l^4}{1 \times 2 \times 3 \times 4} +$ &c. ——— *Quod regressum ex arcubus attinet, incideram ego directe in Regulam quæ ex dato arcu sinum complementi exhibet. Nempe sinus complementi* $= 1 - \frac{a^2}{1 \times 2} + \frac{a^4}{1 \times 2 \times 3 \times 4} -$ &c. *Sed postea quoque deprehendi ex ea illam nobis communicatam pro inveniendo sinu recto, qui est* $\frac{a}{1} - \frac{a^3}{1 \times 2 \times 3} + \frac{a^5}{1 \times 2 \times 3 \times 4 \times 5} -$ &c. *posse demonstrari.* Thus Mr. *Leibnitz* put in his Claim for the Co-invention of these four Series, tho' the Method of finding them was sent him at his own Request, and he did not yet understand it. For in this same Letter of *August* 27 1676. he desired Mr. *Newton* to explain it further. His Words are. *Sed desideraverim ut Clarissimus* Newtonus *nonnulla quoque amplius explicet; ut originem Theorematis quod initio ponit: Item modum quo quantitates* p, q, r, *in suis Operationibus invenit: Ac denique quomodo in Methodo regressuum se gerat, ut cum ex Logarithmo quærit Numerum. Neque enim explicat quomodo id ex methodo sua derivetur.* He pretended to have found two Series for the Number whose Logarithm was given, and yet in the same Letter desired

ſired Mr. *Newton* to explain to him the Method of finding thoſe very two Series.

When Mr. *Newton* had received this Letter, he wrote back that all the ſaid four Series had been communicated by him to Mr. *Leibnitz*; the two firſt being one and the ſame Series in which the Letter *l* was put for the Logarithm with its Sign + or —; and the third being the Exceſs of the Radius above the verſed Sine, for which a Series had been ſent to him. Whereupon Mr. *Leibnitz* deſiſted from his Claim. Mr. *Newton* alſo in the ſame Letter dated *Octob.* 24. 1676. further explained his Methods of Regreſſion, as Mr. *Leibnitz* had deſired. And Mr. *Leibnitz* in his Letter of *June* 21. 1677. deſired a further Explication: but ſoon after, upon reading Mr. *Newton*'s Letter a ſecond time, wrote back *July* 12. 1677. that he now underſtood what he wanted; and found by his old Papers that he had formerly uſed one of Mr. *Newton*'s Methods of Regreſſion, but in the Example which he had then by chance made uſe of, there being produced nothing elegant, he had, out of his uſual Impatience, neglected to uſe it any further. He had therefore ſeveral direct Series, and by conſequence a Method of finding them, before he invented and forgot the inverſe Method. And if he had ſearched his old Papers diligently, he might have found this Method alſo there; but having forgot his own Methods he wrote for Mr. *Newton*'s.

When Mr. *Newton* in his Letter dated *June* 13. 1676. had explained his Method of Series, he added: *Ex his videre eſt quantum fines Analyſeos per hujuſmodi infinitas æquationes ampliantur: quippe quæ earum beneficio ad omnia pene dixerim problemata (ſi numeralia* Diophanti *& ſimilia excipias) ſeſe extendit. Non tamen omnino univerſalis evadit, niſi per ulteriores quaſdam Methodos eliciendi Series infinitas. Sunt enim quædam Problemata in quibus non licet ad Series infinitas per Diviſionem vel Extractionem radicum ſimplicium affectarumve pervenire. Sed quomodo in iſtis caſibus procedendum ſit jam non vacat dicere; ut neque alia quædam tradere, quæ circa Reductionem infinitarum Serierum*

in finitas, ubi rei natura tulerit, excogitavi. Nam parcius scribo, quod hæ speculationes diu mihi fastidio esse cœperunt; adeo ut ab iisdem jam per quinque fere annos abstinuerim. To this Mr. *Leibnitz* in his Letter of *August* 27. 1676. answered: *Quod dicere videmini plerasque difficultates (exceptis Problematibus* Diophantæis*) ad series Infinitas reduci; id mihi non videtur. Sunt enim multa usque adeo mira & implexa ut neque ab æquationibus pendeant neque ex Quadraturis. Qualia sunt (ex multis aliis) Problemata methodi Tangentium inversæ.* And Mr. *Newton* in his Letter of *Octob.* 24. 1676, replied: *Ubi dixi omnia pene Problemata solubilia existere; volui de iis præsertim intelligi circa quæ Mathematici se hactenus occuparunt, vel saltem in quibus Ratiocinia Mathematica locum aliquem obtinere possunt. Nam alia sane adeo perplexis conditionibus implicata excogitare liceat, ut non satis comprehendere valeamus: & multo minus tantarum computationum onus sustinere quod ista requirerent. Attamen ne nimium dixisse videar, inversa de Tangentibus Problemata sunt in potestate, aliaque illis difficiliora. Ad quæ solvenda usus sum duplici methodo, una concinniori, altera generaliori. Utramque visum est impræsentia literis transpositis consignare, ne propter alios idem obtinentes, institutum in aliquibus mutare cogerer.* 5 a cc d æ 10 e ff h, &c. id est, *Una methodus consistit in extractione fluentis quantitatis ex æquatione simul involvente fluxionem ejus: altera tantum in assumptione seriei pro quantitate qualibet incognita, ex qua cætera commode derivari possunt; & in collatione terminorum homologorum æquationis resultantis ad eruendos terminos assumptæ seriei.* By Mr. *Newton*'s two Letters, its certain that he had then (or rather above five Years before) found out the Reduction of Problems to fluxional Equations and converging Series: and by the Answer of Mr. *Leibnitz* to the first of those Letters, its as certain that he had not then found out the Reduction of Problems either to differential Equations or to converging Series.

And the same is manifest also by what Mr. *Leibnitz* wrote in the *Acta Eruditorum, Anno* 1691, concerning this Matter.

Jam

Jam anno 1675, ſaith he, *compoſitum habebam opuſculum Quadraturæ Arithmeticæ ab amicis ab illo tempore lectum, ſed quod, materia ſub manibus creſcente, limare ad Editionem non vacavit, poſtquam aliæ occupationes ſupervenere; præſertim cum nunc prolixius exponere vulgari more quæ Analyſis noſtra paucis exhibet, non ſatis operæ pretium videatur.* This Quadrature compoſed *vulgari more* he began to communicate at *Paris* in the Year 1675. The next Year he was poliſhing the Demonſtration thereof, to ſend it to Mr. *Oldenburgh* in Recompence for Mr. *Newton*'s Method, as he wrote to him *May* 12. 1676; and accordingly in his Letter of *Auguſt* 27. 1676. he ſent it compoſed and poliſhed *vulgari more.* The Winter following he returned into *Germany*, by *England* and *Holland*, to enter upon publick Buſineſs, and had no longer any Leiſure to fit it for the Preſs, nor thought it afterwards worth his while to explain thoſe Things prolixly in the vulgar manner which his new *Analyſis* exhibited in ſhort. He found out this new *Analyſis* therefore after his Return into *Germany*, and by conſequence not before the Year 1677.

The ſame is further manifeſt by the following Conſideration. Dr. *Barrow* publiſhed his *Method of Tangents* in the Year 1670. Mr. *Newton* in his Letter dated *December* 10. 1672. communicated his *Method of Tangents* to Mr. *Collins*, and added: *Hoc eſt unum particulare vel Corollarium potius Methodi generalis, quæ extendit ſe citra moleſtum ullum calculum, non modo ad ducendum Tangentes ad quaſvis Curvas ſive Geometricas ſive Mechanicas, vel quomodocunque rectas Lineas aliaſve Curvas reſpicientes; verum etiam ad reſolvendum alia abſtruſiora Problematum genera de Curvitatibus, Areis, Longitudinibus, Centris Gravitatis Curvarum,* &c. *Neque (quemadmodum* Huddenii *methodus de* Maximis *&* Minimis) *ad ſolas reſtringitur æquationes illas, quæ quantitatibus ſurdis ſunt immunes. Hanc methodum intertexui alteri iſti qua Æquationum Exegeſin inſtituo, reducendo eas ad ſeries infinitas.* Mr. *Sluſius* ſent his Method of Tangents to Mr. *Oldenburgh Jan.* 17. 167$\frac{2}{3}$, and the ſame was

ſoon aſter publiſhed in the *Tranſactions* It proved to be the ſame with that of Mr *Newton*. It was founded upon three *Lemmas*, the firſt of which was this, *Differentia duarum dignitatum ejuſdem gradus applicata ad differentiam laterum dat partes ſingulares gradus inferioris ex binomio laterum, ut* $\frac{y^3 - x^3}{y - x} = yy + yx + xx$, that is, in the Notation of Mr. *Leibnitz* $\frac{dy^3}{dy} = 3yy$. A Copy of Mr. *Newton*'s Letter of *Decemb.* 10. 1672 was ſent to Mr. *Leibnitz* by Mr. *Oldenburg* amongſt the Papers of Mr *James Gregory*, at the ſame time with Mr. *Newton*'s Letter of *June* 13. 1676. And Mr. *Newton* having deſcribed in theſe two Letters that he had a very general *Analyſis*, conſiſting partly of the Method of converging Series, partly of another Method, by which he applied thoſe Series to the Solution of almoſt all Problems (except perhaps ſome numeral ones like thoſe of *Diophantus*) and found the Tangents, Areas, Lengths, ſolid Contents, Centers of Gravity, and Curvities of Curves, and curvilinear Figures Geometrical or Mechanical, without ſticking at Surds; and that the Method of Tangents of *Sluſius* was but a Branch or Corollary of this other Method: Mr. *Leibnitz* in his returning Home through *Holland*, was meditating upon the Improvement of the Method of *Sluſius*. For in a Letter to Mr. *Oldenburgh*, dated from *Amſterdam Nov.* $\frac{13}{23}$ 1676, he wrote thus. *Methodus Tangentium a* Sluſio *publicata nondum rei faſtigium tenet. Poteſt aliquid amplius præſtari in eo genere quod maximi foret uſus ad omnis generis Problemata: etiam ad meam (ſine extractionibus) Æquationum ad ſeries reductionem. Nimirum poſſet brevis quædam calculari circa Tangentes Tabula, eouſque continuanda donec progreſſio Tabulæ apparet; ut eam ſcilicet quiſque quouſque libuerit ſine calculo continuare poſſit.* This was the Improvement of the Method of *Sluſius* into a general Method, which Mr. *Leibnitz* was then thinking upon, and by his Words,

Poteſt

Potest aliquid amplius præstari in eo genere quod maximi foret usus ad omnis generis Problemata, it seems to be the only Improvement which he had then in his Mind for extending the Method to all sorts of Problems. The Improvement by the differential Calculus was not yet in his Mind, but must be referred to the next Year.

Mr. *Newton* in his next Letter, dated *Octob.* 24. 1676, mentioned the *Analysis* communicated by Dr. *Barrow* to Mr. *Collins* in the Year 1669, and also another Tract written in 1671. about converging Series, and about the other Method by which Tangents were drawn after the Method of *Slusius*, and *Maxima* and *Minima* were determined, and the Quadrature of Curves was made more easy, and this without sticking at Radicals, and by which Series were invented which brake off and gave the Quadrature of Curves in finite Equations when it might be. And the Foundation of these Operations he comprehended in this Sentence exprest enigmatically as above. *Data æquatione fluentes quotcunque quantitates involvente fluxiones invenire, & vice versa.* Which puts it past all Dispute that he had invented the Method of Fluxions before that time. And if other things in that Letter be considered, it will appear that he had then brought it to great Perfection, and made it exceeding general; the Propositions in his Book of Quadratures, and the Methods of converging Series and of drawing a Curve Line through any Number of given Points, being then known to him. For when the Method of Fluxions proceeds not in finite Equations, he reduces the Equations into converging Series by the binomial Theoreme, and by the Extraction of Fluents out of Equations involving or not involving their Fluxions. And when finite Equations are wanting, he deduces converging Series from the Conditions of the Probleme, by assuming the Terms of the Series gradually, and determining them by those Conditions. And when Fluents are to be derived from Fluxions, and the Law of the Fluxions is wanting, he finds

that Law *quam proxime*, by drawing a Parabolick Line through any Number of given Points. And by these Improvements Mr. *Newton* had in those Days made his Method of Fluxions much more universal than the Differential Method of Mr. *Leibnitz* is at present.

This Letter of Mr. *Newton*'s, dated *Octob.* 24. 1676, came to the Hands of Mr. *Leibnitz* in the End of the Winter or Beginning of the Spring following; and Mr. *Leibnitz* soon after, *viz.* in a Letter dated *June* 21. 1677, wrote back: *Clarissimi* Slusii *methodum Tangentium nondum esse absolutam Celeberrimo* Newtono *assentior. Et jam à multo tempore rem Tangentium generalius tractavi, scilicet per differentias Ordinatarum.* ——— *Hinc nominando*, in posterum, dy *differentiam duarum proximarum* y *&c.* Here Mr. *Leibnitz* began first to propose his Differential Method, and there is not the least Evidence that he knew it before the Receipt of Mr. *Newton*'s last Letter. He saith indeed, *Jam à multo tempore rem Tangentium generalius tractavi, scilicet per differentias Ordinatarum*: and so he affirmed in other Letters, that he had invented several converging Series direct and inverse before he had the Method of inventing them; and had forgot an inverse Method of Series before he knew what use to make of it. But no Man is a Witness in his own Cause. A Judge would be very unjust, and act contrary to the Laws of all Nations, who should admit any Man to be a Witness in his own Cause. And therefore it lies upon Mr. *Leibnitz* to prove that he found out this Method long before the Receipt of Mr. *Newton*'s Letters. And if he cannot prove this, the Question, Who was the first Inventor of the Method, is decided.

The Marquis *De l' Hospital* (a Person of very great Candour) in the Preface to his Book *De Analysi quantitatum infinitè 'parvarum*, published *A. C.* 1696. tells us, that a little after 'the Publication of the Method of Tangents of *Des Cartes*, 'Mr. *Fermat* found also a Method, which *Des Cartes* himself 'at length allowed to be, for the most part, more simple than his

'his own. But it was not yet so simple as Mr. *Barrow* afterwards made it, by considering more nearly the nature of Polygons, which offers naturally to the Mind a little Triangle, compos'd of a Particle of the Curve lying between two Ordinates infinitely near one another, and of the Difference of these two Ordinates, and of that of the two correspondent *Abscissa*'s. And this Triangle is like that which ought to be made by the Tangent, the Ordinate, and the Sub-tangent: so that by one simple Analogy, this last Method saves all the Calculation which was requisite either in the Method of *Des Cartes*, or in this same Method before. Mr. *Barrow* stopt not here, he invented also a sort of Calculation proper for this Method. But it was necessary in this as well as in that of *Des Cartes*, to take away Fractions and Radicals for making it useful. Upon the Defect of this Calculus, that of the celebrated Mr. *Leibnitz* was introduced, and this learned Geometer began where Mr. *Barrow* and others left off. This his *Calculus* led into Regions hitherto unknown, and there made Discoveries which astonished the most able Mathematicians of *Europe*,' &c. Thus far the Marquiss. He had not seen Mr. *Newton*'s *Analysis*, nor his Letters of *Decem.* 10. 1672. *June* 13. 1676, and *Octob.* 24. 1676: and so not knowing that Mr. *Newton* had done all this and signified it to Mr. *Leibnitz*, he reckoned that Mr. *Leibnitz* began where Mr. *Barrow* left off, and by teaching how to apply Mr. *Barrow*'s Method without sticking at Fractions and Surds, had enlarged the Method wonderfully. And Mr. *James Bernoulli*, in the *Acta Eruditorum* of *January* 1691 *pag.* 14. writes thus: *Qui calculum* Barrovianum *(quem in Lectionibus suis Geometricis adumbravit Auctor, cujusque Specimina sunt tota illa Propositionum inibi contentarum farrago,) intellexerit,* [calculum] *alterum à Domino* Leibnitio *inventum, ignorare vix poterit; utpote qui in priori illo fundatus est, & nisi forte in Differentialium notatione & operationis aliquo compendio ab eo non differt.*

Now

Now Dr. *Barrow*, in his Method of Tangents, draws two Ordinates indefinitely near to one another, and puts the Letter *a* for the Difference of the Ordinates, and the Letter *e* for the Difference of the *Abscissa*'s, and for drawing the Tangent gives these Three Rules 1. *Inter computandum*, saith he, *omnes abjicio terminos in quibus ipsarum* a *vel* e *potestas habeatur, vel in quibus ipsæ ducuntur in se. Etenim isti termini nihil valebunt.* 2. *Post æquationem constitutam omnes abjicio terminos literis constantes quantitates notas seu determinatas significantibus, aut in quibus non habentur* a *vel* e. *Etenim illi termini semper ad unam æquationis partem adducti nihilum adæquabunt.* 3. *Pro* a *Ordinatam, & pro* e *Subtangentem substituo. Hinc demum Subtangentis quantitas dignoscetur.* Thus far Dr. *Barrow*.

And Mr. *Leibnitz* in his Letter of *June* 21. 1677 above-mentioned, wherein he first began to propose his Differential Method, has followed this Method of Tangents exactly, excepting that he has changed the Letters *a* and *e* of Dr. *Barrow* into *dx* and *dy*. For in the Example which he there gives, he draws two parallel Lines and sets all the Terms below the under Line, in which *dx* and *dy* are (severally or jointly) of more than one Dimension, and all the Terms above the upper Line, in which *dx* and *dy* are wanting, and for the Reasons given by Dr. *Barrow*, makes all these Terms vanish. And by the Terms in which *dx* and *dy* are but of one Dimension, and which he sets between the two Lines, he determines the Proportion of the Subtangent to the Ordinate. Well therefore did the Marquiss *de l'Hospital* observe that where Dr. *Barrow* left off Mr. *Leibnitz* began: for their Methods of Tangents are exactly the same.

But Mr. *Leibnitz* adds this Improvement of the Method, that the Conclusion of this Calculus is coincident with the Rule of *Slusius*, and shews how that Rule presently occurs to any one who understands this Method. For Mr. *Newton* had represented in his Letters, that this Rule was a Corollary of his general Method.

And

And whereas Mr. *Newton* had ſaid that his Method in drawing of Tangents, and determining *Maxima* and *Minima*, &c. proceeded without ſticking at Surds: Mr. *Leibnitz* in the next Place, ſhews how this Method of Tangents may be improved ſo as not to ſtick at Surds or Fractions, and then adds: *Arbitror quæ celare voluit* Newtonus *de Tangentibus ducendis ab his non abludere. Quod addit, ex hoc eodem fundamento Quadraturas quoque reddi faciliores me in hac ſententia confirmat; nimirum ſemper figuræ illæ ſunt quadrabiles quæ ſunt ad æquationem differentialem.* By which Words, compared with the preceding Calculation, its manifeſt that Mr. *Leibnitz* at this time underſtood that Mr. *Newton* had a Method which would do all theſe things, and had been examining whether Dr. *Barrow*'s Differential Method of Tangents might not be extended to the ſame Performances.

In *November* 1684 Mr. *Leibnitz* publiſhed the Elements of this Differential Method in the *Acta Eruditorum*, and illuſtrated it with Examples of drawing Tangents and determining *Maxima* and *Minima*, and then added. *Et hæc quidem initia ſunt Geometriæ cujuſdam multo ſublimioris, ad difficillima & pulcherrima quæque etiam miſtæ* Matheſeos *Problemata pertingentis, quæ ſine calculo differentiali* AUT SIMILI *non temere quiſquam pari facilitate tractabit.* The Words AUT SIMILI plainly relate to Mr. *Newton*'s Method. And the whole Sentence contains nothing more than what Mr. *Newton* had affirmed of his general Method in his Letters of 1672 and 1676.

And in the *Acta Eruditorum* of *June* 1686, *pag.* 297. Mr. *Leibnitz* added: *Malo autem* dx *& ſimilia adhibere quam literas pro illis, quia iſtud* dx *eſt modificatio quædam ipſius* x, &c. He knew that in this Method he might have uſed Letters with Dr. *Barrow*, but choſe rather to uſe the new Symbols *dx* and *dy*, though there is nothing which can be done by theſe Symbols, but may be done by ſingle Letters with more brevity.

The next Year Mr. *Newton's Principia Philosophiæ* came abroad, a Book full of such Problemes as Mr. *Leibnitz* had called *difficillima & pulcherrima etiam mistæ Matheseos problemata, quæ sine calculo differentiali aut* SIMILI *non temere quisquam pari facilitate tractabit.* And the Marquess *de L' Hospital* has represented this Book *presque tout de ce calcul*; composed almost wholly of this Calculus. And Mr. *Leibnitz* himself in a Letter to Mr. *Newton*, dated from *Hannover, March* $\frac{7}{17}$ 1693, and still extant in his own Hand-writing, and upon a late Occasion communicated to the Royal Society, acknowledged the same thing in these Words: *Mirifice ampliaveras Geometriam tuis Seriebus, sed edito* Principiorum *opere ostendisti patere tibi etiam quæ Analysi receptæ non subsunt. Conatus sum ego quoque, notis commodis adhibitis quæ differentias & summas exhibeant, Geometriam illam quam Transcendentem appello Analysi quodammodo subjicere, nec res male processit*; And again in his Answer to Mr. *Fatio*, printed in the *Acta Eruditorum* of *May* 1700. *pag.* 203. *lin.* 21. he acknowledged the same thing. In the second Lemma of the second Book of these *Principles*, the Elements of this Calculus are demonstrated synthetically, and at the End of the Lemma there is a Scholium in these Words. *In Literis quæ mihi cum Geometra peritissimo* G. G. Leibnitio *annis abhinc decem intercedebant, cum significarem me compotem esse methodi determinandi Maximas & Minimas, ducendi Tangentes & similia peragendi, quæ in terminis surdis æque ac in rationalibus procederet; & literis transpositis hanc sententiam involventibus* [Data æquatione quotcunque fluentes quantitates involvente, fluxiones invenire, & vice versa] *eandem celarem: rescripsit Vir clarissimus se quoque in ejusmodi methodum incidisse, & methodum suam communicavit à mea vix abludentem præterquam in verborum & notarum formulis. Utriusque fundamentum continetur in hoc Lemmate.* In those Letters, and in another dated *Decem.* 10. 1672, a Copy of which, at that time, was sent to Mr. *Leibnitz* by Mr. *Oldenburgh*, as is mentioned above, Mr. *Newton* had so far explained his Method, that it was not difficult for Mr.

Mr. *Leibnitz*, by the Help of Dr. *Barrow*'s Method of Tangents, to collect it from those Letters. And its certain, by the Arguments above-mentioned, that he did not know it before the writing of those Letters.

Dr. *Wallis* had received Copies of Mr. *Newton*'s two Letters of *June* 13. and *Octob.* 24. 1676 from Mr. *Oldenburgh*, and published several things out of them in his *Algebra*, printed in *English* 1683, and in Latin 1693; and soon after had Intimation from *Holland* to print the Letters entire, because Mr. *Newton*'s Notions of Fluxions passed there with Applause by the Name of the Differential Method of Mr. *Leibnitz*. And thereupon he took notice of this Matter in the Preface to the first Volume of his Works published *A. C.* 1695. And in a Letter to Mr. *Leibnitz* dated *Decemb.* 1. 1696, he gave the Account of it. *Cum Præfationis (præfigendæ) postremum folium erat sub prælo, ejusque typos jam posuerant Typothetæ; me monuit amicus quidam (harum rerum gnarus) qui peregre fuerat, tum talem methodum in Belgio prædicari, tum illam cum* Newtoni *methodo Fluxionum quasi coincidere. Quod fecit ut (translatis typis jam positis) id monitum interseruerim.* And in a Letter dated *April* 10. 1695, and lately communicated to the Royal-Society, he wrote thus about it. *I wish you would print the two large Letters of* June *and* August [he means *June* and *October*] 1676. *I had intimation from* Holland, *as desired there by your Friends, that somewhat of that kind were done; because your Notions (of Fluxions) pass there with great Applause by the Name of* Leibnitz's Calculus Differentialis. *I had* this intimation when all but part of the Preface to this Volume was printed off; so that I could only insert (while the Press stay'd) that short Intimation thereof which you there find. You are not so kind to your Reputation (and that of the Nation) as you might be, when you let things of worth lye by you so long, till others carry away*

* Extat hæc Epistola in tertio volumine operum *Wallisii*.

the Reputation that is due to you. I have endeavoured to do you Justice in that Point, and am now sorry that I did not print those two Letters verbatim.

The short Intimation of this Matter, which Dr. *Wallis* inserted into the said Preface, was in these Words. *In secundo Volumine (inter alia) habetur* Newtoni *Methodus de Fluxionibus (ut ille loquitur) consimilis naturæ cum* Leibnitii (*ut hic loquitur*) *Calculo Differentiali (quod qui utramque methodum contulerit satis animadvertat, ut ut sub loquendi formulis diversis) quam ego descripsi (Algebræ* cap. 91. &c. *præsertim* cap 95) *ex binis* Newtoni *Literis, aut earum alteris,* Junii 13. *&* Octob. 24. 1676 *ad* Oldenburgum *datis, cum* Leibnitio *tum communicandis (iisdem fere verbis, saltem leviter mutatis, quæ in illis literis habentur,) ubi* METHODUM HANC *LEIBNITIO* EXPONIT, *tum ante* DECEM ANNOS *nedum plures* [id est, anno 1666 vel 1665] *ab ipso excogitatam. Quod moneo, nequis causetur de hoc Calculo Differentiali nihil à nobis dictum esse.*

Hereupon the Editors of the *Acta Lipsiensia*, the next Year in *June*, in the Style of Mr. *Leibnitz*, in giving an Account of these two first Volumes of Dr. *Wallis*, took notice of this Clause of the Doctor's Preface, and complained, not of his saying that Mr. *Newton* in his two Letters above-mentioned explained to Mr *Leibnitz* the Method of Fluxions found by him Ten Years before or above; but that while the Doctor mentioned the Differential Calculus, and said that he did it *nequis causetur de calculo differentiali nihil ab ipso dictum fuisse*, he did not tell the Reader that Mr. *Leibnitz* had this Calculus at that time when those Letters passed between him and Mr. *Newton*, by means of Mr. *Oldenburgh*. And in several Letters which followed hereupon, between Mr. *Leibnitz* and Dr. *Wallis*, concerning this Matter, Mr. *Leibnitz* denied not that Mr. *Newton* had the Method Ten Years before the writing of those Letters, as Dr. *Wallis* had affirmed; pretended not that he himself had the Method so early; brought no Proof that he had it before the Year

1677;

1677; no other Proof besides the Concession of Mr. *Newton* that he had it so early; affirmed not that he had it earlier; commended Mr. *Newton* for his Candour in this Matter; allowed that the Methods agreed in the main, and said that he therefore used to call them by the common Name of his *Infinitesimal Analysis*; represented, that as the Methods of *Vieta* and *Cartes* were called by the common Name of *Analysis Speciosa*, and yet differed in some things; so perhaps the Methods of Mr. *Newton* and himself might differ in some things, and challenged to himself only those things wherein, as he conceived, they might differ, naming the Notation, the differential Equations and the Exponential Equations. But in his Letter of *June* 21. 1677 he reckon'd differential Equations common to Mr. *Newton* and himself.

This was the State of the Dispute between Dr. *Wallis* and Mr. *Leibnitz* at that time And Four years after, when Mr. *Fatio* suggested that Mr. *Leibnitz*, the second Inventor of this Calculus, might borrow something from Mr. *Newton*, the oldest Inventor by many Years: Mr. *Leibnitz* in his Answer, published in the *Acta Eruditorum* of *May* 1700, allowed that Mr. *Newton* had found the Method apart, and did not deny that Mr. *Newton* was the oldest Inventor by many Years, nor asserted any thing more to himself, than that he also had found the Method apart, or without the Assistance of Mr. *Newton*, and pretended that when he first published it, he knew not that Mr. *Newton* had found any thing more of it than the Method of Tangents. And in making this Defence he added: *Quam* [methodum] *ante Dominum* Newtonum *&* Me *nullus quod sciam Geometra habuit; uti ante hunc maximi nominis Geometram N E M O specimine publice dato se habere probavit, ante Dominos* Bernoullios *&* Me *nullus communicavit*. Hitherto therefore Mr. *Leibnitz* did not pretend to be the first Inventor. He did not begin to put in such a Claim till after the Death of Dr. *Wallis*, the last of the old Men who were acquainted with what had passed between the *English* and Mr. *Leibnitz*

Forty Years ago. The Doctor died in *October A. C.* 1703, and Mr. *Leibnitz* began not to put in this new Claim before *January* 1705.

Mr. *Newton* publiſhed his *Treatiſe of Quadratures* in the Year 1704. This Treatiſe was written long before, many things being cited out of it in his Letters of *Octob.* 24. and *Novemb.* 8. 1676. It relates to the Method of Fluxions, and that it might not be taken for a new Piece, Mr. *Newton* repeated what Dr. *Wallis* had publiſhed Nine Years before without being then contradicted, namely, that this Method was invented by Degrees in the Years 1665 and 1666. Hereupon the Editors of the *Acta Lipſienſia* in *January* 1705. in the Style of Mr. *Leibnitz*, in giving an Account of this Book, repreſented that Mr. *Leibnitz* was the firſt Inventor of the Method, and that Mr. *Newton* had ſubſtituted Fluxions for Differences. And this Accuſation gave a Beginning to this preſent Controverſy.

For Mr. *Keill*, in an Epiſtle publiſhed in the *Philoſophical Tranſactions* for *Sept.* and *Octob.* 1708, retorted the Accuſation, ſaying: *Fluxionum Arithmeticam ſine omni dubio primus invenit D.* Newtonus, *ut cuilibet ejus Epiſtolas à* Walliſio *editas legenti facile conſtabit. Eadem tamen Arithmetica poſtea mutatis nomine & notationis modo à Domino* Leibnitio *in* Actis Eruditorum *edita eſt.*

Before Mr. *Newton* ſaw what had been publiſhed in the *Acta Leipſica*, he expreſs'd himſelf offended at the printing of this Paragraph of Mr. *Keill*'s Letter, leaſt it ſhould create a Controverſy. And Mr. *Leibnitz*, underſtanding it in a ſtronger Senſe than Mr. *Keill* intended it, complain'd of it as a Calumny, in a Letter to Dr. *Sloane* dated *March* 4. 1711 *N.S.* and moved that the Royal-Society would cauſe Mr. *Keill* to make a publick Recantation. Mr. *Keill* choſe rather to explain and defend what he had written; and Mr. *Newton*, upon being ſhewed the Accuſation in the *Acta Lipſica*, gave him leave to do ſo. And Mr. *Leibnitz* in a ſecond Letter to Dr. *Sloane*, dated *Decem.* 29. 1711, inſtead of making good his Accuſation

ſation, as he was bound to do that it might not be deem'd a Calumny, inſiſted only upon his own Candour, as if it would be Injuſtice to queſtion it; and refus'd to tell how he came by the Method; and ſaid that the *Acta Lipſica* had given every Man his due, and that he had concealed the Invention above Nine Years, (he ſhould have ſaid Seven Years) that No body might pretend (he means that Mr. *Newton* might not pretend) to have been before him in it; and called Mr. *Keill* a Novice unacquainted with things paſt, and one that acted without Authority from Mr. *Newton*, and a clamorous Man who deſerved to be ſilenced, and deſired that Mr. *Newton* himſelf would give his Opinion in the Matter. He knew that Mr. *Keill* affirmed nothing more than what Dr. *Wallis* had publiſhed thirteen Years before, without being then contradicted. He knew that Mr. *Newton* had given his Opinion in this matter in the Introduction to his Book of *Quadratures*, publiſhed before this Controverſy began: but Dr. *Wallis* was dead; the Mathematicians which remained in *England* were Novices; Mr. *Leibnitz* may Queſtion any Man's Candour without Injuſtice, and Mr. *Newton* muſt now retract what he had publiſhed or not be quiet.

The Royal-Society therefore, having as much Authority over Mr. *Leibnitz* as over Mr. *Keill*, and being now twice preſſed by Mr. *Leibnitz* to interpoſe, and ſeeing no reaſon to condemn or cenſure Mr. *Keill* without enquiring into the matter; and that neither Mr. *Newton* nor Mr. *Leibnitz* (the only Perſons alive who knew and remembred any thing of what had paſſed in theſe matters Forty Years ago) could be Witneſſes for or againſt Mr. *Keill*; appointed a numerous Committee to ſearch old Letters and Papers, and report their Opinion upon what they found; and ordered the Letters and Papers, with the Report of their Committee to be publiſhed. And by theſe Letters and Papers it appear'd to them, that Mr. *Newton* had the Method in or before the Year 1669, and it did not appear to them that Mr. *Leibnitz* had it before the Year 1677.

For

For making himself the first Inventor of the Differential Method, he has represented that Mr. *Newton* at first used the Letter *o* in the vulgar manner for the given Increment of *x*; which destroys the Advantages of the Differential Method; but after the writing of his Principles, changed *o* into $\dot{x}$; substituting $\dot{x}$ for dx. It lies upon him to prove that Mr. *Newton* ever changed *o* into $\dot{x}$, or used $\dot{x}$ for dx, or left off the Use of the Letter *o*. Mr. *Newton* used the Letter *o* in his *Analysis* written in or before the Years 1669, and in his Book of *Quadratures*, and in his *Principia Philosophiæ*, and still uses it in the very same Sense as at first. In his Book of Quadratures he used it in conjunction with the Symbol $\dot{x}$, and therefore did not use that Symbol in its Room. These Symbols *o* and $\dot{x}$ are put for things of a different kind. The one is a Moment; the other a Fluxion or Velocity as has been explained above. When the Letter *x* is put for a Quantity which flows uniformly, the Symbol $\dot{x}$ is an Unit, and the Letter *o* a Moment, and $\dot{x}o$ and dx signify the same Moment. Prickt Letters never signify Moments, unless when they are multiplied by the Moment *o* either exprest or understood to make them infinitely little, and then the Rectangles are put for Moments.

Mr. *Newton* doth not place his Method in Forms of Symbols, nor confine himself to any particular Sort of Symbols for Fluents and Fluxions. Where he puts the Areas of Curves for Fluents, he frequently puts the Ordinates for Fluxions, and denotes the Fluxions by the Symbols of the Ordinates, as in his *Analysis*. Where he puts Lines for Fluents, he puts any Symbols for the Velocities of the Points which describe the Lines, that is, for the first Fluxions; and any other Symbols for the Increase of those Velocities, that is, for the second Fluxions, as is frequently done in his *Principia Philosophiæ*. And where he puts the Letters *x*, *y*, *z* for Fluents, he denotes their Fluxions, either by other Letters as *p*, *q*, *r*; or by the same Letters in other Forms as *X*, *Y*, *Z* or $\dot{x}$, $\dot{y}$, $\dot{z}$; or by any

any Lines as *D E*, *FG*, *H I*, considered as their Exponents. And this is evident by his Book of *Quadratures*, where he represents Fluxions by prickt Letters in the first Proposition, by Ordinates of Curves in the last Proposition, and by other Symbols, in explaining the Method and illustrating it with Examples, in the Introduction. Mr. *Leibnitz* hath no Symbols of Fluxions in his Method, and therefore Mr. *Newton*'s Symbols of Fluxions are the oldest in the kind. Mr. *Leibnitz* began to use the Symbols of Moments or Differences *dx*, *dy*, *dz* in the Year 1677. Mr. *Newton* represented Moments by the Rectangles under the Fluxions and the Moment *o*, when he wrote his *Analysis*, which was at least Forty Six Years ago. Mr. *Leibnitz* has used the Symbols *∫x*, *∫y*, *∫z* for the Sums of Ordinates ever since the Year 1686; Mr. *Newton* represented the same thing in his *Analysis*, by inscribing the Ordinate in a Square or Rectangle. All Mr. *Newton*'s Symbols are the oldest in their several Kinds by many Years.

And whereas it has been represented that the use of the Letter *o* is vulgar, and destroys the Advantages of the Differential Method: on the contrary, the Method of Fluxions, as used by Mr. *Newton*, has all the Advantages of ths Differential, and some others. It is more elegant, because in his Calculus there is but one infinitely little Quantity represented by a Symbol, the Symbol *o*. We have no Ideas of infinitely little Quantities, and therefore Mr. *Newton* introduced Fluxions into his Method, that it might proceed by finite Quantities as much as possible. It is more Natural and Geometrical, because founded upon the *primæ quantitatum nascentium rationes*, which have a Being in Geometry, whilst *Indivisibles*, upon which the Differential Method is founded, have no Being either in Geometry or in Nature. There are *rationes primæ quantitatum nascentium*, but not *quantitates primæ nascentes*. Nature generates Quantities by continual Flux or Increase; and the ancient Geometers admitted such a Generation of Areas and Solids, when they drew one Line into another by local Motion

to generate an Area, and the Area into a Line by local Motion to generate a Solid. But the ſumming up of Indiviſibles to compoſe an Area or Solid was never yet admitted into Geometry. Mr. *Newton*'s Method is alſo of greater Uſe and Certainty, being adapted either to the ready finding out of a Propoſition by ſuch Approximations as will create no Error in the Concluſion, or to the demonſtrating it exactly: Mr. *Leibnitz*'s is only for finding it out. When the Work ſucceeds not in finite Equations Mr. *Newton* has recourſe to converging Series, and thereby his Method becomes incomparably more univerſal than that of Mr. *Leibnitz*, which is confin'd to finite Equations: for he has no Share in the Method of infinite Series. Some Years after the Method of Series was invented, Mr. *Leibnitz* invented a Propoſition for tranſmuting curvilinear Figures into other curvilinear Figures of equal Areas, in order to ſquare them by converging Series: but the Methods of ſquaring thoſe other Figures by ſuch Series were not his. By the help of the new *Analyſis* Mr. *Newton* found out moſt of the Propoſitions in his *Principia Philoſophiæ*: but becauſe the Ancients for making things certain admitted nothing into Geometry before it was demonſtrated ſynthetically, he demonſtrated the Propoſitions ſynthetically, that the Syſteme of the Heavens might be founded upon good Geometry. And this makes it now difficult for unskilful Men to ſee the Analyſis by which thoſe Propoſitions were found out.

It has been repreſented that Mr. *Newton*, in the Scholium at the End of his Book of Quadratures, has put the third, fourth, and fifth Terms of a converging Series reſpectively equal to the ſecond, third, and fourth Differences of the firſt Term, and therefore did not then underſtand the Method of ſecond, third, and fourth Differences. But in the firſt Propoſition of that Book he ſhewed how to find the firſt, ſecond, third and following Fluxions *in infinitum*; and therefore when he wrote that Book, which was before the Year 1676, he did underſtand the Method of all the Fluxions, and by con-

ſequence of all the Differences. And if he did not underſtand it when he added that Scholium to the End of the Book, which was in the Year 1704, it muſt have been becauſe he had then forgot it. And ſo the Queſtion is only whether he had forgot the Method of ſecond and third Differences before the Year 1704.

In the Tenth Propoſition of the ſecond Book of his *Principia Philoſophiæ*, in deſcribing ſome of the Uſes of the Terms of a converging Series for ſolving of Problemes, he tells us that if the firſt Term of the Series repreſents the Ordinate *BC* of any Curve Line *ACG*, and *CBDI* be a Parallelogram infinitely narrow, whoſe Side *DI* cuts the Curve in *G* and its Tangent *CF* in *F*, the ſecond Term of the Series will repreſent the Line *IF*, and the third Term the Line *FG*. Now the Line *FG* is but half the ſecond Difference of the Ordinate: and therefore Mr. *Newton* when he wrote his *Principia*, put the third Term of the Series equal to half of the ſecond Difference of the firſt Term, and by conſequence had not then forgotten the Method of ſecond Differences.

In writing that Book, he had frequent occaſion to conſider the Increaſe or Decreaſe of the Velocities with which Quantities are generated, and argues right about it. That Increaſe or Decreaſe is the ſecond Fluxion of the Quantity, and therefore he had not then forgotten the Method of ſecond Fluxions.

In the Year 1692, Mr. *Newton*, at the Requeſt of Dr. *Wallis*, ſent to him a Copy of the firſt Propoſition of the Book of Quadratures, with Examples thereof in firſt, ſecond and third Fluxions: as you may ſee in the ſecond Volume of the Doctor's Works, *pag.* 391, 392, 393 and 396. And therefore he had not then forgotten the Method of ſecond Fluxions.

Nor is it likely, that in the Year 1704. when he added the aforesaid Scholium to the End of the Book of Quadratures, he had forgotten not only the first Proposition of that Book, but also the last Proposition upon which that Scholium was written. If the Word [*ut*], which in that Scholium may have been accidentally omitted between the Words [*erit*] and [*ejus*,] be restor'd, that Scholium will agree with the two Propositions and with the rest of his Writings, and the Objection will vanish.

Thus much concerning the Nature and History of these Methods, it will not be amiss to make some Observations thereupon.

In the *Commercium Epistolicum*, mention is made of three Tracts written by Mr. *Leibnitz*, after a Copy of Mr. *Newton*'s *Principia Philosophiæ* had been sent to *Hannover* for him, and after he had seen an Account of that Book published in the *Acta Eruditorum* for *January* and *February* 1689. And in those Tracts the principal Propositions of that Book are composed in a new manner, and claimed by Mr. *Leibnitz* as if he had found them himself before the publishing of the said Book. But Mr. *Leibnitz* cannot be a Witness in his own Cause. It lies upon him either to prove that he found them before Mr. *Newton*, or to quit his claim.

In the last of those three Tracts, the 20*th* Proposition (which is the chief of Mr. *Newton*'s Propositions) is made a Corollary of the 19*th* Proposition, and the 19*th* Proposition has an erroneous Demonstration adapted to it. It lies upon him either to satisfy the World that the Demonstration is not erroneous, or to acknowledge that he did not find that and the 20*th* Proposition thereby, but tried to adapt a Demonstration to Mr. *Newton*'s Proposition to make it his own. For he represents in his 20*th* Proposition that he knew not how Mr. *Newton* came by it, and by consequence that he found it himself without the Assistance of Mr. *Newton*.

By the Errors in the 15*th* and 19*th* Proposition of the third Tract, Dr. *Keill* hath shewed that when Mr. *Leibnitz* wrote these

these three Tracts, he did not well understand the Ways of working in second Differences. And this is further manifest by the 10*th*, 11*th*, and 12*th* Propositions of this third Tract. For these he lays down as the Foundation of his infinitesimal Analysis in arguing about centrifugal Forces, and proposes the first of them with relation to the Center of Curvity of the Orb, but uses this Proposition in the two next, with Relation to the Center of Circulation. And by confounding these two Centers with one another in the fundamental Propositions upon which he grounds this *Calculus*, he erred in the Superstructure, and for want of Skill in second and third Differences, was not able to extricate himself from the Errors. And this is further confirmed by the sixth Article of the second Tract. For that Article is erroneous, and the Error arises from his not knowing how to argue well about second and third Differences. When therefore he wrote those Tracts he was but a Learner, and this he ought in candour to acknowledge.

It seems therefore that as he learnt the Differential Method by means of Mr. *Newton*'s aforesaid three Letters compared with Dr. *Barrow*'s Method of Tangents; so Ten Years after, when Mr. *Newton*'s *Principia Philosophiæ* came abroad, he improved his Knowledge in these Matters, by trying to extend this Method to the Principal Propositions in that Book, and by this means composed the said three Tracts. For the Propositions contained in them (Errors and Trifles excepted) are Mr. *Newton*'s (or easy Corollaries from them) being published by him in other Forms of Words before. And yet Mr. *Leibnitz* published them as invented by himself long before they were published by Mr. *Newton*. For in the End of the first Tract, he represents that he invented them all before Mr. *Newton*'s *Principia Philosophiæ* came abroad, and some of them before he left *Paris*, that is before *October* 1676. And the second Tract he concludes with these Words: *Multa ex his deduci possent praxi accommodata, sed nobis*

his nunc fundamenta Geometrica jecisse suffecerit, in quibus maxima consistebat difficultas. Et fortassis attente consideranti vias quasdam novas satis antea impeditas aperuisse videbimur. Omnia autem respondent nostræ Analysi Infinitorum, hoc est calculo Summarum & Differentiarum (cujus elementa quædam in his Actis dedimus) communibus quoad licuit verbis hic expresso. He pretends here that the *Fundamenta Geometrica in quibus maxima consistebat difficultas* were first laid by himself in this very Tract, and that he himself had in this very Tract opened *vias quasdam novas satis antea impeditas.* And yet Mr. *Newton*'s *Principia Philosophiæ* came abroad almost two Years before, and gave occasion to the Writing of this Tract, and was written *communibus quoad licuit verbis,* and contains all these Principles and all these new Ways. And Mr. *Leibnitz,* when he published that Tract, knew all this, and therefore ought then to have acknowledged that Mr. *Newton* was the first who laid the *Fundamenta Geometrica in quibus maxima consistebat Difficultas,* and opened the *vias novas satis antea impeditas.* In his Answer to Mr. *Fatio* he acknowledged all this, saying *Quam* [methodum] *ante Dominum* Newtonum *& me nullus quod sciam Geometra habuit; uti ante hunc maximi nominis Geometram, NEMO SPECIMINE PUBLICE DATO se habere PROBAVIT.* And what he then acknowledged he ought in Candour and Honour to acknowledge still upon all Occasions.

Mr. *Leibnitz* in his Letter of *May* 28. 1697, wrote thus to Dr. *Wallis. Methodum Fluxionum profundissimi* Newtoni *cognatam esse methodo meæ differentiali non tantum animadverti postquam opus ejus* [Principiorum scilicet] *& tuum prodiit; sed etiam professus sum in Actis Eruditorum, & alias quoque monui. Id enim candori meo convenire judicavi, non minus quam ipsius merito. Itaque communi nomine designare soleo Analyseos infinitesimalis; quæ latius quam Tetragonistica patet. Interim quemadmodum &* Vietæa *&* Cartesiana *methodus Analyseos speciosæ nomine venit; discrimina tamen nonnulla supersunt: ita fortasse*

&

& Newtoniana *&* Mea *differunt in nonnullis.* Here alſo Mr. *Leibnitz* allows that when Mr. *Newton*'s Principles of Philoſophy came abroad, he underſtood thereby the Affinity that there was between the Methods, and therefore called them both by the common Name of the infiniteſimal Method, and thought himſelf bound in candour to acknowledge this Affinity: and there is ſtill the ſame Obligation upon him in point of Candour. And beſides this Acknowledgment, he here gives the Preference to Mr. *Newton*'s Method in Antiquity. For he repreſents that as the vulgar Analyſis in *Species* was invented by *Vieta*, and augmented by *Cartes*, which made ſome Differences between their Methods: ſo Mr. *Newton*'s Method and his own might differ in ſome things. And then he goes on to enumerate the Differences by which he had improved Mr. *Newton*'s Method as we mentioned above. And this Subordination of his Method to Mr. *Newton*'s, which he then acknowledged to Dr. *Wallis*, he ought ſtill to acknowledge.

In enumerating the Differences or Improvements which he had added to Mr. *Newton*'s Method; he names in the ſecond Place Differential Equations: but the Letters which paſſed between them in the Year 1676, do ſhow that Mr. *Newton* had ſuch Equations at that time, and Mr. *Leibnitz* had them not. He names in the third Place Exponential Equations: but theſe Equations are owing to his Correſpondence with the *Engliſh*. Dr. *Wallis*, in the Interpolation of Series, conſidered Fract and Negative Indices of Dignities. Mr. *Newton* introduced into his Analytical Computations, the Fract, Surd, Negative and Indefinitive Indices of Dignities; and in his Letter of *October* 24. 1676, repreſented to Mr. *Leibnitz* that his Method extended to the Reſolution of affected Equations involving Dignities whoſe Indices were Fract or Surd. Mr. *Leibnitz* in his Anſwer dated *June* 21. 1677, mutually deſired Mr. *Newton* to tell him what he thought of the Reſolution of Equations involving

Dig-

Dignities whose Indices were undetermined, such as were these $x^y + y^x = xy$, $x^x + y^y = x + y$. And these Equations he now calls Exponential, and represents to the World that he was the first Inventor thereof. and magnifies the Invention as a great Discovery. But he has not yet made a publick Acknowledgment of the Light which Mr. *Newton* gave him into it, nor produced any one Instance of the use that he has been able to make of it where the Indices of Dignities are Fluents. And since he has not yet rejected it with his usual Impatience for want of such an Instance, we have reason to expect that he will at length explain its Usefulness to the World.

Mr. *Newton* in his Letter of *October* 24. 1676 wrote that he had two Methods of resolving the Inverse Problems of Tangents, and such like difficult ones; one of which consisted *in assuming a Series for any unknown Quantity from which all the rest might conveniently be deduced, and in collating the homologous Terms of the resulting Equation, for determining the Terms of the assumed Series.* Mr. *Leibnitz* many Years after published this Method as his own, claiming to himself the first Invention thereof. It remains that he either renounce his Claim publickly, or prove that he invented it before Mr. *Newton* wrote his said Letter.

It lies upon him also to make a publick Acknowledgment of his Receipt of Mr. *Oldenburgh*'s Letter of *April* 15. 1675, wherein several converging Series for squaring of Curves, and particularly that of Mr. *James Gregory* for finding the Arc by the given Tangent, and thereby squaring the Circle, were communicated to him. He acknowledged it privately in his Letter to Mr. *Oldenburg* dated *May* 20. 1675 still extant in his own Hand-writing, and by Mr *Oldenburg* left entred in the Letter-Book of the *Royal-Society*. But he has not yet acknowledged it publickly, as he ought to have done when he published that Series as his own.

It

It lies upon him alſo to make a publick Acknowledgment of his having received the Extracts of Mr. *James Gregory*'s Letters, which, at his own Requeſt, were ſent to him at *Paris* in *June* 1676 by Mr. *Oldenburgh* to peruſe: amongſt which was Mr. *James Gregory*'s Letter of *Feb.* 15. 1671, concerning that Series, and Mr. *Newton*'s Letter of *December* 10. 1672 concerning the Method of Fluxions.

And whereas in his Letter of *Decem.* 28. 1675 he wrote to Mr. *Oldenburgh*, that he had communicated that Series above two Years before to his Friends at *Paris*, and had written to him ſometimes about it; and in his Letter of *May* 12. 1676 ſaid to Mr. *Oldenburgh* that he had written to him about that Series ſome Years before; and in his Letter to Mr. *Oldenburgh* dated *Aug.* 27. 1676, that he had communicated that Series to his Friends above three Years before; that is, upon his firſt coming from *London* to *Paris*: He is deſired to tell us how it came to paſs, that when he received Mr. *Oldenburgh*'s Letter of *Apr.* 15. 1675 he did not know that Series to be his own.

In his Letters of *July* 15. and *Octob.* 26. 1674, he tells us of but one Series for the circumference of a Circle, and ſaith that the Method which gave him this Series, gave him alſo a Series for any Arc whoſe Sine was given, tho' the Proportion of the Arc to the whole Circumference be not known. This Method therefore, by the given Sine of 30 Degrees, gave him a Series for the whole Circumference. If he had alſo a Series for the whole Circumference deduced from the Tangent of 45 Degrees, he is deſired to tell the World what Method he had in thoſe Days, which could give him both thoſe Series. For the Method by the Tranſmutation of Figures will not do it. He is deſired alſo to tell us why in his ſaid Letters he did not mention more Quadratures of the Circle than one.

And if in the Year 1674 he had the Demonſtration of a Series for finding any Arc whoſe Sine is given, he is deſired

 to

to tell the World what it was; and why in his Letter of *May* 12. 1676 he desired Mr. *Oldenburgh* to procure from Mr. *Collins* the Demonstration of Mr. *Newton*'s Series for doing the same thing; and wherein his own Series differed from Mr. *Newton*'s. For upon all these Considerations there is a Suspicion that Mr. *Newton*'s Series for finding the Arc whose Sine is given, was communicated to him in *England*; and that in the Year 1673 he began to communicate it as his own to some of his Friends at *Paris*, and the next Year wrote of it as his own in his Letters to Mr. *Oldenburgh*, in order to get the Demonstration or Method of finding such Series. But the Year following, when Mr. *Oldenburgh* sent him this Series and the Series of Mr. *Gregory* and Six other Series, he dropt his Pretence to this Series for want of a Demonstration, and took time to consider the Series sent him, and to compare them with his own, as if his Series were others different from those sent him. And when he had found a Demonstration of *Gregory*'s Series by a Transmutation of Figures, he began to communicate it as his own to his Friends at *Paris*, as he represents in the *Acta Eruditorum* for *April* 1691. *pag.* 178, saying; *Jam Anno* 1675 *compositum habebam opusculum Quadraturæ Arithmeticæ ab Amicis ab illo tempore lectum*, &c. But the Letter by which he had received this Series from Mr. *Oldenburgh* he concealed from his Friends, and pretended to Mr. *Oldenburgh* that he had this Series a Year or two before the Receipt of that Letter. And the next Year, upon receiving two of Mr. *Newton*'s Series again by one *George Mohr*, he wrote to Mr. *Oldenburgh* in such a manner as if he had never seen them before, and upon Pretence of their Novelty, desired Mr. *Oldenburgh* to procure from Mr. *Collins* Mr. *Newton*'s Method of finding them. If Mr. *Leibnitz* thinks fit to obviate this Suspicion, he is in the first Place to prove that he had Mr. *Gregory*'s Series before he received it from Mr. *Oldenburgh*.

It

It lies upon him alſo to tell the World what was the Method by which the ſeveral Series of Regreſſion for the Circle and Hyperbola, ſent to him by Mr. *Newton June* 13. 1676, and claimed as his own by his Letter of *Auguſt* 27. following, were found by him before he received them from Mr. *Newton*.

And whereas Mr. *Newton* ſent him, at his own Requeſt, a Method of Regreſſion, which upon the firſt reading he did not know to be his own, nor underſtood it; but ſo ſoon as he underſtood it he claimed as his own, by pretending that he had found it long before, and had forgot it, as he perceived by his old Papers: it lies upon him, in point of Candor and Juſtice, either to prove that he was the firſt Inventor of this Method, or to renounce his Claim to it for preventing future Diſputes.

Mr. *Leibnitz* in his Letter to Mr. *Oldenburgh* dated *Feb.* 3. 167⅔ claimed a Right to a certain Property of a Series of Numbers Natural, Triangular, Pyramidal, Triangulo-Triangular, *&c.* and to make it his own, repreſented that he wondred that Monſieur *Paſcal*, in his Book entituled *Triangulum Arithmeticum*, ſhould omit it. That Book was publiſhed in the Year 1665, and contains this Property of the Series; and Mr. *Leibnitz* has not yet done him the Juſtice to acknowledge that he did not omit it. It lies upon him therefore in Candor and Juſtice, to renounce his Claim to this Property, and acknowledge Mr. *Paſchal* the firſt Inventor.

He is alſo to renounce all Right to the Differential Method of *Mouton* as ſecond Inventor: for ſecond Inventors have no Right. The ſole Right is in the firſt Inventor until another finds out the ſame thing apart. In which caſe to take away the Right of the firſt Inventor, and divide it between him and that other, would be an Act of Injuſtice.

In his Letter to Dr. *Sloane* dated *Decem.* 29. 1711. he has told us that his Friends know how he came by the Differential Method. It lies upon him, in point of Candor, openly and plainly, and without further Heſitation, to ſatisfy the World how he came by it.

In the same Letter he has told us that he had this Method above Nine Years before he published it, and it follows from thence that he had it in the Year 1675 or before. And yet its certain that he had it not when he wrote his Letter to Mr. *Oldenburgh* dated *Aug.* 27. 1676, wherein he affirmed that Problems of the Inverse Method of Tangents and many others, could not be reduced to infinite Series, nor to Equations or Quadratures. It lies upon him therefore, in point of Candor, to tell us what he means by pretending to have found the Method before he had found it.

We have shewed that Mr. *Leibnitz* in the End of the Year 1676, in returning home from *France* through *England* and *Holland*, was meditating how to improve the Method of *Slusius* for Tangents, and extend it to all sorts of Problems, and for this end proposed the making of a general Table of Tangents; and therefore had not yet found out the true Improvement. But about half a Year after, when he was newly fallen upon the true Improvement, he wrote back; *Clariss.* Slusii *Methodum Tangentium nondum esse absolutam Celeberrimo* Newtono *assentior. Et jam A MULTO TEMPORE rem Tangentium generalius tractavi, scilicet per differentias Ordinatarum.* Which is as much as to say that he had this Improvement long before those Days. It lies upon him, in point of Candor, to make us understand that he pretended to this Antiquity of his Invention with some other Design than to rival and supplant Mr. *Newton*, and to make us believe that he had the Differential Method before Mr. *Newton* explained it to him by his Letters of *June* 13. and *Octob.* 24. 1676, and before Mr. *Oldenburgh* sent him a Copy of Mr. *Newton*'s Letter of *Decem* 10. 1672 concerning it.

The Editors of the *Acta Eruditorum* in *June* 1696, in giving an Account of the two first Volumes of the Mathematical Works of Dr. *Wallis*, wrote thus, in the Style of Mr. *Leibnitz*: *Cæterum ipse* Newtonus, *non minus Candore quam præclaris in rem Mathematicam meritis insignis, publice & pri-*

privatim agnovit Leibnitium, *tum cum (interveniente celeberrimo Viro* Henrico Oldenburgo *Bremensi, Societatis Regiæ* Anglicanæ *tunc Secretario) inter ipsos (ejusdem jam tum Societatis Socios) Commercium intercederet, id est jam fere ante annos viginti & amplius, Calculum suum differentialem, Seriesque infinitas, & pro iis quoque Methodos generales habuisse; quod* Wallisius *in Præfatione Operum, factæ inter eos communicationis mentionem faciens, præteriit, quoniam de eo fortasse non satis ipsi constabat. Cæterum Differentiarum consideratio* Leibnitiana, *cujus mentionem facit* Wallisius *(ne quis scilicet, ut ipse ait, causeretur de Calculo Differentiali nihil ab ipso dictum fuisse) meditationes aperuit, quæ aliunde non æque nascebantur, &c.* By the Words here cited out of the Preface to the two first Volumes of Dr. *Wallis*'s Works, it appears that Mr. *Leibnitz* had seen that Part of the Preface, where Mr. *Newton* is said to have explained to him (in the Year 1676) the Method of Fluxions found by him Ten Years before or above. Mr. *Newton* never allowed that Mr. *Leibnitz* had the Differential Method before the Year 1677. And Mr. *Leibnitz* himself in the *Acta Eruditorum* for *April* 1691. pag. 178. acknowledged that he found it after he returned home from *Paris* to enter upon Business, that is, after the Year 1676. And as for his pretended general Method of infinite Series, it is so far from being general, that it is of little or no use. I do not know that any other Use hath been made of it, than to colour over the Pretence of Mr. *Leibnitz* to the Series of Mr. *Gregory* for squaring the Circle.

Mr. *Leibnitz*, in his Answer to Mr. *Fatio* printed in the *Acta Eruditorum* for the Year 1700. pag. 203. wrote thus. *Ipse* [Newtonus] *scit unus omnium optime, satisque indicavit publice cum sua* Mathematica Naturæ Principia *publicaret, Anno* 1687, *nova quædam inventa Geometrica, quæ ipsi communia mecum fuere, NEUTRUM LUCI AB ALTERO ACCEPTÆ, sed meditationibus quemque suis debere, & a me decennio ante* [i. e. anno 1677] *exposita fuisse.* In the

Book of *Principles* here referred unto, Mr. *Newton* did not acknowledge that Mr. *Leibnitz* found this Method without receiving Light into it from Mr. *Newton*'s Letters above-mentioned; and Dr. *Wallis* had lately told him the contrary without being then confuted or contradicted. And if Mr. *Leibnitz* had found the Method without the Assistance of Mr. *Newton*, yet second Inventors have no Right.

Mr. *Leibnitz* in his aforesaid Answer to Mr. *Fatio*, wrote further: *Certe cum elementa Calculi mea edidi anno* 1684, *ne constabat quidem mihi aliud de inventis ejus* [sc. Newtoni] *in hoc genere, quam quod ipse olim significaverat in literis, posse se Tangentes invenire non sublatis irrationalibus, quod* Hugenius *quoque se posse mihi significavit postea, etsi cæterorum ejus Calculi adhuc expers. Sed majora multo consecutum* Newtonum, *viso demum libro* Principiorum *ejus, satis intellexi.* Here he again acknowledged that the Book of *Principles* gave him great Light into Mr. *Newton*'s Method: and yet he now denies that this Book contains any thing of that Method in it. Here he pretended that before that Book came abroad he knew nothing more of Mr. *Newton*'s Inventions of this kind, than that he had a certain Method of Tangents. and that by that Book he received the first Light into Mr. *Newton*'s Method of Fluxions: but in his Letter of *June* 21. 1677 he acknowledged that Mr. *Newton*'s Method extended also to Quadratures of curvilinear Figures, and was like his own. His Words are; *Arbitror quæ celare voluit* Newtonus *de Tangentibus ducendis ab his non abludere. Quod addit, ex hoc eodem fundamento Quadraturas quoque reddi faciliores me in sententia hac confirmat; nimirum semper figuræ illæ sunt quadrabiles quæ sunt ad æquationem differentialem.*

Mr. *Newton* had in his three Letters above-mentioned (copies of which Mr. *Leibnitz* had received from Mr. *Oldenbergh*) represented his Method so general, as by the Help of Equations, finite and infinite, to determin *Maxima* and *Minima*, Tangents, Areas, solid Contents, Centers of Gravity,

Lengths and Curvities of curve Lines and curvilinear Figures, and this without taking away Radicals, and to extend to the like Problems in Curves usually called Mechanical, and to inverse Problems of Tangents and others more difficult, and to almost all Problems, except perhaps some Numeral ones like those of *Diophantus*. And Mr. *Leibnitz* in his Letter of *Aug.* 27. 1676, represented that he could not believe that Mr. *Newton*'s Method was so general. Mr. *Newton* in the First of his three Letters set down his Method of Tangents deduced from this general Method, and illustrated it with an Example, and said that this Method of Tangents was but a Branch or Corollary of his General Method, and that he took the Method of Tangents of *Slusius* to be of the same kind: and thereupon Mr. *Leibnitz*, in his Return from *Paris* through *England* and *Holland* into *Germany*, was considering how to improve the Method of Tangents of *Slusius*, and extend it to all sorts of Problems, as we shewed above out of his Letters. And in his third Letter Mr. *Newton* illustrated his Method with Theorems for Quadratures and Examples thereof. And when he had made so large an Explanation of his Method, that Mr. *Leibnitz* had got Light into it, and had in his Letter of *June* 21. 1677 explained how the Method which he was fallen into answered to the Description which Mr. *Newton* had given of his Method, in drawing of Tangents giving the Method of *Slusius*, proceeding without taking away Fractions and Surds, and facilitating Quadratures; for him to tell the *Germans* that in the Year 1684, when he first published his Differential Method, he knew nothing more of Mr. *Newton*'s Invention, than that he had a certain Method of Tangents, is very extraordinary and wants an Explanation.

At that time he explained nothing more concerning his own Method, than how to draw Tangents and determin *Maxima* and *Minima* without taking away Fractions or Surds. He certainly knew that Mr. *Newton*'s Method would do all

this

this, and therefore ought in Candor to have acknowledged it. After he had thus far explained his own Method, he added that what he had there laid down were the Principles of a much ſublimer Geometry, reaching to the moſt difficult and valuable Problems, which were ſcarce to be reſolved without the Differential Calculus, *AUT SIMILI*, or another like it. What he meant by the Words *AUT SIMILI* was impoſſible for the *Germans* to underſtand without an Interpreter. He ought to have done Mr. *Newton* juſtice in plain intelligible Language, and told the *Germans* whoſe was the *Methodus SIMILIS*, and of what Extent and Antiquity it was, according to the Notices he had received from *England*; and to have acknowledged that his own Method was not ſo ancient. This would have prevented Diſputes, and nothing leſs than this could fully deſerve the Name of Candor and Juſtice. But afterwards, in his Anſwer to Mr. *Fatio*, to tell the *Germans* that in the Year 1684, when he firſt publiſhed the Elements of his Calculus, he knew nothing of a *Methodus SIMILIS*, nothing of any other Method than for drawing Tangents, was very ſtrange and wants an Explanation:

It lies upon him alſo to ſatisfy the World why, in his Anſwer to Dr. *Wallis* and Mr. *Fatio*, who had publiſhed that Mr. *Newton* was the oldeſt Inventor of that Method by many Years, he did not put in his Claim of being the oldeſt Inventor thereof, but ſtaid till the old Mathematicians were dead, and then complained of the new Mathematicians as Novices; attacked Mr. *Newton* himſelf, and declined to contend with any Body elſe, notwithſtanding that Mr. *Newton* in his Letter of *Octob.* 24. 1676 had told him, that for the ſake of Quiet, he had Five Years before that time laid aſide his Deſign of publiſhing what he had then written on this Subject, and has ever ſince induſtriouſly avoided all Diſputes about Philoſophical and Mathematical Subjects, and all Correſpondence by Letters about thoſe Matters, as tending to Diſ-

putes;

putes; and for the ſame Reaſon has forborn to complain of Mr. *Leibnitz*, untill it was ſhewed him that he ſtood accuſed of Plagiary in the *Acta Lipſiæ*, and that what Mr *Keill* had publiſhed was only in his Defence from the Guilt of that Crime.

It has been ſaid the Royal-Society gave judgment againſt Mr. *Leibnitz* without hearing both Parties. But this is a Miſtake. They have not yet given judgment in the Matter. Mr *Leibnitz* indeed deſired the Royal-Society to condemn Mr. *Keill* without hearing both Parties; and by the ſame ſort of Juſtice they might have condemned Mr. *Leibnitz* without hearing both Parties; for they have an equal Authority over them both. And when Mr. *Leibnitz* declined to make good his Charge againſt Mr. *Keill*, the Royal-Society might in juſtice have cenſured him for not making it good. But they only appointed a Committee to ſearch out and examin ſuch old Letters and Papers as were ſtill extant about theſe Matters, and report their Opinion how the Matter ſtood according to thoſe Letters and Papers. They were not appointed to examin Mr. *Leibnitz* or Mr. *Keill*, but only to report what they found in the ancient Letters and Papers: and he that compares their Report therewith will find it juſt. The Committee was numerous and skilful and compoſed of Gentlemen of ſeveral Nations, and the Society are ſatisfied in their Fidelity in examining the Hands and other Circumſtances, and in printing what they found in the ancient Letters and Papers ſo examined, without adding, omitting or altering any thing in favour of either Party. And the Letters and Papers are by order of the Royal-Society preſerved, that they may be conſulted and compared with the *Commercium Epiſtolicum*, whenever it ſhall be deſired by Perſons of Note. And in the mean time I take the Liberty to acquaint him, that by taxing the Royal-Society with Injuſtice in giving Sentence againſt him without hearing both Parties, he has tranſgreſſed one of their Statutes which makes it Expulſion to defame them.

 The

The Philosophy which Mr. *Newton* in his *Principles* and *Optiques* has pursued is Experimental; and it is not the Business of Experimental Philosophy to teach the Causes of things any further than they can be proved by Experiments. We are not to fill this Philosophy with Opinions which cannot be proved by Phænomena. In this Philosophy Hypotheses have no place, unless as Conjectures or Questions proposed to be examined by Experiments. For this Reason Mr. *Newton* in his Optiques distinguished those things which were made certain by Experiments from those things which remained uncertain, and which he therefore proposed in the End of his Optiques in the Form of Queries. For this Reason, in the Preface to his *Principles*, when he had mention'd the Motions of the Planets, Comets, Moon and Sea as deduced in this Book from Gravity, he added: *Utinam cætera Naturæ Phænomena ex Principiis Mechanicis eodem argumentandi genere derivare liceret. Nam multa me movent ut nonnihil suspicer ea omnia ex viribus quibusdam pendere posse, quibus corporum particulæ per causas nondum cognitas vel in se mutuo impelluntur & secundum figuras regulares coharent, vel ab invicem fugantur & recedunt: quibus viribus ignotis Philosophi hactenus Naturam frustra tentarunt.* And in the End of this Book in the second Edition, he said that for want of a sufficient Number of Experiments, he forbore to describe the Laws of the Actions of the Spirit or Agent by which this Attraction is performed. And for the same Reason he is silent about the Cause of Gravity, there occurring no Experiments or Phænomena by which he might prove what was the Cause thereof. And this he hath abundantly declared in his *Principles*, near the Beginning thereof, in these Words; *Virium causas & sedes Physicas jam non expendo.* And a little after: *Voces Attractionis, Impulsus, vel Propensionis cujuscunque in centrum indifferenter & pro se mutuo promiscue usurpo, has Vires non Physice sed Mathematice tantum considerando. Unde caveat Lector ne per hujusmodi voces cogitet me*

 speciem

speciemvel modum actionis, causamve aut rationem physicam alicubi definire, vel Centris (quæ sunt puncta Mathematica) vires verè & physicè tribuere, si forte aut Centra trahere aut vires Centrorum esse dixero. And in the End of his Opticks: *Qua causa efficiente hæ attractiones* [sc. gravitas, visque magnetica & electrica] *peragantur, hic non inquiro. Quam ego Attractionem appello, fieri sane potest ut ea efficiatur impulsu vel alio aliquo modo nobis incognito. Hanc vocem Attractionis ita hic accipi velim ut in universum solummodo vim aliquam significare intelligatur qua corpora ad se mutuo tendant, cuicunque demum causæ attribuenda sit illa vis. Nam ex Phænomenis Naturæ illud nos prius edoctos oportet quænam corpora se invicem attrahant, & quænam sint leges & proprietates istius attractionis, quam in id inquirere par sit quanam efficiente causa peragatur attractio.* And a little after he mentions the same Attractions as Forces which by Phænomena appear to have a Being in Nature, tho' their Causes be not yet known; and distinguishes them from occult Qualities which are supposed to flow from the specifick Forms of things. And in the Scholium at the End of his *Principles*, after he had mentioned the Properties of Gravity, he added: *Rationem vero harum Gravitatis proprietatum ex Phænomenis nondum potui deducere, & Hypotheses non fingo. Quicquid enim ex Phænomenis non deducitur Hypothesis vocanda est; & Hypotheses seu Metaphysicæ seu Physicæ, seu Qualitatum occultarum, seu Mechanicæ, in Philosophia experimentali locum non habent. —— satis est quod Gravitas revera existat & agat secundum leges à nobis expositas, & ad Corporum cœlestium & Maris nostri motus omnes sufficiat.* And after all this, one would wonder that Mr. *Newton* should be reflected upon for not explaining the Causes of Gravity and other Attractions by Hypotheses; as if it were a Crime to content himself with Certainties and let Uncertainties alone. And yet the Editors of the *Acta Eruditorum*, (*a*) have told the World that Mr. *Newton* denies that the cause of Gravity is Mechanical, and that if the Spirit or Agent by which Electrical Attraction is performed, be not the *Ether* or *subtile Matter* of *Cartes*, it is less valuable than an Hypothesis, and perhaps may be the Hylarchic Principle of Dr. *Henry Moor*: and Mr. *Leibnitz* (*b*) hath accused him of making Gravity a natural or essential Property of Bodies, and an occult Quality and Miracle. And by this sort of Railery they are perswading the *Germans* that Mr. *Newton* wants Judgment, and was not able to invent the Infinitesimal Method.

(*a*) *Anno* 1714, *mense Martio*, p. 141. 143. (*b*) *In tractatu de Bonitate Dei & in Epistolis ad D.* Hartsoeker *& alibi.*

It must be allowed that these two Gentlemen differ very much in Philosophy. The one proceeds upon the Evidence arising from Experiments and Phænomena, and stops where such Evidence is wanting; the other is taken up with Hypotheses, and propounds them, not to be examined by Experiments, but to be believed without Examination. The one for want of Experiments to decide the Question, doth not affirm whether the Cause of Gravity be Mechanical or not Mechanical: the other that it is a perpetual Miracle if it be not Mechanical. The one (by way of Enquiry) attributes it to the Power of the Creator that the least Particles of Matter are hard: the other attributes the Hardness of Matter to conspiring Motions, and calls it a perpetual Miracle if the Cause of this Hardness be other than Mechanical. The one doth not affirm that animal Motion in Man is purely mechanical: the other teaches that it is purely mechanical, the Soul or Mind (according to the Hypothesis of an *Harmonia Præstabilita*) never acting upon the Body so as to alter or influence its Motions. The one teaches that God (the God in whom we live and move and have our Being) is Omnipresent; but not as a Soul of the World: the other that he is not the Soul of the World, but *INTELLIGENTIA SUPRAMUNDANA*, an Intelligence above the Bounds of the World; whence it seems to follow that he cannot do any thing within the Bounds of the World, unless by an incredible Miracle. The one teaches that Philosophers are to argue from Phænomena and *Experiments* to the Causes thereof, and thence to the Causes of those Causes, and so on till we come to the first Cause: the other that all the Actions of the first Cause are Miracles, and all the Laws impreſt on Nature by the Will of God are perpetual Miracles and occult Qualities, and therefore not to be considered in Philosophy. But must the constant and universal Laws of Nature, if derived from the Power of God or the Action of a Cause not yet known to us, be called Miracles and occult Qualities, that is to say, *Wonders* and *Absurdities*? Must all the Arguments for a God taken from the Phænomena of Nature be exploded by *new hard Names*? And must Experimental Philosophy be exploded as *miraculous* and *absurd*, because it asserts nothing more than can be proved by Experiments, and we cannot yet prove by Experiments that all the Phænomena in Nature can be solved by meer Mechanical Causes? Certainly these things deserve to be better considered.

ERRATA. *Pag.* 199. *lin.* 14. *put an* Asterisk (*) *after the Word* Letter.


Pectus Cygni
γ
Philosoph. Trans.
N.b 343
Nova Anni 1600
b
p
η
ν
χ
Nova Anni 1686
m
φ
α
Nova Anni 1670
Fig. A
c
e
f
h
g
d
χ
a
b
20
15
10
5
55
50
I. Senex Sculp.

Numb. 343.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *March*, *April* and *May*, 1715.

The CONTENTS.

I. *D.* Gothofredi Kirchii *Aſtronomi & Obſervatoris præſtantiſſimi, & Societatis Regiæ* Berolinenſis *dum vixit Socii, De varia apparentia* Stellæ novæ *in Collo Cygni Narratio: è Miſcellaneis* Berolinenſibus *anno* MDCCX *editis deſumpta.* pag. 208.

QUanquam multæ varietates mutationeſque inter ſtellas fixas, quoad apparentem earum magnitudinem in cœlo contingant, nulla tamen inter omnes mutabilis apparentiæ ſtellas fixas mirabilior habita eſt, illâ quam *Fabricius* Anno 1596. in collo Ceti primus obſervavit. Licet enim primò pro ejuſmodi nova Stella habita fuerit quæ nunquam antea extiterit, & poſtquam diſparuit non amplius eſſet reditura: nunc tamen experientia ſatis comprobavit eam conſtanter exiſtere, & abſque omni dubio à principio mundi in eo loco, quem nunc quoque obtinet, exſtitiſſe. Hoc ſolummodo in ea mirandum, quod ſeſe quotannis in varia magnitudine ſpectandam exhibeat, & plerumque certis temporibus nudis oculis plane videri nequeat; qua de cauſa etiam a Domino *Hevelio* Stella Mira vocata eſt.

Aliam huic ſimilem ego quoque in collo Cygni deprehendi, multo autem minorem, & quotannis breviori ſpatio temporis conſpiciendam. Unde mirum non eſt eam tamdiu incognitam manſiſſe: Imo pro ſingulari felicitate reputandum, quod eo ipſo tempore viſibilis fuerit, ac in ſua maxima magnitudine apparuerit, quo *Bayerus* ſtellas in Cygno conſiderabat & delineabat, ubi eam notâ χ deſignavit, & inter conſtanter apparentes fixas 5^{tæ} magnitudinis recenſuit. Quemadmodum etiam ſupra memoratam in collo Ceti, cum hocce Sydus conſideraret & delinearet, in quarta magnitudine deprehendit & literâ o deſignavit, eam-

eamque pro cōnſtanter apparente Stella fixa habuit.

Ut mutabilem apparentiam ſtellæ χ in collo Cygni deprehenderem, occaſionem dedit ſtella capiti Cygni vicina, quam *Hevelius*, Aſtronomus ſolertiſſimus, Anno 1670 & 71. obſervavit. Cum enim ſpem conciperem fore ut eadem ſtella nunc iterum ſæpius eſſet apparitura, non ſecus ac ſtella in collo Ceti, quam *Hevelio* poſt primam diſparitionem mox iterum apparuiſſe conſtabat, quærebam eam 1° & 6to (11° & 16°) Julii 1686. noctibus ſerenis, non autem reperiebam; ſed potius animadvertebam ſtellam illam 5^{tæ} magnitudinis in collo Cygni, à *Bayero* græca litera χ ſignatam, deſiderari. Die vero 9 (19.) Octobris deprehendi eam nudis oculis diſtinctè omnino. Et quia facile adducebar ut exiſtimarem eandem nudis noſtris oculis iterum diſpariturum, delineavi aliquot ipſam circumſtantes ſtellulas, ope bipedalis magnæque capacitatis Tubi, ut ex harum cum illa comparatione magnitudinem ejus, cum decreſceret, expenderem, uti Fig. A. exhibetur.

Deprehenſum eſt quoque, ſtellam iſtam paulatim decreviſſe, donec eam Tubo 8 pedum non potuerim amplius aſſequi, cum tamen aliam illam in collo Ceti, quando nudo oculo non amplius patet, per tubum 4 pedum ſemper dignoſcere poſſim.

Ab illo tempore quærebam quidem ſtellam iſtam variis noctibus fruſtra, tandem verò tamen eam reperi 6to (16°) Aug. 1687. iterum ope octopedalis Tubi, at vero exiguam valde. Inde de die in diem creviſſe deprehendebatur; & factum eſt, ut die 23 Octob. (2 Novemb.) iterum primâ vice, nudo oculo, ſe ſe conſpiciendam præberet, valde licet adhuc dum exiguam. Die 2do (12°) Nov. optime erat conſpicua, etiam poſt 26^{tum} Nov. (6. Dec.) ut ut hoc ultimo die jam iterum in ſtatu decreſcendi exiſteret. Poſtmodum non niſi per Tubos dignoſci potuit, tandemque adeò exigua evaſit, ut iterum Tubo 8 pedum eam deprehendere non potuerim. Atque ita animadverſum eſt hac vice, ab

una disparitione usque ad alteram, annum unum, mensem unum, unamque hebdomadam circiter effluxisse. Sequentes quoque observationes docuerunt hancce stellam tempus satis constans in sua apparitione servare, non tamen quavis vice ad æqualem magnitudinem pervenire. Imo aliquando accidit, ut nudo oculo planè invisibilis maneat, dum per Tubum est conspicua, & maximam suam magnitudinem assecuta est; prout Anno 1688 in fine, & 1689 in principio anni accidit. E contrario, sequente anno 1690, stella hæc eo melius videri poterat, & quidem notabiliter major quam sua vicina, quam *Bayerus* juxta χ collocavit extra collum Cygni, nullaque literâ notavit, quam ego solius memoriæ juvandæ gratia, Hebraica litera ג notavi. Et postquam hujus stellæ apparentiam & disparentiam sæpius observavi, comperi eam valde esse regularem, revolutionemque 404½ dierum servare.

Cum Miscellanea Berolinensia, *serius ad nos perlata sint, non ante annum ultimo elapsum hanc Stellam novam secundum D.* Kirchii *monitum conspeximus; idque juxta Idus Julii,* st. vet *cum multo clarior quam vicina* ג, *ac fere æqualis mediæ in collo Cygni* (Bayero n) *apparuit: sed post mensem nudis oculis inconspicua facta, tandem etiam Telescopio evanuit. Juxta periodum qua revolvi dicitur, mense saltem Augusto currentis anni 1715. maximam suam claritatem adipisci debet: unde curiosis syderum scrutatoribus de hacre sat sane notabili certa capi poterint documenta.*

Ut autem facilius in Cœlo inveniatur, duo Schemata adjecimus, quorum alterum exhibet Cygni collum, cum Fixis huic Novæ adjunctis, Novisque duabus aliis intra seculum in ejus vicinia emersis; quarum quæ ante Pectus Cygni etiamnum conspicitur quasi quinti honoris: quæ vero sub Capite per biennium tantum visa hactenus latet. Altera Figura, quæ est Kirchii A, *Telescopicas Novæ proximas demonstrat, ut sciatur quo præcise loco primum redeuntis radium seduli Cœlispices Tubo armati præstolari debeant.*

II.

II. BOTANICUM Hortense IV.

Giving an Account *of divers* Rare Plants, *Observed the last* Summer *A. D.* 1714. *in several* Curious Gardens *about* London, *and particularly* the Society of Apothecaries *Physick-Garden at* Chelsea. *By* James Petiver, F. R. S.

SECT. I. *EUROPEAN* Plants.

1. VAlentia *Knotgrass.* Ray's *English Herball.* Tab. x. fig. 8.

Anthyllis *Valentina* Clus. *Hist.* 186. c. 9. Fig. *id.* Hisp. 480. Fig. *Park.* 446. fig. 3.

Anthyllis maritima, *Chamæsyce* similis C B. 282. 1. *Phyt.* 552. 1.

Anthyllidis species quibusdam *Chabr.* 452. *Ic.* 5. I. B. Vol 3. L. 29. p. 374. Fig.

This is a low spreading Ground Plant, with roundish small *Leaves*, and very little four leaved blush *Flowers.*

Dr. *John Placa* M. D. and publick Professor of *Valentia*, first observed this Plant about the Ditches of that City, and there shewed it to that accurate Botanist *Carolus Clusius* who has given us a very good Figure of it.

Dr. *Magnol* hath also found it on the Coasts of *Languedock.*

Mons. *Riqueur* Apothecary *to the late Queen* of *Spain*, sent me the Seed of this and many other curious Plants, which he collected about *Madrid*, several of which were the last Summer raised in our *Physick Garden* at *Chelsea*, where this *Flowred.*

2.

2. Annual *Fleawort*. Ray *Hist*. Plant. 881. 1.

Psyllium majus erectum C B. pin. 191.3. alterum C B phyt. 353. 2.

Psyllium sive *Pulicaris* Herba *Ger*. 471. fig. 1. *Jonst*. 587. fig. 1.

Pulicaris Herba *Lobel*. Icon. 436.2. *id*. Belg. 523. Obs. 239.

Its Top Branches and Stalks are somewhat fat or clamy, its *Leaves* are like *Hyssop* and broader than the *Perennial*. *Grows* plentifully in the Fields about *Montpelier*.

3. Notcht-leaved Fleawort. Ray H. Pl. 882. 2.

Psyllium *Dioscoridis* sive *Indicum* foliis crenatis C.B. 199.1. prodr. 99. 1.

Psyllium *Indicum* foliis crenatis *Park*. 277. 3.

Psyllium laciniatis foliis *Bocc*. 8. Tab. 4.

This differs from the common *Annual*, only in having notcht or indented Leaves.

4. Perennial Fleawort Ray 882. 3.

Psyllium *Camer*. Epit. 811. Fig. *Chabr*. 501, Ic. 3. I B. 3. l. 31. p. 513. fig.

Psyllium majus supinum C. B. 191. 2. majus C. B. phyt. 353. 1.

Psyllium maj. sempervirens *Park*. 277. 2. & vulg. 278. *secund*. Fig.

Psyllium *Plinianum* fortè, *radice* perenni, supinum *Lobel*. Icon. 437. 1. *id*. Belg. 523. *id*. Obs. 239. fig.

Grows frequently in *Italy* and about *Montpelier*.

Vertues. The Seed of this Plant evacuates yellow Choler, and by its Mucilage, blunts the Acrimony of the Humors, and is therefore commended in Dysenteries and other Corrosions of the Gutts.

Dr. *Sloane* has experieneed it in Excoriations of the Uvula or Palat, and where the Tongue is parcht.

H. Reusnerus says a Mucilage of its *Seeds*, in *Rose-water* with *Vinegar*, has cured great Pains in the Head, proceeding from a Hot Cause, when other Medicines have failed.

The

The ſame with *Camphire* has been ſucceſsfully applied to inflamed Eyes.

5. Maple *Blite*. Ray's Engliſh Herbal Tab. 8. Fig. 7.

Atriplex odore & folio *Daturæ*, minori tamen, *Triumphet*. 65.

Blitum *Aceris* folio *Cat*. Herbar. *Britan*. Tab. 8. fig. 7.

Blitum ſeu Atriplex *Pes Anſerinus* dicta, *Stramonii* acutiore folio ramoſum *Pluk. Mantiſ.*

Chenopodio affinis, folio lato laciniato in longiſſimum mucronem procurrente, florum racemis ſparſis *Ray* H. Pl. Vol. 3. p. 123.

Mr. *Dale* was the firſt that obſerved this in *England*, viz. about *Colcheſter*. I find it the ſame with that of *Triumphetti*, a Specimen of it being lately ſent me from *Peter Antony Micheli* Botaniſt to his Royal Highneſs the *Duke* of *Florence*.

6. Thorney *Burnett* Ray 1492. cap. 7.

Pimpinella ſpinoſa *Park* 998. fig.

Poterion Lob. Ic. T. 2. p. 26. Fig. 2. *Belg*. 2. p. 30. Obſ. 491. fig.

Poterion *Lob*. ſive *Pimpinella* ſpinoſa C B. 388. 2.

Poterio *affinis* folio *Pimpinellæ*, ſpinoſa C B. 382. 2.

Rawolph firſt obſerved this Plant on the Sides of *Mount Libanus*, and from whom all our Figures are copied.

Dalechamp has ſince found it in the *Valleys* about *Marra* near *Gratianople* in *Dauphiny*.

Honorius Bellus a learned Phyſitian in *Candy*, ſays the Ruſticks of that Iſland make a Tea of this Plant, which cures them of all Sorts of Fluxes.

Its called *Stoibeda* in moſt parts of *Greece*.

7. Blew *Cat-Succory* Ray 257. c. 6.

Catanance *Dalech*. fl. Cyani, fol, *Coronopi* Chabr. 342. Ic. opt. 2. I. B. 3. l. 25. p. 26. Fig.

Chondrilla *Seſamoides* dicta *Park* 786. fig. 5.

Chondrilla *Seſamoides* dicta cærulea C B phyt. 217. 14.

Chondrilla cærulea, *Cyani* capitulis C B. 131. 6.

Seſamoides parvum, *Matth*. Ger. 397. Ic. 4. Jonſt. 493. fig. 3.

Its blew *Succory*-like *Flowers*, with narrow dented *Leaves* diſtinguiſh it from all others.

Grows

Grows very common near *Narbone*, and in *Savoy* on dry ſtony Hills.

I gathered this elegant Plant in *Flower* this Summer in His Majeſty's Gardens at *Hampton Court*, under the care of Mr. *Wiſe*, King *George*'s Gardiner.

8. Yellow *Cat-Succory*.

Catanance *Cretica* fl. luteo.

Stæbe *Plantaginis* folio. *Alpin.* Exot. 286. fig. *Park.* 477. fig. 7.

Stæbe *Plantaginis* folio, fl. luteo H. *Oxon.* Vol. 2. p. 137. 4.

Mr. *Jacob Bobart* Botanick Profeſſor at *Oxford*, ſent me the firſt Specimen of this, which has lately Flowred very well with us in *Chelſea* Garden.

9. Sea *Ragwort*. Ray 286. 6.

Jacobea marina *Jonſt.* 280. Ic. 4. C B. *phyt.* 218. 1.

Jacobea *marina* ſive *Cineraria* Chabr. 330. Ic. 6. 1. B. 2. l. 24. p. 1056. fig.

Jacobea marina ſive *Cineraria* vulg. *Park.* 669. fig. 7.

Jacobea maritima C B. 131. 3.

This has been long cultivated as a great Ornament in *Gardens*.

Vertues. *Alpinus* ſays the *Ægyptians* uſe this as a very ſovereign Plant, drinking a Tea of it for the Stone, and to open Obſtructions of the Bowels and Womb.

Grows on the Coaſts of *Tuſcany*.

10. Sicilian *Ragwort*, Ray 286, 9.

Jacobea Sicula *Chryſanthemi* facie *Bocc.* 66. Tab. 36.

It leaves like our *Corn Marygold*, flowers in *Chelſea* Garden even till *Chriſtmas*. *Grows* wild about *Catania*, &c.

11. Common Narrow *Caſſidony* Ray 281. 4.

Elychryſon ſive *Stæchas* citrina anguſtifolia C B. 264. 4. vel *Gallica* phyt. 513. 4.

Chryſocome vulg. 1. Cluſ. 326. fig.

Chryſocome media, ſ. *Stachas* citrina vulg. *Barrelier.* 974. Ic. 409.

Stæchas

Stæchas Citrina *Dod.* 268.
Stæchas citrina ſive *Amaranthus* luteus *Jonſt.* 646. *Ic.* 1.
Stæchas *citrina* ſive *Coma aurea* Park. 68. fig. 7.
Stæchas citrina, tenuifolia *Narbonenſis* I B. 3. l. 26. p. 154. fig. *ead.* flore luteo palleſcente *Chabr.* 369. Ic. 5.

The Leaves of this Plant are beſt repreſented in I B. and *Chabreus*, being much narrower than thoſe *Figured* by *Cluſius.*

Grows plentifully about *Montpelier*, where it *Flowers* in *April* and *May.*

12. Candy *Caſſidony*, Ray 282. 8.

Elychryſum Creticum C B. 264. 6.
Chryſocome 5 quæ *Cretica* Cluſ. 327.
Chryſocome ſive *Stæchas* citrina *Cretica* Park. 69. 8.
Stæchas citrina globoſo & amplo flore *Cretica*, Barrelier *pl.* 987. Ic. *opt.* 814.

This laſt Author has given a very accurate Figure of this Plant, which is ſo beautiful an Ornament in our moſt curious Gardens.

13. Stif-rim'd *Mary-gold*, Ray 338 c. 4. pl. 2.

Aſter Atticus *Cæſalp.* 495. c. 30. *Ger.* 392. Ic. 1. *Jonſt.* 486. Ic. 1.
Aſter *Atticus* 1. Cluſ. 13. Fig. 1. *Maſſiloticus Tabern.* Icon. 361. 2.
Aſter *Att.* luteus vulg. *Park.* 128. fig. 1.
Aſter Atticus & *Inguinaria* ſ. *Inguinalis* Lob. Ic. 348. 2. *Belg.* 423. Obſ. 188. fig.
Aſter luteus, foliis ad florem rigidis C B. 266. 1. *Phyt.* 518. 1.
Chryſanthemum *Aſteris* facie, foliis ad florem rigidis H. *Leyd* 144.

Its Rim of yellow Flowers is beſet with ſtiff, long, pointed green Leaves, by which its diſtinguiſhed from all others.

Its common in *Sicily*, *Italy*, *Narbon* and *Spain*, Flowring in *May* and *June.*

14. *Bobart's* Venice *Chamomil* Ray 3. p. 223. 15.
Cotula Veneta *Sophiæ* folio *Nobis.*
Chamæmelum annuum ramosum *Cotulæ fætidæ* fol. ampliori-
bus capitulis spinosis *Bob.* H. Ox. 3. p. 36. 12. *Sect.*
VI. Tab. 8. fig.

We are obliged to Mr. *Jacob Bobart* for the first Knowledge of this Plant.

15. *Distaff-Thistle* Ray 304. 4.
Atractylis *Offic.* Dale 168. 3. *Ger.* 1008. Ic. 1. *Jonst.* 1171.
Ic. 1.
Atractylis lutea C. B. 379. 1. fl. luteo *Park.* 963. Ic. 1.
Atractylis *veterum* f. vera, fl. luteo *Chabr.* 353. Ic. 4. I B. 3. l.
25. p. 85. fig.
Atractylis *Theophrasti* & *Diosc.* sanguineo succo *Col.* 19.
fig. 23.

Mr. *Ray* has given a large *Description* of this *Thistle* p. 304. from the accurate *Columna,* and it is remarkable for its bloody Juice.

Its said to have the same *Vertues* with the *Carduus Benedictus.*

Grows in *France, Spain* and *Italy* as also about *Geneva* in Path-ways and Borders of Fields.

16. Cobweb *Distaff-Thistle.*
Atractylis ramulis araneosis.
an *Chameleon* niger verus *Park.*?

This differs from the *Distaff-Thistle* in having its upper Stalks woolly like *Cobwebs.* It was many Years since raised in Mr. *Charles Dubois* his Garden at *Mitcham,* from Seed I gave him brought me by Mr. *Samuel Daniel,* Surgeon, from the *Island* Coos.

17. *Clusius* his *Salamanca* Welted Thistle, Ray 315.
Acarna major caule folioso C B. 379. 6. *Park.* 966. f. 6.
Acarnæ similis fl. purp. *Chameleon* Salmant *Clus.* I. B. 3. l.
25. p. 91. fig. *Chabr.* 155. Ic. 6.
Chameleon *Salmanticensis* Clus. *Hist.* 155. f. 1. *Jonst.* 1160.
fig. 2.

Clusius

Clusius first observed this about *Salamanca* in *Spain*, it hath since been found in *Languedoc* and other places.

18. *Theophrastus* his *Fish Thistle* Ray 315. 4. *Dale* Suppl. 74. 4.

Acarna *Theophrasti* Ger. 1012. fig. 7. Jonst. 1175. f. 7.
Acarna di *Theophrasto* Imperati 669. fig. opt.
Acarna major caule non folioso C. B. 273. 7. *Park.* 966. f. 7.
Acarna *Theophr.* Imperati *Ilvensis* f. *Italica* Barrel. 912. Ic. 1211.
Acarnæ similis, *Carduus* polyacanthus *Chabr.* 356. Ic. 2.
Polyacanthus Causabona *Acarnæ* similis I. B. 3. l. 25. p. 92. fig.

Imperatus his *Figure*, which *Barrelier* has copied, very well represents this elegant Thistle. *Chabr.* & I B. are also better than *Lobel's*, which *Park.* and most others have followed.

Grows on the Hills, North of *Rio* near the Iron Mines in the Island of *Ilva*.

19. Dwarf *Narbone* Artichoke. Ray 329. 29.

Centaurium majus incanum humile, cap. *Pini.* El. Bot. 355. *Instit.* 449.
Chameleon non aculeatus *Lob.* Ic. p. 2. *p.* 7. *Ad.* 367. fig.
Jacea montana incana *Pini* capite C. B. 272.
- humilis mont. cap. *Pino* simili C. B. phyt. 531. 13.
Jacea pumila *Narbonensis* Park. 471. fig. 6.
Jacea mont. capite magno *Stroboli* I. B. 3. l. 25. p. 30. fig. Chabr. 343. *Ic.* 3.
Stœbe *Pinea* amplo capite *Barrel.* 970. Ic. opt. 138.

Some of the bottom *Leaves* of this are whole, which are not exprest in any Figure yet extant. I have received very fair Specimens of this elegant Plant from that *Accurate Botanist* Dr. *John Salvadore* at *Barcelona*. It *Grows* plentifully about *Narbone* and *Montpelier*, where it *Flowers* in *June*, as it did this Summer in *Chelsea* Garden.

20. *Cobweb-headed* Yellow *Portugal* Knapweed *Ray* Vol. 3. p. 204. 28.

Carduus *Lusitan.* canescens, alato caule, capite lanuginoso *El. Bot.* 350. *Inst.* 441.

Jacea *Lusit.* canescens alato caule, capite spinoso & lanuginoso *Ray* Vol. 3. p. 204. pl. 28.

Its *Root*-Leaves like *Scabious*, but on the *Stalk* whole and narrow, its *Head* woolly like a *Cobweb*, beset with long Thorns, in the midst of which comes a yellow *Flower*. Raised this Summer in *Chelsea* Garden.

21. Succory leaved, *Yellow* Oriental *Knap-weed.*

Jacea lutea *Oriental.* capite spinis simplicibus armato.

The lower *Leaves* are lobated like the *Stæbe* Salam. 1. *Clus.* but on the Stalks they are plain and narrow. At the Top grow specious yellow *Flowers* like the *Sultan*, of that Colour, set in scaley Heads, each ending in a single longish Prickle.

I have as yet seen this only with Mr. *Fairchild* at *Hoxton*, raised from Seed which Dr. *Sherard* sent to Mr. *Stonestreet*.

22. Purple *Knapweed* with black edged Scales *Ray* 322. 21.

Jacea carnea, marginibus squamarum nigris *Nobis.*

Jacea humilis, *Hieracii* folio *Park.* 471. 5. H. Lugd. 1193. fig.

Jacea humilis alba, *Hieracii* folio C. B. 271. 2. phyt. 530. 12.

Jacea pumila *Ad.* 235. *fig.* pumila serpens acaulis ferme *Lob.* Ic. 542. 2.

Jacea *Monspeliaca* cui in squamis fibræ nigræ, interdum acaulis I. B. 2. l. 25. p. 29. *Chabr.* 343. Ic. 1.

Lobel first observed this at *Montpelier*, where it is more commonly found with a white Flower than a purple. Mr. *Jezreel Jones* gathered it about *Lisbon*, a very fair Specimen of which *Monsieur Vaillant* sent me from *Paris.*

23. Austrian and Spanish *Stœbe* Ray. 324. 4.

Stœbe *Gallica* & *Austriaca* elatior Clus. l. 4. p. 10.

Stœbe

Stæbe *Austriaca* elatior *Park.* 476.
Stæbe major calyculis non ſplendentibus C B. 273. 3.
Stæbe Salmantica alterius, altera ſpecies *Cluſ.* Hiſp. 362.
Centaurium majus in Muris *Geſn.* Hort. 252.
- - ſpecies tenuifolia *Chabr.* 345. Ic. 6. I. B. 3. l. 25. p. 31. fig.
Jacea alba *Lugd.* 1192. Ic. 2.
Jacea *Stæbe* dicta 4. C B. phyt. 532. 19.
Jacea non ſpinoſa, fol. magis diviſis elatior, capitulis minoribus non ſplendentibus *Bob.* Oxon 140. 15.

Its lower Leaves ſmall and deeply cut, its *Flowers* purple like the *Common*, with ſmall half ſtarr'd hairy *Scales*.

Monſ. *Riqueur* ſent me the *Seed* of this from *Madrid*, which Flowred in *Chelſea Garden* this *Autumn*.

24. *Pona*'s Pine-leaved *Candy* Knapweed.
Chamæpeuce Pr. Alpin. *Exot.* 76. fig. ex ſententia G. *Sherard.*
Chamæpitys *Berthiolo.*
Chamæpitys fruticoſa *Cretica* Belli.
Jacea fruticans *Pini* folio C. B. 271. 3. *Pluk.* Tab. 94. fig. 3.
Jacea Cretica fruteſcens, *Elychriſi* folio, *fl.* magno purpuraſcente *T. Coral.* 32.
Stæbe *Roriſmarini* folio *Jonſt.* 731. fig. 4.
Stæbe capitata *Roriſmarini* folio *Pona* 329. fig. *Chabr.* 344. Ic. 4. I. B. 3. l. 25. p. 36. fig.
Stæbe capit. overo *Chamæpino* frutticoſo di *Candia* Ponæ *Ital.* 75. fig.
Stæbe *Cretica* fruticans, *Piceæ* aut potius *Pini* anguſtis foliis crebrius ſtipatis *Bob. Oxon.* 137. 8. *Ray* 3. p. 204. 29.

This is not the *Cyanus* arboreſcens longifolia *Pr. Alp.* Exot. p. 30. as *Parkinſon* and ſome others aſſert.

Dr. *Plukenet*'s Figure (which he took from Sir *George Wheeler*'s Specimen) very well agrees with the Pattern which Dr. *Sherard* ſent me from *Smyrna* A. D. 1705. *Proſper Alpinus*'s alſo is well cut. *Pona*'s amongſt thoſe of

Mount

Mount Baldus was taken from a Garden Plant, but that in the *Italian Edition* from a Native.

This elegant Plant I have only ſeen with Mr. *Fairchild* at *Hoxton*, raiſed from the Seed which Dr. *Sherard* ſent to the Reverend Mr. *Stoneſtreet*.

25. Aſh-leaved *Scabious* Ray Vol. 3. p. 236. pl. 30. Scabioſa *Fraxinellæ* folio *Inſtitut*. Rei Herbar. 666.

This is a ſpecious Plant and grows in *Chelſea Garden* near two Foot high, its lower *Leaves* are much deeper dented than the *Fraxinella*, and more reſembles our *Manna* Aſh. It *Flowers* in *July* and *Auguſt*.

Umbelliferous Plants, &c.

26. Arch-Angelica *Ray* 434. 3. *Bob*. H. Oxon. 281. 5. *Archangelica* Cluſ. 114. Ic. *Pan*. 694. *Chabr*. 400. Ic. 6. *Dod*. 318. fig. *Jonſt*. 1000. fig. 3. *Park*. 940. fig. 4. I. B. 3. l. 27. p. 143. fig.

Angelica *Cæſalp*. 307. c. 48.

Angelica ſylv. montana C. B. 156. 5. *phyt*. 273. 4. Moriſſ. *Umb*. 9. pl. 3.

Angelica mont. maxima, floſculis candicantibus, ad caulium nodos umbellifera *Pluk*. Tab. 134. fig. opt. 1. *Alm*. Bot. 30.

Imperatoria *Archangelica* dicta *El. Bot*. 267. *Inſt*. 317. *Grows* on the *Alps* and other Mountains.

27. Round Parſley *Ray* 462. c. 18. 1. H. Ox. B. 293. 13, *Apium* peregrinum foliis ſubrotundis C. B. 153. 9. *Prodr*. 81. fig. *phyt*. 269. 7.

Daucus 3. *Dioſc*. 2. *Plinii* Col. 109, fig.

Selinum montanum *Offic*. Dale *Suppl*. 103. 2.

Selinum peregrinum 1 Cluſ. 199. *c*. 21. Hiſp. 431.

Selinum

Selinum ſive *Apium* peregrinum 1. *Cluſ.* Park. 929. fig.
Saxifraga 3 *Cæſalp.* 315.
Viſnaga minor quorundam, *Selinum* peregrinum *Cluſ.* ſemine hirſuto I. B. 3. l. 27. p 94. fig. *Chabr.* 396. Ic. 2.
Cluſius obſerved this about *Salamanca* in *Spain*, *Columna* in *Italy*, and Mr. *Ray* in the Hedges about *Meſſina* in *Sicily*.

28. Geneva *Laſerwort.* Ray 427. 5. *Bob.* H. Oxon. 321. 6.

Laſerpitium fol. latioribus lobatis *Moriſ.* Umb. 29.
- - - - majus *Almageſt.* Botan. 207.
Libanotis latifolia altera C. B. *Phyt.* 277. 3.
- - - ſive *vulgatior* C B. pin. 157. 2.
Libanotis *Theophraſti* Lob. Ic. 704. 1. *Belg.* 857. Obſ. 402.
Libanotis *Theophr.* major Jonſt. 1010. Ic. 1.
Seſeli *Æthiopicum* Herba Dod. 313 Fig.
This *Grows* plentifully on the Hills about *Geneva*.

29. *Great* black *Maſter-wort.* Ray 475. 1.

Aſtrantia Cluſ. 194. fig. major *Moriſ.* Umbell. 7. & 10.
- maj. coronâ floris purpuraſcente *Inſtit.* Rei Herbar. 314.
Aſtrantia *nigra* Ger. 828. Ic. *Jonſt.* 978. fig. *Lob.* Ic. 681. 2. Belg. 829. *Obſ.* 388.
Aſtrantia nigra major *Bob.* H. Oxon. 279. 1.
Helleborus niger *Saniculæ* folio major C. B. 186. 5. *phyt.* 340. 4.
Imperatoria nigra *Tab.* Hiſt. 300. fig. 1. Sanicula fœm. Ic. 831.
- Ranunculoides *Saniculæ* folio major *Alm.* Botan. 198.
Sanicula fæmina Fuchſii 670. fig.
- - quibuſdam *aliis* Elleborus *niger* I. B 3. l. 34. p. 638. fig.
Veratrum nigrum *Dioſc.* Dod. 38. fig.
I have ſeen the Tops of this mixt with ſome vulnerary Herbs from *Germany*.
It *Grows* on the *Alps* and the *Hills* about *Geneva*.

30. Shrub *Hartwort.* Ray 476. c. 5

Seſeli *Æthiopicum* Offic. *Dale* ſupl. 104. 45. *Lob.* Ic. 634. 1. Ad. 284. *Belg.* 771. fig.

Seſeli

Seſeli *Æthiopicum* frutex *Bob.* H. Ox. 298. c. 27. *Dod.* 312. fig. *Ger.* 1233. Ic. *Jonſt.* 1421. Ic. *Park.* 907. fig. 14.

Seſeli *Æthiopicum* fruticoſum, folio *Periclymeni* Chabr. 406. Ic. 4. I. B. 3. l. 27. p. 197. fig.

Seſeli Æthiopicum *Salicis* folio C. B. 161. 7.

- - Herbariorum C. B. *phyt.* 286. fig.

Bupleurum arboreſcens *Salicis* folio E. B. 260. *Inſt.* 310.

This *Grows* on the Sea-Coaſt at *Marſeilles* and about *Montpelier.*

31. Great *Turnſole* Ray 501.

Heliotropium *Dod* 70. fig.

Heliotropium majus *Dioſc.* C. B. 253. 1. phyt. 487. 1.

Heliotropium majus G. 264. Ic. 1. *Jonſt.* 334. Ic. 1. *Park.* 438. f. 1.

Heliotropium majus fl. albo I. B. 3. l. 33. p. 60. fig.

Heliotropium, Herba Cancri *Chabr.* 521. Ic. 1.

Heliotropium majus & *Herba Cancri* Lob. Ic. 260. 2. *Belg.* 313. *Obſ.* 132. Ic.

Grows wild in many Places of *France, Italy, Germany,* &c.

32. Galen's *Horehound* Ray. 557. 9.

Alyſſum Galeni *Cluſ.* 35. fig. *Hiſp.* 387. *Dod.* 88. *Park.* 590. f. 4.

Alyſſum *Galeni* Ger. 379. fig. *Jonſt.* 465. Ic. 1.

Alyſſum Galeni *Cluſii* & Herbariorum *Lob.* Ic. 524. 1. Belg. 620. *Obſ.* 283.

Alyſſum verticillatum, foliis profundè inciſis C. B. 232.

Marrubium album, fol. profundius inciſis, fl. cæruleo *Bob.* Ox 377. 12. Sect. xi. Tab. x. fig.

Marrubium *Hiſpan.* ſupinum, *calyce* ſtellato & aculeato E. B. 161. *Inſt.* 192.

Dr. *Salvadore* hath ſent me this from *Barcelona:* It grows alſo about *Madrid* and other parts of *Spain.*

33. Galen's *Horehound* with more deep cut Leaves.

Alyſſum *Galeni* foliis altiùs inciſis *Nobis.*

Like the *Common,* but the Leaves much deeper cut, and ſtand

ſtand on longer footſtalks. Both theſe I have obſerved in *Chelſea* Garden.

34. *Spaniſh* Silver *Horehound*.

Marrubium *Hiſp.* ſupinum, fol. ſericeis argenteis E. B. 161. *Inſt.* 192.

Marrubium album *Hiſpan.* majus *Barrel.* 263. Ic. 686.

This was raiſed in *Chelſea Garden* from Seed which Monſieur *Ricqueur* ſent me from *Madrid*, and the Plant is very well expreſt in *Barrelier*'s Icons.

35. Anguillara's *Horehound* Ray 3. p. 303 11. & 304. 8.

Pſeudo-Dictamnus Hiſp *Scrophulariæ* folio. E.B 157. *Inſt.* 188.

Galeopſis *Anguillaræ* 278. ſive *Pſeudo-Dictamnum* nigrum *Siculum* Boc. *Muſ.* 151. Tab. 114.

Dr. *Laurence Heiſter* Profeſſor of Anatomy at *Altorf* ſent me formerly a Specimen of this, amongſt divers curious Plants he had gathered in the Phyſick-Gardens at *Amſterdam* and *Leyden*.

Dr. *Herman*'s *Figure* very accurately agrees with this Plant.

Marrubium album rotundifolium *Hiſpanicum* maximum *Schol. Bot.* 60. Parad. *Batav.* 201. fig. *opt.*

36. *Herman*'s Cupt Hore-hound Ray 3. p. 303. 10.

Marrubium *Dictamni ſpurii* foliis & facie *Parad. Bat.* 200. fig.

Pſeudo-Dictamnus *Hiſp.* folio rugoſiore *Schol.* Bot. 61. *Bob.* Oxon. 380. 4.

Pſeudodictamnus *Hiſp.* fol. criſpis & rugoſis E. B. 157. *Inſt.* 188.

Pſeudo-Dictamnus nigro rotundo criſpo folio *Bocc.* Muſ. 152. Tab. 1.

This chiefly differs from the Common in having thinner and larger Flower-cups; its *Leaves* more pointed and ſomewhat dented.

37. Common *Cupt Horehound Ray* 557. XI.

Pſeudo-Dictamnus Park. 28. fig. 2.

Pſeudo-Dictamnus verticillatus inodorus C. B. 222. 2. *phyt.* 424 2.

Pseudodictamnus fol. non crenatis, verticillatus inodorus *Bob.* H. Ox. 379. 1.

Pseudodictamnum *Cam.* Epit. 474 fig. opt *Dod.* 281. fig. *Ger.* 651. f. 2. *Jonst.* 795. f. 1.

Pseudodictamnum floribus verticillatis *Lob.* 502. Ic. 2. *Belg.* 592. *Obs.* 267. fig.

This is known from the last, in having smaller *Cupps*, plain and rounder *Leaves* on very woolly *Stalks*.

38. Broad *Phlome*, Yellow *or* French *Sage*, Ray 511. 13.

Phlomis fruticosa, *Salviæ* folio latiore & rotundiore *Instit.* 177.

Salvia frut. lutea, latifolia, *sive* Verbascum *sylv. &c.* Park. 52. fig xi.

Verbascum latis Salviæ foliis C. B. 240. 1. phyt. 455. 1.

Verbascum sylv. *Matth.* Clus. 28. fig. 1.

Verbascum 4 Matth. *Lob.* Ic. 56 B. 661. Obs. 302.

The *French* call this Plant, *Sauge Sauvage* or *Wild Sage*.

It *Grows* plentifully on *Sierra morena* or the *black mountain* supposed the *Mons* Marianus *of the* Antients, situate between *Portugal* and *Andalusia*, where the Natives call this Plant *Matulera*. My worthy Friend Mr. *Charles du Bois* tells me the Country People about *Mitcham* use this as a certain Remedy in the *Quinsey*.

39. Narrow *Phlome*.

Phlomis fruticosa, *Salviæ* folio longiore & angustiore *Instit.* 177.

The *Leaves* of this are very like *Common Sage*, but paler above and whiter underneath, and much lesser than the last and narrower. That accurate Botanist Dr. *Salvadore* hath sent me this from *Barcelona*.

40. Aleppo *Phlome*.

Pseudo-Salvia *Chalepensis* ampliore folio cordiformi *Bobart.* H. Ox. 397. 2. *Sect.* xi. Tab. 16. fig.

These

Theſe *Leaves* differ from the *Broad Phlome* in being thicker, more rugged and cordated at the Footſtalk : I am obliged to Mr. *Jacob Bobart* for the firſt Knowledge of this Plant, which I have ſince obſerved with Mr. *Thomas Fairchild* at *Hoxton*.

41. Samos *Phlome*.

Phlomis Samia *Lunariæ* folio, Boer. p. 62.
Phlomis Samia *Herbacea*, folio *Lunariæ* T. Coral. 10.

The *Flowers* pale, buff or whitiſh, the inſide or lower Lip pounct or ſhaded with brown, the Bottoms of each Calyx are guarded with two or three long ſlender Thorns: its Root or lower Leaves, in Shape, reſemble *Garden Honeſty*, but are ſtiffer, and underneath ſoft and whitiſh.

I have as yet obſerved this Plant only in *Chelſea Garden* where it *Floured* in *July*.

42. True *Old* Time *Ray* 519. 3. c. 7. *Lecaan* 43. p. 80.
Thymus *Capitatus* qui *Dioſcoridis* C.B. 219. 3. *phyt.* 414. 3.
Thymum legitimum *Cluſ.* 357. fig. opt.
Thymum legitimum capitatum *Park.* 7. fig. 1.
Thymum *Creticum* Jonſt. 574. fig. 3. opt.
Thymum *Cret.* ſ. Antiquorum I. B. 3. l. 28. p. 262.

This fragrant *Time* of the *Antients* I firſt received from *Coos*; it grows alſo about *Sevill* and *Cales*.

43. Broad *Candy* Savory, *Ray* 519. 4.
Satureia *Cretica* C. B. 218. 4. phyt. 413. 4. *Jonſt.* 576. f. 4.
Satureia *Cretica* Jonſt. 576. f. 4. latiore folio *Bob.* H. Ox. 412. 6.
Thymbra legitima *Cluſ.* 358. fig. 1. opt.
Thymbra legitima *Dioſcoridis* Ponæ 104.
Thymbra ſ. Satureia *Cretica* legitima *Park.* 5. fig. 4.
Thymum Creticum *Ponæ* verticillatum *Barrel.* 278. Ic. 898.
Tragoriganum *Cluſii* Ger. 543. fig.

It's diſtinguiſh'd by its broad *Time* leaves and cloſe Whorles.

44. Black rough *Goat Succory* Ray 523. 3. *Lecaan.* 37. p. 77.
Tragoriganum *P. Alp.* 78. fig. c 36. Dod.
Tragoriganum *Creticum* C. B. 223. 4. *Park.* 17. fig. 1.
Tragoriganum *Cretense Jonst.* 668. Ic. 3.
Tragoriganum 2 altera ſpecies *Cluſ.* 355. fig. 3.

Alpinus and *Cluſius* his *Figures*, which are both *Originals*, very well agree with this Plant, which *Jacob Bobart* not long ſince ſent me a Sample of, and has much narrower and ſmaller Leaves than the *broad* Candy *Savory*.

45. Narrow-leaved *Goat-Savory* Ray 523. 1. *Lecaan* 36. p. 76.

Tragoriganum Ger. 543. fig. 1.
Tragoriganum *Cluſ.* Jonſt. 668. fig. 2.
Tragoriganum alterum *Cluſ.* 355. fig. 2. *Hiſp.* 240. fig. Dod. 286.
Tragoriganum anguſtifolium C. B. 223. 3.
- - 2. C. B. phyt. 422. x, fl. albo *Cluſ.* Lob. *Obſ.* 264. fig. Ic. 494. 1.
Tragoriganum *Hiſpanicum* Park. 17. f. 3.
Tragoriganum tenuioribus foliis fl. candido Chabr. 421. Ic. 4. I. B. 3. l. 28. p. 261. fig.
Sideritis *Hiſpanica* erecta fol. anguſtiore E. B. 160. *Inſt.* 191.

Monſ. *Ricqueur* ſent me the Seed of this elegant Plant from *Madrid*, which *Flowred* with us in *Chelſea* Garden.

46. Sage *Iron-wort*, Ray 566. 17.

Sideritis marina *Salvifolia* noſtra *Donati* 84.
Sideritis Heraclea *Dioſcoridis*, ſive marina *Salvifolia* noſtra *Donato* Park. 1681. fig. 16.
Betonica maritima, flore ex luteo palleſcente *Inſtit.* 203.

Dr. *Magnol* found this on the Stoney Sea Shores in *Languedoc*: and Dr. *Salvadore* hath ſent it me from *Barcelona*: It much reſembles the *Sideritis* glabra arvenſis *Chabr.* 473. Ic. 1. but has yellowiſh *Flowers* and ſofter *Leaves*. It *flowers* with us in *June* and *July*.

III.

N. B. The *Reſt* will be incerted in the next *Tranſactions*.

III. *Obſervations of the late* Total Eclipſe *of the* Sun *on the* 22d *of* April *laſt paſt, made before the* Royal Society *at their Houſe in* Crane-Court *in* Fleet-ſtreet, London. *By Dr.* Edmund Halley, *Reg. Soc. Secr. With an Account of what has been communicated from abroad concerning the ſame.*

THough it be certain from the Principles of Aſtronomy, that there happens neceſſarily a Central Eclipſe of the Sun in ſome part or other of the Terraqueous Globe, about Twenty Eight times in each Period of Eighteen Years; and that of theſe no leſs than Eight do paſs over the Parallel of *London*, Three of which Eight are Total with continuance: yet, from the great Variety of the Elements whereof the *Calculus* of Eclipſes conſiſts, it has ſo happened that ſince the 20*th* of *March*, *Anno Chriſti* 1140, I cannot find that there has been ſuch a thing as a Total Eclipſe of the Sun ſeen at *London*, though in the mean time the Shade of the Moon has often paſt over other Parts of *Great Britain*.

The Novelty of the thing being likely to excite a general Curioſity, and having found, by comparing what had been formerly obſerved of Solar Eclipſes, that the whole Shadow would fall upon *England*, I thought it a very proper Opportunity to get the Dimenſions of the Shade aſcertained by Obſervation; and accordingly I cauſed a ſmall Map of *England*, deſcribing the Track and Bounds thereof, to be diſperſed all over the Kingdom, with a Requeſt to the Curious to obſerve what they could:

could about it, but more eſpecially to note the Time of Continuance of total Darkneſs, as requiring no other Inſtrument than a *Pendulum Clock* with which moſt Perſons are furniſh'd, and as being determinable with the utmoſt Exactneſs, by reaſon of the momentaneous Occultation and Emerſion of the luminous Edge of the Sun, whoſe leaſt part makes Day. Nor has this Advertiſement failed of the deſired Effect, for the Heavens having proved generally favourable, we have received from ſo many Places ſo good Accounts, that they fully anſwer all our Expectations, and are ſufficient to eſtabliſh ſeveral of the Elements of the *Calculus* of Eclipſes, ſo as for the future we may more ſecurely rely on our Predictions; though it muſt be granted, that in this our Aſtronomy has loſt no Credit.

The Day of the Eclipſe approaching, I received the Orders of the Society to provide for the Obſervation to be made at their Houſe in *Crane-Court*, and accordingly I procured a *Quadrant* of near 30 Inches *Radius*, exceedingly well fixt with Teleſcope Sights, and moved with Screws ſo as to follow the Sun with great Nicety; as alſo a very good *Pendulum Clock* well adjuſted to the mean Time, and ſeveral Teleſcopes to accomodate the more Obſervers.

In order to examine both Clock and Quadrant, I, on the 20*th* of *April*, obſerved the Diſtance of the upper Limb of the Sun from the Zenith 36°. 16′, and the next Day 35°. 58′; by which it appeared that the Diſtances from the *Zenith* taken by this Quadrant ought to be encreaſed by about one Minute: and that Allowance being made, by ſeveral Obſervations taken before and after Noon on the ſaid 21*ſt* Day, the Clock was found to anſwer the apparent Time or Hour of the Sun with ſufficient Exactneſs, as not going above 10″ too faſt. The next Day *April* 22°, juſt before the Eclipſe began, we took three Diſtances of the Sun from the Zenith, *viz.* at 7^{h}. 42′. 52″. A. M. the

correct

correct Diſtance of the Sun's Center *à vertice* was 62°. 1′. 40″. At 7h. 45′. 48″. it was 61°. 34′. 40″. And again at 7h. 48′. 55″ it was 61°. 6′. 40″: which with the given Declination of the Sun and Latitude of the Place ſhew the true Times reſpectively to have been 7h. 42′. 38″, 7h. 45′. 35″. and 7h. 48′ 39″: all concurring that the Clock was only 14 Seconds too faſt, and had gained ſcarce any thing ſenſible in a Day's time: ſo that it might be entirely depended upon during the Continuance of the Eclipſe.

Having computed that the Eclipſe would begin at 8h. 7′, I attended ſoon after Eight with a very good Teleſcope of about Six Foot, without ſtirring my Eye from that part of the Sun whereat the Eclipſe was to begin: and at 8h. 6′. 20″. by the Clock, I began to perceive a ſmall Depreſſion made in the Sun's Weſtern Limb, which immediately became more conſpicuous; ſo that I concluded the juſt Beginning not to have been above five Seconds ſooner; that is, exactly at 8h. 6′ 00″ correct Time.

From this time the Eclipſe advanced, and by Nine of the Clock was about Ten Digits, when the Face and Colour of the Sky began to change from perfect ſerene azure blew, to a more dusky livid Colour having an eye of Purple intermixt, and grew darker and darker till the total Immerſion of the Sun, which hapened at 9h. 9′. 17″. by the Clock, or 9h. 9′. 3″. true time. This Moment was determinable with great nicety, the Sun's light being extinguiſh'd at once; and yet more ſo was that of the Emerſion, for the Sun came out in an Inſtant with ſo much Luſtre that it ſurprized the Beholders, and in a Moment reſtored the Day, *viz.* at 9h. 12′. 26″. true time, after he had been totally obſcured for 3′, 23″ of Time. And as near as I could eſtimate the Points on the Moon's Limb; where the laſt Particle of the Sun vaniſhed was about the middle of the *South Eaſt* Quadrant of her Limb, or about 45 Degrees from her *Nadir* to the Left-Hand: And the firſt

Emerſi-

Emerſion was about Ten Degrees below the Horizontal Line through the Moon's Center on the Weſt ſide; and at 14 Minutes paſt Nine, correct Time, I judged the Horns of the Eclipſe to have been exactly perpendicular, and by conſequence, the Centers of the Sun and Moon to be in equal Altitude.

It was univerſally remarked, that when the laſt part of the Sun remained on his Eaſt ſide, it grew very faint, and was eaſily ſupportable to the naked Eye, even through the Teleſcope, for above a Minute of Time before the total Darkneſs; whereas on the contrary, my Eye could not endure the Splendour of the emerging Beams in the Teleſcope from the firſt Moment. To this perhaps two Cauſes concurred; the one, that the Pupil of the Eye did neceſſarily dilate it ſelf during the Darkneſs, which before had been much contracted by looking on the Sun. The other, that the Eaſtern parts of the Moon, having been heated with a Day near as long as Thirty of ours, could not fail of having that part of its Atmoſphere replete with Vapours, raiſed by the ſo long continued action of the Sun; and by conſequence it was more denſe near the Moons Surface, and more capable of obſtructing the Luſtre of the Sun's Beams. Whereas at the ſame time the Weſtern Edge of the Moon had ſuffered as long a Night, during which there might fall in Dews all the Vapours that were raiſed in the preceeding long Day; and for that reaſon, that part of its Atmoſphere might be ſeen much more pure and tranſparent. But from whatever cauſe it proceeded, the thing it ſelf was very manifeſt and noted by every one.

About two Minutes before the Total Immerſion, the remaining part of the Sun was reduced to a very fine Horn, whoſe Extremities ſeemed to loſe their Acuteneſs, and to become round like Stars. And for the Space of about a Quarter of a Minute, a ſmall Piece of the Southern Horn

of

of the Eclipſe ſeemed to be cut off from the reſt by a good interval, and appeared like an oblong Star rounded at both Ends, in this Form ⬭ : which Appearance could proceed from no other Cauſe but the Inequalities of the Moon's Surface, there being ſome elevated parts thereof near the Moon's Southern Pole, by whoſe Interpoſition part of that exceedingly fine Filament of Light was intercepted.

A few Seconds before the Sun was all hid, there diſcovered it ſelf round the Moon a luminous Ring, about a Digit or perhaps a tenth Part of the Moons Diameter in Breadth. It was of a pale whiteneſs or rather Pearl colour. ſeeming to me a little tinged with the Colours of the *Iris*, and to be concentrick with the Moon, whence I concluded it the Moon's Atmoſphere. But the great height thereof far exceeding that of our Earth's Atmoſphere; and the Obſervations of ſome, who found the Breadth of the Ring to encreaſe on the Weſt Side of the Moon, as the Emerſion approached; together with the contrary Sentiments of thoſe whoſe Judgment I ſhall always revere, makes me leſs confident, eſpecially in a Matter whereto, I muſt confeſs, I gave not all the Attention requiſite.

Whatever it was, this Ring appeared much brighter and whiter near the Body of the Moon than at a Diſtance from it; and its outward Circumference, which was ill defined, ſeemed terminated only by the extream Rarity of the Matter it was compoſed of; and in all Reſpects reſembled the Appearance of an enlightned Atmoſphere viewed from far: but whether it belonged to the Sun or Moon I ſhall not at preſent undertake to decide.

During the whole time of the Total Eclipſe I kept my Teleſcope conſtantly fixt on the Moon, in order to obſerve what might occur in this uncommon Appearance: and I found that there were perpetual Flaſhes or Coruſcations of Light, which ſeemed for a Moment to dart out

 from

from behind the Moon, now here, now there, on all Sides; but more eſpecially on the Weſtern Side a little before the Emerſion: And about two or three Seconds before it; on the ſame Weſtern Side where the Sun was juſt coming out, a long and very narrow Streak of a dusky but ſtrong Red Light ſeemed to colour the dark Edge of the Moon; tho' nothing like it had been ſeen immediately after the Immerſion. But this inſtantly vaniſhed upon the firſt Appearance of the Sun, as did alſo the aforeſaid luminous Ring.

As to the Degree of Darkneſs, it was ſuch that one might have expected to have ſeen many more Stars than I find were ſeen at *London*: The three Planets, *Jupiter*, *Mercury* and *Venus* were all that were ſeen by the Gentlemen of the Society from the Top of their Houſe, where they had a free Horizon: and I do not hear that any one in Town ſaw more than *Capella* and *Aldebaran* of the Fixed Stars. Nor was the Light of the Ring round the Moon capable of effacing the Luſtre of the Stars, for it was vaſtly inferiour to that of the full Moon, and ſo weak that I did not obſerve that it caſt a Shade. But the under Parts of the Hemiſphere, eſpecially in the *South Eaſt*, under the Sun, had a crepuſcular brightneſs: and all round us, ſo much of the Segment of our Atmoſphere as was above the Horizon and was without the Cone of the Moon's Shadow, was more or leſs enlightened by the Sun's Beams: and its Reflection gave a diffuſed Light which made the Air ſeem hazey, and hindred the Appearance of the Stars. And that this was the real Cauſe thereof, appears by the Darkneſs being more perfect in thoſe Places near which the Center of the Shade paſt, where many more Stars were ſeen, and in ſome not leſs than Twenty; though the Light of the Ring was to all alike.

During the Time whilſt the Sun recovered his Light, ſeveral Altitudes were taken to examine the Regularity of the

the Clock's Motion; and though the Sun now rose much slower than at the beginning, yet they all conspired within a very few Seconds that the Clock went still one Quarter of a Minute too fast. And the End of the Eclipse approaching, I attended the Moment thereof with all the Care I could, and concluded the compleat Separation of the Sun and Moon at $10^h. 20'. 15''$. by the Clock, or exactly $10^h. 20'$. correct time.

Hitherto I exhibit only what my self saw, but there were with us a great many of the Members of the Society; and the Right Honourable the *Earl* of *Abingdon* and the Lord Chief Justice *Parker* were of the Number: the latter of which shewed an uncommon Curiosity and Desire of Exactness, his Lordship doing us the Honour to assist at most of the Observations made for determining the Error of the Clock; and did himself, at the Moment of the Emersion from total Darkness, observe the Distance of the Planet *Jupiter* from the Zenith 48°. 29'. by which the Time thereof is verified.

There were also present several foreign Gentlemen, and among them Monsieur *le Chevalier de Louville* and Mr. *Monmort*, both of them Members of the *Royal Academy* of *Sciences* at *Paris*: the first whereof came purposely to observe this Eclipse with us, and having seen the Beginning applyed himself to take Digits with his Micrometer, and to observe the Occultations of three Spots at that time seen in the Sun; and he was pleased to communicate the following Notes, *viz.*

	h.	′	″	
At	8	28	20	Four Digits were Eclipsed.
	8	32	57	The First and bigger Spot touched the Moon.
	8	33	18	The same was wholly hid.
	8	34	08	The first of the two lesser Spots was hid.
	8	34	58	The Second of them was hid.

	h.	′	″	
At	9.	36.	01	Emerſion of the greater Spot.
	9.	38.	26	Emerſion of the firſt leſſer Spot.
	9.	40.	25	Emerſion of the ſecond leſſer Spot.
	10.	20.	04	The End of the Eclipſe.

And he determined the time of the total Datkneſs 3′.22″, or one Second leſs than by my Account.

The Heavens were all the while very propitious to us, and there was very little or no Wind, and not ſo much as one Cloud interrupted our View from the Beginning to the End; but no ſooner was the Eclipſe over, but a great Body of Clouds hid the Sun for many Hours after.

Theſe Obſervations having been made with all the Care we could, are not, 'tis hoped, far from the Truth.

What we have received from other Places is as follows,

The Reverend Mr. *James Pound* Rector of *Wanſted* in *Eſſex* and R. S. S. gives the following Account of the principal Phænomena obſerved there; he being furniſh'd with very curious Inſtruments, and well skill'd in the Matter of Obſervation, and having rectified his Clock by ſeveral Altitudes of the Sun taken both before and after, *viz.*

	h	′	″	
At	8.	6.	37	The Eclipſe firſt perceived.
	9.	9.	28	The Total Immerſion.
	9.	12.	48	The Emerſion.
	10.	20.	32	The juſt End of the Eclipſe.
	c.	3.	20	The Continuance of total Darkneſs.

The near Agreement of this Obſervation with our own (the Difference being only what is due to the Difference of our Meridians) makes us the leſs ſolicitous for what was noted at the *Royal Obſervatory* at *Greenwich*, from whence we can only learn that the Duration of Total Darkneſs was 3′, 11″.

The

The Reverend Mr. *William Derham* Rector of *Upminster* in *Essex* and *Reg. Soc. Sod.* assisted by *Samuel Molyneux* Esq; Secretary to his *Royal Highness* the *Prince*, and other Persons of Quality, made the following Observations there, which he has lately communicated, *viz*,

	h.	′	″	
At	8.	7.	41	The Eclipse began.
	8.	33.	46	The Moon touched the greater Spot.
	8.	34.	36	She touched the middle Spot.
	8.	35.	41	She touched the third Spot.
	9.	10.	58	The total Darkness began on a sudden, and *Aldebaran* appeared.
	9.	14.	6	The Emersion or End of total Darkness.
	0.	3.	8	Continuance of total Darkness.
	9.	42.	41	The third and last Spot discovered.
	10.	21.	45	The End of the Eclipse, by a 13½ Foot Glass.

And a little before the Beginning of the Eclipse, he found the greater and preceeding Spot to be more Northerly than the Sun's Center 373½ such Parts as the Sun's Diameter was 1647, and that it followed his Western Limb 0′. 43″ of Time: by which *data* the Situation of that Spot is well determined.

Our Professors of Astronomy in both Universities were not so fortunate: My worthy Collegue Dr. *John Keill* by reason of Clouds saw nothing distinctly at *Oxford* but the End, which he observed at 10h. 15′. 10″. As to the total Darkness, he could only estimate it by the sudden Change of the Light of the Sky; and reckoned its Continuance but 3′. 30″; which was certainly too little, the Center of the Shadow having without doubt past very near *Oxford*. And the Reverend Mr. *Roger Cotes* at *Cambridge* had the misfortune to be oppreſt by too much Company, so that, though the Heavens were very favourable, yet he miss'd both the time of the Beginning of the Eclipse and that of total

total Darkneſs. But he obſerved the Occultations of the three Spots, *viz*, of the firſt and greateſt at $8^h. 34'. 11''$. of the ſecond at $8^h. 35'. 15''$, and of the laſt at $8^h. 36'. 55''$. He noted alſo the End of total Darkneſs at $9^h. 14'. 37''$. and the exact End of the Eclipſe at $10^h. 21'. 57''$.

We have received ſeveral Accounts from ſome Places which lay near the Track of the Center of the Shade, and which might have been very proper to determine the greateſt Continuance of the Darkneſs; as from *Plymouth*, *Exeter*, *Weymouth*, *Daventry*, *Northampton* and *Lynn regis*, all agreeing that the whole Sun was obſcured at thoſe Places full four Minutes, and at ſome of them rather more. But theſe Obſervers give us no Account how they meaſured this Time, and therefore it may well be ſuppoſed they took it in a round Number, and perhaps from pocket Minute-Watches. What I think may beſt be relied on for this Purpoſe, are two correſponding Obſervations made, the one at *Barton* near *Kettering* in *Northamptonſhire*, where by the Obſervation of *John Bridges* Eſq; Treaſurer of his Majeſty's Revenue of Exciſe, and R. S. S. with a good Pendulum-Clock and all due Care, the whole Sun was hid no more than $3'. 53''$. The other was by Mr. *John Whiteſide*, A. M. Keeper of the *Aſhmolean Muſeum* at *Oxford*, and a skilful Mathematician, who obſerved after the ſame manner, at *King's-Walden* in *Hertfordſhire* near *Hitchin*, that the total Eclipſe continued but $3'. 52''$. Hence it follows that the Center of the Shade paſt near the middle between theſe two Places, which are but 30 Geographical Miles aſunder, and ſituate near at right Angles to the Way of the Shade, and therefore that the total Obſcurity, where longeſt, could laſt but about $3'. 57''$, or perhaps a ſecond or two more at *Lynn* and leſs at *Plymouth*: the Velocity of the Progreſs of the Shade gradually decreaſing, and its Diameter encreaſing as it paſt on to the Eaſtwards. And this Situation of the middle Line is confirmed by an Obſerva-

tion

tion made at the Seat of the Right Honourable the Lord *Foley* at *Witley* eight Miles beyond *Worcester*, by his Order, and communicated by his Lordship to the Royal Society; whereby it appears that the total Darkness lasted there 3'. 15". Hence it follows that *Witley* was about three or 4 Miles farther from the Center of the Shade on the North-side than *London* on the South; and *Witley* being by *Ogilby*'s Mensurations, 118 measured Miles from *London*, it is plain that the Center past over *Islip*, which is, by the same Admeasurement, 57 such Miles on that Road, and about five Miles almost due North from *Oxford*; so that the Center of the Shade left *Oxford* but very little upon the right Hand. This Situation agrees perfectly well with the former between *Barton* and *King*'s *Walden*, and as far as the Geography of our Country may be relied on, I conclude the Center to have entred upon *England* about *Plymouth*, and to have past over *Exeter*, the *Devizes*, *Islip*, *Buckingham* and *Huntington*, leaving *Oxford* and *Bedford* on the Right, and *Lynn* on the Left, and to have quitted the Coast of *Norfolk* about *Wells* and *Blakeney*.

As for the Limits of the Shade, both on the North and South side, we have by Enquiry gotten them with all the Exactness the thing is capable of, and we should have been glad the *French Astronomers* had done the like for the Total Eclipse that past over *Languedoc*, *Provence* and *Dauphiny* on the First of *May* 1706. But as this is the first Eclipse of this kind that has been observed with the Attention the Dignity of the Phænomenon requires, we hope those which may happen for the future to traverse *Europe*, may not pass by so little regarded as hitherto.

As to the Southern Limit or Term where the Eclipse ceased to be Total on the South side of the Sun, we have received an Account of an Observation made at *Norton-court* about Ten Miles on this side *Canterbury*, by the Reverend Dr. *John Harris*, S. T. P. Prebendary of *Rochester*

and

and R S. S. assisted by that accurate Observer Mr. *Stephen Gray*; by which we learn that the Eclipse began there at 8h. 8′. 55″. and ended at 10h. 24′. 47″; and that the Total Darkness continued but about one Minute or rather less, the middle thereof being at 9h. 13′. 52″. From this Duration it will follow that *Norton-court* was but about 3 or 4 Miles within the Shade. And that it was really so is confirmed by the Relation of the Inhabitants of *Boctton*, about Midway between *Norton-court* and *Canterbury*, who assured Mr. *Gray*, as he was returning home that same Day, that the Eclipse was not Total there, but, as one of them exprest it, before the Sun had quite lost his Light on the East-side he recovered it on the West: and that there was a small Light left on the lower part of the Sun that appeared like a Star. And from *Cranbrook* in *Kent*, we are informed, by the Relation of the curious *William Tempest* Esq; R. S. S. that he observed there the Sun to be extinguished but for a Moment, and instantly to emerge again: So that the Limit past exactly over this Town, which is about 38 Geographical Miles from *London*, and very near the right Angle where the Perpendicular from *London* falls on the Line of the Limit, being 3′. 00 of Time to the Eastwards of *London* in the Latitude of 51°. 6′, as near as I can gather.

How it past over *Sussex* we have not so authentick Relations, but have learnt that it was Total at *Wadhurst* beyond *Tunbridge-wells*, as also for some short time at *Lewis*; but that it was not so at *Brightling*, which Place being situated on an Eminence that has a commanding Prospect, all the Country to the Northward was seen in Darkness, whilst they there had some Benefit of a small Remainder of the Sun.

From these Observations we may conclude that this Limit came upon the Coast of *England*, about the middle between *Newhaven* and *Brighthelmston* in *Sussex*, and passing by

by *Cranbrook* and *Boston*, left *Canterbury* about 4 Miles on the Right hand, and quitted the Coast of *Kent*, not far from *Hern* toward the antient *Regulbium*, now called *Reculver*. So that it seems scarce one third part of *Kent*, and not so much of *Sussex*, out of all the South Coast of *Great-Britain*, escaped being involved in this Darkness.

The Northern Limit, having past over a much greater Space, has had more Observers, and is not less curiously determined than the other. We find by the Account given by the Reverend Mr. *Roger Prosser*, Rector of *Haverford-West*, that the Eclipse was total there a Minute and half, whence it follows that *Haverford* was but about 6 Miles within the Limit; and therefore that it entred on *Pembrokeshire* about the middle of St. *Brides* Bay, leaving St. *David*'s and *Cardigan* on the left Hand: and having traversed those two Counties and *Montgomery-shire*, it entred on *Shropshire*, leaving the Town of *Shrewsbury* 1′. 40″. in the Shadow, as was observed there by Dr. *Hollings*: whereby it appears that *Shrewsbury* was about 8 Miles within the Limit. Thence it proceeded by the East-side of *Cheshire*, leaving *Whitchurch* and *Nantwich* a very little without; and passing by *Congleton* went over the *Peak* of *Darbyshire* into *Yorkshire*, and croft the great Northern Road between *Pontefract* and *Doncaster*, somewhat nearer the former than the latter. For by the Observations of that curious Gentleman *Theophilus Shelton* Esq; at *Darrington* about two Miles on this side *Pontefract*, (in Lat. 53°. 40′ and Long. West from *London* 4′. 40″. of time, as may be concluded from *Norwood*'s Measure of a Degree) the Sun at 9^h. 11′. was reduced almost to a Point, which both in Colour and Size resembled the Planet *Mars*; but whilst he watched for the Total Eclipse, that Point grew bigger and the Darkness diminished; whence he argued the Limit to have been very little more Southerly. And since he has been informed that it was just Total in *Barnsdale*, three Miles South from thence.

 And

And that it was ſo at *Badſworth* about the ſame Diſtance from *Darrington*, we are told by a Letter of the Reverend and Learned Mr. *Daubuz*, that he has a certain Account from that Place, that the luminous Ring round the Moon was ſeen there, which was no where viſible but while the Eclipſe was Total. From theſe *Data* we may ſecurely determine the Remainder of this Track, and that the Edge of the Shadow having paſt over the reſt of *Yorkſhire* went off to Sea about *Flamborough* head.

So that of the forty Counties into which *England* is ſubdivided, only the five moſt Northerly have not had the Sun wholly hid from them; and ſix others have eſcaped but in part, *viz.* *Shropſhire*, *Cheſhire* and *Yorkſhire*, and the extream part of *Darbyſhire* on the *North*, and *Kent* and *Suſſex* on the *South*; all the reſt of the Kingdom having more or leſs ſuffered an Interval of Total Darkneſs.

I ſhall not at preſent conſider this Eclipſe as univerſal, but only as it related to *England*; and it ſhall ſuffice to ſay, that the Shadow came out of the *Atlantick* Ocean, having paſt over the Iſlands *Azores*; and that the Southern Limit thereof reach'd the Iſle of *Uſhant*, and the Northweſt Coaſts of *Britanny* between *Breſt* and *Morlaix*? and dividing our Iſlands of *Guernſey* and *Jerſey*, juſt touched upon the Promontory of *Normandy* called *Cape de Hague*. And that after it had quitted *England* and traverſed the *German Ocean*, it fell on *Jutland* on the Southſide, and *Norway* on the *North*; and thence proceeded to the Eaſtwards over *Sweden*, *Finland*, &c.

It remains now to conſider the Figure, Poſition, Direction, Velocity and Magnitude of the Shadow at is paſt over us. And firſt as to the Figure, 'tis obvious that the Shadow of the Moon being a Cone and the Earth's Surface ſufficiently Spherical, the apparent Shadow on the Earth will be the common Interſection of a Cone and Sphere, which is a Figure hitherto little conſidered by Geometers;

and

and not being *in Plano* is not to be exactly described but in the Spherical or Conical Surface. How to find the Points of this Curve in all Cases is taught by *P. Coursier*, in a very scarce Latin Book printed at *Dijon* in *Burgundy*, and published at *Paris* in the Year 1663: nor do I hear of any other Author that has handled the same Subject since, though capable and worthy of further Improvement. By what he there delivers, *Prop.* 11. 12. Lib. I. it will be easily understood, that the Convexity of so small a part of the Earths Surface as the Shadow commonly occupies, can produce but an inconsiderable Effect; so that without sensible Errour we may take it for a Plain, and the Section for a true *Apollonian Ellipsis*, whose transverse *Axis*, by reason of the smallness of the Angle of the Cone, will be to its Conjugate nearly as *Radius* to the *Sine* of the Sun's Altitude at its Center, especially if he be considerably elevated. But when he is near the Horizon, it will be necessary to have regard to the true Figure, by reason of the great Length to which the Transverse Axe is extended, and particularly when the Shade is entring upon or leaving the Earth's Disk. Of these perhaps a fuller Account may be given upon a further occasion.

As to the Position of the *Axis* of the Shadow, it is manifest that it must always lie in the Plane of a great Circle of the Earth passing through the *Axis* of the Cone of the Shade: and therefore it will be only requisite to obtain the Azimuth and Altitude of the Sun at the Place where the Center of the Shade at any time is found, to determine the Situation of the Axe and *Species* of the Ellipse required. Thus the middle of the Eclipse at *London* having been observed at $9^h. 10'. 45''$, by the given Latitude and Declination we find his Azimuth about $59°. 00'$. and Altitude $40°. 46'$. that is just 40 Degrees high at the Center of the Shadow. Wherefore the Transverse Axe of the Ellipse was to its Conjugate very near as *Rad.* to the *Sine* of 40°,

or as 1000 to 643 *proximè*; and did make an Angle of 59°, or very little more, with the Meridian passing at that time through the Center of the Shade.

Next the Direction and the Velocity of the Motion wherewith the Center of the Shade past over *England* comes to be considered, wherein the Reader is to be told that the Shadow passes in a very compound Curve, which, as the former, is not *in plano*, and only describable on the Surface of the Sphere: nor is its Motion equable, but compounded of very many Elements producing a great Variety. By what Method its Points, and its Tangents in those Points, are to be obtained, I reserve to the next Opportunity; this Account being designed for the Curious in general: only I must acquaint them, that for so small a part of the Curve as went over *England*, it may be esteemed a right Line, with more Exactness than we usually find in most of our Geographical Charts. And the like may be said for the Velocity, which, though in our present Instance it was continually decreasing, may, for so short a time, be supposed to have been the same without sensible Errour.

By a careful Calculation I have determined the Velocity of the Motion, at the Time of the Middle of the Eclipse at *London*, to have been 29 Geographical Miles in a Minute of Time *quam proximè*: and that its Way made an Angle of 52°. 45′ with the Meridian towards the Eastwards of the North; wherefore the said Way made an Angle with the Axis of the Ellipsis of 68°. 15′. And the greatest Duration of Total Darkness having been 3′,57″, (as was before shewn) it will follow, that that Diameter of the Elliptick Figure according to which the Shade past, was no less than 114½ Geogr. Miles. And from the Elements of the Conicks 'tis easy to be proved, that supposing the Figure of the Shade a true Ellipse, whose Axes are as *Radius* to the *Sine* of 40 Degrees, the greater Axis would be 171 Geographical Miles, and the lesser 110; and the nearest distance between the Limits supposed Parallel 164 such Miles.

And

And this Length of the *Axis* of the Shade, derived purely from the Continuance of Total Darkness, is fully confirmed by the observed Distance of the Parallel Limits; the one passing by *Badsworth* in *Yorkshire*, the other by *Cranbrook* in *Kent*. For by the two Latitudes 53°. 37′ and 51°. 6′, with the Difference of Longitude 7′ and 40″ of Time, or 1°.55′, the Distance of these two Places is given 166½ Geogr. Miles; with the mean Angle of Position 25 Degrees from the North Westwards; wherefore this Arch makes an Angle with the Track of the Shade of 77°¾: and hence the nearest Distance of the Parallels becomes 163 such Miles, which by the other Way was found 164.

If therefore we conclude the *Axis* of the Shadow, when the Sun was just 40 Degrees high, to have extended over 2°.50′ of a great Circle, we may securely determine the Difference of the Sun and Moon's Diameters at this time. For the Difference of the Horizontal Parallaxes of the Sun and Moon being found to be 60′.38″. (as shall be hereafter shewn, but is not required with extream exactness for this Purpose) the Difference of the Parallaxes in Altitude at both Ends of the Axis, will be found to be 1′. 56″; and by so much did the Diameter of the Moon when forty Degrees high exceed that of the Sun: Hence the Horizontal Diameter of the Moon in this Anomaly is found 33′.27″, which may serve for a Rule in all other Cases.

I forbear to mention the *Chill* and *Damp* which attended the Darkness of this Eclipse, of which most Spectators were sensible, and equally Judges. Nor shall I trouble you with the Concern that appear'd in all Sorts of Animals, *Birds*, *Beasts* and *Fishes* upon the Extinction of the Sun, since our selves could not behold it without some sense of Horror.

Lastly, I have added the following *Synopsis* of such Observations as have hitherto come to my Hands; acknowledging the Favour of all those, who have been willing to promote our Endeavours to perfect the Doctrine of Eclipses.

Place	Obſerver	Beginn. h. ′ ″	Immerſ. h. ′ ″	Emerſ. h. ′ ″	Tot. ′	End. h. ′ ″
Barton	M. *Bridges*				3.53	
Bell-bar	M *Jones*	8. 6.25	9. 9.45	9.13.27	3.42	
Broadway } *Carmarth.*			8.47.00	8.49.30	2.30	
Cambridge	M. *Cotes*			9.14.37		10.21.57
Canterbury	M. *Gray*	8.10.00				10.24.30
Cheſter	M. *Ward*	7.57.40				10. 6.35
Crew	M. *Wright.*		9. 2. 8		2.00	10. 9.00
Dublin	*L. Arch Biſh.*	7.42.11				9.49.4●
Dublin	M. *Hawkins*	7.41.30				9.48.45
Exon	*L. Biſhop*		8.55. 0	8.59. 0	4.00	10. 0.00
Exon	M. *Hudſon*	7.47.30			3.30	10. 0.30
Greenwich	M. *Flamſteed*				3.11	
King's Wald.	M. *Whitſide*				3.52	
Llanidan } *Angleſey*	M. *Rowland*	7.52.30				
London	*R. Society*	8. 6 00	9. 9. 3	9.12.26	3.23	10.20.00
Northampt.	M. *Hawkins*		9. 5.22	9. 9.24	4. 2	10.15.35
Norton-court	D. *Harris*	8, 8.55	9.13.23	9.14.22	0.59	10.24.47
Oxon	D. *Keill*				3.30	10.15.10
Paris	R. *Academy*	8.11.00				10.28.00
Plymouth	M. *Heines*	7.41.00	8.45.30	8.50 00	4.30	9.54.30
Portcheſter	C. *Candler*		9. 2.25	9. 6.15	3.50	
Salop	D. *Hollings*	7.58. 0			1.40	10. 6.00
Upminſter	M. *Derham*	8. 7.41	9.10.58	9.14. 63. 8		10.21.45
Wanſted	M. *Pound*	8. 6.37	9. 9.28	9.12.48	3.20	10 20.32
Weymouth	M. *Hobbs*		8.54.00	8.58.00	4.00	
Witley	M. *Baxter*	7.59. 0			3.15	10.13.00

IV.

IV. *An Account of a Book.* Bibliographiæ Anatomicæ Specimen, *sive Catalogus omnium pene Auctorum, qui ab HIPPOCRATE ad HARVEIUM Rem Anatomicam ex professo vel obiter scriptis illustrarunt,* &c. *Curâ & Studio JACOBI DOUGLAS,* M. D. *Reg. Soc. S. & in Colleg. Chirurg. Londinensi Prælect. Anatom.* 8*vo. Londini* 1715.

THE Author of this Treatise, whose admirable Skill in the Practice of Dissections, as well as in the Theory of the Structure of the Parts, leaves him not many Equals, in order to discover what Improvements and Progress Anatomy has met with, and with what Industry the Study of it has been cultivated, has with much Application perused a very great Number of Authors who have advanced the Science; observing therein who have the Honour of being the first Discoverers, and who have unjustly arrogated to themselves that Title, that each may receive a due Proportion of Praise according to his Merit. And in this Decision he has impartially weighed their Deserts, the better to lay before the Reader the Increase of these Studies, and to determine more exactly the Differences that have arisen about who are first Inventors; which the Book, Chapter and Page where they are treated of will easily manifest.

The History, Lives and Elogies ascrib'd to Anatomists, which he has inserted either from their own Writings, or their Editors, or Commentators, will afford a great Variety of Pleasure, in which he has been particulary careful

ful to ſet down the Names, Sur-Names, Country, Time of their Birth, what Year they died in, under what Maſters educated, where they flouriſh'd, and in what part of Anatomy they excell'd.

Nor has he been leſs diligent in the Account he has given of the Books of *Anatomy*, which his Friends ſupply'd him with in great Number. The Reader will ſee here laid before him, all the ſeveral Editions, in what Language, what Volume they were printed in, with the Place and date of the Year they were publiſhed at; and which are the firſt Impreſſions, and which copied from them. Nor has he judged it improper to give ſome Account of the Figures diſpers'd up and down in Anatomical Books; as whether they were Originals or Copies, cut in Wood or Copper, *&c.* To theſe he has added three Indexes, whoſe Uſe will be ſeen by the Titles. As for the difference of Style remarkable in this Treatiſe, It is chiefly owing to the Variety of Authors made uſe of, he thinking himſelf not at Liberty to vary the Expreſſion of them whoſe Authority he quotes.

He ſays he would willingly have recounted the great Advantage Anatomy has received from the *Engliſh* Nation: but out of juſt Regard to their Merits, he has reſign'd this Province to his Friend Mr. *William Becket*, whoſe Induſtry in collecting their Writings will not in his Opinion exceed his Talent and Abilities to recommend them to the World.

He hopes the Reader will pardon him in this, that as ſeveral Books and Editions came late to his Hands, he was forc'd to add the *omiſſa* ſeparately: which being in greater Number than at firſt expected, the Author earneſtly deſires the Favour of thoſe who have in their Collections any thing of this kind here omitted, that they would pleaſe to communicate the ſame, in order to render this firſt *Specimen* ſtill more complete.

F I N I S.

Numb. 344.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *June*, *July* and *August*, 1715.

The CONTENTS.

I. *Experimenta Barometrica pro varia diversi Aeris Elasticitate exploranda, in variis* Helvetiæ *locis, occasione Excursionis Alpinæ, mense Sept. Anni præteriti susceptæ. Ope Tubi* 32 *digit.* Parisin. *longi,* 2 *linn. diametro. à* Johanne Jacobo Scheuchzero, M. D. Math. P. Tigurino. & R. S. S.

COlumna *prima* notat Aerem in Tubo relictum. *Secunda* altitudinem Mercurii supra Argenti vivi superficiem; *Tertia* spatia Aeris expansi. *Quarta* notat descensum Mercurii propter Aerem relictum.

D. 6. Septembr. *Tiguri,* Barometri totius altitudo hor. 8. ant. erat 26 digit. *Parisiens.* 4 linn. H. 9½ vero 26 digg. 4½ linn.

Col. I.	Col. II.		Col. III.		Col. IV.	
digg.	digg.	lin.	digg.	lin.	digg.	lin.
3	19	9 bis	12	6½ bis	6	7½
6	16	8	15	7½	9	8½
	16	7½	15	8	9	9
9						
12	11	11 bis	20	3 bis	14	5½
15	9	9 bis	22	6 bis	16	7½
18	7	5½ }	24	8½ }	18	11
	7	6 }	24	8 }	18	10½
21	5	3	27	0 bis	21	1½
24	3	3	28	11 bis	23	1½
27	1	6	30	7½ bis	24	10½
30	0	4	31	10½ bis	26	0

D. 11. Sept. in Pascuo *Alpino* Ennenseweni gen Averen Montis Liberi, *Glaronensis* ditionis, h. 1. pom. cœlo sereno Altitudo totius Barometri 23. 10 bis.

Col. I.	Col. II.		Col. III.		Col. IV.	
digg.	digg.	lin.	digg.	lin.	digg.	lin.
3	18	7	13	6	5	3
6	15	7½	16	4	8	2½
9	13	3	18	7	10	7
12	11	1½	20	9	12	8½
15	9	0	22	9	14	10
18	6	11	25	0	16	11
21	4	11	26	10	18	11
24	3	0	28	10	20	10
27	1	4	30	5	22	6
30	0	2	31	8	23	8

D. 12 Sept. h. 7. ant. cœlo sereno, auff Scherf jugo editiore Montis Liberi. Altitudo totius Barometri 21. 8.

3	17	6	14	6	4	2
6	14	7	17	3	7	1
9	12	6	19	6	9	2
12	10	5	21	6	11	3
15	8	5	23	6	13	3
18	6	5	25	3	15	3
21	4	7	27	1	17	1
24	2	9½	29	0½	18	10½
27	1	4	30	6	20	4
30	0	2	31	8	21	6

D. 12. Sept. h. 9. ant. cœlo sereno, auff dem Blattenstock jugo editiore Montis Liberi. Altitudo tot. Barom. 21. 6.

3	17	2½	14	6	4	3½
6	14	5	17	5	7	1
9	12	4	19	6	9	2
12	10	4½	21	5	11	1½

Col. I.	Col. II.		Col. III.		Col. IV.	
digg.	digg.	lin.	digg.	lin.	digg.	lin.
15	8	7	23	4½	12	11
18	6	7	25	3	14	11
21	4	8	27	3	16	10
24	2	9	29	0	18	9
27	1	3	30	5	20	3
30	0	3	31	6	21	3

D.14. Sept. h.12. intra ipſam Venam Chalybis *Sarunetanam*, 300. incirca paſſus ab oſtio, cœlo foris ſereno. Barometri totius altitudo 24. 4. & 24. 3.

3	18	9	13	1	5	7
6	15	9	16	1	8	7
9	13	5	18	5	10	11
12	11	3	20	7	13	1
15	9	1	22	9	15	3
18	7	0	24	10	17	4
21	4	11	27	0	19	5
24	3	0	28	10	21	4
27	1	4	30	6	23	0
30	0	3	31	6	24	1

Extra hanc Venam metallicam ſub dio eandem altitudinem obſervavi Mercurii in Barometro integro, item in 3 & 9 digg. Aeris in tubo relicti Sed notandum eſt aerem in intimis fodinæ partibus, ubi experimenta feci, fuiſſe ob ignem præterito die accenſum (quo venam duriſſimam coquunt foſſores) rarefactum, & locum hypocauſti inſtar moderatè calefactum.

N. B. *Multis experimentis coram R. Societate factis compertum eſt, Aeris compreſſi vires Elaſticas eſſe ut pondera comprimentia directè. His Cl.* Scheuchzeri *obſervatis patet eandem in Aere rarifacto obtinere regulam quam proxime; Nam licet differentia aliqua reperiatur, tanta non eſt, ut ab inæqualite diametri Tubi non facile oriatur. Ut autem experimenta hæc rite fiant, oportebit Tubi capacitatem, immiſſo unciatim Mercurio, in æquales partes dividi, loco partium longitudine æqualium.*

Bo-

BOTANICUM Hortense IV.

Giving an Account *of divers* Rare Plants, *Observed the last* Summer *A. D.* 1714. *in several* Curious Gardens *about* London *and particularly* the Society *of* Apothecaries *Physick-Garden at* Chelsea.

By James Petiver, F. R. S.

Continued from the last *Transactions.*

47. *Plantæ* Nasifloræ. *Snout-Flowers.*

ALehoof Snapdragon. Ray Hist. Plantar. p. 567. pl. 3.

Cymbalaria *Hederæ terrestris* folio, *flore* maximo. *Antirrhino* seu *Linariæ* affinis fol. *Hederæ* terrestris. *Flor.* Batav.

Asarina *Lob.* Lugd. 915. fig. 2.

Asarina Lob. *flore* Hederæ terrestris I. B. 3. App. 840. fig.

Asarina sterilis, *Sevenæ* Narbonensis agri *Lob.* Obs. 329. Ic.

Asarina sive saxatilis *Hederula* Lob. *Icon.* 601.

Hedera saxatilis *Jonst.* 856. fig. 2.

Hedera terrestris, magno *flore* C. B. 306. Phyt. 609. 3.

Its *Flowers, Seeds* and *Capsules* plainly shew it to be a kind of *Snapdragon. Lobel* says this grows wild in *Narbone* and *Provence* (from whence I should be glad to see it,) but his *Figure* does not express the Hoariness with which our *Leaves* are endowed, nor are its *Flowers* so large or exact. I first saw this elegant Plant in the

 Amster-

Amsterdam Physick-Garden, *Flowring* in *July*, and since at *Chelsea*.

48. English *Yellow Pansey*. Ray's *British* Herbal. *Tab*. 37. fig. 10.

Viola montana lutea grandiflora *nostras* Ray 1052. 5. *Syn*. ed. 2 15. 8.

Viola 5. Jonst. 853. *Ger*. 704.

Mr. *Ray* observed this in divers boggy mountainous Pastures in *Darby-shire*, *Yorkshire*, and *Wales*. It *Flowers* very well in our *Gardens*, and makes a beautiful edging.

49. *Bog* Violet. *Rays* British *Herball* Tab. 37. Fig. 5.

Viola palustris rotundifolia *Plot*, Hist. Oxon. 144. 4. Tab. 9. fig. 2. *Ray* 1050. 4. Synopf. 152. 4. edit. 2. p. 214. 4.

Viola palustris rotundifo ia glabra *Moris*. Oxon. 475. 5. *sect*. 5. Tab. 35. *fig*.

Dr. *Plot* first discovered this in the Bogs about *Stowwood* in *Oxfordshire*, and *Chiswell* in *Barkshire*, since which it hath been found in the like Places between *Wickam* and *Croydon* in *Surry* plentifully, from whence it was transplanted into *Chelsea Garden*. It *Flowers* in *April* and *May*.

50. *Hairy Wood* Violet. *Rays* British Herball Tab. 37. fig. 7.

Viola *Trachelii* folio *Morret*. pin. 125.

Viola *Martia* hirsuta, major, inodora Plot. Oxon. 144. 3. Ray 1051. 5. Synopf. 152. 6. ed. 2. p. 215. 6.

Viola *Martia* major, hirsuta inodora *Moris*. Ox. 475. 4. S. 5 *Tab*. 35. fig.

Grows plentifully in *Charlton* and other *Woods* in *Kent*, &c. It *Flowers* in *March* and *April*.

51. Upright *Tree-Violet* Ray 1052. 1.

Viola *arborescens* vel potius erecta *Camer*. Epit. 511. fig. opt.

Viola assurgens tricolor *Ger*. 703. Ic. 2. *Jonst*. 854. fig.

Viola elatior *Clus*. 309.

Viola Martia *arborescens* purpurea C. B. 199. 2.

Viola *Martia* surrectis cauliculis *Lob*. Ic. 610. 2.

Jacea

Jacea *tricolor*, surrectis caulibus, quibusdam *arborea* dicta *Chabr.* 510. Ic. 3. I. B 3. l. 32. p. 547. fig.

Clusius observed this to grow common in the *Woods* of *Austria* and *Styria*. Dr. *Morison* in his *Hist. Oxon.* p. 476. says this is found in *England* on the Tops of *Mountains* and in *Woods*, which has not as yet been observed, either by the inquisitive Mr. *Ray* or any other.

Plantæ Bacciferæ. Berry-bearing Plants.

52. Long *Horse-tongue* Ray 663. 1.

Bonifacia sive *Bislingua* Chabr. 45. Ic. 5. & 6. I. B. 1. l. 5. p. 575. fig.

Hippoglossum *Camer.* Epit. 919. fig.

Hippoglossum *Dioscoridis* & Laurotaxa *Plinii* Col. 165. fig. opt.

Hippoglossum sive *Bislingua* Barrel. pl. 5. Ic. 249. Park. 702. Ic.

Hippoglossum mas & feminæ Ger. 761. Ic. 1. & 2. Jonst. 908. Ic. 1. & 2.

Radix Idea, Hippoglossum *Col.* phyt. 64. fig.

Ruscus angustifolius, *fructu* folio innascente *El. Bot.* 70. *Inst.* 79.

This and the next are accurately *Figured* by *Columna* & *Barrellier*. The peculiarity of this Plant is to have a small tongue like the *Key* or *Fruit* of the *Ash-tree*, growing from the middle Rib on the under side of each Leaf. *Grows* on shady *Mountains* in *Italy* and *Hungary*.

53. Round *Horse-tongue* Ray 663. 2.

Laurus Alexandrina *Camer.* Epit 936. Ic. *Chabr.* 46. Ic. 1. I. B. 1. l. 5. p. 574. fig.

Laurus *Alexandrina* & Chamædaphne *Col.* 165. *fig.* opt.

Laurus *Alexandrina* genuina *Park.* 700. fig. 1.

Laurus *Alexandrina* vera *Diosc.* & *Theophr.* Barrel. 6. Ic. 250.

Laurus *Alex.* fructu folio insidente C B. 305.

Ranunculus mont. subhirsutus, *Geranii* folio C B. 182. 13.
Ranunculus montanus subhirsutus, *Geranii* folio C B. *pr.* 96. 6. fig.
Ranunculus mont latifol. hirsutus alter C B. *phyt.* 323. 15.

C B. in his *Phytopinax* says he first found this on Mount *Mutet* near *Basil*, and afterwards on *Rosberg* near *Tubing* in *Wirtenbergh*. I received the first Specimen of this from Mr. *Jacob Bobart* at *Oxford*.

Herbæ *Tetrapetalæ*. Quaterpetals.

60. Babylonion *Cress*, Ray 821. 1.

Draba prima umbellata C B. *phyt.* 174. 1.
Draba vulgaris *Park.* 849. fig. 1.
Draba *Dioscoridis* Jonst. 274. fig. 1.
Draba sive *Arabis* Chabr. 295. Ic. 6.
Draba multis fl. albo. I B. 2. l. 22. p. 939. fig.
Draba *umbellata* vel *Draba* maj. capitulis donata C B. 109. 1.
Arabis sive *Draba* & Nasturtium *Babylonicum* Lob. Ic. 224. 1 *Belg.* 267. fig *Obs.* 111.

Grows in the Borders of Fields about *Vienna* plentifully, as also in the like Places in *Italy* and *France*. Flowring in *May* and *June*.

61. Bur *Rocket* Ray 804. 4.

Erucago segetum *El. Bot.* 199. Inst. 152.
Eruca *Monspeliaca* siliquâ quadrangulâ *Park.* 821. fig.
Eruca *Monsp.* sil. quadrangulâ echinatâ C B. 99. 14. *phyt.* 149. 15. *prodr.* 41. 7. *fig.* opt.
Raphanistrnm dispermon *Monspel.* sil. quadrangulâ echinatâ *H. Leyd.* 520.
Sinapi echinatum *Lugd.* 647. fig. *Chabr.* 275. Ic. 3. I. B 2. l. 21. p. 858. fig.

Grows about *Montpelier* in Corn. It *Flowers* and *Seeds* in *June*, &c.

62. Ever-green *Persian* Tufts *Ray* 837. 7.

Thlaspi fruticosum *Leucoii* folio latifol. C B. 108. 1. *prodr.* 49. 9. *Park.*

Thlaspi

Thlaspi frut. umbellatum *Persicum*, fol. *Leucoii* instar sempervirentibus H. *Oxon* 296. 23.

Thlaspi latifol. platycarpon, *Leucoii* foliis *Bocc.* 55. fig. opt.

Thlaspi sempervirens & florens *Dodart.* 115. fig.

Thlaspidium frutic. *Leucoii* folio semperflorens E. B. 183. *Inst.* 214.

Thlaspio fruticoso di *Persia*, con foglia di *Keiri*, di fiore odorato *Zanon* 106. Tab. 74. fig.

This is always green, *Flowers* long and especially towards the *Winter*, its said to come from *Persia*, Boccone *found* it wild about *Palermo* in *Sicily*.

63. Garlick *Cress* Ray 834. 19.

Scorodo-thaspi *Ulissis Aldrovandi* Chabr. 294. Ic. 1. I. B. 2. l. 22. p. 932. fig.

Thlaspi *Allium* redolens H. *Oxon.* 297. 28. Tab. 18. fig.

That very great and laborious Naturalist *Ulisses Aldrovandus* first discovered this Plant growing about *Bononia*. It *Flowers* and Seeds in *Chelsea-Garden* early in the *Spring*.

Plantæ fl. pentapetalo. *Cinquepetals.*

64. Dwarf *Gentian* with a large Flower *Ray* 718. 2.

Gentiana *Alpina* magno flore I B. 3. l. 31. p. 523. fig.

Gentiana *Alp.* pumila, *latifol.* magno *flore* Elem. *Botan.* 96.

Gentiana *Alp.* magno *fl.* cæruleo violaceo *Marianæ* Chabr. 503. Ic. 6.

Gentiana 5. *Gentianella* major verna *Clus.* 314. fig.

Gentianella *Alpina* verna major *Park.* 402. fig. 1.

Gentianella *Alp.* latifol. magno *flore* C B. 187. 1. phyt. 347. 16 *prodr*, 97. c. x. 1.

Gentianella *Helvetica* Lob. *Icon.* 310. 1. Belg. 381.

Gentianella *Campanulæ* flore purp. *Helveticæ* Adv. 130. fig.

I B. makes this and the *Angustifolia* to be the same Plant.

Grows

Grows on most *Mountains* in *Austria*, *Styria* and *Swisserland*, where it *Flowers* in their Spring, *viz.* in *June*, or as soon as the Snow there melts.

This elegant Plant is pretty *common* in most of our *Countrey* Gardens.

65. Pona's *blew Valerian* Ray 745. 6.
Cervicaria *Valerianoides* cærulea C B 95. 20.
Rapunculus *Valerianoides* cæruleus umbellatus *Boer.* 104. 3.
Rapuntium umbellatum *Col.* phyt. 2. p. 22.
Trachelio azuro umbellifero *Ponæ* Ital. 44. fig.
Trachelium umbelliferum cæruleum *Park.* 645. 8.
Valerianoides Alpin. *Trachelii* folio, *fl.* coccineis *Alm. Bot.* 379.

This is very different from the *Telephium* fl. purpureis *Lobel.* Ic. 389. 2. as *Columna* and C B. from him take to be the same.

Pona, in his *Italian Edition* of the Plants on *Mount Baldus*, gives the first *Figure* and *Description* of this elegant Plant, which he received from *Signior Contareni* at *Venice*, and supposes it to come from *Candy*; but *Pere Barrelier* says it grows in moist shady Places not only about *Rome* but *Valentia* in *Spain*, where its also sometimes found with a *white Flower*. Its blew umbelliferous Tufts make a beautiful Shew in our *Physick Garden* where it *Flowers* from *Midsummer* till the end of *September*.

66. Childing *Pink* Ray 990. 13. H. *Ox* 563. 21.
Caryophyllus sylv. prolifer C B. 209. 6. *phyt.* 393. 18.
Caryophyllus prolifera *Park.* 1338. fig. 1.
Betonica *Coronaria* squamosa sylv. *Chabr.* 446. Ic. 6. I B. 3. l. 29. p. 335. fig.
Armeria prolifera *Lob.* Ic. 449. 1. *Belg.* 534. *Jonst.* 599. fig. 5.

Grows in barren *Pastures* in many Parts of *France*, *Italy*, *Germany*, &c.

67. *Gray*

67. Gray *Stitchwort* Ray 1020. 7. H. Ox. 546. 44.
Holosteum *Hort.* incanum vulgare *Nobis.*
Caryophyllus Holosteus *Ger.* 277. xi. *Jonst.* 595. fig. 15.
Caryophyllus *Holosteus* tomentosus I B. 3 l. 29. p. 360. fig. *Park.* 1339. fig. 7.
Caryophyllus *Holost.* tomentosus latifolius C B. 210. 2. *pr.* 104. 5.
Lychnis maritima incana & tomentosa *H. Oxon.* 546. 44.
Myosotis tomentosa *Linaria* folio amplo. E B. 211. *Inst.* 244.

The *Description* of I B. and others much better agree with this Plant than that of C B. prodr. p. 104. 5.

There is no good *Figure* of this Herb altho a *Common Edging* in every *Countrey* Garden.

68. Penny's *Myrtle Cistus* Ray 1011. 13.
Ascyrum *Balearicum* frutescens, magno *flore* luteo, *fol.* minoribus lucidis subtus verrucosis *J. Salvadore.*
Myrto-Cistus Pennei *Clus.* 68. fig. *Pan.* 67. fig. *Park.* 665. fig. *Ger.* 1098. fig. 16. *Jonst.* 1279. fig. 16. *Tab.* Ic. 1054. 2.
Myrto-Cestus Pennei, fl. luteis oblongis *Chabr.* 103. Ic. 2. I B. 2 l. 13. p. 21. fig.

Dr *Penny* a famous Physitian of *London* and a very curious Naturalist of that time, first communicated this Plant A. D. 1580. which he had gathered on the Island *Majorca* to *Carolus Clusius*, who gave a *Figure* of it, and from whence all other Authors have since copied it. We are lately obliged to Dr. *John Salvadore* at *Barcelona* for the more perfect Knowledge of it, who amongst many other very *curious* and *Rare Plants* gathered in that Island and *Minorca*, sent me this under the Name as above, of which it is a true *Species.*

69. *Matthiolus* his *Coris* or St. *John's-wort* Ray 1018. 4.
Coris C B in Matth. 669. fig opt. *Cam.* 678. fig. *Park.* 570. fig. 1. *Jonst.* 544. fig. 1. *Lob.* Ic. 403. 1. *Belg.* 489. *Obs.* 220. fig.
Coris lutea C B. 280. 1. lutea major *phyt.* 548. 1.

Grows on most *Mountains* in *Austria*, *Styria* and *Swisserland*, where it *Flowers* in their Spring, *viz.* in *June*, or as soon as the Snow there melts.

This elegant Plant is pretty *common* in most of our *Countrey* Gardens.

65. Pona's *blew Valerian* Ray 745. 6.
Cervicaria *Valerianoides* cærulea C B 95. 20.
Rapunculus *Valerianoides* cæruleus umbellatus *Boer.* 104. 3.
Rapuntium umbellatum *Col.* phyt. 2. p. 22.
Trachelio azuro umbellifero *Ponæ* Ital. 44. fig.
Trachelium umbelliferum cæruleum *Park.* 645. 8.
Valerianoides Alpin. *Trachelii* folio, *fl.* coccineis *Alm. Bot.* 379.

This is very different from the *Telephium* fl. purpureis *Lobel.* Ic. 389. 2. as *Columna* and C B. from him take to be the same.

Pona, in his *Italian Edition* of the Plants on *Mount Baldus*, gives the first *Figure* and *Description* of this elegant Plant, which he received from *Signior Contareni* at *Venice*, and supposes it to come from *Candy*; but *Pere Barrelier* says it grows in moist shady Places not only about *Rome*, but *Valentia* in *Spain*, where its also sometimes found with a *white Flower*. Its blew umbelliferous Tufts make a beautiful Shew in our *Physick Garden* where it *Flowers* from *Midsummer* till the end of *September*.

66. Childing *Pink* Ray 990. 13. H. *Ox.* 563. 21.
Caryophyllus sylv. prolifer C B. 209. 6. *phyt.* 393. 18.
Caryophyllus prolifera *Park.* 1338. fig. 1.
Betonica *Coronaria* squamosa sylv. *Chabr.* 446. Ic. 6. I B. 3. l. 29. p. 335. fig.
Armeria prolifera *Lob.* Ic. 449. 1. *Belg.* 534. *Jonst.* 599. fig. 5.

Grows in barren *Pastures* in many Parts of *France*, *Italy*, *Germany*, &c.

67. *Gray*

67. Gray *Stitchwort* Ray 1020. 7. H. Ox. 546. 44.
Holosteum *Hort.* incanum vulgare *Nobis.*
Caryophyllus Holosteus *Ger.* 277. xi. *Jonst.* 595. fig. 15.
Caryophyllus *Holosteus* tomentosus I B. 3 l. 29. p. 360. fig. *Park.* 1339. fig. 7.
Caryophyllus *Holost.* tomentosus latifolius C B. 210. 2. *pr.* 104. 5.
Lychnis maritima incana & tomentosa *H. Oxon.* 546. 44.
Myosotis tomentosa *Linaria* folio amplo. E B. 211. *Inst.* 244.

The *Description* of I B. and others much better agree with this Plant than that of C B. prodr. p. 104. 5.

There is no good *Figure* of this Herb altho a *Common Edging* in every *Countrey* Garden.

68. Penny's *Myrtle Cistus* Ray 1011. 13.
Ascyrum *Balearicum* frutescens, magno *flore* luteo, *fol.* minoribus lucidis subtus verrucosis *J. Salvadore.*
Myrto-Cistus Pennei *Clus* 68. fig. *Pan.* 67. fig. *Park.* 665. fig. *Ger.* 1098. fig. 16. *Jonst.* 1279. fig. 16. *Tab.* Ic. 1054. 2.
Myrto-Cestus Pennei, fl. luteis oblongis *Chabr.* 103. Ic. 2. I B. 2 l. 13. p. 21. fig.

Dr *Penny* a famous Physitian of *London* and a very curious Naturalist of that time, first communicated this Plant A. D. 1580. which he had gathered on the Island *Majorca* to *Carolus Clusius*, who gave a *Figure* of it, and from whence all other Authors have since copied it. We are lately obliged to Dr. *John Salvadore* at *Barcelona* for the more perfect Knowledge of it, who amongst many other very *curious* and *Rare Plants* gathered in that Island and *Minorca*, sent me this under the Name as above, of which it is a true *Species.*

69. *Matthiolus* his *Coris* or St. *John's-wort* Ray 1018. 4.
Coris C B in Matth. 669. fig opt. *Cam.* 678. fig. *Park.* 570. fig. 1. *Jonst.* 544. fig. 1. *Lob.* Ic. 403. 1. *Belg.* 489. *Obs.* 220. fig.
Coris lutea C B. 280. 1. lutea major *phyt.* 548. 1.

Hypericoides *Coris* quorundam & *Coris* legitima *Cretica* Chabr. 456. Ic. 5. l. B. 3. l. 29. p. 384. fig

Hypericum seu *Coris* legitima *Ericæ* similis *H. Oxon* 469. 4.

Hypericum *Syriacum* tenuissimo & glauco folio *El. Bot.* 222 *Inst.* 255.

Matthiolus says this grows in dry Grounds and Hills in many Places of *Italy*, and by *Carara* a Village of *Liguria*, as *Camerarius* asserts.

I take this by its Figure to be different from *Bellus* and *Pona* their *Coris* legitima *Cretica*.

70. Succory *Mullein*.

Blattaria *Cichorei* folio villoso *Nobis*.

an *Blattaria* Orient. *Agrimoniæ* folio T. Coral.

Its *Leaves* spread on the Ground, are deeply cut long and hoary, from the midst of these rise slender Stalks, much branched and full of small yellow *Flowers* which soon fall off.

I have as yet seen this in *Chelsea Garden* only.

71. Great *Pile Trefoil* Ray 968. 12. *H. Ox.* 177. 14.

Lotus Hæmorrhoidalis *sive* Trifolium *Hæmorroid.* majus *Park.* 1102. fig. 8.

Lotus pentaphyllos siliquosus villosus C B. 332. 9.

Lotus polyceratos frutescens incana alba, *siliquis* curtis crassioribus & brevioribus erectis *H. Ox.* 177. 14.

Oxytriphyllum alterum *Scribonii* Herbariorum *Lob.* Adv. 381. fig.

Trifolium album, rectum hirsutum valde *Chabr.* 160. Ic. 1. l B 2. l. 17. p. 360. fig.

Trifolium *Hæmorrhoidale* Lob. *illustr.* 151.

Grows plentifully about *Montpelier*, Lobel *says* its also found about *Lyons* and *Turin*.

The *Leaves* or rather the *Seed* given in *Powder* from 30 Grains to 60 in *Red Wine*, is said to be very effectual in curing the *Piles*.

72. Lesser *Pile Trefoil* Ray 968 xi.

Lotus

Lotus *Hæmorrhoidalis* alter minus ſive Lotus *Lybica* Dal. *Park.* 1102. f. 9.

Lotus ſiliquoſus glaber, *flore* rotundo C B. 332. 10.

Lotus *Lybica* Dalechampii *Lugd.* 509. fig.

Lotus *Lybica* Dalech. *vulgò* H. Leyd. 384.

Trifolium rectum *Monſpeſſulanum* I B. 2. l. 17. p. 359. *ſine* fig.

Trifolium rectum *Monſpeſ.* fl. rotundo C B *phyt.* 666. 46.

Grows by River ſides and moiſt Places about *Meſſina, Calabria* and *Montpelier.*

73. *Camerarius* his *Birds Foot* Trefoil. Ray 970. 20.

Lotus ſiliquis *Ornithopodii* C B. 332. 2. *phyt.* 667. 36. I B. 2. l. 17. p. 359. fig.

Lotus peculiaris, ſiliquis *Ornithopodii* Camer. Hort. 91. Ic. 25. opt.

Lotus *Ornithopodii* ſiliquis, *fl.* luteo, nigris lineis ſtriato *Chabr.* 158. Ic. 1.

Lotus polyceratos annuus procumbens latifolius, ſiliquis *Ornithopodii* H. Ox. 176. 8.

Lotus ſylveſtris *Creticus* Park. 1101. fig. 4. ex *Pona* Ital. 153.

That Reverend Divine and Botaniſt Sir *George Wheeler*, obſerved this in the *Iſland* Corfu.

74. Fine *Fumitory* Ray 406. 3. H. Ox. 261. xi.

Fumaria tenuifolia *Ger.* 928. fig. 3. *Jonſt.* 1088. fig. 3.

Fumaria minor ſive tenuifolia *Park.* 287. fig. 2.

Fumaria minor tenuifolia C B. 143. 3. *phyt.* 245. 2.

Fumaria minor *ſive* tenuifolia ſurrecta *Chabr.* 277. Ic. 6. I B. 3. l. 26 p. 203. fig.

Capnos tenuifolia *Cluſ.* 208. fig. 2. *Hiſp.* 375. fig. *Lob.* Ic. 757. 2. *Belg.* 914 *Obſ.* 438.

Grows amongſt Corn and by Pathways about *Montpelier*, and in divers parts of *Spain* where its called *Palomilla*, and flowers in *April* and *May*.

75. Barcelona *Hart Fumitory.*

Fumaria henneaphyllos *Hiſpanica* ſaxatilis *Bocc.* Muſ. 83. Tab. 73.

Fumaria heneaphyllos *Hispan.* saxatilis, *fl.* vario *Barrel.* 866. Ic. 42.

Fumaria *Hisp.* saxatilis, *fol.* amplioribus cordiformibus, *sem.* compressis *El. Bot.* 334. *Inst.* 422

Dr. *Salvadore* observed this in the Fissures of *Mount Serrat*, and in the Hermits Walls about *Barcelona* where it *Flowers* in *May*, *June*, and *July*.

The curious *Dutchess Dowager* of *Beaufort* shewed me very fair Specimens of this Plant, which her *Grace* raised from *Seed* I had given her.

Plantæ *fructu* tricocco. *Plants* with Treble Husks and Seed.

76. Garden *Spurge* Ray 866. 18.

Lathyris C B. in Matth. 868. fig. 1. *Cam.* 968. *fig.* opt.

Lathyris major C B. 293. 1. *phyt.* 577. 1.

Lathyris major *Hortensis Park.* 191. fig. 1.

Lathyris sive *Cataputia* minor *Chabr.* 535. Ic. 4. I B. 3. p. 880. fig. *Jonst.* 503. fig. 18.

Cataputia minor *Lob.* Ic. 362. 1. *Belg.* 439. *Obs.* 197. *Ger.* 405. fig. 14.

Cataputia minor sive *Lathyris* angustifolia *Tab.* 587. Ic 2.

Tithymalus latifolius *Cataputia* dictus H. *Leyd.*

Grows spontaneously in many *Gardens*, where it *Flowers* and *Seeds* in *July* and *August*.

77. Red Flowred *Spurge* Ray 864. 2.

Characias *Monspeliensium* Clus. & Matth *Lob.* Ic. 359. 2. Belg 43. Obs. 194. fig. 2. Adv. 152.

Tithymalus Amygdaloides *sive* Characias *Chabr.* 534. Ic. 2. I B. 3. l. 34. p 672. *fig.*

Tithymalus Characias 1. *Clus.* 188. fig. 1. *Hisp.* 436. fig.

Tithymalus Characias *Monspeliensium* Park. 186. 2. *Jonst.* 499. fig. 8.

Tithymalus *Characias* rubens peregrinus C B. 290. 2. *phyt.* 572. 1

Clusius observed this on the rough Mountains of *Spain*, its also found in *Italy* and about *Montpelier* in stoney Places.

Its dark red *Flowers*, distinguish it from others like it.

78. Montpelier *Meadow* Spurge.

Tithymalus *Lithospermi* majoris folio *Bot. Monsp.* 305.
An *Tithymalus* Tingitanus *Linariæ* folio, lunato *flore* H. *Leyd.*
An *Tithymalus* Linariæ *folio*, lunato *flore* H. Bles. 313.

Dr. *Nissole* that curious Botanist sent me this Plant among many others gathered about *Montpelier*.

79. Soft *Spurge* of *Mount Polline*.

Tithymalus *Montis Pollini*, fol. & fructu pubescentibus.

Its *Stalks* brown and hoary as are its *Leaves* which grow alternately and not thick set. It *Flowers* and *Seeds* in *July* and *August*.

Mons. *Vaillant* sent me a *Specimen* and *Seed* of this from *Paris*.

80. Cambridge *Corn* Spurge *Ray* 868. 22.

Tithymalus *Segetum* longifolius *Cat. Cant.* app.

It grows in the *Corn* beyond *Kingston-wood* in the Road to *Gamlingay* in *Cambridge-shire*, where I found it in company with Mr. *James Sherard*, an accurate *Botanist* and well verst in the Knowledge of our *English* Plants

81. Cypress *Spurge* Ray 865. 19. *H. Ox.* Bob. 335. 4.
Tithymalus Cupressinus *Jonst.* 449 fig. 5. H. Ox. *Bob.* 335. 4. 1. Tab. Ic. 594. fig. 2. Hist. 2. p. 293. f. 3. opt.
Tithymalus *Pinea* Ger. 402. fig. 6. opt.
Tithymalus Cyparissias C B. 291. 2. T. Esula *Chabr.* 532. Ic. 6. I B. 3. l. 34 p. 663. fig. *Cam.* 964. fig. *Lob.* Obs. 192. fig. 3. Adv. 151. C B in *Matth.* 865. fig. 5. *Lugd.* 1644. fig. 2. opt.
Tithymalus *Cyparissias* minor C B *phyt.* 575. 15.
Tithymalus *Cyparissias* vulg. *Park.* 193. *descr.* 3 but its *Figure* is of the *Pytiusa* or *Esula* major.

Vertues. Dr. *Tournefort* says the *Root* of this *Spurge* is a proper *Cathartick* in *Hydropick* and *Cachectic* Bodies, as also for intermittent *Feavers* given from ʒss to ℈ij with 20 Grains

Grains of *Mercurius dulcis* and 30 of *Cream* of *Tartar*, 10, 15 or 20 of its *Seed* is a ſtrong Purge, and the *Baſis* of *Fernelius* his *Pil. ex Eſula*, its Doſe ℥ij. The *Roots* of this Plant are alſo uſed in the *Benedicta* Laxativa. *Hydragogum* eximium *Renodei*. Extract. *eximium* & Cholagogum *Rolfinchii*.

It *grows* pretty common in many *Gardens*.

82. Common *Sea* Spurge *Ray* Brit. Herbal. *Tab.* 53. fig. 8.

Tithymalus maritimus C B 291. 1. *phyt.* 574. xi.

Tithymalus *Paralius* Chabr. 534. Ic. 6. I B. 3. l 34. p. 674. fig. *Lob.* Ic. 354. 2, Belg. 430. *Obſ.* 191, fig. Camer. 962. *fig.* opt.

Tithymalus paralius *ſeu* maritimus *Park.* 184. fig *Ger.* 401. fig. 1. *Jonſt.* 498. fig 1. *opt.* Matth. *Lugd.* 1643. fig. 2. & 47. fig. 1. *Tab.* 593. Ic 1. Hiſt. 2. p. 292. f. 3. C B in Matth. 864. fig. 3.

This elegant *Spurge* is found on moſt of the *Sea* Coaſts of *Europe*.

83. Small *Portland* Spurge.

Tithymalus *maritimus* minor *Portlandicus*.

From a ſmall downright ſingle *Root*, riſe ſlender *Stalks* with longiſh oval *Leaves* not thick ſet (as in the *common Sea-Spurge*) at the Top come ſmall Umbels of *Flowers* and *Seeds* like other *Spurges*. It *grows* not a Span high even in our *Gardens* where it is very luxuriant.

The Reverend Mr. *William Stoneſtreet*, that accurate *Botaniſt*, firſt diſcovered this about a Year ſince on a narrow Neck of Land covered with *Peebles* which joyns *Portland* with the Coaſt of *Dorſet-ſhire*.

Arbores Piſifloræ. *Trees* with *Pea-blooms*.

84. Bean *Trefoil-tree*. Ray 1721. 1.

Anagyris Cam. 671. fig. *Ger.* 1239. fig. 1. *Jonſt.* 1427. fig. 1.

Anagyris 1. Tabern. 1089. Ic. 2. Hiſt. Vol. 2. p. 794. f. 1.

Anagyris non fætida major vel *Alpina* C B. 391. 3.

Anagyris ſive *Laburnum* majus *Park.* 245 fig. 2. opt.

Anagy-

Anagyris minus fœtens vel *Laburnum* Lob. Ic. 2. p. 49. 2 Belg. 2. p. 56. fig.
Laburnum Chabr. 78. Ic. 1.
Laburnum arbor trifolia *Anagyridi* similis I B. 1. l. xi. p. 361. f.
It *Flowers* in *May*, *June*, &c. On *Saleve* & *Jura* two Mountains near *Geneva* where Mr *Ray* observed it, as also on the *Alps* of *Savoy*.
85. Great podded, *Bastard* Sena *Ray* 923. 1. H Ox. 122. 1.
Colutea *Scorpoides* Lob. Ic. 86. 2. pt. 2. *Belg.* 100. T. 2. *Adv.* 405. fig. *Chabr.* 82. Ic. 1. I B. 1. l. xi. p. 381. fig. *Ger.* 1116. fig. 2. *Jonst.* 1299. fig. 2.
Colutea Scorpioides 1. *Tab.* 1091. Ic. 1 Hist. 695. f. 2.
Colutea *Scorpoides* 1. elatior *Clus* 97. fig.
Colutea *Scorpioides* major *Park.* 227. fig. 2.
Colutea *Scoopioides* elatior & major frutescens *H. Ox.* 122. 1.
Colutea *filiquosa* sive *Scorpioides* major C B 397. 2.
Emerus vulgò *Cæsalp.* 117.
Mr. *Ray* has observed this about *Montpelier*, *Geneva*, &c.
86. Common *Bladder* Bastard *Sena* Ray 1720. 1.
Colutea Ger. 1116. fig. 1. *Jonst.* 1299. fig. 1. *Tab.* 1090. Ic. 2.
Colutea Theoph. *Lob.* Ic. 88. 2. pt. 2. *Belg.* 101. T. 2. *Obs.* 530.
Colutea *Vesicaria* vulgaris *Park* 227. fig. 1. C B. 396. 1.
Colutea Vesicaria *Chabr.* 81. Ic. 6. I B. 1. l. xi. p. 380. fig.
Emeri alterum genus *Cæsalp.* 117.
Grows about *Trent* plentifully, as also on *Mount* Vesuvius, &c.

Arbores Bacciferæ. *Berry-trees.*

87. Widdow-wayl *Ray* 1710. c. 12. Lecaan 83, 50.
Olivella Hispanis. *Garoupe* Narbonens. *Frembd Seidel past.* Germ.
Chamelea Dod. 633. fig. *Cam.* 573. fig. opt.
Chamelea tricoccos. C B. 462. 1 *Clus.* 87. fig. 1 *Hisp.* 170. fig *Chabr.* 46 Ic. 3. I B 1. l. 5. p. 584 fig. *Park.* 202. fig. 5.
Chamelea *Arabum* tricoccos *Ger.* 1215. fig. *Jonst.* 1402. fig. 1. *Tab.* 1075. Ic. 2.
Chamelea *vera* aliis *Tricoccos*, Mezereon *Cam.* Hort. 39.

Chamu-

Chamælea vera, *aliis* Tricoccos *Lob.* Ic. 369. 2. *Adv* 157. fig. *Belg.* 448. fig.

Grows plentifully in Olive Grounds and on the Rocky Hills about *Montpelier.*

88. Plain Ovall *Mock-Privet.*

Phyllyrea folio subrotundo vix serrato.

These *Leaves* much resemble the *Figure* in *Camerarius* his *Epitome* p. 90. marked thus * but its notches less visible.

89. True *Mock-Privet.*

Phyllyrea vera, *folio* acuto, basi lato.

This also is very lightly notcht.

90. Olive *Mock Privet. Ray* Dendr. 53. 2.

Phillyrea *Oleæ Ephesiacæ* folio *Hort.* Med. *Chelf.* Pluk. *Tab.* 310. fig. 5.

These *Leaves* are oblong, pointed and wholly plain.

91. Fine dented, small leaved *Mock-Privet.*

Phyllyrea folio minori argutè serrato.

The only Tree of this I have as yet seen is in the *Wilderness* of the *Bishop* of *London*'s Garden at *Fulham.*

92. Common *Mock-Privet.* Ray 1585. 2.

Phillyrea fol. *Ligustri* C. B. 476. 4.

Phillyrea latiore folio *Ger.* 1209. fig. *Jonst.* 1395. fig. 2.

Phillyrea latiusculo fol. *Chabr.* 42. Ic. 2. I B. 1. l. 5. p. 539. fig.

Phillyrea 3 Clus. 52. fig. 2. *Hisp.* 68. fig.

Phillyrea angustifolia 1. *Park.* 1443 fig. 4. *secundum* Iconem.

Grows wild in *Spain* and about *Montpelier.*

93. Common Narrow *Mock-Privet* Ray. 1585. 1.

Phillyrea angustifolia *Ger.* 1209. fig. 1. *Jonst.* 1595. fig. 1. *Chabr.* 42. Ic. 1. I. B. 1. l. 5. p. 538. fig. *Lob.* Ic. 2. p. 132. Belg. T. 2. p. 154 fig. *Obs.* 565. fig.

Phillyrea angustifolia 1 C B. 476. 5.

Phillyrea 4 Clus. 52. fig. 3. *Hisp.* 64. fig.

Frequent about *Montpelier.*

We shall *Conclude* with the *Indian Herbs and Trees* in the *next* Transactions.

III. *OBSERVATIONES COELESTES* BRITANNICÆ, Grenovici *in Observatorio Regio habitæ, anno* MDCCXIII.

Observationes SATURNI.

Temp. per Horolog.			Tempora correcta.			Die Solis, Januarii 25.	Distantiæ a Vertice		
h.	′	″	h.	′	″		°	′	″
8	30	15	8	28	5	Geminorum πεοπτεs transiit	28	13	20
8	41	7		38	57	Pes Castoris η transiit	28	54	50
8	49	8	8	46	58	Calx ejusdem sive μ transiit	28	50	40
12	10	49	12	8	39	Leonis ψ *Bayero* transiit	36	9	30
12	25	24		23	14	Leonis ν transiit	37	40	30
12	34	21		32	11	Saturni centrum transiit	36	51	45
12	35	44	12	33	34	Cor Leonis transiit	38	7	00
						Ascensio rect. ♄ 147 55 10			
						Dist. a Polo Bor. 75 23 55			
						Longitudo ♌ 25 8 15			
						Latitudo Bor. 1 31 27			

Temp. per Horolog.	Tempora correcta	Die Jovis Februarii 5. 1713.	Distantiæ a Vertice.
h. ′ ″	h. ′ ″		
11 20 52	11 14 10	Leonis ξ *Bayero* transiit --	38 55 20
30 9	23 27	Leonis *o* transiit	40 17 30
35 18	28 36	Leonis 16*ta Cat. Brit.* transf.	38 21 20
47 8	40 26	Leonis ν transiit	37 40 30
52 42	46 00	Saturni centrum transiit	36 32 50
11 57 25	50 43	Cor Leonis transiit	38 7 5
12 0 35	53 53	Leonis 31*ma* transiit	36 42 50
5 40	11 58 58	Leonis 34*ta* transiit	36 19 30
12 10 50	12 4 8	Leonis 38*va* transiit	35 3 50
		Ascensio recta ♄ 147 4 45	
		Distant a Polo 75 5 00	
		Longitudo ♌ 24 14 8	
		Latitudo Bor. 1 32 16	
		Die Veneris Februarii 6.	
8 22 53	8 14 43	Lucidus pes Pollucis, ♊γ tr.	34 52 00
11 29 22	11 21 12	Leonis ψ transiit	36 9 15
11 43 58	11 35 48	Leonis ν transiit	37 40 30
11 49 10	11 41 0	Saturni centrum transiit	36 31 00
		Ascensio recta ♄ 146 59 0	
		Distantia a Polo 75 3 10	
		Longitudo ♌ 24 8 17	
		Latitudo Bor. 1 32 8	
		Die Mercurii Febr. 18.	
11 0 15	10 53 0	Saturni centrum transiit --	36 12 00
11 8 53	11 1 38	Cor Leonis transiit	38 7 5
11 17 7	11 9 52	Leonis 34*ta Cat. Brit.* transf.	36 19 35
11 22 16	11 15 1	Ejusdem 38*va* transiit	35 3 55
		Ascensio rect. ♄ 146 5 00	
		Distantia a Polo 74 44 10	
		Longitudo ♌ 23 12 47	
		Latitudo Bor. 1 32 41	

Die

Temp. per Horolog.	Tempora correcta.	Die Lunæ Martii 2. 1713.	Distantiæ a Vertice
h. ′ ″	h. ′ ″		° ′ ″
10 6 30	9 52 3	Leonis ♆ transiit	36 9 30
19 27	10 5 00	Saturni centrum transiit	35 55 20
31 21	16 54	Cor Leonis transiit	38 7 5
34 31	20 4	Leonis 31*ma* transiit	36 42 50
10 39 35	10 25 8	Leonis 34*ta* transiit.	36 19 40
		Ascensio recta ♄ 145 16 00	
		Distant. a Polo 74 27 30	
		Longitudo ♌ 22 22 40	
		Latitudo Bor. 1 32 52	
		Die Martis Aprilis 7.	
7 43 1	7 41 15	Leonis ♆ transiit	36 9 25
50 46	7 49 00	Saturni centrum transiit	35 31 10
57 36	55 50	Leonis *v Bayero* transiit	37 40 20
8 6 39	8 4 53	Leonis in collo *n* transiit	33 19 25
16 7	14 21	Ejusdam 34*ta Cat. Br.* transi.	36 19 30
8 21 17	8 19 31	Ejusdem 38*va* transiit	35 3 50
		Ascensio recta ♄ 143 57 45	
		Distantia a Polo 74 3 15	
		Longitudo ♌ 21 3 32	
		Latitudo 1 31 20	
		Die Mercurii Aprilis 8.	
7 47 43	7 46 00	Saturni centrum transiit	35 31 15
54 34	52 51	Leonis *v* transiit	37 40 20
8 3 37	8 1 54	Leonis *n* transiit	33 19 20
13 4	11 21	Leonis 34*ta* transiit	36 19 35
8 18 14	8 16 31	Leonis 38*va* transiit	35 3 45
		Ascensio recta ♄ 143 57 30	
		Distantia a Polo 74 3 20	
		Longitudo ♌ 21 3 20	
		Latitudo 1 31 10	
		Saturno pene stationario	Die

Temp. per Horolog.	Tempora correcta.	Die Jovis Novemb. 5. 1713.	Distantiæ à Vertice
h. ′ ″	h. ′ ″		° ′ ″
18 22 00	18 15 37	Leonis π in genu seq. transf	42 4 00
30 8	23 45	Cor Leonis transiit	38 7 5
54 35	48 12	Leonis in Axilla ρ transiit	40 42 0
19 11 5	19 4 42	Leonis in ventre l transiit	39 25 15
19 26 23	19 20 00	Saturni centrum transiit	42 10 40
		Ascensio rect. ♄ 162 23 20	
		Distantia à Polo 80 43 00	
		Longitudo ♍ 10 13 40	
		Latitudo Bor. 1 39 37	

Observationes JOVIS.

Anno MDCCXIII.

Temp. per Horolog.	Tempora correcta.	Die Solis Augusti 9.	Distantiæ a Vertice.
h ′ ″	h ′ ″		° ′ ″
12 40 4	12 37 27	Aquarii λ in effusione Aquæ transiit	60 32 50
12 48 37	12 46 0	Jovis centrum transiit	60 48 35
12 52 36	12 49 59	Aquarii 73ta *Cat. Brit.* prima ad *h* transiit	60 46 20
13 4 21	13 1 44	Aquarii in aqua χ transiit	60 49 10
		Ascens. rect. ♐ 341 33 5	
		Dist. a Polo Bor. 99 21 40	
		Longitudo ♓ 9 26 00	
		Latitudo Aust. 1 25 8	

Die

Temp. per Horolog.	Tempora correcta	Die Lunæ Augusti 10. 1713.	Distantiæ a Vertice.
h. ′ ″	h. ′ ″		° ′ ″
12 36 21	12 33 55	Aquarii λ transiit	60 32 50
12 44 26	12 42 00	Jovis centrum transiit	60 52 00
12 48 53	12 46 27	Aquarii 73*ia* transiit	
		Ascens. rect. ♐ 341 26 5	
		Distant. a Polo 99 25 5	
		Longitud. Jov. ♓ 9 18 17	
		Latitudo Aust. 1 25 40	
		Die Lunæ Octobris 26.	
7 29 16	7 28 42	Aquarii in Clune σ transiit	63 34 40
7 36 34	7 36 0	Jovis centrum transiit	63 00 5
8 14 34	8 14 0	Aquarii 80*ma* prima ad ♆ tr	62 5 20
8 17 45	8 17 11	Aquarii 84*ta* seq. ad ♆ transi.	62 37 5
		Ascensio recta ♐ 335 41 30	
		Distantia a Polo 101 33 20	
		Longitudo Jovis ♓ 3 16 00	
		Latitudo Aust. 1 19 8	
		Die Martis Octob. 27.	
7 25 40	7 23 34	Aquarii Clunis σ transiit	63 34 35
7 33 6	7 31 00	Jovis centrum transiit	62 59 15
8 11 00	8 8 54	Aquarii prima ad ♆ transiit	62 5 15
8 14 10	8 12 4	Sequens ad ♆ transiit	62 37 10
		Ascensio rect. ♐ 335 43 20	
		Distantia a Polo 101 32 30	
		Longitudo ♓ 3 17 58	
		Latitudo Aust. 1 19 00	
		Die Jovis Octob. 29.	
7 18 29	7 15 19	Aquarii σ transiit	63 34 40
26 10	7 23 00	Jovis centrum trasiit	62 57 20
8 3 47	8 0 37	Prima ad ♆ transiit	62 5 20
6 55	8 3 45	Sequens ad ♆ transiit	62 37 10

Die Jovis Octob. 29.

Ascensio recta	♑ 335	47	45
Distantia a Polo	101	30	35
Longitudo Jov.	♐ 3	22	41
Latitudo Aust.	1	18	49

Observationes MARTIS.
Anno MDCCXIII.

Temp. per Horolog.			Tempora correcta.			Die Mercurii Feb. 18. 1713.	Distantiæ a Vertice		
h.	′	″	h.	′	″		°	′	″
12	28	38	12	21	20	Leonis in poplite τ transiit	47	2	15
12	51	6	12	43	48	In ancone Alæ ♍ β transiit	48	5	40
13	10	21	13	3	3	Virginis 10*ma Cat. Brit.* r tr.	47	57	40
13	13	18	13	6	00	Martis centrum transiit	47	2	5
13	21	8	13	13	50	In cervice Virginis c transiit	46	33	30
						Asc. rect. *Martis* 179 29 20			
						Distantia à Polo 85 34 35			
						Longitudo ♍ 27 46 00			
						Latitudo Bor. 3 51 10			

Die Martis Martii 3.

11	55	52	11	53	2	In vertice Virginis r transf.	43	20	00
12	4	50	12	2	00	Martis centrum transiit	45	14	20
	10	54		8	4	In vultu Virginis π transiit	43	15	25
	20	8		17	18	Undecima Virginis s transiit	44	3	30
12	30	25		27	35	Virginis 16. in Cervice c tr.	46	33	25
12	32	37	12	29	47	Virginis 17*ma Cat Br* transf.	44	33	45
						Asc. rect. *Martis* 175 1 15			
						Dist. a Polo Bor. 83 46 45			
						Longitudo ♍ 22 57 33			
						Latitudo Bor. 3 43 37			

Die

Temp. per Horolog.	Tempora correcta.	Die Martis Aprilis 7. 1713.	Distantiæ a Vertice
h. ′ ″	h. ′ ″		° ′ ″
9 4 47	9 3 10	Sub Ventre Leonis ♌ transf.	42 35 25
9 17 37	9 16 00	*Martis* centrum transiit	42 42 50
9 38 10	9 36 33	Prima Virg. *Cat. Br.* ω transf.	41 44 50
9 45 00	9 43 23	Borea in Vertice ♍ & transf.	41 37 15
		Asc. rect. *Martis* 165 45 40	
		Distantia a Polo 81 15 10	
		Longit. *Martis* ♍ 13 30 40	
		Latitudo Bor. 2 26 31	

		Die Mercurii Aprilis 8.	
9 1 44	9 0 29	Leonis ♌ transiit	42 35 30
9 14 15	9 13 00	*Martis* centrum transiit	42 43 40
9 35 7	9 33 52	Virginis ω transiit	41 44 55
9 41 58	9 40 43	Virginis & transiit	41 37 15
		Asc. rect. *Martis* 165 41 00	
		Distantia a Polo 81 16 00	
		Longitud. *Martis* ♍ 13 26 45	
		Latitudo Bor. 2 23 58	

		Die Veneris Maii 1.	
7 55 9	7 50 00	*Martis* centrum transiit	44 17 30
8 18 12	8 13 3	In Vertice Virginis ν transf.	43 20 00
8 33 14	8 28 5	In Vultu Virginis π transf	43 15 30
		Asc. rect. *Martis* 166 59 40	
		Distantia a Polo 82 49 50	
		Longitudo ♍ 15 15 00	
		Latitudo Bor. 1 27 40	

Temp. per Horolog.	Tempora correcta.	Die Saturni Maii 2. 1713.	Distantiæ a Vertice.
h ′ ″	h ′ ″		° ′ ″
7 52 45	7 47 00	*Martis* centrum transiit	44 24 20
8 15 7	8 9 22	Virginis ν transiit	43 20 5
8 30 8	8 24 23	Virginis π transiit	43 15 25
		Asc. rect. *Martis* 167 10 00	
		Distantia a Polo 82 56 40	
		Longitudo ♏ 15 27 5	
		Latitudo Bor. 1 25 20	

Observationes LUNÆ.

Anno MDCCXIII.

		Die Solis Januarii 25.	
8 9 33	8 7 23	Telescopica α transiit	28 26 20
8 15 5	8 12 55	Tauri 123*ia Cat. Brit.* transi.	27 2 30
8 20 20	8 18 10	Lunæ limbus præced. transiit, centro a Vertice	27 32 40
8 21 23	8 19 13	Lunæ centrum transi. limbo remoto a Vertice	27 47 40
8 22 52	8 20 42	Lunæ cuspis Bor. a Vertice	27 17 40
8 30 15	8 28 5	Geminorum πρῶτος transiit	28 13 20
8 41 7	8 38 57	Pes Castoris η transiit	28 54 50
8 49 8	8 46 58	Calx ejusdem μ transiit	28 50 40
		Asc. rect. cent. ☽ 84 26 55	
		Dist. a Polo visa 66 4 40	
		Sed adhibit. Paral. 65 39 50	
		Longit. Lunæ ♊ 24 56 30	
		Latitudo Bor. 0 57 00	

Temp. per Horolog.	Tempora correcta.	Die Lunæ Januarii 26. 1713.	Distantiæ a Vertice
h. ′ ″	h. ′ ″		° ′ ″
8 26 41	8 24 36	Propus transiit	28 13 30
8 37 31	8 35 26	Pes Castoris *n* transiit	28 54 50
8 45 31	8 43 26	Calx Castoris *μ* transiit	28 50 40
9 9 43	9 7 38	Lunæ limbus præcedens transiit, centro a Vertice	28 45 00
9 10 50	9 8 45	Lunæ centrum transf. limbo remoto a Vertice	29 0 5
9 12 30	9 16 25	Lunæ cuspis Bor. a Vertice	28 30 10
9 27 50	9 25 45	♈orum 46*ta Cat. Br.* transf.	28 26 10
9 42 44	9 40 39	♈orum in Inguine ♂ transf.	28 59 30
		Asc. rect. cent. ☽ 97 43 50	
		Dist. a Polo visa 67 17 5	
		Adhibitâ Parallaxi 66 51 15	
		Longit. Lunæ ♋ 7 6 18	
		Latitudo Aust. 0 8 48	

Observationes SATELLITUM JOVIS.

		Die Veneris Octob. 30.
6 56 30	6 52 35	Quartus Satelles visus est emergens ab umbra, diametro Jovis distans a *tertio* ei proximo ad dextram, Tubo scil. octo pedum.
7 4 00	7 00 00	Clare explenduit, & linea ducta a proximo illo per centrum Jovis emergentem reliquit ad Austrum, *situ scilicet inverso.*
7 36 31	7 32 30	Pegasi *μ* transiit per planum Arcus meridionalis.
		Die Saturni Novemb. 7.
7 13 2	7 5 00	Secundus Satelles emergebat, vel potius emergere incipiebat. Tubo octo pedum
2 5 11	8 57 00	Piscium ♂ in Lino australi transiit. *N.B.*

N.B. *Stella illa Telescopica* a *quæ die Januarii* 25° *Lunam præcessit, Ascensionem rectam tunc habuit* 81° 28′ $\frac{1}{2}$, *& distabat a Polo* 66° 58′ 20″, *unde fit Longitudo ejus* ♊ 22° 9′ $\frac{1}{3}$ *cum Latitudine Australi* 0° 13′ $\frac{1}{2}$. *Hæc autem est ea ipsa stella ad quam applicabatur Jupiter in Statione secunda, anno* 1634 *Februarii* 6, *eamque non nisi tribus sui corporis diametris ad Austrum reliquit, observante* Gassendo: *ut habetur inter Observata ejus* pag 174. *Et ad eandem* Mars *observatus est Septembris* 6*to anno* 1644 *mane, ut videre est in Prolegomenis Selenographiæ* Hevelianæ pag. 65 *&* Fig. 1. *Veram multum usui erit, ad accuratam Nodi Jovis determinationem, ejusque motus, si modo inter stellas fixas planum orbitæ Jovialis non hæreat immobile. Etenim post decursum* 83 *annorum, quibus Jupiter satis accurate septem absolvit periodos, anno stil.* 1717. *Januarii* 10. *mane, Planeta stellam illam corporaliter teget vel saltem stringet, spectaculo quidem raro neque hactenus quod sciam Astronomis in Jove concesso.*

Stella autem ipsa, etiamsi Telescopica vocetur, sudo cælo & absente Lunâ inermis oculi aciem non fugit; comitemque habet sequentem ad Austrum, & semidiametro Solis circiter distantem, apud quam conspicietur Jupiter arctissime conjunctus, Die vicesimo Julii anni proximi 1716 *mane.*

IV. *An Account of an Experiment made by Dr.* Brook Taylor *assisted by Mr.* Hawkesbee, *in order to discover the Law of the Magnetical Attraction.*

BY Order of the Royal Society Mr. *Hawkesbee* and my self made an Experiment with the great Loadstone belonging to the Royal Society, in order to discover the Law of the Magnetical Atrraction; and not long

after I gave an account of it to the Society in a Letter to Dr. *Sloane*, (who was then Secretary) dated *June* 25. 1712. Since that, Mr. *Hawkesbee* made another Experiment of the ſame nature with a ſmaller Loadſtone; which he has given an account of in the *Philoſophical Tranſactions* No. 335. But upon comparing the Numbers of that Experiment with thoſe of the other, I find the Numbers of the firſt Experiment to be very much more regular. Wherefore I conclude that to be the beſt Experiment, and ſince no notice has been taken of the Account I gave of it, and I have reaſon to believe Mr. *Hawkesbee* loſt the Table I left with him for the Society, of the Numbers relating to it, I take this occaſion to preſent the Society with the following Account of it.

We placed the great Loadſtone belonging to the Royal Society ſo, that it's two Poles lay in the Plane of the Horizon, and were in a Line exactly at right Angles with the natural Direction of the Needle we made uſe of, (which was that Dr. *Halley* had made to obſerve the Variations with). And by means of a Carriage contrived for that purpoſe, the Stone was eaſily moved to and fro, the Poles continuing always in the ſame Line. The Needle was ſo placed, that the Center it play'd upon was in the ſame Line with the Poles of the Stone; the North Pole being towards the Needle. We meaſured the Diſtances from the Center of the Needle to the Extremity of the Stone; and we found the Variations of the Needle from its natural Poſition to be as in the following Table.

Diſtant. Feet	Variat. ° ′	Diſtant. Feet	Variat. ° ′	Diſtant. Feet	Variat. ° ′
1	81 45	4	16 0	7	3 30
2	58 00	5	9 20	8	2 20
3	30 00	6	5 35	9	1 35

V. A

V. *A short Account of the Cause of the Saltness of the Ocean, and of the several Lakes that emit no Rivers; with a Proposal, by help thereof, to discover the Age of the World. Produced before the* Royal-Society *by* Edmund Halley, R. S. Secr.

THere have been many Attempts made and Proposals offered, to ascertain from the Appearances of Nature, what may have been the Antiquity of this Globe of *Earth*; on which, by the Evidence of Sacred Writ, *Mankind* has dwelt about 6000 Years; or according to the *Septuagint* above 7000. But whereas we are there told that the Formation of *Man* was the last Act of the *Creator*, 'tis no where revealed in Scripture how long the *Earth* had existed before this last Creation, nor how long those five Days that preceeded it may be to be accounted; since we are elsewhere told, that in respect of the Almighty a thousand Years is as one Day, being equally no part of *Eternity*; Nor can it well be conceived how those Days should be to be understood of natural Days, since they are mentioned as Measures of Time before the Creation of the Sun, which was not till the Fourth Day. And 'tis certain *Adam* found the *Eerth*, at his first Production, fully replenished with all sorts of other *Animals*. This Enquiry seeming to me well to deserve Consideration, and worthy the Thoughts of the *Royal Society*, I shall take leave to propose an Expedient for determining the Age of the World by a *Medium*, as I take it, wholly new, and which in my Opinion seems to promise success, though the Event cannot be judged of till after a long Period of Time; submitting the same to their better Judgment.

ment. What ſuggeſted this *Notion* was an Obſervation I had made, that all the *Lakes* in the *World*, properly ſo called, are found to be *Salt*, ſome more ſome leſs than the Ocean Sea, which in the preſent caſe may alſo be eſteemed a *Lake*; ſince by that term I mean ſuch ſtanding Waters as perpetually receive Rivers running into them, and have no Exite or Evacuation.

The Number of theſe Lakes, in the known Parts of the World is exceeding ſmall, and indeed upon Enquiry I cannot be certain there are in all any more than four or five, *viz.* firſt, The *Caſpian Sea*; ſecondly, The *Mare Mortuum* or *Lacus Aſphaltites*; thirdly, The Lake on which ſtands the City of *Mexico*, and fourthly, The Lake of *Titicaca* in *Peru*, which by a Channel of about fifty Leagues communicates with a fifth and ſmaller, call'd the Lake of *Paria*, neither of which have any other Exite. Of theſe the *Caſpian*, which is by much the greateſt, is reported to be ſomewhat leſs ſalt than the Ocean. The *Lacus Aſphaltites* is ſo exceedingly Salt, that its Waters ſeem fully ſated, or ſcarce capable to diſſolve any more; whence in Summer-time its Banks are incruſtated with great Quantities of dry *Salt*, of ſomewhat a more pungent nature than the *Marine*, as having a Reliſh of *Sal Armoniac*; as I was informed by a curious Gentleman that was upon the place.

The *Lake* of *Mexico* properly ſpeaking is two Lakes, divided by the *Cauſways* that lead to the *City*, which is built in Iſlands in the midſt of the *Lake*, undoubtedly for its Security; after the Idea, tis probable, its firſt Founders borrowed from their *Beavers*, who build their Houſes on Damms they make in the Rivers after that manner. Now that part of the *Lake* which is to the Northwards of the *Town* and *Cauſways*, receives a River of a conſiderable magnitude, which being ſomewhat higher than the other, does with a ſmall Fall exonerate it ſelf in the Southern

part, which is lower. Of these the lower is found to be salt, but to what degree I cannot yet learn; though the upper be almost fresh.

And the Lake of *Titicaca*, being near eighty Leagues in circumference, and receiving several considerable fresh Rivers, has its Waters, by the Testimony of *Herrera* and *Acosta*, so brackish as not to be potable, though not fully so salt as that of the Ocean; and the like they affirm of that of *Paria*, into which the Lake of *Titicaca* does in part exonerate it self, and which I doubt not will be found much salter than it, if it were enquired into.

Now I conceive that as all these Lakes do receive Rivers and have no *Exite* or Discharge, so 'twill be necessary that their Waters rise and cover the Land, until such time as their Surfaces are sufficiently extended, so as to exhale in Vapour that Water that is poured in by the Rivers; and consequently that Lakes must be bigger or lesser according to the Quantity of the fresh they receive. But the Vapours thus exhaled are perfectly fresh, so that the saline Particles that are brought in by the Rivers remain behind, whilst the fresh evaporates; and hence 'tis evident that the Salt in the Lakes will be continually augmented, and the Water grow salter and salter. But in Lakes that have an *Exite*, as the Lake of *Genesaret*, otherwise call'd that of *Tiberias*, and the upper Lake of *Mexico*, and indeed in most others, the Water being continually running off, is supply'd by new fresh river Water, in which the saline Particles are so few as by no means to be perceived.

Now if this be the true Reason of the Saltness of these Lakes, tis not improbable but that the Ocean it self is become salt from the same Cause, and we are thereby furnished with an Argument for estimating the Duration of all Things, from an Observation of the Increment of Saltness in their Waters. For if it be observed what Quantity of Salt is at present contained in a certain Weight of the

Water of the *Caspian Sea*, for example, taken at a certain Place, in the dryest Weather; and after some Centurys of Years the same Weight of Water, taken in the same place and under the same Circumstances, be found to contain a sensibly greater Quantity of Salt than at the time of the first Experiment; we may by the Rule of Proportion, take an estimate of the whole time wherein the Water would acquire the Degree of Saltness we at present find in it.

And this Argument would be the more conclusive, if by a like Experiment a simular Encrease in the Saltness of the *Ocean* should be observed: for that, after the same manner as aforesaid, receives innumerable Rivers, all which deposite their saline Particles therein; and are again supplyed, as I have elsewhere shewn, by the *Vapours* of the *Ocean*, which rise therefrom in Atoms of pure Water, without the least admixture of Salt. But the Rivers in their long Passage over the Earth do imbibe some of the saline Particles thereof, though in so small a Quantity as not to be perceived, unless in these their Depositories after a long Tract of time. And if upon repeating the Experiment, after another equal Number of Ages, it shall be found that the Saltness is further encreased with the same Increment as before, then what is now proposed as *Hypotheticall* would appear little less than Demonstrative. But since this Argument can be of no use to Ourselves, it requiring very great Intervals of time to come to our Conclusion, it were to be wished that the ancient *Greek* and *Latin* Authors had delivered down to us the degree of the Saltness of the Sea, as it was about 2000 Years ago: for then it cannot be doubted but that the Difference between what is now found and what then was, would become very sensible. I recommend it therefore to the *Society*, as opportunity shall offer, to procure the Experiments to be made of the present degree of Saltness of the Ocean, and of as many of these Lakes as can be

come at, that they may ſtand upon Record for the benefit of future Ages.

If it be objected that the Water of the *Ocean*, and perhaps of ſome of theſe *Lakes*, might at the firſt Beginning of Things, in ſome meaſure contain Salt, ſo as to diſturb the Proportionality of the Encreaſe of Saltneſs in them, I will not diſpute it: But ſhall obſerve that ſuch a Suppoſition would by ſo much contract the Age of the World, within the Date to be derived from the foregoing Argument, which is chiefly intended to refute the ancient Notion, ſome have of late entertained, of the Eternity of all Things; though perhaps by it the World may be found much older than many have hitherto imagined.

Accounts of BOOKS.

I. Linear Perſpective, *or a New Method of repreſenting juſtly all manner of Objects*, &c. *By* Brook Taylor, *L. L. D. and R. S. Secr.* 8vo. London, 1715.

THE Author of this Book, finding the Art of Perſpective to be very imperfect in the Books that have hitherto been publiſh'd on that Subject, thought it worth his while to conſider the whole matter anew; and from a careful Examination of the Principles this Art is founded upon, he has endeavoured to eſtabliſh ſome Theorems, by means of which the Practice of it might be render'd more general and eaſy than has yet been done. In order to this, at firſt ſight he found it neceſſary to make uſe of new Terms of Art; the old ones ſeeming not to

be expreſſive enough of what is meant by them, and being adapted to too confined an Idea of the Principles of this Art. In the old Perſpective the chiefeſt regard is had to the Ground Plane, that is, the Plane of the Horizon; from whence is derived the *Horizontal Line*, and by means of that Line the Repreſentations of ſome Figures are found by good ſimple Conſtructions. But then the Figures in all other Planes are drawn by reducing them to the Horizontal Plane by means of Perpendiculars; which is an inartificial round-about way, makes a great Confuſion of Lines, and is not capable of ſo much Exactneſs. This confined way of treating this Subject, proceeds from the ſtrong Poſſeſſion the Mind is bred up in of the Notions of Upwards and Downwards, which makes one apt to refer all other irregular Poſitions to thoſe principal ones. But the Minds of all Artiſts ſhould be drawn as much as can be from ſuch confined Ways of thinking, and they ſhould be taught to accuſtom themſelves, as much as may be, to conſider Nature in its general View, without minding thoſe particular Relations which things have with reſpect to themſelves. For this reaſon our Author has rejected the Term of *Horizontal Line*, becauſe it confines the Mind too much to the particular conſideration of the Horizontal Plane: but he conſiders all Planes alike, and all Figures as they are in themſelves, without conſidering their Relation to us; leaving the Artiſt to do that, when he comes to apply the general Rules of practice to any particular Deſign.

This Treatiſe is very ſhort, becauſe the Author has confined himſelf only to give the general Rules of practice, leaving the Reader to himſelf or to a Maſter to find out particular Examples to exerciſe himſelf in. Yet he hopes he has omitted nothing that is material to the underſtanding of this Art in the full extent of it. The whole Book conſiſts of five Sections,

The first Section contains an Explanation of the fundamental Principle of this Art, with the Definitions of the Terms, and four Theorems. The fundamental Principle of this Art, is, that the Representation of any Point is a Point on the Picture where it is cut by a Line drawn from the original Point really placed where it ought to seem to be, to the place of the Spectators Eye; and consequently, the Representation of any Line is the Intersection of the Picture with a Surface made by drawing Lines from the place of the Spectator's Eye, to the several Points of the original Line to be represented, really placed where it ought to seem to be. For these Lines which come from the several Points of the original Object to be placed in its proper Situation, to the Spectator's Eye, are as so many visual Rays which make the Object sensible.

When a Right Line is continued *in infinitum*, the Visual Ray becomes at last parallel to it, and an Object of any given bigness, if it goes still further and further off on that Line, will at last seem to vanish; and at that time the place of its Representation on the Picture is the Point where the Ray parallel to the original Line cuts the Picture. For this reason our Author has thought it proper to call that Point the *Vanishing Point* of such an original Line (and consequently of all others parallel to it (Def. 5.) And for the same Reason he calls that Line on the Picture a *Vanishing Line* (Def. 6) which is produced by the Intersection of the Picture with a plane passing thro' the Spectator's Eye parallel to an original Plane. There are ten Definitions in all, but these are the principal. And in our Author's Method these Vanishing Points and Vanishing Lines are of great use for the Representation of any Line passing through its vanishing Point. (Prop. 1.) Having found the Representation of one Point in any Line, by any Method whatsoever, he finds the Representation of the whole Line by its vanishing Point, which he shews an

easy

eaſy Way to find in Propp 6, 8, 12. which are in the ſecond Section. And by this means he ſolves ſeveral Problems in Perſpective, which it is not poſſible to do by the common Way, at leaſt without a great deal of Difficulty, and a great Confuſion of Lines. And by this Method he ſhews how the compleat Repreſentations of any propoſed Figures may be found, having given the Repreſentation only of ſome principal parts of them. The ſecond Section contains ſeveral Propoſitions to that purpoſe, ſhewing how to find the vaniſhing Points and Lines of propoſed Lines and Planes, according to the ſeveral Circumſtances propoſed; and by the means of them, how to find the Repreſentation of any given Figure. In the End of this Section there are ſome Examples, in the Deſcription of the regular Solids and ſome other Figures.

The third Section ſhews how to find the Repreſentation of the Shadows of all Objects.

The fourth Section ſhews how to find the Repreſentations of the Reflexions of Figures made by poliſh'd Planes.

The fifth Section contains a few Propoſitions relating to the inverſe Method of Perſpective; or the manner of examining a Picture already drawn; ſo as to find out what Point the Picture is to be ſeen from, or having that given, to find what the Figures are which are deſcribed on the Picture.

Our Author has obſerved that there may be a very good Expedient made uſe of in painting of large Rooms and Churches, which is drawn from the Nature of thoſe Rays which produce the Vaniſhing Points. This not being mention'd in the Book it ſelf, he thinks it not improper to take notice of it here: The Expedient is this, Having ſome way or other found the Repreſentation of one Point of a Line that is wanted in the Picture, to find the whole Line, paſs a Thread ſtretch'd through the place of the Spectator's Eye, in a Direction parallel to the Direction the original

ginal Line ought to be in, and the Shadow of that Thread caſt by a Candle, ſo as to paſs through the given Point on the Picture will be the Repreſentation ſought. The reaſon of this Conſtruction is, becauſe the Rays of Light that paſs from the Candle to the Thread ſo ſtretch'd, make the Plane which generates the Repreſentation ſought. (ſee Prop. 1.) And there may be other Expedients of the like nature gather'd from the ſame Principle.

II. *DUCATUS LEODIENSIS*, Or, *The* Topography *of the ancient Town and Pariſh of* LEEDS *and Parts adjacent, in the County of* YORK, &c. *By* Ralph Thoresby, *Eſq; Fellow of the* Royal-Society, *London. Fol.* 1715.

THO' the diligent and curious Author of this Work do not profeſſedly treat of any Place but the ancient Town and Pariſh of *Leedes*, and the *Regio Leodis*, or adjoyning Territory called *Elmet*; yet not only the Preface is more general, relating to the County, but there are many Paſſages in the Book it ſelf, wherein he takes occaſion to inſert the Pedigrees of ſuch of the Nobility and Gentry, as have had any Eſtates within the preſcribed Limits, tho' the chief Seat of the Family be diſtant; as eſteeming all *Provinciales*, who have but *Domicilium in Provincia*: to ſome of theſe he hath premiſed ſeveral Deſcents from ancient Deeds yet remaining in the reſpective Families; and to moſt of thoſe that are inſerted in the Viſitations in the College at Arms, *London*, he hath added the Dates from Original Deeds, Regiſters, *&c.* and continued them to the preſent time, which hath rendred

it

it ſo acceptable to the learned Gentlemen of that Faculty, that Four Kings at Arms, and ſome eminent Heralds, have not only ſubſcribed, but ſince their Peruſal thereof, bought others for their abſent Friends, expreſſing great ſatisfaction in that part of the Performance: as many learned Antiquaries have done in the other Parts relating to the Topography and Etymology of the Names of Places, &c. which he hath been very particular in, as finding the Name to be frequently a brief Deſcription of the Place; and hath been thereby enabled to diſcover the *Veſtigia* of ſome conſiderable Antiquities, in the actual Survey that he made of thoſe Places to render the Work more compleat: He hath, by the ancient Names and the Situation of the places, been enabled to deſcribe, in a very particular manner, the Tranſactions between the *Pagans* and Primitive *Chriſtian Saxons*, relating to that noted Battle upon *Win-moor*, An. Dom 655. There are alſo many very conſiderable Benefactions, and ſtately Edifices erected of later times, particularly a magnificent *Church* built and endowed by Mr. *Harriſon*; whoſe Nephew the Reverend Mr. *Robinſon* hath moſt generouſly promis'd to endow another Church, which, it is hoped, will be ſhortly erected in that populous Town of *Leeds*, to the building of which ſeveral of the Magiſtrates, particularly Mr. *Milner* (who hath adorned the Market-place with a moſt noble Marble Statue of Her late Majeſty placed in the Front of the *Guild-hall*) and other Inhabitants have ſubſcribed very liberally. Here is alſo a Charity-School for an Hundred poor Children, who are cloathed and taught here, *&c.*

But what relates more immediately to theſe *Philoſophical Tranſactions*, is the annexed Catalogue of the Authors *Muſæum*, juſtly celebrated for *Antiquities* and for *natural* and *artificial Curioſities*. The Catalogue of the Coins and

Medals is surprizingly copious and valuable. To the ancient *Greek* and *Consular*, or Family-Monies of the *Romans*, he hath added above a thousand *Imperial*, several of which are noted by the learned Baron *Spanhemius* as very rare; and so likewise are those justly esteemed that relate more immediately to *Britain*, whether minted by the *Romans* or *Britains*. That of *Thor* with *Runic* Letters is inestimable, being the only known Piece in the World with those ancient Characters upon it. This was first deciphered by the Right Reverend Dr. *Nicholson* Lord Bishop of *Carlisle*, and after by Dr. *Hicks*, the two great Revivers of that sort of Literature. Upon which single Medal a learned Foreigner hath printed a distinct Treatise. *And the ingenious Sir *Andrew Fountain* in his *Dissertatio Epistolaris* to the Right Honourable *Thomas* Earl of *Pembroke*, saith expresly "*Numismatum omnium quæ aut* "Anglo-Saxonibus, *aut* Anglo-Danis *in usu fuisse videntur*, "*nullum notatu dignius est, quam id literis* Runicis *inscrip-* "*tum, quod possidet vir genere & ingenio clarus* Radulphus "Thoresbeius, Leodiensis." Those of the *Saxon* Kings begin with a very choice one of *Edwin* the ancientest Coin of the *English* Nation, and of the first *Christian* King of *Northumberland*; and are succeeded by those of the *Danish* and *Norman* Lines, and continued to the present Age, in a great Variety of current Monies and Medals in Gold, Silver and Copper. Those of *Ireland* and the *English Plantations* in *America*, are interspers'd in the several Reigns: but those of *Scotland*, from the first of the *Alexanders*, are so numerous and valuable as to merit a particu-

---

* De Argento insignito *Runis* seu literis Gothicis, *&c.* Sententia *Nicholai Kederi*, Regii Antiquitatum Collegii, quod Helmiæ est, Assessoris. *4to* 1703. Lipsiæ.

lar

lar Deſcription. All along are very inſtructive Directions how to diſtinguiſh the Kings of the ſame Name from one another, before the Numbers were added upon their Monies. The *Roman* Emperours and *Saxon* Kings being well engraved before, the chief Defect and Difficulty is in thoſe from *William* I. to *Henry* VII. which are therefore delineated here from the Originals. To theſe are prefixed the moſt ancient Conſular Monies, which many Ages preceeded the Incarnation of our Bleſſed Saviour, becauſe never yet extant in any *Engliſh* Author. The other Medals and Monies of Popes, Emperours, Kings and Republicks, muſt be omitted for brevity's ſake, tho' ſome of them (particularly that of the Siege of *Leyden* in *Paſtboard*) be very rare.

The *Natural Curioſities* are ranked in the following Method, 1. *Human* Rarities, 2. *Quadrupeds*, Viviparous (multifidous and bifidous) and Oviparous, with an Account of certain Balls and Stones found in the Stomachs of ſeveral Animals. 3. *Serpents*. 4. *Birds*, Land and Water-Fowls with their Eggs. 5. *Fiſhes*, viviperous and oviparous, ſcaled and exanguious. 6. *Shells*, whirled and ſingle, double and multiple. 7. *Inſects*, with naked and with ſheathed Wings, and creeping Inſects. 8. *Plants*, which begin with Dr. *Nicolſon*'s Collection of above 800 dry'd Plants; the reſt are reduced to the accurate Method of Dr. *Sloane*, in his *Cat. Plant. in Inſula Jamaicæ*, proceeding from the Corals and other Submarines to the Fruits and Parts of Trees. 9. *Formed Stones*, which are ranged according to Mr. *Llwyd*'s curious Tract, *Lithophylac. Britan.* only to the Cryſtals and Diamonds are premiſed the *Margaritæ Cumbrenſes*, ſome of which have as good a Water as the Oriental. After the foſſile Shells and Stones of the turbinated Kind, the Bivalves and Shells amaſſed together into great Stones by a petrifyed Cement,

follow the Marbles and other Stones irregular. 10. The *Metals* Ores, Salts and Ambers, of which one with a Fly, another with a Spider enclosed.

The *Artificial* Curiosities relate to *War*, as *Indian* and *Persian* Bows, Arrows, Darts, Armour, Shields, Targets, Tomahaws, poisoned Daggers: to the *Mathematicks*, to *Houshold-stuff*, *Habits*, &c. from the remotest Parts of the habitable World; not neglecting those that are obsolete of our own Nation. Then follow *Statues*, *Bass-Relieves*, *Seals*, Impressions, Copper-plates, Heathen Deities, Amulets, Charms and Matters relating to Romish Superstitions.

Of *enamel'd* Curiosities, that of General *Fairfax* and the fatal Battle at *Naseby* is perform'd with so exquisite Art, that it infinitely transcends the Metal, tho' Gold. And for *Paintings*, the *Misery of War* is admirably express'd, as to the various Passions, upon a Copper-plate about two Foot broad. To these may be added the Collection of printed Heads, and the *Effigies* of illustrious and learned Persons, beginning with the Royal-Family: then the Nobility, Warriours, Gentry, *&c.* in a Chronological Series. In the Ecclesiastical State, the Archbishops and Bishops are introduced by the Martyrs and Confessors of their venerable Order, and succeeded by other learned Dignitaries and pious Divines of both Denominations. The Judges are attended by the *Literati* in all Faculties, Physicians, Philosophers, Historians, Poets, Painters and other Artists. Some learned and pious Ladies are interspers'd. There are Volumes of the Saints, Popes, Emperors, and other Foreigners, amounting to the Number of 15 or 1600, many of which are done by the most celebrated Hands. Original *Designs* drawn by the *Pen* of noted *Virtuoso's*. *Writings* and *Drawings* by the Blind or Lame, as born without Hands. Some by other persons

so

ſo admirably ſmall yet legible, that in one there are 21, in another 28 Lines in the compaſs of an Inch. *Papers* of different Materials, Colour, Fineneſs, *&c.* ancient and modern: one Sheet of tranſparent Indian Paper a Yard in length. *Inkherns* from *Muſcovia*, and *Turkey*, with Reed-pens painted and gilt. A *Turkiſh* Commiſſion and *Seal*, a *Mancks* Warrant, the former impreſs'd with *Ink* not *Wax*, the latter upon blew *Slate* not *Paper*. Books printed in ſeven ſeveral Languages that are ſpoken in the Engliſh Dominions, not including what may now be added by the Acceſſion of His preſent Majeſty. A Catalogue of the various Editions of the *Bible* in this Muſeum; of the *Concordances* alſo, and *Common-Prayer* Books in different Languages; of the *Manuſcripts* alſo, it being conſiderably encreaſed ſince that inſerted in the *Oxford* Catalogue *anno* 1697. To theſe are added a Liſt of Books publiſhed in the Infancy of the Art of *Printing*, and others that later Controverſies have rendred remarkable. And alſo a large Catalogue of *Autographs* begun of late Years by the Author, yet by his general Correſpondence furniſhed with the Signs Manual of many of the Kings of *England* before the Reformation; and the proper Hand-Writing of every one ſince: with thoſe of a vaſt Number of the Lords Spiritual and Temporal in ſeveral Reigns, and of the learned Authors, *&c.* The like alſo of foreign Potentates, Warriours, *Literati &c.* of theſe ſome are very remarkable, being ſubſcribed by the Lords of the Privy Council at *Whitehall*, by the Lord Preſident and Council at *York*, and the Lord Deputy and Council at *Dublin*, from Queen *Elizabeth*'s Reign to the laſt Day of King *James* II. when the Warrant could not be executed. *Oliver Cromwell*'s Inſtructions to the Lord *Faulconberg* when ſent Ambaſſadour to the *French* King. The Warrants of the ſeveral Governments that ſo haſtily ſupplanted one another in that Year of Confuſion 1659, (which occaſio-

ned the Restoration) all under their proper Hands and Seals. To these he hath since added *Richard Cromwell*'s original Letters Patents to dissolve the Parliament; and another rare *Album* with many learned Hands, to those before mentioned. Then followeth a Catalogue of several *Manuscript Rols, Letters Patents, Diploma*'s, *Charters* and ancient Deeds of Gift to Religious Houses, which would be of use towards another Volume of the *Monasticon Anglicanum. Bede-Rolls, Dispensations*, &c. Lastly, a Description of other *Antiquities* here deposited, as *Roman* Deities, Altars, Sepulchral Monuments, Urns of different Forms and Colours, Cornelian Signets, a *Roman* Triumph in *Basse-Relieve*, and the Story of *Adonis* slain by a Boar. Besides these there are Clay Coining-Moulds for counterfeiting the *Roman* Coyns when currant, *Fibulæ Vestiariæ*, Rings or Bracelets of Jett, tessellated Pavements, Lamps, Bricks with Inscriptions, of which one very instructive is mentioned in the *Oxford* Edition of *Livy*. To which are added Brass-Swords found in *England, Ireland* and the Isle of *Man*; *British* Arrowheads of Flint; a *Danish* Sacrificing Mallet of Marble, Antique Spurs, Shields, *&c.* of later Ages, tho' now antiquated. The Figures of many of these are very well engraven, as also the Churches and Prospects in the Book.

By the Appendix it appears what considerable Additions the indefatigable Author is continually making to this Musæum. A Medal of *Jo Kendall* is especially remarkable, because retrieving the Memory of that noted Warriour, representing his Head in a noble *Relievo*, who was *Turcopellerius* or Colonel of the Cavalry (which Office belonged to the *English* Nation) at the memorable Siege of *Rhodes*, when *Mahomet the Great* was worsted. To the *Autographs* is added one impressed with a *Stile* upon a *Palmetto* Leaf, and folded up as a missive Letter in the *East-Indies*

by

by one *Timothy* a converted *Malabarian.* Through the whole Work he is particularly grateful, in writing the Names of his Benefactors that have ſent him any Curioſities. And concludes with an account of *unuſual Accidents* that have attended ſome Perſons in their *Births*, *Lives*, and *Deaths*, of which many are very very remarkable, but I fear to be too tedious.

FINIS.

LONDON: Printed for W. INNYS at the *Prince's Arms* in St. *Paul's Church-yard.* MDCCXV.

ADVERTISEMENT.

Just Publish'd,

METhodus Incrementorum Directa, & Inversa, Auctore *Brook Taylor*, L. L. D. & R. S. Secr. In two Parts. In the first Part are explain'd the Principles of the New Incremental Method, and by the means of that the Method of Fluxions is more fully explain'd than has yet been done; it being shew'd how this Method is deduced from the former, by taking the first and last Ratio's of the nascent and evanescent Increments. In the second Part the Usefulness of these two Methods is set forth by several Examples, viz. 1. In the summing up of Arithmetical Serieses. 2. In finding all the figurate Numbers. 3. In the finding of Tangents, Rays of Concavity, and the Quadrature of all sorts of Curves. 4. The Catenaria. 5. The Velaria. 6. The Fornix. 7. The Vibration of a Musical String. 8. The Centers of Oscillation and Percussion. 9. The Density of the Atmosphere. 10. The Refraction of Light passing thro' the Atmosphere, 4to. — Printed for *W. Innys* at the *Prince's Arms* in St. *Paul's Church Yard*.

An Account of this Book will be given in a following Transaction.

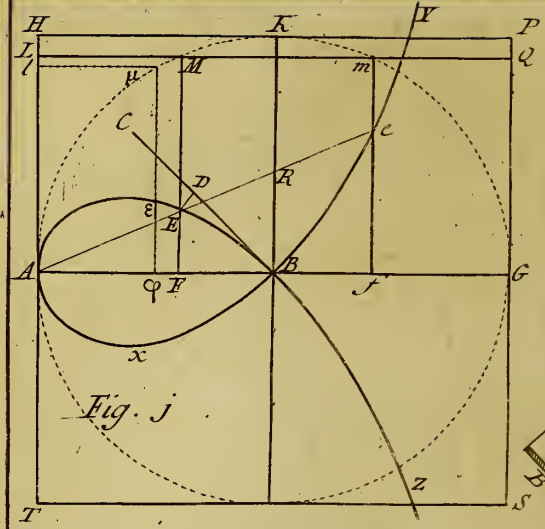


Fig. 1

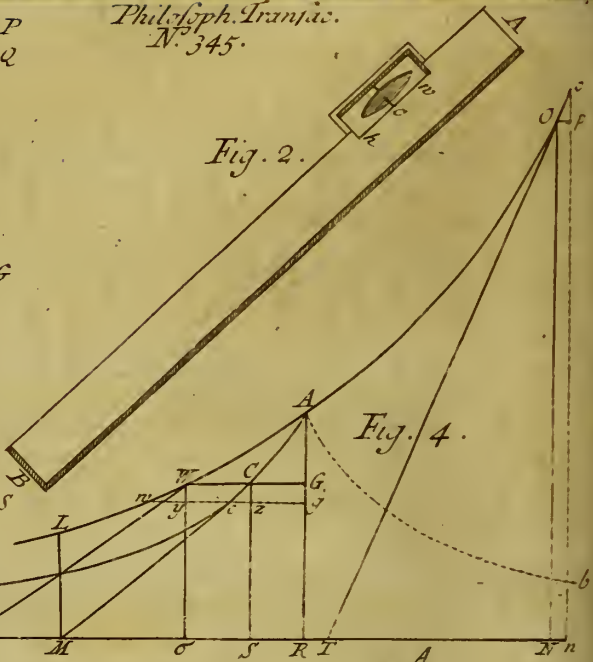


Fig. 2.

Fig. 4.

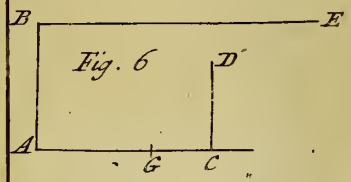


Fig. 6

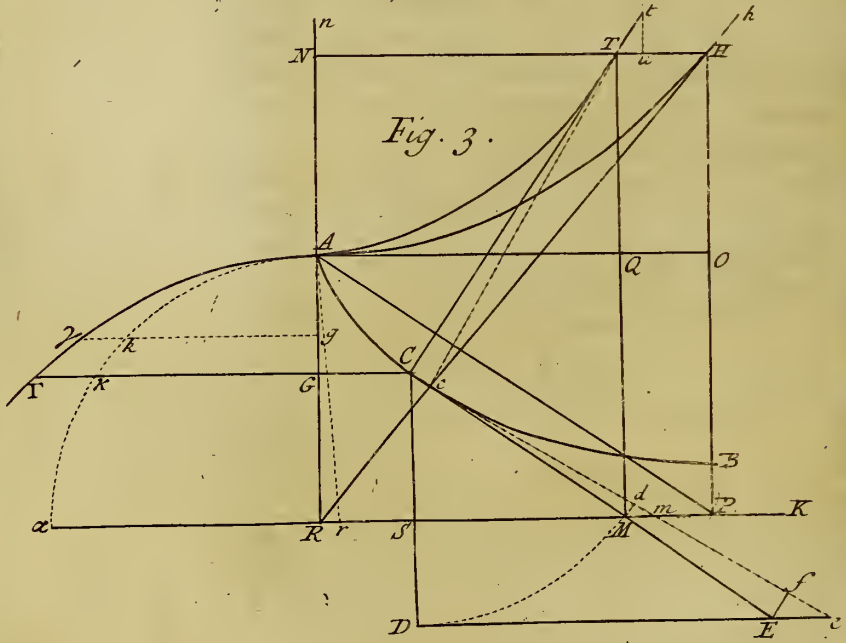


Fig. 3.

Fig. 5.

Numb. 345.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *September* and *October*, 1715.

The CONTENTS.

I. *Some Accounts of the late great Solar Eclipse on* April 22. 1715. mane. *Communicated to the* Royal-Society *from abroad.*

SInce the Publication of the large Account we gave in *Phil. Transf.* No 343. of what was obſerved in *England*, and particularly at *London*, of this Eclipſe, we have received from foreign Parts the following Obſervations; which ſeem not unworthy the Acceptance of the Curious. And firſt Mr. *John Edens*, who has obliged us with the following moſt particular Relation of the Pike of *Teneriff* and of the Aſcent thereto, being on his Voyage to that Iſland, obſerved the Eclipſe at Sea, in Latitude, by Obſervation 34°. 20′, and Longitude 0^{h}. 54′. Weſt from *London*, as he concluded by their Diſtance and Poſition from the Iſland *Forte ventura*, which they ſoon after fell with. He writes that it began at VI^{h}. 49′. and ended at VIII^{h}. 47′. this latter very exactly, tho' not quite ſo nice as to the Beginning.

Had this Obſerver ſignified what Difference of Meridians there was found between the Place of Obſervation and the Weſt End of *Forte ventura*, we might, without ſenſible Error, have concluded the true Longitude, not only of that Iſland, but alſo of the Pike of *Teneriff*, where *Our Geographers* and the *Dutch* have fixed their firſt Meridian. But this Gentleman being both able and deſirous to render the Publick this ſort of Service, we hope from him ſuch further Obſervations as may put the matter paſt diſpute. He adds that the greateſt Darkneſs was about $\frac{3}{4}$ of the Sun's Diameter, or nine Digits on the North ſide.

From

From *Germany* we have received the following Accounts.

At *Nurenburg*.

The Beginning and greateſt Obſcurity could not be ſeen for Clouds, but the End happen'd at $xi^h. 10'.\frac{1}{2}$.

At *Hamburg*.

The Beginning was obſerved at $viii^h. 57'$. The greateſt Obſcurity at $x^h. 5'. 30''$, when $xi\frac{1}{2}$ *digg.* were darkned. The End could not be ſeen for Clouds.

At *Kiel* in *Holſtein*.

The Beginning $ix^h. 4'$. The greateſt Obſcurity $x^h. 19'. 20''$, and the Quantity then eclipſed xi. *digg.* 20′ The End was at $xi^h. 29'$.

At *Berlin*.

The Beginning could not be ſeen for Clouds, but the greateſt Obſcurity was at 22 *min.* paſt Ten, when xi *digg.* were eclipſed. The juſt End was at $xi^h. 34'$.

At *Franckfort* on the *Meine*.

The Eclipſe began at $viii^h. 50'$. The greateſt Darkneſs at $x^h. 11'$, *but perhaps ſhould be* $x^h.$ 01 *min.* the Digits being x. and 34 *min.* The End was obſerved at 10 *min.* paſt Eleven.

By whom theſe Obſervations were made, and with what Inſtruments, we are not as yet informed, but hope they may be exact enough to confirm the *Longitudes* of thoſe ſeveral places, which are at preſent reaſonably well known.

Since theſe there is lately come to Hand a *Dutch* Print entituled *Nouvelles Literaires*, publiſh'd at the *Hague*, wherein, *pag.* 404. 405, there is an Account of the Obſervation of this Eclipſe at *Upſal* in *Sweden*, made by M. *Jo. Waller*, Profeſſor of Mathematicks in that Univerſity, who was very careful to obſerve it exactly; the Times being verified by three Clocks perfectly agreeing with one another and with the Sun: but more eſpecially

by a Quadrant of 5 Foot Radius for taking the Sun's Altitude. By this Instrument he has determined the Height of the Pole at *Upsall* 59°. 51′ 54″. And by the same, a little before the Beginning of the Eclipse he found the Height of Sun 39°. 36′. 42″. his Clocks then shewing the Hour 1x^{h}. 47′. 50″, which proves that they were very near the true Time. At x^{h}. 58′. 15″. the Altitude of the Sun being 44°. 17′. 29″, was the Beginning of the total Darkness, and at x1^{h}. 2′. 24″. was the End thereof, *alto sole* 44°. 29′. 13″. so that here the Duration of the total Eclipse was 4′. 9″, and the Middle thereof but one third of a Minute after Eleven. And lastly the End is said to have happen'd about 4 Minutes before Noon, the Sun being 45°. 42′. 6″. high: But in this is a manifest Mistake, for it makes the Time of Emersion, or from the Middle to the End, but 55′. 20″; whereas being so near the Meridian, 'tis certain that this Emersion was the greater part of the Duration of the whole Eclipse, and consequently more than an Hour. Perhaps the Times might be deduced from the Altitudes only, and then the Mistake might be in supposing the End so much before Noon as it was really after it. However, to prevent all Doubts, we have compared this Observation with what we observed of this Eclipse at *London*, and find that in the Latitude of 59°. 50′, the Place where the Middle of total Darkness was at x1^{h}. 0′. 20″, was near 19 Degrees more Easterly than *London* (that is exactly in the Meridian of *Dantzick*) and that the Eclipse began there at 1x^{h}. 52′ ½, and ended at x11^{h}. 10′. Wherefore the Duration could not be 2^{h}. 7′. 50″, as the *Editor* of the said *Nouvelles* has publish'd; not considering that the Beginning could not be seen for Clouds, as in the very next Words he assures us.

As to the Darkness, it was such that they could scarce distinguish one another: and besides *Jupiter*, *Mercury* and *Venus*; of the Fixt Stars *Cassiopea*, *Capella*, *Oculus Tauri* and *Orion*, (*Sirius not being yet risen*) were visible. II.

II. *An Account of a Journey from the Port of* Oratava *in the Island of* Tenerife *to the Top of the Pike in that Island, in* August *last; with Observations thereon by Mr.* J. Edens.

ON *Tuesday August* the 13th *N. S.* at Half an Hour past Ten in the Evening, I, in company of four more English and one Dutch-Man, with Horses and Servants to carry our Provision, together with a Guide (which is the same that has conducted all those that have been this Journey for many Years) set forward from the Port of *Oratava.* The Night being somewhat cloudy, and the Moon in the full at 12 the Night following.

At half an Hour past Eleven we came to the Town of *Oratava,* which is about two Miles from the Port, where we stopt for about half an Hour, to get walking Staves to assist us in our ascending the Steep of the Pike.

At One a-Clock on Wednesday Morning we came to the Foot of a very steep Rising, about a Mile and half above the Town of *Oratava,* where it began to clear up; and we saw the Pike with a white Cloud covering the Top of it like a Cap.

At Two a-Clock we came to a plain place in the Road which the Spaniards call *Dornajito en el Monte verde* (the little Trough in the green Mountain) so call'd I suppose because a little below this Plain, on the Right hand as we went, there is a deep Hollow; at the upper End of which Hollow, there is a Spout of Wood placed in a Rock, through which there runs very clear and cool Water, which comes from the Mountains; and at a Descent a little lower than the Spout there is a Trough into which the Water comes.

At Three, after travelling a Road, which was ſometimes pretty ſmooth and at other times very rough, we came to a little wooden Croſs, by the Road ſide on the Left-hand, which the Spaniards call *la Cruz de la Solera* (The Croſs of the *Solera*) A *Solera* is a long Pole with a Hole at each end, which the Spaniards uſe to draw Wood with, by faſtning one end to the Wood and the other to the Oxen. This Croſs was made with a Piece of a *Solera*, and for that reaſon is ſo call'd, but why it was ſet up in this place I can't tell, unleſs it was becauſe ſomebody was kill'd thereabouts. At this place we alſo ſaw the Pike before us; and altho' we had come up hill quite from the Port, yet to our thoughts it ſeem'd almoſt as high here as when we were there, the white Cloud ſtill hiding the greateſt part of the Sugar-loaf.

After riding about half a Mile further, we came to the ſide of a Hill which was very rough and ſteep, (the place call'd *Caravala*; where are a great many Pine Trees that grow on both ſides the Road for a great way, both on the Right-hand and the Leſt, one of which that was cloſe to the Road, on the Right-Hand as we went, our Guide deſir'd us to obſerve; it having a great Branch growing out, which with the Boughs that were upon it look'd like the Forepart of a Ship. And from the likeneſs this Tree has to a Ship I ſuppoſe the Place took its Name, for *Caravela* ſignifies an old-faſhioned Veſſel formerly much uſed in *Spain*, ſharp before, ill ſhap'd every Way, and all the Maſts ſtooping forwards; their Sails are all Mizen Sails, that is, Triangular; they will lye nearer the Wind than other Sails, but are not ſo commodious to handle. Amongſt theſe Trees, not a great Height in the Air, we ſaw the Sulphur diſcharge its ſelf like a Squib or Serpent made of Gun-powder, the Fire running downwards in a Stream, and the Smoak aſcending upwards from the place where it firſt took Fire; and like this we ſaw another,

ther, whilſt we lay under the Rocks the next Night at *la Stancha*, part of the way up the Pike; But I could not obſerve whether either of them gave any Report as they diſcharg'd.

At three Quarters after Four we came to the Top of this high rough and ſteep Mountain, where grows a Tree which the Spaniards call *el Pino de la Merenda*, The Pine-Tree of the Afternoon's Meal. This is a large Tree, and is burnt at the Bottom, as having had Fires made againſt it; and in the burnt place there iſſues out Turpentine, a little of which I brought with me. At a few Yards diſtance from this Tree we had a Fire made, where we ſtay'd and baited our Horſes, and breakfaſted our ſelves. Theſe Hills are very ſandy, and there are a great many Rabbets which breed there; There is alſo much Sand found a great way up the Pike it ſelf, and not a great way below the Foot of the Sugar-loaf, ſome of which I brought down with me.

At Three Quarters after Five we ſet forwards again, and at Half an Hour paſt Six came to the *Portillo*, which in Spaniſh ſignifies a Breach or Gap. We ſaw the Pike about two Leagues and a half before us, cover'd ſtill with a Cloud at Top; and the Spaniards told us we were come about two Leagues and a half from the Port.

At half an Hour paſt Seven we came to *las Faldas*, that is the Skirts of the Pike; from whence all the way to *la Stancha*, which is about a Quarter of a Mile up from the Foot of the Pike, we rode upon little light Stones, for the moſt part not much bigger than ones Fiſt; and a great many not much broader than a Shilling: and if we kept the beaten Track which was uſed before, it was not ſo deep, but if we turn'd out of it the Horſes went almoſt over their Feet. I lighted and made a Hole there, thinking to find how deep theſe little Stones lie, but could

not find the Bottom ; which makes me conclude they may cover the Ground for a great thickneſs.

There are a great many vaſt Rocks, ſome of them two Mile or thereabouts from the Foot of the Pike, which the *Pike-Man* told us was caſt out from the Top of the Pike at the time it was a *Vulcano* ; many of them lye in Heaps of above threeſcore Yards long, and I obſerv'd that the further theſe Rocks lye from the Foot of the Pike, the more like they are to the Stone of other common Rocks: But the nearer we went to the Pike we found them more black and ſolid ; and ſome of them, tho' not many, were gloſſy like Flint, and all extream heavy. Thoſe that ſhone ſo, I ſuppoſe, retain'd their natural Colour, but there are ſome that look like Droſs that comes out of a Smith's Forge, which without doubt was occaſioned by the extream Heat of the place they came from.

Some of theſe great Rocks were thrown out of the *Caldera* or Kettle in the Top of the Pike ; and others from a Cave or Ciſtern which is a pretty way up the ſide of the Pike, and has by ſome been thought to have no Bottom, more of which I ſhall ſay anon.

At Nine on *Wedneſday* Morning we arrived at *la Stancha*, about a Quarter of a Mile above the Foot of the Pike on the Eaſt-ſide, where are three or four large hard and ſolid black Rocks lodg'd : under ſome of theſe we put our Horſes, and under others we lay down our ſelves to ſleep, after having refreſh'd our ſelves with a little Wine : and we had a Fire made in order to get our Dinner ready, where a Cook we took along with us both roaſted and boyled our Meat and Fowls very well. We ſlept here for about two Hours, then roſe again, and at about Two in the Afternoon went to dinner.

There are ſeveral Mountains that lye Eaſtward from the Pike at four or five Miles Diſtance, call'd the *Malpeſes*, and one more lying a little more to the Southward call'd

la

la montaña de rejada; all which were formerly *Vulcanos*, tho' not so great as that of the Pike, as appears by the Rocks and small burnt Stones that lye near them, just in the same manner as about the Pike.

Still being at *la Stancha*, after we had dined we lay down again to take a Nap, under the Rocks as before Dinner, but not sleeping very well we all got up again, the rest of them spending the Afternoon at Cards, *&c.* But I made it my business to admire the strangeness and vastness of that great Body, which indeed is very wonderful, insomuch that its impossible to express to one that has never seen it, in what a manner that confused Heap of Rubish lyes; for it may very well be stiled one of the greatest Wonders in the World. About Six at Night we saw *Grand Canaria* from *la Stancha* bearing from us *E.* by *N.*

At Nine at Night, after having had our Suppers, we retired to our former Lodgings, where laying Stones for our Pillows and our Cloaks for Bed-cloaths, we endeavoured to get to sleep, but all in vain for a great while. Some lying pretty nigh a Fire complain'd of being burnt on the one side and froze on the other (for the Air was very cutting and sharp) Others happening to lye in a place where there was a great many Fleas; though it be something strange that Fleas should be found there, the place being so cold in the Night: perhaps the Goats sometimes get under these Rocks and so leave them; and I am inclin'd to believe it, because the Guide and I found a dead Goat in a Cave at the very top of the Pike. I suppose this Goat straggling up here by chance was benighted, and so finding the Cold got into this place for Heat, where meeting with too much of it, and a very strong Sulphurious Vapour it overcame him; for he was almost dryed to Powder. But to proceed, betwixt Eleven and Twelve we got to sleep, and slept till One, when waking, our Guide told

us 'twas time to prepare for our Journey. We immediately rose, and by half an Hour past One we were all upon the march, and leaving our Horses and some of our Men behind, we went away fasting, excepting about two Mouthfulls of Wine apiece, which we took at our uprising. Betwixt *la Stancha* and the Top of the Pike there are two very high Mountains and the Sugar-Loaf, each of which Mountains is almost half a Mile's walking: on the first of them the Rubbish is more small, and we were apt to slip back as we stept upwards. But the uppermost is all composed of hard loose rocky great Stones, cast one among another in a very confused Order. After resting several times, we came to the Top of the first Mountain, where we drank every one of us a little more Wine, and eat each of us a Bit of Ginger-Bread we had amongst us. Then being pretty well refresh'd, we set forwards again to ascend the second Mountain, which is higher than the first, but is better to walk on because of the firmness of the Rocks. After we had travel'd for about half an Hour up the second Mountain, we came within sight of the Sugar-Loaf, which before we could not see by reason of the Interposition of these great Hills. After we were arriv'd to the Top of this second Mountain we came to a way that was almost level, but bearing some small matter up-hill; and about a Furlong farther is the Foot of the Sugar-Loaf, which we soon after came to. Then looking upon our Watches found it to be just three a-Clock. The Night was clear where we were, and the Moon shone very bright, but below over the Sea we could see the Clouds, which look'd like a Valley at a prodigious Depth below us. We had a brisk Air, the Wind being *S. E.* by *S.* as it was for the most part whilst we were upon our Journey.

Whilst we sat at the Foot of the Sugar-Loaf, resting and refreshing our selves as before in other places, we saw the

the Smoak break out in ſeveral places, which at firſt look'd like little Clouds, but they ſoon vaniſh'd, others not long after coming in their room from the ſame or other places.

We ſet forwards to aſcend the laſt and ſteepeſt part of our Journey, *viz.* the Sugar-Loaf, exactly at half an Hour paſt Three, and after we had reſted twice or thrice, I left the Guide and the reſt of my Company, and ran forwards; and when I was got very nigh the Top (which was at three quarters after Three) two more of the Company deſerted, and came up about Five Minutes after me; the reſt of the Company and the Guide coming up to the Top juſt at Four.

The Shape of the Top of the Pike is partly oval, the longeſt Diameter lying *N. N. W.* and *S. S. E,* and is as nigh as I could gueſs, about 140 Yards long; the Breadth the other way being about 110. Within the Top of the Pike is a very deep Hole call'd the *Caldera* (or *Kettle*) the deepeſt part of which lyes at the *South* End: It is I believe 40 Yards deep, reckoning from the higheſt ſide of the Pike: but it is abundance ſhallower reckoning from the ſide oppoſite to *Garachica.* The ſides of this Kettle are very ſteep, in ſome places as ſteep as the Deſcent on the outſide of the Sugar-Loaf. At the Bottom of this Kettle we all were, where lye a great many very large Stones, ſome of them higher than our Heads. The Earth that is within ſide the Kettle, being roll'd up long and put to a Candle, will burn like Brimſtone. Several places within ſide the Top of the Pike are burning, as on the Outſide; and in ſome places if you turn up the Stones you'll find very fine Brimſtone or Sulphur ſticking to them. At theſe Holes where the Smoak comes out there alſo comes forth a great Heat, ſo hot that one cannot endure one's Hand there long. At the *N.* by *E.* ſide within the Top is the Cave where we found the dead Goat; in which Cave

ſometimes the true Spirit of Sulphur diſtills, as they ſay, but it did not drop whilſt I was there.

The Report is falſe about the Difficulty of breathing upon the Top of this place; for we breath'd as well as if we had been below; we eat our Breakfaſt there, and I was up in all for about two Hours and a quarter.

Without doubt the Quickſilver would have fell very much upon this high place, if I had had but the good fortune to have got a couple of Barometers to try. But there is no ſuch thing in this Iſland, and I was fearful of not getting Company in the mind to go up with me another Year (for to go up by ones ſelf is very chargeable) elſe I would have ſent to *England* to have been ſupply'd, tho' the Expence had come all out of my own Pocket.

Before the Sun roſe I think the Air was as cold as I have known it in *England*, in the ſharpeſt Froſt I was ever in; I could ſcarcely endure my Gloves off. There was a great Dew all the while we were there till Sun riſing, which we could find by the Wetneſs of our Cloaths; but the Sky look'd thereabouts as clear as poſſible.

A little after Sun riſing we ſaw the Shadow of the Pike upon the Sea, reaching over the Iſland of *Gomera*; and the Shadow of the upper part, *viz.* of the *Sugar-loaf*, we ſaw imprinted like another Pike in the Sky it ſelf, which look'd very ſurprizing: but the Air being cloudy below us, we ſaw none of the other Iſlands but *Grand Canaria* and *Gomera*.

At ſix on *Thurſday* Morning we came down from the Top of the Sugar-Loaf; at ſeven we came to the Ciſtern of Water which is reported to be without Bottom: this the Guide ſays is falſe, for about ſeven or eight Years ago, when there was a great *Vulcano* in this Country, the Cave was dry and he walk'd all about it, and ſaid that the deepeſt part of Water, when we were there, was not above two Fathoms.

The Dimenſions of this Cave I gueſs to be as follows.

Length about 35 Yards
Breadth —— 12
Ordinary Depth 14 from Top to Bottom.

Upon the furthermoſt ſide grows white Stuff, which the *Pike-man* told us was Salt-Peter. There was both Ice and Snow in it when we were there: and the Ice was of a great Thickneſs covered with Water about Knee deep. We let down a Bottle at the End of a String for ſome of the Water, in which we put ſome Sugar and drank it, but it was the coldeſt I ever drank in my Life. The Ice was broken juſt under the Mouth of it, where we could ſee the Stones lye at the Bottom, for it was very clear. A little to the Right-hand within this Cave the Ice was riſen up in a high heap, in form of a Spire Steeple or like a Sugar-Loaf; and in this place I believe the Water comes in. I ſhould have been glad to have come at it, to let down a Line to try whether there may not be ſome Hole that the Guide knows not of, that may be of a great Depth.

In our way home, we came by a Cave three or four Miles from the Pike, where are a great many Skeletons and Bones of Men; and ſome ſay there are the Bones of Giants in this Cave, but we know not how many Bodies are laid here, nor how far the Cave may go. I intend (God willing) to go again before I leave the Iſland, and then I'll take a Light with me and ſee what Diſcoveries I can make.

We came home to the Port at about ſix a-Clock this Evening, being *Thurſday Auguſt* 15. 1715. *N S.*

Whoever reads this, I hope, will pardon the Faults my Pen may have committed, for I was forc'd to write all Night; the Ship being to ſail the next Morning, and I have not time to examine it.

III. Ventriculus cordis ſiniſter ſtupendæ magnitudinis, *lately communicated to the* Royal-Society *by* James Douglaſs, M. D. *and* R. S. S.

I Lately opened a young Man in St. *Bartholomew's Hoſpital*, that died of the Palpitation of the Heart, whoſe violent beating and prodigious ſubſultory Motion, for ſome Months before his Death, was not only eaſily felt by laying the Hand on the Region of the Heart; but ſeen to riſe and fall by raiſing the Bedcloaths that covered it. And, which is almoſt incredible, at ſometimes the trembling and throbbing made ſuch a Noiſe in his Breaſt, as plainly could be heard at ſome Diſtance from his Bedſide. This was accompanied with frequent *Deliquiums*, ſometimes ſlow, ſometimes ſwift, and often intermitting.

Johannes Fernelius in his *Pathologia* lib. 5. cap. 12. gives us an Obſervation of a very uncommon and ſurpriſing Caſe of this kind; where he ſays the frequent Concuſſion of the Heart was ſo violent and powerfull, as not only to diſplace or luxate, but even to break ſome of the adjoining Ribs.

Franciſcus de la Boe Sylvius, another Writer of unqueſtionable Integrity, has a parallel Obſervation in his Account of this Diſeaſe.

Theodorus Kerkringius relates the Hiſtory of a Woman he opened, whoſe Heart was of a prodigious Bigneſs; in his *Spicilegium Anatomicum*, Obſ. 16.

And to mention no more, *Monſieur Dionis*, at the End of his Anatomy, gives a large Deſcription of a very uncommon Caſe, in which the right Auricle of the Heart was prodigiouſly dilated to the Bigneſs of the Head of a new born child.

In

In the Diſſection of this morbid Heart I obſerved the following remarkable Particulars.

1. That the *Pericardium* or *Capſula Cordis* was very thick, and firmly adhered or grew by a fibrous Connexion to all the outer Surface of the Heart.

2. Inſtead of the Water called *Liquor Pericardii*, there was only in ſome places about the *Baſis* of the Heart a mucilaginous clear Subſtance like a Gelly.

3. In the right Auricle lay'd open there was nothing preternatural. The aſcending and deſcending *Cava* opened into the ſame as uſual. The *Veſtigium* or Mark of the *Foramen ovale* with its ſemicircular *limbus* was very plain. And the *Orificium* of the *Vena Cordis Coronaria* was extreamly large, yet its Valve was leſs than uſual.

4. In the right Ventricle layed open, the *Valvulæ* called *tricuſpides* were configurate after the uſual manner. The ſides of this Cavity were thin and full of ſmall fleſhy *Columnæ* as they commonly are, with great variety of Furrows and little holes. The three *ſigmoide* or *ſemilunar Valves* in the Mouth of the *arteria pulmonalis*, were as they always are in a natural State.

5. The left Auricle was not much bigger than ordinary: but its muſcular Appendage, called the *Bulb* of the *Pulmonary Vein* by the late Mr. *Cowper*, was extraordinarily dilated and enlarged beyond any thing that I ever ſaw.

6. The left Ventricle, whoſe Capacity in a natural State is always leſs than the right, was here conſiderably larger. And if the Experiment had been made, before Diſſection, of filling both with any Liquor, this had certainly contained three times more than the other.

7. The *Valvulæ* called *Mitrales*, placed at the Orifice of this Ventricle, are much thicker in Subſtance than ordinary; and the two fleſhy Columns, called by *Nicolaus Maſſa*, almoſt 200 Years ago, *duo parvi muſculi*, which ſend out

abundance of ſmall Tendons to be inſerted into theſe Valves, were proportionably augmented in Bigneſs.

8: The *ſemilunary Valves* in the Mouth of the *Aorta*, or of that great *Vena pulſatilis* that diſpenſes the Blood to all the ſeveral parts of the human Body, were very much preternaturally affected; as would eaſily appear upon comparing them with thoſe in the Orifice of the *pulmonary Artery*, in which they are thin and very broad, ſo as to be able to ſhut the Cavity of that Veſſel, and hinder the Blood from returning back into the Ventricle, and likewiſe tranſparent: but in this they are very thick, contracted as it were, and furled together, and of a whitiſh Colour; and in all appearance, if the Perſon had lived longer, they had turned boney, or undergone a Petrification.

This uncommon Structure of the Heart being thus demonſtrated, let us endeavour to account for the following Phænomena. The firſt is the Palpitation of the Heart, which was the chief Symptom and Complaint of the ſick Perſon. The ſecond is the preternatural Dilatation and Enlargement of the left Ventricle. It is not improbable but the firm adheſion of the *Capſula Cordis membranoſa* to the ſubſtance of the Heart, occaſioned that uncommon trembling and throbbing thereof: its free and eaſy Motion being hindered by that thick *involucrum* which ſurrounded it ſo cloſe on each ſide. The learned Dr. *Lower*, in his elaborate Treatiſe *de Corde humano*, gives us ſuch an inſtance; and explains the Palpitation after this manner:

As for the ſecond, *viz.* the Dilatation of the left Ventricle and muſcular Bag of the *Pulmonary Vein*; that is altogether owing to the ill configuration of the *Valves* we have now deſcribed: for as the great Artery or *Aorta* ariſes out of this Ventricle, it has three Valves which ſeparating give paſſage to the Blood from the Ventricle into

the Vessel; and in a natural State they shut that Passage, and so prevent the Blood from recoiling into the same, if it should endeavour to return. But in this case, by reason of its contracted Narrowness and Thickness, not being able to close or shut the Passage, the Blood flowd back again into the Cavity, which it had gradually enlarged, and dilated to the Bigness we see. Besides the *Muscular Valves* not being duly qualified for the Performance of their Office, the Blood recoiled into the *Auricle*, which it had distended in the like manner. This constant Regurgitation or Reflux of the Blood is besides sufficient of its self, to produce this extraordinary trembling or παλμὸς καρδίας, as the *Greeks* call it.

IV. *A ready Description and Quadrature of a* Curve *of the* Third Order, *resembling that commonly call'd the* Foliate. *Communicated by Mr.* Abr. de Moivre, *F. R. S.*

I Have look'd a little farther into that Curve which fell lately under my consideration. It is not the *Foliate* as I did at first imagine, but I believe it ought not to make a *Species* distinct from it. *A E B* (*Fig.* 1.) is the Curve I thus describe. Let *A B* and *B K* be perpendicular to each other. From the point *A* draw *A R* cutting *B K* in *R*, and make *R E* = *B R*; the point *E* belongs to the Curve. Draw *B C* making an Angle of 45 *grad.* with *A B*, this Line *B C* touches the Curve in *B*; from the point *E* draw *E D* perpendicular to *B C*, and calling *B D*, x; *D E*, y; *A B*, a; and making $\sqrt{8aa} = n$, the Equation belonging to that Curve is $x^3 + xxy + xyy + y^3 = nxy$ or $\frac{x^4 - y^4}{x - y} = nxy$. Taking *B G* = *A B*, and drawing *G P* perpendicular to *B G*, *P G* is an *Asymptote*. In the *Foliate*

the Equation is $x^3 + y^3 = \frac{1}{2} nxy$. in which the two Terms $xxy + xyy$ of the former Equation are wanting; and its *Asymptote* is distant from B by $\frac{1}{3} BA$. Again draw EF perpendicular to AB: let BF be called z and FE v; the Equation belonging to the Curve AEB is $vv = \frac{azz - z^3}{a + z}$. In the *Foliate* the Equation is $vv = \frac{azz - z^3}{a + 3z}$ From these two last Equations it seems that these Curves differ no more from one another than the *Circle* from the *Ellipsis*. I should be very glad to know your Opinion thereupon.

The Quadrature of the Curve here described has something of Simplicity with which I was well pleased. With the Radius BA and Center B describe a Circle AKG, let the Square $HPST$ circumscribe it, so that HP be parallel to AG: prolong FE till it meet the Circumference of the Circle in M, and through M draw LMQ parallel to HP. The Area BFE is equal to the Area $KHLM$, comprehended by KH, HL, LM and the Arc KM. And the Area Bfe is equal to the Area $KmLH$ or $KMPQ$. Therefore if BF and Bf are equal, the two Areas BFE, Bfe taken together are equal to the Rectangle HQ, and therefore the whole Space comprehended by $BEAXBeYGZ$ (supposing Y and Z to be at an infinite Distance) is equal to the circumscribed Square HS.

N.B. *This Quadrature is easily demonstrated from the Equation: for by it* a + z : a − z :: zz : vv, *that is* AF : EF :: MF : FB, *and so* φF *the* Fluxion *of* AF *to* Ll *the* Fluxion *of* MF. *Hence the* Areola $EF\varphi e$ *will be always equal to the* Areola MLlμ, *and therefore the* Area AEF *always equal to the* Area MAL.

Hence it appears that this Curve requires the Quadrature of the Circle to square it; whereas the Foliate *is exactly quadrable, the whole* Leaf *thereof being but one third of the Square of* AB, *which in this is above three sevenths of the same. Again*

in

in our Curve, the greateſt Breadth is when the Point F *divides the Line* A B *in extream and mean Proportion: whereas in the* Foliate *it is when* A B *is triple in power to* B F. *And the greateſt* E F *or Ordinate in the* Foliate *is to that of our Curve nearly as* 3 *to* 4, *or exactly as* $\sqrt{\frac{2}{3}\sqrt{\frac{1}{3}}-\frac{1}{3}}$ *to* $\sqrt{5\sqrt{\frac{5}{4}}-5\frac{1}{2}}$.

But ſtill theſe Differences are not enough to make them two diſtinct Species, *they being both defined by a like Equation, if the* Aſymptote S G P *be taken for the Diameter. And they are both comprehended under the* fortieth Kind *of the* Curves *of the* third Order, *as they ſtand enumerated by Sir* Iſaac Newton, *in his incomparable Treatiſe on that Subject.*

IV. *An eaſy Mechanical Way to divide the Nautical Meridian Line in* Mercator's *Projection; with an Account of the Relation of the ſame Meridian Line to the* Curva Catenaria. *By* J. Perks, M. A.

THE moſt uſeful Projection of the Spheric Surface of Earth and Sea for Navigation, is that commonly call'd *Mercator's*; tho' its true Nature and Conſtruction is ſaid to be firſt demonſtrated by our Countryman Mr. *Wright*, in his *Correction of the Errors in Navigation.* In this Projection the Meridians are all parallel Lines, not divided *equally*, as in the common plain Chart (which is therefore erroneous,) but the Minutes and Degrees (or ſtrictly, the *Fluxions of the Meridian,*) at every ſeveral Latitude are proportional to their reſpective *Secants.* Or a Degree in the projected Meridian at any Latitude, is to a Degree of Longitude in the Equator, as the *Secant* of the ſame Latitude is to *Radius.*

The

The Reason of which Enlargement of the Elements of Latitude is, to counterbalance the Inlargement of the Degrees of Longitude. For in this Projection, the Meridians being all parallel, a Degree of Longitude at (suppose) 60 Deg. Lat. is become equal to a Degree in the Equator, whereas it really is (on the Globes Surface) but *half* as much, the Radius of the Parallel of 60 Deg (that is its *Cosine*) being but *half* the Radius of the Equator. Therefore to proportion the Degrees of Latitude to those of Longitude, a Degree (or Elemental Particle) in the Meridian, is to be as much greater than a Degree (or like Particle) in the Equator, as the Radius of the Equator is greater than the Radius of the Parallel of Latitude, *viz.* its *Cosine*.

In *Fig.* 3. let the Radius CD represent half of the Equator, DM an Arc of the Meridian; MS its Sine, CE its Secant: then is CS equal to its *Cosine*: and $CS : CM :: CD (= CM) : CE$, that is, as Cosine : to Radius :: so is Radius : to Secant. The Cosines being then, in this Projection, suppos'd all equal to Radius, or (which comes to the same) the Parallels of Latitude being all made equal to the Equator, the Radius of the Globe, at every point of Latitude, (by the precedent Analogy) is supposed equal to the Secant of Latitude; and consequently the Elements (Minutes, *&c.*) of the Meridian must be proportional to their respective Secants.

The Way Mr. *Wright* takes for making his Table of *Meridional Parts*, is by a continual Addition of Natural Secants, beginning at 1 Minute, and so proceeding to 89 Deg. Dr. *Wallis* (in *Phil. Transf.* No. 176.) finds the Meridional Part belonging to any Latitude by this *Series*, putting S for its Natural Sine, *viz.* $S + \frac{1}{3}S^3 + \frac{1}{5}S^5 + \frac{1}{7}S^7 + \frac{1}{9}S^9$ &c. which gives the *Merid. part* required. How to find the same Mechanically by means of an easily-constructed Curve Line, is what I shall now shew.

1.

1. Prepare a Rular *A B* (*Fig.* 2.) of a convenient Length, in which let *B o* be equal to the Radius of the intended Projection. To the Point *o* as a Center (on the narrower Edge of the Rular) faſten a little Plate-Wheel *w h* tight to the Rular, and of a Diameter a little more than the thickneſs of the Rular. Let *K R* (*Fig.* 3.) repreſent another long Rular, to which *A R* is a perpendicular Line. Place the Rular *A B* upon the Line *A R*, with the Center of the Wheel at *A*. Then with one Hand holding faſt the Rular *K R*, with the other Hand ſlide the end *B* of the Rular *A B* by the Edge of *K R*; ſo will the little Wheel *w h* deſcribe on the Paper a Curve Line *A C B*, to be continued as far as is convenient.

2. Having drawn the Curve *A C B*, draw a ſtreight Line *K R* by the Edge of the Rular *K R*: which Line is the *Meridian* to be divided, and alſo an Aſymptote to the Curve *A C B*.

3. In this Meridian, (accounting *R* to be the Point of its Interſection with the Equator,) the Point anſwering to any Degree of Latitude is thus found. In the perpendicular *A R*, make *R G* equal to the *Coſine* of Latitude (Radius being *A R*,) and from *G* draw *G C* parallel to *K R*, and interſecting the Curve in *C*. With Center *C* and Radius *C M* = *A R*, ſtrike an Arc cutting the Meridian at *M*; ſo is *M* the Point deſir'd.

4. In the Curve *A C*, let *c* be a Point infinitely near to *C*, and *c m*, (= *C M*,) a Tangent to the Curve at *c*, making the little Angle *M C m*, to which let the Angle *R A r* be equal: ſo is *R r* = *M d* (a Perpendicular from *M* to *c m*.) Draw *C D* equal and parallel to *A R*, interſecting *K R* in *S*. With Center *C* and Radius *C D* draw the Arc *D M*, and its Tangent *D E* and Secant *C E*.

5. Becauſe of the like Triangles *C D E*, *M d m*; *C D* : *C E* : : *M d* : *M m*, that is, as Radius to Secant of the Arc *D M*, (whoſe Coſine is *C S* = *G R*,) : : ſo is *M d*

($= Rr$ a Degree or Particle of the Equator :) to Mm the Fluxion or correſpondent Particle of the Meridian Line RM. Whence, and from what is premiſed concerning the Nature of this Nautical Projection, 'tis evident that RM is the *meridional Part* anſwering to the Latitude whoſe Coſine is GR. Or thus; With Center R and Radius AR deſcribe the Quadrant $A\varkappa\alpha$, in which let the Arc $A\varkappa$ be equal to the given Lat. From $\varkappa$ draw $\varkappa C$ parallel to KR, and interſecting the Curve in C, ſo is $C\varkappa$ the Meridional Part deſir'd being equal to RM, as is eaſy to ſhew.

As to the other Properties of this Curve, tis evident, from its Conſtruction, that its *Tangent* (as CM) is a *Conſtant Line* every where equal to AR; the Curve being generated by the Motion of the Wheel at the End of the Rular which is its Tangent. And from hence the Curve ACB may, for diſtinction, be call'd the *Equitangential Curve*.

7. The Fluxion of the Area $ARMC$ is the little Sector or Triangle MCd, which ſame is alſo the Fluxion of the Sector CDM: whence the Areas $ARMC$, CDM are equal, and the whole Area $ACB\&c, KMR$ being infinitely continued, is equal to the Quadrant ARa.

8. To find the Radius of Cutvature of any Particle, as Cc, from C draw an indefinite Line CT perpendicular to CM, (on the concave ſide of the Curve) and from c another Line perpendicular to cm, which Lines, (becauſe of the Inclination of CM to cm,) will ſomewhere meet as at T, making an Angle $CTc = MCm$. Theſe Angles being equal, their Radii are proportional to their Arcs: therefore, $Md : Cc :: MC : CT$. But $Cc = dm$ (becauſe of $CM = cm$) ſo that $Md : dm$ ($:: CD : DE$) $:: CM : CT$. But $CD = CM$, therefore $CT = DE =$ Tangent of the Arc DM.

9.

9. So that ſuppoſing ATt a Curve Line in which are all the Centers of Curvature of the Particles of ACB, any point as T being found as before, the Length AT (by the nature of *Evolution of Curves*,) is every where equal to the *Tangent* of its correſpondent Circular Arc DM. The Point T is alſo found by making MT perpendicular to RM, and equal to the Secant CE: for ſo is the Angle $CMT = MCD$; and the Triangle MCT equal to the Triangle CDE.

10. Let AHh be an Equilater Hyperbola whoſe Semi-axis is AR and Center R. In the Meridian let RP be equal to the Tangent DE. Join AP, and draw $PH = AP$ and parallel to AR. Compleat the Parallelogram $HNRP$, ſo will the Point H be in the Hyperbola, and its ordinate $HN (= RP = DE = CT)$ be equal to the Curve ATt. From whence, and from *Prop.* 3 *Coroll.* 2. of Dr. *Gregory's Catenaria* (*Phil. Tranſ.* N. 231.) it appears that the Curve ATt is that call'd the *Catenaria* or *Funicularia*, viz. the Curve into whoſe Figure a *ſlack Cord or Chain* naturally diſpoſes its ſelf by the Gravity of its Particles.

" 11. Hence we have another Property of the *Catenaria* " not hitherto taken notice of (that I know of,) *viz.* that " ſuppoſing AR (= a, the conſtant Line in Dr. *Gregory*) " equal to the *Radius* of the Nautical Projection, and " RN the Secant of a given Latitude, then is NT the " *Catenaria*'s Ordinate at N, equal to RM the Meridio- " nal Part anſwering to the Latitude whoſe Secant is " RN.

12. That TA is the *Catenaria* is alſo demonſtrable from Dr. *Gregory*'s firſt *Prop.* Let Tu be the the Fluxion of the Ordinate NT; and tu ($= Nn$) the Fluxion of the Axe AN. Then becauſe of like Triangles TCM, Tut, $CM : CT (= TA) :: Tu : ut$, that is, as CM a conſtant Line to TA the Curve :: ſo is the Fluxion of the

Ordinate, to that of the Axe ($y : x$) according to *Prop.* 1. *Catenariæ.*

13. From the Premiſes the Conſtruction and ſeveral Properties of the *Catenaria* are eaſily deducible; one or two of which I'll ſet down.

1. The Area $ATMR$ is equal to $AOPR$ a Rectangle contained by Radius AR and RP the Tangent anſwering to Secant $HP = TM$. For becauſe of the like Triangles CMm, CEe; $CM : CE :: Mm : Ee$, that is, putting r, s, t, m for Radius, Secant, Tangent and Meridional part RM.) $r : s :: m : t$ whence $rt = sm$, and all the rt = all the sm, that is $AOPR = ATMR$, which agrees with Dr. *Gregory*'s *Cor.* 5. of *Prop.* 7.

14. Suppoſing the former Conſtruction, let be added the Line RH, including the *Hyperbolic Sector* ARH. I ſay the ſame Sector is equal to half the Rectangle $ARMQ$ contained by Radius AR and the Meridional Part RM, ($= \frac{1}{2} rm$). For the Sector ARH = Triangle RNH wanting the Semiſegment ANH. The Fluxion of the Triangle RNH is $\frac{st + ts}{2}$. The Fluxion of ANH is ts. So the Fluxion of the Sector ARH is $\frac{st + ts}{2} - ts = \frac{st - ts}{2}$. 'Tis found before (*Sect.* 13.) that $r : s (s : \frac{ss}{r}) :: m : t$; whence $st = \frac{ss}{r} m$. And becauſe of the like Triangles CDE, Efe, $CD : DE :: Ef : fe$. But $Ef = Mm = m$, becauſe both Ef and Mm are to Md in the ſame Reaſon, *viz.* as s to r; therefore $r : t (t : \frac{tt}{r}) :: m : s$: whence $ts = \frac{tt}{r} m$, and $\frac{st - ts}{2} =$

$\frac{ss - tt}{2r} m = \frac{rr}{2r} \dot{m} = \frac{1}{2} r \dot{m}$, = the Fluxion of the Hyperbolic Sector $A R H$, whose flowing Quantity is therefore equal to $\frac{1}{2} r m = \frac{1}{2} A R M Q$. *Q. E. D.*

15, This shews another Property of the *Catenaria*, viz. that it squares the Hyperbola; for $R M$ is equal to $N T$ the Ordinate of the *Catenaria*.

16. In *Fig.* 4. Let $A R$ be Radius, $A C B$ the Equitangential Curve; $M R N$ its Asymptote, in which let M, N, be any two Points equally distant from R. Upon M draw $M L$ parallel to $A R$ and equal to the *Difference* of the Secant and Tangent of that Latitude whose Meridional Part is $R M$ (by § 3, 4.) Upon N draw $N O$ parallel to $A R$, and equal to the *Summ* of the forefaid Secant and Tangent. Do thus from as many Points in the Asymptote as is convenient, and a Curve drawn equably through the Points $L - - - A - - - O$, &c. will be a *Logarithmic Curve*, whose *Subtangent* (being constant) is equal to Radius $A R$.

17. Let $n o$ be an Ordinate infinitely near and parallal to $N O$. $O p = N n$ the Fluxion of the Asymptote; $O T$ the Tangent, and $T N$ the Subtangent to the Logarith. Curve in O. Then $o p : p O :: O N : N T$. But $O N = s + t$; therefore $o p = s + t$. $p O = m$ (the Fluxion of the Meridian or Asymptote.) So the Analogy is $s + t : m :: s + t : N T$. By *Sect.* 13, 14, $s : m :: t : r$. also, $t : m :: s : r$: and thence $s + t : m :: t + s : r$. wherefore is $N T$ (the Subtangent to $L A O$) equal to Radius $A R$ a constant Line, and consequently the Curve $L A O$ is the Logarithmic Curve, and its Subtangent known.

18. The same Demonstration serves for $L M$, (any Ordinate on the other Side of $A R$) only changing the Sine + into — ; and then it agrees with Mr. *James Gregory*'s *Prop.* 3. *pag.* 17. of his *Exercitations*, viz. *That*

the Nautical Meridian is a Scale of Logarithms of the Differences whereby the Secants of Latitude exceed their respective Tangents, Radius being Unity. So here RM is the Logarithm of ML, the Difference of the Secant and Tangent of the Latitude whose Meridional part is RM.

19. Supposing the precedent Construction, if through any point C of the Curve ACB be drawn a right Line GCW parallel to MR, terminated with the Logarithmic Curve in W and the Radius AR in G: I say that the same right Line WG is equal to the intercepted part of the Curve Line AC.

20. Let wg be a Line infinitely near and parallel to WG, and terminated by the same Lines; and CS, $W\sigma$, perpendicular to the Meridian; CS intersecting wg in z, and $W\sigma$ in y. Let CM be a Tangent to AC in C; $W\tau$ a Tangent to AW in W; so is $CM = \sigma\tau$. Because of like Triangles Czc, CSM; and Wyw, $W\sigma\tau$; $CS : CM :: Cz : Cc$: also $W\sigma : \sigma\tau :: Wy : yw$. But $W\sigma = CS$; $\sigma\tau = CM$; $Cz = Wy$; therefore is yw the Fluxion of GW, equal to Cc the Fluxion of the Curve AC. Consequently $GW = AC$. *q. e. d.*

It may be noted that this Equitangential Curve gives the Quadrature of a Figure of Tangents standing perpendicular on their Radius. In *Fig.* 3. let $A\gamma\Gamma$ be a Curve whose Ordinates as $g\gamma$, $G\Gamma$, are equal to the Tangents of their respective intercept Arcs Ak, Ax. Let ΓG be produced to touch the Curve AC in C: then is the Area $A\Gamma G$ equal to the Rectangle contained by Radius AR and GC the produced part of the Ordinate; or $A\Gamma G = AR \times GC$. The Demonstration of which, and of the following *Section*, I for Brevity omit.

22. If we suppose the Figure ACB &c. KR (*Fig.* 3.) infinitely continued, to be turned about its Asymptote RK as an Axe, the Solid so generated will be equal to

a

rectangled Cone whose Altitude is equal to *A R*. And its Curve Surface will be equal to half the Surface of a Globe whose Radius is *A R*. So that if the Curve be continued *both ways* infinitely (as its Nature requires) the whole Surface will be equal to that of a Globe of the same Radius *A R*.

The Description of the Rular and Wheel, *Fig.* 2. is sufficient for the Demonstration of the Properties of the Curve: but in order to an actual Construction for Use, I have added *Fig.* 5. where *A B* is a brass Rular; *w h* the little Wheel, which must be made to move freely and tight upon its Axe (like a Watch-Wheel) the Axe being exactly perpendicular to the Edge of the Rular. *s* represents a little Screw-pin to set at several Distances for different Radii, and its under End is to slide by the Edge of the other fixt Rular. *p* is a Stud for convenient holding the Rular in its Motion.

Note, *Most of these Properties of this* Curve *by the Name of* la Tractrice, *are to be found in a* Memoire *of M.* Bomie *among those of the* Royal Academy *of* Sciences *for the Year* 1712. *but not publish'd till* 1715. *Whereas this Paper of Mr.* Perks *was produced before the* Royal Society *in* May 1714, *as appears by their Journal.*

VI. *An Account of a Book entituled* Methodus Incrementorum, *Auctore* Brook Taylor, LL.D. & R. S. Secr. *By the Author.*

WHEN I apply'd my self to consider throughly the Nature of the Method of Fluxions, which has justly been the Occasion of so much Glory to its great Inventor Sir *Isaac Newton* our most worthy President, I fell by degrees into the Method of Increments, which I have endeavour'd to explain in this Treatise. For it being the Foundation of the Method of Fluxions that the Flux-

ions

ions of Quantities are proportional to the nascent Increments of those Quantities; in order to understand that Method throughly, I found it necessary to consider well the Properties of Increments in general. And from those Properties I saw it would be easy to draw a perfect Knowledge of the Method of Fluxions: for if in any case the Increments are supposed to vanish and to become equal to nothing, their Proportions become immediately the same with the Proportions of the Fluxions. In this Method I consider Quantities, as formed by a continual Addition of parts of a finite Magnitude, and those parts I call the Increments of the Quantities they belong to, because that by the Addition of them the Quantities are increased. These parts being consider'd as formed in the same manner by a continual Addition of other parts; thence follows the Consideration of second Increments; and so on to third, fourth, and other Increments of a higher kind. For Example, if x stands for any Number in the Series 0. 1. 4. 10. 20. 35, *&c.* in which the Numbers are formed by a continual Addition of the Numbers in the Series 1. 3. 6. 10. 15, *&c.* then the Numbers in the latter Series are call'd the Increments of the Numbers in the foregoing Series; thus, for Example, if to the third Number (4) in the first Series, I add the corresponding third Number (6) in the second Series, I shall produce the next, that is the fourth Number (10) in the first Series, and so the rest. Any Number in the first Series being call'd x, the corresponding Number (which is its Increment) in the second Series I express by $\dot{x}$. And these Numbers $\dot{x}$ being form'd in the same manner by the Numbers in the Series 1, 2. 3. 4, 5, *&c.* I call these last Numbers $\ddot{x}$, they being the first Increments of the Numbers $\dot{x}$, and the second Increments of the Numbers x; and so on. Hence having given any Series of Numbers that are call'd by a general Character x, their Increments are found by taking their

their Differences; thus in the present Example, the first Increments $\dot{x}$ in the Series 1. 3. 6. 10. 15, *&c.* are found by taking the Differences of the Numbers $\dot{x}$ in the Series 1. 4. 10. 20. 35, *&c.* and the second Increments $\ddot{x}$ in the Series 1. 2. 3. 4. 5, *&c.* are found in the like manner, by taking the Differences of the Numbers $\dot{x}$, and so of the third and other Increments. This Method consists of two parts; One is concerned in shewing how to find the Relations of the Increments of several variable Quantities, having given the Relations of the Quantities themselves; and the Other is concerned in finding the Relations of the Integral Quantities themselves freed from the consideration of their Increments, having given the Relations of the Increments: either simply, or they being any how compounded with their Integral Quantities. In the Method of Fluxions Quantities are not consider'd with their parts, but with the Velocities of the Motions they are supposed to be formed by; or to speak more accurately, they are consider'd with the Quantities of the Motions by which they are supposed to be generated; for the Fluxions are proportional to the Velocities, only when the moving Quantities, which produce the flowing Quantities consider'd, are equal. These Quantities of Motion, or Velocities when the moving Quantities are equal, are what Sir *Isaac Newton* calls Fluxions. As in the Method of Increments there are second, third, and other Increments; so in the Method of Fluxions there are second, third, and other Fluxions; the Fluxions themselves being consider'd as Quantities that are formed by Motion, the Quantity of which Motion is their Fluxions. As the Method of Increments consists of two Parts; one being concern'd in finding the Increments from the Integrals given, and the other in finding the Integrals having given the Increments; so does the Method of Fluxions consist of two Parts; the one shewing how to find the Fluxions, having the

the Fluents given; and the other ſhewing how to find the Fluents freed from Fluxions, having given the Relations of the Fluxions, whether compounded with their Fluents or otherwiſe. The Principles of this Method may all be drawn directly as a Corollary from the Principles of the Method of Increments. For Sir *Iſaac Newton* having demonſtrated (*Phil. Nat. Princ. Math. Sect.* 1. and in the Beginning of his Treatiſe *De Quadratura Curvarum*) that the Fluxions of Quantities are proportional to their naſcent or evanaſcent Increments, if in any Propoſition relating to Increments, you make the Increments to vaniſh and to become equal to nothing, and for their Proportion put the Fluxions, you will have a Propoſition that will be true in the Method of Fluxions. This is but a Corollary to Sir *Iſaac Newton*'s Demonſtration of the Fluxions being proportional to the naſcent Increments. For this reaſon, to make the Method of Fluxions to be underſtood more throughly, I thought it proper to treat of theſe two Methods together, and I have handled them promiſcuouſly as if they were but one Method. Some people, becauſe that the Fluxions are proportional to the naſcent Increments of Quantities, have thought that by the Method of Fluxions Sir *Iſaac Newton* has introduced into Mathematicks the Conſideration of infinitely little Quantities; as if there were any ſuch thing as a real Quantity infinitely little. But in this they are miſtaken, for Sir *Iſaac* does only conſider the firſt or laſt Ratio's of Quantities, when they begin to be, or when they vaniſh, not after they are become ſomething, or juſt before they vaniſh; but in the very moment when they do ſo. In this caſe Quantities are not conſider'd as infinitely little; but they are really nothing at the time that Sir *Iſaac* takes the Proportions of their Fluxions; and the Truth of this Method is demonſtrated from the Principles of the Method of Increments, in the ſame manner as the Ancients demonſtra-

ted

ted their Conclusions in the Method of Exhaustions, by a *Deductio ad Absurdum.*

Having premised thus much in general concerning the two Methods here treated of, to come to a particular Description of this Book; In the Preface I give a short Description of the Method of Increments, and an Account of Sir *Isaac Newton*'s Notion of the Fluxions which I have already spoke of. The Book consists of two Parts, and contains 118 Pages in *Quarto*; the Propositions being number'd throughout from the Beginning. In the first Part I explain the Principles of both Methods: and in the second Part I shew the Usefulness of them in some particular Examples.

After having explain'd the Notation I make use of in the Introduction, in the first Proposition I explain the direct Method both of Increments and of Fluxions. The second Proposition shews how to transform an Equation wherein Integrals and their Increments, or wherein Fluents and their Fluxions are concern'd; so as in the Room of the Integrals or Fluents, to substitute their Compliments to a given Quantity with their Increments or their Fluxions, they increasing in a contrary Sense to the Quantities in the first Supposition. In the third Proposition I shew how to transform a fluxional Equation, so as to change the Characters of the Fluents, making that Quantity to flow uniformly which in the first Supposition flow'd unequally, having second, third and other Fluxions, and making that Quantity which in the first Supposition flow'd uniformly, now to flow unequally, so as to have second and third Fluxions, *&c.* This Proposition is of great use in the inverse Method, when you would invert the Expression of the Relation of the flowing Quantities; for Example, if in the Supposition z flows uniformly and x variably, by the inverse Method of Fluxions you will find x exprest by the Powers of z: but if you would find z

 exprest

expreſt by the Powers of x, you muſt then transform the Equation by this Propoſition. Sir *Iſaac Newton* and Mr. *de Moivre* do this by the Reverſion of Serieſes; but I take this to be the more proper and more genuine Method of doing it directly. In the fourth and fifth Propoſitions are explain'd the Method of judging of the Nature and Number of the Conditions that may accompany an Inctemental or a Fluxional Equation. This is a Circumſtance that I don't find to have been explain'd by any one before, and the Propoſitions are ſomething intricate; wherefore it will not be improper to explain this Matter a little more at large. Suppoſe then that z and x are two variable Quantities, and ſuppoſing z to increaſe uniformly by the continual Addition of its conſtant Increment z, (according to the Notation I make uſe of in this Book) ſuppoſe $x + z = x$. Then if it be propoſed as a Problem to find the Value of x, expreſs'd by the Powers of z, and quite freed from the Increments; by the fourth Propoſition there may be three Conditions added to this Problem. The Demonſtration of this is taken from the Formation of the Integrals by a continual Addition of their Increments. Suppoſe that all the Values of z, x, $\dot{x}$, $\ddot{x}$, $\dddot{x}$, *&c.* were ſet down in order in ſo many Columns, and that at the Head of the Table, the correſponding Values of $z, x, \dot{x}, \ddot{x}, \dddot{x}$, were expreſt by the Symbols a, c, $\dot{c}$, $\ddot{c}$, $\dddot{c}$. Then by means of the given Equation $\dot{x} + z = x$, or $\dot{x} = x - z$, will $\dot{c} = c - a$, whence the ſecond correſponding Values of z, x, $\dot{x}$, $\ddot{x}$, $\dddot{x}$ will be $a + z$, $c + \dot{c}$, $c + \dot{c}$, $c + c - a$ $(= \dot{c} + \ddot{c})$ and $c + \dot{c} - a - z$ (by the Eq.) Whence again the third Values are $a + 2z$, $c + 2\dot{c} + \ddot{c}$, $c + 2\dot{c} + c - a$ $(= \dot{c} + 2\ddot{c} + \dddot{c})$ $\dot{c} + 2c - 2a + \dot{c} - z$, and $c + 2\dot{c} + \ddot{c} - a - 2z$, and ſo you may proceed to find the fourth, fifth, and all the

the other Values of z, $\dot{x}$, $\ddot{x}$, $\dot{\dot{x}}$, $\dot{\dot{\dot{x}}}$. But by what is already set down, it is evident that all their Values will be exprest by a and z, and the three Symbols c, c, c: and consequently all the Values of the rest of the Increments of x, viz. $\dot{\dot{x}}$, $\dot{\dot{\dot{x}}}$, *&c.* will be express'd by the same Symbols. Whence it follows that to determine the Values of the Symbols c, c, c, there may be taken three Conditions relating to the Values of $\dot{x}$, $\ddot{x}$, $\dot{\dot{x}}$, $\dot{\dot{\dot{x}}}$, *&c.* promiscuously, as the fourth Proposition directs. The same Rule holds good in the Method of Fluxions. For Example, having given the Æquation $a z^2 + \dot{x}^2 \dot{x} = 0$, if it be proposed to describe the Curve that it belongs to; by the fourth Proposition it may also be required as a Condition, that the Curve shall pass through two given Points; that it shall touch two given Lines; that it shall pass thro' a given Point, and when it cuts a given Line shall have a given Degree of Curvature; or that it shall have any other two Circumstances that depend upon the Values of the third, fourth or other Fluxions. These Conditions that attend Incremental or Fluxional Equations, I don't know to have been sufficiently taken notice of by any Body: but they ought well to be attended to in the Inverse Methods; the Solutions of particular Problems being never perfect, unless there be provision made for the satisfying of them, by the indetermined Coefficients in the Equation that contains the Solution of the Problem. Examples of this may be seen in Prop. 17 and 18, where I give the Solution of the Problems concerning the *Isoperimeter*, and the *Catenaria*.

The sixth Proposition contains the general Explanation of the Inverse Method both of Fluxions and of Increments, which consists in the Solution of this Problem. Having given the Relations of the Increments, or of the Fluxions of several Quantities, whether they be considered

red with their proper Integrals or with their proper Fluents or not; to find the Relations of the Integrals or of the Fluents, freed from their Increments or from their Fluxions. The Direction I have given for finding the Solution in finite Terms is but tentative. And I muſt confeſs I know of no other Method that is general for all Caſes. For I can find no certain Rule to judge in general, whether any propoſed Equation, involving Increments or involving Fluxions, can be reſolved in finite Terms. For this Reaſon we are obliged to ſeek the general Solution in infinite Serieſes; which when they break off, or when they can any way be reduced to finite Terms, they then contain the Solutions which we always hope for. The Method of finding theſe Serieſes is explain'd in the eighth Propoſition, and that is by means of a Series that is demonſtrated in the ſeventh Propoſition. And this I take to be the only genuine and general Solution of the inverſe Methods. For in this Solution you always have thoſe indetermined Coefficients, which are neceſſary to adapt the Equation that is found to the Conditions of the Problem propoſed. For want of this Circumſtance all other Methods are imperfect; and particularly Sir *Iſaac Newton*'s Method of finding Serieſes by a Rular and Parallelograms labours under this Difficulty, becauſe it brings no new Coefficients into the reſulting Equation, which may afterwards be determined by the Conditions of the Problem. However becauſe this Method is very ingenious and very elegant, I thought it proper to explain it in the following (*viz.* the *9th*) Prop. The *10th*. *11th*, and *12th* Propoſitions conclude the firſt Part, and in them I treat of the manner of finding the Integral or the Fluent, having given the Expreſſion of a particular Increment, or of a particular Fluxion of it; without being involved with the Integrals, or with the Fluents, or with any other Increments, or with any other Fluxions of it.

it. This is a particular Case of the Inverse Method, but for its great usefulness I thought it deserved particularly to be taken notice of. This Problem is treated of in general in the 10*th* Proposition. The Method of solving it in finite Terms is only tentative; and when that does not succeed, recourse must necessarily be had to the Solution by a Series in the 8*th* Proposition. In the 11*th* and 12*th* Propositions I have shew'd how Serieses may be conveniently found, in some particular Cases when Fluxions are proposed.

In the second Part I have endeavour'd to shew the Usefulness of these Methods in the Solution of several Problems; The 13*th* Proposition is much the same with Sir *Isaac Newton's Methodus Differentialis*, when the Ordinates are at equal Distances: and in an Example at the End of this Proposition I have shew'd how easily Sir *Isaac Newton*'s Series for expressing the Dignity of a Binomial may be found by this Incremental Method. The 14*th* Proposition shews in some measure how this Method may be of use in summing up of Arithmetical Serieses. In the 15*th* Proposition I shew by some Examples how the Proportions of the Fluxions are to be found in Geometrical Figures; from whence immediately flows the Method of finding the Radiuses of their inosculating Circles, the Invention of the Points of contrary Flexure, and the Solution of other Problems of the like nature. In the 16*th* Proposition I shew how the Method of Fluxions is to be applied to the Quadrature of all sorts of Curves. In the following Proposition I give a general Solution of the Problem of the *Isoperimeter*, which has been treated of by the two famous Mathematical Brothers the *Bernoulli*'s. In the 18*th* Proposition I give the Solution of the Problem about the *Catenaria*, not only when the Chain is of a given Thickness every where, but in general, when its Thickness alters according to any given Law. In the

fol-

following Propoſition I ſhew the *Fornix* or Arch which ſupports its own Weight to be the ſame with the *Catenaria.* In the two next Propoſitions I ſhew how to find the Figures of pliable Surfaces which are charged with the Weight of a Fluid. In the 22*d* and 23*d* Propoſitions I treat of the Motion of a Muſical String, and give the Solution of this Problem: To find the Number of Vibrations that a String will make in a certain time, having given its Length, its Weight, and the Weight that ſtretches it. This Problem I take to be entirely new, and in the Solution of it (in the laſt part of Prop. 23.) there is a remarkable Inſtance of the Uſefulneſs of the Method of firſt and laſt Ratio's. The 24*th* Propoſition gives the Invention of the Center of Oſcillation of all Bodies; and in the 25*th* Propoſition I have given the Inveſtigation of the Center of Percuſſion. It is known that this Problem is ſolved by the ſame *Calculus* as the foregoing; wherefore it is generally thought that theſe two Centers are the ſame. But that is a Miſtake, becauſe the Center of Oſcillation can be but one Point; but the Center of Percuſſion may be any where in a certain Line, which this Propoſition ſhews how to find. There is an Error in this Propoſition, which I was not ſenſible of till after the Book was publiſh'd, wherefore I take this Opportunity of correcting of it. It does not affect the Reaſoning by which I find the Diſtance of the Center of Percuſſion from the Axis of Rotation; but it is this, that I ſuppoſed the Center of Percuſſion to be in the Plane paſſing thro' the Center of Gravity, and perpendicular to the Axis of Rotation: which is a Miſtake. It is corrected by the following Propoſition.

PROP.

PROP. PROB.

To find the Diſtance of the Center of Percuſſion from the Plane paſſing thro' the Center of Gravity and perpendicular to the Axis of Rotation.

SOLUTION.

Let the ſixth Figure be ſuppoſed in the Plane paſſing thro' the Axis of Rotation, and in which the Center of Percuſſion is ſought.

Let *A B* be the Axis of Rotation, *A G C* be the Interſection of this Figure with the Plane paſſing thro' the Center of Gravity, and perpendicular to the Axis of Rotation. *G* be the Point whereon a Line, rais'd perpendicular to this Figure, will paſs thro' the Center of Gravity; *B E* be a Line parallel to *A G* wherein is the Center of *Percuſſion*. Then to find the Diſtance *A B*, let *p* ſtand for an Element of the Body propoſed ſtanding perpendicularly on any point *D* Draw *D C* perpendicular to *A G C*. and *A B* will be equal to the Summ of all the Quantities $p \times GC \times CD$ taken with their proper Signs, divided by the Body it ſelf multiplied into the Diſtance *A G*.

Having thus found the Diſtance *A B*, ſuppoſe the Plane of the Figure in *Prop.* 25. to cut the preſent Figure at right Angles in the Line *B E*, and the Center of Percuſſion will be rightly determined by that Propoſition.

The 26*th* Propoſition ſhews how to determine the Denſity of the Air at any Diſtance from the Center of the Earth, ſuppoſing the Denſity always to be proportional to the compreſſing Force, and that the Power of Gravitation is reciprocally as the Diſtances from the Center of the Earth.

 The

The last Proposition shews how to find the Refraction of a Ray of Light in its passage thro' the Atmosphere, upon the Supposition that Light is a Body, and that the Refraction of it is caused by the Attraction of the Bodies the Rays approach to. In this Proposition there is a remarkable Instance of the Usefulness of the Method of Increments in finding the Coefficients of a Series, which according to the Values of a certain Symbol, as *n*, expresses both all the Fluents, and all the Fluxions of a certain Quantity.

II. Ludovici Ferdinandi Marsilii *Dissertatio de* Generatione Fungorum. *Rom.* 1714. 4to.

THis Author tells us that he gave his youthful Inclinations to the Study of the Mathematicks and Observations of Nature, under the Tuition of the celebrated *Malpigius*, and *Lelius Triumfettus* Botanick Professor of *Bononia*: and amongst the various Productions of Nature, his chief Delight was in the Contemplation of the sudden Growth and various sorts of *Mushrooms* which both the Earth and Trees brought forth. Of the first Kind he observed the greatest Number to arise in Camps, produced from the Horses Dung, and are commonly called *Prataiouli*.

In the Years 1699 and 1700, being then in *Croatia* and *Transylvania*, in the Armies there, he made a large Volume of Designs of *Fungi*, which he sent to *Triumfetti* to put in order, who added a great Number to them of such as he found about *Bononia*; yet after all the most diligent Search, he could never find them to produce any Seed either in their Gills or other Parts.

The Origin and Generation of *Mushrooms* he says is not easy to demonstrate, since both the Antients and Moderns

disagree very much about it. The late Botanists seem to be of three different Sentiments concerning their Produce. Mr. *Ray*, Dr. *Sherrard*, Mr. *Doody*, *Boccone* and *Mentzelius* having observed some *Mushrooms* to have had Seed, were of opinion others might have the same Original. *Clusius* and *John Baptista Porta* had in some also observed their Seed: Others, *viz. Sharrock* and the accurate *Malpigius*, who could not find any Seed in them, altho' with the Assistance of Microscopes, did suppose they might be produced by Pieces of themselves, carried by the Winds from place to place, as other Plants are by Slips and Offsetts.

The third Opinion, which he says most agree in, is that they arise from Putrefaction, or a Mixture of certain Salts, Sulphur and Earth impregnated with the Dung of Beasts.

The *Fungus seminifer Campaniformis Mentzelii*, &c. being the Mushroom which first gave the occasion of the Opinion of their having Seed, this nice Author has accurately figured and observed, and supposes with others that these seedlike Bodies may be the *Ovaria* of some Insects; and the rather because they are so very large in proportion to the smallness of the *Mushroom*: and that they had often been sowed by Dr. *Amadoes* a curious Botanist, without any Success towards raising them. From whence he concludes these Bodies ought to have another Denomination than Seed; neither is he of the Opinion that they are produced by parts of themselves.

In his Division of *Mushrooms* he first treats of the *Truffles* and their Increase, Situation and Soyl, Colour, Taste and Consistence. He next proceeds to soft Mushrooms, such as he observed in his own Garden; which having in the Spring been meliorated with Horse-dung, about the middle of *June* there sprung up divers of that sort which the *Italians* call *Prataiuoli*, amongst a Bed of *Lettice*. These continued till near the midst of *August* before they went

off. Of these and some other Kinds he accurately figures the first Shootings and Fibres.

His next Tribe are such as grow from Wood, but yet are themselves soft. Of these he observes three Kinds; the first a large one in his Window, out of a piece of Firwood which it had often rained on; with two smaller sorts from some rotten Boards in his Garden. All these he figures both in their natural and divided States, as also Microscopically.

Treating of hard woody *Mushrooms* (of which he also gives you some accurate Figures) he observes they rarely appear on the Trees, in *Germany* and *Croatia*, before they are twenty or thirty Years old; but most commonly when forty or fifty: and the Original of them he attributes chiefly to the Rottenness of the Wood, and says they generally break out in the Spring, when the Leaves begin to shoot. And that usually they grow below the middle of the Trees, and are cause of so much Decay in them, that they often die in three or four Years.

It may not here be amiss to subjoin what Dr. *Lancisius* communicates to our Author, concerning the *Lapis Fungarius*, *viz.* that altho' this *Mushroom-producer* has the Name of a Stone, it ought not to be reckoned of that *Genus*, it being really no other than a Mass or *Congeries* of Roots, Seeds and Juices coagulated with Earth into, as it were, a stony Substance. Upon which pouring Water and setting it in a warm Place, it loosens its hardned Substance; and by mollifying its Fibres and moistning its concrete Juices, out of the Cliffs and Chinks thereof the *Mushrooms* spring, as they do in other places from simple Dung and loose Earth And it is also farther to be noted, that when this stony Mass has thus yielded these its Offspring, the Remains grows light, porous and decay'd, its nutritive Juices being then exhausted.

F-I N I S.


Fig. I.
A
A
b b b b b b b
d d d d d d d


Fig. V.


Fig. III.
G
G

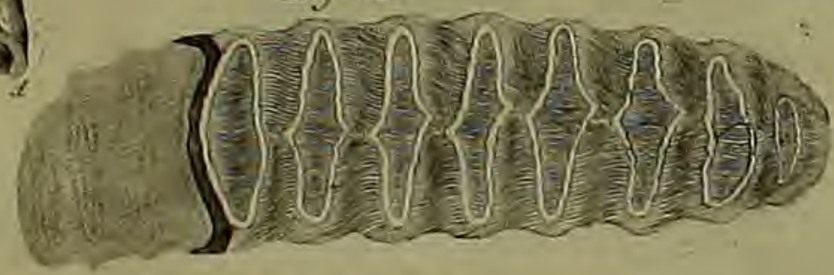


Fig. VI.


A Scale of Inches
1 2 3 4 5 6 7 8 9 10


E
Fig. II.
E
f f f

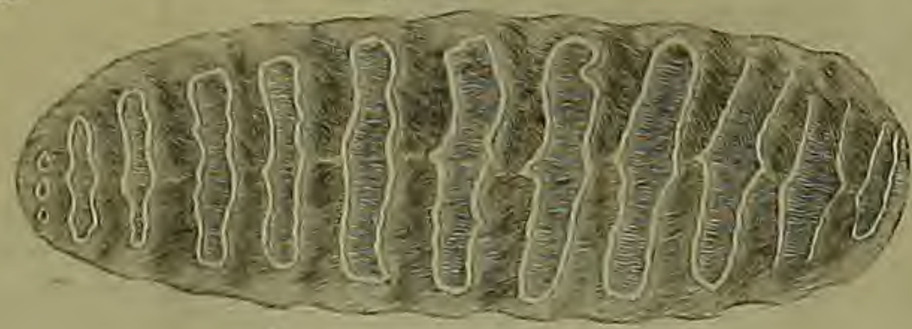


Fig. VII.


Fig. IIII
k
H
H
k

Numb. 346.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *November* and *December*, 1715.

The CONTENTS.

I. *A short History of the several* New-Stars *that have appear'd within these* 150 *Years; with an Account of the Return of that* in Collo Cygni, *and of its Continuance observed this Year* 1715.

WHether it be owing to the greater Diligence of the Moderns, or that in reality no such thing has happen'd for many Ages past, I will not undertake to determine; but this is certain that, within the Space of the last 150 Years, more Discoveries have been made of Changes among the Fixt Starrs, than in all Antiquity before. And tho' it be said that *Hipparchus*, on occasion of a New Star that appeared in his Time, was induced to number the Stars, and make the first Catalogue of them, which was, in the Opinion of *Pliny*, *Res vel Deo improba*; yet neither he or any of the Ancients have left us the Place of that New Star, to compare with those lately seen, one of which might perhaps be the same with it, re-appearing after a long Period of Years. Now though several Authors have severally described those that have been seen nearer to our Times, it may not perhaps be amiss here to give a short Recapitulation of what was principally remarkable in each of them, with the Times of their first Appearance, as far as can be collected.

And first, That in the *Chair* of *Cassiopeia*, was not seen by *Cornelius Gemma* on the Eighth of *November* 1572, who says, he that Night considered that Part of Heaven in a very serene Sky, and saw it not: but that the next Night, *Novemb.* 9°. it appeared with a Splendour surpassing all the fixt Stars and scarce less bright than *Venus*. This was not seen by *Tycho Brahe* before the 11*th* of the same Month, but from thence He assures us that it gradually decreased and died away, so as in *March* 1574,

1574, after sixteen Months, to be no longer visible; and at this Day no Signs of it remain. The Place thereof in the Sphere of Fixt-Stars, by the accurate Observations of the same *Tycho*, was 0s. 9°. 17′. *à* 1ma ✱ ♈b, with 53°. 45′. North Latitude.

Such another Star was seen and observed by the Scholars of *Kepler*, to begin to appear on *Sept.* 30°. *st. vet. anno* 1604, which was not to be seen the Day before: but it broke out at once with a Lustre surpassing that of *Jupiter*; and like the former it died away gradually, and in much about the same time disappear'd totally, there remaining no Footsteps thereof in *January* 160⅞. This was near the Ecliptick, following the Right-Leg of *Serpentarius*; and by the Observations of *Kepler* and others, was in 7s. 20°. 00′ à 1ma ✱. ♈, with North Latitude 1°. 56′. These two seem to be of a distinct *Species* from the rest, and nothing like them has appear'd since.

But between them, *viz.* in the Year 1596, we have the first Account of the wonderful Star *in Collo Ceti*, seen by *David Fabricius* on the third of *August*, st. vet. as bright as a Star of the third Magnitude, which has been since found to appear and disappear periodically: its Period being precisely enough 7 Revolutions in Six Years, tho' it return not always with the same Lustre. Nor is it ever totally extinguish'd, but may at all times be seen with a Six-Foot Tube. This was singular in its Kind, till that in *Collo Cygni* was discovered. It precedes the first Star of *Aries* 1°. 40′, with 15°. 57′ South Latitude.

Another New Star was first observed by *Will. Jansonius* in the Year 1600, *in Pectore* or rather *in eductione Colli Cygni*, which exceeded not the third Magnitude. This having continued some Years, became at length so small as to be thought by some to disappear entirely: but in the Years 1657, 58 and 59, it again arose to the third Magnitude, tho'soon after it decay'd by degrees to the

fifth or ſixth Magnitude, and at this Day is to be ſeen as ſuch in 9ˢ. 18°. 38′. à 1ᵐᵃ ✱. ♈, with 55°. 29′. North Lat.

A fifth New Star was firſt ſeen and obſerved by *Hevelius* in the Year 1670, on *July* 15. *ſt. vet.* as a Star of the third Magnitude, but by the Beginning of *October* was ſcarce to be perceived by the naked Eye. In *April* following it was again as bright as before, or rather greater than of the third Magnitude, yet wholly diſappeared about the middle of *Auguſt*. The next Year, in *March* 1672. it was ſeen again, but not exceeding the ſixth Magnitude: ſince when it has been no further viſible, though we have frequently ſought for its Return; its place is 9ˢ. 3°. 17′. *a* 1ᵐᵃ ✱. ♈. and has Lat. North. 47°. 28′.

The Sixth and laſt is that we deſcribed from the *Acta Berolinenſia*, in No. 343 of theſe Tranſactions; diſcovered by Mr *G. Kirch* in the Year 1686, and its Period determined to be of 404 ½ Days: and though it rarely exceed the fifth Magnitude, yet is it very regular in its Returns, as we found in the year 1714. Since then we have watched, as the Abſence of the Moon and the Clearneſs of Weather would permit, to catch the firſt beginning of its Appearance in a Six-Foot Tube, that bearing a very great Apperture diſcovers moſt minute Stars. And on *June* 15. laſt, it was firſt perceived like one of the very leaſt Teleſcopical Stars; but in the reſt of that Month and *July* it gradually encreaſed, ſo as to become in *Auguſt* viſible to the naked Eye; and ſo it continued all the Month of *September*. After that it again died away by degrees, and on the 8*th* of *December* at Night was ſcarce diſcernible by the Tube, and as near as could be gueſſed, equal to what it was at its firſt Appearance on *June* 15*th*: ſo that this Year it has been ſeen in all near Six Months, which is but little leſs than half its Period: And the middle, and conſequently the greateſt Brightneſs, falls about the 10*th* of *September*. Thoſe that pleaſe to ſeek for it, may expect its firſt Appearance in *July* next, and find it in 9ˢ. 6°. 30′. *circiter* à 1ᵐᵃ ✱. ♈, with *Lat. Bor.* 52°. 40′.

BOTANICUM *Hortense* IV.

II. *Continued from* No. 345. *By* James Petiver, F. R. S.

SECT. II.

Indian Herbs and Trees.

94. A*Ntego* Green *Blite.*
Blitum Antegoanum *viride*, caule & pediculis *rubris.*

This rises about a *Yard* high, its *Leaves* smooth, veined and oval, standing on long slender *Pedicles*, red, as are the large Stalks, from whence grow Tufts of green *Panicles*, which *Flower* and *Seed* from *July* till *Autumn.*

I received *Plants* and *Seed* of this and many others from my worthy Friend Mr. *John Douglass* Surgeon, and *Brother* to his Excellency the *Governour* of *Antego.*

95. Climbing *Virginia* Eupatorium.

Eupatorium *Carolin.* scandens, *Fegopyri* folio *Hort.* nost. Sicc. *Ray* Vol. 3. *Append.* 244. E. 30.

Clematitis novum genus, *Cucumerinis* foliis *Virginianum* Pluk. *Tab.* 163. fig. 3.

Planta peculiaris pappescens non lactescens *Banister* Alm. Botan. p. 109. pl. 6.

This elegant *Plant* is accurately *Figured* in Dr. *Pluknet's* Phytographia, and Mr. *Banister's* Herbarium *Virginianum* which is now in the *Press* from his own *Original* Designs. It is the only *Virginia Climber* of this *Tribe* that has as yet come to our Knowledge, and never raised in any *European Garden* before.

96. Braynes *Cape* Naked Dasie, *Ray* H. Pl. 362. pl. 2. & Vol. 3. p. 220 pl. inter 6 & 7.

Coma Capensis *Coronopi* folio.

Bellis *Afric.* lutea minor, *fl.* nudo cernuo *Alm.* Bot. 66. pl. 6.

Bellis Afric. *Coronopi* folio, *fl.* nudo *Ciassi.*

Bellis *Afric.* capitulo aphyllo luteo, *Coronopi* folio, cauliculis procumbentibus *Herm.* H. Leyd. 86.

Bellis annua, capitulo aphyllo luteo H. *Ox.* Sect. 6. Tab. 7. fig. ult. opt.

Chrysanthemum *Afric.* minimum, *flosculo* nudo cernuo *Ejusd.* p. 30. pl. 24.

Chrysanthemum exoticum minus, capitulo aphyllo, *Chamemeli* nudi facie *Breyn.* Cent. 156. c. 76. fig.

Chrysanthemum *Tingitanum* minimum procumbens, *foliis* versus imum dentatis, *flosculo* nudo cernuo *Moris.*

Its *Leaves*, naked *Flowers*, and Way of *growing* distinguish it from *others.*

97. Trifid *Cape* Gold Tufts, *Ray* 3. p. 173. 11.

Coma aurea *Africana* foliis glaucis in extremitate trifidis *Hort.* Amstelod. Vol. 2.

It *Flowers* in *September*, and the *Seed* is ripe about *Christmas.*

98. Small Headed *American* Live ever.

Elichrysum *Carolin.* Gnaphalii *Americani* facie *Hort.* nost. Sicci *Ray* 3. Append. 244.

Helichrysum annuum majus, erectum *Virginianum* H. Ox. Sect. 7. Tab. x Ser. 2, pl. 1.

Helichrysum sive *Chrysocome* Gnaphaloides *Virginiana* annua, *foliis* obtusioribus, *capitulis* argenteis conglobatis *Bob.* Oxon. 88. pl. 21.

Gnaphalium *Lusit.* comis argenteis *Pluk.* Tab. 31. fig. 5. ex sent. *J. Bobart.*

This rises near a *Yard* high, is somewhat *Woody* with many Branches ending in *Tufts* of small straw coloured *Heads* which rarely or never fully open. The *Indians* rub their *Head* with this Plant, and assert its good for the Eye-sight, as my late curious Friend Mr. *John Lawson* informed me. By another Person I am told they drink a *Tea* of it in *Feavers*.

99. *Fairchild*'s small-flowred *Live-ever*.
Elichrysum fl. pallescentibus minimis, *Spicæ* foliis.

Its *Stalks* are thick set, with whitish pointed *Leaves*; the *Flowers* grow in Tufts of a pale Yellow and very small. It *Flowred* in *July*, &c. in Mr. *Thomas Fairchild*'s Garden at *Hoxton*, the only Place I have yet seen it in.

100. Woolly *Live-ever* with red thready *Flowers*.
Elichrysum Lychnidis *Coronariæ* folio.

This elegant *Plant* and the next, I have with Pleasure seen in the *Bishop* of *London*'s Gardens at *Fulham*, under the Care of Mr. *Millward*.

101. Round Saddle-leaved *Cape* Live-ever.
Elichrysum Capense, *Perfoliatæ* folio.

These *Leaves* are round, somewhat bristle-edged, grow alternate and saddle the Stalk, like our *Perfoliata* or *Thorow Wax*. My late industrious Friend Mr. *William Brown*, Surgeon, brought me from the *Cape of Good Hope* the only Specimens of this singular Plant I have since seen.

102. Welted *Antego* Spike-Cudweed.
Gnaphaloides *Antegoana* spicata, *caule* alato.
an *Amaranthoides* fruticosum *fol.* longis angustis, subtùs niveis *Jam. Cat.* 48. pl. 3. *Hist.* p. 43. *Tab.* 7. *f.* Ray *Vol.* 3. p. 127. 2.

This elegant Plant was raised in *Chelsey-Garden* from Seed sent me by my kind Friend Mr. *John Douglass* at *Antego*.

103. Herman's *Cape* Live-ever with *Sage* Leaves *Ray* 1860 Hist. Ox *Bob.* 905.

Pseudo-Elichrysum Cap. *Salviæ* folio.

Conyza *Afric.* arborescens. *Salicis Capreæ* foliis, odore *Rorismarini* Breyn. Pr 2.

Conyza *Afric.* incana arb odore *Salviæ* & *Rorismarini* Flor. *Pruss.* 149. non occurrit

Elichryso affinis *Afric* arb *fl* purp. violaceis, *Salviæ* foliis, odore *Rorismarini* Herm. H. *Leyd.* 229. fig. *Pluk.* 174. 1.

It *Flowers* in *March* and *April* in our *Stows*, and at the same time at the *Cape of Good Hope.*

104. Herman's *Peruvian* Live-ever *Ray* 1869. pl. ult.

Elichryso affinis *Peruviana* frutescens *Herm.* H. *Leyd.* 666. *Pluk.* 27. fig. 1.

Agerato affinis *Peruviana* frutescens *Par. Bat.* & Prodr.

Pseudo-Helichrysum frut. *Peruv.* fol. longis serratis *Bob.* Ox. 90. 3.

This by some has been erroneously shewn in our *Gardens* for the *true* Cortex Peru.

105. Mary-land *Bobart.* Ray 3. p. 210. pl. 15.

Bobartia lutea hirsuta, *caule* Echii.

Chrysanthemum *Helenii* folio, *umbone* floris grandiusculo prominente *Pluk.* 242. fig. 2. *Bob.* Ox. 23. pl. 65.

Chrysanthemum pilosissimum *umbone* purpurascente, *petalis* extùs villosis *Nob. Act. Phil.* No. 246. p. 401. pl. 26.

Its lower *Leaves* somewhat like *Plantain*, lightly notcht, rough and hairy, the *Stalks* speckled with red, and rough as *Vipers Buglôss*, its *Flowers* composed of 13 yellow *Petalæ* set in a double Row of narrow green hairy pointed *Leaves.* What is remarkable in this *Tribe*, is a large purple *Umbo* or *Disk*, rising in the midst of the *Flower* like a *Button.* A very peculiar sort, of this *Family*, I first saw many Years since with Mr. *Jacob Bobart* in the *Physick-Garden* at *Oxford*; for which reason I have presumed to distin-

ſtinguiſh it by his Name, that it may be the eaſier known from *Chryſanthemum*, *Dracunculus* or *Ptarmica* to which others have ranked it. This *Flowers* moſt part of the *Year*.

106. *Fairchild*'s broad *Bobart*.

Bobartia *Viginiana*, fol. lato ſcabro, baſi alato.

Its Flower-ſtalk ſwells gradually towards the Head, which is compoſed of many regular broad pointed blackiſh green *Scales*, the Rim conſiſts of 15 or more yellow *Petalæ*, out of its purple Disk come many ſmall yellow *Floſculi*, which I have not obſerved in the other. Mr. *Thomas Fairchild* raiſed this Plant from Seed he received, with many others, from that *curious* Botaniſt Mr. *Mark Catesby* of *Virginia*. It *Flowred* at *Hoxton* about the middle of *October*.

107. Cape *Uvedale* with a *Poplar* Leaf, *Ray* 339. 1.

Uvedalia Capenſis *Populi* folio.

Chryſanthemum arboreſcens *Æthiopicum*, foliis *Populi* albæ *Breyn.* Cent. 155. c. 76. fig. *Bob.* Oxon. 23. pl. 58. Sect. 6. Tab. 3. Ser. 3. fig.

an *Chryſanthemum* lanatum, crenatis foliis *Æthiopicum* Pluk. *Tab.* 274. f 5.

This differs from the *Chryſanthemums* in having a *bacciferous* Rim.

108. Wild *Hopps*. Ray 3. p. 307. 16.

Nodiflora Jamaic. *Scrophulariæ* folio.

Sideritis ſpicata *Scrophulariæ* folio, *fl.* albo, *ſpicis* brevibus habitioribus rotundis, pediculis inſidentibus *Sloan.* Cat. Jam. 65. *Hiſt.* Tab. 109. fig. 2.

That curious Naturaliſt Dr. *Hans Sloane*, has well deſcribed this Plant, and given a very accurate *Figure* of it, which truly agrees with a fair Specimen my hearty Friend Capt. *Thomas Walduck* ſent me from *Barbadoes*, from the *Seed* of which this Plant was raiſed. I take the *Nodiflora*

Lu-

Luzon. *Sideritidis* folio *Gazoph.* Naturæ *Tab.* 69. fig. 6. to be a ſort of this with narrower Leaves.

109. Herman's *Virginia* yellow Baſil.

Clinopodium *Virgin.* anguſtifol. fl. luteo *Herm.* H. Leyd. 161.

Clinopodium anguſtifol. *Virg.* Lamii fl. luteo maculato *Bob.* H. Oxon. 375. 8. *Sect.* xi. Tab. 8. Ser. 2. fig. 2.

Clinopodium *Virgin.* anguſtifol. *flor.* amplis luteis, punctis purpureis *Pluk.* Tab. 24. fig. 1.

Origanum flor. amplis luteis purpureo maculatis, cujus caulis ſub quovis verticillo 10 vel 12 foliis eſt circumcinctus *Baniſt.* Ray 1927.

I have ſeen this in *Flower* with Mr. *Fairchild,* raiſed from *Seed* which Mr. *Catesby* ſent him.

110. Canary *Clary.* Ray 3. p. 291. 5.

Horminum haſtatis amplioribus foliis ſ. *Ari* modo alatis, caulibus & pediculis araneosâ lanugine villoſis ex *Inſulâ* Gomera *Pluk* Tab 301. fig. 2.

Muſtazi *Inſulanis* & Salvia arborea *vulgò* Alm. Bot. 185. pl. 5.

Horminum *Canarienſe* tomentoſum haſtato folio, *Muſtazi* & *Salvia arborea* nominata *Bob.* Oxon. 394. 17. *Ray* 3. p. 292. pl. 24.

Sclarea folio triangulari caule tomentoſo *Inſt. Rei Herb.* 180.

This *Flowers* in divers of our *curious Gardens* from *June* till *Autumn.*

111. White *Canary* Horehound. *Ray* 3. p. 302. pl. xi. & 12.

Stachys *Canarienſis* fruteſc. *Verbaſci* folio *Inſtit.* 186.

Stachys ampliſſimis *Verbaſci* foliis, *floribus* albis parvis non galeatis, ſpicâ *Betonicæ* ex Inſula *Canarina* Pluk. Tab. 322. fig. 4. Alm. Bot. 356.

Stachys *Canarica* fruteſcens *Salviæ* folio, fl. candido, *Arvida Salva* incolis nominata *Bob.* Oxon, 382. pl. 6.

Stachys

Stachys *Canariensis* frutescens, *Verbasci* foliis *Hort.* Amst. Vol. 2.

Salvia sylv. amplissimis *Verbasci* foliis graveolens, *fl.* albo parvo *Canariensis* Alm. Bot. 329. pl. 15. *Ray* 274. pl. xi. It holds its *Leaves* all the *Winter* in our *Stowes.*

112. Munting's *yellow* Maracoc. R. 651. 8.

Clematis s. *Flos. Passionis* fl. luteo H. *Pisan.* 18.

Clematis *Passionalis* triphyllos *fl.* luteo *Ox.* Mor. 7. 3. Tab. 2. Ser. 1. f. 3.

Clematis *Passiflora* fl. luteo *Munt.* Prax. 422. *fig.* opt.

Cucumis *Flos Passionis* dictus, *Hederaceo* folio, *fl.* ex luteo viridanti, *Herm.* H. Leyd. 205.

Clematitis s. Flos Passionis *Americana* luteo flore *Vallot.* 53.

an Clematitis *Indica* fl. minimo pallido *Plum.* 73. Tab. 88.

Balsamina altera *Indica* repens, *Hederæ* arboreæ foliis, *fl.* subviridi *Ambros* phyt. 91.

Granadilla folio tricuspidi, *fl.* parvo flavescente *El. Bot.* 206. Inst. 240.

Granadilla folio Hederaceo, *fl.* luteo minore *Flor.* Norib. 200.

Granadilla pumila, *fl.* parvo luteo *D. Alex. Balam.* H. Patav. à *Turre* 55.

Flos Passionis minor, folio in 3 lacinias non serratas minus profundas diviso *Cat.* Jam. 104.

Passiflora *Hepaticæ Nobilis* folio parvo non crenato, *fl.* ex luteo viridante *Alm. Bot.* 282. 1.

This Plant is a Native of *Virginia* from whence Mr. *Catesby* has lately sent it.

113. The *Old* Trefoil *Maracoc* Ray 649. 1.

Balsamina *Indica* repens, Pomifera *Honuphr.* 5.

Clematis *Passiflora*, fl. Roseo triphylla *Munt.* Prax. 423. *f.*

Clematis *Passionalis* triphyllos, *fl.* Roseo *Mor.* Ox. 7. 2. Tab. 1. fig. 9.

Cle-

Clematis trifolia, *fl.* Roſeo clavato C B. 301. xi.
Cucumis triphyllos *fl.* Roſeo clavato *Herm.* Leyd. 205.
Granadilla *Hiſpanis,* Flos Paſſionis *Italis* Col. in *Hern.* 887. & 90. fig. opt.
Granadilla triphylla, *fl.* Roſeo clavato *Fl. Nor.* 199.
Maracoc ſive *Clematis* Virginiana *Park.* Gard. 393. 10. fig. 395. 7.
Maracot Indicum *Aldini* Hort. *Farnes.* 50.
Paſſiflora foliis crenatis tripartito diviſis *Alm. Bot.* 281. pl. 3.

This *beautiful* Old *Plant* has been an Ornament in our *Engliſh Gardens* for near a Hundred Years, and was firſt brought to us from *Virginia.*

114. Common *Fingered* Maracoc.

Clematis *Paſſiflora* pentaphylla, *fl.* cæruleo punctato *Munt.* 421. fig.
Clematis pentaphylla, *fl.* Roſeo clavato *Mor.* Ox. 6. 8. Tab. 1. fig. 8.
Clematis quinquefolia *Americana* ſ. Flos Paſſionis *Roberti* Icon.
Cucumis *Flos Paſſionis* dictus, pentaphyllos, *fl.* clavato *Herm.* H. Leyd. 205.
an *Flos Paſſionis* major pentaphyllos *Cat. Jam.* 104. pl. 1.

This elegant *Climber* is found in divers *Gardens* of the *Curious,* but I could never yet obſerve the *Fiddle-like Dent,* in the middle Leaf, as *Munting, Moriſon,* &c. have *Figured* it.

115. Ever-green *Fingered* Maracoc.

Paſſiflora pentaphylla ſempervirens.

This chiefly differs in being ever-green. I have obſerved it both in *Flower* and *Fruit,* in Mr. *Thomas Fairchild*'s Garden at *Hoxton.*

116. *Yellow* Thorney *Poppey* Ray 856. 8.
Argemone *Mexicana* El. Bot. 204. Inſt. 239.

Papa-

Papaver ſpinoſum C B. 171. 17. *prodr.* 92. fig. *phyt.* 311 15. C B. *Matth.* 748. f. 2. I. B. 3. l. 30. p, 397. fig. *Cluſ.* 93. fig. 1. *Imperat.* 662. fig. 2. *Jonſt.* 371. f. 4.

Papaver campeſtre *ſpinoſum* Chabr. 459. x, Ic.

Papaver ſpinoſum *Americanum* Park. 366. f. 5. opt.

Papaver *ſpinoſum* luteum, *foliis* albis venis notatis *Moriſ.* Ox. 277.

Papaver ſpinoſum, *fl.* luteo ſimplici *Munt.* p. c. 555.

Carduus Chryſanthemus *Peruanus* Ger. 993. f. 2. *Jonſt.* 1155. 2.

Chicallotl *Mexic.* ſeu *Spina* Hernand. 215. c. 60. fig.

Ficus Infernalis *Italorum* an *Glaucium* Dioſc. *Ray* 856. 8.

The *Purging Thiſtle*, i. e. *Carduus Purgans* noſtratibus *dicta* Alm. Bot. 279. pl. 8.

This ſingular Plant is a Native of both the *Eaſt* and *Weſt* Indies.

It *Flowers* and *Seeds* in our Gardens in *July*, *Auguſt*, &c.

117. Hairy *Oriental* Poppy.

Papaver *Oriental.* hirſutiſſimum, *fl.* magno. T. Coral. 17 *Comel.* Plant. rar. 34. fig.

My hearty and very worthy Friend Mr. *George London* was the firſt who ſhew'd me this in *Flower*; ſince which I have obſerved it in our *Phyſick Garden* at *Chelſey*.

118. Spaniſh *Potatoes* Ray 728. 22.

Batata *Hiſpanor.* Camotes ſ. Amotes & Ignanes *Lob.* Ic. 647. *Obſ.* 369. f.

Batatas *Cluſ.* Exot. 341. & *Inhame* Luſitan. *Hiſt.* 78. fig.

Batatas *Occid. Indiæ* & Inhame *Orient.* Luſit. *Park.* 1383. f. 3.

Battades, *Ignames* Adv. 317. *fig.* radicum.

Convolvulus Indicus *Batatas* dictus *Ray* H. Pl. 728. pl 22.

Convolvulus *Ind. Orient.* Inhame *ſeu* Batatas *Mor.* Ox. xi. 4. *Tab.* 3. fig. 4.

Convolvulus Ind. Or *rad.* tuberoſa eduli, *cortice* rubro & albo *Battatas* dictus *Par. Bat.* prodr. 326.

Convolvulus *rad.* tuberosâ esculenti, *Spinachiæ* folio, *fl* albo fundo purpureo, *semine* post singulos Flores singulo *Jam.* Cat 53. *Hist.* 150.

I have gathered this in the *Dutchess of Beaufort*'s Garden at *Chelsea.*

119. White *Virginia* Bindweed, with a blackish bottomed *Flower.*

Convolvulus *Virg.* fol. cordato, *fl.* albo minore, fundo nigricante.

an Convolvulus *Scamonii* folio subrotundo, *fl.* albo, umbone nigro *Curassavicus* Herm. *Par. Bat* Cat. 6.

Its *Leaves* cordated, small and very pointed, the *Flowers* also small, slender and white, but blackish at the Bottom. We owe the Discovery of this to the inquisitive Mr. *Catesby.*

120. Broad-leaved *Cape* Goat-Cranesbill.

Geranium *Tragodes* Capense, *folio* maximo.

an Geranium *Africanum* frutescens, *Malvæ* folio laciniato, odorato instar *Melissæ*, fl. purpurascente *Boerhaave* 6. p. 110.

Its *Flowers* are generally composed out of two small flesh coloured *Petals*, with a Blot of deep scarlet in the Middle, and a Streak of white running from thence down to the Bottom. I first observed this with the Bishop of *London* at *Fulham*, and since in our *Physick Garden* at *Chelsea.* It *Flowers* in *April* and *May.*

121 Cape *Codling-Cranesbill* with bright *Flowers.*

Geranium *Capense* frutesc. *folio* rotundo, Pomaceo odore, *fl.* vivido.

an Geranium *Afric.* arboresc. fol. *Malvæ* plano, lucido, glabro, *fl.* kermesino D. *van Leur.* Boer. 3 p. 109.

I have only seen this at the Bishop of *London's* Garden at *Fulham*; it differs from *Botan. Hortense* No 2. pl. 103. in having a much livelier *red* Flower.

122. Small Smooth curl-leaved *Cape* Cranesbill, Geranium *Capense* folio *Betenicæ* crispo, *fl.* minimo.

Its lower *Leaves* are smooth and more or less round and broad, the upper Leaves are deep cut, with two or three small purple *Flowers* standing on very fine slender Footstalks.

123. Dr. *Uvedale*'s Spot-Flowred *Cape* Cranesbill R. 3. p. 510. pl. 2.

Geranium *Capense* fol. *Betonicæ* molli, *fl.* maculato.

Geranium African. *Betonicæ* folio procumbens, *fl.* parvis eleganter variegatis *Alm. Bot.* 169.

Dr. *Pluknet*, that copious Botanist, first observed this in Dr. *Uvedale*'s curious Garden at *Enfield*. It's *Stalks* and *Leaves* are soft and hoary, the *Flowers* small and deeply spotted like the *Lychnis* hirta minor.

124. *Spotted* American *Arum*, with *whitish* Veins.

Arum *Amer.* majus maculatum, *venis* eleganter albis.

I have seen this beautiful *Arum* in *Flower* in the *Bishop* of *London*'s curious Gardens at *Fulham*.

125. Small *Virginia* Trefoil *Arum*.

Arum humile *Virgin.* triphyllum.

an Arum *Zeylan.* trilobato folio, humilius & minus *Par. Bat.* 79. fig.

Mr. *Thomas Fairchild* gave me the only Specimen of this which I saw growing in his *Garden* at *Hoxton*.

126. Great-Smooth *Cape* Sumac. *Ray* Dendr. 58. 12.

Sumach *Capensis* fol. rotundo splendido majore.

Rhus *Afric.* trifoliatum majus, splendente *folio* rotundo integro *Ray* Dendr. 58. 12.

This and the next I have seen in the *Dutchess* of *Beaufort*'s Gardens at *Badminton*.

127. Small Smooth *Cape* Sumac. R. *Dendr.* 58. xi.

Sumach *Capens.* fol. rotundo splendido minore.

Lentiscus humilis, trifolia *Africana* P. Bat. pr.

Rhus *Afric.* trifoliatum minus glabrum, ſplendente *folio* ſubrotundo integro *Pluk.* Tab. 129. fig. 9. & *Ray* Dendr. 58. xi

This chiefly *differs* from the *laſt* in being *leſs*, particularly in its *Leaves.*

128. Hairy dented *Cape* Sumac. R. *Dendr.* 56. 1.

Sumach. *Cap.* folio dentato hirſuto.

Rhus *ſeu* Sumach *Afric.* trifoliatum hirſutum & crenatum *Herm.* Ray *Dendr.* 56. 1.

Rhus *Afric.* trifol majus, *fol.* obtuſis & inciſis hirſutie pubeſcentibus *Pluk.* 129. f. 7. & *Ray* Dendr. 56. 1.

Theſe *Leaves* are deeply dented and hairy, by which its known from the *reſt.*

129. Narrow white *Cape* Sumac. R. *Dendr.* 57. 8.

Sumach *Cap* fol. anguſto ſubtùs incano, hinc inde dentato.

Rhus *Afric.* trifoliatum *fol.* ſerratis *Herm.* R. Dendr. 57. 8.

Rhus *Afric.* trifol. majus, *fol.* ſubtùs argenteis acutis & margine inciſis *Pluk.* Tab. 129. fig. 6. & *Ray* Dendr. 5. p. 57. 8.

Rhus *Afric.* trifoliatum, folio *Agni Caſti* Herm. *Boerhave* 257. & 8.

Rhus *Afric.* trifol. anguſtiſſimo *folio* ſubtùs incano, ſeu *Agni Caſti* lobis *Herm.* R. Dendr. 56. 2.

This *differs* from the next in having *narrow Leaves* ſomewhat jagged.

130. Round white *Cape* Sumac *Ray* Dendr. 58. 13. & 14.

Sumach *Cap.* folio ſubrotundo integro *ſubtus* incano.

Rhus *Afric.* trifol. majus, *fol.* ſubrotundo integro molli & incano *Pluk.* T. 219. fig. 8. & *Ray* Dendr. 58. 14.

Rhus *Afric.* trifol. *fol.* brevioribus nervoſis ſupernè glabris & ſplendentibus, *ſubtùs* incanis *Ray* Dendr. 58. 13.

All theſe *Trifoliate Sumachs* grow ſpontaneouſly about the fertile *Cape of good Hope*, and from thence brought into the *Gardens* of the moſt *Curious*, where I have ſeen them both in *England* and *Holland.*

III.

III. *An Extract of a Letter from Dr.* Helvetius *at* Paris, *to Monsieur* Duyvenvoorde *Embassador Extraordinary from the States-General, and by him communicated to the* Royal-Society.

I Am extreamly pleased, Sir, that you have applied your self to me, for my Advice about the Use of the *Pareira Brava* which has been recommended to you, because I can give you a very good Account of it, having been one of the first that introduced it in *France*; I have made abundance of lucky Experiments about it, which have made this Medicine very well known to me: wherefore I assure you, you can do nothing better than to make tryal of it.

The *Pareira Brava* is a Root which comes to us from *Brazil* by the way of *Lisbon*, but which the War has rendred pretty scarce; however it is to be found among the good Druggists, and is sold at *Paris* for 40 *Livres* the Pound: 'tis call'd in *Brazil* the Universal Medicine, and is made use of there in all kind of Distempers. A Capuchin Monk, who came from thence, told me that he could not give it a greater Character than by assuring me, that in all their Voyages they carryed the Gospel in one Pocket and the *Pareira Brava* in the other.

'Twill be very easy to convince you, Sir, that the *Pareira Brava* is perfectly good in your Distemper. The Business with you is to restore the Digestions, to the end, that in the first Passages there may not be form'd so much Phlegm and acid Crudities; and it is also necessary to hinder the Serosity of the Blood from spreading it self too much upon the Parts. Now as Experience shows us that

that the *Pareira Brava* does abundantly provoke Urine, it will follow from thence, that it will discharge by the Kidneys the corrosive Acidity of the Mass of Blood ; it is also good to break and thin the pituitous and viscous Humors ; and it cures the Suppressions of Urine occasion'd by Obstructions in the Kidneys.

One may conclude from hence, that the Salts of the *Pareira Brava*, which are moderately Volatile, are proper to dissolve or separate the too thick and too close Texture of the Sulphur of the Lympha Finally this Medicine has a light or gentle Bitterness which corrects the Acids of the Stomach, and renders them more pure and fine.

Hence the Chyle becomes better digested and more Balsamick, and fitted to assimulate it self with the Blood, and to preserve therein that degree of Division and Fluidity which is necessary for it.

The Method of using this Root with success, is to reduce it to an impalpable Powder, and to infuse thereof the weight of a Demi-gros in a Pint of boyling Water, and let it lye in it all Night, and next Morning boyl it one moment. Then pour the Liquor off gently from the Powder, and take of it a Demi-setier in two Cups with a little Sugar as hot as Tea, putting into each Cup 5 Grains of the said Root reduced to an impalpable Powder, which you must stir with a Spoon, that none of it may remain at the Bottom. You may repeat the same Dose about 4 Hours after Dinner, but you must not eat any thing within an Hour after you have taken it.

This Medicine does not oblige you to alter the ordinary Course of your living ; and you may continue the use of it several Months together, in which time also you may discontinue it two or three Days together at a time, if you please ; but you should take some gentle Purge every Fortnight or Three Weeks during the use of the said Medicine.

The

The Preparation of the Pareira Brava, *as Monsieur* Duyvenvoorde *uses it.*

TAke Eleven Grains of this Root, and put it into a Pewter Tea-pot fill'd with boyling Water, and so let it infuse all Night over warm Ashes, or a very small fire; and in the Morning boyl it again, but very gently, till you use it. You must drink it just as you do Tea, and the Liquor which comes from this Infusion must not exceed the Quantity of 5 small Dishes of Tea.

IV. *A Letter of Mr.* Francis Nevile *to the Right Reverend* St. George *Lord Bishop of* Clogher, R. S. S. *Giving an Account of some large Teeth lately dugg up in the North of* Ireland, *and by his Lordship communicated to the* Royal-Society.

Belturbet, July the 29*th.* 1715.

My LORD,

THE Curiosity I here send your Lordship, is so far beyond any thing that I have had the honour to communicate to your Lordship, or that I have ever met with, that I presume your Lordship will think it fit to communicate to the Royal-Society; I have sent the Draught, after the best manner I could draw it, enclosed; it is the Draught of two Teeth lately found within Eight Miles of this Town at a place call'd *Maghery*, in part of the Bishop of *Killmore*'s Lands, sinking the Foundation for a Mill near the Side of a small Brook that parts the Countys of *Cavan* and *Monoghan*.

The

There are in all four Teeth, two of a larger and two of a ſmaller ſort, the largeſt is the fartheſt Tooth in the under Jaw, the other is like it and belongs to the oppoſite Side; the leſſer Tooth I take to be the third or fourth Tooth from it, and has its Fellow: theſe are all that were found, and one of them in a Piece of the Jawbone, which fell to dirt as ſoon as taken out of the Earth; there was part of the Scull found alſo of a very large Size and Thickneſs, but as ſoon as expoſed to the Air that mouldered away as the Jaw had done.

The Account I had led me laſt Week to the Place, where I was reſolved to make the niceſt Search I could; but the Water-wall of the Mill being built, and the Ground all incumbered with the Earth that was thrown up, I could have little Opportunity of doing any thing, but to enquire of the Workmen the manner of finding the Teeth, and where and how they lay. There were ſome few Peices of Bones found, but none entire, yet by thoſe Bits that were found, one might gueſs that they were Parts of thoſe that were of a larger Size.

The Place where this Monſter lay was thus prepared, which makes me believe it had been buried, or that it had lain there ſince the Deluge. It was about four Foot under Ground, with a little Riſing above the Superficies of the Earth, which was a Plain under the Foot of a Hill, and about 30 Yards from the Brook or thereabout. The Bed whereon it lay had been laid with Fern, with that ſort of Ruſhes here call'd *Sprits*, and with Buſhes intermixed. Under this was a ſtiff blew Clay on which the Teeth and Bones were found: Above this was firſt a Mixture of yellow Clay and Sand much of the ſame Colour; under that a fine white ſandy Clay which was next to the Bedd: the Bedd was for the moſt part a Foot thick, and in ſome Places thicker, with a Moiſture clear through it; it lay ſad and cloſe and cut much like Turfe, and

would

would divide into Flakes, thicker or thinner as you would; and in every Layer the Seed of the Rushes was as fresh as if new pull'd, so that it was in the Height of Seed-time that those Bones were lay'd there. The Branches of the Fern, in every Lay as we open'd them, were very distinguishable, as were the Seeds of the Rushes and the Tops of Boughs. The whole Matter smelt very sower as it was dug, and tracing it I found it 34 Foot long, and about 20 or 22 Foot broad.

It will be well worth consideration what sort of a Creature this might be, whether Human or Animal; if Human, there was some reason for the Interrment, and for that Preparation of the Bed it was laid on; if Animal, it was not worth the Trouble: if Human, it must be larger than any Giant we read of; if Animal, it could be no other than an Elephant, and we do not find that those Creatures were ever the Product of this Climate. And considering how long this must have lain here, I do not believe the Inhabitants then had any Curiosity or Conveniency to bring such into this Kingdom; for I suppose the best of their Ships could not carry one. Then if an Elephant, or some other Beast which must have proportion to the Teeth, it must have lain there ever since the Flood; and if so, then the Bed on which it lay must be of its own making: whence it will follow that the Flood coming on him while he lay in his Den, he was there drown'd, and covered with Slime or Mud, which since is turn'd into the Substance of the Earth beforemention'd. I forgot to mention that there was a great many Nutshells found about the Bed, perhaps those might have been on the Bushes which composed part of the Bed.

The two large Teeth are of equal Weight, two Pound three Quarters each; the two little Teeth are six Ounces

 each;

each; but there are ſome of them waſted, and ſome of Holders that go into the Jaw broken off.

I am,

My LORD,

Your Lordſhip's moſt dutiful and obedient Servant.

FRANCIS NEVILE.

V. *Remarks upon the aforeſaid Letter and Teeth, by* Thomas Molyneux, M. D. *and* R.S.S. *Phyſician to the State in* Ireland: *Addreſs'd to his Grace the Lord Archbiſhop of* Dublin.

MY LORD,

WHEN your *Grace* was pleaſed to communicate to me a Letter you received ſome while ſince, containing an Account of an extraordinary Natural Curioſity, lately diſcover'd in the *North* of *Ireland*, in the County of *Cavan*, you deſired I would give you my Thoughts concerning it, and the Purport of the Letter: but truly when firſt your *Grace* gave me the Opportunity of peruſing this Account, and I conſider'd the imperfect Sketches of the *Teeth* annex'd to it, I was not a little concern'd, that upon the making ſo ſurprizing a Diſcovery, I could not command a Sight of the Originals themſelves, from whence the Draughts were taken; or that ſo great a Curioſity ſhould be expreſt by the Hand of an Artiſt that ſhew'd ſo little Skill: however, by the beſt Judgment

I could make from ſo imperfect an Information, I told your *Grace* then, I was pretty well convinced they muſt have been the *Grinding Teeth* of an *Elephant:* Yet I ingaged, if hereafter I might be ſo lucky as to procure a View of the *Teeth* themſelves, I would be more poſitive in my Opinion, and give the Reaſons on which I grounded my Conjecture; as likewiſe I would have the Shape of the Teeth expreſt in their full Dimenſions, by more true and exact Figures.

Since that, the *four Teeth*, with ſome of the Fragments of the Bones that were found with them, have been brought here to *Dublin*, where, by the Favour and Aſſiſtance of my ingenious Friend Sir *Thomas Southwell*, I procured the Loan of them, ſo long as to examine them particularly, make ſome Remarks, and take the following correct Sketches, that expreſs their Form truly, juſt as big as the Life; and your Lordſhip ſeem'd well ſatisfied with the Performance of the Artiſt, when at the ſame time I produced the Draughts and the Originals from whence they were copied, that we might compare them both together.

Upon the whole, I am now fully convinced, and I can upon ſure Grounds affirm to your Lordſhip, that they muſt certainly have been the *Four Grinding Teeth* in the lower Jaw of an *Elephant*: and that the many looſe Fragments of thoſe large Bones that were found with them, muſt have been Remains of the ſame *Animal*. This I take to be one of the greateſt Rarities that has been yet diſcovered in this Country.

In order to clear this Matter 'twill be firſt requiſite to have recourſe to, and explain the annext Figures

Figure the 1*ſt*. *A A* is the large Grinder of the under Jaw on the right Side, weighing two Pounds and three Quarters of a Pound.

b. b. b. b. b. b. b. are white, rough, indented Borders, Seven in Number, of an irregular Shape, rising about the tenth of an Inch higher than the hard black shining Surface of the Tooth; this rough raised Work serves for the bruising and grinding the *Animal*'s Food, the tough Grains of Rize, Leaves, Fruits and the Boughs of Trees; and is made of so extream an hard Texture, that it resembles large knotted Threads of white *Glass*, laid on and closely fastned to the dark Superficies of the *Tooth*: and answers that glassy Surface wherewith Nature has armed the Outside of the *Teeth* of most *Animals*, to prevent their wearing from the constant Attrition in Chewing of their Foods.

c. c. c. c. c. is that part of the Tooth which rises above the Gumms, and continues even now distinguish'd from the rest of the Bone, by having its Colour of a different Shade.

d. d. d. d. d. d. d. are many strong Tangs or Roots, seemingly united altogether, by which the Tooth received its Sense and Nourishment, and tho' it was so large and ponderous, by these it kept firmly fixt into the Jaw.

For the Mechanism Nature shews it self to have followed in framing the *Teeth* of this *Animal*, is no more than this: whereas in other Creatures, she has divided that bony Substance wherewith they chew their Food, each having its peculiar Roots to secure its Articulation in the Jawbone: she has in this of so great Bulk (As *Pliny* the *Naturalist* stiles it *Terrestrium maximum Elephas*,) for the greater Strength, Stabiliment, and Duration of it's *Teeth*, and the better to provide for a compleat Attrition of the Aliment, in order to perfect the Digestion so thoroughly, as to sustain the Life of the Animal for two or three hundred Years, (as it is a common received Opinion in the *East*) She has, I say, contrived to make the Substance of the *Teeth* in their Roots below, and in their upper

per parts above the Gumms, closely unite together; and coalescing thus, form a few large massy Teeth instead of many small ones.

As for instance, in *Man's Body*, that is of so much a less Size, the Number of the Teeth, (when the whole set is compleat) reckons to thirty two, whereas in the large *Elephant*, the Teeth of both the Jaws amount in all but to Eight, besides it's two great Tusks, which rather serve as Horns for its defence than Teeth to prepare it's Food, and therefore I think not so very properly call'd Teeth.

Figure the 2*d*. *E. E.* is the smaller *Grinding Tooth* of the under Jaw on the same side: it's Surface covered over with the same white indented Work, as before describ'd for grinding of the Food.

f.f.f. are three large Roots that kept it firmly fixt in the Jaw Bone.

This smaller Tooth weighed full six Ounces.

Figure the 3*d* *G. G.* is the large *Grinder* of the under Jaw on the left side, much of the Size and Shape and Weight with it's fellow Tooth describ'd Figure the *st*. It shews its Roots and all its parts, with the rough protuberant white Work on its upper Surface made after the same Contrivance, and formed after the same strong Model as the former.

And truly if one considers it, 'tis plain that were not the *Teeth* of this Creature made of so large a Size, and withal of so massy and firm a Substance, 'twere absolutely impossible they could resist the Force, and bear all that Pressure wherewith those vast Muscles exert themselves, that move the lower Jaw in Mastication in this so strong an Animal.

Figure the 4*th*. *H. H.* is the smaller *Grinding Tooth* of the under Jaw on the same side; it is less compleat than the small Tooth describ'd before in Figure 2*d*. for some of the Root is wanting, and part of its outward grinding Surface

Surface is broke off at *k. k.* ſo that it weighs ſomewhat leſs; yet what remains exactly ſhews the ſame kind of Work and Shape of the other Tooth, that anſwer'd it on the right Side.

Theſe *Four Teeth* here deſcrib'd, fully compleat the Sett of the Teeth, wherewith Nature has furniſhed the lower Jaw of the *Elephant*; and are anſwered by juſt as many more, formed after the ſame manner in the upper Jaw, as Dr. *Moulins* informs us, who diſſected the *Elephant* that was burnt here at *Dublin* in 1681. In it's Anatomy *p.* 40. ſpeaking of the Teeth he aſſures, there were beſides the Tusks only four Teeth in each Jaw, two in every ſide: and that theſe eight Teeth were all *Molares*, ſo that he had no *Inciſores*.

But notwithſtanding this, perhaps it will be ſaid, we may not haſtily conclude from hence, that our *Great Teeth* dug up in *Ireland* muſt certainly have been the *Four Grinders* of an *Elephant*, ſince they might as well belong to ſome other large kind of *Terreſtrial* or *Marine Animal*. As for the Hint of their being *human* or *gigantick*, 'tis ſo groundleſs a Thought, and ſo contradictory to *comparative Anatomy* and all *Natural Hiſtory*, it does not deſerve our Conſideration.

To obviate this, I ſhall take notice firſt in general, that the differing Kinds of living Creatures, wherewith Nature has ſtock'd the World, are not more diſtinguiſh'd by the Make of any part of their Bodies from one another than by the various Shape and Diſpoſition of their *Teeth*: and hence it is, we ſhall not find any two diſtinct *Claſſes* of *Animals* that do exactly agree in the ſame Make and Ranging of their Teeth.

But yet to be more particular, and make this Point ſo plain, I hope, as that it may admit of no Controverſy, I ſhall here ſet down at length, as I find them, the

Words

Words of two late Authors, that purposely have described the *Teeth* of the *Elephant*.

The first I shall mention is Mr. *Patrick Blair*, who has publish'd a Treatise he calls *Osteographia Elephantina*, or a Description of the Bones and other Parts of an *Elephant*, that died and was dissected near *Dundee* in *Scotland*, *anno* 1706. in the *London Philosophical Transactions* for *April*, *May*, *June*, *July*, *August* and *September*, 1710. *Numb.* 326. and 327. Here giving us a Description of the Teeth of this *Animal* pag. 110. he says, *Dr.* Moulins *well observes that they are all* Molares, *being two Inches broad in that part of them wherewith they grind, and six Inches and a half long on the Right Side, and five Inches and a half on the Left: Their Surface, tho' flat, is yet very unequal, for they have alternately placed, running from the Right to the Left Side, an Hollowness and then an Eminence; and this Eminence is surrounded by a rough protuberant Border. There are Nine of these Hollownesses and as many Eminences, undulated as they paint Sea Waves,*

'Tis remarkable how very exactly all this agrees with our Figures; 'tis true those *Hollownesses* and *Eminences* which he mentions to be *Nine*, do not so nicely hit with the Number of those in our *Teeth*: but this Difference proceeds from hence, that he describes here the Grinders of the upper, whereas ours are the *Teeth* of the lower Jaw; tho' such a Distinction as this, I am apt to think, may very well arise even in those of the same Jaw, in various Animals, from some peculiar Disposition in one from another, nay and perhaps in the same Animal, at differing times, according as it happens to be older or younger, but this by the bye.

A little farther *pag.* 114. and 115. where he gives an Account of those of the under Jaw, he says

The hind Tooth of the Right Side is four Inches, and that on the Left five: the one half of their Surface, where they begin

gin to appear above the Gumms, is ſemicircular, with the forementioned Ridges and Sulci *running tranſverſly, four on the Right Side and five on the Left, the other half* (or Tooth I ſuppoſe he means) *has five of theſe Eminences where it grinds on the Right, and four on the Left: each of the four Teeth is ſix Inches long, and has ſix or ſeven of the forementioned Eminences and as many Depreſſions: theſe Teeth are the moſt firm, ſolid and weighty Bones of any Animal yet known.*

So much from Mr. *Blair*.

The other *Author* I ſhall produce for the further Illuſtration of this Matter, is the laborious and accurate *Naturaliſt* Mr. *Ray*, who, in his *Synopſis Animalium Quadrupedum*, when he comes to give us the Deſcription of the *Elephant*, has the following Words. *Os pro mole Bellua parvum, quatuor in utrâque maxillâ Dentibus molaribus ſeu Dentium molarium Maſſis inſtructum; ſi quidem* plurimi Dentes in Os ſolidum *&* durum *ita infixi ſunt, ut cum eo & inter ſe* unum & continuum Corpus *efficiant. Dentes hi lineas parallelas undulatas octo vel novem in ſuperficie maſſæ efficiunt; ſuntque reliquo oſſe* candidiores: *Maſſæ integræ, Dentium ſingularium modo, per Gomphoſin maxillis inſeruntur. Inciſoribus omnino caret.*

Thus Mr. *Ray* in very proper and expreſſive Terms deſcribes the *Teeth* of this *Animal*: and truly if your Grace will but compare Mr. *Blair*'s Words with his, and the Particulars of both Accounts with the Deſcription and Figures we have before given of the *Teeth* dug up in *Ireland*, and obſerve how they all agree exactly, even ſo as one may ſay they tally together, I think it will amount to nothing leſs than Demonſtration, and that all our *Ideas* have been taken from one and the ſame *Natural Object*; and as they, ſo we, muſt certainly have deſcribed no other *Teeth* but thoſe of the *Elephant*.

But then perhaps it will be ask'd what is become of all the reſt of the *Teeth* that were in the upper Jaw, which being

being as firm and ſolid Bones as thoſe that are here preſerved, might for the ſame reaſon have ſtill remained intire.

But ſince we find it otherwiſe, 'tis obvious to imagine a probable Conjecture how this might come about. From what Mr. *Nevil* mentions in his Letter, 'tis plain that the Bed where all theſe Bones were found, muſt once have been the outward Surface of the *Earth*, the *Green-Sod*, producing *Ruſhes*, *Fern* and *Nutts*: and when the heavy Beaſt firſt fell dead upon this Spot, the Scull, with all the Bones and Teeth of the upper Jaw, being the higheſt Parts of the *Animal*, might likely fall in ſuch a Poſture, as to be expoſed ſome while above the *Earth*; tho' thoſe of the under Jaw firſt coming to the Ground, might make themſelves a Bed, and being covered with the Mould remain preſerv'd, whilſt the upper Teeth, and moſt of the other Bones, lying expoſed to the Injuries of the Air and Weather, before they got a Covering, might rot and quickly moulder all away.

But tho' this be allowed, yet ſtill a greater Difficulty remains unſolv'd; how this large Body'd *Animal*, a *Native* of the remote warm *Climates* of the *World*, ſhould be depoſited in this wild *Northern Iſland*, (where *Greeks* or *Romans* never had a footing) ſo many Miles from Sea, and diſtant from thoſe Places of the Iſle where People might moſt probably reſort.

And ſtill to make the Difficulty yet greater, we muſt conſider, not only from the dark black Colour of the Teeth, contracted by their lying long under Ground, and the remarkable Alteration wrought on their bony Subſtance, which (by the mineral Streams and Exhalations it has imbib'd whilſt it was in the Earth) is now become more ſolid, hard, and ponderous, than it was naturally at firſt, (nay in ſome Parts we find it plainly petrified) but alſo from the periſhing of all the other Bones of the Animal's Body, and from the conſiderable *Depth* of

Earth that covered those that were found : we must conclude, I say, from hence, that they have lain in this Place for many Centuries : I won't say with Mr. *Nevil* ever since the *Flood*, because I can't suppose that the slight Texture of vegetable Substances, *Nutts* and the Seeds of *Rushes*, could possibly have been preserv'd so long : But this, at least, may safely be affirmed, that these Remains must be Cotemporaries with some of the remote Ages of the *World* ; which carries us so far back into the earliest Times, that we can ne'er imagine the rude Inhabitants of *Ireland*, or any of their neighbouring Countries, were Masters of so much Art, in those Days of Ignorance and Darkness, as to make Carriages by Sea strong and capable or of Curiosity and Politeness enough, to transport a Beast of this large Size from those far distant Countries where 'twas bred ; which they that now attempt do find a Work of vast Care, Trouble and Expence, even in this Age wherein *Navigation* is brought to such perfection.

These Considerations, my Lord, grounded on other Instances of the like kind, make me inclined to think this *Elephant* we are speaking of, might not be brought hither by any Care or Industry of *Man* : but the Surface of this Terraqueous Globe might, in the earliest Ages of the *World*, after the *Deluge*, but before all Records of our oldest *Histories*, differ widely from its present *Geography*, as to the Distribution of the *Ocean* and *Dry-land*, its *Islands*, *Continents* and *Shores*; so as to allow this Beast, and others of its Kind, for ought I know, that may by some such Accident hereafter be luckily discoverd, a free and open Passage into this Country from the Continent.

For otherwise, how can we e're explain that that other vast large stately *Animal* the *Moose-Deer*, little inferior to the *Elephant* it self, could have been brought to *Ireland*, (where elsewhere I have shewn it formerly was common) from distant *North America*, even long before that Quarter

ter of the World was known, and is the only Region I can hear, where this great Beaſt is found at preſent.

And can we well imagine that *Foxes*, *Otters*, *Badgers*, *Tigers*, *Wolves*, with *Linxes* and ſuch ravenous *Animals* as we have been told, have lately been diſcovered by the great Snows that fell this preſent Winter in the *Iſland* of *Sardinia* and other Places, ſhould ever be imported (being uſeleſs noxious Beaſts of Prey) by the Induſtry of *Man*, to propagate in *Iſlands*, that they might deſtroy *Men's* Food and Flocks, and make their Lives not only uneaſy but unſafe?

Nay how can we ſuppoſe that *Birds* of ſhorteſt Flight, the various Sorts of poiſonous *Serpents*, and of offenſive *Creeping Vermin*, with all the various Tribes of ſmaller *Inſects*, could poſſibly be found in *Iſlands*, unleſs they had been ſtock'd with thoſe Inhabitants when the Intercourſe between them and the Continent was free and open.

But in whatever manner this *Elephant* (to return to our Subject) might firſt have made its way for *Ireland*; this is beyond diſpute, that the *Bones* of *Elephants* have been diſcovered deep under Ground, in other Places as well as this Kingdom: and thoſe too out of the way, far diſtant from the Native Countries of this Animal.

For not many Years ago, in a Hill near *Erfurt*, a Town of the *Upper Saxony* in *Germany*, ſeveral Parts of the *Skeleton* of an *Elephant* were Dug up: on which Occaſion *Wilhelmus Erneſtus Tentzelius* Hiſtoriographer to the *Duke* of *Saxony*, writ a Letter to the very learned *Antonio Magliabechi*, Library Keeper to the great *Duke* of *Florence*: This Treatiſe is publiſhed, but I have not been ſo lucky as to procure a Sight of it, and know no more but juſt the Title-page *Wilhelmi Erneſti Tentzelii Hiſtoriographi Ducalis Saxoniæ Epiſtola, de Sceleto Elephantino Tonnæ nuper effoſſo, ad Antonium Magliabechium, Magni Ducis Hetruriæ Bibliothecarium.*

And I am well perſuaded, by the beſt Conſtruction I can make of thoſe imperfect and obſcure Accounts, we have in *Evert Isbrand Iddes* curious Travels from *Muſcovy* to *China* over Land; *Chap.* the *6th*, (which he confeſſes he only gathered from the barbarous *Oſtiacks* Inhabitants of that Country) concerning the vaſt *Teeth* and *Bones* and *Limbs* of Mammuths as he calls them, frequently found (and diligently ſought after to make profit of them) in the Hills, and Banks of ſeveral Rivers in *Siberia*, the *Keta*, *Jenize*, *Trugan*, *Montgamſea* and *Lena*; that they are nothing elſe but the Remains and *Skeletons* of *Elephants* buried there, and accidentally diſcovered by the *Earth*'s opening, and falling down on the ſudden Thaws, after ſevere long Froſts. But of this, pleaſe to conſult the Author, whoſe Words are too prolix to be inſerted here.

But to bring this Matter ſtill nearer home to our ſelves, Mr. *Cambden* in his *Britannia* is of opinion, that thoſe great monſtrous *Teeth* and *Bones*, which he takes notice to have been at ſeveral times dug up in many parts of *Great Britain*, muſt have been the Remains of *Elephants*; but then he thinks, they muſt be of thoſe that *Dion Caſſius* the Hiſtorian tells us, the *Roman* Emperor *Claudius* brought over, when he made his Expedition into that *Iſland*. But that this truly is ſo, I own is but Surmiſe as yet, and has not been ſo fairly proved by him or any other, as that we can rely upon't with ſatisfaction.

What Mr. *William Somner* the learned Antiquary has publiſhed in his Diſcourſe of *Chartham News* is more remarkable; (this is reprinted lately in the *Philoſophical Tranſactions* for *July* 1701. No. 272.) where he informs us, that in the Year 1668 in the Village of *Chartham* near *Canterbury* in *England*, *digging within* 12 *Rods of a River, they found a Parcel of ſtrange monſtrous Bones, ſome whole, ſome broken, together with four Teeth perfect and ſound, each weighing ſomething above half a Pound, and ſome of them al-*

moſt

most as big as a Man's Fist. They are all Cheek-Teeth or Grinders; the Earth in which they lay being like a Sea Earth, or Fulling Earth with not a Stone in it.

'Tis observable how this Account in many of it's Circumstances, agrees with that of Mr *Nevil* in his Letter to your *Grace*: as that the Teeth were all Grinders, Four in number, found with other large broken Bones near a Brook, and in a Claiey Earth, without a Stone: but then the weight and Magnitude of our largest *Teeth*, so far surpass those that were found in *England*, that these did not come up to a fifth Part of those, which shows they could not be the Teeth of the same Animal. I must confess the Author does not so much as suspect they were *Elephant's Teeth*, but on the contrary is of opinion that they belong'd to another *Species*, the *Hippopotamus* or *River-Horse*, a Beast that's yet a greater Stranger in these Parts of the World, than the *Elephant* it self; and therefore it's Passage hither can never be accounted for, but by some such like Supposition as we have made.

However Mr. *John Luffkins* in his Letter, wherein he designs to have reference to that Discourse; and which is inserted in the *Philosophical Transactions* for *Sept.* 1701. No. 274. differs in his Judgments from Mr. *Somners* about these Teeth, which he thinks must have been *Elephant's Teeth*; as he is positive those large Bones he describes in the same Letter, and found near *Harwich* in *Essex*, certainly must have been.

Not having seen, much less examined, any of the Bones or Teeth concern'd in this Controversy; either those that were found in *Kent*, or those in *Essex*; I cannot well take upon me to determine any thing in this matter; tho' those dug up at *Chartham*, as I understand, may still be perused by the Curious among the Natural Rarities of the *Royal Society* in their Repository at *London*. But this at present I can safely say, that if the *Figures* of the *Teeth* given

given us by Mr. *Somner*, and repreſented in the Plate of the forementionʼd *Tranſaction* No. 272. be genuine and well expreſt (as I have no reaſon to doubt, as coming from one ſo skilful and ſo accurate) they no way ſeem to agree either in Shape or Make, or in that particular and *Characteriſtick* Work on the grinding Superficie, with the *Teeth* of the *Elephant*; or with the Deſcription and Figures we have given, which I am ſure are both correct and natural.

I ſhould now, my Lord, make ſome Apology for detaining your *Grace* ſo long upon what may ſeem ſo light and trivial a Subject, a Piece of meer Curioſity: but I am ſo vain as to hope, whatever others may fancy, it may not appear ſo inconſiderable altogether to your Lordſhip's more diſcerning Judgment.

For I am inclined to think, (even from theſe Imperfect Hints) that if we had more correct Hiſtories and Obſervations of this kind, made in diſtant Countries, and skilfully regiſtered, with all their inſtructive Circumſtances, they might lead us into great and momentous *Truths* relating to the *Deluge*; to the wiſe Methods of Providence, in repleniſhing all Regions of the *World* with *Animal Beings* ſoon after the *Flood*; and to the Knowledge of ſeveral important Changes that may have happenʼd on the Surface of this our *Terraqueous Globe*: Inquiries that are truly worthy the utmoſt Application of the moſt learned *Divine* and the moſt ſagacious *Philoſopher*.

But I ſhall ſtop here, and only beg leave to ſubſcribe my ſelf, with the utmoſt Reſpect,

My LORD,

Your Graces moſt devoted
faithful and humble Servant.

T. MOLYNEUX.

This

This Letter of Mr. Nevile *with Dr.* Molineux's *curious Draughts of the Teeth, and his learned Remarks upon them, having been produced and read before the* Royal Society, *they ordered that what Teeth they had of like sort should be look'd out and laid before them; to which Sir* Hans Sloane *was pleased to furnish a yet greater Variety, out of his incomparable Collection of* Natural Rarities. *And to obviate all Doubts, there being at this time in* Westminster *the entire Skull of a large Elephant with the Teeth in it, That was likewise ordered to be viewed and compared with the Figures: which done, it appeared that the Teeth in question could be no other than those of an Elephant.*

By this Enquiry we were likewise satisfied, that the Number of Teeth found, being but four, was no Objection: it appearing that the Number of Molares *in this Animal is not certain.* Pliny *Lib.* XI. *Cap.* 37. *says expresly* Dentes Elephanto intus ad mandendum quatuor, præter eos qui prominent. *And in the Remains of that mighty Elephant described by* Tenzelius. Phil. Trans. No. 236. *there were no more than four Teeth found. In that at* Westminster *there are Six,* viz. *One in each lower Jaw, and Two in each of the Upper, whereof the inner Tooth is about three times as long as the other, and both together longer than those of the under Jaw by about an Inch; the upper small Teeth being much worn by grinding. These we have thought fit to represent by* Fig. 5. *shewing the rough grinding Surface of the left under Tooth, being considerably* Concave; *and by* Fig. 6. *the same Roughness on the upper Teeth is shewn, having a* Convexity *tallying with the* Concavity *of the under, which is a Circumstance not observed by any of those that have described them.*

And altho, by the Observation of Mr. Du Verney, *Dr.* Moulins, *and Mr.* Blaire, *who dissected three different Elephants, it appear that each of them had eight* Molares: *yet from them it is also evident that in the division of them Nature observes no Rule, For Dr.* Moulins *found the two Teeth in each of the upper Jaws of that he dissected, to be divided*

ded after a different manner; so that the inner Tooth on the one side, and the outer on the other, was bigger than its adjoining Fellow, yet not so as to be very unequal: and Mr. Du Verney *and Mr.* Blaire *had on both sides the much greater Tooth outwards: whereas the* Westminster-*Skull, on the contrary, has only a small one outwards, and the much greater Grinder within. All which considered, we may with Assurance conclude, that this Elephant found in* Ireland *had but four Teeth in his Head when he died; and that the two Greater were those of the upper Jaws, and the other two those of the Under.*

Again, by the Size of the grinding Part, we may conclude these to be the Teeth of a very young and small Elephant; since they are not much above half the Length of those that are to be seen at Westminster, *which belonged to a Beast of not more than between* 10 *and* 11 *Foot high; nor much above one Third of the Length of a fossile Elephant's Grinder in the* Royal Society's *Repository, the which is here represented by* Fig. 7. (*all the* Figures *being drawn to the Scale of half their true Dimensions*). *Hence it is not to be marvelled that the Bones of so young an Animal, having not acquired their Firmity, as being in a growing State, should be dissolved by long lying in the Earth, as also the Roots of the Teeth.*

On this Occasion, perhaps it may not be amiss to quote a Passage out of Mathew Paris *his History, who assures us, that in his Time* Louis IX. (*afterwards St.* Louis) *King of* France, *made a Present of an Elephant to his Cotemporary* Henry III. *of* England; *and that in the Year* 1255, *after the* English *had been fourscore Years Masters of* Ireland. *Of this says* Mathew, Nec credimus quod unquam aliquis Elephas visus est in *Anglia* præter illum.

VI.

VI. *An Account of a* BOOK. Guilhelmi Musgrave *Reg. Societ.* utriusque *Socii*, *GETA BRITANNICUS. Accedit Domus* Severianæ *Synopsis Chronologica: & de Icuncula quondam M. Regis* Ælfredi *Dissertatio.* 8vo. Iscæ Dumnoniorum. MDCCXV.

THE Author, having some [1] Years since publish'd a Comment on *Julius Vitalis* his Epitaph, which, (together with his Monument) is to be seen at *Bath*; does now present the publick with another Volume of *Belgic* Antiquities; intending hereby, to illustrate part of a Statue, which was found likewise near that City, and is at this time immured near the Monument aforesaid, at the eastern End of the Abby-Church, looking toward the Grove.

This Fragment of an Equestrian Statue, is in *Basse Releife*: The Rider has in his right Hand a *Hasta pura*, and a *Parma* in his left; as in *Fig.* 1. of the Book. It appears from [2] *Dio* that *Caius* and *Lucius*, Cæsars, (the Nephews, and adopted Sons of *Augustus*) had each of them a *Parma* and an *Hasta* given him: and there being no Instance of this Honour paid to any of an inferiour Rank among the *Romans*, but only to such as were of very great Quality; if not to *Cæsars* only; we are from hence be allowed to think, *That this Statue represented some Person of that Quality.*

But to discover the particular Person, (if it might be done) the Author compared a very good Draught he had procured of this Horseman, with such *Roman* Coyns,

[1] Iscæ Dunmon: MDCCXI.
[2] Lib. LV.

as He could meet with. This Comparison shewed a great Resemblance between the Face in the Statue, and that in two of *Geta*'s Coyns.

This Argument, drawn from the Similitude of Faces (of great force to determine the Reader's Judgment in favour of *Geta*) is farther confirmed by the Horse; a Creature of which *Geta* was very fond; insomuch, as that He affected to be represented under the Figure of *Castor*, (as the *Roman* Emperors often were under the Figures of their Gods) of whom it is said, *Castor gaudet Equis*; — Of this Figure there is in [3] *Oiselius*, a Coyn of *Geta*'s, very much to this purpose; represented *Tab.* IV. fig. 5. of this Book.

These things bring to mind, the Authority which *Geta* had in *South-Britain*: where (as [4] *Herodian* affirms) all matters were under his Administration, during the Stay which *Severus* and *Caracalla* made in the North; which was a Year, or more. In this time, *Geta* had it in his Power, to do many things, in favour of Cities and Countreys, here in the South. The great Generosity of his Mind prompted Him to publick Works; such as are, to this day, attested by [5] Inscriptions, with his Name in them: and it is highly probable, [That this Statue was erected to *Geta* on some such account.]

If this be granted, (as from the concurrence of so much, and so good Testimony, it seems highly probable) here is a large and pleasant View opened into Antiquity; not of late taken notice of by any Writer: It shews, that *Geta* was a great Benefactor to *old Bath*; either by laying, in a perfect Morass, the Foundation of that Town; or by preserving the *Hot-Springs* entire, from the Influx of

---

[3] Tab. XLII. 1.
[4] Lib. III. Sect. XLVIII.
[5] *Gruteri* Thes. Inscriptionum, pag. CLVII. 2, 3, 4, 5.

other

other Waters; or both: Works of great Munificence, and becoming *Geta*'s Spirit. By these, or some such Ways, it is probable, this People was obliged to *Geta*; but no one is more probable, than that of preserving the *Aquæ Calidæ*; which were in those days so famous, as to give a Denomination to the place. It is well known, That *Rome* had her *Thermæ Severianæ* and *Antoninianæ*, so called from their respective Founders; the former being built by *Severus*, the Father; the latter by *Antoninus*, the Brother, of *Geta*: so that *to take care of Baths*, was a sort of Greatness, that Family seemed to delight in; and *Geta* may reasonably be supposed, to have his share of this Delight.

From the great probability of this Opinion, the Author has, out of Love to his native Country, and the Honour due to *Geta*, collected and put together, what He can meet with relating to that Emperor. He has made a new Edition of *Geta*'s Life, from the *Historiæ Augustæ Scriptores*; restoring it to its true Author, *Julius Capitolinus*; and explaining it, with the Notes of *Casaubon*, *Gruter*, and *Salmasius*; to which he has added some of his own. He has reprinted all the Inscriptions he can meet with, of *Geta*'s, and many of his Coyns; with short Notes on both.

After all this, He is not so far engaged in this Opinion, but that if, (by any Inscription on the Basis of this Statue, or any other testimony) it shall hereafter appear, that this Fragment deserves another Explication, he shall readily comply with any such clearer Testimony: being no way disposed, to give farther credit to this broken Monument, than shall answer the imperfect Condition it is now in.

To this Dissertation, *de Geta Britannico*, He has added the Chronology of his illustrious House; shewing, how his Father, *Severus*, from a private Gentleman in *Africa*,

 came

came by degrees to be Emperor of *Rome*; and indeed one of the Greatest, that ever *Rome* had: How He, with his two Sons, *Bassianus* and *Geta*, (three *Roman* Emperors) resided, at one and the same time, here in *Britain)* and from hence sent their Imperial Edicts, Orders, and Dispatches, into all parts of the Empire: and after an amazing Greatness of about twenty four Years, and a Course of almost all Virtues and Vices, at length tumbled down; submitting to the Accidents and Fate of other Men; and were all buried at *Rome*, in the *Septizodium* built by *Severus*.

To these Memoirs of *Geta*, the Author has subjoyned a Discourse, concerning that curious *Cimelium*, which was, some Years since, found at *Athelney* in *Somerset*. It did belong to K. *Ælfred*, and is now in the Possession of Col. *Palmer* of *Fairfield*, in that County. Beside the critical use made of it, by the learned[1] Dr. *Hickes*, our Author writes of it, as *an undeniable Instance of the use of Images, coming from the Heathens into the Christian Church.*

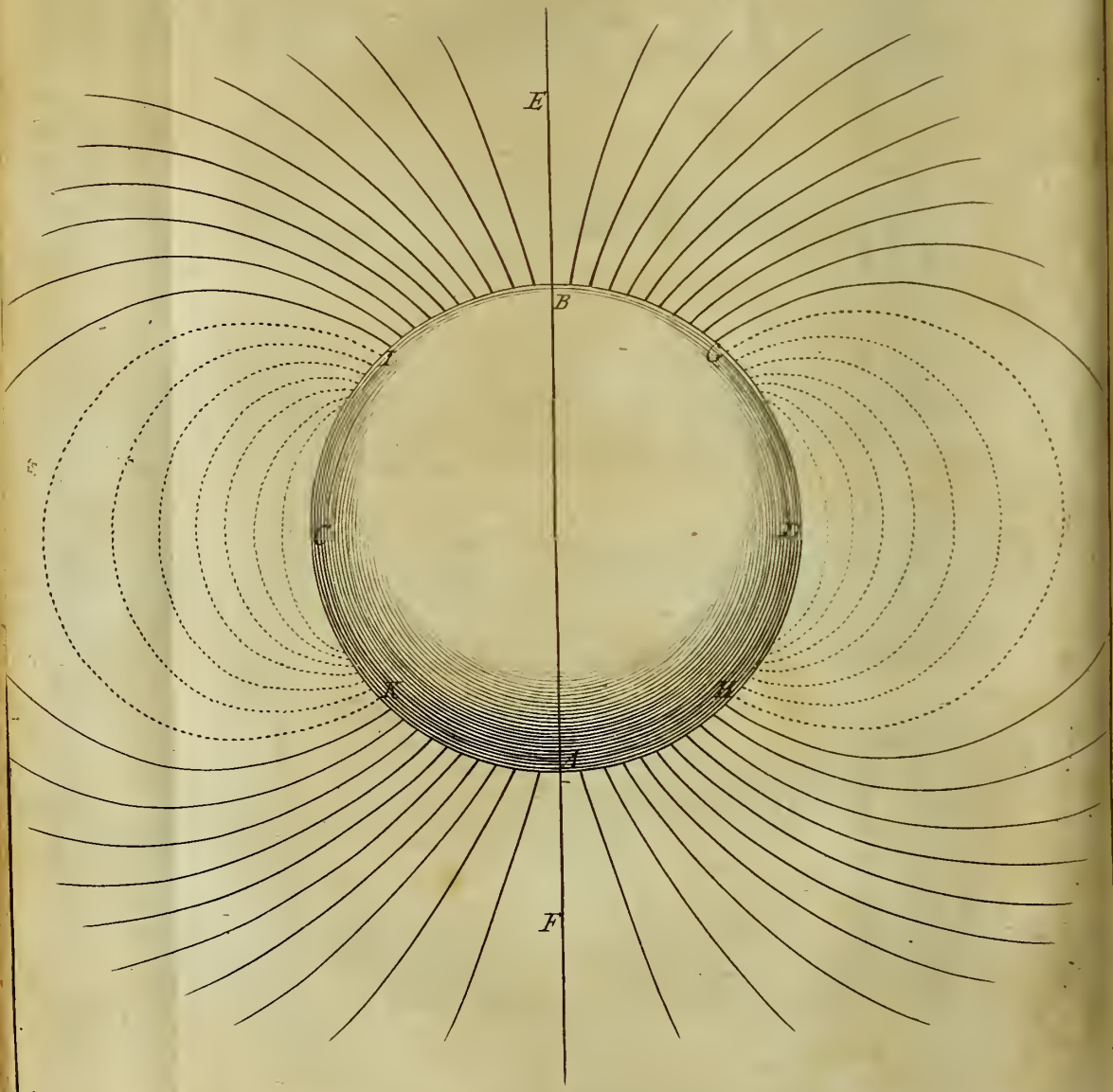
The Book is adorned with several Cuts, of *the Broken Statue* at *Bath*, of *two of* Geta's *Silver-Coyns*, of the *Septizodium Severi*, (out of *Perac*) and three sides of the κειμήλιον *Ælfredi*.

---

[1] In Thesauro Ling. Septentrionalium non ita pridem edito.

FINIS.

LONDON: Printed for W. INNYS at the *Prince's Arms* in St. *Paul's Church-yard*. MDCCXVI.


E
B
I
G
C
D
K
H
A
F

Numb. 347.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *Jan. Febr.* and *March* 1716.

The CONTENTS.

I. *An Account of ſeveral* Nebulæ *or lucid Spots like Clouds, lately diſcovered among the Fixt Stars by help of the Teleſcope.*

IN our laſt we gave a ſhort Account of the ſeveral New-Stars that have appeared in the Heavens, within the laſt 150 Years, ſome of which afford very ſurprizing Phænomena. But not leſs wonderful are certain luminous Spots or Patches, which diſcover themſelves only by the Teleſcope, and appear to the naked Eye like ſmall Fixt Stars; but in reality are nothing elſe but the Light coming from an extraordinary great Space in the Ether; through which a lucid *Medium* is diffuſed, that ſhines with its own proper Luſtre. This ſeems fully to reconcile that Difficulty which ſome have moved againſt the Deſcription *Moſes* gives of the Creation, alledging that Light could not be created without the Sun. But in the following Inſtances the contrary is manifeſt; for ſome of theſe bright Spots diſcover no ſign of a Star in the middle of them; and the irregular Form of thoſe that have, ſhews them not to proceed from the Illumination of a Central Body. Theſe are, as the aforeſaid New Stars, Six in Number, all which we will deſcribe in the order of time, as they were diſcovered; giving their Places in the Sphere of Fixt Stars, to enable the Curious, who are furniſhed with good Teleſcopes, to take the Satisfaction of contemplating them.

The firſt and moſt conſiderable is that in the Middle of *Orion*'s Sword, marked with θ by *Bayer* in his *Uranometria*, as a ſingle Star of the third Magnitude; and is ſo accounted by *Ptolemy*, *Tycho Brahe* and *Hevelius*: but is in reality

reality two very contiguous Stars environed with a very large transparent bright Spot, through which they appear with several others. These are curiously described by *Hugenius* in his *Systema Saturnium* pag. 8. who there calls this brightness, *Portentum, cui certe simile aliud nusquam apud reliquas Fixas potuit animadvertere :* affirming that he found it by chance in the Year 1656. The Middle of this is at present in ♊ 19°. 00, with South Lat. 28°$\frac{3}{4}$.

About the Year 1661 another of this sort was discovered (if I mistake not) by *Bullialdus, in Cingulo Andromedæ.* This is neither in *Tycho* nor *Bayer*, having been omitted, as are many others because of its smallness : But it is inserted into the Catalogue of *Hevelius*, who has improperly call'd it *Nebulosa* instead of *Nebula* ; it has no sign of a Star in it, but appears like a pale Cloud, and seems to emit a radiant Beam into the North East, as that in *Orion* does into the South East. It precedes in Right Ascension the Northern in the Girdle or *ν Bayero*, about a Degree and three Quarters, and has Longitude at this time ♈. 24°. 00′ with Lat North 33°$\frac{1}{3}$.

The Third is near the Ecliptick between the *Head* and *Bow* of *Sagittary*, not far from the Point of the Winter Solstice. This it seems was found in the Year 1665. by a *German* Gentleman *M. J. Abraham Ihle*, whilst he attended the Motion of *Saturn* then near his *Aphelion.* This is small but very luminous, and emits a Ray like the former. Its Place at this time is ♑ 4°$\frac{1}{2}$ with about half a Degree South Lat.

A fourth was found by M. *Edm. Halley* in the Year 1677, when he was making the Catalogue of the Southern Stars. It is in the *Centaur*, that which *Ptolemy* calls ὁ ἐπὶ τῆς ἔ νώτου ἐκφύσεως, which He names *in dorso Equino Nebula* and is *Bayers ω* ; It is in appearance between the fourth and fifth Magnitude, and emits but a small Light for its Breadth,

and is without a radiant Beam; this never rises in *England*, but at this time its Place is ♏ $5^{\circ}\frac{3}{4}$ with $35^{\circ}\frac{1}{5}$ South Lat.

A Fifth was discovered by Mr. *G. Kirch* in the Year 1681, preceding the Right Foot of *Antinous:* It is of its self but a small obscure Spot, but has a Star that shines through it, which makes it the more luminous. The Longitude of this is at present ♑. 9°. *circiter*, with $17^{\circ}\frac{1}{6}$. North Latitude.

The Sixth and last was accidentally hit upon by M. *Edm. Halley* in the Constellation of *Hercules*, in the Year 1714. It is nearly in a Right Line with ζ and *n* of *Bayer*, somewhat nearer to ζ than *n*: and by comparing its Situation among the Stars, its Place is sufficiently near in ♏ $26^{\circ}\frac{1}{2}$. with 57°. 00. North. Lat. This is but a little Patch, but it shews it self to the naked Eye, when the Sky is serene and the Moon absent.

There are undoubtedly more of these which have not yet come to our Knowledge, and some perhaps bigger but though all these Spots are in Appearence but little, and most of them but of few Minutes in Diameter; yet since they are among the Fixt Srars, that is, since they have no Annual Parallax, they cannot fail to occupy Spaces immensely great, and perhaps not less than our whole Solar System. In all these so vast Spaces it should seem that there is a perpetual uninterrupted Day, which may furnish Matter of Speculation, as well to the curious Naturalist as to the Astronomer.

II.

II. *Nova & tuta* Variolas *excitandi per Transplantationem Methodus, nuper inventa & in usum tracta: Per* Jacobum Pylarinum, Venetum, M. D. *& Reipublicæ* Venetæ *apud* Smyrnenses *nuper Consulem.*

OPerationem Medicam inventu non minus quàm eventu mirandam Orbi literario pandimus; Non à Physicæ cultoribus, aut à doctis in Apollinea arte viris, sed à plebea rudique gente in humani generis adjumentum, in sævissimi morbi solamen detectam, vetustis Scholarum Lucubrationibus, sedulis recentiorum investigationibus ignotam; sed quæ ex innocentioris & ἀφιλοσόφου familiæ penu defluxit. Verus ignoratur ejus adinventor: in Græcia tamen, præcisè in Thessalia, primò invaluisse certissimum est; hinc in propinqua successivè serpendo loca & civitates, in Byzantinam tandem irrepsit urbem; ubi latuit quidem initiò per aliquot annos, rarò quoque, & inter humiliores dumtaxat recepta: Immaniter autem grassante nuper Variolarum epidemia, latiùs innotescere cœpit; Numquam tamen sublimiores ausa est ingredi aulas; donec Nobilis quidam nec obscurus inter præstantiores Græcos, & ex antiquo Caryophyllorum stipite clarus, mihi verò intimiori amicitiæ titulo notus, anno salutis 1701. sub hyemis finem, serio mè quidnam de hac insitione sentirem, consuluit; & an ad eandem in quatuor propriis filiis celebrandam præstarem assensum: Nam tùm temporis lethaliter totam ferè civitatem morbus hic invaserat; qui summum ei de natorum salute metum incutiendo, anxium valdè reddiderat. Verùm quid ipse super ignota re decernerem, nulla præcedente novissimæ hujusce methodi notitia, penitus ignorare me dixi; ac simul Operatoris conveniendi copiam petii. Triduo peracto, cum ad amicum denuò accessissem, & de eadem

materia ſermo iteratò inter nos eſſet initus; Ecce paulò poſt mulier Græca ad decentiam ſatis compoſita cubiculum intravit; quæ totam operationis ſeriem, modum, locum, tempus, cæteraſque circumſtantias, ut ego deinceps aperiam, clarè ſatis latéque nobis expoſuit; quamquàm ipſa veram ex inſitione excitationis Variolarum cauſam haud intellexerit: His experimenta & caſus innumeros tutò ſemper & ad ſalutarem eventum perductos annexit; è quibus aliquot (omnes enim quis potuiſſet in urbe ampliſſima exquirere?) veriſſimos eſſe ex aſſertoribus fide dignis deprehendi: Idcircò re bene penſitata, rationi & naturæ haud abſonam omninò comperi: præſertim autem caſibus jam dictis permotus, amico jam fervidius poſt aliquot dies conſilium iterùm expetenti, mè haud alienum, ſub levi tamen hæſitantia, præbui; Quâ, tamquam dato aſſenſu, arreptâ, & de ſervando regimine toto ægrotationis tempore ſatis edoctus, inſitionem per mulierem Græcam in quatuor filiis audacter inſtituit: quorum tres natu minores (quinquennes, & vix ſeptennes) leviter ægrotarunt; pauciſque apparentibus puſtulis poſt hebdomadam, febre penitus & periculo evaſerunt: Ætate verò major, octavum ſuper decimum agens annum, graviter decubuit: Nam continenti febre & malignante correptus, ſuperveniente difficilium ſymptomatum ſyndrome, pluſculiſque quamquàm non copioſis exanthematibus obrutus, vix poſt decimum quartum diem morbum eluſit: Quod ego atrabilari ejus temperamento, ſucciſque pravis, ut & neglectæ prius (contrà datam admonitionem) corporis expiationi tribuendum velim. Felix operationis eventus mirum quam multas nobiliorum familias ad imitationem traxit; Ut hodie ſine hæſitantia, præter timidiuſculos aliquos, unuſquiſque tranſplantationis emolumentum ſentire velit. Soli Turcæ, utpote Fati decretis addicti minuſque dociles, hanc neglexerunt hucuſque.

Naturalis eſt penitùs hæc Operatio, nulloque obducta ſuperſtitionis fuco; quamvis ipſum Tranſplantationis Nomen

men prima facie hæſitantiam pariat. Differt toto cœlo à Curationibus Sympatheticis; eò magis à Magnetiſmo illo tranſplantatorio, per quem Morbi ex uno ſubjecto (mediante imaginaria quadam, gratis efficta, & imperceptibili Mumia) in aliud traduci dicuntur; qua de re Tenzelius, Bartholinus, Maxuellus, Etmullerus, aliique ex Recentioribus, docti aliàs Viri, agunt ſedulò; qui vetuſtas Antiquorum in Medicina quiſquilias expurgandas eſſe cum jactitent; inter quiſquilias ipſi quandoque ſe volutant, noviſſima vanitatis amurca venerandam hactenus puriſſimamque Scientiam deturpantes. Quare, ut verum fatear, quemadmodum Operationes hæ Magneticæ, vel Sympatheticæ ſuperſtitioſæ vanitatis ſuſpicione non carent (ut in Unguento Armario, Pulvere ſympathetico, & ſimilibus), utpotè extra activitatis ſphæram in diſtans agentes; ità Variolarum Tranſplantatio, vera, mera, pura Phyſica eſt; quia puris mediis Phyſicis, & ad oculum patentibus, nec non ipſo ſenſibili contactu completur; ut è mox dicendis clariùs eluceſcet.

Hujuſmodi igitur Variolarum excitatio fit per Metaphoricè ſic dictam Inſitionem ſive Tranſplantationem; quæ nihil aliud eſt, quam fermenti morbifici ſeu puris ex Variolis extracti in corpus ſanum, per vulnuſcula ad hoc facta, inſertio.

Phyſicus excitationis modus hoc pacto ſuccedit. Intruſum in vulnuſcula pus veri fermenti ſuſcipit indolem; Hinc beneficio circulationis per vaſa & canales proprios ad ſanguinis maſſam delatum, diſpoſitas ad hoc particulas, & deliteſcente vitio turgidas ſtatim aggreditur, inficit, venenumque communicando latitans in illis ſeminarium fermentativum excitat, agitat, actuat, inque motum ciet; Unde ebullitio univerſalis, ſeu fermentatio oritur; Vi cujus impuriores & heterogeneæ partes ſeparatæ criticè protruduntur ad cutem; natura interim placidè virtute talis operationis totum opus moderante.

Sed ad Transplantationem ipsam, & ejus celebrandæ formam properandum; modum quò eadem Mulier Transplantatrix tutissimè operabatur fideliter describendo, ordinem, cæteraque omnia; ex quibus regula quædam institui poterit pro operatione hac obeunda. Cæterùm quamvis de omnibus oculatus Testis, ut ingenuè fatear, esse non possim; ex Operatricis tamen ore multa, plura ex fideli relatione transumpsi; plurima & potiora ipsemet observavi. Omitto quæ ad majorem confirmationem universalis fama canit. Quibus omnibus præstandam esse fidem candidè ac strenuè assevero.

Ergo primò tempus ad celebrandam insitionem opportunum seligendum est; Quod secundùm Operatricem hybernum desideratur; & non nisi tali tempore ipsa insitionem instituebat: Vernum ego pariter idoneum credidérim, propter clementiorem aeris temperiem.

Secundò selectissimum adhibet fermentum; Pus scilicèt inserendum non ex quolibet subjecto recipit; sed Variolis epidemicè grassantibus, è pustulis jam maturis decumbentis alicujus pueruli alias ἐυσάρκυ, iisque benignis, punctione illud extrahit exprimitque; & in conchulam aliquam vel vitreum vasculum mundissimum, nec nimis actu frigidum reponit reconditque; quod Vasculum bene sartum tectum in pedissequi sinum fovendum intrudit; mox sine mora ad operationem properat: Pus ex insititiis rejicit, ut inefficax. Quod tamen ego benignioris indolis, nec minoris interim energiæ fecerim: qua in re experientia consulenda.

Tertiò, temperatissimum vult patientis inhabitandum cubiculum quoad aeris modificationes.

Quartò, ad operationem jam celebrandam accedens mulier, frontem in confinio capillorum & quidem medio in loco; mentum & utrasque genas acu ferrea vel aurea pungit; non rectà, sed obliquè impingendo, cutemque acuta cuspide à subjecta carne aliquantillùm separando.

hinc

hinc eadem acu pus jam præparatum in vulnusculum instillat è vasculo, intruditque; superinducta per fasciam ligatura: Manus item ambas in metacarpis, pedes in metatarsis eodem modo ferit, pusque inserit, fasciaque leniter stringit; serio imponendo patienti, ne partes illas scalpat madefaciatve. Potiùs carnosiora pungerem loca, quatenus inflammationibus dolorique minus obnoxia, nec tendinibus intertexta.

Præter hanc operandi formam cæteri omnes rejiciuntur modi, utpotè absoni, inusitati, malè succedentes, infelicisque exitus.

Interim lectulo moderatè manendum, neque plus quàm opus fuerit jacendum.

Quintò bonum in sex rebus non naturalibus regimen, præcipue in victù, injungit; Non solum enim vino & carnibus, verum etiam earumdem jusculo rigorosè patientes ad quadragesimum diem interdicit: Pluribusque monitum non curantibus sæpius malè successit; nam ad oculos, pœna erroris, novas erupisse pustulas, aliaque non parvi discriminis symptomata supervenisse, visum est.

Sic ritè peracta transplantatione, non omnibus eodem temporis intervallo suscitari solent variolarum symptomata; Variè enim fermentum agit; serius vel citius, prout unicuique proprium favet temperamentum, ætas, robur: Quamquàm Variolæ ipsæ, in septimo ferè semper apparere incipiant, qui dies vere criticus est. Nec defuere, quod ratò contingere solet, quibus statim primo die effloruerint.

Symptomata ægrotantibus evenientia variant secundùm temperamentorum diversitatem, succorum in massa sanguinea habitudinem, & particularem in singulis naturæ dispositionem: remissiora nempè vel intensiora ingruunt; At communiter grassantibus non dissimilia, quamvis clementiori ut plurimum facie: Plures vix alterationem læsionemve sentiunt aliquam.

Exci-

Excitatæ Variolæ ferè ſemper ſunt de genere Diſtinctarum; nec numero multæ; Decem, ut plurimum viginti, triginta, rarò ad centum, rariſſimè ad ducentas erumpunt.

Notandum primò nonnullos, unico dumtaxat vulnuſculo ad brachium inflicto contentos, excitaſſe variolas; pauciſque apparentibus puſtulis præſervatos tamen impoſterum fuiſſe à contagio.

Notandum ſecundò accidiſſe interdùm, ut ex inſitione nullæ penitus excitatæ fuerint Variolæ, vel ob non præexiſtentem ullam prorsùs variolicam in corpore diſpoſitionem, vel ob enervatum infractumque fermenti contagium: At poſtea graſſante populariter morbo, correpta ſunt promiſcuè ejuſmodi corpora jam inſitionem paſſa, communi cæterorum ſorte.

Tertiò, Inſertionis loca ſeù vulnuſcula in puſtulas ſemper evadere ſolent: Quibuſdam verò excreſcunt in purulenta tubercula, nullis interim apparentibus puſtulis; Nonnullis in apoſtemata quidem majora degenerant magnam puris copiam effundentia: Non ſemel eadem loca, in pedibus præcipuè manibuſque, ſummo cum dolore intumeſcunt; pureque effuſo ſubſident, iterumque in tumorem attolluntur. Quibuſdam, rariſſimè tamen, ad glanduloſas partes & emunctoria, poſt aliquod tempus, abſceſſus emergunt, ac ſuppurantur paulatim; ludente ſic in diverſiformi corporum craſi natura.

Poſtremò, nunquam ferè ex tranſplantatione hac funeſti quid accidiſſe obſervatum fuit hactenus; etiamſi in quocumque ſexu, temperamento, ætate celebrata fuerit; quinimò, ritè recteque tractata, & in corporibus peritum Medicum aptè præparatis, certiſſimam promittit ſalutem. Variolæ enim hoc modo excitatæ benignioris ſunt indolis, quam ſunt illæ quæ populariter graſſantur; Utpotè ex fermento, ſeù contagio, omni malignitate carente promotæ: Ebullitio, per quam maſſa ſanguinis agitatur ac totum opus perficitur, blandè non violenter, mo-

moderante natura, conamina ſua molitur; Sed præter hæc, tempus ad operationem atque anni tempeſtas magis idonea pro tranſplantatione ad libitum eligi, ut & corpus inſitioni ſubjiciendum congruis adminiculis ad recipiendam illam ex arte præparari diſponique poterit; Quod revera maximi ad ſalutarem fauſtumque morbi ſucceſſum momenti cenſeri debet.

De hac re vide etiam *Philoſ. Tranſ.* No. 339.

III. *Problematis olim in* Actis Eruditorum Lipſiæ *propoſiti Solutio Generalis.*

IN Actis Eruditorum pro menſe *Octobri* Anni 1698. pag. 471. D. *Johannes Bernoullius* hæc ſcripſit. " Methodum quam optaveram generalem ſecandi " [Curvas] ordinatim poſitione datas, ſive algebraicas ſive " tranſcendentales, in angulo recto ſive obliquo, invaria- " bili ſive data lege variabili, tandem ex voto erui : cui, " *Leibnitio* approbatore, ne γρῦ addi poſſet ad ulterio- " rem perfectionem, & vel ideo tantum quod perpetuo " ad æquationem deducat : in qua ſi interdum indetermi- " natæ ſunt inſeparabiles, methodus non ideo imperfecti- " or eſt, non enim hujus ſed alius eſt methodi indetermi- " natas ſeparare. Rogamus igitur fratrem ut velit ſuas " quoque vires exercere in re tanti momenti. Suſcepti " laboris non pœnitebit, ſi felix ſucceſſus fructu jucundo " compenſaverit. Scio relicturum ſuum quem nunc fovet " modum, qui in pauciſſimis tantum exemplis adhiberi " poteſt.

Hi tres Viri celeberrimi ſeſe, jam ab annis quatuor vel quinque circiter, in ſolvendis hujuſmodi Problematibus exercuerant. Abſque ſpiritu divinandi eandem ſolutionem cum *Bernoulliana* tradere difficile fuerit. Sufficit quod Solutio ſequens ſit generalis, & ad æquationem ſemper deducat.

PROB.

Problema.

Quæritur Methodus generalis inveniendi Seriem Curvarum, quæ Curvas in serie alia quacumque data constitutas, ad angulum vel datum vel data lege variabilem secabunt.

Solutio.

Natura Curvarum secandarum dat Tangentes earundem ad intersectionum puncta quæcumque; & anguli intersectionum dant perpendicula Curvarum secantium; & perpendicula duo coeuntia, per concursum suum ultimum, dant centrum Curvaminis Curvæ secantis ad punctum intersectionis cujuscumque. Ducatur Abscissa in situ quocumque commodo, & sit ejus Fluxio Unitas; & positio perpendiculi dabit Fluxionem primam Ordinatæ ad Curvam quæsitam pertinentis; & Curvamen hujus Curvæ dabit Fluxionem secundam ejusdem Ordinatæ. Et sic Problema semper deducetur ad æquationes. Quod erat faciendum.

Scholium.

Non hujus sed alius est methodi æquationes reducere, & indeterminatas separare, absolutè si fieri possit, sin minus per Series infinitas. Problema hocce, cum nullius fere sit usus, in Actis Eruditorum annos plures neglectum & insolutum mansit. Et eadem de causa solutionem ejus non ulterius prosequor.

IV.

IV. *Some late curious Astronomical Observations communicated by the Reverend and Learned Mr.* James Pound, *Rector of* Wansted, *and* R. Soc. Soc.

The Occultation of Jupiter *by the Moon observed at* Wansted *the* 14th *of* July *in the Morning*, 1715.

HAving after Midnight carefully corrected the Clock by no less than ten Observations of the Altitude of the *Lucida Arietis*, the Error thereof was found 5′. 13″. too fast, the extreams not differing above 6″: And in the morning about 7h, by as many Altitudes of the *Sun*, with a like Agreement, the same Error was found 5′. 14″, to be deducted from the Times shewn by the Clock.

Julii 13°. P. M. N.	Time by the Clock			Time correct.		
	h.	′	″	h.	′	″
The Third Satellite of *Jupiter* was hid by the Moon	13	27	33	13	22	20
The first Satellite was hid	13	32	35	13	27	22
The Second Satellite was hid	13	34	25	13	29	11
The first Contact of the Limbs of ♃ and ☾	13	34	54	13	29	41
Jupiter wholly hid	13	36	23	13	31	10
The Third Satellite came out from behind the dark side of the Moon	14	7	25	14	2	12
The first Satellite	14	12	25	14	7	12
The Second Satellite	14	14	38	14	9	25
The first Limb of *Jupiter* came out	14	14	45	14	9	32
The following Limb of *Jupiter*, or last Contact	14	16	15	14	11	2
The fourth Satellite emerged	14	18	49	14	13	36

Jupiter and the Satellites were to the Northward of the visible Way of the Moon's Center.

This Occultation was observed through a Telescope, in which the Focal Length of the Object Glass was $14\frac{1}{2}$ Feet, and of the Eye Glass $2\frac{3}{4}$ Inches. And the Aperture of the Object Glass was $1\frac{1}{10}$ Inch.

I could perceive no Colours on *Jupiter*'s Limb, either at his Immersion or Emersion, when the Axis of the Tube was directed to him.

Observ.	Apparent Time			*An Eclipse of the Moon observed at* Wansted October 30. 1715.		
	h.	′	″		′	″
1	15	09	00	The Eclipse had been for some time begun		
2		17	00	The Moons Diameter measured by a Micrometer was	34	04
3		22	25	The Chord connecting the Horns	30	28
4		35	45	The inlightned Part of the Diameter continued to the Chord between the Horns	19	58
5		43	24	The inlightned Part of the Diameter	13	52
6		49	50	The same repeated	12	02
7		52	43	The same repeated	11	44
8		56	51	The inlightned part of the Diameter continued to the Chord between the Horns	15	22
9		59	27	The inlightned Part of the Diameter	10	35
10	16	04	04	The same repeated	9	43
11		18	34	The same again repeated	9	07
12		23	45	The Chord between the Horns	32	35
13		26	30	The same repeated	33	07

	h.	′	″		′	″
14	16	31	16	The ſame again At which time alſo the Shade paſſed thro' the middle of *Schikardus.*	33	19
15		37	15	The Chord between the Horns, agreeing with the ☽'s Diameter	33	57
16		40	45	The inlightned part of the Diameter	11	56
17		43	40	The ſame produced to the Chord between the Horns	16	13
18		46	55	The ſame repeated	17	28
19		47	57	The inlightned part of the Diameter	13	38
20		52	57	The ſame	15	30
21		55	27	The Edge of the Shadow paſſed thro' the Middle of *Gaſſendus.*		
22		56	12	The inlightned part produced to the Chord between the Horns	19	58
23	17	02	45	The Chord between the Horns	32	12
24		8	20	The ſame repeated	30	28
25		10	39		29	56
26		13	00		28	31
27		15	29	The ſame again	27	33
28		17	37		26	35
29		19	35		25	36
30		21	47	The ſame again	24	38
31		23	24		23	39
32		24	54		22	40
33		26	27	The ſame again	21	41
34		27	57		20	42
35		29	08		19	43
36		30	20	The ſame again	18	44
37		31	07		17	45
38		32	04		16	46
39		32	50	The ſame again	15	47
40		34	12		13	48
41	17	35	20	The ſame again repeated	11	42

At $17^h. 39'.$ the Eclipſe was thought to be ended; and was viſibly ſo at $17^h. 41'$: But by comparing the laſt Obſervations of the Chords between the Horns, it follows that the true End of the Eclipſe was at $17^h. 38'. 20''$. At $17^h. 43$ the Moon's Diameter was $33'. 40''$.

The Middle cannot be ſuppoſed to be very accurately determined by theſe Obſervations, which were not ſufficiently diſtant from the time of the greateſt Obſcuration. However by comparing ſeveral of them together, the Middle will be obtained, *viz.*

	h.	′	″
By Obſ. 3. compared with Obſ. 24. at	16	15	21
By Obſ. 4. compared with Obſ. 22. at	16	15	58
By Obſ. 5. compared with 19. and 20 at	16	16	00
By Obſ. 6. and 7. compared with 16. at	16	15	48

By reaſon of Clouds I could not ſee the Beginning of the Eclipſe, nor make ſuch Obſervations of the Moon's immerging into the Shadow as I did of her emerging out of it.

By Obſervation 11. compared with Obſervation 15. the Digits Eclipſed were $8\frac{3}{4}$.

The Angles were meaſured by a *Micrometer* in a 15 Foot Teleſcope. I have not conſidered how far they are conſiſtent with one another; they being ſet down here exactly as they were firſt taken.

This Eclipſe is the more conſiderable, as happening very near the Moon's *Perigee*, and therefore uſeful to verify her *Anomaly*; as alſo to limit the greateſt Diameter of the Shadow of the Earth, and conſequently the Parallax of the Moon. This may very properly be compared with that of the 19*th* of *October*, 1697, whoſe middle was at $7^h. 41'$. P. M. at *London*, and Quantity the ſame as now.

The

The Times by the Clock were 17′. 45″. ſooner than the apparent time, as was found by the following Obſervations of *Cor Leonis* and *Arcturus*, which through the Clouds were but juſt diſcernible.

Apparent Zenith Diſtance	Time by the Clock	Apparent Time by Calculat.	The Difference	
of Cor ♌				
° ′	h. ′ ″	h. ′ ″	′ ″	Mean Diff.
70 16 ½	13 32 43	13 50 35	17 52	′ ″
69 38	36 50	54 44	17 54	17 50
69 09	40 06	57 51	17 45	
68 40	43 09	14 00 59	17 50	
68 08	46 37	04 26	17 59	
of *Arctur.*				
65 19	17 37 40	17 55 24	17 44	
65 06	39 12	56 48	17 36	
64 41	41 49	59 29	17 40	17 40
63 47	47 40	18 05 17	17 37	
		Clock too ſlow		17 45

The Latitude of *Wanſted* is 51°. 34′. Its Longitude is 8″ in time Eaſtward from the *Obſervatory* at *Greenwich.*

The Account given of this Eclipſe by the Reverend Mr. William Derham, *who obſerv'd it at* Upminſter, *is agreeable to this, as far as Clouds would permit him to obſerve.*

V.

V. *An Account of the late ſurprizing Appearance of the* Lights *ſeen in the* Air, *on the ſixth of* March *laſt; with an Attempt to explain the Principal* Phænomena *thereof; As it was laid before the* Royal Society *by* Edmund Halley, J. V. D. Savilian *Profeſſor of* Geom. Oxon, *and* Reg. Soc. Secr.

THE *Royal Society*, having received Accounts from very many Parts of *Great Britain*, of the unuſual *Lights* which have of late appeared in the *Heavens*; were pleaſed to ſignify their Deſires to me, that I ſhould draw up a general Relation of the *Fact*, and explain more at large ſome Conceptions of mine I had propoſed to them about it, as ſeeming to ſome of them to render a tollerable Solution of the very ſtrange and ſurprizing *Phænomena* thereof. The Deſires of the *Society* having with me the force of Commands, I ſhall not decline the Task: only premiſing that if, in delivering the *Etiology* of a Matter ſo uncommon, never before ſeen by my ſelf, nor fully deſcribed by any either of the Ancients or Moderns, I fail to anſwer their Expectation or my own Deſires; yet 'tis hoped a good Hiſtory of the *Fact*, deduced partly from our own Obſervations, and partly collected from the uniform Relations of credible Perſons, or from the Letters of ſuch, may not be unacceptable to the Curious; and may perhaps excite the *Genius* of ſome more able *Meteorologiſt* to a more ſatisfactory Enquiry. The Account of this Appearance take as follows.

On

On *Tuesday* the sixth of *March, st. vet.* in the current Year 1716, (the Afternoon having been very serene and calm, and somewhat warmer than ordinary) about the Time it began to grow dark, that is much about 7 of the Clock, not only in *London*, but in all Parts of *England*, where the Beginning of this wonderful Sight was seen; out of what seemed a dusky Cloud, in the N. E. parts of the Heaven and scarce ten Degrees high, the Edges whereof were tinged with a reddish Yellow like as if the Moon had been hid behind it, there arose very long, luminous Rays or Streaks perpendicular to the Horizon, some of which seem'd nearly to ascend to the Zenith. Presently after, that reddish Cloud was swiftly propagated along the Northern Horizon, into the N. W. and still farther Westerly; and immediately sent forth its Rays after the same manner from all Parts, now here, now there, they observing no Rule or Order in their rising. Many of these Rays seeming to concur near the Zenith, formed there a *Corona*, or Image which drew the Attention of all Spectators, who according to their several Conceptions made very differing Resemblances thereof; but by which compared together, those that saw it not, may well comprehend after what manner it appeared. Some likened it to that Representation of *Glory* wherewith our Painters in Churches surround the Holy *Name* of *God*. Others to those radiating *Starrs* wherewith the Breasts of the *Knights* of the most Noble Order of the *Garter* are adorned. Many compared it to the *Concave* of the great *Cupola* of St. *Paul's Church*, distinguisht with Streaks alternably Light and obscure, and having in the middle a Space less bright than the rest, resembling the Lantern. Whilst others, to express as well the Motion as Figure thereof, would have it to be like the Flame in an Oven reverberated and rouling against the arched Roof thereof: And some there were that thought it liker to that tremulous

Light

Light which is caſt againſt a Ceiling by the Beams of the Sun, reflected from the Surface of Water in a Baſon that's a little ſhaken; whoſe reciprocal vibrating Motion it very much imitated. But all agree that this *Spectrum* laſted only a few Minutes, and ſhow'd it ſelf variouſly tinged with Colours, Yellow, Red and a dusky Green: Nor did it keep in the ſame Place; for when firſt it began to appear, it was ſeen a little to the Northwards of the *Zenith*, but by degrees declining towards the South, the long *Striæ* of Light, which aroſe from all Parts of the Northern Semicircle of the Horizon, ſeemed to meet together, not much above the Head of *Caſtor* or the Northern *Twin*, and there ſoon diſappeared.

After the firſt *Impetus* of this aſcending Vapour was over, the *Corona* we have been deſcribing appeared no more; but ſtill, without any order as to Time, or Place, or Size, luminous *Radii* like the former continued to ariſe perpendicularly, now oftner and again ſeldomer, now here, now there, now longer, now ſhorter. Nor did they proceed at firſt out of a Cloud, but oftner would emerge at once out of the pure Sky, which was at that time more than ordinary ſerene and ſtill. Nor were they all of the ſame Form. Moſt of them ſeemed to end in a Point upwards, like erect Cones; others like truncate Cones or Cylinders, ſo much reſembled the long Tails of Comets, that at firſt ſight they might well be taken for ſuch. Again, ſome of theſe Rays would continue viſible for ſeveral Minutes; when others, and thoſe the much greater part, juſt ſhew'd themſelves and died away. Some ſeem'd to have little Motion, and to ſtand as it were fix'd among the Stars, whilſt other with a very perceptible Tranſlation moved from Eaſt to Weſt under the Pole, contrary to the Motion of the Heavens; by which means they would ſometimes ſeem to run together, and

at

at other times to fly one another; affording thereby a surprizing Spectacle to the Beholders.

After this Sight had continued about an Hour and a Half, those Beams began to rise much fewer in Number and not near so high, and by degrees that diffused Light, which had illustrated the Northern Parts of the Hemisphere, seemed to subside, and settling on the Horizon formed the Resemblance of a very bright *Crepusculum*: That this was the State of this *Phænomenon*, in the first Hours, is abundantly confirmed by the unanimous Consent and concurring Testimony of several very worthy Persons no ways enclined to deceive. For by the Letters we have received from almost all the extream Parts of the Kingdom, there is found very little Difference in the Description from what appeared at *London* and *Oxford*; unless that in the North of *England*, and in *Scotland*, the Light seemed somewhat stronger and brighter.

Hitherto I am forced to relate the Observations of others, wherein I fear many very material Circumstances may be omitted: and assuredly I am not a little concern'd that I had no Notice of this Matter, till between Nine and Ten of the Clock, being at that Time at a Friend's House, and no ways suspecting what past without Doors. But upon the first Information of the thing, we immediately ran to the Windows, which hapned to regard the *South* and *South-West* Quarter; and soon perceived, that though the Sky was very clear, yet it was tinged with a strange sort of Light; so that the smaller Stars were scarce to be seen, and much as it is when the Moon of four Days old appears after Twilight. And whilst we regarded the Heavens with attention, we perceived a very thin Vapour to pass before us, which arose from the precise *East* part of the *Horizon*, ascending obliquely, so as to leave the Zenith about 15 or 20 Degrees to the Northward. But the swiftness wherewith it procee-

ded was ſcarce to be believed, ſeeming not inferiour to that of Lightning; and exhibiting, as it paſt on, a ſort of momentaneous *Nubecula*, which diſcovered it ſelf by a very diluted and faint Whiteneſs; and was no ſooner formed, but before the Eye could well take it, it was gone, and left no Signs behind it. Nor was this a ſingle appearance; but for ſeveral Minutes that we regarded it, about ſix or ſeven times in a Minute, the ſame was again and again repeated; theſe Waves of Vapour (if I may be allowed to uſe the Word) regularly ſucceeding one another, and, as we gueſt, at intervals very nearly equal; all of them in their Aſcent producing a like tranſient *Nubecula*.

By this particular we were firſt aſſured, that the Vapour we ſaw, whatever it were, became conſpicuous by its own proper Light, without help of the Suns Beams: for theſe *Nubeculæ* did not diſcover themſelves in any other part of their paſſage, but only between the *South-Eaſt*, and *South*, where being oppoſite to the Sun they were deepeſt immerſt in the Cone of the Earths Shadow; nor were they viſible before or after. Whereas the contrary muſt have happened, had they borrowed their Light from the Sun.

We then made all the haſt we could to a place where there is a free Proſpect of the Northern Horizon. Being come there, not much paſt Ten of the Clock, we found, on the Weſtern Side, *viz.* between the W. and N. W. the Repreſentation of a very bright *Twilight*, contiguous to the Horizon; out of which there aroſe very long Beams of Light, not exactly erect toward the *Vertex*, but ſomething declining to the South, which aſcending by a quick and undulating Motion to a conſiderable Height, vaniſhed in a little time, whilſt others, tho' at uncertain Intervals, ſupply'd their Place. But at the ſame time, through all the reſt of the Northern Horizon, *viz.* from the North-

Weſt

Weſt to the true Eaſt, there did not appear any ſign of Light to ariſe from, or joyn to, the Horizon; but on the contrary, what appeared to be an exceeding black and diſmal Cloud ſeem'd to hang over all that part of it. Yet was it no Cloud, but only the ſerene Sky more than ordinary pure and limpid, ſo that the bright Stars ſhone clearly in it, and particularly *Cauda Cygni* then very low in the *North*; the great Blackneſs manifeſtly proceeding from the Neighbourhood of the Light which was collected above it. For the Light had now put on a Form quite different from all that we have hitherto deſcribed, and had faſhioned it ſelf into the Shape of two *Laminæ* or Streaks, lying in a Poſition parallel to the Horizon, whoſe Edges were but ill terminated. They extended themſelves from the *N.* by *E.* to the *North Eaſt*, and were each about a Degree broad; the undermoſt about eight or nine Degrees high, and the other about four or five Degrees over it; theſe kept their Places for a long time, and made the Sky ſo light, that I believe a Man might eaſily have read an ordinary Print by the Help thereof.

Whilſt we ſtood aſtoniſhed at this ſurprizing Sight, and expecting what was further to come, the Northern End of the upper *Lamina* by degrees bent downwards, and at length cloſed with the End of the other that was under it, ſo as to ſhut up on the Northſide an intermediate Space, which ſtill continued open to the Eaſt. Not long after this, in the ſaid included Space, we ſaw a great Number of ſmall Columns or whitiſh Streaks to appear ſuddenly, erect to the Horizon, and reaching from the one *Lamina* to the other; which inſtantly diſappearing were too quick for the Eye, ſo that we could not judge whether they aroſe from the Under or fell from the Upper, but by their ſudden Alterations they made ſuch an Appearance, as might well be taken to reſemble the Conflicts of Men in Battle.

And much about the ſame time, to encreaſe our Wonder, there began on a ſudden to appear, low under the Pole and very near due North, three or four lucid *Areas* like Clouds, diſcovering themſelves, in the pure but very black Sky, by their yellowiſh Light. Theſe, as they broke out at once, ſo after they had continued a few Minutes, diſappeared as quick as if a Curtain had been drawn over them: Nor were they of any determined Figure, but both in Shape and Size might properly be compared to ſmall Clouds illuminated by the full Moon, but brighter.

Not long after this, from above the aforeſaid two *Laminæ*, there aroſe a very great *Pyramidal* Figure, like a *Spear*, ſharp at the Top, whoſe Sides were inclined to each other with an Angle of about four or five Degrees, and which ſeemed to reach up to the Zenith or beyond it. This was carried with an equable and not very ſlow Motion, from the N. E. where it aroſe, into the N. W. where it diſappeared, ſtill keeping in a perpendicular Situation, or very near it; and paſſing ſucceſſively over all the Stars of the *Little Bear*, did not efface the ſmaller ones in the Tail, which are but of the Fifth Magnitude; ſuch was the extream Rarity and Perſpicuity of the Matter whereof it conſiſted.

This ſingle Beam was ſo far remarkable above all thoſe that for a great while before had preceeded it, or that followed it, that if the Situation thereof among the Circumpolar Stars had at the ſame Inſtant been accurately noted, for Example, at *London* and *Oxford*, whoſe Difference of *Longitude* is well known, we might be enabled thereby with ſome certainty to pronounce, by its *diverſitas Aſpectûs*, concerning the Diſtance and Height thereof; which were undoubtedly very great, tho' as yet we can no ways determine them. But as this Phænomenon found all thoſe that are skill'd in the Obſervation of the Heavens unprepared, and unacquainted with what

was

was to be expected; so it left all of them surprized and astonished at the Novelty thereof. When therefore for the future any such thing shall happen, all those that are curious in Astronomical Matters, are hereby admonished and entreated to set their Clocks to the apparent Time at *London*, for Example, by allowing so many Minutes as is the Difference of Meridians; and then to note at the End of every half Hour precisely, the exact Situation of what at that time appears remarkable in the Sky; and particularly the *Azimuths* of those very tall *Pyramids* so eminent above the rest, and therefore likely to be seen furthest: to the intent that by comparing those Observations taken in the same Moment in distant Places, the Difference of their *Azimuths* may serve to determine how far those *Pyramids* are from us.

It being now past Eleven of the Clock, and nothing new offering it self to our View, but repeated *Phases* of the same Spectacle; we thought it no longer worth while to bear the Chill of the night-Air *sub dio*. Wherefore being returned to my House, I made haste to my upper Windows, which conveniently enough regard the N. E. Parts of Heaven, and soon found that the two *Laminæ* or Streaks parallel to the Horizon, of which we have been speaking, had now wholly disappeared; and the whole Spectacle reduced it self to the Resemblance of a very bright *Crepusculum* setling on the Northern Horizon, so as to be brightest and highest under the Pole it self; from whence it spread both Ways, into the N. E. and N. W. Under this, in the middle thereof, there appeared a very black Space, as it were the Segment of a lesser Circle of the Sphere cut off by the Horizon. It seemed to the Eye like a dark Cloud, but was not so; for by the Telescope the small Stars appeared through it more clearly than usual, considering how low they were: and upon this as a *Basis* our *Lumen Auroriforme* rested,

which

which was no other than a Segment of a Ring or Zone of the Sphere, intercepted between two parallel lesser Circles, cut off likewise by the Horizon; or, if you please, the Segment of a very broad *Iris*, but of one uniform Colour; *viz.* a Flame-colour inclining to yellow, the Center thereof being about forty Degrees below the Horizon. And above this there were seen some Rudiments of a much larger Segment, with an Interval of dark Sky between, but this was so exceeding faint and uncertain that I could make no proper estimate thereof.

I was very desirous to have seen how this Phænomenon would end, and attended it till near Three in the Morning, and the rising of the Moon: but for above two Hours together it had no manner of Change in its Appearance, nor Diminution nor Encrease of light; only sometimes for very short Intervals, as if new Fuel had been cast on a Fire, the Light seem'd to undulate and sparkle, not unlike the rising of vaporous Smoak out of a great Blaze when agitated: But one thing I assured myself of by this Attendance and Watching, *viz.* that this *Iris*-like Figure did by no means owe its Origine to the Sun's Beams: for that about Three in the Morning, the Sun being in the Middle between the North and East, our *Aurora* had not follow'd him, but ended in that very Point where he then was: whereas in the true North, which the Sun had long past, the Light remained unchanged and in its full Lustre.

Hitherto I have endeavoured by Words to represent what I saw, but being sensible how insufficient such a verbal Description of a thing so extraordinary and unknown may be to most Readers, I have thought fit to annex a Figure exhibiting that particular Appearance of the two *Laminæ*, which I saw at *London* between the Hours of Ten and Eleven: more especially, because I do not find, among the many Relations I have seen, any one

one that has taken notice of it. In this Figure *A B* is the under *Lamina*, ſomewhat broader and brighter than the upper *C D*: it had near its under Edge the *Lucida Lyræ*, and below its Northern Extremity, on the Left-hand, *Cauda Cygni*: and as well above and below theſe, as in the intermediate Space between them, and indeed all round about that Part of the Heavens, the Sky was ſo unuſually dark and black, as if all that *exotick Light* that had ſhew'd itſelf before, had been then collected into thoſe two Streaks. Only at *Q* between the Weſt and Northweſt and no where elſe, out of a Brightneſs adjoining to the Horizon, there aroſe conical Beams as *M, L, N*, after the ſame manner as at firſt.

Whilſt we ſtood looking on, the Streak *C D* at its Northern End bent downward, and joyned with the Under *A B* at *E*, and included the Space *D C E A B*, which ſtill kept open at the other End towards the Eaſt. And in the mean time, out of the very clear Sky, ſome luminous Spots, ſituated and figured as in the Scheme at *G, G, G, G*, preſented themſelves to the Eye, in Colour much like the *Laminæ*. Theſe did not ſhew themſelves all together, but came ſucceſſively, yet ſo as two or three of them were ſeen at a time; and as their coming was inſtantaneous, ſo they went away in a Moment. At the ſame time likewiſe, the ſeveral little white Columns marked *F, F, F, F*, occupied that Part of the Space between the two Streaks next to *E*, and by their ſudden and very irregular Motion, and the vaniſhing of ſome whilſt others at the ſame time emerged, gave occaſion to the Conception of thoſe that fancy'd Battles fought in the Air. Laſtly from about the middle of *C D*, there aroſe ſuddenly a Cone or Obelisk of a pale whitiſh Light, greater than any we had yet ſeen, as *H*; which moving from Eaſt to Weſt, with a Motion ſufficiently regular, was tranſlated to *K*, in the *North Weſt*, and there diſappeared.

That

That we might by the ſame Scheme ſhew the Appearance of the laſt Hours, after Midnight; the Reader is deſired to take notice that we have made the Light at *Q*, much bigger than what appeared in the Weſt about Ten of the Clock; ſo as to repreſent truly that other. In this Caſe the Point *Q* muſt, by the Imagination, be ſuppoſed transferred to the Interſection of the Horizon and Meridian under the Pole. And that we might the better be underſtood in what follows, we have made this ſhort Recapitulation as annex'd to, and explicative of, the Scheme, which could by no means be contrived to anſwer the wonderful Variety this Phænomenon afforded; ſince even the Eye of no one ſingle Obſerver, was ſufficient to follow it in the Suddenneſs and Frequency of its Alterations.

Thus far I have attempted to deſcribe what was ſeen, and am heartily ſorry I can ſay no more as to the firſt and moſt ſurprizing Part thereof, which however frightful and amazing it might ſeem to the vulgar Beholder, would have been to me a moſt agreeable and wiſh'd for Spectacle; for I then ſhould have contemplated *propriis oculis* all the ſeveral Sorts of Meteors I remember to have hitherto heard or read of. This was the only one I had not as yet ſeen, and of which I began to deſpair, ſince it is certain it hath not happen'd to any remarkable Degree in this Part of *England* ſince I was born; nor is the like recorded in the *Engliſh Annals* ſince the Year of our Lord 1574, that is above One Hundred and Forty Years ago, in the Reign of Queen *Elizabeth*. Then, as we are told by the Hiſtorians of thoſe times, *Cambden* and *Stow*, Eye-Witneſſes of ſufficient credit, for two Nights ſucceſſively, *viz.* on the 14*th* and 15*th* of *November* that Year, much the ſame wonderful Phænomena were ſeen, with almoſt all the ſame Circumſtances as now.

Nor

Nor indeed, during the Reign of that glorious Princeſs, was this ſo rare a Sight as it has been ſince. For we find in a Book entituled *a Deſcription* of *Meteors*, reprinted at *London* in the Year 1654, whoſe Author writes himſelf *W.F.* D.D. that the ſame thing, which he there calls *Burning Spears*, was ſeen at *London* on *January* 30. 1560; and again by the Teſtimony of *Stow*, on the *7th* of *October* 1564. And from foreign Authors we learn, that in the Year 1575, the ſame was twice repeated in *Brabant*, *viz.* on the *13th* of *February* and *28th* of *September*; and ſeen and deſcribed by *Cornelius Gemma*, Profeſſor of Medicine in the Univerſity of *Lovain*, and Son of *Gemma Friſius* the Mathematician. He, in a Diſcourſe he wrote of the Prodigies of thoſe Times, after ſeveral ill-boding Prognoſticks, thus very properly deſcribes the *Cupola* and *Corona* that he ſaw in the *Chaſma* (as he calls it) of *February* *Paulo poſt undecunque ſurgentibus Haſtis & flammis novis, flagrare cœlum à Borea parte uſque ad verticem videbatur: ac denique ne nihil quæ contigerunt hactenus præfiguratum antea videretur, converſa eſt Cœli facies, per horæ ſpatium, in Fritilli aleatorii ſpeciem peregrinam; alternantibus ſeſe cæruleo & candido, non minore vertigine motuſque celeritate, quam ſolares radii ſolent, quoties ab objecto ſpeculo regeruntur.* Here it is not a little remarkable, that all theſe four already mentioned fell exactly upon the ſame Age of the Moon, *viz.* about two Days after the Change.

As to the other of *September* in the ſame Year 1575, theſe are the Words of *Gemma*. *Minus quidem horrendum, ſed varia tamen magiſque confuſa nobis apparuit alterius Chaſmatis forma, quarto Calendas* Octobreis *ſubſecuti, ſtatim ab occaſu Solis. Nam in illo viſi ſunt arcus illuſtres plurimi, ex quibus Haſtæ ſenſim eminentes, Urbeſque turritæ & Acies militares. Erant hinc radiorum excurſus quaquaverſum, & nubium fluctus & prælia: inſectabantur invicem & fugiebant, facta in orbem converſione mirabili.* From hence 'tis ma-

nifest that this *Phænomenon* appeared in our Neighbourhood three several Times, and that with considerable Intervals, within the Campass of one Year; though our *English* Historians have not recorded the two latter; nor did *Gemma* see that of *November* 1574, as 'tis most likely by reason of Clouds. After this, in the Year 1580. we have the Authority of *Michael Mæstlin*, * (himself a good Astronomer, and still more famous for having had the honour to be the great *Kepler*'s Tutor in the Sciences) that at *Baknang* in the Country of *Wirtemburg* in *Germany*, these *Chasmata*, as he likewise stiles them, were seen by himself no less than seven times within the Space of twelve Months. The first of these, and most considerable, fell out on the very same Day of the Month with ours, *viz.* on *Sunday* the sixth of *March*, and was attended with much the same Circumstances, which, for Brevity's sake, I omit. And again the same things were seen in a very extraordinary manner on the 9*th* of *April* and 10*th* of *September* following: but in a less degree, on the 6*th* of *April*, 21*st* of *September*, 26*th* of *December* and 16*th* of *February*, 1581: the last of which, and that of the 21*st* of *September* must needs have been more considerable than they then appeared, because the Moon being near the Full, necessarily effaced all the fainter Lights. Of all these however no one is mentioned in our Annals to have been seen in *England*, nor in any other place that I can find; such was the neglect of curious matters in those Days.

The next in order that we heat of, was that of the Year 1621, on *September* the 2*d. st. vet.* seen all over *France*, and well described by *Gassendus* in his *Physicks*, who gives it the Name of *Aurora Borealis*. This, tho' little inferiour to what we lately saw, and appearing to the Northwards both of *Rouen* and *Paris*, is no where said to have been observed in *England*, over which the Light seemed to lie. And since then for above 80 Years, we have no Account

* M. Mæstlin. lib. de Cometa 1580.

of

of any ſuch Sight either from home or abroad; notwithſtanding that for above half that time, theſe *Philoſophical Tranſactions* have been a conſtant Regiſter of all ſuch extraordinary Occurrences. The firſt we find on our Books, was one of ſmall Continuance ſeen in *Ireland* by Mr. *Neve* on the 16*th* of *November* 1707. of which ſee *Phil. Tranſ.* No. 320. And in the *Miſcellanea Berolinenſia* publiſhed in 1710, we learn that in the ſame Year 1707, both on the 24*th* of *January* and 18*th* of *February*, *ſt. vet.* ſomething of this kind was ſeen by *M. Olaus Römer* at *Copenhagen*: and again on the 23*d* of *February*, the ſame excellent *Aſtronomer* obſerved there ſuch another Appearence, but much more conſiderable; of which yet he only ſaw the Beginning, Clouds interpoſing. But the ſame was ſeen that Night by Mr. *Gotfried Kirch*, at *Berlin* above 200 Miles from *Copenhagen*, and laſted there till paſt Ten at Night. To theſe add another ſmall one of ſhort Duration, ſeen near *London*, a little before Midnight between the Ninth and Tenth of *Auguſt* 1708, by the Right Reverend *Philip* Lord Biſhop of *Hereford*, and by his Lordſhip communicated to the *Royal Society*: ſo that, it ſeems, in little more than eighteen Months this ſort of Light has been ſeen in the Sky, no leſs than five times; in the Years 1707 and 1708.

Hence we may reaſonably conclude that the Air, or Earth, or both, are ſometimes, though but ſeldom and with great Intervals, diſpoſed to produce this Phænomenon: for though it be probable that many times, when it happens, it may not be obſerved, as falling out in the Day-time, or in cloudy Weather, or bright Moon-ſhine: yet that it ſhould be ſo very often ſeen at ſome times and ſo ſeldom at others, is what cannot well be that way accounted for. Wherefore caſting about and conſidering what might be moſt probably the *Material Cauſe* of theſe Appearances; what firſt occurr'd was the Vapour of Water

rarified exceedingly by ſubterraneous Fire, and tinged with ſulfureous Steams; which Vapour is now generally taken by our Naturaliſts to be the Cauſe of *Earthquakes*. And as Earthquakes happen with great Uncertainty, and have been ſometimes frequent in Places, where for many Years before and after they have not been felt; ſo Theſe, which we might be allowed to ſuppoſe produced by the Eruption of the pent Vapour through the Pores of the Earth, when it is not in ſufficient Quantity, nor ſudden enough to ſhake its Surface, or to open it ſelf a Paſſage by rending it. And as theſe Vapours are ſuddenly produced by the Fall of Water upon the nitro-ſulphurous Fires under Ground, they might well be thought to get from thence a Tincture which might diſpoſe them to ſhine in the Night, and a Tendency contrary to that of Gravity; as we find the Vapours of *Gun-powder*, when heated in *Vacuo*, to ſhine in the Dark, and aſcend to the Top of the Receiver though exhauſted: the Experiment of which I ſaw very neatly performed by Mr. *J. Whiteſide* Keeper of *Aſhmole's Muſeum* in *Oxford*.

Nor ſhould I ſeek for any other Cauſe than this, if in ſome of theſe Inſtances, and particularly this whereof we treat, the Appearance had not been ſeen over a much greater Part of the Earth's ſurface that can be thus accounted for. It having in this laſt been viſible from the Weſt Side of *Ireland* to the Confines of *Ruſſia* and *Poland* on the Eaſt (nor do we yet know its Limits on that Side) extending over at leaſt thirty Degrees of Longitude; and in Latitude, from about fifty Degrees over almoſt all the North of *Europe*; and in all Places exhibiting at the ſame time the ſame wonderous Circumſtances, as we are informed by the Publick News. Now this is a Space much too wide to be ſhaken at any one Time by the greateſt of Earthquakes, or to be affected by the Perſpiration of that Vapour, which being included and wanting vent, might

might have occaſioned the Earth to tremble. Nor can we this Way account for that remarkable Particular attending theſe Lights, of being always ſeen on the Northſide of the Horizon, and never to the South.

Wherefore laying aſide all hopes of being able to explain theſe Things by the ordinary Vapours or Exhalations of the Earth or Waters, we are forced to have recourſe to other ſorts of *Effluvia* of a much more ſubtile Nature, and which perhaps may ſeem more adapted to bring about thoſe wonderful and ſurprizingly quick Motions we have ſeen. Such are the *Magnetical Effluvia*, whoſe Atoms freely permeate the Pores of the moſt ſolid Bodies, meeting with no Obſtacle from the Interpoſition of Glaſs or Marble or even Gold it ſelf. Theſe by a perpetual Efflux do, ſome of them, ariſe from the Parts near the Poles of the Magnet, whilſt others of the like Kind of Atoms, but with a contrary Tendency, enter in at the ſame Parts of the Stone, through which they freely paſs; and by a kind of Circulation ſurround it on all Sides, as with an Atmoſphere, to the Diſtance of ſome Diameters of the Body. This thing *des Cartes* has endeavoured to explain (*Princip. Philoſoph.* Lib IV.) by the *Hypotheſis* of the Circulation of certain skrewed or *ſtriate Particles*, adapted to the Pores they are to enter.

But without enquiring how ſufficient the *Carteſian Hypotheſis* may be for anſwering the ſeveral Phænomena of the *Magnet*: that the Fact may be the better comprehended we ſhall endeavour to exhibit the manner of the Circulation of the Atoms concerned therein, as they are expoſed to view, by placing the Poles of a *Terrella* or *Spherical Magnet* on a Plane, as the Globe on the Horizon of a Right Sphere: Then ſtrewing fine Steel duſt or Filings very thin on the Plain all round it, the Particles of Steel, upon a continued gentle knocking on the underſide of the Plain, will by degrees conform themſelves

to

to the Figures in which the Circulation is performed. Thus in *Fig.* II. Let *A B C D* be a *Terrella*, and its Poles *A* the South, and *B* the North ; and by doing as prescribed, it will be found that the Filings will lie in a Right Line perpendicular to the Surface of the Ball, when in the Line of the Magnetical Axis continued. But for about forty five Degrees on either side, from *B* to *G* or *I*, and from *A* to *H* or *K*, they will form themselves into Curves, more and more crooked as they are remoter from the Poles ; and withall more and more oblique to the Surface of the Stone : as our Figure truly represents, and as may readily be shewn by the *Terrella* and *Apparatus* for that Purpose in the *Repository* of the *Royal Society* Hence it may appear how this exceeding subtile Matter revolves; and particularly how it permeates the Magnet with more force and in greater Quantity in the circumpolar Parts, entring into it on the one side, and emerging from it on the other, under the same oblique Angles : whilst in the middle Zone about *C* and *D*, near the Magnet's Equator (if I may use the Word) very few if any of these Particles do impinge, and those very obliquely.

Now by many and very evident Arguments it appears that our *Globe* of *Earth* is no other than one great Magnet, or (if I may be allowed to alledge an Invention of my own) rather two ; the one including the other as the *Shell* includes the *Kernel* (for so and not otherwise we may explain the changes of the Variation of the Magnetical Needle) but to our present Purpose the Result is the same. It suffices that we may suppose the same sort of Circulation of such an exceeding fine Matter to be perpetually performed in the *Earth*, as we observe in the *Terella*; which subtile Matter freely pervading the Pores of the Earth, and entring into it near its Southern Pole, may pass out again into the Ether, at the same Distance from the Northern, and with a like Force ; its Direction being still

ſtill more and more oblique, as the Diſtance from the Poles is greater. To this we beg leave to ſuppoſe, that this ſubtile Matter, no otherways diſcovering it ſelf but by its Effects on the Magnetick Needle, wholly imperceptible and at other times inviſible, may now and then, by the Concourſe of ſeveral Cauſes very rarely coincident, and to us as yet unknown, be capable of producing a ſmall Degree of Light; perhaps from the greater Denſity of the Matter, or the greater Velocity of its Motion: after the ſame manner as we ſee the *Effluvia* of *Electrick* Bodies by a ſtrong and quick Friction emit Light in the Dark: to which ſort of Light this ſeems to have a great Affinity.

This being allowed me, I think we may readily aſſign a Cauſe for many of the ſtrange Appearances we have been treating of, and for ſome of the moſt difficult to account for otherwiſe; as why theſe Lights are rarely ſeen any where elſe but in the North, and never, that we hear of, near the Equator: as alſo why they are more frequently ſeen in *Iceland* and *Groenland*, than in *Norway*, though nearer the *Pole* of the *World*. For the Magnetical Poles, in this Age, are to the Weſtward of our Meridian, and more ſo of that of *Norway*, and not far from *Groenland*; as appears by the Variation of the Needle this Year obſerved, full twelve Degrees at *London* to the Weſt.

The erect Poſition of the luminous Beams or *Striæ* ſo often repeated that Night, was occaſioned by the riſing of the Vapour or lucid Matter nearly perpendicular to the Earth's Surface. For that any Line erected perpendicularly upon the Surface of the Globe, will appear erect to the Horizon of an Eye placed any where in the ſame ſpherical Superficies; as *Euclid* demonſtrates in a Plain, that any Line erected at Right Angles to it, will appear to be perpendicular to that Plain from any Point thereof. That it ſhould be ſo in the Sphere is a very pretty Propoſition,

not

not very obvious, but demonſtrated from *Prop.* 5. *Lib.* I. *Theodoſii Spharic.* For by it all Lines erect on the Surface paſs through the Center, where meeting with thoſe from the Eye, they form the Plains of Vertical Circles thereto. And by the Converſe hereof it is evident, that this luminous Matter aroſe nearly perpendicular to the Earth's Surface, becauſe it appeared in this erect Poſition. And whereas in this Appearance (and perhaps in all others of the Kind) thoſe Beams which aroſe near the *Eaſt* and *Weſt*, as *L*, *M*, *N*, were furtheſt from the Perpendicular, on both ſides inclining towards the *South*, whilſt thoſe in the *North* were directly upright: the cauſe thereof may well be explained by the Obliquity of the Magnetical Curves, making ſtill obtuſer Angles with the Meridians of the *Terrella*, as they are further from its Poles.

Hence alſo it is manifeſt how that wonderful *Corona* that was ſeen to the Southwards of the *Vertex*, in the Beginning of the Night, and ſo very remarkable for it's tremulous and vibrating Light, was produced; to wit, by the Concourſe of many of thoſe Beams ariſing very high out of the circumjacent Regions, and meeting near the Zenith: the *Effluvia* whereof they conſiſted mixing and interfering one with another, and thereby occaſioning a much ſtronger but uncertain wavering Light. And ſince it is agreed by all our Accounts that this *Corona* was tinged with various Colours, 'tis more than probable that theſe Vapours were carried up to ſuch a Height, as to emerge out of the Shadow of the Earth, and to be illuſtrated by the direct Beams of the Sun: whence it might come to paſs that this firſt *Corona* was ſeen coloured and much brighter than what appeared afterwards in ſome Places, where the Sight thereof was more than once repeated, after the Sun was gone down much lower under the Horizon. Hence alſo it will be eaſily underſtood that this *Corona* was not one and the ſame in all Places, but was diffe-

different in every differing Horizon; exactly after the ſame manner as the Rainbow ſeen in the ſame Cloud is not the ſame Bow, but different to every ſeveral Eye.

Nor is it to be doubted, but the Pyramidical Figure of theſe Aſcending Beams is Opticall: ſince according to all likelyhood they are parallel-ſided, or rather taporing the otherway. But by the Rules of Perſpective, their Sides ought to converge to a Point, as we ſee in Pictures the Parallel Borders of ſtreight Walks, and all other Lines parallel to the Axis of Viſion, meet as in a Center. Wherefore thoſe Rays which aroſe higheſt above the Earth and were nereſt the Eye, ſeemed to terminate in Cuſps ſufficiently acute, and have been for that reaſon ſuppoſed by the Vulgar to repreſent *Spears*. Others ſeen from afar, and perhaps not riſing ſo high as the former, would terminate as if cut off with Plains parallel to the Horizon, like truncate Cones or Cylinders: theſe have been taken to look like the Battlements and Towers on the Walls of Cities fortified after the ancient manner. Whilſt others yet further off, by reaſon of their great Diſtance, good part of them being intercepted by the Interpoſition of the Convexity of the Earth, would only ſhew their pointed Tops, and becauſe of their Shortneſs have gotten the Name of *Swords*.

Next the Motion of theſe Beams, furniſhes us with a new and, as it ſeems to me, moſt evident Argument to prove the diurnal Rotation of the Earth: (though that be a matter which, at preſent, is generally taken by the Learned to be paſt diſpute) For thoſe Beams which roſe up to a Point, and did not preſently diſappear, but continued for ſome time, had moſt of them a ſenſible Motion from Eaſt to Weſt, contrary to that of the Heavens; the biggeſt and talleſt of them, as being nereſt, ſwifteſt; and the more remote and ſhorter, ſlower. By which means, the one overtaking the other, they would ſome-

times ſeem to meet and joſtle ; and at other times to ſeparate, and fly one another. But this Motion was only Optical, and occaſioned by the Eye of the Spectator being carried away with the Earth into the Eaſt ; whilſt the exceeding rare Vapour of which thoſe Beams did conſiſt, being, as I take it, raiſed far above the Atmoſphere, was either wholly left behind, or elſe followed with but part of its Velocity, and therefore could not but ſeem to recede and move the contrary Way. And after the ſame manner as the Stars that go near the *Zenith*, paſs over thoſe Vertical Circles which border on the Meridian, much ſwifter than thoſe Stars which are more diſtant therefrom ; ſo theſe luminous Rays would ſeem to recede faſter from Eaſt to Weſt, as their Baſes were nearer the Eye of the Spectator ; and *è contra*, ſlower as they were further off.

Nor are we to think it ſtrange, if after ſo great a Quantity of luminous Vapour had been carried up into the Ether out of the Pores of the Earth, the Cauſe of its Effervescence at length abating, or perhaps the Matter thereof conſumed ; theſe *Effluvia* ſhould at length ſubſide, and form thoſe two bright *Laminæ* which we have deſcribed, and whoſe Edges being turn'd to us were capable to emit ſo much Light that we might read by them. I chooſe to call them *Laminæ*, becauſe, without doubt, though they were but thin, they ſpread Horizontally over a large Tract of the Earth Surface. And whilſt this luminous Matter dropt down from the upper Plate to the under, the many little white Columns were formed between them by its Deſcent, only viſible for the Moment of their Fall. Theſe by the Swiftneſs with which they vaniſhed and their great Number, ſhewing themſelves and diſappearing without any order, exhibited a very odd Appearence ; thoſe on the Right ſeeming ſometimes to drive and puſh thoſe on the left, and *vice verſa*.

I

I have been obliged to omit ſeveral Particulars of leſs moment: But theſe are the principal Phænomena; of whoſe Cauſes I ſhould have more willingly and with more certainty given my Thoughts, if I had had the good luck to have ſeen the whole from Beginning to end; and to have added my own Remarks to the Relations of others: and eſpecially if we could by any means have come at the Diſtances thereof. If it ſhall by any be thought a hard Suppoſition that I aſſume the *Effluvia* of the Magnetical Matter for this purpoſe, which in certain Caſes may themſelves become luminous, or rather may ſometimes carry with them out of the Bowels of the Earth a ſort of Atoms proper to produce Light in the Ether. I anſwer that we are not as yet informed of any other Kinds of *Effluvia* of terreſtrial Matter which may ſerve for our purpoſe, than thoſe we have here conſidered, *viz.* the Magnetical Atoms, and thoſe of Water highly rarified into Vapour. Nor do we find any thing like it in what we ſee of the Celeſtial Bodies, unleſs it be the *Effluvia* projected out of the Bodies of Comets to a vaſt Height, and which ſeem by a *Vis centrifuga* to fly with an incredible Swiftneſs the Centers both of the Sun and Comet, and to go off into Tails of a ſcarce conceivable Length. What may be the Conſtitution of theſe Cometical Vapours, we Inhabitants of the *Earth* can know but little, and only that they are evidently excited by the Heat of the Sun; whereas this Meteor, if I may ſo call it, ſeldom is ſeen but in the polar Regions of the World, and that moſt commonly in the Winter Months. But whatever may be the Cauſe thereof, if this be not, I have followed the old Axiom of the Schools. *Entia non eſſe temere neque abſque neceſſitate multiplicanda.*

Laſtly I beg leave on this Occaſion to mention what, near 25 Years ſince, I publiſh'd in No. 195. of theſe *Transactions*, *viz.* That ſuppoſing the *Earth* to be concave,

with a leſſer Globe included, in order to make that inner Globe capable of being inhabited, there might not improbably be contained ſome luminous *Medium* between the Balls, ſo as to make a perpetual Day below. That very great Tracts of the Etherial Space are occupied by ſuch a ſhining *Medium* is evident from the Inſtances given in the firſt Paper of this *Tranſaction*: And if ſuch a *Medium* ſhould be thus incloſed within us; what ſhould hinder but we may be allowed to ſuppoſe that ſome parts of this lucid Subſtance may, on very rare and extraordinary Occaſions, tranſude through and penetrate the *Cortex* of our *Earth*, and being got looſe may afford the Matter whereof this our *Meteor* conſiſts. This ſeems favoured by one conſiderable Circumſtance, *viz.* that the *Earth*, becauſe of its diurnal Rotation, being neceſſarily of the Figure of a Flat *Spheroid*, the thickneſs of the *Cortex*, in the *Polar* Parts of the Globe, is conſiderably leſs than towards the *Equator*; and therefore more likely to give Paſſage to theſe Vapours; whence a reaſon may be given why theſe Lights are always ſeen in the North. But I deſire to lay no more ſtreſs upon this Conceit than it will bear.

It having been noted that in the Years 1575 and 1580, wherein this Appearance was frequent, that it was ſeen not far from the Times of the two Equinoxes; it may be worth while for the Curious, to beſtow ſome Attention on the Heavens in the Months of *September* and *October* next; and in caſe it ſhould again happen, to endeavour to obſerve, by the Method I have here laid down, what may determine, with ſome degree of Exactneſs, the Diſtance and Height thereof; without which we can ſcarce come to any juſt Concluſion.

F I N I S.

Errata. No. 346. *p.* 383. *l.* 18. read 234. *p.* 408. *l.* 20. read, *proceed, as at firſt.*

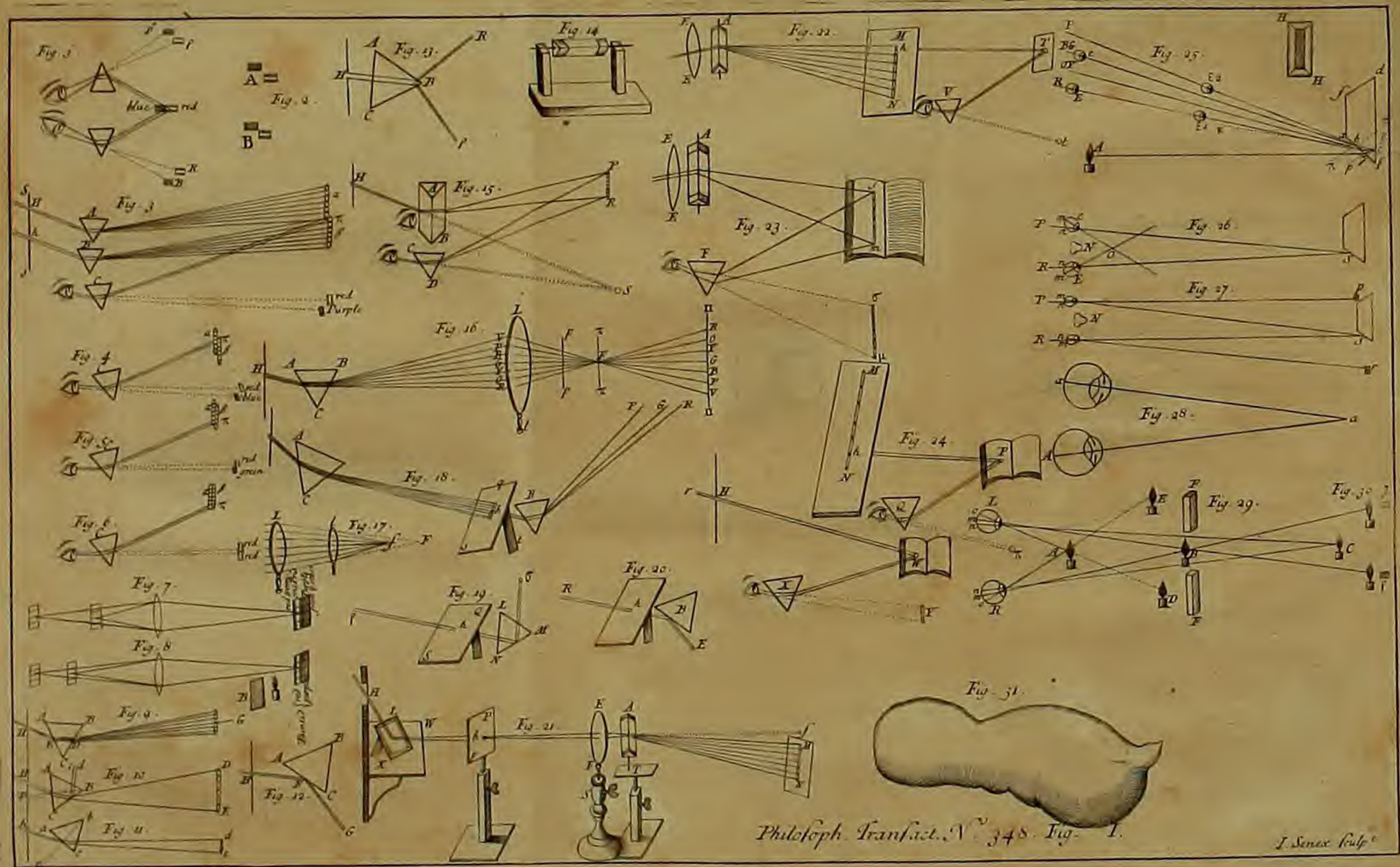
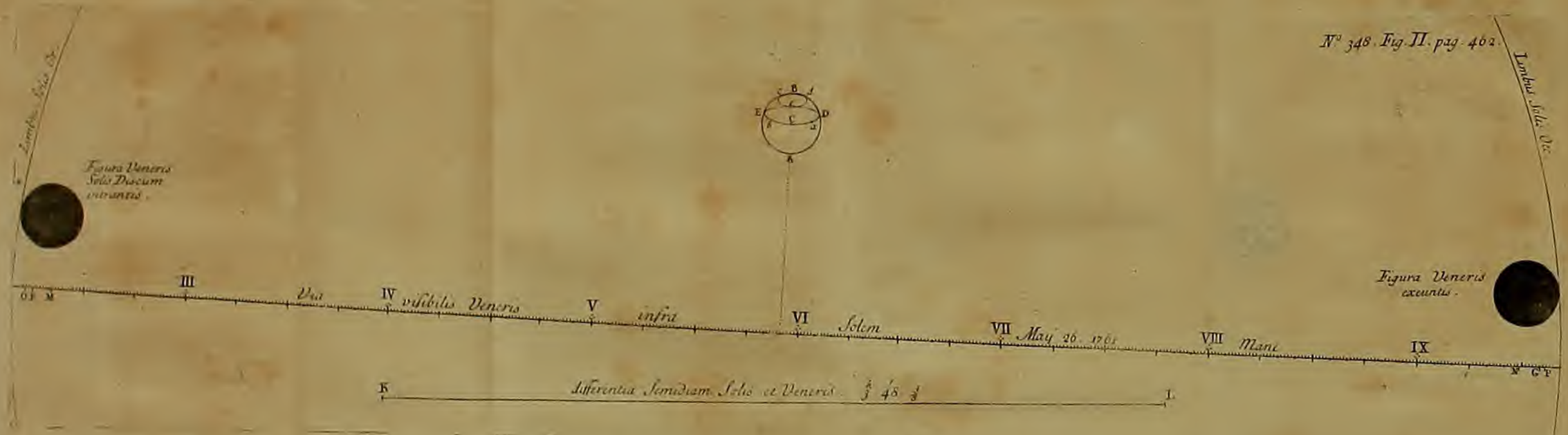


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
Fig. 19.
Fig. 20.
Fig. 21.
Fig. 22.
Fig. 23.
Fig. 24.
Fig. 25.
Fig. 26.
Fig. 27.
Fig. 28.
Fig. 29.
Fig. 30.
Fig. 31.
Philosoph. Transact. N° 348. Fig. 1.
J. Senex sculp.


Limbus Solis Oc.
Figura Veneris Solis Discum intrantis.
Limbus Solis Oc.
Figura Veneris exeuntis.
O I M
III
Via
IV visibilis Veneris
V
infra
VI
Solem
VII May 26. 1761
VIII Mani
IX
N G T
K
Differentia Semidiam. Solis et Veneris ∴ 48 ∴
L

PHILOSOPHICAL TRANSACTIONS.

For the Months of *April*, *May* and *June*, 1716.

The CONTENTS.

I. *A description of the Phænomenon of* March 6. *last, as it was seen on the Ocean, near the Coast of* Spain. *With an Account of the return of the same sort of Appearance, on* March 31, *and* April 1. *and* 2. *following.*

IN our last, we endeavour'd to give the Publick as good an Account of the late surprizing *Meteor*, seen in the Heavens, on the sixth of *March* last, as could be gathered from the several Relations of very distant Spectators, which had then come to the *Royal Society*'s Notice. And since then, we can only add thereto, that at *Paris*, the Light was so inconsiderable, that it was not regarded: But from a Letter to Mr. *Alexander Geekie*, Surgeon, dated on Board a Ship in *Nevis* Road, in *America*, *April* 19. 1716. we have copied the following Passage. "On the sixth of *March*, at 9 a Clock "in the Evening, we being then in the Latitude of 45°. 36'. "(off of the *N.W.* Coast of *Spain*). A clear Cloud appea-"red *East* of us, not far distant from our Zenith, which "afterwards darted it self forth into a number of Rays of "Light, every way like the Tail of a Comet, of such a "great Length, that it reach'd within a short way of the "Horizon. There likewise appear'd a Body of Light, "*N. N. E.* of us, and continued as Light almost as Day, till "after 12 a Clock. It appear'd at a good distance from us, "and darkned on a sudden."

Hence it should seem, that the Vapour which caused this Appearance, arose indifferently out of the deep Ocean Sea, as well as from the Land; by which we may conclude the great Subtilty of the Matter thereof, since it could permeate so great a Quantity of Water, and yet retain its Velocity; which is a Circumstance deserving the further consideration of the Curious.

But since this, most of the same *Phænomena* have been repeated three several Nights successively, *viz.* On the last of *March*, and first and second of *April*. The best and fullest Descrip-

tion

tion of the two firſt, is, from a Letter of Dr. *Brook Taylor*, *LL. D.* and Secretary to the *Royal Society*, dated *April* 2, from *Cotterſtock*, near *Oundle* in *Northampton-ſhire*, who thus deſcribes them. " On *Saturday* Night laſt, and laſt Night, I ſaw Appearances of the ſame kind, with thoſe of *March* 6. but not to compare for Extent and Strength to the other. They both began ſoon after Sun ſet, and continued till after 12, but how much longer I cannot tell. They were both about 10 or 15 degrees to the Weſtward of the *North*, and took up about 80 degrees of the Horizon; and the *Aurora* roſe about 30 gr. high, with a dark Bottom; like what was ſeen in the Firſt; and from whence there ſprung out ſeveral Bodies of Light, which immediately run into Streams, aſcending about 30, or at moſt 40 gr. high. There was no flaſhing nor waving Light, but in all other reſpects, theſe Lights were of the ſame kind with what we ſaw at *London*. Indeed in that laſt Night, there was one *Phænomenon* like the flaſhing Light; for a Body of Light about 15 or 20 degrees long, parallel to the Horizon, roſe till it came about 6 degrees above the black Baſis, and then ſent up two ſtrong Streams of Light about 40 gr. high, which at top daſht againſt one another, and diſappear'd."

At *London*, the firſt Night, *March* 31. It did not begin to radiate, till towards Mid-night, and was ſeen but by few curious Perſons; the Beams not riſing very high, and ſcarce appearing over the Houſes, were little taken notice of: but by the Relation of thoſe that ſaw it, it was much more conſiderable than the next Night following *Eaſter-day*, for it then ſent out but few and very ſhort Beams, moſtly terminating in a ſharp Point, and preſently diſappearing. Only it beginning to ſtream ſo ſoon as it became Dusky, it was very obſervable, that thoſe Rays which aroſe out of the Weſt end of the Luminous Arch, next the Sun, were enlightned by its Beams, and ſhew'd themſelves much brighter than thoſe which ſprung up under the Pole, or to the Eaſtward thereof. And after Nine, till Mid-night, no more Beams aroſe; and the Luminous Arch with its black *Baſis*, ſettled down very low in the Northern Horizon.

The ſame two Nights, by the Obſervation of Mr. *William Lingen*, the like Appearance was ſeen at *Dublin*, about the hours

hours of Nine or Ten; at which time, in the former Night, it was near as Light as in a Moon-light Night. And from *France* we have an Account, that both those Nights, the same was seen at *Paris*, with much the same Circumstances as at *Dublin*. So that it seems this Meteor, though no ways comparable to that of the 6th of *March*. was seen not less than 150 Leagues, and probably much further.

The following Night, *April* 2. When it began to be dark, a Luminous Arch appear'd in the *North*, with a very narrow black Bottom under it, very low, and depress'd to the Horizon; nor was it seen at, or about *London*, to project any pointed Rays as the former.

But what was most remarkable that Evening, was, what was seen at *London*, by that ingenious Gentleman *Martin Foulks*, Esq; *R. S. S.* about Nine that Night. He being in the open Air at that time, saw in an Instant, a bright Ray of very white Light, appear in the East, out of the pure Sky, then very serene and still; it very much resembled the Tail of a Comet, and was about 20 gr. inclined from the Perpendicular to the Right, beginning about γ of *Bayer* in the *Corona Borea*, and terminating about the *Informis* by some call'd *Cor Caroli*. This having appear'd but a very short time, disappear'd at once, as in a Moment. When on a sudden, such another Beam was instantly produced, not exactly in the same place, but in the same Situation. Its lower end being about 20 gr. high, was terminated exactly between κ and γ, in the Right Hand and Arm of *Hercules*, and the Middle of it past over σ and ϱ in the Girdle of *Bootes*, and thence proceeded Westwards, leaving *Cor Caroli* four or five degrees to the Northwards. After it had continued in this posture near 10 Minutes immoveable among the Stars, it began to move slowly towards the North: and the lower end passing over the Northern Edge of the *Crown*, and the Ray it self over *Cor Caroli*, it grew fainter, and vanished, having continued in all about 20 Minutes. This latter with some Interruptions was extended between *Castor* and *Pollux*, very far into the West. And about that time, the same, or such another Beam, was seen at St. *Asaph*, by Doctor *Stanley*, the Reverend Dean of that Church.

II. *An*

II. *An Account of some Experiments of Light and Colours, formerly made by Sir* Isaac Newton, *and mention'd in his Opticks, lately repeated before the* Royal Society *by* J. T. Desaguliers, F. R. S.

THE manner of separating the primitive Colours of Light to such a Degree, that if any one of the separated Lights be taken apart, its Colour shall be found unchangeable, was not published before Sir Is. Newton's *Opticks came abroad. For want of knowing how this was to be done, some Gentlemen of the English College at* Liege, *and Monsieur* Mariotte *in* France, *and some others took those for primitive Colours, which are made by immitting a Beam of the Sun's Light into a dark Room through a small round Hole, and refracting the Beam by a triangular Prism of Glass placed at the Hole. And by trying the Experiment in this manner, they found that the Colours thus made were capable of change, and thereupon reported that the Experiment did not succeed. And lately the Editor of the* Acta Eruditorum *for* October 1713. pag. 447, *desired that Sir* Is. Newton *would remove this Difficulty.* Objectiones, *inquit,* quas Viri docti tum in Gallia tum in Anglia contra illam [de Coloribus] Theoriam fecere, felicissime diluit Vir perspicacissimus *Newtonus,* quemadmodum ex Transactionibus Anglicanis N. 84, 85, 88, 96, 97, 121, 123, 128 abunde constat. Unde multi optant ut mentem suam aperire dignetur, de difficultate ab ingeniosissimo *Mariotto,* rerum naturalium (dum viveret) scrutatore indefesso nec infelici, in Tractatu de Coloribus p. 207. & seq. contra eam mota. In distantia scilicet 25 circiter aut 30 pedum, charta excepit radium solidum per exiguum foramen in cameram obscuram immissum,

& per Prisma vitreum trigonum transmissum, coloremque violaceum spatium majus quam trium linearum occupantem per crenam duarum linearum trajectum, alio Prismate excepit admodum oblique opposito : quo facto quandam ejus partem in Rubrum & Flavum transmutari observavit. Similiter luminis rubri partem in cœruleum & violaceum transmutari expertus est. Hac verò transmutatione admissa corruere Theoriam Newtonianam, ex Actis A. 1706. p. 60. & seq. manifestum est. Assumpsit autem *Mariottus* distantiam 30 pedum, ne quis exciperet in minori distantia nondum factam esse plenariam radiorum heterogeneorum separationem. Nobis experimentum *Mariotti* tum demum videretur decisivum, si lumen cœruleum integrum in aliud mutatum fuisset. *Thus far the Editor of the* Acta. *In answer to which it is to be observed that the Red and Yellow which came out of the Violet, and the Blew and Violet which came out of the Red, might proceed from the very bright Light of the Sky next encompassing the Sun, and that several sorts of Rays which come from several Parts of the Sun's Body are intermixt in all parts of the coloured* Spectrum *which falls upon a Paper at any Distance from the Prism. In this manner of trial, for making the Experiment succeed, the Light of the bright Clouds, immediately surrounding the Sun, should be intercepted by an opake Skreen placed in the open Air without, at the distance of Ten or Twenty Foot from the Hole through which the Sun shines into the dark Room. And in the Skreen there should be a small Hole for the Sun to shine through. The Hole may be either round or oblong, and not above one eighth or one tenth Part of an Inch broad; so that the Skreen may intercept not only the bright Light of the Clouds next encompassing the Sun's Body, but also the greatest Part of the Sun's Light. For thereby the Colours will become less mixed. The Beam of Light which passes through this Hole must afterwards pass through the other Hole into the dark Room, and the Prism must be placed parallel to the oblong Hole in the*

Skreen

Skreen, and the refracting Angle thereof be sixty Degrees or above. In this manner the Experiment may be tried with Success, but the Trial will be less troublesome if it be made in such a manner as is described in the fourth Proposition of the first Book of Sir Is. Newton's *Opticks.*

Sir Isaac Newton *therefore, upon reading what has been cited out of the* Acta Eruditorum, *desired Mr.* Desaguliers *to try the Experiment in the manner described in the said Proposition; and he tried it accordingly with Success before several Gentlemen of the* Royal Society, *and afterwards before Monsieur* Monmort *and others of the* Royal Academy *of* Sciences; *and still shews it to those who desire to see it. How this and other concomitant Experiments were tried and succeeded, is described as follows.*

EXPERIMENT I.

Having sew'd together end-wise two Pieces of Ribbon four Inches long each, the one blew and the other red, whose common Breadth was $\frac{3}{4}$ of an Inch; I caus'd it to be held in such manner, that the Light which fell from the Clouds thro' the Window was so reflected, that the Angle made by the Rays of Light, which came in at the Middle of the Window, with the Plane of the Ribbon produced, was equal to the Angle made by a Line drawn from the Ribbon to my Eye and the said Plain of the Ribbon. My Eye was plac'd as far behind the Ribbon as the Window was before it, the Distance from which to me was about 12 Feet. Then looking thro' a Prism at the Ribbon, it appear'd broken asunder in the Place where the blew and red Half join'd. If the Prism was held with the refracting Angle downwards (or laid with one of its Planes flat upon the Nose) the blew Half of the Ribbon appear'd to be carried down lower than the red, as at *B*, *R* in *Fig.* 1. but if the refracting Angle of the Prism was turn'd up-

wards (as when the Prism has one of its Planes laid flat to the Forehead) then the blew Half of the Ribbon was lifted up, as at 6 *p*.

The Prism was of white Glass, having every Angle of 60 Degrees: but when instead of it, one of a greenish sort of Glass, such as Object Glasses of Telescopes are made of, was used, having the refracting Angle which I look'd thro' of about 48 Degrees; the same Phænomenon was more distinct, this Glass having no Veins, but the Red and Blue were nearer to a streight Line: in such manner that if *A* represent the Ribbon seen through the first Prism, *B* will represent the Ribbon seen thro' the second Prism, *Fig.* 2. If the refracting Angle of the last Prism had been as great as that of the first, the Light being transmitted thro' too great a Body of greenish Glass, the Phænomenon would not have succeeded so well.

The blue Ribbon being somewhat too pale, and the red a little dull; I repeated the Experiment with a Skeen of blue, and one of red Worsted join'd together in the Middle as the Ribbons were before; and, the Colours of both being very intense, the Experiment succeeded better with both Prisms. All that were present trying the Experiment found it to succeed, and that every Circumstance answer'd to the Account given in *Prop.* 1. *Theor.* 1. *Book* 1. of Sir *Isaac Newton's Optics*, as far as the Directions there given were followed. So that it appear'd that the Blue being carried lower than the Red in the first case, and lifted higher in the second, was owing to the greater Refraction of the blue Ray: for tho' each Part of the Ribbon or Worsted reflected all manner of Rays, yet the Phænomenon was very apparent; as also that the blue Ribbon or Worsted reflected the blue Rays more copiously than the red Rays, and that the red Ribbon or Worsted reflected the red Rays more than the blue ones, because the Red of the blue Half seen thro' the Prism was less intense than

than that of the red Half, and the Blue or Purple of the red Half ſeen thro' the Priſm was leſs intenſe than that of the blue Half.

N. B. If the Ribbon or Worſted is laid upon any enlightned Body, the Phænomenon will not ſucceed ſo well; the Colours of the Body ſeen thro' the Priſm mixing with thoſe of the Ribbon or Worſted. Even a black Body will not do, if Light falls upon it: but there muſt be a black Cloath behind, in ſuch manner that no Light falling upon it can be reflected ſo as to diſturb the Phænomenon. And if a ſhort-ſighted Perſon looks through the Priſm, a concave Lens between his Eye and the Priſm will render the Phænomenon more diſtinct than it wou'd otherwiſe be.

EXPERIMENT II.

Some Days after, the Sun ſhining, I made two Holes *H*, *h*, in the Window Shut *S s*, of a darkned Room; thro' which letting the Suns Beams paſs, by means of two Priſms *A*, *B*, (one near each Hole) I open'd the Rays coming from the Sun into the two colour'd *Spectra* α, β, where the following Colours were very diſtinct, *viz.* Red, Orange, Yellow, Green, Blue, Purple and Violet. Now the Reaſon of their being more diſtinct than ordinary, was, that the Priſms which I made uſe of were made of the greeniſh Glaſs mentioned before; which is very free from thoſe Veins by which the Colours are too much thrown into one another, by the beſt white Priſms of the common ſort.

The forementioned colour'd *Spectra* being thrown into the Room, to the Diſtance of about 20 Feet from the Window where the Sun's Light came in, I caus'd a Piece of white Paper π, $\frac{3}{4}$ Inch broad and 5 Inches long, to be held within the refracted Rays, (at the Diſtance of 10 Feet from the Windows,) which produc'd theſe Colours in ſuch manner, that by turning the Priſms round their

Axes,

Axes, I cou'd make the red Ray of the *Spectrum*, made by the one Prism fall upon one half of the Paper, and the purple Ray of the *Spectrum* made by the other Prism fall upon the other Half; for the *Spectra* were both vertical, the Lines which terminated the long Sides of them towards each other just touching, as appears in *Fig.* 3. Then at the Distance of 9 Foot, looking thro' the Prism *C* at the Paper thus colour'd, the red Half appear'd very much separated from the Purple, the one seeming lifted up from the other; the Red or the Purple appearing the highest, according as the refracting Angle of the Prism was either held upwards or downwards. The Phænomenon is much more distinct this way than any other; for the Paper not only seems divided into two, when it is coloured by a red and a purple Ray, but also by a Red and Blue, (*Fig.*4.) by a red and a green Ray, (*Fig.*5.) or indeed by any two Colours that are different, how near soever their Places in the *Spectra* be to each other. The Halves of the Paper appear, when view'd thro' the Prism, to be farther from each other, when the Paper is ting'd with such Colours as are farther from each other in the Series of Colours in the *Spectrum*: and nearest, tho' still divided, when neighbouring Colours fall upon the Paper, as Yellow and Green, or a light and a deep Green. But the Paper appears no way divided, when colour'd with the Red of the two *Spectra*, (*Fig.* 6.) if those Reds are equally intense: and so of the other Colours.

EXPERIMENT III.

I held a Lens of about 3 Foot Radius at the Distance of Six Feet from the oblong Paper (on which a red and purple Ray falling, made it look half Red and half Purple) and I projected the Image of the said colour'd Paper at the Distance of about Six Foot on the other Side of the Lens, on

on a white Sheet of Paper; where it was obſerveable, that when the red Half was diſtinctly painted on the white Paper (which was known by the Edges of the Image being regularly terminated) then the blue Half of the Image was confus'd: but if the white Paper was brought about two Inches nearer to the Lens, the Image of the blue Half became diſtinct, and that of the red Half confus'd.

I try'd the Experiment with a Paper colour'd half red and half blue, the red with Carmine and the blew with Smalt, making the Candle to enlighten the Paper (the Room being otherwiſe dark) and the Experiment ſucceeded in the ſame manner. The Experiment thus made is the ſame that Sir *Iſaac Newton* gives an Account of, *Book* 1. *Part.* 1. *Theor.* 1. *of his Optics.* Only it is to be obſerv'd that when the oblong Paper is coloured with red and blue from the Priſms, the focal place, where the red part of the Image is diſtinct, is more diſtant from the place where the blue part of the Image is diſtinct, than when the Paper is colour'd with the Painter's Powders, and much more vivid.

The *7th* Figure ſhews the Projection of the Paper ting'd with the Rays; and *Fig.*8. the Projection of it when painted: where a black Thread is wrapp'd round the red and the blue part, that the Diſtinctneſs of the Image of the Thread may ſhew when the red or when the blue part of the Image of the Paper is moſt diſtinct.

N. B. When the Candle enlightens the painted Paper, ſet an opaque Body as *B* between the Candle and Lens; leſt the Image of the Candle being alſo projected ſhould diſturb the Experiment.

EXPERIMENT IV.

Having made an Hole of $\frac{1}{4}$ Inch Diameter in the Window-Shut of the darkned Room, I ſuffer'd a Sun-Beam to come

come into the Room, which I intercepted with a Prism at the Distance of 5 Inches from the Hole; and after its Refraction in passing thro' the Prism, I receiv'd it upon a Sheet of white Paper, where it was colour'd, making an oblong Image of the Sun or *Spectrum* of about 9 Inches in length and 2 in breadth, which Breadth was nearly equal to the Diameter of the round Image of the Sun received upon a Paper at the same Distance from the Hole, which here was 18 Foot. Or if the Sun be too high, a Looking-Glass being put in the room of the Prism will throw a white round *Spectrum* upon the Paper, which held at the said Distance of 18 Foot, will have its Diameter equal to the Breadth of the colour'd *Spectrum*.

The Colours of the *Spectrum* were these; Red, Orange, Yellow, Green, Blue, Purple and Violet, tho' the Violet was so faint in this as to be scarce perceivable. *See Fig.* 9.

N. B. The Axis of the Prism in this, and all the other Experiments hereafter mention'd must be perpendicular to the Ray that falls on it; and the Plane into which the Ray enters must be held in such a Position, that the Angle which such a Ray makes with that Plane when it enters, may be equal to the Angle made by the middle Line of those Rays which emerge after Refraction, on the other Side of the refracting Angle of the Prism, with the Plane out of which they emerge. That is $\angle BDG = \angle AEH$.

If the Plane AC, on which the Sun-Beam falls, be turned nearer to a perpendicular to the Sun Beam than before, the *Spectrum* will be much longer: if it be more inclin'd to the said Beam, the *Spectrum* will be shorter, and in both Cases less distinct. See the *Spectrum* DE and the *Spectrum* de in *Fig.* 10. and 11. where H, h, represents the Hole in the Window-Shut in each Case; AC, ac the Plane of the Prism on which the Rays enter; BC, bc that out of which they emerge; P, p the perpendicular, and C, c the refracting Angle. If

If the Plane *A C* be ſtill more oblique to *H F*, all the Light will be reflected, and there will be no colour'd Image or *Spectrum* made by Refraction at all. *Fig.* 12.

But if it be held ſo as to be more nearly perpendicular to the Sun Beam than in *Fig* 10. the whole Beam will indeed enter the Priſm; but meeting with *B C* the lower Surface of the Priſm, or rather the Surface of the Air contiguous to it, ſome of the Light will by the Plane *B C* be reflected to *d e*, paſſing almoſt perpendicularly thro' *A B*; and the reſt will emerge thro' *B C*, and by Refraction make the imperfect Spectrum *D E*. See *Fig.* 10.

If the Sun-Beam enter *A C* perpendicularly and in the middle of it, the Light will be all reflected as in *Fig.* 13. ſome of it by the Plane *B C* to *R*, and the reſt by the Plane *A B* to *ρ*. But if the Beam fall nearer to *A* (ſtill perpendicularly) it will all be reflected by the Plane *A B*; if nearer to *B*, it will be all reflected by the Plane *B C*.

In order therefore to have the colour'd *Spectrum* as it ought to be, care muſt be taken that the emerging coloured Light may make the ſame Angle with the Plane *B C*, as the immerging Light does with the Plane *A C*; that is, the Angle *A E H* muſt be equal to *B D G*, as was ſaid before, *Fig.* 9. which may alſo be ſeen on the enlightned Duſt in the Air. But the beſt way is to turn the Priſm on its Axis, and at the ſame time look at the colour'd *Spectrum*, which will riſe and fall and become longer or ſhorter as you turn the Priſm; and between the Aſcent and Deſcent of the Image, it will appear ſtationary: there ſtop the Priſm, and the Reflection will be ſuch as is required for all the Experiments hereafter mention'd.

In order to have the Priſm move freely on its Axis, and ſtop any where, I fix'd each End of it into a triangular Collar of Tin, from the End of which came a Wire, which was the Axis of the Priſm produc'd; and ſo I laid it on two wooden Pillars, with a Notch on the Top to receive

the Wires, and fix'd it to a ſmall Board juſt broad enough to ſtand faſt. See *Fig.* 14.

EXPERIMENT V.

I took the Priſm *C D*, and thro' it look'd at the coloured *Spectrum R P*, which appear'd then round and white as at *S*, juſt as if it had been the Sun's Light received on a Paper from the Hole *H*, and ſeen with the naked Eye. In this caſe the Priſm *C D* muſt be held *in directum* with *A B*, and the refracting Angles in the two Priſms muſt be equal. This *Spectrum* appearing white but juſt in one Point, is not ſo readily found; but the beſt way is to look thro' the ſame Priſm *A B* which makes the *Spectrum*, which may eaſily be done if it be pretty long, and then *R P* will be ſeen white and round, and as at *S*, as if coming directly from *H*. See Fig. 15.

EXPERIMENT VI.

I held a broad *Lens L l*, ground to a Radius of 2 $\frac{1}{2}$ Feet, in ſuch manner that the whole colour'd *Spectrum* fell upon it; and after Refraction all the Colours appear'd to converge, if receiv'd on a Paper at *p p*; but when the Paper was held in the Focus at *F* in the poſition π *F* π, the *Spectrum* was round and perfectly white by the Union of all the colour'd Rays. If the Paper was held at Π Π, the Colours appear'd to diverge from each other, but then the Red was uppermoſt, which before us'd to be the loweſt, and ſo on in an inverted Order.

I try'd the ſame Experiment with a *Lens* of one Foot Radius, with one of 9 Inches, and with another of 7, and the Succeſs was the ſame. See the 16*th* Figure, where the *R*, *O*, *Y*, *G*, *B*, *P*, *V*, expreſs the Colours.

N. B.

N. B. Care muſt be taken that the very end of the Red, and the Extremity of the Violet be taken in by the *Lens*; otherwiſe the *Spectrum* will not be perfectly white at the Glaſs's Focus.

There is no fix'd Diſtance of the Priſm from the Lens, but it ought to be brought ſo near the Priſm that the two Ends of the *Spectrum* may fall nearer the Axis of the Lens than the Edges of the Lens; becauſe there the Refraction is not ſo regular.

Behind the Lens *L*, which made the Colours converge into White at the diſtinct Baſe or Focus *F*, I plac'd the Lens *l*, which made the White be at *f* the diſtinct Baſe of the two Glaſſes combin'd; and the Experiment ſucceeded as before. Fig. 17.

When the Paper was held in the Focus of the Lens, ſo as to receive the white Image of the colour'd *Spectrum* projected by the Lens; if with a Card I intercepted the red Ray, the White appear'd ting'd with Purple, and if I intercepted the Violet or purple Ray, or both, the White appear'd ting'd with Red; and if the Red was intercepted at the ſame time, the *Spectrum* appear'd to be a Mixture of Yellow, Green and Blue. If any ſingle Colour was ſuffer'd to fall upon the Lens, the reſt being intercepted, that Colour wou'd continue the ſame; only it wou'd be more intenſe in the Focus of the Lens.

EXPERIMENT VII.

I took a Board (*Fig.* 18.) *q h s* which ſtood reclining on a Prop *t*, having an Hole of a Quarter of an Inch Diameter at *h*, and behind it a Priſm *B* ſupported on two Props, as above-mention'd, ſo as to turn eaſily about its Axis; and having ſet this Board on the Ground with the Priſm behind it at *B*; by turning the Priſm *A C* about its Axis, I firſt made the red Ray of the colour'd *Spectrum*

paſs thro' the Hole *h*, and fall obliquely upon the ſecond Priſm *B*. This Ray after its Refraction in paſſing thro' the ſecond Priſm, was carried up to the Ceiling of the Room at the place mark'd *R*: then I made the purple Ray fall upon the Board, and paſs thro' the Hole *h*, as the Red had done before; and after Refraction thro' the Priſm *B* it was carried up to the Ceiling at *P*. And the green Ray being afterwards made to paſs the ſecond Priſm in the ſame manner, went up to *G*: and ſo of all the inrermediate Rays, which were by this ſecond Refraction thrown to the intermediate places on the Ceiling between *R* and *P*.

Care is to be taken that the ſecond Priſm be plac'd oblique to the Rays which come thro' the Hole *h*, leaſt they be reflected, as they wou'd be, if the Board being in the Poſition *Q S*, and the ſecond Priſm in the Poſition *L N M*, the Ray from the firſt Priſm be *p h*; for then it will be reflected upwards to *σ* inſtead of being refracted (*Fig.* 19.). Neither muſt the Plane of Immerſion be too oblique, leaſt the Incident Ray be reflected downwards by it, as the Ray *R h* is by the Priſm *B* thrown to *E*, in *Fig.* 20. Several have confeſs'd to me that they at firſt us'd to fail in this Experiment, for want of ſetting the ſecond Priſm in a due Inclination.

Tho' the Colours by the ſecond Refraction on the Ceiling appear'd unchang'd, when ſeen by the naked Eye, yet if view'd thro' a Priſm, they afforded new Colours (except ſome part of the Red, and ſome part of the Violet); which was owing to their not being fully ſeparated; for which reaſon I made the following Experiment, to prove that if the Colours be well ſeparated, they are truly homogeneal and unchangeable.

N. B. When the Priſms are good, and no Clouds are near the Sun, the Extremity of the Red or Violet will afford unmix'd Colours in this Experiment; otherwiſe not.

EX-

EXPERIMENT VIII.

Having made a Hole in the Window-Shut 2 Inches wide (*Fig.* 21.) I applied to it a Tin Plate, which sliding up and down hid all this Hole in the Wood, and only transmitted a small Beam thro' it's own Hole *H*, whose Diameter was $= \frac{1}{16}$ Inch. This Beam, by means of the Looking Glass *L*, plac'd on the Board of the Window *XW*, I reflected horizontally to the other end of the Room. But to correct the Irregularity of the Reflection of the Looking-Glass, I made use of the Frame of Past-Board *Pp*, which had an Hole in it *h* of $\frac{1}{16}$ Inch likewise: and placing it at *Pp* I suffer'd some of the reflected Beams to pass thro' it, so as to fall upon the Lens *FE* (convex on both Sides, and ground to a Radius of $4\frac{1}{2}$ Feet) at the Distance of 9 Feet, so that the Image of the Hole *h* was projected to *f* on the other Side of the Glass, at the Distance of 9 Feet more. Just behind the Lens, which by a Screw in the Stand *S* might be rais'd or let down, so as always to receive the Beam along its Axis, I plac'd a Prism *A* (upright on one of its Ends and easily moveable about its Axis, by reason of its Wire turning freely in an Hole in the solid piece of Wood *T*, which stood on another Stand behind the Lens) as near as I cou'd to the Lens *EF*, so that the Image of *h* instead of being round, white, and projected to *f*, was cast sidewise on a white Paper stretch'd on a Frame, and appear'd colour'd, and 30 or 40 times its Breadth, as at *MN*. The Colours in this Case were very vivid and well separated, only the Violet had some pale Light darting from its End, upon account of some Veins in the Prism *A*, and the Light not coming directly from the Sun, but reflected; which ought not to have been, if the Sun had been low enough to have thrown the Rays a good way into the Room without the Help of a Looking Glass.

To

To ſhew that the Colours in this *Spectrum* were ſimple and homogeneal Lights, I made the following Experiments.

EXPERIMENT IX.

Having made an Hole *h* in the Paper which receiv'd the colour'd *Spectrum*, I ſuffer'd the red Light to paſs; which being refracted by a ſecond Priſm, fell upon another Paper at *T*, where it appear'd ſtill Red whether ſeen with the naked Eye or Priſms of different refracting Angles. To the Eye which ſaw it thro' the Priſm *V*, it appear'd indeed lower as at *t*, but red, round and unchang'd. I made the Experiment upon all the Colours, which by this means appear'd to be ſimple and homogeneal. See *Fig.* 22. Where the ſame Letters denote the Lens, Priſm and firſt Paper.

Thro' the ſame Lens and Priſm the *Spectrum* was made to fall on a Book; then thro' the Priſm *F* it appear'd unchang'd; and the Letters in the Book which croſs'd the *Spectrum*, were as diſtinct as when ſeen with the naked Eye. See *Fig.* 23.

N. B. The Axis of the Priſm *F* ought to be perpendicular to the long Axis of the *Spectrum s m* thrown on the Book, which will appear as at $\sigma\mu$; and the Priſm in the Poſition repreſented at *F*, with its flat Side towards the Noſe: for that is the moſt convenient Poſition for looking at the *Spectrum* in theſe Experiments.

I ſuffer'd the purple Ray only to paſs thro' the Hole *h* and fall upon a Book at *P*, the Letters of which appear'd at π, and were as diſtinct thro' the Priſm *Q* as when ſeen with the naked Eye: and I had the ſame Succeſs with all the other Rays. See *Fig* 24.

But if a Sun-Beam as *r* comes thro' the Hole *H* directly upon the Book at *W*, an Eye looking at it thro' a Priſm

at

at *X* will ſee this Beam at *Y* oblong and colour'd, and the Letters on which it falls, confus'd. See *Fig.* 24.

N. B. The Lens ought to be very good, without Veins or Blebs, and ground to no leſs a Radius than I mention'd in the Experiment; tho' a Radius of a Foot or two longer is not amiſs. The Priſm ought to be of the ſame Glaſs as the Object-Glaſſes of Teleſcopes, the white Glaſs, of which Priſms are uſually made, being commonly full of Veins. And the Room in theſe laſt Experiments ought to be very dark.

A few Days after, having got very good Priſms made for the purpoſe of the above mention'd Glaſs, I made all the Experiments over again before ſeveral Members of the *Royal Society* with better Succeſs; and had the *Spectrum* very regularly terminated, without any pale Light darting from the Ends of it.

For a further Account of Experiments to this purpoſe, ſee Sir Iſaac Newton's Opticks. B. 1. Part. 1. to which I might have referr'd the Reader altogether; but that I was willing to be particular in mentioning ſuch things as ought to be avoided in making the Experiments above-mention'd; ſome Gentlemen abroad having complained that they had not found the Experiments anſwer, for want of ſufficient Directions in Sir *Iſaac Newton's Opticks*; tho' I had no other Directions than what I found there.

III.

III. *A plain and easy Experiment to confirm Sir* Isaac Newton's *Doctrine of the different Refrangibility of the Rays of Light. By the same.*

AFter the *Experimentum Crucis* made by two Prisms, I shou'd not give the following Experiment, but that it is so easy to be made, that by it those who want the *Apparatus* (or are unwilling to be at the pains) to make the *Experimentum Crucis*, may at any time satisfy themselves of the Truth of the fore-mention'd Doctrine.

Let the Candle *A* be set before the Bar of a Chimney Looking-Glass, such as is represented by *H H* (*Fig.* 25) which is a Piece of Looking-Glass Plate consisting of four Planes, seen in the Section of it $\alpha f d \beta$, *viz.* $d \beta$ which is quick-silver'd behind, $f \alpha$ a Plane parallel to it, $f d$ one of the Side-planes bezell'd towards $d \beta$, or inclin'd to it in an Angle of about 40 Degrees (tho' from 30 to 40 will do, but the greater the Angle the better, if it does not exceed 45°.) $\alpha \beta$ the other Side-Plane inclin'd in the same Angle to βd.

The Rays of the Candle which come from *A* to γ fall obliquely on the Plane $\alpha \beta$, so that instead of going on to a, they are by Refraction made to incline more towards the Perpendicular $p p$, namely to go on in the Line γc, and then are reflected from the Point c on the quick-silver'd Surface, in the Direction $c \varkappa$, so as to make the Angle $\varkappa c d = \gamma c \beta$. Now as the Rays which wou'd go to $\varkappa$, if not refracted, emerge obliquely from the Plane $\alpha \beta$, they leave the Direction $c \varkappa$, and decline from the Perpendicular $\pi \pi$, and, being differently refracted, open into four differently colour'd Rays; *viz.* $b R$ a red Ray, $b T O$ a Ray made

made up of Orange and Yellow; *b G B* a Ray made up of Green and Blue or a Sea-Green, and *b P* a purple Ray.

If from the place *E e* you look full upon the Point *b*, the *Spectrum* or Image of the Candle at *b* will appear double, but not mix'd; that is, there will appear a Sea-green Spot and a red Spot, as it were, one upon another; but not so as to produce a mix'd or intermediate Colour. Then if the right Eye or Eye at *E* be shut, there will appear only a green Spot to the Eye at *e*; if the Eye at *e* be shut, the Eye at *E* will see only a red Spot.

If you come nearer to *b*, so that the Eyes at ε1, ε2 receive the most and the least refrangible Rays, there will be a double *Spectrum*, *viz.* a red and a purple one just touching, or upon one another: and the Phænomenon will answer as before. (*Fig.* 25.).

If keeping both Eyes open, you direct their Axes towards *O* a Point nearer than the usual place of the compound *Spectrum S*, (*Fig.* 26.) which Point is in a Line from the Nose *N* to the Point *S*; or in other Words, if you look full at *O*, or at the End of your Finger held in *O*, the red and the blue (or purple Spot) will appear to be divided from each other after the manner represented at *p r* (in *Fig.* 27) where the Red will appear to be on the Right-hand, and the Blue on the Left.

To make plain what is meant by *seeing* the *Spectra p* and *r* whilst we look full at *O*, I beg leave to explain the Distinction between *looking* and *seeing*; that I may the better shew how this Phænomenon proves that the Sensation of different Colours is caus'd by Rays differently refracted.

I. *Definition.*

The *Optic Axis* is a Line which going thro' the Center of the Convexity of all the Coats and Humours of the

Eye, falls upon the Middle of the *Retina*, as *a a* or *A a* *Fig.* 28.

II. *Definition.*

To *look* at any Point, is to turn both Eyes towards it in ſuch manner, that the *Optic Axes* making an Angle at the ſaid Point as *a*, the Rays from *a* may have the *Optic Axis* for their Axis, and (by their Convergence upon the *Retina* after Refraction in the *Eye*) may paint the Image of the ſaid Point upon the Middle of the *Retina* of each Eye, where the Optic Axis in each Eye falls.

III. *Definition.*

To *ſee* without *looking*, is to direct the Optic Axes to ſome other place than to the Point which is then ſeen; and in ſuch a caſe, the Image of the Point ſeen will be projected upon a part of the *Retina* of each Eye, where the Optic Axis does not fall, namely either nearer to the Noſe *N* as in (*Fig.* 26.) at the Points of the *Retina* mark'd *n n*; or farther from the Noſe than the Middle of the *Retina*, as at *o o* in *Fig.* 29.

Whatever is *ſeen*, by being *look'd* at with both Eyes, always appears ſingle, by reaſon of the Communication between the Middle of the *Retina* in one Eye, and the Middle of the *Retina* of the other: there being no ſuch Communication between any other part of the *Retina* in one Eye, and the Correſpondent part of the *Retina* in the other, when theſe correſpondent parts are equally diſtant from the Noſe.

There is indeed a Communication between the Nervous Fibres on the Right-ſide of the Retina *of one Eye, and the nervous Fibres on the Right-ſide of the* Retina *of the other Eye, and ſo of thoſe on the Left: but no ſingle Object can be ſo painted in each Eye, as to have its Image on the Right or Left Part of one*

one Retina *that communicates with the Right or Left part of the other, of the ſame bigneſs and at the ſame time as in the other; becauſe in whatever Poſition the Object is, it muſt be nearer to one Eye than to the other, except it be juſt in a Line from the Noſe betwixt the two Eyes ſtreight forward.*

Hence it is that if there be two Candles ſet before any one, the Firſt at the Diſtance of one Foot, and the Second at the Diſtance of two Feet, from the Eyes; he that looks at the ſecond Candle at *B* will ſee it ſingle, but ſee the firſt Candle or the Candle *A* double; one Appearance being in the Line *A D* γ, the other in *o A E*, becauſe it paints it ſelf upon *o o* in the Retina of each Eye, which Points are not the middle Points, but farther from the Noſe than the middles *m m*.

So if *B* be the firſt Candle, and *C* the ſecond, he that looks at *B* will ſee *C* double, becauſe it is painted in the *Retina* at the Points *n n* nearer the Noſe than *m m*; and ſo will appear to be in the ſame Poſition as *p r*. in *Fig*. 27.

If γ ρ be two Candles ſo diſpoſed, *Fig*. 30. that by the Interpoſition of a perforated Board *F F*, γ can paint it ſelf only in the Eye *R*, and ρ in the Eye *L*. Upon making the Optic Axes meet at *B* and to tend towards ρ and γ, ρ and γ will each paint an Image on the Middle of the *Retina* of each Eye, by croſſing their Rays at *B*: and thus the two Candles will appear to be but One, or rather to be in one Place, upon the account of the Communication of the Middle of each *Retina*. But if inſtead of the Candles, ρ be a piece of red Silk, and γ a piece of green Silk, the ſame Poſition of the Eyes will make an Image at *B*, appearing like a red and green Spot together without a Mixture of the Colours. If ρ be a red hot Iron, and γ a Candle of Sulphur, the Phænomenon will be more diſtinct. If the Optic Axes be turn'd directly towards γ and ρ, as if there was no Board *F F* in the way, there will appear two

Holes in the Board, the one having the red hot Iron in it, the other the Candle.

Now if, of the refracted Rays of the Candle in the first Case (*Fig.*25.) those which diverge from each other, so as to fall into each Eye, cause the same Sensations respectively, as the Rays which come from a red hot Iron and those which come from a blue Candle; it is evident that the Candle in the first case affords red-making and blue-making Rays after Refraction, and that those Rays are differently refrangible; the red *b R* (*Fig.* 25.) the least refrangible, as declining less from the Perpendicular ϖ ϖ; and the Purple as *b P* declining most from the said Perpendicular.

The same will (*cæteris paribus*) be found true in the intermediate Rays; and to be certain that the Experiment is as I have related it, the Planes α *f* and *f d* of the Barr may be covered with Paper.

IV. *An Account of what appear'd on opening the* big-belly'd Woman *near* Haman *in* Shropshire, *who was suppos'd to have continued many Years with Child. Communicated by Dr.* Hollings M.D. *from* Shrewsbury.

A Marry'd Woman, near *Haman* about Three Miles from *Shrewsbury*, about the 40*th* Year of her Age, had then first the common Reasons to believe she was with Child: at the Time of her Account she had the usual Signs of Labour, and a good Midwife, tho' mistaken, assur'd her it was so, but that the Child was so big she could not be delivered without bringing it away in pieces. She not submitting to that, her Pains went soon off, and she continued without any other Disorders Nine Months longer, when

when ſhe had again the Signs of Labour; and the ſame Midwife aſſur'd her as before, and ſhe perſiſting in her former Reſolution, her Pains, after a Day or two went off. Soon after her Belly ſwell'd to a ſurprizing Size, by which ſhe got Subſiſtance for her Family by being ſeen as a Shew. I ſaw her firſt above twenty Years ſince, when her Belly was almoſt even with her Chin, the Weight of it ſo great, that ſhe was oblig'd to ſupport it with a Stool. She could not ſtand without the help of a Rope from the Cieling, which aſſiſted her in changing her Poſture of ſitting She ſlept commonly with her Arms folded on her Belly, and her Head reſted between them. She had no ſwelling in her Legs: every other Part emaciated as uſual in the like Caſes. Thus this poor Creature liv'd without any other conſiderable Complaint above Thirty Years, the moſt remarkable Circumſtance, I think, in her Caſe. She died in *May* 1715, when this appear'd to be an *Aſcites*.

I need not mention the State the common Teguments muſt neceſſarily be in from ſo great a Diſtention, which had diſtorted many of her Ribs, and forc'd the Diaphragm ſo high, that it was ſurprizing to find her breathing could be ſo long continu'd. The Water was all contain'd in the Duplicature of the *Peritonæum*, 13 Gallons beſides a Quart that was ſpilt: it was ſaltiſh, with ſome little fat upon it, and towards the latter Running ting'd with Blood as uſual. There was not any Water in the Cavity of the *Abdomen*, except what was contain'd in a kind of Bladder of the Shape I have ſent, *Fig.* 31. which lay a-croſs the *Fundus Uteri*. This was divided by a Cartilaginous Subſtance into two Cavities; in one there was a Pint and a half, in the other three Parts of a Pint of Water. I believe it was this (I know not how) impos'd on the Midwife. The *Uterus* was of the natural Size without any Alteration, except that the *Os Tincæ* and *Collum minus* were fill'd with a gritty Subſtance, hard as Stone, which I take to be the Humour

ſepa-

ſeparated there, and coagulated by Time. Mr. *Cooper Tab.* 15. *Fig.* 4. ſays he found the ſame Parts fill'd with a glutinous Matter, which he thinks is uſeful to prevent Abortion; which if vitiated, Impregnation is hinder'd.

The Liver and other parts contain'd in the *Abdomen*, were forc'd into an incredible ſmall Compaſs (and by that Preſſure a little chang'd in Shape) to perform their Office ſo long; to which the Muſcles of the *Abdomen*, diſtended ſo as to be ſcarce diſcernible, could give but little, if any, Aſſiſtance.

The Awe that People have here for dead Bodies, tho' never ſo prejudicial to the Living, would not ſuffer her Friends to let me make any farther Enquiry; ſo that I can ſend no Account of any other Part. The ſame Error hindered me examining another Woman, who died here about a Week after, of an *Aſcites* which ſhe had had Forty Years, any farther than to be ſatisfied ſhe had Seven Gallons of Water contain'd between the Duplicatures of the *Peritonæum*, and none in the Cavity of the *Abdomen*.

IV. *Methodus ſingularis quâ* Solis *Parallaxis ſive diſtantia à* Terra, *ope* Veneris *intra* Solem *conſpiciendæ, tuto determinari poterit: propoſita coram* Regia Societate *ab* Edm. Halleio J. U. D. *ejuſdem Societatis Secretario.*

PLurima ſunt maxime quidem paradoxa, omnemque fidem apud vulgus ſuperantia, quæ tamen adhibitis Mathematicarum Scientiarum principiis levi negotio enodantur. Ac ſane nullum problema magis arduum ac difficile videbitur, quam eſt *Solis* à *Terra* diſtantiam vero proximam determinare; quod tamen obtentis accuratis qui-

quibusdam observationibus, ad electa & prævisa tempora peractis, non multo opere efficietur. Id quod inclytæ huic Societati, quam immortalem fore auguror, in hac dissertatione ob oculos ponere libet, ut junioribus nostris Astronomis, quibus forsan hæc observare ob minorem ætatem obtingere potest, viam præmonstrem, quâ immensam Solis distantiam intra quingentesimam sui partem rite dimetiri poterint.

Notum autem vobis est hanc distantiam à diversis Astronomiæ authoribus diversam fingi, prout cuique ex conjecturâ probabile visum est: à *Ptolemæo* quidem ejusque asseclis, uti & *Copernico* & *Tychoni Brahæo*, *Terræ* semidiametris mille & ducentis, *Keplero* ter mille quingentis fere. *Ricciolus* distantiam *Keplerianam* duplicat, quam tamen *Hevelius* dimidio tantum auget. At vero visis in *Solis* disco ope Telescopii Planetis *Venere* & *Mercurio* mutuato fulgore nudatis, tandem compertum est Planetarum diametros visibiles multo minores esse quam eatenus haberentur; *Venerisque* Semidiametrum è *Sole* visam, non nisi quartam minuti primi partem vel quindecim secunda subtendere; *Mercuriique* semidiametrum, ad mediam ipsius à *Sole* distantiam, sub angulo decem tantum secundorum conspici; atque sub eodem etiam *Saturni* semidiametrum è *Sole* videri. *Jovis* autem Planetarum maximi semidiametrum non nisi tertiam minuti primi partem apud *Solem* subtendere. Unde, servatâ analogiâ, nonnullis è modernis Astronomis visum est, *Terræ* quoque semidiametrum è *Sole* conspectam, medio loco inter *Jovis* majorem & *Saturni* & *Mercurii* minorem angulum subtendere, *Venerisque* æqualem, nempe quindecim secundorum: adeoque *Solem* à *Terra* quatuordecim fere millibus semidiametrorum *Terræ* distare. Iisdem autem Authoribus, aliud argumentum paulo ampliavit hanc distantiam: quoniam enim *Lunæ* diameter paulo major est quartâ parte diametri *Terræ*, si Parallaxis *Solis* ponatur quindecim minutorum secundorum, fieret

Lunæ

Lunæ corpus corpore *Mercurii* majus, Planeta ſcil. ſecundarius primario major ; quod concinnitati Syſtematis mundani contrariari videretur. E contra vero *Venerem* inferiorem & Satellitio deſtitutam, majorem eſſe *Terrâ* noſtrâ ſuperiori & tam inſignem comitem nactâ, vix concedere videtur eadem concinnitas. Ut itaque medio loco incedamus, ponatur *Terræ* ſemidiameter è *Sole* viſa, ſeu quod idem eſt, *Solis* Parallaxis horizontalis, duodecim ſecundorum cum ſemiſſe: unde *Luna* minor erit *Mercurio* & *Terra Venere* major ; ac proveniet *Solis* a *Terra* diſtantia ſedecies mille cum quingentis *Terræ* ſemidiametris proxime. Huic autem diſtantiæ in præſentiarum aſſenſum præbeo, uſq; dum Experimento quod proponimus quanta ſit certius conſtet. Nec moror authoritatem quantumvis gravem eorum qui *Solem* ultra hos terminos in immenſum evehunt, freti obſervationibus vibrantis Penduli, determinandis his angulorum minutiis, uti videtur, haud ſatis fidis: ſaltem hâc methodo tentanti Parallaxis aliquando nulla, aliquando etiam negativa occurret ; hoc eſt diſtantia vel infinita fiet, vel infinito major : quod abſurdum. Et, ut verum fatear, minuta ſecunda vel etiam dena ſecunda inſtrumentis quantumvis affabre factis certo diſtinguere vix homini datum eſt ; atque adeo minime mirandum, ſi tantorum Artifitum multos & ingenioſos conatus hactenus eluſerit rei ipſius maxima ſubtilitas.

Dum autem, ante 40 fere annos, in Inſula *Sanctæ Helenæ*, ſyderum polum Auſtralem ambientium obſervationibus operam darem ; contigit mihi *Mercurium* ſub *Solis* diſco tranſeuntem omni adhibitâ diligentiâ obſervare: quodque mihi præter ſpem feliciter ſucceſſit, momentum quo *Mercurius* ingrediens *Solis* limbum interius contingere viſus eſt, pariterque momentum quo egrediens limbum Solis ſtrinxit, facto angulo contactûs interioris, Tubo optimo viginti quatuor pedum accuratiſſime obtinui. Unde pro comperto habui intervallum quo *Mercurius* totus intra *Solis*

diſcum

diſcum tum temporis apparuit, etiam abſque errore unius minuti ſecundi temporis: Nam filum luminis Solaris, inter limbum planetæ obſcurum & *Solis* lucidum interceptum, quantumvis tenue in oculos incurrere viſum eſt; & in ictu oculi, denticulus in limbo *Solis* a *Mercurio* ingrediente factus evaneſcere, uti ab egrediente factus quaſi momento incipere. Hoc autem perſpecto ſtatim intellexi *Solis* Parallaxin ex hujuſmodi obſervationibus rite concludi poſſe, ſi modo *Mercurius Terris* vicinior majorem haberet parallaxin à *Sole*; etenim hæc parallaxium differentia tantilla eſt, ut ſemper minor ſit ipsâ Solari quam quærimus; proinde *Mercurius*, licet frequenter intra *Solem* videndus, huic noſtro negotio vix ſatis aptus habebitur.

Reſtat itaque *Veneris* tranſitus per *Solis* diſcum, cujus parallaxis quadruplo fere major Solari, maxime ſenſibiles efficiet differentias, inter ſpatia temporis quibus *Venus Solem* perambulare videbitur, in diverſis *Terræ* noſtræ regionibus. Ex his autem differentiis debito modo obſervatis, dico determinari poſſe *Solis* parallaxin etiam intra ſcrupuli ſecundi exiguam partem. Neque alia inſtrumenta poſtulamus præter *Teleſcopia* & *Horologia* vulgaria ſed bona: & in Obſervatoribus non niſi fides & diligentia, cum modica rerum Aſtronomicarum peritia deſiderantur. Non enim opus eſt ut Latitudo Loci ſcrupuloſè inquiratur, nec ut Horæ ipſæ reſpectu meridiani accurate determinentur: ſufficit, Horologiis ad Cæli revolutiones probe correctis, ſi numerentur tempora à totali Ingreſſu *Veneris* infra diſcum *Solis*, ad principium Egreſſus ex eodem; cum ſcilicet primum incipiat Globus *Veneris* opacus limbum *Solis* lucidum attingere; quæ quidem momenta, propria experientia novi, ad ipſum ſecundum temporis minutum obſervari poſſe.

Ob leges autem motuum admodum arctas, rariſſime intra *Solis* orbem conſpicitur *Venus*, ac per plus quam centum & viginti annorum decurſum, ne ſemel

 quidem

quidem ibidem videbitur; nempe ab anno 1639. (cum præclaro Juveni *Horroxio* nostro, eique primo & soli à rerum conditu, jucundissimum hoc spectaculum obtigit,) usquè in annum 1761, quo juxta Theorias quas hactenus cœlo conformes experimur, Stella *Veneris* iterum subtercurret *Solem, Maii* 26. mane; * ita ut *Londini*, horâ fere sextâ matutinâ in medio disci Solaris expectanda sit, nec nisi quatuor minutis centro *Solis* Australior. Duratio autem hujus transitus erit octo fere Horarum, nempe à secunda usque in decimam fere matutinam. Atque adeo ingressus minime *Anglis* conspicuus erit: cum autem Sol tum temporis occupaturus sit 16. *Geminorum* gradum, viginti tres ferme gradus in Boream declinans; per totam quasi Zonam frigidam Septentrionalem inocciduus conspicietur: ac proinde qui littus *Norwegiæ* incolunt ultra Urbem *Nidrosiam*, quam *Drontem* vocant, usque ad Promontorium ejus *Boreale*, *Venerem Solis* discum subingredientem observare poterunt; ac fortasse *Scotis* Borealioribus & Insulæ *Hetlandiæ*, olim *Thylen* dictæ, incolis, in oriente Sole ingressus ille conspici poterit. Quo tempore vero *Venus Solis* centro proxima erit, *Sol* verticalis erit supra littora Borealia sinus *Gangetici*, vel potius regni *Peguani*; ac proinde in Regionibus circumvicinis, cum *Sol* in ingressu *Veneris* quatuor fere horis distabit ad ortum, & in egressu totidem fere ad occasum, accelerabitur motus apparens *Veneris* intra *Solem* duplo fere parallaxeos horizontalis *Veneris* à *Sole*; quia *Venus* tunc ab ortu in occasum fertur retrogradè, interea dum oculus ad *Terræ* superficiem positus in contrarias partes ab occasu in ortum gyratur.

Positâ autem parallaxi *Solis*, uti diximus, duodecim secundorum cum semisse, erit parallaxis *Veneris* 43um secundorum; & sublatâ parallaxi *Solis*, restabit saltem semiminutum pro parallaxi Horizontali *Veneris* à *Sole*, ac proinde dodrante saltem minuti promovebitur *Veneris* motus à parallaxi illa, interea dum *Solis* discum percurrit, in iis scili-

* Vide *Phil. Transact.* No. 193.

ſcilicet Poli altitudinibus quæ Tropico vicinæ ſunt; atque adhuc amplius in vicinia Æquatoris. *Venus* autem tum temporis ſatis accurate quatuor minuta prima ſingulis horis intra *Solem* conficiet; ac propterea dodranti minuti undecim ſaltem temporis minuta prima competunt, quibus duratio *Eclipſeos* hujus *Venereæ* ob parallaxin contrahetur. Atque ex hac contractione ſolâ liceret de parallaxi quam quærimus tutò pronunciare, ſi modo darentur *Solis* diameter *Veneriſque* Latitudo in minimis accuratæ; quas tamen ad computum poſtulare, in re tam ſubtili, haud integrum eſt.

Procuranda eſt igitur alia obſervatio, ſi fieri poſſit, in locis illis ubi medium *Solis* occupat *Venus* in ipſo Medinoctio; nempe ſub Meridiano priori oppoſito, *i. e.* ſex quaſi horis vel 90 gradibus *Londino* occidentaliore, & ubi *Venus* paulo ante occaſum *Solem* ſubintrat, paulo poſt ortum, exit; id quod fiet in dicto Meridiano, ſub altitudine Poli Borei quinquaginta ſex circiter graduum: hoc eſt, in eo Sinu qui *Hudſoni* dicitur, ad Portum ejus cui nomen *Nelſoni* inditum. In locis enim huic circumvicinis parallaxis *Veneris* durationem tranſitus protraher, & ſex ſaltem temporis minutis longiorem efficiet; quia dum *Sol* ab occaſu in ortum ſub Polo tendere videtur, ea loca in diſco *Terræ*, motu contrario in occaſum ferri videbuntur, hoc eſt motu cum motu proprio *Veneris* conſpirante; proinde tardius moveri videbitur *Venus* intra *Solem*, ac cum diuturniore mora diſcum ejus pertranſire.

Si itaque in utroque loco hic tranſitus ab Artificibus idoneis contigerit debite obſervari, manifeſtum eſt totis ſeptendecim minutis longiorem futuram eſſe moram in portu *Nelſoni* obſervabilem, quam quæ apud *Indos* orientales expectanda eſt: nec multum refert an ad Fortalitium *Sancti Georgii* vulgo *Maderás* dictum, vel ad *Bencoulam* in litore occiduo Inſulæ *Sumatræ* prope æquatorem capiatur obſervatio, ſi *Anglis* tum temporis hæc ſtudia curæ fu-

erint. Si vero *Gallis* his rebus invigilare placuerit, non incommode apud *Poudechery* se sistet Observator in litore *Sinus Gangetici* occidentali, sub altitudine Poli duodecim fere graduum. *Batavis* autem celeberrimum *Bataviæ* suæ Emporium Observatorium huic negotio satis aptum ministrat, si modo illis etiam animus fuerit hac in parte cœlorum scientiam promovere. Ac sane vellem diversis in locis ejusdem Phænomeni observationes à pluribus institui, tum ad majorem adstruendam ex consensu fidem, tum ne Nubium interventu frustraretur singularis Spectator, eo spectaculo quod nescio an denuo visuri sunt hujus & subsequentis seculi Mortales; & à quo pendet Problematis nobilissimi & aliunde inaccessi solutio certa & adæquata. Curiosis igitur syderum scrutatoribus, quibus, nobis vita functis, hæc observanda reservantur, iterum iterumque commendamus ut, moniti hujus nostri memores, observationi peragendæ strenue totisque viribus incumbant; iisque fausta omnia exoptamus & vovemus, præprimis ne nubili cœli importuna obscuritate exoptatissimo spectaculo priventur; utque tandem Orbium cœlestium magnitudines intra arctiores limites coercitæ in eorum gloriam famamque sempiternam cedant.

Diximus autem hac ratione Solis Parallaxin intra quingentesimam sui partem investigari posse, id quod nonnullis mirum sine dubio videbitur. Veruntamen si in utroque è locis nuper designatis accurata habeatur observatio; jam monstravimus, totis septendecim minutis differre inter se durationes *Eclipseων* harum *Venerearum*, ex Hypothesi scilicet quod Solis parallaxis fuerit duodecim cum dimidio minutorum secundorum. Quod si major vel minor reperiatur ex observatione hæc differentia, in eadem fere ratione major vel minor erit Solis parallaxis. Cumque 17 minuta prima temporis competant duodecim secundis cum dimidio parallaxeos Solaris; pro unoquoque parallaxeos minuto secundo, orietur differentia plusquam 80 secundorum

dorum minutorum temporis; adeoque ſi habeatur differentia hæc intra bina ſecunda vera & comprobata, intra quadrageſimam partem unius ſecundi minuti conſtabit quanta ſit Solis Parallaxis; ac proinde diſtantia ejus determinabitur intra quingenteſimam ſui partem, ſaltem ſi parallaxis non minor reperiatur eâ quam ſuppoſuimus: quadragies enim duodecim cum dimidio fiunt quingenti.

Hactenus Aſtronomice doctis ſatis ſuperque rem indicaſſe mihi videor, quos etiam monitos velim, me in hoc argumento, Latitudinis Planetæ rationem non habuiſſe, tum ad vitandas calculi intricatioris moleſtias, concluſionem etiam minus evidentem reddituras; tum ob motum Nodorum *Veneris* nondum compertum, nec niſi ex hujuſmodi corporalibus Planetæ cum *Sole* Conjunctionibus rite determinandum. Non enim concluſum eſt *Venerem* quatuor minuta infra *Solis* centrum tranſituram, niſi ex Hypotheſi quod Planum Orbitæ *Veneris*, in Sphæra ſtellarum fixarum immobile, Nodos ſuos iiſdem in locis habiturum ſit, ubi anno 1639 inventi ſunt. Quod ſi tramite Auſtraliori tranſeat anno 1761, liquido patebit Nodos regredi; ſi vero Borealiori, progredi inter Fixas; idque in ratione $5\frac{1}{2}$ *min.* in centum annis *Julianis*, pro unoquoque minuto, quo via *Veneris* tum temporis plus vel minus diſtabit à *Solis* centro quam dictis quatuor minutis. Differentia autem inter durationes harum Eclipſium paulò minor fiet ſeptendecim minutis, ob Latitudinem *Veneris* Auſtralem; major vero futura, ſi, procedentibus Nodis, ad Boream centri *Solem* tranſierit.

In eorum autem gratiam, qui cum obſervandis ſyderibus oblectentur, nondum tamen integram Parallaxium doctrinam hauſerint, libet Schemate ſimulque Calculo paulo accuratiore, rem plenius exponere.

Ponamus igitur, anno 1761, *Maii* 25°. 17^{h}. 55′. *Londini*, *Solem* occupaturum ♊ 15°. 37′. ac proinde ad centrum ejus Eclipticam tendere in Boream angulo 6°. 10′.

Veneris

Veneris autem visibilem intra *Solis* discum Viam tum temporis descendere in Austrum, facto angulo cum Ecliptica 8°. 28′: proinde via *Veneris* tendet parum in Austrum respectu æquatoris, intersecans declinationis parallelos angulo 2°. 18′. Ponamus etiam *Venerem* ad dictum tempus *Solis* centro proximam fore, ac ab eodem quatuor minutis distare ad Austrum; singulisque horis etiam quatuor minuta prima intra *Solem* motu retrogrado describere. Erit autem *Solis* Semidiameter 15′. 51″. proxime, *Veneris* vero 0′. 37″ $\frac{1}{2}$. Ac supponamus, experimenti gratia, differentiam parallaxium Horizontalium *Veneris* & *Solis*, quam quærimus, 0′. 31″ esse, qualis ex supposita *Solis* Parallaxi 0′. 12″ $\frac{1}{2}$ elicitur. Describatur itaque (*Fig.* II.) centro *C* circellus *A E B D*, cujus semidiameter sit 0′. 31″. discum Terræ repræsentans, & in eo Ellipses parallelorum 22 & 56 *grad.* Latitudinis Borealis, modo jam ad construendas Eclipses Solares ab Astronomis usitato, ut *D a b E*, *c d e*: sit autem *B C A* Meridianus in quo *Sol*; ad quem inclinetur recta *F H G* Viam *Veneris* designans angulo 2°. 18′, quæque distet à centro *C* 240 partibus qualium *B C* est 31; & de *C* cadat recta *C H* ipsi *F G* perpendicularis. Ac posito planeta in *H* ad 17 h. 55′, vel 5 h. 55′ mane, dividatur recta *F H G* in spatia Horaria III. IV, IV. V, V. VI, *&c.* ipsi *C H*, hoc est quatuor minutis æqualia. Fiat etiam recta *K L*, æqualis differentiæ apparentium Semidiametrorum *Solis* & *Veneris* sive 15′. 13″ $\frac{1}{2}$. Et Circulus radio *K L*, centro vero quolibet puncto intra circellum Disci *Terræ* descriptus, occurret rectæ *F G* in puncto denotante quota hora *Londini* numerabitur, cum in eo *Terræ* superficiei loco, qui sumpto in disco puncto subjacet, *Venus* angulo contactus interioris *Solis* limbum continget. Ac si centro *C* radio *K L* descriptus circulus occurrat ipsi *F G* in punctis *F* & *G* erunt recte *F H*, *H G* = 14′. 41″, id quod percurrere videbitur Venus tribus horis cum 40 min. Cadet igitur *F* in

F in IIh. 15′, *Londini*; *G* vero in IXh 35′ mane. Unde manifestum est quod, si Terræ magnitudo, ob immensam distantiam, quasi in punctum evanesceret; vel si motu diurno destituta *Solem* haberet eidem puncto *C* semper verticalem, Eclipseos hujus Mora integra per septem horas cum triente duraret. Verum Terrâ interea motu motui *Veneris* contrario gyratâ per 110 *grad.* Longitudinis suæ, ac proinde contractâ dictæ moræ duratione, puta 12 min. proveniet ea 7h. 8′. proximè, sive 107 *grad.*

Jam in ipso Meridiano *Venus Solis* centro proxima erit ad Ostium orientale fluminis *Gangis*, ubi poli altitudo est 22 *grad.* circiter. Locus igitur ille utrinque æqualiter distabit à Sole, in momentis introitus & exitus planetæ, nempe 53° ½ *grad.* ut sunt puncta *a*, *b*, in parallelo majore *D a b E*. Erit autem Diameter *A B* ad distantiam *a b* ut quadratum Radii ad contentum sub Sinubus 53 ½ & 68° *grad.* hoc est, ut 1′. 02″ ad 0′. 46″. 13‴; ac calculo rite instituto (quem ne Lectori tædio sit, omittere præstat) invenio quod circulus centro *a* & radio *K L* descriptus occurret rectæ *F H*, in puncto *M*, ad IIh. 20′. 40″; centro vero *b* descriptus occurret ipsi *H G* in *N*, ad IXh 29′ 22″; horis scilicet *Londini* numeratis; proinde tota *Venus* intra *Solem* conspicietur ad *Gangis* ripas, per 7h. 8′. 42″. Rectè igitur posuimus durationem fore 7h. 8′; cum pars minuti hic nullius sit momenti.

Aptato autem calculo ad *Portum Nelsoni*, invenio, quod Sole jamjam occasuro, discum ejus subitura sit Venus; statim vero ab ortu ejus exitura ab eodem; Loco illo interea per Hemisphærium à Sole aversum de *c* ad *d* translato, motu motui *Veneris* conspirante. Mora igitur *Veneris* intra *Solem* diuturnior fiet ob Parallaxin, puta quatuor minutis; ut sit omnino 7h. 24′. sive 111 *grad.* æquatoris. Cumque Latitudo Loci sit 56 *gr*, erit ut Quadratum Radii ad contentum sub Sinubus 55 ½ & 34 *grad.* ita *A B* = 1′, 02″ ad *c d* = 28″. 33‴. Ac calculo rite peracto

racto constabit, circulum centro *c* radio *K L* descriptum rectæ *F H* occursurum in *O*, ad IIh. 12′ 45″, centro vero *d* descriptum ipsi *H G* in *P*, ad IXh. 36″. 37″. Quocirca duratio Moræ ad *Nelsoni* portum erit 7h. 23′. 52″; major scilicet quam ad ostia *Gangis* totis 15′. 10″ temporis. Quod si *Venus* absque Latitudine transierit, fiet dicta differentia 18′. 40″; Si vero quatuor minutis *Solis* centro fuerit Borealior, ad 21′. 40″ augebitur eadem differentia, multo major futura auctâ Planetæ Latitudine Borea.

Londini autem, ex prædictis Hypothesibus, consequitur *Venerem* jam tum infra *Solem* ingressam orituram; & ad 9h. 37′ mane, in Egressu *Solis* limbum interius contacturam; ac denique non nisi horâ 9h. 56′, orbem ejus integrum relicturam esse.

Iisdem etiam Hypothesibus constat *Venerem* extremum *Solis* limbum Boreum quasi centro suo stringere debere, Anno 1769, *Maii*, 23°. 11h. 00′, ita ut, ob Parallaxin, in Borealibus *Norwegiæ* partibus, tota intra *Solem* inocciduum apparere poterit: dum in litoribus *Peruviæ* & *Chili*, vix exiguo sui segmento cadentis *Solis* disco quasi inequitare videbitur; uti in Insulis *Moluccis* earumque viciniâ, oriente *Sole*. Quod si Nodi *Veneris* retrocedere reperiantur (ut ob nuperas quasdam observationes suspicio est) tum toto corpore intra orbem *Solis* ubique conspicua, maximâ harum Eclipseωn differentiâ argumentum Parallaxeos *Solaris* præbebit adhuc multo luculentius.

Quomodo autem ex observatis alicubi apud *Indos* Orientales, anno 1761, Ingressu & Egressu *Veneris*, & cum Exitu ejus apud Nos observabili collatis, eadem Parallaxis derivari poterit; aptando scilicet angulos Trianguli specie dati in trium Circulorum æqualium circumferentias, alia occasione docebitur.

F I N I S.

helmo *Musgrave* G. F. è Coll. *Exon Oxonii* : Typis *L. Lichfield*: Impensis *Phil. Teo*, Bibliopolæ *Exon.* Anno MDCCXVI.

I. *An*

Fig. 1.

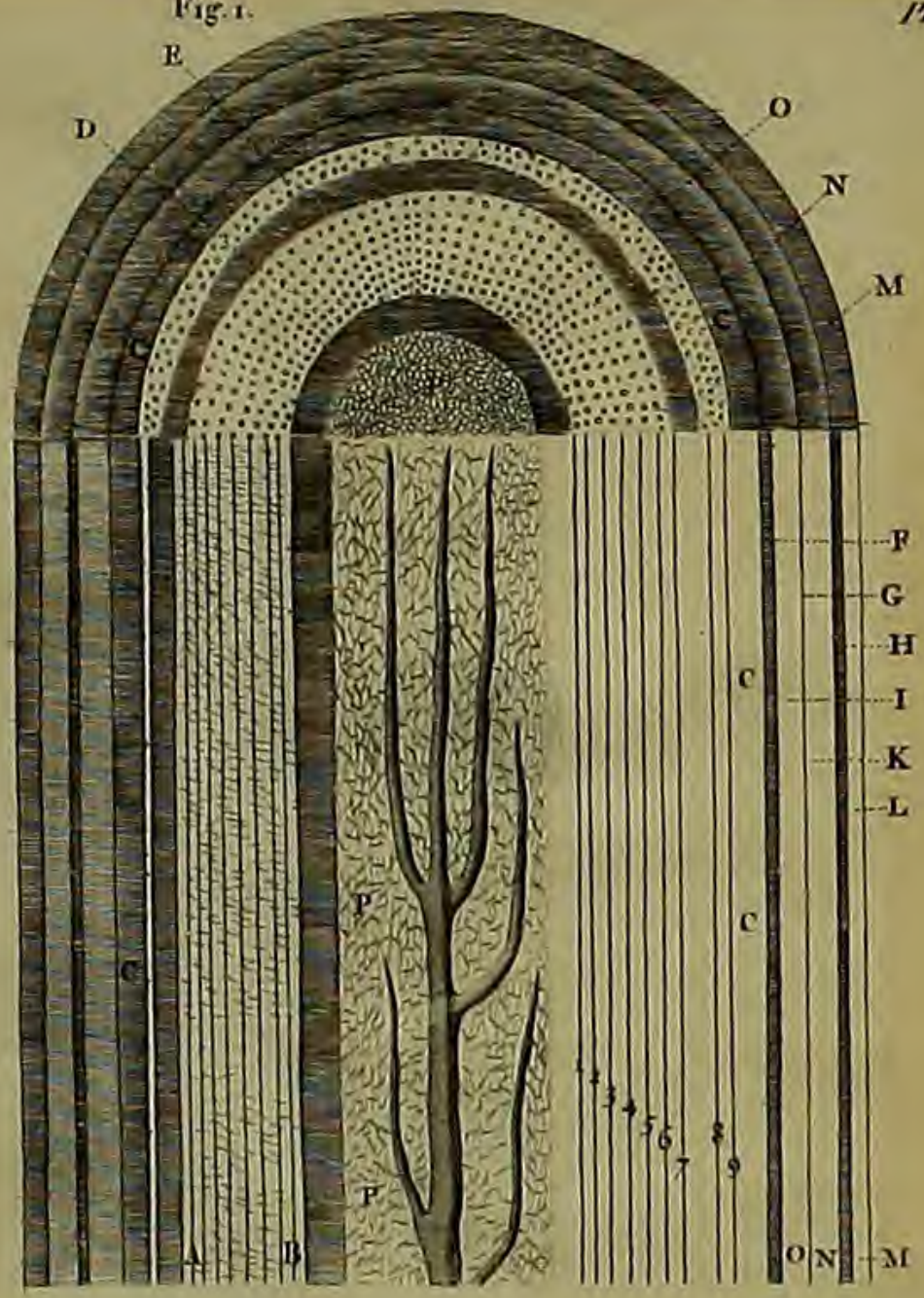


Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *July*, *August* and *September*, 1716.

The CONTENTS.

I. *An Account of the Cause of the late remarkable Appearance of the Planet* Venus, *seen this Summer, for many Days together, in the Day time.* *By* Edm. Halley, *R. S. Secr.*

IT may justly be reckoned one of the principal Uses of the Mathematical Sciences, that they are in many Cases able to prevent the Superstition of the unskilful Vulgar; and by shewing the genuine Causes of rare Appearances, to deliver them from the vain apprehensions they are apt to entertain of what they call *Prodigies*; which sometimes, by the Artifices of designing Men, have been made use of to very evil purposes.

Of this kind was the late Appearance of *Venus* in the Day time, generally taken notice of about *London* and elsewhere; and by some reckoned to be Prodigious. This put me upon the enquiry, how it came to pass that at that time the *Planet* should be so plainly seen by Day, whereas she rarely shews her self so, unless to those who know exactly where to look for her. To resolve this, the following Problem arose, *viz.* To find the Situation of the *Planet* in respect of the *Earth*, when the *Area* of the illuminated part of her Disk is a *Maximum*.

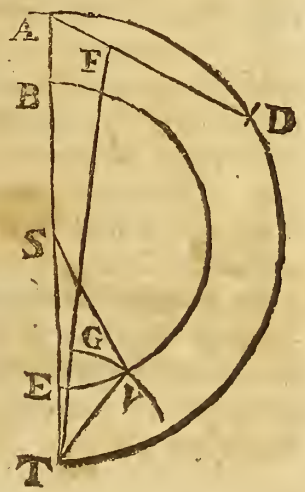
To investigate this *Maximum*, I found it requisite to assume the following *Lemmata*. I. That the visible *Areas* of the Disk of the same *Planet*, at differing Distances, are always reciprocally as the Squares of those Distances; which is evident from the first Principles of *Opticks*. II. That the *Area* of the whole Disk of the *Planet* is to the *Area* of the illuminated Part thereof, as the Diameter of a Circle to the Versed-Sine of the exteriour Angle at the *Planet*, in the Triangle at whose Angles are the *Sun*, *Earth*, and *Planet*. III. That in all plain Triangles, four times the Rectangle of the Sides containing any Angle, is to the excess of the Square of the Sum of the Sides above the Square of the Base, as the Diameter is to the Versed-Sine of the

the Complement of the contained Angle to a Semicircle, which I call the exteriour Angle: This is a new *Theorem* of good use in *Trigonometry*, and easily proved from the 12*th* and 13*th* of the II. *Elem. Euclid.*

This premised, putting m for the Distance of the *Sun* and *Earth*, and n for that of the *Sun* and *Venus*, and x for the Distance of the *Earth* and *Venus*, or the third Side of the Triangle which we seek; by the third *Lemma*, $4nx$, will be to the excess of the Square of $n+x$ above the Square of m, as the *Area* of the whole Disk of *Venus* to the *Area* of the part illuminated; and by the first *Lemma*, the *Area*'s of her whole Disk are at all times as the Squares of x reciprocally; whence the Quantity $\frac{nn+2nx+xx-mm}{4nx^3}$ will in all Cases be proportional to the *Area* of the illuminated part.

Now that this should be a *Maximum*, it is required that the Fluxion thereof be equal to 0, or that the Negative parts thereof be equal to the Affirmative, that is, that $\overline{2n\dot{x}+2x\dot{x}} \times 4nx^3 = 12nx^2\dot{x} \times \overline{nn+2nx+xx-mm}$; and dividing all by $4nx^2\dot{x}$, the Equation becomes $2nx+2xx = 3nn+6nx+3xx-3mm$. Consequently $3nn+4nx+xx = 3mm$, and therefore $x = \sqrt{3mm+nn} - 2n$.

From hence a ready and not inelegant Geometrical Construction (if I may be allowed to say so) becomes obvious; for with the Center S and Radius $ST = m$, describe the Semicircle TDA; and with the same Center and Radius $SE = n$, the Semicircle EVB; which two Semicircles shall represent the Orbs of the *Earth* and *Venus*. Make the chord AD equal to the Radius ST, and from D towards A, lay off $DF = SE$; draw TF, and thereon place $FG = BE = 2n$, and with the Center T and *Radius*



T G deſcribe the arch *G V*, cutting the Semicircle *B V E* in *V*; and draw the lines *S V*, *T V*: I ſay the Triangle *S T V* is Similar to that at whoſe Angles are the *Sun*, *Earth* and *Venus*, at the time when the *Area* of the enlightned part of that Planet's Disk, as ſeen from the *Earth*, is greateſt. How this Geometrical Effection follows from the Equation is too evident to need repetition.

In conſequence of this Solution, I find this *Maximum* always to happen, when the Planet is about forty Degrees diſtant from the *Sun*; and the times thereof, about the middle between her greateſt Elongations on both ſides from him, and her retrograde Conjunctions with him; when little more than a quarter of her viſible Disk is luminous, and reſembling the Moon of about five Days old; and notwithſtanding that her Diameter is at that time but 50 Seconds, yet ſhe ſhines with ſo ſtrong a Beam, as to ſurpaſs the united light of all the fixt Stars that appear with her, and caſts a very ſtrong Shade on the Horizontal plain whereon they all ſhine: an irrefragable Argument to prove that the Disks of the fixt Stars are unconceivably ſmall, and next to nothing; ſince ſhining with a native Light, ſo many of them do not equal the reflex Light of one quarter of a Disk of leſs than a Minute Diameter.

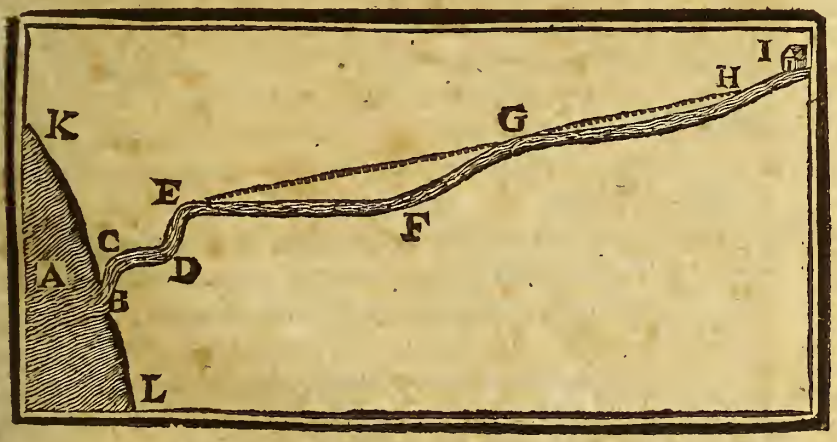
In this ſituation *Venus* was found in *July* laſt, on the tenth Day, about which time, when the *Sun* grew low, ſhe was very plainly ſeen in the Day time, for many Days together; as ſhe might have been in the Mornings, about the latter end of *September*. But this, ariſing from the Cauſes we have now ſhewn, is nothing uncommon; for every eighth Year it returns again, ſo that the Planet may be ſeen on the ſame Day of the Month and Hour, very nearly in the ſame place; as all acquainted with the Heavenly Motions muſt know.

Laſtly, it may not be amiſs to note that the Equation $x = \sqrt{3mm + nn} - 2n$ has a Limit; for if n be equal to $\frac{1}{4}m$, the point *V* will fall on *B*; and the whole Disk of a Planet at that diſtance from the Sun would be the *Maximum*, *viz.* when in its ſuperior Conjunction with the *Sun*. And the like if n were leſs than $\frac{1}{4}m$; the Arch *G V* in ſuch Caſe not interſecting the Semicircle *B E*.

II. *A Letter of the Reverend Mr.* John Sackette, A. M. *to Dr.* Brook Taylor, Reg. Soc. Secr. *Giving an Account of a very uncommon ſinking of the Earth, near* Folkeſtone *in* Kent.

SIR,

I Am about to give you the beſt Account I can of what is remarkable, and known almoſt to all hereabouts, concerning the preſſing forward of the Cliffs, and ſinking of the Hills in the Neighbourhood of our Town of *Folkeſtone.* I begin with giving you a Sketch of the Situation of the Country. This I ſhall do by deſcribing a ſtrait Road from what we call the *Mooring-Rock*, to *Tarlingham-Houſe*; the manner of the Country, as to the Riſing and Falling, being much the ſame, for about a Mile on either Hand of the Road deſcribed.



A. The *Mooring-Rock*, about half-way between high and low Water-Mark.

B. The Foot of the Cliff, 50 Yards from the Rock.

C. The Top of the Cliff, about 6 Yards high.

C D. A Plain of 50 Yards.

D E. A cragged Cliff, of 60 Yards high.

E F. A Plain above a Mile long.

F G. An Hill of ſteep Aſcent, near half a Mile.

G H. The Land from the Top of the Hill to the Houſe, near a Mile.

I. Tarlingham Houſe, lying near 2 Miles and a half *N. N. W.* from the Rock.

E G H. A Line of Sight.

K B L. The Shore at High-Water Mark.

I hope Sir, you will underſtand the Situation of the Place pretty well, tho' I have not obſerved exact Proportion in the Sketch; which the Paper would not allow after I had taken the Riſe of the Cliffs ſo high, which I thought proper for the more particular Deſcribing of them.

The *Mooring-Rock* (tho' it lies ſurrounded with great numbers of other Rocks) is it ſelf a moſt noted one, known by this Name, time out of mind. At this Veſſels uſe to be moored, while they are loading other Rocks; which they take from hence, not only for our own *Pier-Heads*, but for thoſe of *Dover Pier*; and a very great Quantity of them were Shipt, in the time of *Oliver*'s Uſurpation, and carried to *Dunkirk*, for the Service of that Harbour.

This Rock has remain'd fixt thus, for the memory of Man; and old Men have obſerved, that, for forty Years and upwards, the diſtance between it and the Foot of the leſſer Cliff *A B.* has been much the ſame; neither can they be much out in their Gueſs, the Diſtance being ſo ſmall. Tho' there ſeems nothing extraordinary in this, yet its what they take ſpecial Notice of, to their great ſurprize: for they ſay, and prove by good Marks and Tokens,

kens, that the leſſer Cliff *B C* has been conſtantly falling in, inſomuch, that from time to time, in their Memory, near 10 Rods forward to the Land has been carried away by the Sea. From whence, as it appears that the Plain between the Top of the leſſer Cliff and the Foot of the higher *C D* has been formerly double the Breadth that it is at preſent, ſo the diſtance been the Rock and the Foot of the leſſer or lower Cliff *A B*. ſhould have increaſed in Proportion, and would have been double at preſent, to what it has been formerly: But this Diſtance remaining the ſame (as is above noted) or rather leſs (in the Opinion of many) is what is greatly wonder'd at: nor can it be accounted for otherwiſe, than by ſuppoſing that the Land preſſing forward into the Sea is waſhed away by the high Tides; and, as often as this happens, preſſes forward again. This preſſing forward of the Land into the Sea, would be incredible, were it not ſhewn to be matter of Fact; and that not only at this one Place of Obſervation, but by like Obſervations all along this Coaſt, as far as the Situation continues the ſame.

Now, Sir, let us climb both theſe cragged Cliffs, and place our ſelves at the Top of the higher One, at the Point *E*. And here we are to obſerve, that (as old Men inform us) upward of forty Years ago, not ſo much as the Top of *Tarlingham-Houſe* could be diſcern'd, neither from hence, nor yet a good Diſtance off at Sea; but it diſcover'd it ſelf by degrees, till at this Day, not only the whole Houſe, but a great Tract of Land below it, is plainly to be ſeen, as in the Line of Sight *E G H*. The Tract of Land is more in Proportion than deſcrib'd in the Sketch, between the Point at *H* and the Houſe. In this there can be no Fallacy; and we can aſcribe it to nothing leſs than the ſinking of the Hills (for their Tops could never wear

away conſiderably, being always cover'd with Graſs, and never broken up by the Plough or otherwiſe). Theſe Hills are all of Chalk, and have probably very large Caverns within, Springs of Water always flowing plentifully from the foot of them; and I have had it obſerv'd to me, that upon their Tops frequent Cracks have been taken notice of. Whatever be the Cauſe of it, 'tis not to be doubted but that theſe Hills are greatly ſunk. And this ſinking of the Hills, the People at this Place believe, forces the Cliffs and all the Land forward into the Sea. The Cliffs conſiſt of great ragged Sand-Stones till we come to near a Yard (at ſome places more) of the Bottom; then we meet with what they call a Slipe, *i. e.* a ſlippery ſort of Clay always wet. Upon this Slipe at the bottom, they preſume that the hard Stony Land above ſlides forwards toward the Sea, as a Ship is launch'd upon tallow'd Planks. I thought it proper to give you this account of the Nature of the Earth; and withal to mingle with it the Opinion of the People, that you might perceive they are ſo far from doubting of the Truth of what is abovewritten, that they endeavour to find ſome Solution of it, as being a thing not more ſtrange than true. If I ſhould take all the Hands that can be got to teſtifie the Truth of this, it would make too large a Roll, ſo I ſhall chuſe only a few of the moſt antient and of beſt Credit.

I aſſure my ſelf that I have Credit enough with you to be believed upon my own ſingle Subſcription, that I am,

SIR, Yours,

Folkeſtone in *Kent*,
February 24, 1715-16.

John Sackette.

We

We whose Names are underwritten do hereby testifie the Truth of the Matters of Fact in the within written Letter related,

Benjamin Master, a Jurat of the Town, aged 74.
Robert Hammond, Senior, a Jurat of the Town, aged 77.
William Godden, a Fisherman, aged 74.
Thomas Marsh, a Fisherman, aged 72.
William Hall, a Fisherman, aged 73.
James Godden, a Fisherman, upward of 60.

III. *Miscellaneous Observations made about* Rome, Naples *and some other Countries, in the Year* 1683 *and* 1684; *and communicated to the Publisher by* Tancred Robinson M. D. R. S. S.

SIR,

YOU having been pleas'd to think some of my Observations might be agreeable to the Publick, I shall here freely give you them (such as they are) omitting those that were formerly extracted out of the *MSS.* Diaries of my Travels, some of which are Printed in several *Philosophical Transactions*, and others in some of Mr. *Ray's* English Tracts.

In my Journey from *Rome* to *Naples* I observ'd on the Rubbish of the *Tre Taberne* an unusual Vegetable for that place, remote from Town or House, which was the *Ficus Indica Spinosa* commonly call'd the *Opuntia* or *Tuna*, and by our Writers of *America* the *Prickly-Pear*, whose Juice gives the Urine a red Colour; when I came to *Naples*, I found it there near the Rocks, and in some wild solitary Places like a Native. If the *Spaniards* planted it, they chose desert Situations. On this Plant the *Cochineel*

Ver-

Vermiculus is ſaid to feed in great Numbers, before it changes into the *Chryſalis* or *Aurelia* of a *Lady-Cow*: but the Colour lies in the *Nymph Worm* before it turns a *Beetle*. This gives me occaſion to reflect upon the many Species of our *Europæan Vermiculi*, ſome of which might be found to yield rich Colours (if try'd): We are certain the Maggot of our *Ilex* gives the *Kermes*, and a noble Scarlet Dye before it turns into a Fly. Many Shell-Fiſh (which are a ſort of Inſect) contain Purple Juices.

This brings on another Remark I made in paſſing the *Apennines* and *Alps*, where I noted in ſome Beds or *Strata*, and even in the midſt of the hardeſt Rocks, great varieties of perfect Shells, that never occurr'd to me on the *Italian* Shores, nor in any of the numerous *Muſeums* of that Country: ſo I gueſs they might be *Exotick*.

Going further on the *Via Appia*, I obſerv'd abundance of the *Siliqua Arbor* or *Carob Tree*, commonly call'd *Panis S. Joannis Baptiſtæ*; on the Pulp whereof many poor People were feeding. The Husks taſted like *Manna* to me. Near them grew plenty of the *Arbor Judæ*.

The *Arbutus*, or *Strawberry Tree*, was common in the woody places; if this grows wild in the South Weſt parts of *Ireland*, as ſome affirm, I ſhall think them much warmer than any Counties of *England*.

Before I enter'd the beautiful *Campania* of *Naples* large Woods of *Cork Trees* grew on each ſide the Road, where the Inhabitants were decorticating them. I ask'd if the Trees did not periſh: they anſwer'd, ſome did, but the Acorns return'd annual Supplies. The Women and Children wore Shoes made of the Bark.

Coming near *Capua* I obſerv'd a Species of Aſh, or *Ornus*, on the Trunk whereof many Saccharin Concretions were viſible. This prov'd the true *Manna*, that iſſues out thro' the Inciſions made in this Tree by the Inhabitants of *Calabria*. Swarms of *Cicada*'s were ſucking the

Body

Body and Boughs, and perhaps by wounding then made way for fresh *Manna*. Here I may note, that many Insects have not only a *Proboscis* to bore and draw out the Juices of Plants for Aliment, but other proper Instruments to convey their Eggs into Vegetables and Animals, where they may find Covert and Food when they come to hatch, in the Gall-Tumours, and other Excrescences occasion'd by the Wounds of the Parent Insects, that make such variety of *Cuniculi* in all parts of Plants, and even in the cutaneous parts of living Creatures and in dead Flesh.

This confirms me, that many Gums and Exudations find their way out of Vegetables thro' the Wounds of Insects and other Apertures. Most *Voyagers* thro' the *East Indies* affirm, that *Gum Lack* is work'd and made by large *Ants* that cover the Trees. I rather think the Insects suck and terebrate the Tree, and so give vent to that peculiar Sap that hardens in the Sun. This may extend to most Balsamiferous, Gummiferous, and Saccharine Plants, especially in hot Climates where Insects abound, and are more active. In cold Climates the Saps of many Vegetables will boyl into Sugars, as that of Maple, Birch, Reeds, *&c.* Not but that the Fluids of Plants (like those of Animals) will spontaneously break thro' their Vessels in a Plethory, and make on the superficial Parts various Eruptions and Congestions.

Discoursing of *Manna* I may here take Notice there are many adulterations of this Drug: all passes for the *Calabrian*, whereas that of *Brianson* is from the *Larix*, that of *Persia* from the *Myrica*, and these frequently mixt with the Juices of Spurges, and other Purgative Ingredients. I must not here deny that Dew will sometimes in cool Mornings shoot, and congeal into a solid, sweet, white Substance, which I once observ'd in very hot Weather before Sun-rise.

Upon viewing the *Vulcano's* about *Naples*, *Vesuvius* on the East side, the *Solfatara* and *Monte di Cinere* on the West near *Puzzuolo* and *Bajæ*; I observ'd the same Face of Nature, which I believe runs thro' all the other *Vulcano's* of our Globe, *viz.* heaps of Pumice Stones and Cinders of *Marchasites* on the sides, with Beds of Flower of Brimstone on the tops. The Holes and Cavities in those calcin'd Minerals seem to be the *Nidus* of the *Sulphur*, which hath been sublim'd by the Heat and Fire of that vast Mass of *Pyrites*, that compose the Bowels of those *Vulcano's*, and lye scatter'd thro' many parts of the Earth, even under the Sea, where they sometimes germinate, ferment, and take Fire, throwing up little *Islands*. Earthquakes and other *Choc's* of the Globe may spring from the Mines of these combustible and explosive Minerals, loaden with Brimstone and Elastic Salts. Hence some Account may be given of *Thermæ* or hot Baths, whose Waters gliding thro' these hot Beds take their *Gas*. Of such Medicinal boiling Waters and Stoves, there are more about *Naples* than in any place I ever saw or heard of, the whole Country being continually pervaded by hot Steams.

Walking round this City I found *Palm Trees*, some with unripe *Dates* hanging down, others without any Fruit: and there was another Species of *Palm* that sweats out the *Gum Dragon*: I suppose the Monks had transplanted them out of *Africa*. I saw growing here many *Sugar-Canes*, *Rice*, *Maiz*, abundance of the purging *Senna*, and *Cummin* Seed. Thro' the whole *Campania* of *Naples* I observ'd the same Vegetables to be larger and more proud than in other parts of *Italy*, as the *Platanus*, the *Lentiscus*, the *Terebinthus*, the *Pistaches*, the *Oleanders*, *Agnus Castus*, *Barba Jovis*, the *Tragacanth*, the *Styrax*, the *Capers*, *&c.* The *Melons*, *Jujubes*, the *Azaroles*, and other Fruits were of a better Taste. The *Gossypium*, with the Cotton

ton breaking out of the husks, adorn'd ſome of the Fields; the Hedges full of *Pomegranats*, *Almonds*, *Tamarisk*, *Sumach*, *Cedrus Lycia* (a ſort of Juniper or Savin) abundance of *Phillyrea*, *Alaternus*, *Ciſti*, *Cytiſi*, *Myrtles*, *Spaniſh Broom*, *Bays*, *Lauruſtines*, *&c.* all wild, Indigenous of that warm Soil and kind Climate. The Water-Melons, the Olives, the Oranges, Lemons and Citrons were better than about *Genoa* or in *Provence*.

The *Lotus Arbor* or *Nettle Tree*, the *Paliurus* or *Chriſt Thorn*, the *Ricinus* or *Palma Chriſti*, common in the Hedges, with ſeveral *Thymelæa's*.

I ſaw them fiſhing for *Coral*, and *Hippocampi*: the firſt did not come ſoft out of the Sea; the hard Incruſtation covers the Vegetable part that bears Seed, as the *Alga's* and *Fuci* do. They take the *Sword-Fiſh* by darting a Spear into him, as they do the *Whales* in the *Greenland* Fiſhery.

When dark Night came on, I could ſee Multitudes of Luminous Flies thro' the *Campania* of *Naples*: perhaps our Male *Gleworm*, or flying *Cicindela*, may abound there; not but that many other Inſects may carry ſuch Lanthorns about them. The *Scorpions* creep out about that time; and I have found them often in Bed, with the *Punaiſes*.

The Hedges are full of *Lizards* of various Colours; and the *Cicada's* chirp and ſing towards Evening. I obſerv'd ſeveral Species of ſtinging Spiders in the Corn Fields, ſome of which, in hot Harveſts, may prove *Tarantula's*; the Poyſons of Animals and Plants increaſing with the approach of the Sun, and the Heats of Climates. Abundance of Silk-Worms were ſpinning on the Trees and Shrubs; the Birds prey'd upon them, before they could change into *Papilio's*, as they do upon ſwarms of *Locuſts*.

I eat often their young Frogs, Tortoiſes and Snails, ſerved up with Oyl and Pepper, which agreed well with me: ſo did their *Sea Urchins*, and the *Urtica Marina*, (called *Sea Gelly* or *Blubber*, tho' it be an Animal, having a true Heart, and Veſſels for the Circulation of Fluids) Some of their Thiſtles are no ungrateful Sallet.

I ſaw ſome Vitriol Works about *Siena*, *Rome* and *Puzzuolo*; thoſe of *Alum* only about *Civita Vecchia*. Amongſt the Sands of the *Adriatic* Sea I obſerv'd many white, clear, ſhining Flints; which they told me were carried to *Venice*, to make the fine Chryſtal Glaſs at *Muran*.

Upon reading our ingenious Dr. *Muſgrave*, *de Geta Britan. & Synop. Chronolog. Dom. Sever.* I conſulted my Diary taken at *Rome*. The Magnificent *Septizonium* figur'd by him ſtood near the Foot of the *Palatine* Hill, on the *E.S.E.* ſide, overlooking the *Via Appia* and the *Circus Maximus*, the *Amphitheatre* of *Titus* being near on the other Side. By the number of *Portico's* (which were Seven) it might contain Multitudes of People, as Spectators of the Triumphal Entries and the publick Games. But I would not be thought to differ from our *Learned Countryman*, who with good Authority, thinks it the *Sepulchretum* of that Imperial Family; tho' moſt of the Ancient *Mauſolæum's*, (at leaſt thoſe I ſaw) were *Rotonda's*, or *Columbaria's*, for the more convenient placing the *Urns* of the Kindred; as that of *Auguſtus* near the *Campus Martius*; that of *Adrian* on the other Bank of the *Tyber*; thoſe ſaid to be of *Scipio*, of *Cicero*, and *Munatius Plancus*, near *Gaeta* and the *Via Appia*; that of *Virgil* on the ſide of Mount *Pauſilippus*; that of *C. Metella* and ſome others on the *Via Flaminia*. Some were Pyramidal as that of *Ceſtius* in the Wall of *Rome*, and a few others on the public Roads. This *Septizonium Severi* ſeems to differ from the reſt of thoſe Ancient *Sepulchretum's*, which

might

might be varied according to the Fancy and Humour of great Families.

This Urn Burial was only in Fashion amongst the *Gentes Majores*: as for the dead Bodies of the *Plebeians* and Slaves, they were generally laid in places where they had dug Stone; and those Quarries became *Catacombes*. The Laws prohibited them to bury within a City, unless the Bodies were first reduc'd to Ashes.

I observ'd in many of the Ruins about *Rome* and *Naples*, great Stones laid close, and wedged very fast with little or no Cement; the Bricks, towards the middle of a Building, were generally of a *Rhomboidal* Figure, very Smooth, Shining and Hard, laid in Plaister as firm as Marble. Their Mortar was much more durable then ours, as appears at this Day by their *Aquæducts* and *Piscina's*, the *Cento Camare*, and *Caligula's* Bridge under Water at *Bajæ*. *Pliny* says, they made use of the *Terra Puteolana*, but the present Inhabitants have lost the way of tempering it.

During my abode at *Genoa*, *Leghorn*, *Ostia* and *Civita Vecchia*, I observ'd many *Torpedo's* or *Cramp Fishes*, most accurately Anatomized by *S. Lorenzini*; plenty of *Sphyræna's*, (a Species of Sea Pike, a-kin to the Needle-Fishes) The *Uranoscopus*, call'd *Bocca in Capo* and *Prete*. The *Mola* or Sun Fish. The *Dentex* or *Pentalis*, *Altavela's* a sort of *Pastinaca*. The *Pesce Balestra* or *Capriscus*. The *Pesce Pettine* or *Novacula*. The *Zygæna* or Ballance Fish, as large as the Saw-Fish or most Sharks. The *Scolopax* or *Trombetta*, call'd by our Seamen the Bellows or Trumpet-Fish. The *Draco Marinus*: The Tunny-Fish. The *Centrina* or *Pesce Porco*. The *Aquila*. The *Scorpius Major*, with Varieties of *Turdi* in the Markets. But what pleas'd me most, were some odd Sea Animals, as the *Lepus Marinus*, (a Species of naked Snail) the *Hystrix Marinus*, or *Eruca*, call'd by the Seamen *Pincio*, with a Brush hanging out of the Tail, like the *Byssus* or

Silk of the *Pinna*. Many *Tamburo's* or Drum-Fiſhes; Plenty of *Muræna's*. I obſerv'd a ſtrange Sea Animal, call'd the *Microcoſmo marino*, with many Shells, *Tubuli* and Vegetables growing or ſticking to the Back of it, this appear'd to me a-kin to the *Echini Marini*, or rather to the *Stellæ Marinæ*, being Triangular, and ſometimes *Pentadactylous*.

I embark't once with the Fiſhermen, who ſhew'd me ſeveral *Loligo's*, *Polypi*, and *Sepia's*, or Cuttle-Fiſhes, (all *Cruſtaceous*) ſome of them were caſting out their Ink in the Water: I ſuppoſed ſome Sharks, Dog-Fiſhes, or other Enemies, were near them; this black Liquor may be the Gall of thoſe Animals. In the Nets, I often found Sea Inſects, and Vegetables; and indeed a new World, undeſcrib'd by natural Writers, at leaſt unknown to me: but for want of the Art of Deſigning or Drawing, abundance of things eſcap'd me, and were utterly loſt; therefore I would adviſe all Travellers to be converſant in that moſt uſeful Science.

I obſerv'd the *Italians* near the *Alps* and *Appennines*, call'd ſeveral Birds *Francolino's*, as our Red, Grey and Black Game; and even their red and white Patridges; the different Colours of the Hens from the Cocks, the many Variegations in Feathers, the different Ages and Places, have all given occaſion to multiply Names and Species; the ſame may happen in Fiſhes, Quadrupeds, Inſects, and all the Diviſions of *Zoology*; and even in *Botany* and *Minerology*.

The *Italians* call many of their little fat Birds *Beccafigo's*, that feed upon Figs, Grapes, and other ſweet Fruits. So the *French* multiply their *Ortulans*, taken in the Vineyards and Gardens. Some of the Antient Writers take Notice that the *Romans* uſed to feed their Geeſe and other Birds with Figs, when they intended to ſwell their Livers to a monſtrous Bigneſs.

The

The *Merops* or *Apiaster* is common on their Brooks; it flies like our Kings-Fisher, and preys not only upon Insects but Fish. There is a very beautiful Bird in *Italy*, that suspends its Nest down from the Boughs of Trees. When I saw it fly by me, I took it for an *Indian*, from the brightness of its Colours; it is as large as our Missel-Bird and Thrush *an Icterus Plinii*?

The great *Cock of the Wood* (said to be found in *Ireland*) is common on the sides of the *Italian* Hills, and brought frequently to the Markets. I saw twice or thrice the *Himantopus*, and the *Phænicopterus* or *Flamingo*, (whose Tongue was a Dainty amongst the *Romans*, when they grew Luxurious). I observ'd some *Spoon-bills*: these three last Birds were wading in the Rivers and Marshes, near the Sea. Once I spy'd some *Pelecans* on the *Adriatic*, near the mouth of the *Po*. The *Avis Diomedæa* was hung up dry'd in one of the *Museums* at *Florence*, but they told me it had been taken on some of the Isles of the *Archipelago*.

On the *Laguna* of *Venice*, I saw several Species of *Mergi*, *Lari*, *Colymbi*, and other Water Fowls, most of which Div'd. I was surpriz'd with the Variety of them, having not seen so many on other Coasts: perhaps the hard Winter had forc'd some unusual Birds thither. The Monks and Fryers told me, they eat some of those Sea Birds in *Lent* and on Fast Days, because they liv'd upon Fish, and had a piscose Taste, as the *French* pretend their *Macreuse* to have, which is a sort of Sea Duck, common on the Coast of *Normandy*, and brought to the Markets, even at *Paris* on *Maigre* Days; of which I gave a long History in the *Philos. Transact. An.* 1685. N°. 172.

Buffalo's are common in the Kingdom of *Naples*, and in some parts of *Lombardy*, where they plough and draw

with

with them. A peculiar Cheese is made of their Milk (call'd *Casio di Cavallo*) rowl'd up like stiff pieces of Ribbon. Out of their black shining Horns they make Snuff-boxes and Combs. The Creature is unruly, and therefore they lead them with Iron or Brass Rings drawn thro' their Noses. They make a Buff Leather of their Skins. I once saw some hairy Sheep feeding on a Common; perhaps they had been brought from *Africa*.

In passing the high *Alps*, I had a View of the *Ibex* or *Steinbock*, whose large Horns are recurvated almost as far back as the Tail; they are very ponderous for the bulk of the Animal, having many knotty Rings, that may help them in climbing. They are rarely taken.

The *Rupicapra* or *Chamois*, is very common on the sides of the Cliffs, whose Skins afford the soft Leather. The *Mus Alpinus*, or *Marmota*, is as large as a Rabbet, will soon grow tame in Houses, tho' brought down from the Summits of the highest Mountains, where it will grow fat.

I have seen in several Towns of *Italy* fresh strong *Porcupines*, which the Inhabitants told me were taken in the Hedges and Ditches thereabouts, tho' much more rare than our Land *Urchins*. In the *Grisons* Country, and in some Cantons of *Switzerland*, I have often observ'd the *Ranunculus Viridis* or small Tree-Frog, perching on the Boughs and Leaves.

In the Northern Parts of *Germany* I saw several *Elk*-Skins, and those of the *Rhin-Deer* stuffed, and set up in *Museum*'s, but never alive: tho' the Animals are said to be common in *Muscovy* and *Lapland*, and sometimes seen in the Forests of *Prussia*.

The Skins of *Hippopotami* (said to be the *Behemoth*) are in some Collections of Curiosities in *Italy* and *Holland*: so are those of the *Musk-Deer*, one of which is in the *Museum* of our *Royal Society*.

Give

Give me leave here to reflect a little upon the late *Aurora Borealis*, whose *Phænomena* you have so well describ'd and explain'd in your late *Philosophical Transaction*, No. 347. I am of your Opinion, that those Phosphorous or Luminous Appearances in the Firmament, proceed from the various *Effluvia* perspir'd out of our Globe, or passing thro' it; for I have seen those Lights over *Vesuvius*, the *Strombulo* Islands, and towards *Ætna* in dark Nights, when those *Vulcano's* were not flaming nor burning, their Sides and Tops being passable to Travellers at that time, and all their outward parts quiet. We are certain that *Iceland* and *Greenland* abound with *Vulcano*'s; so may North East *Lapland*, North *Russia* and *Tartary*, where vast Chains of Mountains are said to run. The *Jesuits*, and other Travellers, relate many prodigious Eruptions of Fires, and Earthquakes towards the North of *China*; but nearer the Pole the Earth must be clos'd and pent up many Months, by the long severe freezings and continual Snow and Ice, which relaxing towards Spring, may give vent to that vast Mass or Magazine of perspirable Matter, that had been kept so long in hot Subterraneous Prisons. This may be one Reason why Animal Bodies themselves are often sensible of Changes at that Season in our Climate, when Perspiration is upon such an Increase; but I will not take up your time any longer, especially upon a Subject that you understand so well.

SIR,

Your most humble Servant,

Tancred Robinson.

IV. *An*

IV. *An Account of the Mischiefs ensuing the swallowing of the Stones of* Bullace *and* Sloes. *By the Reverend* William Derham, *Prebend of* Windsor, *and* F. R. S.

AMong the Accounts which the Royal Society hath had of the Mischiefs ensuing the swallowing of divers sorts of Stones, I do not remember any Case wherein the lesser Stones of Fruits (such as *Sloes* particularly and *Bullace)* have produced any dangerous Symptoms, especially in the Stomach alone. The larger Stones, I know, of *Prunes*, and other great *Plumbs*, have produced very fatal Effects; but the lesser Stones of *Sloes, Cherries, &c.* many swallow rather out of choice, than with any apprehensions of Danger, thinking them useful in preventing a Surfeit from the Fruit. But the following Case will shew the Danger even of these lesser Stones. And I have acquainted the Society with it, on purpose to prevent Dangers, if it should be thought fit to publish it in the *Transactions*, for a warning to others.

The Case is this. About two Years ago the Man-Servant of a Neighbouring Clergyman complained to me of excessive Pains in and about his Stomach; that he lay under a great Dejection of Appetite; and whenever he eat, that he could not retain it, but in a little time vomited it up. By which means he was, in a short time, reduced to a very low and languishing Condition, insomuch as they began to despair of his Life.

Upon this he applied himself to some Practitioners in Physick: One of which ply'd him with strong Vomits

eight

eight Days together, with very little Signs of Succeſs. But ſome time after, having Occaſion to ride ſomewhat more than ordinary, he found himſelf very ſore in his Stomach, and Sick ; which ending in violent Vomiting and Straining, brought up the firſt Stones he ever perceived to come from him, which were about Twenty in number.

After this he had frequent returns of the Vomiting up of *Bullace* and *Sloe-Stones*, eſpecially upon ſtrong Exerciſes ; particularly moving and ſtooping much in Weeding in the Garden ; in Riding alſo, although it was only to water his Maſter's Horſe. Upon theſe Occaſions he would be ſeized with acute Pains in his Stomach, and ſoon after Vomit up more of thoſe Stones.

He hath counted above One hundred and twenty *Bullace* and *Sloe-Stones* that have been diſcharged ; and many others he could not number, by reaſon they came up when he was in Riding or in his Buſineſs. He is not yet free of them, but is in Pain oftentimes, and Vomits them up, eſpecially in Riding ; but after he hath diſcharged them, he is much eaſier for a while. He commonly brings up a ſlimy Matter with them, mixed with Blood or ſomething very like Blood.

The Cauſe of all this Diſaſter the Man aſſures himſelf was this, namely, being in his Youth a great lover of Fruit, he uſed greedily to devour all ſorts he could come at, and *Bullace* and *Sloes* being the eaſieſt to be gotten, he uſed to ingurgitate great quantities of them, without evacuating many of the Stones by Stool, as he well remembers, and as he obſerved others did. Theſe Stones he thinks have lain in his Stomach (ſome of them at leaſt) above ten Years ; but he felt no Pains till about four Years ago. And thoſe at firſt were not ſo violent, nor attended with ſuch ſevere Fits of vomiting, and

loſs of Appetite, as they by degrees came to be afterwards.

Thus having related the Caſe as the Man told it me, I ſhall leave the Ætiology of it to the learned Phyſicians, it being ſufficient for me to relate the matters of Fact, and thereby teſtifie the Duty and Reſpects owing to the Society by

Their moſt obedient
Humble Servant,
W. Derham.

V. *Obſervations and Experiments relating to the Motion of the Sap in Vegetables.* By *Mr.* Richard Bradley, R. S. S.

OF Plants in general we may firſt obſerve, that they are either *Terreſtrial, Amphibious*, or *Aquatick*; and ſo nearly do Vegetables agree with Animals in moſt points, except Local Motion and its Conſequences, that from the Knowledge of the one we are reaſonably led to the Diſcovery of the other.

Thoſe Plants which I call *Terreſtrial* are ſuch as *Trees, Shrubs* and *Herbs*, which grow only on the Land. Theſe like Land-Animals have diverſities of Food, a Method of Generating, and certain Periods of Life.

Of the *Amphibious* race, which live as well on Land as in the Waters, are the *Willows, Ruſhes, Minths, &c.* theſe are not unlike in many reſpects to the *Otter, Tortoiſe, Frog, &c.*

The *Aquaticks*, whether of *Lakes, Rivers*, or the *Sea*, are very numerous; theſe may be compared with the Fiſh-kind, and like them will not live out of their proper

per Element. In *Fresh Waters* are the *Water-Lilly's, Plantains, &c.* and in the *Sea, Corals, Fuci, &c.*

Plants seem to possess only the next degree of Life below the most stupid *Animal*; or where Animal Life leaves off the Vegetable Life seems to begin.

The Seasons of Motion in *Plants* are the same with those of *Animals*, which sleep during the Winter. An Artificial Heat will give Motion to either of these in the Coldest time.

The common Opinions relating to the *Saps* Motion are as follows. First, The *Sap* does not rise by the *Pith*; because some have observ'd the Trunks of large Trees to be without that part, and yet the same Trees have continued to put forth Fruit, and Branches on their Tops. I have observ'd, that the *Pith* is not found in those Branches of a Tree which exceed two or three Years growth; and it is certain, that the *Pith* which is in a Branch of this Year, will (the greatest part of it) be distributed into those Boughs which form themselves the next Season.

It is said by some, that the Tree does not receive its Nourishment by the *Bark*, for that Trees having lost that part, will still continue their Growth. Others tell us, that if the *Bark* be cut away round the Trunk of a Tree, it will presently die. These various opinions seem to have been set on foot without extraordinary Consideration, upon the belief that a Tree has but one *Bark*: Whereas, upon Examination with the Microscope, we find four distinct Coverings to each Branch, without the woody parts. The two outermost *Barks* may be taken from a Tree without great Damage, but the other two which lye nearer the Wood being strip'd off will kill the Tree.

Some affirm, that the *Sap* doth neither rise nor fall in the woody part of a Tree, because they have not been

able to discern any *Sap* to issue out of that part, when a Branch has been cut. The Microscope plainly shews us the Vessels in the Wood, through which the *Sap* riseth from the Root; but as these Tubes are not large enough to admit into them any thing more gross than Vapour, so they have not been esteem'd to be of any great Use. But I hope the Explanation of the adjoyn'd Figure will in some measure discover the Office of these, and of such other parts of a *Plant* as are severally design'd for the Growth of Vegetables; but it will first be convenient to enquire a little into the Nature of the *Root*.

The *Root* of a Tree is chiefly composed of a *Parenchyma*; more gross than that in the Stem or Body of the Tree; it has likewise Vessels and a Covering, which I shall better explain in another Paper. The Root, that is, the principal part of it, receives into it such Juices of the Earth as are proper for it; and no other. Somewhat like a Weck of Cotton, which having been impregnated with Oil, will only admit Oil into it. This Provision being made in the *Stomach* of the *Plant* (as I call it) chiefly in the Autumn Months; the Tree is prepared for Germination so soon as the Earth is sufficiently warm'd, either by the Sun's Beams, or an artificial Heat, such as Horse dung, Bran and Water, or other such like Ferments. These Heats raise into Vapour the Juices contained in the Root, and by that means cause Vegetation.

Figure I. which I am about to explain, is part of the Branch of an *Apple Tree* made in *May* 1715, and cut in *April* 1716. It was cut in figure of a half Cylinder, the length somewhat more than the Diameter, which was about a quarter of an Inch. This being magnified with one of *Campani*'s Microscopes, discovers the following parts, *viz.*

1, 2,

1, 2, 3, 4, 5, 6, 7. are Capillary Veſſels, which run longitudinally through the Branch, in the Ligneous part, which was made in the Year 1715. Through theſe Tubes, the Steam riſeth from the Root; the ſtrength of which is well explain'd by the Engine for raiſing Water by Fire, invented by the late Captain *Savory*.

From A to B, we may view Veſſels of the ſame ſort, made at the ſame time.

8, 9. are Veſſels of the ſame uſe with the former, now forming themſelves for the uſe of the Year 1716.

By this means the Diameter of the Branch is increas'd, and additional Nouriſhment ſuffer'd to paſs into thoſe Buds which are to make new Branches. Theſe are made out of the Fourth or innermoſt Bark, markt C, C.

The Mouths of the Capillary Tubes of the Years 1715, and 1716. are D, E. The Vapour which riſeth from the Root, is continued in theſe Veſſels, to the extremities of the Branches; where it meets with parts (not here deſcrib'd) like to *Glands*; which *Glands*, if we may ſo call them, are likewiſe found at every Knot or Joynt. At theſe places, the Vapour coming near the Air is condenſed, and returns between the Barks, by means of its own weight, down F, G, H, leaving in each Bark mark'd I, K, L, ſuch Juices as each of them naturally is inclin'd to ſeparate from it; till at laſt, the more Oyly part paſſing to the Root, may lengthen the *Fibres* thereof, as Icicles are lengthned; and by its Oleous Particles, preſerve them from Rotting by the Wet. The parts which compoſe the ſeveral Barks, are *Parenchymous* or *Spongey*.

The firſt mark'd M, is of a cloſer Texture than the ſecond N, and the ſecond cloſer than the third O, and ſo on till theſe Parenchymous Parts are interwoven with the longitudinal Wood-Veſſels, where they are ſomewhat conſtrain'd, till they come to make the Pith mark'd P. Then they are much Larger than in any

other

other part of the Tree; and by what I have obſerv'd, ſeem to contain a more finiſh'd Juice than the reſt, and may well enough be ſtiled the *Medulla.*

We may note, that when the fourth or innermoſt Bark C, has once compleated its Sap-Veſſels, and is firmly join'd to the Wooden Part, then the third Bark O takes its place for the ſucceeding Year; and ſo the reſt, except that the firſt mark'd M, ſplits and divides its ſelf, to ſupply the place of the ſecond, as I ſhall demonſtrate hereafter.

Before I conclude, I ſhall beg leave to recommend the following Enquiry to the Curious, *viz.*

If the ſeveral Barks, having different Texture of Parts, admit into each ſeparate and different Juices from the reſt: Whether thoſe Juices may not be of very Different Vertues; the firſt more *Aſtringent* than the others, the ſecond perhaps *Emetick*; and the third *Cathartick.* This ſeems to be worth Enquiry.

VI. *Some Microſcopical Obſervations, and Curious Remarks on the Vegetation, and exceeding quick Propagation of Moldineſs, on the Subſtance of a Melon. Communicated by the ſame.*

I Had lately a large Melon-Fruit, which I ſplit lengthways thro' the Middle, in order to obſerve the Veſſels which compoſed the Membrane or Tunick of each Ovary; but my affairs at that time not permitting me to continue the Work I had began, I lay'd by the one half of the Melon, to be examin'd when I might have more Leiſure.

At

At the end of four Days, I found ſeveral Spots of Moldineſs began to appear on the fleſhy Part of the Fruit, ſomewhat Green towards the Rind; and of a paler Colour towards the Middle of the Fruit. Theſe Spots grew larger every Hour, for the ſpace of five Days; at which time the whole Fruit was quite cover'd.

This ſurpriſing Vegetation made me Curious to examine, if there was any difference between thoſe Parts which were Green and the others, beſides their Colour. The firſt being ſeen with the Microſcope, appear'd to be a *Fungus*, (See Fig. 2.) whoſe Cap was fill'd with little Seeds, to the number of about Five Hundred; which ſhed themſelves in two Minutes after they had been in the Glaſſes.

The other Sort had many Graſs-like Leaves, among which appear'd ſome Stalks with Fruit on their Top. Each Plant might well enough be compared to a ſort of *Bull-Ruſh*. (Fig 3.) They had their Seed in great Quantities, which I believe were not longer than three Hours before they began to Vegetate; and it was about ſix Hours more, before the Plants were wholly perfected: for, about ſeven of the Clock one Morning, I found three Plants at ſome Diſtance from any others; and about four the ſame Day, I could diſcern above Five Hundred more growing in a Cluſter with them, which I ſuppoſed were Seedling-Plants of that day. The Seed of all theſe were then Ripe and Falling.

When the whole Fruit had been thus cover'd with Mold for ſix Days, this Vegetable Quality began to abate, and was entirely gone in two Days more. Then was the Fruit putrified, and its fleſhy Parts now yielded no more than a ſtinking Water, which began to have a gentle motion on its Surface, that continued for two Days without any other Appearance. I found then ſeveral ſmall Maggots (Fig. 4.) to move in it, which

grew

grew for the ſpace of ſix Days; after which they laid themſelves up in their Bags. Thus they remain'd for two Days more without Motion, and then came forth in the Shape of Flies. (Fig. 5.) The Water at that time was all gone, and there remain'd no more of the Fruit than the Seeds, the Veſſels which compoſed the Tunicks of the Ovarys, the outward Rind, and the Excrement of the Maggots; all which together weigh'd about an Ounce. So that there was loſt of the firſt weight of the Fruit when it was cut, above twenty Ounces.

We may Judge from this, and other Caſes of the like nature, how much Vegetable Life is dependent on Fermentation, and Animal Life on Putrifaction.

VII. *The Art of* Living *under* Water: *Or, a Diſcourſe concerning the Means of furniſhing* Air *at the Bottom of the* Sea, *in any ordinary Depths. By* Edm. Halley, LL. D. *Secretary to the* Royal Society.

THere have been many Methods propoſed, and Engines contrived, for enabling *Men* to abide a competent while under *Water:* And the Reſpiring freſh *Air* being found to be abſolutely neceſſary to maintain Life in all that breath, ſeveral ways have been thought of, for carrying this *Pabulum Vitæ* down to the *Diver*, who muſt, without being ſomehow ſupplied therewith, return very ſoon, or periſh.

We have heard of the *Divers* for Spunges in the *Archipelago*, helping themſelves by carrying down *Spunges* dipt in Oyl in their Mouths: but conſidering how ſmall a Quantity of *Air* can be ſuppoſed to be contained in

the Pores or Interſtices of a *Spunge*, and how much that little will be contracted by the Preſſure of the *incumbent Water*, it cannot be believed that a Supply, by this means obtained, can long ſubſiſt a Diver. Since by Experiment it is found that a *Gallon* of *Air*, included in a *Bladder*, and by a Pipe reciprocally inſpired and expired by the *Lungs* of a Man, will become unfit for any further Reſpiration, in little more than one Minute of Time; and though its Elaſticity be but little altered, yet in paſſing the *Lungs*, it loſes its *vivifying Spirit*, and is rendred effete, not unlike the *Medium* found in *Damps*, which is preſent Death to thoſe that breath it; and which in an inſtant extinguiſhes the brighteſt *Flame*, or the ſhining of glowing *Coals* or red hot *Iron*, if put into it. I ſhall not go about to ſhew what it is the *Air* loſes by being taken into the *Lungs*, or what it communicates to the *Blood* by the extream ramifications of the *Aſpera Arteria*, ſo intimately interwoven with the Capillary *Blood-Veſſels*; much leſs to explain how 'tis performed, ſince no diſcovery has yet been made, to prove that the ultimate Branches of the *Veins* and *Arteries* there, have any *Anaſtomoſes* with thoſe of the *Trachæa*; as by the *Microſcope* they are found to have with one another. But I rather chooſe to leave this Enquiry to the Curious *Anatomiſt*, to whom the Structure of the *Lungs* is better underſtood; and ſhall only conclude from the aforeſaid Experiment, that a naked Diver, without a *Spunge*, may not be above a couple of Minutes encloſed in Water, (as I once ſaw a *Florida-Indian* at *Bermudas*) nor much longer with a *Spunge*, without Suffocating; and not near ſo long without great Uſe and Practice: ordinary Perſons generally beginning to ſtifle in about half a Minute of Time. Beſides if the Depth be conſiderable, the preſſure of the *Water* on the Veſſels is found by Experience to make

the Eyes Blood-ſhot, and frequently to occaſion ſpitting of Blood.

When therefore there has been occaſion to continue long at the *Bottom*, ſome have contrived double flexible Pipes, to circulate *Air* down into a Cavity encloſing the *Diver* as with Armour, to bear off this preſſure of the *Water*, and to give leave to his Breaſt to dilate upon *Inſpiration*: the freſh *Air* being forced down by one of the Pipes with *Bellowes* or otherwiſe, and returning by the other of them; not unlike to an *Artery* and *Vein*. This has indeed been found ſufficient for ſmall Depths, not exceeding twelve or fifteen Foot: but when the Depth ſurpaſſes three Fathoms, Experience teaches us that this Method becomes impracticable: for though the Pipes and the reſt of the *Apparatus* may be contrived to perform their Office duly, yet the *Water*, its weight being now become conſiderable, does ſo cloſely embrace and claſp the Limbs that are bare, or covered with a flexible Covering, that it obſtructs the Circulation of the *Blood* in them; and preſſes with ſo much force on all the Juntures, where the *Armour* is made tight with *Leather*, *Skins* or ſuch like, that if there be the leaſt defect in any of them, the whole Engine will inſtantly fill with Water, which will ruſh in with ſo much violence, as to endanger the Life of the Man below, who may be drown'd before he can be drawn up. Upon both which accounts, the danger encreaſes with the Depth. Beſides a Man thus ſhut up in a weighty *Caſe*, as this muſt needs be, cannot but be very unwieldy and unactive, and therefore unfit to execute what he is deſigned to do at the Bottom.

To remedy theſe Inconveniences, the *Diving-Bell* was next thought of; wherein the *Diver* is ſafely conveyed into any reaſonable Depth, and may ſtay more or leſs time under *Water*, according as the *Bell* is of greater or leſſer Capacity. This is moſt conveniently made in form

of

of a *Truncate Cone*, the ſmaller *Baſis* being cloſed, and the larger open; and ought to be ſo poiſed with *Lead*, and ſo ſuſpended, that the Veſſel may ſink full of *Air*, with its greater or open *Baſis* downwards, and as near as may be in a ſituation parallel to the *Horizon*, ſo as to cloſe with the Surface of the Water all at once. Under this Couvercle the Diver ſetting, ſinks down together with the included *Air* into the Depth deſired; and if the Cavity of the Veſſel may contain a Tun of *Water*, a ſingle Man may remain therein at leaſt an Hour, without much inconvenience, at five or ſix Fathoms Deep. But this included Air, as it deſcends lower, does contract it ſelf according to the weight of the Water that compreſſes it; ſo as at thirty three Foot deep or thereabouts, the Bell will be half full of Water, the Preſſure of it being then equal to that of the whole Atmoſphere: and at all other Depths, the ſpace occupied by the compreſt Air in the upper part of the *Bell*, will be to the under part of its Capacity fill'd with Water, as thirty three Feet to the depth of the Surface of the Water in the Bell below the common Surface thereof. And this condenſed Air, being taken in with the Breath, ſoon inſinuates itſelf into all the Cavities of the Body, and has no ſenſible effect, if the Bell be permitted to deſcend ſo ſlowly as to allow time for that purpoſe. The only inconvenience that attends it, is found in the Ears, within which there are Cavities opening only outwards, and that by Pores ſo ſmall as not to give admiſſion even to the Air itſelf, unleſs they be dilated and diſtended by a conſiderable Force. Hence on the firſt deſcent of the Bell, a Preſſure begins to be felt on each Ear, which by degrees grows painful, like as if a Quill were forcibly thruſt into the Hole of the Ear; till at length, the force overcoming the Obſtacle, that which conſtringes theſe Pores yields to the Preſſure, and letting ſome condenſed Air ſlip in, preſent Eaſe enſues.

But the Bell descending still lower, the Pain is renewed, and again eased after the same manner. On the contrary, when the Engine is drawn up again, the condensed Air finds a much easier Passage out of those Cavities, and even without Pain. This Force on the auditory Passages might possibly be suspected to be prejudicial to the Organs of Hearing, but that Experience teaches otherwise. But what is more inconvenient in this Engine, is the Water entring into it, so as to contract the bulk of Air (according to the aforesaid Rule) into so small a space, as that it soon heats and becomes unfit for Respiration, for which reason it must be often drawn up to recruit it: and besides the Diver being almost covered with the Water thus entring into his Receptacle, will not be long able to endure the Cold thereof.

Being engaged in an Affair that required the Skill of continuing under Water, I found it necessary to obviate these Difficulties which attend the use of the common Diving-Bell, by inventing some means to convey Air down to it, whilst below; whereby not only the Air included therein, would be refresh'd and recruited, but also the Water wholly driven out, in whatever Depth it was. This I effected by a Contrivance so easy, that it may be wondred it should not have been thought of sooner, and capable of furnishing Air at the bottom of the Sea in any quantity desired. The description of my *Apparatus*, take as follows.

The Bell I made use of was of Wood, containing about 60 Cubick Foot in its Concavity, and was of the form of a Truncate-Cone, whose Diameter at Top was three Foot, and at Bottom five. This I coated with Lead so heavy that it would sink empty, and I distributed the weight so about its bottom, that it would go down in a perpendicular Situation and no other. In the Top, I fixed a strong but clear Glass, as a Window to let in the

Light

Light from above; and likewiſe a Cock to let out the hot Air that had been Breathed; and below, about a Yard under the Bell, I placed a *Stage* which hung by three Ropes, each of which was charged with about one Hundred Weight, to keep it ſteddy. This Machine I ſuſpended from the Maſt of a Ship, by a *Spritt* which was ſufficiently ſecured by *Stays* to the Maſt-head, and was directed by *Braces* to carry it over-board clear of the Ship ſide, and to bring it again within-board as occaſion required.

To ſupply Air to this Bell when under Water, I cauſed a couple of Barrels, of about 36 Gallons each, to be caſed with Lead, ſo as to ſink empty; each having a Bung-hole in its loweſt Part to let in the Water, as the Air in them condenſed on their deſcent; and to let it out again, when they were drawn up full from below. And to a Hole in the uppermoſt Part of theſe Barrels I fixed a Leathern Trunk or Hoſe, well liquored with Bees-Wax and Oyl, and long enough to fall below the Bung-hole, being kept down by a Weight appended; ſo that the Air in the upper Part of the Barrels could not eſcape, unleſs the lower ends of theſe Hoſe were firſt lifted up.

The Air-Barrels being thus prepared, I fitted them with Tackle proper to make them riſe and fall alternately, after the manner of two Buckets in a Well; which was done with ſo much eaſe, that two Men, with leſs than half their Strength, could perform all the Labour required: and in their deſcent they were directed by Lines faſtned to the under edge of the Bell, the which paſt through Rings placed on both ſides the Leathern *Hoſe* in each Barrel; ſo that ſliding down by thoſe Lines, they came readily to the Hand of a Man, who ſtood on the Stage on purpoſe to receive them, and to take up the ends of the *Hoſe* into the *Bell*. Through theſe Hoſe, as ſoon as their ends came above the Surface of the Water in

the

the Barrels, all the Air that was included in the upper Parts of them was blown with great force into the Bell, whilſt the Water entred at the Bung-holes below, and fill'd them: and ſo ſoon as the Air of the one Barrel had been thus received; upon a ſignal given, That was drawn up, and at the ſame time the Other deſcended: and by an alternate Succeſſion furniſhed Air ſo quick, and in ſo great Plenty, that I my ſelf have been One of Five who have been together at the Bottom, in nine or ten Fathoms Water, for above an Hour and half at a time, without any ſort of ill conſequence: and I might have continued there as long as I pleaſed, for any thing that appeared to the contrary. Beſides the whole Cavity of the Bell was kept entirely free from Water, ſo that I ſat on a Bench, which was diametrically placed near the Bottom, wholly dreſt with all my Cloaths on. I only obſerved, that it was neceſſary to be let down gradually at firſt, as about 12 Foot at a time; and then to ſtop and drive out the Water that entred, by receiving three or four Barrels of freſh Air, before I deſcended further. But being arrived at the Depth deſigned, I then let out as much of the hot Air that had been Breathed, as each Barrel would repleniſh with Cool, by means of the Cock at the Top of the Bell; through whoſe Aperture, though very ſmall, the Air would ruſh with ſo much violence, as to make the Surface of the Sea boyle, and to cover it with a white Foam, notwithſtanding the great weight of Water over us.

Thus I found I could do any thing that was required to be done juſt under us; and that, by taking off the Stage, I could, for a ſpace as wide as the Circuit of the Bell, lay the Bottom of the Sea ſo far Dry, as not to be over-ſhoes thereon. And by the Glaſs Window, ſo much Light was tranſmitted, that, when the Sea was clear, and eſpecially when the Sun ſhone, I could ſee perfectly well to Write or Read, much more to faſten or lay hold on any thing

thing under us, that was to be taken up. And by the return of the Air-Barrels, I often ſent up Orders, written with an Iron Pen on ſmall Plates of Lead, directing how to move us from Place to Place as occaſion required. At other times when the Water was troubled and thick, it would be dark as Night below; but in ſuch Caſe, I have been able to keep a Candle burning in the Bell as long as I pleas'd, notwithſtanding the great expence of Air requiſite to maintain Flame.

This I take to be an Invention applicable to various Uſes; ſuch as *Fiſhing* for *Pearl*, Diving for Coral, Spunges and the like, in far greater Depths than has hitherto been thought poſſible. Alſo for the fitting and plaining of the Foundations of Moles, Bridges, *&c.* upon Rocky Bottoms; and for the cleaning and ſcrubbing of Ships Bottoms when foul, in calm Weather at Sea. But as I have no experience of theſe matters, I leave them to thoſe that pleaſe to try. I ſhall only intimate, that by an additional Contrivance, I have found it not impracticable for a Diver to go out of our Engine, to a good diſtance from it, the Air being conveyed to him with a continued Stream by ſmall flexible Pipes; which Pipes may ſerve as a Clew to direct him back again, when he would return to the Bell. But of this perhaps more hereafter.

VIII. *Obſervations on the* Glands *in the Human* Spleen; *and on a Fracture in the upper part of the* Thigh-bone. *By* J. Douglaſs, M.D. *and* R. S. S.

THat Anatomy, as well as Phyſick and Surgery, has received much improvement from a careful and true obſervation of what was found in the Diſſection of morbid

morbid Bodies, will appear from the two following Instances, among many more that might be adduced for that purpose. For it is certain, that nothing has contributed so much towards forming a right Notion of the Nature of the several Diseases, and a true knowledge of the Structure of many Parts of the Human Body, as their appearance in a preternatural State.

My first Observation is of the Glands visible to the naked Eye, that appear dispersed thro' the Fibrous Substance of the Human *Spleen.* The Subject I found them in, was a Boy of about 4 or 5 Years Old, that died of a general *Atrophy*, or Consumption of all the Muscular Fleshy Parts of the Body, occasioned without all doubt from the numerous Glandulous Swellings scattered up and down the whole Mesentery; which by compressing the Lymphatick Vessels, called in this place *Vasa lactea*, prevented the access and supply of the Chyle, so necessary for the continued nourishment and increase of the Parts. For without the constant Recruit of this whitish Balsamick Liquor, the Mass of Blood will in a short time be unfit to perform any of those good Offices, which a fresh accession of Chyle qualifies it for.

In a piece of this *Spleen* we might see, without the assistance of a Glass, several round whitish Bodies of a pretty hard Consistence, and abundance of small white and softer specks; but both of the same nature. These, to me at least, appear to be so many distinct Glands become visible; which in a Natural State are only to be seen by a fine Glass, as the curious *Malpighius* first observed. *Vid.* his Treatise de *Liene, Cap.* V. *De quibusdam corporibus per Lienem dispersis. Minimæ hæ glandulæ*, says he, *non æque facile sese produnt in quocunque animalium Liene: imo solâ Lienis laceratione innotescunt in Bove, Ove, &c. In Homine vero difficilius emergunt: si tamen ex morbo universum glandularum genus turgeat, manifestiores redduntur, aucta*

ipsarum

ipsarum magnitudine, ut in defuncta puella observavi : in quâ Lien globulis conspicuis racematim dispersis totus scatebat. Which Case was the very same with mine.

The second Observation. We had still been in the Dark, about the nature of a Luxation of the Head of the Thigh Bone, had we not carefully examined the Part in the dead Body. For by that sort of enquiry, the common mistake of Surgeons was detected, and what was esteemed and treated by them as a Luxation of the Head of the *Femur*, was discover'd to be nothing else but a Fracture of the same Bone, near its Neck; the globular Head being still retained close in its own Socket, called the *Acetabulum Coxendicis*.

Amongst all the Writers of Surgery and Anatomy, I know but three that were apprised of this mistake: the first was *Ambrose Paree*, the second Dr. *Ruysch* at *Amsterdam*, and Mr *Cheselden*, a Member of the *Royal Society*; whose Observations on this Subject I intend to communicate at another time, together with an account of the true Structure of this Joint; in which I will consider the depth of the Articulation; the wonderful strength of the Muscles that surround it; the many strong ligaments that bind the Head within the Socket; the smallness of the Neck of the Bone; its poreous and spungy Substance, which makes it much weaker than the rest; and last of all the disadvantageous oblique position of this Neck, which exposes it the more to outward accidents. From a review of such like Considerations, it will plainly appear that a Fracture can much more easily happen, than a Dislocation in that Part from an external Cause.

This *Os femoris* belonged to an old Woman turn'd of Fourscore, who only fell from her Chair whereon she was sitting, and thereby suffered this breach of continuity in the Substance of the Bone. She lived three Weeks after it; and tho' it never was reduc'd, yet she complained of

very little or no pain, which may ſeem very extraordinary. It is obſervable that the Fracture is not only Oblique, near the Neck of the Bone; but that each *Trochanter*, *i. e.* the two *proceſſes* near its *Cervix*, are likewiſe broke ſhort off; and that they were both drawn up almoſt as high as the Head of the Bone it ſelf, by the ſtrong contraction of the *Glutei* and other Muſcles.

IX. *An Account of a Book.* DISSERTATIO de DEA SALUTE, In qua illius Symbola, Templa, Statuæ, Nummi, Inſcriptiones exhibentur, illuſtrantur. Auctore *Guilhelmo Muſgrave* G. F. è Coll. *Exon. Oxonii*: Typis *Leon. Lichfield*: Impenſis *Phil. Yeo*, Bibliopolæ *Exon.* Anno MDCCXVI.

THe Author of this Treatiſe, as the occaſion of it, obſerves that the little God *Teleſphorus* had juſt cauſe to complain, that ſo much reſpect was paid to *Dea Febris*, and a Book lately publiſht *de Dea Podagra*, yet no ſuch Honour was done his Mother Ὑγίεια, (who certainly was more to be eſteemed, than all the Tribes of Diſeaſes). Upon this Conceit, he took what Books he had in his reach, of the antient Latin and Greek, and having collected out of them, what he met with relating to this Goddeſs, put it together, as now it appears in Print.

It conſiſts of VI Chapters: of which the firſt is Introductory, ſpeaks of Health in general, has, in praiſe of it, that memorable Ode of *Ariphron* the *Sicyonian*, publiſht by *Athenæus*, and tranſlated by *Sennertus*; together with a Hymn, ſaid to be compoſed by *Orpheus*, on the ſame Subject;

Subject; he ranks this Goddess among the *Dii Medioxumi*, and gives an account of Her from the Mythologists.

Chap. II. de *Salutis Symbolo*, which he takes to be a *Serpent*, an *Omen* of good things, and a frequent Companion of the Gods; as appears from *Virgil*, *Valerius Flaccus*, *Statius*, and *Macrobius*. He mentions another *Symbolum Salutis*, used by *Antiochus Soter*, now to be seen in some of his Coins, and sometimes printed in Physick Books in the following form.

The III. Chapter treats of the *Temples* erected to this Goddess; in which Prayers were offer'd up to Her, sometimes for the Health of private Persons, and often for the welfare of the Publick: of which many Instances are here produced. To the account of *Temples* is subjoyned the Divination, known by the name of *Salutis Augurium*, which is often mentioned by Roman Authors, as *Dion Cassius*, *Tully*, and *Tacitus*.

In the IV. Chapter, the *Statues* of this Goddess are consider'd. Some of these represent her and *Æsculapius* together, *tamquam* Θεὸς Συμβώμος. *Pliny*, *Pausanias*, *Lucian*, *Plutarch* and *Monfaucon* afford instances of this kind.

Coins relating to this Goddess, come next in view, Chap. V. These either express her *Effigies*, or her *Worship* under some Symbol or other. Of the first order, one out of *Fulvius Ursinus* has the Head of the Goddess, with SALUS inscribed. Another like this, is in *Gevartius*. Some, together with this Goddess have also her Father *Æsculapius*; as a *Coin* of *Trajan*; and in one of *Aurelius Antoninus*, struck in memory of the Remedies reveal'd to him in a Dream, which cured the Emperor of

a *Sputum Sanguinis* and *Vertigo.* As indeed most of these Coins were (in all likelihood) struck on some such occasion, *viz.* the Recovery of some great Person. *A noble Expression of Gratitude, fit and worthy of imitation.*

Of the second Order is the Coin of *Dossenus*, having an Altar with a Serpent, taken from *Ursinus.* Another of *Tiberius*, with an Altar and SAL. AUG. Another of *Nero* in which is a *Serpens Tortuosus*; with many others.

The *Gemmæ* of the Antients, according to *Leonardus Augustinus*, are of use to set forth the Sacrifices made of old to this Goddess. One of these *Gemmæ* represents *Æsculapius*, his Daughter *Hygieia* and Grand-Son *Telesphorus* so call'd ἀπὸ τοῦ τέλους φέρειν, *à Valetudine post morbum confirmata.* This God, being Young and Tender, had (I suppose, by the care of his Mother *Hygiæa*) a *Bardocucullus*, or Cloket, to keep him from taking Cold. These three Gods are represented in one Figure, with the following Inscription under them, ΕΥΖΕΤΕ ΜΕ, i. e. *Salvere me Jubete*, which *Augustinus* happily conjectures to have been a Form of Prayer offer'd up to them.

In the last Chapter come the Inscriptions, which are taken out of *Gruter* and *Reinesius.* They are chiefly to *Æsculapius* and *Hygiæa*; but to confirm the Divinity of *Telesphorus* the little God of the *Pergameni*, he is mentioned in one of their Inscriptions dug up at *Verona.*

The Author makes no manner of doubt, but there are many more Coins and Inscriptions relating to this Goddess to be found in other Books. But these being all, or most of such as came in his way, and enough to give a Specimen of the Devotion paid by the Antients to this Goddess, he has contented himself with this small Number; leaving it to others to make such Additions, as from greater Opportunities and Abilities, they shall think fit.

LONDON, Printed for W. INNYS, at the *Princes-Arms* in St. *Paul's Church-Yard.* MDCCXVII.

M. Ste Guche Sculp

N. Bundock delin.

Fr. Gucht Sculp.

PHILOSOPHICAL TRANSACTIONS.

For the Months of *Octob. Novem.* and *Decem.* 1716.

The CONTENTS.

I. *Obſervationes nonnullæ Planetarum Primariorum, ac ſpeciatim Stellæ cujuſdam fixæ corporis* Jovis *interpoſitu occultatæ; a R. D.* Jacobo Pound, *R. S. Soc. nuper habitæ, & ab eodem cum Regia Societate communicatæ.*

ABſque Obſervationibus accurate inſtitutis, irrito ſane conamine veræ motuum Cœleſtium Theoriæ Calculuſque cælo conformis expeterentur. Quod autem tam diu latuerit Veritas Aſtronomica ex eo factum eſt, quod Veteres Artifices, & imprimis *Ptolemæus*, nulla omninò nobis tradiderint Obſervata, præter ea quibus ad ſtabiliendas Hypotheſes Tabulaſque ſuas Principiis uſi ſunt: Cum tamen multo magis ex re fuiſſet *Timocharidis*, *Ariſtilli*, *Hipparchi* ſuaſque *Ptolemæi* ipſius τηρήσεις debitâ fide ad nos tranſmiſiſſe, & numerorum ſuorum à Cælo diſſenſus ingenuè annotaſſe (ad exemplum magni *Hippocratis*, cui minime pudori fuit ſub curâ ſuâ mortuos æque ac ſanitati reſtitutos Poſteritati conſignaſſe) potius quam vanâ quâdam gloriolæ ſpecie, Sphalmata ſua ſibi ipſis perſpecta tacuiſſe; celatis ſc. Obſervationibus iis quibus Tabulas ſuas male reſpondentes experti ſunt. Hoc autem ante *Tychonem Brahe* omnium pene gentium Aſtronomis commune vitium.

Ex quo autem à ſagaciſſimo *Keplero* adinventa eſt genuina motûs planetarii Theoria, ejuſdemque etiam Geometrica veritas à *Newtono* magno nuper patefacta, nonnullos ingens inceſſit cupido calculum Aſtronomicum Cœlo omnino conſentaneum exhiberi poſſe; & in eum finem, præter Obſervatores Principum, Privati etiam plurimi apud nos obſervandis Sideribus inhiant: Horum autem nemini arte vel induſtriâ ſecundus Rever. D. *Jacobus Pound*, Reg. Soc. Socius, Obſervationes

ſequentes

ſequentes à ſe nuper apud *Wanſted* habitas, Tubiſque longiſſimis & Micrometro quantum fieri potuit accuratè captas, cum publico communicandas obtulit.

Anno 1715. *Auguſti* 21°. 8ʰ. 25′½ Temp. æq. *Mars* præcedebat. ſcil. Aſcenſione rectâ, Mediam frontis Scorpii (*Bayero* δ) 6′. 54″ Borealior Fixâ 9′. 47″.

Sept. 18°. 7ʰ. 30′. *Mars* præcedebat Claram in pede Serpentarii (*Bayero* θ) 17′. 48″. & eandem habuit Declinationem accuratè.

Novemb. 30. 18ʰ. 8′. *Saturnus* præcedebat γ, ſive ſecundam Alæ Virginis 23′. 19″ & erat Fixâ Auſtralior 25′. 3″. *Decembris* autem 4ᵗᵒ 17ʰ. 25′. præcedebat eam 10′. 50″ & Auſtralior erat 29′. 00″.

Anno autem 1716. *Feb.* 22°. 7ʰ. 23′ T. æq. *Mars* præcedebat ζ Piſcium, ſive Sequentem trium clariorum in Lino Auſt. Piſcium 3′. 35″. eâdemque Auſtralior erat 1′. 23″, quam proinde obtegere debuit ante bihorium, forſan corporaliter.

Junii 22°. 8ʰ. 52′. T. æq. *Venus* ſequebatur Cor Leonis 34′. 50″, & fixâ Auſtralior erat 7′. 23″.

Aug. 14°. 15ʰ. 00′. *Jupiter* præcedebat *Propoda* uno tantum minuto, cum Declinatione Bor. minore 14′. 26″.

Aug. 19°. 13ʰ. 2′ *Jupiter* præcedebat fixam Teleſcopicam, quæ vocetur *b*, 50′, 08″ eandem habens Declinationem accuratè.

Aug. 24°. 12ʰ. 19′ *Jupiter* Micrometro diſtabat a prædictâ *b*, 5′. 54″, ſimulque ab aliâ Fixâ clariore *a* 7′. 17″. Diſtantia fixarum 12′. 31″. Tunc minor *Jovis* diameter 0′. 38″.

Sept. 12°. 17ʰ. 00 *Venus* recens a Statione ſecunda ſequebatur Teleſcopicam 17′. 40″, eâque Auſtralior erat 5′. 30″. Hæc autem Fixa tunc occupavit ♌ 27°. 44′½ cum Lat. Auſt. 5°. 39.

Octob. 15°. 17ʰ. 12½ *Venus* diſtabat Microm. a Fixâ τ in Crure Leonis 27′. 55″.

Novemb.

Novemb. 20°. 6h. 18′½ *Jupiter* regressus est ad stellas *a* & *b*, ad quas observatus est *Aug.* 24°. & distabat à *b* 6′. 21″, ab *a* vero 11′. 36″.

Novemb. 21. 7h. 38′, *Jupiter* distabat à *b* 9′. 19″, & ab *a* 3′. 48″. Fixæ inter se 12′. 30″. *Jovis* diameter minor sive Axis 0′. 44″. Deinde hora 18h. 50′. visa est stella *a* limbo *Jovis* quasi adhærere, eratque quasi ⅔ semidiametri vel 0′. 15″. centro *Jovis* Borealior. Juxta has autem observationes constat medium Occultationis Fixæ, interposito *Jovis* corpore, contigisse *Nov.* 21°. 19h. 55′. vel proximè. Deinde

Nov. 30. 5h. 41′ ♑.	præced. *Propoda*	12′. 36″	*Australior*	7′. 36″
Dec. 4. 6. 0 ♑.	Sequeb. eam	22. 49	——	7. 47
Dec. 5. 6. 0	Repet.	31. 35	——	7. 50
Dec. 6. 6. 0	Repet.	40. 30	——	7. 52
Dec. 7. 6. 0	Iterum	49. 15	——	7. 54

Ex his ultimis Observationibus liquet *Jovem* & *Propoda* eandem habuisse Longitudinem *Dec.* 1°. 15h. 29′, quo tempore *Jupiter* Australior erat stella 7′ 40′. Ex iisdem etiam constabit *Jovem* in opposito Solis fuisse, quoad Longitudinem, *Dec.* 6°. 12h. 46′.

N. B. Stellas illas Telescopicas *a* & *b* vocatas, haberi in Catalogo Fixarum Britannico D. *Flamsteedii*, ubi ipsi *a* Locus datur, ad annum Scil. 1690 ineuntem, ♊ 27°. 54′. 29″ cum Lat. Aust. 0 21′. 55″; alteri vero *b* ♊ 28°. 5′. 24″ cum Lat. Aust. 28′. 5″. Neque aliam novimus Fixam à corpore *Jovis* occultatam & ab invento Telescopio observatam, præter jam dictam Stellam *a*; ad quam olim arctissime applicabatur *Jupiter*, ante annos 83. *Decembris* nono St. nov. Anni 1633. Vesp. cum *Gassendus Diniæ* vidit *Jovem* huic Fixæ conjunctum, nec nisi quinque semidiametris corporis sui superiorem. Unde calculo debitè inito, constabit Nodos hujus Planetæ Planumque Orbis ejus, situm in Sphæra Fixarum servare immobilem, vel saltem lentissimo motu cieri. Vide *Gassendi* Observ. Tom IV. p. 162.

II. *A Description of that curious Natural Machine, the* Wood-Peckers Tongue, *&c. By* Richard Waller *Esq; late Secretary to the* Royal Society.

THE *Picus Martius* or Wood-Pecker has several Particularities in the Structure and Mechanism of its whole Body, which may deserve a nice and accurate Observation and Description: all which are wisely contrived and adapted, either for catching the Food and Sustenance of the Individual, or continuing the Species.

That this Bird makes a round Hole even in sound and hard Trees, such as the Oak, Horn-beame, Beech and the like, is commonly observed; and that within these Holes, the Hollow being enlarged, the Nest is made, the Eggs laid and Hatch'd; and the young Brood fed, as by other Birds.

For this purpose, that he may be enabled to perform such hard work, the Muscles of his Neck, Breast, and Thighs, are exceeding strong in proportion to the bigness of the Bird: he has also a very firm strong sharp Bill, his Legs are strengthned with very strong Tendons; and his Toes, which are two before and two behind, (as it is in some other Birds) are provided with sharp strong hooked Claws or Talons: Besides this, his Tail consists of ten very stiff large and strong Quills, firmly set into a robust strong *Uropygium* or Rump; so that when he has fastened his Claws and Feet into the Clefts and Inequalities of the Bark of the Tree, he claps his strong Tail-Feathers against the Body of the Tree; and so stands with his Head erect, to give the strokes with his Bill with the greater Force.

That he is of the Insectivorous kind is certain, and lives not only upon Insects catcht creeping on the outside of Trees, but also on such as are under the Bark between the Bark and Wood, as likewise on those in rotten Wood; and as I am very confident on Worms and other Insects in the Ground: for I have frequently observed the Roots of their Bills very dirty, as it is in *Crows* and *Rooks, &c.* Whence I suppose he strikes his long sharp Bill into the soft Earth to take the Worms out of it. I have also found their Craws full of small Ants.

But the Contrivance and Mechanism of the Tongue in this Bird being the most Remarkable, I shall presume to lay before this Illustrious Society some few Remarks of this curious Contrivance of Nature, with some Figures I have drawn by the Parts themselves, in order to explain the whole.

This Bird is known to throw out a long, slender, round Tongue, to a considerable distance beyond the End of his Bill; and to draw it in again very quick into his Mouth or Bill, with the caught Insect spitted on the Tip of it.

The *Chameleon* indeed darts out its Tongue to a considerable length; and having intangled the Fly in the glutinous Matter at the End of it, draws it into its Mouth, together with the Prey; but the Mechanism in that Animal is wholly different from that of the present Subject: as may be seen by the Account the Gentlemen of the *Academy Royal* give thereof, in their *Memoirs* for a History of *Animals.*

The Protrusion therefore of the Tongue to the length even of three or four Inches in this Bird, being very extraordinary, and the Mechanism of the several Parts for that end no less Curious; several learned and diligent Enquirers have attempted to explain it; but I am of opinion

opinion they have been, in ſome Particulars at leaſt, miſtaken. I ſhall mention ſome of theſe.

The learned and curious Enquirer into Nature, Monſ. *Perault*, deſcribes it after this manner *.

This long Tongue he throws out by the means of two ſmall bony Cartilages, about ſeven Inches long, and of the thickneſs each of a middling Pin, which are perfectly Smooth and Slippery. Theſe two Cartilages are united at the End, and being in this place covered with Fleſh make the fore-part of the Tongue. The reſt of theſe Cartilages are ſeparated from each other, and paſs turning round under the Ears; and then riſing up behind the Head, where they meet again, they paſs over the Top of the Head, and ſo extend themſelves to the Root of the Beak. Theſe Cartilages which make the hinder part of the Tongue, are alſo incloſed in a Channel Fleſhy on the out-ſide, and whoſe inſide is covered with a very ſmooth ſlippery Membrane.

Now theſe Fleſhy Channels, which incompaſs and keep in theſe Cartilages, are the Muſcles by which the Tongue is moved: for having their origine at the *Larynx*, and their inſertion at the extremities of the Cartilages, it comes to paſs, that when thoſe Muſcles of the two Fleſhy Channels, which make the hinder part of the Tongue are ſhortned, they force the fore-part of the Tongue out of the Beak, by drawing the poſterior or fartheſt end nearer to the *Larynx*: and on the contrary, when the Fleſhy Channel which makes the anterior Part acts, it draws the fore-part of the Tongue into the Bill towards the *Larynx*.

This Mechaniſm of making a hard part, ſuch as the bony Cartilages are, to come out and return into another,

---

* *Eſſays de Phyſique*, Tom. 3. Part 2. p. 148.

ſuch as the Canals are, by the means of Cords drawing them, which are the Muſcles, is made uſe of in Coaches to pull up the Glaſſes of the Doors; for the String, being faſten'd to the lower part of the Glaſs-Frame, makes it riſe when drawn, which reſembles that action of the Muſcles by which this Tongue is moved.

Of theſe Cartilages and other Parts, and of the Head of the Bird, Mr. *Perault* gives the Figures.

Either the Wood-peckers in *France* are different from ours in *England*; or this Figure of the Head is very ill deſigned; it being much too broad and large, and the Beak too ſhort. Beſides he makes the two Cartilages to come to the Root of the Beak ſeparately, one on one ſide, the other on the other ſide of it; whereas in all the Wood-peckers Heads I have met with, the two Cartilages joyn cloſe together about the Top of the Head, and thence proceed joyned, tho' not faſtned to one another, a little ſlanting towards the right Noſe-hole, where they end together.

Beſides upon viewing and examining ſeveral Subjects, I could not find them agree in divers particulars with his Account and Explication. For the Muſcles which are faſtened to the end of the Cartilages at the Root of the upper Beak, are not inſerted at the *Larynx*, but paſs on and are faſtened to the lower Bill. This pair I take to be the Muſcles chiefly concerned in forcing the Tongue out of the Bill. There is another pair of Muſcles, which, being faſtened to the place where the two bony Cartilages are articulated with one ſingle Bone in the fore-part of the Tongue; (as will be ſhewn in the 4th Figure) is, as I apprehend, the chief pair concerned in the drawing the Tongue with its Prey into the Mouth. Theſe proceeding from that articulation of the Cartilages as far as the *Larynx*, (each of them ſending a Branch to the *Cartilago Scutiformis*) from thence go on along with

the Neck, (tho' not faſtned to it) till they come within the Cavity of the *Thorax*, where they are inſerted under the *Clavicula* or *Merry-thought-bone*, as 'tis called. This pair is repreſented by *k. k.* in the ſecond Figure; and by *q. q.* in the Firſt.

There is likewiſe a very ſlender white Thread, (whether Tendon or Nerve, I am uncertain) which accompanies this Muſcle its whole length; and which drawn gently, (for fear of breaking) pulls in with it the end of the Tongue. As there is ſuch another all along the *Vagina* to the End at *c*.

Volker Coiterus, as he is mentioned by *Gerard Bläſius*, in his *Anatome Animalium*, *Cap.* 24. *p.* 64. treating of the Tongue of this Bird, makes it to be made of three ſlender Bones, round, and as he ſays bound together, (*invicem colligatis*) which is a Miſtake; for tho' reckoning the two bony Cartilages for *Oſſicula*, yet the third is not bound up with them, but articulated to the End of them. The ſame Perſon ſays the Tongue may be thruſt out to the length of an Inch and a half, whereas when drawn in, it is ſcarce half an Inch long; when in reality it may be thrown out near four Inches; and I believe cannot be drawn in, ſo as to be leſs than an Inch and quarter, *viz.* to that place where the two Cartilages are articulated with the ſingle Bone. Beſides he makes the uſe of the long flat Muſcle running over the Top of the Head, to be (if I rightly apprehend his meaning) to draw the Tongue to the upper Jaw, whereas their uſe is for thruſting the Tongue out of the Birds Mouth.

But this Perſon having given no Figures, has rendred what he ſays leſs intelligible; tho' indeed he mentions two pair of Muſcles, as there are ſo many chiefly concerned, yet there are at leaſt two other pair, that aſſiſt the Performance.

Where-

Wherefore I ſhall leave him, and proceed to the Account given by *Alphonſus Borellus* in his Treatiſe *de Motu Animalium*, *part.* 2. *pag.* 24. which is in ſeveral Reſpects likewiſe unſatisfactory, and the Figure given by him to explain it very defective and ill deſigned.

He makes the pair of Muſcles concerned in thruſting the Tongue out, to be faſtened indeed as they are to the lower Beak towards the Point; but then he makes their Inſertion at their other End to be at the extremities of the two *Oſſa Hyoïdea*; whereas they really reach to the very end of the long Cartilages that go round the Head: Theſe by another Miſtake, he makes to be the Retractors of the Tongue, and joins another pair as Aſſiſtants in the ſame Action, which he makes to be twiſted ſpirally about the *Trachia.* None of all which agree with the Subjects I have met with, as will be ſeen by the deſcriptions of my Figures.

In the Hiſtory of the *Academie Royale des Sciences*, publiſh'd in Latin by Monſ. *du Hamel*, 1698. *Lib.* 4. *Cap.* 5. There is another Deſcription of this admirable Contrivance of Nature, by Monſ. *Mery*, read at a Meeting of the *Academie*, *November* 16, 1695.

In this he differs from both *Perault* and *Borelli*, taking the Horny End and Bone to which it is joined, to be only the Tongue properly ſo called, and that the next two Bones anſwer the *Hyoïdes* with the long Cartilages annexed to them. But even in this he ſeems to me not to be ſo clear; confounding, as I apprehend, the two Bones with the Cartilages. He deſcribes the *Vagina*, in which the Bones and part of the Cartilages are encompaſſed, and which is faſtned to the Horny end, and is protruded and drawn back with the Tongue: he takes notice of the little ſharp Points or Prickles on the Horny Part being moveable, and with their Points bending towards the Throat; but I apprehend it is a Miſtake to make the

Mucous matter Glutinous which is furnished by the two *Pyramidal Glands*; for I take the use of that *Mucus* to be chiefly, if not only, to lubricate the Passage in the *Vagina*, for the more easy slipping of the Cartilages therein.

He describes the Muscles for exerting the Tongue, and extends them from their Insertion at the lower Beak, to the End of the springy *Ligaments*, as he terms what I call *Cartilages*; to which he adds another small Ligament capable of Extension, at the End of the other two next the Nose, which when the Tongue is thrust out is relaxt and stretch'd. He also describes the pair of Muscles fastned to the Root of the Tongue and *Os Hyoïdes*, serving to draw the Tongue into the Mouth: these he makes to be wound round about the *Aspera Arteria* once or twice, in which I think there is some Mistake; being of opinion the Mechanism for this Action of drawing in of the Tongue, is different from what is here described, as in the Explication of the Figures I shall endeavour to shew. But not to insist on all the Particularities mentioned in this Description, which, for want of more Figures to explain the several Parts in so curiously contrived an Organ, is not so clear as might be desired (there being but One, and that a wooden Cut of the Head, Tongue, Bones, Muscles, *&c.*) I shall now proceed to the Explication of the several Draughts I made, with what exactness and care I could, in 8 or 10 several Subjects.

FIGURE the First.

Represents the Head with part of the Neck of this Bird, the Skin being taken off, in which,

A. The Scull, having two shallow Grouves or Channels, or rather one broad one with a small Rising in the midst, on the *Sinciput* or Back-part, from each side of the Neck to the Top of the Head, where they unite into one.

one, which paſſes ſlanting towards the right Side, and ends at the Hole for the Noſtril on that ſide at *c*.

b. Is the Hole or paſſage for Hearing.

d. A large white Gland, containing a glutinous Liquor, almoſt like Cream as to Colour and Conſiſtence, which empties it ſelf into the Mouth; I ſuppoſe to lubricate the Cartilages.

e. The Eye, which has a Bony Ring, encompaſſing the *Iris*.

f. Part of the Tongue, which in this Figure is repreſented as almoſt all drawn into the Mouth, of which more when I come to deſcribe the Cartilages, *&c*. In the 2d Fig.

g. Part of the Neck, which is large and furniſhed with very ſtrong Muſcles.

h. The *Oeſophagus*, opening very wide at the *Fauces*, and wholly Muſculous.

i. i. i. A long, but thin and flat Muſcle in reſpect of its breadth, which is about $\frac{1}{8}$ of an Inch, reaching from the end of the Cartilage at *c*, to the under Bill or Beak at *k*, to the inſide of which it is very firmly faſtned; as is ſuch another on the other ſide.

k. The under Bill very ſtrong and ſharp pointed, articulated with the Scull a little behind the Ear-hole *b*.

l. l. l. The Cartilage on one ſide, the other being exactly the ſame. This Cartilage is Round, very Smooth, Even and Slippery, about the ſize of a pretty large Pin; and reaches, when the Tongue is drawn in and the Muſcle *i. i*. relaxed, from the Root of the upper Beak at *c*, to the Root of the Tongue properly ſo called, or to the Bones of the Tongue where they are articulated, being bent like a Hoop as in the Figure, ſlipping very freely in a Sheath or Membranous *ductus* faſtned on the outward or convex Edge of the flat Muſcle *i. i. i*. which Muſcle accompanies it from its end at *c*, almoſt to the

end

end of the Canal or Sheath, which opens at a Hole a little before the *Larynx*; (as will be ſhewn in the third Figure) and thence the Muſcle proceeds to its Inſertion into the lower Beak at *k*. From the concave Edge of this Muſcle there is a thin and tranſparent but very ſtrong Membrane, ſtrained like a Drum-head to the Scull at *m*, where it is very ſtrongly faſtned; this Membrane is furniſhed with Capillary Veins and Arteries, and doubtleſs is Nervous. *n. n.* repreſents this Membrane. This Cartilage, when the Tongue is exerted, parts about half an Inch from the Root of the Beak at *c*.

o. o A pretty large Vein and Artery.

p. p. A Muſcle reaching from one Jaw to the other, under the Throat, ſerving as a Bandage to keep in the Cartilages, and the Root and *os Hyoïdes* of the Tongue, as I may call it, from ſtarting out at that Part where are the articulations of the Cartilages with the Bones, when by the Muſcles, inſerted into the Sheath at or near *p* and thence paſſing to the end of the Tongue, it is drawn into the Mouth.

q. q. One of the laſt mentioned Muſcles, which is round, of the ſize in the Figure, and faſtned to the Breaſt of the Bird, cut off at *r*.

s. The *Aſpera Arteria* conſiſting of perfect Rings.

t. t. A Muſcle accompanying the *Aſpera Arteria*.

FIGURE the Second.

A. A. The under part of the lower Bill.

B b. The Tongue.

b. The Place where the two Cartilages and two Bones repreſented by *f. f.* in Fig. 4. are brought into and incloſed in one Tube or Membranous Sheath.

C. C. Two Glands diſplaced in this Figure.

c. c. Two Muſcles attending theſe Glands, and faſtned near the end of the Bill.

 D. D. The

d. d. The two Bony Cartilages, bent, and paſſing on each ſide of the Neck, but united at *b*.

e e e, e e e. The pair of Muſcles, one attending each Cartilage from the End of it at the upper Beak, and firmly adhering to the *Vagina*, in which it ſlips, till about *f f*.

f. f. The place where theſe Muſcles leave the *Vagina*, and paſs on to the inſide of the Bill, where they are inſerted. Their Action is to thruſt the Tongue forward, or out of the Mouth.

g. g. A pair of Muſcles faſtned a little below the *Larynx*, to the Muſculous part of the *Aſpera Arteria*, at *i*; the other end of them going up to the place *b* at the Root of the Tongue, whence they go on incompaſſed by the *Vagina* to the articulation of the Cartilages with the two Bones. I take their Action to be to draw the end of the Tongue towards the *Larynx*.

k. k. Two Muſcles faſtned at one end within the *Thorax*, under the Merry-thought or *Clavicula*; and at the other Ends to the articulation of the Cartilages with the two Bones of the Tongue, marked *f. f.* in Fig. 4. Theſe have the forementioned Nerves accompanying them. I take theſe to be chiefly concerned in drawing in the Tongue; each of theſe ſends a Branch to the Griſle at the Top of the *Aſpera Arteria* at *n*.

l. l. l. l. Two Muſcles running along and faſtned to the Sides of the *Aſpera Arteria*, from the *Thorax* to the place where they are united, where each of them ſends a Branch; which binding over the Bones and Cartilages goes on to the *Fauces*, where they are inſerted.

m. Part of the *Gula*.

n. A Cartilage at the Top of the *Aſpera Arteria*.

o. o. The *Aſpera Arteria*.

p. The Neck bending like an *S*. The Wind-Pipe

and

and *Gula* in this Bird paſs always on the right ſide of the Neck.

FIGURE the Third.

A. A. The two long flat Muſcles repreſented by *i. i.* in the firſt Figure. Theſe join cloſe to one another at the Top of the Head, and ſo paſs on together to the end of the Cartilages; to the end of which, as I take it, they are faſtned: from whence a ſlender weak kind of Ligament reaches to, and is inſerted at, the right Noſe-hole, at the Root of the upper Beak. This Ligament is relaxt when the Tongue is thruſt out.

b. b. The Cartilages running in their *Vagina* on the out-ſide of the ſaid Muſcles.

c. The *Larynx* or paſſage to the *Aſpera Arteria.* I obſerved no *Epiglottis.*

d d. Two Articulations or Joints in the under Beak or Bill.

e. The Hole or Paſſage, whereby the Tongue in its *Vagina* comes out and is drawn in again.

f. What I call the Tongue, in the inſide of which the two Cartilages are brought together, till they are both articulated to one ſingle Bone, at the end of which is the Horny barbed Tip.

g. One of the Pyramidal Glands.

h. The lower Bill.

FIGURE the Fourth.

A. That part which I think may moſt properly be called the Tongue; a ſmall Bone running thro' it: This, as far as *c*, is Flat and Thin at the Sides. It is cut away at *d*, to ſhew the Bones within it.

b. The Horny Tip of the Tongue, about a quarter of an Inch long, ſtrong and ſharp, furniſhed with four or five Barbs on each ſide; (not with an infinite Number as

Coiterus ſays) Theſe Barbs are ſharp and moveable, like the ſmall Teeth at the Root of the Tongue, and beginning of the *Gula*, in the *Pike* and *Jack-Fiſhes*, in that of Eagles and the like; ſo as to let the Prey ſlip eaſily on, but not ſo eaſily get off again.

c. The End of the Bone of the Tongue where the two bony Cartilages are articulated.

d. The place where the upper part of the Tongue is cut away to ſhew the Bone.

e. Several ſmall Tendons, or rather, as I take them to be, Nerves running thro' the Tongue. Of theſe ſome go to the End of the Cartilages, others accompany the Muſcles to the Neck.

f. f. Two Bones or Cartilages, which in the Bird, are united by a thin Membrane as far as the next joynt, ſo as to open aſunder to ſome diſtance, but not to ſeparate quite. Theſe two Bones ſeem to anſwer to the *oſſa Hyoïdea* in other Creatures. At the Place marked *g. g.* the Muſcle that draws the Tongue into the Mouth is faſtned, or rather leaves the Tongue at that place; it having its Inſertion near to the End of it. This Muſcle is repreſented by *q. q.* in the firſt Figure.

h. h. The two bony and ſpringy Cartilages running on each ſide of the Neck; which being joyned cloſe together on the Top of the Head, paſs ſo joyned to the Noſtril, or Noſe-Hole on the right Side.

From the Conſideration and comparing of theſe four Figures, the true Mechaniſm and Motion of the Tongue, ſeems to be in ſhort thus: The two long Muſcles inſerted near the End of this lower Beak, and reaching to the End of the Cartilages, being contracted, the round Hoop of the Cartilages is drawn up, from each ſide of the Neck, cloſe to the Pyramidal Glands; and at the ſame time the Muſcles that draw the Tongue into the the Mouth being relaxed, and the Articulations at *c*

and

and *g. g.* in the 4th Figure, brought near to a ſtraight Line, the Tongue is thrown out to the length of 4 or 5 Inches.

But when thoſe long Muſcles are relaxed, the pair of Muſcles repreſented by *k k.* in the ſecond Figure, being contracted, draw the Articulations *g. g.* where they are faſtned, down into the Throat or wide looſe Skin of the Neck; and at the ſame time the Cartilages opening into a wide Hoop, the whole Tongue is drawn into the Mouth.

FIGURE the Fifth.

A. The Scull.

b. The ſhallow *Crena* or Groove, for the Cartilages.

c. The Place of their Ending at the right Noſe-Hole.

d. The Orbite of the Eye.

e. The Hole for the Optic Nerve.

f. A Hole or paſſage thro' from one Orbite to the other.

g. A Bone covering the Hole to the Ear.

h. The lower Jaw and Bill.

i. A Ridge or *Proceſſus* in the Scull, beginning at the Root of the upper Bill, and keeping the two Ends of the bony Cartilages in their place on the right Side.

k. The *Os Jugale.*

l. The upper Bill.

FIGURE the Sixth.

Repreſents the right Leg and Foot, in which there are two *Digiti* before, and two behind. The Strength, Largeneſs, and Sharpneſs of the hooked Claws or Talons are remarkable.

FIGURE

FIGURE the Seventh.

A. The *Oesophagus.*

B. The *Ingluvies* or Crop, partly Musculous, and lined with a Glandulous Coat. This I found quite filled with small black Pismires; as also

C. The *Ventriculus* or Gizzard, which joyned close to the *Ingluvies.*

d. d. d. The Intestines nearly of the same bigness for the whole Length.

e. The beginning of the *Rectum.*

f. The *Pancreas.*

FIGURE the Eighth.

One of the middle pair of Feathers of the Tail, in which the great Strength of the Quill for so small a Feather, and its bifurcate End, are very remarkable.

FIGURE the Ninth.

The Roof of the Mouth, where 'tis observable, that the *Rima* or Passage for the Air to the Nostrils, is beset on each side with a Row of 10 or 12 little sharp Teeth, with their Points standing inwards, towards the *Gula.* These take the Prey from the end of the Tongue whose Barbs or Prickles are moveable, and are to keep it from going out of the Beak again with the Tongue, and from hence it is conveyed to the Swallow.

III. *The*

III. *The Natural History and Description of the* Phœnicopterus *or* Flamingo; *with two Views of the Head, and three of the Tongue, of that beautiful and uncommon Bird. By* James Douglas, *M. D. Reg. Soc. S.*

NOMEN.

IT was the famous Comical Poet *Aristophanes*, that first makes mention of this Bird by the Name of Φοινικόπτερος (*a*), and not long after it is called ὄρνις Φοινικος (*b*) by *Philostratus* in his Life of *Apollonius Tyaneus*. *Apicius*, *Plinius*, *Suetonius*, *Juvenal*, and other Latin Writers, retain the Greek Word, and call it *Phœnicopterus*. *Bellonius* (*c*) says that in French it is named *le Flement* or *Flambant*. *Scaliger* affirms, that in *Provence* they call it *Flammant*: And (*d*) *Gesner* says it may be called *Avis Rubra* per excellentiam. (*e*) *Aldrovandus* writes, that in *Sardinia* it goes by the Name of *Fiamingo*; and *de Laet* tells us, the *Spaniards* in the *West-Indies* call it *Flamenco*. Dr. *Charlton* and Dr. *Grew* convert the Greek Appellation into English, naming it the *Phœnicopter*: And Sir *Hans Sloane*, in his Catalogue of *Jamaica Birds* annexed to Mr. *Ray*'s *Synopsis Avium*, stiles it the *Flamingo*. (*f*) *Du Tertre* calls it *le Flamand*, which differs but little from the Name given it by *Bellonius*: And

(*a*) *Aristoph. Aves. Sc.* 4. (*b*) *Philost. Lib.* 8. *Pag.* 387. *Edit. Paris.* 1605. *fol.* (*c*) *Bellon. Histoire des Oyseaux, Lib.* 8. *Cap.* 8. (*d*) *Gesner Hist. Anim. Lib.* 3. (*e*) *Aldrov. Ornithol. Tom.* III. *Lib.* 20. *Cap.* 4. (*f*) *Hist. des Isles*, &c. *p.* 300.

to

to mention no more. (g) *Du Hamel* says 'tis commonly call'd *Becharu* in *France*.

Etymologia.

All these differing Names may be easily accounted for, from the Colour most prædominant in its Wings. Thus *Martial* (Epigram 58. Lib. III.) says of this Bird:

Nomenque debet quæ rubentibus pennis.

And again (Epigr. 71. Lib. XIII.) he makes it give the true Derivation of its own Name;

Dat mihi penna rubens nomen.

The Greek Name is compounded of two, *viz* φοινικεος, *puniceus*, *ruber*, and πτερον, *Ala*, a Wing, *quod sit rubentibus Alis*; which thing in different Words is expreſt as follows, by the several Authors I have consulted. *Bellonius* says 'tis called in French *Flambant*, not only from the Date-Colour of its Wings, *à Dactylorum colore*, i. e. a Scarlet or light red, like the Fruit of the Palm or Date-Tree called in Greek φοῖνιξ; but also from the Lustre of the Colour resembling Flame: or as *Aldrovandus* has it, *quod velut ignis instar ejus rubedo emicet.* The Words of *Gesner* are, *Ego Gallicum nomen à rubro & flammeo rostri, crurum, pennarumque in aliquibus partibus colore inditum esse conjecerim: aut forte quoniam ex Flandria hyeme ad* Narbonensis *Provinciæ maritima volat; nam Flandrum Galli* Flammant *appellant: vel à corporis proceritate, quales solent esse Flandri.* Mr. *Willoughby* (h) says the French name it thus rather from the flammeous Colour of the Wings and Feet, than that it comes in the Winter Time

(g) *Hist. de l'Acad. Royale, p.* 213. (h) *Ornithologiæ, Lib.* III. *Sect.* 2. *Cap.* 1.

from

from *Flanders*. For he believes there was ſcarce ever ſeen in *Flanders* a Bird of that kind; ſo far are they from being common there, and flying from thence into other Countries. (*k*) Dr. *Grew* believes it named in Greek from the Scarlet Colour of its Wings; and *Flamment* in French for the ſame Reaſon. *Du Hamel* explains its Name *Becharu* by *Aratri-roſtrum*, (*quaſi Bec-Charrüe*) *quoniam roſtrum ejus aratri inſtar inflectitur.*

Genus.

All Authors, from *Ariſtophanes* down to *Aldrovandus*, have accounted the *Phœnicopterus* a Bird of the *Palmipede* or web-footed kind; and tho' this laſt named Author will not allow it to be ſo, yet he is forced to own that it is not a true *Fiſſipede* or digitated Fowl; *nam & membranæ digitos ſepientis quoddam habet rudimentum*, are his own Words. Dr. *Charlton* only, among all the later Natural Hiſtorians, has approved of his Diviſion, and accordingly ranked the *Phœnicopter* in the Claſs of *Aquatick Fiſſipedes*. But that it is a Water-Fowl all agree; *Ariſtophanes* calls it λιμνᾶῶ, i. e. *paluſtris*; and *Aldrovandus* ſays of it, *Avis eſt aquas amans*: not to mention others.

Differentiæ.

I find Authors are ſilent as to the different Sorts of this Bird, only *Aldrovandus* gives us two Figures thereof that are not alike.

Locus Natalis.

This Bird is found in three of the Principal Parts of the World, that is, in *Africa*, *America* and *Europe*. *Heliodorus* (*Æthiop.* Lib. 6.) calls it Νειλῶον φοινικόπτερον, a Bird of the *Nile*; and the old *Scholiaſt* upon *Juvenal* (Sat. xi. ver. 139.) affirms, that *abundans eſt in Africa*; and *Du Hamel*'s Words are, *Inter animantes*

(*k*) *Muſæum Reg. Soc. p.* 67.

quæ ſua mole commendantur, Avis illa ex Ægypto allata eſt, quam Veteres ob plumas in Alis rubeas Phœnicopterum *dixere.* *John de Laet* writes, that there is an Abundance of them in the Iſland of *Cuba*, as alſo at the Iſle called *Rocca*, lying on the Coaſt of the Province of *Venezuela* in *South America*; and *Rochfort* ſays the ſame thing of the Iſland of *St. Domingo.*

(l) Dampier ſaw ſome few of them at *Sal*, one of the *Cape Virde* Iſlands: He hath likewiſe ſeen ſome of them at *Rio la Hacha*; alſo at an Iſland near the Main of *America* right againſt *Queriſao*, call'd by the Privateers *Flamingo-Key*, from the Multitude of theſe Fowls that breed there; and he never ſaw of their Neſts and Young but there only.

Tho' theſe winged Creatures live for the moſt part in thoſe hot Countries, yet they ſometimes viſit us here in *Europe*, and ſo may be accounted amongſt the Migratory Kind, or Birds of Paſſage, which is confirmed by the Teſtimonies of ſeveral Authors: For,

Bellonius told us long ago, *migrant ultrà mare*, and are often taken in *Italy*, and oftner in *Spain.*

(m) Gaſſendus ſays they are frequently catched in the fenny Grounds and Marſhes about *Arles* in *Provence*, upon the *Rhone.*

Geſnerus. Quidam mihi retulit avem hanc non procul à Monte-Peſſulano *capi* He ſays in another Place, that they ſwim in Flocks not far from the Shore *in Mediterraneo Mari Gallico.*

Willoughby writes, that in hard Weather in the Winter Time, it comes over to the Coaſt of *Provence* (and is often taken about *Martiquez*, a Sea-Port Town in that Country) and in *Languedoc*, and is frequently found about *Montpelier:* But whence it comes and where it is bred,

(*l*) *Damp. New Voyage round the World, p.* 67. (*m*) *Gaſſ. Vita Peireſc. Lib.* II. *in fine.*

to

to me, ſays he, is unknown. *N.B.* This Paſſage is not in the Latin Edition of his Works, but added to the Engliſh which was publiſhed two Years after the firſt. However, he ſays poſitively, that they don't come from *Flanders*, where they are ſo far from being common, as ſome alledge, that there never was one ſeen in that Country.

(*n*) Dr. *Charleton* informs his Reader that he was preſented with the Skin of one of theſe Birds, well ſtuffed and dried, by a Gentleman at his Return from the Univerſity of *Montpelier*, near which Place it had been taken. *Hujus exuvias ritè conditas infertaſque mihi dono dedit prænobilis Juvenis D.* Thomas Crew, *Eq. Aurat.*

(*o*) Dr. *Liſter* ſays, *Frequens eſt Phœnicopterus in paludibus maritimis ad mare Mediterraneum Provinciæ & Languedociæ.*

Whether this Bird were known to *Ariſtotle* is a Queſtion; for all our Writers of Natural Hiſtory agree, that the *Phœnicopterus* is no where mentioned by Name by the Philoſopher; yet they can hardly believe that he was ignorant of a Bird ſo clearly deſcribed by his Contemporary *Ariſtophanes. Mirum eſt*, ſays *Geſner, hujus tam pulchræ & eximiæ Avis nomen ab Ariſtotele taceri, cum Ariſtophanes, qui vixit eadem ætatè, meminerit. Sed Græcis etiam raram eſſe hanc avem puto.*

Bellonius thinks that *Ariſtotle* deſcribed this Bird under the Name of *Glottis* or *Lingulaca*, as *Theodorus Gaza* tranſlates it. *Aldrovandus* is of the ſame Opinion, but *Geſner* and *Scaliger* are not; for the firſt ſays, *Ego verò iis quas Gallinulas aquaticas noſtri vocant avibus Glottidem adnumero, quæ omnes fiſſipedes ſunt:* And the latter in his Commentary upon this Paſſage ſays, *Glottis autem quæ ſit nondum mihi conſtat. Ridiculum quod quidam de* Phœnicoptero *auſus eſt pronuntiare.*

(*n*) *Charlton de Differentiis & Nom. Animalium.* (*o*) *Liſter Annot. in Apicium Cœlium, Lib.* VI. *cap.* 7.

Victus Ratio, Nidificatio, Volatus, &c.

Gesner says, *circa lacus & paludes victitat*, and that it feeds on Perwinkles and Fish: And by *Dampier*'s Account we learn, that they delight to keep together in Flocks, and feed in Mud and Ponds, or in such Places where there is not much Water; that they are very shy, and therefore it is hard to shoot them; that they build their Nests in shallow Ponds, where there is much Mud, which they scrape together, making little Hillocks, like small Islands, appearing out of the Water, a Foot and an half from the Bottom: They make the Foundation of these Hillocks broad, bringing them up tapering to the Top, where they leave a small hollow Pit to lay their Eggs in. And when they either lay their Eggs or hatch them, they stand all the while, not on the Hillock, but over it, with their Legs on the Ground in the Water, resting themselves against the Hillock, and covering the hollow Nest upon it with their Wings: For their Legs are very long, and building thus, as they do, upon the Ground, they could neither draw their Legs conveniently into their Nests, nor sit down upon them otherwise than by resting their whole Bodies there, to the Prejudice of their Eggs or Young, were it not for this admirable Contrivance, which they have by natural Instinct. They never lay more than three Eggs, and seldom fewer. The young ones cannot fly till they are almost full grown; but will run prodigiously fast. Thus far *Dampier*.

Du Tertre, in his History of the Isles, *&c.* gives these further Circumstances. *Ces oyseaux, dit il, ont le ton de la voix si fort, qu'il n'y à personne, en les entendant, qui ne creust que ce sont des trompettes qui sonnent. Ils sont toujours en bandes, & pendant qu'ils ont la teste cachée, barbotant dans l' eau comme les Cygnes, pour trouver leur mangeaille, il y a toujours un en sentinelle tout de bout, le col éten-*

du:

dù, l'oeil circonspect, & la teste inquiete. Si tost qu'il appercoit quelqu'un, il sonne la trompette, donne l'alarme au quartier, prend le vol tout le premier, & tous les autres le suivent. Ils volent en ordre comme les Grües; que si l'on les peut surprendre, ils sont si facile à tuer, que les moindres blessures les font demeurer sur la place. Ils sont rares & ne se voyent jamais, si non dans les salines le plus éloignees du Peuple.

On les écorche, & de leur peaux on fait de fourreurs, que l'on dit etre tres utile a ceux qui sont travaillez des froideurs & debilitè d' estomac.

(p) *Rochfort* likewise informs us, That *Ils ont l' Ouye & l' Odorat si subtile, qu'ils eventent de loin les chasseurs, & les armes à feu. Pour eviter aussi tout surprise, ils se posent volontiers en dés lieux découverts, & au milieu de marécage, d'ou ils peuvent appercevoir de loin leurs ennemis; & il y en a toujours un de la bande qui fait le guet. Ils sont gras & ont la chaire assez delicate. On conserve leur peau qui est couvert d'un mol duvet, pour étre employé aux mesmes usages que celles du Cygne & du Vautour.*

De Laet observes, that these Birds are so accustomed to Salt Water, that the *Indians*, when they tame them, mix Salt with the fresh Water for them; else they pine away and die. And though *Aristophanes* says it is ἐ τῶν ἠθάδων, or not used to be tame; yet *Gassendus* writes, that *M. Varius*, President of the Parliament at *Aix* in *Provence*, and a great Friend of *M. Peiresc*, used to divert himself with feeding them with Bread moistned with Water, which they commonly eat in the Night and not in the Day Time. The same learned Person observed, that they could discern the Approach of cold Weather, and would come to the Fire, so as sometimes to burn their Feet; and that when one Foot pained them, they would go upon the other, using their Bill instead

(p) *Histoire des Antilles.*

of

of the burnt Foot; That they ſlept ſtanding upright on one Foot, with the other drawn up to their Breaſt among their Feathers: And laſtly, that very little Sleep ſerved their Turn.

Uſus.

This beautiful and ſcarce Bird was much eſteemed by the *Romans*, and frequently made uſe of in their coſtly Sacrifices and ſumptuous Entertainments. Thus *Suetonius (q)* deſcribing the exquiſite Sacrifices which were appointed by the mad Emperor *Caligula* to be offered to himſelf as a Divinity, ſays of them, *Hoſtiæ erant Phœnicopteri, Pavones, Tetraones, Numidicæ, Meliagrides, Phaſianæ, quæ generatim per ſingulos dies immolarentur.* And the ſame Hiſtorian relates further *(r)*, that this Emperor *pridiè quam periret ſacrificans reſperſus eſt Phœnicopteri ſanguine.*

That the Tongue of this Volatile was much commended, and in great Eſteem, for its excellent Taſte and moſt delicious Reliſh, will appear from the following Quotations. And firſt we read in *Pliny (s)*, that *Apicius* ſaid the Tongue of this Bird was a delicious and ſavory Bit, *Phœnicopteri linguam præcipui eſſe ſaporis Apicius docuit, nepotum omnium altiſſimus gurges.*

The Poet *Martial* ſays the ſame thing in the aforecited Epigram:

Dat mihi penna rubens nomen: ſed lingua guloſis
Noſtra ſapit;

And *Juvenal (t)* in that Satyr where he expoſes the extravagant Luxury and Gluttony of the *Romans*, men-

(q) *Suetonii Caligula*, §. 22. (r) *Scalig.* §. 57. (s) *Plinii Nat. Hiſt. Lib.* X. *cap.* 48. (t) *Juvenal. Sat.* XI.

tions

tions this Fowl, amongst some others equally rare, that they made use of in their Feasts.

Et Scythicæ volucres & Phœnicopterus ingens.

We read in *Suetonius* how the Emperor *Vitellius* had them often served at his Table, with a great many more Varieties brought from the most distant Parts of the Universe; his Words are, *(u) In hâc Scarorum jocinera, Phasianorum cerebella, linguas Phœnicopterûm, Murænarum lactes à Carpathio usque fretoque Hispaniæ per Navarchos ac Triremes petitarum commiscuit*; hoc est, ab extremis imperii finibus Orientem versus & Occidentem. And *Heliogabalus*, another of the *Roman* Emperors, as *Lampridius* writes, treated his Courtiers with sumptuous nice Dishes made of the Inwards and Brains of *Phœnicopters*, *exhibuit Palatinis ingentes dapes extis & cerebellis Phœnicopterorum refertas.*

What is related by *Gassendus*, in the Life of that learned Nobleman *Peireskius*, is no Argument against the excellent Relish of the Tongue of this Bird: For his Friend *Varius*, who therein seems to contradict the received Opinion, was at that Time just upon the Recovery from a long Illness; he had no Appetite, loathed all Sorts of Meats, and mended but very slowly; so that its no Wonder if he did not perceive all the Relish of that nice Bit, for which of old it was so much commended. Besides, his Answer is not as to the Tongue, which was owned to be much sweeter than that of a Kid, but to the Flesh of this Bird, (as will appear from the Original.) *Rogatus subinde fuit de sapore carnis Phœnicopteri. Excepit autem mirari se, cur illam Apicius apud Plinium, & Imperatores Caligula & Vitellius apud Suetonium, Heliogabalus apud Lampridium, & nonnulli alii tantis in deliciis habuissent. Esse enim eam injucundam, aut saporis certe non exquisiti, aquaticarum aliarum instar, cum etiam piscem oleat; unde à*

(u) *Suetonii Vitell.* §. 13.

Fro-

Provincialibus utplurimum abjicitur, exuviæque solum sunt usui in fastuosis conviviis, carnibus aliarum avium obtegendis.

The Way to dress the *Phœnicopter*, and how to make a Sawce fit for it, we may read in *Apicius*'s Book *de Obsoniis & Condimentis, seu de Arte coquinariâ*, Lib. VI. c. 7.

Phœnicopterum elixas. lavas, ornas; includi in cacabum, adjicies aquam, salem & aceti modicum; dimidiâ coctura alligas fasciculum porri & coriandri ut coquatur: Prope coctura defrutum mittis, coloras: adjicies in mortarium piper, cuminum, coriandrum, laseris radicem, mentham, rutam: fricabis: suffundis acetum: adjicies caryotam. Jus de suo sibi perfundis, reexinanies in eundem cacabum, amilo obligas, jus perfundis & inferes. Aliter. *Assas avem, teres piper, ligusticum, apii semen, sesamum, defrutum, petroselinum, mentham, cepam siccam, caryotam; melle, vino, liquamine, aceto, oleo & defruto temperabis.*

Philostratus *Puniceam Avem*, i. e. *Phœnicopterum, inter mensarum delicias numerat*, Lib. VIII. Vitæ Apoll.

(x) Wormius. *Linguam hujus avis veteribus Romanis in deliciis olim fuisse docent cupediarum magistri Apicius & alii.*

Dr. *Grew*. The Tongue of this Bird, as *Apicius* said, was a delicious Morsel amonst the *Romans*.

N. B. In the Treatise *de Obsoniis & Condimentis*, that goes under the Name of *Apicius*, there's no mention made of the Tongue of this Fowl: For as Dr. *Lister* well observes, *Apicius noster hîc silet de linguæ præcipuo sapore:* Which is a pretty convincing Proof, that this Book *de re coquinaria*, is only a Collection made by some modern *Roman*; the Name of the old *Apicius*, that great Master of the Art of Eating, being only prefixed to it, for the Benefit of the Bookseller.

(x) *Musæum.*

Dampier

Dampier. The Flesh of both young and old is lean and black, yet very good Meat, tasting neither fishy nor unsavory: A Dish of *Flamingo*'s Tongues being fit for a Prince's Table. They are large, having a large Knob of Fat at the Root, which is an excellent Bit.

Du Tertre. La chair en est excellente, quoy qu'elle sent un peu la marine: mais sur tout la langue passe pour le plus friand morceau qui puisse etre mangé.

Descriptio Partium.

Magnitudo.

According to *Bellonius* this Bird is of the Bigness of the Fowl he calls *Elorius*, which is our *Curliew*.

Scaliger compares it to the *Heron, magnitudo ei Ardeæ.*

Gesner says it is as big as a *Ciconia* or *Stork*, or rather bigger.

Aldrovandus writes, *de magnitudine ejus ego nihil certi assero, quia Avem nunquam vidi.*

Dampier. The *Flamingo* is a sort of large Fowl much like the Heron in Shape, but bigger and of a reddish Colour.

Du Tertre. *Le Flamand est un oiseau gros comme une Oye sauvage.*

Collum.

It hath an extraordinary long Neck according to Mr. *Willoughby.*

Du Tertre. *Il a le cou rouge, fort menu pour la grandeur de l' oyseau, & long d'une demy Toise.*

Cauda.

Scaliger. *Caudam habet brevissimam ac veluti præcisam.*

Rostrum & Caput.

Scaliger writes, that the Bill of this Fowl is neither streight nor altogether crooked: *Rostrum neque rectum*

planè, sed neque aduncum habet, Scythici arcus partem potiùs imitatur.

Gesner, who compares this Bird to the Crane for Bigness, adds, *Rostro sesquialtera fere longitudine ad Ciconiæ rostrum, superius crasso & tuberculis quibusdam aspero.*

Aldrovandus commends the Account *Scaliger* gives of the Bill, and then adds, *in Rostri autem conformatione non parum lusit Natura; non enim ut Anatûm aut Anserûm planum est, cum alioqui sit latum, neque ut Ardearum rectum & rotundum, neque denique ut rapacium Aquilarum aut Accipitrum aduncum; cum tamen sit curvum quidem & deorsum inflexum, sed in medio superioris mandibulæ notabili extuberantia insigne, sex digitos longum, intus cavum & canaliculatum media sui parte. Superior etiam mandibula inferiori longior est, & in acutissimam aciem desinit; contra vero inferior longè crassior.*

Du Tertre. *Il a la teste ronde & petite, à laquelle est attache un gros bec, long de quatre pouces, moitiè rouge & moitiè noire, & recourbè en forme de cüeilliere.*

Olaus Wormius gives the following Description of the Head and Bill of the *Phœnicopter*, which he had sent him from a Friend, *viz.*

Caput longitudine uncias octo superabat, ipsum caput, excepto rostro, trium erat. Rostrum ipsum figuram à Scaligero *delineatam obtinet, in medio crassum satis sed cavum, superiore ejus parte utrinque ad latera caniculatum; ad sui exortum duobus grandibus foraminibus olfactui deputatis præditum, in extremitate aduncum, internè denticulatum cum costâ seu eminentiâ in medio. Pars vero inferior nigra, frequentibus prædita striis ad extremum excurrentibus; longitudine vero cedit superiori, sed ampla est & capax, crassæ linguæ, quæ aberat, excipiendæ apta.*

Dr. *Grew* has obliged us with a very curious Account of the Bill of this Bird, for which he says it is most remark-

remarkable. The Figure of each Beak is truly Hyperbolical: The upper Jaw is ridged behind, before plain or flat, and pointed like a Sword, with the Extremity bended a little downwards: Within it hath an Angle or ſharp Ridge, which runs all along the Middle, at the Top of the Hyperbole, not above a quarter of an Inch high: The lower Beak in the ſame Place above one Inch high, hollow, and the Margins ſtrangely expanded inward, for the Breadth of above a quarter of an Inch, and ſomewhat convexly. They are both furniſhed with black Teeth, as I call them from their Uſe, of an unuſual Figure, *ſcil.* ſlender, numerous, and parallel as in Ivory Combs; but alſo very ſhort, ſcarce the eighth Part of an Inch deep. An admirable Invention of Nature, by the Help of which and of the ſharp Ridge above-mentioned, this Bird holds his ſlippery Prey the faſter.

Mr. *Ray* deſcribes the *Roſtrum* to be *latiuſculum, ſingularis & inſolitæ figuræ; mandibula nempe ſuperior incurva, depreſſa, dentata; inferior craſſior.*

Menippus, the Cynick Philoſopher, in a Fragment of his *de Homine* (which however at this Time is either loſt, or at leaſt hard to be come at) affirms this Bird to move its upper Jaw, as we find him quoted by (*y*) *Rondeletius*, where he is talking of the Crocodile's moving that Mandible: His Words are, *ſed id non ſoli ex omnibus animalibus Crocodilo peculiare, nam intra Aves,* Phœnicopterus *ſuperiorem partem Roſtri movet, ut annotavit* Menippus *Philoſophus, Libro* de Homine.

Geſner makes the very ſame Quotation from *Rondeletius.*

(*z*) *Cardanus* repeats the ſame thing without Mention of any Author, *nam quidam exiſtimant etiam Phœni-*

(*y*) *Rondel. Lib. de Amphibiis, Chap.* 5.
(*z*) *Cardan. de Varietate Rerum, Lib.* 7. *Cap.* 37.

copteros aves Mandibulam movere superiorem; but subjoins, *sed non adeo manifesta causa est in Ave ut in Crocodilo.*

Wormius is of the same Opinion; but with *Cardan*, he thinks the Cause is not so manifest as in the Crocodile.

Dr. *Charleton* says, that it was *Cardanus* that first made that Observation.

Dr. *Grew* argues for this Movement from the peculiar Structure of the *Rostrum*; alledging, however, that there can be no Determination of these Matters, without Inspection into the Muscles, and the Articulation of the Bones. As for the *Phœnicopter*, says he, it must needs be said, that the Shape and Bigness of the upper Beak (which here, contrary to what it is in all other Birds that I have seen, is thinner and far less than the neither) speaks it to be the more fit for Motion, or to make the Appulse, and the neither to receive it.

Crura & Pedes.

Bellonius remarks, that the Legs of this Volatile are very long. And on the contrary,

* *Scaliger* writes, *Crura pedesque sunt adeo breves, ut cùm in Homine* Galenus *agnoverit longissimos, huic omnium, quæ nota sunt nobis, animalium brevissimos attribuere potuerit.* For this he is severely taxed by Dr. *Charlton, hic nobis candidè notandus occurrit error quidam* Scaligeri *egregius: Is nimirùm in Exercit. in* Arist. *Hist. Animal.* (this is wrong quoted, the Place being in *Arist. Lib. de Generat. Anim.*) *peculiares* Phœnicopteri *notas satis prolixè describens, crura ei brevissima curtosque pedes attribuit, (verbis supra citatis) atque Avem hanc & cruribus & suris gradiri longissimis, omnium quotquot seu vivam seu mortuam contemplati sunt oculi confirmant. Et quis precor, ullam aliam, ex Aquaticarum fissipedum & piscivorarum classe, volucrem unquam conspexit brevibus pedibus in-*

* Scalig. *Exercit.* 233. § 2. *de Subtilitate, ad* Cardanum.

structam?

ſtructam? Certè nemo. Neque conſentaneum eſt, ut quas Naturæ conſilium ad piſces in ſtagnis & fluviorum vadis, non natando ſed grallatoriis veluti gradibus vadando, captandos deſtinaverit, eis crura concederentur tam neceſſario ad victum quærendum officio imparia. Ad hæc, non aliunde fuit quàm à proceritate crurum & colli, quod Phœnicopterus *olim à* Juvenale ingens *diceretur.*

Dr. *Grew* obſerves the ſame Miſtake, but in fewer Words, which are the following; *when* Scaliger *therefore ſaith that this Bird hath the ſhorteſt Legs of any Animal yet known, he would have ſaid the longeſt.*

Geſner ſays, it is *Cruribus rubris, ea proceritate qua in Ciconia ſunt, vel procerioribus.*

Du Tertre. *C'eſt le plus haut monté de tous les oyſeaux que J'ay jamais veu en ma vie. Il a les Jambs tout rouges, & les pieds a demi marin.*

(*y*) Rochfort. *Ils ont les jambs & les cuiſſes ſi hautes, que le reſte de leur corps eſt elevé de terre de deux bons pieds ou environ.*

Color Plumarum.

Scaliger thus elegantly expreſſes the fine Colour of its Wings. *Cinereum colorem nobilitant alarum puniceæ pennæ.*

Aldrovandus. *Mirum eſt cur nigrum alarum colorem non annotavit* Scaliger. *Cætera tota Avis ex cinereo, phœniceo & albo coloribus miſtis ſpectatur.*

Geſner ſays, *Pennis albis parte prona; rubentibus per Collum, Pectus, Ventrem & Alas.* And ſpeaking of one taken near *Montpellier*, he ſays, *tota alba, præter illas in alis partes quæ nigræ ſunt in Ciconiis.*

(*z*) *De Laet* obſerves, that while they are young their Feathers are chiefly white; but as they grow up, they are painted with an Infinity of Colours. *Mais ils different en couleur, d'autant qu'ils ont le plumage blanc quand ils*

(*y*) *Hiſt. des Antilles Edit. Rottord.* p. 583.
(*z*) *Hiſt. du Nouveau Monde, Lib.* 1. *Cap.* 9. & *Lib.* 18. *Cap.* 15.

ſont

ſont jeunes; puis apres à meſure qu'ils croiſſent, ils devient de couleur de roſe, & en fin quand ils ſont agez, il eſt tout incarnat. Il ſe trouve de ce mêmes oiſeaux, pres de Montpelier, *qui ont ſeulement le deſſous des ailes & du corps incarnat, & le deſſus noir. Il s'en voit auſſi aux Iſles, qui ont les ailes mêlees de quelques plumes blanches & noires.*

Du Tertre gives much the ſame account. *Les Jeunes ſont beaucoup plus blancs que les vieux, & ils rougiſſent à meſure qu'ils avancent en age. J'en ay veu auſſi quelqueſuns qui avoient les ailes meſlees de plumes rouges, noires & blanches, & je croy que ce ſont les mâles.*

(†) Conſtantinus. *Roſtrum, & crura, & pars alarum, puniceo colore rutilat.*

Willoughby ſays, the Neck and Body is white: The *Alarum Remiges*, or Quill-Feathers of the Wings, are black; but the *Veſtitrices*, or Covert-Feathers, are wholly dyed with a moſt beautiful, bright purple, or flame-Colour, *unde ei nomen.*

Dampier. The young ones at firſt are of a light grey; and as their Wing-Feathers ſpring out, they grow darker, and never come to their right Colour, or any beautiful Shape, under ten or eleven Months old. When many of them are ſtanding together by a Pond's Side, being half a Mile diſtant from a man, they appear to him like a Brick Wall; their Feathers being of the Colour of a new red Brick: And they commonly ſtand upright and ſingle, one by one exactly in a Row, except when feeding, and cloſe by each other.

Color Pedum.

All Authors agree in the red Colour of its Legs and Feet. Thus *Scaliger*, *Crura pedeſque alis habet concolores.*

(†) *Conſtant. Lexicon Græco-Latin.*

Color

Color Rostri.

Gesner says, *Colore rubro instar sanguinis.*

Aldrovandus writes, *Pars quæ spectat frontem ex albo ad Castaneæ colorem vergit, cætero nigrum.*

Willoughby affirms, that the Tip of the Bill is black, or of a dark blue.

Figuræ.

The whole Fowl is delineated by *Gesner* and *Aldrovandus*; and Dr. *Grew* has given us the Figure of the Head and Bill, as he found it amongst the Rarities in *Gresham-College*. *N. B.* The Figure of the *Phœnicopter* in *Willoughby*, is copyed from the second of *Aldrovandus*. *Gesner* says the *Phœnicopterus* whence his Figure was taken, was sent to him by *Rondeletius*. *Aldrovandus* had the first of his Figures from *Sardinia*; and the second, which he calls *Phœnicopterus alter rostro lato*, was given him by that famous Botanist *Carolus Clusius*: He owns that he never saw the Bird himself.

In *de Rochfort*, the Body and Neck of the *Flammant* is pretty well delineated; but the Legs are not, neither is the Bill, nor the Claws.

(*) *Du Hamel* gives a very exact Account of this Bird in the following Words, with which I'll close this tedious and prolix Description, collected from all the Authors that have made any mention of the *Phœnicopter* or *Red-Wing*. *Collo prælongo, cruribus productis, exili pede sed firmo donatur; oculi itidem angusti sunt & rubei; cystis fellea è parte inferiore hepatis est pensilis. Vas ipsum è quo suspenditur, quove bilem excipit, amplum est, contra atque in Homine & in Quadrupedibus observatur; in his*

(*) *Hist. Acad. Royale Edit.* Paris, p. 213.

enim

enim radices vesiculæ sunt admodum exiles. Oesophagus in sui initio valde angustus, paulatim latior factus in ingluviem seu in saccum ampliorem desinit: Ventriculus fere ut in Gallina, tametsi Granis non vescitur, sed parvis Conchyliis, quæ ventriculi musculis teruntur ut Grana. This Author tells us that the *Phœnicopter* was dissected by Monsieur *Perrault* at *Paris.*

Alæ ejus diductæ colorem illum rutilum exhibent, unde vulgo Flamand *dici solet, non quod in* Belgio *reperiatur, sed quia ejus Plumæ per membranam pellucidam visæ colorem flammeum præbent: Vix ulla est Avis major: Rostrum ex utraque parte deorsum incurvatum, quod in ea Ave omnino est singulare; aratri enim instar inflectitur, unde & vulgo* Becharü, *quasi Aratrirostrum, appellatur.* Hæc ille.

When I get the Opportunity, which I do very much long for, of dissecting this fine Volatile with my own Hand, I shall then more particularly insist on the Anatomical Part; and, with all the Exactness I can, give a true Description of each *Viscus.* I proceed to the Explanation of the Figures, which were drawn from a *Flamingo* that was sent to Mr. *Botley* to be stuffe.

The Explanation of the Figures. Tab. II.

Fig. I. Gives a Side-view of the Head and Bill.

Fig. II. In this is represented a Front-view of the same Parts.

Fig III. Exhibits the under-side of the Tongue next the under Bill. In which *a* denotes a Cartilaginous Substance that covers the Tip or extremity of the Tongue; *b* a Glandulous Substance at its Basis; *c* the Horns of the *Os Hyoeides.*

Fig. IV. In this the upper side of the Tongue is fairly delineated, upon which we see two Rows of strong *Papillæ Nerveæ*; their *Apices* or Points turning inwards, for the better retention of the Prey.

Fig. V.

Fig. V. In this the Tongue is drawn in a lateral View, that we may have a Prospect of the true Figure of these *Papillæ*, which being hooked and turn'd backwards prevent, in a great Measure, the return of any little Animal swallow'd alive, which they feed upon.

Fig. VI. The *Cornua* or Horns of the *Os Hyoidæum* are drawn in this last Figure, as all the other Parts are, as big as the Life.

Advertisement.

WHEREAS some of our practising Surgeons, as I am inform'd, have taken Offence at a Passage in a Paper of mine, publish'd in the last *Philosophical Transactions*, viz. *By that sort of Enquiry the common Mistake of Surgeons was detected, and what was esteem'd and treated by them as a Luxation of the Head of the* Os femoris, *was discover'd to be nothing else but a fracture of the same Bone near it's Neck*; I take this first Opportunity to explain my self, that by the preceding Words I only meant the Writers of Surgery now extant, as may appear by my Quotations, and not in the least intended any Reflection on the present Practitioners in it. That skilful Society I not only Respect and Esteem for their great Merit and extensive Knowledge in the several Parts of their Profession, but to them am under many Obligations also, for the Honour they have done me in twice choosing me one of their Lecturers. I have this farther to add, that that Observation was publish'd entirely without my Knowledge about a Year after it had been read at a Meeting of the *Royal Society*.

James Douglas.

An INDEX to this Nine and Twentieth Volume of the *Philosophical Transactions*. *n.* denotes the Number of the Tract, *p.* the Page.

Barometrica

A

I S.